KINETO-ELASTODYNAMIC ANALYSIS OF A FOUR-BAR AND ITS COGNATE MECHANISM

By

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AND ITS COGNATE MECHANISM

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure		Page

CHAPTER I

INTRODUCTION

A detailed survey of the existing literature in the field of kinematic reveals the fact that the rigidity assumption in the design of mechanisms which are composed of links, gears, sliders, etc., fails to supply the need of accuracy in the output function of a mechanism wherever high speed is a criterion for fast production.

The simplest and most useful mechanism is a four-bar linkage, the application of which is extensive such as in a printing machine or a gripping device for speed packaging or labelling, etc. At a high operating speed, the mechanism designed on the basis of rigidity may fail to accomplish the goal because of the inertial and external forces inducing elastic deflections in the links.

Kineto-elasto dynamics (K.E.D.) is the study of mechanisms in motion consisting of deformable elastic elements which may deflect due to external loads or internal body forces.

Several authors have dealt with "elastic-complex system," i.e., the mixed elastic and non-elastic members (1)(2). Because of the complexity in obtaining the solution, usually one element in the mechanism members is treated as elastic, thereby treating only one degree of elastic freedom in deformation, i.e., torsion, extension, or flexure alone. The most adequate technique often employed is the Lagrangian-Mechanics to derive equations of motion, but unfortunately, the assumptions for simplifying sacrifice the reality of the problem.

Burns and Crossley (3) performed a kineto-elasto static synthesis on a four-bar function generator with a flexible coupler. Kohli, Hunter, and Sandor (4) presented elasto dynamic analysis of a slider-crank mechanism using Euler-Lagrange Differential Equations of motion, which is an extension of the Lagrangian Mechanics mentioned above.

Notable contribution is made by Erdman (5), who presented for the first time the KEDSRO (Kineto-Elastodynamic-Stretch Rotation Operator) for the synthesis of a completely elastic model. Synthesis of planar four-bar Crank Rocker mechanism with elastic links using Stiffness-Approach is investigated by Patwardhan and Soni (6).

The above cited literature survey reveals that the designers have treated the effect of elasticity in linkages by simply over-designing the mechanism with a few exceptions of synthesis considering elasticity in the mechanism members (4)(6). No further attention was focussed on analyzing the cognate mechanisms which are an alternate answer to a source mechanism. The search for accurate synthesis procedures where-ever high speed and accuracy is the objective requires first a complete and accurate K.E.D. analysis of the mechanism where all of the links are considered to be elastic.

This thesis presents a generalized approach where four-bar path generating source and coupler-cognate mechanisms are analyzed with all of their links regarded as elastic, and are examined based on the flexibility method of structural analysis. The mechanism is frozen in various configurations and analyzed as an instantaneous structure (with elastic members) to determine the elastic displacements of its path generating coupler point. Since the cognate mechanism can be a substitute for the source mechanism whose coupler point generates the same curve in rigid mode, analysis is done for one of its cognate mechanisms.

The procedure involves the following three cases in increasing level of accuracy for both source and its cognate mechanism.

1) Completely elastic moving system where the links are assumed to be mass-less compared to an inertial mass located at the path point.

2) Each element having a concentrated or disc mass located at each joint.

3) Mass of each element is distributed along the element in the form of sub-elements.

A brief discussion about the coupler cognate mehcanisms of a four-bar is presented in Chapter II. The structural analysis based on flexibility approach is applied to mechanisms in Chapter III. The necessary equations for computing the K.E.D. deflections for the source and its cognate mechanisms are developed for the three above mentioned cases in Chapter IV. The results and conclusions are presented in Chapter V.

CHAPTER II

FOUR-BAR COGNATE MECHANISMS

Alternate mechanisms that differ in dimensions but have the same kinematic performance are called cognate mechanisms. If the three four-bars as shown in Figure 1a are examined, all three produce the same coupler curve generated by a common coupler point of the coupler. The four-bars (not being identical) are called Robert's cognate mechanisms or cognate to each other (8). These cognate mechanisms are built using the construction of parallelograms and similar triangles. They have a common frame as well as a common coupler point. Figure 1a demonstrates the construction of the cognates as follows:

1) The source mechanism with the coupler point "P" is constructed to a suitable scale as MAPBQ.

2) The parallelograms BQEP and AME'P are constructed on either side of the source four-bar linkage.

3) PED and E'PD' are similar triangles both similar to the coupler triangle PBA.

4) The construction of the parallelogram PDOD' locates "O" the other fixed point for the cognates.

A close consideration reveals that the two cognates, namely, ODEQ and OD'E'M, are obtained by geometric stretch rotation. The operation of Stretch-Rotation is a spiral similarity transformation, which is a combination of central dilatation and rotation about the centers Q and

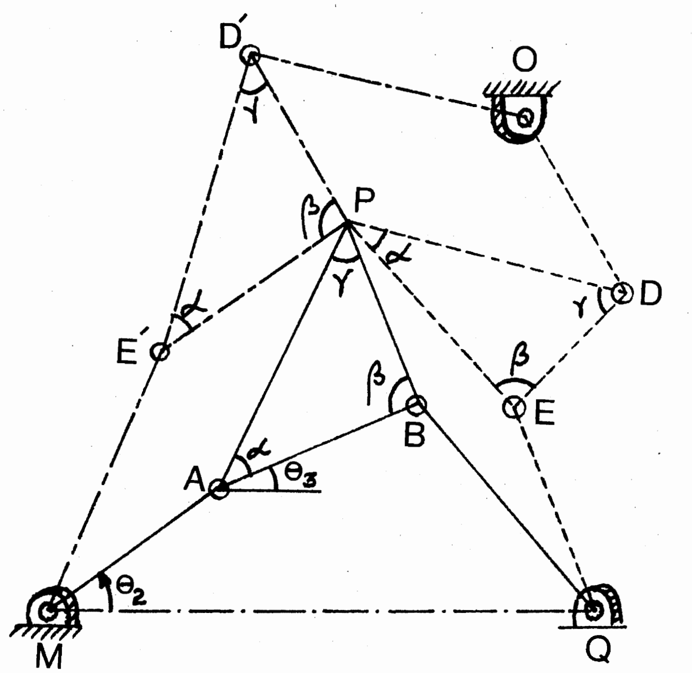


Figure 1a. Cognate Mechanisms of a Source Four-Bar Linkage

PE = BQ
MA = E'P
PA = E'M
EQ = PB

Triangles PAB, DPE, AND D'E'P are similar triangles

Angle OMQ = α
Angle OQM = β
Angle MOQ = γ

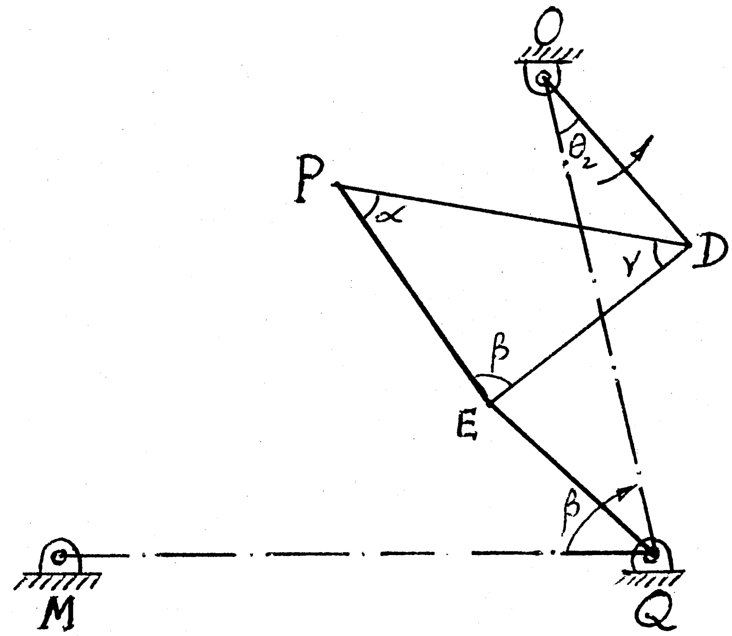


Figure 1b. Right Side Cognate (showing the fixed link MQ rotated about Q and stretched by a factor $K = \frac{OM}{MQ}$)

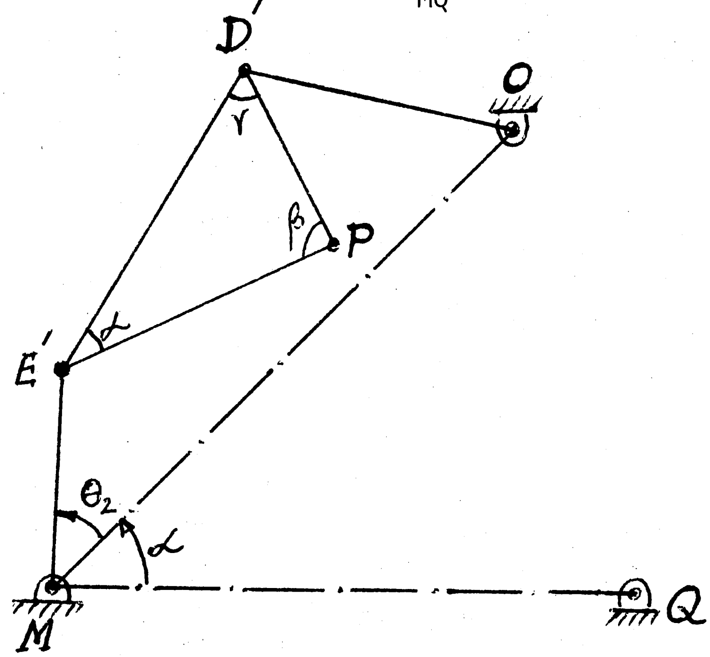


Figure 1c. Left Side Cognate (showing the link MQ rotated about M and stretched by a factor $K = \frac{OM}{MQ}$)

M for the two cognates ODEQ and OD'E'M, respectively. The new link lengths are a multiple of the dilatation factor, K, and the arguement is the rotation of the fixed link MQ to QO (about the point Q) through a fixed angle β (as shown in Figure 1b) for the right side cognate ODEQ. For the left side cognate, the fixed link MQ of the source mechanism has rotated through an angle α about the point M and stretched by a factor K (as shown in Figure 1c).

Since both the dilatation factor K and the arguements MQO and QMO are independent of time for a rigid transformation, the input angular displacements of the links do not change. Further, the link OD makes the same input angle with the fixed link OQ as the source mechanism link MA makes with its fixed link MQ. Since the angular displacements for the input links of the source and its cognates do not change, the input velocity for all of the input links remains the same.

A quick way to find the link-lengths of the coupler-cognate mechanisms is the usage of Caley's diagram, as shown in Figure 2. Caley's diagram is obtained by making the coupler links AB, DE, E'D' coincide with the fixed links MQ, QO, MO of the three four-bars as shown in Figure 1a. Using the properties of the similar coupler triangles PAB, DPE, D'E'P, the link lengths of the cognate mechanisms are obtained.

To determine the deflections of the coupler point of both the source and its cognate mechanism, considering all links to be elastic, a Crank-Rocker mechanism is selected as a source linkage which has a cognate of crossed configuration. However, the methodology developed is for any four-bar linkage. K.E.D. analysis for both the source and its cognate is performed by the method of Structural analysis using the flexibility approach discussed in the next chapter.

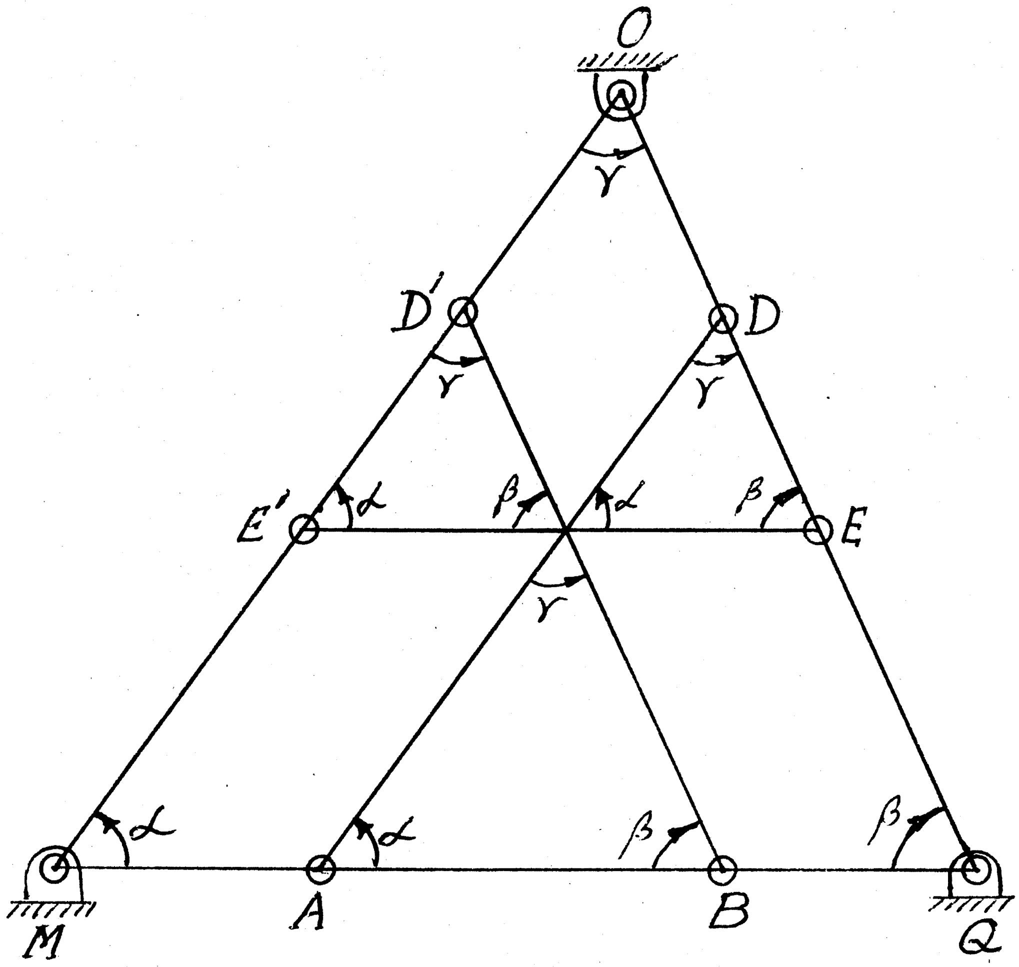


Figure 2. Caley's Diagram for Cognate Link-Lengths

CHAPTER III

STRUCTURAL ANALYSIS APPROACH APPLIED TO MECHANISMS

This section of the thesis demonstrates how the method of structural analysis may be applied to the analysis of mechanisms in motion. A structure can be changed into a mechanism by removing one or more physical constraints of the structure thus allowing rigid body motion of its members. For example, a rectangular pinned frame within a diagonal bar may be transformed from a structure of rigid body components having zero degree of freedom into a mechanism with a degree of freedom of one by the removal of the diagonal bar as shown in Figures 3a, 3b.

The above transformation is reversible. A mechanism can be reduced to a structure by adding physical constraints; that is, by reducing its degrees of freedom to at least zero.

This thesis is based partially on the representation of a mechanism as a statically "Instantaneous Structure" by adding one or more mobile constraints. For example, the configuration of a four-link mechanism is determined at a particular angle of the input-link. Once this angle is set, the whole mechanism can be frozen for that instant as a structure.

For a particular set of the input angle of the four link mechanism, the input-link (Element 1) is modelled as a cantelever-beam or free-fixed beam (as shown in Figure 4a). For this cantelever beam of

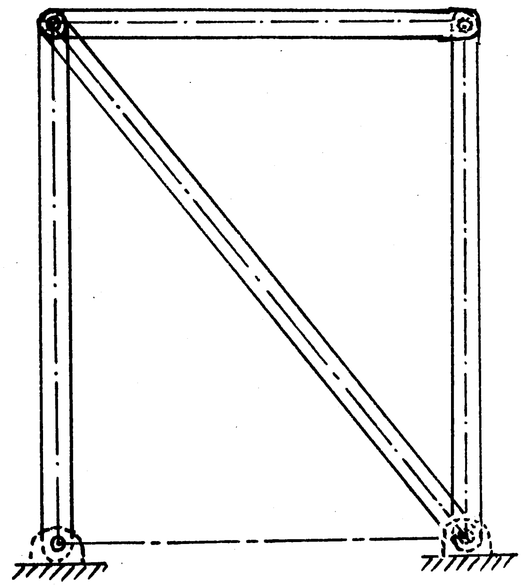


Figure 3a. Structure With Degree of Freedom Zero

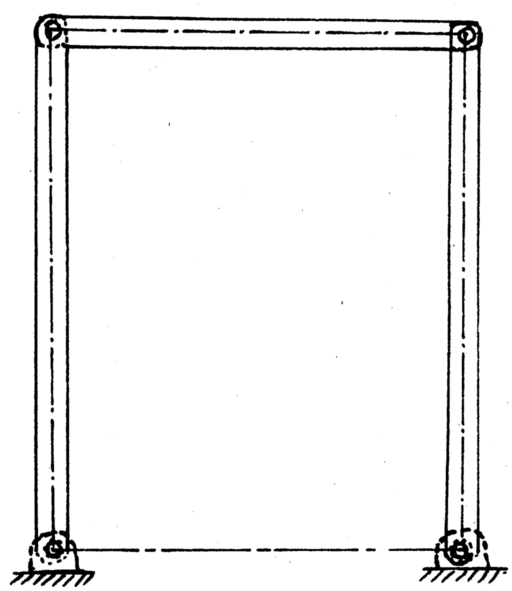


Figure 3b. Four-Link Mechanism With Degree of Freedom One

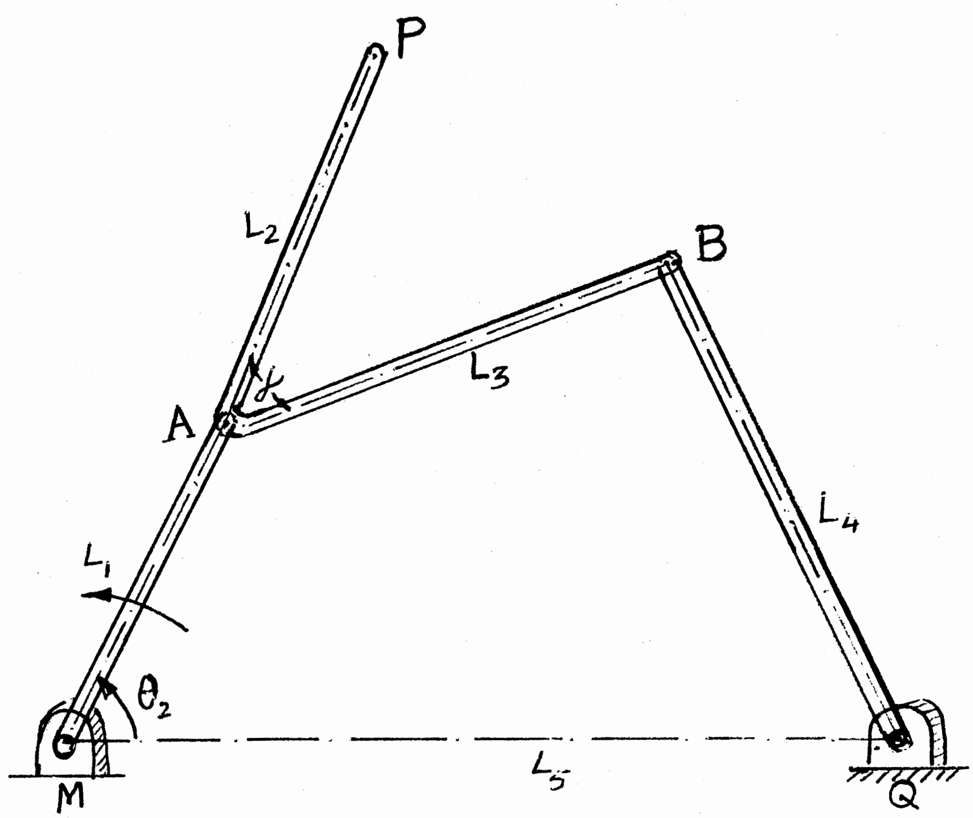


Figure 4. Four-Bar Mechanism

MA - input-link of length - L_1
AB - coupler link of length = L_3
AP - coupler extender of length L_2
making a rigid angle 'α'
with AB
BQ - follower link of length L_4
MQ - Fixed or grounded link of length L_5

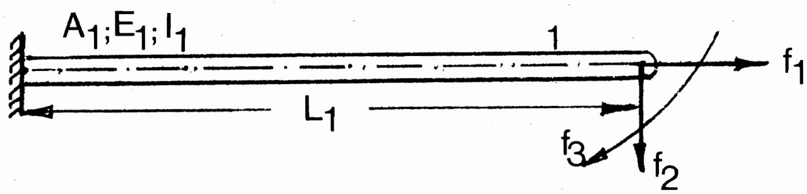


Figure 4a. Input Element as a Cantilever Beam

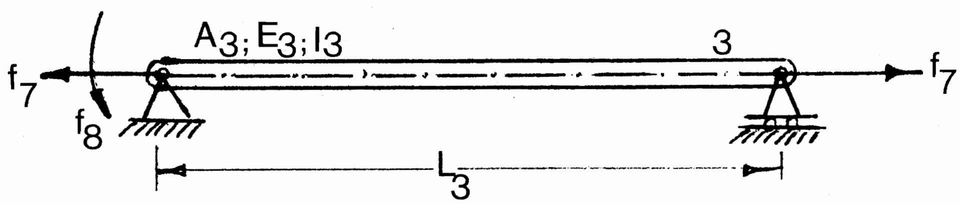


Figure 4b. Coupler Link Modelled as Simply Supported Beam With End Moments

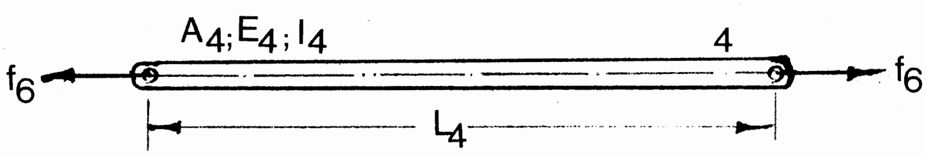


Figure 4c. Follower Link Behaving as a Two-force Member

length L1, the cross-sectional area A, Modulus of elasticity E, cross-sectional moment of inertia I (about an axis-Z normal to the plane of the mechanism), the internal element forces f_1, f_2, and the internal element moment f_3 cause corresponding translations d_1, d_2, and angular deflection d_3 at the end of the element.

These forces and displacements can be expressed as follows:

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} F \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \qquad (1)$$

where F is the element flexibility matrix given as

$$\begin{bmatrix} F \end{bmatrix} = \begin{bmatrix} L_1/AE & 0 & 0 \\ 0 & L_1^3/3EI & L_1^2/2EI \\ 0 & L_1^2/2EI & L_1/EI \end{bmatrix} \qquad (2)$$

The out-put link of the four-bar linkage (Element 4) is a two force member. A two force member with two pin joints can transmit only longitudinal force as shown in Figure 4c. Thus, the link 4 has one elastic degree of freedom (extensibility) and its element flexibility matrix has only one term

$$\begin{bmatrix} F \end{bmatrix} = \begin{bmatrix} L_4/AE \end{bmatrix} \tag{3}$$

and the element deflection is given as

$$\begin{bmatrix} d_1 \end{bmatrix} = \begin{bmatrix} L_4/AE \end{bmatrix} \begin{bmatrix} f_1 \end{bmatrix} \tag{4}$$

for some cases where a mechanism link is not just a simple straight beam, for example, in the four-bar linkage of Figure 4 the coupler link is composed of two elements rigidly fixed at an angle α, the extender element 2 may be treated as a simple cantelever beam with three elastic degrees of freedom while element 3 behaves like a simply supported beam with a moment on the left end due to element 2 and a longitudinal force as shown in Figure 4b.

The element flexibility matrix for the element 3 is

$$\begin{bmatrix} F \end{bmatrix} = \begin{bmatrix} L_3/AE & 0 \\ 0 & L_3/3EI \end{bmatrix} \tag{5}$$

The above method of modelling the elastic motion of a mechanism is not limited to only a four-bar mechanism but can be extended to a multi-link mechanism. Gears teeth are investigated by considering the tooth as a cantilever beam (9). The total deformation of the tooth can be calculated which is a result of direct compression at the point of contact between teeth, beam deflection, and shear. This theory can also be extended to spatial-linkages considering proper degrees of freedom and the forces induced in the mechanism members.

CHAPTER IV

ELASTO-DYNAMIC DEFLECTION ANALYSIS APPROACH

The flexibility approach of structural analysis to the individual element was demonstrated in the previous section. This section deals with the total setup of the whole mechanism under consideration.

Considering that the mechanism has several external "system forces" or generalized forces acting on it (including inertia moments and forces), a deflected configuration of the instantaneous structure is desired.

The flexibility approach permits in determining the deformations in the direction of any desired set of system coordinates. If the system forces are represented by a column matrix P_j, $j = 1,\ldots.n$ where n is the number of system forces and system coordinates. Since the number of elastic degrees of freedom of the mechanism system is the sum of the independent internal forces of its elements, every independent internal force has a corresponding element coordinate X_i, $i = 1,\ldots.m$, where m is the number of element coordinates.

The system forces may be transormed into element or internal forces f_i, $i = 1,\ldots.m$ each acting in the respective element coordinate direction by deriving an (mXn) force transformation matrix by the method of rigid member static analysis (7).

The matrix described above is dependent on the configuration of the system. It is thus a function of the reference variables of the mechanism. Since a four-bar mechanism has only one reference variable, namely the input angle, the force transformation matrix is a function of the input angle.

The flexibility matrices of the elements can be assembled to form an element flexibility matrix for the whole system. This is derived in a later section. The element flexibility matrix is independent of the configuration of the mechanism position.

The element deformation matrix is thus the product of the element flexibility matrix with the force transfer matrix and the element force matrix, as

$$d = \left[F \right] \left[f \right] = \left[F \right] \left[\beta \right] \left[P \right]$$

where $\left[\beta \right] \left[P \right]$ gives the forces acting on the mechanism in a particular configuration.

These element deformations will have a resulting effect for the whole system of the mechanism. Thus, these element deflections are transferred to system deflections by pre-multiplying these element deflection matrices by the transposed force transfer matrix. The conversion of element deflections to system deflections is described in reference (7).

The system deflections are given as:

$$\left[\delta \right] = \left[\beta \right]^{t} \left[F \right] \left[\beta \right] \left[P \right] \tag{6}$$

where the element force vector is

$$\left[f \right] = \left[\beta \right] \left[P \right] \tag{7}$$

and the element deflection vector is $\left[d \right] = \left[F \right] \left[f \right]$

The flexibility approach described above is demonstrated on a planar four-bar and its cognate mechanism to determine the displacements (elastic) of the path point through its cycle of motion.

K.E.D. Assumptions

The following assumptions are made:

1) All deformations are in the elastic range.

2) Joints between the links are non-elastic, have no play, they are mass-less and frictionless compared to the rest of the mechanism.

3) The input angular-velocity of the input-link is constant.

4) The coupler link has an extender which makes a rigid angle α with the coupler link of the source four-bar and a rigid angle γ with the coupler link of its cognate. The four links, i.e., the input link, extender, coupler link, and the follower link are assumed to be flexible in the plane of motion and extensible. The same assumption applies for its cognate.

5) Since the path-point deviation of both source and cognate mechanisms is under consideration, each mechanism has three system coordinates. This system is an ortho-normal translating coordinate system in which X and Y coo-dinate systems remain parallel to an inertial system and are located in the plane of motion and Z coordinate expresses the angular orientation located at the path-point.

6) The mechanism motion is considered to be in the horizontal plane; thereby effect of gravity is eliminated.

Three cases are considered in an increasing level of accuracy:

Case I: Completely elastic moving system where the links are assumed mass-less compared to an inertial mass at the path-point.

Case II: Each element has a concentrated or a disc mass located at each joint.

Case III: Mass of each element is distributed along the element in the form of sub elements.

Case I: Lumped Mass at Path Point

K.E.D. Analysis is performed for the source and its cognate mechanism where links are considered to be mass-less compared to an inertial mass at the coupler point. Equation (6) derived above is used to determine the elastic displacement of the coupler point shown in Figure 5. The deflections are given as

$$[\delta] = [\beta]^t [F][\beta][P]$$

The force transformation matrix $[\beta]$, the element flexibility matrix $[F]$, and the system force matrix $[P]$ are calculated for this case.

Derivation of the Force Transformation Matrix for Case I. The force transformation matrix for the source four link mechanism (Figure 5) is derived in this section as follows (referring to Figure 6b):

$$f_1 = P_1\cos(\theta_3+\alpha)+P_2\sin(\theta_3+\alpha)$$

$$f_2 = -P_1\sin(\theta_3+\alpha)+P_2\cos(\theta_3+\alpha) \qquad (9)$$

$$f_3 = P_3$$

Referring to Figure 6 for the coupler and the extender, the sum of

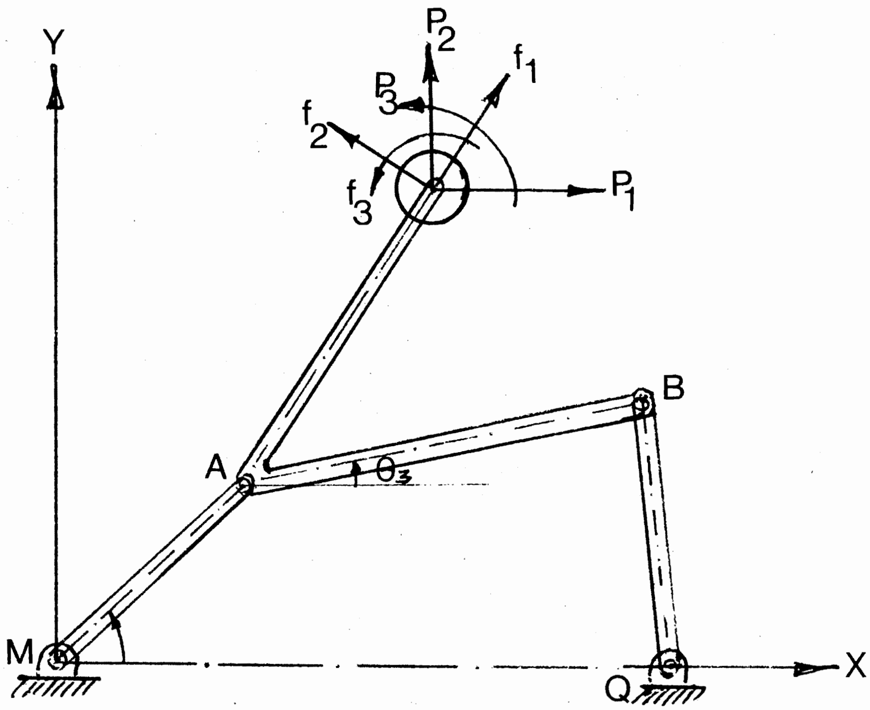


Figure 5. Source Four Bar Showing System Forces P_1, P_2, P_3

Force Transfer Matrix for Source Four-Bar Mechanism
(Case I)

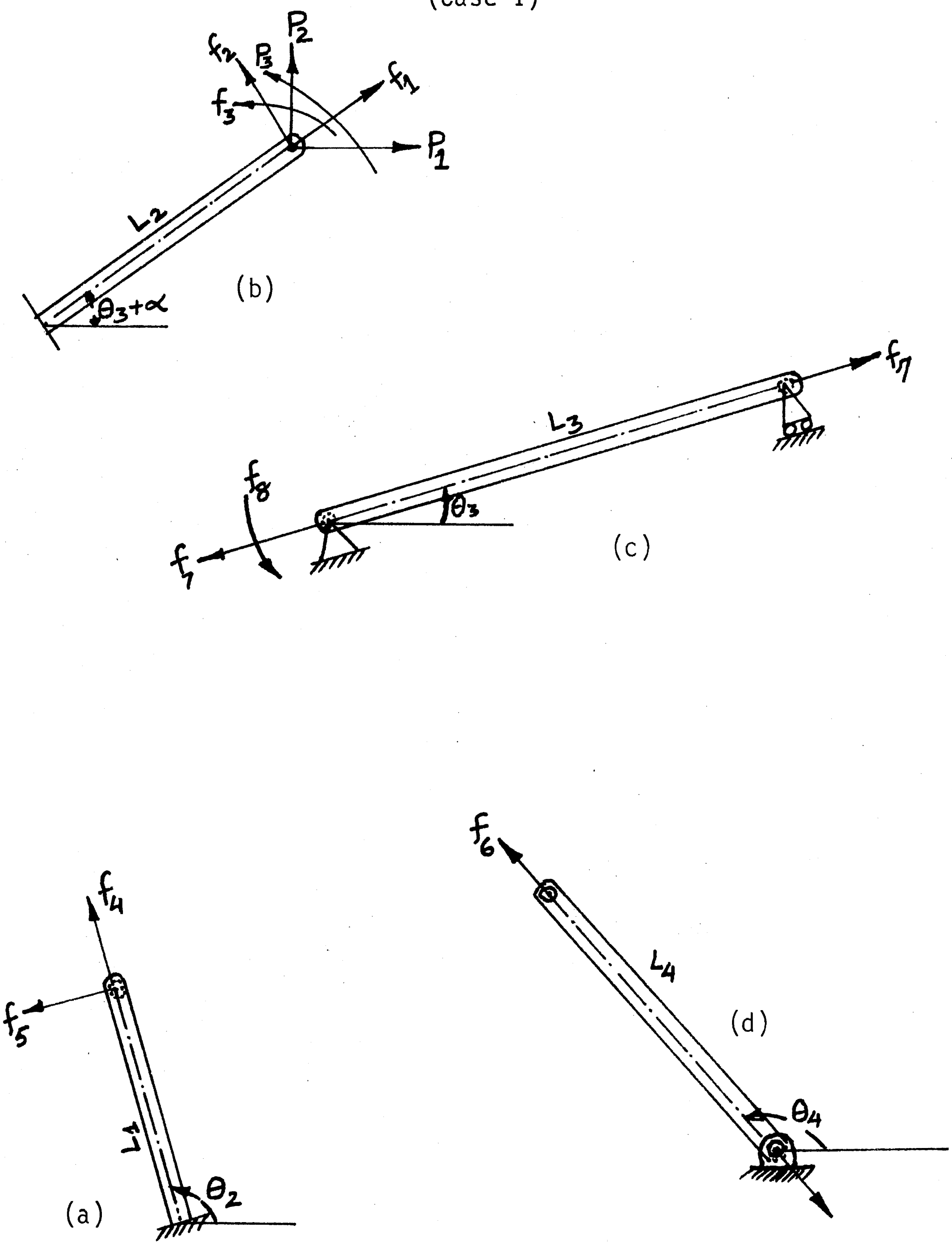


Figure 6. Diagram Showing the Element Coordinates and Their System Forces of the Source Four-Bar

the horizontal forces on the coupler is:

$$P_1 - f_4\cos(\Theta_2) + f_5\sin(\Theta_2) - f_6\cos(\Theta_4) = 0$$

i.e.,

$$P_1 = f_4\cos(\Theta_2) - f_5\sin(\Theta_2) + f_6\cos(\Theta_4) \tag{10}$$

The sum of the vertical forces on the coupler is:

$$P_2 = f_4\sin(\Theta_2) + f_5\cos(\Theta_2) + f_6\sin(\Theta_4) \tag{11}$$

The moments about the coupler point P are

$$\begin{aligned} P_3 = &\left\{L_2\cos(\Theta_3+\alpha)(-\sin(\Theta_2)) + L_2\sin(\Theta_3+\alpha)\cos(\Theta_2)\right\} f_4 \\ &+\left\{L_2\cos(\Theta_3+\alpha)(-\cos(\Theta_2)) - L_2\sin(\Theta_3+\alpha)\sin(\Theta_2)\right\} f_5 \\ &+\left\{(L_3\cos(\Theta_3) - L_2\cos(\Theta_3+\alpha))(\sin(\Theta_4))\right. \\ &\left.+ (L_2\sin(\Theta_3+\alpha) - L_3\sin(\Theta_3))(\cos(\Theta_4))\right\} f_6 \end{aligned} \tag{12}$$

