This dissertation has been 65-10,420
microfilmed exactly as received

OGDEN, Burton Ray, 1932-
OSCILLATORY FLOW PHENOMENA.

The University of Oklahoma, Ph.D., 1965
Engineering, chemical

University Microfilms, Inc., Ann Arbor, Michigan

THE UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

OSCILLATORY FLOW PHENOMENA

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

degree of

DOCTOR OF PHILOSOPHY

BY

BURTON RAY OGDEN

Norman, Oklahoma

1965

OSCILLATORY FLOW PHENOMENA

APPROVED BY

Orrin K. Crosser

Norman H. Packer

G. N. Murphy

Gerard Timma

C M Sliepcevich

F. M. Townsend

DISSERTATION COMMITTEE

ACKNOWLEDGMENTS

The writer wishes to thank Professor John E. Powers, who directed the research, for much valuable advice and guidance, and Professor O. K. Crosser who continued in Dr. Powers' place. Professor Gerald Tuma provided practical advice in the installation and calibration of the electronics equipment. Mr. Francis (Frank) Maginnis provided useful information to the construction of the Hartmann whistles, and first queried about the behavior of the whistle at low nozzle pressures, leading to the discovery of the low pressure oscillation. McDonnell Aircraft Corporation provided the instrumentation. Finally the writer thanks his wife, Opal, who typed the manuscript.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS

Figure | Page

OSCILLATORY FLOW PHENOMENA

CHAPTER I

INTRODUCTION

Oscillatory flow phenomena are a mid-ground between acoustics and steady state flow.

Both acoustics and steady state flow are well developed disciplines. However, fluid dynamics encompasses not only the phenomena of sound propagation and steady flow but also transient flow and oscillatory flow.

Oscillatory flow differs from sound in that bulk flow is involved. The phenomena of sound propagation and sound generation are usually treated separately. However, in the oscillatory flow system treated here the oscillation is self-generated in the flow system, and there is no clear cut distinction between generation and propagation.

While the theory of sound propagation is well-developed, the theory of sound generation in flow systems (oscillatory flow) is by comparison poorly developed. The contrast between oscillatory flow theory development and oscillatory electrical circuit theory is distinct.

In A.C. electrical circuitry we have such sophisticated hardware as television and radar, while in oscillatory fluid flow the automobile muffler compares poorly in the degree of sophistication. And it is only a passive device.

There are a number of practical active oscillatory flow devices, such as the wind musical instruments, whistles, and the pulse jet engine.

Perhaps the reason so few oscillatory flow devices have been developed is that oscillatory flow is usually regarded as an objectionable noise source.

From the chattering water faucet to the screaming rocket engine, oscillatory flow is regarded as a malfunction. Yet the phenomenon is widespread in nature, as common as the spoken word.

Regardless of whether one seeks to accentuate or eliminate the phenomenon, it is important to have a workable theory for it.

At present, however, acoustics and fluid flow are practiced as two different disciplines. A consideration of oscillatory fluid flow requires a merger of these two separated subjects.

The specific system chosen for study was the Hartmann whistle. This device was chosen for several reasons. It is a true oscillatory flow device, it is relatively simple in geometry while permitting a diversity of variations within that geometry, and consequently produces a wide range of intensity and frequency of the output. Basically, the

Hartmann whistle consists of a nozzle discharging directly at a cavity.

The experimental work was conducted as part of a study on high intensity sound generation under contract through the University of Oklahoma Research Institute with the McDonnell Aircraft Corporation. This study was directed by Dr. John E. Powers.

The contract study involved experimental studies of several sound generating systems, including the refractory burner, as well as the Hartmann whistle. The Hartmann whistle was chosen as the simplest system for which a quantitative theory was lacking.

It was expected that the data would corroborate the qualitative literature explanation of the Hartmann whistle behavior and that the theoretical analysis would consist of the first application of classical acoustical lumped system analysis to the Hartmann whistle. Surprises were in store on both counts.

The experimental data revealed a low pressure range of operation for the whistle, which was not predicted by the literature explanation of its mechanism of operation.

The application of classical acoustical theory to the Hartmann whistle resulted in a decisive failure of that discipline to predict that the device would oscillate at all, much less predict the conditions for and nature of the oscillation.

However the failure of the literature explanation to delineate the behavior of the Hartmann whistle, and the failure of acoustical theory to predict any oscillation at all, presented this investigation with a unique opportunity. If the acoustical theory could be generalized sufficiently to treat flow devices of this type, a theory for the operation of the Hartmann whistle would be provided. At the same time a more general method for the analysis of oscillatory flow phenomena would be evolved. The Hartmann whistle experiments would serve as the important crucial experiment.

This dissertation is an account of the sequence in which these things were done. Revealed here are data showing that the literature explanation for the oscillation mechanism, which is contingent on the requirement that the nozzle be choked, is incorrect. The oscillation mechanism was found to persist for nozzle pressures well below the critical pressure. Indeed, a Hartmann whistle may be adjusted so that it may be blown with the human mouth.

However, this experimental extension of the data on a little-known whistle device is not very significant. Even the fact that this data repudiates the accepted theory for the Hartmann whistle is of no great consequence. The accepted theory was little more than a verbal description of the oscillation phenomenon augmented with photographs and a few simple correlational expressions for the data. The theory was not a theory derived from basic physical laws: the mass, energy, momentum, and entropy balances. Therefore it is not vitally significant that such a limited theory has been overthrown.

It is significant that no successful mathematical model exists in the literature for so simple a system as the Hartmann whistle, or for any of the self-driven oscillatory fluid flow devices. It is significant, by contrast, that self-driven oscillatory electrical phenomena are successfully analyzed with the electrical circuit theory. Lumped system analysis has provided circuit representations for mechanical, magnetic, thermal and acoustic systems as well. Circuit analysis is a useful tool for the analysis and synthesis of systems involving one or more types of energy.

The thesis of this dissertation is the foundation of lumped system analysis on thermodynamic principles which permit the circuit technique to be applied to a wider range of phenomena: in particular, oscillatory flow phenomena.

CHAPTER II

THE HARTMANN WHISTLE

The phenomenon of the Hartmann whistle was accidentally discovered in 1916 by Professor Julius Hartmann of the Royal Technical College in Copenhagen. While comparing the velocity distribution of a jet of mercury with a jet of air, by measuring stagnation pressures, he found regions in the air jet in which his pitot gauge readings were unreadable due to a marked oscillation of the impact pressure. The stagnation pressure downstream from a sharp-edged nozzle supplied with a pressure greater than critical pressure was observed to vary as a damped cosine wave with increasing downstream distance. In those zones where stagnation pressure increased with downstream distance (the instability zones), Professor Hartmann observed violent oscillation of the pressure gauge needle with comparatively large diameter pitot probes, while the use of very small diameter probes permitted the measurement of the complete damped cosine curve.

Intrigued with this phenomenon, Professor Hartmann replaced the pitot gauge with a nearly spherical wide-mouthed flask with a short neck converging to an opening approximately the diameter of the nozzle orifice. This cavity, when axially located with the opening in a zone of instability, produced a relatively pure tone. The frequency decreased as the volume of the flask was increased. Apparently the flask behaved as a Helmholtz resonator. Cylindrical cavities were also used.

These produced a tone corresponding to the quarter wave frequency of the cavity. In the discussion to follow, relating to Hartmann whistles with cylindrical cavity pulsators, it may be helpful to refer to Figure 2 in Chapter IV. Professor Hartmann investigated the device over a period of years, announced his work to the scientific community in 1927,[1] and described and demonstrated the device in 1936 at Blackpool in England before Section A (Mathematical and Physical Sciences) of the British Association. An anonymously authored report of that paper followed.[2] Further data were published in 1939,[3] and also in 1939 a final report was published, and made available in the English language.[4] His

---

[1]J. Hartmann and B. Trolle, "A New Acoustic Generator: The Air Jet Generator," Journal of Scientific Instruments, vol. 4, (1927), pp. 101-111.

[2]"The Hartmann Acoustic Generator," Engineering (London), vol. 142, (1936), pp. 491-492.

[3]J. Hartmann, "Construction, Performance, and Design of Acoustic Air Jet Generator," Journal of Scientific Instruments, vol. 16, (1939), pp. 140-149.

[4]Julius Hartmann, in co-operation with Peter and Elisabeth V. Mathes and Freimut Lazarus, "The Acoustic Air-Jet Generator," Ingeniorvidenskabelige Skrifter, 1939, Nr. 4, Akademiet for Tekniske Videnskaber og Dansk Ingeniørforening.

work was surveyed by another anonymous report in 1940.[5]

Interest in the Hartmann whistle as a potential source of industrial ultrasonic radiation developed in the United States.

In 1944, at the Institute of Gas Technology, in Chicago, Leonard E. Savory experimented with Hartmann whistles. He noted that the sound output was sensitive to the nozzle pressure, and to the distance between the nozzle and the oscillating cavity (pulsator). Savory noted that the sound intensity (at a microphone 24 to 27 inches downstream from the nozzle on a line at an angle of 30 degrees from the whistle axis) could be increased by holding solid objects on two sides of the air stream. These objects also "stabilized" the air jet, or decreased the sensitivity of the sound output to the distance between the nozzle and the pulsator.

Pursuing this effect, Savory added "regenerator pads" to his Hartmann whistle. The experiments progressed to the use of three configurations of "regenerator cylinders," which in effect partially enclosed the air jet. Believing the sound-producing mechanism to be a surface effect of the air jet, Savory reasoned that the insertion of a small rod down the axis of the whistle would decrease the cross sectional area of the nozzle and leave the surface area of the air jet unchanged, thus hopefully decreasing the air rate and leaving

---

[5]"The Acoustic Air-Jet Generator," Engineering (London), vol. 150, (1940), p. 314.

the sound power unchanged. Three sizes of these "stabilizing rods" were tested. For nozzle pressures of 10 and 25 psig., the effects of the rods were indeed beneficial. The data at 50 psig. showed a decrease in intensity with increasing rod diameter.

Although Savory's concept of the Hartmann whistle oscillation mechanism was vague, he did show that improvements could apparently be made to the basic design. He also mentioned, in passing, that one of his regenerative cylinders, a conical shaped one, allowed low intensity sound to be produced to very low nozzle pressures (3 to 5 psig.). He did not comment that even though this low pressure operation was obtained with a rather specially shaped addition to the basic configuration, it was still inconsistent with the mechanism proposed by Hartmann. His work with the conventional Hartmann configuration was reported first[6] with the investigation of regenerator pads and cylinders and stabilizing rods being reported a week later.[7]

Interest had developed in the use of the Hartmann whistle as a means of smoke coagulation.[8] Additional Hartmann

---

[6] L. E. Savory, "Experiments with the Hartmann Acoustic Generator," Engineering, vol. 170, no. 4410, Aug 4 (1950), pp. 99-100.

[7] L. E. Savory, "Experiments with the Hartmann Acoustic Generator," Engineering, vol. 170, no. 4411, Aug 11 (1950), pp. 136-138.

[8] H. O. Monson, "Investigation of Ultrasonic Smoke Coagulation," Ph.D. Thesis, February, 1950.

whistle data were accumulated for that purpose. Hartmann's data were limited primarily to unity values of the pulsator depth to pulsator diameter ratio, and also to unity values of the pulsator diameter to nozzle diameter ratio. Savory varied the pulsator depth to pulsator diameter ratio, but remained restricted to equal values of nozzle diameter and pulsator diameter.

However, both ratios were varied by Monson and Binder.[9] They found important increases in intensity by increasing the pulsator diameter to nozzle diameter ratio up to 1.27. Their optimized whistle yielded 153.9 decibels at 6 inches radially as compared with 149.5 decibels for the original Hartmann configuration, which corresponds to an increase in sound power by a factor of 2.8. Monson and Binder made no attempt at any theoretical explanation, but they did contribute important data for the empirical design of Hartmann whistles.

Monson and Binder remarked that there was need for more data on the Hartmann whistle at the time of their investigation. Their discovery of the strong effect of the pulsator diameter to nozzle diameter ratio on power output confirmed their judgement.

The data obtained by Hartmann, Savory, Monson and Binder in the literature cited provided an adequate basis for the empirical design of workable Hartmann whistles.

---

[9] H. O. Monson and R. C. Binder, "Intensities Produced by Jet-Type Ultrasonic Vibrators," Journal of the Acoustical Society of America, vol. 25, no. 5, Sep (1953).

But, like the previous investigations, the experimental phase of this investigation was conducted without the benefit of any guiding theory.

CHAPTER III

THE LABORATORY EQUIPMENT

At the time Professor Hartmann conducted his researches, there was not much sound measurement equipment to choose from. He measured sound-pressure intensities with Rayleigh disks and a form of Westphal balance. Frequencies were obtained by shining a pencil of light on the Riemann shock standing between the pulsator and the nozzle. Since this shock represents a density discontinuity, it served as a mirror. It oscillated at the frequency of the sound emission, and the light beam reflected from this moving Riemann mirror permitted an oscillogram to be traced. This oscillogram provided a visualization of the wave shape, and a means of measuring the frequency. His whistles, designed with adjustable geometry, were mounted in an acoustic cabinet lined with sound-absorbing material to approximate free field conditions.

L. E. Savory had a somewhat more sophisticated, but probably no more effective, instrumentation. A microphone was connected to a sound level meter (with a flat response from 20 to 15,000 cps) and the vertical plane of a cathode ray oscillograph. The horizontal plane of the oscillograph was driven by an electric audio oscillator (20 to 20,000 cps). By dialing the oscillator until a circle was obtained on the oscillograph, the frequency could be read. The wave form was obtained by using the standard oscillograph sawtooth sweep. His microphone was located 24 to 27 inches downstream from the nozzle on a line inclined 30 degrees to the generator axis. The apparatus was mounted in a room with a background sound level of 72 decibels. Savory's air supply consisted of an air pressure tank connected through $\frac{1}{2}$-inch pipe to a standard pipe tee which branched to a calibrated 100 psig. pressure gauge and a $\frac{1}{2}$-inch pipe nipple about $3\frac{1}{2}$ inches long threaded into the nozzle piece. The inherent lack of pressure control in Savory's air supply probably accounts for his concern with decreasing the pressure sensitivity of the Hartmann whistle. His pulsator could be backed off from the nozzle as much or little as he would desire, and the pulsator depth was also adjustable.

Monson and Binder had a somewhat more modern instrumentation than Savory, which is to be expected since their work was done at a later date. The principle difference was the use of a Massa-type microphone (an armored quartz crystal). This permitted them to measure at six inches radially without fear of damaging the microphone. Apparently their air supply was more stable, although no details are given in the cited reference.

At the beginning of this investigation, sound measuring equipment was available throughout a wide range of price, accuracy, versatility and convenience. The equipment for this investigation was purchased in 1957 and 1958. A tabulation of the laboratory equipment used for the Hartmann whistle investigation is presented in Table 1.

TABLE 1.-Sonics Instrumentation for the Hartmann Whistle Investigation

Equipment Designation	Price	Purchase Date	Source
Sound Level Meter, Type 1551-A..	$ 370.00	4-13-57	GRC
Low Frequency Oscillator, 202C..	300.00	6-16-58	HPC
Power Supply, Type 1262-A (for above)....................	70.00	4-13-57	GRC
Attenuator Pad, Type 1551-P11 (for above)....................	15.00	4-13-57	GRC
Tripod and Extension Cable, Type 759-P21 (for above)............	33.50	4-13-57	GRC
Sound Level Calibrator, Type 1552-B (for above)..............	47.50	4-13-57	GRC
Transistor Oscillator, Type 1307-A (for above)..............	80.00	5-19-58	GRC
Sound Analyzer, Type 760-B......	537.10	5-23-57	GRC
Recorder, SL-4, with Link Unit (for above)....................	1140.00	7-22-57	SAC
Microphone, Massa M-141B, with cable..........................	190.00	5-28-57	MLI

Source Codes	Source
GRC	General Radio Company, 275 Massachusetts Avenue, Cambridge 39, Mass.
SAC	Sound Apparatus Company, Stirling, New Jersey
MLI	Massa Laboratories, Inc., Hingham, Massachusetts
HPC	Hewlett-Packard Co., 275 Page Mill Road, Palo Alto, California

The equipment listed in Table 1 were purchased through the University of Oklahoma Research Institute Project 1147 (initially designated 147) by the McDonnell Aircraft Corporation. Other equipment was also purchased under that contract, but was not used specifically in the Hartmann whistle investigation. Some equipment owned by the Chemical Engineering Department of the University of Oklahoma was also used, including a General Radio Type 1555-A Sound-Survey meter used for monitoring noise leakage from the laboratory and for mapping sound fields; and an Eico oscilloscope, for observation of wave forms.

The equipment was used to measure overall intensity, the intensity and frequency of individual sound components, and to record sound spectra.

