A SYSTEMS APPROACH TO THE DETERMINATION OF OPTIMAL BEEF HERD CULLING AND REPLACEMENT RATE STRATEGIES

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PREFACE

The study is concerned with the analysis of different replacement and culling rate strategies for a typical cow-calf operation. The primary objective is to provide appropriate herd management criteria at the firm level that will enable producers to optimize their returns under conditions of highly variable prices and rapidly rising costs. A system model is used in this study to incorporate all factors which are necessary to provide a "good" solution to the above problem.

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CHAPTER I

INTRODUCTION

Current Conditions

Cattle producers operate under conditions of highly variable product prices. Cattle price variations seem to occur in a rather cyclical pattern as demonstrated by Figure 1. Observing Figure 1, we see the average cattle price rises from $14.90 in 1956 to $22.60 in 1959; it then drops passing through the fitted trend in 1962, continues to drop until reaching a low of $18.00 in 1964. From there the price rises steadily passing back through the trend again in 1971 and continuing to a peak of $42.80 in 1973. Price then falls sharply to $32.30 in 1975 and rises slightly again to $33.70 in 1976. Similar cyclical price variation have been fairly predominate in the cattle industry over the last 50 years.

Cyclical price variations are even more evident in certain sections of the industry. Franzmann (1) demonstrated this in 1971 when he plotted average monthly cost per 100 pounds of federally inspected cattle slaughter from 1921 to 1969. A reproduction of this graph is provided in Figure 2. The cyclical pattern of the monthly cattle slaughter in Figure 2 is much more pronounced than the average annual cattle price cycles of Figure 1.

Cattle price cycles have also shown a strong trend in past years (Figure 1). Most agricultural economists agree that these types of

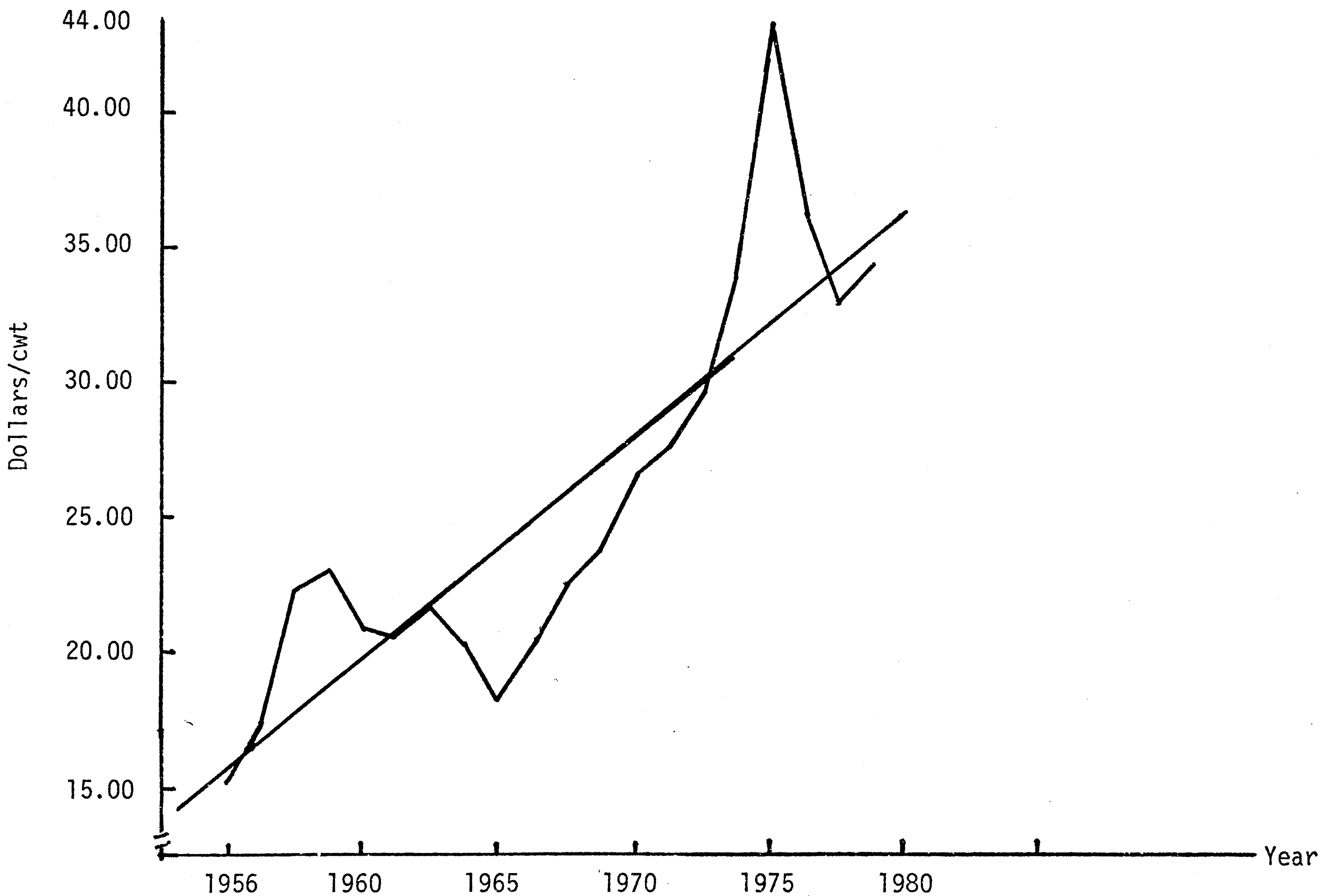

Figure 1. Average Cattle Price, 1956-1976

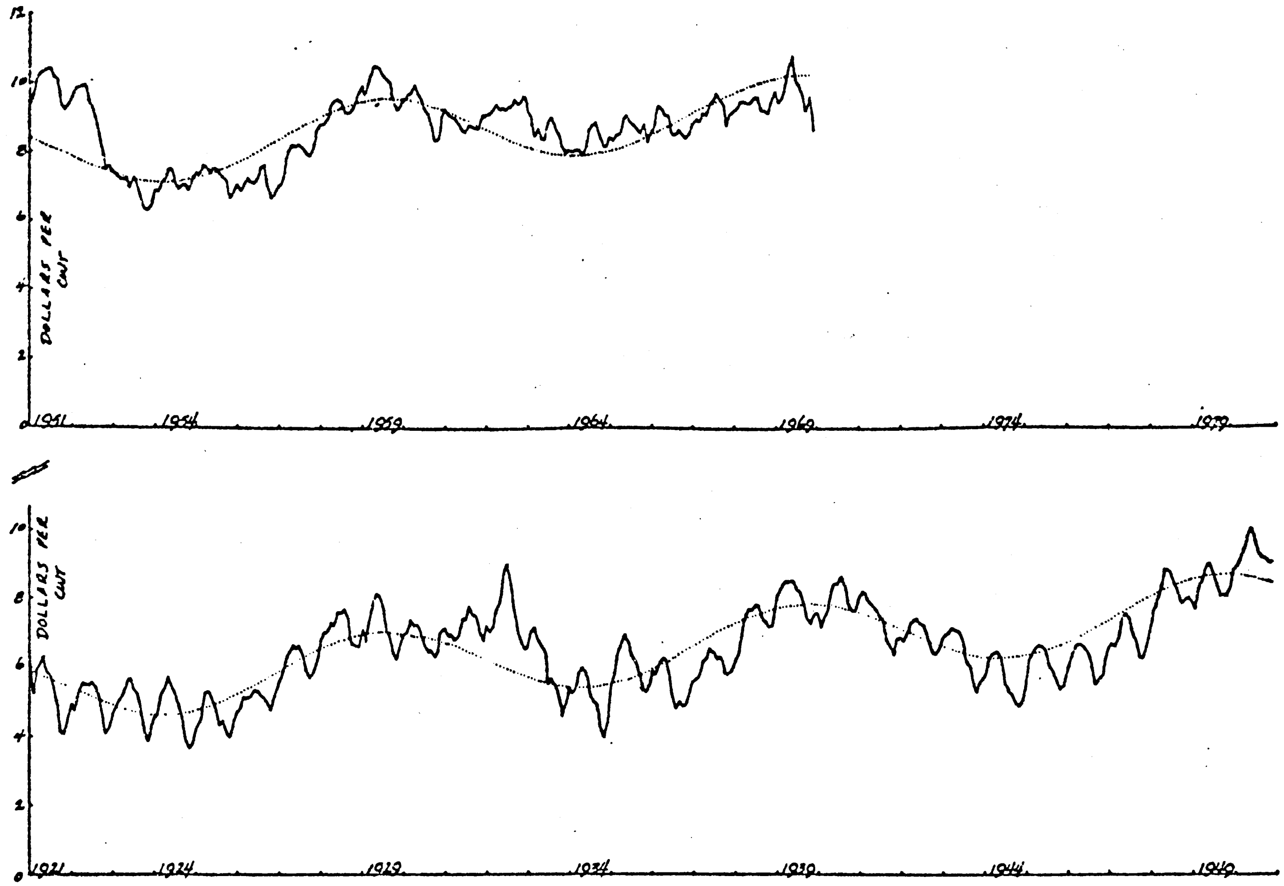

Figure 2. Average Monthly Cost Per 100 Pounds of Cattle Slaughtered Under Federal Inspection, Deflated, Jan., 1921 - Dec., 1969 and Estimated Cyclical Trend

cyclical and trend movements will continue in the future.

The moving forces behind the cattle price cycle are the supply-demand equilibrium seeking process of the cattle industry and the biological, behavior and physical lags inherent in the production of cattle at the farm level. These factors merit a short explanation.

Equilibrium is simply defined as the equating of supply and demand. It is rarely totally obtained but is important because of process by which equilibrium is sought. With perfect mobility of resources, once the equilibrium is disturbed both the quantity supplied and quantity demanded would instantly converge to a new equilibrium. However, beef production cannot change instantaneously, hence price rises or declines must continue for sometime before beef supply is adjusted. This often leads to exaggerated changes in price and quantity and overshoting or undershoting of the equilibrium price and quantity. Hence, a second and perhaps even third series of adjustments may be required to reach equilibrium.

The quantity of beef demanded is determined by beef prices, consumers' incomes, price of other products (substitutes and compliments) and consumers' tastes and preferences, etc. It is not the purpose of this thesis to examine the nature of the demand for beef. An understanding of the nature of the demand for beef is necessary to substantiate the logic of the model and data used in this thesis. For a more detailed discussion of beef demand, the reader is referred to unpublished thesis written by Hamilton (2) on the demand aspects. Hamilton's thesis is a part of the same four-year research project from which this thesis also stems. This thesis is largely devoted to the supply side of this research project.

The phenomenon of the cattle cycle rests largely on the supply side of the cattle industry. More specifically it is hypothesized to depend upon the management decision methods of the cow-calf producer. As cattle prices change, the cow-calf producer's response is initially slow (maybe 2 or 3 months) until he becomes confident of the price change. He then adjusts his production strategy depending upon the type of price change--holding back more heifers and/or keeping brood cows to older ages for an increase in price with opposite changes for price decreases. As all producers make similar production changes, the cattle supply is changed in such a way so that there is reinforcement of the present price change in the short run and reversal of the present price pattern in the long run (3, 4 or 5 years). This reverse price movement does not affect the short-run because of lags inherent in cattle production that were mentioned earlier.

As management decisions are made to increase production, heifers are held off the market and the supply of cattle is reduced. Increased production will occur only after these heifers are bred, produce a calf and their calves are raised to a marketable size. When these calves reach a marketable age, supply is increased sharply because of similar action by all producers. Although some high profit are made by producers in the short run, cattle numbers eventually become oversupplied and price begins to drop.

At this point, producers should cut back production but many do not do so because (1) they feel price will soon rise again and (2) they have large amounts of capital tied up in the cows, land and productive facilities. As the price drops even more, producers are eventually forced to sell off part of their herd or liquidate all together. At this point although the number of cattle on farms may be decreasing, the total supply appears to increase because of the sell off of large herds built during favorable prices. This increase in supply pushes prices even lower and these prices will not rise again until excess cattle supply is liquidated and consumed. Once the excess supply is gone what is left is production from the cattle on farms at present and since the cattle on farms is decreased (and probably still declining), the supply of cattle is low. This leads to rising prices and repetition of the same process.

From the above, two very important points should be made. One is that price cycle exist in a large part because there is a cattle numbers cycle and vice versa. This numbers cycle is very evident from Figure 3 where total annual cattle numbers are plotted over time. It is significant to compare Figures 1 and 3 and to note the fairly regular offsetting movements of these two cycles. The numbers cycle does appear to lag the price cycle slightly.

A second point is the importance of individual producers culling, replacement, and herd sizes policies. These three management decisions taken collectively for all cattle producers determine supply and influence greatly the cyclical variation seen in both cattle numbers and cattle prices.

The Problem

Historically, producers have found themselves having to operate under conditions of "booms and busts." These conditions of "booms and busts" necessitate that short-run production adjustments be made on a rather regular basis. Producers, though, have found that when they

Figure 3. January 1 Inventory of Total Cattle Numbers, 1950-1978

make short-run adjustment in production (culling and replacement rates and herd sizes), that undesired long-run consequences are often the result (because of the long-run lag inherent in cattle production). For the producer, this means uncertainty as to what decisions to make under such conditions.

Not only have variable prices been a major problem to producers, in recent years they have also seen significant rises in the costs of raising cattle. This fact is strikingly clear when the composite variable cost and composite fixed cost indices are examined over time as in Figure 4. (An example of how these indices are derived is provided in Chapter III.) On a relative basis the variable cost of producing cattle has more than doubled between 1969 and 1977, while fixed cost has quadrupled in the same time period.

Under these conditions of rapidly rising costs, the producer finds his management decisions concerning short-run adjustments in production even more difficult and is even more uncertain as to what the results of these decisions will be. Although many producers recognize these "boom and bust" situations and rising costs, they have no effective decision making criteria for such conditions. Neither has the large amount of research devoted to these problems provided any such criteria. What producers need now are better management rules as to what to do under conditions of variable prices and rapidly rising costs.

Objectives

The overall objective of this study is to provide producers with management decision criteria which under conditions of uncertainty due

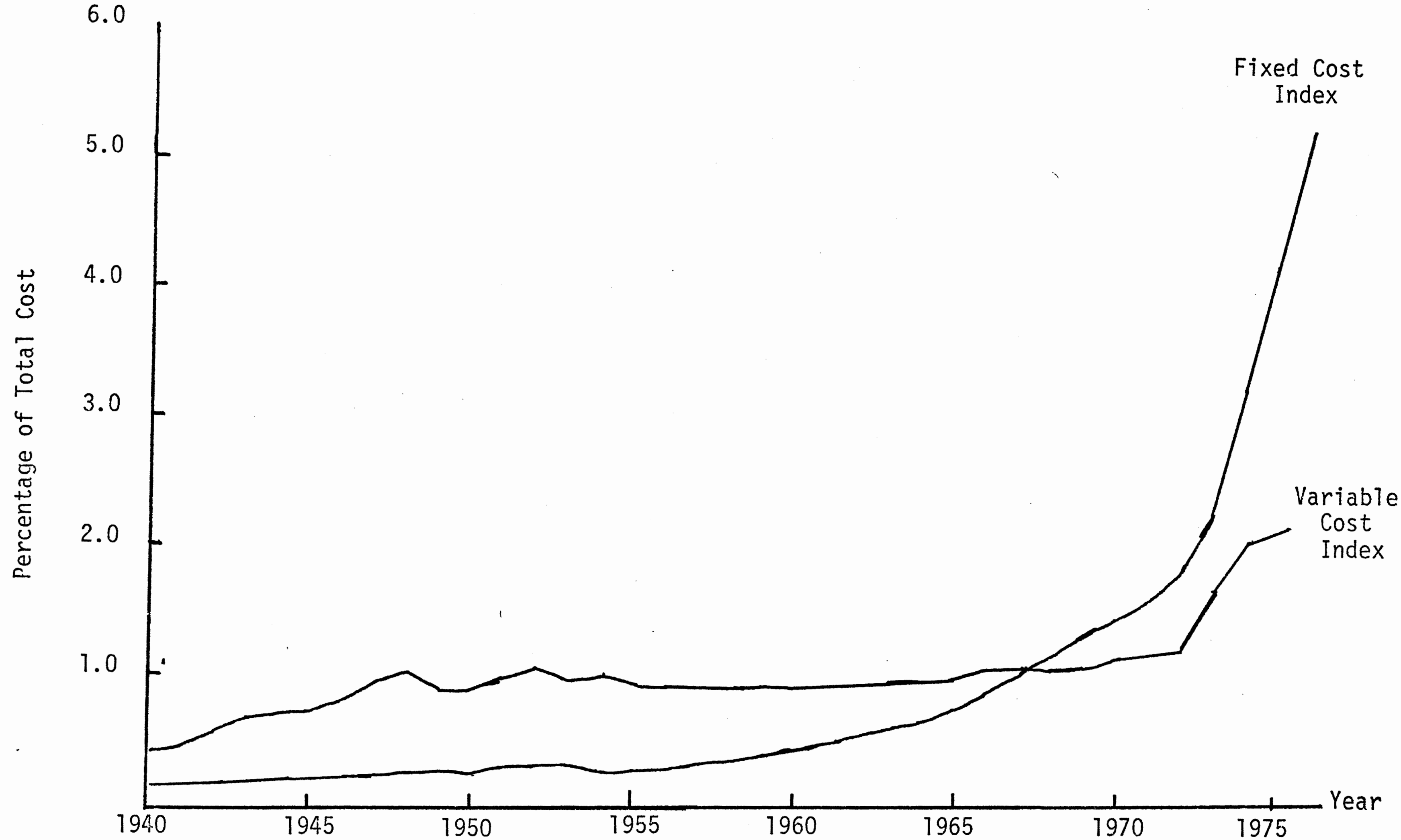

Figure 4. Composite Variable Cost Indices and Composite Fixed Cost Indices, 1940-1977

to variable prices and rising cost, will maximize the producer's net revenue over time.

