SYNTHESIS OF AN R.S.S.R MECHANISM FOR FUNCTION GENERATION AND FOR REPLACING HYPOID GEARS USING HIGHER ORDER SPACE PATH CURVATURE THEORY

By

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Thesis Approved:

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CHAPTER I

INTRODUCTION

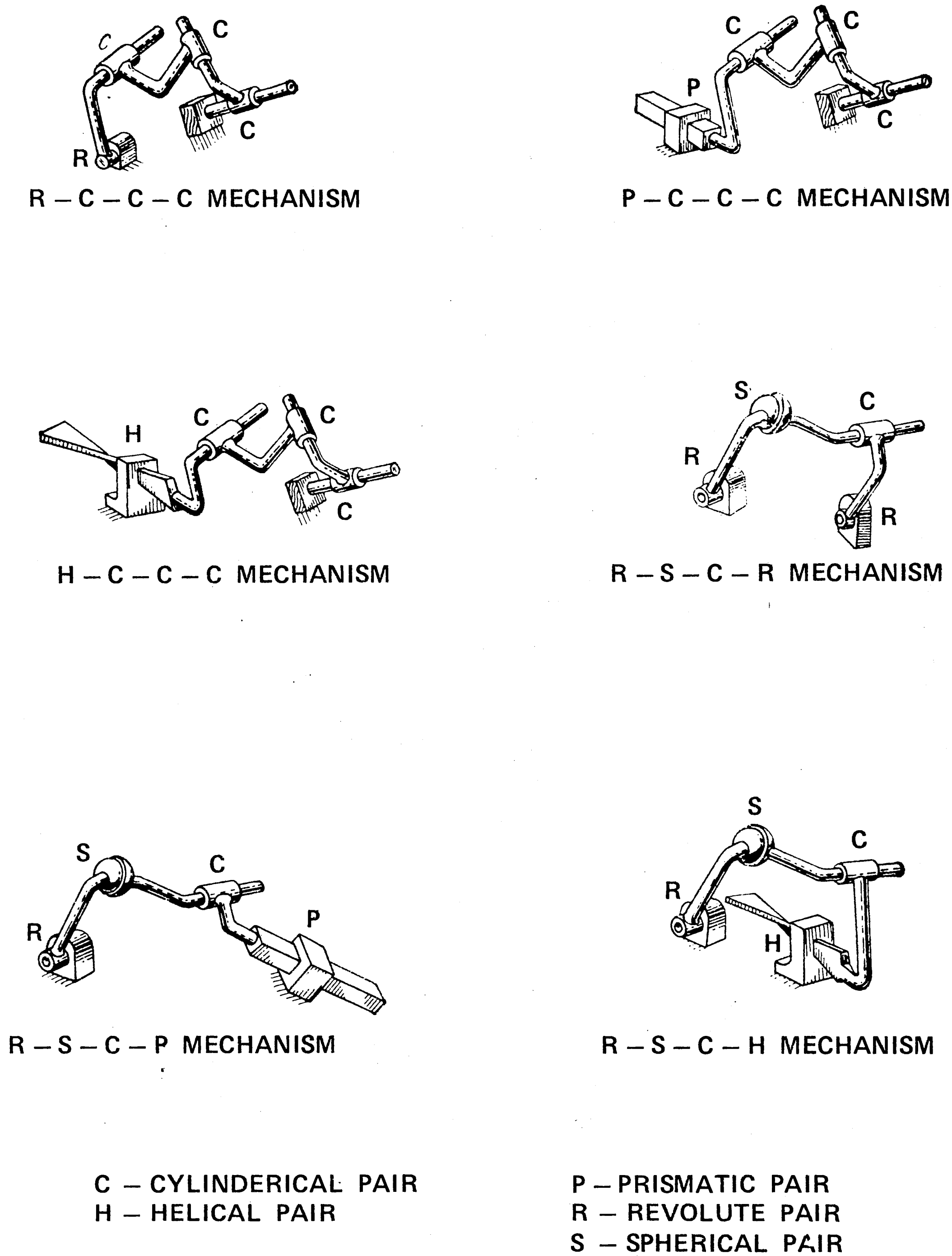
1.1 General

One of the current areas of keen interest in mechanism research is the vast domain of three dimensional linkages, frequently called the space mechanisms. There are potentially hundreds of them. But only a few kinds have been investigated or described.

A space mechanism can exist with a wide variety of connecting joints or pair combinations. Detailed examination of the various kinds of space mechanisms showed many of these to be mechanically complex and of limited adaptability. But the four link mechanisms have particular appeal because of their mechanical simplicity. Figures 1a and 1b show the best of a class of four link space mechanisms according to Harrisberger (1).

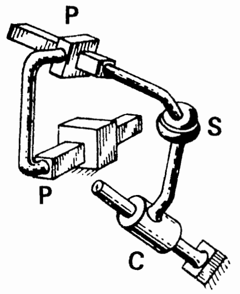
Among four link space mechanisms it is well known that the R.S.S.R mechanism (see Figure 1b) is an outstanding choice as the most versatile and practical configuration capable of giving double crank motion requirements.

An R.S.S.R crank rocker mechanism consists of three movable links mounted to a fixed link or frame. The driver is the rotating link. The follower is the oscillating link. The coupler is the moving link between the driver and the follower connected to them by spherical joints. The driver and the follower are fixed to the frame by means to two revolute

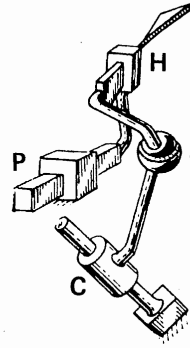


A

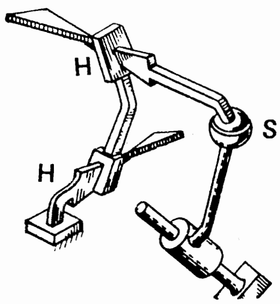
Figure 1. Examples of Space Mechanisms According to Harrisberger



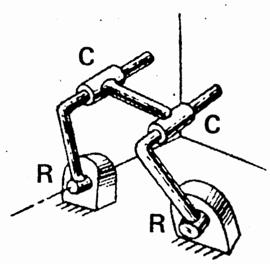
P – P – S – C MECHANISM



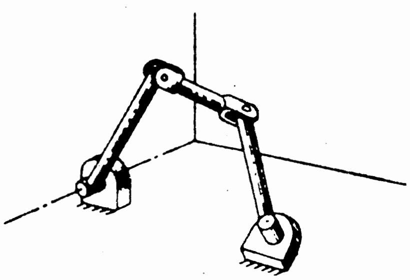
P – H – S – C MECHANISM



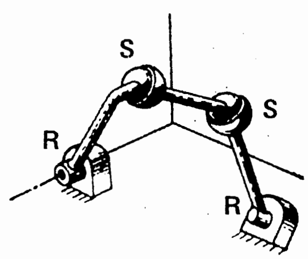
H – H – S – C MECHANISM



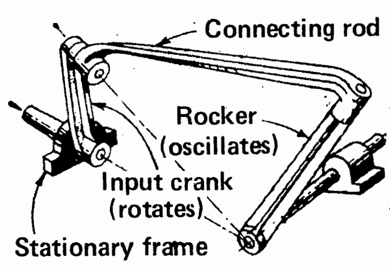
R – C – C – R MECHANISM



BENNET R – R – R – R MECHANISM



R – S – S – R MECHANISM



SPACE CRANK MECHANISM

B

Figure 1. (Continued)

joints. Basically it is a two degree of freedom mechanism with a passive degree of freedom of the coupler motion about its own axis. The practical simplicity of the mechanism and its unlimited geometric adaptability justifies its importance as a practical and useful space mechanism.

1.2 Problem Statement

Function generation is one of the important purposes for which a mechanism is often synthesized. For function generation a mechanism is so synthesized that the output motion is a desired function of the input motion, both motions being rotary or oscillatary in most cases. Thus, a function generating mechanism essentially converts a uniform motion into another uniform motion or nonuniform motion. Circular gears, chains, belts, and the like comprise most of the uniform motion convertors. Nonuniform conversions are made with noncircular gears, cam follower systems, ratchets and linkages.

In this thesis, the synthesis of a spatial R.S.S.R linkage for function generation between two links mounted on nonintersecting skew shafts is presented. The accuracy achieved is up to fourth order while in the existing literature it is only up to third order as discussed in the later sections of this thesis. A simple method of analysis up to fourth order is also presented to check the correctness of the synthesized linkage and its accuracy in neighboring positions.

1.3 Literature Review

In most engineering fields gears and cams are used for motion transmission with uniform and nonuniform ratios of transmission. It has been an old dream of the kinematicians to find equivalent substitutes of gears and cams by linkages. They came to the conclusion that it is possible to replace gears and cams by linkages if small deviations from the ratios of transmission are permissible. The mechanism can be manufactured cheaper and easier.

Hall and Dunk (2) developed procedure for designing planar four bar linkages as a simple and effective substitute for more expensive gears. This design was suitable for transmission of a substantial constant angular velocity ratio for a limited range of angular motion.

Freudenstein (3) presented time saving self-explanatory tables on planar four bar function generators, illustrating the linkage types, functions, ranges and accuracies possible.

Hain (4) presented a practical method for designing planar four bar linkages for oscillatary motion with approximate constant transmission ratio within prescribed tolerances.

Harrisberger (5) described a simple method for synthesizing an R.S.S.R mechanism for finite displacements.

Scroggin and Morse (6) presented the relationships for the synthesis of R.S.S.R mechanism up to second order.

Suh (7) (8) presented matrix methods for the synthesis of R.S.S.R function generators up to third order. A special case of the function generators serve to replace hypoid gears up to third order.

Mohanrao, Sandor, Kohli, and Soni (9) presented methods to synthesize R.S.S.R mechanism for seven finite positions.

Recently, Chunsiripong and Soni (10) developed mathematical procedures to synthesize R.S.S.R mechanism to coordinate motions of input and output links for their finite and infinitesmal displacements.

However, the infinitesmal synthesis is carried out up to third order only.

The existing literature shows the following:

1. The kinematic analysis was carried out up to third order only.

2. In most cases the synthesis was done for finite displacements.

3. In case of higher order synthesis, the order achieved was up to three only.

4. Function generation synthesis was carried out using inversion techniques.

5. The design equations were obtained using the constant length constraint on the coupler.

To further the state of art in R.S.S.R mechanism synthesis, the following objectives are determined in this study.

1.4 Objectives of the Study

1. To synthesize the R.S.S.R mechanism up to fourth order.

2. To use a new approach other than the constant length criterian of the coupler length.

3. To use the higher order path curvature theory to determine locus of points that describe spherical paths up to fourth order.

4. To use the above locus to determine the spherical pairs on the coupler.

5. To synthesize R.S.S.R mechanism to replace gears up to fourth order.

6. To analyze the R.S.S.R mechanism up to fourth order to determine the correctness of the synthesized mechanism and its accuracy in neighboring positions.

The above objectives of the study are made possible by the most recent studies by Siddhanty and Soni (11) on higher order path curvature theory.

CHAPTER II

SYNTHESIS

2.1 Configuration

Figure 2 shows the configuration of an R.S.S.R mechanism. $\overline{A}$ and $\overline{B}$ are the input and the output rotation axes which are fixed on a frame. A_oB_o is the shortest distance and α is the angle between them.

Let A_oAS_A be the input crank rotating about axis $\overline{A}$. A_o is the location of revolute pair and S_A is the location of spherical pair. We denote AS_A as the input crank length and A_oA as the input crank offset length.

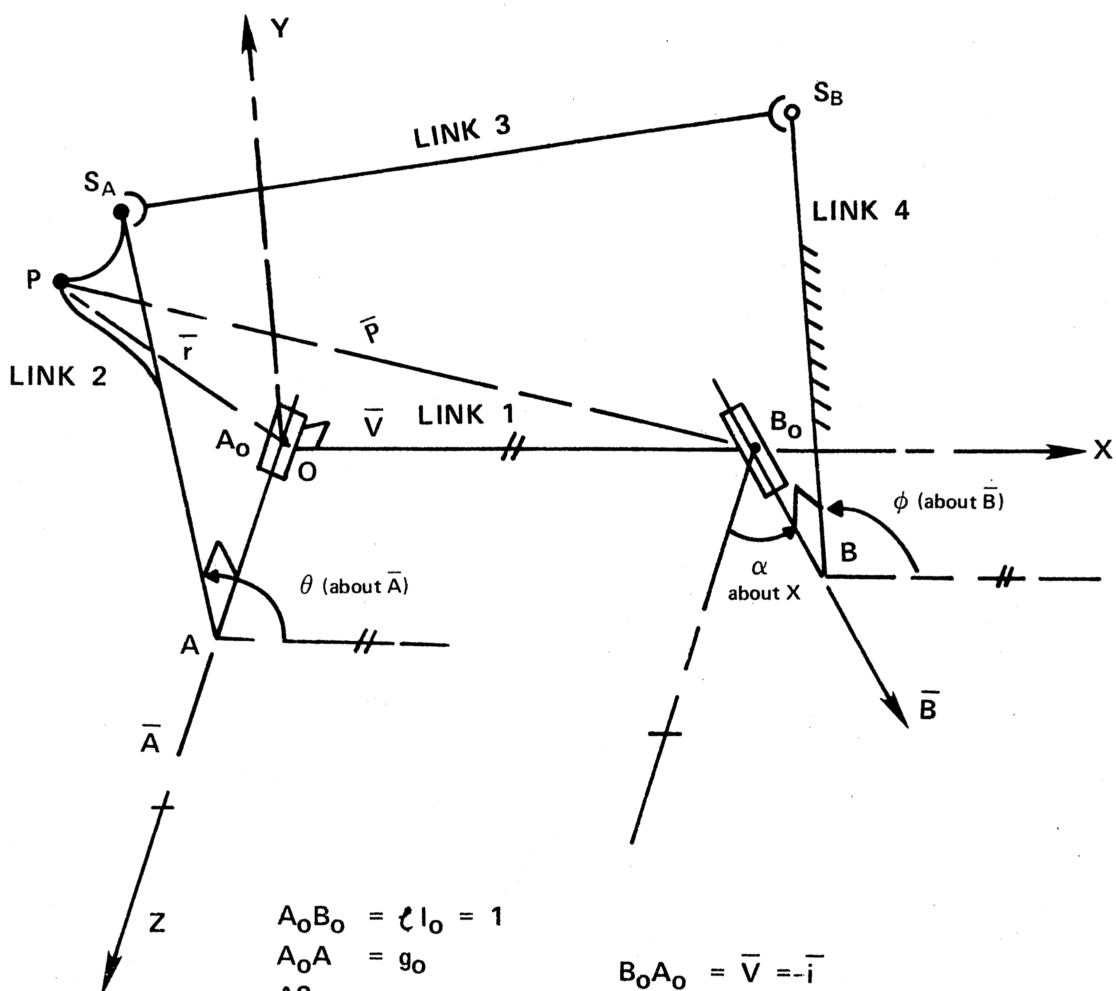
Let B_oBS_B be the output crank rotating about axis $\overline{B}$. B_o is the location of revolute pair and S_B is the location of spherical pair. We denote BS_B as the output crank length and B_oB as the output crank offset length.

Let S_AS_B be the coupler connecting the input and output cranks at spherical pairs S_A and S_B.

For simplicity we establish the following coordinate system.

Let OXYZ be the fixed coordinate system with its origin at A_o, X-axis along A_oB_o and Z-axis along $\overline{A}$. Y-axis is determined by the right hand rule.

Let $\overline{i}$, $\overline{j}$, $\overline{k}$ be the unit vectors along X, Y, and Z-axis. Let α be the angle of skew of the output axis $\overline{B}$ relative to the input axis $\overline{A}$



$A_oB_o = \ell l_o = 1$
$A_oA = g_o$
$AS_A = g$
$S_AS_B = \ell$
$B_oB = h_o$
$BS_B = h$

$B_oA_o = \bar{V} = -\bar{i}$
$A_oP = \bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$
$B_oP = \bar{P} = \bar{V} + \bar{r}$

Figure 2. R.S.S.R Mechanism

measured about the X-axis. It is the angle from vector $\overline{A_oA}$ to vector $\overline{B_oB}$.

