A NEW SEQUENTIAL ALLOCATION METHOD

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION AND LITERATURE REVIEW

Estimating the Mean

It is well known that when estimating the mean, μ, of a population, the estimate can be improved by judicious blocking. For finite populations, this can be accomplished by partitioning the population into k sub populations, called strata, in such a way that the variation within each stratum is small compared to the variation among the strata. Once the strata have been defined, how should the observations be allocated among the strata so that the variance of the estimator of the mean is minimized?

Suppose that for i = 1, 2, ..., k, the i^{th} stratum is of size N_i with mean μ_i. Denote by Y_{ij} the j^{th} element from the i^{th} stratum where j = 1, 2, ..., N_i. Let $N = \sum_{i=1}^{k} N_i$.

Neyman Allocation

A stratified random sample is drawn with n_i observations from the i^{th} stratum, i = 1, ..., k. Let y_{ij} be the j^{th} sample observation from stratum i and let $\bar{y}_i$ denote the arithmetic mean of these n_i observations. Neyman(1934) showed that the estimator:

$$\bar{y}_N = \frac{1}{N}\sum_{i=1}^{k} N_i \bar{y}_i \qquad (1)$$

is the best linear unbiased estimator of μ and that for a given total sample size $n=\sum_{i=1}^{k} n_i$, $V(\bar{y}_N)$ is minimized when the n_i are chosen by the following rule:

$$n_i = \frac{N_i S_i}{\sum_i N_i S_i} n \qquad (2)$$

where

$$S_i^2 = \frac{\sum_{j=1}^{N_i}(Y_{ij} - \bar{Y}_i)^2}{N_i - 1} = \sigma_i^2 \frac{N_i}{N_i - 1} \qquad (3)$$

This rule, known today as Neyman allocation, can only be used when the stratum sizes, N_i, and the stratum variances, σ_i^2, are known, for all i = 1, ..., k. In practice, the stratum sizes are often known; however, the stratum variances are rarely known.

Allocation When Population Variances Are Unknown

Without knowledge of the σ_i^2 values, Equation (2) cannot be used to evaluate the appropriate sample sizes, n_i. In such instances, researchers are forced to pursue other alternatives. When no information about the relative sizes of the stratum variances is known, it has been common practice to use proportional allocation, setting $n_i = n \frac{N_i}{N}$, or to replace the S_i^2 values in (2) by variance estimates. These variance estimates are provided by guesses, by data from preliminary studies, or from previous studies on similar data. In a recent journal article, Deshpande and

Prabhu-Ajgaonkar(1989) suggested other alternatives when some knowledge about the stratum variances is known. In particular, they considered two important cases: If the $(N_i\sigma_i)$'s are approximately equal, they showed that the number of observations should be divided equally among the strata. If the $\left[\frac{N_i}{\sigma_i}\right]$'s are approximately equal, then the stratum sizes should be taken proportional to the N_i^2.

It was Sukhatme(1935), who suggested using estimates of S_i^2 obtained from a pilot study involving a small number of observations. He showed that in certain examples the probability is high that the variance of $\bar{y}$ for this method is less than the variance of $\bar{y}$ for proportional allocation. For the same examples, he estimated the average gain in efficiency when using his method instead of the method of proportional allocation. He showed that there was an average gain in efficiency, that is on the average, the variance of the estimator using his allocation would be smaller than the variance of the estimator under proportional allocation. The fact that his results were based on examples was only one limitation of the study. He assumed normality of the stratum populations, and he assumed that the stratum population sizes were large.

Armitage(1947) showed that when the S_i^2 are known, the variance of $\bar{y}$ under Neyman allocation is no larger than the variance of $\bar{y}$ with proportional allocation. He also showed that this latter variance can be larger than the variance of $\bar{y}$ under simple random sampling. This undesirable situation occurs when the S_i^2 are equal.

Evans(1951) extended Sukhatme's work by deriving a lower bound for the sample sizes of the strata in the preliminary study. This lower bound ensures that the variance of this estimator would be less than the variance of the estimator with proportional allocation.

Sukhatme and Sukhatme(1970) developed a lower bound similar to Evans, except that it did not depend on knowledge of the coefficient of variation or of the Pearson kurtosis criterion. This lower bound for the sample sizes in the initial study was, however, equally impractical in that it requires knowledge of the stratum variances.

Multivariate Allocation

The problem of allocation becomes even more difficult if we consider multivariate estimators. It is very possible that the optimal allocation for one of the variates does not give minimum variance for the estimators of the other variates. One approach is to use a Neyman allocation for the variate deemed the most important, thereby reducing it to a univariate problem. Various approaches to allocation in the multivariate case have been suggested by Yates, Chatterjee, Kokan, and others.

Yates(1960) assigned a cost to each unit of variance, say c_j, for the estimator of j^{th} variate. He then attempted to choose the sample sizes n_i that minimize the total cost:

$$C = \sum_{j=1}^{L} c_j \ V(\bar{y}_j) \quad . \tag{4}$$

He showed that the appropriate n_i are given by:

$$n_i = \frac{\theta_i}{\sum_{i=1}^{k} \theta_i} n \tag{5}$$

where

$$\theta_i = N_i \sqrt{\frac{N_i}{N_i - 1} \sum_{j=1}^{L} c_j \sigma_{ij}^2} \tag{6}$$

and σ_{ij}^2 is the variance of the j^{th} variate in the i^{th} stratum. This solution is once again Neyman's solution with the stratum variances being replaced by the cost of one observation in that stratum, i.e., by

$$\sum_{j=1}^{L} c_j S_{ij}^2 = \frac{N_i}{N_i - 1} \sum_{j=1}^{L} c_j \sigma_{ij}^2 \tag{7}$$

Chatterjee's criterion(1967) was to allocate observations in such a way that the average relative increase in variance is minimized. He showed that for a given sample size n, this optimal allocation is given by:

$$n_i = \frac{n \sqrt{\sum_{j=1}^{L} (n_{ij}^N)^2}}{\sum_{i=1}^{k} \sqrt{\sum_{j=1}^{L} (n_{ij}^N)^2}} \tag{8}$$

where n_{ij}^N is the number of observations in stratum i using the Neyman allocation for the j^{th} variate.

In 1968, Chatterjee again tackled the problem of optimum allocation for such multivariate stratified surveys. His criteria makes it necessary to use linear or nonlinear programming. Others, such as Kokan(1963), Kokan and Khan(1967), and Huddleston, Claypool, and Hocking(1970), have also provided their versions of optimal allocation in the multivariate context using programming methods.

CHAPTER II

A NEW SEQUENTIAL ALLOCATION METHOD

The Procedure

There are many other parameters, besides the mean, which may be of interest to the researcher. One such parameter is p, the proportion of the measurements of a population which possess some specified attribute. It is therefore important to consider the general problem of estimating a parameter θ of a population partitioned into k strata in such a way that the variance of the estimator is minimized. The new procedure is a sequential procedure for which a closed formula for the variance is not known, and consequently, for which the allocation yielding minimum variance is not known. The goal of the procedure is to strive for the allocation yielding minimum variance for fixed sample size problems. For many problems of this type, Equation (9) provides a general description.

$$n_i = \frac{n \, f_i(\sigma_i^2)}{\Sigma \, f_i(\sigma_i^2)} \tag{9}$$

where the f_i are continuous functions defined from the positive reals into the positive reals. See, for example, Equations (2), (5), and (8).

Now let $$\lambda_i^n = \frac{f_i(\sigma_i^2)}{\Sigma f_i(\sigma_i^2)} - \frac{n_i}{n}. \qquad (10)$$

$\lambda_i^n = 0$ is equivalent to (9), thus optimal allocation is attained when $\lambda_i^n = 0$. If one additional observation is allocated to stratum i, then the new expression, denoted by λ_i^{n+1}, is given by:

$$\frac{f_i(\sigma_i^2)}{\Sigma f_i(\sigma_i^2)} - \frac{n_i+1}{n+1} \qquad (11)$$

which is smaller than λ_i^n. Note also that if instead, an additional observation was allocated to a stratum other than i, then λ_i^{n+1} would be:

$$\frac{f_i(\sigma_i^2)}{\Sigma f_i(\sigma_i^2)} - \frac{n_i}{n+1} \qquad (12)$$

which is larger than λ_i^n. It will also be shown that:

$$\sum_{i=1}^{k} \lambda_i^n = 0. \qquad (13)$$

Unless all λ_i^n are 0, in which case optimal allocation has been attained, there are at any given time, necessarily strata with a negative λ and strata with a positive λ. If the next observation is not added to the strata with negative λ's, then those λ will increase toward 0. These remarks point to a natural rule for allocating observations in a sequential manner, namely, that the next observation should be allocated to the stratum with the maximum λ. By doing so, the λ_i^n which are the furthest from 0 have been forced to be closer to 0.

Sequential Allocation With Unknown Variances

Since in practice, σ_i^2 for the ith stratum i = 1, 2, ..., k is unknown, it is replaced by a consistent variance estimate. This estimate, denoted by $\hat{\sigma}_{i,n}^2$, uses all observations allocated to the ith stratum up to that point in time. Substituting into λ_i^n, we have:

$$\hat{\lambda}_i^n = \frac{f_i(\hat{\sigma}_{i,n}^2)}{\Sigma\, f_i(\hat{\sigma}_{i,n}^2)} - \frac{n_i}{n}$$

Note that to distinguish between different points in the process, $\hat{\sigma}_{i,n}^2$ and $\hat{\lambda}_i^n$ are written using the total number of observations allocated to all strata, n, rather than n_i, the number of observations allocated to the ith stratum alone.

Consistency

Almost sure convergence of $\hat{\lambda}_i^n$ to 0 for all i = 1, ..., k, would be equivalent to Neyman allocation when the total number of observations to be allocated is sufficiently large. This almost sure convergence will be proved in Theorem 1 with the help of six lemmas.

The consistency of the allocation assures that for a sufficiently large sample size, the allocation scheme from the new sequential allocation will be the same as Neyman's allocation. But, it will be shown that this consistency also assures the researcher that the variance of the estimator of the new sequential procedure is as small as the minimum variance among unbiased fixed sample size estimators of θ.

The variance of $\hat{\theta}^n$, the new sequential estimator of θ at stage n is given by:

$$\int_{\hat{\theta}^n} \int_{n_k} \cdots \int_{n_1} (\hat{\theta}^n - E(\hat{\theta}^n))^2 \, g(\hat{\theta}^n/n_1,\ldots,n_k) \, g(n_1,\ldots,n_k)dn_1\ldots dn_k \, d\hat{\theta}^n \qquad (14)$$

But the n_i converge to the Neyman allocation as n approaches infinity. Thus the joint density $g(n_1,\ldots,n_k)$ converges in law to a degenerate distribution where the $n_1,\ldots,n_k$ identically take on the Neyman allocation sample sizes with probability 1. Therefore, the variance of $\hat{\theta}^n$, as defined in (14), converges almost surely to

$$\int_{\hat{\theta}^n} (\hat{\theta}^n - E(\hat{\theta}^n))^2 \, g(\hat{\theta}^n/n_1,\ldots,n_k) \, d\hat{\theta}^n \qquad (15)$$

where $n_1,\ldots,n_k$ are now fixed Neyman allocation sample sizes when n observations are available for allocation. Finally since the formula for $\hat{\theta}^n$ using the new sequential allocation procedure is the same as the formula for the Neyman allocation estimator of θ, Equation (15) is the minimum variance among estimators of θ among fixed sample size methods. Thus the variance of the estimator of the new sequential procedure is as small as the minimum variance among unbiased fixed sample size estimators of θ.

Lemma 1(Standard Convergence Results)

Since the following are standard results, references will be given in Appendix A.

Part A) The i^{th} central sample moment, m_i, converges a.s. to the i^{th} central population moment, μ_i. In particular, the sample variance, $m_2 = \hat{\sigma}^2$ converges a.s. to $\mu_2 = \sigma^2$. (16)

Part B) Let $X_n \xrightarrow{d} X$ and let $Y_n \xrightarrow{d} c$, where c is a finite constant. Then $X_n Y_n \longrightarrow cX$

Part C) if $X_n \xrightarrow{a.s.} X$ and $Y_n \xrightarrow{a.s.} Y$ then

i) $X_n + Y_n \xrightarrow{a.s.} X + Y$ (17)

ii) $X_n\ Y_n \xrightarrow{a.s.} X\ Y$ (18)

iii) Suppose that $X_n \neq 0$ a.s. and $X \neq 0$ a.s.

$$\text{Then } \frac{1}{X_n} \xrightarrow{a.s.} \frac{1}{X}$$

$$\text{and } \frac{Y_n}{X_n} \xrightarrow{a.s.} \frac{Y}{X} \qquad (19)$$

iv) Let f be any continuous function defined from the positive reals into the positive reals and suppose that $X_n \xrightarrow{a.s.} X$. Then

$$f(X_n) \xrightarrow{a.s.} f(X) \qquad (20)$$

Part D) Let f be any continuous function. Then for any i= 1, ..., k:

$$f_i(\hat{\sigma}^2_{i,n})/\Sigma f_i(\hat{\sigma}^2_{i,n}) \xrightarrow{a.s.} f_i(\sigma^2_i)\ /\ \Sigma f_i(\sigma^2_i) \qquad (21)$$

Lemma 2

Suppose that an arbitrary number, n, of observations have been allocated. Then it follows that

$$\max_{i=1,...,k} \left[\hat{\lambda}_i^n \right] \geq 0 \qquad \text{and} \qquad \min_{i=1,...,k} \left[\hat{\lambda}_i^n \right] \leq 0$$

<u>Proof</u>. $\sum_{i=1}^{k} \hat{\lambda}_i^n = \sum_{i=1}^{k} \left[\frac{f_i(\hat{\sigma}^2_{i,n})}{\Sigma f_i(\hat{\sigma}^2_{i,n})} - \frac{n_i}{n} \right]$

$$= \sum_{i=1}^{k} \left[\frac{f_i(\hat{\sigma}^2_{i,n})}{\Sigma f_i(\hat{\sigma}^2_{i,n})} \right] - \sum_{i=1}^{k} \left[\frac{n_i}{n} \right] = 0 \tag{22}$$

So $0 = \sum_{i=1}^{k} \hat{\lambda}_i^n \leq \sum_{i=1}^{k} \left[\max_{i=1,...,k} \left[\hat{\lambda}_i^n \right] \right]$

$$= k \left[\max_{i=1,...,k} \left[\hat{\lambda}_i^n \right] \right] \tag{23}$$

I.e. $0 \leq \max_{i=1,...,k} \left[\hat{\lambda}_i^n \right]$

But since $\sum_{i=1}^{k} \hat{\lambda}_i^n = 0$, the previous inequality implies that

$0 \geq \min_{i=1,...,k} \left[\hat{\lambda}_i^n \right]$ Q.E.D.

Consider an arbitrary stratum i and any sequence of consecutive steps of the process during which observations are not allocated to that stratum. Is the distance between λ_i^n and λ_i^m for any $n \neq m$ of the sequence bounded by some small bound δ? The next lemma states that for any small δ, there exists a point in the allocation process after which all such sequences have this property.

<u>Lemma 3</u>

Consider an arbitrary stratum i from among the k strata. Let δ be any arbitrary positive number. Then for any n sufficiently large for which,

$$\hat{\lambda}_i^m < \max_{j=1,\dots,k} \left[\hat{\lambda}_j^m \right] \tag{24}$$

for each m = n, n+1,..., n+t < ∞, it follows that

$$\left| \hat{\lambda}_i^m - \hat{\lambda}_i^n \right| < \delta \text{ a.s.} \tag{25}$$

<u>Proof</u>. Result (21) implies that ∃ P_1 s.t. for $n > P_1$:

$$\left| \frac{f_i(\hat{\sigma}^2_{i,n})}{\Sigma f_i(\hat{\sigma}^2_{i,n})} - \frac{f_i(\sigma^2_i)}{\Sigma f_i(\sigma^2_i)} \right| < \delta/3 \text{ a.s.} \tag{26}$$

Let $P = \max\{P_1, \frac{3t}{\delta}\}$ and let $n > P$. (27)

Then

$$\left| \hat{\lambda}_i^m - \hat{\lambda}_i^n \right|$$

$$= \left| \frac{f_i(\hat{\sigma}^2_{i,m})}{\Sigma f_i(\hat{\sigma}^2_{i,m})} - \frac{n_i}{m} - \left[\frac{f_i(\hat{\sigma}^2_{i,n})}{\Sigma f_i(\hat{\sigma}^2_{i,n})} - \frac{n_i}{n} \right] \right|$$

$$\leq \left| \frac{f_i(\hat{\sigma}^2_{i,m})}{\Sigma f_i(\hat{\sigma}^2_{i,m})} - \frac{f_i(\sigma^2_{i,m})}{\Sigma f_i(\sigma^2_{i,m})} \right|$$

$$+ \left| \frac{f_i(\hat{\sigma}^2_{i,n})}{\Sigma f_i(\hat{\sigma}^2_{i,n})} - \frac{f_i(\sigma^2_{i,n})}{\Sigma f_i(\sigma^2_{i,n})} \right| + \left| \frac{n_i}{n} - \frac{n_i}{m} \right|$$

which from (26) is bounded a.s. by:

$$\delta/3 + \delta/3 + \left| \frac{n_i}{n} - \frac{n_i}{m} \right| \tag{28}$$

But

$$\left| \frac{n_i}{n} - \frac{n_i}{m} \right| = \left| \frac{n_i\ t}{n\ m} \right| \leq \left| \frac{n\ t}{n\ m} \right| = \left| \frac{t}{m} \right|$$

$$\leq \left| \frac{t}{n} \right| \tag{29}$$

$$\leq \left| \frac{t}{\frac{3t}{\delta}} \right| . \tag{30}$$

Equation (29) is a result of the fact that m is an integer larger than n. Equation (30) follows from the restriction placed on n, namely that $n > 3t/\delta$.

Thus

$$\left| \frac{n_i}{n} - \frac{n_i}{m} \right| < \delta/3 \tag{31}$$

Combining (28) and (31) yields

$$\left| \hat{\lambda}_i^m - \hat{\lambda}_i^n \right| \leq 2\delta/3 + \delta/3 = \delta \quad \text{a.s.} \qquad \text{Q.E.D.}$$

For an arbitrary stratum i, consider the values of $\hat{\lambda}_i$ before and after the n+1st observation is allocated. One type of transition of $\hat{\lambda}_i^n$ to $\hat{\lambda}_i^{n+1}$ is the following:

$\hat{\lambda}_i^n$ is the maximum among the $\hat{\lambda}_j^n$, $j = 1,\ldots, k$.

$\hat{\lambda}_i^{n+1}$ is not the maximum among the $\hat{\lambda}_j^{n+1}$, $j = 1,\ldots,k$.

The next two lemmas deal with the existence of a lower bound for the $\hat{\lambda}_i$'s after this type of transition. Lemma 4 deals with a lower bound for the n+1st step only, while Lemma 5 deals with a sequence of consecutive points beginning with the n+1st step and ending before another observation can be allocated to the ith stratum.

Lemma 4

Let $\delta < 0$ be arbitrary and let j be an arbitrary stratum.

Then there exists $P > 0$ s.t. for any $n > P$ for which

$$\hat{\lambda}_j^n = \max_{i=1,\ldots,k} \left[\hat{\lambda}_i^n \right] \tag{32}$$

it follows that $\hat{\lambda}_j^{n+1} > \delta$ a.s.

<u>Proof</u>. Let δ assume an arbitrary negative value and consider an arbitrary stratum, say stratum j.

From (32) and LEMMA 2

$$\hat{\lambda}_j^n > 0 \tag{33}$$

and

the next observation will be allocated to stratum j. (34)

Now if $\hat{\lambda}_j^n \le \hat{\lambda}_j^{n+1}$, then $\delta < 0 < \hat{\lambda}_j^n < \hat{\lambda}_j^{n+1}$

which implies that:

$$\delta < \hat{\lambda}_j^{n+1} \tag{35}$$

so it only remains to consider $\hat{\lambda}_j^n > \hat{\lambda}_j^{n+1}$. (36)

$$\hat{\lambda}_j^n - \hat{\lambda}_j^{n+1}$$

$$= \left[\frac{f_j(\hat{\sigma}^2_{j,n})}{\Sigma f_j(\hat{\sigma}^2_{j,n})} - \frac{n_j}{n}\right] - \left[\frac{f_j(\hat{\sigma}^2_{j,n+1})}{\Sigma f_j(\hat{\sigma}^2_{j,n+1})} - \frac{n_j + 1}{n+1}\right]$$

$$= \left[\frac{f_j(\hat{\sigma}^2_{j,n})}{\Sigma f_j(\hat{\sigma}^2_{j,n})} - \frac{f_j(\hat{\sigma}^2_{j,n+1})}{\Sigma f_j(\hat{\sigma}^2_{j,n+1})}\right] + \frac{n_j + 1}{n+1} - \frac{n_j}{n}$$

$$= \left[\frac{f_j(\hat{\sigma}^2_{j,n})}{\Sigma f_j(\hat{\sigma}^2_{j,n})} - \frac{f_j(\hat{\sigma}^2_{j,n+1})}{\Sigma f_j(\hat{\sigma}^2_{j,n+1})}\right] + \frac{(n_j+1)\ n - (n+1)n_j}{n\ (n+1)}$$

$$= \left[\frac{f_j(\hat{\sigma}^2_{j,n})}{\Sigma f_j(\hat{\sigma}^2_{j,n})} - \frac{f_j(\hat{\sigma}^2_{j,n+1})}{\Sigma f_j(\hat{\sigma}^2_{j,n+1})}\right] + \frac{n - n_j}{n\ (n + 1)}$$

$$\leq \left[\frac{f_j(\hat{\sigma}^2_{j,n})}{\Sigma f_j(\hat{\sigma}^2_{j,n})} - \frac{f_j(\hat{\sigma}^2_{j,n+1})}{\Sigma f_j(\hat{\sigma}^2_{j,n+1})}\right] + \frac{n}{n\ (n + 1)}$$

$$= \left[\frac{f_j(\hat{\sigma}^2_{j,n})}{\Sigma f_j(\hat{\sigma}^2_{j,n})} - \frac{f_j(\hat{\sigma}^2_{j,n+1})}{\Sigma f_j(\hat{\sigma}^2_{j,n+1})}\right] + \frac{1}{n + 1} \tag{37}$$

Combining the first and last line in the previous string of inequalities and moving some terms to the opposite side of the inequality, yields

$$\hat{\lambda}^{n+1}_j \geq \hat{\lambda}^n_j - \frac{1}{n + 1} - \left[\frac{f_j(\hat{\sigma}^2_{j,n})}{\Sigma f_j(\hat{\sigma}^2_{j,n})} - \frac{f_j(\hat{\sigma}^2_{j,n+1})}{\Sigma f_j(\hat{\sigma}^2_{i,n+1})}\right] \tag{38}$$

But taking advantage of (33),

$$\hat{\lambda}^{n+1}_j \geq - \frac{1}{n + 1} - \left[\frac{f_j(\hat{\sigma}^2_{j,n})}{\Sigma f_j(\hat{\sigma}^2_{j,n})} - \frac{f_j(\hat{\sigma}^2_{j,n+1})}{\Sigma f_j(\hat{\sigma}^2_{j,n+1})}\right] \tag{39}$$

Now (21) is equivalent to saying that there exists a positive constant M_1 s.t. for $n > M_1$

$$\left| \frac{f_j(\hat{\sigma}^2_{j,n})}{\Sigma f_j(\hat{\sigma}^2_{j,n})} - \frac{f_j(\sigma^2_j)}{\Sigma f_j(\sigma^2_j)} \right| < -\delta/4 \quad \text{a.s.} \tag{40}$$

But if the previous inequality holds for all $n > M_1$, then it also holds for n+1:

$$\left| \frac{f_j(\hat{\sigma}^2_{j,n+1})}{\Sigma f_j(\hat{\sigma}^2_{j,n+1})} - \frac{f_j(\sigma^2_j)}{\Sigma f_j(\sigma^2_j)} \right| < -\delta/4 \quad \text{a.s.} \tag{41}$$

and (40) and (41) imply the following:

$$\left| \frac{f_j(\hat{\sigma}^2_{j,n})}{\Sigma f_j(\hat{\sigma}^2_{j,n})} - \frac{f_j(\hat{\sigma}^2_{j,n+1})}{\Sigma f_j(\hat{\sigma}^2_{j,n+1})} \right|$$

$$< \left| \frac{f_j(\hat{\sigma}^2_{j,n})}{\Sigma f_j(\hat{\sigma}^2_{j,n})} - \frac{f_j(\sigma^2_j)}{\Sigma f_j(\sigma^2_j)} \right| + \left| \frac{f_j(\hat{\sigma}^2_{j,n+1})}{\Sigma f_j(\hat{\sigma}^2_{j,n+1})} - \frac{f_j(\sigma^2_j)}{\Sigma f_j(\sigma^2_j)} \right|$$

$$< -\delta/4 + -\delta/4 = -\delta/2 \qquad \text{a.s.} \qquad (42)$$

and thus $\dfrac{f_j(\hat{\sigma}^2_{j,n})}{\Sigma f_j(\hat{\sigma}^2_{j,n})} - \dfrac{f_j(\hat{\sigma}^2_{j,n+1})}{\Sigma f_j(\hat{\sigma}^2_{j,n+1})} < -\delta/2$ a.s.

So (39) becomes

$$\hat{\lambda}^{n+1}_j > -\frac{1}{n+1} + \delta/2 \qquad \text{a.s.} \qquad (43)$$

and since $-\dfrac{1}{n+1} > \delta/2$ is equivalent to $n > -2/\delta - 1$,

choosing $n > \max(M_1, M_2 = -2/\delta - 1\}$, transforms (43) into the desired result, namely that $\hat{\lambda}^{n+1}_j > \delta$ a.s. Q.E.D.

Lemma 5

Let $\varepsilon < 0$ be arbitrary and let j be an arbitrary stratum. Then there exists a positive integer P s.t. for any n > P for which:

$$\hat{\lambda}^P_j = \max_{i=1,...,k} \left[\hat{\lambda}^P_i\right] \qquad (44)$$

and

$$\hat{\lambda}^{P+t}_j < \max_{i=1,...,k} \left[\hat{\lambda}^{P+t}_i\right] \quad \text{for } t = 1, 2, ..., T \qquad (45)$$

it follows that $\hat{\lambda}^{P+t}_j > \varepsilon$ a.s. for t = 1, 2, ...,T (46)

Proof. Let $\varepsilon < 0$ be arbitrary.

Let $P = \max\{P_1, P_2\}$ where P_1 and P_2 are integers for which

$n \geq P_1$ ===> Lemma 4 holds for $\delta = \varepsilon/2$

$n \geq P_2$===> Lemma 3 holds for $\delta = -\varepsilon/2$

Then by Lemma 4 and (44), $\hat{\lambda}_j^{P+1} > \varepsilon/2$ a.s. (47)

and since (45) satisfies Lemma 3, it follows that

$$|\hat{\lambda}_j^{P+t} - \hat{\lambda}_j^{P+1}| < -\varepsilon/2 \quad \text{a.s.} \tag{48}$$

Recall that $\varepsilon < 0$, hence $-\varepsilon/2$ is a positive constant. Ignoring the upper bound of (48), and adding through by $\hat{\lambda}_j^{P+1}$,

$$\hat{\lambda}_j^{P+t} > \hat{\lambda}_j^{P+1} -(- \varepsilon/2) = \hat{\lambda}_j^{P+1} + \varepsilon/2 \quad \text{a.s.}$$

which with the help of (47) gives

$\hat{\lambda}_j^{P+t} > \varepsilon/2 + \varepsilon/2 = \varepsilon$ a.s. Q.E.D

Lemma 6

Let j be an arbitrary stratum. Then at each of infinitely many points in the process, $\hat{\lambda}_j$ will be the maximum $\hat{\lambda}$ of all strata at that step in the process.

I.e.

$$\forall P > 0 \ \exists n \geq P \text{ s.t. } \hat{\lambda}_j^n = \max_{i=1,\dots,k} \left[\hat{\lambda}_i^n \right] \tag{49}$$

Proof. Let j be an arbitrary stratum. Suppose that there exists $P > 0$, s.t. for all $t = 1, 2, \dots$

$$\hat{\lambda}_j^{P+t} < \max_{i=1,\dots,k} \left[\hat{\lambda}_i^{P+t} \right]. \tag{50}$$

Let $(P+t)_j$ denotes the number of observations allocated to stratum j when at the $P+j^{th}$ step. Thus for example, $(P+1)_j$ denotes the number of observations allocated to stratum j after P+1 steps. If no observations are allocated to the j^{th} stratum over a sequence of t consecutive steps beginning with step P+1, then it follows that $(P+1)_j = (P+t)_j$. Now (50) implies that no observations are allocated to stratum j after step P. As a direct result, for t = 1, 2,...:

$$\hat{\lambda}_j^{P+t} = \frac{f_j(\hat{\sigma}^2_{j,P+t})}{\Sigma f_j(\hat{\sigma}^2_{j,P+t})} - \frac{(P+t)_j}{P+t} \qquad (51)$$

can be written as:

$$\hat{\lambda}_j^{P+t} = \frac{f_j(\hat{\sigma}^2_{j,P})}{\Sigma f_j(\hat{\sigma}^2_{j,P+t})} - \frac{(P+1)_j}{P+t} \qquad (52)$$

Thus as t gets larger and larger, the first term of (52) approaches a positive constant, while the second term approaches 0. Then, $\hat{\lambda}_j^{P+t}$ must converge to a positive constant, which by the definition of convergence, says that there exists a positive lower bound ν for $\hat{\lambda}_j^{P+t}$ for all n sufficiently large, say for $t > T_j$. (53)

Let T be the maximum of the T_j.

It may be assumed from (50), that there are strata which will receive only a finite number of observations. In each of these strata, there exists a point in the allocation process after which no more observations will be allocated to the strata. If the $\hat{\lambda}$'s of the other strata, i.e. the strata where an infinite number of observations are allocated, never become the minimum $\hat{\lambda}$, then the minimum $\hat{\lambda}$ must eventually belong to the strata where no new observations are added. This would imply that the minimum $\hat{\lambda}$

would converge to a positive constant(see (53)), a clear violation of Lemma 2). Let T' be some point in the process after T where the minimum $\hat{\lambda}$ corresponds to one of the infinitely sampled strata, say of stratum i.

Being the minimum $\hat{\lambda}$, $\hat{\lambda}_i^{T'} < 0.$ (54)

But observations are only drawn in the stratum corresponding to the maximum $\hat{\lambda}$. Thus since an infinite number of observations are drawn from stratum i, it follows that the $\hat{\lambda}$ of stratum i must become the maximum $\hat{\lambda}$ at some future step in the process. Let step T'+t' be the first such occurrence after T'. Being the maximum $\hat{\lambda}$,

$$\hat{\lambda}_i^{T'+t'} = \max_{i=1,\dots,k} \left[\hat{\lambda}_i^{T'+t'} \right] > \hat{\lambda}_j^{T'+t'} \tag{55}$$

$$> \nu \, . \tag{56}$$

(56) follows since T'+t' > T. See (53) for more details. From (54) and (56) it follows that $\hat{\lambda}_i^{T'+t'} - \hat{\lambda}_i^{T'} > \nu - 0$, which implies that

$$\left| \hat{\lambda}_i^{T'+t'} - \hat{\lambda}_i^{T'} \right| > \nu$$

This contradicts Lemma 3, hence the assumption about stratum j being allocated a finite number of observations must be false. Q.E.D.

Lemmas 5 and 6 will be used in the proof of the theorem.

Theorem 1

Consider an arbitrary stratum, say stratum j. Then for any j = 1, ..., k, $\hat{\lambda}_j^n \xrightarrow{a.s.} 0$ as $n \longrightarrow \infty$.

Proof of Theorem 1. Now, $\sum_{i=1}^{k} \hat{\lambda}_i^n = 0$. See (22).

Thus it suffices to show that $\min_{i=1,\dots,k}\left[\hat{\lambda}_i^n\right]$ converges to 0 a.s. when the allocation plan is used.

That is, it must be shown that for any arbitrary $\varepsilon < 0$ there exists a positive integer P s.t. for every $n \geq P$

$$\min_{i=1,\dots,k}\left[\hat{\lambda}_i^n\right] > \varepsilon \text{ a.s.} \tag{57}$$

Let $\varepsilon < 0$ be arbitrary.

