PREDICTING THE DEPTH OF PENETRATION OF A BUOYANT PLUME IN A STRATIFIED LAKE

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NOMENCLATURE

C_n	Constants; where n = 0, 1, 2, ..., 9
D	Diameter of jet
E	Increased energy content in pumped fluid, or head
Fr_d	Densimetric Froude number
g	Local gravitational force
g'	Characteristic gravitational force
H_e	Epilimnion penetration length
H_h	Hypolimnion penetration length
H_p	Length of plume
L	Characteristic length
N	Rotative speed of propeller
P	Power input to propeller
Q	Pump flowrate
R^2	Coefficient of Determination Statistic
S	Standard Deviation Statistic
T	Temperature of water, ^oC
V	Characteristic jet velocity
V_o	Initial jet velocity
V_p	Jet velocity at penetration depth

GREEK LETTERS

ρ	Density of water, kg/cu m
ρ_h	Density of water at some depth, h
ρ_o	Average density of pumped water
ρ_n	Density of water at depth, n; where n = 0, 1, 2
$\Delta\rho$	Density difference between ρ_h and ρ_o

CHAPTER I

INTRODUCTION

Problem Synopsis

The necessity of establishing design criteria is intrinsic to new applications of advancing technology. The Garton pump was developed specifically to enhance lake water quality by propelling large volumes of surface waters downward to mix with lower strata in stratified lakes. This method of lake mixing has proven to be successful in improving the quality of water within the lake and in the releases from the lake for downstream needs. However, early development and prototype testing of the Garton pump revealed the importance and need for design criteria relative to the depth of penetration of the pumped waters in a stratified lake.

As lake surface waters are warmed during spring and summer months, thermal-density stratification prevents holomixis (the surface to bottom mixing of the lake water). Typically, three distinguishable strata form. The epilimnion, the upper stratum of the lake, is characterized by warm temperatures, oxygenated, good quality water. The thermocline (or metalimnion), located below the epilimnion, is characterized by declining temperatures and often decreasing oxygen concentrations. Without the benefit of circulation or oxygen production, the lower stratum of the lake, the hypolimnion, generally becomes anoxic due to biological respiration. Concentrations of hydrogen sulfide and ammonia nitrate, both products of biological activities, increase in the anoxic hypolimnion as do the concentrations of dissolved metals such as iron and manganese. Thus, under anoxic conditions, the hypolimnion becomes putrid and incapable of supporting fish life.

Various methods have been employed to improve the lake water quality during periods of stratification. Air-bubbling, liquid oxygen injection, and mechanical mixing are examples. Mechanical mixing can be an effective and economical method for mixing a lake when the Garton pump is used to induce lake destratification. The objective of destratification is to eliminate the anoxic hypolimnion by induced holomixis; this process occurs naturally in the fall as the result of the cooling of lake surface temperatures. The elimination of the anoxic hypolimnion has two major benefits: an overall improvement in water quality and the preclusion of whole lake degradation that is associated with sudden fall turnovers.

Although the power consumption of the Garton pump is quite modest, especially when compared to other methods, the volumetric flowrate of pumped water is large. The Garton pump has been successful in lake mixing tests where the pumped waters have penetrated to the lake bottom. A failure of pumped waters to penetrate to the bottom of the lake has resulted in only partial lake mixing. An inability to predict the depth of penetration of the pumped waters hampered the success of prototype testing in some lake destratification studies.

Many man-made lakes release water from the bottom of the lake. While this is hydraulically useful to minimize the size of the outlet structure for large release rates, it commonly results in the downstream release of poor quality water during stratified periods. With increased water quality awareness and enforcement of point discharge standards, the need for an economical alternative to outlet modification arose. When whole lake destratification was not possible or desired, the Garton pump has been used to achieve local destratification for the improvement of the release water quality (6, 13, and 18). By locating the pump over the intake port and forcing good quality surface water downward to displace the anoxic water at the intake level, the release water quality was improved. If the pumped water does not penetrate to the withdrawal zone of the intake, no benefit to water quality will be realized in the released water. If the pumped water penetrates too deeply or in too great of volume, then excessive mixing can cause a decrease in the benefit of this method.

For either localized or global destratification, the prediction of the depth of penetration of the pumped water into lake hypolimnia was demonstrated to be the single most important design factor; yet, no accurate prediction methods were known.

Objective

The objective of this study was to develop a non-dimensional equation to predict the depth of penetration of a buoyant plume produced by a Garton pump in a stratified lake as a function of pump diameter, pumping velocity, and density profile of the lake.

Limitations of the Study

The ranges of the prediction parameters to be investigated were limited to normal design characteristics of the Garton pump. The characteristics of the density profiles investigated were limited to those that naturally occurred in several Oklahoma lakes during statification periods. Transportation problems limited (to three) the number of pumps that could be hauled to a test site.

CHAPTER II

REVIEW OF LITERATURE

Thermal-density Stratification

The temperature profile associated with summer stratification is the result of a complex interaction of radiation input, air temperature, wind, and geometric characteristics of the lake and basin. The thermal-density differences within a lake profile can effectively prevent holomixis. A density difference as small as 0.0004g/cc can cause stable stratification (11); this is approximately equal to the temperature difference between 18.0 and 20.0^{o}C. In Oklahoma, epilimnion temperatures of 25 to 33^{o}C are common while hypolimnion temperatures may vary between 10 to 20^{o}C. A thermal difference of less than 10^{o}C has been reported to stop the hypolimnetic penetration of a downward flowing jet of surface water (having a velocity of 0.67m/s) in less than 2m (16).

To characterize the resistance to mixing of the density layers within a thermally stratified lake, a unit of relative thermal resistance (RTR) has been defined (24). A unit of RTR is equal to the density difference between water at 5 and 4^{o}C (i.e. 8×10^{-5}gm/cc) for a column of water 0.5m long. Figure 1 (page 40) shows the RTR for a typical summer profile of a lake where the RTR is expressed as the ratio of the density difference between water at the top and bottom of each column to one unit of RTR. The density-related resistance to penetration of a downward flow of surface water is particularly strong in the thermocline region. Similar figures have been presented by Birge (1) and Vallentine (23).

The thermocline is generally referred to as the stratum in a lake that has $\geq 1^{o}C$ change per one meter depth (24); however, it is sometimes referred to as the point in the lake profile where the maximum temperature gradient occurs. The distinction between the gradient description and the single-point description is important because: definitions of the bottom of the epilimnion and the beginning of the hypolimnion are dependent upon the thermocline definition; and a precise description of the thermocline depth is necessary for prediction equation analyses. In this report, the single-point definition of the thermocline will be assumed unless the gradient is specifically referenced.

To determine the density of water as a function of temperature, where dissolved and suspended solids do not contribute to the density, the following equation was reported (5):

$$\rho = 1000. - \frac{(T - 3.98)^2 \ (T + 283)}{(503.57)\ (T + 67.26)} \tag{2.1}$$

where ρ = density of water in kg/cu m

T = temperature of water in ^{o}C.

Axial Flow Pump

The use of an axial flow pump to mix a lake by propelling a large volume of fluid at low velocities, low heads, and low power requirements was first reported by Quintero and Garton (17). Since that report, the pump (Figure 2, page 41) has been referenced in literature as a Garton pump. Although a summation of the notations used in this report is given on page ix, Figure 3 (page 42) depicts the notation that applies to the operation of the pump within a stratified lake. Presentations of reported equations and figures, from other authors, have been converted to one consistent set of notation as used in this report.

Early work by Steichen (21) and Strecker (22) concluded that basic fan laws provided an effective means of predicting the pump performance in water from the available data based on air tests. The fan laws for a constant diameter are:

$$\frac{Q_1}{Q_2} = \frac{N_1}{N_2} \qquad (2.2)$$

$$\frac{E_1}{E_2} = \frac{N^2{}_1 \ \rho_1}{N^2{}_2 \ \rho_2} \qquad (2.3)$$

$$\frac{P_1}{P_2} = \frac{N^3{}_1 \ \rho_1}{N^3{}_2 \ \rho_2} \qquad (2.4)$$

where Q = pump flowrate

N = rotative speed

E = increased energy content in the fluid pumped, or head

P = input power.

These equations are valid for low heads. When no fluid is delivered from the propeller, the fluid flow consists of eddies entering near the hub and leaving near the tips. This action alters the propeller performance and increases both pressure and horsepower (20). In model studies of the Garton pump, Hart (8) concluded that the propeller performance is reduced by a downstream obstruction such as a high thermocline (about 3 pump diameters below the pump). A suggested relationship was not given for the correction of propeller performance due to a high thermocline. Hart also reported unpublished findings of others (namely M. A. Haywood, T. F. Maloney, B. E. Mckillop, and G. F. Sander) that indicated the propeller performance was reduced by a solid obstruction within 2 pump diameters, and that penetration was reduced by viscous dissipation as the thermocline depth increased beyond 2 pump diameters below the pump.

Forced Plumes

The term "forced plumes" was used by Morton (15), and other more recent researchers, to describe the resultant shape of a body of fluid pumped into a fluid of dissimilar density. Although the forced plume has been modelled and studied by various researchers, prototype study of penetration characteristics was limited to a single reference (16). In this study, a cluster of sixteen 1.83m diameter pumps was used and no data relative to the effect of diameter and varied stratification patterns were available. The diameter of the pump has been determined to be an important parameter in penetration (7, 9, and 12).

The densimetric Froude number, Fr_d, has been established as the most important non-dimensional relationship associated with the forced plume. The Fr_d number is the inverse square root of the densimetric Richardson number, Ri_d. The Fr_d number represents the ratio of inertial forces (jet momentum flux) to the buoyancy forces (a product of the jet volume flux and the kinematic buoyancy); the generalized form is:

$$Fr_d = \frac{V}{\sqrt{g' L}} \tag{2.5}$$

where V = characteristic jet velocity

g′ = characteristic gravitational effect

L = characteristic length.

Although the importance of the Fr_d number in plume studies was undisputed in the literature, there were differences with regards to the specific definitions of the three variables involved.

Characteristic Velocity

The characteristic velocity was defined in most model studies as the maximum or mean jet velocity measured some distance below the initial jet. Generally, the propeller flow characteristics were unknown in model studies and therefore required flow measurements. In studies where the pump performance was known, the characteristic velocity was a calculated mean velocity of the initial jet (4, 21, and 22). The actual definition of velocity appears to be less critical than choosing a characteristic velocity that can be accurately and consistently determined for the particular study.

Characteristic Gravitational Effect

The gravitational effect must be sensitive to the differing densities associated with a stratified body of fluid. An application of Bernoulli´s equation (Appendix C) showed the gravitational effect in a stratified fluid to be:

$$g' = g \int_0^{H_p} \frac{\rho_h - \rho_o}{\rho_o} \, dh \qquad (2.6)$$

where g = local gravitational force

H_p = length of the plume

ρ_h = density of the water at some depth, h

ρ_o = average density of the pumped water.

Using a series of mathematical substitutions, Moretti showed (Appendix C) that the density difference ratio (the integral expressing of Equation 2.6) simplifies to the more common definition which has been given as:

$$g' = g(\Delta\rho/\rho_o) \text{ ; for the plume length} \qquad (2.7)$$

where $\Delta\rho = \rho_h - \rho_o$.

In reported model studies, the values of ρ_o and ρ were constant and selectable for the initial conditions. In lakes, neither ρ_o nor ρ are constant and therefore the values require more specific definitions.

Equation 2.6 does not distinguish between the depth of the thermocline and the density difference. Thus, equivalent values of g′ can be calculated for either: (1) a shallow thermocline and large density difference, or (2) a deep thermocline and small density difference. An assumption inherent to Bernoulli´s equation is that the flow is inviscid; therefore there is no accounting for flux losses in epilimnion penetration (i.e. penetration to the depth of the thermocline). Equation 2.7 is totally insensitive to the thermocline depth.

Characteristic Length

The characteristic length in Equation 2.5 has been defined as the diameter of the jet, or a representative length of the plume. The derivation from Bernoulli´s equation (Appendix C) substantiates the latter definition. Considering the definition of Fr_d number from dimensional analysis, the first definition seems appropriate since both the jet momentum flux and the buoyancy flux are functions of the jet diameter (2, 4, 10, and 12). Thus, reasonable arguments have been made to support either definition of the characteristic length.

Prediction of Penetration

An application of Bernoulli's equation was used to develop an equation which is a function of hypolimnetic penetration (Appendix C). The equation has the form of the Fr_d number squared:

$$C_0 = \frac{V^2}{g(\Delta\rho/\rho_o)H_h} \qquad (2.8)$$

where $C_0 = 2.0$

H_h = hypolimnetic penetration length.

This equation is also know as the inverse Richardson number and has been used to predict plume rises in the atmosphere. The equation is independent of the jet diameter which was assumed to be of secondary importance. By solving for H_h, the equation can be used to predict the plume penetration into the hypolimnion. The total plume length would be the sum of H_h and the distance from the pump to the hypolimnion. By dividing each term by jet diameter to obtain a non-dimensional form, the equation becomes:

$$\frac{H_p}{D} = \frac{V^2}{g(\Delta\rho/\rho_o)D} + \frac{H_e}{D} \qquad (2.9)$$

where H_e = epilimnion penetration length

D = diameter of jet.

Using the Bernoulli approach and adding an empirical term for the viscous dissipation of the jet in the epilimnion, the following empirical expression was derived (3):

$$V_p^{\,2} = V_o^{\,2}\,\frac{(CD)^2}{(H_p)^2} - 2g\int_0^{H_p} \frac{\rho_h - \rho_o}{\rho_o}\,dh \qquad (2.10)$$

where V_p = jet velocity at H_p

V_o = initial jet velocity

C = 3.5 (from experimental data)

$H_p \geq CD$ for validity.

The equation implicitly determines the maximum depth of penetration by trial and error substitutions for H_p (when V_p equals zero).

From dimensional analysis, the non-dimensional penetration depth can be expected to be a function of several parameters:

$$\frac{H_p}{D} = f\left\{Fr_d,\ \frac{H_e}{D},\ \frac{H_h}{D},\ Re\right\} \qquad (2.11)$$

where Re = Reynolds number.

Results from modelling studies have shown that jet penetration is a strong function of the Fr_d and the thermocline location, and a relatively weak function of the Re number (14).

