THE LEAST SQUARED APPROACH TO THE TRAVELING SALESMAN PROBLEM

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PREFACE

A method for solving the distribution problem known as the traveling salesman problem is presented. The convex hull, cluster analysis and regression line mathematical theories provide the basis of the method. The method has application to any delivery problem that routes from a distribution location out to other locations and back to the distribution location.

The method is applied to a thirteen point randomly generated distribution problem and route solutions are generated. These solutions are compared to solutions founded by another distribution method known as lockset. Solution routes for this particular example indicate equivalent and possible advantagous use of this method as an alternative to lockset or other methods.

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CHAPTER I

INTRODUCTION

In today's modern industrial markets with their variety of distribution channels, business is faced with the question, "What is the best distribution mode?". With increased domestic and foreign competition and recent recession battle wounds fresh in mind, the most efficient utilization of this best mode is also becoming ever increasingly important. The selection of a firm's distribution channel and its most efficient use can be the deciding factor in the firm's survival in a market place. In many firms the distribution costs can constitute a significant portion of the product cost and thus present necessary opportunities for cost savings to the firm. The ever increasing need to trim costs and maintain service in foreign and domestic markets requires that the firm management understands the operation and theory of its distribution method. Then they must service these markets at the most economical and acceptable quality level.

Even after the painstaking effort of selecting a distribution mode, the manager's work has only begun. In each mode one can see there are an unlimited number of potential solutions and a large variety of methods available. In addition the most efficient use of the mode requires balancing a large variety of parameters. To further frustrate the unwary manager, not all of the solutions result in efficient, much less optimum, use of available resources. To help find and select possible solutions, computers are being utilized and the logistic problems of the past are being simplified and automated. The high speed data manipulations allow for calculation of the large number of permutations and the automatic selection of the best solution using some method and selection criteria. Whereas these computers reduce the mechanical gymnastics involved, the theory and equations underlying the decision criteria in a number of the methods are of such complexity so as to elude understanding by the average manager. This lack of understanding can discourage use of any proven method by small firms and encourages a laid back approach by management of larger firms. The result is that research into improved distribution methods is left in the academic arena and not in the business arena. One additional characteristic of the problem that discourages human intervention is that as the problem size increases, the calculations and solutions can become overwhelming, increasing at an exponential rate. Also, numerous methods do not strive for optimum solutions; better, they provide a starting solution upon which improvements can be made. It is for this end, to provide a starting solution upon which improvements can be made, that this method is presented.

The above arguments hold even for a basic problem of scheduling and routing deliveries to a plurality of locations from a single distribution point. Almost all firms are faced to some extent with this problem and many either cannot afford the expenditure or feel that they need to optimize their use. Most cannot understand and do not get involved with the principles underlying the methods. Therefore alternative models based on sound logical principles readily available to the average firm are needed. The following discussion is designed to present an understanding of and a workable method to solve this type of routing problem. The optimum solution as stated above is not guaranteed, but the procedure gives solutions comparable to other methods and provides a starting point for subsequent improvements. The basic principles are such that the average manager should not be afraid to get his hands "wet" with them.

The specific problem area to which this method is directed is the routing of transportation units (trucks for example) to a plurality of locations away from and back to a single original distribution point. The objective is to minimize the distance while balancing as best as possible other characteristics such as work load among the resources (trucks). The selection of the channel is not examined in this text; however the method should apply to any distribution mode or channel that exhibits the above characteristics and following assumptions. The objective function comprises three basic goals to arrive at the solution to the distribution problem: first, to determine the minimum amount (number) of transportation units required to service all locations within the problem constraints such as time; second, equal distribution of the work load between the units so as to minimize resource idle time; and finally, to determine the individual unit routing path that minimizes parameters such as distance or time.

The paper begins with a statement of the specific problem and a review of some present methods found in the literature. Interesting similarities between these methods and the author's method are noted. A brief discussion of their main underlying principles is presented so the reader can contrast the differing methods. Next, a discussion of the author's method, which involves the use of what is known as a convex hull (Carlson, 1977) and principle axis approach, associated assumptions and the detailed explanation of the theoretical principles and calculations involved is then given. An evaluation in terms of an actual example compares it with another method known as the lockset method using a simple randomly-generated distribution problem. Finally, conclusions and implications of the method to the business world completes the presentation of the method. In reading this paper one must remember that this example is only a minute section of a much larger distribution problem that has been reduced in complexity to better present the main principles. It is presented to provide an alternative method that will provide a starting solution to which improvement can be made with potential cost savings for any sized firm through the understanding of its underlying principles and application to their distribution situation .

STATEMENT OF THE PROBLEM

The problem is to determine the minimum number of transportation units needed (in this discussion trucks will be used to designate the transportation units), and the actual distribution (routing) path for each truck to service an array of locations spaced away from an original distribution point. The trucks will be routed from the original distribution point, through their designated routes and return to the original point. If necessary this path may cover several days, but the route is set as determined in the method. The objective as stated above requires determining the routing paths so as to minimize the distance traveled and the number of trucks used while maintaining as much as possible an equal work load among the trucks. A twelve point (called stations or points hereafter) and single original distribution point (called the origin) problem as given in Figure 1 in Appendix D and Table I in Appendix E is used to demonstrate the application of the method. An alternate method known as lockset is also applied to this problem so as to provide a comparative solution.

CHAPTER II

REVIEW OF THE LITERATURE

The above problem is what is referred to as the traveling salesman problem (TSP), a very common problem encountered in the area of logistics and operation research. Thus, there is no lack for methods to solve the problem, each one embodying an algorithm and other associated parameters (such as boundary conditions). According to Ballou (1973), these methods can be divided into four types. Similar classifications are also emphasized by Mole (1979). The four methods types comprise: branch and bound (integer programming), dynamic programming, graphics and heuristics. The choice of and trade off between these methods rests in the solution quality (optimality) versus the computation time required by the user (Ballou 1973).

Integer programming can handle TSP and among the branch and bound procedures, the most common integer programming algorithm, is the shortest route tree approach. This problem is described as "routing through a network where the origin and destination points are not the same" by Ballou (1973). This problem is also referred to as the minimum spanning tree by Hillier and Lieberman (1980). The object is to minimize the travel time between the origin and destination by selecting the shortest route. It also can be used to find the shortest route to all points between the origin and destination. The procedure selects the minimum distance (or time) between the origin and a first node. In the second step it selects another link between the origin and a second node. The third step sums the distance from the first node to the second node with the first found shortest distance. The fourth step then compares the length of the link from the origin to the second node to the summed lengths. If the link from the origin to the second node is less than the summed links, the two links involved in the sum are eliminated from further consideration. This is repeated for each successive nodal pair until the shortest route is determined. As a matter of fact, the shortest route to all points in the network can be found if the procedure is carried to its limit. For a more detailed explanation of this solution see Ballou (1973).

A second type of method to solve the separate origin/ destination problem is dynamic programming (Hillier and Lieberman, 1980). Being less straightforward than linear programming, dynamic programming requires insight and ingenuity since each problem is considered independently. It essentially starts with a subset, solving it first and then working towards solving the larger problem by successive additions to the smaller problem. Working backwards from the end product or destination of the problem, the method accumulates alternative decision values until the beginning or origin is attained. The ingenuity enters when one must recognize a "recursive relationship" that will identify the optimum decision policy for all states in the problems (Hillier and Lieberman, 1980).

A second type of TSP is one that has the origin and destination points at the same place. The detailed specifics such as time away, in terms of a day or several days, are not critical when examining this problem. The main restriction is that the two points are the same. Again, minimizing the travel time (distance) is the objective. Returning to the integer programming type methods, Ballou (1973) sees the branch and bound method also applied to this problem as a "sequence of linear programming problems" which are successively solved until a final optimum solution is found. As is evident, as the size of the problem increases, the number of successive linear program problems also increase rapidly. Per Schruben and Clifton (1968), the total possible number of routes is (1/2)N! where N is the number of stations to be visited. The reader is referred to Efroymson and Ray (1966) for a more in-depth discussion of this branch and bound method.

The graphic approach generally deals with a visual interpretation of the problem and resultant solution. The problem is physically laid out on graph paper and alternative solutions chosen. While not very sophisticated, an experienced route scheduler can select very efficient routes. The limitation of this method is self evident in reference to problem size. For a more detailed description of the graphic approach, the reader is referred to Barachet (1975).

The last classification by Ballou (1973) is the heuristics type. Ballou's (1973) discusses the heuristics algorithm of Karg and Thompson (1964). The algorithm basically randomly selects two starting points and combines them in a route. The next point also randomly selected is then inserted in the path so as to minimize the increased distance of adding the third point to the two originally selected points. This is accomplished by adding the lengths of the legs between the new point and each of the other points and subtracting the length of the leg between the previously selected points. For example, with three previously selected points, three possible sets of two points are formed with a new point. From the sum of the length of the two sides connecting the newly selected point and each other point of the set, the distance between the previous pair of points is subtracted. This value is representative of the additional distance traveled if the newly selected point is placed between these two points in the path. This is repeated for all point pair combinations. The minimum value then determines where the point should be placed in the path to add the minimum distance to the path. The computations in this method are quite simple; however the trade-off is that the likelihood of an optimum solution is low and decreases with increasing problem size.

In discussing the Karg and Thompson (1964) method Ballou (1973) suggests that for a problem with a large number of points, a number of subproblems should be separated and solved individually. Ballou (1973) notes that this subdivision is best applied to areas of the problem that have a convex surface. Karg and Thompson (1964) subdivided a boomerang shaped multipoint problem by selecting one end of the boomerang that had a generally convex curved surface. Then they applied the above algorithm to this subdivision. This convexed curved surface can be compared to the author's convex hull as explained in further detail later. However, the author's method utilizes it as a means to estimate an initial value for (maximum) the number of trucks necessary.

The final heuristic method presented is known as the lockset method by Schruben and Clifton (1968). It is quite similar to the Karg and Thompson (1964) method above. It starts out with the solutions of routing to each station individually from the origin. This would be the maximum distance involved. The second iteration then finds the distance from one route to all other points using a distance saved coefficient. The point having the highest value is selected and locked in. The process is repeated for all points. This method is used as a comparative example in the discussion in Chapter III and Appendix B.

In closing the review of the literature, a second common aspect of the author's approach to another method is noted. Mole (1979) referenced a theory by Gaskell (1967) that placed emphasis on the spatial distribution of customers that would tend to generate routes having a generally narrow petal shape. For larger sized problems and large cluster arrangements, the use of the regression line by the author minimizes the distance the points are away from the line. Thus the distance away from the line (width) should be less than the length of the line (length) giving a narrow longer length route result. Remember this is viewed in a very general sense. A problem associated with Gaskell's (1967) theory, however, is that subtle changes in the grouping or ranking result in totally altered routes. This stimulated others to try to modify the method with a savings function so as to save the good features and correct this shortcoming. One effect mentioned by Mole (1979) of these saving functions was the generation of routes too short to implement but too complex to simply be combined. Experience could be used to give a better combined route and cost savings than the use of these saving functions. The author's method uses a cluster operation designed to provide the grouping of the points and thus reduce the need to alter these groupings. Small cluster changes result in only small group changes and therefore do not result in gross route alterations. With these brief descriptions of several common methods for the TSP scenario, the author's method is now presented.

CHAPTER III

STATEMENT OF THE METHOD

Several assumptions are utilized to simplify the presentation of the procedures involved in the method.

The first assumption is that the trucks travel at a constant (average) speed so as to equate distance and time and thus work. Work in this context is the amount of time a truck is enroute. From the speed of the truck and the length of the work day, a total distance for each truck is found. This distance is related to an area of equivalent value for the trucks. The importance of this will be explained later.

The second assumption is that all stops are of equal duration or work load. For this theory, a zero stop duration is used so as to remove unnecessary constant factors from the presentation.

The final assumption or restriction is that each location is to be serviced only once over the time of the route. In other words, a single location will not be routed to more than one time.

There are several other characteristics of the problem not included in the analysis. For example, all trucks are of equal volume, they do not need to return to the origin during the routes (this will become more clear later), and all locations receive the same or same type product being transported in equal amounts (this is not critical). As these parameters and others are added to the problem, its complexity will increase. This will be discussed later in the paper.

