TWO ESSAYS ON THE MISPRICING OF THE S&P 500 FUTURES CONTRACT

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TWO ESSAYS ON THE MISPRICING OF

THE S&P 500 FUTURES

CONTRACT

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PREFACE

This dissertation demonstrates that mispricing of the S&P 500 futures contract can result from structural dissimilarities between the New York Stock Exchange and the Chicago Mercantile Exchange. These related markets are linked through the agency of stock index arbitrage.

Methodologies based upon the emerging science of nonlinear dynamics and system simulation are used to support this proposition. Because of the underlying deterministic (as opposed to a stochastic) slant of the methodologies, it is possible to generate policy implications regarding the regulation of the financial markets. The conclusions from this dissertation can be used to support a free market regulatory stance.

I wish to express my deep gratitude to Dr. Timothy Krehbiel who was my thesis adviser for a testing two years. His contribution goes well beyond the boundaries of this dissertation. My thanks also go to the rest of my thesis committee for being helpful and supportive of this research and to Dr. Joe Mize for his encouragement and assistance with the simulation methodology. To my dear friend and fellow graduate student Doug Kern whose compassion is blind to the distinctions of color, culture and creed, I dedicate this dissertation.

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CHAPTER I

INTRODUCTION

The Standard and Poor's 500 (S&P 500) futures contract was introduced in 1982 at the Chicago Mercantile Exchange. Since then the trading volume on the contract has steadily increased and today represents two thirds of all index futures trading. The S&P 500 futures contract provides unique advantages for institutional investors who wish to hedge diversified holdings and for those who choose to speculate on broad market movements. For such purposes the index futures contract and the underlying stock basket are perfect substitutes and the pricing of the futures contract would reflect this fact. Empirical analysis of the S&P 500 futures contract by Modest and Sundaresan (1983), Cornell and French (1983), Figlewski (1984(a), 1984(b)), Merrick (1987), Arditti, Ayadin, Mattu and Rigsbee (1986), MacKinlay and Ramaswamy (1987) document significant and sustained deviation of the index futures price from its theoretical value. Extending these findings, Arditti etal. (1986), Merrick (1989), Finnerty and Park (1988) show that index arbitrage could have been profitable through these periods. Yadav and Pope (1990) corroborate these results by finding evidence of mispricing in the UK Financial Times Stock Exchange (FTSE - 100) stock index futures contract traded on the London International Financial Futures Exchange as well.

Extensive mispricing was also observed during the period of the 1987 stock market crash when computerized stock index arbitrage was believed by many to have destabilized the stock market[1]. Such findings of sustained mispricing and the availability of a "free lunch" by way of risk free arbitrage trades are not consistent with the hypothesis of market efficiency. Following the stock market crash of 1987 policy-makers focused upon the cash and futures market linkages and many commissions were appointed to examine the events surrounding the crash period[2]. Academic interest has also been drawn to the mispricing anomaly and there have been attempts to statistically analyze and model the process generating the index futures mispricing[3]. Brennan and Schwartz (1990) use a Brownian Bridge process to represent the evolving arbitrage opportunity (mispricing) for a trader. At the same time they recognize that the Markov nature of the Brownian Bridge process is not consistent with the observed serial correlation or path dependence in the mispricing series.

This dissertation is in the form of two essays that investigate the issue of index futures mispricing. The first essay titled "Does the S&P 500 Mispricing Series Exhibit Nonlinear Serial Dependence" (Chapter 2) takes a completely different tack

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[1] Mr. Charles Schwab in an article in the Wall Street Journal of April 25, 1989 states: "When program trading is roiling the water, there is no safe haven. At such times the futures market drives the stock market..." "To regain the confidence of individual investors, policy makers and the security industry need to take the lead in corralling the destructive aspects of program trading."

[2] Mackay (1988) summarizes the recommendation of the many commissions set up to investigate the crash.

[3] MacKinlay and Ramaswamy (1988); Brennan and Schwartz (1990); Yadav and Pope (1990) represent some of this work.

from previous studies by hypothesizing and testing for the existence of nonlinear time dependence in the S&P 500 futures mispricing series. The presence of such nonlinear structure in the mispricing series would be consistent with a deterministic as opposed to a stochastic explanation for the mispricing anomaly. Nonlinearities are also a necessary condition for the existence of deterministic chaos. Systems in chaos display time paths that appear random even to many statistical tests. The observed irregular fluctuations in index futures mispricing levels and the steep discounts observed during the crash period are typical of a system displaying chaotic dynamics. A procedure to test for nonlinear structure developed by Brock, Dechert and Schienkman (BDS) (1986) is applied in this study. In addition to testing for nonlinear structure this essay also identifies some aspects of market microstructure that can generate chaotic dynamics in the mispricing series.

The second essay titled "Market Microstructure Issues in the Mispricing of the S&P 500 Futures Contract" (Chapter 3) builds upon the first through a computer simulation of the role of differential trading delays in producing the observed mispricing dynamics. The System Dynamics simulation methodology based upon DYNAMO is used for this purpose. The use of a simulation methodology presupposes an endogenous view of system behavior. The literature review in Chapter 3 establishes the justification for use of the simulation methodology for the study of nonlinear systems in general and market microstructure issues in particular. Differential trading delays are shown to be a potential cause of the overshootings and path dependence that have been identified in the mispricing series.

Chapter 4 both summarizes and links together the results from the two essays. The benefits and limitations of the simulation methodology are discussed and the conclusions draw out the policy implications that follow from this research.

CHAPTER II

DOES THE S&P 500 FUTURES MISPRICING SERIES EXHIBIT NONLINEAR SERIAL DEPENDENCE?

Theoretical Background

Index futures mispricing is defined as the difference between the market price of the index futures contract and its theoretical value:

$$f(t) = S(t).\ \exp.(r\text{-}d)(T\text{-}t) \qquad (1)$$

The right hand side of equation (1) expresses the fair (theoretical) futures price as the cost of carrying the index basket of stocks to maturity. Here f(t) is the theoretical value of the index futures contract at time t. S(t) is the level of the underlying index at time t; ‘r’ represents the risk free interest rate and ‘d’ is the dividend rate on the underlying basket of stock. Both rates are assumed constant for the duration of the futures contract. T is the maturity date of the futures contract.

In a continuous efficient market and in the absence of transaction costs, there should be no deviation of the index futures price from its theoretical value. Any deviation would result in a profitable, risk free, arbitrage opportunity. For instance, if F(t), the futures price is greater than f(t), its theoretical value, the appropriate arbitrage action would be to sell the index futures contract and buy the underlying basket of stocks. The stock purchase is financed by borrowing at the risk free rate. The riskless arbitrage profit would then amount to F(t) - S(t) exp. (r-d)(T-t)

The opposite arbitrage action, purchasing the index futures contract and selling the underlying index basket of stock, would occur if the index futures price is below its theoretical value. In this case, the sale proceeds are assumed to be invested at the risk free rate thereby providing an arbitrage profit of S(t) exp (r-d)(T-t) - F(t).

Introducing transactions costs into the model provides bounds within which arbitrage would not be profitable. Since such costs depend upon the size of the arbitrage trade, the practice has been to express the index futures mispricing and the associated transaction costs as a percentage of the index value. If $M_{t,T}$ denotes the percentage mispricing at time t for the contract expiring in time period T, then its value is given by;

$$M_{t,T} = \frac{F_{t,T} - S(t)\exp(r-d)(T-t)}{S(t)}$$

The corresponding transaction costs bounds would be determined by

$$|M_{t,T}| = (T_s + I_s + T_f + I_f)$$

where

T_s = Percentage round trip stock commission

T_f = Percentage round trip futures commission

I_s = Percentage market impact in stocks and

I_f = Percentage market impact in futures

The market impact costs should be negligible in an efficiently functioning market since the demand for both the stock basket and the related futures contract would be perfectly elastic at the full information price.

Literature Review

Empirical Findings on Index Futures Mispricing

The existence of transactions costs would imply that mispricing within transaction cost bounds would still be consistent with market efficiency. However index futures mispricing in excess of transactions cost bounds have been observed throughout the history of the S&P 500 futures contract. In an early study, using data for the June and December 1982 contracts, Modest and Sundaresan (1983) conclude that the futures price violates the theoretical bounds if market participants were assumed to have full use of short sale proceeds. Even so, as Figlewski (1984a) notes, well diversified institutional investors need not resort to short sales to take advantage of profitable arbitrage opportunities. Evidence of mispricing for different time periods has also been documented by other researchers including Figlewski (1984(a), 1984(b)), Merrick (1987, 1989) and Finnerty and Park (1988). Figlewski (1984a) examines the various reasons put forward to explain the empirical observations of index futures mispricing and concludes that.. "the (futures) discount represented a situation of disequilibrium - a transitory phenomenon caused by unfamiliarity with the new markets and institutional inertia in developing systems to take advantage of the opportunities presented." (p.43). Merrick (1987) examines the implications of observed index futures mispricing for institutions who hedge their stock portfolios with stock index futures. Mispricing of the futures contract will affect the optimal hedge ratio for such investors. Merrick (1989) attempts to demonstrate the profitability of arbitrage related program trading strategies. Yadav and Pope (1988) extend the .."broad consensus that observed mispricing is often sufficient to span the transaction cost bounds and offer arbitrage possibilities.." to the analysis of the U.K FTSE 100 futures contract traded on the London International Financial Futures Exchange. (p.573). Yadav and Pope (1988) explicitly consider transactions costs while computing the mispricing of the futures contract and conclude that the mispricing is too large to be accounted for by transactions costs. In the most recent and comprehensive study yet of the extent of mispricing in the S&P 500 futures contract, MacKinlay and Ramaswamy (1988) compute intra-day mispricing levels at 15 minute intervals. The price histories studied stretch from 1983 through 1987. Transaction cost boundaries are conservatively estimated at +0.6 percent and -0.6 percent of the cash index. Their results show that over the 16 contracts examined, the average mispricing amounted to 0.12 percent of the index value, with a high for the December 1986 contract of 0.78 percent. The authors record that out of 26070 observations over all contracts there were 3149 upper bound transactions cost violations and 602 lower bound transactions cost violations. On average, therefore, the transaction cost bounds were violated 14.3 percent of the time.

A number of reasons have been put forward to explain the mispricing puzzle. Almost all of these can be seen as variants of the transaction cost argument, that in some way transaction costs are greater or that index arbitrage is riskier, than is assumed by the cost of carry model. These include the beneficial tax timing option that is available only to the spot position and not the futures position, Cornell and French (1983); the real world stochasticity of dividend and interest rates, in violation of the constant dividend and interest rate assumptions of the pricing model, (Cox, Ingersoll and Ross (1981)) and the difficulty of tracking the S&P 500 index with only a subset of stocks (Figlewski (1984a)).

In conflict with these explanations is the observation by Rubinstein (1987) that arbitrageurs comprise large institutional investors with broad based portfolio holdings. Institutional investors are not affected by tax considerations nor are they constrained to track the index with only a subset of stocks. A subsequent empirical study by Cornell (1985) designed to follow up on the Cornell and French (1983) paper failed to find supporting evidence for the tax argument.

The observed mispricing could result from using a misspecified pricing model. The cost of carry pricing model used in computing the theoretical forward price could be misspecified due to factors such as stochastic interest rates and the daily marking to market for the futures contract (Cox, Ingersoll and Ross (1981); Jarrow and Oldfield (1981); French (1983)).

If the stochasticity of interest rates is a factor in the mispricing of the index futures contract, then the same extent of mispricing should be observed in the case of other financial instruments that have futures contracts traded as well. An investigation of mispricing in the foreign exchange futures contract by Cornell and Reinganum (1981) does not support this explanation. Simulation of the impact of stochastic interest rates and marking to market on equilibrium stock index futures price by Modest (1984) showed that this effect would be minimal. These studies suggest that the observed mispricing is related to market features specific to the S&P 500 index futures contract.

Yadav and Pope (1990) argue that transactions costs may well be lower than is commonly estimated. In an efficient market, it would be the transaction costs of the lowest cost trader that would be binding and not those of the marginal operator with the highest cost. It should also be recognized that arbitrageurs have the option to reverse their positions even prior to expiration if the mispricing level overshoots and changes sign. Such overshootings have the potential to increase arbitrage profits because the only cost of an early unwinding is the market impact cost of closing the futures position. The market impact cost should be negligible in an efficient market. MacKinlay and Ramaswamy (1988) show that such overshootings have been common in the history of the contract. This factor could induce arbitrage trades at levels of mispricing within transactions cost bounds.

In reality stocks do not trade continuously and the stock index value often reflects stale prices. The existence of stale prices or nonsynchronous trading can cause autocorrelations in the index prices. As Harris (1989) points out, "If the nonsynchronous trading problem is ignored, spurious conclusions about volatility, market efficiency, and the relation between the futures and the cash market can be obtained and arbitrage opportunities can be falsely identified." (p.78). However, an empirical investigation by Harris (1989) of the October 1987 S&P 500 stock futures basis show that "portfolio returns are autocorrelated even after the effects of nonsynchronous trading are explicitly removed." (p.78). MacKinlay and Ramaswamy

(1988) come to a similar conclusion using intraday prices for the period September 1983 through June 1987.

The stock market crash of 1987, followed by the minibreak of 1989, graphically underscored the importance of index futures mispricing in the overall functioning of the market. It also served to bring the issue of index arbitrage into the focus of the public and the policy makers. On both occasions, the S&P 500 futures contract was observed to trade at a considerable discount, not only to the theoretical value but also to the S&P 500 index itself. Such a phenomenon should be a near impossibility in an efficient market. Specifically, for such a large discount to occur, given the cost of carry pricing relation, the expected dividend earnings from holding the index basket of stocks should exceed the interest cost, thereby enabling a negative cost of carry for the arbitrage transaction.

The stock market crash and the resulting public outcry against computer based index arbitrage, led to a number of studies[1] that focused upon the nexus between index arbitrage and stock market volatility. The results from these studies document the association between severe market volatility and the S&P 500 futures mispricing. The formulation of a regulatory framework to stabilize markets would be improved by analysis, understanding and modeling of index futures mispricing.

[1] Mackay (1988)

Modeling Index Futures Mispricing:

Randomness vs Determinism

In a recent paper, Stoll and Whaley (1990) demonstrate that if the cost of carry pricing model were to hold there should be no evidence of either linear or nonlinear serial dependence in the mispricing series. The path of mispricing observations should be random. The empirical observations to date, however, contrast sharply with the theoretical extensions.

The MacKinlay and Ramaswamy (1988) paper takes the first step toward examining the statistical characteristics of the mispricing series. Some of their observations relevant to this research paper can be summarized as follows:

(1) The mispricing series display sharp reversals with a tendency for the mispricing levels to stay above or below zero for considerable periods of time.

(2) The series was found to be extremely autocorrelated. In this context, MacKinlay and Ramaswamy (1988) suggest that the option to unwind arbitrage positions prematurely (prior to maturity) brings about the observed path dependence in the data.

(3) The mispricing series does not appear to fluctuate randomly around zero.

MacKinlay and Ramaswamy (1988) conclude their study, by recommending that future research should aim at modeling the arbitrage process and the mispricing series.

In a recent study, using the same data, Brennan and Schwartz (1990) use a Brownian Bridge process to represent the evolving arbitrage opportunity (mispricing) in order to determine optimal arbitrage strategies for a trader. The authors note, however, that the Markov (path independent) nature of the Brownian Bridge would not be in accordance with the observed path dependence in the mispricing series. In their conclusion Brennan and Schwartz (1990) observe that the .."real challenge remains to **endogenize** (emphasis added) the stochastic behavior of the simple arbitrage opportunity given the nature of transaction costs and the structure of the market." (p. s.22). An endogenous explanation would require a deterministic as opposed to a stochastic view of the intermarket linkages and the arbitrage process. As Yadav and Pope (1990) point out, "It is difficult to select a stochastic process to model "mispricing"....when the empirical evidence by MacKinlay and Ramaswamy (1987) indicates that the mispricing series is path dependent." (p.577).

The Hypothesis of Nonlinear Dependence

The objective of this research is to test for the presence of nonlinear dependencies in the S&P 500 mispricing series. Findings of nonlinear, low dimensional structure in the data series has some significant implications for the modeling of the mispricing series.

(1) Nonlinear dependence is a necessary condition for the existence of deterministic chaos which is commonly interpreted to be the random appearing trajectory of a purely deterministic system (Savit (1988)). The observed irregular fluctuations of the index futures mispricing series suggest a chaotic (deterministic) explanation to the dynamic behavior of index futures mispricing.

(2) A low order (deterministic) dimensional reading as opposed to a large or infinite (random) dimensional series would suggest that the irregular fluctuations in the mispricing series have a deterministic (structural) explanation. A finding of a large or

infinite dimension, on the other hand, would imply that the dynamic behavior of the mispricing series is caused by very many factors and the appropriate modeling approach would be to determine the stochastic process that best explains its dynamic features.

(3) Nonlinearities are also a necessary condition for catastrophes. A catastrophe in this context according to Rahn (1981) represents a sudden change in the state of a system in response to a smooth change in the parameters of the independent variables. An example would be a sudden change from a bull market to a bear market (stock market crash) that cannot be associated with discontinuous changes in any of the relevant environmental parameters. It is precisely this kind of turning point that modelers find difficult to forecast and, as Forrester (1987) points out, may well be a characteristic of nonlinear systems.

The case for the existence of nonlinear dependencies in the index futures mispricing series can be made using a mix of empirical findings and observations of market microstructure. There have been a series of research studies that document the evidence of nonlinear dependence in economic and financial data. Brock (1988) surveys some of this work. Scheinkman and LeBaron (1989) examine a time history of daily returns on the value weighted portfolio provided by CRSP, and conclude that the portfolio returns in the sample exhibit nonlinear dependence across time. Of particular interest to the present research project is a study by Blank (1990), which finds strong evidence of nonlinear dependence and deterministic chaos in the S&P 500 futures price series. Such findings of nonlinear dependence in stock and futures market data motivate, in part, this attempt to test for nonlinearities in the S&P 500 futures mispricing series as well.

Chaotic fluctuations could arise in the mispricing series due to the widely differing microstructure of the futures market and the stock market. Arbitrage serves to couple these two very different markets. The index futures market and the stock market differ functionally due to their methods of trading: an open outcry system in the Chicago Mercantile Exchange as compared to the specialist system that operates in the New York Stock Exchange. Information is quickly and fully reflected in futures market prices even if it involves large discontinuous jumps. In contrast, the specialist system is designed to cushion shocks, slow down price changes and maintain continuity in price movements[2]. Stoll and Whaley (1990) conclude that the S&P 500 futures price leads the index by as much as ten minutes. It would appear that price discovery takes place in the futures market and the information is carried via arbitrage to the stock market. Because of the difference in the microstructure of the two markets, an arbitrageur, while responding to a mispricing, is likely to face a delay in putting through the stock market leg of the arbitrage. The immediate response in the mispricing level would be only partial, reflecting the change in the futures price alone. With more than one arbitrageur in the market, the partial response in the mispricing level may induce further arbitrage activity and could actually result in the overshooting of the arbitrage bounds.

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[2] Report of the Committee on Market Volatility and Investor Confidence. (1990): New York Stock Exchange Publication

The existence of the uptick rule in the stock market and its absence in the futures market is another microstructure difference which may induce a delay in the response of the stock market price to arbitrage. The existence of the rule has been cited by Edwards (1988) as one factor that exacerbated the market crash. The uptick rule restricts short sales on a stock below the price at which the last sale was effected. The purpose of this rule is to prevent panic selling in a declining market. It also serves as a deterrent against any bear raids[3]. In the context of futures mispricing, the uptick rule effectively delays the stock market leg of an arbitrage trade in an asymmetric manner: the time delay will be greater when the index futures is trading at a discount to the spot index because the appropriate arbitrage response in this case requires the purchase of the futures contract in the CME and the short sale of the underlying basket of stocks in the NYSE. The delays that arise in executing the stock market leg of an arbitrage trade, for both of the above reasons, are capable of causing chaotic fluctuations and instabilities in the observed mispricing series.

The observation made above, that response delays within coupled nonlinear systems can cause chaotic fluctuations in observed time histories of system variables, is well established in system dynamics literature.

Rasmussen and Mosekilde (1988) use system simulation to show how chaotic behavior can arise in a firm that allocates its resources between production and marketing depending upon its order backlog/inventory of finished goods. Faced with a backlog of orders, the company would shift its resources to production. If the inventory build up becomes excessive, the company would transfer resources to

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[3] Report of the Committee on Market Volatility and Investor Confidence. (1990)

marketing so as to drum up sales. Nonlinear responses in this model are coupled with delays in generating production/sales in order to achieve a desired inventory. This results in irregular cyclical variation in sales. When select system parameters such as the customer defect rate are changed, the entire system shifts into chaos. The arbitrageurs' response to perceived mispricing is analogous to the foregoing description of a typical firm's response to unsatisfactory levels of inventory. In both cases a perfectly rational response by participants when channeled through a system characterized by nonlinearities and adjustment delays can cause very complex dynamical behavior.

Sterman (1989) investigated the capital investment decisions of experimental subjects operating in a simple multiplier accelerator (nonlinear) economy. Here too the time delay between capital investment and generation of production capacity generates instabilities and chaos in system levels.

Using a Kaldor type business cycle model, Lorenz (1987) showed that the coupling of three regularly oscillating economic sectors can lead to irregular chaotic behavior in the entire system. The comparison here would be to the coupling of the dissimilar futures and cash markets by arbitrage activity.

Modeling of predator prey relationships in ecology[4] provide the clearest picture of the route to nonlinearities and chaotic fluctuations in the population level of the two species. There is a natural equilibrating mechanism at work since an overly

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[4] Robert Pool (1989), "Ecologists Flirt with Chaos," Science vol.243, January, (310-313)

Johan Swart (1990), "A System Dynamics Approach to Predator-Prey Modeling," System Dynamics Review, 6:1, (94-110) large predator population would decimate the population level of prey. The shortage of food would in time reduce the predator population enabling the prey to flourish once again. Surprisingly however, the population levels of the predator and prey oscillate continuously and never reach a long term equilibrium. The different gestation periods of the two species introduces a differential delay in the population adjustment mechanism causing continuous irregular cycles in the populations. The differential trading delays in the futures and the stock markets is analogous to the differential gestation periods of the predator/prey populations.

These studies identify two characteristics common to many systems that can occur together and bring about chaotic dynamics in the time values of the system variables:

(1) When two or more dissimilar systems characterized by nonlinear relationships among variables are coupled through some form of feedback linkage.

(2) When there are time delays in adjusting to system changes.

The market for the S&P 500 futures contract and the stock market are structurally dissimilar systems and arbitrage activity links the two. This fact in combination with system delays in putting through arbitrage trades could cause chaotic fluctuations in the mispricing series. An empirical verification of nonlinear dependence and a low order dimension in the mispricing series would indicate that endogenous factors and system microstructure are one likely source of observed futures mispricing. Nonlinearities are a necessary condition for the generation of chaotic dynamics.

Methodology

This paper follows the testing procedure first laid out by Brock, Dechert and Scheinkman (BDS) (1986). The test relies on two concepts of dynamical systems; attractors and dimension. This section begins with a review of these relevant concepts.

The Concept of an Attractor

An attractor is a set of points towards which the dynamical path of a system will converge. An attractor is a useful concept because it can be visualized as a geometric form. A formal definition of an attractor is found in Ramsey (1990):
" An attractor is a compact set A, such that the limit set of the orbit $\{x_n\}$ or $\{x_t\}$, as n or t -> ∞ is A, for almost all initial conditions within a neighborhood of A." (p.125)

An attractor can have different forms: a single point, a limit or periodic orbit, a torus (doughnut shaped attractor), or a strange (chaotic) attractor. The form of the attractor according to Radzicki (1990) can be used to classify dynamical systems:

(1) Linear dynamical systems have restricted time paths. The attractor for linear dynamical systems is always a single point.

(2) Nonlinear dynamical systems on the other hand correspond to all four forms of attractors of which the strange attractor alone is associated with deterministic chaos. Nonlinearity is therefore a necessary but not a sufficient condition for chaos. Dynamical systems represented by torus shaped attractors display time paths that are aperiodic as in the case of chaotic attractors.

However sensitive dependence on initial conditions are characteristic only of chaotic attractors.

The Concept of Dimension

The aim of modeling is to represent the given data set with only a few dimensions or degrees of freedom. In mathematical terms a single point is said to have dimension zero, a line has dimension one, a square has two dimensions and a solid is three dimensional. A random series by definition cannot be modeled with a finite number of variables. It is said to have an infinite number of dimensions.

The most popular method of determining the dimension of a data series is the correlation dimension attributed to Grassberger and Procaccia (1983).

$$C_m(L)=\lim_{N\to\infty}\left[\frac{1}{N(N-1)}\right]\sum_{i\neq j}^{n}(L,Y_i,Y_j)$$

C_m, the correlation integral is a measure of the number of points in embedded space ' m ' whose distance between each other is less than a predetermined length ' L '. N is the number of observations in the sample. Y_i and Y_j are vector valued observations and $(L, Y_i, Y_j) = (1, \text{if } | Y_i - Y_j | < L ; 0, \text{otherwise})$

One difficulty with this procedure is that the number of available observations decreases with each increase in embedding dimension. With an embedding dimension of 10 for instance a continuous set of ten original observations would denote a single point in ten dimensions. This would mean that a data series as large as 1000 observations will provide only 100 data points in ten dimensions. It is important therefore to have a sufficiently large data set in order to estimate the Grassberger-Proccacia correlation dimension.

The dimension of the dynamical system is determined by first estimating the slope of the regression line of log C_m (L) versus log (L) for each embedded dimension ' m '.The value of the slope where it stabilizes as the embedding dimension is increased gives the estimate of the correlation dimension. If the data is generated by a random process, then the slope would continually increase with the embedding dimension ' m '. This fact is made use of by Brock and Dechert (1986) to develop a test for serial independence in a data series[5].

The Brock Dechert Schienkman (BDS)

Procedure and Statistic

If the data series is independently, identically distributed (iid), then the dimension ' d ' would never stabilize at any embedding dimension ' m ' but would scale upward as ' m ' is increased. The test statistic for independence based upon the correlation dimension is then given by:

$$B\ (m,L,N) = N^{0\,5}\ [\ C_m\ (L) - C_1\ (L)^m\ ]$$

The statistic B (m, L, N) converges to a normal distribution with zero mean and variance V. The variance V can also be consistently estimated from the sample data as V (m, L, N). Dividing the statistic by the estimate of the standard deviation gives:

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[5] For a derivation of the statistic and its properties see Brock, Dechert and Scheinkman (1986).

$$W(m,L,N) = B(m,L,n) / V(m,L,n)^{0.5}$$

The W statistic converges to a normal distribution with unit variance, i.e., N (0,1), which implies that inference based upon the standard normal distribution is possible. Rejecting the null hypothesis provides evidence of serial dependence in the data.

The alternate hypothesis can imply either linear or nonlinear dependence in the data series. To eliminate the possibility of linear dependence, the methodology used (Scheinkman and LeBaron (1989); Brock, Hsieh and LeBaron (1991)) is to first fit an autoregressive (AR) process to the data and test the residuals from such a fit using the BDS statistic. A rejection by the BDS test applied to the filtered data would then support the hypothesis of nonlinear serial dependence in the mispricing series.

Testing Procedure. (1) Take the individual mispricing series (Y_t) and compute the BDS statistic for various embedding dimensions.[6] Determine whether the null hypothesis of iid innovations is rejected in favor of the alternate hypothesis of serial dependence in the data series.

(2) For the same raw series (Y_t) fit the best linear autoregressive model to the data. Denote the number of lags as L.

(3) Filter the raw series using the autoregressive filter of lag (L) and extract the residuals. Designate filtered residuals so obtained as U_t.

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[6] This study uses a computer algorithm to determine the correlation integral and the BDS statistic for every embedding dimension. The software was provided by Dr. William Brock of the University of Wisconsin (Madison). His assistance is gratefully acknowledged.

(4) Compute the BDS statistic for the filtered residuals U_t. Rejection of the null hypothesis would support the alternative of nonlinear serial dependence in the mispricing series.

(5) As a control measure, scramble the residuals U_t to eliminate any nonlinear structure. Denote the scrambled series S_t. Compute the BDS statistic for S_t. The null hypothesis of iid innovations should fail to be rejected in this case.

(6) For the raw series, regress the Ln Cm(L) against the Ln(L) for each embedding dimension (m). Denote the value of the slope of each regression line as 'd'. Plot the slope 'd' against the embedding dimension 'm' and determine whether the slope stabilizes (becomes horizontal) at any point. The value of the slope at this point is the correlation dimension of the data series. If the series is random (infinite or large dimension) the slope would never stabilize at any point but would instead scale upward with every increase in 'm'. On the other hand, the finding of a low order correlation dimension in combination with a finding of nonlinearity is evidence supporting the hypothesis of chaotic structure in the mispricing series.

Data

The data for this study have been obtained from, and is used with the permission of, MacKinlay and Ramaswamy (1987,1988). Each data series represents the futures mispricing for the S&P 500 futures contract which is computed as the difference between the S&P 500 index futures price and the theoretical forward price. It is expressed as a percentage of the index value. The data series starts with the September 1983 S&P 500 contract and follows the December, March, June, September cycle with the last series relating to the June 1987 S&P 500 contract. Each of the series has approximately 1600 observations. The stock index quotes are time stamped approximately one minute apart while the futures are time stamped transaction data. MacKinlay and Ramaswamy (1987, 1988) use the futures quotes at fifteen minute intervals to compute the mispricing series. The cost of carry is computed using the daily dividend yield of the value weighted index of all NYSE stocks supplied by the Center for Research in Security Prices as a proxy for the dividend yield on the S&P 500. The daily interest rates on certificates of deposit is the remaining input used to compute the theoretical forward price and the mispricing series.

