SPRING STIFFNESSES FOR BEAM-COLUMN ANALYSIS OF SOIL-STRUCTURE INTERACTION PROBLEMS

By

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NOMENCLATURE

A	coefficients for exponential equation
B	coefficients for exponential equation
C	coefficients for exponential equation
D	coefficients for exponential equation
D_s	second soil parameter
E	approximate interface stiffness matrix
F_B	force applied to beam-column
K	modulus of subgrade reaction
K_{BB}	beam-column stiffness matrix
K_{II}	stiffness term for nodes on interface between FEM foundation and beam-column beam
K^*_{II}	interface stiffness matrix which contains all characteristics of the two-dimensional foundation
K_{IR}	stiffness terms coupling the interface to the remainder of the foundation
K_{RI}	stiffness terms coupling the interface to the remainder of the foundation
K_{RR}	stiffness terms of remainder of foundation
U_B	beam-column (interface nodes) displacements
U_R	displacement of remainder of foundation
q	surface pressure
w	surface displacements

X number of nodes from major diagonal

Y magnitude of interface stiffness term

CHAPTER I

INTRODUCTION

Structures of Interest

Design engineers have long been concerned with the interaction between soils and structures. The design of slabs on grade and U-frame structures are controlled by the soil pressure acting on them. These structures must be designed to account for the interaction between the structure and soil. The problem of determining the relationship between the structure displacements and the soil pressures is referred to as a soil-structure interaction problem.

Potential Methods

The soil-structure interaction problem in general is a nonlinear, three-dimensional problem. The load-deformation curves for all soils are nonlinear and are affected by many factors such as changes in pore water pressures or cyclic loading. Many structures have changes in geometry which can be analyzed only by considering a three-dimensional behavior. Three-dimensional finite element analyses are the only types of analyses which can address these complicated problems. When both the three-dimensional geometry and nonlinear soil behavior are considered, time required for preparation of data for a finite element model and computer time costs become prohibitive. For these reasons three-dimensional nonlinear finite element analyses are impractical for preliminary designs.

Because the cross sections of slabs on grade and U-frame structures are relatively constant along the length, analysis for preliminary design can be made using a two-dimensional plane strain slice of the structure. Two-dimensional finite element analyses including nonlinear soil behavior have been performed; however, these analyses are still impractical for preliminary design because of both cost and time.

For many types of soil-structure systems, the structure may be modeled as an assemblage of beam-column elements in contact with the soil. With this simplification, the major problem is to represent the pressures at the soil-structure interface as nonlinear functions of the interface displacements. Winkler's hypothesis has been used for analysis of horizontal beam-column/soil systems (2). However, Winkler's hypothesis does not account for the two-dimensional behavior of the soil. The solution of a prismatic beam-column beam on a Winkler foundation loaded with a uniform load yields a constant displacement across the beam. The beam from this model has no rotations or moments and results in a simple one-dimensional problem. A complete two-dimensional analysis of the beam-on-foundation indicates that displacements as well as moments are not constant.

Although the soil-structure interaction behavior is generally nonlinear, for preliminary design or for short-term service loads, the soil-structure interaction behavior may be assumed to be a linear relationship between displacements and pressures. Although much experimental data have been accumulated from plate bearing tests to produce "coefficients of subgrade reactions," these coefficients are usable only with Winkler's hypothesis.

Statement of the Problem

The objective of this study was to provide a simple means of combining a two-dimensional foundation with a beam-column structure which can be used for preliminary design of slabs on grade or the base slab of a U-frame structure. The study examines the characteristics of the soil-structure interface and develops a procedure for coupling the two-dimensional foundation to a beam-column model. The problem then becomes one of understanding the changes of the characteristics of the model's stiffness terms and being able to reproduce these characteristics in a simple and accurate procedure.

CHAPTER II

PREVIOUS WORK

The solution of soil-structure problems begins in 1776, with the work of Coulomb (1). Coulomb's theory provides a means of calculating earth pressures against retaining structures. His theory assumes that the structure is free to move a sufficient amount to either produce full active or passive earth pressures. Consequently, this theory is useful for calculating only maximum pressures. This early work is presently being used for limit state analysis for design of earth retaining structures. Thus it is valid for predicting failure loads, but gives no information about displacements or stresses between extremes.

Winkler (2), in 1867, developed a soil model for evaluating forces on the structure which are dependent on the displacements of the structure. Winkler assumed that the stress at any point on the surface of the soil is directly proportional to the structural displacement only at that particular point. This hypothesis has been used to aid in analyzing many soil-structure interaction problems but fails to account for continuity of the subgrade.

Subsequently, a procedure was developed by Biot (3) for analysis of an infinite beam on an elastic two- or three-dimensional foundation. This procedure was based on Winkler's hypothesis and had no provisions for plates or finite length beams and considered only concentrated loads.

Vesic (5) extended Biot's work to include finite length beams and loading by a couple. Vesic (4) also studied the validity of Winkler's hypothesis and concluded that the hypothesis is valid for infinite beams resting on a semi-infinite elastic subgrade. Vesic stated that the use of Winkler's hypothesis leads to an overestimation of bending moments and an underestimation of contact pressures and displacements. Vesic also noted that conventional analysis using Winkler's hypothesis could be used for finite beams and recommended procedures for beams with different length characteristics.

Reese and Matlock (6) used the Winkler hypothesis with a nonlinear foundation for analysis of laterally loaded piles. While the foundation was a nonlinear Winkler model, the structure was modeled with beam-column elements. The finite difference technique was used to formulate the beam-column equations.

Haliburton (7) extended the procedure used by Matlock et al. (8) for analyzing flexible retaining structures. Haliburton discussed nonlinear soil response, anchor and brace supports, and procedures for developing nonlinear soil load-deflection curves.

Dawkins (9) combined the nonlinear Winkler foundation with a stiffness method approach to the beam column model. This work also allowed the use of linearly varying distributed springs as well as point linear/nonlinear springs.

Two-parameter models were developed in order to overcome deficiencies inherent in the Winkler hypothesis in modeling the continuous behavior of the subgrade. The two-parameter models improved on Winkler's model by adding a continuous layer between the structure and the Winkler springs. The Filonenko-Borodich model (10, 11, 12) adds a membrane under tension between the structure and the Winkler spring. The Hetenyi (12, 13) model adds an elastic beam for two-dimensional problems in place of the Filonenko-Borodich model. The Pasternak (12, 14) model uses a shear layer to interact between the Winkler springs and the structure. Vlasov and Leont'ev (15, 16) developed a two-parameter foundation model from the continuum point of view.

All two-parameter models can, in general, be represented by the following equation:

$$q(x) = K\, w(x) - D_s \frac{d^2 w(x)}{dx^2}$$

where

$q(x)$ = surface pressure;

$w(x)$ = surface displacements;

K = modulus of subgrade reaction; and

D_s = second soil parameter.

The two parameters, K and D, are difficult to determine. Presently, these parameters are being determined from extensive field tests. Although the parameter K has the same name as the K for Winkler's model, i.e., modulus of subgrade reaction, the values for K for a Winkler model cannot be used for a two-parameter model. This foundation model, when combined with a structural model, leads to a very complex mathematical problem. Because of the difficulties in calculations and in determining the two parameters, this is presently not a practical tool for preliminary design.

Elastic continuum models allow for a continuous description of the soil beneath the structure as well as displacements of the soil away from the structure. Flamant (17) and Cheung (18) calculated a flexibility matrix for the interface. The inverse of this flexibility matrix yields a stiffness matrix for the foundation which could be combined with a beam-column stiffness matrix. A comparison of this approach with a two-dimensional finite element model of the foundation indicates that the moments, rotations, and relative displacements are accurately predicted by the continuum model. However, the absolute displacements are substantially in error.

The finite element method has proven to be a valid tool for analyzing soil-structure interaction problems. Clough and Duncan (19) used the finite element method to analyze sheet pile walls, and U-frame structures that were a part of the Port Allen and Old River Locks. The procedure accounts for nonlinear stress-strain behavior of the soil medium and uses an incremental loading technique to simulate a construction sequence. The favorable comparison of analytical and experimental results for the Port Allen and Old River Locks demonstrated the usefulness of this method in the soil-structure interaction area (19, 20). However, the use of this method requires the tedious generation of complicated grids for modeling the problem, large amounts of computer time, and many hours of interpreting the results of analysis. Therefore, the high cost of this analysis technique makes it impractical for use by most designers.

CHAPTER III

TWO-DIMENSIONAL FINITE ELEMENT ANALYSIS OF THE STRUCTURE SOIL SYSTEM

Objective

The objective of this study is to provide a simple means of combining a two-dimensional foundation with a beam-column structure which can be used for preliminary design purposes. To accomplish this goal, a procedure to produce an interface model which approximates the results obtained from a finite element solution was developed.

Beam-Column/Finite Element Method

Assumptions

As stated previously, attention is limited to those structures such as slabs-on-grade or U-frame base slabs for which a two-dimensional slice of the soil-structure system adequately represents the behavior for preliminary design. It is further assumed that the dimensions of the structure allow a model composed of beam-column elements to be used to represent this part of the system.

Inherent in the two-dimensional model of the soil-structure system is the assumption that the soil foundation is in a state of plane strain. Although the extent of the soil foundation away from the structure will influence the soil-structure interaction, it is assumed for this investigation that the soil can be represented as a homogeneous isotropic semi-infinite half space. Because preliminary designs are frequently based on short-term service loads which result in small displacements, it is further assumed that the soil is linearly elastic. Under these assumptions the soil foundation is modeled initially as an assemblage of two-dimensional finite elements.

A final simplification is made at the interface between the beam-column and the soil foundation. Enforcement of compatibility of nodal displacements on the interface would result in shear stresses in the soil along the interface and a corresponding axial stress resultant in the beam-column. This effect would represent frictional interaction between the base of the structure and the foundation. The magnitude of this friction resistance is dependent on roughness of the structure as well as the cohesion and internal friction of the soil. The friction effect could be accounted for by introduction of an interface element (21) between the structure and foundation. However, preliminary studies indicated that the interface friction had negligible influence on the response of the system. Consequently, the friction effect is neglected and compatibility of vertical displacements only is enforced at the interface, which allows the interface nodes to be placed at the longitudinal axis of the beam-column rather than at the base of the beam with no loss in accuracy.

Finite Element Model

Figure 1 displays the interface area between the beam-column beam and finite element foundation; the entire finite element foundation model is shown in Figure 2. The grid contains 2139 nodes and 2124 elements. The quadrilaterals are four node isoparametric elements using the

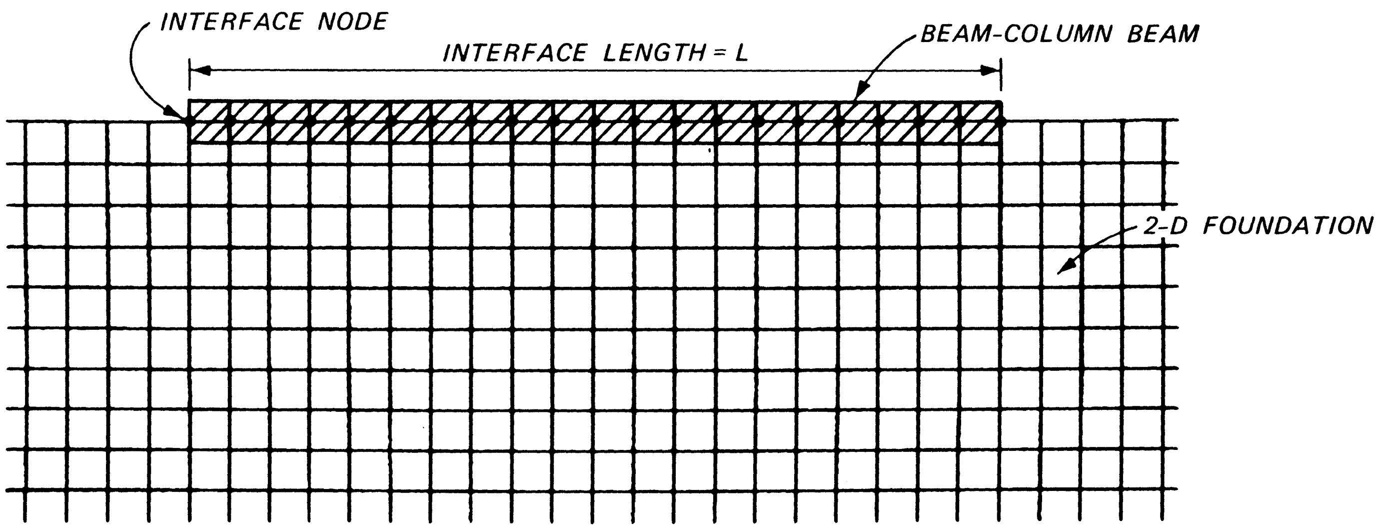

Figure 1. Beam-Column Beam With Two-Dimensional Foundation

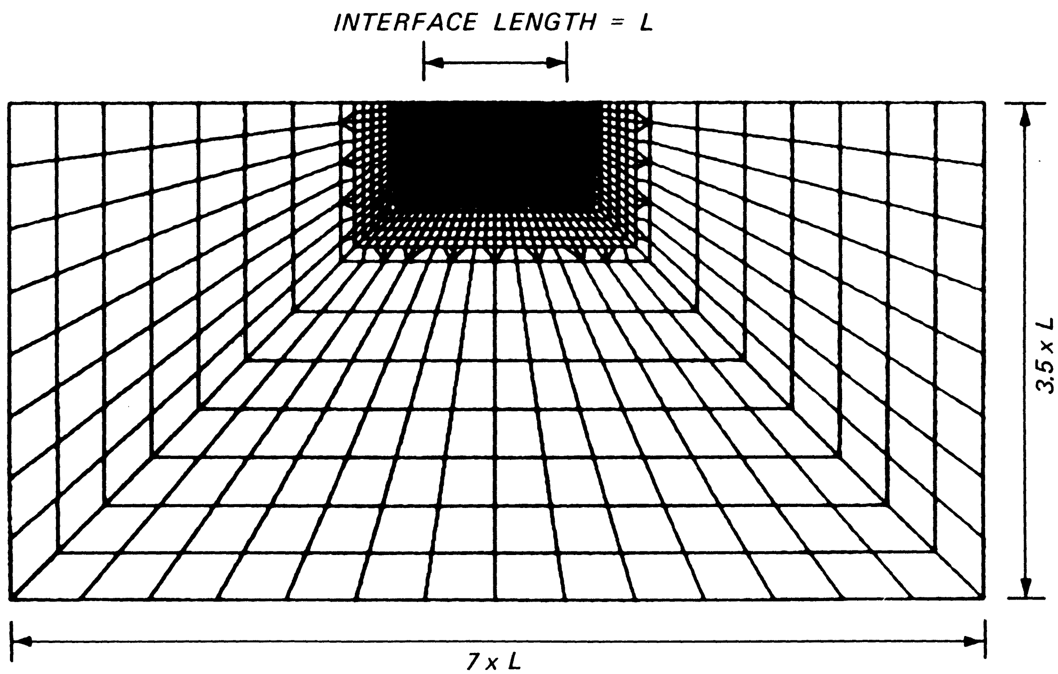

Figure 2. FEM Foundation Model for 20 Foot Interface

incompatible mode to eliminate parasitic shear. The triangles are three node constant strain elements. Figure 3 shows a window display of the grid below the structure. For a beam with a length of 20 feet, the grid models an area with a height of 3.5 times and a width of 7 times the length of the beam.

Substructure Equations

The nodal force-nodal displacement relationship for a beam-column on a two-dimensional finite element method (FEM) foundation may be expressed as:

$$\left\{ \underset{\text{Beam}}{\begin{bmatrix} K_{BB} & 0 \\ 0 & 0 \end{bmatrix}} + \underset{\text{Foundation}}{\begin{bmatrix} K_{II} & K_{IR} \\ K_{RI} & K_{RR} \end{bmatrix}} \right\} \begin{Bmatrix} U_B \\ U_R \end{Bmatrix} = \begin{Bmatrix} F_B \\ 0 \end{Bmatrix} \qquad 3.1$$

where

K_{BB} = beam-column stiffness matrix;

K_{II} = stiffness terms for nodes on interface between FEM foundation and beam-column beam;

$K_{IR} = K_{RI}$ = stiffness terms coupling the interface to the remainder of the foundation;

K_{RR} = stiffness terms of remainder of foundation;

U_B = beam-column (interface nodes) displacements;

U_R = displacement of remainder of foundation; and

F_B = force applied to beam-column.

Expanding Equation (3.1) yields:

$$(K_{BB} + K_{II})\, U_B + K_{IR} U_R = F_B \qquad 3.2a$$

and

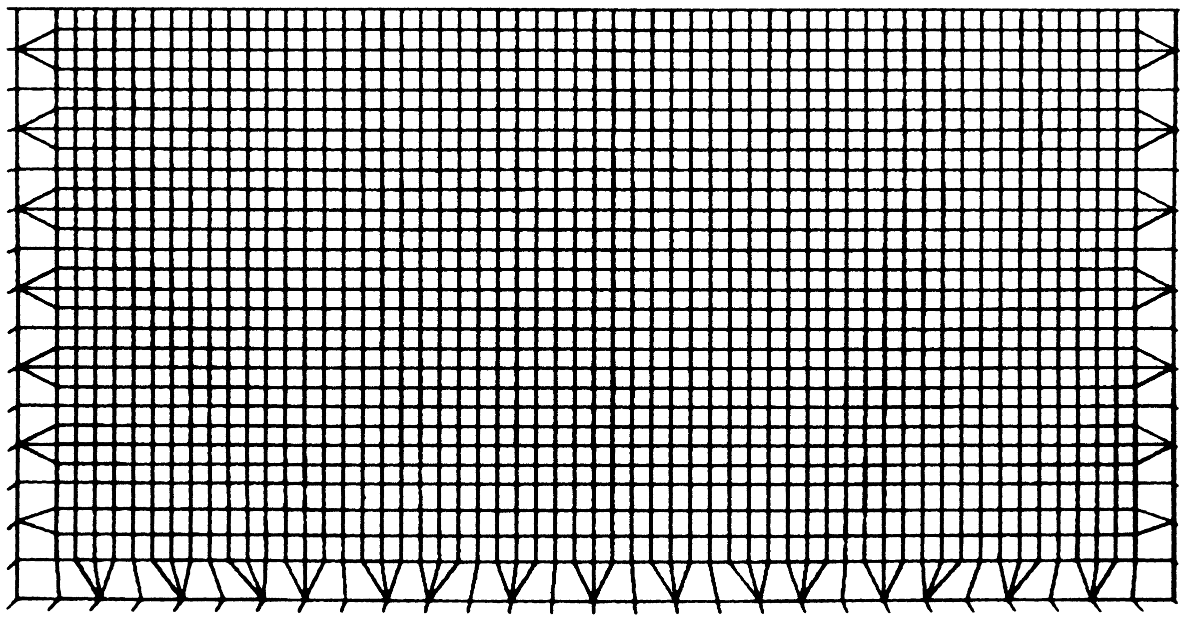

Figure 3. Window at Interface of FEM Foundation Model

$$K_{RI}U_B + K_{RR}U_R = 0 \qquad 3.2b$$

Equation (3.2b) may be written as:

$$U_R = -K_{RR}^{-1} K_{RI}U_B \qquad 3.3$$

or

$$(K_{BB} + K_{II})\ U_B - K_{IR}K_{RR}^{-1} K_{RI}U_B = F_B \qquad 3.4a$$

$$[(K_{BB} + K_{II} - K_{IR}K_{RR}^{-1} K_{RI})]\ U_B = F_B \qquad 3.4b$$

$$[(K_{BB} + K_{II}^{*})]\ U_B = F_B \qquad 3.4c$$

where K_{II}^{*} is the interface stiffness matrix which contains all characteristics of the two-dimensional foundation.

The solution of Equation (3.4) is inexpensive since the number of simultaneous equations is the same as for the solution of the beam-column. Once the interface stiffness matrix, K_{II}^{*}, has been determined, the beam-column stiffness matrix, K_{BB}, and load array, F_B, can be changed. This allows the solution of any load case or changes in the beam without re-solving for K_{II}^{*}.

Factors Affecting Interface Matrix K_{II}^{*}

Because the foundation is assumed to be linearly elastic, homogeneous, and isotropic, the interface matrix K_{II}^{*} is directly proportional to the modulus of elasticity of the foundation material. Consequently, interface matrices need be developed only for a unit value of foundation modulus of elasticity. Other factors which influence the interface stiffness matrix are number of nodes on the interface and Poisson's ratio for the foundation.

Poisson's ratio for soils varies from near zero to near one-half. To examine the influence of Poisson's ratio, interface matrices were developed for Poisson's ratio ranging from 0.1 to 0.4. Although magnitudes of elements of stiffness matrices depend on the value of Poisson's ratio, characteristics of matrices to be discussed later are the same. Hence, only matrices for Poisson's ratios of 0.2 and 0.3 are presented and discussed in detail. To determine the effect of the number of nodes on the interface, complete interface matrices were evaluated for 11, 21, 31, and 41 interface nodes.

Method of Extracting Interface Matrix K_{II}

By definition, any element $K_{i,j}$ of a stiffness matrix is the force corresponding to the degree of freedom (DOF) _i_ due to a unit displacement of DOF _j_ with all other displacements equal to zero. Because the finite element model of the foundation is symmetric, it is only required that the interface matrix be developed for nodes on and to one side of the axis of symmetry. The interface matrix was developed by sequentially applying a unit vertical displacement at one node with displacements of other nodes on the interface equal to zero. The reactions generated at the restrained interface nodes comprise one column (and row) of the desired matrix.

Characteristics of K^*_{II}

To study K^*_{II}, interface matrices were generated for 11, 21, 31, and 41 nodes on the interface. The off diagonal terms for each row of each matrix were plotted as shown in Figures 4, 5, 6, and 7. The terms below the major diagonal term to the end of each column of the first rows

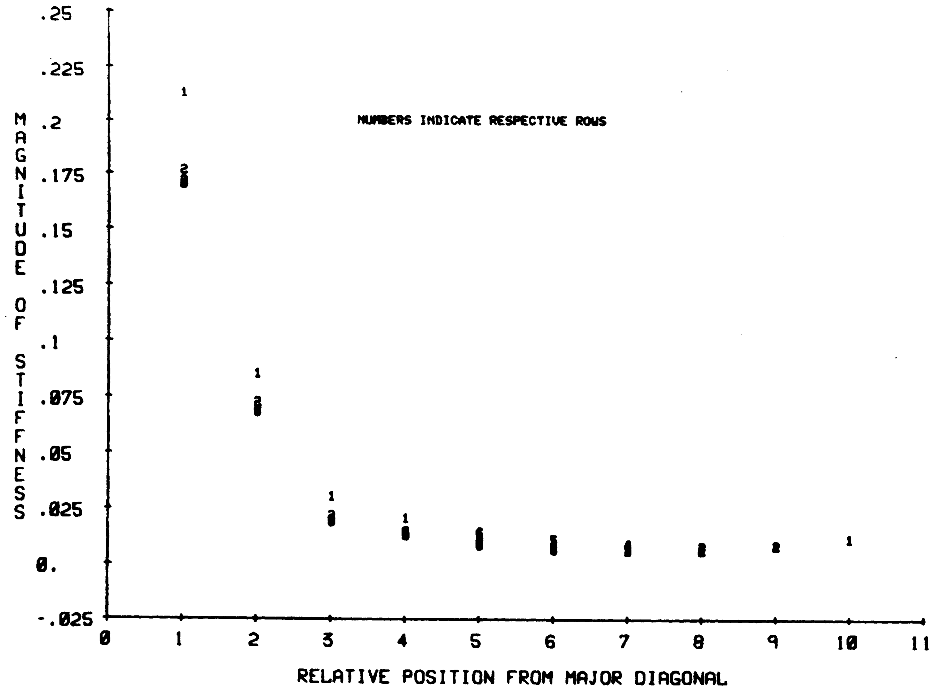

Figure 4. Off Diagonal Terms for 11 Node Interface

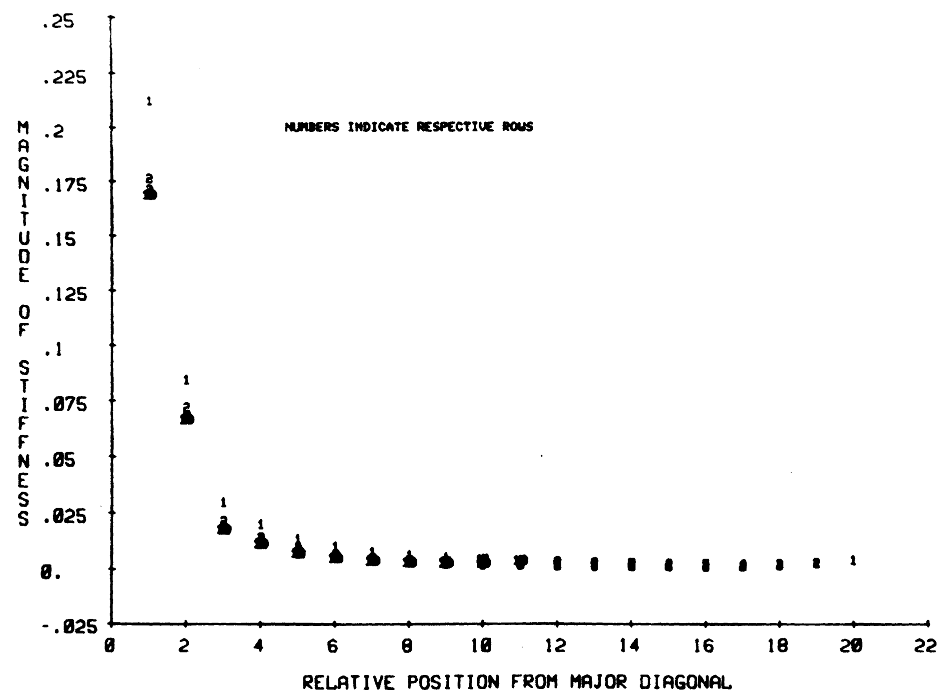

Figure 5. Off Diagonal Terms for 21-Node Interface

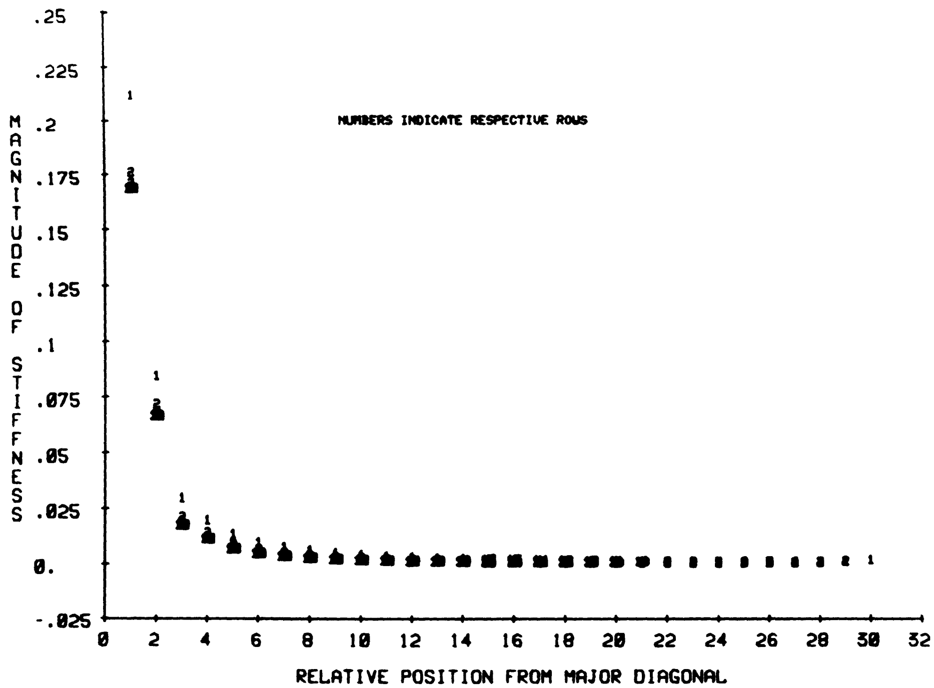

Figure 6. Off Diagonal Terms for 31 Node Interface

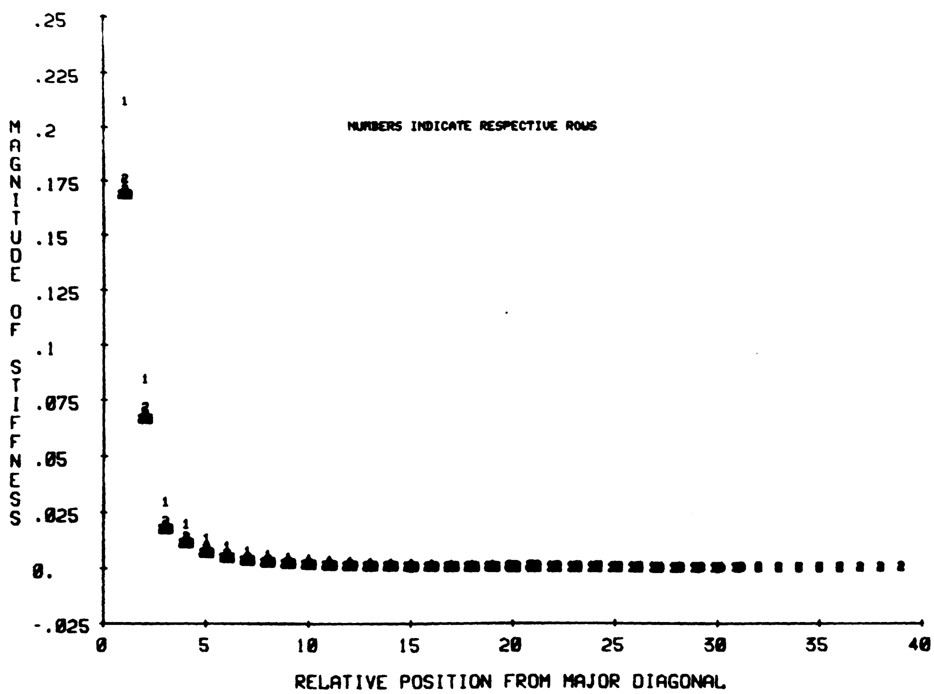

Figure 7. Off Diagonal Terms for 41 Node Interface

(i.e., the nodes on the interface) through the middle nodes are plotted. These figures demonstrate that all relative terms of the interface stiffness matrices are very close in magnitude except for the first two (i.e., the end) rows. These figures also demonstrate that the terms change only slightly for different numbers of interface nodes. These plots were for a foundation with a Poisson's ratio of 0.3. The same characteristics for other values of Poisson were observed.

Figures 8, 9, 10, and 11 are plots of the major diagonal terms of the four interface stiffness matrices. These plots show little change in the major diagonal terms after the first two columns and little effect from change of number of interface nodes.

Figure 12 is a plot of the centerline row of the four interface stiffness matrices. Figure 13 is a plot of the first column of the four interface stiffness matrices. Except for the 11-node matrix, these plots indicate that the magnitude of the stiffness terms are dependent on relative position from the main diagonal and not dependent on number of nodes of the interface. These similarities of the curves suggest that an interface stiffness matrix for a two-dimensional foundation could be generated without resorting to a finite element solution.

The following is a summary of the characteristics of the curves shown in Figures 4 through 13:

1. OFF diagonal terms approach infinity as $x \to 0$ (i.e., as x approaches the major diagonal).

2. OFF diagonal terms approach zero as $x \to \infty$.

This suggests that the magnitude of the stiffness terms could be calculated as:

$$y = Ae^{-f(x)}$$

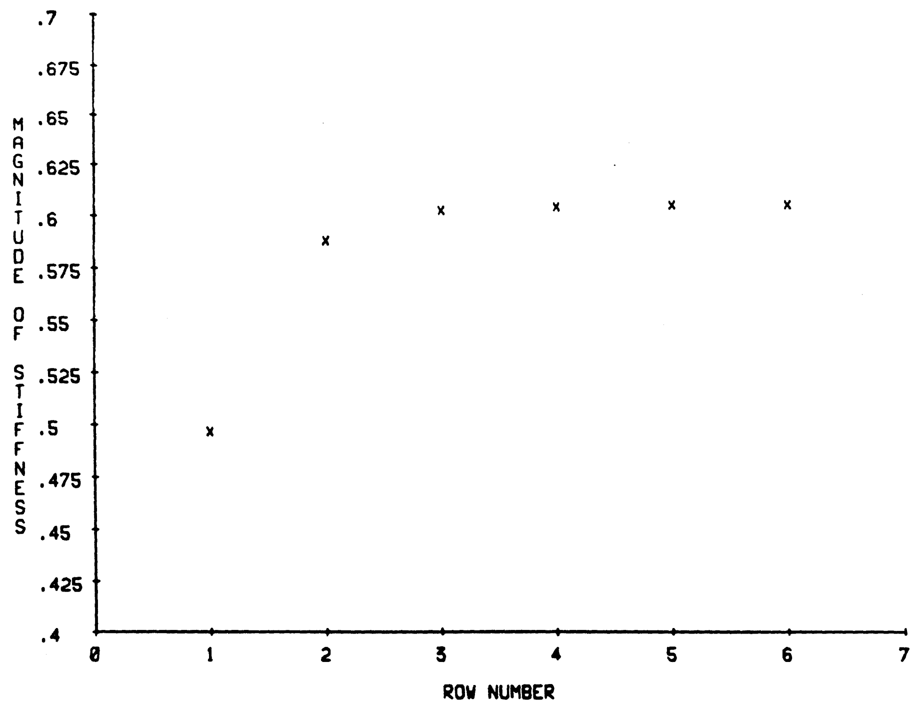

Figure 8. Major Diagonal Terms for 11 Node Interface

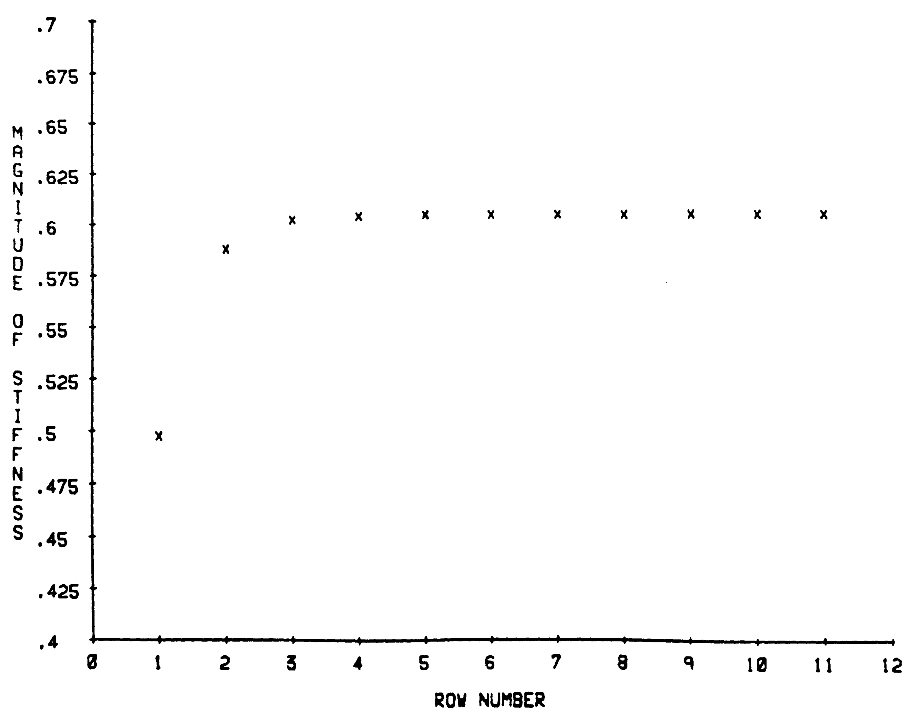

Figure 9. Major Diagonal Terms for 21 Node Interface

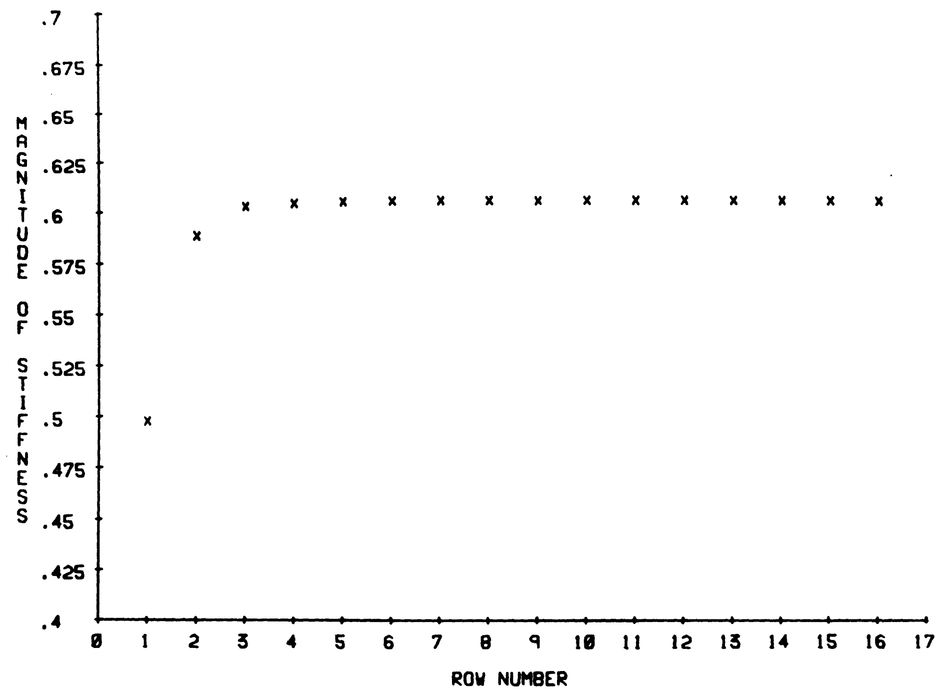

Figure 10. Major Diagonal Terms for 31 Node Interface

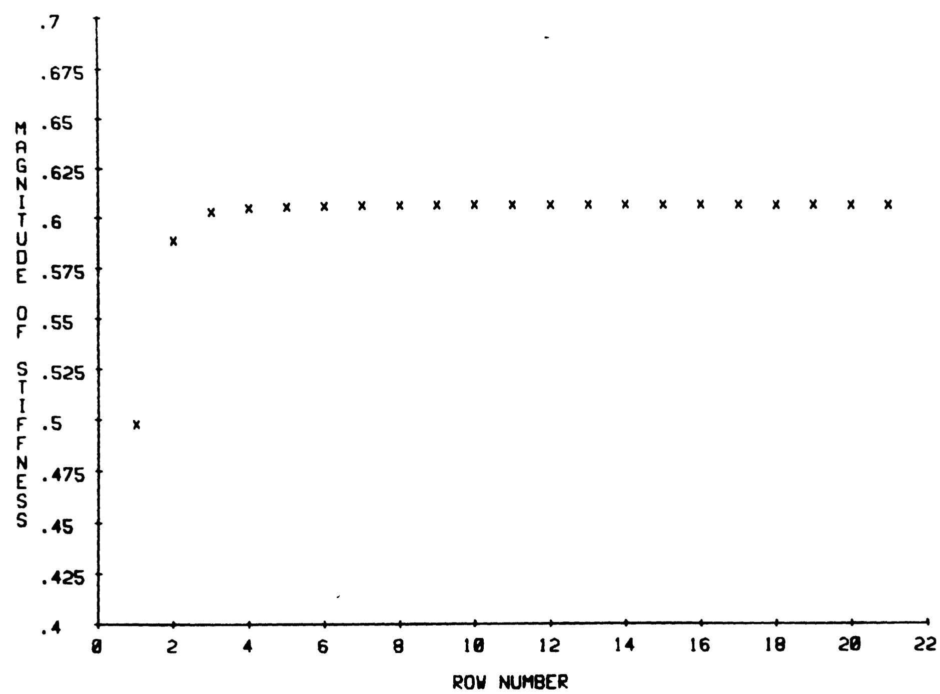

Figure 11. Major Diagonal Terms for 41 Node Interface

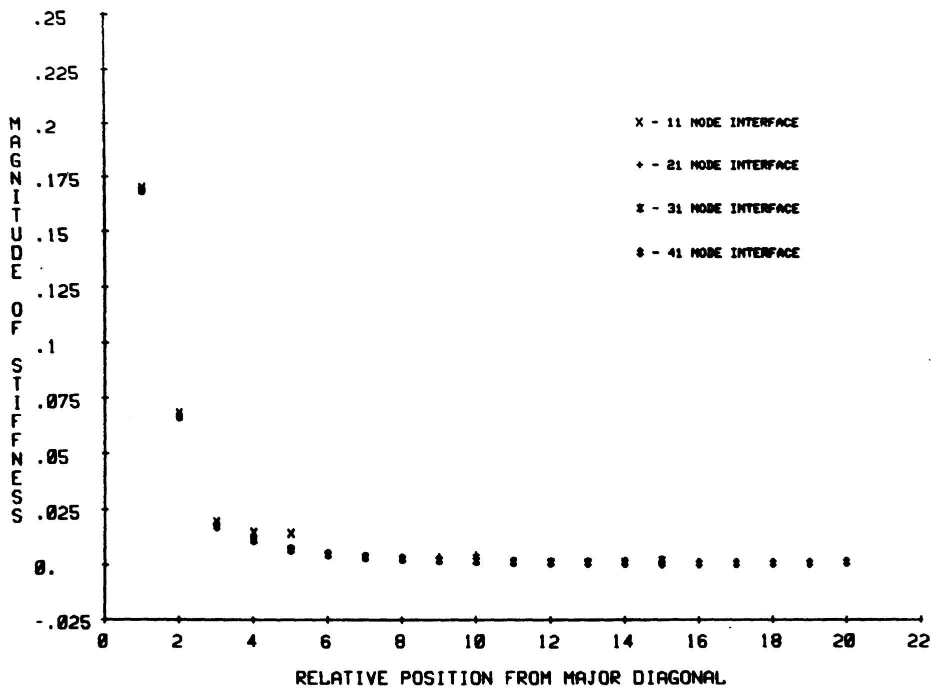

Figure 12. Centerline Rows, K*

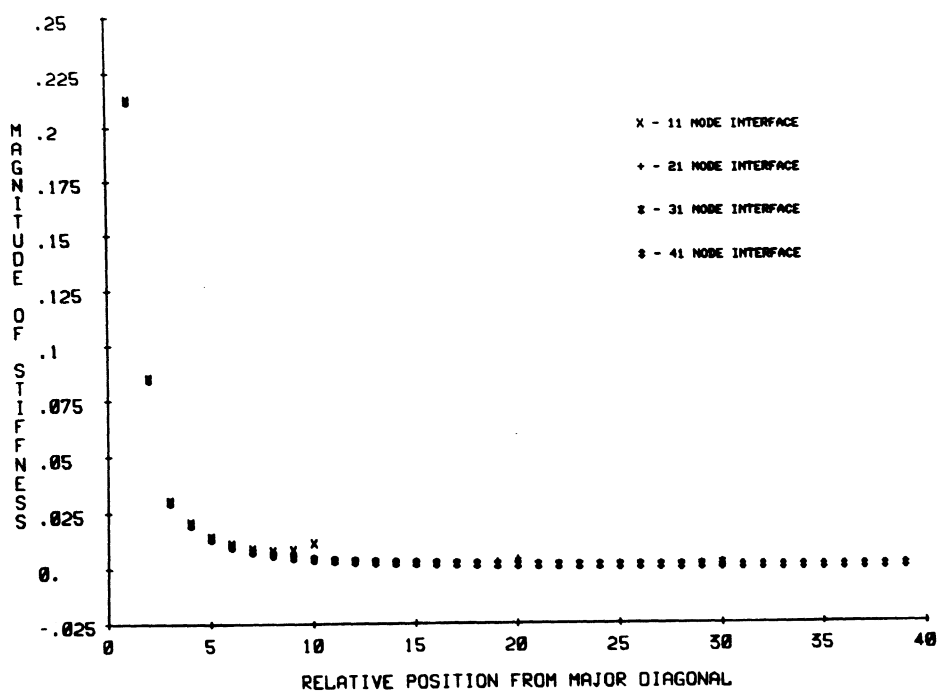

Figure 13. First Rows, K*

where y is the magnitude of the interface stiffness term, and x is the number of terms from the major diagonal;

$$f(x) \rightarrow 0 \quad \text{as} \quad x \rightarrow 0$$

$$f(x) \rightarrow 0 \quad \text{as} \quad x \rightarrow 0$$

$$f(x) = Bx^{C}$$

where A, B, C are functions of Poisson's ratio.

Equilibrium Requirement

Each row of K^{*}_{II} is a system of forces produced by a unit displacement of a node on the interface with all other nodes of the interface node having no displacements. The forces produced on the interface nodes, together with the reactions produced at the boundary nodes of the complete finite element model, constitute a system of forces in equilibrium. If the rows of the interface matrix are approximated, the terms must continue to satisfy overall equilibrium. Therefore,

$$\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} K_{ij} \qquad j = 1, 2, 3, \ldots, N$$

where

n = number of node on interface;

y = approximated row; and

$K_{i,j}$ = finite element interface matrix stiffness coefficient.

CHAPTER IV

GENERATION OF APPROXIMATE INTERFACE AND COMPARISON OF RESULTS

Introduction

Because the stiffness matrix of the beam column is independent of the interface stiffness matrix, any beam can be used for comparison of results. For the following comparison, a beam 12 inches high and 1 inch thick will be used. Poisson's ratio of 0.3 and Young's modulus of 3×10^6 will be used for the material properties of the beam. The four beams will have length-to-depth ratios of 5, 10, 15, and 20. The node spacing for all beams will be six inches in order to accurately represent the beam. Because each beam node is connected to the interface, the interface node spacing is also six inches. This node spacing will allow for accurate modeling of the soil-structure interaction behavior. The following is a list of the five different load cases used for comparison:

Load Case 1--Uniform load over entire beam

Load Case 2--Point load at centerline

Load Case 3--Point load at left quarter point

Load Case 4--Uniform load over center one-half of beam

Load Case 5--Uniform load over left half of beam.

Basis of Comparison

To provide a basis for comparison with approximate interface matrices, displacements and rotations using the exact interface matrices were obtained for Poisson's ratios of 0.1, 0.2, 0.3, and 0.4 for 11 and 21 node beams using all load cases. The exact displacements for the 31 and 41 were calculated for Poisson's ratios of 0.2 and 0.3 using all five load cases.

For reinforcement, complete solutions of the beam-soil system using two-dimensional finite elements for both beam and foundation were obtained for 11 and 21 node beams with a foundation having a Poisson's ratio of 0.3 and a Young's modulus of 7000 pounds per square inch (psi). The 7000 psi represents a dense sand (22). The results were compared with solutions for a beam-column model of the beam with a two-dimensional finite element foundation. As expected, these solutions produced identical results for both models. Additional reinforcement resulted from the solutions for 11, 21, and 31 node beams coupled to a 41 node exact interface matrix. As expected, these solutions produced the same results as when each beam was coupled with an interface matrix containing the same number of nodes. An example of a 21 beam on a 21 node foundation compared with a 21 beam on a 41 node foundation for a uniform load over the entire beam is shown in Table I. A Poisson's ratio of 0.3 was used for the foundations for this comparison.