Specific sub-objectives are:

(1) To develop a conceptual system model of a typical cow-calf operation,

(2) To estimate the required parameter of the systems model and to validate the model,

(3) To evaluate the impact of cycles (prices and numbers) and rising cost on producers,

(4) To evaluate the profitability of certain individual producer strategies for a typical cow-calf operation.

Procedures

The "systems approach" will be used as the basic tool to achieve the overall objective. It is used because of its ability to include all factors important in reaching a "good solution" to a given problem. Implimentation of the "systems approach" will be achieved by using a variety of appropriate quantitative tools and in light of knowledge gained from an extensive review of previous studies on this topic. This systems model of a representative cow-calf operation will then be combined with a systems optimizing routine to determine what adjustment strategies will optimize the producer's return over time.

A Summary of Specific Procedures:

1) Based on existing knowledge (literature review) a conceptual model of a typical cow-calf operation will be formulated and implemented into a computerized model.

2) Historical price data for feeder calves, cull cows, yearling heifers, and livestock production costs will be assembled. Econometric methods will then be used to estimate the cyclically and/or trendular pattern in these data. The estimated relations will then be used as input into the model to describe "typical" beef market conditions over time.

3 & 4) The computerized model of the cow-calf system will be run using estimated typical cyclical price series, actual historic prices and/or other scenarios of prices. Producer's incomes and risks will be evaluated with the model under each of these price conditions and in conjunction with the use of alternative management strategies. System optimization techniques will be used in an effort to determine "optimal" management strategies under alternative expected price conditions and firm conditions.

Literature Review

From the problem faced by producers and the current conditions under which they must operate, it seems worthwhile to review both the previous studies of the cattle cycle and the previous studies on optimal herd replacement policy. For this reason the literature review consists of two sections--one devoted to the cattle cycle and the other dealing with herd replacement policy.

The Cattle Cycle

The cyclical variation seen both in cattle numbers and cattle prices has been recognized in the literature as an integral part of the cattle industry. One of the earlier studies of the cattle cycle was done by R. J. Foote in 1953 (3) when he developed a four equation (one equation each for the price of corn, number of grain-consuming animal units fed, price of livestock and production of livestock) model to predict cycles. He did not solve these equations simultaneously but rather in sequence. He concluded that there was an inner mechanism which seemed to generate series of prices and quantities itself in the absence of other factors such as weather. Of his model, Foote said that it would always have more cyclical fluctuations than the actual, and that these fluctuations would tend to increase in amplitude and ultimately explode but these explosive tendencies would manifest themselves slowly.

Two years later in 1955, Breimyer (4) developed arguments leading to similar conclusions. Breimyer's work proposes that there is no simple price-supply relationship to explain the cattle cycle as there is to explain the hog cycle (meaning the hog-corn price ration). Because of this Breimyer felt that most analysts have turned toward empirical manifestation to explain the cattle cycle. In his work, Breimyer also discussed the two schools of thought concerning the cattle cycles. One school sees the cattle cycle as self-generating and the other feels that it is caused by outside influences. Breimyer, himself, felt that the cycle was self-generating but that outside factors do affect the cycle. His model, which involved the estimation of four relationships, (inventory numbers, slaughter numbers, price ratios, and price levels) tried to incorporate some of these factors and showed the need for more work on predicting the effect of outside influences. He finally proposed that a change in the number of cattle on farms is inaugurated by the holding not only of steers and heifers but all classes of cattle a longer or shorter period of time.

Later in 1958, Nerlove (5) advanced the idea that suitable dynamic models of consumer or producer behavior have been shown to lead to distributed lags. He incorporated distributed lags into a model predicting the cattle cycle and compared this model to models without distributed lags. The results showed that the dynamic distributed lag model explained the data better and had coefficients more reasonable in size and magnitude than the traditional static models.

In 1965, Walters (6) tried a different approach to predicting the cattle cycle. He predicted the cattle cycle by considering four separate classes of cattle and summing their predicted values. The four classes he used were:

1) Cows and heifers two years and older.
2) Heifers one to two years old and steers one year and older.
3) Calves.
4) Bulls one year and older.

Walters found that the four class approach worked very well but never exactly equaled the estimates under the balance sheet approach. He explained this difference as being due mostly to the fact that in the balance sheet approach, the inventory, supply and disappearance numbers are determined by independent survey.

Using a twelve relationship model in 1967, Langemeier and Thompson (7) tried not only to predict the cattle cycle but to predict supplies and demands and allow for simultaneity between supplies and demands. Their model was not the first to be solved simultaneously but was probably the best and most thorough at that time.

Langemeier and Thompson felt that to some extent supply had been overlooked and assumed to be predetermined by others who had attempted predicting the cattle cycle, and thus they were very much concerned with the simultaneous prediction of supply and demand. Also in their model, to isolate the income effect and minimize problems of multicollinearity, they expressed the demand and margins relationship on a per capita basis and supply relationship on a total basis. From their twelve relationship model, they concluded that the excess demand functions for beef are relatively stable, income elasticities for beef have not decreased overtime and beef supplies have not been noticeably effected by technology.

Crom (8) developed a recursive model in 1970 based on the cobweb theorem to estimate both the beef and pork industries. He divided his model into three components--fed beef, non-fed beef, and pork. Crom's model was a quarterly model and was estimated using data over the period 1955 to 1969. Whenever a substantial error occurred, Crom introduced operating rules to change the model to be more in line with observed values, thus the final model's deviations from observed values were relatively small.

Two years later in 1972, Crom (9) refined and improved this earlier model into a behavioral model to explain the cattle cycle. Crom felt that this behavioral dimension was necessary because as he stated, "Models which quantify the economic structure of a sector of the economy describe that economic structure which evolves from the institutional structure and human behavior of that period. Thus, any projection of such a model implies that the institutional structure and human behavior which existed in the past will be invariant in the future." To compensate for this assumption, Crom incorporated operating rules which change various behavioral parameters of the model as specified condition arise in the model's simulated future.

Crom's works demonstrated that there is still a great need for more work on the effects of human behavior on the cattle cycle.

In 1971, Franzmann (1) took a little different approach to the cattle cycle as he pointed out how stable the cycle was by plotting the average monthly cost per hundred pounds of cattle slaughter against trend values which had been adjusted for cyclical and seasonal fluctuations (Figure 2). Franzman shows the time period of each cycle to be relatively unchanged over the last fifty years. He also concludes from his study that there is no observed dampening in the cycle over time.

Franzman does admit that on occasions the observed values do substantially vary from the predicted values. But he feels there is even a stronger case for the existence of the constant cyclical pattern as these departures from the cycle have always dropped sharply back to resume the cyclical pattern. Franzmann feels that more work needs to be done to find why these departures happen and how they could possibly be explained by activities in the hog sector or feedlot activities.

Franzmann's model proves the consistency of the cycle and is a very good predictor of cycle turning points but is a poor predictor of price levels. However, his model might be utilized to form models which are better price level predictors.

The theory of price dependency was advanced by Heien (10) in his study in 1976. Heien looked at price on an annual basis versus a monthly or quarterly basis for almost all sectors of the agriculture industry. For his annual model, Heien found that he had causative specification problems which he solved by normalizing one equation on price. The idea underlying this price dependency or price normalization theory is that stocks and output are fairly constant on an annual basis and therefore, most of the variation is contained within the prices. For each agricultural sector Heien predicted prices by estimating retail demand as a function of farm output of that product and the ratio of retail price to processing cost. The procedure works very well on an annual basis and given a static equilibrium model.

On a monthly and quarterly basis, Heien found different results. Here there is a state of disequilibrium rather than equilibrium. Heien used the Phillips curve to explain price determination in the disequilibrium short-run model. By substituting the rate of change of stocks for the unemployment rate and using the rate of change of price for the rate of change of wages, Heien developed an excess demand-type model which worked better in the short-run.

More work on the behavioral aspect of the cycle was done in the early 1970's by Keith and Purcell (11). They fitted a model predicting quarterly per capita beef production for 1975 and 1976 and identified significant behavioral reactions involved in the beef cycle. They obtained an idea of probable behavior occurances in near future by sending survey questionnaires to Oklahoma cow-calf men. Their research showed that there was actually a behavior change in decision-making processes for the upswing of the cycle and also for the downswing of the cycle.

Keith and Purcell's work again pointed out the lack of uniform data on slaughter in predicting the cycle. But even with limited data, they concluded the following about seasonal cattle slaughter--seasonal slaughter, which is uptrending annually and therefore heaviest in the last half of the year, is amplified during years of cyclical price bottoms, thus shifting an even larger proportion of slaughter into the last half of the year. This makes slaughter an even more important variable in the predicting of the cycle when the cycle is in the low phase.

In summary, the studies reviewed do a very good job of analyzing certain aspects of the cycle and also add much to the pool of knowledge concerning this phenomenon. They have shown that the cycle is basically a "biological-lag-type phenomenon" that is influenced by many factors. A synthesis of these articles leads to the conclusion that three types of factors are critical to the explanation of the cattle cycle. First, delays are created between the decision to increase production and actual resulting production due to biological factors such as pregnancy period, growth periods etc.; second, the current condition of the cow-calf firm effect his ability to react to price changes, i.e., what is his current herd size, production capacity, cost of production etc.; and third, the economic behavioral response of farmers to price changes in the process of making decisions. All of the studies reviewed have considered economic theory in relation to one or more of these three types of factors. It is contended, however, that none of the studies reviewed has adequately separated each of these effects apart so each can be quantified and then applied simultaneously to determine the cyclical response under specific conditions.

It should be noted that random factors such as weather, unusual changes in feed price, will affect the cycle also, but there is no way of measuring the random components. So we must rely on the behavior decision patterns of farmers to reflect the end results of their effects.

What must be done now is to develop a model which incorporates and explains to the fullest extent possible all three components and their economic interaction that we call the cattle cycle.

Optimal Herd Replacement Theory

In recent years, much of the research conducted around the cattle cycle has centered on what is an optimal cow replacement policy for a typical cow-calf operation. This is a very logical approach since over 90 percent of U.S. beef supplies originate from U.S. cow-calf herds (historically we have imported less than 10 percent of our beef supply). The Theory of Optimal Herd Replacement is concerned with determining when to replace an asset (cattle in our case) with another newer asset so that the returns from a perpetual stream of assets is maximized, rather than the return to a single asset being maximized.

Faris (12) made one of the earlier studies of replacement decision in 1960 when he examined optimal replacement policy for assets of three types. These being assets having a:

I) Short production period with revenue being realized by the sale of the asset,

II) Long production period with revenue being realized by the sale of the asset,

III) Long production period with revenues being realized throughout the life of the asset.

Faris examined these three types of assets in relation to when to replace a stand of trees. For the first type, Faris concluded the present stand should be replaced when the marginal net revenue of the present stand (MNR_n) falls below the anticipated average net revenue possible (ANR_{n+1}) from the stand to immediately following the present one. The rule is stated in equation 1.

1) a. $MNR_n > \text{Max } ANR_{n+1}$, hold the present asset.

b. $MNR_n \leq \text{Max } ANR_{n+1}$, replace the present asset with asset to follow.

For the second and third types of assets rules similar to the first were used by Faris except because of uncertainty and time preference, the ANR_{n+1} is no longer valid. In the later cases he used the anticipated amortized present value of assets to immediately follow the present asset.

Faris also found that under the policy of maximizing perpetual returns (or revenues) that an asset would be replaced earlier if we are concerned with the present asset and future replacement assets returns than if only the returns from the present asset are considered. Further, Faris feels that fixed costs can be deleted from the cost calculations without changing the optimum replacement pattern.

Five years later Burt (13) examined optimum replacement under conditions of uncertainty due to risks. Using an infinite planning horizon, he divided replacement into planned and random replacements and assigned probabilities for both types of replacements and used the probabilities to compute expected value. He then added the expected present value of returns from the first asset held to the expected present value of returns from all future assets to get the expected value of the discounted net revenues from an infinite planning horizon. Next, he applied a marginal approach to determine an optimum replacement policy.

His conclusion was that the optimal replacement policy is to continue with currently held asset until the expected marginal net revenue minus expected marginal cost of planned replacement is less than weighted average net revenue from the potential replacement.

One year later Chisholm (14) in a review of articles on optimal replacement by Faris and others concluded that previous material about replacement pattern for long-lived asset have overlooked an important item of marginal cost: namely interest on the total revenue obtainable from the sale of the asset. Chisholm used Faris's forestry example, but included in his criteria for replacement, annual running cost, interest on the total revenue obtainable from the sale of the asset and the amortized value of the net returns from the following relation. Chisholm then maximized for a perpetual sequence of production periods and derived rules similar to previous rules (those rules stated in 1 above) on replacement with the exception of the inclusion of the opportunity cost.

In 1971, Rogers (15) examined five factors which he thought affected the replacement decisions of commercial beef herds. Of the five, four are age related. These are calving percentages, weaning weight of calves, sale value of cows, and death loss of cows. The remaining factor deals with cattle price movements.

Of the four age related factors, Rogers found that the first three each increased to certain age and declined through the rest of the cow's life. The years in which each of these factors peaked are; 5 to 6 years for calving percentage; 4 years for weaning weight; and maturity (at 2 years old) for the sales value of a cow (price per lb.). Rogers also concluded that the sales value of a cow is based on both her productivity and ultimate salvage value for meat up to 5 or 6 years of age and thereafter is related primarily to her marketability for slaughter. For death loss, he concluded that it held constant for the first 6 years at slightly over 2 percent, increased rapidly until the 12th year and then held constant between 6 and 7 percent per year.

Using these four age related factors and a variety of cattle price data scenarios, Rogers found optimum replacement policy to be an alternative year replacement scheme where in twice the normal replacements were retained every other year with the entire calf crop sold in intervening years. But because Rogers also felt that a uniform system would be more suitable to family consumption needs, stable income taxes, provide selectivity in choosing replacements, and be easier to manage, he concluded that a uniform system was more feasible. Further he found that a 15 percent uniform replacement policy yielded only 25¢ less net return, i.e., \$7.70 versus \$7.45 per animal per year than the optimal alternating years system. He concluded the uniform system would be the most logical one to use.

One year later Rogers (16) again used the same four age related factors mentioned above. This time he combined these factors with four different price levels and two different interest rates. He concluded that although more profit could be made at higher prices and less profit at lower prices that the level of prices did not change the optimum replacement policy of culling cows at 8 to 9 years of age. He did conclude though that higher interest rates would increase the time cows are kept before being replaced.

In that same year Perrin (17) developed replacement principles for both appreciating assets and depreciating assets while incorporating opportunity cost appropriate for the replacement decision. He developed these principles for both the continuous time case and discrete time case. The principles are basicly the same as earlier theories, but cover a more diversified group of costs and returns.

Perrin's conclusions that were different from or new to the body of replacement theory already known are:

1) If an asset is to be replaced with a technologically improved asset it will be replaced sooner than if it is not replaced with an improved asset.
2) A higher interest rate in some cases calls for earlier replacement age. The effects of changes in discount rate on replacement ages varied with different assets.

Perrin then pointed out the importance of choosing the appropriate discount rate and that the discount rate could be chosen on the basis of one of three different criterias. These being:

1) Cost of capital (perfect capital market),
2) Timing of personal consumption (destitute owner who values future earnings quite low relative to present earnings) and
3) Return on alternative investment (internal rate of return).

In 1972, Nelson and Purcell (18) in an article entitled "A Quantitative Approach to the Feedlot Replacement Decision" used the Compertz Curve to determine the growth function for cattle. They established cost relationship based on the net energy requirements of a certain level of maintenance and gain per day. They then established revenue relationships based on weights and grades. The growth function and cost and revenue relationships were used to develop a replacement model. Since a feedlot feeding period is normally on an annual basis, they expected a replacement criteria similar to Faris's (12) type one asset and the replacement model did provide an appropriate replacement criteria.

They did, however, cite three limitations in using their model and the Gompertz curve. First, it is desirable that birth weight and weaning weight at early age and other early observations on weight be included to provide a better sampling and more confidence in the estimated parameters. Secondly, observations on time and weight used to estimate the curve should be relatively large and spaced throughout the growth period. And finally, there should be no outstanding environmental factors that would affect the growth of the cattle in question.

The theory of optimal replacement was advanced even further in 1976 by Bentley, Waters and Shumway (19) when they adapted a linear programming model and used it to determine an optimal herd replacement criteria. Using expected values and factors relating age of the cow to optimal replacement policy, they found a solution specifying replacement at eight years of age. They then tested the solution's sensitivity to cattle prices. Their results were that the optimal replacement pattern of 7 to 8 years did not change in response to changes in either cattle prices or feed costs. However, a higher calving percentage caused a later replacement age and vice versa. They also found that as cull cow prices increased, the optimal replacement age decreased.

In 1977, Kay and Rister (20) researched the effects of taxes on replacement policy. Since they wanted to maximize returns after taxes they considered strategies under which replacement heifers are either purchased or self-produced. Investment credit and accelerated depreciation can be used when replacement heifers are purchased. When replacement heifers are internally produced their production expense can be deducted and long term capital gains claimed when they are sold.

Although their results, in general, favored self-production strategies, they found higher marginal tax rates favored the raise strategy even more while the discount rate had little effect on which strategy was used. The condition under which buy strategies were predominant is that of a restricted herd size due to limited resources, i.e., pasture.