The futures mispricing has been computed through the entire calendar period, September 1983 through June 1987 using futures prices from the nearest contract at any point. The nearest contract is also the most frequently traded contract. Observations for the June 1984 mispricing series, for example, use prices from commencement of trading on March 16, 1984 the day after expiry of the March 1984 contract. This makes it possible to link the mispricing observations into a single, continuous data series stretching across contract periods as illustrated in Figure 1.

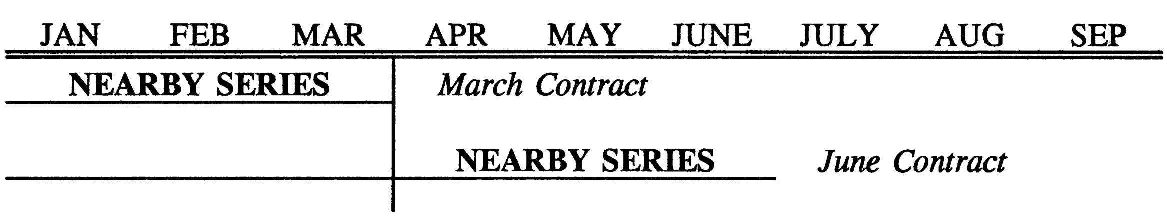

Figure 1. Construction of Nearby Series

The June 1987 mispricing series with 1675 observations is used for computing the BDS statistic and the combined March and June 1987 mispricing series with 3348 observations is used for estimating the Grassberger - Procaccia correlation dimension which requires a larger number of data points. Combining the June 1987 mispricing series with the preceding March 1987 series serves this purpose. At the same time with only one contract rollover, the risk of importing excessive noise into the data is kept to a minimum. The June 1987 mispricing series is chosen because it is the latest series available. Persistance of arbitrage opportunities in the June 1987 mispricing series cannot be attributed to market immaturity which might have been the case when the S&P 500 futures contract first started trading. Index arbitrage strategies using computerized programs were well in place there is no evidence of a lack of arbitrage capital during this period.

Results

Identifying Nonlinear Dependence

The testing procedure described in the methodology section are performed on the June 1987 contract which is the most recent data series available.

Table 1 gives the results of the BDS test for independence. The lengths used for the BDS test are defined as various proportions of the standard deviation of the series. The results of the test for independence is reported for embedding dimensions 1 through 4. The null hypothesis of an independent, identically distributed times series can be rejected at the 5% level if the W statistic value exceeds 1.96. As can be observed from the last column of Table 1, the null is rejected in every case; this

TABLE 1

BDS STATISTICS FOR THE JUNE 1987
MISPRICING SERIES

LENGTH (STD.DEV)	EMBEDDING DIMENSION	BDS B(m,L,N)	STD.DEV (BDS)	W STATISTIC BDS/SD
0.50	2.00	1.3950	0.0300	46.49
0.50	3.00	1.2620	0.0230	54.93
0.50	4.00	0.8510	0.0130	64.52
0.40	2.00	0.9790	0.0200	48.76
0.40	3.00	0.7110	0.0120	58.00
0.40	4.00	0.3880	0.0060	69.10
0.30	2.00	0.6280	0.0120	51.05
0.30	3.00	0.3560	0.0060	61.57
0.30	4.00	0.1510	0.0020	74.19
0.20	2.00	0.2960	0.0060	53.41
0.20	3.00	0.1130	0.0020	65.67
0.20	4.00	0.0330	0.0004	82.43

supports the alternate hypothesis of serial dependence in the mispricing innovations.

In order to eliminate the possibility of linear dependence in the mispricing series, the procedure adopted is to fit the best AR process to the data. The AR order is determined using Parzen's Criterion Autoregressive Transfer Function (CAT) criterion. The CAT criterion chooses the order that results asymptotically in the spectral estimator that is closest in the sense of the integrated relative mean square error to the true AR (∞) transfer function. For the June 1987 mispricing series, the CAT criterion finds an AR order of 10 to be the best linear fit to the data.

Table 2 gives the AR values and the corresponding lags. The residuals obtained using this AR filter are used for the second stage of the testing.

TABLE 2

AUTOREGRESSIVE LAGS AND COEFFICIENTS

LAGS	1	2	3	4	5
COEFFICIENT	-0.349	-0.089	-0.119	-0.031	-0.060
LAGS	6	7	8	9	10
COEFFICIENT	-0.056	-0.030	-0.024	-0.038	-0.071

TABLE 3

BDS STATISTIC FOR FILTERED RESIDUALS

LENGTH (STD.DEV)	EMBEDDING DIMENSION	BDS B(m,L,N)	STD.DEV (BDS)	W STATISTIC BDS/SD
0.50	2.00	0.3225	0.0585	5.51
0.50	3.00	0.3462	0.0534	6.49
0.50	4.00	0.2519	0.0365	6.89
0.40	2.00	0.2074	0.0421	4.92
0.40	3.00	0.1743	0.0311	5.61
0.40	4.00	0.0987	0.0172	5.74
0.30	2.00	0.1214	0.0260	4.65
0.30	3.00	0.0761	0.0146	5.23
0.30	4.00	0.0322	0.0061	5.27
0.20	2.00	0.0581	0.0130	4.48
0.20	3.00	0.0283	0.0050	5.67
0.20	4.00	0.0100	0.0014	6.91
0.10	2.00	0.0179	0.0035	5.10
0.10	3.00	0.0050	0.0007	7.24
0.10	4.00	0.0010	0.0001	9.33

Table 3 shows the results of the BDS test for the filtered residuals using the linear filter. Even though the value of the W statistic in every case is lower than the raw series, the null hypothesis of iid innovations is clearly rejected supporting the alternate hypothesis of nonlinear dependence in the mispricing series.

As a further control the filtered residuals are scrambled to eliminate any deterministic structure. The BDS test should fail to reject the null hypothesis of a random iid series. This proves to be the case as evidenced by the W statistics for the scrambled series shown in Table 4.

TABLE 4

BDS STATISTICS FOR SCRAMBLED RESIDUALS

LENGTH (STD.DEV)	EMBEDDING DIMENSION	BDS B(m,L,N)	STD.DEV (BDS)	W STATISTIC BDS/SD
0.5	2.0	-0.0057	0.0610	-0.09
0.5	3.0	0.0037	0.0577	0.06
0.5	4.0	0.0157	0.0409	0.38
0.4	2.0	-0.0021	0.0417	-0.05
0.4	3.0	0.0006	0.0307	0.02
0.4	4.0	0.0048	0.0170	0.28
0.3	3.0	0.0036	0.0150	0.24
0.3	4.0	0.0016	0.0064	0.25
0.2	2.0	0.0037	0.0125	0.29
0.2	3.0	0.0032	0.0047	0.69
0.2	4.0	0.0020	0.0013	1.49

As can be seen the null hypothesis of iid innovations is not rejected in all cases. The wide difference in the value of the W statistic for the filtered residuals series as compared to the scrambled residuals further supports the hypothesis of nonlinear structure in the mispricing series. Recently, it has been shown that the BDS statistic may not have adequate power against the ARCH / GARCH formulations which are nonlinear stochastic processes. (Brock, Hsieh and LeBaron (1991)). An ARCH process was fitted to the June 1987 mispricing series and the results of applying the BDS procedure to the residuals show that the null hypothesis of independence is rejected in three out of five cases.[7] This strengthens the case for nonlinear determinism in the S&P 500 mispricing series.

Identifying the Correlation Dimension

A finding of low order dimension would be strong evidence in favor of a deterministic explanation for the persistence of index futures mispricing. The objective here is to identify the Grassberger-Proccacia correlation dimension by searching across 15 embedded dimensions. The combined March and June 1987 mispricing series is used for this purpose.

Table 5 shows the results of the regression of Ln(Cm) against Ln(L) for 15 embedding dimensions. The X coefficients are the slope of the regression line which provides an excellent fit as evidenced by the regression coefficient (R^2) which in all cases is greater than 0.9. The value of the X coefficient (slope of regression line) where it stabilizes is the correlation dimension of the mispricing series. As can be observed from Table 5, the slope coefficient increases for initial embedded dimensions and then the rate of increase decreases around a value of 6 even as the embedded dimension is increased.

Figure 2 shows a plot of the slope coefficient of the regression equations against the embedding dimension. The 45 degree line represents the theoretical plot of a random process which is shown for comparison. As explained earlier, for a random

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[7] Vaidyanathan and Krehbiel (forthcoming 1992); The Journal of Futures Markets.

TABLE 5

ESTIMATING THE CORRELATION DIMENSION
MARCH + JUNE 1987 MISPRICING SERIES

EMB. DIM	1	2	3	4	5
SLOPE COEFF.	0.970	1.860	2.730	3.620	4.580
STD ERROR	0.002	0.008	0.014	0.022	0.038
R SQUARED	0.999	0.998	0.997	0.996	0.993
EMB. DIM	6	7	8	9	10
SLOPE COEFF.	5.230	5.630	6.020	6.30	6.500
STD ERROR	0.059	0.042	0.039	0.040	0.056
R SQUARED	0.990	0.996	0.997	0.997	0.995
EMB. DIM	11	12	13	14	15
SLOPE COEFF.	6.580	6.660	6.660	6.730	6.960
STD ERROR	0.056	0.068	0.081	0.093	0.122
R SQUARED	0.991	0.987	0.981	0.976	0.967

process the slope coefficient would increase continuously with an increase in the embedding dimension. The slope coefficient for a deterministic process on the other hand would stabilize at some point as the embedded dimension is increased. The value of the slope coefficient where it stabilizes is the correlation dimension of the series. For the combined mispricing series, the slope coefficient stabilizes at around a value of 6 consistent with a low order nonlinear deterministic process. This strengthens the case for a deterministic explanation to the mispricing anomaly as opposed to a stochastic approach that had been adopted earlier.

The value of the correlation dimension obtained here has to be viewed with caution however. Empirical estimates are sensitive to sample size and the presence of noise in the data set. Results from Ramsey and Yuan (1989) indicate that dimension

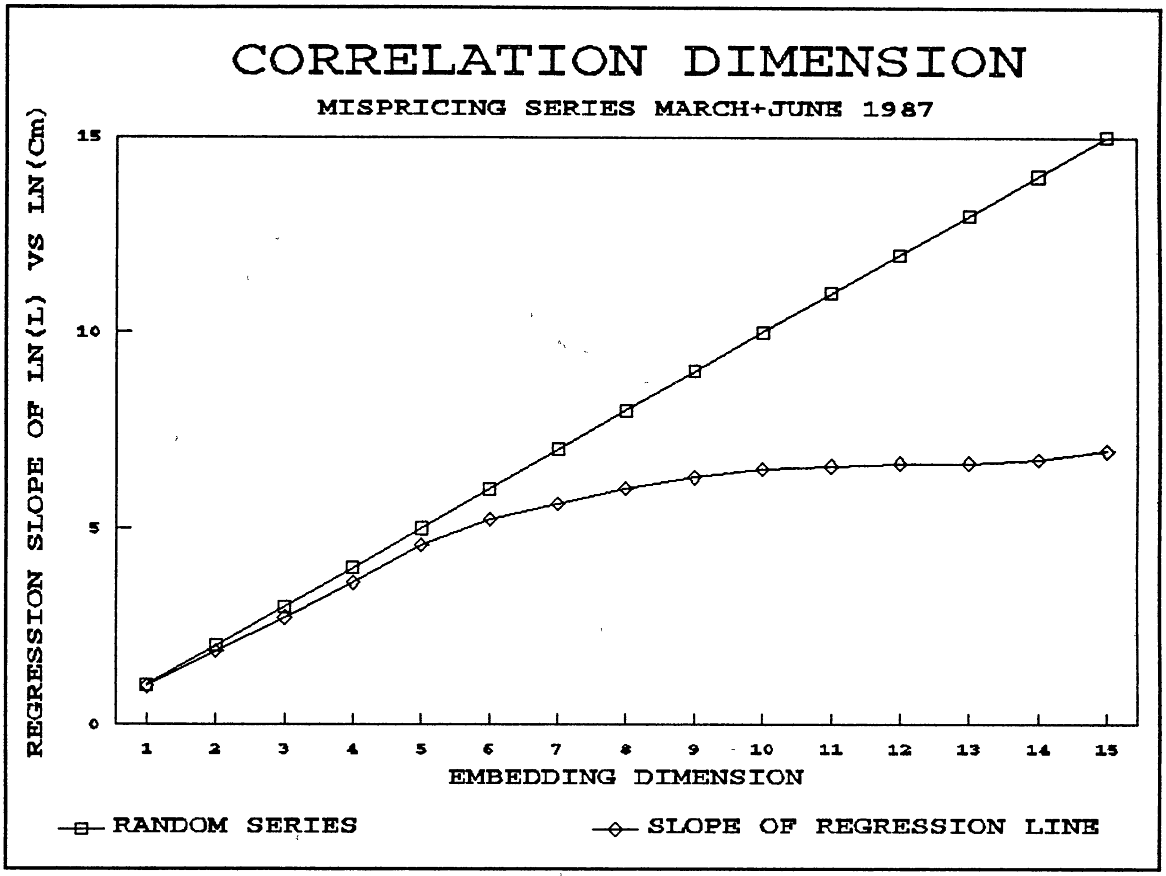

Figure 2. Identifying the Correlation Dimension

estimates are biased downward for random noise and biased upward for attractors. As Sayers (1990) states, ..."a correlation dimension estimate of 0.214 could imply an actual correlation dimension value of as high as 1.68." (p.185). The emphasis of this paper is limited to the issue of whether a deterministic explanation exists for the mispricing puzzle. The finding of a low order correlation dimension as opposed to the precise determination of the correlation dimension is sufficient to support the hypothesis of nonlinear dependence in the mispricing series.

CHAPTER III

MARKET MICROSTRUCTURE ISSUES IN THE MISPRICING OF THE S&P 500 FUTURES CONTRACT

Background

This essay applies simulation techniques to examine the impact of select market microstructural features such as trading delays on the arbitrage process. A number of researchers have observed that the S&P 500 futures contract has been significantly mispriced in the past and attempts have also been made to model the mispricing series[1]. Along the same lines researchers have also noted that the differences in the microstructure of the futures market with its speedy open outcry method of trading as opposed to the slower specialist oriented NYSE system causes information to be incorporated earlier in the futures prices[2]. This brings about a change in the S&P 500 futures price which causes it to be mispriced relative to the cash index value. The mispricing is corrected when the information is transmitted through the process of index arbitrage to the stock market. The object here is to study

[1] Modest and Sundaresan (1983); Cornell and French (1983); Figlewski (1984a), MacKinlay and Ramaswamy (1988); Yadav and Pope (1990); Brennan and Schwartz (1990). These papers among others are reviewed in the previous essay.

[2] Garbade and Silber (1983); Ng (1987); Kawaller, Koch and Koch (1987); Herbst, McCormack and West (1987); Stoll and Whaley (1990).

the process of adjustment to an observed mispricing by developing a simulated arbitrage system that incorporates the differential trading delays in the futures and the stock market.

The foregoing essay brought out some aspects of the mispricing of the S&P 500 futures contract that motivates the current investigation. Evidence obtained from applying the BDS methodology to the mispricing series supports the hypothesis of nonlinear dependence in the data. Separately the Grassberger-Procaccia measure of the correlation dimension suggests the possibility of a deterministic explanation for the process that generates the index futures mispricing. This essay builds upon these results by hypothesizing an endogenous explanation to the mispricing puzzle, i.e., structural features of the futures and the stock market, namely, the trading delays are capable of shifting the system behavior from one of speedy elimination of mispricing to a chaotic system in which mispricing persists.

Simulation is the methodology of choice for such research since it is impossible to conduct a controlled experiment in real world markets that will focus upon the effect of microstructural differences upon the arbitrage process. Furthermore, the nonlinearity inherent in real world systems (Forrester (1987)) and the absence of closed form solutions to most nonlinear systems (Radzici (1990)) makes simulation the only way of analyzing the behavior of such systems.

Because of these advantages, simulation techniques have been applied to many other studies of market microstructure as well. Cohen, Maier, Schwartz and Whitcomb (1984) adopt a simulation methodology to examine various market stabilization policies including the role of the specialist. As the authors observe, "Simulation has been used because the complexity of actual security markets makes it virtually impossible to infer meaningful ceteris paribus relationships using the mathematics of comparative statics or standard multivariate statistical tests. In our simulated environment, we have been able to contrast runs, that are, by construction identical except for whether or not there is a market maker and if so whether he is a pure stabilizer or a speculating stabilizer." (p. 190). Cohen etal (1984) also point out that the simulation methodology provides the closest possible approximation in market studies to the laboratory experimental methodology pursued in the pure sciences. By doing so it offers a way to experiment with different policy options prior to their adoption.

Literature Review

Simulation in Chaotic Systems and Financial Markets

Simulation techniques have gained increasing popularity during the last two decades primarily due to the exponential increase in computing power. This new found capability has allowed researchers to simulate the behavior of complex nonlinear systems that had until recently been ignored. The difficulties encountered by researchers in most part stem from the mathematical intractability of nonlinear equations which in almost all cases do not possess unique or exact solutions. It is now recognized that nonlinear relationships among variables are the norm in social systems because there exists inherent physical, social and psychological limits that constrain the behavior of the elements within them (Forrester (1987)). The lack of unique solution paths for nonlinear systems is in some ways an advantage. The current academic interest in nonlinear dynamics and deterministic chaos is motivated by the recognition that nonlinear systems are capable of depicting a wide range of behaviors as select parametric values are changed (Mosekilde, etal. (1988)). Consequently, nonlinear models afford researchers ways to simulate evolutionary and adaptive systems that change their behavior over time. Radzicki (1990) makes this reasoning clear when he states that ".....the evolutionary behavior of nonlinear socioeconomic systems can only be revealed through computer simulation for in almost all instances, they do not possess exact analytical solutions. The reason for this lack of mathematical elegance is that, unlike most linear models, nonlinear patterns cannot be broken down into pieces that are solved and then added together to reveal the behavior of the whole. Much to the contrary their behavior can only be uncovered through examinations involving their entire structure as a unit. This is another way of saying that the behavior of nonlinear systems is something more that just the sum of the behavior of their parts." (p. 60).

Early work on nonlinear systems by Lorenz (1963), May (1976) and Fiegenbaum (1983) involved the simulation of select nonlinear functions and systems of equations as a means of understanding their behavior. One of the surprising results of these simulations was the nonrepetitive and unpredictable time paths of variables that were generated by such deterministic functions. The term deterministic chaos was used to describe the apparently random behavior of purely deterministic functions. (Baumol and Benhabib (1990)). Interest in nonlinear dynamics rose when Fiegenbaum (1983) demonstrated that the route to chaos for nonlinear functions was through a series of bifurcations. For some parametric values the system converges to a single point called an attractor. As the parametric value is, increased the system becomes unstable and shifts to a new attractor by doubling the number of points visited by the system. The series of bifurcations continues as the tunable parameter is increased until the system shifts to a chaotic attractor, a behavior mode whose true period is infinite since the behavior never repeats itself. At the same time the motion is bounded and appears to have a certain recurrence. An important characteristic of a system in a chaotic mode is that two trajectories (with starting points that are infinitesimally apart) very soon evolve in a completely different manner. This sensitivity to initial conditions of chaotic systems makes point prediction of future system states impossible even though the equations of motion are described by perfectly deterministic and specified parameters. Fiegenbaum (1983) then went on to demonstrate that the bifurcation route to chaos possesses certain invariant characteristics and is the same for a universal class of functions that have a single peak or maxima.

The four graphs (Figures 3,4,5, and 6) shown below reproduce the Fiegenbaum (1983) cascade of bifurcations for the nonlinear function $X(n+1) = F(X(n)) = 4bX(n)(1-X(n))$ where $0<X<1$. For an initial value of the tunable parameter $b = 0.1$ the dynamic path converges to a single point. When $b = 0.8$ the system shifts attractors and cycles between two points. At $b = 0.88$ the system bifurcates and cycles between four points. This process continues until the system shifts to a chaotic attractor shown in Figure 6 at $b = 0.9936$.

The recognition that nonlinear systems can exhibit chaotic behavior led researchers to empirically investigate a wide collection of data sets for evidence of

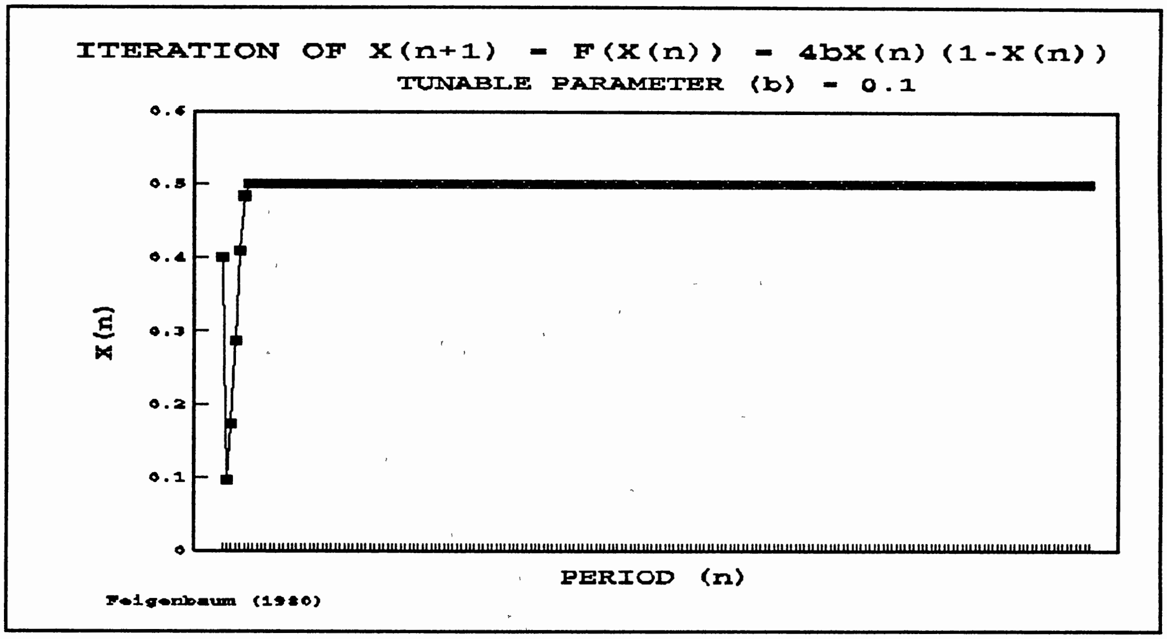

Figure 3. Single Point Attractor

nonlinear structure and deterministic chaos. In the general area of financial markets such empirical research includes Scheinkman and LeBaron (1989) on stock returns, Hsieh (1989) on foreign exchange rates, Blank (1990) on futures prices, Barnett and Chen (1987) on monetary aggregates, Frank and Stengos (1989) on gold and silver returns and Larrain (1991) on Treasury Bill rates. Much of this empirical research is reviewed in the previous essay.

A finding of nonlinear dependence in a data series, or even the detection of a chaotic attractor, says nothing about the precise deterministic or structural element in a real financial or other social system that is responsible for driving the system into a chaotic mode. In other words, from the viewpoint of understanding and control, it becomes necessary to ask what particular structural parameter/s can be capable of shifting a system such as the stock market, for instance, into a chaotic phase. In this

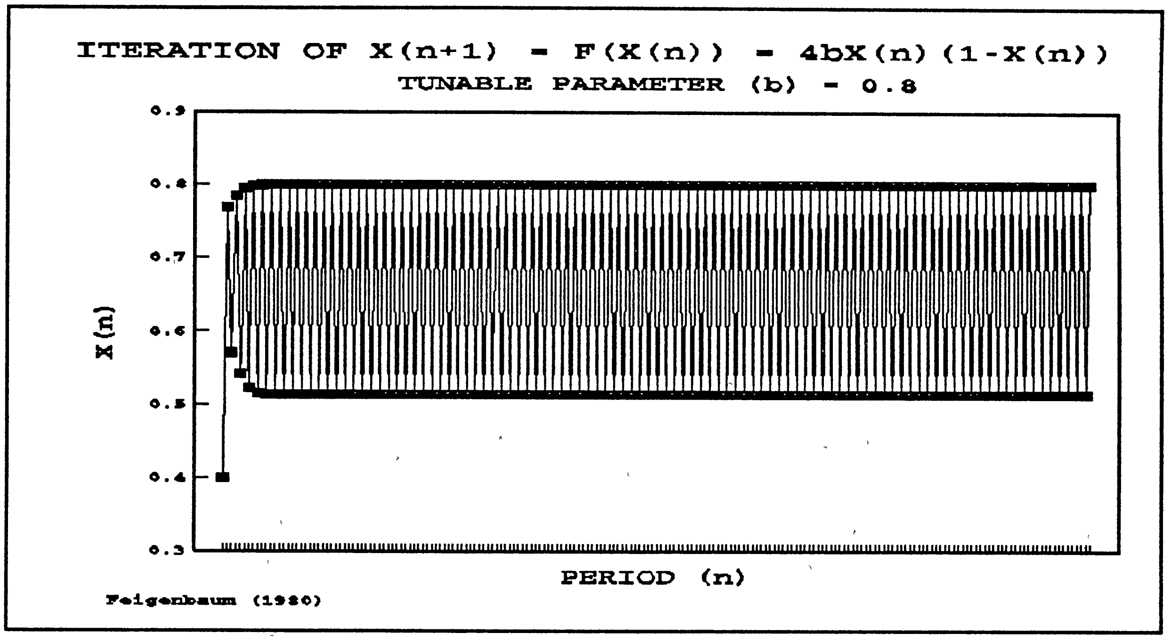

Figure 4. Two Point Cycle

context Radzicki (1990) notes that "...the study of how a system can change its behavior from one mode to another, including from a non-chaotic mode to a chaotic mode is considered as important as chaos itself." (p. 67). This genre of experimental research is now fairly widespread. Rasmussen and Mosekilde (1988) use a simulation model to show that chaos can arise in a firm that shifts its resources between production and marketing in response to inventory levels. Lorenz (1989) demonstrates that the coupling of three regularly oscillating economic sectors can bring about chaotic behavior in the entire system. Sterman (1989) shows how the investment decisions of experimental subjects operating in a simple multiplier-accelerator economy are capable of generating the complex bifurcation structure found in nonlinear systems. In doing so Sterman (1989) follows a two step approach. In the first stage experimental subjects make investment decisions in a laboratory setting

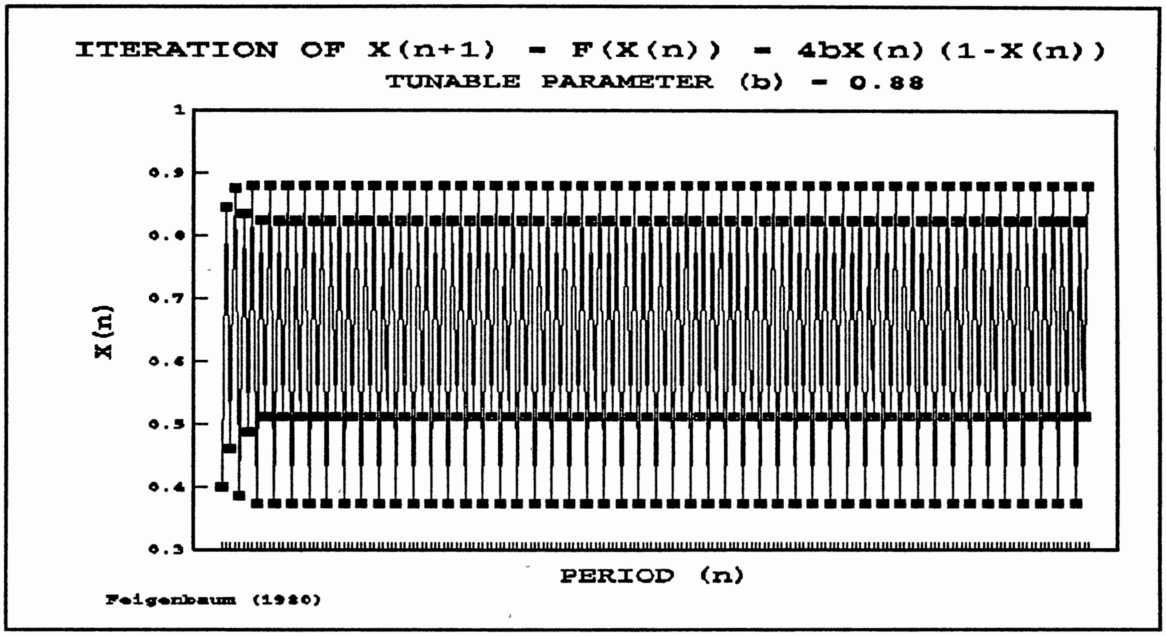

Figure 5. Four Point Cycle

which reflect the decision rules (heuristics) used by managers in day-to-day investment decisions. The estimated decision rules so obtained are formalized and used in the second stage as an input to a computer simulation model. Repeated simulation under different conditions showed that " approximately 40% yield a variety of dynamics including limit cycles, period multiples and chaos." (Sterman (1989), p. 1).