Equations (10), (11), and (12) are of the form:

$$\begin{aligned} P_1 &= af_4 + bf_5 + cf_6 \\ P_2 &= df_4 + ef_5 + ff_6 \\ P_3 &= gf_4 + hf_5 + if_6 \end{aligned} \tag{13}$$

Solution of equation (13) by Cramer's rule yields:

$$f_4 = \frac{(ei-fh)P_1+(ch-bi)P_2+bf-ce)P_3}{r}$$

$$f_5 = \frac{(fg-di)P_1+(ai-cg)P_2+(cd-af)P_3}{r} \qquad (14)$$

$$f_6 = \frac{(dh-eg)P_1+(bg-ah)P_2+(ae-bd)P_3}{r}$$

where

$$r = a(ei-fh)-b(di-fg)+c(dh-eg)$$

The free body diagram for the element 3 as shown in Figure 7 helps in determining the element forces f_7 and f_8.

$$f_7 = -f_6\cos(\Theta_4-\Theta_3)$$

$$f_8 = L_3 f_6 \sin(\Theta_4-\Theta_3) \qquad (15)$$

Referring to Figure 8, for both coupler and extender summing moments about the point A, f_6 can be expressed in terms of the system forces P_1, P_2, P_3, as follows:

$$f_6 = \frac{-L_2\sin(\Theta_3+\alpha)P_1+L_2\cos(\Theta_3+\alpha)P_2+P_3}{L_3\sin(\Theta_4-\Theta_3)}$$

Thus

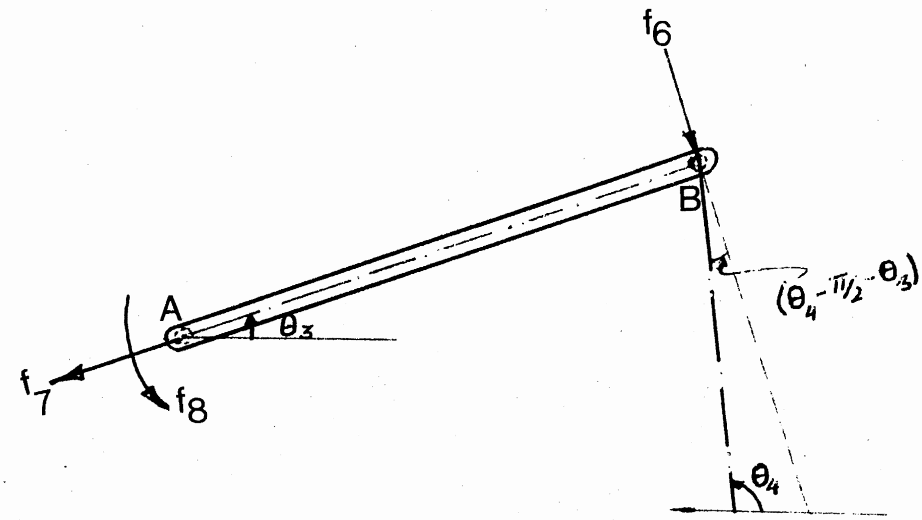
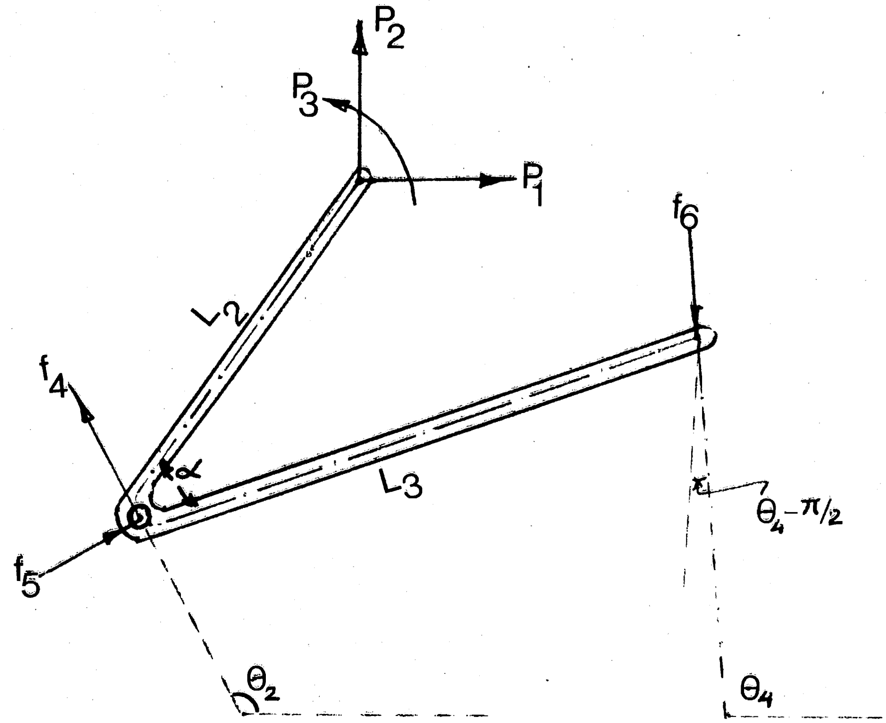


Figure 7. Free Body Diagram for Element 3



Case I

Figure 8. Force Diagram for Coupler and Coupler Extender

$$f_7=(L_2/L_3)\cot(\Theta_4-\Theta_3)\sin(\Theta_3+\alpha)P_1-(L_2/L_3)\cot(\Theta_4-\Theta_3)\cos(\Theta_3+\alpha)P_2$$
$$-\cot(\Theta_4-\Theta_3)(1/L_3) \tag{16}$$

and

$$f_8=-L_2\sin(\Theta_3+\alpha)P_1+L_2\cos(\Theta_3+\alpha)P_2+P_3$$

Combining equations (9), (14), (16), and expressing in a matrix form, the element forces f_1, f_2,....,f_8 are obtained from the system forces P_2, P_2, P_3.

The following symbols are used for space limitation:

$S\alpha\Theta_3 = \sin(\Theta_3+\alpha)$

$C\alpha\Theta_3 = \cos(\Theta_3+\alpha)$

$T\Theta_4\Theta_3 = \cot(\Theta_4-\Theta_3)$

$S\Theta_2 = \sin(\Theta_2)$

$C\Theta_2 = \cos(\Theta_2)$

$S\Theta_3 = \sin(\Theta_3)$

$C\Theta_3 = \cos(\Theta_3)$

$S\Theta_4 = \sin(\Theta_4)$

$C\Theta_4 = \cos(\Theta_4)$

Thus, the force transformation matrix is expressed using the relation

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \end{bmatrix} = \begin{bmatrix} \\ \beta \\ \\ \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} \qquad (17)$$

where

$$\begin{bmatrix} C\alpha\Theta_3 & S\alpha\Theta_3 & 0 \\ -S\alpha\Theta_3 & C\alpha\Theta_3 & 0 \\ 0 & 0 & 1 \\ (ei-fh)/r & (ch-bi)/r & (bf-ce)/r \\ (fg-di)/r & (ai-cg)/r & (cd-af)/r \\ (dh-eg)/r & (bg-ah)/r & (ae-bd)/r \\ (T\Theta_4\Theta_3)(S\alpha\Theta_3)L_2/L_3 & (-C\alpha\Theta_3)(T\Theta_4\Theta_3)L_2/L_3 & (-T\Theta_4\Theta_3)1/L_3 \\ -S\alpha\Theta_3L_2 & C\alpha\Theta_3L_1 & 1 \end{bmatrix} \qquad (18)$$

where

$a = C\Theta_2$

$b = -S\Theta_2$

$c = C\Theta_4$

$d = S\Theta_2$

$e = C\Theta_2$

$f = S\Theta_4$

$g = -L_2C\alpha\Theta_3S\Theta_2+L_2S\alpha\Theta_3C\Theta_2$

$h = -L_2C\alpha\Theta_3C\Theta_2-L_2S\alpha\Theta_3S\Theta_2$

$i = S(L_3C\Theta_3-L_2C\alpha\Theta_3)+C(L_2S\alpha\Theta_3-L_3S\Theta_3)$

$r = a(ei-fh)-b(di-fg)+c(dh-eg)$

Note that the matrix $\left[\ \beta\ \right]$ is a function of the link-lengths and the input angle Θ_2.

<u>Derivation of Force Transformation Matrix for the Cross-Cognate Mechanism</u>. One of the possibilities for the cognate of a source four-bar (Crank-Rocker) mechanism is that it can be a crossed four-bar (Crank Rocker) linkage. In such case, the orientation of the system forces and element forces vary because of the changed configuration of the cognate as shown in Figure 9. Thus, the force transformation matrix will vary for such a cognate. Referring to the free-body diagrams of the elements of the cognate from Figure 9b, the force transfer matrix is derived as follows:

$f_1 = -P_1\cos(\gamma-\Theta_3)+P_2\sin(\gamma-\Theta_3)$

$f_2 = -P_1\cos(\pi/2-(\gamma-\Theta_3)-)_2\sin(\pi/2-(\gamma-\Theta_3)$

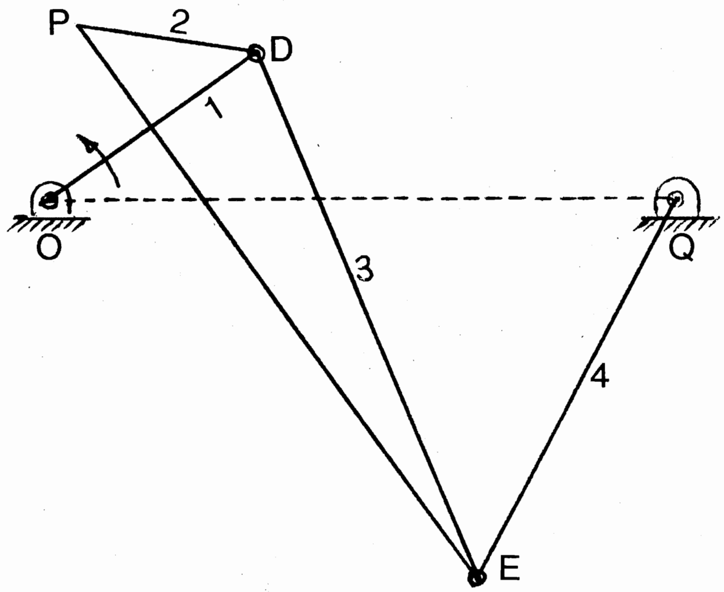


Figure 9. Crossed Cognate Four-Bar

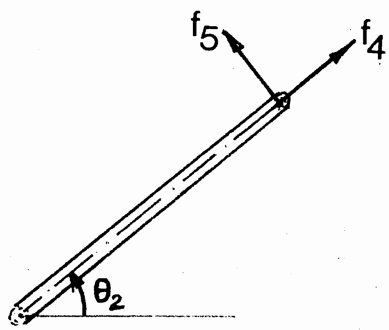


Figure 9a. Element 1 of the Cognate

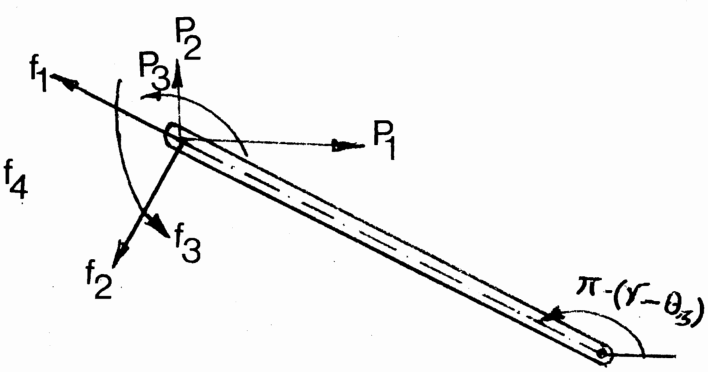


Figure 9b. Element 2 Coupler Extender of the Cognate

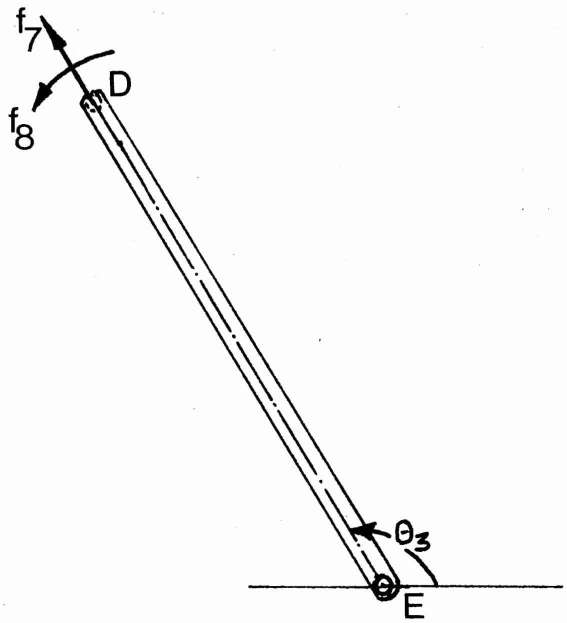


Figure 9c. Element 3, the Coupler Link of the Cognate

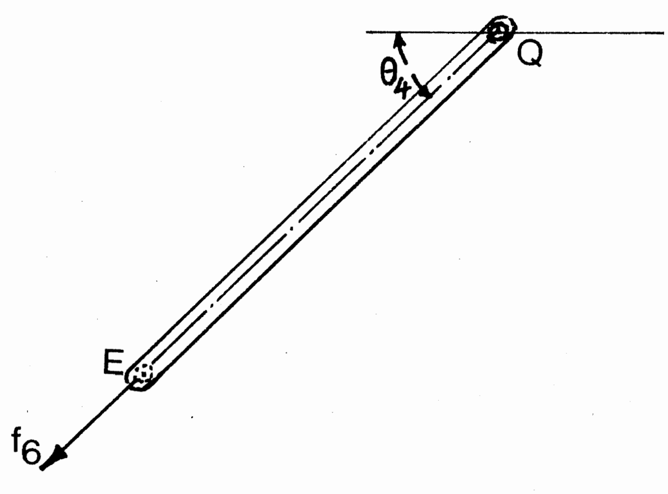


Figure 9d. Element 4, the Follower Link of the Cognate

$$f_3 = P_3$$

or

$$f_1 = -P_1\cos(\gamma-\Theta_3)+P_2\sin(\gamma-\Theta_3)$$

$$f_2 = -P_1\sin(\gamma-\Theta_3)-P_2\cos(\gamma-\Theta_3) \tag{19}$$

$$f_3 = P_3$$

Summing the horizontal forces of the coupler point:

$$P_1 = -f_4\cos(\Theta_2)+f_5\sin(\Theta_2)+f_6\cos(\Theta_4) \tag{20}$$

Summing the vertical forces on the coupler point:

$$P_2 = -f_4\sin(\Theta_2)-f_5\cos(\Theta_2)+f_6\sin(\Theta_4) \tag{21}$$

Taking moments about the point P we have

$$P_3 = \left\{\cos(\Theta_2)L_2\sin(\gamma-\Theta_3)+\sin(\Theta_2)L_2\cos(\gamma-\Theta_3)\right\}f_4$$

$$+\left\{\cos(\Theta_2)L_2\cos(\gamma-\Theta_3)-L_2\sin(\Theta_2)\sin(\gamma-\Theta_3)\right\}f_5 \tag{22}$$

$$-\left\{\cos(\Theta_4)L_2\sin(\gamma-\Theta_3)+\cos(\Theta_4)L_3\cos\Theta_3+\sin(\Theta_4)L_2\cos(\gamma-\Theta_3)+\sin\Theta_4\right.$$

$$\left.L_3\sin\Theta_3\right\}f_6$$

Equations (20), (21), and (22) are of the form

$$P_1 = af_4+bf_5+cf_6$$

$$P_2 = df_4+ef_5+ff_6 \tag{23}$$

$$P_3 = gf_4+hf_5+rf_6$$

Solution of the set of equations yields:

$$f_4 = \frac{(er-fh)P_1+(ch-br)P_2+(Pf-ce)P_3}{K}$$

$$f_5 = \frac{(fg-dr)P_1+(ar-cg)P_2+(cd-af)P_3}{K} \qquad (24)$$

$$f_6 = \frac{(dh-eg)P_1+(bg-ah)P_2+(ac-bd)P_3}{K}$$

where

$$K = a(er-fh)-b(dr-fg)+c(dh-eg)$$

Considering the free-body diagram of the coupler alone as shown in Figure 10:

$$f_7 = f_6\cos(\Theta_3 - \Theta_4)$$

$$f_8 = f_6L_3\sin(\Theta_3 - \Theta_4) \qquad (25)$$

may also be written in terms of P_1, P_2, P_3 by summing the moments about the point D, as shown in Figure 10.

$$f_6L_3 = \sin(\Theta_3 - \Theta_4) = -L_2\sin(\gamma-\Theta_3)P_1-P_2L_2\cos(\gamma-\Theta_3)+P_3$$

$$\therefore f_6 = \frac{-L_2\sin(\gamma-\Theta_3)P_1-L_2\cos(\gamma-\Theta_3)P_2+P_3}{L_3\sin(\Theta_3-\Theta_4)}$$

Thus

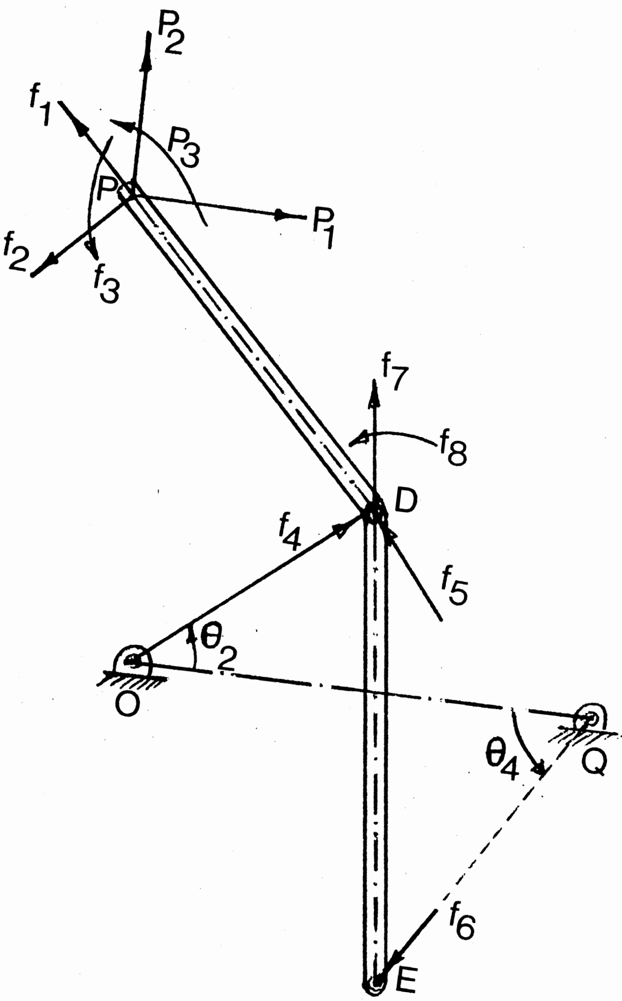


Figure 10. Figure Showing the Coupler Link With Extender, of the Cognate Four-Bar

$$f_7 = (-L_2/L_3)\cot(\Theta_3-\Theta_4)\sin(\gamma-\Theta_3)P_1-(L_2/L_3)\cot(\Theta_3-\Theta_4)\cos$$

$$(\gamma-\Theta_3)P_2+\cot-(\Theta_3-\Theta_4)P_3/L_3$$

and

$$f_8 = -L_2\sin(\gamma-\Theta_3)P_1-L_2\cos(\gamma-\Theta_3)P_2+P_3 \qquad (26)$$

Combining equations (19)(24)(26) into a matrix form, the element forces $f_1, f_2, \ldots. f_8$ may be derived from the system forces P_1, P_2, P_3. The following symbolic notations are used:

$$C\gamma\Theta_3 = \cos(\gamma-\Theta_3), \quad S\ell\Theta_3 = \sin(\gamma-\Theta_3)$$

$$C\Theta_2 = \cos(\Theta_2), \quad S\Theta_2 = \sin(\Theta_2)$$

$$C\Theta_2 = \cos(\Theta_3), \quad S\Theta_3 = \sin(\Theta_3)$$

$$C\Theta_4 = \cos(\Theta_4), \quad S\Theta_4 = \sin(\Theta_4)$$

$$C\Theta_3\Theta_4 = \cot-(\Theta_3-\Theta_4)$$

The force transformation matrix may be represented as

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \cdot \\ \cdot \\ \cdot \\ f_8 \end{bmatrix} = \begin{bmatrix} \beta_c \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} \qquad (27)$$

where β_c is given as:

$$\begin{bmatrix} -C\gamma\Theta_3 & S\gamma\Theta_3 & 0 \\ -S\gamma\Theta_3 & -C\gamma\Theta_3 & 0 \\ 0 & 0 & 1 \\ (er-fh)/K & (ch-br)/K & (bf-ce)/K \\ (fg-dr)/K & (ar-cg)/K & (cd-af)/K \\ (dh-eg)/K & (bg-ah)/K & (ae-bd)/K \\ -C\Theta_3\Theta_4\gamma S\gamma\Theta_3\ L_2/L_3 & -C\Theta_3\Theta_4 . C\gamma\Theta_3\ L_2/L_3 & C\Theta_3\Theta_4/L_3 \\ -S\gamma\Theta_3\ L_2 & -C\gamma\Theta_3 L_2 & 1 \end{bmatrix} \quad (28)$$

where

$a = -C\Theta_2$

$b = S\Theta_2$

$c = C\Theta_4$

$d = -\ S\Theta_2$

$e = -C\Theta_2$

$f = S\Theta_4$

$g = C\Theta_2 L_2\ S\gamma\Theta_3 + S\Theta_2 L_2\ C\gamma\Theta_3$

$h = C\Theta_4 L_2\ C\gamma\Theta_3 - S\Theta_2 L_2\ S\gamma\Theta_3$

$\gamma = C\Theta_4 L_2\ S\gamma\Theta_3 + C\Theta_4 L_3\ C\Theta_3 + S\Theta_4 L_2\ C\gamma\Theta_3 + S\Theta_4 L_3\ S\Theta_3$

$K = a(er-fh) - b(dr-fg) + c(dh-eg)$

Element Flexibility Matrix of the System. The element flexibility matrix for the four link mechanism can be expressed in a diagonal super matrix consisting of the individual element flexibilities as derived in Chapter III.

The element flexibility matrix for a four-link mechanism is an (8x8) diagonal matrix as shown below:

$$
\left[F \right] = \left[\begin{array}{ccc|cc|c|cc}
L_2/A_2E_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & L_2^3/3E_2I_2 & L_2^2/2E_2I_2 & 0 & 0 & 0 & 0 & 0 \\
0 & L_2^2/2E_2I_2 & L_2/E_2I_2 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & L_1/A_1E_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & L_1^3/3E_1I_1 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & L_4/A_4E_4 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & L_3/A_3E_3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & L_3/3E_3I_3
\end{array} \right] \qquad (29)
$$

The following notations are used:

L_1 = input-link length of the source four-bar

L_2 = the coupler extender length

L_3 = the coupler link length

L_4 = the follower link length

A_i = area of cross-section of corresponding link

E_i = modulus of elasticity of corresponding link

I_i = cross-sectional moment of inertia about an axis normal to the plane of the mechanism of the corresponding link.

It is evident that the element flexibility matrix is independent of the configuration of the mechanism, i.e., the input angle.

The element of flexibility matrix for the cognate linkage remains the same except for the following link dimensions:

cL_1 = input link length of the cognate

cL_2 = coupler extender length of the cognate

cL_3 = coupler link length of the cognate

cL_4 = follower link length of the cognate

For both the source and the cognate mechanisms, the base link (fixed link) is considered as rigid since it is grounded. The four-bar and its cognate are considered to be made up of homogeneous metal (aluminum) and each link is of uniform cross-section.

Determining the System Forces. The key interest of the problem is to compute the deflections of the coupler point of the source and its cognate mechanism. The links are assumed mass-less compared to the inertial mass located at path point "P." The external or the generalized forces acting on the system are the horizontal and the vertical inertial forces P_1 and P_2 and an inertial torque P_3, all located at the path point "P."

The computation of these forces requires first the complete kinematic analysis of the four-bar and its cognate mechanism. A complete kinematic analysis of the source four-bar and its cognate is performed using the "Complex Number Approach" (8).

Using equation (6), i.e.,

$$[\delta] = [\beta]^t \ [F] \ [\beta] \ [P]$$

the deflections of the coupler point for the source and its cognate are determined for this case.

A computer program is developed for this case, and is given in Appendix B. A numerical example problem for this case is presented.

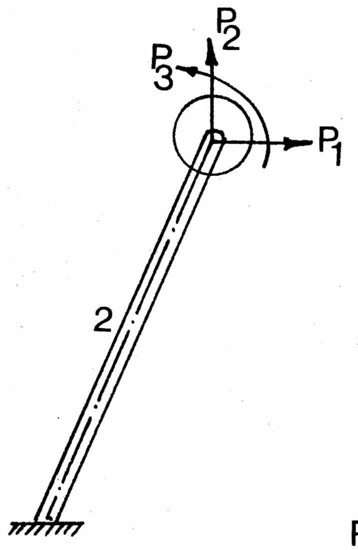
Case II: Mass at Each Joint of the Linkages

The second case differs from the first case in the respect that now each element has a concentrated or disc mass located at each joint as shown in Figure 11. There are eight system forces (P_1, P_2,....,P_8) instead of the three system forces (P_1, P_2, P_3) in the first case. The other five system forces directed in the five element coordinate directions are associated with elements 1, 3, 4. These eight system forces represent the inertia forces of each respective element.

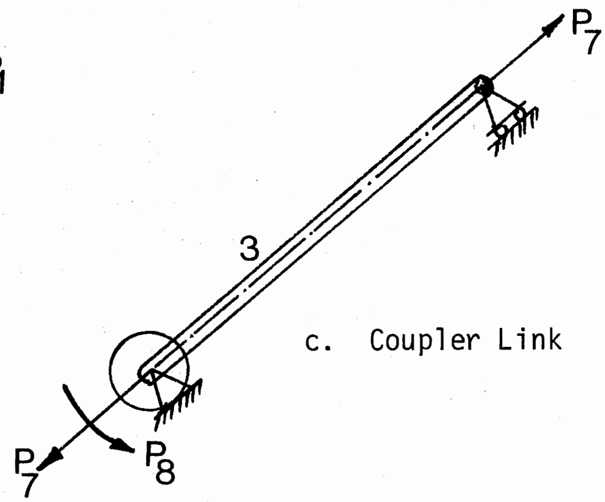
The objective is to compute the deflections at the path-point of the four-bar and its cognate mechanism. For this purpose, equation (6) is still valid excep with the following change in matrix dimensions:

P: the system force matrix is an (8x1) matrix instead of (3x1) as in Case I. However, the force-transformation matrix will vary and is a new (8x8) matrix. The flexibility matrix for the elements is independent of the configuration of the mechanism and thus remains the same as derived for Case I, since deflections at the coupler point of interest $[\beta]^t$ matrix remains the same.

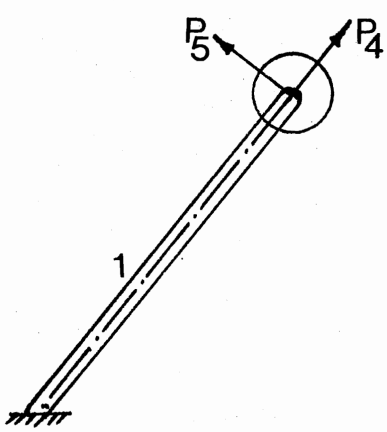
The new force transformation matrix for the source four-bar is of the form:



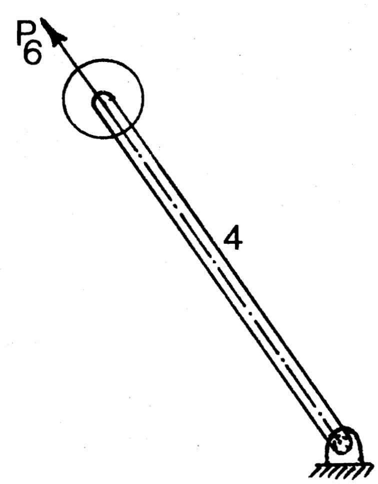
a. Coupler Extender



c. Coupler Link



b. Input Link



d. Follower Link

Figure 11. Showing the Eight System Forces $P_1,\ldots.,P_8$

$$\left[\beta_2\right] = \left[\begin{array}{c|ccccc} & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ \left[\beta_1\right] & 1 & 0 & 0 & -\cos(\Theta_2-\Theta_3) & 0 \\ & 0 & 1 & 0 & \sin(\Theta_2-\Theta_3) & 0 \\ & 0 & 0 & 1 & 0 & 0 \\ & 0 & 0 & 0 & 1 & 0 \\ & 0 & 0 & 0 & 0 & 1 \end{array}\right] \tag{30}$$

where $\left[\beta_1\right]$ is the force transformation matrix from Case I.

Since the cognate configuration is crossed, the new force transformation matrix for this case differs from the source linkage and is derived as follows:

$$\left[\beta_2\right]_{cog} = \left[\begin{array}{c|ccccc} & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ & 1 & 0 & 0 & \cos(\Theta_3^*-\Theta_2) & 0 \\ \left[\beta_1\right]_{cog} & 0 & 1 & 0 & \sin(\Theta_3^*-\Theta_2) & 0 \\ & 0 & 0 & 1 & 0 & 0 \\ & 0 & 0 & 0 & 1 & 0 \\ & 0 & 0 & 0 & 0 & 1 \end{array}\right] \tag{31}$$

where Θ_3^* - is the coupler angle for the cognate.

Where $\left[\beta_1\right]_{cog}$ is the force-transfer matrix for the cognate from Case I.

The deflections at the coupler point can be represented as:

$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \vdots \\ \delta_8 \end{bmatrix} = \begin{bmatrix} (3x8) \\ \beta_2 \end{bmatrix}^t \begin{bmatrix} (8x8) \\ F \end{bmatrix} \begin{bmatrix} (8x8) \\ \beta_2 \end{bmatrix} \begin{bmatrix} (8x1) \\ P \end{bmatrix} \qquad (32)$$

where $\left[\beta_2\right]^t$ and $\left[\beta_2\right]$ and $\left[P\right]$ are different for source and cognate as derived above.

The method of computing the deflection is general. This can be demonstrated by adding any number of inertia and/or external forces to the system. For example, if there are twenty system forces and fifteen element coordinates, then the system force matrix is (20x1), i.e., P_j, j=1,2,3,...,20. The corresponding force transformation matrix transferring fifteen element forces to system forces becomes (15x20). The fifteen element flexibilities can be coupled to form a (15x15) element flexibility matrix. Then, the fifteen element deflections can be expressed as:

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_{15} \end{bmatrix} = \begin{bmatrix} F \\ (15x15) \end{bmatrix} \begin{bmatrix} \beta \\ (15x20) \end{bmatrix} \begin{bmatrix} P \\ (20x1) \end{bmatrix} \qquad (33)$$

In the example problem, the mass of each link is computed and assumed to be lumped at each joint. The deflections of the coupler point are calculated; however, by this approach, the deflections of each element can be calculated. Listing of the computer program for the second case is given in Appendix C.

Case III: Distributed Mass Model

The third case describes the computation of the deflections induced in the members of the mechanism by considering the mass of each link to be distributed in the form of sub-elements. For the four-bar under consideration, the deformation of the coupler link is due to its own inertia since Elements 1 and 4 (the input and the follower links) may only cause the coupler to deflect as a rigid body.

Considering the mass to be distributed in the form of sub-elements in the coupler extender which is of primary importance, deflections at each point can be calculated by considering the mass being made up of elemental masses located at the infinite tips in an increasing trend of length of the extender, as in Figures 12a and 8b. The system forces P_j, $j=1,\ldots,n$ (where n is the number of system forces) can be computed.

However, for practical computation, if the extender is divided into five parts with mass located at each node as shown in Figure 12c, there are fifteen system forces. The number of system forces increases with the number of nodes selected for study.

The system force matrix $\left[P_j \right]$ is a (15x1) matrix for this case.