Expedient Approach

Because the beam-column matrices and the foundation matrices are independent, it is possible to produce a beam-column computer program which stores complete interface matrices for various Poisson's ratios. Only one interface matrix for each Poisson's ratio is required, provided the matrix is sufficiently large to accommodate a wide variety of beam

TABLE I

21 NODE BEAM ON 41 NODE INTERFACE

Node Number From Left End of Beam	Beam on 41 Node Foundation		Beam on 21 Node Foundation	
	Deflection (in.)	Rotation	Deflection (in.)	Rotation
1	-0.00245166	-0.312634×10^{-5}	-0.00245166	-0.312637×10^{-5}
2	-0.00247034	-0.308966×10^{-5}	-0.00247034	-0.308967×10^{-5}
3	-0.00248859	-0.298111×10^{-5}	-0.00248859	-0.298112×10^{-5}
4	-0.00250596	-0.279648×10^{-5}	-0.00250597	-0.279649×10^{-5}
5	-0.00252201	-0.254003×10^{-5}	-0.00252201	-0.254004×10^{-5}
6	-0.00253631	-0.221918×10^{-5}	-0.00253632	-0.221919×10^{-5}
7	-0.00254853	-0.184316×10^{-5}	-0.00254853	-0.184317×10^{-5}
8	-0.00255834	-0.142222×10^{-5}	-0.00255835	-0.142223×10^{-5}
9	-0.00256552	-0.967215×10^{-6}	-0.00256553	-0.967217×10^{-6}
10	-0.00256990	-0.489344×10^{-6}	-0.00256991	-0.489345×10^{-6}
℄	-0.00257137	0.0	-0.00257138	0.0

lengths. A 61 node interface would cover a majority of beam lengths. The user of such a program would be required to provide only the modulus of elasticity and Poisson's ratio for the beam and foundation, length of the beam, moment of inertia of the beam, and loading on the beam. This seems feasible since only 961 terms are needed to produce a 61 node interface matrix, as the matrices are symmetric about both diagonals. The studies of the interface matrices indicated that the magnitude of terms in any row exhibited identical variations with position from the major diagonal as shown in Figures 4 through 7. Consequently, this indicates the possibility of reproducing the interface behavior with fewer than 961 terms.

Generation of Matrix From Centerline Row

Introduction

This centerline row method develops the entire interface matrix using only the centerline row of a 61 node exact interface matrix and 30 terms to force equilibrium. This procedure generates an approximate interface matrix producing results sufficiently accurate for preliminary designs for beams with up to 41 nodes. Beams with more than 41 nodes will have larger errors because the beam action will be influenced by the outer rows. As seen in Appendix A, the changes in any interface matrix from one row to the next are small, except for the outside few rows. This end effect of the outside row is difficult to predict and is not modeled by this procedure. This method models the matrix by using the centerline row and forcing equilibrium of all rows.

Equilibrium

As previously stated, each row of the interface matrix represents a set of reactions resulting from a single interface node being displaced a unit value. The sum of all vertical reactions is zero, i.e., the sum of vertical reactions of the interface nodes and of the boundary of the finite element grid. The sum of all horizontal forces is also zero. Thus, the problem is in a state of equilibrium.

The approximate matrix must also satisfy equilibrium. If the approximated interface matrix does not satisfy equilibrium, the matrix will not produce reasonable results. For example, a 10 percent change of the smallest off diagonal term of a 21 node interface matrix connected to a 21 node beam will result in a 3 percent error of all displacements. This indicates that the results produced from these interface matrices are very sensitive to the requirement for satisfying of equilibrium. A matrix which does not satisfy equilibrium results in a change in the internal strain energy of the foundation and misrepresents the true system of forces. Thus, each approximate interface matrix must be generated such that each row satisfies equilibrium.

Each row can be forced to satisfy equilibrium by making the sum of each approximate row equal the sum of the corresponding row of the exact interface matrix. The tables in Appendix B give the sums of terms on each row of the interface stiffness matrices listed in Appendix A. The sum of each row was divided by the sum of terms of the centerline row to produce factors which can be used to force equilibrium of generated matrices by knowing the sum of the centerline row for a given Poisson's ratio. Figure 14 displays the ratios of the sums of the rows plotted for each row for a Poisson's ratio of 0.2. Figure 15 displays the ratios of the first term

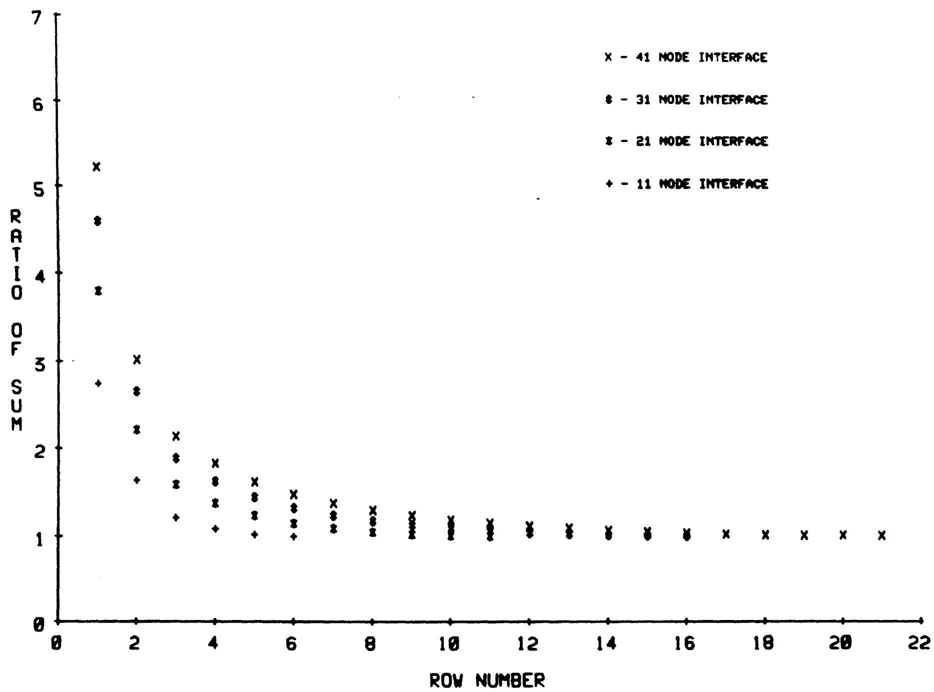

Figure 14. Ratio of Sum of Rows of K*

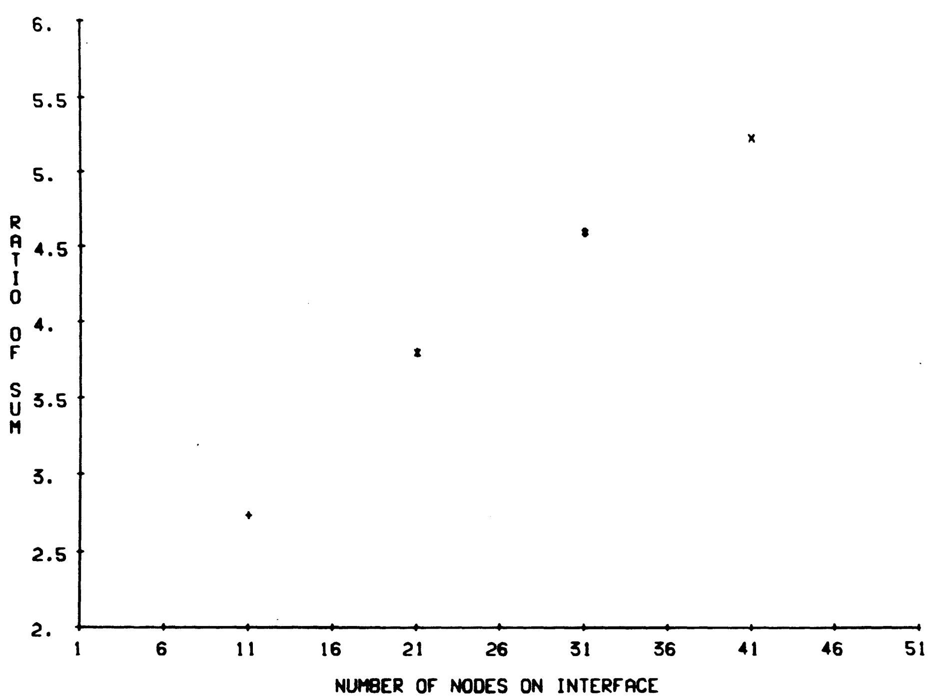

Figure 15. Ratio of Sums for First Row of Four K*

plotted against the number of nodes on the interface. This illustrates that the relationship (i.e., ratio) of the sum of the first row with the sum of the centerline row changes very little with increased nodes on the interface. These figures suggest that equilibrium factors could be extrapolated from these data. However, extrapolation of these data would produce results with errors greater than those which will be reported later in this study.

41 Node Foundation

Table II lists the results of using the previously described beam with 21 nodes on a 41 node interface matrix, [E], for Poisson's ratio of 0.3 which was generated by the centerline row method. The maximum displacement due to a uniform load is 1.1 percent in error, while the maximum moment is 2.0 percent in error. These results indicate that the centerline row method of producing an approximate interface method can be used for preliminary design. The following is the centerline row method for generating the interface matrix:

1. Place the centerline row, which depends on the desired Poisson's ratio, in every row starting at the major diagonal (Figure 16). The placement of the centerline row results in a banded matrix with a half band width equal to half the number of terms of any row plus one, since all interface matrices have an odd number of nodes. The odd number of nodes for each interface forces a node to be at the axis of symmetry.

2. Force symmetry about the major diagonal.

3. Satisfy equilibrium on the first row above the centerline row (Row 20). This row contains one unknown term, $E_{20,41}$, where

TABLE II

RESULTS OF 21 NODE BEAM ON GENERATED 41 NODE FOUNDATION WITH EQUILIBRIUM SATISFIED

Node	Y FEM	Y-Cal Centerline Row Method	Percent Error	Node	MOM FEM	Mom-Cal Centerline Row Method	Percent Error
1	-0.2241E-02	-0.2269E-02	1.26	1	0.	0.	0
2	-0.2259E-02	-0.2287E-02	1.24	2	-0.4753E 01	-0.4701E 01	1.11
3	-0.2277E-02	-0.2305E-02	1.21	3	-0.1028E 02	-0.1014E 02	1.34
4	-0.2295E-02	-0.2322E-02	1.19	4	-0.1562E 02	-0.1539E 02	1.49
5	-0.2310E-02	-0.2338E-02	1.17	5	-0.2051E 02	-0.2018E 02	1.59
6	-0.2325E-02	-0.2351E-02	1.15	6	-0.2478E 02	-0.2436E 02	1.71
7	-0.2337E-02	-0.2363E-02	1.14	7	-0.2836E 02	-0.2784E 02	1.82
8	-0.2346E-02	-0.2373E-02	1.12	8	-0.3116E 02	-0.3057E 02	1.89
9	-0.2354E-02	-0.2380E-02	1.11	9	-0.3319E 02	-0.3253E 02	1.97
10	-0.2358E-02	-0.2384E-02	1.11	10	-0.3441E 02	-0.3371E 02	2.02
11	-0.2359E-02	-0.2385E-02	1.11	11	-0.3482E 02	-0.3411E 02	2.03
12	-0.2358E-02	-0.2384E-02	1.11	12	-0.3441E 02	-0.3371E 02	2.02
13	-0.2354E-02	-0.2380E-02	1.11	13	-0.3319E 02	-0.3253E 02	1.98
14	-0.2346E-02	-0.2373E-02	1.12	14	-0.3116E 02	-0.3057E 02	1.90
15	-0.2337E-02	-0.2363E-02	1.14	15	-0.2836E 02	-0.2784E 02	1.83
16	-0.2325E-02	-0.2351E-02	1.15	16	-0.2479E 02	-0.2436E 02	1.73
17	-0.2310E-02	-0.2338E-02	1.17	17	-0.2051E 02	-0.2018E 02	1.59
18	-0.2295E-02	-0.2322E-02	1.19	18	-0.1562E 02	-0.1539E 02	1.46
19	-0.2277E-02	-0.2305E-02	1.21	19	-0.1028E 02	-0.1014E 02	1.33
20	-0.2259E-02	-0.2287E-02	1.24	20	-0.4754E 01	-0.4701E 01	1.12
21	-0.2241E-02	-0.2269E-02	1.26	21	0.	0.	0.

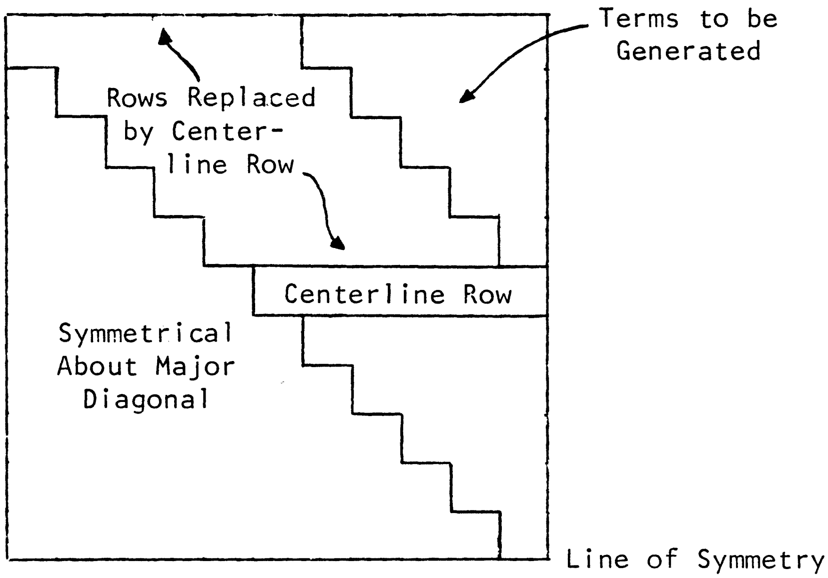

Figure 16. Interface Matrix Generated With Centerline Row

$$E_{20,41} = \sum_{i=1}^{41} \text{Exact } E_{20,i} - \sum_{i=1}^{40} \text{Approx. } E_{20,i}$$

The exact matrix [E] has the same Poisson's ratio as the approximate matrix.

4. Force symmetry about the minor diagonal:

$$E_{1,22} = E_{20,41}$$

5. Satisfy equilibrium on row 19. The row contains two unknowns, $E_{19,40}$ and $E_{19,41}$:

$$\sum_{i=1}^{41} \text{Exact } E_{19,i} - \sum_{i=1}^{39} \text{Approx. } E_{19,i} = E_{19,40} + E_{19,41}$$

Because values at the end of each row in the matrix become essentially constant (Figures 4 through 7), it is assumed that $E_{19,40} = E_{19,41}$.

6. Force symmetry about minor diagonal:

$$E_{1,23} = E_{19,41} \quad \text{and} \quad E_{2,23} = E_{19,40}$$

7. The process is continued for each successive row above the centerline row. Each row contains one or more unknown terms. When more than one unknown is involved, all the involved terms are assigned the same value for that row.

8. Each term which has been previously determined by forcing symmetry about the minor diagonal must not be changed. For example, row two has 19 unknown terms, after placement of centerline row and forcing symmetry about the major diagonals:

$E_{2,2}$ through $E_{2,22}$ = Centerline row

$E_{2,1} = E_{1,2}$ step 2

Therefore, $E_{2,23}$ through $E_{2,41}$ are unknown after forcing symmetry about the major diagonal. Terms $E_{2,23}$ through $E_{2,39}$ will be determined from forcing symmetry about minor diagonal. $E_{2,23} = E_{19,40}$, $E_{2,24} = E_{18,40}$, $E_{2,25} = E_{17,40}$ . . . $E_{2,39} = E_{3,40}$. Therefore, only terms $E_{2,40}$ and $E_{2,41}$ are unknown when approximate row 2 is forced to satisfy equilibrium.

9. Multiply the entire matrix by the desired modulus of elasticity. This 41 node approximated matrix can be used only with beams of a limited number of nodes. Beams with more than 21 nodes will have greater errors than the 2 percent error in moment reported above. This is a result of improper modeling of the outside rows of the interface matrix.

61 Node Foundation

By using a 61 node foundation, the troublesome interface end effects are moved further away from the end of the beam. Appendix C gives the centerline row for Poisson's ratio of 0.1, 0.2, 0.3, and 0.4. Appendices D through J give the results of the five load cases for Young's modulus of 7000 psi with Poisson's ratios of 0.2 and 0.3 for 21, 31, and 41 node beams. Table III gives a summary of the errors of maximum deflection, moment, and shear for two different Poisson's ratios and the five load cases. The 61 node interface matrix generated for these results was produced using the steps outlined for the 41 node generated interface.

Appendix D presents the results of the 21 node beam on a 61 node centerline row interface matrix for Poisson's ratio of 0.3. As seen in Table III, the percentage error of the maximum displacement for a uniform load over the entire beam is 0.91 percent. This percentage error is better than the 1.1 percent error reported for the same beam on a 41 node

TABLE III

SUMMARY OF RESULTS

Appendix	Poisson's Ratio	No. of Nodes for Beam Load Case	Percent Error at Maximum Displacement					Percent Error at Maximum Moment					Percent Error at Maximum Shear				
			1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
D	0.3	21	0.91	0.84	0.18	0.86	0.38	0.84	0.17	0.14	0.31	0.45	0.52	0.02	0.13	0.24	0.20
E	0.3	31	0.99	0.80	1.71	0.85	2.13	1.86	0.30	0.13	0.56	0.66	1.04	0.02	0.16	0.32	0.20
F	0.3	41	1.29	0.92	1.93	1.02	2.67	2.35	0.31	0.10	0.75	0.10	0.76	0.01	0.14	0.46	0.03
G	0.2	21	0.91	0.85	0.38	0.87	0.37	0.90	0.19	0.15	0.33	0.50	0.58	0.01	0.16	0.26	0.07
H	0.2	31	1.13	0.93	0.75	0.98	1.98	1.95	0.31	0.18	0.60	0.72	0.93	0.03	0.18	0.35	0.12
I	0.2	41	1.29	0.93	2.17	1.04	3.20	2.46	0.34	0.08	0.79	0.17	0.65	0.03	0.25	0.64	0.82

Young's modulus, 7000 psi.

foundation. Also, the percentage in the maximum moment has decreased from 2.0 percent error for a 41 node centerline row interface to 0.84 percent error for a 61 node centerline row interface. This reduction in error is due to the effect of the outside row of the approximate interface being further away from the end of the beam.

The shear diagrams in Appendices D through I have a stair-stepped shape for sections of the beam where loads have not been applied, and have a saw-toothed shape for sections which have been loaded with a uniformly distributed load. These discontinuities occur at each beam node and the magnitude of these discontinuities are the reactions produced by the foundation at that particular node. The elastic half space foundation has been discretized and is represented by the interface matrix which acts as a series of interconnected linear springs attached to each beam node. These discontinuities will become smaller as the node spacing of the foundation becomes smaller.

As seen in Table III, this method produces good results for beams with up to 41 nodes. Also, this table illustrates that as the number of nodes increases, so does the percentage error. The 61 node centerline row interface is limited to a maximum of approximately 41 nodes for two reasons. The first reason is demonstrated in Table III, which shows greater errors with the 41 nodes, particularly with unsymmetric loads near the edge of the beam (Load Case 5). Appendices F and I illustrate that for 41 node beams the deformed shape for Load Case 5 (uniform load over left half of the beam) is different from the correct deformed shape. The difference in this case is not constant; some displacements are above the correct value while others are below. The difference in the deformed shape for smaller beams for other load cases are in error by a constant displacement, rigid body rotation (Appendices D, E, G, and H). The second reason is the depth of foundation to the length of the beam becomes smaller with larger beams. The depth-to-length ratio for the 41 node beam is 3.5, which means the interface is still an approximation for an elastic half space. When the depth-to-length ratio becomes smaller, the assumption of an elastic half space is not valid and the interface will begin to be influenced by the boundary of the finite element grid.

Conclusion

This procedure illustrates that an approximate interface stiffness matrix can be developed from the centerline row of the exact matrix and the ratios required to satisfy equilibrium. Therefore, a 61 node interface matrix can be produced from 61 terms (31 term centerline row of matrix and 30 ratios). The procedure is more acceptable for inclusion with a beam-column program than the previous procedure because of the fewer terms required to produce the interface matrices. The procedure still has the limitation of not accounting for foundations with specific depths rather than the half space as used herein.

Generation of Matrix With Approximated Centerline Row

Introduction

It has been shown that the 61 node interface stiffness matrix generated from the centerline row can be used for beams up to 41 nodes and thus eliminates the need for developing interface stiffness matrices for each beam with a different number of nodes. The following procedure makes one additional approximation in that procedure. An equation is used to calculate the off diagonal terms of the centerline row of the interface stiffness matrix.

Figures 4 through 7 display the characteristic shape of the rows of each interface stiffness matrix. After examination of several equations, the one found to best fit the data is of the form:

$$y_{(stiffness)} = Ae^{-BX^C} + D$$

where X is the number of nodes from major diagonal, and B, C, and D are constant. An example of this fit is shown in Figure 17.

The above equation cannot be fitted to the stiffness values by conventional least squares method. Therefore, a sequential search method, described in Appendix J, was used to find the coefficients A, B, C, and D of the equation. The fit is accomplished by letting the sequential method, a simplex method by Nelder and Mead (23), vary A, B, C, and D until the sum of the squares of differences between the FEM stiffness values and the calculated stiffness values becomes a minimum.

Procedure

This procedure, the coefficient method, is identical to the centerline row method except for the approximation of the off diagonal terms of the centerline row. By knowing the general shape of the variation of the off diagonal terms ($y = Ae^{-BX^C} + D$), the following procedure can be used to develop an approximate 61 node interface stiffness matrix:

1. Make all major diagonal terms a single value, i.e., the first term of the centerline row listed in Appendix C. As with the centerline

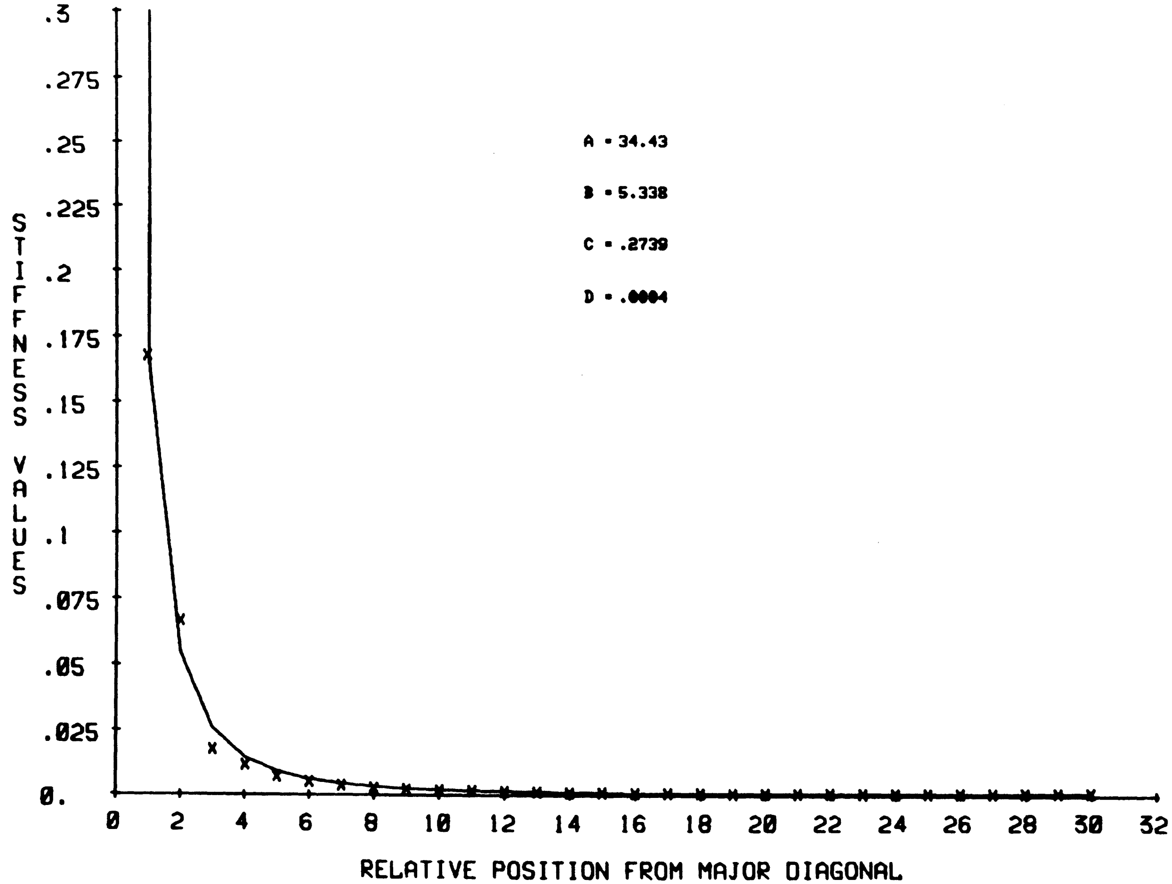

Figure 17. Fit of Curve to Centerline Row of K*

row procedure, the selection of the major diagonal term depends only on the desired Poisson's ratio.

2. Use the equation ($y = Ae^{-BX^C} + D$) to calculate the first 30 off diagonal terms. The values for the coefficients A, B, C, and D for Poisson's ratios of 0.1, 0.2, 0.3, and 0.4 are given in Table IV.

3. Force symmetry about the major diagonal.

4. Force every row of the generated stiffness matrix to satisfy equilibrium according to the sums in Appendix B. The sums chosen from this appendix depend on the value of Poisson's ratio. The procedure was described in detail for the 41 node matrix.

5. Construct the matrix to be symmetrical about the minor diagonal. The procedure for forcing symmetry was also previously described for the 41 node interface.

6. Multiply the entire matrix by the desired modulus of elasticity.

Appendices K through M give the results of the five load cases for a Poisson's ratio of 0.3 and a Young's modulus of 7000 psi for 21, 31, and 41 node beams using the procedure described above. Table V gives a summary of the errors in maximum deflection, moment, and shear using Poisson's ratios, F.3, and the five load cases.

These results demonstrate the same characteristics as the results from the centerline row method. The errors increase as the number of beam nodes (length of beam) increase. This increase in error for this procedure is a result of the end effects from interface matrices as was the reason for the centerline row method. However, the errors in the results for the coefficient method are greater than those from the previous results, since the procedure has an added assumption of the approximation of the off diagonal terms.

TABLE IV

COEFFICIENTS OF EXPONENTIAL EQUATIONS

Poisson's Ratio	Coefficients A	B	C	D
0.1	34.26	5.381	0.2793	0.0004
0.2	34.22	5.376	0.2768	0.0004
0.3	34.43	5.338	0.2739	0.0004
0.4	34.41	5.322	0.2440	0.0007

TABLE V

SUMMARY OF RESULTS

Appendix	Poisson's Ratio	No. of Nodes for Beam Load Case	Percent Error at Maximum Displacement					Percent Error at Maximum Moment					Percent Error at Maximum Shear				
			1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
J	0.3	21	3.90	3.73	1.15	3.78	1.12	6.69	0.70	0.76	1.56	3.56	1.80	0.21	1.20	1.53	0.03
K	0.3	31	4.18	3.76	2.28	3.88	0.42	9.30	0.70	1.96	1.80	4.97	3.02	0.17	0.83	0.84	1.09
L	0.3	41	4.17	3.75	0.09	3.92	1.49	12.89	1.12	1.37	3.27	6.37	4.39	0.14	0.61	1.31	3.10

Conclusion

This procedure illustrates that an interface stiffness matrix can be developed from 36 terms ([four coefficients, A, B, C, and D], 31 sums of rows, and the major diagonal term of centerline row). These 36 terms are dependent only on Poisson's ratio and the modulus of elasticity. This procedure produces sufficiently accurate results for preliminary designs with 25 fewer terms by assuming an equation for the off diagonal terms. This procedure could easily be included into a beam-column analysis computer program for analysis and design of slabs on grade and U-frame structures. The procedure has the same limitation as the previous procedure of not accounting for foundations with specific depths instead of the results of a half space solution.

CHAPTER V

SUMMARY, CONCLUSION, AND RECOMMENDATIONS

Summary

The intent of this research was to investigate the soil-structure interaction for a beam-column structure on a continuous linearly elastic subgrade, and to retain the simplicity of the beam-column analysis while providing a foundation model which includes the two-dimensional behavior of the foundation. The goal was to develop a simple method to produce a two-dimensional foundation behavior which can be used for preliminary design and analysis of slabs on grade and U-frame structures.

This research showed that the foundation could be investigated and formed independent of the structural beam-column matrix. Also, it was shown that the interface matrix, which represents a semi-infinite half space, could be generated from a few terms. The terms can be determined from known values of Poisson's ratio and the modulus of elasticity of the foundation. Two procedures to generate a soil-structure interaction stiffness matrix were illustrated.

Conclusion

The first procedure, which generates the interface stiffness matrix from the centerline row, produces the best results and has one less approximation. Since this procedure satisfies the goal of producing an easily constructed, two-dimensional foundation useful for preliminary design and analysis, it is preferable over the second procedure which approximates the off diagonal terms with an exponential equation. The second procedure produces acceptable results but has at this point little additional benefits other than requiring slightly fewer numerical values.

Recommendations

Both procedures developed an approximate semi-infinite half space. The effect of finite depths on this soil-structure interaction problem was not investigated. The effect for shallow depth is a common problem for foundations and needs to be investigated. The procedure which uses an equation to produce the off diagonal terms would be more beneficial if the terms of the equation could be related directly to depth, Poisson's ratio, and the modulus of elasticity. This goal could possibly be achieved by investigating other equations with more coefficients. Further studies based on the theory of elasticity solutions, such as Flamant's, could lead to finding the correct relationship between the coefficients of the equation and the physical properties of the foundation.

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APPENDIX A

INTERFACE STIFFNESS MATRICES

Poisson Ratio 0.2

-0.4687746	0.2014856	0.0801580	0.0289753	0.0200524	0.0138527	0.0107073	0.0088199	0.0077963	0.0082193	0.0113046
-0.5551036	0.1672307	0.0678919	0.0205690	0.0143566	0.0095844	0.0073999	0.0061676	0.0061966	0.0082193	
-0.5686661	0.1624127	0.0646176	0.0183880	0.0127684	0.0084279	0.0065749	0.0061676	0.0077963		
-0.5703274	0.1613199	0.0639394	0.0179566	0.0125569	0.0084279	0.0073999	0.0088199			
-0.5710193	0.1609287	0.0637420	0.0179566	0.0127684	0.0095844	0.0107073				
-0.5711841	0.1609287	0.0639394	0.0183880	0.0143566	0.0138527					

Poisson Ratio 0.3

-0.4966217	0.2130380	0.0852976	0.0305564	0.0207889	0.0144729	0.0111426	0.0091479	0.0080547	0.0084601	0.0115681
-0.5877406	0.1766444	0.0723771	0.0218514	0.0148399	0.0100300	0.0077026	0.0064041	0.0064124	0.0084601	
-0.6022470	0.1715333	0.0689623	0.0195554	0.0131726	0.0088129	0.0068396	0.0064041	0.0080547		
-0.6039966	0.1704024	0.0682524	0.0191044	0.0129469	0.0088129	0.0077026	0.0091479			
-0.6046983	0.1700019	0.0680526	0.0191044	0.0131726	0.0100300	0.0111426				
-0.6048700	0.1700019	0.0682524	0.0195554	0.0148399	0.0144729					

A1--11 Node Interface

-0.4697093	0.2008391	0.0795837	0.0283696	0.0193791	0.0130671	0.0097499	0.0075756	0.0061079	0.0050566
0.0042791	0.0036886	0.0032324	0.0028769	0.0026011	0.0023934	0.0022496	0.0021827	0.0022039	0.0026336
0.0039857									
-0.5555509	0.1668329	0.0674720	0.0201019	0.0138109	0.0089184	0.0065326	0.0049883	0.0039730	0.0032580
0.0027381	0.0023493	0.0020540	0.0018289	0.0016604	0.0015423	0.0014807	0.0014810	0.0017531	0.0026336
-0.5690201	0.1620386	0.0642007	0.0179004	0.0121723	0.0076494	0.0055139	0.0041496	0.0032686	0.0026570
0.0022187	0.0018960	0.0016557	0.0014780	0.0013526	0.0012811	0.0012663	0.0014810	0.0022039	
-0.5707233	0.1608783	0.0634220	0.0173227	0.0117269	0.0072940	0.0052236	0.0039084	0.0030659	0.0024854
0.0020736	0.0017744	0.0015569	0.0014034	0.0013116	0.0012811	0.0014807	0.0021827		
-0.5715127	0.1603496	0.0630309	0.0170223	0.0114881	0.0071003	0.0050641	0.0037761	0.0029560	0.0023951
0.0020014	0.0017210	0.0015247	0.0014034	0.0013526	0.0015423	0.0022496			
-0.5718656	0.1600897	0.0628326	0.0168660	0.0113630	0.0069989	0.0049820	0.0037104	0.0029051	0.0023590
0.0019816	0.0017210	0.0015569	0.0014780	0.0016604	0.0023934				
-0.5720557	0.1599457	0.0627204	0.0167776	0.0112930	0.0069441	0.0049406	0.0036819	0.0028897	0.0023590
0.0020014	0.0017744	0.0016557	0.0018289	0.0026011					
-0.5721636	0.1598631	0.0626567	0.0167290	0.0112571	0.0069200	0.0049280	0.0036819	0.0029051	0.0023951
0.0020736	0.0018960	0.0020540	0.0028769						
-0.5722254	0.1598171	0.0626236	0.0167071	0.0112461	0.0069200	0.0049406	0.0037104	0.0029560	0.0024854
0.0022187	0.0023493	0.0032324							
-0.5722576	0.1597964	0.0626134	0.0167071	0.0112571	0.0069441	0.0049820	0.0037761	0.0030659	0.0026570
0.0027381	0.0036886								
-0.5722676	0.1597964	0.0626236	0.0167290	0.0112930	0.0069989	0.0050641	0.0039084	0.0032686	0.0032580
0.0042791									

A2--21 Node Interface With Poisson Ratio of 0.2

```
-0.4975209 0.2124117 0.0847381 0.0299631 0.0201257 0.0136959 0.0101914 0.0079113 0.0063659 0.0052590
 0.0044394 0.0038163 0.0033337 0.0029567 0.0026626 0.0024387 0.0022801 0.0021971 0.0022043 0.0026160
 0.0039241
-0.5881771 0.1762541 0.0719629 0.0213879 0.0142961 0.0093637 0.0068347 0.0052166 0.0041467 0.0033936
 0.0028450 0.0024343 0.0021214 0.0018821 0.0017017 0.0015731 0.0015007 0.0014923 0.0017557 0.0026160
-0.6025963 0.1711623 0.0685466 0.0190671 0.0125731 0.0080301 0.0057659 0.0043383 0.0034106 0.0027669
 0.0023049 0.0019640 0.0017096 0.0015203 0.0013853 0.0013047 0.0012829 0.0014923 0.0022043
-0.6043910 0.1699599 0.0677317 0.0184640 0.0121084 0.0076599 0.0054639 0.0040879 0.0032003 0.0025893
 0.0021549 0.0018387 0.0016077 0.0014437 0.0013421 0.0013047 0.0015007 0.0021971
-0.6051956 0.1694159 0.0673303 0.0181557 0.0118639 0.0074617 0.0053011 0.0039530 0.0030887 0.0024979
 0.0020821 0.0017850 0.0015760 0.0014437 0.0013853 0.0015731 0.0022801
-0.6055621 0.1691464 0.0671246 0.0179940 0.0117343 0.0073570 0.0052164 0.0038853 0.0030364 0.0024607
 0.0020617 0.0017850 0.0016077 0.0015203 0.0017017 0.0024387
-0.6057590 0.1689974 0.0670087 0.0179026 0.0116621 0.0073006 0.0051739 0.0038560 0.0030207 0.0024607
 0.0020821 0.0018387 0.0017096 0.0018821 0.0026626
-0.6058706 0.1689120 0.0669429 0.0178524 0.0116251 0.0072757 0.0051609 0.0038560 0.0030364 0.0024979
 0.0021549 0.0019640 0.0021214 0.0029567
-0.6059344 0.1688644 0.0669087 0.0178300 0.0116137 0.0072757 0.0051739 0.0038853 0.0030887 0.0025893
 0.0023049 0.0024343 0.0033337
-0.6059677 0.1688431 0.0668981 0.0178300 0.0116251 0.0073006 0.0052164 0.0039530 0.0032003 0.0027669
 0.0028450 0.0038163
-0.6059781 0.1688431 0.0669087 0.0178524 0.0116621 0.0073570 0.0053011 0.0040879 0.0034106 0.0033936
 0.0044394
```

A3--21 Node Interface With Poisson Ratio of 0.3

-0.4698703	0.2007350	0.0794986	0.0282869	0.0192956	0.0129801	0.0096576	0.0074760	0.0059991	0.0049364
0.0041450	0.0035370	0.0030586	0.0026749	0.0023619	0.0021033	0.0018871	0.0017049	0.0015501	0.0014181
0.0013053	0.0012090	0.0011276	0.0010597	0.0010053	0.0009654	0.0009429	0.0009471	0.0009863	0.0012154
0.0018839									
-0.5556184	0.1667774	0.0674180	0.0200473	0.0137540	0.0088581	0.0064676	0.0049173	0.0038944	0.0031704
0.0026390	0.0022357	0.0019219	0.0016724	0.0014707	0.0013053	0.0011681	0.0010531	0.0009564	0.0008744
0.0008053	0.0007470	0.0006987	0.0006600	0.0006314	0.0006146	0.0006154	0.0006391	0.0007857	0.0012154
-0.5690657	0.1619941	0.0641559	0.0178536	0.0121226	0.0075957	0.0054551	0.0040849	0.0031963	0.0025753
0.0021250	0.0017871	0.0015266	0.0013214	0.0011569	0.0010229	0.0009126	0.0008211	0.0007446	0.0006806
0.0006271	0.0005831	0.0005479	0.0005216	0.0005054	0.0005041	0.0005217	0.0006391	0.0009863	
-0.5707666	0.1608344	0.0633763	0.0172741	0.0116746	0.0072367	0.0051603	0.0038379	0.0029860	0.0023940
0.0019673	0.0016484	0.0014039	0.0012121	0.0010590	0.0009351	0.0008336	0.0007499	0.0006804	0.0006230
0.0005760	0.0005383	0.0005100	0.0004921	0.0004889	0.0005041	0.0006154	0.0009471		
-0.5715571	0.1603033	0.0629817	0.0169691	0.0114303	0.0070363	0.0049926	0.0036951	0.0028631	0.0022873
0.0018736	0.0015659	0.0013306	0.0011469	0.0010007	0.0008830	0.0007871	0.0007086	0.0006441	0.0005916
0.0005497	0.0005181	0.0004974	0.0004921	0.0005054	0.0006146	0.0009429			
-0.5719137	0.1600384	0.0627771	0.0168056	0.0112961	0.0069241	0.0048974	0.0036134	0.0027924	0.0022254
0.0018193	0.0015179	0.0012881	0.0011093	0.0009677	0.0008540	0.0007620	0.0006873	0.0006269	0.0005789
0.0005424	0.0005181	0.0005100	0.0005216	0.0006314	0.0009654				
-0.5721103	0.1598869	0.0626560	0.0167064	0.0112134	0.0068541	0.0048374	0.0035617	0.0027473	0.0021860
0.0017847	0.0014876	0.0012619	0.0010864	0.0009481	0.0008377	0.0007490	0.0006779	0.0006216	0.0005789
0.0005497	0.0005383	0.0005479	0.0006600	0.0010053					
-0.5722271	0.1597936	0.0625799	0.0166430	0.0111599	0.0068084	0.0047981	0.0035277	0.0027179	0.0021604
0.0017627	0.0014687	0.0012457	0.0010733	0.0009377	0.0008303	0.0007450	0.0006779	0.0006269	0.0005916
0.0005760	0.0005831	0.0006987	0.0010597						
-0.5723013	0.1597331	0.0625296	0.0166007	0.0111240	0.0067779	0.0047719	0.0035051	0.0026984	0.0021440
0.0017489	0.0014574	0.0012370	0.0010671	0.0009344	0.0008303	0.0007490	0.0006873	0.0006441	0.0006230
0.0006271	0.0007470	0.0011276							
-0.5723506	0.1596924	0.0624956	0.0165720	0.0110996	0.0067570	0.0047543	0.0034903	0.0026863	0.0021341
0.0017413	0.0014523	0.0012343	0.0010671	0.0009377	0.0008377	0.0007620	0.0007086	0.0006804	0.0006806
0.0008053	0.0012090								
-0.5723840	0.1596646	0.0624723	0.0165524	0.0110831	0.0067433	0.0047430	0.0034813	0.0026794	0.0021294
0.0017390	0.0014523	0.0012370	0.0010733	0.0009481	0.0008540	0.0007871	0.0007499	0.0007446	0.0008744
0.0013053									
-0.5724070	0.1596454	0.0624564	0.0165393	0.0110726	0.0067350	0.0047367	0.0034771	0.0026773	0.0021294
0.0017413	0.0014574	0.0012457	0.0010864	0.0009677	0.0008830	0.0008336	0.0008211	0.0009564	0.0014181
-0.5724227	0.1596327	0.0624461	0.0165313	0.0110666	0.0067310	0.0047347	0.0034771	0.0026794	0.0021341
0.0017489	0.0014687	0.0012619	0.0011093	0.0010007	0.0009351	0.0009126	0.0010531	0.0015501	
-0.5724327	0.1596249	0.0624404	0.0165276	0.0110647	0.0067310	0.0047367	0.0034813	0.0026863	0.0021440
0.0017627	0.0014876	0.0012881	0.0011469	0.0010590	0.0010229	0.0011681	0.0017049		
-0.5724384	0.1596211	0.0624386	0.0165276	0.0110666	0.0067350	0.0047430	0.0034903	0.0026984	0.0021604
0.0017847	0.0015179	0.0013306	0.0012121	0.0011569	0.0013053	0.0018871			
-0.5724403	0.1596211	0.0624404	0.0165313	0.0110726	0.0067433	0.0047543	0.0035051	0.0027179	0.0021860
0.0018193	0.0015659	0.0014039	0.0013214	0.0014707	0.0021033				

A4--31 Node Interface With Poisson Ratio of 0.2

```
-0.4976613 0.2123200 0.0846626 0.0298893 0.0200504 0.0136170 0.0101073 0.0078200 0.0062656 0.0051477
 0.0043146 0.0036744 0.0031707 0.0027664 0.0024366 0.0021639 0.0019356 0.0017429 0.0015790 0.0014387
 0.0013184 0.0012151 0.0011270 0.0010526 0.0009916 0.0009447 0.0009141 0.0009077 0.0009343 0.0011356
 0.0017324
-0.5882374 0.1762043 0.0719141 0.0213381 0.0142440 0.0093080 0.0067743 0.0051503 0.0040731 0.0033110
 0.0027513 0.0023264 0.0019957 0.0017327 0.0015200 0.0013453 0.0012003 0.0010787 0.0009760 0.0008889
 0.0008147 0.0007520 0.0006996 0.0006566 0.0006236 0.0006020 0.0005966 0.0006131 0.0007446 0.0011356
-0.6026376 0.1711217 0.0685051 0.0190237 0.0125267 0.0079799 0.0057106 0.0042770 0.0033417 0.0026887
 0.0022150 0.0018593 0.0015850 0.0013689 0.0011954 0.0010543 0.0009379 0.0008410 0.0007600 0.0006919
 0.0006347 0.0005871 0.0005486 0.0005189 0.0004991 0.0004931 0.0005057 0.0006131 0.0009343
-0.6044309 0.1699191 0.0676890 0.0184184 0.0120589 0.0076054 0.0054034 0.0040200 0.0031234 0.0025010
 0.0020517 0.0017161 0.0014587 0.0012566 0.0010953 0.0009646 0.0008573 0.0007687 0.0006950 0.0006337
 0.0005831 0.0005423 0.0005109 0.0004896 0.0004823 0.0004931 0.0005966 0.0009077
-0.6052371 0.1693721 0.0672836 0.0181051 0.0118083 0.0074000 0.0052319 0.0038743 0.0029983 0.0023923
 0.0019567 0.0016324 0.0013846 0.0011909 0.0010367 0.0009123 0.0008109 0.0007276 0.0006590 0.0006027
 0.0005574 0.0005226 0.0004989 0.0004896 0.0004991 0.0006020 0.0009141
-0.6056080 0.1690973 0.0670713 0.0179356 0.0116694 0.0072841 0.0051337 0.0037901 0.0029254 0.0023287
 0.0019010 0.0015834 0.0013413 0.0011527 0.0010031 0.0008830 0.0007854 0.0007061 0.0006416 0.0005900
 0.0005501 0.0005226 0.0005109 0.0005189 0.0006236 0.0009447
-0.6058116 0.1689404 0.0669460 0.0178331 0.0115840 0.0072120 0.0050720 0.0037370 0.0028791 0.0022884
 0.0018657 0.0015526 0.0013144 0.0011294 0.0009833 0.0008664 0.0007724 0.0006966 0.0006363 0.0005900
 0.0005574 0.0005423 0.0005486 0.0006566 0.0009916
-0.6059324 0.1688440 0.0668674 0.0177677 0.0115290 0.0071650 0.0050316 0.0037021 0.0028490 0.0022623
 0.0018433 0.0015333 0.0012981 0.0011160 0.0009727 0.0008590 0.0007683 0.0006966 0.0006416 0.0006027
 0.0005831 0.0005871 0.0006996 0.0010526
-0.6060091 0.1687816 0.0668156 0.0177241 0.0114920 0.0071336 0.0050046 0.0036790 0.0028293 0.0022456
 0.0018291 0.0015219 0.0012893 0.0011099 0.0009694 0.0008590 0.0007724 0.0007061 0.0006590 0.0006337
 0.0006347 0.0007520 0.0011270
-0.6060599 0.1687397 0.0667806 0.0176947 0.0114670 0.0071123 0.0049866 0.0036639 0.0028167 0.0022354
 0.0018214 0.0015166 0.0012866 0.0011099 0.0009727 0.0008664 0.0007854 0.0007276 0.0006950 0.0006919
 0.0008147 0.0012151
-0.6060943 0.1687110 0.0667566 0.0176746 0.0114501 0.0070981 0.0049751 0.0036547 0.0028097 0.0022307
 0.0018190 0.0015166 0.0012893 0.0011160 0.0009833 0.0008830 0.0008109 0.0007687 0.0007600 0.0008889
 0.0013184
-0.6061180 0.1686914 0.0667403 0.0176611 0.0114393 0.0070896 0.0049687 0.0036503 0.0028076 0.0022307
 0.0018214 0.0015219 0.0012981 0.0011294 0.0010031 0.0009123 0.0008573 0.0008410 0.0009760 0.0014387
-0.6061340 0.1686783 0.0667299 0.0176529 0.0114331 0.0070856 0.0049666 0.0036503 0.0028097 0.0022354
 0.0018291 0.0015333 0.0013144 0.0011527 0.0010367 0.0009646 0.0009379 0.0010787 0.0015790
-0.6061444 0.1686703 0.0667240 0.0176490 0.0114311 0.0070856 0.0049687 0.0036547 0.0028167 0.0022456
 0.0018433 0.0015526 0.0013413 0.0011909 0.0010953 0.0010543 0.0012003 0.0017429
-0.6061503 0.1686664 0.0667220 0.0176490 0.0114331 0.0070896 0.0049751 0.0036639 0.0028293 0.0022623
 0.0018657 0.0015834 0.0013846 0.0012566 0.0011954 0.0013453 0.0019356
-0.6061521 0.1686664 0.0667240 0.0176529 0.0114393 0.0070981 0.0049866 0.0036790 0.0028490 0.0022884
 0.0019010 0.0016324 0.0014587 0.0013689 0.0015200 0.0021639
```

