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THE EFFECT OF WAVE INDUCED TURBULENCE ON THE RATE OF ABSORPTION OF GASES IN FALLING LIQUID FILMS

A DISSERTATION SUBMITTED TO THE GRADUATE FACULTY in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

BY

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Norman, Oklahoma

THE EFFECT OF WAVE INDUCED TURBULENCE ON THE RATE OF ABSORPTION OF GASES IN FALLING LIQUID FILMS

APPROVED BY over iene

DISSERTATION COMMITTEE i congress a

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My wife Joan whose help made this dissertation possible.

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THE EFFECT OF WAVE INDUCED TURBULENCE ON THE RATE OF ABSORPTION OF GASES IN FALLING LIQUID FILMS

CHAPTER I

SUMMARY

Prediction of the rate at which mass is transferred from a gas phase into a liquid film flowing down an inclined plate requires, in part, a knowledge of the hydrodynamics of the liquid film and of the surface area between the two phases. In this work, the surface area was measured by the use of a capacitometer which gave a continuous trace of the film thickness at a known point in the contacting cell. At Re_L up to 1732, the maximum flow rate studied, the increase in surface area was only 2.5%.

Wave characteristics such as mean wave amplitude and mean wave frequency were also obtained.

To measure the effect of the waves on the concentration profile in a liquid film, carbon dioxide was absorbed into water and the resulting concentration profiles measured by use of an interferometer. Evaluation of these profiles in terms of the eddy diffusivity showed that the diffusivity was a function of the liquid flow rate and of location in the

film. It was found that the diffusivity was a maximum in the center of the film and decreased as the boundaries of the film were approached. The value of the diffusivity tended toward the molecular diffusivity at the boundaries. There seemed to be no effect on the diffusivity with location in the film at a Re_{L} of 732. However, when the liquid rate was increased the value of the diffusivity at the center of the film also increased. The effect of the waves on the concentration profile was to compress and expand the profile, indicating that the waves do not directly affect the concentration profile.

From these results a model is proposed in which the main resistance to mass transfer is in a pseudo stagnant film at the surface of the film in which eddies from the bulk of the film are damped out. For design purposes empirical equations are presented that will allow the mass transfer equations to be solved using the results of the diffusivity measurements.

CHAPTER II

INTRODUCTION

The removal of constituents from gas streams is generally carried out industrially by contacting the gas stream with a liquid in which certain components of the gas stream are preferentially soluble. Some methods used in contacting the gas with the liquid are: packed columns, agitated vessels, plate columns, wetted-walled columns and spray towers. In all these methods the primary considerations are maximizing the contact surface area between the gas and the liquid phases and minimizing the resistance to mass transfer within the phases. The diffusion rates of the solute in the phases, particularly the liquid phase, are generally low, and agitation of the phase or a chemical reaction which removes the diffusing species will enhance the rate of mass transfer. Studies of this increase in mass transfer with agitation is complicated unless accurate information is also available for the surface area between the gas and liquid phases. However, accurate prediction of the surface area in most of the contacting methods given above is not possible as it involves a knowledge of the size and distribution of gas bubbles and/or liquid drops that form in the contacting device.

Of the contacting devices mentioned, the wetted-walled column has been used most extensively in the study of experimental gas absorption because its contact area is the most easily calculated. This apparatus usually consists of a large cylindrical tube or a flat inclined plate down which a thin layer of liquid flows. The gas stream flows above the surface of the liquid and the contact surface area is assumed to be the surface area of the liquid film. Because the surface area is known, studies of methods for improving the rate of mass transfer by dynamic action in either the gas or liquid film are possible.

Application of the wetted-walled column to studies of absorption when the main resistance to mass transfer is in the liquid film have been made by many investigators. One of the first results published was that of Johnstone and Pigford. (27) They assumed that the surface of the film was saturated, the velocity profile in the liquid film was described by the laminar film theory and that transfer of the solute to the film was by molecular diffusion then analytically solved the resulting differential equations. They reported that their model predicted the rate of mass transfer as long as the surface of the film was smooth. But, when the liquid Reynolds number exceeded 25 the surface of the film became unstable with ripples forming near the top of the cell and moving down the film. Under these conditions the heights of liquid film transfer units, $H_{\rm L}$, were reported to be $\frac{1}{2}$ to $\frac{1}{4}$ the values

-4

predicted by their equations. This effect of ripples was also noted by other investigators. (12, 18, 38 and 41)

In an attempt to better understand this dynamic action a number of investigators have made experimental and theoretical studies on:

- a. flow conditions when ripples form
- b. velocity profiles in the film
- c. surface area under flow conditions when ripples occur.

Analysis of the problem is complicated because any hydrodynamic theory which explains waves contains simplifications and cannot explain the enhanced mass transfer. For example, Lamb's (34) analysis of waves clearly shows a local circulation. However, this result is an ideal action, with each particle moving in the same way relative to its neighbor and would not result in increased mass transfer. Similarly Portalski's (44) modification of Kapitsa's (30) results for vertical films probably describes the dynamics in the most complete fashion. However, it is not at all clear how such an essentially ideal motion would enhance the rate of mass transfer. Levich (35) has included a wave motion in the only attempt to use such dynamics combined with mass transfer to obtain the result that about 15% more transfer will occur with waves. This enhancement appears as the result of an additional velocity term in the differential equation describing the system. This theory assumes very small penetrations,

and further the main question of the nature of the effect involved cannot be known, as the existence of eddy irreversibility cannot be accommodated by his theory.

Flow conditions under which waves form on the surface of liquid films flowing down vertical plates has also been studied with some detail. The values for the "minimum critical Reynolds number" at which waves form quoted in the literature vary, but in general they are small. Binnie (5, 6) reported a value of 4.7 for the critical Reynolds number, Friedman and Miller (17) a value of 25 and Brauer (8) a value of 8.

Film thickness and the onset of instability have been measured using a variety of experimental techniques. These include direct weighing (29), radioactive tracer (25), photographic (7, 8) and electrical capacitance (14, 50). Results of these measurements indicated that in spite of the appearance of ripples at a Reynolds number of about 25, the average film thickness was quite well predicted ty Nusselt's laminar film theory up to a Reynolds number of about 250. The Reynolds number-friction factor correlation for true streamline flow was found to be applicable for Reynolds numbers of less than 1500 (17), indicating that even though the total flow as calculated from the average film thickness is affected by the presence of ripples, the total shear in the film is not changed until turbulent flow conditions are reached. Brauer (8), reported that the average film thickness was

successfully predicted by Nusselt's laminar film theory up to a Reynolds number value of 400, but that the measurement of the surface velocity of the film indicated that the deviation from Nusselt's theory occurred at a Reynolds number of 8, the transition point where ripples start to form. To characterize the flow patterns, Brauer defined a friction factor f by

$$f = \frac{\tau_w}{\rho \overline{v}^2/2} \tag{1}$$

where τ_W is the wall shear stress, ρ the density of the liquid and \overline{V} the average film velocity. Plotting f vs. the Reynolds number, he found definite breaks in the curve which he interpreted to be transition points in the flow pattern. From this interpretation, empirical equations relating the wall shear stress to the Reynolds number and physical properties of the liquid were calculated for each region defined by the transition points.

More recently, experimental investigations have elucidated the structure of the wave flow. Tailby and Portalski (50) measured instantaneous film thicknesses using a small capacitance probe. The measurements were used to determine the increase in surface area as a result of the waves. Lilleht and Hanratty (36) measured the instantaneous film thickness of stratified liquid-gas flow with the liquid Reynolds number in the range 200 to 500 and the gas Reynolds number 4,000 to 10,000. The film thickness was measured by passing a narrow light beam through the film onto a photomultiplier tube. Under these conditions, the waves were three dimensional. Analysis of the data consisted in a statistical interpretation of the fluctuating film thickness. Grimley (20) measured velocity profiles in films flowing on vertical walls with a modified ultra-microscope. He concluded that the maximum velocity did not occur at the interface but slightly inward from the surface. Wilkes and Hedderman (57) measured velocity profiles in falling films by stereoscopically photographing tiny air bubbles in the liquid. They were able to verify Nusselt's parabolic profile under conditions of no waves and to get average profiles in wavy flow. The shape of the velocity profiles under the condition of waves was still very nearly parabolic. Residence times of a tracer (salt solutions) injected into a falling liquid film have been measured by Asbjørnse (2) in the region of Reynolds numbers 50 to 600. From the resulting distribution functions he proposed a double layer flow model consisting of an outer turbulent layer superimposed on an inner laminar layer. The turbulent layer is described by a diffusion model similar to that applied by Taylor and Tichacek (53) to turbulent flow in pipes.

Yih (60) and Brooke Benjamin (9) have theoretically investigated the stability of laminar flow down an inclined plane with two-dimensional infinitesimal disturbances. In the case of a fluid flowing down a vertical wall, Brooke Benjamin found that a minimum critical Reynolds number does

not exist "in the usual sense", but that the flow is unstable for all finite Reynolds numbers, a result independent of the presence or absence of surface tension. For Reynolds numbers less than 4 there is only very slight amplification of infinitesimal disturbances, but for Reynolds numbers greater than 4 the amplification increases considerably with increasing Reynolds numbers. Yih's calculations indicated a minimum critical Reynolds number of approximately 1.5 for the case of a fluid with zero surface tension. However, his results are very sensitive to small computational errors and this result is presented mainly to show that the flow is unstable at low Reynolds numbers. In a more recent publication Yih (61) has presented a new and simpler method of solving the resulting equations. With the new method he is able to duplicate Brooke Benjamin's results. Sternling and Berr-David (48) solved the stability equations based on the linearized stability theory by use of a digital computer. Their results support Brooke Benjamin's conclusion that flow down a vertical wall is unstable for all Reynolds numbers and that waves are not seen at the very low Reynolds number because of the small amplification rate. Tailby and Portalski (51) integrated the equations of motion assuming the velocity parallel to the wall is distributed parabolically and the pressure gradient across the film is zero. Their analysis was an extension of an analysis made by Kapitza (31) and minimum critical Reynolds numbers and surface enlargements

due to rippling were calculated. Henratty and Hershman (21) using a similar integral approach found results which agreed in form with those of Brooke Benjamin.

The use of surface active agents to prevent rippling has been considered by several investigators. Tailby and Portalski (50) and Grimley (20) have found that the use of these agents has a strong stabilizing effect on the film which prevents the formation of waves until much larger Reynolds numbers.

Studies of the absorption of gases into falling liquid films when the liquid phase is controlling have been made with the liquid film in laminar flow, wave flow and wavy flow with surface active agents. Emmert and Pigford (16) studied the adsorption and desorption of oxygen and carbon dioxide in water. Their experiments confirmed Pigford's (43, 27) theoretical calculations for mass transfer of gases into laminar films. The formation of waves was prevented by the use of surface active agents. When rippling occurred in the film, the mass transfer rates became much larger. From experiments with and without surface active agents they concluded that surface active agents did not appreciably affect the surface resistance to mass transfer. Lynn, Straatemeier and Kramers (38) studied the absorption of sulphur dioxide into water, and aqueous solutions of HCl, NaHSO3 and NaCl. Surface active agents were used to hinder the formation of waves. Their experiments confirmed the results that rippling

significantly increased transfer rates. Stirba and Hurt (49) measured the rate of solution of various organic acids from the wall into the falling film and Garwin and Key (18) and Kirkbride (32) did the same for heat transfer from the wall into the film. Their experimental results also confirmed the increase of mass or heat transfer with the presence of waves in the film.

In an attempt to prevent rippling without using surface active agents experimental data has been taken in short wetted-walled columns of about 2 to 5 cm in length. Vivian and Peaceman (54) measured the desorption of CO₂ from water and Clp from dilute HCl solutions. Their experimental results were 10 to 30% below the predictions of theoretical model assuming a flat velocity profile and using the penetration theory (24). Perry (47) investigated the absorption of CO2 in water and KOH solutions. He also found that the experimental adsorption rates were low compared to the theoretical model used by Vivian and Peaceman. To account for the discrepancy he assumed that the interface was not saturated with CO2 and defined this nonequilibrium by an accommodation coefficient. Scriven and Pigford (47) studied the phase equilibrium between gas-liquid interfaces during absorption. They concluded that equilibrium is established almost instantaneously for solutes that have low solubilities when only physical absorption occurs. Cullen and Davidson (12) investigated the effect of surface active agents on the surface

resistance and found that the resistance to mass transfer fell to zero at very low and very high concentrations of these agents. They found that the agents can reduce the rate of absorption up to 25% in a system in which they do not influence the hydrodynamics. Harvey (23) verified the effect of the surface active agents found by Cullen and Davidson by measuring concentration profiles of CO_2 diffusing into water with surface active agents present. The concentration profiles were measured with a Mach-Zehnder interferometer. Goodridge and Brickwell (19) and Scriven and Pigford (47) have also studied the effects of these agents and have reached the same conclusion that surface active agents can appreciably increase the surface resistance to mass transfer.

Several theoretical models have been proposed to explain the effect of waves on the mass transfer rate. Danckwerts (13) proposed a surface renewal model in which the surface of the liquid film is continually being replaced by fresh liquid. Harriott (22) proposed a model for mass transfer from a turbulent fluid to an interface in which eddies arriving at random times come to within random distances from the surface sweeping away the accumulated solute. Transfer is assumed to be by molecular diffusion in the interval between eddies. Brauer (8) assumed that the main resistance to mass transfer is in a laminar layer of liquid next to the wall and that mass transfer in this region is only by molecular diffusion. The thickness of the layer is defined in

terms of wall shear stresses which he determined experimentally from the film thickness. Stirba and Hurt (49) proposed that the effect of waves could be expressed in terms of an eddy diffusion coefficient. Using the analytical solution of Johnstone and Pigford (27), for the case of molecular diffusion of a gas into laminar film, they assumed that the thickness of the film with waves was the laminar film thickness and calculated the diffusion coefficient which predicted the experimental values for the average solute concentration in the film. Perhaps one of the more sophisticated theoretical analyses published at the present time is the work of Levich (35). Using theoretical equations for the velocity components perpendicular and parallel to the direction of flow of the film he derives equations for the mass transfer of gas in a liquid film by considering bulk flow both perpendicular and parallel to the direction of flow and molecular diffusion from the interface towards the wall. Results of his analysis indicated that the flux into the film is increased by 15% over that expected with laminar flow. He also states that this result was experimentally verified.

These results of the experimental and theoretical studies indicate that the ripples are a symptom of some dynamic action which enhances the mass transfer. In mose cases one postulates that this occurs near the surface. However, a ripple action obviously refers to a large dynamic effect. Ideal waves on the surface of a quiet pool do not increase

the mass transfer rate; thus, in a flowing system, some other mechanism must cause both the formation of the waves and the increase in mass transfer. Attempts to explain their effect through theoretical studies of hydrodynamics have not been fruitful as the resulting equations are too complicated to be solved.

The results of this discussion indicate that before more theoretical analyses are made, experimental data on concentration profiles in the film must be obtained. Data on the amplitude and frequency of the ripples moving on the film is also necessary in order that the mass transfer characteristics might be related to the film characteristics. This data will determine which theories should be pursued further and suggest modifications that will improve them.

Research Objectives

This research was the initial phase of a study of the absorption of gases into falling liquid films. The ultimate object was to obtain a better understanding of the phenomena of waves which form on the surface of the film and how they affect the mass transfer rates in the film.

The initial phase of this program is to develop experimental techniques for measuring concentration profiles of the solute in the film and the amplitude and frequency of the waves which form on the surface of the liquid film.

Data from these measurements was to be used to determine whether a correlation exists between the wave properties

and the mass transfer properties. If a correlation exists, equations could be developed relating the two phenomena. Results from these experiments will also be used to indicate what hydrodynamic data is required in future work to elucidate our understanding of the phenomena of mass transfer in falling films.

Experimental Methods

Up to the present time, as far as the author knows, the only experimental method used in measuring the absorption of gases in falling liquid films measures the liquid flow rate and determines the average concentration of the solute in the film from liquid samples taken at the exit of the contacting plate or tube by chemical analysis. This method is not particularly satisfactory as nothing is known about the concentration profiles in the film. As a result several theoretical models can be fitted to the same data. The measurement of the concentration profile has not been accomplished because of the small thickness of the falling film. The thickness is generally of the order of 1 mm or less and samples cannot be taken at different points in the film. To be able to measure the profile, some sort of an analytical system is required that will allow measurement of the concentration profiles in the film without disturbing the liquid flow pattern. One possible method is, if the optical properties of the film are a function of the concentration of the solute in the film, to measure the index of refraction of the film.

Watson (55) measured the change in the index of refraction of water as a function of concentration of carbon dioxide and found that the change was directly proportional to the amount of carbon dioxide present:

$$\eta_{c} - \eta_{o} = A(C - C_{o}) \tag{2}$$

This system is particularly desirable as even though carbon dioxide reacts with water to form carbonic acid the absorption is considered to be physical absorption, and the heat of reaction is sufficiently small that the temperature of the water is not changed.

Harvey (23), utilizing the above characteristics of the carbon dioxide water system, calculated the diffusion coefficients of carbon dioxide in quiescent water by measuring the change in index of refraction of the water with a Mach-Zehnder interferometer. This was accomplished by photographing the interference patterns formed by the interferometer as carbon dioxide was absorbed into the water. Photographs were taken at known time intervals after carbon dioxide entered the cell above the surface of the liquid. From the resulting photographs he was able to measure at known distances from the interface the change in concentration of the solute as a function of time and to calculate the second derivative of the change in concentration as a function of distance from the interface. Using Fick's law of diffusion he then evaluated the diffusion coefficients. Caldwell, Hall, and Babb (10, 11) used the interferometer to measure mutual diffusivities of volatile liquid systems and Lin, Moulton, and Putnam (37) have measured the concentration of an electrolyte near the surface of a polarized electrode in turbulent liquid streams. From the results of these experimental investigations it was concluded that the interferometer offered a practical means of evaluating concentration profiles in falling liquid films. The system chosen to be used with the interferometer was carbon dioxide and water.

The theory of the interferometer and application has been discussed in many publications (3, 4, 10, 11, 15, 23, 28, 33, 37, 39, 45, 56, 58, 59). Of these references, the discussion by Ladenburg (33) probably is the most informative. These references are presented for the reader if further information is desired about the interferometer, but will not be discussed unless applicable to the discussion on the Mach-Zehnder interferometer which follows.

The Mach Zehnder interferometer consisted of 2 onehalf silvered beam splitter plates and 2 full silvered mirrors located at opposite corners of a parallelogram or rectangle. (see Figure 4) A light beam from a monochromatic light source was centered on one of the beam splitter plates. One-half of the light beam was reflected onto a full silvered mirror which reflected this beam parallel to the original light beam and onto the second beam splitter plate. Again one half of the beam was reflected away from the

interferometer and the other half passed through the plate. The light beam that passed through the first beam splitter plate was reflected from the surface of the second full silver mirror and on to the surface of the second beam splitter plate. One-half of this beam passed through the plate and away from the interferometer. The other half was reflected by the plate and emerged from the interferometer parallel to the first light beam. The purpose of splitting a light beam emitted from a single light source was to obtain two light beams with the same characteristics that could be made to intersect and form interference patterns. The bright bands in the resulting interference pattern were formed when the light wave trains of the two beams were in phase and the dark bands, when they were out of phase. The relationship between the spacing of the fringes $\boldsymbol{\xi}$ and angle of intersection of the light beam γ is given by (23, 33)

$$\xi = \frac{\lambda}{2\eta \sin \frac{\gamma}{2}}$$
(3)

where η is the index of refraction of the media through which the light of wave length λ is passing. The difference in the optical path lengths of the two light beams must be less than about $\pi/31,400$ inches where π is the total number of fringes obtainable when the two light paths are exactly equal (23).

The optical path length through a body is its physical length multiplied by its index of refraction. Applying this

relationship to a light beam passing through a liquid with some concentration C, of a solute, the increase in the optical path length is given by

$$(\eta_{c} - \eta_{o})L = \Delta N \lambda$$
⁽⁴⁾

where $\eta_c - \eta_0$ is the change in the index of refraction of the liquid due to the presence of solute, L is the width of the liquid through which the light beam is passing, ΔN is the number of wave lengths or fringe shifts corresponding to the change in the index of refraction and λ is the wave length of the light source. Substituting equation (2) into equation (4) equation (5) is obtained which relates the change in concentration of the solute to the fringe shift.

$$A(C - C_{O})L = \Delta N \lambda$$
(5)

Dividing this expression by the expression for saturation conditions results in

$$\frac{C - C_{o}}{C_{s} - C_{o}} = \frac{\Delta N}{\Delta N_{s}}$$
(6)

This equation is very useful in determining concentration values from the interferometric data since the total fringe shift ΔN_S which corresponds to the saturation concentration of the solute C_S is easily determined experimentally.

CHAPTER III

EXPERIMENTAL EQUIPMENT

Interferometer

Description of the Interferometer

The interferometer was designed and constructed at the University of Oklahoma. All parts except the optical pieces were machined and fabricated by the Physics Department Machine Shop. The optical mirrors, beam splitter plates, compensating plates and lenses were purchased from the J. Unertl Optical Company of Pittsburgh, Pennsylvania.

The interferometer was designed in such a manner that a plane formed by the center points of the mirrors and plates could be rotated from 0° to 90° . The purpose of this was twofold. The first was to increase the versatility of the equipment. The second was to allow the inclined plate contacting cell to be mounted to the interferometer thus allowing the angle of inclination of the cell to be changed without disturbing the interferometer adjustment.

The basic support of the interferometer consisted of a cast iron sheet, 24" x 46" x 2-1/4". (Figure 1) Bolted at each end of this sheet were steel support arms 2-1/2" x



FIGURE NO. 1 PHOTOGRAPH OF INTERFEROMETER BASE

1-7/8"x 11" which were used to support the base for the mirrors, plates and lenses. The interferometer base was a hollow, ridge supported aluminum block 19" x 38" x 3" which was cast and subsequently heat treated to reduce internal stresses. Mounted at the center point of the 19" sides, parallel to the long axis of the base were 3/4" steel rods. These rods were used to support the base and were mounted on the support arms in such a manner that the base could be rotated and locked at any angle between 0° and 90°. The interferometer base was cut out on one side so that equipment could be mounted in the light path of the interferometer regardless of its angle of inclination (Figure 2). To adjust the level of the interferometer 3/8" x 1" machine bolts were mounted on the bottom of each corner of the basic support.

To isolate the interferometer from external vibrations, the basic support was set on two 6" I beams 12' long. The I beams were set 18" apart and were supported by concrete pillars 6-1/2' apart. The pillars were buried 6' in the ground and were completely isolated from the building. (Figure 3)

The mirrors, beam splitters and compensating plates were mounted on the machined surface of the interferometer base in a 60° parallelogram configuration. (Figures 2 and 4) The mirrors and beam splitters were 2" square and were mounted by a 3 point suspension in rectangular mounting brackets. These brackets were constructed so that the mirrors and beam



FIGURE NO. 2 PHOTOGRAPH OF THE INTERFEROMETER



FIGURE NO. 3 PHOTOGRAPH OF I BEAM SUPPORTS AND CAMERA



- 1. High Pressure Mercury Vapor Lamp
- 2. Condensing Lens
- 3. Filter
- 4. Slit
- 5. Iris Diaphram
- 6. Collimating Lens
- M1 & M3. Full Silvered Mirrors
- M2 & M4. Beam Splitter Plates
- 9. Test Cell
- 11. Compensator Plates
- 13. Photographic Objective Lens
- 14. Camera

Figure 4. SCHEMATIC DIAGRAM OF INTERFEROMETER

splitters could be rotated around 2 separate axes of rotation. These axes of rotation were perpendicular and parallel to the plane formed by the center points of the mirrors and plates. The compensating plates were 60 mm square and were also mounted by 3 point suspension in rectangular brackets similar to the ones used for the mirrors and plates. However, only one axis of rotation, perpendicular to the plane of the center points of the mirrors, was available for adjustment.

Adjustment of the mirrors, beam splitter plates and compensating plates around their axis of rotation was made by a double lever arm type adjustment. This consisted of two lever arms. The first was attached to the mirror or plate mounting bracket perpendicular to the axis of rotation. The second was pivoted at the end of the first arm and extended back towards the axis of rotation parallel to the first arm. To move the lever arm an adjustable spring plunger was mounted to the base of the mounting bracket and was adjusted so that it rested on the end of the first arm on the side opposite the pivot point of the second arm. An adjustable screw was also mounted to the base of the mounting bracket in such a manner that it rested on the second arm about 1/4" from the arms pivot point. A second adjustable screw was mounted at the end of the second arm and rested on the first arm. With this arrangement the screw at the end of the first arm was used as a coarse adjustment and the screw located at the end of the second arm was used as a fine adjustment. Figure 5 is


Figure 5. MIRROR AND BEAM SPLITTER PLATE LEVER ARM ADJUSTMENT

a schematic drawing of the lever arm adjustment.

To permit adjustment of the optical path length between the two beams of the interferometer, the base of mirror M3, (see figure 4) could be translated in a direction which bisected the angle formed by M2, M3, and M4. The same double lever arm and spring plunger type adjustment was used for this translation.

The mirrors and beam splitters were 1/2" thick. The mirrors were made from pyrex glass and the plates from interferometer quality borcsilicate crown glass. The mirrors had all surfaces fine ground and the rear surfaces parallel to the reflecting side with a surface accuracy of 1/10 wave green light. The surfaces were aluminized and had a silicon monoxide overcoating. The beam splitter plates had a surface accuracy of 1/10 wave green light and were parallel to 1/5 wave length. One surface had a high efficiency dielectric beam splitting coating with a reflectance of 50% at 60° angle of incidence. The rear surface had a low reflectance coating. The compensating plates were 45.7 and 22.8 mm thick respectively. They were made from borosilicate crown glass with a surface accuracy of 1/10 wave length green light and were parallel to 1/5 wave length.

The inclined plate contacting cell support was mounted to the interferometer base in such a manner that the cell was centered in the top light beam of the interferometer. The support consisted of a flat plate approximately 1-1/2 times

longer than the cell. The support was mounted at an angle to the interferometer base so that when the cell was positioned such that the light beam passed through the cell near the water inlet, the lower part of the cell did not disturb the lower beam of light. Metal guides were used on the plate to keep the cell aligned when it was moved to a new position. (Figures 1 and 2)

To minimize the effect of air at different temperatures passing through the light beams of the interferometer and causing distortion in the interference fringes, the interferometer was placed in a square polyvinyl bag that was supported from the ceiling and sealed around the base support of the interferometer. (Figures 3 and 6)

Principles of Adjustment

There were four degrees of freedom in the optical system. Referring to figure 4 these degrees of freedom were: 1) and 2)--Mirrors M1 and M3 and plates M2 and M4 could be rotated about axes perpendicular and parallel to the plane formed by the center points of the mirrors and plates. The effect of rotation around the perpendicular axis was to cause the emergent beams to intersect in a vertical plane causing horizontal fringes. The effect of rotation around the parallel axis was to cause the emergent beams to intersect in a horizontal plane causing vertical fringes. 3)--Rotating the compensating plates around an axis perpendicular to the plane formed by the center points of the mirrors and plates caused



FIGURE NO. 6 PHOTOGRAPH OF INTERFEROMETER TENT

the fringes to move parallel to themselves. This resulted from the increase or decrease of the optical path length in this path and thus changed the plane of localization of the beams. 4)--Mirror M3 could be moved along a line formed by the bisector of the angle formed by M2, M3 and M4. (Figure 4) Translation of M3 along this line changed the difference in the optical path lengths between the two emergent beams causing the fringes to move sideways parallel to themselves and together.

