

I. IN SITU TRANSFORMATION OF SPRING-APPLIED
AMMONIUM-¹⁵N

II. MAXPROFIT - A SOFTWARE TOOL FOR
TEACHING PRINCIPLES OF ECONOMIC
FERTILIZER USE

By

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INTRODUCTION

There are two parts to this dissertation pertaining to two projects that were completed for my doctoral degree. Both parts are presented in a format suitable for publication in a professional journal.

A common practice in winter wheat production is surface-application of ammoniacal fertilizer in the early spring. This practice requires that NH_4^+ be transformed to NO_3^- and move into the root zone before this fertilizer N is available to the crop. The objective of the research in Chapter I was to determine the transformation of spring-applied ammonium- ^{15}N throughout the growing season at two locations in Oklahoma when no crop uptake was allowed.

Chapter II presents "MaxProfit", a computer program that is designed to be used in classroom instruction of economic fertilizer use. The objective of this project was to develop a software tool that will assist in studying economic fertilizer use when considering more than one crop nutrient applied to more than one crop simultaneously, the difference between maximizing crop yield and profit with regard to nutrient requirements, crop yield, cost, and profit, and how nutrient usage can be distributed among crops when available capital is limited.

CHAPTER I
IN SITU TRANSFORMATION OF SPRING-APPLIED
AMMONIUM-¹⁵N

ABSTRACT

In regions of winter wheat production, topdress application of ammoniacal N sources is a common practice. From a soil fertility perspective, there is reasonable interest in determining how quickly surface applied ammoniacal N sources move into the root zone for crop uptake as well as other transformations of N over time. The objective of this research was to determine the transformation of spring-applied ammonium-N throughout the wheat growing-season when no crop uptake was allowed. Two winter wheat soil fertility experiments were selected at Lahoma and Stillwater, Oklahoma. Prior to fertilizer application, galvanized metal tubes (5-cm diameter) were inserted into plots between drill rows 1 m apart within a 1 m-wide strip down the center of the 0 and 45 kg N ha⁻¹ plots in 3 replications. On 28 Feb. and 1 Mar. 1989, at Lahoma and Stillwater, respectively, (¹⁵NH₄)SO₄ (15 atom %) was surface-applied to the tubes in the 45 kg N ha⁻¹ plots at a rate of 22.5 kg N ha⁻¹. Beginning on the day of fertilization and continuing through post harvest tubes were removed at approximately one month intervals (six sample dates total). Tubes were sectioned into depth increments of 0 to 15, 15 to 30, and 30 to 45 cm, freeze dried, and ground. Nitrogen content and isotope-ratio analysis were conducted on all samples from the fertilized plots for total-, NH₄⁺-, NO₃⁻-, and organic-N fractions. Organic N was determined as the difference between total and mineral N. Only samples from the first sample date in the unfertilized plots were utilized to determine the natural ¹⁵N abundance in each N fraction at both locations. By the final sample date only

35 and 48% of the total fertilizer N remained in the 0 to 45 cm soil profile at Stillwater and Lahoma, respectively. Of the ^{15}N remaining in the 45 cm profile, the majority remained in the 0 to 15 cm soil depth as organic N. Spring-applied ammonium- ^{15}N not immobilized underwent nitrification, and then under the conditions of this experiment, NO_3^- was lost via leaching and/or denitrification, since conditions were conducive to both. Soil compaction during installation of tubes likely enhanced leaching and/or denitrification and therefore, fertilizer N loss from the 0 to 45 cm profile. Fertilizer N loss was further exacerbated since there was no crop uptake of N and all precipitation was trapped by the tube enclosing the soil cores. Regression analysis was performed by depth for each N fraction with only the results for the 0 to 15 cm soil depth reported. The environmental variables of cumulative heat units and precipitation after fertilization (CHUAF and CPAF, respectively) were found to describe a significant proportion of the variability in the data for all fertilizer N fractions. The models developed also described fertilizer N transformations in line with physical and biological expectations. A combined location model for NH_4^+ was developed which showed a logarithmic decrease in fertilizer NH_4^+ as a function of CHUAF. Using this model it was found that 50% of the fertilizer NH_4^+ was transformed within 10 days of application. This indicates that broadcast application of ammoniacal fertilizer sources in the spring will be readily nitrified and available for crop uptake.

INTRODUCTION

In recent years there has been increased emphasis placed upon managing fertilizer N to reduce the likelihood of NO_3^- contaminating groundwater. Within a given cropping system, N management should be examined to determine ways of reducing potential pollution and increasing its efficient use by the crop. One N-management practice available to producers of winter wheat is to split the application of fertilizer N between preplant, in the fall, and a topdress application in the spring. Previous work by Olson et

al. (1979) showed that spring application of N to winter wheat resulted in more of the fertilizer N taken up in the crop compared to fall application. Harper et al. (1987) and Belford et al. (1987) have shown the importance of spring N applications in amount of fertilizer N translocated to grain and its effects on increased root development, respectively. Currently in Oklahoma, there is interest in managing winter wheat for forage and grain production due to the economic benefits of wheat pasture for beef production (Trapp, 1984). According to Krenzer et al. (1991) there are producers who apply only enough N to achieve a moderate yield goal in the fall and then, based on the amount of forage removal, determine if additional N is needed as a topdress application in the spring. From a soil fertility perspective, there is reasonable interest in determining how quickly surface applied ammoniacal N sources move into the root zone for crop uptake as well as other transformations of N over time.

The objective of this research was to determine the transformation of spring-applied ammonium- ^{15}N throughout the wheat growing-season in Oklahoma when no crop uptake was allowed. Transformation of N in the soil is a biological process conducted by various heterotrophic and autotrophic microorganisms. As such, soil temperature and moisture have an affect upon these processes. Therefore, these environmental variables were utilized to model changes in the total-, NH_4^+ -, NO_3^- -, and organic- ^{15}N fractions after $(^{15}\text{NH}_4^+)_2\text{SO}_4$ application in early spring in two wheat fertility experiments.

MATERIALS AND METHODS

Using the general procedure of Raison et al. (1987), galvanized metal tubes (5-cm diameter) were inserted into plots in long-term (18+ years) winter wheat fertility experiments at Lahoma and Stillwater, Oklahoma on 14 and 29 Nov. 1988, respectively. The Stillwater soil is a Kirkland silt loam, (fine, mixed, thermic Udertic Paleustoll) and the Lahoma soil is a Grant silt loam (fine-silty, mixed, thermic Udic Argiustoll). Twelve tubes were placed between drill rows 1 m apart within a 1 m-wide strip down the center of the 0

and 45 kg N ha⁻¹ plots in 3 replications. All tubes in the 45 kg N ha⁻¹ plots were placed within a 1 m-wide strip that had not received previous fertilization. Tubes were driven into the soil with a post driver to a depth of 45 cm leaving 5 cm protruding from the soil surface to facilitate their extraction and prevent runoff or run-on of precipitation. When soil compaction was obvious, as evidenced by a lower soil surface inside the tube compared to the surrounding soil, another tube was driven into the soil to the side of the failed tube one drill row away. The 12 tubes formed two groups of six, one group in the east, the other in the west half of each plot, since there was another ¹⁵N experiment being conducted in the center of the same plots.

After tubes were in place, they were temporarily capped, and the previously unfertilized 1 m-wide strip in the 45 kg N ha⁻¹ plots was fertilized with a small turf-grass spreader, calibrated at the proper N rate. After the fertilization of the center strip, all stoppers were removed from the tubes. Thermocouples were installed at 7.5, 22.5, and 37.5 cm soil depths in two plots (a fertilized and a control plot) at each location for continuous monitoring of soil temperature. Daily Precipitation was also recorded at each location.