A5--31 Node Interface With Poisson Ratio of 0.3

-0.4699266	0.2007033	0.0794759	0.0282667	0.0192767	0.0129616	0.0096386	0.0074566	0.0059786	0.0049147
0.0041216	0.0035117	0.0030314	0.0026451	0.0023294	0.0020676	0.0018477	0.0016611	0.0015013	0.0013631
0.0012430	0.0011379	0.0010451	0.0009633	0.0008906	0.0008257	0.0007679	0.0007160	0.0006696	0.0006281
0.0005913	0.0005587	0.0005303	0.0005061	0.0004867	0.0004729	0.0004663	0.0004721	0.0004944	0.0006114
0.0009486									
-0.5556373	0.1667634	0.0674053	0.0200351	0.0137419	0.0088457	0.0064547	0.0049037	0.0038801	0.0031550
0.0026224	0.0022179	0.0019026	0.0016513	0.0014476	0.0012797	0.0011397	0.0010214	0.0009207	0.0008341
0.0007591	0.0006939	0.0006366	0.0005860	0.0005413	0.0005017	0.0004664	0.0004351	0.0004071	0.0003826
0.0003609	0.0003420	0.0003260	0.0003131	0.0003041	0.0002997	0.0003034	0.0003180	0.0003936	0.0006114
-0.5690767	0.1619839	0.0641459	0.0178436	0.0121123	0.0075851	0.0054440	0.0040730	0.0031836	0.0025617
0.0021104	0.0017711	0.0015093	0.0013024	0.0011359	0.0009997	0.0008869	0.0007921	0.0007119	0.0006431
0.0005840	0.0005327	0.0004879	0.0004486	0.0004140	0.0003836	0.0003566	0.0003327	0.0003117	0.0002933
0.0002774	0.0002640	0.0002533	0.0002457	0.0002420	0.0002450	0.0002566	0.0003180	0.0004944	
-0.5707763	0.1608250	0.0633669	0.0172644	0.0116643	0.0072260	0.0051489	0.0038256	0.0029729	0.0023799
0.0019519	0.0016317	0.0013856	0.0011920	0.0010369	0.0009103	0.0008059	0.0007184	0.0006446	0.0005817
0.0005276	0.0004809	0.0004401	0.0004046	0.0003733	0.0003460	0.0003219	0.0003007	0.0002823	0.0002666
0.0002533	0.0002426	0.0002350	0.0002313	0.0002339	0.0002450	0.0003034	0.0004721		
-0.5715664	0.1602939	0.0629719	0.0169590	0.0114196	0.0070249	0.0049804	0.0036821	0.0028491	0.0022720
0.0018570	0.0015476	0.0013106	0.0011247	0.0009761	0.0008554	0.0007559	0.0006730	0.0006030	0.0005436
0.0004927	0.0004487	0.0004106	0.0003774	0.0003484	0.0003231	0.0003011	0.0002820	0.0002656	0.0002517
0.0002407	0.0002329	0.0002289	0.0002313	0.0002420	0.0002997	0.0004663			
-0.5719234	0.1600284	0.0627667	0.0167946	0.0112844	0.0069117	0.0048840	0.0035991	0.0027769	0.0022084
0.0018007	0.0014974	0.0012656	0.0010841	0.0009396	0.0008221	0.0007257	0.0006454	0.0005779	0.0005206
0.0004717	0.0004296	0.0003931	0.0003616	0.0003341	0.0003103	0.0002897	0.0002721	0.0002574	0.0002457
0.0002373	0.0002329	0.0002350	0.0002457	0.0003041	0.0004729				
-0.5721206	0.1598760	0.0626446	0.0166943	0.0112006	0.0068403	0.0048226	0.0035456	0.0027297	0.0021667
0.0017636	0.0014643	0.0012357	0.0010573	0.0009151	0.0008000	0.0007056	0.0006271	0.0005613	0.0005056
0.0004580	0.0004171	0.0003820	0.0003517	0.0003256	0.0003030	0.0002839	0.0002679	0.0002550	0.0002457
0.0002407	0.0002426	0.0002533	0.0003131	0.0004867					
-0.5722384	0.1597817	0.0625671	0.0166296	0.0111454	0.0067929	0.0047811	0.0035093	0.0026977	0.0021383
0.0017381	0.0014414	0.0012151	0.0010387	0.0008983	0.0007849	0.0006919	0.0006147	0.0005500	0.0004954
0.0004490	0.0004093	0.0003751	0.0003460	0.0003209	0.0002997	0.0002820	0.0002679	0.0002574	0.0002517
0.0002533	0.0002640	0.0003260	0.0005061						
-0.5723140	0.1597197	0.0625154	0.0165854	0.0111076	0.0067599	0.0047523	0.0034837	0.0026750	0.0021181
0.0017201	0.0014251	0.0012006	0.0010256	0.0008864	0.0007741	0.0006823	0.0006061	0.0005426	0.0004889
0.0004433	0.0004046	0.0003714	0.0003433	0.0003194	0.0002997	0.0002839	0.0002721	0.0002656	0.0002666
0.0002774	0.0003420	0.0005303							
-0.5723647	0.1596773	0.0624793	0.0165544	0.0110806	0.0067363	0.0047316	0.0034654	0.0026587	0.0021034
0.0017070	0.0014134	0.0011900	0.0010161	0.0008781	0.0007667	0.0006757	0.0006004	0.0005376	0.0004849
0.0004401	0.0004023	0.0003703	0.0003433	0.0003209	0.0003030	0.0002897	0.0002820	0.0002823	0.0002933
0.0003609	0.0005587								

A6--41 Node Interface With Poisson Ratio of 0.2

-0.5724001	0.1596471	0.0624534	0.0165320	0.0110610	0.0067191	0.0047163	0.0034519	0.0026467	0.0020927
0.0016974	0.0014049	0.0011824	0.0010094	0.0008723	0.0007617	0.0006714	0.0005970	0.0005349	0.0004829
0.0004391	0.0004023	0.0003714	0.0003460	0.0003256	0.0003103	0.0003011	0.0003007	0.0003117	0.0003826
0.0005913									
-0.5724257	0.1596253	0.0624346	0.0165156	0.0110466	0.0067063	0.0047050	0.0034419	0.0026377	0.0020849
0.0016904	0.0013989	0.0011771	0.0010049	0.0008684	0.0007586	0.0006690	0.0005953	0.0005340	0.0004829
0.0004401	0.0004046	0.0003751	0.0003517	0.0003341	0.0003231	0.0003219	0.0003327	0.0004071	0.0006281
-0.5724444	0.1596091	0.0624204	0.0165033	0.0110357	0.0066969	0.0046967	0.0034344	0.0026313	0.0020791
0.0016856	0.0013946	0.0011736	0.0010020	0.0008661	0.0007570	0.0006683	0.0005953	0.0005349	0.0004849
0.0004433	0.0004093	0.0003820	0.0003616	0.0003484	0.0003460	0.0003566	0.0004351	0.0006696	
-0.5724583	0.1595970	0.0624099	0.0164940	0.0110276	0.0066897	0.0046904	0.0034290	0.0026266	0.0020753
0.0016823	0.0013920	0.0011716	0.0010006	0.0008654	0.0007570	0.0006690	0.0005970	0.0005376	0.0004889
0.0004490	0.0004171	0.0003931	0.0003774	0.0003733	0.0003836	0.0004664	0.0007160		
-0.5724689	0.1595880	0.0624020	0.0164871	0.0110217	0.0066846	0.0046860	0.0034253	0.0026236	0.0020727
0.0016804	0.0013907	0.0011710	0.0010006	0.0008661	0.0007586	0.0006714	0.0006004	0.0005426	0.0004954
0.0004580	0.0004296	0.0004106	0.0004046	0.0004140	0.0005017	0.0007679			
-0.5724766	0.1595811	0.0623961	0.0164821	0.0110173	0.0066810	0.0046830	0.0034230	0.0026217	0.0020716
0.0016797	0.0013907	0.0011716	0.0010020	0.0008684	0.0007617	0.0006757	0.0006061	0.0005500	0.0005056
0.0004717	0.0004487	0.0004401	0.0004486	0.0005413	0.0008257				
-0.5724824	0.1595763	0.0623920	0.0164786	0.0110144	0.0066787	0.0046813	0.0034219	0.0026211	0.0020716
0.0016804	0.0013920	0.0011736	0.0010049	0.0008723	0.0007667	0.0006823	0.0006147	0.0005613	0.0005206
0.0004927	0.0004809	0.0004879	0.0005860	0.0008906					
-0.5724866	0.1595727	0.0623891	0.0164763	0.0110129	0.0066776	0.0046807	0.0034219	0.0026217	0.0020727
0.0016823	0.0013946	0.0011771	0.0010094	0.0008781	0.0007741	0.0006919	0.0006271	0.0005779	0.0005436
0.0005276	0.0005327	0.0006366	0.0009633						
-0.5724894	0.1595706	0.0623874	0.0164753	0.0110123	0.0066776	0.0046813	0.0034230	0.0026236	0.0020753
0.0016856	0.0013989	0.0011824	0.0010161	0.0008864	0.0007849	0.0007056	0.0006454	0.0006030	0.0005817
0.0005840	0.0006939	0.0010451							
-0.5724911	0.1595694	0.0623870	0.0164753	0.0110129	0.0066787	0.0046830	0.0034253	0.0026266	0.0020791
0.0016904	0.0014049	0.0011900	0.0010256	0.0008983	0.0008000	0.0007257	0.0006730	0.0006446	0.0006431
0.0007591	0.0011379								
-0.5724916	0.1595694	0.0623874	0.0164763	0.0110144	0.0066810	0.0046860	0.0034290	0.0026313	0.0020849
0.0016974	0.0014134	0.0012006	0.0010387	0.0009151	0.0008221	0.0007559	0.0007184	0.0007119	0.0008341
0.0012430									

A7--41 Node Interface With Poisson Ratio of 0.2 (Cont.)

-0.4977104	0.2122934	0.0846444	0.0298736	0.0200360	0.0136027	0.0100929	0.0078050	0.0062497	0.0051307
0.0042963	0.0036546	0.0031490	0.0027426	0.0024101	0.0021346	0.0019030	0.0017064	0.0015380	0.0013924
0.0012657	0.0011546	0.0010567	0.0009699	0.0008927	0.0008237	0.0007619	0.0007063	0.0006563	0.0006113
0.0005707	0.0005344	0.0005021	0.0004737	0.0004494	0.0004300	0.0004164	0.0004123	0.0004213	0.0005044
0.0007557									
-0.5882529	0.1761930	0.0719040	0.0213284	0.0142344	0.0092981	0.0067640	0.0051394	0.0040614	0.0032984
0.0027377	0.0023117	0.0019796	0.0017150	0.0015003	0.0013236	0.0011760	0.0010514	0.0009451	0.0008539
0.0007747	0.0007056	0.0006449	0.0005913	0.0005439	0.0005016	0.0004640	0.0004303	0.0004000	0.0003731
0.0003490	0.0003279	0.0003093	0.0002937	0.0002813	0.0002729	0.0002709	0.0002779	0.0003346	0.0005044
-0.6026463	0.1711134	0.0684971	0.0190156	0.0125183	0.0079710	0.0057013	0.0042671	0.0033311	0.0026773
0.0022026	0.0018457	0.0015701	0.0013524	0.0011773	0.0010340	0.0009151	0.0008154	0.0007309	0.0006586
0.0005961	0.0005420	0.0004946	0.0004530	0.0004163	0.0003839	0.0003550	0.0003293	0.0003066	0.0002864
0.0002687	0.0002534	0.0002407	0.0002307	0.0002241	0.0002230	0.0002294	0.0002779	0.0004213	
-0.6044387	0.1699114	0.0676811	0.0184101	0.0120501	0.0075963	0.0053937	0.0040096	0.0031121	0.0024887
0.0020384	0.0017016	0.0014426	0.0012389	0.0010754	0.0009423	0.0008323	0.0007403	0.0006626	0.0005961
0.0005391	0.0004897	0.0004467	0.0004090	0.0003759	0.0003466	0.0003209	0.0002981	0.0002780	0.0002606
0.0002456	0.0002331	0.0002234	0.0002171	0.0002164	0.0002230	0.0002709	0.0004123		
-0.6052449	0.1693641	0.0672751	0.0180963	0.0117990	0.0073901	0.0052211	0.0038629	0.0029859	0.0023787
0.0019419	0.0016161	0.0013666	0.0011707	0.0010143	0.0008871	0.0007823	0.0006949	0.0006210	0.0005583
0.0005046	0.0004580	0.0004177	0.0003824	0.0003516	0.0003246	0.0003009	0.0002801	0.0002620	0.0002467
0.0002339	0.0002241	0.0002179	0.0002171	0.0002241	0.0002729	0.0004164			
-0.6056163	0.1690887	0.0670623	0.0179259	0.0116591	0.0072731	0.0051217	0.0037773	0.0029113	0.0023134
0.0018841	0.0015647	0.0013206	0.0011294	0.0009770	0.0008534	0.0007517	0.0006670	0.0005957	0.0005353
0.0004836	0.0004390	0.0004003	0.0003667	0.0003376	0.0003120	0.0002899	0.0002706	0.0002543	0.0002409
0.0002306	0.0002241	0.0002234	0.0002307	0.0002813	0.0004300				
-0.6058206	0.1689309	0.0669360	0.0178223	0.0115724	0.0071994	0.0050584	0.0037221	0.0028630	0.0022707
0.0018461	0.0015309	0.0012901	0.0011021	0.0009523	0.0008310	0.0007314	0.0006486	0.0005791	0.0005201
0.0004699	0.0004267	0.0003894	0.0003570	0.0003290	0.0003049	0.0002840	0.0002664	0.0002519	0.0002409
0.0002339	0.0002331	0.0002407	0.0002937	0.0004494					
-0.6059424	0.1688333	0.0668560	0.0177554	0.0115157	0.0071507	0.0050160	0.0036850	0.0028303	0.0022416
0.0018201	0.0015076	0.0012693	0.0010833	0.0009353	0.0008157	0.0007176	0.0006361	0.0005679	0.0005101
0.0004610	0.0004189	0.0003826	0.0003514	0.0003246	0.0003016	0.0002823	0.0002664	0.0002543	0.0002467
0.0002456	0.0002534	0.0003093	0.0004737						
-0.6060206	0.1687694	0.0668026	0.0177101	0.0114767	0.0071169	0.0049863	0.0036589	0.0028071	0.0022210
0.0018017	0.0014911	0.0012544	0.0010699	0.0009233	0.0008049	0.0007080	0.0006276	0.0005603	0.0005036

A8--41 Node Interface With Poisson Ratio 0.3

0.0004553	0.0004141	0.0003789	0.0003487	0.0003231	0.0003016	0.0002840	0.0002706	0.0002620	0.0002606
0.0002687	0.0003279	0.0005021							
-0.6060729	0.1687256	0.0667654	0.0176783	0.0114491	0.0070927	0.0049651	0.0036401	0.0027904	0.0022061
0.0017884	0.0014791	0.0012437	0.0010604	0.0009149	0.0007974	0.0007013	0.0006219	0.0005554	0.0004996
0.0004521	0.0004120	0.0003777	0.0003487	0.0003246	0.0003049	0.0002899	0.0002801	0.0002780	0.0002864
0.0003490	0.0005344								
-0.6061094	0.1686947	0.0667389	0.0176553	0.0114290	0.0070751	0.0049496	0.0036263	0.0027781	0.0021951
0.0017787	0.0014706	0.0012361	0.0010537	0.0009090	0.0007923	0.0006970	0.0006183	0.0005527	0.0004976
0.0004511	0.0004120	0.0003789	0.0003514	0.0003290	0.0003120	0.0003009	0.0002981	0.0003066	0.0003731
0.0005707									
-0.6061356	0.1686723	0.0667196	0.0176384	0.0114143	0.0070620	0.0049380	0.0036161	0.0027691	0.0021873
0.0017717	0.0014644	0.0012307	0.0010491	0.0009051	0.0007891	0.0006946	0.0006167	0.0005519	0.0004976
0.0004521	0.0004141	0.0003826	0.0003570	0.0003376	0.0003246	0.0003209	0.0003293	0.0004000	0.0006113
-0.6061547	0.1686557	0.0667051	0.0176259	0.0114033	0.0070523	0.0049296	0.0036085	0.0027626	0.0021814
0.0017667	0.0014601	0.0012271	0.0010461	0.0009029	0.0007877	0.0006939	0.0006167	0.0005527	0.0004996
0.0004553	0.0004189	0.0003894	0.0003667	0.0003516	0.0003466	0.0003550	0.0004303	0.0006563	
-0.6061690	0.1686434	0.0666944	0.0176164	0.0113950	0.0070451	0.0049233	0.0036031	0.0027579	0.0021774
0.0017634	0.0014574	0.0012251	0.0010449	0.0009021	0.0007877	0.0006946	0.0006183	0.0005554	0.0005036
0.0004610	0.0004267	0.0004003	0.0003824	0.0003759	0.0003839	0.0004640	0.0007063		
-0.6061797	0.1686341	0.0666864	0.0176094	0.0113889	0.0070399	0.0049187	0.0035994	0.0027547	0.0021750
0.0017616	0.0014561	0.0012246	0.0010449	0.0009029	0.0007891	0.0006970	0.0006219	0.0005603	0.0005101
0.0004699	0.0004390	0.0004177	0.0004090	0.0004163	0.0005016	0.0007619			
-0.6061876	0.1686273	0.0666804	0.0176043	0.0113846	0.0070363	0.0049157	0.0035970	0.0027529	0.0021737
0.0017609	0.0014561	0.0012251	0.0010461	0.0009051	0.0007923	0.0007013	0.0006276	0.0005679	0.0005201
0.0004836	0.0004580	0.0004467	0.0004530	0.0005439	0.0008237				
-0.6061936	0.1686221	0.0666761	0.0176007	0.0113816	0.0070339	0.0049140	0.0035959	0.0027523	0.0021737
0.0017616	0.0014574	0.0012271	0.0010491	0.0009090	0.0007974	0.0007080	0.0006361	0.0005791	0.0005353
0.0005046	0.0004897	0.0004946	0.0005913	0.0008927					
-0.6061979	0.1686187	0.0666733	0.0175984	0.0113800	0.0070329	0.0049134	0.0035959	0.0027529	0.0021750
0.0017634	0.0014601	0.0012307	0.0010537	0.0009149	0.0008049	0.0007176	0.0006486	0.0005957	0.0005583
0.0005391	0.0005420	0.0006449	0.0009699						
-0.6062007	0.1686164	0.0666716	0.0175973	0.0113794	0.0070329	0.0049140	0.0035970	0.0027547	0.0021774
0.0017667	0.0014644	0.0012361	0.0010604	0.0009233	0.0008157	0.0007314	0.0006670	0.000621[illegible]	0.0005961
0.0005961	0.0007056	0.0010567							
-0.6062023	0.1686153	0.0666710	0.0175973	0.0113800	0.0070339	0.0049157	0.0035994	0.0027579	0.0021814
0.0017717	0.0014706	0.0012437	0.0010699	0.0009353	0.0008310	0.0007517	0.0006949	0.0006626	0.0006586
0.0007747	0.0011546								
-0.6062029	0.1686153	0.0666716	0.0175984	0.0113816	0.0070363	0.0049187	0.0036031	0.0027626	0.0021873
0.0017787	0.0014791	0.0012544	0.0010833	0.0009523	0.0008534	0.0007823	0.0007403	0.0007309	0.0008539
0.0012657									

A9--41 Node Interface Node With Poisson Ratio of 0.3 (Cont.)

APPENDIX B

SUMS OF RUNS TO SATISFY EQUILIBRIUM

POISSON'S RATIO = .2

SUM	FACTOR
-0.77403204E-01	0.27396167E 01
-0.46002001E-01	0.16281994E 01
-0.34124003E-01	0.12077884E 01
-0.30627002E-01	0.10840152E 01
-0.28772999E-01	0.10183944E 01
-0.28253297E-01	0.10000000E 01

* B1--11 Node Interface

POISSON'S RATIO = .3

SUM	FACTOR
-0.84094501E-01	0.27459428E 01
-0.49980605E-01	0.16320197E 01
-0.36970101E-01	0.12071869E 01
-0.33160296E-01	0.10827851E 01
-0.31189200E-01	0.10184228E 01
-0.30625001E-01	0.10000000E 01

* B2--11 Node Interface

POISSON'S RATIO = .2

SUM	FACTOR
-0.67653701E-01	0.37945027E 01
-0.39302405E-01	0.22043596E 01
-0.28219701E-01	0.15827624E 01
-0.24451503E-01	0.13714149E 01
-0.21975196E-01	0.12325259E 01
-0.20417603E-01	0.11451650E 01
-0.19358295E-01	0.10857515E 01
-0.18641996E-01	0.10455763E 01
-0.18177000E-01	0.10194960E 01
-0.17914302E-01	0.10047620E 01
-0.17829398E-01	0.10000000E 01

* B3--21 Node Interface

POISSON'S RATIO = .3

SUM	FACTOR
-0.73989805E-01	0.38009180E 01
-0.42986803E-01	0.22082680E 01
-0.30774900E-01	0.15809323E 01
-0.26647799E-01	0.13689197E 01
-0.23981994E-01	0.12319750E 01
-0.22285200E-01	0.11448093E 01
-0.21132303E-01	0.10855841E 01
-0.20352098E-01	0.10455043E 01
-0.19844897E-01	0.10194489E 01
-0.19559101E-01	0.10047673E 01
-0.19466298E-01	0.10000000E 01

* B4--21 Node Interface

POISSON'S RATIO = .2

SUM	FACTOR
-0.63916201E-01	0.45892756E 01
-0.36869300E-01	0.26472690E 01
-0.26221099E-01	0.18827128E 01
-0.22508696E-01	0.16161570E 01
-0.20012798E-01	0.14369479E 01
-0.18376198E-01	0.13194376E 01
-0.17193603E-01	0.12345256E 01
-0.16313897E-01	0.11713614E 01
-0.15642697E-01	0.11231682E 01
-0.15124901E-01	0.10859897E 01
-0.14725500E-01	0.10573122E 01
-0.14422403E-01	0.10355494E 01
-0.14200000E-01	0.10195805E 01
-0.14045894E-01	0.10085155E 01
-0.13957194E-01	0.10021467E 01
-0.13927296E-01	0.10000000E 01

* B5--31 Node Interface

POISSON'S RATIO = .3

SUM	FACTOR
-0.70285104E-01	0.45953288E 01
-0.40546500E-01	0.26509813E 01
-0.28752298E-01	0.18798615E 01
-0.24663899E-01	0.16125569E 01
-0.21960996E-01	0.14358376E 01
-0.20168197E-01	0.13186222E 01
-0.18874702E-01	0.12340518E 01
-0.17910402E-01	0.11710047E 01
-0.17174405E-01	0.11228843E 01
-0.16606999E-01	0.10857866E 01
-0.16169300E-01	0.10571693E 01
-0.15837301E-01	0.10354627E 01
-0.15592595E-01	0.10194635E 01
-0.15424595E-01	0.10084795E 01
-0.15327699E-01	0.10021443E 01
-0.15294902E-01	0.10000000E 01

* B6--31 Node Interface

POISSON'S RATIO = .2

SUM	FACTOR
-0.62147898E-01	0.52352692E 01
-0.35729698E-01	0.30098297E 01
-0.25600002E-01	0.21312438E 01
-0.21626901E-01	0.18218259E 01
-0.19140803E-01	0.16123998E 01
-0.17488998E-01	0.14732535E 01
-0.16276998E-01	0.13711561E 01
-0.15355998E-01	0.12935720E 01
-0.14632699E-01	0.12326422E 01
-0.14053903E-01	0.11838850E 01
-0.13582701E-01	0.11441915E 01
-0.13197502E-01	0.11117427E 01
-0.12879303E-01	0.10849381E 01
-0.12618998E-01	0.10630102E 01
-0.12405600E-01	0.10450338E 01
-0.12234002E-01	0.10305786E 01
-0.12098596E-01	0.10191721E 01
-0.11996478E-01	0.10106053E 01
-0.11925802E-01	0.10046162E 01
-0.11884502E-01	0.10011371E 01
-0.11871003E-01	0.10000000E 01

B7--41 Node Interface

POISSON'S RATIO = .3

SUM	FACTOR
-0.68685503E-01	0.52413678E 01
-0.39492804E-01	0.30136827E 01
-0.27882198E-01	0.21276812E 01
-0.23817701E-01	0.18175208E 01
-0.21110598E-01	0.16109427E 01
-0.19291699E-01	0.14721431E 01
-0.17959602E-01	0.13704912E 01
-0.16944000E-01	0.12929910E 01
-0.16148296E-01	0.12322711E 01
-0.15510401E-01	0.11835935E 01
-0.14951902E-01	0.11440270E 01
-0.14566499E-01	0.11115647E 01
-0.14216101E-01	0.10848259E 01
-0.13920999E-01	0.10629173E 01
-0.13694299E-01	0.10450073E 01
-0.13505098E-01	0.10305695E 01
-0.13356303E-01	0.10192150E 01
-0.13244599E-01	0.10106909E 01
-0.13166601E-01	0.10047389E 01
-0.13119897E-01	0.10011750E 01
-0.13104500E-01	0.10000000E 01

B8--41 Node Interface

POISSON'S RATIO = .1

SUM	FACTOR
-0.58260105E-01	0.62988669E 01
-0.32858204E-01	0.35525074E 01
-0.23354205E-01	0.25249702E 01
-0.19879703E-01	0.21493199E 01
-0.17515498E-01	0.18937108E 01
-0.15939900E-01	0.17233630E 01
-0.14771599E-01	0.15970506E 01
-0.13872196E-01	0.14998104E 01
-0.13156096E-01	0.14223885E 01
-0.12570994E-01	0.13591294E 01
-0.12084198E-01	0.13064988E 01
-0.11673501E-01	0.12620958E 01
-0.11323301E-01	0.12242334E 01
-0.11021498E-01	0.11916036E 01
-0.10760195E-01	0.11633525E 01
-0.10531797E-01	0.11386589E 01
-0.10331997E-01	0.11170573E 01
-0.10157101E-01	0.10981482E 01
-0.10003600E-01	0.10815522E 01
-0.98689959E-02	0.10669993E 01
-0.97513980E-02	0.10542851E 01
-0.96485981E-02	0.10431707E 01
-0.95601009E-02	0.10336027E 01
-0.94835011E-02	0.10253210E 01
-0.94196973E-02	0.10184228E 01
-0.93666017E-02	0.10126823E 01
-0.93240957E-02	0.10080867E 01
-0.92909007E-02	0.10044978E 01
-0.92674046E-02	0.10019575E 01
-0.92535006E-02	0.10004542E 01
-0.92492992E-02	0.10000000E 01

B9--61 Node Interface

POISSON'S RATIO = .2

SUM	FACTOR
-0.61039506E-01	0.62613580E 01
-0.34925701E-01	0.35826358E 01
-0.24608100E-01	0.25242689E 01
-0.20952693E-01	0.21493016E 01
-0.18467903E-01	0.18944149E 01
-0.16803899E-01	0.17237235E 01
-0.15571999E-01	0.15973567E 01
-0.14624101E-01	0.15001224E 01
-0.13868602E-01	0.14226242E 01
-0.13251396E-01	0.13593121E 01
-0.12738799E-01	0.13067304E 01
-0.12306004E-01	0.12623349E 01
-0.11936399E-01	0.12244212E 01
-0.11617803E-01	0.11917401E 01
-0.11342095E-01	0.11634583E 01
-0.11101401E-01	0.11387682E 01
-0.10891202E-01	0.11172062E 01
-0.10704494E-01	0.10982590E 01
-0.10544601E-01	0.10816523E 01
-0.10402895E-01	0.10671163E 01
-0.10278000E-01	0.10543047E 01
-0.10169901E-01	0.10432161E 01
-0.10076600E-01	0.10336454E 01
-0.99958020E-02	0.10253572E 01
-0.99260035E-02	0.10184025E 01
-0.98713987E-02	0.10126576E 01
-0.98273013E-02	0.10080726E 01
-0.97920989E-02	0.10044616E 01
-0.97674970E-02	0.10019379E 01
-0.97527999E-02	0.10004303E 01
-0.97486050E-02	0.10000000E 01

B10--61 Node Interface

POISSON'S RATIO = .3

SUM	FACTOR
-0.68132407E-01	0.62734147E 01
-0.38989005E-01	0.35899832E 01
-0.27380700E-01	0.25211275E 01
-0.23294598E-01	0.21448923E 01
-0.20561401E-01	0.18932282E 01
-0.18711304E-01	0.17228772E 01
-0.17342900E-01	0.15968789E 01
-0.16288803E-01	0.14998210E 01
-0.15449199E-01	0.14225130E 01
-0.14762896E-01	0.13593204E 01
-0.14192199E-01	0.13067724E 01
-0.13710599E-01	0.12624282E 01
-0.13299201E-01	0.12245480E 01
-0.12945098E-01	0.11919433E 01
-0.12637797E-01	0.11636480E 01
-0.12369993E-01	0.11389895E 01
-0.12135195E-01	0.11173701E 01
-0.11929700E-01	0.10984487E 01
-0.11749698E-01	0.10818747E 01
-0.11590598E-01	0.10672253E 01
-0.11452395E-01	0.10545000E 01
-0.11331901E-01	0.10434053E 01
-0.11227396E-01	0.10337828E 01
-0.11137497E-01	0.10255052E 01
-0.11061702E-01	0.10185262E 01
-0.10999301E-01	0.10127805E 01
-0.10948676E-01	0.10081210E 01
-0.10909701E-01	0.10045304E 01
-0.10882700E-01	0.10020443E 01
-0.10866202E-01	0.10005252E 01
-0.10860498E-01	0.10000000E 01

B11-61 Node Interface

POISSON'S RATIO = .4

SUM	FACTOR
-0.86405101E-01	0.64508367E 01
-0.47167201E-01	0.35214114E 01
-0.33568205E-01	0.25061369E 01
-0.28484597E-01	0.21266046E 01
-0.25231001E-01	0.18836974E 01
-0.22993705E-01	0.17166652E 01
-0.21332900E-01	0.15926728E 01
-0.20049705E-01	0.14968719E 01
-0.19025499E-01	0.14204067E 01
-0.18187402E-01	0.13578360E 01
-0.17489002E-01	0.13056949E 01
-0.16899697E-01	0.12616985E 01
-0.16395503E-01	0.12240564E 01
-0.15961201E-01	0.11916322E 01
-0.15583902E-01	0.11634638E 01
-0.15254998E-01	0.11389085E 01
-0.14966696E-01	0.11173844E 01
-0.14713902E-01	0.10985113E 01
-0.14491297E-01	0.10818921E 01
-0.14296098E-01	0.10673189E 01
-0.14125500E-01	0.10545823E 01
-0.13976597E-01	0.10434655E 01
-0.13847095E-01	0.10337972E 01
-0.13736195E-01	0.10255176E 01
-0.13642694E-01	0.10185370E 01
-0.13564898E-01	0.10127289E 01
-0.13502400E-01	0.10080629E 01
-0.13454899E-01	0.10045166E 01
-0.13420794E-01	0.10019703E 01
-0.13400995E-01	0.10004922E 01
-0.13394402E-01	0.10000000E 01

B12--61 Node Interface

APPENDIX C

CENTERLINE ROWS FOR 61 NODE INTERFACE

-0.56404965E 00	0.16103446E 00	0.58458386E-01	0.15996674E-01	0.10742534E-01	0.64344953E-02	0.45185527E-02
0.32922967E-02	0.25169003E-02	0.19836183E-02	0.16040107E-02	0.13236628E-02	0.11109951E-02	0.94591898E-03
0.81534584E-03	0.71041205E-03	0.62495845E-03	0.55460253E-03	0.49616803E-03	0.44732110E-03	0.40633504E-03
0.37193824E-03	0.34321996E-03	0.31958583E-03	0.30075588E-03	0.28688510E-03	0.27863818E-03	0.27888179E-03
0.28978657E-03	0.35464108E-03	0.55824179E-03				

Centerline Row--Poisson's Ratio of 0.1

-0.57252066E 00	0.15954030E 00	0.62357947E-01	0.16446164E-01	0.10983349E-01	0.66485856E-02	0.46520390E-02
0.33930236E-02	0.25926504E-02	0.20430340E-02	0.16515721E-02	0.13625006E-02	0.11431951E-02	0.97295993E-03
0.83829574E-03	0.73006310E-03	0.64190983E-03	0.56931557E-03	0.50900340E-03	0.45856463E-03	0.41621600E-03
0.38064253E-03	0.35090003E-03	0.32636774E-03	0.30674670E-03	0.29217100E-03	0.28332206E-03	0.28294496E-03
0.29331201E-03	0.36050987E-03	0.55861670E-03				

Centerline Row--Poisson's Ratio of 0.2

-0.60623068E 00	0.16858737E 00	0.66643267E-01	0.17569504E-01	0.11351780E-01	0.70051563E-02	0.48860573E-02
0.35685006E-02	0.27253194E-02	0.21468363E-02	0.17344057E-02	0.14297923E-02	0.11986187E-02	0.10191242E-02
0.87709209E-03	0.76289592E-03	0.66984123E-03	0.59316168E-03	0.52939954E-03	0.47601031E-03	0.43110663E-03
0.39329166E-03	0.36155490E-03	0.33521916E-03	0.31393544E-03	0.29777816E-03	0.28739340E-03	0.28519216E-03
0.29396814E-03	0.35925670E-03	0.55214670E-03				

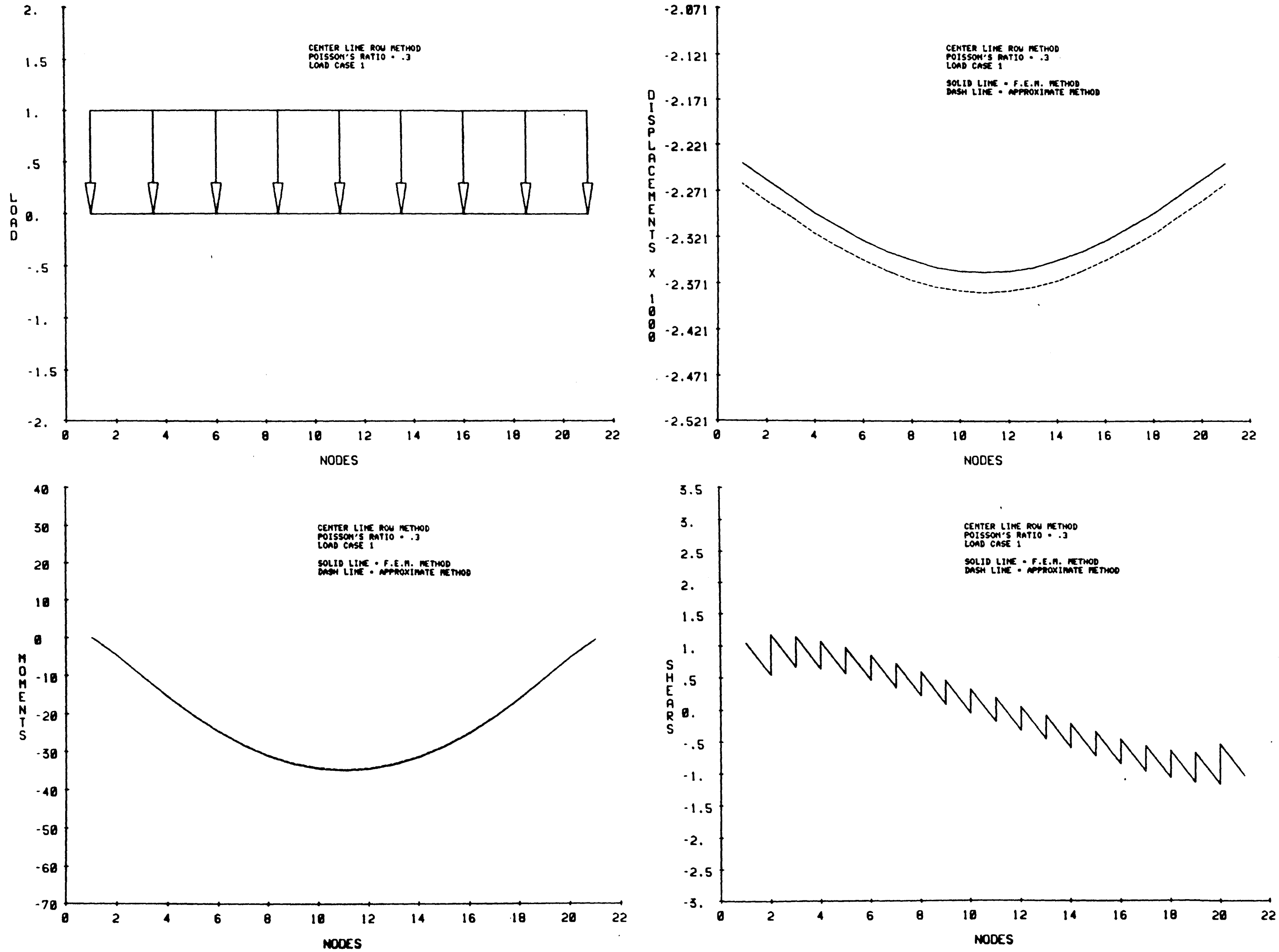
Centerline Row--Poisson's Ratio of 0.3

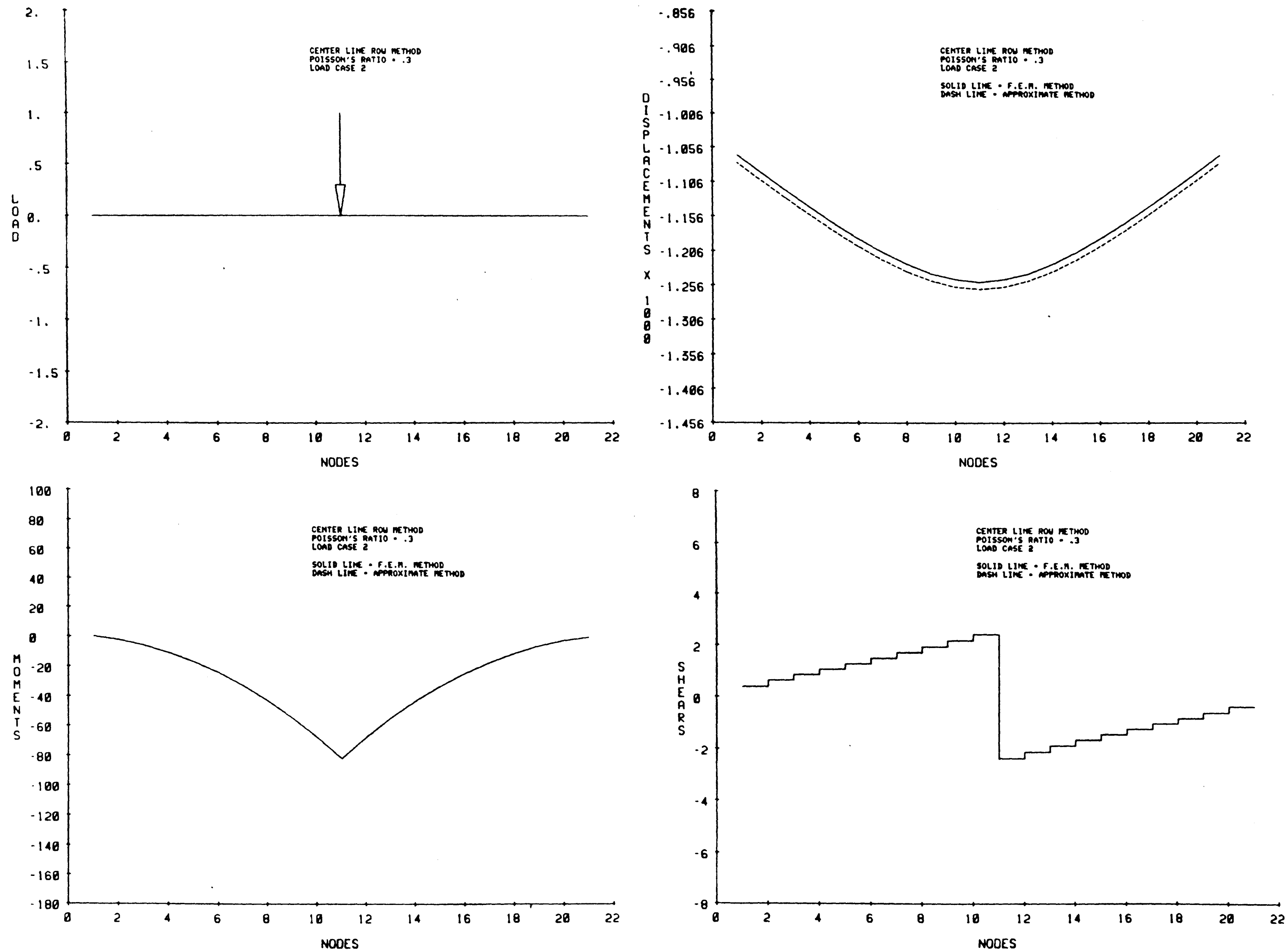
-0.70393959E 00	0.20941784E 00	0.68819424E-01	0.19880224E-01	0.11798353E-01	0.74582253E-02	0.52106690E-02
0.38194684E-02	0.29176806E-02	0.22973648E-02	0.18537494E-02	0.15255991E-02	0.12762382E-02	0.10824183E-02
0.92889124E-03	0.80531280E-03	0.70447465E-03	0.62123506E-03	0.55185527E-03	0.49357517E-03	0.44433658E-03
0.40260161E-03	0.36723604E-03	0.33744333E-03	0.31274534E-03	0.29303844E-03	0.27873184E-03	0.27150266E-03
0.27492553E-03	0.32354127E-03	0.50417816E-03				

Centerline Row--Poisson's Ratio of 0.4

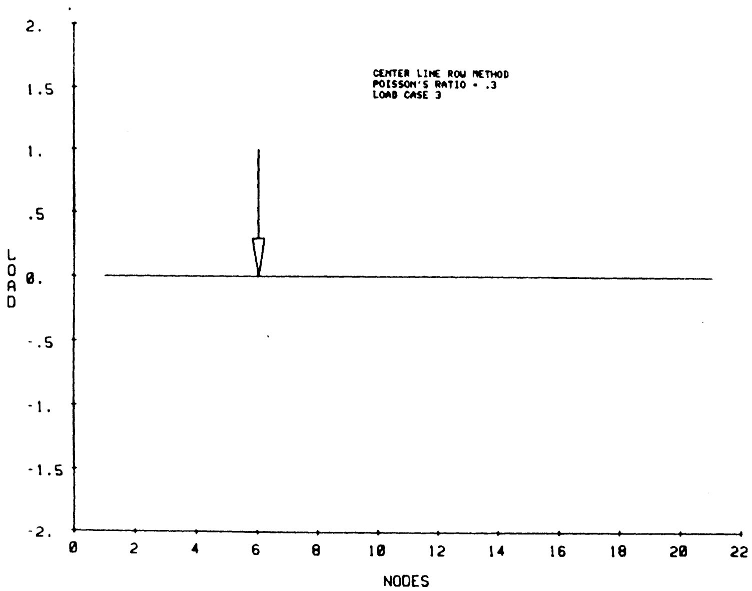
APPENDIX D

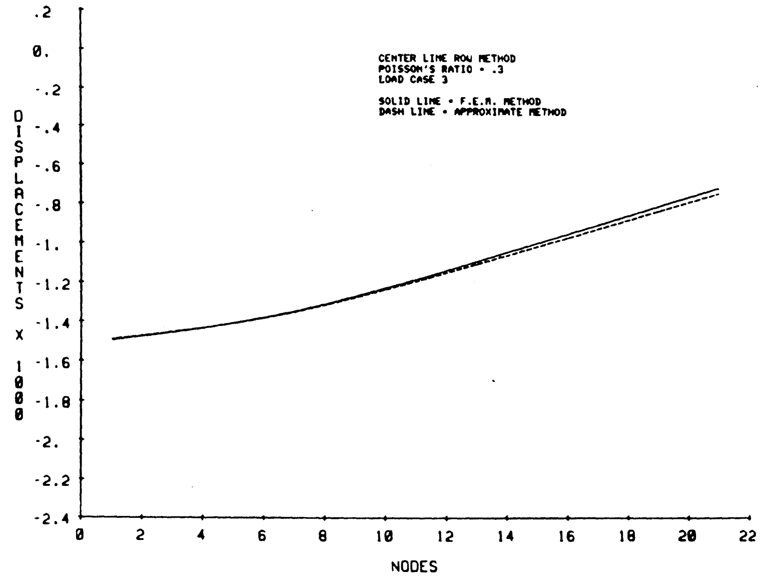
21 NODE BEAM ON FOUNDATION WITH POISSON'S RATIO OF 0.3 AND MODULUS OF ELASTICITY EQUAL TO 7000 PSI

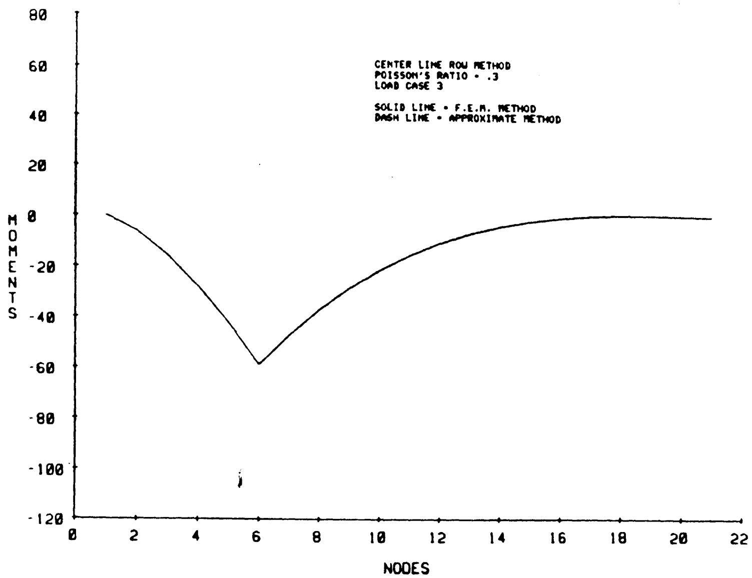

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 1
LOAD
NODES
DISPLACEMENTS X 1000
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
SHEARS