Since Element 2 is subdivided into five elements, the element flexibility matrix $\left[F^* \right]$ varies for both source and cognate mechanism, and is a (40x40) matrix, as given below:

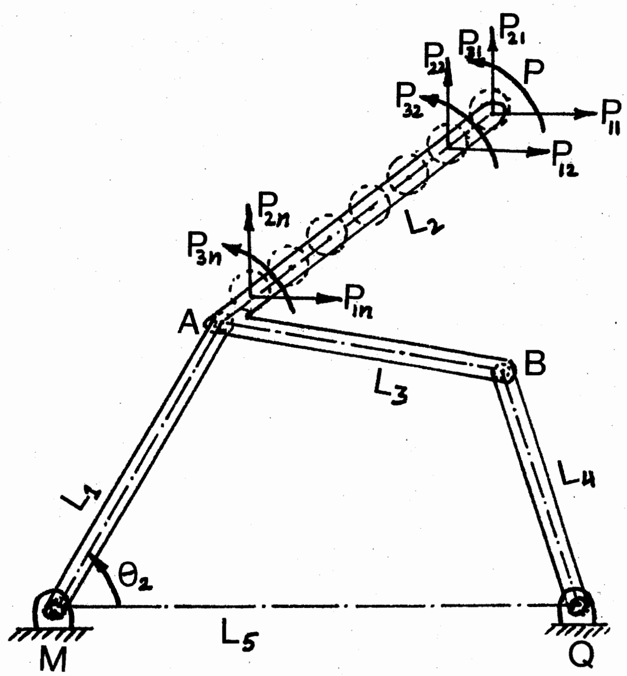


Figure 12a. Distributed Mass Model for Coupler

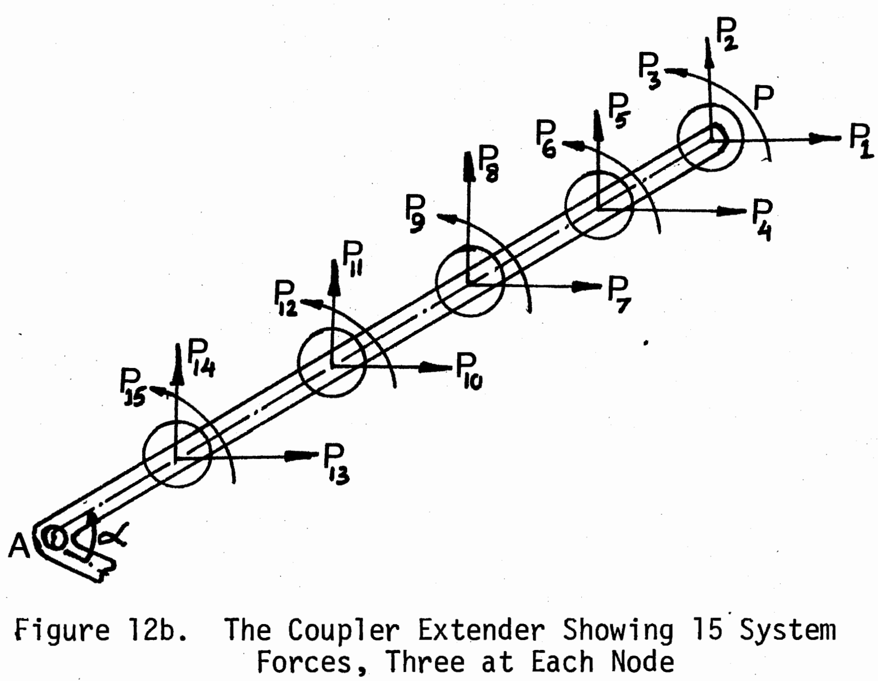


Figure 12b. The Coupler Extender Showing 15 System Forces, Three at Each Node

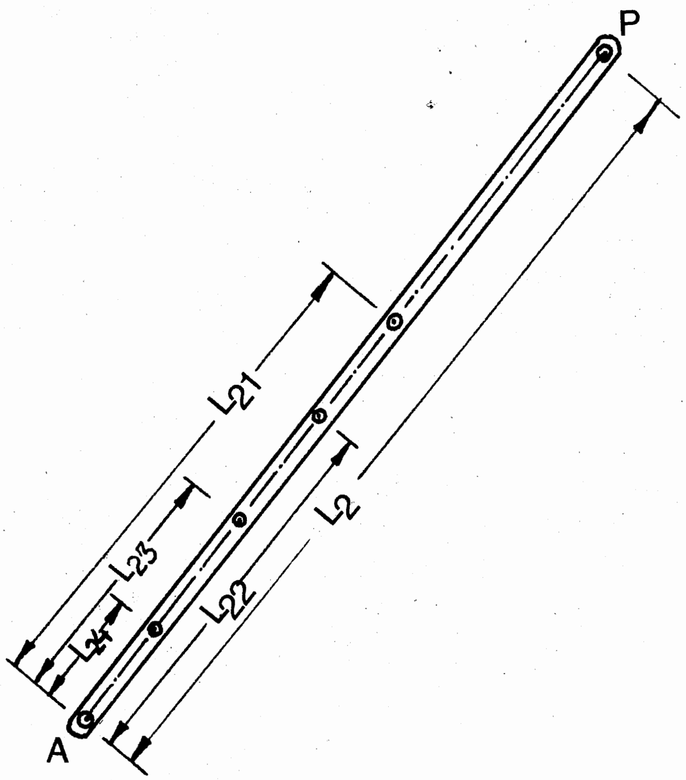


Figure 12c. Location of Masses and Corresponding Subelement Lengths

$$[F^*] = \begin{bmatrix} \overset{(8x8)}{F_1} & 0 & 0 & 0 & 0 \\ 0 & \overset{(8x8)}{F_2} & 0 & 0 & 0 \\ \vdots & & & & \\ 0 & 0 & \overset{(8x8)}{F_3} & 0 & 0 \\ 0 & 0 & 0 & \overset{(8x8)}{F_4} & 0 \\ 0 & 0 & 0 & 0 & \overset{(8x8)}{F_5} \end{bmatrix}$$

where F_1 is the same as $[F]$ in Case I

where F_2 is the same as $[F]$ in Case I but with L_{21}, similarly

where F_5 is the same as $[F]$ in Case I but with L_{24}

The division of Element 2 into five sub-elements contributes in the increase in number of elemental forces, $f_1, f_2, f_3, \ldots, f_{40}$. The force transformation matrix is a (40x15) matrix. For the source four-bar mechanism, the force transfer matrix is a diagonal matrix with five sub-force transfer matrices as shown:

$$[\beta^*] = \begin{bmatrix} \beta_1 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & 0 \\ 0 & 0 & \beta_3 & 0 & 0 \\ 0 & 0 & 0 & \beta_4 & 0 \\ 0 & 0 & 0 & 0 & \beta_5 \end{bmatrix}$$

where $[\beta_1]$ to $[\beta_5]$ are similar to $[\beta]$ in Case I, except with lengths L_2, $L_{21}, \ldots, L_{24}$ β_1 to β_5 each time calculated with different lengths

L_2, L_{21}, L_{22}, L_{23}, L_{24}, respectively.

Thus, for each source and its cognate, the deflections of the coupler extender at the five selected points are evaluated by the relation:

$$\begin{bmatrix} \delta_1 \\ \vdots \\ \delta_{15} \end{bmatrix} = \begin{bmatrix} (15\text{x}40) \\ \beta \end{bmatrix}^t \begin{bmatrix} (40\text{x}40) \\ F \end{bmatrix} \begin{bmatrix} (40\text{x}15) \\ \beta \end{bmatrix} \begin{bmatrix} (15\text{x}1) \\ P \end{bmatrix}$$

The methodology developed for the three cases to calculate the deflections of the coupler point is demonstrated on a four-bar (Crank-Rocker) mechanism and its crossed cognate is shown in Figures 13a and 13b. The computer program for this case is given in Appendix D.

The results and conclusions are presented in the next chapter.

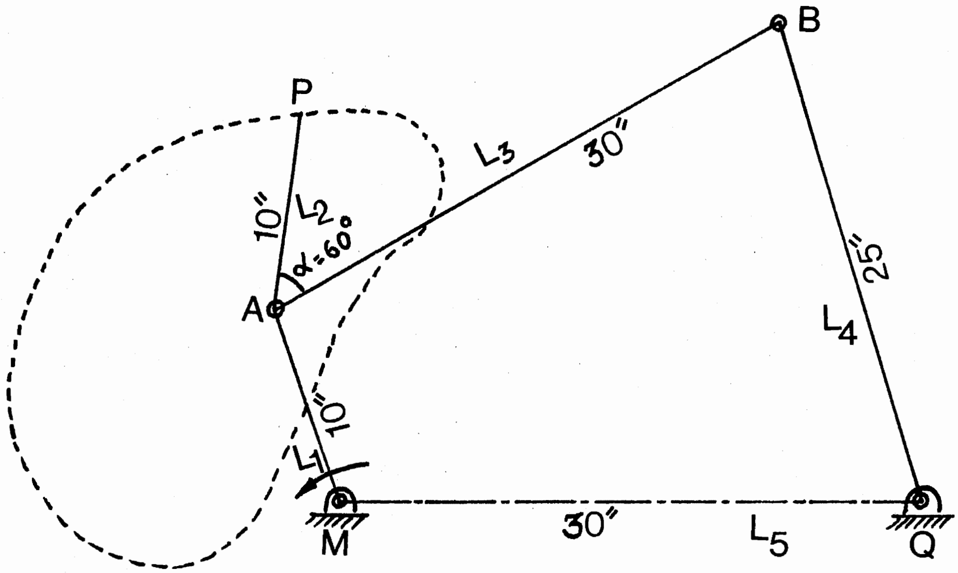


Figure 13a. Four-Bar Path Generator Source Mechanism

Data for Numerical Example

Input-link length	L_1	= 10 in
Coupler link length	L_3	= 30 in
Coupler extender length	L_2	= 10 in
Follower link length	L_4	= 25 in
Fixed or grounded link length	L_5	= 30 in
Uniform circular cross-sectional area of all links	A	= 0.19634 sq in
Rigid angle between coupler and extender	α	= 60 degrees
Modulus of elasticity for aluminum	E	= $10x10^6$ psi
Cross-sectional moment of inertia	I	= 0.00306 in^4
Input link velocity	ω_2	= 300 rpm
Mass at coupler Point for Case I	M_g	= 2 lbf

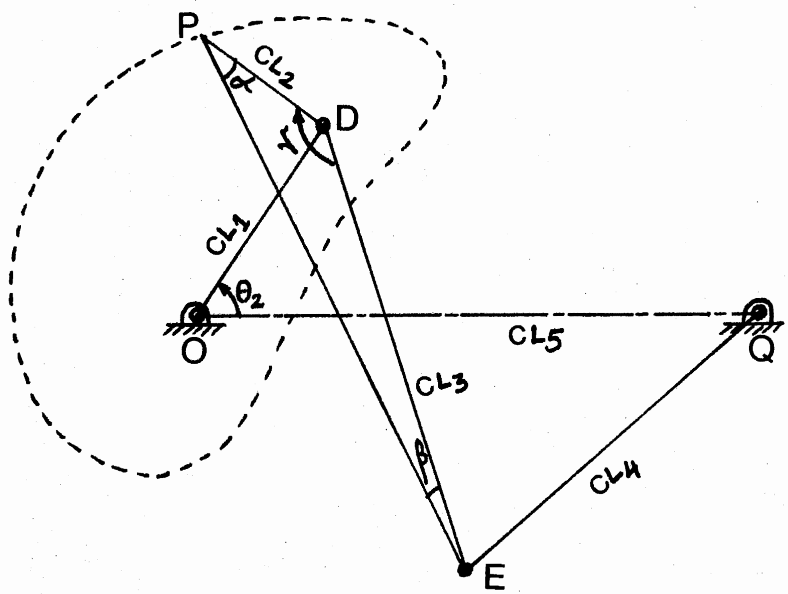


Figure 13b. The Coupler Cognate of the Source Four-Bar

<u>Data for Numerical Example</u>

Input link length	cL_1
Coupler extender length	cL_2
Coupler link length	cL_3
Follower link length	cL_4
Ground link length	cL_5
Rigid coupler angle	γ

From Figure 13a, if $K = L_2/L_3$ using the parallelogram properties, the link lengths and the angles are calculated as follows:

$$CL_4 = \text{SQRT } L_2^2 + L_3^2 - 2L_2L_3\cos(L)$$
$$CL_3 = L_4 \times CL_4/CL_3$$
$$CL_2 = K.L_4$$
$$CL_1 = L_1 \times CL_4/CL_3$$
$$CL_5 = \sqrt{(KL_5)^2 - 2K.L_5.L_5.\cos(\alpha)}$$
$$\gamma = 101 \text{ degrees}$$

CHAPTER V

RESULTS AND CONCLUSIONS

This thesis presents a general method of kineto-elasto dynamic analysis, which may be applied to various planar mechanisms with elastic links. The flexibility method of structural analysis is applied to mechanisms. The mechanism is frozen in various configurations and analyzed as an instantaneous structure with elastic members.

The flexibility approach described above is demonstrated on a planar four-bar linkage and its coupler cognate mechanism to determine the elastic deflections of the coupler point through a steady state cycle of motion. Three cases of increasing level of accuracy are considered:

1) Completely elastic system where the mass of links is negligible in comparison to the inertial mass at the coupler point.

2) Each element has its mass located at the joints of the mechanisms.

3) Mass of each element distributed in the form of sub-elements.

Computer programs are developed for the above three cases given in Appendices B, C, and D.

For the above three cases, a planar four-bar Crank-Rocker mechan ism which has a crossed cognate is selected as an example problem. Tables I, II, and III present the rigid path of the coupler course and the actual path when elastic deflections are added to it. For all of

TABLE I

MASS AT PATH POINT
(Case I)

Input Link Rotation	Coupler Point Coordinates in Rigid Mode		Source-Linkage Deflections		Source Coupler Points in K.E.D. Mode		Cognate-Linkage Deflections		Cognate Coupler Points in K.E.D. Mode	
Degrees	X	Y	ΔX	ΔY	X_{new}	Y_{new}	ΔX	ΔY	X_{new}	Y_{new}
0	5.648	9.004	-0.238	-0.539	5.410	8.465	-0.037	-0.567	5.611	8.437
20	6.637	13.033	-2.255	-3.486	4.382	9.547	-0.265	-1.311	6.372	11.722
40	6.234	16.328	-3.589	-4.197	2.645	12.131	-0.360	-1.474	5.874	14.854
60	4.470	18.648	-3.791	-3.670	0.679	14.978	-0.277	-1.395	4.193	17.253
80	1.710	19.849	-3.215	-2.804	-2.045	17.045	-0.122	-1.265	1.588	18.584
100	-1.571	19.845	-2.255	-1.891	-3.826	17.954	0.038	-1.113	-1.533	18.732
120	-4.904	18.655	-1.179	-1.003	-6.083	17.652	0.166	-0.933	-4.738	17.722
140	-7.871	16.418	-0.171	-0.156	-8.042	15.992	0.235	-0.723	-7.636	15.695
160	-10.145	13.382	-0.655	-0.660	-10.800	12.722	0.231	-0.493	-9.914	12.889
180	-11.507	9.873	1.244	1.465	-10.263	11.338	0.151	-0.254	-11.356	9.619
200	-11.846	6.261	1.579	2.276	-10.267	8.537	0.007	-0.008	-11.839	6.253
220	-11.163	2.923	1.664	3.079	-9.519	6.002	-0.188	0.249	-11.351	3.172
240	-9.561	0.223	1.526	3.823	-8.035	4.046	-0.405	0.525	-9.966	0.748
260	-7.242	-1.515	1.230	4.433	-6.012	2.918	-0.597	0.814	-7.839	-0.701
280	-4.478	-2.026	0.893	4.820	-3.585	2.794	-0.701	1.087	-5.179	-0.939
300	-1.574	-1.131	0.669	4.867	-9.905	3.736	-0.643	1.268	-2.217	0.137
320	1.209	1.218	0.671	4.333	1.880	5.551	-0.385	1.680	0.824	2.898
340	3.695	4.812	0.671	2.641	4.366	7.453	-0.066	0.513	3.629	5.325
360	5.648	9.004	-0.238	-0.539	5.410	8.465	-0.037	-0.567	5.611	8.437

TABLE II

MASS AT EACH JOINT
(Case II)

Input Link Rotation	Coupler Point Coordinates in Rigid Mode		Source-Linkage Deflections		Source Coupler Points in K.E.D. Mode		Cognate-Linkage Deflections		Cognate Coupler Points in K.E.D. Mode	
Degrees	X	Y	ΔX	ΔY	X_{new}	Y_{new}	ΔX	ΔY	X_{new}	Y_{new}
0	5.648	9.004	-0.126	-0.062	5.522	8.942	-0.003	-0.046	5.645	8.958
20	6.637	13.033	-0.251	-0.074	6.386	12.959	-0.021	-0.106	6.616	12.928
40	6.234	16.328	-0.265	-0.039	5.969	16.289	-0.029	-0.119	6.205	16.208
60	4.470	18.648	-0.196	-0.011	4.274	18.637	-0.022	-0.112	4.448	18.536
80	1.710	19.849	-0.101	-0.001	1.609	19.848	-0.010	-0.101	1.700	19.748
100	-1.571	19.845	-0.009	0.000	-1.562	19.845	0.003	-0.089	-1.569	19.756
120	-4.904	18.655	0.066	0.001	-4.898	18.656	0.013	-0.075	-4.891	18.581
140	-7.871	16.418	0.121	0.004	-7.750	16.422	0.019	-0.058	-7.852	16.360
160	-10.145	13.382	0.156	0.012	-9.989	13.394	0.019	-0.039	-10.127	13.342
180	-11.507	9.873	0.175	0.027	-11.332	9.900	0.012	-0.020	-11.495	9.853
200	-11.846	6.261	0.182	0.046	-11.664	6.307	0.000	0.000	-11.846	6.261
220	-11.163	2.923	0.179	0.067	-10.984	2.99	-0.016	0.021	-11.179	2.944
240	-9.561	0.223	0.169	0.088	-9.392	0.311	-0.033	0.043	-9.594	0.266
260	-7.242	-1.515	0.152	0.101	-7.090	-1.414	-0.048	0.065	-7.290	-1.449
280	-4.478	-2.026	0.129	0.104	-4.349	-1.922	-0.056	0.087	-4.535	-1.939
300	-1.574	-1.131	0.102	0.090	-1.472	-1.041	-0.051	0.102	-1.626	-1.029
320	1.209	1.218	0.067	0.057	1.276	1.275	-0.031	0.094	1.179	1.312
340	3.695	4.812	0.001	0.000	3.696	4.812	-0.005	0.041	3.690	4.853
360	5.648	9.004	-0.126	-0.062	5.522	8.942	-3.003	-0.046	5.645	8.956

TABLE III

DISTRIBUTED MASS MODEL
(Case III)

Input Link Rotation	Coupler Point Coordinates in Rigid Mode		Source-Linkage Deflections		Source Coupler Points in K.E.D. Mode		Cognate-Linkage Deflections		Cognate Coupler Points in K.E.D. Mode	
Degrees	X	Y	ΔX	ΔY	X_{new}	Y_{new}	ΔX	ΔY	X_{new}	Y_{new}
0	5.648	9.004	-0.141	-0.090	5.507	8.914	0.001	0.072	5.649	9.076
20	6.634	13.033	-0.255	-0.115	6.382	12.918	-0.004	-0.044	6.630	12.989
40	6.234	16.328	-0.259	-0.051	5.975	16.277	-0.019	-0.118	6.215	16.010
60	4.470	18.648	-0.199	-0.008	4.271	18.640	-0.043	-0.112	4.427	18.536
80	1.710	19.849	-0.113	0.002	1.597	19.851	-0.057	-0.091	1.653	19.758
100	-1.571	19.845	-0.025	-0.002	-1.596	19.843	-0.053	-0.068	-1.624	19.777
120	-4.904	18.655	0.053	-0.007	-4.851	18.649	-0.038	-0.046	-4.942	18.609
140	-7.871	16.418	0.113	-0.003	-7.757	16.415	-0.019	-0.024	-7.890	16.394
160	-10.145	13.382	0.154	0.010	-9.991	13.392	-0.001	-0.003	-10.146	13.379
180	-11.507	9.873	0.176	0.032	-11.331	9.905	0.012	0.020	-11.495	9.893
200	-11.846	6.261	0.181	0.057	-11.665	6.318	0.018	0.042	-11.828	6.303
220	-11.163	2.923	0.176	0.078	-10.987	3.001	0.017	0.061	-11.146	2.984
240	-9.561	0.223	0.166	0.091	-9.395	0.314	0.012	0.076	-9.549	0.299
260	-7.242	-1.515	0.157	0.100	-7.085	-1.415	0.005	0.089	-7.237	-1.426
280	-4.478	-2.026	0.145	0.107	-4.334	-1.919	-0.004	0.101	-4.482	-2.025
300	-1.574	-1.131	0.125	0.104	-1.450	-1.027	-0.014	0.111	-1.588	-1.020
320	1.209	1.218	0.091	0.080	1.301	1.298	-0.019	0.117	1.190	1.335
340	3.695	4.812	0.014	0.017	3.710	4.829	-0.012	0.114	3.683	4.926
360	5.648	9.004	-0.141	-0.090	5.507	8.914	0.001	0.072	5.649	9.076

the cases described, graphs are plotted for two different speeds of the input link, i.e., at 300 rpm and 400 rpm to observe the major change in deflections with an increasing speed (Appendix A).

The following are the observations:

1) It is observed that with an increase of 100 rpm in the speed of the input link, the elastic deflections were nearly doubled.

2) Maximum deflections occur in the second half of the cycle of motion of the input link.

3) The increase in deflections with the increase in speed of the input rotation shows that there is a critical speed where links of a mechanism assembly fail to obey Hook's law, and the deformations induced will be permanent and will not balance with the cycle of motion.

4) The angular velocity of the input links for both the source and its cognate is assumed to be constant. Any fluctuation in speed causing acceleration will affect the deflections of the coupler path in X and Y directions considerably and the rotation in Z direction, with reference to a fixed reference plane.

5) The difference in the deflections of the source and its cognate for each case (Tables I, II, and III) clearly justifies that the parallelogram property of the construction of cognates does not hold good in K.E.D. mode. Thus, finding a coupler cognate in K.E.D. mode becomes a synthesis problem.

6) The accuracy of the computed deflections depends on the following factors:

a) the number and choice of the mechanism elements.

b) the size of the increment between each successive input rotation.

c) the mass model utilized.

d) the accuracy of the system forces.

The first three factors depend on the time available to the designer and the computer time.

The results of this thesis demonstrate the need for incorporating kineto-elastodynamic effects in overall mechanical design analysis. Further, the effects of induced elasticity in linkages can be overcome by the K.E.D. "re-synthesis" procedure. These considerations are of utmost importance wherever high speed and accuracy are the criteria for design.

This thesis sets a base for undertaking some of the possible research studies.

1) K.E.D. analysis and synthesis of elastic four-bar linkage with arbitrary mass assigned to each link.

2) K.E.D. analysis and synthesis of planar four-bar with a variable mass, where the mass is added and removed during certain parts of the mechanism cycle.

3. K.E.D. analysis based on the fluctuating angular velocity of the input link.

4) Extension of the idea of flexibility approach to spatial linkages.

5) K.E.D. re-synthesis of mechanisms to account for weight minimization, balancing, and stability.

6) Compilation of K.E.D. coupler-curve atlas which will be an improvement over the Hrones and Nelson atlas, accounting for the elasticity.

7) The effect of clearance in the joints of the mechanisms considering the joints to be elastic.

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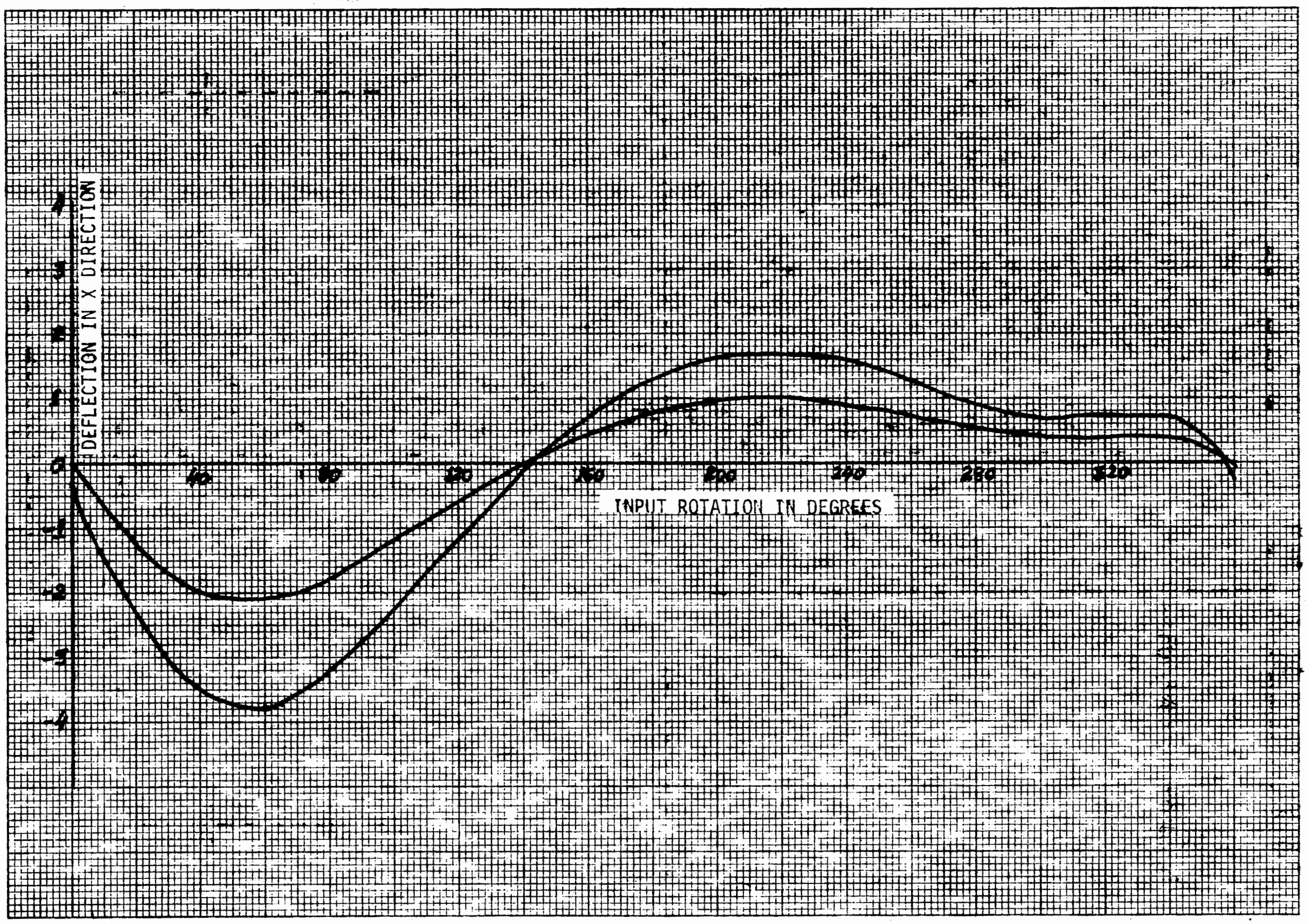
APPENDIX A

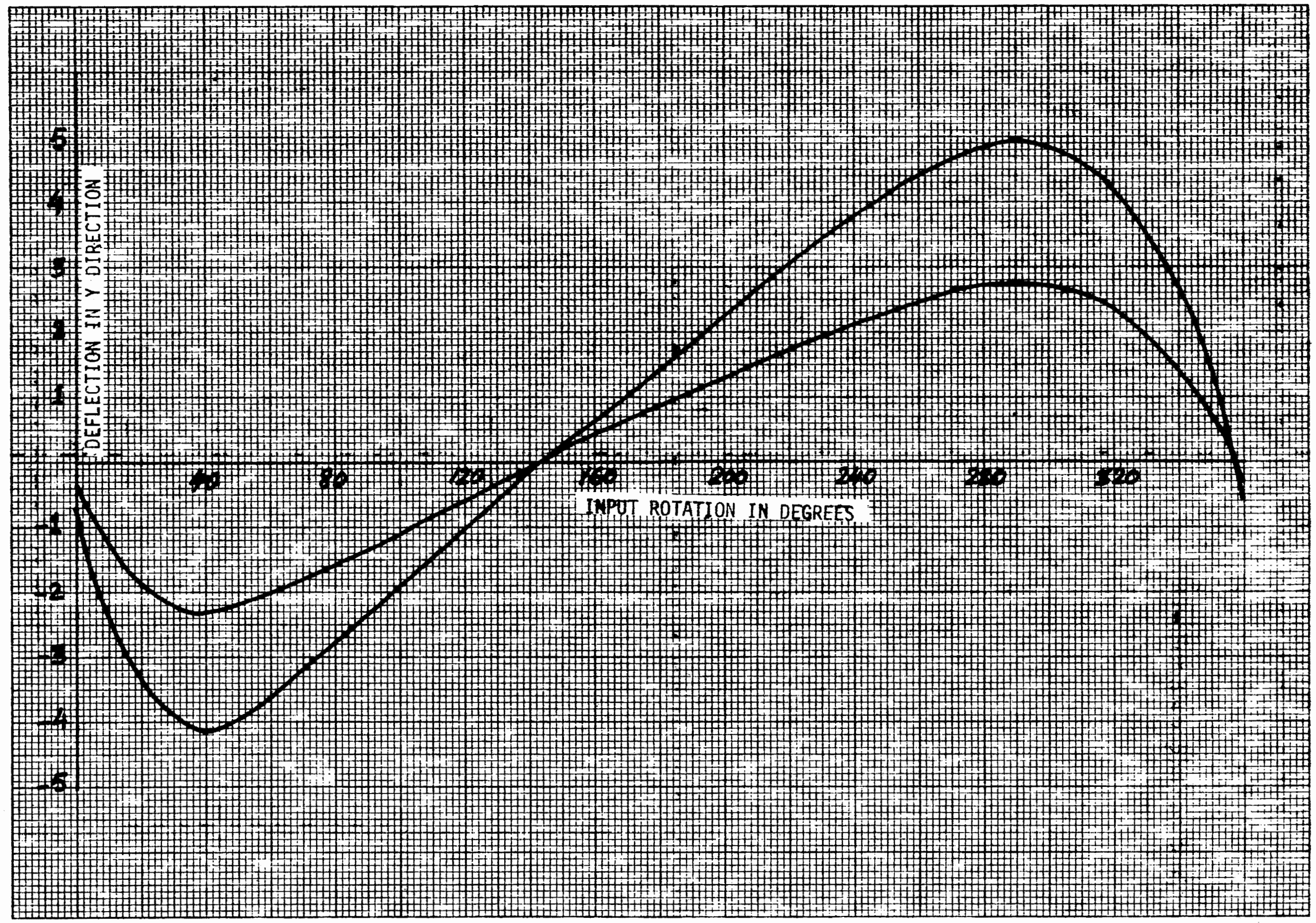
GRAPHS - INPUT-LINK ROTATION VS. ELASTIC DEFLECTIONS

SOURCE MECHANISM - CASE I



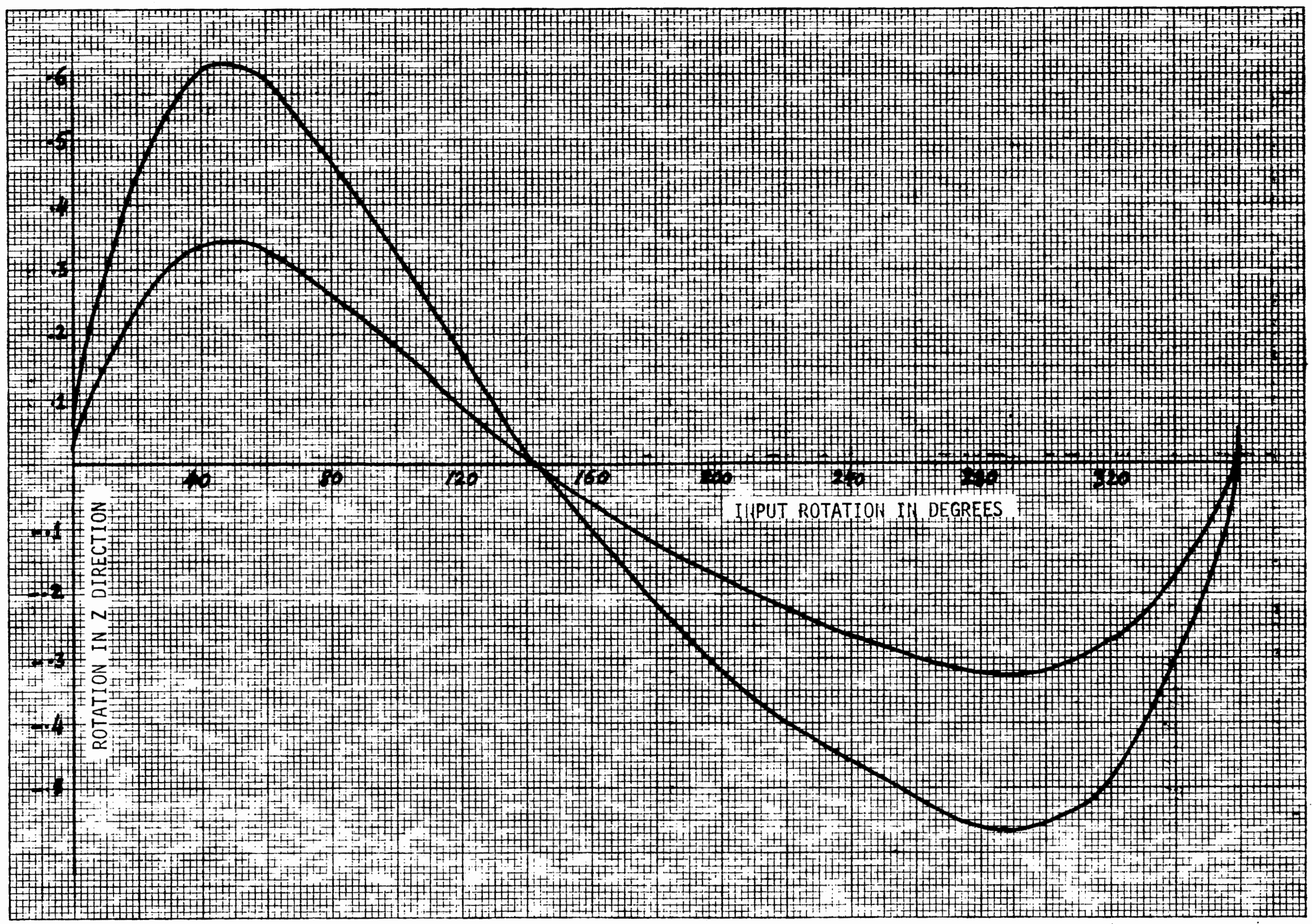
Input Link Rotation vs. Elastic Displacement of the Coupler Point in X-Direction

SOURCE MECHANISM - CASE I



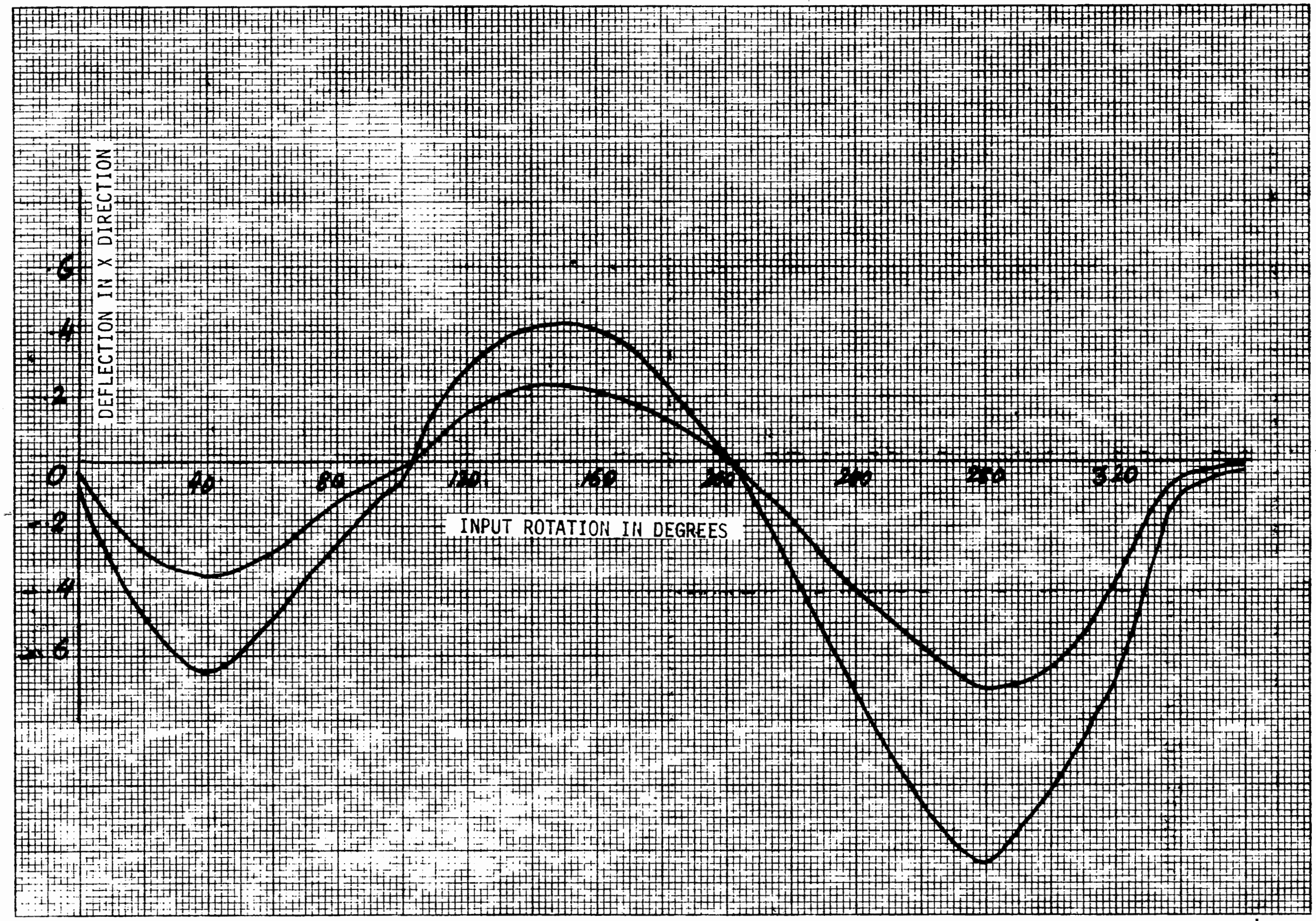
Input Link Rotation vs. Elastic Displacement of the Coupler Point in Y-Direction