The previous studies of optimal herd replacement have developed rules to determine replacements (e.g., 1) and have showed that modifications must be made for the specific situation under study. They have also demonstrated the need for the researcher to have a thorough knowledge of the problem or situation being studied. The previous studies have, however, not solved the whole issue of herd replacement policy but they have provided a very sound foundation for further studies in this area.

Specifically, previous studies of optimal beef cow replacement strategies have assumed a constant herd size is desired or required. If the assumption of a constant herd size is relaxed the problem of optimal replacement and culling becomes dynamic and more complex. In the light of cyclical prices in the beef industry it would appear that the assumption of constant herd size and the replacement and culling criteria it generates may not be an optimal criteria. Hence, development of a culling and replacement model which simultaneously considers culling and replacement decisions which do not necessarily generate a constant herd size appears to be needed.

CHAPTER II

CONCEPTUAL FRAMEWORK

Methodology

In order to analyze the problem posed in this research effort a conceptual abstract model of the "real world" conditions being studied was developed. Economic theory provides a set of guidelines for developing abstract representations of the "real world." In this study a computerized economic systems model of a cow-calf firm will be developed. Although the model will be an abstract simplification of the "real world," the economic and biological data it is based upon will be incorporated so as to convey a realistic description of those aspects of cow-calf production being analyzed.

Once the model is developed, it must first be verified and then simulated to obtain results. At this point, economic logic must be used again to interpret these results and evaluate their meaning in a real-world setting.

Previous studies have attempted to develop conceptually sound models that describes the cattle industry as it operates in real life. These studies have added significantly to the theory of optimal cow herd replacement. The present research, though, is intended to add even more to the existing pool of knowledge on cow herd replacement by doing two things. First, this research should provide a description of the changing cost structures associated with a typical cattle operation and the influence of alternative herd sizes upon average per head cost of production. Secondly, this research should enable the examination of the impact of varying herd sizes on returns, cost and culling and replacement decisions and vice versa.

The cow-calf simulation model developed for this study is actually two parallel models. One model is a physically oriented model describing the calf production process, aging of the cow etc. while the other model is an economically oriented model describing the cost and revenue flows, prices received, current expected net present value of a cow etc. The structure of the physical model is depicted in Figure 5 and will be discussed first.

The cow-calf model developed is essentially a population model which describes the nature of the cow herd population at any point in time based upon the birth rates, death rates, replacement rates and culling rates given. Cows are assumed to have a maximum productive life of 15 calving years as this is consistant with prior literature and are culled after their 15th calf regardless of previous performance. Each age group of cows has a specific set parameter associated with it which describes the calving rate, weight of the calf produced at weaning, average death rate of the cows and average weight of the cows. These parameters have been taken from previous studies and are presented in Chapter III. The assumption has been made that cows in each cohort which fail to calf, i.e. that average percent which fail to produce a calf, are culled from the herd as a standard managerment practice. It has also been assumed that replacements enter the herd only as heifers. The ability to buy mature cows as replacements has not been considered. Hence the number of cows in each cohort depicted in Figure 5 is deter-

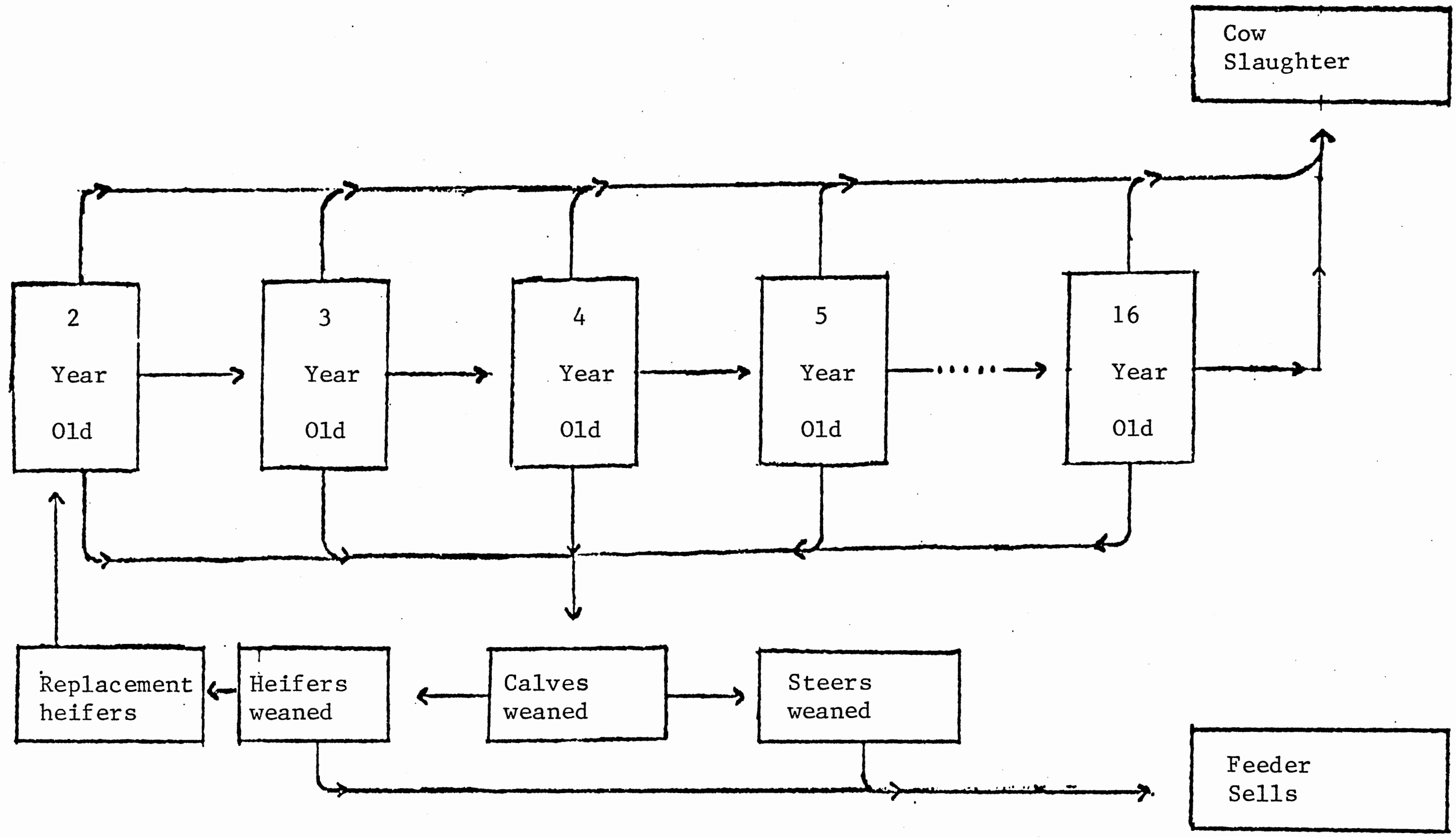

Figure 5. Model Representation of a Typical Beef Cow-Calf Herd

mined for each succeeding year by considering the number of cows currently in the cohort immediately proceeding it and then subtracting from it the number of cows which die or are culled due to failure to calf. Culling of cows from a given cohort above the standard rate due to the failure to produce is carried out when the culling strategy being studied calls for it. Hence during simulation of alternative culling strategies culling for performance failure and additional selected culling will occur. Culling is never permitted to decline below that due to failure to produce a calf.

Heifers held for replacement to the herd are assumed to be removed from the calf population just before the sale of the calves as feeder calves. These heifers are then held in a replacement pool for one year. At the end of this year they enter the cow herd as first calf cows. Expense of obtaining a replacement heifer in this manner is equal to the revenue foregone by selling her as a feeder plus the cost of maintaining her for one year until she is ready to enter the herd. Costing and pricing procedures will be explained further in Chapter III and in the discussion of the economic portion of the model.

The model described in Figure 5 can also be summarized in mathematical notation. Equations 1 through 5, and 11 through 13 and 31 through 36 describe the physical attributes of the herd as just discussed and presented in Figure 5. The remainder of the equations describe the economic attributes of the herd that are being modeled. To aid the reader to understand the model, the equations will be listed in groups, each group explained, and then the next group listed and explained.

(1) $BR(t,s) = f(s)$

(2) $DR(t,s) = f(s)$

(3) $SWW(t,s) - f(s)$

(4) $FWW(t,s) = f(s)$

(5) $CW(t,s) = f(s)$

(6) $CV(t,s) = f(CCP(t), SLP(t), s)$

(7) $FSP(t) = f(t)$

(8) $FHP(t) = f(t)$

(9) $CCP(t) = f(t)$

(10) $SLP(t) = f(t)$

where:

t = time subscript

s = cow age subscript

BR(t,s) = birth rate percentage for an s age cow

DR(t,s) = death rate percentage for an s age cow

SWW(t,s) = feeder steer calf weaning weight for an s year old cow

FWW(t,s) = feeder heifer calf weaning weight for an s year old cow

CW(t,s) = brood cow weight for s year old cow

CV(t,s) = brood cow value in year t and at age s

FSP(t) = feeder steer calf price in year t

FHP(t) = feeder heifer calf price in year t

CCP(t) = cull cow price in year t

SLP(t) = slaughter heifer price in year t

Equations 1 through 5 describe the physical change associated with each cow as she ages. These being the changes in weaning weight of the calf she produced, her own weight, the probability of her dying and the probability of her successfully raising a calf to weaning age. Equation 6 describes the cow's market value associated with her age and is dependent both upon her age and the average cull cow price for that year. Equations 7, 8, 9 and 10 are the price series used in the model, each varies varies cyclical with time. The nature of this variation will be discussed in Chapter III.

(11a) $A(t,s) = (1-BR(t,s))* HS(t,s)$

(11b) $A(t) = \sum_{i=1}^{15} A(t,i)$

(12a) $D(t,s) = DR(t,s)* HS(t,s)$

(12b) $D(t) = \sum_{i=1}^{15} D(t,i)$

(13a) $HS(t,s) = HS(t-1,s) - D(t,s) - A(t,s)$

(13b) $HS(t) = \sum_{i=1}^{15} HS(t-1,i) - D(t) - A(t)$

(14) $FSS(t,s) = FSP(t)* SWW(t,s)$

(15) $FHS(t,s) = FHP(t)* FWW(t,s)$

(16) $TCS(t,s) = (FSS(t,s) + FHS(t,s))/2.0$

(17) $CULLS(t,s) = A(t,s)* CW(t,s)* CV(t,s)$

(18) $I(t,s) = TCS(t,s) + CULLS(t,s)$

(19) $FC(t) = {}^{K}/_{HS}(t)$, where k = constant

where:

A(t,s) = number of s age cows culled because of failure to give birth to a calf in year t

A(t) = total number of cows culled due to calf failure in year t

D(t,s) = number of s age cows that died in year t

D(t) = total number of cows that died in year t

HS(t,s) = number of s age cows living in year t

HS(t) = herd size in year t

FSS(t,s) = market value of weaned steer calf of an s age cow in year t

FSH(t,s) = market value of weaned heifer calf of an s age cow in year t

TCS(t,s) = average market value of weaned feeder calf of age s cow in year t

CULLS(t,s) = value of cull sales of cow age s in year t

I(t,s) = revenue received from sales of culls and calves of cows of age group s in year t

FC(t) = fixed cost in year t.

Equation 11 calculates the number of cows of each age group that will be culled due to calving failure in a given year. The total number of cows lost through this process each year is dependent upon the age of the cows in the herd, since calving success is a function of age, birth rate percentages and the current herd size. Given the death rate distribution, age structure of the herd and herd size, the total number of cows of each age that die are computed by equation 12. Equation 13 computes the current herd size as a function of last year's herd size less the deaths and unsatisfactory performance culls. Equation 14 and 15 are identities used to compute the market value of male and female calf sales, respectively. Equation 16 uses the calf sales of 14 and 15 to compute an average value of calf sales for both male and female calves. This represents the average revenue received of each cow of each group from the sale of her calf. The revenue generated from unsatisfactory performance culling is computed in Equation 17. It is important to note that for each age group, the number of cows culled is multiplied by the appropriate cow weight for a cow of that age group and the appropriate cow value per pound given the cow's age and the year she is sold. Equation 18 sums equations 16 and 17 to arrive at the average total revenue accruable to each cow of a certain age group. Revenue generated by each age group consist of revenue from calf sales as reflected by Equation 16 and revenue from the sale of cull cows as reflected by Equation 17. These two sources of revenue divided amongst the number of cows in the group provide the average revenue per cow reflected by Equation 18. Equation 19 shows the total fixed cost of the herd to be exogenous and a function of time. Further discussion of trends in cost of production will be presented in Chapter III.

(20) $VC(t) = f(HS(t),t)$

(21) $C(t,s) = (FC(t)tVS(t))* (HS(t,s) + .75A(t,s) + .5D(t,s))$

(22) $RCS(t,s) = TCS(t,s) - C(t,s)$

(23) $RCF(t,s) = I(t,s) - C(t,s)$

(24) $SR(t,s) = CW(t,s)*CV(t,s)*HS(t,s)$

(25) $NR(t,s) = RCF(t,s) + SR(t,s)$

(26) $ANPV_{CF}(t,s,r) = \sum_{i=s}^{n} RCS(t,s)/(1+c)^{(i-s)}$, where $n=r+s \leq 15$

(27) $ANPV_{TF}(t,s,r) = \sum_{L=S}^{n} RCF(t,s)/(1+c)^{i-s}$, where $n=r+s \leq 15$

(28) $ANPV_{SF} = \sum_{i=s}^{n} NR(t,s)/(1+c)^{(i-s)}$, where $n=r+s \leq 15$

(29) $N(t) = f(t,ANPV_{TF}^{(t,s,r)}, (V(t,s), CV(t,s))$

(30) $M(t) = f(t,ANPV_{TF}^{(t,o,r)}, FHP(t), \frac{\sum_{1=1}^{15} FWW(t,I)}{15}$

where:

o = age subscript for replacement heifers not yet added to herd

r = time counter representing the additional years the cow will be retained as part of the herd

c = discount rate

n = r + s and is the maximum number of years of useful life any cow can produce

VC(t) = variable cost in year t

C(t,s) = total cost of s age cow in year t

RCF(t,s) = net revenue received from the sales of culls and calves of cows of age group s in year t

RCS(t,s) = net revenue received from the sales of calves of cows of age group s in year t

$ANPV_{CF}^{(t,s,r)}$ = discounted cash calf flows in year t of s age cow that will be kept r years

$ANPV_{TF}^{(t,s,r)}$ = discounted total cash flows (culls and calves) in year t of s age cow that will be kept r years

SR(t,s) = revenue received when cow is culled under planned program in year t at age s

NR(t,s) = net revenue from sales if the cow is also sold in year t

$ANPV_{SF}$ = discounted net revenue from sales when cow is sold under planned culling policy

M = Replacement rate in year t

N = Cull rate in year t

Equation 20 shows variable cost to be both a function of herd size and time. The effect of herd size upon variable cost will also be discussed in Chapter III. Briefly stated, changing herd size is specified to cause movement along a u-shaped cost curve, while progression through time causes the curve to shift upward as cost increases.

Equation 21 calculates total cost as the sum of total fixed cost and total variable cost. Total variable cost is determined by multiplying the average variable cost per head times the average herd size for the year. The average herd size for the year is calculated as the beginning herd size minus 25 percent of those cows culled for failure to produce a calf (this assumes the cow is in the herd three-fourths of a year before she loses her calf and is then partially fattened for sale) and minus 50 percent of those cows who die during the year, i.e. this assumes deaths are distributed uniformly through the year. Equation 22 is an identity stating that the net return to cows of certain age just from the sale of their calves is simply a matter of subtracting the cost of production from gross receipts for the sale of calves.

Equation 23 is another measure of net revenue. It includes revenue from both the sale of calves and revenue obtained from the sale of cows which fail to produce a calf and are culled. The cost subtracted is the same as in Equation 22. Equation 24 calculates the current market value of the herd. This value is dependent upon the current price level and the size and age distribution of the herd, i.e. cull cow price is a function of cow age as well as current prices. Equation 25 is a third and final measure of net revenue. It includes revenue from calf sales and the sale of culls due to failure to calf but also includes revenue that could be obtained by liquidating the herd. The cost subtracted is the same as described in Equation 21 and used in the other two net revenue equations.

Equations 26, 27 and 28 each compute the amortized present value of the revenue flows from a cow of age s, that has r years of useful life left in the herd. Equations 26, 27 and 28 are simply the amortized net present value of the stream of net revenues that would be generated overtime as calculated from Equations 22, 23 and 25, respectively.

Equations 29 and 30 indicate that culling and replacement rates are hypothesized to be a function of the net present value of cows of various ages, the net present value of feeder heifers if placed in the herd, the current value of feeder heifers if sold as feeders and the current value of cows if sold as slaughter cows. The market value of any cow in the herd equals CV(t,s) * CW(t,s) while the market value of a typical replacement heifer is calculated as $FHP_t * (\sum_{i=1}^{n} FWN(t,i)/15)$.