Shaffer (1991) extends this form of experimental simulation to the stock market and focuses upon the factors responsible for the stock market crash of October 1987. His study illustrates the application of simulation techniques to nonlinear systems in general and the stock market in particular. Shaffer (1991) puts forward the hypothesis that the steep drop in stock prices on Black Monday 1987 may have been the **consequence** of a sharp increase in volatility and not the reverse. The argument is

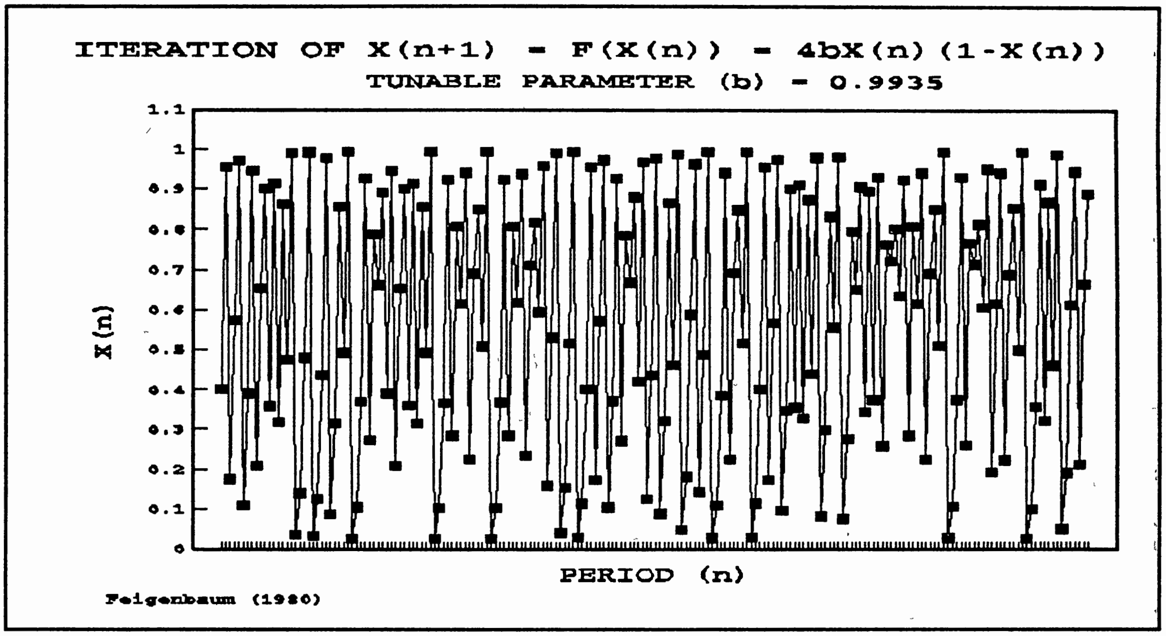

Figure 6. Chaotic Attractor

built up as follows: A sharp increase in volatility would lead investors to demand greater returns consequent to the increased risk of their portfolios. Existing stock prices would appear overvalued under this changed scenario thus bringing about a discontinuous fall in stock prices overall. Such a discontinuous jump in volatility is a possibility in a chaotic market which by definition is extremely sensitive to small changes in parameters. Shaffer generates a chaotic path of stock prices using a declining marginal efficiency of investment curve in combination with a fixed dividend payout ratio. Within certain parametric regions, the level of volatility in this model is found to be quite sensitive to small changes in structural parameters. Interestingly enough the changes in volatility are found to be of a magnitude sufficient to cause the stock market crash.

In developing the model Shaffer (1991) notes that in order to analyze crash behavior the researcher must explicitly "...probe inside the 'black box' of volatility and try to understand volatility as an endogenous variable." (p. 202). This approach contrasts sharply with the stochastic view of stock prices wherein volatility is exogenous to the model. Shaffer (1991) justifies the endogenous approach as follows: "To begin with, it is formally incorrect to model the stock market as purely random. Each transaction along with its price is the result of conscious decisions by a buyer and seller. Conventional microeconomic theory treats such decisions as the outcome of an optimization process and hence as deterministic. Stochastic models have been applied to describe this process because of their tractability and because they closely approximate the observed patterns of price movements over time. However when there is a question of causal factors, a deterministic model must be sought." (p.203).

A similar argument can be made to justify the endogenous approach that is used in this study to examine the problem of index futures mispricing. Focusing upon specific microstructural issues allows the researcher to identify the separate market features that can shift the arbitrage system to a chaotic mode. The findings of nonlinear dependence and a low order correlation dimension relative to the mispricing observations tilts the balance toward a deterministic as opposed to a stochastic explanation for the mispricing puzzle.

The simulation approach to problems in finance are not limited to investigations of nonlinear or chaotic systems but have been used by researchers in the field of market microstructure as well. There have been two major reasons for adopting the simulation methodology to research questions relating to market microstructure. The first has to do with the complexity of financial markets which makes formal mathematical modeling and theorizing extremely difficult. This is even more true for studies of market dynamics as compared to static or even comparative static analysis. The second motivation is due to the fact that it is impossible to conduct controlled experiments under ceteris paribus conditions in real world financial markets. Research into market microstructure that has utilized the simulation methodology include an early paper by Garman (1976) who is credited with coining the term 'market microstructure'. Garman's (1976) simulation approach compared the bid-ask spread in a market dominated by a central market maker with that of a double auction market system. Cohen, Maier, et al. (1978) investigate serial correlation patterns in stock returns in the absence of stabilizers, arbitrageurs or large market traders. Their model simulates an atomistic continuous auction market.

Many simulation studies in the area of market microstructure have examined the monopoly status of the stock exchange specialist and the profitability of that function. Hakansson, Beja and Kale (1981) for instance examine the feasibility of automating the function of the specialist. Cohen, Maier et al. (1983) note that the positive profits earned by stock exchange specialist is derived partly from the bid-ask spread and commissions and that nothing definite can be said about the profitability of the stabilization function per se. Their experimental simulation system allows them to separate the components of a specialist's earnings and investigate the profitability of stabilization alone. The simulation model is also used to investigate the experimental impact on the bid-ask spreads faced by an investor consequent to the adoption of different policy options. These include the benefits from tightening or loosening the stabilization constraints that are binding on the specialist; the result of denying monopoly knowledge of the order book to the specialist; and the gains from substituting a mechanical stabilizer in lieu of the stock exchange specialist. Cohen, Maier et al. (1983) conclude from their simulation runs that the specialist's activity reduces bid-ask spreads and quite surprisingly that pure stabilization combined with knowledge of the order book leads to a greater improvement in the bid-ask spreads. Cohen et al. (1983) also identify various issues relating to securities markets that are especially amenable to modeling through simulation: " In general we find that simulation is a useful tool for dealing with a variety of complex, interrelated issues concerning the functioning of a securities market. ...[T]he (simulation) model should yield additional insights into areas which do not lend themselves to pure mathematical analysis, for instance the dynamic adjustments of stock prices to informational change **when traders reactions are not instantaneous**; the relationship between stock prices and option price movements when **arbitrage mechanisms are imperfect;** or the effect of a stock's order flow and trading characteristics of **interlinking various markets that have different features.**" (p. 190: emphasis added). The very same microstructural features identified by Cohen etal. (1983) are the focus of this study as well.

This essay examines the dynamical repercussions of a delayed trading response in the stock market upon mispricing levels. This delay is the consequence of linking the open outcry trading system of the futures exchange with the specialist dominated stock exchange through arbitrage mechanisms. This reasoning is in agreement with the conclusions of Cohen, Hawawini, etal. (1980) who identify the frictions in the trading process (price adjustment delays) as causing the observed serial correlation in individual stock price returns as well as in market index returns. The authors also cite specialist intervention to ensure an orderly continuous market as being one of the factors that causes a delay in the adjustment of prices.

Simulation Methodology Using System Dynamics

A formal methodology of simulating complex systems currently known as system dynamics was developed at the Massachusetts Institute of Technology by Jay W. Forrester. This methodology utilized concepts from control engineering, cybernetics and organizational theory and includes a library of symbols and the rules for connecting them. (Meadows (1984)). Together these constitute a map of the dynamical system. The system portrait so developed highlights the interconnectedness of the system variables and guides the development of the algebraic equations that represent these relationships. Forrester's contribution included the development of a simulation language (DYNAMO) which is used to code the symbols from the feedback diagram into algebraic equations. Morecroft (1988) summarizes Forrester's contribution to the field of system dynamics by stating: "Forrester's reshaping of methods from control engineering led to a visual representation of feedback systems, and through simulations to a visual representation of feedback dynamics. These graphics provide a conduit for a policymaker's knowledge and a basis for policy debate." Over the last two decades, the system dynamics simulation methodology has been applied to a wide range of managerial, economic and social problems. Some of the better known applications include a world model (Meadows and Meadows (1972, 1974)) that examined the time paths and interrelations between world population, resource utilization, pollution and economic growth; a model of urban growth and decay (Forrester (1969)); a simulation of the problems of heroin addiction in a large city (Levin, Hirsch and Roberts (1975)); an examination of the fluctuation of commodity prices (Meadows (1970)); the causes of business cycles (Mass (1975)); economic development (Meadows, Behren, Meadows et al. (1974)) and various corporate policy studies (Roberts (1978), Lyneis (1980)). More recently system dynamics has been used in modelling chaotic fluctuations in the inventory and production cycles of a firm (Rasmussen and Mosekilde (1988)); in the study of self organizing and evolutionary systems (Radzicki (1990)) and the spread of the AIDS epidemic (Ahlgren and Stein (1990).

The primary orientation of the system dynamics paradigm is toward the understanding of system behavior as an outcome of the underlying system structure made up of interrelated feedback loops. The relationships between the system variables are very often nonlinear reflecting the emerging nonlinear view of complex natural and social systems (Forrester (1987)). The system dynamics slant is toward an endogenous explanation of the dynamic behavior of the system and there are benefits to maintaining such a view. If the causes of problematic system behavior can be found within the causal feedback loops that make up the system structure of the system, then therein will also lie the remedies to those problems. By understanding the role of feedback mechanisms in explaining behavior, the modeler can generate workable policy measures that will help solve social and economic problems. Without this emphasis on causal structure a modeler will be forced to look for the causes outside the system thereby imposing an exogenous and necessarily stochastic understanding of system behavior. This belief that macrobehavior arises from microstructure is brought out in the writings of many system dynamics researchers namely Forrester (1977), Richardson (1984), Richardson and Pugh (1981).

A system dynamics model has some unique characteristics according to Richardson (1984) for it is almost always described in terms of closed boundaries and the feedback loops that are contained within them. It is the closed system boundary that reflects the endogenous bias of the system dynamics methodology. The feedback loops link the level and rate variables and the flow of material and information within these loops generate the dynamic path of the system variables over time.

Feedback Loops

Feedback loops represent the circular causality within the system. The emphasis inherent in the word feedback rests equally on both the transmission and return of information. (Richardson and Pugh (1980)). For example, the thermostat in a room is connected to the heating system and transmits information about the room's temperature back to the system thereby either switching on or shutting off the heat. The thermostat is therefore the feedback device and together with the heating system makes up the feedback system.

Feedback loops are classified according to the positive or negative polarities associated with them. Positive feedback loops amplify any change within the system and are also characterized as destabilizing, disequilibrating or self-reinforcing.

Negative feedback loops are classified as goal seeking or equilibrating in their effect. The thermostat controlled heating system is a negative feedback system.

Levels and Rate Variables

The system variables are categorized as levels (stocks) that represent accumulations within the system and rates (flows) that embody decisions or activity that change the levels. The term level invokes the image of a liquid accumulating in a container. Some examples of levels would include population, inventories, capital stock and even psychological variables such as perceptions that in reality represent accumulations of information. (Meadows (1980)). The flows that add to or decrease the level are the rates and are the elements that represent the decisions, action or change in the levels. Examples include the birth rate that leads to an increase in the population level; the death rate that depletes the population level, investment rates that increase the capital stock (level) in an economy or rates of depletion of inventory through sales.

Positive and Negative Feedback

The concept of a negative and a positive feedback loop are important elements of the system dynamics methodology. Together they represent respectively the goal seeking or disequilibrating potential of a dynamical system. The pollution control feedback diagram is an example of a negative feedback loop. In the simple pollution control loop, (Figure 6) the amount of pollution is the important level variable. An increase in the amount (level) of pollution leads to an increase in the citizenry's concern about the ill effects of pollution. Thus the level of pollution and the concern for pollution are directly related and this is represented by a positive sign in the arrow connecting the two elements. An increase in the concern for pollution brings about political action and a resulting increase in the number of pollution controls. Once again the relationship is direct as evidenced by the positive sign at the arrowhead. An increase in the number of pollution controls leads to a decrease in emissions of polluting substances that decreases the pollution level. The number of pollution controls is inversely related to the amount of pollution and this is represented by a negative sign in the arrow connecting the two variables. Overall the system is goal-seeking and is called a negative feedback system since any increase in pollution will cause activity around the loop that moves the pollution level to an equilibrium. The number of negative signs around the loop determines the polarity of the entire loop. An odd number of negative signs represents a negative feedback loop and an even number would imply a positive feedback loop. In the pollution example above, there is one negative sign (an odd number) confirming the goal seeking property of the system.

The model of job stress in Figure 7 is an example of a positive feedback loop. It is a simple model of stress in the workplace. The worker in this model has a job backlog that is increased as new assignments land upon her table. An increase in the job backlog causes an increase in the anxiety level of the worker represented by the positive sign on the arrow head. In her case an increase in anxiety is disabling and causes an increase in the average time taken to complete the task. An increase in task completion time causes a fall in the task completion rate which is an inverse

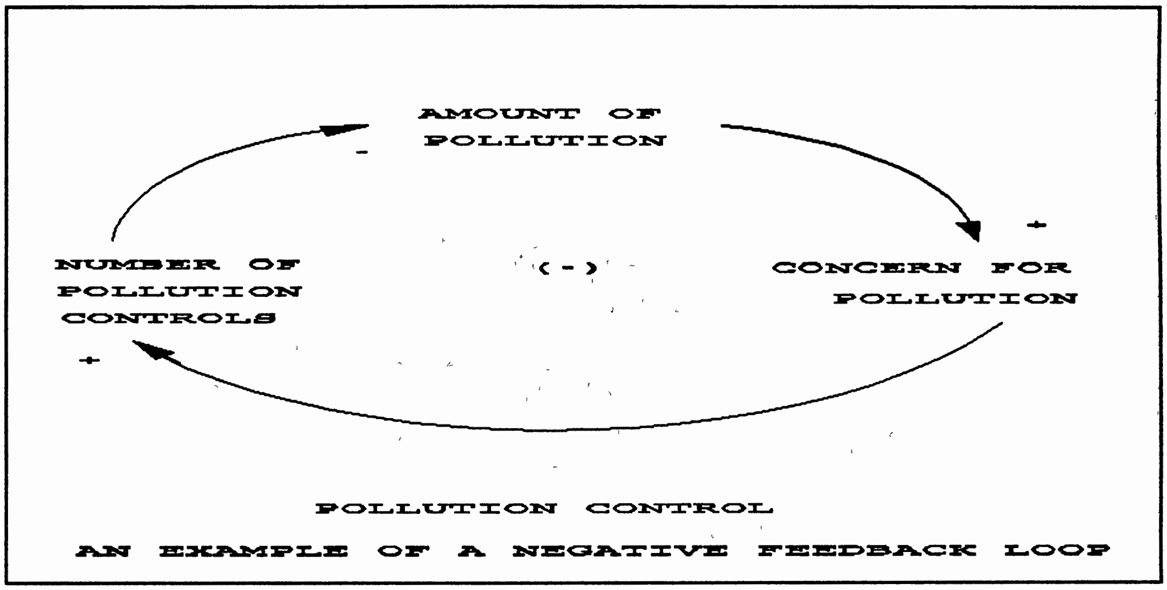

Figure 7. Flow Chart of Negative Feedback System

relationship represented by the negative sign on the arrow. A fall in the task completion rate causes an increase in the job backlog which is once again an inverse relationship. The net result is that the job backlog continues to grow causing even greater anxiety in ever greater proportions. This is a positive feedback loop because it causes the job backlog to grow with each iteration around the loop. The loop has an even number (two) negative signs on the arrowheads which confirms the self reinforcing nature of the model.

This study uses the system dynamics simulation methodology to develop a model of the arbitrage process. The first step in the exercise is to develop a " partial model" as it is called by Morecroft (1988). The partial model is a deliberately simple and uncomplicated representation of reality which makes specific some part of the subsystem that is of interest. It is designed to depict the arbitrage process under the

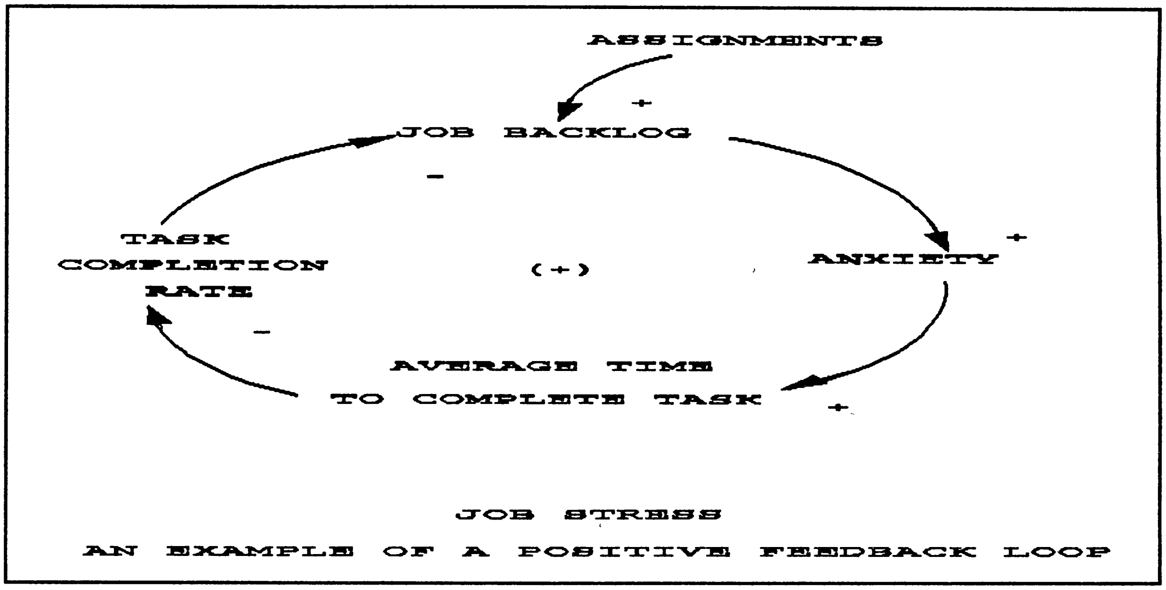

Figure 8. Flow Chart of Positive Feedback Behavior

most ideal conditions, i.e., with sufficient arbitrage capital and the absence of any market frictions such as trading delays. By design, there are no other players in the arbitrage system. The test of this partial model is that it should eliminate any mispricing as it occurs by appropriate adjustments to the cash and futures price. In the next stage, specific market features such as price adjustment delays in the stock market and the uptick rule are introduced into the system. The path of adjustment to an initial mispricing can then be observed under these added circumstances and contrasted with the base case. This helps provide a better understanding of the effect of select microstructural elements viewed in isolation. In effect, this methodology formalizes through simulation the ceteris paribus approach that is universally adopted in theoretical economics.

The Arbitrage Model

The simulation model for the index arbitrage system as shown in Figure 9A consists of two level (state) variables; the spot index price and the corresponding index futures value. As can be observed from this flow chart, the spot index value at any point in time determines the corresponding forward price. The forward price is the sum of the cash index value and the cost of carrying the index basket to maturity. The interest rate and the dividend rates are inputs to this formulation and are assumed constant for the simulation exercise. The difference between the forward price and the futures price gives the index futures mispricing which is computed as a percentage of the cash index value in accordance with the empirical definition given by MacKinlay and Ramaswamy (1988). The observed mispricing (positive or negative) induces arbitrage activity causing equilibrating changes in both the futures price and the cash index value. If the futures price exceeds the forward price the arbitrage action would involve the sale of the futures contract and the purchase of the underlying basket of stocks. The result would be a fall in the futures price and an increase in the cash index value. Both actions drive the level of mispricing to zero. The opposite arbitrage effects would occur if the futures price falls below the forward price.

The arbitrage price change in both markets will drive the mispricing level toward zero as can be seen by the two arrows in Figure 9A that lead from the arbitrage price change back once again to the spot price and the futures price respectively. The combined index arbitrage mechanism can therefore be viewed as a feedback system comprising two negative feedback (equilibrating) loops; the stock market loop and the futures market loop. Arbitrage causes the two prices to change in

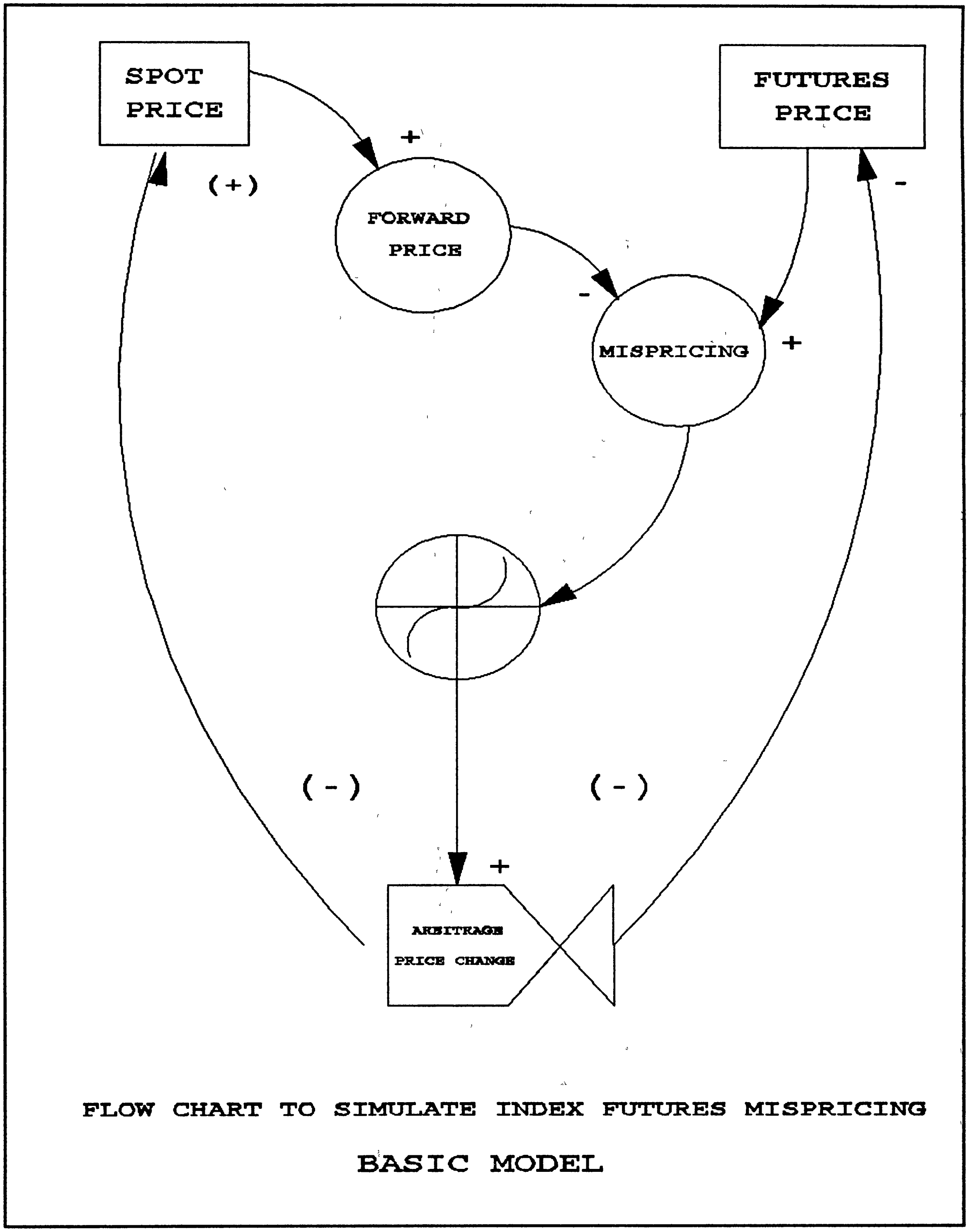

Figure 9A. System Dynamics Model of the Arbitrage System without Delays opposite directions thereby reducing the mispricing to zero. In Figure 9B, the same feedback mechanism is shown with only the market structural feature of trading delays added in.

Calibrating the Arbitrage Price Reaction Function

The arbitrage price reaction (5) is a rate equation which maps the corresponding price change to different levels of mispricing. The price reaction parameters used in this exercise are graphed in Fig 9. The x-axis shows the mispricing level which ranges from -3 percent to +3 percent of index value. These high and low values of mispricing (rounded up) are obtained from the MacKinlay and Ramaswamy (1988) data series. Corresponding to each of the mispricing levels are the arbitrage price reactions shown in the y-axis. The high (+3.75%) and (-3.75%) low bounds for the arbitrage price reactions are also based upon the MacKinlay and Ramaswamy (1988) data series.

The summary statistics of mispricing and price change for the March and June 1987 series is shown in Table (6).

TABLE 6

MISPRICING AND PRICE CHANGES

DATA SERIES	MISPRICING MAXIMUM	FUTURES PRICE CHANGE (MAX)	SPOT PRICE CHANGE (MAX)
June 1987	2.60%	1.99%	1.80%
March 1987	1.70%	3.75%	2.00%

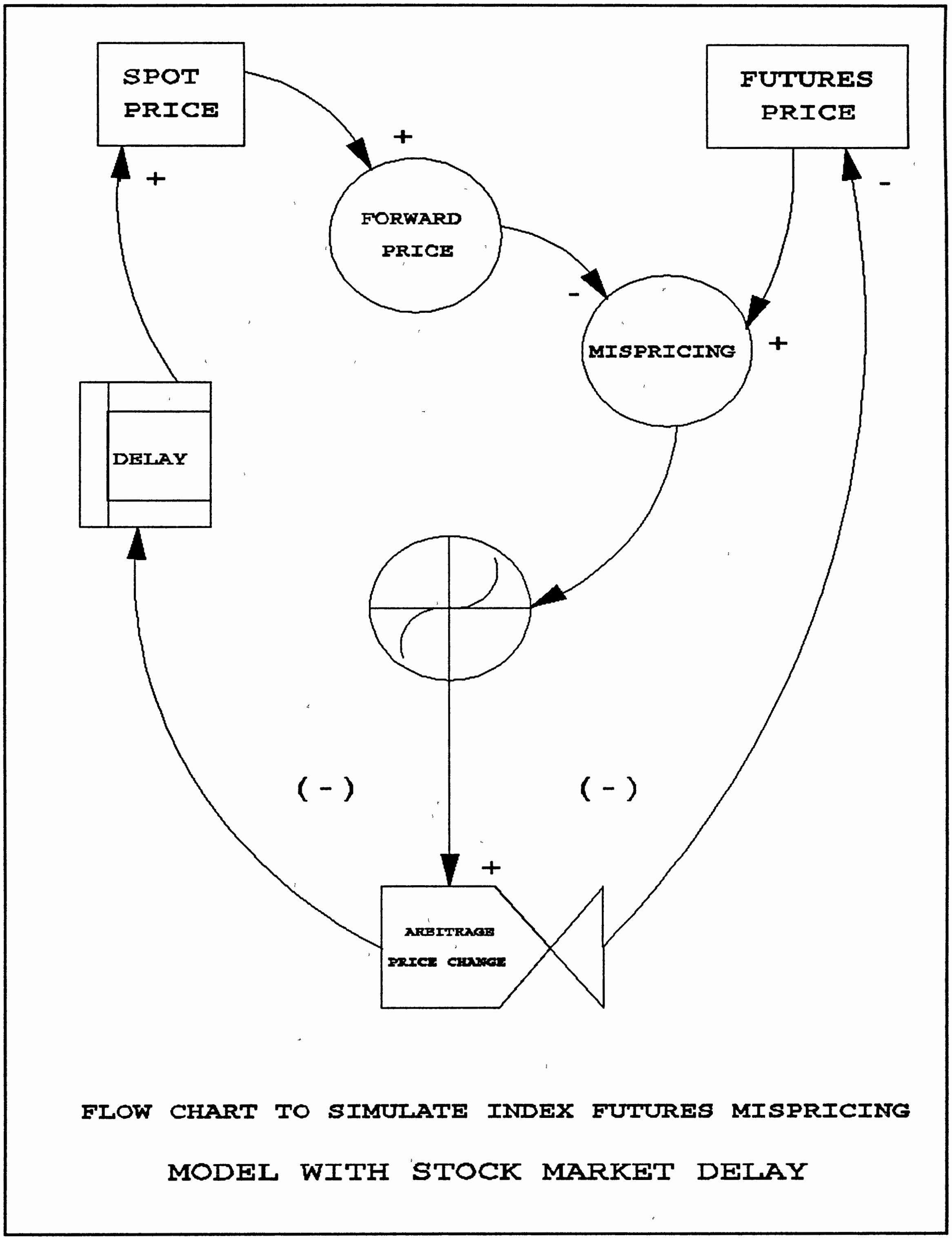

Figure 9B. System Dynamics of the Arbitrage System with Stock Market Trading Delay

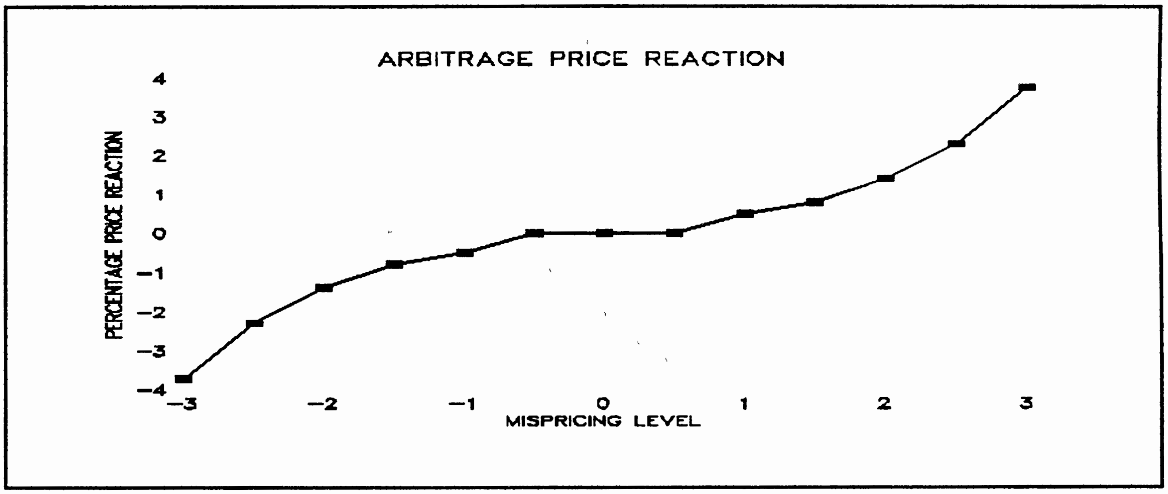

Figure 10. Table Function Relating Mispricing and Arbitrage Price Response

This serves to provide realistic bounds for the mispricing levels and the corresponding arbitrage price reactions. Unfortunately, the exact mathematical function that will relate each observed mispricing level to the corresponding price change cannot be observed empirically. This is because the price changes in either market are a result not only of arbitrage activities but also include an amalgam of price effects caused by hedgers, speculators, noise traders, and other classes of market players. Isolating the arbitrage price reaction from the mix of other influences can be very difficult if not impossible. Therefore, some logical assumptions need to be made in order to specify the relationship between observed mispricing and the arbitrage related price change in the futures and cash market. The arbitrage price reaction function as graphed in Figure 10 slopes upwards and has a positive second derivative. The justification for these assumptions are as follows. At low levels of mispricing only the most efficient arbitrageurs with the lowest transaction costs would find it profitable to arbitrage. At higher levels of mispricing however even marginal arbitrageurs with higher transaction costs would be induced to invest in arbitrage. This would draw more investment capital to the arbitrage arena and cause a larger price reaction. At very high levels of mispricing it is conceivable that many other classes of market players (non-arbitrageurs) would be attracted by the profitable arbitrage opportunity. The price reaction function under such assumptions would have a positive slope that would increase at an increasing rate.

