# SYNTHETIC RAINFALL AND ITS USE IN <br> HYDROLOGIC MODELING 

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## HYDROLOGIC MODELING

Thesis Approved


## PREFACE

This study was stimulated by the need to overcome the problems associated with the prediction of watershed runoff in rural areas for which there is little or no climatic data available. The project was financed by the Oklahoma State University Agricultural Experiment Station under project R1632, "Development of Hydrologic and Water Quality Models for Agriculture and Forestry".

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## CHAPTER 1

## INTRODUCTION

The term stochastic, in the hydrological context, refers to the random nature of a variate such as rainfall, stream flow, or wind velocity. Runoff modeling refers to the analytical simulation of runoff processes that take place in natural watersheds with a view to the prediction of runoff and the effect that changes in the watershed characteristics may have on the runoff on an annual, monthly, daily or storm basis.

The principal input for watershed models is rainfall data which is most costly and time consuming to collect. In remote or rural areas this data is often not available, is unreliable, or the records are of short duration. Furthermore, observed rainfall data, although essential, give the researcher the opportunity to study the hydrology of watersheds based upon only one realization of a rainfall sequence. The use of other rainfall sequences, having the same (or similar) properties as the observed sequence, could yield a range of useful runoff results that would be produced by equally likely rainfall series. Synthetic sequences of rainfall based upon the stochastic structure of the historic series are useful for this purpose.

Various methods have been used over the past two and a half decades to generate rainfall data stochastically. Two major techniques have emerged. One is to generate an annual rainfall sequence assuming a normal distribution about a long term mean. The annual rainfall amount is then disaggregated into monthly, biweekly or weekly values based upon annual fragment sets determined from observed records (Srikanthan and McMahon, 1980; Lane, 1982). The process is repeated for each year of the simulated annual time series. This process follows the principle of working from the whole to the part.

The second technique follows the principle of working from the part to the whole. Daily rainfall events are generated by way of a Monte Carlo process to determine the rainfall state and/or rainfall amount. Typically a Markov process is used to determine the dry or wet state of a day given the state of the previous day (Cole and Sherriff, 1972; Buishand, 1978; Nicks and Harp, 1980) and the $2 x 2$ transitional probability matrix describing the probability of a wet or dry day occurring after the occurrence of a wet or dry day. The determination of the rainfall amount accumulated on a wet day is usually based upon the assumption that the daily rainfall amounts fit a predetermined distribution.

The choice of the generation technique would be governed in part by the purpose for which the rainfall data will be used. Such generated rainfall data may be used to supplement limited historical records or provide long term
synthetic records which, together with a rainfall-runoff model, can be used:
a. to determine watershed yields for irrigation, urban or industrial use,
b. to generate stream flow records,
c. to determine the effect of land use or other hydrologic changes on watershed yield,
d. to design water storage structures for a particular assured water supply,
e. as a watershed management tool for erosion control,
f. to establish standards for agricultural practices to ensure hydrologic stability and agricultural productivity over the long term,
g. in the design of water resources systems which often require long term records of daily rainfall data.

Objectives

The objectives of this study were to
a. develop a stochastic daily rainfall model and
b. evaluate the use of simulated rainfall data and a runoff model to study watershed hydrologic responses.

Scope of the Study

The research covered two main aspects of hydrologic research - the generation of synthetic daily rainfall data and the use of this data to predict runoff from agricultural watersheds using an hydrologic model.

A stochastic model was developed to generate daily rainfall data assuming stationarity within each month. The similarity of the simulated and historic records were assessed with respect to the relative frequency of rainfall amounts, the number of consecutive wet and dry day runs, monthly accumulated rainfall and annual accumulated rainfall.

The hydrologic response of an agricultural watershed to the synthetic and historic rainfall data was examined by applying a rainfall-runoff model to a watershed (R-7) of 19.5 acres located at Chickasha, about 100 miles Southwest of Stillwater. This watershed, operated by the USDA-ARS Water Quality and Watershed Research Laboratory from 1966 to 1978, was used by Pathak (1983) to asses the performance of the CREAMS hydrologic model (Knisel, 1980) to predict runoff from a grassland watershed. The applicable soil profile data and watershed parameters established by Pathak (1983) were used with the CREAMS model on the Chickasha $R-7$ watershed on the strength of his findings. The predicted runoff produced by the model using the synthetic and observed rainfall input data respectively were compared in terms of the mean monthly runoff, mean annual runoff, accumulated annual runoff and frequency of monthly runoff amounts.

## CHAPTER II

## LITERATURE REVIEW

In accordance with the objectives of this study, literature in two distinct fields of hydrologic research were examined - rainfall simulation and runoff prediction. These two fields cannot, however, be divorced from each other and be studied independently. Rainfall data is the principle input of any runoff prediction model and the form in which it is available (or is synthesized) has a major bearing on the runoff model to which it can be applied. Previous work relating to the generation of daily rainfall and the prediction of daily runoff, aggregated to obtain weekly, biweekly, monthly and annual runoff values, was reviewed, adhering to the principle (adopted by Diskin et al. 1973) of working from the part to the whole.

## Daily Rainfall Models

Most techniques for generating daily rainfall sequences use a separate process for the simulation of a rainfall occurrence (wet days or dry days) and another process to simulate the rainfall amount on a wet day (Buishand, 1978). The probability of the occurrence of a wet day appears to have been studied first by Newham (1916) in England. He
concluded that wet (and dry) weather is persistent and that the probability of a wet day occurring is related to the number of preceding wet days. Although this was confirmed by Lawrence (1954) it was not supported by Longley (1953) in his studies in Canada. The latter showed that a wet day following a wet day (or a dry day following a dry day) is almost independent of the number of preceding wet (or dry) days. Gabriel and Neumann (1962) have been cited as being the first to use the Markov chain to describe the occurrence of daily rainfall events. Chin (1977) investigated the use of higher order Markov chains to model daily rainfall occurrence. He voiced doubt about the application of a 1 st order Markov chain for this purpose due to the persistence of daily rainfall events. Evidence indicating the feasibility of using a 1 st order Markov chain to describe a sequence of daily rainfall records has, however, been presented by other authors such as Gabriel and Neumann, 1962; Cole and Sherriff, 1972; Buishand, 1978; Nicks and Harp, 1980. The Markov chain used in hydrologic simulation is a special application of the more general Markov process.

The Markov Chain

Markov processes have been used by most researchers in developing stochastic rainfall models for more than twenty years (Buishand, 1978). A Markov process can be described as a process for generating a value ( $X_{n}$ ) of a variable at the nth time interval while taking into account the value of
the variable at each of the i preceding time intervals. A factor $r(i)$ describes the relative influence of the value at the ith preceding time interval on the value of $X_{n}$. The maximum value of $i$ describes the 'order' of the process.

The mathematical relationships defining the 1 st order Markov process can be found in Haan (1977), Linsley et al. (1982) and others in the following form.

$$
x_{n+1}=\mu_{x}+r_{x}(1)\left(X_{n}-\mu_{x}\right)+\varepsilon_{n+1}
$$

where $\quad X_{n}=$ value of the process at time $n$

$$
\begin{aligned}
\mu_{x} & =\text { mean value of } X \\
r_{x}(1) & =\text { first order serial correlation } \\
\varepsilon_{n+1} & =\text { random component }
\end{aligned}
$$

If $\varepsilon_{n+1}$ is selected from a distribution which is normally distributed with a mean equal to zero and variance equal to $\sigma_{x}^{2}$, then the above relationship can be written as

$$
X_{n+1}=\mu_{x}+r_{x}(1)\left(x_{n}-\mu_{x}\right)+R_{n+1} \sigma_{x} \sqrt{1-r_{x}^{2}(1)}
$$

where $\sigma_{x}^{2}=$ variance of $X$
$R_{n+1}=$ random component which is normally distributed with a mean equal to zero and variance equal to one.

A 1 st order Markov chain is a special case of a Markov process in which the value of the variable $\left(X_{n}\right)$ at time $n$
depends only on its value $\left(X_{n-1}\right)$ at time $n-1$ and is independent of the sequence of values $X_{n-2}, X_{n-2}, \ldots, X_{0}$ that the variable takes on before arriving at its value at time $n$ (Haan, 1977). The variable $\left(X_{n}\right)$ is further usually classified into an arbitrarily chosen number of classes (C) or ranges called 'states' of the variable. A CxC transitional probability matrix is required to describe the probability of occurrence of any state given an existing state. The probability ( $p_{i j}$ ) of a transition from state $i$ to $j$ can be represented as

$$
p_{i j}=\operatorname{prob}\left(X_{n}=c_{j} \mid X_{n-1}=c_{i}\right) \quad i, j>0
$$

The CxC transitional probability matrix $P$ is written as follows:

The Markov chain has become the tool most often used in modeling to generate rainfall events.

Rainfall Occurrence

Gabriel and Neumann (1962) found that a simple Markov chain probability model fitted the occurrence of daily rainfall in Tel Aviv. Caskey (1963) fitted a first order Markov
chain model to the occurrence of wet and dry days at Denver, Colorado, for each of four seasons into which he divided the year. Weiss (1964) showed that a two state Markov chain probability model could fit sequences of wet and dry days in records of various lengths and for climatically different areas. Hopkins and Robillard (1964) also found that a two state Markov chain model gave acceptable approximations to the April-September frequency statistics for durations of dry spells recorded in 45 years of observations at three cities in Canada. The model was less satisfactory in predicting the total number of rainy days per month, tending to underestimate the frequency of months with few rainy days.

Feyerherm and Bark (1965) developed a procedure based upon a first order Markov chain to estimate the probability of occurrence of a given consecutive sequence of wet (and dry) days, beginning with any day of the year. In 1967 they reported the adequacy of a first order Markov chain for computing probabilities but found that it may not be satisfactory for long sequences. Research by Lowry and Gutherie (1968) showed that first order Markov chain models of daily rainfall occurrence are adequate for the prediction of the probability of wet or dry spells. They suggested that the threshold value indicating a wet day should be relatively small.

Selvalingam and Miura (1978) used a two state first order Markov chain model to generate daily rainfall events for each calendar month. They assumed that the system was
stationary during each month. Snyder (1976) presented methods for estimating continuous, seasonally varying, transition probabilities using non-linear least squares methods. Richardson (1981), while also using a two state Markov chain to model the occurrence of rain, used a Fourier series to describe the seasonal nature of the transitional probabilities.

Rainfall Amount

In rainfall modeling, rainfall amounts can be determined only after the sequence of wet and dry spells have been generated (Cole and Sherriff, 1972; Selvalgingan and Miura, 1978). Besides multi-state Markov chain models the most common approach is to assume that daily rainfall amounts on successive days are independent and to fit some recognized probability distribution (Tordorovic and Woolhiser, 1974, 1975; Woolhiser et al 1973). Another approach is to assume that rainfall amounts are independent but that the distribution function depends upon whether the previous day was wet or dry (Katz, 1977). Buishand (1977) distinguished three different types of wet days, namely, solitary wet days, wet days bounded on one side by a wet day and by a dry day on the other side and a wet day bounded on both sides by a wet day.

While researchers have generally ignored any persistence in rainfaill amounts on successive wet day, no single distribution has been shown to be universally suitable for
the simulation of rainfall amounts (Skees and Shenton, 1974). Jones, Colwick and Threadgill (1972) obtained rainfall amounts by Monte Carlo sampling from a two parameter Gamma distribution. The Gamma distribution parameters were based upon data for the year ignoring persistence in rainfall amounts on successive days.

Cole and Sherriff (1972) made three distinct analyses of rainfall amounts based upon three criteria. These were (a) a solitary wet day, (b) the first day of a wet spell, (c) the remaining days of a wet spell. Empirical distributions and transitional probabilities were then used to generate rainfall amounts.

Allen and Haan (1975) used a multi-state (7x7) Markov chain model and a uniform distribution within each of the wet states except for the last one. An exponential distribution was used in the last state to generate rainfall amounts. Twelve transition probability matrices were estimated, one for each calendar month. Due to sparseness of data in the last class for each month the values in this class were lumped together. Only one value of the exponential parameter was estimated to generate the rainfall amount in this class for all months. The simulated mean monthly rainfall amounts calculated from the generated daily rainfall data were in agreement with the historical mean monthly amounts. Simulated average annual rainfall was, however, always greater than the historical value (by approximately $2.5 \%$ ) and there was a slight trend towards underestimating
the largest rainfall. A large number of parameters (505) had to be estimated and the model appeared to require at least 40 years of historical data at the Kentucky location for satisfactory parameter estimation.

Selvalingan and Miura (1978) modified the multistate 1st order Markov chain model of Allen and Haan (1975). Separate parameters were estimated for the exponential distribution for each monthly season. These parameters were, however, determined by trial and error making the model unsuitable as a general model for the generation of daily rainfall amounts. The same authors also reported the performance of a model in which a three parameter Gamma distribution was fitted to the square root of the daily rainfall amounts for each month. The rainfall on wet days generated in this way did not preserve the correlation between rainfall amount and the duration of the rainfall event.

Carey and Haan (1978) also modified the Markovian Model of Allen and Haan so that it could be used when limited historical daily rainfall data were available. The daily rainfall amounts were divided into three states. State $1=<0.005$ inches (assumed dry), state $2=0.005-0.145$ inches, state $3=>0.145$ inches. The last two states contained approximately the same number of observations. Transitional probabilities were used to describe the occurrence of any one of the states on a particular day in a season given the state on the preceeding day. A two parameter Gamma distribution was fitted to the rainfall amounts within
each state for each month. To reduce the number of parameters that needed to be estimated they showed that a single distribution could be fitted to the rainfall from all three states. Thus a total of 60 parameters (5 per season two for the Gamma distribution and three for the occurrence of a dry day (or wet day) following each wet state) were required for the model. This model proved to be superior to the Allen and Haan Model (1975) with respect to the rainfall amount simulated, the number of parameters to be estimated and historical record required for stable parameter estimates. The daily rainfall data generated by the modified model reduced the error in simulated annual rainfall from 2.5 percent to 0.5 percent and about 150 historical rainfall events per season were required for stable estimates of the distribution parameters.

Bridges and Haan (1972) showed that only as the number of observations approached 100 would the estimated values of the parameters of the Gamma distribution approximate the population values. They produced tables for the evaluation of the adequacy of a rainfall record that may be used to determine the parameters of a Gamma distribution. Matalas (1967) presented evidence on the limitations on the use of a Gamma distribution to generate synthetic rainfall when the skewness coefficient of the historic record used to estimate the distribution parameters is greater than $2 \sqrt{2}$.

In his paper more recently, McMahon and Miller (1971) supported this inconsistency of the Gamma function to
preserve all the lower moments of historical data. He showed that for a skewness coefficient between $\pm 2$ the Gamma transformation of a normal variable successfully preserves the moments of the historical data. Beyond these limits, however, no moment preservation is assured. Todorovic and Woolhiser (1974) found the application of the exponential distribution very promising in describing daily rainfall amounts and suggested that further investigations were warranted. Woolhiser and Roldan (1982) compared the use of the exponential, Gamma and mixed-exponential distributions as potential models for the distribution of daily rainfall. Using the maximum likelihood method to estimate the parameters for each distribution they found that the mixed exponential distribution was the best on the basis of the Akaike information criterion (Akaike, 1974). Richardson (1982), however, found that all three of the above distributions were capable of reproducing the historical distribution of annual and monthly rainfall data.

