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SYNTHESIS OF INVARIANCE PRINCIPLE CONTROL

SYSTEMS FOR CHEMICAL PROCESSES

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ABSTRACT

The invariance principle for designing control systems has been developed in Russia during the past twenty-five years. Most investigators have been concerned with the mathematical aspects of the theory. Little regard has been given to the problems of practical applications and no experimental data have been published for the purpose of delineating the problems of applying the theory to actual chemical processes.

In this work the characteristics of chemical processes are discussed in relation to the present state of process control design techniques. Invariance theory is shown to be particularly suited to the synthesis of control systems for chemical processes.

The theory is presented first in a tutorial manner and examples are taken from the analysis of the experimental system in the laboratory. The theory is presented then in general notation which is applicable to most chemical processes. The two conditions for the mathematical attainment of invariance are discussed: the invariance condition and the dual channel condition. The satisfaction of these conditions is discussed for linear and nonlinear systems. The dual channel condition is shown to be verified by any of three methods--physical, mathematical, or topological. The use of linear operators and matrices for linear systems and time-domain analysis for nonlinear systems is presented. The choice of model types is discussed in relation to the ultimate use of the model.

Analog computer simulation studies demonstrate the theoretical application of the theory to a well-behaved, accurately modeled system and illustrate the use of analog computers for the study of invariance control systems. The laboratory process and the controllers which are designed using both linearized and nonlinear models are analog simulated. The efficiency of a controller is defined and the linear and nonlinear controller efficiencies are compared for twelve configurations.

The most important part of the study is the application of invariance theory to an actual chemical process which is a completely mixed, stirred tank reactor. The process is coolant flow-forced and controlled by the reactant flow rate. The control of any one of three output temperature variables by feedback control on either of the other two outputs or by feedforward control on the disturbance is discussed and demonstrated. Electronic controllers are assembled from analog computing elements according to the invariance theory design specifications for twelve configurations.

The results of experimental data and simulation studies indicate the feasibility of practical application of invariance theory for the design of high quality control systems and suggest areas for future investigations for improving the synthesis procedure. Recommendations for synthesis procedures using this theory are given.

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David Emerson Haskins

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SYNTHESIS OF INVARIANCE PRINCIPLE CONTROL SYSTEMS FOR CHEMICAL PROCESSES

CHAPTER I

INTRODUCTION

Although systems engineering as a separate discipline has much to offer the longer established engineering fields, the chemical process industries have yet to realize its full potential. Most large chemical companies have some group which functions in systems engineering of the type discussed by Wherry (88) or Williams (91), but the levels of effort as measured in terms of investments and operating budgets vary considerably among them. This difference can be attributed to the difficulty of establishing a firm dollar return on investment; consequently expenditures for systems engineering research and development are subjected to continuing review and evaluation by management.

Ledgerwood (49) has addressed this problem of proving the worth of control systems. It appears that the economics of utilizing control concepts in the chemical industries are rarely publicized by leaders in the field, undoubtedly because of the highly competitive nature of the business. This secrecy is largely responsible for the general lack of incentive in the chemical industry as a whole to make the necessary investments to exploit the full economic potential in process control.

Another related problem is the scarcity of engineers with the formal interdisciplinary training required in systems engineering as applied to chemical processing. Engineers in the chemical industry have relied heavily on theories which have been developed either generally by mathematicians or specifically by electrical, aero-space, or servosystem engineers. While some of the advanced theories and their subsequent applications have made significant contributions to problems in other disciplines, these same theories have not always proved to be applicable to chemical processes since the problems are somewhat different. Consequently the more recent trends in the chemical industry have been to define the characteristics of chemical processes in order to develop theories which are more applicable to them. It is certain that chemical systems engineers can no longer rely solely on theories and techniques developed in other disciplines.

Across various fields the theories of dynamic analysis, control, and optimization are quite advanced compared to their practical applications. When attempts are made to put some of the more advanced theories into practice to solve specific problems, the solutions appear to be either physically or economically unrealizable. For this reason, systems engineering is frequently subject to criticism. Chestnut (18) speaks of the control gap between theory and practice as a vector quantity because progress in the two is not only different in magnitude, but also it appears to be pulling in different directions. The academically oriented theoreticians are interested in understanding the mathematics of control with little concern for eventual applications. On the other hand, economically motivated practitioners are designing complex control systems with little concern for the theoretical foundations.

There are a number of unique, dynamic characteristics about chemical processes which identify them from other applications for control theory. Calvert and Coulman (16) have discussed the major differences between chemical and other process systems. Their objective was to illustrate the advantages of considering a control system design philosophy more general than feedback--specifically, feedforward control.

The important characteristics are that chemical processes are (a) multivariable, (b) optimizable on a steady-state basis, (c) nonlinear, (d) describable by mathematical models, (e) subject to disturbances of known origin, (f) characterized by large response times, and (g) often describable by distributed-parameter models.

Each of these points will be discussed to indicate where chemical process systems engineering needs to depart from the approaches developed in other disciplines.

a) Multivariable systems theory has recently become a very popular topic for investigation as evidenced by the work of Hammond (36) which covers 206 references. Within the theory of multivariable systems are various methods for attacking problems which may arise: non-interaction, minimizing the effect of parameter variation, dynamic optimization, and minimizing the output error.

The terms interaction and coupling in multivariable systems refer specifically to a phenomenon which may occur in systems which have more than one controller. Usually each variable which requires controlling will need one controller. It is quite possible that corrections for errors in one variable will have effects on other system variables, requiring further control effort. The result is a total system which spends considerable control effort correcting its own disturbances and, in extreme cases, a poorly controlled or unstable system. It is possible to synthesize additional controllers for these interacting systems which will make each controlled variable subject to only its own controller's actions, not to the other controllers' actions. Such a system is then called "noninteracting" or often "decoupled." In servosystems, which are mechanical systems, these interactions can be serious and the added complexity and expense of noninteracting controllers is sometimes warranted. For example, for turbojet engine control it is desirable to control aircraft speed and tail-pipe temperature independently by manipulating fuel flow, exhaust cross-section area, and propeller pitch. All three inputs can change the two outputs with the result that control is difficult. Noninteracting control systems have been built which allow the aircraft speed to be controlled without affecting the tail-pipe temperature and vice versa.

Chemical processes, which are complex multivariable systems, have been treated in the past as collections of single variable processes. Each variable requiring control has been treated as a separate system and the necessary controller attached. Not much has been mentioned in the literature about interactions of these separate univariable control systems because the interactions are often weak in chemical processes and probably unrecognized. While it is possible to have interaction in chemical processes--and this fact should not be overlooked in system design--it is not a difficult problem. However, in the more general sense, the kind of non-interaction which is needed in chemical systems is the decoupling of certain output variables from disturbance variables.

While a few chemical processes may be described by linear mathematical models which have parameters that are functions of the independent variable time, it is usually preferable to use nonlinear models. If these time-varying parameters are important, they should be treated as separate variables or related to some other dependent variable. If it were possible to find a parameter which is purely a function of time, there would be two alternatives. Either the time variation is so slow that in the process dynamic study it can be considered constant, or the variation is so fast compared to the process dynamics that it needs to be considered. For the latter case, if the variation is periodic, then some minimization of its effect may be realized, but if it is not periodic, then its specific functional form must be known for all time even to get a model for the system. Although there are techniques available for design of control systems for time-varying processes, examples in the chemical industry are hard to find. Thus, minimizing the effects of parameter variations which are not output variables is not considered to be important here.

b) There are excellent methods for treating dynamic optimization problems: variational calculus, Bellman's dynamic programming, and Pontrjagin's maximum principle. These techniques have proved to be useful for optimizing missile and space-capsule trajectories. However, these methods become difficult when systems are subject to external disturbances. It is still being debated whether or not dynamic optimization by on-line computers is economically feasible for chemical processes. As Ledgerwood (49) has pointed out, there are not much economic data available to support either side of the debate. If the complete system dynamics were accurately known and if enough measuring and controlling devices were available, the plant could be controlled to operate at conditions that are dynamically optimum. An on-line hybrid analog-digital computer could continuously sample process conditions, make dynamic optimization calculations, and force the process to carry out the optimum strategy. However, the incremental gains from installing computer control may be small in proportion to the expenditures. A more economical--though less glamorous--method is to optimize a plant periodically with a general purpose off-line computer and to use less expensive control schemes to keep the process variables at the calculated optimum conditions. This off-line optimization can be either static or dynamic, depending on plant economic criteria.

The foregoing arguments lead to the conclusion that the primary problem in multivariable chemical processes is the minimization of the errors in certain output variables. This assumption predicates the research in invariance principle control systems for chemical processes.

c) Above all other types of systems, the nonlinear nature of chemical processes must be considered. Not only does the process itself usually contain nonlinearities, such as the product and Arrhenius (exponential) types, but the control systems superimposed add nonlinear effects, such as hysteresis, backlash, and saturation. Models, which describe the process dynamic behavior, may be linear or nonlinear--the choice being dictated by the model's ultimate use. If the assumed control quality criterion is satisfied by a controller synthesized by using a linear model and one of many linear system techniques, there is no need for a more accurate model. On the other hand, if better control is required, it may be necessary to consider the more accurate nonlinear model. A general process control theory applicable to chemical systems should provide for treatment of nonlinear systems when it is necessary.

d) It should be recognized that some kind of model is generally available for the system being studied. However, a chemical process cannot be simply regarded as a "black box" with inputs and outputs, but with no knowledge of internal structure. As more system types are investigated and described, and better methods of system identification become available, the importance of internal structure becomes more evident.

e) Included in the process description (or model) is some variable (or variables) which constitutes the disturbance to the process. Usually the source of this disturbance is known; therefore it should be possible to use this information to minimize the output errors of chemical systems.

f) Chemical systems normally are characterized by large response times as opposed to electrical systems, for example. In a process which is reasonably described by response times of minutes and hours, it is important to consider disturbance compensation in addition to feedback control. In systems with time lags of the order of seconds or fractions of seconds, common feedback control methods are sufficient. MacMullan and Shinskey (57) have reported a system which required feedforward control to overcome excessive process time lags when feedback control was unsatisfactory. The superfractionator with which they were concerned is perhaps an extreme example, but is is a good indication of the problems which may be encountered in chemical processes.

g) Many chemical processes demand description by models which are dependent on both time and space coordinates. These models consist of partial differential equations which usually must be simplified before they are useful for design methods. These "distributed-parameter" systems constitute a very difficult segment of control theory which needs more attention. A completely general chemical process control theory should provide for such systems, though they will not be treated in this work. The reason for this omission is the need for fundamental work in the unidimensional time domain before proceeding to the more complicated four-dimensional time and space domain.

These characterizations of chemical processes have led to the following conclusions: (a) the process is described as a nonlinear multivariable system which is subject to disturbances for which compensation is required to keep certain output variables under close control, (b) a mathematical model can be devised, and (c) measurements can be made of the several process variables and disturbances. The question is whether or not in all the literature on such systems there is a theory which is general enough for the design of an efficient control system.

Feedback and Feedforward Control

Investigations of feedback control theory, which is very extensive, reveal numerous techniques for the design of feedback systems and various additional feedback compensation methods when ordinary feedback control is not acceptable. When feedback control was developed--at a time when process dynamics were relatively unknown--the best procedure was to wait for a variation in the controlled variable and then to bring about some corrective action to return the process controlled variable to its desired setpoint. Presently, more is known about process dynamics, modeling, and sources of disturbances. Measuring devices are much improved and some variables which were formerly measured by laboratory batch methods of analysis are now available on a continuous on-line basis. However, continued application of feedback control methods alone does not make effective use of recent technological advances and information available from the process.

Some years ago it was recognized that control could be improved by sensing disturbances. As a result cascade control, ratio control, and other schemes were developed; however, no theoretical design procedures were available. The early empirical design methods have gradually been replaced by techniques which use process models to assist the designer. A good example of a recent control system for disturbance compensation is the Phillips Petroleum Company Feed Enthalpy Computer. This device measures process disturbances, calculates the feed enthalpy, and provides heat compensation to keep the fractionator feed stream enthalpy as constant as possible.

Process models have proved to be useful for the synthesis of feedback and feedforward control systems. These models also may be useful for yet another kind of control system if they include variables which are neither disturbances nor controlled variables. The possibility exists of measuring these additional variables and using them for control action. There is not much in the literature about this concept, but this work will consider the feasibility of using such additional variables for control purposes.

From the foregoing discussion it appears that a theory which would incorporate all of the information available about the process through all of its variables and disturbances is needed for the proper design of a control system. In searching the systems engineering literature it soon became evident that the best approach to the synthesis of control systems for chemical processes is provided by the invariance theory which has evolved in Russia over the past twenty-five years.

Invariance Theory

This theory incorporates the process model description with a generalized control equation involving all the process variables and disturbances. It is able to treat feedforward control as well as feedback control on variables other than the ones of primary interest (the controlled variables). While it provides no new approach to the usual feedback control methods, it still allows the designer to use any of the available procedures for feedback controller design.

The theory was originally developed for linear systems, which allow the designer to use Laplace transformations or differential operators to reduce the differential equations to algebraic equations and then to use matrix algebra to simplify the notation and solution of multivariable system controller design. At first the characteristic nonlinearity of chemical processes appeared to place the most severe restriction on application of the theory, but it will be shown here that nonlinear systems may be treated by this theory. Of course, in nonlinear systems it will not be possible to use Laplace transformations and matrix algebra; it will be necessary to remain in the time domain and to manipulate differential equations. However, it will not be necessary to obtain solutions to nonlinear sets of differential equations, which is often impossible.

It is easy to illustrate the application of invariance theory to systems, linear or nonlinear, on an analog computer. Therefore, it seems odd that there is a serious lack of successful applications to physical systems even in the Russian schools that are the chief proponents of these synthesis procedures. It is realized that there are limitations to the successful application of the theory--the major one being the possibility of physically unrealizable devices requisite for invariance. However, the fact that it may not be possible to practically realize a controller, which is optimum in the sense of being an absolute disturbance compensator, does not detract from the usefulness of knowing the optimum control law. This knowledge allows the designer to make the best compromise, if necessary, between theory and practice to obtain efficient control.

Research Purposes

The purpose of this research is to formulate a theory which is convenient for application to invariant control of chemical processes. The theory will be developed which is not only mathematically sound, but practically significant. This effort represents an attempt to "add" vectorially the progress vector quantities of theory and practice in order to reduce the control gap discussed by Chestnut (18).

An analog computer will be used to illustrate many points of the theory and to provide some necessary information about the theory and the specific system investigated. The nature of the physical system experimentation will be exploratory; it will provide some data to delineate the problems of practical application and to indicate directions for future investigations of invariance theory.

Semantics

As a final introductory point, the problem of semantics in this particular area of control theory should be mentioned. The general approach which is to be discussed in this work has been called the invariance principle for several reasons. The term invariance is taken from English translations of Russian papers on the same general subject, though there seems to be some question as to whether it is properly a theory, a principle, or something else again. Unfortunately, some of the invariance concepts appear in other references and have been given assorted names. In a field which is already burdened with excessive use of jargon*, this duplication of names is another handicap. For clarification purposes and to assist others who may wish to investigate similar studies, other terminology which has been used will be discussed briefly. However, none of these other terms is judged to be any better. They are all too limiting, cumbersome, or ambiguous.

The term multivariable system describes the type of system to which this theory applies, but it implies nothing about a control philosophy. Adding the word control is helpful, but the total of three words is still vague and rather cumbersome. Complex control is sometimes used for this type of approach, but the term does not mean much by itself and is considered ambiguous. Decoupling and noninteraction have been discussed previously. These terms usually have a specific meaning concerning which variables are to be noninteracting. Since the type of noninteraction considered here is different from the ordinary type, these terms will not be used in order to avoid misunderstanding. Autonomy is a term which is rather vague, but since it is thought to refer to conditions similar to certain types of noninteraction, its usage will be avoided. The term feedforward control may represent a specific form of invariance; hence it is too limiting. Combined feedforward and feedback control adequately describes the approach, but it is very cumbersome for continued use.

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*Cf. Appendix B for a glossary.

Disturbance compensation is the term which most closely approaches invariance. Therefore, when used in this work occasionally as an alternate expression, it will have the equivalent meaning of invariance.

Rather than invent a new term, it was decided to return to the original "Russian" designation, invariance principle, which probably predates all others. It is a convenient term from the standpoint of easy transformations to verb, noun, adjective, and adverb forms, and it is very explicit in its aims. It is not likely to be confused with anything except a minor mathematical concept in matrix theory. It certainly does not arouse any preconceived notions about the theory's content since the term has rarely appeared in the literature of systems engineering in this country even as late as 1964.

CHAPTER II

REVIEW OF PREVIOUS WORK AND CONTROL PHILOSOPHY

Invariance Theory

Invariance theory has had a curious history both in- and outside Russia. The theory was first formulated by Luzin in 1940 as a result of some investigations by Shchipanov two years earlier. During the past twenty-five years the work has been continued primarily by the section of the Russian Academy at Kiev. The foremost author has been Ivakhnenko, and other contributors to the literature have been Kulebakin, Petrov, Kuznetsov, and Doganovskii.

There appears to be some evidence of varying Russian opinion as to the merits of the theory. It is believed to be generally accepted in the Soviet Union as a useful technique although much of the Russian work is not readily available. Some of the difficulties that have been encountered in acceptance of invariance theory can be gleaned from a chronicle of a conference held in Kiev in October, 1958 (20). The summary of that meeting contains an odd mixture of politics and science, and it includes a reference to a formal disagreement between the

Russian Academy and Shchipanov over invariance theory. Most of the Russian work from Kiev is concerned with theoretical considerations with only brief references to applications. There are very few references to chemical process applications.

The major source of references which have been translated is the journal, Automation and Remote Control. Practically all of the articles appearing in this source are theoretical, as are the articles which appear in the Proceedings of the Moscow International Federation of Automatic Control Conference of 1960. Abstracts of papers from the Russian journal Avtomatika appear in the IRE Transactions on Automatic Control periodically. The most important translations will be summarized below.

A general discussion of invariance theory is given by Kulebakin (47), though some of the limitations to a successful application of the theory are not adequately discussed. As a result, this article is considered to be a little misleading to the reader first becoming acquainted with this approach. A much better paper is that presented by Petrov (69), who also includes a brief discussion of some limited nonlinear applications. Chinaev (19) presents a good general discussion which is oriented towards servomechanism theory; physical realizability in the mathematical sense is discussed. Dudnikov (26) presents a very good paper and his general discussion refers to both invariance and autonomy of systems. Vershinin (87) considers some of the problems in the mathematical matrix manipulations. Doganovskii has written several papers (21, 22, 23) on an invariant system which is adaptive and uses a statistical performance criterion. Two applications to nonlinear systems of special types are discussed by Doganovskii (24) and by Menskii and Pavlichuk (62). A practical application to a multivariable servomotor system is analyzed by Dunaev (27). Dozorov's paper (25) is one in Automation and Remote Control which deals theoretically with a chemical process specifically (chemical reactor). He prefers to speak of autonomy of control, but the approach is similar to invariance theory. Meerov (61) discusses the problem of stability in multiloop combined-control systems.

Outside of the Soviet Union interest in invariance theory is remarkably lacking. Attention was first drawn to the theory at the Moscow International Federation of Automatic Control Conference in 1960 where the papers of Kulebakin and Petrov, which were mentioned above, were presented. While some interest was generated at the time, very few investigators appeared to be interested enough to undertake much extended research or to consider applications. Some critics dismissed the theory as being impractical or else merely a statement of what "everybody" already knew. However, there has been no experimental evidence of the impracticality of the theory. It seems that few people understood the theory enough to know whether it was a restatement of other work or not. At any rate a new approach, which is more general in some respects, should not be dismissed because some of its results can be generated by different, established theories.

The foregoing emphasis on the Russian work is not intended to imply that no one outside the Soviet Union has done work in invariance types of control. However, there does not appear to be a comparable general theory for invariance such as that formulated in Russia. General texts on control theory make no reference to methods of invariance; the closest approach is given by Tsien (85) in his book on engineering cybernetics.

The British are aware of invariance theory through tutorial articles by Finkelstein (29) and G. M. E. Williams (89), but original research publications have not appeared as yet. Finkelstein presented a simplified restatement of the theory as presented by Kulebakin and Petrov. He did not appear to be of the opinion that the theory had much to offer by itself, but he states that it has "given impetus to much rigorous study of the theory of control systems" and has "drawn renewed attention to the advantages of the more general use of feedforward in combination with feedback." Williams (89) has reviewed a very interesting lecture by Ivakhnenko about invariance theory with emphasis on servomechanisms. Williams (90) also reviewed three papers on invariance theory that were presented by Russian authors at the Basel conference of the International Federation of Automatic Control in September, 1963. He was critical, as were listeners at Basel, of the Russian overemphasis on theory with little regard for possible applications.

In the United States there have been several reviewers as well who have passed lightly on invariance theory. Only at the Universities of Delaware and Texas have investigators given much attention to this type of approach. However, even in these places, little credit is given to any work of Russian origin, and concepts which are similar to Russian invariance are apparently passed on as something new.

The investigators at the University of Delaware are to be commended for a good multivariable approach to chemical process problems. Bollinger and Lamb (12) have done a service by restating invariance theory in a language which is more familiar to systems engineers in the western world and have included examples of chemical systems. However, their matrix nomenclature is rather involved in an attempt to be completely general, and their theory is applicable only to linear systems. Although other investigators at the University of Delaware have considered difficult identification and control problems, it is not always best to tackle the most difficult problems first in the course of investigating possible applications of a different approach, such as invariance theory. The problems studied have been distributed-parameter systems, such as the fixed-bed reactors of Tinkler and Lamb (83) and Luyben and Gerster (55), and highly multivariable systems, such as the distillation columns of Rippin and Lamb (75) and the reactor-regenerator of Luyben and Lamb (56). These systems have been so complicated that the authors have had to make limiting assumptions, such as linearity, and to use approximate methods (frequency response) for identifying usable process models. Most of the work at Delaware has been directed toward applications of only feedforward control of specific systems.

Although the work at the University of Delaware has been very worthwhile, the advantages of generality available in invariance theory have not been manifested because of very specific applications. Furthermore, most of the experimentation was done on analog computers, sometimes using real system experimental data to verify a mathematical model. This comment is not meant to be an unqualified criticism because modeling is a proven valuable tool for studying system dynamics. Yet, the ultimate purpose of the design of a control system is the actual application to a real system and not to the model of it. Invariance theory has suffered because of the lack of demonstrated applications to simple real systems.

The work by Harris and Schechter at the University of Texas contains a similar approach to that at the University of Delaware. It is applied to the design of a feedforward controller for a chemical reactor. It is not known from their publication (37) whether or not they were influenced by any of the previous works mentioned. Although the approaches are similar, their theory lacks the generality of invariance theory. They also imply that it is necessary to solve differential equations and obtain solutions in the time domain in order to use this theory. Even though they have the controller dynamics specified in the Laplace domain (transfer function), they feel it is necessary to return to the time domain, from where it is more difficult to extract the feedforward control law and to synthesize a controller. The dependence on solutions of equations might mislead the reader into believing that the functional form of disturbances has to be known before invariance theory can be put into practice.

In all the investigations in this country the models are linear, usually being obtained from frequency response tests of nonlinear and sometimes distributed-parameter systems with subsequent fitting of standard first and second order frequency response magnitude ratio and phase angle curves. Most authors have recognized the restrictions of the linearity assumption, but they have not suggested any alternatives to design on a linearized model basis.

The literature relating to the invariance principle--outside the Russian school--consists mainly of statements and restatements of the theory, reviews, criticisms, and comments. Generalizations have been made about whether practical systems can be realized without any supporting evidence. Until concrete data on specific systems become available, there does not seem to be any justification for continued speculation.

Since other system design techniques used in process control have been able to provide some of the same information as invariance theory, the remainder of this chapter will cover those aspects of multivariable system theory which relate to invariance theory. Some attention will be focused on proven techniques in the chemical and process industries which tend to indicate the validity and value of such a general approach to multivariable chemical systems as invariance theory provides. Finally, a pertinent discussion of general control philosophies will conclude the chapter.

Multivariable Systems.

A good review of the general theory of multivariable systems and a fairly complete bibliography have been collected by Hammond (36). He covered the early history of this theory, which is about twenty-five years old. The majority of the early papers were oriented towards servomechanism systems and many of them tended to rely heavily on unidimensional system approaches. Two such papers were by Golomb and Usdin (34) and Povejsil and Fuchs (73). Multivariable systems have tended to be considered as collections of unidimensional parts, which has limited the scope of design methods. However, in some of the early papers some good ideas were presented which bear on invariance theory.

Kavanagh (43-45), Freeman (31, 32), and more recently Loomis (50) have extended and generalized the conditional feedback concept of Lang and Ham (48), which was originally applied to a unidimensional system. The idea of a disturbance-response feedback system as presented by Reswick (74) was close to invariance theory, but it apparently was not pursued much further. Bohn (8, 9, 10) has continued the work of Kavanagh and Freeman; he has been concerned with stability and output variable noninteraction of control systems. Horowitz (39) has considered the problem of plant parameter variations. McBride and Narendra (60) spoke of an expanded matrix representation of multivariable systems and mentioned invariance and other control concepts. Chatterjee (17) has given a good multivariable process design discussion with some pertinent remarks about practical limitations and the general state of process control theory. For a more complete discussion of these works the reader is referred to Hammond.

Successful applications of multivariable system theory to practical systems have been discussed by Boksenbom and Hood (11) and by Shull and Russell (79). Both of these problems were concerned with noninteraction of system output variables.

Amara (1) has treated multivariable processes with stochastic inputs. Shevelev (77) and Belen'kii (6) discussed the relationship between invariance theory and the Wiener-Hopf statistical approach, and work has proceeded since then in these parallel approaches. In this country some of the investigators have been Hsieh and Leondes (40) who studied optimum filter synthesis; Brockett and Mesarovic (14) who studied a general approach to control synthesis on a statistical basis; and Narendra and Goldwyn (67) who investigated a Wiener-Hopf method with constraints. The Wiener-Hopf methods are necessary for some purposes, especially the treatment of noise problems. However, the mathematics is difficult for the uninitiated and practical applications are rather obscure, especially for chemical processes. Nevertheless, a few of the statistical ideas have been useful, especially the treatment of random inputs to processes which can be characterized by spectral densities. The reader is referred to Bollinger and Lamb (13) for an example.

Feedforward Control

Because feedforward control design methods are one of the important results of invariance theory, a review of the pertinent work in this area is presented. Duthie (28) has shown how feedforward can improve feedback control for servosystems and has called for more general use of it. Pink (71) has indicated the possibility of using a feedforward optimizing computer. T. J. Williams (91) in his series of articles on systems engineering drew attention to the possibilities of feedforward control, including some objections to it. Calvert and Coulman (16) have delineated the differences between chemical and mechanical systems and have called for more general use of feedforward control of chemical processes. These papers were all general in nature and no specific techniques were suggested.

