

**THE ECONOMIC DESIGN AND EVALUATION  
OF THREE VARIABLES CONTROL CHARTS**

**By**

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THE ECONOMIC DESIGN AND EVALUATION  
OF THREE VARIABLES CONTROL CHARTS

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## PREFACE

The objective of this research is to provide optimal economically-based control charts for monitoring a process in a realistic environment. Three variables control charts are considered. They are the (1) X-bar control chart with AT&T runs rules, (2) Exponentially Weighted Moving Average chart, and (3) Zone control chart. The economic models of these three variables control charts are developed. The cost structure of these models follows Duncan's approach to the economic design of the X-bar control chart. Interactive computer programs are developed to help theoreticians and practitioners in design and evaluation of these three charts. Economic comparisons, analyses, and sensitivity analyses are then performed. Some useful guidelines in the economic selection and use of the control charts are provided.

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## NOMENCLATURE

ARL	= Average Run Length of a control chart
ARL1	= ARL when the process mean is in a SOSC
ARL2	= ARL when the process mean is OOC
b	= fixed cost per subgroup taken
c	= variable cost per unit sampled
CL	= Center line
D	= Expected time required to identify a special cause
e	= delay factor
E(.)	= An expectation
EFAC	= Expected false alarm cost per hour of operation
ELOPC	= Expected length of a production cycle
ELOSS	= Expected loss (cost) per hour of operation
EMC	= Expected control chart maintenance cost
ENFA	= Expected number of false alarm
EPC	= Expected penalty cost
ETAC	= Expected true alarm cost per hour of operation
EWMA	= Exponentially Weighted Moving Average
EWMA <sub>t</sub>	= The value of EWMA at time t
G	= A random variable
h	= Sampling interval
I	= Identity matrix
k	= Width of control limits

LCL	= Lower control limits
L(u)	= ARL of an EWMA chart given that the EWMA starts with u
M	= $V_0 - V_1$ ; expected penalty cost per hour of operation
n	= Subgroup size
OOC	= Out-of-control
P	= One-step transition probability matrix
$\pi_i$	= probabilities associated with each zone of a Zone Control Chart
Q	= The same probability matrix as P except crossing out the last row and the last column of P
RULE	= A decision variable of the economic design of the X-bar control chart with AT&T runs rules. It represents one of the eight combinations of the four AT&T runs rules
S1	= Zone score 1
S2	= Zone score 2
S3	= Zone score 3
S4	= Zone score 4
SOSC	= State of statistical control
T	= Expected cost to search for a false alarm
$T_a$	= The length of time when the process is in a SOSC
$T_r$	= The length of time when the process is OOC
$T_1$	= The length of time from the last subgroup taken while the process is in a SOSC to the time a special cause occurs
$T_2$	= The length of time from the occurrence of a special cause to the first subgroup taken after the occurrence of a special cause
$T_3$	= The length of time from the first subgroup taken after the occurrence of a special cause to the "detecting" subgroup taken

- $T_4$  = The length of time from the "detecting" subgroup taken to the identification of the special cause
- $T(k_1, m_1, a_1, b_1)$  = A runs rule  $i$  which represents that  $k_1$  out of  $m_1$  consecutive points fall in region  $(a_1, b_1)$
- UCL = Upper control limit
- $V_0$  = Average income per hour of operation when the process is in a SOSC
- $V_1$  = Average income per hour of operation when the process is OOC
- $W$  = Expected cost to search for a true alarm
- $W'_1$  = Vector which represents the true status of a control chart
- $X'_1$  = Vector which represents the status of a control chart that may contribute to an OOC condition
- $Y$  = Expected number of subgroups taken when the process is in a SOSC
- $y_t$  = sample average observed at time  $t$
- $Z$  = A random variable
- $\alpha$  = Weight; smoothing constant
- $\sigma$  = Process standard deviation
- $\sigma_{EWMA}$  = The standard deviation of EWMA
- $\theta$  = The occurrence rate per hour of the process failure mechanism
- $\delta$  = Magnitude of the shift in the process mean
- $\beta$  = Proportion of time a process is in control

## CHAPTER I

### THE RESEARCH PROBLEM

#### Purpose

Concepts of control charting are formally introduced in the documents prepared by Dr. Walter Shewhart in 1924 (1931). They differentiate between the common causes (random causes, chance causes) and the special causes (nonrandom causes, assignable causes) affecting a process. If common causes only are at work, the process is stable and statistically predictable. If special causes are also present, the process is unstable and unpredictable; the special causes should be detected and eliminated.

The most commonly used control chart is the Shewhart X-bar control chart with 3-sigma control limits used to monitor the process mean. A subgroup is sampled from a process over time and the sample mean is calculated and plotted on the X-bar control chart. If a plotted point falls outside either one of the control limits, it is assumed that a special cause affects the process. An attempt is made to identify and remove the cause(s).

It is well known that the Shewhart X-bar chart is not sensitive enough to detect a small shift in the process



mean. Therefore, modifications and extensions to Shewhart control charts have been developed to improve their sensitivity to detect a shift in the process mean. Three of the most important modifications and extensions are the X-bar control chart using AT&T runs rules, the Exponentially Weighted Moving Average (EWMA) chart, and the Zone control chart (ZCC). These three variables control charting techniques have been declared to possess better statistical performance than the standard Shewhart X-bar control scheme.

On the other hand, the economic performance is of much interest to practitioners and researchers. The use of any control chart is basically an economic activity. The design of a control chart which is statistically desirable may not necessarily be economically optimal. Therefore, the economic aspects of a process should be explicitly considered when statistical process control procedures are utilized.

The main purpose of this research is to economically design and evaluate the

- a. X-bar control chart using AT&T runs rules,
- b. Exponentially Weighted Moving Average chart, and
- c. Zone control chart,

used for monitoring the shifts in the process mean, assuming that the process variance remains the same throughout production. It is important to note that the basic difference between these three variables control

charts and the standard X-bar control chart is that these three charts utilize, more or less, historical data in making a decision instead of using only the current observation. For a standard X-bar control chart, each subgroup taken is assumed independent of the previous subgroups. A decision is made about the stability of the process based only on the most recent observation. This is not the case in the three variables control charts considered here; rather, historical data are part of their decision making process. An effort is needed to develop the economic models for these charts and to provide insights, from an economic viewpoint, for their selection and use.

**Problems Of The Economic Design  
Of Quality Control Charts When  
Historical Data Are Part Of  
The Decision Making Process**

It was not until 1956 that Duncan introduced the profit maximization concept into control charting techniques. The design of an economically-based control chart considers the following economic consequences: (1) the cost of operating the process under an out-of-control condition; (2) the cost of looking for a special cause when one does not exist (false alarm cost); (3) the cost of looking for a special cause when one exists (true alarm cost); and, (4) the cost of sampling, inspection, and plotting a point. All of these factors are affected by

selection of the control chart design parameters. Duncan's (1956) pioneering work on the economic design of X-bar control charts provides an approach for determining the control chart design parameters of subgroup size ( $n$ ), sampling interval ( $h$ ), and width of control limits ( $k$ ) which maximize the average net income of a process.

An approximation method which determines the optimal values of  $n$ ,  $h$ , and  $k$  for the economic design of the X-bar control chart is developed by Duncan (1956). He shows, by giving 25 examples, that the designs of the control charts deviate considerably from Shewhart's recommendations. Control charts based on economic models can therefore result in substantial cost savings. Considerable work has since focused on the economic designs of process control charts, but none of them are designs for the X-bar control chart using AT&T runs rules, the Exponentially Weighted Moving Average chart, or the Zone control chart.

The reason for this void is that the Type I error and Type II error probabilities associated with these three types of variables control charts are unclear and have never been formally defined. This causes the difficulty in estimating the expected number of false alarms and, hence, the false alarm cost.

A possible solution to this difficulty is to use the average run length (ARL), which is defined as the expected number of subgroups inspected until a process mean shift

signal is given. There are two ARLs which need to be distinguished. The first is the ARL when the process is really in a state of statistical control (SOSC). The second is the ARL when the process actually goes out-of-control (OOC); that is, when the process mean actually shifts away from its target value. Ideally, it is desirable to have an infinite ARL when the process is in a SOSC in order to have no false alarms (signals). It is desirable to have an ARL of one when the process is OOC in order to immediately detect the shift in the process mean. In actual practice, these ideals cannot be achieved.

The voids and problems described above lead to the need for further research on economically designed process control models. The aim of this research is to fill these voids.

#### Research Objectives

The primary objective of this research is:

**Objective:** To provide optimal economically-based control charts, including the (1) X-bar chart using AT&T runs rules, (2) Exponentially Weighted Moving Average chart, and (3) Zone control chart for monitoring a process in a realistic environment.

In order to accomplish this objective, several subobjectives must be met. The subobjectives are:

(1) To develop an analytical model to evaluate and optimize, from an economic viewpoint, the

- a. X-bar control chart using AT&T runs rules,
  - b. Exponentially Weighted Moving Average chart, and,
  - c. Zone control chart.
- (2) To increase knowledge concerning the relationships between the statistical performance and economic performance of a control chart.
  - (3) To economically compare the performance of the three control charts to gain insights into applying these control charting techniques.
  - (4) To develop computer programs to economically design and evaluate these three variables control charts.
  - (5) To conduct sensitivity analyses to systematically study the effects of the costs and operating parameters on both the control chart design parameters and the resulting operating loss, using a design of experiments (DOE) approach.

#### Contribution

The contributions of this research are as follows.

- (1) This research becomes the first of its kind to provide an economic design of (a) the X-bar control chart using AT&T runs rules, (b) the Exponentially Weighted Moving Average chart, and (c) the Zone control chart. Both theoreticians and practitioners can benefit from this research and its results.
- (2) This research provides guidelines on how to construct prediction equations for both the control chart design

parameters and the resulting loss associated with operating the process.

- (3) The prediction equations provided in this research clearly indicate the magnitude and direction of the effect when one or more factors are misspecified.
- (4) The prediction equations provided in this research help the user to (a) determine the initial values of the control chart design parameters, and (b) provide an estimated value of the resulting operating loss.
- (5) The results of the sensitivity analyses provide guidance for the selection of search regions for design parameters which need to be optimized.
- (6) Suggestions are provided regarding the selection of (a) the runs rules used in combination with an X-bar chart, and (b) the  $\alpha$  value for an Exponentially Weighted Moving Average chart. These aid the user in selection of the initial values of the design parameters.
- (7) This research provides the relationships between the statistical performance and economic performance of a control chart. This helps the user in the selection of (a) a better set of design parameters within a control scheme, and/or (b) a control scheme (chart), which possesses better statistical (power of detection) and economic performance.
- (8) This research identifies, from an economic viewpoint, the minimum magnitude of shift in the process mean which is of real concern.

- (9) Computer programs are developed to help the theoreticians and practitioners in the design and evaluation of the economic models of the three variable control charts addressed.
- (10) This research extends Duncan's (1956) economic model of the X-bar chart to the (a) X-bar chart with AT&T runs rules, (b) Exponentially Weighted Moving Average chart, and (c) Zone control chart. Average run length is used to construct the fully economic model so that Duncan's approach is adaptive to these three variables control charts.

## CHAPTER II

### LITERATURE SURVEY

#### Introduction

Dr. Shewhart (1931) developed the quality control chart in 1924. Since then, various techniques have evolved to deal with different process control situations. Most of the existing control charting techniques are based on the assumption of a normal process generating independent and identically distributed (iid) observations. The basic principle is that the variation in measurement data pertaining to a process can be separated into two sources: inherent process variation due to chance (common) causes and variation due to special (assignable) causes. For each technique, criteria are established to determine if the process is in a state of statistical control.

This chapter reviews the literature which relates to the three variables control charts studied in this research. This chapter is divided into two sections:

- (1) Statistical design of variables control charts; and,
- (2) Economic design of X-bar control charts.



Statistical Design of Variables  
Control Charts

Shewhart X-bar Control Chart

And Its Enhancements

Shewhart's (1931) recommendations for the three parameters of the X-bar control chart are (1) subgroup size,  $n$ , equal to 4 or 5, (2) the factor for control limit spread,  $k$ , equal to 3, and (3) the sampling interval,  $h$ , not specified, leaving this as a choice for the practitioner at a site. Assuming normality of the production process, there is a probability of 0.0027 that a plotted point will fall outside either of the control limits. If a plotted point falls outside either of the control limits, it is inferred that one or more special causes exist in the process.

The ARL is used to evaluate the performance of a control chart and is defined as the expected number of subgroups inspected until a shift signal is given. Due to the assumption of a normal process and independence of the subgroups taken, the underlying distribution of run length of the X-bar control chart is the geometric distribution when a stability decision is made based only on the current observation.

It is found that the Shewhart X-bar control chart is not sensitive in detecting small to moderate amounts of shift in the process mean. Thus, many enhancements have

been suggested during the past four decades. Weiler (1953) suggests, in order to detect small changes in the process mean, that control by runs of the sample means above or below certain control limits makes it possible to use small subgroups and yet maintain the advantage of a reduced amount of inspection.

Page (1955) develops the control charting technique using both warning limits and action (control) limits. Samples of fixed size are taken at regular intervals and a statistic of the sample (for example, the sample mean) is plotted on the chart. If a sample point falls outside control limit(s) drawn on the chart, a corrective action is taken. He gives four different runs rules to determine if the process is in a state of statistical control. The ARLs of the control charts, combined with these four rules, are developed and evaluated using discrete Markov chains. As Page points out, the ARL of the runs rules can be evaluated by enumerating the possible combinations of the  $(n-1)$  points on the chart such that action has not been required, and treating the combinations as the states of a discrete Markov chain. This leads the way to the study and evaluation of the ARLs of the Shewhart X-bar control chart with supplementary runs rules (which includes AT&T runs rules) by Champ and Woodall (1987).

Weindling, Littauer, and Oliveira (1970) suggest that the Shewhart control chart for the sample mean can be made more sensitive to small changes by adding a pair of warning

limits, located inside the action limits, and taking action when a run of a specified number of consecutive sample means falls between the warning and action limits. They recognize that when the cost of searching for the cause of a shift is high, a chart having a large value of the ARL (or Mean Action Time, MAT) in control is preferred. When the cost of producing off-spec items is high, a small value of the ARL is desired when some certain amount of shift in the process mean is most likely to occur.

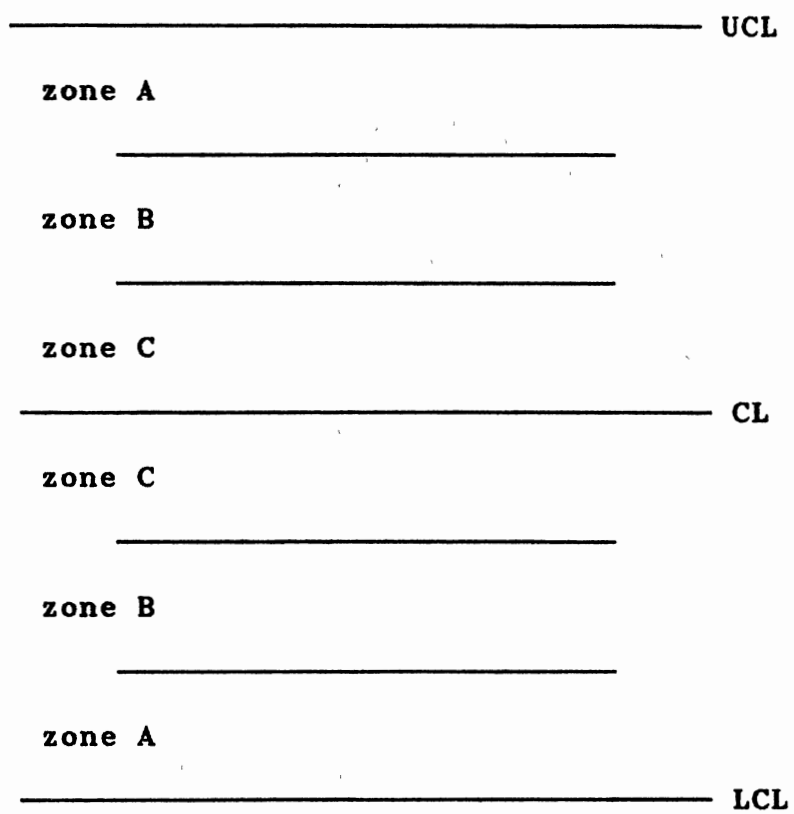
### The X-bar Control Chart

#### With AT&T Runs Rules

Western Electric Company (now AT&T, 1958) presents four runs rules to improve the sensitivity of quality control charts. The X-bar control chart with AT&T runs rules is employed to maintain the production process in a state of statistical control. The statistic of interest is the sample average which is assumed to be normally distributed. The region from the lower control limit (LCL) to the upper control limit (UCL) of a control chart is divided into six equal zones, shown as Figure 2.1.

An out-of-control signal is given when a specified situation is met, which depends on the rules that are used. The AT&T runs rules are summarized below for a one-sided control chart. The rules apply equally to each side.

Rule 1: A single point falls outside of the 3-sigma control limit (beyond zone A).



**Figure 2.1 Zones Of A Control Chart**

Rule 2: Two out of three successive points fall in zone A or beyond; the other point may be anywhere.

Rule 3: Four out of five successive points fall in zone B or beyond; the other point may be anywhere.

Rule 4: Eight successive points fall in zone C or beyond.

Wheeler (1983) provides expressions for up to 10 run-length probabilities for some one-sided Shewhart X-bar charts with supplementary runs rules. Champ and Woodall (1987) provide a recursive method using Markov chains, which can be used with a one-sided or a two-sided chart, to evaluate the ARLs of the X-bar control chart with supplementary runs rules. Their method can be applied to calculate any number of run-length probabilities. Champ and Woodall are the first two researchers to use an exact method to evaluate the ARLs of the X-bar control chart with AT&T runs rules.

#### The EWMA Chart

Roberts (1959) develops a control chart using the Exponentially Weighted Moving Average (EWMA, there called the Geometric Moving Average) technique. It gives the most recent observation the greatest weight with all previous observations weights decreasing in a geometric progression from the most recent back to the first. The basic formulae for calculating the EWMA and the control limits are listed as follows:

## (1) Calculation of the EWMA's

$$EWMA_t = (1 - \alpha) * EWMA_{t-1} + \alpha * y_t$$

where,

$$0 < \alpha \leq 1, \quad t = 1, 2, \dots,$$

$\alpha$ : smoothing constant ,

$y_t$ : sample average observed at time t, and

$EWMA_0$ : the nominal value of the process mean.

## (2) Calculation of the control limits (CLs)

$$CLs = \text{Nominal} \pm k * \sigma_{EWMA}$$

where,

$$\sigma_{EWMA} = \sigma * \{\alpha/[n*(2-\alpha)]\}^{0.5}$$

$\sigma$ : process standard deviation

Roberts evaluates the ARLs of the EWMA control scheme using simulation. A comparison of the properties of control chart tests based on the EWMA's and the ordinary moving averages is performed. He concludes that tests based on the EWMA's compare most favorably with multiple run tests and moving average tests with regard to simplicity and statistical properties. Roberts also realizes that the use of the EWMA control scheme is an economic one due to the complexity of the EWMA chart compared to the standard Shewhart X-bar control chart.

Robinson and Ho (1978) develop numerical procedures utilizing recursive techniques and an Edgeworth expansion to formulate the probability law for the time of the first passage of the EWMA variable across either the upper or lower control limit. Both one- and two-sided ARLs are

calculated and tabulated for various settings of the control limits, smoothing constant, and shifts in the nominal level of the process mean.

Crowder (1987a, 1987b, 1989) presents a general methodology for studying the EWMA procedures assuming an iid normal process. The approach uses a Fredholm integral equation of the second kind for moments of run-length distribution with an EWMA chart. An intensive study of the ARLs of the EWMA charts has been carried out and the ARLs are tabulated for various settings of control chart parameters. A set of procedures is given for the statistical designs of the EWMA charts. Crowder (1989) declares that his design procedures are optimal because, for a given in-control ARL, the parameters chosen by his procedures minimize the out-of-control ARL for a specified shift in the process mean.

Ng and Case (1989) propose methodologies to construct the EWMA control charts used for monitoring the sample means (SM), sample ranges (SR), individual observations (ID), and moving ranges (MR). Four control charts are developed; they are EWMASM, EWMASR, EWMAID, and EWMAMR. Extensive tables of factors for control limits of each chart are given. They find that a systematic and consistent derivation of the EWMA of variables is possible and may be more easily understood.

Lucas and Saccucci (1990) evaluate, using Markov chains, the ARLs of the EWMA chart used to monitor the mean

of a normally distributed process that may experience shifts away from the target value. They give detailed discussions about the zero-state ARLs and the steady-state ARLs. Several enhancements are given, such as the Fast Initial Response (FIR) feature which makes the scheme more sensitive at start-up; a combined Shewhart-EWMA scheme that provides protection against both large and small shifts in the process mean; and, a robust EWMA scheme that provides extra protection against outliers. A set of the statistical design procedures for the EWMA control scheme is presented. Basically, their design procedures are the same as Crowder's. Their results show that the properties of EWMA's are very close to those of the Cumulative Sum (CUSUM) schemes.

#### The Zone Control Chart

Jaehn (1987a, 1987b, 1987c, 1989) proposes the Zone control chart technique as a further development in the area of sensitizing control charts, but with a minimum of mathematical analysis. The Zone control chart, shown in Figure 2.2, looks like a Shewhart control chart with AT&T runs rules.

In this technique, either side (from center line to UCL or LCL) of the control chart is divided into three equal zones. Zone scores of 1, 2, 4, and 8 are employed with critical values being set equal to the score of the outermost zone. A special cause is considered existing in



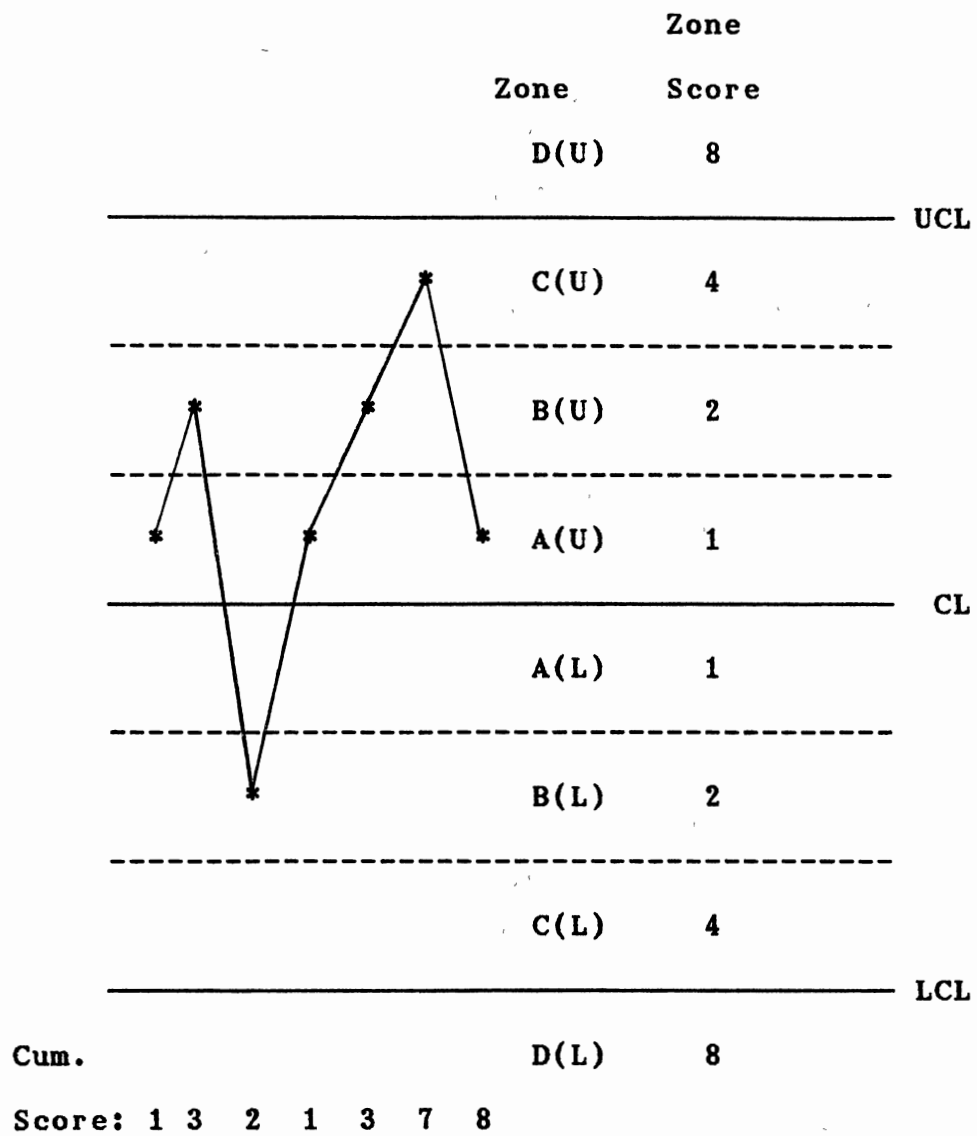


Figure 2.2 Example Of A Zone Control Chart

the process when the cumulative zone score is greater than or equal to the critical value. It is reported that the Zone control chart is simpler and more sensitive than the Shewhart X-bar control chart when the shift in the process mean is small. The ARLs of this particular scheme are obtained by simulation.

Hendrix (1989) introduces the use of the Zone control charts. He uses simulation to obtain the ARLs of the Zone control charts with different sets of zone scores and compares them against the Shewhart X-bar control charts. No mathematical analysis and evaluation of the performance of the Zone control charts are given by either Jaehn or Hendrix.

The problem of the Zone control chart proposed by Jaehn is that it gives a high false alarm rate when the process is in a SOSC. Recall that one of the properties desired when constructing a control chart is the low (nearly zero) false alarm rate when the process is really stable. Therefore, an improvement in the Zone control chart is needed.

In order to solve the problem described above, Fang and Case (1990) mathematically formulate the Zone control chart. An analytical model using Markov chains is given to evaluate the ARLs of the Zone control chart. A suggested improvement to the Zone control chart is given.

Independently, Davis, Homer, and Woodall (1990) also mathematically evaluate the performance of the Zone control

charts using Markov chains. They conclude that the assigned zone scores can greatly affect the performance of the Zone control charts. When zone scores are properly assigned, the Zone control charts outperform, based on the ARLs, the competing Shewhart X-bar control charts with supplementary runs rules.

### Economic Design Of X-bar Control Charts

Duncan (1956) is the first to introduce profit maximization concepts into control charting techniques. His pioneering work leads the way in the study of this area. In his procedures, subgroups of size  $n$  are taken from the production process every  $h$  hours. The sample means of these subgroups are calculated and plotted on the X-bar control chart with control limits symmetrically placed  $\pm k$ -sigma away from the center line. The subgroup size ( $n$ ), sampling interval ( $h$ ), and the spread of the control limits ( $k$ ) are the control chart design parameters which need to be optimized. A loss function in terms of expected loss per hour is constructed to evaluate the optimal design.

The expected loss per hour is defined as the expected loss per production cycle divided by the expected length of a production cycle. The expected loss per production cycle consists of four elements as follows:

- (1) Penalty cost: The cost due to operating the process under an OOC condition.

- (2) False alarm cost: The cost due to looking for a special cause when none exists;
- (3) True alarm cost: The cost due to looking for a special cause when one exists;
- (4) Control chart maintenance cost: The cost due to sampling, inspection, and plotting on the control chart.

The length of a production cycle is defined as the total time from which the process starts in a SOSC, shifts to an OOC condition, the OOC condition is detected, and the special cause is identified.

An approximation method is developed to evaluate the optimal values of the control chart design parameters. Twenty five numerical examples are given and evaluated which essentially represent a one-factor-at-a-time type of sensitivity analysis. In addition to his original work, Duncan (1971) develops the economic model for the X-bar control chart subject to a multiplicity of special causes.

Goel, Jain, and Wu (1968) develop an algorithm for the determination of the economic design of the X-bar control chart based on Duncan's model. The algorithm consists of solving an implicit equation in  $n$  and  $k$ , and an explicit equation in  $h$ . Duncan's (1956) assumptions to simplifying the calculation of the expected length of a production cycle are still applied to Goel, Jain, and Wu's model. Therefore, their model is still an approximation. They declare that their algorithm yields designs with smaller

cost and, in many cases, the differences from Duncan's (1956) results are quite significant.

Chiu and Wetherill (1974) propose a simple semi-economic scheme for the design of the X-bar control chart. In this scheme, control chart design parameters ( $n$ ,  $h$ ,  $k$ ) are determined under the condition that a consumer's risk point on the OC curve must be selected to protect against inferior quality. One may then determine the value of  $k$  and  $n$  from a table by rule of thumb. The value of  $h$  is calculated by a very simple algebraic formula. Chiu and Wetherill declare that their method permits a rapid determination of the control parameters which generally yield an average cost close to the exact minimum. They also demonstrate that, in most cases, despite its simplification of the problem, their method gives better solutions than Duncan's with the advantage that the OC-curve can be partially controlled by the user.

Gordon and Weindling (1975) propose a cost model for the economic design of control charts with warning limits. The production rate is assumed to be constant in their model. The average number of good products being produced during a production cycle and the expected costs generated within a production cycle can then be calculated. Gordon and Weindling build their model based on the average cost per good part produced, instead of using Duncan's (1956) approach. The reason is to avoid the difficulty of neglecting the effects of lost production when the process

is stopped to search for the possible special cause(s).

Chiu and Cheung (1977) investigate the economic design of X-bar control charts with both warning and action limits. Various comparisons are performed among the minimum cost designs of the X-bar control charts (with and without warning limits) and CUSUM charts. They declare that, when each of these three charts has the minimum cost design for the same cost and operating parameters, the X-bar control chart with warning limits and the CUSUM charts are almost identically efficient in most economic respects; and, both are only slightly better than the ordinary X-bar control chart. He thus recommends the X-bar control chart with warning limits for practical application as they are much easier to handle than the CUSUM charts. Chiu and Cheung also find, from an economic viewpoint, that the warning limits should be placed at about 0.85 times the distance between the center line and the upper or lower control limit, instead of the commonly used two thirds of that distance.

Krishnamoorthi (1985) points out that economic control charts are not well accepted by QC professionals in the field due to the complexity of the economic models and the way they are presented in the literature. Therefore, Krishmoorthi (1985) presents a tutorial paper to introduce the concepts and use of the economically-based X-bar control chart, necessary data requirements, and the benefit

of using economically designed control charts. Duncan's (1956) pioneering economic model of the X-bar chart is employed for illustration. Montgomery's (1982) computer program is used to determine the optimal design parameters (subgroup size, sampling interval, and width of the control limits) and the resulting operating loss.

Krishnamoorthi (1985) also presents a simple method to estimate the magnitude of shift in the process mean, utilizing the data obtained for the control chart. The X-bar values, which are obtained from previously maintained X-bar chart with 3-sigma control limits, that are outside the control limits are used. False alarms are omitted. If the process mean shifts above the nominal value, then, the average value of those X-bar's above the control limits is given by

$$\bar{x} + \delta\sigma + (\sigma/\sqrt{n}) * \{[\phi(3-\delta\sqrt{n})] / [1-\Phi(3-\delta\sqrt{n})]\}.$$

If the process mean shifts below the nominal value, then, it is given by

$$\bar{x} - \delta\sigma - (\sigma/\sqrt{n}) * \{[\phi(\delta(\sqrt{n})-3)] / [1-\Phi(\delta(\sqrt{n})-3)]\},$$

where  $\phi(\cdot)$  and  $\Phi(\cdot)$  are the probability density function and distribution function of the standard normal distribution, respectively, and  $\delta$  is the magnitude of shift in the process mean measured in number of process standard deviations.

As stated previously, the economic models of process control charts are complex. Therefore, one direction of the

study of economic designs is to find simpler ways to determine optimal control chart design parameters. Chung (1990) adopts McWilliams' (1989) suggestion to use [(expected time the process is in control / sampling interval) - 0.5] to approximate the quantity of the expected number of subgroups taken while the process is in control. This leads Chung to derive a simplified procedure for solving the optimal design parameters of an economically-based X-bar chart. An explicit equation for solving the sampling interval is then obtained. By solving this equation, close-to-optimal control chart design parameters and lower operating loss are obtained. The assumptions made in Duncan's (1956) paper in order to solve for near-optimal design parameters are avoided. The results are compared to those of Goel, Jain, and Wu's (1968) and it is reported that Chung's results are better than Goel's et al.

In the literature of the economic design of process control charts, there are two different manufacturing process models often cited. Duncan's (1956) original paper assumes that the production process is not stopped while the investigation of a possible special cause is undertaken; some others assume the process is stopped. Panagos, Heikes, and Montgomery (1985) investigate the effects of these two assumptions. They designate their "continuous" model as the one without stopping the process while an investigation of a special cause is in progress;



the "discontinuous" model assumes the process is stopped. The hourly-based expected loss is used as a criterion for determining the optimal designs.

Panagos et al. use a designed experiment and the approach of analysis of variance (ANOVA) to conduct sensitivity analyses regarding the effects of the cost and operating parameters on the optimal design parameters and the resulting operating loss. In their continuous model, there are 9 cost and operating parameters considered and a  $2^{9-4}$  fractional factorial (FF) experiment with resolution IV is carried out. In their discontinuous model, 13 factors are considered and a  $2^{13-8}$  FF experiment also with resolution IV is conducted. Their work concentrates on the main effects only, due to the fact that the resolution of the experiments is IV and the two-way linear interactions are confounded with each other. They show that the choice of the proper manufacturing process model is critical because selection of an inappropriate model may result in significant economic penalties. They also observe that to stop the process while investigation of a special cause is in progress results in larger subgroup sizes, wider control limits and longer sampling intervals.

Other than the fully economic models developed (Montgomery, 1980; Panagos, Heikes, and Montgomery, 1985), Montgomery and Storer (1986) also develop an alternative approach to economically design process control charts. Instead of using 9 cost and operating parameters, a

simplified model with only 5 cost and operating parameters is proposed. An example is demonstrated to show the little difference in loss between the optimal design from the full economic model and the simplified one.

Various assumptions regarding the manufacturing processes and the cost and operating parameters have been made since 1956. For example, as shown previously, some authors assume that the process is stopped when the investigation of a possible special cause is in progress, others do not; some authors choose to include the down-time cost and repairing cost in their models, while others do not. Also, notation used is not unified.

Lorenzen and Vance (1986) provide a unified approach to the economic design of process control charts. A general process model is considered. An effort is made to unify the notation used. Their model includes 12 cost and operating parameters, 2 indicator variables which determine if the manufacturing activities continue during the search or repair stage, and 3 control chart design parameters (subgroup size, sampling interval, and width of the control limits) which need to be optimized in order to minimize the hourly-based expected loss. An example is given and a sensitivity analysis is conducted. They find that the expected minimum loss per hour is sensitive to the change of the magnitude of the process mean shifts ( $\delta$ ); however, the sampling plan itself is not sensitive to the change of  $\delta$ . Therefore, a crude approximation of the process

parameters can be made to design a good sampling plan.

Collani (1988) also proposes a unified approach to the optimal design of process control charts. A different approach is adopted, however. In his model, the process is assumed to operate under one of two states. State I represents "satisfactory", in which no corrective action is thought to be necessary. State II represents "unsatisfactory", in which a corrective action is thought to be necessary. Three different policies (monitoring, inspecting, and renewal/ replacement) are defined and incorporated into his model. An example using the X-bar chart is given. The objective is to find the optimal design parameters in order to maximize the net profit per item produced. This model is reported to be applicable to both the statistical quality control and the reliability areas.

Collani (1986) also proposes a different procedure to determine the economic design of the X-bar control chart. In this procedure, other than employing the regular X-bar chart, the author also includes periodic inspection of the process without performing sampling inspection as an alternative. Therefore, there are two strategies in determination of the optimal design. The first one is to use the regular X-bar chart procedure in which a subgroup of size  $n \geq 1$  is taken from the process every  $h$  hours. The quality characteristic of this subgroup is then computed and plotted on the X-bar chart with control limits placed at  $\pm k$ -sigma away from the center line. The second strategy

calls for inspection of the process every  $h$  hours without sampling a subgroup. Therefore,  $n$  and  $k$  are zeroes.

In Collani's model, it is assumed that (1) the production rate is constant; (2) the process is shut down during search and repair operations; (3) the overall loss due to the down time of the process is considered; and (4) the time required to sample and interpret one item is negligible. The optimal design for each strategy is determined. The overall optimal design is then determined by selecting the smaller loss per item of these two strategies.

Comparisons are made against Montgomery's (1982) results using the economic design of the X-bar chart and Chiu and Wetherill's (1974) results using semi-economic design of the X-bar chart. Collani (1986) reports that his results are very close to the optimal designs in terms of minimum cost. In some cases, this holds for even sub-optimal designs. It is also reported that his procedure is superior to Chiu and Wetherill's semi-economic scheme when sampling is expensive.

Traditional economic design of the X-bar control charts use equidistant control limits. This is due to the assumptions of (1) constant process variance, (2) perfect measurement of the quality characteristic, and (3) equal probabilities of upward and downward shifts in the process mean. Tagaras (1989a) relaxes these three assumptions in developing and studying, from both the statistical and

economic viewpoints, the X-bar chart with asymmetric control limits. He assumes that the process variance changes with the process mean and the coefficient of variation of the process remains constant throughout production. The hourly-based expected loss is employed for determining the optimal design.

Comparisons between X-bar charts with symmetric and asymmetric control limits are performed. A sensitivity analysis is conducted regarding the effects of misspecification of the cost and operating parameters and the model parameters (probabilities of shifts, rate of error of constant variance, and rate of error of measurement) on the optimal design parameters and the resulting operating loss. It is reported that the probability of shift in the process mean and the accuracy of measurement have noticeable effects on the optimal design and the resulting loss; however, the assumption of constant variance is shown to be relatively unimportant. Tagaras (1989a) also provides some advice on estimating the model parameters: (1) if uncertainty exists about the accuracy of the estimate of the probabilities of shifts in the process mean, a value close to 0.5 should be used from the economic viewpoint; and (2) it is better to assume a large value of coefficient of variation.

The basic assumptions when constructing control chart limits are: (1) the distribution of the measurable quality characteristic is normal; and (2) the inspection of the

quality characteristic is not subject to measurement error. These assumptions, however, may be violated in the physical environment. Rahim (1985) explores the effects of non-normality and measurement error on the design of the X-bar chart. The underlying distribution of the measurable quality characteristic is assumed to be non-normal by explicitly considering the skewness and kurtosis of the distribution. The measurement error is considered to be normally distributed. An economic model is developed in which the subgroup size, sampling interval, and the control limits are determined based on minimizing the expected loss. Rahim (1985) shows, by giving some numerical examples, that the conventional control plans with normality assumption will result in misleading values of the optimal design parameters and a resulting operating loss when the process is markedly non-normal and subject to measurement errors.

Most of the work of the economic design of quality control charts assumes that the underlying distribution of the process failure mechanism is exponential. That is, the times between occurrences of successive special causes are exponentially distributed with a specified mean value. The exponential distribution has the "memoryless" property. Therefore, it is a truly random shock because by assuming an exponential distribution, a constant failure rate for the process is implied. For some processes which deteriorate with time, the exponential assumption may not

be appropriate. A rich distribution must be employed for more complex situations.

Hu (1984, 1986) modifies Duncan's (1956) model to employ the Weibull distribution as the underlying distribution of the process failure mechanism. The process deterioration can then be simulated by varying the shape parameter of the distribution. The situations with shape parameter set from 1 to 4 are selected for study, while the scale parameter is adjusted to maintain the same mean duration of the in-control period. The control chart design parameters (subgroup size, sampling interval, and width of the control limits) are kept constant throughout production. The objective is to optimize the design parameters in order to minimize the expected loss. It is reported that Duncan's (1956) economic model is insensitive to misspecification of the process failure rate.

Banerjee and Rahim (1988) point out that the assumption of a constant sampling interval is counterintuitive in the case of an increasing failure rate of the process. A more realistic approach is to shorten the sampling interval because the process deteriorates further as time goes by. Therefore, they propose an economic model of the X-bar chart under Weibull shock using a varying sampling interval. They define the sampling interval to keep the probability of a shift in an interval, given no shift up to the start of the interval, constant for all intervals.

Comparisons are made among three cases: (1) a Weibull shock model with a varying sampling interval scheme; (2) a Weibull shock model under a constant sampling interval scheme; and (3) an exponential shock model under a constant sampling interval scheme. It is found, based on the expected loss per hour, that the results of case 1 are superior to both case 2 and 3. The differences of the losses between case 2 and 3 are negligible. This means that if a constant sampling scheme is employed, then, different assumptions regarding the process failure mechanism do not affect the expected loss much. If a varying sampling interval scheme is employed, then, the proper process failure mechanism should be carefully investigated and determined because a substantial loss will incur if the wrong distribution is assumed.

A sensitivity analysis is also conducted regarding the effects of the variation of the Weibull parameters on the optimal control design parameters and the resulting operating loss. They find that the optimal design parameters are not sensitive to a moderate degree of misspecification of the Weibull parameters.

Banerjee and Rahim (1987) also propose another approach to design and evaluate economically-based control charts. Their purpose is to study the role of the Markovian assumption. A renewal theory approach is employed to formulate and calculate the expected cycle length and the expected loss per cycle. In their model, the possible



system states are viewed at the end of the first sampling interval. At that point of time, the expected residual cycle length and the associated probability for each possible state of the system is determined. The renewal equation is formulated and then solved to obtain the expected cycle length. The expected loss per cycle is obtained using a similar approach. Examples are given for the situations where the distributions of the process failure mechanism are geometric and Poisson. The case of the Gamma shock model is thoroughly discussed. They show that certain non-Markovian models can be analyzed by adopting a renewal equation approach.

McWilliams (1989) conducts a sensitivity analysis of the effects of misspecification of the underlying distribution of the process failure mechanism on the optimal control chart design parameters and the resulting operating loss. The Weibull distribution is selected to represent the underlying distribution of the process failure mechanism and it is implemented in Lorenzen and Vance's (1986) model. He finds that, by assuming that the mean value of the in-control time is correctly specified, the economic control chart design is not sensitive to the shape of the Weibull distribution.

Due to the fact that the Weibull distribution is a rich distribution, McWilliams concludes that the above result will occur in general when considering the various economic control chart models and other distributions for

the in-control time; hence, the existing economic models are more widely applicable than their assumptions would indicate. He then emphasizes that (1) the expected time of occurrence of a special cause, within a sampling interval, should be approximated by one-half of the sampling interval; and (2) the expected number of subgroups taken while the process is in control be approximated by  $[(\text{expected time the process is in-control}/\text{sampling interval}) - 0.5]$  in order to simplify the economic models. Note, however, in this study, the control chart design parameters are kept constant throughout production.

Parkhideh and Case (1989) develop a more general economic model for the design of an X-bar chart. They, in addition to adopting the rich Weibull failure mechanism, allow the control chart design parameters to vary over time. Therefore, it is an economically-based dynamic X-bar chart. Duncan's (1956) approach to the economic design of an X-bar chart is employed. The subsequent values of the control chart design parameters (subgroup size, sampling interval, and width of the control limits) are assumed to be functions of their initial values. Therefore, the objective is to find the optimal initial values of the design parameters in order to minimize the expected loss per time unit. Comparisons are made between the dynamic X-bar chart and the traditional X-bar chart under a wide range of situations. They report that the dynamic X-bar chart is always superior to Duncan's (1956) X-bar chart

when the distribution of the process failure is Weibull.

Duncan (1971) develops an economic model for the X-bar chart subject to a multiplicity of special causes. This problem has also been addressed by several other researchers. All of them use only one set of control limits to maintain the process under control. There are situations, however, where different special causes will shift the process mean by different amounts; also, different cost and restoration procedures are required to repair the process for different shifts. Therefore, there is a need to develop a model which can distinguish between different status of the process and thus reduce the resulting loss.

Jaraied and Zhuang (1991) provide a computer program to economically determine the optimal control chart design parameters and the resulting operating loss when the process is subject to a multiplicity of special causes. This program is developed based on Duncan's (1971) model. The partial derivative of the loss function with respect to  $h$  (sampling interval) is set equal to zero to solve for  $h$ . A Fibonacci search technique is then applied to the subgroup size and the width of the control limits to determine the optimal values.

Tagaras and Lee (1988) propose an economic model of multiple control limits with multiple corresponding levels of process shifts. The design parameters that need to be optimized are the subgroup size, sampling interval, and

multiple sets of control limits. The criterion used for determining the design parameters is the expected loss per time unit. A large number of numerical examples are presented and a sensitivity analysis is performed on these examples. A comparison is made between the proposed model and an approximately matched single cause model. It is reported that the proposed model shows a significant improvement.

As mentioned before, even though much of the work of the economic designs reports the benefits and savings through the use of economically-based control charts, their implementation is still limited in the practical environment due to the complexity of the economic models and the optimization techniques required. Therefore, several efforts have been devoted to developing the approximation methods.

One of the approximation methods is developed by Tagaras (1989b) who proposes a log-power function to estimate the power of detection of the control chart at optimality. Multiple linear regression is employed for the derivation of the approximate formula expressing the power of the control chart as a function of the cost and operating parameters. Duncan's (1956, 1971) models with single and multiple special causes are studied. It is shown that, in the case of the X-bar chart and a single special cause, the approximation provides solutions which are very close to the true optima.

In almost all of the literature of the economic design of process control charts, it is assumed that the cost and operating parameters of the process are known or can be precisely estimated. In many cases, this information is not available or is difficult to obtain. Therefore, Pignatiello and Tsai (1988) propose an economic model which explicitly considers the imprecision of the estimation of the cost and operating parameters.

Duncan's (1956) economic X-bar chart is selected to implement this idea. An approach similar to the use of a Taguchi robust designed experiment (Kackar, 1985) is employed. The subgroup size, sampling interval, and width of the control limits are treated as controllable variables and the cost and operating parameters are treated as noise factors. The precise values of the noise factors are not known; however, it is assumed that a prior distribution can be specified for these factors. The low-cost, robust design for the X-bar chart is then formulated. It is reported that the loss function formulated under this new approach performs markedly better than the one without considering the implementation of a measure of the imprecision, especially when the rates of error of estimation of the noise factors are greater than 20%.

Most of the applications of the X-bar chart are in a piece-part manufacturing environment. Koo and Case (1990) apply the X-bar control chart procedure to a continuous flow process and develop an economic model. In their

procedure, a sample of size 1 is taken from the process every  $h$  hours. A subgroup of size  $n$  is formed from these samples. Therefore, a subgroup consists of  $n$  samples each taken  $h$  hours apart. The control chart design parameters which need to be optimized are the subgroup size,  $n$ , sampling interval,  $h$ , and width of the control limits,  $k$ . The objective is to minimize the hourly-based expected loss. A detailed derivation for the expected loss function is given. The Nelder-Mead search technique is employed to determine the optimal design parameters and the loss. A sensitivity analysis regarding the effects of the cost and operating parameters on the optimal designs is carried out.

Montgomery (1980) contains references to earlier work on economic design of control charts. Vance (1983) provides a bibliography for economic design of control charts of the period 1970-1980. Both are good references for the economic design of control charts.

#### Summary

A literature survey of the problems, contributions, and needs related to the objectives of this research is presented. It is obvious that there has been no work done for the economic designs of the  $\bar{X}$ -bar control chart with AT&T runs rules, the EWMA chart, and the Zone control chart. All three of these control schemes are used in industry. But, the tasks of formulating an economic model for each one of the three control schemes is yet to be

accomplished.

One of the key elements in gaining competitiveness in the international marketplace is to produce products of higher quality at lower costs. As mentioned before, the use of quality control charts for process control is an economic activity. Therefore, this survey indicates that a need exists to:

- (1) Provide economic models for the proposed
  - a. X-bar control chart with AT&T runs rules,
  - b. EWMA chart, and,
  - c. Zone control chart.
- (2) Provide guidelines as to which cost and operating parameters have effects on the design parameters and the resulting operating loss.
- (3) Gain insights of the economic use of the proposed three variables control charts.
- (4) Develop computer programs to help the design and evaluation of the economic models of the proposed three variables control charts.

## CHAPTER III

### DEVELOPMENT OF THE ECONOMIC MODELS OF THREE VARIABLES CONTROL CHARTS

#### Introduction

The Shewhart X-bar control chart with AT&T runs rules has been available since its development in 1956. However, its statistical performance was not evaluated until 1987. The EWMA control scheme was introduced in 1959. Yet, it received little attention until 1986. The Zone control chart is a new development in the area of sensitizing the performance of quality control charts. The Zone control chart is reported to be widely used in different areas (Jaehn, 1989).