Experience has shown that the lognormal distribution is particularly suited to modeling daily rainfall amounts (Haan, 1977; Nicks, 1984). Three techniques can be used to determine the distribution parameters of the lognormal distribution. One method is to transform the data ( $X_{i}$ ) to some concomitant values ( $Y_{i}$ ) using the transformation

$$
Y_{i}=\ln \left(X_{i}\right)
$$

If the historic data ( $X_{i}$ ) are lognormally distributed then by the Central Limit Theorem the $Y_{i}$ 's will be normally distributed with mean $\mu_{Y}$ and variance $\sigma_{Y}^{2}$. The parameters of $\mu_{Y}$ and $\sigma \frac{2}{Y}$ can be estimated by $\bar{Y}$ and $S_{Y}^{2}$ using standard statistical procedures.

A second method, present by Chow (1954) provides for the calculation of $\bar{Y}$ and $S_{Y}^{2}$ without taking the logarithms of all the data using the relationships

$$
\begin{aligned}
\bar{Y} & =1 / 2 \ln \left(X^{2} / C v^{2}+1\right) \\
S & =\ln \left(C v^{2}+1\right)
\end{aligned}
$$

where $C v=$ coefficient of variation of the original data.
A third method presented by Brakensiek (1958) uses the least squares method for estimating the parameters of a lognormal distribution.

Snyder and Wallace (1974) show how the nonlinear least squares method of fitting a three parameter lognormal distribution could be executed but suggested that one could not distinguish whether a gamma or lognormal distribution was the best distribution to apply to hydrologic data. In a later paper, Snyder (1975, 1976) further showed how this method can be used to adapt the lognormal distribution to a seasonally continuous distribution by making two of its three parameters cyclic functions of annual time.

Hansen (1982) showed that the two parameter lognormal distribution could be used to generate synthetic annual
rainfall series. Using a limited record of annual averages, he estimated the distribution parameters from the logtransformed observations of an annual rainfall series.

Srikanthan and McMahon (1978) used the two-parameter and three-parameter lognormal distributions to model hydrologic data in Australia. Determining the distribution parameters from the log-transformed data they found that the two-parameter distribution overestimated the skewness and did not preserve the lag-1 serial correlation. They nevertheless recommended that when the coefficient of skewness exceeded 1.0 the two-parameter lognormal distribution gave the best results. This recommendation was, however, reversed in a later paper (Srikanthan and McMahon, 1980) in which the value of the skewness coefficient was not mentioned.

Haan (1977) and Matalas (1967) both commented on the inability of the lognormal and power transformation to preserve the mean, variance, coefficient of skewness and lag-1 serial correlation. They both pointed out that the distribution characteristics could not be carried through from the original data to the transformed data with the non-linear transformations. In order to retain the original distribution characteristics in a synthetically generated series using a log-transformation, the technique proposed by Chow (1954), or a more sophisticated method of Matalas (1967) was recommended.

The choice of one of the forementioned models (or any other model) for the generation of synthetic rainfall data will be dictated by (a) the sequence (annual, monthly, daily, hourly, etc.) that is to be generated, (b) the historical record available from which the distribution parameters have to be estimated and (c) the purpose for which the synthetic series is to be used. In this study synthetic rainfall data was required to examine the effect of applying historical, or statistically similar synthetic, rainfall series to a watershed model for the prediction of runoff.

## Watershed Models

Watershed or hydrologic models can be classified as either material or formal. A material model is a simpler physical representation of a more complex system and may be an iconic (look alike) system or an analog system. That is a system in which physical phenomena, difficult to measure, are substituted by measurable quantities such as voltage, current or deflection. Eagleson (1970) suggested that material models have limited application in watershed modeling and favored the use of formal or mathematical models.

The advances in computer technology has stimulated the development of mathematical watershed models. Renard et al (1982) lists 175 models currently available. Woolhiser and Brakensiek (1982) give a comprehensive description of six classes of hydrologic models. Haan (1977) notes that most
quantitative hydrologic models can be identified as deterministic, parametric, stochastic or a combination of these. There is no distinct division among these three basic types of models. Such hydrologic models, used to predict runoff, are either event simulation models or continuous simulation models (Nicks, 1982). Rainfall data is the most important and sensitive input required by runoff models and may be required in daily, hourly or smaller time increments. Some of the major hydrologic models and their required rainfall inputs are given in Table I. The rainfall increments required by the models tabulated, range from breakpoint (short unequal time intervals bounded by slope breakpoints on the rainfall recorder chart trace) for the CREAMS (Knisel, 1980) and USDAHL (Holtan et al. 1975) models to daily (accumulated in 24 hours) rainfall for the majority of the other models.

Each individual component in a complex watershed system is described in an hydrologic model, in varying degrees of detail. These components may include surface storage, infiltration, evapotranspiration, geomorphology, surface runoff, snowmelt, ground water flow, water quality, sediment yield and nutrient transport. Model parameters may be lumped or distributed. The simpler models with lumped parameters require less input data than the more complex models with spatially distributed parameters. The parameters of the latter models may be more physically based but require more computer time for simulations. Such distributed parameter hydrologic models, although normally too

TABLE I
MAJOR HYDROLOGIC MODELS

| MODEL | AUTHOR | DATE | INPUT |
| :---: | :---: | :---: | :---: |
| U.S. Soil <br> Conservation <br> Service | Mockus | 1964 | by storm daily |
| Stanford Mark IV | Crawford and Linsley | 1966 | $\begin{aligned} & \text { 15-min } \\ & \text { hourly } \\ & \text { daily } \end{aligned}$ |
| USDAHL | Holtan et al. | 1975 | break-point hourly daily |
| Kentucky | Haan | 1972 | daily |
| HEC-1 | U.S.A.C.E | 1973 | incremental |
| SSARR | U.S.A.C.E | 1974 | 10-24 hour |
| ARM | Donigian \& Crawford | 1976 (a) | $5 \mathrm{~min}, 15 \mathrm{~min}$ |
| NPS | Donigian \& Crawford | 1976(b) | $5 \mathrm{~min}, 15 \mathrm{~min}$ |
| ANSWERS | Beasley | 1977 | 1-24 hour |
| CREAMS | Knisel | 1980 | $\begin{aligned} & \text { daily } \\ & \text { break point } \end{aligned}$ |
| SMAP | Lopes et al. | 1982 | daily |
| SWRRB | Williams and Nicks | 1983 | daily |

time consuming for engineering applications, can be useful for research purposes (Linsley et al. 1982).

The rainfall data (input) available and the purpose for which the runoff (output) is required are usually the major factors that dictate which hydrologic model will be used. For field scale applications of the CREAMS, ARM or NPS models, for example, a single gage or point rainfall amount is considered adequate. For basin size watersheds, especially if the watersheds are large, several raingage stations around and within the basin should be considered. The Thiesen weighted mean of such rainfall amounts has been shown to be the best estimate of basin rainfall amount (Nicks, 1982) A single centrally located gage in a watershed will tend toward the Thiesen rainfall average from multipoints.

## Rainfall Data Source

There are two major sources of rainfall data available. 1. Hydrological Data for Experimental Agricultural Watersheds in the United States (operated by the United States Federal and State agencies, universities and private organizations. USDA-ARS has operated networks for rainfall data collection for research purposes for more than 40 years at many locations in the United States of America (Burford et al. 1980).
2. United States National Weather Service Co-operative observers and first order weather stations.

Nicks (1982) noted that rainfall data are available in time increments and spatial distribution ranging from one minute, from several gages in a single watershed, to daily, from a single gage located outside the watershed of interest.

## CHAPTER III

RAINFALL MODEL DEVELOPMENT AND EVALUATION

Daily rainfall data for Stillwater, Oklahoma were used in developing a stochastic daily rainfall model. The data was collected over 80 years from 1900 to 1979 under the auspices of the Oklahoma State University. Although originally collated on magnetic tape and archived at the National Climatic Center in Ashville, North Carolina, these data were made available through the Oklahoma State University Water Research Institute (Stadler et al. 1982). The data consisted of the daily accumulated rainfall amounts in one-hundredths of an inch. The smallest rainfall amount in the record was 0.01 inches. An analysis of the data showed that 5800 wet days occurred over the period 1900 to 1979 in Stillwater. The results of a statistical analysis of the 5800 observations are shown in Table II. All the data analyses and model development were done using the SAS language (SAS, 1982) on the Oklahoma State University IBM-3081D computer.

## TABLE II

## STATISTICAL ANALYSIS OF THE 5800 WET DAYS THAT OCCURRED OVER THE PERIOD 1900 TO 1979 <br> IN STILLWATER, OKLAHOMA

| MONTH |  | STD ${ }^{1}$ | VAR ${ }^{2}$ | SKEWNESS | KURTOSIS | $\begin{aligned} & \text { MONTHLY } \\ & \text { MEAN } \\ & \text { (inch) } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 24.06 | 34.28 | 1175.68 | 2.65 | 8.26 | 1.04 |
| 2 | 26.78 | 34.25 | 1173.68 | 2.85 | 11.48 | 1.27 |
| 3 | 38.16 | 43.85 | 1923.10 | 1.95 | 4.40 | 2.18 |
| 4 | 44.36 | 57.61 | 3319.25 | 2.70 | 11.44 | 3.34 |
| 5 | 50.47 | 70.07 | 4910.68 | 3.33 | 18.06 | 4.77 |
| 6 | 50.23 | 58.88 | 3467.36 | 2.28 | 8.18 | 3.94 |
| 7 | 49.97 | 70.36 | 4950.77 | 2.90 | 11.62 | 2.98 |
| 8 | 48.03 | 67.80 | 4597.65 | 3.28 | 18.50 | 2.96 |
| 9 | 59.45 | 79.89 | 6382.82 | 2.46 | 8.00 | 3.75 |
| 10 | 51.29 | 66.32 | 4399.06 | 2.97 | 15.21 | 2.83 |
| 11 | 44.29 | 58.59 | 3432.98 | 2.64 | 9.36 | 2.08 |
| 12 | 29.86 | 35.97 | 1293.91 | 1.90 | 4.03 | 1.28 |
| Annual Total |  |  |  |  |  | 32.42 |

The rainfall model developed consisted of two distinct stages. The first stage generated the occurrence of a rainfall event. A 1st order, two state Markov chain was used in this stage following the recommendations of Gabriel and Neumann (1962), Gringorton (1966), Smith and Schreiber (1974), Haan (1977), Nicks et al. (1980) and Richardson (1981). The second stage of the model generated the amount of rainfall accumulated in a day given that a wet day occurred. This daily rainfall amount was generated using a lognormal distribution found to be applicable by Srikanthan and

McMahon (1978), and Nicks (1984). The Gamma distribution was not considered on the recommendation of Matalas (1967) and Srikanthan and McMahon (1978). Both authors suggested that the Gamma distribution should not be used to describe data if the coefficient of skewness of the data was not within the interval of $\pm 2$. The skewness coefficients of the daily rainfall amounts for most months did not fall within this range (Table II).

Following this two stage procedure, the model generates observations only for wet days.

## Model for Rainfall Occurrence

A 1st order, two state, Markov chain was used to describe the occurrence of wet days and dry days. The notation $P(W \mid W)$ was used to describe the probability of a wet day occurring given that the previous day was wet and $P(W \mid D)$ was used to describe the probability that a wet day occurred given that the previous day was dry.

Using the above probabilities, the probability of a dry day occurring given the previous day was wet, $P(D \mid W)$, and the probability of a dry day occurring given that the previous day was dry, $P(D \mid D)$, was determined from

$$
\begin{aligned}
& P(D \mid W)=1-P(W \mid W) \\
& P(D \mid D)=1-P(W \mid D)
\end{aligned}
$$

Thus calculating the probabilities $P(W \mid W)$ and $P(W \mid D)$ fully defined the $2 x 2$ transitional probability matrix required to implement the Markov chain model. The matrix can be written as

|  | $e_{i}$ |  |
| :--- | :---: | :---: |
| $e_{i-1}$ | $D$ | $W$ |
| $D$ | $P(D \mid D)$ | $P(W \mid D)$ |
| $W$ | $P(D \mid W)$ | $P(W \mid W)$ |

where $e_{i}$ is the occurrence of event $e$ on day $i$. The transitional probability matrix shown in Table III was determined from the eighty years of daily rainfall data for Stillwater, Oklahoma. In calculating these transitional probabilities it was noted that the number of transition counts $\mathbb{N}(W \mid D)$ from a dry to wet state was equal to the number transition counts $N(D \mid W)$ from a wet to dry state. It was therefore only necessary to count the number of transitions from a dry to wet state $\mathbb{N}(W \mid D)$ and from a wet to wet state $\mathbb{N}(W \mid W)$.

Since

$$
N(W \mid D)=N(D \mid W)
$$

The count

$$
N(D \mid D)=T-2 N(W \mid D)+N(W \mid W)
$$

where $\mathbb{T}=$ total number of days in 80 years.

## TABLE III

OVERALL TRANSITIONAL PROBABILITY MATRIX, FROM 80 YEARS OF DAILY RAINFALL DATA FOR STILLWATER, OKLAHOMA

|  | $D$ | $W$ |
| :---: | :---: | :---: |
|  | D | .846 |
|  | .154 |  |

Since the transitions to a wet or dry state from a given state are mutually exclusive, the sum of the transitional probabilities to the two states from a given state is equal to one. That is since

$$
P(D \mid D)+P(W \mid D)=1
$$

Then,

$$
P(D \mid D)=N(D \mid D) /(N(D \mid D)+N(W \mid D))
$$

and

$$
P(W \mid D)=N(W \mid D) /(N(D \mid D)+N(W \mid D))
$$

Also, since

$$
P(D \mid W)+P(W \mid W)=1
$$

Then,

$$
P(D \mid W)=N(D \mid W) /(N(D \mid W)+N(W \mid W))
$$

and

$$
P(W \mid W)=N(W \mid W) /(N(D \mid W)+N(W \mid W))
$$

Applying the above procedures to the monthly data for Stillwater, aggregated over the 80 years of record, twelve monthly transitional probability matrices were calculated. These matrices are shown in Table IV.

The above estimation procedure is a maximum likelihood procedure and can be expressed in the following terms (Allen and Haan, 1975).

$$
P_{i j}^{(k)}=f_{i j}^{(k)} / \sum_{j=0}^{i} f_{i j}^{(k)} \quad(i, j=0,1 \text { and } k=1, \ldots 12)
$$

where $P_{i j}^{(k)}$ is the probability, for season $k$ of the transition from state $i$ to state $j$,
$f_{i}\left(\frac{k}{j}\right)$ is the transition count from state $i$ to state $j$,
i,j=0 represents a dry day,
$i, j=1$ represents wet day and
$k=1,2, \ldots 12$ denotes month of the year.

## TABLE IV

MONTHLY TRANSITIONAL PROBABILITY MATRICES CALCULATED FROM 80 YEARS OF DAILY RAINFALI FOR STILLWATER, OKLAHOMA

|  | D | W | D | W | D | W |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | JANUARY |  | FEBRUARY |  | MARCH |  |
| D | . 895 | .105 | . 870 | .130 | . 852 | . 148 |
| W | . 651 | . 349 | . 646 | . 354 | . 656 | . 344 |
|  | APRIL |  | MAY |  | JUNE |  |
| D | . 799 | . 201 | . 746 | . 254 | . 792 | . 208 |
| W | . 600 | . 400 | . 579 | . 421 | . 589 | . 411 |
|  | JULY |  | AUGUST |  | SEPTEMBER |  |
| D | . 845 | . 155 | . 838 | .162 | . 840 | . 160 |
| W | . 652 | . 348 | . 653 | . 347 | . 600 | . 400 |
|  | OCTOBER |  | NOVEMBER |  | DECEMBER |  |
| D | . 826 | . 134 | . 884 | . 116 | . 890 | . 110 |
| W | . 621 | . 379 | . 622 | . 378 | . 683 | . 317 |

A first order, two state Markov chain model to generate rainfall occurrence was built around the transitional probabilities $\mathrm{P}(\mathrm{W} \mid \mathrm{W}), \mathrm{P}(\mathrm{W} \mid \mathrm{D})$ and a uniform random number generator. The flowchart for the model is shown in Figure 1. The SAS (SAS, 1982) program is included in the program listed in Appendix A.