Let us denote A_oB_o as the Link 1 which is fixed, A_oAS_A as Link 2, S_AS_B as Link 3 and B_oBS_B as Link 4.

Let Θ be the input crank rotation angle measured about the axis $\overline{A}$. It is the angle from vector $\overline{A_oB_o}$ to vector $\overline{AS_A}$.

Let ϕ be the output crank rotation angle measured about the axis $\overline{B}$; it is the angle from vector $\overline{A_oB_o}$ to vector $\overline{BS_B}$.

In the given coordinate system we have

$$\overline{A} = \overline{K} \tag{2.1}$$

$$\overline{B} = - \text{Sin } \alpha\overline{j} + \text{Cos } \alpha\overline{k} \tag{2.2}$$

We note $\overline{A}$ and $\overline{B}$ are unit vectors.

2.2 Function Generation Relationships

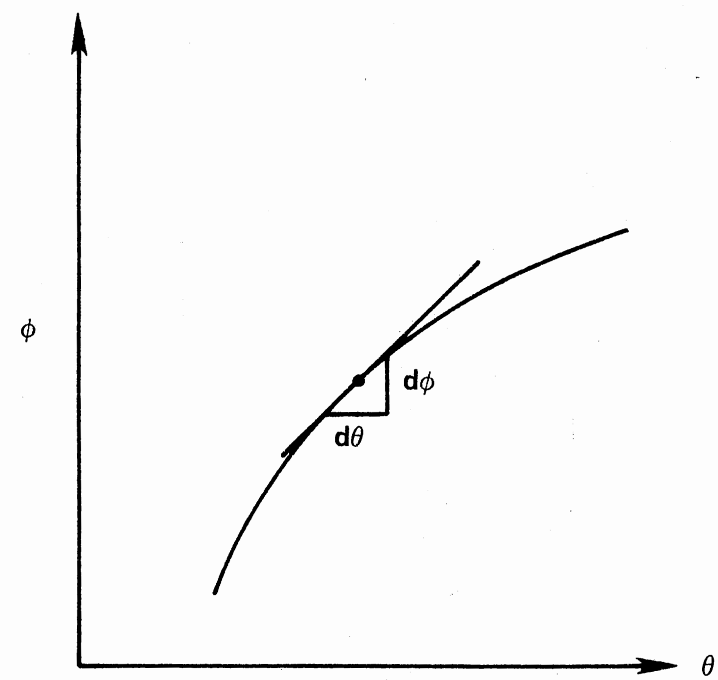
Figure 3 shows the functional relationship between input and output crank motions.

Let $\phi = f(\Theta)$. (2.3)

Since the number of links in the mechanism are limited, the synthesized mechanism in general can satisfy the function (2.3) only at finite number of points in case of finite synthesis, or up to a finite number of derivatives in the case of derivative synthesis. In this thesis we are concerned with derivative synthesis of the R.S.S.R mechanism up to fourth order. This means we are going to synthesize the R.S.S.R mechanism so that the derivatives $\frac{d\phi}{d\Theta}$, $\frac{d^2\phi}{d\Theta^2}$, $\frac{d^3\phi}{d\Theta^3}$, and $\frac{d^4\phi}{d\Theta^4}$ are realized in the synthesized mechanism at a given instant.

Let

$$\frac{d\phi}{d\Theta} = n_1$$



$\phi = f(\theta)$

Figure 3. Functional Relationship
Between Θ and ϕ

$$\frac{d^2\phi}{d\Theta^2} = n_2$$

$$\frac{d^3\phi}{d\Theta^3} = n_3$$

$$\frac{d^4\phi}{d\Theta^4} = n_4 \qquad (2.4)$$

The case, in which n_1 is less than zero and $n_2 = n_3 = n_4 = 0$, represents a gearing relationship up to fourth order. A mechanism for this gear relationship can replace a set of hypoid gears with sufficient accuracy in the neighborhood of the synthesized position of the linkage. Higher the order of derivative synthesis, more will be the accuracy of gearing ratio in the neighborhood.

In the synthesis procedure we take time as the independent motion parameter. Hence, input and output motions are expressed as functions of time. So we can express the derivatives in Equation (2.4) as function of time as follows. The dots represent differentiation with respect to time.

$$n_1 = \frac{d\phi}{d\Theta} = \frac{\dot{\phi}}{\dot{\Theta}} \qquad (2.5)$$

$$n_2 = \frac{d^2\phi}{d\Theta^2} = (\ddot{\phi}\,\dot{\Theta} - \dot{\phi}\,\ddot{\Theta})/\dot{\Theta}^3 \qquad (2.6)$$

$$n_3 = \frac{d^3\phi}{d\Theta^3} = [(\dddot{\phi}\,\dot{\Theta} - \dot{\phi}\,\dddot{\Theta})\dot{\Theta} - 3(\ddot{\phi}\,\dot{\Theta} - \dot{\phi}\,\ddot{\Theta})\,\ddot{\Theta}\,]/\dot{\Theta}^5 \qquad (2.7)$$

$$n_4 = \frac{d^4\phi}{d\Theta^4} = [(\ddddot{\phi}\,\dot{\Theta} + \dddot{\phi}\,\ddot{\Theta} - \ddot{\phi}\,\dddot{\Theta} - \dot{\phi}\,\ddddot{\Theta})\dot{\Theta}^2 -$$

$$7(\dddot{\phi}\,\dot{\Theta} - \dot{\phi}\,\dddot{\Theta})\,\ddot{\Theta}\,\dot{\Theta} + (\ddot{\phi}\,\dot{\Theta} - \dot{\phi}\,\ddot{\Theta})(15\,\ddot{\Theta}^2 -$$

$$3\,\dddot{\Theta}\,\dot{\theta})]/\dot{\Theta}^7 \qquad (2.8)$$

For the sake of simplicity, we can assume

$$\dot{\Theta} = 1, \ \ddot{\Theta} = 0, \ \dddot{\Theta} = 0, \ \ddddot{\Theta} = 0. \tag{2.9}$$

Then we have

$$\begin{aligned} n_1 &= \dot{\phi} \\ n_2 &= \ddot{\phi} \\ n_3 &= \dddot{\phi} \\ n_4 &= \ddddot{\phi} \end{aligned} \tag{2.10}$$

2.3 The Principle of Inversion

In Figure 2 we have Link 1 fixed. However, if we fix Link 4 instead of Link 1 the synthesis procedure will be easier. A mechanism obtained by fixing an alternate link is known as the inversion of the original mechanism.

Figure 4 shows an inversion of the mechanism in Figure 2. We note that Link 4 is fixed in the inversion. It is to be noted that the relative motion of a link with respect to any other link in the mechanism is not altered in its inversion. Further, we can assume A_oB_o as unity without loss of generality in the functional relationship between Θ and ϕ. Now when n_1, n_2, n_3, n_4 and α are specified we can determine the locations A_o, and B_o, directions $\overline{A}$ and $\overline{B}$ and the relative motions of Link 1 with respect to Link 4 and Link 2 with respect to Link 4. The mechanism is synthesized when we determine the locations of S_A and S_B. In the inversion S_B is a point on Link 4 which is fixed. S_A is a point moving on a sphere with S_AS_B as the radius. We use this property of S_A

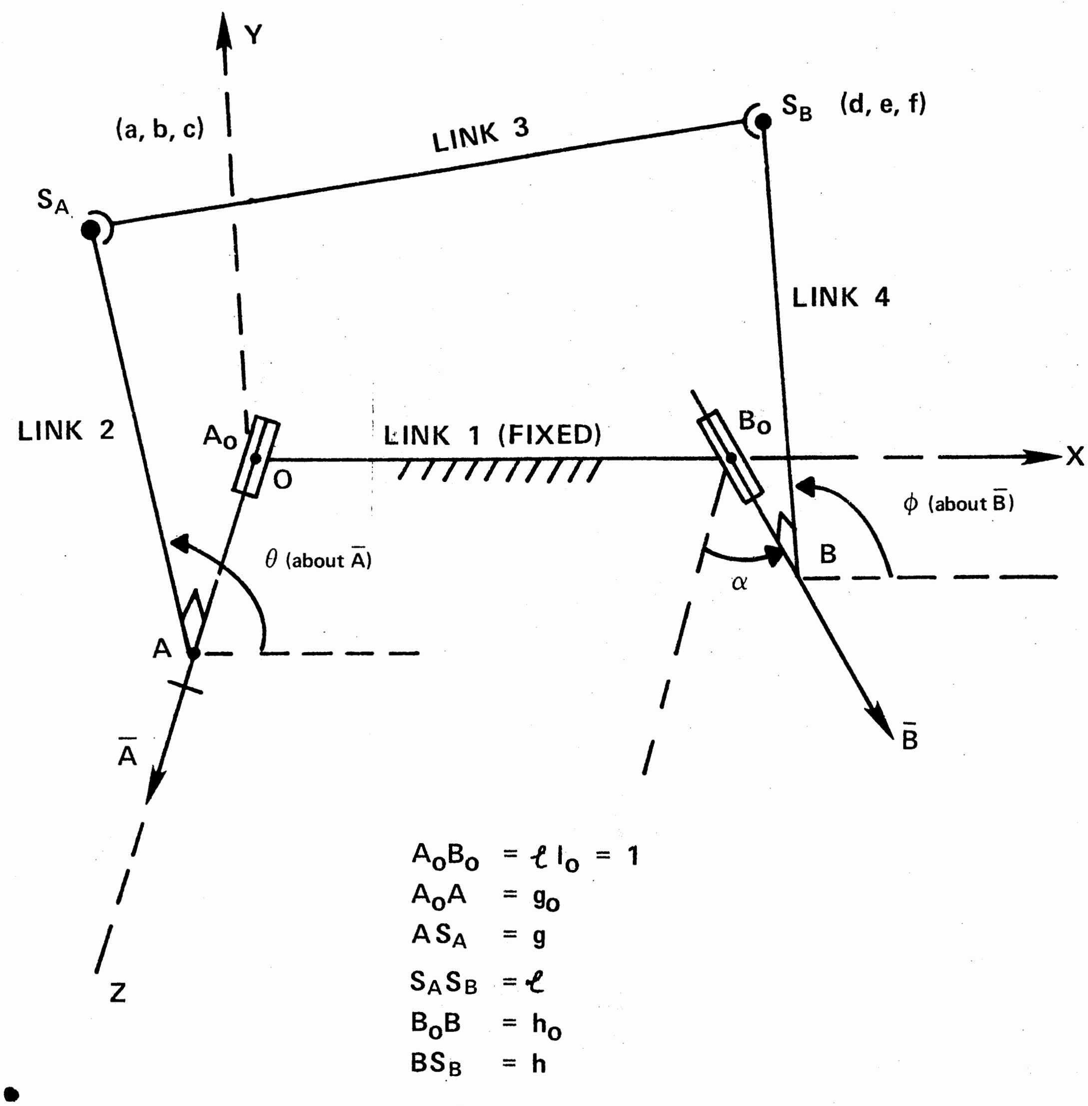


Figure 4. Inversion of R.S.S.R Mechanism

having a spherical path as the synthesis technique. We note S_A, being a pair, is also considered as a point on Link 2. Further, we can find the locus of points in Link 2 that describe spherical paths up to fourth order. Any point on this locus can serve as a spherical joint S_A. Having determined S_A, we can determine also the center of sphere of the spherical path which will yield S_B. These are derived in the following sections.

2.4 Relative Angular Motions

Referring to Figure 4, we have the instantaneous angular motion of Link 1, with respect to Link 4 is given up to fourth order as follows:

$$
\begin{array}{lll}
\text{First Order} & \overline{\omega}_{1/4} = -\dot{\phi}\,\overline{B} & \text{Velocity} \\
\text{Second Order} & \dot{\overline{\omega}}_{1/4} = -\ddot{\phi}\,\overline{B} & \text{Acceleration} \\
\text{Third Order} & \ddot{\overline{\omega}}_{1/4} = -\dddot{\phi}\,\overline{B} & \text{Jerk} \\
\text{Fourth Order} & \dddot{\overline{\omega}}_{1/4} = -\ddddot{\phi}\,\overline{B} & \text{Kerk}
\end{array}
\qquad (2.11)
$$

Relative motion of Link 2 with respect to Link 1 is given by

$$
\begin{array}{lll}
\text{First Order} & \overline{\omega}_{2/1} = \dot{\Theta}\,\overline{A} & \text{Velocity} \\
\text{Second Order} & \dot{\overline{\omega}}_{2/1} = \ddot{\Theta}\,\overline{A} & \text{Acceleration} \\
\text{Third Order} & \ddot{\overline{\omega}}_{2/1} = \dddot{\Theta}\,\overline{A} & \text{Jerk} \\
\text{Fourth Order} & \dddot{\overline{\omega}}_{2/1} = \ddddot{\Theta}\,\overline{A} & \text{Kerk}
\end{array}
\qquad (2.12)
$$

Motion of Link 2 with respect to Link 4 is given by

$$\overline{\omega}_{2/4} = \overline{\omega}_{2/1} - \overline{\omega}_{4/1}$$

$$= \overline{\omega}_{2/1} - (-\overline{\omega}_{1/4}) \text{ (since } \overline{\omega}_{4/1} = -\overline{\omega}_{1/4})$$

$$= \overline{\omega}_{2/1} + \overline{\omega}_{1/4}$$

$$= \dot{\Theta}\,\overline{A} - \dot{\phi}\,\overline{B} \qquad (2.13)$$

$$\dot{\overline{\omega}}_{2/4} = \ddot{\Theta}\,\overline{A} + \dot{\Theta}\,\dot{\overline{A}} - \ddot{\phi}\,\overline{B} \qquad (2.14)$$

$$\ddot{\overline{\omega}}_{2/4} = \dddot{\Theta}\,\overline{A} + 2\,\ddot{\Theta}\,\dot{\overline{A}} + \dot{\Theta}\,\ddot{\overline{A}} - \dddot{\phi}\,\overline{B} \qquad (2.15)$$

$$\dddot{\overline{\omega}}_{2/4} = \ddddot{\Theta}\,\overline{A} + \dddot{\Theta}\,\dot{\overline{A}} + 2\,\dddot{\Theta}\,\dot{\overline{A}} + 2\,\ddot{\Theta}\,\ddot{\overline{A}} + \ddot{\Theta}\,\ddot{\overline{A}} + \dot{\Theta}\,\dddot{\overline{A}} - \ddddot{\phi}\,\overline{B}$$

$$= \ddddot{\Theta}\,\overline{A} + 3\,\dddot{\Theta}\,\dot{\overline{A}} + 3\,\ddot{\Theta}\,\ddot{\overline{A}} + \dot{\Theta}\,\dddot{\overline{A}} - \ddddot{\phi}\,\overline{B} \qquad (2.16)$$

In this inversion case $\overline{A}$ is a rotating vector about $\overline{B}$. Therefore,

$$\dot{\overline{A}} = -\dot{\phi}\,\overline{B} \times \overline{A} = -[\dot{\phi}\,(\overline{B} \times \overline{A})] \qquad (2.17)$$

$$\ddot{\overline{A}} = -\ddot{\phi}\,\overline{B} \times \overline{A} + (-\dot{\phi}\,\overline{B} \times \dot{\overline{A}})$$

$$= -[\ddot{\phi}\,(\overline{B} \times \overline{A}) + \dot{\phi}(\overline{B} \times \dot{\overline{A}})] \qquad (2.18)$$

$$\dddot{\overline{A}} = -\dddot{\phi}\,\overline{B} \times \overline{A} - \ddot{\phi}\,\overline{B} \times \dot{\overline{A}} + (-\ddot{\phi}\,\overline{B} \times \dot{\overline{A}}) + (-\dot{\phi}\,\overline{B} \times \ddot{\overline{A}})$$

$$= -[\dddot{\phi}\,(\overline{B} \times \overline{A}) + 2\,\ddot{\phi}\,(\overline{B} \times \dot{\overline{A}}) + \dot{\phi}\,(\overline{B} \times \ddot{\overline{A}})] \qquad (2.19)$$

2.5 Analysis of Point Motion

Referring to Figure 4, let P be a point on Link 2 noting B_o is a fixed point in the inversion. Link 1 rotates about $\overline{B}$ and Link 2 rotates about the moving axis $\overline{A}$ in Link 1.