By Lemma 5) there exists for each stratum j = 1, 2,..., N an integer P_j such that if

i) $n > P_j$ (58)

ii) $\hat{\lambda}_j^n = \max_{i=1,\dots,k}\left[\hat{\lambda}_i^n\right]$ (59)

and

iii) $\hat{\lambda}_j^{n+t_j} < \max_{i=1,\dots,k}\left[\hat{\lambda}_i^{n+t_j}\right]$ (60)

for $t_j = 1, 2, \dots, T_j$, for some T_j,

it follows that $\hat{\lambda}_j^{n+t_j} > \varepsilon$ (61)

for $t_j = 1, 2, \dots, T_j$, for some T_j.

Let $P = \max_{i=1,\dots,k}\left[P_i\right]$. (62)

Then Lemma 5) holds for any stratum j whenever $n > P$

By Lemma 6), there exists for every stratum j, a sequence of positive integers $P \leq M_{j1} < M_{j2} < M_{j3} < \dots$ s.t.

$$\hat{\lambda}_j^{M_{ji}} = \max_{c=1,\dots,k}\left[\hat{\lambda}_c^{M_{ji}}\right] \tag{63}$$

for j = 1, 2, ..., k and i = 1, 2, ...

$$\text{Let } M = \max_{j=1,\dots,k}\left[M_{j1}\right], \tag{64}$$

that is, if sampling continues until at least one observation has been allocated to each stratum after point P in the process, this step is referred to as point M. Then after an arbitrary number of additional observations have been allocated among the strata, say n observations, let j represent the stratum with the minimum $\hat{\lambda}$. That is, $\hat{\lambda}_j^{M+n} = \min_{i=1,\ldots,k} \left[\hat{\lambda}_i^{M+n}\right]$. Let M_{jL} be the most recent point of the process where an observation was allocated to stratum j. So,

1) $\hat{\lambda}_j^{M_{jL}} = \max_{i=1,\ldots,k} \left[\hat{\lambda}_i^{M_{jL}}\right]$ (65)

2) $\hat{\lambda}_j^{M_{jL}+t} < \max_{i=1,\ldots,k} \left[\hat{\lambda}_i^{M_{jL}+t}\right]$

for $t = 1, 2, \ldots, n-M_{jL}$ (66)

3) $M_{jL} > P > P_j$ (67)

But (65), (66), (67) are precisely the conditions necessary for LEMMA 5), hence $\min_{i=1,\ldots,k} \left[\hat{\lambda}_i^n\right] > \varepsilon$ a.s. Q.E.D.

CHAPTER III

THE PROCESS AS A MODIFIED ROBBINS-MONRO PROCESS

Introduction

There exists a class of sequences of random variables, known as Robbins-Monro Processes, with numerous beneficial asymptotic properties. The intent of this chapter is to present a brief discussion of the similarities and dissimilarities of the new sequential process with the Robbins-Monro process. This chapter is not intended to be a rigorous and complete treatise.

A one dimensional Robbins-Monro process is a procedure for estimating a single root L_p of a function M(x) where $M(x_o)$ is the expectation of a specified random variable $Y(x_o)$. That procedure consists of deriving a recursive sequence of random variables X_n by the following formula:

$$X_{n+1} = X_n - a_n(Y_n - p) \qquad (68)$$

where p is the root that is being estimated, Y_n is the observed response associated with the corresponding level of X_n and a_n is a fixed sequence of constants.

Robbins and Monro(1951) showed that under certain conditions on the $\{a_n\}$, X_n converges to L_p in L^2. Almost sure convergence was later proved by Blum(1954) and Goodsell and Hanson(1976). Sachs(1958) considered the special case where a_n is proportional to 1/n. Under certain conditions, he showed that $\sqrt{n}\ (X_n - L_p)$ converges in distribution to a normal distribution with mean 0 and variance:

$$\frac{a^2\ \left[\lim_{x \to L_p} \sigma^2_{Y(x)}\right]}{2aM'(L_p) - 1} .$$

Anbar(1978), proposed an estimator for $M'(L_p)$ and Moser and Fei(1991) extended the procedure to k dimensions.

Another Look at the New Sequential Process

Consider again k disjoint populations and recall that when $\hat{\lambda}_i^n = \max_{j=1,\dots,k}\left[\hat{\lambda}_j^n\right]$, $\hat{\lambda}_i^{n+1}$ is defined by:

$$\hat{\lambda}_i^{n+1} = \frac{f_i(\hat{\sigma}^2_{i,n+1})}{\Sigma\ f_i(\hat{\sigma}^2_{i,n+1})} - \frac{n_i + 1}{n + 1} \tag{69}$$

since the new observation is allocated to the i^{th} population. Otherwise:

$$\hat{\lambda}_i^{n+1} = \frac{f_i(\hat{\sigma}^2_{i,n+1})}{\Sigma\ f_i(\hat{\sigma}^2_{i,n+1})} - \frac{n_i}{n + 1} . \tag{70}$$

Now, $$\hat{\lambda}_i^n = \frac{f_i(\hat{\sigma}^2_{i,n})}{\Sigma\ f_i(\hat{\sigma}^2_{i,n})} - \frac{n_i}{n} .$$

It follows that when $\hat{\lambda}_i^n = \max_{j=1,\dots,k}\left[\hat{\lambda}_j^n\right]$,

$$\hat{\lambda}_i^{n+1} - \hat{\lambda}_i^n = \frac{f_i(\hat{\sigma}^2_{i,n+1})}{\Sigma\ f_i(\hat{\sigma}^2_{i,n+1})} - \frac{n_i + 1}{n + 1} - \left[\frac{f_i(\hat{\sigma}^2_{i,n})}{\Sigma\ f_i(\hat{\sigma}^2_{i,n})} - \frac{n_i}{n}\right]$$

$$= \frac{f_i(\hat{\sigma}^2_{i,n+1})}{\Sigma\, f_i(\hat{\sigma}^2_{i,n+1})} - \frac{f_i(\hat{\sigma}^2_{i,n})}{\Sigma\, f_i(\hat{\sigma}^2_{i,n})} + \frac{n_i}{n} - \frac{n_i + 1}{n + 1}$$

$$= \frac{f_i(\hat{\sigma}^2_{i,n+1})}{\Sigma\, f_i(\hat{\sigma}^2_{i,n+1})} - \frac{f_i(\hat{\sigma}^2_{i,n})}{\Sigma\, f_i(\hat{\sigma}^2_{i,n})} + \frac{1}{n}\left[\frac{n_i - n}{(n + 1)}\right].$$

$$= \frac{1}{n}\left[\frac{-\,n}{(n + 1)}\right] + \frac{f_i(\hat{\sigma}^2_{i,n+1})}{\Sigma\, f_i(\hat{\sigma}^2_{i,n+1})} - \frac{f_i(\hat{\sigma}^2_{i,n})}{\Sigma\, f_i(\hat{\sigma}^2_{i,n})} + \frac{1}{n}\left[\frac{n_i}{(n + 1)}\right]. \qquad (71)$$

When $\hat{\lambda}^n_i < \max_{j=1,\dots,k}\left[\hat{\lambda}^n_j\right]$, it follows from (69) and (70) that

$$\hat{\lambda}^{n+1}_i - \hat{\lambda}^n_i = \frac{f_i(\hat{\sigma}^2_{i,n+1})}{\Sigma\, f_i(\hat{\sigma}^2_{i,n+1})} - \frac{n_i}{n + 1} - \left[\frac{f_i(\hat{\sigma}^2_{i,n})}{\Sigma\, f_i(\hat{\sigma}^2_{i,n})} - \frac{n_i}{n}\right]$$

$$= \frac{f_i(\hat{\sigma}^2_{i,n+1})}{\Sigma\, f_i(\hat{\sigma}^2_{i,n+1})} - \frac{f_i(\hat{\sigma}^2_{i,n})}{\Sigma\, f_i(\hat{\sigma}^2_{i,n})} + \frac{n_i}{n} - \frac{n_i}{n + 1}$$

$$= \frac{f_i(\hat{\sigma}^2_{i,n+1})}{\Sigma\, f_i(\hat{\sigma}^2_{i,n+1})} - \frac{f_i(\hat{\sigma}^2_{i,n})}{\Sigma\, f_i(\hat{\sigma}^2_{i,n})} + \frac{n_i}{n(n+1)}\,. \qquad (72)$$

For each i = 1, ..., k, define the random variable $Y_{i,n}$ in the following manner: (73)

$$Y_{i,n} = \begin{cases} \frac{n}{n+1} & \text{if the next observation is to be taken from the } i^{\text{th}} \text{ population.} \\ 0 & \text{otherwise} \end{cases}$$

Then note that (71) and (72) can both be written as:

$$\hat{\lambda}^{n+1}_i = \hat{\lambda}^n_i - \frac{1}{n}\, Y_{i,n} + \frac{f_i(\hat{\sigma}^2_{i,n+1})}{\Sigma\, f_i(\hat{\sigma}^2_{i,n+1})} - \frac{f_i(\hat{\sigma}^2_{i,n})}{\Sigma\, f_i(\hat{\sigma}^2_{i,n})} + \frac{n_i}{n(n+1)}. \qquad (74)$$

This is true for (72), since the next observation will not be added to cell i, resulting in $Y_{i,n} = 0$.

Consider the last three terms of (74). Since $0 < \frac{n_i}{n(n+1)} < \frac{1}{n+1}$, it follows that $\frac{n_i}{n(n+1)}$ converges to 0 as n tends to infinity.

Let $$\vec{f}_{i,n}(\hat{\sigma}^2_{i,n}, \hat{\sigma}^2_{i,n+1}) = \frac{f_i(\hat{\sigma}^2_{i,n+1})}{\Sigma\, f_i(\hat{\sigma}^2_{i,n+1})} - \frac{f_i(\hat{\sigma}^2_{i,n})}{\Sigma\, f_i(\hat{\sigma}^2_{i,n})} . \qquad (75)$$

Then, $\vec{f}_{i,n}(\hat{\sigma}^2_{i,n}, \hat{\sigma}^2_{i,n+1})$ converges to 0, since it consists of consecutive terms of a sequence of convergent random variables. (See Lemma 1.)

But $\hat{\lambda}^{n+1}_i = \hat{\lambda}^n_i - \frac{1}{n} Y_{i,n}$ is a Robbins-Monro process, thus it is evident that the sequence of random variables, $\hat{\lambda}^n_i$, can be represented as a Robbins-Monro process plus these extra terms. Combining the first k-1 scalars $\hat{\lambda}^{n+1}_i$, i = 1,..., k-1 into one vector, $\underset{\sim}{\hat{\lambda}}_{n+1}$, yields the following: (76)

$$\underset{\sim}{\hat{\lambda}}_{n+1} = \begin{bmatrix} \hat{\lambda}^{n+1}_1 \\ \hat{\lambda}^{n+1}_2 \\ \cdot \\ \cdot \\ \cdot \\ \hat{\lambda}^{n+1}_{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} \hat{\lambda}^n_1 \\ \hat{\lambda}^n_{2}, \\ \cdot \\ \cdot \\ \cdot \\ \hat{\lambda}^n_{k-1} \end{bmatrix} - \begin{bmatrix} \frac{1}{n} Y_{1,n} \\ \frac{1}{n} Y_{2,n} \\ \cdot \\ \cdot \\ \cdot \\ \frac{1}{n} Y_{k-1,n} \end{bmatrix} + \begin{bmatrix} \vec{f}_{1,n}(\hat{\sigma}^2_{1,n}, \hat{\sigma}^2_{1,n+1}) \\ \vec{f}_{2,n}(\hat{\sigma}^2_{2,n}, \hat{\sigma}^2_{2,n+1}) \\ \cdot \\ \cdot \\ \cdot \\ \vec{f}_{k-1,n}(\hat{\sigma}^2_{k-1,n}, \hat{\sigma}^2_{k-1,n+1}) \end{bmatrix} + \begin{bmatrix} \frac{n_1}{n(n+1)} \\ \frac{n_2}{n(n+1)} \\ \cdot \\ \cdot \\ \cdot \\ \frac{n_{k-1}}{n(n+1)} \end{bmatrix}$$

Recall that:

$$\sum_{i=1}^{k} \hat{\lambda}_i^{n+1} = 0, \tag{77}$$

which was verified in lemma 2. A natural consequence of this fact is that the k^{th} component can be dropped from the vector, since its value can be determined from the other k-1 components, as illustrated in (76). Of great importance, is what transpires for n sufficiently large.

The Case of n Sufficiently Large

For large n,

$$\frac{f_i(\hat{\sigma}^2_{i,n+1})}{\Sigma\, f_i(\hat{\sigma}^2_{i,n+1})} \quad \text{closely approximates} \quad \frac{f_i(\sigma^2_{i,n+1})}{\Sigma\, f_i(\sigma^2_{i,n+1})},$$

we will write λ_i^n instead of $\hat{\lambda}_i^n$. It has already been shown that (76) reduces to

$$\underset{\sim}{\lambda}_{n+1} = \begin{bmatrix} \lambda^{n+1}_1 \\ \lambda^{n+1}_2 \\ \cdot \\ \cdot \\ \cdot \\ \lambda^{n+1}_{k-1} \end{bmatrix} = \begin{bmatrix} \lambda^n_1 \\ \lambda^n_2 \\ \cdot \\ \cdot \\ \cdot \\ \lambda^n_{k-1} \end{bmatrix} - \begin{bmatrix} \frac{1}{n} Y_{1,n} \\ \frac{1}{n} Y_{2,n} \\ \cdot \\ \cdot \\ \cdot \\ \frac{1}{n} Y_{k-1,n} \end{bmatrix} . \tag{78}$$

But recall from (73) that if the next observation is to be drawn from the i^{th} population, then $Y_{i,n} = \frac{n}{n+1}$ is approximately 1 for large n. $Y_{i,n}$ can for all practical purposes be replaced by: (79)

$$Y_{i,n} = \begin{cases} 1 & \text{if the next observation is to be taken from the } i^{th} \text{ population.} \\ 0 & \text{otherwise} \end{cases}$$

Now recall the following property:

$$\max_{i=1,\dots,k} \left[\lambda_i^n\right] > 0.$$

Since observations are drawn from the population with the maximum λ, only populations with positive λ's are sampled. If $\lambda_i^n < 0$, it follows that $P(Y_{i,n} = 1) = 0$ and in the case that $\lambda_i^n \geq 0$, the larger λ_i^n becomes, the higher the probability is that $Y_{i,n} = 1$. Now for binomial random variables, $M(\lambda_i^n) = P(Y_{i,n} = 1)$. $M(\lambda_i^n)$ of $Y_{i,n}$ for $k \geq 3$ is a non decreasing function and is illustrated in Figure 1.

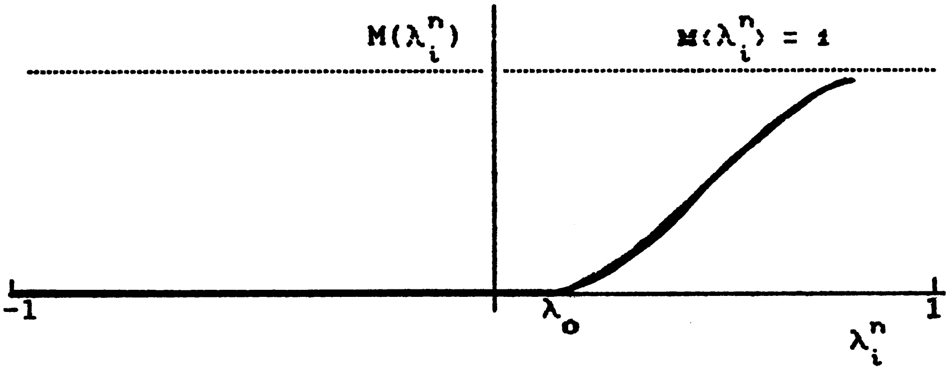

Figure 1. $M(\lambda_i^n)$ for large n and $k > 2$

In the case of two populations, $\lambda_1^n = -\lambda_2^n$, which forces $M(\lambda_i^n)$ to be a step function with the point of discontinuity occurring at $\lambda_i^n = 0$.

Consistency

A solution to:

$$M(L_p) = p$$

can be found by a sufficiently large number of iterations of:

$$\lambda_i^{n+1} = \lambda_i^n - a_n(Y_{i,n} - p) \tag{80}$$

Robbins and Monro showed that a consistent solution occurs when the following conditions are met:

i) $P\left[\,\left|\,Y(\lambda_i^n)\,\right| \leq C\right] = 1,$

ii) $0 < \sum_{n=1}^{\infty} a_n^2 < \infty,$

iii) $\sum_{n=1}^{\infty} \dfrac{a_n}{a_1 + \ldots + a_{n-1}} = \infty,$

iv) $M(\lambda_i^n)$ is nondecreasing,

v) $M(L_p) = p,$

vi) $M'(L_p) > 0.$

In the case of the new sequential procedure,

$$\left|\lambda_i^n\right| = \left|\frac{f_i(\sigma_{i,n}^2)}{\Sigma\, f_i(\sigma_{i,n}^2)} - \frac{n_i}{n}\right| \tag{81}$$

$$\leq \left|\frac{f_i(\sigma_{i,n}^2)}{\Sigma\, f_i(\sigma_{i,n}^2)}\right| \tag{82}$$

since both terms of (81) are positive. But, (82) ≤ 1, producing upper and lower bounds equal to 1. Conditions ii) and iii) follow since $a_n = 1/n$. It has already been argued that condition iv) holds. $M(\lambda_i^n)$ is a continuous function for $k \ge 3$, so for $0 < L_p < 1$, condition v) holds. Finally, it has been argued that for $L_p > 0$, $M'(p) > 0$.

Comparing (68) to the iterative equation of the process presented by the new sequential procedure when n is large:

$$\lambda_i^{n+1} = \lambda_i^n - \frac{1}{n} Y_{i,n} = \lambda_i^n - \frac{1}{n} (Y_{i,n} - 0) \ . \tag{83}$$

it is evident that $p = 0$. This creates a minor problem in that the last condition for a consistent solution holds only when $p > 0$. A possible approach is to consider only the subsequence of positive λ_i^n's. This approach is reasonable since Lemma 6 guarantees that an infinite number of observations will be allocated to all strata, and those observations are allocated to strata with positive λ's. This subsequence meets the conditions for a sequential solution, i.e., $\lambda_i^n \xrightarrow{a.s.} 0$.

CHAPTER IV

SIMULATION STUDY

Introduction

Asymptotic properties of the new allocation method have been discussed in the previous chapters. This chapter will focus on the question of how well the sequential method compares against Sukhatme's approach using small sample sizes.

Recall that Sukhatme's approach consists of sampling in two stages. In the first stage, the observations in each stratum are used to estimate the variance of that stratum. These estimates then replace the variances in Neyman's formula to determine the allocation of the remaining observations among the strata in the second stage.

Sukhatme's Examples

To support his conjecture, Sukhatme included three numerical examples, obtained by a poll conducted by the Polish Institute for Social Problems. In each example, Sukhatme attempted to estimate the following:

i) the probability that the standard error from his approach would be smaller than the standard error from the proportional method of allocation,

and

ii) the average gain in precision:

$$\frac{V(\bar{y}_{Prop}) - (V(\bar{y}_{suk}))}{V(\bar{y}_{suk})} * 100 \qquad (84)$$

The details of his examples follow. The number of strata for the three examples were 5, 10, and 20 respectively. Instead of specifying the exact stratum sizes and stratum variances, he specified values to which they were proportional. The stratum means were not given, because these are inconsequential to the problem, since a shift in location of a distribution does not affect the spread. He made the assumption that the stratum sizes were sufficiently large so that the multiplier $\frac{N_i - n_i}{N_i}$ could be ignored, and he also assumed that the stratum sub-populations were distributed normally. Tables I, II, and III give the respective stratum information for the three examples.

TABLE I

STRATUM SIZES AND VARIANCES FOR EXAMPLE I

Stratum	Size ∝	Variance ∝
1	5	1
2	5	1
3	1	4
4	2	9
5	3	16

TABLE II

STRATUM SIZES AND VARIANCES FOR EXAMPLE II

Stratum	Size $\propto$	Variance $\propto$
1	1	1
2	1	1
3	1	1
4	2	1
5	1.5	2.75
6	1.25	4
7	1	9
8	3	1
9	1.5	4
10	1	36

TABLE III

STRATUM SIZES AND VARIANCES FOR EXAMPLE II

Stratum	Size $\propto$	Variance $\propto$
1	1	1
2	1	1
3	1	1
4	2	1
5	1	4
6	1.5	4
7	3	1
8	2	4
9	4	1
10	1	16
11	2.5	4
12	1	25
13	3	4
14	1	36
15	3.5	4
16	2	16
17	1	64
18	0	1
19	2	25
20	1	100

To estimate the probability that the standard error from his approach is smaller than the standard error from the proportional method of allocation, Sukhatme first used a Type III Pearson curve with 300 trials to estimate the distribution of $\hat{V}(\bar{y}_{suk})$, the estimated variance of his estimator. The variance of the estimator using proportional allocation is:

$$V(\bar{y}_{Prop}) = \frac{1}{n} \sum_{h=1}^{k} \frac{N_h}{N} \sigma_h^2 \tag{85}$$

under the assumption of large stratum sizes.

In order to estimate *ii)*, the average gain in precision, recall that:

$$V(\bar{y}_{suk}) = E\left[V\left(\bar{y}_{suk}/\hat{\sigma}_1^2, \hat{\sigma}_2^2, \ldots, \hat{\sigma}_k^2\right)\right] + V\left[E\left(\bar{y}_{suk}/\hat{\sigma}_1^2, \hat{\sigma}_2^2, \ldots, \hat{\sigma}_k^2\right)\right]. \tag{86}$$

But the second term of (86) is 0 since:

$$E\left(\bar{y}_{suk}/\hat{\sigma}_1^2, \hat{\sigma}_2^2, \ldots, \hat{\sigma}_k^2\right)$$

is independent of the $\hat{\sigma}_i^2$, $i = 1, \ldots, k$. Thus,

$$V(\bar{y}_{suk}) = E\left[V\left(\bar{y}_{suk}/\hat{\sigma}_1^2, \hat{\sigma}_2^2, \ldots, \hat{\sigma}_k^2\right)\right] \tag{87}$$

$$= E\left[\frac{1}{n}\left[\frac{1}{N^2} \sum_{h=1}^{k} \sum_{j=1}^{k} \frac{\hat{\sigma}_j}{\hat{\sigma}_h} N_j N_h \sigma_j^2\right]\right] \tag{88}$$

$$= E\left[\frac{1}{n}\left[\frac{1}{N^2} \sum_{h=1}^{k}\left(N_h \hat{\sigma}_h \sum_{j=1}^{k} N_j \frac{\sigma_j^2}{\hat{\sigma}_j}\right)\right]\right] \tag{89}$$

Sukhatme averaged

$$\frac{1}{n} \left[\frac{1}{N^2} \sum_{h=1}^{k} \left(N_h \hat{\sigma}_h \sum_{j=1}^{k} N_j \frac{\sigma_j^2}{\hat{\sigma}_j} \right) \right]$$

over the 300 trials, obtaining an estimate of (87), which was then used to estimate the average gain in precision. Note that through this approach, Sukhatme was able to obtain his results without simulating the second stage of his process, a real bonus considering the limited computing facilities at his disposal.

The New Simulations

Introduction

Our approach takes advantage of the vastly superior computing power that is presently at our disposal, thus eliminating the need to use Pearson curves to estimate the distribution. This approach is also necessary since the derivation of the variance of the estimator for the sequential procedure, $V(\bar{y}_{seq})$, is intractable.

The Simulation Plans

The simulation plans can be divided into three categories:

i) in the case of the three numerical examples summarized in Tables I, II, and III.

ii) when observations are drawn from other normal distributions.

iii) when observations are drawn from Gamma distributions.

The Normal distributions will be partitioned into two or three strata. The three Gamma distributions will each be partitioned into two strata.

The simulation plans in each category will then be simulated with different combinations of 1) initial observations per stratum and 2) total number of observations allocated. In any given simulation, each stratum will receive the same number of observations initially. Except for Sukhatme's examples, each plan will be simulated with twenty different combinations: initial observations per stratum of 2, 5, 10, 15, 20 combined with total allocated observations of 50, 100, 150, and 200. For Sukhatme's examples, only fifteen initial observations per stratum will be considered. Bias, variance, and mean square error(mse), will be estimated for both Sukhatme's procedure and the new sequential procedure. The proportion of trials that the sequential estimator produces smaller bias, and the proportion of trials that the allocation from the sequential procedure is closer to the Neyman allocation, will be computed. Finally, the average gain in precision from using the sequential approach rather than Sukhatme's two stage process will be studied. Precision is a measure of how much the estimate varies about its mean. For all simulations, 500 trials will be used.

i) Sukhatme's Examples

As previously mentioned, simulations with 15 initial observations per stratum will be combined with setups of total observations. These three setups will change from example to example since the number of strata varies, requiring a different number of observations to be allocated to all the strata combined. In fact, the initial number of observations double from one example to the next, since the number of strata double. For the purposes of these simulations, the situation when the number of initial observations allocated to a stratum exceeds the number of observations to be allocated to that stratum will be avoided. Recall that stratum information can be found in Tables I, II, and III. Sukhatme did not reveal the values of the means for the various strata, information which is needed for our simulations. Because actual values do not affect the procedure, means of 1000 per stratum will be assumed.

ii) Other Normal Distributions

In this category, Sukhatme's examples are extended to other examples with data from normal strata. Note that for larger stratum sizes, the stratum sizes can be taken as constants since Neyman sample sizes:

$$n_i = \frac{N_i S_i}{\sum_{i=1}^{k} \left[N_i S_i \right]} n \tag{90}$$

depend on N_i only through the product $N_i S_i$. It is sufficient to vary the S_i^2 alone. In fact, since (90) can be written as:

$$n_i = \frac{n \frac{N_i S_i}{N}}{\sum_{i=1}^{k} \frac{N_i S_i}{N}} = \frac{n \frac{N_i}{N} S_i}{\sum_{i=1}^{k} \left[\frac{N_i}{N} S_i \right]} = \frac{n W_i S_i}{\sum_{i=1}^{k} \left[W_i S_i \right]} , \qquad (91)$$

stratum sizes N_i can be replaced by stratum proportions W_i. Because stratum sizes can be taken as constant, all stratum proportions will be chosen to be equal, i.e., 1/k. Tables IV) and V) give the variances for the different plans.

TABLE IV

STRATUM VARIANCES FOR THE NORMAL DISTRIBUTION WITH TWO STRATA.

	Plan Number			
Stratum	1	2	3	4
1	1	1	1	1
2	100	10	4	1

TABLE V

STRATUM VARIANCES FOR THE NORMAL DISTRIBUTION WITH THREE STRATA.

	Plan Number				
Stratum	1	2	3	4	5
1	1	1	1	1	1
2	100	10	1	4	1
3	10000	100	10	9	1

iii) Gamma Distributions

In most studies, the data are assumed to be normally distributed. This assumption is often, in practice, not valid. In fact, many positive valued variates in sample surveys are right skewed, e.g., income, number of brothers and sisters, etc.

The Gamma distributions, whose probability density functions(pdf) are given by:

$$f_{\alpha,\beta}(x) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \exp(-\beta x)\ x^{\alpha-1}\ I_{(0,\infty)}(x), \qquad (92)$$

where $\alpha > 0$ and $\beta > 0$, make up a family of distributions which are right skewed. In addition, they represent diverse shapes and central locations.

The Choice of Distributions. The location parameter β will be assumed to be 1 for this study, since the center of the distribution does not affect the optimal allocation. Three members of the family of these distributions will be selected, namely those with respective shape parameters α = 1, 2, and 5. A quick look at their probability density functions(pdf's), given in Figure 2, shows the shapes of these three distributions.

The strata in this study will consist of intervals of the form (a, b), where b>a and b is possibly infinite. Two strata will be selected for the purpose of the study and the boundary between the strata will be chosen so that the proportion of the population in the first stratum will meet a specified level. Four such levels will be considered, namely 0.5, 0.6, 0.7, and 0.8. Table VI lists the different plans.

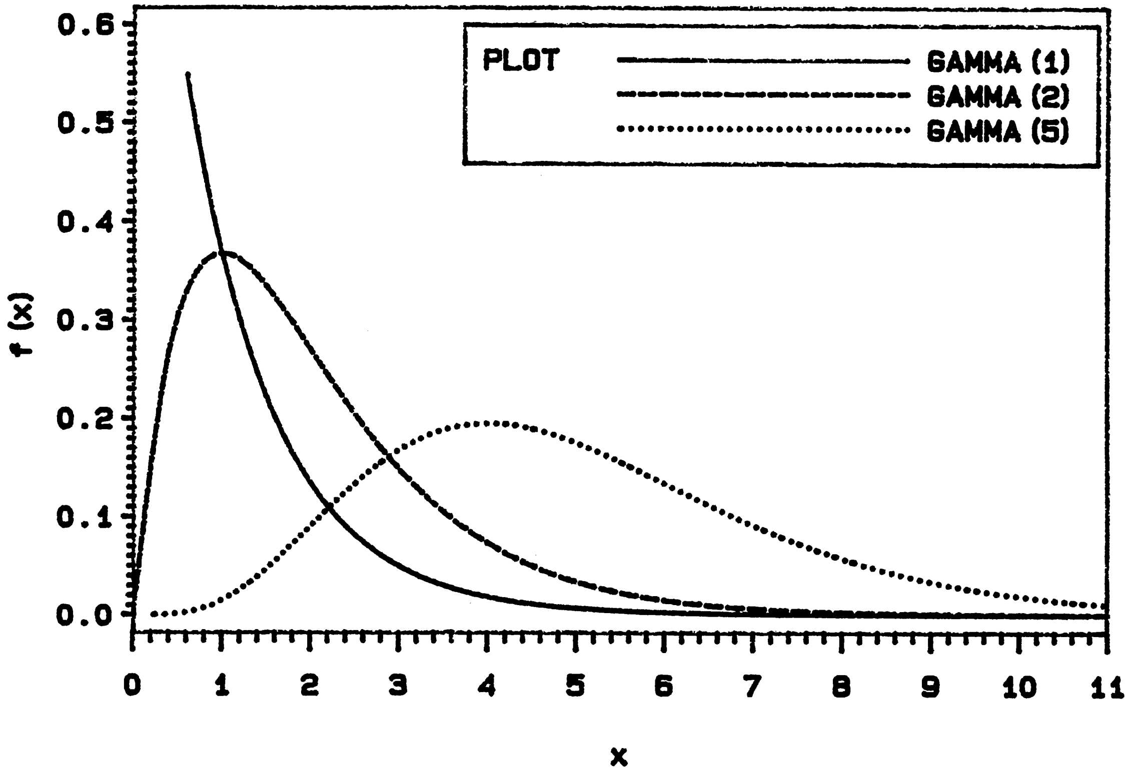

Figure 2. Pdf's of the Gamma Distributions

TABLE VI

SAMPLING PLANS FOR THE GAMMA SIMULATIONS

plan	Number of Strata	Proportion in stratum 1	2
1	2	.8	.2
2	2	.7	.3
3	2	.6	.4
4	2	.5	.5

Derivation of Means, and Variances. All simulations require that the variances and means of the strata be known. Consider an arbitrary stratum, say stratum i, consisting of the interval (a, b), where $0<a<b$. The mean and variance of stratum i are the first and second central moments of the population distribution conditioned on (a, b). Then it follows that the proportion of the population found in stratum i, is given by:

$$W_i = \int_a^b f(x)dx, \tag{93}$$

It also follows that the mean of the stratum is given by:

$$\mu_i = \left[\int_a^b xf(x)dx \right] / W_i \tag{94}$$

and the variance of the stratum is given by:

$$\sigma_i^2 = \frac{\left[\int_a^b x^2 f(x)dx \right]}{W_i} - [\mu_i]^2 \tag{95}$$

The mean and variance of the Gamma distribution conditioned on (a, b) can be written as multiples of integrals of other

Gamma pdf's. From (94) and (95), and assuming $\beta = 1$,

$$\mu_i = \int_a^b x \frac{1}{\Gamma(\alpha)} \exp(-x)\ x^{\alpha-1}\ I_{(0,\infty)}(x)dx \ /\ W_i \tag{96}$$

$$= \int_a^b \frac{1}{\Gamma(\alpha)} \exp(-x)\ x^{\alpha+1-1}\ dx \ /\ W_i \tag{97}$$

$$= \frac{\Gamma(\alpha+1)}{\Gamma(\alpha)} \int_a^b \frac{1}{\Gamma(\alpha+1)} \exp(-x)\ x^{\alpha+1-1}\ dx \ /\ W_i \tag{98}$$

$$= \frac{\Gamma(\alpha+1)}{\Gamma(\alpha)} \int_a^b f_{\alpha+1,\beta}(x)dx \ /\ W_i \tag{99}$$

$$= \alpha \int_a^b f_{\alpha+1,\beta}(x)dx \ /\ W_i\ , \tag{100}$$

since $\dfrac{\Gamma(\alpha+1)}{\Gamma(\alpha)} = \dfrac{\alpha\ \Gamma(\alpha)}{\Gamma(\alpha)} = \alpha$.