Alignment of the cell windows was critical in that the windows must be parallel both vertically and horizontally. The effect of nonvertical alignment was to cause curved fringes as a result of the unequal optical path length along the vertical width of the cell windows. Longitudinal alignment was necessary to insure that the interference pattern remained visible when the cell was translated along the cell support.

Method of Adjustment of the Interferometer

Before the interferometer was aligned, the mirrors, plates and compensating plates were cleaned. To clean the mirrors and plates the surfaces were blotted with ethyl alcohol using "Batiste" as the applicator. After the alcohol was applied to the surfaces, the excess was removed by blotting the surfaces with another clean dry piece of "Batiste". Extreme care had to be used in cleaning the surfaces as the soft overcoating of silicon monoxide scratched easily.

Alignment of the interferometer was done in two stages. The first adjustment or coarse adjustment consisted in orienting the mirrors and plates so that their surfaces were parallel and adjusting the mirrors and plates until the interference pattern became visible. The second or fine adjustment consisted in orienting the fringes and adjusting for maximum fringe definition.

Coarse adjustment. The interference filter was removed from the interferometer and mirror M3 translated until it was located at its center point which was marked on its The compensating plates were rotated until their surbase. faces were perpendicular to a line between the centers of M3 and M4. The interferometer base was then rotated until the cell was in a horizontal position. The cell was filled 3/4full with distilled water and leveled with a spirit level so that the cell windows were approximately perpendicular to the plane of the center points of the mirrors and plates. The light source was turned on and adjusted so that the parallel light beam emerging from the collimating lens passed through the center of M4 and centered on M3. M4 was then rotated until the light beam reflected from its surface centered on M2. The same procedure was used to adjust M3 by centering its reflected light beam on M2. It was necessary that the light beam passing through the contacting cell be parallel with the surface of the water in the cell. To make this adjustment a mirror was placed between the objective lens and

the camera. It was positioned such that it would reflect the emergent light beam from the object lens on to a telescope mounted near the interferometer. The telescope was focused on the surface of the water in the cell. The light beam was then adjusted so that it was parallel to the surface of the water. Looking through the telescope Ml was rotated around its axis perpendicular to the plane formed by the center points of the mirrors and plates until a dark band appeared at the surface of the water. MI was then rotated in the opposite direction until the band disappeared. The light beam was then approximately parallel to the surface. If the light beam emerging from Ml and passing through the cell did not center on M2 after it had been made parallel to the surface of the water, the adjustment screws on the base support of the interferometer were adjusted and the above procedure repeated until it was centered on M2.

The next step was to adjust the mirrors and plates so that all surfaces were in the same plane. The photographic objective lens and collimating lens were removed from the light path and the slit was adjusted to give a point source of light. The mirror between M2 and the camera was rotated so that the emergent beam from M2 reflected back on to the surface of M2. The telescope was positioned such that it was looking at M1 through M4. Four points of light should have been visible when the telescope was focused. If they were not visible, the mirror between M2 and the camera was

adjusted until they became visible. Double reflections occurred in this method of adjustment and care had to be taken that the four brightest points of light were used. M3 and M2 were rotated around their axis of rotation until the 4 points of light were <u>superimposed</u> on each other. The collimating lens was then replaced and the centering of the light beams on the mirrors and plates checked. If they were not centered, they were readjusted and the superimposing of the points of light repeated.

After completing the above alignment, fringes should have been visible. To check for fringes, the collimating lens and photographic objective lens were replaced. The telescope was again mounted so that it was looking at the mirror between M2 and the camera. The mirror was rotated until its reflected beam was centered on the telescope. The telescope was then focused on the surface of the water, the interference filter replaced on the optical bench and the slit opened until the surface of the water became visible. It was important that the slit opening be kept as small as possible during this stage of adjustment as the number of fringes that were visible was inversely proportional to the width of the slit opening.

Looking through the telescope fringes should have been visible if the alignment was done carefully. If fringes were not visible, M3 was translated along the bisector of the angle formed by M2, M3 and M4 until they became visible. If

no fringes were found after the translation of M3, M3 was rotated slightly around either of its axes of rotation and again translated. By repeating this procedure fringes were brought into focus.

Fine adjustment. At this point fringes were visible with the slit opening at its smallest setting. It was now necessary to adjust for maximum fringe definition, intensity and orientation. Mirror M3 was rotated around its horizontal axis until the fringes were correctly oriented. During this orientation it was occasionally necessary to rotate M2 around both axes in order to maintain fringe definition. When the orientation was complete, M3 was translated and M2 again rotated around both axes of rotation until the maximum fringe definition was obtained. To adjust the fringe spacing M3 was rotated around its perpendicular axis. Again fringe definition and orientation were controlled by rotating M2 around both axes of rotation and M3 around its horizontal axis of rotation. The source size was now increased. The desired slit size was about 3 mm x 2 mm. However this was not obtained directly. The slit was opened until the fringes became faint. The fringes were then brought back to maximum definition in the same manner as described above. This procedure was repeated until the desired slit opening was obtained.

To complete the alignment, the mirror between the camera and M2 was removed and the camera shutter opened. A ground glass screen was placed in the photographic plate holder and the camera focused on the surface of the water in the cell. A mirror was placed behind the camera and positioned so that the image of the ground glass screen could be seen with the telescope mounted at the interferometer. Looking through the telescope, Ml was again rotated so that the light beam made a positive angle with the surface of the water and a dark band formed at the surface of the water. Ml was then rotated in the opposite direction until the dark band disappeared and no distortion of the fringes occurred at the interface. M1 and M2 were then locked in position. If the fringes had lost some of their definition they were again brought back to maximum definition by the same procedure as outlined above but using only M3 and M4. To check for maximum fringe definition the interference filter was removed and the white light fringes adjusted until 3 or 4 black fringes occurred with greenish colored fringes fading out on the left and reddish colored fringes fading out on the right of the black fringes.

The Contacting Cell

Two conditions governed the design of the contacting cell:

1. The increase in the optical path length caused by the contacting cell should equal that caused by the compensating plates.

2. The glass windows of the cell must be parallel to

one another within 1/4000" over any 1/2" square of the cell windows.

Compensating plates instead of a duplicate cell were chosen for two reasons. First, the use of a duplicate cell would limit the versatility of the interferometer and second, duplication of the cell accurately would be difficult.

Other conditions which were considered in the design were:

1. The cell windows must be capable of accurate adjustment.

2. Width of the cell must be sufficient so that the change in the optical path length due to absorption of CO_2 into the water film causes a fringe shift which can be measured accurately. Also the width should be sufficient to minimize shear effects in the water film due to the wall without being so wide that continuous waves across the cell cannot exist.

3. Length of the contacting cell must be sufficient to allow the CO_2 to diffuse at least 1/3 of the way into the film.

4. The cell windows should be sealed sufficiently so that a liquid seal is maintained along the glass plate and a positive pressure of carbon dioxide maintained in the cell.

5. The entrance and exit of the water from the cell should be such that a liquid seal can be maintained at all times.

Design of the Optical Components of the Cell

The optical path length of a light beam through the interferometer was defined as:

$$L = \sum_{i=1}^{N} \eta_{i} L_{i}$$
 (7)

where η_i is the index of refraction of the media through which the light was passed and L_i was the length of the material of index of refraction η_i .

Equating the optical path lengths as defined by equation 7 to the two light beams of the interferometer, the following optical balance between the compensating plates and the contacting cell was obtained:

$$T_{c}(\eta_{g} - 1) = 2T_{w}(\eta_{g} - 1) + W(\eta_{w} - 1)$$
 (8)

where T_c and T_w were the thicknesses of the compensating plate and the cell windows respectively; W was the width of the cell, and η_g and η_w were the index of refraction of glass and water respectively. For design purposes 1.517 was chosen for the index of refraction of glass and 1.333 for water. To be sure that the cell windows would seal without distortion, the windows were arbitrarily chosen to be 0.4 inches thick. To determine the cell width, the change in index of refraction of water when saturated with carbon dioxide was taken to be 0.00003 at 20°C and 1 atm pressure (23). The corresponding fringe shift calculated from equation (4) with a light source of 5460 Å and an assumed cell width of 7.5 cm gave 4.13 fringes. This was sufficient to obtain good accuracy in determining the fringe shift. The assumed cell width was also found satisfactory for the formation of continuous waves across the cell.

Substituting 7.5 for W in equation (8) and the values for the indexes of refraction and cell window thickness given above, the compensating plate was calculated to be 68.5 mm thick. To reduce the size of the plate it was divided into two plates of 22.8 mm and 45.7 mm thickness.

Design of the Contacting Cell

In order to estimate the cell length to be used, the average concentration in the case of a vertical plate with laminar flow was calculated using the analytical solution of Johnstone and Pigford (27).

$$\frac{c_z - c_1}{c_0 - c_1} = 0.7857 \ e^{-5.1213} \ \pi' + 0.1001 \ e^{-39.318} \ \pi' + 0.03599 \ e^{-105.64} \ \pi' + 0.01811 \ e^{-204.75} \ \pi'$$
(9)

where $\mathtt{C}_{\mathbf{Z}}$ is the average concentration at \mathbf{Z}

 ${\tt C}_{\rm i}$ is the saturation concentration at the interface

 C_0 is the initial concentration at Z equal to 0 and

$$\pi' = \frac{M^{4} 2DZ \mu}{(Y_{\rm L})^{4} \rho g \sin \theta}$$
(10)

where θ is the angle of inclination of the cell. For the case where the water temperature was 22.2°C, the Reynolds number was 1834, θ was 90° and the cell length was 36 inches, the average concentration at the end of the plate was 12.2% of the saturation concentration. This percentage saturation would have been satisfactory; however, several other factors were considered along with the average percent saturation. To measure the concentration profile in the film, the light beam from mirror Ml had to pass through the liquid film, parallel to the bottom of the cell and as close to parallel as possible to the surface of the flowing film. To meet this condition the contacting cell was chosen to be a flat plate with optically flat glass windows on both sides of the plate which acted as sides to the cell as well as allowing the light beam to pass through the film. Increasing the length of the cell thus required longer optically flat cell windows which are not readily available. Secondly, the film thickness as calculated from the laminar film theory was only 0.5 mm at a Reynolds number of 1834. As this was the maximum flow rate of interest in this study, smaller flow rates resulted in much smaller film thicknesses. From data reported by Brauer (8), the troughs of the waves which occur on the surface of the film existed at film thickness of one-half the calculated laminar film thickness, thus reducing the usable film thickness to 0.25 mm. This film thickness was not sufficient to obtain accurate interferometric data.

To overcome this problem, it was decided to use an inclined plate. At an angle of inclination $9^{\circ}40'$ the laminar film thickness was calculated to be 0.92 mm and the concentration at the bottom of the cell 10.3% of the saturation value. Comparison of these results to those calculated for $\theta = 90^{\circ}$ indicated the desirability of using an inclined plate because of the increased film thickness.

Based on the above calculations the following cell design was chosen:

The cell was rectangular in shape with an internal width of 2.95", height of 7/16" and length of 35.33". It was constructed from aluminum with a 1/16" thick glass plate 2.95" x 34.83" cemented to the bottom plate to act as the surface for the water flow. The sides of the cell were optically flat glass windows 1" x 0.4" x 36" made from shadowgraph quality crown glass with a surface accuracy of 1/4 wave green light with surface parallelism to within 30 seconds of arc along any 2 inch section. To allow the windows to be placed as close to the bottom glass plate as possible and still allow adjustment for parallelism, the aluminum support for the glass plate was approximately 1/4" smaller in width than the glass plate and the gaskets extended 1/16" beyond the edge of the glass plate. To prevent water from flowing between the glass plate and the gasket, the space between them was filled with silastic and the surface tapered from the surface of the gasket to the edge of the glass plate.

The windows were supported against the silastic gasket by four 1" x 1/2" x 1/4" plastic strips mounted to the cell by knurled screws at the top and bottom of the strips. Alignment of the windows was made by adjustment of the mounting screws. (Figures 7, 8 and 9)

Entrance of water into the cell was through a 1/4" x 2.95" slot located at the top edge of the inclined plate. At this entrance, the water first passed into a 2-1/4" x 3-1/4" x 1" calming chamber. The exit slot was located at the upper end of the chamber so that a liquid seal could be maintained when water was flowing onto the glass plate. Water was removed from the cell through another 1/4" x 2.95" slot cut at a 45° angle to the base located at the end of the glass plate. The water leaving this slot entered a 1-1/2" x 3-1/4" x l" chamber. The exit slot was located at the top edge of the chamber so that carbon dioxide could not be trapped inside the chamber. The water level was maintained at the top edge of the slot in order to have a liquid seal. Carbon dioxide entered and left the cell through 1/4" x 2.95" slots at the upper and lower ends of the top plate of the cell. In both cases the gas first entered a $1" \times 1" \times 3 - 1/4"$ chamber before entering the cell. In addition to the gas inlet and outlet, four additional 1/2" diameter holes were located on the top plate, 10", 19", 25" and 31" respectively from the entrance slot. These holes were sealed except when wave data was being taken. (Figures 8 and 10)





FIGURE NO. 8 PHOTOGRAPH OF CONTACTING CELL



Figure 9. CONTACTING CELL LEVEL ADJUSTMENT

To allow horizontal adjustment of the bottom plate, the cell was mounted on a 1/2" diameter aluminum rod. (Figure This rod was in turn mounted onto a 2-3/4" x 1/4" x 32" 10) flat plate by 2 brass supports which allowed the cell to be rotated approximately $\pm 10^{\circ}$ from the horizontal position. The bottom support was machined flat along its edges and, when mounted on the cell support located on the interferometer, allowed the cell to be moved along its long axis without changing its orientation. Rotation of the cell around its long axis was controlled by a lever arm type adjustment mounted on the side of the upper cell support (Figure 9). A spirit level was also mounted on the cell so that it could be leveled. To prevent water from being carried out by the exit gas stream in the case of co-current flow a wedged shaped piece of aluminum was located at the lower end of the cell. The purpose of the wedge was to separate the two phases before they passed through the discharge slots. (Figure 10)

Alignment of the Cell Windows

For maximum fringe definition and orientation the surfaces of the windows had to be parallel along the long axis of the cell and perpendicular to this axis. In addition, the windows had to be parallel to the sides of the cell support which guided the cell on the interferometer cell support. Non-parallelism of the windows along the perpendicular axis resulted in the loss of fringe definition when the cell was

translated. Alignment along the long axis was not so critical as the perpendicular axis because the interferometer could be adjusted to give maximum fringe definition.

The windows were aligned by the following procedure. The cell and cell support were placed on a flat piece of aluminum with the flat edge of the cell support against an aluminum strip mounted to the piece of aluminum. A second strip of aluminum was mounted 3" from the first strip with its outside edge parallel to the edge of the first strip against which the cell support was placed. The cell was then leveled using the spirit level mounted on the cell. A Starrett dial micrometer with 1/1000 inch scale divisions was mounted on a flat base and placed against the second strip. The dial micrometer was adjusted so that it rested against the cell window. The micrometer and base were then moved along the aluminum strip and the cell window adjusted until the micrometer indicated no change along the entire length of the window. To obtain alignment along the vertical axis the micrometer reading at the top edge of the window was recorded. The micrometer and base were then removed from the reference strip and placed against an accurately machined surface which was perpendicular to the base and the micrometer reading again recorded. The micrometer was then lowered to the height of the bottom edge of the cell window and the micrometer adjusted so that it indicated the same reading as before it was lowered. The micrometer was then placed against the second

strip and the dial reading recorded. If the reading at the top and bottom of the window were different the window was adjusted till the readings were the same. This procedure was repeated until the same reading was obtained at the top and bottom edges of the window. The alignment along the long axis of the cell was also checked periodically to be sure that this alignment had not changed. Alignment of the opposite window was done in the same manner. The cell was rotated 180° , leveled and the procedure outlined above repeated. In the initial step of the alignment, the windows were adjusted so that they were flush with the bottom glass plate. If it was not possible to align the windows to the reference strip after this adjustment, the cell position with reference to the 1/2" diameter rod upon which it was mounted was changed. This was easily accomplished by loosening the mounting screws and moving the cell to the correct position.

Final alignment was made when the cell was placed on the interferometer. If the cell windows were not parallel along the vertical axis, the resulting fringes were curved. Where this occurred, the windows were readjusted until straight fringes were obtained. Parallelism along the horizontal axis was also checked at this time by moving the cell along the interferometer cell support and checking the resultant fringes. During this check it was important to check the cell level each time the cell was translated to see that it remained level. As was mentioned earlier, loss of some

of the fringe definition during the translation could be corrected by adjustment of the interferometer and parallelism along the horizontal axis required only that the fringes remained visible.

Photographic Methods

Light Source

Because of the existance of waves on the surface of the water film, the height of the film was not constant at a given cell position and water flow rate, but varied depending on the amplitude and frequency of the waves. To be able to photograph the film height accurately an exposure time of 1/100 second or less was necessary. To obtain sufficient light for the photographic plates used, a Westinghouse SAH 1000A, 1000 watt high pressure mercury vapor lamp was used as the light source. The mercury vapor lamp was chosen as it emitted a high intensity band of light around 5460 Å which was the light wave length that was desired for photographing the fringes. The light from mercury vapor lamp was focused on a Gaertner Model 160A adjustable slit by two 60 mm diameter plano-convex lenses with focal lengths of 140 and 250 mm respectively. The height of the slit was controlled by a disk mounted on the front edge of the slit. The disk contained various width slots ranging from 1 mm to 4 mm. By rotating the disk, the height of the slit could be changed. The light beam leaving the condensing lenses passed through

a Bausch and Lomb 5460 Å interference filter and an infra red filter before passing through the slit. The emergent beam from the slit passed through an iris diaphragm and a 300 mm focal length collimating lens. The parallel light beam leaving the collimating lens was 60 mm in diameter and was centered on plate M4 of the interferometer. All of the optical components mentioned above were mounted on an optical bench which was supported by four hydraulic jacks. The hydraulic jacks were used to support the light source and component items so that the parallel light beam emerging from the collimating lens could be centered on M4 of the interferometer regardless of the angle of inclination of the interferometer. (Figure 1)

Camera

The emergent beam from plate M2 passed through a 250 mm focal length, 60 mm diameter Triplett photographic objective lens and an Ilex Universal Shutter No. 1 mounted with a 3" focal length lens. The shutter was mounted on the front of a 3" diameter brass tube 4 feet long. On the opposite end of the tube was located a mounting bracket for the photographic plate holders. (Figure 2) The mounting bracket was constructed so that the plate holder could be moved the full 10" length of the photographic plates to allow four pictures to be taken on one plate. The diameter of the camera at the location of the photographic plates was 2-1/4". The objective lens and camera lens were positioned such that a

magnification of approximately 14 diameters was obtained.

Film

2" x 10" Kodak spectroscopic plates type 1-N were used to photograph the interference patterns. These plates were developed with Kodak D-19 developer. Satisfactory photographs were obtained with these plates using an exposure of 1/100 second with the filters mentioned above and a slit opening of 2 mm x 3 mm.

Alignment

The mercury vapor lamp and its housing were mounted on an optical bench and the bench leveled with a spirit level. The plano-convex condensing lenses were placed on the bench in their approximate location and their centers set to the same height as the center of the light opening of the mercury vapor lamp housing by use of a cathotometer. Similarly, the slit and collimating lens were placed on the bench and set to the same height as the center of the light opening. The mercury vapor lamp was then started and the light positioned in the housing such that the condensed beam emerging from the condensing lenses was centered on the slit. The condensing lenses and slit were then translated along the optical bench until the beam was focused on the slit. Positioning of the collimating lens was determined by turning the optical bench so that the beam emerging from the collimating lens could be seen on a wall approximately 20 feet from the

collimating lens. The collimating lens was translated on the optical bench until the diameter of the beam of light on the wall was the same diameter as the collimating lens. The optical bench was then returned to its original position and adjusted so that the emergent beam from the collimating lens centered on plate M4 and mirror M3 of the interferometer.

The camera was mounted on an optical bench and the optical bench supported by two hydraulic jacks. The hydraulic jacks allowed the camera height to be changed and were mounted to the I beams so that the camera could be moved horizontally. To align the camera, the image of the surface of the water film was centered on the shutter and the camera adjusted until the image on the ground glass screen was centered and the camera horizontal. The camera was focused by translating it along the optical bench.

Once the camera was positioned and focused, the magnification ratio of the camera could be determined. The contacting cell was removed from the interferometer and was replaced by a micrometer eye piece disk with 0.05 mm scale divisions. The eye piece was positioned so that it was in focus for the camera. The eye piece was then photographed and the magnification determined by measuring the distance between the scale divisions on the photograph and comparing it to the actual scale spacing.

Auxiliary Equipment

Figure 10 is a flow chart of the overall experimental



apparatus. To provide a constant head system for the water, the water was pumped from a 30 gallon reservoir to a 5 gallon tank approximately 15 feet above the reservoir by a 1/20 hp centrifugal pump. The discharge from the pump was filtered through a cheese cloth filter to prevent foreign particles from entering the overhead tank. Dissolved impurities were kept at a minimum by passing a small stream from the outlet of the pump through an ion exchange column and back into the main reservoir. The overhead tank contained two outlets. The first, located near the top of the tank, was the overflow and returned the water to the main reservoir. The second was located at the bottom of the tank and was connected to the contacting cell through a needle valve and rotometer.

As the index of refraction of water was a strong function of temperature, it was imperative that the water temperature be held constant and a constant temperature control system was placed in the overhead tank. The water temperature was held constant to better than $\pm 0.01^{\circ}$ C determined from photographs of the fringe pattern during the heating and cooling cycle. All tanks, reservoirs and water lines were insulated with fiber glass insulation backed by an aluminum foil cover to minimize heat transfer.

Water flow rates were measured by a calibrated rotometer and controlled by a needle valve. The water level in the exit slot of the cell was controlled by forming a syphon from the exit chamber through a needle valve to the

floor drain. Gas flow rates were controlled by needle valves and measured by a capillary orifice. The gas streams after leaving the orifice passed through coils in both the main reservoir and the overhead tank. The purpose of this was to adjust the gas temperature as closely as possible to that of the liquid before it entered the cell.

The cell pressure was measured by a manometer filled with "Hi Vacuum" oil connected to the cell through one of the holes on the top plate of the cell.

Capacitometer

To obtain information about the amplitude and frequency of the waves which form and move down the surface of the liquid film, a capacitometer was built to measure the capacitance between the surface of the film and a small copper plate mounted a known distance above the bottom plate. If the liquid flowing down the plate were a good conductor of electricity, the capacitance would be a function only of the air space between the copper plate and the surface of the liquid and the surface area of the copper strip:

$$\hat{C} = \delta \frac{A}{d}$$
 (11)

where $\boldsymbol{\delta}$ is the dielectric constant of air

- A is the surface area of the probe and
- d is the distance between the probe and the surface of the water film

If the liquid were not a good conductor for electricity, the

capacitance of the film must also be considered along with that of the air gap. In either case, calibration curves of air gap vs. capacitometer readings must be made with known film thickness as the parameter.

The width of the probe in the direction of flow should be at least one order of magnitude smaller than the wave length of the waves on the surface of the liquid film. If the width is larger than this, a point value for the film thickness will not be obtained but rather some average value of the film thickness due to several waves.

Design

The capacitometer used was a modification of a capacitometer built by Dukler and Bergelin (14) using a frequency modulation method of detection (Figure 11). It consisted of a 6AU6 oscillator circuit, a discriminatory network, a two stage amplifier and power supplies. The output from the second stage amplifier instead of being recorded on an oscilloscope (as was used in the reference model) was recorded on a Minneapolis-Honeywell Model 906C Visicorder using a mechanically damped galvanometer with a maximum frequency response of 270 cps \pm 10%. The ground side of the galvanometer was connected through a 50K variable resistor to the output of a VR105 constant voltage tube. By changing the variable resistor, the capacitometer could be adjusted for maximum galvanometer deflection.

The capacitance measured by the capacitometer was



between a copper strip $1/8" \ge 3/16"$ oriented so that its long axis was perpendicular to the direction of flow and the surface of the film. To be able to adjust the position of the strip, it was mounted on the end of a shaft of a depth micrometer with scale divisions of 1/1000". The strip was centered on the end of the shaft with the long axis of the strip perpendicular to the shaft. The micrometer was mounted in one of the 1/2" diameter holes located in the top plate of the contacting cell with the bottom surface of the copper strip parallel to the surface of the glass plate. A shielded conductor was used to connect the capacitometer to the micrometer and surface of the liquid. The shielding was used as the ground wire and was connected to the bottom plate of the cell. The conductor was connected to the micrometer. To insulate the micrometer from the cell, strips of polyethylene 2-1/4" x 1/2" x .005" were placed between the micrometer and the cell and the micrometer was held in place by plastic clamps. Figure 11 is a schematic diagram of the capacitometer and figure 12 a photograph of the capacitometer, the visicorder and the micrometer mounted to the cell.

To correct for drift that might occur in the capacitometer after the calibration was completed, a trim capacitor of range 12 to 62 mm was connected to the bottom plate of the cell and the capacitometer conductor through a two pole micro switch. When the switch was in the closed position the trim capacitor was connected to the capacitometer and the micrometer



FIGURE NO. 12 PHOTOGRAPH OF CAPACITOMETER INSTALLATION

disconnected. Similarly, when the switch was in the open position, the micrometer was connected and the trim capacitor disconnected. To correct for drift the switch was closed and the trim capacitor adjusted until the galvanometer of the visicorder was on scale. This scale reading was recorded and used as a reference value whenever the drift was to be checked. If some drift occurred, the capacitometer was adjusted until the correct scale reading was obtained.

Calibration

Calibration of the capacitometer was relatively simple when the liquid in the cell was a good conductor. However in the case of distilled water the liquid was not a good conductor and the change in capacitance due to the film thickness had to be considered.

To calibrate the capacitometer, the cell was placed horizontally and filled with distilled water to a level which was equal to the maximum film thickness anticipated. The micrometer was then extended until the copper strip was approximately 0.01 inches above the surface of the water. The variable capacitor of the capacitometer was then adjusted until a maximum sensitivity was obtained on the microammeter. Next, the micrometer was extended until the copper strip touched the surface of water. The microammeter should have read zero at this point. If it did not, the variable capacitor was adjusted until it did. The visicorder was next connected to the capacitometer and the variable resistor

connected in series with the galvanometer adjusted until the current through the galvanometer was between 0.2 and 0.3 milliamps. The galvanometer was adjusted so that when the microammeter read zero the scale reading of the visicorder was 6.0. The copper strip was then raised above the surface of the water and its surface dried. The adjustment of the capacitometer was next checked to be sure that the output of the capacitometer as shown on the micrometer or visicorder went to zero when the copper strip touched the surface of the water. The reference capacitor was then connected and the visicorder reading recorded. Before each calibration run, the adjustment of the capacitometer was checked against this reading and if some drift had occurred, the variable capacitor of the capacitometer was adjusted.