These three variables control charts are employed for study in this research. A general economic model is developed for these three variables control charts. The ARLs are used to estimate both the expected length of a production cycle and the expected false alarm cost per hour of operation. The ARLs are functions of control chart design parameters except  $h$  (sampling interval) for each type of control chart. Therefore, the number of design



parameters, which need to be optimized in order to minimize the loss, depends on the number of parameters needed to calculate the ARLs.

For the Shewhart X-bar control chart with AT&T runs rules, the ARL is a function of subgroup size and width of the control limits for a specified runs rule. Therefore, four variables need to be optimized in the economic design in order to minimize the loss. They are subgroup size ( $n$ ), sampling interval ( $h$ ), width of the control limits ( $k$ ), and the combination of the four AT&T runs rules (RULE).

For the EWMA chart, the ARL is a function of subgroup size ( $n$ ), width of the control limits ( $k$ ), and the weight ( $\alpha$ ) used on the current observation. Therefore, there are four variables which need to be optimized in the economic design in order to minimize the loss. They are  $n$ ,  $h$ ,  $k$ , and  $\alpha$ .

For the Zone control chart, the ARL is a function of the subgroup size ( $n$ ), width of the control limits ( $k$ ), and the (four) zone scores ( $S_1$ ,  $S_2$ ,  $S_3$ , and  $S_4$ ). Therefore, there are seven variables which need to be optimized in the economic design. They are  $n$ ,  $h$ ,  $k$ ,  $S_1$ ,  $S_2$ ,  $S_3$ , and  $S_4$ .

Due to the complexity of the calculations of both the ARLs and the loss function, nice expressions of the partial derivatives of the loss function with respect to control chart design parameters are not available. Thus, computer search procedures must be developed to optimize the design

parameters for each of these three variables control charts.

Duncan's (1956) cost assumptions and approach have proven to be the most practical and appealing (Parkhideh and Case, 1989). Therefore, the economic model developed in this research uses the same cost structure as Duncan's economic design to the X-bar control chart. It is Duncan's economically-based X-bar control chart that is used to compare the proposed economic designs of these three variables control charts.

#### Assumptions

The basic assumptions underlying this research are as follows:

- (1) The measurable quality characteristic (it can be the subgroup mean or individual observations) of interest is normally distributed.
- (2) There is only a single special cause. The occurrence of the special cause shifts the process mean to a known value.
- (3) The shift in the process mean is instantaneous.
- (4) The process is not self-correcting. That is, once the process mean has shifted, it stays there until being detected. The process can only be brought back to a state of statistical control by management intervention.

- (5) The occurrence time of the special cause is exponentially distributed with mean  $1/\theta$ .
- (6) The process standard deviation remains unchanged throughout production.
- (7) The process is not shut down while the investigation of a possible special cause is in progress.
- (8) Sampling inspection is not subject to measurement error.
- (9) Action is taken when a pre-specified criterion of an OOC condition is met.
- (10) The costs of adjustment, repair, and bringing the process back to a state of statistical control when it shifts are not considered. This is because all these three costs are assumed to be fixed and they apply to all three charts.
- (11) The time required to take, inspect, compute, and plot a point is proportional to subgroup size.
- (12) The process is properly centered originally so that no matter whether the process shifts upward or downward, the average loss per hour of operation is the same.
- (13) The occurrences of successive false alarms are independent of each other.
- (14) It is assumed that the probability of occurrence of a special cause during the taking of a subgroup can be neglected. This is due to the requirement of homogeneity within a subgroup in order to construct meaningful control chart limits.

- (15) Subgroups are taken from the production process at intervals of  $h$  hours, where  $h$  is a constant throughout production.
- (16) For the Zone control chart, the critical value is set equal to the zone score of the outermost zone.

#### Formulation Of The Economic Model

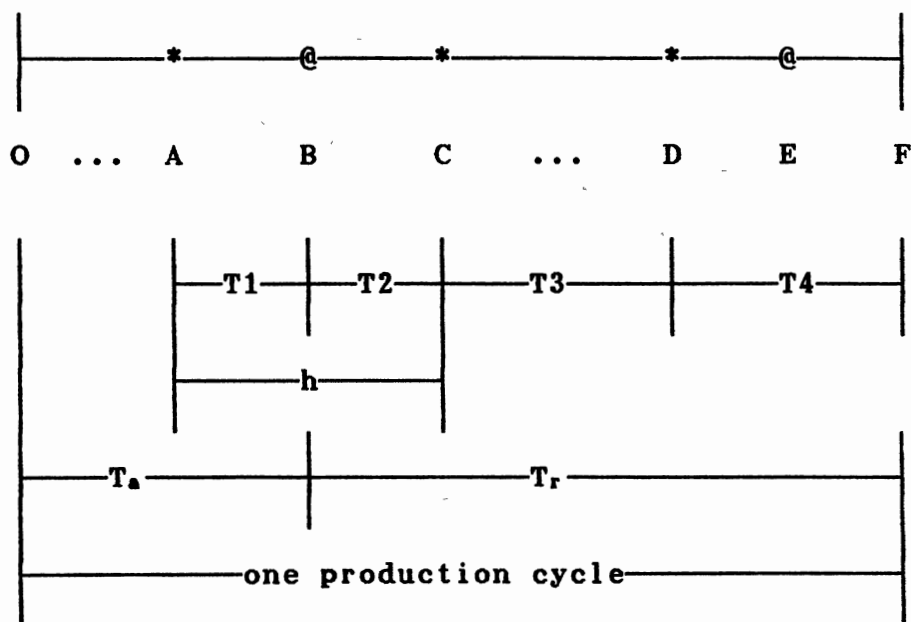
Duncan's (1956) approach to the economic design of the X-bar control chart is adopted for the development of the economic models of the (1) X-bar control chart with AT&T runs rules, (2) EWMA chart, and (3) Zone control chart. The criterion used to evaluate the optimal designs is the expected loss per hour of operation. Duncan (1956) expresses this as

$$E[\text{loss/hour of operation}] = \frac{E[\text{costs generated within a production cycle}]}{E[\text{length of a production cycle}]} \quad (3.1)$$

The length of a production cycle is defined as the total time from which the process starts in a SOSC, shifts to an OOC condition, the OOC condition is detected, and the special cause is identified. Figure 3.1 shows the length of a production cycle.

The expected costs generated within a production cycle can be categorized into four elements. They are:

- (1) Penalty cost: the cost due to operating the process under an OOC condition;
- (2) False alarm cost: the cost of searching for a special



Where,

- O : process begins in control
- A : last subgroup taken while the process is in control
- B : special cause occurs
- C : first subgroup taken after special cause occurs
- D : last subgroup taken after the special cause occurs
- E : special cause is detected
- F : special cause is identified
- T<sub>a</sub> : length of time the process is in-control
- T<sub>r</sub> : length of time the process is out-of-control

Legends:

- \* : a subgroup is drawn
- @ : a point in time; no subgroup is drawn

Figure 3.1 A Production Cycle

- cause when none exists;
- (3) True alarm cost: the cost of searching for a special cause when one exists; and,
- (4) Control chart maintenance cost: the cost of taking, inspecting, calculating, and plotting on the control chart.

### Discussion Of The Expected Loss

#### Per Hour Of Operation

It is well known that if X and Y represent two random variables, then  $E[Y/X]$  is not equivalent to  $E[Y]/E[X]$ . Therefore, the exact expression for the expected cost per hour of operation should be given as

$$E[\text{loss/hr}] = E \left[ \frac{\text{costs generated within a production cycle}}{\text{length of a production cycle}} \right] \quad (3.2)$$

instead of that given in (3.1).

A quality control process, however, is a "renewal reward process". In a quality control system, every production cycle is a renewal event (because whenever a special cause is identified, management intervention can bring the process back to a SOSC). The length of a production cycle is the renewal time. A loss function can be associated with each production cycle. This loss function is crucial in the operation of the quality monitoring process. Bhat (1984) proves that the average

loss of a quality monitoring process (a renewal reward process) can be expressed as

$$\text{Average Loss} = E[G/Z] = E[G] / E[Z],$$

where G represents the expected loss generated within a production cycle and Z represents the expected length of a production cycle. That is, Eq. (3.1) is equivalent to Eq. (3.2).

#### Derivation Of The Economic Model

The economic model consists of two important elements. One is the expected length of a production cycle measured in hours; the other is the expected costs generated within a production cycle. After the expected length of a production cycle is determined, the costs can be converted to an hourly-based loss.

Expected Length Of A Production Cycle. From Figure 3.1, it is observed that a production cycle consists of two portions. One is the length of time when the process is in control, denoted by  $T_a$ ; the other is the length of time when the process is out of control, denoted by  $T_r$ . Time  $T_r$  can be divided into three parts. The first is the length of time from the occurrence of a special cause to the first subgroup taken after the occurrence of the special cause, denoted by  $T_2$ . The second is the length of time between the first subgroup taken after the occurrence of a special cause and the "detecting" subgroup, denoted by  $T_3$ . The

third is the time required to identify the special cause, denoted by  $T_4$ .

From assumption 5, the time of the occurrence of a special cause is exponentially distributed with mean  $1/\theta$ . The expected length of time when the process is in control is given by

$$E[T_a] = 1/\theta.$$

According to assumption 14, the probability of an occurrence of a special cause during the time a subgroup is taken can be neglected. Therefore, a special cause will occur only between subgroups. From assumption 15, subgroups are taken from the production process every  $h$  hours. The average time of the occurrence of a special cause within an interval between subgroups, given that the occurrence of the special cause is between  $n$ th and  $n+1$ st subgroup, is given by (Duncan, 1956)

$$E[T_1] = \frac{\int_{nh}^{(n+1)h} \theta(t-nh)e^{-\theta t} dt}{\int_{nh}^{(n+1)h} \theta e^{-\theta t} dt}$$

Let  $x = t - nh \Rightarrow t = x + nh \Rightarrow dt = dx$ .

Substituting into  $E[T_1]$  gives

$$E[T_1] = \frac{\int_0^h \theta x e^{-\theta(x+nh)} dx}{\int_0^h \theta e^{-\theta(x+nh)} dx}$$



$$\begin{aligned}
 & \frac{e^{-\theta nh} \int_0^h \theta x e^{-\theta x} dx}{e^{-\theta nh} \int_0^h \theta e^{-\theta x} dx} \\
 &= \frac{\int_0^h \theta x e^{-\theta x} d(\theta x)}{\theta(1 - e^{-\theta h})}
 \end{aligned}$$

Applying integration by parts to the numerator, let

$$u = \theta x, \quad dv = e^{-\theta x} d(\theta x).$$

The numerator becomes  $1 - (1+\theta h)e^{-\theta h}$ , therefore,

$$E[T_1] = \frac{1 - (1+\theta h)e^{-\theta h}}{\theta(1 - e^{-\theta h})}$$

$$E[T_2] = h - E[T_1]$$

$$\begin{aligned}
 &= h - \frac{1 - (1+\theta h)e^{-\theta h}}{\theta(1 - e^{-\theta h})} \\
 &= \frac{\theta h(1 - e^{-\theta h}) - [1 - (1+\theta h)e^{-\theta h}]}{\theta(1 - e^{-\theta h})} \\
 &= \frac{\theta h - [1 - e^{-\theta h}]}{\theta(1 - e^{-\theta h})} \\
 &= \frac{h}{1 - e^{-\theta h}} - \frac{1}{\theta}
 \end{aligned}$$

Recall that the definition of the ARL is the expected number of subgroups inspected until the process signals an OOC condition. The expected length of time from the occurrence of a special cause to the "detecting" subgroup is  $h * (ARL2 - 1) + E[T_2]$ , where ARL2 represents the ARL

when the process mean has shifted. Therefore,

$$E[T_3] = h * (ARL2 - 1).$$

According to assumption 11, the time required to take, inspect, calculate, and plot on the control chart is proportional to the subgroup size. Let this be  $e*n$ . Suppose after plotting, the point is identified as a true signal. It then takes, on the average,  $D$  hours to identify the special cause. Then,

$$E[T_4] = en + D.$$

The expected length of a production cycle (ELOPC) then becomes

$$ELOPC = E[T_1] + E[T_2] + E[T_3] + E[T_4]$$

$$= \frac{1}{\theta} + \left[ \frac{h}{1 - e^{-\theta h}} - \frac{1}{\theta} \right] + h(ARL2 - 1) + en$$

$$+ D$$

$$= \frac{h}{1 - e^{-\theta h}} + h * ARL2 - h + en + D$$

$$= h * \left[ \frac{1}{1 - e^{-\theta h}} - 1 + ARL2 \right] + en + D$$

$$= h * \left[ \frac{e^{-\theta h}}{1 - e^{-\theta h}} + ARL2 \right] + en + D$$

#### Expected Costs Within A Production Cycle

The economic model considered consists of four cost

elements. They are the (1) penalty cost, (2) false alarm cost, (3) true alarm cost, and (4) control chart maintenance cost.

Expected Penalty Cost Per Hour Of Operation (EPC).

The penalty cost is generated due to operating the process under an OOC condition. Let  $V_0$  denote the average income per hour when the process operates under a SOSC and  $V_1$  denote the average income per hour when the process operates under an OOC condition. Then,  $M = (V_0 - V_1)$  denotes the expected cost per hour when the process operates under an OOC condition.

The expected length of a production cycle is ELOPC. On the average, the process will be in a SOSC for  $1/\theta$  hours. Thus, the proportion of time a process is in control per hour of operation is given by

$$\beta = (1/\theta) / \text{ELOPC}.$$

The expected penalty cost per hour of operation is given by

$$\begin{aligned} \text{EPC} &= M * (1 - \beta) \\ &= M * \left(1 - \frac{1}{\theta * \text{ELOPC}}\right) \end{aligned}$$

Expected False Alarm Cost Per Hour Of Operation (EFAC). The EFAC is defined as the multiplication of the expected number of false alarms (ENFA) and the cost of searching for a false alarm (T). A false alarm is defined as the process signals an OOC condition when, in fact, it is in a SOSC.

The expected number of subgroups taken between two successive false alarms is called the ARL in control, denoted as ARL1. The proportion of time a process will signal a false alarm is approximately  $1/ARL1$ , given that the process operates under a SOSC.

The process may go out of control at any interval between subgroups. The underlying distribution of the occurrence of a special cause is exponential with rate parameter  $\theta$ . Thus, the expected number of subgroups taken while the process is in control ( $Y$ ) is given by

$$\begin{aligned}
 Y &= \sum_{i=0}^{\infty} i \int_{ih}^{(i+1)h} \theta e^{-\theta t} dt \\
 &= \sum_{i=0}^{\infty} i \left[ -e^{-\theta t} \right]_{ih}^{(i+1)h} \\
 &= \sum_{i=0}^{\infty} i \left[ -e^{-\theta(i+1)h} - (-e^{-\theta ih}) \right] \\
 &= \sum_{i=0}^{\infty} i \left[ e^{-\theta ih} - (-e^{-\theta(i+1)h}) \right] \\
 &= (e^{-\theta h} - e^{-2\theta h}) + 2(e^{-2\theta h} - e^{-3\theta h}) \\
 &\quad + 3(e^{-3\theta h} - e^{-4\theta h}) + \dots \\
 &= e^{-\theta h} + e^{-2\theta h} + e^{-3\theta h} + \dots \\
 &= e^{-\theta h} * [1 + e^{-\theta h} + e^{-2\theta h} + e^{-3\theta h} + \dots] \\
 &= \frac{e^{-\theta h}}{1 - e^{-\theta h}}
 \end{aligned}$$

Therefore,

$$ENFA = Y / ARL1,$$

and, the false alarm cost generated within a production

cycle is  $(T * Y / ARL1)$ . The EFAC is given by

$$EFAC = (T * Y) / (ARL1 * ELOPC).$$

Expected True Alarm Cost Per Hour Of Operation (ETAC).

The expected length of a production cycle is ELOPC. The average number of times per hour that the process actually goes out of control is  $1 / ELOPC$ . Let W denote the expected cost of looking for a special cause when one exists. Then,

$$ETAC = W / ELOPC.$$

Expected Control Chart Maintenance Cost Per Hour Of Operation (EMC). Two types of cost are considered. One is the fixed cost (b) associated with a subgroup taken; the other is the variable cost per unit sampled (c). The sampling interval is h hours. Therefore,

$$EMC = (b + cn) / h.$$

The expected loss per hour of operation of a process (ELOSS) is given by

$$ELOSS = EPC + EFAC + ETAC + EMC$$

$$= M * \left[ 1 - \frac{1}{\theta * ELOPC} \right] + \frac{T * Y}{ARL1 * ELOPC} + \frac{W}{ELOPC} + \frac{b + cn}{h}$$

This ELOSS is used as the criterion for determining the optimal economic design of these three variables control charts. Note that all three of these charts use the

same economic model, the only difference is the calculation of the ARLs.

### Average Run Length

Average run length (ARL) has widely been used as a criterion for statistically comparing the performance of control charts. The economic model proposed in this research employs the ARL as part of the calculation of the ELOSS. The ARL is a function of control chart design parameters, except  $h$ . The number of parameters needed to compute the ARLs is different for each type of control chart. This makes the number of parameters which need to be optimized in each of these three economically-based control charts different.

#### Shewhart X-bar Control Chart

##### With AT&T Runs Rules

Champ and Woodall (1987) evaluate the ARLs of the Shewhart X-bar control chart with supplementary runs rules using Markov chains. Suppose rule  $i$  is used, they denote this rule as  $T(k_1, m_1, a_1, b_1)$ ,  $m_1 > 1$ , which means that an OOC condition is assumed if  $k_1$  out of  $m_1$  consecutive points fall in region  $(a_1, b_1)$ . Using AT&T rule 2 as an example and let  $a_1$  and  $b_1$  represent the 2-sigma and 3-sigma limits, respectively. The AT&T rule 2, for the upper part of the control chart, can be expressed as  $T(2, 3, 2, 3)$ .

The states of the Markov chains indicate the status of the chart with respect to each runs rules. Only one absorbing state is used which corresponds to the OOC condition. Champ and Woodall (1987) define the vectors

$$W'_i = (W_{i,1}, \dots, W_{i,m_i - 1})$$

and,

$$X'_i = (X_{i,1}, \dots, X_{i,m_i - 1})$$

where,

$$\begin{aligned} W_{i,j} &= 1 && \text{if the } j\text{th previous observation was in} \\ & && (a_i, b_i) \\ &= 0 && \text{otherwise} \end{aligned}$$

and,

$$\begin{aligned} X_{i,j} &= W_{i,j} && \text{if } \sum_{h=1}^j (1 - W_{i,h}) < m_i - k_i + 1 \\ &= 0 && \text{otherwise} \end{aligned}$$

The vector  $X'_i$  indicates by 1s only those observations falling in  $(a_i, b_i)$  that may contribute to an OOC condition. Therefore, a transient state of a chart using  $t$  rules can be represented by  $S' = (X'_1, \dots, X'_t)$ , where the subvector  $X'_i$  is defined as previously for the rule  $T(k_i, m_i, a_i, b_i)$ ,  $i = 1, 2, \dots, t$ .

The one-step transition probability matrix can then be constructed as  $P = [P_{i,j}]$ , where  $P_{i,j}$  is the one-step transition probability of moving from state  $i$  to state  $j$ . The states are numbered from 1 to  $s$ , where state 1 is the initial state and state  $s$  is the absorbing state. After the transition probability matrix,  $P$ , has been constructed,

cross out the last row and last column of  $P$  and define this new matrix as  $Q$ . Let  $I$  denote the identity matrix. The ARL1 is obtained by summing the elements of the first row of the matrix  $(I - Q)^{-1}$ , suppose there is no shift in the process mean. Different ARL2s corresponding to different amounts of shift in the process mean are obtained by changing the probability that a point will fall in  $(a_1, b_1)$ . Note, always use the first row of the matrix  $(I - Q)^{-1}$  to calculate the ARLs regardless of whether it is ARL1 or ARL2. Also note that the ARL is equivalent to the average time to absorption in a Markov chains process. For more information regarding the calculation of the ARL, see Brook and Evans (1972).

This research uses Champ and Woodall's approach in calculating the ARLs used for calculation of the ELOSS. The objective function to be minimized in the economic design of the X-bar control chart with AT&T runs rules is the ELOSS, which is a function of the subgroup size ( $n$ ), sampling interval ( $h$ ), width of control limits ( $k$ ), and the runs rules (RULE) used. Therefore, there are four design parameters which need to be optimized.

Note that the design parameter, RULE, is not a quantitative variable. There are eight combinations of the four AT&T runs rules which are commonly used in industry. They are, in Champ and Woodall's (1987) notation, C1, C12, C13, C14, C123, C124, C134, and C1234, where, for example,



C1234 represents that all four AT&T runs rules are employed at the same time.

### The EWMA Chart

Champ and Rigdon (1991) compare the Markov chains and the integral equation approaches for evaluating the run length distribution of quality control charts. They conclude that these two approaches are equivalent. Usually, there exists more than one way to numerically approximate an integral equation. Therefore, Champ and Rigdon suggest the use of the integral equation approach to solve for the run length and average run length of a control chart, whenever an integral equation is available. Hence, Crowder's (1987a) approach is employed to calculate the ARL of an EWMA chart.

Crowder (1987a) gives the following equation and uses numerical approximation to obtain the ARL of an EWMA chart,

$$L(u) = 1 + (1/\alpha) \int_{-q}^q L(w) f \left[ \frac{w - (1-\alpha)u}{\alpha} \right] dw$$

where,

$L(u)$ : the ARL given that the EWMA starts with value  $u$ ;

$\alpha$ : the weight for the current observation.

The equation can be obtained by the following reasoning.

Let the probability density function of the random variable  $Y = \{y_i\}$ ,  $i = 1, 2, \dots$ , be  $f(y)$ . Recall that the calculation of the EWMA is given by

$$EWMA_t = (1-\alpha)EWMA_{t-1} + \alpha y_t, \quad 0 < \alpha \leq 1, \quad t = 1, 2, \dots$$

Let  $L(u)$  be the ARL, given that the EWMA starts with value  $u$ . Let  $EWMA_0$  be  $u$ . If the first observation  $y_1$  is such that  $|(1-\alpha)u + \alpha y_1|$  is greater than a specified value  $q$  ( $q = k\sigma_{EWMA}$ ), an OOC signal is given. Otherwise, the run continues from  $(1-\alpha)u + \alpha y_1$  with  $L[(1-\alpha)u + \alpha y_1]$  representing the additional expected run length. Therefore,

$$\begin{aligned} L(u) &= 1 * \Pr(|(1-\alpha)u + \alpha y_1| > q) \\ &+ \int_{\{|(1-\alpha)u + \alpha y_1| \leq q\}} (1 + L[(1-\alpha)u + \alpha y_1])f(y) dy \\ &= \Pr(|(1-\alpha)u + \alpha y_1| > q) + \Pr(|(1-\alpha)u + \alpha y_1| \leq q) \\ &+ \int_{\{|(1-\alpha)u + \alpha y_1| \leq q\}} L[(1-\alpha)u + \alpha y_1]f(y) dy \\ &= 1 + \int_{\{|(1-\alpha)u + \alpha y_1| \leq q\}} L[(1-\alpha)u + \alpha y_1]f(y) dy \end{aligned}$$

Let  $w = (1-\alpha)u + \alpha y$

then,  $dw = \alpha dy$

and,  $y = [w - (1-\alpha)u] / \alpha$

The above equation then becomes

$$\begin{aligned} L(u) &= 1 + \int_{|w| \leq q} L[w]f([w - (1-\alpha)u] / \alpha) (1/\alpha) dw \\ &= 1 + (1/\alpha) \int_{-q}^q L[w]f([w - (1-\alpha)u] / \alpha) dw \end{aligned}$$

The equation is called Fredholm integral equation of the second kind (Crowder, 1987a). This is an approximation because  $L(\cdot)$  is approximated numerically. The objective function to be minimized in the economic design of an EWMA chart is the ELOSS, which is a function of  $n$ ,  $h$ ,  $k$ , and  $\alpha$

(weight). Therefore, there are four design variables that need to be optimized.

### The Zone Control Chart

Figure 3.2 shows the structure of a Zone control chart. The region between LCL and UCL are divided into six equal zones. Zone scores (S1, S2, S3, and S4) are assigned to each of the four zones at the same side of the center line. The critical value which determines the criterion of an OOC condition is set equal to the zone score of the outermost zone. Probabilities of each zone (p1, ..., p8) are determined depending on the amounts of shift in the process mean.

Fang and Case (1990) develop a set of simultaneous linear equations to evaluate the ARLs of the Zone control chart. The simultaneous linear equations are

$$E(i) = 1 + p_1 * E(i+S_1) + p_3 * E(i+S_2) + p_5 * E(i+S_3) \\ + p_2 * F(S_1) + p_4 * F(S_2) + p_6 * F(S_3)$$

$$F(i) = 1 + p_1 * E(S_1) + p_3 * E(S_2) + p_5 * E(S_3) \\ + p_2 * F(i+S_1) + p_4 * F(i+S_2) + p_6 * F(i+S_3)$$

$$i = 0, 1, 2, \dots, S_4-1$$

$$E(0) = F(0)$$

where,  $E(i)$  represents the expected number of additional subgroups required until an OOC condition is signaled, given that the current cumulative score is  $i$  and the currently plotted point is above the center line. The

	Zone	Score	Probability
	D(U)	S4	p7
UCL	<hr/>		
	C(U)	S3	p5
	<hr style="border-top: 1px dashed black;"/>		
	B(U)	S2	p3
	<hr style="border-top: 1px dashed black;"/>		
	A(U)	S1	p1
CL	<hr/>		
	A(L)	S1	p2
	<hr style="border-top: 1px dashed black;"/>		
	B(L)	S2	p4
	<hr style="border-top: 1px dashed black;"/>		
	C(L)	S3	p6
LCL	<hr/>		
	D(L)	S4	p8

**Figure 3.2 Structure Of A Zone Control Chart**

definition of  $F(i)$  is the same as  $E(i)$  except that it is for the situation when the currently plotted point is below the center line.

The objective function to be optimized in the economic design of the Zone control chart is ELOSS, which is a function of  $n$ ,  $h$ ,  $k$ , and the four zone scores. Therefore, there are seven variables which need to be optimized.

#### Optimization Search Technique

The objective function to be minimized in the economic designs of the three variables control charts is ELOSS, which is a non-linear function of multiple control chart design parameters. Due to the complexity of the calculations involved, nice expressions of the first derivatives of the objective function with respect to the control chart design parameters are not available. Therefore, a direct search technique must be employed to help identify the economic design.

The Nelder-Mead simplex procedure (Nelder and Mead, 1965; it is also known as the Flexible Polyhedron Search, see Himmelblau, 1972) is utilized as the search algorithm. Olsson and Nelson (1975) show the generality of the Nelder-Mead simplex method, its accuracy, and the simplicity of the information required for the computer input statement. This method is developed for minimization of a multivariable function without constraints. No derivatives of the objective function are required.

The simplex procedure forms a simplex and moves along the response surface in search of the minimum. It approaches the minimum by moving away from the highest value of the objective function rather than by trying to move in a line toward the minimum. The procedure is operated by reflection, extension, or contraction so as to conform the characteristics of the response surface. The operation continues until either a specified number of evaluations has been reached or successive function values differ by less than a specified amount.

In this research, three variables control charts are studied. Each has a different number of variables which need to be optimized. Some of them are integers (for example,  $n$ ,  $S_1$ ,  $S_2$ ,  $S_3$ , and  $S_4$ ), some are real values. Even more, there is a non-quantitative variable, which is RULE. The search methods employed by each type of control charts are described as follows.

#### Shewhart X-bar Control Chart With AT&T Runs Rules

There are four variables to be optimized, they are ( $n$ ,  $h$ ,  $k$ , RULE). The RULE is not a quantitative variable. Therefore, there are three quantitative variables, and one of them,  $n$ , is an integer. In order to deal with an integer and a non-quantitative variable, and to find the optimal solution, the following search methods are employed.

A. The user selects a RULE to be used in combination with

the X-bar control chart.

- B. With a starting point and step sizes, do a three variable search. Find the optimal point of real values of  $n$ ,  $h$ , and  $k$ .
- C. The  $n$  found in step B is truncated to an integer and treated as a constant. Do a two variable search on  $h$  and  $k$ . The optimal point  $(h, k)$  found with this truncated  $n$  forms the best solution so far.
- D. Do a line search on  $n$ , employing the Nelder-Mead algorithm, to find the minimum loss. For each  $n$  considered, optimize  $(h, k)$ . The  $(n, h, k)$  point found with the minimum loss is the optimal economic design of the X-bar control chart with a combination of some specified AT&T runs rules.

#### The EWMA Chart

There are four variables,  $(n, h, k, \alpha)$ , to be optimized. The subgroup size ( $n$ ) is an integer, others are real values. The following procedures are adopted to minimize the objective function.

- A. With a starting point and step sizes, do a four variable search. Find the optimal point of real values of  $n$ ,  $h$ ,  $k$ , and  $\alpha$ .
- B. The  $n$  found in step A is truncated to an integer and treated as a constant. Do a three variable search on  $h$ ,  $k$ , and  $\alpha$ . The optimal point  $(h, k, \alpha)$  found with this truncated  $n$  forms the best solution so far.

C. Do a line search on  $n$ , employing the Nelder-Mead algorithm, to find the minimum loss. For each  $n$  considered, optimize  $(h, k, \alpha)$ . The  $(n, h, k, \alpha)$  point found with the minimum loss is the optimal economic design of the EWMA chart.

#### The Zone Control Chart

There are seven variables,  $(n, h, k, S_1, S_2, S_3, S_4)$ , to be optimized in the economic design of the Zone control chart. The subgroup size  $(n)$  and the four zone scores  $(S_1, S_2, S_3, S_4)$  are integers, others are real values. In order to simplify the computer search technique and use integer zone scores to calculate the ARLs, the real value zone scores found using the Nelder-Mead technique are truncated to integers. No line search on zone scores is performed. An implicit assumption is also made such that  $0 \leq S_1 < S_2 < S_3 < S_4$ . This assumption has never been explicitly cited but is always employed in existing literature.

The following procedures are adopted to optimize the economic design of the Zone control chart.

- A. With a starting point and step sizes, do a seven variables search. Find the optimal point of real values of  $(n, h, k, S_1, S_2, S_3, S_4)$ .
- B. The  $n$  found in step A is truncated to an integer and treated as a constant. Do a six variables search on  $h, k, S_1, S_2, S_3,$  and  $S_4$ . Note when applying the Nelder-Mead algorithm, the zone scores are treated as real



values. They are truncated to integers only when they are employed in the ARL calculation subroutine to calculate the ARLs. The optimal point  $(h, k, S1, S2, S3, S4)$  found with this truncated  $n$  forms the best solution so far.

- C. Do a line search on  $n$ , employing the Nelder-Mead algorithm, to find the minimum cost. For each  $n$  considered, optimize  $(h, k, S1, S2, S3, S4)$ . The  $(n, h, k, S1, S2, S3, S4)$  point found with the minimum cost is the optimal economic design of the Zone control chart.

#### Summary

An economic model of the X-bar control chart with AT&T runs rules, the EWMA chart, and the Zone control chart is developed. The model is developed using Duncan's (1956) approach to the economic design of the X-bar control chart. The mathematical development and derivation of the hourly-based loss for these three variables control charts are discussed. The expression of the loss functions of these three variable charts are the same. The only difference is in the calculation of the ARLs for each chart. The ARL calculation for each chart is briefly introduced.

The Nelder-Mead simplex search technique is employed to optimize the economic design of these three variables control charts. Based on the experience gained in this research, multiple starting points are suggested when using

the direct search technique in order to lend confidence to that the optimal or near-optimal point has been reached. For example, the user might want to repeat the search technique at least twice by starting at an arbitrary point, and then, repeat the search technique using the optimal point found in the first run as the inputs of the second run. Also, the user might want to start from multiple arbitrary points.

## CHAPTER IV

### USING THE INTERACTIVE COMPUTER PROGRAMS

#### Introduction

This chapter illustrates the use of the interactive computer programs which permit easy economic design and evaluation of the (1) X-bar control chart with AT&T runs rules, (2) EWMA chart, and (3) Zone control chart. The FORTRAN programs appear in Appendices A, B, and C. These programs are developed and implemented on an IBM PC.

There are three common control chart design parameters which need to be optimized in the economic design of these three variable control charts. They are the subgroup size ( $n$ ), the sampling interval ( $h$ ), and the width of control limits ( $k$ ). In addition to these, the user has to select the runs rules (RULE) used in combination with the X-bar control chart in order to minimize the loss. The weight ( $\alpha$ ) needs to be optimized in order to minimize the loss in the economic design of the EWMA chart. The four zone scores need to be optimized in order to minimize the loss in the economic design of the Zone control chart. Evaluations of the (1) X-bar control chart with AT&T runs rules, (2) EWMA chart, and (3) Zone control chart refer to the calculation

of the loss for a set of specified design parameters for each type of control chart.

The programs are interactive. The user is prompted for all necessary inputs by the programs. The user can choose to enter the cost and operating parameters either from an existing file or manually. Other typical and/or often-used values are pre-programmed. All these values are presented to the user for either verification or change. Only when a set of values has been verified by the user does the program proceed.

Economic Design And Evaluation Of  
The X-bar Control Chart With  
AT&T Runs Rules

The program prompts the user for the main menu:

```
*****
***   MAIN   MENU   ***
*****
```

- (1) ECONOMIC DESIGN OF X-BAR CONTROL CHARTS  
WITH AT&T RUNS RULES,
- (2) EVALUATION OF X-BAR CONTROL CHARTS  
WITH AT&T RUNS RULES,
- (3) EXIT THE PROGRAM.

==> PLEASE ENTER YOUR OPTION (1, 2, 3)! <<<

1 By selecting "1" from this menu, the program leads to the economic design of the X-bar control chart with AT&T runs rules.

## Economic Design Of The X-bar Control

### Chart With AT&T Runs Rules

The program prompts the user to enter the cost and operating parameters. They are the (1) amount of shift in the process mean measured in number of process standard deviations ( $\delta$ ), (2) occurrence rate of the special cause ( $\theta$ ), (3) penalty cost per hour of operation ( $M$ ), (4) expected time required to sample a unit ( $e$ ), (5) expected time required to identify the special cause ( $D$ ), (6) expected cost of searching for a false alarm ( $T$ ), (7) expected cost of searching for a true alarm ( $W$ ), (8) expected fixed cost per subgroup taken ( $b$ ), and (9) expected variable cost per unit sampled.

The user can choose either to enter these values from an existing file or to enter them manually. The user has to build a file storing the cost and operating parameters before he can choose to enter the data from an existing file. An example of the data file which contains the 9 cost and operating parameters from Duncan's (1956) example 1 is given as follows. The name of this file is "CASE1".

```
2 0.01 100 0.05 2 50 25 0.5 0.1
```

A space is needed to separate each individual data value. This set of data has to be saved as an ASCII file for later use.

By selecting "1", the program then prompts the user to enter the filename that contains the cost and operating

parameters. A selection of "2" will lead to asking the user to enter the values manually.

\*\*\* INPUT COST PARAMETERS \*\*\*

DO YOU WANT TO INPUT FROM A FILE OR MANUALLY?  
 ==> PLEASE ENTER 1 = FILE, 2 = MANUALLY. <<<

1

\*\* PLEASE INPUT THE FILENAME THAT  
 CONTAINS THE COST PARAMETERS.

CASE1

In this example, Duncan's (1956) example 1 is selected for trial and this set of parameters has been built as a data file. After entering the filename that contains the cost and operating parameters of Duncan's example 1, the program then prompts the user to verify the values received. A selection of "1" leads the program to continue. A selection of "2" leads the program to prompt the user to enter the filename which contains the desired values of the cost and operating parameters.

\*\* VALUES RECEIVED ARE:

DELTA =	2.0000	THETA =	.0100
M =	100.0000	E =	.0500
D =	2.0000	T =	50.0000
W =	25.0000	B =	.5000
C =	.1000		

==> ARE THESE DATA CORRECT?

==> PLEASE ENTER 1 = YES, 2 = NO. <<<

1

The program then suggests a starting point used to execute the Nelder-Mead search method. Here, the user is asked to accept or reject the suggested point. If the user rejects the suggestion, the program then prompts the user

to enter a new starting point. After the starting point has been accepted and verified, the program then suggested step sizes for each variable which needs to be optimized. The user is also asked to accept or reject the suggestions. A verification of the step sizes is also desired. Finally, the runs rules used in combination with the X-bar control chart need to be selected.

The suggested starting point and step sizes are accepted in this example. The rule C1 (which corresponds to the standard Shewhart X-bar control chart) is selected.

\*\*\* THE SUGGESTED STARTING POINT IS:

N = 5, H = 1.00, K = 3.00

==> DO YOU ACCEPT THIS POINT?

==> ENTER 1 = YES, 2 = NO. <<<

1

THE SUGGESTED STEP SIZES ARE:

N = 1.00, H = .50, K = .50

==> DO YOU ACCEPT THESE SUGGESTIONS?

==> PLEASE ENTER 1 = YES, 2 = NO. <<<

1

\*\*\* PLEASE SELECT RUNS RULES. \*\*\*

- (1) C1,
- (2) C12,
- (3) C13,
- (4) C14,
- (5) C123,
- (6) C124,
- (7) C134,
- (8) C1234.

\*\*\* PLEASE ENTER YOUR OPTION (1-8)! \*\*\*

1

The optimization is then performed and the optimal solution is printed.

\*\*\* THE OPTIMAL POINT FOUND IS \*\*\*

N = 4.8188, H = 1.4222, K = 3.0357  
 LOSS = 4.012059

\*\*\* OPTIMIZATION ITERATION \*\*\*

I	N	H	K	LOSS
1	4	1.2945	2.9651	4.036269
2	5	1.3953	3.0701	4.012948
3	6	1.5103	3.2182	4.047357

\*\*\* THE OPTIMAL DESIGN IS:

N = 5, H = 1.39531, K = 3.07102

\*\*\* THE MINIMUM LOSS PER HOUR IS: 4.012948

An experiment is carried out by manually entering the cost and operating parameters, a new starting point, and step sizes. The interactive procedures are shown as follows.

\*\*\* INPUT COST PARAMETERS \*\*\*

DO YOU WANT TO INPUT FROM A FILE OR MANUALLY?  
 ==> PLEASE ENTER 1 = FILE, 2 = MANUALLY. <<<

2

\*\* INPUT VALUES OF DELTA, THETA, M, E, D, T, W, B, C \*\*  
 2 0.01 100 0.05 2 50 25 0.5 0.1

\*\* VALUES RECEIVED ARE:

DELTA =	2.0000	THETA =	.0100
M =	100.0000	E =	.0500
D =	2.0000	T =	50.0000
W =	25.0000	B =	.5000
C =	.1000		

==> ARE THESE DATA CORRECT?

==> PLEASE ENTER 1 = YES, 2 = NO. <<<

1

\*\*\* THE SUGGESTED STARTING POINT IS:

N = 5, H = 1.00, K = 3.00

==> DO YOU ACCEPT THIS POINT?

==> ENTER 1 = YES, 2 = NO. <<<

2

\*\*\* PLEASE INPUT NEW STARTING POINT \*\*\*



==> KEY IN VALUES FOR N, H, K  
4 2 2

\*\*\* NEW STARTING POINT IS:  
N = 4 H = 2.0000 K = 2.0000

==> ARE THEY CORRECT?  
==> PLEASE ENTER 1 = YES, 2 = NO. <<<

1

THE SUGGESTED STEP SIZES ARE:  
N = 1.00, H = .50, K = .50

==> DO YOU ACCEPT THESE SUGGESTIONS?  
==> PLEASE ENTER 1 = YES, 2 = NO. <<<

2

\*\*\* PLEASE INPUT NEW STEP SIZES \*\*\*  
==> PLEASE ENTER VALUES FOR N, H, K. <<<

1 1 0.5

\*\*\* NEW STEP SIZES ARE:  
N = 1.00, H = 1.00, K = .50

==> ARE THEY CORRECT?  
==> PLEASE ENTER 1 = YES, 2 = NO. <<<

1

\*\*\* PLEASE SELECT RUNS RULES. \*\*\*  
(1) C1,  
(2) C12,  
(3) C13,  
(4) C14,  
(5) C123,  
(6) C124,  
(7) C134,  
(8) C1234.

\*\*\* PLEASE ENTER YOUR OPTION (1-8)! \*\*\*

1

\*\*\* THE OPTIMAL POINT FOUND IS \*\*\*  
N = 4.7953, H = 1.3650, K = 3.0578  
LOSS = 4.011755

\*\*\* OPTIMIZATION ITERATION \*\*\*

I	N	H	K	LOSS
1	4	1.2618	2.9700	4.036453
2	5	1.4230	3.0887	4.012947
3	6	1.4941	3.2448	4.047721

\*\*\* THE OPTIMAL DESIGN IS:

N = 5, H = 1.42303, K = 3.08868  
 \*\*\* THE MINIMUM LOSS PER HOUR IS: 4.012947  
 -----

Economic Evaluation Of The X-bar Control

Chart With AT&T Runs Rules

A selection of "2" from the main menu leads to the economic evaluation of the X-bar control chart with AT&T runs rules. The interactive procedures are shown as follows.

\*\*\*\*\*  
 \*\*\* MAIN MENU \*\*\*  
 \*\*\*\*\*

- (1) ECONOMIC DESIGN OF X-BAR CONTROL CHARTS WITH AT&T RUNS RULES,
- (2) EVALUATION OF X-BAR CONTROL CHARTS WITH AT&T RUNS RULES,
- (3) EXIT THE PROGRAM.

==> PLEASE ENTER YOUR OPTION (1, 2, 3)! <<<

2

\*\*\* INPUT COST PARAMETERS \*\*\*

DO YOU WANT TO INPUT FROM A FILE OR MANUALLY?  
 ==> PLEASE ENTER 1 = FILE, 2 = MANUALLY. <<<

1

\*\* PLEASE INPUT THE FILENAME THAT CONTAINS THE COST PARAMETERS.

CASE1

\*\* VALUES RECEIVED ARE:

DELTA =	2.0000	THETA =	.0100
M =	100.0000	E =	.0500
D =	2.0000	T =	50.0000
W =	25.0000	B =	.5000
C =	.1000		

==> ARE THESE DATA CORRECT?  
 ==> PLEASE ENTER 1 = YES, 2 = NO. <<<

1

\*\*\* PLEASE INPUT N, H, K. \*\*\*

5 1.41 3.2

\*\*\* THE CONTROL CHART PARAMETERS ARE:

N = 5, H = 1.4100, K = 3.2000

==> ARE THEY CORRECT?

==> ENTER 1 = YES, 2 = NO. <<<

1

\*\*\* PLEASE INPUT # OF RUNS RULES! \*\*\*

4

\*\*\* PLEASE INPUT THE RULES! \*\*\*

\*\* INPUT K, M, A, B, FOR RULE 1:

1 1 3.2 9

\*\* INPUT K, M, A, B, FOR RULE 2:

2 3 2.134 3.2

\*\* INPUT K, M, A, B, FOR RULE 3:

2 3 -3.2 -2.134

\*\* INPUT K, M, A, B, FOR RULE 4:

1 1 -9 -3.2

\*\*\* THE FOLLOWING RULES ARE USED:

T( 1, 1, 3.0000, 9.0000)

T( 2, 3, 2.1340, 3.2000)

T( 2, 3, -3.2000, -2.1340)

T( 2, 3, -9.0000, -3.2000)

==> ARE THEY CORRECT?

==> PLEASE ENTER 1 = YES, 2 = NO. <<<

1

\*\*\* THE LOSS OF THE CURRENT DESIGN IS:

4.040455

One thing to note. When the computer program prompts the user to enter the runs rules used, it is noted that "9" is used to represent infinity ( $\infty$ ).

Economic Design And Evaluation  
Of The EWMA Chart

The interactive procedures of the economic design and evaluation of the EWMA chart follow are very similar to those of the X-bar control chart with AT&T runs rules. The procedures and result are shown as follows.

Economic Design Of The EWMA Chart

\*\*\*\*\*  
\*\*\* MAIN MENU \*\*\*  
\*\*\*\*\*

- (1) ECONOMIC DESIGN OF EWMA CONTROL CHART,
- (2) EVALUATION OF EWMA CONTROL CHART,
- (3) EXIT THE PROGRAM.

==> PLEASE ENTER YOUR OPTION (1, 2, 3)! <<<

1

\*\*\* INPUT COST PARAMETERS \*\*\*

DO YOU WANT TO INPUT FROM A FILE OR MANUALLY?

==> PLEASE ENTER 1 = FILE, 2 = MANUALLY. <<<

1

\*\* PLEASE INPUT THE FILENAME THAT  
CONTAINS THE COST PARAMETERS.

CASE1

\*\* VALUES RECEIVED ARE:

DELTA =	2.0000	THETA =	.0100
M =	100.0000	E =	.0500
D =	2.0000	T =	50.0000
W =	25.0000	B =	.5000
C =	.1000		

==> ARE THESE DATA CORRECT?

==> PLEASE ENTER 1 = YES, 2 = NO. <<<

1

\*\*\* THE SUGGESTED STARTING POINT IS:

N = 5, H = 1.00, K = 3.00, ALPHA = .5

==> DO YOU ACCEPT THIS POINT?  
 ==> ENTER 1 = YES, 2 = NO. <<<

1

THE SUGGESTED STEP SIZES ARE:  
 N = 1.00, H = .50, K = .50, ALPHA = .10

==> DO YOU ACCEPT THESE STEP SIZES?  
 ==> PLEASE ENTER 1 = YES, 2 = NO. <<<

1

\*\*\* THE OPTIMAL POINT FOUND IS \*\*\*  
 N = 4.7647, H = 1.4128, K = 3.0390, ALPHA = .9094  
 LOSS = 4.010300

\*\*\* OPTIMIZATION ITERATION \*\*\*

I	N	H	K	ALPHA	LOSS
1	4	1.3044	2.9515	.8895	4.029860
2	5	1.3956	3.1047	.9343	4.011464
3	6	1.5137	3.2176	.9576	4.047058

\*\*\* THE OPTIMAL DESIGN IS:  
 N = 5, H = 1.39564, K = 3.10466, ALPHA = .9343  
 \*\*\* THE MINIMUM LOSS PER HOUR IS: 4.011464

### Economic Evaluation Of The EWMA Chart

\*\*\*\*\*  
 \*\*\* MAIN MENU \*\*\*  
 \*\*\*\*\*

- (1) ECONOMIC DESIGN OF EWMA CONTROL CHART,
- (2) EVALUATION OF EWMA CONTROL CHART,
- (3) EXIT THE PROGRAM.

==> PLEASE ENTER YOUR OPTION (1, 2, 3)! <<<

2

\*\*\* INPUT COST PARAMETERS \*\*\*

DO YOU WANT TO INPUT FROM A FILE OR MANUALLY?  
 ==> PLEASE ENTER 1 = FILE, 2 = MANUALLY. <<<

1

\*\* PLEASE INPUT THE FILENAME THAT  
 CONTAINS THE COST PARAMETERS.

CASE1

\*\* VALUES RECEIVED ARE:

DELTA =	2.0000	THETA =	.0100
M =	100.0000	E =	.0500
D =	2.0000	T =	50.0000
W =	25.0000	B =	.5000
C =	.1000		

=> ARE THESE DATA CORRECT?

=> PLEASE ENTER 1 = YES, 2 = NO. <<<

1

\*\*\* PLEASE INPUT N, H, K, ALPHA. \*\*\*

5 1.3956 3.1047 0.9343

\*\*\* THE CONTROL CHART PARAMETERS ARE:

N =	5,	H =	1.3956,
K =	3.1047	ALPHA =	0.9343

=> ARE THEY CORRECT?

=> ENTER 1 = YES, 2 = NO. <<<

1

\*\*\* THE LOSS OF THE CURRENT DESIGN IS:

4.011464

Economic Design And Evaluation

Of The Zone Control Chart

Economic Design Of The

Zone Control Chart

\*\*\*\*\*  
 \*\*\* MAIN MENU \*\*\*  
 \*\*\*\*\*

- (1) ECONOMIC DESIGN OF ZONE CONTROL CHART,
- (2) EVALUATION OF ZONE CONTROL CHART,
- (3) EXIT THE PROGRAM.

=> PLEASE ENTER YOUR OPTION (1, 2, 3)! <<<

1

\*\*\* INPUT COST PARAMETERS \*\*\*

DO YOU WANT TO INPUT FROM A FILE OR MANUALLY?

=> PLEASE ENTER 1 = FILE, 2 = MANUALLY. <<<

1

\*\* PLEASE INPUT THE FILENAME THAT  
CONTAINS THE COST PARAMETERS.

CASE1

\*\* VALUES RECEIVED ARE:

DELTA =	2.0000	THETA =	.0100
M =	100.0000	E =	.0500
D =	2.0000	T =	50.0000
W =	25.0000	B =	.5000
C =	.1000		

=> ARE THESE DATA CORRECT?