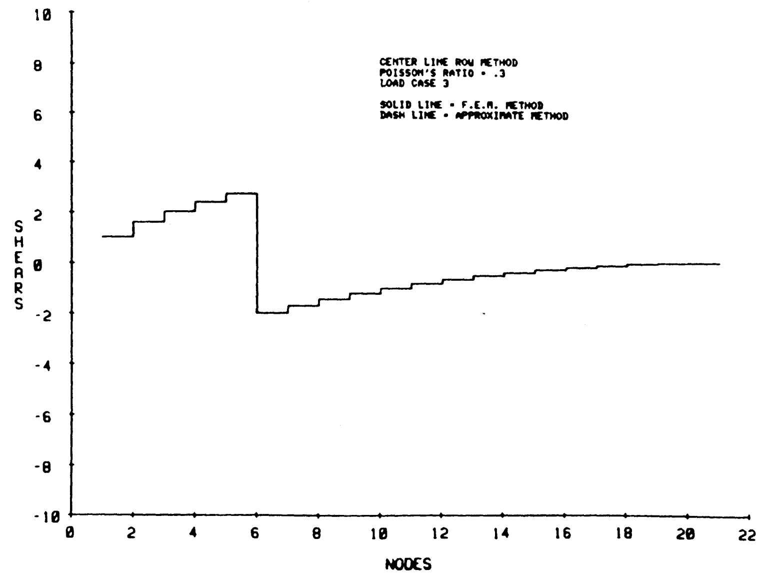


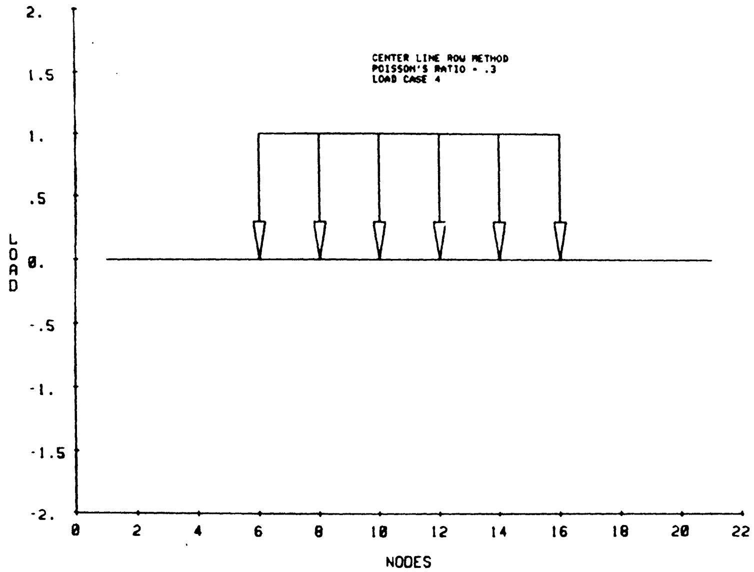
CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 2
LOAD
NODES
CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES
CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES
CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES

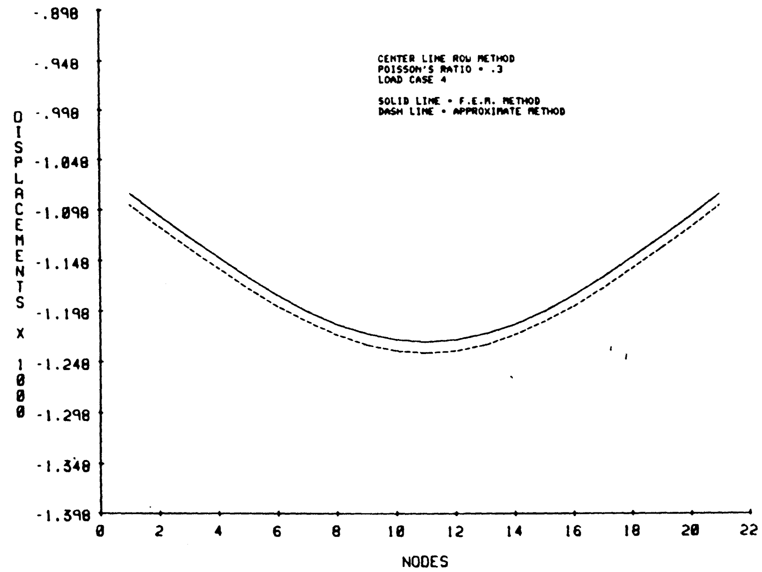

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 3
LOAD
NODES

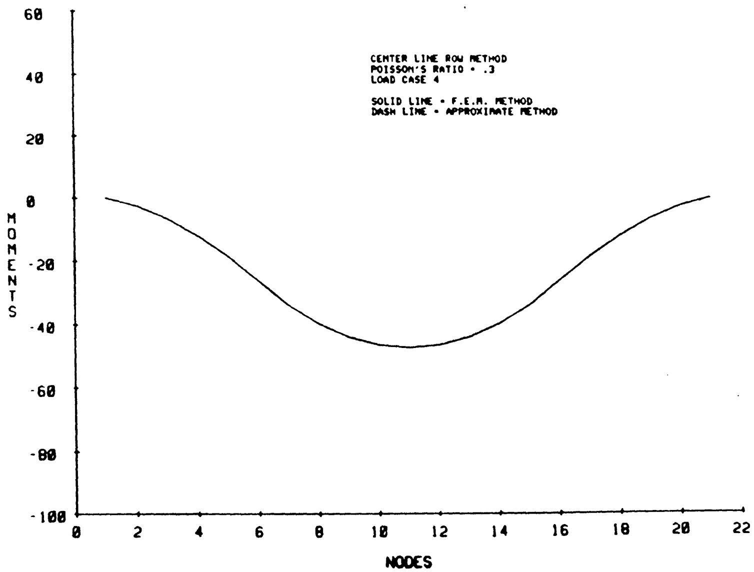

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES

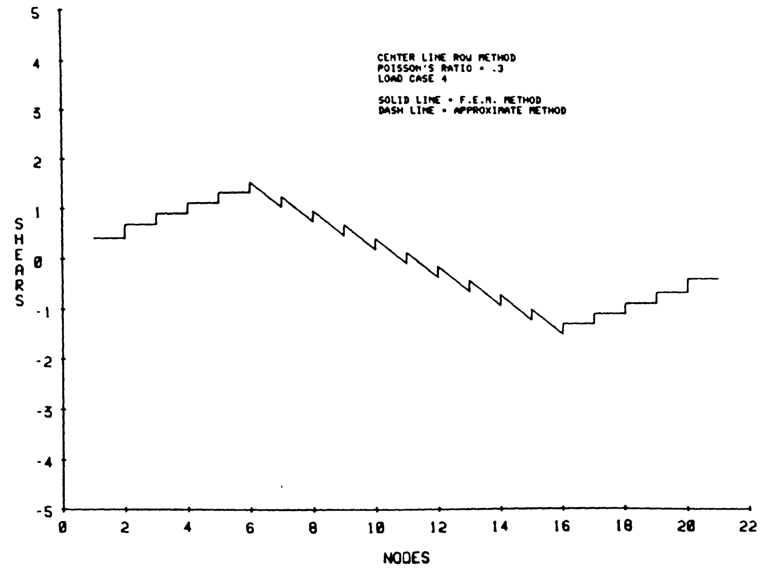

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES

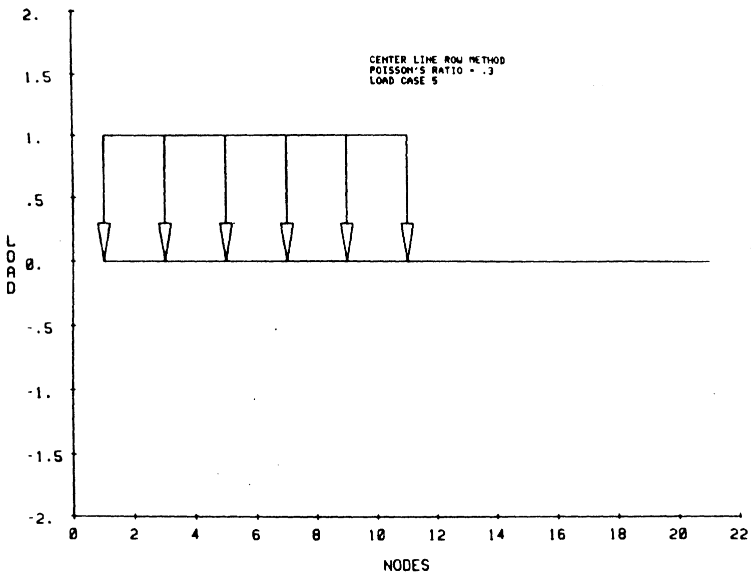

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES

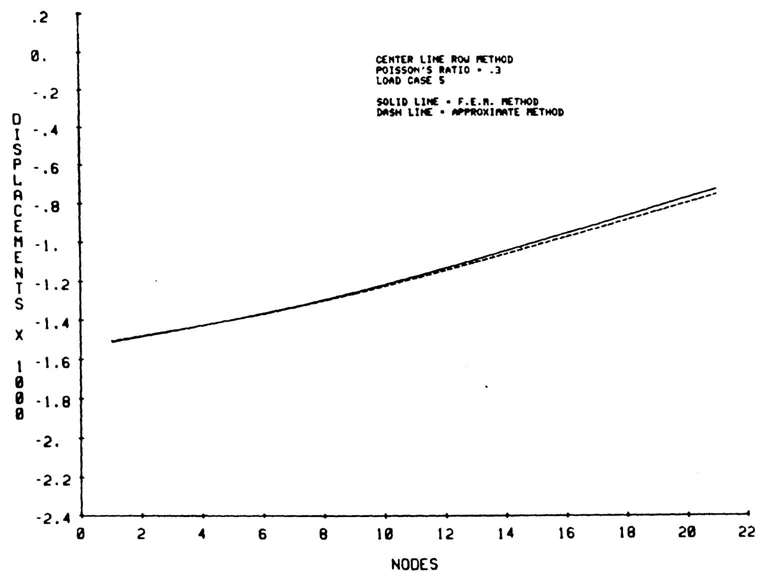

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 4
LOAD
NODES

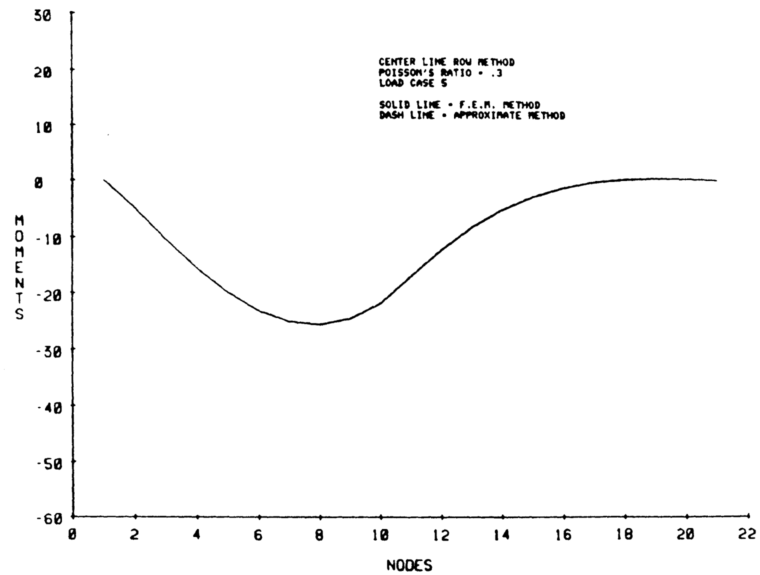

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES

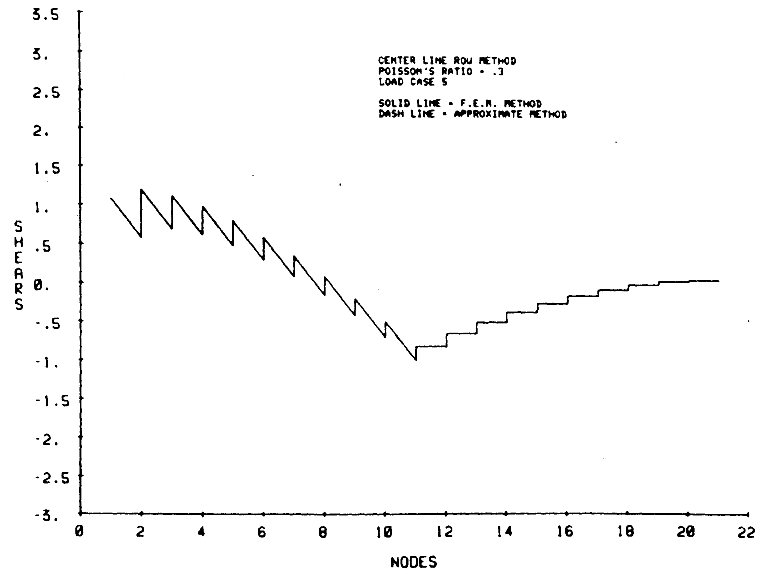

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES


CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES


CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 5
LOAD
NODES
2.
1.5
1.
.5
0.
-.5
-1.
-1.5
-2.
0 2 4 6 8 10 12 14 16 18 20 22

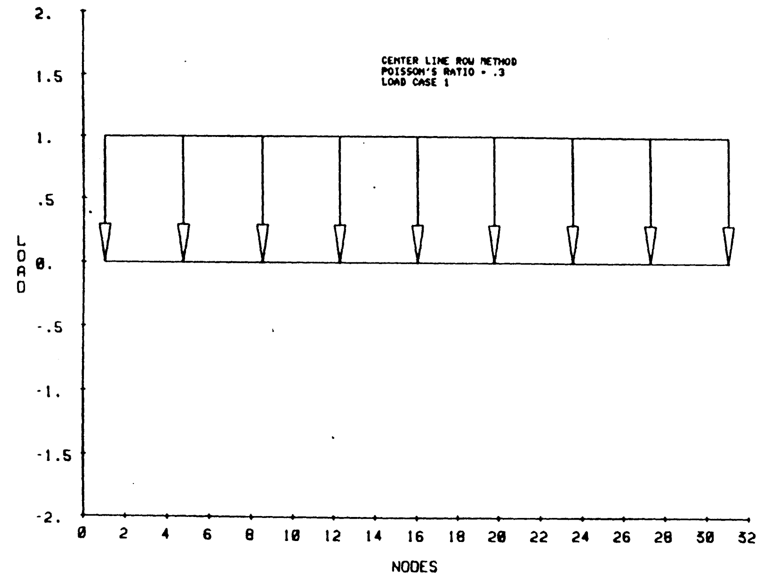

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 5
SOLID LINE = F.E.R. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES
.2
0.
-.2
-.4
-.6
-.8
-1.
-1.2
-1.4
-1.6
-1.8
-2.
-2.2
-2.4
0 2 4 6 8 10 12 14 16 18 20 22

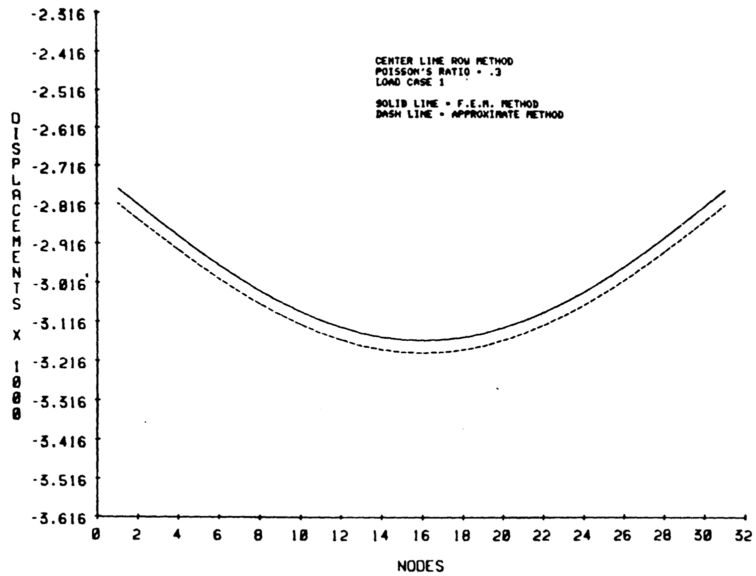

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 5
SOLID LINE = F.E.R. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES
30
20
10
0
-10
-20
-30
-40
-50
-60
0 2 4 6 8 10 12 14 16 18 20 22

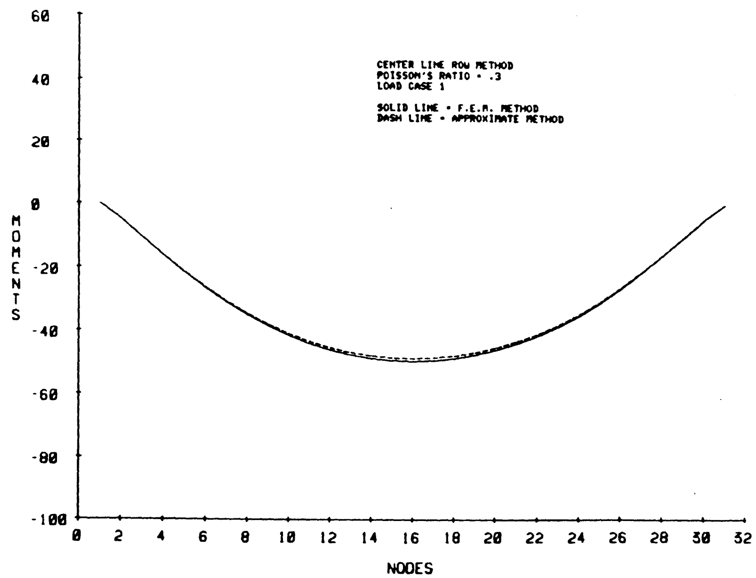

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 5
SOLID LINE = F.E.R. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES
3.5
3.
2.5
2.
1.5
1.
.5
0.
-.5
-1.
-1.5
-2.
-2.5
-3.
0 2 4 6 8 10 12 14 16 18 20 22

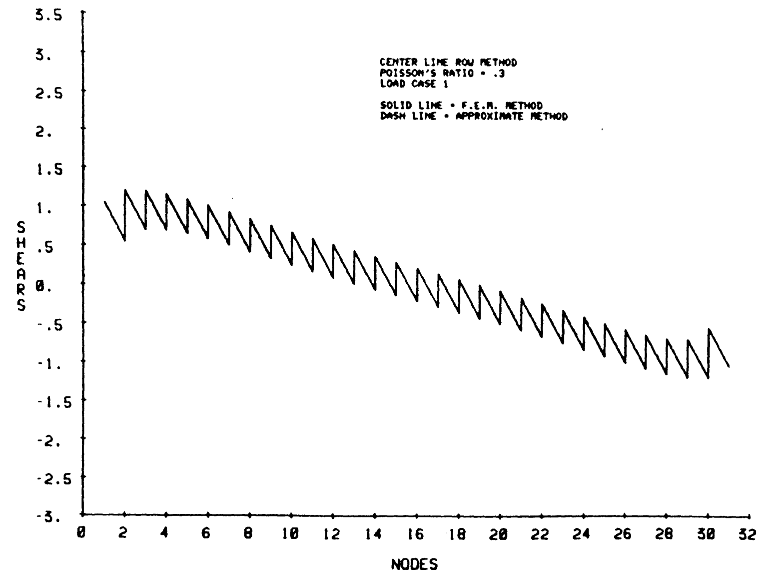
APPENDIX E

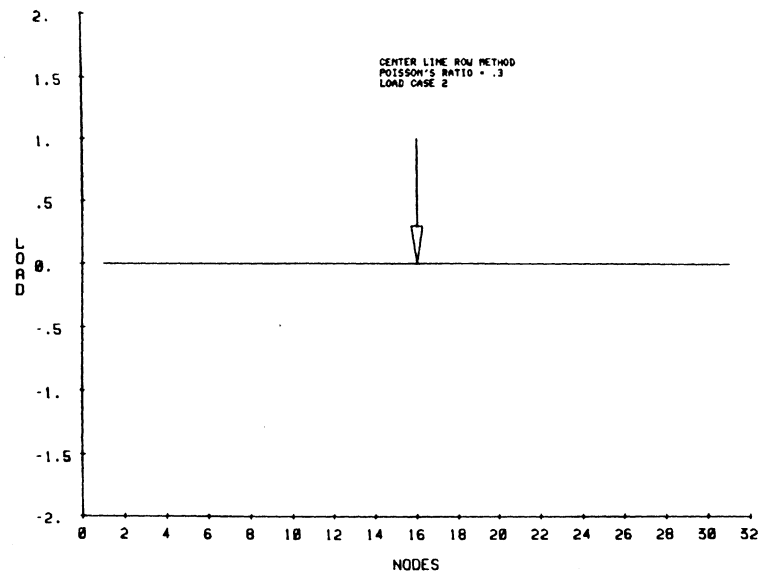
31 NODE BEAM ON FOUNDATION WITH POISSON'S RATIO OF 0.3 AND MODULUS OF ELASTICITY EQUAL TO 7000 PSI

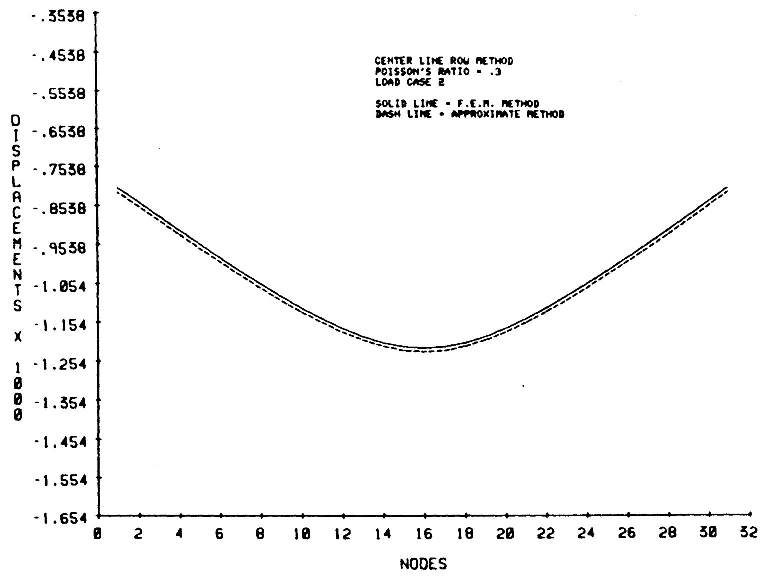

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 1
LOAD
NODES

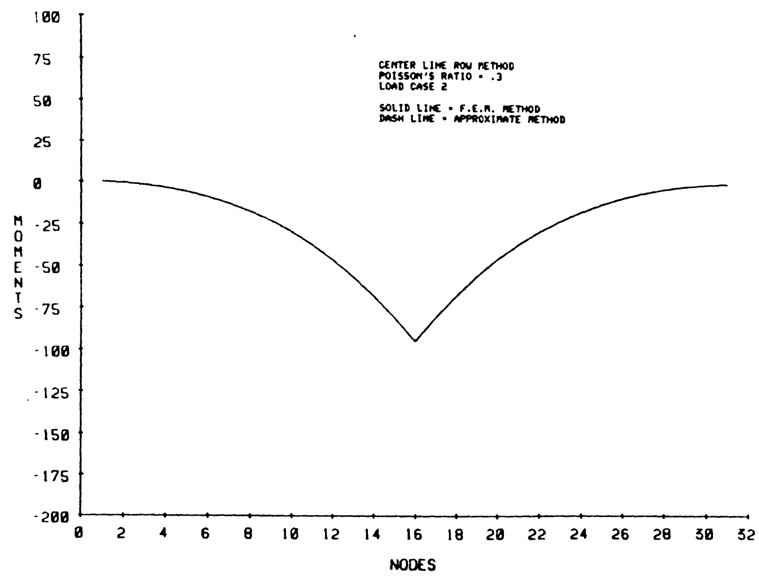

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES

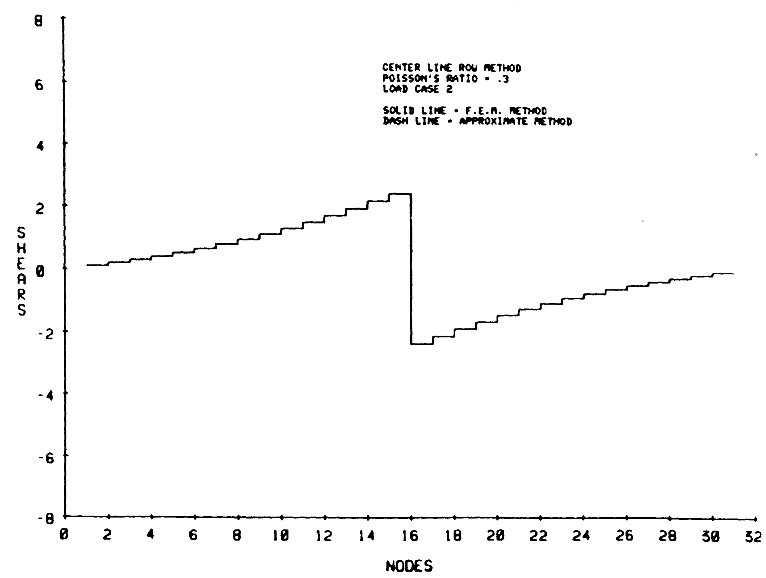

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES

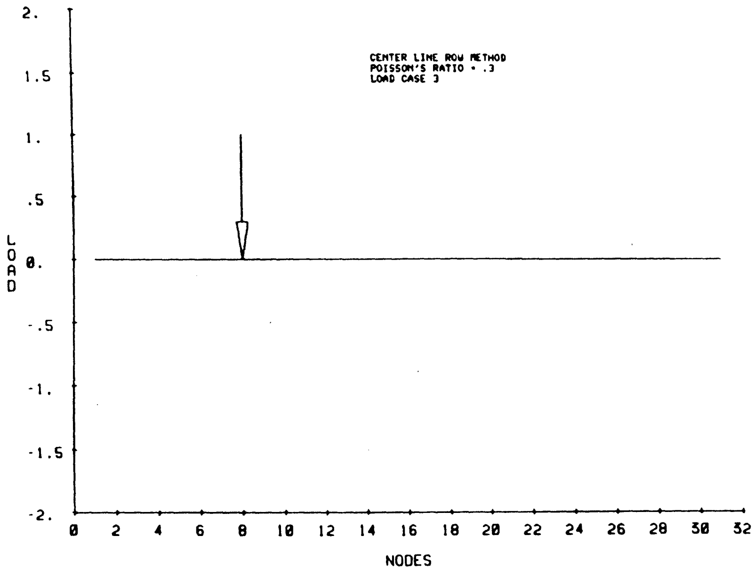

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES

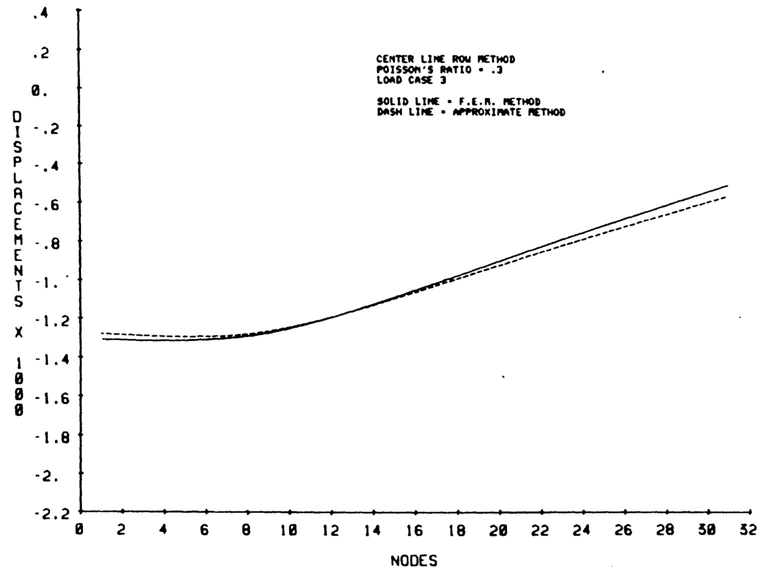

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 2
LOAD
NODES

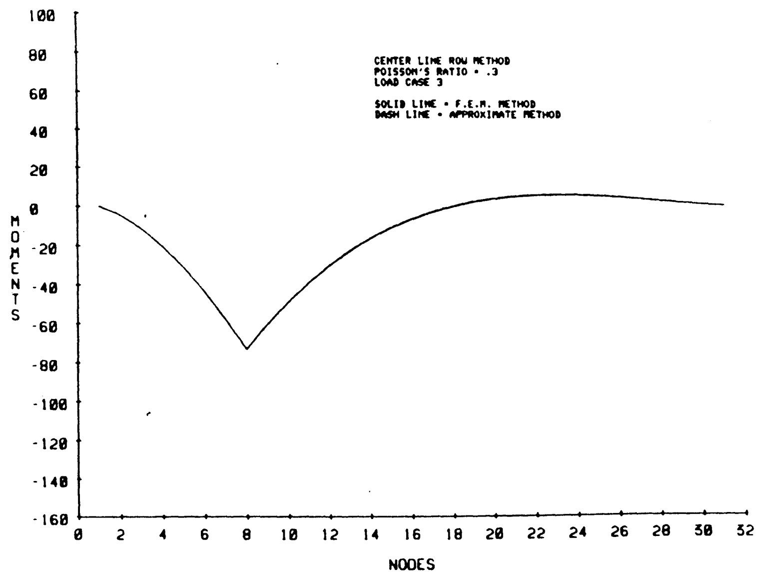

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES

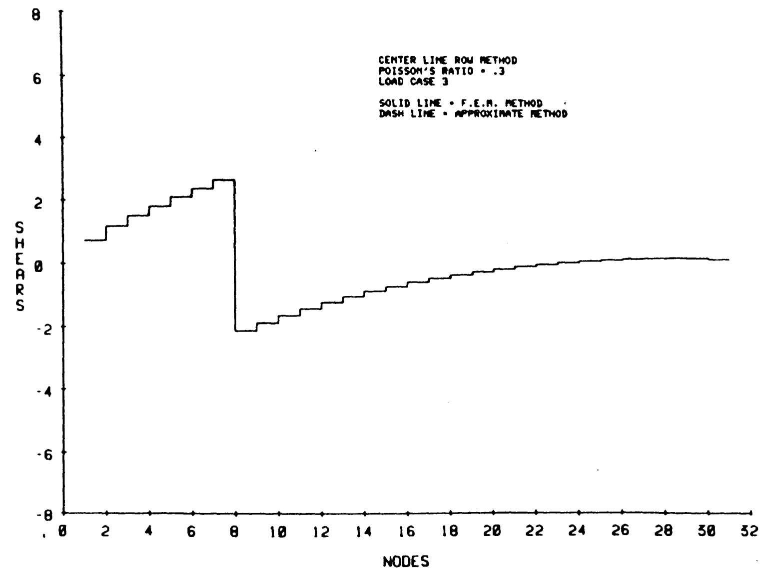

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES

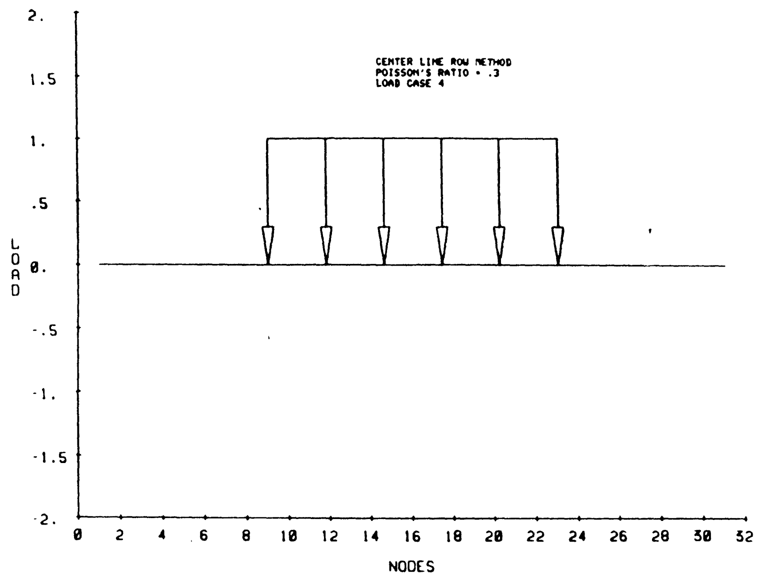

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES


CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 3
LOAD
NODES

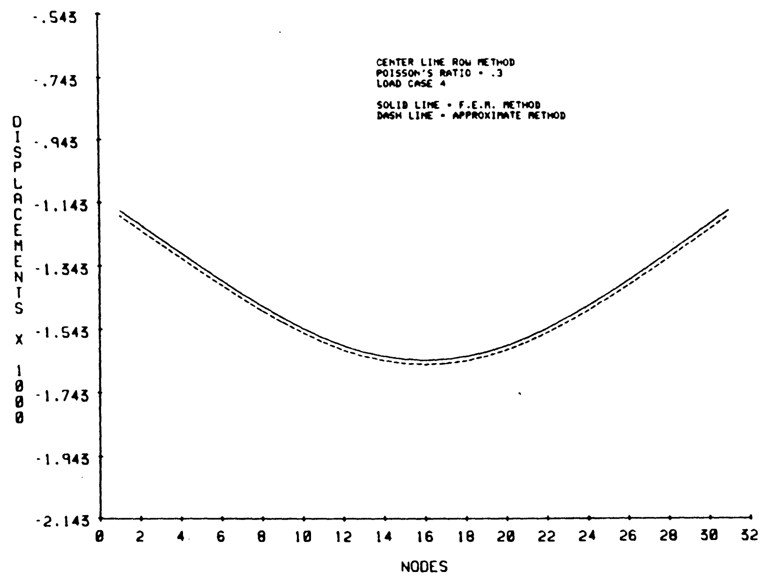

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES


CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES

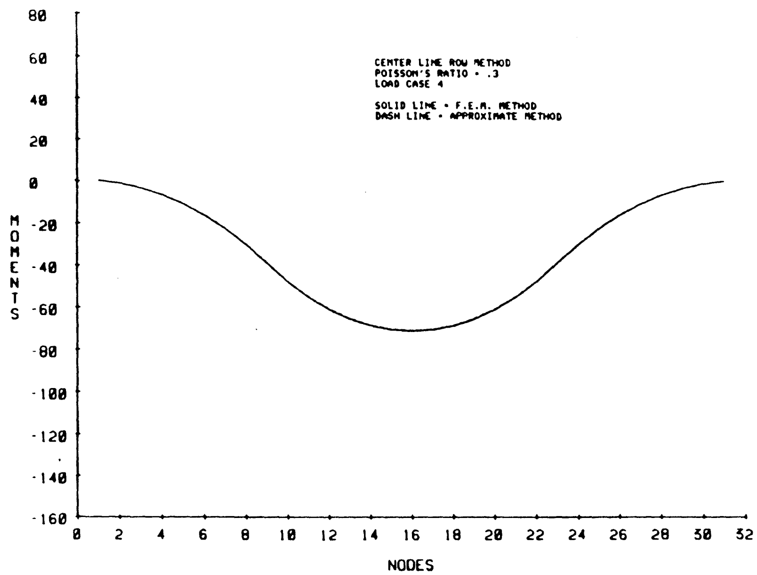

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES



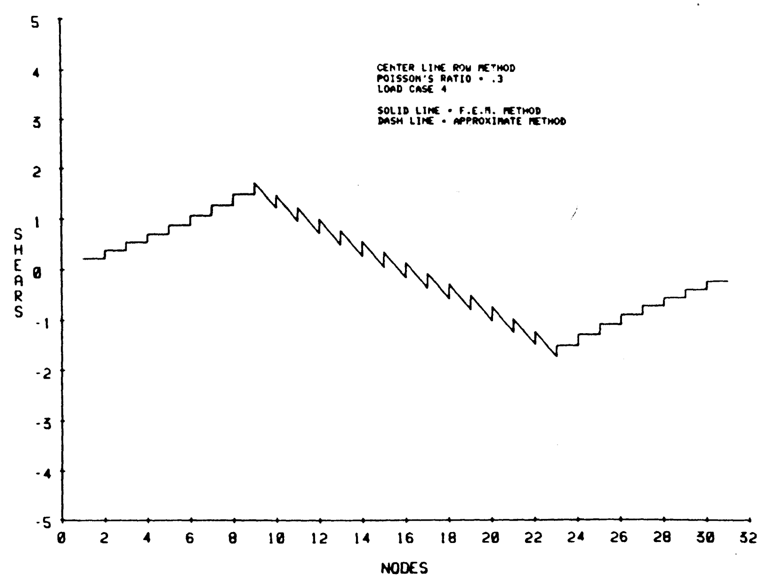
CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 4
LOAD
2.
1.5
1.
.5
0.
-.5
-1.
-1.5
-2.
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
NODES



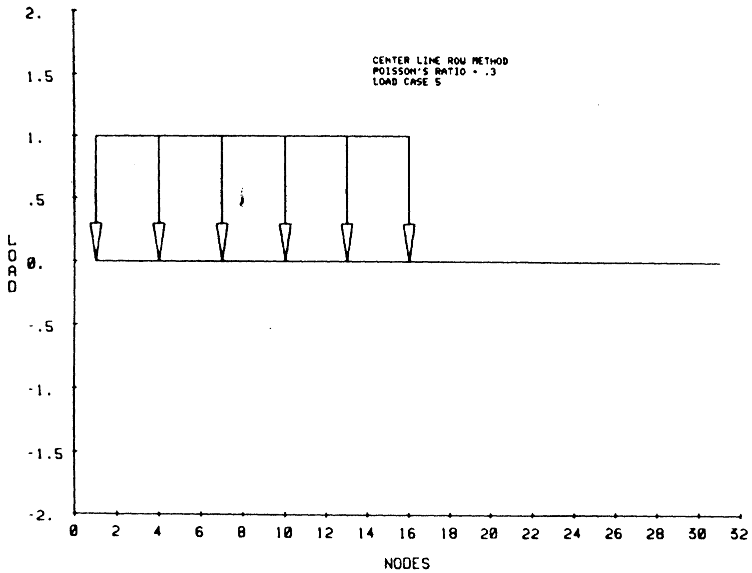
CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
-.543
-.743
-.943
-1.143
-1.343
-1.543
-1.743
-1.943
-2.143
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
NODES

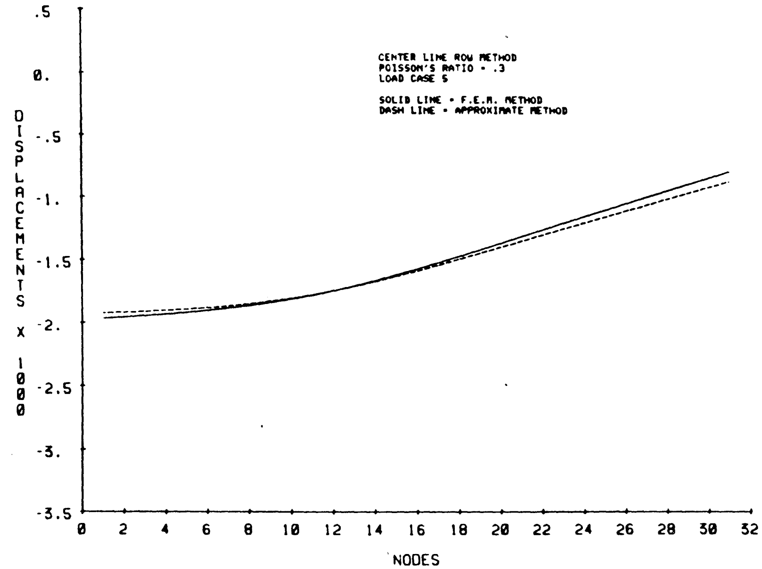


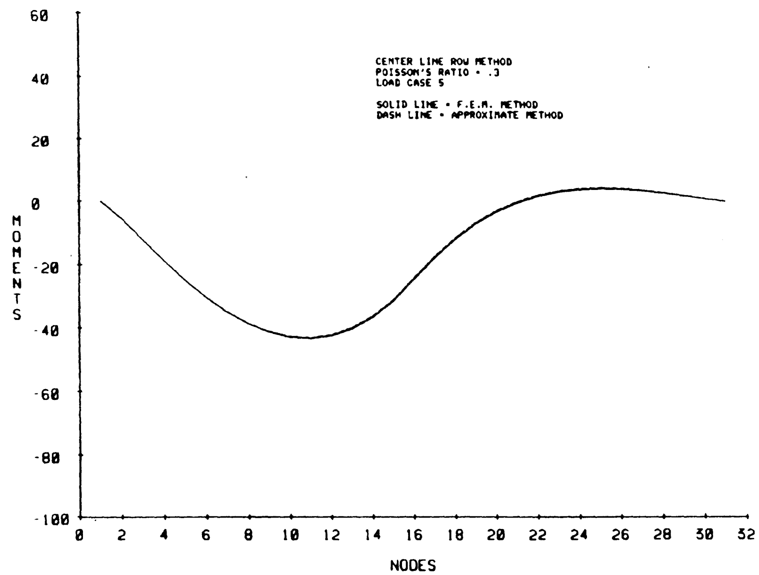
CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
NODES

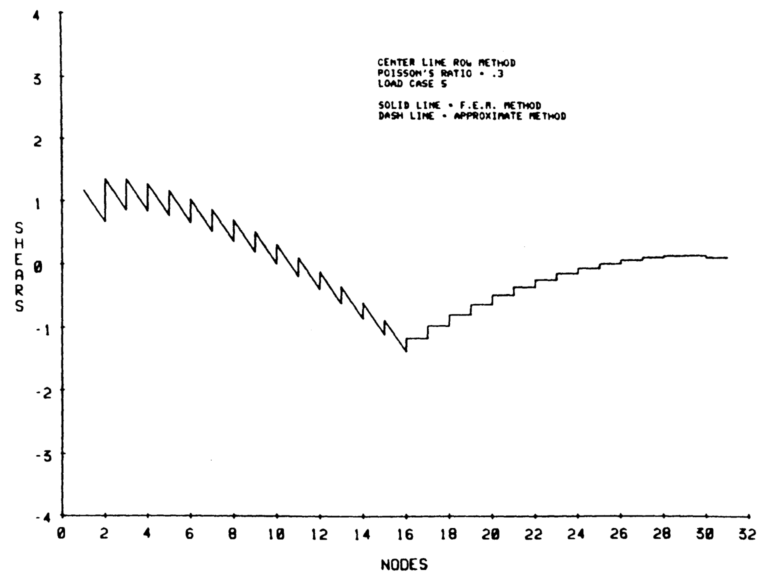


CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
5
4
3
2
1
0
-1
-2
-3
-4
-5
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
NODES


CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 5
LOAD
NODES

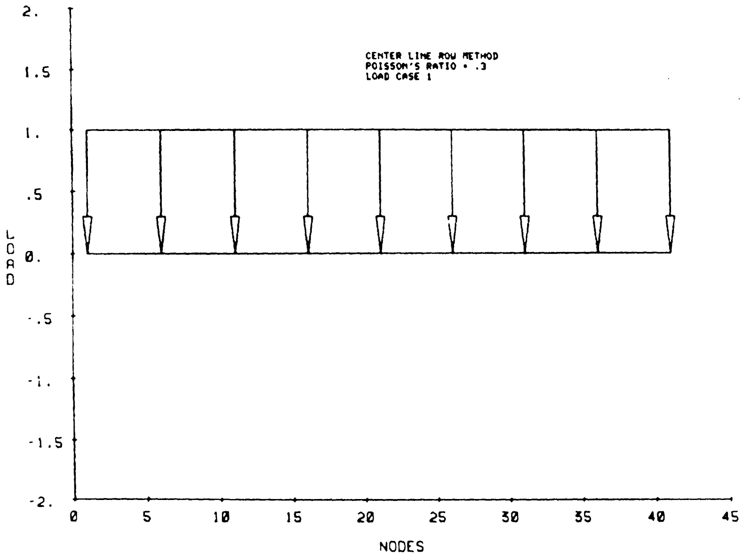

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES

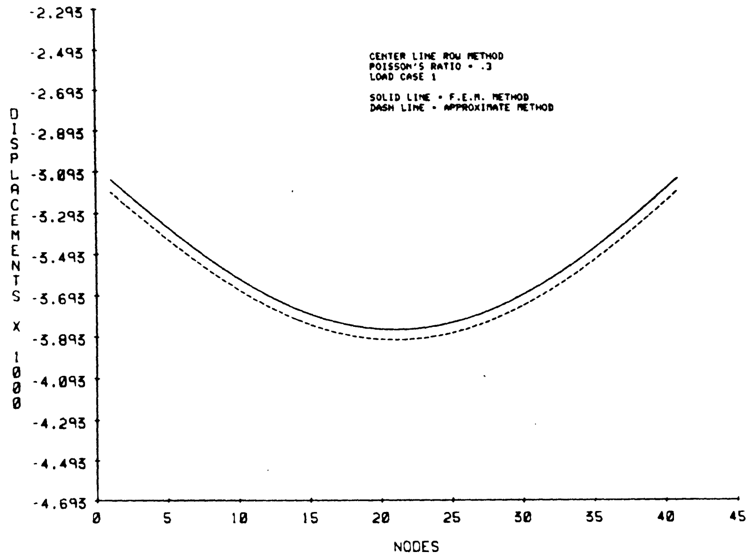

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES

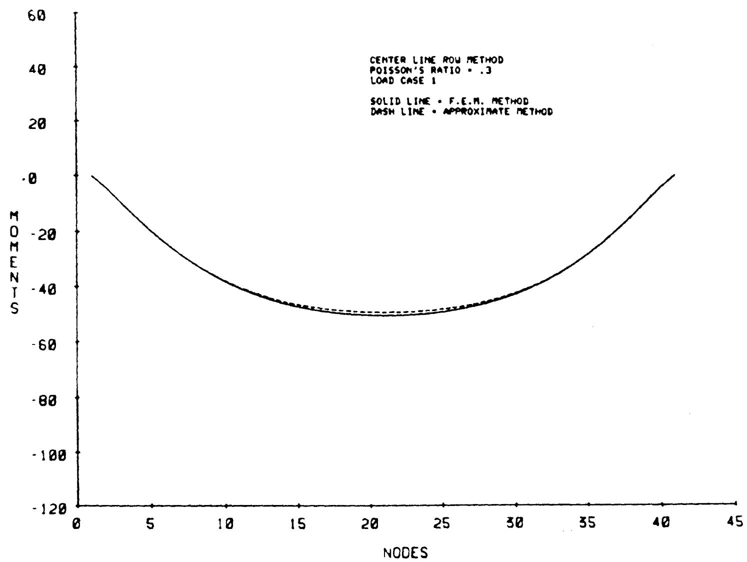

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES

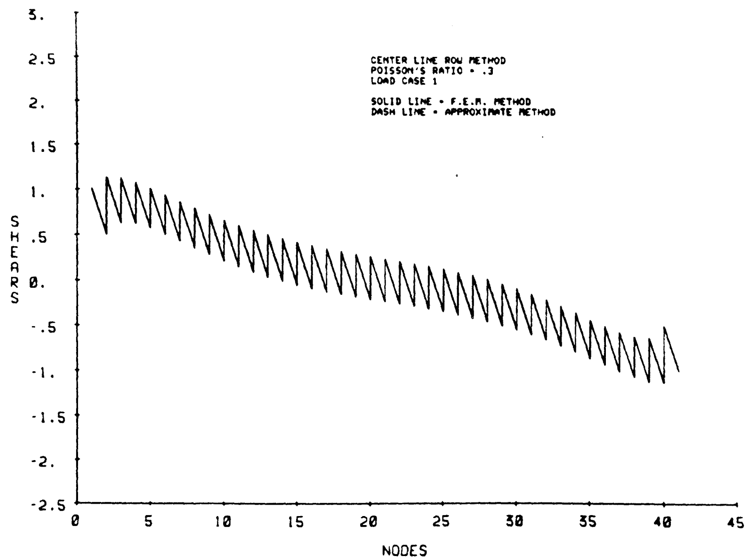
APPENDIX F

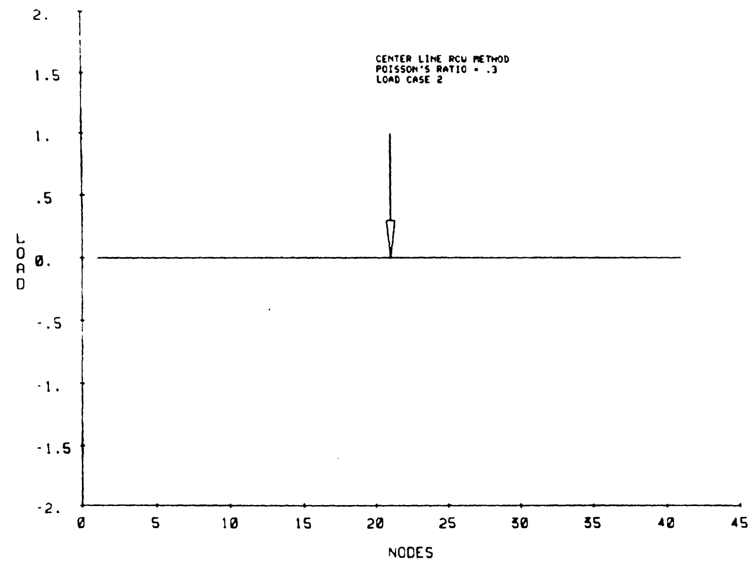
41 NODE BEAM ON FOUNDATION WITH POISSON'S RATIO OF 0.3 AND MODULUS OF ELASTICITY EQUAL TO 7000 PSI