As was mentioned earlier, since distilled water was not a good conductor it was necessary to consider the effect of the liquid film as well as the air gap. To obtain this information calibration curves of air gap vs. visicorder readings were taken for various film thicknesses starting at the maximum film thickness expected and ending at the minimum film thickness. In a typical calibration run the copper strip was dried and then extended until the visicorder came on scale. Drift was checked and corrected using the reference capacitor. The calibration was then begun. Micrometer readings and visicorder readings were taken until the copper strip touched the surface of the water. The film thickness for

that particular run was calculated by recording the micrometer reading when the strip hit the surface of the water. The film thickness was then decreased and the procedure repeated. At the completion of all the calibration runs the copper strip was extended until it hit the surface of the glass plate. The micrometer reading was recorded and used to calculate the film thickness for each run and the air gap which corresponded to the visicorder readings. From this data plots of micrometer readings vs. visicorder readings were prepared using the film thickness as the parameter for the curves.

After experimental data was taken and the micrometer settings recorded, cross plots were prepared in which the visicorder readings were plotted against film thickness for the micrometer settings recorded. From these plots the film thickness could be determined directly from the visicorder recordings (Figure 15).
CHAPTER IV

EXPERIMENTAL PROCEDURE

In the preceding chapter the alignment of the interferometer and capacitometer were discussed. In this chapter the techniques used in the experimental application will be discussed.

Application of the Interferometer

The interferometer was rotated from the horizontal position to the desired angle of inclination. The base was locked in position and the angle of inclination determined by using a cathetometer to measure the vertical height of the contacting plate whose length was known. The cell was then translated until the light beam passed through the cell about 2" from the bottom end of the contacting plate and leveled. The cell was filled with distilled water above the point where the light beam passed through the cell. The light source was then adjusted so that it centered on plate M4 and mirror M3 of the interferometer. The camera was positioned and the ground glass screen placed in the camera. The shutter was opened and a mirror mounted behind the camera positioned so that a telescope mounted at the interferometer

could see the reflected image of the ground glass screen. The cell was leveled using the spirit level mounted on the cell and the image of the cell on the ground glass screen checked for parallelism of the light beam with the bottom glass plate. If they were not parallel, the 3/8" bolts on the bottom of the interferometer basic support were adjusted so that when the light beam from the collimating lens was centered on M4 and M3 the beam of light passing through the cell was parallel to the glass plate and was centered on M2.

In order to measure the fringe shifts that occurred when the water film absorbed carbon dioxide, a reference line was placed in the light beam which was independent of the camera or any other part of equipment that might have moved. This reference line was obtained by mounting a spider web vertically between two support arms that were mounted between the camera and the objective lens on the I beams support. The position of the web was set so that it was in focus along with the fringes and surface of the water film when viewed on the ground glass screen.

In all the experimental runs, carbon dioxide diffused completely to the bottom plate. In order to be able to measure the fringe shift, photographs of the carbon dioxide-water fringe patterns had to be compared with photographs of the case where no carbon dioxide was present in the film. The location of the fringes was not a function of the liquid flow rate and remained constant once the interferometer was

adjusted. This allowed the fringe shift to be measured by using a reference line that was independent of the fringe system. By measuring the distance at a given point in the film between the reference line and a fringe when air was in the cell and subtracting this from the distance between the reference line and the same fringe when carbon dioxide was present, the actual fringe shift was determined. The reference photograph used to determine the original location of the fringes was made when air was in the cell as the presence of air did not affect the fringe system. Only one photograph was necessary at any one cell position since the fringe system was not affected by the liquid flow rate and remained constant with respect to the reference line.

The maximum number of fringe shifts that occurred when the liquid film became saturated with carbon dioxide was around 2.67. The distance the fringes moved, however, was a function of the fringe spacing and the number of fringe shifts. Thus:

$$X = \Delta N \cdot X_{fs}$$
(12)

where X was the distance the fringe moved, ΔN was the number of fringe shifts calculated from equation (4) and X_{fs} was the fringe spacing. The optimum fringe spacing was found experimentally to be about 0.27 mm. With this fringe spacing about 9 to 11 fringes were included in the field of vision of the camera. This fringe spacing of 0.27 mm corresponded to a

fringe spacing of about 3.25 cm on photographs of the interference pattern used to determine the fringe shift. Larger fringe spacings increased the total fringe shift, but fringe definition was lost resulting in larger errors in measuring the fringe shift. Smaller fringe spacings gave satisfactory fringe definition but the total fringe shift was reduced thus again increasing the measurement error.

The locations of the fringes in relation to the reference line in the film were measured with a cathetometer by mounting the photographs in a holder and placing the holder on a wall about 3 feet from the cathetometer. The holder was rotated on the wall until the reference line in the photograph was parallel to the horizontal cross hair of the cathetometer. The distance from the reference line to the center of a fringe was measured and marked. The cathetometer was then lowered and a new location in the film measured. This was done for both air-water and carbon dioxide water fringe patterns. Points were selected about every 0.5 cm from the bottom of the film to the interface. After the center points of the fringes were measured and marked, the photographs were rotated until the horizontal cross hair of the cathetometer was parallel to the bottom of the film. The distance from the bottom of the film to the points marked on the fringes was measured and recorded with the appropriate fringe-reference line distances. Accuracy in measuring the fringe shift distances was found to be \pm 0.02 cm. This

resulted in an accuracy of ± 0.04 cm for the actual fringe shift distance and ± 0.02 cm for the location of the fringes in the film. Values of the experimental fringe shifts range from $\Delta N = 0.1$ to 1.2. With a fringe spacing of 3.25 this was equivalent to measured distances of 0.325 to 3.90 cm. Based on these calculations the measurement error at the bottom of the film was $\pm 12.5\%$ and near the interface $\pm 1.2\%$

Upon completion of the centering of the light beam and adjustment for parallelism with the bottom plate, the water was removed from the cell and a water film started flowing down the glass plate. Because of the waves that occurred on the surface of the film the air-water interface was not flat. As a result some distortion of the fringes occurred near the interface. To correct for this distortion the camera was translated on its optical bench until the fringe distortion was minimized. This resulted in the camera being focused on the inside surface of the cell window closest to the camera. The cell position with respect to the camera was determined by placing a wire perpendicular to the glass plate on the outside surface of the cell window closest to the camera. The wire was set at the desired cell position and its value read from a scale on the bottom of the cell. The cell was next translated until the wire was centered on the reference line when viewed on the ground glass screen. The wire was then removed and the reference line used to mark the exact cell position in the photograph. The interferometer

was adjusted for maximum fringe definition, orientation and the fringe spacing set so that 9 to 11 fringes were visible on the ground glass screen. The final adjustments of the camera were made and the camera locked into position.

The temperature controller on the overhead tank was checked and the heating and cooling cycle adjusted so that they were of about equal duration. This adjustment was made by controlling the flow rate of tap water through the cooling coils. When properly adjusted, the heating and cooling cycles were approximately 20 seconds in length. Temperature control of better than $\pm 0.01^{\circ}$ C was necessary as a temperature change of 0.01° C corresponded to a fringe shift of 0.026or a shift of 0.0822 cm on the enlarged photographs of the interference pattern.

The cell was purged with air and then the by-pass valve on the gas inlet stream opened and the valve on cell gas exit line closed. This resulted in a positive pressure of about 0.26 mm mercury gauge in the cell. The water flow rate was set near its maximum value (Re = 1834) so that as large a film as possible flowed down the inclined glass plate. The barometric pressure and cell pressure were recorded so that the total pressure in the cell was known. The camera shutter speed was set at 1/100 second and a photographic plate placed in the camera. The air-water fringe system was now photographed. The photographic plate was moved to the next position and the cell purged and filled with carbon

dioxide. As the fringe location with respect to the reference line with no carbon dioxide present in the cell had been photographed it was absolutely necessary that extreme caution be used in filling the cell with carbon dioxide. If the cell pressure was allowed to build up, the windows moved and changed the location of the fringes. To fill the cell, the air stream was shut off and the carbon dioxide stream started. The carbon dioxide volumetric flow rate was set the same as that of the air stream to insure the same static cell pressure when the gas flow was stopped. The water flow rate was reduced to prevent water from being trapped in the gas exit line during purging. The exit gas stream valve was opened and the gas by-pass valve closed. The water level at the bottom of the cell was controlled so that no water was carried out by the exit gas stream. If water got into the gas lines, pressure fluctuations occurred in the cell and affected the fringe reference line location. Approximately 3 minutes were allowed for the cell to purge. The by-pass valve was then opened and the exit valve closed. During the purging of the cell, the cell pressure increased about 1 to 2 mm of mercury. If pressure increased above this value, the run was re-started. The water flow rate was set at the maximum value desired and the water level at the bottom of the cell adjusted so that a liquid seal was maintained. Four photographs were taken at each flow rate. This was necessary as the film thickness varied depending on the location of the

waves when the picture was taken. After the four photographs were taken, the water flow rate was changed and four more pictures taken. This procedure was repeated until all the desired flow rates were photographed. The photographic plate was removed and the ground glass screen placed in the camera. The water flow rate was set at the lowest rate for which measurements were made. The cell was then purged with air and the number of fringe shifts that occurred when air entered the cell estimated from visual observation on the ground glass screen. Estimates of ±0.5 fringe shifts were possible and this data was used to determine the integral number of fringe shifts measured from the photographs. The cell was next translated to a new position and the above procedure repeated. After the experimental data was taken, the magnification ratio of the camera was determined as outlined in a preceding section.

Translation of the cell was necessary because although the width of the light beam passing through the cell was 2", the width of the cell photographed was only 0.1". Thus, in order to obtain information on how the fringe shift changed with distance the cell was translated and photographs taken at each cell position. The interferometer was realigned at each new cell position because of slight non-alignment of the cell windows. The realignment of the cell did not affect analysis of the data because the concentration of carbon dioxide in the water was expressed in terms of fringe shifts

and the distance a fringe was shifted was a function only of the fringe shift and the fringe spacing which was obtained from the air-water photograph.

To insure the same flow pattern in the liquid film after translating the cells, the cell was leveled with the spirit level. Sometimes, the bottom glass plate was not completely parallel with the light beam after the cell was leveled because of slight bending of the glass plate. If this occurred, the light source was raised or lowered by use of the hydraulic jacks until the light beam was parallel to the glass plate. This method of adjustment was very helpful because it was done quickly without realignment of the interferometer. A uniform water flow rate in the cell was obtained by taking the flow from a constant head tank and controlling with a needle valve. The flow rate was measured by a rotameter with a calibration accuracy of $\pm 1.3\%$ at Re_L in the cell of 732 and $\pm 0.4\%$ at Re_L equal to 1834.

Analysis of Carbon Dioxide Concentration by Analytical Techniques

To check the results of the interferometric data, water samples were taken from the exit water stream and the average carbon dioxide concentration determined by adding an excess of NaOH and BaCl₂ to the samples to precipitate BaCO₃. The excess NaOH was then back-titrated with HCl to a phenolphthalein end-point.

To be able to obtain average carbon dioxide

concentrations at the same location in the cell as the interferometric data, the water level at the bottom of the celi was adjusted so that the edge of the gas-water interface at the top plate of the cell was centered at one of the locations where the interferometric data was taken. Slight errors in the concentration measured occurred because of the end effects where the air-water interface ended. The length of the cell was sufficient however to minimize this effect. The minimum contact length measured was 22.5" from the entrance of the water onto the glass plate. The water temperature was the same as that measured with the interferometric data. To duplicate the total cell pressure, the barometric pressure was determined and the resulting pressure difference between the cell pressure desired and the barometric pressure obtained by partially closing the gas inlet by-pass valve until the cell manometer indicated the desired pressure difference.

Water samples were taken by a 10 cc hypodermic needle from a gum rubber hose used to connect the water level control valve to the water discharge chamber of the cell. The water was allowed to flow for a minimum of 3 minutes at a given water level in the cell and liquid flow rate before data was taken.

Application of the Capacitometer

The angle of inclination of the cell was set to the desired value and the micrometer positioned in a hole in the

top plate of the cell 31.5 inches from the water inlet. The capacitometer was adjusted as outlined in Chapter III and the copper strip lowered until it was just above the surface of the film with the long axis of the strip perpendicular to the direction of flow of the film. For each flow rate measured the copper strip was set as close to the surface of the water film as possible without the waves touching the strip. The chart speed of the visicorder was set to 2 inches/sec and the time marker set to 0.1 second. A wave trace of about 60 seconds duration was taken. The chart speed was then changed to 10 inches/sec and about a 6 second run was made at this chart speed. The micrometer setting for this run was recorded in order that the calibration data and curves might be completed later. The water flow rate was then changed and the procedure repeated. Before each run, the reference capacitor was checked so that drift in the capacitometer could be corrected. After the runs were completed, the cell was placed horizontal and the calibration started. Care was taken to maintain a water film on the glass plate at all times until the calibration was completed.

All data was taken with quiescent air in the cell. It would have been better to have had carbon dioxide in the cell but this complicated the experimental procedure and it was felt that the effect of the carbon dioxide on the physical properties of the water was not sufficient to warrant the additional experimental problems.

The reason for maintaining a liquid film on the glass plate at all times until the calibration run was completed was that water got under the glass plate and affected the measured capacitance. However, once the water got under the plate the capacitance remained constant unless the water film was removed.

Calibration of the capacitometer indicated that with an air gap of about 0.003", a 0.003" change in film thickness corresponded to a 1" deflection on the visicorder. While with an air gap of 0.035", a 0.0205" change in film thickness was required for the same visicorder deflection.

CHAPTER V

EXPERIMENTAL RESULTS

Interferometric Data

Appendices A and B are tabulations of the cathetometer measurements taken from photographs of the fringe patterns. The data reported under column X_{CO_2} give the measured distances from the center points of the fringes to an arbitrary reference line photographed with the interference patterns of the CO_2 - water system. The data under column $X_{\mbox{Air}}$ give the measured distance from the center points of the same fringes to a reference line photographed with the interference patterns of the air-water system. Column Y is the distance from the surface of the glass plate to the point where the fringe shifts were measured. The actual fringe shift that occurs at a specified Y is calculated by subtracting X_{Air} from $X_{CO_{c}}$ at this Y and dividing it by the fringe spacing given for that particular set of data. It is important to note that each run has a different fringe spacing and that comparison of the data can only be made after the fringe shifts have been calculated.

The fringe locations next to the surface of the glass plate were not measured because of a linear distortion of the

fringe pattern in this region. This distortion was caused by a slight curvature of the glass plate resulting from the method by which it was mounted to the cell. This distortion occurred in all the interference patterns photographed and was not a function of the operating conditions. To determine if any fringe shifts were occurring in this region $X_{\rm CO_2}$ and ${\rm X}_{\mbox{\rm Air}}$ were measured for several sets of data. Results of these measurements indicated that the fringe shift that occurred in this distortion region was the same as that which was measured directly above the distorted pattern. Figure 13 is a photograph of the fringe patterns of the CO2-water system. The distortion that was discussed above can be seen at the bottom of the photograph. Figure 14 is a photograph of the fringe pattern of the air-water system at the flow rate and cell position as figure 13. The same distortion is present in this photograph as was found in figure 13. Determination of the location of the surface of the glass plate in the interference patterns was made in the following manner:

The depth micrometer used in measuring the wave data was mounted on the top plate of the cell and the shaft extended until it touched the surface of the glass plate. The cell was filled with water and the interference patterns observed on the camera mounted with a ground glass screen. The cell was positioned so that the shaft of the micrometer was also visible with the interference pattern. The micrometer shaft was raised until light could be seen under the end of



FIGURE NO. 13 PHOTOGRAPH OF CO₂-WATER INTERFERENCE PATTERN



FIGURE NO. 14 PHOTOGRAPH OF AIR-WATER INTERFERENCE PATTERN

the shaft. It was then extended carefully until the light under the shaft disappeared. The difference between the micrometer reading at this point and on the surface of the plate was the error in determining the surface of the glass plate. It was found that the surface of the plate was located at the bottom edge of the linear distortions in the interference pattern. The maximum error measured in determining the location of the surface was 0.0005".

Column C/C_S in Appendix A is the ratio of the calculated carbon dioxide concentration corrected for pressure effects to the saturation concentration. The values for C/C_S were calculated in the following manner.

From equation (5) the relationship between ΔN and the actual carbon dioxide concentration was given as

$$C - C_{C} = \frac{\lambda}{LA} \Delta N \qquad (5)$$

Where A is the proportionality constant that relates the change in concentration of carbon dioxide with the corresponding change in the index of refraction. The value of \neq A was determined experimentally by saturating a film of water in the cell with carbon dioxide and measuring the resulting fringe shift. The carbon dioxide concentration at saturation was measured by taking samples of the water film with a hypodermic needle and determining the concentration by chemical analysis. Saturation was determined by observing the interference pattern of the water film. Saturation was assumed

when the fringes stopped shifting and had resumed the same orientation with the reference line as when no carbon dioxide was in the cell. The temperature of the film was 22.2° C and the cell pressure 739.7 mm Hg. The initial carbon dioxide concentration in the water film was zero gm/liter. The fringe shift measured at saturation was 2.6726 which corresponded to a measured saturated carbon dioxide concentration of 1.4668 gm/liter. Using these values in equation (5) with λ equal to 5460 Å and with L equal to 7.5692 cm, the value for A was determined to be 3.962 x 10^{-6} . With equation (5) carbon dioxide concentration could be calculated. However, to simplify the calculations equation (6) was used instead.

$$\Delta N / \Delta N_{\rm S} = C / C_{\rm S} \tag{6}$$

The concentration profiles can now be calculated from the data given in columns X_{CO_2} and X_{Air} . To determine the accuracy of these carbon dioxide concentrations, the average concentration from the fringe patterns was calculated and compared to the average carbon dioxide concentration determined by titration of water samples taken from the cell. (Figures 29 and 30) The water samples were taken with a hypodermic needle in the water outlet tube from the water exit chamber of the cell. The water level in the cell was varied so that the carbon dioxide water interface ended at the locations in the cell where the interferometric data was taken, thus allowing the water samples to be taken at the same cell locations as the interferometric data. The method used in calculating the average carbon dioxide concentration from the interferometric data will be deferred for detailed discussion in the next chapter.

Comparison of the average carbon dioxide concentration determined by titration expressed as $\overline{C}/C_{\mathbf{S}}$ with the interferometric average concentration $\Delta \overline{N} / \Delta N_{\rm s}$ was not satisfactory. The deviations between the values at the same cell location and flow rate were larger than could be explained by the method used to calculate $\Delta \overline{N}$. Examination of the original data showed that the discrepancy was a result of shifting of the cell windows during the change from air to carbon dioxide. This shifting was caused by a pressure increase in the cell during the purging operation. An additional factor was the effect of the liquid flow rate on the location of the fringes with respect to the reference line. It was found that a change in the flow rate shifted the fringes in a direction parallel to themselves, but did not affect their orientation with respect to the reference line. The apparent cause of this shift was also a pressure effect on the cell windows as the shift increased with increased flow rates. This shift becomes important in the analysis of the data as the airwater fringe patterns were taken at only one flow rate and thus caused an error when applied to flow rates other than the one at which they were measured. The interferometric data was corrected so that it agreed with the water samples

defining a correction factor $\Delta \overline{N}/\Delta N_{\rm S} - \overline{C}/C_{\rm S}$ for each cell position and flow rate and subtracting from C/C_S calculated from the interferometric data. The correction factor was smallest at the largest flow rate as this was the flow rate used in determining the fringe pattern for the air-water system.

With the above change, the data in column C/C_s was calculated. The figures were calculated from X_{CO_2} and X_{Air} and the correction factors by the following equation:

$$\frac{C}{C_{s}} = \frac{X_{CO_{2}} - X_{Air}}{X_{Fs} \cdot \Delta N_{s}} - \left(\frac{\Delta \overline{N}}{\Delta N_{s}} - \frac{\overline{C}}{C_{s}}\right)$$
(13)

It should be pointed out that the effect of the correction is only to shift the curves parallel to themselves and does not change the shape of the measured profile.

The fringe patterns were measured in most cases to about 0.0005" of the surface of the film. The controlling factor in this measurement was the loss of fringe definition near the surface of the film.

Wave Data

Appendix C is a tabulation of the wave amplitude and frequency data. The amplitude data is reported as the number of waves counted on the visicorder wave trace that have amplitudes less than or equal to specified distances above the surface of the bottom plate ranging from the maximum amplitude observed to the minimum. The fraction of the total number of waves counted at each distance is given in column P_A . The frequency of the waves was measured by dividing the visicorder wave trace into one second time intervals and counting the number of waves in each interval. The data reported in the appendix is the number of time intervals having frequencies less than or equal to specified frequencies ranging from the maximum frequency counted to the minimum. The fraction of the total number of time intervals counted for each frequency was also included and tabulated under column P_w .

To obtain the film thicknesses that corresponded to the visicorder scale readings, calibration data was taken in which visicorder readings vs. air gap distances were measured for different water film thicknesses. The data was taken using the procedure outlined in chapter III. When the micrometer readings which correspond to the air gap distances were plotted against the visicorder readings, it was found that a line drawn through the data points did not pass through 6.0 on the visicorder scale at the micrometer reading which corresponded to the surface of the water film. Since this condition was required, the curve was shifted parallel to visicorder scale axis until the curve passed through 6.0. This was done for each calibration curve measured. The effect of this shift was to correct the calibration data so that all data for the different film thicknesses could be evaluated under the same capacitometer conditions. This shift of the reference point is appropriate because the output of the

capacitometer was linear over the range involved. Deviations in the calibration data were caused by the presence of water under the glass plate.

It was found that the film thicknesses determined from the wave traces were too large when compared to the photographs of the films. The value of the laminar film thickness for the flow rates being measured when plotted on the visicorder tracer indicated the same result. The reason for this error is the same as for the error in the calibration data. That is, the presence of water under the glass plate caused the setting of the capacitometer to change thus causing the deviation noted. It should be noted here that over the time period used in the experimental runs the drift in the capacitometer output was negligible.

Since the distilled water which was used in the cell was not a good conductor, the bottom plate of the capacitor was in actuality the metal plate upon which the glass plate was mounted and not the surface of the water. As a result very slight changes in the amount of water between the glass plate and the metal plate changed the capacitometer adjustment. To correct for this error, the average film thickness as measured by the visicorder was determined by graphically determining the area under the curves plotted by the visicorder and dividing it by the length of the tracer. (Figures 20, 21, 22 and 23) To be able to measure this area accurately the visicorder tracer was run at 10"/sec as compared to 2"/sec

during the experimental runs. The average film thickness in terms of the visicorder scale was then assumed to be equal to the laminar film thickness as determined from the laminar film theory. The calibration curves were corrected by shifting them along their abscissa until the calibration curves gave the laminar film thickness when the average film thickness from the visicorder trace was used as the visicorder reading. Figure 15 is the corrected calibration curve for data taken at an angle of inclination of the plate of $9^{\circ}44'$.

To make this correction, two assumptions were made. First was that the wave profile measured at the center of the glass plate was assumed to be an accurate representation of the wave profile across the full width of the cell. Second was that the output of the capacitometer was assumed to be linear over the range of film thickness measured in the visicorder tracer. The first assumption was satisfactory as from visual observations of the wave pattern the waves appeared to be almost symmetrical across the plate. The second assumption was not so satisfactory. The evaluation of the average visicorder reading from the wave trace assumed that the proportionality between the film thickness and the visicorder scale was constant for all values of the visicorder readings. Figure 15 shows this is not correct. However, it was nearly constant over the range 0 to 2. Examination of the wave trace for run 48 gave the total area under the curve as 142. To estimate the error, the area under the wave trace



above 2 was measured and found to be 4.77. This indicated that the error in assuming a constant proportionality constant was only about 4% which was acceptable.

Results of the wave data analysis are presented in figures (24, 25, 26 and 27) and Appendix C.

CHAPTER VI

ANALYSIS OF THE EXPERIMENTAL DATA

Wave Data

The wave phenomena that occurs on the surface of a falling liquid film must be described by the hydrodynamics of the film. The most readily measurable characteristics are the Reynolds number, the wave frequency and the wave ampli-The Reynolds number was obtained by measuring the flow tude. rate, the width and angle of inclination of the plate upon which it flows and the temperature of the flowing media. The wave frequency and amplitude are more difficult. By use of the capacitometer described in preceding chapters, traces of the surface of the film at the center of the plate were ob-Inspection of these traces (figures 20, 21, 22 and tained. 23) showed that both the amplitude and frequency of the waves were of a random nature, especially above Reynolds numbers of 732, and could best be expressed by some type of statistical correlation. To determine a correlation for the wave amplitude the following probability function P_A was defined as the ratio of the number of waves having a film thickness equal to or less than a given film thickness divided by the total number of waves observed. The value of P_A was unity at the

maximum film thickness and zero at the minimum. Plotting P_A versus the film thickness, a normal distribution skewed to the right was obtained. To determine if the wave data could be represented by a normal distribution, the Probit diagram method was used. (1) This method is extremely valuable if the distribution function desired is of the Kapteyn (1) class of distributions which are generalizations of the normal distribution. Kapteyn's class of distributions is given as:

$$d\Phi_{x} = d\Psi \left(\frac{G(t) - \chi}{\sigma} \right) = \frac{1}{\sqrt{2\pi} \sigma} \exp \left[- \frac{\left[G(t) - \chi \right]^{2}}{2\sigma^{2}} \right] \left| \frac{dG(t)}{dt} \right| dt$$

where $M{G(X)} = \chi$ is the mean and $\sigma{G(X)} = \sigma$ is the dispersion. Since $u = \Psi(t)$ is always increasing in the whole interval $-\infty < t < \infty$ t is a function of u: $t = \Psi^{-1}(u)$. For the general normal distribution, $u = \Psi\left(\frac{t-\chi}{\sigma}\right)$, thus we have $(t - \chi)/\sigma = \Psi^{-1}(\chi)$. Inserting here $u = \Psi(t)$ we obtain a function, $v_1 = \Psi^{-1}(\Psi(t))$, which is equal to $v_1 = (t - \chi)/\sigma$. If we now form $v_2 = \Psi^{-1}(F(t))$, where $F(t) = u = N(t)/\eta$ we shall expect this function of t to lie close to a straight line $v_1 = (t - \chi)/\sigma$ passing through the point $(\chi, 0)$ and having the slope $1/\sigma$. If not x, but a certain function of x, G(x) is normally distributed then $v_2 = \Psi^{-1}(F(t))$ should also lie very close to the curve $v_1 = \frac{G(t) - \chi}{\sigma}$. This method allows the function of t to be chosen and furthermore, it may be used for obtaining numerical values of the parameters χ and σ graphically.