=> PLEASE ENTER 1 = YES, 2 = NO. <<<

1

\*\*\* THE SUGGESTED STARTING POINT IS:

N = 5, H = 1.00, K = 3.00,  
S(1) = .0, S(2) = 1.0, S(3) = 2.0, S(4) = 15.0

=> DO YOU ACCEPT THIS POINT?

=> ENTER 1 = YES, 2 = NO. <<<

1

THE SUGGESTED STEP SIZES ARE:

N = 1.00, H = .50, K = .50,  
S(1) = 1.0, S(2) = 1.0, S(3) = 1.0, S(4) = 1.0

=> DO YOU ACCEPT THESE STEP SIZES?

=> PLEASE ENTER 1 = YES, 2 = NO. <<<

1

\*\*\* THE OPTIMAL POINT FOUND IS \*\*\*

N = 4.8105, H = 1.3688, K = 3.0620,  
SCORE 1 = .0, SCORE 2 = 1.2,  
SCORE 3 = 2.3, SCORE 5 = 15.1,  
LOSS = 4.011720

\*\*\* OPTIMIZATION ITERATION \*\*\*

I	N	H	K	S(1)	S(2)	S(3)	S(4)	LOSS
1	4	1.2803	2.9432	0	1	2	15	4.036212
2	5	1.4256	3.0853	0	1	2	15	4.012943
3	6	1.4973	3.2057	0	1	2	16	4.047313

\*\*\* THE OPTIMAL DESIGN IS:

N = 5, H = 1.42556, K = 3.08534,  
SCORE 1 = 0, SCORE 2 = 1,  
SCORE 3 = 2, SCORE 4 = 15,

\*\*\* THE MINIMUM LOSS PER HOUR IS: 4.012943

Economic Evaluation Of The

Zone Control Chart

\*\*\*\*\*  
 \*\*\* MAIN MENU \*\*\*  
 \*\*\*\*\*

- (1) ECONOMIC DESIGN OF ZONE CONTROL CHART,  
 (2) EVALUATION OF ZONE CONTROL CHART,  
 (3) EXIT THE PROGRAM.

==> PLEASE ENTER YOUR OPTION (1, 2, 3)! <<<

2

\*\*\* INPUT COST PARAMETERS \*\*\*

DO YOU WANT TO INPUT FROM A FILE OR MANUALLY?

==> PLEASE ENTER 1 = FILE, 2 = MANUALLY. <<<

1

\*\* PLEASE INPUT THE FILENAME THAT  
 CONTAINS THE COST PARAMETERS.

CASE1

\*\* VALUES RECEIVED ARE:

DELTA =	2.0000	THETA =	.0100
M =	100.0000	E =	.0500
D =	2.0000	T =	50.0000
W =	25.0000	B =	.5000
C =	.1000		

==> ARE THESE DATA CORRECT?

==> PLEASE ENTER 1 = YES, 2 = NO. <<<

1

\*\*\* PLEASE INPUT N, H, K, SCORE 1,  
 SCORE 2, SCORE 3, SCORE 4.\*\*\*

5 1.42 3.1 0 1 2 15

\*\*\* THE CONTROL CHART PARAMETERS ARE:

N =	5,	H =	1.4200,	K =	3.1000
SCORE 1 =	0,	SCORE 2 =	1,		
SCORE 3 =	2,	SCORE 4 =	15,		

==> ARE THEY CORRECT?

==> ENTER 1 = YES, 2 = NO. <<<

1

\*\*\* THE LOSS OF THE CURRENT DESIGN IS:  
 4.013132



### Summary

Most of the features of the interactive computer programs of this research have been illustrated in this chapter. Examples are given to describe the capabilities of the programs. These programs provide easy and convenient approaches to the design and evaluation of the (1) X-bar control chart with AT&T runs rules, (2) EWMA chart, and (3) Zone control chart. Therefore, these programs are useful tools for both practitioners and theoreticians.

## CHAPTER V

### RESULTS COMPARISON AND ANALYSES

#### Introduction

This chapter provides the economic comparisons and analyses among the (1) X-bar control chart with AT&T runs rules, (2) EWMA chart, and (3) Zone control chart. In order to economically compare the optimal designs of these three variables charts, 22 out of 25 examples from Duncan's (1956) paper are selected for study. These 22 examples are listed in Table 5.1. The example numbers used in this research correspond to the original example numbers in Duncan's paper.

Sensitivity analyses are performed using designed experiments (DOE). The Plackett-Burman design with 12 runs (L12), the Taguchi L12 design, the  $2^{9-1}$  fractional factorial (FF) design, and the Central Composite Faced Design (CCFD) (Schmidt and Launsby, 1989) are employed to compare and verify the validity of the analyses and to obtain the prediction equations for the optimal design parameters and the resulting operating loss. Duncan's (1956) example 1 is chosen as the basis or real environment for study and illustration.

TABLE 5.1

## COST AND OPERATING PARAMETERS OF DUNCAN'S 22 EXAMPLES

Ex no.	Cost and Operation Parameters								
	$\delta$	$\theta$	M	e	D	T	W	b	c
1	2	0.01	100	0.05	2	50	25	0.5	0.1
2	2	0.02	100	0.05	2	50	25	0.5	0.1
3	2	0.03	100	0.05	2	50	25	0.5	0.1
4	2	0.01	50	0.05	2	50	25	0.5	0.1
5	2	0.01	1000	0.05	2	50	25	0.5	0.1
6	2	0.01	10000	0.05	2	50	25	0.5	0.1
7	2	0.01	100	0.50	2	50	25	0.5	0.1
8	2	0.01	100	0.05	20	50	25	0.5	0.1
9	2	0.01	100	0.05	2	5	2.5	0.5	0.1
10	2	0.01	100	0.05	2	500	250	0.5	0.1
12	2	0.01	100	0.05	2	50	25	5.0	0.1
13	2	0.01	100	0.05	2	50	25	0.5	1.0
14	2	0.01	100	0.05	2	50	25	0.5	10
15	2	0.01	1000	0.05	2	50	25	0.5	1.0
16	1	0.01	12.87	0.05	2	50	25	0.5	0.1
17	1	0.01	128.7	0.05	2	50	25	0.5	0.1
18	1	0.01	12.87	0.05	2	500	250	0.5	0.1
19	1	0.01	12.87	0.05	2	50	25	5.0	0.1
20	1	0.01	12.87	0.05	2	50	25	0.5	1.0
21	.5	0.01	2.25	0.05	2	50	25	0.5	0.1
22	.5	0.01	225	0.05	2	50	25	0.5	0.1
24	.5	0.01	2.25	0.05	2	50	25	5.0	0.1

TABLE 5.1 (Continued)

Ex no.	Remarks
1	Basis
2	Same as #1 except theta increased
3	Same as #1 except theta increased
4	Same as #1 except M decreased
5	Same as #1 except M increased
6	Same as #1 except M increased
7	Same as #1 except e increased
8	Same as #1 except D increased
9	Same as #1 except T & W decreased
10	Same as #1 except T & W increased
12	Same as #1 except b increased
13	Same as #1 except c increased
14	Same as #1 except c increased
15	Same as #1 except M & c increased
16	Same as #1 except $\delta$ & M decreased
17	Same as #16 except M increased
18	Same as #16 except T & W increased
19	Same as #16 except b increased
20	Same as #16 except c increased
21	Same as #1 except $\delta$ & M decreased
22	Same as #21 except M increased
24	Same as #21 except b increased

## Verification Of The Employed Search Technique

It is noted that the Shewhart X-bar control chart is equivalent to the X-bar control chart with AT&T rule one (i.e., RULE C1) only. The economic design of these three variables control charts in this research uses Duncan's (1956) approach to the economic design of the X-bar control chart. Therefore, the results of the economically-based X-bar control chart with RULE C1 obtained from the economic model developed in this research should be at least as good as those of Duncan's.

In order to get his results, Duncan (1956) simplifies his model by making some assumptions. This makes Duncan's model become an approximation. However, Duncan does provide exact solutions for some of his examples. Goel, Jain, and Wu (1968), and Koo (1987) provide the losses of part or all of those 25 examples and show that there are errors in Duncan's (1956) calculations. Even though Goel, Jain, and Wu (1968) provide an algorithm to calculate the loss of the economically-based X-bar control chart, Duncan's (1956) assumptions in estimating the expected length of a production cycle are still applied in their algorithm. Only Koo (1987) provides the results of the economically based X-bar control chart using Duncan's (1956) exact model. Therefore, Koo's calculations are used to compare against the results obtained from the proposed model in

order to verify the adequacy of the search technique discussed in chapter III.

The losses of these 22 examples obtained from Koo's calculations and the calculations using the proposed model are listed in Table 5.2. It is observed from Table 5.2 that the results obtained are very close. The percentages of differences for all 22 examples are less than 0.005% which is small enough to be neglected. All the results obtained from the proposed model show smaller losses than both Duncan's and Goel, Jain, and Wu's results (Note that both Koo and Goel et al. show that the exact result of example 8 given by Duncan is not correct. The correct value is given by Koo). Therefore, it can be said that the search procedures employed in this research are adequate.

Economic Comparison Among The X-bar  
Control Chart With AT&T Runs Rules,  
The EWMA Chart, And The  
Zone Control Chart

In order to provide an economic comparison among the (1) X-bar control chart with AT&T runs rules, (2) EWMA chart, and (3) Zone control chart, the 22 examples listed in Table 5.1 are considered. Table 5.3 shows the optimal design parameters and the losses of the economically-based X-bar control chart with AT&T runs rules; Table 5.4 shows the results of the EWMA chart; and Table 5.5 shows the results of the Zone control chart. Table 5.6 shows the

TABLE 5.2

COMPARISON OF THE LOSSES OBTAINED FROM KOO'S CALCULATION  
AND THE CALCULATION FROM THE PROPOSED MODEL(\*)

Ex. no.	n	h	k	Loss
1	5	1.4032	3.0853	4.0128 *
	5	1.3953	3.0701	4.0129
2	5	1.0216	3.0787	6.9460 *
	5	1.0045	3.0695	6.9464
3	4	0.7832	2.9366	9.5924 *
	4	0.7920	2.9448	9.5926
4	5	1.4617	3.0713	4.1527 *
	5	1.4678	3.0877	4.1529
5	4	0.4050	2.9574	26.9753 *
	4	0.3970	2.9705	26.9763
6	2	0.0913	2.6914	228.8060 *
	2	0.0903	2.7005	228.8069
7	2	0.9385	2.6856	5.4005 *
	2	0.9491	2.6859	5.4007
8	5	1.6554	3.0575	18.3716 *
	5	1.6988	3.0522	18.3720
9	3	1.2650	2.2082	3.6087 *
	3	1.2600	2.2000	3.6087
10	6	1.4527	3.6731	6.3670 *
	6	1.4709	3.6744	6.3671
12	6	3.4650	2.8777	5.8669 *
	6	3.4309	2.8831	5.8670

Note: "\*" represent results from Koo (1987).

TABLE 5.2 (Continued)

Ex. no.	n	h	k	Loss
13	3	2.5963	2.4243	5.6313 *
	3	2.5973	2.4237	5.6313
14	1	4.6928	1.4424	9.8733 *
	1	4.7086	1.4474	9.8733
15	3	0.8120	2.4257	31.7500 *
	3	0.8299	2.4305	31.7524
16	14	5.4897	2.6754	1.4159 *
	14	5.5014	2.6671	1.4159
17	11	1.4552	2.5962	6.2759 *
	11	1.4579	2.5948	6.2759
18	21	7.1429	3.3953	3.6409 *
	21	7.1431	3.3957	3.6409
19	18	11.0205	2.5451	1.9551 *
	18	11.1019	2.5556	1.9551
20	8	12.3708	1.8864	2.4207 *
	8	12.2994	1.8440	2.4207
21	38	23.5481	2.1258	0.8308 *
	38	23.1217	2.1700	0.8309
22	20	1.2451	2.1053	13.5571 *
	20	1.2556	2.1073	13.5571
24	45	37.4997	2.0253	0.9772 *
	46	38.2675	2.0250	0.9772

Note: "\*" represent results from Koo (1987).



TABLE 5.3

RESULTS OF THE 22 EXAMPLES OF THE ECONOMICALLY-BASED  
X-BAR CONTROL CHART WITH AT&T RUNS RULES

Ex. no.	n	h	k	RULE	Loss
1	5	1.3953	3.0701	C1	4.0129
2	5	1.0045	3.0695	C1	6.9464
3	4	0.7920	2.9448	C1	9.5926
4	5	1.4678	3.0877	C1	4.1529
5	4	0.3970	2.9705	C1	26.9763
6	2	0.0900	2.9200	C12	227.7351
7	2	0.8608	2.9952	C12	5.2894
8	5	1.6988	3.0522	C1	18.3720
9	3	1.2600	2.2000	C1	3.6087
10	6	1.4709	3.6744	C1	6.3671
12	6	3.4309	2.8831	C1	5.8670
13	3	2.5970	2.4237	C1	5.6313
14	1	4.7086	1.4474	C1	9.8734
15	3	0.8299	2.4305	C1	31.7524
16	14	5.5014	2.6671	C1	1.4159
17	11	1.4579	2.5948	C1	6.2759
18	21	7.1431	3.3957	C1	3.6409
19	18	11.1019	2.5556	C1	1.9551
20	8	12.2994	1.8440	C1	2.4207
21	38	23.1217	2.1700	C1	0.8309
22	17	0.9519	2.4344	C12	13.3473
24	46	38.2675	2.0250	C1	0.9772

TABLE 5.4  
RESULTS OF THE 22 EXAMPLES OF THE  
ECONOMICALLY-BASED EWMA CHART

Ex. no.	n	h	k	$\alpha$	Loss
1	5	1.3956	3.1047	0.9394	4.0114
2	4	0.9228	2.9895	0.9099	6.9434
3	4	0.7930	2.9491	0.9053	9.5823
4	5	1.4361	3.0902	0.9490	4.1514
5	4	0.4102	2.9671	0.9032	26.9545
6	2	0.0817	2.8540	0.6938	227.3700
7	2	0.8665	2.8441	0.6836	5.2616
8	5	1.6900	3.0500	0.9500	18.3708
9	3	1.2696	2.2152	0.9042	3.6067
10	6	1.4729	3.6877	0.9175	6.3633
12	6	3.4342	2.8676	0.9855	5.8670
13	3	2.5955	2.4318	0.8950	5.6196
14	1	4.4659	1.4920	0.7073	9.7683
15	3	0.7968	2.4609	0.8930	31.7120
16	14	5.4500	2.7000	0.9000	1.4131
17	11	1.4171	2.6412	0.8267	6.2431
18	20	6.8600	3.4000	0.9100	3.6379
19	18	11.0911	2.5398	0.9593	1.9549
20	7	11.1981	1.8823	0.8327	2.4098
21	37	23.2301	2.1827	0.8531	0.8283
22	11	0.6139	2.4288	0.3976	12.9841
24	45	37.9068	2.0168	0.9016	0.9768

TABLE 5.5  
RESULTS OF THE 22 EXAMPLES OF THE ECONOMICALLY-BASED  
ZONE CONTROL CHART

Ex. no.	n	h	k	S1	S2	S3	S4	Loss
1	5	1.4256	3.0853	0	1	2	15	4.0129
2	5	1.0255	3.0693	0	1	2	19	6.9460
3	4	0.7863	2.9440	0	1	4	16	9.5922
4	5	1.4588	3.0739	0	1	2	19	4.1526
5	4	0.4131	2.9443	0	1	4	16	26.9756
6	2	0.0836	2.8615	0	1	8	16	227.9304
7	2	0.8906	2.7880	0	1	6	18	5.3370
8	5	1.6603	3.0568	0	1	2	23	18.3716
9	3	1.2652	2.2088	0	1	2	21	3.6087
10	6	1.4619	3.6870	0	1	8	24	6.3663
12	6	3.4294	2.8872	0	1	2	20	5.8670
13	3	2.6143	2.4218	0	1	2	20	5.6313
14	1	4.7352	1.4374	0	1	2	23	9.8733
15	3	0.8290	2.4259	0	1	2	22	31.7518
16	14	5.4827	2.6764	0	1	2	19	1.4159
17	11	1.4319	2.6232	0	1	7	21	6.2700
18	20	7.1315	3.3616	0	1	7	21	3.6406
19	18	11.1092	2.5563	0	1	2	21	1.9551
20	8	12.2589	1.8849	0	1	2	24	2.4208
21	38	23.1331	2.1688	0	1	2	23	0.8309
22	16	0.9258	2.4383	0	1	8	16	13.2636
24	45	38.0622	2.0122	0	1	2	24	0.9772

TABLE 5.6  
LOSSES OF THREE VARIABLES CONTROL CHARTS

Ex. No.	X-bar Chart With AT&T Runs Rules	EWMA Chart	Zone Chart
1	4.0129	4.0114	4.0129
2	6.9464	6.9434	6.9460
3	9.5926	9.5823	9.5922
4	4.1529	4.1514	4.1526
5	26.9763	26.9545	26.9756
6	227.7351	227.3700	227.9304
7	5.2894	5.2616	5.3370
8	18.3720	18.3708	18.3716
9	3.6087	3.6067	3.6087
10	6.3671	6.3633	6.3663
12	5.8670	5.8670	5.8670
13	5.6313	5.6196	5.6313
14	9.8734	9.7683	9.8733
15	31.7524	31.7120	31.7518
16	1.4159	1.4131	1.4159
17	6.2759	6.2431	6.2700
18	3.6409	3.6379	3.6406
19	1.9551	1.9549	1.9551
20	2.4207	2.4098	2.4208
21	0.8309	0.8283	0.8309
22	13.3473	12.9841	13.2636
24	0.9772	0.9768	0.9772

losses of these three variables control charts in order to make a clear comparison.

It is observed from Tables 5.3 to 5.6 that

- (1) The economically-based EWMA chart is superior to both the economically-based X-bar control chart with AT&T runs rules and the economically-based Zone control chart.
- (2) The economically-based Zone control chart performs better than, or as good as, the economically-based X-bar control chart with AT&T runs rules in 16 out of 22 examples.
- (3) In the economic design of the X-bar control chart with AT&T runs rules, 19 out of 22 examples which yield smaller losses use RULE C1. The other three use C12.
- (4) The  $\alpha$  values in all 22 examples are large. The smallest  $\alpha$  value is approximately 0.4, which occurs in example 22.
- (5) In the economic design of the Zone control chart, 0 and 1 are used as zone score 1 (S1) and zone score 2 (S2), respectively, in all 22 examples.

Some other observations are:

- (1) Three out of 22 examples of the economically-based X-bar chart use RULE C12; others use C1, as shown in Table 5.3. Among these three examples, two of them have relatively high penalty cost, M. This can be seen by comparing example 1 with example 6 and example 21 with example 22. The increase in M is 100 times. The other

example using C12 has a large value for the delay factor ( $e$ ). For example, comparing example 1 with example 7, the increase in  $e$  is 10 times. Therefore, it can be said that if the value of the penalty cost or delay factor is relatively large, more rules should be considered for use in combination with an X-bar chart.

- (2) Examining Table 5.3, it is found that those examples using C12 have values of  $\delta\sqrt{n}$  fairly close to the width of the control limits. Others have values of  $\delta\sqrt{n}$  greater than the width of the control limits.

Examining the statistical property (power of detection of a shift in the process mean) of those three examples using C12, it is found that, based on the same ARL1, RULE C12 produces the smallest ARL2 among all 8 combinations of runs rules.

- (3) The  $\alpha$  values of the economically-based EWMA chart are large in all 22 examples. Even though there is not any recommendation being made regarding the selection of an  $\alpha$  value, the most often used  $\alpha$  values in industry ranges from 0.15 to 0.3. Table 5.4 shows that the economically optimal  $\alpha$  values differ noticeably from the commonly used values.

- (4) The economically-based ZCC has similar performance to the economically-based X-bar control chart with AT&T runs rules. But, the ZCC has the advantage of flexibility. The number of the combinations of the four AT&T runs rules is limited; however, the combinations

of the four zone scores are virtually infinite.

Sensitivity Analyses Of The Effects  
Of Variation In Cost And  
Operating Parameters On  
The Optimal Design

In the last four decades, researchers have conducted sensitivity analyses regarding the effects of variation in the cost and operating parameters on the economic design parameters and the resulting operating loss, following Duncan's (1956) approach. Duncan's approach is a one-factor-at-a-time type of analysis. This type of analysis is known to be highly inefficient because only one factor is changed at one run while all the others are kept constant. Once an optimal solution is found for that factor, it is held at that value and another factor is manipulated. Therefore, the design is not orthogonal and the traditional statistical methods can not be employed for analysis. The one-factor-at-a-time analysis also assumes no interaction between variables, and prediction equations for responses are not available.

Panagos, Heikes, and Montgomery (1985) conduct a  $2^9-4$  fractional factorial experiment to study the effects of the cost and operating parameters. The traditional analysis of variance (ANOVA) approach is adopted. Panagos et al. assume that the interactions between variables can be neglected. The emphasis is put on the study of main effects.

Therefore, all the unused terms are pooled together to estimate the error term. This error term is then used to evaluate the significance of main effects. No prediction equations are given. One question then arises. Since the experiment is conducted using a computer and the economic model is fixed, there is no variation in the responses if replications are considered. Therefore, there is no meaningful explanation as what the error term means. Also, regardless of whether it is a one-factor-at-a-time type of analysis or a fractional factorial analysis, the sensitivity analysis is always conducted using the following procedures. After the level of each factor is determined, the associated optimal design parameters and loss associated with this particular design are obtained. The sensitivity analysis is then carried out using all the data so obtained. This approach is appropriate for the optimal design parameters; however, it is not appropriate for the resulting operating loss.

Collins, Case, and Bennett (1978) propose a more realistic and reasonable approach for conducting the sensitivity analysis regarding the resulting operating loss. They propose that whenever the level of each factor is determined, a set of optimal design parameters can be obtained. This set of design parameters is then implemented into the "real" environment and an associated resulting operating loss is obtained. The sensitivity analysis is then conducted based on the losses so obtained. This



research adopts this latter approach to sensitivity analysis.

#### Procedures For Sensitivity Analysis

The procedures for conducting the sensitivity analyses are outlined as follows:

- (1) Determine a designed experiment to examine sensitivity.
- (2) Determine the levels for each factor (cost and operating parameters) and the total number of experimental runs.
- (3) Find optimal design parameters for each run.
- (4) Implement the optimal design parameters found in (3) into the "real" environment and determine the loss associated with operating the system.
- (5) Conduct the sensitivity analysis based on the data obtained in (3) and (4).

#### DOE Techniques Employed

Duncan's (1956) example 1 is chosen as the basis or "real" environment. There are 9 factors to be studied in this research. Following Panagos, Heikes, and Montgomery's (1985) work, main effects are of concern during the first stage of the sensitivity study. Therefore, some simple designs are employed initially. Schmidt and Launsby (1989) point out that the Plackett-Burman (P-B) designs are developed for evaluating main effects with few or no interactions of interest. Also, if one's objective is to

screen out factors which are thought to be important using 2-level unreplicated designs, then the non-geometric P-B designs are more efficient than the geometric P-B designs. The disadvantage of the non-geometric P-B designs is that the confounding patterns are not available. Since it is assumed initially that the interaction between variables are not of interest, a non-geometric P-B design with 12 runs (L12) (Plackett and Burman, 1946; Schmidt and Launsby, 1989) is employed for study.

In order to ensure that the results from different analysis techniques are indifferent, a theoretically equivalent Taguchi L12 design (Kacker, Lagergren, and Filliben, 1991) is also employed. It is shown later that the conclusions regarding the control chart design parameters are almost the same as those obtained from the P-B L12 design; however, the conclusion regarding the loss is different. This suggests that the interactions or non-linear effects should be explicitly considered.

The disadvantage of the P-B L12 and Taguchi L12 designs is that the confounding patterns are not available. Therefore, the information obtained previously can not be utilized for further experimentation. In order to study the effects of all the 2-way interactions, a design with resolution V is desired. Therefore, a  $2^{9-1}$  FF experiment is carried out at the second stage. A central composite faced design (CCFD) is also carried out to improve the analysis and prediction equations obtained in the  $2^{9-1}$  FF

experiment. The results indicate that a designed experiment with 2 levels for each factor is sufficient for conducting the sensitivity analysis.

For the P-B L12 design and Taguchi L12 design, the rates of error of estimation of the 9 cost and operating parameters are assumed to be  $\pm 10\%$ ,  $\pm 30\%$ , and  $\pm 50\%$  away from the true values. Due to the fact that the conclusions regarding the effects of the cost and operating parameters on the design parameters are the same no matter what rate of error of estimation is used, only the example with  $\pm 30\%$  error of estimation is used for illustration. When the  $2^{9-1}$  FF design and CCFD (using the results of the  $2^{9-1}$  experiment in combination with 19 more runs) are employed, only the case with  $\pm 30\%$  error of estimation on the cost and operating parameters is conducted.

#### Plackett-Burman L12 Design

Table 5.7 shows the design matrix of the P-B L12 design with both coded and non-coded values. Duncan's (1956) example 1 is chosen for demonstration. Therefore, the true values of the 9 cost and operating parameters are:  $\delta=2.0$ ,  $\theta=0.01$ ,  $M=100$ ,  $e=0.05$ ,  $D=2.0$ ,  $T=50$ ,  $W=25$ ,  $b=0.5$ , and  $c=0.1$ .

The X-bar Control Chart With AT&T Runs Rules. Table 5.8 shows the optimal design parameters and resulting operating losses in the "real" environment of these 12



TABLE 5.8

THE OPTIMAL DESIGN PARAMETERS AND THE RESULTING  
OPERATING LOSSES OF THE X-BAR CHART  
WITH AT&T RUNS RULES

#	n	h	k	RULE	Loss	% increase in loss
1	3	1.5049	3.0244	C1	4.3696	8.88883
2	3	1.516	2.9988	C1	4.3538	8.49510
3	7	1.2117	2.6879	C1	4.3936	9.48690
4	3	1.0737	3.2898	C1	4.4136	9.98529
5	3	1.1183	3.1179	C1	4.2788	6.62613
6	4	1.0827	3.4203	C1	4.1699	3.91238
7	7	1.1532	2.8341	C1	4.328	7.85217
8	8	1.6149	2.7827	C1	4.3205	7.66527
9	8	2.5642	2.8347	C1	4.3954	9.53176
10	3	1.7163	3.2233	C1	4.7518	18.4131
11	9	1.6949	3.0092	C1	4.3388	8.12130
12	9	1.9655	2.9434	C1	4.3569	8.57235
basis	5	1.3953	3.0701	C1	4.0129	0

Note: The resulting operating losses are obtained by implementing the optimal design parameters into the "real" environment.

runs. Instead of applying the traditional analysis of variance (ANOVA) approach and using the F-test, this research uses half effect plots (Schmidt and Launsby, 1989) to identify the important effects. The 9 cost and operating parameters are treated as the independent variables in the experimentation. The optimal design parameters and the resulting operating losses are the responses (dependent variables). The half effects are obtained by calculating half the difference between the mean responses of the high levels and low levels for each factor. The half effect plot is obtained by first taking the absolute values of those 9 half effects, then plotting the half effects versus the corresponding factors in a descending order.

Figures 5.1 - 5.4 show the half effect plots of  $n$ ,  $h$ ,  $k$ , and loss, respectively, of the X-bar chart with AT&T runs rules. It is observed that

- (1) The optimal subgroup size,  $n$ , is primarily determined by  $\delta$ , the magnitude of shift in the process mean measured in terms of the number of process standard deviations.
- (2) The values of  $\delta$ ,  $\theta$ ,  $M$ ,  $b$ , and  $c$  have significant effects on the optimal sampling interval,  $h$ .
- (3) The value of  $\delta$ ,  $e$ ,  $T$ , and  $c$  have effects on the optimal width of control limits,  $k$ .
- (4) All 9 factors have effects on the resulting loss.
- (5) RULE C1 is used (optimal) in all 12 runs.

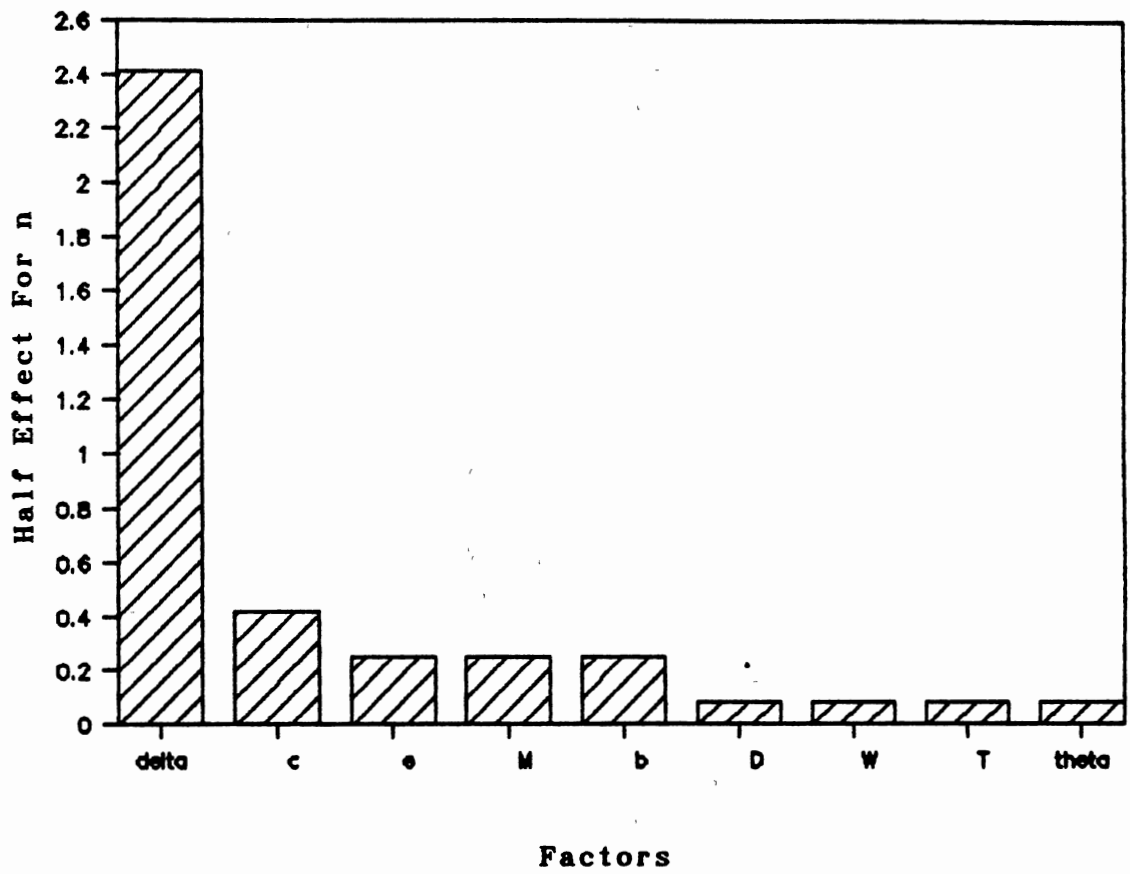


Figure 5.1 Half Effect Plot For n Using The X-bar Chart With AT&T Runs Rules

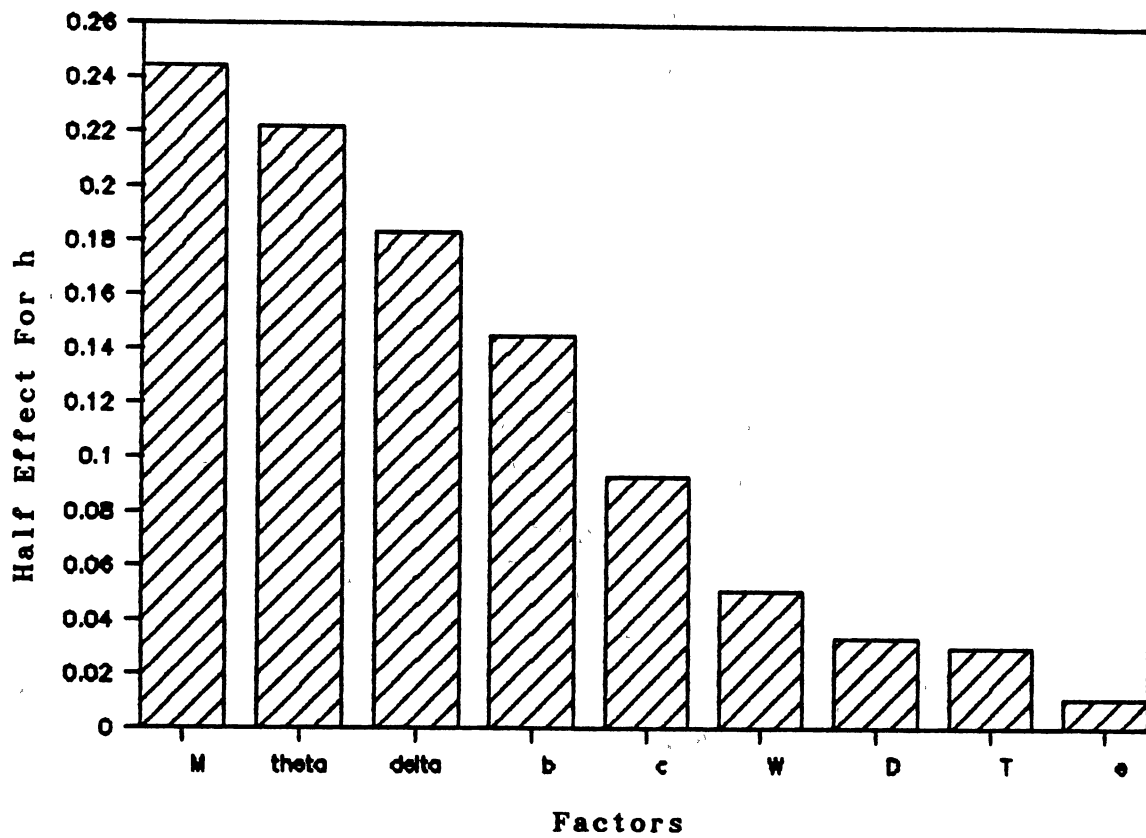


Figure 5.2 Half Effect Plot For h Using The X-bar Chart With AT&T Runs Rules



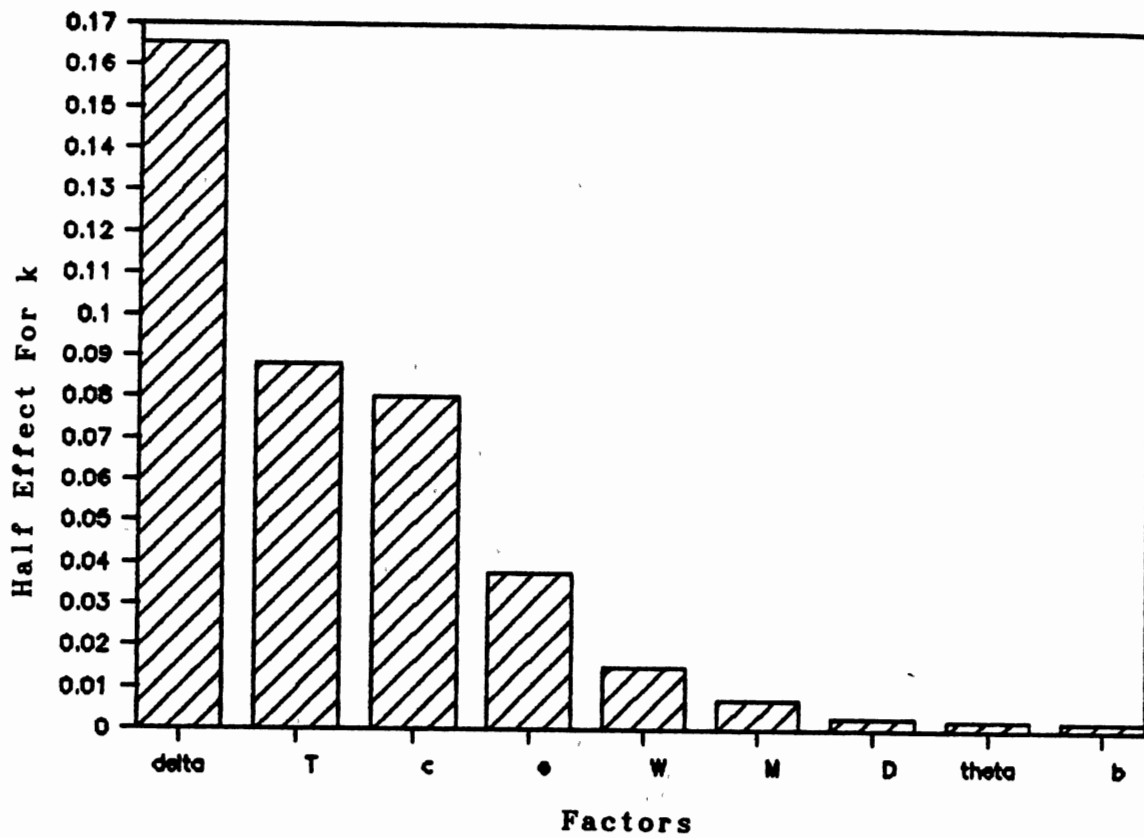


Figure 5.3 Half Effect Plot For k Using The X-bar Chart With AT&T Runs Rules

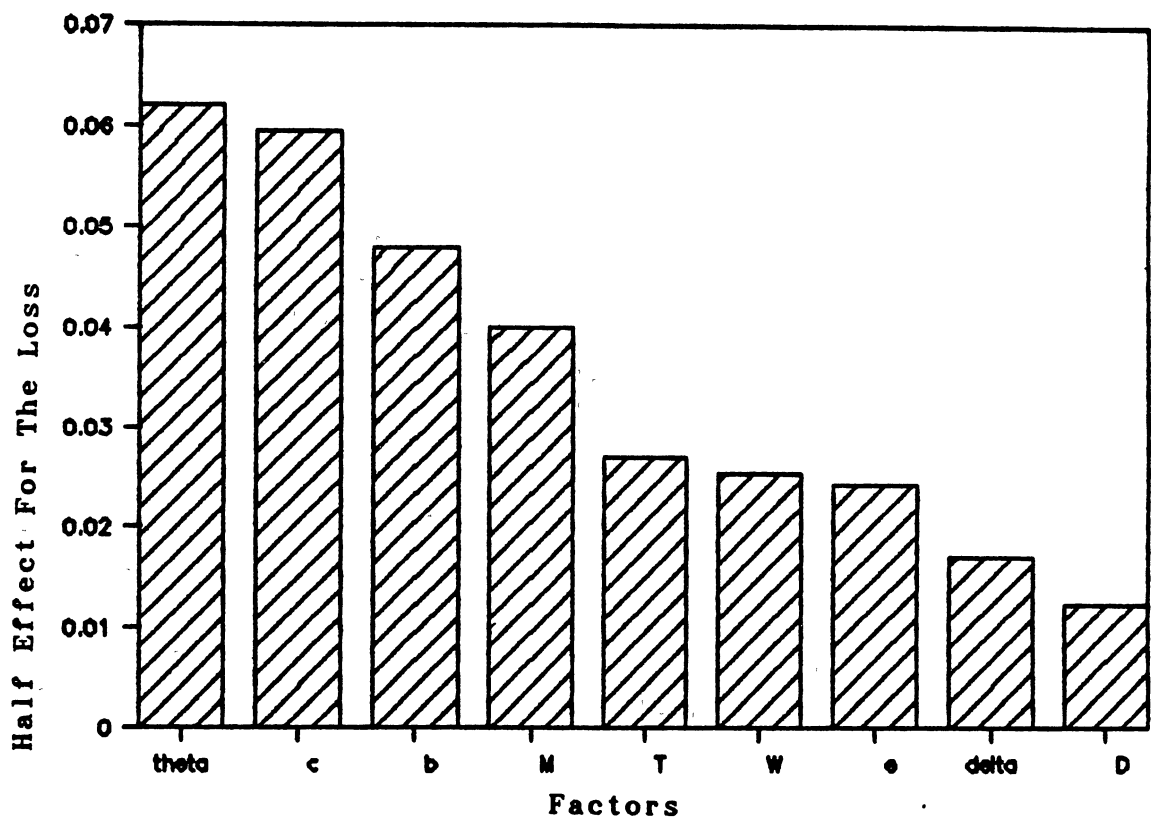


Figure 5.4 Half Effect Plot For The Losses Using The X-bar Chart With AT&T Runs Rules

The EWMA Chart. Table 5.9 shows the optimal design parameters and the resulting losses of the economically-based EWMA chart. Figures 5.5 - 5.9 show the half effect plots of  $n$ ,  $h$ ,  $k$ ,  $\alpha$ , and loss, respectively. The conclusions are:

- (1) The value of  $\delta$  has a major effect on subgroup size,  $n$ .
- (2) The values of  $\delta$ ,  $\theta$ ,  $M$ ,  $b$ , and  $c$  have significant effects on the optimal sampling interval,  $h$ .
- (3) The values of  $\delta$ ,  $e$ ,  $T$ , and  $c$  have effects on the optimal width of control limits,  $k$ .
- (4) Except for  $D$  and  $\theta$ , all the other 7 factors have effects on the weight,  $\alpha$ . However,  $\delta$  has a major effect on  $\alpha$ .
- (5) Except for  $\delta$ , all the other 8 factors have effects on the resulting loss.

Other than the observations above, it is also found that  $\alpha$  increases when  $\delta\sqrt{n}$  increases.

The Zone Control Chart. Table 5.10 shows the optimal design parameters and the resulting losses of the economically-based ZCC. Figures 5.10 - 5.13 show the half effect plots for  $n$ ,  $h$ ,  $k$ , and loss. The conclusions are:

- (1) The value of  $\delta$  has a major effect on subgroup size,  $n$ .
- (2) The values of  $\delta$ ,  $\theta$ ,  $M$ ,  $b$ , and  $c$  have noticeable effects on the optimal sampling interval,  $h$ .

TABLE 5.9

THE OPTIMAL DESIGN PARAMETERS AND THE RESULTING  
OPERATING LOSSES OF THE EWMA CHART

#	n	h	k	alpha	Loss	% increase in loss
1	3	1.499	2.9996	0.9295	4.302	7.24435
2	3	1.5201	3.0017	0.9525	4.3252	7.82270
3	7	1.1594	2.7191	0.885	4.4028	9.75719
4	3	1.0549	3.3046	0.9209	4.3381	8.14428
5	3	1.1092	3.1565	0.9319	4.2535	6.03529
6	4	1.0635	3.3861	0.9329	4.1285	2.91918
7	6	1.0343	2.8523	0.82	4.2715	6.48402
8	7	1.5222	2.7365	0.8931	4.2535	6.03529
9	8	2.5443	2.8743	0.9011	4.3815	9.22620
10	3	1.7241	3.2328	0.9034	4.6478	15.8647
11	9	1.6628	3.0133	0.916	4.3401	8.19414
12	8	1.8478	2.8606	0.9194	4.2793	6.67846
base	5	1.3956	3.1047	0.9343	4.0114	0

Note: The resulting operating losses are obtained by implementing the optimal design parameters into the "real" environment.