On 28 Feb. and 1 Mar. 1989 at Lahoma and Stillwater, respectively, (¹⁵NH₄)SO₄ (15 atom %) was applied to the tubes in the 45 kg N ha⁻¹ plots at a rate of 22.5 kg N ha⁻¹. Fertilizer solution was surface-applied using a 25 mL pipette. An equal aliquot of distilled H₂O was also applied to tubes in the unfertilized plots. Enough tubes were placed into each plot to allow removal of two on each of six sampling dates. One tube from each group (east and west) was randomly removed beginning on the day of fertilization and continuing through post harvest. The remaining five sample dates following initiation were 7 Apr., 24 Apr., 11 May, 2 Jun., and 20 Jun. at Lahoma, and at Stillwater they were 6 Apr., 21 Apr., 10 May, 31 May, and 21 Jun. After extraction, the ends of each tube were sealed with plastic. Tubes were placed in chest-type coolers containing dry-ice in the field and transported back to the laboratory where they were stored in a horizontal

position at 2°C. All samples were then sectioned into depth increments of 0 to 15, 15 to 30, and 30 to 45 cm, freeze dried, and ground, taking care not to allow cross contamination. Bulk density was calculated using the volume and weight of soil removed from each depth increment.

Two soil cores (one from the east and west half of each plot) were sectioned and each depth increment analyzed for total- and inorganic-N. Inorganic-N included NH_4^+ -, NO_2^- -, and NO_3^- -N. In all cases NO_2^- -N was considered insignificant and was included with all NO_3^- -N determinations. Organic-N was computed as the difference between total- and inorganic-N. Total-N samples were prepared using modified Kjeldahl digestion which included permanganate-reduced iron to include NO_2^- - and NO_3^- -N (Bremner and Mulvaney, 1982). Inorganic-N samples were prepared as 2 M KCl extracts (Bremner and Edwards, 1965). Quantitative N determinations were made using a Lachat QuickChem AE auto-analyzer (Lachat Instruments, Inc., Milwaukee, WI). Isotope-ratio analysis of total-N samples were completed using the diffusion process outlined by Liu and Mulvaney (1992). Samples from the east and west halves of the experiment units were treated as subsamples and analysis of variance was performed by subsample group (two groups, east and west). No differences were found between either mean squared errors or treatment means for total-, or inorganic-N, or isotopic-ratio analysis of the total-N fraction when comparing groups. Therefore, east and west samples were combined for use in subsequent analyses.

To reduce the number of analyses that remained, inorganic-N isotope-ratio analyses were completed on composite samples. Soil samples taken on the same date from the same plot (east and west sides) were combined to form a composite sample for each depth. Steam distillation was used to prepare samples for isotope-ratio analysis of inorganic-N. Distillations were completed on two 100 mL aliquots of the same soil extract (250 mL of 2 M KCl was used to extract 25 g soil) using 300 mL distillation flasks. One aliquot was distilled with MgO and one with MgO + Devarda's alloy

(Bremner and Edwards, 1965) so that samples including NH_4^+ -N and $(\text{NH}_4^+ + \text{NO}_3^-)$ -N, respectively could be prepared for isotope-ratio analysis. A wash procedure, including back flushing the entire system with one 100 mL aliquot of 0.1 M HCl and two 100 mL aliquots of deionized-distilled H_2O , followed by steaming 20 mL of deionized-distilled H_2O for 5 min was completed between each sample. Tests with ^{15}N showed this to be effective. Potential cross-contamination was further reduced by arranging the order of the analysis of samples according to the expected increasing ^{15}N enrichment. All steam distilled samples included a 5 mL addition of $20 \mu\text{g NH}_4^+$ -N mL^{-1} to ensure adequate N for isotope-ratio analysis that was performed using an automated mass spectrometer (Nuclide/MAAS Model 3-60-RMS) as described by Mulvaney et al. (1990). Atom % ^{15}N for the NH_4^+ - and NO_3^- -N fractions were computed as described by Hauck (1982) and the quantity of fertilizer N in each fraction was computed using equations given by Westerman et al. (1972). Only samples from the first sample date in the unfertilized treatment underwent ^{15}N analysis as described above to establish the natural ^{15}N abundance for each N fraction at both locations. In this paper none of the other measured parameters from the unfertilized plots were utilized.

Soil pH was measured with a glass electrode (1:1 soil-to-water ratio), organic C by the Walkley-Black procedure (Nelson and Sommers, 1982), and CEC by the method described by Polemio and Rhoades (1977). This information along with soil bulk density and texture are reported in Table 1.

Cumulative heat units after fertilization (CHUAF) were calculated as described by Gomes and Loynachan (1983) using daily maximum and minimum soil temperatures. Cumulative precipitation after fertilization (CPAF) was calculated as the sum of daily rainfall events. These cumulative parameters are plotted in Fig. 1 (CHUAF) and 2 (CPAF).

Table 1. Selected chemical and physical soil properties at Stillwater and Lahoma, Oklahoma.

Soil Depth cm	pH	Organic Carbon g kg ⁻¹	Exchange Capacity cmol kg ⁻¹	Bulk Density Mg m ⁻³	Sand	Texture Silt Clay %	
Stillwater							
0-15	5.8	8.03	27.03	1.70	20	54	25
15-30	6.4	6.88	32.17	1.65	16	51	32
30-45	7.1	5.36	39.19	1.65	12	46	43
Lahoma							
0-15	5.7	7.76	25.28	1.58	16	66	18
15-30	6.7	7.14	33.40	1.59	14	65	21
30-45	7.3	6.62	37.41	1.63	12	58	30

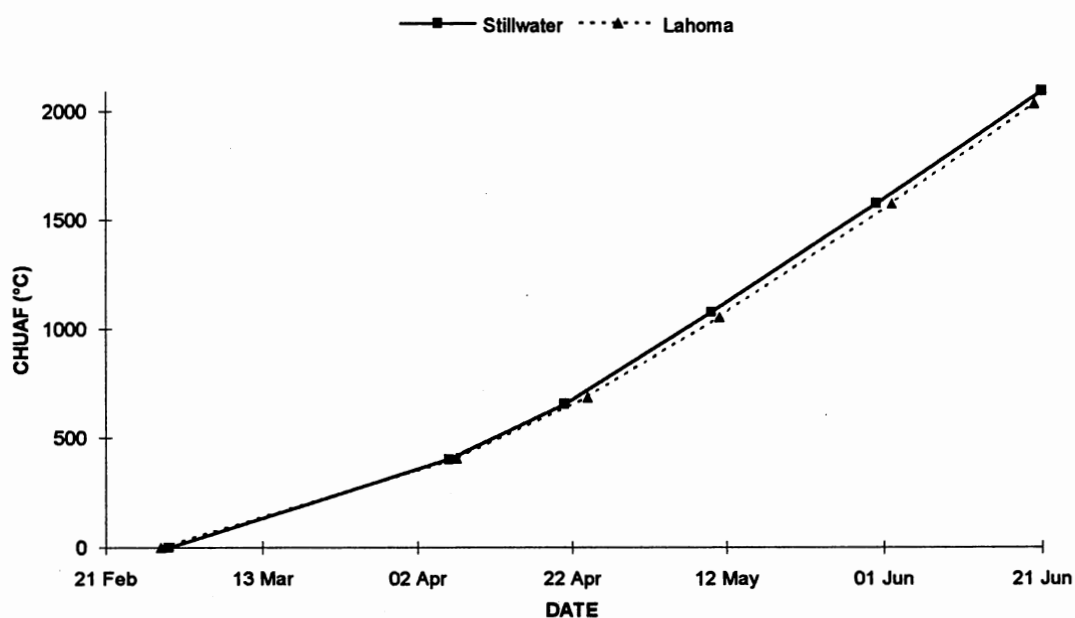


Fig. 1. Cumulative heat units after fertilization (CHUAF) at Stillwater and Lahoma Oklahoma, 1989.