Applying the Probit method to the wave amplitude data given in Appendix C, it was found that all the wave amplitude data could be represented by a logarithmic normal distribution:

$$d\Phi_{\mathbf{X}}(t) = d\Psi \left(\frac{\ln t - \chi}{\sigma}\right) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\left(\ln t - \chi\right)^2}{2\sigma^2}\right] \frac{dt}{t} \quad (15)$$

where $M\{\ln x\} = \chi$ and $\sigma\{\ln x\} = \sigma$

The correlation of the wave frequency data was made in a manner similar to that used for the amplitude data. Choosing an arbitrary time interval of 1 second, the number of waves per time interval was determined (see Appendix C). A probability P_w was defined as the ratio of the number of time intervals having a wave frequency less than or equal to a given wave frequency divided by the total number of intervals. Applying the "Probit method" it was found that all the wave frequency data were correlated by a normal distribution

$$d\Phi_{\mathbf{X}}(t) = \frac{1}{\sqrt{2\pi} \sigma} \exp\left[-\frac{(t-\chi)^2}{2\sigma^2}\right] dt$$
(16)

Examples of the analysis of the wave amplitude and wave frequency data by the Probit method are given in figures 16 and 17 respectively. The mean wave frequency and the antilog of the mean wave amplitude and mean film thickness are given in figures 24, 25, 26 and 27. Also included in these figures are values for the maximum wave amplitude, maximum film thickness, average trough height and minimum trough height





determined from the wave traces and the calculated laminar film thickness.

Surface Area

The wave traces recorded by the visicorder were plots of the location of the surface of the film for a given cell position as a function of the velocity of the waves. To calculate the surface area the relationship between the visicorder chart speed and the velocity of the waves had to be known. The velocity of the waves was not known, so the velocity of the waves was assumed to be equal to the surface velocity of an equivalent laminar film. That is $V_s = 3/2 V_A$ where ${\tt V}_{\tt A}$ is the average bulk velocity. The use of 3/2 ${\tt V}_{\tt A}$ for the velocity of the waves was not correct as it has been experimentally determined that the surface velocity of such films is closer to 2 V_{A} (25). From hydrodynamic considerations, the velocity of the waves should be at least equal to the surface velocity. The value of $3/2 V_A$ however, was used as it resulted in an overestimate of the surface area, thus insuring that the estimate was conservative.

Measurement of the surface area was made by representing the surface of the waves as the hypotenuse of right triangles. The altitude of these triangles was assumed to be the change in film thickness. The base was the measured length of the wave multiplied by the ratio of the wave velocity to the visicorder chart speed. Based on this method the ratio of areas with waves to the area without waves was given by

$$\frac{A_{w}}{A_{L}} = \frac{\sum_{i=1}^{N} \left[\left(Y_{2} - Y_{1} \right)_{i}^{2} + \left(\Delta X_{i}^{'} \right)^{2} \right]^{\frac{1}{2}}}{\sum_{i=1}^{N} \Delta X_{i}^{'}}$$
(17)

where N is the maximum number of straight line segments. $Y_2 - Y_1$ is the change in film thickness and $\Delta X_1'$ is the measured length of the horizontal side multiplied by the velocity ratio, $\frac{V_{surface} (ripples)}{Visicorder Chart Speed}$.

The increase in area calculated was so small that a second approximation was made to decrease the computation time. This approximation was

$$\frac{A_{w}}{A_{L}} = \frac{\sum_{i=1}^{N} [(Y_{2} - Y_{1})_{i} + \Delta X_{i}^{i}]}{\sum_{i=1}^{N} \Delta X_{i}^{i}}$$
(18)

Results of the evaluation of surface area are given in Table 1.

Re	θ	Laminar Film Thickness cm	V _L cm/sec	2 V _A cm/sec	Mean Film Thickness cm	w sec ⁻¹	V sMF cm/sec	Average Trough Height cm	% Increase in Surface Area
732	9° 44 '	.0 67 3	39.3	52.40	.0919	5.95	7 3. 4	.0425	1.449
	18° 27 '	.0542	47.84	63.70	.0714	8.40	83.20	.0419	.902
	25° 42 '	.0489	53.50	71.40	.0648	8.80	94.10	.0406	.690
1099	9° 44 '	.0768	51.0	68.0	.09 6 5	8.3	80.88	.0521	1.232
	18° 27 '	.0623	63.19	84.4	.0801	8.6	105.0	.0450	.909
	25° 42 '	.0560	70.25	93.9	.0746	11.2	125.0	.0472	.93
1482	9°44'	.0848	62.3	83.30	.0984	13.1	84.15	.0596	1.702
	18°27'	.0 6 88	77.06	102.6	.0876	10.0	125.2	.0407	.945
	25°42'	.0 6 19	85.60	114.0	.0795	12.35	140.0	.0445	1.1317
1834	9°44'	.0909	71.6	95.60	.1068	15.8	91.8	.0638	2.2558
	18°27'	.0738	88.68	118.0	.0953	10.8	148.0	.0445	1.224
	25°42'	.0664	98.70	131.5	.0811	14.9	148.0	.0394	1.415

TABLE 1. SURFACE VELOCITY AND AREA MEASUREMENTS

surface velocity. From this method the effect of conservatively estimating the wave velocity was to over estimate the surface area. It was found that the largest increase in surface area occurred at the lowest angle of inclination which corresponded to the largest film thickness. This is apparently the result of a low surface velocity and thick film which allows larger amplitude waves and thus more surface area. It would seem from the wave traces that the percentage increase in surface area as the flow rate is increased should be larger than the calculated values. However, as the flow rate is increased, the velocity of the waves is also increased while the speed of the visicorder remained constant. The result of this is to compress the shape of the waves on the visicorder output thus making it appear that the surface area is much larger than actually occurs.

Wave Model

To calculate the average carbon dioxide concentration in the film from the interferometric data the following model of the film was proposed:

The film was divided into two regions. The first was a homogeneous region consisting of that part of the film from the surface of the bottom glass plate to the average trough height (Y_T) determined from the visicorder traces. The second region was that part of the film from the average trough height to the surface of the film. The waves were assumed to be triangular in shape with a uniform height defined by the

mean film thickness $(Y_{\rm MF})$ determined from the statistical correlation of the wave data. The number of waves in a one second time interval was set equal to the mean wave frequency also determined from the wave data. The velocity profile in the film was assumed to be parabolic in shape (57) and was set equal to the velocity profile (equation (19)) calculated by assuming a laminar film of thickness $Y_{\rm MF}$ flowing at the same angle of inclination,

$$V_{(\underline{Y})} = \frac{\rho g \sin \theta (\underline{Y}_{MF})^{2}}{2 \mu M^{2}} \left[1 - \left(\frac{\underline{Y}_{MF} - \underline{Y}}{\underline{Y}_{MF}} \right)^{2} \right]$$
(19)
$$0 \le \underline{Y} \le \underline{Y}_{MF}$$

The width of the triangular waves at the average trough height was determined by assuming a base width and calculating the total mass flow rate. The correct width was determined when the calculated flow rate equaled the measured flow rate. These calculations are represented by the following equation:

$$\frac{MQ}{W} = \sum_{i=0}^{Y_{T}} \Delta Y_{i}V_{i} + \omega \sum_{i=Y_{T}}^{Y_{MF}} \Delta Y_{i}V_{i}b_{i}$$
(20)

where b_1 is the width of the triangular wave at Y_1 and is equal to the calculated base width b_T at $Y \leq Y_T$ and 0 at Y_{MF} , V_1 is the velocity at Y_1 calculated from equation (19) and ΔY_1 is the value of the step taken along the Y axes. The accuracy of this model can be estimated by comparing the proposed wave model in figure 18 with the actual wave pattern,
























the visicorder wave trace given in figure 20.

Interferometric Data

Using the proposed wave model, the average carbon dioxide concentration was determined from the interferometric data. To obtain an average concentration which included the effects of the waves, the flow pattern of the liquid film for a time period of one second was used as the basis for the calculations. The interferometric data for the cell position in question, even though accurate only at the film thickness measured, was used to predict concentration profiles at other film thicknesses. It was found from the experimental data and visual observation of the interference patterns that the concentration profile for a film of the thickness Y_2 was predicted reasonably well from the experimental data at film thickness Y_1 by multiplying the Y values by the ratio Y_2/Y_1 .

Using this method of predicting concentration profiles the average fringe shift $\Delta \overline{N}$ which corresponded to the average concentration of carbon dioxide in the film was calculated. $\Delta \overline{N}$ was calculated in the same manner as the mass flow rate in the wave model. The equations used were

$$\Delta \overline{N} = \frac{\begin{array}{c} \begin{array}{c} Y_{T} \\ \mathbf{a} \sum \\ \mathbf{i} = 0 \end{array} & \begin{array}{c} V_{i} \quad \Delta Y_{i} \quad \Delta N_{i} + w \sum \\ \begin{array}{c} V_{MF} \\ \mathbf{i} = 0 \end{array} & \begin{array}{c} V_{i} \quad \Delta Y_{i} \quad \Delta N_{i} \quad b_{i} \end{array} \\ \begin{array}{c} Y_{T} \\ \sum \\ \mathbf{i} = 0 \end{array} & \begin{array}{c} Y_{MF} \\ V_{i} \quad \Delta Y_{i} + w \sum \\ \begin{array}{c} Y_{T} \\ Y_{T} \end{array} & \begin{array}{c} V_{i} \quad \Delta Y_{i} \quad b_{i} \end{array} \end{array}$$
(21)

where "a" is the fraction of the film surface at ${\tt Y}={\tt Y}_{\rm T}$

passing a given point in one second and is equal to $1-\omega_{V_{T}}$. The values of ΔN_{i} in the first term of equation (21) were obtained from the measured concentration profile with the Y axis expanded to $Y_{F} = Y_{T}$. Similarly ΔN_{i} for the second term was obtained from the same concentration profile except that the Y axis was expanded to $Y_{F} = Y_{T}$. Comparison of ΔN with \overline{C} determined from liquid samples was made with equation (6)

$$C/C_{S} = \Delta N / \Delta N_{S} \tag{6}$$

Results of the comparison are discussed in Chapter V.

Evaluation of the Eddy Diffusion Coefficients

Mass transfer in the flowing film was assumed to be described by the equation

$$\Lambda^{(\Lambda)} \frac{q_{\rm C}}{q_{\rm C}} = \frac{g_{\rm A}}{g} \left[\left(D + \epsilon \right) \frac{g_{\rm A}}{g_{\rm C}} \right]$$
(55)

Assuming that $\frac{d}{dY} (D + E) \left[\frac{dC}{dY}\right]$ could be neglected for small ΔY the values of D + E were evaluated by measuring d^2C/dY^2 and dC/dZ from the experimental data and substituting them into equation (22). The second derivative was readily determined from the concentration profiles measured by the interferometer. The velocity was obtained from the calculated velocity profile (equation (19)) based on the mean film thickness $(Y_{\rm MF})$. The first derivative of the change in concentration as a function of cell length, however, was not readily available. Because of the pressure effects in the cell the absolute concentration was not measured at each cell position

directly, but calculated by the method outlined in Chapter V. Using these concentration profiles, the change in concentration as a function of length was calculated in the following manner. Photographs were chosen at each cell position that had as close to the same film thickness as possible. An average film thickness, Y_F , was then chosen and the profiles corrected to this film thickness. Values of AN at given Y values were plotted as a function of Z and the slope measured. The data were fitted by a least squares fit to a straight line and the slope determined for each value of Y.

The values of the second derivative were calculated from the concentration profiles at two values of Z for each flow rate. To determine the second derivative the plot of ΔN_Y was fitted by a fourth order polynomial using the method of least squares. The first derivative was obtained by differentiating the 4th order polynomial. The plot of the first derivative was then smoothed and fitted again by another 4th order polynomial. The first derivative of this polynomial was plotted against Y. The curve was smoothed and the values of the second derivative read from the plot. Values for (D + ε) were then calculated using the values for d²C/dY², dC/dZ and V_(Y). Appendix D contains the results of this method of analysis.

The first method did not prove successful in determining how the eddy diffusivity was related to location in the film.

-available from the results. In order to determine the effect of film location on the diffusivity the following method of analysis was used.

Assuming that equation (22) described the mass transfer process in the film

$$V_{(Y)} \frac{dC}{dZ} = \frac{d}{dY} \left((D + \epsilon) \frac{dC}{dY} \right)$$
(22)

A mass balance was made on the solute in the film at various locations in the film. Choosing as a system a cube of water of unit width, length $Z_2 - Z_1$, and thickness Y where Y = 0 at the wall, the following equation was obtained:

$$-(Z_{2} - Z_{1})(D + \epsilon) \frac{dC}{dY} \Big|_{\frac{Z_{1}+Z_{2}}{2}} = \int_{0}^{Y} (Y)^{C}(Y)^{dY} \Big|_{Z_{2}} - \int_{0}^{Y} (Y)^{C}(Y)^{dY} \Big|_{Z_{1}}$$
(23)

Solving for $(D + \varepsilon)$ and substituting in equation (19) for the velocity profile equation (23) became

$$(D + \epsilon)_{Y} = \frac{V_{S}}{g_{2} - Z_{1}} \left(\frac{dY}{dC} \right)_{\underline{Z_{1} + Z_{2}}} \left[\int_{0}^{Y} \left\{ 1 - \left(\frac{Y_{MF} - Y}{Y_{MF}} \right)^{2} \right\} C_{(Y)} dY \Big|_{Z_{2}} - \int_{0}^{Y} \left\{ 1 - \left(\frac{Y_{MF} - Y}{Y_{MF}} \right)^{2} \right\} C_{(Y)} dY \Big|_{Z_{1}}$$
(24)
where
$$V_{S} = \frac{\rho \left(Y_{MF} \right)^{2} g \sin \theta}{2 \mu M^{2}}$$

To eliminate entrance effects equation (24) was evaluated by assuming the concentration profile at Z = 57.15 cm was the initial profile. Values of the integral were determined by

graphical integration where $C_{(Y)}$ was the concentration profile as determined from the interferometric data. Values of dY/dC were also obtained from the concentration profiles by curve fitting the data with a 4th order polynomial and differentiating the resulting equation. In determining $C_{(Y)}$ and dY/dC no corrections were made on the data for film thickness. It was thought that a wiser method was to ignore the correction in the calculation and then average the results. In the case where more than one set of data was available at a given cell position and flow rate, the value of $C_{(Y)}$ and dY/dC used were the average values of the data available.

Appendix E contains the tabulated data used to evaluate equation (24) and the value of $(D + \varepsilon)$ as a function of film location and cell position. Figures 33, 34, 35 and 36 are graphical representations of the diffusivities as a function of film location. The solid line drawn through the points represents the arithmetic mean for each set of points.

CHAPTER VII

DISCUSSION

Wave Hydrodynamics

The surface profile of the falling films as recorded by the visicorder (figures 20, 21, 22 and 23) shows that at low flow rates when ripples first form on the surface of a falling film they are of a fairly uniform frequency and amplitude. As the liquid flow rate is increased the frequency and amplitude of the waves become irregular and statistically The effect of increasing the angle of inclination of random. the cell while maintaining the same liquid flow rates was to decrease the film thickness and change the flow pattern. Figures 24, 25 and 26 show how the maximum film thickness, maximum wave amplitude, average trough height, minimum trough height, mean film thickness and mean wave amplitude are affected by the angle of inclination of the cell. The laminar film thickness calculated as

$$Y_{L} = \left(\frac{3\mu^{2} Re}{4\rho^{2}g \sin\theta}\right)^{1/3}$$
(25)

is also included. Figure 27 shows how the mean wave frequency varies with angle of inclination. There appears to be no pattern as to how the wave frequency changes with the angle of inclination of the cell.

Values of the mean wave amplitude, mean film thickness and mean wave frequency as shown in figures 24, 25 and 26 were calculated from statistical models of the wave data. Using the probability function described in Chapter VI and tabulated in Appendix C at the maximum film thickness or wave amplitude the probability is 1, that all waves or film thicknesses are less than or equal to the maximum amplitude and film thickness and below the minimum trough height the probability is zero that any waves or troughs will appear. This probability distribution of the waves was found to be described by a logarithmic normal distribution.

$$d\Phi_{\mathbf{x}}(t) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\left(\ln t - \chi\right)^2}{2\sigma^2}\right] \frac{dt}{t}$$
(15)

where $M(\ln x) = \chi$ is the mean and $\sigma(\ln x) = \sigma$ the dispersion

The distribution function described the wave amplitude and film thickness data satisfactorily for all flow rates except Re = 732 at θ = 9°44' indicating that the flow is still in the transition region and wave characteristics are neither uniformly nor randomly distributed. The mean, M(ln x), of the distribution was chosen to represent the wave amplitude and film thickness for each flow rate and angle of inclination. Since the mean of the logarithmic normal distribution is in logarithmic units, the antilog of the mean was used and was defined as the mean wave amplitude and mean

film thickness (Y_{MF}) . The dispersions of the distribution function were also calculated. It was found that values of the dispersion scattered badly with no apparent relation to flow rate or angle of inclination and was thus neglected.

The probability function for the wave frequency was also discussed in Chapter VI and is tabulated in Appendix C (i.e., for a given time period the probability is one that at all time periods have less than or equal to the maximum number of waves observed for any one time period and that the probability is zero that any time period will occur with a wave frequency less than the maximum observed). This variable is described by a normal distribution. Good agreement was obtained at all flow rates and angles of inclination measured. As with the amplitude data the mean wave frequency was chosen to represent the frequency of the film. The dispersion of the frequency data was also calculated, but not reported as no correlation with flow rate was found.

The use of a logarithmic normal distribution to describe the wave amplitude and film thickness data differs from the approach of Hanratty (36). He assumed that the probability function defined by the fraction of time the film height has a value between h and h - dh was represented by a normal distribution for the case where waves are formed on the surface of a horizontal stagnant liquid film by air passing over the surface.

Using the value of the mean film thickness determined

from the statistical model of the wave data, $a \not A$ velocity profile in the film may be calculated. To do this the following assumptions are made.

1. The velocity profile is parabolic in shape.

2. The surface velocity is determined by the mean film thickness calculated from the statistical models of the wave data.

3. The velocity profile in the film above the mean film thickness is flat with the velocity being the same as that calculated at the mean film thickness.

The resulting velocity profile is given by equations (19) and (26)

$$V_{(Y)} = \frac{\rho (Y_{MF})^2 g \sin \theta}{M^2 2 \mu} \left[1 - \left(\frac{Y_{MF} - Y}{Y_{MF}} \right)^2 \right]$$
(19)

$$0 \leq Y \leq Y_{MF}$$

and

$$V_{sMF} = \frac{\rho (Y_{MF})^2 g \sin \theta}{M^2 2 \mu}$$
(26)
$$Y \ge Y_{MF}$$

Surface velocities calculated in this manner may be compared to those predicted by the laminar film theory (3/2 average bulk velocity) or to experimentally measured values of surface velocity. These measurements of other investigators indicate that above a Reynolds number of 80 the surface velocity is twice the average bulk velocity. Figure 28 presents a comparison of surface velocity calculated from the



wave data against surface velocities calculated as twice the average bulk velocity. The best agreement is obtained at the lowest angle of inclination. Discussion of the model used in calculation of the mean film surface velocity is given in the chapter on analysis of experimental data.

The effect of the wall on the shape of the waves as they move down the plate was not measured with the capacitometer. However, visual observations indicated a retarding of the velocity of the waves at the wall with the long axis of the wave being generally perpendicular to the direction of flow at the center of the plate and slightly bowed at the walls. The increase in surface area when ripples formed was also measured for the 3 angles of inclination. (Table 1) For the measurement to be exact, the velocity of the ripples had to be known. Since this information was not available, it was assumed that the velocity of the ripples was equal to the surface velocity predicted by the laminar film theory. This estimate of the wave velocity was conservative as from experimental measurements the surface velocity of the film was found to be larger than the predicted laminar surface velocity. From hydrodynamic considerations the velocity of the waves must at least be equal to the film surface velocity. The method of measurement of the surface area is discussed in Chapter VI. From this method the effect of conservatively estimating the wave velocity was to over estimate the surface area. This arose from the fact that the wave traces figures

20, 21, 22 and 23 were taken at one chart speed. To correct the length of the wave shown on the trace the measured wave length was multiplied by the ratio of the estimated wave velocity to the chart speed. Thus the larger the ratio, the larger the wave and the smaller the increase in area. It was found that the largest increase in surface area occurred at the lowest angle of inclination which corresponded to the largest film thickness. This is apparently the result of a low surface velocity and thick film which allows larger amplitude waves and thus more surface area. It would seem from the wave traces that the percentage increase in surface area as the flow rate is increased should be larger than the calculated values. However, as the flow rate is increased, the velocity of the waves is also increased while the speed of the visicorder remained constant. The result of this is to compress the shape of the waves on the visicorder output thus making it appear that the surface area is much larger than actually occurs.

Mass Transfer Results

To determine the effect of waves on the liquid phase rate of absorption of a gas into a falling liquid film, carbon dioxide was absorbed into a water film. To eliminate any gas phase resistance pure carbon dioxide gas was used as the gas

phase. All data were taken at zero gas velocity. Results of the experimental data consisted of interferometric pictures of the concentration profiles of carbon dioxide in the water film and average concentrations determined from liquid samples at various liquid flow rates and distances from the entrance. Data was taken at two angles of inclination, $9^{\circ}44'$ and $18^{\circ}27'$.

Because of the design of the cell the liquid flow pattern was altered at the end of the contacting zone when liquid samples were taken. However, the same end effect occurred at each cell position so that if the end effect were large it would be constant and thus not affect the measured rate of change of concentration with distance from the inlet. Results of the experimentally determined average concentrations are presented in figures 29 and 30 as a function of cell position. Based on this data the height of a transfer unit H_T defined as

$$H_{L} = \frac{Z_{2} - Z_{1}}{\sum_{C_{Z_{1}}} \frac{dC}{C_{s} - C}}$$
(27)

Results of this evaluation are given in figure 31. To compare the above results to that predicted by the laminar film theory, the Johnstone-Pigford (27) equation for absorption into a laminar film (Appendix F) was used to determine the length of cell necessary to absorb the same amount of carbon dioxide as was experimentally determined. Based on this





length a theoretical H_L was calculated from equation (27) and plotted in figure 31. It is important to note here that entrance effects have been eliminated as all measurements were made at a sufficient distance down stream from the entrance that stable flow conditions existed. The entrance effect as discussed here includes the effect of flow irregularities at the water inlet and the acceleration of the liquid film as the water flowed down the inclined plate.

Evaluation of the interferometric data was complicated by a pressure effect on the cell windows which affected the position of the fringe pattern and made an absolute measurement of concentration impossible. The fringe shift resulting from this effect was uniform throughout the film and did not affect the fringe shift due to the presence of carbon dioxide. This was verified both by photographs of the fringe pattern and visual observations. Using a ground glass screen the fringe pattern was observed through the complete cycle of adjustment. Under static gas conditions the fringe pattern remained stationary. However, when the cell was purged, the fringe pattern shifted, the degree of shift being dependent on the increase in pressure in the cell. This pressure effect during purging was easily distinguished from that caused by the absorption of carbon dioxide as it took approximately 15 seconds for carbon dioxide to reach the cell after purging was started.

Visual observations of the interference pattern in



the cell at the region next to the water inlet when carbon dioxide was being absorbed indicated that mass transfer in this region was by molecular diffusion. However, as the liquid moved down the cell, the film accelerated and became unstable forming waves. Coincident with formation of waves, the rate of absorption of carbon dioxide into the film increased. The shape of the interference pattern near the interface did not change significantly at first, but the interference pattern near the bottom of the film indicated that the carbon dioxide penetrated immediately to the bottom of the film. As the film moved down the inclined plate the interference pattern for the region near the bottom of the film continued to shift showing that the concentration was increasing. However, the orientation of the interference pattern in this region with respect to a reference line changed very little from the air-water orientation indicating a very small concentration gradient in the film. If one compares these observations to the concentration profile predicted by assuming a laminar film with molecular diffusion (Appendix D), one finds that the increase in concentration at the bottom of the film is much greater than predicted and the concentration gradient much smaller. For this to be true turbulence in the film must be complementing a molecular diffusion mechanism. The observation that a very small concentration gradient exists in the film near the bottom indicates that turbulence exists throughout the film.

The fringe shift within 0.8 mm of the glass plate on the bottom of the cell was affected by a secondary fringe shift caused by a slight non-alignment of the bottom plate. This secondary fringe shift was constant and was a function only of the geometry of the contacting cell. Measurements were not made in this region because of the poor fringe definition. However, to determine how the concentration changed in the region a few measurements were randomly made. The secondary fringe shift was measured from the air-water photographs and subtracted from the carbon dioxide-water measurements. From these results it was found that the concentration at the surface of the inclined plate was the same as that measured above the secondary fringe pattern within the experimental accuracy of the measurements.

The effect of the waves on the concentration profile in the region below the average trough height was negligible. That is, the concentration profile was the same whether a large wave or trough was passing over the point of observation. In the liquid film above the average trough height the following characteristics were observed: The concentration near the surface of the waves appeared to be nearly constant regardless of the film thickness. As the waves moved past a given point in the cell the interference pattern, proportional to the concentration profile, was compressed and expanded following the surface of the film as produced by the waves. Figure 19 shows some experimentally measured concentration

profiles taken at the same cell position, angle of inclination and liquid flow rate but at different film thicknesses resulting from the presence or absence of waves. In general, it was found that the concentration profile was proportional only to the wave amplitude and cell position and a concentration profile measured at a given wave amplitude could be used to predict concentration profiles for other wave amplitudes by proportionally expanding or contracting the film thickness scale of the experimental curve to a reference or standard film thickness. These observations are particularly important as they mean that the waves do not directly reflect the amount of turbulence in the film. The use of a dimensionless length for the film thickness in the mass transfer equations are also valid even when the film thickness varies because of waves.

Calculation of the eddy diffusivity in the bulk of the film was based on equation (22). Data used was for an angle of inclination of $9^{\circ}44$ ' and is tabulated in Appendix A. Data for an angle of inclination of $18^{\circ}27$ ', tabulated in Appendix B, was not evaluated. However, preliminary examination of the data in Appendix B indicated all comments made about the characteristic of the wave data for $\theta = 9^{\circ}44$ ' are applicable. Data for $\theta = 18^{\circ}27$ ' was included so that the effect of the angle of inclination could be determined.

Figure 32 is a plot of the average values of the diffusivities for each location in the film as a function of the











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liquid flow rate. It can be seen that the value of total diffusivity varies considerably with the location in the film. The eddy diffusivity is a maximum at the center of the film and decreases as the bottom plate and interface are approached. The effect of liquid flow rate is also apparent in that the value of the diffusivity increases as the liquid flow rate is increased. At a Reynolds number of 732 the diffusivity does not appear to be a function of film location. However, the scatter in the data at this flow rate may be sufficient to mask out the effect.

From these results, the following statements can be made:

1. The eddy diffusivity in a falling film is a function of location in the film and of the liquid flow rate.

2. The diffusivity is a maximum at the center of the film and decreases approaching the value of the molecular diffusivity at the interface and at the surface of the inclined plate.

3. The value of the diffusivity at the center of the film is a function of the liquid flow rate and increases as the liquid flow rate is increased.