It can be similarly shown that (95) can be written as:

$$\sigma_i^2 = \frac{\left[\alpha(\alpha+1) \int_a^b f_{\alpha+2,\beta}(x)dx\right]}{W_i} - [\mu_i]^2 \tag{101}$$

The stratum means and variances were computed using SAS. They are displayed in Table VII. The SAS code used to obtain these values is found in Appendix B.

TABLE VII

STRATUM BOUNDARIES, MEANS, AND VARIANCES

Proportion for stratum 1	2	α	Boundary between strata	stratum 1 mean	var	stratum 2 mean	var
		1	1.61	0.60	0.19	2.61	1.00
.8	.2	2	2.99	1.44	0.57	4.24	1.44
		5	6.72	4.14	1.92	8.44	2.51
		1	1.20	0.48	0.11	2.20	1.00
.7	.3	2	2.44	1.26	0.38	3.73	1.50
		5	5.89	3.83	1.43	7.72	2.73
		1	0.92	0.39	0.07	1.92	1.00
.6	.4	2	2.02	1.10	0.26	3.35	1.55
		5	5.24	3.55	1.09	7.18	2.94
		1	0.69	0.31	0.04	1.69	1.00
.5	.5	2	1.68	0.95	0.18	3.05	1.61
		5	4.67	3.27	0.83	6.73	3.15

Results

Results of the simulations are summarized in Tables X through LXXIII, all of which are in Appendix E. The order of the tables follows the discussion of the results. Thus, the first group of tables covers the estimates of μ, the second group covers the proportion of trials where the sequential procedure did better, and the last group summarizes the proportion of observations allocated to the first stratum.

In the last group, there are tables corresponding to simulations involving two strata only. In examples involving more than two strata, the analysis of the proportion of observations allocated to a single stratum can be very misleading without some multivariate analysis.

Within each group of tables, the order is always the following:

1. Sukhatme examples, for first two groups,
2. the other normal distributions,
3. the Gamma distributions.

Most results hold for all the simulations, in which case the reader will be referred to a single representative table.

Estimates of μ.

For all the simulations, Tables X through XXXIII show the estimates of bias, variance, and mean square error(MSE) for both procedures. As a point of reference, the variance of the estimator of the mean under Neyman allocation is given at the bottom of the tables. This variance, when a total of n observations are available for allocation, is given by equation (102).

$$V(\hat{y}_{NEY}/n) = \frac{1}{n}\left[\frac{1}{N}\sum_{h=1}^{k} N_h S_h\right]^2 = \frac{1}{n}\left[\sum_{h=1}^{k} W_h S_h\right]^2 \qquad (102)$$

Note that all three of the statistics: bias, variance, and MSE; become smaller with the increase in initial sample sizes and/or number of observations. See for example table XXII. These results make good sense. The more data you have, the better will be the estimate. In fact, it is well known that with a fixed sample size problem, the variance of the estimator decreases by a factor of c when the sample size is increased by a factor of c. $V(\hat{y}_{NEY})$ is a good example of this phenomenon. If sample size is increased from n in equation (102) to cn for some positive constant c, then the variance becomes:

$$V(\hat{y}_{NEY}/cn) = \frac{1}{cn}\left[\frac{1}{N}\sum_{h=1}^{k} N_h\ S_h\right]^2 = \frac{1}{c}\ V(\hat{y}_{NEY}/n) \qquad (103)$$

The variance estimates from both Sukhatme's method and the sequential methods stay fairly close to this rule, although in certain cases, they do a little better. Consider for example the Gamma distribution with shape parameter $\alpha = 1$ partitioned into two equally weighted strata(Table XXII). Figure 3 extracts the case of initial sample sizes of 2 observations per stratum. The necessary information can be determined by a comparison of the magnitude of the slopes of the line segments. Since the slope of the line segments of Neyman's allocation is equivalent to the average improvement due to adding additional observations, the line segments of the estimated variances of the Sukhatme and Sequential estimators, $\hat{V}(\hat{y}_{SUK})$ and $\hat{V}(\hat{y}_{SEQ})$ respectively, should be compared against the slope of the corresponding line segment of $V(\hat{y}_{NEY})$. The line segments of $\hat{V}(\hat{y}_{SUK})$ or $\hat{V}(\hat{y}_{SEQ})$ are steeper than the line segment corresponding to $V(\hat{y}_{NEY})$, when the total number of observations increases from 100 to 150 in this example. When the total number of observations increases from 50 to 100, the improvement in $\hat{V}(\hat{y}_{SUK})$ and $\hat{V}(\hat{y}_{SEQ})$ is approximately equivalent to the improvement due to the increase in sample size, whereas the improvement is

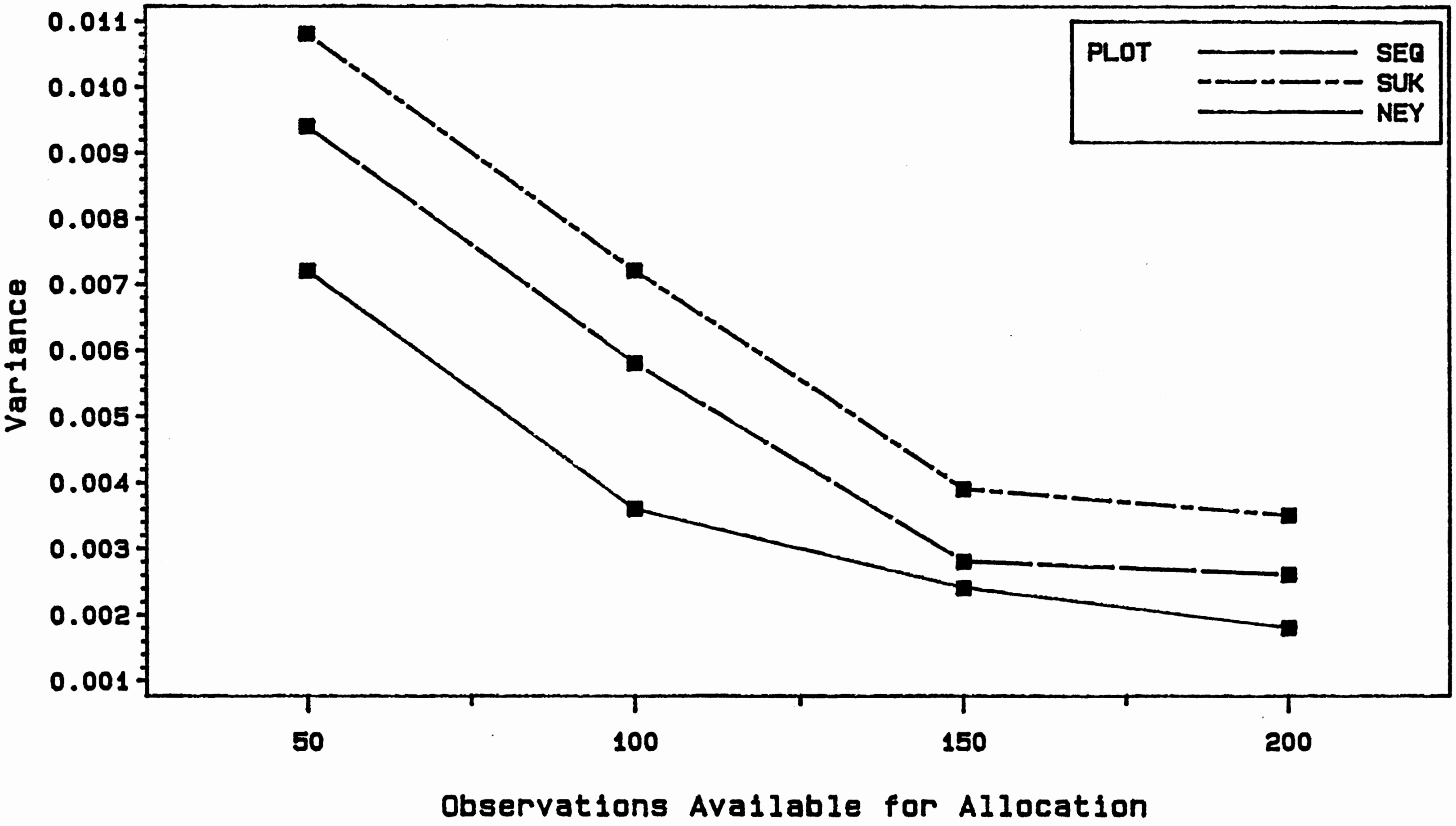

Figure 3. Variance Estimates for Gamma With Alpha=1, 50% of the Population in Each Stratum, and 2 Initial Obs per Stratum.

slower when sample size increases from 150 to 200. The strong improvement going from 100 to 150 observations more than offsets the sluggish improvement during the next 50 observations. There is a direct relationship between $\hat{V}(\hat{y}_{SUK})$ or $\hat{V}(\hat{y}_{SEQ})$ and $V(\hat{y}_{SUK})$, which transcends the changes in initial sample sizes and/or total observations available for allocation. Generally, the larger $V(\hat{y}_{NEY})$ is, the larger are $\hat{V}(\hat{y}_{SUK})$ and $\hat{V}(\hat{y}_{SEQ})$. In the case of examples involving the normal distributions, $V(\hat{y}_{NEY})$ decreases as one cycles from plan I to plan IV. See Table VIII on the following page. Recall that in each of the plans, the population is divided equally among the two strata and the variance of the first stratum is 1. With these values being fixed, it can be deduced from equation (102), that $V(\hat{y}_{NEY})$ is an increasing function of the variance of the second of the two strata. The variance of the second stratum decreases as the plan number increases, which translates into a decrease in $V(\hat{y}_{NEY})$ as the plan number increases.

When the proportion of the population in the first stratum is held constant, a decrease in the size of $V(\hat{y}_{NEY})$ occurs each time a more skewed Gamma distribution is selected, i.e. each time α increases. Refer to Table IX. Once again, the increase in $V(\hat{y}_{NEY})$ is coupled with increases in $\hat{V}(\hat{y}_{SUK})$ and $\hat{V}(\hat{y}_{SEQ})$. As an example, refer to the Gamma distributions with the population divided evenly between the two strata. Table IX shows that when 50 observations are available for allocation, $V(\hat{y}_{NEY})$ goes from

.007 to .014 and then to .036 when α increases from 1 to 2 and finally to 5. Tables XXII, XXVI, and XXX show the corresponding increase in $\hat{V}(\hat{y}_{SUK})$ and $\hat{V}(\hat{y}_{SEQ})$.

TABLE VIII

STRATUM 1 PROPORTIONS FOR THE NORMAL DISTRIBUTIONS WITH TWO STRATA

PLAN	PROPORTION OF OBS ALLOCATED TO FIRST STRATUM UNDER NEYMAN ALLOCATION	VARIANCE OF NEYMAN ESTIMATOR WHEN 50 TOTAL OBSERVATIONS AVAILABLE FOR ALLOCATION
I	.0909	.605
II	.2402	.087
III	.3333	.043
IV	.5000	.020

TABLE IX

INFORMATION ON THE NEYMAN ALLOCATION FOR THE GAMMA DISTRIBUTIONS WITH 2 STRATA

	PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM 1 UNDER NEYMAN ALLOCATION			VARIANCE OF NEYMAN ESTIMATOR WHEN 50 OBS. ALLOCATED		
	ALPHA			ALPHA		
PROPORTION OF POP. IN FIRST STRATUM	1	2	5	1	2	5
.5	.1644	.2512	.3385	.007	.014	.036
.6	.2806	.3818	.4772	.006	.013	.034
.7	.4383	.5415	.6281	.006	.013	.036
.8	.6359	.7148	.7775	.006	.014	.041

In almost all the simulations, the estimated variance of the new sequential method, $\hat{V}(\hat{y}_{SEQ})$ is at least as small as the estimated variance of Sukhatme's method, $\hat{V}(\hat{y}_{SUK})$, in almost all the simulations. In the few instances where this is not true, they are very close. See for example the estimated variances from plan II of the Normal distribution simulations, which are summarized in Table XIV. For an initial allocation of 15 in each of two strata, when a total of 50, 150, or 200 observations are allocated, $\hat{V}(\hat{y}_{SEQ})$ is larger than $\hat{V}(\hat{y}_{SUK})$ but not by much. In fact $\hat{V}(\hat{y}_{SEQ})$ is at most 4% larger than $\hat{V}(\hat{y}_{SUK})$. There is not much difference in the estimated variances for initial sample sizes exceeding 5. However with extremely small sample sizes, especially initial sample sizes of 2 per stratum, there is often a fairly significant difference.

The estimated biases for Sukhatme's method and for the sequential method tend to be very small and contribute very little to the mean square error. In the case of the Gamma distributions, the bias is also invariably negative. Refer to Table XXII for an example. Does this suggest that the procedure tends to have a negative bias for right skewed distributions? I am not sure, although the simulations support it in the cases studied here. In all the simulations involving the asymmetric distributions, i.e. the Gamma distributions, the absolute bias gets smaller as the distribution becomes more symmetric. Consider, for example, the sequence of Tables XXII, XXVI, and XXX. These were chosen because the proportion of the distribution found in the first stratum remains fixed, while the shape parameter is increasing, or equivalently, the skewness is decreasing.

Proportion of Time That the New Sequential Method Did Better

Tables XXXIV through LIX give the proportion of the trials in which the new sequential method did better in terms of smaller bias, and also the proportion of the trials in which the sequential procedure did better in terms of conditional variance. Two different definitions of better will be considered, namely, with and without ties. "With ties" means that if the trial ends in a tie, it is counted in favor of the sequential method. "Without ties" means it is not accounted towards the sequential method. This is particularly critical due to the way that the simulations were constructed. If n_i and n_i' observations are respectively allocated to stratum i by Sukhatme's method and the Sequential method, then for n_i'' trials, where n_i'' is the smaller of n_i and n_i' , both methods have the same value for y. This means of course that if both methods allocate the same number of observations in all strata, then $\bar{y}$ will be the same for both methods, resulting in a tie. Both methods attempt to allocate in such a way that the resulting allocation will come as close as possible to the Neyman allocation, hence this approach has an intuitive appeal since the same allocation should be a tie. Sometimes, of course, the bizarre situation arises where all the trials result in a tie. See for example Table XXXVIII. Here this occurs when 20 observations are allocated to each of the two strata and 50 total observations are allocated. A quick glance at the bottom of Table VIII reveals that 24% of the 50 observations should be allocated to the first stratum. Consider that 12 observations is close to the initial 10 which were allocated to that stratum. It appears that the estimates on the stratum variances vary so little that both methods produce the same results with the little breathing room that are given.

There appears to be little difference in the two methods in terms of the proportion of time that one method comes closer than the other to the true mean, except for a slight advantage when initial sample sizes are small. Note for example in Table XXXVII that with initial stratum sample sizes of 2, the two methods do about the same, but as total sample size increases, the sequential method improves slightly in comparison to Sukhatme's method until a total of 150 observations have been allocated. With a total of 200 observations, it appears that the the methods tend to perform more alike again.

There is, however, a clear advantage to using the sequential method when the concern is the proportion of trials for which $\hat{V}(\hat{y}_{SEQ})$ has smaller conditional variance than $\hat{V}(\hat{y}_{SUK})$. Conditional variance in this case means the variance conditioned on the same allocation in repeated trials:

$$V(\bar{Y}/\ n_1,\ \ldots,\ n_k,\ \Sigma\ n_i\ =\ n\ )\ =\ \Sigma\ \frac{W_i^2 S_i^2}{n_i} \tag{104}$$

(104) is, of course, minimized with the use of Neyman's allocation. The relative increase in variance from using an allocation of n_i, i = 1, ..., k, rather than the Neyman's allocation: n_i^N, i= 1,..., k is given by:

$$\Sigma\ \frac{(n_i - n_i^N)^2}{n_i^N}\ . \tag{105}$$

Thus the sequential estimator is better if its relative conditional increase is smaller. This can also be thought of as the allocation on that particular trial which was closer to the true allocation.

There are clear and definite patterns which hold for both the normal distributions and the Gamma distributions.

Regardless of the distribution or the number of observations initially allocated to the strata, as more and more observations are allocated, the sequential method results with greater frequency in an allocation which is closer to the Neyman allocation. This is not terribly surprising. As more observations are allocated, the sequential method is taking advantageous of more and more information on the stratum variances. In contrast, Sukhatme's method allocates observations based on old information only, namely the information resulting from the initial observations have been allocated.

The second result is that there is negative correlation between initial sample size and the proportion of trials in

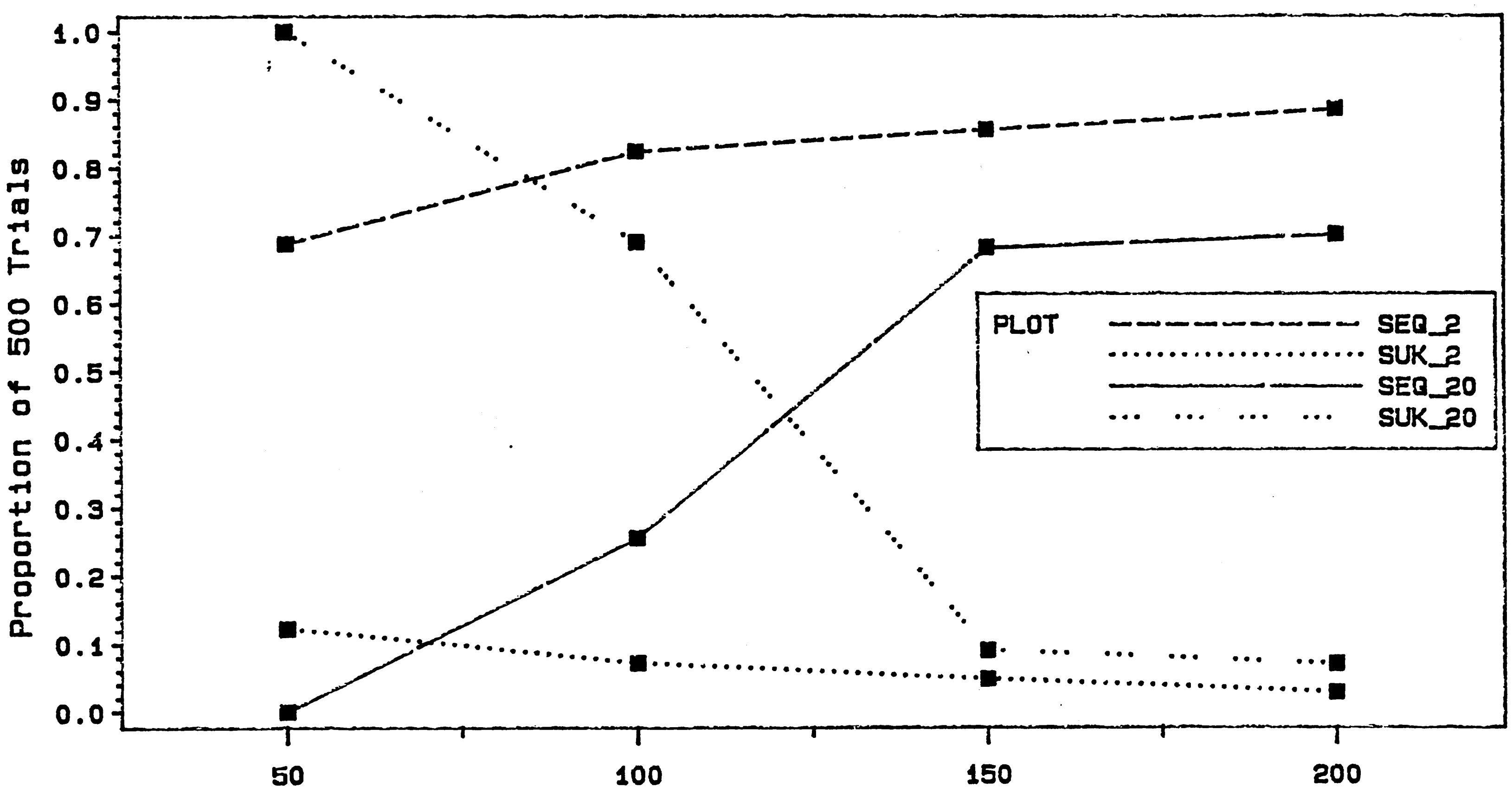

Figure 4. Proportion of Trials Where Methods Resulted in Smaller Conditional Variance. Table XLVI 2 and 20 Initial Observations Per Stratum

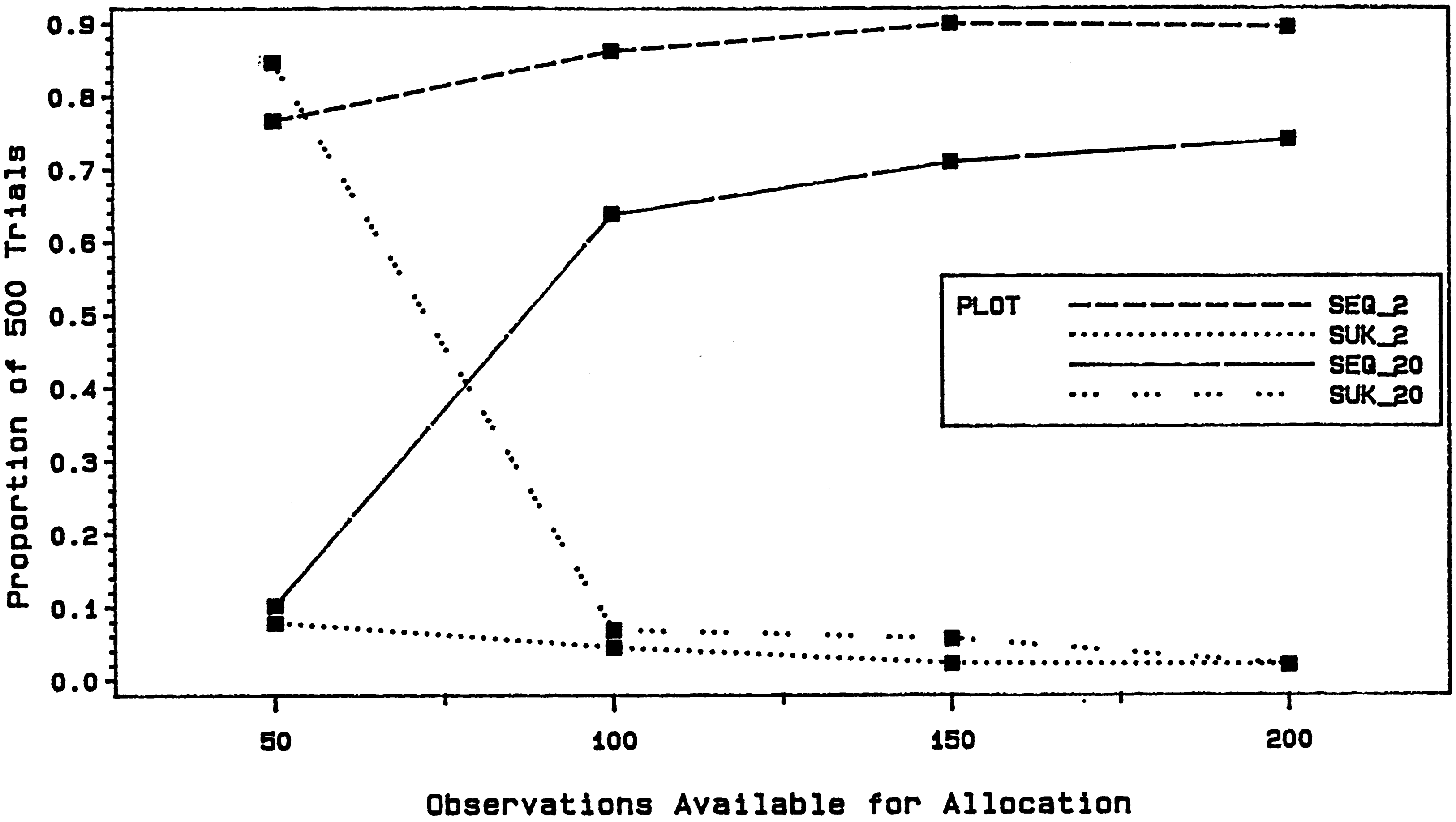

Figure 5. Proportion of Trials Where Methods Resulted
in Smaller Conditional Variance. Table LIV
2 and 20 Initial Observations Per Stratum

which the new method did strictly better, in terms of conditional variance. Sukhatme's method benefits by acquiring additional information on the stratum variances, hence it is more competitive when initial sample sizes are larger.

It also appears that as $V(\hat{y}_{NEY})$ increases, this proportion also increases. This is most easily seen by looking at the three Gamma distributions and holding the distribution of the population among the strata fixed. Recall that as α increases, $V(\hat{y}_{NEY})$ decreases. Four situations, which epitomize the three results described above, will be examined. They consist of initial sample sizes of 2 and 20 crossed with the extreme values of α considered in this dissertation, 1 and 5. Figures 4-5, found on the last two pages, illustrate the four situations. The results can also be gleaned from Tables XLVI and LIV. In each figure, the proportions of trials in which each method does strictly better are plotted against the number of observations available for allocation.

Proportion of Observations Allocated to Stratum 1

The last group of tables, Tables LVIII through LXXIII, is restricted to the 2 strata cases. These tables give the minimum, the three quartiles, and the maximum of the number of observations allocated to stratum 1. At the bottom of the tables, the reader will find the proportion of the observations which has been allocated to the first stratum when the Neyman allocation is used. This value is the goal that both of the methods are striving to obtain.

Again the results seem consistent across the different simulations. As a result, we will examine Table LIX, which corresponds to plan 2 of the Normal distributions partitioned into 2 strata.

Consider the worst possible case! More precisely, the case of the trial which produced the allocation the furthest from the goal. With only two initial observations per stratum, the minimum and maximum don't improve even after 200 observations are taken! This is true for both Sukhatme's method and the new sequential method. As the initial sample size increases, more improvement in the extreme values is seen for the new sequential method. As the number of total observations increases, the proportion of observations being allocated to the first stratum by Sukhatme's method does not change much. This arises because Sukhatme computes the proportion after the initial observations are allocated. The new Approach continues to use new information on the stratum variances in order to allocate the remaining observations.

Examine the median values for Table LIX. Note that for all combinations of initial sample size and total sample size, the median is fairly close to the true proportion of 24%. Figures 6 and 7 both show a series of box and whisker plots which summarize the progression of distributions, as total sample size is increased from 50 to 200 in steps of 50. The two figures are for initial sample sizes of 2 and 20 respectively. Note with the Sequential method, the improvement in the median and quartiles.

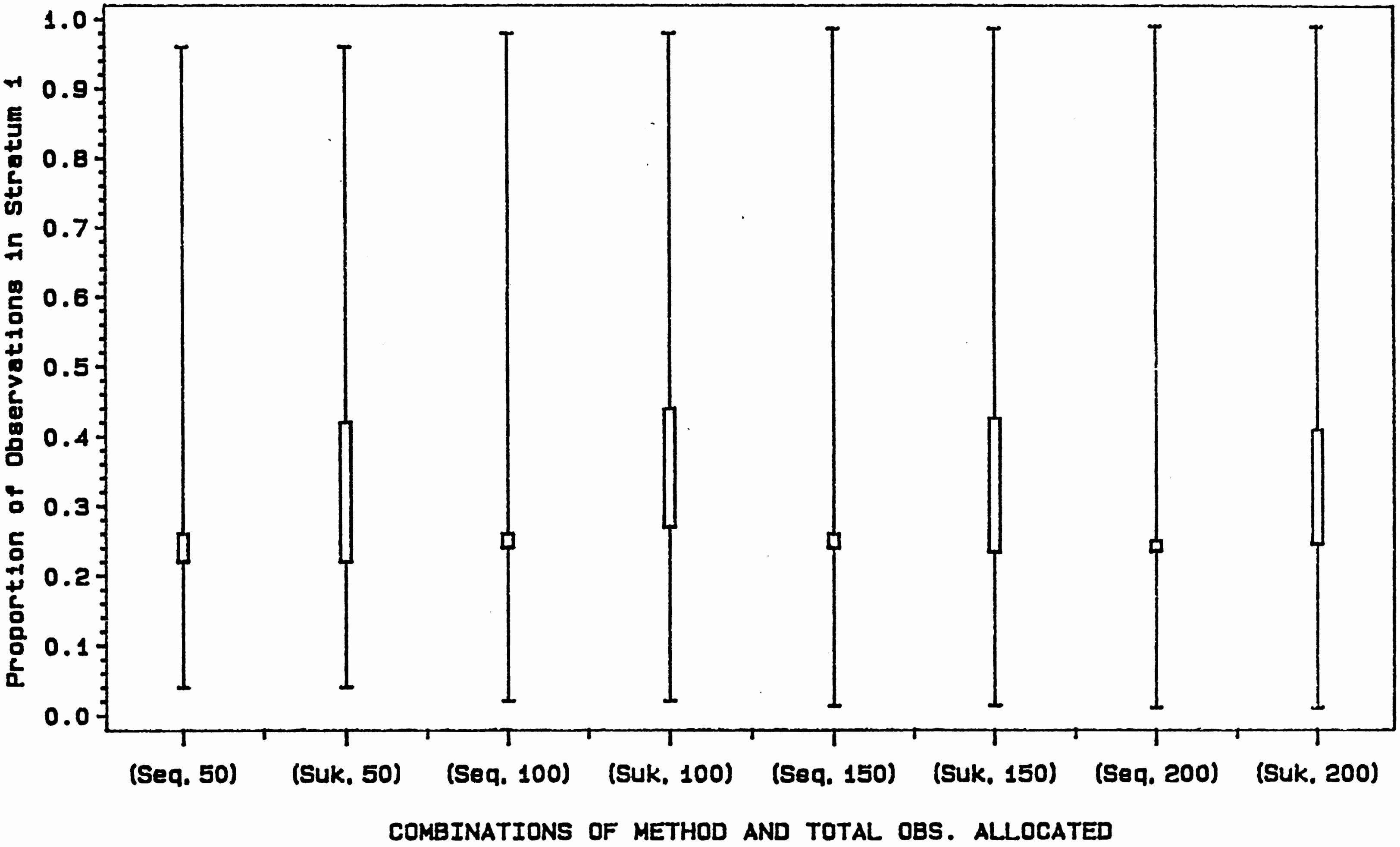

Figure 6. Box and Whisker Plots Table LIX
Case of 2 Initial Observations per Stratum

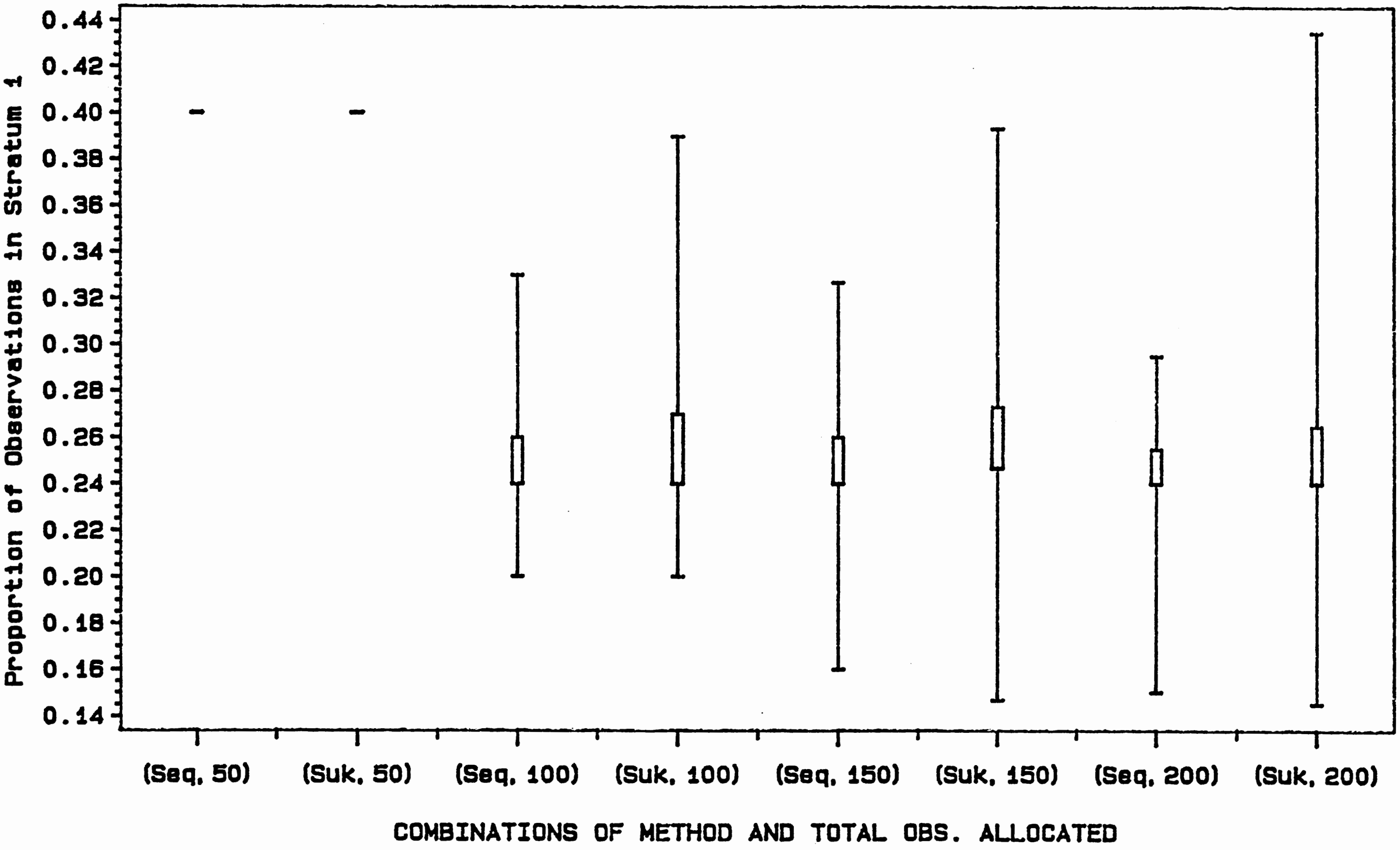

Figure 7. Box and Whisker Plots Table LIX
Case of 20 Initial Observations per Stratum

CHAPTER V

COMPUTER ASSISTED TELEPHONE INTERVIEWS

Introduction

Since its introduction in the early seventies, the use of computer assisted telephone interviewing, commonly abbreviated as CATI, has become steadily more widespread. In this data collection method, questions are displayed to the interviewer on the computer screen. These questions are read over the phone to the person being interviewed, and the interviewee's response to a question is then typed directly into the computer by the interviewer.