SOURCE MECHANISM - CASE I



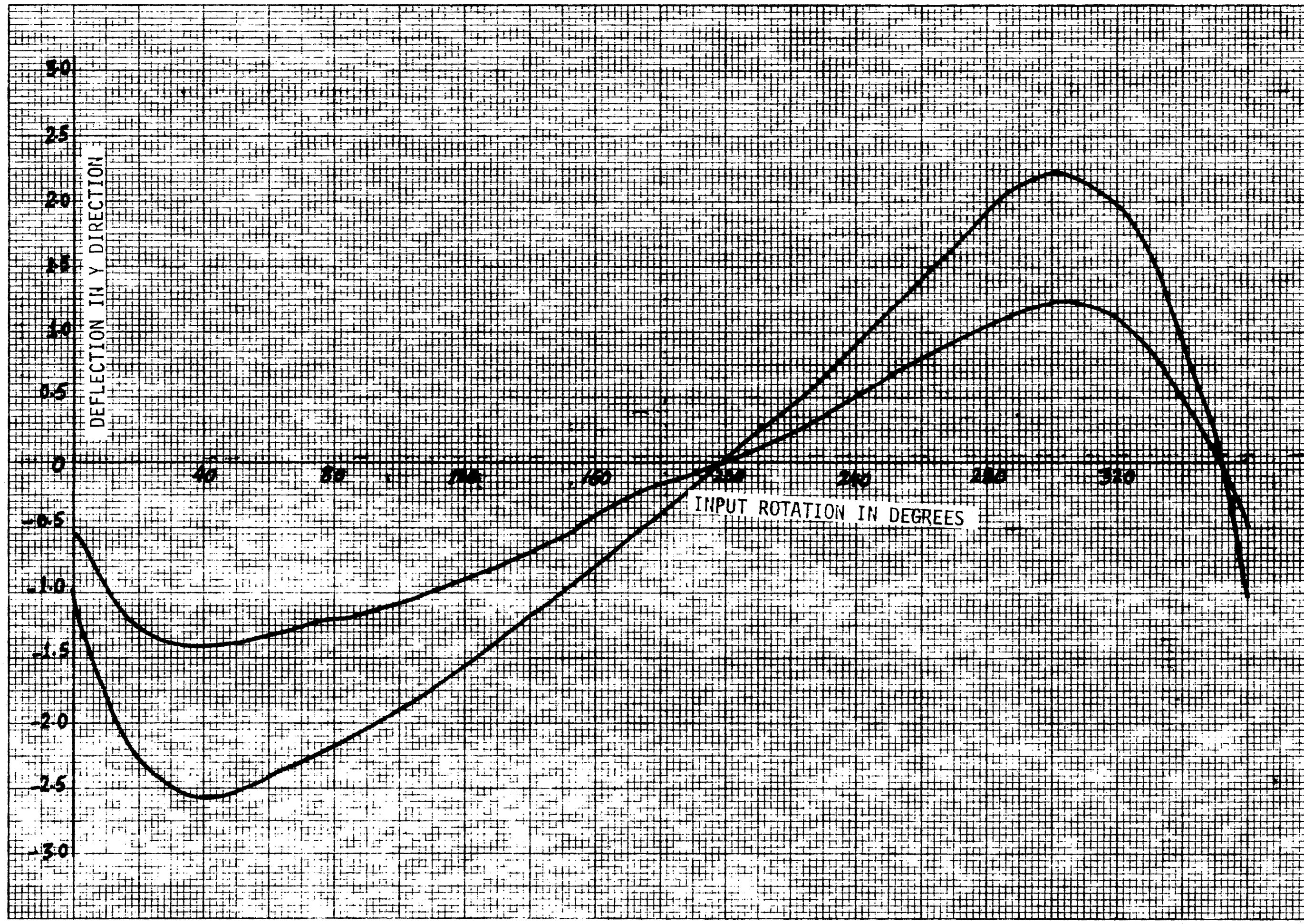
Input Link Rotation vs. Elastic Rotation in Z Direction

COGNATE MECHANISM - CASE I



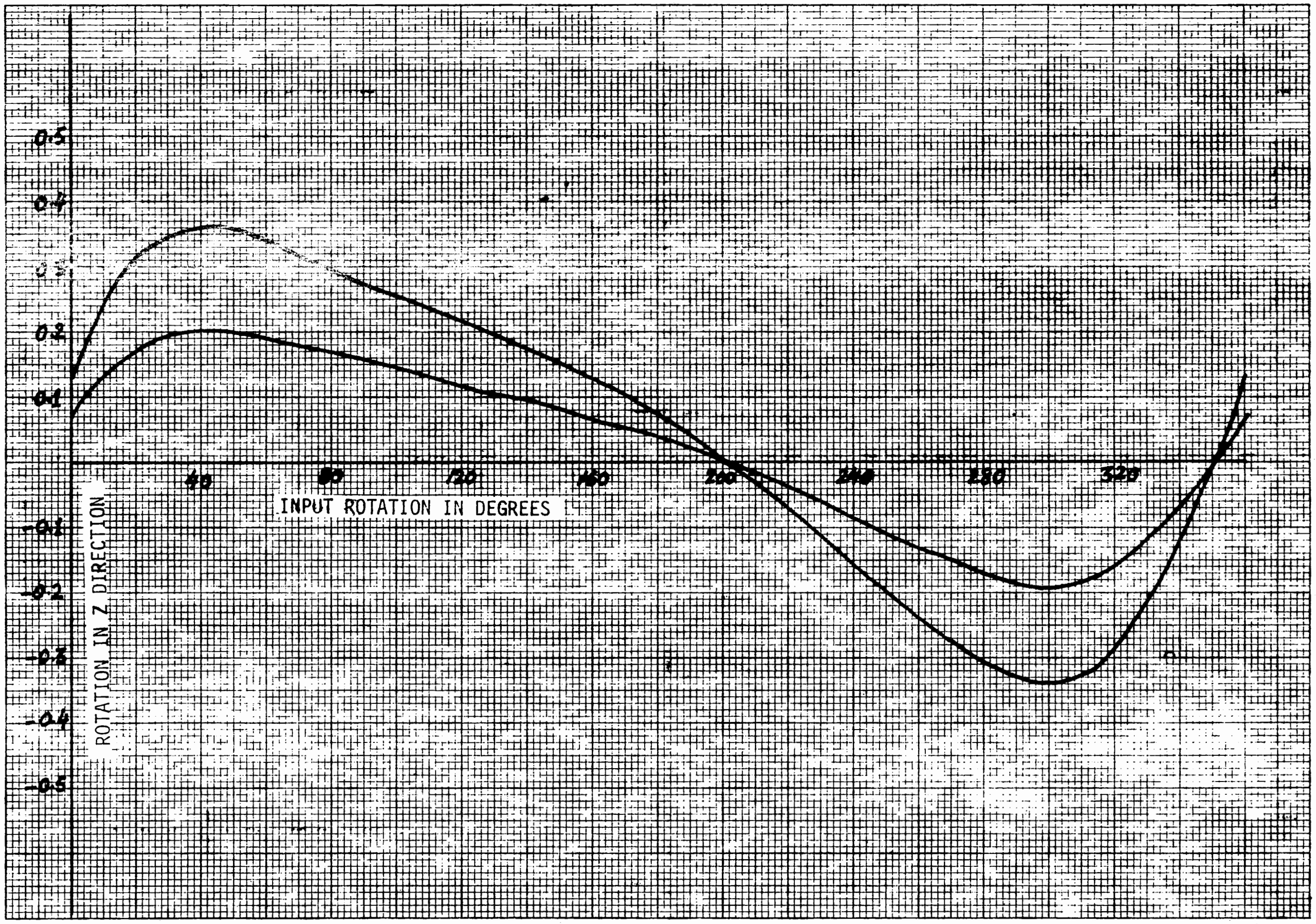
Input Link Rotation vs. Elastic Displacement of the Coupler Point in X Direction

COGNATE MECHANISM - CASE I



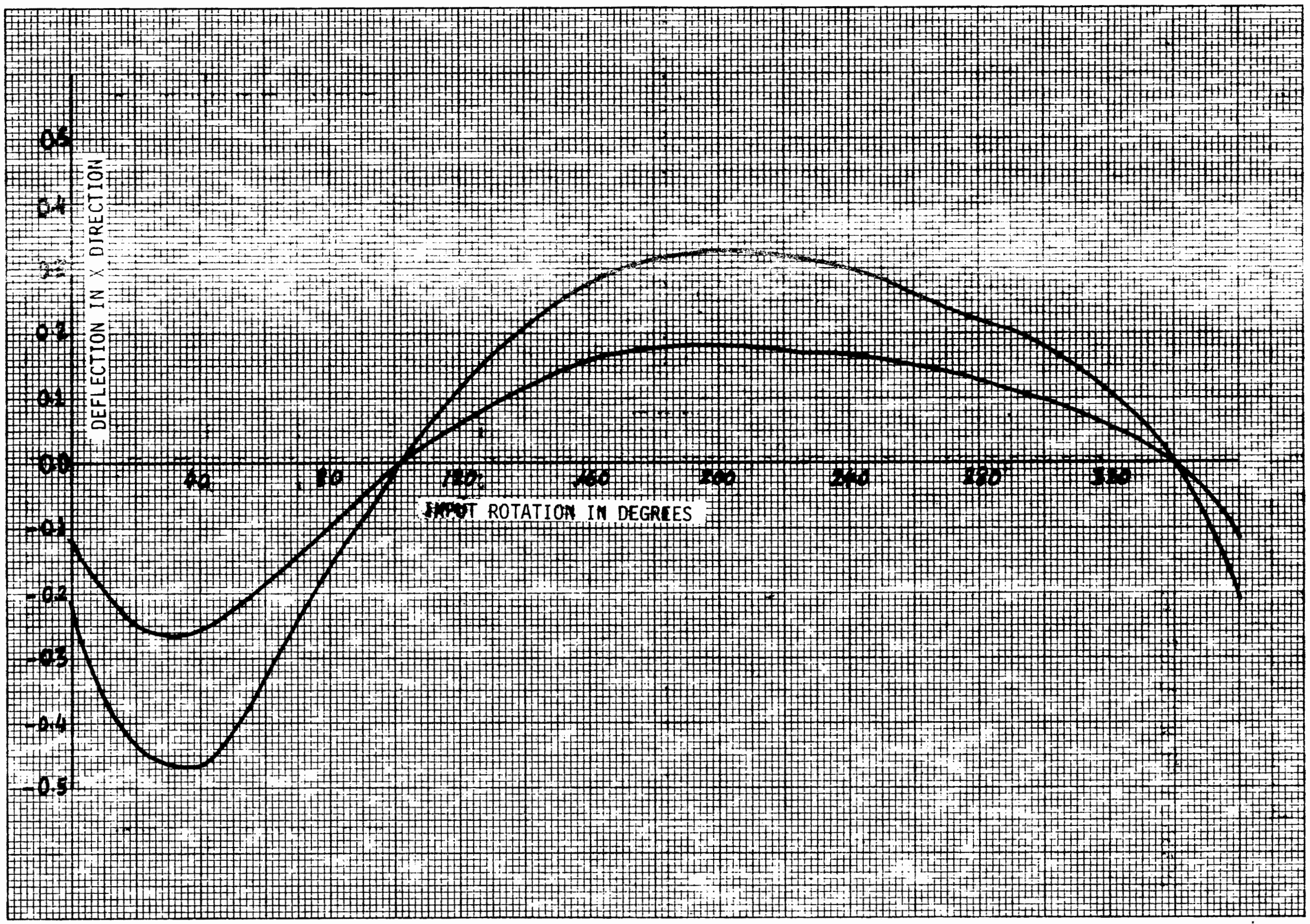
Input Link Rotation vs. Elastic Displacement of the Coupler Point in Y Direction

COGNATE MECHANISM - CASE I



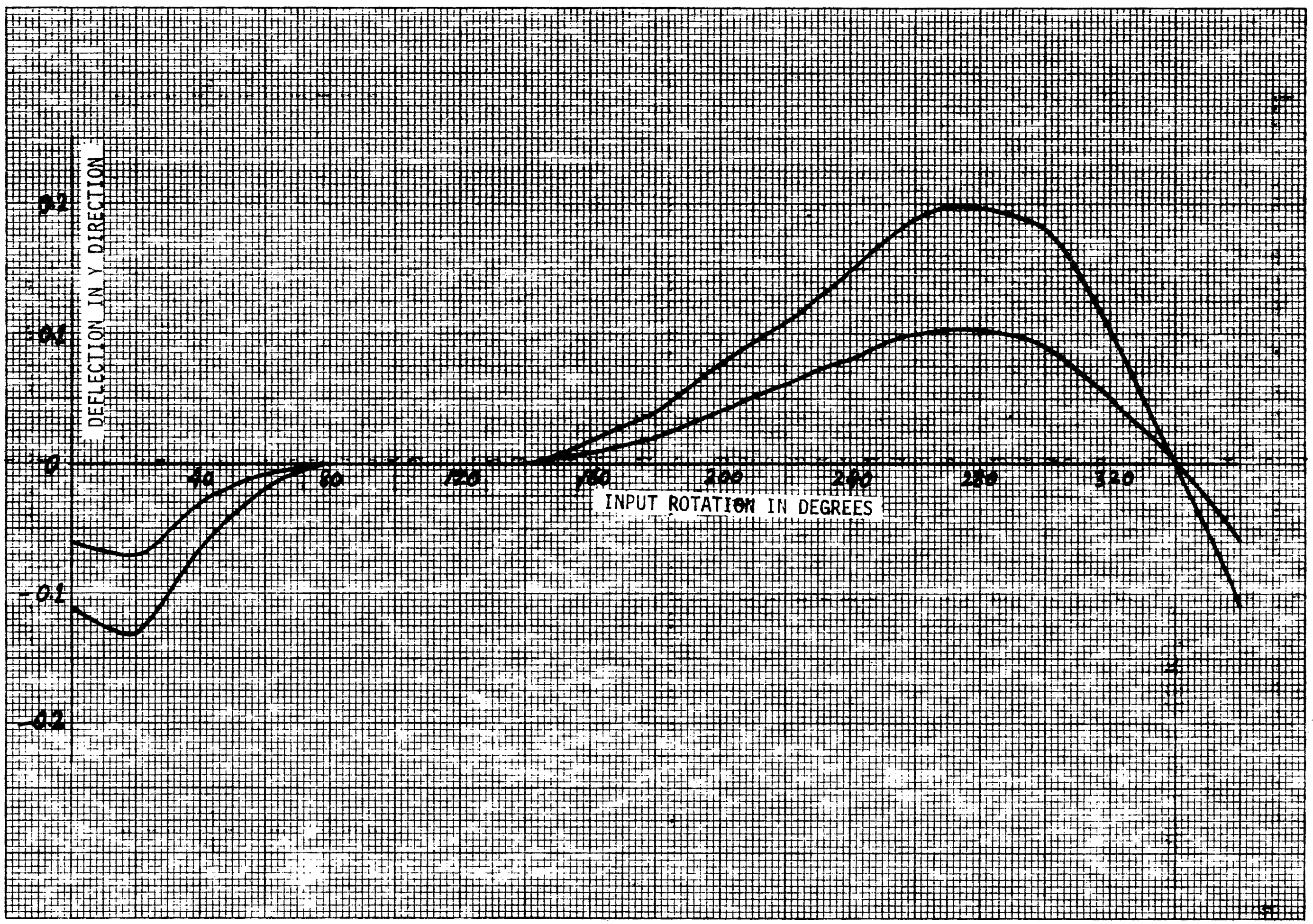
Input Link Rotation vs. Elastic Rotation in Z Direction

SOURCE MECHANISM - CASE II



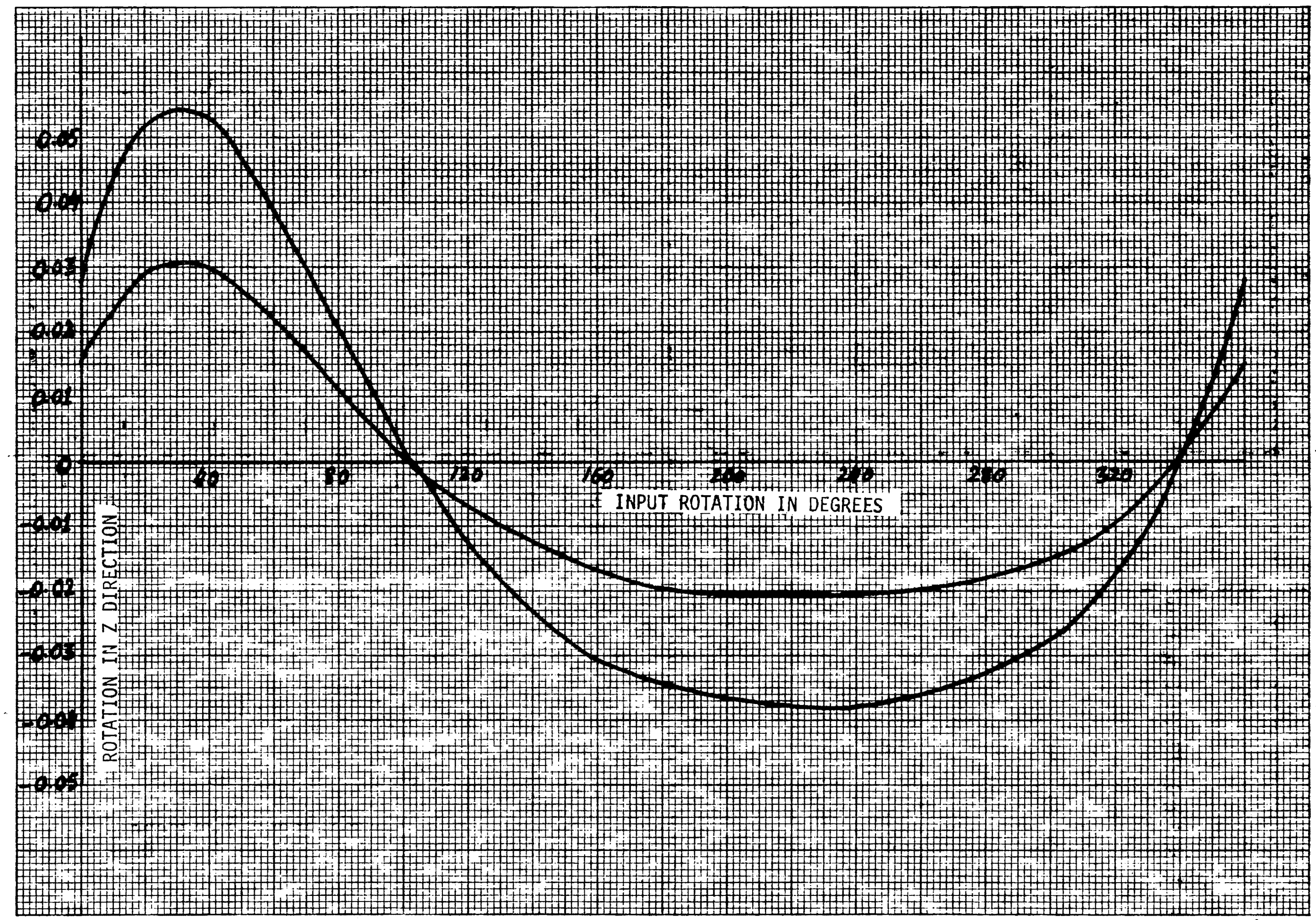
Input Link Rotation vs. Elastic Displacement of the Coupler Point in X-Direction

SOURCE MECHANISM - CASE II



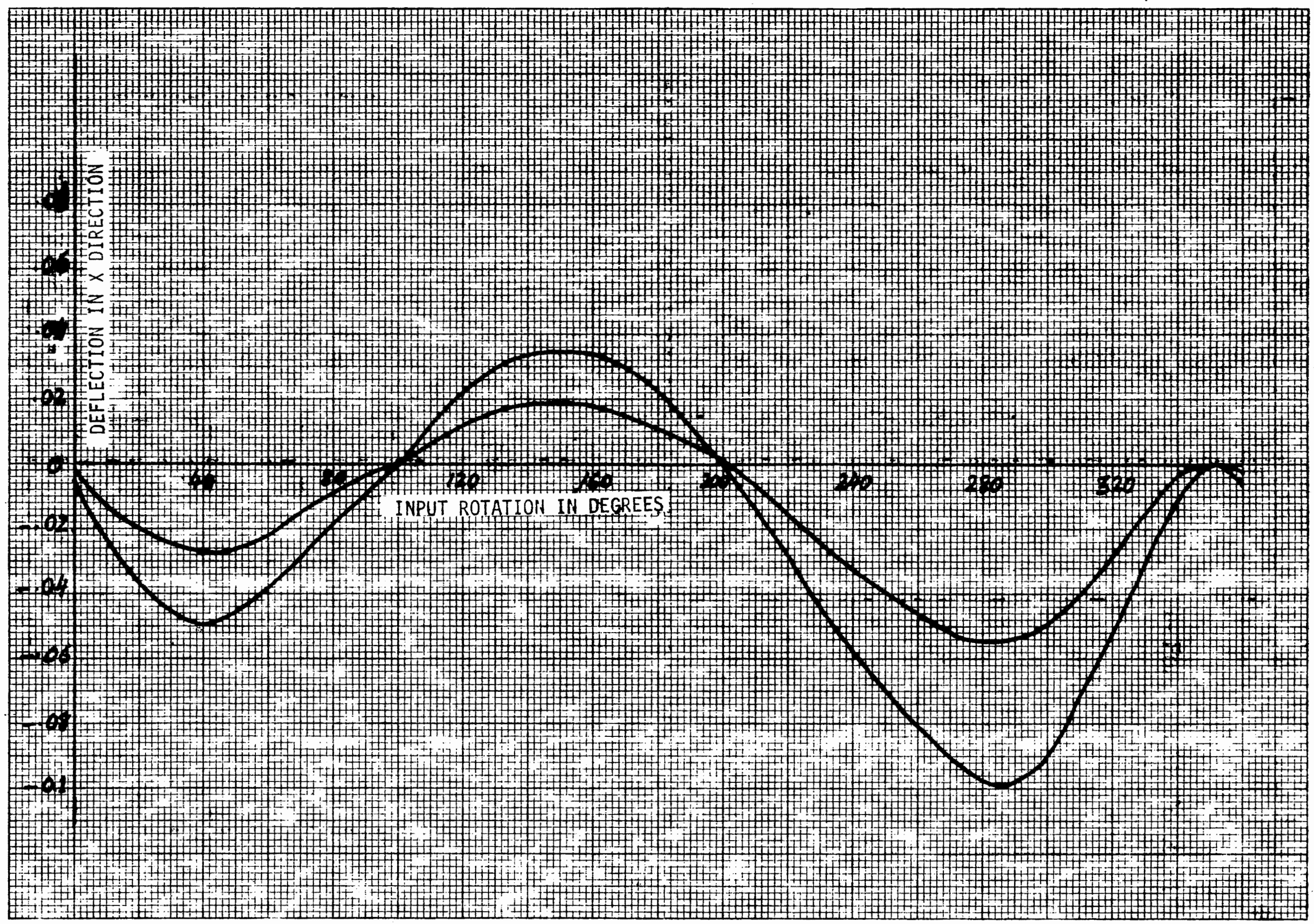
Input Link Rotation vs. Elastic Displacement of the Coupler Point in Y-Direction

SOURCE MECHANISM - CASE II



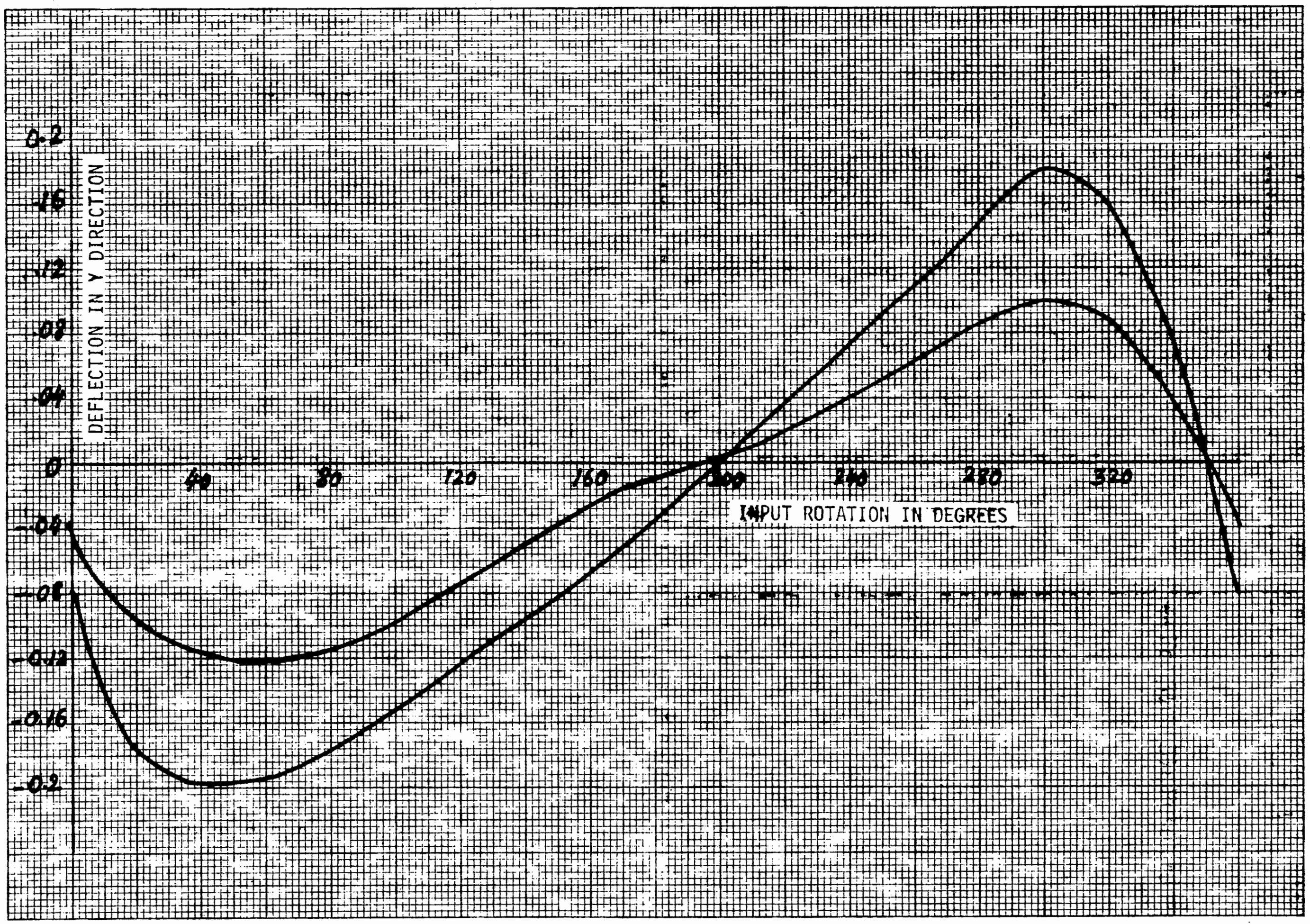
Input Link Rotation vs. Elastic Rotation in Z Direction

COGNATE MECHANISM - CASE II



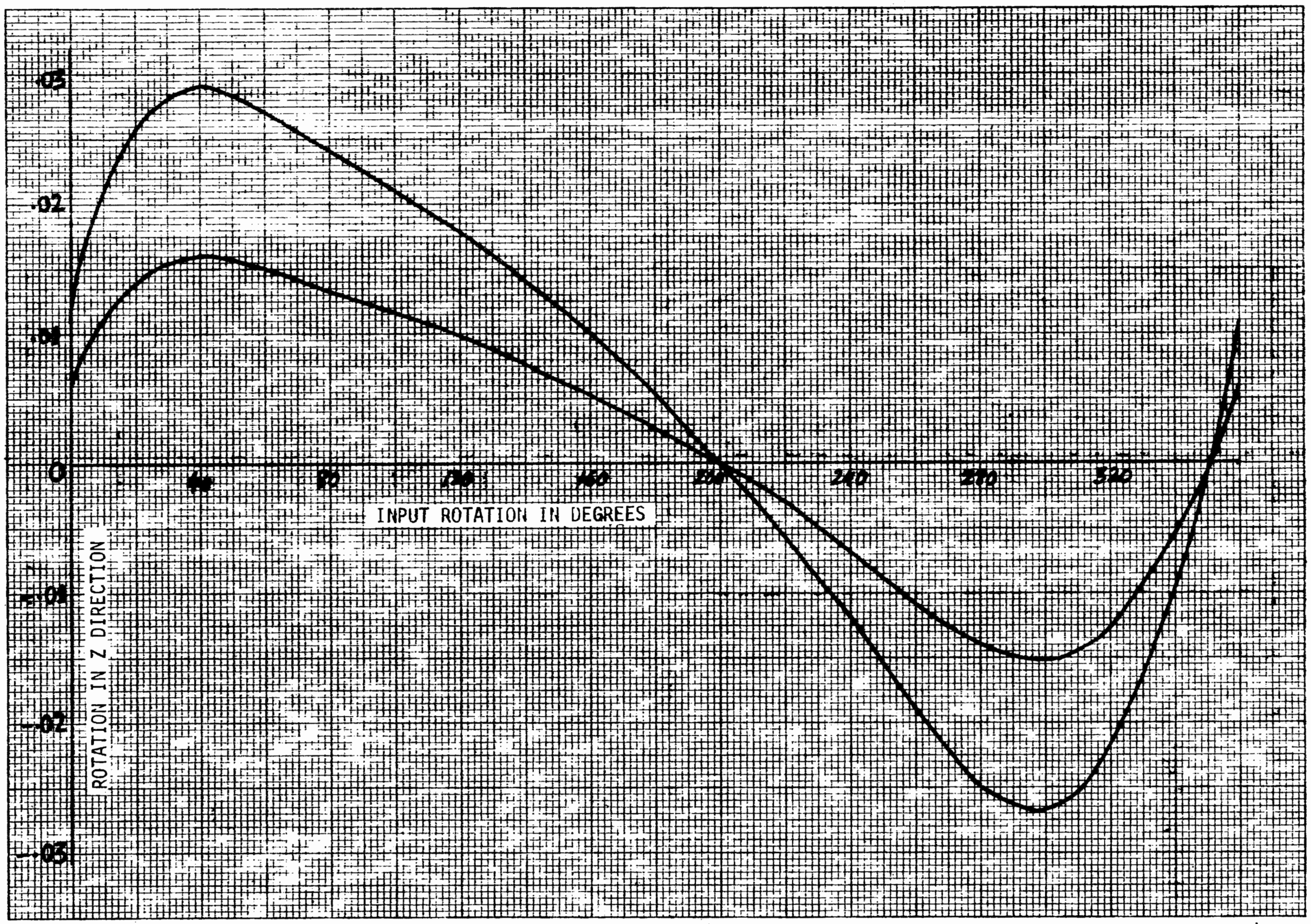
Input Link Rotation vs. Elastic Displacement of the Coupler Point in X-Direction

COGNATE MECHANISM - CASE II



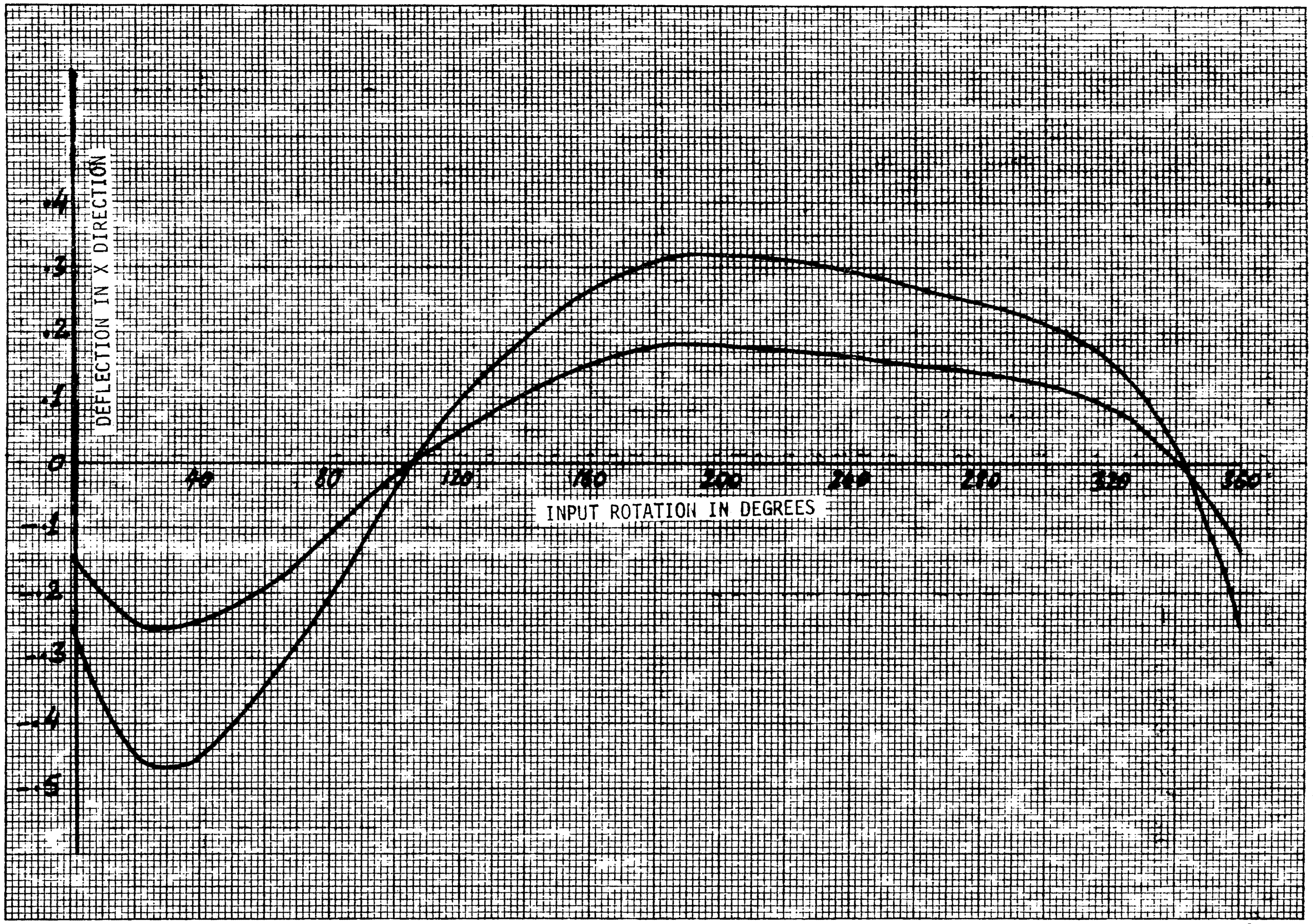
Input Link Rotation vs. Elastic Displacement of the Coupler Point in Y-Direction