4. Below a Reynolds number of 732 the diffusivity is not a strong function of location in the film.

The decrease of the eddy diffusivity in a falling film as the bottom plate and interface are approached is extremely interesting as it offers an explanation of the mechanism of mass transfer when ripples form on the surface of the film even though the increase in surface area may be negligible. Referring to Table 2 which gives the pseudo diffusivity calculated from the Johnstone Pigford equation using the experimentally determined average concentrations, it can be seen that these apparent diffusivities are much smaller than the diffusivities calculated from the concentration profiles. (Figure 32) Even if the mean film thickness

Table 2

Apparent Diffusivities from Pigford and Johnstone Equation

Re	θ	D _l cm ² /sec	D ₂ cm ² /sec	
732	9° 44 '	2.20	3.52	
732	18°27'	2.46	3.31	
1099	9°44'	2.30	3.87	
1099	18°27'	2.62	3.62	
1482	9° 44 '	2.26	4.05	
1482	18° 27 '	3.03	4.23	
1834	9° 44 '	1.34	5.34	
1834	18°27'	2.89	4.59	

 D_1 includes entrance effect. D_2 measurements started 57.15 cm from water inlet.

and surface velocity are used in this solution, the value of the diffusivity calculated is still much smaller than the measured values. For this to be true some mechanism other
than turbulent diffusion in the bulk of the film must be controlling the rate of mass transfer. It is proposed that this controlling mechanism is that the eddy diffusivity approaches the molecular diffusivity at the interface of the film. The degree of approach to the molecular value is not known, but it is believed that at the saturated interface there exists a thin layer where the eddies are completely damped out. The mechanism of mass transfer through this layer is by molecular diffusion. The rate at which the eddy diffusivity increases within the film depends on the liquid flow rate. In extremely turbulent systems the layer may be very small while in the flow region where waves form but are well behaved the layer may be relatively thick.

The value of the eddy diffusivity in the film is related in some way to the waves that move down the film. The relationship at this time is, however, not known. Experimental observations on the effect of waves on the concentration profile in the film have shown that the concentration profile expands as a wave passes and compresses when a trough passes. For this to be true the waves must not be directly affecting the diffusivity. One possible explanation is suggested by visual flow studies made by Kline and Runstadler (33A). In their study of turbulence in the sublaminar region of a water film flowing over a horizontal plate, they found that turbulence in the film was induced at the wall. The degree of turbulence increased with the distance from the

wall and a maximum occurred somewhere in the middle of the film. As the free surface was approached the degree of turbulence again decreased. It has also been noted that when a dye is placed on the surface of the film in the presence of waves little or no spreading of the dye is noted as it moves down the cell (19). This also is an indication that at the surface there exists a region of little turbulence even in the presence of waves.

Applying the experimentally determined diffusivity data to the case where a solute is absorbed from the wall of the cell into the film the same controlling mechanism is suggested. That is, the diffusivity is a maximum at the center of the film and decreases in value to the molecular diffusivity at the wall of the cell. This model explains why Brauer's (8) laminar film theory correlates data for mass transfer at the interface and at the wall. Brauer's model, although assuming a laminar film at the wall, really cannot locate the film, but only requires that such a film exists. Thus in the case of absorption of a gas through the interface, Brauer's model can also describe a pseudo laminar layer at the interface even though he refers to it as a laminar layer next to the wall.

The eddy diffusivity curves shown in figure 32 can be described mathematically by the following expression

$$D + \epsilon = D + 4\epsilon_{m} \left[e^{-\psi \left(Y/Y_{T} \right)^{2}} \left\{ 1 - e^{-\psi \left(Y/Y_{T} \right)^{2}} \right\} \right]$$
(27)

where ψ is a parameter which determines the location of the maximum point of the curve, Y_T is the average trough height and ε_m is the maximum eddy diffusivity measured from the experimental data. The value of ε_m can be predicted from the empirical expression

$$\epsilon_{\rm m} = 2.36 \ {\rm x} \ 10^{-9} \ {\rm Re}_{\rm L}^{3.3}$$
 (28)

which is based on experimental results from the 9°44' angle of inclination (Figure 37). Figure 38 is a plot of the predicted (D + ϵ) from equations (27) and (28) with $\psi = 4$ showing how it compares with the experimental values.

Substitution of equation (27) and (28) and the velocity profile

$$V_{(\Upsilon)} = \frac{\rho (\Upsilon_{MF})^2 g \sin \theta}{M^2 2 \mu} \left[1 - \left(\frac{\Upsilon_{MF} - \Upsilon}{\Upsilon_{MF}} \right)^2 \right]$$
(19)

into equation (22) results in

$$\frac{dC}{dZ} = \frac{1}{\frac{\rho \left(Y_{MF}\right)^2 g \sin \theta}{M^2 2 \mu} \left[1 - \left(\frac{Y_{MF} - Y}{Y_{MF}}\right)^2\right]} \frac{d}{dY} \left[\left\{ D + \frac{1}{M^2 2 \mu} \right\} \right]$$

$$4(2.36 \times 10^{-9}) \operatorname{Re}^{3.3} \left[e^{-4(Y/Y_T)^2} (1 - e^{-4(Y/Y_T)^2}) \right] \frac{dC}{dY} \right]$$
(29)

which is the partial differential equation that describes the rate of mass transfer of a gas into a falling liquid film when the gas velocity and gas phase resistance are zero.

Values of $\mathtt{Y}_{MF},\ \mathtt{V}_{s}$ and \mathtt{Y}_{T} which are required for design

calculations, for flow conditions for which experimental data are not available, may be predicted in the following manner:

The surface velocity V_s may be approximated as twice the average bulk velocity of the liquid film, $Y_{\rm MF}$ by assuming that V_s calculated above represents the surface velocity of a laminar film thickness, and $Y_{\rm T}$, which is the average trough height, may be estimated by determining the film thickness which, based on the predicted velocity profile, satisfies the mass balance on the water film assuming no waves.





CHAPTER VIII

CONCLUSIONS

It has been shown that when ripples or waves are present in a falling liquid film the rate of mass transfer is increased by as much as 50 to 100%. The consequent increase in surface area is no more than 2.5% and thus cannot account for such high mass transfer retes. Calculation of the eddy diffusivities in the film indicates that as the flow rate is increased the eddy diffusivity is increased. The maximum diffusivity occurs in the central region of the film. As the interface and bottom plate are approached, the value of the eddy diffusivity decreases and approaches the molecular diffusivity value. The rate at which the eddy diffusivity decreases is a function of liquid flow rate with the rate decreasing as the flow rate is reduced. Diffusion through the laminar film next to the interface controls the rate of mass transfer. Since data were not obtained in the region next to the interface the exact mechanism is not known, but it appears that a pseudo stagnant film exists at the interface in which the eddy motion of the bulk of the film is damped out. Experiments performed where a dye is placed on the surface of a falling film (17) show little dispersion of

the dye as the film moves down the plate. This is a verification that the surface of the film is not subjected to turbulence from within the film. Experiments on the time required for the interface to reach equilibrium in the system used (25) indicate that less than 0.001 seconds are required.

The appearance of waves and ripples on the surface of the film is believed to be the result of turbulence generated in the bulk of the film. From the results of Kline and Runstadler it appears that the instability occurs near the bottom plate of the cell and that the turbulence grows and reaches a maximum value in the central region of the film with its magnitude again decreasing as the interface is approached.

The effect of waves on the concentration profile was to compress and expand the profile with the concentration near the interface remaining constant. These results indicate that the waves do not have a direct effect on rate of mass transfer in the film but are a result of the same mechanism as controls the diffusivity.

Measurement of the amplitude and frequency of the waves was useful in correcting the experimental data but did not appear to be the correct variables to use in correlating the eddy diffusivity. The amplitude of the waves and their frequency are not consistent with the increase in angle of inclination which indicates that other variables are also important. It appears now that an important variable not

measured was the velocity profile in the film. Values of the eddy diffusivity were calculated using an assumed profile.

An equation is proposed to describe the rate of mass transfer into the film which however does not apply in the entrance region where the liquid is accelerating and ripples have not formed.

CHAPTER IX

FUTURE RESEARCH

More data are necessary on the effect of liquid flow rates and angles of inclination on the concentration profile of gases being absorbed into falling films. The empirical relationship between the maximum eddy diffusivity and the liquid Reynolds number is based on only four data points taken at one angle of inclination and additional data is needed to substantiate this relationship. Measurements of the velocity profile are also needed to cross correlate with the eddy diffusivity.

Measurement of the increase in surface area when waves form on the surface of the film indicates that the percentage increase is small. Measurements using the capacitance method are dependent on knowledge of the velocity of the waves as they move down the film. At the present time information is available only on the surface velocity of the film. The relationship between the velocity of the waves and the surface velocity has not been determined. This information is required for more accurate calculation of the interfacial area and to allow an analysis of the hydrodynamics of the film in the region next to the interface.

At the present time no explanation is available as to why the eddy diffusivity is a maximum in the central region of the film and decreases as the boundaries of the film are approached. The explanation may lie in second order wave effects, boundary layer turbulence or perhaps in both. The important point is that the problem has now been defined and experimental data made available for comparison with theoretical results.

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NOMENCLATURE

- a The fraction of the film surface passing a given cell position in one second that is at a film thickness of $Y_{\rm T}$
- A Proportionality constant relating the fringe shift to the carbon dioxide concentration in g/liter
- A Surface area of capacitance probe
- b Time width of the triangular shaped waves, sec
- C Concentration of carbon dioxide in water, g/liter
- $\mathtt{C}_{\mathtt{S}}$ Saturated concentration of carbon dioxide in water, g/liter
- \overline{C} Average film concentration of carbon dioxide in water, g/liter

$$c_{s} \left[\frac{x_{CO_{2}} - x_{Air}}{x_{Fs}} \frac{1}{\Delta N_{s}} - \left(\frac{\Delta \overline{N}}{\Delta N_{s}} - \frac{c}{c_{s}} \right) \right]$$

Ĉ Capacitance, farads

--

- d Distance between the capacitance probe and the surface of the water film, cm
- D Diffusion coefficient, cm²/sec

$$E = \frac{\Delta N}{\Delta N_{S}} - \frac{C}{C_{S}}$$

- g Acceleration of gravity, cm/sec²
- G(X) Some function of X
- L Optical path length, cm
- M Magnification ratio of the photograph of the interference patterns

- $M{G(X)} = \chi$ the mean or center of gravity of the whole probability
- ΔN Number of fringe shifts, (X/X_s)
- ΔN_s Number of fringe shifts that correspond to water saturated with CO_2
- $\Delta \overline{N}$ Number of fringe shifts that correspond to the average carbon dioxide concentration in the film
- P_{Δ} Wave amplitude probability
- P_w Wave frequency probability
- Q Volumetric flow rate, cm³/sec
- Re Reynolds number, $\frac{4Q\rho}{W\mu}$
- t Intervals in which the observations X were measured
- T. Thickness of compensating plates, cm
- T_w Thickness of cell windows, cm
- V_A Bulk average film velocity, cm/sec
- V Laminar film surface velocity, cm/sec
- V_s Surface velocity of a liquid film with waves, cm/sec
- ${\tt V}_{_{\rm SMF}}$ Surface velocity of a film calculated from ${\tt Y}_{\rm MF}$
- W Width of the glass contacting plate, cm
- X Observational values used to fit the distribution function
- X_{Air} Distance from the reference line to the center point of a fringe at some Y value for air-water photographs
- X_{CO_2} Distance from the reference line to the center point of a fringe at some Y value for CO₂-water photographs
- X_{FS} Fringe spacing as measured from the photographs of the fringe pattern
- Y Distance from the surface of the contacting plate to some point in the film times the magnification ratio, cm
- Y_F Film thickness times the magnification ratio, cm

- ${\rm Y}_{\rm L}$ $$\rm Laminar\ film\ thickness\ times\ the\ magnification\ ratio, cm$
- Y_{MF} Mean film thickness as determined from the true data times the magnification ratio, cm
- \mathbf{Y}_{T} . Average trough height times the magnification ratio, cm
- Z Distance from the top of the contacting plate, cm
- Y Angle of interaction of the light beams of the interferometer
- δ Dielectric constant

- Eddy diffusivity, cm²/sec
- $\epsilon_{\rm m}$ Maximum eddy diffusivity, cm²/sec
- η_c Index of refraction of water with carbon dioxide
- η_0 Index of refraction of water
- θ Angle of inclination of the cell
- λ Wave length of light source used with the interferometer

$$K \qquad \frac{(D + \epsilon)Z}{V_s(Y_F)^2} \qquad TT' = \frac{M^4 2 D \neq M}{(Y_L)^4 (P95in\Theta)}$$

- μ Viscosity, poises
- 5 Fringe spacing, cm (X_{FS}/M)
- π Total number of fringes visible when the optical path lengths of the interferometer are equal
- ρ Density of liquid, gr/cm³
- σ $\sigma\{G(X)\}$ The dispersion or the square root of the moment of inertia of the total probability mass with respect to X
- $\Phi_{\mathbf{x}}(t)$ Theoretical distribution function
- $\Psi(t)$ The normalized distribution function $\chi = 0$ and $\sigma = 1$
- ω Mean wave frequency, sec⁻¹

APPENDIX

.

APPENDIX A

CONCENTRATION PROFILES $\theta = 9^{\circ}44'$

Barometric pressure = 739.4 mm Hg Cell pressure = 739.7 mm Hg Cell angle of inclination = $\theta = 9^{\circ}44^{\circ}$ Water temperature = 22.2°C Magnification ratio = 118.9 Inlet water CO₂ concentration = 0.0046 g/liter Experimentally determined saturated concentration CO₂ in water = 1.4764 g/liter Number of fringe shift corresponding to saturation = 2.6726 Cell width = 7.5692 cm

	Run 18-1-3 Cell position 32.5 i Re = 1834 Fringe space = 3.2412 Film thickness = 6.86 $\overline{C/C_s} = 0.101$ $\Delta \overline{N}/\Delta N_s = 0.122$	nches from inlet cm 76	
<u>Y, cm</u>	x_{CO_2}, cm	X _{Air} , cm	C/C _s
6.8676 6.3019 5.6964 5.1546 4.5276 3.9420 3.1756 2.2604 1.1109	3.2263 2.9226 2.6329 2.4547 2.3821 2.3343 2.2898 2.2496 2.2335	1.312 1.311 1.309 1.308 1.305 1.302 1.301 1.300 1.296	0.2016 0.1661 0.1328 0.1115 0.1036 0.0986 0.0931 0.0886 0.0868

	Run 18-1-4 Cell position 32.5 inc Re = 1834 Fringe space = 3.2412 c Film thickness = 6.5528 $\overline{C/C_s} = 0.101$ $\Delta \overline{N}/\Delta N_s = 0.1182$	hes from inlet m cm	
Y, cm	x_{CO_2}, cm	X _{Air} , cm	C/C _s
6.5528 6.0741 5.5121 4.9960 3.8942 2.8586 1.9417 1.0852	3.0767 2.8400 2.5893 2.4423 2.2768 2.1737 2.1678 2.1726	1.311 1.310 1.309 1.307 1.302 1.300 1.299 1.297	0.1866 0.1594 0.1305 0.1138 0.0953 0.0836 0.0831 0.0839

	Run 18-1-5 Cell position 32.5 incres Re = 1834 Fringe space = 3.2412 cm Film thickness = 5.5577 cm $\overline{C}/C_{s} = 0.101$ $\Delta \overline{N}/\Delta N_{s} = 0.0894$	irom inist	
Y, cm	x_{CO_2} , cm	X _{Air} , cm	C/C _s
5.5577 5.0065 4.4149 3.8857 3.2887 2.5086 1.7377 1.0893	2.7409 2.4247 2.2039 2.1636 2.0591 1.9790 1.9674 1.9668	1,309 1.307 1.305 1.302 1.301 1.300 1.298 1.297	0.1762 0.1406 0.1152 0.1059 0.0987 0.0894 0.0890 0.0883

	Run 18-3-2 Cell position 30 inches from inlet Re = 1834 Fringe space = 3.4132 cm Film thickness = 6.7853 cm $\overline{C}/C_{s} = 0.094$ $\Delta \overline{N}/\Delta N_{s} = 0.1156$		
Y, cm	X _{CO2} , cm	X _{Air} , cm	c/c _s
6.7853 6.3216 5.8295 5.3022 4.7572 4.1538 3.3677 2.5930 1.8209	4.5037 4.2727 4.0201 3.8137 3.6757 3.6379 3.6286 3.628 3.6143 3.6143	2.478 2.485 2.514 2.545 2.578 2.614 2.660 2.710 2.758 2.796	0.2005 0.1960 0.1435 0.1175 0.0987 0.0906 0.0846 0.0791 0.0727 0.0699

	Run 18-3-5 Cell position 30 inc Re = 1834 Fringe space = 3.4132 Film thickness = 6.96 $\overline{C}/C_{s} = 0.094$ $\Delta \overline{N}/\Delta N_{s} = 0.1156$	hes from inlet cm 46 cm	
Y, cm	$\frac{X_{CO_2}, cm}{cm}$	X _{Air} , cm	c/c _s
6.9646 6.4867 6.0070 5.4301 4.8569 4.1150 3.0029 2.0340 0.9916	4.6770 4.2765 3.9855 3.7937 3.6814 3.6754 3.6897 3.6920 3.7137	2.447 2.472 2.504 2.538 2.572 2.616 2.682 2.740 2.801	0.2432 0.1927 0.1493 0.1238 0.1078 0.1040 0.0980 0.0909 0.0853

Run 18-5-2 Cell position 27.5 inches from inlet
Re = 1834
Fringe space = 3.1702 cm
Film thickness = 5.6394 cm
$\overline{C}/C_{c} = 0.087$
$\Delta \overline{N} / \Delta N_{s} = 0.076$

Y, cm	$\frac{x_{CO_2}, \text{ cm}}{2}$	X _{Air} , cm	C/C _s
5.6394 5.0787 4.5842 4.0650 3.5227 2.6963 1.9771	4.7136 4.5654 4.2743 4.2048 4.0945 4.0701 4.0577	3.5102 3.5062 3.5042 3.5022 3.4972 3.4952 3.4932 3.4932	0.1530 0.1360 0.1019 0.0939 0.0815 0.0789 0.0776

	Run 18-5-3 Cell position 27.5 inc Re = 1834 Fringe space = 3.1702 c Film thickness = 7.6216 $\overline{C/C}_{s} = 0.087$ $\Delta \overline{N}/\Delta N_{s} = 0.0908$	hes from inlet m cm	
Y, cm	\sum_{CO_2} , cm	X _{Air} , cm	s
7.6216 7.1629 6.5808 5.9937 5.4105 4.6669 3.9858 2.9657 1.9632 1.0675	5.2672 4.9676 4.7582 4.5142 4.3679 4.2980 4.2161 4.1140 4.0977 4.0953	3.537 3.528 3.524 3.518 3.510 3.510 3.505 3.497 3.488 3.479	0.1983 0.1627 0.1392 0.1103 0.0946 0.0872 0.0778 0.0673 0.0673 0.0673

Y, cm	X _{CO2} , cm	XAir, cm	C/Cs
6.4402	4.9747	3.514	0.1680
5.8298	4.6638	3.510	0.1325
5.3060	4.5115	3.508	0.1145
4.7417	4.4001	3.505	0.1025
4.2094	4.2965	3.503	0.0887
3.6439	4.2274	3.499	0.0808
2.7653	4.1423	3.496	0.0718
2.0031	4.1328	3.494	0.0711
1.2022	4.1216	3.492	0.0707

Run 18-5-5
Cell position 27.5 inches from inlet
Re = 1834
Fringe space = 3.1702 cm
Film thickness = 5.7328 cm
$\bar{C}/C_{0} = 0.087$
$\Delta \overline{N} / \Delta N_{\alpha} = 0.0797$

Y, cm	x_{CO_2} , cm	XAir, cm	C/Cs
5.7328	4.8061	3.521	0.1567
5.2619	4.5311	3.517	0.1249
4.7676	4.3681	3.512	0.1061
4.2675	4.2647	3.510	0.0949
3.7194	4.2020	3.502	0.0874
2.7675	4.1086	3.496	0.0781
1.8632	4.0984	3.490	0.0781
0.9628	4.1014	3.479	0.0781

	Run 18-7-2 Cell position 25 inches Re = 1834 Fringe space = 2.8500 o Film thickness = 6.0538 $\overline{C}/C_{s} = 0.08$ $\Delta \overline{N}/\Delta N_{s} = 0.0770$	s from inlet em	
Y, cm	x_{CO_2} , cm	X _{Air} , cm	C/C _s
6.0538 5.5711 5.0649 4.5071 3.7913 2.7297 1.7240 0.9750	3.9430 3.8487 3.7482 3.6903 3.6476 3.6760 3.6820 3.6764	2.945 2.970 2.998 3.029 3.016 3.126 3.184 3.226	0.1280 0.1124 0.0955 0.0838 0.0799 0.0692 0.0692 0.0561

	Run 18-7-3 Cell position 25 inche Re = 1834 Fringe space = 2.8500 Film thickness = 5.902 $\overline{C}/C_{s} = 0.080$ $\Delta \overline{N}/\Delta N_{s} = 0.1032$	s from inlet cm 3 cm	
Y, cm	$\frac{X_{CO_2}, cm}{cm}$	X _{Air} , cm	C/Cs
5.9023 5.3477 4.7391 4.1389 3.3110 2.5736 1.8557 1.1318	4.1920 4.0005 3.9241 3.8746 3.8675 3.8613 3.8656 3.8603	2.952 2.981 3.017 3.049 3.092 3.135 3.175 3.218	0.1407 0.1114 0.0965 0.0864 0.0797 0.0722 0.0684 0.0610

	Run 18-7-5 Cell position 25 inche Re = 1834 Fringe space = 2.8500 Film thickness = 6.805 $\overline{C}/C_s = 0.080$ $\Delta \overline{N}/\Delta N_s = 0.0917$	s from inlet cm 7 cm	
Y, cm	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	c/c _s
6.8057 6.2799 5.7422 5.1823 4.5668 3.7769 2.9188 2.0496 1.1319	4.1759 3.9939 3.8514 3.7760 3.7419 3.7402 3.7381 3.7588 3.7538	2.902 2.931 2.960 2.993 3.023 3.067 3.115 3.164 3.217	0.1553 0.1276 0.1040 0.0909 0.0827 0.0763 0.0699 0.0662 0.0584

Run 18-9-2 Cell position 22.5 inches from inlet Re = 1834 Fringe space = 2.8149 cm Film thickness = 7.9266 cm $\overline{C}/C_s = 0.072$ $\Delta N/\Delta N_s = 0.0609$

Y, cm	x_{CO_2} , cm	X _{Air} , cm	C/Cs
7.9266 7.4032 6.9656 6.5093 5.7792 4.7406 3.2719 2.1797 0.9631	3.5201 3.2242 3.1682 3.0767 2.9882 2.9832 3.0004 3.0635 3.0901	2.217 2.267 2.311 2.354 2.425 2.525 2.525 2.665 2.775 2.895	0.1840 0.1421 0.1234 0.1066 0.0871 0.0718 0.0579 0.0486 0.0373

	Run 18-9-3 Cell position 22.5 in Re = 1834 Fringe space = 2.8149 Film thickness = 6.94 $\overline{C/C_s} = 0.072$ $\Delta N/\Delta N_s = 0.0636$	ches from inlet cm 88 cm	
em_	x_{CO_2} , cm	X _{Air} , cm	
488 276	3.4513 3.3080	2.313 2.365	

Y, cm	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	c/c _s
6.9488 6.4076 5.8144 5.2632 4.6564 3.8582 2.8974 2.0185 1.1769	3.4513 3.3080 3.1642 3.0878 3.0730 3.0675 3.0731 3.0969 3.0919	2.313 2.365 2.423 2.474 2.532 2.604 2.705 2.790 2.874	0.1598 0.1338 0.1069 0.0900 0.0803 0.0700 0.0573 0.0492 0.0374

Run 18-9-5 Cell position 22.5 inches from inlet Re = 1834 Fringe space = 2.8147 cm Film thickness = 7.0350 cm $\overline{C/C_s} = 0.072$ $\Delta \overline{N}/\Delta N_s = 0.0692$

<u>Y, cm</u>	x_{CO_2} , cm	X _{Air} , cm	C/C _s
7.0350	3.4423	2.304	0.15453
6.5254	3.3053	2.355	0.12983
6.0653	3.2042	2.398	0.10963
5.4790	3.1784	2.455	0.10027
4.5968	3.1225	2.537	0.08156
3.6077	3.0818	2.633	0.06286
2.4300	3.1152	2.750	0.05163
0.9449	3.1740	2.895	0.0404

Run 18-1-6 Cell position 32.5 inches from inlet Re = 1482 Fringe space = 3.2412 cm Film thickness = 5.8530 cm $C/C_{\rm S}$ = 0.121 $\Delta N/\Delta N_{\rm S}$ = 0.1459

5.85303.28801.3090.25.24063.02591.3080.14.74072.76921.3060.14.21542.61081.3040.13.70622.55381.3020.1	C/C _s
2.8203 2.5286 1.300 0.1 2.0046 2.4980 1.298 0.1 1.2318 2.4037 1.296 0.1	0.2035 0.1734 0.14401 0.1260 0.1196 0.1169 0.1136

	Run 18-1-7 Cell position 32.5 inc Re = 1482 Fringe space = 3.2412 Film thickness = 5.931 $\overline{C/C}_{s} = 0.121$ $\Delta N/\Delta N_{s} = 0.168$	ches from inlet cm ll cm	
<u>Y, cm</u>	$\frac{X_{CO2}, cm}{cm}$	XAir, Cm	C/Cs
5.9311 5.4307 4.8803 4.2755 3.5713 2.7205 1.9196 1.1180	3.8019 3.4270 3.0586 2.7942 2.6804 2.5944 2.5494 2.5491	1.309 1.308 1.306 1.303 1.301 1.300 1.299 1.297	0.2407 0.1976 0.1553 0.1251 0.1122 0.1024 0.0870 0.0975

	Run 18-1-8 Cell position 32.5 inches from inlet Re = 1482 Fringe space = 3.2412 cm Film thickness = 5.8213 cm $\overline{C}/C_s = 0.121$ $\Delta N/\Delta N_s = 0.168$	
<u>n</u>	X_{CO_2}, cm X_{Air}, cm	

Y, cm	x_{CO_2} , cm	X _{Air} , cm	c/c _s
5.8213	3.6125	1.309	0.2185
5.2989	3.3052	1.308	0.1829
4.7691	3.0299	1.305	0.1515
4.2744	2.8561	1.303	0.1317
3.6996	2.7509	1.301	0.1201
2.9052	2.6895	1.300	0.1122
2.0969	2.5900	1.299	0.1006
1.2166	2.5266	1.297	0.0950

	Run 18-3-7 Cell position 30 inches Re = 1482 Fringe space = 3.4132 of Film thickness = 4.6330 $\overline{C}/C_s = 0.115$ $\Delta \overline{N}/\Delta N_s = 0.167$	s from inlet cm D cm	
Y, cm	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	C/C _s
4.6330 4.2436 3.8102 3.3222 2.5565 1.7408 1.0008	4.7667 4.5997 4.4023 4.2747 4.2566 4.2753 4.2762	2.582 2.606 2.634 2.664 2.712 2.763 2.800	0.1875 0.1666 0.1418 0.1236 0.1163 0.1128 0.1098

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Y, cm	x_{CO_2} , cm	X _{Air} , cm	c/c _s
5.0599	4.8796	2.560	0.1908
4.6340	4.6233	2.584	0.1635
4.2086	4.3894	2.609	0.1399
3.7228	4.2386	2.640	0.1197
3.0667	4.1955	2.680	0.1141
2.5266	4.1989	2.711	0.1103
1.7406	4.2111	2.758	0.1032
0.9836	4.2275	2.822	0.0973