The use of CATI has numerous significant advantages, many of which are discussed by Groves and Nicholls II(1986). The elimination of the intermediate pencil and paper stage results in a considerable savings in time and a reduction in data entry errors. The computer can be programmed to randomly choose and display the name and phone number of the subsequent interviewee, thus reducing the work of the interviewer. Since the response to a question is immediately entered into the computer, the wording of following questions, as well as the selection of questions, can be easily tailored to each individual. Responses may be automatically checked for unreasonableness or for information which conflicts with information given earlier in the interview. Appropriate questions can then be displayed to permit the interviewer to clarify the situation.

The Use of Cati With Sequential Procedures

CATI has another significant advantage. The use of sequential designs, such as the procedure introduced in this dissertation, involves tedious calculations before each and every observation is drawn. These calculations are simply too complex for even the most mathematically skilled operator. CATI, by its very ability to do repetitious calculations, can be programmed to automatically perform this task.

Consider, as an example of this type of interactive program, the QBASIC code presented in Appendix D. Questions are supplied to the program in an ASCII file named "QUESTION.DAT". These questions may be entered by means of any word processor which has the capability of creating ASCII files. The program shows the interviewer the questions before the interviews begin and requests the following information:

1) Is the response to the question numeric or alphanumeric?

2) Is the distribution of responses for the question the distribution used to determine optimal allocation?

Names and the corresponding phone numbers are supplied to the program in ASCII files named "STRATUM1.DAT", and

"STRATUM2.DAT" for strata 1 and 2, respectively. The inteviewer enters the number of interviews which will be carried out.

Although some of the advantages of CATI are highlighted in this example, it is intended primarily to illustrate the ease with which the new sequential allocation procedure can be incorporated into the framework of CATI.

CHAPTER VI

CONCLUSIONS

The new sequential allocation method provides a competitive alternative for estimating certain parameters of a population partitioned into subpopulations, whose variances are unknown. It is also the first approach which uses a sequential approach, although its major competitor is a two stage process. In addition, the allocation from this method has been proven to converge almost surely to Neyman's allocation, which for known stratum variances provides minimum variance among unbiased fixed sample estimators of the parameter. Our only restriction is that the estimators of the stratum variances be consistent.

It was shown that the process is similar to a Robbins-Monro process, processes which have already been extensively researched. This similarity opens the door to further research to determine which of the myriad beneficial properties of Robbins-Monro procedures are applicable to this allocation procedure.

It was demonstrated that this method is competitive with Sukhatme's two stage procedure for initial sample sizes of under 20 per stratum. Furthermore, where the researcher is limited to extremely small initial sample sizes of at most 5 per stratum, the allocation of this procedure is on average much closer to the desired Neyman allocation, and the variance of the estimator appears to be smaller than its competition. All results hold even for skewed distributions such as the family of Gamma distributions.

Although choosing the next stratum involves tedious calculations, this encumbrance can be easily eliminated by programming the computer to perform the necessary calculations. Indeed, computers are frequently used to assist the researcher in carrying out telephone interviews anyway, so only a slight modification of the programming code is necessary to use this new approach.

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APPENDIX A

REFERENCES AND PROOFS FOR LEMMA 1

Parts A) and B)

A) Serfling p. 69

B) Serfling p. 19

Parts C) i) & ii)

C) i) Serfling p. 26

ii) Serfling p. 26

Part C) iii)

Suppose that $X_n \neq 0$ a.s., $X \neq 0$ a.s. and $X_n \xrightarrow{a.s.} X$

Then there exists a set A s.t. $P(A) = 1$ and for every $\omega \in A$ $X_n(\omega) \longrightarrow X(\omega)$.

Furthermore there exists sets B_1 and B_2
s.t. $P(B_i) = 1 \quad i = 1, 2$

and for every $\omega \in B_1 \quad X_n(\omega) \neq 0$

and for every $\omega \in B_2 \quad X(\omega) \neq 0$
Let $B = A \cap B_1 \cap B_2$.

$P[(A \cap B_1 \cap B_2)] = 1 - P(A^c \cup B_1^c \cup B_2^c)$

$\geq 1 - [P(A^c) + P(B_1^c) + P(B_2^c)] = 1$
so $P(B) = 1$

Let $\omega \in B$
Then $\omega \in A$, $\omega \in B_1$.and $\omega \in B_2$

$\omega \in A \Longrightarrow X_n(\omega) \longrightarrow X(\omega)$.

$\omega \in B_1 \Longrightarrow X_n(\omega) \neq 0. \Longrightarrow 1/X_n(\omega) < \infty$

$\omega \in B_2 \Longrightarrow X(\omega) \neq 0 \quad \Longrightarrow 1/X(\omega) < \infty$

It follows that $1/X_n(\omega) \longrightarrow 1/X(\omega)$ for all $\omega \in B$

and since $P(B) = 1$, $1/X_n \xrightarrow{a.s.} 1/X$ QED

Part C iv)

C) iv) Serfling p .24

Part D

Consider N independent populations defined on Ω.

Let $\hat{\sigma}^2_{i,n}/n_i$ be the estimated variance of the estimator in population i, i = 1, ... N.

Then

$\hat{\sigma}^2_{i,n} \xrightarrow{a.s.} \sigma^2_i$ where σ^2_i is the variance of the population in population i.

Then for any continuous functions f_i i = 1, 2, ..., N it follows by (20) that for i = 1, 2, ..., N

$f_i(\hat{\sigma}^2_{i,n}) \xrightarrow{a.s.} f_i(\sigma^2_i)$

By (17) $\sum_{i=1}^{N} f_i(\hat{\sigma}^2_{i,n}) \xrightarrow{a.s.} f_i(\sum_{i=1}^{N} \sigma^2_i)$

Assume that $f_i(\hat{\sigma}^2_n(\omega)) \neq 0$ and $f_i(\sigma^2) \neq 0$

for all $\omega \in A \subset \Omega$ s.t. $P(A)= 1$

By (19), $1/\sum_{i=1}^{N} f_i(\hat{\sigma}^2_{i,n}) \xrightarrow{a.s.} 1/f_i(\sum_{i=1}^{N} \sigma^2_i)$

Then by (19)

$f_i(\hat{\sigma}^2_{i,n})/ \sum_{i=1}^{N} f_i(\hat{\sigma}^2_{i,n}) \xrightarrow{a.s.} f_i(\sigma^2_i) / f_i(\sum_{i=1}^{N} \sigma^2_i)$ QED

APPENDIX B

SAS CODE TO CALCULATE BOUNDARIES,
MEANS, AND VARIANCES

```
data gamstats;
  array boundary {4} b1-b4;
  array p        {4} p1-p4;
  input k@@ ;
  kminus1 = k - 1;
  pcum = 0;
  do stratum = 1 to kminus1;
    input ptemp @@;
    pcum + ptemp;
    p(stratum)= pcum;
  end;

*****************************************
**  Do loop for three values of alpha  **
*****************************************;
  do alpha = 1, 2, 5;
     do stratum = 1 to kminus1;
       boundary(stratum) = gaminv(p(stratum), alpha);
     end;

*****************************************
**  Calculate F(a)                     **
*****************************************;

     do i = 1 to k;
        if i = 1 then do;
           ln=0; lnp1=0;lnp2=0;
        end;
        else do;
          a = boundary(i-1);
          ln = probgam(a,alpha);
          lnp1 = probgam(a,alpha+1);
          lnp2 = probgam(a,alpha+2);
        end;

*****************************************
**  Calculate F(b)                     **
*****************************************;

        if i = k then do;
```

```
          un = 1; unp1 = 1; unp2 = 1;
        end;
        else do;
          b = boundary(i);
          un = probgam(b,alpha);
          unp1 = probgam(b,alpha+1);
          unp2 = probgam(b,alpha+2);
        end;
*****************************************
**  Calculate Mean and Variance        **
*****************************************;

          falpha = un-ln;
          falphap1 = unp1 - lnp1;
          falphap2 = unp2 - lnp2;
          E = alpha*falphap1/falpha;
          V = (alpha+1)*alpha*falphap2/falpha - E**2;
          output;
   end;end;
cards;
2 .8
2 .7
2 .6
2 .5
3 .7 .2
3 .6 .3
3 .6 .2
3 .5 .3
3 .5 .4
3 .4 .3
;
```

APPENDIX C

SAS CODE FOR THE GAMMA SIMULATION STUDY

The following SAS code(SAS Institute, Inc.) was used to simulate the sequential sampling from a gamma distribution with location parameter β = 1 and shape parameter α.

```
LIBNAME TABLE 'U12617A.GAMMATAB.SAS' DISP=(OLD,KEEP);
LIBNAME SASDD 'U12617A.GAMMAOUT.SAS' DISP=(OLD,KEEP);
***********************************************************
**  PROGRAM TO SIMULATE THE COMPARISON OF SUKHATME'S     **
**  PROCEDURE AND THE NEWSEQUENTIAL ALLOCATION PROC-     **
**  DURE IN THE CONTEXT OF A GAMMA DISTRIBUTION          **
**  PARTITIONED INTO TWO OR THREE STRATA.                **
**-----------------------------------------------------**
**  INPUT:                                               **
**     BY ASSIGNMENT STATEMENT:                          **
**        K        = THE NUMBER OF STRATA                **
**        NOTRIALS = THE NUMBER OF TRIALS                **
**     BY DO LOOP STATEMENTS:                            **
**        NINIT1   = THE NUMBER OF OBS INITIALLY         **
**                   ALLOCATED TO EACH STRATUM           **
**        NSEQMX   = TOTAL NUMBER OF OBS TO ALLOCATE     **
**     BY THE INPUT STATEMENT:                           **
**        1) ALPHA  2) PROPORTION OF POP IN STRATUM 1    **
**        3) MEAN OF STRATUM 1  4) VARIANCE IN STRATUM 1 **
**        5) MEAN OF STRATUM 2  6) VARIANCE IN STRATUM 2 **
**        7) PROPORTION OF POP IN STRATUM 2              **
**        8) MEAN OF STRATUM 3  9) VARIANCE IN STRATUM 3 **
**     IF K = 2, THEN ENTER "." FOR EACH OF 7), 8), 9)   **
***********************************************************;

DATA SASDD.GNEW2;
                                 /***************************/
 ARRAY BOUND {3} BOUND1-BOUND3; /*  STRATUM BOUNDARIES      */
 ARRAY FS2H{3} FS2HOF1-FS2HOF3; /*  F(SIGMA HAT SQUARED)    */
 ARRAY LHAT{3} LHAT1-LHAT3;     /*  LAMBDA HATS             */
 ARRAY MU{3} MU1-MU3;           /*  STRATUM MEANS           */
 ARRAY NINIT{3} NINIT1-NINIT3;  /*  INITIAL SAMPLE SIZES    */
 ARRAY NNEY{3} NNEY1-NNEY3;     /*  OPTIMUM SAMPLE SIZES    */
 ARRAY NSEQ{3} NSEQ1-NSEQ3;     /*  SAMPLE SIZES            */
 ARRAY OK{3}   OK1-OK3;         /*  UTILITY ARRAY           */
```

```
ARRAY WPOPCUM{3} WPOPCUM1-      /*                           */
                 WPOPCUM3;      /*  STRATUM PROPORTIONS      */
ARRAY WPOP{3} WPOP1-WPOP3;      /*  STRATUM SIZES            */
ARRAY NSUK{3} NSUK1-NSUK3;      /*  SUKHATME'S SAMPLE SIZE*/
ARRAY S2{3} S2OF1-S2OF3;        /*  STRATUM VARIANCES        */
ARRAY SH2{3} SH2OF1-SH2OF3;     /*  ESTIMATED VARIANCES      */
ARRAY SUKTEMP{3} SUKTEMP1-      /*                           */
                 SUKTEMP3;      /*  CUMULATED X'S FOR SUK */
ARRAY XCUM {3} XCUM1-XCUM3;     /*  CUMULATED X'S            */
ARRAY X2CUM {3} X2CUM1-X2CUM3; /*  CUMULATED SQUARED X'S */
                                /*************************/
*******************************
***  INPUT INFORMATION      ***
*******************************;
K=2;
NOTRIAL=100;
INPUT ALPHA WPOP1 MU1 S2OF1 MU2 S2OF2 WPOP2 MU3 S2OF3;
TRUEMU=ALPHA;
WPOP2 = 1 - WPOP1;
WPOPCUM1 = 0;
WPOPCUM2 = WPOP1;
WPOPCUM3 = WPOP1+WPOP2;
BOUND1=GAMINV(WPOPCUM2,ALPHA);
DO SEQMX = 50, 100, 150, 200;
  *****************************************************
  ***** CALCULATE THE OPTIMUM NEYMAN ESTIMATOR  *****
  *****************************************************;
  DO STRATUM = 1 TO K;
    NNEY(STRATUM) = WPOP(STRATUM)*SQRT(S2(STRATUM));
  END;
  S= SUM(OF NNEY1-NNEY3);
  DO STRATUM = 1 TO K;
    NNEY(STRATUM) = NSEQMX*NNEY(STRATUM)/S;
  END;
  VNEY = 0;
  DO STRATUM = 1 TO K;
    VNEY + WPOP(STRATUM)**2*S2(STRATUM)
             / NNEY(STRATUM);
  END;

  *********************************************
  *****   ESTABLISH INITAL SAMPLE SIZES   *****
  *********************************************;
  DO NINIT1 = 2 , 5, 10, 15, 20;
    IF NINIT1*K > NSEQMX THEN GOTO BOTTOM;
    DO STRATUM = 2 TO K;
      NINIT(STRATUM) = NINIT1;
    END;

    DO TRIAL = 1 TO NOTRIAL;
      DO STRATUM = 1 TO K;
        NSEQ(STRATUM) = NINIT(STRATUM);
      END;
      NSEQT = SUM(OF NSEQ1-NSEQ3);
```

```
    *************************************
    ***** TAKE INITIAL OBSERVATIONS *****
    *************************************;
    DO STRATUM = 1 TO K;
      U=STRATUM;
      XCUM(STRATUM) = 0; X2CUM(STRATUM)= 0;
      DO J = 1 TO NSEQ(STRATUM);

        ***************************************
        **  SET PARAMETERS, CALL GAMMA, AND   **
        **  TRANSFER RESULTS TO THE VARIABLES**
        ***************************************;
        R = RANUNI(982)*WPOP(U)+WPOPCUM(U);
        PNT = INT(R*1000); IF PNT = 0 THEN PNT = 1;
        SET TABLE.GAMMA2 POINT=PNT;
        XCUM(STRATUM) = XCUM(STRATUM) + Y;
        X2CUM(STRATUM) =X2CUM(STRATUM) + Y **2;
      END;

      ***********************************
      ** CALCULATE VARIANCE ESTIMATES  **
      ***********************************;
      SH2(STRATUM) = (X2CUM(STRATUM)-XCUM(STRATUM)**2
                  /NSEQ(STRATUM))/(NSEQ(STRATUM)-1);
      FS2H(STRATUM) = SQRT(SH2(STRATUM))*WPOP(STRATUM);
    END;
    FS2HT=SUM(OF FS2HOF1-FS2HOF3);

    *********************************************
    ***** CALCULATE SUKHATME'S STATISTICS   *****
    *********************************************;
    DO STRATUM = 1 TO K;
      TEMP = NSEQMX*FS2H(STRATUM)/FS2HT;
      NSUK(STRATUM) = ROUND(TEMP, 1);
      OK(STRATUM) = 1;
    END;
    *********************************
    ** CORRECT FOR FEWER OBS THAN  **
    ** INITIALLY ALLOCATED         **
    *********************************;

    CNT = 0;
TOP: ;
    DO STRATUM = 1 TO K;
      IF NSUK(STRATUM) < NINIT1 THEN DO;
        CNT = CNT + 1;
        OK(STRATUM) = 0;
        NSUK(STRATUM) = NINIT1;
        UCUM = 0;
        DO J = 1 TO K;
          UCUM + OK(J)*FS2H(J);
        END;
        DO J = 1 TO K;
          IF OK(J) = 1 THEN DO;
            NSUK(J) =
```

```
                (NSEQMX-NINIT1*CNT)*FS2H(J)*OK(J)/UCUM;
          NSUK(J) =ROUND(NSUK(J),1);
        END;
      END;
      GOTO TOP;
    END;
  END;

  RINCSUK = 0;
  DO STRATUM = 1 TO K;
    RINCSUK + (NNEY(STRATUM)-NSUK(STRATUM))**2
       /(NSEQMX*NSUK(STRATUM));
  END;

  *********************************
  ***** SEQUENTIAL SAMPLING   *****
  *********************************;
  NSEQT = SUM(OF NINIT1-NINIT3);
  LASTSTEP=NSEQMX - 1;
  DO STEP = NSEQT TO LASTSTEP;
    DO STRATUM = 1 TO K;
      IF NSUK(STRATUM)=NSEQ(STRATUM)
      THEN SUKTEMP(STRATUM) = XCUM(STRATUM);
    END;

    *************************************
    ***** CALCULATE THE LAMBDA HATS *****
    *************************************;
    DO STRATUM = 1 TO K;
      LHAT(STRATUM)=FS2H(STRATUM)
                   /FS2HT-NSEQ(STRATUM)/NSEQT;
    END;

    ************************************************
    ***** FIND STRATUM WITH MAXIMUM LAMBDA HAT *****
    ************************************************;
    MAXLH = MAX(OF LHAT1-LHAT3);
    DO STRATUM = 1 TO K;
      IF MAXLH = LHAT(STRATUM) THEN MAXPOS = STRATUM;
    END;

    **************************************
    ***** CHOOSE THE NEXT OBSERVATION ****
    **************************************;
    NSEQ(MAXPOS) +1;
    NSEQT +1;
    U=MAXPOS;
    R = RANUNI(1082)*WPOP(U)+WPOPCUM(U);
    PNT = INT(R*1000); IF PNT = 0 THEN PNT = 1;
    SET TABLE.GAMMA2 POINT=PNT;
    XCUM(MAXPOS) + Y;
    X2CUM(MAXPOS) + Y**2;

    ************************************
    ** UPDATE THE STRATUM INFORMATION **
```

```
          ***********************************;
          SH2(MAXPOS) = (X2CUM(MAXPOS)-XCUM(MAXPOS)**2
                      /NSEQ(MAXPOS))/(NSEQ(MAXPOS)-1);
          FS2H(MAXPOS) = SQRT(SH2(MAXPOS))*WPOP(MAXPOS);
          FS2HT = SUM(OF FS2HOF1-FS2HOF3);
        END;

        ***************************************
        **  TAKE THE REST OF THE OBSERVATIONS**
        **  FOR SUKHATME                     **
        ***************************************;
        SUKMEAN=0;
        DO STRATUM = 1 TO K;
          IF NSEQ(STRATUM) > NSUK(STRATUM)
          THEN UCUM = SUKTEMP(STRATUM);
          ELSE DO;
            UCUM= XCUM(STRATUM);
            START = NSEQ(STRATUM) + 1;
            DO OBS =  START TO NSUK(STRATUM);
              R = RANUNI(932)*WPOP(STRATUM)+WPOPCUM(STRATUM);
              PNT = INT(R*1000); IF PNT = 0 THEN PNT = 1;
              SET TABLE.GAMMA2 POINT=PNT;
              UCUM + Y;
            END;
          END;
          SUKMEAN + WPOP(STRATUM)*UCUM/NSUK(STRATUM);
        END;
        SUKBIAS = SUKMEAN - TRUEMU;

        *************************************
        ***** CALCULATE SEQUENTIAL STATS ****
        *************************************;
        SEQMEAN = 0;
        RINCSEQ = 0;
        DO STRATUM = 1 TO K;
          RINCSEQ + (NNEY(STRATUM)-NSEQ(STRATUM))**2
                /(NSEQMX*NSEQ(STRATUM));
          SEQMEAN +XCUM(STRATUM)*WPOP(STRATUM)/NSEQ(STRATUM);
        END;
        SEQBIAS = SEQMEAN - TRUEMU;

        **************************************
        **  CALCULATE COMPARISON STATISTICS **
        **************************************;
        IF RINCSEQ <= RINCSUK THEN VARFLAG = 1;
        ELSE VARFLAG = 0;
        IF ABS(SEQBIAS) <= ABS(SUKBIAS) THEN BIASFLAG = 1;
        ELSE BIASFLAG = 0;
        VNEY = ROUND(VNEY, .0001);
        RINCSEQ = ROUND(RINCSEQ, .0001);
        RINCSUK = ROUND(RINCSUK, .0001);
        OUTPUT;
      END; /* TRIAL LOOP */
    BOTTOM: END; /* NINIT1 LOOP */
  END /* NSEQMX LOOP */ ;
```

APPENDIX D

QBASIC CODE SHOWING AN EXAMPLE OF THE SEQUENTIAL PROCEDURE INCORPORATED INTO A COMPUTER ASSISTED TELEPHONE INTERVIEW

```
DECLARE SUB CALCW (NUMREC1%, NUMREC2%)
DECLARE SUB DETSTRATUM (NUMSTR%, MAXP%)
DECLARE SUB ENDQMESSAGE ()
DECLARE SUB INTERVIEW (NUMBER, STR%, Y)
DECLARE SUB MAXIMUM (STR%, MAXP%)
DECLARE SUB QINFO (FLAG, I, NUMLINES, OPTIMALQ, TOFQ)
DECLARE SUB READYMESSAGE (MININT%, NUMINT%)
DECLARE SUB STARTMESSAGE ()

'FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
'FFF  FUNCTION DEFINITION FNSUM      FFF
'FFF==================================FFF
'FFF  Finds the sum of an array      FFF
'FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF

DEF FNSUM (SIZE%)
   SUM = 0
   FOR I% = 1 TO SIZE%
     SUM = SUM + FS2H(I%)
     PRINT "FNSUM"; SUM; FS2H(I%)
   NEXT
  FNSUM = SUM
END DEF
'MMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMM
'MMM  MAIN MODULE                                      MMM
'MMM  programmer: Christoph Maier  date: July 2, 1992  MMM
'MMM===================================================MMM
'MMM  queries interviewer about number of strata,      MMM
'MMM  calls QINFO, opens name/telephone files,  MMM
'MMM  randomly chooses names from the correct stratum, MMM
'MMM  calls INTERVIEW, and updates information used    MMM
'MMM  for the sequential sampling.                     MMM
'MMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMM

DIM SHARED LHAT(1 TO 2)
DIM MU(1 TO 2)
```

```
DIM NINIT(1 TO 2)
DIM RESPONSE AS STRING * 10
DIM SHARED FS2H(1 TO 2)
DIM SHARED NUMQUESTIONS AS INTEGER
DIM SHARED NSEQ(1 TO 2)
DIM QUESTION AS STRING * 80
DIM SH2(1 TO 2)
DIM SHARED WPOP(1 TO 2)
DIM XCUM(1 TO 2)
DIM X2CUM(1 TO 2)
TIMER ON
NUMINIT% = 2
NUMSTR% = 2;
CLS
MININT% = NUMSTR% * NUMINIT%
NUMINT% = NUMSTR%
NUMBER = 1
FLAG = 0
I = 1
OPEN "QUESTION.DAT" FOR INPUT AS #10
OPEN "QINFO.DAT" FOR OUTPUT AS #20
LINE INPUT #10, QUESTION
DO
 '------------ SELECT-----------------
 SELECT CASE UCASE$(LEFT$(QUESTION, 3))
 '-------------------------------------
   CASE "   "
  '----------
      NUMLINES = 1
      DO WHILE UCASE$(LEFT$(QUESTION, 3)) = "   "
        PRINT QUESTION
        LINE INPUT #10, QUESTION
        NUMLINES = NUMLINES + 1
      LOOP
      CALL QINFO(FLAG, I, NUMLINES, OPTIMALQ, TOFQ)
      WRITE #20, I, TOFQ, NUMLINES, OPTIMALQ
      PRINT "TYPE"; TOFQ;
            "NUMLINES"; NUMLINES; "OPTIMALQ"; OPTIMALQ
      I = I + 1

   CASE "END"
  '----------
      NUMQUESTIONS = I - 1
      Y = 0
      CLOSE #10
      CLOSE #20
      GOTO 100

   CASE ELSE
  '---------
      PRINT "   "
      PRINT "****** QUESTION ********"
      PRINT QUESTION
      LINE INPUT #10, QUESTION
   END SELECT
```

```
  '-----------
LOOP

100  CALL READYMESSAGE(MININT%, NUMINT%)

OPEN "A:STRATUM1.DAT" FOR RANDOM AS #1 LEN = 40
FIELD #1, 25 AS FULLNAME1$, 13 AS PHONE1$
NUMREC1% = LOF(1) / 40

OPEN "A:STRATUM2.DAT" FOR RANDOM AS #2 LEN = 40
FIELD #2, 25 AS FULLNAME2$, 13 AS PHONE2$
NUMREC2% = LOF(2) / 40

CALL CALCW(NUMREC1%, NUMREC2%)
RANDOMIZE (TIMER)
FOR STR% = 1 TO NUMSTR%
  XCUM(STR%) = 0
  X2CUM(STR%) = 0
  FOR j = 1 TO NUMINIT%

    SELECT CASE STR%
   '-----------------
    CASE 1
   '------
     INPUT "CASE 1", T
     REC% = RND * NUMREC1% + 1
     PRINT "GETNAME"; NUMREC1%, REC%
     GET #1, REC%
     CLS
     PRINT "RECORD FROM STRATUM 1:  "; FULLNAME1$; PHONE1$
     INPUT "Hit any key when ready to proceed ", T
    CASE 2
   '------
     INPUT "CASE2", T
     REC% = RND * NUMREC2% + 1
     GET #2, REC%
     CLS
     PRINT "RECORD FROM STRATUM 2:  "; FULLNAME2$; PHONE2$
     INPUT "Hit any key when ready to proceed ", T
    END SELECT
   '----------
    DO
       PRINT "Here are the questions!!!"
       CALL INTERVIEW(NUMBER, STR%, Y)
    LOOP UNTIL UCASE$(LEFT$(RESPONSE, 3)) = "YES"

    NUMBER = NUMBER + 1
    XCUM(STR%) = XCUM(STR%) + Y
    X2CUM(STR%) = X2CUM(STR%) + Y * Y
  NEXT

  '************************************
  '** CALCULATE VARIANCE ESTIMATES   **
  '************************************
```

```
  PRINT STR%
  NSEQ(STR%) = NUMINIT%
  SH2(STR%) = (X2CUM(STR%)-XCUM(STR%)^2
             /NUMINIT%)/(NUMINIT%-1)
  PRINT "SH2"; SH2(STR%)
  PRINT WPOP(STR%)
  FS2H(STR%) = SQR(SH2(STR%)) * WPOP(STR%)
NEXT
NSEQT = NUMINIT% * NUMSTR%
FS2HT = FNSUM(NUMSTR%)
PRINT "FS2HT"; FS2HT

'*********************************
'***** SEQUENTIAL SAMPLING   *****
'*********************************

  INPUT "SEQUENTIAL SAMPLING", T
  LASTSTEP% = NUMINT% - 1
  PRINT MINT%, LASTSTEP%
  FOR RSTEP = MINT% TO LASTSTEP%
    CALL DETSTRATUM(NUMSTR%, MAXP%)

    SELECT CASE MAXP%
   '-----------------
    CASE 1
   '------
      INPUT "CASE 1", T
      REC% = RND * NUMREC1% + 1
      PRINT "GETNAME"; NUMREC1%, REC%
      GET #1, REC%
      PRINT "RECORD FROM STRATUM 1:  "; FULLNAME1$; PHONE1$
    CASE 2
   '------
      INPUT "CASE2", T
      REC% = RND * NUMREC2% + 1
      GET #2, REC%
      PRINT "RECORD FROM STRATUM 2:  "; FULLNAME2$; PHONE2$
   END SELECT
  '----------

  CALL INTERVIEW(NUMBER, MAXP%, Y)
  NUMBER = NUMBER + 1
  XCUM(MAXP%) = XCUM(MAXP%) + Y
  X2CUM(MAXP%) = X2CUM(MAXP%) + Y * Y
  PRINT MAXP$, "CUMS", Y, XCUM(MAXP%), X2CUM(MAXP%)
  '************************************
  ' ** UPDATE THE STRATUM INFORMATION **
  ' ************************************
  SH2(MAXP%) = (X2CUM(MAXP%) - XCUM(MAXP%) ^ 2
             / NSEQ(MAXP%)) / (NSEQ(MAXP%) - 1
  FS2H(MAXP%) = SQR(SH2(MAXP%)) * WPOP(MAXP%)
  FS2HT = FNSUM(NUMSTR%)
NEXT

CLOSE
```

```
'SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
'SSS  CALCW                    SSS
'SSS==========================SSS
'SSS  converts stratum sizes   SSS
'SSS  to stratum proportions   SSS
'SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
'
SUB CALCW (N1%, N2%)
    NTOT% = N1% + N2%
    WPOP(1) = N1% / NTOT%
    WPOP(2) = N2% / NTOT%
    PRINT "NUMRECORDS", N1%; N2%
    PRINT "WPOPS"; WPOP(1), WPOP(2)
END SUB

'SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
'SSS  DETSTRATUM                  SSS
'SSS=============================SSS
'SSS  calculates the lambdas and SSS
'SSS  chooses the next stratum   SSS
'SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
'
SUB DETSTRATUM (NUMSTR%, MAXP%)

     INPUT "LAMBDA HATS", T
     FOR STR% = 1 TO NUMSTR%
       LHAT(STR%) = FS2H(STR%) / FS2HT - NSEQ(STR%) / NSEQT
     NEXT
     INPUT "CHOOSE THE NEXT OBS", T
     PRINT MAXP%
     NSEQ(MAXP%) = NSEQ(MAXP%) + 1
     NSEQT = NSEQT + 1

END SUB


'SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
'SSS  SUBROUTINE INTERVIEW                SSS
'SSS=====================================SSS
'SSS  opens question and answer files   SSS
'SSS  and records person's answers      SSS
'SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
'
SUB INTERVIEW (NUMBER, STR%, Y) STATIC

 DIM QUESTION AS STRING * 80
 DIM RESPONSE AS STRING * 10
 DIM ARESPONSE AS STRING * 20

 OPEN "ANSWERS.DAT" FOR OUTPUT AS #30
 OPEN "QUESTION.DAT" FOR INPUT AS #10 LEN = 80
 OPEN "QINFO.DAT" FOR INPUT AS #20 LEN = 80

 PRINT "INTERVIEW"
```

```
 FOR I = 1 TO NUMQUESTIONS
   INPUT #20, I, TOFQ, NUMLINES, OPTIMALQ
   PRINT "TOFQ", I, TOFQ, NUMLINES, OPTIMALQ
   FOR j = 1 TO NUMLINES
     LINE INPUT #10, QUESTION
     PRINT QUESTION
   NEXT
   IF TOFQ = 1 THEN
     INPUT "ANSWER? ", NRESPONSE
     PRINT #30, NRESPONSE;
     IF OPTIMALQ = I THEN Y = NRESPONSE
   ELSE
     INPUT ARESPONSE
     PRINT #30, AREPONSE
   END IF
 NEXT

 NUMBER = NUMBER + 1
 CLOSE #10
 CLOSE #20
 CLOSE #30

END SUB

'SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
'SSS  SUBROUTINE MAXIMUM              SSS
'SSS==================================SSS
'SSS  finds the index of the maximum SSS
'SSS  in an array                     SSS
'SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
'
SUB MAXIMUM (SIZE%, MAX%)

  MAX% = SIZE%
  PRINT "MAXIMUM"; SIZE%
  FOR I% = 1 TO SIZE% - 1
    IF LHAT(MAX%) < LHAT(I%) THEN MAX% = I%
  NEXT

END SUB

'SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
'SSS  QINFO                           SSS
'SSS==================================SSS
'SSS  queries the interviewer about  SSS
'SSS  the nature of the questions    SSS
'SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
'
SUB QINFO (FLAG, I, NUMLINES, OPTIMALQ, TOFQ)
   DIM RESPONSE AS STRING * 10
   PRINT "   "
   INPUT "Will the response to this question be numeric?"
         , RESPONSE
   TOFQ = 0
   IF UCASE$(LEFT$(RESPONSE, 1)) = "Y" THEN TOFQ = 1
```

```
   IF TOFQ = 1 AND FLAG = 0 THEN
      INPUT "Is this the question used for stratification?"
            , RESPONSE
   END IF
   IF UCASE$(LEFT$(RESPONSE, 1)) = "Y" THEN
       OPTIMALQ = I
       FLAG = 1
   END IF

END SUB



'SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
'SSS  SUBROUTINE READYMESSAGE               SSS
'SSS========================================SSS
'SSS  clears screen, displays message, and  SSS
'SSS  queries the inteviewer to find the    SSS
'SSS  number of people to be interviewed    SSS
'SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
'
SUB READYMESSAGE (MININT%, NUMINT%)
 DIM RESPONSE AS STRING * 10
 CLS
 PRINT "READY FOR THE INTERVIEWING PROCESS!!                  "
 PRINT "Randomly chosen individuals and their phone numbers"
 PRINT "will be displayed on the screen.  These individuals"
 PRINT "are chosen from the best stratum, based on the     "
 PRINT "optimality criterion of this sequential procedure. "
 PRINT
 PRINT "It is time to determine the number of interviews. "
 PRINT "This number must exceed "; MININT%

 DO
   FLAG = 0
   INPUT "How many interviews? ", NUMINT%
   PRINT "You requested "; NUMINT%; " interviews"
   IF MININT% > NUMINT% THEN
      FLAG = 1
      PRINT "The number of interviews must exceed "; MININT%
   ELSE
      INPUT "Is this the correct number? ", RESPONSE
      IF UCASE$(LEFT$(RESPONSE, 1)) = "N" THEN FLAG = 1
   END IF
   IF FLAG = 1 THEN PRINT "Try again!"
 LOOP UNTIL FLAG = 0

END SUB
```