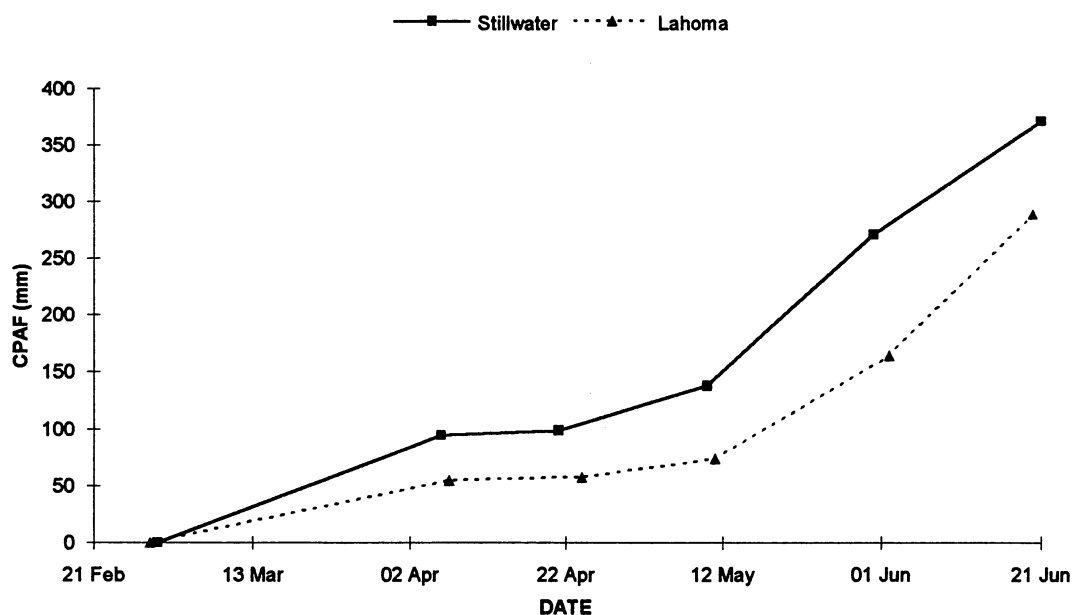


Fig. 2. Cumulative precipitation after fertilization (CPAF) at Stillwater and Lahoma Oklahoma, 1989.

RESULTS AND DISCUSSION

After the last sample date and before any of the soil cores had been sectioned or freeze-dried, the refrigeration unit in which the samples were being stored malfunctioned. This resulted in warming of samples for 24 to 36 h before the problem was detected. All samples were then transferred to another cold storage facility until sectioning and freeze-drying could be accomplished. Thus, the amount of N in a given fraction is the result of the sample status in the field at the time of sampling plus any transformations that occurred during storage.

Recovery of total fertilizer N was not affected on the first sample date (104 and 89% at Stillwater and Lahoma, respectively), but there was transformation of N at both locations in the 0 to 15 cm soil depth. At Stillwater the fertilizer N recovered was partitioned as 50% NH_4^+ , 2% NO_3^- , and 48% organic-N. At Lahoma the recovered

fertilizer N was found to be 21% NH_4^+ , 50% NO_3^- , and 30% organic-N. Since the samples were taken from the field within an hour of $(^{15}\text{NH}_4^+)_2\text{SO}_4$ application, the labeled-N should have been mostly in the NH_4^+ form. The only logical explanation for the amount of immobilization at Stillwater and nitrification and immobilization at Lahoma is microbial transformation during sample storage.

It is obvious that immobilization and nitrification proceeded rapidly, although differentially, with regard to nitrification for Stillwater and Lahoma samples. The factors that limit nitrification are NH_4^+ , O_2 , CO_2 , pH, and temperature (Schmidt, 1982). Anderson and Boswell (1964), working with several different soils, showed markedly different nitrification rates when soil temperature increased above 8 to 10° C. Apparently samples from Lahoma differed from Stillwater in either the degree of warming or nitrifying organisms and their activity. In addition, NH_4^+ (as well as NO_3^- in the absence of NH_4^+) is subject to immobilization by heterotrophic organisms and this can be stimulated by N fertilizer addition (Jansson and Persson, 1982).

Because of the stimulating effect of recently added $(^{15}\text{NH}_4^+)_2\text{SO}_4$, we believe that N transformations were significant for the first sample date but minimal for later sample dates. Samples taken after the first date had five or more weeks to establish a pseudo-equilibrium between added fertilizer N and microbial activity. Therefore, a short period of warming would result in a continuation of microbial activity without any dramatic changes as was apparent for the first sample date. The results from Stillwater and Lahoma do show N transformations in line with expected results in samples after the first sample date. In order to correct for changes during sample storage, the amount of N for the first sample date was adjusted to reflect the actual amount of N applied as NH_4^+ -N. This was accomplished by making the NH_4^+ - and total- ^{15}N for the first sample date equal the 22.5 kg $^{15}\text{N ha}^{-1}$ added. This corrected data is used in all the statistical analysis and discussion. Figures 3 and 4 show the means of each fertilizer N fraction for both locations plotted with the modified data for the first sample date.

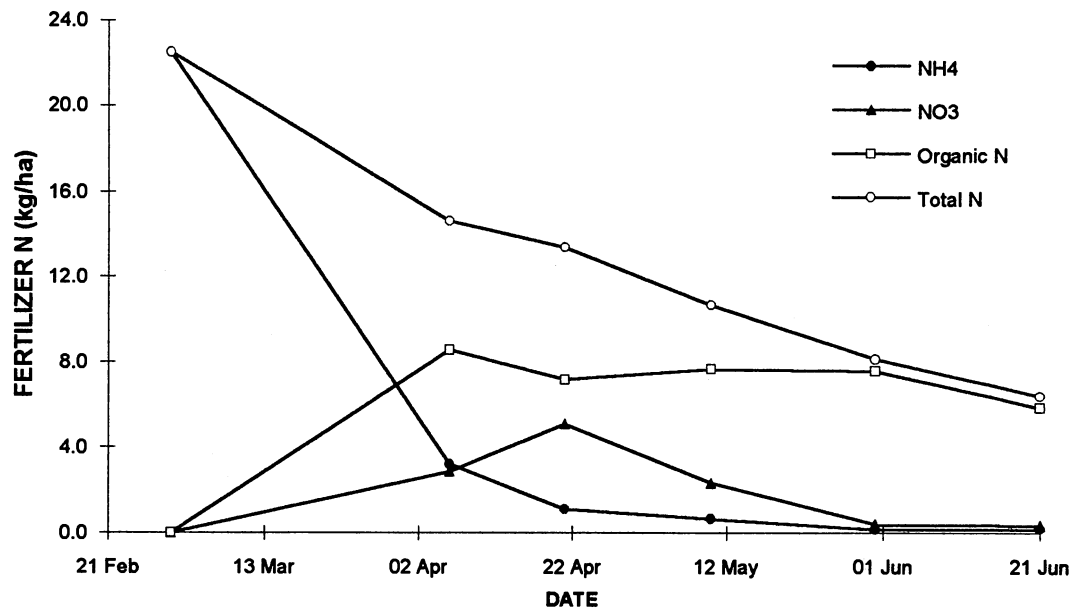


Fig. 3. Recovery of fertilizer N in each fraction in the 0 to 15 cm soil depth at Stillwater Oklahoma, 1989.

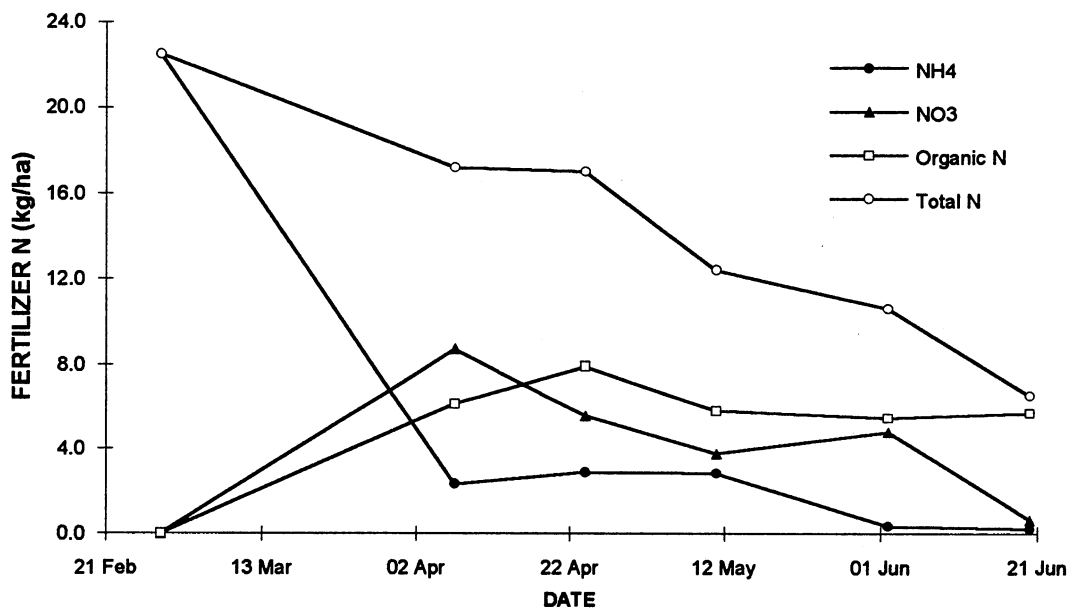


Fig. 4. Recovery of fertilizer N in each fraction in the 0 to 15 cm soil depth at Lahoma Oklahoma, 1989.