The air supply system, diagrammed in Figure 1, provided a steady and adjustable nozzle pressure to the Hartmann whistles. The $1\frac{1}{2}$-inch (nominal) pipe from the university 110 psig. air line led to a knockout drum at the ground level on the outside of the building. From the knockout drum, $1\frac{1}{2}$-inch pipe led to the sonics laboratory (on the second floor) to a Kimray pressure reducer, where the pipe diameter increased to 2 inches. The 2-inch pipe was brought to a valve manifold in the control room. The valve manifold consisted of a 1-inch Marsh needle valve, a $\frac{1}{2}$-inch Crane needle valve, and a $\frac{1}{4}$-inch Hoke needle valve with a micrometer handwheel. These valves were all connected in parallel, and permitted

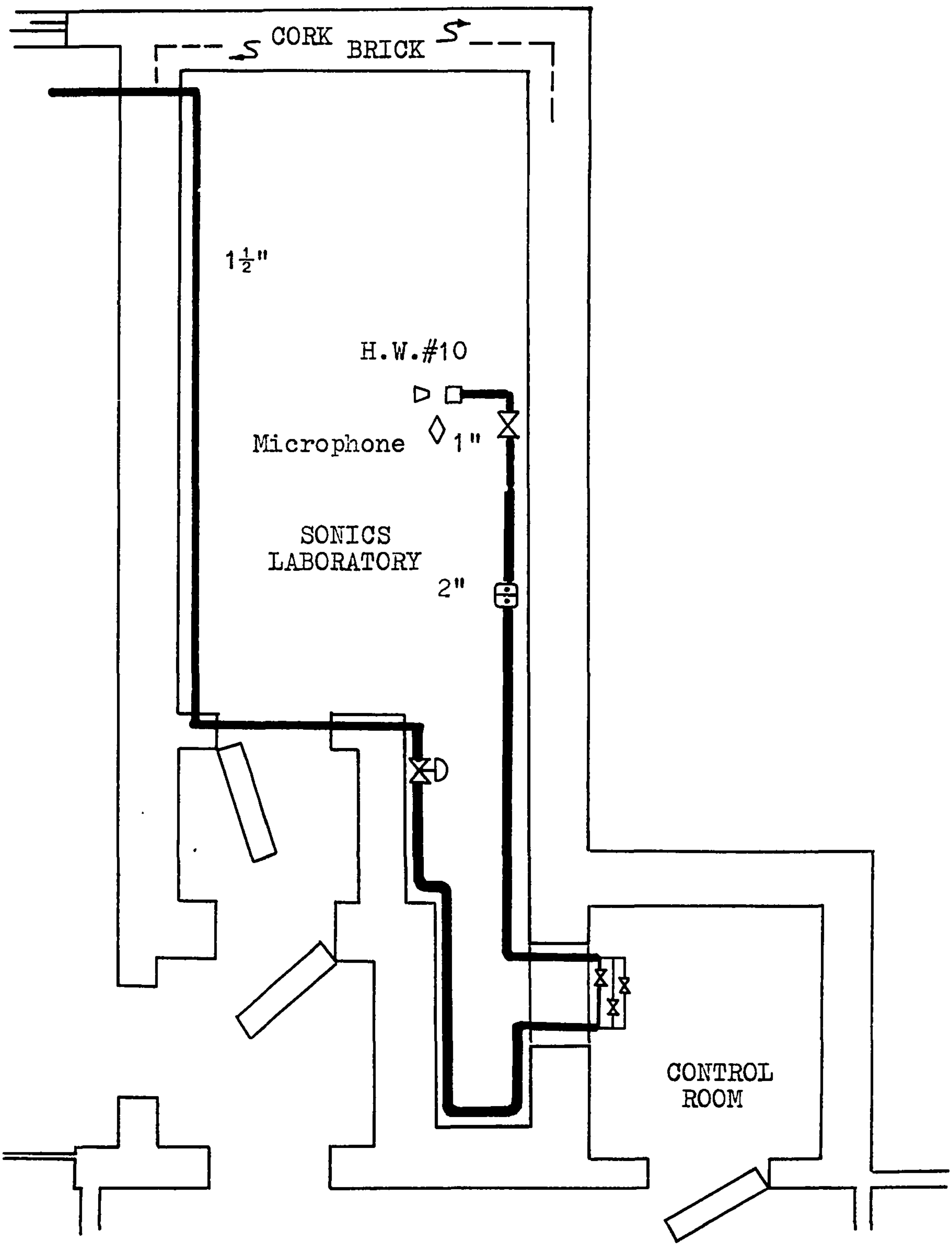

Figure 1.-Air Supply System Layout Scale: ¼" = 1'

precise throttling over a wide flow range. The pressure to the valve manifold could be set to within 1 psig. at the pressure reducer, and the pressure fluctuations were unreadably small. However, it was found for some of the larger Hartmann whistles an attempt to maintain a nozzle pressure in excess of 95 psig. would cause the pressure upstream of the Kimray pressure reducer to fall below the preset downstream pressure. When this occurred, pressure would drift uncontrollably. This was the only situation in which the pressures were not rock-steady. Fortunately, it was not necessary to take sonic data in this region.

The valve manifold led to a standard ASME code 2-inch orifice run with flange taps and without straightening vanes, then through a 2-inch gate valve (for normal shutoff) to a 2-inch "cross" containing a thermowell and a bushing to 1-inch pipe and a normally open solenoid valve (for emergency shutoff) and then to the whistle. A pressure tap just upstream of the nozzle led the nozzle pressure signal through a $\frac{1}{4}$-inch copper tube to a pressure gauge in the control room. The 2-inch diameter orifice meter run was selected, even though it was oversize, because that was the smallest size at that time for which standard orifice plates and flange taps were commercially available.

The instrumentation was connected according to the manufacturer's directions. The microphone (either the Rochelle salt microphone which came with the Type 1551-A sound level meter, or the Massa microphone, depending on whether the sound level was below or above 120 decibels) was mounted in the sonics laboratory on a tripod. The microphone cable passed into the control room through an opening under the sound proof window between the two rooms.

Through this opening passed the air line to and from the throttling needle valve manifold, a gas line to a similar valve manifold, copper tubing from the air and gas orifice runs to their respective mercury manometers in the control room, and shielded 110-volt electrical lines from switches in the control room to the air and gas quick shutdown solenoid valves. The spaces around the lines were blocked by $\frac{1}{2}$-inch thick wooden covers, specially cut with holes just the size of these various lines. The voids within this passage were packed with rags and shipping packing to absorb sound which would be transmitted through the wooden barriers.

The microphone cable passed through this barrier into the control room to the sound level meter. The sound level meter provided a visual reading of the overall sound intensity in decibels. From the output (labeled "PHONES") of the sound level meter a shielded cable (all electrical conductors used were shielded) carried the signal to the Type 760-B sound analyzer.

This analyzer converted the overall signal to a signal proportional to the intensity of the frequency set on its rotatable dial. Range buttons on the face of the analyzer determined the frequency scale and the power of 10 to be applied to the basic frequency setting. The frequency ranges, as cycles per second, were: 25 to 75, 75 to 250, 250 to 750, 750 to 2500, and 2500 to 7500. The intensity at the indicated frequency was readable on a meter on the analyzer (for manual operation of the frequency setting), and was also available as an electrical signal at an output jack.

This output signal was connected to the SL-4 strip chart recorder. The paper drive of the recorder was linked by low-backlash gears and chain to the frequency dial of the sound analyzer. The strip chart paper was ruled for the frequency ranges of the analyzer. This permitted a semi-automatic recording of a sound spectrum throughout the frequency range of the analyzer, by simply pushing the appropriate range button at the right time.

All of the instrumentation could be calibrated at any frequency from 25 to 7500 cps by placing the Type 1552-B sound level calibrator on the microphone (the calibrator was compatible with both microphones), driving the calibrator with the 202C low frequency oscillator, set at 2.0 volts and the desired frequency. It was found to be more convenient to drive the calibrator with the Type 1307-A transistor oscillator, designed for the calibrator. This permitted calibration at only two frequencies, 400 and 1000 cps. However the flat frequency response curves of all the instruments made this type of quick calibration quite adequate.

Although the apparatus was calibrated at the beginning and end of each day on which data were taken (after a recommended warm-up time), the necessary sensitivity adjustments were very small or zero. It was only over a period of months that the cumulative sensitivity adjustments indicated any significant aging of the instrument circuitry. Battery checks on the Type 760-B sound analyzer were accomplished by pushing a button. The current drain on these batteries was quite low, and they lasted many months between changes.

It was quite necessary that the instruments were isolated from strong acoustic exposure in the control room, as the light bulbs in the sonics laboratory had remarkably short filament lives during high intensity sonic experimentation.

CHAPTER IV

EXPERIMENTAL INVESTIGATION OF THE HARTMANN WHISTLE

A Hartmann whistle was fabricated with an adjustable pulsator depth and retreat distance. There were three independent variables in this investigation, retreat distance (the distance between the nozzle and the pulsator), the pulsator depth, and the nozzle pressure. The pulsator was of the quarter wave tube type, consisting of a simple tubular cavity. The whistle, designated as Hartmann Whistle Number 10, is diagrammed in Figure 2.

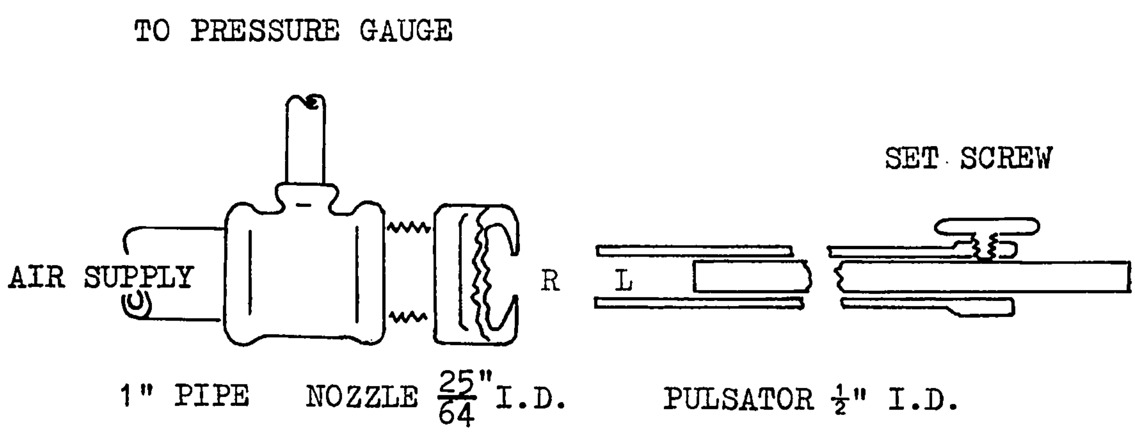

Figure 2.-Hartmann Whistle Number 10.

The nozzle of the whistle was machined from a black one-inch pipe cap, and was sharp-edged (it was about 1/64 inch "deep"). An aluminum rod sliding snugly in a brass tube equipped with a set screw constituted the pulsator. Lanco laboratory clamps and one half inch aluminum tubing were assembled in a framework which was attached to the one eighth inch galvanized pipe pressure tap and also supported from the floor. This framework rigidly positioned the pulsator.

The data were taken by first setting the pulsator depth, then the retreat distance, and finally the nozzle pressure. Thus, a complete set of nozzle pressures would be run before resetting the retreat distance. A complete set of retreat distances would be run before resetting the pulsator depth.

The data, tabulated in the appendix, show the fundamental frequency and intensity, and the total sound level (all measured at 6 inches radially) as functions of the independent variables: pulsator depth, retreat distance, and nozzle pressure. Pulsator depth settings were 0.5, 0.75, 1.0, 2.0, and 4.0 inches. Retreat distance settings, expressed as the ratio of the retreat distance to the nozzle orifice inside diameter, were: 0.9, 1.0, 1.3, 1.5 and 1.6. Nozzle pressures ranged from 3.0 to 85.0 psig. Fundamental frequency ranged from 705 to 5470 cps. Fundamental intensity ranged from 80. decibels (effectively no oscillation) up to

153. db. Total sound levels ranged from 100.3 to 156.3 db.

Although the data permit a wide variety of presentation, the most interesting plot for this study shows the limits of oscillation. These limits were defined arbitrarily in this case as those points for which the fundamental intensity was 100 db, or 25 db below the overall level, whichever appeared first, moving from oscillation to non-oscillation. These limits were defined within 1 psig. for the nozzle pressure. Such a plot is shown in Figure 3, where the previously unobserved low pressure range of oscillation of the Hartmann whistle lies below the line at 13.1 psig.

This figure clearly documents that the Hartmann whistle oscillates below the critical pressure necessary to choke the nozzle.

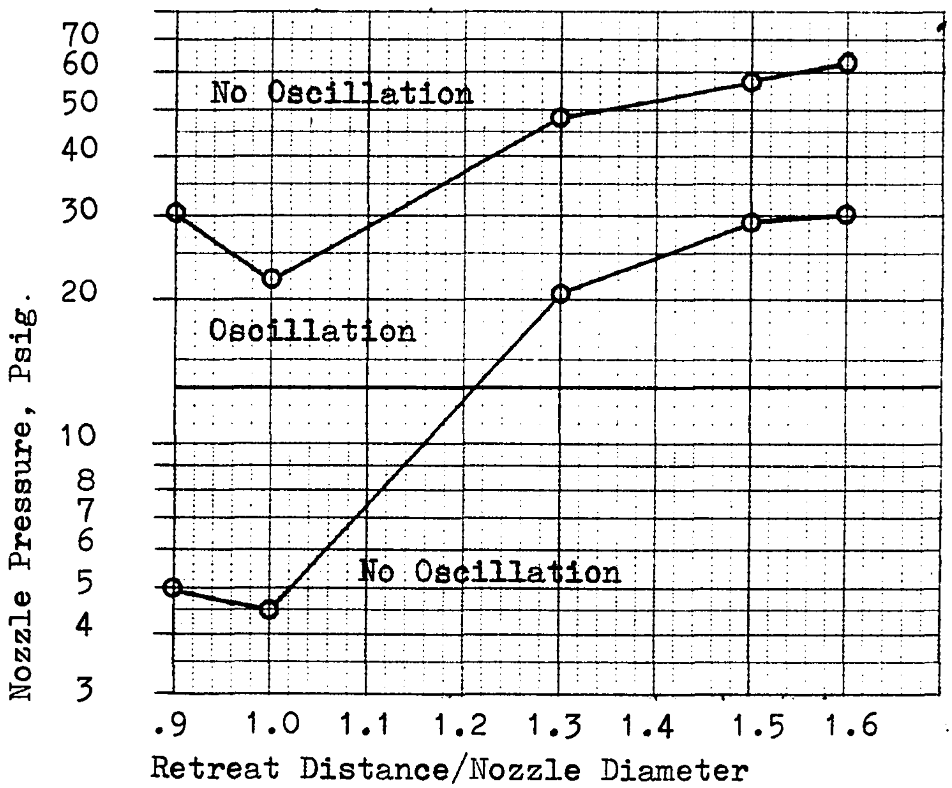

Figure 3.-Zone of Oscillation of Hartmann Whistle Number 10.
Pulsator Depth: 0.75 inch

CHAPTER V

THERMODYNAMIC FOUNDATIONS FOR GENERALIZED NETWORK ANALYSIS

Network analysis is a useful model for complicated systems and processes. The important concepts in network analysis are potential, current and impedance or its reciprocal, mobility.

No general thermodynamic foundation has been previously required by network analysis. It has been so successful in its applications that no real need seemed to exist for examining its foundations. No logical criterion to guide the selection of what quantities should be currents and what quantities should be potentials has been required either. These selections have been made instead by tradition and, when necessary, by the democratic process of voting.

However, the application of the contemporary network analysis for fluid flow to the Hartmann whistle resulted in a failure to predict self-driven oscillation.

An RLC network was constructed for the Hartmann whistle, representing the nozzle as an inductance and a resistance, the pulsator as an inductance, resistance and capacitance in series, and the impedance to ground seen by the escaping air as an inductance and a resistance in series. The response of this network to a step function in nozzle pressure was oscillatory in the steady state <u>only</u> if the pulsator resistance was zero. The presence of even a small pulsator resistance imposed exponential damping on the sinusoidal terms such that the steady state current was constant. If the resistance magnitudes were larger yet, even the initial transient damped oscillation was absent. This behavior is generally characteristic of linear RLC networks. One cannot make an oscillator with only linear RLC elements. In addition to this, the volumetric flow, as current, fails to be conservative around a simple circuit loop. These failures suggested that network analysis does need more rigorous foundations and a logical criterion for current and potential selection.

Historically, network theory was developed for the analysis of electrical networks. The widest contemporary application of the discipline is still electrical. There is a natural tendency to associate network theory with electricity. Indeed, Beranek, whose work represents perhaps the most advanced non-electrical application of network theory, writes, "The subject of electro-mechano-acoustics (sometimes called dynamical analogies) is the *application of electrical-circuit theory* [italics mine] to the solution of mechanical and acoustical problems."[1]

---

[1]L. L. Beranek, "Acoustics," McGraw-Hill Book Co., Inc., 1954, p. 47.

This tendency to equate network theory with electrical network theory should be avoided. Electrical network theory was derived for the special needs of electrical flow, making some assumptions (for example neglecting the kinetic energy of the electrons in motion) which do not necessarily hold when some other type of flow (fluid flow in a rocket engine, for example) is considered. However, the basic philosophy and structure of the lumped-system approximation is common to all forms of network theory.

The prime objective of network theory is, as defined in a more general context, to establish a satisfactory lumped system model. The one-to-one correspondence between electrical, acoustical, mechanical, magnetic, and thermal circuits, emphasized by the dynamical analogies discipline, is not sought here. These close correspondences may not always exist. If in the unbiased development of different networks such correspondences appear fortuitously, then an interesting analogy exists. These analogies should not be forced. The usual price for obtaining a tight analogy between two systems is a loss of fidelity in the model for one or more of the systems.

The terms, "network" and "circuit" will be used more or less interchangeably here. Many authors tend to use one to the exclusion of the other. Semantically, a network is more complicated than a circuit, but a precise distinction between the two words is not practiced in the literature.

The formal application of circuit methods to fluid flow has been mostly acoustical. For acoustic circuits, several simplifying assumptions are made. For bulk flow these assumptions are frequently not justified. The neglect of kinetic energy is one such assumption.

In acoustical circuits, involving sound without bulk flow, the specific volume does not change appreciably. The classical selection of total volume flux as current and static pressure as potential has proven successful in acoustic circuits. Circuit analysis requires that the current selected must be conservative around a loop. And yet Olson[2] has applied the acoustic analogy to a ductwork system in which a purely steady flow without acoustic transfer occurs with a steady pressure rise source (a blower or fan) driving a volumetric current through a passive network of fluid flow resistances (the viscous dissipation in the various branches and legs of the ductwork system). The circuit theory method is quite practical for designing complicated ductwork layouts. Since the blower discharge pressure is normally measured in inches of water, the inherent assumption that volume is conserved is at least approximately obeyed.