CHAPTER III

EQUIPMENT AND INSTRUMENTATION

General

Three prototype pumps were designed so that important prediction parameters could be varied over a wide range of normal operating values. The pump propellers, purchased from Aerovent Manufacturing, Inc., were as geometrically similar as possible as off-the-shelf items. Using the Garton pump design, three pumps having 1.22, 1.83, and 2.44m propeller diameters were constructed and used for the penetration tests. Although some tests were conducted with other existing Garton pumps, the three pumps were designed and built specifically for this study to reduce variations in pump designs.

Pump Design and Construction

The major components of a Garton pump are the propeller and shroud, drive shaft and bearings, right-angle drive gear box, motor, and a flotation platform. Because of the logistic problems of traveling to several data collection sites, the pumps were designed to be easily transported, maneuverable on the lakes, and loaded and unloaded readily. Special design considerations included an outboard engine mount, a splashboard assembly, and wheels. The pumps were transported on a single goose-neck trailer and were easily unloaded and loaded at public boatramps with the aid of a winch (Figure 4). For each of the pumps, a maximum pumping velocity of about 1m/s was determined to be the stress limitation for the propeller blades. Since the design medium of the propellers was air, consultations with the manufacturing engineers was necessary to establish the maximum safe velocity. Using the velocity limitation as a design factor, the shaft diameter, bearings, gearbox ratings, and engines were sized for each pump. Gasoline engines were used to operate the pumps because of the need for a variable drive speed source and the lack of electrical source at remote test sites. The three gasoline engines selected produced a maximum power of 2.24kW, 3.73kW, and 5.97kW. Belt and pulley arrangements were used to reduce the engine drive speed to the desired speed of the gearbox input shaft. The gearboxes were designed by Faulk, Inc., and had input:output ratios of 15.5:1, 20.5:1, and 29.5:1 for the 1.22m, 1.83m, and 2.44m diameter pumps. A clutch assembly was included on the 2.44m pump to reduce the starting torque.

For each of the pumps, the support frame was 2.0m square, and the propeller and shroud were suspended 1.4m below the flotation platform. Tiedown rings were welded to the corners of the platforms to allow the pumps to be lashed together in a barge arrangement for movement in the lakes. A single, 5.2kW outboard engine, could be mounted on one of the platforms, to provide the needed propulsion to move all three pumps to the test site within each lake.

Instrumentation

To determine the lake temperature profiles, *in situ* thermal monitors were used. The primary instrument used was a Yellow Springs Temperature and Dissolved Oxygen Meter. A Martek Mark V Digital Water Quality Analyzer System was used as a backup for temperature sensing and for determining the conductivity profiles. The temperatures were measured to the nearest 0.2^{o}C and the conductivity was measured to the nearest whole umho/cm. The conductivity profiles were used to check for possible density gradients due to dissolved solids. The operating rpm of the pump was measured at the input shaft of the gearbox using a self-timing tachometer.

CHAPTER IV

PROCEDURES AND TEST SITES

Data Collection Procedures

The most important water quality parameter affecting plume penetration in a stratified lake is the density profile. However, routine field collection, _in situ_ testing, and multiple sampling to determine a lake profile are reasons why direct density measurements were not feasible. Using an established temperature-density relationship (Equation 2.1), the density profile was determined from the temperature profiles.

Prior to operating the pumps for the penetration tests, the thermal profile of the lake was determined. To measure the depth of penetration of the pump plume, a temperature sensing probe was lowered into the plume. The temperature in the center of the plume was indicative of lake surface values and relatively constant. As the sensing probe passed through the plume, a sudden decrease in temperature indicated the limit of penetration and entry into the unperturbed portion of the lake profile. Although the plume bottom is characterized by turbulent mixing, fluctuations in plume depth was generally observed to be less than 0.5m. During the pump operation, the revolutions per minute (rpm) of the gearbox input shaft were monitored. A single pump penetration test, at a constant propeller rpm, generally required about 10 minutes to complete. Depending upon the stratification pattern and the depth at the test site, up to five tests were conducted for a single pump diameter. After a series of tests, the thermal profile of the lake was redetermined prior to a new series of tests.

Determination of Prediction Variables

Critical prediction variables such as the characteristic jet velocity, the single-point description of the depth of the density gradient, the non-dimensional density difference, and the characteristic density of the pumped water did not have clear definitions in naturally occurring lake conditions. For examples: should the characteristic velocity be defined as the mean velocity of some plume cross-section, the centerline velocity, or mean velocity through the pump propellers; what is the proper representation of the characteristic density of the pumped water in a system where density varies with depth; and how should the characteristic depth of the density gradient be defined? Since a review of the literature only provided general guidance, specific definitions for the variables were determined as part of this study.

Physical constraints and reasonable considerations, limited the kinds of velocity descriptions that were available. The turbulent nature of the jet obviated the ability to measure the velocity profile below the propeller. The low velocities associated with the flowfield around the perimeter of the pump (above the propeller) are difficult to determine under *in situ* conditions. More importantly, the prediction of penetration becomes less important if a pump needs to be built in order to determine the jet velocities that can be achieved. The characteristic velocity used in this study was the mean water velocity through the propeller as calculated from the performance literature for air. Knowing the pump rotative speed, the flowrate through the propeller was calculated using the fan laws. By dividing the flowrate by the area of the propeller swath, the mean pump velocity was calculated.

To determine the best definition of the characteristic depth of the density gradient and the characteristic jet density, a statistical evaluation of the theoretical equations was used. The Statistical Analysis System, SAS, (19) was used for regression analyses.

Location of Test Sites

Five Oklahoma lakes were selected for pump test sites: Lake of the Arbuckles (24m deep), Pine Creek Lake (14m deep), Lake Texoma (24m deep), Wister Lake (8m deep), and Birch Lake (12m deep). Lake of the Arbuckles was constructed by the Bureau of Reclamation and the other lakes were constructed by the US Army Corps of Engineers. A single penetration test was taken at Lake of the Ozarks (23m deep), Missouri. Because of the nature of the tests, the areal extent and capacity of the lakes were unimportant as long as the pump tests did not alter the thermal profile for the duration of the tests.

At each lake, the test site was located in the deepest portion of the lake. Generally, the pumps were secured to the line of "no boating" buoys near the outlet works. According to profile information on file at the Corps of Engineers, Tulsa District, no known chemical-density gradients existed in the lakes chosen as test sites.

CHAPTER V

PRESENTATION AND ANALYSIS OF DATA

General

Penetration tests were conducted as part of lake mixing studies as early as August of 1977; however, the pumps used in 1977 and 1978 were different than the pumps designed specifically for this study. The tests conducted in 1977 and 1978, using two pump diameters of 1.07 and 1.83m, were not included in the analysis of prediction equations (because of different propeller designs) but were used for validation purposes. Data for penetration prediction analyses were collected during the summer 1979 and 1980 with the three pumps designed for this study. Unfortunately, an incorrect setting of the blade angle for the 1.83m pump and a belt slippage problem on the 2.44m pump rendered the 1979 data unusable except for the 1.22m pump tests. From the 1979 effort, 16 penetration tests were used. During the stratification period of 1980, a total of 55 penetration tests were completed using the three pumps. Data relative to the plume penetration tests are given in Appendix B.

An additional penetration test, using a 5.18m diameter pump (6), was included for verification of the prediction equations. The test was conducted at Lake of the Ozarks, Missouri, using the test guidelines established for this study. An aircraft propeller, without a shroud, was used in the pump design.

Temperature and Density Profiles

A wide variety of thermal profiles was encountered. Although classical thermal profiles were common, a few profiles defied classical descriptions. The thermal profiles, the thermal-density profiles, and plots of the third-degree polynomials which mathematically describe the density profiles are given in Appendix A. A mathematical description of the exact depth of the thermocline proved to be difficult because of the various shapes of the profiles.

The "densicline" is herein defined as the density gradient (or single point) associated with the previously defined thermocline. As shown in the profiles (Appendix A, pages 44 through 52), the densicline gradient was slightly more pronounced than the thermal gradient of the thermocline. For the analysis of the prediction equations, a single point in the lake profile was needed to represent the characteristic depth of the densicline. In model studies, this point was generally well defined at the interface of the two fluids of dissimilar densities. In the naturally occurring stratification patterns, this point was difficult to define. Ideally, this single depth should be mathematically determined from a fitted polynomial to eliminate the error induced by personal judgment; several approaches were investigated.

Characteristic Depth of the Densicline

The first approach for defining a single-point densicline depth was to use the inflection point of the polynomial fitted to the density profile. For the classical profiles, the inflection point of the fitted polynomial seemed to be a consistent single-point describer of the depth of the densicline. The inflection point was determined by setting the second derivative of the third-degree polynomial to zero and explicitly solving for the depth. For the profile measured at Pine Creek Lake on August 9, 1979 (see page 46), the classical thermocline gradient was not clearly defined; the inflection point of the polynomial was at a depth of 7.2m which seemed too deep to be appropriate. For the profile of Lake Texoma on August 8, 1980 (see page 52), the inflection point clearly proved to be a poor describer; the inflection point was at depth of 7.4m while the densicline was clearly in the range of from 17 to 19m.

Another approach to determine a mathematical describer was to calculate the average of the two points of maximum slope of the fitted polynomial. By setting the first derivative of the polynomial to zero and then solving, the depths of the maximum slope of the profile were determined. This method also proved faulty because both the points of maximum slope of the polynomial could occur above (or below) the obvious densicline (see page 52). The nature of the fitted polynomial caused this method to be inaccurate.

The method selected for describing a single point to represent the characteristic densicline depth was based upon defining the beginning of the thermocline gradient. First, the changes in temperature at one meter intervals were computed for the density profile in the region of the thermocline. Then, the depth prior to the first large increase in temperature (relative to the thermocline gradient) was selected as the single-point describer of the densicline depth. In cases where no obvious thermocline existed (e.g. Pine Creek Lake, August 9, 1979, page 46), densicline depth was defined as the mathematical midpoint between the propeller and the penetration depth; this yielded better correlations than using the value at an endpoint. The locations of the single-point densicline depth (denoted as D_d) that were used in the analyses of the prediction equations are shown with the profiles in Appendix A (pages 44 through 52).

Characteristic Density Difference

To account for the characteristic density difference between the pumped water and the water at the penetration depth, either Equation 2.6 or 2.7 could be used; however, both were dependent upon a definition of the average density of the pumped water, ρ_o. The definition of ρ_o, which consistently yielded the best correlations, was:

$$\rho_o = \frac{\rho_0 + \rho_1 + \rho_2}{3} \qquad (5.1)$$

where ρ_0 = density of lake surface

ρ_1 = density at one meter below surface

ρ_2 = density at two meters below surface.

Representing ρ_o by the density at any single depth (above the propeller) did not correlate with various prediction equations as well as the average ρ_o concept.

The definition of the characteristic density difference which yielded the best correlations was described by Equation 2.7 even though there is no sensitivity to the densicline depth. This finding supports the arguments (presented in Appendix C) for reducing the integral form of Equation 2.6 to the simple difference form of Equation 2.7. The error induced by fitting a polynomial to the density profile may have been sufficient reason for the simple difference form to yield better correlations than did the integral form.

Evaluations of Prediction Equations

The coefficient of determination, R^2, and standard deviation, S, were used in the statistical evaluation of the equations. Because non-dimensional equations contain a common variable on each side of the equation, some amount of correlation will be inherent in regression analysis. In order to compare the goodness of fit of the various equations, the left-hand term of each equation was consistently divided by the diameter. As a basis of comparison, a regression was done to evaluate the plume penetration as a function of the epilimnion penetration alone (i.e. the distance from the pump to the single-point densicline depth). The results were:

$$\frac{H_p}{D} = C_1 \frac{H_e}{D} \qquad (5.2)$$

where

$C_1 = 1.469$

$R^2 = 0.795$

$S = 1.79$

This indicates the plume penetration was strongly dependent upon the depth of the densicline. No other single variable was as statistically significant in correlations with the penetration depth. Clearly, a significant source of error will be intrinsic to any prediction equation that does not account for epilimnion penetration (i.e. the densicline depth).

A regression analysis of the Fr_d squared, as defined in Equation 2.8, yielded the following (after rearranging):

$$\frac{H_h}{D} = C_2 \frac{V^2}{g(\Delta\rho/\rho_o)D} \qquad (5.3)$$

where

$C_2 = 0.138$

$R^2 = 0.819$

$S = 1.18$

For this relation, C_2 was expected to be about 0.5. This equation only predicts the hypolimnetic penetration and the correlation noted is largely due to the diameter being present in the denominator of each side.

The total plume penetration is the sum of the epilimnion and hypolimnion penetrations. Equation 2.9, which includes the form of Equation 5.3 to predict hypolimnion penetration, also accounts for epilimnion penetration. The regression analysis yielded:

$$\frac{H_p}{D} = C_3 \frac{V^2}{g(\Delta\rho/\rho_o)D} + C_4 \frac{H_e}{D} \qquad (5.4)$$

where

$C_3 = 0.176$

$C_4 = 0.756$

$R^2 = 0.937$

$S = 0.70$

Therefore, the inclusion of the pump diameter and the term for epilimnion penetration greatly enhanced the Bernoulli-based prediction equation. C_3 was expected to be about 0.5, and C_4 was expected to be about 1.0. A plot of the measured penetration versus the predicted penetration is shown in Figure 14 (page 53).

Before a regression analysis of the empirical equation (Equation 2.10) could be accomplished, it was necessary to rewrite the form using equation 2.7 to replace the right-hand term:

$$V_p^2 = V_o^2 \frac{(CD)^2}{(H_p)^2} - 2g(\Delta\rho/\rho_o)H_p \qquad (5.5)$$

At the maximum penetration point, V_p equals zero. After rearranging and taking the square root, the derived equation had the form of the Fr_d number where the characteristic length is the plume penetration depth:

$$\frac{H_p}{D} = C_5 \frac{V_o}{\sqrt{g(\Delta\rho/\rho_o)H_p}} \tag{5.6}$$

When evaluated for the cases where $H_p \geq CD$, the statistical result was:

$$C_5 = 4.271$$
$$R^2 = 0.329$$
$$S = 2.28$$

When Equation 5.5 was simplified into Equation 5.6, a poor statistical significance was shown and the penetration predictions would be inferior to the predictions of Equation 5.4.