Tables 2 and 3 give the amount of fertilizer N recovered within each depth increment and for the 45 cm soil profile. Also given is the distribution (% of the 0 to 45 cm profile) of fertilizer N for each depth increment and N fraction for Stillwater and Lahoma. By the final sample date only 35 and 48% of the total fertilizer N remained in the 0 to 45 cm soil profile at Stillwater and Lahoma, respectively. Of the ^{15}N remaining in the entire 45 cm profile, the majority remained in the 0 to 15 cm soil depth at each location as organic N. A larger proportion of NO_3^- -N remained in 45 cm soil profile at Lahoma than at Stillwater.

Table 2. Fertilizer N recovered as total-, inorganic-, and organic-N at Stillwater Oklahoma, 1989.

Date	Soil Depth Interval (cm)			
	0-15	15-30	30-45	0-45
	kg ¹⁵ N ha ⁻¹			
Total N				
01 Mar	22.5 (100)†	0.0 (0)	0.0 (0)	22.5 (100)
06 Apr	14.6 (65)	1.5 (7)	0.9 (4)	17.0 (76)
21 Apr	13.4 (60)	1.1 (5)	0.8 (4)	15.3 (68)
10 May	10.7 (47)	1.2 (5)	0.6 (3)	12.4 (55)
31 May	8.1 (36)	0.6 (3)	0.5 (2)	9.3 (41)
21 Jun	6.4 (28)	0.7 (3)	0.8 (3)	7.8 (35)
NH ₄ ⁺				
01 Mar	22.5 (100)‡	0.0 (0)	0.0 (0)	22.5 (100)
06 Apr	3.2 (19)	0.1 (0)	0.0 (0)	3.3 (19)
21 Apr	1.1 (7)	0.1 (0)	0.1 (0)	1.2 (8)
10 May	0.7 (5)	0.1 (1)	0.0 (0)	0.8 (6)
31 May	0.2 (2)	0.0 (0)	0.0 (0)	0.2 (2)
21 Jun	0.2 (2)	0.1 (1)	0.1 (1)	0.3 (4)
NO ₃ ⁻				
01 Mar	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)
06 Apr	2.9 (17)	0.9 (5)	0.4 (3)	4.2 (25)
21 Apr	5.1 (33)	0.6 (4)	0.2 (2)	5.9 (39)
10 May	2.3 (19)	0.4 (3)	0.1 (1)	2.9 (23)
31 May	0.4 (4)	0.1 (1)	0.0 (0)	0.5 (5)
21 Jun	0.4 (5)	0.1 (1)	0.0 (0)	0.5 (6)
Organic-N				
01 Mar	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)
06 Apr	8.6 (50)	0.5 (3)	0.4 (3)	9.5 (56)
21 Apr	7.2 (47)	0.5 (3)	0.5 (3)	8.2 (53)
10 May	7.7 (62)	0.7 (5)	0.5 (4)	8.8 (71)
31 May	7.6 (82)	0.5 (5)	0.5 (5)	8.6 (92)
21 Jun	5.9 (75)	0.5 (7)	0.7 (9)	7.1 (90)

† Values in parenthesis for Total N are % of the 22.5 kg ¹⁵N ha⁻¹ applied.

‡ Values in parenthesis for NH₄⁺, NO₃⁻, and Organic N are % of the Total N in the 0 to 45 cm profile for each date.

Table 3. Fertilizer N recovered as total-, inorganic-, and organic-N at Lahoma Oklahoma, 1989.

Date	Soil Depth Interval (cm)			
	0-15	15-30	30-45	0-45
	kg ¹⁵ N ha ⁻¹			
Total N				
28 Feb	22.5 (100)†	0.0 (0)	0.0 (0)	22.5 (100)
07 Apr	17.2 (76)	1.7 (8)	0.8 (3)	19.6 (87)
24 Apr	17.0 (76)	1.1 (5)	1.4 (6)	19.6 (87)
11 May	12.4 (55)	1.2 (6)	0.5 (2)	14.1 (63)
02 Jun	10.6 (47)	2.2 (10)	1.9 (8)	14.6 (65)
20 Jun	6.5 (29)	2.0 (9)	2.3 (10)	10.8 (48)
NH ₄ ⁺				
28 Feb	22.5 (100)‡	0.0 (0)	0.0 (0)	22.5 (100)
07 Apr	2.3 (12)	0.3 (1)	0.1 (0)	2.7 (14)
24 Apr	2.9 (15)	0.1 (1)	0.1 (1)	3.2 (16)
11 May	2.8 (20)	0.2 (1)	0.1 (0)	3.0 (22)
02 Jun	0.3 (2)	0.1 (0)	0.1 (1)	0.5 (4)
20 Jun	0.2 (2)	0.1 (1)	0.2 (2)	0.5 (5)
NO ₃ ⁻				
28 Feb	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)
07 Apr	8.7 (45)	0.9 (4)	0.5 (2)	10.1 (51)
24 Apr	5.6 (28)	0.7 (3)	0.2 (1)	6.5 (33)
11 May	3.8 (27)	0.6 (4)	0.1 (1)	4.5 (32)
02 Jun	4.8 (33)	2.1 (15)	0.8 (5)	7.7 (53)
20 Jun	0.6 (6)	1.2 (11)	0.8 (8)	2.7 (25)
Organic N				
28 Feb	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)
07 Apr	6.1 (31)	0.5 (3)	0.2 (1)	6.9 (35)
24 Apr	7.9 (40)	0.3 (2)	1.1 (5)	9.3 (48)
11 May	5.8 (41)	0.5 (4)	0.3 (2)	6.6 (47)
02 Jun	5.5 (37)	0.1 (1)	0.9 (6)	6.5 (45)
20 Jun	5.7 (53)	0.6 (6)	1.2 (11)	7.5 (70)

† Values in parenthesis for Total N are % of the 22.5 kg ¹⁵N ha⁻¹ applied.

‡ Values in parenthesis for NH₄⁺, NO₃⁻, and Organic N are % of the Total N in the 0 to 45 cm profile for each date.

The data in Tables 2 and 3 illustrate that loss of fertilizer N from the profile occurred. To assist in determination of the most probable mechanism of fertilizer N loss, Chemical Movement in Layered Soils (CMLS), a computer model by Nofziger and Hornsby (1986) was used to estimate leaching potential of surface applied NO_3^- -N. The soil parameters required by CMLS include volumetric water content at the permanent wilting point, field capacity, and saturation. These were estimated from the bulk density and texture at each soil depth (Table 1) using procedures described by Cambell (1985) for water content at the wilting point and field capacity. Saturated water content was computed using bulk density from relationships given by Hillel (1980). Daily precipitation and soil parameters were used by CMLS to predict movement of NO_3^- , surface applied on 1 Mar. and 28 Feb. at Stillwater and Lahoma, respectively. Since there was no crop uptake of fertilizer N in the field, rooting depth was entered as zero for each location. This resulted in no evaporational loss of water in the simulation. The depth to which CMLS predicted NO_3^- -N movement on each sample date is shown in Table 4 for both locations. When the simulated NO_3^- -N movement and the fertilizer NO_3^- -N recovered at Stillwater (Table 2) is compared, there is agreement that by 6 Apr., NO_3^- would have moved to within the 15 to 30 cm soil depth. However, there was also an increase in NO_3^- on 6 Apr. at the 30 to 45 cm soil depth and CMLS did not predict NO_3^- movement to that depth until 10 May. After 31 May, according to CMLS, all NO_3^- would be below the 45 cm sampling depth at Stillwater. At Lahoma, the amount of fertilizer NO_3^- -N which moved to the 15 to 30 and 30 to 45 cm soil depths by the second and third sample dates is nearly the same as found at the same depths and sample dates at Stillwater (Tables 2 and 3). There was considerably greater movement of NO_3^- to lower depths when compared to simulated movement at Lahoma (Table 4). CMLS predicted that NO_3^- would move to the 30 to 45 cm soil depth by 2 June, when there was an observed increase in NO_3^- at this depth by 7 Apr. and in total N by 24 Apr.