A valid circuit representation should involve a current which is a total flux of a conservative extensive property. Currents should be chosen as conservative species fluxes.

---

[2] H. F. Olson, "Dynamical Analogies," 2d ed., D. Van Nostrand Co., 1958, p. 201.

With this criterion for defining current, there remains the question as to what a potential should be. It may be noticed that classical dynamical analogies have selected currents and potentials such that the product of current and potential represents an energy total flux. Moreover, the definition of electrical potential identifies electrical potential (the volt is a common unit) as an amount of work per unit charge necessary to bring that charge from some reference level up to the point at which the potential is measured.

The reference level in circuits is chosen as some ground state or dump reservoir big enough so that enough electrons are freely available to charge whatever capacitances may accumulate them. This restriction to the ground is stated because the ground may be only a metallic chassis. The ground is thus assumed not to "run out" of electrons during an accumulation of electrons in some part of the circuit.

The potential at a point with respect to some "dump," ground, or reference state may be defined as the work investment per unit current species necessary to bring that species from the dump state to the specified point. The concepts of work and the dump or ground state reveal the thermodynamic basis of circuit analysis.

Potential, the specific work investment (availability), is an intensive property. The current is a flux of an extensive property. The circuit loops represent cyclic processes. Thermodynamics, before Gibbs, analyzed processes primarily by the construction of equivalent cycles. The work of J. W. Gibbs and E. A. Guggenheim popularized the more convenient use of potentials. The methods are complementary, and aspects of both are clearly evident in circuit analysis.

The thermodynamics referred to is classical macroscopic open system thermodynamics. Recently, the word thermostatics has been proposed to denote classical thermodynamics, with the word thermodynamics to be used for either the discipline now called irreversible thermodynamics (or, less frequently, non-equilibrium thermodynamics),[3] or for non-steady state transport phenomena.[4] Veinik seeks to use the term thermo-kinetics for the irreversible thermodynamic discipline. However, this discussion will attempt to retain the accepted designations, thermodynamics, irreversible thermodynamics, and transport phenomena.

The generalized concept of thermodynamics which corresponds to the work potential of electrical or mechanical circuits is that of maximum useful work or availability. This represents the total work obtainable from a system in terms of the difference between the values of its thermodynamic potentials and those values at a specified ground state.

At this point it would be possible to synthesize a fluid flow circuitry based on mass flow rate as current, and

---

[3]M. Tribus, "Thermostatics and Thermodynamics," D. Van Nostrand Co., Inc., 1961, p. 383.

[4]A. I. Veinik, "Thermodynamics," NASA TT F-148, 1965, p. ix.

the availability per unit mass as potential. However, the use of the availability concept to define potentials requires a discussion in some detail.

Availability is not a new concept. Schottky, Ulich, and Wagner[5] made use of the reversible work between two states as a criterion for equilibrium and as a state function as early as 1929. Their work considered not only pressure-volume and temperature-entropy effects, but chemical reactions as well. In 1932, Keenan[6] defined an availability for closed systems accounting for Carnot and pressure-volume work investments only.

Then, in 1938, George Granger Brown[7] presented a paper before the American Institute of Chemical Engineers formulating a general availability in the tradition of Gibbs, including Carnot, pressure-volume, surface energy, electrostatic, gravitational, diffusional, and "etc." terms. Brown used his availability to define equilibrium in an unconventional (and more modern) way, pointing out that equilibrium between two states with macroscopically different pressures, temperatures, electrostatic potentials, etc., may exist, just so the reversible work between the two states is zero. Thus equilibrium was defined as the condition of balanced

---

[5]W. Schottky, H. Ulich, and C. Wagner, "Thermodynamik," Berlin, 1929.

[6]J. H. Keenan, _Mech_. _Eng_., 54, (1932), pp. 195-204.

[7]G. G. Brown, _Trans_. _Am_. _Inst_. _Chem_. _Engrs_., 34, (1938), p. 489.

general availability.

Somewhat later, Barnett F. Dodge[8] presented a definition of availability, and, using the symbol B, obtained:

$$-\Delta B = [\Delta H - T_o \Delta S]_{T_1}^{T_2} \tag{5.1}$$

where T_o denotes the dump temperature.

The availability concept has not been universally popular. In 1950 in the French language "Thermodynamique Chimique" (later translated into English by D. H. Everett), Prigogine and Defay treat chemical reactions by the explicit use of the entropy production. They regard the use of maximum work as being "inconvenient" and subject to "obscurities and complications."[9] Prigogine and Defay refer in particular to the application by Schottky, Ulich and Wagner[10] of the "loss of useful work" as a criterion for irreversibility, maintaining that "the concept of loss of useful work has no clear physical interpretation."[11] Prigogine and Defay show that the reversible work concept yields the same result as the entropy production approach for <u>isothermal</u> chemical

---

[8]B. F. Dodge, "Chemical Engineering Thermodynamics," McGraw-Hill Book Company, Inc., 1944, pp. 74-76.

[9]I. Prigogine and R. Defay, "Chemical Thermodynamics," translated by D. H. Everett, John Wiley and Sons, 1954, p. xvi.

[10]W. Schottky, H. Ulich and C. Wagner, "Thermodynamik," Berlin, 1929, <u>et passim</u>.

[11]I. Prigogine and R. Defay, op. cit., pp. xvii-xviii.

reactions, but they state that the entropy production concept is required if the process is nonisothermal. Prigogine and Defay do not include an entry for availability in their index.

C. M. Sliepcevich and D. Finn credit Bryan with "presumedly the first complete and precise treatment of the availability of energy and its relationship to the entropy concept"[12] in 1907, predating any of the work cited here.

Brown[13] writes an expression for the differential change in availability due to a number of currents passing through potential drops. It is abridged here to a form similar to Sliepcevich's expression.

$$-\delta W_{max} = \delta S dT + \delta V dP + \delta\left(\frac{mg}{g_c}\right) dh + \delta(n\mathcal{F}) d\xi$$
$$+\delta m_A d\mu_A + \delta m_B d\mu_B + \delta\sigma\, d\gamma + \text{etc.} \qquad (5.2)$$

Sliepcevich[14] expresses the Brown availability as

$$-\delta W_{max} = \delta\left(\frac{Q}{T}\right) dT - \delta V dP + \delta\left(\frac{mg}{g_c}\right) dz + \delta(n\mathcal{F}) d\xi$$
$$+ \delta m_A d\overline{G}_A + \delta m_B d\overline{G}_B + \delta\sigma\, d\gamma + \text{etc} \qquad (5.3)$$

The expressions are similar except that the sign on the term δVdP differs. Sliepcevich uses Q/T in place of

---

[12] C. M. Sliepcevich and D. Finn, "Thermodynamics," Perry's Chemical Engineers' Handbook, Fourth Edition, McGraw-Hill Book Co., Inc., 1963, p. 4-38.

[13] G. G. Brown, op. cit.

[14] C. M. Sliepcevich and D. Finn, op. cit., p. 4-44.

entropy, and partial molal Gibbs free energy in place of chemical potential, and z in place of h for elevation (Brown used the symbol G for availability - which will be avoided here to prevent confusion with Gibbs free energy). All these differences are trivial matters of notation, with one important exception, the sign on the term δVdP, which was reversed in order to make Brown's expression compatible with the Gibbs equation for internal energy.[15]

The desired availability function satisfies two requirements. It must be a state function, and it must be sufficiently general to apply to whatever currents are required. That is, it must define an availability for each extensive quantity in transition.

Although the availability concept can apparently serve as a basis for potential selection, it is not the one most frequently applied, at least recently. The modern literature uses irreversible thermodynamics as the fundamental description of fluxes and their conjugate potential forces. Therefore one must consider the application of irreversible thermodynamics to generalized circuit theory.

Irreversible thermodynamics begins with an expression for the rate of dissipation. The dissipation rate may be denoted as D for the time being. The basic equation expresses D as the sum of a number of flux-force products.

---

[15] C. M. Sliepcevich, <u>op. cit</u>.

$$D = J_1X_1 + J_2X_2 + J_3X_3 + \cdots \tag{5.4}$$

Everything from this point is straight forward and consistently practiced in the discipline. Each flux, J, is related to all the forces via phenomenological coefficients.

$$J_1 = L_{11}X_1 + L_{12}X_2 + L_{13}X_3 + \cdots \tag{5.5}$$

$$J_2 = L_{21}X_1 + L_{22}X_2 + L_{23}X_3 + \cdots \tag{5.6}$$

The Onsager relations are accepted, or derived

$$L_{12} = L_{21}, \quad L_{23} = L_{32}, \quad L_{31} = L_{13}, \quad \cdots \tag{5.7}$$

The selection of D in the dissipation expression automatically then determines the phenomenological equations (rate equations or transport equations), the definition of the phenomenological coefficients, and their corresponding Onsager relations. The whole thing hinges upon what selection of D is made. Some of the more popular options are tabulated in Table 2. Some authors use two forms, and change from one to the other with little or no explanation. All the dissipation expressions involve the extensive rate of entropy production, dS_i/dt. The structure of irreversible thermodynamic theory is uniform regardless of what form of D is selected. Therefore the heat conduction Onsager coefficients (L_{11}) generated by each form of the dissipation rate represent a conflicting version of either what component of

TABLE 2.-Comparison of Different Dissipation Equations Used in the Literature of Irreversible Thermodynamics

Selected Form of Rate, D	Authors Who Use the Selected Form	The Resulting Expression for One-Dimensional Conductive Heat Transfer	References	Pages
$\frac{dS_i}{dt}$	S. R. de Groot.....	$\frac{\dot{Q}}{A} = -\frac{L_{11}V}{T^2}\frac{\partial T}{\partial x}$	16	7,22,28
	I. Prigogine.......		20	40
	G. N. Lewis, M. Randall, *et al*..		21	454
	P. Van Rysselberghe		22	37
	M. Tribus..........		23	619
$T\frac{dS_i}{dt}$	I. Prigogine.......	$\frac{\dot{Q}}{A} = -\frac{L_{11}V}{T}\frac{\partial T}{\partial x}$	20	41
$\frac{1}{V}\frac{dS_i}{dt}$	S. R. de Groot.....	$\frac{\dot{Q}}{A} = -\frac{L_{11}}{T^2}\frac{\partial T}{\partial x}$	16	40
	S. R. de Groot and P. Mazur........		18	65
	K. G. Denbigh......		17	29
$\frac{T}{V}\frac{dS_i}{dt}$	K. G. Denbigh......	$\frac{\dot{Q}}{A} = -\frac{L_{11}}{T}\frac{\partial T}{\partial x}$	17	29
	D. D. Fitts........		19	153
	S. R. de Groot and P. Mazur........		18	344

[16]S. R. de Groot, "Thermodynamics of Irreversible Processes," North-Holland Publishing Company, Amsterdam, 1958, pp. 7, 22, 28, 40.

[17]K. G. Denbigh, "The Thermodynamics of the Steady State," John Wiley and Sons, Inc. (Methuen Monographs on Chemical Subjects), 1951, p. 29.

[18]S. R. de Groot and P. Mazur, "Non-Equilibrium Thermodynamics," North-Holland Publishing Company, Amsterdam, 1962, pp. 21, 65, 344.

[19]D. D. Fitts, "Nonequilibrium Thermodynamics," McGraw-Hill Book Co., Inc., 1962, pp. xvii, 153.

[20]I. Prigogine, "Introduction to Thermodynamics of Irreversible Processes," Second Revised Edition, John Wiley and Sons (Interscience), 1961, pp. 40, 41.

thermal conductivity approaches a constant or, more exactly, what form the heat conduction rate equation approaches as the departure from equilibrium becomes small. For heat conduction the gradient of the temperature or the gradient of some function of the temperature may be taken as a measure of the departure from equilibrium.

It is apparent that none of the dissipation rates tabulated yields the Fourier law of heat conduction. Rather, they suggest $1/T$ or $\ln(T)$ in place of T in Fourier's law. To be sure, Fourier's law is not a law, but it is an implicit definition of a property, the thermal conductivity. Since thermal conductivity is a different function of temperature (and pressure) for each substance, the temperature functionality of thermal conductivity is not an adequate criterion to distinguish between the various candidate forms of the dissipation rate. Indeed, no such criterion exists in the literature.

Since irreversible thermodynamics does not provide an unique selection of potentials and currents, the availability concept is instead chosen arbitrarily as a guide.

Availability-based potentials should be datum level

---

[21] G. N. Lewis, M. Randall, K. S. Pitzer and L. Brewer, "Thermodynamics," McGraw-Hill Book Co., Inc., 1961, p. 454.

[22] P. Van Rysselberghe, "Thermodynamics - of Irreversible Processes," Blaisdell Publishing Co., 1963, p. 37.

[23] M. Tribus, "Thermostatics and Thermodynamics," D. Van Nostrand Co., Inc., 1961, p. 619.

invariant. They should also be state properties (exact differentials). The selection of potentials for circuit analysis effectively is also a selection of the dissipation function, D, on which irreversible thermodynamics is based. All of the dissipation functions tabulated were based on some expression involving the entropy production, which for a non-differential potential drop, cannot be evaluated directly, but instead is determined from an entropy balance in which all other terms are known for the steady state. All the other terms in the entropy balance, except the entropy accumulation term, are system boundary terms which are inherently known. The reason for not using lost work as a basis for potential is that it is not generally a state function.

Sliepcevich[24] expresses the maximum useful work as

$$-(W_{max})_u = (H_{P_D,T_D} - H_{P,T}) - T_D(S_{P_D,T_D} - S_{P,T}) \qquad (5.8)$$

where kinetic and potential energies have been neglected. This expression is a form of the availability. A potential for fluid flow may be arbitrarily defined as the maximum useful work recoverable from a unit mass of the fluid by adiabatic reversible operations which leave the fluid sample at the datum pressure, velocity, and elevation. Since the operations which define this potential are constrained to be adiabatic, the final temperature of the sampled mass will not

---

[24] C. M. Sliepcevich, op. cit., p. 4-42.

necessarily be the datum temperature, but may be greater or less than the datum temperature depending on the state of the sampled fluid mass and the datum state. The adiabatic constraint on the measuring operation is not entirely arbitrary, since it causes differences of the defined potential to be datum level invariant. Eqn. (5.8), subjected to the measuring operation constraint which has been selected, and augmented with kinetic and potential energy terms, permits the flow potential just described to be written as

$$(\underline{W}_{max})_S = \int_{P_D}^{P} [\underline{V}dP]_{\underline{S}} + v^2/2g_c + (g/g_c)z \tag{5.9}$$

where the sub-bars refer to extensive properties per unit mass.

Note that the term "adiabatic" refers to the measurement operation, and not to the flow in a network. For example, in an actual flow process, heat may be added to an element in a flow circuit such that the pressure drop across the element is zero, and the kinetic and gravitational drops across the flow element may be negligible. However, the adiabatic flow potential normally increases since the volume integral in Eqn. (5.9) normally increases under these conditions. There are exceptional cases, where a fluid may have a negative value of the partial of specific volume with respect to entropy at constant pressure (water, for example, near its freezing point) in which a small heat addition might decrease the adiabatic flow potential under these conditions. In any case, Eqn. (5.9) represents the maximum specific availability which can be measured by an adiabatic operation. The remainder of the availability is measured in a Carnot or heat-pump type operation, and is attributed to the thermal circuit.

The end result of this chapter is the adiabatic availability flow potential as given by Eqn. (5.9). This flow potential will be represented by the symbol b. It is used throughout the rest of this dissertation as the potential associated with mass current.

CHAPTER VI

CIRCUIT THEORY AND THE HARTMANN WHISTLE

As has been mentioned, the Hartmann whistle yields an oscillatory output from a constant input, displaying self-excited oscillation in ranges of its parameters. Self-excited oscillation is not unusual in electric circuits. Electric network analysis has circuit elements for the vacuum tubes and transistors normally responsible for self-excited oscillation.

However, acoustic circuit analysis does not have circuit elements analogous to the vacuum tube or the transistor. Although, from the viewpoint of the dynamical analogy discipline, it might be tempting to search for acoustic analogs to the vacuum tube and the transistor (the methods presented by Trent[1] are a potent tool for such a search), the strategy in this investigation is to derive independently circuit elements for a one-mass-component fluid flow circuit, letting the analogies fall where they may.

First a purely dissipative fluid flow element, the fluid resistance, will be examined. In fluid flow as opposed

---

[1] H. M. Trent, "Isomorphisms between Oriented Linear Graphs and Lumped Physical Systems," J. Acoust. Soc. Am., 27, (1955), pp. 500-527.

to electrical flow, there are two common types of dissipation. Laminar flow dissipation will be considered first. If the fluid resistor is a circular straight pipe with a very short length, ΔX, a momentum balance yields the Hagen-Poiseuille equation, presented here in a form modified to include variable density, ρ. The viscosity is μ, velocity is v, and pressure is P.

$$\dot{M} = -g_c\rho^2A^2[(\Delta P)/\rho + (\Delta\rho v^2)/g_c\rho]/8\pi\mu\Delta X \tag{6.1}$$

Since the flow is assumed to be steady state, and the cross sectional area, A, is constant, ρv is constant, and the expression above for the mass flow rate, $\dot{M}$, becomes

$$\dot{M} = -g_c\rho^2A^2[(\Delta P)/\rho + (v\Delta v)/g_c]/8\pi\mu\Delta X \tag{6.2}$$

The adiabatic flow potential will now be labeled as "b." A differential of the flow potential appears within the square brackets of the last equation.

$$\dot{M} = -g_c\rho^2A^2[\Delta b]/8\pi\mu\Delta X \tag{6.3}$$

This last equation may be integrated

$$\dot{M}X = -(g_cA^2/8\pi)\int_{b_{in}}^{b_{out}}(\rho^2/\mu)db \tag{6.4}$$

Now, in general both density, ρ, and viscosity, μ, will be functions of the flow potential. However, mean values may be defined

$$\dot{M}X = [g_c A^2 \rho_m^2 / 8\pi\mu_m](b_{in} - b_{out}) \qquad (6.5)$$

This last expression may be put in the form corresponding to the expression for a resistance

$$(b_{in} - b_{out}) = [8\pi\mu_m X / g_c A^2 \rho_m^2]\dot{M} \qquad (6.6)$$

$$(b_{in} - b_{out}) = \dot{M}\ R \qquad (6.7)$$

This last expression is equivalent to E=IR, Ohm's law, in electrical circuits. The expression for laminar fluid resistance in a circular pipe is a function of the pipe geometry and the flowing fluid properties. A nonlinear expression was avoided by taking "mean" values of the fluid properties.