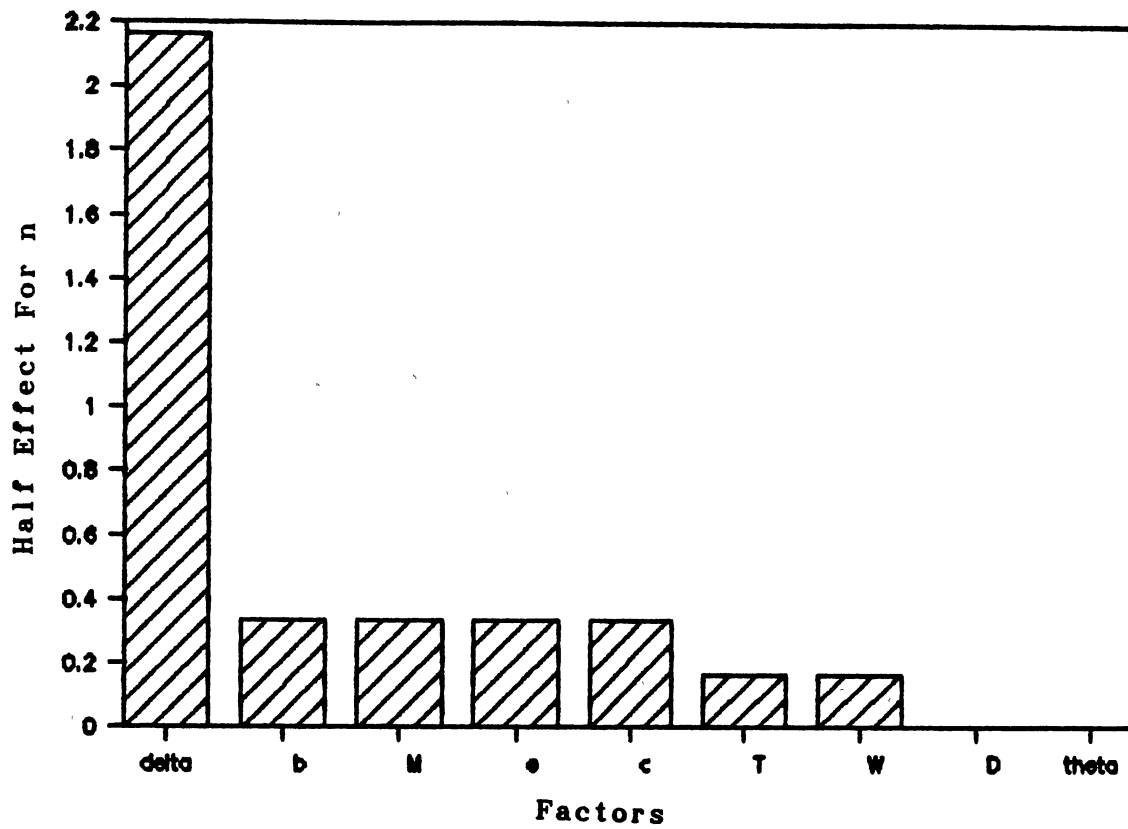


Figure 5.5 Half Effect Plot For n Using The EWMA Chart

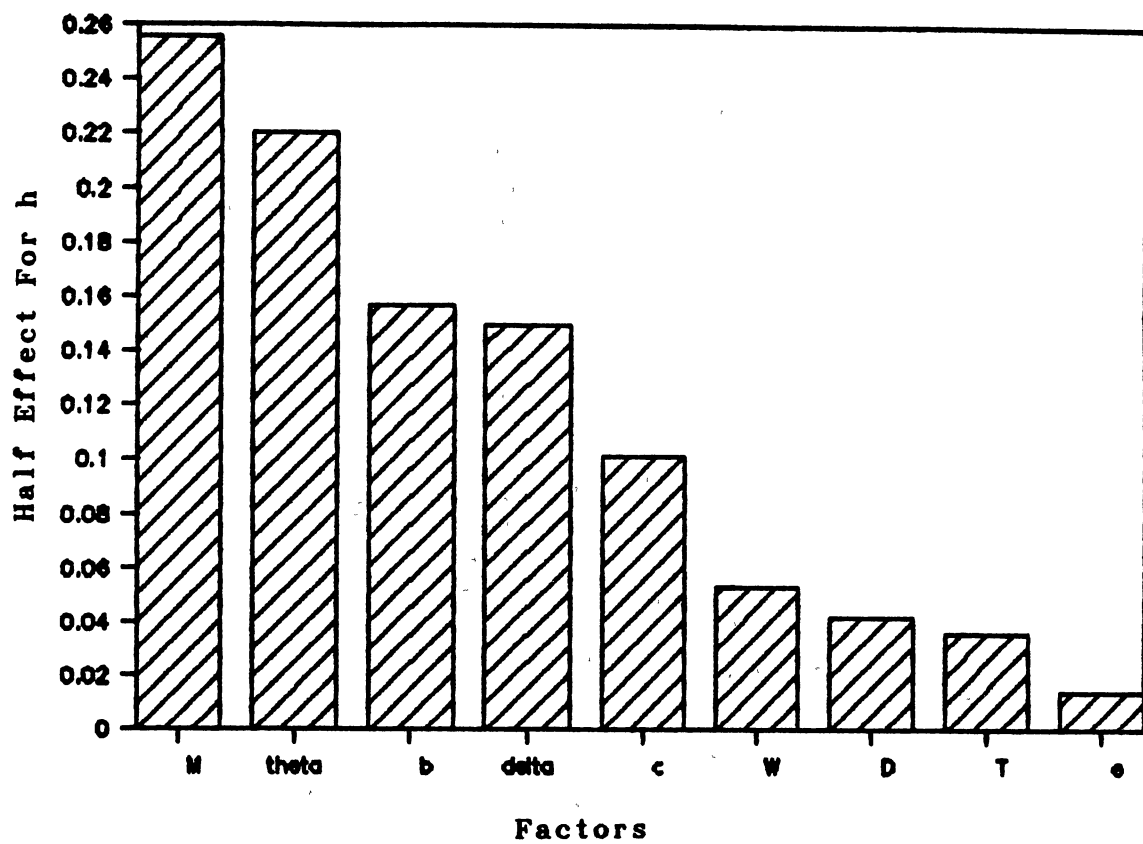


Figure 5.6 Half Effect Plot For h Using The EWMA Chart

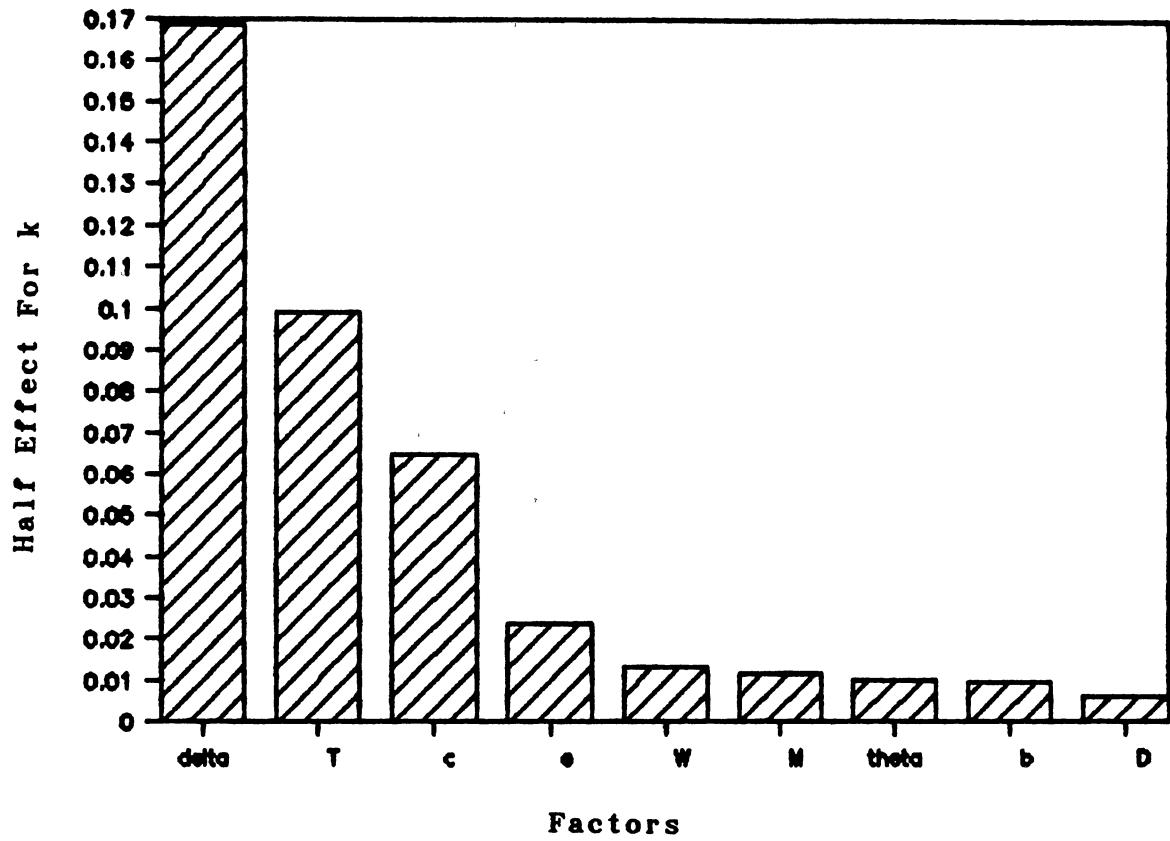


Figure 5.7 Half Effect Plot For k Using The EWMA Chart

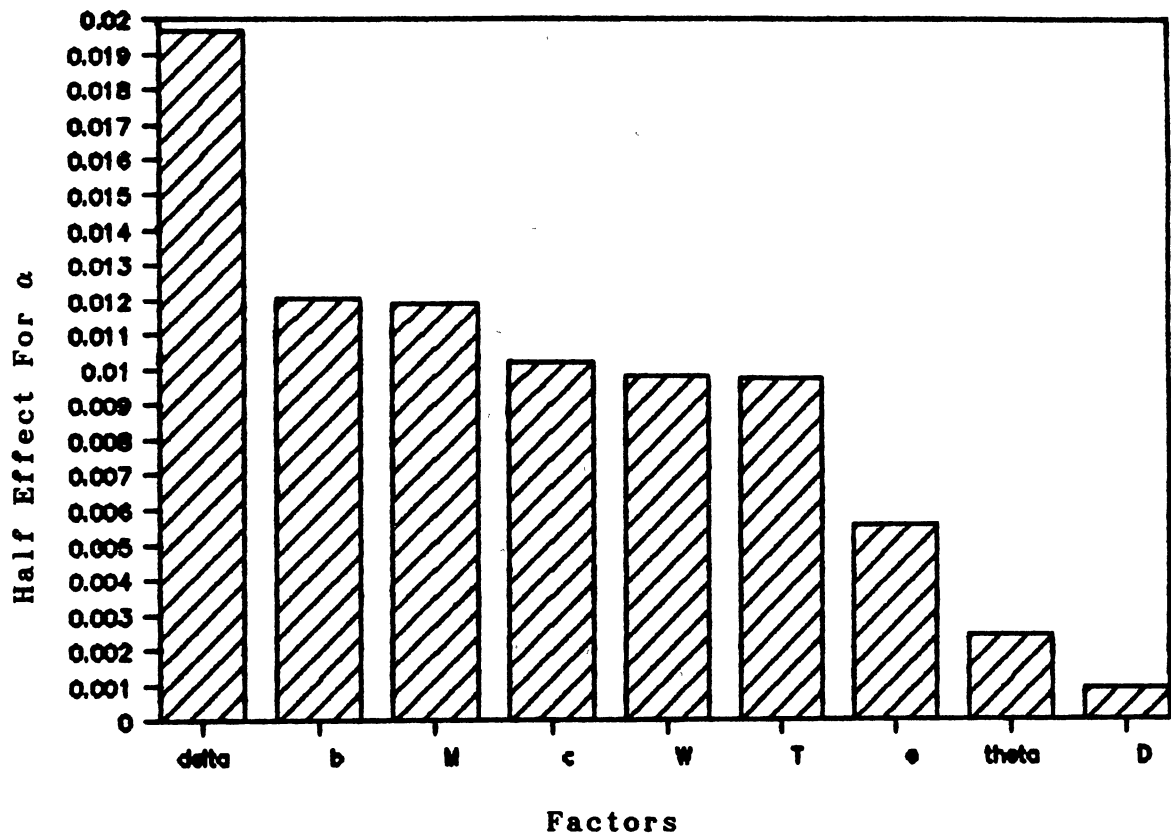


Figure 5.8 Half Effect Plot For  $\alpha$  Using The EWMA Chart



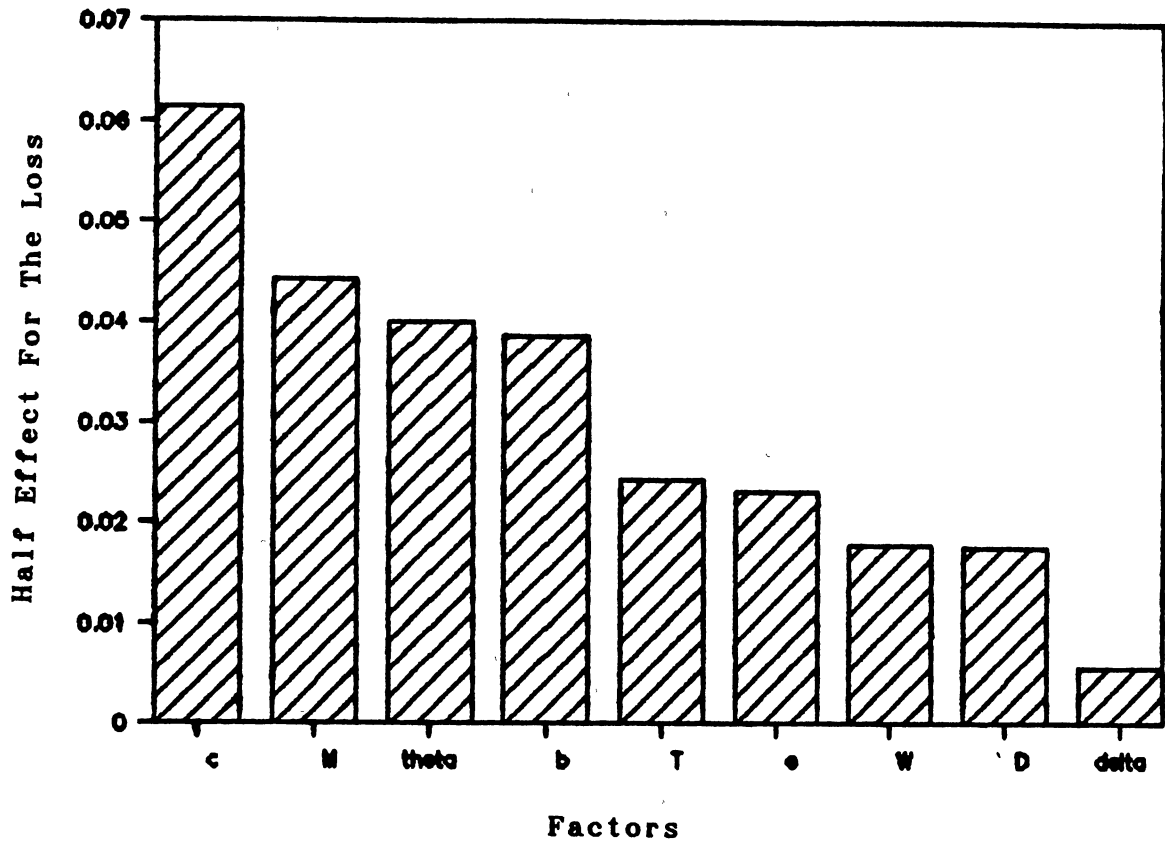


Figure 5.9 Half Effect Plot For The Losses Using The EWMA Chart

TABLE 5.10

THE OPTIMAL DESIGN PARAMETERS AND THE RESULTING  
OPERATING LOSSES OF THE ZONE CONTROL CHART

#	n	h	k	scores	Loss	% increase in loss
1	3	1.4913	2.9986	0,1,2,16	4.3414	8.18609
2	3	1.5243	3.0179	0,1,2,15	4.374	8.99847
3	7	1.2012	2.6728	0,1,2,15	4.2392	5.63931
4	3	1.0757	3.2677	0,1,2,15	4.3907	9.41463
5	3	1.1121	3.1228	0,1,2,15	4.2801	6.65852
6	4	1.067	3.4221	0,1,2,15	4.1704	3.92484
7	7	1.1471	2.8355	0,1,2,14	4.3308	7.92195
8	8	1.6137	2.7845	0,1,2,16	4.3198	7.64783
9	8	2.5596	2.8466	0,1,2,16	4.3913	9.42958
10	3	1.7177	3.2247	0,1,2,15	4.754	18.4679
11	9	1.6374	2.999	0,1,2,15	4.3459	8.29823
12	9	1.9831	2.9112	0,1,2,15	4.3665	8.81158
base	5	1.4256	3.0853	0,1,2,15	4.0129	0

Note: The resulting operating losses are obtained by implementing the optimal design parameters into the "real" environment.

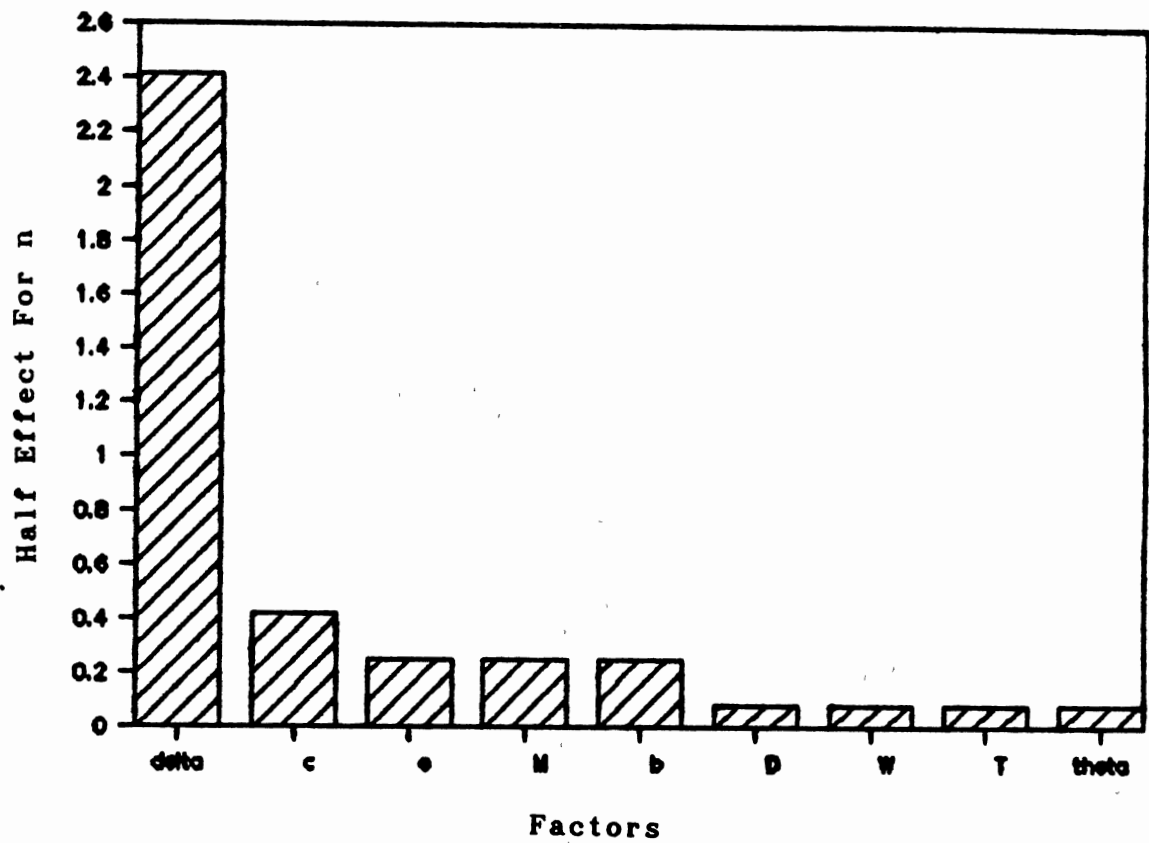


Figure 5.10 Half Effect Plot For n Using The ZCC

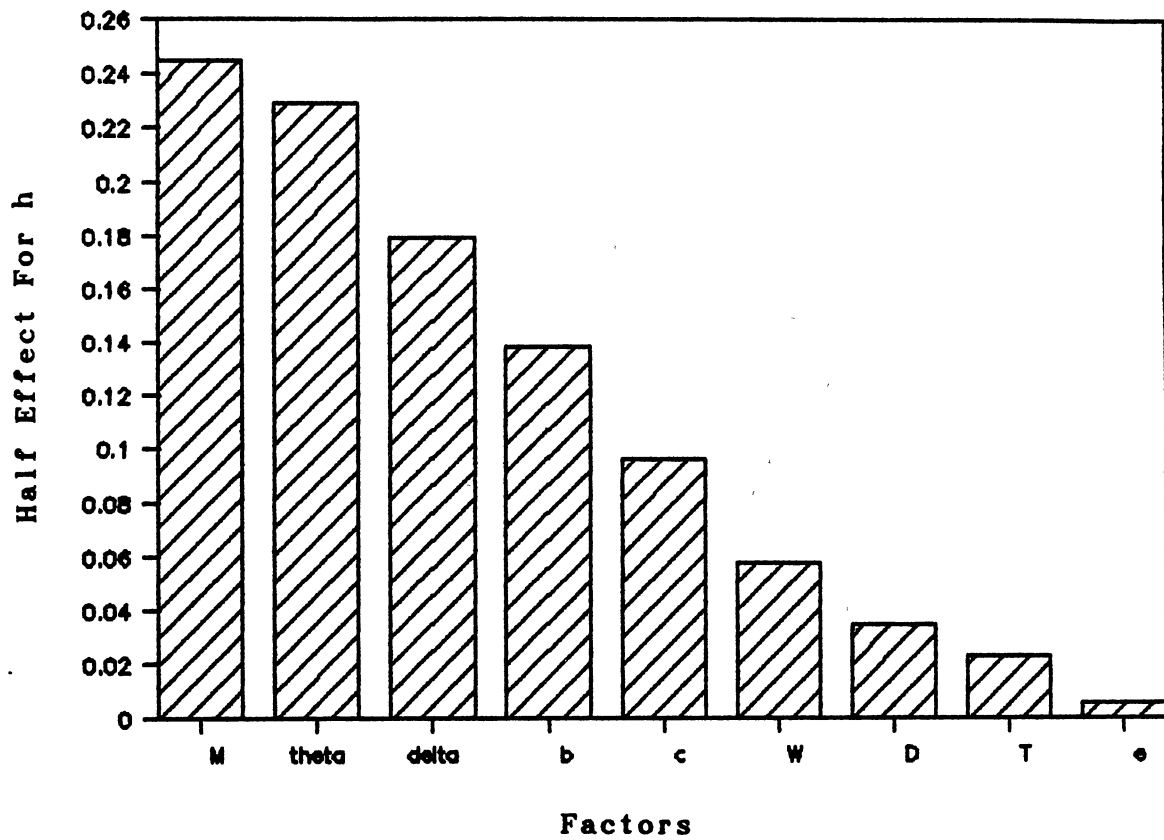


Figure 5.11 Half Effect Plot For h Using The ZCC

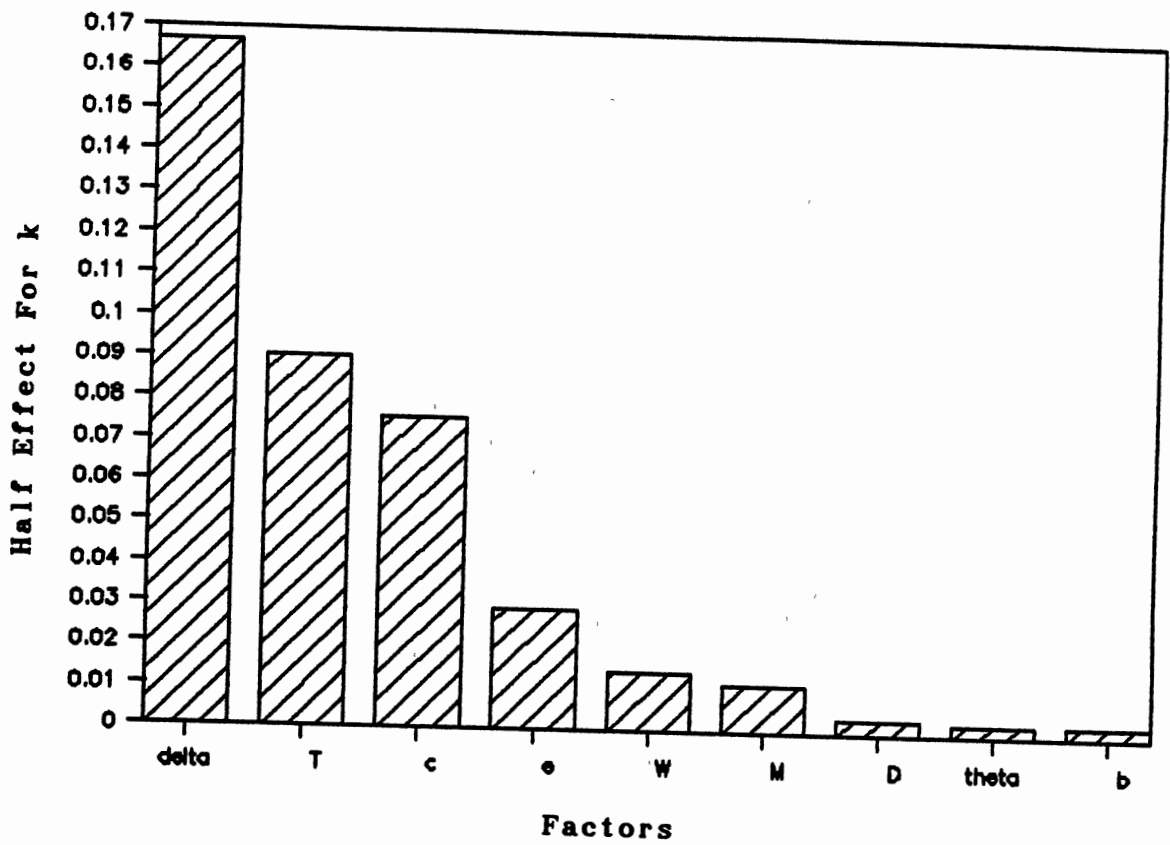


Figure 5.12 Half Effect Plot For k Using The ZCC

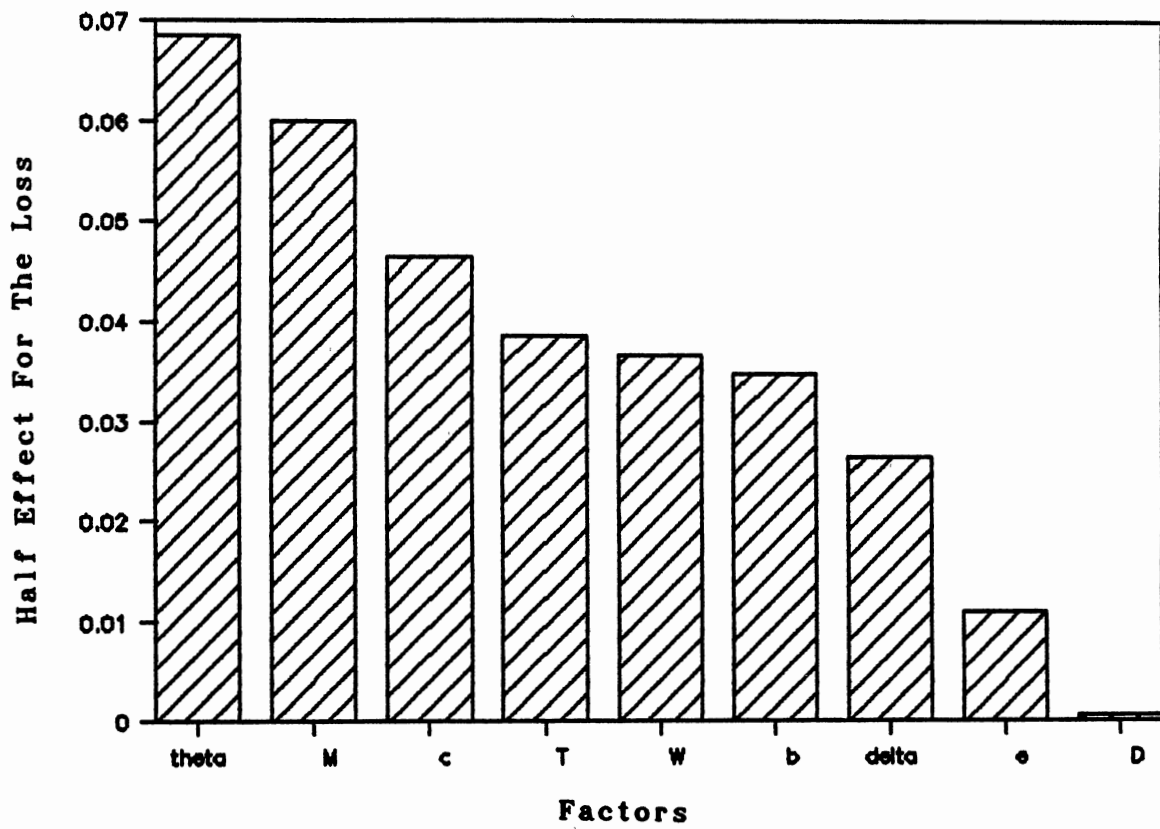


Figure 5.13 Half Effect Plot For The Losses Using The ZCC

- (3) The values of  $\delta$ ,  $e$ ,  $T$ , and  $c$  have effects on the optimal width of control limits,  $k$ .
- (4) Except for  $D$ , all the other 8 factors have effects on the resulting operating losses.
- (5) The first three zone scores are not affected by any of the 9 factors; however, the fourth zone score has slight variation.

Comparing the conclusions from all three types of control charts, it is observed that the conclusions regarding the control chart design parameters  $n$ ,  $h$ , and  $k$  are consistent; however, it is not the case for the resulting operating loss. The results suggest that the interaction between variables and/or non-linear effects must be explicitly considered.

In order to study the effects of the rates of error of estimation of those 9 cost and operating parameters, the experiments are also conducted with the rates of error of estimation being  $\pm 10\%$  and  $\pm 50\%$ . The results show indifference in the conclusions regarding the design parameters; however, they do show differences regarding the resulting operating loss.

### Taguchi Design

In order to ensure that the results of analyses obtained are not affected by design techniques employed, a Taguchi L12 designed experiment is also carried out. Note that the Taguchi L12 design is theoretically equivalent to

the P-B L12 design. The layout of the design matrix of the Taguchi L12 design is different from that of the P-B L12 design. However, by changing the low levels to high levels and vice versa in certain columns, and then manipulating the rows and columns, the Taguchi L12 design becomes exactly the same as the P-B L12 design. The details are given by Kacker, Lagergren, and Filliben (1991).

The layout of this designed matrix is shown in Table 5.11. The procedures for the sensitivity analysis in this experiment are the same as those in the P-B L12 design. The results show the same conclusions, regarding the design parameters, as those of the P-B L12 design; however, different conclusions are obtained regarding the resulting operating loss.

## 9-1 2 Fractional Factorial Design

The confounding patterns of the Taguchi L12 design and the P-B L12 design are not available. Therefore, in order to explicitly consider all the 2-way linear interaction between variables, a  $2^{9-1}$  fractional factorial (FF) design is employed. The defining relationship (Schmidt and Launsby, 1989) is  $I = \delta\theta MeDTWbc$ .

The cost and operating parameters, optimal design parameters, and resulting operating loss of these 256 runs using the X-bar control chart with AT&T runs rules are listed in Appendix D. It is interesting to note that the



TABLE 5.11  
 DESIGN MATRIX OF THE TAGUCHI L12 EXPERIMENT

No. of Run	$\delta$	$\theta$	M	e	D	T	W	b	c
1	-1	-1	-1	-1	-1	-1	-1	-1	-1
2	-1	-1	-1	-1	-1	1	1	1	1
3	-1	-1	1	1	1	-1	-1	-1	1
4	-1	1	-1	1	1	-1	1	1	-1
5	-1	1	1	-1	1	1	-1	1	-1
6	-1	1	1	1	-1	1	1	-1	1
7	1	-1	1	1	-1	-1	1	1	-1
8	1	-1	1	-1	1	1	1	-1	-1
9	1	-1	-1	1	1	1	-1	1	1
10	1	1	1	-1	-1	-1	-1	1	1
11	1	1	-1	1	-1	1	-1	-1	-1
12	1	1	-1	-1	1	-1	1	-1	1

optimal RULE used in all 256 runs is C1, which is not listed in Appendix D. The procedures employed for the sensitivity study are exactly the same as those in the Taguchi L12 and P-B L12 experiments. The rates of error of estimation of the 9 cost and operating parameters are assumed to be  $\pm 30\%$ .

The results regarding the control chart design parameters  $n$  and  $h$  show the same conclusions as those obtained in the P-B L12 and Taguchi L12 designs. A slight difference exists in the analysis of  $k$ . It is noted that  $\theta$  and  $M$  (and  $b$  in the EWMA chart) have an effect on  $k$ . The result of the analysis regarding the resulting operating loss shows noticeable difference from the previous conclusions. The effects which show significant in the analysis of the loss are  $\delta M$ ,  $\delta \theta$ ,  $\delta c$ ,  $\theta M$ ,  $M$ ,  $\delta e$ ,  $\theta$ ,  $M c$ ,  $\theta c$ ,  $c$ ,  $\delta T$ ,  $M b$ ,  $\theta b$ ,  $b c$ , and  $\delta b$  (in the EWMA chart,  $\delta b$  is not included), in a descending order of importance. Due to the hierarchy rule, the main effects  $\delta$ ,  $e$ ,  $T$  and  $b$  are also included when constructing the prediction equations in later sections. Note that  $W$  and  $D$  are not included.

#### Central Composite Faced Design

In order to improve the results of the sensitivity analyses and prediction equations, a CCFD using all the results from those 256 runs of the  $2^{9-1}$  design, plus one run of the center point, and 18 more runs of the axial points (because there are 9 factors) is carried out. Since

all the factors have been set at their extreme values when conducting the  $2^{9-1}$  FF experiment, the " $\alpha$ " (this is not the same  $\alpha$  as that in the economic design of the EWMA chart) values for the axial points are  $\pm 1$  in this experiment. This is why it is called a central composite "faced" design. One thing to note is that since this analysis is conducted using SAS (Schlotzhauer and Littell, 1987), the unused columns are pooled to estimate the error term. This is different from previous analyses in this research.

The conclusions are close to those obtained in the  $2^{9-1}$  FF experiment, as shown in Table 5.12. The 3-level design does not improve the results much. Therefore, a 2-level design is sufficient for conducting the sensitivity analysis. All the significant effects identified by each of these four designs are tabulated in Table 5.12.

#### Prediction Equations

Previous work of the economic design of quality control charts use one-factor-at-a-time (Duncan, 1956) or fractional factorial (Panagos, Heikes, and Montgomery, 1985) experiment to conduct the sensitivity analyses. All previous research identifies only the direction of the important factors (cost and operating parameters) which show significant effects on the optimal control chart design parameters and the resulting loss. The magnitude of the effects cannot be obtained. The prediction equations constructed in this research provide the

TABLE 5.12

SUMMARY OF CONCLUSIONS OF THE SENSITIVITY ANALYSES  
OF THE P-B L12 DESIGN, TAGUCHI L12 DESIGN,  
 $2^{9-1}$  FF DESIGN, AND CCFD

Response	Significant Effects			
	P-B	Taguchi	$2^{9-1}$ FF	CCFD (for <1> only)
n	$\delta$	$\delta$	$\delta$	$\delta$
h	$\delta, \theta, M, b, c$	$\delta, \theta, M, b, c$	$\delta, \theta, M, b, c$	$\delta, \theta, M, b, c$
k	$\delta, e, T, c$	$\delta, e, T, c$	$\delta, \theta, M, e, T, c$ (and b in <2>)	$\delta, \theta, M, e, T, c$
loss	<1> $\delta, \theta, M, e,$ $D, T, W, b,$ c	$\theta, M, e, D,$ $T, W, b, c$	$\delta, \theta, M, e,$ $T, b, c,$ $\delta M, \delta \theta, \delta c,$ $\delta e, \delta T, \delta b,$ $\theta M, \theta c, \theta b,$ $M c, M b, b c$	The same as those in $2^{9-1}$ FF design plus $\delta^2$
	<2> $\theta, M, e, D, T$ $W, b, c$	$\theta, M, e, D, T$ $W, b, c$	same as <1> except $\delta b$	N/A
	<3> $\delta, \theta, M, e, T$ $W, b, c$	$\theta, M, e, D, T$ $W, b, c$	same as <1>	
% of Difference	The same conclusions as those for loss			

Note: A. "<1>" represents the X-bar control chart with AT&T runs rules;  
B. "<2>" represents the EWMA chart;  
C. "<3>" represents the ZCC.

following advantages:

- (1) They identify the important factors;
- (2) They indicate both the magnitude and direction of the important factors;
- (3) They provide easy identification of joint effect when more than one factors are misspecified at the same time;
- (4) They help the user to (i) determine the initial values for the control chart design parameters; and, (ii) provide regions for search of the optimal values of the design parameters. For example, if the levels of each factor are determined, and if the user wishes to optimize the design parameter, say  $k$ , then, he can search within the range of  $[k_{pred} * (1 \pm 0.05)]$ ; and,
- (5) Substantial savings can be obtained by using the predicted values as initial values, instead of using the commonly used values (e.g., for the X-bar chart,  $n=5$ ,  $h=1$ ,  $k=3$ , see Duncan, 1956) as the control design parameters. For example, suppose  $\delta=2$ ,  $\theta=0.01$ ,  $M=100$ ,  $e=0.05$ ,  $D=2$ ,  $T=50$ ,  $W=25$ ,  $b=0.5$ , and  $c=0.1$  (Duncan's example 1). Then, the coded values for all 9 cost and operating parameters are zeroes. Take the X-bar control chart using RULE C1 as an example. The loss using the commonly used design values is 4.1234. The losses using the predicted values, (i)  $n=5$  (truncated),  $h=1.51898$ ,  $k=3.02165$ , and (ii)  $n=6$  (rounding),  $h=1.51898$ ,  $k=3.02165$ , are 4.0171 and 4.0613,

respectively. The savings are 2.65% and 1.53%, respectively.

The prediction equations for the optimal design parameters and the resulting loss for each chart are obtained using the results of the  $2^{9-1}$  fractional factorial experiment. Note that the rate of error of estimation of the 9 cost and operating parameters is  $\pm 30\%$ . Therefore, the ranges of the values for each factor (non-coded) are:

$\delta$ : 1.4 - 2.6,  
 $\theta$ : 0.007 - 0.013,  
M: 70 - 130,  
e: 0.035 - 0.065,  
D: 1.4 - 2.6,  
T: 35 - 65,  
W: 17.5 - 32.5,  
b: 0.35 - 0.65, and,  
c: 0.07 - 0.13.

The coded values are +1 and -1 for the highest and the lowest values of each factor, respectively. The experiment is set up within the operating range.

The calculated half effects for the optimal design parameters and the resulting loss for each chart are provided in Appendix E. Also provided are the comparisons of the true values and the predicted values for the first 40 runs of the  $2^{9-1}$  experiment for each chart.

The X-bar Control Chart With AT&T Runs Rules. The prediction equations are:

- (1) Prediction equation for the design parameter  $n$ ,  

$$n_{pred} = 5.60546 - 2.27734\delta$$
- (2) Prediction equation for the design parameter  $h$ ,  

$$h_{pred} = 1.51898 - 0.14694\delta - 0.23629\theta - 0.25168M$$

$$+ 0.13811b + 0.09446c$$
- (3) Prediction equation for the design parameter  $k$ ,  

$$k_{pred} = 3.02165 + 0.18345\delta - 0.01603\theta - 0.01683M$$

$$+ 0.03396e - 0.09415T - 0.07298c$$
- (4) Prediction equation for the resulting operating loss,  

$$ELOSS_{pred} = 4.35245 + 0.00379\delta - 0.02738\theta - 0.03143M$$

$$+ 0.00215e + 0.0077T + 0.00533b$$

$$+ 0.01896c - 0.042\delta\theta - 0.04563\delta M$$

$$+ 0.02912\delta e + 0.01824\delta T - 0.00931\delta b$$

$$+ 0.04099\delta c + 0.04093\theta M - 0.01489\theta b$$

$$- 0.02268\theta c - 0.01807Mb - 0.02564Mc$$

$$+ 0.00988bc$$
- (5) Prediction equation for the percentage of increase in  
the true minimum loss,  

$$\%_{pred} = 8.46151 + 0.09458\delta - 0.68253\theta - 0.78342M$$

$$+ 0.0538e + 0.19208T + 0.13297b$$

$$+ 0.47252c - 1.0468\delta\theta - 1.13708\delta M$$

$$+ 0.72589\delta e + 0.45471\delta T - 0.23205\delta b$$

$$+ 1.02146\delta c + 1.02017\theta M - 0.37109\theta b$$

$$- 0.56517\theta c - 0.45043Mb - 0.63894Mc$$

$$+ 0.24642bc$$

The maximum percentages of deviation between the optimal values and the predicted values for  $n$ ,  $h$ ,  $k$ , and

the loss, in all 256 runs, are 21.17%, 26%, 4.3%, and 2.82%, respectively.

The EWMA Chart. The prediction equations are:

- (1) Prediction equation for the design parameter  $n$ ,  

$$n_{pred} = 5.46093 - 2.14843\delta$$
- (2) Prediction equation for the design parameter  $h$ ,  

$$h_{pred} = 1.49475 - 0.12688\delta - 0.23315\theta - 0.24943M$$

$$+ 0.14133b + 0.09202c$$
- (3) Prediction equation for the design parameter  $k$ ,  

$$k_{pred} = 3.02435 + 0.17996\delta - 0.01487\theta - 0.01345M$$

$$+ 0.02944e - 0.10214T - 0.01865b$$

$$+ 0.06957c$$
- (4) Prediction equation for the design parameter  $\alpha$ ,  

$$\alpha_{pred} = 0.91505 + 0.01943\delta$$
- (5) Prediction equation for the resulting operating loss,  

$$ELOSS_{pred} = 4.32175 + 0.00191\delta - 0.02576\theta - 0.02793M$$

$$+ 0.00283e + 0.00712T + 0.01163b$$

$$+ 0.01031c - 0.04245\delta\theta - 0.04503\delta M$$

$$+ 0.03244\delta e + 0.01355\delta T + 0.03672\delta c +$$

$$0.04401\theta M - 0.02092\theta b - 0.02165\theta c -$$

$$0.02079Mb - 0.02239Mc + 0.01044bc$$

The maximum percentages of deviation between the optimal values and the predicted values for  $n$ ,  $h$ ,  $k$ ,  $\alpha$ , and the loss, in all 256 runs, are 23.9%, 25.24%, 4.24%, 2.6%, and 2.7%, respectively.

The Zone Control Chart. The prediction equations are:



- (1) Prediction equation for the design parameter n,  

$$n_{pred} = 5.60156 - 2.28906\delta$$
- (2) Prediction equation for the design parameter h,  

$$h_{pred} = 1.51891 - 0.14096\delta - 0.23815\theta - 0.25462M$$

$$+ 0.13953b + 0.09582c$$
- (3) Prediction equation for the design parameter k,  

$$k_{pred} = 3.01846 + 0.18043\delta - 0.02098\theta - 0.01619M$$

$$+ 0.02927e - 0.09587T - 0.06909c$$
- (4) Prediction equations for the four zone scores,  

$$S_{i,pred} = S_i\text{-bar}$$
 where  $i=1, 2, 3, 4$ ; and,  $S1\text{-bar} = 0$ ,  $S2\text{-bar} = 1$ ,  $S3\text{-bar} = 2.05859$ , and  $S4\text{-bar} = 17.2421$ .
- (5) Prediction equation for the resulting operating loss,  

$$ELOSS_{pred} = 4.35523 + 0.00537\delta - 0.02834\theta - 0.02898M$$

$$+ 0.00024e + 0.00647T + 0.00692b$$

$$+ 0.01904c - 0.04239\delta\theta - 0.04413\delta M$$

$$+ 0.0293\delta e + 0.0167\delta T - 0.00863\delta b$$

$$+ 0.0407\delta c + 0.04133\theta M - 0.01593\theta b$$

$$- 0.02409\theta c - 0.01833Mb - 0.02372Mc$$

$$+ 0.00943bc$$

The maximum percentages of deviation between the optimal values and the predicted values for n, h, k, S1, S2, S3, S4, and the loss, in all 256 runs, are 21.09%, 29%, 3.73%, 0%, 0%, 74%, 29% and 3.9%, respectively. For S3, there are only 2 (out of 256) cases which deviate from the optimal values with 74%, others are 0%. For S4, there are also 2 cases which deviate from the optimal value with 29%;

others are within 13.33%.

The prediction equations obtained in combination with Appendix E provide guidelines for practitioners and theoreticians as to how to conduct the sensitivity analysis. The following information is provided.

- (1) A 2-level designed experiment is sufficient to conduct the sensitivity analysis; even though the true relationships between the cost and operating parameters and the optimal design parameters are not known, and the relationship between the cost and operating parameters and the resulting operating loss is non-linear.
- (2) Some simple designed experiments, such as the P-B L12 and Taguchi L12 designs, can be employed to study and build the predictions for the optimal design parameters.
- (3) A smaller designed experiment can be selected for an initial study of the effects of the cost and operating parameters on the resulting operating loss. Also, prediction equations can be built. For example, a  $2^{9-4}$  FF experiment can be employed, including those 12 2-way linear interactions in the design matrix, to conduct the analysis.

Some important conclusions are also obtained from the prediction equations. Keep in mind, however, that the conclusions are drawn under the assumption that the rate of error of estimation is  $\pm 30\%$ . The conclusions are:

- (1) The optimal subgroup size is primarily determined by the magnitude of the shift in the process mean ( $\delta$ ) measured in the number of process standard deviations. When  $\delta$  increases,  $n$  decreases.
- (2) The magnitude of the shift in the process mean ( $\delta$ ), the rate of occurrence of the special cause ( $\theta$ ), the penalty cost ( $M$ ), the fixed cost per subgroup taken ( $b$ ), and the variable cost per unit sampled ( $c$ ) have their effects on the optimal sampling interval,  $h$ . An increase in  $\delta$ ,  $\theta$ , or  $M$  will decrease  $h$ , and an increase in  $b$  or  $c$  will increase  $h$ .
- (3) The optimal width of control limits ( $k$ ) is affected by variations in  $\delta$ ,  $M$ ,  $e$  (delay factor),  $T$  (false alarm cost), and  $c$ . An increase in  $\delta$  or  $T$  will widen  $k$ , and an increase in  $M$ ,  $e$ , or  $c$  will narrow down  $k$ . In the case of the EWMA chart,  $b$  also shows noticeable effect on  $k$ . Wider  $k$  is preferred if  $b$  decreases.
- (4) The effects of the cost and operating parameters on the resulting operating loss are not precisely known because interactions between variables are present.
- (5) In the economically-based EWMA chart, the weight,  $\alpha$ , is primarily determined by the amount of shift in the process mean,  $\delta$ .
- (6) In the economically-based Zone control chart, the four zone scores are not affected by the variation of the cost and operating parameters.

### Some Comments

There are two questions which have been asked frequently. One is "Under what conditions will one control scheme perform better than the others from an economic viewpoint?"; the other is "What is the minimum magnitude of shift in the process mean that is of real concern?" In this section, the above questions are answered.

#### Analysis Of The Relationship Between The Statistical Performance And The Economic Performance Of Control Charts

A study of the relationship between the cost and operating parameters and the ARLs is carried out in this section. This study reveals that if several types of control charting techniques are presented for selection under the situation of (approximately) the same  $n$ ,  $h$ , ARL in control, and cost and operating parameters, the one which possesses the smallest ARL when the process mean shifts by a certain amount is preferred when a certain condition is met. This condition is that  $\{(M/\theta)+W+[(T*Y)/ARL_1]\}$  must be less than zero.

Recall that the loss function is given by

$$L = \left(1 - \frac{1}{\theta * B}\right)M + \frac{T * Y}{B * ARL_1} + \frac{W}{B} + \frac{b + cn}{h}$$

where  $B = h(\text{ARL2}+Y)+en+D$  is the expected length of a production cycle. Let

$L_o$ : the original loss

$L_x$ : the new loss

$\text{ARL1}_o$ : the original ARL in control

$\text{ARL2}_o$ : the original ARL when the process mean has shifted by a specified amount

$\text{ARL1}_x$ : the new ARL in control

$\text{ARL2}_x$ : the new ARL when the process mean has shifted by a specified amount

$B_o = h(\text{ARL2}_o+Y)+en+D$ : the original expected length of a production cycle

$B_x = h(\text{ARL2}_x+Y)+en+D$ : the new expected length of a production cycle

Based on the same design parameters  $n$  and  $h$ , and the same cost and operating parameters,  $L_o$  and  $L_x$  can be expressed as follows.

$$L_o = \left(1 - \frac{1}{\theta \cdot B_o}\right)M + \frac{T \cdot Y}{B_o \cdot \text{ARL1}_o} + \frac{W}{B_o} + \frac{b + cn}{h}$$

$$L_x = \left(1 - \frac{1}{\theta \cdot B_x}\right)M + \frac{T \cdot Y}{B_x \cdot \text{ARL1}_x} + \frac{W}{B_x} + \frac{b + cn}{h}$$

It is desired that  $L_x - L_o < 0$ . Therefore,

$$L_x - L_o =$$

$$\left[ \left(1 - \frac{1}{\theta \cdot B_x}\right) - \left(1 - \frac{1}{\theta \cdot B_o}\right) \right] M$$

$$\begin{aligned}
& + \left[ \frac{T * Y}{B_x * ARL1_x} - \frac{T * Y}{B_o * ARL1_o} \right] \\
& + \left[ \frac{W}{B_x} - \frac{W}{B_o} \right] \\
& = [(1/B_x) - (1/B_o)] [(M/\theta) + W] \\
& \quad + T * Y \{ [1/(B_x * ARL1_x)] - [1/(B_o * ARL1_o)] \} \\
& < 0
\end{aligned}$$

Let the following comparison be based on the same ARL in control. That is, let  $ARL1_x = ARL1_o = ARL1$ . Then, the desired situation is

$$\begin{aligned}
L_x - L_o & = [(1/B_x) - (1/B_o)] [(M/\theta) + W] \\
& \quad + T * Y \{ [1/(B_x * ARL1_x)] - [1/(B_o * ARL1_o)] \} \\
& = [(1/B_x) - (1/B_o)] [(M/\theta) + W + (T * Y / ARL1)] \\
& < 0
\end{aligned}$$

Let  $DET = (M/\theta) + W + (T * Y / ARL1)$ , then,

(A) If  $DET > 0$ , then  $[(1/B_x) - (1/B_o)] < 0$ .

This implies that  $B_x > B_o$ , which also implies that  $ARL2_x > ARL2_o$ . That is, given the same  $n$ ,  $h$ ,  $ARL1$ , and cost and operating parameters, a new plan will yield a smaller loss if it produces a larger ARL when the process mean shifts to a specified amount, under the condition that  $DET$  is greater than zero.

(B) If  $DET < 0$ , then  $[(1/B_x) - (1/B_o)] > 0$ .

This implies  $B_x < B_o$ , which also implies that  $ARL2_x < ARL2_o$ . That is, given the same  $n$ ,  $h$ ,  $ARL1$ , and cost and

operating parameters, a new plan will yield a smaller loss if it produces a smaller ARL when the process mean shifts to a specified amount, under the condition that DET is less than zero.

These results are applicable both "within" a control chart for selection of a different set of design parameters (for example, the  $(\alpha, k)$  combination in an EWMA chart) which yields a smaller ARL2 (i.e., better statistical performance), and "between" control charts. These results reveal the relationship between the statistical performance and economic performance of control charts. In all 22 examples employed in this research, the DETs are all negative. This indicates that a smaller loss is obtained if a control chart (scheme) possesses better statistical performance (power of detection) when the mean shifts by a certain amount, given (approximately) the same  $n$ ,  $h$ , ARL1, and cost and operating parameters.

#### Minimum Magnitude Of Shift

##### In The Process Mean

Examples 1, 16, and 21 have been selected for study. Table 5.13 presents the optimal results of these three examples for the X-bar chart with AT&T runs rules. It is observed that when  $\delta$  is really small, such as 0.1, the optimal design parameters indicate that the best policy is to leave the process alone. Note that in example 21, the optimal design parameters also indicate, to some extent, to

**TABLE 5.13**  
**OPTIMAL DESIGN PARAMETERS OF EXAMPLES 1, 16, AND 21**  
**WHEN  $\delta$  IS SMALL**

Ex. no.	$\delta$	n	h	k	RULE
1	0.1	1	69.9995	0.0009	C1
	0.2	77	51.0911	1.1036	C1
16	0.1	1	69.9989	0.0838	C1
	0.2	77	56.7808	1.0747	C1
21	0.1	1	69.9749	0.5623	C1
	0.2	59	69.9963	1.04	C1



leave the process alone when  $\delta$  is 0.2. This is because the penalty cost,  $M$ , in this example is very small (2.25) compared to the false alarm cost (50) and the true alarm cost (25). Therefore, it makes sense to leave the process alone when  $\delta$  is small, because if one tries to interrupt the process, the loss will increase due to both the increase in false alarm cost and the expense involved in finding a true alarm. The same conclusions are also observed in the economically-based EWMA chart and ZCC.

For the above conclusions, two conditions need to be clarified. The first is that, in the computer programs, the upper limit on  $h$  is specified to be 70 hours. That is, if  $h=70$ , it implies infinity. The second is that, when searching for the optimal design parameters, the penalty cost is assumed to be proportional to the number of nonconforming items produced, and the specs of the products are assumed to be placed at  $\pm 3\sigma$  away from the nominal value.