METHOD OVERVIEW

The method starts out by determining the maximum number of trucks required to service the distribution area. An area equivalent mentioned earlier is used. Given a set of stations (an array of points), the convex hull (as explained later) about these stations is found. The area of the convex hull is then determined and by finding the area equivalent of a truck from the distance traveled, the maximum number of trucks is determined. To the original array of stations, a cluster analysis (station grouping) is performed to determine the best grouping arrangement, with the number of clusters initially equal to the maximum number of trucks. The set of stations within each cluster is treated as a separate problem. A least squared line (principle axis, regression) for each cluster set is determined. The cluster then consists of two groups of stations, one group on each side of the regression line. Each station on one side of the line is then projected on to the least squared line with a coordinate value. Moving along the line, preferably in increasing coordinate value, determines the sequence (order) of the stations to be visited. The route for one side of the line is then determined by connecting a line to each station location in the order just determined. The route for the group of stations on the opposite side of the line is determined in a like manner and the routes are connected at their end points. The origin, if it has been omitted from a cluster set, is then connected to and from the route by selecting two adjacent stations within the cluster, the first of which is nearest the origin point. This is repeated in turn for each cluster set to determine the best route. As one will see, due to the inaccuracies in relating the distance traveled and area equivalent, the actual subroutes for the cluster groupings will result in less travel time than the initial convex hull area subdivision, therefore the cluster analysis is repeated for successively less numbers of clusters (trucks) until actual cluster routes are sufficiently long to utilize the trucks at their desired level. It is also within the scope of this method to increase the number of trucks (clusters) if desired.

The method begins with an arbitrarily chosen contention that the distance traveled is related to the area of a polygon that encloses all the points. The proposed relationship is discussed later. Also, since the trucks are traveling at a constant average speed, the distance traveled can be related to the work load which the method is striving to balance among the trucks.

DETAILED DESCRIPTION

A detailed description of the author's method along with its application to a randomly generated thirteen (13) point distribution problem is given. The same problem is also solved using the lockset method of Schruben and Clifton (1968) in Appendix B.

Table I in Appendix E and Figure 1 in Appendix D give the location of the stations in terms of their X Y coordinates, and graphical presentation using station one as the coordinate axial origin. The selection of the axis is arbitrary and has no affect on the outcome of the problem. The first step in the method is to determine the number of trucks necessary by applying the convex hull process.

The convex hull polygon is a relatively simple mathematical relationship to visualize, but a rather complex concept to mathematically describe. In essence, it is the least sided polygon that totally includes or encloses all points of an array. A characteristic of the hull is that all the interior angles between two sides are less than 180°. To determine the convex hull, it is only necessary to find those points that lie on the perimeter. By finding these points in order around the perimeter, the area of the hull can be determined very simply using a summation step. In any array of points, those points that establish the sides of the convex hull are determined by a series of tests and a simple relationship that is satisfied as one moves from one point to the next adjacent point on the polygon's perimeter. The sequence of tests applied to each point within the array are both necessary and sufficient to determine the successive perimeter points. In addition, they reduce the number of points considered to generate the convex hull. In this presentation the points are described or characterized by their X Y coordinates referenced to an arbitrary coordinate axis. The convex hull starting point is at the point having the minimum X and maximum associated Y value. This point is always on the hull. The relationship as called for in the tests is:

$$\text{Next point } X_i = \text{min. of } \frac{X_i - X_0}{Y_i - Y_0} \qquad (1)$$

Next point X_i = next point on the convex hull

X_i = X value of the next considered point

Y_i = Y value of the next considered point

X_0 = X value of last selected hull point

Y_0 = Y value of last selected hull point

i = 1 to total number of points in the array that satisfies the applicable test.

A series of tests is used to determine either the set of points to be considered in applying equation 1 or the next point on the hull. The tests are applied sequentially as follows:

Test 1 Apply equation (1) to all points where $Y_i > Y_0$

2 Points where $Y_i = Y_0$ are the next points in ascending order of X value on the hull

3 Apply equation (1) to all points where $X_i > X_0$

4 Points where $X_i = X_0$ are the next points in descending order of Y value on the hull

5 Apply equation (1) to all points where $Y_i < Y_0$

6 Points where $Y_i = Y_0$ are the next points in descending order of X value on the hull

7 Apply equation (1) to all points where $X_i < X_0$

8 Points where $X_i = X_0$ are the next points in ascending order of Y value on the hull

For each successive point determination it is important that the tests are applied sequentially eliminating prior tests only after no points satisfy their conditions. Also it is important to observe the rule of signs, higher value negative numbers are actually lower (minimum) than low value negative numbers (-6 is less than -4). To see how this works, let's apply it to the problem in Figure 1 in Appendix D. The hull starting point is the station having the minimum X value and maximum associated Y value. This is station 3 with an X value of zero and Y value of 16. These are X_0 and Y_0 respectively. To determine the next hull station we apply Test 1. According to Test 1, stations 5 and 6 are the only stations to be considered as the next possible hull point X_i. Applying equation 1 to stations 3 and 5, with $X_i = 8$, $Y_i = 19$ gives:

$$X_i = \frac{8-0}{19-16} = 2.67$$

Next, applying equation 1 to stations 3 and 6 gives a X_i value of 5. Therefore station 5 is selected as hull point 2 since it has the minimum X_i value. Now $X_0 = 8$ and $Y_0 = 19$ since station 5 is the last selected hull point. Test 1 is again applied with the result that no points satisfy Test 1. Therefore, Test 2 is applied. Only station 6 satisfies Test 2 so station 6 becomes the third hull point. The coordinates of station 2 now become X_0 and Y_0. Following this hull point selection, the tests are again applied in order and the procedure is repeated until the initial convex hull point, station 3 is selected as the next hull point by the tests. The results of this procedure applied to our example are found in Table II of Appendix E, and Figure 2 in Appendix D.

The use of this test procedure accomplishes two results. First, it determines those points that make up the hull, and second, it selects them in order. Though not absolutely essential, as stated earlier, the selection of the points in order can be used to reduce the area determination process. This is accomplished by determining the area of each section simultaneously with the point selection of the hull and retaining the sum. A more general approach, however, to the area is by the use of the summation process and is given by

$$\text{Area} = \Sigma \tfrac{1}{2}(X_i - X_0)(Y_i + Y_0) \tag{2}$$

Σ = summation from $i = 1$ to n

i = next station on the hull

n = number of stations on the hull

The variables X_i, X_0, Y_i and Y_0 are the same as described earlier. This formula is more fully explained and developed in Appendix A. The sum value over all the hull points is the total area of the convex hull which is used with the area equivalent (examined shortly) of the trucks to find the maximum number of trucks needed.

The above area for our problem is found using equation (2) and the stations given in Table II in Appendix E as follows. For example, starting at station 5, the second point on the convex hull with $X_i = 8$, $Y_i = 19$, $X_0 = 0$ and $Y_0 = 16$, the first iteration ($i = 1$) of equation (2) gives

$$\begin{aligned} \text{Area} &= \tfrac{1}{2}(8 - 0)(16 + 19) \\ &= 140 \text{ units.} \end{aligned}$$

For the second iteration ($i = 2$) and station 6 with $X_i = 15$, $Y_i = 19$, $X_0 = 8$ and $Y_0 = 19$ the area is

$$\begin{aligned} \text{Area} &= \tfrac{1}{2}(15 - 8)(19 + 19) \\ &= 133 \text{ units.} \end{aligned}$$

The total area to this point using the summation according to equation (2) for the three stations mentions is

$$\text{Total area} = 140 + 133 = 273 \text{ units.}$$

(Remember, the term units is used here since the actual units from the equation will be based on the user selection such as time, distance, etc.) The total area of the convex hull is determined by repeating the above process for each convex hull point and summing the total over these points. As shown in Appendix A as the area calculation progresses around the hull perimeter, those areas outside of the hull found in the earlier iterations will be subtracted from the total area in the later iterations. In this particular example, however, this is not readily evident since the convex hull extends below the X axis. The area extending below the X axis and above the convex hull perimeter line in this case is added to the total area (see stations 13 and 1, Table II, Appendix E). If the convex hull fell above the X axis the area between the X axis and the convex hull perimeter would be subtracted from the total area. As equation (2) is applied to all the points on the convex hull for the above problem, the area of each section is found and summed. These along with the total area are given in Table II of Appendix E. The total area is 391.5 units. Also a graphical representation of the convex hull is found in Figure 2 of Appendix D.

To find the maximum number of trucks needed it remains only to determine the area equivalent of each truck. The area equivalent for a truck is selected (arbitrarily) as the product of its average speed and length of the work scheduled. This product is the maximum distance that a truck can travel in a particular work schedule. The effect of this area equivalent selection is to equate an area to a distance. This relationship, lacking proof, though works well in the number of truck determination process. This area equivalent value (distance) is divided into the total area of the convex hull (area) to give the maximum number of trucks needed to service that area. For this example the total area of the hull is 391.5 units. The units can represent distance, or time, or some other parameter being considered. For purposes of simplicity and demonstration, let us assume each unit represents approximately two and one half miles and we are trying to minimize distance. Also let us assume the trucks average 40 miles per hour for 8 hours per day. The area equivalent of a truck is therefore equal to 320 miles (40 mph x 8 hours). To convert from miles to units so that the area of the hull is in the same units of measure, we divide by the conversion factor 2.5 miles per unit. Therefore the area equivalent of each truck becomes 128 units. The total number of trucks needed then is found by dividing this area equivalent into the convex hull's total area of 391.5. This problem calls for just over three trucks (391.5 divided by 128) so we select three trucks. This number is the starting place for the remainder of the method.

Before progressing further with development of the method, it is with the area equivalent value that the effects of other parameters can be integrated into the problem. For example, differing truck volumes can be considered by varying the area equivalent of the trucks. In adding just this one parameter, the user must be aware of possible complications; for example, the addition of variable truck volumes can cause possible problems in the clustering analysis part of the method since the clustering method generally tries to group the stations equally among the trucks. Thus, even though the truck size is varied, the number of stations assigned a truck tries to remain equal. On the other hand, as the number of cluster groups are decreased to lengthen the individual truck route, a mismatch of work load between the trucks can also result. A second parameter such as variable speed among the different routes can also be compensated for by the area equivalent of the trucks. Slower speeds would result in smaller area equivalents. Finally, varying unloading times can also be included in the analysis by varying the area equivalent of the truck, the longer unloading times, the smaller area equivalents. It is important to remember that the method does not try to solve these potential conflicts. It is provided as a starting place in a routing problem. After application of the method, it is to the user's advantage to refine the solution by examining the results and modifying them to achieve an improved ultimate solution. This can be done by subjective evaluation or through the use of more sophisticated methods. This is left for future investigation and not considered further in this paper.

Now let's return to the explanation of the second step of the process. In most instances this step requires use of a computer to perform a cluster analysis. Cluster analysis groups the stations by some common parameter into subgroups. For example, the Ward method used in the SAS cluster procedure groups the points in a cluster by minimizing the distance between the points and placing the associated point in the desired number of clusters. The complexity of the cluster analysis is such that a mathematical explanation and presentation of an example is outside the scope of this paper (Milligan, 1980). Since the particular cluster analysis selected is not critical to the application of the method, further explanation of the processes involved in the actual cluster analysis is omitted. As a matter of fact, a subjective visual clustering of the stations is possible and in smaller sized problems possibly preferable. The only thing to remember is the clustering is divided initially into n different clusters and then successively reduced, with n equal to the number of trucks found above. For our example where n = 3, the first clustering has three groups, the second clustering has two groups and the final clustering has one group (all stations). As is evident, the clustering is performed so as to divide the stations as equitably as possible among the trucks and should result in as well balanced a distribution as possible. The Ward method referenced above is used in this example for several reasons. First, its results are fairly well balanced clusters. Second, it is available in the statistical analysis system (SAS) package widely available to many firms. And finally, it provides all possible clustering combinations in the output (n cluster combinations where n equals the total number of stations in the original cluster). The application of the Ward method on the above problem was performed using "cluster W" command in the SAS program on a TSO terminal. The cluster analysis gave the stations in each cluster (truck) for the 3, 2 and 1 clustering scenerios. The results are given in Table III of Appendix E. The one cluster problem is not listed since it includes all stations listed in the original problem. Also, the order of stations listed is unimportant at this time.

One point that should be made at this time is the way that the origin point is handled. In the cluster analysis there are two ways to handle the original distribution point. It can be included in the clustering or it can be excluded and integrated into the problem after the clustering. This is discussed in more detail later along with the preferred approach and reasons for its preference.