The motion of P up to fourth order is considered as a vector sum of motion of point A_o with respect to fixed point B_o and relative motion of P with respect to moving point A_o. It is given by

$$\bar{P} = \bar{V} + \bar{r}$$

$$\dot{\bar{P}} = \dot{\bar{V}} + \dot{\bar{r}}$$

$$\ddot{\bar{P}} = \ddot{\bar{V}} + \ddot{\bar{r}}$$

$$\dddot{\bar{P}} = \dddot{\bar{V}} + \dddot{\bar{r}}$$

$$\ddddot{\bar{P}} = \ddddot{\bar{V}} + \ddddot{\bar{r}} \qquad (2.20)$$

where the position and derivative motion of point A_o with respect to fixed point B_o is given by

$$\bar{V} = -\bar{i}$$

$$\dot{\bar{V}} = \bar{\omega}_{1/4} \times \bar{V}$$

$$\ddot{\bar{V}} = \dot{\bar{\omega}}_{1/4} \times \bar{V} + \bar{\omega}_{1/4} \times \dot{\bar{V}}$$

$$\dddot{\bar{V}} = \ddot{\bar{\omega}}_{1/4} \times \bar{V} + 2(\dot{\bar{\omega}}_{1/4} \times \dot{\bar{V}}) + \bar{\omega}_{1/4} \times \ddot{\bar{V}}$$

$$\ddddot{\bar{V}} = \dddot{\bar{\omega}}_{1/4} \times \bar{V} + 3\, \ddot{\bar{\omega}}_{1/4} \times \dot{\bar{V}} + 3\, \dot{\bar{\omega}}_{1/4} \times \ddot{\bar{V}} + \bar{\omega}_{1/4} \times \dddot{\bar{V}} \qquad (2.21)$$

and relative derivative motion of P with respect to the moving point A_o is given by

$$\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$$

$$\dot{\bar{r}} = \bar{\omega}_{2/4} \times \bar{r}$$

$$\ddot{\bar{r}} = \dot{\bar{\omega}}_{2/4} \times \bar{r} + \bar{\omega}_{2/4} \times \dot{\bar{r}}$$

$$\dddot{\bar{r}} = \ddot{\bar{\omega}}_{2/4} \times \bar{r} + 2(\dot{\bar{\omega}}_{2/4} \times \dot{\bar{r}}) + \bar{\omega}_{2/4} \times \ddot{\bar{r}}$$

$$\ddddot{\bar{r}} = \dddot{\bar{\omega}}_{2/4} \times \bar{r} + 3(\ddot{\bar{\omega}}_{2/4} \times \dot{\bar{r}}) + 3(\dot{\bar{\omega}}_{2/4} \times \ddot{\bar{r}}) + \bar{\omega}_{2/4} \times \dddot{\bar{r}} \qquad (2.22)$$

So the path of any general point P is determined by its coordinates X, Y, Z and the other known parameters obtained from the synthesis specifications, and the determined mechanism parameters.

2.6 Differential Geometry of the Point Path

From differential geometry (12) and referring to Figure 5, we have the following relationships for the path of point $\bar{P}$.

Tangent $$\bar{t} = \frac{\dot{\bar{P}}}{|\dot{\bar{P}}|} \tag{2.23}$$

Bi-Normal $$\bar{b} = \frac{\dot{\bar{P}} \times \ddot{\bar{P}}}{|\dot{\bar{P}} \times \ddot{\bar{P}}|} \tag{2.24}$$

Normal $$\bar{n} = \bar{b} \times \bar{t} \tag{2.25}$$

The radius of curvature $$\rho = \left[\frac{B^3}{A}\right]^{1/2} \tag{2.26}$$

where

$$B = \dot{\bar{P}} \cdot \dot{\bar{P}} \quad \text{(dot product of } \bar{P} \text{ with } \bar{P}) \tag{2.27}$$

$$A = (\dot{\bar{P}} \times \ddot{\bar{P}}) \cdot (\dot{\bar{P}} \times \ddot{\bar{P}}) \tag{2.28}$$

Rate of change of radius of curvature with respect to arc length s along the path of P is given by

$$\frac{d\rho}{ds} = \frac{\dot{\rho}}{\dot{s}} \tag{2.29}$$

where

$$\dot{\rho} = \frac{3}{2} \times \left[\frac{B^{1/2}(\dot{B})}{A^{1/2}}\right] + \left(-\frac{1}{2}\right)\left[\frac{B^{3/2}\ \dot{A}}{A^{3/2}}\right] \tag{2.30}$$

and

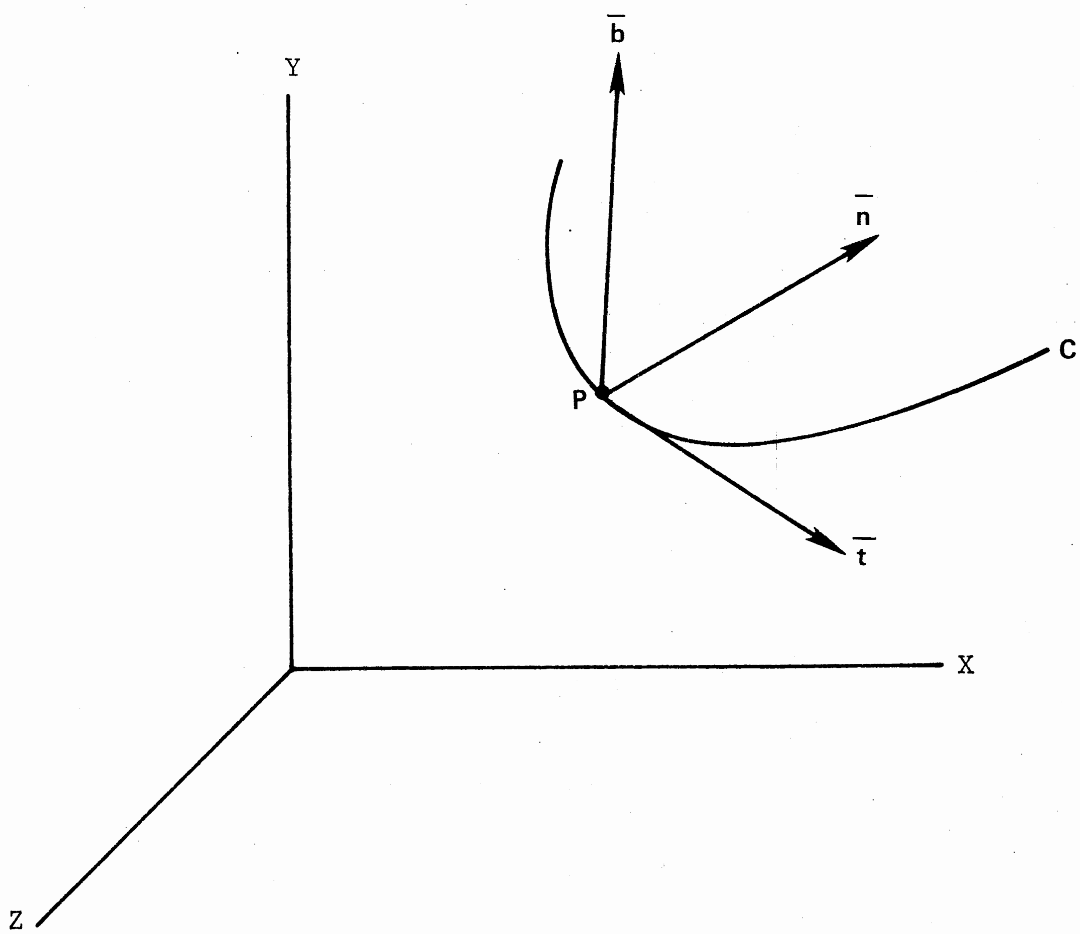


Figure 5. Space Curve C with Frenet Frame of Reference at Point P

$$\dot{s} = B^{1/2} \tag{2.31}$$

$$\frac{d\rho}{ds} = \frac{3}{2} \frac{\dot{B}}{A^{1/2}} + (-\frac{1}{2}) \frac{B\dot{A}}{A^{3/2}} \tag{2.32}$$

The second rate of change of radius of curvature with respect to arc length s along the path of P is given by

$$\frac{d^2\rho}{ds^2} = \frac{d}{dt}\left[\frac{\dot{\rho}}{\dot{s}}\right]\frac{dt}{ds}$$

$$= \frac{d}{dt}\left[\frac{\dot{\rho}}{\dot{s}}\right]\frac{1}{\dot{s}}$$

$$= \frac{\dot{s}\ddot{\rho} - \dot{\rho}\ddot{s}}{(\dot{s})^2} \cdot \frac{1}{\dot{s}}$$

$$= \frac{\dot{s}\ddot{\rho} - \dot{\rho}\ddot{s}}{(\dot{s})^3} \tag{2.33}$$

The radius of torsion of the path

$$\sigma = \frac{A}{C} \tag{2.34}$$

where

$$C = (\overline{\dot{P}} \times \overline{\ddot{P}}) \cdot \overline{\dddot{P}} \tag{2.35}$$

The rate of change of radius of torsion of this path with respect to the arc length s along the path P is given by

$$\frac{d\sigma}{ds} = \frac{d\sigma}{dt} \cdot \frac{dt}{ds} = \frac{\dot{\sigma}}{\dot{s}} \tag{2.36}$$

$$\dot{\sigma} = (C\dot{A} - A\dot{C})/C^2 \tag{2.37}$$

$$\frac{d\sigma}{ds} = (C\dot{A} - A\dot{C})/C^2\dot{s} \tag{2.38}$$

The equation for the radius of sphere for the path is given by

$$R^2 = \rho^2 + (\sigma)^2\left[\frac{d\rho}{ds}\right]^2 \tag{2.39}$$

The condition for the radius of the sphere of the path to be constant is

$$\frac{dR}{ds} = 0 \tag{2.40}$$

This is a fourth order condition.

Substituting for R in Equation (2.40) and simplifying we get

$$\rho + \sigma\left[\frac{d\sigma}{ds}\right]\left[\frac{d\rho}{ds}\right] + (\sigma)^2\left[\frac{d^2\rho}{ds^2}\right] = 0 \tag{2.41}$$

The terms in Equation (2.41) are functions of X, Y and Z. Hence, Equation (2.41) represents a characteristic surface. The points on this surface have the property that their paths are spherical up to fourth order. The highest power of the terms in the equation is 20. The equation has many terms having multiples of half power. Hence, it is a very complex surface to visualize.

2.7 Synthesis of Spherical Joint S_A and Spherical Joint S_B

Since Equation (2.41) is a characteristic surface of points with spherical paths up to fourth order, we can pick any point on it and consider that as a spherical joint S_A. To do this we employ a numerical technique.

In Equation (2.41) let us assume Y and Z. Then we find Equation (2.41) reduces to a function of X, i.e.,

$$f(x) = \rho + \sigma\left[\frac{d\sigma}{ds}\right]\left[\frac{d\rho}{ds}\right] + (\sigma)^2\left[\frac{d^2\rho}{ds^2}\right] \tag{2.42}$$

With known values of Y and Z, the above equation is solved using Newton Raphson method. Thus, the position of spherical joint S_A is obtained.

Having found the spherical joint S_A, the spherical joint S_B can be determined as described below. Let S_B be the center of the sphere. That means S_AS_B forms the radius. The vector from S_A to S_B is given by

$$\overline{S_AS_B} = \rho\ \overline{n} + \sigma\left[\frac{d\rho}{ds}\right]\overline{b} \tag{2.43}$$

where $\overline{n}$ is the normal, $\overline{b}$ is the bi-normal at S_A as defined before. Therefore,

$$\overline{A_oS_B} = \overline{A_oS_A} + \overline{S_AS_B} \tag{2.44}$$

By changing the values of Y and Z we can obtain other solutions for X. In this way there are theoretically ∞^2 solutions possible to synthesize a mechanism up to fourth order.

2.8 Determination of the R.S.S.R Configuration

In order to construct a physical mechanism, we should derive from the X, Y, Z coordinates of S_A and S_B, the crank lengths, offset lengths, input and output angles, etc. These are done as follows.

The angle between $\overline{A}$ and $\overline{B}$ is assumed to be α and the distance between $\overline{A}$ and $\overline{B}$ is assumed to be unity, i.e., $A_oB_o = 1$.

Let a, b, c be the coordinates of S_A and d, e, f be the coordinates of S_B. Coupler link length ℓ is given by

$$\ell^2 = (d - a)^2 + (e - b)^2 + (f - c)^2 \tag{2.45}$$

input crank offset length A_oA along $\overline{A}$ is given by

$$g_o = c \tag{2.46}$$

The input crank length is given by

$$g = [a^2 + b^2]^{1/2} \tag{2.47}$$

Angle Θ_o is determined by

$$\Theta_o = \text{Tan}^{-1} \frac{b}{a} \tag{2.48}$$

The output crank offset length along $\overline{B}$, of the output crank B_oB is given by

$$h_o = - e \sin \alpha + f \cos \alpha \tag{2.49}$$

The output crank length is given by

$$h = [(d - 1)^2 + e^2 + f^2 - h_o^2]^{1/2} \tag{2.50}$$

Angle ϕ_o is given by

$$\text{Tan}^{-1} \frac{h_2}{h_1} \tag{2.51}$$

ϕ_o is measured from X about $\overline{B}$, where

$$h_1 = d - 1 \tag{2.52}$$

$$h_2 = e \cos \alpha + f \sin \alpha \tag{2.53}$$

CHAPTER III

ANALYSIS

Displacement analysis, and derivative analysis up to the fourth order, for the R.S.S.R mechanism have been worked out as per details shown. This analysis program serves as a check on the synthesis results.

3.1 Displacement Analysis

Displacement analysis, of the R.S.S.R mechanism shown in Figure 2, is obtained by expressing the coordinates of spherical joints S_A and S_B with respect to a fixed set of axis OXYZ. Expressing the coordinates of the spherical joint S_A and S_B as a function of rotation angle we have

$$a = g \cos \Theta$$

$$b = g \sin \Theta$$

$$c = g_o \qquad (3.1)$$

Let

$$h_1 = h \cos \alpha \qquad (3.2)$$

and

$$h_2 = h \sin \alpha \qquad (3.3)$$

then

$$h = [h_1^{\,2} + h_2^{\,2}]^{1/2} \qquad (3.4)$$

$$d = h \cos \phi + 1$$

$$e = h_1 \sin \phi + h_o \sin \alpha$$

$$f = h_2 \sin \phi + h_o \cos \alpha \qquad (3.5)$$

where

g = input crank length

h = output crank length

g_o = input crank offset length

h_o = output crank offset length

$S_A S_B = \ell$ = coupler link length

Θ = input angle of Link g relative to plane perpendicular to common normal

ϕ = output angle of Link h relative to plane perpendicular to common normal (3.6)

As the links are assumed to be rigid the coupler link length $S_A S_B$ is constant. Hence,

$$(a - d)^2 + (b - e)^2 + (c - f)^2 = \ell^2. \quad (3.7)$$

Substituting for a, b, c, d, e and f leads to

$$(g \cos \Theta - h \cos \phi - 1)^2 + (g \sin \Theta - h_o)^2 + (g_o - h \sin \phi)^2 = \ell^2. \quad (3.8)$$

Simplifying the above equation we get

$$2\, g_o h \sin \phi + 2\, K_1 h \cos \phi = K_1^2 + K_2^2 + g_o^2 + h^2 - \ell^2 \quad (3.9)$$

where

$$K_1 = g \cos \Theta - 1 \quad (3.10)$$

$$K_2 = g \sin \Theta - h_o. \quad (3.11)$$

Equation (3.9) can be written as

$$A \sin \phi + B \cos \phi = C \quad (3.12)$$

where

$$A = 2\, gah \quad (3.13)$$

$$B = 2\, K_1 h \quad (3.14)$$

$$C = K_1^2 + K_2^2 + g_o^2 + h^2 - \ell^2. \quad (3.15)$$

Using the trigonometric identities

$$\sin \phi = 2 \text{ Tan } \frac{\phi}{2}/(1 + \text{Tan}^2 \frac{\phi}{2}) \tag{3.16}$$

$$\cos \phi = (1 - \text{Tan}^2 \frac{\phi}{2})/(1 + \text{Tan}^2 \frac{\phi}{2}). \tag{3.17}$$

And substituting in Equation (3.12)

$$2 \text{ A Tan } \frac{\phi}{2} + B(1 - \text{Tan}^2 \frac{\phi}{2}) = C(1 + \text{Tan}^2 \frac{\phi}{2}) \tag{3.18}$$

or

$$(B + C) \text{ Tan}^2 \frac{\phi}{2} - (2A) \text{ Tan } \frac{\phi}{2} + (C - B) = 0. \tag{3.19}$$

From which

$$\text{Tan } \frac{\phi}{2} = [A \pm (A^2 + B^2 - C^2)^{1/2}]/(B + C). \tag{3.20}$$

This leads to two distinct values of ϕ as

$$\phi_1 = 2 \text{ arc Tan } \frac{A + (A^2 + B^2 - C^2)^{1/2}}{B + C} \tag{3.21}$$

$$\phi_2 = 2 \text{ arc Tan } \frac{A - (A^2 + B^2 - C^2)^{1/2}}{B + C}. \tag{3.22}$$

The two values correspond to the two ways in which a four bar linkage may be closed.