## Model for Rainfall Amount

The amount of rainfall accumulated on a wet day was assumed to be independent of the amount accumulated on the previous day. This assumption was verified by examing the relative frequency of the occurrence of rainfall amounts on all the wet days (total data), on wet days following dry days (dry data) and on wet days following wet days (wet data). Daily rainfall amounts were catagorized into 0.1 inch classes for the analyses which were performed on the monthly aggregated data for the 80 year record. The relationship of the relative frequency versus daily rainfall amount was plotted, for each month, for the total data, the dry data and the wet data. The graphical comparison of these curves for the month of December, illustrated in Figure 2, show that there is no marked difference among the rainfall amounts on the three types of wet days. Similar plots for the other months in the year can be seen in Appendix B. In these plots there is no evidence to reject the assumption of independence stated above.


Figure 1. Flow Chart for a lst Order, Two State Markov Chain Model for Rainfall Occurrence.


[^0]With the above substantiating evidence, a single distribution could be fitted to the daily rainfall amounts with confidence. The single parameter exponential and two parameter lognormal distribution were selected for examination. The parameters for these two distributions were determined by the method of moments. Haan (1977) and De Coursey et al. (1982) showed that the method of moments and the generally prefered maximum likelihood procedure yield the same parameter estimates for the exponential distribution. It is shown in Appendix $C$, that for the lognormal distribution, however, the maximum likelihood procedure yields parameter estimates which are quite different to the moment method developed by Chow (1954). The use of this moment method for parameter estimates is recommended by De Coursey (1982), Selvalingam and Miura (1978) and McMahon (1971) as it leads to the better preservation of the lower order moments of the historical data in subsequently simulated data. The degree of bias in the estimate of the variance of the lognormal distribution resulting from the maximum likelihood procedure approaches zero as the sample size from which it is estimated is increased. Increasing the sample size when using the maximum likelihood procedure nevertheless, does not improve the preservation of the historical moments in the simulated data as Chow's (1954) method does.

The means and variances for the daily rainfall amounts from the historical record were determined, on a monthly
basis, (see Table II) using SAS (SAS, 1982). The total rainfall record was used in the calculation of these moment estimates in an attempt to obtain the closest approximation of the true population values.

Exponential Model

The exponential distribution was selected for possible use because of its simplicity and ease of application. The single parameter exponential distribution has a density function given by

$$
p_{x}(X)=\lambda e^{-\lambda X} \quad X>0, \lambda>0
$$

where $\hat{\lambda}=1 / \bar{X}$ and can be estimated by the reciprocal of the sample mean. A value for $\hat{\lambda}$ was calculated for each month from the eighty year historical daily rainfall record. The daily rainfall amounts on wet days ( $X_{i}$ ) were simulated using the SAS (SAS, 1982) procedure to generate a random exponential deviate using the appropriate values for $\lambda$. The generating function is

$$
X_{i}=-\ln (R) / \lambda_{i}
$$

where $X_{i}=r a i n f a l l$ amount generated for the $i t h$ month, $R=a \operatorname{random}$ number uniformly distributed between zero and one, and
$\lambda_{i}=$ the reciprical of the mean daily rainfall for the ith month.

A separate seed was used to initiate the random number streams for the exponential model and the Markov chain to ensure that the random nature of each stream was retained.

## Lognormal Model

The two parameter lognormal distribution was selected as an alternative to the exponential distribution to examine whether the greater flexibility it offers was meaningful. The lognormal density function is given by

$$
P_{X}(X)=\left(2 \pi X^{2} \sigma_{Y}^{2}\right)^{-1 / 2} \exp \left(-\left(Y-\mu_{u}\right)^{2} / 2 \sigma_{y}^{2}\right)
$$

where $Y=\ln (X)$

$$
\begin{aligned}
& \mu_{y}=\text { mean of the logarithms of the data } \\
& \sigma_{y}^{2}=\text { variance of the logarithms of the data. }
\end{aligned}
$$

Using the lognormal distribution, daily rainfall amounts on wet days $\left(X_{i}\right)$ were generated using the SAS (SAS, 1982) procedure to generate a random lognormal deviate. The generating function is

$$
X_{i}=\exp \left(M_{i}+S_{i}(R)\right)
$$

where $X_{i}=$ rainfall amount generated for the ith month, $\mathrm{R}=\mathrm{a}$ random number, normally distributed with mean equal to zero and variance equal to one, $S_{i}=$ a standard deviation for the ith month calculated in one of two ways described below, and $M_{i}=a$ mean for the ith month calculated in one of two ways described below.

As with the exponential model, separate seeds were used to initiate the random number streams for the lognormal model and the Markov chain. Two methods can be used to determine the lognormal distribution parameters.

The first method of parameter estimation for a lognormal distribution involves the transformation of each observation of the historical data ( $X$ ) using the relationship

$$
Y=\ln (X) .
$$

The mean and variance of the transformed data are determined and used as estimates for the parameters $M$ and $S$ in the generating function.

The second method of lognormal parameter estimation utilizes the mean and standard deviation of the historical data shown in Table II. The logarithms of the data are not required and estimates of the parameters used in the
generating function are determined using the parameter transformation relationships

$$
\begin{aligned}
M & =1 / 2 \ln \left(\overline{\mathrm{X}}^{2} /\left(\mathrm{C}_{\mathrm{v}}^{2}+1\right)\right) \\
S^{2} & =\ln \left(C_{v}^{2}+1\right)
\end{aligned}
$$

$$
\text { where } \begin{aligned}
\mathrm{C}_{\mathrm{V}}= & \mathrm{S}_{\mathrm{x}} / \overline{\mathrm{X}} \text { (coefficient of variations of the original } \\
& \text { data) } \\
S_{\mathrm{x}}= & \text { standard deviations of the original data } \\
\overline{\mathrm{X}}= & \text { mean of the original data. }
\end{aligned}
$$

The parameters calculated using the two methods above are shown in Table V. The table shows that values of the means determined from the log-transformed data (1st method) are smaller and the variances larger than the values of the same parameters determined using the parameter transformation relationships (2nd method). Forty years of daily rainfall were simulated using the distribution parameters calculated using the first method (log-transformed data). Five such simulations were performed. The mean annual rainfall calculated for each simulation, shown in Table VI, was consistently larger than the historical mean annual rainfall of 32.42 inches (Table II). The mean of the five mean annual rainfall amounts, 40.05 inches, was approximately twenty four percent greater than the historical value.

Similar simulations were performed using the parameters calculated using the second method (parameter transformation). The mean annual rainfall amounts of the five simulations (Table VI) were well distributed about the historical mean annual rainfall of 32.42 inches. The mean of the five mean annual rainfall amounts was within one percent of the historical value. The parameter transformation relationships thus yielded the best lognormal distribution parameter estimates with respect to the preservation of the means.

## TABLE V

LOGNORMAL DISTRIBUTION PARAMETERS DETERMINED FROM LOG-TRANSFORMED DATA AND BY USING THE PARAMETER TRANSFORMATION RELATIONSHIPS

| Month | M |  | $S^{2}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | LogTransformed Data | Parameter Transformation | LogTransformed Data | Parameter <br> Transformation |
| 1 | 2.35 | 2.63 | 1.78 | 1.35 |
| 2 | 2.55 | 2.80 | 1.77 | 1.22 |
| 3 | 2.93 | 3.22 | 1.78 | 1.14 |
| 4 | 2.96 | 3.30 | 2.03 | 1.26 |
| 5 | 3.07 | 3.38 | 2.04 | 1.33 |
| 6 | 3.17 | 3.48 | 1.93 | 1.59 |
| 7 | 3.01 | 3.37 | 2.15 | 1.34 |
| 8 | 3.00 | 3.32 | 2.09 | 1.35 |
| 9 | 3.18 | 3.45 | 2.23 | 1.29 |
| 10 | 3.11 | 3.29 | 2.11 | 1.26 |
| 11 | 2.98 | 3.28 | 1.96 | 1.28 |
| 12 | 2.61 | 2.95 | 1.94 | 1.19 |

## TABLE VI

MEAN ANNUAL RAINFALL AMOUNTS FROM FORTY YEARS OF DATA SIMULATED USING THE LOGNORMAL DISTRIBUTION PARAMETERS CALCULATED BY THE TRANSFORMED DATA AND PARAMETER TRANSFORMATION METHODS

|  | MEAN ANNUAL RAINFALL (INCH) |  |
| :---: | :---: | :---: |
| SIMULATION <br> RUN | LOG-TRANSFORMED <br> DATA | PARAMETER <br> TRANSFORMATION |
| 1 | 40.29 | 30.88 |
| 2 | 39.56 | 33.98 |
| 3 | 38.74 | 32.13 |
| 4 | 39.75 | 32.97 |
| 5 | 41.80 | 33.29 |
| MEAN | 40.05 | 32.65 |

Historical mean annual rainfall $=32.42$ inches

The expected relative frequencies for each month of the year, for the exponential and lognormal distribution, based upon the parameter estimates in Table II were calculated using the approximation

$$
f_{x i}=\Delta X_{i} \quad p_{x}\left(X_{i}\right)
$$

where $\quad f_{x i}=$ expected relative frequency for the ith class interval,
$\Delta X_{i}=$ the midpoint of the ith class interval.
$X_{i}=$ range of the ith class interval ( 0.09 for the first class interval and 0.10 for all sub sequent class intervals).
$p_{x}\left(X_{i}\right)=$ the probability density function evaluated at the midpoint of the ith class interval.

The frequencies of daily rainfall amounts for each month of the year for the historical data over the eighty year record were also calculated using 0.10 inch class intervals (Appendix D). The Kolmogorof-Smirnof test and the Chi-square test (two sample tests) were performed on the historical frequencies and the exponential and lognormal distribution relative frequencies respectively, to determine whether the relative frequencies and the historical data could be from the same population.

The monthly Kolmogorof-Smirnof test statistics shown in Table VII for both the exponential and lognormal distributions were all less than the tabulated values of Seigel (1954). This shows that there is no evidence to indicate that the historical data cannot be described equally well by both the distributions. The Chi-square test statistics shown in Table VIII, however, indicated that the exponential distribution did not describe the historical data for three and ten months at the 0.10 and 0.01 levels respectively. The Chi-square tests for the lognormal distribution were not significant for any month at the 0.005 level. This indicates that we have to reject the hypothesis that the historical data can be described by the exponential distribution for all months. There is no evidence to make the same conclusion for the lognormal distribution.

The plots of the relative frequencies of the historical data, the exponential probability density function and the lognormal probability density function for the month of December are shown in Figure 3. This graphical comparison supports the above conclusion as the two parameter, lognormal distribution fits the historical data better than the one parameter exponential distribution. The same conclusion can be drawn from similar plots for the other months of the year shown in Appendix E. The lognormal distribution was therefore the distribution chosen for inclusion in the daily rainfall simulation model.

## TABLE VII

KOLMOGOROF-SMIRNOF TEST OF EXPONENTIAL AND LOGNORMAL DISTRIBUTIONS WITH THE HISTORICAL DAILY RAINFALL AMOUNTS

| MONTH | LOGNORMAL | EXPONENTIAL |
| :---: | :---: | :---: |
| 1 | 4.75847 |  |
| 2 | 6.91527 | 13.0493 |
| 3 | 7.79362 | 7.6481 |
| 4 | 4.84700 | 11.1554 |
| 5 | 4.36175 | 11.6524 |
| 6 | 6.68631 | 6.4628 |
| 7 | 4.41388 | 13.6132 |
| 8 | 4.69974 | 12.4578 |
| 9 | 5.26779 | 13.8454 |
| 10 | 5.10661 | 10.8213 |
| 11 | 5.00746 | 10.8160 |
| 12 | 5.65610 | 8.8881 |

Critical value at 0.01 level = 22

TABLE VIII

## CHI-SQUARE TEST OF EXPONENTIAL AND LOGNORMAL DISTRIBUTIONS WITH THE HISTORICAL DAILY RAINFALL AMOUNTS

| MONTH | LOGNORMAL | EXPONENTIAL |
| :---: | :---: | :---: |
| 1 | 7.9934 | $73.4008^{*}$ |
| 2 | 7.6083 | $68.2607^{*}$ |
| 3 | 9.9285 | $13.7595^{* *}$ |
| 4 | 9.1776 | $17.5343^{*}$ |
| 5 | 8.5141 | $20.7507^{*}$ |
| 6 | 9.3257 | $9.7913^{*}$ |
| 7 | 11.8849 | $34.2788^{*}$ |
| 8 | 7.7609 | $23.6096^{*}$ |
| 9 | 12.4910 | $15.6565^{* *}$ |
| 10 | 9.7854 | $18.2602^{* *}$ |
| 11 | 7.0361 | $21.6251^{*}$ |
| 12 | 12.7495 |  |

*Chi-square value at 0.10 level $=39.1$
** Chi-square value at 0.01 level $=14.3$


Figure 3. Relative Frequency Curves of Daily Rainfall Amounts for the Historical Data
and the Exponential and Lognormal Probability Density Functions for December.

Description of Model Developed

The daily rainfall model developed incorporates two sub-models: (a) rainfall event model, to generate the occurrence of a rainfall event (wet day) and (b) a rainfall amount model to generate the amount of rainfall that would accumulate on a wet day. The rainfall event model consists of a first order, two state, Markov chain in which a one and zero denote an event (wet day) and nonevent (dry day) respectively. The model requires twelve $2 x 2$ transitional probability matrices, each of the form

> event on day i

|  | 1 | 0 |
| :---: | :---: | :---: |
| event on day | $1 P(1 \mid 1)$ | $P(0 \mid 1)$ |
| i-1 | $0 P(1 \mid 0)$ | $P(0 \mid 0)$ |

These matrices describe the occurrence of a wet day or dry day occurring given the state of the previous day for each month of the year. The twelve transitional matrices are calculated from the historical data assuming that the transitional probabilities are stationary within each month. The transitional probabilities $P(1 \mid 0)$ and $P(1 \mid 1)$ for the twelve months of the year are entered into the model together with the record length (years) to be simulated. An initial dry state is assumed. For each day of the synthetic record, a random number, uniformly distributed between zero and one, is generated and compared with $P(1 \mid 0)$ or $P(1 \mid 1)$
depending upon the state of the previous day. If the random number is larger than the appropriate transitional probability a dry day results, otherwise a wet day is generated and the second sub-model is invoked.

The rainfall amount model is based upon two assumptions: (a) there is no persistence in daily rainfall and (b) the daily rainfall amounts are lognormally distributed. The two distribution parameters ( $M$ and $S^{2}$ ) are calculated from the mean and the variance of the historical data using the following moment relationships developed by Chow (1954).

$$
\begin{aligned}
& M=1 / 2 \ln \left(\bar{X}^{2} / C_{V}^{2}+1\right) \\
& S^{2}=\ln \left(C_{v}^{2}+1\right)
\end{aligned}
$$

where $C_{V}=S / \bar{X}$ coefficient of variation of the original data $S=$ standard deviation of the original data and $\overline{\mathrm{X}}=$ mean of the original data.