Equations 5.2 through 5.6 were based on the characteristic length definition as being a plume length (as derived from Bernoulli´s equation). Defining the characteristic length to be the jet diameter (from dimensional analysis), and using the simplest form of Equation 2.11 (similar to the form of Equation 5.6), the following results were obtained:

$$\frac{H_p}{D} = C_6 \frac{V}{\sqrt{g(\Delta\rho/\rho_o)D}} \tag{5.7}$$

where

$$C_6 = 1.693$$
$$R^2 = 0.795$$
$$S = 1.26$$

A plot of the measured penetration versus the predicted penetration is shown in Figure 15 (page 54). This equation does not correlate with the data as well as Equation 5.4.

Correction Functions

Several reasons existed for inclusion of some sort of a correction function. Firstly, the definitions of some variables (i.e. characteristic velocity and density difference) were based upon an interpretation of available data. Secondly, with the exception of Equation 5.5, the previous equations did not account for the variations in the thermocline depths; the evaluation of Equation 5.2 suggested that this variable should be included in prediction equations. Thirdly, model studies have verified reductions in the characteristic velocity induced by either a shallow thermocline or by viscous dissipation for deep thermoclines.

From inferences made (by the author) on data presented by Hart (8), a second-degree polynomial equation seemed be an appropriate correction factor to improve the characteristic velocity definition above a shallow thermocline. To extend the function to include a correction for deep thermoclines, a third-degree (or more) polynomial would be required. The form of Equation 2.11, which is consistent with dimensional analysis and used to derive Equation 5.7, was redefined as:

$$\frac{H_p}{D} = f\{\frac{H_e}{D}, \frac{H_e^2}{D}, \frac{H_e^3}{D}, Fr_d\} \quad (5.8)$$

Using a statistical evaluation to determine the constants of the polynomial, and then performing the regression for the final equation, the results were:

$$\frac{H_p}{D} = P_3 \frac{V}{\sqrt{g(\Delta\rho/\rho_o)D}} \quad (5.9)$$

where $P_3 = 1.758 - 0.110\frac{H_e}{D} + 0.00313\frac{H_e^2}{D^2} - 0.00145\frac{H_e^3}{D^3}$

$R^2 = 0.932$

$S = 0.73$

Statistically, this is a good correlation. A plot of the measured penetration versus the predicted penetration is shown in Figure 16 (page 55).

After applying the same treatment of correction to Equation 5.4, the following results were obtained:

$$\frac{H_p}{D} = P_{3'} \frac{V^2}{g(\Delta\rho/\rho_o)D} + C_7 \frac{H_p}{D} \quad (5.10)$$

where $P_{3'} = 0.439 - 0.120\frac{H_e}{D} + 0.0123\frac{H_e^2}{D^2} - 0.00041\frac{H_e^3}{D^3}$

$$C_7 = 1.080$$

$$R^2 = 0.943$$

$$S = 0.66$$

This equation produced the best statistical correlation; Figure 17 (page 56) shows the plot of the predicted plume penetration versus the predicted values. Equations 5.9 and 5.10 should be used only in cases where H_e/D is less than 4.0; the polynomial correction functions are only beneficial for the range of H_e/D values used in the regressions.

Predictions for Dissimilar Pumps

Seven plume penetration tests were conducted using Garton pumps other than those designed specifically for this study. Three pump diameters were tested. Three tests were conducted with a seven-bladed, 1.07m diameter pump that did not have a shroud. Three tests were conducted with a six-bladed, 1.83m diameter pump that had a shroud. Although the 1.83m diameter pump did have the same diameter as one of the pumps built for specifically for this study, the blade design was different. One test was conducted using a three-bladed, 5.18m diameter pump that did not have a shroud. The 1.07 and 1.83m pump tests were conducted in open water (away from any structures). The 5.18m pump test was conducted with the pump located immediately adjacent to the face of Bagnell Dam, Missouri. Because of the location, the penetration depth may have been influenced.

The data from the tests are presented in Table IV (page 61). The data were used to test the validity of Equations

5.4, 5.7, 5.9, and 5.10. The results of the calculations are presented in Table V (page 61).

The theoretical predictions of Equations 5.4 and 5.7 were in good agreement with the observed penetrations. Although Equation 5.7 did not correlate with the test data as well as Equation 5.4, the penetration predictions of Equation 5.7 for other pumps were as good as those of Equation 5.4. Because Equation 5.4 consistently underpredicted the penetration while Equation 5.7 tended to overpredict, Equation 5.4 would provide better design criteria (penetration that is too shallow tends to limit the success of either local or global destratification more than penetration that is too deep).

Penetration predictions of Equation 5.10 were superior to all other predictions for four out of seven cases. Although the predictions of Equation 5.9 were in good agreement with the penetrations, none of the predictions were superior to all other predictions. Equation 5.9 tended to overpredict penetration while Equation 5.10 tended to neither overpredict nor underpredict.

Prediction Depth of Penetration

The non-dimensional, theoretical form of Equation 5.4, derived from a Bernoullian approach, can be used to predict accurately the depth of penetration of a low-velocity, downward-flowing jet of lake surface water. The semi-empirical form of Equation 5.10 yielded the most consistent penetration predictions for the available range of parameter combinations.

To obtain the plume penetration depth in a lake relative to the lake surface, the depth of the propeller (i.e. the origin of the jet) should be added to the calculated penetration depth.

CHAPTER VI

SUMMARY AND CONCLUSIONS

Summary

The objective of this study was to develop a non-dimensional equation to predict the depth of penetration of a buoyant plume in a stratified lake as a function of pump diameter, pumping velocity, and density profile of the lake. The range of the variables investigated were limited to the normal design characteristics of the Garton pump. Three pumps, with variable speed drives, were designed and built specifically for this study. Pump diameters of 1.27, 1.83, and 2.43m were studied.

Penetration tests were conducted on several Oklahoma lakes over two summer stratification periods. Penetration test data collected from lake mixing studies, using Garton pumps of different ropeller designs, were used to verify the prediction equations developed in this study.

Conclusions

1. The plume penetration depth of a low-velocity, downward-flowing, buoyant jet in a stratified lake can be predicted accurately using a non-dimensional equation.

2. For the plume velocities studied, the thermocline

depth was an important variable in predicting the depth of plume penetration.

3. The densimetric Froude number squared, as derived from Bernoulli´s equation, can be used to predict accurately the plume penetration depth when terms are included that account for variations in pump diameter and epilimnion penetration.

4. The densimetric Froude number, as derived from dimensional analysis, is proportional to the plume length divided by the jet diameter, and can be used to predict the plume penetration depth.

5. An empirical correction function, based on the distance from the pump to the thermocline, improved the prediction of plume penetration for the range of the parameters tested.

6. Penetration depths were accurately predicted for tests conducted with Garton pumps having different diameters and propeller designs than those constructed for this study.

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APPENDIX A

FIGURES

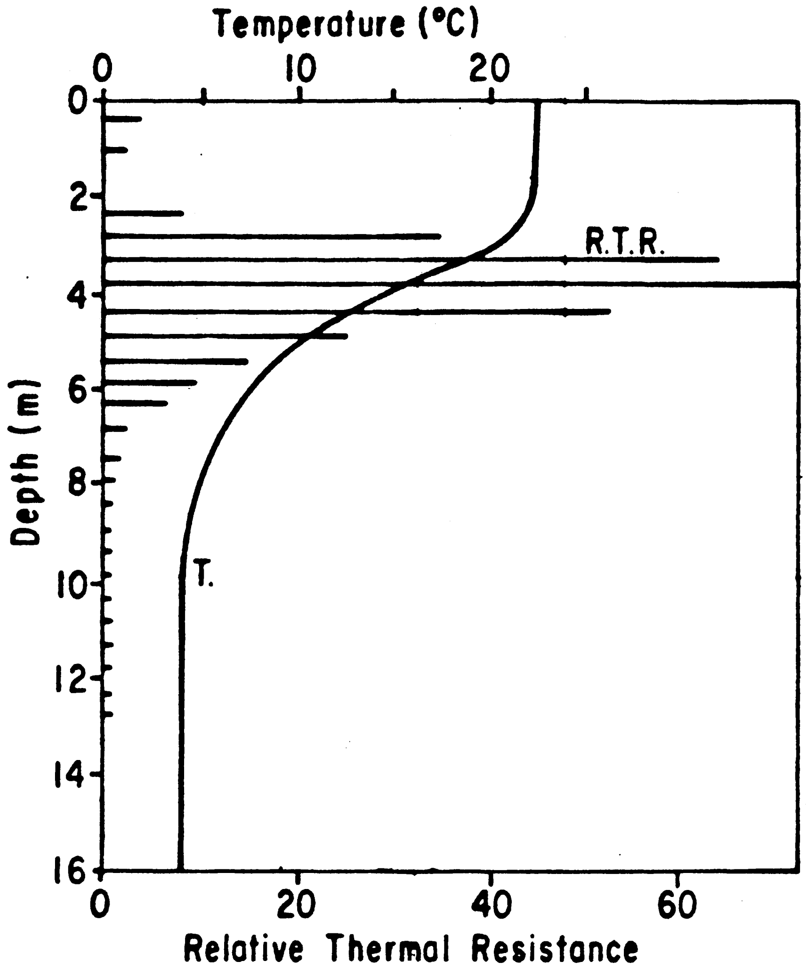


Figure 1. Summer Temperature Profile and Relative Thermal Resistance to Mixing (after Wetzel, 1975)