Feedforward control is not very recent. There have been references to an open-loop controller as early as 1934 applied by the British Admiralty (cf. Duthie). Graham (35) gave a brief discussion of a feedforward application to a servosystem, and Moore (66) discussed what he referred to as combination open-cycle, closed-cycle control systems.

Much of the work on feedforward control has been concerned with unidimensional systems and transfer function techniques. Mamzic (58), and more recently, Forman (30) have discussed various schemes for process control and have included several different types of feedforward control. Phillips (70) has applied disturbance compensation by feedforward control to several processes which were simulated on an analog computer. The processes which he investigated were simple liquid level and liquid flow control.

For complex multidimensional systems design techniques have been naturally slower in appearing. Tierney et al (82) have suggested the use of a digital computer as a feedforward controller with standard feedback devices added for fine control. The work under Lamb and Gerster, by Bollinger, Luyben, Rippin, and Tinkler at the University of Delaware, and the work by Harris and Schechter at the University of Texas--all of which are concerned with feedforward control--have been previously mentioned. Zahradnik, Archer, and Rothfus (93) have discussed the feedforward dynamic optimization of a distillation column using variational techniques.

Feedforward compensation in the presence of dead times in the process has been discussed by Buckley (15) and by Lupfer and Oglesby (51). Buckley covers a number of techniques including the linear predictor concept of Smith (80) and an approach something like that of Phillips (70). Lupfer and Oglesby (51) extended the work of Smith to apply to a chemical reactor.

The important advantages of feedforward analog computer control of a slowly responding chemical process are evident from an article by MacMullan and Shinskey (57).

The system which they considered was a 100 tray superfractionator. The two, first-order lag times in their model were sixty and ninety-five minutes long, and the two, pure dead-times were twenty and thirty minutes long. Conventional feedback control of the output variable of interest proved to be very unsatisfactory; the controlled variable deviated from the set-point about thirty minutes after a step disturbance and did not cross the set-point again for approximately two-hundred minutes. Feedforward control, designed on the basis of a very simple process model, provided a substantial improvement. This type of control allowed not only less variation in product, but it also allowed operation at more profitable conditions. An investment of $10,000 was paid out in three months. The authors also mentioned an additional economic incentive to feedforward control in the design of new plants. The advantage lies in the possibility of eliminating surge and storage vessels which have been used in the past primarily to eliminate disturbances. This same advantage is afforded by invariance theory or any other good systems engineering approach to design.

Analog Computer Control

The general use of special purpose analog devices for control purposes has been covered by Plant (72) and Shinskey (78) and the specific devices by Mamzic (59) and Ryan (76). This approach has been applied with good success by Phillips Petroleum Company. These special purpose computers, which possess some feedforward qualities, have been described in several papers. Tolin and Fluegel (84) have analyzed an analog computer which scans fourteen variables, computes production rates, and exercises control capabilities. Lupfer, Oglesby, and Parsons (52-54) in several papers have discussed the feed enthalpy, internal reflux to feed ratio, and bottom product flow computers. Parsons and Tolin (68) have recently presented an excellent analog computing control approach which is similar in many ways to invariance theory. The article by MacMullan and Shinskey (57) which discusses analog computer control of a specific system has been mentioned. Also a little information is available for use in industry from the Bailey Meter Company (2), The Foxboro Company, and Electronics Associated, Incorporated.

General Control Philosophies

In the past few years there have been several attempts to develop a general theory of control systems. Perhaps the author who has had the largest impact on these theoreticians is Kalman (41, 42). His works are characterized by mathematical excellence and some good general comments on the ideas of observability and controllability. Gilbert (33) has extended Kalman's ideas; both of them use state-space techniques, which have been discussed by Zadeh (92). State-space methods and the associated techniques of dynamic programming (Bellman) and the maximum principle (Pontrjagin) are not often as useful in chemical process design as they are in other fields. To understand this statement one must examine the reasons for the use of state-space methods and the nature of chemical process models. State-space variables are artificially introduced into the model as a mathematical convenience in order to eliminate what Bellman refers to as the "curse of dimensionality." The aim of these changes in variables is the reduction of a system which is "n"th order in one variable (e.g. the controlled variable) to a system which is first order in "n" variables (the state-space variables). In state space these "n" variables may or may not have physical significance and, accordingly, may or may not be directly measurable. For instance, the state-space variables of a temperature controller system might be the derivatives of the controlled temperature with respect to time. In chemical systems the equations are frequently first order in "n" output variables, all of which have physical significance and are usually measurable. The system used in this research is a good example of such a case. Since chemical process system model equations are frequently in state-space notation to begin with, the advantages of state-space transformations are not realized.

Furthermore, state-space methods become quite difficult when the systems are affected by disturbances which cannot be ignored; for example in chemical systems. The resulting control laws are not easy to implement as indicated by Zahradnik, Archer, and Rothfus (93). Kipiniak (46) has also discussed a general control theory based on variational approaches using state-space equations. His theory is able to treat nonlinear systems and linear, time-varying systems, as well as linear ones, but the method is quite difficult. Kipiniak cites a chemical reactor as one example, but the resulting optimization procedure makes excessive demands on both the number of system measurements and the required on-line computing ability (IBM 7090).

Horowitz (38) has taken a somewhat different step towards a general control theory based on the concept of degrees of freedom. He has developed a good feedback control philosophy, and he contributes the useful term of "sensitivity." Mesarovic (63-65) has discussed the topology of systems and speaks of design in terms of specified "canonical forms" for systems. His rearrangement of linear systems into different forms is allowable, mathematically speaking, but as will be discussed later, there are difficulties which may be overlooked in such transformations. Tu Xu-Yen (86) has contributed an approach which speaks of harmonically acting control systems, but this theory is rather obscure.

Balchen has stated his control philosophy in several places (3-5), and it is closest to invariance of all those mentioned previously. The biggest advance that he has made over the others is a convenient method for treating disturbances. However, he does not consider all the various possible types of control (combination feedforward and feedbacks). While most of the forms of invariance controllers can be extracted from Balchen's theory, it is only done with difficulty. His articles contain many interesting and important comments about multivariable control of systems with characteristics like chemical processes.

Invariance Control Philosophy

At this point, having discussed other control philosophies, it is constructive to consider the philosophy used in this research. The present state of chemical process control theory is still predominantly feedback oriented because of the prior lack of knowledge of the dynamic behavior of chemical systems and the advanced state of feedback control theory. Most of the theory which is applied to chemical process systems has come from servosystem applications, and many processes with numerous controlled quantities are still considered as collections of univariable control systems. In many instances this approach is adequate for accepted control criteria, and consequently, there is some tendency to consider the regulator problem as being solved. However when control is not adequate, present theory is incapable of leading to a good design unless a more general view of multivariable processes and their control is taken. The alternative to such a general theory is expensive elaboration on feedback control schemes in which the cost is out of proportion to the quality of control.

The problem that will be considered is the control of one output of a multivariable system. The system has measurable output variables and two kinds of input variables: control variables which can be manipulated and disturbance variables which can be measured. Such a system can be diagrammed in the form of Figure 1.

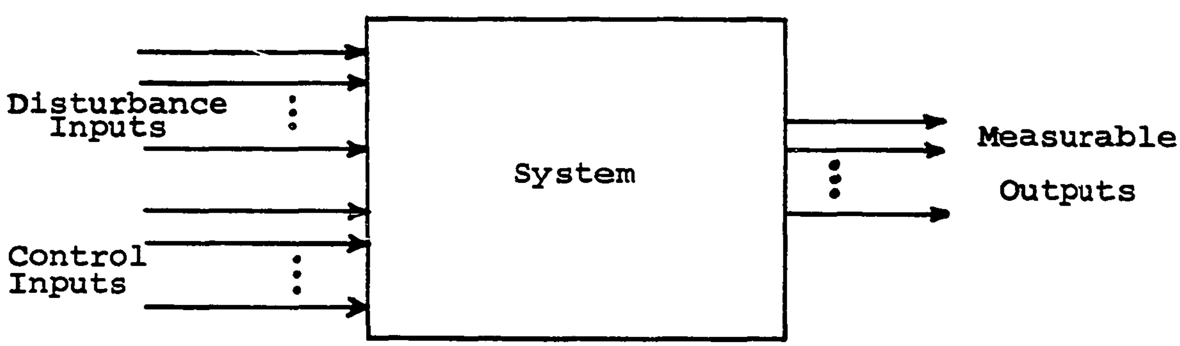

Figure 1. Multivariable System

In the past, when processes were considered as "black boxes" in which relationships between inputs and outputs were not known, a feedback control system would have been installed in the manner of Figure 2.

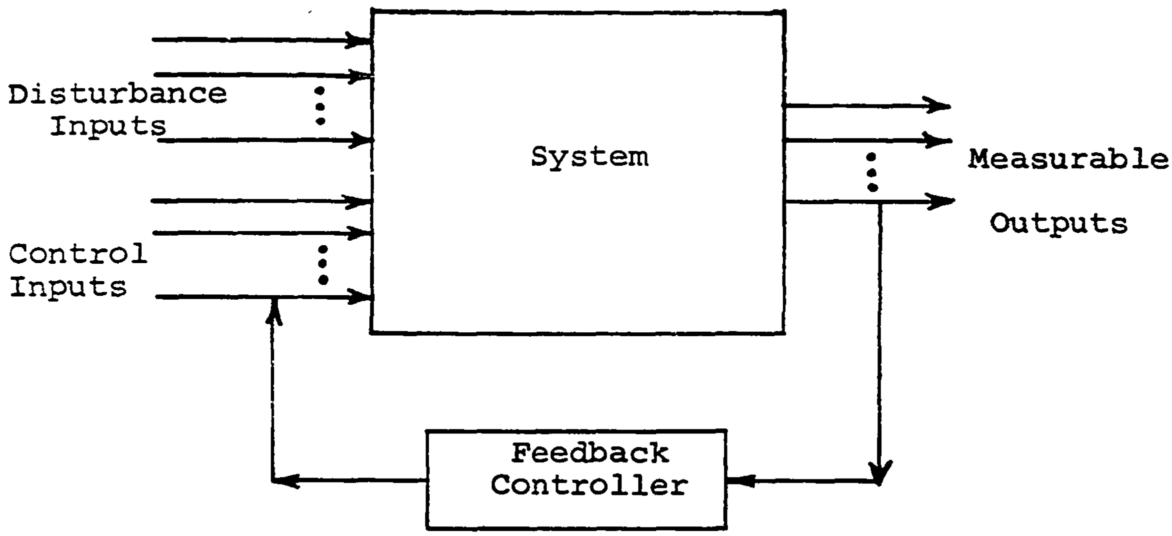

Figure 2. Feedback Control System

The principle advantage of these conventional feedback control systems is their ability to correct the controlled variable for any disturbance that might upset it regardless of the origin of the disturbance. Thus in a feedback control system it is not of primary importance to know exactly what caused the upset; however the convenient adaptability of feedback has the disadvantage in that some error in the controlled variable must be tolerated since no correction can be provided until an error has appeared. This error is accentuated when the process contains times lags or dead times.

The feedback concept often implies that the internal mechanisms of the process are undetermined and that the only reliable information available is the current state of the controlled output variable.

In chemical processing the best operating conditions are frequently obtained from bench-scale and pilot-plant studies, which yield the essential relationships among the internal process variables. Measuring devices have improved to the extent that there are few process variables which cannot be measured. Even if such small-scale studies are not made, various system identification methods can provide the necessary information about internal structure. Since a great deal of information about the process can be obtained, and assuming there is a variable which requires close control, the question is whether there is a better control scheme than conventional feedback control. In order to attempt to reach an answer a general control theory, such as invariance theory, is needed. This general theory stems from a consideration of a control system which can be diagrammed as Figure 3.

Invariance theory provides a systematic and theoretical, rather than empirical, procedure for design of control systems. The theory allows the selection of the minimum number of outputs and control inputs that will accomplish the control purpose. It is not implied that invariance theory is any more general than the theories of Bellman, Pontrjagin, Kalman, or Balchen, if one is concerned with control of all types of systems.

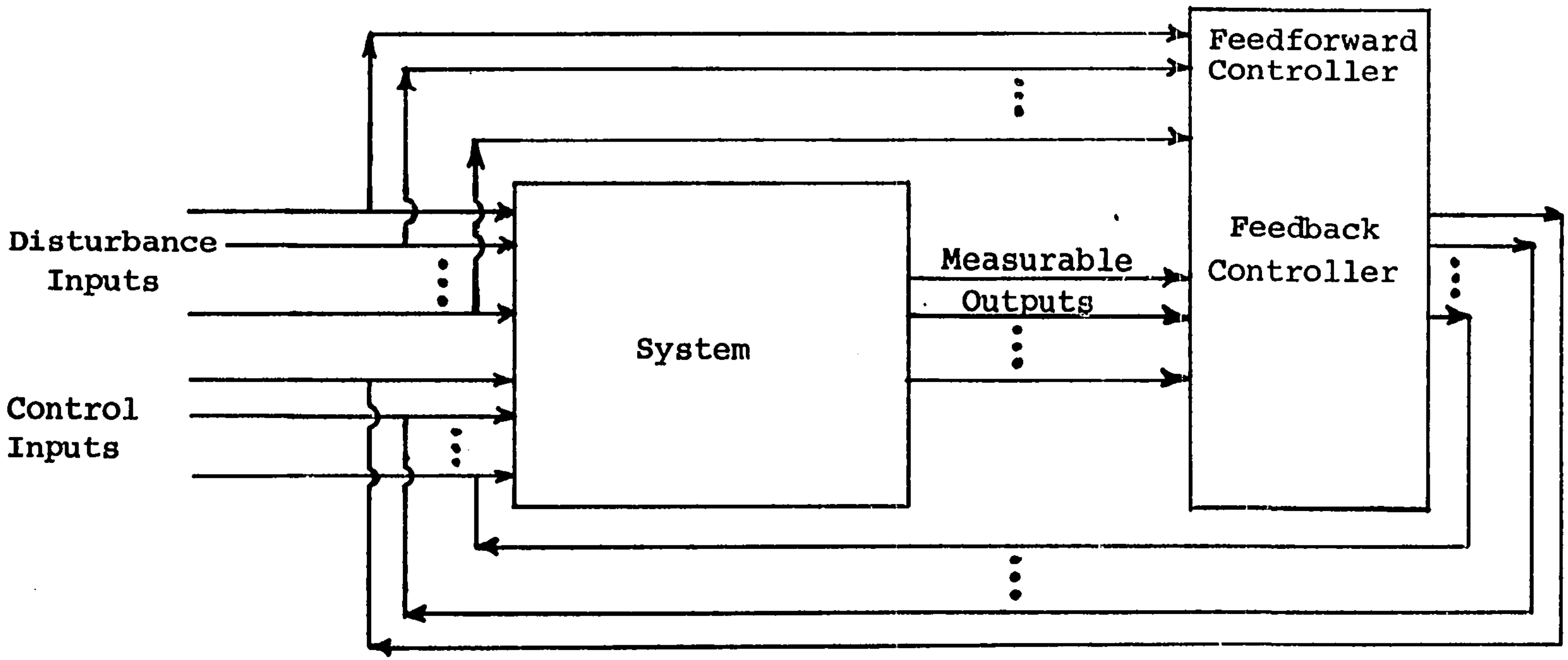

Figure 3. General Invariance Principle Control System

However, it is a general theory which is most suitable to the chemical process characteristics discussed in Chapter I.

The advantages of invariance theory can be summarized.

a) The aim of invariance is the aim of any good control system--to make a given variable invariant to disturbances.
b) The theory uses all of the available information about the system.
c) The theory can use disturbance information of the type usually encountered in chemical processes more conveniently than can the general theories of Bellman, Pontrjagin, and Kalman.
d) The controllers which result from the theory are usually simple.
e) The theory is able to treat nonlinear system.
f) The designer usually has the choice of several options for control, which can be selected according to convenience and economic constraints.

CHAPTER III

HEURISTIC PRESENTATION OF THE THEORY

Multivariable system theories are usually formulated with vector equations for a concise and general presentation. The price of brevity and generality is a more complex nomenclature with many subscripts and superscripts and an increased abstraction of the problem. These abstract presentations often hide the true mathematical processes and can in some cases practically eliminate the physical meaning of the problem. To place the reader in a better position with regard to the fine points of the theoretical considerations and practical applications, the theory will be presented first in this chapter as an example using the experimental equipment for the system of interest. Once the procedure has been established, a more formal and general theory can profitably be presented.

The System

The physical system which is to be investigated is a completely mixed, stirred tank chemical reactor. No actual reaction occurs in the reactor; the reaction is simulated by a heat transfer process. A schematic diagram of a jacketed, well-agitated continuous reactor is shown in Figure 4. Hot oil enters the reactor at a constant temperature, and a cold solution of water and ethylene glycol (hereafter called the coolant) enters the jacket at a constant temperature also. The oil is mechanically agitated, but the coolant is only mixed by its natural circulation towards the reactor coolant outlet.

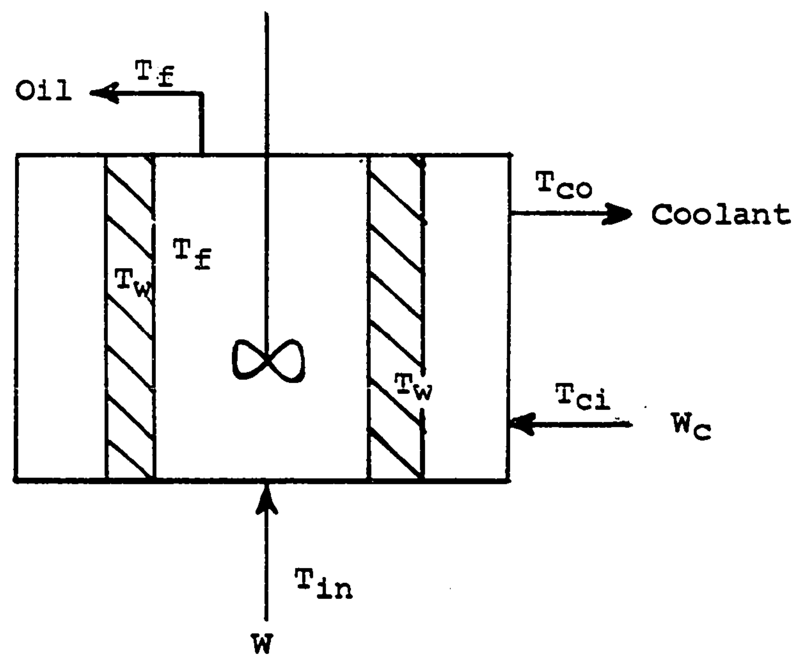

Figure 4. Schematic Diagram of a Jacketed, Well-agitated Continuous Reactor

The Mathematical Model

The following assumptions for the development of the mathematical model are made.

1. The oil is perfectly mixed; that is, the temperature of the oil at any place in the reactor is equal to the outlet temperature.

2. The coolant is <u>not</u> perfectly mixed. Heat is considered to be transferred from the oil temperature to a temperature which is assumed to be the arithmetic mean of the coolant inlet and outlet temperatures. This assumption of the mean greatly simplifies the model. The mean coolant temperature is assumed to be that used in the accumulation term in the coolant energy balance.
3. The jacket wall between the coolant and oil has a large enough thermal capacitance that a dynamic energy balance must be written for the wall.
4. The coolant and oil inlet temperatures are constant.
5. Heat losses are lumped and included in the energy balance for the oil. Although this simplification is not necessarily correct, it is expedient. The alternative to this assumption is a rather ambitious experimental program to determine individual heat transfer coefficients and local temperatures. A heat loss term is required to satisfy the steady state experimental data. The actual location of the heat loss term in the model does not affect the dynamic controller equations which will be derived from the model.
6. Heat capacities and densities of the oil, coolant, and wall metal are assumed to be constant.

Expected small fluctuations in system temperatures justify the evaluation of these constants under steady-state operating conditions.

7. The temperature of the wall is considered to be proportional to the average of the electromotive forces produced by four thermocouples which were imbedded in the wall during its casting. This temperature is assumed to be constant throughout the wall at any instant in time for heat transfer purposes.

8. The heat transfer coefficients were assumed to be constant with respect to operating temperatures and flow rates. This assumption was checked and justified by steady-state experimental data taken under various conditions in flow rates and temperatures (cf. Appendix C). The heat transfer coefficients are "overall" in so far as they are functions of the liquid film resistance and the metal wall resistance between the liquid surface and the location of the thermocouples.

The model equations are derived by writing dynamic energy balances on the oil, coolant, and wall in the manner of Stewart (81, pp. 34-35).

$$(\rho VCp)_f \dot{T}_f = (h_i A_i) T_w - (h_i A_i) T_f + (Cp_f T_{in}) W - (Cp_f) W T_f - (Q_L)$$

$$(\rho VCp)_w \dot{T}_w = (h_i A_i) T_f + (h_o A_o) T_{cm} - (h_i A_i + h_o A_o) T_w \qquad (1)$$

$$(\rho VCp)_c\dot{T}_{cm} = (h_oA_o)T_w - (h_oA_o)T_{cm} + (T_{ci}Cp_c)W_c - (Cp_c)W_cT_{co}$$

T_{cm} = Mean coolant temperature, °F

T_{co} = Coolant outlet temperature, °F

T_f = Bulk oil temperature, °F

T_w = Wall temperature, °F

W = Oil Flow rate, lbs. per hour

W_c = Coolant flow rate, lbs. per hour

dot $(\dot{\ })$ = Time derivative

The nomenclature for the constants is given in Table 1. The second assumption allows the substitution:

$$T_{cm} = \frac{T_{co} + T_{ci}}{2} \tag{2}$$

The final model equations are the following:

$$\begin{aligned}
(\rho VCp)_f\dot{T}_f &= (h_iA_i)T_w - (h_iA_i)T_f + (Cp_fT_{in})W - (Cp_f)WT_f - (Q_L) \\
(\rho VCp)_w\dot{T}_w &= (h_iA_i)T_f + \left(\frac{h_oA_o}{2}\right)T_{co} - (h_iA_i + h_oA_o)T_w + \left(\frac{h_oA_oT_{ci}}{2}\right) \\
(\rho VCp)_c\dot{T}_{co} &= (2h_oA_o)T_w - (h_oA_o)T_{co} + (2Cp_cT_{ci})W_c \\
&\quad - (2Cp_c)W_cT_{co} - (h_oA_oT_{ci})
\end{aligned} \tag{3}$$

Table 1 lists the values of the system constants, steady-state temperatures, nomenclature, and the sources of information. Substitution of these values into Equations (3) gives the mathematical model for normal operating conditions:

$$\begin{aligned}
\dot{T}_f &= -31.7T_f + 31.7T_w + 102.5W - 0.666WT_f - 186 \\
\dot{T}_w &= -83.3T_w + 41.1T_f + 21.0T_{co} + 658 \\
\dot{T}_{co} &= -41.4T_{co} + 82.8T_w + 75.2W_c - 2.4W_cT_{co} - 1296
\end{aligned} \tag{4}$$

TABLE 1

LIST OF SYSTEM CONSTANTS: NOMENCLATURE, VALUES, UNITS, AND SOURCES

Symbol	Nomenclature	Value	Units	Source
Cp_c	Coolant heat capacity	0.763	BTU/lb°F	Handbook data
Cp_f	Oil heat capacity	0.538	BTU/lb°F	Lab. measurement
Cp_w	Wall metal heat capacity	0.042	BTU/lb°F	Estimate from handbook data
h_iA_i	Oil side heat transfer coefficient x area	25.6	BTU/hr°F	Steady-state data
h_oA_o	Coolant side heat transfer coefficient x area	26.3	BTU/hr°F	Steady-state data
T_{ci}	Coolant inlet temperature	31.3	°F	Steady-state data
$T_{co_{ss}}$	Steady-state coolant outlet temp.	59.5	°F	Steady-state data
$T_{f_{ss}}$	Steady-state oil temperature	138.	°F	Steady-state data
T_{in}	Reactor oil inlet temperature	154.0	°F	Steady-state data
$T_{w_{ss}}$	Steady-state wall temperature	91.0	°F	Steady-state data
Q_L	Heat loss term	150.0	BTU/hr	Steady-state data
V_c	Reactor coolant volume	.0124	cu.ft	Lab. measurement
V_f	Reactor oil volume	.0287	cu.ft	Stewart (81)
V_w	Reactor wall volume	.0246	cu.ft	Lab. measurement
W_{ss}	Steady-state oil flow rate	157.0	lb/hr	Steady-state data
$W_{c_{ss}}$	Steady-state coolant flow rate	54.0	lb/hr	Steady-state data
ρ_c	Coolant density	67.1	lb/cu.ft	Handbook data
ρ_f	Oil density	52.3	lb/cu.ft	Lab. measurement
ρ_w	Wall metal density	603.2	lb/cu.ft	Handbook data

Equations (4) will be referred to as the "nonlinear total variable model."

For convenience Equations (4) can be simplified by changing the variables to perturbation (or transient) variables. This change is accomplished by assuming that each "total" variable in Equations (4) is composed of a steady-state plus a transient portion. Thus

$$
\begin{aligned}
T_f &= T_f{}^* + T_{f_{ss}} \\
T_w &= T_w{}^* + T_{w_{ss}} \\
T_{co} &= T_{co}{}^* + T_{co_{ss}} \\
W &= W^* + W_{ss} \\
W_c &= W_c{}^* + W_{c_{ss}}
\end{aligned}
\tag{5}
$$

These equations are substituted into Equations (4). The steady-state equations corresponding to Equations (4) (time derivatives set equal to zero) are then subtracted from the previous equations--Equations (4) and (5) combined--to give:

$$
\begin{aligned}
\dot{T}_f{}^* &= (-31.7-0.666W_{ss})T_f{}^* + 31.7T_w{}^* + (102.5-0.666T_{f_{ss}})W^* \\
&\quad - 0.666W^*T_f{}^* \\
\dot{T}_w{}^* &= -83.3T_w{}^* + 41.1T_f{}^* + 21.0T_{co}{}^* \\
\dot{T}_{co}{}^* &= (-41.4-2.4W_{c_{ss}})T_{co}{}^* + 82.8T_w{}^* + (75.2-2.4T_{co_{ss}})W_c{}^* \\
&\quad -2.4W_c{}^*T_{co}{}^*
\end{aligned}
\tag{6}
$$

When the steady-state values of the flow rates and temperatures, which are given in Table 1, are substituted into Equations (6), perturbation model equations result.

$$\dot{T}_f^* = -136.3T_f^* + 31.7T_w^* + 10.6W^* - 0.666W^*T_f^*$$

$$\dot{T}_w^* = -83.3T_w^* + 41.1T_f^* + 21.0T_{co}^* \qquad (7)$$

$$\dot{T}_{co}^* = -171T_{co}^* + 82.8T_w^* - 67.6W_c^* - 2.4W_c^*T_{co}^*$$

These equations will be referred to as the "nonlinear perturbation model."