A second point to be discussed deals with the successively fewer number of clusters. There is a very sound reason for interest in the clustering arrangement having fewer clusters than the original number of trucks. Remembering that the distance traveled is arbitrarily equated to the area of the polygon, so as the stations are subdivided into smaller number of clusters, smaller polygons around these clusters can be found and, significant area amounts (distances) between clusters are eliminated from the distance traveled by the trucks. Thus, the individual lengths of the routes due to the clustering, generally would be quite less than the route lengths determined from the convex hull. For example, the convex hull gave a total area (distance) of 391.5 units. If we were to look at the area of each cluster, say from the three cluster scenerio analysis (Figure 3, Appendix D), we can see the total area of the three individual clusters is significantly less than the total area of the convex hull. Therefore, a single truck may be able to handle two clusters instead of one. For our example we will examine scenerios for 3, 2 and 1 truck solutions. In other words, the truck numbers are the same as the cluster numbers in Table III of Appendix E, and thus would service those stations listed adjacent the cluster number. For example, under a three-truck scenerio, truck 1 is responsible for stations 1, 2 and 13; truck 2 stations 3, 4, 5 and 6, etc. Under a two-truck scenerio, truck 1 would be responsible for stations 3, 4, 5 and 6, etc. The same explanation applies to Table VIII of Appendix E.

With the stations now divided among the trucks it remains to define the individual truck routes. This is accomplished using a least squared (regression) line approach for each cluster. The least squared line is the line that best describes the set of points. It is also a line that minimizes the distance each point is located from the line. The formula for finding the least squared line is

$$Y = a + bX \tag{3}$$

where

$$b = n \frac{\Sigma(X_i)(Y_i) - \Sigma(X_i)\ \Sigma(Y_i)}{n\ \Sigma(X_i)^2 - (\Sigma\ X_i)^2} \tag{4}$$

$$a = \frac{\Sigma(Y_i) \quad - b \quad \Sigma(X_i)}{n} \tag{5}$$

n = number of points in the cluster

X_i = X value of the i point in the cluster

Y_i = Y value of the i point in the cluster

a = the Y- intercept of the regression line

b = the slope of the regression line

For a more in depth explanation and derivation of the least squared regression line, the reader is referred to Winkler and Hays (1975). The regression line approach is chosen for two reasons. First, from intuitive observation, a route that progresses around the outer perimeter of an array appears to give the shortest routes connecting the points. Therefore dividing the array in about half (distance-wise) using the regression line establishes an outer perimeter type route for the outward bound and the inward bound legs, the above line and below line points respectively. The second reason is that the projection of the points of the array onto the regression line gives a fairly good order of points along the line and is fairly easy to calculate. The result is a route that tends to follow the perimeter of the array fairly well.

A regression line is fit to each cluster separately; that is one is fit to each cluster in the differing scenerio problems having successively lower number of clusters. In applying the regression line fitting to the clusters, the author assumes the reader is familiar with regression analysis to the extent of at least finding and understanding the regression equation. Therefore, the actual calculations necessary for finding the equations are omitted.

Starting at the three-cluster scenerio problem, a regression line is fit to each cluster using equations 3, 4 and 5. For cluster 1, the regression line is Y = 1.68-.23X. For cluster 2, Y = 15.99 +.23X, etc. The same procedure is repeated for the two cluster and single cluster cases. The results are summarized in Table IV of Appendix E. Graphical representations of the cluster and their respective regression lines are shown in Figure 3 for the three cluster, in Figure 4 for the two cluster, and in Figure 5 for the single cluster problems in Appendix D.

At this point it is important to discuss several alternative aspects of the process in dealing with the original distribution point. For this particular problem, the original distribution point was arbitrarily chosen as station 13. As mentioned earlier, this point can be included or excluded from the clustering analysis; the preference is that of the user. The best approach would be to examine both alternatives and select the better. In this example the author selected to include the origin in the cluster analysis due to the minor alterations with its exclusion. For example, under the three cluster scenerio if the origin was excluded, a very differing sloped regression line would result for cluster 1. The route though would still be as found with its inclusion.

The next question encountered in reference to the original distribution point is in the regression line determination. Again the origin can be included or excluded in the stations making up the cluster array for determining the regression line. As can be seen from Table III in Appendix E, the origin falls in only one cluster in each of the different problem scenerios and therefore would only affect one regression line in each problem. Again, the preference of including or excluding the origin point is left to the user's descretion with the preferred method being to examine both alternatives and select the better one. In this problem the author included the origin in the regression line determination process since again it did not affect the regression line significantly.

Returning to the explanation of the process, the regression lines, as stated earlier, divide the clusters into two groups of stations, one on each side of the line. By projecting the stations on one side of the line onto the regression line, a fairly systematic order of points along that side of the line is found. This projection is good at arranging points that are relatively close on that side of the line in order on the regression line. There are exceptions (such as very far spaced points) that are discussed later. This projection, however, is used to arrange the stations in order of service. By using the X value of their projection on the regression line, the stations are listed in order of ascending X value. To determine the order in which the stations are visited by the trucks, each point in a group (that is the points that are on one side of the line) in a cluster is projected onto the regression line, the projection being along the path perpendicular to the regression line. The X value of this projection, used to assign the station order, onto the regression line by station i is found by

$$X = X_i \pm \left[\frac{(Y_i - (a+bX_i))^2}{2 + 1/b^2 + b^2} \right]^{\frac{1}{2}} \qquad (6)$$

where

X = projected station's X-value on the regression line

X_i = X value of i station

Y_i = Y value of i station

a = Y intercept of regression line for the cluster containing station i

b = slope of regression line for the cluster containing station i

A derivation of equation (6) is given in Appendix C. The ± sign found in equation (6) is necessary since the stations are located above and below the regression line and since the slope of the regression line can be positive or negative. The positive sign (+) is used when either the slope is positive and the stations are above the line <u>or</u> the slope of the line is negative and the points are below the line. The negative sign (-) is used for opposite conditions.

From these projection values the station sequence is then the ascending or descending order of the X value for each point as it is projected on the line. On the inward leg of the route, the descending order will be used and on the outward leg of the route, the ascending order will be used. The actual direction is dependent on the direction of the route. This will become more clear later. For our example, using equation (6) with the three cluster scenerio and for cluster number 3, station 7 and 8 are above the line as can be seen in Figure 3 of Appendix D, and stations 9, 10, 11 and 12 are below. Applying equation (6) to station 7 using a = -5.50, b = 0.5, Y_i = 10 and X_i = 21 from Table IV in Appendix E, gives the X value of station 7 of 23.0.

$$X = 21 + \left[\frac{(10-(-5.50 + (.5)21))^2}{2 + (1/(.5)^2 + (.5)^2}\right]^{1/2}$$

$$= 23.0$$

Again, applying equation (6) to station 8 using the same values for a and b but Y_i = 7 and X_i = 17 gives X value for station 8 = 18.6. Thus station 8 is visited first and station 7 second, on this side of the line. Applying the X_i, Y_i, a and b values into equation (6) for stations 9, 10, 11 and 12 give X values respectively equal 20.6, 16.6, 14.2 and 19. Therefore, the lower side of the line route order is stations 11, 10, 12 and then 9. Finally, the two routes are completed by connecting the end points of the two routes, one from above and one from below the line. That is, the station above the line having the greatest project X value is ordered (serviced) just before or after depending on the travel direction, the station below the line having the greatest projected X value. To join the above routes from the example connect the stations having the highest projected X values on opposing sides of the line, in this case, stations 7 and 9 and for the opposite end, the stations having the lowest projected X value on opposing sides of the line, in this case stations 11 and 8. The proposed route now is, say starting at station 7: 7, 8, 11, 10, 12, 9 and back to 7. The final step in this method is now to connect the original distribution point to the above determined routes since it is not included in this cluster. With some routes such as cluster 1 of the three cluster scenerio, this is not necessary, since the origin is included in the cluster and subsequent route determining process. The origin inclusion is accomplished by selecting the station in each cluster that minimizes or is the minimum distance from the origin. This will be the first station on the route from the origin. The exit station back to the origin is the adjacent station having the shortest distance to the origin. The direction of travel along the route now must proceed in the direction opposite the exit station so that the exit station is serviced last. The final route selection initiates at the origin, progresses to the nearest station on the route, sequences through the route, exits at the selected adjacent station to the entrance and returns to the origin. In our problem since the origin is not included in the cluster, the final step requires connecting the distribution station 13 with the above determined route. This is done by finding two adjacent stations that include the shortest distance from the origin and the shortest adjacent distance from the origin. Using Table V of Appendix E, the distance between stations and the origin station 13 are given in column 13. Examining only those stations found in this cluster, stations 7, 8, 9, 10, 11 and 12, the shortest distance is between stations 13 and 11. The next adjacent station on the route with the shortest distance to the origin is between stations 13 and 10. Therefore, station 13 connects to the route at station 11 and from the route at station 10. The final solution is to enter along the shortest distance and exit the other. Therefore, the final route is stations 13, 11, 8, 7, 9, 12, 10 and 13. (See Table VI in Appendix E and Figure 6 in Appendix D.) Notice, it is important that direction along the route is observed. In including station 13, we cannot sequence station 10 and 11 consectively since we enter at one and exit at the other. Therefore, we progress in the opposite direction of station 10, that is, to station 8.

The above procedure is applied to all the clusters within the various scenerios and the resultant routes are summarized in Table VI of Appendix E. Individual routes and their distances as well as total distances (in units) are also given in Table VI of Appendix E. Graphical representations of Table VI in Appendix E are given in Figure 6, 8 and 10 in Appendix D. This finishes the steps involved in determining the routes according to the author's method. The steps are summarized as follows:

Step 1 Find the convex hull of the array of stations and its associated area. Using the average speed and work schedule length, find an area equivalent for the trucks. Finally, find the maximum number of trucks by dividing the convex hull area by the area equivalent of the trucks.

Step 2 Perform a cluster analysis on the array of points with the cluster number initially equal to the number of trucks. Successively fewer cluster numbers can be used to obtain the desired length route. Clustering may or may not include the origin.

Step 3 For each cluster determine a regression line.

Step 4 Project each point onto the regression line and arrange the points on each side of the line in order of descending or ascending X values. Connect the end points of the points above the line with the end points below the line.

Step 5 If the origin is not in the cluster connect the origin to two adjacent points on the route which include the shortest distance from the origin to a cluster point.

The above method with only slight modifications can be used to encompass a variety of additional problem scenerios, features and constraints. If one wishes to allow for expansion of the distribution sequence by only partial utilization of the trucks, it requires only increasing the number of clusters which results in shorter truck routes. If a problem requires servicing a single location at two different times along the route, the single cluster, including this repeated serviced station, can be divided into two clusters, one cluster of stations serviced before the repeated station and a second cluster of stations after the repeated station. The repeated station is then included in each cluster for route determination.

Before comparing the results of this method to another alternative method, the lockset method, it is important to reemphasize several aspects of this method. In performing both the cluster analysis and the regression analysis, it is within the scope of this method to either include or exclude the original distribution station. It can be included at the cluster analysis or at the regression analysis. One reason for including the origin may be to reduce the route selection process. If the origin is near one cluster its inclusion automatically places it in the determined route. On the other hand, if it is spaced away from a cluster, its inclusion could totally alter the route selection sequence since the regression line would be affected significantly by this point.

In the above example, the original distribution station is included in the cluster analysis since it is located reasonably close to the cluster as a whole. The result is that it reduces the route determination process by not requiring connecting the origin to a cluster in each scenerio. This is seen in cluster 1 of the three cluster problem (See Table III in Appendix E). One disadvantage to including the origin is that the origin can bias the clustering analysis and cause the resultant clusters to be unbalanced in terms of work load. The actual resultant effect can be quite significant in terms of route determination and therefore it is recommended that the method be run with the original distribution station, both included and excluded, and the best solution selected therefrom. The effect of excluding the origin in our problem is not examined in this paper. It is left for further study later. The reason for not examining its exclusion is that this paper is presented as a description of the proposed process and not as an optimal solution to a particular problem.

COMPARISON OF METHODS

To provide a comparative selection method the same problem is solved using the lockset method. The lockset method utilizes what is called the distance saved coefficient (DSC) to select the order of stations. The actual coefficients and the station sequence is dependent on a series of tests. Example calculations and tests involved in generating the solution of the lockset method are given in Appendix B. The DSC for the thirteen-point problem is given in Table VII of Appendix E. Other values such as distance from the distribution station to each station used in the lockset method are given in Table V of Appendix E. The generation of the DSC values are explained in more detail in Appendix B. The station selection sequence of the lockset method is by descending order of DSC value in combination with the series of tests also referenced in Appendix B. For example, a station is added to a route only if each station involved in the selected DSC was previously on different routes. Again for a more in-depth description of lockset, one is referred to Schruben and Clifton (1968). Utilizing the lockset method, 1, 2 and 3 route solutions were found and are summarized in Table VIII of Appendix E along with their associated lengths. The lockset method was divided into the two and three routes by restricting the distance traveled by each truck. For the two route problem the distance was held at least equal to and as near 30 units as possible; for the three route problem, a distance at least equal to and as near as possible 15 units, units again being convertable to miles.