#### Summary

An economic comparison among the (1) X-bar control chart with AT&T runs rules, (2) EWMA chart, and (3) Zone control chart is performed. Twenty two examples from Duncan's (1956) paper are used in this comparison. The results are shown in Tables 5.3 to 5.6. An analysis of these results shows, from the economic viewpoint, that the EWMA chart is superior to both the X-bar control chart with

AT&T runs rules and the Zone control chart. The Zone control chart is slightly better than the X-bar control chart with AT&T runs rules.

Sensitivity analyses have been carried out using the (1) Plackett-Burman L12 design, (2) Taguchi L12 design, (3)  $2^{9-1}$  fractional factorial design, and (4) central composite faced design. The results indicate that a fractional factorial designed experiment with 2 levels for each factor is sufficient for conducting the sensitivity analysis. The prediction equations for the optimal design parameters and the resulting operating losses are given. The analysis regarding the effects of the cost and operating parameters to the resulting operating loss adopts the approach proposed by Collins, Case, and Bennett (1978).

## CHAPTER VI

### SUMMARY, CONCLUSIONS, AND FUTURE WORK

This chapter summarizes all the steps carried out in order to fulfill the objective and subobjectives of this research. Conclusions are then provided, and finally, possible future work and extensions of this research are outlined.

#### Summary

Chapter I of this research provides the problem statements. It includes the purpose of this research, the problems of the economic design of quality control charts when historical data are part of the decision making process, the research objective and the contributions of this research.

Chapter II provides an extensive literature survey of statistically- and economically-based control charts used to monitor the process mean. In chapter III, the economic models of the (1) X-bar control chart with AT&T runs rules, (2) EWMA chart, and (3) Zone control chart are developed. Chapter IV introduces the use of interactive computer programs which help theoreticians and practitioners in design and evaluation of the economically-based (1) X-bar

control chart with AT&T runs rules, (2) EWMA chart, and (3) Zone control chart. Chapter V provides economic comparisons and sensitivity analyses among these three variables control charts.

The following accomplishments have been achieved:

- (1) An analytical model to economically optimize and evaluate the (i) X-bar control chart with AT&T runs rules, (ii) EWMA chart, and (iii) Zone control chart is developed.
- (2) Economic comparisons are performed among these three variables control charts.
- (3) Sensitivity analyses are carried out for all three types of control charts. Prediction equations are provided for the optimal design parameters and the resulting operating loss.
- (4) An analysis of the relationship between the statistical performance and economic performance is carried out. The result of this analysis explicitly reveals that a control scheme which has a better statistical performance also has a better economic performance.
- (5) Interactive computer programs are developed and implemented to help theoreticians and practitioners in the design and evaluation of the proposed economically-based three variables control charts.

#### Conclusions

Based on the results obtained in this research, the

conclusions are as follows:

- (1) The EWMA chart is superior to, from an economic viewpoint, both the X-bar control chart with AT&T runs rules and the Zone control chart. The superiority is especially significant when the amount of shift in the process mean is small to moderate.
- (2) The economically-based Zone control chart is slightly better than the economically-based X-bar control chart with AT&T runs rules.
- (3) If the economically-based X-bar control chart with AT&T runs rules is to be used for monitoring the process mean, then RULE C1 is recommended for use unless (i) the penalty cost is relatively high compared to the false alarm cost and true alarm cost, and/or (ii) the value of delay factor is relatively large, in which cases RULE C12 is recommended.
- (4) If the shift in the process mean is small, such as 0.1 process standard deviation, then the optimal policy is to leave the process alone except under most unusual conditions.
- (5) A fractional factorial experiment with 2 levels for each factor is sufficient to conduct a sensitivity analysis and construct prediction equations.
- (6) The optimal subgroup size is primarily determined by the magnitude of shift in the process mean ( $\delta$ ) measured in number of process standard deviations. When  $\delta$  increases,  $n$  decreases. That is, if the shift in the

process mean becomes large, a smaller subgroup size can be used to catch the shift.

- (7) The magnitude of shift in the process mean ( $\delta$ ), the rate of occurrence of the special cause ( $\theta$ ), the penalty cost ( $M$ ), the fixed cost per subgroup taken ( $b$ ), and the variable cost per unit sampled ( $c$ ) have their effects on the optimal sampling interval,  $h$ . An increase in  $\delta$ ,  $\theta$ , or  $M$  will decrease  $h$ , and an increase in  $b$  or  $c$  will increase  $h$ . That is, if the shift in process mean is large, or the occurrence rate of the special cause is high, or the cost of operating the process under an OOC condition is high, then shorter sampling intervals are preferred in order to catch the changes earlier and reduce the loss. If the fixed cost per subgroup taken, or the variable cost per unit sampled is high, then a longer sampling interval is preferred in order to reduce the loss.
- (8) The optimal width of control limits ( $k$ ) is affected by variations in  $\delta$ ,  $M$ ,  $e$  (delay factor),  $T$  (false alarm cost), and  $c$ . The increase in  $\delta$  or  $T$  will widen  $k$  and the increase in  $M$ ,  $e$ , or  $c$  will narrow down  $k$ . That is, wider control limits are preferred if the shift in the process mean is large, or the false alarm cost is high. Wider control limits reduce the number of false alarms and hence false alarm cost. Tighter control limits are preferred if the penalty cost is high, or the time required to sample, compute, and plot on the control

chart is long, or variable cost per unit sampled is high.

- (9) The effects of the cost and operating parameters on the resulting loss are not precisely known because interaction between variables is present.

#### Future Work

Possible future work related to extensions of this research are as follows:

- (1) Multiple special causes may be considered. In this research, only a single special cause is considered.
- (2) Other types of process failure mechanism can be assumed. In this research, the underlying distribution of the process failure mechanism is assumed to be exponential.
- (3) Different types of shift, for example linear trend, in the process mean may be considered. In this research, the shift in the process mean is assumed to be instantaneous and persistent.
- (4) A joint design of economically-based control charts used to simultaneously monitor both process mean and process variation may be developed. In this research, the process variation is assumed to remain unchanged throughout production.
- (5) Economic models when the process characteristic is normally/non-normally distributed and subject to measurement error can be developed. In this research,

it is assumed that the underlying distribution of the process characteristic is normal and the measurement process is error-free.



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## **APPENDIXES**

**APPENDIX A**

**FORTRAN PROGRAM FOR DESIGN AND EVALUATION  
OF THE X-BAR CONTROL CHART  
WITH AT&T RUNS RULES**



```

C*****
C  MAIN PROGRAM
C*****
$DEBUG
1  WRITE(*,2)
2  FORMAT(1H1,12X,24(1H*),/,13X,'***  MAIN  MENU  '
   &'*** ',/,13X,24(1H*),/,/,
   &5X,'(1) ECONOMIC DESIGN OF X-BAR CONTROL CHARTS',
   &/,5X,'    WITH AT&T RUNS RULES',/,/,
   &5X,'(2) EVALUATION OF X-BAR CONTROL CHARTS',/,
   &5X,'    WITH AT&T RUNS RULES.',/,
   &5X,'(3) EXIT THE PROGRAM',/,/,
   &5X,'==> PLEASE ENTER YOUR OPTION (1, 2, OR 3)! '
   &' <<<' )
   READ(*,*) IANS
   GO TO (4,5,6) IANS
   WRITE(*,3)
3  FORMAT(/,5X,'*** ERROR INPUT! PLEASE TRY AGAIN!'
   &' ***' )
   GO TO 1
4  CALL ECXBATT
   GO TO 1
5  CALL EVXBATT
   GO TO 1
6  STOP
   END
C
C*****
      SUBROUTINE ECXBATT
C*****
C-----
C  ECONOMIC DESIGN OF THE XBAR CHART WITH AT&T RUNS
C  RULES
C-----
      IMPLICIT REAL*8(A-H,O-Z)
      CHARACTER IFILE*12
      COMMON/EC1/THETA
      COMMON/EC2/N,NOPT
      COMMON/EC3/H,EK,HOPT,EKOPT,FOPT,STEP(20)
      COMMON/EC4/DELTA,B,C,D,E,EM,T,W
      COMMON/XBATT1/K(20),M(20),A1(20),B1(20),NT
C-----
C  INPUT COST AND OPERATION PARAMETERS
C-----
1  WRITE(*,2)
2  FORMAT(/,5X,'*** INPUT COST PARAMETERS. ***',/,/,
   &5X,'DO YOU WANT TO INPUT FROM A FILE OR '
   &'MANUALLY?',/,5X,'==> PLEASE ENTER 1 = FILE, 2 = '
   &'MANUALLY. ')
   READ (*,*) IANS
   GO TO (500,501) IANS
   WRITE (*,502)
502 FORMAT(/,5X,'** ERROR INPUT! PLEASE TRY AGAIN!'
   &'**' )

```

```

      GO TO 1
C-----
C INPUT THE COST PARAMETERS FROM USER'S FILE
C-----
500 WRITE(*,503)
503 FORMAT(/,5X,'** PLEASE INPUT THE FILENAME THAT',/,
&5X,' CONTAINS THE COST PARAMETERS.')
      READ(*,31) IFILE
31  FORMAT(A12)
      OPEN(50,FILE=IFILE,STATUS='OLD')
      READ(50,*) DELTA,THETA,EM,E,D,T,W,B,C
      CLOSE(50)
      GO TO 3
C-----
C INPUT THE COST PARAMETERS MANUALLY
C-----
501 WRITE(*,504)
504 FORMAT(/,5X,'** INPUT VALUES OF DELTA,THETA,M,E,D,
&T,W,B,C **')
      READ(*,*) DELTA,THETA,EM,E,D,T,W,B,C
3  WRITE(*,4) DELTA,THETA,EM,E,D,T,W,B,C
4  FORMAT(/,5X,' ** VALUES RECEIVED ARE:',/,
&5X,' DELTA = ',F10.4,7X,' THETA = ',F10.4,/,
&5X,' M = ',F10.4,7X,' E = ',F10.4,/,
&5X,' D = ',F10.4,7X,' T = ',F10.4,/,
&5X,' W = ',F10.4,7X,' B = ',F10.4,/,
&5X,' C = ',F10.4,/,/,
&5X,' ==> ARE THESE DATA CORRECT? ',/,
&5X,' ==> PLEASE ENTER 1 = YES, 2 = NO <<<')
      READ(*,*) IANS
      GO TO (5,1) IANS
      GO TO 3
C-----
C SELECT THE STARTING POINT
C-----
5  N=5
      H=1.0D0
      EK=3.0D0
6  WRITE(*,7) N,H,EK
7  FORMAT(/,5X,' *** THE SUGGESTED STARTING POINT '
&' IS: ', &/,5X,' N = ',I3,', H = ',F6.2,', K = ',
&F6.2,/,/,
&5X,' ==> DO YOU ACCEPT THIS POINT?',/,
&5X,' ==> ENTER 1 = YES, 2 = NO. <<<')
      READ(*,*) IANS
      GO TO (8,14), IANS
      GO TO 6
C-----
C SELECT THE STEP SIZE
C-----
8  STEP(1)=0.5
      STEP(2)=0.5
      STEP(3)=1.0
9  WRITE(*,10) STEP(3),STEP(1),STEP(2)

```

```

10  FORMAT(/,5X,'*** THE SUGGESTED STEP SIZES ARE: '
    &/,5X,'N = ',F5.2,', ',3X,' H = ',F6.2,', ',3X,
    &' K = ',F6.2,/,/,
    &5X,'==> DO YOU ACCEPT THESE SUGGESTIONS?',/,
    &5X,'==> PLEASE ENTER 1 = YES, 2 = NO. <<<')
    READ(*,*) IANS
    GO TO (11,18) IANS
    GO TO 9

C-----
C  PERFORM THE ECON DESIGN OF THE XBATT
C-----
11  CALL OPXBATT
C-----
C  PRINT THE OPTIMAL DESIGN
C-----
12  WRITE(*,13) NOPT,HOPT,EKOPT,FOPT
13  FORMAT(/,5X,58(1H-),
    &/,5X,' *** THE OPTIMAL DESIGN IS: ',
    &/,5X,' N = ',I4,', ',3X,' H = ',F10.5,', ',
    &3X,' K = ',F10.5,
    &/,5X,' *** THE MINIMUM LOSS PER HOUR IS: ',
    &F14.6,/,/,5X,58(1H-))
    GO TO 22

C-----
C  USER SELECTS THE STARTING POINT
C-----
14  WRITE(*,15)
15  FORMAT(/,5X,'*** PLEASE INPUT NEW STARTING POINT'
    &'***',/,5X,' ==> KEY IN VALUES FOR N, H, K')
    READ(*,*) N,H,EK
16  WRITE(*,17) N,H,EK
17  FORMAT(/,5X,' *** NEW STARTING POINT IS:',/,
    &5X,' N = ',I4,5X,' H = ',F8.4,' K = ',F8.4,/,
    &5X,' ==> ARE THEY CORRECT?',/,
    &5X,' ==> PLEASE ENTER 1 = YES, 2 = NO. <<<')
    READ(*,*) IANS
    GO TO (8,14), IANS
    GO TO 16

C-----
C  USER SELECTS THE STEP SIZES
C-----
18  WRITE(*,19)
19  FORMAT(/,5X,'*** PLEASE INPUT NEW STEP SIZES ***',
    &/,5X,'==> PLEASE ENTER VALUES FOR N, H, K. <<<')
    READ (*,*) STEP(3),STEP(1),STEP(2)
20  WRITE (*,21) STEP(3),STEP(1),STEP(2)
21  FORMAT(/,5X,'*** NEW STEP SIZES ARE:',/,
    &5X,'N = ',F5.2,', ',3X,' H = ',F6.2,', ',3X,
    &' K = ',F6.2,/,/,
    &5X,'==> ARE THEY CORRECT?',/,
    &5X,'==> PLEASE ENTER 1 = YES, 2 = NO. <<<')
    READ (*,*) IANS
    GO TO (11,18) IANS
    GO TO 20

```

```

22      RETURN
      END
C*****
      SUBROUTINE OPXBATT
C*****
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/EC2/N,NOPT
      COMMON/EC3/H,EK,HOPT,EKOPT,FOPT,STEP(20)
      COMMON/XBATT1/K(20),M(20),A1(20),B1(20),NT
      COMMON/SAMPLE/IX
      COMMON IRULE
      EXTERNAL XBATT3
      EXTERNAL XBATT2
      REAL*8 FMIN(10),X(10),XMIN(20),XSEC(20),F
C-----
C  SELECT RUNS RULES USED IN COMBINATION WITH
C  THE X-BAR CONTROL CHART
C-----
19      WRITE (*,20)
20      FORMAT(/,5X,'*** PLEASE SELECT RUNS RULES. ***',/,
&5X,'(1) C1,',/,5X,'(2) C12,',/,5X,'(3) C13,',/,
&5X,'(4) C14,',/,5X,'(5) C123,',/,5X,'(6) C124,',/,
&5X,'(7) C134,',/,5X,'(8) C1234.',/,/,
&5X,'*** PLEASE ENTER YOUR OPTION (1-8)! ***')
      READ (*,*) IRULE
C-----
C  ASSIGN NUMBER OF VARIABLES TO BE OPTIMIZED,
C  SEARCH STEP, AND TERMINATE VALUE. (TREAT
C  N AS A REAL VALUE)
C-----
      ND=3
      ICOUNT=700
      REQMIN=0.0001
C-----
C  ASSIGN STARTING POINT
C-----
      X(1)=H
      X(2)=EK
      X(3)=DBLE(N)
      CALL NELMIN(XBATT3,ND,X,XMIN,XSEC,YNEWLO,YSEC,
&REQMIN,STEP,ICOUNT)
C-----
C  PRINT OUT THE OPTIMAL POINT FOUND
C-----
      WRITE(*,1) XMIN(3),XMIN(1),XMIN(2),YNEWLO
1      FORMAT(/,5X,' *** THE OPTIMAL POINT FOUND IS ***',
&/,/,5X,' N = ',F7.4,' , H = ',F7.4,' , K = ',
&F7.4,/,5X,' LOSS = ',F14.6,/)
C-----
C  ASSIGN NUMBER OF VARIABLES TO BE OPTIMIZED,
C  SEARCH STEP, AND TERMINATE VALUE. (TRUNCATE
C  N TO AN INTEGER VALUE)
C-----
      ND=2

```

```

      ICOUNT=700
      REQMIN=0.0001
C-----
C  ASSIGN STARTING POINT
C-----
      IX=XMIN(3)
      X(1)=XMIN(1)
      X(2)=XMIN(2)
      CALL NELMIN(XBATT2,ND,X,XMIN,XSEC,YNEWLO,
&YSEC,REQMIN,STEP,ICOUNT)
      I=1
      WRITE(*,2)
2     FORMAT(/,5X,' *** OPTIMIZATION ITERATION ***',/,/,
&5X,' I ',T12,' N ',T18,' H ',T28,' K ',T48,
&'      LOSS ',/,5X,58(1H-),/)
      WRITE(*,3) I,IX,XMIN(1),XMIN(2),YNEWLO
3     FORMAT(5X,I2,T10,I3,T15,F7.4,T25,F7.4,T45,F14.6)
      INCR=1
      ITIME=0
C-----
C  KEEP THE POINT AS CURRENT OPTIMAL
C-----
4     FMIN(5)=YNEWLO
      DO 5 L=1,2
        FMIN(L)=XMIN(L)
5     CONTINUE
C-----
C  DO A LINE SEARCH ON N
C-----
6     IX=IX+INCR
7     X(1)=XMIN(1)
      X(2)=XMIN(2)
      ND=2
      ICOUNT=700
      REQMIN=0.0001
      CALL NELMIN(XBATT2,ND,X,XMIN,XSEC,YNEWLO,
&YSEC,REQMIN,STEP,ICOUNT)
      I=I+1
      WRITE(*,3) I,IX,XMIN(1),XMIN(2),YNEWLO
      IF(ITIME.EQ.1) GO TO 10
      IF(YNEWLO.GT.FMIN(5)) GO TO 9
        ITIME=1
        FMIN(5)=YNEWLO
        DO 8 L=1,2
          FMIN(L)=XMIN(L)
8     CONTINUE
      GO TO 6
9     INCR=-INCR
      IX=IX-2
      ITIME=1
      GO TO 7
10    IF(YNEWLO.LE.FMIN(5)) GO TO 4
      IXMIN=IX-INCR
      NOPT=IXMIN

```

```

HOPT=FMIN(1)
EKOPT=FMIN(2)
FOPT=FMIN(5)
RETURN
END
C*****
SUBROUTINE XBATT3(X,F)
C*****
IMPLICIT REAL*8(A-H,O-Z)
REAL*8 THETA,ARL
COMMON/EC1/THETA
COMMON/EC4/DELTA,B,C,D,E,EM,T,W
COMMON/XBATT1/K(20),M(20),A1(20),B1(20),NT
COMMON IRULE
REAL*8 X(10)
H=X(1)
DK=X(2)
DN=X(3)
IF(H.GT.70..OR.H.LE.0.) F=9999999
IF(H.GT.70..OR.H.LE.0.) RETURN
IF(DK.GT.6..OR.DK.LE.0.) F=9999999
IF(DK.GT.6..OR.DK.LE.0.) RETURN
IF(DN.LT.1) F=9999999
IF(DN.LT.1) RETURN
1 GO TO (21,22,23,24,25,26,27,28) IRULE
WRITE(*,2)
2 FORMAT(/,5X,'*** ERROR INPUT! SELECT (1-8)! ***')
READ(*,*) IRULE
GO TO 1
21 NT=2
K(1)=1
M(1)=1
A1(1)=DK
B1(1)=9.
K(2)=1
M(2)=1
A1(2)=-9.
B1(2)=-DK
GO TO 8
22 NT=4
K(1)=1
M(1)=1
A1(1)=DK
B1(1)=9.
K(2)=2
M(2)=3
A1(2)=2.*DK/3.
B1(2)=DK
K(3)=2
M(3)=3
A1(3)=-B1(2)
B1(3)=-A1(2)
K(4)=1
M(4)=1

```

```
23  A1(4)=-9.  
    B1(4)=-DK  
    GO TO 8  
    NT=4  
    K(1)=1  
    M(1)=1  
    A1(1)=DK  
    B1(1)=9.  
    K(2)=4  
    M(2)=5  
    A1(2)=DK/3.  
    B1(2)=DK  
    K(3)=4  
    M(3)=5  
    A1(3)=-B1(2)  
    B1(3)=-A1(2)  
    K(4)=1  
    M(4)=1  
    A1(4)=-9.  
    B1(4)=-DK  
    GO TO 8  
24  NT=4  
    K(1)=1  
    M(1)=1  
    A1(1)=DK  
    B1(1)=9.  
    K(2)=8  
    M(2)=8  
    A1(2)=0.  
    B1(2)=DK  
    K(3)=8  
    M(3)=8  
    A1(3)=-B1(2)  
    B1(3)=-A1(2)  
    K(4)=1  
    M(4)=1  
    A1(4)=-9.  
    B1(4)=-DK  
    GO TO 8  
25  NT=6  
    K(1)=1  
    M(1)=1  
    A1(1)=DK  
    B1(1)=9.  
    K(2)=2  
    M(2)=3  
    A1(2)=2.*DK/3.  
    B1(2)=DK  
    K(3)=4  
    M(3)=5  
    A1(3)=DK/3.  
    B1(3)=DK  
    K(4)=K(3)  
    M(4)=M(3)
```

```

A1(4)=-B1(3)
B1(4)=-A1(3)
K(5)=K(2)
M(5)=M(2)
A1(5)=-B1(2)
B1(5)=-A1(2)
K(6)=K(1)
M(6)=M(1)
A1(6)=-B1(1)
B1(6)=-A1(1)
GO TO 8
26 NT=6
K(1)=1
M(1)=1
A1(1)=DK
B1(1)=9.
K(2)=2
M(2)=3
A1(2)=2.*DK/3.
B1(2)=DK
K(3)=8
M(3)=8
A1(3)=0.
B1(3)=DK
K(4)=K(3)
M(4)=M(3)
A1(4)=-B1(3)
B1(4)=-A1(3)
K(5)=K(2)
M(5)=M(2)
A1(5)=-B1(2)
B1(5)=-A1(2)
K(6)=K(1)
M(6)=M(1)
A1(6)=-B1(1)
B1(6)=-A1(1)
GO TO 8
27 NT=6
K(1)=1
M(1)=1
A1(1)=DK
B1(1)=9.
K(2)=4
M(2)=5
A1(2)=DK/3.
B1(2)=DK
K(3)=8
M(3)=8
A1(3)=0.
B1(3)=DK
K(4)=K(3)
M(4)=M(3)
A1(4)=-B1(3)
B1(4)=-A1(3)

```



```

K(5)=K(2)
M(5)=M(2)
A1(5)=-B1(2)
B1(5)=-A1(2)
K(6)=K(1)
M(6)=M(1)
A1(6)=-B1(1)
B1(6)=-A1(1)
GO TO 8
28 NT=8
K(1)=1
M(1)=1
A1(1)=DK
B1(1)=9.
K(2)=2
M(2)=3
A1(2)=2.*DK/3.
B1(2)=DK
K(3)=4
M(3)=5
A1(3)=DK/3.
B1(3)=DK
K(4)=8
M(4)=8
A1(4)=0.
B1(4)=DK
K(5)=K(4)
M(5)=M(4)
A1(5)=-B1(4)
B1(5)=-A1(4)
K(6)=K(3)
M(6)=M(3)
A1(6)=-B1(3)
B1(6)=-A1(3)
K(7)=K(2)
M(7)=M(2)
A1(7)=-B1(2)
B1(7)=-A1(2)
K(8)=K(1)
M(8)=M(1)
A1(8)=-B1(1)
B1(8)=-A1(1)
8 IN=X(3)
SHIFT=0.0D0
CALL ARLXBATT(NT,K,M,A1,B1,SHIFT,ARL)
ARL1=ARL
IF (ARL1.EQ.0.0D0) THEN
  F=9999999.
  GO TO 1000
END IF
SHIFT=DELTA*DSQRT(DN)
CALL ARLXBATT(NT,K,M,A1,B1,SHIFT,ARL)
ARL2=ARL
Y1=DEXP(-1.0D0*THETA*H)

```

```

Y2=1-Y1
Y=Y1/Y2
CYCTIME=H*(ARL2+Y)+E*DN+D
PC=(1.0D0-(1.0D0/(THETA*CYCTIME)))*EM
FC=T*Y/ARL1/CYCTIME
TC=W/CYCTIME
SC=(B+C*DN)/H
ELOSS=PC+FC+TC+SC
F=ELOSS
1000 RETURN
END
C*****
SUBROUTINE XBATT2(X,F)
C*****
IMPLICIT REAL*8(A-H,O-Z)
REAL*8 THETA,ARL
COMMON/EC1/THETA
COMMON/EC4/DELTA,B,C,D,E,EM,T,W
COMMON/SAMPLE/IX
COMMON/XBATT1/K(20),M(20),A1(20),B1(20),NT
COMMON IRULE
REAL*8 X(10)
H=X(1)
DK=X(2)
DN=DBLE(IX)
IF(H.GT.70..OR.H.LE.0.) F=9999999
IF(H.GT.70..OR.H.LE.0.) RETURN
IF(DK.GT.6..OR.DK.LE.0.) F=9999999
IF(DK.GT.6..OR.DK.LE.0.) RETURN
IF(DN.LT.1) F=9999999
IF(DN.LT.1) RETURN
1 GO TO (21,22,23,24,25,26,27,28) IRULE
21 NT=2
K(1)=1
M(1)=1
A1(1)=DK
B1(1)=9.
K(2)=1
M(2)=1
A1(2)=-9.
B1(2)=-DK
GO TO 8
22 NT=4
K(1)=1
M(1)=1
A1(1)=DK
B1(1)=9.
K(2)=2
M(2)=3
A1(2)=2.*DK/3.
B1(2)=DK
K(3)=2
M(3)=3
A1(3)=-B1(2)

```

```

      B1(3)=-A1(2)
      K(4)=1
      M(4)=1
      A1(4)=-9.
      B1(4)=-DK
      GO TO 8
23   NT=4
      K(1)=1
      M(1)=1
      A1(1)=DK
      B1(1)=9.
      K(2)=4
      M(2)=5
      A1(2)=DK/3.
      B1(2)=DK
      K(3)=4
      M(3)=5
      A1(3)=-B1(2)
      B1(3)=-A1(2)
      K(4)=1
      M(4)=1
      A1(4)=-9.
      B1(4)=-DK
      GO TO 8
24   NT=4
      K(1)=1
      M(1)=1
      A1(1)=DK
      B1(1)=9.
      K(2)=8
      M(2)=8
      A1(2)=0.
      B1(2)=DK
      K(3)=8
      M(3)=8
      A1(3)=-B1(2)
      B1(3)=-A1(2)
      K(4)=1
      M(4)=1
      A1(4)=-9.
      B1(4)=-DK
      GO TO 8
25   NT=6
      K(1)=1
      M(1)=1
      A1(1)=DK
      B1(1)=9.
      K(2)=2
      M(2)=3
      A1(2)=2.*DK/3.
      B1(2)=DK
      K(3)=4
      M(3)=5
      A1(3)=DK/3.

```

```

B1(3)=DK
K(4)=K(3)
M(4)=M(3)
A1(4)=-B1(3)
B1(4)=-A1(3)
K(5)=K(2)
M(5)=M(2)
A1(5)=-B1(2)
B1(5)=-A1(2)
K(6)=K(1)
M(6)=M(1)
A1(6)=-B1(1)
B1(6)=-A1(1)
GO TO 8
26 NT=6
K(1)=1
M(1)=1
A1(1)=DK
B1(1)=9.
K(2)=2
M(2)=3
A1(2)=2.*DK/3.
B1(2)=DK
K(3)=8
M(3)=8
A1(3)=0.
B1(3)=DK
K(4)=K(3)
M(4)=M(3)
A1(4)=-B1(3)
B1(4)=-A1(3)
K(5)=K(2)
M(5)=M(2)
A1(5)=-B1(2)
B1(5)=-A1(2)
K(6)=K(1)
M(6)=M(1)
A1(6)=-B1(1)
B1(6)=-A1(1)
GO TO 8
27 NT=6
K(1)=1
M(1)=1
A1(1)=DK
B1(1)=9.
K(2)=4
M(2)=5
A1(2)=DK/3.
B1(2)=DK
K(3)=8
M(3)=8
A1(3)=0.
B1(3)=DK
K(4)=K(3)

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M(4)=M(3)
A1(4)=-B1(3)
B1(4)=-A1(3)
K(5)=K(2)
M(5)=M(2)
A1(5)=-B1(2)
B1(5)=-A1(2)
K(6)=K(1)
M(6)=M(1)
A1(6)=-B1(1)
B1(6)=-A1(1)
GO TO 8
28 NT=8
K(1)=1
M(1)=1
A1(1)=DK
B1(1)=9.
K(2)=2
M(2)=3
A1(2)=2.*DK/3.
B1(2)=DK
K(3)=4
M(3)=5
A1(3)=DK/3.
B1(3)=DK
K(4)=8
M(4)=8
A1(4)=0.
B1(4)=DK
K(5)=K(4)
M(5)=M(4)
A1(5)=-B1(4)
B1(5)=-A1(4)
K(6)=K(3)
M(6)=M(3)
A1(6)=-B1(3)
B1(6)=-A1(3)
K(7)=K(2)
M(7)=M(2)
A1(7)=-B1(2)
B1(7)=-A1(2)
K(8)=K(1)
M(8)=M(1)
A1(8)=-B1(1)
B1(8)=-A1(1)
8 IN=IX
SHIFT=0.0D0
CALL ARLXBATT(NT,K,M,A1,B1,SHIFT,ARL)
ARL1=ARL
IF (ARL1.EQ.0.0D0) THEN
  F=9999999.
  GO TO 1000
END IF
SHIFT=DELTA*DSQRT(DN)

```

```

CALL ARLXBATT(NT,K,M,A1,B1,SHIFT,ARL)
ARL2=ARL
Y1=DEXP(-1.0D0*THETA*H)
Y2=1-Y1
Y=Y1/Y2
CYCTIME=H*(ARL2+Y)+E*DN+D
PC=(1.0D0-(1.0D0/(THETA*CYCTIME)))*EM
FC=T*Y/ARL1/CYCTIME
TC=W/CYCTIME
SC=(B+C*DN)/H
ELOSS=PC+FC+TC+SC
F=ELOSS
1000 RETURN
END
C*****
SUBROUTINE EVXBATT
C*****
IMPLICIT REAL*8(A-H,O-Z)
CHARACTER IFILE*12
REAL*8 ARL,SHIFT
DIMENSION K(20),M(20),A1(20),B1(20)
C-----
C INPUT COST AND OPERATION PARAMETERS
C-----
1 WRITE(*,2)
2 FORMAT(/,5X,'*** INPUT COST PARAMETERS. ***',/,/,
&5X,'DO YOU WANT TO INPUT FROM A FILE OR '
&'MANUALLY?',/5X,'PLEASE ENTER 1 = FILE, 2 = '
&'MANUALLY. ')
READ(*,*) IANS
GO TO (500,501) IANS
WRITE(*,502)
502 FORMAT(/,5X,'** ERROR INPUT! PLEASE TRY AGAIN! '
&'**')
GO TO 1
500 WRITE(*,503)
503 FORMAT(/,5X,'** PLEASE INPUT THE FILENAME THAT',/,
&5X,' CONTAINS THE COST PARAMETERS. ')
READ(*,31) IFILE
31 FORMAT(A12)
OPEN(50,FILE=IFILE,STATUS='OLD')
READ(50,*) DELTA,THETA,EM,E,D,T,W,B,C
CLOSE(50)
GO TO 3
501 WRITE(*,504)
504 FORMAT(/,5X,'=> INPUT VALUES OF',
&'DELTA,THETA,M,E,D,T,W,B,C')
READ(*,*) DELTA,THETA,EM,E,D,T,W,B,C
3 WRITE(*,4) DELTA,THETA,EM,E,D,T,W,B,C
4 FORMAT(/,5X,' ** VALUES RECEIVED ARE:',/,
&5X,' DELTA = ',F10.4,7X,' THETA = ',F10.4,/,
&5X,' M = ',F10.4,7X,' E = ',F10.4,/,
&5X,' D = ',F10.4,7X,' T = ',F10.4,/,
&5X,' W = ',F10.4,7X,' B = ',F10.4,/,

```

```

&5X, '    C    = ',F10.4,/,/,
&5X, ' ==> ARE THESE DATA CORRECT? ',/,
&5X, ' ==> PLEASE ENTER 1 = YES, 2 = NO <<<')
  READ (*,*) IANS
  GO TO (11,1), IANS
  GO TO 3
11  WRITE(*,12)
12  FORMAT(/,5X,'*** PLEASE INPUT N, H, K. ***')
  READ(*,*) N,H,DK
13  WRITE(*,14) N,H,DK
14  FORMAT(/,5X,'*** THE CONTROL CHART PARAMETERS '
&'ARE:',/,5X,'N = ',I4,',',',11X,' H = ',F10.4,',',',
&/,5X,'K = ',F10.4,',',',
&/,'==> ARE THEY CORRECT? ',
&/,'==> PLEASE ENTER 1 = YES, 2 = NO <<<')
  READ (*,*) IANS
  GO TO (15,11) IANS
  GO TO 13
15  WRITE(*,16)
16  FORMAT(/,5X,'*** PLEASE INPUT # OF RUNS RULES! '
&'***')
  READ(*,*) NT
  WRITE(*,17)
17  FORMAT(/,5X,'*** PLEASE INPUT THE RULES! ***')
  DO 19 I=1,NT
    WRITE(*,18) I
18    FORMAT(/,5X,'** INPUT K,M,A,B FOR RULE',I3,': ')
    READ(*,*) K(I),M(I),A1(I),B1(I)
19  CONTINUE
21  WRITE(*,22)
22  FORMAT(/,5X,'*** THE FOLLOWINF RULES ARE USED:')
  DO 24 I=1,NT
    WRITE(*,23) K(I),M(I),A1(I),B1(I)
23  FORMAT(/,5X,'T(',I3,',',',I3,',',',F7.4,',',',F7.4,')')
24  CONTINUE
25  WRITE(*,26)
26  FORMAT(/,5X,'==> ARE THEY CORRECT?',/,
&5X,'==> PLEASE ENTER 1 = YES, 2 = NO. <<<')
  READ (*,*) IANS
  GO TO (27,15) IANS
  GO TO 25
27  DN=FLOAT(N)
  SHIFT=0.0D0
  CALL ARLXBATT(NT,K,M,A1,B1,SHIFT,ARL)
  ARL1=ARL
  SHIFT=DELTA*DSQRT(DN)
  CALL ARLXBATT(NT,K,M,A1,B1,SHIFT,ARL)
  ARL2=ARL
  Y1=DEXP(-1.0D0*THETA*H)
  Y2=1-Y1
  Y=Y1/Y2
  CYCTIME=H*(ARL2+Y)+E*DN+D
  PC=(1.0D0-(1.0D0/(THETA*CYCTIME)))*EM
  FC=T*Y/ARL1/CYCTIME

```

```

TC=W/CYCTIME
SC=(B+C*DN)/H
ELOSS=PC+FC+TC+SC
F=ELOSS
WRITE(*,20) F
20  FORMAT(/,5X,'*** THE LOSS OF THE CURRENT DESIGN '
    &' IS:',/,5X,F14.6)
    RETURN
    END
C*****
      SUBROUTINE NELMIN(FN,N,START,XMIN,XSEC,YNEWLO,
        &YSEC,REQMIN,STEP,ICOUNT)
C*****
C THIS SUBROUTINE IS MODIFIED FROM:
C OLSSON, D. M., "A SEQUENTIAL SIMPLEX PROGRAM FOR
C SOLVING MINIMIZATION PROBLEM," JQT, VOL. 6,
C NO. 1, PP. 53-57, JAN. 1974.
C*****
      REAL*8 START(N),STEP(N),XMIN(N),XSEC(N),YNEWLO,
        &YSEC,REQMIN,P(20,21),PSTAR(20),P2STAR(20),
        &PBAR(20),Y(20),DN,Z,YLO,RCOEFF,YSTAR,ECOEFF,
        &Y2STAR,CCOEFF,F,DABIT,DCHK,COORD1,COORD2
      DATA RCOEFF/1.0D0/,ECOEFF/2.0D0/,CCOEFF/0.5D0/
      KCOUNT=ICOUNT
      ICOUNT=0

C-----
C  INITIALIZATION
C-----
      DO 60 I=1,N
        XMIN(I)=0.0D0
        XSEC(I)=0.0D0
60  CONTINUE
      YNEWLO=0.0D0
      YSEC=0.0D0
      IF (REQMIN.LE.0.0D0) ICOUNT=ICOUNT-1
      IF (N.LE.0) ICOUNT=ICOUNT-10
      IF (N.GT.20) ICOUNT=ICOUNT-10
      IF (ICOUNT.LT.0) RETURN
      DABIT=2.04607D-35
      BIGNUM=1.0D30
      KONVGE=5
      XN=FLOAT(N)
      NN=N+1

C-----
C  CONSTRUCTION OF INITIAL SIMPLEX
C-----
1001 DO 1 I=1,N
1    P(I,NN)=START(I)
      CALL FN(START,F)
      Y(NN)=F
      ICOUNT=ICOUNT+1
      DO 2 J=1,N
        DCHK=START(J)
        START(J)=DCHK+STEP(J)

```



```

      DO 3 I=1,N
3     P(I,J)=START(I)
      CALL FN(START,F)
      Y(J)=F
      ICOUNT=ICOUNT+1
2     START(J)=DCHK
C-----
C   SIMPLEX CONSTRUCTION COMPLETE
C-----
C   FIND HIGHEST AND LOWEST Y VALUE
C   YNEWLO INDICATES THE VERTEX OF THE
C   SIMPLEX TO BE REPLACED
C-----
1000  YLO=Y(1)
      YNEWLO=YLO
      ILO=1
      IHI=1
      DO 5 I=2,NN
          IF (Y(I).GE.YLO) GO TO 4
          YLO=Y(I)
          ILO=I
4     IF (Y(I).LE.YNEWLO) GO TO 5
      YNEWLO=Y(I)
      IHI=I
5     CONTINUE
C-----
C   PERFORM CONVERGENCE CHECKS ON FUNCTION
C-----
      DCHK=(YNEWLO+DABIT)/(YLO+DABIT)-1.0D0
      IF (DABS(DCHK).LT.REQMIN) GO TO 900
      KONVGE=KONVGE-1
      IF (KONVGE.NE.0) GO TO 2020
      KONVGE=5
C-----
C   CHECK CONVERGENCE OF COORDINATE
C   ONLY EVERY 5 SIMPLEX
C-----
      DO 2015 I=1,N
          COORD1=P(I,1)
          COORD2=COORD1
          DO 2010 J=2,NN
              IF (P(I,J).GE.COORD1) GO TO 2005
              COORD1=P(I,J)
2005          IF (P(I,J).LE.COORD2) GO TO 2010
              COORD2=P(I,J)
2010          CONTINUE
          DCHK=(COORD2+DABIT)/(COORD1+DABIT)-1.0D0
          IF (DABS(DCHK).GT.REQMIN) GO TO 2020
2015          CONTINUE
          GO TO 900
2020          IF (ICOUNT.GE.KCOUNT) GO TO 900
C-----
C   CALCULATE PBAR, THE CENTROID OF THE SIMPLEX
C   VERTICES EXCEPT THAT WITH Y VALUE YNEWLO

```

```

C-----
      DO 7 I=1,N
        Z=0.0D0
        DO 6 J=1,NN
          Z=Z+P(I,J)
6        CONTINUE
        Z=Z-P(I,IHI)
7      PBAR(I)=Z/FLOAT(N)
C-----
C REFLECTION THROUGH THE CENTROID
C-----
      DO 8 I=1,N
8      PSTAR(I)=(1.0D0+RCOEFF)*PBAR(I)-RCOEFF*P(I,IHI)
        CALL FN(PSTAR,F)
        YSTAR=F
        ICOUNT=ICOUNT+1
        IF (YSTAR.GT.YLO) GO TO 12
        IF (ICOUNT.GE.KCOUNT) GO TO 19
C-----
C SUCCESSFUL REFLECTION, SO EXTENTION
C-----
      DO 9 I=1,N
9      P2STAR(I)=ECOEFF*PSTAR(I)+(1.0D0-ECOEFF)*PBAR(I)
        CALL FN(P2STAR,F)
        Y2STAR=F
        ICOUNT=ICOUNT+1
C-----
C RETAIN EXTENSION OR CONTRACTION
C-----
        IF (Y2STAR.GE.YSTAR) GO TO 19
10     DO 11 I=1,N
11     P(I,IHI)=P2STAR(I)
        Y(IHI)=Y2STAR
        GO TO 1000
C-----
C NO EXTENSION
C-----
12     L=0
        DO 13 I=1,NN
          IF (Y(I).GT.YSTAR) L=L+1
13     CONTINUE
        IF (L.GT.1) GO TO 19
        IF (L.EQ.0) GO TO 15
C-----
C CONTRACTION ON THE REFLECTION SIDE OF THE CENTROID
C-----
      DO 14 I=1,N
14     P(I,IHI)=PSTAR(I)
        Y(IHI)=YSTAR
C-----
C CONTRACTION ON THE Y(IHI) SIDE OF THE CENTROID
C-----
15     IF (ICOUNT.GE.KCOUNT) GO TO 900
        DO 16 I=1,N

```

```

16  P2STAR(I)=CCOEFF*P(I,IHI)+(1.0D0-CCOEFF)*PBAR(I)
    CALL FN(P2STAR,F)
    Y2STAR=F
    ICOUNT=ICOUNT+1
    IF (Y2STAR.LT.Y(IHI)) GO TO 10

```

```

C-----
C  CONTRACT THE WHOLE SIMPLEX
C-----

```

```

    DO 18 J=1,NN
      DO 17 I=1,N
        P(I,J)=(P(I,J)+P(I,ILO))*0.5D0
17   XMIN(I)=P(I,J)
        CALL FN(XMIN,F)
        Y(J)=F
18   CONTINUE
      ICOUNT=ICOUNT+NN
      IF (ICOUNT.LT.KCOUNT) GO TO 1000
      GO TO 900

```

```

C-----
C  RETAIN REFLECTION
C-----

```

```

19   CONTINUE
    DO 20 I=1,N
20   P(I,IHI)=PSTAR(I)
      Y(IHI)=YSTAR
      GO TO 1000
900  DO 23 J=1,NN
      DO 22 I=1,N
22   XMIN(I)=P(I,J)
      CALL FN(XMIN,F)
      Y(J)=F
23   CONTINUE
      YNEWLO=BIGNUM
      DO 24 J=1,NN
        IF (Y(J).GE.YNEWLO) GO TO 24
        YNEWLO=Y(J)
        IBEST=J
24   CONTINUE
      Y(IBEST)=BIGNUM
      YSEC=BIGNUM
      DO 25 J=1,NN
        IF (Y(J).GE.YSEC) GO TO 25
        YSEC=Y(J)
        ISEC=J
25   CONTINUE
      DO 26 I=1,N
        XMIN(I)=P(I,IBEST)
        XSEC(I)=P(I,ISEC)
26   CONTINUE
      RETURN
      END

```

```

C*****
C  SUBROUTINE ARLXBATT(NT,K,M,A,B,SHIFT,ARL)
C*****

```

```

C THIS SUBROUTINE IS A MODIFICATION FROM:
C CHAMP, W. C. AND WOODALL, W. H., "A PROGRAM
C TO EVALUATE THE RUN LENGTH DISTRIBUTION OF
C A SHEWHART CONTROL CHART WITH SUPPLEMENTARY
C RUNS RULES", JQT, VOL.22, NO.1, PP. 68-73,
C JAN. 1990.
C*****
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION K(20),M(20),A(20),B(20),NX(58)
      INTEGER H,CK,CX,QH,SG,TMP,D,QQNS,DI(20),PS(58)
      INTEGER X(20,10),Q(400,10),QQ(400),S(20)
      DOUBLE PRECISION CDF(41),CUM,LR(400),LH,LP,
&P(10),R(41)
      DOUBLE PRECISION ARL,STD,U(400),ZA,ZB,SHIFT,ZCDF
      R(1)=-9
      DO 2 I=1,NT
        R(2*I)=A(I)
2       R(2*I+1)=B(I)
      R(2*NT+2)=9
      MR=2*NT+1
      NR=MR
3     CK=0
      DO 5 J=1,MR
        IF (R(J).EQ.R(J+1).AND.J.LE.NR) THEN
          DO 4 L=J,NR
            R(L)=R(L+1)
4           NR=NR-1
            CK=1
          ENDIF
          IF(R(J).GT.R(J+1)) THEN
            TP=R(J)
            R(J)=R(J+1)
            R(J+1)=TP
            CK=1
          ENDIF
5         CONTINUE
        MR=MR-1
        IF (MR.GT.NR) MR=NR
        IF (CK.EQ.1.AND.MR.GE.1) GO TO 3
        CK=0
        NV=0
        DO 6 I=1,NT
          NV=NV+M(I)-1
          IF (K(I).LT.M(I)) CK=1
6         CONTINUE
          IF (CK.EQ.0) NV=NV+1
          DI(1)=M(1)-1
          DO 7 I=2,NT
            DI(I)=DI(I-1)+M(I)-1
            IF(M(I).EQ.1) DI(I)=0
7         CONTINUE
          DO 9 I=1,NT
            DO 8 J=1,NR
              X(I,J)=0

```

```

      IF(A(I).LE.R(J).AND.R(J+1).LE.B(I)) X(I,J)=1
8     CONTINUE
9     CONTINUE
      QQ(1)=0
      QQNS=2**NV-1
      NS=1
      H=1
10    QH=QQ(H)
      DO 11 L=1,NV
        PS(L)=QH-2*(QH/2)
11    QH=QH/2
      DO 13 I=1,NT
        S(I)=0
        IF(M(I).GT.1) THEN
          DO 12 L=DI(I)-M(I)+2,DI(I)
12    S(I)=S(I)+PS(L)
          ENDIF
13    CONTINUE
      DO 19 J=1,NR
        SG=0
        DO 16 I=1,NT
          IF(SG.EQ.0) THEN
            IF(S(I)+X(I,J).GE.K(I)) THEN
              SG=1
            ELSE
              IF(M(I).GT.1) NX(DI(I)-M(I)+2)=X(I,J)
              IF(M(I).GT.2) THEN
                DO 14 L=DI(I)-M(I)+3,DI(I)
14    NX(L)=PS(L-1)
                ENDIF
              ENDIF
              IF(X(I,J).EQ.0.AND.M(I).GT.1) THEN
                TMP=S(I)-PS(DI(I))+1
                L=DI(I)
                CK=0
15    IF(NX(L).EQ.1) THEN
                  CK=1
                  IF(TMP.LT.K(I)) THEN
                    NX(L)=0
                    TMP=TMP-1
                    CK=0
                  ENDIF
                ENDIF
                L=L-1
                TMP=TMP+1
                IF(CK.EQ.0.AND.L.GE.DI(I)-M(I)+2) GO TO 15
              ENDIF
            ENDIF
          CONTINUE
16    IF(SG.EQ.0) THEN
            QH=NX(1)
            DO 17 L=2,NV
17    QH=QH+NX(L)*(2**(L-1))
            CK=0

```

```

DO 18 L=1,NS
  IF(CK.EQ.0.AND.QH.EQ.QQ(L)) THEN
    Q(H,J)=QQ(L)
    CK=1
  ENDIF
18 CONTINUE
  IF(CK.EQ.0) THEN
    NS=NS+1
    QQ(NS)=QH
    Q(H,J)=QH
  ENDIF
  ELSE
    Q(H,J)=QQNS
  ENDIF
19 CONTINUE
  H=H+1
  IF(H.LE.NS) GO TO 10
  NS=NS+1
  QQ(NS)=QQNS
  DO 20 J=1,NR
20   Q(NS,J)=QQNS
21  H=0
    CK=0
    DO 23 I=2,NS-H
      IF(QQ(I-1).GT.QQ(I)) THEN
        CK=1
        TMP=QQ(I-1)
        QQ(I-1)=QQ(I)
        QQ(I)=TMP
        DO 22 J=1,NR
          TMP=Q(I-1,J)
          Q(I-1,J)=Q(I,J)
22   Q(I,J)=TMP
        ENDIF
23 CONTINUE
      H=H+1
      IF(CK.EQ.1) GO TO 21
24 CK=0
      I=1
25 H=I+1
26 CX=0
      DO 27 J=1,NR
        IF(Q(I,J).NE.Q(H,J)) CX=1
27 CONTINUE
      IF(CX.EQ.0) THEN
        TMP=QQ(H)
        DO 29 L=1,H-1
          DO 28 J=1,NR
            IF(Q(L,J).EQ.TMP) Q(L,J)=QQ(I)
28 CONTINUE
29 CONTINUE
        DO 31 L=H,NS-1
          QQ(L)=QQ(L+1)
          DO 30 J=1,NR