As Stewart (81, pp. 51-53) has shown, linearization of perturbation equations containing product-type nonlinearities, using Taylor series expansions and retaining only the constant and first order terms, is equivalent to assuming that the product terms are negligible. Thus, Equations (7) can be linearized by the Taylor series expansion method by simply eliminating the product terms. The results--Equations (8)--will be called the "linear perturbation model."

$$\dot{T}_f^* = -136.3T_f^* + 31.7T_w^* + 10.6W^*$$

$$\dot{T}_w^* = -83.3T_w^* + 41.1T_f^* + 21.0T_{co}^* \qquad (8)$$

$$\dot{T}_{co}^* = -171.0T_{co}^* + 82.8T_w^* - 67.6W_c^*$$

The Problem

The system which has been described and modeled is known to be subject to changes in the coolant flow rate. It is desired to eliminate the effects of this disturbance on the wall temperature in the reactor. The oil flow rate can be varied to control this temperature. The problem is knowing specifically how the oil flow rate should be changed. The usual method of control would suggest the measurement of the wall temperature and the application of proportional, reset, or rate feedback operations on the error signal to provide the specification of the necessary correcting oil flow rate. Knowing that deviations from the desired wall temperature must exist even with conventional feedback control, are there any other methods of control that could eliminate these deviations?

The Invariance Principle

If the linear perturbation model is adequate to describe the behavior of the system, Equations (8) can be used. Intuitively, in order to make T_w invariant the first operations for the model equations are the setting of T_w^* and $\dot{T}_w^*$ to zero. Thus, the invariance conditions are

$$T_w^* = \dot{T}_w^* = 0 \tag{9}$$

Substituting Equation (9) into (8) results in the following set:

$$\dot{T}_f^* = -136.3T_f^* + 10.6W^* \tag{10a}$$

$$0 = +41.1T_f^* + 21.0T_{co}^* \tag{10b}$$

$$\dot{T}_{co}^* = -171.0T_{co}^* - 67.6W_c^* \tag{10c}$$

Since these three equations involve four variables, it is possible to determine relationships between any two variables. For the problem considered it is desired to solve Equations (10) for W* in terms of any one of the other three variables.

Equation (10a) can be rearranged.

$$W^* = 0.0943\dot{T}_f^* +12.86T_f^* \tag{11}$$

Substituting Equation (10b) into (11) gives

$$W^* = -0.0482\dot{T}_{co}^* - 6.57T_{co}^* \tag{12}$$

Substituting Equation (10c) into (12) using the differential operator (D = d/dt) results in

$$W^* = \frac{3.26\dot{W}_c^* + 444.0W_c^*}{D + 171} \tag{13}$$

Equations (11), (12), and (13) suggest that there are three methods of obtaining invariance of T_w: making W* proportional to T_f^* and $\dot{T}_f^*$, making W* proportional to T_{co}^* and $\dot{T}_{co}^*$, and making W* proportional to W_c^* and $\dot{W}_c^*$ with a dynamic term represented by the operator polynomial (D + 171). However, there is one additional requirement which must be met for the mathematical attainment of invariance. This requirement is satisfied by meeting conditions of a theorem which Petrov (69) refers* to as the "dual channel" theorem.

Dual Channel Theorem

The principle of dual channels has been given by Petrov (69, p. 120) along with its equivalent mathematical statement (69, p. 118). The concept of dual channels will be discussed first from a mathematical point of view, using the model equations, and then from a physical point of view referring to the actual system. Finally, the block diagram (hereafter called topological) approach will be discussed. The last approach will clarify the choice of name, "dual channel."

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*In English translation.

Because Equations (11), (12), and (13) were developed from Equations (8) simply by making the assumption of Equation (9), it should be possible to obtain Equation (9), the invariance condition, by substituting Equation (11), (12), <u>or</u> (13) into (8). The apparently obvious reversibility is the mathematical requirement which must be met for a successful attainment of invariance. The important point to remember--which is the purpose of the dual channel theorem--is that $T_w{}^*$ can provide no information for the dynamic equations describing $T_f{}^*$ and $T_{co}{}^*$ because $T_w{}^*$ is restricted from varying. Thus, the $T_w{}^*$ terms in Equations (8a) and (8c) should be eliminated in checking the validity of the invariance control Equations (11), (12), and (13). Equations (8) then becomes the following set.

$$\dot{T}_f{}^* = -136.3T_f{}^* + 10.6W^* \qquad (14a)$$

$$\dot{T}_w{}^* = -83.3T_w{}^* + 41.1T_f{}^* + 21.0T_{co}{}^* \qquad (14b)$$

$$\dot{T}_{co}{}^* = -171.0T_{co}{}^* - 67.6W_c{}^* \qquad (14c)$$

<u>Case I</u>

Substituting Equation (12) into (14a) results in the following:

$$T_f{}^* = -136.3T_f{}^* + 10.6(-0.0482\dot{T}_{co}{}^* - 6.57T_{co}{}^*) \qquad (15)$$

$$T_f{}^* = \frac{(10.6)(-0.0482)(D + 136.3)T_{co}{}^*}{(D + 136.3)} = -0.511T_{co}{}^* \qquad (16)$$

Substituting Equation (16) into (14b) gives the relationship

$$\dot{T}_w{}^* = -83.3T_w{}^* \qquad (17)$$

If the initial condition of T_w*, at a time before any disturbance F_c*, is equal to zero, the solution of Equation (17) is

$$T_w^* = 0 \qquad (18)$$

Invariance is assured.

Case II

Substituting Equation (11) into (14a) results in the identity, zero equals zero: no unique relation between T_{co}* and T_f* is found. Therefore, there is no unique solution to Equation (14b) and no invariance condition like Equation (18) can be assured. Invariance is not possible for this case.

A similar substitution of Equation (13) into (14) shows that invariance is possible for the control law represented by Equation (13). The dual channel concept has reduced the mathematical methods for attaining invariance from three to two. Because all operations have been abstractly performed on a mathematical model, it may not be entirely clear why Equation (11) does not work. If one considers the physical system at this point, the failure can be explained.

Equation (11) suggests that W* be changed according to the measured transient temperature T_f*; however, referring to Figure 4, it is seen that a nonzero transient temperature T_f* cannot even exist unless there is a nonzero transient temperature T_w*. Since $T_w^* \neq 0$ is not wanted, this control scheme has to be rejected because it demands an impossibility. The other control laws represented by Equations (12) and (13) are acceptable because they depend upon measuring variables which are affected by the disturbance W_c*, but not through the invariant variable T_w*. The dual channel requirement can be satisfied from a purely physical standpoint in many cases and also can be satisfied by mathematical proofs. The best method is probably the topological, which will now be discussed.

The procedure is to place the model equations in block diagram form, using linear operators, and to check for the existence of dual channels of information. Equations (8) in block diagram form, using differential operators, can be shown as Figure 5. Also in this figure are included the blocks for the different controllers described by Equations (11), (12), and (13). The circuits are broken at points b because, during the fulfillment of invariance conditions, T_w* cannot affect those portions of the block diagram which represent the equations for T_f* and T_{co}*. Each control scheme is selected in turn by selector a and the circuits are checked for the existence of two channels. These channels must be able to convey information in two different paths from the disturbance to the summing junction which affects the variable to be made invariant. In this example both channels must extend from W_c* to summing junction (S2).

<u>Case I</u>

Check of Equation (12).

First channel: W_c* through S3 and T_{co}* to S2.

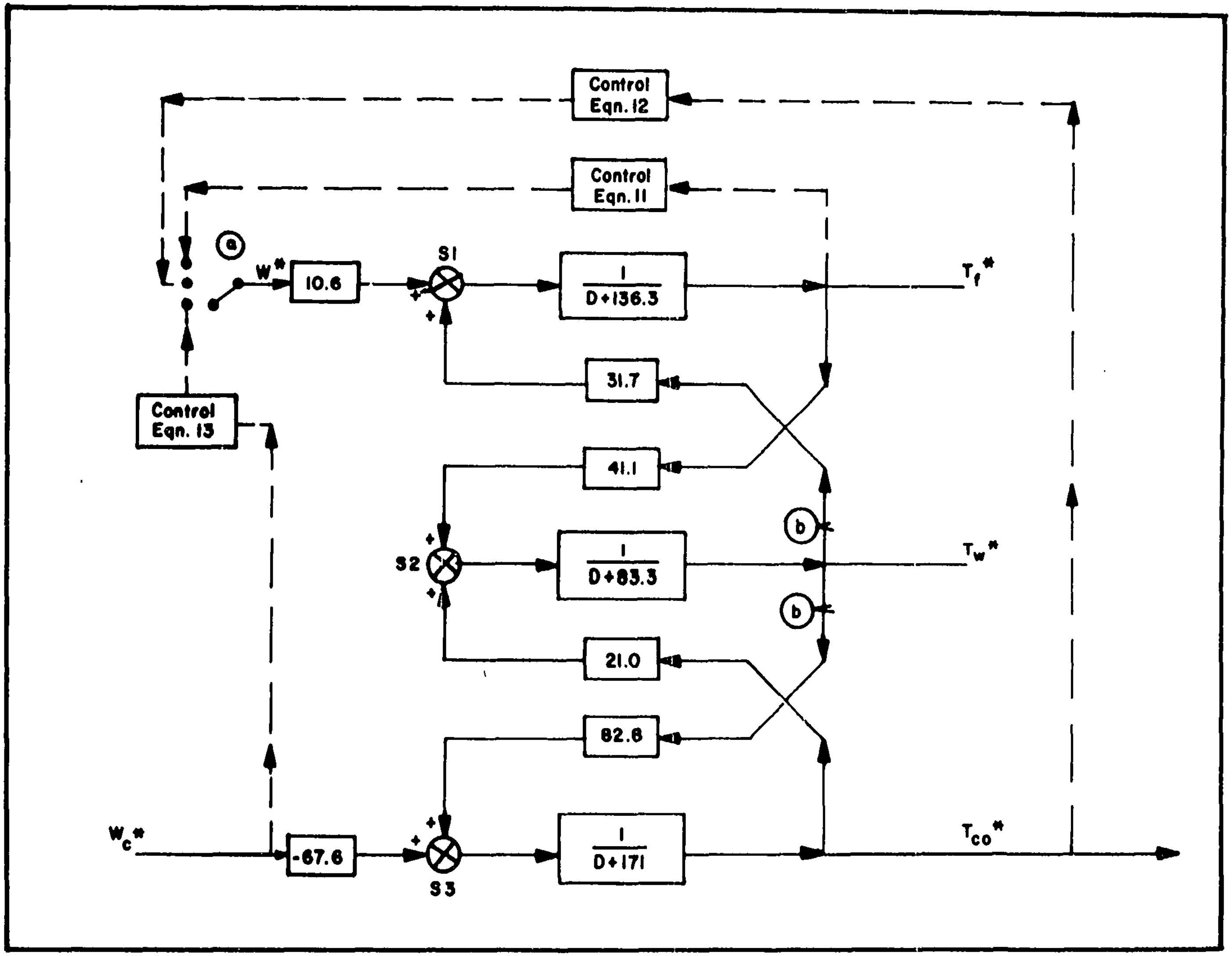

Figure 5. Topological Representation of a Linear Perturbation Model and Controllers for the Invariance of T_w.

Second channel: W_c* through S3, T_{co}*, controller, W*, S1, and T_f* to S2.

Case II

Check of Equation (11).

First channel: W_c* through S3 and T_{co}* to S2.

Second channel: W_c* through S3, T_{co}* and S2 to T_w*. At this point the channel is broken and information cannot get to the controller to complete the circuit. A complete second channel does not exist.

Case III

Check of Equation (13).

First channel: W_c* through S3 and T_{co}* to S2.

Second channel: W_c* through controller, W*, S1 and T_f* to S2.

Again it is seen that only two of the three controllers provide two channels. That there is a correspondence between the mathematical and topological methods has not been formally proven. However, the same information concerning the attainment of invariance is obtained by either method, and, therefore, the entire concept of the second condition for invariance (dual channels) is referred to as the dual channel theorem. The connection between the two methods is still intuitive.

The topological description of the system should be applied carefully. The block diagram should be drawn directly as the model equations appear with no topological transformations, such as moving the summing junctions or take-off points and combination of transfer functions or loops. Such commonly used transformations, although mathematically correct for linear systems, rapidly obscure the real physical relationships and result in artificial variables and transfer functions which do not correspond to physical operations. The result of such transformations is usually a system in which it is difficult to ascertain whether two channels exist or not. For invariance purposes it is recommended that no rearrangement of the topology be allowed.

The three methods of checking the dual channel condition--mathematical, topological, and physical--are equivalent. Any one of them is sufficient to check the existence of two information paths. The topological method is recommended as the simplest and safest in most cases.

Thus far in this chapter the two central ideas of invariance theory have been presented: the sufficient condition for invariance--the equating to zero of the controlled variable and its derivatives, and the necessary condition--the satisfaction of the dual channel theorem. The remainder of this chapter will be devoted to refinements of the theory for the convenient analysis of systems.

Use of Matrices

Because this theory is concerned with multivariable systems, matrix methods can be useful when used with linear operators. The mathematical operations are somewhat simplified, especially in the case of high order systems. Furthermore, because this theory is ultimately used for determining functional relationships between variables rather than for finding unique solutions to sets of equations and because several control schemes are often possible for solutions to specific control problems, matrix methods prove to be ideal. These reasons can best be manifested by reconsidering the same problem which has just been solved and by reworking the theory using matrices.

First of all, Equations (8) can be Laplace transformed conveniently because the initial conditions of T_f^*, T_w^*, and T_{co}^* are zero. The equations are written in matrix form.

$$\begin{vmatrix} s+136.3 & -31.7 & 0 & -10.6 \\ -41.1 & s+83.3 & -21 & 0 \\ 0 & -82.8 & s+171 & 0 \end{vmatrix} \begin{vmatrix} T_f^*(s) \\ T_w^*(s) \\ T_{co}^*(s) \\ W^*(s) \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ -67.6W_c^*(s) \end{vmatrix} \tag{19}$$

It is convenient to add a general control equation to Equation (19). This general equation is assumed to be

$$\begin{aligned} W^*(s) = {} & K_f(s)T_f^*(s) + K_w(s)T_w^*(s) + K_{co}(s)T_{co}^*(s) \\ & + K_c(s)W_c^*(s) \end{aligned} \tag{20}$$

where each of K(s) terms is an undetermined function of s which is equivalent to the transfer function of the controller.

Equation (20) is combined with Equation (19) to form the matrix equation:

$$\begin{vmatrix} s+136.3 & -31.7 & 0 & -10.6 \\ -41.1 & s+83.3 & -21 & 0 \\ 0 & -82.8 & s+171 & 0 \\ -K_f(s) & -K_w(s) & -K_{co}(s) & 1 \end{vmatrix} \begin{vmatrix} T_f^*(s) \\ T_w^*(s) \\ T_{co}^*(s) \\ W^*(s) \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ -67.6W_c^*(s) \\ K_c(s)W_c^*(s) \end{vmatrix} \tag{21}$$

Since T_w^* is the variable that is to be controlled, the set of Equations (21) is solved for $T_w^*(s)$.

$$T_w^*(s) = \left.\begin{vmatrix} s+136.3 & 0 & 0 & -10.6 \\ -41.1 & 0 & -21 & 0 \\ 0 & -67.6W_c^*(s) & s+171 & 0 \\ -K_f(s) & K_c(s)W_c^*(s) & -K_{co}(s) & 1 \end{vmatrix} \middle/ |\underline{\Delta}| \right. \qquad (22)$$

$$\text{where } |\underline{\Delta}| = \begin{vmatrix} s+136.3 & -31.7 & 0 & -10.6 \\ -41.1 & s+83.3 & -21 & 0 \\ 0 & -82.8 & s+171 & 0 \\ -K_f(s) & -K_w(s) & -K_{co}(s) & 1 \end{vmatrix}$$

Expanding the numerator of Equation (22) gives

$$T_w^*(s) = \frac{W_c^*(s)\left[(21)(-67.6)(s+136.3) - (10.6)(41.1)(67.6)K_{co}(s) + (10.6)(41.1)(s+171)K_c(s) + (10.6)(21)(67.6)K_f(s)\right]}{|\underline{\Delta}|} \qquad (23)$$

The invariance condition is T_w^* equals zero. The equivalent condition in operator notation is $T_w^*(s)$ equals zero. Assuming that $W_c^*(s)$ and det$|\underline{\Delta}|$ do not equal zero, Equation (24) is the first condition for invariance.

$$(21)(-67.6)(s+136.3) - (10.6)(41.1)(67.6)K_{co}(s) + (10.6)(41.1)(s+171)K_c(s) + (10.6)(21)(67.6)K_f(s) = 0 \qquad (24)$$

Because the K(s) terms are still completely arbitrary, there are infinitely many ways to satisfy Equation (24). Three examples are given.

1. $K_{co}(s) = K_c(s) = 0$; $K_f(s) = 0.0943\ s + 12.86$

   Substituting these values into Equation (20) gives the control law.

   $$W^*(s) = 0.0943\ s\ T_f^*(s) + 12.86\ T_f^*(s) \qquad (25)$$

   This result is the Laplace transformed Equation (11).

2. $K_f(s) = K_c(s) = 0$; $K_{co}(s) = -0.0482\ s - 6.57$

   From Equation (20)

$$W^*(s) = -0.0482\ s\ T_{co}^*(s) - 6.57\ T_{co}^*(s) \qquad (26)$$

This result is the Laplace transformed Equation (12).

3. $K_f(s) = K_{co}(s) = 0;\ K_c(s) = 3.26(s+136.3)/(s+171)$

From Equation (20)

$$W^*(s) = \frac{3.26\ s\ W_c^*(s) + 444\ W_c^*(s)}{s + 171} \qquad (27)$$

This result is the Laplace transformed Equation (13).

In addition to these three cases there are infinite combinations of controllers K(s) which will satisfy Equation (24).

The dual channel theorem must be satisfied as before. It can be checked physically and topologically as shown previously. The mathematical checking procedure is convenient in matrix notation because it consists of examining the determinant $|\underline{\Delta}|$. For example, for Equation (25)

$$|\underline{\Delta}| = \begin{vmatrix} s+136.3 & -31.7 & 0 & -10.6 \\ -41.1 & s+83.3 & -21 & 0 \\ 0 & -82.8 & s+171 & 0 \\ -.0943s+12.86 & 0 & 0 & 1 \end{vmatrix} \qquad (28)$$

If the validity of control law Equation (25) is desired, the dual channel condition must be checked in a manner similar to that done before. Since $T_W^*(s)$ cannot affect the equations for $T_f^*(s)$ and $T_{co}^*(s)$, the cross terms in the matrix can be equated to zero, whereupon Equation (28) becomes

$$|\underline{\Delta}| = \begin{vmatrix} s+136.3 & 0 & 0 & -10.6 \\ -41.1 & s+83.3 & -21 & 0 \\ 0 & 0 & s+171 & 0 \\ -.0943\ s+12.86 & 0 & 0 & 1 \end{vmatrix} \qquad (29)$$

Expansion of Equation (29) results in Equation (30).

$$\det|\underline{\Delta}| = (s+136.3)(s+83.3)(s+171) - (10.6)(s+83.3)(s+171)(.0943\ s + 12.86) \qquad (30)$$

This equation reduces to

$$\det\left|\underline{\Delta}\right| = 0 \qquad (31)$$

Thus, the mathematical interpretation of the dual channel theorem in matrix form is the following: two channels do not exist (invariance is not possible) when $\det\left|\underline{\Delta}\right| = 0$ for the system model containing the invariance controller with controlled variable "cross-terms" equated to zero.

Examination of Equation (24) quickly indicates that $K_w(s)$ has no effect on the invariance conditions. This fact reinforces the statement that conventional feedback cannot provide invariance, and the system with feedback control has to tolerate some error in the controlled variable.

The advantages of linear operator matrix notation can be listed.

1. Differential equations are transformed to algebraic ones.
2. High order systems can be conveniently and methodically analyzed.
3. One equation defines all invariance control conditions; e.g. Equation (24).
4. Control equations are more easily found and all possible controller combinations are more evident.
5. The mathematical form of the dual channel theorem is convenient and concise.
6. Control equations appear in transfer function form in terms of the operational variable. Control synthesis is convenient because of the numerous techniques for constructing active and passive networks having desired transfer functions.

Choice of Model Types

The entire discussion in this chapter has been concerned with one kind of mathematical model--the linear perturbation Equations (8). Another type of linear model is the total variable model, in which the steady-state solutions have not been subtracted out. For example, such a model is the set of Equations (4) with the product nonlinearity term linearized by a Taylor series expansion.

$$\begin{aligned} \dot{T}_f &= -136.3\ T_f + 31.7\ T_w + 10.6\ W + 14{,}244 \\ \dot{T}_w &= -83.3\ T_w + 41.1\ T_f + 21.0\ T_{co} + 658 \\ \dot{T}_{co} &= -171\ T_{co} + 82.8\ T_w - 67.6\ W_c + 6415 \end{aligned} \tag{32}$$

The same procedure for the application of the invariance conditions and for the check for the presence of two channels can be followed. Any constants which may appear, as in Equation (32), are placed on the right-hand side of the invariance matrix Equation (21). These constants then enter into the calculations of the controllers and ultimately appear in the control law equations.

The choice of which type of model to use is normally dictated by convenience and by the final use of the derived control equations. Several comments about each type of model should be made. Further examples of the perturbation model usage and examples of the total variable model usage will appear in later chapters.

Linear Perturbation Model

1. These equations are convenient for Laplace transformation because the initial conditions are zero.
2. There are no constant terms to complicate the algebra.
3. These equations are very convenient for analog computer simulation because of the absence of constant terms and initial conditions. Furthermore, the analog computer scaling problem is simplified by the removal of large constants such as appear in Equation (32). Perturbation equations also increase the accuracy of analog computation.
4. The objection to perturbation equations is that only the dynamic controller equations result from an application of invariance theory. The physical application of Equation (12) would be something like Figure 6a. In a process the temperature which is measured is normally a total variable and not just the transient portion of it, and the output signal usually wanted is proportional to the total variable also. Therefore, Equation (12) would have to be implemented in the manner of Figure 6b. This configuration requires the addition to the control system of two set-points, $W_{c_{ss}}$ and $T_{co_{ss}}$, both of which must be determined in some way from the desired wall temperature (e.g.

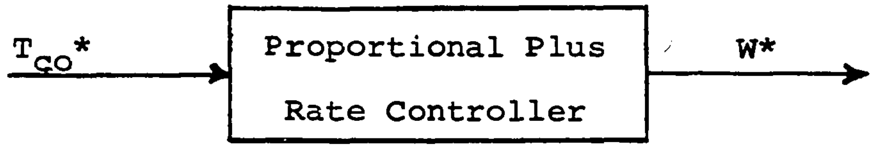

Figure 6a

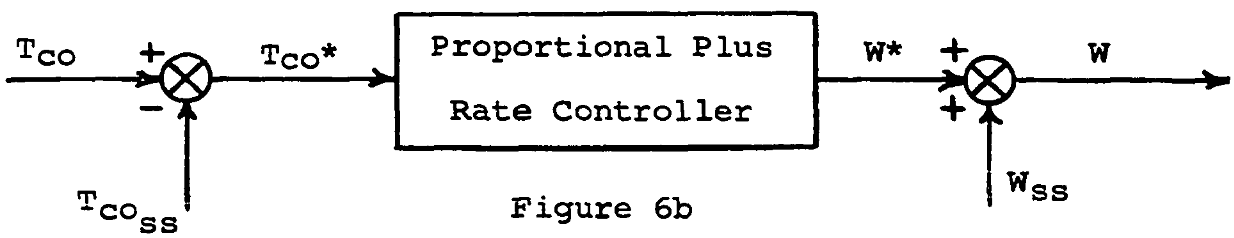

Figure 6b

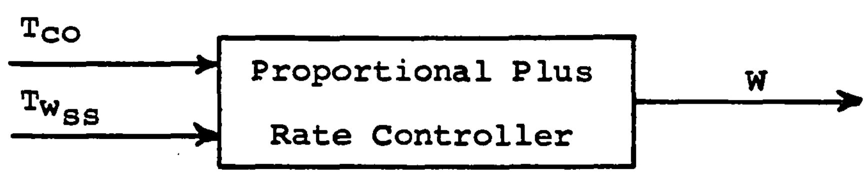

Figure 6c

Figure 6. Physical Application of a Feedback Controller on T_{co} for the Control of T_w Using (a) Perturbation Model and Transient Variables, (b) Perturbation Model and Total Variables, and (c) Total Variable Model and Total Variables.

by the steady-state model equations). Furthermore, these set-points are not of particular interest to the control user. The set-point of interest is normally that for the controlled variable--in this case, $T_{w_{ss}}$.

Linear Total Variable Model

1. Although Laplace transforms can be used for these equations, the nonzero initial conditions make such transformations more complicated. Differential operators are about as convenient.
2. These equations can be programmed for the analog computer, but several complications are encountered: computer variable scaling, introduction of constants and initial conditions, and decreased accuracy.
3. The primary advantage of using the total variable model is that the controller equations resulting from the theory are more useful for application to real systems. If invariance theory is applied to Equations (32) for the purpose of making T_w invariant with respect to F_c by measuring T_{co}, the resulting controller equation would be diagrammed as Figure 6c. The set-point $T_{w_{ss}}$ is now the one desired. The total variable model has provided within itself the mechanism for calculating the two steady-state set-points which had

to be calculated externally when the perturbation model was used for synthesis. The price of this convenience is a more complex controller because, as will be shown later, the set-point is not always used in the controller in a simple manner.

4. In using the total variable model there are two slight modifications to the theory as presented. The invariance condition is no longer $\dot{T}_w{}^*$ and $T_w{}^*$ equal to zero, but is now

$$\dot{T}_w = 0; \; T_w = T_{w_{ss}} \qquad (33)$$

The dual channel condition which formerly was determined while the cross terms from $T_w{}^*$ in the model were zero, should now be checked when the cross terms from T_w are equal to $T_{w_{ss}}$.

Choice of Operators

For either of the linear model types the choice of linear operator is simply a matter of personal preference. It makes no difference whether the Laplacian operator or the differential operator is used. Because functional relationships are looked-for rather than unique solutions to sets of differential equations, the usual arguments about the merits of Laplacian or differential operators are not pertinent. The important point to remember--whatever operator is selected--is that the proper mathematical rules for operators be followed. Some comments concerning two common types of operators follow.

Laplacian Operator (s)

1. Laplace transformations require introduction of initial conditions in specified ways.
2. Constants, such as those in Equation (32), are required to be transformed properly. The Laplace transformation of constants tends to increase the order of algebraic polynomials in s. This result complicates the use of Laplacian operators for total variable equations.
3. Matrix algebra can proceed rapidly once the equations are transformed correctly, and the order of operations is not important.
4. The controller transfer functions appear in Laplacian operator form after the theoretical synthesis procedure. Many techniques are available for synthesizing networks to approximate these transfer functions.

Differential Operator (D)

1. Initial conditions are not important for this theory. This fact simplifies the theoretical synthesis.
2. Constants are no problem with these operators.
3. Matrix algebra is more difficult because the order of operation is important. Particular care must be exercised in performing the algebraic equation operations, such as expanding

a determinant. Serious errors will result if the differential operator rules are not followed.