APPENDIX E

TABLES OF SIMULATION RESULTS

TABLE X

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
SUKHATME'S EXAMPLE NUMBER 1

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED					
		100		200		300	
		SEQ	SUK	SEQ	SUK	SEQ	SUK
15	BIAS	-0.0054	-0.0125	-0.0086	-0.0037	0.0069	0.0070
	VARIANCE	0.0349	0.0376	0.0186	0.0217	0.0124	0.0144
	MSE	0.0350	0.0377	0.0187	0.0217	0.0124	0.0145
VARIANCE OF NEYMAN		0.0188		0.0167		0.0150	

TABLE XI

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
SUKHATME'S EXAMPLE NUMBER 2

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED					
		200		250		300	
		SEQ	SUK	SEQ	SUK	SEQ	SUK
15	BIAS	-0.0007	-0.0008	-0.0007	-0.0008	-0.0007	-0.0008
	VARIANCE	0.0144	0.0145	0.0144	0.0145	0.0144	0.0145
	MSE	0.0144	0.0145	0.0144	0.0145	0.0144	0.0145
VARIANCE OF NEYMAN		0.0154		0.0137		0.0123	

TABLE XII

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
SUKHATME'S EXAMPLE NUMBER 3

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED					
		400		450		500	
		SEQ	SUK	SEQ	SUK	SEQ	SUK
15	BIAS	-0.0035	-0.0016	-0.0035	-0.0016	-0.0035	-0.0016
	VARIANCE	0.0127	0.0125	0.0127	0.0125	0.0127	0.0125
	MSE	0.0127	0.0125	0.0127	0.0125	0.0127	0.0125
VARIANCE OF NEYMAN		0.0126		0.0112		0.0101	

TABLE XIII

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
NORMAL: STRATA=2 PLAN=I

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	BIAS	0.0066	0.0097	0.0001	-0.0253	0.0069	0.0146	0.0152	0.0227
	VARIANCE	0.7212	0.7912	0.3389	0.4585	0.2418	0.3002	0.1651	0.2267
	MSE	0.7212	0.7913	0.3389	0.4592	0.2418	0.3004	0.1653	0.2272
5	BIAS	-0.0796	-0.0796	-0.0214	-0.0194	0.0210	0.0239	-0.0021	-0.0030
	VARIANCE	0.5971	0.5964	0.2935	0.3017	0.2028	0.2100	0.1599	0.1595
	MSE	0.6034	0.6027	0.2939	0.3021	0.2032	0.2106	0.1599	0.1595
10	BIAS	-0.0766	-0.0764	-0.0292	-0.0302	0.0403	0.0435	-0.0207	-0.0183
	VARIANCE	0.6693	0.6699	0.3237	0.3212	0.1993	0.2000	0.1635	0.1647
	MSE	0.6752	0.6757	0.3246	0.3221	0.2010	0.2019	0.1639	0.1650
15	BIAS	-0.0203	-0.0203	0.0086	0.0081	0.0146	0.0139	0.0194	0.0179
	VARIANCE	0.6593	0.6593	0.3318	0.3323	0.1771	0.1774	0.1569	0.1572
	MSE	0.6597	0.6597	0.3319	0.3323	0.1773	0.1775	0.1573	0.1575
20	BIAS	-0.0041	-0.0041	0.0069	0.0069	0.0004	-0.0003	0.0031	0.0037
	VARIANCE	0.7486	0.7486	0.3258	0.3258	0.2108	0.2100	0.1404	0.1408
	MSE	0.7486	0.7486	0.3259	0.3259	0.2108	0.2100	0.1405	0.1408
VARIANCE OF NEYMAN		0.60500		0.30250		0.20167		0.15125	

TABLE XIV

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
NORMAL: STRATA=2 PLAN=II

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	BIAS	0.0013	0.0155	-0.0006	-0.0051	0.0049	0.0127	-0.0072	-0.0019
	VARIANCE	0.1152	0.1232	0.0577	0.0783	0.0423	0.0730	0.0316	0.0419
	MSE	0.1152	0.1235	0.0577	0.0783	0.0424	0.0731	0.0317	0.0419
5	BIAS	-0.0246	-0.0357	-0.0040	-0.0103	-0.0001	0.0084	0.0046	0.0065
	VARIANCE	0.1027	0.1005	0.0456	0.0510	0.0275	0.0289	0.0213	0.0229
	MSE	0.1033	0.1018	0.0456	0.0511	0.0275	0.0290	0.0213	0.0229
10	BIAS	0.0194	0.0181	-0.0035	-0.0066	-0.0001	-0.0004	-0.0197	-0.0178
	VARIANCE	0.0930	0.0939	0.0412	0.0428	0.0307	0.0323	0.0225	0.0227
	MSE	0.0934	0.0942	0.0413	0.0428	0.0307	0.0323	0.0229	0.0230
15	BIAS	0.0190	0.0189	-0.0057	-0.0086	-0.0012	-0.0003	0.0061	0.0059
	VARIANCE	0.0855	0.0852	0.0435	0.0435	0.0298	0.0288	0.0250	0.0246
	MSE	0.0858	0.0855	0.0435	0.0436	0.0298	0.0288	0.0250	0.0246
20	BIAS	0.0183	0.0183	0.0057	0.0081	0.0020	0.0012	-0.0097	-0.0094
	VARIANCE	0.0945	0.0945	0.0445	0.0461	0.0277	0.0281	0.0219	0.0220
	MSE	0.0949	0.0949	0.0445	0.0462	0.0277	0.0281	0.0219	0.0221
VARIANCE OF NEYMAN		0.08662		0.04331		0.02887		0.02166	

TABLE XV

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
NORMAL: STRATA=2 PLAN=III

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	BIAS	0.0207	0.0144	0.0097	0.0113	0.0059	0.0057	-0.0026	-0.0040
	VARIANCE	0.0542	0.0642	0.0313	0.0433	0.0236	0.0314	0.0137	0.0285
	MSE	0.0546	0.0644	0.0314	0.0434	0.0236	0.0315	0.0137	0.0285
5	BIAS	0.0165	0.0164	-0.0051	-0.0045	-0.0032	0.0005	-0.0076	-0.0037
	VARIANCE	0.0517	0.0591	0.0214	0.0222	0.0152	0.0163	0.0118	0.0132
	MSE	0.0520	0.0594	0.0214	0.0222	0.0152	0.0163	0.0119	0.0132
10	BIAS	-0.0232	-0.0163	-0.0071	-0.0031	0.0002	-0.0005	-0.0015	0.0020
	VARIANCE	0.0436	0.0426	0.0213	0.0232	0.0161	0.0160	0.0106	0.0115
	MSE	0.0442	0.0428	0.0213	0.0232	0.0161	0.0160	0.0106	0.0115
15	BIAS	-0.0133	-0.0168	0.0029	0.0018	0.0032	0.0030	-0.0097	-0.0099
	VARIANCE	0.0474	0.0474	0.0211	0.0223	0.0153	0.0158	0.0110	0.0112
	MSE	0.0476	0.0477	0.0211	0.0223	0.0153	0.0158	0.0111	0.0113
20	BIAS	0.0086	0.0088	0.0077	0.0058	-0.0059	-0.0076	0.0074	0.0095
	VARIANCE	0.0478	0.0484	0.0225	0.0227	0.0139	0.0146	0.0103	0.0104
	MSE	0.0479	0.0485	0.0226	0.0227	0.0139	0.0146	0.0104	0.0105
VARIANCE OF NEYMAN		0.04500		0.02250		0.01500		0.01125	

TABLE XVI

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
NORMAL: STRATA=2 PLAN=IV

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED 50 SEQ	50 SUK	100 SEQ	100 SUK	150 SEQ	150 SUK	200 SEQ	200 SUK
2	BIAS	-0.0046	-0.0024	0.0057	0.0088	-0.0013	-0.0034	0.0039	0.0055
	VARIANCE	0.0317	0.0416	0.0134	0.0196	0.0096	0.0151	0.0072	0.0104
	MSE	0.0317	0.0416	0.0134	0.0197	0.0096	0.0152	0.0073	0.0104
5	BIAS	-0.0011	-0.0023	0.0070	0.0019	0.0022	0.0035	0.0046	0.0044
	VARIANCE	0.0217	0.0222	0.0104	0.0111	0.0063	0.0071	0.0046	0.0049
	MSE	0.0217	0.0222	0.0104	0.0111	0.0063	0.0071	0.0047	0.0049
10	BIAS	-0.0012	-0.0028	0.0053	0.0046	-0.0079	-0.0073	-0.0045	-0.0033
	VARIANCE	0.0202	0.0194	0.0108	0.0110	0.0062	0.0063	0.0048	0.0049
	MSE	0.0202	0.0194	0.0108	0.0110	0.0063	0.0064	0.0048	0.0049
15	BIAS	0.0002	0.0016	0.0038	0.0069	0.0050	0.0064	-0.0032	-0.0037
	VARIANCE	0.0195	0.0197	0.0092	0.0092	0.0063	0.0063	0.0047	0.0051
	MSE	0.0195	0.0197	0.0092	0.0093	0.0063	0.0063	0.0047	0.0051
20	BIAS	-0.0003	0.0003	-0.0008	-0.0022	-0.0003	-0.0004	-0.0007	-0.0006
	VARIANCE	0.0204	0.0201	0.0092	0.0092	0.0068	0.0069	0.0048	0.0047
	MSE	0.0204	0.0201	0.0092	0.0093	0.0068	0.0069	0.0048	0.0047
VARIANCE OF NEYMAN		0.02000		0.01000		0.00667		0.00500	

TABLE XVII

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
NORMAL: STRATA=3 PLAN=I

		TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
INITIAL OBS PER STRATUM	STATISTIC	SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	BIAS	-0.0094	-0.0219	-0.2117	-0.3091	0.1138	0.1278	-0.2275	-0.3347
	VARIANCE	30.2803	34.5770	15.8888	19.5242	10.1876	12.3007	9.0794	10.6446
	MSE	30.2804	34.5775	15.9336	19.6198	10.2006	12.3170	9.1312	10.7567
5	BIAS	0.6971	0.6786	-0.3040	-0.2602	0.1616	0.2462	-0.0434	-0.0045
	VARIANCE	28.1681	28.4436	14.4464	14.8161	9.6186	9.8725	7.5841	7.6472
	MSE	28.6540	28.9041	14.5389	14.8838	9.6447	9.9331	7.5860	7.6472
10	BIAS	-0.4297	-0.4297	-0.2164	-0.2261	0.1328	0.1255	0.0198	0.0460
	VARIANCE	38.1274	38.1274	15.8749	15.9188	10.4271	10.5761	7.2723	7.2031
	MSE	38.3120	38.3120	15.9218	15.9699	10.4447	10.5919	7.2727	7.2052
15	BIAS	-0.1566	-0.1566	-0.0615	-0.0615	0.1750	0.1903	-0.0762	-0.0362
	VARIANCE	63.3721	63.3721	15.5163	15.5163	8.9844	9.0152	7.0485	7.0353
	MSE	63.3967	63.3967	15.5201	15.5201	9.0150	9.0514	7.0543	7.0366
20	BIAS	.	.	0.1072	0.1081	-0.1895	-0.1990	-0.0232	-0.0051
	VARIANCE	.	.	11.0900	11.0887	7.6194	7.5874	0.5922	0.7295
	MSE	.	.	11.1015	11.1004	7.6553	7.6270	0.5927	0.7295
VARIANCE OF NEYMAN		27.38000		13.69000		9.12667		6.84500	

TABLE XVIII

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
NORMAL: STRATA=3 PLAN=II

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	BIAS	-0.0232	-0.0051	0.0196	0.0482	-0.0107	-0.0137	0.0038	0.0045
	VARIANCE	0.5922	0.7295	0.3472	0.4170	0.2207	0.3200	0.1326	0.1978
	MSE	0.5927	0.7295	0.3476	0.4193	0.2208	0.3202	0.1326	0.1978
5	BIAS	0.0322	0.0463	0.0366	0.0331	-0.0232	-0.0303	0.0199	0.0233
	VARIANCE	0.4524	0.4876	0.2296	0.2303	0.1437	0.1595	0.1167	0.1192
	MSE	0.4534	0.4898	0.2309	0.2314	0.1442	0.1604	0.1171	0.1197
10	BIAS	-0.0422	-0.0426	0.0239	0.0120	-0.0083	-0.0120	0.0021	-0.0048
	VARIANCE	0.5062	0.5175	0.2225	0.2268	0.1506	0.1530	0.0984	0.1018
	MSE	0.5080	0.5193	0.2231	0.2270	0.1507	0.1532	0.0984	0.1018
15	BIAS	0.0397	0.0401	0.0168	0.0115	0.0076	0.0082	-0.0078	-0.0035
	VARIANCE	0.6712	0.6709	0.2471	0.2439	0.1412	0.1402	0.1102	0.1094
	MSE	0.6727	0.6725	0.2474	0.2440	0.1412	0.1403	0.1102	0.1094
20	BIAS	.	.	-0.0117	-0.0068	-0.0014	0.0007	-0.0053	-0.0075
	VARIANCE	.	.	0.1505	0.1503	0.1171	0.1167	0.0725	0.0979
	MSE	.	.	0.1506	0.1503	0.1171	0.1167	0.0725	0.0979
VARIANCE OF NEYMAN		0.44570		0.22285		0.14857		0.11143	

TABLE XIX

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
NORMAL: STRATA=3 PLAN=III

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	BIAS	-0.0053	-0.0075	0.0030	0.0051	0.0015	-0.0026	-0.0060	0.0004
	VARIANCE	0.0725	0.0979	0.0415	0.0544	0.0359	0.0398	0.0183	0.0357
	MSE	0.0725	0.0979	0.0415	0.0544	0.0359	0.0399	0.0183	0.0357
5	BIAS	0.0253	0.0216	0.0116	0.0106	-0.0012	-0.0020	-0.0052	-0.0049
	VARIANCE	0.0620	0.0652	0.0301	0.0323	0.0202	0.0233	0.0173	0.0183
	MSE	0.0626	0.0657	0.0302	0.0324	0.0202	0.0233	0.0173	0.0183
10	BIAS	0.0111	0.0087	-0.0010	0.0000	-0.0124	-0.0127	0.0064	0.0061
	VARIANCE	0.0554	0.0558	0.0320	0.0339	0.0201	0.0200	0.0148	0.0147
	MSE	0.0555	0.0559	0.0320	0.0339	0.0203	0.0202	0.0148	0.0148
15	BIAS	-0.0076	-0.0076	-0.0084	-0.0096	-0.0061	-0.0077	-0.0044	-0.0050
	VARIANCE	0.0690	0.0690	0.0316	0.0321	0.0196	0.0200	0.0140	0.0148
	MSE	0.0691	0.0691	0.0317	0.0322	0.0196	0.0200	0.0140	0.0148
20	BIAS	.	.	0.0086	0.0073	0.0023	0.0014	0.0007	-0.0162
	VARIANCE	.	.	0.0197	0.0200	0.0148	0.0146	0.1136	0.1412
	MSE	.	.	0.0198	0.0200	0.0148	0.0146	0.1136	0.1415
VARIANCE OF NEYMAN		0.01480		0.00740		0.00493		0.00370	

TABLE XX

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
NORMAL: STRATA=3 PLAN=IV

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	BIAS	0.0007	-0.0162	0.0255	0.0147	-0.0132	-0.0132	0.0186	0.0197
	VARIANCE	0.1136	0.1412	0.0547	0.0835	0.0377	0.0612	0.0345	0.0489
	MSE	0.1136	0.1415	0.0554	0.0838	0.0378	0.0614	0.0348	0.0493
5	BIAS	-0.0113	-0.0097	-0.0042	-0.0010	-0.0039	0.0016	-0.0020	-0.0016
	VARIANCE	0.0888	0.1002	0.0465	0.0491	0.0246	0.0272	0.0175	0.0206
	MSE	0.0889	0.1003	0.0465	0.0491	0.0247	0.0272	0.0175	0.0206
10	BIAS	-0.0089	-0.0093	-0.0052	-0.0056	-0.0022	-0.0080	0.0068	0.0056
	VARIANCE	0.0888	0.0918	0.0405	0.0418	0.0250	0.0252	0.0207	0.0214
	MSE	0.0889	0.0919	0.0405	0.0418	0.0250	0.0252	0.0207	0.0214
15	BIAS	-0.0288	-0.0277	-0.0044	-0.0042	0.0139	0.0081	-0.0027	-0.0021
	VARIANCE	0.0828	0.0829	0.0341	0.0352	0.0276	0.0289	0.0244	0.0240
	MSE	0.0836	0.0836	0.0341	0.0352	0.0278	0.0289	0.0244	0.0240
20	BIAS	.	.	-0.0033	-0.0057	-0.0008	0.0020	0.0093	0.0094
	VARIANCE	.	.	0.0279	0.0278	0.0181	0.0186	0.0336	0.0407
	MSE	.	.	0.0279	0.0279	0.0181	0.0186	0.0337	0.0408
VARIANCE OF NEYMAN		0.02000		0.01000		0.00667		0.00500	

TABLE XXI

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
NORMAL: STRATA=3 PLAN=V

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	BIAS	0.0093	0.0094	-0.0045	-0.0079	0.0016	0.0027	0.0007	0.0004
	VARIANCE	0.0336	0.0407	0.0144	0.0226	0.0094	0.0165	0.0070	0.0109
	MSE	0.0337	0.0408	0.0144	0.0227	0.0094	0.0165	0.0070	0.0109
5	BIAS	-0.0015	-0.0018	0.0048	0.0084	-0.0000	-0.0041	-0.0058	-0.0052
	VARIANCE	0.0208	0.0219	0.0094	0.0108	0.0069	0.0076	0.0051	0.0055
	MSE	0.0208	0.0219	0.0094	0.0109	0.0069	0.0077	0.0052	0.0055
10	BIAS	0.0055	0.0070	-0.0077	-0.0087	-0.0012	0.0005	-0.0000	-0.0001
	VARIANCE	0.0205	0.0203	0.0101	0.0102	0.0063	0.0067	0.0047	0.0050
	MSE	0.0205	0.0203	0.0101	0.0103	0.0063	0.0067	0.0047	0.0050
15	BIAS	-0.0033	-0.0057	-0.0082	-0.0065	0.0037	0.0056	0.0031	0.0046
	VARIANCE	0.0194	0.0196	0.0115	0.0112	0.0064	0.0066	0.0050	0.0053
	MSE	0.0194	0.0196	0.0116	0.0113	0.0064	0.0066	0.0050	0.0053
20	BIAS	.	.	0.0005	0.0013	-0.0057	-0.0044	-0.0057	-0.0044
	VARIANCE	.	.	0.0071	0.0073	0.0050	0.0052	0.0050	0.0052
	MSE	.	.	0.0071	0.0073	0.0051	0.0052	0.0051	0.0052
VARIANCE OF NEYMAN		0.00125		0.00063		0.00042		0.00031	

TABLE XXII

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
GAMMA: ALPHA=1 STRATA=2 PROP1=.5

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	BIAS	-0.0195	-0.0140	-0.0086	-0.0087	-0.0069	-0.0094	-0.0039	-0.0059
	VARIANCE	0.0094	0.0108	0.0058	0.0072	0.0028	0.0039	0.0026	0.0035
	MSE	0.0097	0.0110	0.0059	0.0072	0.0028	0.0040	0.0026	0.0035
5	BIAS	-0.0140	-0.0147	-0.0119	-0.0129	-0.0064	-0.0059	-0.0056	-0.0053
	VARIANCE	0.0072	0.0075	0.0034	0.0036	0.0023	0.0024	0.0018	0.0020
	MSE	0.0074	0.0078	0.0035	0.0037	0.0023	0.0025	0.0019	0.0020
10	BIAS	-0.0027	-0.0037	-0.0064	-0.0074	-0.0071	-0.0079	-0.0050	-0.0055
	VARIANCE	0.0075	0.0076	0.0037	0.0038	0.0022	0.0024	0.0018	0.0018
	MSE	0.0075	0.0076	0.0037	0.0038	0.0023	0.0024	0.0018	0.0019
15	BIAS	-0.0097	-0.0098	-0.0032	-0.0047	-0.0064	-0.0055	-0.0028	-0.0017
	VARIANCE	0.0068	0.0068	0.0035	0.0035	0.0022	0.0022	0.0017	0.0017
	MSE	0.0069	0.0069	0.0035	0.0035	0.0022	0.0022	0.0017	0.0017
20	BIAS	-0.0036	-0.0036	-0.0052	-0.0053	-0.0043	-0.0045	-0.0050	-0.0050
	VARIANCE	0.0081	0.0081	0.0036	0.0036	0.0024	0.0023	0.0015	0.0014
	MSE	0.0081	0.0081	0.0036	0.0036	0.0024	0.0023	0.0015	0.0015
VARIANCE OF NEYMAN		0.0072		0.0036		0.0024		0.0018	

TABLE XXIII

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
GAMMA: ALPHA=1 STRATA=2 PROP1=.6

		TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
INITIAL OBS PER STRATUM	STATISTIC	SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	BIAS	-0.0172	-0.0185	-0.0108	-0.0172	-0.0069	-0.0067	-0.0095	-0.0131
	VARIANCE	0.0096	0.0123	0.0039	0.0075	0.0037	0.0048	0.0022	0.0034
	MSE	0.0099	0.0127	0.0040	0.0078	0.0038	0.0048	0.0023	0.0035
5	BIAS	-0.0105	-0.0127	-0.0031	-0.0046	-0.0061	-0.0055	-0.0040	-0.0041
	VARIANCE	0.0069	0.0071	0.0034	0.0038	0.0021	0.0024	0.0015	0.0016
	MSE	0.0070	0.0073	0.0034	0.0038	0.0021	0.0024	0.0015	0.0017
10	BIAS	0.0009	0.0003	-0.0070	-0.0074	-0.0073	-0.0079	-0.0049	-0.0039
	VARIANCE	0.0062	0.0066	0.0030	0.0032	0.0021	0.0023	0.0015	0.0016
	MSE	0.0062	0.0066	0.0031	0.0033	0.0022	0.0023	0.0015	0.0016
15	BIAS	-0.0103	-0.0107	-0.0031	-0.0035	-0.0094	-0.0092	-0.0039	-0.0034
	VARIANCE	0.0064	0.0065	0.0032	0.0033	0.0021	0.0021	0.0016	0.0017
	MSE	0.0065	0.0066	0.0032	0.0034	0.0021	0.0022	0.0017	0.0017
20	BIAS	0.0029	0.0028	-0.0037	-0.0049	-0.0039	-0.0040	-0.0054	-0.0060
	VARIANCE	0.0066	0.0066	0.0032	0.0032	0.0020	0.0020	0.0016	0.0016
	MSE	0.0066	0.0066	0.0032	0.0033	0.0020	0.0020	0.0016	0.0016
VARIANCE OF NEYMAN		0.0062		0.0031		0.0021		0.0016	

TABLE XXIV

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
GAMMA: ALPHA=1 STRATA=2 PROP1=.7

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	BIAS	-0.0242	-0.0251	-0.0139	-0.0209	-0.0075	-0.0165	-0.0115	-0.0153
	VARIANCE	0.0087	0.0098	0.0040	0.0064	0.0030	0.0047	0.0023	0.0045
	MSE	0.0093	0.0104	0.0042	0.0068	0.0030	0.0049	0.0024	0.0047
5	BIAS	-0.0112	-0.0117	-0.0055	-0.0066	-0.0098	-0.0097	-0.0076	-0.0069
	VARIANCE	0.0055	0.0064	0.0029	0.0034	0.0020	0.0022	0.0012	0.0014
	MSE	0.0056	0.0065	0.0029	0.0034	0.0021	0.0023	0.0013	0.0015
10	BIAS	-0.0085	-0.0098	-0.0079	-0.0093	-0.0046	-0.0050	-0.0069	-0.0080
	VARIANCE	0.0055	0.0056	0.0032	0.0030	0.0017	0.0018	0.0013	0.0013
	MSE	0.0056	0.0057	0.0032	0.0031	0.0018	0.0018	0.0014	0.0014
15	BIAS	-0.0115	-0.0120	-0.0101	-0.0105	-0.0089	-0.0091	-0.0049	-0.0049
	VARIANCE	0.0051	0.0051	0.0032	0.0033	0.0018	0.0017	0.0014	0.0014
	MSE	0.0053	0.0053	0.0033	0.0034	0.0018	0.0018	0.0014	0.0014
20	BIAS	-0.0143	-0.0133	-0.0099	-0.0093	-0.0098	-0.0102	-0.0063	-0.0072
	VARIANCE	0.0059	0.0059	0.0026	0.0026	0.0017	0.0018	0.0014	0.0015
	MSE	0.0061	0.0061	0.0027	0.0027	0.0018	0.0019	0.0014	0.0015
VARIANCE OF NEYMAN		0.0057		0.0029		0.0019		0.0014	

TABLE XXV

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
GAMMA: ALPHA=1 STRATA=2 PROP1=.8

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	BIAS	-0.0243	-0.0161	-0.0176	-0.0183	-0.0078	-0.0085	-0.0045	-0.0052
	VARIANCE	0.0099	0.0147	0.0040	0.0074	0.0033	0.0044	0.0039	0.0052
	MSE	0.0105	0.0150	0.0043	0.0078	0.0034	0.0044	0.0039	0.0052
5	BIAS	-0.0199	-0.0190	-0.0101	-0.0098	-0.0073	-0.0078	-0.0052	-0.0074
	VARIANCE	0.0067	0.0069	0.0031	0.0030	0.0021	0.0021	0.0015	0.0019
	MSE	0.0071	0.0073	0.0032	0.0031	0.0021	0.0022	0.0015	0.0019
10	BIAS	-0.0111	-0.0102	-0.0060	-0.0060	-0.0098	-0.0110	-0.0057	-0.0070
	VARIANCE	0.0055	0.0056	0.0031	0.0032	0.0021	0.0021	0.0015	0.0016
	MSE	0.0056	0.0057	0.0031	0.0032	0.0022	0.0022	0.0015	0.0017
15	BIAS	-0.0050	-0.0062	-0.0077	-0.0080	-0.0079	-0.0083	-0.0055	-0.0053
	VARIANCE	0.0062	0.0064	0.0029	0.0030	0.0020	0.0020	0.0015	0.0015
	MSE	0.0062	0.0064	0.0029	0.0031	0.0021	0.0021	0.0015	0.0015
20	BIAS	-0.0116	-0.0119	-0.0082	-0.0082	-0.0101	-0.0102	-0.0057	-0.0056
	VARIANCE	0.0059	0.0059	0.0031	0.0032	0.0019	0.0020	0.0015	0.0015
	MSE	0.0060	0.0060	0.0031	0.0032	0.0020	0.0021	0.0015	0.0016
VARIANCE OF NEYMAN		0.0060		0.0030		0.0020		0.0015	

TABLE XXVI

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
GAMMA: ALPHA=2 STRATA=2 PROP1=.5