As can be seen in Tables VI and VIII in Appendix E and Figures 6-11 in Appendix D, the author's method and the lockset method result in very closely similar solutions to the problems. A closer look reveals only a single scenerio having a better solution with the author's method to this particular problem. Under the three cluster arrangement, the author's method's overall total path length (sum of all the paths) is shorter than lockset. Due to the closeness of the solutions the author hesitates to claim superiority for either this method or lockset with only a single test of the methods; however, on the surface, this method and lockset appear to be viable alternatives to each other as starting points. Between the methods, the author's method results in shorter routes on some routes while lockset has shorter routes on others. For example, in the three cluster problem, one of the author's routes is shorter and two of the lockset routes are shorter. Another very important aspect of the two methods that should be examined is the balance of routes. As can be seen, the lockset method results in better balanced routes than the author's method. This is due to the clustering step. Even though clustering under the Ward method tries to result in fairly well balanced clusters, this is not always the best balance of routes. The clustering method used therefore can have a significant impact on the ultimate solution. Remember though, that this is only a starting solution. While the overall length savings is in favor of the lockset method, caution is recommended in claiming this for all problem scenerios.

The last point of the methods to be discussed centers around the number of trucks that are finally needed. As can be seen in Tables VI and VIII in Appendix E, the single route with all the points can be serviced by a singular truck (93.5 or 87.7 units on the method's routes with 128 units possible by the trucks). This is the reason for examining solutions with successively fewer trucks. Therefore, utilization of the above method as an alternative starting point in route selection methods for TSP type problems is proposed with possibilities of improved distribution route selections. It is important to remember that these claims are made from the result of only one randomly applied problem. Proof of these claims can result only from further extensive testing of the method applied to a wide variety of problem scenerios. These are not examined in this paper and the method is left to stand on its own.

In closing, it is interesting to note several aspects of this method that can be related to other methods. First, an optimal solution is not necessarily found similar to the lockset method. This is due to the sequence process which does not examine or strive to select the minimum distance between stations, as it miminizes the overall distance from all the stations to a common reference line. The result of this tends to give routes that generally run along the perimeter of a set of points.

Second, the reader will remember that in discussing the Gaskell (1967) method above, long narrow petal-shaped routes were preferred as to broad round petal-shaped routes. By minimizing the distance from the regression line, the routes tend to stay near the regression line and therefore should result in fairly narrow routes in a general sense.

Third, to reemphasize the flexibility of the method as a starting point to solving a distribution problem, the method can be modified (without significant diversion from the original steps) to overcome possible inefficient inclusion of widely spaced apart groups or single stations. For example, instead of servicing these widely spaced stations normally by means of the shortest distance, the author's method sequences them in order by their location value on the regression line as compared to the other stations. It would be better to omit these initially from the route selection sequence (the regression analysis) using the projection on the regression line and incorporate them later in the route using the same rules that are applied to include the original distribution point originally not found in a cluster. This also is left to future investigation.

CHAPTER IV

CONCLUSIONS

The method utilizing the convex hull, cluster analysis, and the regression line to select the sequence of station in a distribution problem is presented to be a viable alternative starting point to the lockset method. Savings in distance may be possible with substitution of this method. In addition to alternate route selections to TSP type problems, the method allows for inclusion of varying problem characteristics into the model. The method is presented only as an alternate method and not as a superior or preferred method to be used.

IMPLICATIONS

The author's method provides an alternative approach to the common distribution problem faced by all firms. Potential use includes all firms that distribute goods from a distribution source to a plurality of locations. Also it would apply to service firms or any other types of firms that distribute. It is not presented as a replacement for other methods especially more sophisticated and optimal solution methods, but more as an alternate initial starting solution, especially for those firms not presently utilizing any route selection methods. It can result in potential savings, and its use is encouraged as a check to presently used method or as a starting point. As pointed out above, modifications can be made to the method to fit it to a variety of scenerios and the author sees its potential application to a large variety of firms.

A SELECTED BIBLIOGRAPHY

(1) Ballou, R. H. Business Logistic Management. N. J.: Prentice Hall Inc., 1973, pp. 345-354.

(2) Barachet, L. L. "Graphic Solution of The Traveling Salesman Problem." Operation Research. Volume 5, (December, 1975), pp. 841.

(3) Carlson, B. C. Special Functions of Applied Mathematics. New York: Academic Press Inc., 1977, pp. 75.

(4) Efroymson, M. A. and T. L. Ray. "A Branch-Bound Algorithm For Plant Location." Operation Research, (May-June, 1966), pp. 361.

(5) Gaskell, T. J. "Bases For Vehicle Fleet Scheduling." Operation Research Quarterly, (1967):18, pp. 281.

(6) Graduate College. Thesis Writing Manual. Stillwater, Oklahoma: Oklahoma State University, Revised, 1982.

(7) Hillier, F. S. and G. J. Lieberman. Introduction To Operation Research. 3rd Ed. San Francisco: Holden Day Inc., 1980, pp. 237-241, 266-272.

(8) Karg, R. L. and G. L. Thompson. "A Heuristic Approach to Solving Traveling Salesman Problem." Management Science, Volume 10 (January, 1964), pp. 225-230.

(9) Milligan, G. W. "An Examination of The Effects of Six Types of Error Perturbations on Fifteen Clustering Algorithms." Psychometrika, (1980):45, pp. 325.

(10) Mole, R. H. "A Survey of Local Delivery Vehicle Routing Methodology." Journal of Operation Research Society, Volume 3, No. 3 (1979), pp. 245-252.

(11) Statistical Analysis System, User Guide, Statistics, 1982 Ed. Cary, N.C.: SAS Institute.

(12) Schruben, L. W. and R. E. Clifton. "The Lockset Method of Sequential Programming Applied to Routing Delivery and Pickup Trucks", American Journal of Agricultural Economics, 50, 1968, pp. 854-866.

(13) Winkler, R. L. and W. L. Hays. Statistics, Probability, Inference, and Decision. 2nd Ed. New York: Holt Rinehart and Winston, 1975, pp. 665.

APPENDIXES

APPENDIX A

CONVEX HULL AREA FORMULA

If we have a convex hull such as in Figure X below and wish to determine

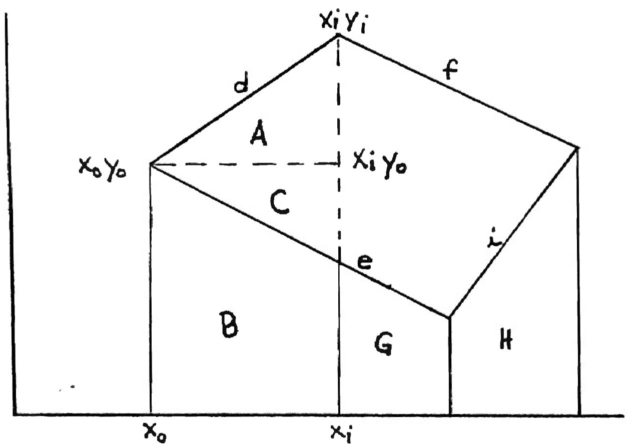

FIGURE X

the area, it can be found by the following argument. The area of A is equal to 1/2 $(X_i - X_0)(Y_i - Y_0)$ and the area of B+C is $(X_i - X_0)(Y_0)$ so the total area under line d of the convex hull is $\frac{1}{2}(X_i - X_0)(Y_i - Y_0) + (X_i - X_0)Y_0$. This reduces to

$$\frac{1}{2}(X_i - X_0)(Y_i + Y_0) = \text{Area under d}$$

The area under f is found in a like manner using the next point around the perimeter. As one continues to find the areas under each perimeter section the areas G, H, and B not within the polygon, will be removed when the areas below lines e and i are found. This is the result of summing in one direction around the perimeter that gives these areas a negative value. The sign of these areas subtract them from the total summed area.

APPENDIX B

LOCKSET CALCULATIONS

The lockset method is one approach to solving the traveling salesman problem. It has several advantages and disadvantages. It is fairly easy to understand and the computations involved are not complex. The decisions utilized in selecting the paths, however, can be rather tedious, especially for large number of point problems. Finally, the solution found is not necessarily the optimal one. The following is an example of the calculations involved in the lockset method as applied to the randomly generated distribution problem shown in Figure 1 of Appendix D. The first necessary data for a decision process is the distance between the origin and each station. This is found in Column 13 of Table V of Appendix E. The first step in the lockset method is to find the list of all possible pair of stations excluding the origin and their corresponding distances. For instance, in the thirteen point problem, we need the distances between stations 1 and 2, 1 and 3, 1 and 4, etc. up to stations 11 and 12. These values are tabulated in Table V of Appendix E, columns 2-12. For example, the distance between stations 1 and 2 is 7.1, 1 and 3 is 16.0, and so on. The next step is to find the distance saved coefficient (DSC) described in the presentation. The DSC is found for every pair of stations in the array from the values in Table V of Appendix E. The general equation is

$$DSC - P_1P_0 + P_2P_0 - P_1P_2$$

where P_1P_0 is the distance from the origin to station 1; P_2P_0 is the distance from the origin to station 2; and P_1P_2 the distance between stations 1 and 2. The subscripts 1 and 2 can be any pair of stations. For example, the DSC for stations 3 and 4 is found by adding the distance between the origin and station 3 with the distance between the origin and station 4. The distance between station 3 and 4 is then subtracted from this sum to give the DSC. Mathematically from Table V of Appendix E the distance from the origin to station 3 is 20.6, from the origin to station 4 is 19.7, and between stations 3 and 4 is 3.0. Thus the DSC for stations 3 and 4 is

$$20.6 + 19.7 - 3 = 37.3 \text{ (3,4 DSC)}$$

This is repeated for each station pair in the problem and these results are listed in Table VII in Appendix E. Once the DSC for each pair is found the third step involves joining the pairs with the highest DSC values into the same route. In the given problem the highest DSC pair is stations 5 and 6 with DSC = 38.1. As stated earlier these pairs can only be merged if they satisfy two tests.

1) Each station must have one leg connected to the origin;

2) Each station must have been on different routes.

It is important to point out here that initially each point (station) is connected to the origin. This is the first solution to the problem and establishes n different routes, n equalling the number of stations. Therefore to join this pair of stations they must satisfy the above tests. The first test is satisfied since initially station 5 and station 6 are both connected to the origin. The second test also is satisfied since each are on separate routes. Thus a new route now is from the origin to station 5, to station 6, and back to the origin. It replaces the two above mentioned single routes. The next highest DSC is then selected and the procedure repeated until all stations are inserted in the route. This is performed in our problem and the resultant route selections for each scenerio are given in Table VIII of Appendix E. In addition to the single route problem, the author was interested in the two route and three route problem. These were found in a similar fashion as described except as the route was being generated the length of it was continuously monitored by adding the distances between the points on the route. When the route length was at least equal and as near as possible to 30 units it was terminated and a new route began. The length used above was exclusive of the distance to and from the origin. The 30 unit length gave the two route solution and a 15 unit length gave the three route solution. All routes are summarized in Table VIII in Appendix E. These are the solution routes utilizing the lockset method and used to compare to the author's method. Graphical representations of the routes are given in Figures 7, 9, and 11 in Appendix D.

APPENDIX C

X PROJECTION VALUE FORMULA

The development of the X projected value formula (equation (6)) in the text is found utilizing two very basic mathematical relationships found in any basic trigonometry text. The first relationship is the sum of the squares of the sides of a right triangle is equal to the square of the hypotenus. The second relationship is the slope of a line is equal to the rise over the run. This is also the inverse of the tangent of the angle adjacent the run leg.

Looking at Figure T below which represents a point i located above a line (regression line), the goal is to find the length of "O". This length is added to or subtracted from the X value of point j to find the desired projected X value. Knowing the X value of point j is equal to the X value of point i, the proof is as follows.