3.2 Derivative Analysis Up to Fourth Order

To derive the equations for derivative analysis up to fourth order of the R.S.S.R mechanism we express the coordinates of the spherical joints S_A and S_B with respect to a fixed set of axis OXYZ. We repeat the following relations already obtained.

$$a = g \cos \Theta$$

$$b = g \sin \Theta$$

$$c = g_o \tag{3.23}$$

Let

$$h_1 = h \cos \alpha$$

$$h_2 = h \sin \alpha$$

$$h = (h_1{}^2 + h_2{}^2)^{1/2} \tag{3.24}$$

$$d = h \cos \phi + 1$$

$$e = h_1 \sin \phi + h_o \sin \alpha$$

$$f = h_2 \sin \phi + h_o \cos \alpha \tag{3.25}$$

Let us get the derivatives of cos Θ and sin Θ with respect to time up to the fourth order. Let

$$\cos \Theta = g_3$$

$$\sin \Theta = h_3$$

Then

$$\frac{d}{dt} (\cos \Theta) = \dot{g}_3 = - h_3 \dot{\Theta}$$

$$\frac{d}{dt} (\sin \Theta) = \dot{h}_3 = g_3 \dot{\Theta}$$

$$\frac{d^2}{dt^2} (\cos \Theta) = \ddot{g}_3 = -\dot{h}_3 \dot{\Theta} - h_3 \ddot{\Theta}$$

$$\frac{d^2}{dt^2} (\sin \Theta) = \ddot{h}_3 = \dot{g}_3 \dot{\Theta} + g_3 \ddot{\Theta}$$

$$\frac{d^3}{dt^3} (\cos \Theta) = \dddot{g}_3 = - \ddot{h}_3 \dot{\Theta} - 2 \dot{h}_3 \ddot{\Theta} - h_3 \dddot{\Theta}$$

$$\frac{d^3}{dt^3} (\sin \Theta) = \dddot{h}_3 = \ddot{g}_3 \dot{\Theta} + 2 \dot{g}_3 \ddot{\Theta} + g_3 \dddot{\Theta}$$

$$\frac{d^4}{dt^4} (\cos \Theta) = \ddddot{g}_3 = - \dddot{h}_3 \dot{\Theta} - 3 \ddot{h}_3 \ddot{\Theta} - 3 \dot{h}_3 \dddot{\Theta} - h_3 \ddddot{\Theta}$$

$$\frac{d^4}{dt^4} (\sin \Theta) = \ddddot{h}_3 = \dddot{g}_3 \dot{\Theta} + 3 \ddot{g}_3 \ddot{\Theta} + 3 \dot{g}_3 \dddot{\Theta} + g_3 \ddddot{\Theta} \tag{3.26}$$

Similarly, let us get the derivatives of cos ϕ and sin ϕ with respect to time up to fourth order. Let

$$\cos\phi = g_4$$

$$\sin\phi = h_4$$

Then

$$\frac{d}{dt}(\cos\phi) = \dot{g}_4 = -h_4\,\dot{\phi}$$

$$\frac{d}{dt}(\sin\phi) = \dot{h}_4 = g_4\,\dot{\phi}$$

$$\frac{d^2}{dt^2}(\cos\phi) = \ddot{g}_4 = -\dot{h}_4\,\dot{\phi} - h_4\,\ddot{\phi}$$

$$\frac{d^2}{dt^2}(\sin\phi) = \ddot{h}_4 = \dot{g}_4\,\dot{\phi} + g_4\,\ddot{\phi}$$

$$\frac{d^3}{dt^3}(\cos\phi) = \dddot{g}_4 = -\ddot{h}_4\,\dot{\phi} - 2\,\dot{h}_4\,\ddot{\phi} - h_4\,\dddot{\phi}$$

$$\frac{d^3}{dt^3}(\sin\phi) = \dddot{h}_4 = \ddot{g}_4\,\dot{\phi} + 2\,\dot{g}_4\,\ddot{\phi} + g_4\,\dddot{\phi}$$

$$\frac{d^4}{dt^4}(\cos\phi) = \ddddot{g}_4 = -\dddot{h}_4\,\dot{\phi} - 3\,\ddot{h}_4\,\ddot{\phi} - 3\,\dot{h}_4\,\dddot{\phi} - h_4\,\ddddot{\phi}$$

$$\frac{d^4}{dt^4}(\sin\phi) = \ddddot{h}_4 = \dddot{g}_4\,\dot{\phi} + 3\,\ddot{g}_4\,\ddot{\phi} + 3\,\dot{g}_4\,\dddot{\phi} + g_4\,\ddddot{\phi} \tag{3.27}$$

In the analysis problem we know Θ, $\dot{\Theta}$, $\ddot{\Theta}$, $\dddot{\Theta}$, and $\ddddot{\Theta}$ knowing ϕ from the displacement analysis we determine $\dot{\phi}$, $\ddot{\phi}$, $\dddot{\phi}$, and $\ddddot{\phi}$ from the following procedure.

Differentiating the expressions of Equation (3.23) with respect to time we get

$$a = g\,g_1$$

$$\dot{a} = g\,\dot{g}_1$$

$$\ddot{a} = g\ \ddot{g}_1$$

$$\dddot{a} = g\ \dddot{g}_1$$

$$\ddddot{a} = g\ \ddddot{g}_1 \qquad (3.28)$$

$$b = g\ h_1$$

$$\dot{b} = g\ \dot{h}_1$$

$$\ddot{b} = g\ \ddot{h}_1$$

$$\dddot{b} = g\ \dddot{h}_1$$

$$\ddddot{b} = g\ \ddddot{h}_1 \qquad (3.29)$$

$$c = g_o$$

$$\dot{c} = 0$$

$$\ddot{c} = 0$$

$$\dddot{c} = 0$$

$$\ddddot{c} = 0 \qquad (3.30)$$

The constraint equation is

$$(a - d)^2 + (b - e)^2 + (c - f)^2 = \ell^2. \qquad (3.31)$$

Differentiating (3.31) we get

$$(a - d)\ (\dot{a} - \dot{d}) + (b - e)\ (\dot{b} - \dot{e}) + (c - f)\ (\dot{c} - \dot{f}) = 0. \qquad (3.32)$$

Let

$$a - d = u$$

$$b - e = v$$

$$c - f = w. \qquad (3.33)$$

Differentiating (3.25) we get

$$\dot{d} = h\ \dot{g}_4$$

$$\dot{e} = h_1 \dot{h}_4$$

$$\dot{f} = h_2 \dot{h}_4 \qquad (3.34)$$

Substituting the above values in Equation (3.32) we have

$$\dot{\phi} \, [- h(uh_4) + (h_1 v + h_2 w) g_4] = u \dot{a} + v \dot{b} + w \dot{c}. \qquad (3.35)$$

Let

$$R_1 = - h(uh_4) + (h_1 v + h_2 w) g_4 \qquad (3.36)$$

and

$$S_1 = u \dot{a} + v \dot{b} + w \dot{c}. \qquad (3.37)$$

Then

$$\dot{\phi} \, R_1 = S_1 \qquad (3.38)$$

or

$$\dot{\phi} = S_1 / R_1. \qquad (3.39)$$

In Equation (3.39) $\dot{\phi}$ can easily be calculated since all the other quantities contained in R_1 and S_1 are known.

Next we differentiate (3.33), (3.36), (3.37) and (3.38) and obtain

$$\dot{u} = \dot{a} - \dot{d}$$

$$\dot{v} = \dot{b} - \dot{e}$$

$$\dot{w} = \dot{c} - \dot{f} \qquad (3.40)$$

$$\dot{R}_1 = - h(\dot{u} \, h_4 + u \, \dot{h}_4) + (h_1 \, \dot{v} + h_2 \, \dot{w}) g_4 +$$

$$(h_1 \, v + h_2 \, w) \dot{g}_4 \qquad (3.41)$$

$$\dot{S}_1 = \dot{u} \, \dot{a} + u \, \ddot{a} + \dot{v} \, \dot{b} + v \, \ddot{b} + \dot{w} \, \dot{c} + w \, \ddot{c} \qquad (3.42)$$

Then

$$\ddot{\phi} \, R_1 + \dot{\phi} \, \dot{R}_1 = \dot{S}_1 \qquad (3.43)$$

or

$$\ddot{\phi} = \frac{\dot{S}_1 - \dot{\phi}\,\dot{R}_1}{R_1} \tag{3.44}$$

$\ddot{\phi}$ can be calculated since all the other quantities in Equation (3.44) are known.

Next, differentiating the expressions of Equations (3.34), (3.40), (3.41), (3.42), and (3.43) we get

$$\begin{aligned} \ddot{d} &= h\,\ddot{g}_4 \\ \ddot{e} &= h_1\,\ddot{h}_4 \\ \ddot{f} &= h_2\,\ddot{h}_4 \end{aligned} \tag{3.45}$$

$$\begin{aligned} \ddot{u} &= \ddot{a} - \ddot{d} \\ \ddot{v} &= \ddot{b} - \ddot{e} \\ \ddot{w} &= \ddot{c} - \ddot{f} \end{aligned} \tag{3.46}$$

$$\begin{aligned} \ddot{R}_1 = &- h(\ddot{u}\,h_4 + 2\,\ddot{u}\,h_4) + (h_1\,\ddot{v} + h_2\,\ddot{w})g_4 + \\ &2(h_1\,\dot{v} + h_2\,\dot{w})\dot{g}_4 + (h_1 v + h_2 w)\,\ddot{g}_4 \end{aligned} \tag{3.47}$$

$$\begin{aligned} \ddot{S}_1 = &\ddot{u}\,\dot{a} + 2\,\dot{u}\,\ddot{a} + u\,\dddot{a} + \ddot{v}\,\dot{b} + 2\,\dot{v}\,\ddot{b} + v\,\dddot{b} + \\ &\ddot{w}\,\dot{c} + \dot{w}\,\ddot{c} + w\,\dddot{c} \end{aligned} \tag{3.48}$$

Then

$$\dddot{\phi}\,R_1 + 2\,\ddot{\phi}\,\dot{R}_1 + \dot{\phi}\,\ddot{R}_1 = \ddot{S}_1 \tag{3.49}$$

or

$$\dddot{\phi} = \frac{\ddot{S}_1 - 2\,\ddot{\phi}\,\dot{R}_1 - \dot{\phi}\,\ddot{R}_1}{R_1}\,. \tag{3.50}$$

All the quantities on the R H S of Equation (3.50) are known. Hence $\dddot{\phi}$ can be obtained.

Once again we differentiate the expression in Equations (3.45), (3.46), (3.47), (3.48), and (3.49) and get

$$
\begin{aligned}
\dddot{d} &= h\,\dddot{g}_4 \\
\dddot{e} &= h_1\,\dddot{h}_4 \\
\dddot{f} &= h_2\,\dddot{h}_4
\end{aligned}
\qquad (3.51)
$$

$$
\begin{aligned}
\dddot{u} &= \dddot{a} - \dddot{d} \\
\dddot{v} &= \dddot{b} - \dddot{e} \\
\dddot{w} &= \dddot{c} - \dddot{f}
\end{aligned}
\qquad (3.52)
$$

$$
\begin{aligned}
\dddot{R}_1 = {} & -h(\dddot{u}\,h_4 + 3\,\ddot{u}\,\dot{h}_4 + 3\,\dot{u}\,\ddot{h}_4 + u\,\dddot{h}_4) + \\
& (h_1\,\dddot{v} + h_2\,\dddot{w})g_4 + 3(h_1\,\ddot{v} + h_2\,\ddot{w})\dot{g}_4 + \\
& 3(h_1\,\dot{v} + h_2\,\dot{w})\,\ddot{g}_4 + (h_1 v + h_2 w)\,\dddot{g}_4
\end{aligned}
\qquad (3.53)
$$

$$
\begin{aligned}
\dddot{S}_1 = {} & \dddot{u}\,\dot{a} + 3\,\ddot{u}\,\ddot{a} + 3\,\dot{u}\,\dddot{a} + u\,\ddddot{a} + \dddot{v}\,\dot{b} + \\
& 3\,\ddot{v}\,\ddot{b} + 3\,\dot{v}\,\dddot{b} + v\,\ddddot{b} + \dddot{w}\,\dot{c} + 3\,\ddot{w}\,\ddot{c} + \\
& 3\,\dot{w}\,\dddot{c} + w\,\ddddot{c}
\end{aligned}
\qquad (3.54)
$$

Then

$$
\ddddot{\phi}\,R_1 + 3\,\dddot{\phi}\,\dot{R}_1 + 3\,\ddot{\phi}\,\ddot{R}_1 + \dot{\phi}\,\dddot{R}_1 = \dddot{S}_1 \qquad (3.55)
$$

or

$$
\ddddot{\phi} = \frac{\dddot{S}_1 - 3\,\dddot{\phi}\,\dot{R}_1 - 3\,\ddot{\phi}\,\ddot{R}_1 - \dot{\phi}\,\dddot{R}_1}{R_1} \qquad (3.56)
$$

Knowing all the quantities on R H S of Equation (3.56) $\ddddot{\phi}$ can be obtained. Thus we have obtained the values of $\dot{\phi}$, $\ddot{\phi}$, $\dddot{\phi}$, and $\ddddot{\phi}$ by knowing the values of $\dot{\Theta}$, $\ddot{\Theta}$, $\dddot{\Theta}$ and $\ddddot{\Theta}$.

We find according to the equations given below

$$n_1 = \frac{d\phi}{d\Theta} = \dot{\phi}/\dot{\Theta} \tag{3.57}$$

$$n_2 = \frac{d^2\phi}{d\Theta^2} = (\ddot{\phi}\ \dot{\Theta} - \dot{\phi}\ \ddot{\Theta})/\dot{\Theta}^3 \tag{3.58}$$

$$n_3 = \frac{d^3\phi}{d\Theta^3} = [(\dddot{\phi}\ \dot{\Theta} - \dot{\phi}\ \dddot{\Theta})\dot{\Theta} - 3(\ddot{\phi}\ \dot{\Theta} - \dot{\phi}\ \ddot{\Theta})\ddot{\Theta}]/(\dot{\Theta})^5 \tag{3.59}$$

$$\begin{aligned} n_4 = \frac{d^4\phi}{d\Theta^4} = [(&\ddddot{\phi}\ \dot{\Theta} - \dddot{\phi}\ \ddot{\Theta} - \ddot{\phi}\ \dddot{\Theta} - \dot{\phi}\ \ddddot{\Theta})(\dot{\Theta})^2 - \\ &7(\dddot{\phi}\ \dot{\Theta} - \dot{\phi}\ \dddot{\Theta})\ddot{\Theta}\ \dot{\Theta} + \\ &(\ddot{\Theta}\ \dot{\Theta} - \dot{\phi}\ \ddot{\Theta})(15\ \ddot{\Theta}^2 - 3\ \dddot{\Theta}\ \dot{\Theta})]/(\dot{\Theta})^7 \end{aligned} \tag{3.60}$$

The synthesized mechanism can be analyzed in the above manner to determine the accuracy of synthesis. WHen the synthesis is correct the analysis of the mechanism yields the same n_1, n_2, n_3 and n_4 values.