(31) COWS(t,s) = L, L is a constant, or L may vary with age of the cow

(32) $CALVES(t) = \sum_{i=1}^{n} BR(t,i)*cows(t,i)$

(33) SCALF(t) = CALVES(t)/2.0

(34) HCALF(t) = CALVES(t)/2.0 - REPH(t)

(35) TMCS(t) = SCALF(t)*FSP(t)

(36) TFCS(t) = HCALF(t)*FHP(t)

(37) TCS(t) = TMCC(t) + TFCC(t)

(38) INC(t,s) = (1-BR(t,s))* COWS(s)

(39) $TINC(t) = \sum_{i=1}^{n} INC(t,i)$

(40) REPH(t) = M*TCOWS, M is the replacement rate

(41) PCULL(t) = N * TCOWS, N is the culling rate

(42) DIF(t) = PCULL(t) - TINC(t)

(43a) TCULL(t) = TINC(t) + DIF(t), $DIF(t) \geq 0$

(43b) TCULL(t) = TINC(t), $DIF(t) < 0$

(44) TCOWS(t) = TCOWS(t-1) + REPH(t-1) - TCULL(t)

where:

COWS(t,s) = number of cows of each age group s

CALVES(t) = total number of calves born in year t

SCALF(t) = steer calves available for sale in year t

HCALF(t) = heifer calves available for sale in year t

TMCS(t) = gross receipts from steer calves sold in year t

TFCS(t) = gross receipts from heifer calves sold in year t

TCS(t) = gross receipts from all calves' sales

INC(t,s) = involuntary culls in year t of cows age s

TINC(t) = total involuntary culls for entire herd in year t

TCOWS(t) = herd size in year t

REPH(t) = replacement heifers held back in year t

PCULL(t) = preplanned number of culls in year t

DIF(t) = difference between planned and involuntary culls in year t

TCULL(t) = total number of culls in year t

Equation 31 establishes the beginning number of cows for each age group. Each age group is assigned a constant number in the beginning of each simulation and then allowed to vary during the simulation run. Equation 32 is an identify stating that the total number of calves born is equal to the sum of the calves born of each age group. Equations 33 and 34 divide the calf crop into female and male calves. The replacements are held back from the female calf pool in Equation 34. Equations 35, 36 and 37 compute the revenue generated from the sale of steer calf, heifer calf, and total calf sales, respectively. Equation 38 calculates the number of cows culled (unplanned culls) from each age group because of failure to bear or raise a calf to marketable age. Equation 39 sums the culls calculated in Equation 38. Equations 40 and 41 calculate the number of replacements and planned culls (those cows culled by a predetermined replacement policy), respectively. The replacement rate and culling rate required to obtain the desired number of culls and replacements, given the herd size, is defined in Equations 40 and 41 also as M and N respectively.

The percentages represented by M and N, may be constant, cyclical, or a function of the cow's net present values. Equation 42 finds the difference between planned and unplanned culls in any year. Equations 43a and 43b define the total number of culls in any year as being equal to involuntary culls if involuntary culls are greater than planned culls or equal to planned culls if the reverse is true. Equation 44 is an accounting rule which computes the number of total cows or herd size. It states the herd size is a function of last year's herd size, last year's replacement stock and the current year's culling number. Equation 44 is stated as a function because the herd is culled in the following manner:

(1) All involuntary culls are culled from the appropriate age group,

(2) If involuntary culls exceed planned culls, the culling process stopped,

(3a) Otherwise, the cows are culled from the oldest age groups until the desired number of cows are culled, or

(3b) Alternatively, the cows are culled on the basis of which are the least profitable.

$$(45)\quad TICULL(t) = \sum_{i=1}^{15} INC(t,i)*CW(t,i)*CV(t,i)$$

(46) $TCULLS(t) = TICULL(t) + DIF(t) * CCP(T)*CW(t,p)$:
$DIF(t) \geq 0.$

(47) $TGR(t) = TCS(t) + TCULLS(t)$

(48) $FC(t) = k/TCOWS(t)$ where k is exogenous and constant

(49) $VC(t) = f(TCOWS(t), t)$

(50) $C(t) = (FC(t) + VC(t))* TCOWS$

(51) $NR(t) = TGR(t) - C(t)$

(52) $CNR = \sum_{i=1}^{J} NR(i)$, where J is length of planning horizon

(53) $FCNET(t) = REPH * FHP(t)$

(54) $SCNFT(t) = \sum_{i=1}^{15} COWS(i) * CW(t,s) *CCP(t)$

(55) $TCNR = CNR + FCNET + SCNET$

(56) The FCNET, CNR, SCNET have been discounted appropriately

where:

p = subscript of those cohorts to be culled

TICULL(t) = value of involuntary culls in year t

TCULLS(t) = market value culled cows in year t

TGR(t) = gross revenue in year t

C(t) = total cost in year t

NR(t) = net revenue in year t

CNR = cummulative net revenue over time

TCNR = cummulative net revenue over time including ending stock value

FCNET(t) = market value of the replacement stock in year t

SCNET(t) = market value of the brood cows on hand in year t

TCNET(t) = total value of cows on hand and sales in year t

Equation 45 shows total involuntary cull cow sales to be the sum of the number of involuntary culls of each age group times the appropriate cow weight and cow price for the appropriate age group. Equation 46 adds the market value of the additional cows culled if planned culling exceeds involuntary culls. This equation also used the weight and price of whichever age group is appropriate. Total gross revenues is just the sum of the calf sales and culled cow sales as stated in Equation 47. Equation 48 shows fixed cost per head as the result of an exogenous constant total fixed cost divided by the herd size. Equation 49 defines variable cost to be a function of herd size and time. The herd size parameter allows movement along any one cost curve, and enables shifts from one cost curve to another over time. Equation 50 figures total cost as the composite of average fixed and variable cost times herd size. Equation 51 is an identity showing net revenue in year t to be the difference between total gross revenue and total cost. Equation 52 specifies cumulative net revenue as the summation of the annual net revenues. Equations 53, 54 and 55 assign a slaughter value to each of the cows in the herd and adds these values to the net return generated by the herd to determine a total current net revenue realizeable.

CHAPTER III

PROGRAMMING AND DATA COLLECTION

In Chapters I and II, the nature of the cattle industry and the abstract conceptual framework to be used to analyze the culling and replacement decisions faced by cow-calf herd managers were discussed. In this chapter a detailed discussion and description of the computerized model developed to provide an abstract representaion of the cow-calf system being studied will be presented. A thorough understanding of the model is essential to understanding and interpreting the results to be presented in Chapter IV. The discussion of the model will be divided into three parts; input requirements, the structure of the model through which the inputs are processed; and the types of output information generated by the model.

Input

Input required by the computer model can be classified into three broad groups of data series. The first group consists of two cost indices, a composite variable cost index and a composite fixed cost index. The second data series group contains four cattle price series. The last group is formed by six sets of age related physical characteristics describing the physical changes in cows and their calves as the cows age. Table I presents the fixed and variable cost indices (1967-100) and cattle price series used. Table II reports the age

TABLE I

COST INDICES AND PRICE SERIES

	Group I - Indices				Group II - Prices		
Year	Variable Cost[1]	Fixed Cost[1]	Cull Cattle[2]	Slaughter Steers[3]	Slaughter Heifers[4]	Feeder Steers[5]	Feeder Heifers[6]
1940	.4039	.1293	.0465	.1043		.1020	
1941	.4391	.1346	.0568	.1133		.1195	
1942	.5265	.1483	.0722	.1379		.1325	
1943	.6187	.1602	.0818	.1530		.1427	
1944	.6841	.1684	.0732	.1544		.1305	
1945	.6808	.1712	.0830	.1618		.1394	
1946	.7846	.1859	.0912	.1916		.1611	
1947	.9171	.2175	.1148	.2583		.2078	
1948	.9747	.2400	.1615	.3088		.2721	
1949	.8385	.2382	.1395	.2580		.2481	
1950	.8475	.2471	.1648	.2788		.2999	
1951	.9519	.2837	.1986	.3418		.3786	
1952	1.0083	.3013	.1522	.3102		.3158	
1953	.9266	.2990	.1019	.2191		.2055	
1954	.9419	.2491	.0916	.2267		.2021	
1955	.8933	.2630	.0927	.2296		.2104	
1956	.8854	.2878	.0930	.2193		.1957	
1957	.8731	.3185	.1150	.2316		.2336	
1958	.8726	.3466	.1588	.2807	.2545	.3168	.2848
1959	.8789	.3897	.1547	.2767	.2549	.3265	.2958
1960	.8688	.4263	.1408	.2590	.2384	.2788	.2459
1961	.8823	.4631	.1474	.2443	.2301	.2777	.2484
1962	.8949	.5104	.1423	.2692	.2509	.2769	.2422
1963	.9298	.5728	.1372	.2358	.2243	.2702	.2383
1964	.9290	.6506	.1266	.2241	.2137	.2267	.1988
1965	.9481	.7456	.1360	.2499	.2346	.2370	.2009
1966	.9938	.8756	.1693	.2571	.2440	.2838	.2425

TABLE I (Continued)

	Group I - Indices		Group II - Prices				
Year	Variable Cost[1]	Fixed Cost[1]	Cull Cattle[2]	Slaughter Steers[3]	Slaughter Heifers[4]	Feeder Steers[5]	Feeder Heifers[6]
1967	1.0000	1.0000	.1666	.2529	.2425	.2800	.2408
1968	.9756	1.1564	.1711	.2687	.2558	.2910	.2506
1969	1.0149	1.3315	.1917	.2945	.2796	.3289	.2864
1970	1.0722	1.4628	.2036	.2936	.2825	.3673	.3163
1971	1.1223	1.5867	.2040	.3239	.3121	.3684	.3213
1972	1.1639	1.8049	.2378	.3578	.3428	.4654	.3866
1973	1.6203	2.2717	.3111	.4454	.4243	.5973	.4961
1974	1.9817	3.2295	.2390	.4189	.4052	.3923	.3359
1975	2.0569	4.1859	.1843	.4461	.4036	.2948	.2348
1976	2.1736	5.1994	.2306	.3911	.3604	.3882	.3100
1977	2.2419	6.7392	.2378	.4038	.3708	.4141	.3364

[1]Original data from Indices of Prices Paid, Ag. Prices.

[2]Cow Prices: Canner and Cutter grade Slaughter cows, Chicago, Livestock and Meat Statistics.

[3]Slaughter Steer Prices: Good grades, Chicago, Livestock and Meat Statistics.

[4]Slaughter Heifer Prices: Good grades, Chicago, Livestock and Meat Statistics.

[5]Feeder Calf Prices: Good and Choice Steer Calves, Kansas City, Livestock and Meat Statistics.

[6]Feeder Calf Prices: Good and Choice Heifer Calves, Kansas City, Livestock and Meat Statistics.

TABLE II

AGE RELATED PHYSICAL CHANGES IN COWS' PRODUCTIVITY

Cow' Age	Calving Year	Average[1] Weaning Weight	Birth[2] Rate %	Death[2] Rate %	Cow[3] Weight	Steer[4] Weaning Weight	Heifer[4] Weaning Weight	Δ in Cow[5] Slaughter Price
2	1	425	85.5	2.25	821	439.45	410.55	.787
3	2	444	89.0	2.25	905	459.096	428.904	.817
4	3	465	92.7	2.30	986	480.81	449.19	.847
5	4	488	94.5	2.35	1041	504.592	471.408	.877
6	5	488	94.3	2.45	1100	504.592	471.408	.907
7	6	488	93.0	2.8	1100	504.592	471.408	.937
8	7	488	90.8	3.25	1100	504.592	471.408	.967
9	8	488	87.0	3.7	1100	504.592	471.408	.997
10	9	488	82.0	4.35	1100	504.592	471.408	1.0
11	10	465	76.6	5.8	1100	480.81	449.19	1.0
12	11	465	70.0	6.3	1075	480.81	449.19	1.0
13	12'	465	63.6	6.5	1050*	480.81	449.19	1.0
14	13	465	56.2*	6.6*	1025*	480.81	449.19	1.0
15	14	465*	45.0*	6.6*	1000*	480.81*	449.19*	1.0
16	15	465*	41.0*	6.6*	1000*	480.81*	449.19*	1.0

*Values for these characteristics could not be found in literature. Estimated values were provided based upon extrapolation of the preceeding series and the author's knowledge of the cattle industry.

[1]Earnest, Waters, and Shumway.

[2]Rogers, 1971.

[3]Kay and Rister.

[4]Average Weaning Weight from Earnest, Shumway and Waters, and percent breakdown heifer and steer weaning from Rogers, 1971.

[5]Rogers, 1971 and extrapolations by author.

dependent physical characteristics of cows that are used.

The Cost Indices

There are no reported series of either a composite variable or a composite fixed cost index for cow-calf firms. However, the USDA does collect and publish indices of prices paid by farmers for most resources used by cow-calf producing firms. Therefore, indices of individual items which were representative of resources used by cow-calf firms were first collected as a starting point to compute composite fixed and variable cost indices. The indices collected included ones for feed, fuel and energy, farm and motor supplies, autos and trucks, all items used for production, interest rates and wages. These indices were collected for the years 1940-1977 from the publication "Agricultural Prices." A land prices index was also found for the same time period in the "Agricultural Finance Databook." All indices were converted to a 1967 base period. The indices were then weighted each year according to the cost of production figures reported in Table III to form composite fixed and variable cost indices. Cost of production figures reported in Table III were obtained from a 1977 Oklahoma State University Extension budget for a 100 head cow-calf operation in Northeastern Oklahoma with spring calving and on dry-grass pasture. After the two cost index series were computed, linear and trigometric time series functions were fitted against the series. It was found that little harmonic type fluctuation existed in the series. Thus, it was elected to use linear time trend functions in the model where extrapolation of the data beyond the sample period was required or general patterns of changes in the cost of production were desired rather than specific

TABLE III

THE COMPOSITE VARIABLE COST AND COMPOSITE FIXED COST INDICES

Variable Cost Index for 1977 (1967 based)			
Item	Dollar Amount	Individual Item Percentage of Total Cost	Corresponding USDA Production Component Index
Feed	78.36	66.52	Feed
Fuel	2.52	2.14	Fuel and Energy
Repairs and Supplies	8.58	7.28	Farm and Motor Supply
Other (Vet, Hauling, Taxes)	12.34	10.48	Items of Production
Operating Capital	8.78	7.45	(Items of Prod.) times Interest Rates
Labor (¼ of total labor used)	7.22	6.13	Wages
Total Cost	117.80	100.00	Composite Variable Cost

Fixed Cost Index for 1977 (1967 based)			
Item	Dollar Amount	Individual Item Percentage of Total Cost	Corresponding USDA Production Component Index
Machinery Investment & Ownership	2.96	2.77	(Autos & Trucks) times Interest Rates
Equipment Investment & Ownership	11.93	11.16	(Farms and Motor Supplies) times Interest Rates
Labor (3/4 of Total Labor Used)	21.65	20.25	Wages
Land (Rental Rate per Cow-Calf Unit)	70.36	65.82	(Land Prices) time Interest Rates
Total Cost	106.90	100.00	Composite Fixed Cost

actual estimates. The linear equations estimated from the fixed and variable cost series and their associated statistical properties are reported in Table IV and plotted in Figure 6.

The cost indices computed and the linear time trend functions fitted to them describe the change in average per head fixed and variable cost over time. However, they do not explain changes in average per head cost as the herd size is changed. These changes in per head cost of production are tied to the shape of the average variable cost curve (AVC), and the number of animals total fixed cost is being spread over.

The Cost of Production Curves

For any one cattle producer, there is a given herd size at which the cost of production for each animal is at a minimum (Figure 7). At this point either increasing the herd size or decreasing it will cause the cost per head to rise. Costs rise when the herd size is increased beyond the minimum point because of the additional need for nutrients, hay and machinery and equipment, fencing and truck usage (Figure 8). At the optimal or minimum cost herd size, machinery and equipment are being used to full capacity and an increase in herd size requires the addition of more of both inputs. As herd size are decreased, machinery and equipment are not used to full capacity and therefore the producer pays for the inputs while not using it fully. The same is true of his pasture. When the herd size is reduced there are fewer cows than the land will support, but the same costs are still associated with the land (assuming land is a fixed resource). The point should be made that he can substitute this intense grazing for

TABLE IV

THE FITTED EQUATIONS OF THE COST INDICES AND PRICES SERIES

Variable Cost Equation

$VC(t) = 0.4260 + 0.03055303 * t$ 1940-1977

$T'_s = (4.76572960), (7,65734569)$ $R^2 = .618975$ $F = 58.482$

$N = 38$ $k = 4$

Fixed Cost Equation

$FC(t) = -.8869 + .09810347 * t$ 1940-1977

$T'_s = (-2.62), (6.49)$ $R^2 = .539248$ $F = 42.133$

$N = 38$ $k = 4$

Prices Equations

CULL Cow Price

$CULLP(t) = 0.0703 + .00400650 * t - .02498561 * TSIN + .01074626 * TCOS$

$T'_s = (7.15), (9.07), (-3.68), (1.58)$ 1940-1977

$R^2 = .774111$ $F = 38.839$

$N = 38$ $k = 4$

Slaughter Steer Price

$SLP2(t) = 0.1590 + .00546352 * t - .03405209 * TSIN + .02662572 * TCOS$

$T'_s = (10.63), (8.13), (-3.29), (2.57),$ 1940-1977

$R^2 = .747710$ $F = 33.588$

$N = 38$ $k = 4$

Slaughter Heifer Price

$SLP(t) = -.1027 + .00760805 * t - .00012894 * t^2 + .92459153 * SLP2(t)$

$T'_s = (-2.29), (2.61), (-2.26), (17.63)$

$-.00352453 * TSIN - .00113870 * TCØS$

$T'_s = (-1.49), (-.62)$ (1958-1977

$R^2 = .995626$ $F = 637.279$

$N = 20$ $k = 6$

Feeder Steer Price

$STEERP(t) = 0.1454 + .00660405 * t - .05021905 * TSIN + .01961992 * TCOS$

$T'_s = (7.66), (7.75), (-3.83), (1.49)$

$R^2 = .730529$ $F = 30.724$

$N = 38$ $k = 4$

TABLE IV (Continued)