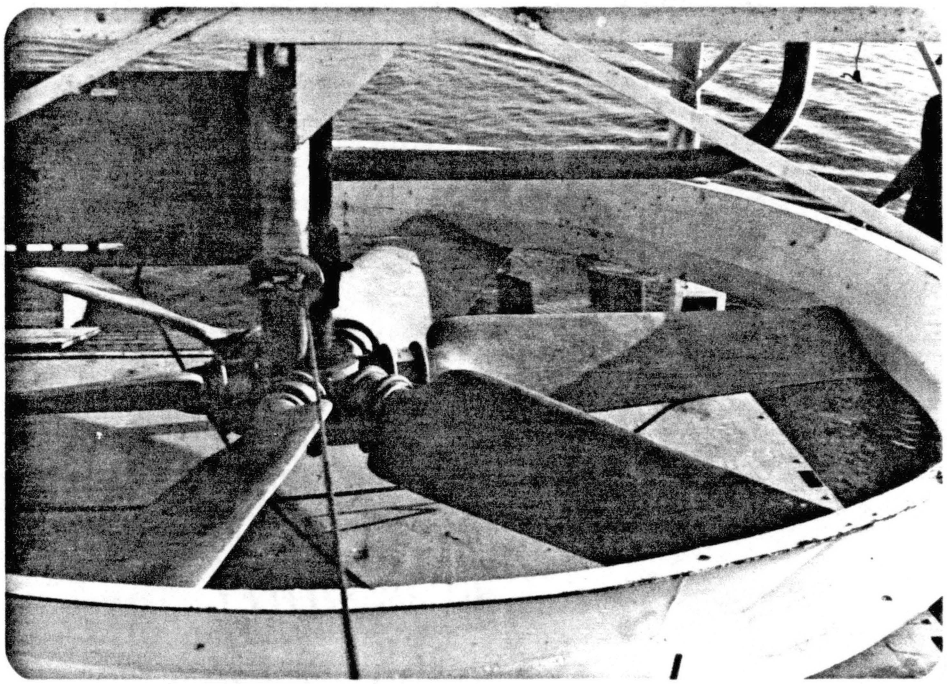


Figure 2. Pump Propeller and Shroud

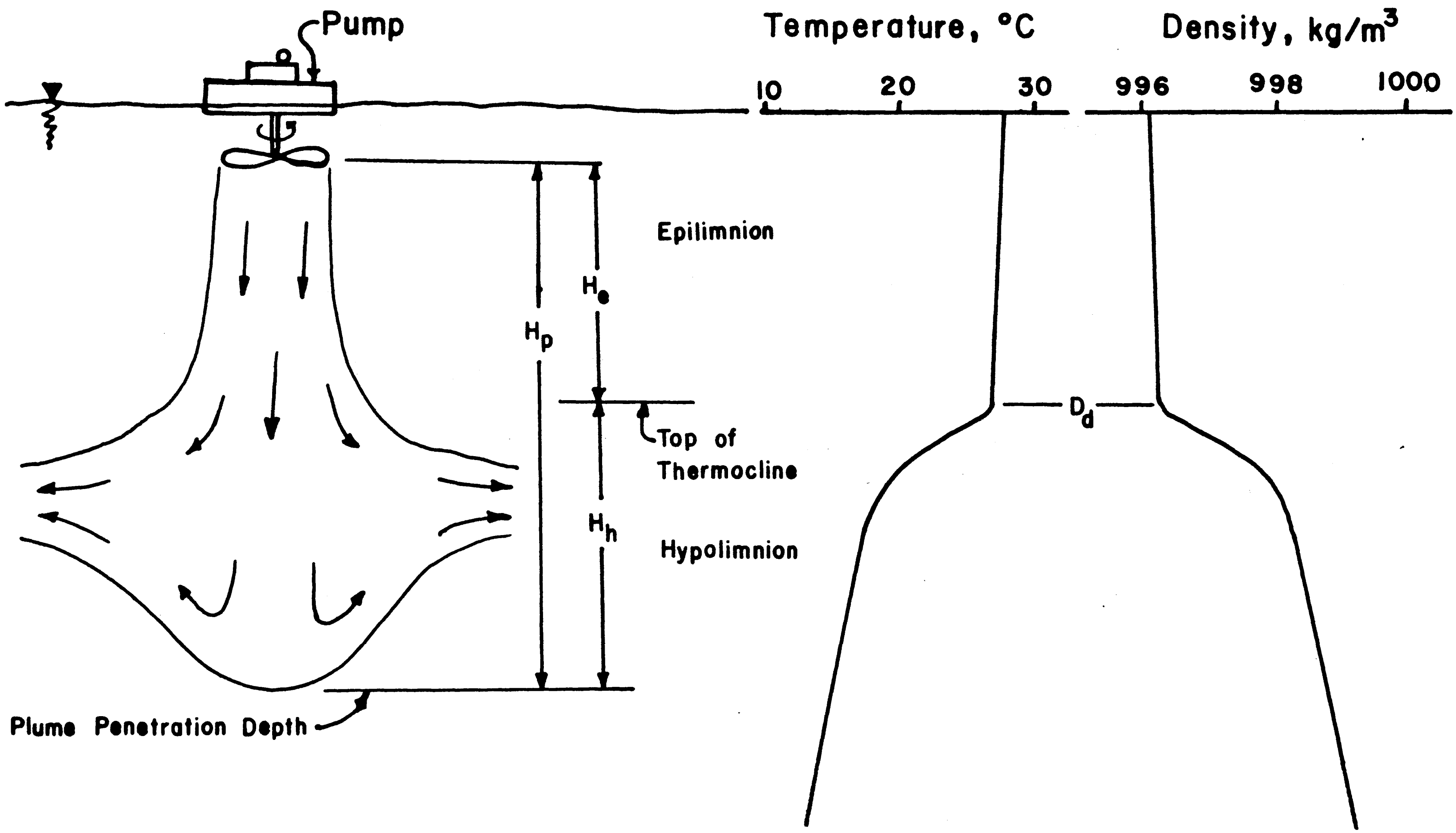
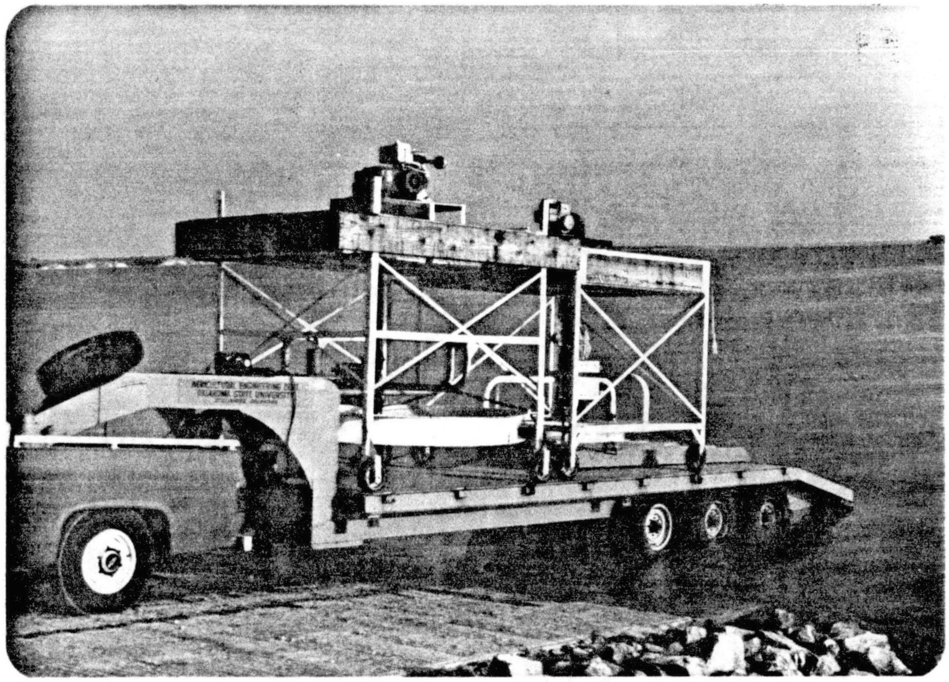
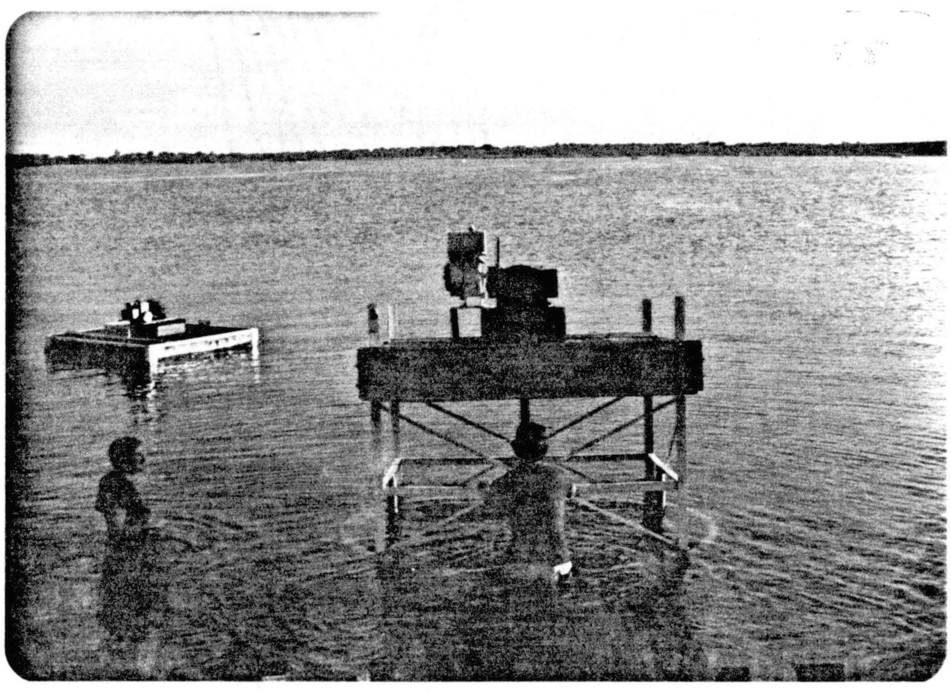