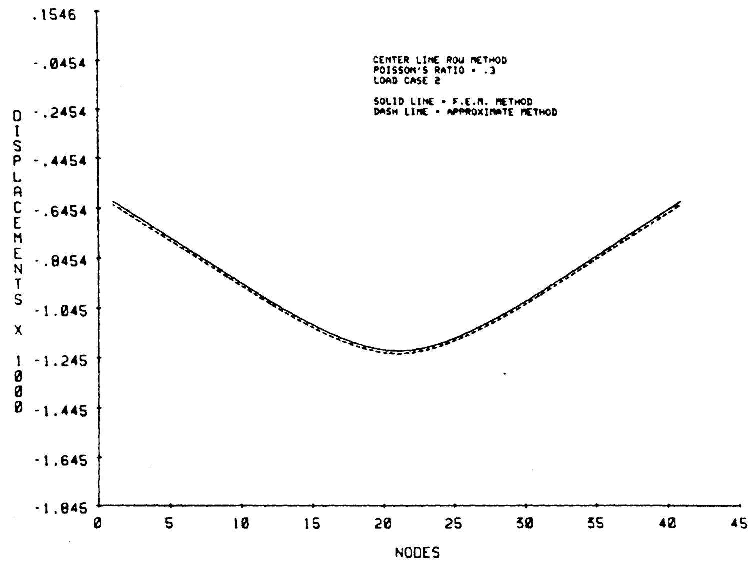

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 1
LOAD
NODES

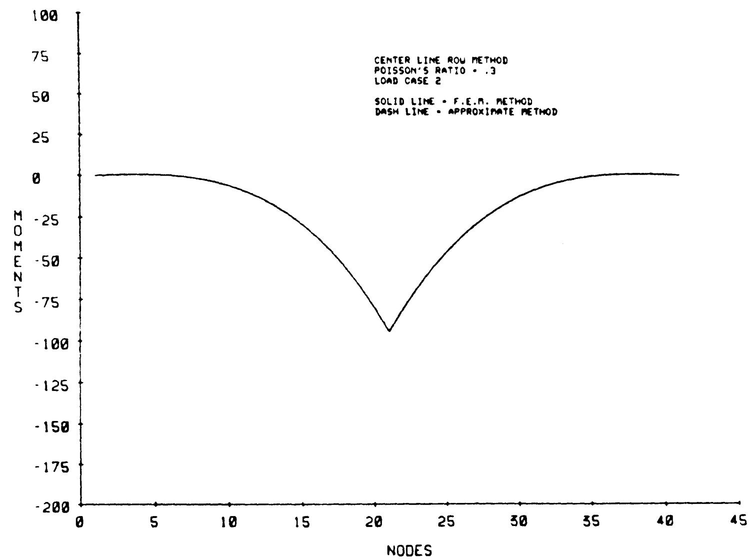

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES

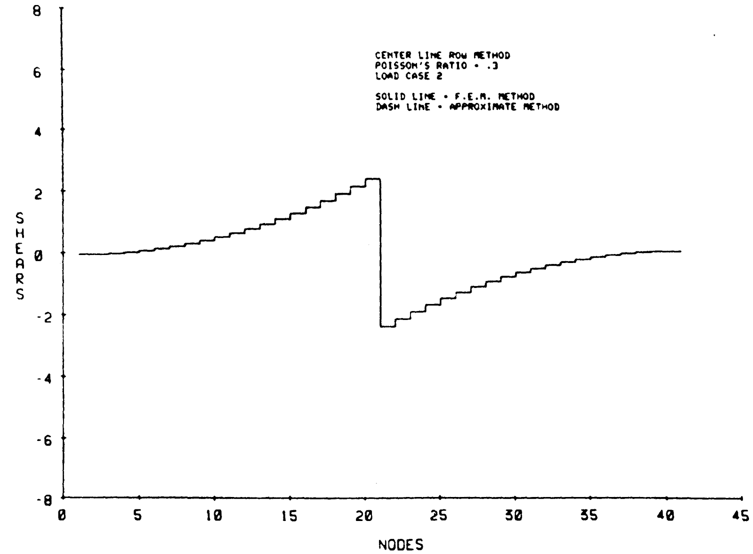

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES

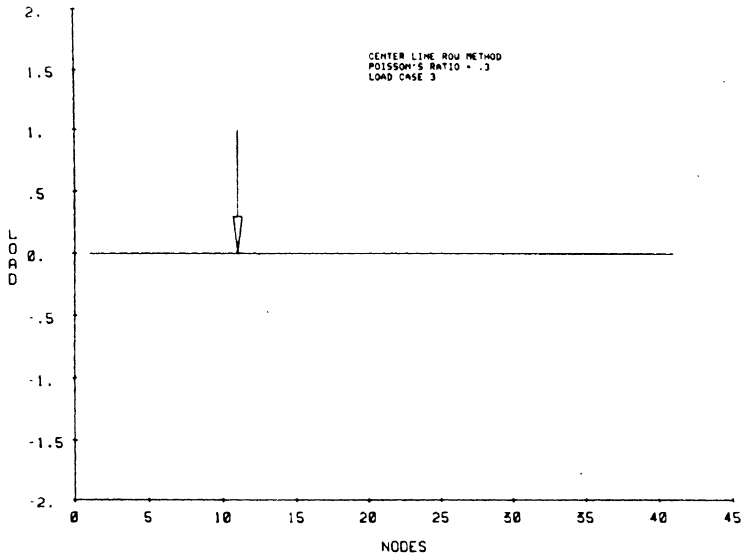

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES

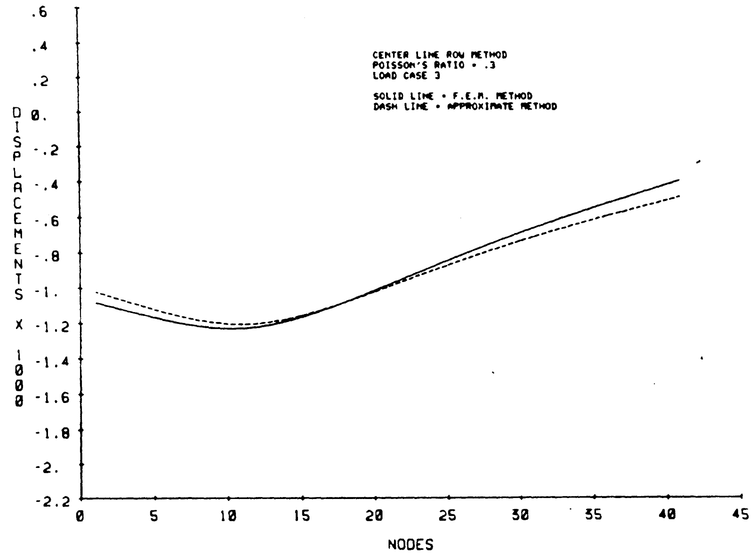

CENTER LINE RCU METHOD
POISSON'S RATIO = .3
LOAD CASE 2
LOAD
2.
1.5
1.
.5
0.
-.5
-1.
-1.5
-2.
0
5
10
15
20
25
30
35
40
45
NODES

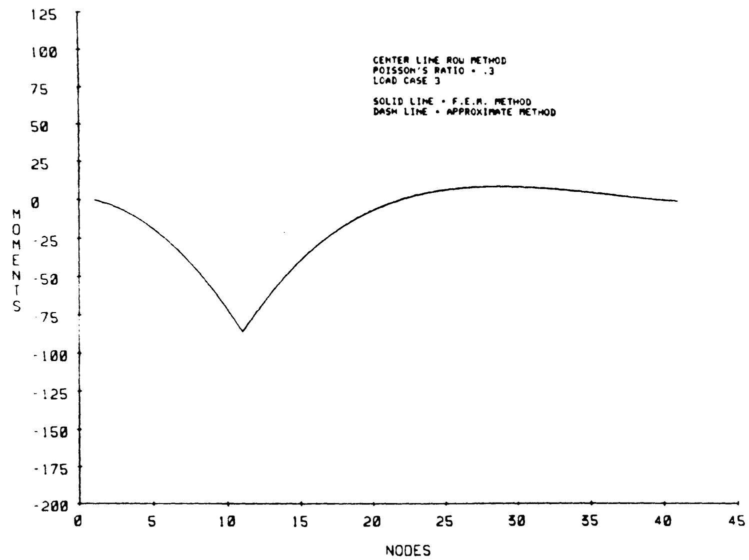

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
.1546
-.0454
-.2454
-.4454
-.6454
-.8454
-1.045
-1.245
-1.445
-1.645
-1.845
0
5
10
15
20
25
30
35
40
45
NODES

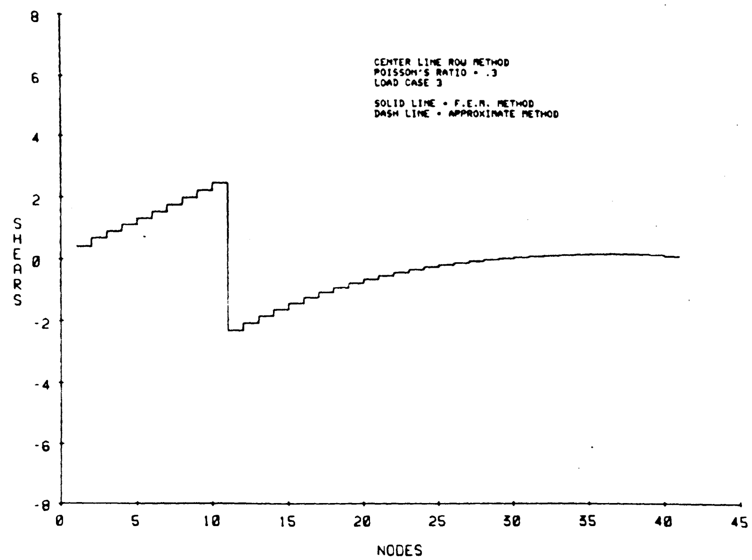

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
100
75
50
25
0
-25
-50
-75
-100
-125
-150
-175
-200
0
5
10
15
20
25
30
35
40
45
NODES

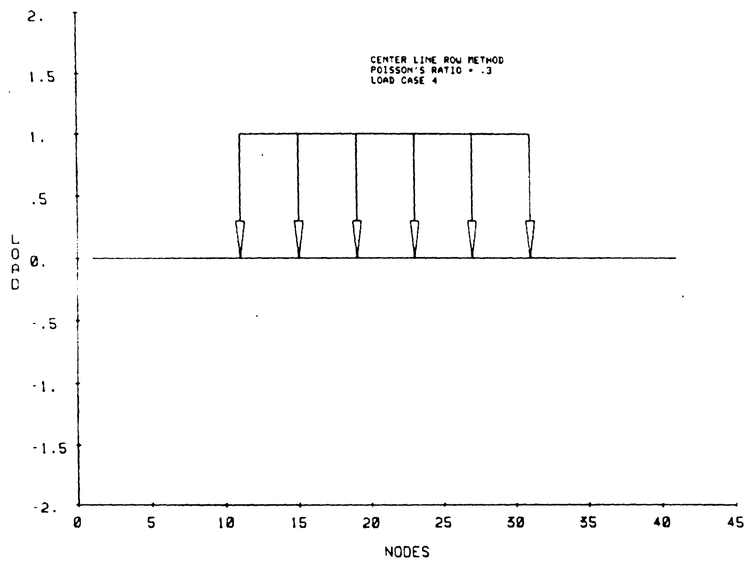

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
8
6
4
2
0
-2
-4
-6
-8
0
5
10
15
20
25
30
35
40
45
NODES

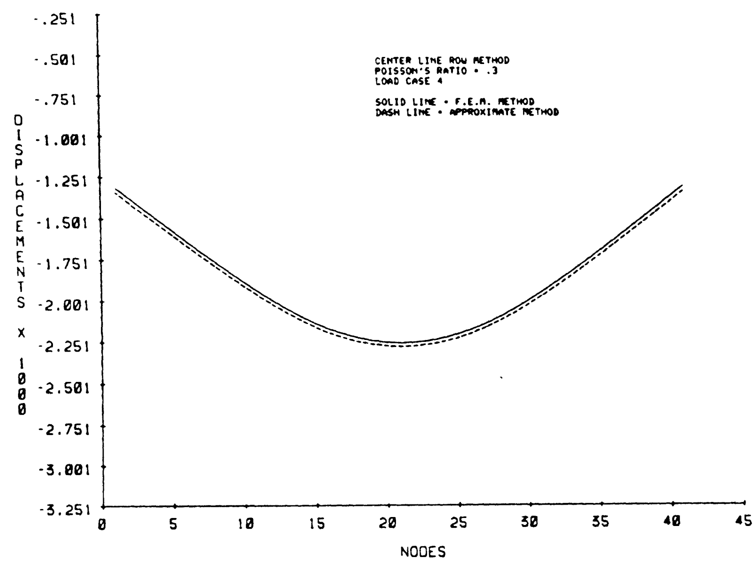

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 3
LOAD
NODES

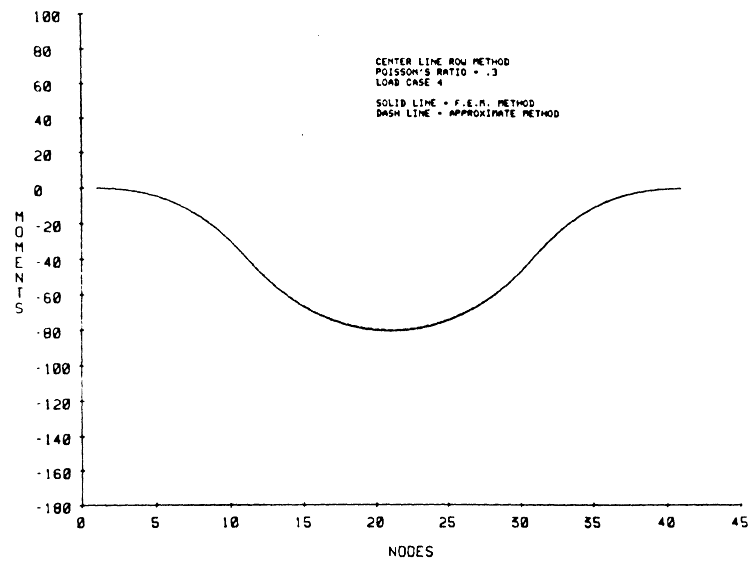

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES

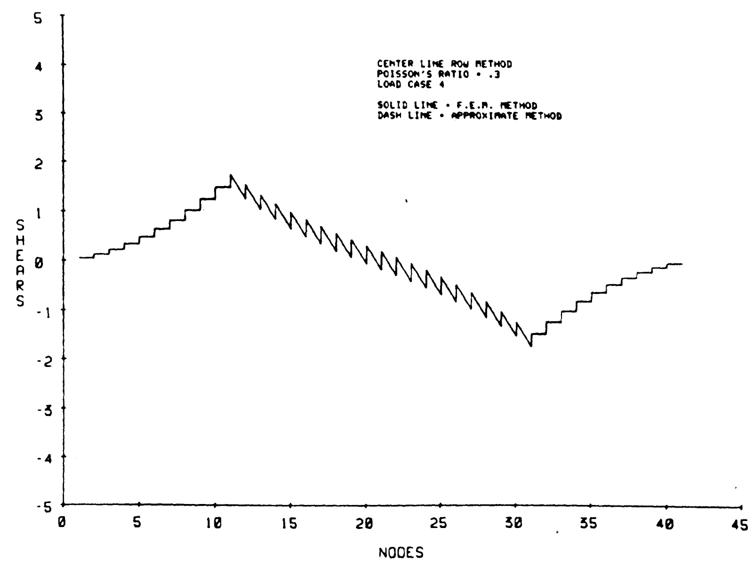

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES

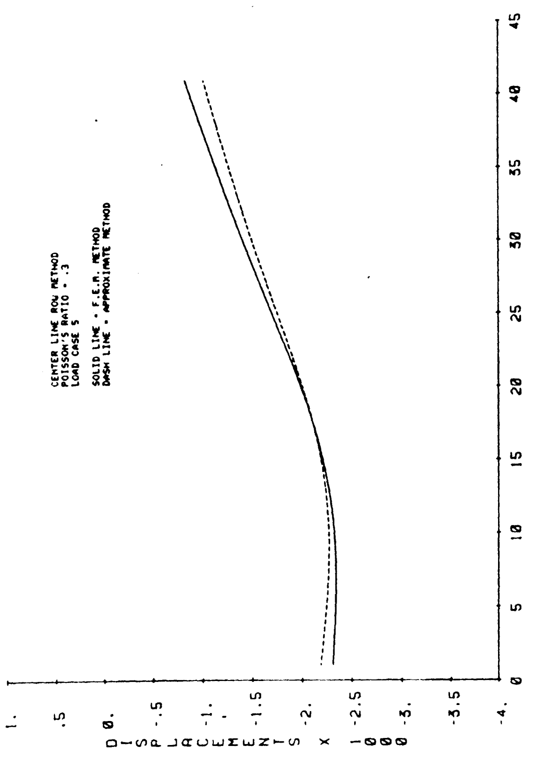

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES

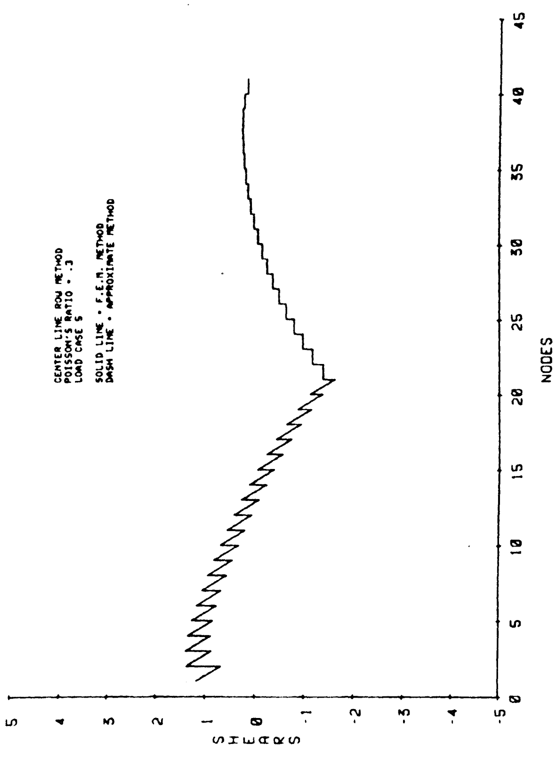

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 4
LOAD
NODES

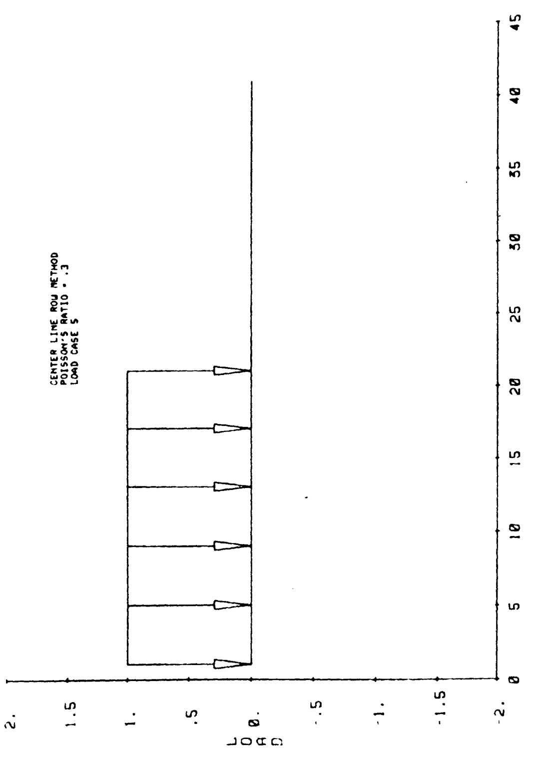

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES

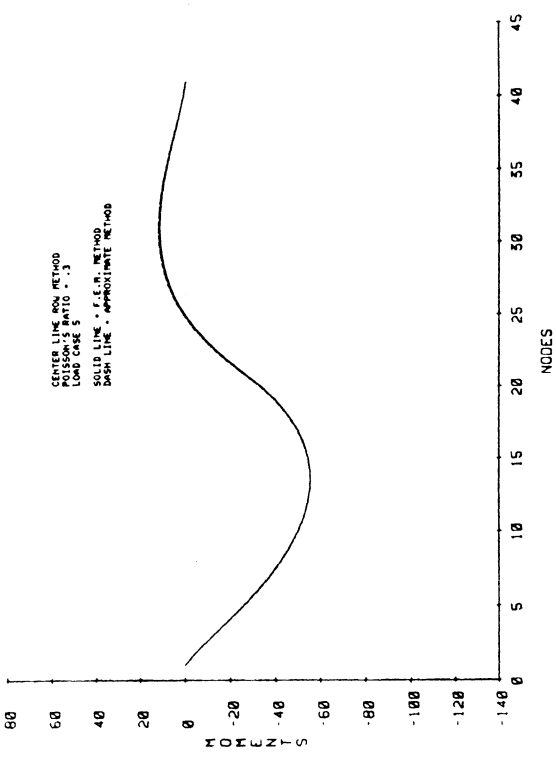

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES


CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES


CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES

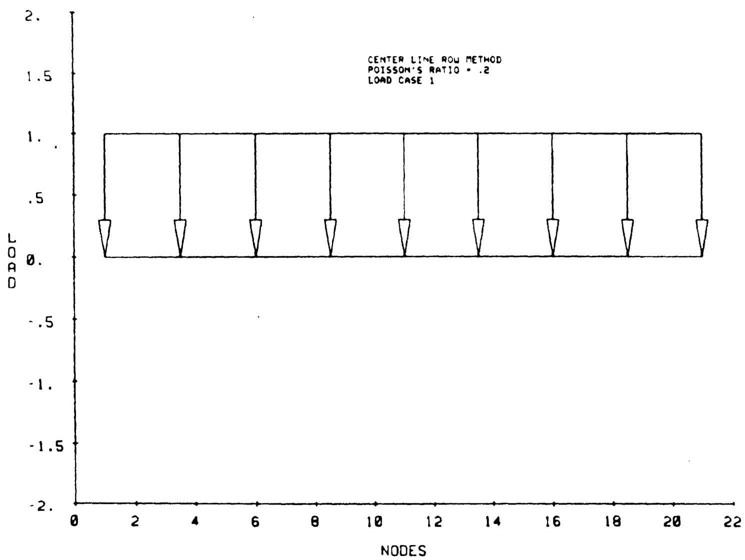

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES

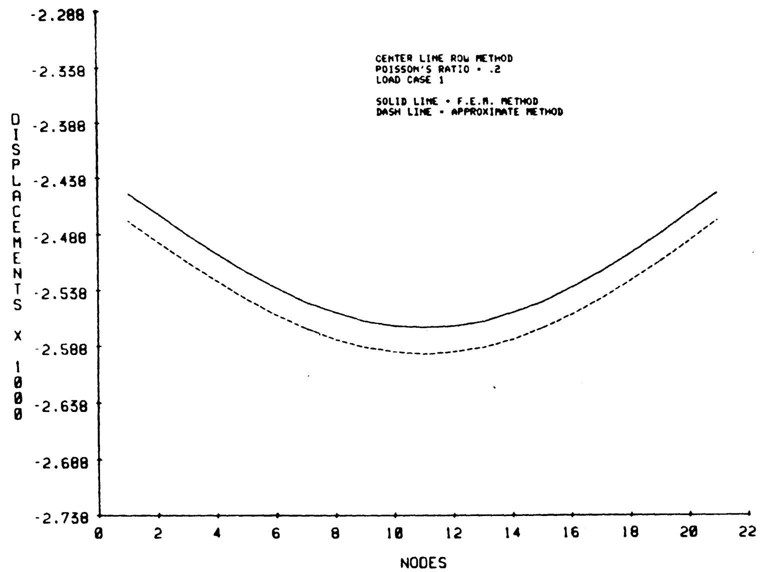

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 5
LOAD
NODES

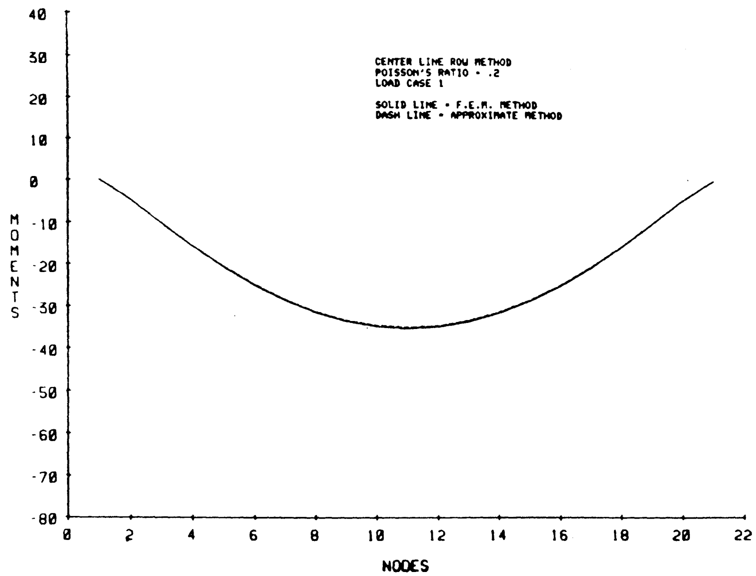

CENTER LINE ROW METHOD
POISSON'S RATIO = .3
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES

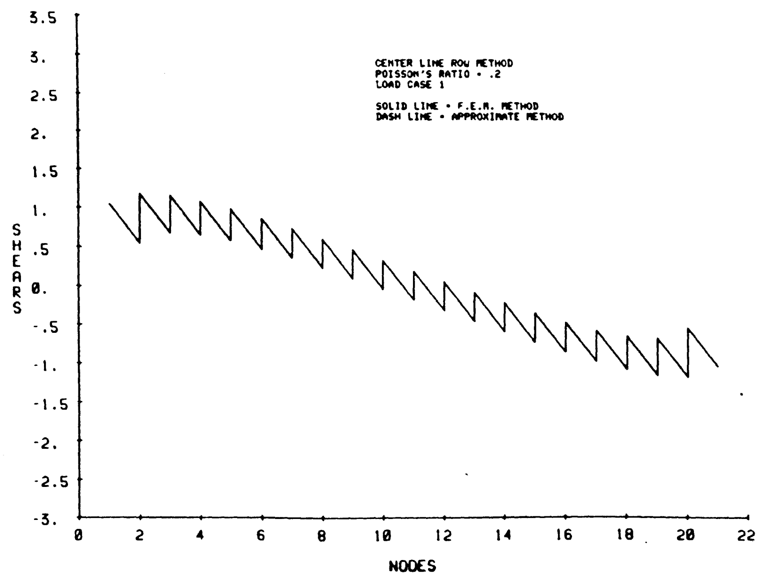
APPENDIX G

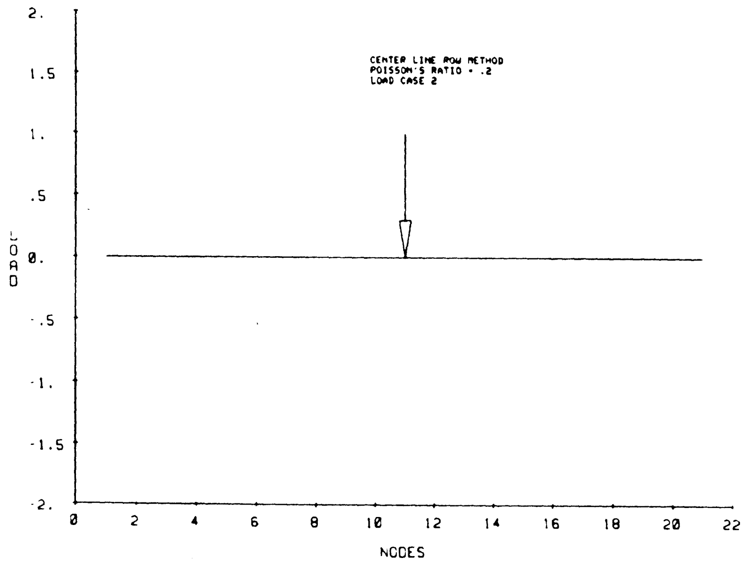
21 NODE BEAM ON FOUNDATION WITH POISSON'S RATIO OF 0.2 AND MODULUS OF ELASTICITY EQUAL TO 7000 PSI

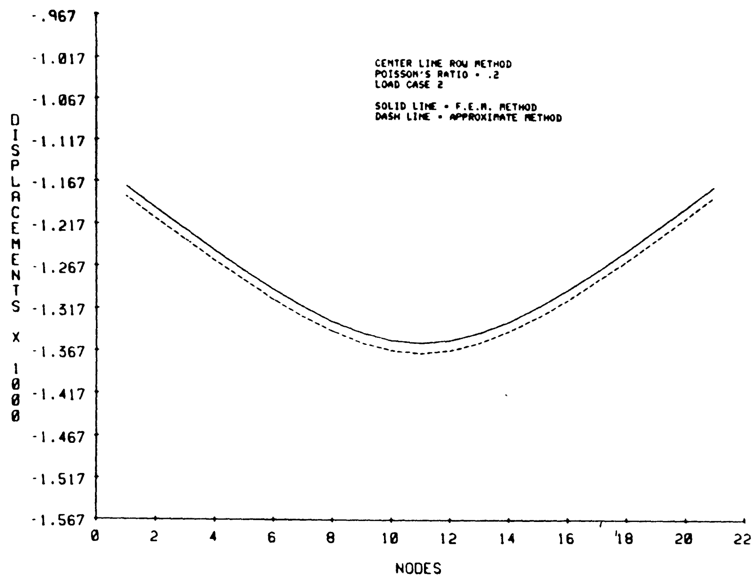

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 1
LOAD
NODES

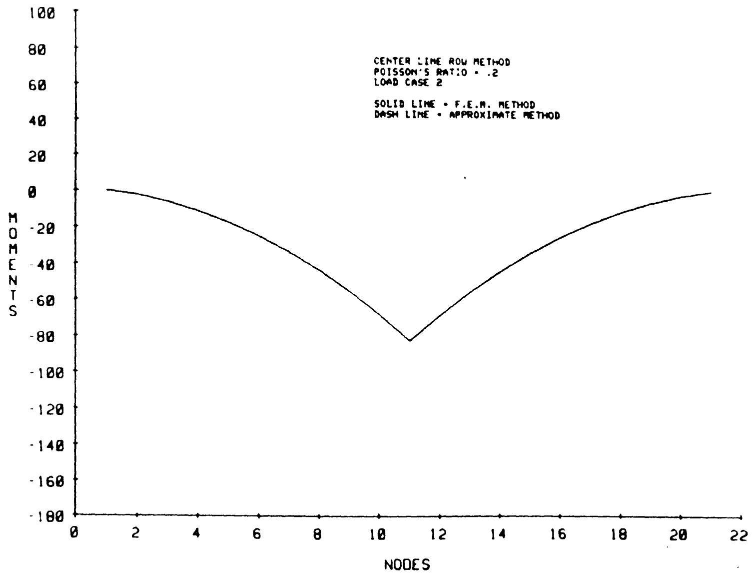

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES

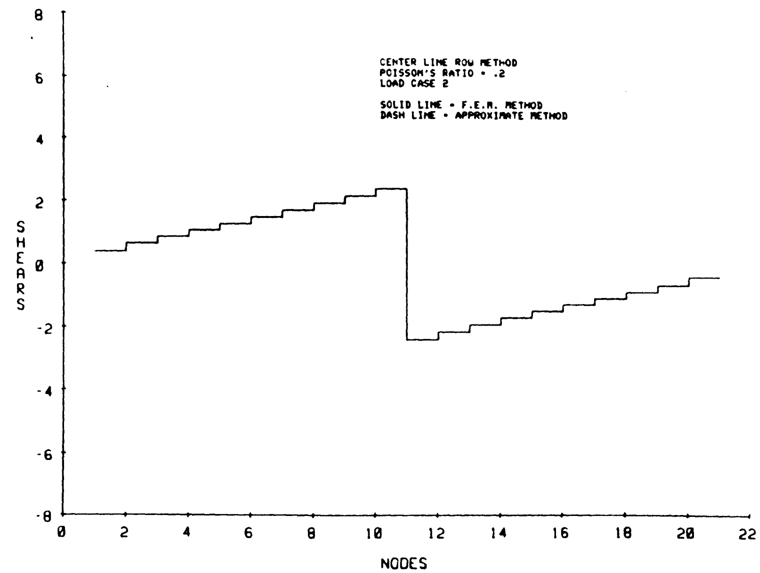

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES

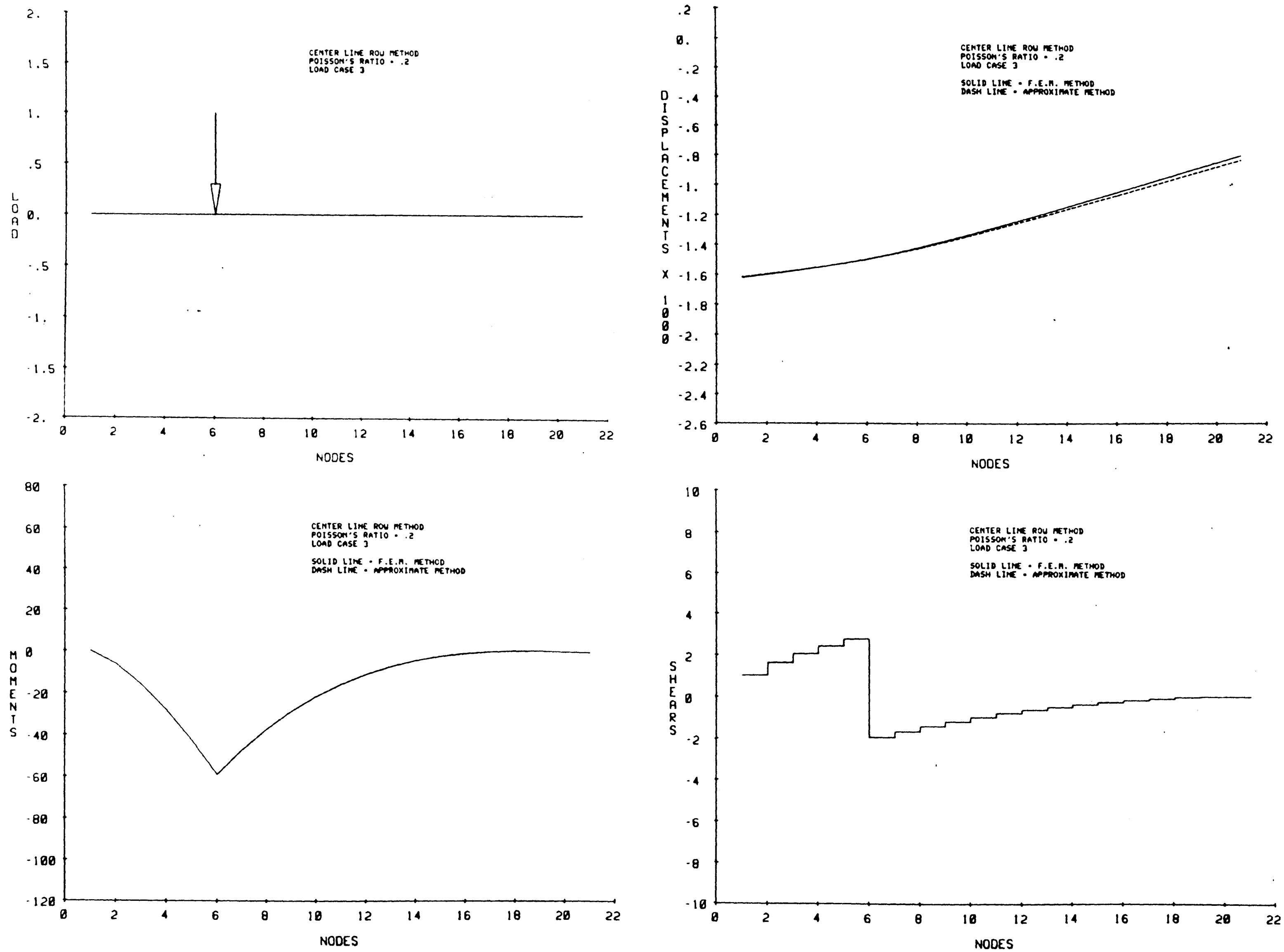

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES

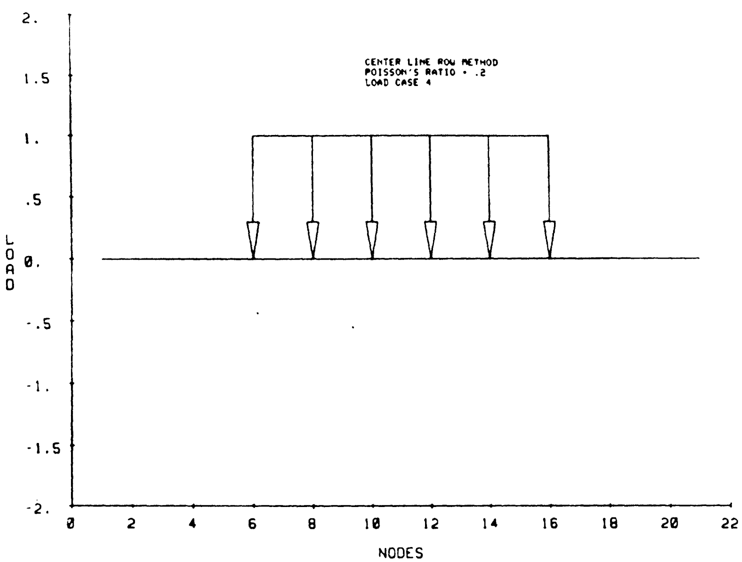

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 2
LOAD
2.
1.5
1.
.5
0.
-.5
-1.
-1.5
-2.
0 2 4 6 8 10 12 14 16 18 20 22
NODES

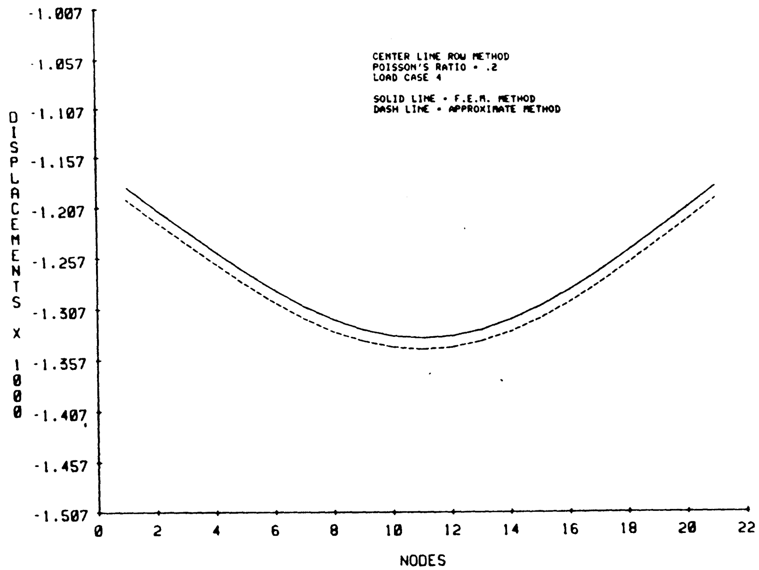

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
-.967
-1.017
-1.067
-1.117
-1.167
-1.217
-1.267
-1.317
-1.367
-1.417
-1.467
-1.517
-1.567
0 2 4 6 8 10 12 14 16 18 20 22
NODES

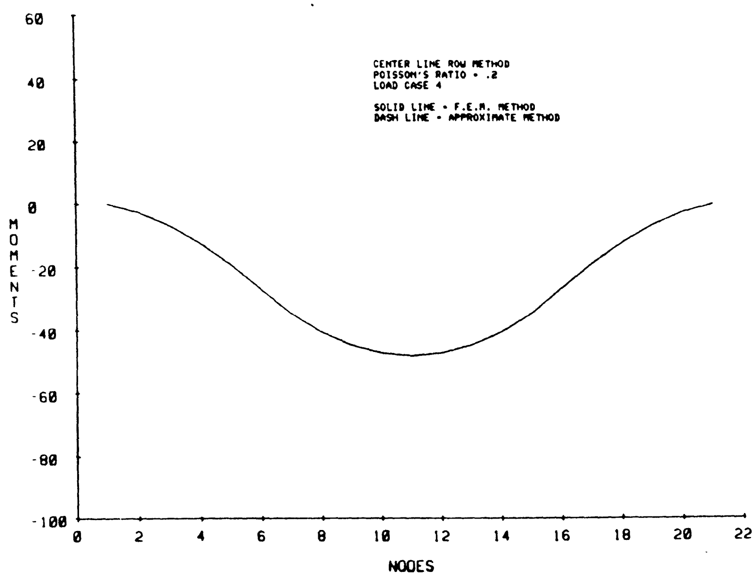

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
0 2 4 6 8 10 12 14 16 18 20 22
NODES

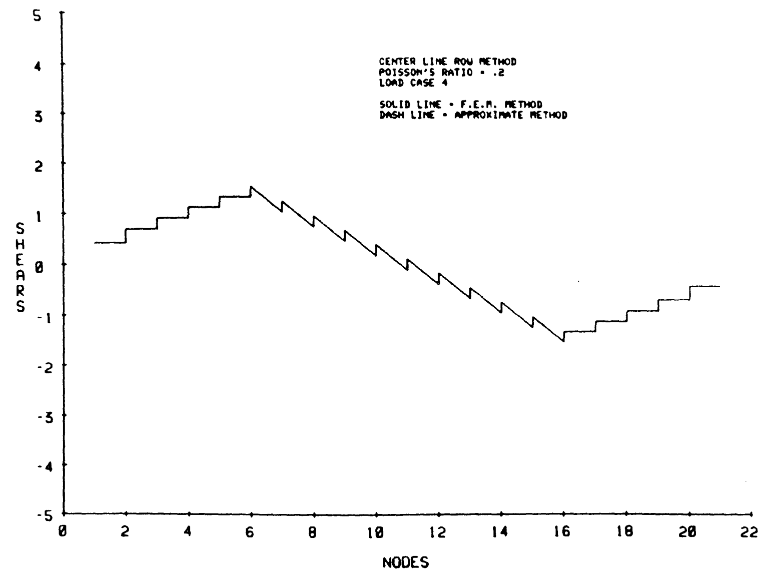

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
8
6
4
2
0
-2
-4
-6
-8
0 2 4 6 8 10 12 14 16 18 20 22
NODES

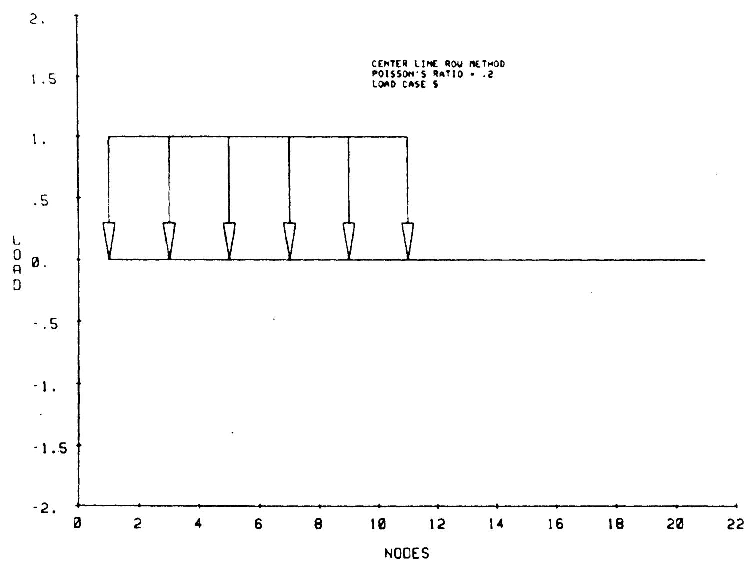

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 3
LOAD
NODES
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
MOMENTS
SHEARS

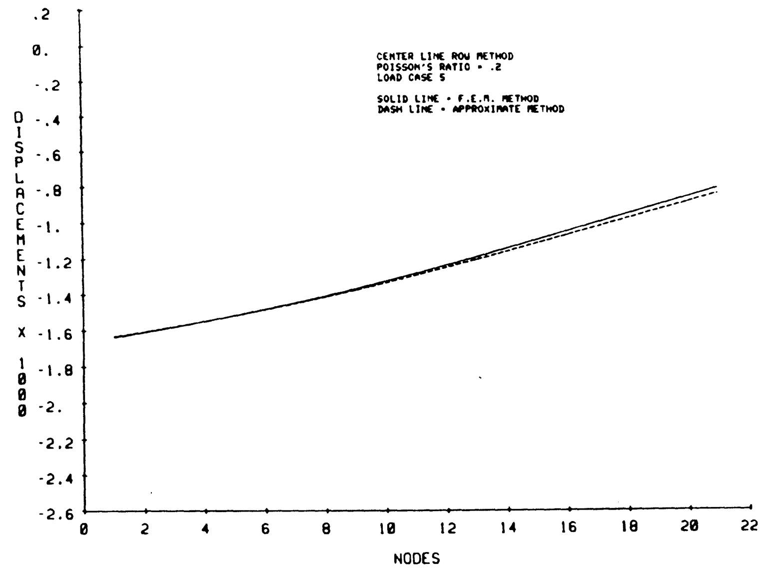

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 4
LOAD
2.
1.5
1.
.5
0.
-.5
-1.
-1.5
-2.
0 2 4 6 8 10 12 14 16 18 20 22
NODES