Feeder Heifer Price

$$HEIFRP(t) = 0.0420 - .00100353^{*}\ t + .81479899^{*}\ STEERP(L)$$
$$T'_s = \quad (7.26), \quad (-3.27) \quad (38.14)$$

$$-.00427589^{*}\ TSIN - .00422076^{*}\ TC\emptyset S$$
$$T'_s = \quad (-2.01), \quad (-2.57) \qquad 1958\text{-}1977$$

$R^2 = .996105$ $\quad F = 959.140$

$N = 20$ $\quad k = 5$

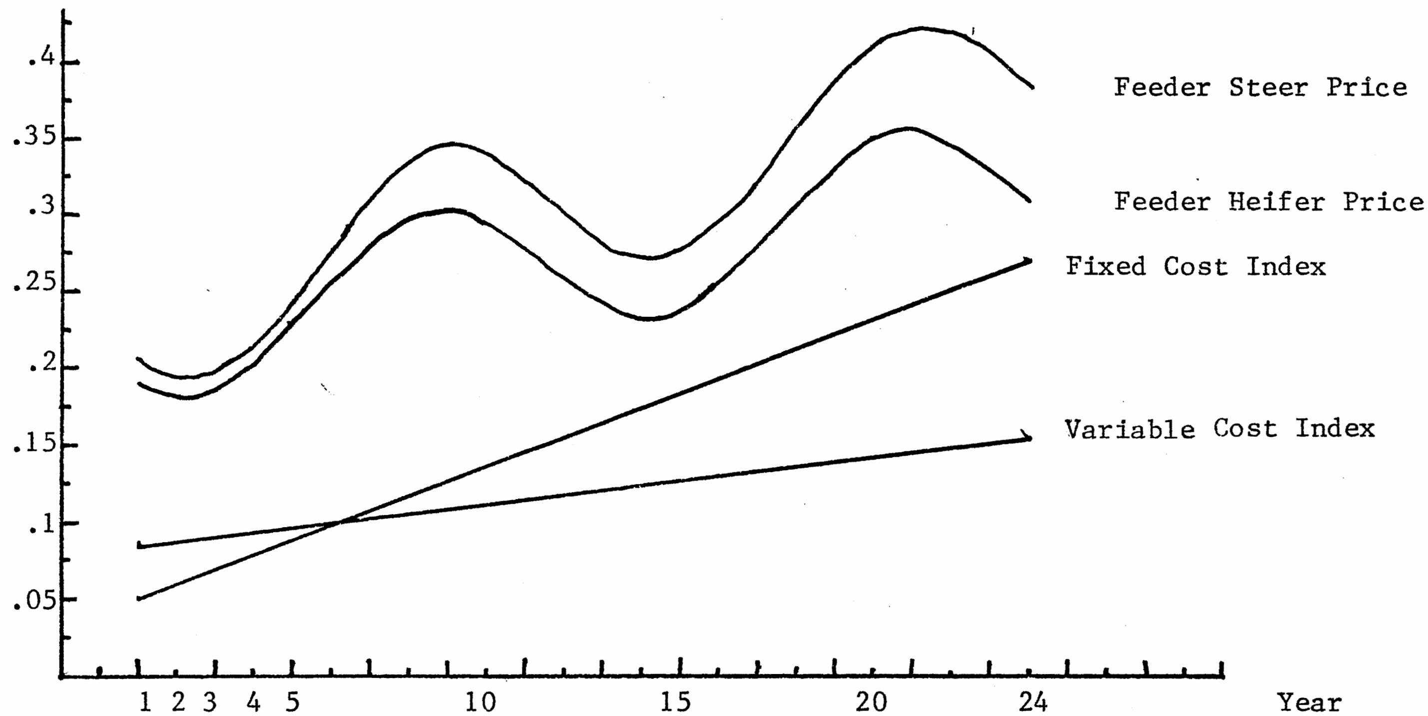

Figure 6. The Predicted Equation for the Feeder Steer and Feeder Heifer Price Series and Fixed and Variable Cost Indices

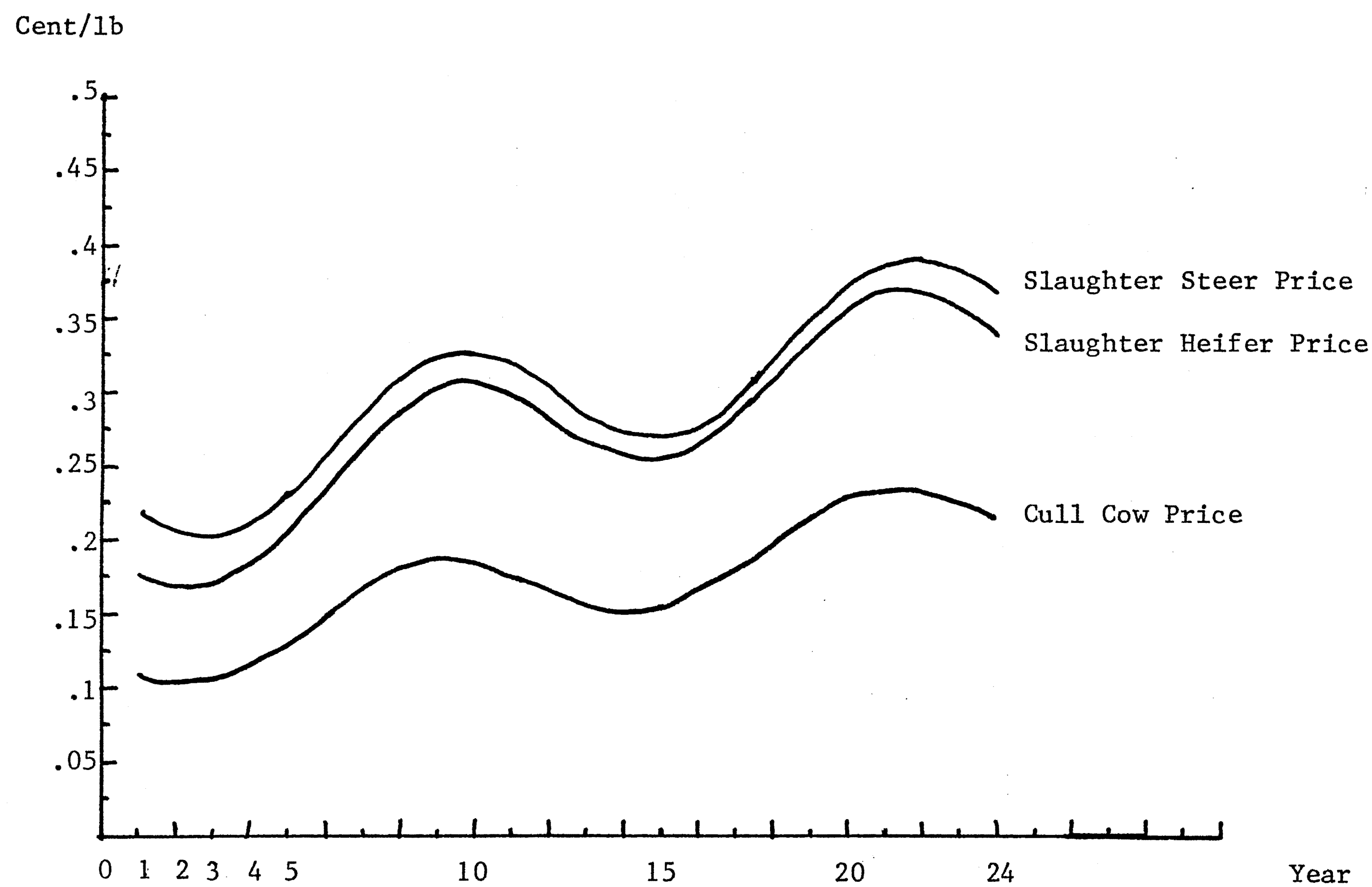

Figure 7. The Predicted Equation for the Cull Cows, Slaughter Heifer and Slaughter Steer Price Series

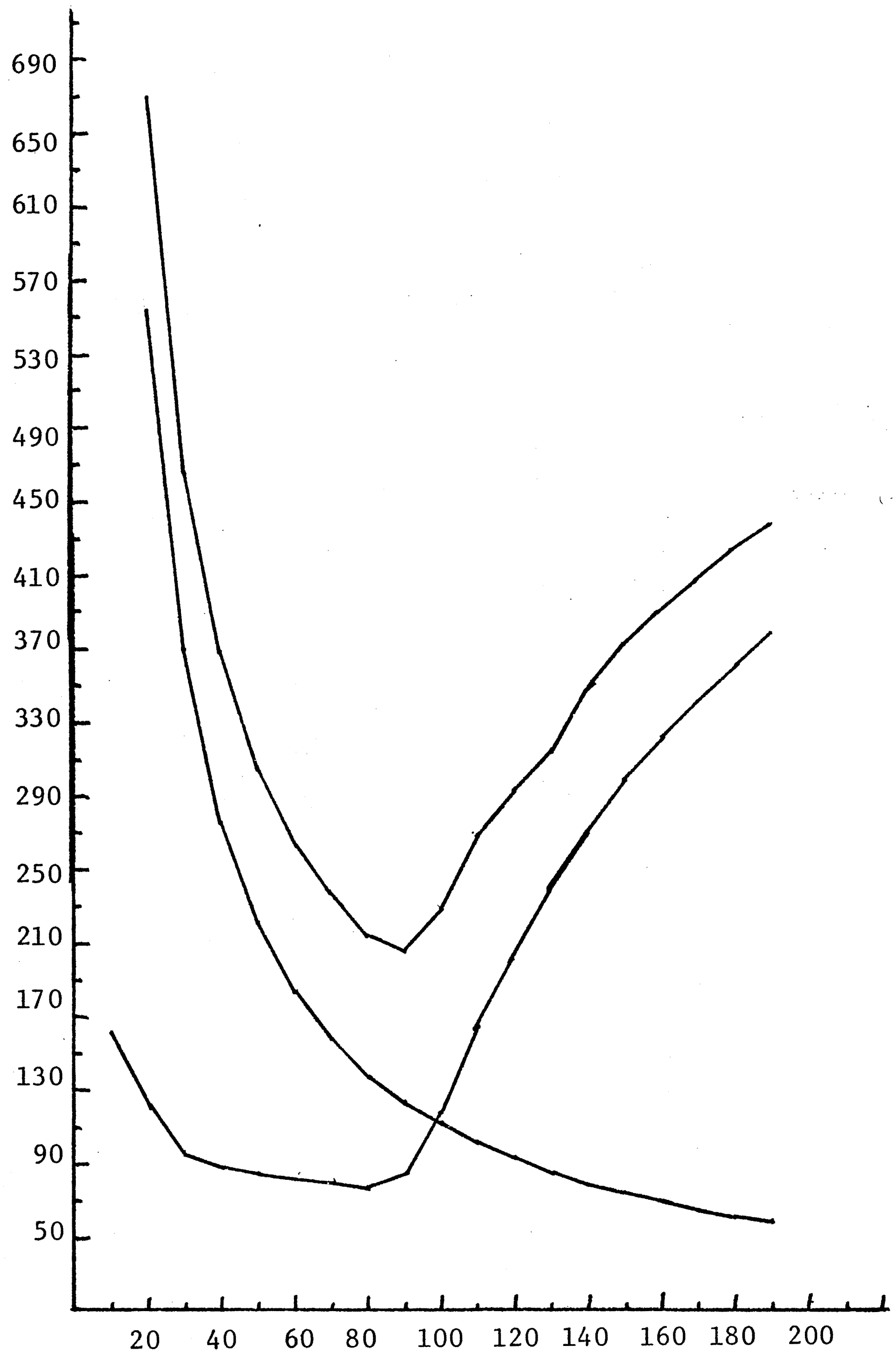

Figure 8. Average Total Cost, Average Variable Cost and Average Fixed Cost

hay and use less hay but only to a certain point because of the need to feed some hay in winter and dry months. Alternatively when heavy concentrations of cattle are grazed supplementary feeds, and the equipment to provide this feed, must be increased significantly.

To map the changes in cost of production per head for different herd sizes, the Oklahoma State University Agricultural Economics Extension budgets were used. A budget for a 100 herd was used as the base budget and then ran for different herd sizes. As the herd sizes were changed, so too were the inputs mentioned above. The cost of production figures generated by these budget runs are graphed in Figure 8.

Price Data

Four types of beef prices of significance to the cow-calf producer, are used in the model; the price received for steer calves, the price received for heifer calves, the price of two year old grass fed slaughter heifers, and the price of cull cows. The calf price series are used to determine the revenue from the sales of steer calves and all heifer calves not held for replacements. Calf sales are assumed to be made soon after the calves are weaned. Therefore, the model uses weaning weights and feeder calf prices representative of prices received for weaned calves. After consulting several data sources it was decided that the most complete and appropriate feeder calf price data series to use was that for good and choice 400-500 pound steer calves sold at Kansas City and for good and choice 400 pound heifer calves sold at Kansas City. Both feeder calf price series are reported in the publication Livestock and Meat Statistics. The weaning weights assumed for calves are dependent upon the sex of the calf and the age of the brood cow producing the calf. Weaning weights used are reported in Table II. In general they average between 400 to 500 pounds.

Price data for utility slaughter cows and good grade slaughter heifers were used to determine the per pound slaughter price or salvage value of cull cows. Previous literature and observation of the cull cow market indicated that the per pound value of cull slaughter cows changes with the age and condition of the cow. The precise nature of this relation is not known. However, it was felt some discounting of the per pound value of cull cows as they aged should be made. The highest price which could be expected for cull cows was hypothesized to be that of a good grade slaughter heifer, while the lowest price which could be expected was that for canner and cutter cows. Two year old heifers would likely sell for the good slaughter heifer price while cows approximately ten years of age and older would likely sell as canner and cutter cows. Based upon these two prices, guidelines presented in previous literature and personal observations of the beef industry a function was developed to approximate the hypothesized decline in slaughter price per pound as a cow aged from a two year old heifer to a ten year and older canner-cutter type slaughter cow. This function will be discussed presently in the discussion of the changes in physical characteristics assumed for cows of differing ages. The prices derived from this function times the weight reported in Table II for cows of a given age was used to determine the salvage or slaughter value of a cull cow. Economic theory would suggest that as long as a cow's net present value as a brood animal exceeds its slaughter value it should be retained in the herd.

It should be noted that while the two year old heifer price series is instrumental in determining cull cow slaughter prices, it also measures the opportunity cost of two year old replacement heifers. As such it reflects a key value to consider in the decision of whether or not to breed or sell two year old heifers ready to enter the herd.

Each of the cattle price series reported in Table I was regressed against time. Several functional forms were tested. Distinct cyclical and time trend pattern were detected. The functional form finally chosen to describe the basic pattern of beef prices over time was one including a linear time trend variable and sine and cosine time variables. The functions chosen are reported in Table IV. Period lengths for the sine and cosine variables ranging from seven to sixteen years were tested. A cycle length of twelve years was chosen based upon the criteria that a twelve year cycle yielded the highest average R^2 value for all the beef price series being considered. A twelve year cycle for beef is also consistent with the conclusions of previous studies of the beef cycle.

Due to the belief that the feeder steer and slaughter steer price series were more accurate than the feeder heifer and slaughter heifer price series and also due to the desire to assure that consistent price spreads were maintained between male and female animals, a different approach was taken to estimate functions describing the feeder heifer and slaughter heifer price series. In essence, these equations become price spread relations. Significant trend and oscillatory patterns were found in these price spread relations (Figures 6 and 7).

Price Spreads

In general the spread between heifer prices and steer prices (both slaughter and feeder) narrows when prices have bottomed and are rising and widens once the prices have peaked and started to drop once again. This is economically reasonable. As the general price level of both steers and heifers rise, and producers begin to expand their herds, heifer prices must also rise relative to steer prices due to the high demand for heifers as both replacement stock and slaughter animals. Also fewer feeder heifers and slaughter heifers are supplied to the market as more and more of these animals are kept for replacements. At this time, the only demand on the steers (both feeder and slaughter) is as market animals. Conversely, once the price peaks and starts to drop, producers wish to contract the herd. There is no exact demand for heifers as replacement stock and more and more heifers are now sold rather than being held back as replacements thus generating a large supply of both feeder and slaughter heifers. This forces the heifer prices to drop away from the steer prices.

It is also interesting to note that on the period depicted, in Figures 6 and 7 (over two complete cycles) that the size of the spreads between feeder steer and feeder heifer prices has increased while the spread between slaughter steer and slaughter heifer prices has decreased. This also is reasonable as feeder prices reflect the price feedlots are willing to pay for these animals. Feedlots will generally pay more for steers as they have higher rate of gain. The slaughter price, on the other hand, reflects the price received by the feedlot for the finished animal. The feedlot can obtain the same quality of finish on both the heifers and steers being fed by simply feeding the heifer animal a longer period of time. Thus the price spread between slaughter steers and heifers decreases over time while the spread in price between feeder steers and heifers increases.

Age Related Changes in Physical Characteristics of Cows

Six physically related factors which vary with the age of the cow and greatly influence the return generated by the herd are considered in the model. These factors are birth rates, cow death rates, cow weight, weaning weights of steer and heifer calves produced and the slaughter price per pound for cows of different ages. Rogers (15) noted these same six factors in his 1971 publication. Each of these factors as they are related to the age of the cow are reported in Table II. The data series for each of these factors, with the exception of the slaughter cow value series, is relatively self explanatory. The birth rate parameter refers to the percentage of cows by age which are successfully bred and raise a calf to weaning age. The birth rate parameters used are those reported by Rogers (15). The death rate parameter series describes the percentage of cows of a given age which die each year. This parameter series was also obtained from Rogers (15). The cow weight series describes the average weight of cows by age and was obtained from Kay and Rister's (20) article. The steer and heifer weaning weight series was developed from data present by Rogers (15) and Earnest, Waters and Shumway (19). Earnest, Waters and Shumway reported an average weaning weight series for steer and heifer calves combined which was felt to be quite consistant with current conditions in the industry. Rogers reported weaning weight series for both steers and heifers. However, his weights were believed to be slightly lighter than current weaning weights in the industry. Hence, the ratio between steer and heifer weaning weights expressed in Rogers' two weaning weight series were used to derive a steer and heifer weaning weight series based upon Earnest, Waters and Shumway's average weaning weight series derived are reported in Table II.