Figure 3. Graphic Presentation of Lake Mixing Notation



(A) Before Unloading



(B) After Unloading

Figure 4. Unloading Operation

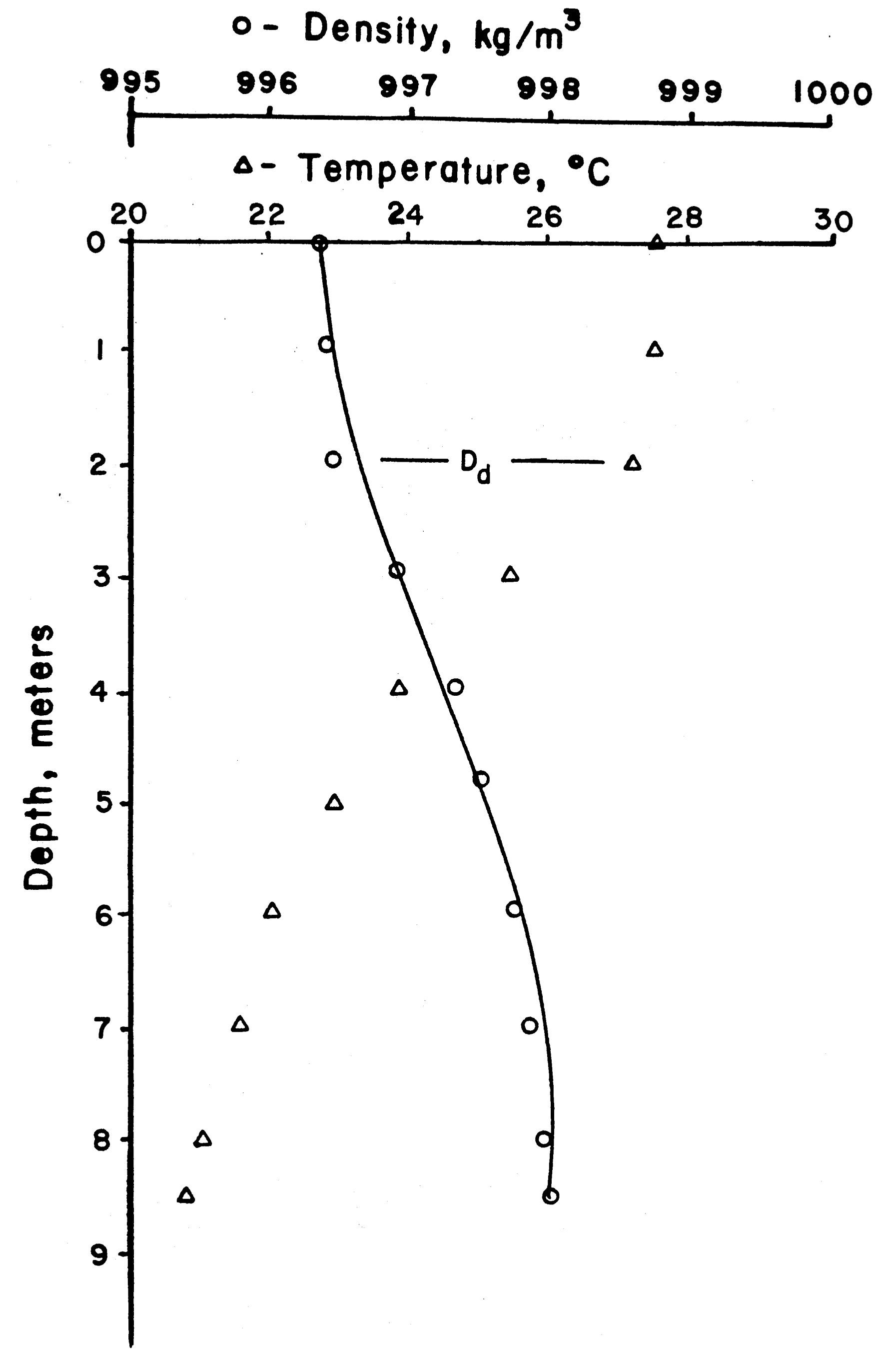


Figure 5. Wister Lake Profile:
Jul 18, 1979

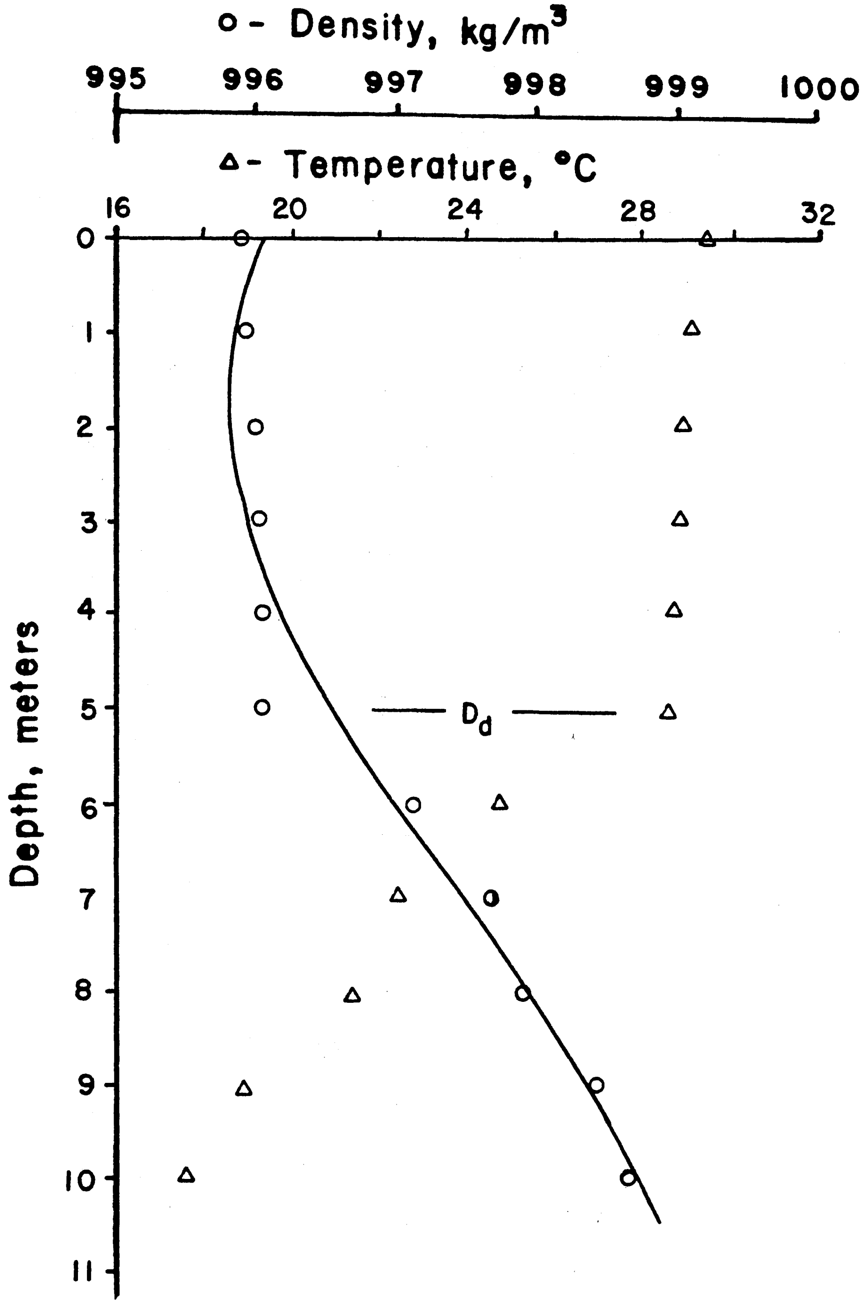


Figure 6. Birch Lake Profile:
Jul 24, 1979

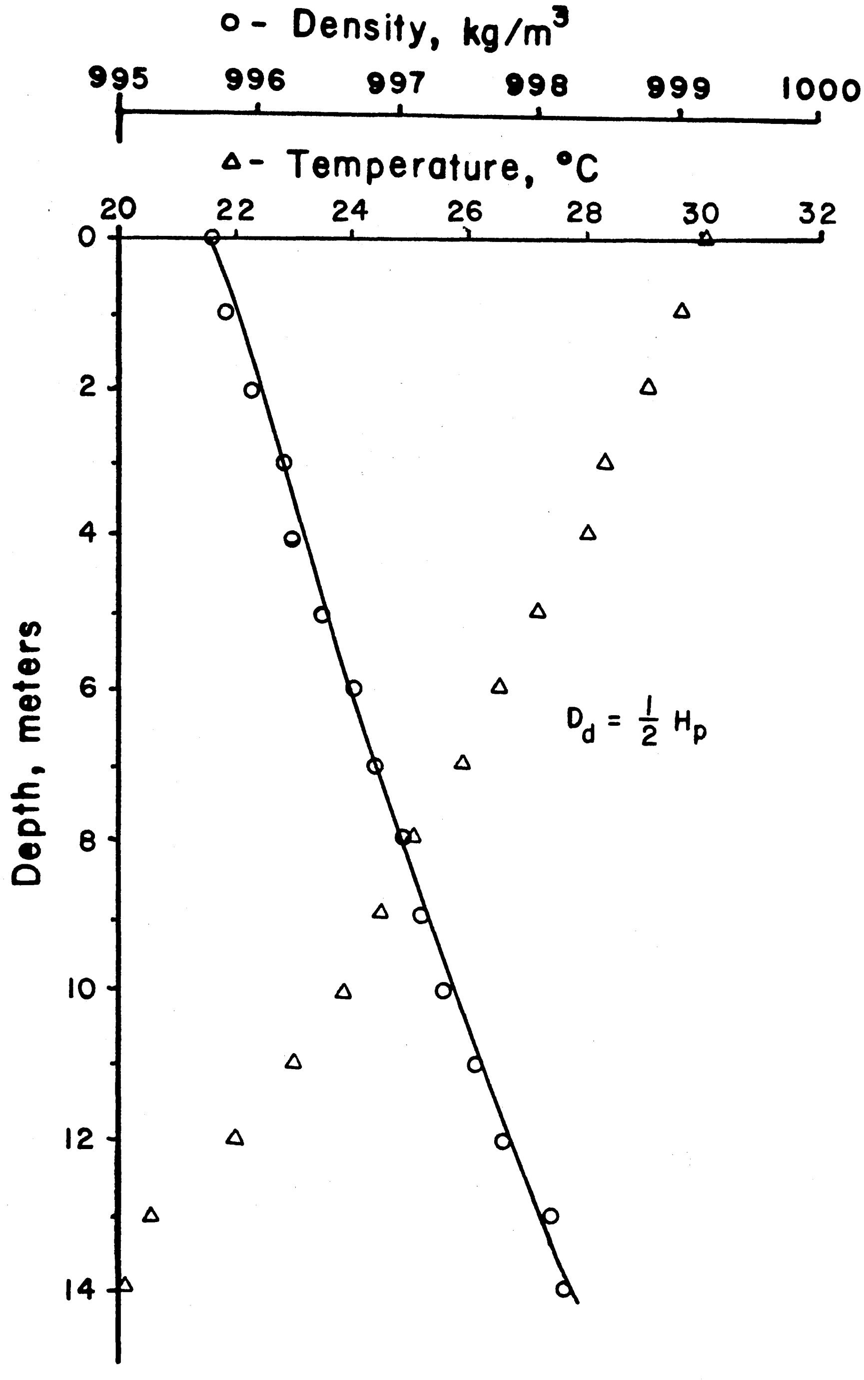


Figure 7. Pine Creek Lake Profile: Aug 9, 1979

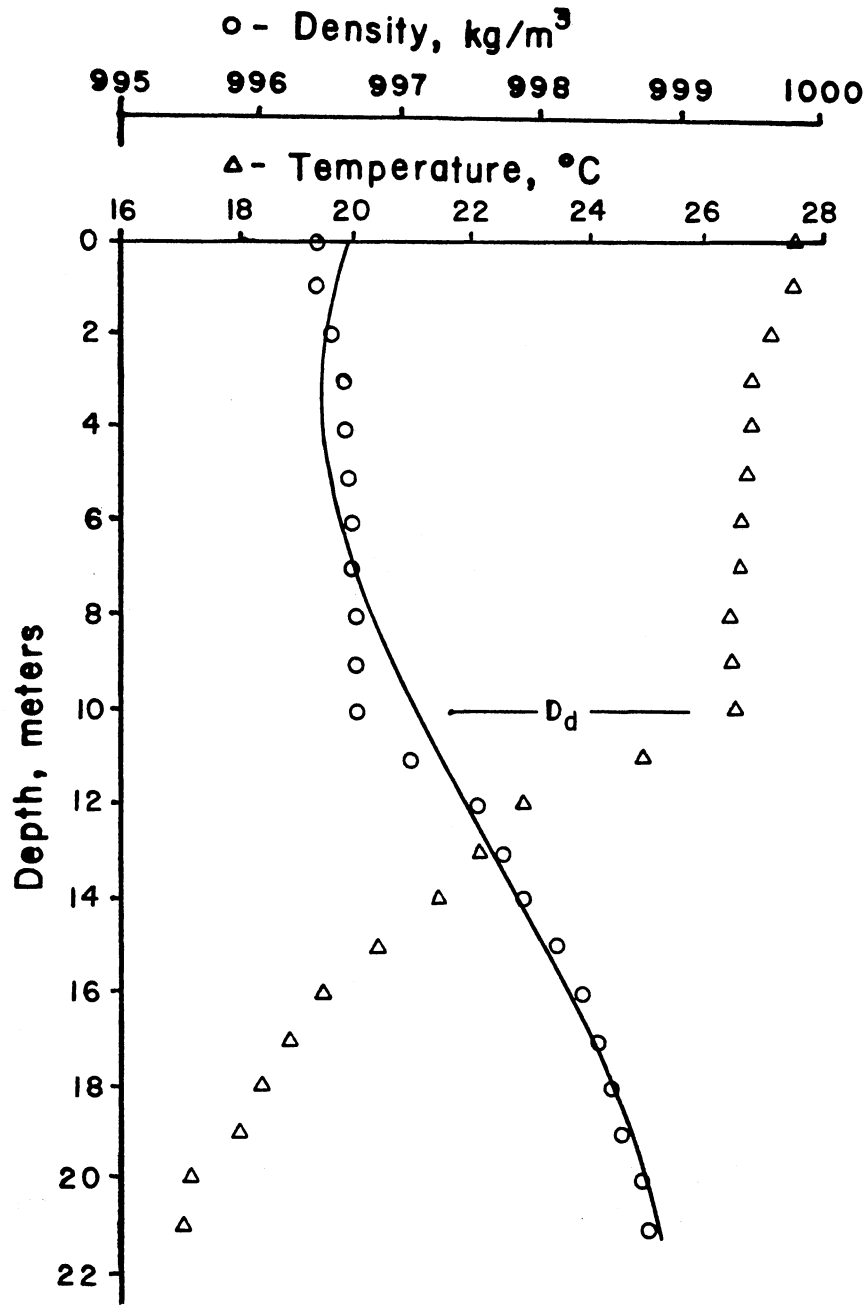


Figure 8. Lake of the Arbuckles Profile: Aug 22, 1979

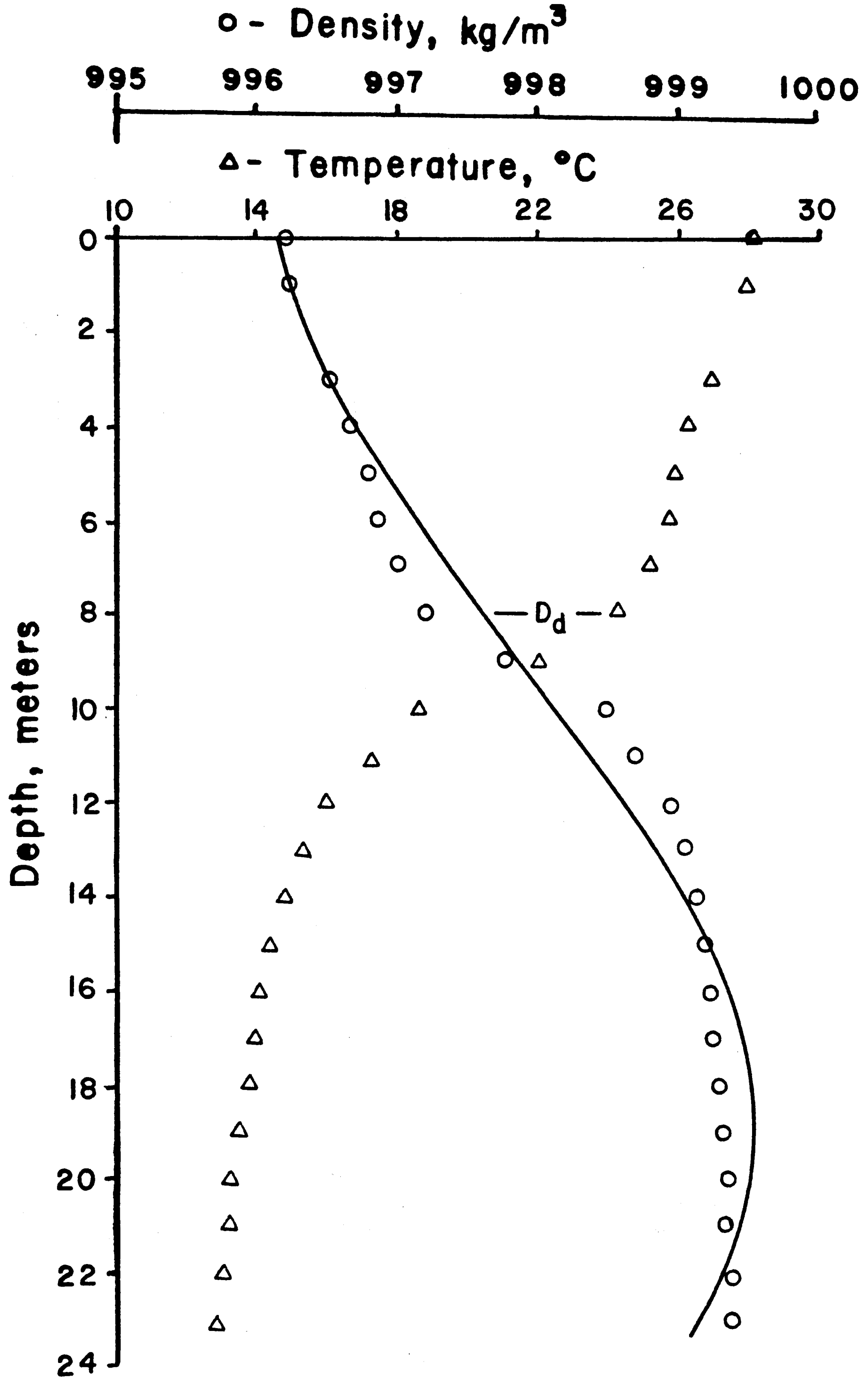