COGNATE MECHANISM - CASE II



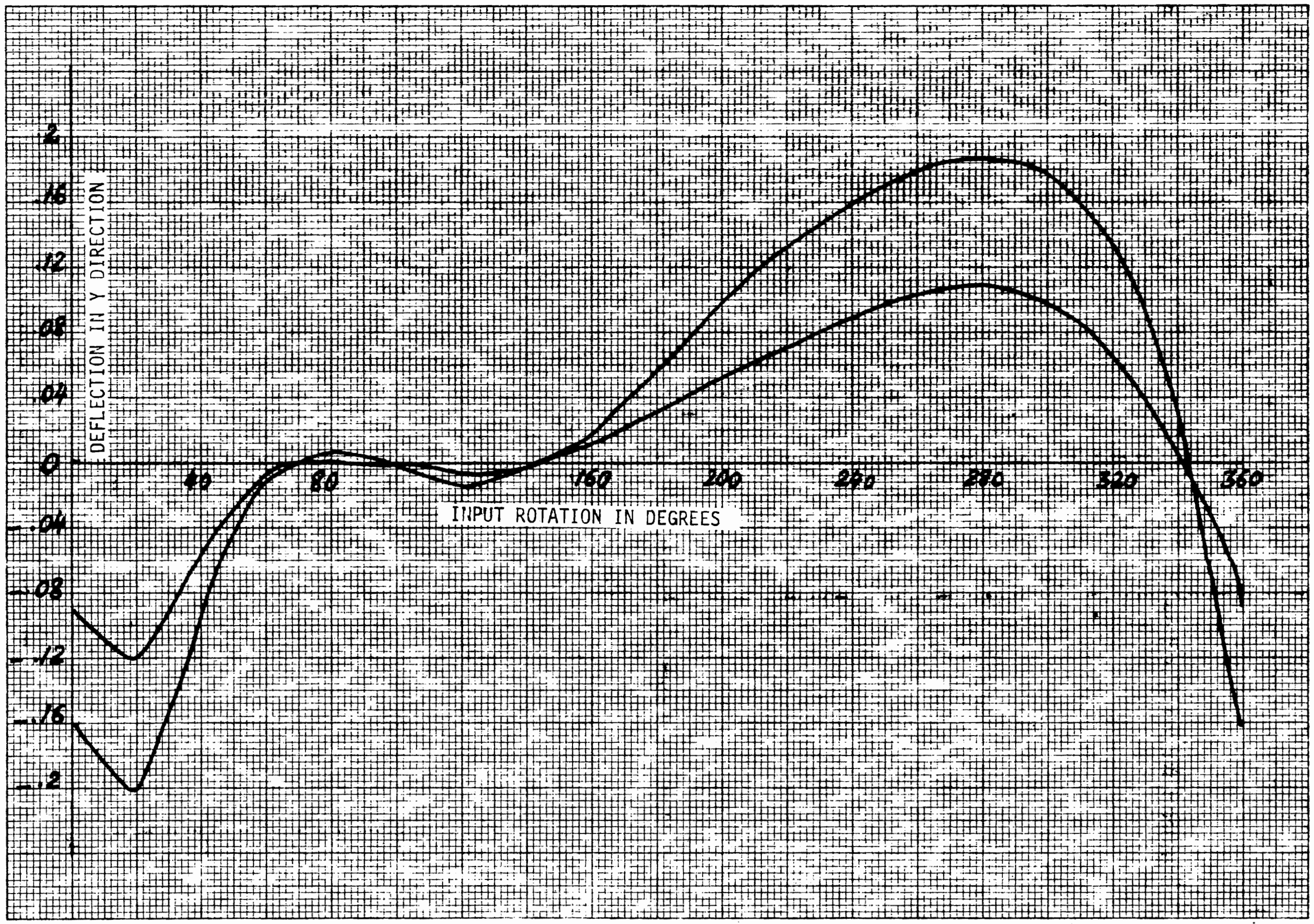
Input Link Rotation vs. Elastic Rotation in Z-Direction

SOURCE MECHANISM - CASE III



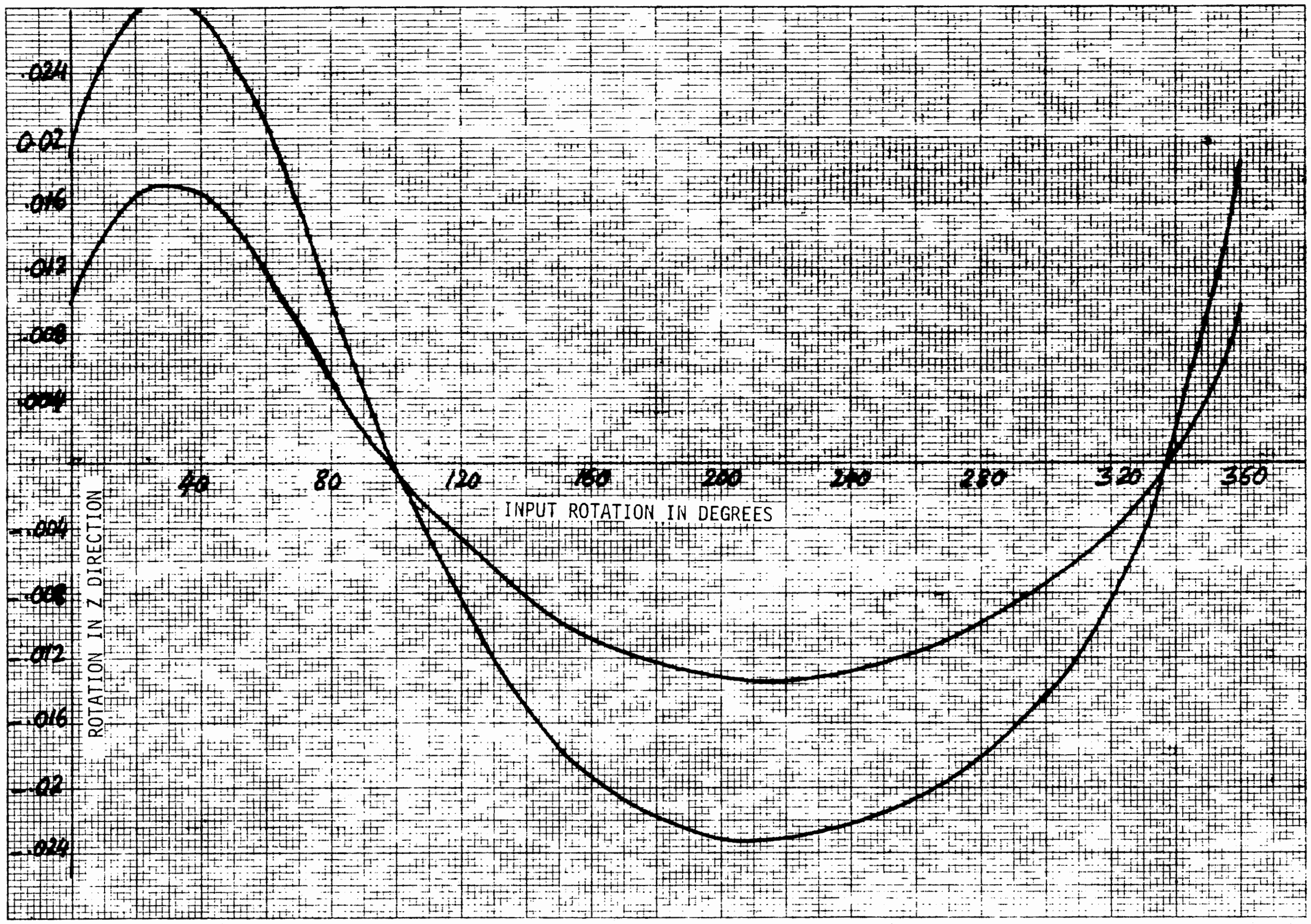
Input Link Rotation vs. Elastic Displacement of the Coupler Point in X-Direction

SOURCE MECHANISM - CASE III



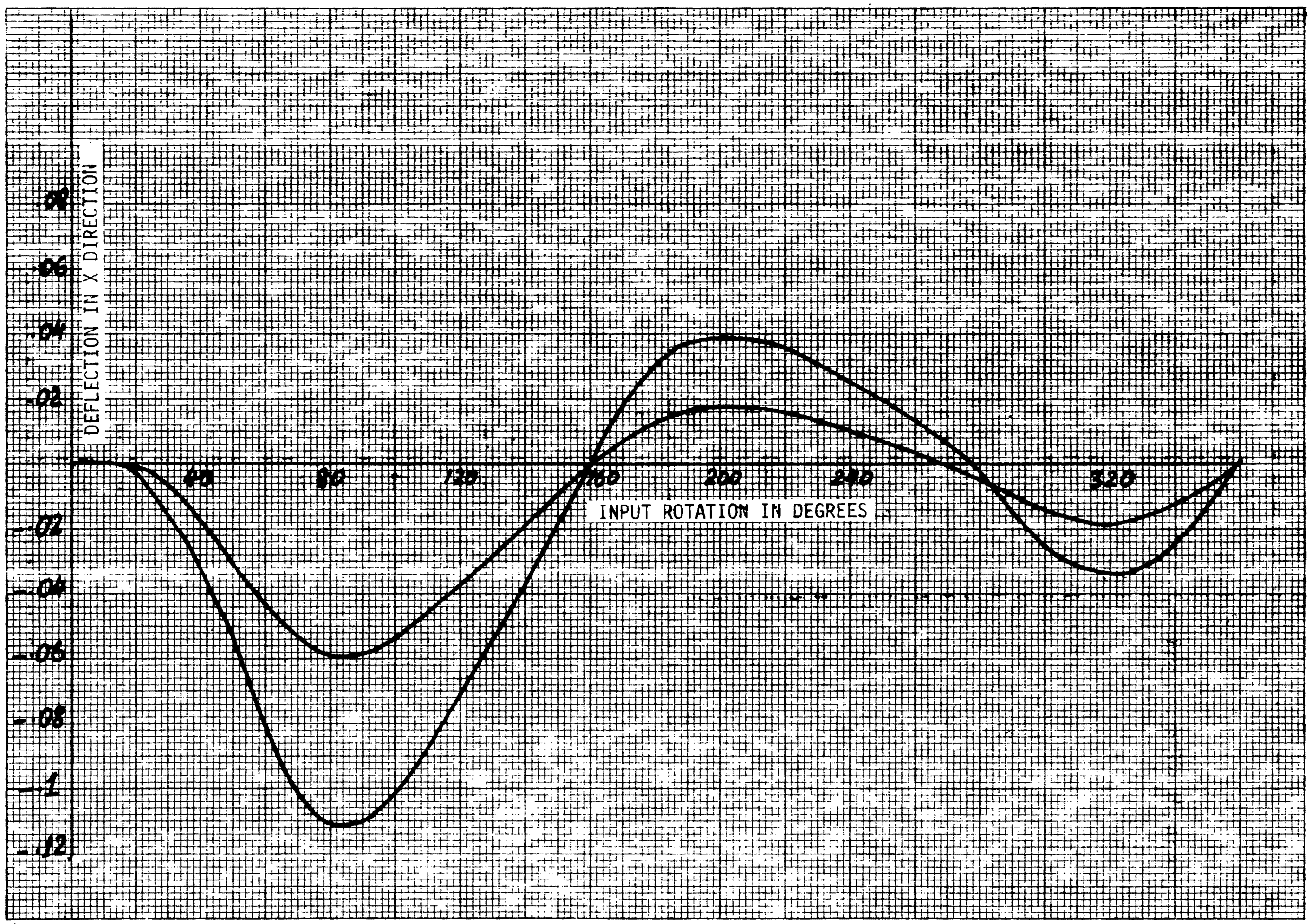
Input Link Rotation vs. Elastic Displacement of the Coupler Point in Y-Direction

SOURCE MECHANISM - CASE III



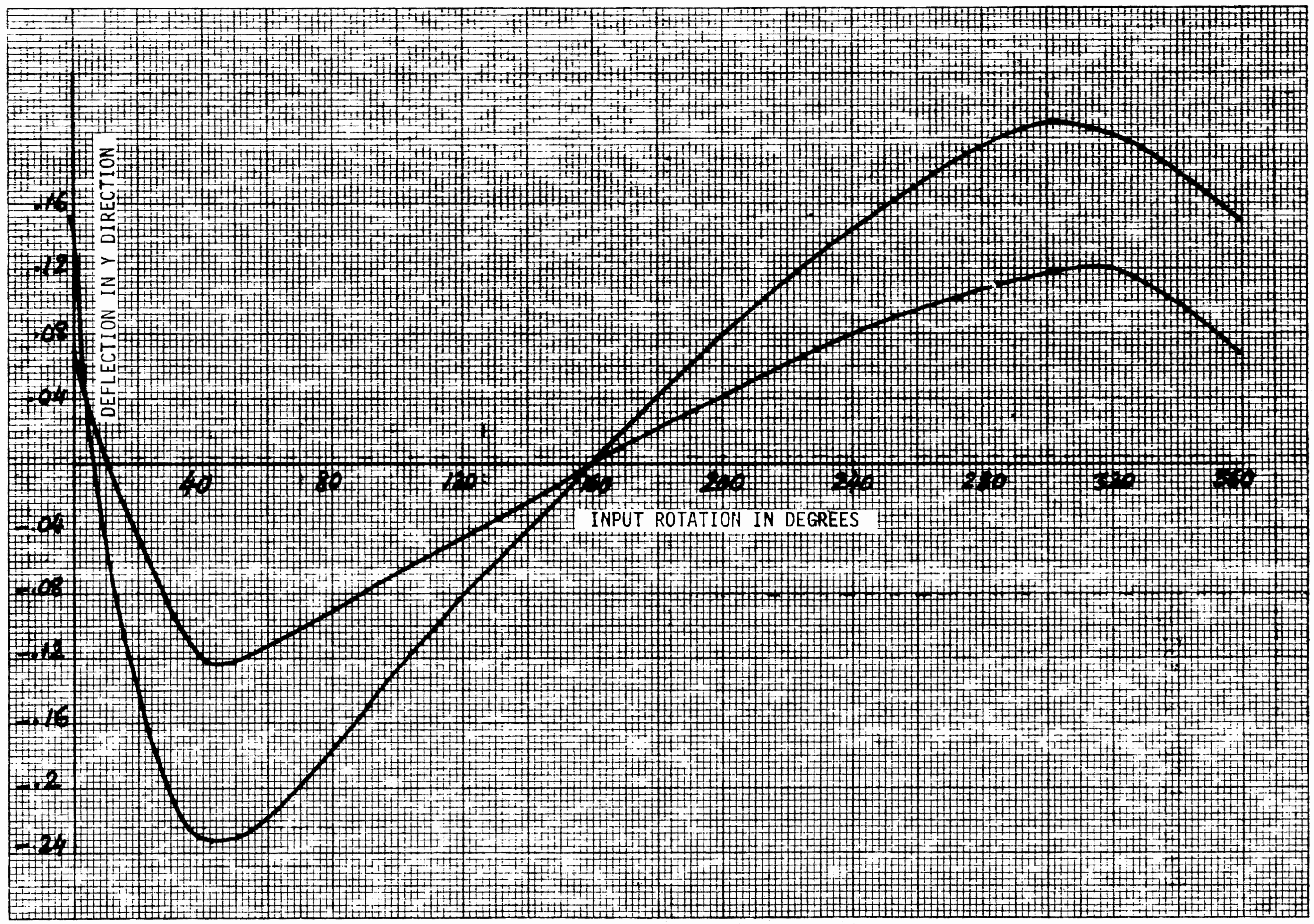
Input Link Rotation vs. Elastic Rotation in Z-Direction

COGNATE MECHANISM - CASE III



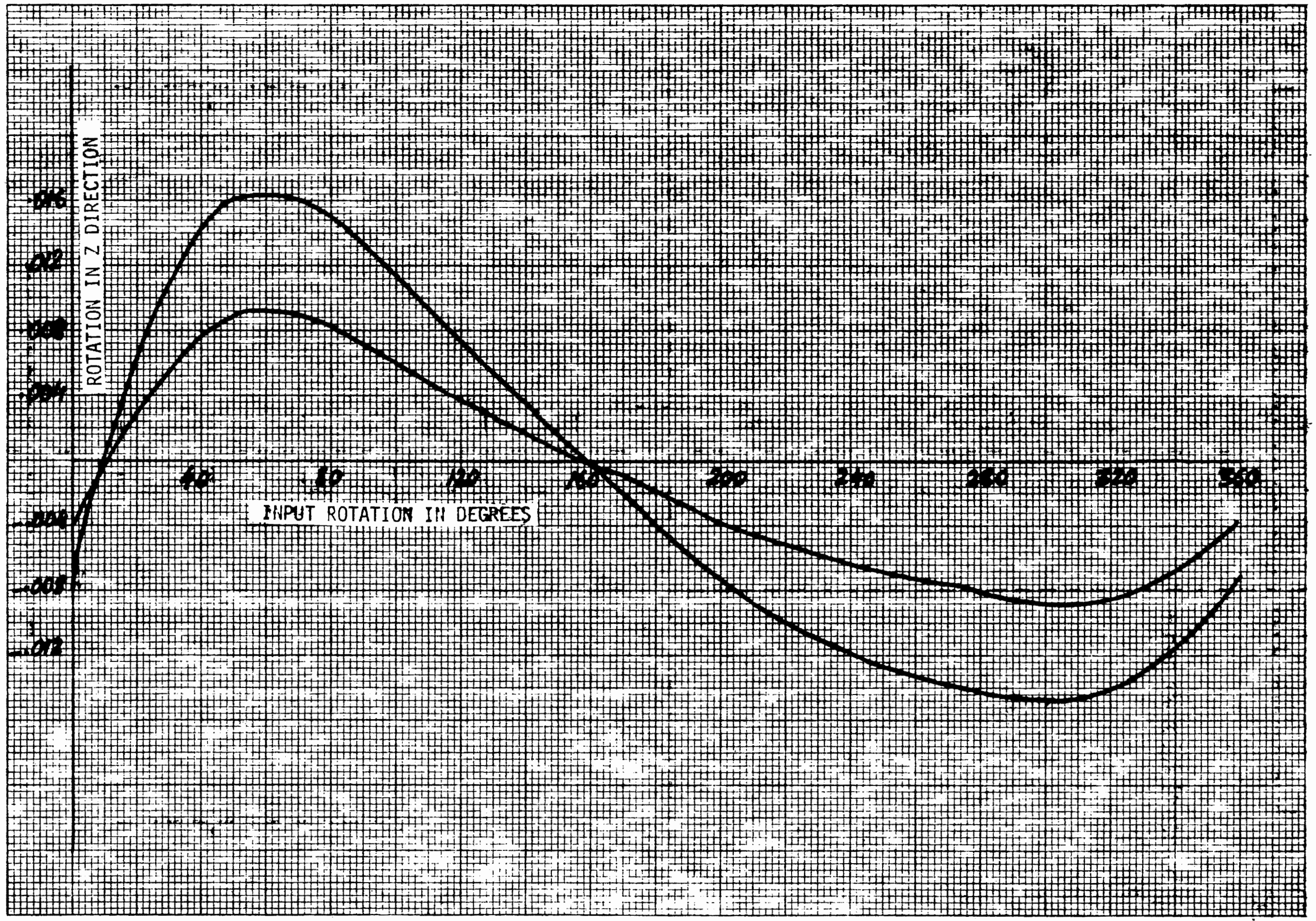
Input Link Rotation vs. Elastic Displacement of the Coupler Point in X-Direction

COGNATE MECHANISM - CASE III



Input Link Rotation vs. Elastic Displacement of the Coupler Point in Y-Direction

COGNATE MECHANISM - CASE III



Input Link Rotation vs. Elastic Rotation in Z-Direction

APPENDIX B

CASE I: LUMPED MASS AT THE PATH POINT

```
$JOB  TIME=15,NOSUBCHK
C     ****************************************************************
C     ****************************************************************
C     *                                                              *
C     ******** K.E.D. ANALYSIS OF THE SOURCE FOUR-BAR MECHANISM. ********
C     *                                                              *
C     *                            CASE I                            *
C     *                            ---- -                            *
C     *                                                              *
C     ****************************************************************
C     *                                                              *
C     *   THIS PROCEDURE COMPUTES THE DEFLECTIONS AT THE COUPLER POINT. *
C     *THE MASS OF THE FOUR-BAR IS ASSUMED TO BE NEGLIGIBLE COMPARED TO *
C     *THE INERTIAL MASS AT THE COUPLER POINT.                       *
C     * THE MASS 'M'=2 POUNDS IS LOCATED AT THE POINT 'P'.           *
C     *                                                              *
C     ****************************************************************
C     ****************************************************************
C     *                                                              *
C      --- THE FOLLOWING  ARE THE DIMENSIONS OF THE FOUR-BAR:----
C     *                                                              *
C     *L1---- THE INPUT LINK LENGTH OF THE SOURCE FOUR-BAR MECHANISM.  *
C     *L2---- THE COUPLER EXTENDER LENGTH ATTACHED RIGIDLY TO COUPLER.  *
C     *L3---- THE COUPLER LINK LENGTH OF THE SOURCE FOUR-BAR MECHANISM. *
C     *L4---- THE OUT PUT LINK LENGTH OF THE SOURCE FOUR-BAR MECHANISM. *
C     *L5---- THE FIXED LINK LENGTH OF THE SOURCE FOUR-BAR MECHANISM.  *
C     *                                                              *
C     *ALP---- THE RIGID ANGLE BETWEEN THE COUPLER AND EXTENDER.       *
C     *T2---- THE INPUT ANGLE IN RADIANS.                            *
C     *T3---- THE COUPLER ANGLE IN RADIANS.                          *
C     *T4---- THE OUT-PUT ANGLE IN RADIANS.                          *
C     *OM2---- THE ANGULAR VELOCITY OF THE INPUT LINK IN RAD/SEC.     *
C     *OM3---- THE ANGULAR VELOCITY OF THE COUPLER IN RADIANS/SEC.    *
C     *OM4---- THE ANGULAR VELOCITY OF THE OUTPUT LINK IN RADIANS/SEC.  *
C     *ALPH2---- THE ANGULAR ACC. OF INPUT IS CONSIDERED AS ZERO.     *
C     *ALPH3---- THE ANGULAR ACC. OF THE COUPLER LINK IN RAD/SEC/SEC.  *
C     *ALPH4---- THE ANGULAR ACC. OF THE OUTPUT LINK IN RAD/SEC/SEC.   *
C     *XPA---- THE HORIZANTAL COMPONENT OF ACC. OF THE PT."P".       *
C     *YPA---- THE VERTICAL COMPONENT OF THE ACC. OF PT."P".         *
C     *XPF---- THE HORIZANTAL COMP. OF THE FORCE AT THE PT."P".      *
C     *YPF---- THE VERTICAL COMP. OF THE FORCE AT THE PT."P".        *
C     *                                                              *
C     ****************************************************************
C     ****************************************************************
C     *                                                              *
C     *THE FOLLOWING DATA MUST BE SUPPLIED TO THE PROGRAM            *
C     *                                                              *
C     *1. THE LINK LENGTHS: L1,L2,L3,L4,L5                           *
C     *2. THE ANGULAR VELOCITY OF THE INPUT LINK "OM2"               *
C     *3. THE ANGULAR ACCELERATION OF THE INPUT LINK "ALPH2"         *
C     *4. THE CROSS-SECTIONAL AREA OD THE LINKS "CA"                 *
C     *5. THE CROSS-SECTIONAL MOMENT OF INERTIA "MI"                 *
C     *6. THE MODULUS OF ELASTICITY OF THE LINK MATARIAL "ME"        *
C     *                                                              *
C     *NOTE: THE LINK LENGTH L2 IS THE COUPLER EXTENDER              *
C     *----                                                          *
C     *NOTE: THE SUBROUTINES BMTRA&MPRD ARE TO BE EXTERNALY SUPPLIED.  *
C     *----                                                          *
C     *                                                              *
C     *..............................................................*
```

```
      DOUBLE PRECISION L1,L2,L3,L4,L5,K1,K2,K3,K4,K5,DCOS,DSIN,DATAN,
     *AA,BB,CC,DD,EE,FF,A1,B1,C1,D1,E1,F1,G1,H1,I1,P1,CX,SX,CTX,DSQRT,
     *T2,T3,T4,ALP,LA,MI,ME,OM2,OM3,OM4,AJ,BJ,CJ,DJ,EJ,FJ,ALPH2,ALPH3,
     *ALPH4,DCOT,PI,XP,YP,XX,YY,NXP,NYP
      DIMENSION A(64),B(64),R(64),S(37,2),Q(37,2)
      L1=10.0
      L2=10.0
      L3=30.0
      L4=25.0
      L5=30.0
      K1=L5/L1
      K2=L5/L4
      K3=(L1*L1-L3*L3+L4*L4+L5*L5)/(2.0*L1*L4)
      K4=L5/L3
      K5=(L4*L4-L5*L5-L1*L1-L3*L3)/(2.0*L1*L3)
      PI=3.142857143
C     N---- THE SPEED OF ROTATION OF THE INPUT LINK.
      N=400.0
      OM2=(2.0*PI*N)/60.0
      WRITE(6,100)
  100 FORMAT(1H1,9X,'DEGREES',13X,'X-DISP.OF P',3X,'Y-DISP.OF P',10X,'Z-
     *ROT.OF P')
      P2=-10.0
    5 P2=P2+10.0
      T2=(P2*2.0*PI)/(360.0)
      AA=DCOS(T2)+K3-K1-(K2*DCOS(T2))
      BB=-2.0*DSIN(T2)
      CC=K1+K3-(1.0+K2)*DCOS(T2)
      DD=(K4*DCOS(T2))+DCOS(T2)+K5-K1
      EE=-2.0*DSIN(T2)
      FF=(K4*DCOS(T2))-DCOS(T2)+K5+K1
      T3=2.0*(DATAN((-EE-DSQRT(EE*EE-4.0*DD*FF))/(2.0*DD)))
      T4=2.0*(DATAN((-BB-DSQRT(BB*BB-4.0*AA*CC))/(2.0*AA)))
      CX=DCOS(T4-T3)
      SX=DSIN(T4-T3)
      CTX=CX/SX
      ALP=1.762782545
      XP=(L1*DCOS(T2))+(L2*DCOS(ALP+T3))
      YP=(L1*DSIN(T2))+(L2*DSIN(ALP+T3))
      OM3=(L1*OM2*DSIN(T4-T3))/(L3*DSIN(T3-T4))
      OM4=(L1*OM2*DSIN(T2-T3))/(L4*DSIN(T4-T3))
      AJ=L4*DSIN(T4)
      BJ=L3*DSIN(T3)
      CJ=(L1*OM2*OM2*DCOS(T2))+(L3*OM3*OM3*DCOS(T3))-(L4*OM4*OM4*DCOS(T4
     *))
      DJ=L4*DCOS(T4)
      EJ=L3*DCOS(T3)
      FJ=(L4*OM4*OM4*DSIN(T4))-(L1*OM2*OM2*DSIN(T2))-(L3*OM3*OM3*DSIN(T3
     *))
      ALPH2=0.0
      ALPH3=(CJ*DJ-AJ*FJ)/(AJ*EJ-BJ*DJ)
      ALPH4=(CJ*EJ-BJ*FJ)/(AJ*EJ-BJ*DJ)
      XPA=-(L1*OM2*OM2*DCOS(T2))-(L1*ALPH2*DSIN(T2))-(L2*OM3*OM3*DCOS(
     *ALP+T3))-(L2*ALPH3*DSIN(ALP+T3))
      YPA=(L1*ALPH2*DCOS(T2))-(L1*OM2*OM2*DSIN(T2))+(L2*ALPH3*DCOS(ALP+
     *T3))-(L2*OM3*OM3*DSIN(ALP+T3))
      XPF=XPA*2.0/(12.0*32.178)
      YPF=YPA*2.0/(12.0*32.178)
      A1=DCOS(T2)
      B1=-DSIN(T2)
```

```
      C1=DCOS(T4)
      D1=DSIN(T2)
      E1=DCOS(T2)
      F1=DSIN(T4)
      G1=-(L2*DCOS(ALP+T3)*DCOS(T2))+(L2*DSIN(ALP+T3)*DCOS(T2))
      H1=-(L2*DCOS(ALP+T3)*DCOS(T2))-(L2*DSIN(ALP+T3)*DSIN(T2))
      I1=DSIN(T4)*((L2*DCOS(T3))-(L2*DCOS(ALP+T3)))+DCOS(T4)*((L2*DSIN(
     #ALP+T3))-(L3*DSIN(T3)))
      R1=(A1*(E1*I1-F1*H1))-(B1*(D1*I1-F1*G1))+(C1*(D1*H1-E1*G1))
C     THE FORCE TRANSFORMATION MATRIX.
      A(1)=DCOS(T3+ALP)
      A(2)=-DSIN(T3+ALP)
      A(3)=0.0
      A(4)=((F1*I1)-(F1*H1))/R1
      A(5)=((F1*G1)-(D1*I1))/R1
      A(6)=((D1*H1)-(E1*G1))/R1
      A(7)=CTX*DSIN(T3+ALP)*(L2/L3)
      A(8)=-(DSIN(T3+ALP))*L2
      A(9)=DSIN(T3+ALP)
      A(10)=DCOS(T3+ALP)
      A(11)=0.0
      A(12)=((C1*H1-B1*I1))/R1
      A(13)=((A1*I1)-(C1*G1))/R1
      A(14)=((B1*G1)-(A1*H1))/R1
      A(15)=-(DCOS(T3+ALP))*CTX*(L2/L3)
      A(16)=DCOS(T3+ALP)*L2
      A(17)=0.0
      A(18)=0.0
      A(19)=1.0
      A(20)=((B1*F1)-(C1*F1))/R1
      A(21)=((C1*D1)-(A1*F1))/R1
      A(22)=((A1*E1)-(B1*D1))/R1
      A(23)=-CTX/L3
      A(24)=1.0
      N=3
      M=3
C    *TRANSPOSING THE FORCE TRANSFORMATION MATRIX(BETA).
      CALL GMTRA(A,R,N,M)
C    *TRANSPOSED FORCE TRANSFER MATRIX IS MULTIPLIED BY FLEXIBILITY MAT*
C    *RIX AND THE RESULT IS STORED IN R.                                *
      DO 55 I=1,24
   55 A(I)=R(I)
C
C    *ALL LINKS ARE OF UNIFORM CIRCULAR CROSS-SECTION OF DIA.=0.5 IN.  *
C
C    *"CA"-CROSS-SECTIONAL AREA OF ALL LINKS.                          *
C
      CA=0.1963495408
C
C    *"ME"-YOUNGS MODULUS OF ELASTICITY FOR THE MATARIAL ALUMINIUM.    *
C
      ME=10000000.0
C
C    *"MI"-CROSS-SECTIONAL MOMENT OF INERTIA.                          *
C
      MI=0.003067961 5
C    *THE FLEXIBILITY MATRIX"(F)".                                     *
      B(1)=L2/(CA*ME)
      DO 10 I=2,9
   10 B(I)=0.0
```

```
      B(10)=(L2*L2*L2)/(3.0*ME*MI)
      B(11)=(L2*L2)/(2.0*ME*MI)
      DO 11 I=12,17
   11 B(I)=0.0
      B(18)=(L2*L2)/(2.0*ME*MI)
      B(19)=L2/(ME*MI)
      DO 12 I=20,27
   12 B(I)=0.0
      B(28)=L1/(CA*ME)
      DO 13 I=29,36
   13 B(I)=0.0
      B(37)=(L1*L1*L1)/(3.0*CA*ME)
      DO 14 I=38,45
   14 B(I)=0.0
      B(46)=L4/(CA*ME)
      DO 15 I=47,54
   15 B(I)=0.0
      B(55)=L3/(CA*ME)
      DO 16 I=56,63
   16 B(I)=0.0
      B(64)=L3/(3.0*ME*MI)
      N=3
      M=8
      MSA=0
      MSB=0
      L=8
      CALL MPRD(A,B,R,N,M,MSA,MSB,L)
C
C    *THE PRODUCT R IS MULTIPLIED BY THE FORCE TRANSFER MATRIX.          *
      DO 66 I=1,24
   66 A(I)=R(I)
C
C    *THE FORCE TRANSFER MATRIX
C
      B(1)=DCOS(T3+ALP)
      B(2)=-DSIN(T3+ALP)
      B(3)=0.0
      B(4)=((E1*I1)-(F1*H1))/R1
      B(5)=((F1*G1)-(D1*I1))/R1
      B(6)=((D1*H1)-(E1*G1))/R1
      B(7)=CTX*DSIN(T3+ALP)*(L2/L3)
      B(8)=-(DSIN(T3+ALP))*L2
      B(9)=DSIN(T3+ALP)
      B(10)=DCOS(T3+ALP)
      B(11)=0.0
      B(12)=((C1*H1)-(B1*I1))/R1
      B(13)=((A1*I1)-(C1*G1))/R1
      B(14)=((B1*G1)-(A1*H1))/R1
      B(15)=-(DCOS(T3+ALP))*CTX*(L2/L3)
      B(16)=DCOS(T3+ALP)*L2
      B(17)=0.0
      B(18)=0.0
      B(19)=1.0
      B(20)=((B1*F1)-(C1*E1))/R1
      B(21)-((C1*D1)-(A1*F1))/R1
      B(22)=((A1*E1)-(B1*D1))/R1
      B(23)=-CTX/L3
      B(24)=1.0
      N=3
      M=8
```

```
      MSA=0
      MSB=0
      L=5
      CALL MPRD(A,B,R,N,M,MSA,MSB,L)
      DO 77 I=1,9
   77 A(I)=R(I)
      B(1)=XPF
      B(2)=YPF
      B(3)=0.0
      N=3
      M=3
      MSA=0
      MSB=0
      L=1
      CALL MPRD(A,B,R,N,M,MSA,MSB,L)
      XX=R(1)
      YY=R(2)
      RT=R(3)
      IF(P2.EQ.360.0) GO TO 999
      WRITE(6,200)P2,XX,YY,RT
  200 FORMAT(1H0,5X,F12.6,8X,F12.6,8X,F12.6,8X,F12.6)
      GO TO 5
  999 STOP
      END

$ENTRY
```

```
$JOB  TIME=20,NOSUBCHK
C      ******************************************************************
C      ******************************************************************
C      *                                                                *
C      *                            CASE I                              *
C      *                            ---- -                              *
C      *                                                                *
C      ******************************************************************
C      *                     K.E.D. ANALYSIS OF                         *
C      *                     - - -  -------- --                         *
C      *       THE COGNATE MECHANISM OF THE SOURCE FOUR-BAR LINKAGE     *
C      *       --- ------- --------- -- --- ------ ---- --- -------     *
C      *                                                                *
C      ******************************************************************
C      ******************************************************************
C      *   THE LINK LENGTHS OF THE COGNATE MECHANISM IS COMPUTED USING  *
C      *THE PROPERTY FROM THE CALEY'S DIAGRAM AND DISPLAYED AS FOLLOWS:- *
C      *CL1---- THE INPUT-LINK OF THE COGNATE MECHANISM IN INCHES.      *
C      *CL2---- THE COUPLER EXTENDER LENGTH IN INCHES.                  *
C      *CL3---- THE COUPLER LINK LENGTH OF COGNATE IN INCHES.           *
C      *CL4---- THE FOLLOWER LENGTH OF THE COGNATE IN INCHES.           *
C      *CL5---- THE GROUND LINK LENGTH OF THE COGNATE IN INCHES.        *
C      *                                                                *
C      *NOTE:                                                           *
C      *-----                                                           *
C      *THE COGNATE MECHANISM IS A CROSSED FOUR-BAR LINKAGE.            *
C      *--- ------- --------- -- - ------- ---- --- -------             *
C      *                                                                *
C      *CALP---- THE RIGID ANGLE IN RAD. BETWEEN COUPLER AND EXTENDER.  *
C      *COM2---- THE ANGULAR VELOCITY OF THE INPUT-LINK RAD/SEC.        *
C      *COM3---- THE ANGULAR VELOCITY OF THE COUPLER LINK RAD/SEC.      *
C      *COM4---- THE ANGULAR VELOCITY OF THE FOLLOWER LINK              *
C      *CALPH2---- THE ANGULAR ACC. OF THE INPUT LINK IS ZERO.          *
C      *CALPH3---- THE ANGULAR ACC. OF THE COUPLER RAD/SEC/SEC.         *
C      *CALPH4---- THE ANGULAR ACC. OF THE FOLLOWER IN RAD/SEC/SEC.     *
C      *                                                                *
C      *   THE FOLLOWING INFORMATION IS REQUIRED FOR THE PROGRAM:       *
C      *   --- --------- ----------- -- -------- --- --- -------        *
C      *THE LINK LENGTHS OF THE SOURCE FOUR-BAR ( L1,L2,L3,L4,L5 ) AND  *
C      *THE RIGID ANGLE ALP ASSOCIATED WITH THE COUPLER AND EXTENDER OF *
C      *THE SOURCE LINKAGE AND THE INPUT LINK VELOCITY.                 *
C      *                                                                *
C      ******************************************************************
       DOUBLE PRECISION L1,CL1,L2,CL2,L3,CL3,L4,CL4,L5,CL5,K1,CK1,K2,CK2,
      *K3,CK3,K4,CK4,K5,CK5,AA,CAA,BB,CBB,CC,CCC,DD,CDD,EE,CEE,FF,CFF,T2,
      *T3,CT3,T4,CT4,OM2,COM2,OM3,COM3,OM4,COM4,AJ,CAJ,BJ,CBJ,CJ,CCJ,DJ,C
      *DJ,FJ,CEJ,FJ,CFJ,ALP,CALP,ALPH2,CALPH2,ALPH3,CALPH3,ALPH4,CALPH4,D
      *COS,DSIN,DATAN,DSQRT,DCOT,CX,CCX,SX,CSX,CTX,CCTX,A1,CA1,B1,CB1,C1,
      *CC1,D1,CD1,E1,CE1,F1,CF1,G1,CG1,H1,CH1,I1,CI1,R1,CR1,PI,CA,CCA,M1,
      *CM1,ME,CME,XP,CXP,YP,CYP,XX,CXX,YY,CYY,NXP,NCXP,NYP,NCYP
      *,XPP,YPP,BETA,Z1,Z2,K,CBTA,SBTA,TBTA
      *,DCXX,RCXX,TCPPX,TCPPY
      *,S
       DIMENSION A(64),B(64),R(64),U(64),F(64)
       L1=10.0
       L2=10.0
       L3=30.0
       L4=25.0
       L5=30.0
       K1=L5/L1
```

```
      K2=L5/L4
      K3=(L1*L1-L3*L3+L4*L4+L5*L5)/(2.0*L1*L4)
      K4=L5/L3
      K5=(L4*L4-L5*L5-L1*L1-L3*L3)/(2.0*L1*L3)
      PI=3.142857143
C
C     S---- THE SPEED OF ROTATION OF THE INPUT LINK =300 R.P.M.
C
      S=300.0
      OM2=(2.0*PI*S)/60.0
      WRITE(6,100)
  100 FORMAT(1H1,9X,'DEGREES',13X,'X-DISP.OF P',8X,'Y-DISP.OF P',10X,'Z-
     *ROT.OF P')
      P2=-10.0
    5 P2=P2+10.0
      T2=(P2*2.0*PI)/(360.0)
      AA=DCOS(T2)+K3-K1-(K2*DCOS(T2))
      BB=-2.0*DSIN(T2)
      CC=K1+K3-(1.0+K2)*DCOS(T2)
      DD=(K4*DCOS(T2))+DCOS(T2)+K5-K1
      EE=-2.0*DSIN(T2)
      FF=(K4*DCOS(T2))-DCOS(T2)+K5+K1
      T3=2.0*(DATAN((-EE-DSQRT(EE*EE-4.0*DD*FF))/(2.0*DD)))
      T4=2.0*(DATAN((-BB-DSQRT(BB*BB-4.0*AA*CC))/(2.0*AA)))
      ALP=PI/3.0
      XP=(L1*DCOS(T2))+(L2*DCOS(ALP+T3))
      YP=(L1*DSIN(T2))+(L2*DSIN(ALP+T3))
C    *THE COUPLER COGNATE DIMENSIONS ARE AS FOLLOWS:-                  *
C
      K=L2/L3
      CL4=DSQRT((L2*L2)+(L3*L3)-(2.0*L2*L3*DCOS(ALP)))
      CL3=L4*CL4/L3
      CL2=K*L4
      CL5=DSQRT(((K*L5)**2)+(L5*L5)-(2.0*L5*L5*K*DCOS(ALP)))
      CL1=L1*CL4/L3
      CBTA=((L3*L3)+(CL4*CL4)-(L2*L2))/(2.0*L3*CL4)
      SBTA=DSQRT(1.0-(CBTA)**2)
      TBTA=SBTA/CBTA
      BETA=DATAN(TBTA)
      CALP=PI-(BETA+ALP)
      CK1=CL5/CL1
      CK2=-CL5/CL4
      CK3=(CL3*CL3-CL1*CL1-CL4*CL4-CL5*CL5)/(2.0*CL1*CL4)
      CK4=-CL5/CL3
      CK5=(CL1*CL1+CL3*CL3+CL5*CL5-CL4*CL4)/(2.0*CL1*CL3)
      CAA=DCOS(T2)+CK3-CK1-(CK2*DCOS(T2))
      CBB=-2.0*DSIN(T2)
      CCC=CK1+CK3-(1.0+CK2)*DCOS(T2)
      CDD=(CK4*DCOS(T2))+DCOS(T2)+CK5-CK1
      CEE=-2.0*DSIN(T2)
      CFF=(CK4*DCOS(T2))-DCOS(T2)+CK5+CK1
      CT3=2.0*(DATAN((-CEE-DSQRT(CEE*CEE-4.0*CDD*CFF))/(2.0*CDD)))
      CT4=2.0*(DATAN((-CBB-DSQRT(CBB*CBB-4.0*CAA*CCC))/(2.0*CAA)))
      XPP=(CL1*DCOS(T2))+(CL2*DCOS(PI-(CALP-CT3)))
      YPP=(CL1*DSIN(T2))+(CL2*DSIN(PI-(CALP-CT3)))
C     COM2---- THE ANGULAR VELOCITY OF THE INPUT LINK OF THE COGNATE.
      COM2=(2.0*PI*S)/60.0
      CALPH2=0.0
      COM3=(CL1*COM2*DSIN(T2-CT4))/(CL3*DSIN(CT3-CT4))
      COM4=(CL1*COM2*DSIN(CT3-T2))/(CL4*DSIN(CT4-CT3))
```

```
62        CAJ=CL4*DSIN(CT4)
63        CBJ=CL3*DSIN(CT3)
64        CCJ=-(CL1*CALPH2*DSIN(T2))-(CL1*COM2*COM2*DCOS(T2))+(CL3*COM3*COM3
         **DCOS(CT3))-(CL4*COM4*COM4*DCOS(CT4))
65        CDJ=CL4*DCOS(CT4)
66        CEJ=CL3*DCOS(CT3)
67        CFJ=-(CL1*CALPH2*DCOS(T2))+(CL1*COM2*COM2*DSIN(T2))-(CL3*COM3*COM3
         **DSIN(CT3))+(CL4*COM4*COM4*DSIN(CT4))
68        CALPH3=(CCJ*CDJ-CAJ*CFJ)/(CAJ*CEJ-CBJ*CDJ)
69        CALPH4=(CCJ*CEJ-CBJ*CFJ)/(CAJ*CEJ-CBJ*CDJ)
70        CXPA=-(CL1*COM2*COM2*DCOS(T2))-(CL1*CALPH2*DSIN(T2))+(CL2*COM3*COM
         *3*DCOS(CALP-CT3))-(CL2*CALPH3*DSIN(CALP-CT3))
71        CYPA=-(CL1*COM2*COM2*DSIN(T2))+(CL1*CALPH2*DCOS(T2))-(CL2*COM3*COM
         *3*DSIN(CALP-CT3))-(CL2*CALPH3*DCOS(CALP-CT3))
72        CXPF=CXPA*2.0/(12.0*32.178)
73        CYPF=CYPA*2.0/(12.0*32.178)
74        CA1=-DCOS(T2)
75        CB1=DSIN(T2)
76        CC1=DCOS(CT4)
77        CD1=-DSIN(T2)
78        CE1=-DCOS(T2)
79        CF1=DSIN(CT4)
80        CG1=(CL2*DCOS(T2)*DSIN(CALP-CT3))+(CL2*DSIN(T2)*DCOS(CALP-CT3))
81        CH1=(CL2*DCOS(T2)*DCOS(CALP-CT3))-(CL2*DSIN(T2)*DSIN(CALP-CT3))
82        CI1=(CL2*DCOS(T2)*DSIN(CALP-CT3))+(CL3*DCOS(CT3)*DCOS(CT4))+(CL2*D
         *SIN(CT4)*DCOS(CALP-CT3))+(CL3*DSIN(CT4)*DSIN(CT3))
83        CR1=(CA1*((CE1*CI1)-(CF1*CH1)))-(CB1*((CD1*CI1)-(CF1*CG1)))+(CC1*(
         *(CD1*CH1)-(CE1*CG1)))
84        CCX=DCOS(CT3-CT4)
85        CSX=DSIN(CT3-CT4)
86        CCTX=CCX/CSX
      C
      C   THE FORCE-TRANSFORMATION MATRIX FOR THE COGNATE MECHANISM.
      C
87        U(1)=-DCOS(CALP-CT3)
88        U(2)=-DSIN(CALP-CT3)
89        U(3)=0.0
90        U(4)=((CE1*CI1)-(CF1*CH1))/CR1
91        U(5)=((CF1*CG1)-(CD1*CI1))/CR1
92        U(6)=((CD1*CH1)-(CE1*CG1))/CR1
93        U(7)=-(CL2*DSIN(CALP-CT3))*CCTX/CL3
94        U(8)=-CL2*DSIN(CALP-CT3)
95        U(9)=DSIN(CALP-CT3)
96        U(10)=-DCOS(CALP-CT3)
97        U(11)=0.0
98        U(12)=((CC1*CH1)-(CB1*CI1))/CR1
99        U(13)=((CA1*CI1)-(CC1*CG1))/CR1
100       U(14)=((CB1*CG1)-(CA1*CH1))/CR1
101       U(15)=-CL2*DCOS(CALP-CT3)*CCTX/CL3
102       U(16)=-CL2*DCOS(CALP-CT3)
103       U(17)=0.0
104       U(18)=0.0
105       U(19)=1.0
106       U(20)=((CB1*CF1)-(CC1*CE1))/CR1
107       U(21)=((CC1*CD1)-(CA1*CF1))/CR1
108       U(22)=((CA1*CE1)-(CB1*CD1))/CR1
109       U(23)=CCTX/CL3
110       U(24)=1.0
      C
      C   TRANSPOSING THE FORCE-TRANSFER MATRX
```

```
C     TRANSPOSING THE FORCE TRANSFER MATRIX (CBETA) OF THE COGNATE.
C
      N=8
      M=3
      CALL GMTRA(U,R,N,M)
C
C     THE TRANSPOSED FORCE-TRANSFER MATRIX IS MULTIPLIED BY THE
C     COGNATE FLEXIBILITY MATRIX .
C
      DO 88  I=1,24
   88 A(I)=R(I)
C
C    *ALL LINKS ARE OF UNIFORM CROSS-SECTION OF DIA.=0.5".            *
C
C    *''CCA''-CROSS-SECTIONAL AREA OF ALL LINKS.                      *
      CCA=0.1963495408
C
C    *''CME''-YOUNG'S MODULUS OF ELASTICITY FOR MATERIAL ALUMUNIUM.   *
      CME=10000000.0
C
C    *''CMI''-CROSS-SECTIONAL MOMENT OF INERTIA.                      *
      CMI=0.0030679615
C
C    THE FLEXIBILITY MATRIX FOR THE COGNATE MECHANISM ''CF''.
C
      F(1)=CL2/(CCA*CME)
      F(2)=0.0
      F(3)=F(2)
      F(4)=F(2)
      F(5)=F(2)
      F(6)=F(2)
      F(7)=F(2)
      F(8)=F(2)
      F(9)=F(2)
      F(10)=(CL2*CL2*CL2)/(3.0*CME*CMI)
      F(11)=(CL2*CL2)/(2.0*CME*CMI)
      F(12)=F(2)
      F(13)=F(2)
      F(14)=F(2)
      F(15)=F(2)
      F(16)=F(2)
      F(17)=F(2)
      F(18)=(CL2*CL2)/(2.0*CME*CMI)
      F(19)=CL2/(CME*CMI)
      F(20)=F(2)
      F(21)=F(2)
      F(22)=F(2)
      F(23)=F(2)
      F(24)=F(2)
      F(25)=F(2)
      F(26)=F(2)
      F(27)=F(2)
      F(28)=CL1/(CCA*CME)
      F(29)=F(2)
      F(30)=F(2)
      F(31)=F(2)
      F(32)=F(2)
      F(33)=F(2)
      F(34)=F(2)
      F(35)=F(2)
```

```
      F(36)=F(2)
      F(37)=(CL1*CL1*CL1)/(3.0*CCA*CME)
      F(38)=F(2)
      F(39)=F(2)
      F(40)=F(2)
      F(41)=F(2)
      F(42)=F(2)
      F(43)=F(2)
      F(44)=F(2)
      F(45)=F(2)
      F(46)=CL4/(CCA*CME)
      F(47)=F(2)
      F(48)=F(2)
      F(49)=F(2)
      F(50)=F(2)
      F(51)=F(2)
      F(52)=F(2)
      F(53)=F(2)
      F(54)=F(2)
      F(55)=CL3/(CCA*CME)
      F(56)=F(2)
      F(57)=F(2)
      F(58)=F(2)
      F(59)=F(2)
      F(60)=F(2)
      F(61)=F(2)
      F(62)=F(2)
      F(63)=F(2)
      F(64)=CL3/(3.0*CME*CMI)
      N=3
      M=8
      MSA=0
      MSB=0
      L=8
      CALL MPRD(A,F,R,N,M,MSA,MSB,L)
C
C     THE PRODUCT R IS MULTIPLIED BY THE FORCE-TRANSFER MATRIX.
C
      DO 99  I=1,24
   99 A(I)=R(I)
      U(1)=-DCOS(CALP-CT3)
      U(2)=-DSIN(CALP-CT3)
      U(3)=0.0
      U(4)=((CE1*CI1)-(CF1*CH1))/CR1
      U(5)=((CF1*CG1)-(CD1*CI1))/CR1
      U(6)=((CD1*CH1)-(CE1*CG1))/CR1
      U(7)=-(CL2*DSIN(CALP-CT3))*CCTX/CL3
      U(8)=-CL2*DSIN(CALP-CT3)
      U(9)=DSIN(CALP-CT3)
      U(10)=-DCOS(CALP-CT3)
      U(11)=0.0
      U(12)=((CC1*CH1)-(CB1*CI1))/CR1
      U(13)=((CA1*CI1)-(CC1*CG1))/CR1
      U(14)=((CB1*CG1)-(CA1*CH1))/CR1
      U(15)=-CL2*DCOS(CALP-CT3)*CCTX/CL3
      U(16)=-CL2*DCOS(CALP-CT3)
      U(17)=0.0
      U(18)=0.0
      U(19)=1.0
      U(20)=((CB1*CF1)-(CC1*CE1))/CR1
```

```
      U(21)=((CL1*CD1)-(CA1*CF1))/CR1
      U(22)=((CA1*CE1)-(CB1*CD1))/CR1
      U(23)=CCTX/CL3
      U(24)=1.0
      N=3
      M=8
      MSA=0
      MSB=0
      L=3
      CALL MPRD(A,U,R,N,M,MSA,MSB,L)
      DO 111 I=1,9
  111 A(I)=R(I)
C
C     THE PRODUCT IS MULTIPLIED BY THE EXTERNAL FORCE MATRIX ''P''.
C
      B(1)=CXPF
      B(2)=CYPF
      B(3)=0.0
      N=3
      M=3
      MSA=0
      MSB=0
      L=1
      CALL MPRD(A,B,R,N,M,MSA,MSB,L)
      CXX=R(1)
      CYY=R(2)
      CRT=R(3)
C
C                  THE FOLLOWING TRANSFORMATION LOCATES
C                  --- --------- -------------- -------
C
C                        THE COGNATE IN ITS TRUE
C                        --- ------- -- --- ----
C
C                                POSITION
C                                --------
C
      XDP=XPP-CL5
      XRP=(XDP*DCOS(-BETA))-(YPP*DSIN(-BETA))
      CXP=XRP+L5
      CYP=(XDP*DSIN(-BETA))+(YPP*DCOS(-BETA))
      NCXP=CXP+CXX
      NCYP=CYP+CYY
      NCXP=CXP+CXX
      NCYP=CYP+CYY
      WRITE(6,200)P2,CXX,CYY,CRT
  200 FORMAT(1H0,5X,F12.6,8X,F12.6,8X,F12.6,8X,F12.6)
      IF(P2.GT.360.0) GO TO 1
      GO TO 5
    1 STOP
      END

$ENTRY
```