	Run 18-5-6 Cell position 27.5 inc Re = 1482 Fringe space = 3.1702 Film thickness = 5.411 $\overline{C}/C_{s} = 0.108$ $\Delta \overline{N}/\Delta N_{s} = 0.118$	hes from inlet cm 5 cm	
<u>Y, cm</u>	<u>x_{CO2}, cm</u>	X _{Air} , cm	c/c _s
5.4115 4.9568 4.5301 4.0544 3.5844 2.5304 1.7702 0.9810	5.2182 4.8893 4.7003 4.5856 4.4891 4.3922 4.3689 4.3467	3.518 3.513 3.509 3.506 3.503 3.494 3.487 3.479	0.1889 0.1507 0.1290 0.1144 0.1040 0.0935 0.0916 0.0901

	Run 18-5-8 Cell position 27.5 incl Re = 1482 Fringe space = 3.1702 G Film thickness = 5.3223 $\overline{C/C}_{S} = 0.108$ $\Delta \overline{N}/\Delta N_{S} = 0.118$	nes from inlet cm 3 cm	
Y, cm	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	C/Cs
5.3223 4.9004 4.4102 3.8745 3.2579 2.4386 1.6776 0.8758	2.0360 1.7374 1.5332 1.3475 1.2765 1.2245 1.2037 1.1691	0.314 0.309 0.304 0.295 0.289 0.281 0.274	0.1932 0.1586 0.1351 0.1135 0.1058 0.1058 0.1004 0.0989 0.0956

	Run 18-7-6 Cell position 25 inches Re = 1482 Fringe space = 2.8500 c Film thickness = 5.9009 $\overline{C/C_s} = 0.101$ $\Delta \overline{N}/\Delta N_s = 0.1336$	from inlet m cm	
<u>Y, cm</u>	X_{CO_2} , cm	X _{Air} , cm	C/Cs
5.9009 5.4191 4.9271 4.3858 3.7949 2.9187 2.0284 1.1370	1.6673 1.5076 1.3720 1.2826 1.2370 1.2084 1.2306 1.2401	0.110 0.136 0.162 0.195 0.224 0.270 0.320 0.374	0.1717 0.1474 0.1261 0.1100 0.1003 0.0902 0.0868 0.0812

	Run 18-7-7 Cell position 25 inches Re = 1482 Fringe space = 2.8500 of Film thickness = 5.4547 $\overline{C}/C_{s} = 0.101$ $\Delta \overline{N}/\Delta N_{s} = 0.1220$	s from inlet Cm ⁷ cm	
<u>Y, cm</u>	X _{CO2} , cm	X _{Air} , cm	c/c _s
5.4547 5.0165 4.5624 4.0777 3.4901 3.7359 1.8684 1.0466	1.7324 1.5345 1.3492 1.1568 1.1354 1.1523 1.1779 1.1575	0.134 0.158 0.185 0.210 0.240 0.280 0.330 0.366	0.1885 0.1604 0.1305 0.1062 0.0968 0.0931 0.0875 0.0819

	Run 18-8-1 Cell position 25 inches Re = 1482 Fringe space = 2.8500 c Film thickness = 5.7158 $\overline{C/C_s} = 0.101$ $\Delta \overline{N}/\Delta N_s = 0.0962$	s from inlet em 3 cm	
<u>Y, cm</u>	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	C/C_s
5.7158 5.0745 4.6073 3.8514 3.1766 2.5514 1.9645 1.2600	4.1263 3.8975 3.7602 3.7590 3.7575 3.7615 3.7705 3.7604	2.960 2.998 3.023 3.061 3.099 3.137 3.169 3.210	0.1580 0.1229 0.1016 0.0965 0.0913 0.0868 0.0838 0.0771

Run 18-9-6	
Cell position 22.5 inches from t	inlet
Re = 1482	
Fringe space = 2.8419 cm	
Film thickness = 5.0132 cm	
$\overline{C}/C_{s} = 0.094$	
$\Delta \overline{N} / \Delta N_{\rm S} = 0.0841$	

Y, cm	x_{CO_2} , cm	X _{Air} , cm	c/c _s
5.0132	3.4944	2.498	0.1427
4.4672	3.4098	2.550	0.1240
3.9669	3.3325	2.599	0.1107
3.4395	3.3346	2.650	0.1012
2.7651	3.3363	2.717	0.0922
2.0216	3.3446	2.790	0.0840
1.1867	3.3341	2.824	0.0780

	Run 18-9-7 Cell position 22.5 incl Re = 1482 Fringe space = 2.8419 Film thickness = 6.9120 $\overline{C/C_s} = 0.094$ $\Delta \overline{N}/\Delta N_s = 0.0988$	nes from inlet cm) cm	
Y, cm	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	C/C _s
6.9120 6.3539 5.8648 5.3650 4.5568 3.7535 2.9612 2.0860 1.1225	3.7058 3.5012 3.4246 3.3596 3.2981 3.3172 3.3218 3.3914 3.4037	2.315 2.370 2.417 2.465 2.541 2.617 2.698 2.785 2.878	0.1805 0.1468 0.1270 0.1150 0.0981 0.0880 0.0787 0.0738 0.0645

	Run 18-9-8 Cell position 22.5 inch Re = 1482 Fringe space = 2.8419 c Film thickness = 6.3727 $\overline{C/C_s} = 0.094$ $\Delta \overline{N}/\Delta N_s = 0.098$	nes from inlet em ' cm	
Y, cm	x_{CO_2} , cm	X _{Air} , cm	c/c _s
6.3727 5.9225 5.4116 4.9145 4.2445 3.2895 1.9857 0.9073	3.5734 3.4897 3.4495 3.3920 3.3477 3.3299 3.3945 3.4032	2.367 2.410 2.461 2.508 2.570 2.665 2.795 2.898	0.1552 0.1388 0.1287 0.1134 0.1025 0.0838 0.0748 0.0 6 32

	Run 18-2-3 Cell position 32.5 inc Re = 1099 Fringe space = 3.2412 Film thickness = 5.322 $\overline{C/C_s} = 0.1459$ $\Delta \overline{N}/\Delta N_s = 0.1949$	hes from inlet cm 3 cm	
Y, cm	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	c/c _s
5.3223 4.8768 4.3684 3.8086 3.2983 2.6235 1.8815 1.0885	3.9688 3.6113 3.3356 3.1111 2.9430 2.7949 2.7031 2.6697	1.308 1.306 1.304 1.302 1.300 1.299 1.298 1.297	0.2578 0.2166 0.1837 0.1597 0.1403 0.1231 0.1126 0.1097

	Run 18-2-4 Cell position 32.5 inches from inlet Re = 1099 Fringe space = 3.2412 cm Film thickness = 4.2868 cm $\overline{C/C_s} = 0.1459$ $\Delta \overline{N}/\Delta N_s = 0.1855$		
<u>Y, cm</u>	x_{CO_2} , cm	X _{Air} , cm	C/C_s
4.2868 3.7834 3.2750 2.7847 1.2368 1.1500 1.1020	3.8987 3.4577 3.0694 2.8690 2.7615 2.7069 2.6979	1.304 1.302 1.301 1.300 1.299 1.298 1.297	0.2597 0.2074 0.1662 0.1423 0.1314 0.1232 0.1213
	Run 18-4-2 Cell position 30 inches from inlet Re = 1099 Fringe space = 3.4132 cm Film thickness = 4.9469 cm $\overline{C/C_s} = 0.137$ $\overline{\Delta N}/\Delta N_s = 0.215$		
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Y, cm	x _{CO2} , cm	X _{Air} , cm	c/c _s
4.9460 4.4058 3.8934 3.3565 2.6684 1.8260 0.8310	5.7337 5.2931 4.8912 4.6004 4.5271 4.5401 4.5359	2.566 2.598 2.629 2.661 2.740 2.757 2.809	0.2693 0.2174 0.1700 0.1346 0.1179 0.1174 0.1113

	Run 18-4-5 Cell position 30 inches Re = 1099 Fringe space 3.4132 cm Film thickness = 5.3812 $\overline{C/C_s} = 0.137$ $\Delta \overline{N}/\Delta N_s = 0.215$	from inlet cm	
Y, cm	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	C/Cs
5.3812 4.8051 4.2712 3.6737 3.1315 2.5037 2.7106 0.9110	5.6042 5.1758 4.9036 4.6848 4.5916 4.5906 4.6028 4.6017	2.541 2.573 2.603 2.640 2.673 2.711 2.759 2.808	0.2510 0.1986 0.1612 0.1328 0.1231 0.1174 0.1148 0.1077

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	Run 18-6-2 Cell position 27.5 incl Re = 1099 Fringe space = 3.1702 Film thickness = 4.9314 $\overline{C}/C_{\rm S}$ = 0.1282 $\Delta \overline{N}/\Delta N_{\rm S}$ = 0.1664	nes from inlet cm 4 cm	
Y, cm	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	C/C _s
4.9314 4.4796 3.9783 3.4240 2.9683 2.2525 1.5688 0.9726	2.6245 2.2267 1.9102 1.6706 1.5173 1.1574 1.5083 1.5016	0.309 0.304 0.301 0.295 0.292 0.286 0.280 0.274	0.2331 0.1878 0.1507 0.1275 0.1062 0.1062 0.1043 0.1040

	Run 18-6-3 Cell position 27.5 i Re = 1099 Fringe space = 3.170 Film thickness = 4.3 $\overline{C/C_s} = 0.1282$ $\Delta \overline{N}/\Delta N_s = 0.1371$	nches from inlet 2 cm 929 cm	
Y, cm	$\frac{X_{CO_2}, cm}{cm}$	X _{Air} , cm	c/c _s
4.3929 3.9335 3.4585 2.9546 2.3678 1.7911 1.1753	1.8979 1.6982 1.5126 1.4288 1.3927 1.3881 1.3777	0.298 0.296 0.291 0.290 0.288 0.286 0.285	0.1782 0.1539 0.1352 0.1221 0.1191 0.1191 0.1191

	Run 18-6-4 Cell position 27.5 inches Re = 1099 Fringe space = 3.1702 cm Film thickness = 2.8150 cm C/C_s = 0.1282 $\Delta N/\Delta N_s$ = 0.1370	from inlet
Y, cm	x_{CO_2} , cm	X _{Air} , cm

2.8150	1.8075	0.299	0.1690
2.4132	1.6297	0.295	0.1485
1.9896	1.5116	0.293	0.1338
1.5570	1.3844	0.289	0.1203
0.7384	1.3359	0.282	0.1154

C/C_s

	Run 18-8-2 Cell position 25 inche Re = 1099 Fringe space = 2.8500 Film thickness = 5.77^4 $\overline{C}/C_{\rm S}$ = 0.1193 $\Delta \overline{N}/\Delta N_{\rm S}$ = 0.1916	es from inlet cm 43 cm	
Y, cm	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	C/Cs
5.7743 5.3091 4.8243 4.2525 3.7181 3.0760 2.3262 1.6380 0.9600	2.4951 2.1811 1.9525 1.8158 1.6462 1.5848 1.5842 1.5913 1.5893	0.115 0.143 0.170 0.202 0.227 0.263 0.310 0.343 0.385	0.2402 0.1953 0.1616 0.1391 0.1148 0.1013 0.0950 0.0916 0.0864

Run 18-8-3 Cell position 25 inches from inlet Re = 1099 Fringe space = 2.8500 cm Film thickness = 5.8973 cm $\overline{C/C_s} = 0.1193$ $\Delta \overline{N}/\Delta N_s = 0.1902$

Y, cm	x_{CO_2} , cm	X _{Air} , cm	c/c _s
5.8973	5.6436	2.950	0.2620
5.5026	5.3045	2.970	0.2152
5.0907	4.9852	2.994	0.1703
4.6227	4.6312	3.018	0.1329
4.0563	4.4717	3.050	0.0955
3.3821	4.3762	3.085	0.0768
2.4170	4.3956	3.143	0.0674
1.6896	4.4067	3.185	0.0656
0.9556	4.3871	3.226	0.0626

	Run 18-8-4 Cell position 25 inches from inlet Re = 1099 Fringe space = 2.8500 cm Film thickness = 5.8925 cm $\overline{C/C_8} = 0.1193$ $\Delta \overline{N}/\Delta N_8 = 0.1715$		
Y, cm	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	c/c _s
5.8925 5.3616 4.7630 4.1882 3.5944 2.7686 1.9942	2.4719 2.0638 1.7223 1.5236 1.4450 1.4492 1.4543 1.4543	0.110 0.140 0.171 0.205 0.235 0.279 0.324 0.365	0.2579 0.2004 0.1515 0.1209 0.1067 0.1014 0.0962

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	Run 18-10-2 Cell position 22.5 inc Re = 1099 Fringe space = 2.8149 Film thickness = 4.444 $\overline{C}/C_{s} = 0.1104$ $\Delta \overline{N}/\Delta N_{s} = 0.1392$	hes from inlet cm 8 cm	
Y, cm	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	C/C _s
4.4448 4.0638 3.7101 3.3925 2.8124 2.2474 1.6820 1.0900	3.9853 3.8416 3.7744 3.7103 3.6833 3.6840 3.7328 3.7587	2.551 2.585 2.623 2.654 2.712 2.768 2.825 2.882	0.1619 0.1382 0.1242 0.1116 0.1003 0.0930 0.0919 0.0877

	Run 18-10-3 Cell position 22.5 in Re = 1099 Fringe space = 2.814 Film thickness = 4.8 $\overline{C/C_s} = 0.1104$ $\Delta \overline{N}/\Delta N_s = 0.1429$	nches from inlet 9 cm 330 cm	
<u>Y, cm</u>	$\frac{X_{CO_2}, cm}{cm}$	X _{Air} , cm	C/C _s
4.8330 4.3339 3.8156 3.2383 2.6938 1.9828 1.1701	3.9642 3.8591 3.7876 3.7405 3.7201 3.7196 3.7210	2.515 2.563 2.615 2.671 2.725 2.795 2.865	0.1601 0.1398 0.1234 0.1097 0.0998 0.0904 0.0813

	Run 18-10-5 Cell position 22.5 inches from inlet Re = 1099 Fringe space = 2.8149 cm Film thickness = 5.0070 cm \overline{C}/C_{s} = 0.1104 $\Delta \overline{N}/\Delta N_{s}$ = 0.1485		
Y, cm	x_{CO_2}, cm	X _{Air} , cm	C/C _S
5.0070 4.6241 4.2071 3.8037 3.41385 2.9783 2.3286 1.6900 1.0380	1.4135 1.2133 1.0983 1.0118 0.9529 0.9502 0.9769 0.9836 0.9953	2.492 2.535 2.574 2.613 2.653 2.695 2.760 2.823 2.885	0.1930 0.1601 0.1375 0.1227 0.1107 0.1040 0.0991 0.0927 0.0853

V cm	Run 18-2-6 Cell position 32.5 inch Re = 732 Fringe space = 3.2412 c Film thickness = 4.0432 $\overline{C/C_s} = 0.1820$ $\Delta \overline{N}/\Delta N_s = 0.3139$ X_{CO_2} , cm	nes from inlet em e cm X _{Air} , cm	C∕C,
<u>Y, Cm</u>	2		
4.0432 3.5788 3.1309 2.6151 2.0459 1.5166 0.9990	5.3126 4.9503 4.5337 4.1900 3.8966 3.6680 3.6403	1.303 1.302 1.301 1.300 1.298 1.296 1.295	0.3310 0.2893 0.2412 0.2017 0.1681 0.1419 0.1388

 Run 18-2-8
 Cell position 32.5 inches from inlet

 Re = 732
 Fringe space = 3.2412 cm

 Film thickness = 4.1140 cm $\overline{C/C_s} = 0.1820$
 $\Delta \overline{N}/\Delta N_s = 0.3139$ $\underline{X_{CO_2}, \text{ cm}}$
 $\underline{Y, \text{ cm}}$ $\underline{X_{CO_2}, \text{ cm}}$

 4.1140 5.2793

4.1140	5.2793	1.303	0.3450
3.6558	4.8720	1.302	0.3180
3.1730	4.4994	1.301	0.2370
2.6986	4.0361	1.300	0.1836
2.2585	3.7827	1.298	0.1558
1.7816	3.5370	1.296	0.1274
1.2487	3.4829	1.295	0.1210

C/C_s

	Run 18-4-7 Cell position 30 inches Re = 732 Fringe space = 3.4132 G Film thickness = 3.784 $\overline{C/C_{s}} = 0.171$ $\Delta \overline{N}/\Delta N_{s} = 0.327$	s from inlet cm cm	
Y, cm	X _{CO2} , cm	X _{Air} , cm	C/C _s
3.7840 3.3338 2.8611 2.3264 1.6410 0.8759	7.0366 6.5145 6.0388 5.6169 5.3647 5.3632	2.640 2.665 2.692 2.722 2.762 2.762 2.810	0.3170 0.2610 0.2110 0.1600 0.1252 0.1222

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	Run 18-4-8 Cell position 30 inches Re = 732	from inlet	
,	Fringe space 3.4132 cm Film thickness = 3.5657 $\overline{C}/C_s = 0.171$ $\Delta N/\Delta N_s = 0.342$	cm	
Y, cm	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	c/c _s
3.5657 3.1257 2.6983 2.1913 1.4387 0.8675	7.0179 6.4818 6.0441 5.7411 5.3822 5.3892	2.658 2.676 2.703 2.734 2.780 2.807	0.3069 0.2462 0.1953 0.1586 0.1143 0.1121

Cell position 27.5 inches from inlet Re = 732 Fringe space = 3.1702 cm Film thickness = 4.2694 cm $\overline{C}/C_{s} = 0.1595$ $\overline{AN}/AN = 0.2416$	
$\frac{x_{\text{CO}_2}, \text{ cm}}{x_{\text{Air}}, \text{ cm}}$	

<u>Y, cm</u>	$\frac{X_{CO_2}, cm}{cm}$	X _{Air} , cm	C∕C _s
4.2694	3.6894	0.302	0.3177
3.8786	3.1592	0.298	0.2556
3.4360	2.7414	0.295	0.2066
2.9894	2.4752	0.293	0.1755
2.2673	2.2208	0.284	0.1465
1.5721	1.9987	0.277	0.1211
1.1006	1.9530	0.274	0.1161

,	Run 18-6-7 Cell position 27.5 inches from inlet Re = 732 Fringe space = 3.1702 cm Film thickness = 4.5351 cm $\overline{C}/C_{s} = 0.1595$ $\Delta \overline{N}/\Delta N_{s} = 0.673$			
Y, cm	$\frac{x_{CO_2}, cm}{co_2}$	X _{Air} , cm	C/Cs	
4.5351 4.0724 3.5938 3.0729 2.3794 1.6815 1.0148	3.7717 3.3411 2.8780 2.5409 2.2684 2.0246 1.9193	0.304 0.302 0.296 0.293 0.286 0.281 0.274	0.3210 0.2640 0.2090 0.1710 0.1380 0.1110 0.0995	

	Run 18-6-8 Cell position 27.5 inches from inlet Re = 732 Fringe space = 3.1702 cm Film thickness = 3.7950 cm $\overline{C}/C_{s} = 0.1595$ $\Delta \overline{N}/\Delta N_{s} = 0.2416$				
Y, cm	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	C/C _s		
3.7950 3.4260 3.0431 2.6440 2.1481 1.7075 1.1902	3.3469 3.0065 2.6578 2.4200 2.1940 2.1059 2.0866	0.29 6 0.290 0.289 0.288 0.287 0.286 0.285	0.2790 0.2284 0.1948 0.1648 0.1405 0.1282 0.1244		

	Run 18-8-6 Cell position 25 inches from inlet Re = 732 Fringe space = 2.8500 cm Film thickness = 4.3710 $\overline{C}/C_{S} = 0.1485$ $\Delta \overline{N}/\Delta N_{S} = 0.2638$		
Y, cm	x _{CO2} , cm	X _{Air} , cm	C/C _s
4.3710 4.0366 3.6554 3.3239 2.8116 2.2532 1.1765 1.1774	3.2204 2.9424 2.6106 2.4096 2.2087 2.1030 2.0796 2.0773	0.194 0.214 0.230 0.249 0.273 0.308 0.338 0.370	0.2820 0.2430 0.1972 0.1684 0.1388 0.1204 0.1133 0.1088

	Run 18-8-7 Cell position 25 incher Re = 732 Fringe space = 2.8500 Film thickness = 3.993 $\overline{C}/C_{s} = 0.1485$ $\Delta \overline{N}/\Delta N_{s} = 0.2638$	es from inlet cm 37 cm	
<u>Y, cm</u>	x_{CO_2} , cm	X _{Air} , cm	c/c _s
3.9937 3.6196 3.1045 2.6605 2.1874 1.6589 1.0879	2.9160 2.6044 2.3034 2.1792 2.0601 2.0092 2.0016	0.212 0.233 0.261 0.285 0.314 0.344 0.375	0.2397 0.1960 0.1528 0.1334 0.1139 0.1033 0.0983

	Run 18-8-8 Cell position 25 inches from inlet Re = 732 Fringe space = 2.8500 cm Film thickness = 4.5236 cm $\overline{C}/C_{s} = 0.1485$ $\Delta \overline{N}/\Delta N_{s} = 0.2638$			
<u>Y, cm</u>	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	C/C _s	
4.5236 4.0999 3.7013 3.2432 2.5983 2.0787 1.5085 1.0275	3.2165 2.9353 2.6261 2.3809 2.1894 2.0917 2.0797 2.0737	0.185 0.209 0.227 0.254 0.287 0.317 0.352 0.377	0.2840 0.2435 0.2001 0.1646 0.1350 0.1180 0.1121 0.1080	

	Run 18-10-6 Cell position 22.5 inc Re = 732 Fringe space = 2.8149 Film thickness = 4.170 $\overline{C}/C_{s} = 0.1374$ $\Delta \overline{N}/\Delta N_{s} = 0.571$	ches from inlet cm)2 cm	
<u>Y, cm</u>	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	c/c _s
4.1702 3.7618 3.3419 2.9310 2.4354 1.9234 1.4499 0.9700	5.1070 4.7236 4.5050 4.3819 4.2371 4.2325 4.2350 4.2129	2.575 2.618 2.660 2.700 2.750 2.800 2.845 2.894	0.2590 0.2030 0.1690 0.1466 0.1213 0.1140 0.1082 0.1015

	Run 18-10-8 Cell position 22.5 inches from inlet Re = 732 Fringe space = 2.8149 cm Film thickness = 4.0208 cm $\overline{C/C_s} = 0.1374$ $\Delta N/\Delta N_s = 0.0751$		
Y, cm_	x_{CO_2}, cm	X _{Air} , cm	c/c _s
4.0208 3.67755 3.3051 2.9841 2.3438 1.8491 1.4373 0.9777	4.6681 4.5306 4.4364 4.3694 4.3010 4.2432 4.2226 4.2100	2.590 2.624 2.665 2.695 2.757 2.806 2.847 2.892	0.2018 0.1794 0.1607 0.1487 0.1289 0.1173 0.1083 0.1008

APPENDIX B

CONCENTRATION PROFILES $\theta = 18^{\circ}27'$

Barometric pressure = 732.4 mm Hg Cell pressure = 732.7 mm Hg Cell angle of inclination = θ = 18°27' Water temperature = 22.22°C Magnification ratio = 133.78 Inlet water CO₂ concentration = 0.0 g/liter Experimentally determined saturated concentration CO₂ in water = 1.4914 g/liter Number of fringe shifts corresponding to saturation = 2.678 Cell width = 7.5692 cm

	Run 20-1-3 Cell position 32.5 inches from inlet Re = 1834 Fringe spacing = 3.6749 cm Film thickness = 3.9884 cm $\overline{C}/C_{s} = 0.1302$ $\Delta \overline{N}/\Delta N_{c} = 0.0565$		
Y, cm	x_{CO_2} , cm	X _{Air} , cm	c/c _s
3.9884 3.5523 3.1245 2.6367 2.0675 1.4553	2.975 3.0700 3.157 3.260 3.3775 3.509	2.1015 2.3099 2.5440 2.7077 2.8181 3.0029	0.1645 0.1530 0.1386 0.1327 0.1332 0.1284

	Run 20-1-5 Cell position 32.5 inches from inlet Re = 1834 Fringe space = 3.7222 cm Film thickness = 4.1362 cm $\overline{C}/C_8 = 0.1302$ $\Delta \overline{N}/\Delta N_8 = 0.0525$		
Y, cm	x _{CO2} , cm	X _{Air} , cm	c/c _s
4.1362 3.6902 3.2077 2.7253 2.0777 1.4274	2.949 3.039 3.1375 3.2415 3.370 3.513	2.071 2.2573 2.3913 2.548 2.669 2.8858	0.1649 0.1550 0.1519 0.1467 0.1470 0.1470

	Run 20-3-2 Cell position 30.5 inch Re = 1834 Fringe spacing = 3.348 Film thickness = 5.9262 $\overline{C/C_s} = 0.1241$ $\Delta \overline{N}/\Delta N_s = 0.0436$	es from inlet cm cm	
Y, cm	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	c/c _s
5.9262 5.3837 4.7703 4.2057 3.5950 2.9531 2.1594 1.3075	3.5386 3.5289 3.5920 3.6309 3.7328 3.8533 3.9661 4.1622	2,630 2,780 2,95 3,105 3,270 3,450 3,660 3,910	0.1929 0.1648 0.1529 0.1398 0.1327 0.1259 0.1149 0.1089

	Run 20-3-5 Cell position 30.5 incl Re = 1834 Fringe spacing = 3.230 Film thickness = 6.198 $\overline{C/C_s} = 0.1241$ $\Delta \overline{N}/\Delta N_s = 0.0436$	hes from inlet O cm O cm	
Y, cm	x _{CO2} , em	X _{Air} , cm	c/c _s
6.1980 5.5693 4.9680 4.2871 3.6148 2.9563 2.1628 1.2738	3.179 3.1162 3.0756 3.1748 3.3229 3.4559 3.6979 4.0291	2.550 2.725 2.900 2.942 3.268 3.451 3.670 3.917	0.1532 0.1258 0.1008 0.1074 0.0869 0.0812 0.0837 0.0935

	18	6	
	Run 20-5-3 Cell position 28.50 in Re = 1834 Fringe spacing = 3.3023 Film thickness = 5.5034 $\overline{C}/C_{S} = 0.1185$ $\overline{\Delta N}/\Delta N_{S} = 0.0961$	ches from inlet 3 cm 4 cm	
Y, cm	$\frac{X_{CO_2}}{2}$, cm	X _{Air} , cm	C/C _s
5.3534 4.8382 4.3217 3.8303 3.2830 2.6272 2.0115 1.4044	1.9522 2.0268 2.0635 2.2308 2.3703 2.5592 2.7035 2.9150	0.790 0.960 1.12 1.27 1.46 1.66 1.86 2.13	0.1494 0.1388 0.1253 0.1274 0.1219 0.12065 0.1146 0.1081