Table 4. Cumulative infiltration between sample dates and movement of surface-applied NO_3^- as predicted by CMLS (Nofziger and Hornsby, 1986).

Location	Date	Cumulative Infiltration	Predicted Depth of NO_3^-
		mm	cm
Stillwater	01 Mar	0	0
	06 Apr	95	26
	21 Apr	4	27
	10 May	39	37
	31 May	133	73†
	21 Jun	100	99
Lahoma	28 Feb	0	0
	07 Apr	55	14
	24 Apr	3	15
	11 May	16	19
	02 Jun	90	42
	20 Jun	125	74

† Predictions of NO_3^- movement beyond 45 cm depth assumes same soil properties as in the 30 to 45 cm soil depth.

Denitrification is another potential mechanism of N loss from these samples. There was evidence of restricted water percolation within some of the tubes at both locations. Field notes taken frequently at Stillwater and once at Lahoma revealed that water would pond at the surface of some of the tubes. For instance, at Lahoma there was 9, 36 and 10 mm of precipitation on 22, 28, and 30 Mar., respectively, and notes on 1 Apr. record from 1.5 to 13 mm of water standing on the surface of 8 tubes (only two of these were in fertilized plots). Similarly, at Stillwater on 4 and 6 Mar., there was 20 and 3 mm of precipitation, respectively, and on 13 Mar. it was noted that 9 tubes had from 1 to 13 mm of water on the soil surface (4 were in fertilized plots). Although it is difficult to quantify loss of N due to denitrification, its occurrence seems likely.

Analysis of variance for both locations revealed a significant depth by date interaction for all ^{15}N fractions. Therefore, regression analysis was performed by depth

for each fertilizer N fraction. No significant relationships were found when modeling subsurface samples and therefore only results from the 0 to 15 cm soil depth are reported. Table 5 shows these results for both locations and each fertilizer N fraction, except for a combined location model for NH_4^+ -N. The combined location model for NH_4^+ -N replaced separate models for each location after models containing identical independent variables and homogeneity of error variance for both locations were determined.

These models (Table 5) not only describe a significant proportion of the variability in the data using the environmental variables of cumulative heat units and precipitation after fertilization (CHUAF and CPAF), but they also describe the fertilizer N transformations in line with physical and biological expectations. For instance, the formation and loss of fertilizer NO_3^- is described by its substrate requirement (NH_4^+), the biological requirement of heat (CHUAF) and moisture (CPAF) for stimulated nitrification, and CPAF which promotes leaching and/or denitrification. Gomes and Loynachan (1984) reported a linear decrease in percent NH_4^+ recovered with increasing CHUAF. In this study there was a logarithmic decrease in ammonium- ^{15}N as a function of CHUAF as shown in Fig. 5.

Table 5. Regression models for each fertilizer N fraction in the 0 to 15 cm soil depth at Stillwater and Lahoma Oklahoma, 1989.

Location	Dependent Variable	Independent Variable	Coefficient Estimate	R ²	MSE†
Stillwater	Total N	Constant	22.26393 ***	0.95	1.7
		‡ CHUAF	-0.01782 ***		
		§ CPAF ²	-0.00052 **		
		CHUAF × CPAF	-0.00012 **		
	NO ₃ ⁻	Constant	8.86955 ***	0.96	0.2
		NH ₄ ⁺	-0.39282 ***		
		CHUAF	0.01631 ***		
		CHUAF ²	-0.00002 ***		
		CPAF	-0.13467 ***		
		CHUAF × CPAF	0.00013 ***		
	Organic N	Constant	0.03878 NS	0.91	1.2
		CHUAF	0.01680 ***		
		CHUAF ²	0.00004 *		
		CPAF ²	0.00238 **		
		CHUAF × CPAF	-0.00068 **		
Lahoma	Total N	Constant	22.53424 ***	0.89	4.9
		CHUAF ²	-0.00002 *		
		CPAF	-0.10344 *		
		CPAF ²	-0.00123 +		
		CHUAF × CPAF	0.00034 *		
	NO ₃ ⁻	Constant	21.38010 ***	0.84	2.9
		NH ₄ ⁺	-0.95227 ***		
		CHUAF	-0.03675 *		
		CPAF ²	-0.00174 *		
		NH ₄ × CHUAF	0.00229 +		
		CHUAF × CPAF	0.00034 *		
	Organic N	Constant	0.06442 NS	0.81	1.8
		CHUAF	0.01793 ***		
		CHUAF ²	-0.00001 ***		
		CHUAF × CPAF	0.00005 **		
Combined¶	NH ₄ ⁺	Constant	22.16212 ***	0.98	1.4
		# LN(CHUAF+1)	-3.00318 ***		

*, **, ***, + Significance at the 0.05, 0.01, 0.001, and 0.10 probability levels, respectively. NS Not significant.

† Model mean square error.

‡ Cumulative heat units after fertilization (CHUAF).

§ Cumulative precipitation after fertilization (CPAF).

¶ Combined location model.

Natural logarithm of CHUAF. A constant of one is added to each since the natural logarithm of zero is undefined.

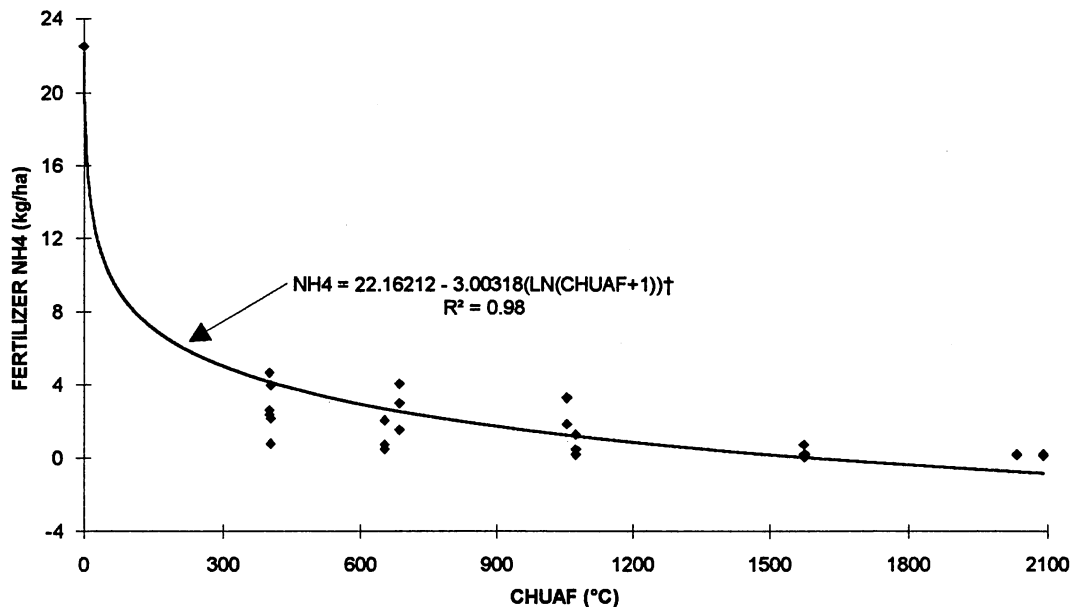


Fig. 5. The disappearance of fertilizer NH_4^+ -N as a function of cumulative heat units after fertilization (CHUAF).

† A constant of one is added to CHUAF since the natural logarithm of zero is undefined.

CONCLUSIONS

Difficulties encountered during cold storage of soil samples encumbered interpretation of the data relative to N transformations. Proper sample pretreatment procedures and analysis of a "time-zero" sample to check ^{15}N recovery are clearly a high priority for this type of research, to detect problems such as those that arose in this study. By making the assumption that only the first sample date was significantly altered in storage due to the stimulating effect of recently added N fertilizer, analysis of data followed soil-biological expectations. Ammonium- ^{15}N was rapidly immobilized in the surface 15 cm soil depth by the second sample date. This immobilized ^{15}N then remained at relatively stable levels for the duration of sampling. The portion of ^{15}N not immobilized was nitrified and lost from the 0 to 45 cm soil profile. By the last sample date only 35 and

48% of the total fertilizer N could be accounted for in the 0 to 45 cm profile (28 and 29% of the total N was found in the 0 to 15 cm soil depth) at Stillwater and Lahoma, respectively. This is equivalent to 82 and 60% of the fertilizer N remaining in the 0 to 15 cm soil depth at Stillwater and Lahoma, respectively. Ninety and 70% of the fertilizer N remaining in the 0 to 45 cm profile was organic N and 75 and 53% of that organic N was in the 0 to 15 cm depth at Stillwater and Lahoma, respectively. Apparently, if fertilizer N was not immobilized it underwent nitrification and then under the conditions of this experiment, was lost via leaching and/or denitrification.