APPENDIX C

CASE II: MASS AT EACH JOINT

```
$JOB TIME=55,NOSUBCHK,LIBLIST
C      ***********************************************************************
C      ***********************************************************************
C      *                                                                     *
C      *                              CASE  II                               *
C      *                              ----  --                               *
C      *                                                                     *
C      ***********************************************************************
C      *                                                                     *
C      *   EACH ELEMENT HAS A CONCENTRATED MASS LOCATED AT EACH JOINT.       *
C      *   ---- ------- --- - ------------ ---- ------- -- ---- -----        *
C      *                                                                     *
C      ***********************************************************************
C      ***********************************************************************
       DOUBLE PRECISION L1,L2,L3,L4,L5,K1,K2,K3,K4,K5,DCOS,DSIN,DATAN,
      *AA,BB,CC,DD,EE,FF,A1,B1,C1,D1,E1,F1,G1,H1,I1,R1,CX,SX,CTX,DSQRT,
      *T2,T3,T4,ALP,CA,MI,ME,OM2,OM3,OM4,AJ,BJ,CJ,DJ,EJ,FJ,ALPH2,ALPH3,
      *ALPH4,DCOT,PI,XP,YP,XX,YY,NXP,NYP
      *,DNIY,VOL1,VOL2,VOL3,VOL4,XAA,XBA,YAA,YBA,M1,M2,M3,M4
       DIMENSION A(64),B(64),R(64),S(37,2),Q(37,2)
       L1=10.0
       L2=10.0
       L3=30.0
       L4=25.0
       L5=30.0
       K1=L5/L1
       K2=L5/L4
       K3=(L1*L1-L3*L3+L4*L4+L5*L5)/(2.0*L1*L4)
       K4=L5/L3
       K5=(L4*L4-L5*L5-L1*L1-L3*L3)/(2.0*L1*L3)
       PI=3.142857143
C
C      SPEED ---- THE SPEED OF THE INPUT LINK IN R.P.M.
C
       SPEED=300.0
       OM2=(2.0*PI*SPEED)/60.0
       WRITE(6,100)
   100 FORMAT(1H1,9X,'DEGREES',13X,'X-DISP.OF P',3X,'Y-DISP.OF P',10X,'Z-
      *RJT.OF P')
       P2=-10.0
     5 P2=P2+10.0
       T2=(P2*2.0*PI)/(360.0)
       AA=DCOS(T2)+K3-K1-(K2*DCOS(T2))
       BB=-2.0*DSIN(T2)
       CC=K1+K3-(1.0+K2)*DCOS(T2)
       DD=(K4*DCOS(T2))+DCOS(T2)+K5-K1
       EE=-2.0*DSIN(T2)
       FF=(K4*DCOS(T2))-DCOS(T2)+K5+K1
       T3=2.0*(DATAN((-EE-DSQRT(EE*EE-4.0*DD*FF))/(2.0*DD)))
       T4=2.0*(DATAN((-BB-DSQRT(BB*BB-4.0*AA*CC))/(2.0*AA)))
       CX=DCOS(T4-T3)
       SX=DSIN(T4-T3)
       CTX=CX/SX
       ALP=PI/3.0
       XP=(L1*DCOS(T2))+(L2*DCOS(ALP+T3))
       YP=(L1*DSIN(T2))+(L2*DSIN(ALP+T3))
       OM3=(L1*OM2*DSIN(T4-T3))/(L3*DSIN(T3-T4))
       OM4=(L1*OM2*DSIN(T2-T3))/(L4*DSIN(T4-T3))
       AJ=L4*DSIN(T4)
       BJ=L3*DSIN(T3)
```

```
39        CJ=(L1*OM2*OM2*DCOS(T2))+(L3*OM3*OM3*DCOS(T3))-(L4*OM4*OM4*DCOS(T4
         *))
40        DJ=L4*DCOS(T4)
41        EJ=L3*DCOS(T3)
42        FJ=(L4*OM4*OM4*DSIN(T4))-(L1*OM2*OM2*DSIN(T2))-(L3*OM3*OM3*DSIN(T3
         *))
43        ALPH2=0.0
44        ALPH3=(CJ*DJ-AJ*FJ)/(AJ*EJ-BJ*DJ)
45        ALPH4=(CJ*FJ-BJ*FJ)/(AJ*EJ-BJ*DJ)
46        XPA=-(L1*OM2*OM2*DCOS(T2))-(L1*ALPH2*DSIN(T2))-(L2*OM3*OM3*DCOS(
         *ALP+T3))-(L2*ALPH3*DSIN(ALP+T3))
47        YPA=(L1*ALPH2*DCOS(T2))-(L1*OM2*OM2*DSIN(T2))+(L2*ALPH3*DCOS(ALP+
         *T3))-(L2*OM3*OM3*DSIN(ALP+T3))
48        XAA=-(L1*DSIN(T2)*ALPH2)-(L1*DCOS(T2)*OM2*OM2)
49        YAA=(L1*DCOS(T2)*ALPH2)-(L1*DSIN(T2)*OM2*OM2)
50        XBA=-(L1*ALPH2*DSIN(T2))-(L1*OM2*OM2*DCOS(T2))-(L3*OM3*OM3*DCOS(T3
         *))+(L3*ALPH3*DSIN(T3))
51        YBA=-(L1*OM2*OM2*DSIN(T2))+(L1*ALPH2*DCOS(T2))-(L3*OM3*OM3*DSIN(T3
         *))+(L3*ALPH3*DCOS(T3))
      C
      C   *ALL LINKS ARE OF UNIFORM CIRCULAR CROSS-SECTION OF DIA.=0.5 IN.   *
      C
      C   *"CA"-CROSS-SECTIONAL AREA OF ALL LINKS.                           *
      C
52        CA=0.1963495408
      C
      C   *"ME"-YOUNGS MODULUS OF ELASTICITY FOR THE MATARIAL ALUMINIUM.     *
      C
53        ME=10000000.0
      C
      C   *"MI"-CROSS-SECTIONAL MOMENT OF INERTIA.                           *
      C
54        MI=0.0030679615
      C
      C   DENSITY OF ALUMUNIUM = 0.098.
      C
55        DNTY=0.098
56        GC=32.178*12.0
57        VOL1=CA*L1
58        VOL2=CA*L2
59        VOL3=CA*L3
60        VOL4=CA*L4
61        M1=DNTY*VOL1
62        M2=DNTY*VOL2
63        M3=DNTY*VOL3
64        M4=DNTY*VOL4
65        P1=(M2*XPA)/(32.178*12.0)
66        Z2=(M2*YPA)/(32.178*12.0)
67        P3=0.0
68        P4=(M1*(XAA*DCOS(T2)+YAA*DSIN(T2)))/GC
69        P5=(M1*(YAA*DSIN(T2)-XAA*DCOS(T2)))/GC
70        P6=(M4*(YBA*DSIN(T4)-XBA*DCOS(T4)))/GC
71        P7=(M3*(YAA*DSIN(T3)-XAA*DCOS(T3)))/GC
72        P8=0.0
73        A1=DCOS(T2)
74        B1=-DSIN(T2)
75        C1=DCOS(T4)
76        D1=DSIN(T2)
77        E1=DCOS(T2)
78        F1=DSIN(T4)
```

```
      G1=-(L2*DCOS(ALP+T3)*DCOS(T2))+(L2*DSIN(ALP+T3)*DCOS(T2))
      H1=-(L2*DCOS(ALP+T3)*DCOS(T2))-(L2*DSIN(ALP+T3)*DSIN(T2))
      I1=DSIN(T4)*((L2*DCOS(T3))-(L2*DCOS(ALP+T3)))+DCOS(T4)*((L2*DSIN(
     #ALP+T3))-(L3*DSIN(T3)))
      R1=(A1*(E1*I1-F1*H1))-(B1*(D1*I1-F1*G1))+(C1*(D1*H1-E1*G1))
C     THE FORCE TRANSFORMATION MATRIX.
      A(1)=DCOS(T3+ALP)
      A(2)=-DSIN(T3+ALP)
      A(3)=0.0
      A(4)=((F1*I1)-(F1*H1))/R1
      A(5)-((F1*G1)-(D1*I1))/R1
      A(6)=((D1*H1)-(E1*G1))/R1
      A(7)=CTX*DSIN(T3+ALP)*(L2/L3)
      A(8)=-(DSIN(T3+ALP))*L2
      A(9)=DSIN(T3+ALP)
      A(10)=DCOS(T3+ALP)
      A(11)=0.0
      A(12)=((C1*H1-B1*I1))/R1
      A(13)=((A1*I1)-(C1*G1))/R1
      A(14)=((B1*G1)-(A1*H1))/R1
      A(15)=-(DCOS(T3+ALP))*CTX*(L2/L3)
      A(16)=DCOS(T3+ALP)*L2
      A(17)=0.0
      A(18)=0.0
      A(19)=1.0
      A(20)=((B1*F1)-(C1*E1))/R1
      A(21)=((C1*D1)-(A1*F1))/R1
      A(22)=((A1*E1)-(B1*D1))/R1
      A(23)=-CTX/L3
      A(24)=1.0
      N=8
      M=3
C     *TRANSPOSING THE FORCE TRANSFORMATION MATRIX(BETA).
      CALL GMTRA(A,R,N,M)
C     *TRANSPOSED FORCE TRANSFER MATRIX IS MULTIPLIED BY FLEXIBILITY MAT*
C     *RIX AND THE RESULT IS STORED IN R.                               *
      DO 55 I=1,24
   55 A(I)=R(I)
C     *THE FLEXIBILITY MATRIX"(F)".                                     *
      B(1)=L2/(CA*ME)
      B(2)=0.0
      B(3)=0.0
      B(4)=0.0
      B(5)=0.0
      B(6)=0.0
      B(7)=0.0
      B(8)-0.0
      B(9)=0.0
      B(10)=(L2*L2*L2)/(3.0*ME*MI)
      B(11)=(L2*L2)/(2.0*ME*MI)
      B(12)=0.0
      B(13)=0.0
      B(14)=0.0
      B(15)=0.0
      B(16)=0.0
      B(17)=0.0
      B(18)=(L2*L2)/(2.0*ME*MI)
      B(19)=L2/(ME*MI)
      B(20)=0.0
      B(21)=0.0
```

```
      B(22)=0.0
      B(23)=0.0
      B(24)=0.0
      B(25)=0.0
      B(26)=0.0
      B(27)=0.0
      B(28)=L1/(CA*ME)
      B(29)=0.0
      B(30)=0.0
      B(31)=0.0
      B(32)=0.0
      B(33)=0.0
      B(34)=0.0
      B(35)=0.0
      B(36)=0.0
      B(37)=(L1*L1*L1)/(3.0*CA*ME)
      B(38)=0.0
      B(39)=0.0
      B(40)=0.0
      B(41)=0.0
      B(42)=0.0
      B(43)=0.0
      B(44)=0.0
      B(45)=0.0
      B(46)=L4/(CA*ME)
      B(47)=0.0
      B(48)=0.0
      B(49)=0.0
      B(50)=0.0
      B(51)=0.0
      B(52)=0.0
      B(53)=0.0
      B(54)=0.0
      B(55)=L3/(CA*ME)
      B(56)=0.0
      B(57)=0.0
      B(58)=0.0
      B(59)=0.0
      B(60)=0.0
      B(61)=0.0
      B(62)=0.0
      B(63)=0.0
      B(64)=L3/(3.0*ME*MI)
      N=3
      M=3
      MSA=0
      MSB=0
      L=8
      CALL MPRD(A,B,R,N,M,MSA,MSB,L)
C
C     *THE PRODUCT R IS MULTIPLIED BY THE FORCE TRANSFER MATRIX.          *
      DO 66 I=1,24
   66 A(I)=R(I)
C
C     *THE FORCE TRANSFER MATRIX
C
      B(1)=DCOS(T3+ALP)
      B(2)=-DSIN(T3+ALP)
      B(3)=0.0
      B(4)=((F1*T1)-(F1*H1))/R1
```

```
      B(5)=((F1*G1)-(D1*I1))/R1
      B(6)=((D1*H1)-(E1*G1))/R1
      B(7)=CTX*DSIN(T3+ALP)*(L2/L3)
      B(8)=-(DSIN(T3+ALP))*L2
      B(9)=DSIN(T3+ALP)
      B(10)=DCOS(T3+ALP)
      B(11)=0.0
      B(12)=((C1*H1)-(B1*I1))/R1
      B(13)=((A1*I1)-(C1*G1))/R1
      B(14)=((B1*G1)-(A1*H1))/R1
      B(15)=-(DCOS(T3+ALP))*CTX*(L2/L3)
      B(16)=DCOS(T3+ALP)*L2
      B(17)=0.0
      B(18)=0.0
      B(19)=1.0
      B(20)=((B1*F1)-(C1*E1))/R1
      B(21)=((C1*D1)-(A1*F1))/R1
      B(22)=((A1*E1)-(B1*D1))/R1
      B(23)=-CTX/L3
      B(24)=1.0
      B(25)=B(3)
      B(26)=B(3)
      B(27)=B(3)
      B(28)=1.0
      B(29)=B(3)
      B(30)=B(3)
      B(31)=B(3)
      B(32)=B(3)
      B(33)=B(3)
      B(34)=B(3)
      B(35)=B(3)
      B(36)=B(3)
      B(37)=1.0
      B(38)=B(3)
      B(39)=B(3)
      B(40)=B(3)
      B(41)=B(3)
      B(42)=B(3)
      B(43)=B(3)
      B(44)=B(3)
      B(45)=B(3)
      B(46)=1.0
      B(47)=B(3)
      B(48)=B(3)
      B(49)=B(3)
      B(50)=B(3)
      B(51)=B(3)
      B(52)=-DCOS(T2-T3)
      B(53)=DSIN(T2-T3)
      B(54)=B(3)
      B(55)=1.0
      B(56)=B(3)
      B(57)=B(3)
      B(58)=B(3)
      B(59)=B(3)
      B(60)=B(3)
      B(61)=B(3)
      B(62)=B(3)
      B(63)=B(3)
      B(64)=1.0
```

```
      N=3
      M=8
      MSA=0
      MSB=0
      L=8
      CALL MPRD(A,B,R,N,M,MSA,MSB,L)
      DO 77 I=1,24
   77 A(I)=R(I)
C
C     THE SYSTEM FORCE MATRIX FOR THE SOURCE MECHANISM.
C
      B(1)=P1
      B(2)=Z2
      B(3)=P3
      B(4)=P4
      B(5)=P5
      B(6)=P6
      B(7)=P7
      B(8)=P8
      N=3
      M=8
      MSA=0
      MSB=0
      L=1
      CALL MPRD(A,B,R,N,M,MSA,MSB,L)
      XX=R(1)
      YY=R(2)
      RT=R(3)
      WRITE(6,200)P2,XX,YY,RT
  200 FORMAT(1H0,5X,F12.6,8X,F12.6,8X,F12.6,8X,F12.6)
      IF(P2.LT.360.0) GO TO 6
      GO TO 5
    6 STOP
      END
```

```
$ENTRY
C
C     ...............................................................
C
C        SUBROUTINE GMTRA
C
C        PURPOSE
C           TRANSPOSE A GENERAL MATRIX
C
C        USAGE
C           CALL GMTRA(A,R,N,M)
C
C        DESCRIPTION OF PARAMETERS
C           A - NAME OF MATRIX TO BE TRANSPOSED
C           R - NAME OF RESULTANT MATRIX
C           N - NUMBER OF ROWS OF A AND COLUMNS OF R
C           M - NUMBER OF COLUMNS OF A AND ROWS OF R
C
C        REMARKS
C           MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A
C           MATRICES A AND R MUST BE STORED AS GENERAL MATRICES
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
```

```
C        METHOD                                                         GMTR 250
C           TRANSPOSE N BY M MATRIX A TO FORM M BY N MATRIX R           GMTR 260
C                                                                       GMTR 270
C     ..................................................................GMTR 280
C                                                                       GMTR 290
      SUBROUTINE GMTRA(A,R,N,M)                                         GMTR 300
      DIMENSION A(1),R(1)                                               GMTR 310
C                                                                       GMTR 320
      IR=0                                                              GMTR 330
      DO 10 I=1,N                                                       GMTR 340
      IJ=I-N                                                            GMTR 350
      DO 10 J=1,M                                                       GMTR 360
      IJ=IJ+N                                                           GMTR 370
      IR=IR+1                                                           GMTR 380
   10 R(IR)=A(IJ)                                                       GMTR 390
      RETURN                                                            GMTR 400
      END                                                               GMTR 410

C                                                                       MPRD  10
C     ..................................................................MPRD  20
C                                                                       MPRD  30
C        SUBROUTINE MPRD                                                MPRD  40
C                                                                       MPRD  50
C        PURPOSE                                                        MPRD  60
C           MULTIPLY TWO MATRICES TO FORM A RESULTANT MATRIX            MPRD  70
C                                                                       MPRD  80
C        USAGE                                                          MPRD  90
C           CALL MPRD(A,B,R,N,M,MSA,MSB,L)                              MPRD 100
C                                                                       MPRD 110
C        DESCRIPTION OF PARAMETERS                                      MPRD 120
C           A - NAME OF FIRST INPUT MATRIX                              MPRD 130
C           B - NAME OF SECOND INPUT MATRIX                             MPRD 140
C           R - NAME OF OUTPUT MATRIX                                   MPRD 150
C           N - NUMBER OF ROWS IN A AND R                               MPRD 160
C           M - NUMBER OF COLUMNS IN A AND ROWS IN B                    MPRD 170
C           MSA - ONE DIGIT NUMBER FOR STORAGE MODE OF MATRIX A         MPRD 180
C                  0 - GENERAL                                          MPRD 190
C                  1 - SYMMETRIC                                        MPRD 200
C                  2 - DIAGONAL                                         MPRD 210
C           MSB - SAME AS MSA EXCEPT FOR MATRIX B                       MPRD 220
C           L - NUMBER OF COLUMNS IN B AND R                            MPRD 230
C                                                                       MPRD 240
C        REMARKS                                                        MPRD 250
C           MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRICES A OR B  MPRD 260
C           NUMBER OF COLUMNS OF MATRIX A MUST BE EQUAL TO NUMBER OF ROWMPRD 270
C           OF MATRIX B                                                 MPRD 280
C                                                                       MPRD 290
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  MPRD 300
C           LOC                                                         MPRD 310
C                                                                       MPRD 320
C        METHOD                                                         MPRD 330
C           THE M BY L MATRIX B IS PREMULTIPLIED BY THE N BY M MATRIX A MPRD 340
C           AND THE RESULT IS STORED IN THE N BY L MATRIX R. THIS IS A  MPRD 350
C           ROW INTO COLUMN PRODUCT.                                    MPRD 360
C           THE FOLLOWING TABLE SHOWS THE STORAGE MODE OF THE OUTPUT    MPRD 370
C           MATRIX FOR ALL COMBINATIONS OF INPUT MATRICES               MPRD 380
C                         A                B                R           MPRD 390
C                      GENERAL          GENERAL          GENERAL        MPRD 400
C                      GENERAL          SYMMETRIC        GENERAL        MPRD 410
C                      GENERAL          DIAGONAL         GENERAL        MPRD 420
```

```
C                                 SYMMETRIC          GENERAL            GENERAL          MPRD 430
C                                 SYMMETRIC          SYMMETRIC          GENERAL          MPRD 440
C                                 SYMMETRIC          DIAGONAL           GENERAL          MPRD 450
C                                 DIAGONAL           GENERAL            GENERAL          MPRD 460
C                                 DIAGONAL           SYMMETRIC          GENERAL          MPRD 470
C                                 DIAGONAL           DIAGONAL           DIAGONAL         MPRD 480
C                                                                                        MPRD 490
C        ...............................................................................MPRD 500
C                                                                                        MPRD 510
      SUBROUTINE MPRD(A,B,R,N,M,MSA,MSB,L)                                               MPRD 520
      DIMENSION A(1),B(1),R(1)                                                           MPRD 530
C                                                                                        MPRD 540
C        SPECIAL CASE FOR DIAGONAL BY DIAGONAL                                           MPRD 550
C                                                                                        MPRD 560
      MS=MSA*10+MSB                                                                      MPRD 570
      IF(MS-22) 30,10,30                                                                 MPRD 580
   10 DO 20 I=1,N                                                                        MPRD 590
   20 R(I)=A(I)*B(I)                                                                     MPRD 600
      RETURN                                                                             MPRD 610
C                                                                                        MPRD 620
C        ALL OTHER CASES                                                                 MPRD 630
C                                                                                        MPRD 640
   30 IR=1                                                                               MPRD 650
      DO 90 K=1,L                                                                        MPRD 660
      DO 90 J=1,N                                                                        MPRD 670
      R(IR)=0                                                                            MPRD 680
      DO 80 I=1,M                                                                        MPRD 690
      IF(MS) 40,60,40                                                                    MPRD 700
   40 CALL LOC(J,I,IA,N,M,MSA)                                                           MPRD 710
      CALL LOC(I,K,IB,M,L,MSB)                                                           MPRD 720
      IF(IA) 50,80,50                                                                    MPRD 730
   50 IF(IB) 70,80,70                                                                    MPRD 740
   60 IA=N*(I-1)+J                                                                       MPRD 750
      IB=M*(K-1)+I                                                                       MPRD 760
   70 R(IR)=R(IR)+A(IA)*B(IB)                                                            MPRD 770
   80 CONTINUE                                                                           MPRD 780
   90 IR=IR+1                                                                            MPRD 790
      RETURN                                                                             MPRD 800
      END                                                                                MPRD 810

C                                                                                        LOC   10
C        ...............................................................................LOC   20
C                                                                                        LOC   30
C        SUBROUTINE LOC                                                                  LOC   40
C                                                                                        LOC   50
C        PURPOSE                                                                         LOC   60
C           COMPUTE A VECTOR SUBSCRIPT FOR AN ELEMENT IN A MATRIX OF                     LOC   70
C           SPECIFIED STORAGE MODE                                                       LOC   80
C                                                                                        LOC   90
C        USAGE                                                                           LOC  100
C           CALL LOC (I,J,IR,N,M,MS)                                                     LOC  110
C                                                                                        LOC  120
C        DESCRIPTION OF PARAMETERS                                                       LOC  130
C           I    - ROW NUMBER OF ELEMENT                                                 LOC  140
C           J    - COLUMN NUMBER  OF ELEMENT                                             LOC  150
C           IR   - RESULTANT VECTOR SUBSCRIPT                                            LOC  160
C           N    - NUMBER OF ROWS IN MATRIX                                              LOC  170
C           M    - NUMBER OF COLUMNS IN MATRIX                                           LOC  180
C           MS   - ONE DIGIT NUMBER FOR STORAGE MODE OF MATRIX                           LOC  190
C                   0 - GENERAL                                                          LOC  200
```

```
C                     1 - SYMMETRIC                                         LOC  210
C                     2 - DIAGONAL                                          LOC  220
C                                                                           LOC  230
C        REMARKS                                                            LOC  240
C           NONE                                                            LOC  250
C                                                                           LOC  260
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                      LOC  270
C           NONE                                                            LOC  280
C                                                                           LOC  290
C        METHOD                                                             LOC  300
C           MS=0   SUBSCRIPT IS COMPUTED FOR A MATRIX WITH N*M ELEMENTS     LOC  310
C                  IN STORAGE (GENERAL MATRIX)                              LOC  320
C           MS=1   SUBSCRIPT IS COMPUTED FOR A MATRIX WITH N*(N+1)/2 IN     LOC  330
C                  STORAGE (UPPER TRIANGLE OF SYMMETRIC MATRIX). IF         LOC  340
C                  ELEMENT IS IN LOWER TRIANGULAR PORTION, SUBSCRIPT IS     LOC  350
C                  CORRESPONDING ELEMENT IN UPPER TRIANGLE.                 LOC  360
C           MS=2   SUBSCRIPT IS COMPUTED FOR A MATRIX WITH N ELEMENTS       LOC  370
C                  IN STORAGE (DIAGONAL ELEMENTS OF DIAGONAL MATRIX).       LOC  380
C                  IF ELEMENT IS NOT ON DIAGONAL (AND THEREFORE NOT IN      LOC  390
C                  STORAGE), IR IS SET TO ZERO.                             LOC  400
C                                                                           LOC  410
C     ..................................................................LOC  420
C                                                                           LOC  430
      SUBROUTINE LOC(I,J,IR,N,M,MS)                                         LOC  440
C                                                                           LOC  450
      IX=I                                                                  LOC  460
      JX=J                                                                  LOC  470
      IF(MS-1) 10,20,30                                                     LOC  480
   10 IRX=N*(JX-1)+IX                                                       LOC  490
      GO TO 36                                                              LOC  500
   20 IF(IX-JX) 22,24,24                                                    LOC  510
   22 IRX=IX+(JX*JX-JX)/2                                                   LOC  520
      GO TO 36                                                              LOC  530
   24 IRX=JX+(IX*IX-IX)/2                                                   LOC  540
      GO TO 36                                                              LOC  550
   30 IRX=0                                                                 LOC  560
      IF(IX-JX) 36,32,36                                                    LOC  570
   32 IRX=IX                                                                LOC  580
   36 IR=IRX                                                                LOC  590
      RETURN                                                                LOC  600