The values of $M$ and $S^{2}$ are calculated in the model from the predetermined values of the monthly means and monthly variances of the historical data as appropriate. The statistics of the historical data are calculated using standard procedures and the model must be modified with respect to these values for each location to which it is applied. The rainfall amount (X) on a simulated wet day is generated using the parameters $M$ and $S^{2}$ for the appropriate month, and a normally distributed random number ( $R$ ) with mean equal to zero and variance equal to one using the relationship.

$$
X=\exp (M+S(R))
$$

The output from the model consists of the date and rainfall amount for each wet day in the synthetically generated record. A flowchart of the daily rainfall simulation model is presented in Figure 4. A listing of the SAS (SAS, 1982) computer program of the model can be found in Appendix A. This program was used to generate the synthetic rainfall data used in this study.

## Simulation of Daily Rainfall Data

A synthetic daily rainfall record of any length can be generated using the daily rainfall model developed in this study. Some historical data are required to determine the model parameters (transitional probabilities and the distribution parameters). The length of historic record available would influence how accurately the model parameters can be determined. The assumptions that rainfall occurrence is weakly persistent and that daily rainfall amounts on consecutive days are independent must be verified before the model is applied to any location other than Stillwater, Oklahoma.

The following steps describe the application of the daily rainfall simulation model for data generation.

1. Determine the $2 \times 2$ transitional probability matrices for each month of the year from the historical daily rainfall record.


Figure 4. Flow Chart for the Daily Rainfall Simulation Model.
2. Enter the transitional probabilities $P(10)$ and $P\left(\begin{array}{ll}1 & 1)\end{array}\right.$ for each month in the model (24 values).
3. Determine the mean and variance of the daily rainfall amounts for each month for the wet days in the historical record.
4. Enter the values of the monthly means and variances of the historical record in the model (24 values).
5. Enter two random number generation seeds for the generation of the uniform and normal random number sequences (2 values).
6. Enter a year, greater than 1900, to indicate the imaginary period, starting at year 1900, for which rainfall data is to be simulated (1 value).

The above steps were executed and the daily rainfall model was used to generate the synthetic rainfall data for this study.

> Evaluation of the Daily Rainfall Model

The rainfall data generated by the daily rainfall model were analyzed and compared with the historical data in terms of (a) consecutive wet and dry days, (b) distribution of daily rainfall amounts, (c) mean monthly rainfall, (d) mean annual rainfall and (e) accumulated annual rainfall.

The curves in Figure 5 indicate that the historical consecutive wet day and dry day runs were well reproduced in the synthetic data.


The results of the statistical analyses of four, forty year synthetic daily rainfall records are shown in Table IX. These results compare favorably with the historical data. The mean monthly and mean annual rainfall amounts from the simulated records are normally distributed about the values of the historical data shown in Table II. The total number of wet days generated in each of the four simulated records compared favorably with the historical number of wet days in forty years. A double mass plot of accumulated annual rainfall for synthetic and historical records for 80 years is shown in Figure 6. The points plotted almost coincice with the equal value line and the slope of a fitted regression line is very close to one. The regression equation fitted was

$$
Q_{h}=56+.996 Q_{S}
$$

where $Q_{h}=$ accumulated annual historical rainfall

$$
Q_{S}=\text { accumulated annual synthetic rainfall. }
$$

These results indicate that a forty year synthetic rainfall record generated with the daily rainfall model developed would be an acceptable realization of a possible record. With this evidence it was assumed that it is not necessary to route a number of synthetic rainfall records through an hydrologic model, in this study, to asses the use of synthetic rainfall and a runoff model to predict watershed runoff.

TABLE IX
STATISTICAL ANALYSES OF RAINFALL ON WET DAYS GENERATED IN THE FOUR, FORTY-YEAR SYNTHETIC RAINFALL RECORDS

| MONTH | $\begin{aligned} & \text { MONTHLY } \\ & \text { MEAN } \\ & \text { (INCH) } \end{aligned}$ | $\begin{gathered} \text { DAILY } \\ \text { MEAN } \\ (1 / 100 \text { INCH }) \end{gathered}$ | STD OF <br> DAILY <br> MEAN |
| :---: | :---: | :---: | :---: |
| 1 | 1.24 | 28.71 | 35.89 |
| 2 | 1.16 | 25.97 | 28.38 |
| 3 | 1.87 | 34.86 | 34.35 |
| 4 | 3.69 | 48.78 | 66.04 |
| 5 | 4.15 | 48.02 | 51.81 |
| 6 | 4.09 | 51.66 | 60.28 |
| 7 | 2.91 | 49.12 | 61.24 |
| 8 | 2.62 | 42.62 | 57.96 |
| 9 | 4.06 | 63.33 | 86.45 |
| 10 | 3.04 | 49.35 | 51.18 |
| 11 | 2.18 | 42.05 | 44.14 |
| 12 | 1.35 | 28.93 | 33.49 |
| $\begin{aligned} & \text { ANNUAL } \\ & \text { TOTAL } \\ & \hline 22.42 \end{aligned}$ |  |  |  |
| MONTHLY | MONTHLY | DAILY | STD OF |
|  | MEAN | MEAN | DAILY |
|  | (INCH) | (1/100 INCH) | MEAN |
| 1 | . 81 | 20.15 | 10.49 |
| 2 | 1.27 | 26.13 | 30.85 |
| 3 | 2.01 | 38.17 | 43.84 |
| 4 | 1.89 | 41.62 | 46.74 |
| 5 | 4.36 | 46.95 | 66.50 |
| 6 | 3.73 | 47.53 | 54.09 |
| 7 | 1.89 | 48.24 | 58.09 |
| 8 | 3.29 | 49.97 | 70.08 |
| 9 | 3.68 | 58.30 | 80.67 |
| 10 | 3.25 | 57.61 | 69.92 |
| 11 | 2.70 | 56.01 | 118.91 |
| 12 | 1.18 | 27.41 | 25.14 |
| ANNUAL TOTAL | 32.11 |  |  |

TABLE IX CONTINUED

| MONTH | $\begin{aligned} & \text { MONTHLY } \\ & \text { MEAN } \\ & \text { (INCH) } \end{aligned}$ | $\begin{gathered} \text { DAILY } \\ \text { MEAN } \\ (1 / 100 \text { INCH }) \end{gathered}$ | $\begin{aligned} & \text { STD OF } \\ & \text { DAILY } \\ & \text { MEAN } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| 1 | 1.08 | 25.38 | 29.39 |
| 2 | 1.30 | 26.01 | 27.21 |
| 3 | 1.92 | 35.75 | 41.73 |
| 4 | 3.07 | 40.86 | 43.71 |
| 5 | 4.96 | 54.28 | 78.54 |
| 6 | 3.98 | 52.56 | 58.38 |
| 7 | 2.97 | 56.93 | 93.30 |
| 8 | 2.59 | 46.84 | 74.34 |
| 9 | 3.90 | 62.26 | 85.49 |
| 10 | 2.77 | 47.97 | 51.88 |
| 11 | 2.47 | 43.73 | 51.94 |
| 12 | 1.04 | 29.31 | 34.87 |
| ANNUAL TOTAL | 32.09 |  |  |
| MONTH | MONTHLY | DAILY | STD OF |
|  | MEAN | MEAN | DAILY |
|  | (INCH) | (1/100 INCH) | MEAN |
| 1 | . 96 | 25.27 | 35.10 |
| 2 | 1.36 | 27.79 | 39.10 |
| 3 | 2.51 | 39.45 | 44.10 |
| 4 | 2.75 | 38.83 | 50.16 |
| 5 | 4.15 | 46.71 | 52.66 |
| 6 | 3.73 | 50.02 | 52.65 |
| 7 | 3.22 | 52.82 | 135.91 |
| 8 | 2.65 | 45.53 | 51.96 |
| 9 | 3.16 | 53.40 | 60.64 |
| 10 | 3.07 | 50.27 | 58.96 |
| 11 | 2.36 | 46.88 | 52.70 |
| 12 | 1.44 | 33.55 | 48.81 |
| ANNUAL TOTAL |  |  |  |
|  | 31.42 |  |  |



Figure 6. Double Mass Plot of Accumulated Annual Rainfall for Synthetic (ASRAIN) and Historic (AHRAIN) Rainfall for 80 Years.

## APPLICATION OF SYNTHETIC AND HISTORICAL DATA TO AN HYDROLOGIC MODEL

The model developed and described in the previous chapter was used to generate forty years of rainfall data. This synthetic rainfall data and the observed rainfall data for Stillwater, Oklahoma, were used independently as input data in an hydrologic model chosen from a list of seventy-five currently available models (Renard et al. 1982) to predict watershed response in terms of runoff.

The USDAHL Model (Holtan and Lopez, 1971) and the CREAMS Model (Knizel, 1980) were subjected to extensive evaluation in the Department of Agricultural Engineering, at Oklahoma State University by Bengston (1980), Crow et al. (1977, 1980), Pathak (1983), Pathak et al. (1984). This previous research and experience served as a basis in deciding which model and watershed would be appropriate for this study.

Choice of Hydrologic Model and Watershed

The CREAMS hydrologic model was chosen to examine watershed response to synthetic rainfall data. This model was developed specifically for research purposes (Knisel,
1980). It was designed for field size watersheds which have single land use, a single management practice, relatively homogeneous soils and uniform rainfall. There are four components in the model, namely, the hydrologic, erosion, nutrient and pesticide components. Only the first, hydrologic component, of the model was used. Of the two model input options available (daily rainfall and break-point rainfall), option one for daily rainfall input was used. This option utilizes the SCS curve number model to estimate runoff.

Pathak (1983) applied the CREAMS Model to four watersheds in central Oklahoma. Of these four watersheds, the model performed most successfully for the 19.5 acre $R-7$ grassland watershed near Chickasha, Oklahoma. The model performance was assessed in terms of the predicted versus observed monthly and annual runoff resulting from observed daily rainfall. The CREAMS Model and the R-7 Chickasha watershed were chosen for use in this study.

The R-7 watershed topographical shape approximated a regular fan shape (Figure 7) with a slope ranging from 2.0 to 2.5 percent. The vegetation cover is blue stem grass and threeawn grass in areal proportions of 69 percent and 31 percent respectively. The soils are described in the soil survey of Grady County (USDA-SCS, 1978) as 38 percent Kingfisher silt loam, 39 percent Renfrow silt loam and 23 percent Kingfisher-Lucien complex. The watershed topographical


Figure 7. Topographic Map for Chickasha R-7 Watershed.
characteristics, soil profile ad plant cover condition parameters determined by Pathak (1983) were used.

Model Inputs

The CREAMS model reads input from two files, namely, the precipitation file and the input parameter file. These files must be prepared in the format specified in the CREAMS manual (Knisel et al. 1980).

The precipitation file contains the daily rainfall data for each year ( 365 values per year, 10 values per line, 37 lines per year) in the period for which runoff is to be determined. A maximum of twenty years data can be included in the file.

The input parameter file contains the title information, option parameters, watershed parameters, climatological data, plant cover data and a line with three instruction codes for each year of simulation. The optimum watershed parameters, established by Pathak (1983), for the R-7 watershed at Chickasha were used. These parameters are shown in Table $X$. Table XI shows the plant cover data included in the input parameter file. The grass cover on the watershed was rated a "good cover" by Pathak (1983) thus one-half of the recommended leaf area index values for a pasture in excellent condition given in the CREAMS manual (Knisel, 1980) were used. The recommended winter cover factor of 0.5 was used.

Table X

## CREAMS MODEL INPUT PARAMETERS FOR R-7 WATERSHED AT CHICKASHA, OKLAHOMA <br> (FROM PATHAK, 1983)

| Field area (acres) | 19.5 |
| :--- | :---: |
| Effective saturated hyd. conductivity (in/hour) | 0.04 |
| Fractions of pore space filled at field capacity | 0.87 |
| Initial fraction of available water storage filled | 0.50 |
| Soil evaporation parameter | 4.5 |
| Soil porosity (in/in) | 0.48 |
| Immobile soil water content (in/in) | 0.22 |
| Depth of surface soil layer (in) | 2 |
| Depth of maximum root growth layer (in) | 36 |
| Effective capillary tension (in) | 16.4 |
| Mannings n for overland flow | 0.03 |
| Effective hydraulic slope (ft/ft) | 0.038 |
| Effective hydraulic slope length (ft) | 290 |

## TABLE XI

LEAF AREA INDEX FOR NATIVE GRASS (FROM PATHAK, 1983)

| Julian Day | Leaf Area Index |
| :--- | :--- |
| 001 | 0.00 |
| 091 | 0.00 |
| 114 | 0.92 |
| 137 | 1.50 |
| 160 | 1.50 |
| 206 | 1.50 |
| 252 | 1.50 |
| 275 | 1.50 |
| 321 | 1.35 |
| 366 | 1.07 |

The mean monthly solar radiation data (Table XII) were taken from the CREAMS manual (Knisel, 1980). The mean monthly temperature data (Table XIII) were compiled from the temperature data used by Pathak (1983).

The above input data were used in the CREAMS model to predict runoff from the Chickasha R-7 watershed using synthetic and historical rainfall respectively.

Predicted Runoff Using Synthetic and<br>Historical Rainfall Data

The CREAMS hydrologic model predicts runoff on a daily, monthly and annual basis from daily rainfall data. The annual and monthly runoff amounts predicted for the Chickasha R-7 watershed from the historical and synthetic rainfall records respectively were used to evaluate the effect of using synthetic rainfall. A frequency analysis (Table XIV) of the annual runoff for an eighty year period was performed using half inch class intervals. This analysis showed that more small runoff events were predicted from the synthetic rainfall and more large runoff events were predicted from the historical rainfall. The frequency analysis on the monthly runoff data (Appendix $F$ ) indicate that the increased number of small runoff events from the synthetic rainfall occurred during the months of March, June, August, September, and October. The increased number of large runoff events from the historical rainfall occurred during the months of May, July, October, and November.

## TABLE XII

## MEAN MONTHLY SOLAR RADIATION FOR OKLAHOMA CITY, OKLAHOMA (FROM, KNISEL, 1980)

| Month | Mean <br> Radiation <br> (Iangleys) |
| :--- | :--- |
| January | 251 |
| March | 319 |
| April | 409 |
| May | 494 |
| June | 536 |
| July | 615 |
| August | 610 |
| September | 593 |
| October | 487 |
| November | 377 |
| December | 291 |

## TABLE XIII

## MEAN MONTHLY TEMPERATURE USED FOR THE R-7 <br> WATERSHED, AT CHICKASHA, OKLAHOMA

| Month | Mean <br> Temperature <br> (of) |
| :--- | :--- |
| January | 40.7 |
| February | 39.9 |
| March | 44.6 |
| April | 53.6 |
| May | 64.5 |
| June | 74.4 |
| July | 80.5 |
| August | 81.3 |
| September | 26.6 |
| October | 67.6 |
| November | 56.8 |
| December | 66.9 |

## TABLE XIV

RELATIVE FREQUENCY TABLE OF ANNUAL RUNOFF

| Runoff (0.5 inches intervals) | Frequency |  | Percent |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Synthetic Data | Historical Data | Synthetic Data | Historical Data |
| . 24 | 10 | 6 | 12.50 | 7.50 |
| . 75 | 13 | 8 | 16.25 | 10.00 |
| 1.25 | 8 | 7 | 10.00 | 8.75 |
| 1.75 | 8 | 7 | 10.00 | 8.75 |
| 2.25 | 10 | 13 | 12.50 | 16.25 |
| 2.75 | 6 | 7 | 7.50 | 8.75 |
| 3.25 | 1 | 8 | 1.25 | 10.00 |
| 3.75 | 4 | 2 | 5.00 | 2.5 |
| 4.25 | 4 | 3 | 5.00 | 3.75 |
| 4.75 | 5 | 2 | 6.25 | 2.5 |
| 5.25 | 3 | 4 | 3.75 | 5.00 |
| 5.75 | 3 | 2 | 3.75 | 2.50 |
| 6.25 | 1 | 0 | 1.25 | 0.00 |
| 6.75 | 0 | 2 | 0.00 | 2.50 |
| 7.25 | 1 | 1 | 1.25 | 1.25 |
| 7.75 | 2 | 0 | 2.5 | 0.00 |
| 8.25 | 0 | 1 | 0.00 | 1.25 |
| 8.75 | 0 | 0 | 0.00 | 0.00 |
| 9.25 | 0 | 3 | 0.00 | 3.75 |
| 9.75 | 0 | 0 | 0.00 | 0.00 |
| 10.00 | 1 | 4 | 1.25 | 5.00 |

The means and the standard deviations of the monthly runoff amounts are shown in Table XV. These results indicate that the means and standard deviation of the monthly runoff were fairly well preserved. Notable differences were found for the months of September, October, and November.