As discussed previously, observation of the beef market indicates that the slaughter value of cull cows changes with their age and condition, with condition being highly related to age. Data to estimate the precise nature of this relation is not available. It is hypothesized that a two year old cow just entering the herd would have a slaughter price nearly equivalent to a good grade slaughter heifer. After one calf or at age three her slaughter value would likely drop to a per pound price roughly equivalent of that for commercial cows. By age ten and older it is hypothesized that most cows would be sold as utility grade cows. If a linear decline in price from commercial to utility grade price is assumed over the ages three to ten years and a standard ratio is assumed between good grade heifer prices and commercial cow prices, a function interpolating by cow age, the decline in price received for slaughter cows can be developed based upon the spread between good slaughter heifer prices and utility cow prices. This price structure is incorporated into the model based upon the following functions:

3-1) DIF = HEIFP(t) - UTILITYP(t)

3-2) CULLP(t,s) = HEIFP(t) - DIF x CPR(s)

where

HEIFP(t) = good grade slaughter heifer price in year t.

UTILITYP(t) = utility cow price in year t.

DIF = the calculated difference between good grade slaughter heifer price and utility cow price in year t.

CPR(s) = the decline in difference or potential premium above utility cow price, i.e., by age tne 100 percent of premium or difference above utility cow price will have been eroded.

CULLP(t,s) = calculate cull price for a cow of age s in year t.

Observation of the typical relation between the price of good slaughter heifers, commercial cows and utility cows indicated that .787 percent of the decline in price between good slaughter heifers and utility cows occurs with the change from good to commercial grade, or as hypothesized here after a cow has her first calf. Thereafter, it is hypothesized that approximately 3 percent of the premium (difference) between the price for good slaughter heifers and utility cows is erroded each year as the cow ages. By age ten 100 percent of the premium (difference) is expected to have been eroded. A schedule of CPR values, as they are referred to in the above equation reflecting the pattern of decline in slaughter cow prices hypothesized here and used in the model is reported in Table II. It should be noted that after age eleven a cow's weight begins to decline, hence, even though her slaughter price is assumed to remain fixed at the Utility cow price her total slaughter value declines due to the decline of her weight.

Model Operation

With the implementation of the three groups of data discussed (cost data, price data and physical characteristics data) into the model structure described in Chapter II the model is ready for simulation. Actual simulation is achieved via use of a computer coded mathematical model of the equations discussed in Chapter II and the data discussed in the previous section of this chapter.

The major purpose of conducting simulation runs with the model is to determine the results of alternative replacement and culling decisions under various sequences of price changes. The model is designed so that any sequence of culling and replacement patterns and initial herd sizes and age distributions can be simulated. The results of a given culling and replacement policy and initial herd distribution can be evaluated using several scenarios of changes in prices and cost of production. Actual price changes and derived production costs over the period 1958 to 1977 can be used; or price and production costs patterns generated by the estimated time series functions for cattle prices and cow-calf production costs can be generated; or a combination of actual prices from 1958 to 1977 and extrapolated prices beyond 1977 based upon the time series function can be used. Finally, if the user wishes to specify some specific series of expected future prices these can also be incorporated into the model.

Each set of culling and replacement decisions and price and cost series considered will generate a unique set of outputs in terms of number of calves produced and sold, cost of production, total revenue, net revenue, herd size etc. In addition to outputting for each year simulated economic and physical activities predicted to have occurred, the model also generates information about expected future economic returns of the herd, i.e., the expected net present value of each cow is computed, etc.

Simulation of an arbitrarily chosen set of culling and replacement decisions, given an initial herd size and age distribution and a set of expected future prices and costs will project the consequences of the chosen set of culling and replacement decisions if the expected prices and costs are realized, but it does not determine the best or profit maximizing culling and replacement strategy. However, by using a system optimization algorithm culling and replacement rates which yield the highest profit over time can be estimated.

Operation of the System Optimization Algorithm

A major part of the effort devoted in this research project has been to adapt a system optimization algorithm to the cow-calf simulation model developed. By using such an algorithm estimates of optimal culling and replacement rates over time, given expected future prices, can be developed. The system optimization algorithm used in this study is titled the "Complex Algorithm" and was developed by M. J. Box (21). A detailed disucssion of the algorithm will not be given here. The reader is referred to the publication Optimization Technique with FORTRAN by Kuester and Mize (22) for a detailed descriptiion of the programming and operation of the optimizer.

The Complex Algorithm is capable of finding a maximum value for a multivariate, nonlinear objective function subject to nonlinear constraints. The objective function specified in this study is defined as the cummulative discounted net profit generated over a specified number of years. The cummulative net profit for a given sequence of culling and replacement rates is calculated by the simulation model and used to determine the objective function value. All of the physical and economic factors discussed previously are brought into consideration in the optimization process to determine the best sequence of culling and replacement rates.

The Complex Algorithm is a "heuristic search" optimization procedure. In essence, the procedure tries alternative culling and replacement patterns and systematically records the profit from each. By relating the change in profit associated with various changes in the culling and replacement rates, which in systems terminology would be referred to as "control variables," the algorithm systematically arrives at a solution for the control variables which maximizes the objective function.

A problem associated with hueristic type optimization algorithm is their inability to determine if the maximum point reached is a local or global maximum. Hence, all solutions must be analyzed with this reservation in mind. This problem is encountered in order to allow nearly complete flexibility in structure of the model or system being optimized. If a continuous differential equation model were specified more powerful control algorithms could be used and global maximums assumed. However, the nature of the cow-calf model required here does not permit such a structure.

To operate the Complex Algorithm the user must supply a model or function capable of generating value for the objective function given any set of control values. Secondly, the user must specify an initial plausible set of control values. The routine will then proceed to find optimal values for the control variables, in this case the culling and replacement rates, and report the objective function value, in this case discounted net cummulative profit, associated with the optimal control values. By requesting a detailed summary of the optimizer's activities the user can also observe the step by step search sequence the optimizer conducted in the determining the optimal conditions.

Output

Output from the model falls into three broad categories; cost and revenue data for each year of operation simulated; physical characteristics of the herd for each year of operation simulated, including the size of the herd, the age distribution of the herd and the replacement rate and culling rate for the current year; and three tables describing the expected net present value of the cummulative cost and revenue flows over various aged cows retained alternative lengths of time. The later set of output information becomes a key element in one approach taken to attempt to devise a set of optimal culling and replacement control functions, e.e., decision criteria, capable of optimizing cummulative net returns.

Revenue and Cost Output

Table V presents a sample set of revenue and cost data outputed by the model each year. Revenue received is broken into that received from the sale of male and female calves and cull cows. Cost is reported in terms of fixed cost and variable cost as well as total cost. The definitional break down of fixed and variable cost was previously outlined in Chapter II. All cost and revenue data are reported in total terms and on a per cow basis. Additional cummulative net revenue over the sequence of years considered is reported each year.

Two additional pieces of information reported in the Revenue and Cost Table are the internal replacement cost and opportunity cost of replacement heifers. The internal replacement cost of a 2-year old heifer is calculated as the sum of revenue foregone by not selling her

TABLE V

TABLE OF REVENUES AND COSTS

REVENUE				
CALF SALES				
MALE	4358.37			
FEMALE	3458.68			
SUBTOTAL		7817.05		
CULL SALES		1344.28		
GROSS (TOTAL)			9161.32	
COSTS				
FIXED		3065.77		
VARIABLE		3336.53		
TOTAL			6402.30	
NET REVENUE			2759.03	
CUMMULATIVE NET REVENUE				13800.50
ADDITIONAL INFORMATION (PER HEAD) ON YEARLY BASIS				
GROSS REVENUE	126.35			
NET REVENUE	39.05			
FIXED COST	42.28			
VARIABLE COST	46.02			
TOTAL COST	88.30			
REPLACEMENT COST				
1--OPPORTUNITY VALUE (950.0 * SLG PRICE)	215.74			
2--INTERNAL REPLACEMENT (CALF SALE + FC + VC)	196.34			

as a feeder heifer plus the fixed and variable cost of maintaining her for one year. The fixed and variable cost of maintaining a replacement heifer for one year until she is mature enough to breed is assumed to be the same as that for maintaining a cow for one year. The opportunity cost of a 2-year old replacement heifer is calculated as the current good grade slaughter heifer price times a weight of 950 pounds.

Net Present Value Table Output

The model outputs three tables describing the net present value of the cost and revenue flows from cows of differing ages and held over different lengths of times. Values contained in the table are sensitive to the discount rate assumed and the series of future prices and costs assumed. The effect of changes in herdsize over time upon the cost of production (due to a U-shaped total cost curve) are not reflected in the net present value tables developed.

Table VI displays a sample table of the net present cummulative value of the differences between the revenue flow generated from calf sales minus the average total cost of maintaining a brood cow one year. The table contains fifteen rows, i.e., one row for each age of cow considered. Cows are grouped by their "calving age." That is, first calving cows are referred to as one year old cows in terms of the number of years they have been in the herd. Likewise, the table has fifteen columns representing the possibility of holding a given aged cow for fifteen years. However, due to the fact that it is assumed that all cows will be culled before producing their sixteenth calf, values for positions in the table where the sum of the row and column index equals sixteen or more have been deleted.

TABLE VI

NET PRESENT VALUE OF A SINGLE BROOD COW-CALF FLOW

	Years Held														
Age[1]	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	17.	40.	74.	117.	163.	207.	245.	274.	292.	300.	302.	302.	301.	301.	300.
2	28.	69.	120.	176.	229.	275.	310.	331.	340.	343.	343.	342.	341.	341.	0.
3	46.	106.	170.	231.	284.	324.	349.	359.	362.	362.	361.	361.	360.	0.	0.
4	66.	137.	204.	263.	307.	335.	346.	349.	350.	349.	348.	347.	0.	0.	0.
5	76.	150.	214.	262.	291.	304.	307.	307.	306.	305.	305.	0.	0.	0.	0.
6	80.	150.	202.	234.	248.	252.	252.	251.	250.	249.	0.	0.	0.	0.	0.
7	77.	135.	171.	186.	190.	190.	189.	188.	167.	0.	0.	0.	0	0.	0.
8	66.	107.	124.	129.	129.	128.	126.	126.	0.	0.	0.	0.	0.	0.	0.
9	49.	70.	76.	76.	74.	73.	72.	0.	0.	0.	0.	0.	0.	0.	0.
10	26.	34.	34.	32.	30.	29.	0.	0.	0.	0.	0.	0.	0.	0.	0.
11	11.	11.	8.	5.	4.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
12	0.	-4.	-9.	-11.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
13	-8.	-16.	-20.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
14	-17.	-24.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
15	-18.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

[1]"Age" refers to the number of years the cow has been in the herd and producing a calf.

The changing probability that a cow will successfully produce a calf as she ages is incorporated into the calculation of the figures in the table.[1] For example consider the net present values in row one. The first value denotes the net present value generated by producing one calf from a typical 2 year old cow. The probability that this cow will produce a calf is 85.5 percent or the birth rate for that age of cow. Hence the expected revenue for that year is 85.5 percent of the weighted average total value of female and male feeder calves that year. In the following year, i.e., for the second column in row one it is expected that .855 cows will remain (the .145 which failed to calf will have been culled) and 89 percent of them will produce a calf, hence, .89 x .855 or .76 calves will be produced from .855 cows. Both the costs and revenues are adjusted in column two of row one to reflect these expectations of number of cows remaining in the herd and their successfully calving rate. Similar adjustments are made in each subsequent period in row one. The same procedure is followed for all rows in the consideration of expected net calf revenue flows from other ages of cows currently held in the herd.

Table VII entitled "Net Present Value of a Single Brood Cow--Flows Only" is similar to Table VI except the annual revenue received from the sale of cull cows as well as calves is now incorporated into the calculation of revenue flows, i.e., in the case previously referred to

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[1]The changing probability that a cow will successfully produce a calf as she ages is incorporated into the calculation of the figures in the table. Although the table is described in terms of a single cow, it is useful to think of it as the average for 100 cows, hence making .8 calves the equivalent of 80 calves per hundred cows, or .02 deaths the equivalent of 2 cows per hundred dying.

TABLE VII

NET PRESENT VALUE OF A SINGLE BROOD COW-FLOWS ONLY

	Years Held														
Age[1]	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	38.	76.	119.	169.	222.	275.	324.	366.	399.	420.	434.	442.	447.	449.	450.
2	46.	97.	157.	221.	285.	344.	394.	433.	459.	476.	485.	491.	494.	495.	0.
3	59.	128.	202.	276.	344.	402.	447.	476.	496.	507.	513.	517.	518.	0.	0.
4	77.	158.	240.	315.	379.	429.	462.	483.	495.	502.	506.	508.	0.	0.	0.
5	89.	177.	258.	328.	382.	418.	441.	454.	462.	466.	468.	0.	0.	0.	0.
6	96.	185.	261.	320.	359.	383.	398.	406.	411.	413.	0.	0.	0.	0.	0.
7	98.	183.	248.	291.	319.	335.	344.	349.	351.	0.	0.	0.	0.	0.	0.
8	96.	171.	220.	251.	270.	281.	286.	289.	0.	0.	0.	0.	0.	0.	0.
9	90.	149.	187.	209.	222.	228.	231.	0.	0.	0.	0.	0.	0.	0.	0.
10	76.	125.	154.	170.	179.	182.	0.	0.	0.	0.	0.	0.	0.	0.	0.
11	69.	110.	132.	145.	150.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
12	64.	100.	119.	127.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
13	63.	97.	112.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
14	68.	98.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
15	77.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

[1]"Age" refers to the number of years the cow has been in the herd and producing a calf.

the revenue generated from the sale of the .145 cows which failed to calf and were culled from the herd is now included in the revenue flow. The price used to calculate the value of the cull cow declines with the age of the cow as previously outlined in the discussion of cull cow prices. Also the weight of the cull cow is adjusted according to those weights specified in Table II. The costs subtracted from revenue flows in this table are the same as those previously subtracted in the calculations of Table VI.

Table VIII, entitled "Net Present Value of a Single Brood Cow," is also similar to Table VI. Revenue flows in Table VIII include the revenue from calf sales, cull cow sales and from sale of the cow herself at the end of the production year in question. Cost considered in Table VIII are the same as those considered in Tables VI and VII. Therefore, the value in column seven of row one would reflect the net present value of the stream of revenues and maintenance cost generated by holding a two year old cow for seven years and then selling her. Actually, the probability that a typical cow will still be in the herd and available for sale at the end of the seven years is much less than one, since a certain percent will have been culled each year leaving only X percent producing and available for sale in the seventh year.

It should be noted that the cost figures used in Table VI through VIII do not include an initial purchase cost of the cow. Hence, economic theory would indicate that the net present values reported in Table VIII for different aged cows held over different time periods is the breakeven purchase price of such a cow. If a cow can be purchased for less than this value a profit will be made, if not, losses will be encountered.

TABLE VIII

NET PRESENT VALUE OF A SINGLE BROOD COW

	Years Held														
Age[1]	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	161.	190.	232.	284.	341.	388.	426.	450.	462.	463.	458.	455.	452.	451.	451.
2	182.	233.	296.	364.	421.	466.	495.	405.	510.	505.	501.	498.	496.	496.	0.
3	216.	288.	367.	432.	484.	518.	535.	535.	530.	524.	521.	519.	519.	0.	0.
4	253.	341.	413.	470.	507.	526.	527.	521.	515.	511.	509.	509.	0.	0.	0.
5	286.	365.	427.	467.	488.	489.	482.	475.	472.	470.	469.	0.	0.	0.	0.
6	301.	369.	412.	435.	435.	428.	421.	417.	415.	414.	0.	0.	0.	0.	0.
7	302.	351.	375.	376.	368.	360.	356.	353.	353.	0.	0.	0.	0.	0.	0.
8	288.	316.	317.	308.	299.	294.	291.	290.	0.	0.	0.	0.	0.	0.	0.
9	264.	266.	254.	244.	238.	234.	234.	0.	0.	0.	0.	0.	0.	0.	0.
10	227.	212.	199.	190.	186.	185.	0.	0.	0.	0.	0.	0.	0.	0.	0.
11	192.	173.	162.	156.	154.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
12	163.	145.	136.	133.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
13	143.	127.	122.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
14	128.	119.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
15	132.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

[1]"Age" refers to the number of years the cow has been in the herd and producing a calf.

Changing Herd Size Effect on Cost

The preceeding net present value tables provide the economic information to theoretically determine if a cow's future flow of discounted net revenue exceeds her current market value. If so she should be retained in the herd, if not she should be sold. Information from these tables will be used in developing the culling and replacement strategies presented in the next chapter. In one case the information in the table will be used directly to determine how many cows to cull and how many replacement heifers to retain. The framework for this application of the table information will be elaborated upon in the next chapter. In all cases the highest discounted net present value of a cow obtainable by holding her an optimum number of additional years as compared to her current slaughter value will be used to determine which cows to cull when culling is called for by the decision strategy. Those cows with the largest negative difference between their expected net present value and current slaughter value, i.e., those expected to lose the most money, will be culled first.