Figure 9. Lake of the Arbuckles Profile: Jun 24, 1979

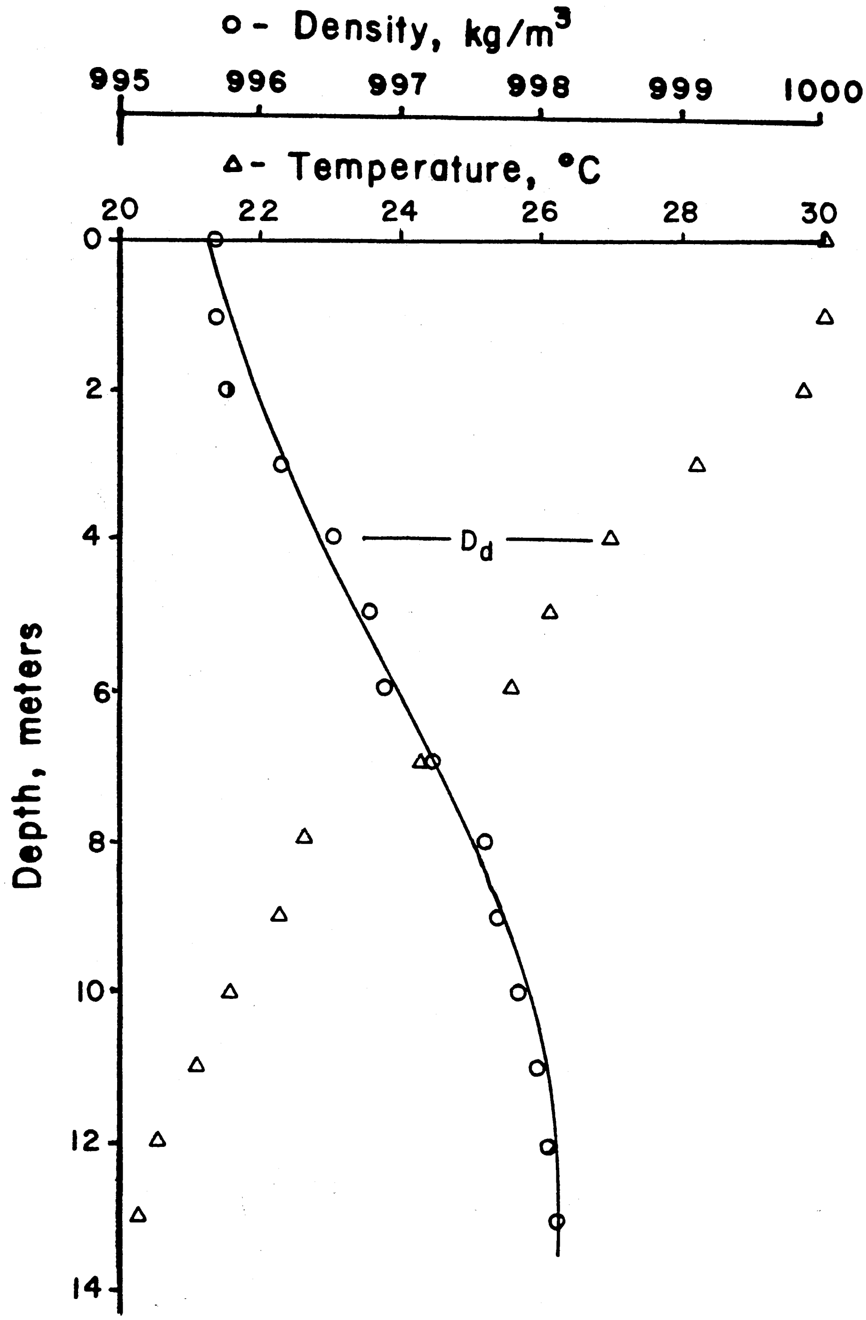


Figure 10. Pine Creek Lake Profile:
Jun 26, 1980

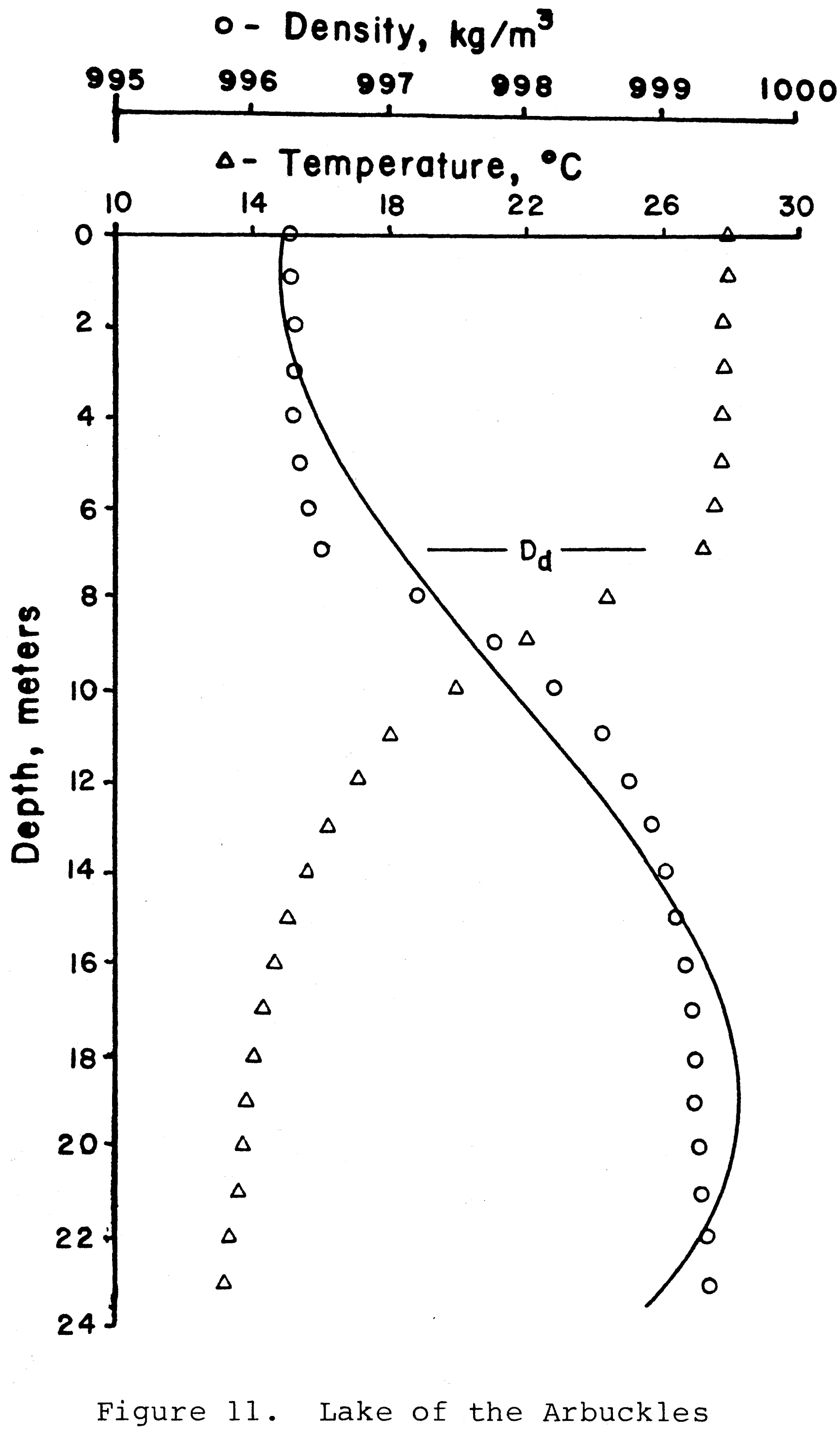


Figure 11. Lake of the Arbuckles Profile: Aug 4, 1980

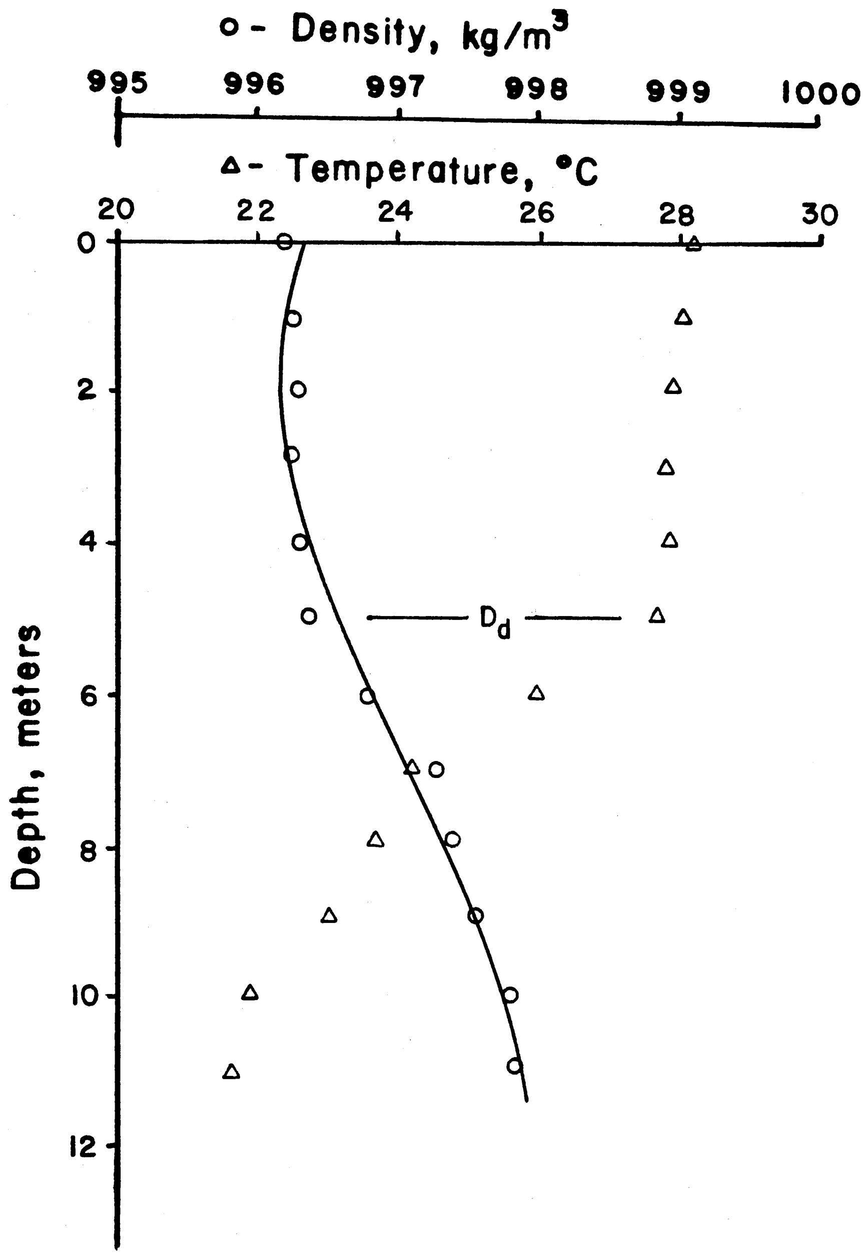


Figure 12. Pine Creek Lake Profile:
Aug 6, 1980

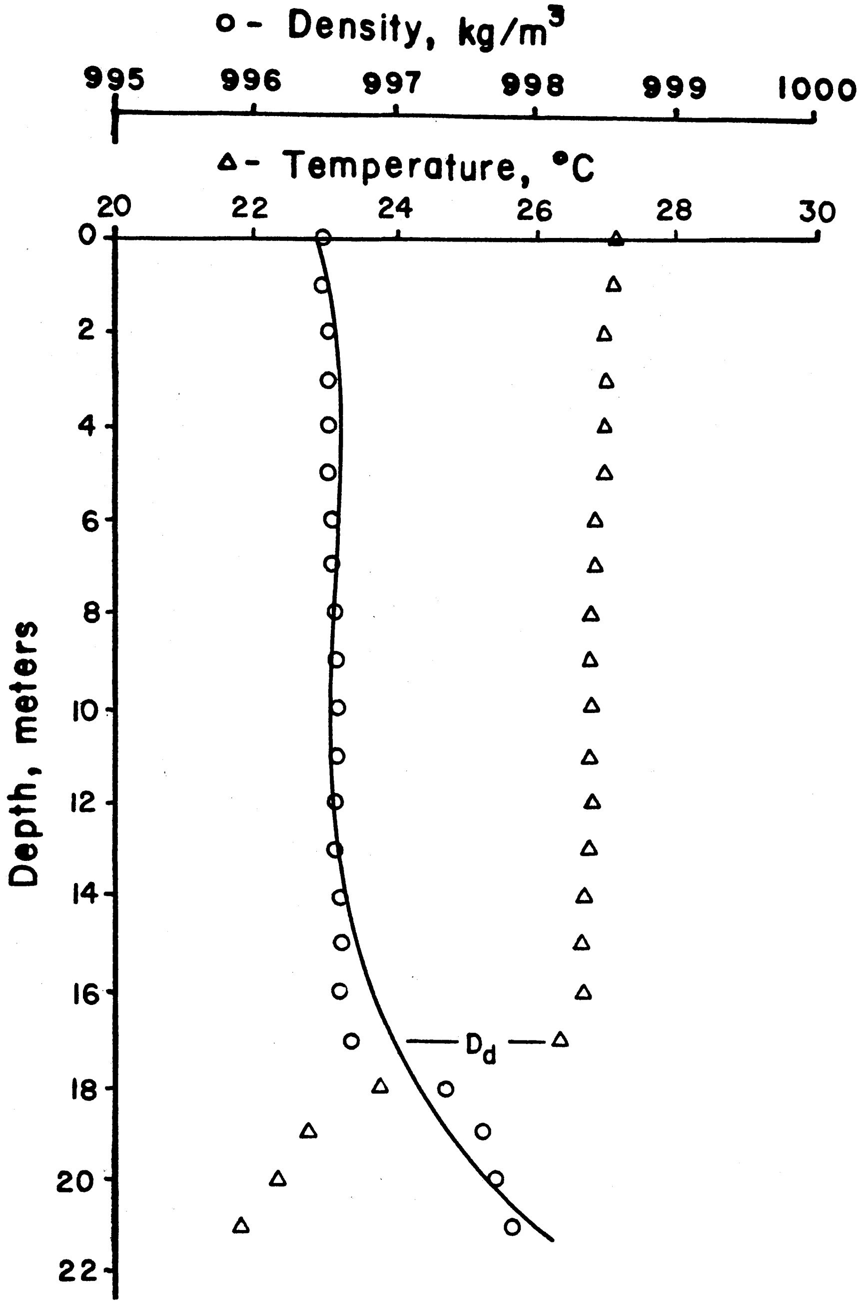


Figure 13. Lake Texoma Profile: Aug 8, 1980

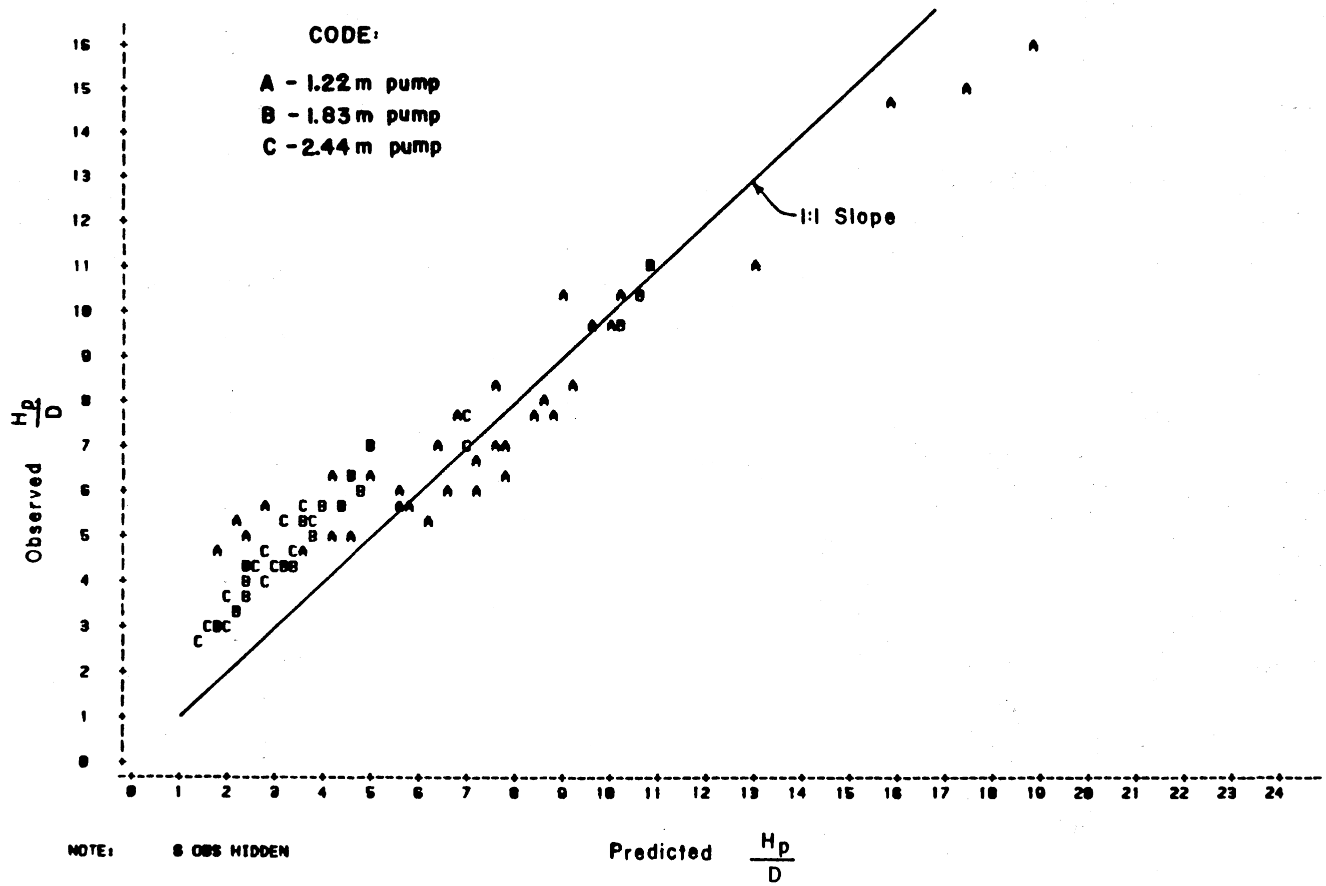


Figure 14. Observed H_p/D versus Predicted H_p/D (Equation 5.4)