Before examining the turbulent resistance, expressions for fluid inductance and fluid capacitance will be derived. Fluid inductance may also be derived from a short piece of circular cross-section pipe. Once again, no mass accumulation will occur, but in this case mass flow rate is assumed to be increasing with time, and viscosity is neglected. A momentum balance, neglecting viscous drag forces, becomes

$$A\Delta X \frac{d\rho v}{dt} = g_c AP_{in} + A(\rho v^2)_{in} - g_c AP_{out} - A(\rho v^2)_{out} \qquad (6.8)$$

Or, if a Δ is now used to represent a potential drop

$$A\Delta X \frac{d\rho v}{dt} = g_c A\Delta P + A\Delta\rho v^2 \qquad (6.9)$$

The last term may be expanded, and the equation divided by density

$$\frac{A\Delta X}{\rho} \frac{d\rho v}{dt} = g_c \frac{A\Delta P}{\rho} + A\ v\Delta v + \frac{Av\Delta\rho v}{\rho} \qquad (6.10)$$

The last term vanishes because the mass flow input is assumed equal to the mass flow out. The flow is being treated as effectively one dimensional. Since the mass flow is $A\rho v$, the equation may be put in the form

$$\frac{\Delta X}{g_c \rho} \frac{d\dot{M}}{dt} = A\Delta b \qquad (6.11)$$

Once again, this equation can be integrated only by assuming an average value of ρ.

$$\Delta b = \frac{X}{g_c A\rho_m} \frac{d\dot{M}}{dt} \qquad (6.12)$$

This last equation has the same form as $\Delta E = LdI/dt$, where the fluid inductance, L, is $X/g_c A\rho_m$.

It will be noticed that this inductance derivation used the same piece of pipe as was used for the laminar resistance. This is possible because of the recognized distinction between a resistor and a resistance (the abstraction). The physical behavior of a resistor (a real device) is represented by abstracting from it a resistance, and a small inductance and capacitance.

In order to derive the expression for fluid capacitance, consider a volume into which fluid flows, and accumulates. Suppose that the process is isentropic. The capacitance, C, is defined by

$$d(b_{in} - b_d)/dt = (1/C)\dot{M} \qquad (6.13)$$

The specific availability, b_{in}, of the fluid flowing into the capacitance may involve a kinetic energy term. Although the fluid in the capacitance is assumed to be at rest, the flow is also assumed to be reversible, and therefore ideal pressure recovery of the kinetic energy occurs. Thus the specific availability, b, inside the volume of the capacitance is equal to the inlet value. Upon further assuming no heat transfer, an entropy balance requires that the specific entropy shall remain constant. The possibility of "charging" the capacitance with a mass flow of varying entropy is excluded by the requirement for reversibility. Eqn. (6.13) may therefore be written.

$$\underline{V}dP/dt = (1/C)\dot{M} \qquad (6.14)$$

If the volume of the enclosure does not change, a mass balance becomes

$$V_{sys}\ d\rho/dt = \dot{M} \tag{6.15}$$

Since entropy is constant, it will be convenient to regard density as a function of pressure and entropy.

$$d\rho = [d\rho/dP]_{\underline{S}}dP + [d\rho/d\underline{S}]_P d\underline{S} \tag{6.16}$$

The last term vanishes, since this process has a constant $\underline{S}$. The velocity of sound, a, is a thermodynamic property. It is given as

$$a^2 = g_c\ [dP/d\rho]_{\underline{S}} \tag{6.17}$$

Therefore, Eqn. (6.16) may be written

$$d\rho = (g_c/a^2)dP \tag{6.18}$$

Substitution of Eqn. (6.18) into Eqn. (6.15) yields

$$V_{sys}g_c/a^2)\ [dP/dt] = \dot{M} \tag{6.19}$$

This equation may be rearranged.

$$\underline{V}[dP/dt] = (a^2\underline{V}/V_{sys}g_c)\dot{M}$$

Now, remembering that Eqns. (6.13) and (6.14) are equivalent,

$$[db_{in}/dt] = (a^2\underline{V}/V_{sys}g_c)\dot{M} \tag{6.21}$$

This equation may be integrated, using mean values of the density and sound velocity.

$$b_{in}(t) - b_{in}(t=0) = (a_m^2/\rho_m\ V_{sys}\ g_c)\int_0^t \dot{M}dt \tag{6.22}$$

This expression indicates that the fluid capacitance, C, of a volume, V_{sys}, is

$$C = g_c \rho_m V_{sys}/a_m^2 \tag{6.23}$$

As in the derivation of the expressions for resistance and inductance, average values of properties which may vary considerably were taken in the integration to yield a linear circuit element. Similar assumptions must be made in the derivation of electrical circuit elements. Circuit analysis, with its lumped system approach, is inherently an approximation. It is dependent on continuum analyses for circuit element expressions. Fluid flow, even without considering several components, heat transfer, or chemical reactions, has more modes of common behavior than electrical flow. This complicates the analysis of fluid flow circuitry, but at the same time gives fluid flow circuits the capability of a wide range of behavior.

These circuit element derivations are far from exhaustive. The laminar resistance can be generalized for ducts of any shape. Since contemporary circuit analysis for fluid flow is almost exclusively limited to acoustics, general fluid flow circuit elements are not available in the literature. However, a number of acoustic circuit element expressions are available in terms of excess pressure, P_e, (of sound) and volume current. These acoustic flow elements can be adapted for use as general flow elements by interpreting acoustic excess pressure as the total pressure <u>above</u> the datum pressure level. In other words, sound propagation is a kind of unsteady fluid flow in which the pressure varies only infinitesimally from the datum pressure. The differential acoustic impedance, $dP_e/d\dot{V} = Z_{acoustic}$, may be converted to the differential fluid flow impedance $d(P/\rho)/d\dot{M} = Z_{acoustic}/\rho_m^2 = Z_{fluid\ flow}$.

The next flow element to be considered is vital for the circuit representation of the Hartmann whistle and many other fluid flow devices. It is the T-junction. Electrical circuits usually have many T-junctions, as do circuits in general. However, since electrical circuit analysis traditionally neglects the kinetic energy of the electron flow, the T-junction is no problem electrically, because the potential is the same all around the junction.

In fluid flow the T-junction exhibits a somewhat more sophisticated behavior. Several steady state fluid flow devices are based on this behavior. These devices include the hose end sprayer, and certain ejectors and siphons. In DC operation the potentials at the two arms of the "T" may be considerably greater than the datum potential, and yet the potential of leg of the "T" may be considerably less. Consequently, even in the steady state, fluid flows from the datum level up the leg of the T. Certain electrical transformers exhibit a similar behavior for AC flow, but in no case does the T-junction in conventional electrical circuits exhibit the hose-end sprayer phenomena for steady DC flow.

Therefore, while the T-junction is not regarded as a circuit element in electrical circuitry, it must be regarded as a circuit element in fluid flow circuitry. With each circuit element, there is an associated equation or equations relating the potential drop(s) across the element to the current(s) through the element. These equations effectively <u>define</u> the circuit element. Simplified relations for the T-junction circuit element may be derived in the following manner.

Consider a T-junction as the junction of three fluid flows. Although the configuration shown in Figure 4 of the junction is selected to correspond to the geometry of an axially symmetric Hartmann whistle, these same methods may be used to derive junction circuit elements for other junction geometries.

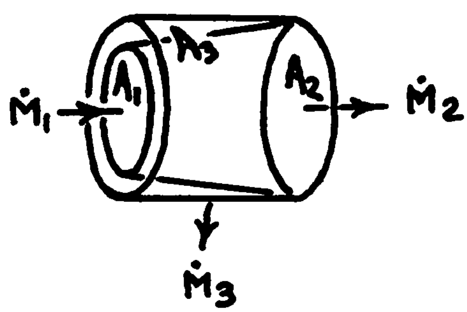

Figure 4.-Axially Symmetric T-junction.

Expressions for capacitance (idealized mass accumulation), resistance (viscous dissipation) and inductance (idealized momentum accumulation) have already been derived. Therefore, only effects other than these will be attributed to the junction circuit element. If the bulk flows are assumed to be normal to the areas through which they flow, the steady-state momentum balance in the axial direction may be written

$$g_c P_1 A_1 + A_1 \rho_1 v_1^2 + g_c P_3 (A_2 - A_1) - g_c P_2 A_2 - A_2 \rho_2 v_2^2 - \dot{M}_3 v_{3x} = 0 \qquad (6.24)$$

The treatment will be vastly simplified by taking a mean density, and by regarding the body of the junction as existing at a uniform static pressure. This, in effect causes static (not stagnation) pressure to be the same at each node of the junction. Under these assumptions, the momentum balance reduces to

$$A_1 \rho_1 v_1^2 - A_2 \rho_2 v_2^2 - \dot{M}_3 \, v_{3x} = 0 \qquad (6.25)$$

The momentum balance effectively determines the axial component of the velocity of the "escaping" mass stream, $\dot{M}_3$. This last equation may be solved for the velocity component.

$$v_{3x} = (\dot{M}_1^2 / A_1 \rho_m - \dot{M}_2^2 / A_2 \rho_m) / \dot{M}_3 \qquad (6.26)$$

The corresponding radial velocity component may be written in terms of the cylindrical area, A_3.

$$v_{3n} = \dot{M}_3/A_3\rho_m \tag{6.27}$$

The specific kinetic energy of the escaping mass stream is

$$v_3^2 = [(\dot{M}_1^2/A_1\rho_m - \dot{M}_2^2/A_2\rho_m)/\dot{M}_3]^2 + [\dot{M}_3/A_3\rho_m]^2 \tag{6.28}$$

Subjected to the assumptions of mean density and uniform static pressure, expressions for the potential drops across the junction become quite simplified.

$$b_1 - b_2 = v_1^2/2g_c - v_2^2/2g_c \tag{6.29}$$

$$b_1 - b_3 = v_1^2/2g_c - v_3^2/2g_c \tag{6.30}$$

$$b_2 - b_3 = v_2^2/2g_c - v_3^2/2g_c \tag{6.31}$$

Kinetic energy is not assumed to be conserved around the junction. The steady state mass balance is simply

$$\dot{M}_1 = \dot{M}_2 + \dot{M}_3$$

Under the rather restrictive assumptions which have been made, the cross-sectional areas effectively control the potentials through the relation

$$\dot{M}_i = A_i\rho_m v_i \tag{6.33}$$

Substitution of Eqns. (6.28), (6.32) and (6.33) into the potential drop expressions, Eqn. (6.29) and (6.30), yields

$$b_1 - b_2 = \dot{M}_1^2/2g_c\rho_m^2A_1^2 - \dot{M}_2^2/2g_c\rho_m^2A_2^2 \tag{6.35}$$

$$b_1-b_3 = \dot{M}_1^2/2g_c\rho_m^2A_1^2-(1/2g_c\rho_m^2)[(\dot{M}_1^2/A_1)-(\dot{M}_2^2/A_2)]^2/[\dot{M}_1-\dot{M}_2]^2$$
$$-(1/2g_c\rho_m^2)\ [\dot{M}_1-\dot{M}_2]^2/A_3^2 \tag{6.36}$$

$$b_3-b_2 = (1/2g_c\rho_m^2)[(\dot{M}_1^2/A_1)-(\dot{M}_2^2/A_2)]^2/[\dot{M}_1-\dot{M}_2]^2$$
$$+\ (1/2g_c\rho_m^2)[\dot{M}_1-\dot{M}_2]^2/A_3^2 - \dot{M}_2^2/2g_c\rho_m^2A_2^2 \tag{6.37}$$

These expressions for the potential drops across the T-junction circuit element become somewhat simpler if $A_1 = A_2$. However, the potential drop expressions remain nonlinear. Eqns. (6.35), (6.36), and (6.37) will be taken as the defining equations for the T-junction circuit element.

Although the T-junction is the only nonlinear circuit element which is <u>required</u> for the circuit approximation of a Hartmann whistle, other nonlinear circuit elements exist in fluid flow. Like the T-junction, these elements do not have electrical counterparts.

For example, the ideal converging-diverging nozzle with isentropic flow has a constant value of the specific availability along its entire length. In the steady state, the de Laval nozzle exhibits no drop in the flow potential. And yet the mass flow current is limited by the nozzle flow element. The current varies with the input potential (because increasing the input pressure increases the density and the sonic velocity at the throat of the nozzle). Electrical circuit theory has no dissipationless circuit element which limits a steady current.

The turbulent dissipation flow element could be considered as a nonlinear resistance whose value is proportional to the current. Since the equation which effectively defines the turbulent resistance involves the second power of current explicitly, rather than the first power, the turbulent resistance is really a different circuit element, in the sense that its potential drop expression is different from laminar resistance. It is the equations which are actually used to model a part of the circuit behavior which determine what circuit elements abstractions are used. A section of pipe in turbulent flow may be approximated by a linear resistance with a mean value. Or a nonlinear circuit element which is a function of current or potential may be described as a linear circuit element with a periodically varying value if the circuit is restricted to steady-state AC operation. Dr. D. G. Tucker has applied this technique to the analysis of modulators and frequency changers.[2] The specific techniques used to solve circuit equations are a mathematical problem. The operation of constructing the circuit model of a system is distinct from the operation of mathematically solving the resulting equations.

---

[2]D. G. Tucker, "Circuits with Periodically-Varying Parameters," D. Van Nostrand Co., Inc., London, 1964, pp. 1-15, <u>et passim</u>.

In order to construct a circuit model for the Hartmann whistle, only the circuit elements which have been introduced will be required. They are resistance, inductance, capacitance, and the T-junction. Of these, only the junction will be accepted as inherently nonlinear. In reality, for every kind of circuit, be it electrical, thermal, fluid dynamic, or whatever, every element is somewhat nonlinear. For many system-processes these nonlinearities have no important effect on system behavior. But for systems which contain only passive elements (no energy input), but which respond to a constant potential driving-function with a self-driven oscillation, a nonlinear element must exist. In the case of the Hartmann whistle, the T-junction is the important nonlinear circuit element. The T-junction is probably responsible for many oscillatory fluid phenomena other than the Hartmann whistle.

The quarter wave pulsator of the Hartmann whistle inherently is an LC circuit leg, since it is closed at one end. Because it is closed at one end, only part of its length is effective in providing inductance (and, we shall assume, resistance). The quarter-wave tube may be thought of as a degenerate form of the Helmholtz resonator (a circuit element which has also successfully served as a pulsator) in which the obviously inductive neck and capacitative bulb have changed shape so that the neck and bulb form a closed-end tube.

The natural frequency (frequency of the response to an impulse) of an LC network leg is $[LC]^{-\frac{1}{2}}$ radians per second. A quarter wave tube will resonate at a frequency of $a/4X_e$ cycles per second, or $\pi a/2X_e$ radians per second, where a is the mean sound velocity in the tube and X_e is the effective tube length, equal to the actual tube length plus an end correction. The representation of a quarter wave tube as a simple series inductance and capacitance is a simplification of the closed-end transmission line characteristics of the tube. Nevertheless it is usually valid for the Hartmann whistle. If the LC-frequency is equated to the quarter wave frequency, and the equation is inverted and squared

$$LC = \frac{[2X_e]^2}{[\pi a]^2} \tag{6.38}$$

The expressions for inductance and capacitance given by Eqns. (6.12) and (6.23) may be used

$$[X_L/g_c A_L \rho_m][g_c \rho_m V_{sys}/a^2] = [2X/\pi a]^2 \tag{6.38a}$$

Cancellations occur, and denoting V_{sys} by AX, where X is the actual tube length, $A_L = A$,

$$X_L = \frac{4[X_e^2]}{\pi^2 X} \tag{6.38b}$$

The effective length, X_e, may be related to the true length, X, and the cross-sectional area, A.

$$X_e = X + n[A/\pi]^{\frac{1}{2}} \tag{6.38c}$$

In this last expression n is a coefficient, less than unity, whose value depends upon the geometry of the tube outlet. If the end effect is neglected, Eqn. (6.38b) becomes

$$X_L = \frac{4}{\pi^2} X = (.4052847)X \tag{6.38d}$$

In this representation of the quarter wave tube the entire volume of the tube is accessible as capacitance (true if a low frequency signal is imposed) but less than half of the length of the tube acts as plug flow inductance. The inductive part of the tube is presumably located near the open end, since the flow must always come to rest at the closed end. It is assumed that laminar resistance is associated only with the inductive segment of the tube.