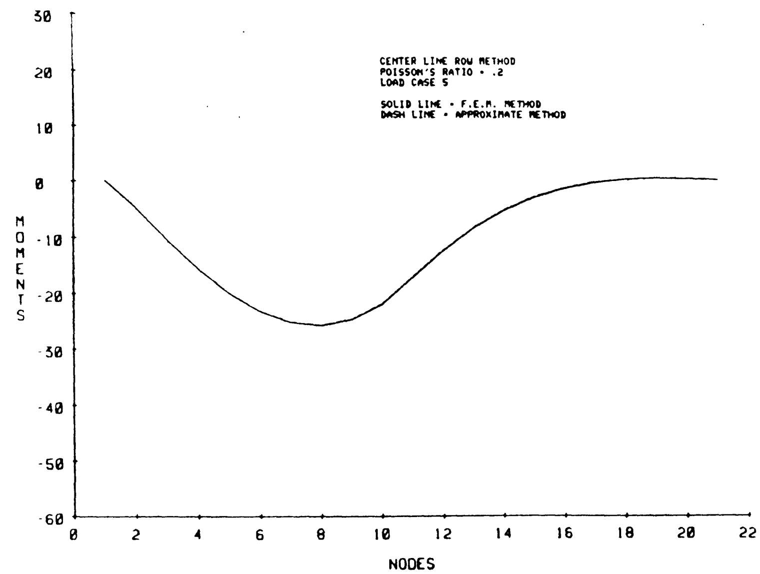

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
-1.007
-1.057
-1.107
-1.157
-1.207
-1.257
-1.307
-1.357
-1.407
-1.457
-1.507
0 2 4 6 8 10 12 14 16 18 20 22
NODES

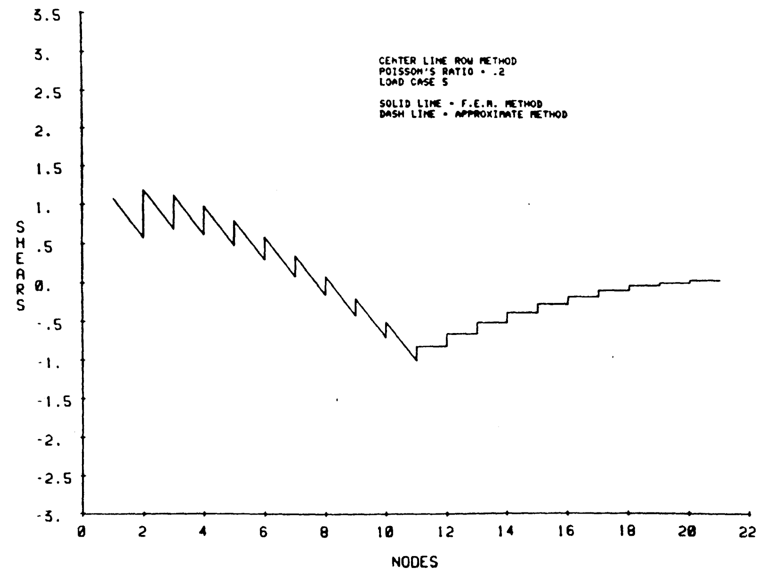

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
60
40
20
0
-20
-40
-60
-80
-100
0 2 4 6 8 10 12 14 16 18 20 22
NODES


CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
5
4
3
2
1
0
-1
-2
-3
-4
-5
0 2 4 6 8 10 12 14 16 18 20 22
NODES


CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 5
LOAD
NODES


CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES


CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES

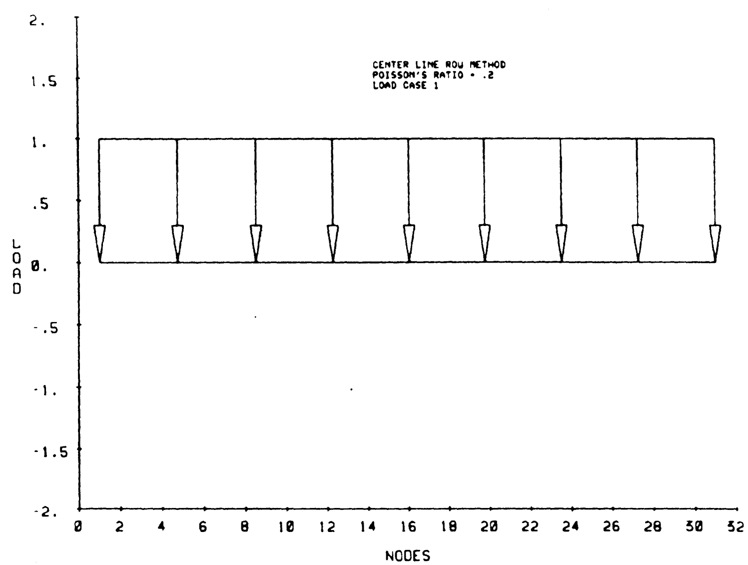

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES

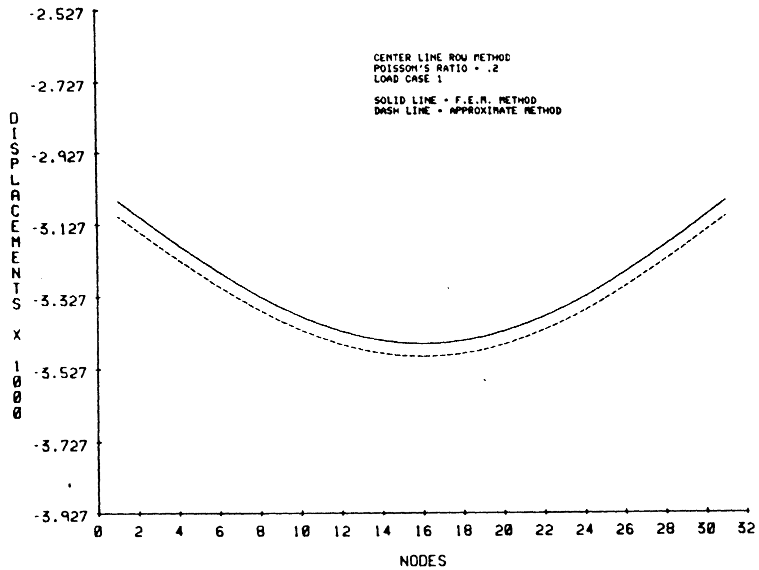
APPENDIX H

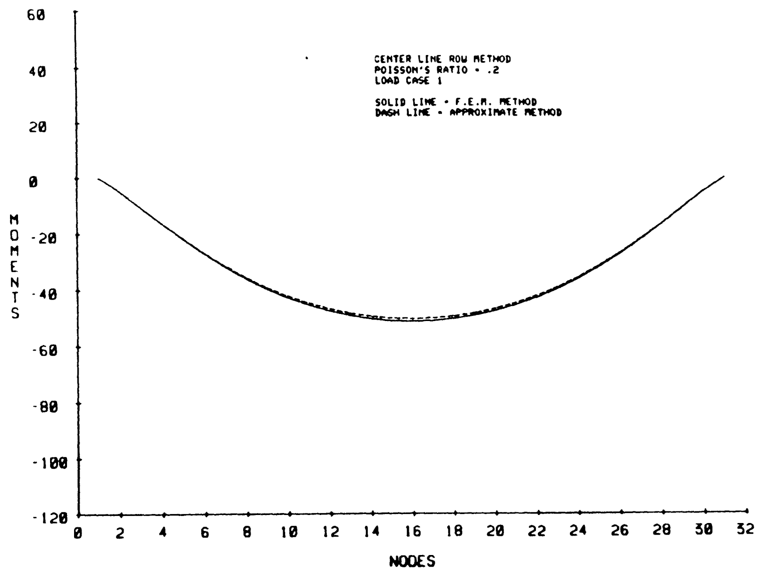
31 NODE BEAM OF FOUNDATION WITH POISSON'S RATIO

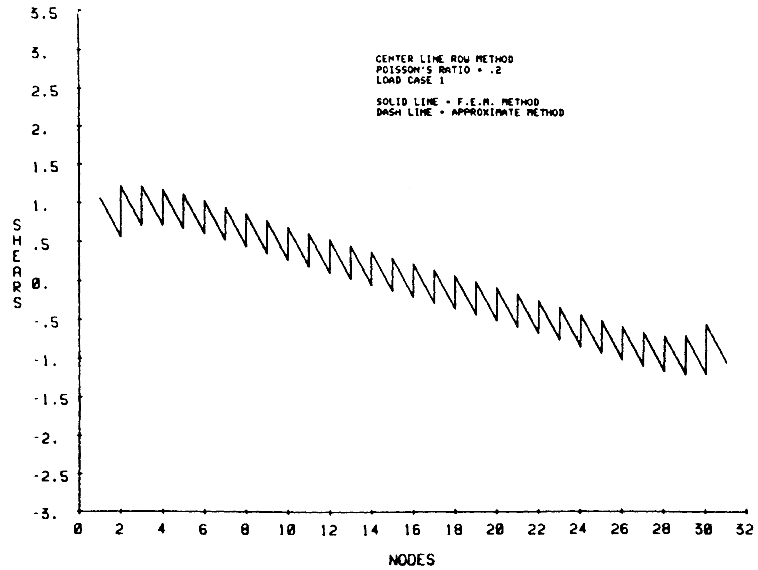
OF 0.2 AND MODULUS OF ELASTICITY

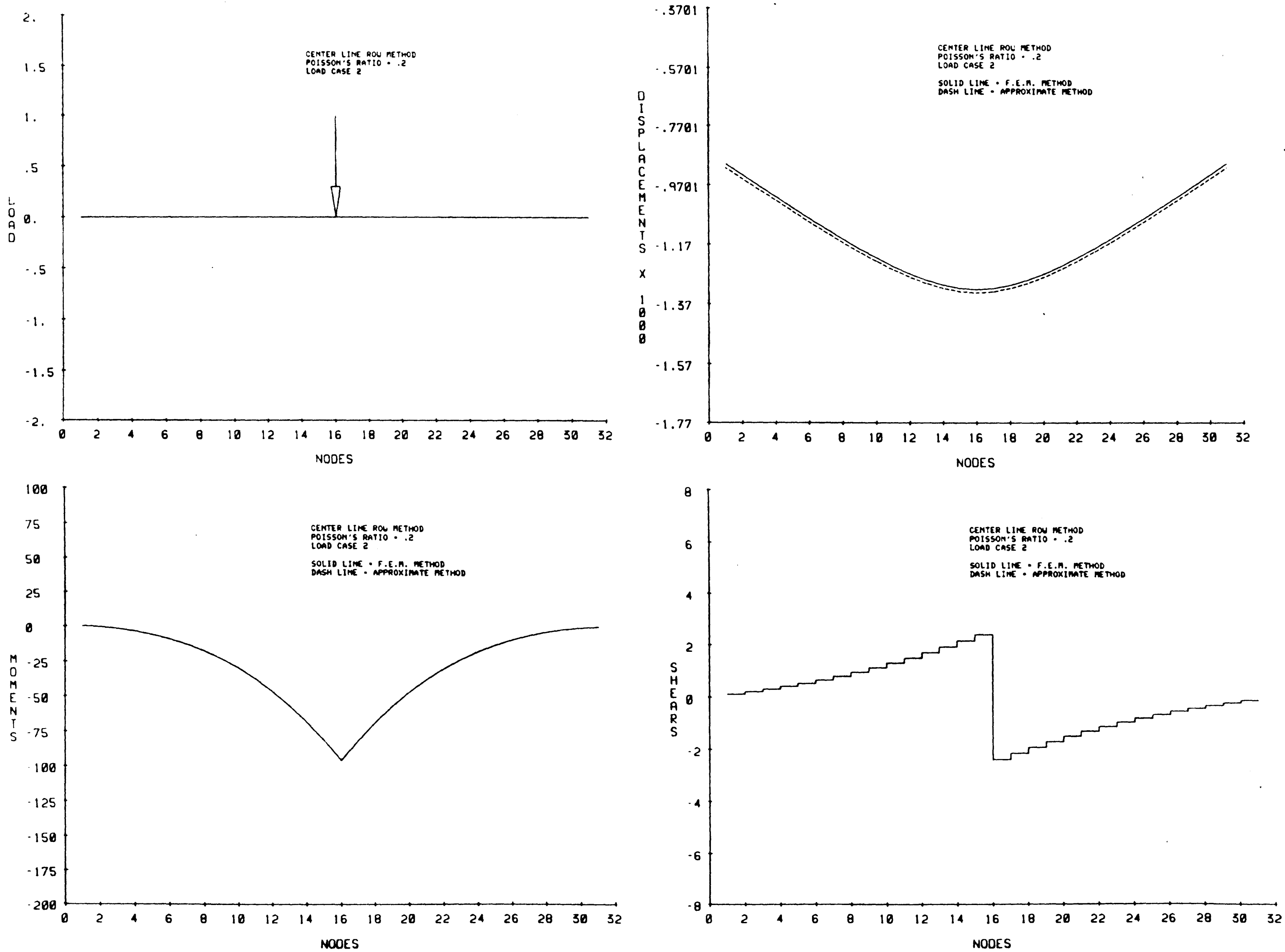
EQUAL TO 7000 PSI

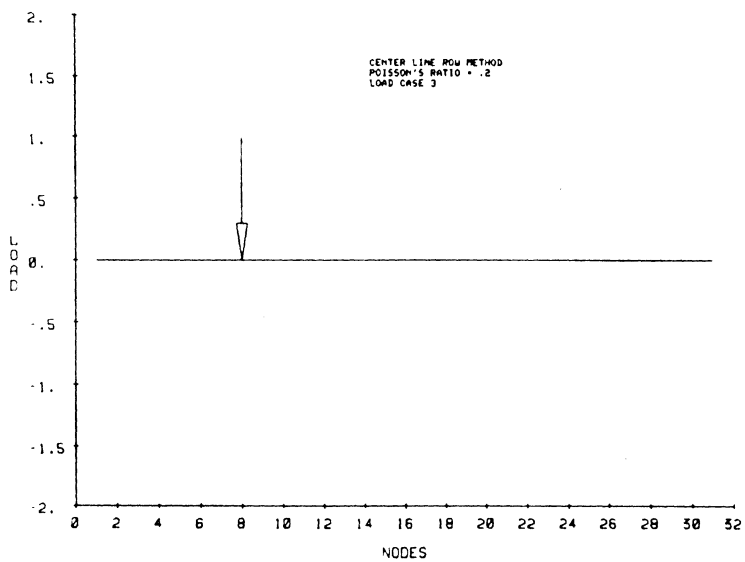

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 1
LOAD
NODES

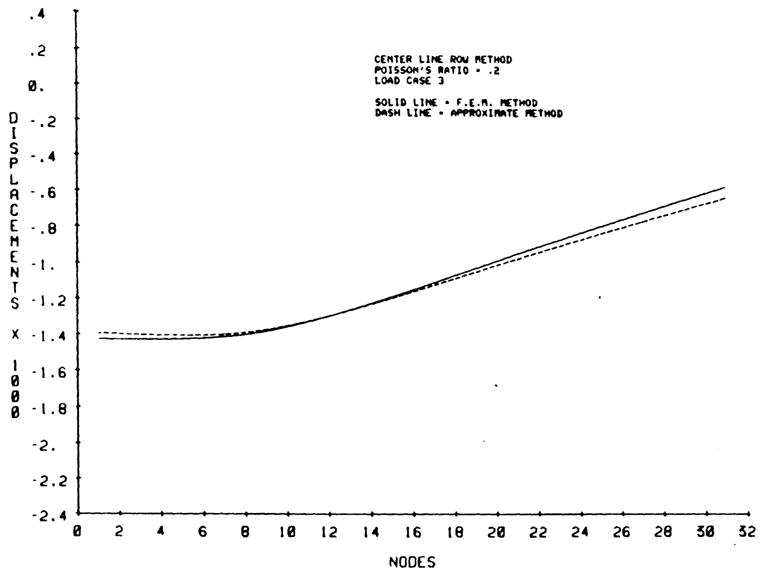

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES

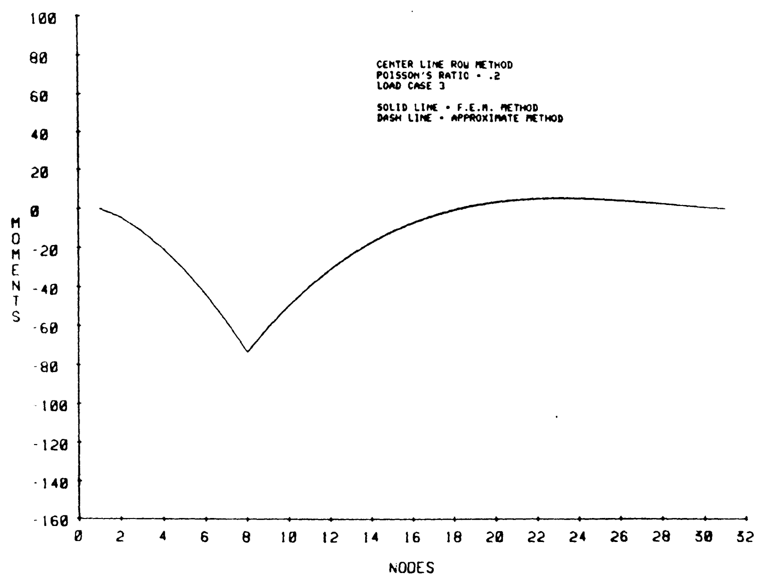

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES

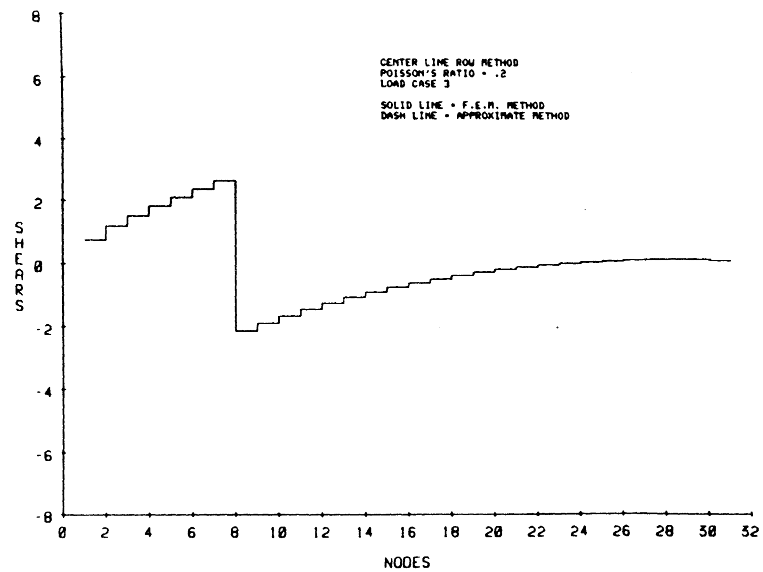

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES

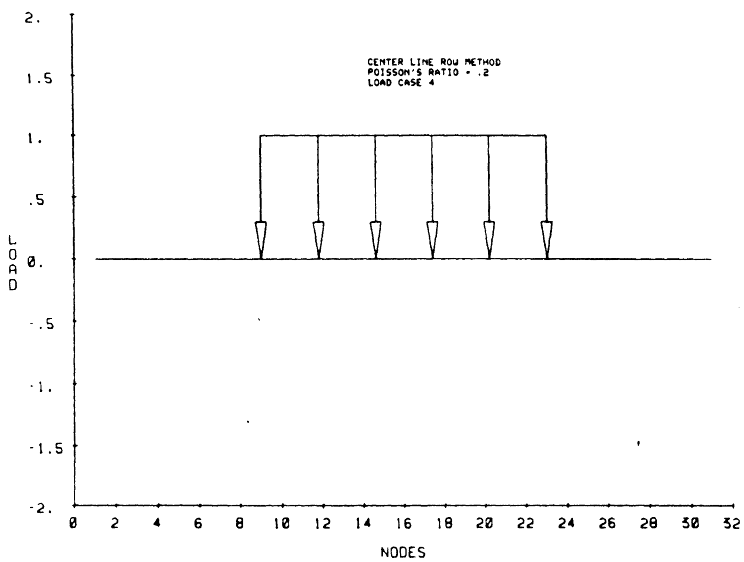

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 2
LOAD
2.
1.5
1.
.5
0.
-.5
-1.
-1.5
-2.
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
NODES
CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
-.3701
-.5701
-.7701
-.9701
-1.17
-1.37
-1.57
-1.77
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
NODES
CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
100
75
50
25
0
-25
-50
-75
-100
-125
-150
-175
-200
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
NODES
CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
8
6
4
2
0
-2
-4
-6
-8
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
NODES

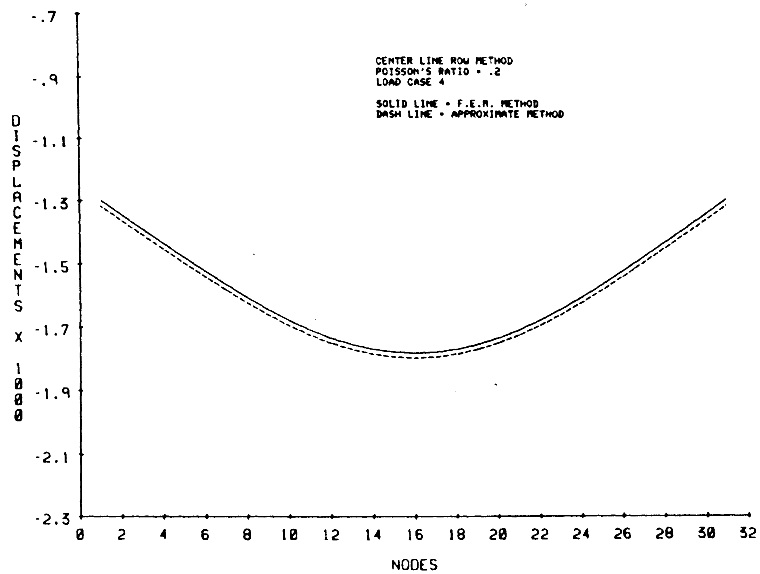

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 3
LOAD
NODES

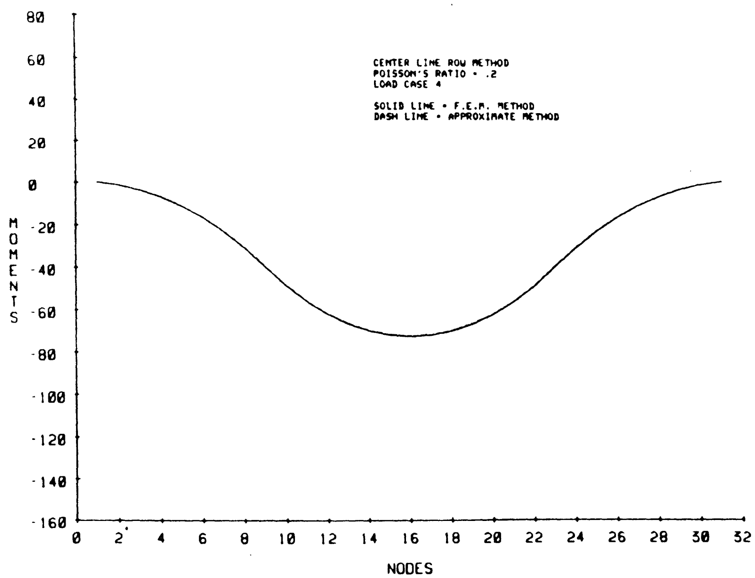

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES

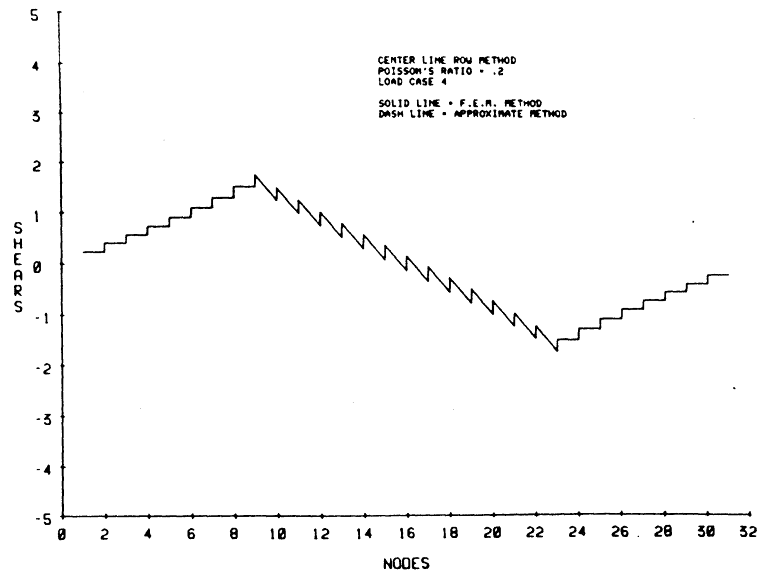

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES

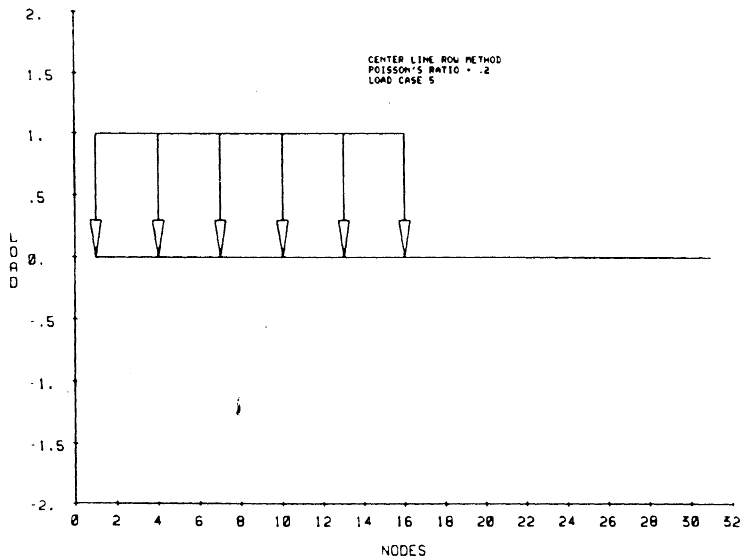

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES

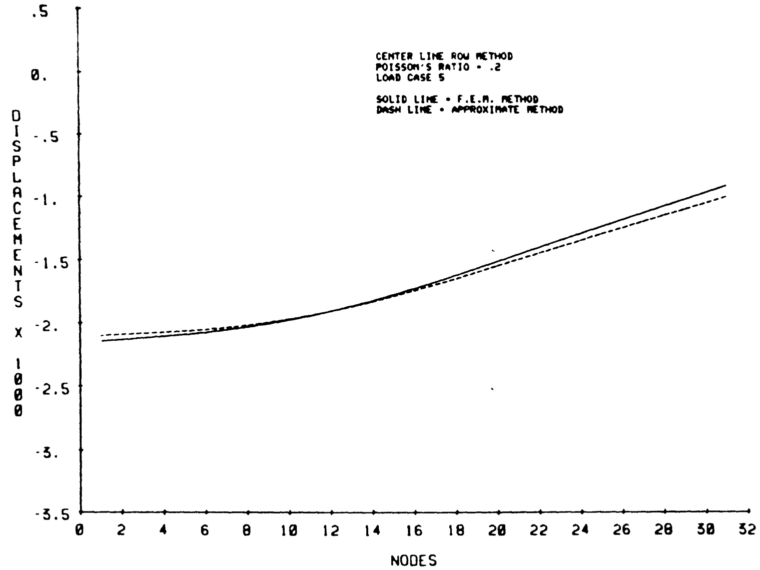

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 4
LOAD
2.
1.5
1.
.5
0.
-.5
-1.
-1.5
-2.
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
NODES

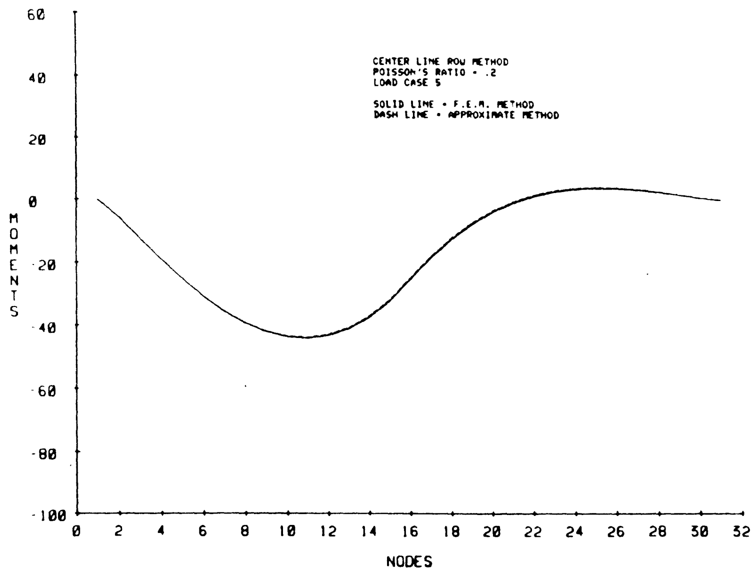

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
-.7
-.9
-1.1
-1.3
-1.5
-1.7
-1.9
-2.1
-2.3
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
NODES

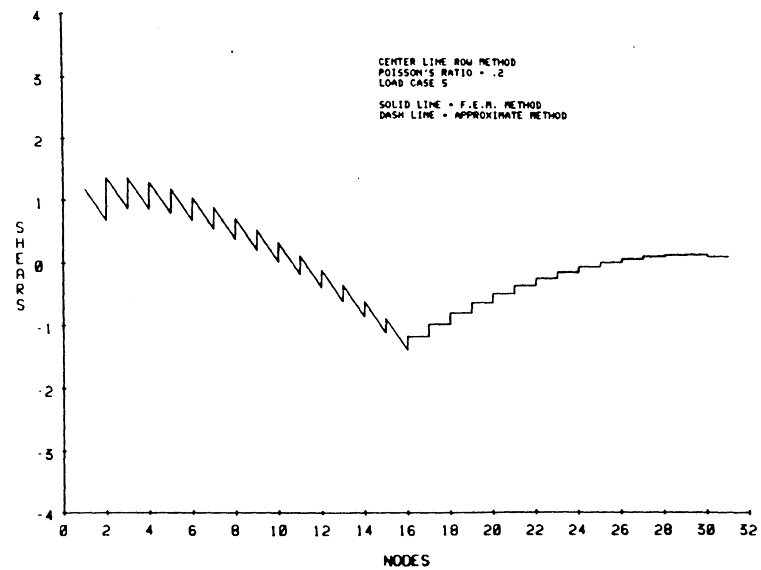

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
NODES


CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
5
4
3
2
1
0
-1
-2
-3
-4
-5
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
NODES


CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 5
LOAD
NODES


CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES


CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES

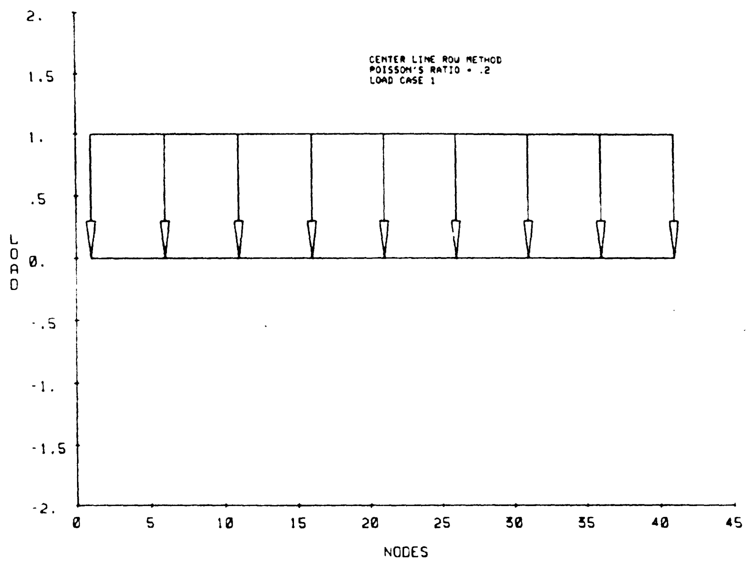

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES

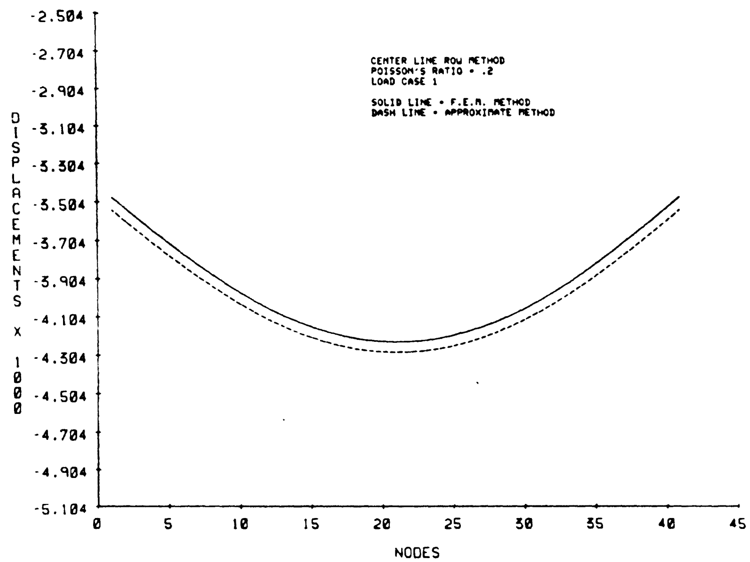
APPENDIX I

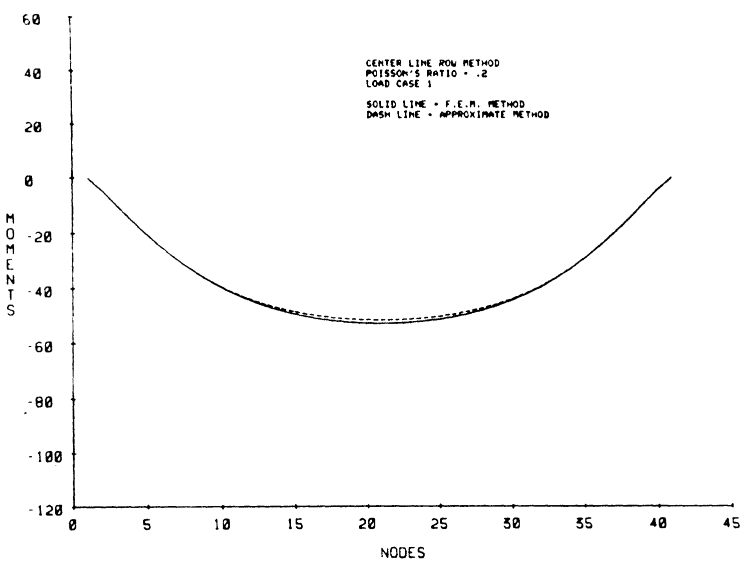
41 NODE BEAM ON FOUNDATION WITH POISSON'S RATIO

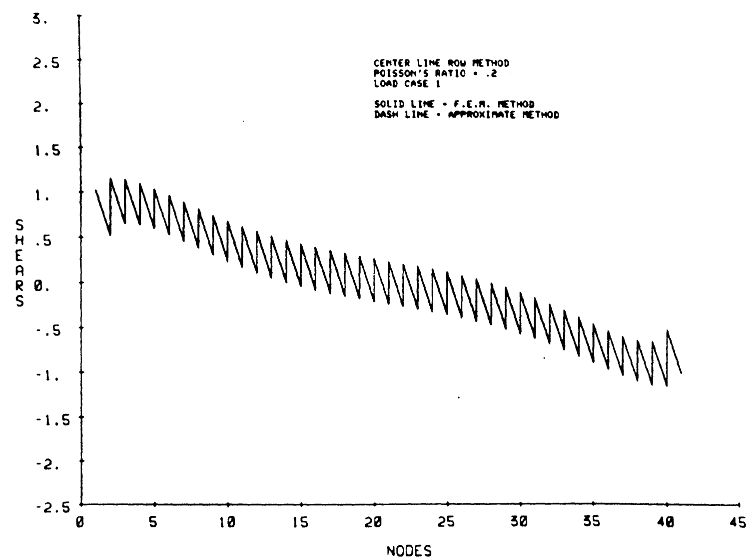
OF 0.2 AND MODULUS OF ELASTICITY

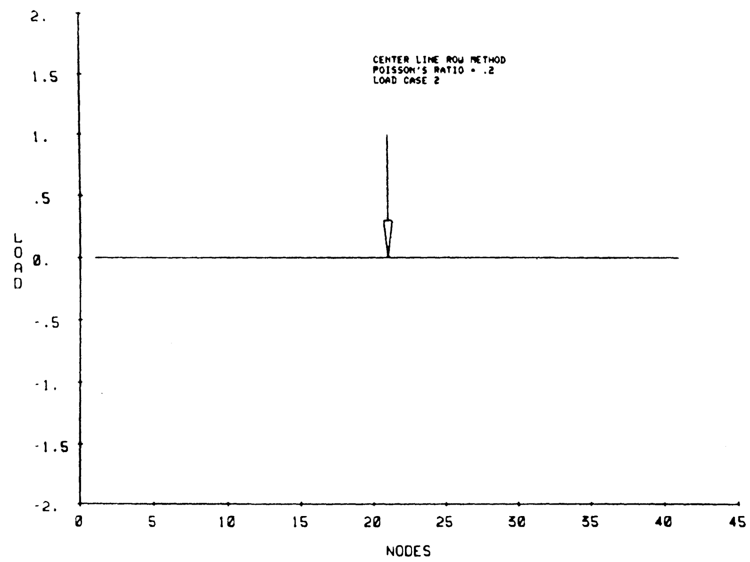
EQUAL TO 7000 PSI

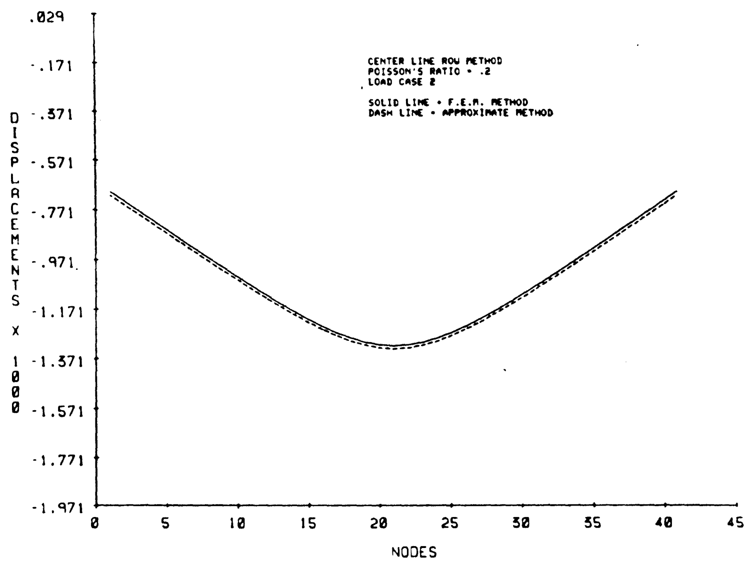

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 1
LOAD
2.
1.5
1.
.5
0.
-.5
-1.
-1.5
-2.
0
5
10
15
20
25
30
35
40
45
NODES

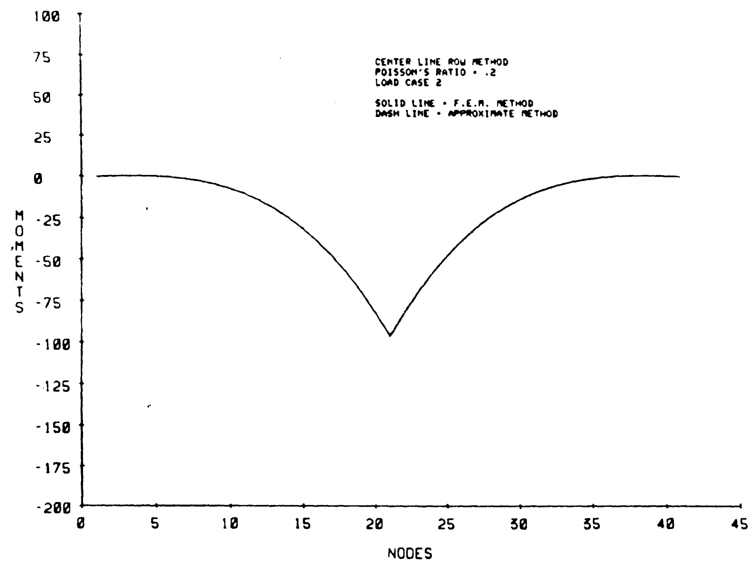

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS x 1000
-2.504
-2.704
-2.904
-3.104
-3.304
-3.504
-3.704
-3.904
-4.104
-4.304
-4.504
-4.704
-4.904
-5.104
0
5
10
15
20
25
30
35
40
45
NODES

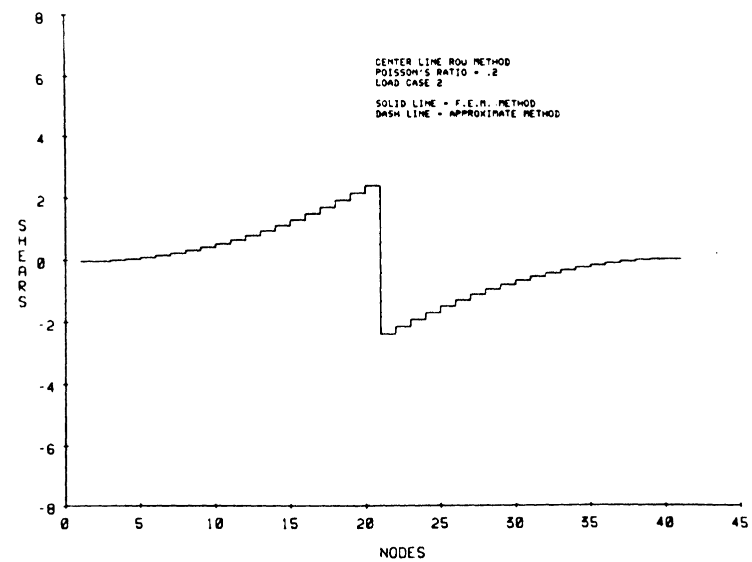

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
60
40
20
0
-20
-40
-60
-80
-100
-120
0
5
10
15
20
25
30
35
40
45
NODES

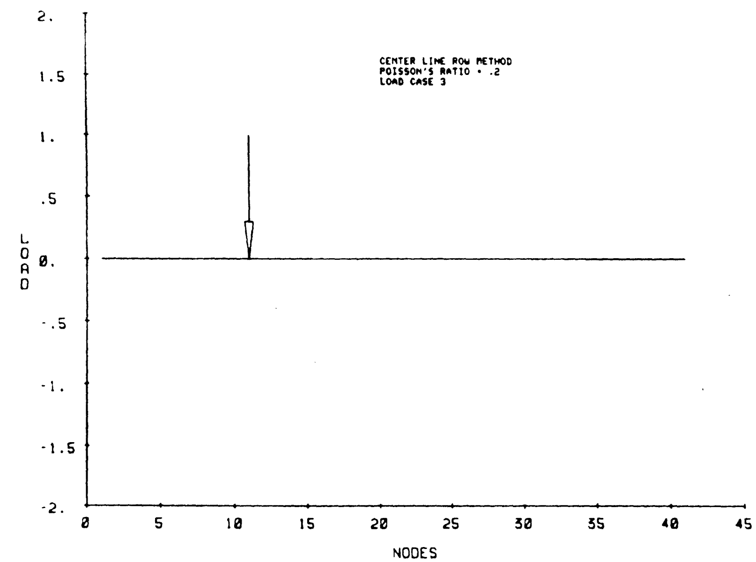

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
3.
2.5
2.
1.5
1.
.5
0.
-.5
-1.
-1.5
-2.
-2.5
0
5
10
15
20
25
30
35
40
45
NODES

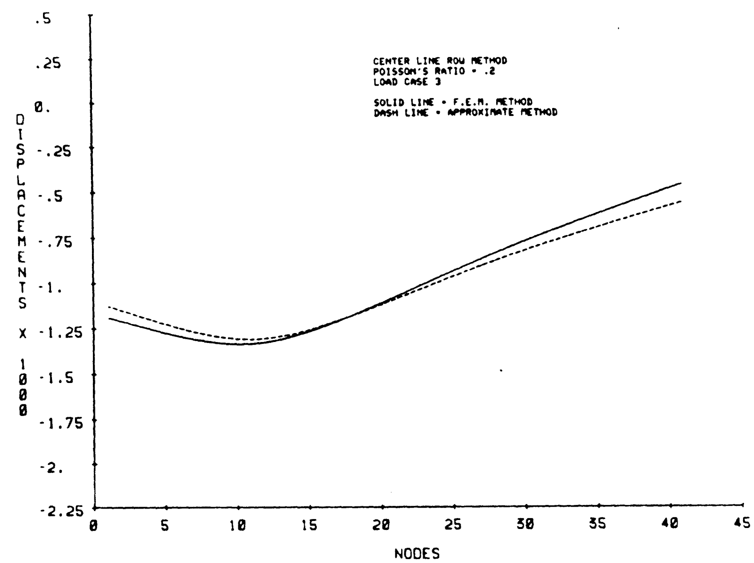

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 2
LOAD
2.
1.5
1.
.5
0.
-.5
-1.
-1.5
-2.
0
5
10
15
20
25
30
35
40
45
NODES

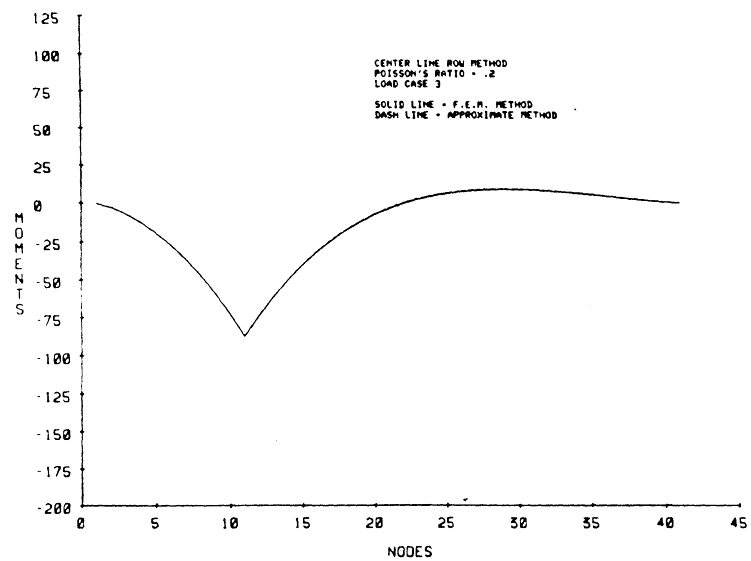

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
.029
-.171
-.371
-.571
-.771
-.971
-1.171
-1.371
-1.571
-1.771
-1.971
0
5
10
15
20
25
30
35
40
45
NODES