	Run 20-5-4 Cell position 28.50 inch Re = 1834 Fringe spacing = 3.3620 Film thickness = 5.6001 $\overline{C}/C_{s} = 0.1185$ $\Delta \overline{N}/\Delta N_{s} = 0.0961$	nes from inlet cm cm	
Y, cm_	x_{CO_2} , cm	X _{Air} , cm	C/C _s
5.6001 5.0036 4.5163 4.0164 3.4642 2.7569 1.9565 1.2971	1.7774 1.8781 1.9851 2.1479 2.2693 2.4536 2.6967 2.8908	0.680 0.870 1.025 1.180 1.355 1.585 1.850 2.140	0.1424 0.1334 0.1274 0.1283 0.1224 0.1173 0.1150 0.1044

	Run 20-7-2 Cell position 26.50 inch Re = 1834 Fringe spacing = 3.0861 Film thickness = 4.4864 $\overline{C}/C_s = 0.1125$ $\Delta \overline{N}/\Delta N_s = 0.1340$	es from inlet cm cm	
<u>Y, cm</u>	x_{CO_2} , cm	X _{Air} , cm	s
4.4868 3.9968 3.5087 2.9686 2.3486 1.5403	2.9943 2.9431 2.9126 2.8635 2.9691 3.2485	1.353 1.485 1.616 1.765 1.950 2.230	0.1775 0.1553 0.1357 0.1115 0.1017 0.1017

	Run 20-7-4 Cell position 26.50 inches Re = 1834 Fringe spacing = 3.0850 cm Film thickness = 5.1370 cm $\overline{C/C_s} = 0.1125$ $\Delta \overline{N}/\Delta N_s = 0.1340$	from inlet	
Y, cm	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	C/C _s
5.1370 4.6694 4.1689 3.6420 2.9921 2.2949 1.5492	2.9238 2.8560 2.8471 2.8444 2.8942 3.0141 3.2496	1.174 1.303 1.442 1.580 1.761 1.960 2.230	0.1905 0.1667 0.1486 0.1313 0.1155 0.1062 0.1017

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	Run 20-9-3 Cell position 24.50 inc Re = 1834 Fringe spacing = 3.1300 Film thickness = 6.3530 $\overline{C/C_s} = 0.107$ $\Delta \overline{N}/\Delta N_s = 0.0296$	hes from inlet) cm) cm
Y, cm	X_{CO_2} , cm	X _{Air} , cm
6.3530	2.1541	2.177

6.3530 5.7481 5.1344 4.4358 3.6127 2.6824 1.6459	2.1541 2.1422 2.1829 2.2745 2.4245 2.5607 2.9652	2.177 2.302 2.432 2.578 2.751 2.947 3.163	0.1338 0.1176 0.1068 0.1304 0.0975 0.0904 0.1130

C/Cs

	Run 20-9-4 Cell position 24.50 inc Re = 1834 Fringe spacing = 3.1300 Film thickness = 8.0974 $\overline{C/C_s} = 0.107$ $\Delta \overline{N}/\Delta N_s = 0.0296$	ehes from inlet) cm 4 cm	
Y, cm	x_{CO_2} , cm	X _{Air} , cm	C∕C _s
8.0974 7.4611 6.7807 6.1116 5.3593 4.4773 3.6891 2.7532 1.8905	1.9398 1.9565 2.0192 2.1224 2.1734 2.3072 2.4252 2.5957 2.9019	1.816 1.947 2.087 2.228 2.383 2.569 2.740 2.930 3.111	0.1502 0.1374 0.1290 0.1253 0.1145 0.1101 0.1050 0.1027 0.1083

	Run 20-2-7 Cell position 32.5 inc Re = 732 Fringe spacing = 3.720 Film thickness = 3.956 $\overline{C}/C_{s} = 0.2125$ $\Delta \overline{N}/\Delta N_{s} = 0.343$	5 inches from inlet 3.7200 cm 3.9567 cm X _{Air} , cm C/C _s	
Y, cm	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	c/c _s
3.9567 3.5489 3.1187 2.6991 2.2235 1.7245	8.4697 3.0289 7.6345 7.2512 6.9350 6.7911	2.995 3.080 3.170 3.260 3.355 3.453	0.4195 0.3665 0.3175 0.2715 0.2285 0.2040

	Run 20-4-7 Cell position 32.5 inch Re = 732 Fringe spacing = 3.3152 Film thickness = 3.8588 $\overline{C}/C_{S} = 0.2035$ $\Delta \overline{N}/\Delta N_{S} = 0.363$	5 inches from inlet 3.3152 cm 3.8588 cm		
Y, cm	X_{CO_2} , cm	X _{Air} , cm	C/C _s	
3.8588 3.4698 3.0891 2.6477 2.0414 1.6285	5.7667 5.1452 5.0750 4.8411 4.7155 4.7249	3.198 3.107 3.412 3.533 3.703 3.820	0,4525 0,4035 0,3265 0,2725 0,2195 0,1945	

	Run 20-6-7 Cell position 28.5 inch Re = 732 Fringe spacing = 3.4301 Film thickness = 2.9003 C/C _s = 0.1940 $\Delta N/\Delta N_s = 0.356$	es from inlet cm cm	
Y, cm	X _{CO2} , cm	X _{Air} , cm	C/C _s
2.9003 2.5470 2.1628 1.7443 1.2997	5.9905 5.7095 5.4813 5.3337 5.3233	1.565 1.678 1.802 1.937 2.076	0.311 0.268 0.229 0.200 0.192

	Run 20-7-6 Cell position 26.5 inc Re = 732 Fringe spacing = 3.086 Film thickness = 3.946 $\overline{C/C_{S}} = 0.1858$ $\Delta \overline{N}/\Delta N_{S} = 0.2845$	hes from inlet 1 cm 5 cm	
<u>Y, cm</u>	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	C/Cs
3.94 65 3.5000 3.0068 2.5090 2.0071 1.4663	5.6250 5.1279 4.7720 4.546 4.5075 4.5142	1.496 1.619 1.754 1.900 2.060 2.270	$\begin{array}{c} 0.4013 \\ 0.3253 \\ 0.2623 \\ 0.2213 \\ 0.1973 \\ 0.1723 \end{array}$

•	$\Delta \overline{N} / \overline{\Delta} N_{\rm s} = 0.2970$
	$C/C_{S} = 0.1745$
	Film thickness = 3.5893 cm
	Fringe spacing = 3.1746 cm
	Re = 732
	Cell position 24.5 inches from inlet
	Run 20-10-7

Y, cm	x_{CO_2} , cm	X _{Air} , cm	c/c _s
3.7893	6.5996	2.756	0.3345
3.3801	6.0575	2.841	0.2595
2.9741	5.8264	2.928	0.2215
2.5101	5.6341	3.023	0.1875
2.0777	5.790	3.114	0.1625
1.4888	5.805	3.241	0.1440

	Run 20-2-5 Cell position 32.5 inch Re = 1099 Fringe spacing = 3.600 Film thickness = 5.1392 $\overline{C}/C_{s} = 0.1675$ $\Delta \overline{N}/\Delta N_{s} = 0.1972$	nes from inlet cm 2 cm	
Y, cm	x _{CO2} , cm	X _{Air} , cm	c/c _s
5.1392 4.6837 4.2379 3.7887 3.3050 2.7812 2.1824 1.6192	5.8970 5.7731 5.6403 5.4849 5.4115 5.3741 5.3081 5.2495	2.750 2.845 2.935 3.030 3.130 3.240 3.365 3.475	0.2863 0.2643 0.2418 0.2163 0.1993 0.1848 0.1653 0.1483

	Run 20-4-4 Cell position 30.5 ind Re = 1099 Fringe spacing = 3.315 Film thickness = 5.231 $\overline{C/C_s} = 0.1600$ $\Delta \overline{N}/\Delta N_s = 0.215$	inches from inlet 3.3152 cm 5.2312 cm		
<u>Y, cm</u>	$\frac{X_{CO_2}, cm}{cm}$	X _{Air} , cm	C/Cs	
5.2312 4.8002 4.3397 3.8572 3.3181 2.5833 1.7227	5.5142 5.4897 5.4594 5.4427 5.4152 5.4482 5.6402	2.820 2.941 3.068 3.198 3.348 3.554 3.792	0.2485 0.2320 0.215 0.198 0.171 0 159 0.1535	

	Run 20-5-4 Cell position 28.5 inc Re = 1099 Fringe spacing = 3.421 Film thickness = 3.332 $\overline{C}/C_{s} = 0.1529$ $\Delta \overline{N}/\Delta N_{s} = 0.1971$	hes from inlet O cm O cm	
<u>Y, cm</u>	X_{CO_2} , cm	X _{Air} , cm	C/C _s
3.3320 2.7646 2.2448 1.7412 1.2905	3.8035 3.7633 3.7805 3.8353 3.9702	1.429 1.610 1.778 1.937 2.080	0.2148 0.1898 0.1718 0.1593 0.1490

	Run 20-8-2 Cell position 26.5 inc Re = 1099 Fringe spacing = 3.086 Film thickness = 4.485 $\overline{C/C_s} = 0.145$ $\Delta \overline{N}/\Delta N_s = 0.340$	5 inches from inlet 3.0861 cm 4.4855 cm	
<u>Y, cm</u>	X_{CO_2} , cm	X _{Air} , cm	C/C _s
4.4855 4.0489 3.5917 3.1306 2.6627 2.1637 1.6079	4.6233 4.5099 4.4683 4.5792 4.6717 4.8327 5.0459	1.353 1.473 1.594 1.720 1.850 2.000 2.210	0.2005 0.1720 0.1530 0.151 0.146 0.148 0.1485

Run 20-10-5 Cell position 24.5 inches from inlet Re = 1099 Fringe spacing = 3.135 cm Film thickness = 4.6408 cm $\overline{C}/C_{s} = 0.1378$ $\Delta \overline{N}/\Delta N_{s} = 0.1302$			
Y, cm	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	c/c _s
4.6408 4.1880 3.6990 3.1760 2.6816 2.1774 1.6120	4.1468 3.9425 3.8841 3.8536 3.8258 3.9135 4.1664	2.536 2.630 2.732 2.840 2.946 3.050 3.171	0.1996 0.1639 0.1452 0.1285 0.1230 0.1107 0.1263

	Run 20-1-7 Cell position 32.5 inch Re = 1482 Fringe spacing = 3.730 Film thickness = 5.3685 \overline{C}/C_{s} = 0.1480 $\Delta \overline{N}/\Delta N_{s}$ = 0.1362	es from inlet cm 5 cm	
<u>Y, cm</u>	x_{CO_2} , cm	X _{Air} , cm	c/c _s
5.3685 4.9318 4.4285 3.9093 3.3910 2.8076 2.2374 1.6043	4.2368 4.2261 4.2555 4.3345 4.4667 4.5774 4.7064 4.8509	2.705 2.790 2.896 3.004 3.110 3.730 3.350 3.480	0.1655 0.1558 0.1480 0.1458 0.1480 0.1568 0.1480 0.1496

	Run 20-5-7 Cell position 30.5 inch Re = 1482 Fringe spacing = 3.3152 Film thickness = 6.0925 $\overline{C/C_S} = 0.1415$ $\Delta \overline{N}/\Delta N_S = 0.1482$	es from inlet cm cm	
Y, cm	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	_C/C _s _
6.0925 5.6734 5.2221 4.7427 4.2588 3.7025 2.9473 2.3625 1.6112	4.8694 4.7769 4.7215 4.6762 4.6672 4.7243 4.7889 4.9097 5.0193	2.581 2.698 2.823 2.954 3.090 3.242 3.451 3.615 3.823	0.2513 0.2275 0.2073 0.1871 0.1714 0.1603 0.1443 0.1394 0.1282

,	Run 20-5-7 Cell position 28.5 incl Re = 1482 Fringe spacing = 3.340 Film thickness = 5.846 $\overline{C}/C_{s} = 0.1355$ $\Delta \overline{N}/\Delta N_{s} = 0.1210$	hes from inlet l cm l cm	
Y, cm	x _{CO2} , cm	X _{Air} , cm	s
5.9461 5.4269 4.9529 4.4250 3.8981 3.260() 2.6211 2.0594 1.4836	2.2338 2.2822 2.3550 2.4526 2.5418 2.6234 2.7198 2.9058 3.0934	0.605 0.770 0.920 1.087 1.280 1.455 1.665 1.845 2.900	0.1925 0.1795 0.1707 0.1636 0.1525 0.1420 0.1297 0.1305 0.1238

	Run 20-7-6 Cell position 26.5 incl Re = 1482 Fringe spacing = 3.086 Film thickness = 4.653 $\overline{C/C_{s}} = 0.1292$ $\Delta \overline{N}/\Delta N_{s} = 0.0278$	nes from inlet 1 cm 1 cm	
Y, cm	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	<u> </u>
4.6531 4.1929 3.6219 2.9979 2.3278 1.5689	1.7208 1.709 6 1.8154 2.0125 2.2294 2.4438	1.310 1.437 1.586 1.758 1.960 2.231	0.1512 0.1344 0.1242 0.1362 0.1340 0.1273

	Run 20-9-8 Cell position 24.5 inches from inlet Re = 1482 Fringe spacing = 3.1746 cm Film thickness = 6.5341 cm $\overline{C/C_s} = 0.1232$ $\Delta \overline{N}/\Delta N_s = 0.0497$				
Y, cm	$\frac{x_{CO_2}, cm}{cm}$	X _{Air} , cm	c/c _s		
6.5341 5.9126 5.2753 4.6395 3.8886 3.1252 2.3286 1.5090	2.8758 2.8702 2.8665 2.9381 3.0876 3.1905 3.3695 3.6354	2.111 2.232 2.356 2.480 2.628 2.781 2.964 3.316	0.1642 0.1493 0.1487 0.1279 0.1281 0.1221 0.1116 0.1125		

APPENDIX C

WAVE DATA

Cell location = 31.5 inches from entrance System = air-water Water temperature = 22.2° C Gas velocity = 0 Cell angle of inclination = $9^{\circ}44^{\circ}$ Time increment for wave frequency analysis = 1 sec

```
Re = 732
Mean film thickness = 0.0362 inches
Mean wave frequency = 5.95 sec<sup>-1</sup>
Y_T = 0.0167 inches
Time length of data sample = 58 sec
Micrometer space above the glass plate = 0.0529 inches
Visicorder chart speed = 2 inches/sec with 0.1 sec time
marking
```

Film Thickness, inches	No. of Waves < to Film Thickness	PA	Wave Freq., l/sec	No. of Time Increments < to Wave Freq.	Pw
0.0506 0.0477 0.0442 0.0422 0.0400 0.0360 0.0345 0.0320 0.0292 0.0257 0.0216	368 367 359 336 259 166 113 85 53 15 0	1.000 0.9971 0.9754 0.913 0.7037 0.4510 0.3070 0.2309 0.1440 0.0408 0	10 98 7 6 5 4 3	58 54 47 32 19 5 0	1.000 0.9654 0.9310 0.8103 0.5517 0.3276 0.0862 0

Re = 1099 Mean film thickness = 0.0382 inches Mean wave frequency = 8.3 sec⁻¹ Y_T = 0.02045 inches Time length of data sample = 58 sec Micrometer space above the glass plate = 0.0598 inches Visicorder chart speed = 2 inches/sec with 0.1 sec time marking

Film Thickness, inches	No. of Waves < to Film Thickness	PA	Wave Freq., l/sec	No. of Time Increments < to Wave Freq.	ĒŴ
0.0497 0.0475 0.0452 0.0438 0.0393 0.0370 0.0341 0.0303 0.0	507 499 478 430 380 251 167 92 19	1.000 0.9842 0.9428 0.8481 0.5917 0.4951 0.3294 0.1815 0.0375	13 12 11 10 9 8 7 6 5 4	58 57 53 47 39 26 17 7 1 0	1.000 0.9827 0.9137 0.8103 0.6724 0.6724 0.4482 0.2931 0.1207 0.0172 0

```
Re = 1482
Mean film thickness = 0.0388 inches
Mean wave frequency = 13.1 sec<sup>-1</sup>
Y_T = 0.02345 inches
Time length of data sample = 58 sec
Micrometer space above the glass plate = 0.0605 inches
Visicorder chart speed = 2 inches/sec with 0.1 sec time
marking
```

Film Thickness, inches	No. of Waves < to Film Thickness	PA	Wave Freq., l/sec	No. of Time Increments < to Wave Freq.	Pw
0.0546 0.0505 0.0455 0.0425 0.0410 0.0390 0.0362 0.0330 0.0288	795 791 746 626 516 353 174 44 0	1.000 0.9950 0.9383 0.7874 0.6490 0.4440 0.2163 0.0553 0	19 18 17 16 15 14 13 12 11 10 9 8 7	58 57 56 53 46 34 25 17 96 31 0	1.000 0.9827 0.9655 0.9138 0.7931 0.5862 0.4310 0.2931 0.1552 0.1034 0.0517 0.0172 0

```
Re = 1834
Mean film thickness = 0.0408 inches
Mean wave frequency = 15.8 sec<sup>-1</sup>
Y_{\rm T} = 0.0251 inches
Time length of data sample = 69 sec
Micrometer space above the glass plate = 0.0605 inches
Visicorder chart speed = 2 inches/sec with 1 sec time marking
```

Film Thickness, inches	No. of Waves < to Film Thickness	PA	Wave Freq., l/sec	No. of Time Increments < to Wave Freq.	Pw
0.0563 0.0524 0.0480 0.0455 0.0433 0.0415 0.0355 0.0370 0.0340 0.0300	1129 1128 1098 1020 875 598 343 129 24 1	1.000 0.9991 0.9725 0.9034 0.7750 0.5297 0.3038 0.1143 0.0213 0.0009	23 22 21 20 19 18 17 16 15 14 13 12 11 10	69 68 65 64 62 56 51 39 26 16 6 3 1 0	1.000 0.9855 0.9420 0.9275 0.8985 0.8116 0.7391 0.5652 0.3768 0.2319 0.0869 0.0435 0.0145 0

Cell location = 31.5 inches from entrance System = air-water Water temperature = 22.2° C Gas velocity = 0 Cell angle of inclination = $18^{\circ}27$ ' Time increment for wave frequency analysis = 1 sec Re = 732 Mean film thickness = 0.0281 inches Mean wave frequency = 8.4 sec⁻¹ Y_T = 0.0165 inches Time length of data sample = 58 sec Micrometer space above the glass plate = 0.0408 inches Visicorder chart speed = 2 inches/sec with 0.1 sec time marking

Film Thickness, inches	No. of Waves < to Film Thickness	P _A	Wa v e Freq., l/sec	No. of Time Increments < to Wave Freq.	Pw
0.0398 0.0397 0.0360 0.0344 0.0322 0.0294 0.0280 0.0262 0.0245 0.0225 0.0217 0.0160	512 508 495 470 408 319 253 185 103 24 5 0	1.000 0.992 0.967 0.918 0.797 0.623 0.494 0.361 0.201 0.047 0.010 0	13 12 11 10 9 8 7 6 5 4	58 55 51 41 23 13 3 1 0	1.000 0.948 0.948 0.879 0.707 0.397 0.224 0.052 0.017 0

```
Re = 1099
Mean film thickness = 0.0315
Mean wave frequency = 8.6 sec<sup>-1</sup>
Y_T = 0.0177 inches
Time length of data sample = 54 sec
Micrometer space above the glass plate = 0.0547 inches
Visicorder chart speed = 2 inches/sec with 0.1 sec time
marking
```

Film Thickness, inches	No, of Waves < to Film Thickness	PA	Wave Freq., l/sec	No. of Time Increments < to Wave Freq.	P _w
0.0521 0.0491 0.0455 0.0410 0.0371 0.0350 0.0324 0.0292 0.0255 0.0230 0.0148	492 492 488 458 386 327 261 157 66 19 0	1.000 1.00 0.9919 0.9309 0.7845 0.6646 0.5305 0.3191 0.1341 0.0386 0	14 13 12 11 10 9 8 7 6 5	54 53 52 50 43 34 18 9 5 0	1.000 0.9815 0.963 0.9259 0.7963 0.6111 0.3333 0.1667 0.0926 0

Re = 1482 Mean film thickness = 0.0345 inches Mean wave frequency = 10.0 sec⁻¹ Y_T = 0.016 inches Time length of data sample = 57 sec Micrometer space above the glass plate = 0.0582 inches Visicorder chart speed = 2 inches/sec with 0.1 sec time marking

Film Thickness, inches	No. of Waves < to Film Thickness	PA	Wave Freq., l/sec	No. of Time Increments < to Wave Freq.	Ĕw
0.057 0.0522 0.0484 0.0436 0.0404 0.0388 0.0370 0.0343 0.0310 0.0270 0.0140	587 584 584 582 568 507 459 389 281 140 28 0	1.000 0.9951 0.9951 0.9917 0.9679 0.8639 0.7821 0.6628 0.4788 0.2386 0.0477 0	18 14 13 12 11 10 9 8 7 6	57 56 54 51 40 33 18 7 2 0	1.000 0.9825 0.9474 0.8947 0.7018 0.5790 0.3158 0.1228 0.0351 0

```
Re = 1834
Mean film thickness = 0.0375 inches
Mean wave frequency = 10.8 \text{ sec} - 1
Y_T = 0.0175 inches
Time length of data sample = 57 sec
Micrometer space above the glass plate = 0.0598 inches
Visicorder chart speed = 2 inches/sec with 0.1 sec time
marking
```

Film Thickness, inches	No. of Waves < to Film Thickness	PA	Wave Freq., l/sec	No. of Time Increments < to Wave Freq.	Pw
0.0590 0.0575 0.0542 0.0503 0.0454 0.0420 0.0405 0.0386 0.0367 0.0336 0.0300 0.0155	643 638 636 634 587 531 467 397 267 112 15 0	1.000 0.9921 0.9890 0.9854 0.9128 0.9257 0.7262 0.6173 0.4152 0.1742 0.0145 0	15 14 13 12 11 10 9 8 7 6	57 53 50 45 30 20 11 2 1 0	1.000 0.9296 0.8770 0.7893 0.5262 0.3508 0.1929 0.0351 0.0175 0
Cell location = 31.5 inches from entrance System = air-water Water temperature = $22.2^{\circ}C$ Gas velocity = 0 Cell angle of inclination = $25^{\circ}42^{\circ}$ Time increment for wave frequency analysis = 1 sec Re = 732 Mean film thickness = 0.0255 inches Mean wave frequency = 8.8 sec^{-1} Y_T = 0.0160 inches Time length of data sample = 59 sec Micrometer space above the glass plate = Visicorder chart speed = 2 inches/sec with 0.1 sec time marking

Film Thickness, inches	No. of Waves < to Film Thickness	PA	Wave Freq., l/sec	No. of Time Increments < to Wave Freq.	Pw
0.0398 0.0387 0.0360 0.0344 0.0322 0.0294 0.0280 0.0262 0.0245 0.0225 0.0217 0.0160	512 508 495 470 408 319 253 185 103 24 5 0	1.000 0.992 0.967 0.918 0.797 0.623 0.494 0.361 0.201 0.047 0.010 0	13 12 11 10 9 8 7 6 5 4	58 55 51 41 23 13 3 1 0	1.000 0.948 0.948 0.879 0.707 0.397 0.224 0.052 0.017

•

```
Re = 1099
Mean film thickness = 0.0294 inches
Mean wave frequency = 11.2 sec<sup>-1</sup>
Y_T = 0.0170 inches
Time length of data sample = 58 sec
Micrometer space above the glass plate =
Visicorder chart speed = 2 inches/sec with 0.1 sec time
marking
```

Film Thickness, inches	No. of Waves < to Film Thickness	PA	Wave Freq., l/sec	No. of Time Increments < to Wave Freq.	Pw
0.0453 0.0423 0.0385 0.0361 0.0325 0.0305 0.0280 0.0267 0.0253 0.0237 0.0223 0.0247	685 684 66 638 577 522 436 297 234 145 84 23 0	1.000 0.9986 0.9636 0.9169 0.8424 0.7621 0.6366 0.4336 0.3416 0.2117 0.1226 0.0336 0	16 15 14 13 12 11 10 9 8	58 58 55 36 23 15 4 0	1.000 1.000 0.948 0.8965 0.6207 0.3965 0.2586 0.0689 0

```
Re = 1482
Mean film thickness = 0.0318 inches
Mean wave frequency = 12.35 sec<sup>-1</sup>
Y_{\rm T} = 0.0175 inches
Time length of data sample = 8 sec
Micrometer space above the glass plate =
Visicorder chart speed = 2 inches/sec with 0.1 sec time
marking
```

Film Thickness, inches	No. of Waves < to Film Thickness	PA	Wave Freq., l/sec	No. of Time Increments < to Wave Freq.	P _w
0.0503 0.0488 0.0458 0.0442 0.0425 0.0406 0.0383 0.0362 0.0342 0.0315 0.0285 0.0250 0.0250 0.0215 0.0215 0.0142	1037 1029 1027 1025 1006 973 876 773 674 528 304 101 36 5 0	1.000 0.9920 0.9903 0.9884 0.9701 0.9383 0.8447 0.7454 0.6499 0.5092 0.2932 0.0974 0.0347 0.0048 0	17 16 15 14 13 12 11 10 9 8 7	81 79 75 67 55 38 19 6 3 2 0	1.000 0.9753 0.9259 0.8272 0.6790 0.4691 0.2346 0.0741 0.0370 0.0247 0

```
Re = 1834

Mean film thickness = 0.0319 inches

Mean wave frequency = 14.9 sec<sup>-1</sup>

Y_T = 0.0155 inches

Time length of data sample = 58 sec

Micrometer space above the glass plate =

Visicorder chart speed = 2 inches/sec with 0.1 sec time

marking
```

Film Thickness, inches	No. of Waves < to Film Thickness	PA	Wave Freq., l/sec	No. of Time Increments < to Wave Freq.	Pw
0.0525	932	1.000	21	58	1.000
0.0480 0.0443 0.0423 0.0402 0.0382 0.0365 0.0345 0.0345 0.0320 0.0284 0.0250 0.0225 0.0110	929 902 901 871 830 771 706 612 470 343 88 39 0	0.9900 0.9678 0.9345 0.8906 0.8273 0.7575 0.6566 0.5043 0.3680 0.0944 0.0418 0	20 19 18 17 16 15 14 13 12 11 10 9	57 54 43 35 31 20 11 7 5 2 0	0.9828 0.9310 0.8275 0.7414 0.6034 0.5344 0.3448 0.1896 0.1207 0.0862 0.0345 0

APPENDIX D

EDDY DIFFUSIVITY CALCULATED FROM MEASUREMENT OF SLOPES

Cell angle of inclination = $9^{\circ}44'$

Data - Appendix A

Concentration profiles corrected to same film thickness

 $d\Delta N/dZ$ = measured from least square fit of experimental data by a straight line

 $d^{2}\Delta N/dY^{2}$ = measured from least square fit of experimental data by a 4th order polynomial