      END                                                                   LOC  610
```

```
$JOB TIME=55,NOSUBCHK,LIBLIST
C      ******************************************************************
C      *                                                                *
C      *                                                                *
C      *                          CASE II                               *
C      *                          ---- --                               *
C      *                                                                *
C      *                                                                *
C      **********************COUPLER- COGNATE MECHANISM*****************
C      *                                                                *
C      *                             OF                                 *
C      *                                                                *
C      *                   FOUR-LINK MECHANISM                          *
C      *
C      ******************************************************************
       DOUBLE PRECISION L1,CL1,L2,CL2,L3,CL3,L4,CL4,L5,CL5,K1,CK1,K2,CK2,
      *K3,CK3,K4,CK4,K5,CK5,AA,CAA,BB,CBB,CC,CCC,DD,CDD,EE,CEE,FF,CFF,T2,
      *T3,CT3,T4,CT4,OM2,COM2,OM3,COM3,OM4,COM4,AJ,CAJ,BJ,CBJ,CJ,CCJ,DJ,C
      *DJ,EJ,CEJ,FJ,CFJ,ALP,CALP,ALPH2,CALPH2,ALPH3,CALPH3,ALPH4,CALPH4,D
      *COS,DSIN,DATAN,DSQRT,DCOT,CX,CCX,SX,CSX,CTX,CCTX,A1,CA1,B1,CB1,C1,
      *CC1,D1,CD1,E1,CE1,F1,CF1,G1,CG1,H1,CH1,I1,CI1,R1,CR1,PI,CA,CCA,M1,
      *CM1,ME,CME,XP,CXP,YP,CYP,XX,CXX,YY,CYY,NXP,NCXP,NYP,NCYP
      *,XPP,YPP,BETA,Z1,Z2,K,CBTA,SBTA,TBTA
      *,DCXX,RCXX,TCPPX,TCPPY
       DIMENSION A(64),B(64),R(64),U(64),F(64)
       L1=10.0
       L2=10.0
       L3=30.0
       L4=25.0
       L5=30.0
       K1=L5/L1
       K2=L5/L4
       K3=(L1*L1-L3*L3+L4*L4+L5*L5)/(2.0*L1*L4)
       K4=L5/L3
       K5=(L4*L4-L5*L5-L1*L1-L3*L3)/(2.0*L1*L3)
       WRITE(6,100)
   100 FORMAT(1H1,9X,'DEGREES',13X,'X-DISP.OF P',8X,'Y-DISP.OF P',10X,'Z-
      *ROT.OF P')
       PI=3.142857143
C
C      S---- THE SPEED OF ROTATION OF THE INPUT LINK =300 R.P.M.
C
       S=300.0
       OM2=(2.0*PI*S)/60.0
       P2=-10.0
     5 P2=P2+10.0
       T2=(P2*2.0*PI)/(360.0)
       AA=DCOS(T2)+K3-K1-(K2*DCOS(T2))
       BB=-2.0*DSIN(T2)
       CC=K1+K3-(1.0+K2)*DCOS(T2)
       DD=(K4*DCOS(T2))+DCOS(T2)+K5-K1
       EE=-2.0*DSIN(T2)
       FF=(K4*DCOS(T2))-DCOS(T2)+K5+K1
       T3=2.0*(DATAN((-EE-DSQRT(EE*EE-4.0*DD*FF))/(2.0*DD)))
       T4=2.0*(DATAN((-BB-DSQRT(BB*BB-4.0*AA*CC))/(2.0*AA)))
       ALP=PI/3.0
       XP=(L1*DCOS(T2))+(L2*DCOS(ALP+T3))
       YP=(L1*DSIN(T2))+(L2*DSIN(ALP+T3))
C      *THE COUPLER COGNATE DIMENSIONS ARE AS FOLLOWS:-                  *
C
```

```
      K=L2/L3
      CL4=DSQRT((L2*L2)+(L3*L3)-(2.0*L2*L3*DCOS(ALP)))
      CL3=L4*CL4/L3
      CL2=K*L4
      CL5=DSQRT(((K*L5)**2)+(L5*L5)-(2.0*L5*L5*K*DCOS(ALP)))
      CL1=L1*CL4/L3
      CBTA=((L3*L3)+(CL4*CL4)-(L2*L2))/(2.0*L3*CL4)
      SBTA=DSQRT(1.0-(CBTA)**2)
      TBTA=SBTA/CBTA
      BETA=DATAN(TBTA)
      CALP=PI-(BETA+ALP)
      CK1=CL5/CL1
      CK2=-CL5/CL4
      CK3=(CL3*CL3-CL1*CL1-CL4*CL4-CL5*CL5)/(2.0*CL1*CL4)
      CK4=-CL5/CL3
      CK5=(CL1*CL1+CL3*CL3+CL5*CL5-CL4*CL4)/(2.0*CL1*CL3)
      CAA=DCOS(T2)+CK3-CK1-(CK2*DCOS(T2))
      CBB=-2.0*DSIN(T2)
      CCC=CK1+CK3-(1.0+CK2)*DCOS(T2)
      CDD=(CK4*DCOS(T2))+DCOS(T2)+CK5-CK1
      CEE=-2.0*DSIN(T2)
      CFF=(CK4*DCOS(T2))-DCOS(T2)+CK5+CK1
      CT3=2.0*(DATAN((-CEE-DSQRT(CEE*CEE-4.0*CDD*CFF))/(2.0*CDD)))
      CT4=2.0*(DATAN((-CBB-DSQRT(CBB*CBB-4.0*CAA*CCC))/(2.0*CAA)))
      XPP=(CL1*DCOS(T2))+(CL2*DCOS(PI-(CALP-CT3)))
      YPP=(CL1*DSIN(T2))+(CL2*DSIN(PI-(CALP-CT3)))
      COM2=(2.0*PI*S)/60.0
      CALPH2=0.0
      COM3=(CL1*COM2*DSIN(T2-CT4))/(CL3*DSIN(CT3-CT4))
      COM4=(CL1*COM2*DSIN(CT3-T2))/(CL4*DSIN(CT4-CT3))
      CAJ=CL4*DSIN(CT4)
      CBJ=CL3*DSIN(CT3)
      CCJ=-(CL1*CALPH2*DSIN(T2))-(CL1*COM2*COM2*DCOS(T2))+(CL3*COM3*COM3
     **DCOS(CT3))-(CL4*COM4*COM4*DCOS(CT4))
      CDJ=CL4*DCOS(CT4)
      CEJ=CL3*DCOS(CT3)
      CFJ=-(CL1*CALPH2*DCOS(T2))+(CL1*COM2*COM2*DSIN(T2))-(CL3*COM3*COM3
     **DSIN(CT3))+(CL4*COM4*COM4*DSIN(CT4))
      CALPH3=(CCJ*CDJ-CAJ*CFJ)/(CAJ*CEJ-CBJ*CDJ)
      CALPH4=(CCJ*CEJ-CBJ*CFJ)/(CAJ*CEJ-CBJ*CDJ)
      CXPA=-(CL1*COM2*COM2*DCOS(T2))-(CL1*CALPH2*DSIN(T2))+(CL2*COM3*COM
     *3*DCOS(CALP-CT3))-(CL2*CALPH3*DSIN(CALP-CT3))
      CYPA=-(CL1*COM2*COM2*DSIN(T2))+(CL1*CALPH2*DCOS(T2))-(CL2*COM3*COM
     *3*DSIN(CALP-CT3))-(CL2*CALPH3*DCOS(CALP-CT3))
      CXPF=CXPA*2.0/(12.0*32.178)
      CYPF=CYPA*2.0/(12.0*32.178)
      CA1=-DCOS(T2)
      CB1=DSIN(T2)
      CC1=DCOS(CT4)
      CD1=-DSIN(T2)
      CE1=-DCOS(T2)
      CF1=DSIN(CT4)
      CG1=(CL2*DCOS(T2)*DSIN(CALP-CT3))+(CL2*DSIN(T2)*DCOS(CALP-CT3))
      CH1=(CL2*DCOS(T2)*DCOS(CALP-CT3))-(CL2*DSIN(T2)*DSIN(CALP-CT3))
      CI1=(CL2*DCOS(T2)*DSIN(CALP-CT3))+(CL3*DCOS(CT3)*DCOS(CT4))+(CL2*D
     *SIN(CT4)*DCOS(CALP-CT3))+(CL3*DSIN(CT4)*DSIN(CT3))
      CR1=(CA1*((CE1*CI1)-(CF1*CH1)))-(CB1*((CD1*CI1)-(CF1*CG1)))+(CC1*(
     *(CD1*CH1)-(CE1*CG1)))
      CCX=DCOS(CT3-CT4)
      CSX=DSIN(CT3-CT4)
```

```
      CCTX=CCX/CSX
C
C
C     THE FORCE TRANSFORMATION MATRIX FOR THE COGNATE FOR CASE#2.
C
      U(1)=-DCOS(CALP-CT3)
      U(2)=-DSIN(CALP-CT3)
      U(3)=0.0
      U(4)=(CE1*CI1-CF1*CH1)/CR1
      U(5)=(CF1*CG1-CD1*CI1)/CR1
      U(6)=(CD1*CH1-CE1*CG1)/CR1
      U(7)=-(CCTX*DSIN(CALP-CT3)*CL2/CL3)
      U(8)=-DSIN(CALP-CT3)*CL2
      U(9)=DSIN(CALP-CT3)
      U(10)=-DCOS(CALP-CT3)
      U(11)=0.0
      U(12)=(CC1*CH1-CB1*CI1)/CR1
      U(13)=(CA1*CI1-CC1*CG1)/CR1
      U(14)=(CB1*CG1-CA1*CH1)/CR1
      U(15)=-(DCOS(CALP-CT3)*CCTX*CL2/CL3)
      U(16)=-DCOS(CALP-CT3)*CL2
      U(17)=0.0
      U(18)=0.0
      U(19)=1.0
      U(20)=(CB1*CF1-CC1*CE1)/CR1
      U(21)=(CC1*CD1-CA1*CF1)/CR1
      U(22)=(CA1*CE1-CB1*CD1)/CR1
      U(23)=CCTX/CL3
      U(24)=1.0
C     TRANSPOSING THE FORCE TRANSFER MATRIX (CBETA) OF THE COGNATE.
C
      N=8
      M=3
      CALL GMTRA(U,R,N,M)
C
C     THE TRANSPOSED FORCE-TRANSFER MATRIX IS MULTIPLIED BY THE
C     COGNATE FLEXIBILITY MATRIX .
C
      DO 88  I=1,24
   88 A(I)=R(I)
C
C    *ALL LINKS ARE OF UNIFORM CROSS-SECTION OF DIA.=0.5".               *
C
C    *''CCA''-CROSS-SECTIONAL AREA OF ALL LINKS.                         *
      CCA=0.1963495408
C
C    *''CME''-YOUNG'S MODULUS OF ELASTICITY FOR MATERIAL ALUMUNIUM.      *
      CME=10000000.0
C
C    *''CMI''-CROSS-SECTIONAL MOMENT OF INERTIA.                         *
      CMI=0.0030679615
C
C    THE FLEXIBILITY MATRIX FOR THE COGNATE MECHANISM ''CF''.
C
      F(1)=CL2/(CCA*CME)
      F(2)=0.0
      F(3)=F(2)
      F(4)=F(2)
      F(5)=F(2)
      F(6)=F(2)
```

```
      F(7)=F(2)
      F(8)=F(2)
      F(9)=F(2)
      F(10)=(CL2*CL2*CL2)/(3.0*CME*CMI)
      F(11)=(CL2*CL2)/(2.0*CME*CMI)
      F(12)=F(2)
      F(13)=F(2)
      F(14)=F(2)
      F(15)=F(2)
      F(16)=F(2)
      F(17)=F(2)
      F(18)=(CL2*CL2)/(2.0*CME*CMI)
      F(19)=CL2/(CME*CMI)
      F(20)=F(2)
      F(21)=F(2)
      F(22)=F(2)
      F(23)=F(2)
      F(24)=F(2)
      F(25)=F(2)
      F(26)=F(2)
      F(27)=F(2)
      F(28)=CL1/(CCA*CME)
      F(29)=F(2)
      F(30)=F(2)
      F(31)=F(2)
      F(32)=F(2)
      F(33)=F(2)
      F(34)=F(2)
      F(35)=F(2)
      F(36)=F(2)
      F(37)=(CL1*CL1*CL1)/(3.0*CCA*CME)
      F(38)=F(2)
      F(39)=F(2)
      F(40)=F(2)
      F(41)=F(2)
      F(42)=F(2)
      F(43)=F(2)
      F(44)=F(2)
      F(45)=F(2)
      F(46)=CL4/(CCA*CME)
      F(47)=F(2)
      F(48)=F(2)
      F(49)=F(2)
      F(50)=F(2)
      F(51)=F(2)
      F(52)=F(2)
      F(53)=F(2)
      F(54)=F(2)
      F(55)=CL3/(CCA*CME)
      F(56)=F(2)
      F(57)=F(2)
      F(58)=F(2)
      F(59)=F(2)
      F(60)=F(2)
      F(61)=F(2)
      F(62)=F(2)
      F(63)=F(2)
      F(64)=CL3/(3.0*CME*CMI)
      N=3
      M=8
```

```
      MSA=0
      MSB=0
      L=8
      CALL MPRD(A,F,R,N,M,MSA,MSB,L)
C
C     THE PRODUCT R IS MULTIPLIED BY THE FORCE-TRANSFER MATRIX.
C
      DO 99  I=1,24
   99 A(I)=R(I)
C     THE FORCE TRANSFER MATRIX FOR COGNATE FOR CASE#2.
      U(1)=-DCOS(CALP-CT3)
      U(2)=-DSIN(CALP-CT3)
      U(3)=0.0
      U(4)=(CE1*CI1-CF1*CH1)/CR1
      U(5)=(CF1*CG1-CD1*CI1)/CR1
      U(6)=(CD1*CH1-CE1*CG1)/CR1
      U(7)=-(CCTX*DSIN(CALP-CT3)*CL2/CL3)
      U(8)=-DSIN(CALP-CT3)*CL2
      U(9)=DSIN(CALP-CT3)
      U(10)=-DCOS(CALP-CT3)
      U(11)=0.0
      U(12)=(CC1*CH1-CB1*CI1)/CR1
      U(13)=(CA1*CI1-CC1*CG1)/CR1
      U(14)=(CB1*CG1-CA1*CH1)/CR1
      U(15)=-(DCOS(CALP-CT3)*CCTX*CL2/CL3)
      U(16)=-DCOS(CALP-CT3)*CL2
      U(17)=0.0
      U(18)=0.0
      U(19)=1.0
      U(20)=(CB1*CF1-CC1*CE1)/CR1
      U(21)=(CC1*CD1-CA1*CF1)/CR1
      U(22)=(CA1*CE1-CB1*CD1)/CR1
      U(23)=CCTX/CL3
      U(24)=1.0
      DO 90 I=25,27
   90 U(I)=0.0
      U(28)=1.0
      DO 91 I=29,36
   91 U(I)=0.0
      U(37)=1.0
      DO 92 I=38,45
   92 U(I)=0.0
      U(46)=1.0
      DO 93 I=47,51
   93 U(I)=0.0
      U(52)=DCOS(CT3-T2)
      U(53)=DSIN(CT3-T2)
      U(54)=0.0
      U(55)=1.0
      DO 94 I=56,63
   94 U(I)=0.0
      U(64)=1.0
      N=3
      M=8
      MSA=0
      MSB=0
      L=8
      CALL MPRD(A,U,R,N,M,MSA,MSB,L)
      DO 111 I=1,24
  111 A(I)=R(I)
```

```
C
C        THE PRODUCT IS MULTIPLIED BY THE EXTERNAL FORCE MATRIX ''P''.
C
         DNTY=0.098
         GC=32.178*12.0
         CXPA=-(CL1*DCOS(T2)*COM2*COM2)-(CL1*DSIN(T2)*CALPH2)+(CL2*DCOS(CAL
        *P-CT3)*COM3*COM3)-(CL2*DSIN(CALP-CT3)*CALPH3)
         CYPA=-(CL1*DSIN(T2)*COM2*COM2)+(CL1*DCOS(T2)*CALPH2)-(CL2*DSIN(CAL
        *P-CT3)*COM3*COM3)-(CL2*DCOS(CALP-CT3)*CALPH3)
         CXDA=-(CL1*DSIN(T2)*CALPH2)-(CL1*DCOS(T2)*COM2*COM2)
         CYDA=(CL1*DCOS(T2)*CALPH2)-(CL1*DSIN(T2)*COM2*COM2)
         CXEA=-(CL1*DCOS(T2)*COM2*COM2)-(CL1*DSIN(T2)*CALPH2)+(CL3*DCOS(CT3
        *)*COM3*COM3)+(CL3*DSIN(CT3)*CALPH3)
         CYEA=-(CL1*DSIN(T2)*COM2*COM2)+(CL1*DCOS(T2)*CALPH2)-(CL3*DSIN(CT3
        *)*COM3*COM3)+(CL3*DCOS(CT3)*CALPH3)
         CVOL1=CCA*CL1
         CVOL2=CCA*CL2
         CVOL3=CCA*CL3
         CVOL4=CCA*CL4
         CM1=DNTY*CVOL1
         CM2=DNTY*CVOL2
         CM3=DNTY*CVOL3
         CM4=DNTY*CVOL4
         CP1=CM2*CXPA/GC
         CP2=CM2*CYPA/GC
         CP3=0.0
         CP4=(CM1*(CXDA*DCOS(T2)+CYDA*DSIN(T2)))/GC
         CP5=(CM1*(CYDA*DSIN(T2)-CXDA*DCOS(T2)))/GC
         CP6=(CM4*(CXEA*DCOS(CT4)+CYEA*DSIN(CT4)))/GC
         CP7=(CM3*(CXDA*DCOS(CT3)-CYDA*DSIN(CT3)))/GC
         CP8=0.0
C
C        THE SYSTEM FORCE MATRIX FOR THE COGNATE MECHANISM.
C
         B(1)=CP1
         B(2)=CP2
         B(3)=CP3
         B(4)=CP4
         B(5)=CP5
         B(6)=CP6
         B(7)=CP7
         B(8)=CP8
         N=3
         M=8
         MSA=0
         MSB=0
         L=1
         CALL MPRD(A,B,R,N,M,MSA,MSB,L)
         CXX=R(1)
         CYY=R(2)
         CXT=R(3)
C
C                    THE FOLLOWING TRANSFORMATION LOCATES
C                    --- --------- -------------- -------
C
C                         THE COGNATE IN ITS TRUE
C                         --- ------- -- --- ----
C
C                                POSITION
C                                --------
```

```
C
      XDP=XPP-CL5
      XRP=(XDP*DCOS(-BETA))-(YPP*DSIN(-BETA))
      CXP=XRP+L5
      CYP=(XDP*DSIN(-BETA))+(YPP*DCOS(-BETA))
      NCXP=CXP+CXX
      NCYP=CYP+CYY
      NCXP=CXP+CXX
      NCYP=CYP+CYY
      WRITE(6,200)P2,CXX,CYY,CRT
  200 FORMAT(1H0,5X,F12.6,8X,F12.6,8X,F12.6,8X,F12.6)
      IF(P2.EQ.360.0) GO TO 1
      GO TO 5
    1 STOP
      END
$ENTRY
C                                                                       GMTR  10
C     ..................................................................GMTR  20
C                                                                       GMTR  30
C        SUBROUTINE GMTRA                                               GMTR  40
C                                                                       GMTR  50
C        PURPOSE                                                        GMTR  60
C           TRANSPOSE A GENERAL MATRIX                                  GMTR  70
C                                                                       GMTR  80
C        USAGE                                                          GMTR  90
C           CALL GMTRA(A,R,N,M)                                         GMTR 100
C                                                                       GMTR 110
C        DESCRIPTION OF PARAMETERS                                      GMTR 120
C           A - NAME OF MATRIX TO BE TRANSPOSED                         GMTR 130
C           R - NAME OF RESULTANT MATRIX                                GMTR 140
C           N - NUMBER OF ROWS OF A AND COLUMNS OF R                    GMTR 150
C           M - NUMBER OF COLUMNS OF A AND ROWS OF R                    GMTR 160
C                                                                       GMTR 170
C        REMARKS                                                        GMTR 180
C           MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A         GMTR 190
C           MATRICES A AND R MUST BE STORED AS GENERAL MATRICES         GMTR 200
C                                                                       GMTR 210
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  GMTR 220
C           NONE                                                        GMTR 230
C                                                                       GMTR 240
C        METHOD                                                         GMTR 250
C           TRANSPOSE N BY M MATRIX A TO FORM M BY N MATRIX R           GMTR 260
C                                                                       GMTR 270
C     ..................................................................GMTR 280
C                                                                       GMTR 290
      SUBROUTINE GMTRA(A,R,N,M)                                         GMTR 300
      DIMENSION A(1),R(1)                                               GMTR 310
C                                                                       GMTR 320
      IR=0                                                              GMTR 330
      DO 10 I=1,N                                                       GMTR 340
      IJ=I-N                                                            GMTR 350
      DO 10 J=1,M                                                       GMTR 360
      IJ=IJ+N                                                           GMTR 370
      IR=IR+1                                                           GMTR 380
   10 R(IR)=A(IJ)                                                       GMTR 390
      RETURN                                                            GMTR 400
      END                                                               GMTR 410
C                                                                       MPRD  10
```

```
C        .................................................................
C
C        SUBROUTINE MPRD
C
C        PURPOSE
C           MULTIPLY TWO MATRICES TO FORM A RESULTANT MATRIX
C
C        USAGE
C           CALL MPRD(A,B,R,N,M,MSA,MSB,L)
C
C        DESCRIPTION OF PARAMETERS
C           A - NAME OF FIRST INPUT MATRIX
C           B - NAME OF SECOND INPUT MATRIX
C           R - NAME OF OUTPUT MATRIX
C           N - NUMBER OF ROWS IN A AND R
C           M - NUMBER OF COLUMNS IN A AND ROWS IN B
C           MSA - ONE DIGIT NUMBER FOR STORAGE MODE OF MATRIX A
C                  0 - GENERAL
C                  1 - SYMMETRIC
C                  2 - DIAGONAL
C           MSB - SAME AS MSA EXCEPT FOR MATRIX B
C           L - NUMBER OF COLUMNS IN B AND R
C
C        REMARKS
C           MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRICES A OR B
C           NUMBER OF COLUMNS OF MATRIX A MUST BE EQUAL TO NUMBER OF ROW
C           OF MATRIX B
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           LOC
C
C        METHOD
C           THE M BY L MATRIX B IS PREMULTIPLIED BY THE N BY M MATRIX A
C           AND THE RESULT IS STORED IN THE N BY L MATRIX R. THIS IS A
C           ROW INTO COLUMN PRODUCT.
C           THE FOLLOWING TABLE SHOWS THE STORAGE MODE OF THE OUTPUT
C           MATRIX FOR ALL COMBINATIONS OF INPUT MATRICES
C                        A                 B                 R
C                     GENERAL           GENERAL           GENERAL
C                     GENERAL           SYMMETRIC         GENERAL
C                     GENERAL           DIAGONAL          GENERAL
C                     SYMMETRIC         GENERAL           GENERAL
C                     SYMMETRIC         SYMMETRIC         GENERAL
C                     SYMMETRIC         DIAGONAL          GENERAL
C                     DIAGONAL          GENERAL           GENERAL
C                     DIAGONAL          SYMMETRIC         GENERAL
C                     DIAGONAL          DIAGONAL          DIAGONAL
C
C        .................................................................
C
307         SUBROUTINE MPRD(A,B,R,N,M,MSA,MSB,L)
308         DIMENSION A(1),B(1),R(1)
C
C        SPECIAL CASE FOR DIAGONAL BY DIAGONAL
C
309         MS=MSA*10+MSB
310         IF(MS-22) 30,10,30
311      10 DO 20 I=1,N
312      20 R(I)=A(I)*B(I)
313         RETURN
```

```
C                                                                        MPRD 620
C         ALL OTHER CASES                                                MPRD 630
C                                                                        MPRD 640
   30 IR=1                                                               MPRD 650
      DO 90 K=1,L                                                        MPRD 660
      DO 90 J=1,N                                                        MPRD 670
      R(IR)=0                                                            MPRD 680
      DO 80 I=1,M                                                        MPRD 690
      IF(MS) 40,60,40                                                    MPRD 700
   40 CALL LOC(J,I,IA,N,M,MSA)                                           MPRD 710
      CALL LOC(I,K,IB,M,L,MSB)                                           MPRD 720
      IF(IA) 50,80,50                                                    MPRD 730
   50 IF(IB) 70,80,70                                                    MPRD 740
   60 IA=N*(I-1)+J                                                       MPRD 750
      IB=M*(K-1)+I                                                       MPRD 760
   70 R(IR)=R(IR)+A(IA)*B(IB)                                            MPRD 770
   80 CONTINUE                                                           MPRD 780
   90 IR=IR+1                                                            MPRD 790
      RETURN                                                             MPRD 800
      END                                                                MPRD 810
C                                                                        LOC   10
C     ...................................................................LOC   20
C                                                                        LOC   30
C        SUBROUTINE LOC                                                  LOC   40
C                                                                        LOC   50
C        PURPOSE                                                         LOC   60
C           COMPUTE A VECTOR SUBSCRIPT FOR AN ELEMENT IN A MATRIX OF     LOC   70
C           SPECIFIED STORAGE MODE                                       LOC   80
C                                                                        LOC   90
C        USAGE                                                           LOC  100
C           CALL LOC (I,J,IR,N,M,MS)                                     LOC  110
C                                                                        LOC  120
C        DESCRIPTION OF PARAMETERS                                       LOC  130
C           I     - ROW NUMBER OF ELEMENT                                LOC  140
C           J     - COLUMN NUMBER  OF ELEMENT                            LOC  150
C           IR    - RESULTANT VECTOR SUBSCRIPT                           LOC  160
C           N     - NUMBER OF ROWS IN MATRIX                             LOC  170
C           M     - NUMBER OF COLUMNS IN MATRIX                          LOC  180
C           MS    - ONE DIGIT NUMBER FOR STORAGE MODE OF MATRIX          LOC  190
C                   0 - GENERAL                                          LOC  200
C                   1 - SYMMETRIC                                        LOC  210
C                   2 - DIAGONAL                                         LOC  220
C                                                                        LOC  230
C        REMARKS                                                         LOC  240
C           NONE                                                         LOC  250
C                                                                        LOC  260
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                   LOC  270
C           NONE                                                         LOC  280
C                                                                        LOC  290
C        METHOD                                                          LOC  300
C           MS=0   SUBSCRIPT IS COMPUTED FOR A MATRIX WITH N*M ELEMENTS  LOC  310
C                  IN STORAGE (GENERAL MATRIX)                           LOC  320
C           MS=1   SUBSCRIPT IS COMPUTED FOR A MATRIX WITH N*(N+1)/2 IN  LOC  330
C                  STORAGE (UPPER TRIANGLE OF SYMMETRIC MATRIX). IF      LOC  340
C                  ELEMENT IS IN LOWER TRIANGULAR PORTION, SUBSCRIPT IS  LOC  350
C                  CORRESPONDING ELEMENT IN UPPER TRIANGLE.              LOC  360
C           MS=2   SUBSCRIPT IS COMPUTED FOR A MATRIX WITH N ELEMENTS    LOC  370
C                  IN STORAGE (DIAGONAL ELEMENTS OF DIAGONAL MATRIX).    LOC  380
C                  IF ELEMENT IS NOT ON DIAGONAL (AND THEREFORE NOT IN   LOC  390
```

```
C                      STORAGE), IR IS SET TO ZERO.                      LOC  400
C                                                                        LOC  410
C     .................................................................LOC  420
C                                                                        LOC  430
      SUBROUTINE LOC(I,J,IR,N,M,MS)                                      LOC  440
C                                                                        LOC  450
      IX=I                                                               LOC  460
      JX=J                                                               LOC  470
      IF(MS-1) 10,20,30                                                  LOC  480
   10 IRX=N*(JX-1)+IX                                                    LOC  490
      GO TO 36                                                           LOC  500
   20 IF(IX-JX) 22,24,24                                                 LOC  510
   22 IRX=IX+(JX*JX-JX)/2                                                LOC  520
      GO TO 36                                                           LOC  530
   24 IRX=JX+(IX*IX-IX)/2                                                LOC  540
      GO TO 36                                                           LOC  550
   30 IRX=0                                                              LOC  560
      IF(IX-JX) 36,32,36                                                 LOC  570
   32 IRX=IX                                                             LOC  580
   36 IR=IRX                                                             LOC  590
      RETURN                                                             LOC  600
      END                                                                LOC  610
```