CHAPTER IV

NUMERICAL EXAMPLES AND DISCUSSION

4.1 Numerical Example of Function Generation

It is desired to synthesize an R.S.S.R mechanism with input and output axes at 90°, the distance between the axes being unity, fulfilling the following function generation requirements.

$$\frac{d\phi}{d\Theta} = -2.0$$

$$\frac{d^2\phi}{d\Theta^2} = -8.5$$

$$\frac{d^3\phi}{d\Theta^3} = -65.0$$

$$\frac{d^4\phi}{d\Theta^4} = -785.0$$

SOLUTIONS: Following the methodology described in Chapter II, a computer program is written for synthesis as given in Appendix A.

Values of Y and Z coordinates and the initial guess values of X coordinate of the spherical pair S_A are assumed. The value of X corresponding to the values of Y and Z is obtained from this program. By changing the values of Y and Z, a new value of X is obtained.

Then, using the computer program as given in Appendix B, the coordinates of spherical pair S_B and various parameters of the mechanism are computed.

Table I shows six solutions fulfilling the same function generation specifications. Theoretically infinite number of solutions are possible by varying Y and Z.

The above results are again fed into the analysis programs given in Appendix C and Appendix D. The results obtained are tabulated in Table II. This proves that the analysis results agree well with the synthesis specifications.

4.2 Numerical Example: Replacement of Gears

It is desired to synthesize an R.S.S.R mechanism to replace a set of hypoid gears. The gear ratio being $-\frac{3}{2}$ and the angle between the shafts being 90°. The distance between the shafts is assumed unity.

SOLUTIONS: For gearing up to fourth order derivative functional relationships between the input and output crank angles are given by $n_1 = -\frac{3}{2}$, $n_2 = 0$, $n_3 = 0$, $n_4 = 0$. As before, the computer program given in Appendix A is used to get the X coordinate of spherical pair S_A by assuming the values of Y and Z coordinates. By changing Y, different values of X are obtained.

Then, using the computer program given in Appendix B, the various other parameters of the mechanism are obtained.

Table III shows six solutions fulfilling the same function generation specifications. These results are fed into the analysis program given in Appendix C and Appendix D. The results obtained are shown in Table IV. The analysis results agree well with the synthesis specifications.

TABLE I

R.S.S.R MECHANISM SYNTHESIS SOLUTIONS FOR FUNCTION GENERATION

Parameters	Solution 1	Solution 2	Solution 3	Solution 4	Solution 5	Solution 6
Normal Distance Between Shafts (ℓ_o)	1.0	1.0	1.0	1.0	1.0	1.0
Angle Between Shafts (α)	90°	90°	90°	90°	90°	90°
Input Crank Offset Length (g_o)	0	0	0	0	0	0
Input Crank Length (g)	1.715301	2.83994	3.140421	3.444008	2.886055	3.134155
Coupler Link Length (ℓ)	1.433977	2.386951	2.637319	2.901628	2.567245	2.811454
Output Crank Offset Length (h_o)	0.04858	-0.405419	-0.392191	-0.381277	0.138747	0.149184
Output Crank Length (h)	0.7326304	1.442359	1.433769	1.427554	0.721997	0.722812
Input Crank Angle in Degrees (Θ)	35.66099	31.88258	33.86584	35.50111	51.22497	52.90754
Output Crank Angle in Degrees (ϕ)	142.7916	60.48786	60.4477	60.41574	145.4357	145.68950

Synthesis Derivatives: $n_1 = -2.0$, $n_2 = -8.5$, $n_3 = -65.0$, $n_4 = -785.0$.

TABLE II

DISPLACEMENT AND DERIVATIVE ANALYSIS OF THE SYNTHESIZED R.S.S.R MECHANISMS FOR FUNCTION GENERATION

Parameters	Solution 1	Solution 2	Solution 3	Solution 4	Solution 5	Solution 6
Input Crank Angle in Degrees (Θ)	35.6609	31.88511	33.86755	35.50048	51.22245	52.90695
Output Crank Angle in Degrees (ϕ)	142.7917	60.48283	60.44442	60.41699	145.4411	145.6905
First Derivative (n_1)	-2.0	-2.0	-2.0	-2.0	-2.0	-2.0
Second Derivative (n_2)	-8.5	-8.503	-8.502	-8.4999	-8.497	-8.499
Third Derivative (n_3)	-65.0	-65.03	-65.02	-64.99	-64.97	-64.99
Fourth Derivative (n_4)	-785.0	-785.6	-785.4	-784.9	-784.5	-784.9

Synthesis Derivatives: $n_1 = -2.0$, $n_2 = -8.5$, $n_3 = -65.0$, $n_4 = -785.0$.

TABLE III

R.S.S.R MECHANISM SYNTHESIS SOLUTIONS FOR REPLACING HYPOID GEARS

Parameters	Solution 1	Solution 2	Solution 3	Solution 4	Solution 5	Solution 6
Normal Distance Between Shafts (ℓ_o)	1.0	1.0	1.0	1.0	1.0	1.0
Angle Between Shafts (α)	90°	90°	90°	90°	90°	90°
Input Crank Offset Length (g_o)	1.5	1.5	1.5	1.5	1.5	1.5
Input Crank Length (g)	0.4735795	0.4559487	0.4415671	0.4286306	0.3737503	0.3298566
Coupler Link Length (ℓ)	1.80229	1.809967	1.82617	1.846641	1.982756	2.121395
Output Crank Offset Length (h_o)	-0.14463	-0.322014	-0.448983	-0.555826	-0.972553	-1.25675
Output Crank Length (h)	0.2030792	0.1944022	0.1901074	0.1874757	0.1815148	0.1743801
Input Crank Angle in Degrees (Θ)	-65.22747	-67.09571	-68.2039	-68.9394	-69.46438	-65.4344
Output Crank Angle in Degrees (ϕ)	8.397328	1.734945	-3.101921	-7.177642	-23.41148	-38.19876

Synthesis Derivatives: $n_1 = -\frac{3}{2}$, $n_2 = 0$, $n_3 = 0$, $n_4 = 0$.

TABLE IV

DISPLACEMENT AND DERIVATIVE ANALYSIS OF THE SYNTHESIZED R.S.S.R MECHANISM FOR REPLACING HYPOID GEARS

Parameters	Solution 1	Solution 2	Solution 3	Solution 4	Solution 5	Solution 6
Input Crank Angle in Degrees (Θ)	-65.22555	-67.09336	-68.20495	-68.93833	-69.4654	-65.43183
Output Crank Angle in Degrees (ϕ)	8.395133	1.731646	-3.099991	-7.179619	-23.4091	-38.20254
First Derivative (n_1)	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5
Second Derivative (n_2)	0.3135×10^{-4}	0.2072×10^{-4}	0.4749×10^{-4}	0.6048×10^{-4}	0.1025×10^{-3}	0.2066×10^{-4}
Third Derivative (n_3)	0.4382×10^{-4}	0.6638×10^{-5}	0.102×10^{-3}	0.1989×10^{-3}	0.548×10^{-3}	0.2066×10^{-3}
Fourth Derivative (n_4)	0.1448×10^{-3}	-0.133×10^{-3}	0.8651×10^{-3}	0.1117×10^{-2}	0.4927×10^{-2}	0.2872×10^{-2}

Synthesis Derivatives: $n_1 = -\frac{3}{2}$, $n_2 = 0$, $n_3 = 0$, $n_4 = 0$.

One solution is picked up to find the deviation in the derivatives over a range of 40° (i.e., 20° below and 20° above the designed input angle). Table V gives the results. As can be seen from the results, the variance is tolerable.

4.3 Discussion

The initial guesses of the coordinates of the spherical joint S_A are found to be very critical. It is essentially a trial and error method. Once a solution is obtained, solutions in the neighborhood are very easy to obtain by assuming proper increments in one of the assumed coordinates Y or Z. Since the surface is very complex, it is found that solution may not exist over a long range of values of a particular coordinate. More difficulty was encountered in obtaining a solution for the gear replacement problem. It may be a good practice to arrive at the gear problem progressively. This means the first trial solution might be attempted with $n_4 = 0$. The second trial solution should be obtained with $n_3 = 0$ and $n_4 = 0$. The third and final solution should be obtained with $n_2 = 0$, $n_3 = 0$ and $n_4 = 0$.

TABLE V

DEVIATIONS OF DERIVATIVES OVER A RANGE OF 40°, 20° BELOW AND 20° ABOVE THE DESIGNED INPUT ANGLE OF ROTATION

Θ in Degrees	ϕ in Degrees	n_1	n_2	n_3	n_4
-45.22552	-21.62835	-1.5	-0.08271	-0.8981	-8.381
-50.2255	-14.10989	-1.5	-0.02941	-0.3866	-3.954
-55.22554	-6.605459	-1.5	-0.007654	-0.1417	-1.889
-60.22547	0.895018	-1.5	-0.0008428	-0.03082	-0.7482
-65.2255	8.395036	-1.5	0.00002787	0.00003355	0.00008837
-70.22547	15.89500	-1.5	0.0008012	-0.02628	0.5888
-75.2255	23.39553	-1.5	0.006038	-0.1019	1.147
-80.22549	30.8990	-1.5	0.02006	-0.2286	1.78
-85.22543	38.41168	-1.5	0.04776	-0.4184	2.62

Synthesis derivatives: $n_1 = -\frac{3}{2}$, $n_2 = 0$, $n_3 = 0$, $n_4 = 0$. Designed input angle of rotation Θ_o is -65.2255°. Output crank angle of rotation corresponding to Θ_o is $\phi_o = 8.395036°$. Other parameters of the mechanism: $\ell_o = 1.0$, $\alpha = 90°$, $g = 0.47358$, $g_o = 1.5$, $h = 0.203079$, $h_o = -0.14463$.

CHAPTER V

SUMMARY AND CONCLUSIONS

The R.S.S.R mechanism is a versatile mechanism, best suited for function generation. The earlier works on the derivative synthesis for function generation were limited up to third power. All the works were based on the concept of constant length constraint on the coupler link and on the principle of inversion. In this thesis the point path properties are studied to find points in the rigid body that have spherical paths up to fourth order. These points were utilized to determine one of the spherical joints, using the principle of inversion. Again using the point path properties, the second spherical joint was determined and the synthesis was completed with known informations. The newness of this thesis is in utilizing the point path properties and extending the derivative synthesis up to fourth order. While obtaining the fourth order synthesis one characteristic equation has been utilized. This left us with the freedom of choosing two of the three coordinates required to locate a spherical joint. The characteristic equation for the fourth order is a condition requiring that the rate of change of radius of the sphere of the point path is zero. Considering second derivatives of the radius of the sphere to be zero, we may obtain another characteristic equation. This equation corresponds to the fifth order properties. Derivative synthesis is possible up to fifth order if we can find the intersection of characteristic equation of the fourth order and the characteristic equation of the fifth order. In this situation we still have the freedom to choose one of the three coordinates of the spherical joint. Extending the same philosophy we may consider the third rate of change of radius of the sphere of the path to be zero for the sixth order derivative synthesis. This condition will yield a third characteristic surface. Intersection of this surface with the previously mentioned two surfaces might yield unique points that can be used as a spherical joint in the synthesis. However, it should be realized that the magnitude of work involved is enormous even for the fourth order and much more so for the higher orders. One of the difficulties that one faces in numerically solving the equation is the initial guess. It is very much a matter of art, patience, and finally, luck. In general, it is more difficult to find a solution for a linkage to replace a gear than to solve for ordinary function generation as has been discussed earlier.

This work may be degenerated to study spherical or planar mechanisms. Then the criterion required to determine a coupler joint in planar and spherical kinematics is that they generate circular paths rather than spherical paths as in R.S.S.R mechanism.

Higher order derivative synthesis for other four bar mechanisms like R.C.C.C., R.S.C.R., etc., might be possible using suitable pair constraint equation. The author cannot readily comment elaborately on them now.

It may be possible to produce design charts for this mechanism. In order to produce design charts we should obtain several solutions with varying shaft angle α, gear ratio, the input angle of crank and compute the relationship with the ratio of the link lengths. Once this is obtained, a design chart can easily be prepared but as pointed out earlier it is very difficult to find a solution. Further, too much time has to be devoted in the process of trials. All the spade work of deriving the synthesis equations and ensuring their correctness is the primary task of this thesis. It will be nice if someone will follow up this study and produce design charts. These design charts are very valuable to the engineers and are an asset to the industry.

It is well known that the higher order synthesis yields higher accuracies over a larger range. At the same time it is to be noted that different solutions yield different accuracies over a given range. Hence, the error is to be checked for the given solution. Keeping this in mind the analysis problem is simultaneously developed to check the correctness of the synthesis and its accuracy in the neighborhood. The results of the third order synthesis have been compared with the results available in the earlier literature and they agree very well. Since this synthesis is one order higher than before, these results are helpful in building linkages with higher accuracies than before.