As mentioned earlier in this chapter, the fluid flow impedance is related to the conventional acoustic impedance by

$$Z_{flow} = Z_{acoustic}/\rho_m^2 \tag{6.39}$$

Therefore the acoustic treatment of the quarter wave tube may be compared with the treatment just given. Beranek[34]

---

[34] L. L. Beranek, op. cit., p. 128.

has derived an expression for the steady state (jw) acoustic impedance seen at the open end of a closed tube. In our notation, it is

$$Z_{acoustic} = - \frac{j\rho_m a}{\pi r^2 g_c} \cot \frac{wX}{a} \qquad (6.39a)$$

The cotangent may be expanded in a Maclaurin series, retaining only the first few terms, yielding, with Eqn. (6.39):

$$Z_{flow} = \frac{a^2}{\pi r^2 X\rho_m (jw) g_c} + \frac{jwX}{3\pi r^2 \rho_m g_c} - \frac{(jw)^3 X^3}{45\pi r^2 a^2 \rho_m g_c} + \cdots \qquad (6.39b)$$

To be consistent with our notation, jw is replaced with the complex variable s associated with the Laplace transform. If more than the first two terms of Eqn. (6.39b) are retained, the equivalent circuit becomes more complicated than a simple LC leg, and approaches a ladder network type of representation.

However, Beranek retains only the first two terms, so that the impedance seen at the mouth of the pulsator is simply an inductance and capacitance in series. The values of the inductance and capacitance may be obtained from Eqn. (6.39b) as

$$L' = \frac{X}{3A\rho_m g_c} \qquad (6.39c)$$

$$C' = \frac{AX\rho_m g_c}{a^2} \tag{6.39d}$$

where the cross sectional area, πr^2, has been replaced with A. These expressions may be compared with the expressions derived from Eqn. (6.38) neglecting the end effect.

$$L = \frac{.4052847X}{A\rho_m g_c} \tag{6.40}$$

$$C = \frac{AX\rho_m g_c}{a^2} \tag{6.40a}$$

The capacitance expressions are identical. The inductance expression derived here contains a factor of 0.4052847 as compared with 0.3333333 in Beranek's inductance. Beranek's LC circuit representation yields a wavelength of $2\pi X/[3]^{\frac{1}{2}}$ or 3.627X as compared with the quarter-wave 4.000X. The difference is due primarily to the approximation introduced by taking only the first two terms in the expansion of the cotangent term in the impedance. More terms in this expansion would yield an increasingly accurate approximation to the natural frequency, but would also yield an increasingly complex equivalent circuit for the tube. Both Beranek's approximation and the one developed here may be improved by replacing X in the effective length, X_e, which includes the end effect. Following Beranek's recommendation, a value of 0.85 is assigned to n in Eqn. (6.38c) if the tube opening terminates in an infinite plate (as would be the case if the pulsator cavity were drilled into a large plate). Beranek recommends a value of 0.613 for n if the tube opening is unflanged (essentially a Borda entrance). For Hartmann Whistle Number 10, the latter value is applicable. The expression for the quarter wave tube pulsator inductance becomes

$$L = \frac{4[X + 0.613r]^2}{\pi^2 X g_c A \rho_m} \tag{6.40b}$$

The expression for the capacitance is still given by Eqn. (6.40a) as no end effect is applied. The equivalent length for resistance is the same as the equivalent length for inductance given by Eqn. (6.38b).

The quarter wave tube may be modeled more faithfully with a ladder network. In this case, the tube is "chopped" into many small segments, and each segment is given a capacitance, inductance, and resistance according to Eqns. (6.6), (6.12) and (6.23). However, the simplified LC model just derived will serve as a more tractable approximation.

The abstraction of circuit elements from specific geometry is always an approximation process, just as is the derivation of the continuum equations from which circuit elements are derived.

The air supply to the Hartmann whistle will be represented as a simple potential source, analogous to an electric battery. The nozzle of the whistle will be represented as a resistance and an inductance in series. The zone between the nozzle outlet and the pulsator inlet is the T-junction. Between the T-junction and ground, a small radiation impedance exists. The inductive and capacitative behavior of radiation impedance will be neglected, and an approximation to the near field flow resistance will consist of a turbulent and a laminar resistance in series. An approximation to turbulent resistance may be obtained from the Fanning equation for specific lost work. The laminar resistance may also be obtained from this equation. Laminar resistance was obtained from the Hagen-Poiseuille equation [Eqn. (6.1)], which may be derived from the Fanning equation by use of the laminar flow relation for the friction factor f. For turbulent flow, the friction factor is roughly a function of geometry only, and varies slowly with Reynolds number. In our notation, the Fanning equation is

$$b_{in} - b_{out} = \frac{f \Delta X v^2}{2 g_c D} \qquad (6.29a)$$

If an average value of area and density is used, and the diameter is expressed in terms of area, this equation may be written as

$$b_{in} - b_{out} = \frac{f \ \Delta X \ M_2}{[4 g_c/(\pi^{\frac{1}{2}})][A^{\frac{1}{2}}]A^2 \rho_m^2} \qquad (6.29b)$$

Now, assuming that the friction factor is not a function of the flow rate, an expression for turbulent resistance, K^2, may be written.

$$K^2 = \frac{f \, \Delta \, X}{[4g_c/(\pi^{\frac{1}{2}})][A^{\frac{1}{2}}]A^2\rho_m^2} \tag{6.29c}$$

with this definition the expression for the potential drop across a turbulent resistance is

$$b_1 - b_2 = [\dot{M}K]^2 \tag{6.29d}$$

The turbulent resistance is represented as a "K" circuit element. The Hartmann whistle pulsator is represented as a resistance (laminar), inductance, and capacitance in series. This Hartmann whistle circuit is shown in Figure 5.

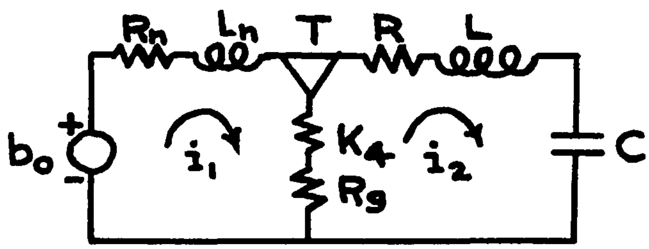

Figure 5.-Approximate Circuit for a Hartmann Whistle

The T-junction current-potential relations given by Eqns. (6.36) and (6.37) may be expanded by long division, with the viewpoint that $\dot{M}_1$ is greater than $\dot{M}_2$. The notation may be simplified by defining

$$K_i^2 = 1/2g_c\rho_m^2A_i^2 \tag{6.41}$$

$$K_i K_j = 1/2 g_c \rho_m^2 A_i A_j \tag{6.42}$$

The potential drop expressions for the T-junction become

$$\begin{aligned} b_1 - b_3 = {} & \dot{M}_1^2 K_1^2 - \dot{M}_1^2 K_1^2 - 2\dot{M}_1 \dot{M}_2 K_1^2 - \dot{M}_2^2[3K_1^2 - 2K_1 K_2] \\ & + (\dot{M}_2^3/\dot{M}_1)[2K_1 K_2 - K_1^2] + (\dot{M}_2^4/\dot{M}_1^2)[2K_1^2 - K_2^2] + \cdots \\ & - (\dot{M}_1 - \dot{M}_2)^2 K_3^2 \end{aligned} \tag{6.43}$$

$$\begin{aligned} b_3 - b_2 = {} & \dot{M}_1^2 K_1^2 + 2\dot{M}_1 \dot{M}_2 K_1^2 + \dot{M}_2^2[3K_1^2 - 2K_1 K_2] \\ & + (\dot{M}_2^3/\dot{M}_1)[K_1^2 - 2K_1 K_2] - (\dot{M}_2^4/\dot{M}_1^2)[2K_1^2 - K_2^2] + \cdots \\ & + (\dot{M}_1 - \dot{M}_2)^2 K_3^2 - \dot{M}_2^2 K_2^2 \end{aligned} \tag{6.44}$$

Upon neglecting the terms involving $(\dot{M}_2^3/\dot{M}_1)$, $(\dot{M}_2^4/\dot{M}_1^2)$, and all higher powers of $(\dot{M}_2/\dot{M}_1)$, and upon letting the cancellations occur, these last expressions simplify to

$$b_1 - b_3 = -(\dot{M}_1 - \dot{M}_2)^2 K_3^2 - 2\dot{M}_1 \dot{M}_2 K_1^2 - \dot{M}_2^2[3K_1^2 - 2K_1 K_2] \tag{6.45}$$

$$\begin{aligned} b_3 - b_2 = {} & +(\dot{M}_1 - \dot{M}_2)^2 K_3^2 + \dot{M}_1^2 K_1^2 + 2\dot{M}_1 \dot{M}_2 K_1^2 \\ & + \dot{M}_2^2[3K_1^2 - 2K_1 K_2 - K_2^2] \end{aligned} \tag{6.46}$$

A simple summation of the last two relations yields

$$b_1 - b_2 = \dot{M}_1^2 K_1^2 - \dot{M}_2^2 K_2^2 \tag{6.47}$$

This expression is simply Eqn. (6.35) with the nomenclature defined in Eqn. (6.41)

Eqns. (6.45) and (6.46) are used as the potential-current relations of the T-junction circuit element for the loop currents selected. The circuit equations for the circuit in Figure 5 may be written from inspection, using Eqns. (6.29d), (6.45) and (6.46). Current will be designated as "i," rather than $\dot{M}$, for convenience.

$$b_o - i_1R_n - L_n(di_1/dt) + (i_1-i_2)^2K_3^2 + 2i_1i_2K_1^2 + i_2^2[3K_1^2-2K_1K_2]$$
$$-(i_1-i_2)^2K_4^2 - (i_1-i_2)R_g = 0 \qquad (6.48)$$

$$(i_1-i_2)^2K_4^2 + (i_1-i_2)R_g - (i_1-i_2)^2K_3^2 - i_1^2K_1^2 - 2i_1i_2K_1^2$$
$$- i_2^2[3K_1^2-2K_1K_2-K_2^2] - i_2R - L(di_2/dt)$$
$$- (1/C)\int i_2dt = 0 \qquad (6.49)$$

The turbulent resistance, K_4, has the same dimensions as the T-junction parameters. This equation set may be regrouped as

$$b_o - i_1(R_n+R_g) - L_n(di_1/dt) + i_1^2[K_3^2-K_4^2]$$
$$+ 2i_1i_2[K_1^2+K_4^2-K_3^2] + i_2^2[K_3^2+3K_1^2-2K_1K_2-K_4^2]$$
$$+ i_2R_g = 0 \qquad (6.50)$$

$$i_2[R_g+R] + L(di_2/dt) + (1/C)\int i_2 dt + i_2^2[3K_1^2-2K_1K_2-K_2^2-K_3^2+K_4^2]$$
$$+ 2i_1i_2[K_4^2-K_3^2+K_1^2] + i_1^2[K_3^2-K_4^2+K_1^2] - i_1R_g = 0 \qquad (6.51)$$

These circuit equations are nonlinear, and consequently the conventional application of the Laplace transformation at this point is ineffective. An approximate solution will be obtained. It will be convenient to linearize the T-junction expressions given by Eqns. (6.45) and (6.46). First, these expressions may be expanded, neglecting products of the pulsator current (already assumed small).

$$b_1 - b_3 = -(\dot{M}_1^2-2\dot{M}_1\dot{M}_2)K_3^2 - 2\dot{M}_1\dot{M}_2K_1^2 \qquad (6.52)$$

$$b_3 - b_2 = + (\dot{M}_1^2-2\dot{M}_1\dot{M}_2)K_3^2 + \dot{M}_1^2K_1^2 + 2\dot{M}_1\dot{M}_2K_1^2 \qquad (6.53)$$

Since the nozzle flow consists mostly of a steady value, these expressions may be linearized by the definition of the circuit parameter, T, which is assumed constant as a mean value.

$$T_i = \dot{M}_1K_i^2 \qquad (6.54)$$

Upon substitution of this equation into Eqns. (6.52) and (6.53)

$$b_1 - b_3 = - T_3\dot{M}_1 + 2T_3\dot{M}_2 - 2T_1\dot{M}_2 \qquad (6.55)$$

$$b_3 - b_2 = T_3\dot{M}_1 - 2T_3\dot{M}_2 + T_1\dot{M}_1 + 2T_1\dot{M}_2 \qquad (6.56)$$

Similarly, the turbulent resistance may be lumped into the laminar resistance. The linearized equation set is

$$b_o - i_1R_n - L_n(di_1/dt) + T_3i_1 - 2T_3i_2 + 2T_1i_2 - (i_1 - i_2)R_a = 0 \qquad (6.57)$$

$$(i_2 - i_1)R_a + T_3i_1 - 2T_3i_2 + T_1i_1 + 2T_1i_2 + i_2R$$
$$+ L(di_2/dt) + (1/C)\int i_2dt = 0 \qquad (6.58)$$

The Laplace transform of these equations takes the form:

$$(b_o/s) - \bar{i}_1[R_n + R_a - T_3 + sL_n] + \bar{i}_2[2T_1 - 2T_3 + R_a] = 0 \qquad (6.59)$$

$$\bar{i}_2[R_a + R + 2T_1 - 2T_3 + sL + (1/sC)] + \bar{i}_1[T_3 + T_1 - R_a] = 0 \qquad (6.60)$$

The expression for the transform of the nozzle current is

$$\bar{i}_1 = \frac{\frac{b_o}{sL_n}\left[s^2 + s(R_a + R + 2T_1 - 2T_3)/L + (1/LC)\right]}{[s + (R_n + R_a - T_3)/L_n][s^2 + s(R_a + R + 2T_1 - 2T_3)/L + (1/LC)] + sB} \qquad (6.61)$$

$$B = [T_3 + T_1 - R_a][2T_1 - 2T_3 + R_a]/LL_n \qquad (6.62)$$

There are various combinations of system parameters which yield oscillatory solutions. The denominator of Eqn. (6.61) may be put into the form

$$D = s^3 + ps^2 + qs + r \qquad (6.63)$$

$$p = \frac{R_a + R + 2T_1 - 2T_3}{L} + \frac{R_n + R_a - T_3}{L_n} \qquad (6.64)$$

$$q = [(R_a+R+2T_1-2T_3)(R_n+R_a-T_3)+(T_3+T_1-R_a)(2T_1-2T_3+R_a)]/LL_n +(1/LC) \tag{6.65}$$

$$r = (R_n+R_a-T_3)/L_nLC \tag{6.66}$$

For an oscillatory solution to exist, two of the roots of the denominator must be complex conjugates. This imposes the condition that

$$b^2/4 + a^3/27 > 0 \tag{6.67}$$

$$a = q - p^2/3 \tag{6.68}$$

$$b = (2p^2-9pq+27r)/27 \tag{6.69}$$

These conditions are sufficient for complex roots to exist. The cubic may be factored into a real root and conjugate imaginaries by noting that

$$(s+p)(s^2+q) = s^3 + ps^2 + qs + pq \tag{6.70}$$

Therefore, a constraint for the oscillatory component to exist without divergence or decay is

$$r = pq \tag{6.71}$$

Expressed in terms of circuit parameters, this constraint is:

$$\frac{R_n+R_a-T_3}{LL_nC} = \left(\frac{R_a+R+2T_1-2T_3}{L} + \frac{R_n+R_a-T_3}{L_n}\right)\left(\frac{1}{LC} + \frac{(T_3+T_1-R_a)(2T_1-2T_3+R_a)}{LL_n} + \frac{(R_a+R+2T_1-2T_3)(R_n+R_a-T_3)}{LL_n}\right) \tag{6.72}$$

If this constraint is applied to Eqn. (6.61) it becomes

$$\bar{i}_1 = \frac{\frac{b_o}{L_n}\left[\, s^2 + s\left(\frac{R_a+R+2T_1-2T_3}{L}\right) + \frac{1}{LC} \,\right]}{s(s+p)(s^2+q)} \tag{6.73}$$

A similar solution for the escaping flow rate is

$$\bar{i}_1 - \bar{i}_2 = \frac{\frac{b_o}{L_n}\left[\, s^2 + s\,\frac{(3T_1-T_3+R)}{L} + \frac{1}{LC} \,\right]}{s(s+p)(s^2+q)} \tag{6.74}$$

These expressions may be conveniently inverted to the real domain by using function-transform pair number 1.218 in Appendix A of Gardner and Barnes.[3] The resulting expressions for the nozzle flow rate are

$$\bar{i}_1 = \frac{b_o}{pqL_nLC} - \frac{b_o[p^2 - [p(R_a+R+2T_1-2T_3)/L] + (1/LC)]}{L_np(p^2+q)}\, e^{-pt}$$

$$- \frac{b_o}{L_nq}\left[\frac{[(1/LC)-q]^2+q[(R_a+R+2T_1-2T_3)/L]^2}{p^2+q}\right]^{\frac{1}{2}} \cos[(q)^{\frac{1}{2}}t+\theta_1] \quad (6.74)$$

$$\theta_1 = \arctan\left[\frac{(q)^{\frac{1}{2}}(R_a+R+2T_1-2T_3)/L}{(1/LC)-q}\right]-\arctan\left[\frac{(q)^{\frac{1}{2}}}{p}\right] \tag{6.75}$$

Similarly, the resulting expression for the flow rate escaping from the system is obtained by inverting Eqn. (6.74), using the same transform pair.