A summary of the input and output data for the CREAMS model is presented in Appendix $G$. In the table, the ratio of the accumulated annual runoff determined from the historical and synthetic rainfall varies from 1.24 to 3.69. This shows that the runoff predicted from the synthetic rainfall record is consistently less than the runoff from the historical rainfall record. The difference is 73.25 inches, or 25.6 percent, less than the runoff from historical rainfall over the eighty year record used. Figure 8 shows the scatter of the double mass plot of the accumulated annual runoff from the synthetic and historical rainfall tabulated in Appendix $G$. The regression equation fitted to the points was found to be

$$
\mathrm{R}_{\mathrm{H}}=20+1.25 \mathrm{R}_{\mathrm{S}}
$$

where $R_{H}=$ accumulated annual historical runoff

$$
\mathrm{R}_{\mathrm{S}}=\text { accumulated annual simulated runoff }
$$

The deviation from the equal value line is significant especially when related to the corresponding plot of the input rainfall data in Figure 6. This result is evidence that the hydrologic model is very sensitive to the rainfall input

TABLE XV
MONTHLY RUNOFF (INCHES) PREDICTED
FROM SYNTHETIC AND HISTORICAL
RAINFALL

|  | Mean |  |  | STD |
| :--- | :---: | :---: | :---: | :---: |
| Month | Synthetic <br> Data | Historical <br> Data | Synthetic <br> Data | Historical <br> Data |
| January | 0.08 | 0.05 | 0.40 | 0.15 |
| February | 0.06 | 0.06 | 0.24 | 0.24 |
| March | 0.17 | 0.23 | 0.43 | 0.50 |
| April | 0.38 | 0.49 | 0.86 | 0.94 |
| May | 0.54 | 0.59 | 0.96 | 1.25 |
| June | 0.17 | 0.26 | 0.50 | 0.56 |
| July | 0.23 | 0.26 | 0.89 | 0.98 |
| August | 0.15 | 0.22 | 0.42 | 0.48 |
| September | 0.46 | 0.58 | 1.67 | 1.22 |
| October | 0.17 | 0.47 | 0.45 | 1.32 |
| November | 0.19 | 0.29 | 0.43 | 0.74 |
| December | 0.06 | 0.08 | 0.23 | 0.25 |



Figure 8. Double Mass Plot of Accumulated Annual Runoff Determined from the Synthetic (ASRUN) and Historic (AHRUN) Rainfall for 80 Years.
data. It further suggests that great caution should be exercised in the use of synthetic rainfall to predict watershed runoff using an hydrologic model. While statistically similar rainfall records can be generated or found from two different locations, the differences in the daily rainfall amounts and the wet day sequences may be significant. These differences can lead to marked differences in predicted runoff when the rainfall is applied to the CREAMS hydrologic model. The under prediction of approximately $25 \%$ resulting from the application of synthetic rainfall to CREAMS is, however, within the acceptable limits for runoff prediction (Beasley et al. 1980).

## CHAPTER V

SUMMARY AND CONCLUSIONS

## Summary

A study was conducted to examine the use of synthetic rainfall in operational hydrology. The objectives of the study were to (a) develop a stochastic daily rainfall model and (b) to evaluate the use of synthetic rainfall data and a runoff model to study watershed hydrologic responses.

The rainfall model developed consisted of a first order, two state Markov chain to generate wet days, and the lognormal distribution to generate a rainfall amount for each wet day. The probabilities describing the four transitions (wet|wet, wet|dry, dry|wet, dry|dry) in the Markov chain were determined for each calendar month using eighty years of observed daily rainfall. The two parameters for the lognormal distribution were also determined for each month using the moment method of Chow (1954) and the observed daily rainfall data. A computer program using the SAS language was developed to generate synthetic daily rainfall. The synthetic rainfall data compared favorably with the historical data in terms of the consecutive wet and dry day runs, frequency of daily rainfall amount, mean monthly
and annual rainfall and accumulated annual rainfall over eighty years.

The synthetic and historical daily rainfall were used independently as input data for the CREAMS hydrologic model. The same watershed parameters, climatological data, soil data, and plant cover data were used in each simulation. The runoff predicted by the CREAMS model using the synthetic and historic rainfall data respectively, were compared in terms of the mean monthly runoff, mean annual runoff and accumulated annual runoff.

The runoff data from the synthetic and historical rainfall data did not compare as favorably as did the two types of rainfall input data itself. Although the means and standard deviations of the monthly runoff data appeared to be well reproduced, the annual runoff from the synthetic rainfall was consistently less than the annual runoff from the historical rainfall for each year in the eighty year record.

Conclusions

A satisfactory daily rainfall simulation model was developed. The analysis of the rainfall data generated by the model indicated that the inclusion of the Markov chain and the lognormal distribution was valid for the Stillwater area. The use of stationary transitional probabilities for each calendar month is not a major limitation of the model. The model could probably be applied to other areas after
making appropriate changes to the monthly transitional probabilities and the lognormal distribution parameters. It is important that Chow's (1954) method be used to determine the lognormal distribution parameters. Although representative synthetic rainfall data can be generated, discretion must be used in the application of such data.

The CREAMS rainfall-runoff model is sensitive to rainfall input data. Even with the marked similarities in the synthetic and historical rainfall data, the runoff predicted by CREAMS, using these two rainfall sequences as input, are somewhat different. The runoff from synthetic rainfall data was substantially less than the runoff from the historical data. From this it could be concluded that the slight differences between the hydrologic model input rainfall data were magnified in the output runoff data. There is insufficient evidence from this study, however, to place great confidence in this conclusion. Further work is needed to determine which components among those of evapotranspiration, antecedent soil moisture and curve number are most sensitive to rainfall and establish possible reasons for the runoff discrepancies.

The sensitivity of the hydrologic model to rainfall data emphasises the point that it is essential to use accurate, representative rainfall data when calibrating a rainfall-runoff model. The stochastic generation of synthetic rainfall data is a useful tool that may be used to extend limited rainfall records. Such extended rainfall
records, used in conjunction with a precalibrated hydrological model could provide valuable information regarding the long-term water resource potential of a watershed.

## Recommendations For Future Research

From the foregoing discussion and conclusions with respect to this study, the following areas for possible future research are identified:
a. Determine whether the rainfall model may be significantly improved through the use of continuously varying transitional probabilities and distribution parameters.
b. Determine the minimum length of rainfall record in arid and humid areas required for stable estimates of the rainfall model parameters.
c. Determine the effect on runoff, predicted by a rainfall-runoff model, when alternate rainfall data, collected from individual gages spatially distributed over the watershed, are used.
d. Determine the cause of the runoff discrepancies in the study reported by monitoring the values of the curve number, the soil moisture and evapotranspiration in the CREAMS model as the synthetic and historical rainfall input data are applied.

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## APPENDIX A

SAS COMPUTER PROGRAM LISTING OF THE DAILY RAINFALL SIMULATION MODEL

```
00010 //UI4520A JOB (14520,442-76-6277),CLASS=A,TIME=(0,40),
00020 // MSGCLASS=X,NOTIFY=*
00030 /*PASSNDRD BREE
00040 /*ROUTE PRINT RMT4
00050 // EXEC SAS
00080 //FREQ DD UNIT=3380,DSN=U14520A.RUNS.FREQ1.DATA,DISP=0LD
00090 //RUN DD UNIT=3380,DSN=U14520A.SAS.RUNS.DATA40,DISP=0LD
00100 //STAT DD UNIT=STORAGE,DSN=U14520A.SAS.STAT.TABLE,DISP=OLD
00110 //SYSIN DD #
0 0 1 2 0
00130 *********************************************************************
00140 * RAINFALL SIMLLATION MODEL *;
00150 * BY *;
00160 * J.E.PETER GREEN *;
00170 * *;
00180 * MARKOU CHAIN - LOGNORMAL FROBABILITY DISTFIBUTION FUNCTION *;
00190 * PROCES5 *;
00200 * *;
00210 #**************************************************F***************;
0 0 2 2 0
00240 SIMDRY(KEEP=YEAR JDAY MONTH PRECIP);
00250 SEED=41011;
00260 MAX=365;
00270 DO YEAR=1900 T0 1939;
00280 IF (YEAR/4-INT(YEAR/4))=0 THEN D=1;
00290 ELSE D=0;
00300 LASTDAY=MAX+D;
00310 DO JDAY=1 TO LASTDAY;
00320 IF YEAR=1900 AND JDAY=1 THEN EVENT=0;
0 0 3 3 0
```



```
00350 * INITIALISE THE MONTHLY TRANSITIONAL PROBABLITIES *;
00360 * P(W/W) AND P(W/D) *;
00370 * FOR THE MARKOV CHAIN PROCESS *;
00380 ********************************************************************;
00390
00400 IF JDAY GE 1 AND JDAY LE 31 THEN
00410 DO; DTW=.105; WTW=.349; LN MEAN=2.3586; MONTH=1;VAR=1.77929; END;
00420 ELSE IF JDAY GE 32 AND JDAY LE (59+D) THEN
00430 DO; DTW=.130; WTW=.354; LN_MEAN=2.5455; MONTH=2;VAR=1.77360;; END;
00440 ELSE IF JDAY GE (60+D) AND JDAY LE (90+D) THEN
00450. DO; DTW=.146; WTW=.353; LN_MEAN=2.9256; MONTH=3;VAR=1.78117; END;
00460 ELSE IF JDAY GE (91+D) AND JDAY LE (120+D) THEN
00470 DO; DTW=.199; WTW=.407; LN_MEAN=2.9627; MONTH=4;VAR=2.03071; END;
00480 ELSE IF JDAY GE (121+D) AND JDAY LE (151+D) THEN
00490 DO; DTW=.249; WTW=.427; LN_MEAN=3.0728; NONTH=5;VAF=2.04375; END;
00500 ELSE IF JDAY GE (152+D) AND JDAY LE (181+D) THEN
00510 DO; DTW=.208; WTW=.411; LN_MEAN=3.1706; MONTH=6; VAR=1.93085; END;
00520 ELSE IF JDAY GE (182+D) AND JDAY LE (212+0) THEN
00530 DO; DTW=.152; WTW=.361; LN MEAN=3.0173; MONTH=7;VAR=2.14966; END;
00540 ELSE IF JDAY GE (213+D) AND JDAY LE (243 +D) THEN
00550 DO; DTW=.162; WTW=.347; LN MEAN=2.9976; MONTH=8;VAR=2.09077; END;
00560 ELSE IF JDAY GE (244+D) AND JDAY LE (273+D) THEN
00570 DO; DTW=.157; WTW=.412; LN.MEAN=3.1821; MONTH=9;VAR=2.23437; END;
```

00580 ELSE IF JDAY GE ( $274+\mathrm{D})$ AND JDAY LE $(304+\mathrm{D})$ THEN
00590 DO; DTW=.132; WTW=.390; LN MEAN=3.1081; MONTH=10;VAR=2.11499; END;
00600 ELSE IF JDAY GE ( $305+\mathrm{D}$ ) AND JDAY LE $(334+\mathrm{D})$ THEN
00610 DO; DTH=.113; WTW=.394; LN MEAN=2.9793; MONTH=11;VAR=1.96010; END;
00620 ELSE IF JDAY GE $(335+D)$ AND JDAY LE (MAX +D$)$ THEN
$00630 \quad D 0 ; D T W=.110 ; W T W=.320 ;$ LN_MEAN=2.6060; MONTH=12;VAR=1.94277; END;
00640
00650 LAMBDA=1/LN_MEAN;

$00670 *$ SIMULATION OF DAILY RAINFALL AMOUNTS *;
00680 * USING *;
$00690 \pm$ LOGNORMAL PROBABLITY DENSITY FUNCTION

00710
00720 IF EVENT=1 THEN
00730 DO; IF RANUNI (SEED) LT WTH THEN
DO; EVENT=1;
RAINL: PRECIP=EXP(LN_MEAN+SQRT(UAR)*RANNOR (SEED)):
IF PRECIP LT 1 THEN PRECIP $=1$ :
ELSE IF PRECIP GT 750 THEN 60 TO RAINI;
END:
00780 ELSE DO; EUENT=0;OUTPUT SIMDRY; END;
00790 END;
00800 ELSE IF EVENT $=0$ THEN
00810 DO; IF RANUNI (SEED) LT DTH THEN
$00820 \quad$ DO; EVENT=1;
00830 RAIN2: PRECIP=EXF (LN MEAN+SQRT(VAR)*RANNOR (SEED));
00832 IF PRECIF LT 1 THEN PRECIP = 1;
00833 ELSE IF PRECIP GT 750 THEN 60 TO RAIN2;
00850 END;
00860 ELSE DO; EVENT=0; OUTPUT SIMDRY: END;
00870 END
00880
00890 RETAIN EVENT;
00900 END;
00910 END;
00920

00940 \# DETERMINATION $*$

00950 * OF $\quad$ :
00960 * COMSECUTIVE WET AND DRY DAY RUNS $\quad$;