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	BIAS	-0.0100	-0.0173	-0.0066	0.0003	-0.0090	-0.0132	-0.0001	-0.0040
	VARIANCE	0.0211	0.0252	0.0113	0.0142	0.0054	0.0090	0.0054	0.0082
	MSE	0.0212	0.0255	0.0113	0.0142	0.0055	0.0092	0.0054	0.0083
5	BIAS	-0.0140	-0.0155	-0.0121	-0.0134	-0.0071	-0.0064	-0.0056	-0.0073
	VARIANCE	0.0142	0.0153	0.0071	0.0081	0.0042	0.0048	0.0036	0.0038
	MSE	0.0144	0.0155	0.0073	0.0082	0.0043	0.0049	0.0036	0.0038
10	BIAS	-0.0077	-0.0089	-0.0101	-0.0094	-0.0089	-0.0095	-0.0052	-0.0062
	VARIANCE	0.0150	0.0150	0.0073	0.0071	0.0045	0.0050	0.0037	0.0037
	MSE	0.0151	0.0150	0.0074	0.0072	0.0045	0.0051	0.0038	0.0037
15	BIAS	-0.0121	-0.0148	-0.0039	-0.0058	-0.0063	-0.0067	-0.0016	-0.0030
	VARIANCE	0.0130	0.0133	0.0068	0.0068	0.0044	0.0045	0.0035	0.0037
	MSE	0.0131	0.0135	0.0068	0.0068	0.0044	0.0045	0.0036	0.0037
20	BIAS	-0.0113	-0.0113	-0.0072	-0.0079	-0.0052	-0.0055	-0.0069	-0.0062
	VARIANCE	0.0131	0.0131	0.0068	0.0068	0.0048	0.0048	0.0029	0.0029
	MSE	0.0132	0.0132	0.0068	0.0068	0.0048	0.0049	0.0029	0.0030
VARIANCE OF NEYMAN		0.0143		0.0071		0.0048		0.0036	

TABLE XXVII

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
GAMMA: ALPHA=2 STRATA=2 PROP1=.6

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	BIAS	-0.0207	-0.0268	-0.0172	-0.0208	-0.0065	-0.0170	-0.0109	-0.0163
	VARIANCE	0.0239	0.0268	0.0108	0.0145	0.0085	0.0137	0.0061	0.0080
	MSE	0.0243	0.0275	0.0111	0.0150	0.0085	0.0140	0.0062	0.0083
5	BIAS	-0.0059	-0.0092	-0.0091	-0.0130	-0.0100	-0.0084	-0.0050	-0.0067
	VARIANCE	0.0141	0.0161	0.0069	0.0079	0.0044	0.0046	0.0031	0.0035
	MSE	0.0141	0.0162	0.0070	0.0081	0.0045	0.0047	0.0032	0.0035
10	BIAS	0.0044	0.0005	-0.0079	-0.0088	-0.0096	-0.0092	-0.0056	-0.0041
	VARIANCE	0.0145	0.0153	0.0069	0.0071	0.0050	0.0048	0.0033	0.0032
	MSE	0.0145	0.0153	0.0069	0.0072	0.0051	0.0049	0.0034	0.0033
15	BIAS	-0.0174	-0.0165	-0.0022	-0.0040	-0.0125	-0.0145	-0.0058	-0.0056
	VARIANCE	0.0136	0.0136	0.0063	0.0063	0.0041	0.0044	0.0034	0.0034
	MSE	0.0139	0.0139	0.0063	0.0064	0.0042	0.0046	0.0035	0.0035
20	BIAS	-0.0023	-0.0024	-0.0056	-0.0062	-0.0074	-0.0072	-0.0072	-0.0077
	VARIANCE	0.0141	0.0140	0.0068	0.0067	0.0043	0.0044	0.0036	0.0037
	MSE	0.0141	0.0140	0.0069	0.0068	0.0043	0.0045	0.0036	0.0038
VARIANCE OF NEYMAN		0.0130		0.0065		0.0043		0.0033	

TABLE XXVIII

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
GAMMA: ALPHA=2 STRATA=2 PROP1=.7

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	BIAS	-0.0248	-0.0260	-0.0095	-0.0184	-0.0100	-0.0149	-0.0059	-0.0088
	VARIANCE	0.0225	0.0289	0.0104	0.0156	0.0062	0.0105	0.0045	0.0069
	MSE	0.0231	0.0296	0.0105	0.0159	0.0063	0.0107	0.0046	0.0069
5	BIAS	-0.0176	-0.0163	-0.0096	-0.0144	-0.0096	-0.0114	-0.0090	-0.0081
	VARIANCE	0.0127	0.0142	0.0068	0.0069	0.0042	0.0044	0.0029	0.0034
	MSE	0.0131	0.0145	0.0069	0.0071	0.0043	0.0045	0.0030	0.0034
10	BIAS	-0.0104	-0.0099	-0.0100	-0.0101	-0.0110	-0.0116	-0.0054	-0.0055
	VARIANCE	0.0125	0.0123	0.0068	0.0072	0.0037	0.0039	0.0034	0.0034
	MSE	0.0126	0.0124	0.0069	0.0073	0.0038	0.0040	0.0034	0.0035
15	BIAS	-0.0182	-0.0175	-0.0130	-0.0123	-0.0080	-0.0083	-0.0057	-0.0057
	VARIANCE	0.0119	0.0123	0.0070	0.0071	0.0039	0.0038	0.0032	0.0033
	MSE	0.0122	0.0126	0.0072	0.0073	0.0040	0.0039	0.0032	0.0033
20	BIAS	-0.0115	-0.0119	-0.0136	-0.0154	-0.0064	-0.0087	-0.0093	-0.0090
	VARIANCE	0.0129	0.0131	0.0060	0.0061	0.0039	0.0038	0.0027	0.0028
	MSE	0.0131	0.0133	0.0062	0.0063	0.0040	0.0039	0.0028	0.0029
VARIANCE OF NEYMAN		0.0128		0.0064		0.0043		0.0032	

TABLE XXIX

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
GAMMA: ALPHA=2 STRATA=2 PROP1=.8

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	BIAS	-0.0285	-0.0218	-0.0334	-0.0327	-0.0124	-0.0120	-0.0136	-0.0129
	VARIANCE	0.0203	0.0261	0.0116	0.0149	0.0063	0.0084	0.0048	0.0070
	MSE	0.0211	0.0265	0.0127	0.0160	0.0064	0.0086	0.0050	0.0071
5	BIAS	-0.0255	-0.0255	-0.0148	-0.0140	-0.0134	-0.0121	-0.0066	-0.0062
	VARIANCE	0.0151	0.0146	0.0082	0.0088	0.0048	0.0054	0.0037	0.0040
	MSE	0.0157	0.0153	0.0085	0.0090	0.0050	0.0056	0.0037	0.0041
10	BIAS	-0.0074	-0.0081	-0.0083	-0.0078	-0.0116	-0.0133	-0.0065	-0.0084
	VARIANCE	0.0117	0.0117	0.0070	0.0075	0.0049	0.0050	0.0034	0.0036
	MSE	0.0118	0.0118	0.0071	0.0076	0.0050	0.0052	0.0034	0.0037
15	BIAS	-0.0009	-0.0015	-0.0112	-0.0112	-0.0145	-0.0147	-0.0053	-0.0063
	VARIANCE	0.0137	0.0138	0.0064	0.0067	0.0045	0.0044	0.0039	0.0039
	MSE	0.0137	0.0138	0.0065	0.0068	0.0047	0.0046	0.0040	0.0039
20	BIAS	-0.0100	-0.0100	-0.0095	-0.0109	-0.0110	-0.0098	-0.0090	-0.0087
	VARIANCE	0.0143	0.0143	0.0081	0.0080	0.0049	0.0048	0.0037	0.0038
	MSE	0.0144	0.0144	0.0082	0.0081	0.0050	0.0049	0.0038	0.0039
VARIANCE OF NEYMAN		0.0141		0.0071		0.0047		0.0035	

TABLE XXX

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
GAMMA: ALPHA=5 STRATA=2 PROP1=.5

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	BIAS	-0.0084	-0.0178	-0.0052	-0.0054	-0.0117	-0.0084	-0.0069	-0.0061
	VARIANCE	0.0837	0.0994	0.0298	0.0432	0.0136	0.0227	0.0141	0.0208
	MSE	0.0837	0.0997	0.0299	0.0432	0.0137	0.0228	0.0141	0.0209
5	BIAS	-0.0195	-0.0272	-0.0168	-0.0179	-0.0085	-0.0117	-0.0159	-0.0170
	VARIANCE	0.0354	0.0404	0.0186	0.0208	0.0124	0.0130	0.0090	0.0097
	MSE	0.0358	0.0411	0.0189	0.0211	0.0125	0.0132	0.0093	0.0100
10	BIAS	-0.0064	-0.0071	-0.0099	-0.0120	-0.0139	-0.0133	-0.0060	-0.0065
	VARIANCE	0.0404	0.0395	0.0200	0.0203	0.0114	0.0119	0.0098	0.0103
	MSE	0.0405	0.0395	0.0201	0.0205	0.0116	0.0120	0.0099	0.0103
15	BIAS	-0.0252	-0.0251	-0.0015	-0.0037	-0.0080	-0.0099	-0.0024	-0.0016
	VARIANCE	0.0312	0.0318	0.0185	0.0186	0.0121	0.0119	0.0087	0.0086
	MSE	0.0318	0.0324	0.0185	0.0186	0.0122	0.0120	0.0087	0.0086
20	BIAS	-0.0099	-0.0102	-0.0139	-0.0136	-0.0072	-0.0083	-0.0110	-0.0101
	VARIANCE	0.0325	0.0327	0.0172	0.0170	0.0121	0.0127	0.0078	0.0079
	MSE	0.0326	0.0328	0.0174	0.0172	0.0122	0.0128	0.0079	0.0080
VARIANCE OF NEYMAN		0.0360		0.0180		0.0120		0.0090	

TABLE XXXI

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
GAMMA: ALPHA=5 STRATA=2 PROP1=.6

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	BIAS	-0.0321	-0.0289	-0.0160	-0.0304	-0.0035	-0.0098	-0.0233	-0.0345
	VARIANCE	0.0573	0.0725	0.0257	0.0420	0.0170	0.0277	0.0161	0.0241
	MSE	0.0583	0.0734	0.0260	0.0429	0.0170	0.0278	0.0167	0.0253
5	BIAS	-0.0035	-0.0066	-0.0160	-0.0196	-0.0147	-0.0192	-0.0053	-0.0066
	VARIANCE	0.0388	0.0405	0.0171	0.0216	0.0122	0.0139	0.0089	0.0098
	MSE	0.0388	0.0406	0.0173	0.0220	0.0124	0.0142	0.0090	0.0099
10	BIAS	0.0003	-0.0025	-0.0091	-0.0094	-0.0133	-0.0144	-0.0072	-0.0076
	VARIANCE	0.0385	0.0390	0.0185	0.0190	0.0120	0.0126	0.0094	0.0091
	MSE	0.0385	0.0390	0.0186	0.0191	0.0122	0.0128	0.0094	0.0091
15	BIAS	-0.0238	-0.0202	-0.0008	-0.0014	-0.0166	-0.0167	-0.0071	-0.0073
	VARIANCE	0.0379	0.0373	0.0185	0.0191	0.0101	0.0105	0.0088	0.0092
	MSE	0.0385	0.0377	0.0185	0.0191	0.0104	0.0108	0.0088	0.0093
20	BIAS	-0.0068	-0.0055	-0.0058	-0.0098	-0.0099	-0.0091	-0.0138	-0.0105
	VARIANCE	0.0351	0.0348	0.0158	0.0161	0.0112	0.0115	0.0095	0.0096
	MSE	0.0351	0.0348	0.0159	0.0162	0.0113	0.0115	0.0096	0.0097
VARIANCE OF NEYMAN		0.0344		0.0172		0.0115		0.0086	

TABLE XXXII

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
GAMMA: ALPHA=5 STRATA=2 PROP1=.7

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	BIAS	-0.0498	-0.0526	-0.0285	-0.0286	-0.0212	-0.0196	-0.0234	-0.0229
	VARIANCE	0.0631	0.0753	0.0276	0.0394	0.0179	0.0299	0.0199	0.0272
	MSE	0.0656	0.0780	0.0284	0.0402	0.0183	0.0303	0.0205	0.0277
5	BIAS	-0.0226	-0.0218	-0.0121	-0.0157	-0.0216	-0.0222	-0.0127	-0.0131
	VARIANCE	0.0387	0.0406	0.0196	0.0200	0.0119	0.0132	0.0084	0.0100
	MSE	0.0392	0.0411	0.0198	0.0202	0.0123	0.0137	0.0085	0.0102
10	BIAS	-0.0119	-0.0144	-0.0141	-0.0192	-0.0061	-0.0081	-0.0140	-0.0128
	VARIANCE	0.0341	0.0356	0.0204	0.0199	0.0119	0.0122	0.0095	0.0099
	MSE	0.0342	0.0358	0.0206	0.0203	0.0119	0.0123	0.0097	0.0101
15	BIAS	-0.0191	-0.0203	-0.0216	-0.0198	-0.0226	-0.0219	-0.0104	-0.0110
	VARIANCE	0.0326	0.0330	0.0206	0.0202	0.0114	0.0118	0.0090	0.0089
	MSE	0.0330	0.0334	0.0211	0.0206	0.0119	0.0123	0.0091	0.0090
20	BIAS	-0.0171	-0.0165	-0.0193	-0.0200	-0.0186	-0.0184	-0.0115	-0.0085
	VARIANCE	0.0343	0.0342	0.0183	0.0185	0.0113	0.0114	0.0092	0.0092
	MSE	0.0346	0.0345	0.0187	0.0189	0.0116	0.0117	0.0093	0.0093
VARIANCE OF NEYMAN		0.0355		0.0178		0.0118		0.0089	

TABLE XXXIII

SUMMARY STATISTICS FOR THE ESTIMATED MEANS
GAMMA: ALPHA=5 STRATA=2 PROP1=.8

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	BIAS	-0.0441	-0.0426	-0.0490	-0.0351	-0.0158	-0.0076	-0.0176	-0.0196
	VARIANCE	0.0779	0.0711	0.0333	0.0410	0.0185	0.0272	0.0127	0.0162
	MSE	0.0799	0.0729	0.0357	0.0422	0.0188	0.0273	0.0130	0.0166
5	BIAS	-0.0338	-0.0362	-0.0236	-0.0224	-0.0164	-0.0196	-0.0115	-0.0111
	VARIANCE	0.0424	0.0431	0.0222	0.0241	0.0136	0.0145	0.0107	0.0108
	MSE	0.0435	0.0444	0.0228	0.0246	0.0139	0.0149	0.0109	0.0109
10	BIAS	-0.0085	-0.0101	-0.0130	-0.0106	-0.0229	-0.0227	-0.0122	-0.0096
	VARIANCE	0.0377	0.0383	0.0195	0.0195	0.0147	0.0147	0.0104	0.0105
	MSE	0.0377	0.0384	0.0196	0.0196	0.0152	0.0152	0.0105	0.0106
15	BIAS	-0.0025	-0.0030	-0.0147	-0.0136	-0.0139	-0.0120	-0.0113	-0.0113
	VARIANCE	0.0424	0.0423	0.0205	0.0208	0.0140	0.0143	0.0103	0.0103
	MSE	0.0424	0.0423	0.0207	0.0210	0.0142	0.0144	0.0104	0.0104
20	BIAS	-0.0115	-0.0115	-0.0092	-0.0082	-0.0195	-0.0189	-0.0107	-0.0115
	VARIANCE	0.0477	0.0477	0.0218	0.0219	0.0132	0.0133	0.0093	0.0094
	MSE	0.0479	0.0479	0.0219	0.0219	0.0136	0.0137	0.0094	0.0095
VARIANCE OF NEYMAN		0.0405		0.0203		0.0135		0.0101	

TABLE XXXIV

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL METHOD DOES BETTER IN BIAS AND VARIANCE SUKHATME'S EXAMPLE NUMBER 1

		TOTAL OBSERVATIONS ALLOCATED					
		100		200		300	
		TIES		TIES		TIES	
INITIAL OBS PER STRATUM	STATISTIC	SEQ	SUK	SEQ	SUK	SEQ	SUK
15	BIAS	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000
	VARIANCE	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000

TABLE XXXV

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL METHOD DOES BETTER IN BIAS AND VARIANCE SUKHATME'S EXAMPLE NUMBER 2

		TOTAL OBSERVATIONS ALLOCATED					
		200		250		300	
		TIES		TIES		TIES	
INITIAL OBS PER STRATUM	STATISTIC	SEQ	SUK	SEQ	SUK	SEQ	SUK
15	BIAS	0.5360	0.5360	0.5400	0.5400	0.5380	0.5380
	VARIANCE	0.8880	0.8880	0.9660	0.9660	0.9840	0.9840

TABLE XXXVI

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL METHOD DOES BETTER IN BIAS AND VARIANCE
SUKHATME'S EXAMPLE NUMBER 3

		TOTAL OBSERVATIONS ALLOCATED					
		400		450		500	
		TIES		TIES		TIES	
INITIAL OBS PER STRATUM	STATISTIC	SEQ	SUK	SEQ	SUK	SEQ	SUK
15	BIAS	0.4780	0.4780	0.4900	0.4900	0.4960	0.4960
	VARIANCE	0.5920	0.5920	0.7640	0.7640	0.8060	0.8060

TABLE XXXVII

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL
METHOD RESPECTIVELY PRODUCES SMALLER
BIAS, SMALLER CONDITIONAL VARIANCE
NORMAL: STRATA=2 PLAN=I

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		TIES		TIES		TIES		TIES	
		WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT
2	BIAS	0.6660	0.3560	0.6160	0.4540	0.5980	0.5040	0.5660	0.4760
	VARIANCE	0.8300	0.5200	0.7920	0.6300	0.8380	0.7440	0.8880	0.7980
5	BIAS	0.8140	0.1800	0.6240	0.4220	0.5500	0.4600	0.5060	0.4580
	VARIANCE	0.9040	0.2700	0.7680	0.5660	0.7600	0.6700	0.7740	0.7260
10	BIAS	1.0000	0.0040	0.8160	0.1640	0.5960	0.3780	0.5680	0.4700
	VARIANCE	1.0000	0.0040	0.8920	0.2400	0.7660	0.5480	0.7520	0.6540
15	BIAS	1.0000	0.0000	0.9940	0.0080	0.8340	0.1680	0.6720	0.3960
	VARIANCE	1.0000	0.0000	1.0000	0.0140	0.9100	0.2440	0.7400	0.4640
20	BIAS	1.0000	0.0000	1.0000	0.0000	0.9900	0.0120	0.8380	0.1820
	VARIANCE	1.0000	0.0000	1.0000	0.0000	1.0000	0.0220	0.9000	0.2440

TABLE XXXVIII

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL METHOD RESPECTIVELY PRODUCES SMALLER BIAS, SMALLER CONDITIONAL VARIANCE
NORMAL: STRATA=2 PLAN=II

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		TIES		TIES		TIES		TIES	
		WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT
2	BIAS	0.5780	0.4540	0.5520	0.4920	0.5720	0.5400	0.5500	0.5160
	VARIANCE	0.8320	0.7080	0.9000	0.8400	0.9360	0.9040	0.9300	0.8960
5	BIAS	0.5640	0.4560	0.5880	0.5340	0.5220	0.4800	0.5140	0.4940
	VARIANCE	0.7540	0.6460	0.8180	0.7640	0.8560	0.8140	0.8680	0.8480
10	BIAS	0.6860	0.3660	0.5300	0.4420	0.5580	0.4920	0.5220	0.4820
	VARIANCE	0.7500	0.4300	0.7660	0.6780	0.8100	0.7440	0.7800	0.7400
15	BIAS	0.9520	0.0400	0.5380	0.4040	0.5080	0.4360	0.5020	0.4480
	VARIANCE	0.9740	0.0620	0.7440	0.6100	0.8120	0.7400	0.7820	0.7280
20	BIAS	1.0000	0.0000	0.6220	0.3800	0.4840	0.4120	0.5100	0.4540
	VARIANCE	1.0000	0.0000	0.7400	0.4980	0.7300	0.6580	0.7340	0.6780

TABLE XXXIX

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL METHOD RESPECTIVELY PRODUCES SMALLER BIAS, SMALLER CONDITIONAL VARIANCE
NORMAL: STRATA=2 PLAN=III

		TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		TIES		TIES		TIES		TIES	
INITIAL OBS PER STRATUM	STATISTIC	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT
2	BIAS	0.5520	0.4560	0.5780	0.5300	0.6020	0.5640	0.5760	0.5560
	VARIANCE	0.8500	0.7540	0.9040	0.8560	0.9460	0.9080	0.9480	0.9280
5	BIAS	0.5900	0.5060	0.5040	0.4540	0.5380	0.5000	0.5340	0.5160
	VARIANCE	0.8000	0.7160	0.8200	0.7700	0.8660	0.8280	0.8640	0.8460
10	BIAS	0.5700	0.4200	0.5080	0.4460	0.4880	0.4420	0.5240	0.4960
	VARIANCE	0.7480	0.5980	0.7660	0.7040	0.8400	0.7940	0.8520	0.8240
15	BIAS	0.7120	0.3160	0.5480	0.4620	0.5200	0.4780	0.5340	0.5020
	VARIANCE	0.7600	0.3640	0.7360	0.6500	0.7660	0.7240	0.7920	0.7600
20	BIAS	0.9720	0.0360	0.5680	0.4760	0.5820	0.5320	0.5260	0.4880
	VARIANCE	0.9800	0.0440	0.7220	0.6300	0.7200	0.6700	0.7440	0.7060

TABLE XL

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL
METHOD RESPECTIVELY PRODUCES SMALLER
BIAS, SMALLER CONDITIONAL VARIANCE
NORMAL: STRATA=2 PLAN=IV

		TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		TIES		TIES		TIES		TIES	
INITIAL OBS PER STRATUM	STATISTIC	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT
2	BIAS	0.6180	0.5480	0.5540	0.5300	0.5960	0.5700	0.5820	0.5600
	VARIANCE	0.8700	0.8000	0.9300	0.9060	0.9320	0.9060	0.9420	0.9200
5	BIAS	0.5260	0.4680	0.5140	0.4900	0.5480	0.5360	0.5300	0.5060
	VARIANCE	0.8080	0.7500	0.8540	0.8300	0.8720	0.8600	0.9020	0.8780
10	BIAS	0.5140	0.4000	0.5280	0.4660	0.5340	0.5000	0.5160	0.4960
	VARIANCE	0.7740	0.6600	0.7540	0.6920	0.8320	0.7980	0.8580	0.8380
15	BIAS	0.6040	0.4100	0.5300	0.4680	0.5460	0.4960	0.5240	0.4900
	VARIANCE	0.7440	0.5500	0.7540	0.6920	0.7540	0.7040	0.8200	0.7860
20	BIAS	0.7200	0.3120	0.5460	0.4460	0.4760	0.4280	0.4860	0.4620
	VARIANCE	0.8120	0.4040	0.7540	0.6540	0.7500	0.7020	0.7400	0.7160

TABLE XLI

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL METHOD RESPECTIVELY PRODUCES SMALLER BIAS, SMALLER CONDITIONAL VARIANCE

NORMAL: STRATA=3 PLAN=I

		TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		TIES		TIES		TIES		TIES	
INITIAL OBS PER STRATUM	STATISTIC	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT
2	BIAS	0.6760	0.3260	0.5980	0.4660	0.6060	0.5040	0.5740	0.5080
	VARIANCE	0.7760	0.4260	0.7580	0.6260	0.8460	0.7440	0.8580	0.7920
5	BIAS	0.8760	0.1240	0.6320	0.3860	0.5800	0.4620	0.5280	0.4600
	VARIANCE	0.9960	0.2440	0.6660	0.4200	0.6440	0.5260	0.7420	0.6740
10	BIAS	1.0000	0.0000	0.8820	0.1000	0.6780	0.3680	0.5200	0.4100
	VARIANCE	1.0000	0.0000	1.0000	0.2180	0.5840	0.2740	0.5040	0.3940
15	BIAS	1.0000	0.0000	1.0000	0.0000	0.9280	0.0840	0.6700	0.3020
	VARIANCE	1.0000	0.0000	1.0000	0.0000	1.0000	0.1560	0.7080	0.3400
20	BIAS	.	.	0.9980	0.0000	0.9220	0.0680	0.5580	0.5020
	VARIANCE	.	.	1.0000	0.0020	1.0000	0.1460	0.8180	0.7620

TABLE XLII

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL METHOD RESPECTIVELY PRODUCES SMALLER BIAS, SMALLER CONDITIONAL VARIANCE
NORMAL: STRATA=3 PLAN=II

		TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
INITIAL OBS PER STRATUM	STATISTIC	TIES WITH	TIES WITHOUT	TIES WITH	TIES WITHOUT	TIES WITH	TIES WITHOUT	TIES WITH	TIES WITHOUT
2	BIAS	0.5580	0.5020	0.5580	0.5500	0.5460	0.5400	0.5780	0.5740
	VARIANCE	0.8180	0.7620	0.8880	0.8800	0.9100	0.9040	0.9500	0.9460
5	BIAS	0.5660	0.4300	0.5180	0.5040	0.5160	0.5140	0.5380	0.5320
	VARIANCE	0.7380	0.6020	0.8000	0.7860	0.8800	0.8780	0.9080	0.9020
10	BIAS	0.8300	0.1740	0.5620	0.4560	0.5180	0.4940	0.5060	0.4980
	VARIANCE	0.9780	0.3220	0.7160	0.6100	0.7520	0.7280	0.8540	0.8460
15	BIAS	0.9980	0.0000	0.5440	0.4020	0.5340	0.4540	0.5160	0.4760
	VARIANCE	1.0000	0.0020	0.5720	0.4300	0.7100	0.6300	0.7640	0.7240
20	BIAS	.	.	0.5460	0.4560	0.5060	0.4540	0.5720	0.5460
	VARIANCE	.	.	0.5300	0.4400	0.7160	0.6640	0.8140	0.7880

TABLE XLIII

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL METHOD RESPECTIVELY PRODUCES SMALLER BIAS, SMALLER CONDITIONAL VARIANCE
NORMAL: STRATA=3 PLAN=III

		TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
INITIAL OBS PER		TIES		TIES		TIES		TIES	
STRATUM	STATISTIC	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT
2	BIAS	0.5720	0.5460	0.5660	0.5560	0.5760	0.5740	0.5740	0.5740
	VARIANCE	0.8140	0.7880	0.9140	0.9040	0.9360	0.9340	0.9560	0.9560
5	BIAS	0.5180	0.4760	0.5100	0.5040	0.4860	0.4860	0.4920	0.4920
	VARIANCE	0.7640	0.7220	0.8560	0.8500	0.9280	0.9280	0.9300	0.9300
10	BIAS	0.7320	0.2480	0.5460	0.5340	0.4960	0.4960	0.4660	0.4640
	VARIANCE	0.8380	0.3540	0.7740	0.7620	0.8440	0.8440	0.8380	0.8360
15	BIAS	1.0000	0.0000	0.5440	0.4800	0.5060	0.5020	0.4980	0.4960
	VARIANCE	1.0000	0.0000	0.7280	0.6640	0.7840	0.7800	0.8040	0.8020
20	BIAS	.	.	0.4920	0.4820	0.5100	0.5060	0.5440	0.5280
	VARIANCE	.	.	0.7380	0.7280	0.7760	0.7720	0.8440	0.8280

TABLE XLIV

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL METHOD RESPECTIVELY PRODUCES SMALLER BIAS, SMALLER CONDITIONAL VARIANCE
NORMAL: STRATA=3 PLAN=IV

		TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		TIES		TIES		TIES		TIES	
INITIAL OBS PER STRATUM	STATISTIC	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT
2	BIAS	0.5440	0.5280	0.5760	0.5740	0.5920	0.5900	0.5720	0.5700
	VARIANCE	0.8440	0.8280	0.9320	0.9300	0.9420	0.9400	0.9620	0.9600
5	BIAS	0.5400	0.5160	0.5360	0.5300	0.5360	0.5360	0.5180	0.5180
	VARIANCE	0.7780	0.7540	0.8800	0.8740	0.9240	0.9240	0.9440	0.9440
10	BIAS	0.5980	0.4340	0.4980	0.4920	0.4560	0.4560	0.4980	0.4940
	VARIANCE	0.7080	0.5440	0.7720	0.7660	0.8100	0.8100	0.8600	0.8560
15	BIAS	0.9300	0.0740	0.5500	0.5260	0.5180	0.5100	0.5020	0.4960
	VARIANCE	0.9580	0.1020	0.7320	0.7080	0.8180	0.8100	0.8400	0.8340
20	BIAS	.	.	0.5140	0.5020	0.5240	0.5120	0.5540	0.5420
	VARIANCE	.	.	0.7420	0.7300	0.7540	0.7420	0.8660	0.8540

TABLE XLV

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL
METHOD RESPECTIVELY PRODUCES SMALLER
BIAS, SMALLER CONDITIONAL VARIANCE
NORMAL: STRATA=3 PLAN=V

		TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		TIES		TIES		TIES		TIES	
INITIAL OBS PER STRATUM	STATISTIC	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT
2	BIAS	0.5540	0.5420	0.5900	0.5900	0.5700	0.5700	0.6060	0.6060
	VARIANCE	0.8660	0.8540	0.9580	0.9580	0.9800	0.9800	0.9900	0.9900
5	BIAS	0.5480	0.5300	0.5160	0.5120	0.4920	0.4920	0.5200	0.5180
	VARIANCE	0.7960	0.7780	0.8940	0.8900	0.9440	0.9440	0.9380	0.9360
10	BIAS	0.5420	0.4620	0.4840	0.4840	0.5100	0.5080	0.5200	0.5180
	VARIANCE	0.6820	0.6020	0.7940	0.7940	0.8640	0.8620	0.9100	0.9080
15	BIAS	0.7500	0.2320	0.5060	0.4940	0.5300	0.5260	0.5220	0.5220
	VARIANCE	0.8260	0.3080	0.7480	0.7360	0.8240	0.8200	0.8420	0.8420
20	BIAS	.	.	0.5380	0.5320	0.4980	0.4960	0.4980	0.4960
	VARIANCE	.	.	0.7640	0.7580	0.8200	0.8180	0.8200	0.8180

TABLE XLVI

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL METHOD RESPECTIVELY PRODUCES SMALLER BIAS, SMALLER CONDITIONAL VARIANCE
GAMMA: ALPHA=1 STRATA=2 PROP1=.5

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		TIES		TIES		TIES		TIES	
		WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT
2	BIAS	0.5920	0.4700	0.5460	0.4740	0.6040	0.5560	0.5820	0.5540
	VARIANCE	0.8100	0.6880	0.8960	0.8240	0.9040	0.8560	0.9140	0.8860
5	BIAS	0.5780	0.4480	0.5480	0.5100	0.5320	0.5120	0.5000	0.4780
	VARIANCE	0.8100	0.6800	0.8320	0.7940	0.8800	0.8600	0.9080	0.8860
10	BIAS	0.8040	0.2080	0.5420	0.4580	0.5380	0.4960	0.5140	0.4720
	VARIANCE	0.9180	0.3220	0.7700	0.6860	0.8340	0.7920	0.8360	0.7940
15	BIAS	0.9880	0.0060	0.6120	0.4100	0.4940	0.4460	0.5200	0.4800
	VARIANCE	1.0000	0.0180	0.7340	0.5320	0.7440	0.6960	0.7740	0.7340
20	BIAS	1.0000	0.0000	0.8380	0.1480	0.5100	0.4200	0.5340	0.4640
	VARIANCE	1.0000	0.0000	0.9460	0.2560	0.7720	0.6820	0.7720	0.7020

TABLE XLVII

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL METHOD RESPECTIVELY PRODUCES SMALLER BIAS, SMALLER CONDITIONAL VARIANCE
GAMMA: ALPHA=1 STRATA=2 PROP1=.6

		TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
INITIAL OBS PER STRATUM		TIES		TIES		TIES		TIES	
	STATISTIC	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT
2	BIAS	0.6060	0.4940	0.6000	0.5660	0.5780	0.5460	0.5680	0.5520
	VARIANCE	0.8440	0.7320	0.9100	0.8760	0.9120	0.8800	0.9220	0.9060
5	BIAS	0.5440	0.4620	0.5340	0.4980	0.5660	0.5520	0.5560	0.5380
	VARIANCE	0.7660	0.6840	0.8400	0.8040	0.8560	0.8420	0.8900	0.8720
10	BIAS	0.5860	0.4460	0.5200	0.4680	0.5260	0.5020	0.5000	0.4840
	VARIANCE	0.7120	0.5720	0.8180	0.7660	0.8300	0.8060	0.8440	0.8280
15	BIAS	0.7820	0.2640	0.5600	0.5080	0.5240	0.4860	0.5280	0.4980
	VARIANCE	0.7960	0.2780	0.8140	0.7620	0.7860	0.7480	0.8140	0.7840
20	BIAS	0.9620	0.0240	0.5000	0.4300	0.5120	0.4640	0.5220	0.4880
	VARIANCE	0.9780	0.0400	0.6760	0.6060	0.7520	0.7040	0.7920	0.7580