4. Although there are few methods of synthesizing networks from differential operator "transfer functions," it should be recognized that the differential operator and Laplacian operator transfer functions are identical. Therefore, there is no problem in finding practical controller devices.

Nonlinear Systems

This chapter is concerned with a heuristic presentation of invariance theory, which is best given initially for linear systems. The reader should <u>not</u> assume that this theory is only applicable to linear systems, despite the preoccupation with them thus far. Unfortunately, the impression received from most of the literature on invariance theory is that invariance is a linear technique. <u>Invariance can be applied to nonlinear systems</u>. Of course, the previously discussed linear operator with matrix algebra techniques cannot be used. The theory must be approached in the same time domain manner that was used for the theory as presented at the beginning of this chapter. Although Laplace transformations cannot be used, differential operators can be used to some extent, so long as solutions to nonlinear differential equations are not required. Either total or perturbation variable models can be used.

A brief example from the original problem of invariance of the reactor wall temperature will illustrate the procedure. As before, the first invariance condition is defined by Equation (9). With this constraint Equations (7) become Equations (34).

$$\dot{T}_f^* = -136.3T_f^* + 10.6W^* - 0.666W^*T_f^* \tag{34a}$$

$$0 = +41.1T_f^* + 21.0T_{co}^* \tag{34b}$$

$$\dot{T}_{co}^* = -171T_{co}^* - 67.6W_c^* - 2.4W_c^*T_{co}^* \tag{34c}$$

Equation (34a) can be solved for W*.

$$W^* = \frac{\dot{T}_f^* + 136.3T_f^*}{10.6 - 0.666T_f^*} \tag{35}$$

Substituting Equation (34b) into (35) yields a nonlinear control law.

$$W^* = -\frac{\dot{T}_{co}^* + 136.3T_{co}^*}{20.7 + 0.666T_{co}^*} \tag{36}$$

The dual channel theorem can be checked physically or topologically in a manner similar to linear system analysis, but a mathematical verification is generally impossible because the analytical solutions to the nonlinear differential equations are unknown. For a nonlinear system whose behavior does not remarkably differ from its linearized model, the verification of the existence of two channels in the linear model also applies to the nonlinear model.

Equation (36) indicates that the nonlinear model has given rise to a nonlinear controller. However, nonlinear models do not always result in nonlinear controllers (examples will be presented in later chapters). The specification of linear or nonlinear controllers depends on the particular configuration of the system, i.e. the specific location of the nonlinearities. It has been found that a nonlinearity in either of the dual channels results in a nonlinear controller, whereas, if both channels are linear, the controller is linear.

Chapter IV will present the essence of invariance theory in more formal notation. The abstract mathematical presentation should be easier to follow now that concepts are understood.

CHAPTER IV

FORMAL PRESENTATION OF THE THEORY

In the previous chapter the basic methods of applying invariance theory have been discussed in relation to a specific system and problem. A comprehension of the invariance approach permits the development in this chapter of a more general theory and the necessary change to more abstract nomenclature.

Consider the generalized chemical process shown in Figure 7.

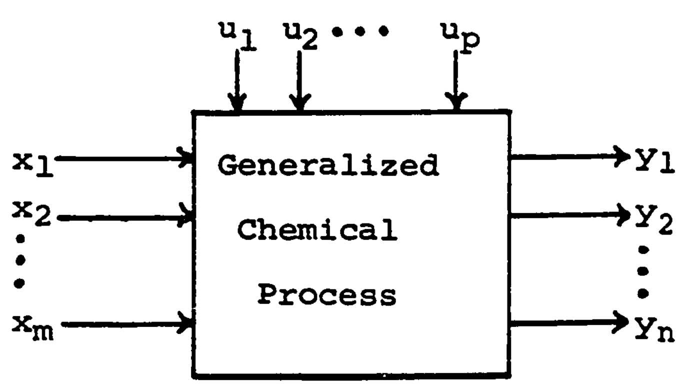

Figure 7. Generalized Chemical Process

There are n measurable outputs, y_i; m controllable inputs, x_j; p measurable and uncontrollable inputs, u_k. The number "n" will be referred to as the order of the system.

It is possible to write a set of equations describing the system in vector notation.

$$\underline{\dot{y}}(t) = \underline{\underline{A}}\underline{y}(t) + \underline{\underline{B}}\underline{x}(t) + \underline{\underline{C}}\underline{u}(t) + \underline{G} + \underline{\Phi}(\underline{y},\underline{x},\underline{u}) \tag{37}$$

where $\underline{\underline{A}}$ is a constant coefficient n x n matrix,
$\underline{\underline{B}}$ is a constant coefficient n x m matrix,
$\underline{\underline{C}}$ is a constant coefficient n x p matrix,
$\underline{\underline{G}}$ is a constant coefficient n x 1 matrix,

$\underline{y}$ (t) is a column n matrix,
$\underline{x}$ (t) is a column m matrix,
$\underline{u}$ (t) is a column p matrix,
$\underline{\Phi}$ is a column n matrix,
and the dot ($\dot{}$) refers to a derivative with respect to time.

Each Φ_i contains the model nonlinear terms, which may be functions of one or more of the y_i, x_j, or u_k variables. The coefficient matrices, $\underline{A}$, $\underline{B}$, $\underline{C}$, and $\underline{G}$, are functions of the constant parameters of the system, such as physical properties of the constituents or system dimensions. Sets of equations like (37) normally arise from material and energy balances made on parts of the system. For most chemical engineering rate operations the equations are in this form initially. For dynamic systems which are usually expressed with equations of higher order time derivatives than first, state-space techniques* can be used to obtain sets of equations in the form of Equation (37). However, for such systems where all the state-space variables cannot be measured, it may be preferable to work with the original sets of higher order time derivative equations. Invariance theory techniques are completely applicable for these cases, but this work will be concerned with equations of type (37).

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*Cf. Zadeh (92).

In this investigation only lumped-parameter systems will be considered. Distributed-parameter systems are best described by partial differential equations which, although frequently containing only first order time derivatives, have first or higher order partial derivatives with respect to one or more dimensional variables. These systems appear to require a great many assumptions including linear models and to demand approximation techniques for identifying simple, useable models. The subsequent results of such assumptions can be a set of differential equations similar to (37), except that $\underline{\Phi}(\underline{x}, \underline{y}, \underline{u})$ contains partial derivatives of variables with respect to dimensions. Linearization techniques for small perturbations can be used*, but the constant coefficients are then functions of the steady-state dimensional derivatives. Such models are difficult to use.

In addition to Equations (37) one can write m general control equations.

$$\underline{x}\ (t) = \underline{\Psi}\ (\underline{u}\ (t),\ \underline{y}\ (t)\ ) \tag{38}$$

where $\underline{\Psi}$ is a linear or nonlinear function of all or part of the vectors $\underline{u}$ (t) and $\underline{y}$ (t).

The control law, $\underline{x}$ (t), which is being sought is best considered to be not a unique function of time, but a function of $\underline{u}$ (t) and $\underline{y}$ (t). This distinction is made because only a relationship between $\underline{x}$ (t), $\underline{u}$ (t), and $\underline{y}$ (t) is needed rather than these variables as time solutions to Equations (37) and

*Cf. Tinkler and Lamb (83).

(38). The vector, $\underline{x}$ (t), is actually determined in practice by measuring the necessary components of $\underline{u}$ (t) and $\underline{y}$ (t), and by transmitting this information through control channels to the point where the input vector, $\underline{x}$ (t), is manipulated.

Whenever the information channel is between an output variable which is to be controlled and an input manipulated variable, it shall be termed <u>primary feedback</u>. When the information channel is between any other output variable and a manipulated input variable, it shall be termed <u>secondary feedback</u>. When the information channel is between a disturbance input variable and a manipulated input variable, it shall be termed <u>feedforward</u>.

Linear Systems

For linear systems $\underline{\Phi}(\underline{x}, \underline{y}, \underline{u})$ in Equation (37) is equal to zero. If all the variables are transients due to subtracting the steady-state solution equations, then $\underline{G} = 0$ and the set of equations can be transformed into the Laplace domain with zero initial conditions.

$$\underline{A}\,(s)\ \underline{Y}^*(s) - \underline{B}\ \underline{X}^*(s) = \underline{C}\ \underline{U}^*(s) \qquad (39)$$

where $\underline{A}\,(s) = s\underline{I} - \underline{A}$, and $\underline{I}$ is the identity (diagonal) matrix, and s is the Laplacian operator.

A general linear form of Equation (38) is transformed to give the Laplacian general control equation.

$$\underline{K}_y(s)\ \underline{Y}^*(s) + \underline{K}_x(s)\ \underline{X}^*(s) = \underline{K}_u(s)\ \underline{U}^*(s) \qquad (40)$$

where the $\underline{K}$ (s) terms are the undetermined controller transfer functions.

Equations (39) and (40) are combined to give

$$\underline{H}\ (s)\ \underline{Z}^*(s) = \underline{C}\ (s)\ \underline{U}^*(s) \tag{41}$$

where $\underline{Z}^*(s) = \begin{bmatrix} \underline{Y}^*(s) \\ \underline{X}^*(s) \end{bmatrix}$ and is $(n + m) \times 1$ dimensional,

$\underline{H}\ (s) = \begin{bmatrix} \underline{A}\ (s) & -\underline{B} \\ \underline{K}_y(s) & \underline{K}_x(s) \end{bmatrix}$ and is $(n + m) \times (n + m)$ dimensional,

$\underline{C}\ (s) = \begin{bmatrix} \underline{C} \\ \underline{K}_u(s) \end{bmatrix}$ and is $(n + m) \times p$ dimensional,

and $\underline{K}_y(s)$ is $m \times n$ dimensional, $\underline{K}_x(s)$ is $m \times m$ dimensional, and $\underline{K}_u(s)$ is $m \times p$ dimensional.

At the moment complete freedom exists to choose the elements in the $\underline{K}$ (s) matrices in order to obtain an output vector which satisfies some performance criterion. Normally, in a chemical process one is not interested in a criterion based on all outputs and inputs, like the one which Balchen (5) suggests. The usual design problem involves one or, at worst, a selected few outputs which for economics, safety, quality, or other reasons are of greatest control interest.

Invariance theory deals with each output of interest in relation to each disturbance input. Thus, for invariance of $y_i(t)$ with respect to $u_k(t)$, by solving Equation (41) for $Y_i^*(s)/U_k^*(s)$, one obtains

$$\frac{Y_i^*(s)}{U_k^*(s)} = \frac{|\underline{H}_c(s)|}{|\underline{H}\ (s)|} \tag{42}$$

where $\underline{H}_c(s)$ is $\underline{H}$ (s) with the i^{th} column replaced by the k^{th} column of $\underline{C}$ (s).

Since it is assumed that $\det\left|\underline{H}\ (s)\right| \neq 0$, $Y_i^*(s)$ and, hence, $y_i^*(t) = 0$ when

$$\det\left|\underline{H}_c(s)\right| = 0 \tag{43}$$

Equation (43) is the first condition for absolute invariance. The designer is free to choose from among many types of control systems (feedback, feedforward, and combined systems) and to select any functions of s, so long as Equation (43) is satisfied. Once the controller transfer functions ($\underline{K}$ (s) terms) are picked, the control law is defined by Equation (40). The major limitation to the <u>mathematical</u> realization of invariance is the necessity of satisfying one other condition. This second condition will be discussed in a later section on the dual channel theorem.

Equation (43) illustrates the point that primary feedback plays no role in the attainment of absolute invariance because $\underline{H}_c(s)$ does not contain the element of the $\underline{K}_y(s)$ matrix which corresponds to the i^{th} (primary) feedback. If absolute invariance cannot be obtained, primary feedback, when present, would affect $y_i(t)$ through the denominator $\left|\underline{H}\ (s)\right|$ in Equation (42). Primary feedback will normally be desired in addition to the invariance controllers to correct for minor disturbances of unspecified origin.

If total variables are used in the linear model, then $\underline{G} \neq 0$, and the equation which is the equivalent of (39) is

$$\underline{A}\ (D)\ \underline{y}\ (t) - \underline{B}\ \underline{x}\ (t) = \underline{C}\ \underline{u}\ (t) + \underline{G} \tag{44}$$

where $\underline{A}(D) = D\,\underline{I} - \underline{A}$ and D is the differential operator.*

The general form of Equation (38) is the following:

$$\underline{K}_y(D)\ \underline{y}\ (t) + \underline{K}_x(D)\ \underline{x}\ (t) = \underline{K}_u(D)\ \underline{u}\ (t) \tag{45}$$

Combining Equations (44) and (45) gives

$$\underline{H}\ (D)\ \underline{z}\ (t) = \underline{C}\ (D)\ \underline{u}\ (t) + \underline{G} \tag{46}$$

where $\underline{z}\ (t) = \begin{bmatrix} \underline{y}\ (t) \\ \underline{x}\ (t) \end{bmatrix}$ and is (n+m) dimensional,

$\underline{H}\ (D) = \begin{bmatrix} \underline{A}(D) & -\underline{B} \\ \underline{K}_y(D) & \underline{K}_x(D) \end{bmatrix}$ and is (n+m) x (n+m) dimensional,

$\underline{C}\ (D) = \begin{bmatrix} \underline{C} \\ \underline{K}_u(D) \end{bmatrix}$ and is (n+m) x p dimensional,

and $\underline{K}_y(D)$ is m x n dimensional, $\underline{K}_x(D)$ is m x m dimensional, and $\underline{K}_u(D)$ is m x p dimensional.

Solving Equation (46) for $y_i(t)$ gives

$$y_i(t) = \frac{|\underline{H}_{cg}(D)|}{|\underline{H}\ (D)|} \tag{47}$$

where $\underline{H}_{cg}(D)$ is $\underline{H}(D)$ with the i^{th} column replaced by the sum of $\underline{G}$ and the product of the k^{th} column of $\underline{C}(D)$ and $u_k(t)$.

Again, assuming that det $|\underline{H}\ (D)| \neq 0$, $y_i(t) = y_{i_{ss}}$ when

$$\frac{|\underline{H}_{cg}(D)|}{|\underline{H}\ (D)|} = y_{i_{ss}} \tag{48}$$

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*The reason for the switch to differential operators from Laplacian operators is simply the preference of the author.

Since $y_{i_{ss}}$ is a constant, $\det|\underline{H}(D)|\, y_{i_{ss}}$ is also a constant. This number is the static controller specification. The dynamic controller specification (feedback or feedforward) is included in $|\underline{H}_{cg}(D)|$ and the control law is defined by Equation (45), subject to the restraints of Equation (48) and the dual channel theorem.

Depending on personal preference, the first condition of invariance can be determined using either the Laplacian or the differential operator technique. It is possible to proceed directly from either operator space controller specification to real or simulated controllers without returning to the time domain.

To indicate the manner of treatment of nonlinear models, it is worthwhile to state the time-domain procedure which is equivalent to the matrix manipulations just discussed. For perturbation equations in the time domain, invariance conditions are obtained by setting $y_i^*(t)$ and its derivative equal to zero, and also setting all of $\underline{u}^*(t)$ and $\underline{x}^*(t)$ equal to zero except $u_k^*(t)$ and $x_j^*(t)$. There are now enough equations to eliminate n-1 of the output variables and to get the feedforward control law

$$x_j^*(t) = \Psi\left(u_k^*(t)\right) . \tag{49}$$

If it is desired to synthesize a secondary feedback controller, the procedure is to set $y_i^*(t)$ and its derivative equal to zero and also all of $\underline{u}^*(t)$. With all of $\underline{x}^*(t)$ except $x_j^*(t)$ equal to zero, there are enough equations to eliminate n-2 of the output variables, giving the control law

$$x_j^*(t) = \Psi\left(y_q^*(t)\right) \tag{50}$$

where $q \neq i$.

For the total variable model the procedure is practically identical, except that $y_i(t)$ is equated to $y_{i_{ss}}$ instead of to zero. There are enough equations to reduce the set to one equation (the control law), which is written in terms of $y_{i_{ss}}$ and a measured input or output and a manipulated input.

Nonlinear Systems

Because $\underline{\Phi}(\underline{x}, \underline{y}, \underline{u})$ in Equation (37) is not zero for nonlinear systems, operational methods have limited use. However, it is possible to determine the invariance control law, at least for some important types of nonlinearities. Although correct functional relationships in the time domain are required, it is not necessary to find solutions to nonlinear differential equations. Thus, this task is not as difficult as it first appears.

As shown in the previous section, the proper procedure is to equate the variable which is to be made invariant to either zero or the steady-state value, depending on whether the perturbation or total variable model is used. With the derivative of the controlled variable equal to zero, all the variables except the measured disturbance or secondary variable and the manipulated input are eliminated from the equations.

Product nonlinearities which commonly occur in chemical processes can be handled quite easily. However, other types of nonlinear functions, such as the Arrhenius (exponential) relationship, cause some difficulties in the elimination of variables. This specific problem will not be discussed in this work. It does not appear to present an insurmountable difficulty, but as the nonlinearities become more complex, the control law may also. The Arrhenius type of nonlinearity could require difficult and costly controllers unless some assumptions can be made practically. The only foreseeable obstacle for the successful applications of invariance to theoretical nonlinear systems is the occurrence of a double-valued function, such as the hysteresis phenomenon. Invariance conceptually appears to demand system relationships which have single-valued inverse relationships. The hysteresis effect presents no such relationship.

Dual Channel Theorem

The control law resulting from equating the invariant variable to its steady-state value and its derivative to zero (and the equivalent matrix operation) in linear and nonlinear systems is not always adequate to insure invariance. For linear systems it can be said that Equation (43) or (48) is a necessary, but not sufficient, condition for absolute invariance. For nonlinear systems the same is true. The mathematical statement of the sufficient condition for invariance has been given by Petrov (69). It is restated here in present terminology.

> An essential requirement for the realization of the conditions of invariance of the variable $y_i(t)$ is that there must be an identical matching of (a) the set of solutions of the system and invariant devices, and (b) the same set, but with the output of the y_ith coordinate fixed at steady-state conditions (zero for perturbation models and a constant for total variable models).

A simple checking procedure for matrix operations is the following: invariance cannot occur when det $|\underline{H}| = 0$.

This sufficient condition requirement is related to the dual channel theorem, which is a topological restatement of the mathematical form just given.

> There must be two channels of disturbance information which reach the controlled variable in the block diagram of the system (process and controller) under conditions of invariance <u>and</u> with the controlled variable output providing no dynamic information.

The phrase "no dynamic information" is the topological equivalent mathematically to setting the output of the controlled variable to zero or to its steady-state value.

The dual channel theorem can be applied from the mathematical, physical, or topological viewpoints. Quite often it will be obvious from the physical system that certain control schemes are not allowed. If more formal proof is necessary, then the mathematical form can be applied; however, this choice demands solutions to sets of differential equations, which are tedious for linear systems of high order and usually impossible for nonlinear systems. The most useful approach is the topological one, if diagram rearrangements are not allowed. Even nonlinear equations can be represented with nonlinear function-generation boxes. Dual channel existence can be visually checked and equation solutions are no longer necessary.

An additional benefit of examining topological structures of nonlinear systems is the indication provided of the type of controller necessary for invariance of nonlinear system variables. An important result, which has been formulated by this author as a hypothesis from examination of specific nonlinear systems is the following:

> If either dual information channel from the measured variable to the variable which is to be made invariant contains a nonlinearity, then the controller to be synthesized must be nonlinear also, while if both channels are linear, then the controller will also be linear.

This hypothesis implies that all nonlinear processes do not require nonlinear controllers. Examples will be given in later chapters. The necessity for certain types of controllers depends, as always, on the particular system topology.

CHAPTER V

ANALOG COMPUTER SIMULATION STUDIES

In this chapter the analog computer simulation studies using the mathematical model derived in Chapter III are described. The purposes of this investigation are the following:

1) To demonstrate the use of linear perturbation models, Laplace transformations, and matrix algebra for the derivation of invariance controller transfer functions.
2) To demonstrate the technique for the derivation of nonlinear invariance control laws using time-domain methods.
3) To illustrate the use of the analog computer as a tool for investigating the feasibility of the linear and nonlinear controllers designed for the purpose of invariance.
4) To compare the quality of control provided by linear and nonlinear controllers.

This last purpose will demonstrate the ideal behavior of an invariance control system because the "process" (analog computer) is "well behaved" and is accurately described by the mathematical model. The term "well behaved" refers to the fact that the analog computer signals have very little noise present and only those dead times, lags, and nonlinearities which were specifically programmed are present.*

The first section will contain the analog computer controller derivations. The second section will cover the analog computer programming techniques, and the last section will describe the experimental procedure for the simulation studies.

Analog Computer Controller Derivations

The model which will be used has been derived in Chapter III. This present chapter will deal exclusively with the perturbation models, leaving the total variable model usage to be described in Chapter VI. For convenience of analog computation, the nonlinear perturbation model Equations (7) will be scaled for magnitude and time. Magnitude scaling is used primarily for the purpose of making all voltages which are analogous to process variables of equal magnitude and large enough for good accuracy on a $\pm$ 100 volt analog computer. Time scaling is used to increase computational speed and to adjust the model coefficients so that they are easily set on analog computer coefficient potentiometers. The chosen time and magnitude scale factors are given in Table 2.

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*There is a tendency for analog programmers to consider only those problems that are convenient.

TABLE 2

ANALOG COMPUTER TIME AND MAGNITUDE SCALE FACTORS

Analog Variable	Process Variable	Conversion Units		
t_f	20 T_f^*	1 °F	equals	20 volts
t_w	10 T_w^*	1 °F	equals	10 volts
t_{co}	5 T_{co}^*	1 °F	equals	5 volts
f	W*	1 lb./hr.	equals	1 volts
f_c	2 W_c^*	1 lb./hr.	equals	2 volts
θ	t/60	1 minute in process	equals	1 second on computer

When the scale factors in Table 2 are introduced into Equations (7), the analog computer equations result.

$$\dot{t}_f = -2.27t_f + 1.06t_w + 3.53f - 0.0111\, f\, t_f$$

$$\dot{t}_w = -1.39t_w + 0.343t_f + 0.700t_{co} \tag{51}$$

$$\dot{t}_{co} = -2.85t_{co} + 0.690t_w - 2.82f_c - 0.0200f_c\, t_{co}$$

For this study the disturbance variable will always be f_c and the manipulated input always f. Equations (51) can be written in the general notation of Chapter IV.

$$\dot{y}_1^* = k_1y_1^* + k_2y_2^* + k_3u_1^* + k_4u_1^*y_1^*$$

$$\dot{y}_2^* = k_5y_2^* + k_6y_1^* + k_7y_3^* \tag{52}$$

$$\dot{y}_3^* = k_8y_3^* + k_9y_2^* + k_{10}x_1^* + k_{11}x_1^*y_3^*$$

where $u_1^* = f_c$ $\quad k_1 = -2.85$ $\quad k_7 = +0.343$

$x_1^* = f$ $\quad k_2 = +0.690$ $\quad k_8 = -2.27$

$y_1^* = t_{co}$ $\quad k_3 = -2.82$ $\quad k_9 = +1.06$

$y_2^* = t_w \qquad k_4 = -0.0200 \qquad k_{10} = +3.53$

$y_3^* = t_f \qquad k_5 = -1.39 \qquad k_{11} = -0.0111$

$k_6 = +0.700$

The topological representation of Equation (52) is Figure 8.

For this system $\underline{G} = 0$ in Equation (37). The general Equation (53) for this system can be compared to the specific Equation (54).

$$\dot{\underline{y}} = \underline{A}\,\underline{y} + \underline{B}\,\underline{x} + \underline{C}\,\underline{u} + \underline{\Phi} \tag{53}$$

$$\begin{vmatrix} \dot{y}_1^* \\ \dot{y}_2^* \\ \dot{y}_3^* \end{vmatrix} = \begin{vmatrix} k_1 & k_2 & 0 \\ k_6 & k_5 & k_7 \\ 0 & k_9 & k_8 \end{vmatrix} \begin{vmatrix} y_1^* \\ y_2^* \\ y_3^* \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ k_{10} \end{vmatrix} \begin{vmatrix} x_1^* \end{vmatrix} + \begin{vmatrix} k_3 \\ 0 \\ 0 \end{vmatrix} \begin{vmatrix} u_1^* \end{vmatrix} + \begin{vmatrix} k_4 y_1^* u_1^* \\ 0 \\ k_{11} y_3^* x_1^* \end{vmatrix} \tag{54}$$

For this system n = 3, m = 1, and p = 1.