00980
00990 DATA RUN.LNIDRY40(KEEP=DRUN) RUN. LNIWET40(KEEP=WRUN);
01010
01020 IF (YEAR/4-INT(YEAR/4))=0 THEN
$01030 \quad D 0 ; D=1 ;$ MAX $=366 ;$ PREMAX $=365$; END;
01040 ELSE IF (YEAR/4-INT(YEAR/4)) $=.25$ THEN
$01050 \quad \mathrm{DO} ; \mathrm{D}=0 ;$ MAX $=365$; PREMAX $=366$; END;
01060 ELSE DO; $D=0 ;$ MAX $=365 ;$ PREMAX $=365$; END;
01070
01080 IF N_ EQ 1 AND JOAY EQ 1 THEN
$01090 \quad$ DO; $W N=1$;
01100 END;

```
01110 ELSE IF N EQ I AND JDAY NE I THEN
01120 DO;盺=1;
01130 DRUN=JDAY-1; OUTPUT RUN.LNIDRY40;
01140 DI=JDAY;
01150 END;
0 1 1 6 0
01170 ELSE IF JDAY EQ DI+1 THEN
01180 DO;WN=WN+1:
01190 DI=JDAY;
01200 END;
01210 ELSE IF JDAY GT DI+1 THEN
01220 DO;WRUN=WN; OUTPUT RUN.LNIWET40;
01230 DRUN=JDAY-(DI+1); DUTPUT RUN.LNIDRY40;
01240 DI=JDAY;
01250 WN=1;
01260 END;
0 1 2 7 0
01280 ELSE IF JDAY LT DI AND DI LT PREMAX THEN
01290 DO;HRUN=WN; OUTPUT RUN.LNINET40;
01300 DRUN=PREMAX-DI+JDAY-1; OUTFUT RUN.LN1DRY40;
01310 DI=JDAY;
01320 WN=1;
01330 END;
01340 ELSE IF JDAY EQ I AND DI EQ PREMAX THEN
01350 DO; WN=WN+1;
01380 DI=JDAY;
01370 END;
01380 ELSE IF JDAY LT DI AND DI EE PREMAX THEN
01390 DO; WRUN=WN; OUTPUT RUN. LNIHET40;
01400 DRUN=JDAY-1; OUTPUT RUN.LNIDRY40;
01410 DI=JDAY;
01420 WN=1;
01430 END:
01440
O1450 ELSE DO;PUT 'CHECK DATA AT 'YEAR JDAY ;
01460 DI=JDAY;
01470 END;
0 1 4 8 0
01490 RETAIN WN;
01500 RETAIN DI;
01510
01520 *****************t*******************************************!*******;
01530 * FREQUENCY ANALYSIS *;
01540 * OF *;
01550 * CONSECUTIVE WET ANI IRY DAY RUNS *;
01560 **********************i゙***********************************************
01570
01580 PROL SREQ OATA=RUN. LNIWET40;
U!590 TABLE HRUN/OUT=FREQ.LNIWET40;
\1600 TITLE FREquENCY TABLE FOR CONSECUTIVE WET DAYS;
01610 TITLE2 40 YEARS - SIMULATED DATA - RUN 1B;
0 1 6 2 0
01630 PROC FREQ DATA=RUN.LNIDRY40;
01640 TABLE DRUN/DUT=FREQ.LNIDRY40;
```

```
01650 TITLE FREQUENCY TABLE FOR CONSECUTIVE DRY DAYS;
01660 TITLE2 40 YEARS - SIMLLATED DATA - RUN 1E:
01670
01680 ***********************************************************************
01690 * FREQUENCY ANALTSIS *
01700 * OF *
01710 * SIMULATED DAILY RAINFALL AMOUNTS *
01720 * (LOGNORMAL DISTRIBUTION) *
01730 *#********************************************************************
01740
01750 DATA ONE ;
01770 PPT=INT (PRECIP/10)*10+5;
0 1 7 8 0
01790 PROC SOFT DATA=ONE ;BY MONTH;
0 1 8 0 0
01810 PROC FREQ DATA=ONE; BY MONTH;
01820 TABLES PPT/OUT=FREQ.LNIMD40;
01830 TITLE FREQUENCY TABLE FOR 40 YEARS OF SIMULATED DATA - RUN IB;
01840 TITLE2 MARKOV CHAIN - LOGNORMAL DISTRIBUTION:
01850
01860 ****************F*************F*******F**F*******************Ft+k*****;
01870 * CALCULATE THE STATISTICAL PARAMETERS *
01875 * FOR THE *
01880 * SIMULATED DAILY RAINFALL AMOUNTS *
01885 * (LOGNORMAL DISTRIBUTION) *
01900 **********************************************************************
0 1 9 1 0
0 1 9 3 0
01950 BY MONTH ; VAR PRECIP;
01960 OUTPUT OUT=STAT1
01970 SUM=SUM MEAN=D_MEAN STD=STD VAR=VAR;
0 1 9 8 0
01990 DATA STAT.LNIDAT40;
02000 SET STATI;
02001 IF MONTH = 1 THEN TOTAL = 0;
02010 YEARS=40;
02015 M_MEAN=SUM/40;
02016 TOTAL=TOTAL +M_MEAN:
02017 DROP SUH:
02018 OUTPUT:
02019 RETAIN TOTAL;
02020 PROC PRINT DATA=STAT.LNIDAT40;
02022 VAR M_MEAN D_MEAN STD VAR TOTAL;
02030 TITLE STATISTICS FOR RAIN EVENTS FOR ;
02040 TITLE2 40 VEARS OF SIMULATED DATA;
02050 TITLES LOGNORMAL DISTRIBUTION - RUN 1B;
02060
READY
```

APPENDIX B
RELATIVE FREQUENCY CURVES OF DAILY RAINFALL AMOUNTS FOR WET DAYS (TOTAL), WET DAYS FOLLOWING DRY DAYS (DRY) AND WET DAYS FOLLOWING WET
DAYS (WET) FOR
EACH MONTH









## APPENDIX C

PARAMETER ESTIMATION FOR THE LOGNORMAL DISTRIBUTION
I. Method of Moments (Haan, 1977) Yields

$$
\begin{aligned}
& \theta_{1}=\bar{Y}=\sum_{i=1}^{n}\left(Y_{i} / n\right) \\
& \theta_{2}^{2}=S y^{2}=\sum_{i=1}^{n}\left(Y_{i}-\bar{Y}\right)^{2} /(n-1)
\end{aligned}
$$

II. Chow's (1954) Method Yields

$$
\begin{aligned}
& \theta_{1}= 1 / 2 \ln \left(\bar{x}^{2} /\left(C_{V}^{2}+1\right)\right) \\
& \theta_{2}^{2}=\ln \left(C_{v}^{2}+1\right) \\
& \text { where } C_{V}= S_{x} / \bar{x} \text { (coefficient of variation of the } \\
& \text { original data) } \\
& S_{X}= \text { standard deviation of the original data and } \\
& \bar{x}= \text { mean of the original data. }
\end{aligned}
$$

III. Method of Maximum Likelihood

The lognormal probability density function is
$p_{x}(x)=\left(2 \pi \theta_{2}^{2}\right)^{-1 / 2} x^{-1} \exp \left(-\left(\ln x-\theta_{1}\right)^{2} / 2 \theta_{2}^{2}\right)$
and the maximum likelihood function is

$$
L\left(\theta_{1}, \theta_{2}^{2}\right)=\left(2 \pi \theta_{2}^{2}\right)^{-n / 2} \prod_{i=1}^{n} x_{i}^{-1} \exp \left(-\sum_{i=1}^{n}\left(\ln x_{i}-\theta_{1}\right)^{2} / 2 \theta_{2}^{2}\right)
$$

Taking the natural logarithms yields
$\ln L\left(\theta_{1}, \theta_{2}^{2}\right)=-n / 2 \ln (2 \pi)-n / 2 \ln \left(\theta_{2}^{2}\right)-$

$$
-\sum_{i=1}^{n} \ln x_{i}-\sum_{i=1}^{n}\left(\ln x_{i}-\theta_{1}\right)^{2} / 2 \theta_{2}^{2}
$$

Maximizing with respect to $\theta_{1}$ yields

$$
\begin{aligned}
0 & =\mathrm{d}\left(\ln L\left(\theta_{1}, \theta_{2}^{2}\right) / \mathrm{d} \theta_{1}\right. \\
& =\sum_{i=1}^{n}\left(\ln x_{i}-\theta_{1}\right) \\
& =\sum_{i=1}^{n} \ln x_{i}-n \theta_{1}
\end{aligned}
$$

Thus $\theta_{I}=\sum_{i=1}^{n}\left(\ln x_{i}\right) / n$
How if $Y_{i}=\ln x_{i}$
Then $\bar{Y}=\sum_{i=1}^{n}\left(\ln x_{i}\right) / n$
and $\theta_{1}=\bar{Y}$
Maximizing with respect to $\theta_{2}^{2}$ yields

$$
\begin{aligned}
0 & =d\left(\ln L\left(\theta_{1}, \theta_{2}^{2}\right) / d \theta_{2}^{2}\right. \\
& =n / 2 \theta_{2}^{2}+\sum_{i=1}^{n}\left(\ln x_{i}-\theta_{1}\right)^{2} / 2 \theta_{2}^{4} \\
& =-\left(n-\sum_{i=1}^{n}\left(\ln x_{i}-\theta_{1}\right)^{2} / \theta_{2}^{2}\right) / 2 \theta_{2}^{2}
\end{aligned}
$$

$$
\text { Thus } \theta_{2}^{2}=\sum_{i=1}^{n^{+}}\left(\ln x_{i}-\theta_{1}\right)^{2} / n
$$

Since $Y_{i}=\ln x_{i}$

$$
\theta_{1}=\bar{Y}
$$

and $S_{y}^{2}=\sum_{i=1}^{n}\left(Y_{i}-\bar{Y}\right)^{2} /(n-1)$

Then $\theta_{2}^{2}=\sum_{i=1}^{n}\left(Y_{i}-\bar{Y}\right)^{2}(n-1) /(n-1) n$

$$
=S_{y}^{2}(n-1) / n
$$

## APPENDIX D

CUMULATIVE FREQUENCY TABLES OF HISTORICAL AND SYNTHETIC DAILY RAINFALL AMOUNTS

COLUMN HEADINGS
P MID POINT OF RAINFALL CLASS

CUM _ PER CUMULATIVE PERCENT OF HISTORICAL DATA
CUM _ LN CUMULATIVE PERCENT OF SYNTHETIC DATA


| MONTH $=2$ |  |  | MONTH $=3$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 BS |  | CUM_PER CUM_LH | 055 | P | CUM_PER CUM_LN |
| 32 | 5 | 38.62442 .1194 | 63 | 5 | 31.14033 .0848 |
| 33 | 15 | $55.026 \quad 61.9417$ | 64 | 15 | 44.95652 .7498 |
| 34 | 25 | 68.78372 .4862 | 65 | 25 | 58.11464 .4215 |
| 35 | 35 | 77.77878 .9064 | bo | 35 | 66.66772 .0524 |
| 36 | 45 | 84.34283 .1578 | 07 | 45 | 72.58877 .4046 |
| 37 | 55 | 88.36086 .1376 | 68 | 55 | 77.63281 .3180 |
| 38 | 65 | 91.00588 .3209 | 69 | 65 | 81.36084 .2865 |
| 39 | 75 | 93.12289 .9090 | 70 | 75 | 85.30786 .6003 |
| 40 | 85 | 94.18091 .2466 | 71 | 85 | 88.15888 .4435 |
| 41 | 95 | 95.23892 .2574 | 72 | 95 | 90.13289 .9384 |
| 42 | 105 | 76.03293 .6726 | 73 | 105 | 92.10591 .1693 |
| 43 | 115 | 96.82593 .7387 | 74 | 115 | 94.29872 .1758 |
| 44 | 125 | 97.619 .94 .2902 | 75 | 125 | 95.61493 .0614 |
| 45 | 135 | 98.41394 .7519 | 75 | 135 | 96.05373 .798 .3 |
| 46 | 145 | 98.41395 .1422 | 77 | 145 | 97.149 94.4312 |
| 47 | 155 | 78.94295 .4750 | 78 | 155 | 97.36894 .9786 |
| 48 | 16.5 | 98.74295 .7610 | 79 | 1.55 | 97.36895 .4558 |
| 49 | 175 | 99.206 76.0084 | 80 | 175 | 97.59895 .8739 |
| 50 | 185 | 99.471.96.2238 | 81 | 185 | 78.46596 .2424 |
| 51 | 145 | 79.471 96.4125 | 82 | 195 | 98.68476 .5689 |
| 52 | 205 | 99.471 76.5785 | 43 | 205 | 97.12376 .8593 |
| 53 | 215 | 99.73596 .7253 | 84 | 215 | 99.34297 .1189 |
| 54 | 225 | 97.735 86.8558 | 85 | 225 | 97.78197 .3518 |
| 55 | 235 | 97.73596 .4721 | 89 | 235 | 97.78197 .5515 |
| 56 | 245 | 97.73597 .076 .3 | 87 | 245 | 94.78197 .7511 |
| 57 | 25.5 | 99.73577 .1698 | 86 | 25.5 | 97.78197 .7224 |
| 58 | 265 | 97.73597 .2542 | 49 | 205 | 100.00098 .0790 |
| 54 | 275 | 100.00097 .3304 | 90 | 275 | 100.00078 .2214 |
| 00 | 285 | 100.00097 .3796 | 71 | 285 | 100.00078 .3514 |
| 01 | 275 | 100.00097 .4824 | 92 | 295 | 100.00078 .4705 |
| 62 | 3 3 51 | 100.000977 .5197 | 93 |  | 10.000 98.5800 |

## MONTH $=4$

## OSS P CIM_PER CUM_LN

$94 \quad 531.561532 .1031$
$\begin{array}{lll}95 & 15 & 46.8439 \\ 50.4712\end{array}$
$96 \quad 2556.976761 .4898$
$97 \quad 3564.784168 .8257$
$98 \quad 4569.435274 .0461$
$99 \quad 5573.089777 .9367$
$100 \quad 6.578 .571480 .9370$
$101 \quad 7582.392083 .3124$
$102 \quad 8584.551585 .2331$
103 斩 86.544986 .8129
10410589.202788 .1312
10511570.531689 .2448
10612571.362190 .1953
10713592.524991 .0142
10814594.352291 .7253
10915595.348892 .3473

110165 95.847292 .8948 11117596.511693 .3795 11218597.010093 .8109 11319597.176194 .1966 11420597.840594 .5430 11521597.840594 .8554 11622598.006695 .1380 11723598.505095 .3746 11824598.837295 .6284 11725597.169495 .8420
12026599.169496 .0376
12127599.169496 .2173
12228599.335596 .3827
12324599.335596 .5353
12436599.335596 .6764
$126 \quad 1544.312247 .7549$
$127 \quad 2554.4974 \quad 56.8591$
$128 \quad 35 \quad 62.037066 .3907$
$129 \quad 4567.989471 .8265$
$130 \quad 5571.957775 .9243$
$131 \quad 6575.925979 .1147$
$132 \quad 7578.8360 \quad 81.6617$
$133 \quad 8582.804283 .7362$ $134 \quad 5585.5820 .85 .4537$ 13510584.904880 .8754 13511585.354886 .1199 13712589.682589 .1702 13813591.005390 .0793 13914591.931290 .8721 14015542.460341 .5684 $14116592,989492.1836$ 14217593.915392 .7302 14318594.576793 .2163 14417595.370473 .6501 14520596.428694 .0505 14621596.693174 .4072 14722596.957794 .7309 14823597.069995 .0256 14924598.148195 .2947 15025598.677295 .5411 15126598.941895 .7675 15227599.074195 .9758 15326599.206376 .1680 15429599.205396 .3457 15530595.206396 .5104

MONTH=6

OBS P CUM FER CUM LN
$56 \quad 524.242427 .4931$
$157 \quad 1538.915545 .6018$
$\begin{array}{lll}158 & 25 & 51.3557 \\ 57.0789\end{array}$
$\begin{array}{lll}59 & 35 & 60.7656 \\ 64.9733\end{array}$
$160 \quad 4566.347770 .7186$
$161 \quad 5570.175475 .0731$
$162 \quad 6573.524778 .4756$
$163 \quad 7577.671581 .1785$
$164 \quad 8581.020783 .4197$
$165 \quad 9583.732185 .2612$
16610585.486486 .8078
16711587.559888 .1216 16812589.473789 .2471 16913591.547090 .2248 17014592.344591 .0755 17115593.779991 .823 $17216594.736892,4820$ 17317595.853393 .0678 17418596.969793 .5905 17519597.767194 .0593 17620598.086194 .4812 17721598.086194 .8625 17822598.405195 .2083 17923599.043195 .5228 19024599.202695 .8077 18125599.202696 .0726 18226599.202676 .3136 18327597.202696 .5353 18428599.362096 .7396
18524597.521576 .7285
18630599.521597 .1033