TABLE XLVIII

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL METHOD RESPECTIVELY PRODUCES SMALLER BIAS, SMALLER CONDITIONAL VARIANCE
GAMMA: ALPHA=1 STRATA=2 PROP1=.7

		TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
INITIAL OBS PER STRATUM		TIES		TIES		TIES		TIES	
	STATISTIC	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT
2	BIAS	0.5580	0.4880	0.5740	0.5380	0.5700	0.5420	0.6180	0.6040
	VARIANCE	0.8080	0.7380	0.8980	0.8620	0.9240	0.8960	0.9340	0.9200
5	BIAS	0.5720	0.4960	0.5400	0.5160	0.4980	0.4860	0.5300	0.5200
	VARIANCE	0.7940	0.7180	0.8500	0.8260	0.8500	0.8380	0.8580	0.8480
10	BIAS	0.5520	0.4400	0.5040	0.4640	0.5260	0.5060	0.4840	0.4500
	VARIANCE	0.7060	0.5940	0.7640	0.7240	0.7900	0.7700	0.8440	0.8100
15	BIAS	0.5800	0.4220	0.5540	0.4920	0.5480	0.5200	0.4580	0.4340
	VARIANCE	0.6900	0.5320	0.7220	0.6600	0.8040	0.7760	0.8080	0.7840
20	BIAS	0.7440	0.2700	0.5300	0.4700	0.5180	0.4760	0.5360	0.4900
	VARIANCE	0.7600	0.2860	0.7060	0.6460	0.7500	0.7080	0.7720	0.7260

TABLE XLIX

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL
METHOD RESPECTIVELY PRODUCES SMALLER
BIAS, SMALLER CONDITIONAL VARIANCE
GAMMA: ALPHA=1 STRATA=2 PROP1=.8

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		TIES		TIES		TIES		TIES	
		WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT
2	BIAS	0.6000	0.4940	0.6300	0.5720	0.5720	0.5400	0.5640	0.5320
	VARIANCE	0.8160	0.7100	0.8780	0.8200	0.9080	0.8760	0.9280	0.8960
5	BIAS	0.5520	0.4520	0.5100	0.4860	0.5040	0.4860	0.5640	0.5460
	VARIANCE	0.7440	0.6440	0.8060	0.7820	0.8480	0.8300	0.8740	0.8560
10	BIAS	0.6020	0.3880	0.5460	0.4940	0.5100	0.4940	0.5600	0.5380
	VARIANCE	0.7420	0.5280	0.7400	0.6880	0.7900	0.7740	0.8020	0.7800
15	BIAS	0.7440	0.2480	0.5920	0.4820	0.5200	0.4720	0.5040	0.4720
	VARIANCE	0.8240	0.3280	0.7460	0.6360	0.7380	0.6900	0.7480	0.7160
20	BIAS	0.9500	0.0620	0.5440	0.4460	0.5540	0.4880	0.5460	0.5180
	VARIANCE	0.9580	0.0700	0.6700	0.5720	0.7020	0.6360	0.7300	0.7020

TABLE L

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL METHOD RESPECTIVELY PRODUCES SMALLER BIAS, SMALLER CONDITIONAL VARIANCE
GAMMA: ALPHA=2 STRATA=2 PROP1=.5

		TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		TIES		TIES		TIES		TIES	
INITIAL OBS PER STRATUM	STATISTIC	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT
2	BIAS	0.5920	0.4740	0.5560	0.5060	0.5900	0.5580	0.5900	0.5560
	VARIANCE	0.8420	0.7240	0.8940	0.8440	0.9300	0.8980	0.9320	0.8980
5	BIAS	0.5940	0.4760	0.5460	0.5180	0.5740	0.5520	0.5380	0.5200
	VARIANCE	0.7860	0.6680	0.8420	0.8140	0.8820	0.8600	0.8800	0.8620
10	BIAS	0.5760	0.4000	0.4920	0.4260	0.5640	0.5300	0.5200	0.4760
	VARIANCE	0.7600	0.5840	0.7840	0.7180	0.8140	0.7800	0.8280	0.7840
15	BIAS	0.8740	0.1940	0.5560	0.4700	0.5480	0.4860	0.5280	0.5000
	VARIANCE	0.8980	0.2180	0.7600	0.6740	0.8020	0.7400	0.7940	0.7660
20	BIAS	0.9880	0.0080	0.5280	0.4080	0.5500	0.4900	0.5320	0.4840
	VARIANCE	0.9980	0.0180	0.7240	0.6040	0.7480	0.6880	0.7660	0.7180

TABLE LI

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL METHOD RESPECTIVELY PRODUCES SMALLER BIAS, SMALLER CONDITIONAL VARIANCE
GAMMA: ALPHA=2 STRATA=2 PROP1=.6

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		TIES		TIES		TIES		TIES	
		WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT
2	BIAS	0.5220	0.4480	0.5920	0.5580	0.5780	0.5520	0.5860	0.5620
	VARIANCE	0.8420	0.7680	0.9080	0.8740	0.9180	0.8920	0.9380	0.9140
5	BIAS	0.5520	0.4760	0.5580	0.5400	0.5100	0.4820	0.5060	0.4880
	VARIANCE	0.7700	0.6940	0.8460	0.8280	0.8840	0.8560	0.8940	0.8760
10	BIAS	0.5840	0.4540	0.5340	0.4900	0.4980	0.4740	0.4680	0.4440
	VARIANCE	0.7620	0.6320	0.7840	0.7400	0.7840	0.7600	0.8460	0.8220
15	BIAS	0.6100	0.4460	0.5340	0.4760	0.5400	0.5060	0.5120	0.4860
	VARIANCE	0.6600	0.4960	0.7140	0.6560	0.8020	0.7680	0.8000	0.7740
20	BIAS	0.7900	0.1660	0.5760	0.5000	0.5580	0.5040	0.5360	0.4920
	VARIANCE	0.8080	0.1840	0.7140	0.6380	0.7140	0.6600	0.7800	0.7360

TABLE LII

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL
METHOD RESPECTIVELY PRODUCES SMALLER
BIAS, SMALLER CONDITIONAL VARIANCE
GAMMA: ALPHA=2 STRATA=2 PROP1=.7

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		TIES		TIES		TIES		TIES	
		WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT
2	BIAS	0.5860	0.5120	0.5700	0.5300	0.5960	0.5580	0.5620	0.5480
	VARIANCE	0.8300	0.7560	0.9160	0.8760	0.9240	0.8860	0.9420	0.9280
5	BIAS	0.5440	0.4700	0.5320	0.5020	0.5000	0.4720	0.5500	0.5400
	VARIANCE	0.7960	0.7220	0.8180	0.7880	0.8700	0.8420	0.8880	0.8780
10	BIAS	0.5520	0.4240	0.5640	0.5220	0.5240	0.4740	0.5240	0.5100
	VARIANCE	0.7180	0.5900	0.7580	0.7160	0.8100	0.7600	0.8020	0.7880
15	BIAS	0.6340	0.3420	0.5140	0.4640	0.5000	0.4720	0.5200	0.4920
	VARIANCE	0.7500	0.4580	0.7340	0.6840	0.7480	0.7200	0.7780	0.7500
20	BIAS	0.8060	0.2480	0.5340	0.4660	0.5320	0.4860	0.5200	0.4940
	VARIANCE	0.8380	0.2800	0.7240	0.6560	0.7500	0.7040	0.7420	0.7160

TABLE LIII

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL METHOD RESPECTIVELY PRODUCES SMALLER BIAS, SMALLER CONDITIONAL VARIANCE
GAMMA: ALPHA=2 STRATA=2 PROP1=.8

		TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		TIES		TIES		TIES		TIES	
INITIAL OBS PER STRATUM	STATISTIC	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT
2	BIAS	0.6080	0.4380	0.5640	0.5080	0.5520	0.5200	0.5740	0.5500
	VARIANCE	0.7980	0.6280	0.8520	0.7960	0.8680	0.8360	0.9200	0.8960
5	BIAS	0.5200	0.3760	0.5420	0.4980	0.5160	0.5080	0.5120	0.4900
	VARIANCE	0.7500	0.6060	0.7780	0.7340	0.7920	0.7840	0.8560	0.8340
10	BIAS	0.6900	0.3720	0.5900	0.5160	0.4980	0.4560	0.5160	0.4780
	VARIANCE	0.7860	0.4680	0.7300	0.6560	0.7460	0.7040	0.7720	0.7340
15	BIAS	0.8920	0.1340	0.5860	0.4640	0.4900	0.4360	0.5060	0.4660
	VARIANCE	0.9140	0.1560	0.6940	0.5720	0.7060	0.6520	0.7460	0.7060
20	BIAS	0.9960	0.0060	0.6120	0.3900	0.5460	0.4820	0.5480	0.5060
	VARIANCE	0.9960	0.0060	0.6700	0.4480	0.6420	0.5780	0.7520	0.7100

TABLE LIV

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL METHOD RESPECTIVELY PRODUCES SMALLER BIAS, SMALLER CONDITIONAL VARIANCE

GAMMA: ALPHA=5 STRATA=2 PROP1=.5

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		TIES		TIES		TIES		TIES	
		WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT
2	BIAS	0.6420	0.5640	0.5600	0.5160	0.6120	0.5900	0.5960	0.5760
	VARIANCE	0.8440	0.7660	0.9060	0.8620	0.9220	0.9000	0.9140	0.8940
5	BIAS	0.5600	0.4740	0.5220	0.4920	0.5540	0.5240	0.4800	0.4680
	VARIANCE	0.7780	0.6920	0.8580	0.8280	0.8740	0.8440	0.8580	0.8460
10	BIAS	0.5460	0.4220	0.5460	0.4760	0.4900	0.4680	0.5200	0.4900
	VARIANCE	0.7240	0.6000	0.7920	0.7220	0.8260	0.8040	0.8440	0.8140
15	BIAS	0.6700	0.3540	0.5420	0.4660	0.5120	0.4720	0.5180	0.4800
	VARIANCE	0.7280	0.4120	0.7500	0.6740	0.7820	0.7420	0.8180	0.7800
20	BIAS	0.9160	0.0700	0.5000	0.4320	0.5820	0.5260	0.5440	0.5240
	VARIANCE	0.9480	0.1020	0.7060	0.6380	0.7660	0.7100	0.7600	0.7400

TABLE LV

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL METHOD RESPECTIVELY PRODUCES SMALLER BIAS, SMALLER CONDITIONAL VARIANCE
GAMMA: ALPHA=5 STRATA=2 PROP1=.6

		TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		TIES		TIES		TIES		TIES	
INITIAL OBS PER STRATUM	STATISTIC	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT
2	BIAS	0.5960	0.5080	0.6180	0.5860	0.5960	0.5680	0.6200	0.6000
	VARIANCE	0.8580	0.7700	0.9020	0.8700	0.9360	0.9080	0.9340	0.9140
5	BIAS	0.5460	0.4720	0.5600	0.5340	0.5640	0.5420	0.5400	0.5260
	VARIANCE	0.8240	0.7500	0.8380	0.8120	0.8440	0.8220	0.8960	0.8820
10	BIAS	0.5700	0.4280	0.5660	0.5140	0.5440	0.5180	0.5280	0.4980
	VARIANCE	0.7400	0.5980	0.8100	0.7580	0.8120	0.7860	0.8300	0.8000
15	BIAS	0.6100	0.4040	0.5280	0.4680	0.4980	0.4600	0.5380	0.5080
	VARIANCE	0.7020	0.4960	0.7320	0.6720	0.7620	0.7240	0.7540	0.7240
20	BIAS	0.7280	0.2700	0.5300	0.4660	0.5380	0.4680	0.5440	0.5100
	VARIANCE	0.8020	0.3440	0.7220	0.6580	0.7740	0.7040	0.7760	0.7420

TABLE LVI

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL METHOD RESPECTIVELY PRODUCES SMALLER BIAS, SMALLER CONDITIONAL VARIANCE
GAMMA: ALPHA=5 STRATA=2 PROP1=.7

		TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
INITIAL OBS PER		TIES		TIES		TIES		TIES	
STRATUM	STATISTIC	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT
2	BIAS	0.6080	0.4900	0.5980	0.5460	0.5980	0.5640	0.5700	0.5440
	VARIANCE	0.8480	0.7300	0.9120	0.8600	0.9280	0.8940	0.9280	0.9020
5	BIAS	0.5520	0.4900	0.5120	0.4680	0.5320	0.5060	0.5520	0.5320
	VARIANCE	0.7700	0.7080	0.8260	0.7820	0.8340	0.8080	0.8800	0.8600
10	BIAS	0.5880	0.4440	0.4900	0.4400	0.5320	0.5060	0.5320	0.5040
	VARIANCE	0.7040	0.5600	0.7740	0.7240	0.7680	0.7420	0.8260	0.7980
15	BIAS	0.7040	0.2820	0.5340	0.4660	0.5600	0.5140	0.4980	0.4620
	VARIANCE	0.8180	0.3960	0.7240	0.6560	0.7900	0.7440	0.7920	0.7560
20	BIAS	0.9320	0.0840	0.5480	0.4360	0.5480	0.5040	0.5140	0.4760
	VARIANCE	0.9480	0.1000	0.6560	0.5440	0.7020	0.6580	0.7540	0.7160

TABLE LVII

PROPORTIONS OF TRIALS IN WHICH THE SEQUENTIAL METHOD RESPECTIVELY PRODUCES SMALLER BIAS, SMALLER CONDITIONAL VARIANCE
GAMMA: ALPHA=5 STRATA=2 PROP1=.8

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		TIES		TIES		TIES		TIES	
		WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT	WITH	WITHOUT
2	BIAS	0.5820	0.3840	0.5580	0.4720	0.5960	0.5460	0.5840	0.5400
	VARIANCE	0.7560	0.5580	0.8380	0.7520	0.8660	0.8160	0.9040	0.8600
5	BIAS	0.6500	0.4340	0.5320	0.4540	0.5600	0.5260	0.5360	0.5120
	VARIANCE	0.7380	0.5220	0.7660	0.6880	0.8100	0.7760	0.8180	0.7940
10	BIAS	0.7820	0.2340	0.5420	0.4420	0.5560	0.4760	0.5280	0.4940
	VARIANCE	0.8660	0.3180	0.6940	0.5940	0.7260	0.6460	0.7680	0.7340
15	BIAS	0.9820	0.0120	0.6280	0.4200	0.5380	0.4580	0.5500	0.4920
	VARIANCE	0.9940	0.0240	0.7260	0.5180	0.6860	0.6060	0.7240	0.6660
20	BIAS	1.0000	0.0000	0.7280	0.2520	0.5460	0.4420	0.5100	0.4580
	VARIANCE	1.0000	0.0000	0.8120	0.3360	0.6740	0.5700	0.6920	0.6400

TABLE LVIII

SUMMARY STATISTICS FOR THE PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM 1 FOR NORMAL: STRATA=2 PLAN=I

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	MAXIMUM	0.9600	0.9400	0.9800	0.9800	0.9867	0.9867	0.2300	0.9750
	Q3	0.1000	0.2000	0.1000	0.2000	0.1000	0.1967	0.1000	0.1900
	MEDIAN	0.0800	0.0800	0.0800	0.0900	0.0867	0.0933	0.0900	0.0950
	Q1	0.0400	0.0400	0.0450	0.0400	0.0667	0.0467	0.0750	0.0400
	MINIMUM	0.0400	0.0400	0.0200	0.0200	0.0133	0.0133	0.0100	0.0100
5	MAXIMUM	0.1800	0.3400	0.1500	0.2700	0.1333	0.5200	0.1300	0.4150
	Q3	0.1200	0.1200	0.1000	0.1200	0.1000	0.1267	0.1000	0.1250
	MEDIAN	0.1000	0.1000	0.0900	0.0900	0.0867	0.0867	0.0900	0.0950
	Q1	0.1000	0.1000	0.0700	0.0700	0.0733	0.0667	0.0800	0.0650
	MINIMUM	0.1000	0.1000	0.0500	0.0500	0.0333	0.0333	0.0250	0.0250
10	MAXIMUM	0.2000	0.2200	0.1500	0.2400	0.1400	0.2133	0.1350	0.2350
	Q3	0.2000	0.2000	0.1100	0.1100	0.1000	0.1133	0.1000	0.1100
	MEDIAN	0.2000	0.2000	0.1000	0.1000	0.0933	0.0933	0.0900	0.0900
	Q1	0.2000	0.2000	0.1000	0.1000	0.0800	0.0733	0.0800	0.0700
	MINIMUM	0.2000	0.2000	0.1000	0.1000	0.0667	0.0667	0.0500	0.0500
15	MAXIMUM	0.3000	0.3000	0.1500	0.1900	0.1400	0.2067	0.1300	0.1850
	Q3	0.3000	0.3000	0.1500	0.1500	0.1000	0.1067	0.1000	0.1050
	MEDIAN	0.3000	0.3000	0.1500	0.1500	0.1000	0.1000	0.0900	0.0900
	Q1	0.3000	0.3000	0.1500	0.1500	0.1000	0.1000	0.0800	0.0750
	MINIMUM	0.3000	0.3000	0.1500	0.1500	0.1000	0.1000	0.0750	0.0750
20	MAXIMUM	0.4000	0.4000	0.2000	0.2000	0.1333	0.1800	0.1350	0.1650
	Q3	0.4000	0.4000	0.2000	0.2000	0.1333	0.1333	0.1000	0.1050
	MEDIAN	0.4000	0.4000	0.2000	0.2000	0.1333	0.1333	0.1000	0.1000
	Q1	0.4000	0.4000	0.2000	0.2000	0.1333	0.1333	0.1000	0.1000
	MINIMUM	0.4000	0.4000	0.2000	0.2000	0.1333	0.1333	0.1000	0.1000

PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM 1 UNDER NEYMAN: 0.0909

TABLE LIX

SUMMARY STATISTICS FOR THE PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM 1 FOR NORMAL: STRATA=2 PLAN=II

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED 50 SEQ	SUK	100 SEQ	SUK	150 SEQ	SUK	200 SEQ	SUK
2	MAXIMUM	0.9600	0.9600	0.9800	0.9800	0.9867	0.9867	0.9900	0.9900
	Q3	0.2600	0.4200	0.2600	0.4400	0.2600	0.4267	0.2500	0.4100
	MEDIAN	0.2200	0.2200	0.2400	0.2700	0.2400	0.2333	0.2350	0.2450
	Q1	0.1600	0.1000	0.2200	0.1300	0.2200	0.1200	0.2200	0.1100
	MINIMUM	0.0400	0.0400	0.0200	0.0200	0.0133	0.0133	0.0100	0.0100
5	MAXIMUM	0.3600	0.6600	0.3400	0.7500	0.3133	0.7133	0.3050	0.7700
	Q3	0.2600	0.3200	0.2600	0.3200	0.2600	0.3167	0.2550	0.3050
	MEDIAN	0.2400	0.2400	0.2400	0.2500	0.2400	0.2333	0.2400	0.2400
	Q1	0.2000	0.1800	0.2200	0.1900	0.2267	0.1800	0.2250	0.1850
	MINIMUM	0.1000	0.1000	0.0600	0.0500	0.1400	0.0467	0.1650	0.0500
10	MAXIMUM	0.3800	0.4600	0.3400	0.4800	0.3200	0.6200	0.3100	0.4500
	Q3	0.2800	0.2800	0.2600	0.2900	0.2533	0.2767	0.2550	0.2800
	MEDIAN	0.2400	0.2400	0.2400	0.2400	0.2400	0.2333	0.2400	0.2400
	Q1	0.2000	0.2000	0.2200	0.2100	0.2233	0.1933	0.2250	0.2000
	MINIMUM	0.2000	0.2000	0.1000	0.1100	0.1200	0.1067	0.1750	0.1000
15	MAXIMUM	0.3800	0.4200	0.3400	0.3900	0.3067	0.4267	0.2950	0.4450
	Q3	0.3000	0.3000	0.2600	0.2700	0.2600	0.2867	0.2550	0.2750
	MEDIAN	0.3000	0.3000	0.2400	0.2400	0.2400	0.2400	0.2400	0.2400
	Q1	0.3000	0.3000	0.2200	0.2100	0.2267	0.2067	0.2250	0.2100
	MINIMUM	0.3000	0.3000	0.1500	0.1500	0.1600	0.1133	0.1450	0.1050
20	MAXIMUM	0.4000	0.4000	0.3300	0.3900	0.3267	0.3933	0.2950	0.4350
	Q3	0.4000	0.4000	0.2600	0.2700	0.2600	0.2733	0.2550	0.2650
	MEDIAN	0.4000	0.4000	0.2400	0.2400	0.2400	0.2467	0.2400	0.2400
	Q1	0.4000	0.4000	0.2200	0.2100	0.2267	0.2200	0.2250	0.2150
	MINIMUM	0.4000	0.4000	0.2000	0.2000	0.1600	0.1467	0.1500	0.1450

PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM 1 UNDER NEYMAN: 0.2403

TABLE LX

SUMMARY STATISTICS FOR THE PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM 1 FOR NORMAL: STRATA=2 PLAN=III

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	MAXIMUM	0.9600	0.9600	0.9800	0.9800	0.9867	0.9867	0.9900	0.9900
	Q3	0.3600	0.5200	0.3600	0.5450	0.3533	0.5167	0.3550	0.5550
	MEDIAN	0.3200	0.3200	0.3300	0.3400	0.3333	0.3067	0.3350	0.3450
	Q1	0.2800	0.1800	0.3100	0.1800	0.3133	0.1600	0.3150	0.1650
	MINIMUM	0.0400	0.0400	0.0200	0.0200	0.0133	0.0133	0.0100	0.0100
5	MAXIMUM	0.4800	0.7400	0.4200	0.7800	0.4000	0.7600	0.4000	0.7600
	Q3	0.3600	0.4200	0.3600	0.4200	0.3533	0.4267	0.3500	0.4000
	MEDIAN	0.3400	0.3200	0.3300	0.3400	0.3333	0.3400	0.3350	0.3350
	Q1	0.3000	0.2400	0.3100	0.2650	0.3133	0.2667	0.3200	0.2500
	MINIMUM	0.1000	0.1000	0.2100	0.0500	0.2467	0.0733	0.2600	0.0500
10	MAXIMUM	0.4800	0.6000	0.4500	0.6200	0.4467	0.6000	0.3950	0.5650
	Q3	0.3600	0.4000	0.3600	0.3900	0.3533	0.3867	0.3500	0.3900
	MEDIAN	0.3400	0.3400	0.3300	0.3300	0.3333	0.3333	0.3350	0.3300
	Q1	0.3000	0.2800	0.3100	0.2800	0.3133	0.2800	0.3150	0.2850
	MINIMUM	0.2000	0.2000	0.2000	0.1300	0.2400	0.1467	0.2550	0.1300
15	MAXIMUM	0.5200	0.5800	0.4500	0.6400	0.4267	0.5400	0.4100	0.5800
	Q3	0.3600	0.3800	0.3600	0.3700	0.3533	0.3733	0.3500	0.3750
	MEDIAN	0.3400	0.3400	0.3400	0.3350	0.3333	0.3333	0.3350	0.3350
	Q1	0.3000	0.3000	0.3100	0.2900	0.3133	0.2933	0.3200	0.3000
	MINIMUM	0.3000	0.3000	0.1800	0.1500	0.2067	0.1733	0.2650	0.1600
20	MAXIMUM	0.4800	0.4800	0.5000	0.5300	0.4133	0.4800	0.4150	0.5200
	Q3	0.4000	0.4000	0.3500	0.3700	0.3533	0.3733	0.3500	0.3750
	MEDIAN	0.4000	0.4000	0.3300	0.3300	0.3333	0.3333	0.3350	0.3400
	Q1	0.4000	0.4000	0.3100	0.3000	0.3167	0.3000	0.3200	0.3050
	MINIMUM	0.4000	0.4000	0.2000	0.2000	0.2467	0.2000	0.2450	0.1900

PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM 1 UNDER NEYMAN: 0.3333

TABLE LXI

SUMMARY STATISTICS FOR THE PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM 1 FOR NORMAL: STRATA=2 PLAN=IV

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	MAXIMUM	0.9600	0.9600	0.9800	0.9800	0.9867	0.9867	0.9900	0.9900
	Q3	0.5400	0.7200	0.5300	0.7050	0.5200	0.7100	0.5200	0.6800
	MEDIAN	0.5000	0.5000	0.5000	0.5200	0.5000	0.4933	0.5000	0.4600
	Q1	0.4600	0.3000	0.4700	0.3000	0.4733	0.2867	0.4850	0.2875
	MINIMUM	0.0400	0.0400	0.0200	0.0200	0.0133	0.0133	0.0100	0.0100
5	MAXIMUM	0.7600	0.8800	0.6200	0.8500	0.5933	0.9200	0.5700	0.8100
	Q3	0.5400	0.5800	0.5300	0.6100	0.5200	0.5900	0.5150	0.6050
	MEDIAN	0.5000	0.5000	0.5000	0.5100	0.5000	0.5000	0.5000	0.5100
	Q1	0.4600	0.4000	0.4800	0.4100	0.4800	0.4200	0.4850	0.4100
	MINIMUM	0.2200	0.1400	0.3900	0.1500	0.4067	0.0933	0.4250	0.0800
10	MAXIMUM	0.7600	0.7200	0.6300	0.7600	0.5800	0.7333	0.5750	0.7700
	Q3	0.5400	0.5600	0.5200	0.5500	0.5200	0.5533	0.5150	0.5550
	MEDIAN	0.5000	0.5000	0.5000	0.5000	0.5000	0.4933	0.5000	0.4950
	Q1	0.4600	0.4400	0.4700	0.4400	0.4800	0.4400	0.4850	0.4350
	MINIMUM	0.3200	0.2600	0.3800	0.2500	0.4133	0.2600	0.4300	0.2500
15	MAXIMUM	0.7000	0.7000	0.6400	0.7000	0.5867	0.6867	0.5800	0.7000
	Q3	0.5400	0.5400	0.5200	0.5500	0.5200	0.5400	0.5175	0.5400
	MEDIAN	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.4950
	Q1	0.4600	0.4600	0.4700	0.4500	0.4800	0.4600	0.4850	0.4550
	MINIMUM	0.3200	0.3000	0.4000	0.2800	0.3933	0.2800	0.4350	0.2450
20	MAXIMUM	0.6000	0.6000	0.6100	0.6700	0.5867	0.6867	0.5800	0.6950
	Q3	0.5400	0.5400	0.5300	0.5450	0.5200	0.5333	0.5200	0.5350
	MEDIAN	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.4950
	Q1	0.4600	0.4600	0.4700	0.4600	0.4800	0.4600	0.4850	0.4600
	MINIMUM	0.4000	0.4000	0.3900	0.2900	0.4200	0.3533	0.4200	0.2450

PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM 1 UNDER NEYMAN: 0.5

TABLE LXII

SUMMARY STATISTICS FOR THE PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM1 FOR GAMMA: ALPHA=1 STRATA=2 PROP1=.5

		TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
INITIAL OBS PER STRATUM	STATISTIC	SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	MAXIMUM	0.9600	0.9600	0.9800	0.9800	0.6000	0.9667	0.9900	0.9900
	Q3	0.2000	0.4000	0.1900	0.4700	0.1867	0.4000	0.1800	0.4425
	MEDIAN	0.1800	0.2200	0.1700	0.2200	0.1733	0.2133	0.1700	0.1950
	Q1	0.1200	0.1000	0.1500	0.0900	0.1533	0.1033	0.1550	0.0850
	MINIMUM	0.0400	0.0400	0.0200	0.0200	0.0133	0.0133	0.0100	0.0100
5	MAXIMUM	0.3200	0.7200	0.2400	0.7600	0.2400	0.6800	0.2250	0.6750
	Q3	0.2000	0.3000	0.1900	0.2800	0.1867	0.2733	0.1800	0.2800
	MEDIAN	0.1800	0.2000	0.1750	0.2000	0.1733	0.2067	0.1700	0.2000
	Q1	0.1400	0.1400	0.1600	0.1400	0.1600	0.1400	0.1600	0.1375
	MINIMUM	0.1000	0.1000	0.0500	0.0500	0.0400	0.0400	0.0700	0.0250
10	MAXIMUM	0.3000	0.4800	0.2500	0.4500	0.2667	0.4333	0.2250	0.4900
	Q3	0.2000	0.2400	0.1900	0.2300	0.1867	0.2400	0.1850	0.2300
	MEDIAN	0.2000	0.2000	0.1700	0.1800	0.1733	0.1867	0.1700	0.1775
	Q1	0.2000	0.2000	0.1500	0.1400	0.1600	0.1467	0.1600	0.1450
	MINIMUM	0.2000	0.2000	0.1000	0.1000	0.0800	0.0667	0.1000	0.0500
15	MAXIMUM	0.3400	0.4000	0.2600	0.4200	0.2467	0.4000	0.2250	0.4650
	Q3	0.3000	0.3000	0.1900	0.2100	0.1867	0.2200	0.1850	0.2150
	MEDIAN	0.3000	0.3000	0.1700	0.1800	0.1733	0.1800	0.1700	0.1800
	Q1	0.3000	0.3000	0.1500	0.1500	0.1600	0.1467	0.1600	0.1500
	MINIMUM	0.3000	0.3000	0.1500	0.1500	0.1133	0.1000	0.1100	0.0900
20	MAXIMUM	0.4000	0.4000	0.2600	0.3700	0.2400	0.3600	0.2200	0.3150
	Q3	0.4000	0.4000	0.2000	0.2100	0.1867	0.2133	0.1800	0.2050
	MEDIAN	0.4000	0.4000	0.2000	0.2000	0.1733	0.1800	0.1700	0.1700
	Q1	0.4000	0.4000	0.2000	0.2000	0.1600	0.1467	0.1550	0.1450
	MINIMUM	0.4000	0.4000	0.2000	0.2000	0.1333	0.1333	0.1200	0.1000

PROPORTION OF OBS ALLOCATED TO STRATUM 1 UNDER NEYMAN: 0.1644

TABLE LXIII

SUMMARY STATISTICS FOR THE PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM1 FOR GAMMA: ALPHA=1 STRATA=2 PROP1=.6

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	MAXIMUM	0.9600	0.9600	0.9800	0.9800	0.9867	0.9867	0.9900	0.9900
	Q3	0.3400	0.6000	0.3200	0.6100	0.3133	0.5867	0.3050	0.6100
	MEDIAN	0.2800	0.3500	0.2900	0.3800	0.2867	0.3600	0.2900	0.3325
	Q1	0.2400	0.1600	0.2600	0.1800	0.2667	0.1600	0.2750	0.1600
	MINIMUM	0.0400	0.0400	0.0200	0.0200	0.0133	0.0133	0.0100	0.0100
5	MAXIMUM	0.5600	0.8200	0.4300	0.8000	0.4000	0.8733	0.3800	0.8300
	Q3	0.3400	0.4200	0.3200	0.4300	0.3067	0.4400	0.3050	0.4175
	MEDIAN	0.3000	0.3200	0.2900	0.3200	0.2867	0.3333	0.2850	0.3200
	Q1	0.2600	0.2200	0.2700	0.2300	0.2667	0.2467	0.2700	0.2350
	MINIMUM	0.1000	0.1000	0.1800	0.0600	0.1800	0.0600	0.2300	0.0450
10	MAXIMUM	0.4800	0.7000	0.3800	0.6700	0.3867	0.6267	0.3850	0.6300
	Q3	0.3400	0.3800	0.3200	0.3700	0.3133	0.3867	0.3050	0.3750
	MEDIAN	0.3000	0.3000	0.2900	0.3100	0.2933	0.3100	0.2875	0.3050
	Q1	0.2600	0.2600	0.2700	0.2400	0.2667	0.2500	0.2700	0.2450
	MINIMUM	0.2000	0.2000	0.1900	0.1100	0.1733	0.0867	0.2150	0.1050
15	MAXIMUM	0.4800	0.5200	0.4100	0.5700	0.3933	0.5733	0.3650	0.5700
	Q3	0.3400	0.3600	0.3200	0.3500	0.3067	0.3600	0.3050	0.3500
	MEDIAN	0.3000	0.3000	0.2900	0.3000	0.2933	0.3067	0.2850	0.3000
	Q1	0.3000	0.3000	0.2700	0.2400	0.2733	0.2533	0.2700	0.2450
	MINIMUM	0.3000	0.3000	0.1700	0.1500	0.1733	0.1600	0.2250	0.1150
20	MAXIMUM	0.5000	0.5400	0.4200	0.5000	0.3800	0.5067	0.3750	0.4850
	Q3	0.4000	0.4000	0.3200	0.3400	0.3067	0.3400	0.3050	0.3375
	MEDIAN	0.4000	0.4000	0.2900	0.3000	0.2867	0.2933	0.2900	0.2950
	Q1	0.4000	0.4000	0.2600	0.2600	0.2667	0.2600	0.2700	0.2550
	MINIMUM	0.4000	0.4000	0.2000	0.2000	0.2200	0.1533	0.2150	0.1550