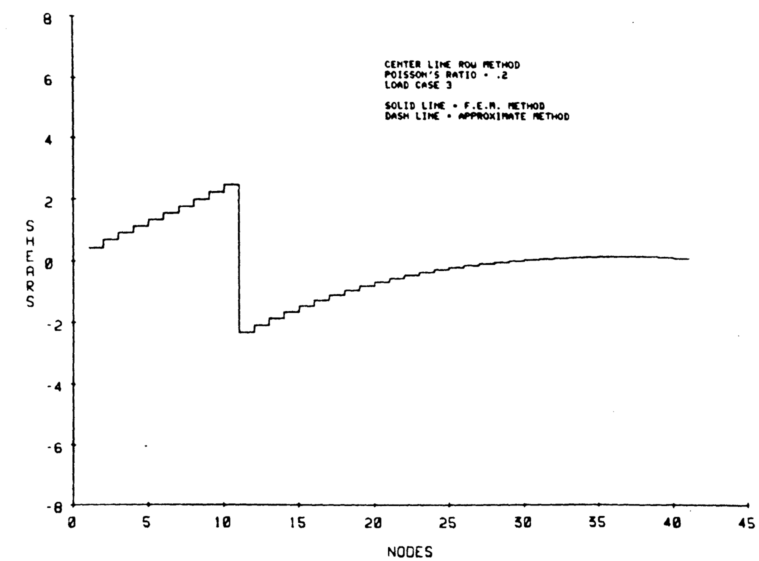

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
100
75
50
25
0
-25
-50
-75
-100
-125
-150
-175
-200
0
5
10
15
20
25
30
35
40
45
NODES

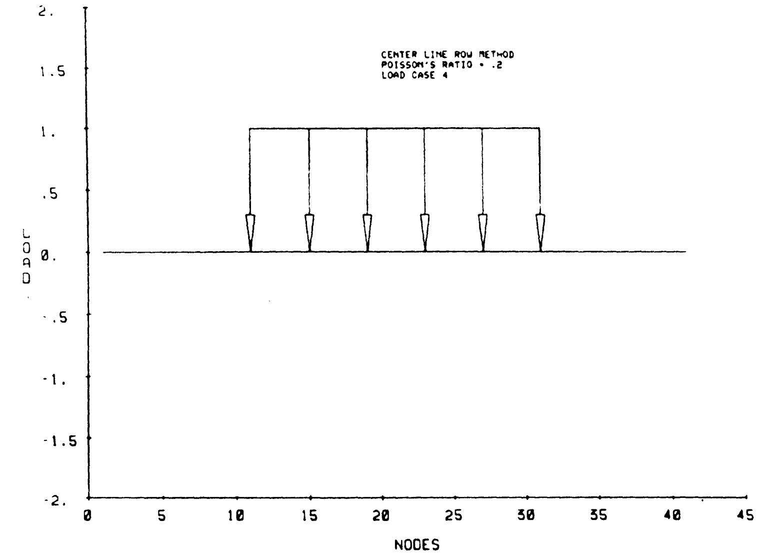

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
8
6
4
2
0
-2
-4
-6
-8
0
5
10
15
20
25
30
35
40
45
NODES

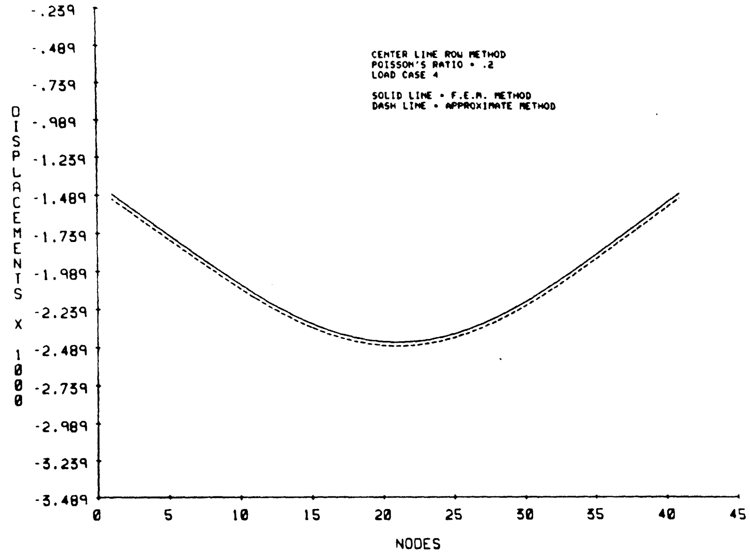

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 3
LOAD
NODES

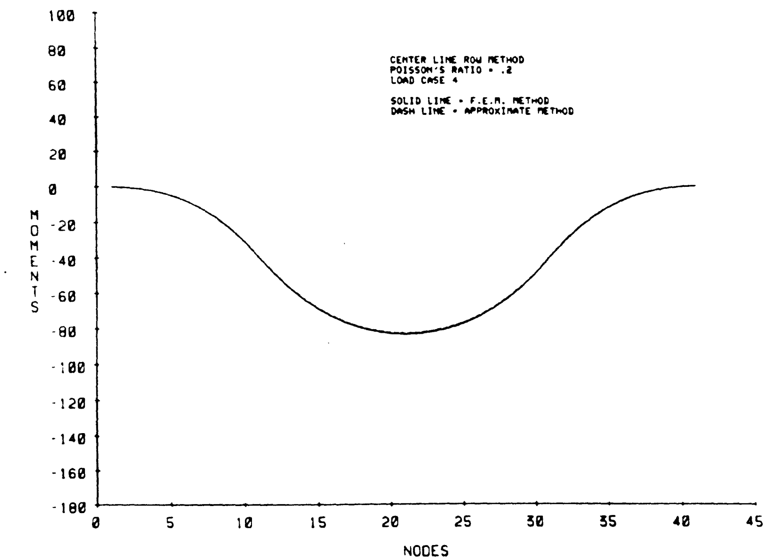

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES

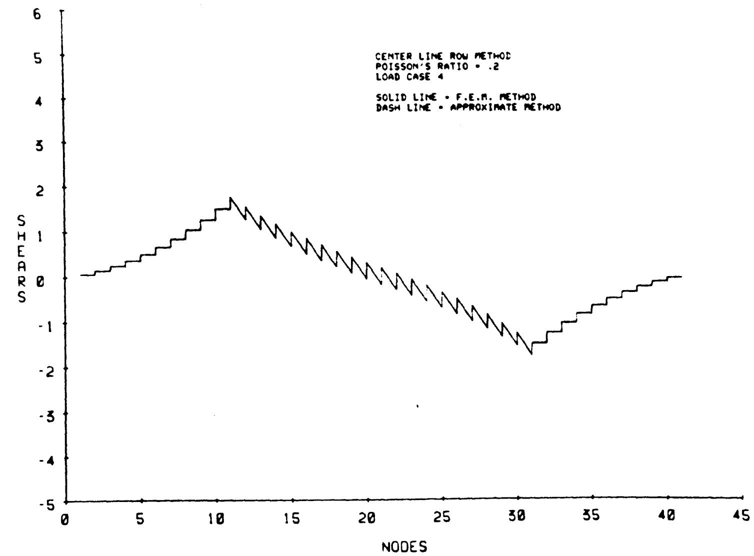

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES

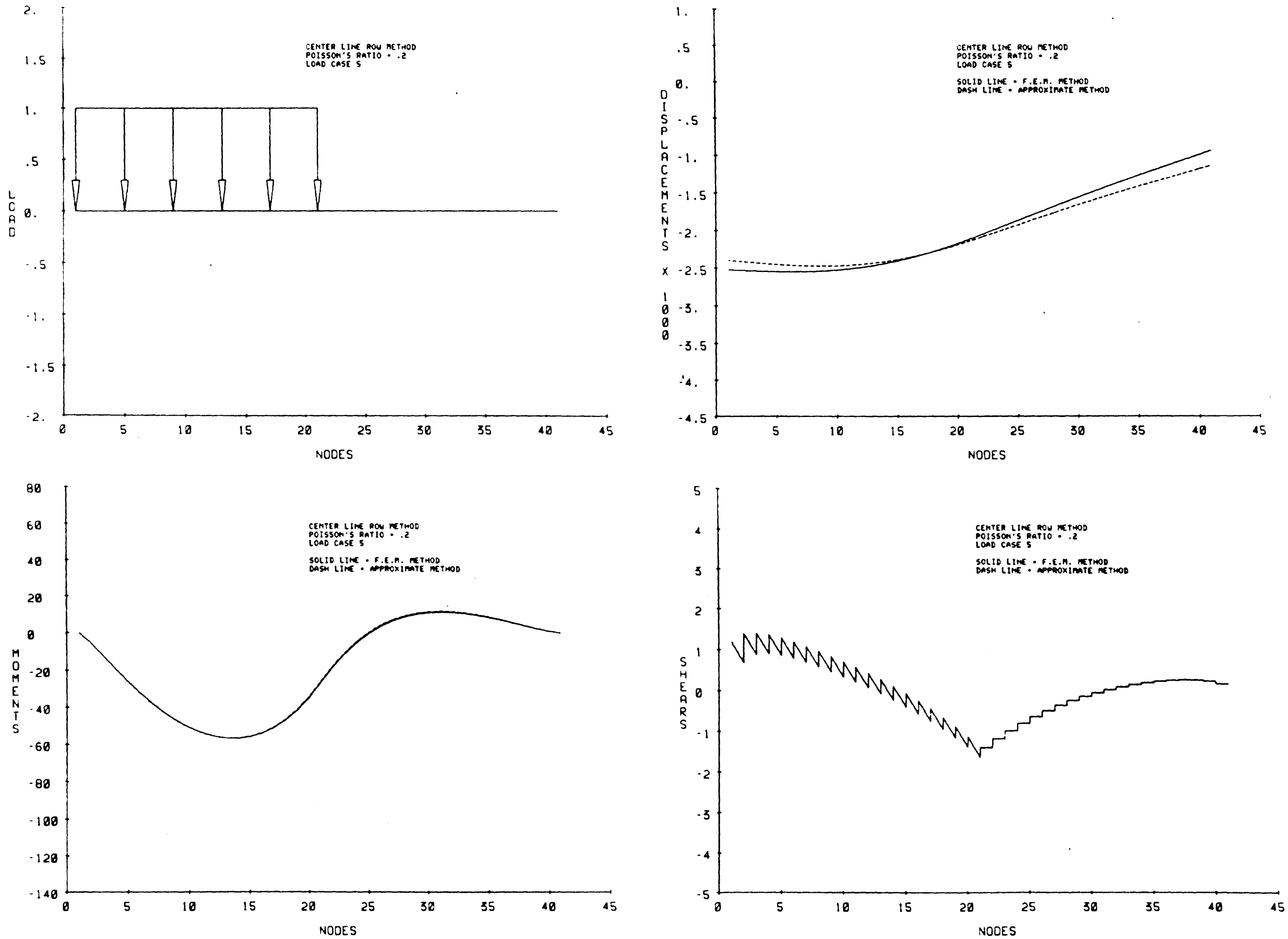

CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES


CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 4
LOAD
NODES


CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES


CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES


CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES


CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 5
LOAD
NODES
CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES
CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES
CENTER LINE ROW METHOD
POISSON'S RATIO = .2
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES

APPENDIX J

SEQUENTIAL SEARCH METHOD BY NELDER AND MEAD

The sequential search method was used to find the coefficients for given functions to fit a row of the foundation stiffness matrix in a least squares sense. This sequential search method is an optimization process known as the flexible simplex search developed by Nelder and Mead (23, 24).

This procedure considers an objective function, f, defined throughout an N-dimensional space with coordinates $(X_1, X_2, X_3, \ldots, X_n)$. The search begins with an initial point and generates N more points with a total of N + 1 corner points (called a simplex). Each corner point defined in the N-dimensional space is systematically replaced by new ones in order to make the objective function produce a minimum value. As the method produces points which are producing smaller objective function evaluations, the simplex becomes smaller until the desired minimum is obtained.

The interactive procedure of replacing the corner points uses three moves called reflection, expansion, and contraction. For each trial point which is a corner to the simplex, the objective function is evaluated. The corners are then numbered $Z_o \ldots Z_n$, where the smallest function value corresponds to F_o and the largest function value corresponds to F_n. The centroid, $\bar{Z}$, is calculated from points $Z_o, \ldots, Z_{n-1}$. A reflected point, Z_r, is calculated as follows:

$$Z_r = \bar{Z} + A(\bar{Z} - Z_n)$$

where A is a reflection coefficient (positive number). The objective function evaluation of this point is labeled F_r:

$$F_r = f(Z_r)$$

The magnitude of F_r is examined to determine further steps according to the following cases:

Case 1: $F_o < F_r < F_{n-1}$

Case 2: $F_r < F_o$

Case 3: $F_{n-1} < F_r$.

For Case 1, replace Z_n with point Z_r and begin iteration. For Case 2, a better point has been found and an attempt is made to expand further along the same direction. The expanded point, Z_e, is calculated as follows:

$$Z_e = \bar{Z} + B(Z_n - \bar{Z})$$

where B is an expansion coefficient (positive number). The objective is evaluated for Z_e to determine F_e. If $F_e < F_o$, the expansion is accepted. Then Z_n is replaced with Z_e and iteration is begun. If $F_e > F_o$, the expansion is rejected and Z_n is replaced with Z_r. Then a new iteration is started.

For Case 3, the simplex is too large and a contraction point, Z_c, is calculated as follows:

For $F_n < F_r$: $Z_c = \bar{Z} + C(Z_n - \bar{Z})$

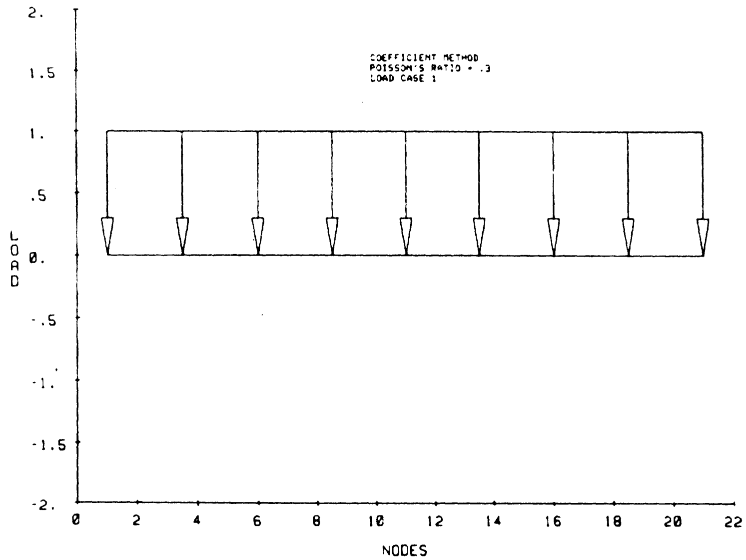
For $F_n > F_r$: $Z_c = \bar{Z} + C(Z_r - \bar{Z})$

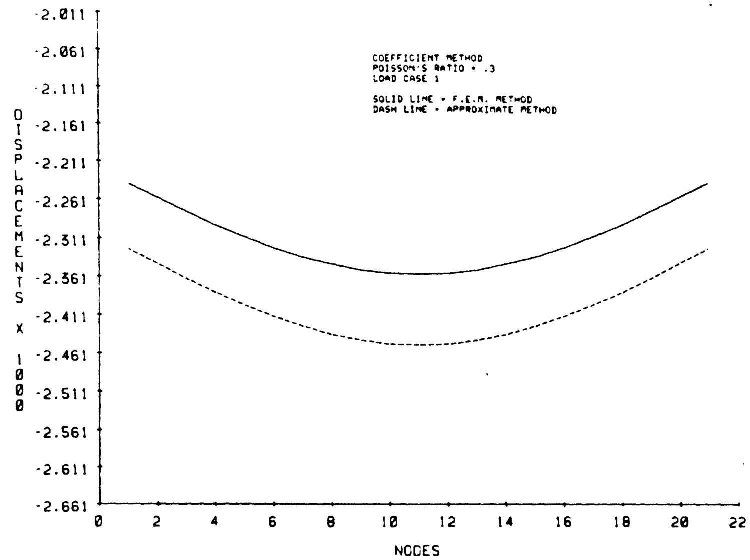
where C are contraction coefficients. The objective function is then evaluated to determine F_c. If $F_c < F_n < F_r$ or $F_c < F_r < F_n$, the contraction is completed by replacing Z_n with Z_c and starting iteration. Otherwise, contraction is made by moving each corner point halfway toward the point with the lowest function evaluation. Then a new iteration begins.

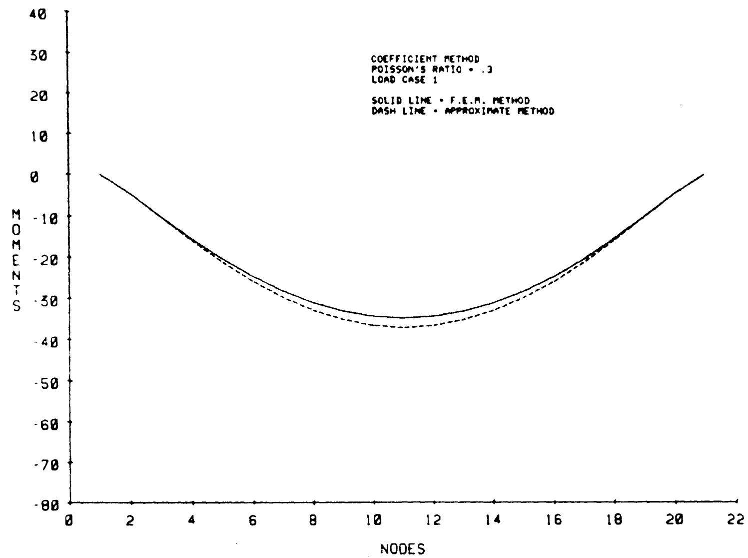
Convergence is met when the standard deviation of the function evaluations of the corner points is a fraction of the mean of the function evaluations of the corner points.

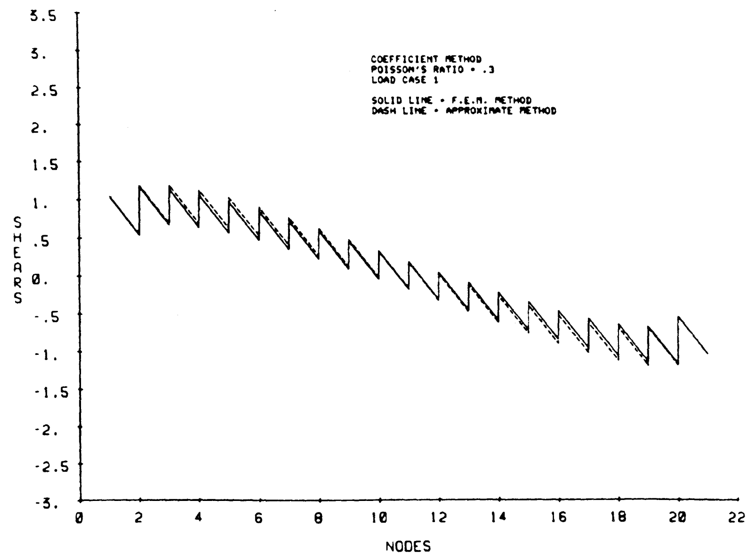
APPENDIX K

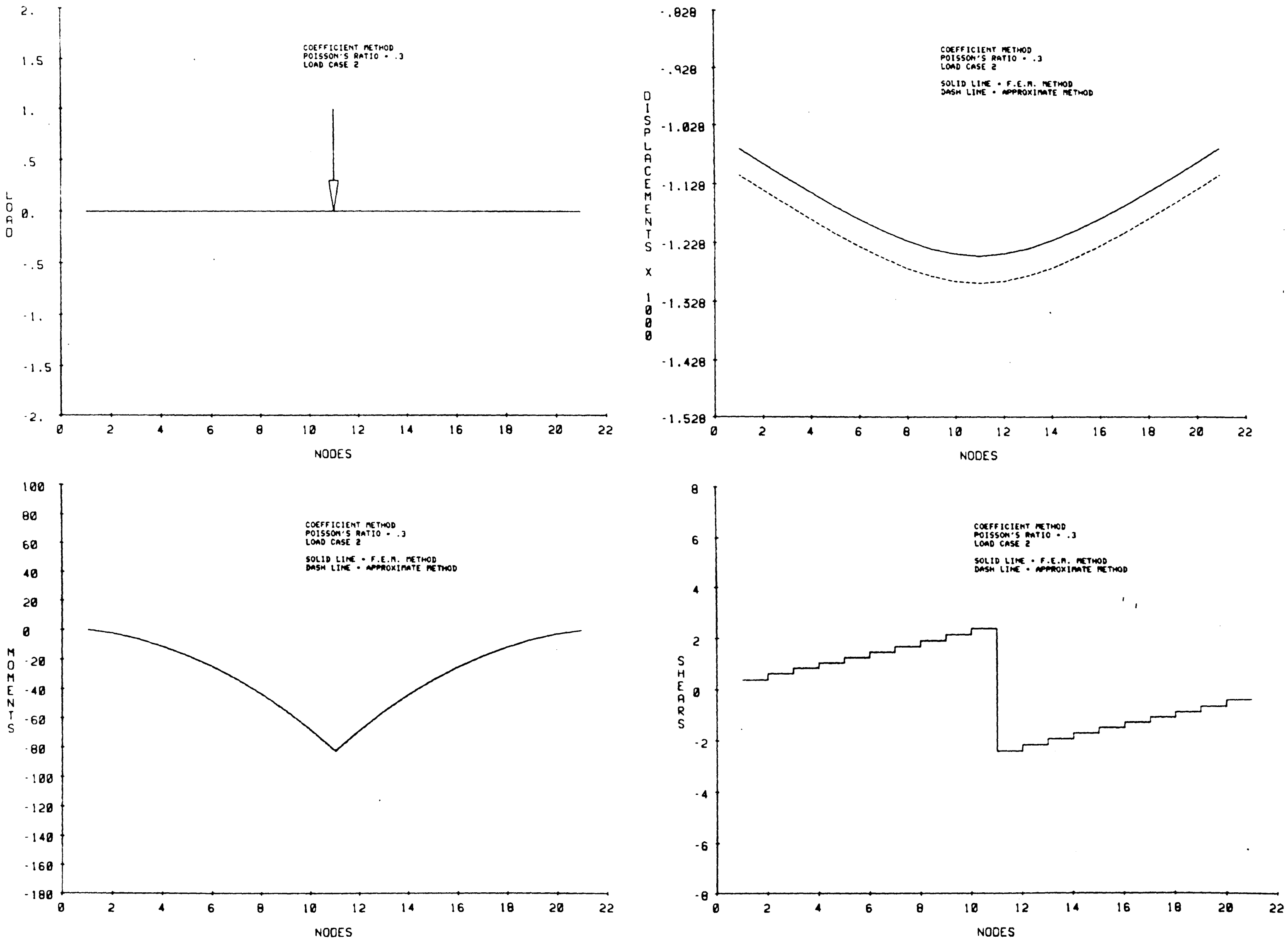
21 NODE BEAM ON FOUNDATION GENERATED FROM
COEFFICIENTS WITH POISSON'S RATIO OF
0.3 AND MODULUS OF ELASTICITY
EQUAL TO 7000 PSI

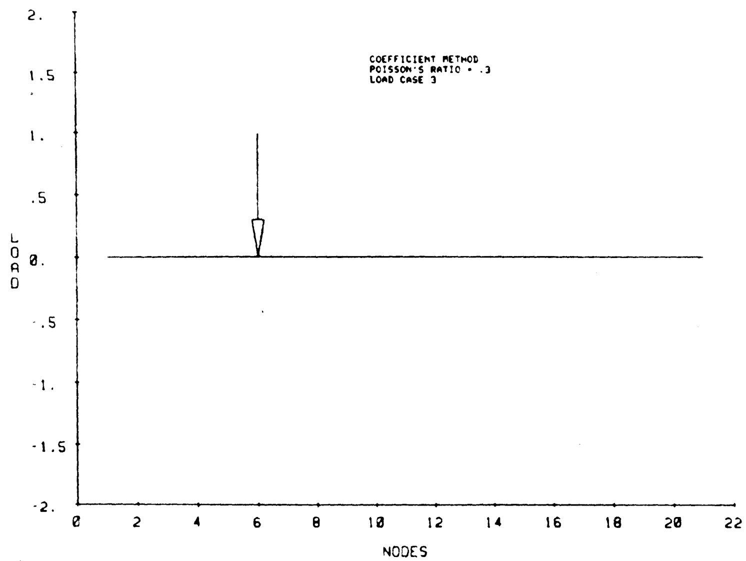

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 1
LOAD
NODES

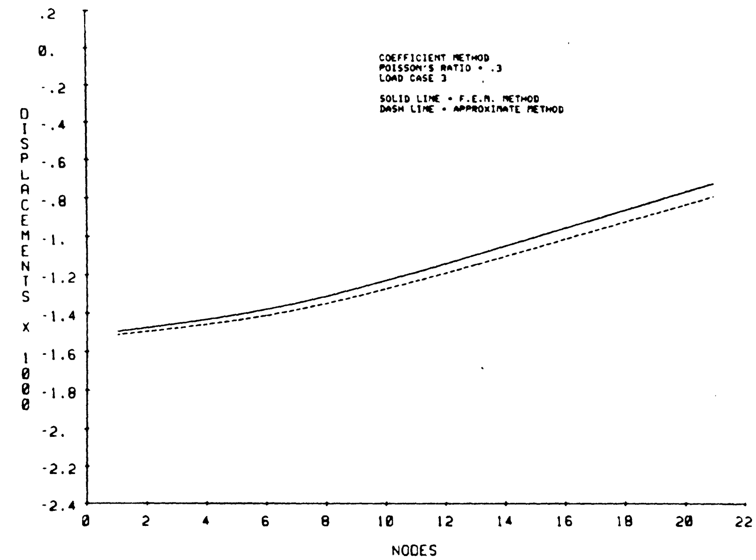

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES

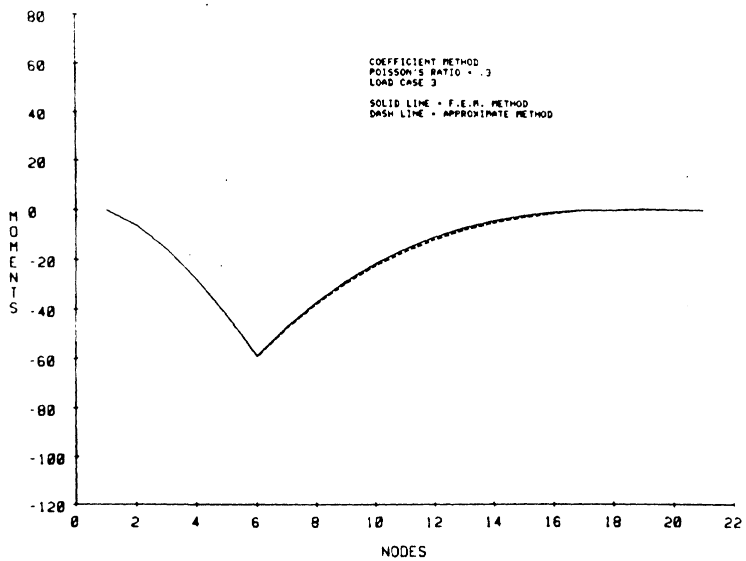

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES

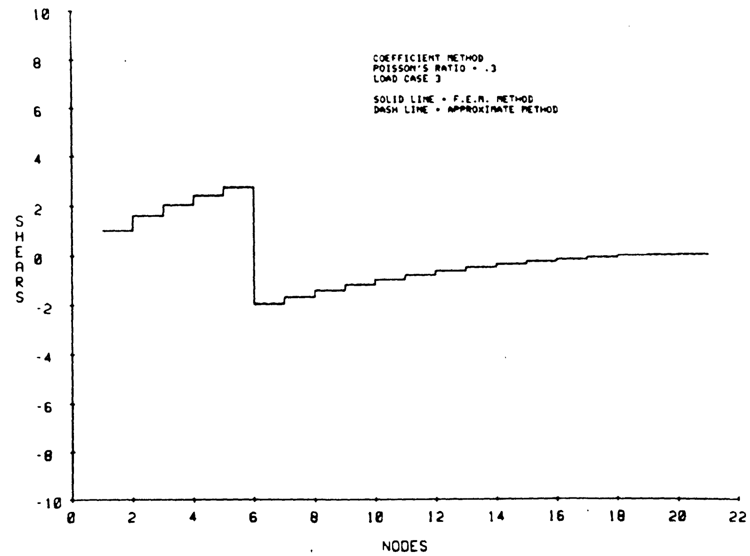

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES

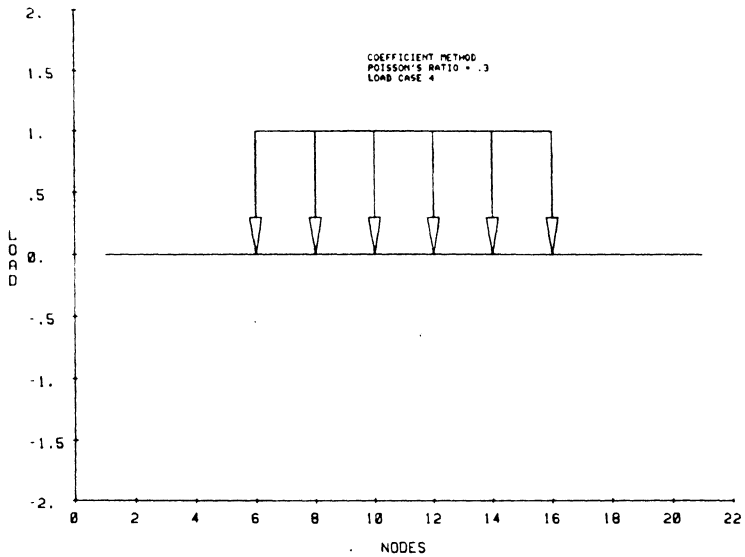

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 2
LOAD
NODES
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
MOMENTS
SHEARS

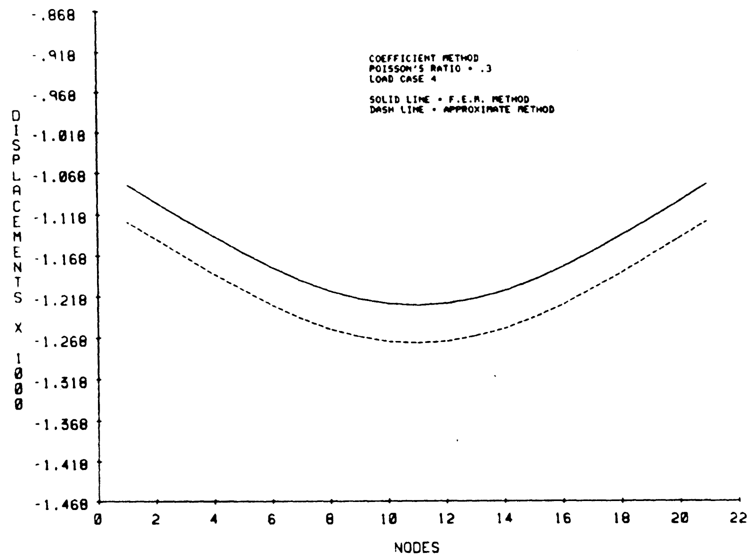

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 3
LOAD
NODES

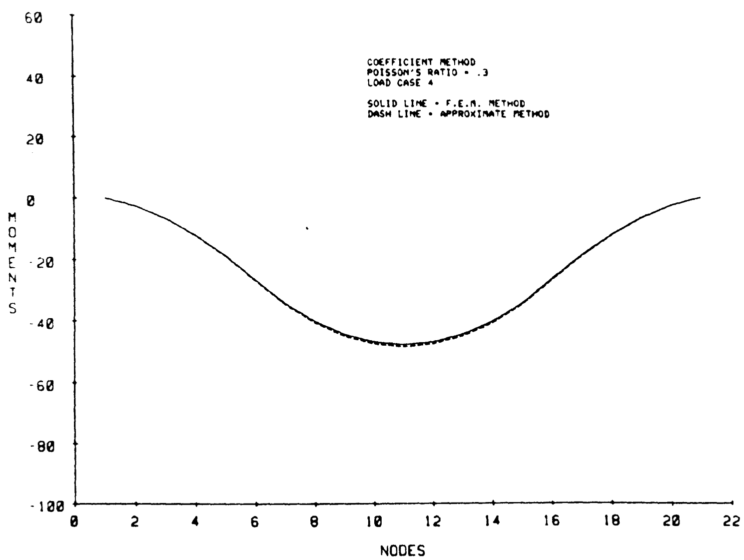

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS x 1000
NODES

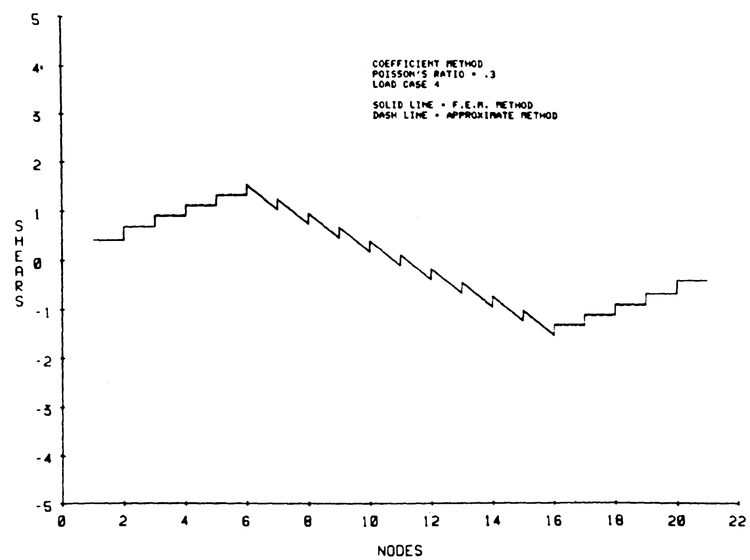

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES

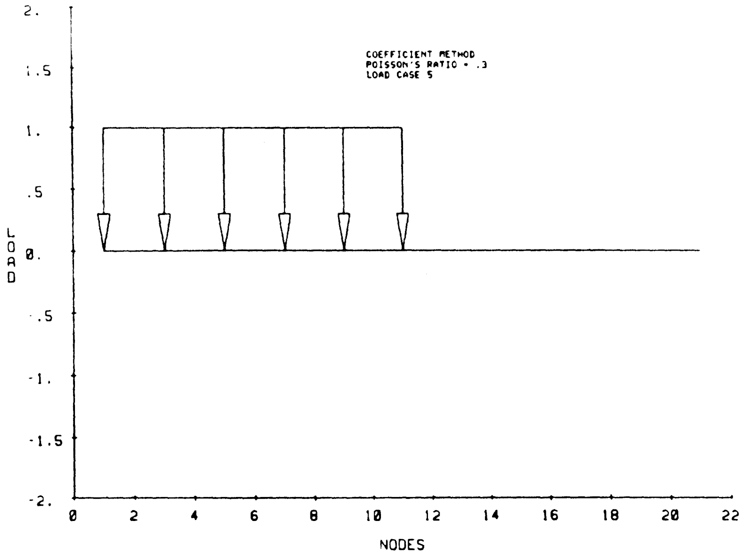

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES

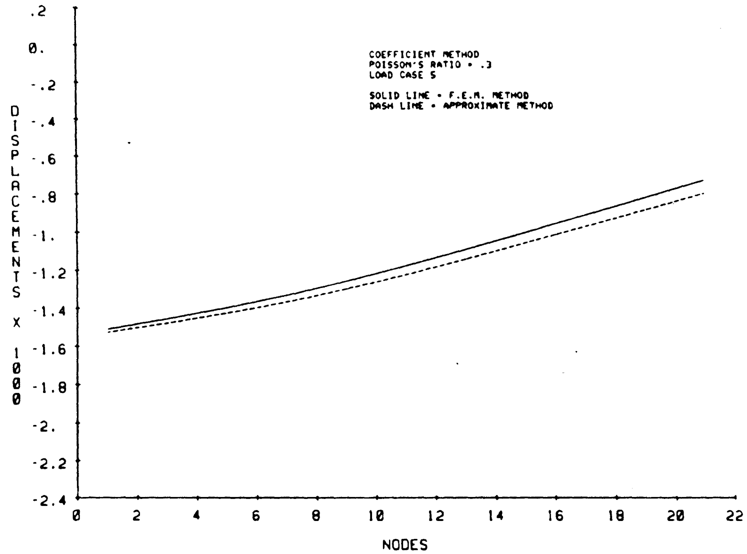

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 4
LOAD
NODES

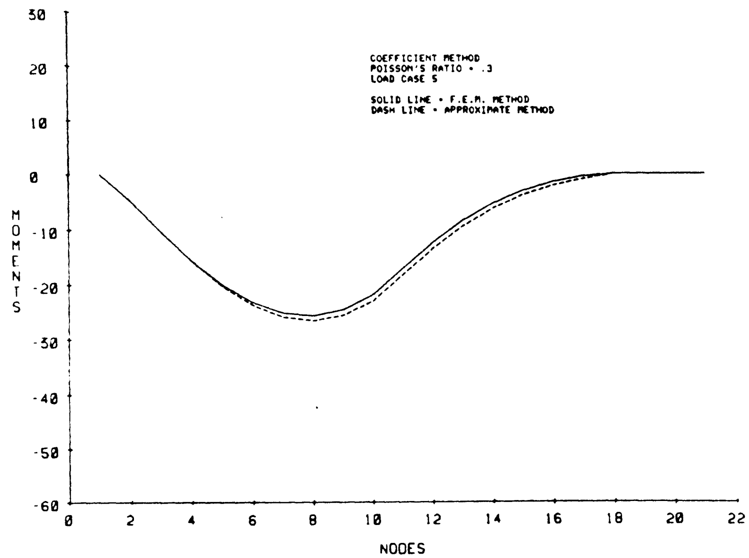

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES

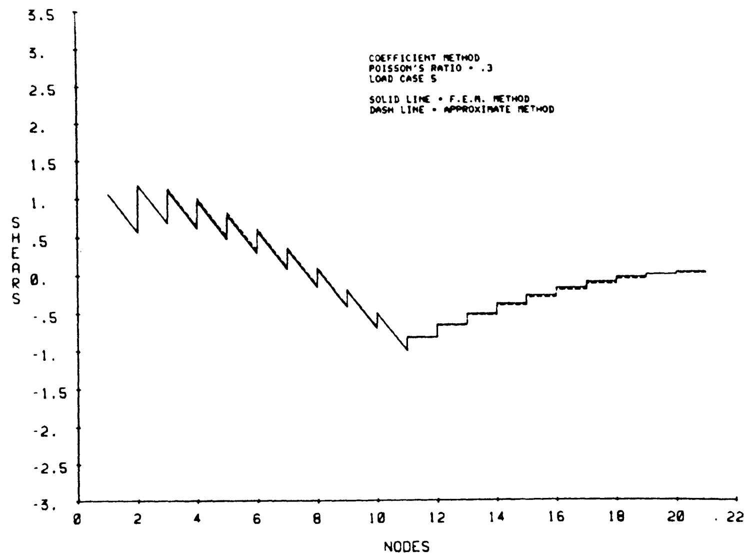

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES


COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES


COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 5
LOAD
NODES

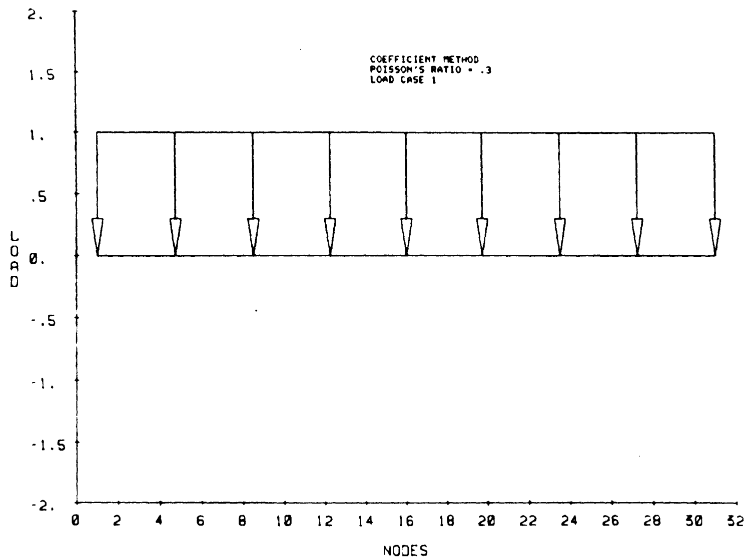

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES

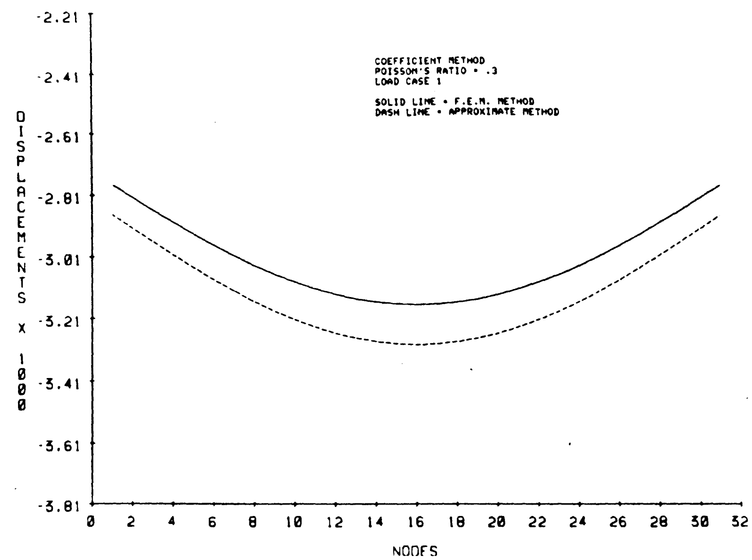

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES

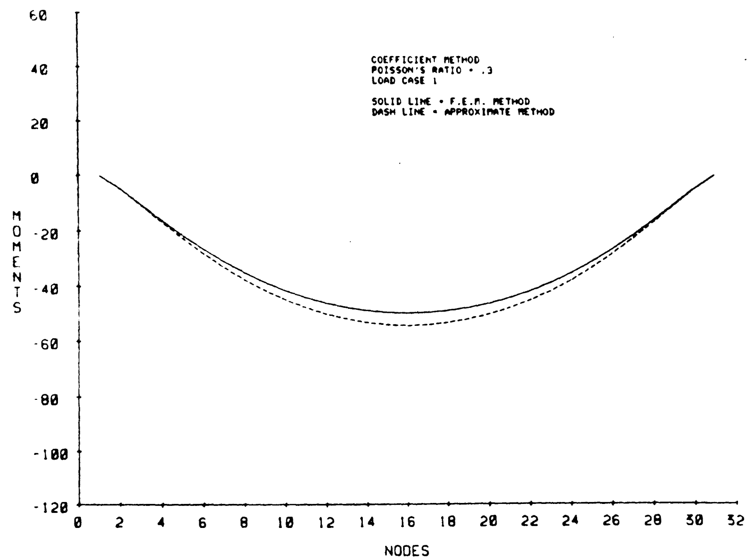

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES

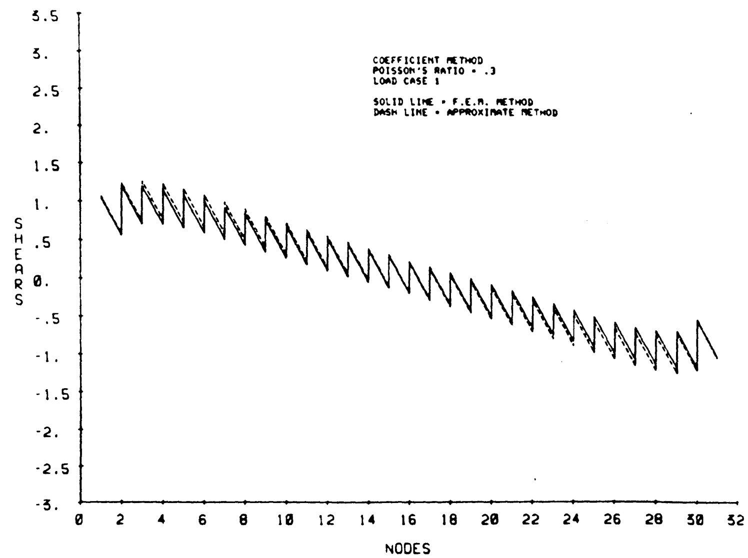
APPENDIX L

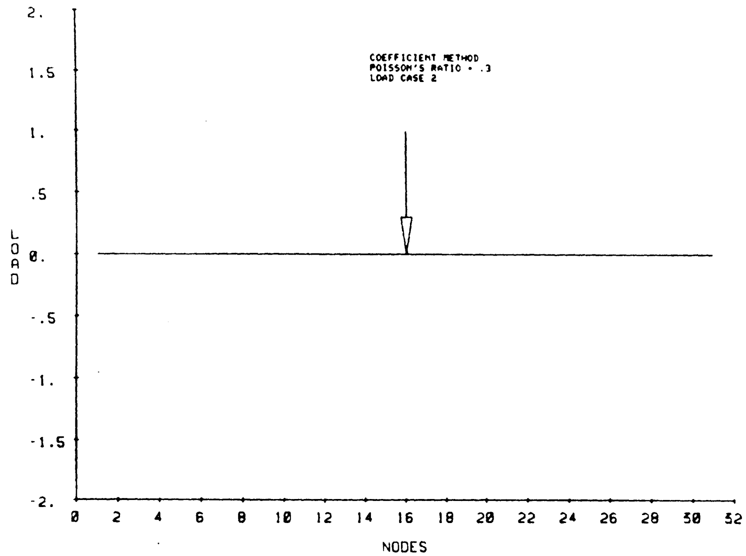
31 NODE BEAM ON FOUNDATION GENERATED FROM COEFFICIENTS WITH POISSON'S RATIO OF 0.3 AND MODULUS OF ELASTICITY EQUAL TO 7000 PSI