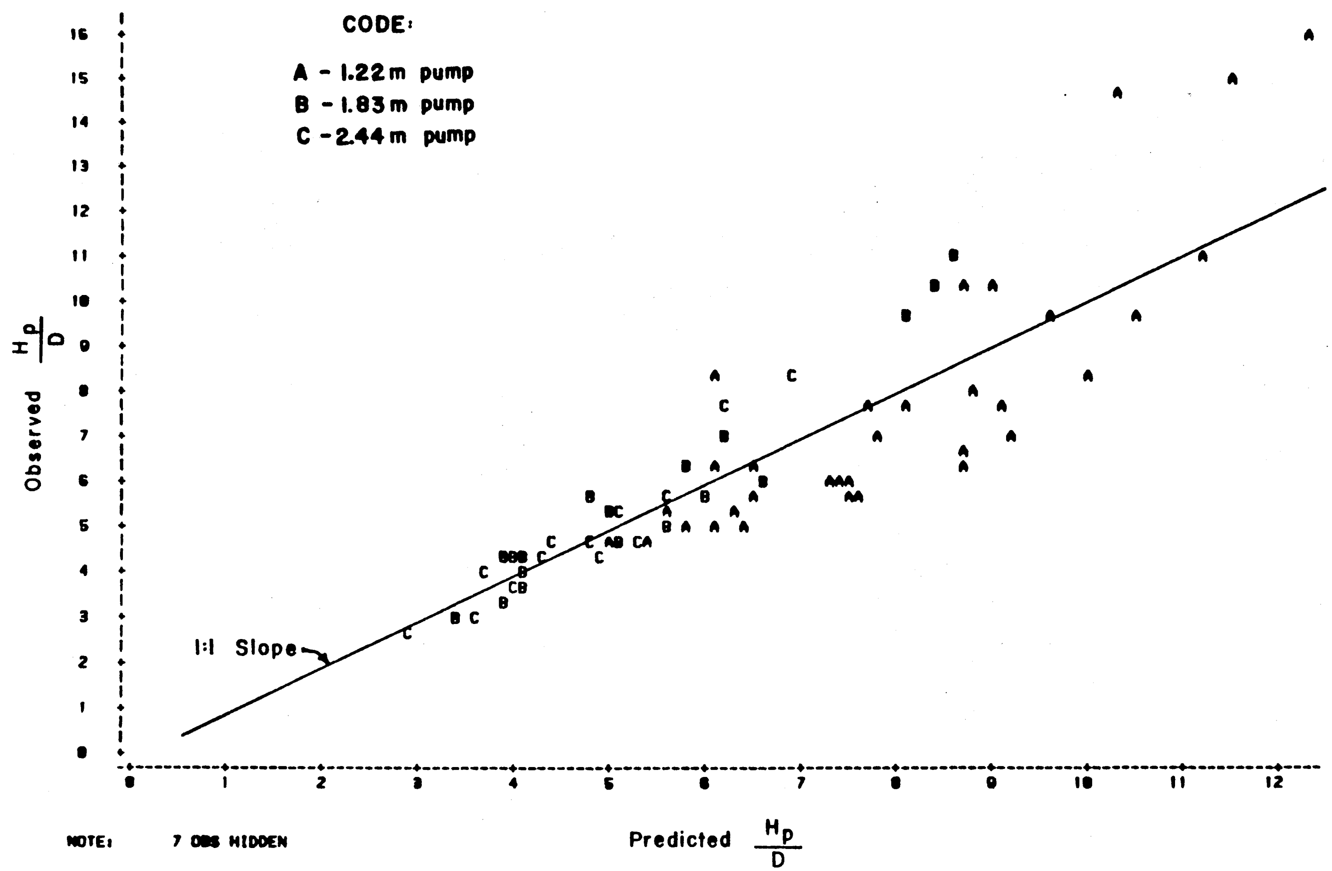


Figure 15. Observed H_p/D versus Predicted H_p/D (Equation 5.7)

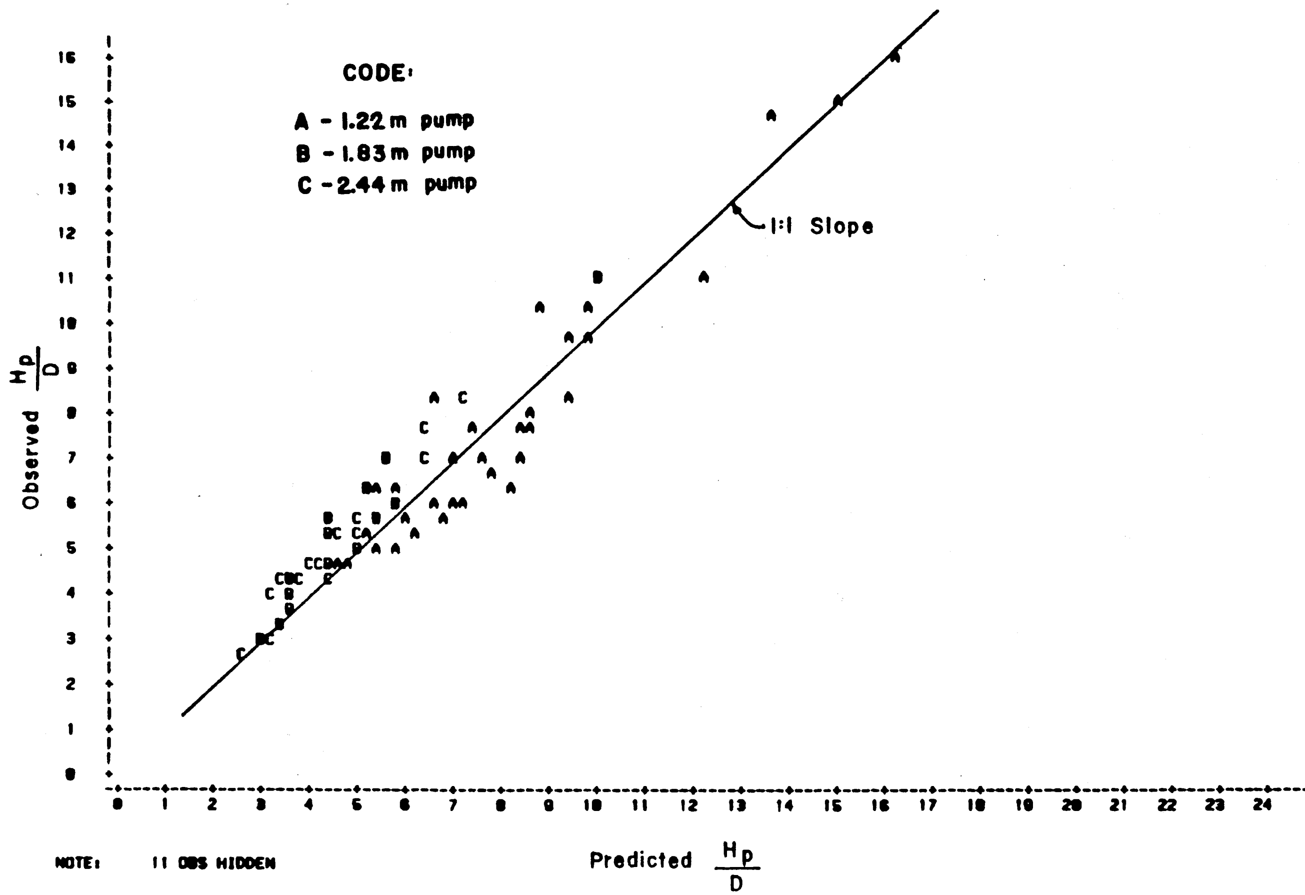


Figure 16. Observed H_p/D versus Predicted H_p/D (Equation 5.9)

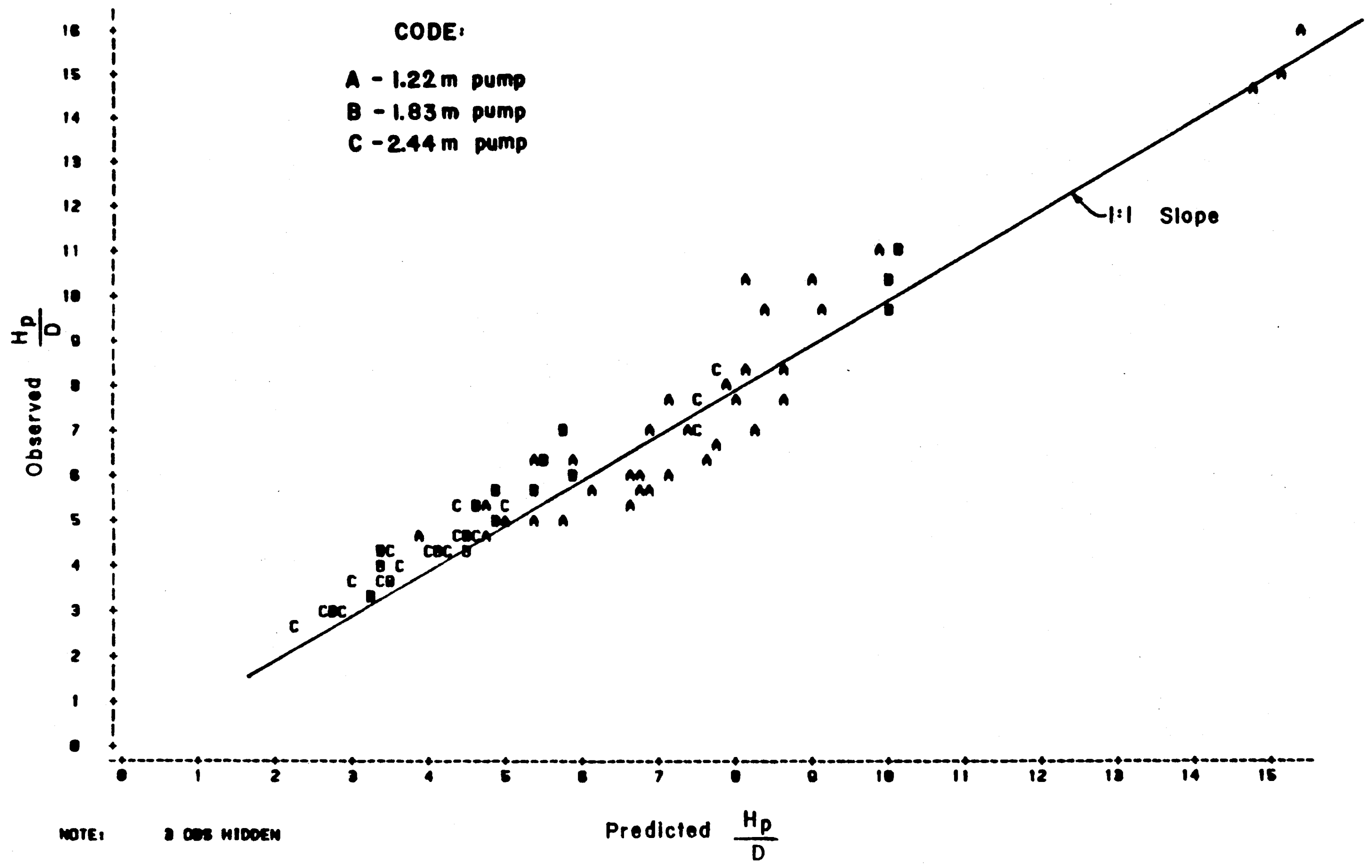


Figure 17. Observed H_p/D versus Predicted H_p/D (Equation 5.10)

APPENDIX B

TABLES

TABLE I

PENETRATION TEST DATA FOR 1.22M PUMP

OBS	DATE	D (m)	V (m/s)	$\Delta\rho/\rho$	H_p (m)	H_h (m)
1	7/18/79	1.22	0.375	0.00136	5.50	5.00
2	7/18/79	1.22	0.448	0.00144	6.00	5.50
3	7/18/79	1.22	0.457	0.00158	6.50	6.00
4	7/18/79	1.22	0.530	0.00161	7.00	6.50
5	7/24/79	1.22	0.548	0.00174	6.00	2.50
6	7/24/79	1.22	0.680	0.00196	7.00	3.50
7	7/24/79	1.22	0.762	0.00249	7.50	4.00
8	8/09/79	1.22	0.396	0.00102	6.00	3.00
9	8/09/79	1.22	0.555	0.00126	7.00	3.50
10	8/09/79	1.22	0.664	0.00139	8.00	4.00
11	8/09/79	1.22	0.741	0.00154	8.50	4.25
12	8/09/79	1.22	0.796	0.00199	12.50	5.50
13	8/22/79	1.22	0.402	0.00065	9.50	1.00
14	8/22/79	1.22	0.421	0.00113	10.00	1.50
15	8/22/79	1.22	0.701	0.00147	12.50	4.00
16	8/22/79	1.22	0.945	0.00170	13.50	5.00
17	6/24/80	1.22	0.466	0.00132	6.67	0.50
18	6/24/80	1.22	0.619	0.00171	7.17	1.00
19	6/24/80	1.22	0.768	0.00232	8.67	2.50
20	6/24/80	1.22	0.911	0.00255	9.67	3.50
21	6/24/80	1.22	1.042	0.00282	11.67	5.50
22	6/26/80	1.22	0.457	0.00174	5.67	2.83
23	6/26/80	1.22	0.613	0.00212	7.67	3.83
24	6/26/80	1.22	0.753	0.00226	8.67	4.33
25	6/26/80	1.22	0.792	0.00231	9.17	4.58
26	8/04/80	1.22	0.625	0.00171	7.17	2.00
27	8/04/80	1.22	0.759	0.00183	7.67	2.50
28	8/04/80	1.22	0.896	0.00231	9.17	4.00
29	8/04/80	1.22	1.015	0.00249	10.17	5.00
30	8/04/80	1.22	1.119	0.00270	11.67	6.50
31	8/06/80	1.22	0.466	0.00141	7.67	4.50
32	8/08/80	1.22	0.710	0.00115	17.67	2.50
33	8/08/80	1.22	0.808	0.00119	18.17	3.00
34	8/08/80	1.22	0.933	0.00137	19.67	4.50