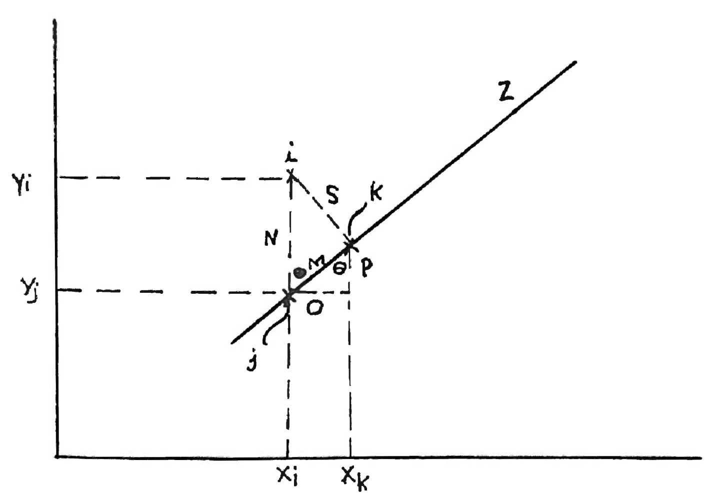

FIGURE T

Proof: The slope b, of the line z, is:

$$b = \frac{P}{O} = \frac{M}{S} \quad \text{(This last equality is found from + tan } \theta = O/P = S/M\text{, the indicated } \theta \text{ are all equal.)} \tag{1}$$

$$P = bO \tag{2}$$

$$S = M/b \tag{3}$$

From our first relationship we find,

$$M^2 = O^2 + P^2 \longrightarrow O^2 = M^2 - P^2 \tag{4}$$

$$N^2 = S^2 + M^2 \longrightarrow M^2 = N^2 - S^2 \tag{5}$$

Combining equations (2) and (4),

$$O^2 = M^2 - b^2O^2 = \frac{M^2}{1+b^2} \tag{6}$$

Combining equatios (3) and (5),

$$M^2 = N^2 - \frac{M^2}{b^2} = \frac{N^2}{1+ (1\div b^2)} \tag{7}$$

Combining equations (6) and (7),

$$O^2 = \frac{N^2}{(1+(1\div b^2))(1+b^2)} = \frac{N^2}{2+(1\div b^2)+b^2} \tag{8}$$

Now to find N so that O is a function of known parameters. We know that point i has coordinates X_i and Y_i. At point j the regression line equation a + bX = Y gives the Y value of point i onto the regression line. In other words $Y_j = a+bX_i$ and thus point j has coordinates X_i, $a+bX_i$. Therefore

$$N = Yi - (a+bX_i') \tag{9}$$

$$N^2 = [Y_i' - (a + bX_i')]^2 \tag{10}$$

Combining equations (10) and (8),

$$0 = \left[\frac{(Y_i' - (a + bX_i'))^2}{2 + (1 \div b^2) + b^2}\right]^{\frac{1}{2}}$$

The formula for "O" then is a function of known variables. Therefore, the X value of point k is found by

$$X_k = X_i' \pm 0$$

The plus or minus sign is due again to the location of the point above or below the line and the slope of the line. See the text for which sign to use. For this particular example the plus sign would be used.