Linear System

For this model $\underline{\Phi}$ is equal to 0. Equation (39)

$$\underline{A}(s)\,\underline{Y}^*(s) - \underline{B}\,\underline{X}^*(s) = \underline{C}\,\underline{U}^*(s) \tag{39}$$

specifically is

$$\begin{vmatrix} s - k_1 & -k_2 & 0 \\ -k_6 & s - k_5 & -k_7 \\ 0 & -k_9 & s - k_8 \end{vmatrix} \begin{vmatrix} Y_1^*(s) \\ Y_2^*(s) \\ Y_3^*(s) \end{vmatrix} - \begin{vmatrix} 0 \\ 0 \\ k_{10} \end{vmatrix} \begin{vmatrix} X_1^*(s) \end{vmatrix} = \begin{vmatrix} k_3 \\ 0 \\ 0 \end{vmatrix} \begin{vmatrix} U_1^*(s) \end{vmatrix} \tag{55}$$

The control equation is written.

$$\underline{K}_y(s)\,\underline{Y}^*(s) + \underline{K}_x(s)\,\underline{X}^*(s) = \underline{K}_u(s)\,\underline{U}^*(s) \tag{40}$$

$$\begin{vmatrix} K_{y1}(s) & K_{y2}(s) & K_{y3}(s) \end{vmatrix} \begin{vmatrix} Y_1^*(s) \\ Y_2^*(s) \\ Y_3^*(s) \end{vmatrix} + \begin{vmatrix} K_{x1}(s) \end{vmatrix} \begin{vmatrix} X_1^*(s) \end{vmatrix} = \begin{vmatrix} K_{u1}(s) \end{vmatrix} \begin{vmatrix} U_1^*(s) \end{vmatrix} \tag{56}$$

Combining Equations (55) and (56) gives the total system

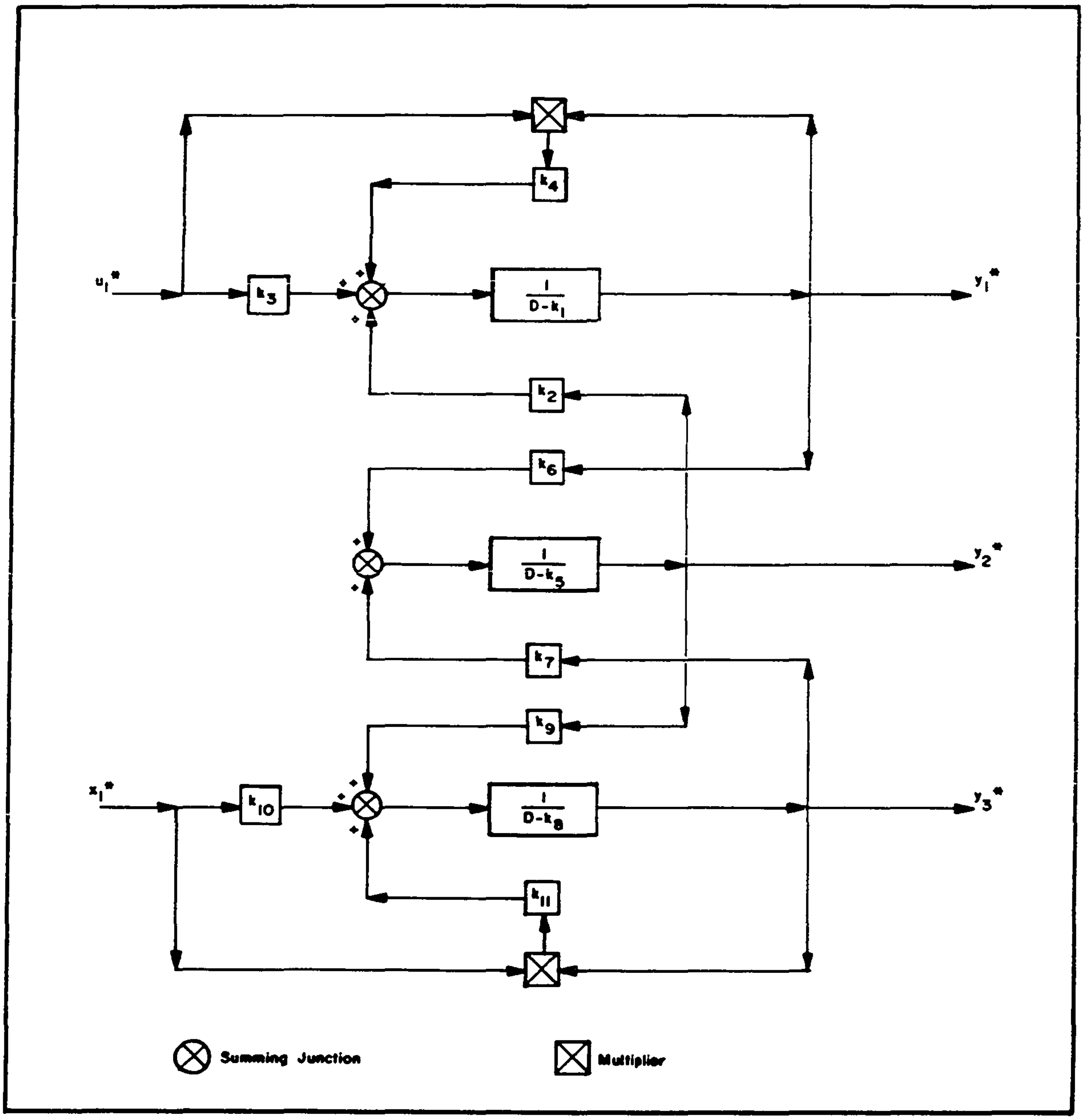


Figure 8. Block Diagram of the Analog Computer Model

matrix equation.

$$\begin{array}{ccc} \underline{H}(s) & \underline{Z}^*(s) & = \underline{C}(s) \quad \underline{U}^*(s) \end{array}$$

$$\begin{vmatrix} s-k_1 & -k_2 & 0 & 0 \\ -k_6 & s-k_5 & -k_7 & 0 \\ 0 & -k_9 & s-k_8 & -k_{10} \\ K_{y1}(s) & K_{y2}(s) & K_{y3}(s) & K_{x1}(s) \end{vmatrix} \begin{vmatrix} Y_1^*(s) \\ Y_2^*(s) \\ Y_3^*(s) \\ X_1^*(s) \end{vmatrix} = \begin{vmatrix} k_3 \\ 0 \\ 0 \\ K_{u1}(s) \end{vmatrix} \begin{vmatrix} U_1^*(s) \end{vmatrix}$$

For invariance of y_1^* with respect to u_1^*, only feed-forward control is possible. This restriction is shown by checking the existence of dual channels in Figure 8. Thus, all of the elements in the $\underline{K}_y(s)$ matrix equal zero. With this simplification of Equation (57), Equation (42) is written in the following form.

$$\frac{Y_1^*(s)}{U_1^*(s)} = \frac{|\underline{H}_c(s)|}{|\underline{H}(s)|} = \frac{\begin{vmatrix} k_3 & -k_2 & 0 & 0 \\ 0 & s-k_5 & -k_7 & 0 \\ 0 & -k_9 & s-k_8 & -k_{10} \\ K_{u1}(s) & 0 & 0 & K_{x1}(s) \end{vmatrix}}{\begin{vmatrix} s-k_1 & -k_2 & 0 & 0 \\ -k_6 & s-k_5 & -k_7 & 0 \\ 0 & -k_9 & s-k_8 & -k_{10} \\ 0 & 0 & 0 & K_{x1}(s) \end{vmatrix}} \tag{58}$$

The determinant $|\underline{H}(s)|$ is known in linear systems theory as the characteristic polynomial. To determine the stability of this control system, the usual practice is to examine the roots of the characteristic equation

$$\det \underline{H}\,|(s)| = 0 \tag{59}$$

The roots can be found by a number of methods, or the number of roots with positive real parts can be found by the Routh stability criterion. For this controller (and all of the controllers in this work) there are no roots with positive real parts: stability is assured.

The invariance condition is given by Equation (43)

$$\det\left|\underline{H}_c(s)\right| = 0 \tag{43}$$

Expanding $\det\left|\underline{H}_c(s)\right|$ results in the control equation

$$0 = k_3\left[(s-k_5)(s-k_8)K_{x1}(s) - k_7k_9K_{x1}(s)\right] + k_2\, k_7k_{10}K_{u1}(s) \tag{60}$$

Although $K_{x1}(s)$ is still arbitrary, it is simplest to set

$$K_{x1}(s) = 1 \tag{61}$$

Solving Equation (60) for $K_{u1}(s)$ gives the controller transfer function.

$$K_{u1}(s) = \frac{-k_3}{k_2k_7k_{10}}\left[(s-k_5)(s-k_8) - k_7k_9\right] \tag{62}$$

From the control Equation (56), it is found that

$$\frac{X_1^*(s)}{U_1^*(s)} = K_{u1}(s) \tag{63}$$

Substituting numerical values and changing back to analog computer variables in Equations (62) and (63) results in the final analog computer controller transfer function for invariance of t_{co} by feedforward control on f_c.

$$\frac{F\ (s)}{F_c(s)} = 3.38(s^2 + 3.66s + 2.79) \tag{64}$$

For invariance of y_2^* with respect to u_1^* the dual channel theorem allows feedforward control on u_1^* and secondary feedback on y_1*. This permits the setting of $k_{y2}(s)$ and $K_{y3}(s)$ equal to zero. Equation (42) takes the following form:

$$\frac{Y_2^*(s)}{U_1^*(s)} = \frac{|\underline{H}_c(s)|}{|\underline{H}\ (s)|} = \frac{\begin{vmatrix} s-k_1 & k_3 & 0 & 0 \\ -k_6 & 0 & -k_7 & 0 \\ 0 & 0 & s-k_8 & -k_{10} \\ K_{y1}(s) & K_{u1}(s) & 0 & K_{x1}(s) \end{vmatrix}}{|\underline{H}\ (s)|} \tag{65}$$

The invariance condition, Equation (43) results in the control equation.

$$(s-k_1)k_7k_{10}K_{u1}(s) - k_3k_7k_{10}K_{y1}(s) - k_6(s-k_8)K_{x1}(s) = 0 \tag{66}$$

With $K_{x1}(s)$ set equal to one, there are an infinite number of possible transfer functions giving invariance. There are infinitely many combinations of $K_{y1}(s)$ and $K_{u1}(s)$ that satisfy Equation (66). All these cases constitute examples of combined feedforward and secondary control. This work is only concerned with pure feedforward or secondary feedback control and none of these combined control systems will be investigated. The feedforward controller transfer function is derived by setting $K_{x1}(s)$ equal to one and $K_{y1}(s)$ equal to zero and solving for $K_{u1}(s)$ in Equation (66).

$$K_{u1}(s) = \frac{-k_3k_6\ (s-k_8)}{k_7k_{10}(s-k_1)} \tag{67}$$

Equation (56) and the numerical substitutions give the final analog computer controller transfer function for invariance of t_w by feedforward control on f_c.

$$\frac{F\ (s)}{F_c(s)} = 1.63\ \frac{(s + 2.27)}{(s + 2.85)} \tag{68}$$

The secondary feedback controller transfer function is obtained by setting $K_{x1}(s)$ equal to one and $K_{u1}(s)$ equal to zero and solving for $K_{y1}(s)$ in Equation (66).

$$K_{y1}(s) = \frac{k_6(s - k_8)}{k_7\ k_{10}} \tag{69}$$

The final analog computer controller transfer function for invariance of t_w by secondary feedback on t_{co} is Equation (70).

$$\frac{F\ (s)}{T_{co}(s)} = -0.578(s + 2.27) \tag{70}$$

For invariance of y_3* with respect to u_1* the dual channel theorem allows feedforward control on u_1* and secondary feedback control on y_1* and y_2*. Equation (42) takes the following form with $K_{y3}(s)$ equal to zero.

$$\frac{Y_3^*(s)}{U_1^*(s)} = \frac{|\underline{H}_c(s)|}{|\underline{H}\ (s)|} = \frac{\begin{vmatrix} s-k_1 & -k_2 & k_3 & 0 \\ -k_6 & s-k_5 & 0 & 0 \\ 0 & -k_9 & 0 & -k_{10} \\ K_{y1}(s) & K_{y2}(s) & K_{u1}(s) & K_{x1}(s) \end{vmatrix}}{|\underline{H}\ (s)|} \tag{71}$$

The invariance condition Equation (43) results in the control equation.

$$(s-k_1)(s-k_5)k_{10}K_{u1}(s) - k_2k_6k_{10}K_{u1}(s) + k_3k_6k_9K_{x1}(s)$$
$$-k_3k_6k_{10}K_{y2}(s) - k_3k_{10}(s-k_5)\ K_{y1}(s) = 0 \qquad (72)$$

The feedforward controller transfer function is obtained by setting $K_{x1}(s)$ equal to one, $K_{y1}(s)$ and $K_{y2}(s)$ equal to zero, and by solving for $K_{u1}(s)$ from Equation (72).

$$K_{u1}(s) = \frac{-k_3\ k_6\ k_9\ /k_{10}}{(s-k_1)(s-k_5)-k_2k_6} \qquad (73)$$

The final analog computer controller transfer function for invariance of t_f by feedforward control on f_c is Equation (74).

$$\frac{F\ (s)}{F_c(s)} = \frac{0.593}{s^2 + 4.24s + 3.48} \qquad (74)$$

The secondary feedback controller on y_2* is obtained by setting $K_{x1}(s)$ equal to one, $K_{u1}(s)$ and $K_{y1}(s)$ equal to zero, and by solving Equation (72) for $K_{y2}(s)$.

$$K_{y2}(s) = \frac{k_9}{k_{10}} \qquad (75)$$

The final analog computer controller transfer function for invariance of t_f by secondary feedback on t_w is Equation (76).

$$\frac{F\ (s)}{T_w(s)} = -0.300 \qquad (76)$$

The secondary feedback controller on y_1* is obtained by setting $K_{x1}(s)$ equal to one, $K_{u1}(s)$ and $K_{y2}(s)$ equal to zero, and by solving Equation (72) for $K_{y1}(s)$.

$$K_{y1}(s) = \frac{k_6k_9}{k_{10}(s-k_5)} \qquad (77)$$

The final analog computer controller transfer function for invariance of t_f by secondary feedback on t_{co} is Equation (78).

$$\frac{F\ (s)}{T_{co}(s)} = \frac{-0.210}{(s + 1.39)} \tag{78}$$

Combination feedbacks and feedforward will not be considered though there are an infinite number of such combinations which are possible.

Nonlinear System

For the synthesis of controller equations using the nonlinear model, Equations (52) are used. Since the allowable control schemes have been determined by application of the dual channel theorem to the linear model, it is not necessary to recheck the existence of dual channels in the nonlinear model. This correspondence between linear and nonlinear topology is not necessarily true for all nonlinear systems, but in the case of product type nonlinearities it is a safe assumption. Rather than attempt to find a general control equation, the procedure is one of finding the specific nonlinear control law which corresponds to each linear transfer function which was found using the linear model.

For invariance of y_1^* with respect to u_1^*, the feedforward control law will be found. The invariance conditions for the perturbation model used are

$$y_1^* = 0$$

$$\text{and} \quad \dot{y}_1^* = 0 \tag{79}$$

Equations (52) become the following set of equations.

$$0 = k_2 y_2^* + k_3 u_1^* \tag{80a}$$

$$\dot{y}_2 = k_5 y_2^* + k_7 y_3^* \tag{80b}$$

$$\dot{y}_3^* = k_8 y_3^* + k_9 y_2^* + k_{10} x_1^* + k_{11} x_1^* y_3^* \tag{80c}$$

In order to obtain x_1^* as a function of u_1^*, the variables y_2^* and y_3^* are eliminated from Equations (80). From Equation (80a)

$$y_2^* = \frac{-k_3}{k_2} u_1^* \tag{81}$$

and from Equations (80b) and (81)

$$y_3^* = \frac{\dot{y}_2^* - k_5 y_2^*}{k_7} = \frac{-k_3(\dot{u}_1^* - k_5 u_1^*)}{k_2\ k_7} \tag{82}$$

Differentiating Equation (82) with respect to time gives

$$\dot{y}_3^* = \frac{-k_3(\ddot{u}_1^* - k_5 \dot{u}_1^*)}{k_2\ k_7} \tag{83}$$

Equations (81), (82), and (83) are substituted into Equation (80c) to give the following relationship.

$$\begin{aligned} \frac{-k_3}{k_2 k_7}\ddot{u}_1^* + \frac{k_3 k_5}{k_2 k_7}\dot{u}_1^* &= \frac{-k_8 k_3}{k_2 k_7}\dot{u}_1^* + \frac{k_8 k_5 k_3}{k_2 k_7} u_1^* - \frac{k_9 k_3}{k_2} u_1^* \\ &\quad + k_{10} x_1^* + k_{11} x_1^* \left[\frac{-k_3}{k_2 k_7}\dot{u}_1^* + \frac{k_3 k_5}{k_2 k_7} u_1^* \right] \end{aligned} \tag{84}$$

The nonlinear control law is obtained by solving Equation (84) for x_1^*.

$$x_1^* = \frac{\dfrac{-k_3}{k_2 k_7}\left[\ddot{u}_1^* - (k_5 + k_8)\dot{u}_1^* + (k_5 k_8 - k_7 k_9) u_1^*\right]}{k_{10} + \dfrac{k_{11} k_3}{k_2\ k_7}\left[k_5 u_1^* - \dot{u}_1^*\right]} \tag{85}$$

The similarity between this Equation (85) and the linear transfer function Equation (62) can be noted. Using the constants given in Equation (52) and changing to analog computer variables convert Equation (85) to the final nonlinear control law for invariance of t_{co} by feedforward control on f_c.

$$f = \frac{\ddot{f}_c + 3.66\ \dot{f}_c + 2.79\ f_c}{0.296 - 0.0154\ f_c - 0.0111\ \dot{f}_c} \tag{86}$$

Comparison of Equation (86) to (64) quickly shows that the assumption of the linear model is the same as the assumption that the terms involving f_c and $\dot{f}_c$ in the denominator of Equation (86) are negligible compared to the constant (0.296) term.

For invariance of y_2* with respect to u_1* the control laws for feedforward control and secondary feedback control will be determined in the same manner. The invariance conditions for this case are

$$\begin{aligned} y_2^* &= 0 \\ \text{and } \dot{y}_2^* &= 0 \end{aligned} \tag{87}$$

Equations (52) become the following set:

$$\dot{y}_1^* = k_1 y_1^* + k_3 u_1^* + k_4 u_1^* y_1^* \tag{88a}$$

$$0 = k_6 y_1^* + k_7 y_3^* \tag{88b}$$

$$\dot{y}_3^* = k_8 y_3^* + k_{10} x_1^* + k_{11} x_1^* y_3^* \tag{88c}$$

Equation (88b) solved for y_3* gives

$$y_3^* = \frac{-k_6}{k_7} y_1^* \tag{89}$$

and substituting Equation (89) into Equation (88c) gives the nonlinear control law for secondary feedback on y_1*.

$$x_1^* = \frac{\frac{-k_6}{k_7}\dot{y}_1^* - k_8 y_1^*}{k_{10} - \frac{k_{11}k_6}{k_7} y_1^*} \tag{90}$$

The numerical values and analog variables are substituted in the previous manner and the result is the final nonlinear control law for invariance of t_w by secondary feedback on t_{co}.

$$f = \frac{-0.578\ (\dot{t}_{co} + 2.27t_{co})}{1 + 0.00642\ t_{co}} \tag{91}$$

Compare this equation with the linear transfer function Equation (70).

For the feedforward controller, it is necessary to find x_1* in terms of u_1*. To find this relationship requires the elimination of y_1* from Equations (88a) and (90). This elimination is not convenient to do analytically, but it certainly presents no problem for analog computation. Equation (88a) with numerical values substituted and variables converted to analog variables is Equation (92):

$$\dot{t}_{co} = -2.85t_{co} - 2.82f_c - 0.0200f_c t_{co} \tag{92}$$

The analog computer program for the nonlinear controller for the invariance of t_w by feedforward control on f_c would appear as Figure 9.

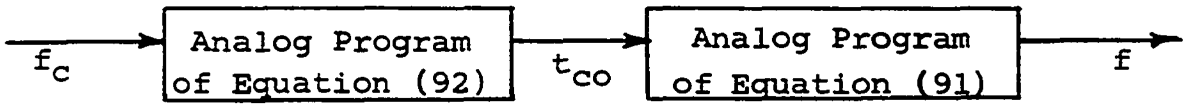


Figure 9. Analog Computer Block Diagram of a Feed-forward Controller for Invariance of t_w.

For invariance of y_3^* with respect to u_1^*, the invariance conditions are

$$y_3^* = 0 \text{ and } \dot{y}_3^* = 0 \tag{93}$$

Equations (52) become the following set:

$$\dot{y}_1^* = k_1 y_1^* + k_2 y_2^* + k_3 u_1^* + k_4 y_1^* u_1^* \tag{94a}$$

$$y_2^* = k_5 y_2^* + k_6 y_1^* \tag{94b}$$

$$0 = k_9 y_2^* + k_{10} x_1^* \tag{94c}$$

Equation (94c) itself is the nonlinear control law for feedback on y_2^*.

$$x_1^* = \frac{-k_9}{k_{10}} y_2^* \tag{95}$$

Notice that this time domain equation is the inverse transform of Equation (75). This invariance control problem is an example of the phenomenon mentioned in the theoretical presentation: analysis of a nonlinear model resulting in a linear controller. Examination of the dual channels of information in Figure 8 leads to the discovery that both channels are composed entirely of linear functions; hence, the linear controller results. For analog computation, the linear transfer function, Equation (76) can be used.

When Equation (94b) is solved for y_2* and the result substituted into Equation (95), the control law which results is the inverse of the transfer function, Equation (77). This case is another example in which the linear and nonlinear models both lead to the same control law. The analog computer controller law is Equation (78).

In order to find the feedforward control law, y_1* and y_2* must be eliminated from Equation (94a), which results in the following:

$$\ddot{x}_1^* - (k_1 + k_5)\dot{x}_1^* + (k_1k_5 - k_2k_6)x_1^* + \frac{k_6k_9k_3}{k_{10}} u_1^* + k_4u_1^*\left[k_5x_1^* - \dot{x}_1^*\right] = 0 \tag{96}$$

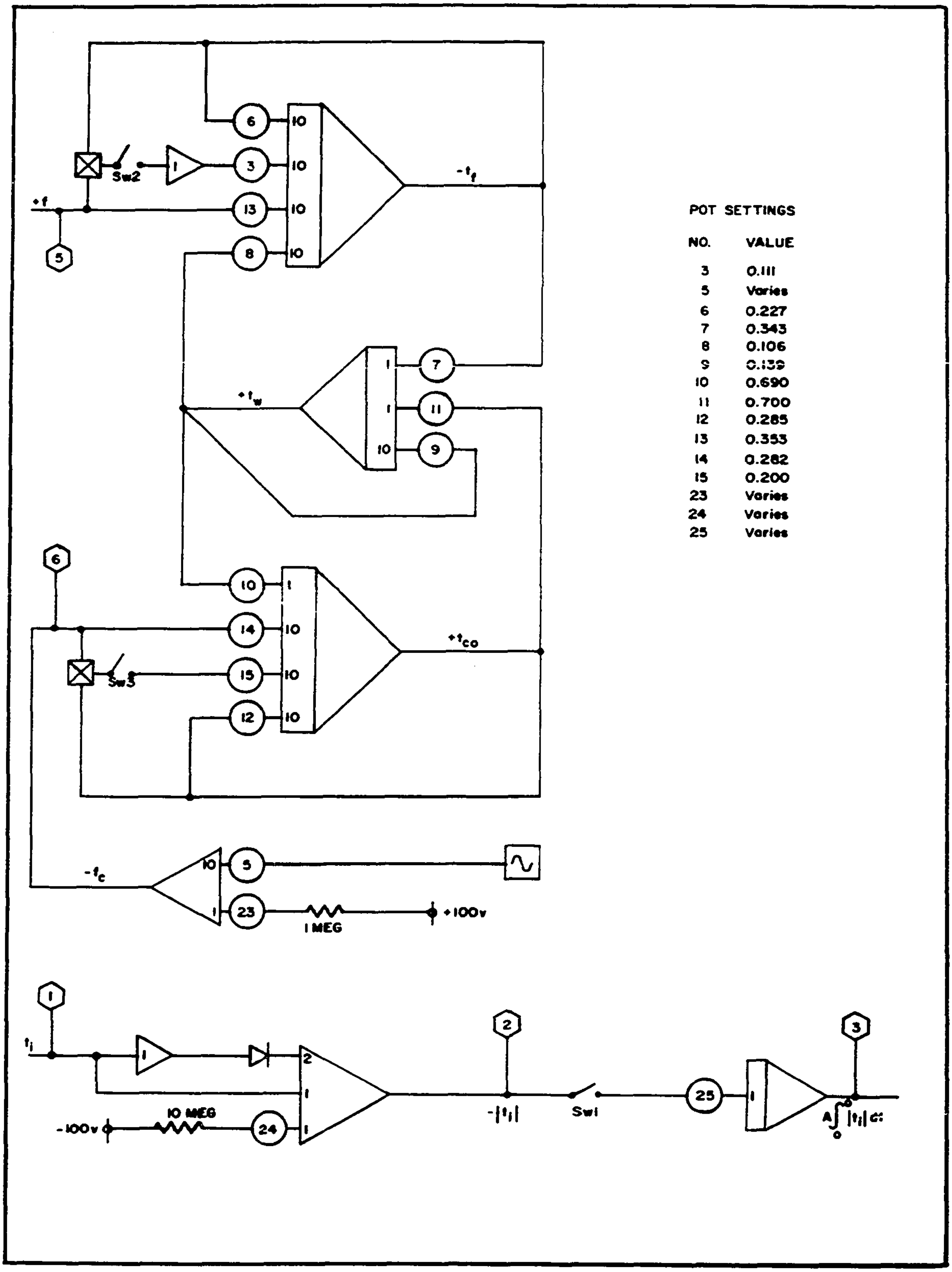
This equation differs from the control law determined from the linear model by the last term on the left-hand side of Equation (96). When the numerical values of the constants are introduced and the variables are changed to analog variables, the result is Equation (97):

$$\ddot{f} + 4.24\dot{f} + 3.48f - 0.593f_c + 0.0200f_c\left[1.39f + \dot{f}\right] = 0 \tag{97}$$

Compare this equation with Equation (74). Although it is not easy to solve explicitly for f, Equation (97) can be programmed on the analog computer. This program will be the analog controller for invariance of t_f by feedforward on f.

Analog Computer Programming

The nonlinear analog computer Equations (51) are in a suitable form for programming. The analog computer circuit for the simulated process is shown in Figure 10. The circuit



POT SETTINGS

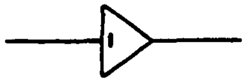
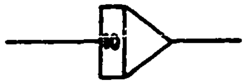
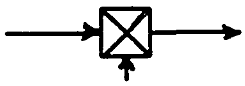
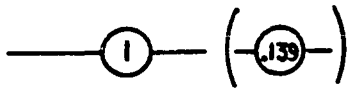
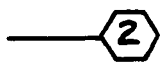
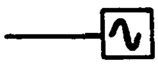
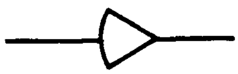
NO.	VALUE
3	0.111
5	Varies
6	0.227
7	0.343
8	0.106
9	0.139
10	0.690
11	0.700
12	0.285
13	0.353
14	0.282
15	0.200
23	Varies
24	Varies
25	Varies

Figure 10. Analog Computer Circuit Diagram for Simulation Studies

diagram for the integral absolute value circuit is also depicted. The circuit diagram symbols are given in Table 3.

TABLE 3

ANALOG CIRCUIT SYMBOLS

Symbol	Description
	Inverting amplifier with gain of one.
	Integrating amplifier with gain of ten.
	Electronic multiplier.
	Switch.
	Coefficient potentiometer (pot) number one (or a pot with a numerical setting of 0.139).
	Recorder channel number two.
	Low frequency function generator.
	Vacuum tube diode.
	Operational amplifier.

Several features of the analog computer circuit in Figure 10 which may not be obvious are the following:

1) Pot number 5 adjusts the gain of the input disturbance and pot number 23 changes the zero level. Both adjustments are not available in the low frequency function generator.

2) Pot number 24 is needed to zero the output of the absolute value circuit. This adjustment eliminates a small bias voltage that otherwise would make the integral absolute value too high.

3) Switch 1 is closed at the time at which the integration is desired to start.

4) Switches 2 and 3 determine whether the simulated system is linear or nonlinear.

The linear controllers are programmed directly from the transfer functions determined in the first section of this chapter. The nonlinear controllers are programmed from the control laws or the differential equations which describe them. The analog computer circuit diagrams for the controllers are shown in Figures 11 through 13. Each of the six controller configurations is given an identifying name for future reference (letters A through F).