The environmental variables CHUAF and CPAF were effectively utilized to model changes in all fertilizer N fractions. Disappearance of ammonium- ^{15}N was found to be a logarithmic function of CHUAF for both locations. This indicates that broadcast application of ammoniacal fertilizer sources in the spring will be readily nitrified and available for crop uptake. To illustrate how rapidly NH_4^+ was initially transformed, the daily CHUAF and predicted disappearance of NH_4^+ -N is plotted for the first 16 days of the study at both locations in Fig. 6. After only 10 days there was about 30 to 35 CHUAF and 50% of fertilizer N was transformed (Fig. 6). Gomes and Loynachan (1983) reported 60% recovery of NH_4^+ -N applied as anhydrous ammonia after an accumulation of 233 heat units in Iowa.

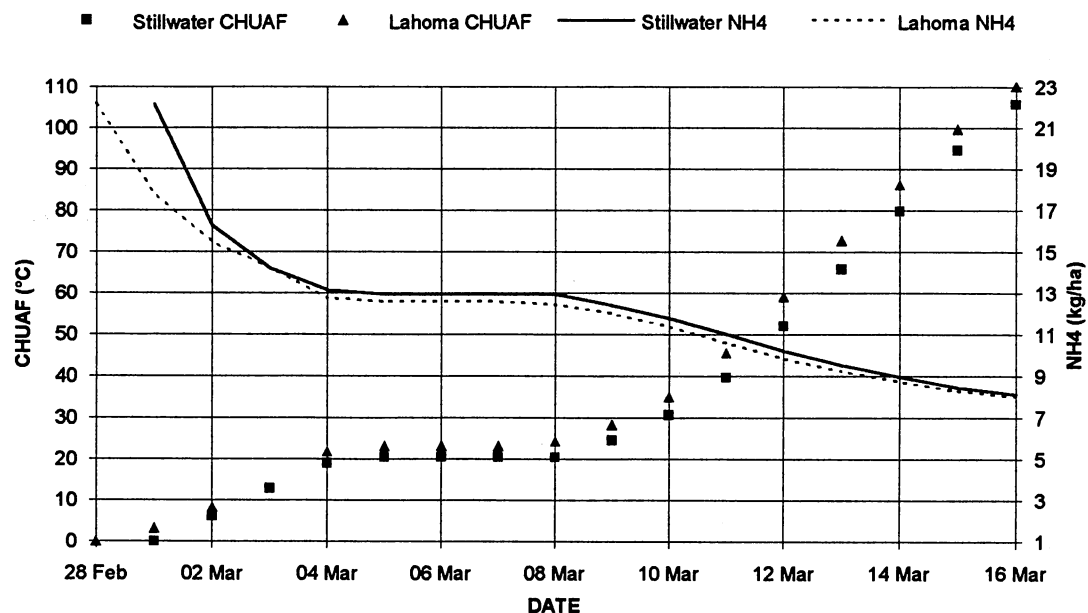


Fig. 6. Daily CHUAF and predicted disappearance of NH_4^+ -N for the first 16 days of the study at Stillwater and Lahoma Oklahoma, 1989.

Total ^{15}N decreased over the entire sampling period due to losses of NO_3^- -N via leaching and/or denitrification. Apparently, after nitrification, $^{15}\text{NO}_3^-$ did not enter the immobilization-mineralization turnover cycle. This was either due to loss of NO_3^- before immobilization was possible, or exclusion of NO_3^- -N from immobilization by microbial preference. Jansson (1958) states that NH_4^+ is preferred over NO_3^- by heterotrophic microorganisms during immobilization. Later Jansson and Persson (1982) reported that nitrification results in withdrawal of inorganic N from the mineralization-immobilization turnover cycle. In other words, once nitrified, NO_3^- is excluded from the turnover pathway as long as net immobilization persists, though it is still available for plant uptake (Jansson and Persson, 1982).

The in situ technique of Raison et al. (1987), was not found to be a satisfactory method of conducting ^{15}N tracer studies in Oklahoma. This method was conducive to soil

compaction during installation of tubes and conditions that enhanced leaching and denitrification. These results likely represent an upper limit of fertilizer N loss from the 0 to 45 cm profile, since there was no crop uptake of N and all precipitation was trapped by the tube enclosing the soil cores. Overall, $^{15}\text{NH}_4^+$ -N was rapidly nitrified which resulted in loss of ^{15}N from the 0 to 45 cm sample profile. Apparently, this loss of NO_3^- was predominantly due to leaching, but conditions were also favorable for denitrification. The majority of fertilizer N which remained at the last sample date in the entire sample profile was found in the 0 to 15 cm depth as immobilized N.

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CHAPTER II

MAXPROFIT - A SOFTWARE TOOL FOR TEACHING PRINCIPLES OF ECONOMIC FERTILIZER USE

ABSTRACT

Teaching principles of economic fertilizer use generally includes a discussion of maximum yield versus maximum profit. This can be straight forward with simple cases of a single crop and nutrient, but if multiple nutrients are applied to more than one crop, the calculation of profitability becomes significantly more difficult. When capital is limited, the maximum economic fertilizer rate may not be feasible and resources must be shared. The objective of this project was to develop a software tool that would assist in studying economic fertilizer use when considering (a) more than one crop nutrient applied to each crop simultaneously, (b) the difference between maximizing crop yield and profit with regard to nutrient requirements, crop yield, cost, and profit, and (c) how nutrient usage can be distributed among crops when the available capital is limited. MaxProfit is a computer program that completes all the above computations for up to five crops and up to five nutrients applied to each crop simultaneously. The student begins by entering yield and nutrient rate data and the software determines a quadratic yield equation for each crop. This crop production function along with crop price, fertilizer price, and fixed cost are used to calculate the required nutrient rates, yield, cost, and profit at the maximum crop yield and maximum economic yield. If the maximum economic yield requires expenditures above available capital then the nutrient rates, yield, cost, and profit required to maximize

profit subject to the capital constraint can be computed. Calculated Results are given in tabular form.

INTRODUCTION

The use of computer software as a teaching tool in classroom instruction can be of great value for demonstrating how fundamental principles can be used to solve difficult problems. Teaching the principles of economic fertilizer use can be included in soil fertility courses and generally includes a discussion of maximum yield versus maximum profit. Barreto and Westerman (1987) utilized computer software to provide insight into the use and application of various proposed yield response models for a single crop and nutrient. Teaching economic fertilizer use can be straight forward with simple cases of a single crop and nutrient. If multiple nutrients are applied to more than one crop, the calculation of profitability becomes significantly more difficult. When capital is limited, the maximum economic fertilizer rate may not be feasible and resources must be shared. Isfan (1986) described a method of sharing a single fertilizer nutrient between two crops in a limited capital situation using a quadratic yield equation.

The objective of this project was to provide a software tool that would assist in studying economic fertilizer use when considering, (a) more than one crop and more than one nutrient applied to each crop simultaneously, (b) the difference between maximizing crop yield and maximizing profit with regard to nutrient requirements, crop yield, cost, and profit, and (c) how nutrient usage can be distributed among crops when the available capital is limited and the maximum economic yield is not feasible.