---

[3] M. F. Gardner and J. L. Barnes, op. cit., p. 341.

$$i_1 - i_2 = \frac{b_o}{pqL_nLC} - \frac{b_o[p^2-[p(R+3T_1-T_3)/L]+(1/LC)]}{L_np(p^2+q)} e^{-pt}$$

$$- \frac{b_o}{L_nq}\left[\frac{[(1/LC)-q]^2+q[(R+3T_1-T_3)/L]^2}{p^2 + q}\right]^{\frac{1}{2}}\cos[(q)^{\frac{1}{2}}t+\theta_2] \qquad (6.76)$$

$$\theta_2 = \arctan\left[\frac{(q)^{\frac{1}{2}}(R+3T_1-T_3)/L}{(1/LC) - q}\right] - \arctan\left[\frac{(q)^{\frac{1}{2}}}{p}\right] \qquad (6.77)$$

From Eqn. (6.74) the steady component of the nozzle flow is

$$i_o = \frac{b_o}{pqL_nLC} \qquad (6.78)$$

Upon insertion of the circuit parameters, as given by Eqns. (6.71) and (6.66), the steady flow component becomes

$$i_o = \frac{b_o}{R_n+R_a-T_3} \qquad (6.78a)$$

The linearized circuit parameter, T_3, may be related to system geometry by approximating Eqn. (6.54) as

$$T_3 = i_oK_3^2 = \frac{i_o}{2g_c\rho_m^2A_3^2} \qquad (6.54a)$$

At this point a discussion of the denominator of Eqn. (6.78a) is in order. As can be seen from Eqn. (6.54a), T_3 approaches zero as A_3 becomes large. However, it would appear at first glance that as A_3 becomes small, as it would if the retreat distance were reduced drastically, that for a given nozzle pressure (or nozzle potential) the steady flow component would diverge. However, the atmospheric resistance will always dissipate whatever kinetic energy availability exists as well as whatever compression energy exists, such that

$$R_a > T_3 \tag{6.79}$$

Or, in effect the atmospheric resistance contains a term which always cancels T_3

$$R_a = R_a^o + T_3 \quad \text{or} \quad R_a - T_3 = R_a^o \tag{6.80}$$

Therefore Eqn. (6.78a) may be written

$$i_o = \frac{b_o}{R_n + R_a^o} \tag{6.81}$$

R_a^o is retained as nonzero since the structure around the exit may involve viscous dissipation of compression energy in addition to the mandatory dissipation of kinetic energy. Therefore the T parameters may be written as

$$T_i = \frac{b_o}{2g_c\rho_m^2 A_i^2 (R_n + R_a^o)} \qquad i = 1, 2, 3 \tag{6.82}$$

The circuit parameter groups p, q, and r may be rewritten by combining Eqns. (6.82) and (6.80) with Eqns. (6.64), (6.65) and (6.66).

$$p = \frac{R_a^o + R + \dfrac{b_o}{g_c\rho_m^2 A_1^2(R_n+R_a^o)} - \dfrac{b_o}{2g_c\rho_m^2 A_3^2(R_n+R_a^o)}}{L} + \frac{R_a^o+R_n}{L_n} \quad (6.64a)$$

$$q = \frac{b_o}{2g_c LL_n\rho_m^2 A_1^2(R_n+R_a^o)}\left[\frac{b_o}{g_c\rho_m^2 A_1^2(R_n+R_a^o)} - \frac{b_o}{2g_c\rho_m^2 A_3^2(R_n+R_a^o)} + R_a^o+2R_n\right]$$

$$- \frac{b_o R_n}{2g_c LL_n\rho_m^2 A_3^2(R_n+R_a^o)} + \frac{R_nR_a^o+R_nR+RR_a^o}{LL_n} + \frac{1}{LC} \quad (6.65a)$$

$$r = \frac{R_n+R_a^o}{L_nLC} \quad (6.66a)$$

At this point, insertion of numerical values for R, R_n, R_a^o, A_1, A_2, A_3, L, L_n, ρ_m, a_m, and C as functions of the retreat distance and the nozzle pressure permits the use of Eqn. (6.72) to determine this linearized treatments prediction of the oscillation boundary. Yet another oscillation singular condition is obtained by setting the amplitude of the cosine term in Eqns. (6.74) and (6.76) equal to zero.

Although symbols rather than numerical values for the circuit parameters have been carried through the inversion process, in more complicated circuits it is necessary to insert numerical values before the inversion is possible, since the denominator of the complex current expression may consist of polynomials of higher order than the cubic encountered here. It is possible to factor analytically a quartic, but it is more practical to factor a quartic numerically. For higher order polynomials it is necessary to use numerical techniques to factor the denominator before the current expressions may be inverted to the real domain. Although substituting numbers into the complex domain flow rate expressions facilitates their inversion, it also causes a loss of information, since it is not evident what circuit elements are contributing to the various terms of the solution.

Approximate values of the circuit parameters for Hartmann whistle number 10 are tabulated in Table 3. The nozzle resistance, R_n, was computed by taking the equivalent circular area for the nozzle as the cross-sectional area of the nozzle exit, and by assuming the equivalent length was equal to the nozzle inside diameter. The viscosity, sound velocity, and density of the flowing fluid correspond to air at $70^{o}F$ and 1 atm. for all the circuit elements.

The nozzle inductance was derived by making the same assumptions as were used to obtain the nozzle resistance. The mean cylindrical area between the nozzle and the pulsator was taken as a cylinder whose diameter was the diameter of the pulsator inlet and whose length was the retreat distance, designated here as R^* to avoid confusion with the resistance designations. The areas A_1 and A_2 are taken as the cross-sectional areas of the nozzle and the pulsator of Hartmann whistle number 10.

The viscous resistance in the exit area was estimated by assuming an equivalent cross-section equal to twice A_3, and an equivalent length equal to the outside radius of the machined pipe cap.

The inductance and capacitance of the pulsator were calculated by using Eqns. (6.40a) and (6.40b). The resistance of the capacitor was estimated by using the effective inductive length, X_L, as the equivalent length, and the cross-sectional area as the equivalent area.

TABLE 3.-Hartmann Whistle Number 10 Circuit Element Nominal Values

Circuit Element	Nominal Value	Units
R_n	104.5	$(ft - lb_f - sec)/(lb_m)^2$
R_a^o	$1.16/(R^*)^2$	$(ft - lb_f - sec)/(lb_m)^2$
R	$40.2[L^*+.153]^2/L^*$	$(ft - lb_f - sec)/(lb_m)^2$
L_n	18.63	$(ft - lb_f - sec^2)/(lb_m)^2$
L	$11.79[L^*+.153]^2/L^*$	$(ft - lb_f - sec^2)/(lb_m)^2$
C	$1.87 \times 10^{-10}(L^*)$	$(lb_m)^2/(ft - lb_f)$

R* is the retreat distance in inches.
L* is the pulsator depth in inches.

Substitution of numerical values of the circuit parameters from Table 3 into the expression for p given by Eqn. (6.64a) results in

$$p = [\frac{1.16}{(R^*)^2} + \frac{40.2[L^*+.153]^2}{L^*} + \frac{b_o}{9.53(10^{-8})[104.5+1.16/(R^*)^2]} - \frac{b_o}{3.27(10^{-5})(R^*)^2[104.5+1.16/(R^*)^2]}]/[\frac{11.79(L^*+.153)^2}{L^*} + \frac{104.5+[1.16/(R^*)^2]}{18.63}] \tag{6.64b}$$

The numerical values of R* and L* are on the order of an inch. Since the nozzle pressures are usually greater than 1 psig. and b_o is in English units of lb_f/ft^2, all but two of the terms in this last equation are negligible.

$$p = [\frac{10^8}{9.53} - \frac{10^5}{3.27(R^*)^2}] \frac{L^*b_o}{11.79(L^*+.153)^2[104.5+1.16/(R^*)^2]} \tag{6.64c}$$

The term, $1.16/(R^*)^2$, is retained since it has a value of 9.4 as compared with 104.5 for the minimum R*/D ratio used on Hartmann Whistle Number 10. The end effect, .153, is retained since the values of L* were as low as 0.5 inch. The circuit equations are not valid for vanishing values of the pulsator depth, L*. The orders of magnitude of the circuit parameters given in Table 3 permit Eqns. (6.64a) and (6.65a) to be written with a number of terms neglected

$$p = \frac{\frac{b_o}{L}}{g_c\rho_m^2A_1^2(R_n+R_a^o)} - \frac{\frac{b_o}{L}}{2g_c\rho_m^2A_3^2(R_n+R_a^o)} \tag{6.64d}$$

$$q = \frac{b_o}{2g_c L_n \rho_m^2 A_1^2 (R_n + R_a^o)} [p] \tag{6.65b}$$

Substitution of Eqns. (6.64d), (6.65b) and (6.66a) into the oscillation criterion, Eqn. (6.71), results in

$$\frac{y^2}{L_n LC} - \frac{y}{LC} - \frac{b_o}{2g_c L_n \rho_m^2 A_1^2} = 0 \tag{6.71a}$$

where

$$y = \left[\frac{b_o/L}{g_c \rho_m^2 A_1^2} - \frac{b_o/L}{2g_c \rho_m^2 A_3^2}\right]^{-1} (R_n + R_a^o)^2 \tag{6.83}$$

Since Eqn. (6.71a) is a cubic in b_o, it will be convenient to compute the oscillation boundaries by assigning the nozzle pressure (or b_o), and the pulsator depth, L*, and then computing R*. The retreat distance, R*, is implicit in only two of these non-negligible circuit parameters.

$$R_a^o = \frac{1.16}{R^{*2}} \tag{6.84}$$

$$A_3^2 = 1.194(10^{-4})R^{*2} \tag{6.85}$$

Solutions to Eqn. (6.71a) are

$$y_i = \frac{L_n}{2} \pm \frac{L_n LC}{2} \left[\frac{1}{L^2C^2} + \frac{2b_o}{L_n^2 LC g_c \rho_m^2 A_1^2}\right]^{\frac{1}{2}}$$

$$i = 1,2 \tag{6.86}$$

Yet another quadratic must be solved to obtain oscillatory boundary values of the retreat distance. Substitution of Eqns. (6.86), (6.84) and (6.85) into Eqn. (6.83) yields

$$\frac{1.345}{(R^{*2})^2} + \left[2.32R_n + \frac{y_i b_o (10^4)}{2g_c \rho_m^2 (1.194L)}\right] \frac{1}{(R^{*2})} + R_n^2 - \frac{y_1 b_o}{g_c \rho_m^2 A_1^2 L} = 0 \qquad (6.87)$$

This equation may be regarded as a quadratic in $(R^*)^{-2}$, and determines values of R^* necessary to satisfy the oscillation boundary criterion. Eqns. (6.86) and (6.87) implicitly give R^* as a function of L^* and b_o. For a pulsator depth of 0.75-inch, Eqns. (6.86) and (6.87) may be put into a computational form, where b_o^* is the nozzle pressure in psig. (a mean density has been used once again).

$$y_i = 9.315\ [1 \pm (1 + .2395b_o^*)^{\frac{1}{2}}] \qquad (6.86a)$$

$$\frac{1}{(R^*)^2} = -\ (90.2+1.958(10^6)b_o^* y_i)$$

$$\pm[(90.2+1.958(10^6)b_o^* y_i)^2 + 1.345(10^9)b_o^* y_i - 8.12(10^3)]^{\frac{1}{2}} \qquad (6.87a)$$

Since imaginary values of the retreat distance indicate no possible oscillation boundary, the following roots may be selected, and conveniently calculated as long as the absolute value of $b_o^* y_i$ is not much less than unity.
Applying the binomial theorem to Eqn. (6.87a) permits the

sign options to yield two simple expressions for R*

$$\frac{1}{R^{*2}} = -[90.2 + 1.958(10^6)b_o^*y_i][2 + \frac{.6725(10^9)b_o^*y_i - 4030.}{(90.2+1.958(10^6)b_o^*y_i)^2}] \tag{6.87b}$$

$$\frac{1}{R^{*2}} = \frac{.6725(10^9)b_o^*y_i - 4030.}{90.2 + 1.958(10^6)b_o^*y_i} \tag{6.87c}$$

It has been assumed that the last ratio in Eqn. (6.87b) is small compared to unity such that the binomial theorem could be used to express $[1 + x]^{\frac{1}{2}} = 1 + \frac{1}{2}x$. The nozzle pressure will be assumed to be positive. However, positive or negative values of y_i will be provided by Eqn. (6.86a). Only negative values will be permissible in Eqn. (6.87). Positive or negative values are useable, within magnitude limits, in Eqn. (6.87c). It is possible to get as many as four oscillatory retreat values from Eqns. (6.86a) and (6.87a). However, Eqn. (6.87b) shows that one pair of these values corresponds to an extremely small retreat distance, and is of doubtful physical significance. The values from Eqn. (6.87c) are also much smaller than is indicated by the data.

Hartmann's explanation for oscillation may be quoted from Reference 2 of Chapter II, which is a technical journalist's account of Hartmann's presentation.

> It was found that when the velocity of the air in the jet exceeded that of sound in air, or, what comes to the same thing, if the pressure in the air receiver supplying the jet was higher than about 1.9 atmospheres, the curve obtained by plotting the gauge readings [of static

> pressure in a capillary Pitot tube] on a base representing the position of the Pitot-tube mouth on the axis of the jet became periodic... Moreover, it was found that if a comparatively wide-mouthed pitot tube were used, it was impossible to obtain satisfactory pressure-gauge readings over the parts of the curve... where the pressure increased in the direction of flow... From what has been said above, it will be clear that *the pulsation phenomenon depends upon the periodic character of the Pitot-tube pressure curve, which, in turn, is dependent upon the periodic structure of a jet having a velocity higher than that of sound* [italics mine].

The article goes on to make a fairly convincing graphical argument showing that the flow in the pulsator should be unstable when the mouth of the pulsator is located in a zone where the static pressure increases with axial distance from the nozzle.

Hartmann[4] gives an empirical expression applicable to the case when the nozzle diameter is equal to the pulsator diameter. His expression may be written

$$(a_2 - a_1)/d = 0.43\ (P - 0.93)^{\frac{1}{2}} \qquad (6.84)$$

where P is gauge pressure in kg/cm^2, d is the nozzle diameter and a_1 and a_2 define a retreat distance zone (centimeters) in which the radiated power is approximately constant. Hartmann has also given an empirical expression for a_1, the retreat distance in centimeters at which oscillation begins. This expression is

$$a_1/d = 1 + 0.04\ (P - 0.93) \qquad (6.85)$$

---

[4] J. Hartmann, *op. cit.*, p. 147.

It should be noted that these expressions given by Hartmann are merely arbitrary curve forms chosen to correlate Hartmann's data. These expressions do serve rather well in that capacity, but they were not "derived" nor do they represent a formal theory. The form of the expressions does embody the assumption that the jet must be choked and that the Hartmann whistle phenomenon ceases to exist at a nozzle pressure below 0.93 kg/cm^2. This statement is supported by the following reasoning. If air is regarded as an ideal diatomic gas with a specific heat ratio of 1.4, the nozzle pressure necessary to achieve sonic velocity upon expanding to one atmosphere is .9159 kg/cm^2 gauge. The presence of the term $(P-.93)^{\frac{1}{2}}$ in Hartmann's correlation for the oscillation zone indicates that the nozzle flow must be slightly supersonic for an oscillation zone to be non-imaginary, although Hartmann probably intended for the 0.93 kg/cm^2 to correspond to sonic exit velocity.

In Table 4 the experimental oscillation limits, the linearized circuit theory oscillation limits [from Eqn. (6.87c)], and Hartmann's correlation oscillation limits are compared. These limits are lower and upper values of the retreat distance in inches at an oscillation boundary. The data and the circuit equation relation are for the 0.75-inch pulsator depth. Hartmann's expression does not depend on pulsator depth.

Hartmann's correlations, even though they apply strictly only to whistles in which the pulsator diameter is equal to the nozzle diameter, are nevertheless a pretty good correlation for Hartmann Whistle Number 10. The linearized circuit criterion, although it has a quadratic-cubic form capable of predicting oscillation criteria, does in fact not come close numerically. The retreat distance predicted by the linearized circuit theory is too low by a factor of ten, and the oscillation zones are orders of magnitude too narrow.

TABLE 4.-Comparison of Experimental, Linearized Circuit Theory, and Hartmann's Correlation Oscillation Limits for Hartmann Whistle Number 10 with a Pulsator Depth of 0.75-Inch

Nozzle Pressure Psig.	Retreat Distance in Inches					
	Linearized Circuit Theory		Hartmann's Correlation		Hartmann Whistle 10	
	Lower Limit (Eqn. 6.87b)	Upper Limit (Eqn. 6.87c)	Lower Limit (Eqn. 6.84)	Upper Limit (Eqn. 6.85)	Lower Limit (Figure 3)	Upper Limit (Figure 3)
10.	.050	.055	.39	*	?	.45
20.	.053	.054	.40	.59	?	.51
29.	.054	.054	.41	.59	.45	.65
100.	.054	.054	.49	.68	?	?

*imaginary number

?data do not permit an interpolation

CHAPTER VII

CONCLUSIONS AND COMMENTS

In this dissertation the Hartmann whistle was presented as a simple configuration for which no satisfactory theory existed. Professor Hartmann and others had taken extensive data on the device, and effective correlations of that data had been prepared by arbitrarily curve fitting plausible curve forms to certain ranges of the data points.

In this work, a new range of operation of the Hartmann whistle was found, in the low pressure range. The fact that the Hartmann whistle will operate at low nozzle pressures was in conflict with the popular opinion as to the mechanism of the whistle.

An attempt was made in this dissertation to establish a theory for this process in terms of fluid flow circuit theory.

This attempt prompted a study of circuit theory and its application to self-excited oscillatory flow phenomena. The thermodynamic foundations of such a circuit theory were examined, with particular emphasis on the availability concept for defining potentials, the requirement that currents be conservative, and that the rate equations describing the circuit elements must be datum level invariant. Irreversible thermodynamics, which deals with currents and potentials, was also examined to see if it would serve as a foundation for the circuit analysis.

A flow potential, adiabatic availability, was defined, and mass flow was selected as current. A number of fluid flow circuit elements were derived, including a new circuit element, the T-junction. A nonlinear circuit representation for the Hartmann whistle was derived and the equations were linearized and solved. The resulting solution demonstrated self-excited oscillation in the presence of dissipation, but did not accurately predict the region of oscillation, possibly because of the linearization approximations which were made. The solution also suggested an unrealistic possibility of divergent oscillation.