One additional adjustment will be made to the calculation of the net present values used for planning culling and replacement rates. Consideration will be taken of planned future changes in the herd size (the tables presented here and printed by the program assume a constant herd size). Hence, the cost figures used for future years will be adjusted as planned herd size changes. Adjustments will be made according to the U-shaped cost curve presented in Chapter II.

In order to accomplish the proper adjustment for herd size in future years during the net present value computation process, the entire sequence of culling and replacement rates, plus the initial herd size, must be known at the beginning of the simulation run. This is not possible since a sequential decision process is programmed which makes culling and replacement decision based upon current information and expected future prices. To obtain knowledge about future herd sizes the program was written such that it initially assumed a constant herd size over the planning period considered and thereafter assumed the herd size pattern overtime to be that generated by the last iteration of the hueristic search optimization algorithm. Actually the assumed herd size time pattern was not changed on every iteration, instead it was changed according to a convergence criteria which allowed the algorithm to partially converge to a solution before the herd size time pattern assumption was changed and the solution value for the control parameters adjusted accordingly. As the algorithm approaches its final solution convergence point, the culling and replacement rate sequences over time do not significantly change between iterations, hence nearly identical herd sizes over time are generated and the assumed future herd size time pattern becomes nearly, if not exactly, equal to the actual herd size time pattern generated. This procedure allows future changes in herd size as well as future changes in prices to be injected into the decision process. This approach, to the best of the author's knowledge, constitutes a form of "adaptive control" based upon "feedback" from the system.

CHAPTER IV

MODEL SIMULATION AND ANALYSIS OF RESULTS

The model developed in Chapters II and III has the capability of calculating the costs and returns generated by a typical cow-calf operation and computing the net present value of a cow in the herd or a replacement heifer being considered as an addition to the herd (the net present value referred to is that associated with the cow's value as a productive member of the herd). To achieve the objectives set forth in Chapter I, the cow-calf model developed must be incorporated into a systems optimization framework as outlined in Chapters II and III.

Use of the systems optimization procedure enables year to year sequences of replacement rates and culling rates to be estimated which will yield maximum profits over a specified period of time. The sequence of replacement and culling rates generated are the solutions to a time varying "control function." The control function may be any of many forms ranging from relatively simplistic to complex. Part of the task of determining an "optimal culling and replacement" strategy is to select the proper type and complexity of control function.

The remaining portions of this chapter will discuss the objective function used and its relation to the cow-calf simulation model, the nature of the control functions selected and the optimal solutions obtained with each control function tested.

The Objective Function in Relation to Model Conditions

The fundamental objective value to be maximized was specified to be the net present value of the accumulated net returns to a cow-calf firm over a specified period of time. Values for calculating the objective function were obtained from the cow-calf simulation model. Several problems arose in specifying and relating the objective function to the operation of the model. First, an initial herd size and age distribution had to be determined and held constant over all alternative strategies tested.[2] Secondly, a question arose as to what monetary value to assign to cattle in the herd at the end of each year of the simulation runs. Thirdly, the questions of what prices, cost structure, discount rate and simulation period length to use had to be resolved.

The initial herd size and age distribution specified affects the revenue flow generated over a number of years. Indeed, if the initial herd size and age distribution are nonoptimal for the economic conditions existing in the first few years of simulated runs very severe and perhaps abnormal culling and replacement patterns will be necessary to correct this situation. It was reasoned that a good initial herd age distribution to use would be one which represents an average distribution for one complete cycle. Such an age distribution was obtained by performing three steps. First, the model was ran for 24 years (two complete cycles) with a uniform initial age distribution[3] and a 20 per-

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[2]All runs would begin with this initial herd distribution and herd size then would be allowed to vary the distribution and herd size to achieve the greatest return.

[3]Each of the 15 age groups was specified to contain 5.555 cows.

cent replacement and culling rate. A 20 percent annual culling and replacement rate was shown by previous studies to be reasonably efficient. The ending age distribution obtained from this run was then used as the initial age distribution for a second run simulated over 100 years. A 20 percent culling and replacement rate was again used. By running the model 100 years with a constant set of culling and replacement rates the age distribution stabilized before the end of the run, i.e., reached equilibrium. The age distribution obtained from this run was then used as the initial age distribution for all subsequent runs. The age distribution obtained and used is reported in Table IX. An initial herd size of 100 was chosen both for convenience of interpreting results and because the budgets used to develop the cost structure of the firm were based upon a 100 head herd.

A second question which arose with regard to setting up the model to determine an optimal culling and replacement strategy was how to value the ending inventory of cattle. It was found that if only the revenue flows generated over a given period were entered into the objective function the optimal culling and replacement strategy became one that liquidated the herd over the last several years of the run. Since it was desired that the program should determine whether liquidation or continued holding of brood cow inventories appeared most desirable some value had to be assigned to the brood cow and replacement stock inventory. The question arises as to what prices to use in this evaluation process, slaughter cow prices or something else? The procedure adopted was to value the cows in the ending inventory at either their expected best possible net present value or their current slaughter value during the current year of the simulation run. The highest

TABLE IX

INITIAL STABILIZED HERD DISTRIBUTION

Year of Production	Cow's Age	Herd Age Distribution
1	2	16.80[a]
2	3	13.98
3	4	12.13
4	5	10.97
5	6	10.10
6	7	9.28
7	8	8.37
8	9	7.33
9	10	6.11
10	11	4.74
11	12	0.19
12	13	0.00
13	14	0.00
14	15	0.00
15	16	0.00

[a]The number of replacement heifers held back each year. This will also equal the number of culls each year.

expected net present value was obtained from the "Brood Cow Net Present Value Table" presented in Chapter III. The "highest net present value" is the net present value associated with retaining the cow in the herd to an age which generates the largest discounted cummulative net return. Heifer replacement stock not yet in the herd and producing calves were always valued at their current market price.

With regard to what price series to use in the model for the optimization simulation runs it was resolved to use the price sequences generated by the time dependent functions reported in Chapter III. One modification was, however, made to these equations. All trend variables were held constant and only cyclically related time variables were allowed to change. This was done in order to hold all average price relations at constant spreads and to essentially remove upward trends in prices due to inflation. 1967 price levels and relationships were used. As will presently be seen in the simulation runs the price spreads existing in 1967 were profitable ones. The conclusion should not be drawn that price spreads remained constant in each year of the simulation. Cyclical variation of prices about horizontal trend lines was present in the model. Since the degree of oscillation and the phase of all cycles simulated were not the same, price spreads and relations did change from year to year but the average relations over a number of cycles would remain constant since no diverging or converging trends were permitted.

The cost structure developed in Chapter III was used in the optimization runs. This cost structure was U-shaped in nature but exhibited a relatively flat slope between 80 and 90 head of cattle. The minimum value of the average total cost curve occurred at 80 head. Again for convenience, the average variable cost curve was shifted to the right by 10 head. This allowed use of the same general shaped average variable and average total cost curves but forced the minimum point of the average variable cost curve to be 90 and the bottom of average total cost curve to be 100. Only one cost structure was tested in this analyses. Other cost structures would likely have generated different results, particularly if the average total cost curve were "flatter" or significantly higher. However, the cost structure used here is felt to be reflective of general cow-calf firm cost structures.

A simulation run length of 24 years (two cycles) was chosen. It was believed that this length would be long enough to see if definite patterns existed in the culling and replacement strategy and would not generate prohibitively large costs in terms of computer expenses.

Finally, a discount rate of 4 percent was selected. This rate is believed to be an appropriate discount rate given the fact that inflation has been removed from the price series by detrending the price data. Several previous studies used a 5 percent discount rate.

The Control Function

The optimization procedure used here (and in all systems optimization, i.e., control procedures) require the user to specify a time varying control function. This function may be solely a function of time, in which case it is referred to as an "open loop control function" or the function may contain time subscripted output from the model, in which case the function is referred to as a "closed loop control function." Both types of control functions will be tested in this analysis.

Ideally, one would like a control function which allowed an independent control value, i.e., culling or replacement rate, for each period to be considered. This is infeasible when many periods are considered or when a continuous system is developed and the number of periods considered becomes infinite. Hence, a time dependent control function with a finite number of parameters must be specified. In this analysis two control functions are specified, one for replacement rates and one for culling rates. Due to the capabilities of the Complex Algorithm and the size of the cow-calf simulation model, solution costs prohibit more than approximately three to four parameters being considered for each control equation.

Five alternative sets of control functions were tested. These functions by no means exhaust all of the potential control functions which could be considered and indeed better functions generating more profitable strategies may be possible. However, the control functions considered do allow a great deal to be learned about the optimal culling and replacement strategies for cow-calf firms faced with cyclical prices and U-shaped cost of production structures.

The nature of the five control function or culling and replacement strategies tested are summarized in Table X. Optimizing simulation runs for each of the strategies were made under identical model conditions with the value of the culling rate and the replacement rate being the only factors allowed to vary. Briefly, the five strategies may be summarized as follows: in the first two strategies, the replacement rate and culling rates are held equal to each other. This will cause a constant herd size to result. In addition to forcing culling and replacement rates to be equal strategy #1 required a constant culling and replacement rate over time. Strategy #2 allowed culling and re-

TABLE X

RATE STRATEGIES

Strategy	Control Function Form	Relationship Between the Culling and Replacement Rate
1	CR[1] = L[3] RR[2] = L	equal
2	CR = f(time, sine, cosine) RR = f(time, sine, cosine)	equal
3	CR = L[3] RR = L	not equal
4	CR = f(time, sine, cosine) RR = f(time, sine, cosine)	not equal
5	CR = f(ANVC)[4] RR = f($HNPV_{FH}$[5] - MV[6]FH)	not equal

[1]CR is the culling rate.

[2]RR is the replacement rate.

[3]L is a constant value between zero and one. In Strategy #1, L's are held equal but allowed to vary independently of one another in Strategy #3.

[4]ANVC equals the average value of the difference between a cow's slaughter market value (MV) and her net present value (NPV) as a brood cow for the subset of cow age groups where MV exceeds NPV.

[5]$HNPV_{FH}$ is net present value (NPV) obtainable by holding a feeder heifer to her optimal age, i.e., the age that generates the highest NPV.

[6]MV_{FH} is current market value of feeder heifer.

placement rates to vary overtime in a regular cyclical pattern. Strategies #3, #4 and #5 permitted replacement and culling rates to vary independent of each other, thus, allowing changes in herd size overtime if such changes resulted in higher profits. Strategy #3 parallel strategy #1 in that it forced constant (but independent) replacement and culling rates overtime. Strategy #4 paralleled strategies #2 in that it allowed cyclical (but independent) variation of replacement and culling rates overtime. The last control function or strategy tested is the most sophisticated. It uses expected net present values, heifer slaughter values and cow slaughter values calculated from the simulation model as variables in the control function. Since these values change according to the culling and replacement rates specified overtime, this form of control function is a "closed loop" control function. The first four control functions are "open loop" control functions since they are based only upon time. The results of the optimization simulation runs for each of these strategies will be presented and discussed in the following sections.

Description of Runs

Strategy #1: Constant and Equal Replacement and Culling Rates

Strategy #1 is a relatively simplictic strategy and resembles the strategy assumed by previous studies of optimal culling and replacement strategies. The culling and replacement rates are restricted to equal each other, hence, a constant herd size will be maintained. Additionally, the culling and replacement rates are not allowed to vary overtime.

Hence, the task of the optimizer is to find the single best average replacement and culling rate to use.

The culling rate found to optimize discounted net cummulative profit with this strategy was .1889. This culling rate, together with the conditions specified in the model resulted in a stable herd size of 102.11 head of cows and an optimal culling age of 9 years, i.e., after the eighth calf. To verify these results with previous studies required the running of the model using strategy #1 at 5% interest rate.

Previous studies by Rogers (16), Bentley, Waters and Shumway (19), and Kay and Rister (20), mathematically derived analogous optimal culling and replacement rates using somewhat different price assumptions, biological parameters, etc. The optimal culling rates and replacement age derived by these studies are reported in Table XI. As can be seen, the results of this study under this control function are very similar to previous results.

TABLE XI

COMPARATION OF OPTIMIZATION MODEL RESULTS TO RESULTS OF PREVIOUS STUDIES CONCERNING OPTIMAL HERD REPLACEMENT

Study	Replacement Age
Rogers (1971)	8 calving years
Bentley, Waters, and Shumway	7 calving years
Kay and Rister	9 calving years
Optimization Model (at 5%)	8 calving years

The question now becomes can a more profitable strategy or control function be specified which does not require equal culling and replacement rates and/or does not require constant culling and replacement rates overtime. Strategies two through five attempt to do this. Previous studies of optimal culling and replacement strategies were unable to consider such strategies due to their failure to use a non-linear optimization algorithm and control function.

Strategy #2: Cyclical and Equal Replacement and Culling Rates

Strategy #2 incorporates a control function capable of generating oscillatory time patterns for culling and replacement rates, i.e., sine and cosine functions of time are incorporated in the control function. Since very little has been done with culling and replacement rates as a time varying value, this strategy proved to be difficult to develop. Two major problems had to be overcome - one, no reasonable series of time varying culling and replacement rates existed to base the initial specification of the function upon the precise form and parameters to use for the oscillatory control function were not known.

The optimizing algorithm was used to solve both problems. The control function was specified as a time dependent function of the type in equation 4-1 below.

equation 4-1) $CR = RR = \text{intercept} + B_1 8\text{sine}\ \frac{2\pi t}{P} + B_2{*}\text{cosine}\ \frac{2\pi t}{P}$

where t is time, t = 1, ..., 24

P = 12, representing the cycle length

Then by specifying some arbitrary value for the initial intercept value and assigning zero values to initial slope parameters, the optimizer could be used to find appropriate parameters. This procedure was used for all runs where the culling and replacement rates were not held constant over time.

Under the conditions assumed in this study, strategy #2 yielded an optimal solution with higher returns than strategy #1. As a matter of fact, the return under this strategy was the highest of the five strategies tried (Table XII). Although strategy #2 optimized returns by maintaining a constant herd size, it did not hold the culling and replacement rates constant, nor did it specify one replacement age to be the optimal (Table XIII). Instead it varied the age of the herd from year to year to take advantage of changing prices (Table XIV and Figure 9). The variation in the culling and replacement rate enabled this strategy to keep relative young cows during times of high or rising prices and older cows when prices were low or dropping (Figure 9).

TABLE XII

RETURNS AND HERD SIZES PER RATE STRATEGY

Strategy	Herd Size	Return over 24 years
1	102.11	$ 89,750.375
2	99.99-100.01	102.639.312
3	97.81-101.67	91,958.25
4	99.90-100.02	100,969,875
5	98.22-102.12	92,026.8125

TABLE XIII

RATES AND HERD AGE BY STRATEGIES

Year	1-Constant, equal rates[1]	2-Cyclical, equal rates CR = RR	2-Cyclical, equal rates Cow Age	3-Constant, unequal rates[1]	4-Cyclical, unequal rates CR	4-Cyclical, unequal rates RR	4-Cyclical, unequal rates Cow Age	5-NPV-based, unequal rates CR	5-NPV-based, unequal rates RR	5-NPV-based, unequal rates Cow Age
1	CR = .18887693 RR = .18887693 Cow Age = 8	.168051	11	CR = .19059861 RR = .18868661 Cow Age = 8	.168037	.167919	11	.190407	.189233	11
2		.364508	11		.178388	.178157	11	'	.189277	11
3		.485917	7		.230314	.230030	10	'	.189315	10
4		.499778	6		.309914	.309655	9	'	.189361	10
5		.402382	5		.395880	.395716	7	'	.189385	9
6		.2198	5		.465201	.465176	5	'	.189384	9
7		.1191	5		.499321.	.499442	4	.196772	.189358	9
8		.1058	6		.489106	.489342	3	.195186	.189317	8
9		.1020	7		.437291	.437579	3	.190407	.189273	'
10		.1079	8		.357746	.358009	3	'	.189237	'
11		.1213	9		.271763	.271932	3	'	.189217	'
12		.1388	10		.202359	.202389	4	'	.189224	'
13		.168051	11		.168113	.167996	5	'	.189255	'
14		.363493	12		.178191	.177960	7	'	.189303	'
15		.485383	10		.229895	.229612	7	'	.189354	'
16		.5	6		.309386	.309127	7	'	.189396	'
17		.403273	5		.395385	.395220	7	'	.189417	'
18		.221079	5		.464871	.464845	6	'	.189411	'
19		.1179	6		.499244	.499364	4	.193093	.133381	'
20		.1078	7		.489303	.489538	3	.192799	.189334	'
21		.1035	8		.437709	.437997	3	.190407	.189282	'
22		.1100	9		.358273	.358273	3	'	.189241	'
23		.1235	10		.272258	.272429	3	'	.189220	'
24		.1405	11		.206910	.202721	4	'	.189225	'

[1]These numbers do not change from year to year.