It is also assumed that the arbitrage price changes in the futures market and the cash market are symmetric since the arbitrageur would buy and sell equal quantities of the two baskets. Furthermore, lower and upper transaction cost bounds have been set for the mispricing levels at -0.5 and +0.5 respectively. Arbitrage will not be profitable within these transaction cost boundaries. Correspondingly, the associated arbitrage price change would be zero.

The flexibility inherent in a simulation experiment permits the scaling of the price reaction to a fraction of the normal to reflect inadequate arbitrage capital. Rubinstein (1987) suggests that the persistence of index futures mispricing may be caused by such capital rationing. Alternatively it is possible to scale the arbitrage price reaction to many times the normal to reflect a flush capital situation. This is achieved by using a scaling factor to represent the strength of arbitrage (SOA). If the scaling factor is set at 0.5 the arbitrage price reaction would be one half the base or normal situation. This would represent a weak price response. At a strength of arbitrage of 2.0 the arbitrage price response would be doubled to twice the norm. A strength of arbitrage of 1.0 would represent the base case. All three situations are investigated in this simulation experiment.

Formulating Trading Delays

Specific market microstructural issues in the form of trading delays are introduced into this simple arbitrage experimental system. In Figure 9B, the flowchart incorporates a trading delay in the stock market leg of the arbitrage transaction. The behavior of the arbitrage system with stock market delays can then be compared with the base case in the absence of delays. The differential delay in the stock market leg is increased in small time increments and the system is simulated repeatedly under different delay conditions.

The Justification for Delays. Sufficient evidence exists to show that it takes longer to trade a basket of stocks in the stock market as opposed to the futures market. The futures leg requires one transaction while the stock market leg would require many more. Stoll and Whaley (1990) conclude that the S&P 500 futures returns leads the cash index by as much as ten minutes. The reasons relate to the different microstructure of the two markets. The futures market uses the open outcry system of trading whereas the stock market is a specialist based system. Information is quickly incorporated in futures prices even if it calls for discontinuous price changes. In the stock market, on the other hand, the specialist's role is to cushion shocks and smooth price changes when necessary by taking the opposite side of the transaction[3]. According to Miller (1990) this delayed price response causes problems because the specialist is forced to take the losing end of every arbitrage transaction. This is always the case since the arbitrageur locks in a profit when making the trade.

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[3] Report of the Committee on Market Volatility and Investor Confidence (1990). New York Stock Exchange Publication.

In this context Miller (1990) notes: "That the **slowness** (emphasis added) in adjusting quotes on the NYSE is the real source of pain inflicted by arbitrage is suggested by the absence of complaints about intermarket arbitrage in the many other futures and options markets currently operating. When a government bond dealer looks at the appropriate window in his quote screen and sees that the futures price has fallen significantly below his own quotes, he doesn't wait around for the arbitrageurs to arrive. He immediately marks down his quote." (p.63)

The existence of the uptick rule in the stock market and its absence in the futures market is yet another microstructural difference that induces a stock market delay. In this case the delay is asymmetric since it limits price declines and not price increases. The uptick rule restricts short sales on a stock below the price at which the last sale was affected. The existence of the uptick rule has been cited by Edwards (1988) as one factor that exacerbated the stock market crash of 1987 since the appropriate arbitrage action at that time would have required the purchase of the futures contract and the sale of the underlying basket of stocks.

The asymmetric delay caused by the uptick rule is explicitly introduced into the simulation model. The response of the arbitrage system to the introduction of the uptick delay is then investigated.

The complete program code in DYNAMO including the feedback equations are set out as appendices to this essay. An explanation of important sections of the program code is also included. All the simulation results reported in this essay can be reproduced by anyone with access to the PROFESSIONAL DYNAMO simulation software. Each of the different microstructure features investigated requires a different program. Consequently the appendices contain the DYNAMO code for an arbitrage model without delays; with stock market delays; and with the uptick rule combined with stock market delays.

Arbitrage System Simulation Results Using Dynamo

The Dynamo model of the arbitrage system is initialized with a cash index value of 100, a corresponding futures price of 106, an interest rate of 10% and a dividend rate of 3%. Given these initial values the model computes the initial mispricing and the arbitrage reactions begin. Dynamo keeps track of the number of trading intervals (minutes) taken for the system to reach equilibrium at which time the arbitrage responses cease. At equilibrium mispricing will lie within the transaction cost bounds.

Experimental Simulation Results without Stock Market Delays

The plot of mispricing against transaction time is shown in Figures 11 and 12, for strength of arbitrage set to 0.5, (weak) and 1.0 (normal) respectively. There are no transaction delays. As can be observed in both cases the system quickly reduces mispricing to the nonarbitrage range.

The robustness of the experimental system would depend upon its response to repeated disequilibrium situations just as in the real world. In order to study the response of the simulation model to frequent mispricing, a predetermined shock term is introduced which shoots the level of mispricing above transaction cost bounds every

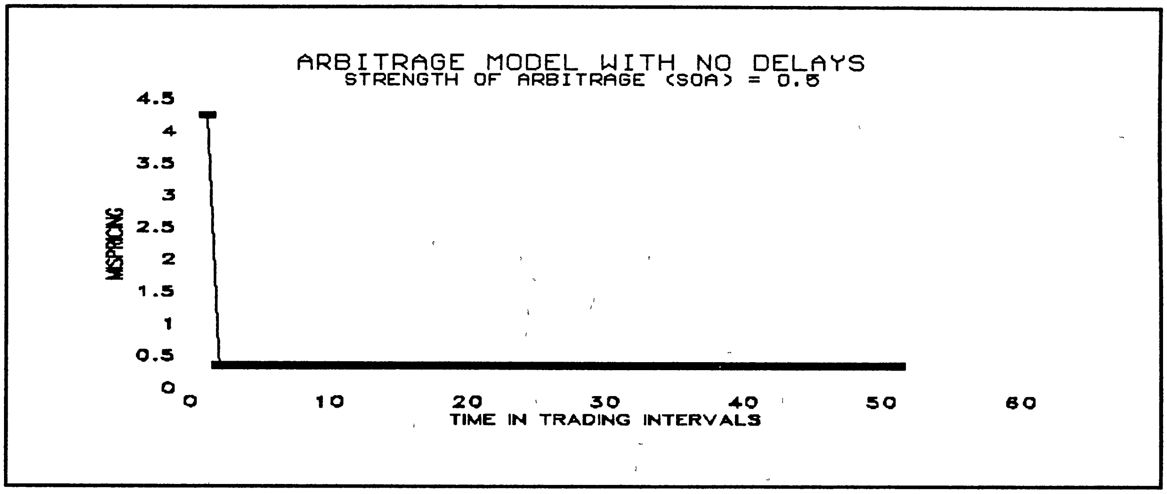

Figure 11. Mispricing with Weak Arbitrage Response

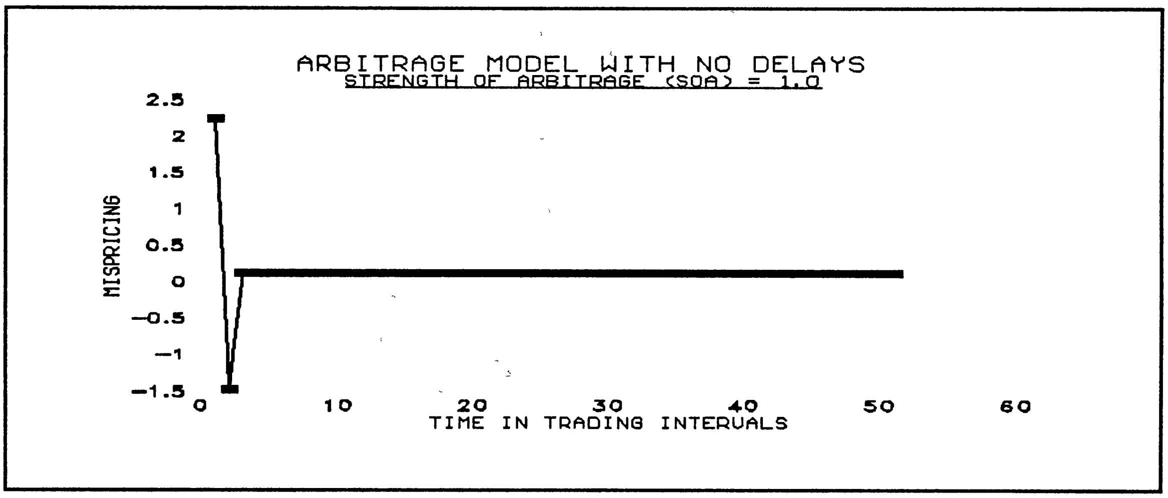

Figure 12. Mispricing with Medium Arbitrage Response

20 minutes. The model's response is plotted in Figure 12 for the slowest arbitrage price reaction (SOA = 0.5). Once again the arbitrage system quickly moves mispricing to the nonarbitrage bounds. The robustness of the simulation model in terms of its response to repeated destabilization is evident from the speed of recovery.

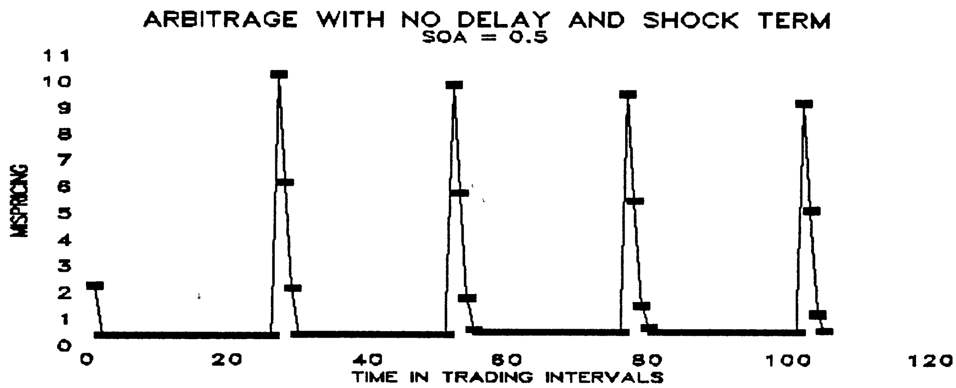

Figure 13. Test of the Simulated Arbitrage System

Experimental Simulation Results with Stock Market Delays

Knowledge that the arbitrage system responds as it should to index futures mispricing makes it is possible to investigate the effect of differential delays upon the arbitrage system. To do so, three levels of delays are introduced; a delay of 3 minutes, of 5 minutes, and of 10 minutes. The selection of these delay parameters are based upon Stoll and Whaley's (1990) observation of up to a ten minute lag in stock market returns. The system response at these delays with SOA = 0.5 is graphed in Figures 14, 15, and 16. As can be seen erratic changes in the mispricing levels are introduced. The mispricing levels overshoot and it takes longer to reach equilibrium. With a delay of 3 minutes (Figure 14) the system takes about 15 minutes to stabilize with two overshootings. As the delay is increased to 5 minutes (Figure 15) the magnitude of the oscillations increase and the system takes 20 minutes to stabilize. The overshootings increase to three with a delay of 10 minutes (Figure 16) and it now takes the system a little more than 50 minutes to stabilize. The erratic moves in the path of the mispricing series are clearly caused by the delayed changes in the cash index price because the system had adjusted speedily in the absence of delays.

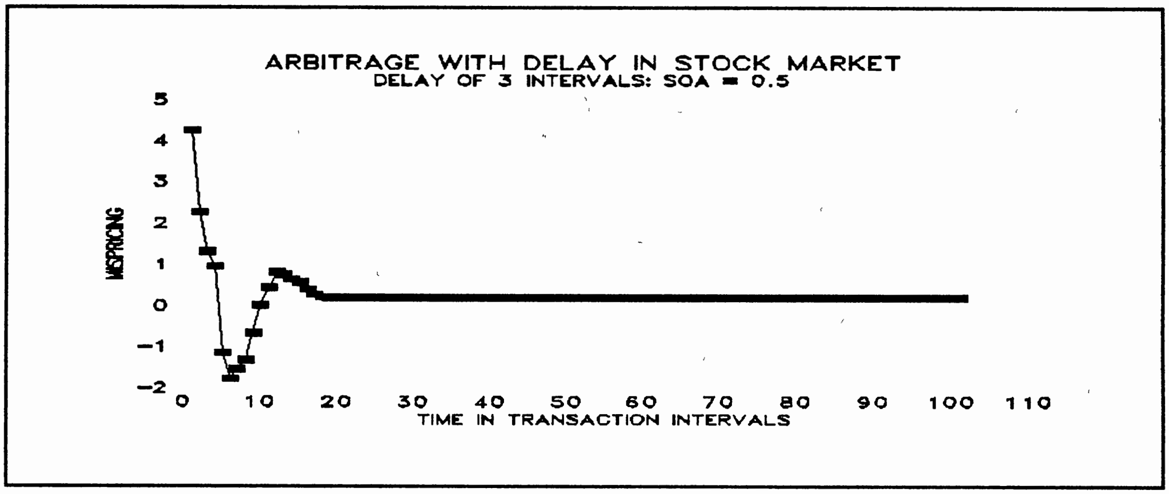

Figure 14. Simulated Mispricing Series with Three Minute Trading Delay

The market scenario that would correspond to these observations may be as follows. On observing the initial level of mispricing arbitrageurs initiate the appropriate arbitrage trade. Under normal circumstances without delays the system would have quickly stabilized within one or two minutes. However, with a delay in the stock market, only the futures leg of the arbitrage trade is captured in the price and the mispricing response is therefore partial and not complete. Observing the continued mispricing, more arbitrage trades are initiated. This is especially likely with many arbitrageurs in the market who have no way of knowing the number of trades already in the pipeline. By this time, however, the delayed stock market reaction kicks in and the mispricing level overshoots.

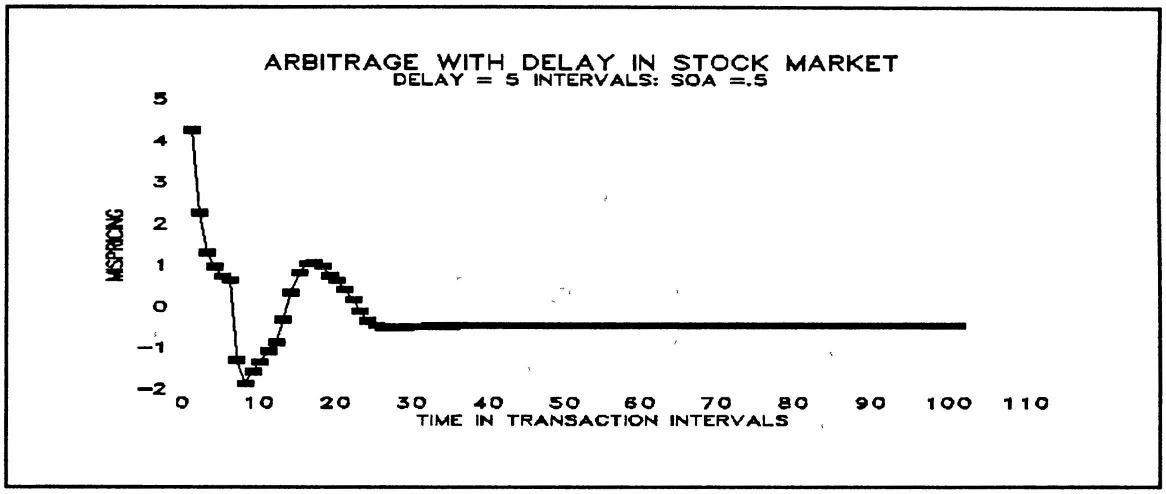

Figure 15. Simulated Mispricing Series with Five Minute Trading Delay

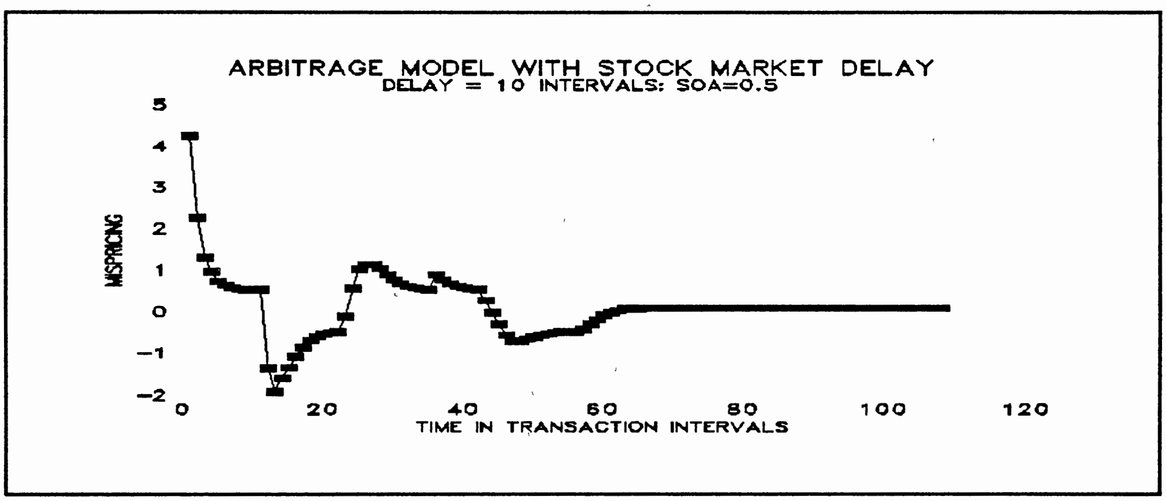

Figure 16. Simulated Mispricing Series With Ten Minute Trading Delay

Figure 17 graphs the results of a simulation run using the same delay parameter (3 minutes) with the strength of arbitrage set at 1.0 (the normal case). Increasing the strength of arbitrage shifts the system into a repeated cycling mode.

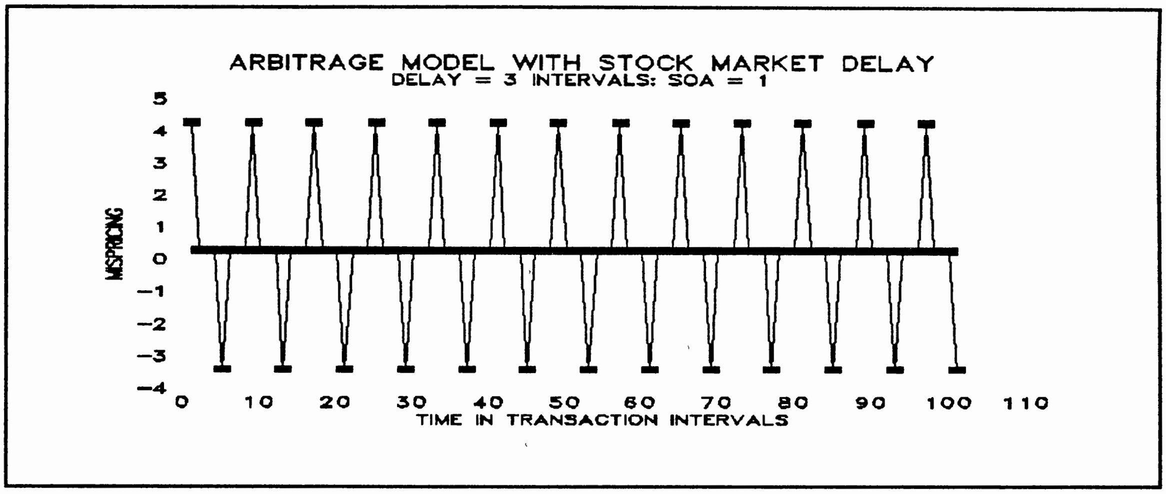

Figure 17. Cycling Behavior in the Simulated Mispricing Series

This is reminiscent of the Fieganbaum bifurcation scenario (shown earlier) that results when select parameters are increased in a nonlinear systems. As can be observed from the plots, the system initially drives the mispricing within transaction bounds. Within a few minutes the delayed stock price reaction hits the market and the mispricing level overshoots. The situation is fundamentally the same for delays of 5 and of 10 minutes and for that reason are not shown.

<u>Experimental Simulation Results with Uptick Rule</u>

Introducing the uptick rule as an additional feature of the market microstructure, in combination with trading delays, brings about a very different time path for the mispricing series. Figures 18, 19,and 20 show the system response to an asymmetric uptick delay of 5 minutes in addition to a stock market trading delay of 10 minutes. With a strength of arbitrage set at half the base case (SOA=0.5), the system goes through three overshootings and takes seventy minutes to stabilize

(Figure 18). When the strength of arbitrage is increased to 1.0 (Figure 19), the system reaches equilibrium after 5 overshootings and takes more than 80 minutes to stabilize. When the strength of arbitrage is doubled to twice the base case (SOA = 2), the system shifts to a chaotic continuous disequilibrium (Figure 20). Here the time path of the mispricing levels looks random despite the fact that this is a perfectly determined system. It must be emphasized that the model does not use any random or stochastic inputs. The chaotic fluctuations are solely caused by the endogenous system structure, specifically by the uptick delay in conjunction with a powerful arbitrage generated price response.

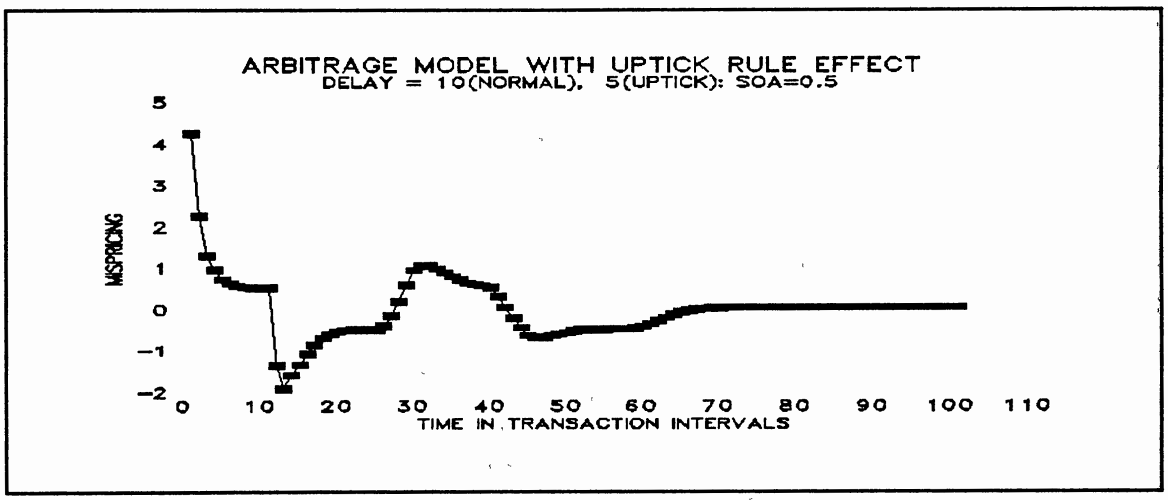

Figure 18. Simulated Market with Uptick Rule and Weak Arbitrage Responses

It may be argued that the delay caused by the uptick rule would not be constant but would vary depending upon the extent of short selling pressure in the stock market. Thus at times such as during Black Monday 1987, the uptick delay would have been very large due to arbitrage related stock sales whereas it would not

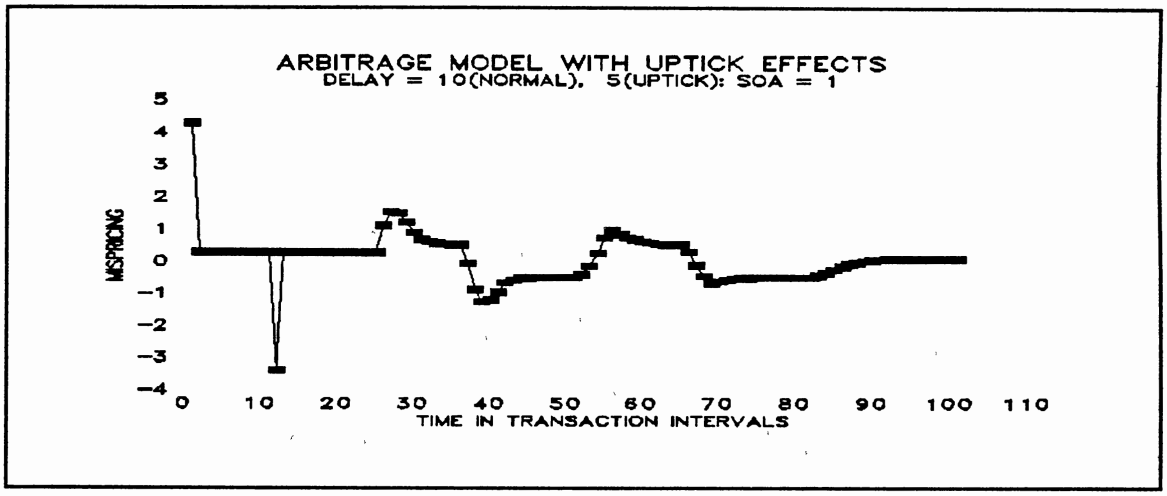

Figure 19. Simulated Market with Uptick Rule and Medium Arbitrage Response

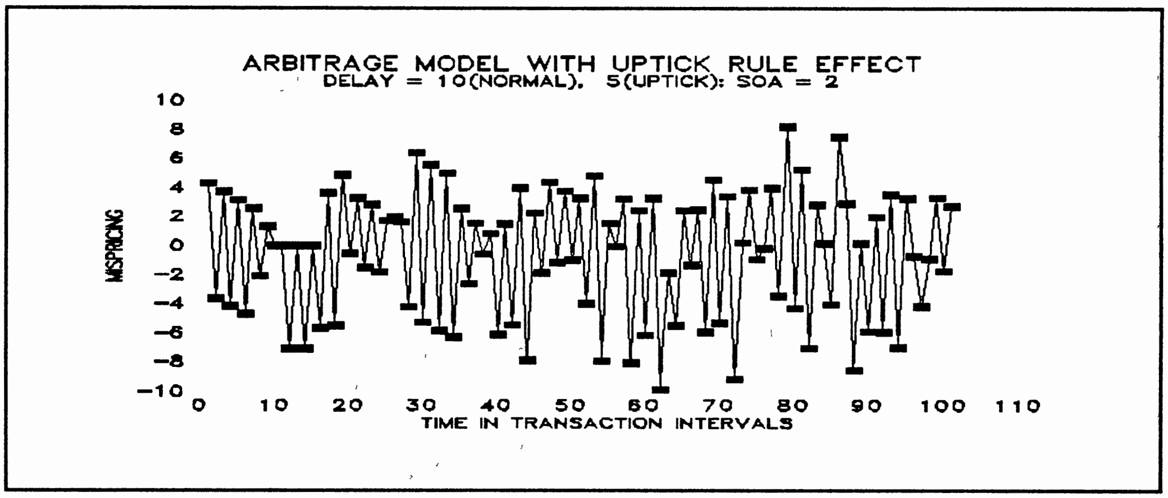

Figure 20. Chaotic Mispricing in the Simulated System

be significant during normal trading days. To investigate such a scenario the system is reconfigured with a variable uptick delay whose magnitude varies with the extent of selling pressure in the stock market. Table 7 shows the parameters of the uptick delay which ranges from a low of 3 minutes associated with a 1 percent decline in the cash index to a high of 30 minutes with a stock index fall of 10%.