Two special features in the controller circuits which are not standard programming techniques are the analog differentiator circuit and the analog electronic division circuit. The differentiator circuit which is shown in Figure 14 is an adaptation of the "implicit differentiator" method. This circuit allows the approximation to the differentiation process to be as exact as desired, depending on the setting of the pot. For pot settings less than one-tenth, the circuit is stable and its action approximates a differentiator increasingly better as one approaches a setting of

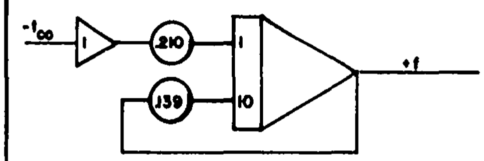
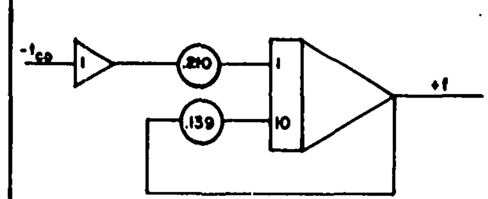
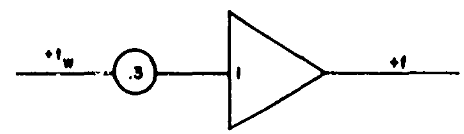
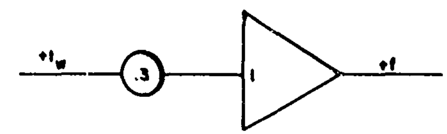
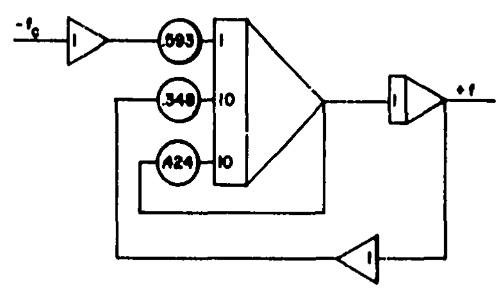
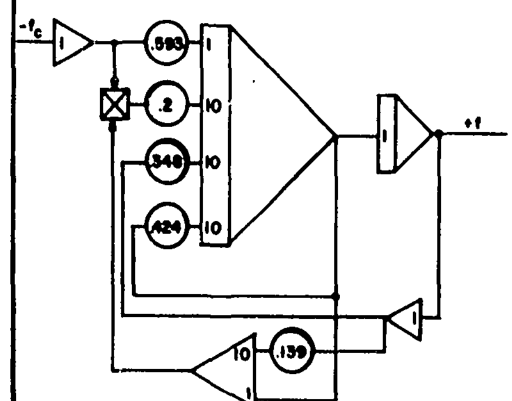
Invariant Variable	Measured Variable	Controller Name and Equations	Linear Model Controller	Nonlinear Model Controller
t_f	$-t_{co}$	A Linear Equation (78) Nonlinear Equation (78)		
t_f	t_w	B Linear Equation (76) Nonlinear Equation (76)		
t_f	$-t_c$	C Linear Equation (74) Nonlinear Equation (97)		

Figure 11. Analog Circuit Diagrams for Controller Configurations A, B, and C.

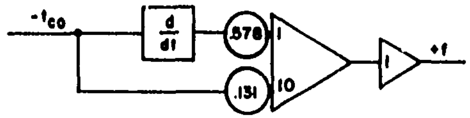
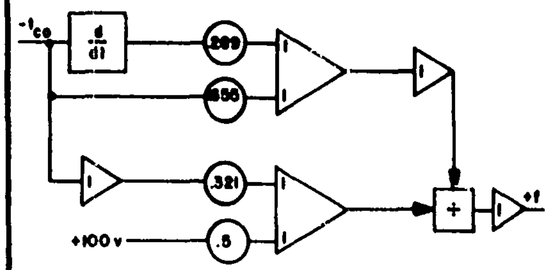
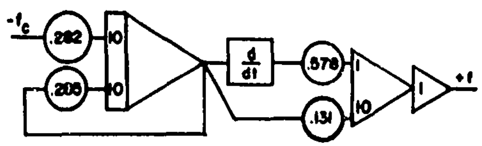
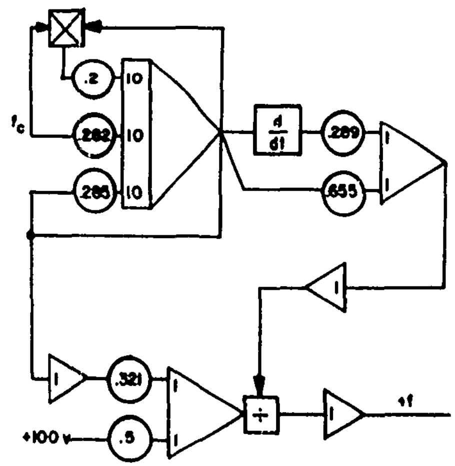
Invariant Variable	Measured Variable	Controller Name and Equations	Linear Model Controller	Nonlinear Model Controller
t_w	$-t_{co}$	D Linear Equation (70) Nonlinear Equation (91)		
t_w	$-f_c$	E Linear Equation (68) Nonlinear Equations (91) & (92)		

Figure 12. Analog Circuit Diagrams for Controller Configurations D and E.

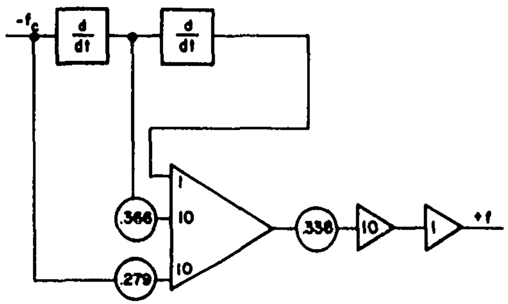
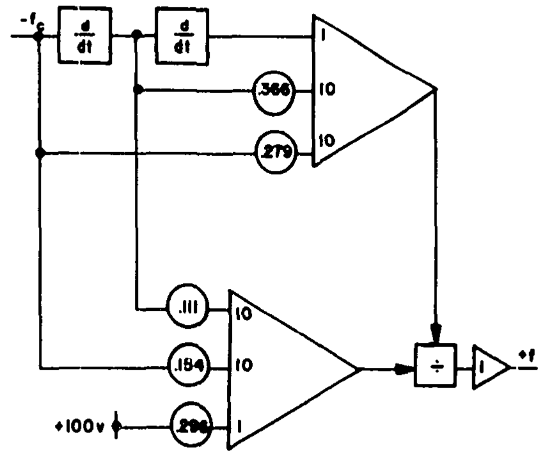
Invariant Variable	Measured Variable	Controller Name and Equations	Linear Model Controller	Nonlinear Model Controller
f_{co}	$-f_c$	F Linear Equation (64) Nonlinear Equation (66)		

Figure 13. Analog Circuit Diagrams for Controller Configuration F.

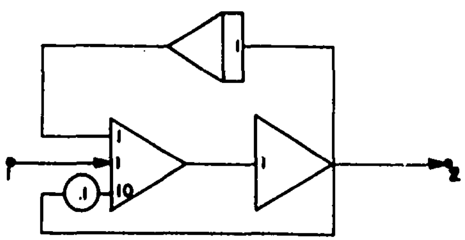
Symbol	Analog Circuit
	

Figure 14. Analog Differentiator Circuit

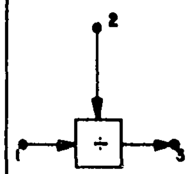
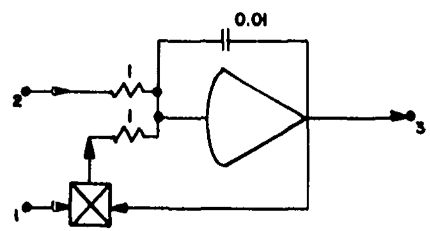
Symbol	Analog Circuit
	

Figure 15. Analog Electronic Division Circuit

one-tenth. The theoretically perfect setting of one-tenth may or may not be perfect in practice due to slight variances in the amplifier input resistances. However, the use of the potentiometer allows the proper setting to be "tuned." This differentiator circuit is the best one for analog computation because it is simple, versatile, and accurate. The electronic division circuit shown in Figure 15 has been briefly mentioned in several analog computer textbooks. The main reason for mentioning it here is to point out the necessity for including a small (0.01μ fd) capacitor in the amplifier feedback loop. This fact is not mentioned in textbooks. Also, it should be noted that the voltage at terminal 1 must always be negative for stability reasons. If the output of the amplifier is -0.01XY, where X and Y are the multiplier inputs, the circuit of Figure 15 has the following relationship:

$$\text{Terminal 3 Voltage} = \frac{-100(\text{Terminal 2 Voltage})}{\text{Terminal 1 Voltage}}$$

Simulation Study Procedure

The analog computer which was used for these studies was a Donner Model 3100D which has been modified by Bishop and Sims (7). The input disturbance f_c was generated by a Hewlett-Packard low-frequency function generator, Model 202A, which offers a square, sinusoidal, or triangular wave output with a frequency range of 0.008 to 1200 cycles per second and a peak-to-peak magnitude of approximately thirty volts. The recorder was a Sanborn, Model 156-1100C, Six-channel recorder. The analog computer variables which were recorded are shown in Figure 10.

After the simulated process was programmed according to Figure 10, each controller configuration was programmed in turn. The variable which was to be made invariant in each case was connected to the input of the integral absolute value circuit. This connection allowed a measurement of the relative quality of control offered by the invariance devices. In order to make the results quantitative the efficiency of a controller was defined:

$$\varepsilon = \frac{I_u - I_c}{I_u} \tag{98}$$

where ε is the controller efficiency, I_u is the integral absolute value for the uncontrolled system variable, and I_c is the integral absolute value for the controlled system variable.

This definition of efficiency is interpreted as the amount of area under the time-temperature curve which is eliminated by the invariance device. Thus, if all the area is eliminated, the controller is 100 per cent efficient, and if three-quarters of the area is eliminated, the controller is 75 per cent efficient, etc. In order to be sure that the comparison is meaningful the integration was made over several disturbance cycles and the integration was always started at the same relative point in time for the controlled and uncontrolled systems. For example, the integration could be started when the sinusoidal disturbance wave passed through zero.

For each controller configuration the following tests were made:

1) Linear process simulation, no control.
2) Linear process simulation, linear model controller.
3) Nonlinear process simulation, no control.
4) Nonlinear process simulation, linear model controller.
5) Nonlinear process simulation, nonlinear model controller.

All testing was done in the frequency range of 0.01 to 0.50 cycles per second (analog time) which proved to be the range of greatest interest. Higher frequencies were not tested because the frequency response characteristics of the process were such that the magnitude of response dropped off sharply at about 0.50 cycles per second. Lower frequencies were not tested because the controller efficiencies were not particularly sensitive to frequency changes and because of numerous realizations of 100 per cent efficiency in the range tested. If a controller exhibits invariance for 0.01 cycles per second, the same behavior is expected for lower frequencies.

Invariance theory makes no demands on the time-functional form of the disturbance. Therefore, square, triangular, and sinusoidal waves were arbitrarily chosen for convenience and for testing the ability of the various controllers to cope with different types of disturbances. The magnitude of the wave disturbances was held constant at $\pm$ 17 volts (peak-to-peak) because the efficiency was insensitive to disturbance magnitude. One series of tests was made to examine the variation in controller efficiency with disturbance magnitude for the linear model approximation controlling a nonlinear process. Configuration F had to be tested at a lower magnitude of disturbance to keep the system under control without saturation of system components.

Although the integrals I_u or I_c could have been determined in the real time-temperature units, it was not necessary to convert all recording areas back to real units. This saving of extensive computations was a result of using consistent units in Equation (98) and because both I_u and I_c were computed for the same number of cycles. Thus, the efficiencies were based in fact on time-voltage areas. The recordings for each test had to be read at two points. The recorded integral absolute value was measured at the beginning of the test period and again after a fixed number of disturbance cycles. The change in the integral absolute value was the area under the time-voltage curve for the number of cycles chosen (cf. Figure 31).

A total of 122 runs were made on the various controller configurations. This number is conservatively estimated to be the equivalent of 200 hours of real system testing.

CHAPTER VI

EXPERIMENTAL APPARATUS AND PROCEDURE

In this chapter the methods of applying invariance theory to the physical system described in Chapter III are presented. The total variable model which was derived in Chapter III is used to establish the control laws for the physical system. The purposes of this part of the investigation are the following:

1) To demonstrate the use of linear and nonlinear total variable models for the derivation of linear and nonlinear control laws using time-domain methods.
2) To demonstrate the feasibility of applying the theory to a real system.
3) To delineate for the first time the actual type of problems that arise during practical applications of the theory.
4) To compare the quality of control provided by linear and nonlinear controllers.

Since the model does not necessarily describe the entire process in detail and because one has to accept the noise, lags, and minor nonlinearities (whether they are included in the model or not), the physical system is not as well behaved as the process simulated on the analog computer.

The first section will contain the derivations of the controller equation, and the second will consist of controller synthesis procedures. The third section will contain a description of the experimental apparatus and testing equipment, and the fourth will consist of a description of the procedure for obtaining experimental data.

Derivation of Controller Equations

The mathematical model which is used for the derivation of controller equations is the set of Equations (4). The unit of time for Equations (4) is an hour. The controllers to be synthesized will be electronic because they will be assembled using analog computer elements. These analog elements are based on a unit of time of one second. Therefore, the coefficients of Equations (4) will have to be based on seconds instead of hours. For real process applications it is not possible to time scale the model equations because the controllers must operate in real time along with the process itself. Equations (4) with coefficients converted to units of time in seconds are the following set.

$$\begin{aligned} \dot{T}_f &= -0.00881\, T_f + 0.00881\, T_w + 0.0285\, W - 0.000185\, W T_f - 0.0517 \\ \dot{T}_w &= -0.0231\, T_w + 0.0114\, T_f + 0.00583\, T_{co} + 0.183 \qquad (99) \\ \dot{T}_{co} &= -0.0115\, T_{co} + 0.0230\, T_w + 0.0209\, W_c - 0.000667\, W_c T_{co} - 0.360 \end{aligned}$$

A linearized model can be obtained from Equations (99) by expanding the product nonlinearities in a Taylor series expansion about the steady-state operating points. The linear model is the following set of equations.

$$\begin{aligned}
\dot{T}_f &= -\ 0.0379\ T_f + 0.00881\ T_w + 0.00297\ W + 3.96 \\
\dot{T}_w &= -\ 0.0231\ T_w + 0.0114\ T_f + 0.00583\ T_{co} + 0.183 \qquad (100) \\
\dot{T}_{co} &= -\ 0.0475\ T_{co} + 0.0230\ T_w - 0.0188\ W_c + 1.78
\end{aligned}$$

Linear Model, Controller Derivations

As outlined in the theoretical presentation, matrix methods are applicable for the derivation of the controller equations using Equations (100); however, for a third order system matrix algebra offers no great advantage. It is simpler to remain in the time domain. For this reason the control laws will be developed directly from Equations (100).

For invariance of T_f with respect to F_c the invariance conditions are

$$\dot{T}_f = 0$$

and (101)

$$T_f = T_{f_{ss}} \ .$$

Substituting Equations (101) into (100) gives the following set of Equations.

$$0 = -0.0379\ T_{f_{ss}} + 0.00881\ T_w + 0.00297\ W + 3.96 \qquad (102a)$$

$$DT_w = -0.231\ T_w + 0.0114\ T_{f_{ss}} + 0.00583\ T_{co} + 0.183 \qquad (102b)$$

$$DT_{co} = -0.0475\ T_{co} + 0.0230\ T_w - 0.0188\ W_c + 1.78 \qquad (102c)$$

The dot notation for time derivative has been replaced by the differential operator D. When Equation (102a) is solved for W, the result is

$$W = -2.97\ T_w + 12.76\ T_{f_{ss}} - 1333 \qquad (103)$$

This equation is the control law for invariance of T_f by secondary feedback on T_w. Notice that the set point $T_{f_{ss}}$ appears in this equation. The existence of dual channels does not need to be reexamined because the analog simulation study for the same controller (but using the perturbation variable model) has shown that the dual channel theorem is satisfied. Because of the simulation study, none of the controllers in this chapter need to be checked for duality. If Equation (102b) is solved for T_w and the result substituted into Equation (103), the control law for invariance of T_f by secondary feedback on T_{co} is obtained.

$$W = \frac{-2.97(0.0114\ T_{f_{ss}} + 0.00583\ T_{co} + 0.183)}{D + 0.0231} + 12.76\ T_{f_{ss}} - 1333 \qquad (104)$$

Notice that the set point appears again but in a slightly more complicated manner. Equation (102b) can be solved for T_{co}. When this expression is substituted into Equation (102c) the result is an equation which gives T_w in terms of W_c. This last expression can then be substituted into Equation (103) to give the control law for invariance of T_f by feedforward control on W_c.

$$W = \frac{-0.00161\ T_{f_{ss}} + 0.0003255\ W_c - 0.0567}{D^2 + 0.0706\ D + 0.0009632} + 12.76\ T_{f_{ss}} - 1333 \qquad (105)$$

For invariance of T_w with respect to F_c the invariance conditions are

$$\dot{T}_w = 0$$
$$T_w = T_{w_{ss}} \quad . \tag{106}$$

By substituting these equations into the set of Equations (100) the following set is obtained.

$$D\ T_f = -0.0379\ T_f + 0.00881\ T_{w_{ss}} + 0.00297\ W + 3.96 \tag{107a}$$

$$0 = -0.0231\ T_{w_{ss}} + 0.0114\ T_f + 0.00583\ T_{co} + 0.183 \tag{107b}$$

$$D\ T_{co} = -0.0475\ T_{co} + 0.0230\ T_{w_{ss}} - 0.0188\ W_c + 178 \tag{107c}$$

Equation (107b) is solved for T_f and DT_f. When Equation (107a) is solved for W and the values of T_f and DT_f just found from Equation (107b) are substituted, the result is the control law for invariance of T_w by secondary feedback on T_{co}.

$$W = -172.2\ DT_{co} - 6.53\ T_{co} + 22.89\ T_{w_{ss}} - 1538 \tag{108}$$

The easiest method of finding the control law for invariance of T_w by feedforward on W_c is the solution of Equation (107c) for T_{co} and the substitution of this relationship into Equation (108).

$$W = \frac{-172.2(D+0.0379)}{(D+0.0475)}\ (0.0230\ T_{w_{ss}} - 0.0188\ W_c + 1.78) + 22.89\ T_{w_{ss}} - 1538 \tag{109}$$

In order to obtain the feedforward control law for invariance of T_{co}, the following procedure is followed. The invariance conditions

$$\dot{T}_{co} = 0$$
$$T_{co} = T_{co_{ss}} \tag{110}$$

change Equations (100) to the following set.

$$D\ T_f = -0.0379\ T_f + 0.00881\ T_w + 0.00297\ W + 3.96 \tag{111a}$$

$$D\ T_w = -0.0231\ T_w + 0.0114\ T_f + 0.00583\ T_{co_{ss}} + 0.183 \tag{111b}$$

$$0 = -0.0475\ T_{co_{ss}} + 0.0230\ T_w - 0.0188\ W_c + 1.78 \tag{111c}$$

Equation (111c) is solved for T_w and substituted into Equation (111b). Equation (111b) is solved for T_f and substituted into Equation (111a). Equation (111a) is solved for W; the result is the control law for invariance of T_{co} by feedforward on W_c.

$$W = (24{,}130\ D^2 + 1472\ D + 18.7)\ W_c + 40.74\ T_{co_{ss}} - 3309 \tag{112}$$

Nonlinear Model, Controller Derivations

The details of these derivations will not be given because the procedure is practically indentical to that for the linear model controller derivations. The main difference is the use of Equations (99) instead of (100). The nonlinear terms present no difficulties in the derivation of the control laws, but they do complicate the expressions somewhat. The final control laws are given below.

T_f invariance by secondary feedback on T_{co}:

$$W = \frac{\frac{-2.97}{D+0.0231}(0.0114\ T_{f_{ss}} + 0.00583\ T_{co} + 0.183) + 2.97\ T_{f_{ss}} + 17.43}{(9.61 - 0.0624\ T_{f_{ss}})} \tag{113}$$

T_f invariance by secondary feedback on T_w:

$$W = \frac{-2.97\ T_w + 2.97\ T_{f_{ss}} + 17.43}{(9.61 - 0.0624\ T_{f_{ss}})} \tag{114}$$

T_f invariance by feedforward on W_c:

$$W = \frac{\left[\dfrac{-(0.000389\ T_{f_{ss}} + 0.0000169 + 0.0000226\ T_{f_{ss}} W_c + 0.000725 W_c)}{(D^2 + 0.0346\ D + 0.0001316) + 0.000667\ W_c (D + 0.0231)}\right] + 2.97\ T_{f_{ss}} + 17.43}{(9.61 - 0.0624\ T_{f_{ss}})} \tag{115}$$

T_w invariance by secondary feedback on T_{co}:

$$W = \frac{-17.94\ D\ T_{co} - 0.158\ T_{co} + 0.317\ T_{w_{ss}} - 3.49}{1.104 - 0.0132\ T_{w_{ss}} + 0.00332\ T_{co}} \tag{116}$$

T_w invariance by feedforward on W_c:

$$W = \frac{-17.94(D+0.00881)\left[\dfrac{0.0230\ T_{w_{ss}} + 0.0209\ W_c - 0.360}{D + 0.0115 + 0.000667\ W_c}\right] + 0.317\ T_{w_{ss}} - 3.49}{1.104 - 0.0132\ T_{w_{ss}} + 0.00332\left[\dfrac{0.0230\ T_{w_{ss}} + 0.0209\ W_c - 0.360}{D + 0.0115 + 0.000667\ W_c}\right]} \tag{117}$$

T_{co} invariance by feedforward on W_c:

$$W = \frac{(3814\ D^2+121.7\ D + 0.393)\ (0.000667\ T_{co_{ss}}-0.0209)W_c + 0.0000159\ T_{co_{ss}} + 0.05061}{0.02563 - 0.00009284\ T_{co_{ss}} - (0.7056)(D + 0.0231)(0.000667\ T_{co_{ss}} - 0.0209)W_c} \tag{118}$$

These equations include nonlinear relationships of the measured variable and also nonlinear combinations of the set points. Notice that there are no instances of identical linear and nonlinear controllers, whereas there were two identical controllers when the perturbation model was used. In the case of the total variable model the difference in linear and nonlinear controllers is due to different methods of entering the set-point.

Controller Synthesis Procedure

Once the control laws have been derived, the next step is the execution of these laws. The control law equations which have been derived in the first section of this chapter are expressed in terms of physical variables: degrees Fahrenheit and pounds per hour. The controller input and output signals are to be electrical because these devices will be assembled on the analog computer. To convert the control law equations to analog computer control equations the relationships between physical variables and electrical variables must be determined. Specifically, the relationships must be found between T_f and T_f^o, T_w and T_w^o, T_{co} and T_{co}^o, W_c and W_c^o, W and W^o. The superscript (^{o}) refers to analog voltage. The ideal relationships between these variables would be linear ones.

Temperature Variables: The conversion from degrees Fahrenheit to voltage is comparatively simple. Thermocouples are temperature measuring devices that do this conversion. Furthermore, the relationship between voltage and temperature is almost linear. For example, the equation

$$T_f = a_1 T_f^o + b_1 \tag{119}$$

can be assumed to apply over a relatively wide range of temperature. In this range the constants a_1 and b_1 are determined by some average temperature. Once a steady-state temperature is chosen, a_1 and b_1 can be calculated from thermocouple conversion tables. Equation (119) should be valid in a temperature range about the steady-state value. The thermocouple electromotive forces are too small for use directly by the analog computer controllers, and they must be amplified. It is convenient to subtract the constant b_1. This subtraction is done either by zero-suppression on a low-level amplifier or by an analog computer amplifier. The final relationship is

$$T_f = a_1 T_f^o \tag{120}$$

In a similar manner

$$T_w = a_2 \; T_w^{\circ} \tag{121}$$

and

$$T_{co} = a_3 \; T_{co}^{\circ} \tag{122}$$

The constants a_1, a_2, and a_3 are evaluated at the steady-state temperatures which are listed in Table 1.

Coolant Flow Rate Variable: The relationship between W_c and W_c° is obtained from a calibration of the coolant flow measuring device. In this system the flow measuring device is a differential pressure transducer. The differential pressure is transduced to an electrical current which causes a voltage drop across a precision resistor. The fault of this device is that the voltage is not linear with respect to flow rate; however, a linear approximation over a limited allowable flow rate range can be assumed. The linear approximation is

$$W_c = a_4 \; W_c^{\circ} + b_4 \tag{123}$$

The actual calibration curve and its linear approximation are shown in Figure 16.