```

```

          Q(L,J)=Q(L+1,J)
          IF(Q(L,J).EQ.TMP) Q(L,J)=QQ(I)
30      CONTINUE
31      CONTINUE
      NS=NS-1
      CK=1
      ENDIF
      H=H+1
      IF(H.LT.NS) GO TO 26
      I=I+1
      IF(I.LT.NS-1) GO TO 25
      IF(CK.EQ.1) GO TO 24
      DO 34 I=1,NS
        DO 33 J=1,NR
          IF(Q(I,J).LT.QQNS) THEN
            CK=0
            L=1
32          IF(Q(I,J).EQ.QQ(L)) THEN
              Q(I,J)=L
              CK=1
            ENDIF
            L=L+1
            IF(CK.EQ.0.AND.L.LT.NS) GO TO 32
          ELSE
            Q(I,J)=NS
          ENDIF
33          CONTINUE
34          CONTINUE
          IST=1
          DO 37 J=1,NR
            ZA=R(J)-SHIFT
            ZB=R(J+1)-SHIFT
37          P(J)=ZCDF(ZB)-ZCDF(ZA)
          DO 39 I=1,NS-1
            U(I)=0.0D0
            DO 38 J=1,NR
              IF(Q(I,J).NE.NS) U(I)=U(I)+P(J)
38          CONTINUE
39          U(I)=1.0D0-U(I)
          N=1
          CUM=U(IST)
          CDF(1)=CUM
          ARL=CUM
          CK=0
40          N=N+1
          DO 42 I=1,NS-1
            LR(I)=0
            DO 41 J=1,NR
              IF(Q(I,J).NE.NS) LR(I)=LR(I)+P(J)*U(Q(I,J))
41          CONTINUE
42          CONTINUE
          IF(U(IST).NE.0.0.AND.CUM.NE.1.0) THEN
            LH=LR(IST)/U(IST)
            LP=(1-CUM-LR(IST))/(1-CUM)

```

```

      TP=ABS(LH-LP)
      IF(N.GT.9.AND.TP.LT.0.000001) CK=1
    ENDIF
    IF(CUM.EQ.1.0D0) CK=1
    IF(N.GT.40) CK=1
    ARL=ARL+N*LR(IST)
    IF(CK.EQ.1) THEN
      TP=N/(1-LH)+1/((1-LH)*(1-LH))
      ARL=ARL+LH*LR(IST)*TP
      TP=1-LH
      TP=N*N/TP+(2*N-1)/(TP*TP)+2/(TP*TP*TP)
    ENDIF
43   DO 43 I=1,NS-1
      U(I)=LR(I)
      CUM=CUM+LR(IST)
      CDF(N)=CUM
      IF(CK.EQ.0) GO TO 40
    RETURN
  END

C*****
C STANDARD NORMAL CUMULATIVE DISTRIBUTION FUNCTION
C REF: SHENTON, L. R., "INEQUALITIES FOR THE NORMAL
C      INTEGRAL INCLUDING A NEW CONTINUED FRACTION,"
C      BIOMETRIKA, VOL. 41, PP. 177-189.
C*****
      REAL*8 FUNCTION ZCDF(Z)
      REAL*8 C
      REAL*8 X,Z,Z1,Z2,R
      INTEGER K
      ZCDF=0.5D0
      IF (Z.EQ.0.0) RETURN
      X=ABS(Z)
      C=0.3989422804014D0
      Z1=0.50D0
      Z2=2.50D0
      IF (X.LE.Z1) THEN
        R=1.0D0
        TT=1.0D0
        DO 300 K=0,5
          U=(-(2*K+1)*X*X)/((2*K+3)*2*(K+1))
          TT=U*TT
          R=R+TT
300      CONTINUE
        ZCDF=ZCDF+X*(R)*C
      ENDIF
      IF ((Z1.LT.X).AND.(X.LE.Z2)) THEN
        R=31
        DO 200 K=15,1,-1
          R=(2*K-1)+((-1)**K)*K*X*X/R
200      CONTINUE
      R=X/R
      ZCDF=ZCDF+C*EXP(-.5*X*X)*R
    ENDIF
    IF ((Z2.LT.X).AND.(X.LT.4.0)) THEN

```



```
      R=X+15
      DO 100 K=15,1,-1
        R=X+(K/R)
100    CONTINUE
      R=1/R
      ZCDF=1.0-C*EXP(-.5*X*X)*R
    ENDF
    IF (X.GE.4.0) ZCDF=1
    IF (Z.LT.0.0) ZCDF=1-ZCDF
    RETURN
  END
```

APPENDIX B

FORTRAN PROGRAM FOR DESIGN AND EVALUATION  
OF THE EWMA CHART

```

C*****
C  MAIN PROGRAM
C*****
$DEBUG
1  WRITE(*,2)
2  FORMAT(/,1H1,12X,24(1H*),/,
&13X,'***  MAIN  MENU  ***',/,13X,24(1H*),/,/,
&5X,'(1) ECONOMIC DESIGN OF EWMA CONTROL '
&'CHARTS',/,5X,'(2) EVALUATION OF EWMA CONTROL '
&'CHARTS',/,5X,'(3) EXIT THE PROGRAM',/,/,
&5X,'==> PLEASE ENTER YOUR OPTION (1, 2, OR 3)!'
&' <<<')
    READ(*,*) IANS
    GO TO (4,5,6) IANS
    WRITE(*,3)
3  FORMAT(/,5X,'*** ERROR INPUT! PLEASE TRY AGAIN! '
&'***')
    GO TO 1
4  CALL ECEWMA
    GO TO 1
5  CALL EVEWMA
    GO TO 1
6  STOP
    END
C*****
    SUBROUTINE ECEWMA
C*****
C  ECONOMIC DESIGN OF THE EWMA CHART
C*****
    IMPLICIT REAL*8(A-H,O-Z)
    CHARACTER IFILE*12
    COMMON/EC1/THETA
    COMMON/EC2/N,NOPT
    COMMON/EC3/H,EK,HOPT,EKOPT,FOPT,STEP(20)
    COMMON/EC4/DELTA,B,C,D,E,EM,T,W
    COMMON/EWMA1/ALPHA,ALPHAOPT
C-----
C  INPUT COST AND OPERATION PARAMETERS
C-----
1  WRITE(*,2)
2  FORMAT(/,5X,'*** INPUT COST PARAMETERS. ***',/,/,
&5X,'DO YOU WANT TO INPUT FROM A FILE OR '
&'MANUALLY?',/,5X,'==> PLEASE ENTER 1 = FILE, 2 = '
&'MANUALLY. ')
    READ (*,*) IANS
    GO TO (500,501) IANS
    WRITE (*,502)
502 FORMAT(/,5X,'** ERROR INPUT! PLEASE TRY AGAIN! '
&'**')
    GO TO 1
C-----
C  INPUT FROM FILE
C-----
500 WRITE(*,503)

```

```

503  FORMAT(/,5X,'** PLEASE INPUT THE FILENAME THAT',/,
      &5X,'  CONTAINS THE COST PARAMETERS.')
```

31 FORMAT(A12)  
 OPEN(50,FILE=IFILE,STATUS='OLD')  
 READ(50,\*) DELTA,THETA,EM,E,D,T,W,B,C  
 CLOSE(50)  
 GO TO 3

C-----  
 C INPUT MANUALLY  
 C-----

```

501  WRITE(*,504)
504  FORMAT(/,5X,'=> INPUT VALUES OF DELTA,THETA,M,
      &' E,D,T,W,B,C')
      READ(*,*) DELTA,THETA,EM,E,D,T,W,B,C
3    WRITE(*,4) DELTA,THETA,EM,E,D,T,W,B,C
4    FORMAT(/,5X,' ** VALUES RECEIVED ARE:',/,
      &5X,' DELTA = ',F10.4,7X,' THETA = ',F10.4,/,
      &5X,' M = ',F10.4,7X,' E = ',F10.4,/,
      &5X,' D = ',F10.4,7X,' T = ',F10.4,/,
      &5X,' W = ',F10.4,7X,' B = ',F10.4,/,
      &5X,' C = ',F10.4,/,/,
      &5X,' => ARE THESE DATA CORRECT? ',/,
      &5X,' => PLEASE ENTER 1 = YES, 2 = NO <<<')
      READ(*,*) IANS
      GO TO (5,1), IANS
      GO TO 3
```

C-----  
 C SELECT THE STARTING POINT  
 C-----

```

5    N=5
      H=1.0D0
      EK=3.0D0
      ALPHA=0.5D0
6    WRITE(*,7) N,H,EK,ALPHA
7    FORMAT(/,5X,'*** THE SUGGESTED STARTING POINT'
      &' IS: ',/,5X,' N = ',I3,', H = ',F6.2,', K = ',
      &F6.2,', ALPHA = ',F6.2,/,/,
      &5X,' => DO YOU ACCEPT THIS POINT?',/,
      &5X,' => ENTER 1 = YES, 2 = NO. <<<')
      READ(*,*) IANS
      GO TO (8,14), IANS
      GO TO 6
```

C-----  
 C SELECT THE STEP SIZE  
 C-----

```

8    STEP(1)=0.5
      STEP(2)=0.5
      STEP(3)=0.1
      STEP(4)=1.0
19   WRITE(*,20) STEP(4),STEP(1),STEP(2),STEP(3)
20   FORMAT(/,5X,'*** THE SUGGESTED STEP SIZES ARE:',/,
      &5X,' N = ',F5.2,', ',3X,' H = ',F6.2,', ',3X,
      &' K = ',F6.2,', ',3X,' ALPHA = ',F4.2,/,/,
```

```

&'==> DO YOU ACCEPT THESE STEP SIZES?',/,
&'==> PLEASE ENTER 1 = YES, 2 = NO. <<<')
  READ (*,*) IANS
  GO TO (11,21) IANS
  GO TO 19
C-----
C  PERFORM THE ECON DESIGN OF THE EWMA
C-----
11  CALL OPEWMA
C-----
C  PRINT THE OPTIMAL DESIGN
C-----
12  WRITE(*,13) NOPT,HOPT,EKOPT,ALPHAOPT,FOPT
13  FORMAT(/,5X,58(1H-),
&/,5X,' *** THE OPTIMAL DESIGN IS: ',
&/,5X,' N = ',I4,',',3X,' H = ',F10.5,',',
&3X,' K = ',F10.5,',',3X,' ALPHA = ',F6.4,
&/,5X,' *** THE MINIMUM LOSS PER HOUR IS: ',
&F14.6,/,/,5X,58(1H-))
  GO TO 18
C-----
C  INPUT NEW STARTING POINT
C-----
14  WRITE(*,15)
15  FORMAT(/,5X,' *** PLEASE INPUT NEW STARTING POINT
***', &/,5X,' ==> KEY IN VALUES FOR N, H, K, ALPHA')
  READ(*,*) N,H,EK,ALPHA
16  WRITE(*,17) N,H,EK,ALPHA
17  FORMAT(/,5X,' *** NEW STARTING POINT IS:',/,
&5X,' N = ',I4,',',3X,' H = ',F8.4,',', ' K =
',F8.4, &',',3X,' ALPHA = ',F6.4,/,
&5X,' ==> ARE THEY CORRECT?',/,
&5X,' ==> PLEASE ENTER 1 = YES, 2 = NO. <<<')
  READ(*,*) IANS
  GO TO (8,14), IANS
  GO TO 16
C-----
C  INPUT NEW STEP SIZES
C-----
21  WRITE(*,22)
22  FORMAT(/,5X,' *** PLEASE INPUT NEW STEP SIZES '
&'***',/,5X,' ==> ENTER VALUES FOR N, H, K, ALPHA.'
&' <<<')
  READ (*,*) STEP(4),STEP(1),STEP(2),STEP(3)
23  WRITE (*,24) STEP(4),STEP(1),STEP(2),STEP(3)
24  FORMAT(/,5X,' *** THE NEW STEP SIZES ARE:',/,
&5X,' N = ',F5.2,',',3X,' H = ',F6.2,',',3X,
&' K = ',F6.2,',',3X,' ALPHA = ',F4.2,/,/,
&'==> ARE THEY CORRECT?',/,
&'==> PLEASE ENTER 1 = YES, 2 = NO. <<<')
  READ (*,*) IANS
  GO TO (11,21) IANS
  GO TO 23
18  RETURN

```

```

END
C*****
SUBROUTINE OPEWMA
C*****
IMPLICIT REAL*8(A-H,O-Z)
COMMON/EC2/N,NOPT
COMMON/EC3/H,EK,HOPT,EKOPT,FOPT,STEP(20)
COMMON/EWMA1/ALPHA,ALPHAOPT
COMMON/SAMPLE/IX
EXTERNAL FUNCT4
EXTERNAL FUNCT3
REAL*8 FMIN(10),X(10),XMIN(20),XSEC(20),F
C-----
C ASSIGN NO. OF VARIABLE,SEARCH STEP,
C AND TERMINATE VALUE
C-----
ND=4
ICOUNT=700
REQMIN=0.0001
C-----
C ASSIGN STARTING POINT
C-----
X(1)=H
X(2)=EK
X(3)=ALPHA
X(4)=DBLE(N)
CALL NELMIN(FUNCT4,ND,X,XMIN,XSEC,YNEWLO,YSEC,
&REQMIN,STEP,ICOUNT)
C-----
C PRINT OUT THE OPTIMAL POINT FOUND
C-----
WRITE(*,1) XMIN(4),XMIN(1),XMIN(2),XMIN(3),YNEWLO
1 FORMAT(/,5X,' *** THE OPTIMAL POINT FOUND IS ***',
&/,/,5X,' N = ',F7.4,' , H = ',F7.4,' , K = ',
&F7.4,' , ALPHA = ',F7.4,/,5X,' LOSS = ',F14.6,/)
C-----
C ASSIGN VARIABLE NO., SEARCH STEP
C AND TERMINATE VALUE
C-----
ND=3
ICOUNT=700
REQMIN=0.0001
C-----
C ASSIGN STARTING POINT
C-----
IX=XMIN(4)
X(1)=XMIN(1)
X(2)=XMIN(2)
X(3)=XMIN(3)
CALL NELMIN(FUNCT3,ND,X,XMIN,XSEC,YNEWLO,
&YSEC,REQMIN,STEP,ICOUNT)
I=1
WRITE(*,2)
2 FORMAT(/,5X,' *** OPTIMIZATION ITERATION ***',/,/,

```

```

&5X, 'I', T12, 'N', T18, 'H', T28, 'K', T38, 'ALPHA', T48,
&'   LOSS', /, 5X, 58(1H-), /)
WRITE(*, 3) I, IX, XMIN(1), XMIN(2), XMIN(3), YNEWLO
3   FORMAT(5X, I2, T10, I3, T15, F7.4, T25, F7.4, T35, F7.4,
&T45, F14.6)
      INCR=1
      ITIME=0
C-----
C  KEEP THE POINT AS CURRENT OPTIMAL
C-----
4     FMIN(5)=YNEWLO
      DO 5 L=1, 3
        FMIN(L)=XMIN(L)
5     CONTINUE
6     IX=IX+INCR
7     X(1)=XMIN(1)
      X(2)=XMIN(2)
      X(3)=XMIN(3)
      ND=3
      ICOUNT=700
      REQMIN=0.0001
      CALL NELMIN(FUNCT3, ND, X, XMIN, XSEC, YNEWLO,
&YSEC, REQMIN, STEP, ICOUNT)
      I=I+1
      WRITE(*, 3) I, IX, XMIN(1), XMIN(2), XMIN(3), YNEWLO
      IF(ITIME.EQ.1) GO TO 10
      IF(YNEWLO.GT.FMIN(5)) GO TO 9
        ITIME=1
        FMIN(5)=YNEWLO
        DO 8 L=1, 3
          FMIN(L)=XMIN(L)
8     CONTINUE
      GO TO 6
9     INCR=-INCR
      IX=IX-2
      ITIME=1
      GO TO 7
10    IF(YNEWLO.LE.FMIN(5)) GO TO 4
      IXMIN=IX-INCR
      NOPT=IXMIN
      HOPT=FMIN(1)
      EKOPT=FMIN(2)
      ALPHAOPT=FMIN(3)
      FOPT=FMIN(5)
      RETURN
      END
C*****
      SUBROUTINE FUNCT4(X, F)
C*****
      IMPLICIT REAL*8(A-H, O-Z)
      REAL*8 THETA, ARL
      COMMON/EC1/THETA
      COMMON/EC4/DELTA, B, C, D, E, EM, T, W
      REAL*8 X(10)

```

```

H=X(1)
DK=X(2)
DA=X(3)
DN=X(4)
IF(H.GT.70..OR.H.LE.0.) F=9999999
IF(H.GT.70..OR.H.LE.0.) RETURN
IF(DK.GT.6..OR.DK.LE.0.) F=9999999
IF(DK.GT.6..OR.DK.LE.0.) RETURN
IF(DA.GT.1..OR.DA.LE.0.) F=9999999
IF(DA.GT.1..OR.DA.LE.0.) RETURN
IF(DN.LT.1) F=9999999
IF(DN.LT.1) RETURN
IN=X(4)
SHIFT=0.0D0
CALL ARLEWMA(DA,DK,SHIFT,ARL)
ARL1=ARL
SHIFT=DELTA*DSQRT(DN)
CALL ARLEWMA(DA,DK,SHIFT,ARL)
ARL2=ARL
Y1=DEXP(-1.0D0*THETA*H)
Y2=1-Y1
Y=Y1/Y2
CYCTIME=H*(ARL2+Y)+E*DN+D
PC=(1.0D0-(1.0D0/(THETA*CYCTIME)))*EM
FC=T*Y/ARL1/CYCTIME
TC=W/CYCTIME
SC=(B+C*DN)/H
ELOSS=PC+FC+TC+SC
F=ELOSS
RETURN
END

```

```

C*****

```

```

SUBROUTINE FUNCT3(X,F)

```

```

C*****

```

```

IMPLICIT REAL*8(A-H,O-Z)
REAL*8 THETA,ARL
COMMON/EC1/THETA
COMMON/EC4/DELTA,B,C,D,E,EM,T,W
COMMON/SAMPLE/IX
REAL*8 X(10)
H=X(1)
DK=X(2)
DA=X(3)
DN=DBLE(IX)
IF(H.GT.70..OR.H.LE.0.) F=9999999
IF(H.GT.70..OR.H.LE.0.) RETURN
IF(DK.GT.6..OR.DK.LE.0.) F=9999999
IF(DK.GT.6..OR.DK.LE.0.) RETURN
IF(DA.GT.1..OR.DA.LE.0.) F=9999999
IF(DA.GT.1..OR.DA.LE.0.) RETURN
IF(DN.LT.1) F=9999999
IF(DN.LT.1) RETURN
IN=IX
SHIFT=0.0D0

```



```

CALL ARLEWMA(DA,DK,SHIFT,ARL)
ARL1=ARL
SHIFT=DELTA*DSQRT(DN)
CALL ARLEWMA(DA,DK,SHIFT,ARL)
ARL2=ARL
Y1=DEXP(-1.0D0*THETA*H)
Y2=1-Y1
Y=Y1/Y2
CYCTIME=H*(ARL2+Y)+E*DN+D
PC=(1.0D0-(1.0D0/(THETA*CYCTIME)))*EM
FC=T*Y/ARL1/CYCTIME
TC=W/CYCTIME
SC=(B+C*DN)/H
ELOSS=PC+FC+TC+SC
F=ELOSS
RETURN
END

```

```
C*****
```

```
  SUBROUTINE ARLEWMA(DA,DK,SHIFT,ARL)
```

```
C*****
```

```

C REF: CROWDER, S. V., "AVERAGE RUN LENGTH OF
C       EXPONENTIALLY WEIGHTED MOVING AVERAGE
C       CONTROL CHARTS," JQT, VOL. 19, NO. 3,
C       PP. 161-164, JUL, 1987.

```

```
C*****
```

```

  REAL*8 ARG,A(24,24),B2(24),W(24),P(24),X1(24),
&F1,H1,WK(24),ARL,DA,DK,SHIFT

```

```
  INTEGER IPIVOT(24),IFLAG
```

```
  P(1)=0.9951872199970213D0
```

```
  P(2)=0.9747285559713095D0
```

```
  P(3)=0.9382745520027327D0
```

```
  P(4)=0.8864155270044010D0
```

```
  P(5)=0.8200019859739029D0
```

```
  P(6)=0.7401241915785543D0
```

```
  P(7)=0.6480936519369755D0
```

```
  P(8)=0.5454214713888395D0
```

```
  P(9)=0.4337935076260451D0
```

```
  P(10)=0.3150426796961634D0
```

```
  P(11)=0.1911188674736163D0
```

```
  P(12)=0.0640568928626056D0
```

```
  W(1)=0.0123412297999872D0
```

```
  W(2)=0.0285313882689337D0
```

```
  W(3)=0.0442774388174198D0
```

```
  W(4)=0.0592985849154368D0
```

```
  W(5)=0.0733464814110803D0
```

```
  W(6)=0.0861901615319533D0
```

```
  W(7)=0.0976186521041139D0
```

```
  W(8)=0.1074442701159656D0
```

```
  W(9)=0.1155056680537256D0
```

```
  W(10)=0.1216704729278034D0
```

```
  W(11)=0.1258374563468283D0
```

```
  W(12)=0.1279381953467521D0
```

```
  H1=DSQRT(DA/(2.0D0-DA))*DK
```

```
  DO 1 I=1,12
```

```

      P(25-I)=-P(I)
      W(25-I)=W(I)
1     CONTINUE
      DO 2 I=1,24
          W(I)=H1*W(I)
          P(I)=H1*P(I)
2     CONTINUE
      DO 10 I=1,24
          B2(I)=-1.0D0
          DO 20 J=1,24
              ARG=(P(J)-(1.0D0-DA)*P(I))/DA
              IF (I.EQ.J) THEN
                  A(I,J)=(1.0D0/DA)*W(I)*F1(ARG-SHIFT)-1.0D0
              ELSE
                  A(I,J)=(1.0D0/DA)*W(J)*F1(ARG-SHIFT)
              END IF
          DO 20 J=1,24
              ARG=(P(J)-(1.0D0-DA)*P(I))/DA
              IF (I.EQ.J) THEN
                  A(I,J)=(1.0D0/DA)*W(I)*F1(ARG-SHIFT)-1.0D0
              ELSE
                  A(I,J)=(1.0D0/DA)*W(J)*F1(ARG-SHIFT)
              END IF
          END IF
20    CONTINUE
10    CONTINUE
      CALL FACTOR(A,24,WK,IPIVOT,IFLAG)
      IF (IFLAG.EQ.0) THEN
          WRITE(*,50)
          RETURN
      END IF
      CALL SUBST(A,IPIVOT,B2,24,X1)
      ARL=0.0D0
      DO 30 I=1,24
          ARG=P(I)/DA
          ARL=ARL+W(I)*X1(I)*F1(ARG-SHIFT)
30    CONTINUE
      ARL=1.0D0+ARL/DA
50    FORMAT(5X,'ZERO, DETERMINANT FOR LINEAR SYSTEM.')
      RETURN
      END
C*****
      DOUBLE PRECISION FUNCTION F1(XX)
C*****
      DOUBLE PRECISION XX
      F1=3.989422804014327D-1*DEXP(-0.5D0*XX*XX)
      RETURN
      END
C*****
      SUBROUTINE SUBST(W1,IPIVOT,B2,NI,X2)
C*****
      INTEGER IPIVOT(24),I,IP,J
      REAL*8 B2(24),W1(24,24),X2(24),SUM
      IF (NI.LE.1) THEN
          X2(1)=B2(1)/W1(1,1)
          RETURN
      END IF
      IP=IPIVOT(1)
      X2(1)=B2(IP)
      DO 15 I=2,NI
          SUM=0.0D0
          I1=I-1

```

```

DO 14 J=1,I1
SUM=W1(I,J)*X2(J)+SUM
14 CONTINUE
IP=IPIVOT(I)
X2(I)=B2(IP)-SUM
15 CONTINUE
X2(NI)=X2(NI)/W1(NI,NI)
I2=NI-1
DO 20 ISTEP=1,I2
I=NI-ISTEP
SUM=0.0D0
I3=I+1
DO 19 J=I3,NI
SUM=W1(I,J)*X2(J)+SUM
19 CONTINUE
X2(I)=(X2(I)-SUM)/W1(I,I)
20 CONTINUE
RETURN
END
C*****
SUBROUTINE FACTOR(W1,NJ,D1,IPIVOT,IFLAG)
C*****
REAL*8
D1(24),W1(24,24),AWIKOD,COLMAX,RATIO,ROWMAX,TEMP
INTEGER IFLAG,IPIVOT(24),I,ISTAR,J,K
IFLAG=1
DO 9 I=1,NJ
IPIVOT(I)=I
ROWMAX=0.0D0
DO 5 J=1,NJ
ROWMAX=DMAX1(ROWMAX,DABS(W1(I,J)))
5 CONTINUE
IF (ROWMAX.EQ.0.0D0) THEN
IFLAG=0
ROWMAX=1.0D0
END IF
D1(I)=ROWMAX
9 CONTINUE
IF (NJ.LE.1) RETURN
N1=NJ-1
DO 20 K=1,N1
COLMAX=DABS(W1(I,K))/D1(K)
ISTAR=K
K1=K+1
DO 13 I=K1,NJ
AWIKOD=DABS(W1(I,K))/D1(K)
IF (AWIKOD.GT.COLMAX) THEN
COLMAX=AWIKOD
ISTAR=I
END IF
13 CONTINUE
IF (COLMAX.EQ.0.0D0) THEN
IFLAG=0
ELSE

```

```

      IF (ISTAR.GT.K) THEN
        IFLAG=-IFLAG
        I=IPIVOT(ISTAR)
        IPIVOT(ISTAR)=IPIVOT(K)
        IPIVOT(K)=I
        TEMP=D1(ISTAR)
        D1(ISTAR)=D1(K)
        D1(K)=TEMP
        DO 15 J=1,NJ
          TEMP=W1(ISTAR,J)
          W1(ISTAR,J)=W1(K,J)
          W1(K,J)=TEMP
15      CONTINUE
        END IF
        K2=K+1
        DO 19 I=K2,NJ
          W1(I,K)=W1(I,K)/W1(K,K)
          RATIO=W1(I,K)
          K3=K+1
          DO 18 J=K3,NJ
            W1(I,J)=W1(I,J)-RATIO*W1(K,J)
18      CONTINUE
19      CONTINUE
        END IF
20      CONTINUE
        IF (W1(NJ,NJ).EQ.0.0D0) IFLAG=0
        RETURN
      END
C*****
      SUBROUTINE EVEWMA
C*****
      IMPLICIT REAL*8(A-H,O-Z)
      CHARACTER IFILE*12
      REAL*8 ARL,SHIFT,DK,DA
C-----
C  INPUT COST AND OPERATION PARAMETERS
C-----
1      WRITE(*,2)
2      FORMAT(/,5X,'*** INPUT COST PARAMETERS. ***',/,/,
&5X,'DO YOU WANT TO INPUT FROM A FILE OR
&'MANUALLY?',/,5X,'==> PLEASE ENTER 1 = FILE, 2 = '
&'MANUALLY.')
```

READ (\*,\*) IANS  
GO TO (500,501) IANS  
WRITE (\*,502)

```

502  FORMAT(/,5X,'** ERROR INPUT! PLEASE TRY AGAIN! '
&'**')
```

GO TO 1

```

500  WRITE(*,503)
503  FORMAT(/,5X,'** PLEASE INPUT THE FILENAME THAT',/,
&5X,'  CONTAINS THE COST PARAMETERS.')
```

READ(\*,31) IFILE

```

31  FORMAT(A12)
      OPEN(50,FILE=IFILE,STATUS='OLD')
```

```

READ(50,*) DELTA,THETA,EM,E,D,T,W,B,C
CLOSE(50)
GO TO 3
501 WRITE(*,504)
504 FORMAT(/,5X,'=> INPUT VALUES OF
DELTA,THETA,M,E,D,T,W, &B,C')
READ(*,*) DELTA,THETA,EM,E,D,T,W,B,C
3 WRITE(*,4) DELTA,THETA,EM,E,D,T,W,B,C
4 FORMAT(/,5X,'** VALUES RECEIVED ARE:',/,
&5X,' DELTA = ',F10.4,7X,' THETA = ',F10.4,/,
&5X,' M = ',F10.4,7X,' E = ',F10.4,/,
&5X,' D = ',F10.4,7X,' T = ',F10.4,/,
&5X,' W = ',F10.4,7X,' B = ',F10.4,/,
&5X,' C = ',F10.4,/,/,
&5X,'=> ARE THESE DATA CORRECT? ',/,
&5X,'=> PLEASE ENTER 1 = YES, 2 = NO <<<')
READ(*,*) IANS
GO TO (11,1), IANS
GO TO 3
11 WRITE(*,12)
12 FORMAT(/,5X,'=> PLEASE INPUT N, H, K, ALPHA.')
READ(*,*) N,H,DK,DA
13 WRITE(*,14) N,H,DK,DA
14 FORMAT(/,5X,'*** THE CONTROL CHART PARAMETERS '
&'ARE:',/,5X,'N = ',I4,',',11X,' H = ',F10.4,',',
&/,5X,'K = ',F10.4,',',5X,'ALPHA = ',F6.4,
&/,'=> ARE THEY CORRECT? ',/,
&/,'=> PLEASE ENTER 1 = YES, 2 = NO <<<')
READ(*,*) IANS
GO TO (15,11) IANS
GO TO 13
15 DN=FLOAT(N)
SHIFT=0.0D0
CALL ARLEWMA(DA,DK,SHIFT,ARL)
ARL1=ARL
SHIFT=DELTA*DSQRT(DN)
CALL ARLEWMA(DA,DK,SHIFT,ARL)
ARL2=ARL
Y1=DEXP(-1.0D0*THETA*H)
Y2=1-Y1
Y=Y1/Y2
CYCTIME=H*(ARL2+Y)+E*DN+D
PC=(1.0D0-(1.0D0/(THETA*CYCTIME)))*EM
FC=T*Y/ARL1/CYCTIME
TC=W/CYCTIME
SC=(B+C*DN)/H
ELOSS=PC+FC+TC+SC
F=ELOSS
WRITE(*,16) F
16 FORMAT(/,5X,'*** THE LOSS OF THE CURRENT DESIGN '
&' IS:',/,5X,F14.6)
RETURN
END

```

```

C*****
  SUBROUTINE NELMIN(FN,N,START,XMIN,XSEC,YNEWLO,
    &YSEC,REQMIN,STEP,ICOUNT)
C*****
  REAL*8 START(N),STEP(N),XMIN(N),XSEC(N),YNEWLO,
    &YSEC,REQMIN,P(20,21),PSTAR(20),P2STAR(20),
    &PBAR(20),Y(20),DN,Z,YLO,RCOEFF,YSTAR,ECOEFF,
    &Y2STAR,CCOEFF,F,DABIT,DCHK,COORD1,COORD2
  DATA RCOEFF/1.0D0/,ECOEFF/2.0D0/,CCOEFF/0.5D0/
  KCOUNT=ICOUNT
  ICOUNT=0

C-----
C  INITIALIZATION
C-----
      DO 60 I=1,N
        XMIN(I)=0.0D0
        XSEC(I)=0.0D0
60    CONTINUE
      YNEWLO=0.0D0
      YSEC=0.0D0
      IF (REQMIN.LE.0.0D0) ICOUNT=ICOUNT-1
      IF (N.LE.0) ICOUNT=ICOUNT-10
      IF (N.GT.20) ICOUNT=ICOUNT-10
      IF (ICOUNT.LT.0) RETURN
      DABIT=2.04607D-35
      BIGNUM=1.0D30
      KONVGE=5
      XN=FLOAT(N)
      NN=N+1

C-----
C  CONSTRUCTION OF INITIAL SIMPLEX
C-----
1001  DO 1 I=1,N
1      P(I,NN)=START(I)
        CALL FN(START,F)
        Y(NN)=F
        ICOUNT=ICOUNT+1
        DO 2 J=1,N
          DCHK=START(J)
          START(J)=DCHK+STEP(J)
          DO 3 I=1,N
3          P(I,J)=START(I)
            CALL FN(START,F)
            Y(J)=F
            ICOUNT=ICOUNT+1
2          START(J)=DCHK
C-----
C  SIMPLEX CONSTRUCTION COMPLETE
C-----
C  FIND HIGHEST AND LOWEST Y VALUE
C  YNEWLO INDICATES THE VERTEX OF THE
C  SIMPLEX TO BE REPLACED
C-----
1000  YLO=Y(1)

```

```

YNEWLO=YLO
ILO=1
IHI=1
DO 5 I=2,NN
  IF (Y(I).GE.YLO) GO TO 4
  YLO=Y(I)
  ILO=I
4  IF (Y(I).LE.YNEWLO) GO TO 5
  YNEWLO=Y(I)
  IHI=I
5  CONTINUE
C-----
C  PERFORM CONVERGENCE CHECKS ON FUNCTION
C-----
  DCHK=(YNEWLO+DABIT)/(YLO+DABIT)-1.0D0
  IF (DABS(DCHK).LT.REQMIN) GO TO 900
  KONVGE=KONVGE-1
  IF (KONVGE.NE.0) GO TO 2020
  KONVGE=5
C-----
C  CHECK CONVERGENCE OF COORDINATE
C  ONLY EVERY 5 SIMPLEX
C-----
  DO 2015 I=1,N
  COORD1=P(I,1)
  COORD2=COORD1
  DO 2010 J=2,NN
  IF (P(I,J).GE.COORD1) GO TO 2005
  COORD1=P(I,J)
2005 IF (P(I,J).LE.COORD2) GO TO 2010
  COORD2=P(I,J)
2010 CONTINUE
  DCHK=(COORD2+DABIT)/(COORD1+DABIT)-1.0D0
  IF (DABS(DCHK).GT.REQMIN) GO TO 2020
2015 CONTINUE
  GO TO 900
2020 IF (ICOUNT.GE.KCOUNT) GO TO 900
C-----
C  CALCULATE PBAR, THE CENTROID OF THE SIMPLEX
C  VERTICES EXCEPT THAT WITH Y VALUE YNEWLO
C-----
  DO 7 I=1,N
  Z=0.0D0
  DO 6 J=1,NN
  Z=Z+P(I,J)
6  CONTINUE
  Z=Z-P(I,IHI)
7  PBAR(I)=Z/FLOAT(N)
C-----
C  REFLECTION THROUGH THE CENTROID
C-----
  DO 8 I=1,N
8  PSTAR(I)=(1.0D0+RCOEFF)*PBAR(I)-RCOEFF*P(I,IHI)
  CALL FN(PSTAR,F)

```

```

YSTAR=F
ICOUNT=ICOUNT+1
IF (YSTAR.GT.YLO) GO TO 12
IF (ICOUNT.GE.KCOUNT) GO TO 19
C-----
C SUCCESSFUL REFLECTION, SO EXTENTION
C-----
      DO 9 I=1,N
9      P2STAR(I)=ECOEFF*PSTAR(I)+(1.0D0-ECOEFF)*PBAR(I)
      CALL FN(P2STAR,F)
      Y2STAR=F
      ICOUNT=ICOUNT+1
C-----
C RETAIN EXTENSION OR CONTRACTION
C-----
      IF (Y2STAR.GE.YSTAR) GO TO 19
10     DO 11 I=1,N
11     P(I,IHI)=P2STAR(I)
      Y(IHI)=Y2STAR
      GO TO 1000
C-----
C NO EXTENSION
C-----
12     L=0
      DO 13 I=1,NN
          IF (Y(I).GT.YSTAR) L=L+1
13     CONTINUE
      IF (L.GT.1) GO TO 19
      IF (L.EQ.0) GO TO 15
C-----
C CONTRACTION ON THE REFLECTION SIDE OF THE CENTROID
C-----
      DO 14 I=1,N
14     P(I,IHI)=PSTAR(I)
      Y(IHI)=YSTAR
C-----
C CONTRACTION ON THE Y(IHI) SIDE OF THE CENTROID
C-----
15     IF (ICOUNT.GE.KCOUNT) GO TO 900
      DO 16 I=1,N
16     P2STAR(I)=CCOEFF*P(I,IHI)+(1.0D0-CCOEFF)*PBAR(I)
      CALL FN(P2STAR,F)
      Y2STAR=F
      ICOUNT=ICOUNT+1
      IF (Y2STAR.LT.Y(IHI)) GO TO 10
C-----
C CONTRACT THE WHOLE SIMPLEX
C-----
      DO 18 J=1,NN
          DO 17 I=1,N
              P(I,J)=(P(I,J)+P(I,ILO))*0.5D0
17     XMIN(I)=P(I,J)
      CALL FN(XMIN,F)
      Y(J)=F

```



```
18 CONTINUE
   ICOUNT=ICOUNT+NN
   IF (ICOUNT.LT.KCOUNT) GO TO 1000
   GO TO 900
```

```
C-----
C  RETAIN REFLECTION
```

```
C-----
19 CONTINUE
   DO 20 I=1,N
20 P(I,IHI)=PSTAR(I)
   Y(IHI)=YSTAR
   GO TO 1000
900 DO 23 J=1,NN
     DO 22 I=1,N
22 XMIN(I)=P(I,J)
     CALL FN(XMIN,F)
     Y(J)=F
23 CONTINUE
   YNEWLO=BIGNUM
   DO 24 J=1,NN
     IF (Y(J).GE.YNEWLO) GO TO 24
     YNEWLO=Y(J)
     IBEST=J
24 CONTINUE
   Y(IBEST)=BIGNUM
   YSEC=BIGNUM
   DO 25 J=1,NN
     IF (Y(J).GE.YSEC) GO TO 25
     YSEC=Y(J)
     ISEC=J
25 CONTINUE
   DO 26 I=1,N
     XMIN(I)=P(I,IBEST)
     XSEC(I)=P(I,ISEC)
26 CONTINUE
   RETURN
   END
```

**APPENDIX C**

**FORTRAN PROGRAM FOR DESIGN AND EVALUATION  
OF THE ZONE CONTROL CHART**

```

C*****
C  MAIN PROGRAM
C*****
$DEBUG
1  WRITE(*,2)
2  FORMAT(1H1,12X,24(1H*),/,13X,'***  MAIN  MENU  '
&'*** ',/,13X,24(1H*),/,/,
&5X,'(1) ECONOMIC DESIGN OF ZONE CONTROL CHARTS,',
&/,5X,'(2) EVALUATION OF ZONE CONTROL CHARTS',/,/,
&5X,'(3) EXIT THE PROGRAM',/,/,
&5X,'=> PLEASE ENTER YOUR OPTION (1, 2, OR 3)! '
&'<<<')
    READ(*,*) IANS
    GO TO (4,5,6) IANS
    WRITE(*,3)
3  FORMAT(/,5X,'*** ERROR INPUT! PLEASE TRY AGAIN! '
&'***')
    GO TO 1
4  CALL ECZCC
    GO TO 1
5  CALL EVZCC
    GO TO 1
6  STOP
    END
C*****
    SUBROUTINE ECZCC
C*****
C  ECONOMIC DESIGN OF THE ZONE CONTROL CHART
C*****
    IMPLICIT REAL*8(A-H,O-Z)
    CHARACTER IFILE*12
    INTEGER IS(4)
    REAL*8 BX(80),A(80,81)
    COMMON/EC1/THETA
    COMMON/EC2/N,NOPT
    COMMON/EC3/H,EK,HOPT,EKOPT,FOPT,STEP(20)
    COMMON/EC4/DELTA,B,C,D,E,EM,T,W
    COMMON/ZCC1/S(4),ISOPT(4)
    COMMON/ZCC2/ARL,SHIFT,DK,IS
    COMMON/ZCC3/P1,P2,P3,P4,P5,P6,BX,A
C-----
C  INPUT COST AND OPERATION PARAMETERS
C-----
1  WRITE(*,2)
2  FORMAT(/,5X,'*** INPUT COST PARAMETERS. ***',/,/,
&5X,'DO YOU WANT TO INPUT FROM A FILE OR'
&'MANUALLY?',/,5X,'=> PLEASE ENTER 1 = FILE, 2 ='
&'MANUALLY. <<<')
    READ(*,*) IANS
    GO TO (500,501) IANS
    WRITE(*,502)
502 FORMAT(/,5X,'** ERROR INPUT! PLEASE TRY AGAIN! '
&'**')
    GO TO 1

```

```

C-----
C INPUT FROM FILE
C-----
500 WRITE(*,503)
503 FORMAT(/,5X,'** PLEASE INPUT THE FILENAME THAT',/,
&5X,' CONTAINS THE COST PARAMETERS. ')
READ(*,31) IFILE
31 FORMAT(A12)
OPEN(50,FILE=IFILE,STATUS='OLD')
READ(50,*) DELTA,THETA,EM,E,D,T,W,B,C
CLOSE(50)
GO TO 3

C-----
C INPUT MANUALLY
C-----
501 WRITE(*,504)
504 FORMAT(/,5X,'==> INPUT VALUES OF
DELTA,THETA,M,E,D, &T,W,B,C')
READ(*,*) DELTA,THETA,EM,E,D,T,W,B,C
3 WRITE(*,4) DELTA,THETA,EM,E,D,T,W,B,C
4 FORMAT(/,5X,' ** VALUES RECEIVED ARE:',/,
&5X,' DELTA = ',F10.4,7X,' THETA = ',F10.4,/,
&5X,' M = ',F10.4,7X,' E = ',F10.4,/,
&5X,' D = ',F10.4,7X,' T = ',F10.4,/,
&5X,' W = ',F10.4,7X,' B = ',F10.4,/,
&5X,' C = ',F10.4,/,/,
&5X,' ==> ARE THESE DATA CORRECT? ',/,
&5X,' ==> PLEASE ENTER 1 = YES, 2 = NO <<<')
READ(*,*) IANS
GO TO (5,1), IANS
GO TO 3

C-----
C SELECT THE STARTING POINT
C-----
5 N=5
H=1.0D0
EK=3.0D0
S(1)=0.0D0
S(2)=1.0D0
S(3)=2.0D0
S(4)=15.0D0
6 WRITE(*,7) N,H,EK,S(1),S(2),S(3),S(4)
7 FORMAT(/,5X,' *** THE SUGGESTED STARTING POINT '
&' IS: ',/,5X,' N = ',I3,', H = ',F6.2,', K = ',
&F6.2,/,5X,' S(1) = ',F4.1,', S(2) = ',F4.1,', S(3)'
&' = ',F4.1,', S(4) = ',F5.1,/,/,
&5X,' ==> DO YOU ACCEPT THIS POINT?',/,
&5X,' ==> ENTER 1 = YES, 2 = NO. <<<')
READ(*,*) IANS
GO TO (8,14), IANS
GO TO 6

C-----
C SELECT THE STEP SIZE
C-----

```

```

8     STEP(1)=0.5
      STEP(2)=0.5
      STEP(3)=1.0
      STEP(4)=1.0
      STEP(5)=1.0
      STEP(6)=1.0
      STEP(7)=1.0
19    WRITE(*,20)
      STEP(7),STEP(1),STEP(2),STEP(3),STEP(4),
      *STEP(5),STEP(6)
20    FORMAT(/,5X,'*** THE SUGGESTED STEP SIZES ARE:',
&/,5X,' N = ',F4.2,',', ' H = ',F6.2,',', ' K = ',
&F6.2, &/,5X,' S(1) = ',F4.1,',', ' S(2) = ',
&F4.1,',', ' S(3) = ', &F4.1,',', ' S(4) = ',F5.1,/,
&5X,'==> DO YOU ACCEPT THESE STEP SIZES?',/,
&5X,'==> PLEASE ENTER 1 = YES, 2 = NO. <<<')
      READ (*,*) IANS
      GO TO (11,21) IANS
      GO TO 19

C-----
C PERFORM THE ECON DESIGN OF THE ZCC
C-----
11    CALL OPZCC
C-----
C PRINT THE OPTIMAL DESIGN
C-----
12    WRITE(*,13) NOPT,HOPT,EKOPT,ISOPT(1),ISOPT(2),
&ISOPT(3),ISOPT(4),FOPT
13    FORMAT(/,5X,65(1H-),
&/,5X,' *** THE OPTIMAL DESIGN IS: ',
&/,5X,' N = ',I4,',',3X,' H = ',F10.5,',',
&3X,' K = ',F10.5,',',/,
&5X,' SCORE 1 = ',I3,',',5X,' SCORE 2 = ',I3,',',/,
&5X,' SCORE 3 = ',I3,',',5X,' SCORE 4 = ',I3,',',/,/,
&5X,' *** THE MINIMUM LOSS PER HOUR IS: ',
&F14.6,/,/,5X,65(1H-))
      GO TO 18

C-----
C INPUT NEW STARTING POINT
C-----
14    WRITE(*,15)
15    FORMAT(/,5X,'*** PLEASE INPUT NEW STARTING POINT '
&'***',/,5X,' ==> KEY IN VALUES FOR N, H, K, S(1), '
&' S(2), S(3), &S(4)')
      READ(*,*) N,H,EK,S(1),S(2),S(3),S(4)
16    WRITE(*,17) N,H,EK,S(1),S(2),S(3),S(4)
17    FORMAT(/,5X,' *** NEW STARTING POINT IS:',
&/,5X,' N = ',I3,',', ' H = ',F6.2,',', ' K = ',F6.2,
&/,5X,' S(1) = ',F4.1,',', ' S(2) = ',F4.1,',', ' S(3)
&' ) = ',F4.1,',', ' S(4) = ',F5.1,/,
&5X,'==> ARE THEY CORRECT?',/,
&5X,'==> PLEASE ENTER 1 = YES, 2 = NO. <<<')
      READ(*,*) IANS
      GO TO (8,14), IANS

```

```

          GO TO 16
C-----
C  INPUT NEW STEP SIZES
C-----
21      WRITE(*,22)
22      FORMAT(/,5X,'*** PLEASE INPUT NEW STEP SIZES. '
&'***',/,5X,'==> ENTER VALUES FOR N, H, K, AND '
&'FOUR SCORES. ')
        READ (*,*) STEP(7),STEP(1),STEP(2),STEP(3),
&STEP(4),STEP(5),STEP(6)
23      WRITE (*,24) STEP(7),STEP(1),STEP(2),STEP(3),
&STEP(4),STEP(5),STEP(6)
24      FORMAT(/,5X,'*** THE NEW STEP SIZES ARE:',
&/,5X,' N = ',F4.2,',', ' H = ',F6.2,',', ' K = ',
&F6.2,/,5X,' S(1) = ',F4.1,',', ' S(2) = ',F4.1,',',
&' S(3) = ',F4.1,',', ' S(4) = ',F5.1,/,
&5X,'==> ARE THEY CORRECT?',/,
&5X,' ==> PLEASE ENTER 1 = YES, 2 = NO. <<<')
        READ (*,*) IANS
        GO TO (11,21) IANS
        GO TO 23
18      RETURN
        END
C*****
      SUBROUTINE OPZCC
C*****
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/EC2/N,NOPT
      COMMON/EC3/H,EK,HOPT,EKOPT,FOPT,STEP(20)
      COMMON/ZCC1/S(4),ISOPT(4)
      COMMON/SAMPLE/IX
      EXTERNAL ZCCF7
      EXTERNAL ZCCF6
      REAL*8 FMIN(10),X(10),XMIN(20),XSEC(20),F
C-----
C  ASSIGN NO. OF VARIABLE,SEARCH STEP,
C  AND TERMINATE VALUE
C-----
        ND=7
        ICOUNT=700
        REQMIN=0.0001
C-----
C  ASSIGN STARTING POINT
C-----
        X(1)=H
        X(2)=EK
        X(3)=S(1)
        X(4)=S(2)
        X(5)=S(3)
        X(6)=S(4)
        X(7)=DBLE(N)
        CALL NELMIN(ZCCF7,ND,X,XMIN,XSEC,YNEWLO,YSEC,
&REQMIN,STEP,ICOUNT)