TABLE 7

VARIABLE DELAY DUE TO UPTICK RULE

% PRICE DECLINE	1	2	3	4	5	6	7	8	9	10
UPTICK DELAY	3	6	9	12	15	18	21	24	27	30

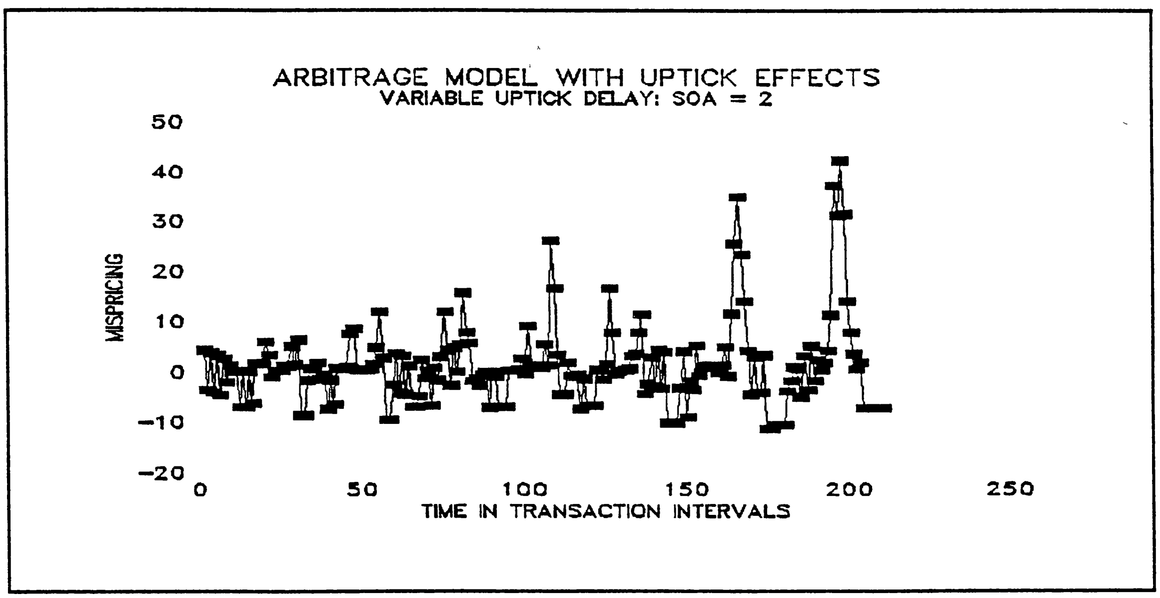

Figure 20. Chaotic Mispricing With Variable Uptick Delay

The outcome with a variable uptick delay is shown in Fig. 20. The strength of arbitrage is maintained at twice the base case (SOA = 2). The time plot looks chaotic with significant peaks and very high mispricing levels, thus demonstrating that even in an otherwise equilibrating arbitrage system trading delays can generate continuous disequilibria.

Such large levels of mispricing in the real system would lead observers to the conclusion that the market is inefficient and/or that market players are irrational. This simulation experiment shows that large mispricing can occur even with perfectly rational arbitrageurs possessing sufficient capital for arbitrage (Note: SOA = 2 in the previous simulation run).

The observed mispricing in this experimental model is a consequence of differing microstructure of the two markets. When arbitrage links two dissimilar markets with different response times, the resulting mispricing innovations can appear chaotic and show all the puzzling characteristics of an inefficient market with untapped potential for supernormal arbitrage profits.

Validating the Simulation Experiment

Despite its obvious simplicity this model of the arbitrage process succeeds in capturing some of the rich patterns in the empirical mispricing series. Figure 22 reproduces the December 1984 mispricing series for purposes of comparison. The simulation model demonstrates how overshootings can occur in an arbitrage system that consists of two negative feedback loops. Such overshootings of transaction cost bounds have been identified by MacKinlay and Ramaswamy (1988) and can be observed very clearly in their plot of the December 1984 mispricing series as well.

TABLE 8

AUTOCORRELATION PATTERNS IN OBSERVED
AND SIMULATED DATA

MISPRICING DATA	MEAN	SD	LAG1	LAG2	LAG3	LAG4	LAG5
March 1987	-0.02	0.21	0.65	0.58	0.60	0.59	0.56
June 1987	-0.11	0.22	0.46	0.34	0.31	0.27	0.25
SIMULATED DATA							
Uptick: SOA =.5	-0.03	0.79	0.74	0.57	0.45	0.34	0.26
Uptick: SOA = 1	0.04	0.58	0.45	0.38	0.31	0.25	0.19

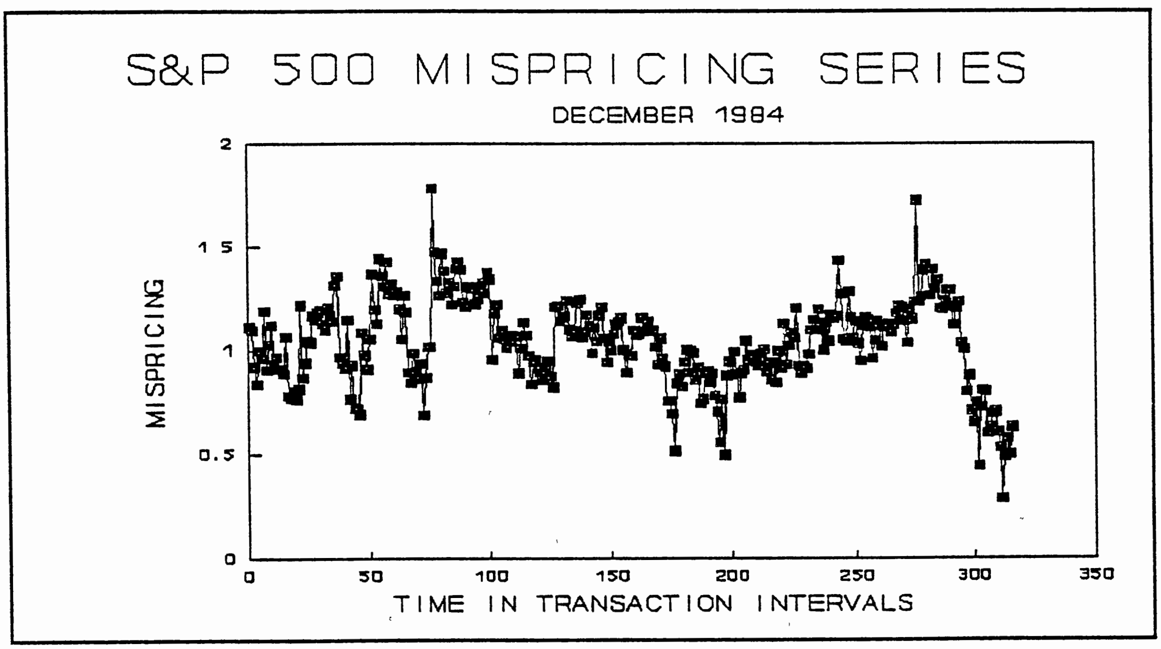

Figure 21. Plot of MacKinlay and Ramaswamy (1988) Mispricing Series

Yet another feature of the mispricing series that has generated interest among researchers is the presence of strong serial correlation patterns in the data. This conflicts with the notion of efficient markets and the resulting random walk price process. The simulation runs identify a possible cause by showing how delayed price adjustments can bring about path dependence in the mispricing series. Table 8 reproduces these autocorrelation patterns for the June 1987 mispricing series and two of the runs with uptick delays for purposes of comparison. All the autocorrelation coefficients are significant at the 5 percent level. As can be observed the simulated data exhibit very similar path dependence in the form of autocorrelation patterns.

To what extent should an experiment replicate the real world data series before researchers can gain confidence in the simulation model? In the presidential address to the Southern Economic Association, Charles Plott (1991) addresses this very complicated issue and discusses the proper role of such experimental simulation in the context of market economies. The following quote from his paper serves as an especially relevant and appropriate postscript to this essay:

"This belief suggested that the only effective way to create an experiment would be to mirror in every detail, to simulate so to speak some ongoing natural process. Early experimenters were guilty of yielding to this belief and described experiments as simulations of a market or attempted to include in their experiments much of the rich and complicating details found in many markets. As a result the experiments tended to be dismissed either because as simulations the experiments were incomplete or because as experiments they were so complicated that tests of models were unconvincing. ...Once models as opposed to economies became the focus of research the simplicity of an experiment and perhaps even the absence of features of more complicated economies became an asset. The experiment should be judged by the lessons it teaches about theory and not by its similarity with what nature might happen to have created." (p. 906)

CHAPTER IV

SUMMARY AND CONCLUSIONS

There have been many findings in recent years of regularities and long term serial dependence in financial time series. Such findings have contradicted the assumption of market efficiency. The nature and form of such regularities are of interest to financial researchers everywhere. In fact the large amounts of money spent by investment institutions on market analysis bear testimony to this. The science of nonlinear dynamics can provide some insight into such questions. In particular, it can be demonstrated that a time history of a nonlinear function (F) of the form,

$$Y(t) = F[Y(t-1), Y(t-2), \ldots Y(t-i)]$$

can appear quite random even though it is completely deterministic[1]. It has been hypothesized that the apparent randomness of economic and financial series may actually be caused by nonlinear relationships among the variables. This may help explain the tantalizing findings of regularities in financial data.

Researchers have identified nonlinearities and deterministic chaos in financial time series. However, a finding of low order dimension and nonlinear structure tell us nothing about the precise nonlinear functional form or even the general class of nonlinear relations among the variables under investigation. Nonetheless, the

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[1] May (1976) illustrates this using select nonlinear functions. So do Baumol and Benhabib (1990)

empirical finding of nonlinear dynamics in the mispricing series is of considerable value even without the specification of the precise nonlinear mathematical relationships. **This is so because market efficiency should imply the absence of any deterministic structure whatsoever.**

The first essay "Does the S&P 500 Mispricing Series Exhibit Nonlinear Serial Dependence?" finds evidence of low order determinism and nonlinearities in the S&P 500 mispricing series. The BDS statistic and its associated testing procedure were used for this purpose (Brock, Hsieh and LeBaron (1991)). In addition the paper identified some structural issues such as the differential delays in trading the index basket in the futures market and the cash market which when combined with nonlinear adjustment processes within the system can generate sustained and irregular fluctuations in mispricing levels. In an intense and fast trading system such wide fluctuations can potentially destabilize the market. A rigorous examination of the dynamics of the arbitrage process in a nonlinear environment with adjustment delays is necessary to validate this hypothesis. This is made the focus of the second essay.

The second essay titled "Market Microstructure Issues in the Mispricing of the S&P 500 Futures Contract" uses the system dynamics simulation methodology to represent the arbitrage linkage between the two markets. Trading delays are then introduced into the system and its effect on the path of index futures mispricing is analyzed. The simulation results demonstrate that introducing differential delays in an otherwise equilibrating system causes sustained fluctuations in mispricing levels. Increasing the strength of arbitrage in this system only serves to increase the fluctuations. This result, in some ways, is counter-intuitive for one would expect that a strong arbitrage response should bring about a quick adjustment. Yet in the presence of a an asymmetric delay caused by the uptick rule the strong arbitrage response is actually destabilizing. These conclusions support the findings of Blank (1990) and Scheinkman and LeBaron (1989) who find evidence of nonlinear structure in the S&P 500 futures and the cash index respectively. This experiment shows that nonlinearities in the mispricing series result from the negative feedback structure of the arbitrage system. The mispricing feeds back to the spot and futures price through arbitrage trades. Consequently both the spot and futures index prices would show evidence of nonlinearities as well.

TABLE 9

SUMMARY OF SIMULATION RESULTS

	MARKET DISSIMILARITY				
	DELAY 0 MIN	DELAY 5 MIN	DELAY 10 MIN	UPTICK 5 MIN	UPTICK VARIABLE
MARKET LINKAGE					
SOA = 0.5	2 TI* 0 OS	25 TI 3 OS	52 TI 3 OS	65 TI 3 OS	60 TI 4 OS
SOA = 1.0	2 TI 1 OS	1 TI CYCLE	1 TI CYCLE	85 TI 5 OS	1 TI CYCLE
SOA = 2.0	CYCLE	CYCLE	CYCLE	CHAOTIC	CHAOTIC

**TI represents the number of transaction intervals for the system to reach equilibrium.*
OS represents the number of overshootings of transaction cost boundaries during the adjustment process.

In Table 9, which summarizes the simulation results, the market dissimilarities are represented in the form of differential trading delays. An increase in the differential delay represented in the table columns implies greater market dissimilarity. The market linkages are represented in this simple model by the strength of arbitrage. An increase in the strength of arbitrage down the table rows represents stronger market linkage.

It can be observed from Table 9 that as dissimilarity increases for a given strength of arbitrage, the arbitrage system takes longer to reach equilibrium. With differential trading delays kept constant, the increase in market linkage in the form of strength of arbitrage initially reduces mispricing and then cycles regularly. With both the market dissimilarity and the market linkage at the highest, the mispricing series exhibits chaotic fluctuations. These simulation results are revealing in the sense that they show how irrational mispricing levels can occur even with perfectly rational arbitrageurs possessing adequate arbitrage capital.

Market linkages and the strength of arbitrage

In the language of system dynamics the ratio of the arbitrage price response to a change in mispricing (SOA) is known as amplification and is sometimes referred to as system gain. Various aspects of the financial markets (other than arbitrage capital) can bring about an increase in amplification of price responses over time. The preponderance of institutional traders with large amounts of trading capital and the use of computerized trading systems that allow them to commit large proportions of their funds almost at will leads to amplified price responses. It would appear therefore that the evolution of financial markets is towards greater amplification of price responses to information and not less. Other forms of positive feedback trading can also amplify price responses. Delong etal. (1990) discuss some forms of positive feedback trading that result in the amplification of trends. Examples are portfolio insurance, frontrunning, limit prices and some forms of technical trading techniques that look for and follow apparent price patterns. This results in the reinforcing of the very same patterns. Such self-reinforcing behavior is not a feature of trader behavior alone. Even structural features such as margin calls in a declining market can result in further price declines through forced liquidations. These are all forms of positive feedback that can amplify an arbitrage response and differential delays under such conditions become important factors in the adjustment of the system to an observed mispricing.

During the stock market crash the combination of arbitrage selling in the stock market and portfolio insurance sales would have caused a large amplification in the price response to an observed mispricing. This may be one reason why index arbitrage reaped much of the blame for the stock market crash of 1987. An expanded simulation model needs to be developed in order to investigate the impact of positive feedback trading on the arbitrage process. If the results of the current simulation are extrapolated to an experimental market that includes positive feedback motivations one could expect to observe chaotic fluctuations at even lower strengths of arbitrage price response.

Policy Implications of Simulation Experiment

These simulation results suggest some guidelines for government policy relating to the stabilization of financial markets. The focus of regulatory response in the form of curbs on program trading, circuit breakers and the uptick rule serves to reduce the extent of market linkage. In terms of the simulation model, this would imply a reduction in the strength of arbitrage. One consequence of this policy is that the markets may at times be completely delinked. This would undermine the very purpose of a derivative market. The futures market serves as a hedging vehicle and delinking the market shortcircuits this very fundamental purpose. Moreover, the simulation results as seen in Table 9 indicate that mispricing for extended periods can occur even at low strength of arbitrage under conditions of differential trading delays.

This simulation experiment brings out the implications of market dissimilarities that result in differential trading delays. If structural differences are indeed a potential cause of market instabilities, then the appropriate policy action should not rely upon different uptick rules and different restrictions on program trading for the cash and futures markets. These only serve to lengthen the differential trading delay in the futures and the cash market. The emphasis of regulatory effort should instead be on reducing the structural differences between the two markets by speeding up trading in the stock market as opposed to slowing it down. These conclusions are in line with the free market position on market regulation and can be used to validate this policy stance.

Of course the appropriate caveats in terms of the limitations of this simulation model should be applied before extending the simulation results to the real world stock and futures markets. This model is limited to the arbitrage process and does not include the wide range of trading motivations represented in the financial markets.

There is no information arrival in this experimental system and microstructural features other than the trading delays have been ignored.

On the other hand, one should recognize that no simulation model can ever completely mirror reality. This is in some ways an advantage. Simulation allows the researcher to focus on specific structural issues and conduct an experiment under ceteris paribus conditions without being bogged down by the complexity of the system. The second essay is designed as such a simulation experiment that focuses on a narrow aspect of market microstructure.

This dissertation makes a contribution by linking the disparate research areas of nonlinear dynamics and market microstructure. It does so by identifying a specific market feature that has the potential to generate bifurcations and chaos in the arbitrage system.

Directions for Further Research

The simulation model developed in this dissertation could be used to examine other aspects of market microstructure. It would be possible to examine the impact of alternate trading motivations on the extent of index futures mispricing. These could include positive feedback trading behavior in the form of portfolio insurance strategies.

An empirical study that compares the extent of mispricing in different pairs of linked markets that do not bear the same extent of microstructural differences as the New York Stock Exchange and the Chicago Mercantile Exchange would further test the conclusions brought out in this dissertation. An examination of index futures mispricing in the London Financial Times Stock Index before and after the Big Bang in 1986 would offer such an opportunity.