Oil Flow Rate Variable: The change of variable for the manipulated input is a different problem because the conversion is the reverse procedure of that for a change of measured variable. The relationship between the controller output W° and the flow rate W can be obtained from a calibration of the transducing, controlling, and measuring elements of the oil flow system. The physical

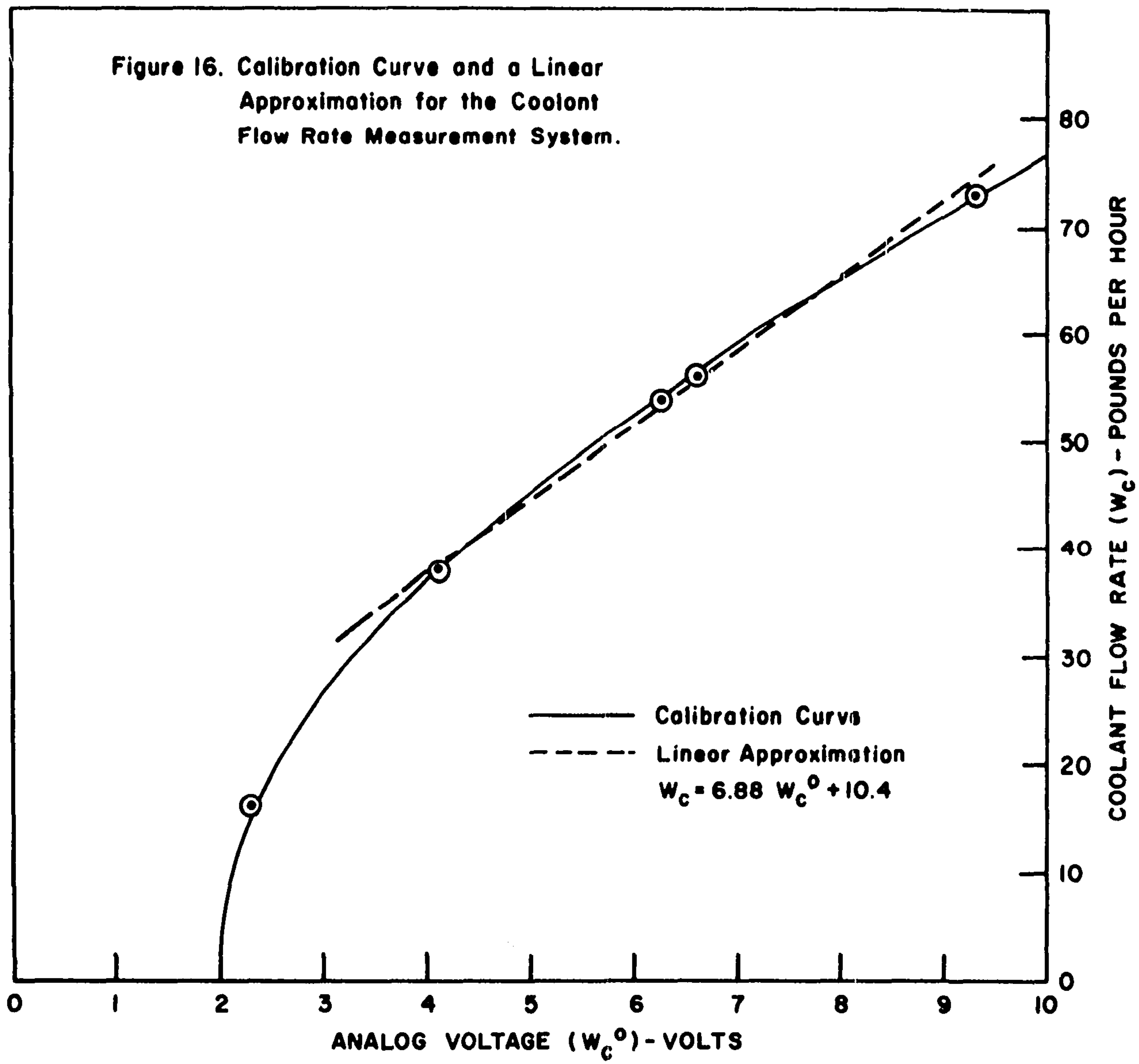


Figure 16. Calibration Curve and a Linear Approximation for the Coolant Flow Rate Measurement System.

arrangement consists of an electro-pneumatic transducer, control valves, and the turbine flowmeter system. All of these devices have approximately linear characteristics. When an attempt is made to determine the relationship between transducer electrical input and flowmeter reading, a severe hysteresis effect is noted (approximately $\pm$ 10 per cent of the steady state flow value). Figure 17a shows a calibration curve with hysteresis present. Because invariance demands accurate execution of the control law, this situation is intolerable. The solution to this problem is a feedback control loop about the flow system. Figure 18 is a block diagram of the feedback system and Figure 19 is the analog circuit diagram. W' is the electrical signal from the flowmeter which is proportional to flow rate. The two pot settings which determine the linear relationship of Equation (124) are taken from the slope and intercept of an approximate median line on the hysteresis curve, Figure 17a.

$$W' = 12 - 1.3\ W^{o} \tag{124}$$

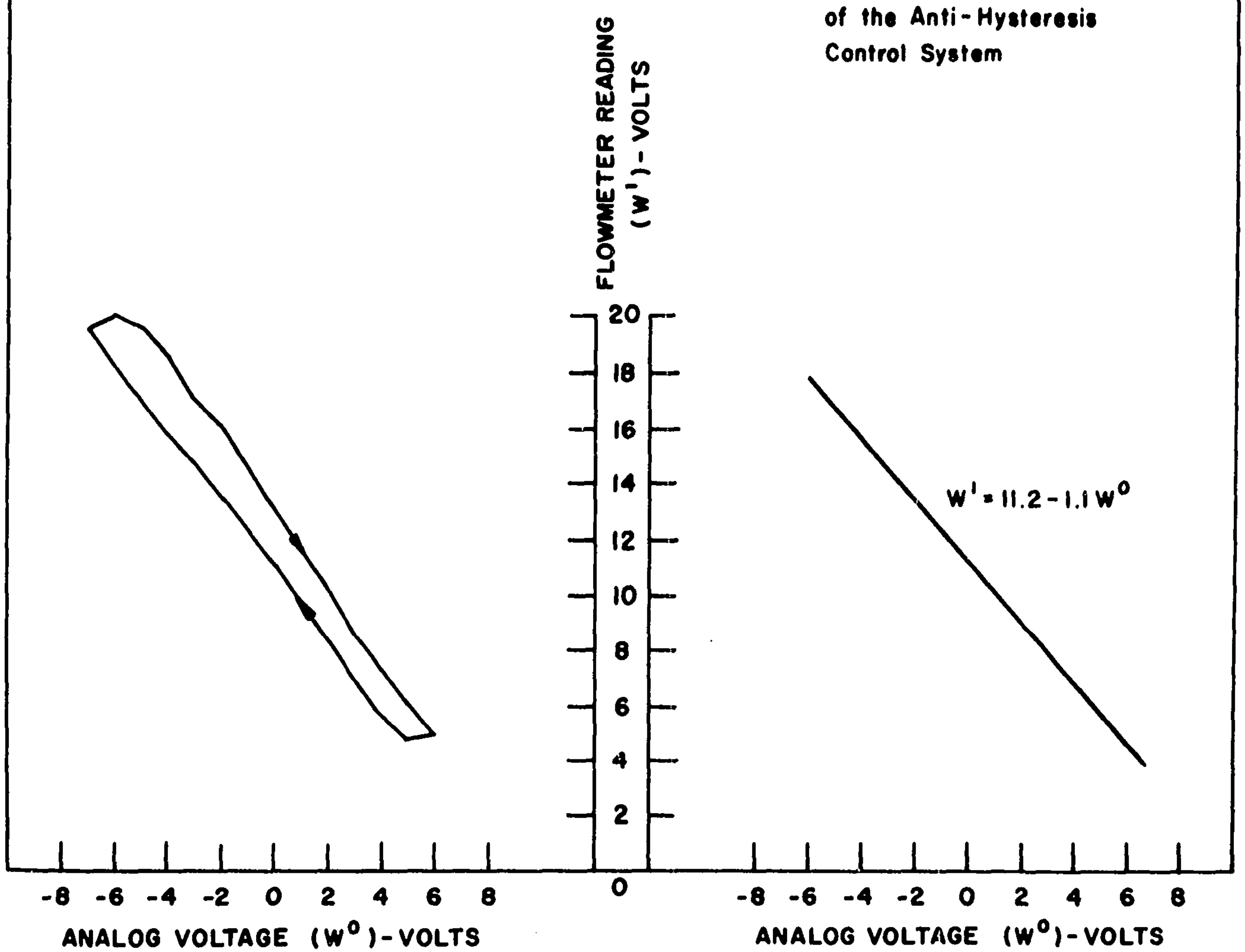
The linear relationship which results from calibration of this controlled flow system is shown in Figure 17b. The equation for this line is

$$W' = 11.2 - 1.1\ W^{o} \tag{125}$$

The difference between Equations (124) and (125) is explained by the offset (droop) characteristic of proportional controllers. The programmed curve, Equation (124),

Figure 17a. Hysteresis Behavior Exhibited by Oil Flow Rate System

Figure 17b. Behavior of Oil Flow Rate System After the Addition of the Anti-Hysteresis Control System



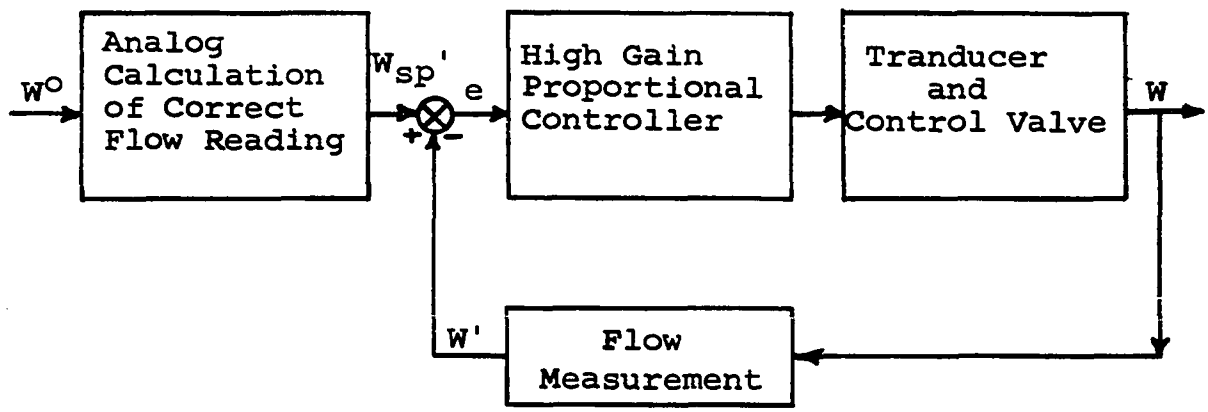


Figure 18. Block Diagram of the Anti-Hysteresis Control System.

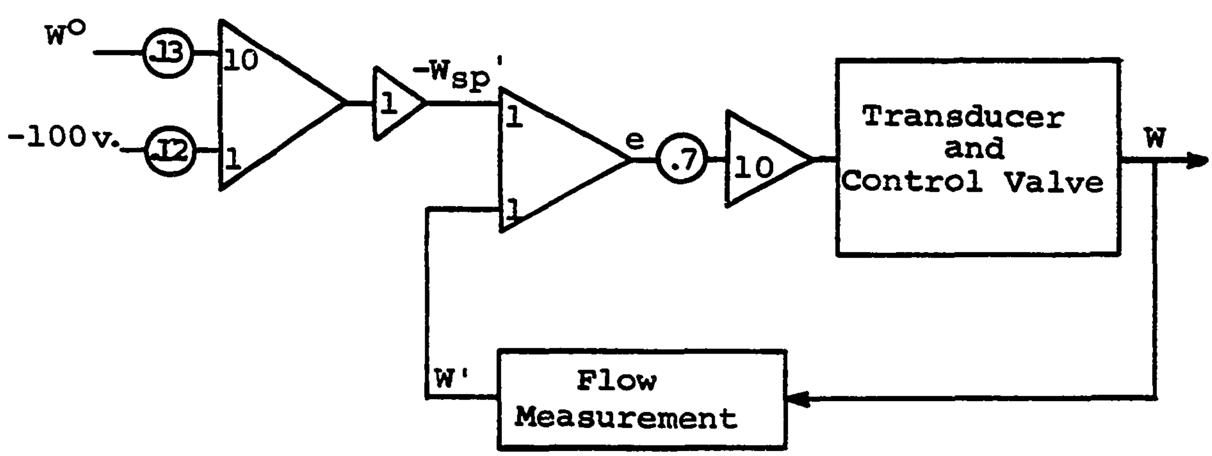


Figure 19. Analog Circuit Diagram of the Anti-Hysteresis Control System.

can be obtained exactly from calibration tests if reset action is added to the proportional controller. However, for purposes of this work it does not matter what linear relationship is obtained so long as it is known. Combining Equation (125) with the linear fit of the calibration data given in Figure 20 results in the final determination of the desired relationship

$$W = a_5 W^o + b_5 \tag{126}$$

Besides eliminating the hysteresis, the feedback control loop has two other advantages. First of all the feedback loop increases the speed of response of the manipulated variable because of the high gain. Secondly, the relationship between W and W^o is linear because it is forced to be linear by the control system. This oil flow rate control system is one of the important solutions to the specific problems of this system when invariance theory is applied in practice.

The final equations for changing variables from physical quantities to analog voltages are given in Table 4.

The changes of variables given in Table 4 are made for the linear model Equations (103), (104), (105), (108), (109), and (112) and the nonlinear model Equations (113) through (118). The resulting equations are programmed on the analog computer to complete the controller synthesis procedure. The major difficulty in programming is that time scaling is not permitted. The controller equations, if they

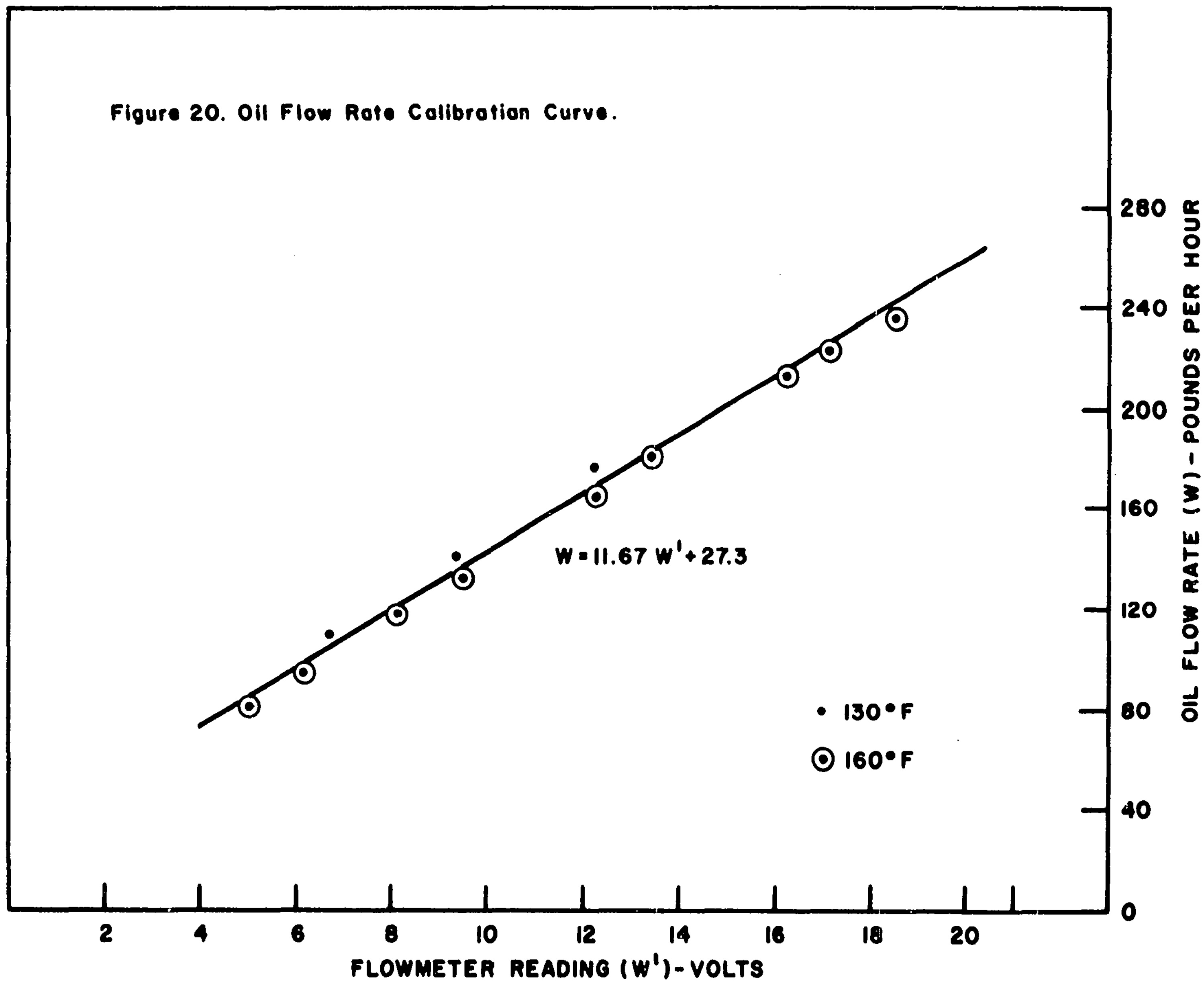


Figure 20. Oil Flow Rate Calibration Curve.

TABLE 4

RELATIONSHIPS BETWEEN PHYSICAL VARIABLES
AND ANALOG CONTROLLER VOLTAGES

Equation Number	Equation
(120)	$T_f = 16.67\ T_f^o$
(121)	$T_w = 17.39\ T_w^o$
(122)	$T_{co} = 18.18\ T_{co}^o$
(123)	$W_c = 6.88\ W_c^o + 10.4$
(126)	$W = -12.84\ W^o + 158$

were used by themselves, would be time scaled to increase the accuracy of computation. However, the controller must operate in real (process) time. Some magnitude scaling is possible, but it is limited by the necessity for remaining within the $\pm$ 100 volt limits of the analog computer. The final analog controller equations are given in Table 5 and the analog computer controller circuit diagrams are shown in Figures 21 through 23. The circuit diagram symbols are the same as those listed in Table 3 in the previous chapter. The capacitors which are shown in the feedback loops of some inverting amplifiers are added to provide some low-pass filtering. The pots are listed either with their numerical setting or with a <u>star</u>. The star means that the setting of this pot is a function of the steady-state value of the invariant variable. Thus, for each controller, all the pots with stars must be changed to specific new values whenever the set-point has to be changed.

TABLE 5

FINAL CONTROL LAW EQUATIONS IN TERMS OF ANALOG COMPUTER VARIABLES

Controller Name	Linear Model Control Law
A	$DW^{o} = - 0.0203\ T_{f_{ss}} + 0.0245\ T_{co}{}^{o} - 0.0231\ W^{o} + 2.72$
B	$W^{o} = - 0.994\ T_{f_{ss}} + 116.1 + 4.02\ T_{w}{}^{o}$
C	$D^{2}W^{o} = - 0.0706\ DW^{o} - 0.000963\ W^{o} - 0.000832\ T_{f_{ss}} + 0.116 - .000174\ W_{c}{}^{o}$
D	$W^{o} = 243.8\ DT_{co}{}^{o} + 9.244\ T_{co}{}^{o} - 1.783\ T_{w_{ss}} + 132.1$
E	$DW^{o} = - 0.0475\ W^{o} - 1.734\ DW_{c}{}^{o} -0.0657\ W_{c}{}^{o} + 7.079 - 0.0730\ T_{w_{ss}}$
F	$W^{o} = - (12{,}929\ D^{2} + 788.7\ D + 10.02)\ W_{c}{}^{o} - 3.173\ T_{co_{ss}} + 255$

TABLE 5--Continued

Controller Name	Nonlinear Model Control Law
A	$DW^{o} = \dfrac{0.02449\ T_{co}{}^{o} - 0.002702\ T_{f_{ss}} + 0.01098}{9.61 - 0.0624\ T_{f_{ss}}} + 0.2843 - 0.0231\ W^{o}$
B	$W^{o} = \dfrac{-\ 51.65\ T_{w}{}^{o} + 2.97\ T_{f_{ss}} + 17.43}{-\ 123.4 + 0.801\ T_{f_{ss}}} + 12.31$
C	$D^{2}W^{o} = -\ 0.0415\ DW^{o} - 0.000292\ W^{o} - 0.00459\ W_{c}{}^{o}\ DW^{o} - 0.0001059\ W^{o}W_{c}{}^{o} + \left[-\ 0.001305 + \dfrac{0.0001594\ T_{f_{ss}} - 0.003137}{+\ 123.4 - 0.801\ T_{f_{ss}}}\right] W_{c}{}^{o} + \left[-\ 0.00359 + \dfrac{0.000241\ T_{f_{ss}} - 0.002465}{123.4 - 0.801\ T_{f_{ss}}}\right]$
D	$W^{o} = \dfrac{25.4\ DT_{co}{}^{o} + .9665\ T_{co}{}^{o} - 0.1871\ T_{w_{ss}} + 13.86}{1.104 - 0.0132\ T_{w_{ss}} + 0.06036\ T_{co}{}^{o}}$

TABLE 5--Continued

Controller Name	Nonlinear Model Control Law
E	$W^o = \dfrac{1.397(D+0.00881)\left[\dfrac{0.0230T_{w_{ss}}-0.143+0.144W_c^o}{D+0.0184+0.00459W_c^o}\right]-0.0247T_{w_{ss}}+0.272}{1.104-0.0132T_{w_{ss}}+0.00332\left[\dfrac{0.0230T_{w_{ss}}-0.143+0.144W_c^o}{D+0.0184+0.00459W_c^o}\right]}+12.31$
F	$W^o = \dfrac{-(3814D^2+121.7D+0.393)a\,\dfrac{6.88}{12.84}\,W_c^o - \dfrac{0.393(10.4)\ a + b}{12.84}}{c-(0.7056)(D+0.0231)a(6.88)W_c^o-(0.7056)(.0231)a(10.4)} + 12.31$ where $a = 0.000667\ T_{co_{ss}} - 0.0209$ $b = 0.0000159\ T_{co_{ss}} + 0.05061$ $c = 0.02563 - 0.00009284\ T_{co_{ss}}$

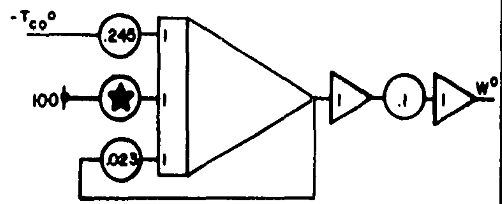
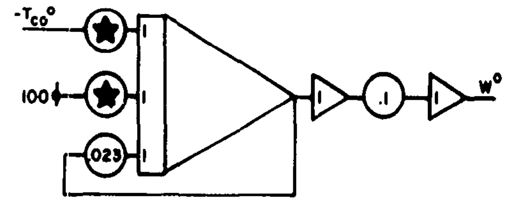
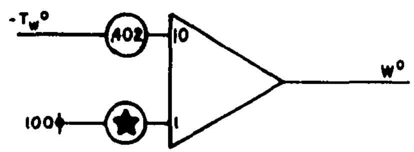
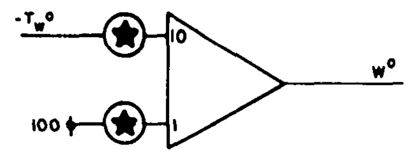
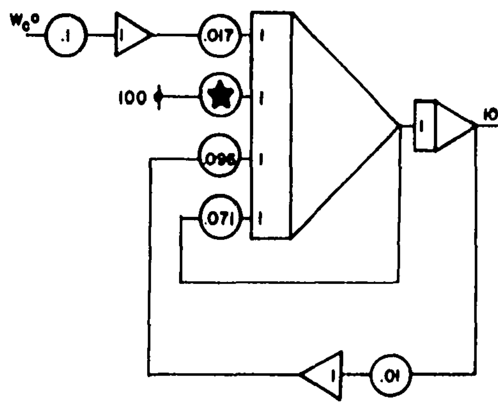
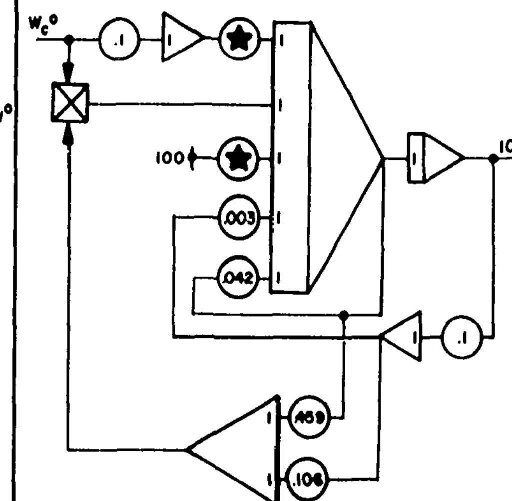
Invariant Variable	Measured Variable	Controller Name	Linear Model Controller	Nonlinear Model Controller
T_f	T_{co}	A		
T_f	T_w	B		
T_f	W_c	C		

Figure 21. Analog Circuit Diagrams for Experimental System Controllers A, B, and C.

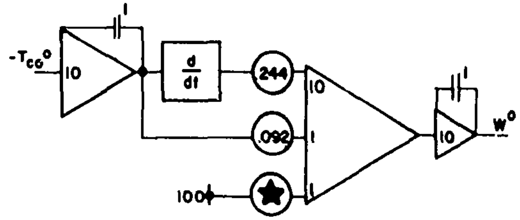
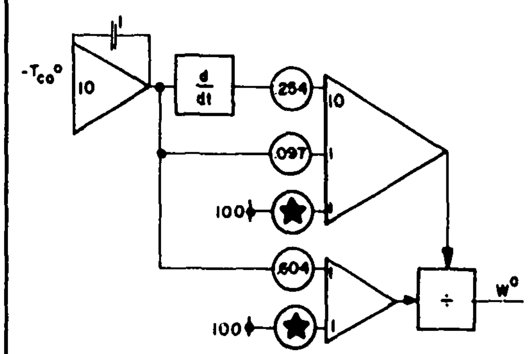
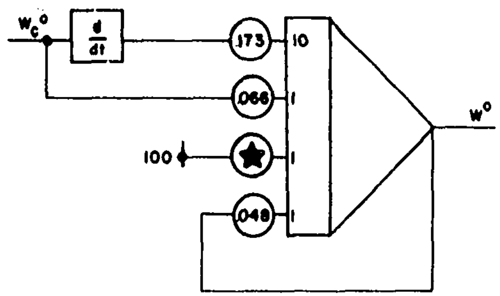
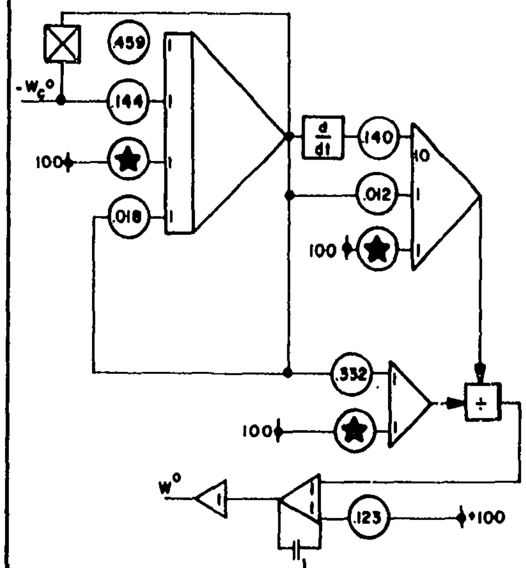
Invariant Variable	Measured Variable	Controller Name	Linear Model Controller	Nonlinear Model Controller
T_w	T_{co}	D		
T_w	W_c	E		

Figure 22. Analog Circuit Diagrams for Experimental System Controllers D and E.

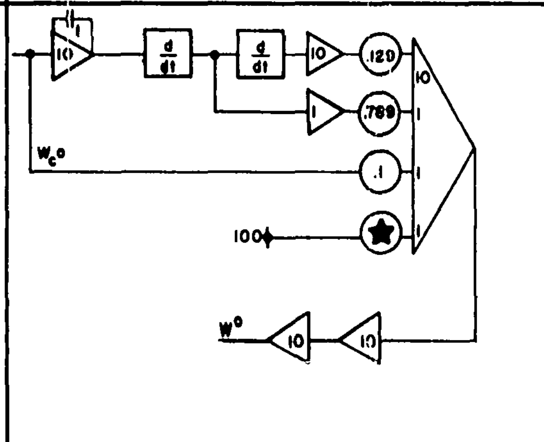
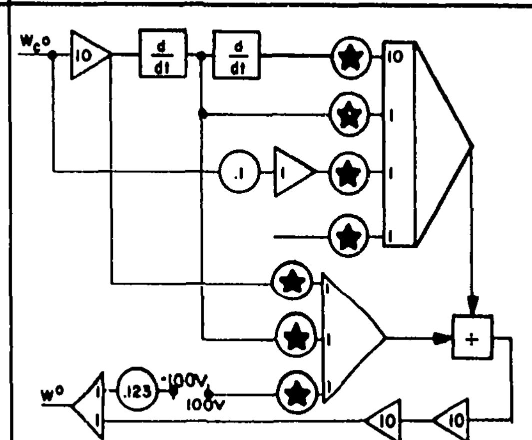
Invariant Variable	Measured Variable	Controller Name	Linear Model Controller	Nonlinear Model Controller
T_{co}	W_c	F		

Figure 23. Analog Circuit Diagram for Experimental System Controller F.