PROGRAM USE AND DATA REQUIREMENTS

The computer program (MaxProfit) was written in the C programming language and compiled for use on IBM personal computers, and is available on request from the authors (see "Software Specifications"). MaxProfit is an interactive computer program

with pop-up menus and data entry forms in which all information is entered. All data entered can be saved in a binary data file and reused or modified. The calculated results can be viewed on the computer's video display, printed, using an attached printer, or saved in a text file. MaxProfit requires the user to enter data for one to five different crops (or locations) and one to five different nutrients applied to those crops simultaneously. Crop yield data for each nutrient applied must be entered for three rates, 0, "medium", and "high", where "medium" and "high" are numeric rates entered by the user. Yield data associated with the three nutrient rates must define a quadratic yield response equation that is concave-down. The program uses the yield data for each crop to compute the quadratic production function and displays the function coefficients for each nutrient. The user must then enter the crop price, fertilizer, and fixed costs. Using the crop production function, crop price, fertilizer price, and fixed cost information, MaxProfit can calculate the required nutrient rates, yield, cost, and profit at the maximum crop yield and maximum economic yield. If the maximum economic yield requires expenditures above available capital then the nutrient rates, yield, cost, and profit required to maximize profit subject to the capital constraint can be computed. Ease of changing previously entered data allows the software-user to quickly see how each input affects the results.

MATHEMATICAL MODELS AND CALCULATIONS

Since this program is designed to be a teaching tool rather than a predictive model in the field, the quadratic yield equation provides a simple expression of decreasing marginal returns with increasing fertilizer input. The production function used to describe crop yield is a quadratic equation with no cross-product terms which takes the following form:

$$Y_i(X_j) = \beta_0 + \beta_1 X_1 + \beta_{12} X_1^2 + \beta_2 X_2 + \beta_{22} X_2^2 + \dots + \beta_m X_m + \beta_{m2} X_m^2 \quad \text{Eq. [1]}$$

where Y is the yield equation for crop i , β_j and β_{j2} are the first- and second-order equation coefficients for nutrient X_j (j from one to five), respectively, that are included in the production of this crop. All second-order coefficients (β_{j2}) must be less than zero.

The quadratic equation with no interaction terms was selected for the yield response equation primarily for its simplicity. Mathematically, the quadratic equation has some nice properties that simplify the calculations that must be made. When expressed as stated above (all second-order coefficients less than zero) the resulting quadratic function is positive-definite. This assures that there exists a unique global maximum. The maximum is found where the first partial derivative of the quadratic equation with respect to each X_j equals zero. The contours of Eq. [1] are ellipsoids (Gill et al., 1981). This means that if the global maximum is outside the domain of the data (outside of the range of nutrient rates entered in the yield data), one variable (the variable outside the domain) can be moved to the nearest boundary (i.e., either 0 or the maximum nutrient rate). If the solution results in more than one X_j outside the bounds of the data then the X_j farthest from its respective bound is set equal to that bound. This effectively removes one variable from the system of equations which then undergoes another solution iteration. This continues until either all remaining nutrients are removed from the system of equations (by setting them equal to the appropriate boundary) or the final iteration results in a solution where the remaining nutrients are within the domain of the data. This will result in a maximum value within the domain of the data (Gill et al., 1981). All of the following types of problems use this property in the calculations.

Maximum Yield

This quantity is calculated by setting the gradient vector (vector containing the first partial derivatives of Y_i (Eq. [1] with respect to each X_j) equal to the zero vector (vector containing only zeros) and solving for each X_j (nutrient rate). As described above, this is repeated until all X_j are within the bounds of the yield and nutrient data.

Maximum Economic Yield

The profit function is the difference between the product of the crop price and the production function and the total cost. This is expressed as follows:

$$\Pi_i(X_j) = P_{Y_i} \cdot Y_i(X_j) - \sum_{j=1}^m (P_{X_j} \cdot X_j) - FC_i \quad \text{Eq. [2]}$$

Where Π is the profit function, Y is the production function, P_Y is the price of the crop, and FC is the fixed cost of producing crop i . The nutrient parameters used in producing crop i are P_{X_j} which is the price of nutrient X_j . Equation [2] reduces to a quadratic equation with the same properties as described for the yield equation. Thus, the maximum profit is found by solving the system of equations for each X_j when the gradient vector of Eq. [2] is equal to the zero vector. The values of each X_j are adjusted to the boundary value if necessary as described for maximum yield.

Maximum Profit Subject to a Capital Constraint

Unlike the maximization of yield or profit, the formulation of this problem requires that the field size (FS) on which each crop is to be grown be taken into account. The equation to be maximized then becomes a sum of the product of FS_i and Π_i . Similarly, the capital constraint becomes the sum of the product of the FS_i and the cost function for crop i . This problem is solved using a quadratic program subject to a linear equality constraint and simple bounds on all the variables (X_j). Note that this can be solved as a linear equality problem because the constraint equation is linear and the value of the constraint entered by the user is bounded below by the cost of production with no added nutrients and above by the cost of producing the maximum economic yield, this ensures that the solution exists. The problem can be stated as follows:

$$\text{Maximize:} \quad f(X_j) = \sum_{i=1}^n [FS_i \cdot \Pi_i(X_j)] \quad \text{Eq. [3]}$$

$$\text{Subject to:} \quad Ax = b \quad \text{Eq. [4]}$$

Where f is the function to be maximized, Π_i is as given in Eq. [2], and FS_i is the field size on which crop i is being grown. Since there is only one constraint equation (the capital constraint equation), Eq. [4] is greatly simplified. When placed in the same form as Eq. [4], A becomes a $1 \times n$ matrix (vector), x is a $n \times 1$ vector containing the unknown X_j 's (nutrient rates), and b is a 1×1 vector (constant). Before moving all constants to the right-hand side of the equation, Eq. [4] is set up as follows:

$$\sum_{i=1}^n \sum_{j=1}^m [FS_i \cdot C_i(X_j)] = CC \quad \text{Eq. [5]}$$

where CC is the specified capital constraint, C_i is the cost function for crop i , which has the following form:

$$C_i(X_j) = \sum_{j=1}^m [P_{X_j} \cdot X_j] + FC_i \quad \text{Eq. [6]}$$

The maximization of $f(X_j)$ is equivalent to the *minimization* of $-f(X_j)$, and since the mathematical theory is based on minimization, this is the form that is used. The sufficient conditions for a minimum of a linear equality constraint problem (LEP) are as follows (Gill et al., 1981):

1. $Ax^* = b$

where the feasible point x^* is a local minimum of LEP.

2. $Z^T g(x^*) = 0$

where Z^T is the transpose of an orthogonal basis of A (Venit and Bishop, 1985),

$g(x^*)$ is the gradient vector of $-f(X_j)$ and 0 is the zero vector.

3. $Z^T G(x^*) Z$ is positive definite.
 where $G(x^*)$ is the Hessian matrix of $-f(X_j)$ (matrix composed of the second partial derivatives of $-f(X_j)$ with respect to each X_j).

The steps that are used to solve LEP are as follows:

1. Put the objective function (Eq. [3]) in proper form and multiply by negative 1 (for maximization, see above).
2. Put the constraint equation in proper form; $Ax = b$.
3. Repeat the following until there is a solution for each X_j (rate of nutrient j) within the domain of the data:
 - A. Find the orthogonal basis (Z) of A .
 - B. Find the gradient vector ($g(x)$) of $-f(X_j)$.
 - C. Combine the equations $Ax = b$ and $Z^T g(x) = 0$ to form a system of n equations in n unknowns.
 - D. Solve the above system of equations for all X_j remaining in the problem.
 - E. For each remaining X_j , determine if any of the X_j are less than 0 or greater than the maximum nutrient rate. This condition indicates that one X_j should be removed from the problem.
 1. If X_j is less than 0, then store the position of the most negative X_j .
 2. If X_j is greater than the its maximum nutrient rate then store the position of the X_j that exceeds the boundary of its maximum nutrient rate by the greatest amount.
 - F. If any X_j are to be removed from the problem, then the X_j that is farthest from the boundary is remove by setting that X_j equal to the exceeded

boundary (0 or its maximum rate)¹.

- G. If no X_j are removed from the problem or there are no remaining X_j to remove then the current values of all X_j compose the solution.