```

```

C-----
C PRINT OUT THE OPTIMAL POINT FOUND
C-----
      WRITE(*,1)
      XMIN(7),XMIN(1),XMIN(2),XMIN(3),XMIN(4),
      &XMIN(5),XMIN(6),YNEWLO
1     FORMAT(/,5X,' *** THE OPTIMAL POINT FOUND IS ***',
      &/,/,5X,' N = ',F7.4,' ',H = ',F7.4,' ',K = ',
      &F7.4,/,
      &5X,' SCORE 1 = ',F5.1,',',5X,' SCORE 2 = ',F5.1,',',
      &/,5X,' SCORE 3 = ',F5.1,',',5X,' SCORE 4 = ',F5.1,
      &',',/,5X,' LOSS = ',F14.6,/)
C-----
C ASSIGN VARIABLE NO., SEARCH STEP
C AND TERMINATE VALUE
C-----
      ND=6
      ICOUNT=700
      REQMIN=0.0001
C-----
C ASSIGN STARTING POINT
C-----
      IX=XMIN(7)
      DO 50 L=1,6
          X(L)=XMIN(L)
50     CONTINUE
      CALL NELMIN(ZCCF6,ND,X,XMIN,XSEC,YNEWLO,
      &YSEC,REQMIN,STEP,ICOUNT)
      I=1
      WRITE(*,2)
2     FORMAT(/,5X,' *** OPTIMIZATION ITERATION ***',/,/,
      &5X,' I ',T10,' N ',T15,' H ',T25,' K ',T35,' S(1) ',T42,
      &' S(2) ',T49,' S(3) ',T56,' S(4) ',T65,' LOSS ',/,
      &5X,65(1H-),/)
      WRITE(*,3) I,IX,XMIN(1),XMIN(2),INT(XMIN(3)),
      &INT(XMIN(4)),INT(XMIN(5)),INT(XMIN(6)),YNEWLO
3     FORMAT(5X,I2,T8,I3,T12,F7.4,T22,F7.4,T33,I3,T40,
      &I3,T46,I3,T53,I3,T59,F14.6)
      INCR=1
      ITIME=0
C-----
C KEEP THE POINT AS CURRENT OPTIMAL
C-----
4     FMIN(8)=YNEWLO
      DO 5 L=1,6
          FMIN(L)=XMIN(L)
5     CONTINUE
6     IX=IX+INCR
7     DO 52 L=1,6
          X(L)=XMIN(L)
52    CONTINUE
      ND=6
      ICOUNT=700
      REQMIN=0.0001

```

```

      CALL NELMIN(ZCCF6,ND,X,XMIN,XSEC,YNEWLO,
&YSEC,REQMIN,STEP,ICOUNT)
      I=I+1
      WRITE(*,3) I,IX,XMIN(1),XMIN(2),INT(XMIN(3)),
&INT(XMIN(4)),INT(XMIN(5)),INT(XMIN(6)),YNEWLO
      IF(ITIME.EQ.1) GO TO 10
      IF(YNEWLO.GT.FMIN(8)) GO TO 9
      ITIME=1
      FMIN(8)=YNEWLO
      DO 8 L=1,6
        FMIN(L)=XMIN(L)
8      CONTINUE
      GO TO 6
9      INCR=-INCR
      IX=IX-2
      ITIME=1
      GO TO 7
10     IF(YNEWLO.LE.FMIN(8)) GO TO 4
      IXMIN=IX-INCR
      NOPT=IXMIN
      HOPT=FMIN(1)
      EKOPT=FMIN(2)
      DO 54 L=1,4
        ISOPT(L)=INT(FMIN(L+2))
54     CONTINUE
      FOPT=FMIN(8)
      RETURN
      END
C*****
      SUBROUTINE ZCCF7(X,F)
C*****
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 THETA,ARL,SHIFT,DK
      INTEGER IS(4)
      COMMON/EC1/THETA
      COMMON/EC4/DELTA,B,C,D,E,EM,T,W
      COMMON/ZCC2/ARL,SHIFT,DK,IS
      REAL*8 X(10)
      H=X(1)
      DK=X(2)
      DO 55 L=1,4
        IS(L)=INT(X(L+2))
55     CONTINUE
      DN=X(7)
      IF(H.GT.70..OR.H.LE.0.) F=9999999.
      IF(H.GT.70..OR.H.LE.0.) RETURN
      IF(DK.GT.6..OR.DK.LE.0.) F=9999999.
      IF(DK.GT.6..OR.DK.LE.0.) RETURN
      IF(DN.LT.1) F=9999999.
      IF(DN.LT.1) RETURN
      IF(IS(1).LT.0) F=9999999.
      IF(IS(1).LT.0) RETURN
      IF(IS(2).LE.IS(1)) F=9999999.
      IF(IS(2).LE.IS(1)) RETURN

```



```

IF (IS(3).LE.IS(2)) F=9999999.
IF (IS(3).LE.IS(2)) RETURN
IF (IS(4).LE.IS(3).OR.IS(4).GE.80) F=9999999.
IF (IS(4).LE.IS(3).OR.IS(4).GE.80) RETURN
IN=X(7)
SHIFT=0.0D0
CALL ARLZCC
ARL1=ARL
SHIFT=DELTA*DSQRT(DN)
CALL ARLZCC
ARL2=ARL
Y1=DEXP(-1.0D0*THETA*H)
Y2=1-Y1
Y=Y1/Y2
CYCTIME=H*(ARL2+Y)+E*DN+D
PC=(1.0D0-(1.0D0/(THETA*CYCTIME)))*EM
FC=T*Y/ARL1/CYCTIME
TC=W/CYCTIME
SC=(B+C*DN)/H
ELOSS=PC+FC+TC+SC
F=ELOSS
RETURN
END

```

```
C*****
```

```
  SUBROUTINE ZCCF6(X,F)
```

```
C*****
```

```

IMPLICIT REAL*8(A-H,O-Z)
REAL*8 THETA,ARL,SHIFT,DK
INTEGER IS(4)
COMMON/EC1/THETA
COMMON/EC3/H,EK,HOPT,EKOPT,FOPT,STEP(20)
COMMON/EC4/DELTA,B,C,D,E,EM,T,W
COMMON/ZCC1/S(4),ISOPT(4)
COMMON/ZCC2/ARL,SHIFT,DK,IS
COMMON/SAMPLE/IX
REAL*8 X(10)
H=X(1)
DK=X(2)
DO 55 L=1,4
  IS(L)=INT(X(L+2))
CONTINUE
DN=DBLE(IX)
IF(H.GT.70..OR.H.LE.0.) F=9999999.
IF(H.GT.70..OR.H.LE.0.) RETURN
IF(DK.GT.6..OR.DK.LE.0.) F=9999999.
IF(DK.GT.6..OR.DK.LE.0.) RETURN
IF(DN.LT.1) F=9999999.
IF(DN.LT.1) RETURN
IF(IS(1).LT.0) F=9999999.
IF(IS(1).LT.0) RETURN
IF(IS(2).LE.IS(1)) F=9999999.
IF(IS(2).LE.IS(1)) RETURN
IF(IS(3).LE.IS(2)) F=9999999.
IF(IS(3).LE.IS(2)) RETURN

```

55

```

IF (IS(4).LE.IS(3).OR.IS(4).GE.80) F=9999999.
IF (IS(4).LE.IS(3).OR.IS(4).GE.80) RETURN
IN=IX
SHIFT=0.0D0
CALL ARLZCC
ARL1=ARL
SHIFT=DELTA*DSQRT(DN)
CALL ARLZCC
ARL2=ARL
Y1=DEXP(-1.0D0*THETA*H)
Y2=1-Y1
Y=Y1/Y2
CYCTIME=H*(ARL2+Y)+E*DN+D
PC=(1.0D0-(1.0D0/(THETA*CYCTIME)))*EM
FC=T*Y/ARL1/CYCTIME
TC=W/CYCTIME
SC=(B+C*DN)/H
ELOSS=PC+FC+TC+SC
F=ELOSS
RETURN
END

```

C\*\*\*\*\*

SUBROUTINE ARLZCC

C\*\*\*\*\*

```

REAL*8
SHIFT,P1,P2,P3,P4,P5,P6,LINE(6),DK,CENL,BX(80),
&ARL,A(80,81)
INTEGER IS(4)
COMMON/ZCC2/ARL,SHIFT,DK,IS
COMMON/ZCC3/P1,P2,P3,P4,P5,P6,BX,A

```

C-----

C SIMPLIFY THE ZONE SCORES

C-----

```

IF (IS(1).GT.0) GO TO 2000
IF (IS(1).EQ.0.AND.IS(2).EQ.1) GO TO 2000
I1=IS(3)-IS(2)*INT(IS(3)/IS(2))
I2=IS(4)-IS(2)*INT(IS(4)/IS(2))
IF (I1.EQ.0.AND.I2.EQ.0) THEN
  IDIV=IS(2)
  IS(1)=0
  IS(2)=1
  IS(3)=IS(3)/IDIV
  IS(4)=IS(4)/IDIV
END IF

```

C-----

C DETERMINE CONTROL LIMITS AND WARNING LIMITS

C-----

```

2000 CENL=0.0D0
LINE(5)=DK
LINE(3)=DK*2./3.
LINE(1)=DK/3.
LINE(2)=-LINE(1)
LINE(4)=-LINE(3)
LINE(6)=-LINE(5)

```

```

C-----
C DETERMINE PROBABILITY FOR EACH ZONE AND CALCULATE ARL
C-----
      P1=ZCDF(LINE(1)-SHIFT)-ZCDF(CENL-SHIFT)
      P3=ZCDF(LINE(3)-SHIFT)-ZCDF(LINE(1)-SHIFT)
      P5=ZCDF(LINE(5)-SHIFT)-ZCDF(LINE(3)-SHIFT)
      P2=ZCDF(CENL-SHIFT)-ZCDF(LINE(2)-SHIFT)
      P4=ZCDF(LINE(2)-SHIFT)-ZCDF(LINE(4)-SHIFT)
      P6=ZCDF(LINE(4)-SHIFT)-ZCDF(LINE(6)-SHIFT)
      CALL MATRIX
      ARL=BX(1)
      RETURN
      END
C*****
C THIS SUBROUTINE TRANSFORMS THE SIMULTANEOUS LINEAR
C EQUATIONS INTO MATRIX FORM.
C*****
      SUBROUTINE MATRIX
      REAL*8 P1,P2,P3,P4,P5,P6,ARL,SHIFT,DK
      DOUBLE PRECISION BX(80),A(80,81)
      INTEGER IS(4)
      COMMON/ZCC2/ARL,SHIFT,DK,IS
      COMMON/ZCC3/P1,P2,P3,P4,P5,P6,BX,A
C-----
C DERTIMINE MATRIX SIZE A(IR,IC)
C-----
      IR=2*IS(4)-1
      IC=IR+1
C-----
C INITIALIZATION
C-----
      DO 31 I=1,IR
        DO 30 J=1,IC
          A(I,J)=0.0D0
30      CONTINUE
        BX(I)=0.0D0
31      CONTINUE
C-----
C DETERMINE AUGMENTED MATRIX A
C THE FIRST PART IS FOR SCORE(1) BEING > 0
C-----
      IF (IS(1).EQ.0) GO TO 100
      A(1,1)=1.0D0
      A(1,IS(1)+1)=-1.0D0*P1
      A(1,IS(2)+1)=-1.0D0*P3
      A(1,IS(3)+1)=-1.0D0*P5
      DO 2 I=2,IS(4)
        DO 1 J=1,IS(4)
          IF (J.LT.I) THEN
            A(I,J)=0.0D0
          ELSE
            IF (J.EQ.I) THEN
              A(I,J)=1.0D0
            ELSE

```

```

        A(I,J)=A(I-1,J-1)
        END IF
    END IF
1    CONTINUE
2    CONTINUE
    DO 3 I=1,IS(4)
        A(I,IS(4)+IS(1))=-1.0D0*P2
        A(I,IS(4)+IS(2))=-1.0D0*P4
        A(I,IS(4)+IS(3))=-1.0D0*P6
3    CONTINUE
    DO 4 I=IS(4)+1,IR
        A(I,IS(1)+1)=-1.0D0*P1
        A(I,IS(2)+1)=-1.0D0*P3
        A(I,IS(3)+1)=-1.0D0*P5
4    CONTINUE
    DO 6 I=IS(4)+1,IR
        DO 5 J=IS(4)+1,IR
            IF (J.LT.I) THEN
                A(I,J)=0.0D0
            ELSE
                IF (J.EQ.I) THEN
                    A(I,J)=1.0D0
                ELSE
                    A(I,J)=A(I-1,J-1)
                END IF
            END IF
        END IF
5    CONTINUE
6    CONTINUE
    DO 7 I=1,IR
        A(I,IC)=1.0D0
7    CONTINUE
    GO TO 101
C-----
C THE SECOND PART IS FOR SCORE(1) = 0
C-----
100  A(1,1)=1.0D0-P1-P2
        A(1,IS(2)+1)=-1.0D0*P3
        A(1,IS(3)+1)=-1.0D0*P5
        DO 15 I=2,IS(4)
            A(I,1)=-1.0D0*P2
15   CONTINUE
        A(2,IS(1)+2)=1.0D0-P1
        A(2,IS(2)+2)=-1.0*P3
        A(2,IS(3)+2)=-1.0*P5
        DO 9 I=3,IS(4)
            DO 8 J=2,IS(4)
                IF (J.LT.I) THEN
                    A(I,J)=0.0D0
                ELSE
                    IF (I.EQ.J) THEN
                        A(I,J)=1.0D0-P1
                    ELSE
                        A(I,J)=A(I-1,J-1)
                    END IF
                END IF
            END IF
        END IF
    END IF

```

```

      END IF
8      CONTINUE
9      CONTINUE
      DO 10 I=1,IS(4)
          A(I,IS(4)+IS(2))=-1.0D0*P4
          A(I,IS(4)+IS(3))=-1.0D0*P6
10     CONTINUE
      DO 11 I=IS(4)+1,IR
          A(I,IS(1)+1)=-1.0D0*P1
          A(I,IS(2)+1)=-1.0D0*P3
          A(I,IS(3)+1)=-1.0D0*P5
11     CONTINUE
      A(IS(4)+1,IS(4)+1)=1.0D0-P2
      A(IS(4)+1,IS(4)+IS(2)+1)=-1.0D0*P4
      A(IS(4)+1,IS(4)+IS(3)+1)=-1.0D0*P6
      DO 13 I=IS(4)+2,IR
          DO 12 J=IS(4)+1,IR
              IF (J.LT.I) THEN
                  A(I,J)=0.0D0
              ELSE
                  A(I,J)=A(I-1,J-1)
              END IF
          END DO
12     CONTINUE
13     CONTINUE
      DO 14 I=1,IR
          A(I,IC)=1.0D0
14     CONTINUE
101    CALL ARLCAL(A,IR,IC,BX)
      RETURN
      END
C*****
C THIS SUBROUTINE SOLVES FOR ARL.
C REF : M.L. JAMES, G.M. SMITH, J.C. WOLFORD, 'APPLIED
C       NUMERICAL METHODS FOR DIGITAL COMPUTATION.'
C       HARPER & ROW, PUBLISHERS, NEW YORK, 1985
C*****
      SUBROUTINE ARLCAL(A,N1,M,BX)
      DOUBLE PRECISION A(80,81),BX(80)
      IROW=1
      BIG=DABS(A(1,1))
      DO 1 I=2,N1
          AB=DABS(A(I,1))
          IF (BIG.GE.AB) GO TO 1
          BIG=AB
          IROW=I
1      CONTINUE
      IF (IROW.EQ.1) GO TO 200
      DO 100 J=1,M
          TEMP=A(IROW,J)
          A(IROW,J)=A(1,J)
100     A(1,J)=TEMP
200     CONTINUE
      DO 2 J=2,M
          A(1,J)=A(1,J)/A(1,1)
2      CONTINUE

```

```

DO 10 I=2,N1
  J=I
  DO 4 II=J,N1
    SUM=0.0D0
    JM1=J-1
    DO 3 K=1,JM1
      SUM=SUM+A(II,K)*A(K,J)
3     A(II,J)=A(II,J)-SUM
4     IF (I.EQ.N1) GO TO 7
      IROW=I
      BIG=DABS(A(I,I))
      IP1=I+1
      DO 5 II=IP1,N1
        AB=DABS(A(II,I))
        IF (BIG.GE.AB) GO TO 5
        BIG=AB
        IROW=II
5     CONTINUE
      IF (IROW.EQ.I) GO TO 7
      DO 6 J=1,M
        TEMP=A(IROW,J)
        A(IROW,J)=A(I,J)
6     A(I,J)=TEMP
7     CONTINUE
      IP1=I+1
      DO 9 J=IP1,M
        SUM=0.0D0
        IM1=I-1
        DO 8 K=1,IM1
8         SUM=SUM+A(I,K)*A(K,J)
9         A(I,J)=(A(I,J)-SUM)/A(I,I)
10    CONTINUE
      BX(N1)=A(N1,N1+1)
      LL=N1-1
      DO 12 NN=1,LL
        SUM=0.0D0
        I=N1-NN
        IP1=I+1
        DO 11 J=IP1,N1
11         SUM=SUM+A(I,J)*BX(J)
12        BX(I)=A(I,M)-SUM
      RETURN
      END
C*****
      SUBROUTINE EVZCC
C*****
      IMPLICIT REAL*8(A-H,O-Z)
      CHARACTER IFILE*12
      REAL*8 ARL,SHIFT,DK,DA
      INTEGER IS(4)
      COMMON/ZCC2/ARL,SHIFT,DK,IS
C-----
C INPUT COST AND OPERATION PARAMETERS
C-----

```

```

1   WRITE(*,2)
2   FORMAT(/,5X,'*** INPUT COST PARAMETERS. ***',/,/,
&5X,'DO YOU WANT TO INPUT FROM A FILE OR MANUALLY'
&'?' ,/,5X,'=> PLEASE ENTER 1 = FILE, 2 = MANUALLY'
&' . <<<')
   READ (*,*) IANS
   GO TO (500,501) IANS
   WRITE (*,502)
502  FORMAT(/,5X,'** ERROR INPUT! PLEASE TRY AGAIN! '
&'**')
   GO TO 1
500  WRITE(*,503)
503  FORMAT(/,5X,'** PLEASE INPUT THE FILENAME THAT',/,
&5X,' CONTAINS THE COST PARAMETERS.')
   READ(*,31) IFILE
31   FORMAT(A12)
   OPEN(50,FILE=IFILE,STATUS='OLD')
   READ(50,*) DELTA,THETA,EM,E,D,T,W,B,C
   CLOSE(50)
   GO TO 3
501  WRITE(*,504)
504  FORMAT(/,5X,'INPUT VALUES OF DELTA,THETA,M,E,D,
&T,W,B,C')
   READ(*,*) DELTA,THETA,EM,E,D,T,W,B,C
3   WRITE(*,4) DELTA,THETA,EM,E,D,T,W,B,C
4   FORMAT(/,5X,' ** VALUES RECEIVED ARE:',/,
&5X,' DELTA = ',F10.4,7X,' THETA = ',F10.4,/,
&5X,' M = ',F10.4,7X,' E = ',F10.4,/,
&5X,' D = ',F10.4,7X,' T = ',F10.4,/,
&5X,' W = ',F10.4,7X,' B = ',F10.4,/,
&5X,' C = ',F10.4,/,/,
&5X,'=> ARE THESE DATA CORRECT?',/,
&5X,'=> PLEASE ENTER 1 = YES, 2 = NO <<<')
   READ(*,*) IANS
   GO TO (11,1), IANS
   GO TO 3
11  WRITE(*,12)
12  FORMAT(/,5X,'*** PLEASE INPUT N, H, K, SCORE 1,',
&/,5X,' SCORE 2, SCORE 3, SCORE 4 ***')
   READ(*,*) N,H,DK,IS(1),IS(2),IS(3),IS(4)
13  WRITE(*,14) N,H,DK,IS(1),IS(2),IS(3),IS(4)
14  FORMAT(/,5X,'*** THE CONTROL CHART PARAMETERS '
&'ARE:',/,5X,'N = ',I4,',',5X,' H = ',F10.4,',',
&5X,'K = ',F7.4,',',/,
&5X,'SCORE 1 = ',I3,',',5X,'SCORE 2 = ',I3,',',/,
&5X,'SCORE 3 = ',I3,',',5X,'SCORE 4 = ',I3,',',/,
&/,'=> ARE THEY CORRECT?',/,
&/,'=> PLEASE ENTER 1 = YES, 2 = NO <<<')
   READ (*,*) IANS
   GO TO (15,11) IANS
   GO TO 13
15  DN=FLOAT(N)
   SHIFT=0.0D0
   CALL ARLZCC

```

```

ARL1=ARL
SHIFT=DELTA*DSQRT(DN)
CALL ARLZCC
ARL2=ARL
Y1=DEXP(-1.0D0*THETA*H)
Y2=1-Y1
Y=Y1/Y2
CYCTIME=H*(ARL2+Y)+E*DN+D
PC=(1.0D0-(1.0D0/(THETA*CYCTIME)))*EM
FC=T*Y/ARL1/CYCTIME
TC=W/CYCTIME
SC=(B+C*DN)/H
ELOSS=PC+FC+TC+SC
F=ELOSS
WRITE(*,16) F
16  FORMAT(/,5X,'*** THE LOSS OF THE CURRENT DESIGN '
    &' IS:',/,5X,F14.6)
RETURN
END

C*****
C STANDARD NORMAL CUMULATIVE DISTRIBUTION FUNCTION
C REF : SHENTON, L.R., "INEQUALITIES FOR THE NORMAL
C       INTEGRAL INCLUDING A NEW CONTINUED FRACTION."
C       BIOMETRIKA, 41, 177-189.
C*****
REAL*8 FUNCTION ZCDF(ZZ)
REAL*8 X1,ZZ,Z1,Z2,R,C1
INTEGER K
ZCDF=0.5D0
IF (ZZ.EQ.0.0) RETURN
X1=ABS(ZZ)
C1=0.3989422804014D0
Z1=0.50D0
Z2=2.50D0
IF (X1.LE.Z1) THEN
    R=1.0D0
    TX=1.0D0
    DO 300 K=0,5
        U=(-(2*K+1)*X1*X1)/((2*K+3)*2*(K+1))
        TX=U*TX
        R=R+TX
300    CONTINUE
    ZCDF=ZCDF+X1*(R)*C1
ENDIF
IF ((Z1.LT.X1).AND.(X1.LE.Z2)) THEN
    R=31
    DO 200 K=15,1,-1
        R=(2*K-1)+((( -1)**K)*K*X1*X1)/R
200    CONTINUE
R=X1/R
ZCDF=ZCDF+C1*EXP(-.5*X1*X1)*R
ENDIF
IF ((Z2.LT.X1).AND.(X1.LT.4.0)) THEN
    R=X1+15

```



```

        DO 100 K=15,1,-1
            R=X1+(K/R)
100     CONTINUE
            R=1/R
            ZCDF=1.0-C1*EXP(-.5*X1*X1)*R
        ENDIF
        IF (X1.GE.4.0) ZCDF=1
        IF (ZZ.LT.0.0) ZCDF=1-ZCDF
        RETURN
        END
C*****
      SUBROUTINE NELMIN(FN,N,START,XMIN,XSEC,YNEWLO,
        &YSEC,REQMIN,STEP,ICOUNT)
C*****
      REAL*8 START(N),STEP(N),XMIN(N),XSEC(N),YNEWLO,
        &YSEC,REQMIN,P(20,21),PSTAR(20),P2STAR(20),
        &PBAR(20),&Y(20),DN,Z,YLO,RCEFF,YSTAR,ECOEFF,
        &Y2STAR,CCOEFF,F,DABIT,DCHK,COORD1,COORD2
      DATA RCEFF/1.0D0/,ECOEFF/2.0D0/,CCOEFF/0.5D0/
      KCOUNT=ICOUNT
      ICOUNT=0
C-----
C  INITIALIZATION
C-----
      DO 60 I=1,N
          XMIN(I)=0.0D0
          XSEC(I)=0.0D0
60     CONTINUE
      YNEWLO=0.0D0
      YSEC=0.0D0
      IF (REQMIN.LE.0.0D0) ICOUNT=ICOUNT-1
      IF (N.LE.0) ICOUNT=ICOUNT-10
      IF (N.GT.20) ICOUNT=ICOUNT-10
      IF (ICOUNT.LT.0) RETURN
      DABIT=2.04607D-35
      BIGNUM=1.0D30
      KONVGE=5
      XN=FLOAT(N)
      NN=N+1
C-----
C  CONSTRUCTION OF INITIAL SIMPLEX
C-----
1001  DO 1 I=1,N
1     P(I,NN)=START(I)
      CALL FN(START,F)
      Y(NN)=F
      ICOUNT=ICOUNT+1
      DO 2 J=1,N
          DCHK=START(J)
          START(J)=DCHK+STEP(J)
          DO 3 I=1,N
3     P(I,J)=START(I)
      CALL FN(START,F)
      Y(J)=F

```

```

        ICOUNT=ICOUNT+1
2       START(J)=DCHK
C-----
C   SIMPLEX CONSTRUCTION COMPLETE
C-----
C   FIND HIGHEST AND LOWEST Y VALUE
C   YNEWLO INDICATES THE VERTEX OF THE
C   SIMPLEX TO BE REPLACED
C-----
1000    YLO=Y(1)
        YNEWLO=YLO
        ILO=1
        IHI=1
        DO 5 I=2,NN
            IF (Y(I).GE.YLO) GO TO 4
            YLO=Y(I)
            ILO=I
4       IF (Y(I).LE.YNEWLO) GO TO 5
        YNEWLO=Y(I)
        IHI=I
5       CONTINUE
C-----
C   PERFORM CONVERGENCE CHECKS ON FUNCTION
C-----
        DCHK=(YNEWLO+DABIT)/(YLO+DABIT)-1.0D0
        IF (DABS(DCHK).LT.REQMIN) GO TO 900
        KONVGE=KONVGE-1
        IF (KONVGE.NE.0) GO TO 2020
        KONVGE=5
C-----
C   CHECK CONVERGENCE OF COORDINATE
C   ONLY EVERY 5 SIMPLEX
C-----
        DO 2015 I=1,N
            COORD1=P(I,1)
            COORD2=COORD1
            DO 2010 J=2,NN
                IF (P(I,J).GE.COORD1) GO TO 2005
                COORD1=P(I,J)
2005    IF (P(I,J).LE.COORD2) GO TO 2010
            COORD2=P(I,J)
2010    CONTINUE
            DCHK=(COORD2+DABIT)/(COORD1+DABIT)-1.0D0
            IF (DABS(DCHK).GT.REQMIN) GO TO 2020
2015    CONTINUE
        GO TO 900
2020    IF (ICOUNT.GE.KCOUNT) GO TO 900
C-----
C   CALCULATE PBAR, THE CENTROID OF THE SIMPLEX
C   VERTICES EXCEPT THAT WITH Y VALUE YNEWLO
C-----
        DO 7 I=1,N
            Z=0.0D0
            DO 6 J=1,NN

```

```

        Z=Z+P(I,J)
6      CONTINUE
        Z=Z-P(I,IHI)
7      PBAR(I)=Z/FLOAT(N)
C-----
C REFLECTION THROUGH THE CENTROID
C-----
        DO 8 I=1,N
8      PSTAR(I)=(1.0D0+RCOEFF)*PBAR(I)-RCOEFF*P(I,IHI)
        CALL FN(PSTAR,F)
        YSTAR=F
        ICOUNT=ICOUNT+1
        IF (YSTAR.GT.YLO) GO TO 12
        IF (ICOUNT.GE.KCOUNT) GO TO 19
C-----
C SUCCESSFUL REFLECTION, SO EXTENTION
C-----
        DO 9 I=1,N
9      P2STAR(I)=ECOEFF*PSTAR(I)+(1.0D0-ECOEFF)*PBAR(I)
        CALL FN(P2STAR,F)
        Y2STAR=F
        ICOUNT=ICOUNT+1
C-----
C RETAIN EXTENSION OR CONTRACTION
C-----
        IF (Y2STAR.GE.YSTAR) GO TO 19
10     DO 11 I=1,N
11     P(I,IHI)=P2STAR(I)
        Y(IHI)=Y2STAR
        GO TO 1000
C-----
C NO EXTENSION
C-----
12     L=0
        DO 13 I=1,NN
            IF (Y(I).GT.YSTAR) L=L+1
13     CONTINUE
        IF (L.GT.1) GO TO 19
        IF (L.EQ.0) GO TO 15
C-----
C CONTRACTION ON THE REFLECTION SIDE OF THE CENTROID
C-----
        DO 14 I=1,N
14     P(I,IHI)=PSTAR(I)
        Y(IHI)=YSTAR
C-----
C CONTRACTION ON THE Y(IHI) SIDE OF THE CENTROID
C-----
15     IF (ICOUNT.GE.KCOUNT) GO TO 900
        DO 16 I=1,N
16     P2STAR(I)=CCOEFF*P(I,IHI)+(1.0D0-CCOEFF)*PBAR(I)
        CALL FN(P2STAR,F)
        Y2STAR=F
        ICOUNT=ICOUNT+1

```

```

      IF (Y2STAR.LT.Y(IHI)) GO TO 10
C-----
C  CONTRACT THE WHOLE SIMPLEX
C-----
      DO 18 J=1,NN
        DO 17 I=1,N
          P(I,J)=(P(I,J)+P(I,ILO))*0.5D0
17      XMIN(I)=P(I,J)
          CALL FN(XMIN,F)
          Y(J)=F
18      CONTINUE
          ICOUNT=ICOUNT+NN
          IF (ICOUNT.LT.KCOUNT) GO TO 1000
          GO TO 900
C-----
C  RETAIN REFLECTION
C-----
19      CONTINUE
      DO 20 I=1,N
20      P(I,IHI)=PSTAR(I)
          Y(IHI)=YSTAR
          GO TO 1000
900     DO 23 J=1,NN
          DO 22 I=1,N
22      XMIN(I)=P(I,J)
          CALL FN(XMIN,F)
          Y(J)=F
23      CONTINUE
          YNEWLO=BIGNUM
          DO 24 J=1,NN
            IF (Y(J).GE.YNEWLO) GO TO 24
            YNEWLO=Y(J)
            IBEST=J
24      CONTINUE
          Y(IBEST)=BIGNUM
          YSEC=BIGNUM
          DO 25 J=1,NN
            IF (Y(J).GE.YSEC) GO TO 25
            YSEC=Y(J)
            ISEC=J
25      CONTINUE
          DO 26 I=1,N
            XMIN(I)=P(I,IBEST)
            XSEC(I)=P(I,ISEC)
26      CONTINUE
          RETURN
          END

```

**APPENDIX D**

**INDEPENDENT VARIABLES AND RESPONSES**

**OF THE  $2^9-1$  FRACTIONAL FACTORIAL**

**EXPERIMENT, ONE CENTER**

**POINT, AND 18 AXIAL**

**POINTS**

Value Of Independent Variables Of The  $2^{9-1}$   
 Fractional Factorial Experiment, One  
 Center Point, And 18 Axial Points

No.	$\delta$	$\theta$	M	e	D	T	W	b	c
1	2.6	0.013	130	0.065	2.6	65	32.5	0.65	0.13
2	2.6	0.013	130	0.065	2.6	65	32.5	0.35	0.07
3	2.6	0.013	130	0.065	2.6	65	17.5	0.65	0.07
4	2.6	0.013	130	0.065	2.6	65	17.5	0.35	0.13
5	2.6	0.013	130	0.065	2.6	35	32.5	0.65	0.07
6	2.6	0.013	130	0.065	2.6	35	32.5	0.35	0.13
7	2.6	0.013	130	0.065	2.6	35	17.5	0.65	0.13
8	2.6	0.013	130	0.065	2.6	35	17.5	0.35	0.07
9	2.6	0.013	130	0.065	1.4	65	32.5	0.65	0.07
10	2.6	0.013	130	0.065	1.4	65	32.5	0.35	0.13
11	2.6	0.013	130	0.065	1.4	65	17.5	0.65	0.13
12	2.6	0.013	130	0.065	1.4	65	17.5	0.35	0.07
13	2.6	0.013	130	0.065	1.4	35	32.5	0.65	0.13
14	2.6	0.013	130	0.065	1.4	35	32.5	0.35	0.07
15	2.6	0.013	130	0.065	1.4	35	17.5	0.65	0.07
16	2.6	0.013	130	0.065	1.4	35	17.5	0.35	0.13
17	2.6	0.013	130	0.035	2.6	65	32.5	0.65	0.07
18	2.6	0.013	130	0.035	2.6	65	32.5	0.35	0.13
19	2.6	0.013	130	0.035	2.6	65	17.5	0.65	0.13
20	2.6	0.013	130	0.035	2.6	65	17.5	0.35	0.07
21	2.6	0.013	130	0.035	2.6	35	32.5	0.65	0.13
22	2.6	0.013	130	0.035	2.6	35	32.5	0.35	0.07
23	2.6	0.013	130	0.035	2.6	35	17.5	0.65	0.07
24	2.6	0.013	130	0.035	2.6	35	17.5	0.35	0.13
25	2.6	0.013	130	0.035	1.4	65	32.5	0.65	0.13
26	2.6	0.013	130	0.035	1.4	65	32.5	0.35	0.07
27	2.6	0.013	130	0.035	1.4	65	17.5	0.65	0.07
28	2.6	0.013	130	0.035	1.4	65	17.5	0.35	0.13
29	2.6	0.013	130	0.035	1.4	35	32.5	0.65	0.07
30	2.6	0.013	130	0.035	1.4	35	32.5	0.35	0.13
31	2.6	0.013	130	0.035	1.4	35	17.5	0.65	0.13
32	2.6	0.013	130	0.035	1.4	35	17.5	0.35	0.07
33	2.6	0.013	70	0.065	2.6	65	32.5	0.65	0.07
34	2.6	0.013	70	0.065	2.6	65	32.5	0.35	0.13
35	2.6	0.013	70	0.065	2.6	65	17.5	0.65	0.13
36	2.6	0.013	70	0.065	2.6	65	17.5	0.35	0.07
37	2.6	0.013	70	0.065	2.6	35	32.5	0.65	0.13
38	2.6	0.013	70	0.065	2.6	35	32.5	0.35	0.07
39	2.6	0.013	70	0.065	2.6	35	17.5	0.65	0.07
40	2.6	0.013	70	0.065	2.6	35	17.5	0.35	0.13
41	2.6	0.013	70	0.065	1.4	65	32.5	0.65	0.13
42	2.6	0.013	70	0.065	1.4	65	32.5	0.35	0.07
43	2.6	0.013	70	0.065	1.4	65	17.5	0.65	0.07
44	2.6	0.013	70	0.065	1.4	65	17.5	0.35	0.13
45	2.6	0.013	70	0.065	1.4	35	32.5	0.65	0.07
46	2.6	0.013	70	0.065	1.4	35	32.5	0.35	0.13
47	2.6	0.013	70	0.065	1.4	35	17.5	0.65	0.13
48	2.6	0.013	70	0.065	1.4	35	17.5	0.35	0.07
49	2.6	0.013	70	0.035	2.6	65	32.5	0.65	0.13

No.	$\delta$	$\theta$	M	e	D	T	W	b	c
50	2.6	0.013	70	0.035	2.6	65	32.5	0.35	0.07
51	2.6	0.013	70	0.035	2.6	65	17.5	0.65	0.07
52	2.6	0.013	70	0.035	2.6	65	17.5	0.35	0.13
53	2.6	0.013	70	0.035	2.6	35	32.5	0.65	0.07
54	2.6	0.013	70	0.035	2.6	35	32.5	0.35	0.13
55	2.6	0.013	70	0.035	2.6	35	17.5	0.65	0.13
56	2.6	0.013	70	0.035	2.6	35	17.5	0.35	0.07
57	2.6	0.013	70	0.035	1.4	65	32.5	0.65	0.07
58	2.6	0.013	70	0.035	1.4	65	32.5	0.35	0.13
59	2.6	0.013	70	0.035	1.4	65	17.5	0.65	0.13
60	2.6	0.013	70	0.035	1.4	65	17.5	0.35	0.07
61	2.6	0.013	70	0.035	1.4	35	32.5	0.65	0.13
62	2.6	0.013	70	0.035	1.4	35	32.5	0.35	0.07
63	2.6	0.013	70	0.035	1.4	35	17.5	0.65	0.07
64	2.6	0.013	70	0.035	1.4	35	17.5	0.35	0.13
65	2.6	0.007	130	0.065	2.6	65	32.5	0.65	0.07
66	2.6	0.007	130	0.065	2.6	65	32.5	0.35	0.13
67	2.6	0.007	130	0.065	2.6	65	17.5	0.65	0.13
68	2.6	0.007	130	0.065	2.6	65	17.5	0.35	0.07
69	2.6	0.007	130	0.065	2.6	35	32.5	0.65	0.13
70	2.6	0.007	130	0.065	2.6	35	32.5	0.35	0.07
71	2.6	0.007	130	0.065	2.6	35	17.5	0.65	0.07
72	2.6	0.007	130	0.065	2.6	35	17.5	0.35	0.13
73	2.6	0.007	130	0.065	1.4	65	32.5	0.65	0.13
74	2.6	0.007	130	0.065	1.4	65	32.5	0.35	0.07
75	2.6	0.007	130	0.065	1.4	65	17.5	0.65	0.07
76	2.6	0.007	130	0.065	1.4	65	17.5	0.35	0.13
77	2.6	0.007	130	0.065	1.4	35	32.5	0.65	0.07
78	2.6	0.007	130	0.065	1.4	35	32.5	0.35	0.13
79	2.6	0.007	130	0.065	1.4	35	17.5	0.65	0.13
80	2.6	0.007	130	0.065	1.4	35	17.5	0.35	0.07
81	2.6	0.007	130	0.035	2.6	65	32.5	0.65	0.13
82	2.6	0.007	130	0.035	2.6	65	32.5	0.35	0.07
83	2.6	0.007	130	0.035	2.6	65	17.5	0.65	0.07
84	2.6	0.007	130	0.035	2.6	65	17.5	0.35	0.13
85	2.6	0.007	130	0.035	2.6	35	32.5	0.65	0.07
86	2.6	0.007	130	0.035	2.6	35	32.5	0.35	0.13
87	2.6	0.007	130	0.035	2.6	35	17.5	0.65	0.13
88	2.6	0.007	130	0.035	2.6	35	17.5	0.35	0.07
89	2.6	0.007	130	0.035	1.4	65	32.5	0.65	0.07
90	2.6	0.007	130	0.035	1.4	65	32.5	0.35	0.13
91	2.6	0.007	130	0.035	1.4	65	17.5	0.65	0.13
92	2.6	0.007	130	0.035	1.4	65	17.5	0.35	0.07
93	2.6	0.007	130	0.035	1.4	35	32.5	0.65	0.13
94	2.6	0.007	130	0.035	1.4	35	32.5	0.35	0.07
95	2.6	0.007	130	0.035	1.4	35	17.5	0.65	0.07
96	2.6	0.007	130	0.035	1.4	35	17.5	0.35	0.13
97	2.6	0.007	70	0.065	2.6	65	32.5	0.65	0.13
98	2.6	0.007	70	0.065	2.6	65	32.5	0.35	0.07
99	2.6	0.007	70	0.065	2.6	65	17.5	0.65	0.07
100	2.6	0.007	70	0.065	2.6	65	17.5	0.35	0.13
101	2.6	0.007	70	0.065	2.6	35	32.5	0.65	0.07
102	2.6	0.007	70	0.065	2.6	35	32.5	0.35	0.13

No.	$\delta$	$\theta$	M	e	D	T	W	b	c
103	2.6	0.007	70	0.065	2.6	35	17.5	0.65	0.13
104	2.6	0.007	70	0.065	2.6	35	17.5	0.35	0.07
105	2.6	0.007	70	0.065	1.4	65	32.5	0.65	0.07
106	2.6	0.007	70	0.065	1.4	65	32.5	0.35	0.13
107	2.6	0.007	70	0.065	1.4	65	17.5	0.65	0.13
108	2.6	0.007	70	0.065	1.4	65	17.5	0.35	0.07
109	2.6	0.007	70	0.065	1.4	35	32.5	0.65	0.13
110	2.6	0.007	70	0.065	1.4	35	32.5	0.35	0.07
111	2.6	0.007	70	0.065	1.4	35	17.5	0.65	0.07
112	2.6	0.007	70	0.065	1.4	35	17.5	0.35	0.13
113	2.6	0.007	70	0.035	2.6	65	32.5	0.65	0.07
114	2.6	0.007	70	0.035	2.6	65	32.5	0.35	0.13
115	2.6	0.007	70	0.035	2.6	65	17.5	0.65	0.13
116	2.6	0.007	70	0.035	2.6	65	17.5	0.35	0.07
117	2.6	0.007	70	0.035	2.6	35	32.5	0.65	0.13
118	2.6	0.007	70	0.035	2.6	35	32.5	0.35	0.07
119	2.6	0.007	70	0.035	2.6	35	17.5	0.65	0.07
120	2.6	0.007	70	0.035	2.6	35	17.5	0.35	0.13
121	2.6	0.007	70	0.035	1.4	65	32.5	0.65	0.13
122	2.6	0.007	70	0.035	1.4	65	32.5	0.35	0.07
123	2.6	0.007	70	0.035	1.4	65	17.5	0.65	0.07
124	2.6	0.007	70	0.035	1.4	65	17.5	0.35	0.13
125	2.6	0.007	70	0.035	1.4	35	32.5	0.65	0.07
126	2.6	0.007	70	0.035	1.4	35	32.5	0.35	0.13
127	2.6	0.007	70	0.035	1.4	35	17.5	0.65	0.13
128	2.6	0.007	70	0.035	1.4	35	17.5	0.35	0.07
129	1.4	0.013	130	0.065	2.6	65	32.5	0.65	0.07
130	1.4	0.013	130	0.065	2.6	65	32.5	0.35	0.13
131	1.4	0.013	130	0.065	2.6	65	17.5	0.65	0.13
132	1.4	0.013	130	0.065	2.6	65	17.5	0.35	0.07
133	1.4	0.013	130	0.065	2.6	35	32.5	0.65	0.13
134	1.4	0.013	130	0.065	2.6	35	32.5	0.35	0.07
135	1.4	0.013	130	0.065	2.6	35	17.5	0.65	0.07
136	1.4	0.013	130	0.065	2.6	35	17.5	0.35	0.13
137	1.4	0.013	130	0.065	1.4	65	32.5	0.65	0.13
138	1.4	0.013	130	0.065	1.4	65	32.5	0.35	0.07
139	1.4	0.013	130	0.065	1.4	65	17.5	0.65	0.07
140	1.4	0.013	130	0.065	1.4	65	17.5	0.35	0.13
141	1.4	0.013	130	0.065	1.4	35	32.5	0.65	0.07
142	1.4	0.013	130	0.065	1.4	35	32.5	0.35	0.13
143	1.4	0.013	130	0.065	1.4	35	17.5	0.65	0.13
144	1.4	0.013	130	0.065	1.4	35	17.5	0.35	0.07
145	1.4	0.013	130	0.035	2.6	65	32.5	0.65	0.13
146	1.4	0.013	130	0.035	2.6	65	32.5	0.35	0.07
147	1.4	0.013	130	0.035	2.6	65	17.5	0.65	0.07
148	1.4	0.013	130	0.035	2.6	65	17.5	0.35	0.13
149	1.4	0.013	130	0.035	2.6	35	32.5	0.65	0.07
150	1.4	0.013	130	0.035	2.6	35	32.5	0.35	0.13
151	1.4	0.013	130	0.035	2.6	35	17.5	0.65	0.13
152	1.4	0.013	130	0.035	2.6	35	17.5	0.35	0.07
153	1.4	0.013	130	0.035	1.4	65	32.5	0.65	0.07
154	1.4	0.013	130	0.035	1.4	65	32.5	0.35	0.13
155	1.4	0.013	130	0.035	1.4	65	17.5	0.65	0.13



No.	$\delta$	$\theta$	M	e	D	T	W	b	c
156	1.4	0.013	130	0.035	1.4	65	17.5	0.35	0.07
157	1.4	0.013	130	0.035	1.4	35	32.5	0.65	0.13
158	1.4	0.013	130	0.035	1.4	35	32.5	0.35	0.07
159	1.4	0.013	130	0.035	1.4	35	17.5	0.65	0.07
160	1.4	0.013	130	0.035	1.4	35	17.5	0.35	0.13
161	1.4	0.013	70	0.065	2.6	65	32.5	0.65	0.13
162	1.4	0.013	70	0.065	2.6	65	32.5	0.35	0.07
163	1.4	0.013	70	0.065	2.6	65	17.5	0.65	0.07
164	1.4	0.013	70	0.065	2.6	65	17.5	0.35	0.13
165	1.4	0.013	70	0.065	2.6	35	32.5	0.65	0.07
166	1.4	0.013	70	0.065	2.6	35	32.5	0.35	0.13
167	1.4	0.013	70	0.065	2.6	35	17.5	0.65	0.13
168	1.4	0.013	70	0.065	2.6	35	17.5	0.35	0.07
169	1.4	0.013	70	0.065	1.4	65	32.5	0.65	0.07
170	1.4	0.013	70	0.065	1.4	65	32.5	0.35	0.13
171	1.4	0.013	70	0.065	1.4	65	17.5	0.65	0.13
172	1.4	0.013	70	0.065	1.4	65	17.5	0.35	0.07
173	1.4	0.013	70	0.065	1.4	35	32.5	0.65	0.13
174	1.4	0.013	70	0.065	1.4	35	32.5	0.35	0.07
175	1.4	0.013	70	0.065	1.4	35	17.5	0.65	0.07
176	1.4	0.013	70	0.065	1.4	35	17.5	0.35	0.13
177	1.4	0.013	70	0.035	2.6	65	32.5	0.65	0.07
178	1.4	0.013	70	0.035	2.6	65	32.5	0.35	0.13
179	1.4	0.013	70	0.035	2.6	65	17.5	0.65	0.13
180	1.4	0.013	70	0.035	2.6	65	17.5	0.35	0.07
181	1.4	0.013	70	0.035	2.6	35	32.5	0.65	0.13
182	1.4	0.013	70	0.035	2.6	35	32.5	0.35	0.07
183	1.4	0.013	70	0.035	2.6	35	17.5	0.65	0.07
184	1.4	0.013	70	0.035	2.6	35	17.5	0.35	0.13
185	1.4	0.013	70	0.035	1.4	65	32.5	0.65	0.13
186	1.4	0.013	70	0.035	1.4	65	32.5	0.35	0.07
187	1.4	0.013	70	0.035	1.4	65	17.5	0.65	0.07
188	1.4	0.013	70	0.035	1.4	65	17.5	0.35	0.13
189	1.4	0.013	70	0.035	1.4	35	32.5	0.65	0.07
190	1.4	0.013	70	0.035	1.4	35	32.5	0.35	0.13
191	1.4	0.013	70	0.035	1.4	35	17.5	0.65	0.13
192	1.4	0.013	70	0.035	1.4	35	17.5	0.35	0.07
193	1.4	0.007	130	0.065	2.6	65	32.5	0.65	0.13
194	1.4	0.007	130	0.065	2.6	65	32.5	0.35	0.07
195	1.4	0.007	130	0.065	2.6	65	17.5	0.65	0.07
196	1.4	0.007	130	0.065	2.6	65	17.5	0.35	0.13
197	1.4	0.007	130	0.065	2.6	35	32.5	0.65	0.07
198	1.4	0.007	130	0.065	2.6	35	32.5	0.35	0.13
199	1.4	0.007	130	0.065	2.6	35	17.5	0.65	0.13
200	1.4	0.007	130	0.065	2.6	35	17.5	0.35	0.07
201	1.4	0.007	130	0.065	1.4	65	32.5	0.65	0.07
202	1.4	0.007	130	0.065	1.4	65	32.5	0.35	0.13
203	1.4	0.007	130	0.065	1.4	65	17.5	0.65	0.13
204	1.4	0.007	130	0.065	1.4	65	17.5	0.35	0.07
205	1.4	0.007	130	0.065	1.4	35	32.5	0.65	0.13
206	1.4	0.007	130	0.065	1.4	35	32.5	0.35	0.07
207	1.4	0.007	130	0.065	1.4	35	17.5	0.65	0.07
208	1.4	0.007	130	0.065	1.4	35	17.5	0.35	0.13

No.	$\delta$	$\theta$	M	e	D	T	W	b	c
209	1.4	0.007	130	0.035	2.6	65	32.5	0.65	0.07
210	1.4	0.007	130	0.035	2.6	65	32.5	0.35	0.13
211	1.4	0.007	130	0.035	2.6	65	17.5	0.65	0.13
212	1.4	0.007	130	0.035	2.6	65	17.5	0.35	0.07
213	1.4	0.007	130	0.035	2.6	35	32.5	0.65	0.13
214	1.4	0.007	130	0.035	2.6	35	32.5	0.35	0.07
215	1.4	0.007	130	0.035	2.6	35	17.5	0.65	0.07
216	1.4	0.007	130	0.035	2.6	35	17.5	0.35	0.13
217	1.4	0.007	130	0.035	1.4	65	32.5	0.65	0.13
218	1.4	0.007	130	0.035	1.4	65	32.5	0.35	0.07
219	1.4	0.007	130	0.035	1.4	65	17.5	0.65	0.07
220	1.4	0.007	130	0.035	1.4	65	17.5	0.35	0.13
221	1.4	0.007	130	0.035	1.4	35	32.5	0.65	0.07
222	1.4	0.007	130	0.035	1.4	35	32.5	0.35	0.13
223	1.4	0.007	130	0.035	1.4	35	17.5	0.65	0.13
224	1.4	0.007	130	0.035	1.4	35	17.5	0.35	0.07
225	1.4	0.007	70	0.065	2.6	65	32.5	0.65	0.07
226	1.4	0.007	70	0.065	2.6	65	32.5	0.35	0.13
227	1.4	0.007	70	0.065	2.6	65	17.5	0.65	0.13
228	1.4	0.007	70	0.065	2.6	65	17.5	0.35	0.07
229	1.4	0.007	70	0.065	2.6	35	32.5	0.65	0.13
230	1.4	0.007	70	0.065	2.6	35	32.5	0.35	0.07
231	1.4	0.007	70	0.065	2.6	35	17.5	0.65	0.07
232	1.4	0.007	70	0.065	2.6	35	17.5	0.35	0.13
233	1.4	0.007	70	0.065	1.4	65	32.5	0.65	0.13
234	1.4	0.007	70	0.065	1.4	65	32.5	0.35	0.07
235	1.4	0.007	70	0.065	1.4	65	17.5	0.65	0.07
236	1.4	0.007	70	0.065	1.4	65	17.5	0.35	0.13
237	1.4	0.007	70	0.065	1.4	35	32.5	0.65	0.07
238	1.4	0.007	70	0.065	1.4	35	32.5	0.35	0.13
239	1.4	0.007	70	0.065	1.4	35	17.5	0.65	0.13
240	1.4	0.007	70	0.065	1.4	35	17.5	0.35	0.07
241	1.4	0.007	70	0.035	2.6	65	32.5	0.65	0.13
242	1.4	0.007	70	0.035	2.6	65	32.5	0.35	0.07
243	1.4	0.007	70	0.035	2.6	65	17.5	0.65	0.07
244	1.4	0.007	70	0.035	2.6	65	17.5	0.35	0.13
245	1.4	0.007	70	0.035	2.6	35	32.5	0.65	0.07
246	1.4	0.007	70	0.035	2.6	35	32.5	0.35	0.13
247	1.4	0.007	70	0.035	2.6	35	17.5	0.65	0.13
248	1.4	0.007	70	0.035	2.6	35	17.5	0.35	0.07
249	1.4	0.007	70	0.035	1.4	65	32.5	0.65	0.07
250	1.4	0.007	70	0.035	1.4	65	32.5	0.35	0.13
251	1.4	0.007	70	0.035	1.4	65	17.5	0.65	0.13
252	1.4	0.007	70	0.035	1.4	65	17.5	0.35	0.07
253	1.4	0.007	70	0.035	1.4	35	32.5	0.65	0.13
254	1.4	0.007	70	0.035	1.4	35	32.5	0.35	0.07
255	1.4	0.007	70	0.035	1.4	35	17.5	0.65	0.07
256	1.4	0.007	70	0.035	1.4	35	17.5	0.35	0.13

\*\*The following 19 cases are also employed, in addition to those 256 runs, to form a CCFD experiment.