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APPENDIXES

APPENDIX A

DYNAMO GRAPHS OF SIMULATION MODEL

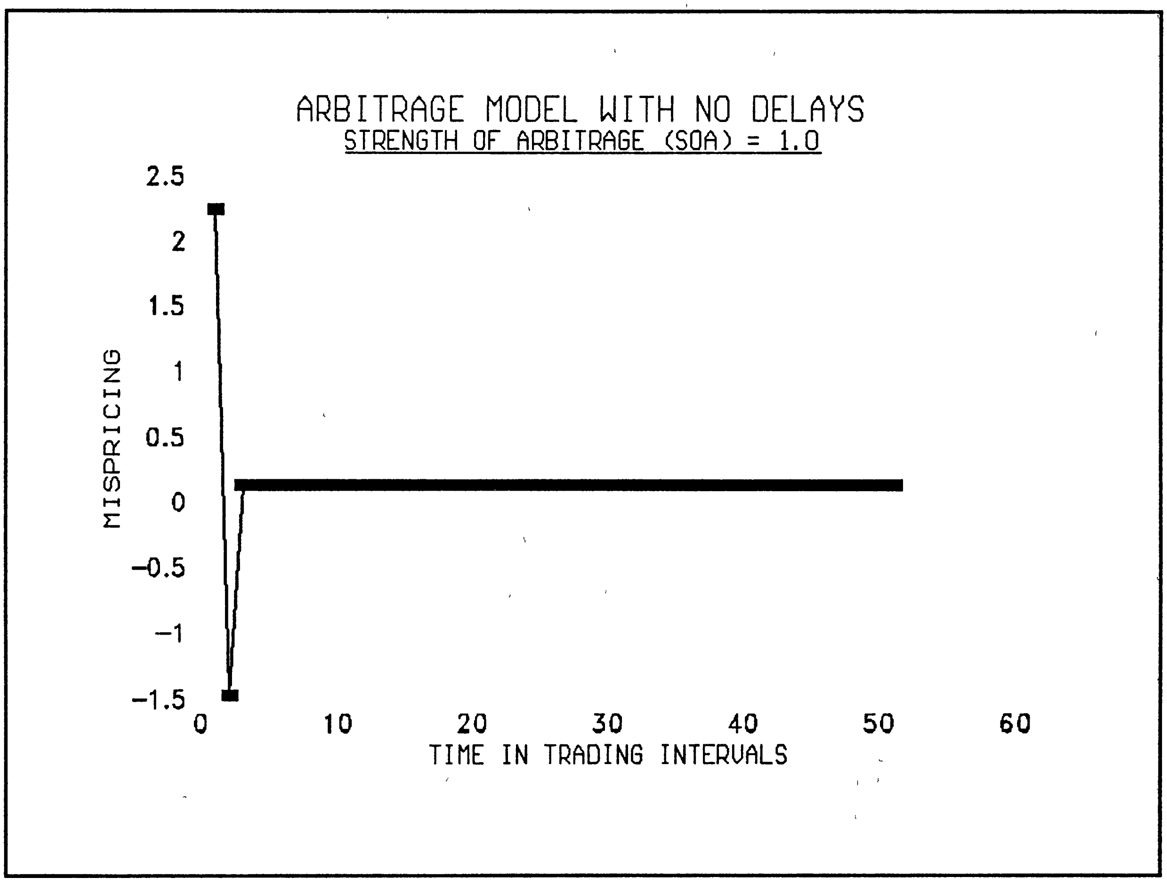

Figure 23. Base Case Strength of Arbitrage with No Delay

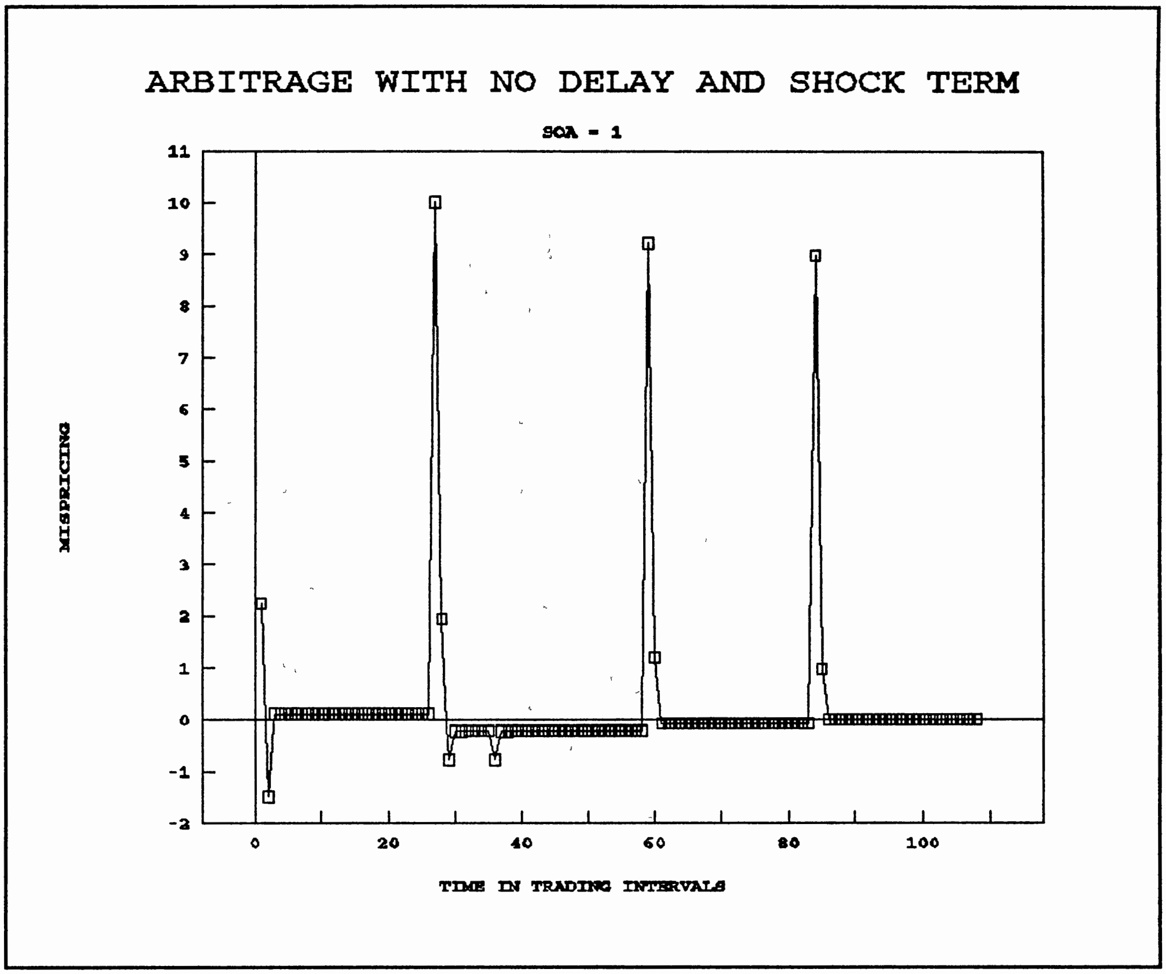

Figure 24. Test of System with Base Level Arbitrage Strength

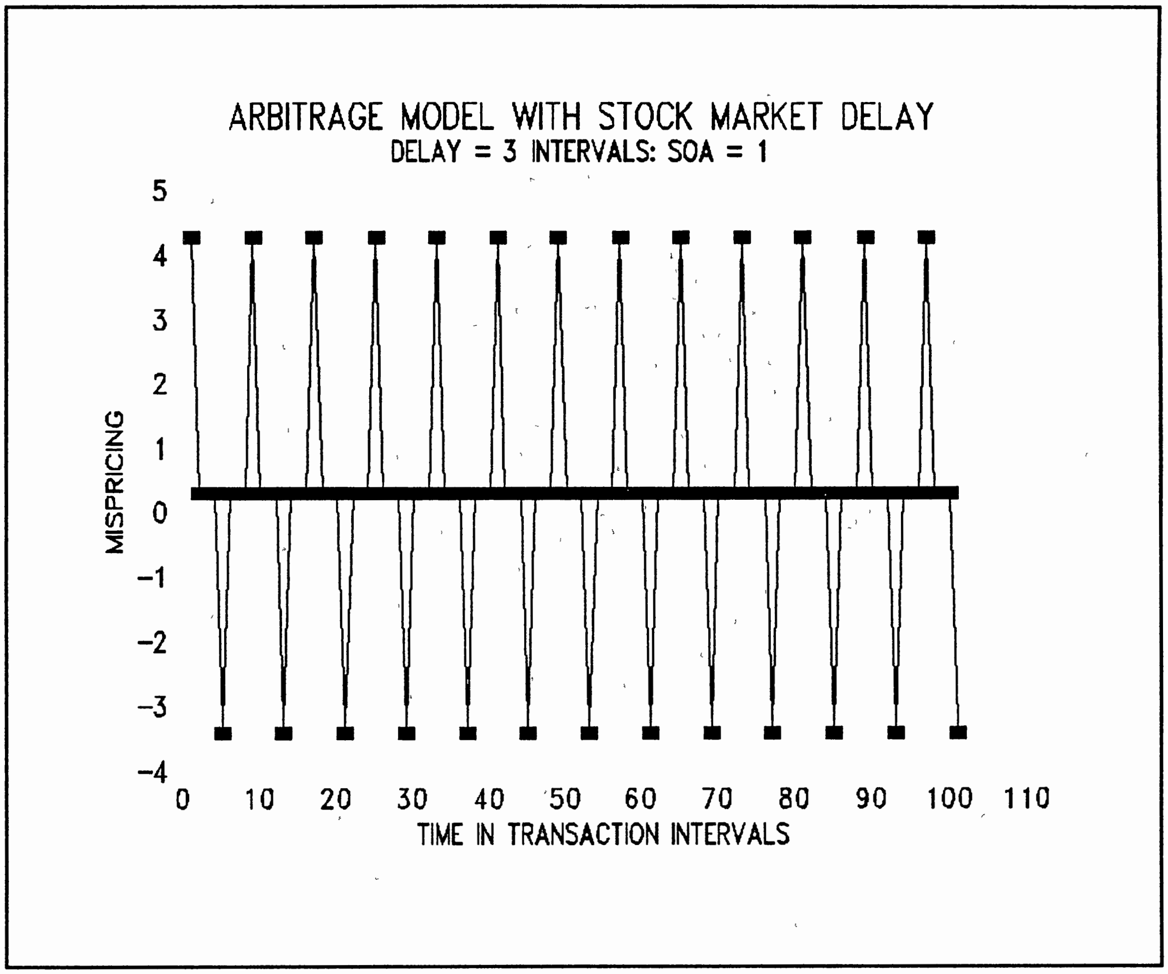

Figure 25. Base Level Arbitrage Strength with Three Minute Delay

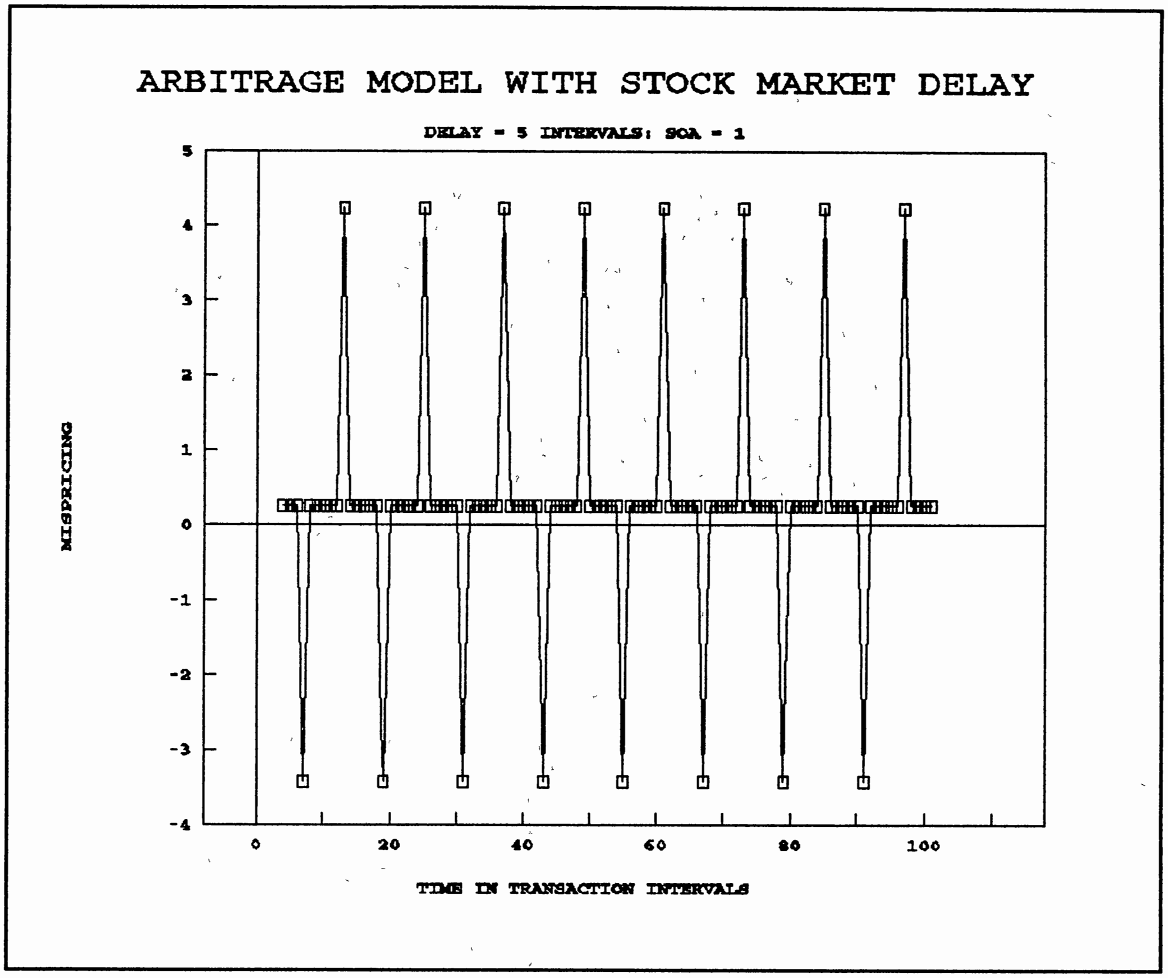

Figure 26. Base Level Arbitrage Strength with Five Minute Delay

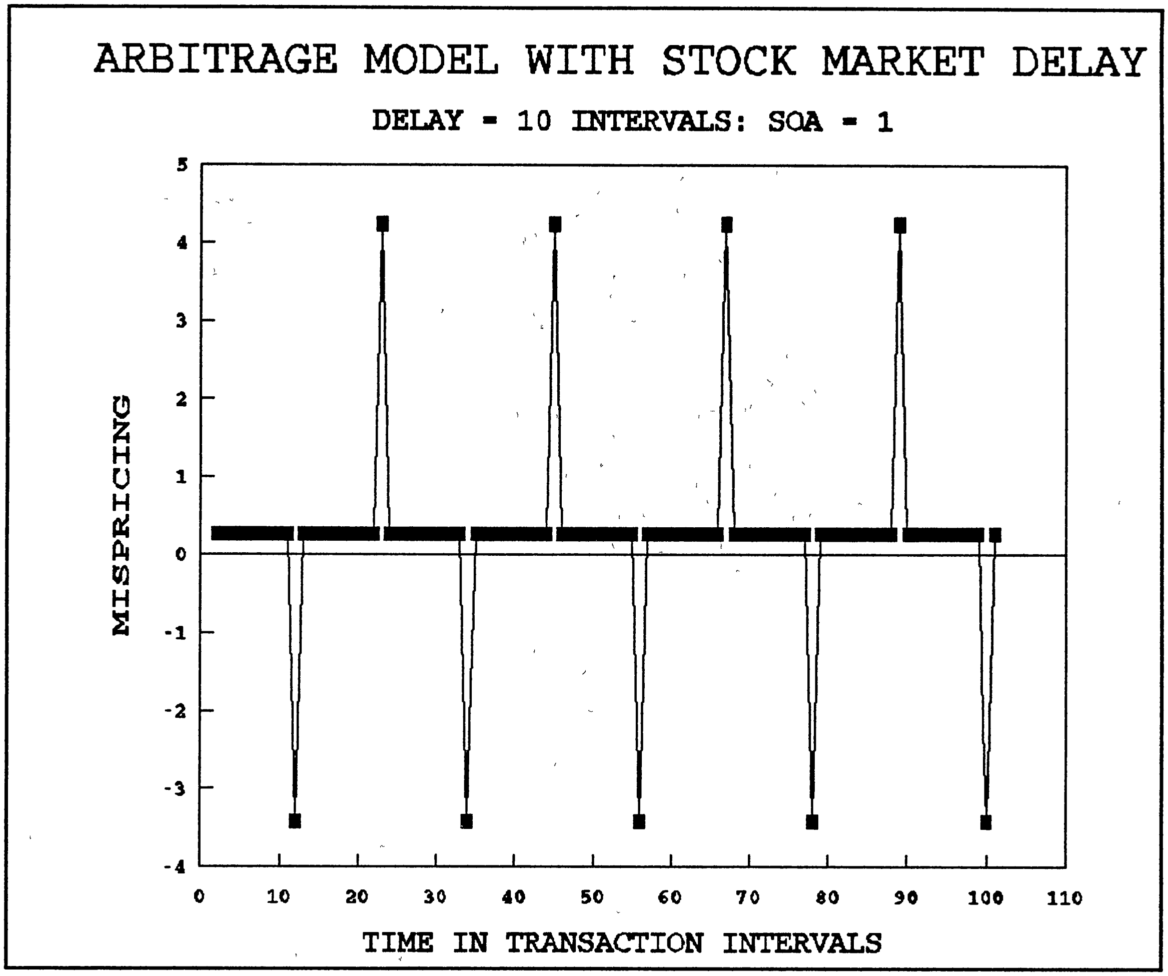

Figure 27. Base Level Arbitrage Strength with Ten Minute Delay

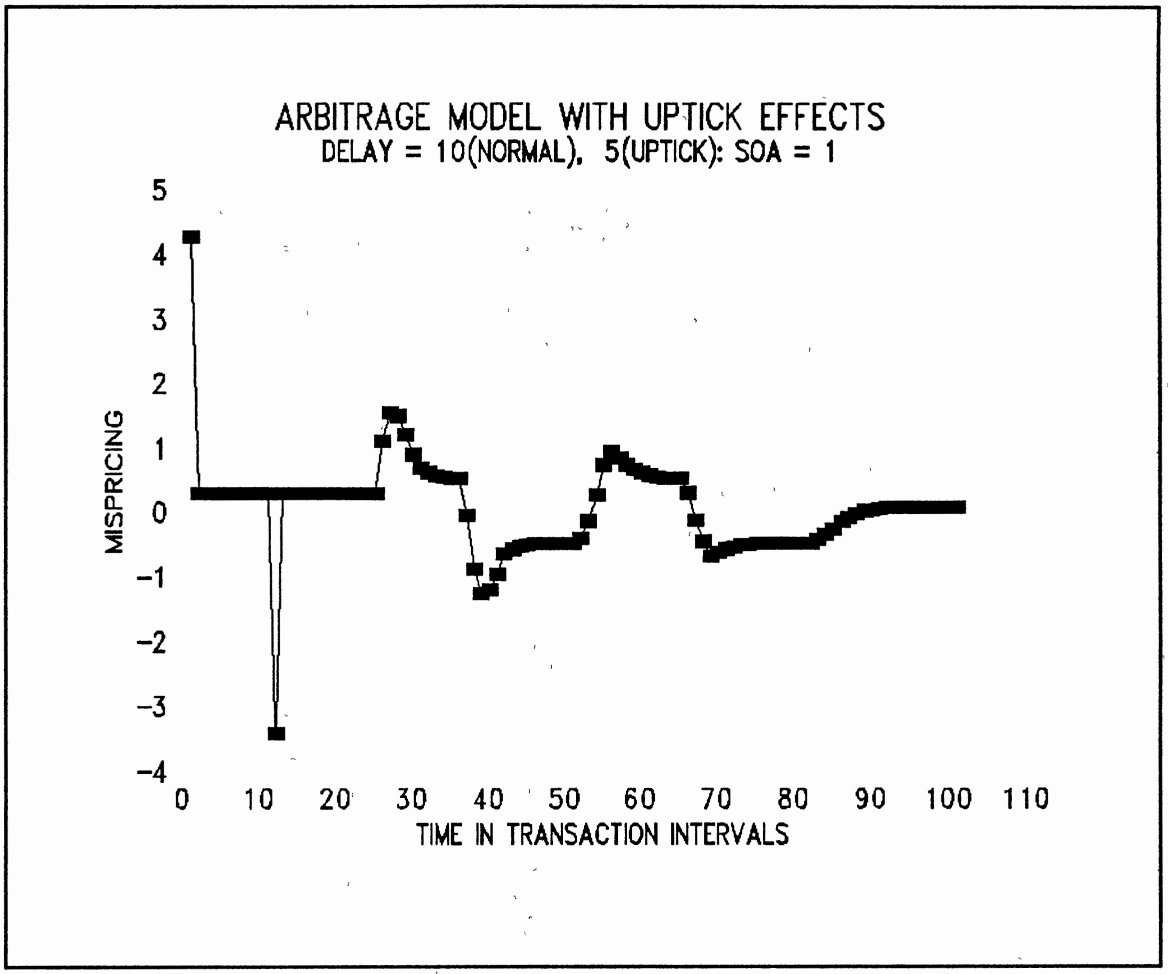

Figure 28. Base Level Arbitrage Strength with Uptick Delay

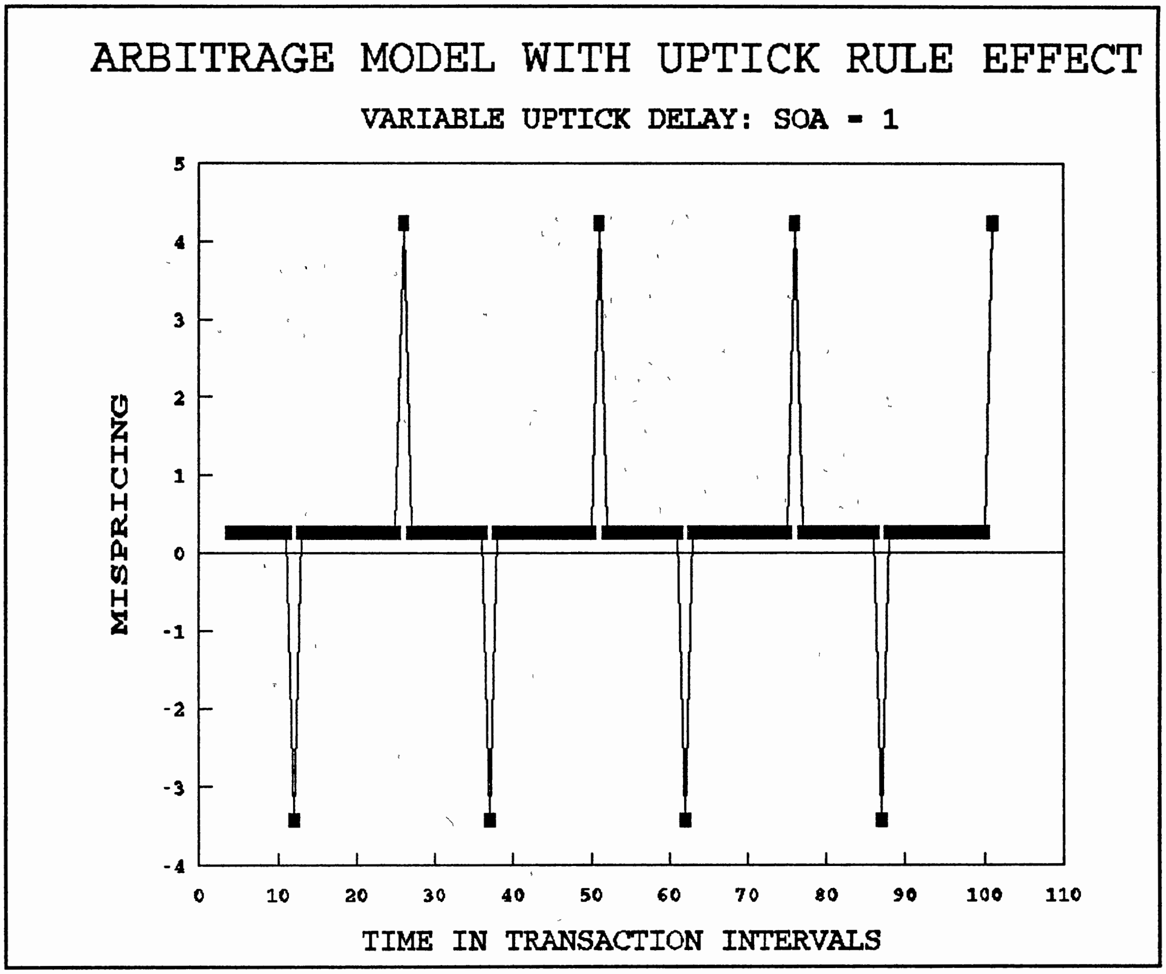

Figure 29. Base Level Arbitrage Strength with Variable Uptick Delay

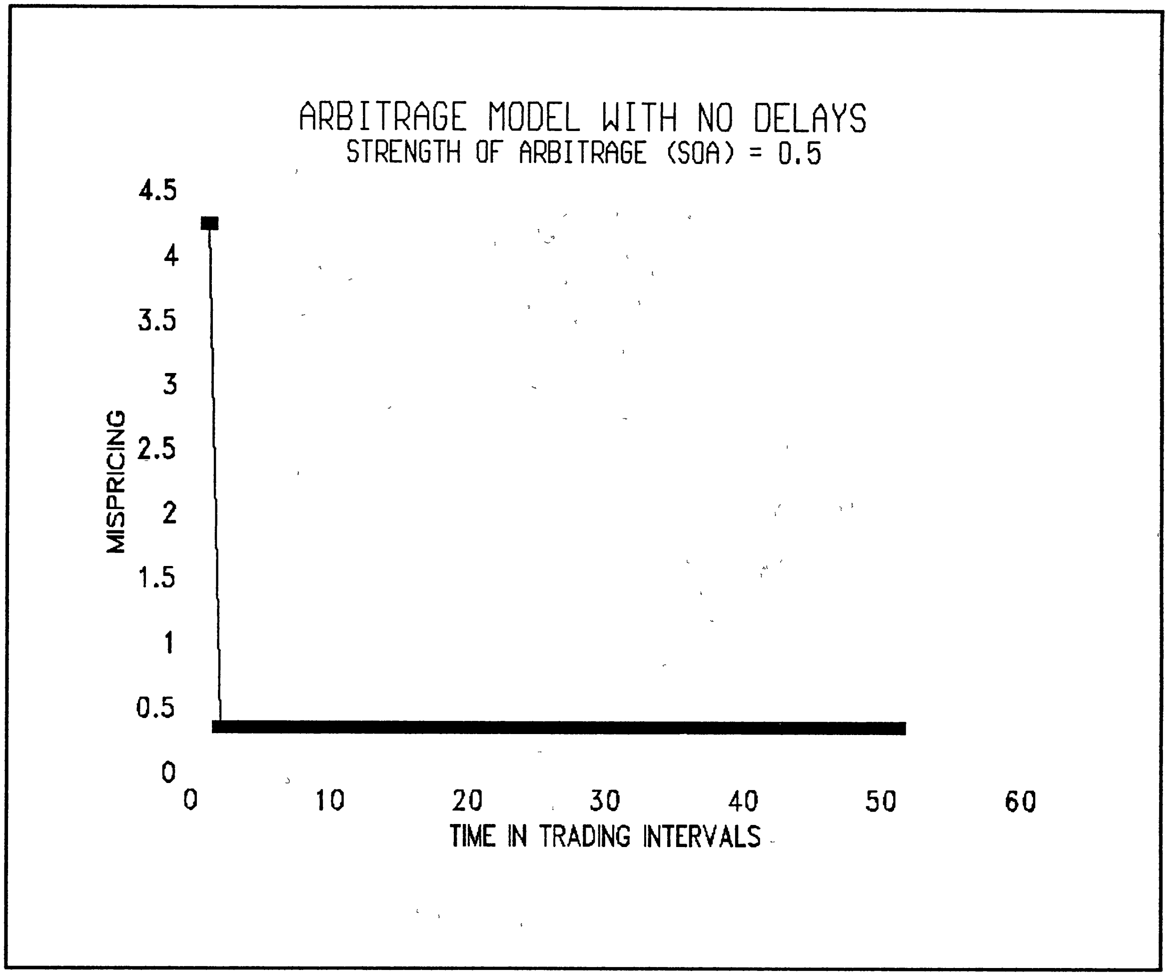

Figure 30. Weak Arbitrage Strength with No Delay

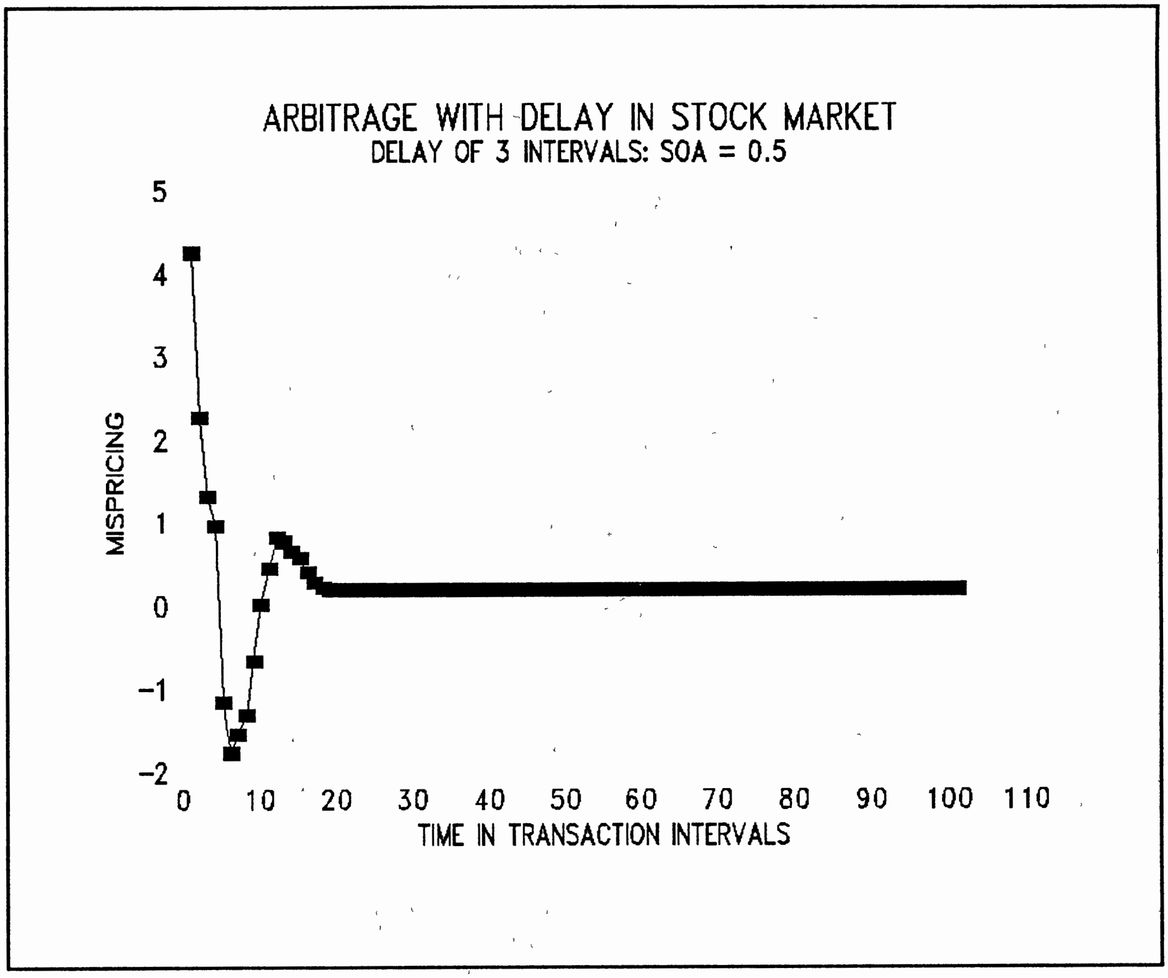

Figure 31. Weak Arbitrage Strength with Three Minute Delay

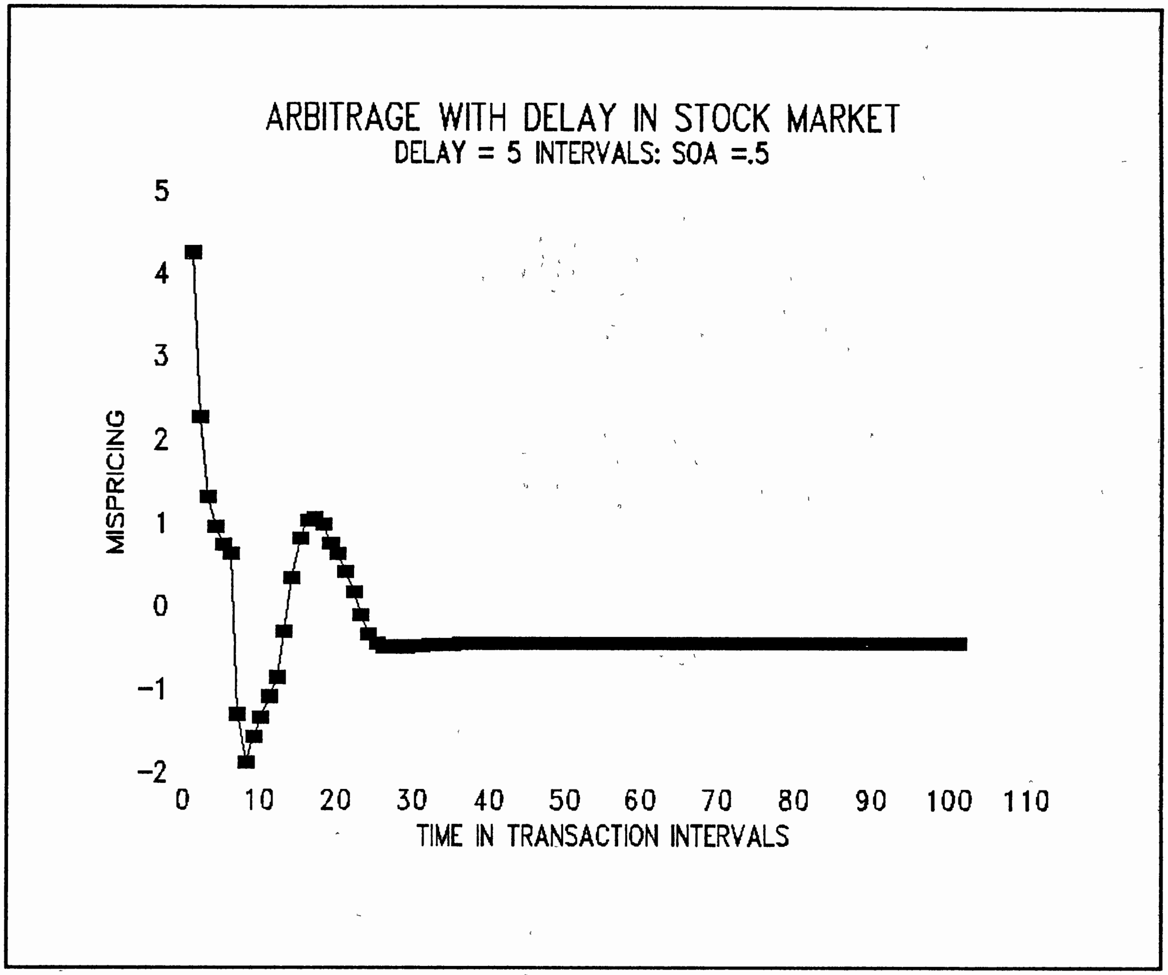

Figure 32. Weak Arbitrage Strength with Five Minute Delay

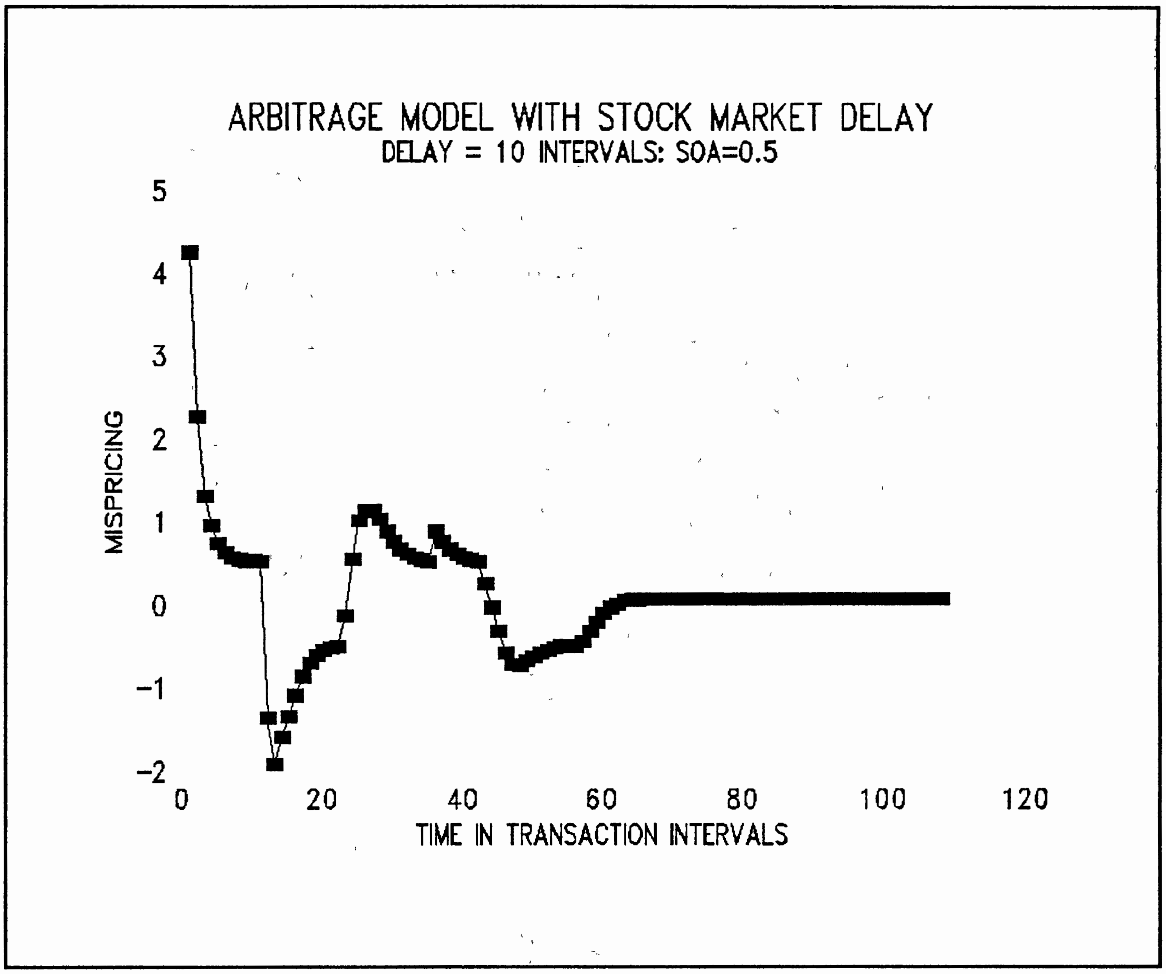

Figure 33. Weak Arbitrage Strength with Ten Minute Delay

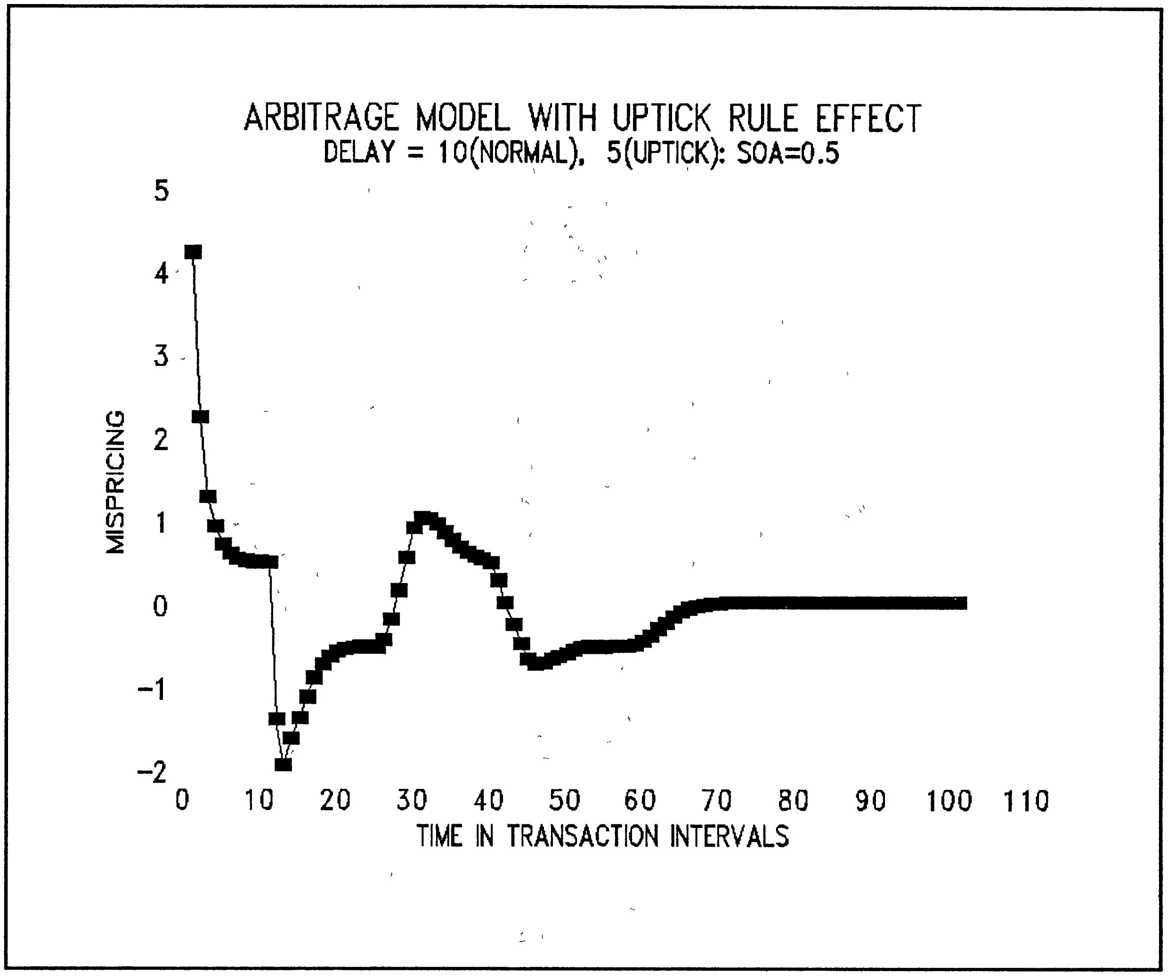

Figure 34. Weak Arbitrage Strength with Uptick Delay

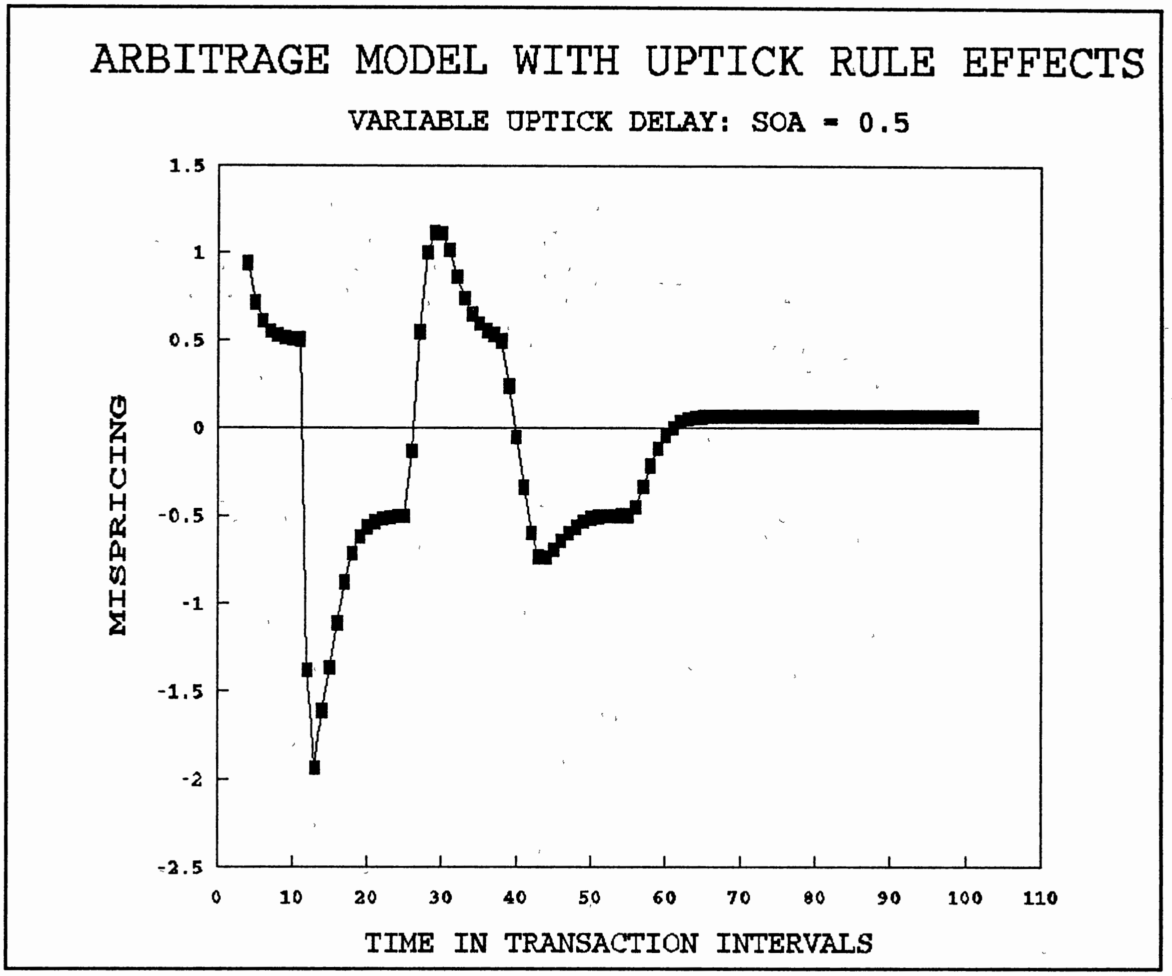

Figure 35. Weak Arbitrage Strength with Variable Uptick Delay

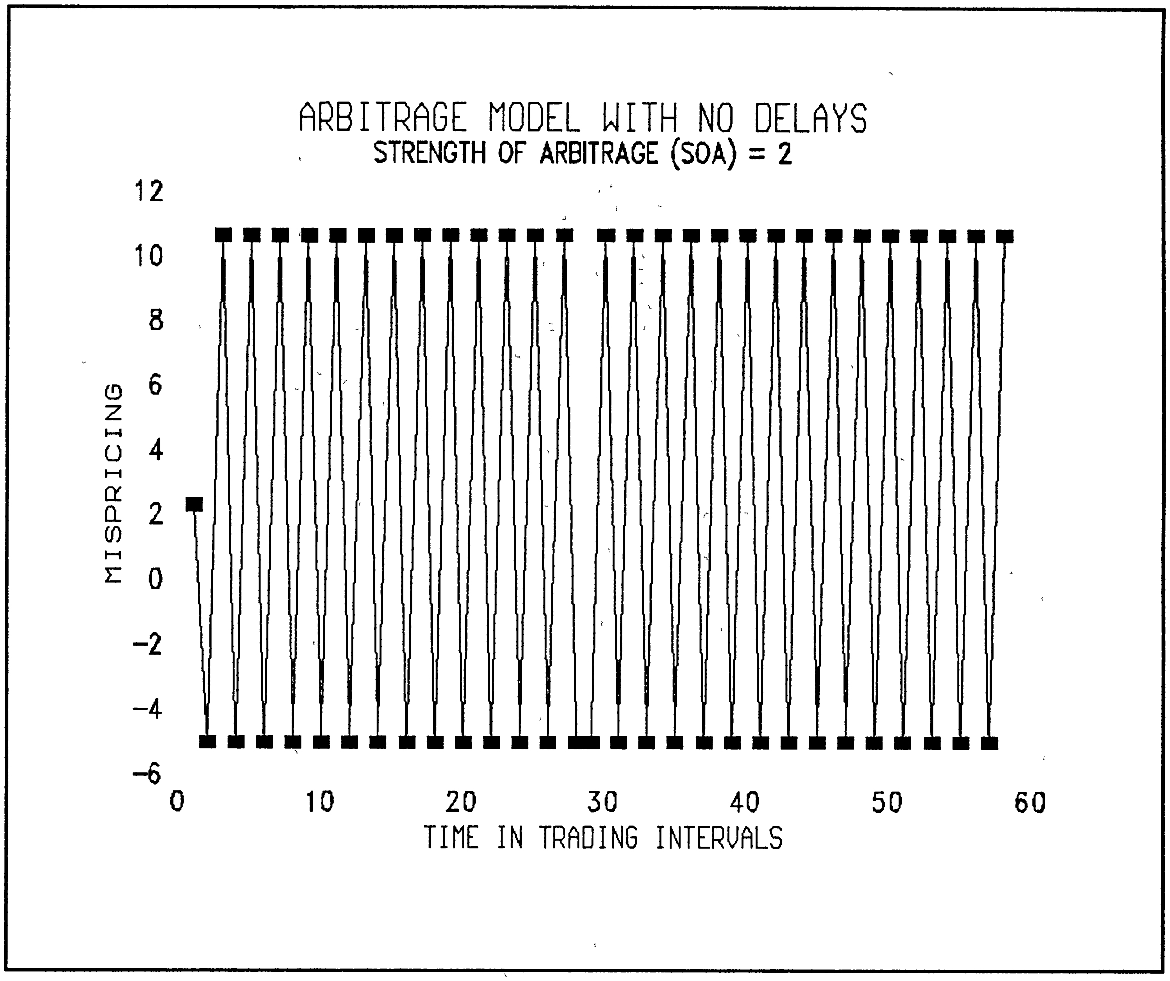

Figure 36. Strong Arbitrage Strength with No Delay

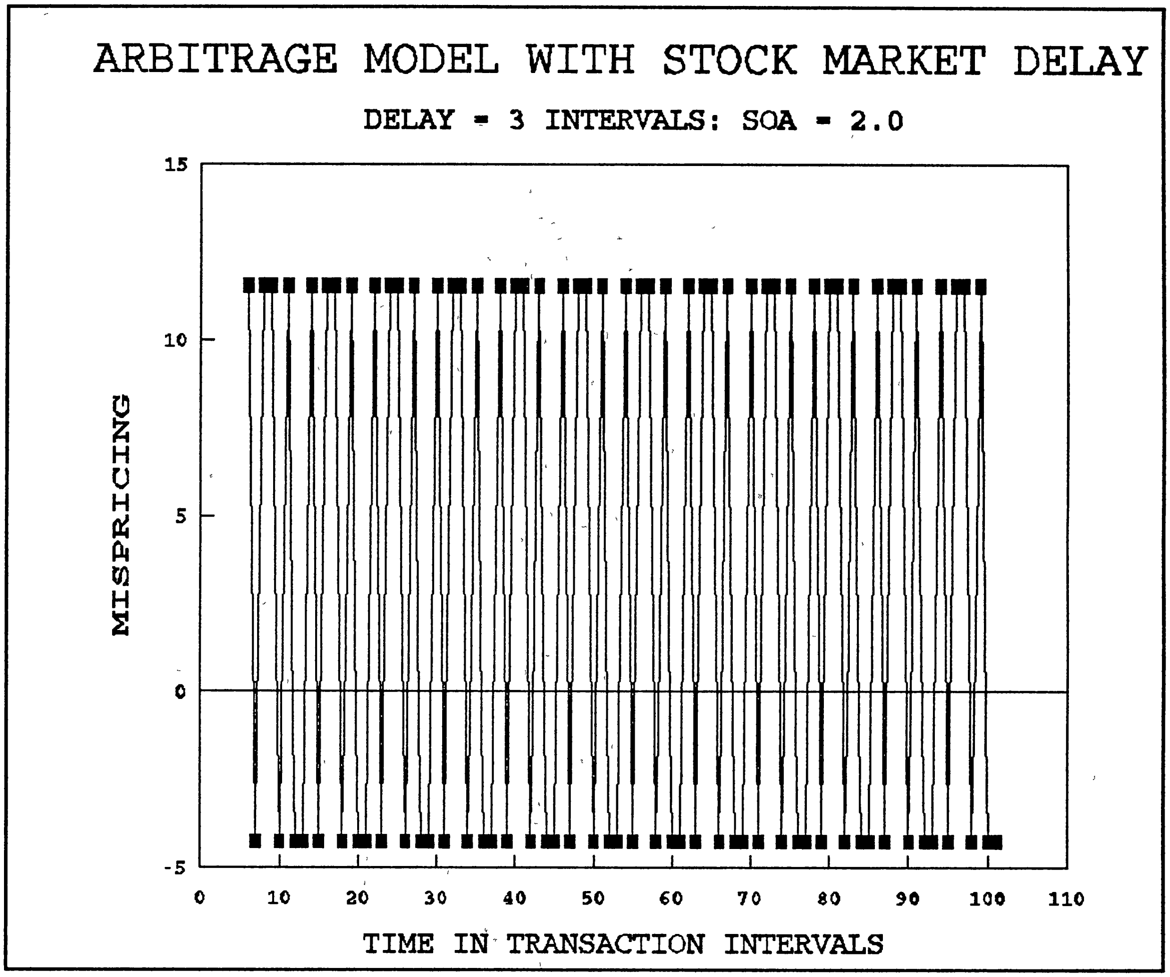

Figure 37. Strong Arbitrage Strength with Three Minute Delay

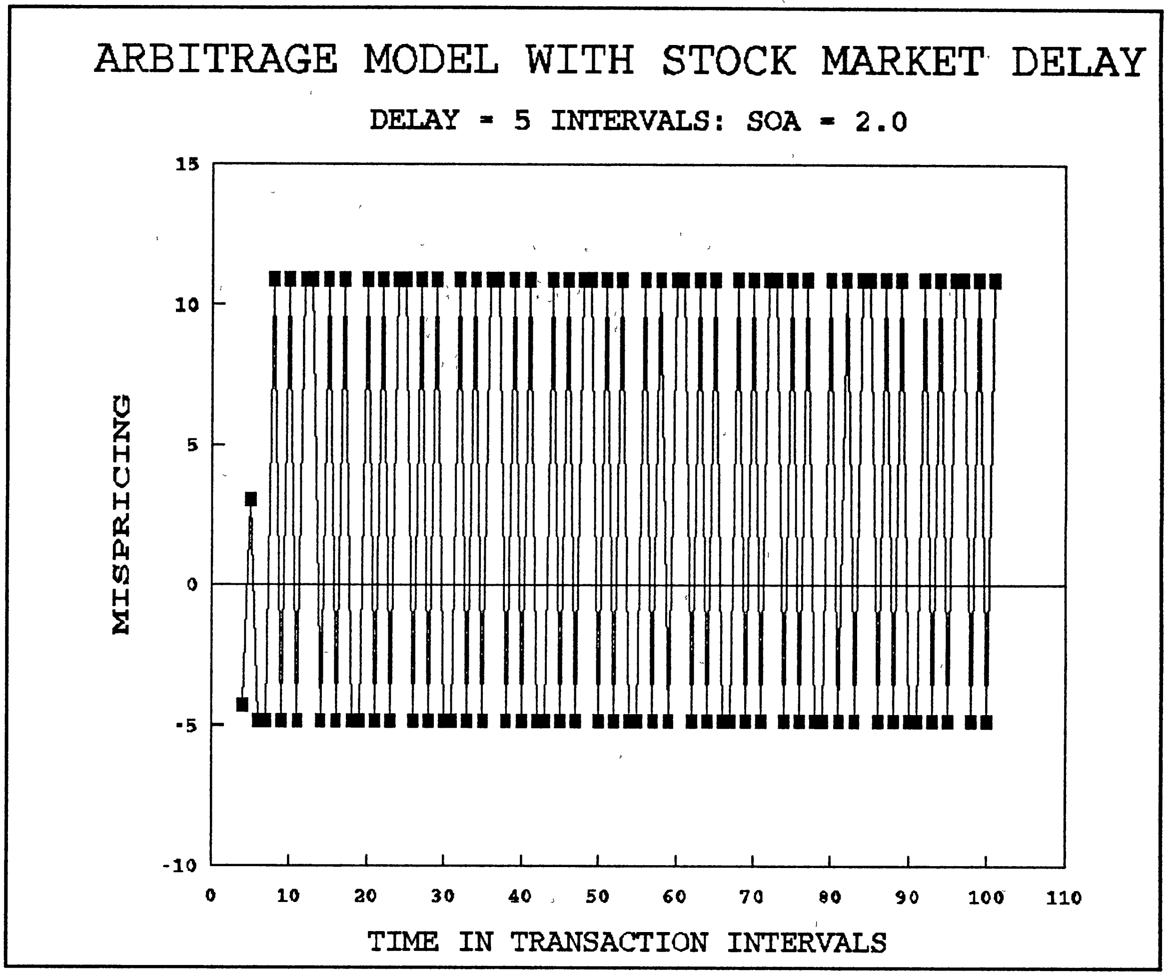

Figure 38. Strong Arbitrage Strength with Five Minute Delay