Contributions here include the experimental and theoretical demonstration that the Hartmann whistle will operate at subcritical nozzle pressures, that a fluid flow circuit analysis can treat active as well as passive systems, and that momentum effects at a fluid junction may be included as a circuit element.

The Hartmann whistle remains a very simple system representative of a class of oscillatory fluid flow devices for which quantitative theories are lacking.

APPENDIX

TABLE 5.-Hartmann Whistle Number 10 Performance Data (Nozzle inside diameter (D) was always 25/64 inch) (Pulsator inside diameter (D_2) was always 32/64 inch)

Pulsator Depth Setting	Retreat Distance Setting (R): Inches	R/D	Nozzle Pressure (psig) Ascending	Nozzle Pressure (psig) Descending	Fundamental Intensity, db	Total Sound Level *	Fundamental Frequency
0.5	22.5/64	0.9	15.0	15.0	110.0	123.0	4050
0.5	22.5/64	0.9	no oscillation at any other nozzle pressures				
0.5	25.0/64	1.0	....	15.0	112.0	123.0	4050
0.5	25.0/64	1.0	....	25.0	92.0	124.0	4150
0.5	25.0/64	1.0	....	24.0	132.0	126.0	4150
0.5	25.0/64	1.0	....	23.0	112.0	122.0	4200
0.5	25.0/64	1.0	....	22.0	102.0	123.0	4200
0.5	25.0/64	1.0	....	21.0	112.0	127.5	4040
0.5	25.0/64	1.0	....	22.0	92.0	123.0	4040
0.5	25.0/64	1.0	33.0	....	90.0	127.0	4050
0.5	25.0/64	1.0	32.0	....	104.0	129.0	4050
0.5	25.0/64	1.0	60.0	....	98.0	139.0	4050
0.5	25.0/64	1.0	negligible oscillation between nozzle pressures of 60 and 100 psig.				
0.5	32.5/64	1.3	15.0	....	105.0	122.0	4100
0.5	32.5/64	1.3	26.0	....	146.0	139.5	4290
0.5	32.5/64	1.3	27.0	....	151.0	142.0	4270
0.5	32.5/64	1.3	25.0	....	110.0	127.0	4260
0.5	32.5/64	1.3	30.0	....	150.0	142.0	4250
0.5	32.5/64	1.3	35.0	....	155.0	146.0	4590
0.5	32.5/64	1.3	49.0	....	155.0	142.0	5010
0.5	32.5/64	1.3	50.0	....	98.0	137.0	5050
0.5	32.5/64	1.3	....	48.0	98.0	136.0	4690
0.5	32.5/64	1.3	....	47.0	157.0	147.0	4960
0.5	32.5/64	1.3	23.0	....	143.0	134.0	4280
0.5	32.5/64	1.3	22.0	....	108.0	126.0	4200
0.5	32.5/64	1.3	32.0	....	153.0	145.0	4410
0.5	32.5/64	1.3	31.0	....	151.0	142.0	4315
0.5	32.5/64	1.3	29.0	....	152.0	142.5	4240
0.5	32.5/64	1.3	28.0	....	151.5	141.5	4250
0.5	32.5/64	1.3	40.0	....	156.5	147.0	4710
0.5	32.5/64	1.3	60.0	....	90.0	138.0	5100
0.5	37.5/64	1.5	15.0	....	117.0	122.0	4000
0.5	37.5/64	1.5	27.0	....	122.0	128.0	4600
0.5	37.5/64	1.5	27.0	....	94.0	127.0	4300
0.5	37.5/64	1.5	28.2	....	143.0	136.0	4310
0.5	37.5/64	1.5	28.0	....	139.0	134.0	4500
0.5	37.5/64	1.5	30.0	....	145.0	139.5	3960
0.5	37.5/64	1.5	32.0	....	149.0	143.0	4030

TABLE 5.-Continued.

Pulsator Depth Setting	Retreat Distance Setting (R): Inches	R/D	Nozzle Pressure (psig) Ascending	Descending	Fundamental Intensity, db	Total Sound Level *	Fundamental Frequency
0.5	37.5/64	1.5	43.0	....	155.0	148.0	4420
0.5	37.5/64	1.5	....	60.0	110.0	138.0	Fund.Zone
0.5	37.5/64	1.5	....	58.0	156.0	146.0	4970
0.5	37.5/64	1.5	....	56.0	158.0	148.0	4820
0.5	37.5/64	1.5	65.0	65.0	105.0	135.0	no oscillation
0.5	37.5/64	1.5	29.0	....	142.0	139.0	4060
0.5	37.5/64	1.5	34.0	....	149.0	142.0	4120
0.5	37.5/64	1.5	36.0	....	152.0	145.0	4200
0.5	37.5/64	1.5	38.0	....	155.0	146.0	4340
0.5	37.5/64	1.5	40.0	....	156.0	146.0	4420
0.5	37.5/64	1.5	27.5	....	145.0	137.0	4480
0.5	37.5/64	1.5	59.0	....	153.0	145.0	5000
0.5	40.0/64	1.6	15.0	....	100.0	121.0	4200
0.5	40.0/64	1.6	35.0	....	147.0	141.0	3870
0.5	40.0/64	1.6	40.0	....	150.0	142.5	4110
0.5	40.0/64	1.6	45.0	....	153.0	145.5	4300
0.5	40.0/64	1.6	50.0	....	156.0	147.0	4400
0.5	40.0/64	1.6	70.0	....	154.0	145.5	5000
0.5	40.0/64	1.6	75.0	....	154.0	145.0	5470
0.5	40.0/64	1.6	76.0	....	98.0	137.0	5470
0.5	40.0/64	1.6	....	70.0	153.0	149.0	4970
0.5	40.0/64	1.6	....	71.0	100.0	140.0	5000
0.5	40.0/64	1.6	30.0	....	90.0	123.0	4150
0.5	40.0/64	1.6	31.0	....	137.0	137.0	4150
0.5	40.0/64	1.6	33.0	....	142.0	140.0	3910
0.5	40.0/64	1.6	34.0	....	145.0	140.5	3870
0.5	40.0/64	1.6	36.0	....	147.0	141.5	3900
0.5	40.0/64	1.6	38.0	....	151.0	142.5	3960
0.5	40.0/64	1.6	55.0	....	157.0	146.0	4510
0.75	22.5/64	0.9	15.0	15.0	138.0	124.0	3240
0.75	22.5/64	0.9	10.0	....	139.0	122.0	3210
0.75	22.5/64	0.9	5.0	....	98.0	107.0	3200
0.75	22.5/64	0.9	6.0	....	106.0	112.0	3210
0.75	22.5/64	0.9	7.0	....	135.0	127.0	3210
0.75	22.5/64	0.9	8.0	....	137.0	129.0	3210
0.75	22.5/64	0.9	9.0	....	139.0	130.0	3205
0.75	22.5/64	0.9	11.0	....	139.5	130.5	3220
0.75	22.5/64	0.9	12.0	....	140.0	132.0	3225
0.75	22.5/64	0.9	20.0	....	138.0	133.0	3170
0.75	22.5/64	0.9	21.0	....	136.0	132.0	3140
0.75	22.5/64	0.9	22.0	....	141.0	134.0	3120

TABLE 5.-Continued.

Pulsator Depth Setting	Retreat Distance Setting (R): Inches	R/D	Nozzle Pressure (psig): Ascending	Descending	Fundamental Intensity, db	Total Sound Level *	Fundamental Frequency
0.75	22.5/64	0.9	25.0	....	135.0	129.0	3100
0.75	22.5/64	0.9	26.0	....	132.0	129.0	3040
0.75	22.5/64	0.9	27.0	....	126.0	129.0	3015
0.75	22.5/64	0.9	29.0	....	124.0	129.0	2920
0.75	22.5/64	0.9	31.0	....	105.0	129.5	2800
0.75	22.5/64	0.9	18.0	....	139.0	132.0	3175
0.75	22.5/64	0.9	....	26.0	129.0	124.0	3340
0.75	22.5/64	0.9	....	25.0	135.0	129.0	3000
0.75	22.5/64	0.9	26.0	....	128.0	129.0	2870
0.75	22.5/64	0.9	....	26.0	134.0	130.0	2910
0.75	22.5/64	0.9	26.0	....	128.0	128.0	2890
0.75	25.0/64	1.0	15.0	. 15.0	127.0	124.5	3220
0.75	25.0/64	1.0	10.0	10.0	127.0	121.0	3240
0.75	25.0/64	1.0	5.0	5.0	103.0	106.0	3260
0.75	25.0/64	1.0	20.0	....	132.0	129.0	3100
0.75	25.0/64	1.0	23.0	....	129.0	128.0	3015
0.75	25.0/64	1.0	....	23.0	110.0	128.0	3015
0.75	25.0/64	1.0	24.0	....	98.0	128.0	3015
0.75	25.0/64	1.0	23.5	23.5	114.0	128.0	3015
0.75	25.0/64	1.0	25.0	....	98.0	128.0	3015
0.75	25.0/64	1.0	60.0	60.0	90.0	137.0	3015
0.75	25.0/64	1.0	....	22.0	106.0	127.0	3040
0.75	25.0/64	1.0	....	23.0	85.0	122.0	3015
0.75	25.0/64	1.0	33.0	....	98.0	129.5	3015
0.75	25.0/64	1.0	34.0	....	90.0	127.0	3015
0.75	25.0/64	1.0	22.0	....	108.0	127.0	3050
0.75	25.0/64	1.0	8.0	....	117.0	114.5	3245
0.75	25.0/64	1.0	13.0	....	123.0	123.0	3270
0.75	25.0/64	1.0	17.0	17.0	131.0	127.0	3180
0.75	32.5/64	1.3	15.0	15.0	108.0	122.5	3220
0.75	32.5/64	1.3	21.0	....	108.0	126.0	3200
0.75	32.5/64	1.3	22.0	....	137.0	130.0	3090
0.75	32.5/64	1.3	23.0	....	137.0	132.0	3060
0.75	32.5/64	1.3	25.0	....	138.0	133.0	3090
0.75	32.5/64	1.3	30.0	....	148.0	139.0	3270
0.75	32.5/64	1.3	35.0	....	149.5	141.0	3360
0.75	32.5/64	1.3	40.0	....	151.0	142.0	3385
0.75	32.5/64	1.3	47.0	....	151.5	142.0	3495
0.75	32.5/64	1.3	48.0	....	107.0	140.0	3495
0.75	32.5/64	1.3	....	39.0	106.0	138.5	3390
0.75	32.5/64	1.3	....	38.0	151.0	142.0	3390
0.75	32.5/64	1.3	45.0	....	147.5	140.5	3440

TABLE 5.-Continued.

Pulsator Depth Setting	Retreat Distance Setting (R): Inches	R/D	Nozzle Pressure (psig) Ascending	Descending	Fundamental Intensity, db	Total Sound Level *	Fundamental Frequency
0.75	32.5/64	1.3	24.0	....	138.0	132.0	3050
0.75	32.5/64	1.3	27.0	....	147.5	139.0	3190
0.75	32.5/64	1.3	26.0	....	140.0	134.5	3130
0.75	32.5/64	1.3	33.0	....	148.5	140.0	3390
0.75	32.5/64	1.3	60.0	....	98.0	140.5	3500
0.75	37.5/64	1.5	15.0	....	114.0	123.0	3200
0.75	37.5/64	1.5	30.0	....	144.0	137.0	2710
0.75	37.5/64	1.5	29.0	29.0	100.0	129.0	3400
0.75	37.5/64	1.5	40.0	....	148.0	142.0	3270
0.75	37.5/64	1.5	50.0	....	152.0	145.0	3320
0.75	37.5/64	1.5	55.0	....	150.5	143.0	3370
0.75	37.5/64	1.5	57.0	....	108.0	138.0	3000-4000
0.75	37.5/64	1.5	....	54.0	108.0	138.0	3000-4000
0.75	37.5/64	1.5	....	51.0	106.0	139.5	3000-4000
0.75	37.5/64	1.5	....	50.0	106.0	139.5	3000-4000
0.75	37.5/64	1.5	....	49.0	152.0	144.0	3300
0.75	37.5/64	1.5	56.0	....	151.0	144.0	3400
0.75	37.5/64	1.5	48.0	....	151.0	143.0	3300
0.75	37.5/64	1.5	45.0	....	151.0	144.0	3250
0.75	37.5/64	1.5	43.0	43.0	152.0	144.5	3240
0.75	37.5/64	1.5	35.0	....	151.0	143.0	3235
0.75	37.5/64	1.5	80.0	80.0	110.0	143.0	3000-4000
0.75	37.5/64	1.5	....	56.0	106.0	139.0	3000-4000
0.75	40.0/64	1.6	15.0	....	106.0	122.5	3150
0.75	40.0/64	1.6	30.0	....	90.0	123.0	3270
0.75	40.0/64	1.6	31.0	....	140.0	136.0	2550
0.75	40.0/64	1.6	35.0	....	145.0	139.0	2895
0.75	40.0/64	1.6	40.0	....	149.0	141.5	3265
0.75	40.0/64	1.6	62.0	....	150.0	143.0	3400
0.75	40.0/64	1.6	63.0	....	115.0	139.0	4900
0.75	40.0/64	1.6	....	59.0	124.0	149.5	5400
0.75	40.0/64	1.6	....	58.0	153.0	145.0	3310
0.75	40.0/64	1.6	85.0	....	138.0	142.5	5000
0.75	40.0/64	1.6	75.0	....	140.0	141.5	4930
0.75	40.0/64	1.6	70.0	....	144.0	141.0	4900
0.75	40.0/64	1.6	65.0	....	120.0	139.0	4800
0.75	40.0/64	1.6	50.0	....	153.0	145.5	3240
0.75	40.0/64	1.6	55.0	....	152.0	145.0	3310
0.75	40.0/64	1.6	60.0	....	147.0	143.0	3390
0.75	40.0/64	1.6	45.0	....	150.0	143.0	3140
0.75	40.0/64	1.6	48.0	....	152.0	144.0	3165

TABLE 5.-Continued.

Pulsator Depth Setting	Retreat Distance Setting (R): Inches	R/D	Nozzle Pressure (psig) Ascending	Descending	Fundamental Intensity, db	Total Sound Level *	Fundamental Frequency
1.0	22.5/64	0.9	15.0	15.0	106.0	123.0	2660
1.0	22.5/64	0.9	20.0	....	135.0	128.0	2650
1.0	22.5/64	0.9	24.0	....	136.0	131.0	2620
1.0	22.5/64	0.9	25.0	....	131.0	128.0	2640
1.0	22.5/64	0.9	26.0	....	130.0	129.0	2630
1.0	22.5/64	0.9	27.0	....	128.0	128.0	2610
1.0	22.5/64	0.9	28.0	....	104.0	128.0	2600
1.0	22.5/64	0.9	60.0	....	85.0	133.0	2600
1.0	22.5/64	0.9	....	26.0	114.0	127.0	2630
1.0	22.5/64	0.9	....	25.0	126.0	127.0	2600
1.0	22.5/64	0.9	....	24.0	135.0	130.5	2620
1.0	22.5/64	0.9	17.0	....	124.0	125.0	2680
1.0	22.5/64	0.9	18.0	....	128.0	125.0	2675
1.0	22.5/64	0.9	19.0	....	134.0	128.0	2655
1.0	22.5/64	0.9	22.0	....	135.0	132.0	2600
1.0	22.5/64	0.9	16.0	....	110.0	124.0	2670
1.0	22.5/64	0.9	5.0	....	120.0	112.0	2690
1.0	25.0/64	1.0	15.0	....	112.0	123.0	2660
1.0	25.0/64	1.0	17.0	....	136.0	129.5	2605
1.0	25.0/64	1.0	16.0	....	132.0	126.0	2630
1.0	25.0/64	1.0	20.0	....	132.0	129.0	2530
1.0	25.0/64	1.0	25.0	....	133.0	132.0	2440
1.0	25.0/64	1.0	26.0	....	132.0	131.0	2430
1.0	25.0/64	1.0	27.0	....	120.0	129.0	2480
1.0	25.0/64	1.0	28.0	....	106.0	129.0	2600
1.0	25.0/64	1.0	30.0	....	104.0	128.5	2600
1.0	25.0/64	1.0	60.0	....	95.0	139.0	2600
1.0	25.0/64	1.0	....	25.0	110.0	128.0	2600
1.0	25.0/64	1.0	....	24.0	133.0	128.0	2460
1.0	25.0/64	1.0	34.0	....	95.0	129.0	2600
1.0	25.0/64	1.0	35.0	....	85.0	126.0	2600
1.0	25.0/64	1.0	10.0	10.0	124.0	120.0	2690
1.0	25.0/64	1.0	8.0	8.0	130.0	121.0	2680
1.0	25.0/64	1.0	6.0	6.0	126.0	118.0	2670
1.0	25.0/64	1.0	5.0	5.0	108.0	106.0	2660
1.0	32.5/64	1.3	15.0	15.0	109.0	122.5	2640
1.0	32.5/64	1.3	20.0	....	106.0	126.0	2620
1.0	32.5/64	1.3	21.0	....	133.5	128.0	2440
1.0	32.5/64	1.3	25.0	....	135.0	130.0	2390
1.0	32.5/64	1.3	30.0	....	137.0	133.0	2500
1.0	32.5/64	1.3	35.0	....	145.0	138.5	2645
1.0	32.5/64	1.3	40.0	....	150.0	142.0	2690

TABLE 5.-Continued.