EXAMPLE

In order to illustrate the use of MaxProfit, the following example is given. In this example there are three hypothetical fields named simply Field 1, Field 2, and Field 3. The first two fields are producing wheat and the third grain sorghum. Note that the following table of information is given only to show the data that was used and does not replicate the format of data entry that is used in the software. Table 1 shows the wheat and grain sorghum yield, fixed cost and crop price data for Fields 1, 2, and 3. The following crop production functions were calculated using the yield data in Table 1 and represent Fields 1, 2, and 3, respectively.

$$Y_1 = 20 + 6.33E - 1N - 3.11E - 3N^2 + 1.78E - 1P - 1.48E - 3P^2 + 2.50E - 2K - 1.56E - 4K^2$$

$$Y_2 = 38 + 9.08E - 1N - 4.86E - 3N^2 + 8.75E - 2P - 9.38E - 4P^2 + 5.83E - 2K - 6.94E - 4K^2$$

$$Y_3 = 35 + 3.20E - 1N - 1.10E - 3N^2 + 4.83E - 1P - 5.00E - 3P^2 + 1.50E - 1K - 1.25E - 3K^2$$

The cost of fertilizer used in this example is the same for all crops grown and must be given on a per unit weight basis of fertilizer material. In this example the price of the fertilizer material was 0.12, 0.06, and 0.12 \$ lb⁻¹ for N, P₂O₅, and K₂O, respectively. The analysis of fertilizer material for each nutrient must be entered separately, and in this example the fertilizer material was 82% N, 46% P₂O₅, and 60% K₂O. Since the calculations are made using the rate of N, P, and K, there is provision made for the user to enter conversion factors if the fertilizer material is given in the oxide form. In this example

¹ There is one exception to this rule. If the capital constraint entered by the user is less than the median of possible values, then the X_j that is most negative will be removed before any X_j that exceed its maximum nutrient rate if both of these conditions exist simultaneously. Thus, X_j are preferentially set to 0 when the capital constraint is small.

$\%P_2O_5$ and $\%K_2O$ are converted to $\%P$ and $\%K$ by multiplying by 0.44 and 0.83, respectively. Note that the user simply enters the conversion factors and MaxProfit does the necessary conversions.

Table 1. Example crop yield, fixed cost, and crop price data for Fields 1, 2, and 3 used by MaxProfit.

Nutrient	Field # and Crop					
	1 Wheat		2 Wheat		3 Grain Sorghum	
	Nutrient Rate	Crop Yield	Nutrient Rate	Crop Yield	Nutrient Rate	Crop Yield
	lb acre ⁻¹	bu acre ⁻¹	lb acre ⁻¹	bu acre ⁻¹	lb acre ⁻¹	cwt acre ⁻¹
-	0	20	0	38	0	35
N	75	50	60	75	100	56
N	150	45	120	77	200	55
P	45	25	40	40	30	45
P	90	24	80	39	60	46
K	80	21	60	39	40	39
K	160	20	120	35	80	38
	Fixed Cost	Crop Price	Fixed Cost	Crop Price	Fixed Cost	Crop Price
	\$ acre ⁻¹	\$ bu ⁻¹	\$ acre ⁻¹	\$ bu ⁻¹	\$ acre ⁻¹	\$ cwt ⁻¹
	70	2.80	60	2.80	55	3.66

After entering the above information, the user can calculate the maximum yield and maximum economic yield. If the user selects the maximum economic yield, the maximum profit subject to a capital constraint can also be computed. First the user must enter the capital constraint which is bounded below by the fixed cost of production (no fertilizer added) and above by the cost of producing the maximum economic yield. Tables 2, 3, and 4 show actual tables that were generated by MaxProfit for maximum yield, maximum economic yield, and maximum profit subject to a capital constraint of 43 thousand dollars. The following tables have the same format as those produced by MaxProfit, giving the

calculated rate of each nutrient required to produce the associated crop yield. From these values the cost and return parameters for each crop are determined on a per-unit-area basis. These include total value of the product (TVP), variable cost (VC), total cost (TC), and profit. The TVP is the gross return on the crop yield. Since the only variable input is the rate of each nutrient, the VC is actually the fertilizer cost. The TC then is the sum of fixed cost and VC. Profit is the difference between TVP and TC. At the bottom of each table is a listing of the cost and profit for the entire enterprise. This takes into account the size of the each field and gives the user a total figure of the cost and profit of growing these crops.

Table 2. Cost and return of crop production based on maximum yield.

Crop 1: 320 acre		Wheat	Field Identifier: Field 1			
Nutrient	Rate	Yield	TVP†	VC‡	TC§	Profit
	lb/ac	bu/ac			\$/ac	
N	101.8	58.6	163.98	55.68	125.68	38.30
P	60.0					
K	80.0					
Crop 2: 160 acre		Wheat	Field Identifier: Field 2			
Nutrient	Rate	Yield	TVP	VC	TC	Profit
	lb/ac	bu/ac			\$/ac	
N	93.4	83.7	234.36	41.05	101.05	133.31
P	46.7					
K	42.0					
Crop 3: 80 acre		Grain Sorghum	Field Identifier: Field 3			
Nutrient	Rate	Yield	TVP	VC	TC	Profit
	lb/ac	cwt/ac			\$/ac	
N	145.5	74.5	268.03	55.39	110.39	157.64
P	48.3					
K	60.0					
Total for Entire Enterprise:						
			Cost	\$ 65217.25		
			Profit	\$ 46197.03		

† Total value of the product.

‡ Variable cost.

§ Total cost.

Table 3. Cost and return of crop production based on maximum economic yield.

Crop 1: 320 acre		Wheat	Field Identifier: Field 1			
Nutrient	Rate	Yield	TVP†	VC‡	TC§	Profit
	lb/ac	bu/ac			\$/ac	
N	91.3	55.3	154.93	23.89	93.89	61.03
P	24.3					
K	0.0					
Crop 2: 160 acre		Wheat	Field Identifier: Field 2			
Nutrient	Rate	Yield	TVP	VC	TC	Profit
	lb/ac	bu/ac			\$/ac	
N	86.7	80.2	224.60	15.86	75.86	148.73
P	0.0					
K	0.0					
Crop 3: 80 acre		Grain Sorghum	Field Identifier: Field 3			
Nutrient	Rate	Yield	TVP	VC	TC	Profit
	lb/ac	cwt/ac			\$/ac	
N	122.4	72.6	261.47	42.28	97.28	164.20
P	40.1					
K	33.2					
Total for Entire Enterprise:						
		Cost	\$ 49965.57			
		Profit	\$ 56464.10			

† Total value of the product.

‡ Variable cost.

§ Total cost.

Table 4. Cost and return of crop production based on maximum profit subject to a capital constraint.

Crop 1: 320 acre		Wheat	Field Identifier:		Field 1	
Nutrient	Rate	Yield	TVP†	VC‡	TC§	Profit
	lb/ac	bu/ac			\$/ac	
N	59.9	46.8	130.95	10.95	80.95	49.99
P	0.0					
K	0.0					
Crop 2: 160 acre		Wheat	Field Identifier:		Field 2	
Nutrient	Rate	Yield	TVP	VC	TC	Profit
	lb/ac	bu/ac			\$/ac	
N	66.6	76.9	215.42	12.18	72.18	143.23
P	0.0					
K	0.0					
Crop 3: 80 acre		Grain Sorghum	Field Identifier:		Field 3	
Nutrient	Rate	Yield	TVP	VC	TC	Profit
	lb/ac	cwt/ac			\$/ac	
N	53.3	55.2	198.72	14.32	69.32	129.39
P	15.5					
K	0.0					
Total for Entire Enterprise:						
		Cost	\$ 43000.00			
		Profit	\$ 49266.58			

† Total value of the product.

‡ Variable cost.

§ Total cost.

SOFTWARE SPECIFICATIONS

MaxProfit requires an IBM² PC, AT, PS/2 or a compatible computer with at least 640K bytes of random access memory and either one 3.5" low- or high-density, or 5.25" high-density floppy disk drive. The operating system must be MS-DOS or PC-DOS 2.01 or later. A printer is useful but is not essential. The program and user's manual are available from the authors for a nominal fee. MaxProfit is public domain software and

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multiple copies for classroom use are permitted. Send requests to Dr. R. L. Westerman, Dep. of Agron., Oklahoma State Univ., Stillwater, OK 74078.

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VITA

Bryan L. Unruh

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Doctor of Philosophy

Thesis: I. IN SITU TRANSFORMATION OF SPRING-APPLIED AMMONIUM-¹⁵N
II. MAXPROFIT - A SOFTWARE TOOL FOR TEACHING PRINCIPLES
OF ECONOMIC FERTILIZER USE

Major Field: Soil Science

Biographical:

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