## $15+5$

MONTH $=7$
OBS P CUM_PER CUM_LN
$187 \quad 531.027330 .8877$
$188 \quad 1546.540948 .6286$
$189 \quad 25 \quad 55.1363 \quad 57.4101$
$\begin{array}{lll}190 & 35 & 62.2642 \\ 66.6780\end{array}$
$19145,67.505271 .9088$
$192 \quad 5573.165675 .8475$
$193 \quad 6576.939278 .9134$
$174 \quad 7581.341781 .3618$
$175 \quad 8583.857483 .3575$
$146 \quad 7584.905785 .0113$
19710585.744286 .4011
19811587.631087 .5828
19912589.517888 .5979
$20013590.3564 \quad 69.4775$
20114590.775790 .2458
20215592.243290 .9214
20316593.081891 .5192
20417594.549392 .0511
20518594.968692 .5268
20619595.387892 .9540
20720596.015893 .3394
20821596.436193 .6885
20922576.855394 .0056
21023547.055054 .2948
21124597.693494 .5592
1225597.903674 .8016
21326596.113295 .0246
21427598.113275 .2301
21528578.113275 .4197
1027598.113295 .5957
21730578.751875 .7588

HUNTH=8

## DBG P CUM PER CUM LN

$218 \quad 530.223131 .3262$
$219 \quad 1546.450349 .3548$
$220 \quad 25 \quad 55.7809 \quad 60.2624$
$221 \quad 35 \quad 62.880367 .5801$
$222 \quad 4569.574072 .8227$
$223 \quad 5574.645076 .7536$
$224 \quad 6578.701879 .8015$
$225 \quad 7581.744482 .2267$
$226 \quad 8583.570084 .1966$
$227 \quad 9586.409785 .8239$ 22810586.815487 .1873
22911588.438188 .3432 23012590.669489 .3334 23113591.277990 .1894 23214591.886490 .9350 23315593.103491 .5892 23416593.5091 .92 .1668 23517593.914892 .6795 23618594.523393 .1370 23719595.131893 .5472 23820595.740473 .9165 23921596.146094 .2502 24022596.754694 .5530 24123597.160294 .8285 24224597.565995 .0800 24325598.174495 .3102 24426598.377395 .5216 24527598.783095 .7161 24628599.188695 .8755 24729599.188696 .0613 248.30599 .371596 .2150

| MONTH=7 |  | MONTH $=10$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
|  |  | 085 | P CUM_fef | CUH_LH |
|  | P CUM_PER CUM_LN |  |  |  |
|  |  | 280 | 528.5068 | 27.0357 |
| 247 | 529.108927 .6212 | 281 | 1541.6290 | 46.6447 |
| 250 | 1542.376244 .5412 | 282 | 2552.7147 | 57.5857 |
|  | 2552.277255 .2136 | 283 | 3557.9548 | 65.0514 |
| 252 | 3558.415862 .6047 | 284 | 4566.2896 | 70.4931 |
| 253 | 4562.772368 .0401 | 285 | 5571.0407 | 74.6129 |
|  | 5568.118872 .2067 | 286 | 65 74.4344 | 77.8365 |
|  | .65 72.871375 .5007 | 287 | 7576.9231 | 80.4268 |
| 256 | 7577.029778 .1675 | 288 | 8581.6742 | 82.5451 |
| 257 | 85 79.009980 .367 日 | 289 | 9583.2577 | 84.3057 |
| 253 | 9580.396082 .2117 | 290 | 10585.2941 | 85.7917 |
| 2591 | 10581.980283 .7770 | 291 | 11587.5566 | 87.0578 |
| 2601 | 11582.970385 .1206 | 292 | 12589.5928 | 88.1481 |
|  | 12585.346586 .2848 | 293 | 13591.1765 | 87.0950 |
| 262 | 13586.534787 .3021 | 294 | 14572.53 .35 | 87.78 .37 |
|  | 14588.316886 .1574 | 295 | 15593.6552 | 90.6538 |
| 26 | 15588.712988 .9704 | 296 | 16594.1176 | 81.3009 |
| 265 | 16589.703089 .6970 | 297 | 17594.3439 | 71.8775 |
| 26 | 17591.485190 .3298 | 298 | 18595.2487 | 72.3939 |
| 26 | 18591.881290 .8991 | 278 | 19595.7014 | 92.8595 |
| 268 | 19573.267391 .4137 | 300 | 20546.6063 | 93.2781 |
| 264 | 20594.059471 .8805 | 301 | 21597.0588 | 73.6595 |
| 270 | 215 75. 949592.3055 | 302 | 22597.2851 | 74.0045 |
|  | 22595.445592 .6938 | 305 | 23597.7636 | 94.3204 |
| 27 | 23595.643673 .0475 | 304 | 24.59 .4163 | 74.6055 |
| 27. | 24596.435693 .3765 | 305 | 25599.6425 | 94.8749 |
|  | 25596.435693 .6777 | 306 | 26578.8688 | 95. 1191 |
| 27 | 26596.831793 .7559 | 307 | 27599.0750 | 95. 3444 |
| 27 | 275 77.4257 94.2135 | 308 | 26597.0950 | 95.5527 |
|  | 28597.821894 .4525 | 309 | 29577.0750 | 95.7457 |
| 27 | 29597.821894 .6747 | 310 | 30577.0750 | 95.7250 |
|  | 30598.217894 .8017 |  |  |  |

MONTH=11
GES F CUM_PEFCUM_LN
$311 \quad 528.989431 .7805$
$312 \quad 1547.0745 \quad 50.4240$
$313 \quad 25 \quad 56.6489 \quad 61.6564$
$314 \quad 3566.223469 .1370$
$315 \quad 4571.542674 .4547$
$316 \quad 5576.329878 .4113$
$317 \quad 6577.255381 .4565$
$318 \quad 7581.914983 .8627$
$319 \quad 8585.372385 .8041$
$320 \quad 9586.702187 .3977$
32110588.031986 .7247
32211589.893689 .8434
32312591.223490 .7904
32413573.085191 .6158
32514594.414992 .3261
$326 \quad 15595.212692 .9461$
32716595.478793 .4909
$32817596.542693 .41 \mathrm{L7}$ 32918596.808594 .4002 33019597.340494 .7820 33120597.606495 .1244 33221597.606495 .4326 33322597.872395 .7110 33423598.138395 .7634 33524598.138376 .1931 33625598.404396 .4025 33726598.670296 .5941 33827598.936276 .7698 33928599.202196 .9313
34029597.202177 .0802
34130577.406197 .2176

MONTH $=12$
$0 B 5$ P CUM PER CUM_LN
$342 \quad 5 \quad 37.209 \quad 37.8986$
$\begin{array}{llll}343 & 15 & 56.686 & 58.9789\end{array}$
$\begin{array}{llll}344 & 25 & 65.698 & 69.3228\end{array}$
$345 \quad 35 \quad 72.674 .75 .8077$
$\begin{array}{llll}346 & 45 & 79.360 & 80.1968\end{array}$
$347 \quad 55 \quad 82.26783 .3363$
$\begin{array}{lllll}348 & 65 & 85.465 & 85.6743\end{array}$
$349 \quad 75 \quad 89.53587 .4701$
$350 \quad 85 \quad 91.86089 .8835$
$351 \quad 95 \quad 93.02390 .0183$
$352105 \quad 95.34990 .9445$
35311595.93091 .7113
$\begin{array}{lll}354 & 125 & 97.093 \\ 72.3537\end{array}$
$355135 \quad 98.54792 .8976$
$356 \quad 145 \quad 98.54793 .3622$
$357155 \quad 99.41993 .7625$
$358 \quad 165 \quad 99.41994 .1098$
$357 \quad 175 \quad 99.709 \quad 94.4131$
$360185 \quad 99.70994 .6795$
$361195 \quad 99.70994 .9149$
$362205 \quad 97.709 \quad 95.1237$
$363215 \quad 97.70995 .3099$
364225100.00095 .4766
365235100.00095 .6264
366245100.00095 .7615
367255100.00095 .8837
368265100.000 .95 .9947
367275100.00096 .0957
370285100.00096 .1878
371275100.00096 .2721
372305100.60076 .3495

## APPENDIX E

PLOTS OF THE MONTHLY RELATIVE FREQUENCIES OF THE HISTORICAL DATA, THE EXPONENTIAL PROBABILITY DENSITY FUNCTION AND THE LOGNORMAL PROBABILITY<br>DENSITY FUNCTION








a




RAINFALL (1/100 INCH)



## APPENDIX $F$

FREQUENCY ANALYSES OF MONTHLY RUNOFF DATA
FFERUENCY TAELES FOF MONTHLY FIUNOFF
MONTH=JAN

| FIUN | FFEQUENCY CUM FFIEQ | FEFICENT | CUM FEFICENT |  |
| ---: | :---: | :---: | :---: | ---: | ---: |
| 25 | 77 | 77 | 96.250 | 96.250 |
| 75 | 3 | 80 | 3.750 | 100.000 |
| SFUN | FFEQUENCY | CUM FFEQ | FEFCENT | CUM FFEFCENT |
| 25 | 77 | 77 | 96.250 |  |
| 125 | 2 | 79 | 2.500 | 96.250 |
| 325 | 1 | 80 | 1.250 | 100.000 |

MONTH=FEE

| FUN | FFFEQUENCY | CUM FFEQ | FEFFCENT | CUM FFFFCENT |
| ---: | :---: | :---: | ---: | ---: | ---: |
| 25 | 78 | 78 | 97.500 | 97.500 |
| 125 | 1 | 79 | 1.250 | 98.750 |
| 175 | 1 | 80 | 1.250 | 100.000 |
| SFUN | FFEEQUENCY | CUM FFEQ | FFEFCENT | CUM FEFICENT |
| 25 | 77 | 77 | 96.250 | 96.250 |
| 75 | 2 | 79 | 2.500 | 98.750 |
| 175 | 1 | 80 | 1.250 | 100.000 |

## MOPTH=MFF:

FUN FFEQUENCY CUM FFEE FEFCENT CUM FEFCENT

| 25 | 69 |  | 69 | 96, 20 |  | 86.250 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\%$ | 5 |  | 74 | $6+250$ |  | 92.500 |
| 125 | :. |  | 76 | 2.500 |  | 95.000 |
| 175 | 3 |  | 79 | 3.750 |  | 98.750 |
| 325 | 1 |  | 80 | 1.250 |  | 100.000 |
| SFUN | FFEQUENCY | CUM | FFiEQ | FEFICENT | CUM | F.EFICENT |
| 25 | 73 |  | 73 | 91.250 |  | 91.250 |
| 75 | 2 |  | 75 | 2.500 |  | 93.750 |
| 12ち | 3 |  | 78 | 3.750 |  | 97.500 |
| 225 | 2 |  | 80 | 2.500 |  | 100.000 |

FFERUENCY TAELES FDF MONTHLY FUUNUFF MOINTH=AFF:

| FiUN | FFEEQUENCY | CUM | FFEE | FEFSENT | CUM | FEFICENT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 25 | 58 |  | 58 | 72.500 |  | 72.500 |
| 75 | 10 |  | 68 | 12.500 |  | 85.000 |
| 125 | 5 |  | 73 | 6.250 |  | 91.250 |
| 175 | 1 |  | 74 | 1.250 |  | 92.500 |
| 225 | 2 |  | 76 | 2.500 |  | 95.000 |
| 275 | 2 |  | 78 | 2.500 |  | 97.500 |
| 425 | 1 |  | 70 | 1. 250 |  | 98.750 |
| 525 | 1. |  | 80 | 1. 250 |  | 100.000 |
| SFIUN | FFEQUENCY | cum | FFEEQ | FEFECENT | CUM | FEFCENT |
| 25 | 65 |  | 65 | 81.250 |  | 81.250 |
| 75 | 6 |  | 71 | 7.500 |  | 88.750 |
| 125 | 6 |  | 77 | 7.500 |  | 96.250 |
| 325 | 1. |  | 78 | 1. 250 |  | 97.500 |
| 375 | 1 |  | 79 | 1.250 |  | 98.750 |
| 525 | 1 |  | 80 | 1.250 |  | 100.000 |
| MUNTH=MAY' |  |  |  |  |  |  |
| FUN | FFEEQUENCY | CUM | FFEQ | FEFECENT | CUH | FEFICENT |
| 25 | 63 |  | 63 | 78.750 |  | 78.750 |
| 75 | 3 |  | 66 | 3.750 |  | 82.500 |
| 125 | 6 |  | 72 | 7.500 |  | 90.000 |
| 225 | 3 |  | 75 | 3.750 |  | 93.750 |
| 2フ5 | 1 |  | 76 | 1.250 |  | 95.000 |
| 425 | 1 |  | 77 | 1.250 |  | 96.250 |
| 525 | 1 |  | 78 | 1.250 |  | 97.500 |
| 575 | 1 |  | 79 | 1.250 |  | 98.750 |
| 625 | 1 |  | 80 | 1.250 |  | 100.000 |
| SFIUN | FFEEQUENCY | CUM | FFiEQ | FEFICENT | CUM | FEFICENT |
| 25 | 54 |  | 54 | 67.500 |  | 67.500 |
| 75 | 12 |  | 66 | 15.000 |  | 82.500 |
| 125 | 5 |  | 71 | 6.250 |  | 88.750 |
| 175 | 3 |  | 74 | 3.750 |  | 92.500 |
| 225 | 2 |  | 76 | 2.500 |  | 95.000 |
| 275 | 2 |  | 78 | 2.500 |  | 97.500 |
| 475 | 2 |  | 80 | 2.500 |  | 100.000 |

FFIEQUENCY TAELEG FDF: MOITTHLY FIUNOFF MONTH = JUN

- FUN

| 25 | 65 | 65 | 81.250 | 81.250 |
| ---: | ---: | ---: | ---: | ---: |
| 75 | 7 | 72 | 8.750 | 90.000 |
| 125 | 4 | 76 | 5.000 | 95.000 |
| 175 | 2 | 78 | 2.500 | 97.500 |
| 225 | 1 | 79 | 1.250 | 98.750 |
| 325 | 1 | 80 | 1.250 | 100.000 |
| SFUR | FFEQUENCY CUM FFEQ | FEFFCENT | CUM FEFCENT |  |
| 25 | 75 | 75 | 93.750 | 93.750 |
| 75 | 1 | 75 | 1.250 | 95.000 |
| 125 | 2 | 78 | 2.500 | 97.500 |
| 225 | 1 | 79 | 1.250 | 98.750 |
| 325 | 1 | 80 | 1.250 | 100.000 |

MINTH = JUL
Futs
FFEEQUENCY
CUM FFEEQ
FEEFCENT
91.250
2.500
1.250
2.500
1.250
1.250

FEFEENT
92.500
1.250
2.500
1.250
1.250
1.250
92.500
$95+750$
96.250
97.500
98.750
100.000

FEEQUENCY TABLES FOF MONTHLY FUNOFF MONTH=AUG

| FUis | FFEEDUENCY | CUM | FFEQ | FEFICENT | CUM | F-EFICENT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 25 | 69 |  | 69 | 86.250 |  | 86.250 |
| 75 | 5 |  | 74 | 6.250 |  | 92.500 |
| 125 | 4 |  | 78 | 5.000 |  | 97.500 |
| 175 | 1 |  | 79 | 1.250 |  | 98.750 |
| 325 | 1 |  | 80 | 1.250 |  | 100.000 |
| SFIJN | FFiEQUENCY | CUM | FFiEQ | FEFCENT | CUM | FEFECENT |
| 25 | 72 |  | 72 | 90.000 |  | 90.000 |
| 75 | 3 |  | 75 | 3.750 |  | 93.750 |
| 125 | 3 |  | 78 | 3.750 |  | 97.500 |
| 175 | 1 |  | 79 | 1.250 |  | 98.750 |
| 275 | 1 |  | 80 | 1.250 |  | 100.000 |
| MONTH=SEF |  |  |  |  |  |  |
| $\because U N$ | FFEEQUENCY | CUM | FFEQ | FEFECENT | CUM | FEFFCENT |
| 25 | 60 |  | 60 | 75.000 |  | 75.000 |
| 75 | 5 |  | 65 | 6.250 |  | 81.250 |
| 125 | 3 |  | 68 | 3.750 |  | 85.000 |
| 175 | 4 |  | 72 | 5.000 |  | 90.000 |
| 225 | 4 |  | 76 | 5.000 |  | 95.000 |
| 325 | 1 |  | 77 | 1.250 |  | 96.250 |
| 425 | 1 |  | 78 | 1.250 |  | 97.500 |
| 525 | 1 |  | 79 | 1.250 |  | 98.750 |
| 675 | 1. |  | 80 | 1.250 |  | 100.000 |
| SFIUN | FFiEQUENCY | CUM | FFER | FEFICENT | CUM | FERCENT |
| 25 | 66 |  | 66 | 82.500 |  | 82.500 |
| 75 | 4 |  | 70 | 5.000 |  | 87.500 |
| 125 | 5 |  | 75 | 6.250 |  | 93.750 |
| 175 | 1 |  | 76 | 1.250 |  | 95.000 |
| 226 | 1 |  | 77 | 1.250 |  | 76.250 |
| 275 | 1 |  | 78 | 1.250 |  | 97.500 |
| . $3 / 5$ | 1 |  | 79 | 1.250 |  | 98.750 |
| 1025 | 1. |  | 80 | 1.200 |  | 100.000 |

 NENTH=OC.