APPENDIX D

CASE III: DISTRIBUTED MASS MODEL

```
C     *****************************************************************
C     *****************************************************************
C     *                                                               *
C     *                                                               *
C                                CASE III
C                                ---- ---
C     *                                                               *
C     *                                                               *
C     *                                                               *
C     *                                                               *
C     *                     DISTRIBUTED MASS MODEL                    *
C     *                     ----------- ---- -----                    *
C     *                               OF                              *
C     *                               --                              *
C     *                THE SOURCE FOUR-BAR MECHANISM                  *
C     *                --- ------ ---- --- ---------                  *
C     *                                                               *
C     *                                                               *
C     *****************************************************************
C     *****************************************************************
C     *****************************************************************
C     *                                                               *
C     *SUBROUTINE MPRD                                                *
C     *                                                               *
C..................................................................
C     *****************************************************************
C     *.............................................................*
C     *                                                               *
C     *PURPOSE                                                        *
C     *                                                               *
C     *   MULTIPLY TWO MATRICES TO FORM A RESULTANT MATRIX.           *
C     *.............................................................*
C     *                                                               *
C     *USAGE                                                          *
C     *                                                               *
C     *   CALL MPRD(A,B,R,N,M,MSA,MSB,L)                              *
C     *.............................................................*
C     *DESCRIPTION                                                    *
C     *   A-NAME OF FIRST MATRIX.                                     *
C     *   B-NAME OF SECOND MATRIX.                                    *
C     *   R-NAME OF OUTPUT MATRIX.                                    *
C     *   N-NUMBER OF ROWS IN A AND R.                                *
C     *   M-NUMBER OF COLUMNS IN A AND ROWS IN B.                     *
C     *   MSA-ONE DIGIT NUMBER FOR STORAGE MODE OF MATRIX A.          *
C     *             0-GENERAL.                                        *
C     *             1-SYMMETRICAL.                                    *
C     *             2-DIAGONAL.                                       *
C     *   MSB- SAME AS MSA EXCEPT FOR MATRIX B.                       *
C     *   L-NUMBER OF COLUMNS IN B AND R.                             *
C     *.............................................................*
C     *                                                               *
C     *DATA                                                           *
C     *                                                               *
C     *GENERAL                                                        *
C     *   TAKES COLUMN WISE                                           *
C     *                                                               *
C     *DIAGONAL                                                       *
C     *   DIAGONAL COLUMN WISE                                        *
```

```
FORTRAN IV G1  RELEASE 2.0          MAIN            DATE = 77074          20/14/13
      C     *                                                                 *
      C     *SYMMETRICAL                                                      *
      C     *   COLUMN WISE                                                   *
      C     *.................................................................*
      C     *                                                                 *
      C     *REMARKS                                                          *
      C     *                                                                 *
      C     *   MATRIX R CANNOT BE IN THE SAME LOCATION AS MATRIX A OR B.     *
      C     *   NUMBER OF COLUMNS OF MATRIX A MUST BE EQUAL TO NUMBER OF ROWS *
      C     *   OF MATRIX B.                                                  *
      C     *.................................................................*
      C     *SUBROUTINES AND SUBPROGRAMS REQUIRED                             *
      C     *   LOC                                                           *
      C     *.................................................................*
      C     *                                                                 *
      C     *METHOD                                                           *
      C     *                                                                 *
      C     *   THE M BY L MATRIX B IS PRE-MULTIPLIED BY THE N BY M MATRIX A  *
      C     *   AND THE RESULT IS STORED IN THE N BY L MATRIX R. THIS IS A ROW*
      C     *   IN TO COLUMN PRODUCT.                                         *
      C     *                                                                 *
      C     *   THE FOLLOWING TABLE SHOWS THE STORAGE MODE OF THE OUTPUT      *
      C     *   MATRIX FOR ALL COMBINATIONS OF INPUT MATRICES.                *
      C     *                                                                 *
      C     *                A                  B                  R          *
      C     *                                                                 *
      C     *           GENERAL            GENERAL            GENERAL         *
      C     *           GENERAL            SYMMETRICAL        GENERAL         *
      C     *           GENERAL            DIAGONAL           GENERAL         *
      C     *           SYMMETRICAL        DIAGONAL           GENERAL         *
      C     *           SYMMETRICAL        GENERAL            GENERAL         *
      C     *           SYMMETRICAL        SYMMETRICAL        GENERAL         *
      C     *           DIAGONAL           GENERAL            GENERAL         *
      C     *           DIAGONAL           SYMMETRICAL        GENERAL         *
      C     *           DIAGONAL           DIAGONAL           DIAGONAL        *
      C     *******************************************************************
            SUBROUTINE MPRD(A,B,R,N,M,MSA,MSB,L)
            DIMENSION A(2000),B(2000),R(2000)
      C     SPECIAL CASE FOR DIAGONAL
            MS=MSA*10+MSB
            IF(MS-22) 30,10,30
         10 DO 20 I=1,N
         20 R(I)=A(I)*B(I)
            RETURN
      C
      C     ALL OTHER CASES
      C
         30 IR=1
            DO 90 K=1,L
            DO 90 J=1,N
            R(IR)=0
            DO 80 I=1,M
            IF(MS) 40,60,40
         40 CALL LOC(J,I,IA,N,M,MSA)
            CALL LOC(I,K,IB,M,L,MSB)
            IF(IA) 50,80,50
         50 IF(IB) 70,80,70
         60 IA=N*(I-1)+J
```

```
                 IB=M*(K-1)+1
              70 R(IR)=R(IR)+A(IA)*B(IB)
              80 CONTINUE
              90 IR=IR+1
                 RETURN
                 END
```

```
C     ******************************************************************
      DOUBLE PRECISION L1,L2,L3,L4,L5,K1,K2,K3,K4,K5,DCOS,DSIN,DATAN,
     *AA,BB,CC,DD,EE,FF,A1,B1,C1,D1,E1,F1,G1,H1,I1,R1,CX,SX,CTX,DSQRT,
     *T2,T3,T4,ALP,CA,MI,ME,OM2,OM3,OM4,AJ,BJ,CJ,DJ,EJ,FJ,ALPH2,ALPH3,
     *ALPH4,DCOT,PI,XP,YP,XX,YY,NXP,NYP
     *,DNTY,VOL1,VOL2,VOL3,VOL4,XAA,XBA,YAA,YBA,M1,M2,M3,M4
     *,G2,G3,G4,G5,H2,H3,H4,H5,I2,I3,I4,I5,R5,R2,R3,R4,L21,L22,L23,L24
     *,CI,CE
     *,P1,Z2,P3,P4,P5,P6,P7,P8,XPA1,YPA1,XPA2,YPA2,XPA3,YPA3,XPA4,YPA4,
     *XPA5,YPA5,M21,M22,M23,M24,GC
      DIMENSION A(1600),B(1600),R(1600),U(1600)
      L1=10.0
      L2=10.0
      L3=30.0
      L4=25.0
      L5=30.0
      K1=L5/L1
      K2=L5/L4
      K3=(L1*L1-L3*L3+L4*L4+L5*L5)/(2.0*L1*L4)
      CI=0.0030679615
      K4=L5/L3
      K5=(L4*L4-L5*L5-L1*L1-L3*L3)/(2.0*L1*L3)
      L21=L2-2.0
      L22=L2-4.0
      L23=L2-6.0
      L24=L2-8.0
      PI=3.142857143
C
C     S---- THE SPEED OF ROTATION OF THE INPUT LINK =300 R.P.M.
C
      S=300.0
      OM2=(2.0*PI*S)/60.0
      P2=-10.0
    5 P2=P2+10.0
      T2=(P2*2.0*PI)/(360.0)
      AA=DCOS(T2)+K3-K1-(K2*DCOS(T2))
      BB=-2.0*DSIN(T2)
      CC=K1+K3-(1.0+K2)*DCOS(T2)
      DD=(K4*DCOS(T2))+DCOS(T2)+K5-K1
      EE=-2.0*DSIN(T2)
      FF=(K4*DCOS(T2))-DCOS(T2)+K5+K1
      T3=2.0*(DATAN((-EE-DSQRT(EE*EE-4.0*DD*FF))/(2.0*DD)))
      T4=2.0*(DATAN((-BB-DSQRT(BB*BB-4.0*AA*CC))/(2.0*AA)))
      CX=DCOS(T4-T3)
      SX=DSIN(T4-T3)
      CTX=CX/SX
      ALP=PI/3.0
      XP=(L1*DCOS(T2))+(L2*DCOS(ALP+T3))
      YP=(L1*DSIN(T2))+(L2*DSIN(ALP+T3))
      OM3=(L1*OM2*DSIN(T4-T3))/(L3*DSIN(T3-T4))
      OM4=(L1*OM2*DSIN(T2-T3))/(L4*DSIN(T4-T3))
      AJ=L4*DSIN(T4)
      BJ=L3*DSIN(T3)
      CJ=(L1*OM2*OM2*DCOS(T2))+(L3*OM3*OM3*DCOS(T3))-(L4*OM4*OM4*DCOS(T4
     *))
      DJ=L4*DCOS(T4)
      EJ=L3*DCOS(T3)
      FJ=(L4*OM4*OM4*)SIN(T4))-(L1*OM2*OM2*DSIN(T2))-(L3*OM3*OM3*DSIN(T3
```

```
                  *))
0046              ALPH2=0.0
0047              ALPH3=(CJ*DJ-AJ*FJ)/(AJ*EJ-BJ*DJ)
0048              ALPH4=(CJ*EJ-BJ*FJ)/(AJ*EJ-BJ*DJ)
0049              XPA=-(L1*OM2*OM2*DCOS(T2))-(L1*ALPH2*DSIN(T2))-(L2*OM3*OM3*DCOS(
                 *ALP+T3))-(L2*ALPH3*DSIN(ALP+T3))
0050              YPA=(L1*ALPH2*DCOS(T2))-(L1*OM2*OM2*DSIN(T2))+(L2*ALPH3*DCOS(ALP+
                 *T3))-(L2*OM3*OM3*DSIN(ALP+T3))
0051              XAA=-(L1*DSIN(T2)*ALPH2)-(L1*DCOS(T2)*OM2*OM2)
0052              YAA=(L1*DCOS(T2)*ALPH2)-(L1*DSIN(T2)*OM2*OM2)
0053              XBA=-(L1*ALPH2*DSIN(T2))-(L1*OM2*OM2*DCOS(T2))-(L3*OM3*OM3*DCOS(T3
                 *))+(L3*ALPH3*DSIN(T3))
0054              YBA=-(L1*OM2*OM2*DSIN(T2))+(L1*ALPH2*DCOS(T2))-(L3*OM3*OM3*DSIN(T3
                 *))+(L3*ALPH3*DCOS(T3))
            C
            C     *ALL LINKS ARE OF UNIFORM CIRCULAR CROSS-SECTION OF DIA.=0.5 IN.  *
            C
            C     *"CA"-CROSS-SECTIONAL AREA OF ALL LINKS.                          *
            C
0055              CA=0.1963495408
            C
            C     *CE---- THE MODULUS OF ELASTICITY.                                *
            C
0056              CE=10000000.0
            C
            C     *"MI"-CROSS-SECTIONAL MOMENT OF INERTIA.                          *
            C
0057              MI=0.0030679615
0058              A1=DCOS(T2)
0059              B1=-DSIN(T2)
0060              C1=DCOS(T4)
0061              D1=DSIN(T2)
0062              E1=DCOS(T2)
0063              F1=DSIN(T4)
0064              G1=-(L2*DCOS(ALP+T3)*DCOS(T2))+(L2*DSIN(ALP+T3)*DCOS(T2))
0065              G2=-(L21*DCOS(ALP+T3)*DCOS(T2))+(L21*DSIN(ALP+T3)*DCOS(T2))
0066              G3=-(L22*DCOS(ALP+T3)*DCOS(T2))+(L22*DSIN(ALP+T3)*DCOS(T2))
0067              G4=-(L23*DCOS(ALP+T3)*DCOS(T2))+(L23*DSIN(ALP+T3)*DCOS(T2))
0068              G5=-(L24*DCOS(ALP+T3)*DCOS(T2))+(L24*DSIN(ALP+T3)*DCOS(T2))
0069              H1=-(L2*DCOS(ALP+T3)*DCOS(T2))-(L2*DSIN(ALP+T3)*DSIN(T2))
0070              H2=-(L21*DCOS(ALP+T3)*DCOS(T2))-(L21*DSIN(ALP+T3)*DSIN(T2))
0071              H3=-(L22*DCOS(ALP+T3)*DCOS(T2))-(L22*DSIN(ALP+T3)*DSIN(T2))
0072              H4=-(L23*DCOS(ALP+T3)*DCOS(T2))-(L23*DSIN(ALP+T3)*DSIN(T2))
0073              H5=-(L24*DCOS(ALP+T3)*DCOS(T2))-(L24*DSIN(ALP+T3)*DSIN(T2))
0074              I1=DSIN(T4)*((L3*DCOS(T3))-(L2*DCOS(ALP+T3)))+DCOS(T4)*((L2*DSIN(A
                 *LP+T3))-(L3*DSIN(T3)))
0075              I2=DSIN(T4)*((L3*DCOS(T3))-(L21*DCOS(ALP+T3)))+DCOS(T4)*((L21*DSIN
                 *(ALP+T3))-(L3*DSIN(T3)))
0076              I3=DSIN(T4)*((L3*DCOS(T3))-(L22*DCOS(ALP+T3)))+DCOS(T4)*((L22*DSIN
                 *(ALP+T3))-(L3*DSIN(T3)))
0077              I4=DSIN(T4)*((L3*DCOS(T3))-(L23*DCOS(ALP+T3)))+DCOS(T4)*((L23*DSIN
                 *(ALP+T3))-(L3*DSIN(T3)))
0078              I5=DSIN(T4)*((L3*DCOS(T3))-(L24*DCOS(ALP+T3)))+DCOS(T4)*((L24*DSIN
                 *(ALP+T3))-(L3*DSIN(T3)))
0079              R1=(A1*(E1*I1-F1*H1))-(B1*(D1*I1-F1*G1))+(C1*(D1*H1-E1*G1))
0080              R2=(A1*(E1*I2-F1*H2))-(B1*(D1*I2-F1*G2))+(C1*(D1*H2-E1*G2))
0081              R3=(A1*(E1*I3-F1*H3))-(B1*(D1*I3-F1*G3))+(C1*(D1*H3-E1*G3))
0082              R4=(A1*(E1*I4-F1*H4))-(B1*(D1*I4-F1*G4))+(C1*(D1*H4-E1*G4))
```

```
0083        R5=(A1*(E1*I5-F1*H5))-(B1*(D1*I5-F1*G5))+(C1*(D1*H5-E1*G5))
0084        XPA1=-(L1*OM2*OM2*DCOS(T2))-(L1*ALPH2*DSIN(T2))-(L2*OM3*OM3*DCOS(A
           *LP+T3))-(L2*ALPH3*DSIN(ALP+T3))
0085        XPA2=-(L1*OM2*OM2*DCOS(T2))-(L1*ALPH2*DSIN(T2))-(L21*OM3*OM3*DCOS(
           *ALP+T3))-(L21*ALPH3*DSIN(ALP+T3))
0086        XPA3=-(L1*OM2*OM2*DCOS(T2))-(L1*ALPH2*DSIN(T2))-(L22*OM3*OM3*DCOS(
           *ALP+T3))-(L22*ALPH3*DSIN(ALP+T3))
0087        XPA4=-(L1*OM2*OM2*DCOS(T2))-(L1*ALPH2*DSIN(T2))-(L23*OM3*OM3*DCOS(
           *ALP+T3))-(L23*ALPH3*DSIN(ALP+T3))
0088        XPA5=-(L1*OM2*OM2*DCOS(T2))-(L1*ALPH2*DSIN(T2))-(L24*OM3*OM3*DCOS(
           *ALP+T3))-(L24*ALPH3*DSIN(ALP+T3))
0089        YPA1=(L1*ALPH2*DCOS(T2))-(L1*OM2*OM2*DSIN(T2))+(L2*ALPH3*DCOS(ALP+
           *T3))-(L2*OM3*OM3*DSIN(ALP+T3))
0090        YPA2=(L1*ALPH2*DCOS(T2))-(L1*OM2*OM2*DSIN(T2))+(L21*ALPH3*DCOS(ALP
           *+T3))-(L21*OM3*OM3*DSIN(ALP+T3))
0091        YPA3=(L1*ALPH2*DCOS(T2))-(L1*OM2*OM2*DSIN(T2))+(L22*ALPH3*DCOS(ALP
           *+T3))-(L22*OM3*OM3*DSIN(ALP+T3))
0092        YPA4=(L1*ALPH2*DCOS(T2))-(L1*OM2*OM2*DSIN(T2))+(L23*ALPH3*DCOS(ALP
           *+T3))-(L23*OM3*OM3*DSIN(ALP+T3))
0093        YPA5=(L1*ALPH2*DCOS(T2))-(L1*OM2*OM2*DSIN(T2))+(L24*ALPH3*DCOS(ALP
           *+T3))-(L24*OM3*OM3*DSIN(ALP+T3))
      C
      C     DENSITY OF ALUMUNIUM IS 0.098 LB/CU.IN.
      C
0094        DNTY=0.098
0095        VOL2=CA*L2
0096        VOL21=CA*L21
0097        VOL22=CA*L22
0098        VOL23=CA*L23
0099        VOL24=CA*L24
0100        M2=VOL2*DNTY
0101        M21=VOL21*DNTY
0102        M22=VOL22*DNTY
0103        M23=VOL23*DNTY
0104        M24=VOL24*DNTY
0105        GC=32.178*12.0
0106        P1=(XPA1*M2)/GC
0107        Z2=(YPA1*M2)/GC
0108        P3=0.0
0109        P4=(XPA2*M21)/GC
0110        P5=(YPA2*M21)/GC
0111        P6=0.0
0112        P7=(XPA3*M22)/GC
0113        P8=(YPA3*M22)/GC
0114        P9=0.0
0115        P10=(XPA4*M23)/GC
0116        P11=(YPA4*M23)/GC
0117        P12=0.0
0118        P13=(XPA5*M24)/GC
0119        P14=(YPA5*M24)/GC
0120        P15=0.0
      C
      C     THE FORCE TRANSFORMATION MATRIX FOR CASE NO.3********************
      C
0121        U(1)=DCOS(T3+ALP)
0122        U(2)=-DSIN(T3+ALP)
0123        U(3)=0.0
0124        U(4)=((E1*I1)-(F1*H1))/R1
```

```
      U(5)=((F1*G1)-(D1*I1))/R1
      U(6)=((D1*H1)-(E1*G1))/R1
      U(7)=CTX*DSIN(T3+ALP)*(L2/L3)
      U(8)=-(DSIN(T3+ALP))*L2
      DO 50 I=9,40
   50 U(I)=0.0
      U(41)=DSIN(T3+ALP)
      U(42)=DCOS(T3+ALP)
      U(43)=0.0
      U(44)=(C1*H1-B1*I1)/R1
      U(45)=(A1*I1-C1*G1)/R1
      U(46)=(B1*G1-A1*H1)/R1
      U(47)=-(DCOS(T3+ALP))*CTX*(L2/L3)
      U(48)=DCOS(T3+ALP)*L2
      DO 51 I=49,90
   51 U(I)=0.0
      U(91)=1.0
      U(92)=(B1*F1-C1*E1)/R1
      U(93)=(C1*D1-A1*F1)/R1
      U(94)=(A1*E1-B1*D1)/R1
      U(95)=-CTX/L3
      U(96)=1.0
      DO 52 I=97,128
   52 U(I)=0.0
      U(129)=U(1)
      U(130)=U(2)
      U(131)=U(3)
      U(132)=(E1*I2-F1*H2)/R2
      U(133)=(F1*G2-D1*I2)/R2
      U(134)=(D1*H2-E1*G2)/R2
      U(135)=CTX*DSIN(T3+ALP)*L21/L3
      U(136)=-DSIN(T3+ALP)*L21
      DO 53 I=137,168
   53 U(I)=0.0
      U(169)=U(41)
      U(170)=U(42)
      U(171)=U(43)
      U(172)=(C1*H2-B1*I2)/R2
      U(173)=(A1*I2-C1*G2)/R2
      U(174)=(B1*G2-A1*H2)/R2
      U(175)=-(CTX*DCOS(T3+ALP)*L21)/L3
      U(176)=DCOS(T3+ALP)*L21
      DO 54 I=177,210
   54 U(I)=0.0
      U(211)=1.0
      U(212)=(B1*F1-C1*E1)/R2
      U(213)=(C1*D1-A1*F1)/R2
      U(214)=(A1*E1-B1*D1)/R2
      U(215)=-CTX/L3
      U(216)=1.0
      DO 55 I=217,256
   55 U(I)=0.0
      U(257)=U(1)
      U(258)=U(2)
      U(259)=U(3)
      U(260)=(E1*I3-F1*H3)/R3
      U(261)=(F1*G3-D1*I3)/R3
      U(262)=(D1*H3-E1*G3)/R3
```

```
      U(263)=CTX*DSIN(T3+ALP)*L22/L3
      U(264)=-DSIN(T3+ALP)*L22
      DO 56 I=265,296
   56 U(I)=0.0
      U(297)=U(41)
      U(298)=U(42)
      U(299)=U(43)
      U(300)=(C1*H3-B1*I3)/R3
      U(301)=(A1*I3-C1*G3)/R3
      U(302)=(B1*G3-A1*H3)/R3
      U(303)=-(CTX*DCOS(ALP+T3)*L22)/L3
      U(304)=DCOS(T3+ALP)*L22
      DO 57 I=305,338
   57 U(I)=0.0
      U(339)=1.0
      U(340)=(B1*F1-C1*E1)/R3
      U(341)=(C1*D1-A1*F1)/R3
      U(342)=(A1*E1-B1*D1)/R3
      U(343)=-CTX/L3
      U(344)=1.0
      DO 58 I=345,384
   58 U(I)=0.0
      U(385)=U(1)
      U(386)=U(2)
      U(387)=U(3)
      U(388)=(E1*I4-F1*H4)/R4
      U(389)=(F1*G4-D1*I4)/R4
      U(390)=(D1*H4-E1*G4)/R4
      U(391)=CTX*DSIN(T3+ALP)*L23/L3
      U(392)=-DSIN(T3+ALP)*L23
      DO 59 I=393,424
   59 U(I)=0.0
      U(425)=U(41)
      U(426)=U(42)
      U(427)=U(43)
      U(428)=(C1*H4-B1*I4)/R4
      U(429)=(A1*I4-C1*G4)/R4
      U(430)=(B1*G4-A1*H4)/R4
      U(431)=-(CTX*DCOS(T3+ALP)*L23)/L3
      U(432)=DCOS(T3+ALP)*L23
      DO 60 I=433,466
   60 U(I)=0.0
      U(467)=1.0
      U(468)=(B1*F1-C1*E1)/R4
      U(469)=(C1*D1-A1*F1)/R4
      U(470)=(A1*E1-B1*D1)/R4
      U(471)=-CTX/L3
      U(472)=1.0
      DO 61 I=473,512
   61 U(I)=0.0
      U(513)=U(1)
      U(514)=U(2)
      U(515)=U(3)
      U(516)=(E1*I5-F1*H5)/R5
      U(517)=(F1*G5-D1*I5)/R5
      U(518)=(D1*H5-E1*G5)/R5
      U(519)=CTX*DSIN(T3+ALP)*L24/L3
      U(520)=-DSIN(T3+ALP)*L24
```

```
          DO 62 I=521,552
       62 U(I)=0.0
          U(553)=U(41)
          U(554)=U(42)
          U(555)=U(43)
          U(556)=(C1*H5-B1*I5)/R5
          U(557)=(A1*I5-C1*G5)/R5
          U(558)=(B1*G5-A1*H5)/R5
          U(559)=-(CTX*DCOS(ALP+T3)*L24)/L3
          U(560)=DCOS(T3+ALP)*L24
          DO 63 I=561,594
       63 U(I)=0.0
          U(595)=1.0
          U(596)=(B1*F1-C1*E1)/R5
          U(597)=(C1*D1-A1*F1)/R5
          U(598)=(A1*E1-B1*D1)/R5
          U(599)=-CTX/L3
          U(600)=1.0
          N=40
          M=15
C
C         FORCE TRANSFER MATRIX IS TRANSPOSED.
C
          CALL GMTRA(U,R,N,M)
          DO 100 I=1,600
      100 A(I)=R(I)
C
C         THE FORCE TRANSFER MATRIX IS MULTIPLIED BY FLEXIBILITY MATRIX.
C
          B(1)=L2/(CA*CE)
          DO 1 J=2,41
        1 B(J)=0.0
          B(42)=(L2*L2*L2)/(3.0*CE*CI)
          B(43)=(L2*L2)/(2.0*CE*CI)
          DO 2 J=44,81
        2 B(J)=0.0
          B(82)=(L2*L2)/(2.0*CE*CI)
          B(83)=L2/(CE*CI)
          DO 3 J=84,123
        3 B(J)=0.0
          B(124)=L1/(CA*CE)
          DO 4 J=125,164
        4 B(J)=0.0
          B(165)=(L1*L1*L1)/(3.0*CE*CI)
          DO 6 J=166,205
        6 B(J)=0.0
          B(206)=L4/(CA*CE)
          DO 7 J=207,246
        7 B(J)=0.0
          B(247)=L3/(CA*CE)
          DO 8 J=248,287
        8 B(J)=0.0
          B(288)=L3/(3.0*CE*CI)
          DO 9 J=289,328
        9 B(J)=0.0
          B(329)=L21/(CA*CE)
          DO 10 J=330,369
       10 B(J)=0.0
```

```
      B(370)=(L21*L21*L21)/(3.0*CE*CI)
      B(371)=(L21*L21)/(2.0*CE*CI)
      DO 11 J=372,409
   11 B(J)=0.0
      B(410)=(L21*L21)/(2.0*CE*CI)
      B(411)=L21/(CE*CI)
      DO 12 J=412,451
   12 B(J)=0.0
      B(452)=L1/(CA*CE)
      DO 13 J=453,492
   13 B(J)=0.0
      B(493)=(L1*L1*L1)/(3.0*CE*CI)
      DO 15 J=494,533
   15 B(J)=0.0
      B(534)=L4/(CA*CE)
      DO 16 J=535,574
   16 B(J)=0.0
      B(575)=L3/(CA*CE)
      DO 17 J=576,615
   17 B(J)=0.0
      B(616)=L3/(3.0*CE*CI)
      DO 18 J=617,656
   18 B(J)=0.0
      B(657)=L22/(CA*CE)
      DO 19 J=658,697
   19 B(J)=0.0
      B(698)=(L22*L22*L22)/(3.0*CE*CI)
      B(699)=(L22*L22)/(2.0*CE*CI)
      DO 20 J=700,737
   20 B(J)=0.0
      B(738)=(L22*L22)/(2.0*CE*CI)
      B(739)=L22/(CE*CI)
      DO 21 J=740,779
   21 B(J)=0.0
      B(780)=L1/(CA*CE)
      DO 22 J=781,820
   22 B(J)=0.0
      B(821)=(L1*L1*L1)/(3.0*CE*CI)
      DO 23 J=822,861
   23 B(J)=0.0
      B(862)=L4/(CA*CE)
      DO 24 J=863,902
   24 B(J)=0.0
      B(903)=L3/(CA*CE)
      DO 25 J=904,943
   25 B(J)=0.0
      B(944)=L3/(3.0*CE*CI)
      DO 26 J=945,984
   26 B(J)=0.0
      B(985)=L23/(CA*CE)
      DO 27 J=986,1025
   27 B(J)=0.0
      B(1026)=(L23*L23*L23)/(3.0*CE*CI)
      B(1027)=(L23*L23)/(2.0*CE*CI)
      DO 28 J=1028,1065
   28 B(J)=0.0
      B(1066)=(L23*L23)/(2.0*CE*CI)
      B(1067)=L23/(CE*CI)
```

```
      DO 29 J=1068,1107
   29 B(J)=0.0
      B(1108)=L1/(CA*CE)
      DO 30 J=1109,1148
   30 B(J)=0.0
      B(1149)=(L1*L1*L1)/(3.0*CE*CI)
      DO 31 J=1150,1189
   31 B(J)=0.0
      B(1190)=L4/(CA*CE)
      DO 32 J=1191,1230
   32 B(J)=0.0
      B(1231)=L3/(CA*CE)
      DO 33 J=1232,1271
   33 B(J)=0.0
      B(1272)=L3/(3.0*CE*CI)
      DO 34 J=1273,1312
   34 B(J)=0.0
      B(1313)=L24/(CA*CE)
      DO 35 J=1314,1353
   35 B(J)=0.0
      B(1354)=(L24*L24*L24)/(3.0*CE*CI)
      B(1355)=(L24*L24)/(2.0*CE*CI)
      DO 36 J=1356,1393
   36 B(J)=0.0
      B(1394)=(L24*L24)/(2.0*CE*CI)
      B(1395)=L24/(CE*CI)
      DO 37 J=1396,1435
   37 B(J)=0.0
      B(1436)=L1/(CA*CE)
      DO 38 J=1437,1476
   38 B(J)=0.0
      B(1477)=(L1*L1*L1)/(3.0*CE*CI)
      DO 39 J=1478,1517
   39 B(J)=0.0
      B(1518)=L4/(CA*CE)
      DO 40 J=1519,1558
   40 B(J)=0.0
      B(1559)=L3/(CA*CE)
      DO 41 J=1560,1599
   41 B(J)=0.0
      B(1600)=L3/(3.0*CE*CI)
      N=15
      M=40
      MSA=0
      MSB=0
      L=40
      CALL MPRD(A,B,R,N,M,MSA,MSB,L)
      DO 200 I=1,600
  200 A(I)=R(I)
C
C     THE RESULTANT IS MULTIPLIED BY THE FORCE TRANSFER MATRIX.
C
      U(1)=DCOS(T3+ALP)
      U(2)=-DSIN(T3+ALP)
      U(3)=0.0
      U(4)=((E1*I1)-(F1*H1))/R1
      U(5)=((F1*G1)-(D1*I1))/R1
      U(6)=((D1*H1)-(E1*G1))/R1
```

```
      U(7)=CTX*DSIN(T3+ALP)*(L2/L3)
      U(8)=-(DSIN(T3+ALP))*L2
      DO 70 I=9,40
   70 U(I)=0.0
      U(41)=DSIN(T3+ALP)
      U(42)=DCOS(T3+ALP)
      U(43)=0.0
      U(44)=(C1*H1-B1*I1)/R1
      U(45)=(A1*I1-C1*G1)/R1
      U(46)=(B1*G1-A1*H1)/R1
      U(47)=-(DCOS(T3+ALP))*CTX*(L2/L3)
      U(48)=DCOS(T3+ALP)*L2
      DO 71 I=49,90
   71 U(I)=0.0
      U(91)=1.0
      U(92)=(B1*F1-C1*E1)/R1
      U(93)=(C1*D1-A1*F1)/R1
      U(94)=(A1*E1-B1*D1)/R1
      U(95)=-CTX/L3
      U(96)=1.0
      DO 72 I=97,128
   72 U(I)=0.0
      U(129)=U(1)
      U(130)=U(2)
      U(131)=U(3)
      U(132)=(E1*I2-F1*H2)/R2
      U(133)=(F1*G2-D1*I2)/R2
      U(134)=(D1*H2-E1*G2)/R2
      U(135)=CTX*DSIN(T3+ALP)*L21/L3
      U(136)=-DSIN(T3+ALP)*L21
      DO 73 I=137,168
   73 U(I)=0.0
      U(169)=U(41)
      U(170)=U(42)
      U(171)=U(43)
      U(172)=(C1*H2-B1*I2)/R2
      U(173)=(A1*I2-C1*G2)/R2
      U(174)=(B1*G2-A1*H2)/R2
      U(175)=-(CTX*DCOS(T3+ALP)*L21)/L3
      U(176)=DCOS(T3+ALP)*L21
      DO 74 I=177,210
   74 U(I)=0.0
      U(211)=1.0
      U(212)=(B1*F1-C1*E1)/R2
      U(213)=(C1*D1-A1*F1)/R2
      U(214)=(A1*E1-B1*D1)/R2
      U(215)=-CTX/L3
      U(216)=1.0
      DO 75 I=217,256
   75 U(I)=0.0
      U(257)=U(1)
      U(258)=U(2)
      U(259)=U(3)
      U(260)=(E1*I3-F1*H3)/R3
      U(261)=(F1*G3-D1*I3)/R3
      U(262)=(D1*H3-E1*G3)/R3
      U(263)=CTX*DSIN(T3+ALP)*L22/L3
      U(264)=-DSIN(T3+ALP)*L22
```

```
      DO 76 I=265,296
   76 U(I)=0.0
      U(297)=U(41)
      U(298)=U(42)
      U(299)=U(43)
      U(300)=(C1*H3-B1*I3)/R3
      U(301)=(A1*I3-C1*G3)/R3
      U(302)=(B1*G3-A1*H3)/R3
      U(303)=-(CTX*DCOS(ALP+T3)*L22)/L3
      U(304)=DCOS(T3+ALP)*L22
      DO 77 I=305,338
   77 U(I)=0.0
      U(339)=1.0
      U(340)=(B1*F1-C1*E1)/R3
      U(341)=(C1*D1-A1*F1)/R3
      U(342)=(A1*E1-B1*D1)/R3
      U(343)=-CTX/L3
      U(344)=1.0
      DO 78 I=345,384
   78 U(I)=0.0
      U(385)=U(1)
      U(386)=U(2)
      U(387)=U(3)
      U(388)=(E1*I4-F1*H4)/R4
      U(389)=(F1*G4-D1*I4)/R4
      U(390)=(D1*H4-E1*G4)/R4
      U(391)=CTX*DSIN(T3+ALP)*L23/L3
      U(392)=-DSIN(T3+ALP)*L23
      DO 79 I=393,424
   79 U(I)=0.0
      U(425)=U(41)
      U(426)=U(42)
      U(427)=U(43)
      U(428)=(C1*H4-B1*I4)/R4
      U(429)=(A1*I4-C1*G4)/R4
      U(430)=(B1*G4-A1*H4)/R4
      U(431)=-(CTX*DCOS(T3+ALP))*L23/L3
      U(432)=DCOS(T3+ALP)*L23
      DO 80 I=433,466
   80 U(I)=0.0
      U(467)=1.0
      U(468)=(B1*F1-C1*E1)/R4
      U(469)=(C1*D1-A1*F1)/R4
      U(470)=(A1*D1-B1*D1)/R4
      U(471)=-CTX/L3
      U(472)=1.0
      DO 81 I=473,512
   81 U(I)=0.0
      U(513)=U(1)
      U(514)=U(2)
      U(515)=U(3)
      U(516)=(E1*I5-F1*H5)/R5
      U(517)=(F1*G5-D1*I5)/R5
      U(518)=(D1*H5-E1*G5)/R5
      U(519)=CTX*DSIN(T3+ALP)*L24/L3
      U(520)=-DSIN(T3+ALP)*L24
      DO 82 I=521,552
   82 U(I)=0.0
```

```
      U(553)=U(41)
      U(554)=U(42)
      U(555)=U(43)
      U(556)=(C1*H5-B1*I5)/R5
      U(557)=(A1*I5-C1*G5)/R5
      U(558)=(B1*G5-A1*H5)/R5
      U(559)=-(CTX*DCOS(ALP+T3)*L24)/L3
      U(560)=DCOS(T3+ALP)*L24
      DO 83 I=561,594
   83 U(I)=0.0
      U(595)=1.0
      U(596)=(B1*F1-C1*E1)/R5
      U(597)=(C1*D1-A1*F1)/R5
      U(598)=(A1*E1-B1*D1)/R5
      U(599)=-CTX/L3
      U(600)=1.0
      N=15
      M=40
      MSA=0
      MSB=0
      L=15
      CALL MPRD(A,U,R,N,M,MSA,MSB,L)
      DO 300 I=1,225
  300 A(I)=R(I)
C
C     THE RESULTANT IS MULTIPLIED BY THE INERTIA MATRIX ''P''.
C
      B(1)=P1
      B(2)=Z2
      B(3)=P3
      B(4)=P4
      B(5)=P5
      B(6)=P6
      B(7)=P7
      B(8)=P8
      B(9)=P9
      B(10)=P10
      B(11)=P11
      B(12)=P12
      B(13)=P13
      B(14)=P14
      B(15)=P15
      CALL MPRD(A,B,R,N,M,MSA,MSB,L)
      N=15
      M=15
      MSA=0
      MSB=0
      L=1
      CALL MPRD(A,B,R,N,M,MSA,MSB,L)
      WRITE(6,500)P2,R(1),R(2),R(3),R(4),R(5),R(6),R(7),R(8),R(9),R(10),
     *R(11),R(12),R(13),R(14),R(15)
  500 FORMAT('1',F18.10,3(8X,F18.10)///'0',18X,3(8X,F18.10)///'0',18X,3(
     *8X,F18.10)///'0',18X,3(8X,F18.10)///'0',18X,3(8X,F18.10))
      IF(P2.GT.360.0) GO TO 999
      GO TO 5
  999 STOP
      END
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VITA

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Master of Science

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