PROPORTION OF OBS ALLOCATED TO STRATUM 1 UNDER NEYMAN: 0.2806

TABLE LXIV

SUMMARY STATISTICS FOR THE PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM1 FOR GAMMA: ALPHA=1 STRATA=2 PROP1=.7

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	MAXIMUM	0.9600	0.9600	0.9800	0.9800	0.9867	0.9867	0.9900	0.9900
	Q3	0.5400	0.7600	0.5000	0.7550	0.4800	0.7500	0.4800	0.7725
	MEDIAN	0.4600	0.5200	0.4600	0.5000	0.4533	0.5167	0.4550	0.5250
	Q1	0.4200	0.3000	0.4300	0.2800	0.4267	0.3133	0.4325	0.2600
	MINIMUM	0.0400	0.0400	0.0200	0.0200	0.0133	0.0133	0.0100	0.0100
5	MAXIMUM	0.9000	0.9000	0.6000	0.8700	0.6067	0.8867	0.5600	0.8850
	Q3	0.5200	0.6200	0.4800	0.6100	0.4800	0.6000	0.4750	0.6000
	MEDIAN	0.4600	0.5000	0.4600	0.5100	0.4533	0.4900	0.4550	0.4700
	Q1	0.4200	0.4000	0.4300	0.3900	0.4333	0.3867	0.4300	0.3750
	MINIMUM	0.1000	0.1000	0.3400	0.0800	0.3467	0.1267	0.3700	0.1000
10	MAXIMUM	0.7800	0.8000	0.6400	0.7800	0.5933	0.7800	0.5550	0.7700
	Q3	0.5200	0.5400	0.4900	0.5500	0.4800	0.5467	0.4750	0.5600
	MEDIAN	0.4600	0.4800	0.4600	0.4700	0.4533	0.4800	0.4525	0.4850
	Q1	0.4200	0.4000	0.4250	0.4000	0.4267	0.4133	0.4300	0.4050
	MINIMUM	0.3000	0.2000	0.3200	0.2100	0.3600	0.1933	0.3500	0.2150
15	MAXIMUM	0.7000	0.7000	0.6700	0.7200	0.5867	0.7267	0.5750	0.7300
	Q3	0.5200	0.5200	0.4900	0.5300	0.4733	0.5267	0.4725	0.5250
	MEDIAN	0.4600	0.4600	0.4500	0.4700	0.4533	0.4700	0.4500	0.4600
	Q1	0.4200	0.4000	0.4200	0.4100	0.4267	0.4033	0.4300	0.4050
	MINIMUM	0.3000	0.3000	0.3400	0.2100	0.3533	0.2200	0.3650	0.2400
20	MAXIMUM	0.6000	0.6000	0.5800	0.7100	0.5733	0.7067	0.5550	0.6850
	Q3	0.5200	0.5200	0.4900	0.5100	0.4800	0.5133	0.4750	0.5100
	MEDIAN	0.4700	0.4600	0.4600	0.4600	0.4533	0.4600	0.4500	0.4600
	Q1	0.4200	0.4200	0.4300	0.4100	0.4300	0.4133	0.4300	0.4050
	MINIMUM	0.4000	0.4000	0.3300	0.2600	0.3533	0.2600	0.3700	0.2650

PROPORTION OF OBS ALLOCATED TO STRATUM 1 UNDER NEYMAN: 0.4383

TABLE LXV

SUMMARY STATISTICS FOR THE PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM1 FOR GAMMA: ALPHA=1 STRATA=2 PROP1=.8

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	MAXIMUM	0.9600	0.9600	0.9800	0.9800	0.9867	0.9867	0.9900	0.9900
	Q3	0.7600	0.8800	0.7100	0.8700	0.6867	0.8467	0.6750	0.8500
	MEDIAN	0.6800	0.7200	0.6600	0.6800	0.6533	0.6867	0.6550	0.7075
	Q1	0.6200	0.4400	0.6300	0.4400	0.6267	0.4467	0.6300	0.4825
	MINIMUM	0.0400	0.0400	0.0200	0.0200	0.0133	0.0133	0.0100	0.0100
5	MAXIMUM	0.9000	0.9000	0.9500	0.9500	0.9067	0.9400	0.7650	0.9650
	Q3	0.7400	0.7800	0.6900	0.7800	0.6800	0.7533	0.6750	0.7800
	MEDIAN	0.6800	0.7000	0.6600	0.7000	0.6533	0.6800	0.6500	0.6950
	Q1	0.6200	0.5800	0.6200	0.6100	0.6267	0.5700	0.6300	0.5900
	MINIMUM	0.4400	0.2200	0.5400	0.2100	0.5600	0.2533	0.5550	0.0850
10	MAXIMUM	0.8000	0.8000	0.8600	0.8600	0.7867	0.8933	0.7800	0.9150
	Q3	0.7200	0.7200	0.6900	0.7350	0.6867	0.7333	0.6750	0.7350
	MEDIAN	0.6600	0.6600	0.6600	0.6600	0.6533	0.6733	0.6500	0.6700
	Q1	0.6200	0.5800	0.6300	0.5900	0.6267	0.6067	0.6250	0.6050
	MINIMUM	0.4800	0.3200	0.5200	0.3300	0.5467	0.3267	0.5450	0.3900
15	MAXIMUM	0.7000	0.7000	0.8500	0.8500	0.8133	0.8467	0.7850	0.8450
	Q3	0.7000	0.7000	0.6900	0.7200	0.6867	0.7167	0.6750	0.7050
	MEDIAN	0.6600	0.6600	0.6600	0.6600	0.6533	0.6600	0.6500	0.6600
	Q1	0.6200	0.6000	0.6200	0.6050	0.6267	0.5933	0.6300	0.6050
	MINIMUM	0.4800	0.4000	0.5400	0.4300	0.5133	0.4133	0.5550	0.4550
20	MAXIMUM	0.6000	0.6000	0.8000	0.8000	0.7933	0.8267	0.8050	0.8350
	Q3	0.6000	0.6000	0.6900	0.7000	0.6867	0.7033	0.6750	0.6950
	MEDIAN	0.6000	0.6000	0.6600	0.6500	0.6533	0.6600	0.6500	0.6550
	Q1	0.6000	0.6000	0.6300	0.6100	0.6267	0.6133	0.6250	0.6100
	MINIMUM	0.4800	0.4800	0.5500	0.4600	0.5533	0.4800	0.5500	0.4750

PROPORTION OF OBS ALLOCATED TO STRATUM 1 UNDER NEYMAN: 0.6359

TABLE LXVI

SUMMARY STATISTICS FOR THE PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM1 FOR GAMMA: ALPHA=2 STRATA=2 PROP1=.5

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	MAXIMUM	0.9600	0.9600	0.9800	0.9800	0.9867	0.9867	0.9900	0.9900
	Q3	0.3000	0.5200	0.2800	0.4700	0.2733	0.5367	0.2750	0.5275
	MEDIAN	0.2600	0.3000	0.2600	0.2800	0.2600	0.2733	0.2550	0.3050
	Q1	0.2000	0.1400	0.2400	0.1350	0.2400	0.1333	0.2400	0.1500
	MINIMUM	0.0400	0.0400	0.0200	0.0200	0.0133	0.0133	0.0100	0.0100
5	MAXIMUM	0.4600	0.7400	0.3600	0.7500	0.3467	0.7533	0.3200	0.7500
	Q3	0.3000	0.3800	0.2900	0.3750	0.2800	0.3733	0.2700	0.3750
	MEDIAN	0.2600	0.3000	0.2600	0.2800	0.2600	0.2867	0.2550	0.2800
	Q1	0.2400	0.2200	0.2400	0.2000	0.2400	0.2067	0.2400	0.2100
	MINIMUM	0.1000	0.1000	0.0500	0.0500	0.1733	0.0467	0.2000	0.0450
10	MAXIMUM	0.4200	0.6000	0.3900	0.6800	0.3467	0.5400	0.3200	0.5850
	Q3	0.3000	0.3400	0.2800	0.3300	0.2800	0.3267	0.2750	0.3250
	MEDIAN	0.2600	0.2800	0.2600	0.2700	0.2600	0.2700	0.2550	0.2650
	Q1	0.2400	0.2200	0.2400	0.2200	0.2400	0.2200	0.2400	0.2200
	MINIMUM	0.2000	0.2000	0.1600	0.1100	0.1867	0.0867	0.1950	0.0900
15	MAXIMUM	0.4000	0.5000	0.3900	0.4700	0.3467	0.4867	0.3250	0.5200
	Q3	0.3000	0.3200	0.2800	0.3100	0.2733	0.3133	0.2750	0.3100
	MEDIAN	0.3000	0.3000	0.2600	0.2600	0.2600	0.2667	0.2550	0.2700
	Q1	0.3000	0.3000	0.2400	0.2200	0.2400	0.2267	0.2400	0.2250
	MINIMUM	0.3000	0.3000	0.1600	0.1500	0.1800	0.1267	0.1950	0.1450
20	MAXIMUM	0.4200	0.4800	0.3600	0.4400	0.3467	0.4467	0.3400	0.5650
	Q3	0.4000	0.4000	0.2800	0.3000	0.2800	0.3067	0.2750	0.3000
	MEDIAN	0.4000	0.4000	0.2600	0.2650	0.2600	0.2667	0.2600	0.2600
	Q1	0.4000	0.4000	0.2400	0.2300	0.2400	0.2267	0.2450	0.2250
	MINIMUM	0.4000	0.4000	0.2000	0.2000	0.1800	0.1467	0.1850	0.1200

PROPORTION OF OBS ALLOCATED TO STRATUM 1 UNDER NEYMAN: 0.2512

TABLE LXVII

SUMMARY STATISTICS FOR THE PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM1 FOR GAMMA: ALPHA=2 STRATA=2 PROP1=.6

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	MAXIMUM	0.9600	0.9600	0.9800	0.9800	0.9867	0.9867	0.9900	0.9900
	Q3	0.4600	0.6600	0.4300	0.6900	0.4200	0.7000	0.4100	0.6575
	MEDIAN	0.4000	0.4600	0.3900	0.4400	0.3933	0.4933	0.3950	0.4150
	Q1	0.3600	0.2400	0.3700	0.2300	0.3733	0.2867	0.3750	0.2125
	MINIMUM	0.0400	0.0400	0.0200	0.0200	0.0133	0.0133	0.0100	0.0100
5	MAXIMUM	0.7600	0.9000	0.5400	0.8900	0.5000	0.8800	0.5050	0.8550
	Q3	0.4400	0.5200	0.4200	0.5450	0.4133	0.5400	0.4125	0.5300
	MEDIAN	0.4000	0.4200	0.3900	0.4300	0.3933	0.4400	0.3900	0.4400
	Q1	0.3600	0.3200	0.3600	0.3300	0.3667	0.3400	0.3750	0.3300
	MINIMUM	0.1000	0.1000	0.2900	0.1300	0.3067	0.1067	0.3200	0.1050
10	MAXIMUM	0.6200	0.7000	0.5700	0.7700	0.5200	0.6867	0.5100	0.7650
	Q3	0.4400	0.4800	0.4200	0.4900	0.4133	0.4667	0.4100	0.4800
	MEDIAN	0.4000	0.4200	0.3900	0.4200	0.3933	0.4067	0.3900	0.4100
	Q1	0.3600	0.3400	0.3700	0.3500	0.3733	0.3467	0.3700	0.3450
	MINIMUM	0.2000	0.2000	0.2800	0.1500	0.3000	0.1733	0.3200	0.1900
15	MAXIMUM	0.6000	0.6200	0.5600	0.6400	0.4933	0.7133	0.4850	0.6350
	Q3	0.4400	0.4400	0.4200	0.4500	0.4200	0.4667	0.4100	0.4575
	MEDIAN	0.4000	0.4000	0.3900	0.4000	0.3933	0.4067	0.3900	0.4050
	Q1	0.3600	0.3400	0.3700	0.3500	0.3733	0.3533	0.3700	0.3500
	MINIMUM	0.3000	0.3000	0.2600	0.2200	0.3000	0.2200	0.3100	0.2050
20	MAXIMUM	0.6000	0.6000	0.5400	0.6400	0.5133	0.6000	0.5000	0.6400
	Q3	0.4400	0.4600	0.4200	0.4400	0.4133	0.4400	0.4100	0.4400
	MEDIAN	0.4000	0.4000	0.3900	0.4000	0.3867	0.3967	0.3900	0.4000
	Q1	0.4000	0.4000	0.3700	0.3500	0.3667	0.3600	0.3750	0.3525
	MINIMUM	0.4000	0.4000	0.3000	0.2400	0.3133	0.2533	0.3200	0.2450

PROPORTION OF OBS ALLOCATED TO STRATUM 1 UNDER NEYMAN: 0.3818

TABLE LXVIII

SUMMARY STATISTICS FOR THE PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM1 FOR GAMMA: ALPHA=2 STRATA=2 PROP1=.7

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	MAXIMUM	0.9600	0.9600	0.9800	0.9800	0.9867	0.9867	0.9900	0.9900
	Q3	0.6600	0.8000	0.5900	0.8100	0.5867	0.8133	0.5750	0.7900
	MEDIAN	0.5800	0.6200	0.5600	0.5900	0.5533	0.5800	0.5500	0.6000
	Q1	0.5200	0.3800	0.5300	0.3400	0.5333	0.3633	0.5300	0.3675
	MINIMUM	0.0400	0.0400	0.0200	0.0200	0.0133	0.0133	0.1000	0.0150
5	MAXIMUM	0.9000	0.9000	0.8700	0.9400	0.6733	0.9400	0.6500	0.9250
	Q3	0.6200	0.7000	0.5900	0.6800	0.5800	0.6900	0.5750	0.6850
	MEDIAN	0.5600	0.5800	0.5600	0.5900	0.5533	0.5933	0.5550	0.5800
	Q1	0.5200	0.4800	0.5300	0.4900	0.5300	0.4933	0.5300	0.4750
	MINIMUM	0.4200	0.1800	0.4600	0.2300	0.4733	0.1200	0.4800	0.1750
10	MAXIMUM	0.7800	0.8000	0.7300	0.8600	0.6867	0.8267	0.6850	0.8050
	Q3	0.6200	0.6400	0.5900	0.6400	0.5800	0.6333	0.5750	0.6300
	MEDIAN	0.5600	0.5600	0.5600	0.5700	0.5533	0.5667	0.5550	0.5550
	Q1	0.5200	0.5000	0.5200	0.5000	0.5267	0.5000	0.5300	0.4850
	MINIMUM	0.4200	0.3000	0.4500	0.2900	0.4533	0.3133	0.4650	0.3100
15	MAXIMUM	0.7000	0.7000	0.8100	0.8000	0.6733	0.7800	0.6850	0.7850
	Q3	0.6200	0.6200	0.5900	0.6200	0.5800	0.6200	0.5750	0.6200
	MEDIAN	0.5600	0.5600	0.5600	0.5700	0.5533	0.5600	0.5500	0.5650
	Q1	0.5200	0.5200	0.5300	0.5100	0.5267	0.5067	0.5300	0.5050
	MINIMUM	0.3400	0.3800	0.4200	0.3500	0.4600	0.3667	0.4700	0.3400
20	MAXIMUM	0.6000	0.6000	0.7400	0.7400	0.6800	0.7533	0.6500	0.7650
	Q3	0.6000	0.6000	0.5900	0.6100	0.5800	0.6100	0.5750	0.6000
	MEDIAN	0.5600	0.5600	0.5600	0.5700	0.5533	0.5600	0.5550	0.5600
	Q1	0.5200	0.5200	0.5300	0.5100	0.5333	0.5100	0.5350	0.5150
	MINIMUM	0.4000	0.4000	0.4300	0.3700	0.4400	0.4000	0.4650	0.3650

PROPORTION OF OBS ALLOCATED TO STRATUM 1 UNDER NEYMAN: 0.5415

TABLE LXIX

SUMMARY STATISTICS FOR THE PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM1 FOR GAMMA: ALPHA=2 STRATA=2 PROP1=.8

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	MAXIMUM	0.9600	0.9600	0.9800	0.9800	0.9867	0.9867	0.9900	0.9900
	Q3	0.8600	0.9000	0.7900	0.9000	0.7667	0.9000	0.7500	0.9000
	MEDIAN	0.7600	0.7800	0.7400	0.7700	0.7333	0.7500	0.7300	0.7800
	Q1	0.7000	0.5200	0.7000	0.5300	0.7000	0.5467	0.7050	0.5275
	MINIMUM	0.0400	0.0400	0.0200	0.0200	0.0133	0.0133	0.0100	0.0100
5	MAXIMUM	0.9000	0.9000	0.9500	0.9500	0.9267	0.9667	0.8750	0.9500
	Q3	0.8000	0.8200	0.7800	0.8200	0.7600	0.8267	0.7550	0.8350
	MEDIAN	0.7400	0.7600	0.7400	0.7500	0.7267	0.7600	0.7300	0.7600
	Q1	0.7000	0.6600	0.7000	0.6600	0.7000	0.6733	0.7100	0.6700
	MINIMUM	0.5800	0.2200	0.5900	0.3300	0.6400	0.3000	0.6500	0.3000
10	MAXIMUM	0.8000	0.8000	0.9000	0.9000	0.9133	0.9267	0.9250	0.9300
	Q3	0.7800	0.7800	0.7700	0.7900	0.7600	0.7867	0.7500	0.7950
	MEDIAN	0.7400	0.7400	0.7300	0.7300	0.7267	0.7333	0.7250	0.7350
	Q1	0.7000	0.6800	0.7000	0.6800	0.7067	0.6800	0.7100	0.6850
	MINIMUM	0.5600	0.4800	0.6100	0.4500	0.6200	0.4600	0.6350	0.4050
15	MAXIMUM	0.7000	0.7000	0.8500	0.8500	0.8867	0.8733	0.8550	0.8900
	Q3	0.7000	0.7000	0.7700	0.7800	0.7600	0.7800	0.7500	0.7750
	MEDIAN	0.7000	0.7000	0.7300	0.7300	0.7333	0.7333	0.7250	0.7350
	Q1	0.6800	0.6900	0.7000	0.6900	0.7067	0.6933	0.7075	0.6900
	MINIMUM	0.5600	0.4200	0.6200	0.5400	0.6333	0.5400	0.6550	0.5200
20	MAXIMUM	0.6000	0.6000	0.8000	0.8000	0.8667	0.8667	0.8800	0.8900
	Q3	0.6000	0.6000	0.7700	0.7700	0.7600	0.7667	0.7500	0.7650
	MEDIAN	0.6000	0.6000	0.7300	0.7400	0.7333	0.7333	0.7250	0.7300
	Q1	0.6000	0.6000	0.7000	0.6900	0.7067	0.7000	0.7050	0.6900
	MINIMUM	0.5200	0.5400	0.6000	0.4800	0.6467	0.4800	0.6500	0.5800

PROPORTION OF OBS ALLOCATED TO STRATUM 1 UNDER NEYMAN: 0.7148

TABLE LXX

SUMMARY STATISTICS FOR THE PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM1 FOR GAMMA: ALPHA=5 STRATA=2 PROP1=.5

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	MAXIMUM	0.9600	0.9600	0.9800	0.9800	0.9867	0.9733	0.9900	0.9850
	Q3	0.3800	0.6400	0.3700	0.5800	0.3667	0.5733	0.3600	0.5625
	MEDIAN	0.3400	0.3800	0.3400	0.3600	0.3467	0.3800	0.3400	0.3650
	Q1	0.3000	0.2000	0.3100	0.1700	0.3200	0.1867	0.3250	0.1850
	MINIMUM	0.0400	0.0400	0.0200	0.0200	0.0133	0.0133	0.0100	0.0100
5	MAXIMUM	0.6200	0.8800	0.4700	0.8200	0.4267	0.8333	0.4400	0.8250
	Q3	0.4000	0.4800	0.3700	0.4800	0.3667	0.4600	0.3650	0.4400
	MEDIAN	0.3400	0.3600	0.3500	0.3600	0.3467	0.3533	0.3450	0.3500
	Q1	0.3100	0.2800	0.3200	0.2700	0.3267	0.2667	0.3300	0.2750
	MINIMUM	0.1000	0.1000	0.2200	0.0600	0.2667	0.0867	0.2750	0.0850
10	MAXIMUM	0.5200	0.7000	0.4800	0.6300	0.4467	0.6000	0.4200	0.6650
	Q3	0.3800	0.4200	0.3700	0.4100	0.3667	0.4200	0.3600	0.4175
	MEDIAN	0.3600	0.3600	0.3400	0.3500	0.3467	0.3600	0.3450	0.3500
	Q1	0.3200	0.3000	0.3200	0.2900	0.3267	0.2933	0.3250	0.2900
	MINIMUM	0.2000	0.2000	0.2100	0.1400	0.2533	0.1333	0.2600	0.1300
15	MAXIMUM	0.5600	0.6000	0.4800	0.5900	0.4600	0.5867	0.4500	0.5900
	Q3	0.4000	0.4000	0.3700	0.4000	0.3667	0.3967	0.3600	0.3950
	MEDIAN	0.3600	0.3600	0.3400	0.3500	0.3467	0.3467	0.3450	0.3600
	Q1	0.3200	0.3000	0.3200	0.3000	0.3267	0.3067	0.3250	0.3100
	MINIMUM	0.3000	0.3000	0.2300	0.1800	0.2467	0.1867	0.2650	0.2000
20	MAXIMUM	0.5600	0.5400	0.4800	0.5400	0.4400	0.5333	0.4450	0.5250
	Q3	0.4000	0.4000	0.3700	0.3950	0.3667	0.3933	0.3600	0.3900
	MEDIAN	0.4000	0.4000	0.3500	0.3500	0.3467	0.3467	0.3450	0.3450
	Q1	0.4000	0.4000	0.3200	0.3100	0.3200	0.3067	0.3250	0.3050
	MINIMUM	0.4000	0.4000	0.2400	0.2000	0.2600	0.2000	0.2650	0.2100

PROPORTION OF OBS ALLOCATED TO STRATUM 1 UNDER NEYMAN: 0.3385

TABLE LXXI

SUMMARY STATISTICS FOR THE PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM1 FOR GAMMA: ALPHA=5 STRATA=2 PROP1=.6

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	MAXIMUM	0.9600	0.9600	0.9800	0.9800	0.9867	0.9867	0.9900	0.9900
	Q3	0.5600	0.7600	0.5250	0.7300	0.5067	0.7067	0.5100	0.7550
	MEDIAN	0.4900	0.5600	0.4900	0.5300	0.4800	0.4867	0.4850	0.5400
	Q1	0.4400	0.3400	0.4600	0.3200	0.4600	0.2767	0.4650	0.3100
	MINIMUM	0.0400	0.0400	0.0200	0.0200	0.0133	0.0133	0.0100	0.0100
5	MAXIMUM	0.8800	0.8800	0.6400	0.8900	0.6400	0.8667	0.5750	0.8900
	Q3	0.5400	0.6200	0.5200	0.6100	0.5067	0.6000	0.5050	0.6100
	MEDIAN	0.4800	0.5000	0.4800	0.4950	0.4800	0.4933	0.4850	0.5200
	Q1	0.4400	0.4000	0.4600	0.4100	0.4567	0.4000	0.4650	0.4150
	MINIMUM	0.2000	0.1400	0.3600	0.1100	0.3600	0.1467	0.4000	0.1450
10	MAXIMUM	0.8000	0.8000	0.6900	0.8300	0.6067	0.7867	0.5750	0.7500
	Q3	0.5400	0.5600	0.5100	0.5550	0.5067	0.5667	0.5075	0.5625
	MEDIAN	0.4800	0.5000	0.4900	0.4900	0.4867	0.5067	0.4850	0.5000
	Q1	0.4400	0.4200	0.4600	0.4200	0.4667	0.4433	0.4625	0.4225
	MINIMUM	0.2800	0.2200	0.3800	0.2000	0.3733	0.2667	0.4050	0.2250
15	MAXIMUM	0.7000	0.7000	0.6300	0.7200	0.6133	0.7133	0.6000	0.7900
	Q3	0.5400	0.5400	0.5100	0.5400	0.5067	0.5533	0.5050	0.5400
	MEDIAN	0.5000	0.5000	0.4900	0.4900	0.4867	0.4867	0.4850	0.4850
	Q1	0.4400	0.4400	0.4600	0.4350	0.4600	0.4400	0.4650	0.4350
	MINIMUM	0.3000	0.3000	0.3400	0.2800	0.3733	0.2667	0.3950	0.2850
20	MAXIMUM	0.6000	0.6000	0.6300	0.7000	0.5867	0.7467	0.6050	0.7400
	Q3	0.5400	0.5400	0.5100	0.5400	0.5067	0.5333	0.5050	0.5350
	MEDIAN	0.5000	0.5000	0.4900	0.4900	0.4867	0.4867	0.4850	0.4850
	Q1	0.4600	0.4400	0.4600	0.4500	0.4667	0.4467	0.4700	0.4450
	MINIMUM	0.4000	0.4000	0.3800	0.2900	0.3800	0.3200	0.4050	0.3400

PROPORTION OF OBS ALLOCATED TO STRATUM 1 UNDER NEYMAN: 0.4772

TABLE LXXII

SUMMARY STATISTICS FOR THE PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM1 FOR GAMMA: ALPHA=5 STRATA=2 PROP1=.7

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	MAXIMUM	0.9600	0.9600	0.9800	0.9800	0.9867	0.9867	0.9900	0.9900
	Q3	0.7300	0.8600	0.6800	0.8400	0.6667	0.8367	0.6650	0.8550
	MEDIAN	0.6600	0.7000	0.6400	0.6700	0.6400	0.6733	0.6400	0.6900
	Q1	0.6000	0.4400	0.6100	0.4200	0.6133	0.4067	0.6150	0.4800
	MINIMUM	0.0400	0.0400	0.0200	0.0200	0.0133	0.0133	0.0100	0.0100
5	MAXIMUM	0.9000	0.9000	0.9500	0.9500	0.7733	0.9533	0.7300	0.9150
	Q3	0.7000	0.7400	0.6700	0.7400	0.6667	0.7467	0.6600	0.7400
	MEDIAN	0.6400	0.6600	0.6400	0.6600	0.6400	0.6567	0.6400	0.6600
	Q1	0.6000	0.5400	0.6100	0.5600	0.6133	0.5533	0.6200	0.5650
	MINIMUM	0.4600	0.1800	0.5200	0.2400	0.5400	0.1933	0.5700	0.2500
10	MAXIMUM	0.8000	0.8000	0.9000	0.9000	0.7467	0.8867	0.7600	0.8750
	Q3	0.7000	0.7200	0.6800	0.7100	0.6600	0.7067	0.6600	0.7100
	MEDIAN	0.6400	0.6400	0.6400	0.6400	0.6400	0.6467	0.6400	0.6450
	Q1	0.6000	0.5800	0.6100	0.5700	0.6133	0.5867	0.6200	0.5825
	MINIMUM	0.4600	0.4000	0.5200	0.3900	0.5333	0.3933	0.5300	0.4100
15	MAXIMUM	0.7000	0.7000	0.8300	0.8500	0.7800	0.8333	0.7150	0.8500
	Q3	0.6800	0.7000	0.6700	0.6900	0.6633	0.7000	0.6550	0.6950
	MEDIAN	0.6400	0.6400	0.6400	0.6500	0.6400	0.6467	0.6350	0.6450
	Q1	0.6000	0.5900	0.6100	0.5900	0.6133	0.5933	0.6150	0.5950
	MINIMUM	0.4200	0.3800	0.5400	0.4000	0.5133	0.3867	0.5400	0.4050
20	MAXIMUM	0.6000	0.6000	0.7900	0.8000	0.7667	0.8000	0.7650	0.8050
	Q3	0.6000	0.6000	0.6800	0.6800	0.6600	0.6800	0.6550	0.6800
	MEDIAN	0.6000	0.6000	0.6400	0.6400	0.6400	0.6400	0.6350	0.6400
	Q1	0.6000	0.6000	0.6100	0.6000	0.6133	0.5933	0.6150	0.5950
	MINIMUM	0.4600	0.4600	0.5300	0.4600	0.5533	0.4267	0.5500	0.4100

PROPORTION OF OBS ALLOCATED TO STRATUM 1 UNDER NEYMAN: 0.6281

TABLE LXXIII

SUMMARY STATISTICS FOR THE PROPORTION OF OBSERVATIONS ALLOCATED TO STRATUM1 FOR GAMMA: ALPHA=5 STRATA=2 PROP1=.8

INITIAL OBS PER STRATUM	STATISTIC	TOTAL OBSERVATIONS ALLOCATED							
		50		100		150		200	
		SEQ	SUK	SEQ	SUK	SEQ	SUK	SEQ	SUK
2	MAXIMUM	0.9600	0.9600	0.9800	0.9800	0.9867	0.9867	0.9900	0.9900
	Q3	0.9000	0.9200	0.8500	0.9200	0.8267	0.9333	0.8125	0.9200
	MEDIAN	0.8200	0.8200	0.8000	0.8400	0.7933	0.8333	0.7850	0.8100
	Q1	0.7800	0.6400	0.7700	0.6400	0.7667	0.6533	0.7700	0.6350
	MINIMUM	0.0400	0.0400	0.0200	0.0200	0.0133	0.0133	0.1150	0.0150
5	MAXIMUM	0.9000	0.9000	0.9500	0.9500	0.9333	0.9600	0.9700	0.9700
	Q3	0.8600	0.8600	0.8300	0.8650	0.8133	0.8600	0.8100	0.8650
	MEDIAN	0.8000	0.8000	0.8000	0.8100	0.7867	0.8133	0.7900	0.8000
	Q1	0.7600	0.7200	0.7700	0.7400	0.7667	0.7467	0.7700	0.7250
	MINIMUM	0.6400	0.4000	0.6900	0.3900	0.7067	0.4333	0.7200	0.2550
10	MAXIMUM	0.8000	0.8000	0.9000	0.9000	0.9333	0.9267	0.9250	0.9400
	Q3	0.8000	0.8000	0.8200	0.8300	0.8133	0.8333	0.8100	0.8350
	MEDIAN	0.7800	0.8000	0.7900	0.7900	0.7900	0.7933	0.7850	0.7950
	Q1	0.7600	0.7400	0.7600	0.7500	0.7667	0.7467	0.7700	0.7500
	MINIMUM	0.6400	0.5400	0.6900	0.5700	0.7000	0.4733	0.7200	0.5050
15	MAXIMUM	0.7000	0.7000	0.8500	0.8500	0.9000	0.9000	0.8850	0.9050
	Q3	0.7000	0.7000	0.8200	0.8200	0.8133	0.8267	0.8100	0.8250
	MEDIAN	0.7000	0.7000	0.7900	0.7900	0.7867	0.7933	0.7850	0.7850
	Q1	0.7000	0.7000	0.7600	0.7500	0.7667	0.7533	0.7650	0.7500
	MINIMUM	0.6400	0.5400	0.7000	0.5700	0.7133	0.6267	0.7150	0.5750
20	MAXIMUM	0.6000	0.6000	0.8000	0.8000	0.8667	0.8667	0.8950	0.9000
	Q3	0.6000	0.6000	0.8000	0.8000	0.8133	0.8200	0.8100	0.8200
	MEDIAN	0.6000	0.6000	0.7900	0.7900	0.7933	0.7933	0.7900	0.7950
	Q1	0.6000	0.6000	0.7600	0.7600	0.7667	0.7533	0.7700	0.7600
	MINIMUM	0.6000	0.6000	0.7000	0.6400	0.7133	0.6400	0.7250	0.6600

PROPORTION OF OBS ALLOCATED TO STRATUM 1 UNDER NEYMAN: 0.7775

VITA

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Candidate for the degree of

Doctor of Philosophy

Thesis: A NEW SEQUENTIAL ALLOCATION METHOD

Major Field: Statistics

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