SUN FFERUENC
CUH FFER

> 66
> 69
> 74
> 75
> 79
> 78
> 79
> 80
82.500
3.750
6.250
1.250
2.500
1.250
1.250
1.250

FHEQLIEMCY

72
3

| $\pm 25$ | $A$ |
| :--- | :--- |
| $2 \%$ |  |

MENTH=NOU
$\because$ FFEECUENCY

| 25 | 69 | 69 |
| :---: | :---: | :---: |
| 75 | 4 | 73 |
| 125 | 2 | 75 |
| 175 | 3 | 78 |
| 325 | 1 | 79 |
| 43 | 1 | 80 |

GFUN FFEOUENCY
CUM FFEEG

67
$7 E$
77
79
80
86. 250
5.000 2.500
3.750

1. 250 1.250

FEFFCENT
83.750
10.000
2.500
2.500
1.250

CUM FEFCENT 90.000 93.750 90.750 100.000
90.000
93.750
98.750
00.000

CUM FEFECEIT

$$
\begin{array}{r}
82 \cdot 500 \\
86 \cdot 250 \\
92.500 \\
93.750 \\
96.250 \\
97.500 \\
96.750 \\
100+000
\end{array}
$$

## CUM FEEFICENT

 86.250 91.250 93.750 97.500 98.750 100.000
## CUH FEFECENT

 83.750 93.750 96.250 98.750 100.000$$
\text { :URT W }=\text { IEC }
$$

FOUR FFEEDUENCY CUH FFEQ FEFCENT CUM FEFICENT

| $\because 5$ | 76 |  | 76 | 95.000 |  | 95.000 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \% | 3 |  | 79 | 3.750 |  | 98.750 |
| 17 | 1 |  | 80 | 1.250 |  | 100.000 |
| G1:UN | FFEEQUENCY | CUM | $F F E Q$ | FEFICEMT | Cum | FEFICENT |
| 25 | $7 \%$ |  | 77 | 96.250 |  | 96.250 |
| 125 | 3 |  | 80 | 3.750 |  | 100.000 |

## APPENDIX G

# SYNTHETIC AND HISTROIC ANNUAL RAINFALL <br> DATA AND RESULTING RUNOFF <br> PREDICTED BY CREAMS 

| HRAIN | HISTORICAL RAINFALL |
| :--- | :--- |
| SRAIN | SYNTHETIC RAINFALI |
| HRUNOFF | HISTORICAL RUNOFF |
| SRUNOFF | SYNTHETIC RUNOFF |
| AHRAIN | ACCUMULATED HISTORICAL RAINFALL |
| ASRAIN | ACCUMULATED SYNTHETIC RAINFALL |
| AHRUNOFF | ACCUMULATED HISTORICAL RUNOFF |
| ASRUNOFF | ACCUMULATED SYNTHETIC RUNOFF |
| R_RUNOFF | RATIO OF ACCUMULATED HISTORIC AND |
|  | SYNTHETIC RUNOFF |
| R_RAIN | RATIO OF ACCUMULATED HISTORIC AND <br> SYNTHETIC RAINFALL |



| 0 | 27.7727 .80 | 2.355 | 1.753 | 29.79 | 29.80 | 2.355 | 1.7530. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 19.9822 .87 | 0.644 | 0. 3.54 | 45.77 | 5.69 | 2.959 | 2.1070 .544581 .42335 |
| 2 | 40.2930 .55 | J. 104 | 1.631 | 70.06 | 83.27 | 8.163 | 3.7381 .081542 .18375 |
| 3 | 31.9823 .71 | 2.834 | 0.234 | 122.04 | 106.58 | 10.997 | 3.9721 .140772 .76863 |
| 4 | 31.3535 .61 | 3.375 | 1.479 | 153.39 | 142.59 | 14.370 | 5.4511 .075742 .63621 |
| J | 38.5924 .64 | 2.474 | 0.063 | 171.98 | 167.23 | 16.844 | 5.5141 .14800 3. 05477 |
| 6 | 41.4719 .90 | 4.017 | 0. | 23 | 18 | 20.863 | $5.65!1.247533 .69191$ |
| 7 | 35.6733 .35 | 3.28 | 5.8 | 26 | 22 | 24. | 2 |
| 8 | 60.2636 .80 | 17.175 | 2.736 | 329.40 | 257.26 | 43.317 | 14.2611 .280423 .03759 |
| $¢$ | 31.7331 .73 | 3.245 | 0.472 | 361.15 | 288.79 | 46.564 | 14.7331 .247633 .16052 |
| 10 | 19.0524 | 0.960 | 0.547 | 380.19 | 31 | 47.524 | 15.2821 .214203 .10980 |
| 11 | 34.4338 | 2.0 | 7. | 414.62 | 351.19 | 47.538 | 22.6311 .180612 .18894 |
| 12 | 27.6040 .08 | 2.9 | 4. | 442.22 | 37 | 52 | 27.1651 .130221 .93304 |
| 13 | 30.5128 .86 | 4.781 | 2.502 | 479.13 | 420.15 | 57.292 | 29.6671 .140351 .43117 |
| 14 | 16.7728 .57 | 0.455 | 1.298 | 495.72 | 448.72 | 57.747 | 30.9651 .105171 .06471 |
| 15 | 45.0243 .54 | 10.203 | 4.715 | 543.74 | 49.66 | 67.750 | 35.6781 .104091 .70454 |
| 16 | 25.01 30.54 | 2.241 | 3.875 | 571.95 | 523.20 | 70.171 | 37.5511 .073181 .77470 |
| 17 | 24.0633 .40 | 1.217 | 3.724 | 59 | 55 | 71.406 | 45.2751 .07150 1.65010 |
| 18 | 37.8028 .47 | 4.014 | 1.857 | 636 | 585 | 75.422 | 45.1721 .087851 .66766 |
| 15 | 5.1022.74 | 1.86 | 1.385 | 665.65 | 607.81 | 77.287 | 45.5571 .101711 .69649 |
| 26 | 47.3433 .21 | 5.87 | 0.837 | 710.97 | 641.02 | 83.114 | 46.3541 .118481 .77146 |
| 21 | 3.8731 .86 | 4.568 | 1.558 | 750.84 | 672.55 | 87.682 | $47.992 \quad 1.1158681 .82701$ |
| 2 | 34.60 \% 3.65 | 4.278 | 5.780 | 785.64 | 710.53 | 71.960 | $53.972 \quad 1.105711 .70422$ |
| 2 | 42.353 .13 | 9.057 | 1.867 | 827.97 | 742.66 | 101.037 | 55.8391 .114871 .80943 |
| 2 | 2.7826 .98 | 0.781 | 0.457 | 851.95 | 769.64 | 101.818 | 56.2961 .106951 .80862 |
| 25 | 22.4442 .02 | 0.685 | 7.954 | 874.39 | 811.66 | 102.506 | 64.2501 .077291 .54542 |
| 26 | 32.0926 .46 | 3.041 | 0.740 | 906.48 | 838.12 | 105.547 | 64.9901 .081561 .62405 |
| 27 | 38.1036 .70 | 5.303 | $0.60_{2}$ | 944.58 | 874.82 | 110.850 | 65.6721 .079741 .68793 |
| 28 | 32.5243 .25 | 2.743 | 5.277 | 977.10 | 918.07 | 113.593 | 70.9691 .064301 .60060 |
| 27 | 37.1425 .40 | 5.963 | 0.548 | 1014.24 | 94.3 .47 | 119.556 | 71.5171 .075011 .67171 |
| 30 | 25.6935 .49 | 2.055 | 2.859 | 1039.93 | 978.96 | 121.611 | 74.4161 .052281 .63421 |
| 31 | 27.3125 .18 | 2.353 | 0.745 | 1067.24 | 1004.14 | 124.004 | 75.1611 .062841 .64984 |
| 32 | 34.9420 .86 | 3.735 | 0.37 | 1102.18 | 1025.00 | 127.737 | 75.5581 .075301 .69061 |
| 3 | 32.3727 .44 | 3.244 | 1.007 | 1134.57 | 1052.44 | 130.983 | 76.5071 .076041 .71070 |
| 34 | 30.5731 .35 | 2.655 | 0.282 | 1165. 24 | 1083.77 | 133.638 | 76.8491 .075171 .73897 |
| 35 | 33.5738 .37 | 2.702 | 4.262 | 1178.83 | 1122.14 | 136.340 | 81.1111 .068341 .68091 |
| 36 | 18.2731 .71 | 0.518 | 2.152 | 1217.12 | 1154.05 | 136.858 | 85.2631 .054651 .64368 |
| 37 | 25.5437 .15 | 1.274 | 2.605 | 1242.71 | 1191.20 | 138.132 | 85.8661 .043241 .60869 |
| 38 | 35.2922 .30 | 2.627 | 1.031 | 1278.00 | 1213.50 | 140.759 | 86.8971 .053151 .61984 |
| 39 | 26.9531 .88 | 1.739 | 2.200 | 1304.95 | 1245. 38 | 142.498 | 89.0971 .047831 .59936 |



|  |  | 2.330 | 2.038 | $1336.98 \cdot 1274.57$ | 144.828 | 91.135 | . 050461.58916 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 41 | 43.68 41.89 | . 86 | 4.962 | 46 | 15 | 96. | 050211.57855 |
| 42 |  | 9.256 | 5.5 | 65 | 160.952 | 101 |  |
| 43 | 31.07 32.0. | . 845 | 4.1 | 458.961379 .68 | 167 | 105 | 461.58660 |
| 44 | 31.2437 .80 | 48 | 2.250 | 20 | 169 | 108 | . 051301.56573 |
| 45 | 34.0441 .22 | 7.317 | 17.520 | . 70 | 176.862 | 125 | 973 1.40893 |
| 46 | 28 | . 196 | 0.7 | 42 | 177.958 |  |  |
| 47 | 27.2130 .13 | 3.022 | 3.060 | 579.631511 .87 | 80.9 | 127.36 | . 044821.39903 |
| 48 | 94 | 748 | 7.710 | . 8 | 182.728 | 137 | 966 |
| 49 | 30.1632 .74 | . 525 | 2.201 | 1582 | 184. | 139.27 | . 37211.32297 |
| 50 |  | 162 | 6.36 | 64.231627 .12 | 184.41 | 145. | 281 |
| 51 | 34.6731 .60 | 948 | 3.65 | 78.801658 .72 | 186.3 | 49. | 22 |
| 5. | 24.1224 .85 | 0.213 | 4.62 | 1723.021683 .55 | 186.576 | 149. | 60 |
| 53 | 32.7128 .71 | 2.712 | 1.5 | 1755.731712 .46 | 189.29 | 151. | .02527 1.24985 |
| 54 | 15.3335 .82 | 0.465 |  | 1774.06 1746.28 | 189.753 |  |  |
| 55 | 27.9826 .77 | 757 | 0.828 |  |  | 153 | . |
| 5 | 42 | 0.183 | 2.488 | 1816.72180 | 93, 85 | 156.212 | 1.0 |
| 5 | 42.7227 .11 | 7.276 | 1.191 | 18 | 20.169 | 157.403 | 1.014641 .27076 |
| 56 | 31.8526 .01 | 1.413 | 2.215 | 1893.29 | 56 | 159.62 z | 1.01758 1.28167 |
| 59 | 25.86 | 23.273 | 547 | 195 | 227.855 | 161.169 | . 03 |
| 60 | 35.95 3.28 | 3.105 | 1.629 | 1991.151719. | 230.960 | 162.798 | 1.03721 .41669 |
| d | 38.89 | 5.488 | 2.244 | 203 | 236.448 | 165.042 | 1.038631 .43265 |
|  | 32.43 30.68 | 2.20 | 0.905 | 2062.471985. | 238.651 | 165.947 | 1.038921 .43812 |
| 63 | .14 38.47 | 0.788 | 2.779 | 2097.612023 | 239.439 | 168.726 | 1.032561 .41910 |
| 64 | 35.01 | 1.070 | 1.224 | 2115.56205 | 240.509 | 169.955 | 1.027621 .41513 |
| 65 | 27.7834 .34 | 2.45 | 2.111 | 2143.3420 | 242.972 | 172.066 | 1.024041 .41209 |
| 66 | 34 37.36 | 2.4 | 3.750 | 2168.732130. | 245.466 | 175.816 | 1.018001 .39615 |
| 57 | 4839.58 | 1.390 | 637 | 2200.212169 | 246 | 180.45 | 3761.36806 |
| 68 | 32.6029 .46 | 1.082 | 2.596 | 223 | 247.944 | 183.051 | 1.015 |
| 69 | 27.8438 .61 | 0.80 | 4.1056 | 2260.6 | 248.7 | 187.107 | 1.010151 .32943 |
| 70 | 28.6915 .43 | 3.13 | 0.153 | 2289.332256 | 251.88 | 187.300 | 1.01459 |
|  | 31.4532 .81 | 2.23 | 1.076 | 2320.782289 | 254.11 | 188.376 | 1.013791. |
| 72 | 27.7632 .57 | 1.701 | 0.935 | 2348.742322. | 255.815 | 169.311 | 1.011421 .35129 |
| 13 | 46.4337 .16 | 8.44 | 5.440 | 2375.172355. | 244.25 | 174.751 | 1.015171 .35699 |
| 74 | 45.7422 .51 | 10.342 | 0.532 | 2440.912351 .88 | 274.598 | 195.283 | 1.024761 .40615 |
| 75 | 39.6542 .94 | 5.224 | 5.210 | 2480.562424. | 279.822 | 200.493 | 1.022791 .39567 |
| 76 | 20.7336 .25 | 0.7 | 4.783 | 2501.272461. | 280.586 | 205.276 | 1.016341 .36687 |
| 77 | 32.4743 .41 | 2.459 | 4.443 | 2533.762504 .4 | 283.045 | 209.719 | 1.01169 |
| 78 | 25.8732 .40 | 0.45 | 0.823 | 2557.632536. | 263.501 | 210.542 | 1.00877 |
|  |  |  |  |  |  |  |  |

VITA<br>James Edward Peter Green<br>Candidate for the Degree of<br>Doctor of Philosophy

Thesis: SYNTHETIC RAINFALE AND ITS USE IN HYDROLOGIC MODELING

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[^0]:    Figure 2. Relative Frequency Curves of Daily Rainfall Amounts for Wet Days (Total), Wet Days Following Dry Days (Dry) and Wet Days Following Wet Days (Wet) for the Month of December.