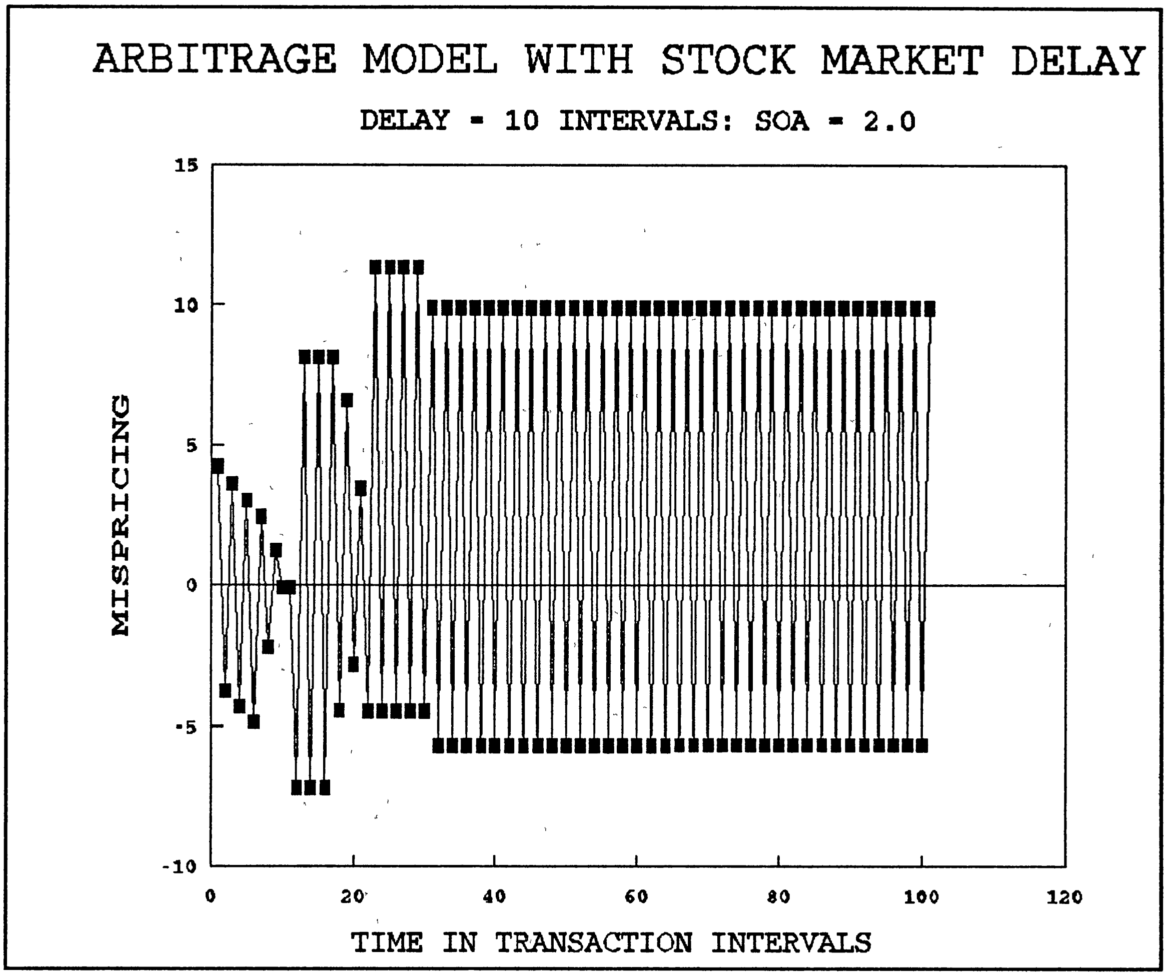

Figure 39. Strong Arbitrage Response With Ten Minute Delay

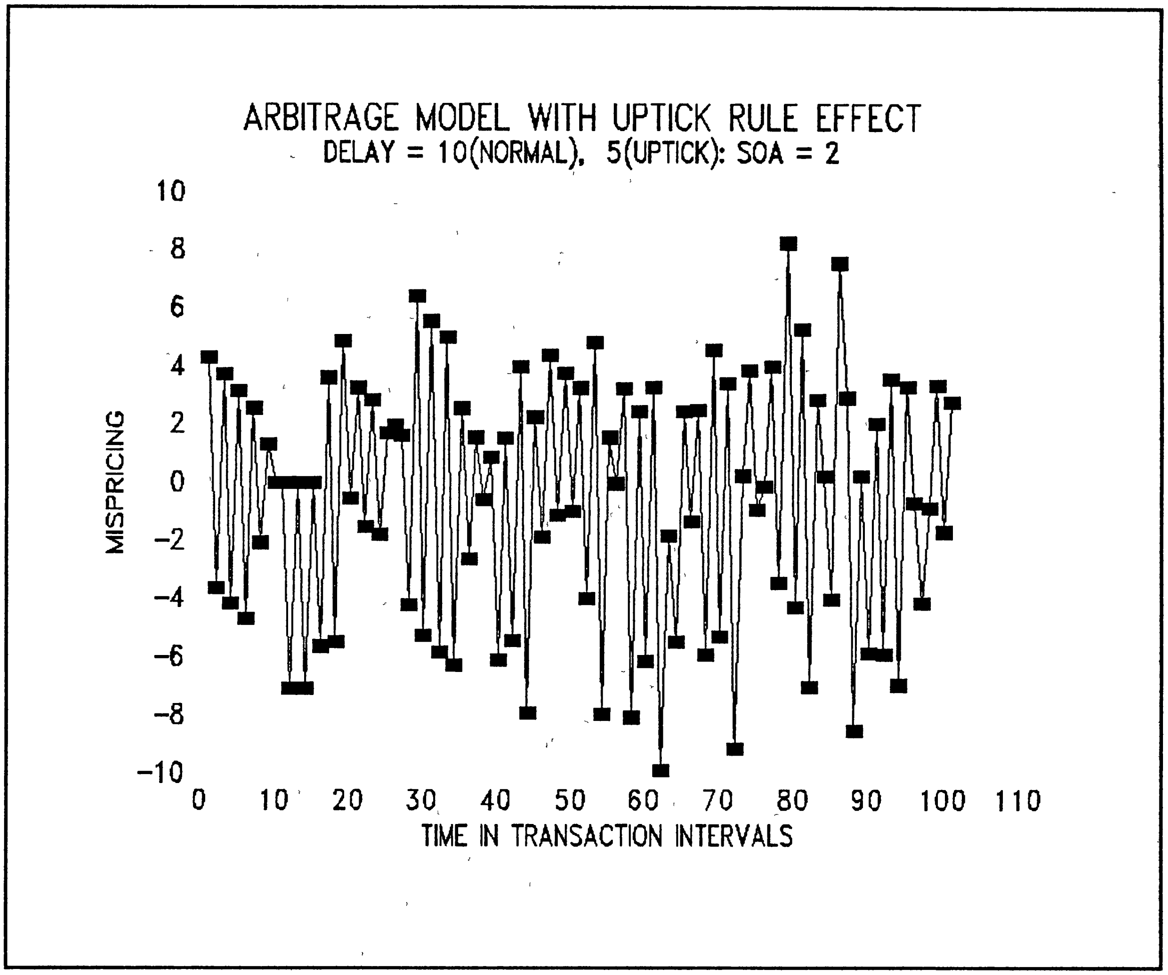

Figure 40. Strong Arbitrage Response with Uptick Delay

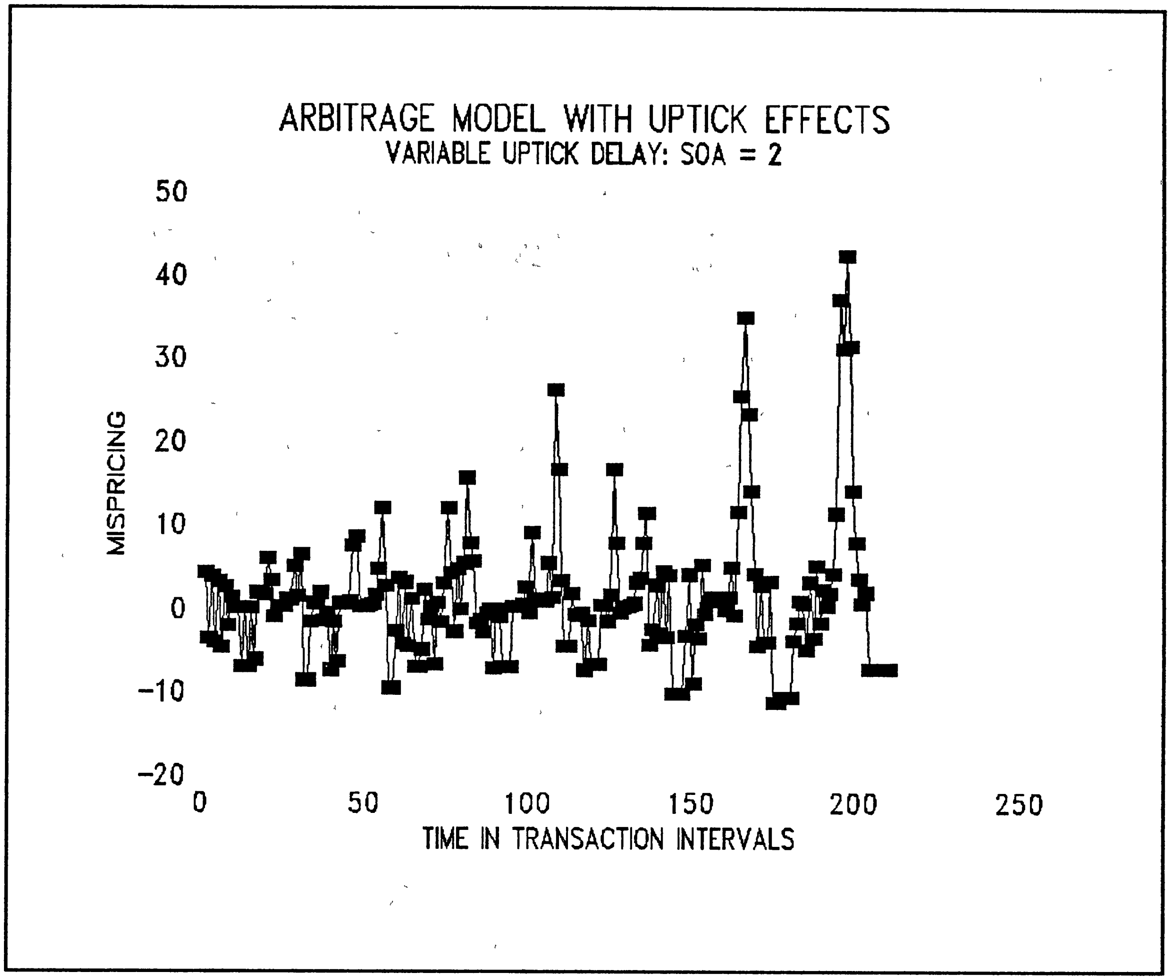

Figure 41. Strong Arbitrage Response with Variable Uptick Delay

APPENDIX B

DYNAMO CODE FOR SIMULATION

Model With No Delays

```
NOTE   SPOT PRICE MODULE
*
L SPOTP.K=SPOTP.J+DT*(SCARB.JK)
  SPOTP = SPOT PRICE ($)
N SPOTP=NSPOT
C NSPOT=100
  SCARB = SPOT PRICE CHANGE DUE TO ARBITRAGE ($/TRANSACTION
INTERVAL)
R SCARB.KL=SPOTP.K*(STPC.KL/100)
R STPC.KL=PPC.KL*SOA
  STPC.KL = PERCENTAGE STOCK PRICE CHANGE
C SOA=1
  SOA = STRENGTH OF ARBITRAGE
  PPC.K = PERCENTAGE PRICE CHANGE
*
NOTE   FUTURES PRICE MODULE
*
L FUTP.K=FUTP.J+DT*(FCARB.JK*(-1))
  FUTP = FUTURES PRICE ($)
  FCARB = FUTURES PRICE CHANGE DUE TO ARBITRAGE
($/TRANSACTION INTERVAL)
N FUTP=NFUT
C NFUT=106
R FCARB.KL=FUTP.K*(PPC.KL/100)*SOA
*
NOTE FORWARD PRICE MODULE
*
A FWDP.K=SPOTP.K*EXP(COC.K*TTM.K)
A COC.K=INT-DIV
C INT=.10
C DIV=.03
  FWDP = FORWARD PRICE ($)
```

```
 COC  = COST OF CARRY (FRACTION/YEAR)
A TTM.K=(MAT-TIME.K)/TIY
C TIY=518400    360 DAYS/YEAR * 24 HOURS/DAY * 60 MIN/HOUR
C MAT=129600    90 DAYS/CONTRACT * 24 HOURS/DAY * 60 MIN/HOUR
 TTM = TIME TO MATURITY (MINUTES REMAINING/CONTRACT)
 TIY = TRANSACTION INTERVALS ( MINUTES IN ONE YEAR)
 MAT = MATURITY (LENGTH OF FUTURES CONTRACT IN MINUTES)
*
NOTE   MISPRICING AND ARBITRAGE MODULE
*
A MISP.K=((FUTP.K-FWDP.K)/SPOTP.K)*100
 MISP = MISPRICING (PERCENTAGE OF INDEX VALUE): NORMAL = 0
R PPC.KL=TABLE(TABPC,MISP.K,-3,3,0.5)
N PPC=0
C TABPC=-3.75/-2.3/-1.4/-0.8/-0.5/0/0/0/0.5/0.8/1.4/2.3/3.75
*
NOTE SYSTEM SPECIFICATIONS
*
SAVE SPOTP/FUTP/FWDP/MISP
SPEC DT=1/LENGTH=300/SAVPER=1
```