M = 118.9

Eddy diffusivity was calculated as

$$(D + \epsilon)_{Y} = \frac{\rho (Y_{MF})^{2} g \sin \theta}{2 \mu M^{2}} \left[1 - \left(\frac{Y}{Y_{MF}}\right)^{2}\right] \frac{\left(\frac{dC}{dZ}\right)_{Y}}{M^{2} \left[\frac{d^{2}C}{dY^{2}}\right]}$$

where Y = 0 at the interface

Re = 732

Run	Y	d∆n dZ	$\frac{d^2 \Delta N}{d(Y)^2}$	$M^2 \frac{d^2 \Delta N}{d(Y)^2} \times 10^{-4}$	V cm/sec	$(D+\varepsilon) \times 10^4$ cm^2/sec
18- 6-7 "	4.53 4.0 3.0 2.0 1.0	0.00867 0.00859 0.00596 0.00188 0.00201	0.21000 0.16503 0.10404 0.06464 0.04117	0.2969 0.2333 0.1471 0.0914 0.0582	48.6 44.0 35.0 24.6 12.8	1.419 1.620 1.418 0.506 0.442
18-8-8 " " "	4.53 4.0 3.0 2.0 1.0	0.00867 0.00859 0.00596 0.00188 0.00201	0.1310 0.13000 0.10874 0.07227 0.02570	0.18519 0.18378 0.15373 0.10217 0.03633	48.6 44.0 35.0 24.6 12.8	2.275 2.057 1.357 0.453 0.708
18-10-6 " "	4.53 4.0 3.0 2.0 1.0	0.008 67 0.00859 0.0059 6 0.00188 0.00201	0.2575 0.1910 0.0950 0.0370 0.0112	0.3640 0.2700 0.1343 0.0523 0.0158	48.6 44.0 35.0 24.6 12.8	1.158 1.400 0.686 0.884 1.628

Average

1.237

Re	-	ا ا	n	a	a
TIC .		`	\sim	7	7

Run	Y	$\frac{\mathrm{d}\Delta \mathrm{N}}{\mathrm{d}Z}$	$\frac{d^2 \Delta N}{d(Y)^2}$	$M^2 \frac{d^2 \Delta II}{d(Y)^2} \times 10^{-4}$	V cm/sec	(D+c) x 10 cm ² /sec
18-2-3	5,35	0.00623	0.1400	0.1979	51.8	1.820
	5.0	0.00618	0.12040	0.1702	55.0	1.997
17	4.0	0.00440	0.07481	0.1058	46.4	1.292
f1	3.0	0.00329	0.0430	0.0608	36.6	1.980
11	2.0	0.00258	0.02600	0.03 6 8	25.4	1.781
11	1.0	0.00296	0.0150	0.0212	13.0	0.815
18-4-5	5.35	0.00623	0.2025	0.2863	57.8	1,258
11	5.0	0.00618	0.17444	0.2466	55.0	1.378
ti	4.0	0.00440	0.10222	0.1445	46.4	1.413
tt	3.0	0.00329	0.04701	0.0665	36.6	1.811
11	2.0	0.00258	0.01201	0.0170	25.4	3.855
11	1.0	0.00296	0.0030	0.0042	13.0	9 .16 2
18-6-2	5.35	0.00623	0.1615	0.2283	57.8	1.577
11	5.0	0.00 6 18	0.1500	0.2121	55.0	1.603
	4.0	0.00440	0.1025	0.1449	46.4	1.409
	3.0	0.00329	0.0500	0.0707	36.6	1.703
	2.0	0.00258	0.0115	0.0163	25.4	4.020
11	1.0	0.00296	0.0040	0.0057	13.0	6.751
18-8-2	5.35	0.00623	0.1775	0.2509	57.8	1.435
**	5.0	0.00618	0.1550	0.2191	55.0	1.551
	4.0	0.00440	0.0970	0.1371	46.4	1.489
	3.0	0.00329	0.0500	0.0707	30.0	1.703
11	2.0	0.00258	0.0200	0.0283	25.4	2.310
	1.0	0.00296	0.00650	0.0092	13.0	4.183
18-10-5	5.35	0.00623	0.1050	0.2015	57.0	1.3//
TI CONTRACTOR	5.0	0.00010	0.1425	0.2015	つつ・U りん り	
ti	4.0	0.00440	0.0000	0.0040	40.4	2.401
11	3.0	0.00329	0.0230		50.0 25 月	5・105
(1	1.0	0.00290	0.0100	0.007	13.0	5,407
Average	<u> </u>	0.00290			<u></u>	2,643

。 我想到了一个人,

Re	Ħ	1482

Run	Y	$\frac{d\Delta N}{dZ}$	$\frac{d^2 \Delta N}{d(Y)^2}$	$M^2 \frac{d^2 \Delta N}{d(Y)^2} \times 10^{-4}$	V cm/sec	$(D + \epsilon) \times 10^4$ cm ² /sec
18-1-8	5.9 5.0 4.0 3.0 2.0 1.0	0.00662 0.00377 0.00247 0.00286 0.00303 0.00844	0.1610 0.09870 0.05047 0.02214 0.00902 0.00300	0.1378 0.0907 0.0502 0.0225 0.0099 0.0042	64.0 57.0 48.2 38.0 26.2 13.4	0.8615 1.5404 1.6697 3.4722 6.2023 26.9286*
18-7-6 " " "	5.9 5.0 4.0 3.0 2.0 1.0	0.00662 0.00377 0.00247 0.00286 0.00303 0.00844	0.0975 0.06419 0.03548 0.01591 0.00200 0.00300	0.1378 0.0907 0.0502 0.0225 0.0099 0.0042	64.0 57.0 48.2 38.0 26.2 13.4	3.075 2.369 2.372 4.830 8.020 26.929*

.

Average

3.541

*Not included in the average

Re	=	1834
ne	.	TO24

Run	Y	$\frac{d\Delta N}{dZ}$	$\frac{\mathrm{d}^{2}\Delta\mathrm{N}}{\mathrm{d}(\mathrm{Y})^{2}}$	$M^2 \frac{d^2 \Delta N}{d(Y)^2} \times 10^{-4}$	V cm/sec	$(D+\epsilon) \times 10^4$ cm ² /sec
18-1-3 " " " "	6.85 6.00 5.0 4.0 3.0 2.0 1.0	0.00528 0.00392 0.00250 0.00278 0.00327 0.00384 0.00433	0.1325 0.08955 0.05102 0.02495 0.00926 0.00186 0.00065	0.1873 0.1266 0.0721 0.0353 0.0131 0.0026 0.0009	74.2 68.2 59.8 50.0 39.4 27.2 13.8	2.092 2.112 2.074 3.938 9.835 40.173* 66.389*
18-7-5 " " "	6.85 6.00 5.00 4.0 3.0 2.0 1.0	0.00528 0.00392 0.00250 0.00278 0.00327 0.00384 0.00433	0.1375 0.07777 0.03786 0.01518 0.00503 0.00272 0.00100	0.1802 0.1100 0.0535 0.0215 0.0071 0.0038 0.0014	74.2 68.2 59.8 50.0 39.4 27.2 13.8	2.174 2.430 2.795 6.146 18.146 27.487* 42.678*

Average

5.206

*Not included in the average

APPENDIX E

EDDY DIFFUSIVITIES CALCULATED FROM MATERIAL BALANCE

Cell angle of inclination = $9^{\circ}44^{\circ}$

Data - Appendix A

M = 118.9

Experimental data at Z = 57.15 cm used as reference point Eddy diffusivity was calculated as

$$(D + \epsilon)_{Y} = \frac{\rho (Y_{MF})^{2} g \sin \theta}{2 \mu Z' M^{2}} \left(\frac{dY}{dC} \right)_{Z_{AVe}} \left[\int_{0}^{Y} C(Y) \left\{ 1 - \left(\frac{Y_{MF} - Y}{Y_{MF}} \right)^{2} \right\} dY \Big|_{Z_{2}} - \int_{0}^{Y} C(Y) \left\{ 1 - \left(\frac{Y_{MF} - Y}{Y_{MF}} \right)^{2} \right\} dY \Big|_{Z_{1}}$$

where Y = 0 at the surface of the contacting plate

$$Z' = Z_2 - Z_1$$

$$Z'_{Ave} = (Z_2 + Z_1)/2$$

$$Z \ge 57.15$$

$$C(Y) = \text{the experimental concentration profile}$$

In the event more than one concentration profile exists at a given cell position values of the integral and derivatives for each curve are averaged together.

Re = 732	$Y_{\rm T}/M = .0424$ cm	$Y_{MF}/M = .092 \text{ cm}$	$V_{sMF} = 73.4 \text{ cm/sec}$	$\omega = 5.95 \text{ sec}^{-1}$
		Z cm		

	82.55		76.2		69.85 63.5			5	57.15		
Y/M cm	$\int dy \times 10^5$	dy dc	∫dy x 10 ⁵	dy dc	∫ay x 10 ⁵	dy dc	∫ċyx 105	dy dc	∫dy x 10 ⁵	dy dc	
.0042 .0084 .0168 .0252 .0294 .0336 .0378	4.0 13.75 54.0 148.0 337.7	3.41 .71 .12 .05 .04	3.90 12.50 52.2 148.0 231.0	2.91 .58 .11 .05 .04	3.67 12.5 51.7 128.0 203.5 268.0 375.0	2.74 .85 .19 .086 .053 .044 .039	2.87 11.3 16.5 111.8 160.3 243.5 345.0	3.74 1.16 .31 .10 .054 .047	2.75 10.75 49.7 110.5 223.0	1.56 .66 .27 .14 .069	

 $D \text{ cm}^2/\text{sec} \times 10^5$

Y/M	cm	

Z ₂ - Z ₁	.0042	.0084	.0168	.0252	.0294	.0336	.0378
82.55 - 76.2 76.2 - 69.85 69.85 - 63.50 63.50 - 57.15 82.55 - 69.85 76.20 - 63.5 69.85 - 57.15 82.55 - 63.50 76.20 - 57.15 82.55 - 57.15	3.0 7.5 3.0 3.7 5.8 19.8 11.4 15.6 9.9 9.0	9.3 9.9 5.8 5.6 6.0 7.6 8.9 4.2 5.9	2.4 .86 15.0 2.1 6.9 2.6 6.3 1.8 2.4	15.6 17.6 1.8 8.2 15.7 11.3 10.9 13.4 10.4	14.3 28.8 20.4	14.5 6.7 16.5 11.4 16.7 15.9	14.8

Re = 1099 Y_T/M = .0521 cm Y_{MF}/M = .097 cm V_{sMF} = 80.9 cm/sec w = 8.3 sec⁻¹ Z cm

	82.55		76.2		69.	69.85		5	57.15	
Y/M cm	\dy x 10 ⁵	$\frac{dy}{dc}$	∫dy x 10 ⁵	dy dc	Sdy x 105	dy dc	Say x 105	dy dc	$\int dy \ge 10^5$	dy dc
.0042 .0084 .0168 .0210	3.8 13.0 46.5	1.34 .92 .46	3.45 11.5 43.7	1.55 1.34 .78	2.83 11.07 47.7 81.6	3.55 2.41 .95	2.23 8.66 33.3	3.33 2.89 1.42	1.90 8.8 36. 0	3.52 2.00 .69
.0252	111.0 1 6 5.0	.18 .076	101.0	.29	91.0 149.0	.32	77.0	.47	83.0	•3 ⁴
.0336 .0378	215.0	.09	190.0 275.0	.11 .06	182.8 230.0	.10	142.0	.17	157.0 210.0	.15 .14
.0421 .04 6 2	351.0	.07	325.9	.06			251.0 334.8	.08 .06	284.0	.06

 $D \text{ cm}^2/\text{sec} \times 10^5$

Y/M cm

Z ₂ - Z ₁	.0042	.0084	.0168	.0252	.0294	.0336	.0378	.0421
82.55 - 76.2 76.2 - 69.85 69.85 - 53.50 63.50 - 57.15	6.43 20.1 26.3	21.6 10.3 813	22.1 21 6 .2	30.1 38.8 69.7		31.0 9.6 70.3	35.0	21.4
82.55 - 69.85 76.2 - 63.5 69.85 - 57.15 82.55 - 63.50 76.20 - 57.15 82.55 - 57.15 82.55 - 57.15	15.1 18.9 20.9 15.5 16.6 14.7	20.4 38.2 31.9 35.0 19.1 19.5	72.5 60.8 52.6 23.9 19.2	31.8 58.0 16.8 46.7 24.2 23.4	13.5	19.6 41.7 21.0 39.5 18.1 22.2	13.2 27.5	33.4 31.4 10.8 12.9

Re =]	1482 Y ₁	M = .0	59 6 cm	Y _{MF}	/M = .1	.00 cm	V _{sMF}	= 84.2 c	em/sec	ω = 13	.1 sec ⁻¹
						Z cm					
	82	2.55		76.2		69.8	35	63.	50	5	7.15
Y/M cm	∫dy x 10 ⁵	; <u>dy</u> dc	∫dy x	$10^5 \frac{d}{d}$	<u>y</u> Sdy	x 10 ⁵	$\frac{dy}{dc}$	$\int dy x \ 10^5$	dy dc	$\int dy x 10$	5 <u>dy</u> dc
.0042 .0084 .0168 .0252 .0336 .0378 .0421 .0462 .0505 .0547	3.33 10.67 40.3 91.3 163.0 275.0 353.0	2.62 1.67 1.19 .65 .26 .11 .079	3. 11. 42. 93. 174. 245. 288.	4 3. 25 2. 4 1. 5 . 0 .	75 83 1 36 3 45 8 20 15 085 068 25	2.85 .0.0 39.5 35.5 53.0	4.50 3.03 2.22 .82 .22 .085	2.33 10.2 35.2 75.0 133.0 181.0 229.0 283.0	3.52 2.18 1.26 .84 .37 .12 .13 .10	1.86 7.66 30.8 70.0 128.0 215.0 310.0	1.99 1.09 .68 53.0 .37 .23 .18
					D cm ² / Y	'sec x : 7/M cm	10 ⁵				
Z ₂	- Z ₁	.0042	.0084	.0168	.0252	.0336	.0378	.0421	.0462	.0505	.0547
76.2 69.85 63.50 82.55 76.2 69.85 82.55 76.2 82.55 76.2 82.55	- 69.85 - 63.50 - 57.15 - 69.85 - 63.5 - 57.15 - 63.50 - 57.15 - 57.15 - 57.15	30.4 27.6 17.1 10.6 25.7 21.3 13.6 19.5 11.2	48.5 55.0 10.4 17.4 32.0 40.0 31.1 13.8	68.9 99.4 56.8 9.1 62.6 83.8 29.5 52.4 29.5	51.5 115.0 28.3 78.7 69.1 53.6 50.6 41.6	58.7 78.3 24.7 15.8 78.2 48.8 41.9 58.5 3 6.6	30.3 42.9	41.3 33.4 11.1 38.8 48.8 24.8 48.0 34.1	28.0		

Re = 1834	$Y_{\rm T}/M = .0637 ~{\rm cm}$	Y _{MF} /M = .1038 cm	V _{sMF} = 91.8 cm/sec	$w = 15.8 \text{ sec}^{-1}$
		Z cm		

	82.	55	76.	20	69.	85	63.	50	57.	15
Y/M cm	$\int dy \ x \ 10^5$	dy dc	$\int dy \ge 10^5$	dy dc	$\int dy x 10^5$	dy dc	$\int dy \ x \ 10^5$	dy dc	$\int dy \ x \ 10^5$	dy dc
.0042 .0084 .0168 .0252 .0336 .0421 .0462 .0505 .0547 .0589 .0631	1.93 7.067 32.0 72.3 125.7 204.0 289.0 300.0 384.5	6.11 5.04 2.94 1.35 .46 .19 .077 .11 .085	1.90 7.0 29.0 68.00 118.5 199.0 310.0 391.0	6.11 5.0 3.67 1.88 .81 .22 .096 .070	1.725 6.75 27.0 61.7 108 184.0 249.0 272.0 388.0	5.56 4.58 2.33 1.03 .43 .20 .10 .15 .10	$1.10 \\ 5.0 \\ 22.7 \\ 54.6 \\ 99.6 \\ 162.0 \\ 224.0 \\ 237.0 \\ 304.0 \\ $	3.24 2.17 1.44 1.03 .64 .27 .13 .14 .10	.67 3.2 16.3 38.2 76.0 134.0 218.0 282.0 329.0 376.0	2.20 1.44 1.10 .77 .55 .36 .20 .11 .12 .091
					<u>^</u>	~				

 $D \text{ cm}^2/\text{sec} \times 10^5$

Y/M	cm
7 1.1	OIII

Z ₂ - Z ₁	.0042	.0084	.0168	.0252	.0336	.0421	.0462	.0505	.0547	.0589
82.55 - 76.20 76.20 - 69.85 69.85 - 63.5 63.5 - 57.15 82.55 - 69.85 76.20 - 63.15 69.85 - 57.15 82.55 - 63.50 76.20 - 57.15 82.55 - 57.15	2.7 14.7 39.7 17.1 8.7 27.0 29.6 18.7 24.9 18.9	48.5 17.3 85.2 47.0 11.0 51.9 75.3 36.0 58.8 45.4	143 86.6 117.2 117.5 91.8 116.2 132.5 98.0 146.0 132.0	100 130.0 105.8 214.0 91.0 141.0 153.2 99.5 190.5 130.2	65.6 94.0 64.8 201.5 56.6 98.5 113.0 68.6 167.0 89.7	15.0 46.6 76.3 128.0 28.6 66.9 101.5 47.2 91.0 69.0	42.2 25.8 32.6	67.0 72.5 46.3 26.6 61.6 67.3 38.4 65.5 46.4	34.5 60.7 36.8 48.2 36.3	46,6

APPENDIX F

SOLUTION TO THE PIGFORD-JOHNSTONE EQUATION

Solution to the mass transfer equation

$$V_{(Y)} \frac{dC}{dZ} = D \frac{d^2C}{dY^2}$$
(22)

where

$$V_{(Y)} = \frac{\rho(Y_L)^2 g \sin \theta}{2 \mu M^2} [1 - (Y/Y_L)^2]$$
(19)

and Y = 0 at the interface

with the boundary conditions

(1) $C = C_s$ at Y = 0 for all Z

(2) $C = C_0$ at $Y \ge 0$ for Z = 0

(3) (dC/dY) = 0 at $Y = Y_{L}$ for all Z

has been solved by Pigford and Johnstone (27) in terms of the average film concentration \overline{C} and the parameter $\frac{DZM^2}{V_L(Y_L)^2} = K$

The solution given is

for large values of Z

Figure 39 is a plot of $\overline{C}/C_{\rm S}$ as a function of K.



APPENDIX G

SAMPLE CALCULATIONS

 Carbon dioxide concentration in the film without correlation

Run 18-6-7

$$X_{FS} = 3.1702 \text{ cm}$$

Re = 732
 $\overline{C}/C_s = 0.1595$
N = 118.9
Y = 4.5351
 $X_{CO_2} = 3.7717$
 $X_{A1r} = 0.304$
 $\Delta N_s = 2.6726$
 $\Delta N = (3.7717 - 0.304)/3.1702 = 1.095$
 $\frac{C}{C_s} = \frac{\Delta N}{\Delta N_s} = \frac{1.095}{2.6726} = .411$
2. Wave model - material balance
Re = 732
Q = 13.2 cm³/sec
Y_{MF} = 10.93
W = 7.5693 cm
Y_T = 5.50
M = 118.9
W = 6 sec⁻¹
MQ/W = 207.5

$$V = 73.44^{+1} + \left(\frac{Y_{\rm M}^{-1} + 1}{Y_{\rm M}^{-1}}\right)$$

$$b = .1021 - .00034Y = 5.05 \le Y \le 10.03$$

$$b = .0585 = Y \le 5.05$$

$$5.95 = 10.03$$

$$E^{2} + \frac{10}{120} = \frac{Y_{\rm A} Y_{\rm A}}{5.05} + \frac{10.23}{5.05}$$

$$10.7.3 = (2.0)(12.8) + (4.0 - 0.0)(35.0) + (5.05 - 4.0)(48) + 5[(7.0 - 5.05)(59.6)(.004)] + (10.93 - 4.0)(38)(.0775) + (10.93 - 4.0)(38)(.0075)(.004) + (10.93 - 4.0)(38)(.0775)(.004) + (10.93 - 4.0)(38)(.0075)(.004) + (10.93 - 4.0)(38)(.0075)(.004) + (10.93 - 4.0)(38)(.0075)(.004) + (10.93 - 4.0)(38)(.0075)(.004) + (10.93 - 4.0)(38)(.0075)(.004) + (10.93 - 4.0)(38)(.0075)(.004) + (10.93 - 4.0)(38)(.0075)(.004) + (10.93 - 4.0)(38)(.0075)(.004) + (10.93 - 4.0)(38)(.0075)(.004) + (10.93 - 4.0)(38)(.0075)(.004) + (10.98)(.005$$

0)(72.4)(.00)1)]

207.24 = 207.5

3. Average fringe snift from insusterometric data

$$\begin{array}{rl} \mathrm{Re} = 732 \\ \mathrm{Run} \ 10^{-3-7} & = \frac{1}{120} & \mathrm{V_{1}} \mathrm{A} \mathrm{V_{1}} \mathrm{A} \mathrm{N_{1}} + 2 & \mathrm{V_{1}} \mathrm{A} \mathrm{V_{1}} \mathrm{A} \mathrm{N_{1}} \mathrm{N_{1}} \\ \mathrm{AN} = & \frac{1}{120} & \mathrm{V_{1}} \mathrm{A} \mathrm{V_{1}} \mathrm{A} \mathrm{N_{1}} + 2 & \mathrm{V_{1}} \mathrm{A} \mathrm{V_{1}} \mathrm{A} \mathrm{N_{1}} \mathrm{N_{1}} \\ \mathrm{N} = 1 - 2 & \mathrm{N} = .37 \ \mathrm{at} \ \mathrm{Y} = \mathrm{X_{T}} \\ \mathrm{AN} = (.377 \ (2.0 - 0) \ (12.5) \ (.52) + (4.0 - 2.0) \ (35.2) \ (.655) + (5.05 - 4.0) \ (.025) \ (48) \] + \\ \mathrm{SI} = (2.0 - 0) \ (12.8) \ (.0555) \ (.51) + \\ \mathrm{SI} = (2.0 - 2.0) \ (35.2) \ (.0555) \ (.52) + \\ \mathrm{SI} = (5.05 - 4.0) \ (48) \ (.0555) \ (.555) + \\ \mathrm{SI} = (7.0 - 5.05) \ (57.6) \ (.044) \ (.63) + \\ \mathrm{SI} = (9.0 - 7.0) \ (68) \ (.0275) \ (.74) + \\ \mathrm{SI} = 139.55/207.24 = .673 \end{array}$$

$$\Delta \overline{N} / \Delta N_s = .673 / 2.6726 = .252$$

4. Absolute concentration from interferometric data

Re = 732
Run 18-6-7
Pressure correction =
$$\Delta \overline{N} / \Delta N_s - \overline{C} / C_s$$

C/C_s = $\Delta N / \Delta N_s - (\Delta \overline{N} / \Delta N_s - \overline{C} / C_s)$
= .411 - (.252 - .159) = .358

5. Surface area from wave data

$$\frac{A}{A_{L}} = \sum_{i=1}^{N} \frac{(Y_{2} - Y_{1})_{i} + \Delta X_{i}}{\Delta X_{i}}$$

A_w area with ripples A_L area without ripples Run 48 Re = 732 $\theta = 9^{\circ}44!$ $\frac{V_L}{V_{1s. chart speed}} = \frac{39.3}{25.4} = 1.545$

- $A_{W}/A_{L} = 2.54 \text{ (cm/in.)}[.0225 + .0050 + .0045 + .01354 + .0095 ...] + 1.545[1.77 + .10 + .5 + 1.3 + 2.65 + ...]/1.545[1.77 + .10 + .5 + 1.3 + 2.68 + ...] = 1.015$
- 6. Eddy diffusivity by measurement of slopes

$$D + \epsilon = \frac{V \frac{d\Delta N}{dZ}}{M^2 \frac{d^2 \Delta N}{dy^2}}$$

Run 18-8-8

$$\theta = 9^{\circ} 44'$$
Re = 732 M = 118.9
Y = 4.53 cm
 $d(\Delta N)/dZ = 0.00867$ $V_{Y} = 73.44[1 - (\frac{10.93 - Y}{10.93})^{2}]$
 $d^{2}(\Delta N)/d(Y)^{2} = 0.1310$
M² $d^{2}(\Delta N)/d(Y)^{2} = .185 \times 10^{4}$ $V_{4.53} = 48.6$ cm/sec
M² $d^{2}(\Delta N)/d(Y)^{2} = .185 \times 10^{4}$ $V_{4.53} = 48.6$ cm/sec
(D + ε) = (48.6 x .00867)/(.185 x 10⁴) = 2.27 x 10⁻⁴ cm²/sec

7. Eddy diffusivity by fitting of experimental concentration profiles Re = 732 K = $\frac{(D - \epsilon)ZM^2}{V_s(Y)^2}$

Reference curve Run 18-10-6 at Z = 57.15 cm Profile fitted with parameter K = .056932, $Y_M = 7.6$ Run 18-4-7 Z = 76.2 $Y_F = 3.775$ $Y_{MF} = 10.93$ Profile fitted with parameter K = .060802 $Y_M = 6.6$ Z' = 76.2 - 57.15 = 19.05 cm K' = .060802 - .056932 = .003870 M = 118.9 V at 3.775 = 73.44 $\left[1 - \left(\frac{10.93 - 3.775}{10.93}\right)^2\right] = 42$ cm/sec B = $Y_F/Y_M = 3.775/6.6 = .571$ $f_2(B) = .654$ $V_S = 42/.654 = 64$ cm/sec D + $\varepsilon = \frac{K' V_S (Y)^2}{Z' M^2} = \frac{.003870 \times 64 \times 43.5}{19.05 \times 1.41 \times 10^4} = 4.02 \times 10^{-5}$ cm²/sec

8. Eddy diffusivity by material balance

$$(D + \varepsilon)_{Y} = -\frac{V_{S}}{Z_{2} - Z_{1}} \left(\frac{dY}{dC}\right)_{Z_{1}+Z_{2}} \left[\int_{0}^{Y} \left\{ 1 - \left(\frac{Y_{MF} - Y}{Y_{MF}}\right)^{2} \right\} c_{(Y)} dY |_{Z_{2}} \right]$$
$$- \int_{0}^{Y} \left\{ 1 - \left(\frac{Y_{MF} - Y}{Y_{MF}}\right)^{2} \right\} c_{(Y)} dY |_{Z_{1}} \right]$$
$$V_{S} = \frac{\rho \left(Y_{MF}\right)^{2} g \sin \theta}{2 \mu M^{2}}$$

Re = 732θ M = 1 Y_{MT} $V_{s} = 73.4$

Z, cm

Y/M, cm

C, g/liter

= 9°44'
1 = 118.9
$X_{\rm MF} = 10.93$
т — 73 Ц

Run 18 - 10 -6	Run 18 -6 -7
57.15	69.85
.0168	.0168
.169	.180

$$1 - \left(\frac{10.93 - Y}{10.93}\right)^2$$
.333..333

$$c \left[1 - \left(\frac{10.93 - Y}{10.93}\right)^{2}\right] .0562 .0600$$

dc/dy -3.40 -5.70

dc/dy

$$\int_{0}^{Y} \left[1 - \left(\frac{10.93 - Y}{10.93} \right)^{2} \right] dy \qquad .000460 \qquad .000510$$

$$(D + \epsilon)_y = + \frac{73.4}{12.70} \frac{2}{3.40 + 5.70} (.000510 - .000460)$$

= 1.27 x .000050 = 6.35 x 10⁻⁵ cm²/sec