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APPENDIXES

APPENDIX A

COMPUTER PROGRAM TO OBTAIN THE COORDINATES OF SPHERICAL JOINT S_A WITH ONE SET OF DATA AND SOLUTION

```
$JOB
C       *************************************************************
C       *  SRI RAMA JEYAM. SRIRAM JEYARAM JEYAJEYA RAM.             *
C       *  SYNTHESIS OF R-S-S-R MECHANISM                            *
C       *  THIS PROGRAM OBTAINS THE CO-ORDINATES OF SPHERICAL        *
C       *  JOINT SA.                                                 *
C       *  DATA CARDS:                                               *
C       *  CARD 1 SPECIFIES THE DERIVATIVES OF FIRST,SECOND,         *
C       *  THIRD, AND FOURTH ORDER.                                  *
C       *  CARD 2: GIVES THE VALUE OF Y AND Z.                       *
C       *  CARD 3: GIVES THE INITIAL GUESS VALUE OF X AND THE        *
C       *  TEST VALUE.                                               *
C       *  ALL THE DATA CARDS ARE PUNCHED IN 12 COLLUMN FIELD        *
C       *  WITH SIX DECIMAL PLACES.                                  *
C       *************************************************************
C
        READ(5,100) AN1,AN2,AN3,AN4
    100 FORMAT(E12.7,E12.7,E12.7,E12.7)
        WRITE(6,105) AN1,AN2,AN3,AN4
    105 FORMAT(//5X,'AN1=',E15.7,5X,'AN2=',E15.7,5X,'AN3=',E15.7,
       15X,'AN4=',E15.7//)
C
     15 READ(5,10) Y,Z
     10 FORMAT(F12.6,F12.6)
C
        AL=0.0
        BL=1.0
C
        P1=AN1*BL
        P2=AN2*BL
        P3=AN3*BL
        P4=AN4*BL
C
        Q1=-AN1*AL
        Q2=-AN2*AL
        Q3=-AN3*AL
        Q4=-AN4*AL
C
        V1=0.0
        V2=-Q1
        V3=P1
C
        V11=P1**2+Q1**2
        V21=-Q2
        V31=P2
C
        V12=3.0*(P2*P1-Q2*Q1)
        V22=-(Q3-Q1**3-P1**2*Q1)
        V32=-(-P3+P1**3+Q1**2*P1)
C
        V13=-(-4.0*P3*P1-4.0*Q3*Q1-3.0*P2**2-3.0*Q2**2+P1**4+Q1**4
       1+2.0*P1**2*Q1**2)
        V23=-(Q4-3.0*P2*P1*Q1-3.0*Q2*P1**2)
        V33=-(-P4-3.0*Q2*Q1*P1+3.0*P2*Q1**2+6.0*P2*P1**2)
C
        AV1=0.0
        AV2=0.0
        AV3=1.0
C
        BV1=0.0
```

```
      BV2=-BL
      BV3=AL
C
      TH1=1.0
      TH2=0.0
      TH3=0.0
      TH4=0.0
C
      PH1=AN1
      PH2=AN2
      PH3=AN3
      PH4=AN4
C
      BAX=BV2*AV3-BV3*AV2
      BAY=BV3*AV1-BV1*AV3
      BAZ=BV1*AV2-BV2*AV1
C
      AV11=-PH1*BAX
      AV21=-PH1*BAY
      AV31=-PH1*BAZ
C
      BA1X=BV2*AV31-BV3*AV21
      BA1Y=BV3*AV11-BV1*AV31
      BA1Z=BV1*AV21-BV2*AV11
C
      AV12=-PH2*BAX-PH1*BA1X
      AV22=-PH2*BAY-PH1*BA1Y
      AV32=-PH2*BAZ-PH1*BA1Z
C
      BA2X=BV2*AV32-BV3*AV22
      BA2Y=BV3*AV12-BV1*AV32
      BA2Z=BV1*AV22-BV2*AV12
C
      AV13=-PH3*BAX-2.0*PH2*BA1X-PH1*BA2X
      AV23=-PH3*BAY-2.0*PH2*BA1Y-PH1*BA2Y
      AV33=-PH3*BAZ-2.0*PH2*BA1Z-PH1*BA2Z
C
      W1=-PH1*BV1+TH1*AV1
      W2=-PH1*BV2+TH1*AV2
      W3=-PH1*BV3+TH1*AV3
C
      W11=-PH2*BV1+TH2*AV1+TH1*AV11
      W21=-PH2*BV2+TH2*AV2+TH1*AV21
      W31=-PH2*BV3+TH2*AV3+TH1*AV31
C
      W12=-PH3*BV1+TH3*AV1+2.0*TH2*AV11+TH1*AV12
      W22=-PH3*BV2+TH3*AV2+2.0*TH2*AV21+TH1*AV22
      W32=-PH3*BV3+TH3*AV3+2.0*TH2*AV31+TH1*AV32
C
      W13=-PH4*BV1+TH4*AV1+3.0*TH3*AV11+3.0*TH2*AV12+TH1*AV13
      W23=-PH4*BV2+TH4*AV2+3.0*TH3*AV21+3.0*TH2*AV22+TH1*AV23
      W33=-PH4*BV3+TH4*AV3+3.0*TH3*AV31+3.0*TH2*AV32+TH1*AV33
C
      A11=0.0
      A21=W3
      A31=-W2
C
      B11=-W3
      B21=0.0
      B31=W1
```

```
C
      C11=W2
      C21=-W1
      C31=0.0
C
      A12=-(W2**2+W3**2)
      A22=W31+W2*W1
      A32=-W21+W3*W1
C
      B12=-W31+W1*W2
      B22=-(W3**2+W1**2)
      B32=W11+W3*W2
C
      C12=W21+W1*W3
      C22=-W11+W2*W3
      C32=-(W1**2+W2**2)
C
      A13=-3.0*(W21*W2+W31*W3)
      A23=W32+W21*W1+2.0*W11*W2-W3**3 -W2**2*W3-W1**2*W3
      A33=-W22+2.0*W11*W3+W31*W1+W2**3+W1**2*W2+W3**2*W2
C
      B13=-W32+W11*W2+2.0*W22*W1+W3**3+W1**2*W3+W2**2*W3
      B23=-3.0*(W31*W3+W11*W1)
      B33=W12+2.0*W21*W3+W31*W2-W1**3-W2**2*W1-W3**2*W1
C
      C13=W22+W11*W3+2.0*W31*W1-W2**3-W1**2*W2-W3**2*W2
      C23=-W12+W21*W3+2.0*W31*W2+W1**3+W2**2*W1+W3**2*W1
      C33=-3.0*(W11*W1+W21*W2)
C
      A14=-4.0*W22*W2-4.0*W32*W3-3.0*W21**2-3.0*W31**2+2.0*W21*W1*W3
     1-2.0*W31*W1*W2+W2**4+W3**4+W1**2*W2**2+2.0*W2**2*W3**2+W3**2*W1**2
C
      A24=W33+W22*W1+3.0*W12*W2+3.0*W21*W11-3.0*W21*W2*W3
     1-3.0*W31*W2**2-W31*W1**2-5.0*W11*W3*W1-W2**3*W1-W2*W1**3
     2-W2*W3**2*W1-6.0*W31*W3**2
C
      A34=-W23+3.0*W12*W3+3.0*W31*W11+3.0*W31*W3*W2+5.0*W11*W1*W2
     1+3.0*W21*W3**2+W21*W1**2+6.0*W21*W2**2-W3**3*W1-W3*W1**3
     2-W3*W1*W2**2+W32*W1
C
      B14=-W33+3.0*W22*W1+3.0*W11*W21+3.0*W11*W1*W3+5.0*W21*W2*W3
     1+3.0*W31*W1**2+W31*W2**2+6.0*W31*W3**2-W1**3*W2
     2-W1*W2**3-W1*W2*W3**2+W12*W2
C
      B24=-4.0*W32*W3-4.0*W12*W1-3.0*W31**2-3.0*W11**2+2.0*W31*W2*W1
     1-2.0*W11*W2*W3+W3**4+W1**4+W2**2*W3**2+2.0*W3**2*W1**2
     2+W1**2*W2**2
C
      B34=W13+W32*W2+3.0*W22*W3+3.0*W31*W21-3.0*W31*W3*W1-3.0*W11*W3**2
     1-W11*W2**2-5.0*W21*W1*W2-W3**3*W2-W3*W2**3-W3*W1**2*W2
     2-6.0*W11*W1**2
C
      C14=W23+W12*W3+3.0*W32*W1+3.0*W11*W31-3.0*W11*W1*W2-3.0*W21*W1**2
     1-W21*W3**2-5.0*W31*W2*W3-W1**3*W3-W1*W3**3-W1*W2**2*W3
     2-6.0*W21*W2**2
C
      C24=-W13+3.0*W32*W2+3.0*W21*W31+3.0*W21*W2*W1+5.0*W31*W3*W1+
     13.0*W11*W2**2+W11*W3**2+6.0*W11*W1**2-W2**3*W3-W2*W3**3
     2-W2*W3*W1**2+W22*W3
C
```

```
      C34=-4.0*W12*W1-4.0*W22*W2-3.0*W11**2-3.0*W21**2+2.0*W11*W3*W2
     1-2.0*W21*W3*W1+W1**4+W2**4+W3**2*W1**2+2.0*W1**2*W2**2
     2+W2**2*W3**2
C
C THE INITIAL GUESS VALUE OF X IS GIVEN HERE
      READ(5,80) X,TEST
   80 FORMAT(F12.7,F12.7)
C
      N=1
      WRITE(6,86) Y,Z
   86 FORMAT(//,5X,'Y=',F12.7,5X,'Z=',F12.6)
      WRITE(6,85) X,TEST
   85 FORMAT(//5X,'X=',F12.7,5X,'TEST=',F12.7)
C
   90 P11=V1+A11*X+B11*Y+C11*Z
      P11X=A11
      P21=V2+A21*X+B21*Y+C21*Z
      P21X=A21
      P31=V3+A31*X+B31*Y+C31*Z
      P31X=A31
C
      P12=V11+A12*X+B12*Y+C12*Z
      P12X=A12
      P22=V21+A22*X+B22*Y+C22*Z
      P22X=A22
      P32=V31+A32*X+B32*Y+C32*Z
      P32X=A32
C
      P13=V12+A13*X+B13*Y+C13*Z
      P13X=A13
      P23=V22+A23*X+B23*Y+C23*Z
      P23X=A23
      P33=V32+A33*X+B33*Y+C33*Z
      P33X=A33
C
      P14=V13+A14*X+B14*Y+C14*Z
      P14X=A14
      P24=V23+A24*X+B24*Y+C24*Z
      P24X=A24
      P34=V33+A34*X+B34*Y+C34*Z
      P34X=A34
C
      AM1=P21*P32-P31*P22
      AM2=P31*P12-P11*P32
      AM3=P11*P22-P21*P12
C
      AM1X=P21X*P32+P21*P32X-P31X*P22-P31*P22X
      AM2X=P31X*P12+P31*P12X-P11X*P32-P11*P32X
      AM3X=P11X*P22+P11*P22X-P21X*P12-P21*P12X
C
      AM11=P21*P33-P31*P23
      AM21=P31*P13-P11*P33
      AM31=P11*P23-P21*P13
C
      AM11X=P21X*P33+P21*P33X-P31X*P23-P31*P23X
      AM21X=P31X*P13+P31*P13X-P11X*P33-P11*P33X
      AM31X=P11X*P23+P11*P23X-P21X*P13-P21*P13X
C
      AM12=P22*P33+P21*P34-P32*P23-P31*P24
      AM22=P32*P13+P31*P14-P12*P33-P11*P34
```

```
154          AM32=P12*P23+P11*P24-P22*P13-P21*P14
        C
155          AM12X=P22X*P33+P22*P33X+P21X*P34+P21*P34X-P32X*P23-P32*P23X
            1-P31X*P24-P31*P24X
156          AM22X=P32X*P13+P32*P13X+P31X*P14+P31*P14X-P12X*P33-P12*P33X
            1-P11X*P34-P11*P34X
157          AM32X=P12X*P23+P12*P23X+P11X*P24+P11*P24X-P22X*P13-P22*P13X
            1-P21X*P14-P21*P14X
        C
158          A=AM1**2+AM2**2+AM3**2
159          A1=2.0*(AM1*AM11+AM2*AM21+AM3*AM31)
160          A2=2.0*(AM11**2+AM21**2+AM31**2+AM1*AM12+AM2*AM22+AM3*AM32)
        C
161          AX=2.0*(AM1*AM1X+AM2*AM2X+AM3*AM3X)
162          A1X=2.0*(AM1X*AM11+AM1*AM11X+AM2X*AM21+AM2*AM21X+AM3X*AM31
            1+AM3*AM31X)
163          A2X=2.0*(2.0*AM11*AM11X+2.0*AM21*AM21X+2.0*AM31*AM31X+AM1X*AM12+
            1AM1*AM12X+AM2X*AM22+AM2*AM22X+AM3X*AM32+AM3*AM32X)
        C
164          B=P11**2+P21**2+P31**2
165          B1=2.0*(P11*P12+P21*P22+P31*P32)
166          B2=2.0*(P12**2+P22**2+P32**2+P11*P13+P21*P23+P31*P33)
        C
167          BX=2.0*(P11*P11X+P21*P21X+P31*P31X)
168          B1X=2.0*(P11X*P12+P11*P12X+P21X*P22+P21*P22X+P31X*P32+P31*P32X)
169          B2X=2.0*(2.0*P12*P12X+2.0*P22*P22X+2.0*P32*P32X+P11X*P13+P11*P13X
            1+P21X*P23+P21*P23X+P31X*P33+P31*P33X)
        C
170          C=P13*AM1+P23*AM2+P33*AM3
171          C1=P14*AM1+P13*AM11+P24*AM2+P23*AM21+P34*AM3+P33*AM31
        C
172          CX=P13X*AM1+P13*AM1X+P23X*AM2+P23*AM2X+P33X*AM3+P33*AM3X
173          C1X=P14X*AM1+P14*AM1X+P13X*AM11+P13*AM11X+P24X*AM2+P24*AM2X
            1+P23X*AM21+P23*AM21X+P34X*AM3+P34*AM3X+P33X*AM31+P33*AM31X
        C
174          R0=B**1.5*A**(-0.5)
175          R01=1.5*B**0.5*B1*A**(-0.5)+B**1.5*(-0.5)*A**(-1.5)*A1
176          R02=1.5*(0.5*B**(-0.5)*B1**2*A**(-0.5)+B**0.5*(B2*A**(-0.5)+
            1B1*(-0.5)*A**(-1.5)*A1))-0.5*(1.5*B**0.5*B1*A**(-1.5)*A1+
            2B**1.5*((-1.5)*A**(-2.5)*A1**2+A**(-1.5)*A2))
        C
177          R0X=1.5*B**0.5*BX*A**(-0.5)+B**1.5*(-0.5)*A**(-1.5)*AX
178          R01X=1.5*(0.5*B**(-0.5)*BX*B1*A**(-0.5)+B**0.5*(B1X*A**(-0.5)+
            1B1*(-0.5)*A**(-1.5)*AX))-0.5*(1.5*B**0.5*BX*A**(-1.5)*A1+
            2B**1.5*((-1.5)*A**(-2.5)*AX*A1+A**(-1.5)*A1X))
        C
179          H1=0.5*B**(-0.5)*B1**2*A**(-0.5)
180          H1X=0.5*((-0.5)*B**(-1.5)*BX*B1**2*A**(-0.5)+
            1B**(-0.5)*(2.0*B1*B1X*A**(-0.5)+B1**2*(-0.5)*A**(-1.5)*AX))
        C
181          H2=B**0.5*(B2*A**(-0.5)+B1*(-0.5)*A**(-1.5)*A1)
182          H2X=0.5*B**(-0.5)*BX*(B2*A**(-0.5)+B1*(-0.5)*A**(-1.5)*A1)+
            1B**0.5*(B2X*A**(-0.5)+B2*(-0.5)*A**(-1.5)*AX+B1X*(-0.5)*A**(-1.5)*
            2A1+B1*(-0.5)*((-1.5)*A**(-2.5)*AX*A1+A**(-1.5)*A1X))
        C
183          H3=1.5*B**0.5*B1*A**(-1.5)*A1
184          H3X=1.5*(0.5*B**(-0.5)*BX*B1*A**(-1.5)*A1+B**0.5*(B1X*A**(-1.5)*A1
            1+B1*((-1.5)*A**(-2.5)*AX*A1+A**(-1.5)*A1X)))
        C
185          H4=B**1.5*((-1.5)*A**(-2.5)*A1**2+A**(-1.5)*A2)
```

```
      H4X=1.5*B**0.5*BX*((-1.5)*A**(-2.5)*A1**2+A**(-1.5)*A2)+B**1.5*
     1((-1.5)*(-2.5)*A**(-3.5)*AX*A1**2+(-1.5)*A**(-2.5)*2.0*A1*A1X+
     2(-1.5)*A**(-2.5)*AX*A2+A**(-1.5)*A2X)
C
      RO2X=1.5*(H1X+H2X)-0.5*(H3X+H4X)
C
      SG=A/C
      SG1=(C*A1-A*C1)/C**2
C
      SGX=(C*AX-A*CX)/C**2
      SG1X=(C**2*(C*A1X+A1*CX-A*C1X-C1*AX)-(C*A1-A*C1)*2.0*C*CX)/C**4
C
      S1=B**0.5
      S2=0.5*B**(-0.5)*B1
C
      S1X=0.5*B**(-0.5)*BX
      S2X=0.5*((-0.5)*B**(-1.5)*BX*B1+B**(-0.5)*B1X)
C
      DRS1=RO1/S1
      DRS2=(S1*RO2-RO1*S2)/S1**3
C
      DRS1X=(RO1X*S1-RO1*S1X)/S1**2
      DRS2X=((S1X*RO2+S1*RO2X-RO1X*S2-RO1*S2X)*S1-
     1(S1*RO2-RO1*S2)*3.0*S1X)/S1**4
C
      DSG1=SG1/S1
      DSG1X=(SG1X*S1-SG1*S1X)/S1**2
C
C
      FX=RO+SG*DSG1*DRS1+SG**2*DRS2
C
      FPX=ROX+SGX*DSG1*DRS1+SG*(DSG1X*DRS1+DSG1*DRS1X)+
     12.0*SG*SGX*DRS2+SG**2*DRS2X
      WRITE(6,225) X,FX,FPX
  225 FORMAT(1H,,'X=',E15.7,5X,'FX=',E15.7,5X,'FPX=',E15.7/)
C
      XA=X-(FX/FPX)
C
      N=N+1
      IF(ABS(XA-X)-TEST)240,240,235
C
  235 X=XA
C
      WRITE(6,230) N,XA
  230 FORMAT(5X,'N=',I8,5X,'XA=',E15.7/)
C
      IF(N.GT.20)GO TO 300
      GO TO 90
C
  300 WRITE(6,310)X
  310 FORMAT(//1X,'NO OF ITERATIONS EXCEED 20. VALUE OF X IS',E11.4//)
C
  240 WRITE(6,245) X
  245 FORMAT(//5X,'X=',E15.7//)
C
  500 STOP
      END

$ENTRY
```

```
    AN1= -0.1500000E 01     AN2=  0.0000000E 00     AN3=  0.0000000E 00     AN4=  0.0000000E 00


    Y=  -0.4000000     Z=    1.500000

    X=    0.0000000     TEST=   0.0000100
X=  0.0000000E 00     FX=  0.6069484E 02     FPX= -0.7381797E 03

    N=       2     XA=  0.8222228E-01

X=  0.8222228E-01     FX=  0.2415315E 02     FPX= -0.2935579E 03

    N=       3     XA=  0.1644996E 00

X=  0.1644996E 00     FX= -0.5801660E 01     FPX= -0.6190525E 03

    N=       4     XA=  0.1551277E 00

X=  0.1551277E 00     FX= -0.5518341E 00     FPX= -0.5076184E 03

    N=       5     XA=  0.1540406E 00

X=  0.1540406E 00     FX= -0.6820679E-02     FPX= -0.4970095E 03

    N=       6     XA=  0.1540268E 00

X=  0.1540268E 00     FX=  0.5035400E-03     FPX= -0.4968782E 03


    X=  0.1540268E 00


STATEMENTS EXECUTED=     682

CORE USAGE       OBJECT CODE=   21832 BYTES,ARRAY AREA=       0 BYTES,TOTAL AREA AVAILABLE=  149504  BYTES

DIAGNOSTICS        NUMBER OF ERRORS=       0, NUMBER OF WARNINGS=       0, NUMBER OF EXTENSIONS=       0

COMPILE TIME=     1.18 SEC,EXECUTION TIME=     0.23 SEC,     20.38.00     WEDNESDAY    24 JAN 79    WATFIV - JUN 1977 V1L6

        C$STOP
```