Model with Three Minute Delay

```
NOTE   SPOT PRICE MODULE
*
L SPOTP.K=SPOTP.J+DT*(SCARB.JK)
 SPOTP = SPOT PRICE ($)
N SPOTP=NSPOT
C NSPOT=100
 SCARB = SPOT PRICE CHANGE DUE TO ARBITRAGE ($/TRANSACTION
INTERVAL)
R SCARB.KL=SPOTP.K*(DPPC3.KL/100)
*
* DELAY MODULE: REPRESENTS PRIMARY DELAY IN STOCK MARKET
TRANSACTION
*
R DPPC10.KL=DELAY1(DPPC9.KL,DET)
R DPPC9.KL=DELAY1(DPPC8.KL,DET)
R DPPC8.KL=DELAY1(DPPC7.KL,DET)
R DPPC7.KL=DELAY1(DPPC6.KL,DET)
R DPPC6.KL=DELAY1(DPPC5.KL,DET)
R DPPC5.KL=DELAY1(DPPC4.KL,DET)
R DPPC4.KL=DELAY1(DPPC3.KL,DET)
R DPPC3.KL=DELAY1(DPPC2.KL,DET)
```

```
R DPPC2.KL=DELAY1(DPPC1.KL,DET)
R DPPC1.KL=DELAY1(STPC.KL,DET)
R STPC.KL=PPC.KL*SOA
C SOA=1
C DET=1
  DPPC'N'= DELAYED PERCENTAGE PRICE CHANGE FOR N DELAYS
  DET = DELAY IN EFFECTING STOCK MARKET TRADES
  PPC.K = PERCENTAGE PRICE CHANGE
*
NOTE   FUTURES PRICE MODULE
*
L FUTP.K=FUTP.J+DT*(FCARB.JK*(-1))
  FUTP = FUTURES PRICE ($)
  FCARB = FUTURES PRICE CHANGE DUE TO ARBITRAGE
($/TRANSACTION INTERVAL)
N FUTP=NFUT
C NFUT=106
R FCARB.KL=FUTP.K*(PPC.KL/100)*SOA
*
NOTE FORWARD PRICE MODULE
*
A FWDP.K=SPOTP.K*EXP(COC.K*TTM.K)
A COC.K=INT-DIV
C INT=.10
C DIV=.03
  FWDP = FORWARD PRICE ($)
  COC  = COST OF CARRY (FRACTION/YEAR)
A TTM.K=(MAT-TIME.K)/TIY
C TIY=518400    360 DAYS/YEAR * 24 HOURS/DAY * 60 MIN/HOUR
C MAT=129600    90 DAYS/CONTRACT * 24 HOURS/DAY * 60 MIN/HOUR
  TTM = TIME TO MATURITY (MINUTES REMAINING/CONTRACT)
  TIY = TRANSACTION INTERVALS ( MINUTES IN ONE YEAR)
  MAT = MATURITY (LENGTH OF FUTURES CONTRACT IN MINUTES)
*
NOTE   MISPRICING AND ARBITRAGE MODULE
*
A MISP.K=((FUTP.K-FWDP.K)/SPOTP.K)*100
  MISP = MISPRICING (PERCENTAGE OF INDEX VALUE): NORMAL = 0
R PPC.KL=TABLE(TABPC,MISP.K,-3,3,0.5)
N PPC=0
C TABPC=-3.75/-2.3/-1.4/-0.8/-0.5/0/0/0/0.5/0.8/1.4/2.3/3.75
*
NOTE SYSTEM SPECIFICATIONS
*
SAVE SPOTP/FUTP/FWDP/MISP
SPEC DT=1/LENGTH=300/SAVPER=1
```

Model with Five Minute Delay

```
NOTE   SPOT PRICE MODULE
*
L SPOTP.K=SPOTP.J+DT*(SCARB.JK)
  SPOTP = SPOT PRICE ($)
N SPOTP=NSPOT
C NSPOT=100
  SCARB = SPOT PRICE CHANGE DUE TO ARBITRAGE ($/TRANSACTION
INTERVAL)
R SCARB.KL=SPOTP.K*(DPPC5.KL/100)
*
* DELAY MODULE: REPRESENTS PRIMARY DELAY IN STOCK MARKET
TRANSACTION
*
R DPPC10.KL=DELAY1(DPPC9.KL,DET)
R DPPC9.KL=DELAY1(DPPC8.KL,DET)
R DPPC8.KL=DELAY1(DPPC7.KL,DET)
R DPPC7.KL=DELAY1(DPPC6.KL,DET)
R DPPC6.KL=DELAY1(DPPC5.KL,DET)
R DPPC5.KL=DELAY1(DPPC4.KL,DET)
R DPPC4.KL=DELAY1(DPPC3.KL,DET)
R DPPC3.KL=DELAY1(DPPC2.KL,DET)
R DPPC2.KL=DELAY1(DPPC1.KL,DET)
R DPPC1.KL=DELAY1(STPC.KL,DET)
R STPC.KL=PPC.KL*SOA
C SOA=1
C DET=1
  DPPC'N'= DELAYED PERCENTAGE PRICE CHANGE FOR N DELAYS
  DET = DELAY IN EFFECTING STOCK MARKET TRADES
  PPC.K = PERCENTAGE PRICE CHANGE
*
NOTE   FUTURES PRICE MODULE
*
L FUTP.K=FUTP.J+DT*(FCARB.JK*(-1))
  FUTP = FUTURES PRICE ($)
  FCARB = FUTURES PRICE CHANGE DUE TO ARBITRAGE
($/TRANSACTION INTERVAL)
N FUTP=NFUT
C NFUT=106
R FCARB.KL=FUTP.K*(PPC.KL/100)*SOA
*
NOTE FORWARD PRICE MODULE
*
```

```
A FWDP.K=SPOTP.K*EXP(COC.K*TTM.K)
A COC.K=INT-DIV
C INT=.10
C DIV=.03
  FWDP = FORWARD PRICE ($)
  COC  = COST OF CARRY (FRACTION/YEAR)
A TTM.K=(MAT-TIME.K)/TIY
C TIY=518400    360 DAYS/YEAR * 24 HOURS/DAY * 60 MIN/HOUR
C MAT=129600    90 DAYS/CONTRACT * 24 HOURS/DAY * 60 MIN/HOUR
  TTM = TIME TO MATURITY (MINUTES REMAINING/CONTRACT)
  TIY = TRANSACTION INTERVALS ( MINUTES IN ONE YEAR)
  MAT = MATURITY (LENGTH OF FUTURES CONTRACT IN MINUTES)
*
NOTE   MISPRICING AND ARBITRAGE MODULE
*
A MISP.K=((FUTP.K-FWDP.K)/SPOTP.K)*100
  MISP = MISPRICING (PERCENTAGE OF INDEX VALUE): NORMAL = 0
R PPC.KL=TABLE(TABPC,MISP.K,-3,3,0.5)
N PPC=0
C TABPC=-3.75/-2.3/-1.4/-0.8/-0.5/0/0/0/0.5/0.8/1.4/2.3/3.75
*
NOTE SYSTEM SPECIFICATIONS
*
SAVE SPOTP/FUTP/FWDP/MISP
SPEC DT=1/LENGTH=300/SAVPER=1
```

Model with Ten Minute Delay

```
NOTE   SPOT PRICE MODULE
*
L SPOTP.K=SPOTP.J+DT*(SCARB.JK)
  SPOTP = SPOT PRICE ($)
N SPOTP=NSPOT
C NSPOT=100
  SCARB = SPOT PRICE CHANGE DUE TO ARBITRAGE ($/TRANSACTION
INTERVAL)
R SCARB.KL=SPOTP.K*(DPPC10.KL/100)
*
* DELAY MODULE: REPRESENTS PRIMARY DELAY IN STOCK MARKET
TRANSACTION
*
R DPPC10.KL=DELAY1(DPPC9.KL,DET)
R DPPC9.KL=DELAY1(DPPC8.KL,DET)
R DPPC8.KL=DELAY1(DPPC7.KL,DET)
```

```
R DPPC7.KL=DELAY1(DPPC6.KL,DET)
R DPPC6.KL=DELAY1(DPPC5.KL,DET)
R DPPC5.KL=DELAY1(DPPC4.KL,DET)
R DPPC4.KL=DELAY1(DPPC3.KL,DET)
R DPPC3.KL=DELAY1(DPPC2.KL,DET)
R DPPC2.KL=DELAY1(DPPC1.KL,DET)
R DPPC1.KL=DELAY1(STPC.KL,DET)
R STPC.KL=PPC.KL*SOA
C SOA=1
C DET=1
  DPPC'N'= DELAYED PERCENTAGE PRICE CHANGE FOR N DELAYS
  DET = DELAY IN EFFECTING STOCK MARKET TRADES
  PPC.K = PERCENTAGE PRICE CHANGE
*
NOTE   FUTURES PRICE MODULE
*
L FUTP.K=FUTP.J+DT*(FCARB.JK*(-1))
  FUTP = FUTURES PRICE ($)
  FCARB = FUTURES PRICE CHANGE DUE TO ARBITRAGE
($/TRANSACTION INTERVAL)
N FUTP=NFUT
C NFUT=106
R FCARB.KL=FUTP.K*(PPC.KL/100)*SOA
*
NOTE FORWARD PRICE MODULE
*
A FWDP.K=SPOTP.K*EXP(COC.K*TTM.K)
A COC.K=INT-DIV
C INT=.10
C DIV=.03
  FWDP = FORWARD PRICE ($)
  COC  = COST OF CARRY (FRACTION/YEAR)
A TTM.K=(MAT-TIME.K)/TIY
C TIY=518400    360 DAYS/YEAR * 24 HOURS/DAY * 60 MIN/HOUR
C MAT=129600    90 DAYS/CONTRACT * 24 HOURS/DAY * 60 MIN/HOUR
  TTM = TIME TO MATURITY (MINUTES REMAINING/CONTRACT)
  TIY = TRANSACTION INTERVALS ( MINUTES IN ONE YEAR)
  MAT = MATURITY (LENGTH OF FUTURES CONTRACT IN MINUTES)
*
NOTE   MISPRICING AND ARBITRAGE MODULE
*
A MISP.K=((FUTP.K-FWDP.K)/SPOTP.K)*100
  MISP = MISPRICING (PERCENTAGE OF INDEX VALUE): NORMAL = 0
R PPC.KL=TABLE(TABPC,MISP.K,-3,3,0.5)
N PPC=0
C TABPC=-3.75/-2.3/-1.4/-0.8/-0.5/0/0/0/0.5/0.8/1.4/2.3/3.75
```

```
*
NOTE SYSTEM SPECIFICATIONS
*
SAVE SPOTP/FUTP/FWDP/MISP
SPEC DT=1/LENGTH=300/SAVPER=1
```

Model with Five Minute Uptick Delay

```
*   STOCK MARKET DELAY = 5: UPTICK DELAY = 5
*
NOTE   SPOT PRICE MODULE
*
L SPOTP.K=SPOTP.J+DT*(SCARB.JK)
  SPOTP = SPOT PRICE ($)
N SPOTP=NSPOT
C NSPOT=100
  SCARB = SPOT PRICE CHANGE DUE TO ARBITRAGE ($/TRANSACTION
INTERVAL)
R SCARB.KL=SPOTP.K*(DSPC.KL/100)
*
NOTE    UPTICK RULE IN STOCK MARKET
*
R PSPC.KL=CLIP(DSPCN.KL,0,DSPCN.KL,0)
  PSPC = POSITIVE SPOT PRICE CHANGE (%)
R NSPC.KL=CLIP(0,DSPCN.KL,DSPCN.KL,0)
  NSPC = NEGATIVE SPOT PRICE CHANGE (%)
R DNEG.KL=DELAY3(NSPC.KL,DSHORT)
C DSHORT=5
  DNEG = DELAY IN NEGATIVE SPOT PRICE CHANGE DUE TO UPTICK
RULE
R DSPC.KL=PSPC.KL+DNEG.KL
R DSPCN.KL=DPPC5.KL
  DSPCN = DELAYED STOCK PRICE CHANGE FOR 'N' DELAYS
*
* DELAY MODULE: REPRESENTS PRIMARY DELAY IN STOCK MARKET
TRANSACTION
*
R DPPC10.KL=DELAY1(DPPC9.KL,DET)
R DPPC9.KL=DELAY1(DPPC8.KL,DET)
R DPPC8.KL=DELAY1(DPPC7.KL,DET)
R DPPC7.KL=DELAY1(DPPC6.KL,DET)
R DPPC6.KL=DELAY1(DPPC5.KL,DET)
R DPPC5.KL=DELAY1(DPPC4.KL,DET)
R DPPC4.KL=DELAY1(DPPC3.KL,DET)
```

```
R DPPC3.KL=DELAY1(DPPC2.KL,DET)
R DPPC2.KL=DELAY1(DPPC1.KL,DET)
R DPPC1.KL=DELAY1(STPC.KL,DET)
R STPC.KL=PPC.KL*SOA
C SOA=1
C DET=1
  DPPC.K=DELAYED PERCENTAGE PRICE CHANGE
  DET = DELAY IN EFFECTING STOCK MARKET TRADES
  PPC.K = PERCENTAGE PRICE CHANGE
*
NOTE   FUTURES PRICE MODULE
*
L FUTP.K=FUTP.J+DT*(FCARB.JK*(-1))
  FUTP = FUTURES PRICE ($)
  FCARB = FUTURES PRICE CHANGE DUE TO ARBITRAGE
($/TRANSACTION INTERVAL)
N FUTP=NFUT
C NFUT=104
R FCARB.KL=FUTP.K*(PPC.KL/100)*SOA
*
NOTE FORWARD PRICE MODULE
*
A FWDP.K=SPOTP.K*EXP(COC.K*TTM.K)
A COC.K=INT-DIV
C INT=.10
C DIV=.03
  FWDP = FORWARD PRICE ($)
  COC  = COST OF CARRY (FRACTION/YEAR)
A TTM.K=(MAT-TIME.K)/TIY
C TIY=518400    360 DAYS/YEAR * 24 HOURS/DAY * 60 MIN/HOUR
C MAT=129600    90 DAYS/CONTRACT * 24 HOURS/DAY * 60 MIN/HOUR
  TTM = TIME TO MATURITY (MINUTES REMAINING/CONTRACT)
  TIY = TRANSACTION INTERVALS ( MINUTES IN ONE YEAR)
  MAT = MATURITY (LENGTH OF FUTURES CONTRACT IN MINUTES)
*
NOTE   MISPRICING AND ARBITRAGE MODULE
*
A MISP.K=((FUTP.K-FWDP.K)/SPOTP.K)*100
  MISP = MISPRICING (PERCENTAGE OF INDEX VALUE): NORMAL = 0
R PPC.KL=TABLE(TABPC,MISP.K,-3,3,0.5)
N PPC=0
C TABPC=-3.75/-2.3/-1.4/-0.8/-0.5/0/0/0/0.5/0.8/1.4/2.3/3.75
*
NOTE SYSTEM SPECIFICATIONS
*
SAVE SPOTP/FUTP/FWDP/MISP
```

```
SPEC DT=1/LENGTH=300/SAVPER=1
```

Model with Uptick Delay

```
* STOCK MARKET DELAY =10: UPTICK DELAY = 5
*
NOTE   SPOT PRICE MODULE
*
L SPOTP.K=SPOTP.J+DT*(SCARB.JK)
  SPOTP = SPOT PRICE ($)
N SPOTP=NSPOT
C NSPOT=100
  SCARB = SPOT PRICE CHANGE DUE TO ARBITRAGE ($/TRANSACTION
INTERVAL)
R SCARB.KL=SPOTP.K*(DSPC.KL/100)
*
NOTE    UPTICK RULE IN STOCK MARKET
*
R PSPC.KL=CLIP(DSPCN.KL,0,DSPCN.KL,0)
  PSPC = POSITIVE SPOT PRICE CHANGE (%)
R NSPC.KL=CLIP(0,DSPCN.KL,DSPCN.KL,0)
  NSPC = NEGATIVE SPOT PRICE CHANGE (%)
R DNEG.KL=DELAY3(NSPC.KL,DSHORT)
C DSHORT=5
  DNEG = DELAY IN NEGATIVE SPOT PRICE CHANGE DUE TO UPTICK
RULE
R DSPC.KL=PSPC.KL+DNEG.KL
R DSPCN.KL=DPPC10.KL
  DSPCN = DELAYED STOCK PRICE CHANGE FOR 'N' DELAYS
*
* DELAY MODULE: REPRESENTS PRIMARY DELAY IN STOCK MARKET
TRANSACTION
*
R DPPC10.KL=DELAY1(DPPC9.KL,DET)
R DPPC9.KL=DELAY1(DPPC8.KL,DET)
R DPPC8.KL=DELAY1(DPPC7.KL,DET)
R DPPC7.KL=DELAY1(DPPC6.KL,DET)
R DPPC6.KL=DELAY1(DPPC5.KL,DET)
R DPPC5.KL=DELAY1(DPPC4.KL,DET)
R DPPC4.KL=DELAY1(DPPC3.KL,DET)
R DPPC3.KL=DELAY1(DPPC2.KL,DET)
R DPPC2.KL=DELAY1(DPPC1.KL,DET)
R DPPC1.KL=DELAY1(STPC.KL,DET)
R STPC.KL=PPC.KL*SOA
```

```
C SOA=1
C DET=1
  DPPC.K=DELAYED PERCENTAGE PRICE CHANGE
  DET = DELAY IN EFFECTING STOCK MARKET TRADES
  PPC.K = PERCENTAGE PRICE CHANGE
*
NOTE   FUTURES PRICE MODULE
*
L FUTP.K=FUTP.J+DT*(FCARB.JK*(-1))
  FUTP = FUTURES PRICE ($)
  FCARB = FUTURES PRICE CHANGE DUE TO ARBITRAGE
($/TRANSACTION INTERVAL)
N FUTP=NFUT
C NFUT=104
R FCARB.KL=FUTP.K*(PPC.KL/100)*SOA
*
NOTE FORWARD PRICE MODULE
*
A FWDP.K=SPOTP.K*EXP(COC.K*TTM.K)
A COC.K=INT-DIV
C INT=.10
C DIV=.03
  FWDP = FORWARD PRICE ($)
  COC  = COST OF CARRY (FRACTION/YEAR)
A TTM.K=(MAT-TIME.K)/TIY
C TIY=518400   360 DAYS/YEAR * 24 HOURS/DAY * 60 MIN/HOUR
C MAT=129600   90 DAYS/CONTRACT * 24 HOURS/DAY * 60 MIN/HOUR
  TTM = TIME TO MATURITY (MINUTES REMAINING/CONTRACT)
  TIY = TRANSACTION INTERVALS ( MINUTES IN ONE YEAR)
  MAT = MATURITY (LENGTH OF FUTURES CONTRACT IN MINUTES)
*
NOTE   MISPRICING AND ARBITRAGE MODULE
*
A MISP.K=((FUTP.K-FWDP.K)/SPOTP.K)*100
  MISP = MISPRICING (PERCENTAGE OF INDEX VALUE): NORMAL = 0
R PPC.KL=TABLE(TABPC,MISP.K,-3,3,0.5)
N PPC=0
C TABPC=-3.75/-2.3/-1.4/-0.8/-0.5/0/0/0/0.5/0.8/1.4/2.3/3.75
*
NOTE SYSTEM SPECIFICATIONS
*
SAVE SPOTP/FUTP/FWDP/MISP
SPEC DT=1/LENGTH=300/SAVPER=1
```

Model with Variable Uptick Delay

```
*   STOCK MARKET DELAY = 10 : UPTICK DELAY IS VARIABLE
*
NOTE   SPOT PRICE MODULE
*
L SPOTP.K=SPOTP.J+DT*(SCARB.JK)
  SPOTP = SPOT PRICE ($)
N SPOTP=NSPOT
C NSPOT=100
  SCARB = SPOT PRICE CHANGE DUE TO ARBITRAGE ($/TRANSACTION
INTERVAL)
R SCARB.KL=SPOTP.K*(DSPC.KL/100)
*
NOTE    UPTICK RULE IN STOCK MARKET
*
R PSPC.KL=CLIP(DSPCN.KL,0,DSPCN.KL,0)
  PSPC = POSITIVE SPOT PRICE CHANGE (%)
R NSPC.KL=CLIP(0,DSPCN.KL,DSPCN.KL,0)
  NSPC = NEGATIVE SPOT PRICE CHANGE (%)
R DNEG.KL=DELAY3(NSPC.KL,DSHORT.K)
A DSHORT.K=TABLE(TABUP,NSPC.KL,-10,-1,1)
C TABUP=30/27/24/21/18/15/12/9/6/3
  DNEG = DELAY IN NEGATIVE SPOT PRICE CHANGE DUE TO UPTICK
RULE
R DSPC.KL=PSPC.KL+DNEG.KL
R DSPCN.KL=DPPC10.KL
  DSPCN = DELAYED STOCK PRICE CHANGE FOR 'N' DELAYS
*
* DELAY MODULE: REPRESENTS PRIMARY DELAY IN STOCK MARKET
TRANSACTION
*
R DPPC10.KL=DELAY1(DPPC9.KL,DET)
R DPPC9.KL=DELAY1(DPPC8.KL,DET)
R DPPC8.KL=DELAY1(DPPC7.KL,DET)
R DPPC7.KL=DELAY1(DPPC6.KL,DET)
R DPPC6.KL=DELAY1(DPPC5.KL,DET)
R DPPC5.KL=DELAY1(DPPC4.KL,DET)
R DPPC4.KL=DELAY1(DPPC3.KL,DET)
R DPPC3.KL=DELAY1(DPPC2.KL,DET)
R DPPC2.KL=DELAY1(DPPC1.KL,DET)
R DPPC1.KL=DELAY1(STPC.KL,DET)
R STPC.KL=PPC.KL*SOA
C SOA=1
C DET=1
```

```
 DPPC.K=DELAYED PERCENTAGE PRICE CHANGE
 DET = DELAY IN EFFECTING STOCK MARKET TRADES
 PPC.K = PERCENTAGE PRICE CHANGE
*
NOTE   FUTURES PRICE MODULE
*
L FUTP.K=FUTP.J+DT*(FCARB.JK*(-1))
 FUTP = FUTURES PRICE ($)
 FCARB = FUTURES PRICE CHANGE DUE TO ARBITRAGE
($/TRANSACTION INTERVAL)
N FUTP=NFUT
C NFUT=106
R FCARB.KL=FUTP.K*(PPC.KL/100)*SOA
*
NOTE FORWARD PRICE MODULE
*
A FWDP.K=SPOTP.K*EXP(COC.K*TTM.K)
A COC.K=INT-DIV
C INT=.10
C DIV=.03
 FWDP = FORWARD PRICE ($)
 COC  = COST OF CARRY (FRACTION/YEAR)
A TTM.K=(MAT-TIME.K)/TIY
C TIY=518400   360 DAYS/YEAR * 24 HOURS/DAY * 60 MIN/HOUR
C MAT=129600   90 DAYS/CONTRACT * 24 HOURS/DAY * 60 MIN/HOUR
 TTM = TIME TO MATURITY (MINUTES REMAINING/CONTRACT)
 TIY = TRANSACTION INTERVALS ( MINUTES IN ONE YEAR)
 MAT = MATURITY (LENGTH OF FUTURES CONTRACT IN MINUTES)
*
NOTE   MISPRICING AND ARBITRAGE MODULE
*
A MISP.K=((FUTP.K-FWDP.K)/SPOTP.K)*100
 MISP = MISPRICING (PERCENTAGE OF INDEX VALUE): NORMAL = 0
R PPC.KL=TABLE(TABPC,MISP.K,-3,3,0.5)
N PPC=0
C TABPC=-3.75/-2.3/-1.4/-0.8/-0.5/0/0/0/0.5/0.8/1.4/2.3/3.75
*
*
NOTE SYSTEM SPECIFICATIONS
*
SAVE SPOTP/FUTP/FWDP/MISP
SPEC DT=1/LENGTH=300/SAVPER=1
```

APPENDIX C

EXPLANATION TO THE DYNAMO MODELS

The Basic Arbitrage Model
Without Delays

The Dynamo model is divided for convenience into four modules. These include,

(1) The mispricing and arbitrage module

(2) The futures price module

(3) The spot price module

(4) The forward price module

The Mispricing Module

The mispricing module may be a convenient place to start the explanation of the program code in Dynamo. Mispricing in the current period is defined as the difference between the futures price and the forward price in the current period and is expressed as a percentage of the current spot price. The suffix "K" indicates that the values are of the current period. The Dynamo statement defining mispricing is an auxiliary equation as denoted by the first letter of the equation below which begins with an "A".

A MISP.K = ((FUTP.K-FWDP.K)/SPOTP.K)*100 (1)

where,

MISP = Mispricing (percentage)

FUTP = Futures Price ($)

SPOTP = Spot Price ($)

FWDP = Forward Price ($)

The observed mispricing will bring about arbitrage activity in both the futures and the cash markets. The result will be a decline in the price of the overpriced asset (futures or cash) due to arbitrage selling and an increase in the price of the relatively underpriced asset. It is assumed that the price change in both markets would be symmetric since the arbitrageur would buy and sell equal proportions of both indices. The percentage price change as a result of mispricing is expressed as a Table function in Dynamo. This maps the corresponding price change in both markets for every level of mispricing. The corresponding Rate equation in Dynamo which begins with the letter "R" is given by:

R PPC.KL = TABLE (TABPC, MISP.K, -3, 3, 0.5) (2)

TABPC = -3.75/-2.3/-1.4/-0.8/-0.5/0/0/0/0.5/0.8/1.4/2.3/3.75 (3)

where,

PPC.KL = Percentage price change caused by arbitrage

TABPC = Table of Price changes for given mispricing

Equation (2) states that the mispricing levels stretch from -3 to +3 at increments of 0.5 and equation (3) expresses the corresponding price changes. The range of mispricing levels is established empirically using the MacKinlay and Ramaswamy (1988) time series. The high and low values of price changes are also obtained from the transaction to transaction price changes in both the index futures and the spot index. The graph of equation (2) shows that the price changes increase exponentially with increased levels of futures mispricing. The justification for this is as follows. At small levels of mispricing only the most cost effective arbitrageurs will enter the markets but at higher levels of mispricing more arbitrageurs would be attracted by the risk free arbitrage profits which would bring about larger equilibriating price changes in the two markets. Conceivably, at very high levels of mispricing even non arbitrage market players would be induced to enter into arbitrage transactions.

THE FORWARD PRICE MODULE

The forward price in the current period is computed as the sum of the spot price and the cost of carrying the basket of stocks to maturity. The corresponding Auxiliary statement in Dynamo would be:

```
A  FWDP.K = SPOTP.K * EXP(COC.K*TTM.K)
A  COC.K = INT - DIV
C  INT = .10
C  DIV = .03
```

where,

COC = Cost of Carry (Fraction/Year)

INT = Assumed interest rate (constant)

DIV = Assumed dividend rate (constant)

TTM = Time to Maturity (Minutes remaining/contract)

The remaining time to maturity is computed as an auxiliary equation in Dynamo:

```
A  TTM.K = (MAT - TIME.K) / TIY
```

Where:

MAT = Maturity (90 days per contract* 24 hours/day* 60 min/hour = 129600 minutes)

TIY = Transaction intervals in one year (360 days/year * 24 hours/day * 60 min/hour = 518400 minutes)

SPOT PRICE MODULE

The current spot price for the purpose of the simulation exercise is given as the sum of the previous spot price and the change in price due to arbitrage activity. The spot price formulation is a level (stock) equation in Dynamo. The Dynamo equation is given by L SPOTP.K = SPOTP.J + DT * (SCARB.JK)

where

SCARB = Stock price change due to arbitrage.

DT = Time increment set equal to 1

The stock price change due to arbitrage (SCARB) is a rate (flow) equation in Dynamo and is computed using the percentage price change from the mispricing module.

R SCARB.KL = SPOTP.K * (PPC.KL / 100) * SOA

Where,

SOA = Strength of Arbitrage

The Strength of arbitrage can be varied during simulation to observe both fast and slow arbitrage price responses. The base case sets SOA equal to one.

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