APPENDIX D

FIGURES

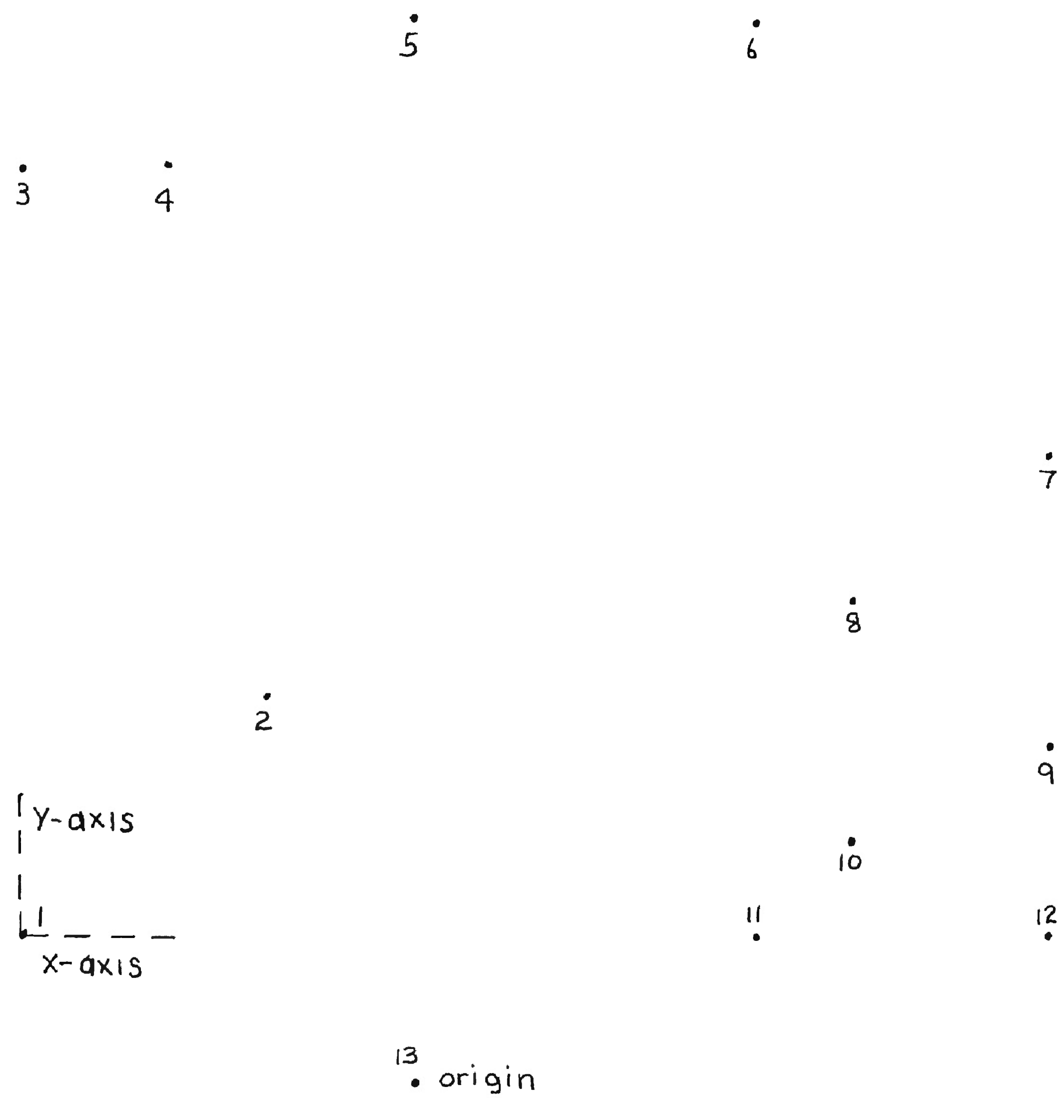

Figure 1. Distribution Array

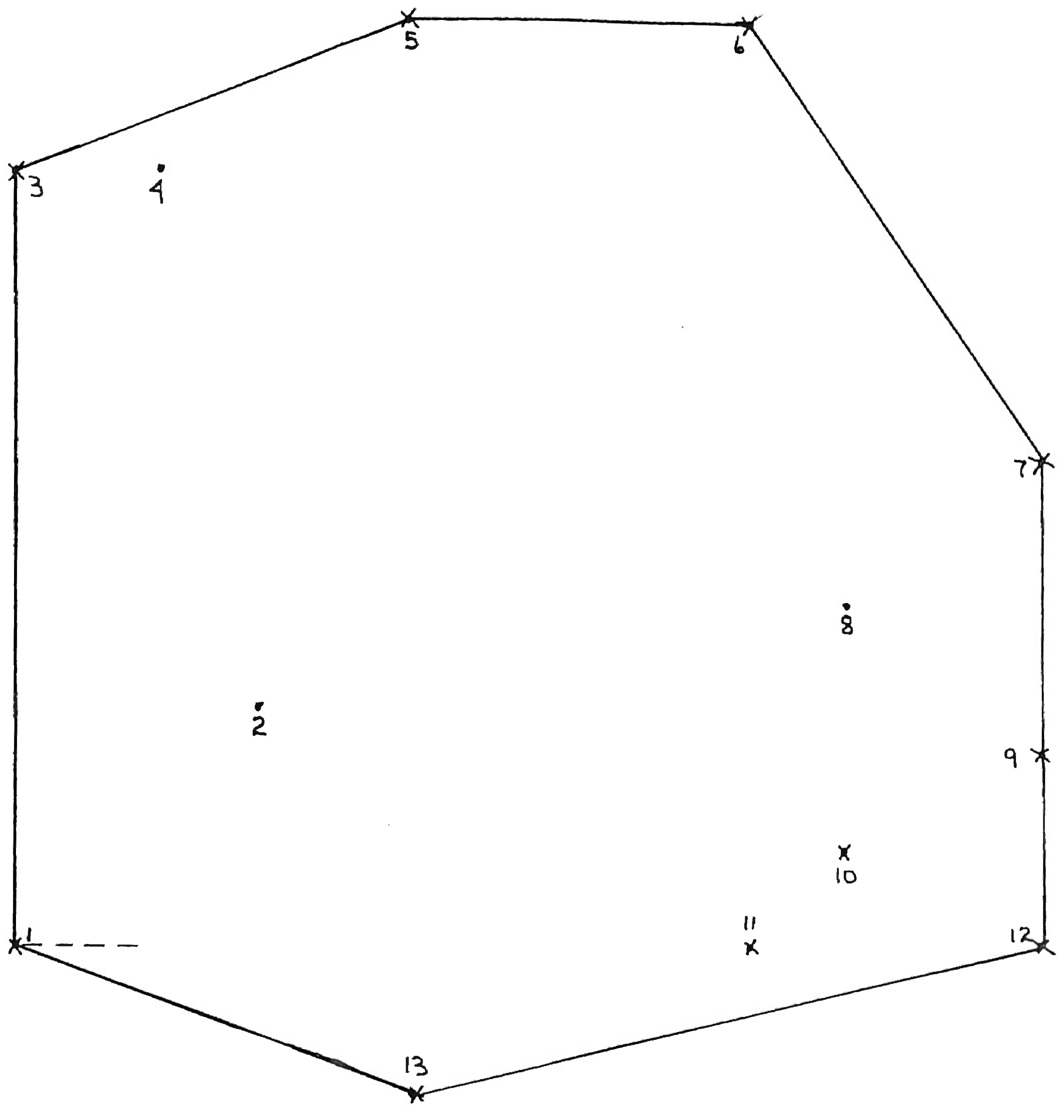

Figure 2. Convex Hull

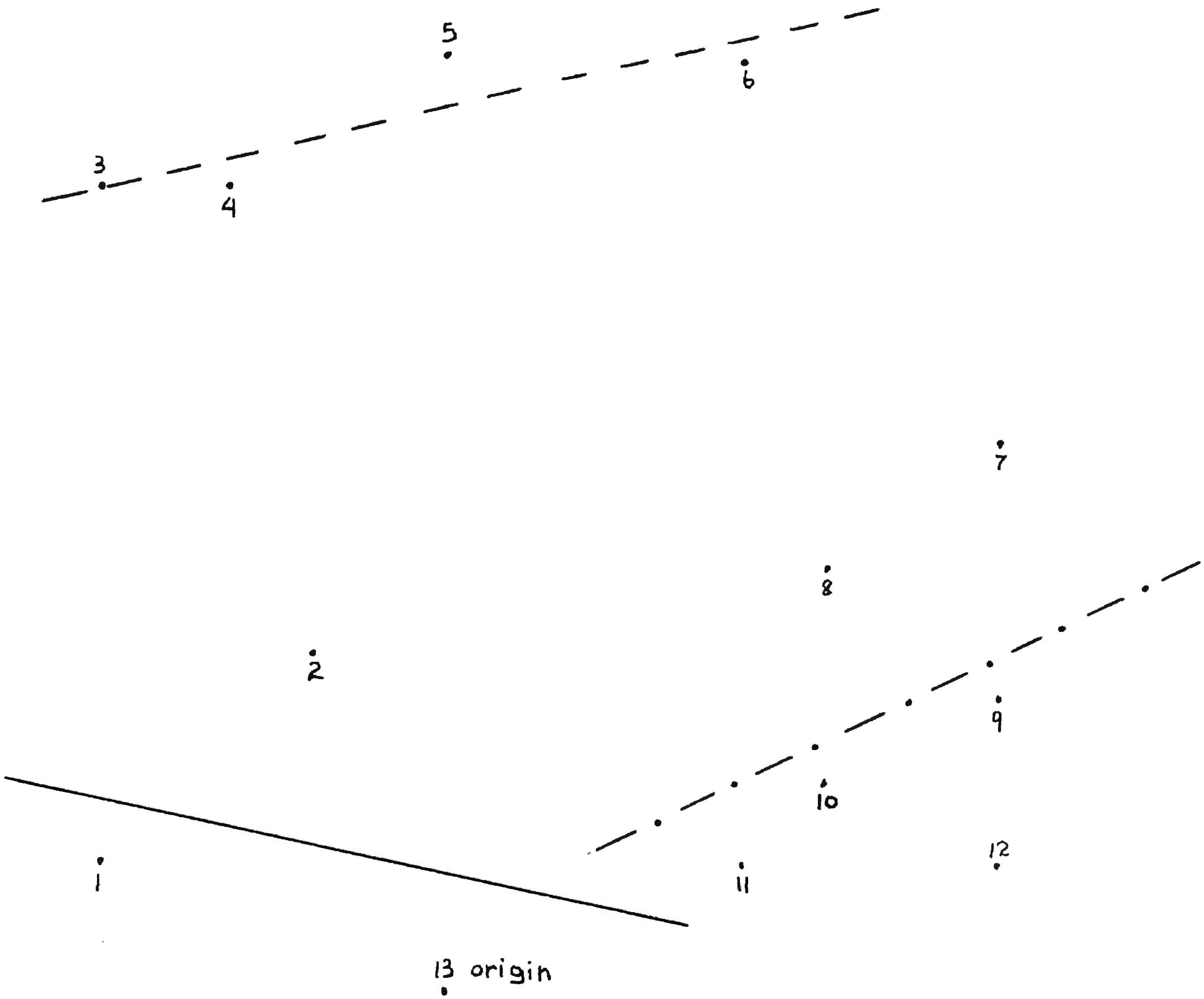

Figure 3. Regression Lines for Three Clusters

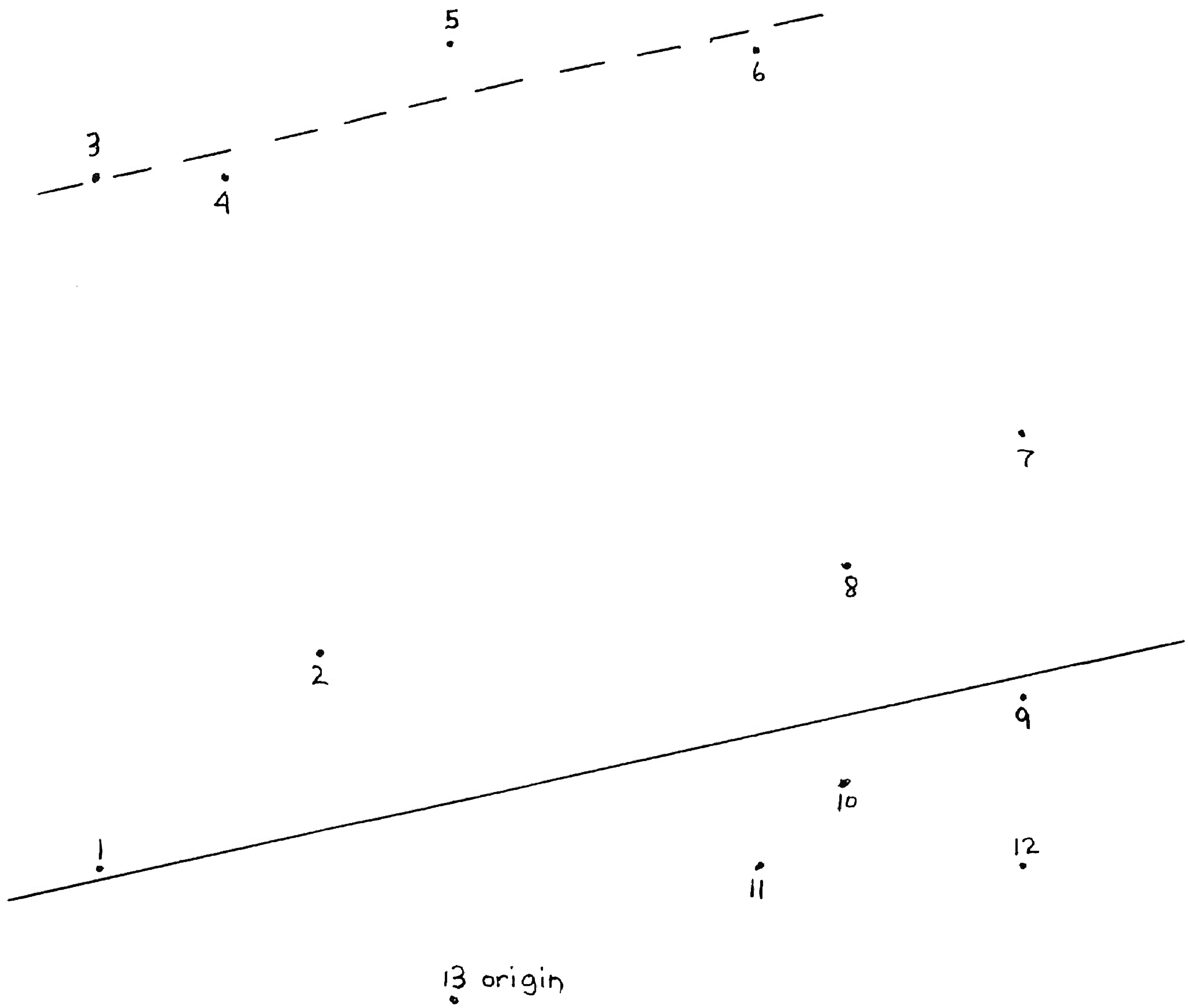

Figure 4. Regression Lines for Two Clusters

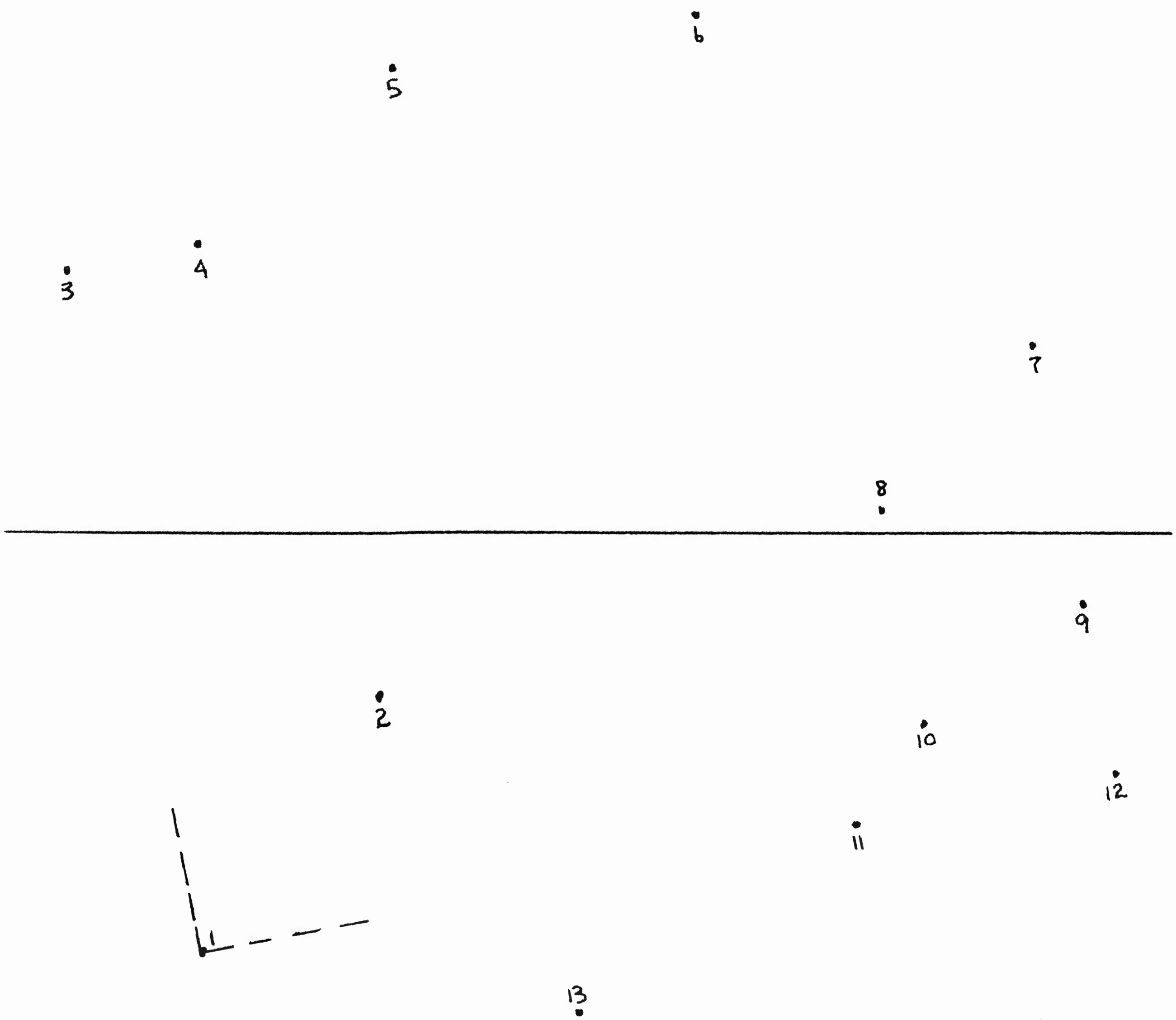

Figure 5. Regression Lines for Single Cluster

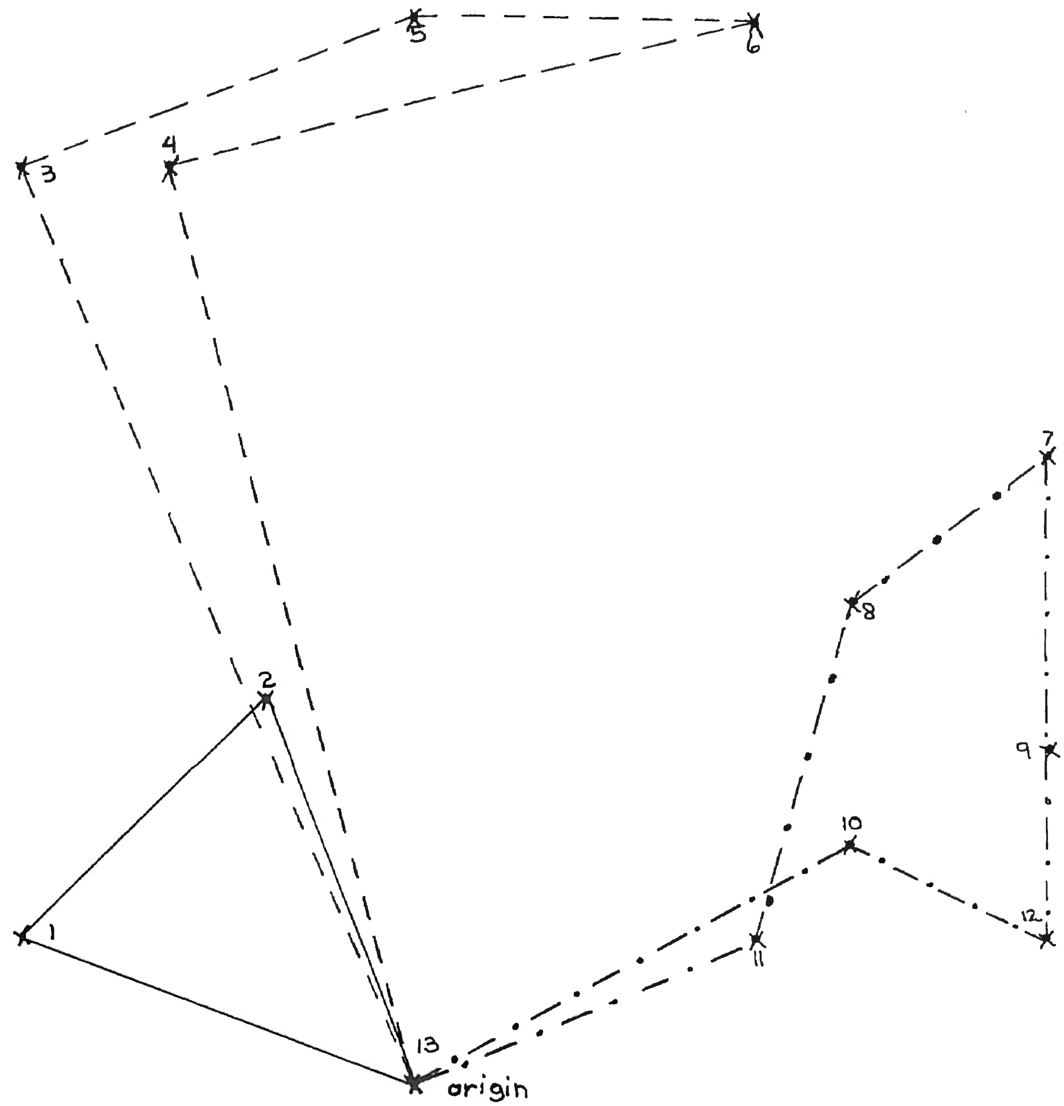

Figure 6. Final Routes for Author's Method Three Clusters

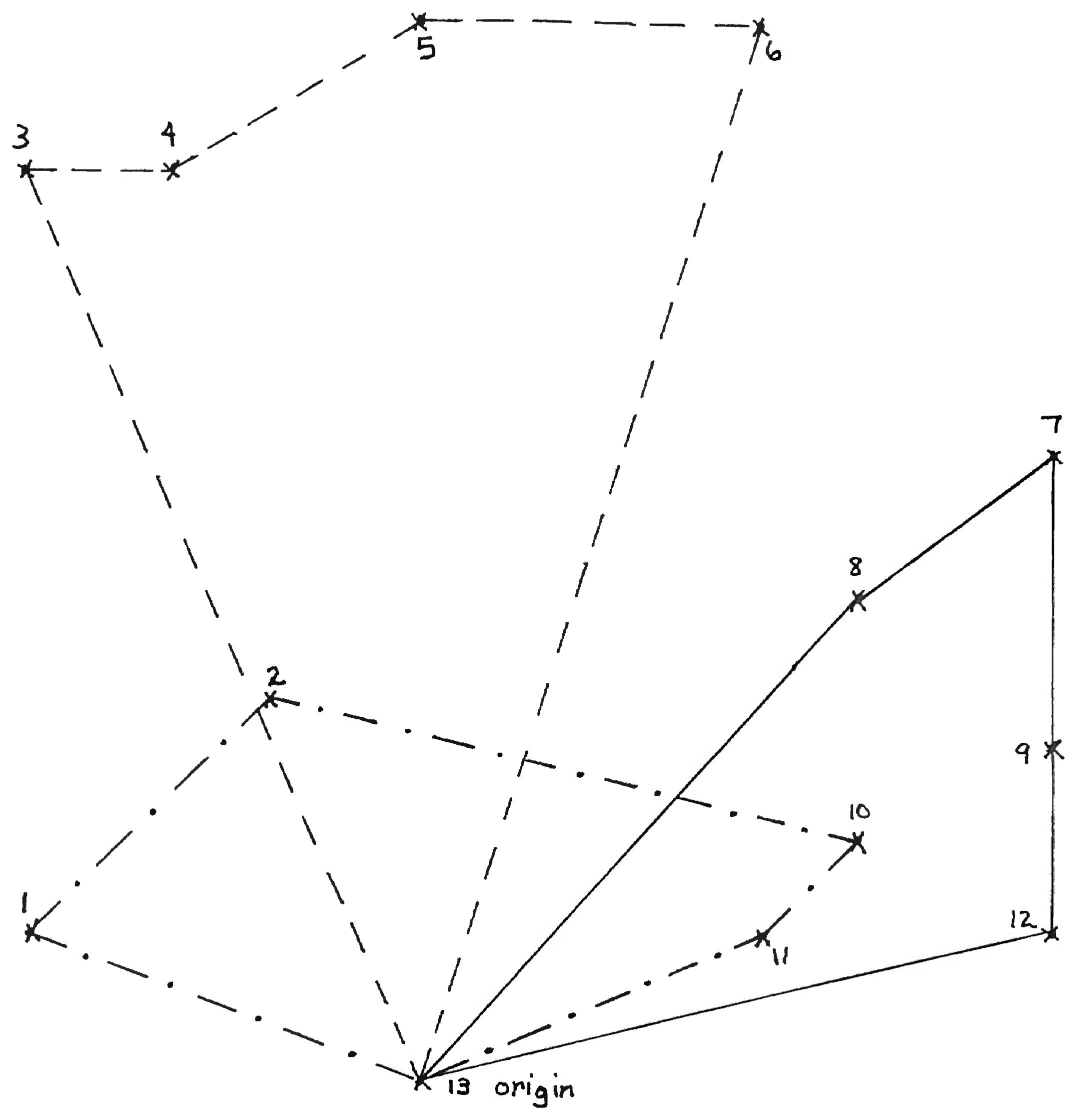

Figure 7. Final Routes for Lockset
Method Three Clusters

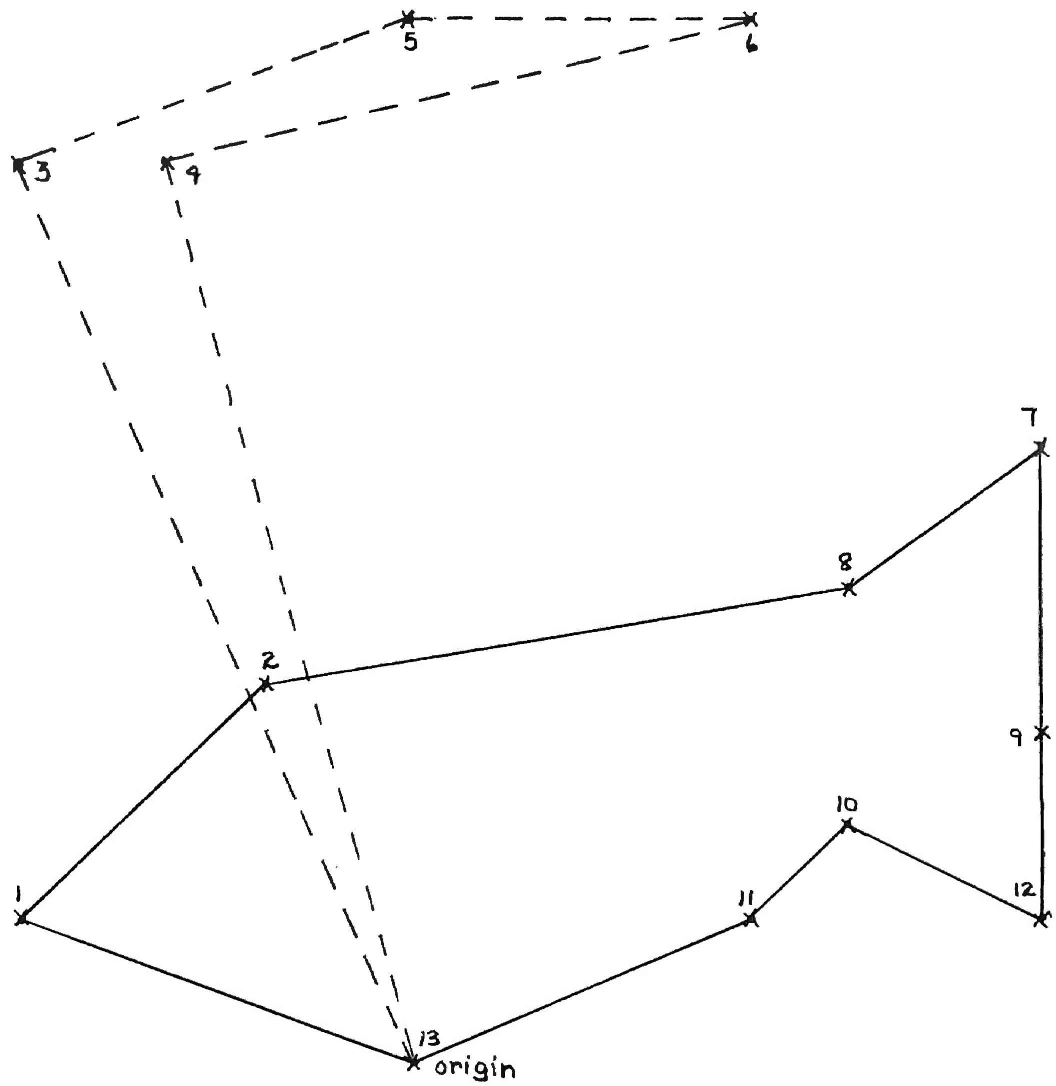

Figure 8. Final Routes for Author's Method Two Clusters

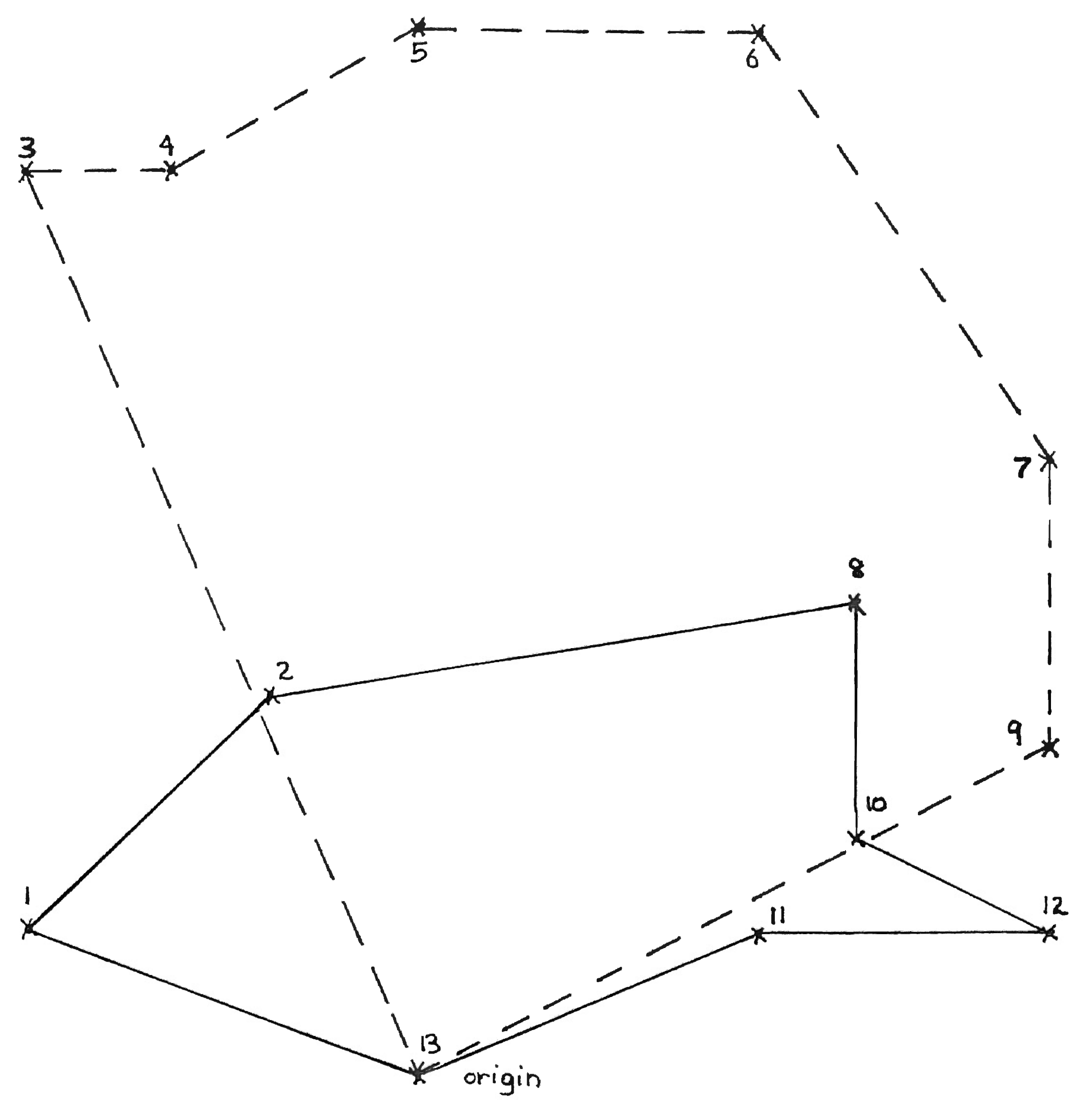

Figure 9. Final Routes for Lockset's Method Two Clusters

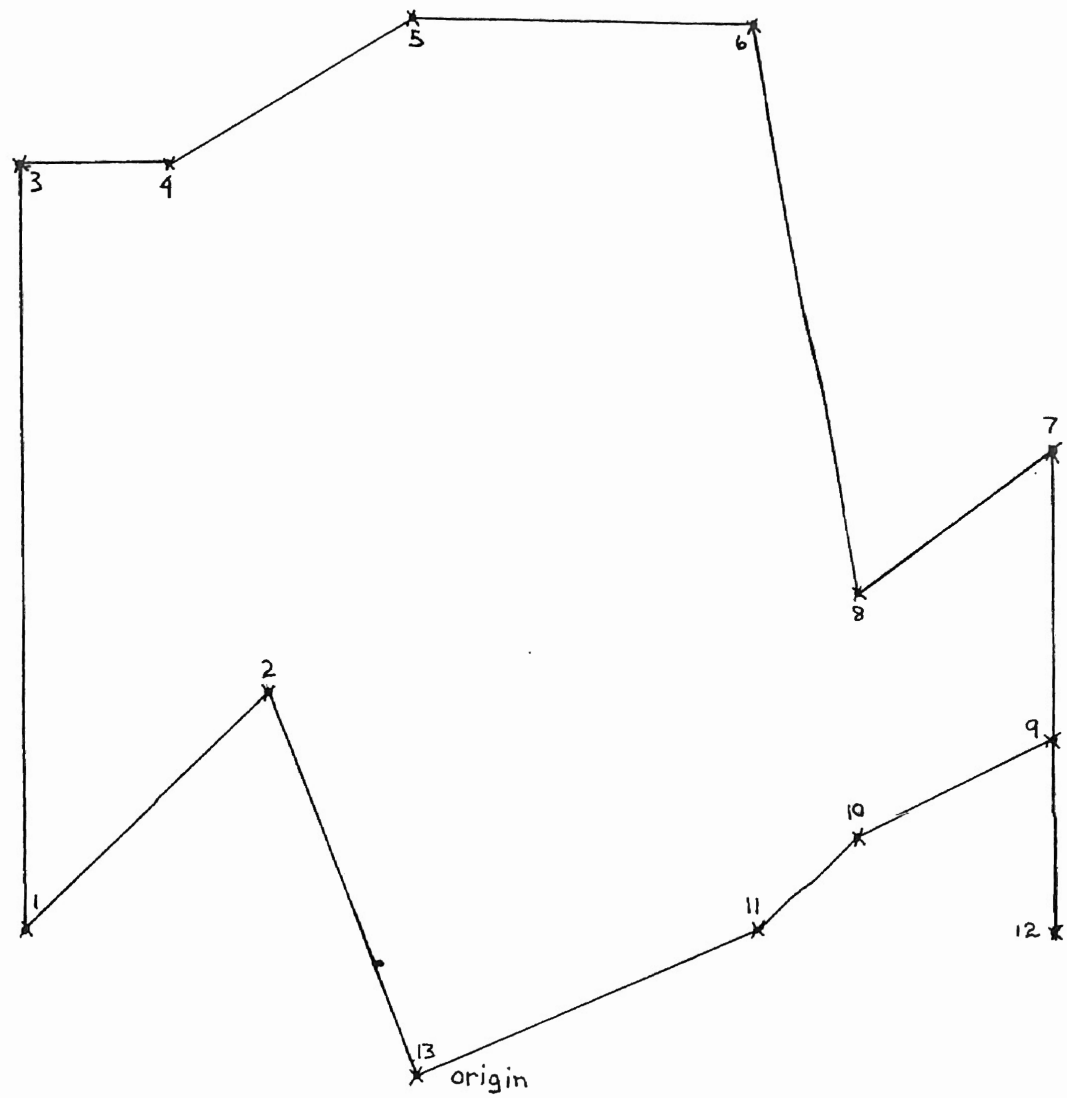

Figure 10. Final Route for Author's Method Single Cluster

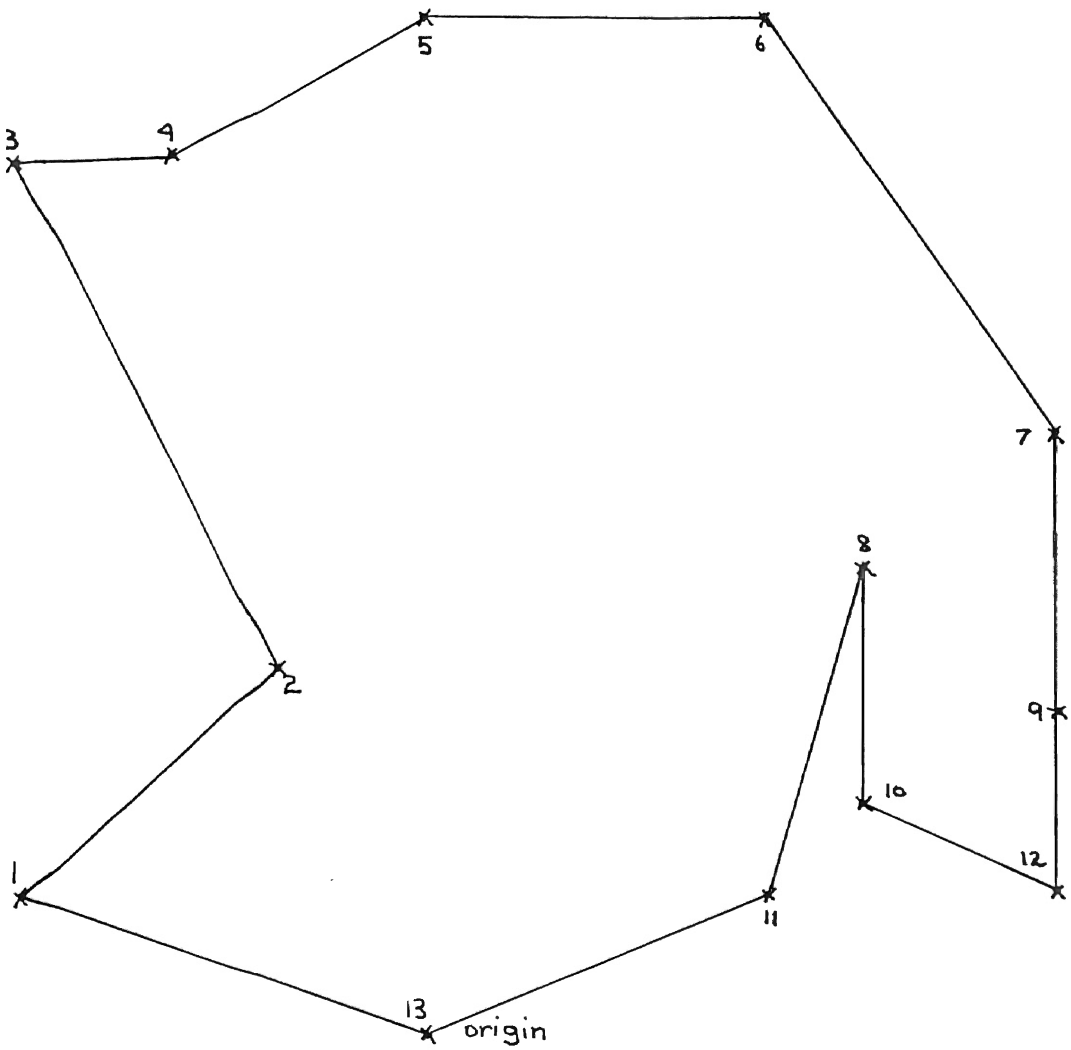

Figure 11. Final Route for Lockset's Method Single Cluster

APPENDIX E

TABLES

TABLE I

STATION COORDINATES

Station Number	X-Coordinate	Y-Coordinate
1	0	0
2	5	5
3	0	16
4	3	16
5	8	19
6	15	19
7	21	10
8	17	7
9	21	4
10	17	2
11	15	0
12	21	0
13	8	-3

TABLE II

CONVEX HULL COORDINATES AND AREA

Point Number	Order	X-Coordinate	Y-Coordinate	Area
1	3	0	16	
2	5	8	19	140
3	6	15	19	133
4	7	21	10	87
5	9	21	4	0
6	12	21	0	0
7	13	8	-3	19.5
8	1	0	0	12
9	3	0	16	0
			TOTAL	391.5

TABLE III

CLUSTERS AND THEIR STATIONS

3-CLUSTER DIVISION

Cluster Number	Station Number
1	1, 2, 13
2	3, 4, 5, 6
3	7, 8, 9, 10, 11, 12

2-CLUSTER DIVISION

Cluster Number	Station Number
1	3, 4, 5, 6
2	1, 2, 7, 8, 9,10, 11, 12, 13

TABLE IV

CLUSTERS REGRESSION LINES

Number of Clusters	Cluster Number	a	b	Formula
3	1	1.68	-.23	Y = 1.68 -.23X
3	2	15.99	.23	Y = 15.99 +.23X
3	3	-5.5	.5	Y = -5.50 +.50X