APPENDIX B

COMPUTER PROGRAM TO OBTAIN THE COORDINATES OF SPHERICAL JOINT S_B AND OTHER PARAMETERS OF THE R.S.S.R MECHANISM WITH ONE SET OF DATA AND SOLUTION

```
$JOB
C
C       ************************************************************
C       *  SRI RAMA JEYAM. SRIRAM JEYARAM JEYAJEYA RAM.            *
C       *  SYNTHESIS OF R-S-S-R MECHANISM.                         *
C       *  THIS PROGRAM OBTAINS THE CO-ORDINATES OF SPHERICAL      *
C       *  JOINT SB AND OTHER PARAMETERS OF THE MECHANISM.         *
C       *  GO= INPUTCRANK OFFSET LENGTH                            *
C       *  HO= OUTPUTCRANK OFFSET LENHTH                           *
C       *  G=INPUTCRANK LENGTH                                     *
C       *  H= OUTPUTCRANK LENGTH                                   *
C       *  S= COUPLER LINK LENGTH                                  *
C       *  THETA= INPUTCRANK ANGLE IN DEGREES                      *
C       *  PHI= OUTPUTCRANK ANGLE IN DEGREES                       *
C       *  DATA CARDS:                                             *
C       *  THIRD, AND FOURTH ORDER.                                *
C       *  CARD 1 SPECIFIES THE DERIVATIVES OF FIRST, SECOND,      *
C       *  CARD 2 SPECIFIES X,Y,Z CO-ORDINATES OF SPHERICAL        *
C       *  JOINT SA OBTAINED FROM PROGRAM A.                       *
C       *  ALL THE DATA CARDS ARE PUNCHED IN 12 COLLUMN FIELD      *
C       *  WITH SIX DECIMAL PLACES.                                *
C       ************************************************************
C
      REAL N1,N2,N3
C
      READ(5,100)AN1,AN2,AN3,AN4
  100 FORMAT(F12.6,F12.6,F12.6,F12.6)
      WRITE(6,105) AN1,AN2,AN3,AN4
  105 FORMAT(//5X,'AN1=',F12.6,5X,'AN2=',F12.6,5X,'AN3=',F12.6,5X,
     1'AN4=',F12.6//)
C
  300 READ(5,90) X,Y,Z
   90 FORMAT(F12.6,F12.6,F12.6)
      WRITE(6,95) X,Y,Z
   95 FORMAT(1H,,'X=',F12.6,5X,'Y=',F12.6,5X,'Z=',F12.6)
C
      AL=0.0
      BL=1.0
C
      P1=AN1*BL
      P2=AN2*BL
      P3=AN3*BL
      P4=AN4*BL
C
      Q1=-AN1*AL
      Q2=-AN2*AL
      Q3=-AN3*AL
      Q4=-AN4*AL
C
      V1=0.0
      V2=-Q1
      V3=P1
C
      V11=P1**2+Q1**2
      V21=-Q2
      V31=P2
C
      V12=3.0*(P2*P1-Q2*Q1)
      V22=-(Q3-Q1**3-P1**2*Q1)
      V32=-(-P3+P1**3+Q1**2*P1)
```

```
C
      V13=-(-4.0*P3*P1-4.0*Q3*Q1-3.0*P2**2-3.0*Q2**2+P1**4+Q1**4
     1+2.0*P1**2*Q1**2)
      V23=-(Q4-3.0*P2*P1*Q1-3.0*Q2*P1**2)
      V33=-(-P4-3.0*Q2*Q1*P1+3.0*P2*Q1**2+6.0*P2*P1**2)
C
      AV1=0.0
      AV2=0.0
      AV3=1.0
C
      BV1=0.0
      BV2=-BL
      BV3=AL
C
      TH1=1.0
      TH2=0.0
      TH3=0.0
      TH4=0.0
C
      PH1=AN1
      PH2=AN2
      PH3=AN3
      PH4=AN4
C
      BAX=BV2*AV3-BV3*AV2
      BAY=BV3*AV1-BV1*AV3
      BAZ=BV1*AV2-BV2*AV1
C
      AV11=-PH1*BAX
      AV21=-PH1*BAY
      AV31=-PH1*BAZ
C
      BA1X=BV2*AV31-BV3*AV21
      BA1Y=BV3*AV11-BV1*AV31
      BA1Z=BV1*AV21-BV2*AV11
C
      AV12=-PH2*BAX-PH1*BA1X
      AV22=-PH2*BAY-PH1*BA1Y
      AV32=-PH2*BAZ-PH1*BA1Z
C
      BA2X=BV2*AV32-BV3*AV22
      BA2Y=BV3*AV12-BV1*AV32
      BA2Z=BV1*AV22-BV2*AV12
C
      AV13=-PH3*BAX-2.0*PH2*BA1X-PH1*BA2X
      AV23=-PH3*BAY-2.0*PH2*BA1Y-PH1*BA2Y
      AV33=-PH3*BAZ-2.0*PH2*BA1Z-PH1*BA2Z
C
      W1=-PH1*BV1+TH1*AV1
      W2=-PH1*BV2+TH1*AV2
      W3=-PH1*BV3+TH1*AV3
C
      W11=-PH2*BV1+TH2*AV1+TH1*AV11
      W21=-PH2*BV2+TH2*AV2+TH1*AV21
      W31=-PH2*BV3+TH2*AV3+TH1*AV31
C
      W12=-PH3*BV1+TH3*AV1+2.0*TH2*AV11+TH1*AV12
      W22=-PH3*BV2+TH3*AV2+2.0*TH2*AV21+TH1*AV22
      W32=-PH3*BV3+TH3*AV3+2.0*TH2*AV31+TH1*AV32
C
```

```
      W13=-PH4*BV1+TH4*AV1+3.0*TH3*AV11+3.0*TH2*AV12+TH1*AV13
      W23=-PH4*BV2+TH4*AV2+3.0*TH3*AV21+3.0*TH2*AV22+TH1*AV23
      W33=-PH4*BV3+TH4*AV3+3.0*TH3*AV31+3.0*TH2*AV32+TH1*AV33
C
      A11=0.0
      A21=W3
      A31=-W2
C
      B11=-W3
      B21=0.0
      B31=W1
C
      C11=W2
      C21=-W1
      C31=0.0
C
      A12=-(W2**2+W3**2)
      A22=W31+W2*W1
      A32=-W21+W3*W1
C
      B12=-W31+W1*W2
      B22=-(W3**2+W1**2)
      B32=W11+W3*W2
C
      C12=W21+W1*W3
      C22=-W11+W2*W3
      C32=-(W1**2+W2**2)
C
      A13=-3.0*(W21*W2+W31*W3)
      A23=W32+W21*W1+2.0*W11*W2-W3**3 -W2**2*W3-W1**2*W3
      A33=-W22+2.0*W11*W3+W31*W1+W2**3+W1**2*W2+W3**2*W2
C
      B13=-W32+W11*W2+2.0*W22*W1+W3**3+W1**2*W3+W2**2*W3
      B23=-3.0*(W31*W3+W11*W1)
      B33=W12+2.0*W21*W3+W31*W2-W1**3-W2**2*W1-W3**2*W1
C
      C13=W22+W11*W3+2.0*W31*W1-W2**3-W1**2*W2-W3**2*W2
      C23=-W12+W21*W3+2.0*W31*W2+W1**3+W2**2*W1+W3**2*W1
      C33=-3.0*(W11*W1+W21*W2)
C
      A14=-4.0*W22*W2-4.0*W32*W3-3.0*W21**2-3.0*W31**2+2.0*W21*W1*W3
     1-2.0*W31*W1*W2+W2**4+W3**4+W1**2*W2**2+2.0*W2**2*W3**2+W3**2*W1**2
      A24=W33+W22*W1+3.0*W12*W2+3.0*W21*W11-3.0*W21*W2*W3
     1-3.0*W31*W2**2-W31*W1**2-5.0*W11*W3*W1-W2**3*W1-W2*W1**3
     2-W2*W3**2*W1-6.0*W31*W3**2
      A34=-W23+3.0*W12*W3+3.0*W31*W11+3.0*W31*W3*W2+5.0*W11*W1*W2
     1+3.0*W21*W3**2+W21*W1**2+6.0*W21*W2**2-W3**3*W1-W3*W1**3
     2-W3*W1*W2**2+W32*W1
C
      B14=-W33+3.0*W22*W1+3.0*W11*W21+3.0*W11*W1*W3+5.0*W21*W2*W3
     1+3.0*W31*W1**2+W31*W2**2+6.0*W31*W3**2-W1**3*W2
     2-W1*W2**3-W1*W2*W3**2+W12*W2
      B24=-4.0*W32*W3-4.0*W12*W1-3.0*W31**2-3.0*W11**2+2.0*W31*W2*W1
     1-2.0*W11*W2*W3+W3**4+W1**4+W2**2*W3**2+2.0*W3**2*W1**2
     2+W1**2*W2**2
      B34=W13+W32*W2+3.0*W22*W3+3.0*W31*W21-3.0*W31*W3*W1-3.0*W11*W3**2
     1-W11*W2**2-5.0*W21*W1*W2-W3**3*W2-W3*W2**3-W3*W1**2*W2
     2-6.0*W11*W1**2
C
      C14=W23+W12*W3+3.0*W32*W1+3.0*W11*W31-3.0*W11*W1*W2-3.0*W21*W1**2
```

```
     1-W21*W3**2-5.0*W31*W2*W3-W1**3*W3-W1*W3**3-W1*W2**2*W3
     2-6.0*W21*W2**2
      C24=-W13+3.0*W32*W2+3.0*W21*W31+3.0*W21*W2*W1+5.0*W31*W3*W1+
     13.0*W11*W2**2+W11*W3**2+6.0*W11*W1**2-W2**3*W3-W2*W3**3
     2-W2*W3*W1**2+W22*W3
      C34=-4.0*W12*W1-4.0*W22*W2-3.0*W11**2-3.0*W21**2+2.0*W11*W3*W2
     1-2.0*W21*W3*W1+W1**4+W2**4+W3**2*W1**2+2.0*W1**2*W2**2
     2+W2**2*W3**2
C
      P11=V1+A11*X+B11*Y+C11*Z
      P11X=A11
      P21=V2+A21*X+B21*Y+C21*Z
      P21X=A21
      P31=V3+A31*X+B31*Y+C31*Z
      P31X=A31
C
      P12=V11+A12*X+B12*Y+C12*Z
      P12X=A12
      P22=V21+A22*X+B22*Y+C22*Z
      P22X=A22
      P32=V31+A32*X+B32*Y+C32*Z
      P32X=A32
C
      P13=V12+A13*X+B13*Y+C13*Z
      P13X=A13
      P23=V22+A23*X+B23*Y+C23*Z
      P23X=A23
      P33=V32+A33*X+B33*Y+C33*Z
      P33X=A33
C
      P14=V13+A14*X+B14*Y+C14*Z
      P14X=A14
      P24=V23+A24*X+B24*Y+C24*Z
      P24X=A24
      P34=V33+A34*X+B34*Y+C34*Z
      P34X=A34
C
      AM1=P21*P32-P31*P22
      AM2=P31*P12-P11*P32
      AM3=P11*P22-P21*P12
C
      AM1X=P21X*P32+P21*P32X-P31X*P22-P31*P22X
      AM2X=P31X*P12+P31*P12X-P11X*P32-P11*P32X
      AM3X=P11X*P22+P11*P22X-P21X*P12-P21*P12X
C
      AM11=P21*P33-P31*P23
      AM21=P31*P13-P11*P33
      AM31=P11*P23-P21*P13
C
      AM11X=P21X*P33+P21*P33X-P31X*P23-P31*P23X
      AM21X=P31X*P13+P31*P13X-P11X*P33-P11*P33X
      AM31X=P11X*P23+P11*P23X-P21X*P13-P21*P13X
C
      AM12=P22*P33+P21*P34-P32*P23-P31*P24
      AM22=P32*P13+P31*P14-P12*P33-P11*P34
      AM32=P12*P23+P11*P24-P22*P13-P21*P14
C
      AM12X=P22X*P33+P22*P33X+P21X*P34+P21*P34X-P32X*P23-P32*P23X
     1-P31X*P24-P31*P24X
      AM22X=P32X*P13+P32*P13X+P31X*P14+P31*P14X-P12X*P33-P12*P33X
```

```
     1-P11X*P34-P11*P34X
      AM32X=P12X*P23+P12*P23X+P11X*P24+P11*P24X-P22X*P13-P22*P13X
     1-P21X*P14-P21*P14X
C
      A=AM1**2+AM2**2+AM3**2
      A1=2.0*(AM1*AM11+AM2*AM21+AM3*AM31)
      A2=2.0*(AM11**2+AM21**2+AM31**2+AM1*AM12+AM2*AM22+AM3*AM32)
C
      AX=2.0*(AM1*AM1X+AM2*AM2X+AM3*AM3X)
      A1X=2.0*(AM1X*AM11+AM1*AM11X+AM2X*AM21+AM2*AM21X+AM3X*AM31
     1+AM3*AM31X)
      A2X=2.0*(2.0*AM11*AM11X+2.0*AM21*AM21X+2.0*AM31*AM31X+AM1X*AM12+
     1AM1*AM12X+AM2X*AM22+AM2*AM22X+AM3X*AM32+AM3*AM32X)
C
      B=P11**2+P21**2+P31**2
      B1=2.0*(P11*P12+P21*P22+P31*P32)
      B2=2.0*(P12**2+P22**2+P32**2+P11*P13+P21*P23+P31*P33)
C
      BX=2.0*(P11*P11X+P21*P21X+P31*P31X)
      B1X=2.0*(P11X*P12+P11*P12X+P21X*P22+P21*P22X+P31X*P32+P31*P32X)
      B2X=2.0*(2.0*P12*P12X+2.0*P22*P22X+2.0*P32*P32X+P11X*P13+P11*P13X
     1+P21X*P23+P21*P23X+P31X*P33+P31*P33X)
C
      C=P13*AM1+P23*AM2+P33*AM3
      C1=P14*AM1+P13*AM11+P24*AM2+P23*AM21+P34*AM3+P33*AM31
C
      CX=P13X*AM1+P13*AM1X+P23X*AM2+P23*AM2X+P33X*AM3+P33*AM3X
      C1X=P14X*AM1+P14*AM1X+P13X*AM11+P13*AM11X+P24X*AM2+P24*AM2X
     1+P23X*AM21+P23*AM21X+P34X*AM3+P34*AM3X+P33X*AM31+P33*AM31X
C
      RO=B**1.5*A**(-0.5)
C
      RO1=1.5*B**0.5*B1*A**(-0.5)+B**1.5*(-0.5)*A**(-1.5)*A1
C
      SG=A/C
C
      S1=B**0.5
C
      DRS1=RO1/S1
C
      R=SQRT(RO**2+SG**2*DRS1**2)
C
      T1=P11/B**0.5
      T2=P21/B**0.5
      T3=P31/B**0.5
C
      BN1=AM1/A**0.5
      BN2=AM2/A**0.5
      BN3=AM3/A**0.5
C
      N1=BN2*T3-BN3*T2
      N2=BN3*T1-BN1*T3
      N3=BN1*T2-BN2*T1
C
      SBX=X+RO*N1+SG*DRS1*BN1
      SBY=Y+RO*N2+SG*DRS1*BN2
      SBZ=Z+RO*N3+SG*DRS1*BN3
C
      AOSA=(X**2+Y**2)**0.5
      BOSB=SQRT((SBX-1.0)**2+(SBZ**2))
```

```
          SASB=SQRT((X-SBX)**2+(Y-SBY)**2+(Z-SBZ)**2)
    C
          G0=Z
          H0=SBY
          G=A0SA
          H=B0SB
          S=SASB
    C
           T=ATAN(Y/X)
          P=ATAN(SBY/(SBX-1.0))
    C
          THETA=T*180./3.14159
          PHI=P*180./3.14159
    C
          WRITE(6,20) SBX,SBY,SBZ
       20 FORMAT(//,5X,'SBX=',F10.6,5X,'SBY=',F10.6,5X,'SBZ=',F10.6)
    C
          WRITE(6,25)G0,H0,G,H,S
       25 FORMAT(//5X,'G0=',E15.7,5X,'H0=',E15.7,5X,'G=',E15.7,5X,
         1'H=',E15.7,5X,'S=',E15.7/)
    C
          WRITE(6,30)T,P,THETA,PHI
       30 FORMAT(5X,'T=',F12.6,5X,'P=',F12.6,5X,'THETA=',F12.6,5X,
         1'PHI=',F12.6/)
      500 STOP
          END
    C
    $ENTRY

 AN1=   -1.500000    AN2=    0.000000     AN3=    0.000000     AN4=    0.000000

X=    0.154027     Y=    -0.400000     Z=    1.500000

 SBX=  1.186007    SBY= -0.555826     SBZ= -0.023425

 G0=  0.1500000E 01    H0= -0.5558261E 00    G=  0.4286306E 00    H=  0.1874757E 00    S=  0.1846641E 01

 T=   -1.205228     P=    -0.125275     THETA=  -68.939940     PHI=   -7.177758

STATEMENTS EXECUTED=      197

CORE USAGE       OBJECT CODE=   19376 BYTES,ARRAY AREA=       0 BYTES,TOTAL AREA AVAILABLE=  149504  BYTES

DIAGNOSTICS       NUMBER OF ERRORS=       0, NUMBER OF WARNINGS=       0, NUMBER OF EXTENSIONS=       0

COMPILE TIME=     0.96 SEC,EXECUTION TIME=     0.04 SEC,     10.55.08    SATURDAY     27 JAN 79    WATFIV - JUN 1977 V1L6

        C$STOP
```