TABLE II

PENETRATION TEST DATA FOR 1.83M PUMP

OBS	DATE	D (m)	V (m/s)	$\Delta\rho/\rho$	H_p (m)	H_h (m)
35	6/24/80	1.83	0.460	0.00220	8.17	2.00
36	6/24/80	1.83	0.625	0.00266	10.17	4.00
37	6/24/80	1.83	0.768	0.00282	11.67	5.50
38	6/24/80	1.83	0.835	0.00289	12.67	6.50
39	6/26/80	1.83	0.332	0.00153	5.17	2.58
40	6/26/80	1.83	0.460	0.00200	6.67	3.33
41	6/26/80	1.83	0.600	0.00226	8.67	4.33
42	6/26/80	1.83	0.668	0.00231	9.17	4.58
43	8/04/80	1.83	0.427	0.00183	7.67	2.50
44	8/04/80	1.83	0.613	0.00240	9.67	4.50
45	8/04/80	1.83	0.747	0.00249	10.17	5.00
46	8/04/80	1.83	0.835	0.00257	10.67	5.50
47	8/06/80	1.83	0.332	0.00116	6.17	3.00
48	8/06/80	1.83	0.375	0.00132	7.17	4.00
49	8/08/80	1.83	0.408	0.00159	8.17	5.00
50	8/08/80	1.83	0.686	0.00115	17.67	2.50
51	8/08/80	1.83	0.747	0.00126	18.67	3.50
52	8/08/80	1.83	0.811	0.00141	20.17	5.00

TABLE III

PENETRATION TEST DATA FOR 2.44M PUMP

OBS	DATE	D (m)	V (m/s)	$\Delta\rho/\rho$	H_p (m)	H_h (m)
53	6/24/80	2.44	0.509	0.00232	9.67	3.50
54	6/24/80	2.44	0.634	0.00266	10.17	4.00
55	6/24/80	2.44	0.732	0.00278	11.17	5.00
56	6/24/80	2.44	0.866	0.00289	12.67	6.50
57	6/26/80	2.44	0.363	0.00194	6.17	3.08
58	6/26/80	2.44	0.472	0.00204	7.17	3.58
59	6/26/80	2.44	0.555	0.00226	8.67	4.33
60	6/26/80	2.44	0.695	0.00241	10.17	5.08
61	6/26/80	2.44	0.768	0.00248	11.17	5.58
62	8/04/80	2.44	0.573	0.00257	10.67	5.50
63	8/04/80	2.44	0.661	0.00270	11.67	6.50
64	8/04/80	2.44	0.777	0.00279	12.67	7.50
65	8/04/80	2.44	0.789	0.00284	13.17	8.00
66	8/04/80	2.44	0.872	0.00290	14.17	9.00
67	8/06/80	2.44	0.366	0.00141	7.67	4.50
68	8/06/80	2.44	0.466	0.00166	9.17	6.00
69	8/08/80	2.44	0.591	0.00110	17.17	2.00
70	8/08/80	2.44	0.637	0.00126	18.67	3.50
71	8/08/80	2.44	0.747	0.00141	20.17	5.00

TABLE IV

PENETRATION TEST DATA FOR OTHER TESTS

OBS	DATE	D (m)	V (m/s)	$\Delta\rho/\rho$	H_p (m)	H_h (m)
1	8/26/77	1.83	0.363	0.00200	5.28	1.50
2	8/26/77	1.83	0.552	0.00260	7.28	3.50
3	8/26/77	1.07	0.591	0.00244	6.27	2.50
4	8/03/78	1.83	0.460	0.00195	6.28	1.50
5	8/03/78	1.07	0.561	0.00216	6.77	2.00
6	8/03/78	1.07	0.719	0.00244	7.77	3.00
7	7/22/80	5.18	0.671	0.00387	17.16	11.00

TABLE V

PENETRATION PREDICTIONS FOR OTHER TESTS

DATE	PUMP	OBSERVED	PREDICTED H_p/D			
	D (m)	H_p/D	Eq. 5.4	Eq. 5.7	Eq. 5.9	Eq. 5.10
8/26/77	1.83	2.88	2.21	3.24	3.17	3.11*
8/26/77	1.83	3.98	2.71	4.33	4.22	3.80*
8/26/77	1.07	5.87	5.07	6.26	6.23	5.87*
8/03/78	1.83	3.43	3.04*	4.16	4.08	4.05
8/03/78	1.07	6.32	5.81	6.31*	6.56	6.37
8/03/78	1.07	7.26	6.93	7.61	7.92	7.08*
7/22/80	5.18	3.31	1.30	2.56*	2.53	2.00

* Indicates most accurate prediction.

APPENDIX C

APPLICATION OF BERNOULLI´S EQUATION

BY MORETTI

APPLICATION OF BERNOULLI´S EQUATION BY MORETTI

Introduction

This section presents an analysis largely carried out by P. M. Moretti[1] as a first-order model of penetration. The nomenclature, as used by Moretti, was changed to be consistent with the main text; any new terms were defined where needed. The analysis of the governing differential equations identifies the key factors to which the results can be correlated, and suggests the forms of the dimensionless parameters which can be used for data plotting and for similitude between models and prototypes. The analysis shows that a integral form of the densimetric Froude number is the primary factor in penetration depth.

First-order Model of Penetration

The governing differential equation for the penetration of a buoyant jet downwards into a stratified lake is the Navier-Stokes equation. When it is put into dimensionless form, the parameters of the equation are the Reynolds number (as the coefficient of the viscosity term) and the Froude

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[1]Professor, Mechanical and Aerospace Engineering Department, Oklahoma State University, Stillwater, Oklahoma.

number, Fr (as the coefficient of the gravity term). These two dimensionless numbers therefore govern the solutions of problems with given geometries and boundary conditions.

For the first-order analysis, the effect of the viscosity term was assumed to be small, so the the Froude number became a weak function of the Reynolds number. The simplest relationships should therefore have the form:

$$Fr = \text{constant} \tag{C.1}$$

where the Froude number is the nondimensional normalized way of writing the penetration depth (or more precisely, the square root of its inverse). Thus, the Froude number is defined on the basis of a reference velocity and the penetration depth:

$$\text{constant} = \frac{V_o}{\sqrt{g' H}} \tag{C.2}$$

or

$$H = \frac{V_o^{\,2}}{(\text{constant})^2 \, g'} \tag{C.3}$$

where V_o = Pump flowrate divided by propeller area.

The proper definition of the effective gravity term, g', presents greater difficulty, because the density may vary throughout the lake; the proper selection is not obvious:

$$g' = g \, \frac{\rho_{(h)} - \rho_o}{\rho_o} \tag{C.4}$$

where $\rho_{(h)}$ = density as a function of depth.

To determine the validity of this equation, the penetration of the central part of the downward jet was analysed (neglecting any viscous effects for this first-order analysis).

The Bernoulli equation was applied to a central streamline from the propeller plane (subscripted by "o") to the stagnation, or penetration, point (subscripted by "p"), using the coordinate "H" in a positive downward direction:

$$P_o + \frac{\rho_o V_o^2}{2} - \rho_o g H_o = P_p + \frac{\rho_o V_p^2}{2} - \rho_o g H_p \qquad (C.5)$$

where P_o = hydrostatic pressure at the propeller depth

P_p = hydrostatic pressure at the penetration depth

Note that $\rho = \rho_o$ throughout the center portion of the jet. Substituting $V_p = 0$, since this is at a stagnation point, and rearranging:

$$\frac{\rho_o V_o^2}{2} = (P_p - P_o) - \rho_o g(H_p - H_o) \qquad (C.6)$$

In the absence of strong induced currents below the stagnation point, the pressure at that level, H_p, is the same as it is at the same depth elsewhere in the lake. Therefore, from hydrostatics:

$$(P_p - P_o) = g\int_{H_o}^{H_p} \rho \, dh \qquad (C.7)$$

$$(P_p - P_o) = g\int_{H_o}^{H_p} \rho_o dh + g\int_{H_o}^{H_p} (\rho - \rho_o)dh \qquad (C.8)$$

$$= g\ \rho_o(H_p - H_o) + g\int_{H_o}^{H_p} (\rho - \rho_o)dh \qquad (C.9)$$

Substituting this in the Bernoulli equation (C.6) yields:

$$\frac{\rho_o V_o^{\ 2}}{2} = g\int_{H_o}^{H_p} (\rho - \rho_o)dh \qquad (C.10)$$

Rearranging this into the dimension of length or "head" gives:

$$\frac{V_o^{\ 2}}{2g} = \frac{1}{\rho_o}\int_{H_o}^{H_p} (\rho - \rho_o)dh \qquad (C.11)$$

Equation C.11 can also be put into the form:

$$\sqrt{2} = \frac{V_o}{\sqrt{g\int_{H_o}^{H_p} \frac{(\rho - \rho_o)}{\rho_o}dh}} \qquad (C.12)$$

Where the right-hand side is an integral formulation of the densimetric Froude number:

$$Fr_d = \frac{V}{\sqrt{g\ \frac{\Delta\rho}{\rho_o}\ H}} \qquad (C.13)$$

This is the appropriate form of the normalized dependent variable H_p, and Equations C.1, C.12, and C.13 imply:

$$Fr_d = 1.4 \tag{C.14}$$

and

$$\frac{\Delta\rho}{\rho_o} = \int_{H_o}^{H_p} \frac{(\rho - \rho_o)}{\rho_o} dh \tag{C.15}$$

The validity of Equation C.15 was explored by analysing two cases of penetration: deep hypolimnetic penetration and shallow hypolimnetic pentration.

Deep Hypolimnetic Penetration

When the jet penetrates deeply into a uniform hypolimnion, the detail features of the thermocline become unimportant, and the density profile may be approximated as a simple step function. The step is assumed to be located at the middle of the thermocline (denoted H_t) which is analagous to the characteristic depth of the densicline. Above the step, the density is taken as ρ_o; below the step is is assumed to have the average hypolimnetic value of ρ_h. Applying these assumptions into Equation C.11 shows:

$$\frac{V_o^2}{2g} = \int_{H_o}^{H_p} \frac{(\rho - \rho_o)}{\rho_o} dh \tag{C.16}$$

$$\text{where } \rho = \begin{cases} \rho_o \text{ for } H < H_t \\ \rho_h \text{ for } H > H_t \end{cases}$$

Further,

$$\frac{V_o^2}{2g} = \int_{H_o}^{H_t} \frac{(\rho_o - \rho_o)}{\rho_o} dh + \int_{H_t}^{H_p} \frac{(\rho_h - \rho_o)}{\rho_o} dh \tag{C.17}$$

$$\frac{V_o^2}{2g} = 0 + \frac{\rho_h - \rho_o}{\rho_o}(H_p - H_t) \qquad \text{(C.18)}$$

$$= \frac{\Delta\rho}{\rho_o} H_h \qquad \text{(C.19)}$$

where $(H_p - H_t) = H_h$, the hypolimnetic penetration.

Using the substitution of C.14, Equation C.19 can then be rearranged into the form of Equation C.13 having the hypolimnetic penetration depth as the characteristic length.

Shallow Hypolimnetic Penetration

When the jet is weak and deflects the top of the thermocline only slightly, the change in density related to the thermocline may be described as a constant slope ($A = d\rho/dh$). Equation C.16 can be transformed to:

$$\frac{V_o^2}{2g} = \int_{H_o}^{H_t} \frac{(\rho - \rho_o)}{\rho_o} dh + \int_{H_t}^{H_p} \frac{(\rho - \rho_o)}{\rho_o} dh \qquad \text{(C.20)}$$

where $\rho = \begin{cases} \rho_o & \text{for } H < H_t \\ \rho_o + A(H - H_t) & \text{for } H > H_t \end{cases}$

Simplifying further,

$$\frac{V_o^2}{2g} = 0 + \int_{H_t}^{H_p} \frac{A(H - H_t)}{\rho_o} dh \qquad \text{(C.21)}$$

$$= \frac{A}{2\rho_o} (H_p - H_t)^2 \qquad \text{(C.22)}$$

$$\frac{V_o^2}{2g} = \frac{A}{2\rho_o} H_h^2 \qquad (C.23)$$

Note that this does not fall into the standard form unless the average density below the top of the thermocline is defined as:

$$\rho_t = \rho_o + \frac{A H_h}{2} \qquad (C.24)$$

where ρ_t = the average density of the thermocline.

Then Equation C.23 can be transformed into:

$$\frac{V_o^2}{2g} = \frac{(\rho_t - \rho_o)}{\rho_o} H_h \qquad (C.25)$$

This form, like Equation C.19, can be put into the form of Equation C.13, thus demonstrating that this integral formulation is the correct way of finding the average effective density difference and that the characteristic length in the densimetric Froude number should be the hypolimnetic penetration length, H_h.

Summary

The interpretation of this analysis is: the pressure in a stratified lake rises more rapidly with depth than the hydrostatic pressure that would arise from the epilimnetic density alone. Thus the jet of epilimnetic water encounters an apparent adverse pressure gradient; the apparent pressure is a function of depth (Equation C.7). The jet penetrates to the depth at which the stagnation pressure head equals the initial velocity head of the jet.

The result of the analysis, Equations C.12, has several interesting features. The depth of penetration is not explicit in this equation, but appears implicitly as the upper limit of the integral. The lower limit of the integral, the depth of the pump, is not critical to this analysis, since the epilimnion is assumed to be well mixed. Two simple cases, deep and shallow penetration, demonstrated how the integral formulation can simplify to the common densimetric Froude number. Although the diameter of the propeller does not appear in the first-order result, second-order corrections (i.e. viscosity terms) would be a function of the diameter.

Because the characteristic length of the densimetric Froude number was determined to be the hypolimnetic penetration length, the epilimnetic penetration length would have to added in order to predict the plume length (i.e. the depth of penetration).

VITA 2

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