2	1	15.99	.23	Y = 15.99 +.23X
2	2	-.15	.21	Y = -0.15 +.21X
1	1	9.92	-.22	Y = 9.92 -.22X

TABLE V

DISTANCES BETWEEN STATIONS

STATION NUMBERS	2	3	4	5	6	7	8	9	10	11	12	13
1	7.1	16.0	16.3	20.6	24.2	23.3	18.4	21.4	17.1	15.0	21.0	8.5
2		12.1	11.2	14.3	17.2	16.8	12.2	16.0	12.4	11.2	16.8	8.5
3			3.0	8.5	15.3	21.8	19.2	24.2	22.0	21.9	26.4	20.6
4				5.8	12.4	19.0	16.7	21.6	19.8	20.0	24.1	19.7
5					7.0	15.8	15.0	19.9	19.2	20.3	23.0	22.0
6						10.8	12.2	16.2	17.1	19.0	19.9	23.1
7							5.0	6.0	8.9	11.7	10.0	18.4
8								5.0	5.0	7.3	8.1	13.5
9									4.5	7.2	4.0	14.8
10										2.8	4.5	10.3
11											6.0	7.6
12												13.3

TABLE VI

ROUTES AND DISTANCES FROM AUTHOR'S METHOD

Problem (Number of Clusters)	Cluster Number	Route (Stations)	Length (Units)
3	1	13, 1, 2, 13	24.2
3	2	13, 4, 6, 5, 3, 13	68.2
3	3	13, 11, 8, 7, 9, 12, 10, 13	44.7
		TOTAL	137.1
2	1	13, 4, 6, 5, 3, 13	68.2
2	2	13, 11, 10, 12, 9, 7, 8, 2, 1, 13	57.7
		TOTAL	125.9
1	1	13, 11, 10, 9, 12, 7, 8, 6, 5, 4, 3, 1, 2, 13	93.5
		TOTAL	93.5

TABLE VII

DISTANCE SAVED COEFFICIENTS (DSC)

X_1 Y_j	2	3	4	5	6	7	8	9	10	11	12
1	10.0	13.2	11.9	9.9	7.4	3.6	3.6	1.9	1.7	1.1	.8
2		17.1	17.0	16.2	14.4	10.1	9.8	7.3	6.4	4.9	5.0
3			37.3	34.1	28.4	17.2	14.9	11.2	8.9	6.3	7.5
4				35.8	30.4	19.1	16.5	12.9	10.2	7.3	8.9
5					38.1	24.6	20.5	16.9	13.1	9.3	12.3
6						30.7	24.4	21.7	16.3	11.7	16.5
7							26.8	27.2	19.8	14.3	21.7
8								23.2	18.8	13.8	18.7
9									20.6	15.2	24.1
10										15.1	19.1
11											15.0

TABLE VIII

ROUTES AND DISTANCES FROM LOCKSET METHOD

Problem (Number of Clusters)	Cluster Number	Route (Stations)	Length (Units)
3	1	13, 3, 4, 5, 6, 13	59.5
3	2	13, 1, 2, 10, 11, 13	38.4
3	3	13, 8, 7, 9, 12, 13	41.8
		TOTAL	139.7
2	1	13, 3, 4, 5, 6, 7, 9, 13	68.0
2	2	13, 1, 2, 8, 10, 12, 11, 13	50.9
		TOTAL	118.9