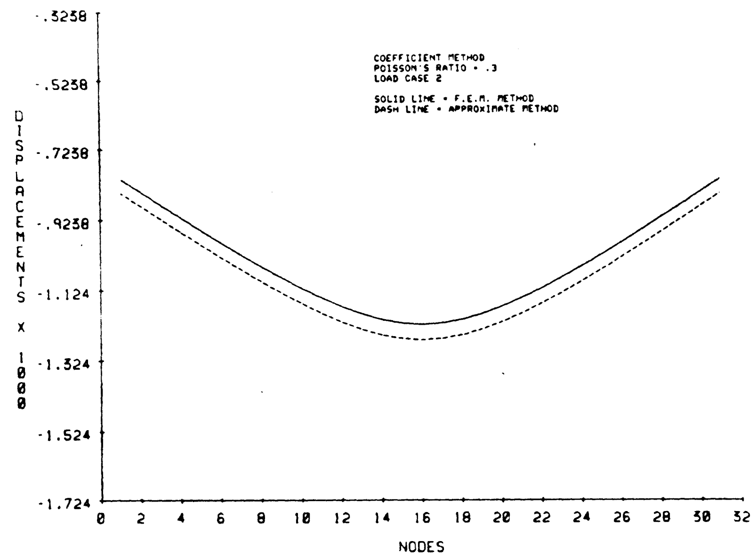

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 1
LOAD
NODES

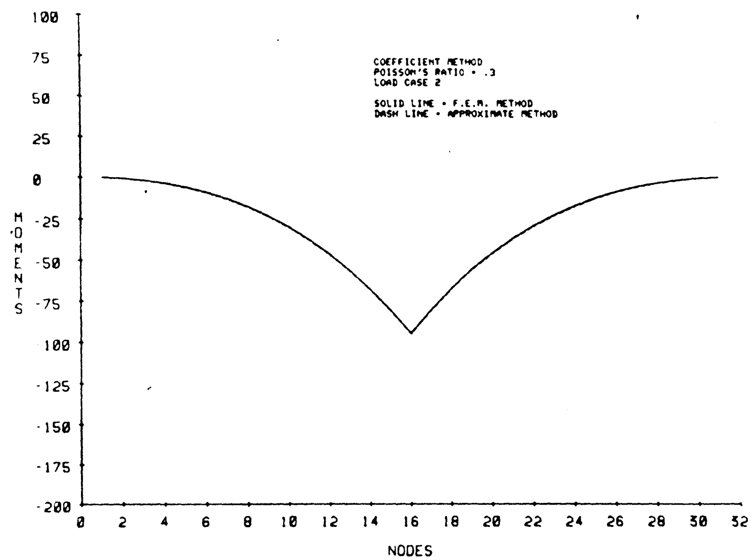

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES

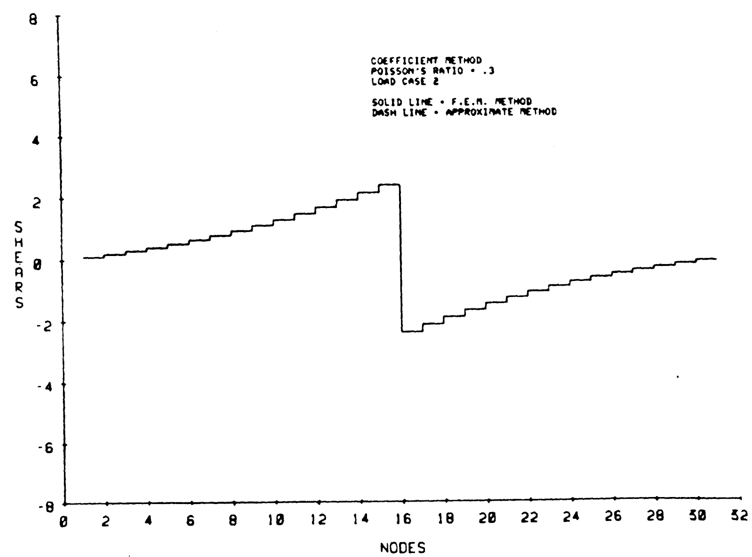

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES

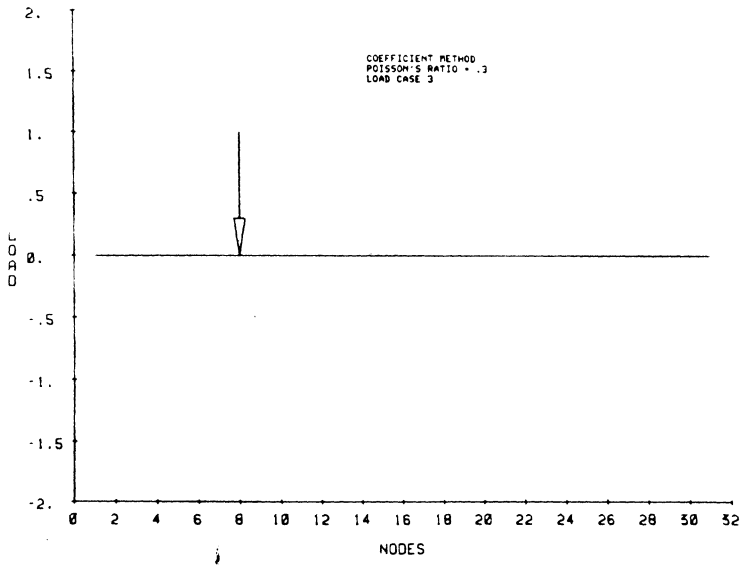

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES

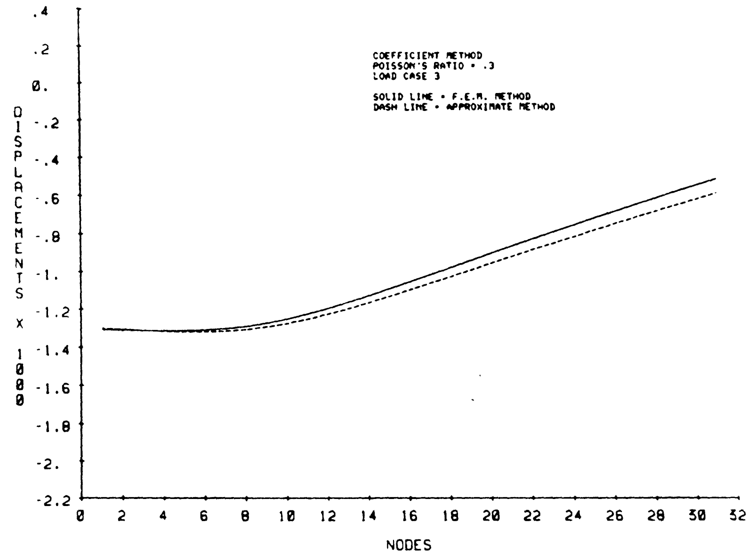

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 2
LOAD
2.
1.5
1.
.5
0.
-.5
-1.
-1.5
-2.
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
NODES

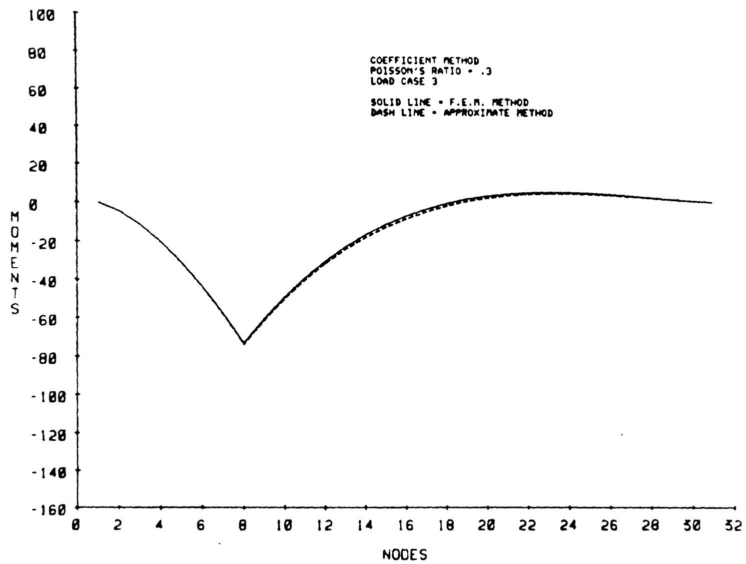

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
-.3258
-.5258
-.7258
-.9258
-1.124
-1.324
-1.524
-1.724
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
NODES

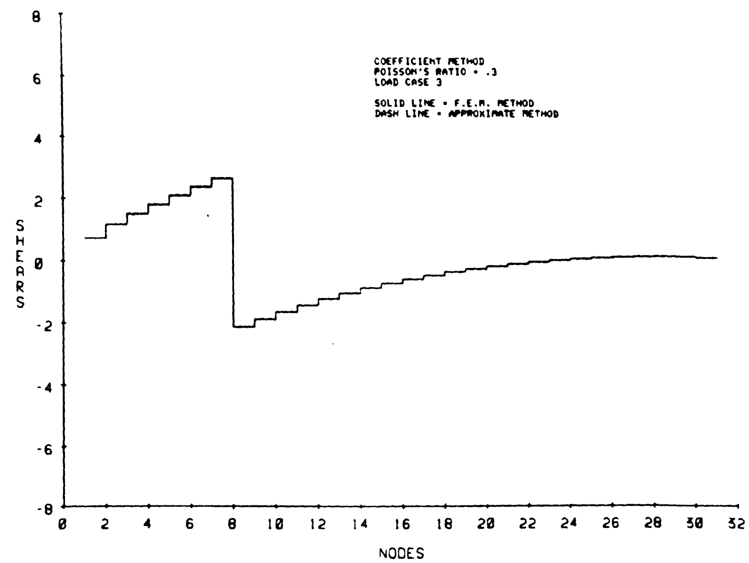

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
100
75
50
25
0
-25
-50
-75
100
-125
-150
-175
-200
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
NODES

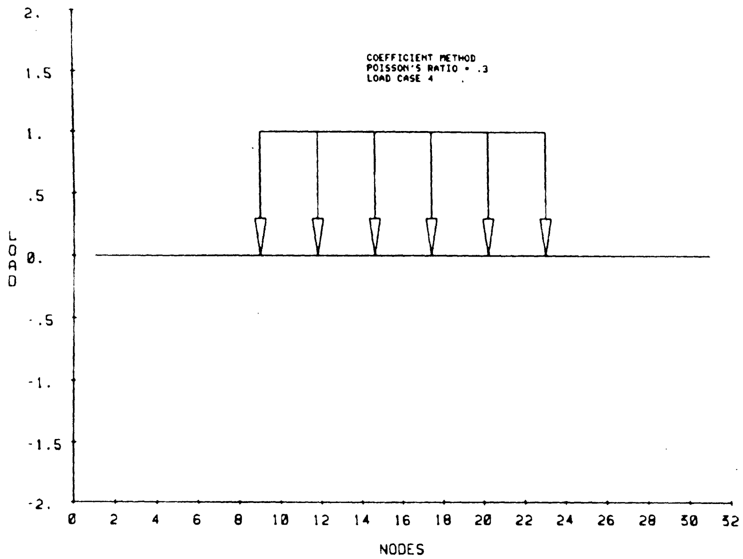

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
8
6
4
2
0
-2
-4
-6
-8
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
NODES

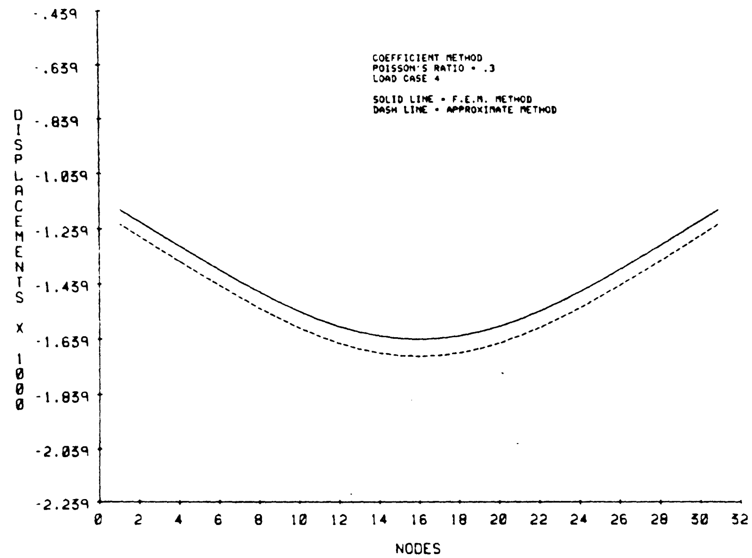

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 3
LOAD
NODES

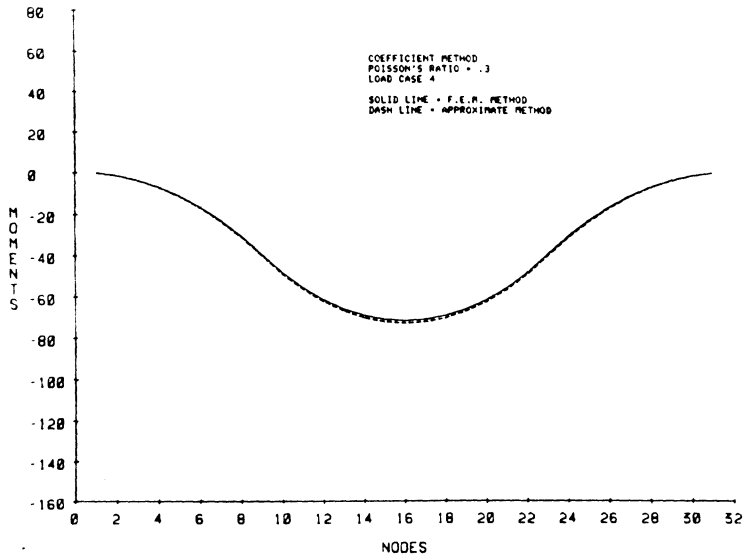

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES

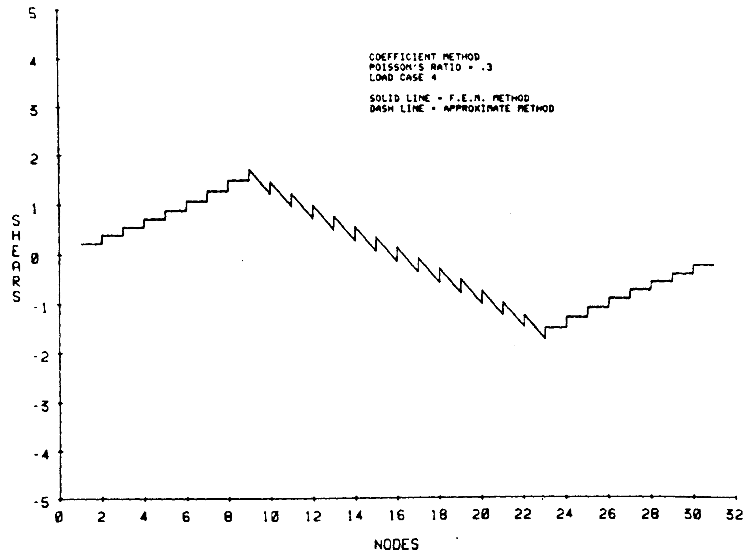

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES

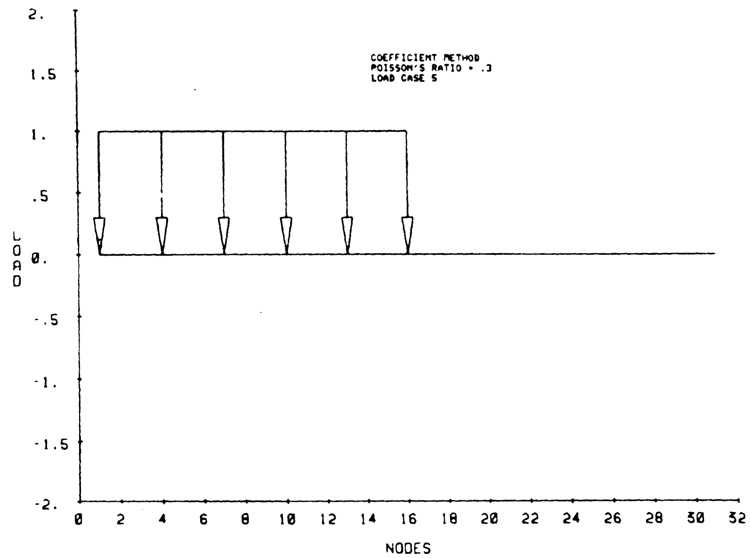

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES

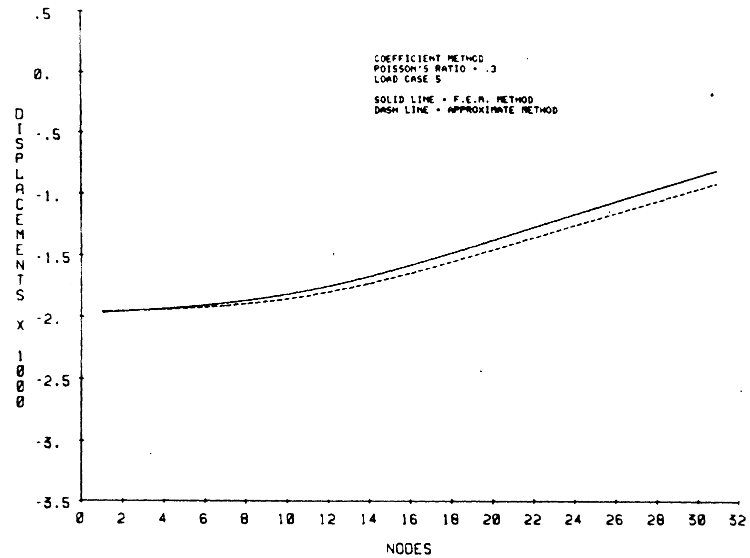

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 4
LOAD
NODES

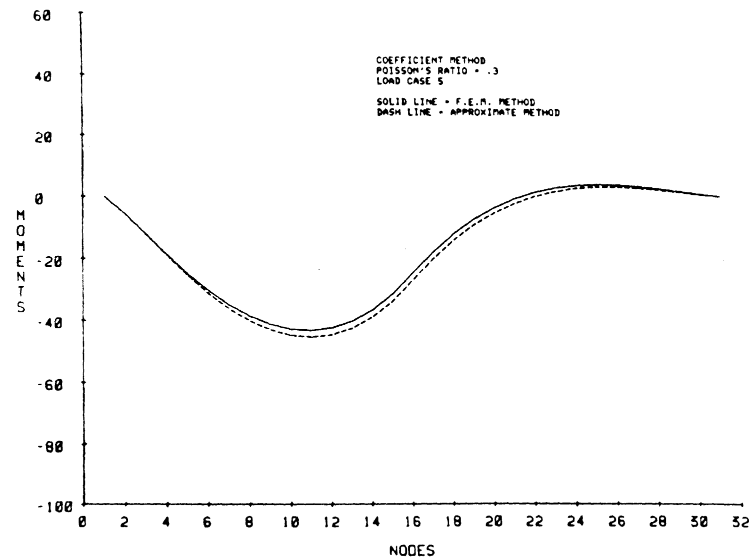

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES

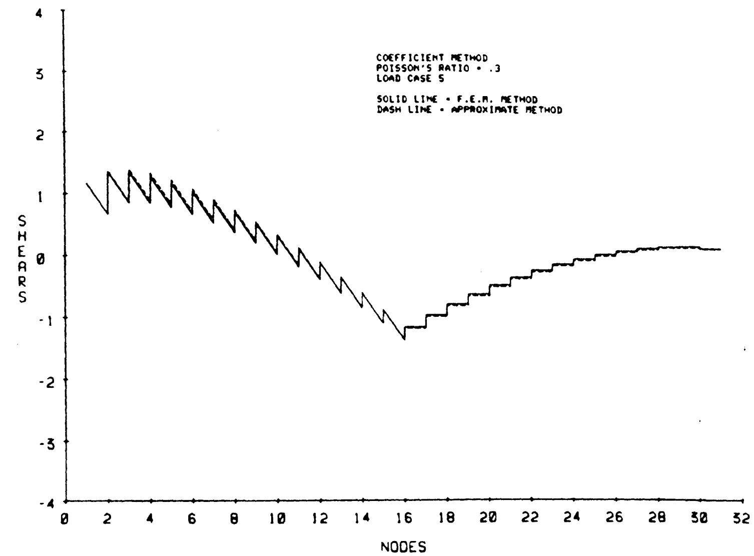

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES


COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES


COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 5
LOAD
2.
1.5
1.
.5
0.
-.5
-1.
-1.5
-2.
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
NODES

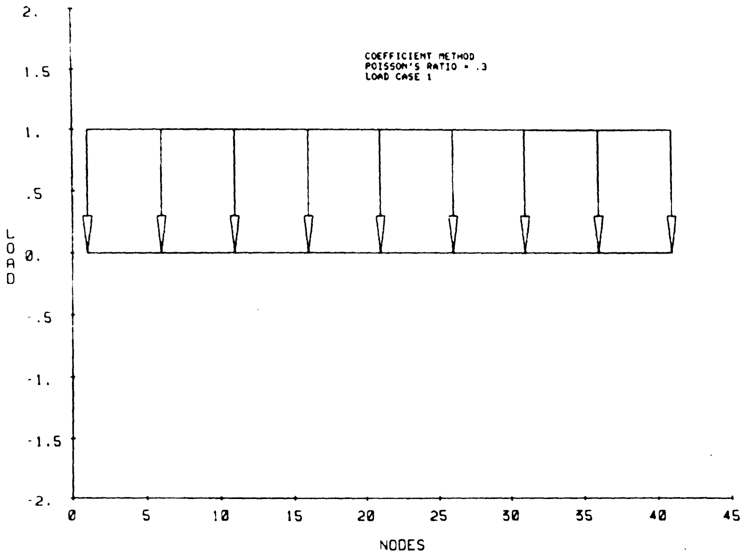

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
.5
0.
-.5
-1.
-1.5
-2.
-2.5
-3.
-3.5
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
NODES

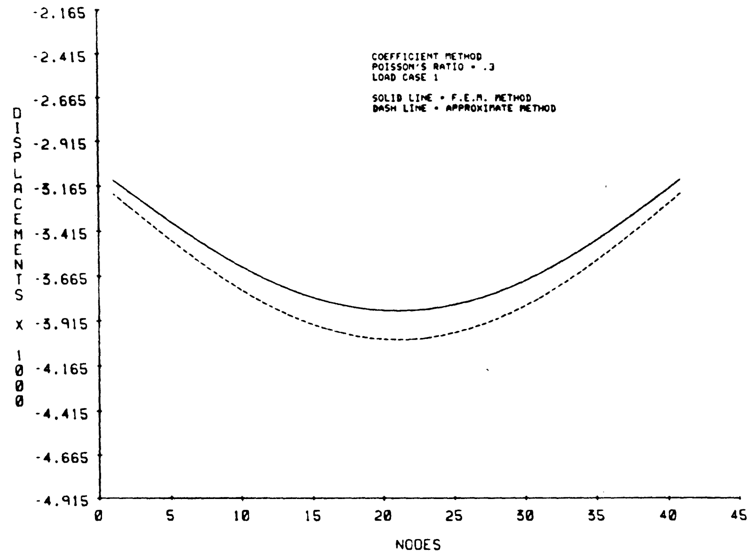

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
60
40
20
0
-20
-40
-60
-80
-100
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
NODES

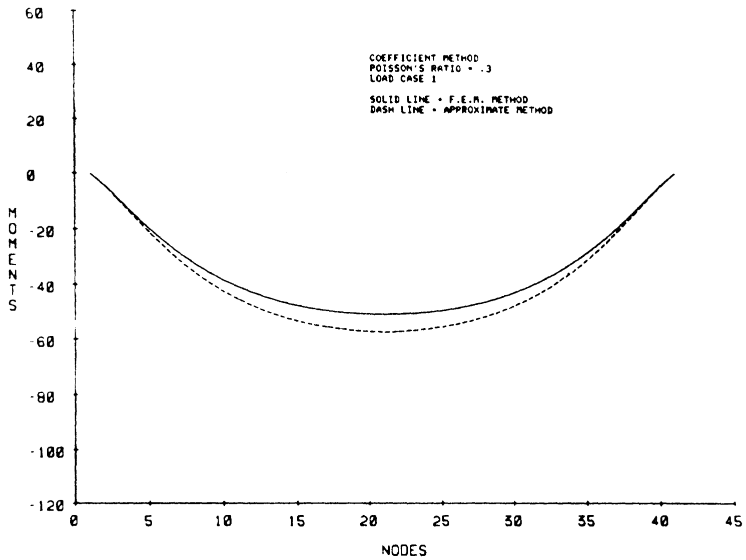

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
4
3
2
1
0
-1
-2
-3
-4
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
NODES

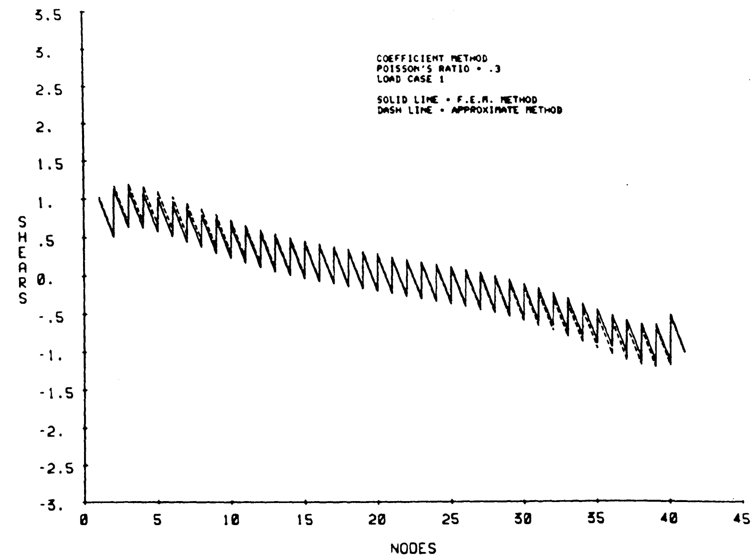
APPENDIX M

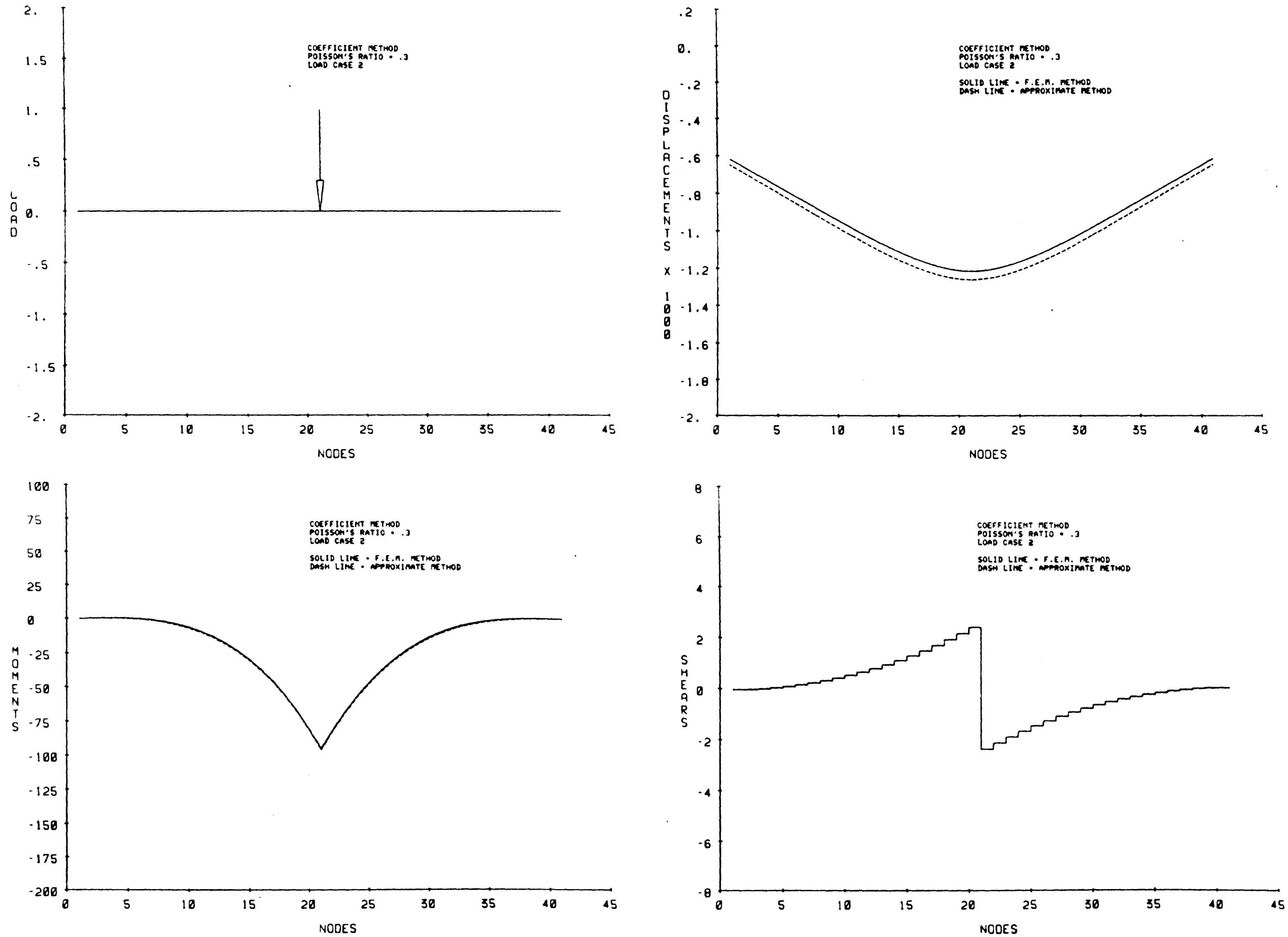
41 NODE BEAM ON FOUNDATION GENERATED FROM COEFFICIENTS WITH POISSON'S RATIO OF 0.3 AND MODULUS OF ELASTICITY EQUAL TO 7000 PSI

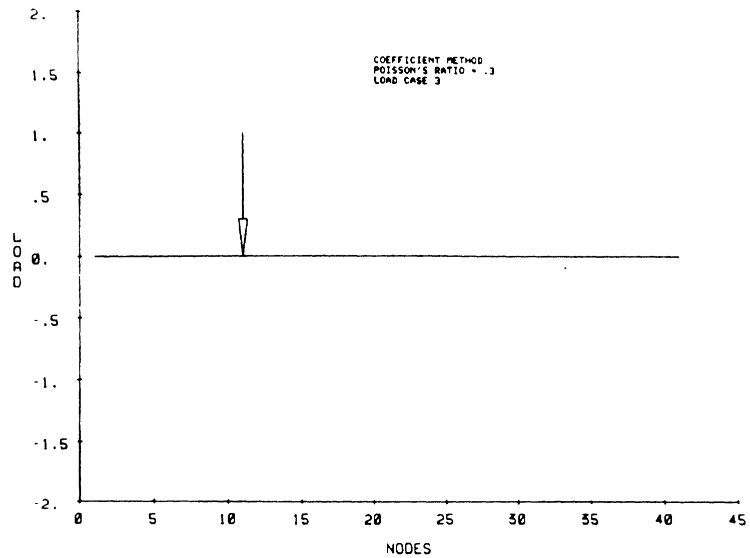

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 1
LOAD
2.
1.5
1.
.5
0.
-.5
-1.
-1.5
-2.
0
5
10
15
20
25
30
35
40
45
NODES

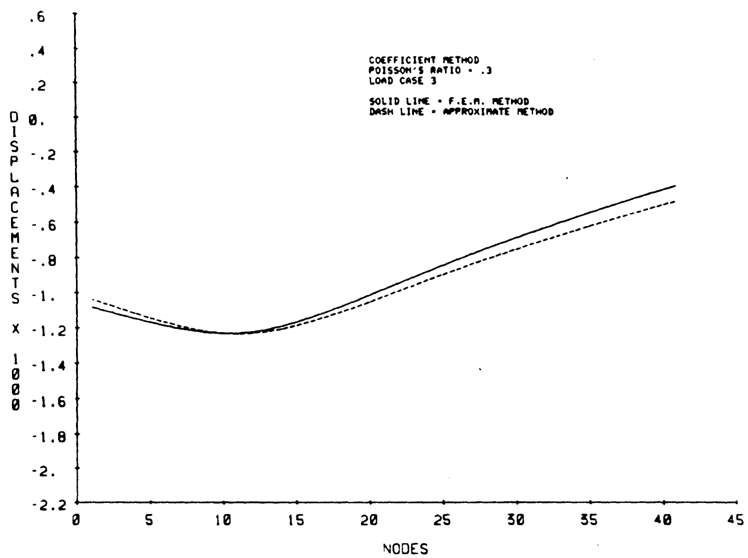

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
-2.165
-2.415
-2.665
-2.915
-3.165
-3.415
-3.665
-3.915
-4.165
-4.415
-4.665
-4.915
0
5
10
15
20
25
30
35
40
45
NODES

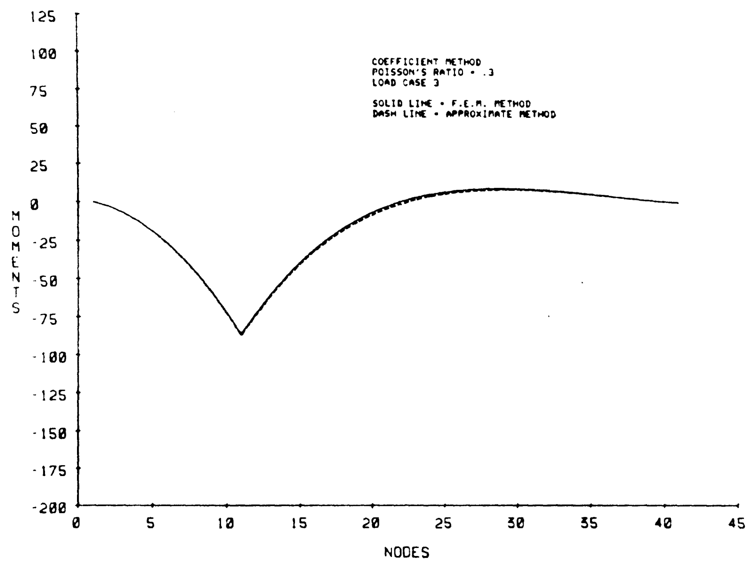

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
60
40
20
0
-20
-40
-60
-80
-100
-120
0
5
10
15
20
25
30
35
40
45
NODES

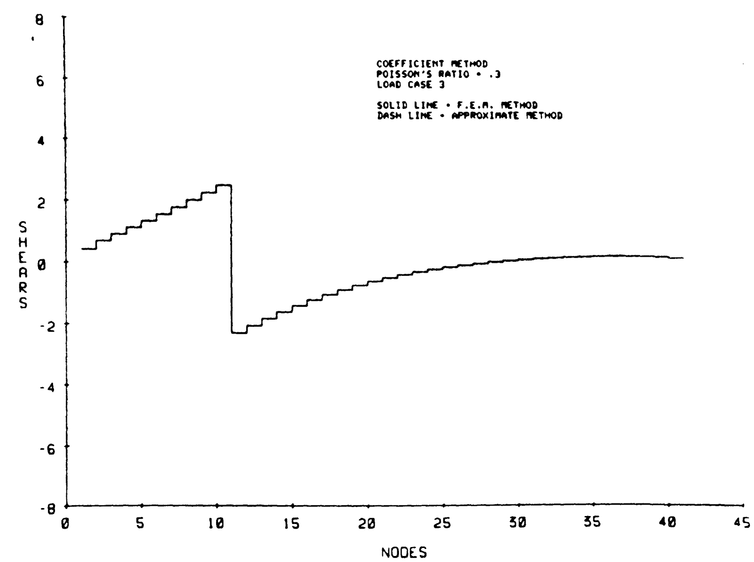

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 1
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
3.5
3.
2.5
2.
1.5
1.
.5
0.
-.5
-1.
-1.5
-2.
-2.5
-3.
0
5
10
15
20
25
30
35
40
45
NODES

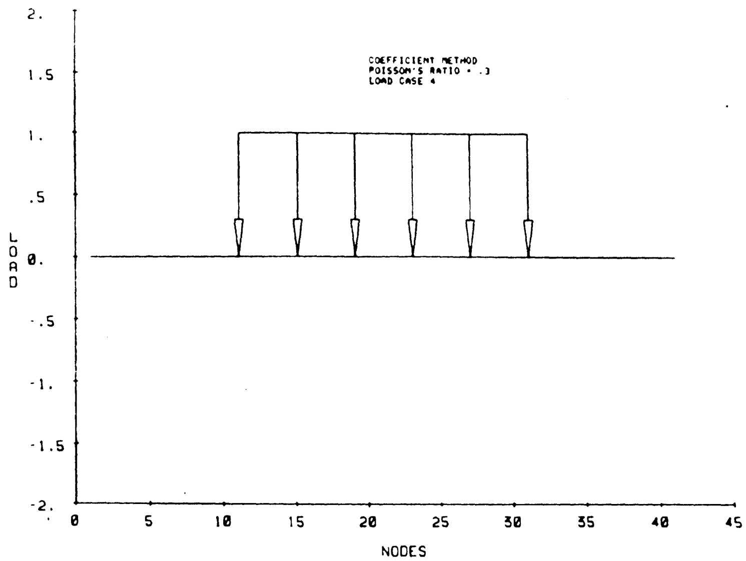

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 2
LOAD
NODES
COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES
COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES
COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 2
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES

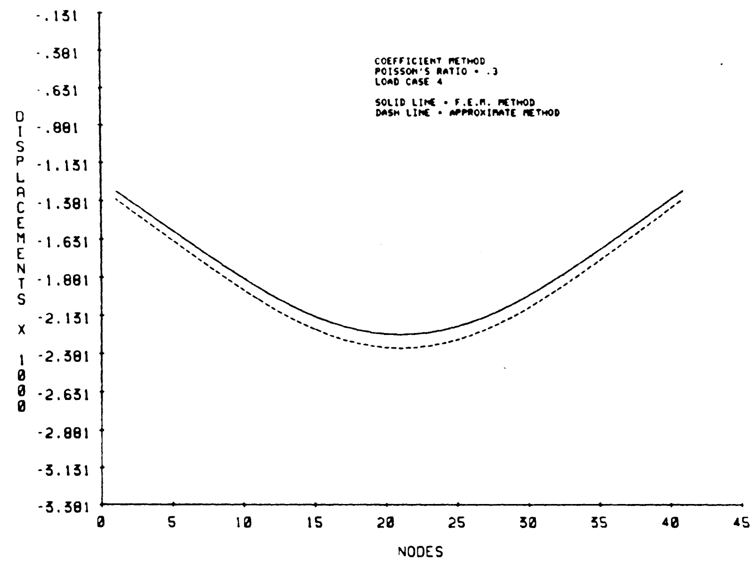

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 3
LOAD
2. 1.5 1. .5 0. -.5 -1. -1.5 -2.
0 5 10 15 20 25 30 35 40 45
NODES

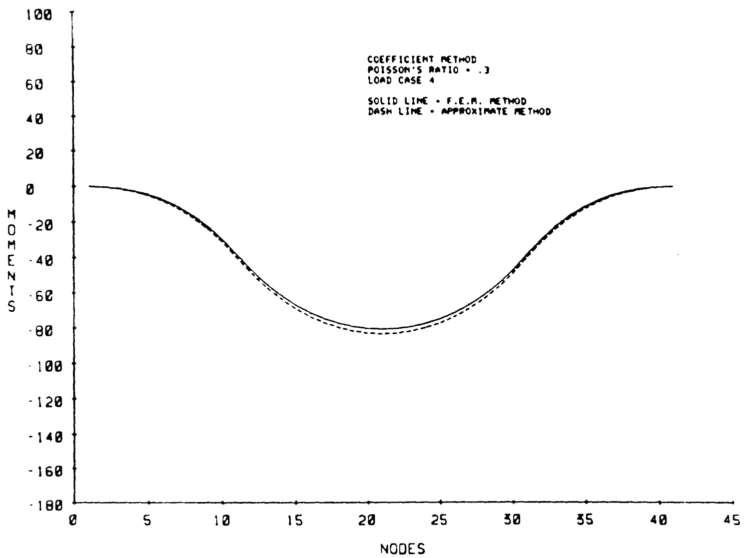

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
.6 .4 .2 0. -.2 -.4 -.6 -.8 -1. -1.2 -1.4 -1.6 -1.8 -2. -2.2
0 5 10 15 20 25 30 35 40 45
NODES

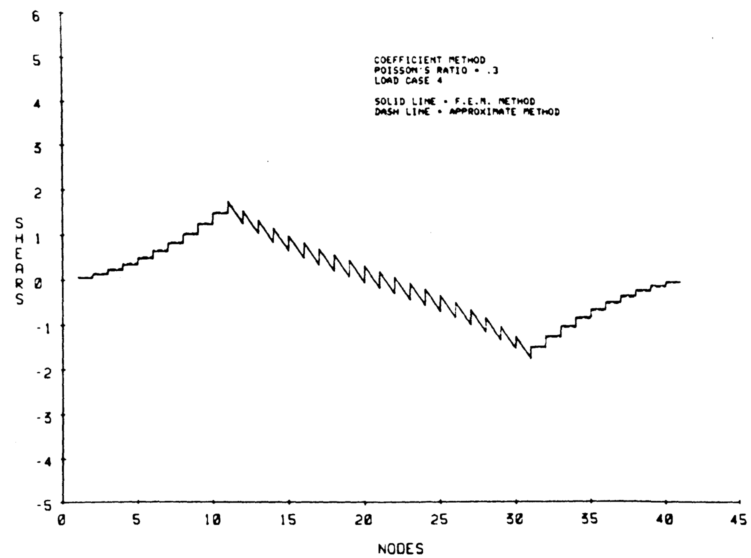

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
125 100 75 50 25 0 -25 -50 -75 -100 -125 -150 -175 -200
0 5 10 15 20 25 30 35 40 45
NODES

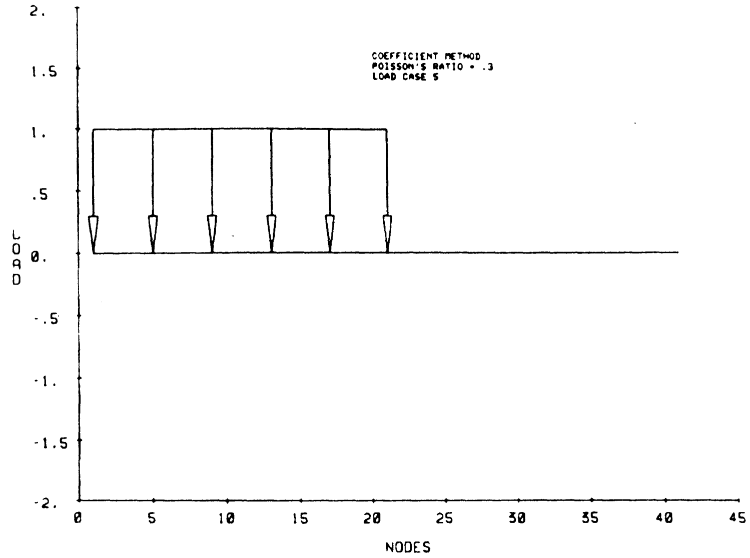

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 3
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
8 6 4 2 0 -2 -4 -6 -8
0 5 10 15 20 25 30 35 40 45
NODES

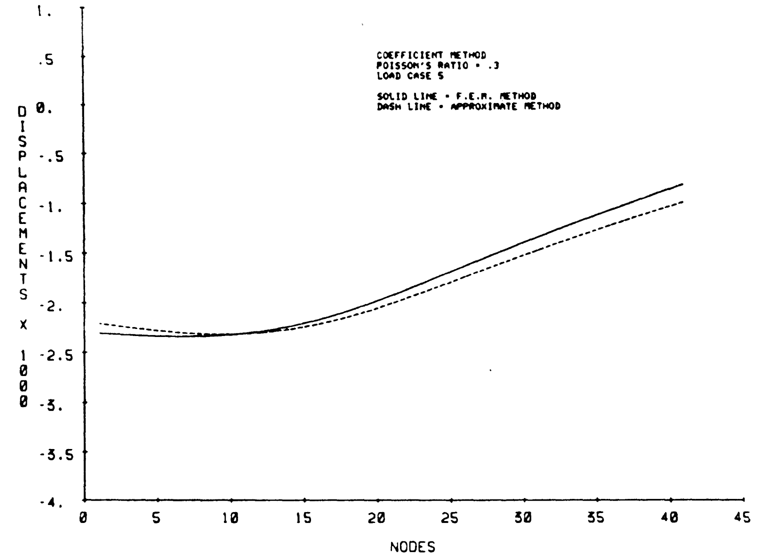

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 4
LOAD
2.
1.5
1.
.5
0.
-.5
-1.
-1.5
-2.
0
5
10
15
20
25
30
35
40
45
NODES

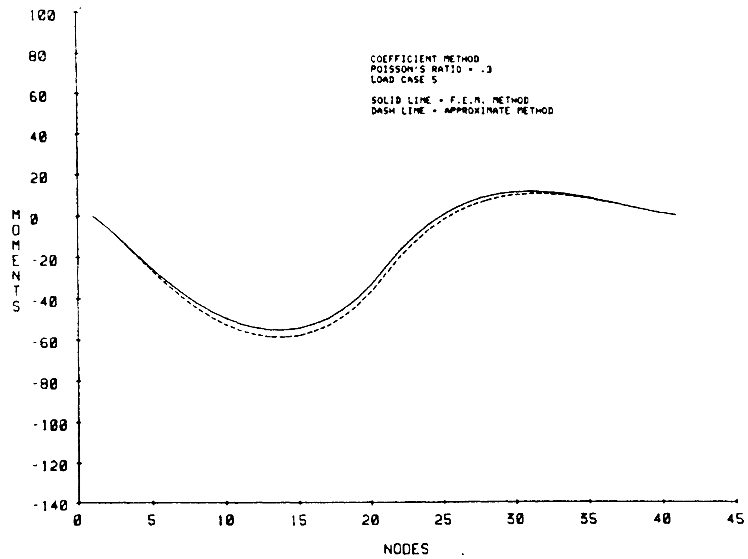

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
-.131
-.381
-.631
-.881
-1.131
-1.381
-1.631
-1.881
-2.131
-2.381
-2.631
-2.881
-3.131
-3.381
0
5
10
15
20
25
30
35
40
45
NODES

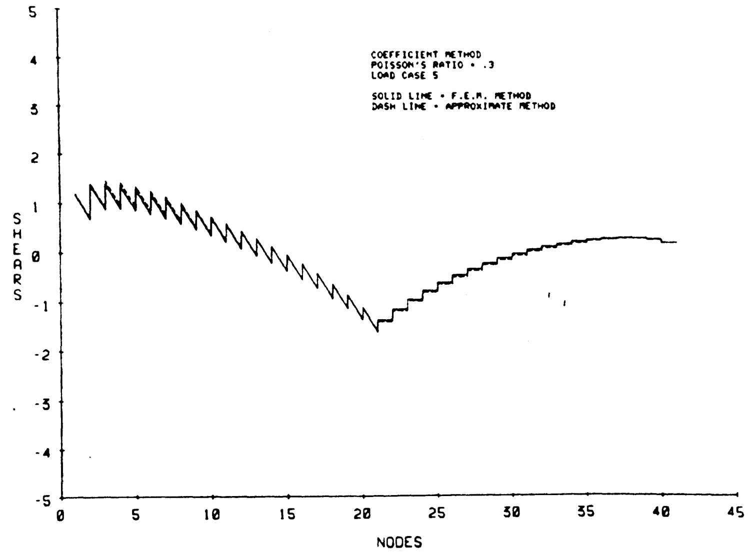

COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
0
5
10
15
20
25
30
35
40
45
NODES


COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 4
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
0
5
10
15
20
25
30
35
40
45
NODES


COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 5
LOAD
NODES


COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
DISPLACEMENTS X 1000
NODES


COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
MOMENTS
NODES


COEFFICIENT METHOD
POISSON'S RATIO = .3
LOAD CASE 5
SOLID LINE = F.E.M. METHOD
DASH LINE = APPROXIMATE METHOD
SHEARS
NODES

VITA

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Doctor of Philosophy

Thesis: SPRING STIFFNESSES FOR BEAM-COLUMN ANALYSIS OF SOIL-STRUCTURE INTERACTION PROBLEMS

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Professional Experience: Civil Engineer, Vicksburg District, Army Corps of Engineers, August, 1971, to June, 1972; Civil Engineer, Waterways Experiment Station, Army Corps of Engineers, June, 1972, to present.