No.	$\delta$	$\theta$	M	e	D	T	W	b	c
257	2	0.01	100	0.05	2	50	25	0.5	0.1
258	2.6	0.01	100	0.05	2	50	25	0.5	0.1
259	1.4	0.01	100	0.05	2	50	25	0.5	0.1
260	2	0.013	100	0.05	2	50	25	0.5	0.1
261	2	0.007	100	0.05	2	50	25	0.5	0.1
262	2	0.01	130	0.05	2	50	25	0.5	0.1
263	2	0.01	70	0.05	2	50	25	0.5	0.1
264	2	0.01	100	0.065	2	50	25	0.5	0.1
265	2	0.01	100	0.035	2	50	25	0.5	0.1
266	2	0.01	100	0.05	2.6	50	25	0.5	0.1
267	2	0.01	100	0.05	1.4	50	25	0.5	0.1
268	2	0.01	100	0.05	2	65	25	0.5	0.1
269	2	0.01	100	0.05	2	35	25	0.5	0.1
270	2	0.01	100	0.05	2	50	32.5	0.5	0.1
271	2	0.01	100	0.05	2	50	17.5	0.5	0.1
272	2	0.01	100	0.05	2	50	25	0.65	0.1
273	2	0.01	100	0.05	2	50	25	0.35	0.1
274	2	0.01	100	0.05	2	50	25	0.5	0.13
275	2	0.01	100	0.05	2	50	25	0.5	0.07

Value Of Responses Of The  $2^{9-1}$  Fractional  
Factorial Experiment, One Center Point,  
And 18 Axial Points

No.	n	h	k	Loss
1	3	1.1183	3.1179	4.2788
2	3	0.8039	3.2505	4.3359
3	3	1.0047	3.1511	4.2749
4	3	0.9298	3.2283	4.3178
5	3	1.0136	3.0441	4.2184
6	3	0.9526	3.064	4.2258
7	3	1.0985	3.0297	4.2219
8	3	0.8181	3.1593	4.2845
9	3	0.9938	3.1959	4.3041
10	3	0.9128	3.223	4.3124
11	3	1.0881	3.1287	4.2779
12	3	0.7999	3.2875	4.3595
13	3	1.1128	2.9945	4.2085
14	3	0.8025	3.1733	4.2951
15	3	0.9967	3.0549	4.2222
16	3	0.9244	3.087	4.2366
17	4	1.0675	3.4001	4.1586
18	3	0.9463	3.1912	4.2936
19	3	1.1104	3.1251	4.2816
20	4	0.8738	3.4833	4.2285
21	3	1.1233	3.0252	4.2243
22	3	0.8193	3.114	4.2664
23	4	1.0727	3.2926	4.1099
24	3	0.9639	3.0617	4.2249
25	3	1.1185	3.1321	4.2891
26	4	0.8866	3.4708	4.2184
27	4	1.0735	3.3874	4.1521
28	3	0.9279	3.2167	4.3093
29	4	1.0787	3.2727	4.1024
30	3	0.9096	3.0964	4.2417
31	3	1.113	2.9924	4.2077
32	3	0.8088	3.1233	4.2731
33	4	1.4984	3.3821	4.2474
34	3	1.2805	3.2288	4.4529
35	3	1.5284	3.151	4.5144
36	3	1.1011	3.2847	4.4205
37	3	1.5289	3.0131	4.3723
38	3	1.1183	3.1179	4.2788
39	3	1.4036	3.0605	4.3496
40	3	1.3045	3.0604	4.3042
41	3	1.5047	3.1249	4.4691
42	3	1.0768	3.2842	4.4091
43	4	1.452	3.3743	4.224
44	3	1.2414	3.2296	4.4318
45	3	1.378	3.0646	4.3407
46	3	1.2654	3.058	4.2866
47	3	1.516	2.9988	4.3538
48	3	1.0967	3.1192	4.2736
49	4	1.6315	3.378	4.3006

No.	n	h	k	Loss
50	4	1.1883	3.4496	4.2009
51	4	1.4779	3.4215	4.2707
52	3	1.2501	3.2303	4.4374
53	4	1.4511	3.326	4.19
54	3	1.2824	3.0841	4.3133
55	3	1.547	2.9841	4.3575
56	3	1.1304	3.0992	4.2694
57	4	1.459	3.4273	4.2679
58	3	1.26	3.2052	4.4154
59	4	1.6028	3.3919	4.2995
60	4	1.1798	3.4659	4.2104
61	3	1.5008	3.0399	4.3812
62	3	1.1172	3.1354	4.2911
63	4	1.46	3.2632	4.1541
64	3	1.2692	3.093	4.3148
65	4	1.5214	3.3524	4.2338
66	3	1.2611	3.18	4.3898
67	3	1.4793	3.1174	4.4456
68	3	1.0828	3.2597	4.3867
69	3	1.5138	3.0249	4.3747
70	3	1.0772	3.1408	4.2835
71	3	1.3685	3.0292	4.3081
72	3	1.2526	3.0929	4.3078
73	3	1.4751	3.152	4.4817
74	3	1.0727	3.2907	4.4141
75	4	1.4331	3.376	4.2185
76	3	1.2379	3.2062	4.4048
77	3	1.3694	3.0537	4.3276
78	3	1.2677	3.0696	4.296
79	3	1.4861	2.9954	4.3365
80	3	1.0748	3.1313	4.2763
81	4	1.6066	3.3336	4.254
82	4	1.1584	3.459	4.2017
83	4	1.4349	3.4227	4.2547
84	3	1.2573	3.2052	4.4139
85	4	1.4442	3.2582	4.1466
86	3	1.2315	3.1072	4.3106
87	3	1.5195	2.9744	4.3365
88	3	1.1004	3.1459	4.2938
89	4	1.4311	3.3773	4.2187
90	3	1.2348	3.1994	4.398
91	4	1.6378	3.324	4.2598
92	4	1.178	3.4902	4.2272
93	3	1.5049	3.0244	4.372
94	3	1.0929	3.1302	4.2803
95	4	1.4288	3.3105	4.1729
96	3	1.247	3.0661	4.2854
97	3	2.039	3.1317	4.8588
98	4	1.5825	3.494	4.3852
99	4	1.9522	3.4266	4.5168
100	3	1.7091	3.2238	4.7469
101	4	1.9796	3.3029	4.4054
102	3	1.7412	3.0953	4.5871

No.	n	h	k	Loss
103	3	2.0757	2.9946	4.6871
104	3	1.4793	3.1174	4.4456
105	4	1.9787	3.3649	4.4659
106	3	1.714	3.2111	4.7307
107	3	2.0242	3.1298	4.8441
108	4	1.6162	3.4722	4.3803
109	3	2.0555	3.0107	4.6939
110	3	1.4893	3.1346	4.4706
111	4	1.9719	3.3077	4.4058
112	3	1.7119	3.1129	4.5895
113	4	1.9694	3.3876	4.4838
114	3	1.684	3.2329	4.7415
115	4	2.2077	3.3503	4.581
116	4	1.6047	3.4561	4.3587
117	3	2.0831	2.9932	4.6903
118	4	1.6076	3.3748	4.2872
119	4	1.9939	3.2645	4.3787
120	3	1.7331	3.0714	4.5524
121	4	2.2015	3.3728	4.6032
122	4	1.5809	3.4817	4.372
123	4	1.947	3.4244	4.5113
124	3	1.6981	3.2079	4.7133
125	4	1.9588	3.2968	4.3891
126	3	1.7186	3.0612	4.5299
127	3	2.0263	3.0022	4.6633
128	4	1.6378	3.324	4.2598
129	8	1.1754	2.9107	4.4023
130	7	1.1511	2.8447	4.3232
131	7	1.3061	2.7874	4.2821
132	7	0.9458	2.9437	4.4325
133	6	1.2376	2.5491	4.4013
134	7	0.941	2.7959	4.534
135	7	1.122	2.7046	4.4417
136	6	1.112	2.6217	4.4023
137	7	1.2969	2.8016	4.2774
138	7	0.9165	2.933	4.47
139	8	1.1513	2.9213	4.4122
140	7	1.1619	2.8378	4.3208
141	7	1.1037	2.7134	4.4479
142	6	1.0919	2.6363	4.4021
143	6	1.2572	2.5602	4.3775
144	7	0.9312	2.7851	4.5545
145	8	1.3817	2.8504	4.3385
146	8	1.0028	3.0046	4.4917
147	9	1.2497	2.9853	4.4579
148	8	1.2668	2.898	4.3606
149	8	1.1924	2.7802	4.4652
150	7	1.2117	2.6879	4.397
151	7	1.3252	2.6302	4.3894
152	8	1.0618	2.8219	4.5351
153	9	1.2106	3	4.474
154	8	1.2109	2.9065	4.3842
155	8	1.3866	2.8596	4.3327

No.	n	h	k	Loss
156	8	0.9863	3.0071	4.5067
157	7	1.3288	2.6399	4.3792
158	8	0.9912	2.8686	4.573
159	8	1.1868	2.7859	4.465
160	7	1.1618	2.692	4.4246
161	8	1.9379	2.8619	4.2835
162	8	1.3671	2.9861	4.2906
163	8	1.6234	2.9327	4.2612
164	7	1.62	2.8298	4.1971
165	8	1.6325	2.7715	4.3239
166	7	1.6411	2.6778	4.2727
167	7	1.831	2.6165	4.3015
168	7	1.3061	2.7874	4.2821
169	8	1.5913	2.9336	4.2639
170	7	1.5839	2.8289	4.2003
171	8	1.9009	2.8382	4.2891
172	8	1.3641	3.0002	4.2867
173	7	1.843	2.6351	4.2895
174	7	1.2969	2.8016	4.2774
175	8	1.6189	2.7719	4.3253
176	7	1.6464	2.6743	4.2743
177	9	1.677	3.0043	4.3412
178	8	1.7164	2.8954	4.2677
179	9	2.0166	2.888	4.3755
180	9	1.4421	3.0403	4.3693
181	8	1.9589	2.7236	4.3384
182	8	1.4205	2.8531	4.3258
183	9	1.6749	2.8556	4.3873
184	7	1.6591	2.6899	4.2636
185	9	1.9884	2.904	4.3688
186	9	1.4287	3.0694	4.3656
187	9	1.6949	3.0092	4.3388
188	8	1.7119	2.8934	4.2685
189	9	1.7023	2.8453	4.3887
190	7	1.6163	2.6714	4.2796
191	8	1.8849	2.7018	4.3468
192	8	1.3634	2.8417	4.3487
193	8	1.8688	2.843	4.286
194	8	1.3059	3.0061	4.3045
195	8	1.5824	2.9409	4.2626
196	7	1.5704	2.8438	4.1955
197	8	1.6149	2.7827	4.32
198	7	1.559	2.6731	4.2869
199	7	1.7977	2.6374	4.2885
200	7	1.2773	2.794	4.2896
201	8	1.5303	2.9381	4.2702
202	7	1.5491	2.8326	4.2024
203	8	1.8396	2.8507	4.2824
204	8	1.3283	2.9868	4.3027
205	7	1.796	2.6394	4.2873
206	7	1.2742	2.782	4.2981
207	8	1.5833	2.7952	4.3188
208	7	1.5884	2.6942	4.2688

No.	n	h	k	Loss
209	9	1.6549	3.0007	4.3439
210	8	1.731	2.8712	4.2757
211	8	1.8665	2.8609	4.2799
212	9	1.4224	3.0669	4.3679
213	8	1.9005	2.7122	4.3418
214	8	1.3611	2.8611	4.3403
215	9	1.6389	2.8583	4.3904
216	7	1.6262	2.6827	4.2713
217	9	1.9655	2.9434	4.3569
218	9	1.4163	3.0557	4.3726
219	9	1.6193	3.0044	4.3465
220	8	1.6752	2.9076	4.2655
221	9	1.6642	2.8632	4.3856
222	7	1.5826	2.6996	4.2663
223	8	1.8689	2.6948	4.3504
224	8	1.3525	2.8544	4.3464
225	9	2.2474	2.9867	4.3812
226	8	2.2816	2.9022	4.3188
227	8	2.5642	2.8347	4.3954
228	8	1.8513	3.0018	4.2418
229	7	2.4637	2.6253	4.3675
230	8	1.8688	2.843	4.286
231	8	2.2053	2.7645	4.3464
232	7	2.1608	2.6988	4.2809
233	8	2.5567	2.8617	4.3871
234	8	1.8312	2.9983	4.2425
235	9	2.2297	2.979	4.3798
236	8	2.303	2.8971	4.324
237	8	2.1812	2.7801	4.3373
238	7	2.177	2.6927	4.286
239	7	2.4543	2.6213	4.3674
240	8	1.87	2.859	4.2806
241	9	2.7186	2.9239	4.4941
242	9	1.9631	3.0648	4.3317
243	10	2.2956	3.05	4.4668
244	8	2.3188	2.9065	4.3249
245	9	2.2716	2.8822	4.4082
246	8	2.3228	2.7368	4.3754
247	8	2.5887	2.7125	4.4378
248	9	1.972	2.928	4.3613
249	10	2.2941	3.0552	4.4657
250	8	2.2874	2.8883	4.3231
251	9	2.6707	2.9194	4.4829
252	9	1.9321	3.0619	4.3299
253	8	2.6037	2.6952	4.4476
254	9	1.9655	2.9434	4.3569
255	9	2.2392	2.8662	4.4074
256	8	2.3201	2.7342	4.3759

\*\*The following 19 results are used in combination with those 256 results to conduct the CCFD analysis.



No.	n	h	k	Loss
257	5	1.3953	3.0701	4.0129
258	3	1.2682	3.1414	4.3561
259	8	1.5602	2.8651	4.2918
260	5	1.265	3.1017	4.0209
261	5	1.6639	3.0717	4.0338
262	5	1.2318	3.0553	4.0287
263	5	1.6942	3.0894	4.0403
264	5	1.3734	3.1029	4.0168
265	5	1.4205	3.0694	4.0129
266	5	1.4165	3.0946	4.013
267	5	1.4039	3.0933	4.0129
268	5	1.4072	3.1448	4.0151
269	5	1.4297	3.0158	4.0149
270	5	1.3953	3.0701	4.0129
271	5	1.4011	3.031	4.0143
272	5	1.5197	3.0674	4.0169
273	5	1.2997	3.1172	4.0171
274	5	1.4959	3.0606	4.0153
275	5	1.3038	3.1283	4.0169

**APPENDIX E**

**CALCULATED HALF EFFECTS FOR THE  
DESIGN PARAMETERS AND THE  
RESULTING OPERATING LOSS  
OF THE THREE VARIABLES  
CONTROL CHARTS**

CALCULATED HALF EFFECTS FOR n AND h USING THE  
X-BAR CONTROL CHART WITH AT&T RUNS RULES

No.	n		h	
	Effect	Half Eff.	Effect	Half Eff.
1	$\delta$	2.277343	M	0.251686
2	e	0.308593	$\theta$	0.236297
3	c	0.300781	$\delta$	0.146946
4	T	0.230468	b	0.138111
5	b	0.183593	c	0.094469
6	M	0.160156	$\delta^*c$	0.036823
7	$\theta$	0.152343	$\theta^*M$	0.036514
8	$\delta^*e$	0.136718	$\delta^*M$	0.031928
9	$\delta^*T$	0.089843	e	0.028540
10	$\delta^*M$	0.082031	$\delta^*\theta$	0.026235
11	$\delta^*\theta$	0.074218	M*b	0.021936
12	D*W	0.066406	$\theta^*b$	0.020596
13	e*c	0.066406	M*c	0.014317
14	$\delta^*c$	0.066406	T	0.013330
15	e*b	0.042968	$\theta^*c$	0.012958
16	$\delta^*b$	0.042968	$\delta^*e$	0.012111
17	$\theta^*c$	0.035156	$\delta^*b$	0.009275
18	b*c	0.035156	b*c	0.008052
19	e*T	0.027343	D*W	0.006747
20	T*b	0.027343	D	0.006566
21	M*c	0.027343	$\delta^*T$	0.004118
22	$\theta^*e$	0.019531	T*b	0.003971
23	M*b	0.019531	W*b	0.003294
24	$\theta^*b$	0.011718	T*c	0.003278
25	$\theta^*M$	0.011718	e*T	0.003034
26	T*c	0.011718	$\theta^*D$	0.002102
27	M*e	0.011718	e*b	0.001921
28	$\delta^*W$	0.003906	$\theta^*e$	0.001654
29	$\delta^*D$	0.003906	$\theta^*T$	0.001432
30	D*b	0.003906	$\theta^*W$	0.001260
31	$\theta^*T$	0.003906	M*T	0.001185
32	D*c	0.003906	e*c	0.001031
33	$\theta^*D$	0.003906	W*c	0.000683
34	T*W	0.003906	M*D	0.000627
35	M*W	0.003906	$\delta^*D$	0.000625
36	W	0.003906	W	0.000569
37	D*T	0.003906	M*e	0.000479
38	M*D	0.003906	D*c	0.000346
39	e*D	0.003906	e*D	0.000321
40	W*b	0.003906	D*b	0.000189
41	M*T	0.003906	e*W	0.000184
42	e*W	0.003906	T*W	0.000125
43	$\theta^*W$	0.003906	D*T	0.000067
44	W*c	0.003906	M*W	0.000033
45	D	0.003906	$\delta^*W$	0.000015

CALCULATED HALF EFFECTS FOR  $k$  AND THE LOSS USING  
THE X-BAR CONTROL CHART WITH AT&T RUNS RULES

No.	$k$		Loss	
	Effect	Half Eff.	Effect	Half Eff.
1	$\delta$	0.183451	$\delta^*M$	0.045630
2	T	0.094154	$\delta^*\theta$	0.042007
3	c	0.072985	$\delta^*c$	0.040990
4	e	0.033966	$\theta^*M$	0.040938
5	M	0.016837	M	0.031437
6	$\theta$	0.016032	$\delta^*e$	0.029129
7	b	0.011277	$\theta$	0.027382
8	$e^*c$	0.010846	$M^*c$	0.025640
9	$e^*b$	0.010063	$\theta^*c$	0.022680
10	$D^*W$	0.009341	c	0.018962
11	$\theta^*c$	0.006709	$\delta^*T$	0.018247
12	$M^*c$	0.005298	$M^*b$	0.018075
13	$e^*T$	0.004875	$\theta^*b$	0.014891
14	$M^*b$	0.004235	$b^*c$	0.009888
15	$\delta^*b$	0.00355	$\delta^*b$	0.009312
16	$b^*c$	0.003300	T	0.007708
17	$\delta^*e$	0.003264	$M^*T$	0.007198
18	$\delta^*c$	0.003068	$D^*W$	0.005562
19	$\delta^*M$	0.002721	$T^*c$	0.005460
20	$M^*T$	0.002413	b	0.005336
21	$\theta^*e$	0.002348	$\theta^*T$	0.004666
22	$T^*b$	0.001929	$\delta$	0.003795
23	$\theta^*b$	0.001868	$T^*b$	0.002638
24	$T^*W$	0.001769	$e^*c$	0.002332
25	$M^*D$	0.001686	e	0.002158
26	D	0.001681	$M^*D$	0.002146
27	W	0.001421	$M^*e$	0.001919
28	$e^*W$	0.001389	$e^*T$	0.001805
29	$M^*e$	0.001346	$e^*b$	0.001623
30	$W^*b$	0.001215	$\delta^*D$	0.001476
31	$\theta^*M$	0.000889	W	0.001100
32	$M^*W$	0.000835	$e^*D$	0.001091
33	$T^*c$	0.000742	$D^*c$	0.000998
34	$\theta^*T$	0.000627	$\theta^*e$	0.000958
35	$\delta^*W$	0.000524	$D^*b$	0.000782
36	$D^*T$	0.000475	$W^*b$	0.000751
37	$W^*c$	0.000396	$\theta^*W$	0.000710
38	$D^*b$	0.000351	$W^*c$	0.000674
39	$e^*D$	0.000342	$M^*W$	0.000637
40	$\delta^*\theta$	0.000321	$T^*W$	0.000535
41	$\delta^*T$	0.000317	$\theta^*D$	0.000347
42	$\theta^*W$	0.000307	D	0.000317
43	$\theta^*D$	0.000199	$D^*T$	0.000264
44	$D^*c$	0.000181	$e^*W$	0.000141
45	$\delta^*D$	0.000166	$\delta^*W$	0.000027

THE TRUE VALUES AND THE PREDICTED VALUES OF THE  
OPTIMAL DESIGN PARAMETERS AND THE RESULTING  
OPERATING LOSSES USING THE X-BAR CONTROL  
CHART WITH AT&T RUNS RULES

No	n	PRE	h	PRE	k	PRE	Loss	PRE
1	3	3.3	1.118	1.116	3.117	3.159	4.278	4.292
2	3	3.3	0.803	0.651	3.250	3.305	4.335	4.343
3	3	3.3	1.004	0.927	3.151	3.305	4.274	4.249
4	3	3.3	0.929	0.840	3.228	3.159	4.317	4.346
5	3	3.3	1.013	0.927	3.044	3.117	4.218	4.197
6	3	3.3	0.952	0.840	3.064	2.971	4.225	4.294
7	3	3.3	1.098	1.116	3.029	2.971	4.221	4.240
8	3	3.3	0.818	0.651	3.159	3.117	4.284	4.291
9	3	3.3	0.993	0.927	3.195	3.305	4.304	4.249
10	3	3.3	0.912	0.840	3.223	3.159	4.312	4.346
11	3	3.3	1.088	1.116	3.128	3.159	4.277	4.292
12	3	3.3	0.799	0.651	3.287	3.305	4.359	4.343
13	3	3.3	1.112	1.116	2.994	2.971	4.208	4.240
14	3	3.3	0.802	0.651	3.173	3.117	4.295	4.291
15	3	3.3	0.996	0.927	3.054	3.117	4.222	4.197
16	3	3.3	0.924	0.840	3.087	2.971	4.236	4.294
17	4	3.3	1.067	0.927	3.400	3.373	4.158	4.186
18	3	3.3	0.946	0.840	3.191	3.227	4.293	4.284
19	3	3.3	1.110	1.116	3.125	3.227	4.281	4.229
20	4	3.3	0.873	0.651	3.483	3.373	4.228	4.280
21	3	3.3	1.123	1.116	3.025	3.039	4.224	4.178
22	3	3.3	0.819	0.651	3.114	3.185	4.266	4.228
23	4	3.3	1.072	0.927	3.292	3.185	4.109	4.135
24	3	3.3	0.963	0.840	3.061	3.039	4.224	4.232
25	3	3.3	1.118	1.116	3.132	3.227	4.289	4.229
26	4	3.3	0.886	0.651	3.470	3.373	4.218	4.280
27	4	3.3	1.073	0.927	3.387	3.373	4.152	4.186
28	3	3.3	0.927	0.840	3.216	3.227	4.309	4.284
29	4	3.3	1.078	0.927	3.272	3.185	4.102	4.135
30	3	3.3	0.909	0.840	3.096	3.039	4.241	4.232
31	3	3.3	1.113	1.116	2.992	3.039	4.207	4.178
32	3	3.3	0.808	0.651	3.123	3.185	4.273	4.228
33	4	3.3	1.498	1.431	3.382	3.339	4.247	4.306
34	3	3.3	1.280	1.343	3.228	3.193	4.452	4.434
35	3	3.3	1.528	1.62	3.151	3.193	4.514	4.452
36	3	3.3	1.101	1.154	3.284	3.339	4.420	4.327
37	3	3.3	1.528	1.62	3.013	3.004	4.372	4.400
38	3	3.3	1.118	1.154	3.117	3.150	4.278	4.276
39	3	3.3	1.403	1.431	3.060	3.150	4.349	4.254
40	3	3.3	1.304	1.343	3.060	3.004	4.304	4.382

CALCULATED HALF EFFECTS FOR n AND h USING  
THE EWMA CHART

No.	n		h	
	Effect	Half Eff.	Effect	Half Eff.
1	$\delta$	2.148437	M	0.249430
2	e	0.34375	$\theta$	0.233151
3	c	0.296875	b	0.141337
4	T	0.257812	$\delta$	0.126886
5	b	0.195312	c	0.092024
6	$\delta*e$	0.1875	e	0.036880
7	M	0.171875	$\theta*M$	0.035005
8	$\theta$	0.164062	$\delta*c$	0.030649
9	$\delta*T$	0.101562	$\delta*M$	0.029978
10	$\delta*c$	0.078125	$\delta*\theta$	0.024936
11	$\delta*M$	0.078125	$\theta*b$	0.024283
12	$\delta*\theta$	0.070312	M*b	0.022189
13	$\delta*b$	0.070312	$\delta*e$	0.018383
14	T*c	0.0625	T	0.015399
15	e*c	0.054687	M*c	0.012030
16	M*c	0.054687	$\theta*c$	0.011029
17	e*T	0.046875	D	0.008949
18	$\theta*c$	0.046875	M*T	0.006126
19	$\theta*b$	0.039062	e*T	0.005542
20	M*b	0.03125	$\theta*T$	0.004816
21	e*b	0.03125	$\delta*b$	0.003512
22	M*e	0.023437	b*c	0.003236
23	M*D	0.015625	$\theta*e$	0.003194
24	M*T	0.015625	T*c	0.003082
25	$\theta*M$	0.015625	$\delta*T$	0.002508
26	$\theta*e$	0.015625	M*e	0.002499
27	D*W	0.015625	D*T	0.002370
28	b*c	0.015625	e*b	0.001641
29	e*W	0.007812	e*W	0.001625
30	D*b	0.007812	T*b	0.001454
31	LAM*T	0.007812	$\delta*D$	0.001338
32	D*T	0.007812	D*b	0.001286
33	$\delta*D$	0.007812	W*c	0.001236
34	W*c	0.007812	W	0.001012
35	M*W	0.007812	e*c	0.000951
36	$\theta*D$	0.007812	$\theta*D$	0.000864
37	T*b	0.007812	$\theta*W$	0.000748
38	D	0.007812	T*W	0.000744
39	$\delta*W$	0	W*b	0.000589
40	W*b	0	D*W	0.000538
41	T*W	0	M*W	0.000400
42	e*D	0	e*D	0.000264
43	D*c	0	M*D	0.000255
44	$\theta*W$	0	D*c	0.000108
45	W	0	$\delta*W$	0.000102

CALCULATED HALF EFFECTS FOR  $k$  AND  $\alpha$  USING  
THE EWMA CHART

No.	$k$		$\alpha$	
	Effect	Half Eff.	Effect	Half Eff.
1	$\delta$	0.179964	$\delta$	0.019435
2	T	0.102141	e	0.008984
3	c	0.069578	$\delta^*e$	0.008428
4	e	0.029449	b	0.008295
5	b	0.018652	M	0.004820
6	$\theta$	0.014877	$\theta$	0.004112
7	M	0.013457	$\delta^*b$	0.003958
8	$e^*c$	0.0105	c	0.003429
9	$T^*c$	0.007453	$\delta^*\theta$	0.003011
10	$e^*b$	0.007252	$M^*T$	0.001921
11	$D^*W$	0.006794	$\delta^*M$	0.001836
12	$e^*T$	0.005050	$e^*c$	0.001555
13	$\theta^*c$	0.004484	$\delta^*c$	0.001442
14	$M^*c$	0.004458	$M^*D$	0.001332
15	$\delta^*e$	0.003980	$M^*e$	0.001196
16	$\theta^*b$	0.003632	$e^*b$	0.001167
17	$\delta^*T$	0.003480	W	0.001130
18	$T^*b$	0.003347	$\delta^*T$	0.001120
19	$M^*b$	0.003084	$\theta^*W$	0.001031
20	$b^*c$	0.002862	$D^*T$	0.001021
21	$\delta^*M$	0.001803	$e^*W$	0.000953
22	$M^*D$	0.001604	$\delta^*W$	0.000932
23	$\theta^*T$	0.001588	$\theta^*b$	0.000925
24	$\delta^*b$	0.001408	$\theta^*D$	0.000895
25	D	0.001392	$M^*c$	0.000764
26	$M^*e$	0.001312	$b^*c$	0.000746
27	$T^*W$	0.001308	$\theta^*T$	0.000645
28	$\delta^*D$	0.001308	$M^*b$	0.000558
29	$M^*T$	0.001178	$T^*b$	0.000426
30	$\theta^*M$	0.001168	$M^*W$	0.000423
31	$W^*b$	0.001097	$T^*c$	0.000421
32	$\delta^*\theta$	0.001069	$D^*c$	0.000354
33	$D^*b$	0.001060	$\theta^*M$	0.000344
34	$\delta^*W$	0.000960	$\theta^*e$	0.000338
35	$\delta^*c$	0.000890	$\theta^*c$	0.000322
36	$D^*c$	0.000790	T	0.000297
37	$M^*W$	0.000757	$D^*b$	0.000293
38	$W^*c$	0.000439	D	0.000246
39	$\theta^*e$	0.000405	$W^*c$	0.000146
40	W	0.000299	$e^*D$	0.000142
41	$\theta^*W$	0.000274	$W^*b$	0.0001
42	$D^*T$	0.000192	$e^*T$	0.0001
43	$\theta^*D$	0.000100	$\delta^*D$	0.000089
44	$e^*D$	0.000025	$T^*W$	0.000086
45	$e^*W$	0.000024	$D^*W$	0.000055

CALCULATED HALF EFFECTS FOR THE LOSS USING  
THE EWMA CHART

No.	Loss	
	Effect	Half Eff.
1	$\delta^*M$	0.045032
2	$\theta^*M$	0.044017
3	$\delta^*\theta$	0.042458
4	$\delta^*c$	0.036722
5	$\delta^*e$	0.032441
6	M	0.027938
7	$\theta$	0.025762
8	M*c	0.022393
9	$\theta^*c$	0.021657
10	$\theta^*b$	0.020929
11	M*b	0.020794
12	$\delta^*T$	0.013556
13	b	0.011631
14	b*c	0.010444
15	c	0.010312
16	T	0.007123
17	M*T	0.006605
18	D*W	0.006501
19	$\theta^*T$	0.006324
20	T*b	0.005929
21	e*c	0.004257
22	$\delta^*b$	0.004088
23	e*b	0.004053
24	e	0.002835
25	$\theta^*D$	0.002090
26	T*c	0.002089
27	$\delta$	0.001915
28	M*e	0.001880
29	D*T	0.00161
30	$\theta^*e$	0.001290
31	D*c	0.001269
32	M*W	0.000930
33	$\delta^*D$	0.000925
34	$\theta^*W$	0.000798
35	e*D	0.000613
36	M*D	0.000540
37	e*W	0.000462
38	$\delta^*W$	0.000394
39	e*T	0.000341
40	D	0.000329
41	W	0.000181
42	D*b	0.000111
43	T*W	0.000102
44	W*c	0.000058
45	W*b	0.000042



THE TRUE VALUES AND THE PREDICTED VALUES OF THE  
OPTIMAL DESIGN PARAMETERS USING THE EWMA CHART

No	n	PRE	h	PRE	k	PRE	$\alpha$	PRE
1	3	3.3	1.121	1.118	3.154	3.164	0.955	0.934
2	3	3.3	0.818	0.652	3.290	3.303	0.949	0.934
3	3	3.3	1.003	0.935	3.208	3.303	0.934	0.934
4	3	3.3	0.920	0.836	3.235	3.164	0.911	0.934
5	3	3.3	0.996	0.935	3.073	3.099	0.959	0.934
6	3	3.3	0.958	0.836	3.062	2.960	0.950	0.934
7	3	3.3	1.135	1.118	3.020	2.960	0.915	0.934
8	3	3.3	0.824	0.652	3.142	3.100	0.932	0.934
9	3	3.3	0.986	0.935	3.194	3.303	0.935	0.934
10	3	3.3	0.899	0.836	3.215	3.164	0.912	0.934
11	3	3.3	1.082	1.118	3.149	3.164	0.925	0.934
12	3	3.3	0.796	0.652	3.278	3.303	0.931	0.934
13	3	3.3	1.121	1.119	3.022	2.960	0.925	0.934
14	3	3.3	0.809	0.652	3.152	3.099	0.929	0.934
15	3	3.3	1.002	0.935	3.071	3.099	0.952	0.934
16	3	3.3	0.934	0.836	3.095	2.960	0.944	0.934
17	4	3.3	1.064	0.935	3.386	3.362	0.933	0.934
18	3	3.3	0.935	0.836	3.231	3.223	0.936	0.934
19	3	3.3	1.136	1.118	3.112	3.223	0.927	0.934
20	4	3.3	0.872	0.652	3.535	3.362	0.925	0.934
21	3	3.3	1.094	1.118	3.002	3.019	0.940	0.934
22	3	3.3	0.813	0.652	3.160	3.158	0.932	0.934
23	3	3.3	1.022	0.935	3.040	3.158	0.932	0.934
24	3	3.3	0.957	0.836	3.058	3.019	0.949	0.934
25	3	3.3	1.123	1.119	3.126	3.223	0.920	0.934
26	4	3.3	0.852	0.652	3.489	3.362	0.914	0.934
27	4	3.3	1.058	0.935	3.426	3.362	0.930	0.934
28	3	3.3	0.932	0.836	3.228	3.223	0.952	0.934
29	4	3.3	0.989	0.935	3.077	3.157	0.950	0.934
30	3	3.3	0.938	0.836	3.070	3.019	0.930	0.934
31	3	3.3	1.114	1.118	2.999	3.019	0.928	0.934
32	3	3.3	0.809	0.652	3.141	3.158	0.915	0.934
33	4	3.3	1.477	1.433	3.375	3.330	0.917	0.934
34	3	3.3	1.288	1.334	3.205	3.191	0.918	0.934
35	3	3.3	1.543	1.617	3.153	3.191	0.913	0.934
36	3	3.3	1.104	1.151	3.292	3.330	0.922	0.934
37	3	3.3	1.521	1.618	3.019	2.987	0.947	0.934
38	3	3.3	1.109	1.151	3.156	3.126	0.932	0.934
39	3	3.3	1.388	1.433	3.041	3.126	0.952	0.934
40	3	3.3	1.282	1.335	3.060	2.987	0.926	0.934

THE TRUE VALUES AND THE PREDICTED VALUES OF THE  
RESULTING OPERATING LOSSES USING THE EWMA CHART

No	Loss	PRE
1	4.278	4.256
2	4.335	4.310
3	4.274	4.229
4	4.317	4.296
5	4.218	4.188
6	4.225	4.254
7	4.221	4.215
8	4.284	4.269
9	4.304	4.229
10	4.312	4.296
11	4.277	4.256
12	4.359	4.310
13	4.208	4.215
14	4.295	4.269
15	4.222	4.188
16	4.236	4.254
17	4.158	4.170
18	4.293	4.236
19	4.281	4.197
20	4.228	4.251
21	4.224	4.156
22	4.266	4.210
23	4.109	4.128
24	4.224	4.195
25	4.289	4.197
26	4.218	4.251
27	4.152	4.170
28	4.309	4.236
29	4.102	4.128
30	4.241	4.195
31	4.207	4.156
32	4.273	4.210
33	4.247	4.284
34	4.452	4.357
35	4.514	4.400
36	4.420	4.282
37	4.372	4.359
38	4.278	4.241
39	4.349	4.243
40	4.304	4.315

CALCULATED HALF EFFECTS FOR n AND h USING  
THE ZCC CHART

No.	n		h	
	Effect	Half Eff.	Effect	Half Eff.
1	$\delta$	2.289062	M	0.254628
2	e	0.304687	$\theta$	0.238158
3	c	0.289062	$\delta$	0.146967
4	T	0.242187	b	0.139537
5	b	0.179687	c	0.095828
6	$\theta$	0.164062	$\theta$ *M	0.036813
7	M	0.164062	$\delta$ *c	0.035968
8	$\delta$ *e	0.148437	e	0.028784
9	$\delta$ *T	0.085937	$\delta$ *M	0.028019
10	$\delta$ *M	0.070312	$\delta$ * $\theta$	0.025762
11	$\delta$ * $\theta$	0.070312	$\theta$ *b	0.020646
12	D*W	0.070312	M*b	0.020526
13	$\delta$ *c	0.070312	T	0.013805
14	$\delta$ *b	0.054687	$\delta$ *e	0.013638
15	e*c	0.054687	D	0.012930
16	e*T	0.039062	M*c	0.011515
17	M*c	0.039062	$\theta$ *c	0.010560
18	e*b	0.039062	D*W	0.009944
19	T*b	0.039062	$\delta$ *b	0.007792
20	$\theta$ *c	0.039062	T*b	0.006241
21	$\theta$ *b	0.023437	$\theta$ *e	0.005505
22	$\theta$ *M	0.023437	M*D	0.005353
23	M*b	0.023437	T*c	0.005207
24	b*c	0.023437	b*c	0.004687
25	T*c	0.007812	e*c	0.004380
26	M*e	0.007812	e*T	0.004368
27	M*T	0.007812	e*W	0.004266
28	$\theta$ *e	0.007812	M*e	0.003735
29	$\theta$ *T	0.007812	D*b	0.003219
30	$\delta$ *W	0	$\theta$ *W	0.002988
31	D*b	0	$\theta$ *T	0.002752
32	D*c	0	W*c	0.002704
33	M*W	0	$\delta$ *T	0.002662
34	T*W	0	W	0.002210
35	$\theta$ *W	0	$\theta$ *D	0.002158
36	D	0	D*T	0.002120
37	D*T	0	$\delta$ *D	0.002096
38	M*D	0	D*c	0.002094
39	e*D	0	T*W	0.002020
40	W*b	0	e*b	0.001808
41	$\delta$ *D	0	$\delta$ *W	0.000915
42	e*W	0	W*b	0.000519
43	W	0	M*W	0.000483
44	W*c	0	M*T	0.000316
45	$\theta$ *D	0	e*D	0.000227

CALCULATED HALF EFFECTS FOR k AND THE LOSS  
USING THE ZCC CHART

No.	k		Loss	
	Effect	Half Eff.	Effect	Half Eff.
1	$\delta$	0.180436	$\delta^*M$	0.044130
2	T	0.095873	$\delta^*\theta$	0.042395
3	c	0.069096	$\theta^*M$	0.041332
4	e	0.029274	$\delta^*c$	0.040709
5	$\theta$	0.020986	$\delta^*e$	0.029303
6	M	0.016196	M	0.028989
7	b	0.014630	$\theta$	0.028342
8	D*W	0.012679	$\theta^*c$	0.024091
9	M*c	0.009328	M*c	0.023723
10	e*c	0.007611	c	0.019044
11	$\theta^*c$	0.007243	M*b	0.018332
12	M*b	0.006852	$\delta^*T$	0.016704
13	$\theta^*b$	0.006311	$\theta^*b$	0.015939
14	e*b	0.004829	b*c	0.009438
15	e*T	0.004708	$\delta^*b$	0.008631
16	$\delta^*\theta$	0.004361	M*T	0.007914
17	D	0.003699	b	0.006925
18	$\delta^*M$	0.002723	T	0.006474
19	b*c	0.002596	$\theta^*T$	0.006228
20	$\theta^*M$	0.002572	T*c	0.005496
21	M*e	0.002421	$\delta$	0.005375
22	$\delta^*e$	0.002396	e*b	0.004958
23	$\delta^*c$	0.002160	D*b	0.004334
24	T*b	0.002147	D*W	0.004294
25	$\theta^*T$	0.002113	T*b	0.004002
26	D*b	0.001534	e*T	0.003271
27	D*c	0.001381	e*c	0.003042
28	W	0.001120	e*W	0.002348
29	$\delta^*b$	0.000987	M*D	0.001907
30	$\theta^*e$	0.000958	$\theta^*D$	0.001878
31	$\delta^*D$	0.000877	T*W	0.001576
32	$\theta^*W$	0.000866	$\theta^*e$	0.001507
33	W*c	0.000712	$\delta^*D$	0.001447
34	$\delta^*T$	0.000608	M*e	0.001364
35	M*W	0.000573	D*c	0.001328
36	D*T	0.000550	$\theta^*W$	0.001156
37	T*c	0.000537	D	0.000890
38	M*D	0.000451	$\delta^*W$	0.000833
39	W*b	0.000415	D*T	0.000804
40	T*W	0.000380	M*W	0.000571
41	M*T	0.000282	W*b	0.0004
42	e*W	0.000276	e*D	0.000382
43	$\theta^*D$	0.000272	W	0.000257
44	$\delta^*W$	0.000210	e	0.000241
45	e*D	0.000012	W*c	0.000101

THE TRUE VALUES AND THE PREDICTED VALUES OF THE  
OPTIMAL DESIGN PARAMETERS AND THE RESULTING  
OPERATING LOSSES USING THE ZCC

No	n	PRE	h	PRE	k	PRE	Loss	PRE
1	3	3.3	1.150	1.115	3.081	3.138	4.263	4.304
2	3	3.3	0.836	0.643	3.235	3.276	4.321	4.335
3	3	3.3	1.028	0.923	3.181	3.276	4.300	4.261
4	3	3.3	0.911	0.835	3.265	3.138	4.336	4.340
5	3	3.3	1.038	0.923	3.038	3.084	4.217	4.215
6	3	3.3	0.898	0.835	3.052	2.946	4.227	4.293
7	3	3.3	1.156	1.114	2.985	2.946	4.212	4.257
8	3	3.3	0.838	0.644	3.079	3.084	4.249	4.288
9	3	3.3	0.964	0.923	3.178	3.276	4.286	4.261
10	3	3.3	0.907	0.835	3.175	3.138	4.281	4.340
11	3	3.3	1.044	1.114	3.127	3.138	4.266	4.304
12	3	3.3	0.795	0.644	3.268	3.276	4.347	4.335
13	3	3.3	1.149	1.114	3.013	2.946	4.223	4.257
14	3	3.3	0.794	0.644	3.187	3.084	4.303	4.288
15	3	3.3	1.019	0.923	3.019	3.084	4.208	4.215
16	3	3.3	0.955	0.844	3.042	2.946	4.217	4.293
17	4	3.3	1.015	0.923	3.379	3.335	4.150	4.203
18	3	3.3	0.879	0.844	3.217	3.197	4.307	4.281
19	3	3.3	1.162	1.114	3.108	3.197	4.285	4.246
20	4	3.3	0.906	0.644	3.507	3.335	4.231	4.276
21	3	3.3	1.136	1.114	2.991	3.005	4.211	4.199
22	3	3.3	0.736	0.644	3.124	3.143	4.307	4.230
23	4	3.3	1.045	0.923	3.057	3.143	4.223	4.157
24	3	3.3	0.971	0.844	3.098	3.005	4.280	4.235
25	3	3.3	1.106	1.114	3.189	3.196	4.330	4.246
26	4	3.3	0.834	0.644	3.451	3.334	4.230	4.276
27	4	3.3	1.040	0.923	3.338	3.334	4.130	4.203
28	3	3.3	0.871	0.844	3.212	3.196	4.304	4.281
29	4	3.3	0.981	0.923	3.035	3.143	4.214	4.157
30	3	3.3	0.916	0.844	3.132	3.005	4.257	4.235
31	3	3.3	1.117	1.114	2.994	3.005	4.209	4.199
32	3	3.3	0.816	0.644	3.148	3.143	4.279	4.230
33	4	3.3	1.573	1.432	3.435	3.309	4.323	4.314
34	3	3.3	1.290	1.344	3.223	3.171	4.451	4.414
35	3	3.3	1.536	1.624	3.174	3.171	4.548	4.451
36	3	3.3	1.100	1.153	3.228	3.308	4.362	4.314
37	3	3.3	1.544	1.624	2.958	2.978	4.336	4.405
38	3	3.3	1.150	1.153	3.082	3.117	4.263	4.268
39	3	3.3	1.372	1.432	3.101	3.117	4.370	4.267
40	3	3.3	1.321	1.344	3.057	2.979	4.309	4.368

THE TRUE VALUES AND THE PREDICTED VALUES OF THE  
OPTIMAL DESIGN PARAMETERS USING THE ZCC(\*)

No.	S1	PRE	S2	PRE	S3	PRE	S4	PRE
1	0	0	1	1	2	2	16	17
2	0	0	1	1	2	2	15	17
3	0	0	1	1	2	2	16	17
4	0	0	1	1	2	2	15	17
5	0	0	1	1	2	2	16	17
6	0	0	1	1	2	2	16	17
7	0	0	1	1	2	2	15	17
8	0	0	1	1	2	2	15	17
9	0	0	1	1	2	2	15	17
10	0	0	1	1	2	2	16	17
11	0	0	1	1	2	2	15	17
12	0	0	1	1	2	2	15	17
13	0	0	1	1	2	2	17	17
14	0	0	1	1	2	2	15	17
15	0	0	1	1	2	2	16	17
16	0	0	1	1	2	2	15	17
17	0	0	1	1	2	2	15	17
18	0	0	1	1	2	2	16	17
19	0	0	1	1	2	2	15	17
20	0	0	1	1	2	2	16	17
21	0	0	1	1	2	2	16	17
22	0	0	1	1	2	2	15	17
23	0	0	1	1	2	2	15	17
24	0	0	1	1	2	2	15	17
25	0	0	1	1	2	2	16	17
26	0	0	1	1	2	2	15	17
27	0	0	1	1	2	2	15	17
28	0	0	1	1	2	2	16	17
29	0	0	1	1	2	2	15	17
30	0	0	1	1	2	2	15	17
31	0	0	1	1	2	2	16	17
32	0	0	1	1	2	2	15	17
33	0	0	1	1	2	2	15	17
34	0	0	1	1	2	2	15	17
35	0	0	1	1	2	2	15	17
36	0	0	1	1	2	2	15	17
37	0	0	1	1	2	2	15	17
38	0	0	1	1	2	2	16	17
39	0	0	1	1	2	2	15	17
40	0	0	1	1	2	2	16	17

\*Note: The predicted zone scores are truncated to nearest integers when necessary.

VITA

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Doctor of Philosophy

**Thesis: THE ECONOMIC DESIGN AND EVALUATION OF THREE  
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