Pulsator Depth Setting	Retreat Distance Setting (R): Inches	R/D	Nozzle Pressure (psig) Ascending	Descending	Fundamental Intensity, db	Total Sound Level *	Fundamental Frequency
1.0	32.5/64	1.3	45.0	....	147.5	141.0	2705
1.0	32.5/64	1.3	48.0	....	149.0	142.0	2725
1.0	32.5/64	1.3	49.0	....	98.0	140.0	2725
1.0	32.5/64	1.3	....	31.0	90.0	140.0	2550
1.0	32.5/64	1.3	....	30.0	136.0	133.0	2500
1.0	32.5/64	1.3	23.0	....	133.0	128.5	2395
1.0	32.5/64	1.3	24.0	....	133.0	129.5	2380
1.0	32.5/64	1.3	26.0	....	137.0	132.0	2390
1.0	32.5/64	1.3	28.0	....	133.0	133.0	2420
1.0	32.5/64	1.3	22.0	22.0	134.0	129.5	2410
1.0	32.5/64	1.3	60.0	....	100.0	140.0	2725
1.0	37.5/64	1.5	15.0	....	112.0	122.0	2620
1.0	37.5/64	1.5	20.0	....	112.0	125.5	2620
1.0	37.5/64	1.5	21.0	....	120.0	127.0	2540
1.0	37.5/64	1.5	22.0	....	134.0	129.0	2450
1.0	37.5/64	1.5	23.0	....	135.0	129.0	2360
1.0	37.5/64	1.5	24.0	....	136.0	132.0	2320
1.0	37.5/64	1.5	25.0	....	137.0	131.0	2260
1.0	37.5/64	1.5	30.0	....	142.0	137.0	2220
1.0	37.5/64	1.5	35.0	....	138.0	138.5	2520
1.0	37.5/64	1.5	32.0	....	138.0	136.0	2350
1.0	37.5/64	1.5	33.0	....	136.0	136.0	2410
1.0	37.5/64	1.5	34.0	....	140.0	136.0	2450
1.0	37.5/64	1.5	40.0	....	146.0	141.0	2530
1.0	37.5/64	1.5	50.0	....	147.0	141.0	2580
1.0	37.5/64	1.5	....	50.0	105.0	140.0	2000-3000
1.0	37.5/64	1.5	....	45.0	146.0	141.0	2560
1.0	37.5/64	1.5	57.0	....	150.0	143.0	2600
1.0	37.5/64	1.5	58.0	....	105.0	141.0	2625
1.0	37.5/64	1.5	....	46.0	105.0	140.0	2580
1.0	40.0/64	1.6	18.0	....	134.0	127.0	2590
1.0	40.0/64	1.6	25.0	....	115.0	126.0	2410
1.0	40.0/64	1.6	26.0	....	131.0	133.0	2310
1.0	40.0/64	1.6	30.0	....	136.0	135.0	2030
1.0	40.0/64	1.6	35.0	....	137.0	137.0	2430
1.0	40.0/64	1.6	40.0	....	140.0	138.0	2490
1.0	40.0/64	1.6	45.0	....	135.0	139.0	2480
1.0	40.0/64	1.6	60.0	....	150.0	141.5	2560
1.0	40.0/64	1.6	66.0	....	146.0	142.0	2595
1.0	40.0/64	1.6	67.0	....	126.0	141.0	3500
1.0	40.0/64	1.6	....	58.0	118.0	139.5	3700
1.0	40.0/64	1.6	....	57.0	146.0	141.0	2530

TABLE 5.—Continued.

Pulsator Depth Setting	Retreat Distance Setting (R): Inches	R/D	Nozzle Pressure (psig): Ascending	Descending	Fundamental Intensity, db	Total Sound Level *	Fundamental Frequency
1.0	40.0/64	1.6	50.0	....	140.0	139.0	2500
1.0	40.0/64	1.6	52.0	....	142.0	140.0	2505
1.0	40.0/64	1.6	48.0	....	145.0	139.0	2500
1.0	40.0/64	1.6	20.0	....	135.0	128.0	2570
1.0	40.0/64	1.6	22.0	....	117.0	126.0	2560
1.0	40.0/64	1.6	17.0	....	112.0	123.0	2600
1.0	40.0/64	1.6	21.0	....	138.0	129.0	2535
2.0	22.5/64	0.9	15.0	15.0	84.0	123.0	1475
2.0	22.5/64	0.9	20.0	....	100.0	127.0	1500
2.0	22.5/64	0.9	25.0	....	106.0	129.0	1520
2.0	22.5/64	0.9	30.0	....	90.0	129.0	1520
2.0	22.5/64	0.9	27.0	....	103.0	128.0	1520
2.0	22.5/64	0.9	23.0	....	100.0	127.5	absent
2.0	22.5/64	0.9	24.0	....	104.0	128.0	1520
2.0	22.5/64	0.9	10.0	....	86.0	117.0	1470
2.0	22.5/64	0.9	3.0	....	100.0	93.0	1500
2.0	25.0/64	1.0	15.0	....	100.0	123.0	1475
2.0	25.0/64	1.0	16.0	....	125.0	124.5	1465
2.0	25.0/64	1.0	17.0	....	130.0	126.0	1465
2.0	25.0/64	1.0	25.0	....	134.0	131.0	1430
2.0	25.0/64	1.0	28.0	....	136.0	132.0	1420
2.0	25.0/64	1.0	29.0	....	98.0	128.0	1550
2.0	25.0/64	1.0	....	26.0	98.0	129.0	1550
2.0	25.0/64	1.0	....	25.0	134.0	131.0	1430
2.0	25.0/64	1.0	33.0	....	90.0	129.0	1500
2.0	25.0/64	1.0	34.0	....	88.0	126.0	1400
2.0	25.0/64	1.0	10.0	....	92.0	116.0	1500
2.0	25.0/64	1.0	5.0	....	92.0	106.0	1520
2.0	25.0/64	1.0	20.0	....	133.0	129.0	1450
2.0	25.0/64	1.0	22.0	....	128.0	129.0	1445
2.0	25.0/64	1.0	23.0	....	120.0	129.0	1440
2.0	25.0/64	1.0	26.0	....	134.0	131.0	1430
2.0	25.0/64	1.0	60.0	....	86.0	139.0	1500
2.0	32.5/64	1.3	15.0	15.0	98.0	123.0	1460
2.0	32.5/64	1.3	17.0	....	105.0	124.0	1450
2.0	32.5/64	1.3	18.0	....	125.0	127.0	1430
2.0	32.5/64	1.3	20.0	....	132.0	128.0	1420
2.0	32.5/64	1.3	25.0	....	134.0	130.0	1380
2.0	32.5/64	1.3	30.0	....	137.5	132.5	1360
2.0	32.5/64	1.3	35.0	....	139.0	135.5	1410
2.0	32.5/64	1.3	40.0	....	143.0	139.0	1480

TABLE 5.-Continued.

Pulsator Depth Setting	Retreat Distance Setting (R): Inches	R/D	Nozzle Pressure (psig) Ascending	Descending	Fundamental Intensity, db	Total Sound Level *	Fundamental Frequency
2.0	32.5/64	1.3	45.0	....	144.0	139.5	1500
2.0	32.5/64	1.3	....	40.0	88.0	140.0	1465
2.0	32.5/64	1.3	....	39.0	143.0	138.0	1465
2.0	32.5/64	1.3	49.0	....	144.0	139.0	1510
2.0	32.5/64	1.3	50.0	....	105.0	139.0	1510
2.0	32.5/64	1.3	28.0	....	138.0	133.0	1345
2.0	32.5/64	1.3	32.0	....	137.0	133.5	1375
2.0	32.5/64	1.3	26.0	....	137.0	132.0	1355
2.0	32.5/64	1.3	27.0	....	135.0	131.0	1350
2.0	32.5/64	1.3	29.0	....	138.5	133.0	1355
2.0	32.5/64	1.3	60.0	....	95.0	138.0	absent
2.0	37.5/64	1.5	15.0	15.0	97.0	123.0	1460
2.0	37.5/64	1.5	21.0	....	111.0	127.0	1460
2.0	37.5/64	1.5	22.0	....	126.0	128.0	1450
2.0	37.5/64	1.5	23.0	....	133.0	129.0	1420
2.0	37.5/64	1.5	24.0	....	132.0	130.0	1400
2.0	37.5/64	1.5	25.0	....	135.0	131.0	1390
2.0	37.5/64	1.5	40.0	....	142.0	138.0	1360
2.0	37.5/64	1.5	63.0	....	145.0	140.0	1460
2.0	37.5/64	1.5	64.0	....	105.0	141.0	1460
2.0	37.5/64	1.5	....	51.0	105.0	140.0	1425
2.0	37.5/64	1.5	....	50.0	144.0	140.0	1425
2.0	37.5/64	1.5	30.0	....	139.0	134.0	1320
2.0	37.5/64	1.5	35.0	....	140.0	135.0	1305
2.0	37.5/64	1.5	45.0	....	139.0	139.0	1400
2.0	37.5/64	1.5	41.0	....	141.0	138.0	1365
2.0	37.5/64	1.5	43.0	....	143.0	139.0	1390
2.0	37.5/64	1.5	42.0	....	141.5	138.5	1380
2.0	37.5/64	1.5	80.0	....	108.0	138.0	1300-1400
2.0	40.0/64	1.6	50.0	....	144.5	140.0	1360
2.0	40.0/64	1.6	23.0	....	114.0	126.0	1440
2.0	40.0/64	1.6	25.0	....	134.0	129.0	1410
2.0	40.0/64	1.6	24.0	....	124.0	127.0	1435
2.0	40.0/64	1.6	40.0	....	141.0	136.0	1275
2.0	40.0/64	1.6	45.0	....	145.0	139.5	1320
2.0	40.0/64	1.6	73.0	....	145.0	142.0	1410
2.0	40.0/64	1.6	74.0	....	100.0	141.0	1410
2.0	40.0/64	1.6	....	59.0	90.0	141.0	1370
2.0	40.0/64	1.6	....	58.0	142.0	140.5	1370
2.0	40.0/64	1.6	30.0	....	140.0	135.0	1260
2.0	40.0/64	1.6	55.0	....	145.0	141.0	1370
2.0	40.0/64	1.6	17.0	....	131.0	125.0	1440

TABLE 5.-Continued.

Pulsator Depth Setting	Retreat Distance Setting (R): Inches	R/D	Nozzle Pressure (psig): Ascending	Descending	Fundamental Intensity, db	Total Sound Level *	Fundamental Frequency
2.0	40.0/64	1.6	16.0	....	116.0	122.5	1430
2.0	40.0/64	1.6	15.0	....	100.0	121.5	1420
2.0	40.0/64	1.6	18.0	....	135.0	126.0	1460
2.0	40.0/64	1.6	19.0	....	102.0	124.5	1500
2.0	40.0/64	1.6	22.0	....	105.0	125.0	1460
4.0	22.5/64	0.9	15.0	15.0	88.0	123.5	790
4.0	22.5/64	0.9	20.0	....	131.0	128.0	2330
4.0	22.5/64	0.9	20.0	....	90.0	128.0	777
4.0	22.5/64	0.9	25.0	....	128.0	128.0	2340
4.0	22.5/64	0.9	25.0	....	114.0	128.0	795
4.0	22.5/64	0.9	26.0	....	90.0	128.0	800
4.0	22.5/64	0.9	26.0	....	135.0	128.0	2330
4.0	22.5/64	0.9	18.0	....	118.0	125.0	2340
4.0	22.5/64	0.9	19.0	....	126.0	127.0	2335
4.0	22.5/64	0.9	22.0	....	134.0	130.0	2320
4.0	22.5/64	0.9	23.0	....	134.0	131.0	2315
4.0	22.5/64	0.9	24.0	....	126.0	128.0	2310
4.0	22.5/64	0.9	....	24.0	98.0	126.0	2310
4.0	22.5/64	0.9	....	23.0	134.0	131.0	2320
4.0	22.5/64	0.9	27.0	....	127.0	128.0	2320
4.0	22.5/64	0.9	27.0	....	110.0	128.0	800
4.0	22.5/64	0.9	28.0	....	90.0	128.0	800 & 2320
4.0	22.5/64	0.9	....	26.0	105.0	128.0	790
4.0	25.0/64	1.0	15.0	....	95.0	123.0	790
4.0	25.0/64	1.0	16.0	....	112.0	124.0	788
4.0	25.0/64	1.0	17.0	....	120.0	126.0	786
4.0	25.0/64	1.0	18.0	....	121.0	127.0	785
4.0	25.0/64	1.0	20.0	....	125.0	128.0	790
4.0	25.0/64	1.0	25.0	....	127.0	131.0	788
4.0	25.0/64	1.0	26.0	....	125.0	132.0	782
4.0	25.0/64	1.0	27.0	....	128.0	132.0	786
4.0	25.0/64	1.0	28.0	....	129.0	132.0	789
4.0	25.0/64	1.0	29.0	....	126.0	131.0	780
4.0	25.0/64	1.0	....	30.0	124.0	130.0	778
4.0	25.0/64	1.0	30.0	....	94.0	128.0	778
4.0	25.0/64	1.0	....	25.0	125.0	129.0	785
4.0	25.0/64	1.0	....	26.0	80.0	132.0	782
4.0	25.0/64	1.0	19.0	....	123.0	128.0	788
4.0	25.0/64	1.0	10.0	....	80.0	116.0	790
4.0	25.0/64	1.0	33.0	....	105.0	128.0	750
4.0	25.0/64	1.0	34.0	....	105.0	126.0	790
4.0	25.0/64	1.0	60.0	....	88.0	137.0	790

TABLE 5.-Continued.

Pulsator Depth Setting	Retreat Distance Setting (R): Inches	R/D	Nozzle Pressure (psig) Ascending	Descending	Fundamental Intensity, db	Total Sound Level *	Fundamental Frequency
4.0	32.5/64	1.3	15.0	15.0	90.0	124.0	775
4.0	32.5/64	1.3	16.0	....	90.0	124.0	775
4.0	32.5/64	1.3	17.0	....	119.0	125.0	775
4.0	32.5/64	1.3	18.0	....	120.0	126.0	775
4.0	32.5/64	1.3	20.0	....	124.0	128.0	780
4.0	32.5/64	1.3	25.0	....	127.5	130.0	750
4.0	32.5/64	1.3	30.0	....	130.0	130.0	748
4.0	32.5/64	1.3	35.0	....	133.0	132.0	770
4.0	32.5/64	1.3	43.0	....	135.0	134.5	790
4.0	32.5/64	1.3	44.0	....	105.0	138.0	790
4.0	32.5/64	1.3	....	39.0	102.0	138.0	780
4.0	32.5/64	1.3	....	38.0	135.0	133.0	780
4.0	32.5/64	1.3	22.0	....	126.0	128.0	770
4.0	32.5/64	1.3	23.0	....	125.0	128.0	760
4.0	32.5/64	1.3	24.0	....	128.0	128.6	755
4.0	32.5/64	1.3	32.0	....	132.0	130.8	753
4.0	32.5/64	1.3	33.0	....	131.0	131.0	760
4.0	32.5/64	1.3	37.0	....	134.0	133.0	785
4.0	32.5/64	1.3	60.0	....	90.0	139.0	790
4.0	37.5/64	1.5	15.0	....	90.0	122.0	780
4.0	37.5/64	1.5	19.0	....	90.0	124.5	780
4.0	37.5/64	1.5	20.0	....	92.0	125.5	780
4.0	37.5/64	1.5	21.0	....	121.0	127.0	780
4.0	37.5/64	1.5	22.0	....	124.0	128.0	775
4.0	37.5/64	1.5	25.0	....	128.0	129.0	750
4.0	37.5/64	1.5	30.0	....	136.5	131.0	727
4.0	37.5/64	1.5	....	52.0	138.0	138.0	745
4.0	37.5/64	1.5	61.0	....	140.0	138.0	755
4.0	37.5/64	1.5	62.0	....	110.0	138.0	755
4.0	37.5/64	1.5	....	53.0	112.0	137.0	745
4.0	37.5/64	1.5	35.0	....	136.0	133.0	720
4.0	37.5/64	1.5	40.0	....	136.0	134.0	725
4.0	37.5/64	1.5	45.0	....	138.0	137.0	735
4.0	37.5/64	1.5	50.0	....	137.5	138.0	748
4.0	37.5/64	1.5	55.0	....	135.5	138.0	750
4.0	37.5/64	1.5	33.0	....	135.0	131.5	722
4.0	37.5/64	1.5	37.0	....	135.0	132.5	722
4.0	42.5/64	1.6	23.0	....	115.0	127.0	773
4.0	42.5/64	1.6	24.0	....	123.0	127.5	776
4.0	42.5/64	1.6	25.0	....	125.0	128.0	775
4.0	42.5/64	1.6	30.0	....	134.0	130.5	726

TABLE 5.-Continued.

Pulsator Depth Setting	Retreat Distance Setting (R): Inches	R/D	Nozzle Pressure (psig) Ascending	Descending	Fundamental Intensity, db	Total Sound Level *	Fundamental Frequency
4.0	42.5/64	1.6	35.0	....	139.0	133.0	705
4.0	42.5/64	1.6	50.0	....	142.0	137.5	706
4.0	42.5/64	1.6	60.0	....	141.0	139.5	723
4.0	42.5/64	1.6	70.0	....	140.0	139.5	740
4.0	42.5/64	1.6	71.0	....	85.0	139.5	740
4.0	42.5/64	1.6	....	60.0	no oscillation		
4.0	42.5/64	1.6	....	59.0	141.0	139.0	725
4.0	42.5/64	1.6	55.0	....	140.5	138.0	724

*The indicated cable correction according to its temperature calibration chart is +7.3 db. An actual calibration of the cable showed a correction of +7.4 at 400. cps. In some cases the fundamental intensity readings, with all corrections applied through the calibration setting, is greater than the value of the Total Sound Level column +7.3 db. This discrepancy is roughly indicative of experimental error when most of the sound energy lies in the fundamental, since the fundamental intensity may not actually exceed the overall intensity.