TABLE XIV

CYCLICAL PRICE SERIES USED IN SIMULATION RUNS

Year	Cull Cow Price	Slaughter Heifer Price	Feeder Prices Steer	Feeder Prices Heifer
1	.1663	.2793	.2968	.2499
2	.1575	.2628	.2802	.2379
3	.1555	.2556	.2770	.2380
4	.1606	.2598	.2881	.2501
5	.1716	.2740	.3105	.2711
6	.1855	.2946	.3382	.2954
7	.1986	.3161	.3638	.3161
8	.2074	.3326	.3804	.3282
9	.2095	.3397	.3837	.3281
10	.2044	.3356	.3726	.3160
11	.1934	.3214	.3503	.2951
12	.1795	.3008	.3225	.2709
13	.1664	.2794	.2969	.2500
14	.1574	.2628	.2802	.2379
15	.1554	.2557	.2770	.2379
16	.1606	.2597	.2880	.2500
17	.1715	.2739	.3103	.2709
18	.1854	.2945	.3380	.2951
19	.1986	.3159	.3636	.3160
20	.2074	.3325	.3804	.3281
21	.2095	.3397	.3837	.3282
22	.2044	.3357	.3727	.3161
23	.1935	.3215	.3504	.2952
24	.1796	.3009	.3227	.2711

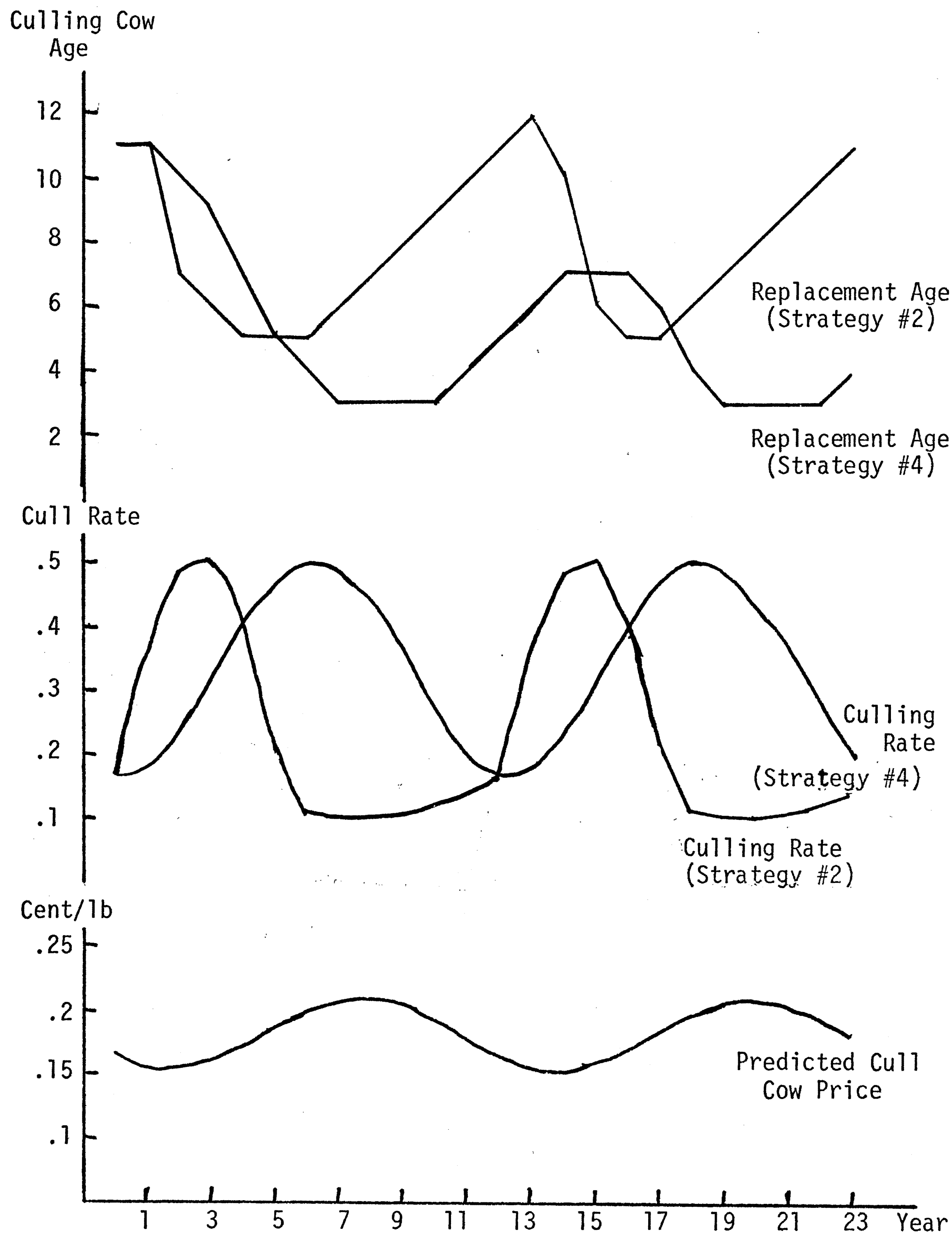

Figure 9. Cull Cow Prices and Culling Rate and Cow Age for Strategies 2 and 4.

The results of strategy #2 indicate the importance of the relationship between the internal make-up of the producer's herd and the beef price cycle. According to this strategy producers should both cull and replace heavily during the three years preceeding and following the bottom of the price cycle. This will enable the approximate 100 head of cows in the herd during the peak of the price cycle to by in large be in their most productive years, i.e., from four to seven years of age. This group of cows is then culled rapidly starting four years after the peak of the price cycle and simultaneously replaced by a group of young cows. In this manner, 100 head of the most productive age cows are available during the peak of the price cycle and a constant herd size near the minimum value of the firm's average total cost curve can be maintained.

It should be pointed out that the culling and replacement rates found under strategy #2 are not regular oscillatory patterns even though a trigometric functional form was specified for the control function. The cause of this is the necessity of holding culling and replacement rates equal to or greater than the uncontrolled culling required due to the failure of some cows to calves. Culling for failure to produce a calf averages about 11 percent of the herd per year but varies as a function of the age structure of the herd. Hence, the cyclical control function solutions are modified by constraints which force culling, and hence replacements in the strategy, to be at least as great as the culling rate due to failure to calf. This leads to the relative flat section of the culling time path curve during the "low portions" of the curve.

Strategy #3: Constant and Unequal Replacement and Culling Rates

Strategy #3 is similar to strategy #1 in that rates are held constant from year to year but unlike strategy #1, the culling rate and replacement rate are not held equal each year (there is one culling rate and a separate replacement rate used in the program). The strategy generates an almost constant herd size and a return higher than that of strategy #1 but lower than that of strategy #2 (Table XII). The culling and replacement rates are almost equal to the rate of strategy #1 with the culling rate slightly above the replacement rate. This causes the herd to be liquidated if the model was simulated over a very long time period (say 200 years) but provided no problems for the length of run used here. The strategy specifies the replacement of a brood cow after her 8th calf to be the optimal policy. This strategy, like the 1st, is limited by the inability to vary the culling and replacement rates from year to year.

Strategy #4: Cyclical and Unequal Replacement and Culling Rates

Strategy #4 is similar to strategy #2 in the respect that triometric functions of time are used as the control function equations. However in the case of strategy #4 the culling and replacement rates can be determined independently. This enables strategy #4 to take advantage of cyclical movements in price levels and to change the herd size over time if it is advantageous to do so.

A critical result of the optimization solution for strategy #4 was that the optimal solution found did not significantly change the herd size over time. Strategy #4 generated nearly equal, culling and replacement rates (which lead to a nearly constant herd size) the culling and replacement rate time pattern generated was cyclical in nature. The cyclical time path of culling and replacement rates generated was similar to that found by Strategy #2 (Table XII). This strategy, like strategy #2, attempts to maintain a constant herd size near the bottom of the U-shaped average total cost curve specified and to have the most productive set of cows possible in the herd when prices are peaking. Although the strategy is capable of varying the herd size (which is a capability strategy #2 did not have) it does not. This is evidently due to the determination by the optimizer that the cost encountered in varying the herd size (due to the U-shaped nature of the firm's cost curve) outweighed the added revenue.

Strategy #5: Closed Loop Control Function

In this strategy the expected net present value of brood cows, the slaughter value of heifers and the slaughter value of brood cows are used as variables in the control function. Since these variables are responsive to the culling and replacement rates specified the control functions specified are "closed loop control functions."

The replacement rate control function is specified as dependent upon the difference between the highest expected net present value of a feeder heifer and her current market value. The culling rate control function is specified to be a function of the average difference between the highest expected net present value and the current slaughter value for all brood cows whose current slaughter value exceeds their best expected net present value, i.e., those cows which are expected to lose money if kept in the herd. Given the total number of cows specified to be culled, those with the largest negative difference between expected net present value and current slaughter value are culled first. This is true for all of the strategies tested.

Strategy #5 generated a return somewhat higher than strategy #1 and #3 where culling and replacement rates were held constant. It did not generate as much profit as strategies #2 and #4 where cyclical culling and replacement rates were used. Strategy #5 resulted in a constant herd size and displayed very little oscillation in culling and replacement rates over time. Indeed the optimal culling and replacement rates found were very similar to those found in strategy #3. After some adjustments of the culling age during the first few years of the simulation, the optimal culling age for strategy #5 stabilized at 8 calving years (Table XIII). This is the same culling age as found in strategy #1 and #3.

It is the author's belief that strategy #5 would perform better relative to the other strategies if random variation were introduced in the price patterns used instead of regular cyclical movements. Irregular price patterns have not been considered in this study.

Summary

Comparing and contrasting the results of strategies 1 through 5, the optimal replacement policy seems to imply keeping a constant herd size while changing the internal make-up of the herd to generate higher returns. By keeping a younger herd during times of high or rising prices, cow-calf producers can take advantage of increased productivity during these periods. As the prices drop, the cows are kept to older ages and more of the heifer calves are sold rather than kept as replacements.

The point should be made that the solution shows much sensitivity to the level of interest rates (the discount rate used), specifying older replacement ages at higher discount rates. Runs were made for strategies 1 and 5 at a 4 percent interest rate (the one used for all simulation runs), a 10 percent interest rate and zero interest rate. In general as the interest rate decreased, the size of both the culling rate and replacement rates increased, thus decreasing the age at which the brood cows are replaced (Table XV). As the interest rates rise, cows will be kept in the herd longer.

TABLE XV

SENSITIVITY OF SOLUTION TO INTEREST RATES

Strategies	Interest Rate	Culling Rate	Replacement Age
1	0.00	.1953	8
	0.04	.1889	8
	0.10	.1767	9
5	0.00	.1933-.1953	8
	0.04	.1889-.1904	8
	0.10	.1767-.1793	10

CHAPTER V

SUMMARY, CONCLUSIONS, AND FUTURE APPLICATION

The results of this study indicate that a producer's best culling and replacement strategies is to maintain a relatively constant herd size but to vary the culling age of his cows between 5 and 11 years of age. Cows should be culled at relatively old ages during the down phase and bottom of the beef price cycle and at younger ages during the up phase and peak of the price cycle. This will lead to relatively heavy replacement rates and culling rates of approximately equal magnitudes, during a six year period centered on or slightly past the bottom of the price cycle. A six year period of relatively light culling and replacement rates should follow. In this manner a constant herd size will be maintained but the age structure of the herd will vary such that a high concentration of 4th through 7th calf bearing cows will be present at the peak of the cycle. These cows are the most productive cows in terms of successful calving rates and weights of calves produced. After the price cycle has peaked this group of cows will begin to age beyond their most productive age. Heavy culling at relatively old ages should then begin simultaneously with equal increases in the replacement rate. This will rid the herd of old unproductive cows and generate a new group of young cows which will reach their productive peak during the peak years of the price cycle.

It is of interest to note that a changing herd size was not found to be optimal in the analysis conducted even though prices received for beef animals were cyclical. This result must be interpreted in the light of the conditions assumed and control functions tested. In essence, this analysis fixed the land, machinery, equipment and a major portion of the labor resources available to the firm and assumed their opportunity cost, if not used, to be zero. Under these assumed conditions, a constant herd size with varying culling ages were found to be the optimal strategy. If a different cost structure were assumed the optimal strategy might be different. Further research investigating this question appears warranted.

Future Applications

This thesis has not exhausted all of the analytical potential offered by the model developed. The analysis conducted here concentrated on one cost structure and one estimated set of cyclical price patterns. Other cost structures reflecting different profit levels and/or with higher degrees of flexibility in resource use, i.e., flatter "U-shaped" cost curves, could be explored. Different price patterns, particularly irregular cyclical price patterns, could be explored. Different age related productive parameters for brood cows could be tested. It is the belief of the author that the optimal culling and replacement strategy will be sensitive to each of these factors and a fuller understanding of how cow-calf firms should react under different conditions can be obtained by analyzing the sensitivity of the control function parameters to changes in such factors.

A second area of further research lies in the area of the effect of price uncertainty and its influence upon culling and replacement decisions. Producers do not have perfect knowledge of future prices as assumed in this study. Rather decisions are based upon a set of expected prices and the profit flows resulting from the actual prices occurring rather than the expected prices. Various expectation models or random errors patterns of expectation could be used and their effect upon efficient culling and replacement decision making analyzed. The question arises as to whether, given unexpected changes in future prices, the same culling and replacement rate patterns found to be optimal under the conditions specified in this study would remain optimal.

A third area of research which may be addressed with the model is that of identifying what causes the cattle cycle and what can be done to alleviate the cycle. The results obtained in this study suggest that rationale cow-calf producers with fixed land resources and perfect knowledge will not create cyclical beef supplies even if prices received are cyclical or unstable in nature. They may, however, produce cyclical or unstable mixes in the type of animals slaughtered. Given this observation, a logical hypothesis to formulate is that the cattle cycle is caused by imperfect knowledge. Either cattle producers expect greater product price variation than occurs or they do not perceive (and may not all have) their cost structure to be as rigid, i.e., as sharply U-shaped as specified here. In this regard further study of the cost structure of the industry and the effect of these structures upon the optimal culling and replacement decision process is warranted.

Limitations

Two major limitations of the research conducted in this thesis should be reiterated. First, the possibility of alternative uses of resources classified as fixed in this study (land, machinery, equipment and most of the labor) and the ability to expand this set of resources temporarily by renting etc. exists and may make the average total cost curve less U-shaped. In this respect, it appears that the option of producing stocker cattle with nearly the same set of resources should be considered as an alternative to cow-calf production, indeed in some periods stocker production may be more profitable than cow-calf production.

A second limitation of the model is the assumption of perfect knowledge. All producers, no matter how well informed, must cope with uncertainty in relation to future prices and physical productivity of their herd. The existance of uncertainty may lead to different types of culling and replacement strategies being optimal rather than those found in this study.

A SELECTED BIBLIOGRAPHY

(1) Franzmann, John R. "Cattle Cycles Revisited." Southern Journal of Agricultural Economics, Vol. 3, 1971, pp. 69-76.

(2) Hamilton, Phillip C. "Fed and Non-fed Retail Beef Demand For 1970's." Unpublished paper, 1979.

(3) Foote, Richard. "Statistical Analysis Relating to the Feed-Livestock Economy." USDA Tech. Bulletin No. 1070, June, 1953.

(4) Breimyer, Harold F. "Observations on the Cattle Cycle." Agr. Econ. Res., Vol. VII, No. 1, January, 1955, pp. 1-11.

(5) Nerlove, Marc. "Distributed Lags and Estimation of Long-run Supply and Demand Elasticities: Theoretical Considerations." Journal of Farm Economics, Vol. 40, pp. 301-311.

(6) Walters, Forrest. "Predicting of the Beef Inventory." Agr. Econ. Res., Vol. XVII, No. 1, January, 1965, pp. 10-18.

(7) Langemeier, Lary and R. G. Thompson. "Demand, Supply, and Price Relationships for the Beef Sector, Post-World II Period." Journal of Farm Economics, Vol. 49, 1967, pp. 169-183.

(8) Crom, Richard. "A Dynamic Price-Output Model of the Beef and Pork Sector." USDA, Econ. Res. Ser., Tech. Bulletin No. 1426, 1970.

(9) Crom, Richard. "Economic Projections Using a Behavorial Model." Agr. Econ. Res., Vol. 24, No. 1, January, 1972, pp. 9-15.

(10) Heien, Dale. "Price Determination Processes for Agricultural Sector Models." Unpublished paper, 1976.

(11) Keith, Kendall and Wayne Purcell. "The Beef Cycle of the 1970's--Analysis, Behaviorial Dimensions, Outlook and Projections." Oklahoma State University Bulletin No. B-721, 1976.

(12) Faris, J. Edwin. "Analytical Techniques Used in Determining the Optimum Replacement Pattern." Journal of Farm Economics, Vol. 42, November, 1960, pp. 755-766.

(13) Burt, Oscar R. "Optimal Replacement Under Risk." Journal of Farm Economics, Vol. 47, May, 1965, pp. 324-346.

(14) Chisholm, Anthony H. "Criteria for Determining the Optimal Replacement Pattern." Journal of Farm Economics, Vol. 48, February, 1966, pp. 107-112.

(15) Rogers, LeRoy F. "Replacement Decision for Commercial Beef Herds." Washington Agricultural Experiment Sta. Bull. 726, 1971.

(16) Rogers, LeRoy F. "Economics of Replacement Rates in Commercial Beef Herds." Journal of Animal Science, Vol. 34 No. 6, 1972, pp. 921-925.

(17) Perrin, R. K. "Asset Replacement Principles." American Journal of Agricultural Economics, Vol. 54, February, 1972, pp. 60-67.

(18) Nelson, Kenneth E. and Wayne D. Purcell. "A Quantitative Approach to the Feedlot Replacement Decision." Southern Journal of Agricultural Economics, Vol. 8, No. 2, Dec., 1976, pp. 13-18.

(19) Bentley, Ernest, James R. Waters and C. Richard Shumway. "Determining Optimal Replacement Age of Beef Cows in the Presence of Stochastic Elements." Southern Journal of Agricultural Economics, Vol. 8, No. 2, Dec. 1976, pp. 13-18.

(20) Kay, Ronald D. and Edward Rister. "Income Tax Effects on Beef Cow Replacement Strategy." Southern Journal of Agricultural Economics, Vol. 9, No. 1, 1977, pp. 169-172.

(21) Box, M. J. "A New Method of Constrained Optimization and a Comparison with Other Methods." Computer Journal, Vol. 8, 1965, pp. 42-52.

(22) Kuester, James L. and Joe H. Mize. Optimization Techniques with Fortran. McGraw Hill Book Co., New York, 1973.

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