1	1	13, 1, 2, 3, 4, 5, 6, 7, 9	
		12, 10, 8, 11, 13	87.7
		TOTAL	87.7

VITA

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Master of Administration

Thesis: THE LEAST SQUARED APPROACH TO THE TRAVELING SALESMAN PROBLEM

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Institution: Oklahoma State University

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Title of Study: THE LEAST SQUARED APPROACH TO THE TRAVELING SALESMAN PROBLEM

Pages in Study: 74 Candidate for Degree of Master of Business Administration

Major Field: Marketing

Scope of Study: This study presents an alternative method to solving multipoint distribution problems. The method provides for potential improved route selection for the distribution problem known as the traveling salesman. The method comprises determining the number of transportation units necessary using the convex hull area, a minimum sided polygon. The individual route selection is found by first performing a cluster analysis to divide the stations into first level groups. Generation of a regression line for each group and projection of each station onto this regression line establishes the route sequence. The distribution point is integrated into the route using the shortest distance path length. The affects of various parameters are also discussed.

Findings and Conclusions: The solution to a randomly generated distribution problem showed a near equivalent solution as compared to another method known as lockset. The method resulted in different solution path lengths. This method provides an alternative approach to solving distribution problems involving a common distribution origin. It also can present potential savings in distance and thus costs by its application. The method provides a viable alternative to other methods.

ADVISER'S APPROVAL James W. Gentry