APPENDIX C

COMPUTER PROGRAM FOR DISPLACEMENT ANALYSIS OF THE R.S.S.R MECHANISM WITH ONE SET OF DATA AND SOLUTION

```
$JOB
C      ***********************************************************
C      *  SRI RAMA JEYAM. SPIRAM JEYARAM JEYAJEYA RAM.           *
C      *  DISPLACEMENT ANALYSIS                                  *
C      *  THIS PROGRAM OBTAINS THE DISPLACEMENT ANALYSIS OF      *
C      *  R-S-S-R MECHANISM.                                     *
C      *  DATA CARDS:                                            *
C      *  CARD 1 SPECIFIES THE PARAMETERS GO,HO,G,H,S,AND T      *
C      *  OF THE MECHANISM OBTAINED FROM PROGRAM B.              *
C      *  THIS PROGRAM GIVES TWO VALUES OF OUTPUT ANGLE PHI      *
C      *  CORRESPONDING TO THE VALUE OF INPUT ANGLE THETA.       *
C      *  ALL THE DATA CARDS ARE PUNCHED UN 12 COLLUMN           *
C      *  FIELD WITH SIX DECIMAL PLACES.                         *
C      ***********************************************************
C
  300 READ(5,10) GO,HO,G,H,S,T
   10 FORMAT(F12.6,F12.6,F12.6,F12.6,F12.6,F12.6)
      WRITE(6,15)GO,HO,G,H,S,T
   15 FORMAT(1H,,'GO=',F12.6,5X,'HO=',F12.6,5X,'G=',F12.6,5X,'H=',
     1F12.6,5X,'S=',F12.6,5X,'T=',F12.6)
C
      CT=COS(T)
      ST=SIN(T)
C
      AK1=G*CT-1.0
      AK2=G*ST-HO
C
      A=2.0*GO*H
      B=2.0*AK1*H
      C=AK1**2+AK2**2+GO**2+H**2-S**2
      D=SQRT(A**2+B**2-C**2)
C
      PHI1=2.0*ATAN((A+D)/(B+C))
      PHI2=2.0*ATAN((A-D)/(B+C))
      PHI1D=PHI1*180.0/3.14159
      PHI2D=PHI2*180.0/3.14159
C
      WRITE(6,20)PHI1,PHI2,PHI1D,PHI2D
   20 FORMAT(//5X,'PHI1=',F12.6,5X,'PHI2=',F12.6,5X,'PHI1D=',F12.6,5X,
     1'PHI2D=',F12.6//)
C
  200 STOP
      END

   $ENTRY
GO=   1.500000    HO=  -0.555826    G=   0.428631    H=   0.187476    S=   1.846641    T=  -1.203228

    PHI1=  -1.989295    PHI2=  -0.125267    PHI1D= -113.978200    PHI2D=  -7.177282


STATEMENTS EXECUTED=       15

CORE USAGE       OBJECT CODE=     2896 BYTES,ARRAY AREA=        0 BYTES,TOTAL AREA AVAILABLE=  149504  BYTES

DIAGNOSTICS        NUMBER OF ERRORS=        0, NUMBER OF WARNINGS=        0, NUMBER OF EXTENSIONS=        0

COMPILE TIME=     0.15 SEC,EXECUTION TIME=      0.01 SEC,      20.20.14      WEDNESDAY     24 JAN 79     WATFIV - JUN 1977 V1L6
```

APPENDIX D

COMPUTER PROGRAM FOR THE DERIVATIVE ANALYSIS

OF THE R.S.S.R MECHANISM WITH ONE SET

OF DATA AND SOLUTION

```
$JOB
C     ***********************************************************
C      *  SRI PAMA JEYAM. SRIRAM JEYARAM JEYAJEYA RAM.             *
C      *  DERIVATIVE ANALYSIS.                                     *
C      *  THIS PROGRAM DOES THE DERIVATIVE ANALYSIS                *
C      *  OF THE R-S-S-R MECHANISM UP TO FOURTH ORDER.             *
C      *  A= X CO-ORDINATE OF SPHERICAL JOINT SA.                  *
C      *  B= Y CO-ORDINATE OF SPHERICAL JOINT SA.                  *
C      *  C= Z CO-ORDINATE OF SPHERICAL JOINT SA.                  *
C      *  D= SBX= X CO-ORDINATE OF SPHERICAL JOINT SB.             *
C      *  E= SBY= Y CO-ORDINATE OF SPHERICAL JOINT SB.             *
C      *  F= SBZ= Z CO-ORDINATE OF SPHERICAL JOINT SB.             *
C      *  DATA CARDS:                                              *
C      *  CARD 1 SPECIFIES GO,HO,G,H,S                             *
C      *  CARD 2 SPECIFIES T AND P.                                *
C      *  SOLUTIONS OBTAINED ARE THE CO-ORDINATES OF SPHERICAL     *
C      *  JOINTS SA, SB AND THE DERIVATIVES                        *
C      *  UP TO THE FOURTH ORDER.                                  *
C      *  ALL THE DATA CARDS ARE PUNCHED IN 12 COLLUMN FIELD       *
C      *  WITH SIX DECIMAL PLACES.                                 *
C     ***********************************************************
C
      READ(5,10) G0,H0,G,H
   10 FORMAT(F12.6,F12.6,F12.6,F12.6)
      WRITE(6,15)G0,H0,G,H
   15 FORMAT(1H,,'G0=',F12.6,5X,'H0=',F12.6,5X,'G=',F12.6,5X,'H=',
     1F12.6)
C
  100 READ(5,20)T,P
   20 FORMAT(F12.6,F12.6)
C
      T1=1.0
      T2=0.0
      T3=0.0
      T4=0.0
C
      H1=0.0
      H2=H
C
      CT=COS(T)
      ST=SIN(T)
C
      CT1=-ST*T1
      ST1=CT*T1
C
      CT2=-ST1*T1-ST*T2
      ST2=CT1*T1+CT*T2
C
      CT3=-ST2*T1-2.0*ST1*T2-ST*T3
      ST3=CT2*T1+2.0*CT1*T2+CT*T3
C
      CT4=-ST3*T1-3.0*ST2*T2-3.0*ST1*T3-ST*T4
      ST4=CT3*T1+3.0*CT2*T2+3.0*CT1*T3+CT*T4
C
      A=G*CT
      A1=G*CT1
      A2=G*CT2
      A3=G*CT3
      A4=G*CT4
C
```

```
28          B=G*ST
29          B1=G*ST1
30          B2=G*ST2
31          B3=G*ST3
32          B4=G*ST4
      C
33          C=G0
34          C1=0.0
35          C2=0.0
36          C3=0.0
37          C4=0.0
      C
38          CP=COS(P)
39          SP=SIN(P)
      C
40          D=H*CP+1.0
41          E=H0
42          F=H2*SP
      C
43          U=A-D
44          V=B-E
45          W=C-F
      C
46          R1=-H*U*SP+(H1*V+H2*W)*CP
47          S1=U*A1+V*B1+W*C1
48          P1=S1/R1
      C
49          CP1=-SP*P1
50          SP1=CP*P1
      C
51          D1=H*CP1
52          E1=H1*SP1
53          F1=H2*SP1
      C
54          U1=A1-D1
55          V1=B1-E1
56          W1=C1-F1
      C
57          S2=U1*A1+U*A2+V1*B1+V*B2+W1*C1+W*C2
58          R2=-H*(U1*SP+U*SP1)+(H1*V1+H2*W1)*CP+(H1*V+H2*W)*CP1
59          P2=(S2-P1*R2)/R1
      C
60          CP2=-SP1*P1-SP*P2
61          SP2=CP1*P1+CP*P2
      C
62          D2=H*CP2
63          E2=H1*SP2
64          F2=H2*SP2
      C
65          U2=A2-D2
66          V2=B2-E2
67          W2=C2-F2
      C
68          S3=U2*A1+2.0*U1*A2+U*A3+V2*B1+2.0*V1*B2+
           1V*B3+W2*C1+2.0*W1*C2+W*C3
69          R3=-H*(U2*SP+2.0*U1*SP1+U*SP2)+(H1*V2+H2*W2)*CP+
           12.0*(H1*V1+H2*W1)*CP1+(H1*V+H2*W)*CP2
70          P3=(S3-2.0*P2*R2-P1*R3)/R1
      C
71          CP3=-SP2*P1-2.0*SP1*P2-SP*P3
```

```
      SP3=CP2*P1+2.0*CP1*P2+CP*P3
C
      D3=H*CP3
      E3=H1*SP3
      F3=H2*SP3
C
      U3=A3-D3
      V3=B3-E3
      W3=C3-F3
C
      S4=U3*A1+3.0*U2*A2+3.0*U1*A3+U*A4+V3*B1+3.0*V2*B2+
     13.0*V1*B3+V*B4+W3*C1+3.0*W2*C2+3.0*W1*C3+W*C4
      R4=-H*(U3*SP+3.0*U2*SP1+3.0*U1*SP2+U*SP3)+
     1(H1*V3+H2*W3)*CP+3.0*(H1*V2+H2*W2)*CP1+
     23.0*(H1*V1+H2*W1)*CP2+(H1*V+H2*W)*CP3
      P4=(S4-3.0*P3*R2-3.0*P2*R3-P1*R4)/R1
C
      AN1=P1/T1
      AN2=(P2*T1-P1*T2)/T1**3
      AN3=((P3*T1-P1*T3)*T1-3.0*(P2*T1-P1*T2)*T2)/T1**5
      AN4=((P4*T1+P3*T2-P2*T3-P1*T4)*T1**2
     1-7.0*(P3*T1-P1*T3)*T2*T1+(P2*T1-P1*T2)*
     2(15.0*T2**2-3.0*T3*T1))/T1**7
C
      TD=T*180.0/3.14159
      PD=P*180.0/3.14159
C
      WRITE(6,30)A,B,C,D,E,F
   30 FORMAT(//5X,'A=',F12.6,5X,'B=',F12.6,5X,'C=',F12.6,/,5X,
     1'D=',F12.6,5X,'E=',F12.6,5X,'F=',F12.6//)
C
      WRITE(6,25)TD,PD,T,P
   25 FORMAT(//5X,'TD=',F12.6,5X,'PD=',F12.6,5X,'T=',F12.6,5X,'P=',
     11F12.6//)
C
      WRITE(6,90) AN1,AN2,AN3,AN4
   90 FORMAT(//5X,'AN1=',E12.4,5X,'AN2=',E12.4,5X,'AN3=',E12.4,
     15X,'AN4=',E12.4//)
C
  500 STOP
      END

$ENTRY
 1.500000     H0=   -0.555826     G=    0.428631     H=    0.187476


A=     0.154027     B=    -0.400000     C=     1.500000
D=     1.186007     E=    -0.555826     F=    -0.023423



TD=  -68.939940     PD=   -7.177278     T=    -1.203228     P=    -0.125267



AN1= -0.1500E 01     AN2=  0.6117E-04     AN3=  0.2024E-03     AN4=  0.1325E-02
```

VITA

B. T. Devanathan

Candidate for the Degree of

Master of Science

Thesis: SYNTHESIS OF AN R.S.S.R MECHANISM FOR FUNCTION GENERATION AND FOR REPLACING HYPOID GEARS USING HIGHER ORDER SPACE PATH CURVATURE THEORY

Major Field: Mechanical Engineering

Biographical:

Personal Data: Born in Madras State, India, February 10, 1931, the son of Sri. T. Thiruvenkatachary and Smti. Kalyani Ammal.

Education: Obtained the Licientiate in Mechanical and Electrical Engineering (L.M.E.E.) in May, 1953, from the Government Technical College, Hyderabad, India; privately studied and passed the sections A and B examinations of the Institute of Engineers, India, in May, 1956; completed requirements for the Master of Science degree at Oklahoma State University in May, 1979.

Professional Experience: Commenced my career as a maintenance engineer with Messers Sirsilk Ltd., Sirpur-Kaghaznagar, October 1, 1954; joined the reputed organization of Tata Chemicals Ltd., Mithapur-Gujarat State, India, in January, 1956; presently designated as senior maintenance superintendent; possess a total of 24 years of experience in the field of maintenance of mechanical equipment in the chemical industry.

Professional Organizations: Member of the Institution of Engineers, India (M.I.E.); designated as chartered engineer of the above institution; represent Tata Chemicals Ltd. as an alternate member in the pressure vessels committee and the pumps committee of the Indian Standards Institution.