If the controllers derived from the perturbation model, which are shown in Figures 11 through 13, are compared with the controllers derived from the total variable model which arc shown in Figures 21 through 23, it is seen that the dynamic portions are the same. However, the latter group of controllers has provided for the set point.

Experimental Apparatus

The Process Dynamics Laboratory at the University of Oklahoma has experimental equipment which has been assembled for general investigations of process identification and control problems. This equipment has sufficient flexibility to simulate a variety of chemical processes.* By appropriate combination of the components of this process, it was possible to devise a system to demonstrate invariance theory, synthesis procedures. Although the system was actually part of the more complicated general system, only the equipment items pertinent to this specific experiment will be mentioned.

Figures 24 through 28 present some views of the Process Dynamics Laboratory. Figure 24 depicts an overall view of the process laboratory. On the table at the left is the equipment for thermocouple voltage measurements and preamplification. In the foreground is a coolant bath and behind it is the reactor with its associated piping

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*Cf. Bishop and Sims (7).

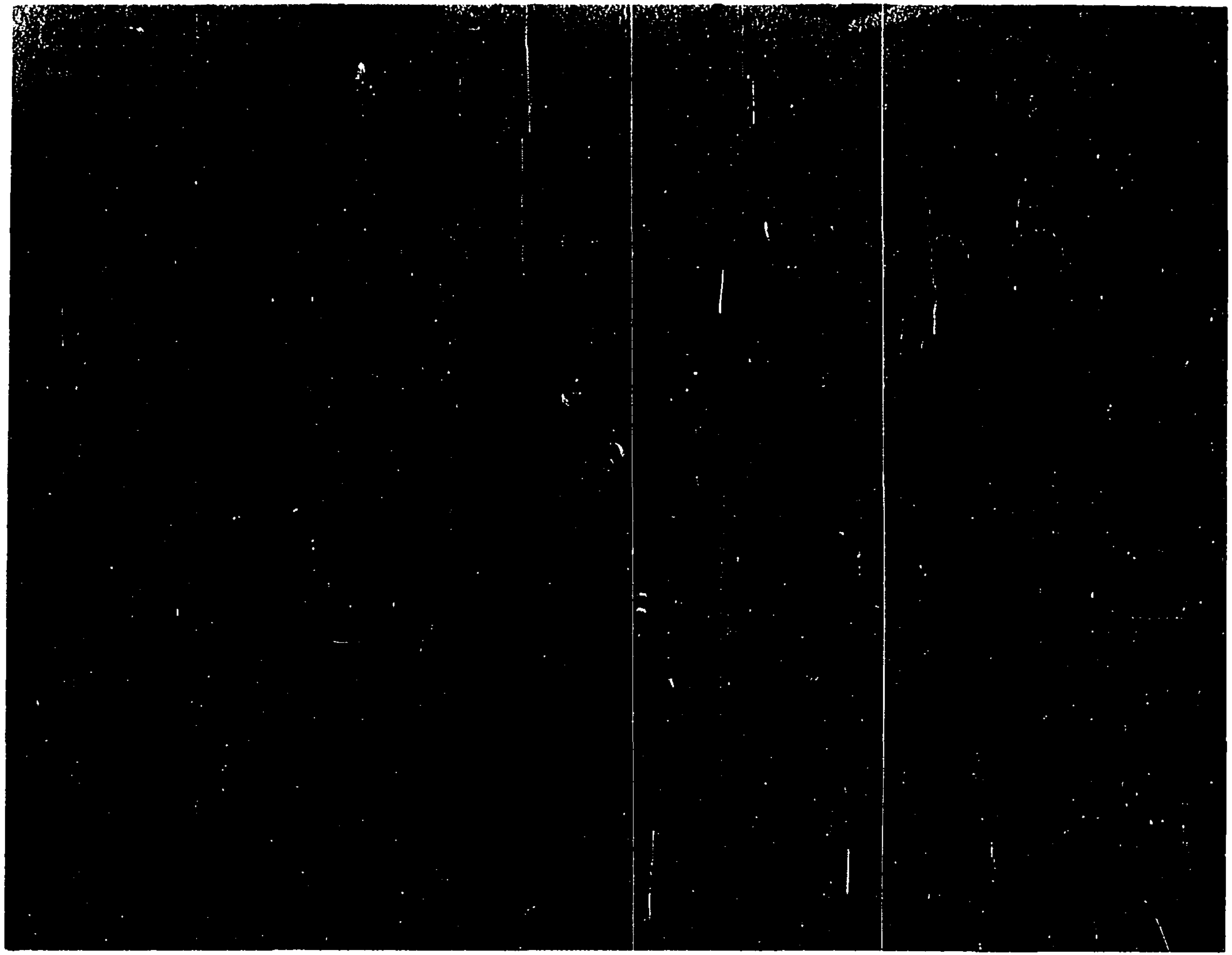


Figure 24. Overall View of Process Laboratory.

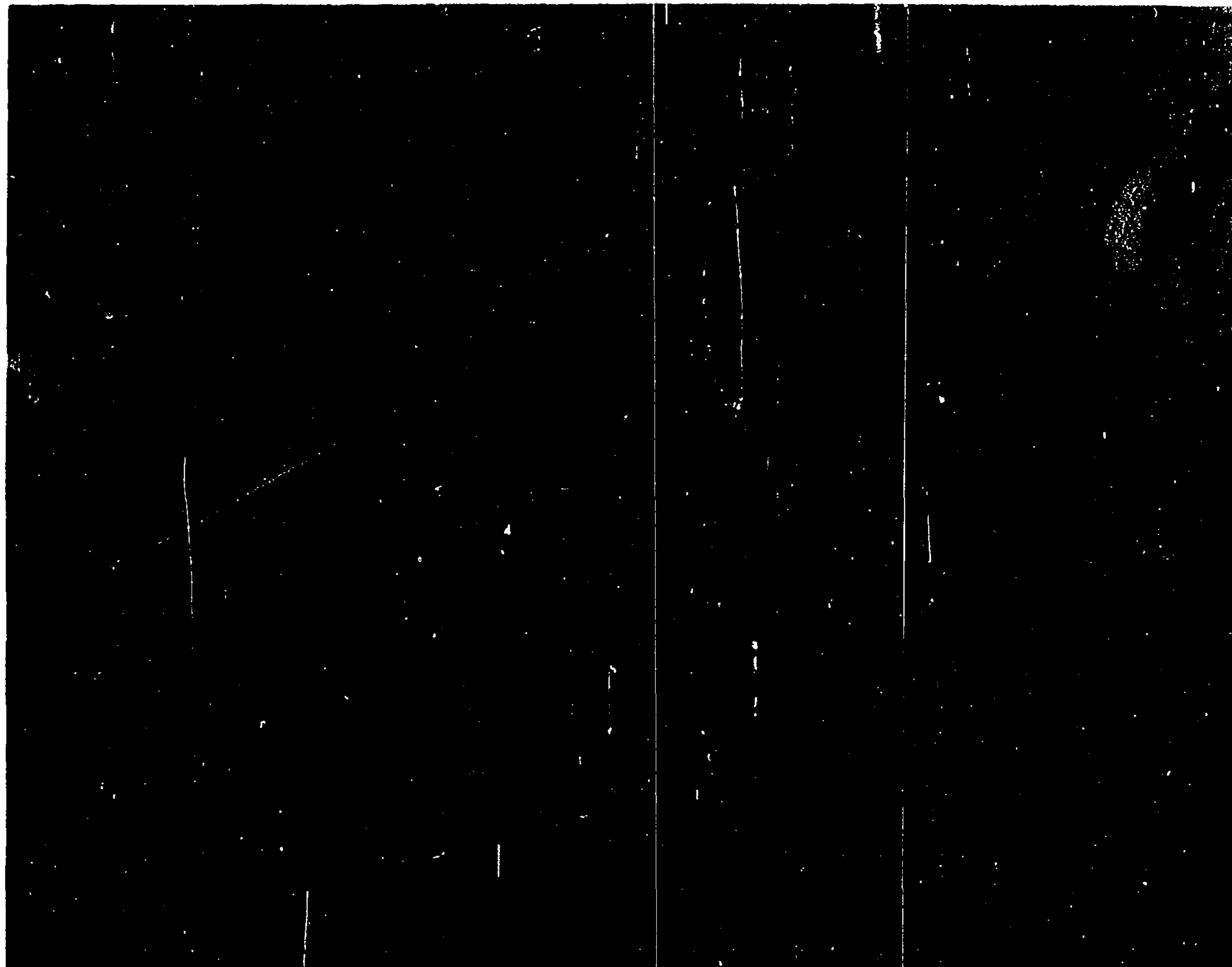


Figure 25. Chemical Reactor and Piping Arrangement.

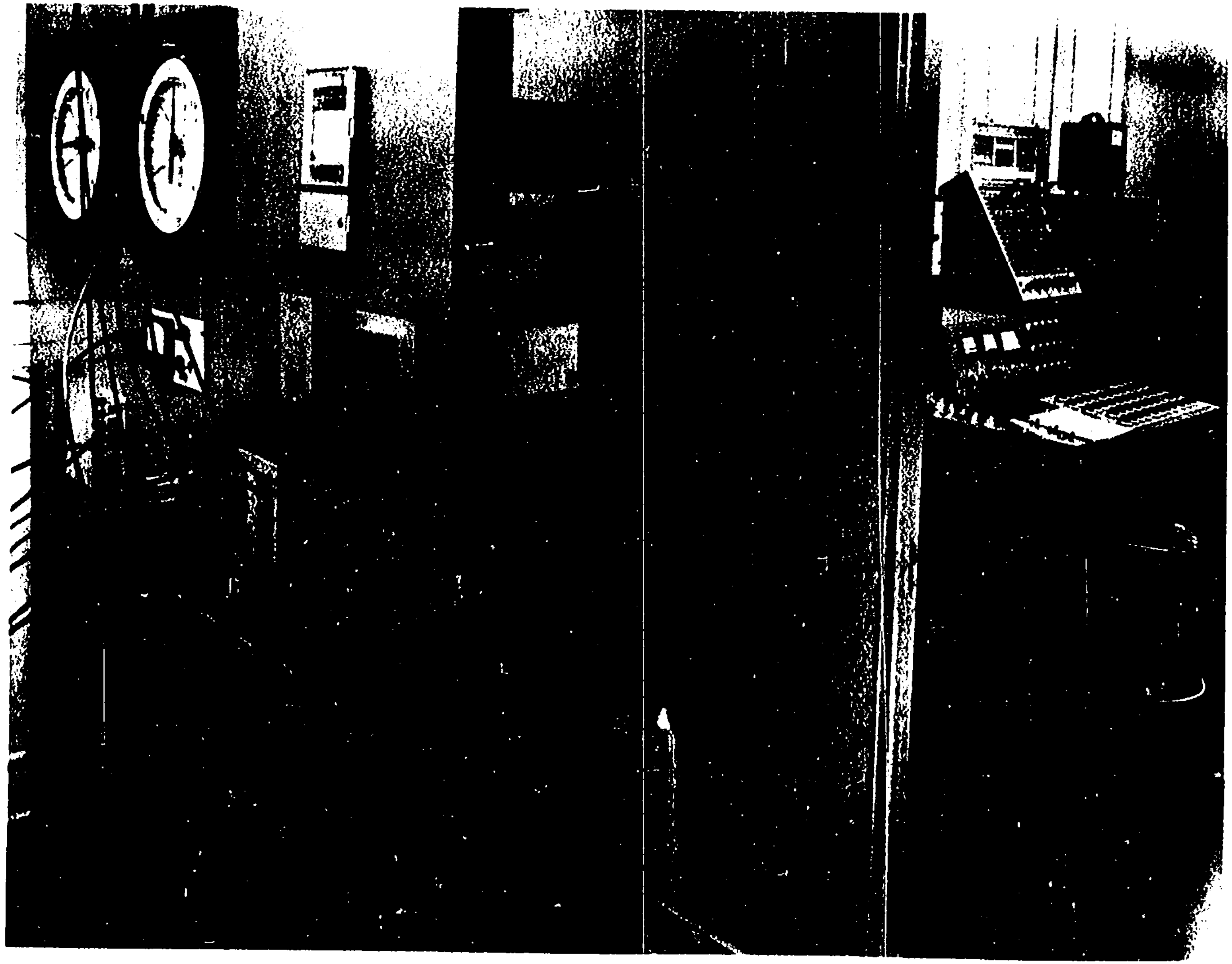


Figure 26. Control Panel for the Generalized Process.

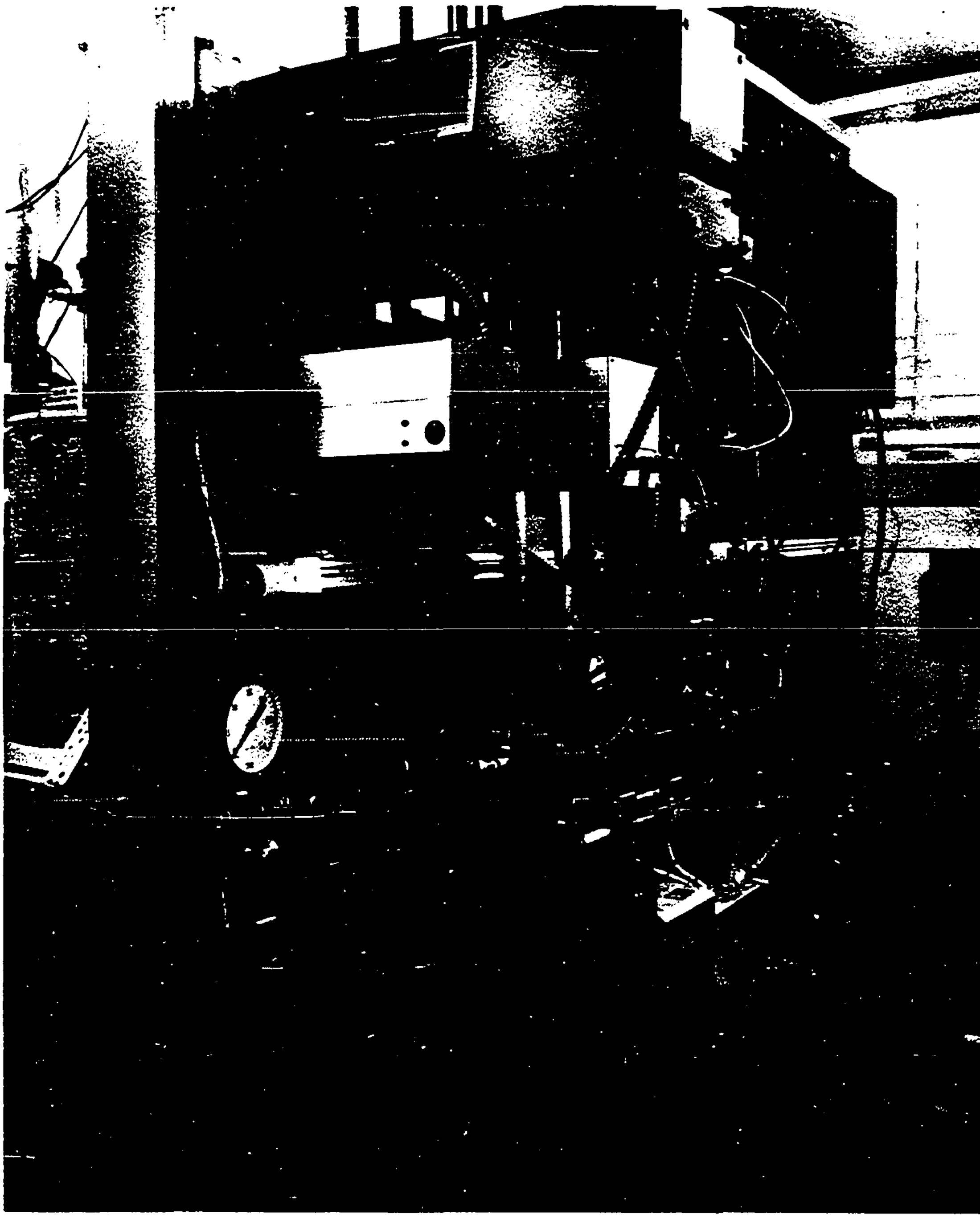


Figure 27. Rear View of Control Panel.

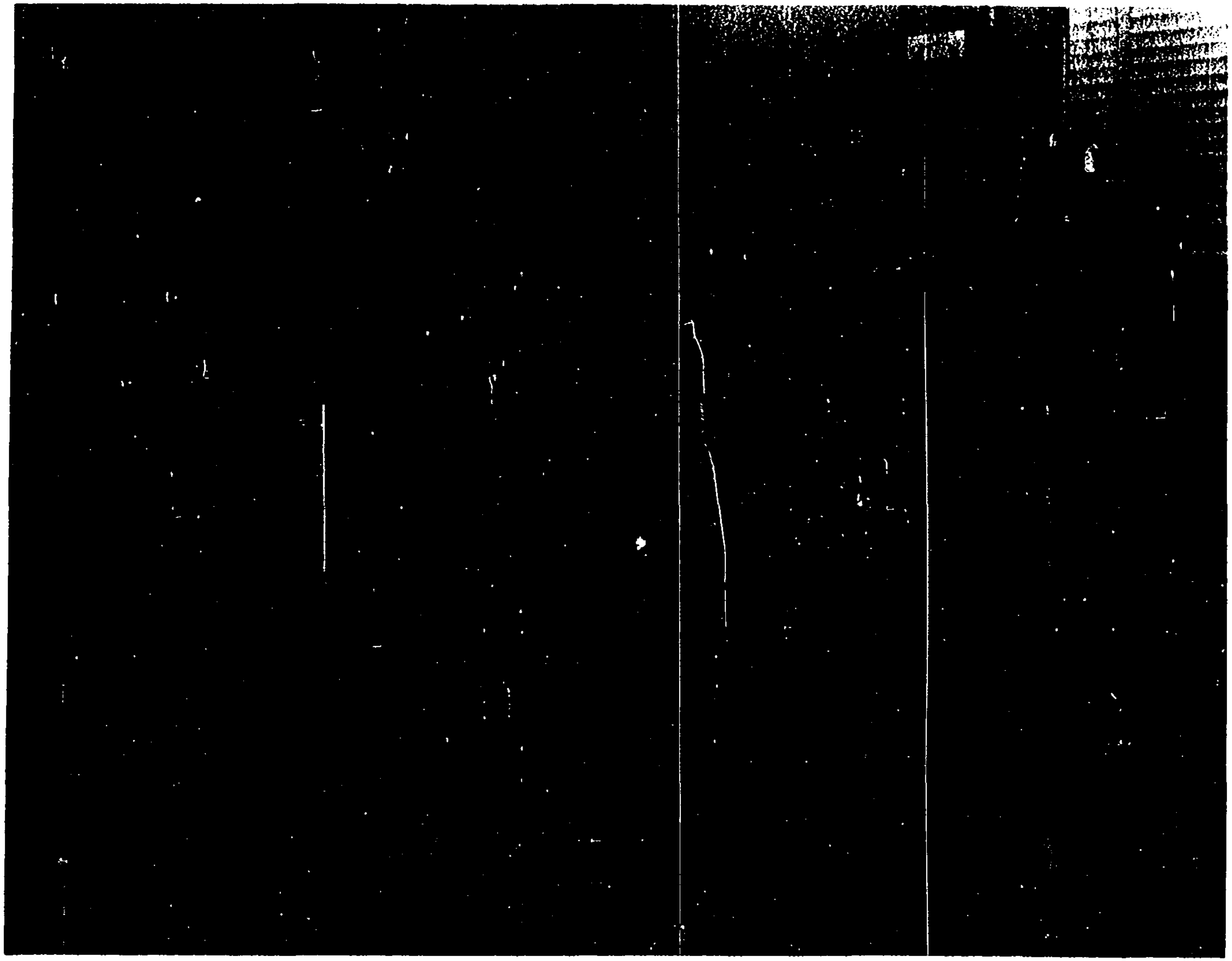


Figure 28. Donner Model 3100 D Analog Computer and Auxiliary Equipment.

configurations. In the background the control panel is visible. Figure 26 shows the control panel and the relationship of the process laboratory and the analog computer laboratory. Figure 27 shows the rear of the control panel where transducers, controllers, electrical and pneumatic signal sources are located. All electrical and pneumatic signals to and from the process pass through the control panel. Telemetry lines from the panel are connected to the analog computer. Figure 28 gives an overall picture of the Donner Model 3100D analog computer and its auxiliary equipment.

For this experiment the process consisted essentially of a simulated stirred tank reactor. Simulated refers to the fact that no actual chemical reaction was carried out in the reactor; the rate operation studied was the transfer of heat from the reactor fluid to the coolant. This simulation had two primary advantages: the process was simple and inexpensive to operate and secondly, the Arrhenius (exponential) type of nonlinearity was not needed in the mathematical model of the system. Since only the product nonlinearities were present, the invariance theory control laws were simplified. However, it should be noted that this system was not so simple that the experiment was trivial; complexity resulted from using a number of measuring, transducing, and controlling devices, as Figure 23 indicates.

The process was similar to that studied by Stewart (81), except that the system as operated demanded a description by a third, rather than second, order model. This complication resulted from using a reactor with walls having significant thermal capacitance, using ethylene glycol-water solutions and oil as fluids, and operating at low flow rates. With these operating conditions it was necessary to consider energy balances on the reactor fluid, coolant, and the wall between them.

The equipment arrangement is shown in Figure 29. Hot oil and cold glycol-water solution entered the reactor at constant temperatures, and both flow rates could be controlled and measured. All of the other system output variables could be measured: the bulk oil temperature, the coolant outlet temperature, and an average wall temperature. Other available measured temperatures were the coolant and oil inlet values.

System Components

Reactor: The simulated reactor was identical to that used by Stewart (81), except that the wall between the oil and coolant was replaced. Instead of steel, the wall was molded of type metal and was approximately one-half inch thick. During the molding process four thermocouples were inserted inside the wall approximately 90° apart. Hot oil entered the one liter reactor at the center of the bottom. The oil was agitated by a stirrer driven by a V-belt and pulley which was connected to a 1/4 horsepower electric motor operated at 1725 RPM. The oil left the

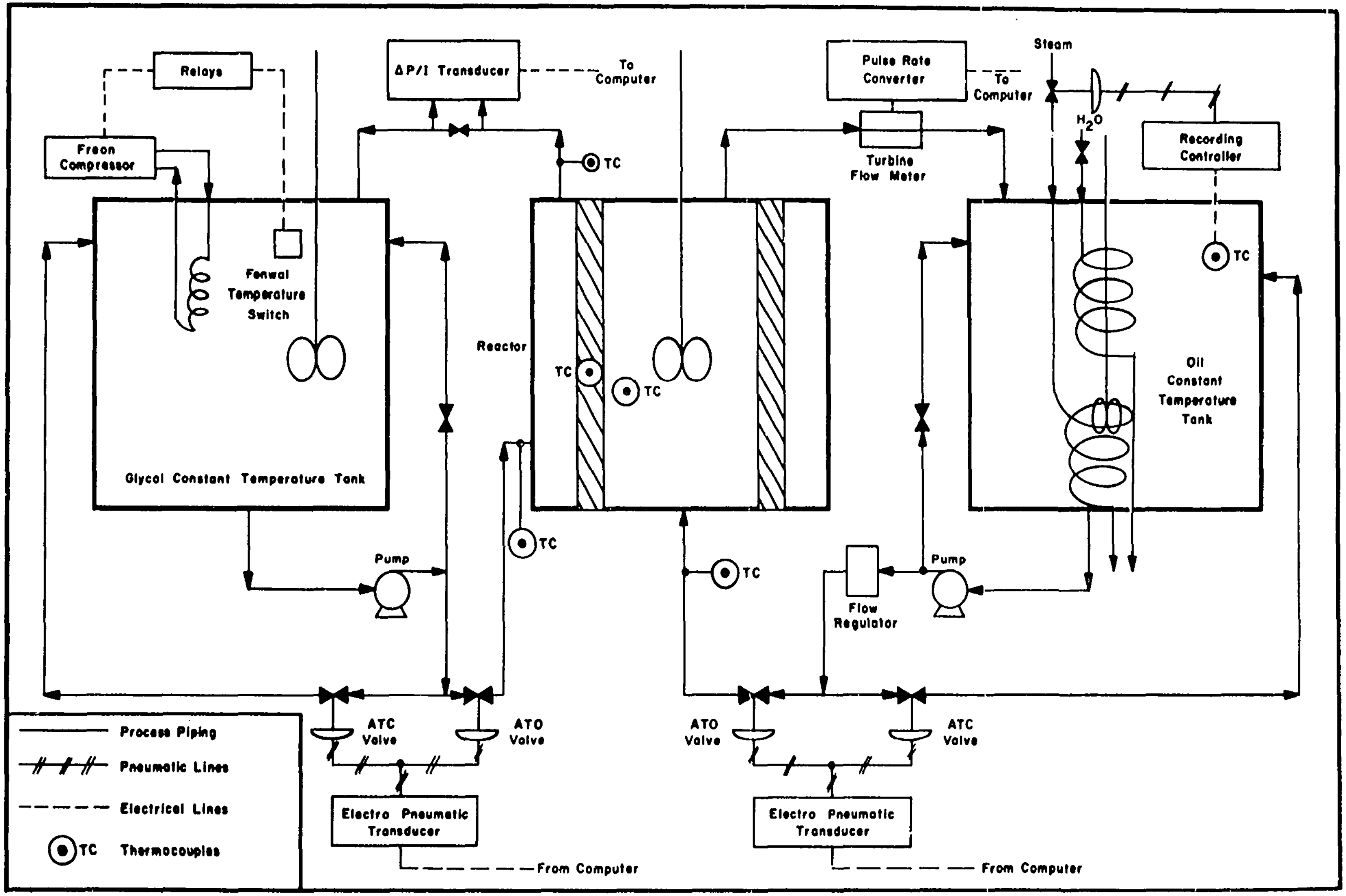


Figure 29. Experimental Apparatus

reactor at the top and off center. The coolant entered the annular space surrounding the reactor wall and left from the top on the opposite side.

<u>Constant Temperature Feed Tanks</u>: The oil was maintained at a constant temperature in a 42-gallon barrel. The oil was agitated by a Lightnin Mixer, Model NC2, rated at 1/8 horsepower and operated at 1725 RPM. The barrel contained both cold water and steam coils. The steam flow rate was controlled by a Research control valve (1/2 inch), Model 75S, air-to-close (C_v = 0.8 and a 3-15 pound range spring). The recording controller was a Minneapolis-Honeywell Brown ElectroniK Potentiometer Pyrometer, Model 152P14P-93-18, with a copper-constantan thermocouple pickup.

The glycol-water solution was maintained at constant temperature in a refrigerated cooler with a capacity of approximately twelve gallons. The glycol solution was agitated by a Precision Scientific Company mixer which was rated at twenty watts and operated at 1525 RPM. The cooler contained freon coils in which the temperature was controlled by cycling the operation of the freon compressor. A Fenwal thermoswitch (Catalog Number 17552-0), having a temperature range of -100 to 600°F, and a series of relays started the freon compressor whenever the temperature rose above the setpoint of the Fenwal thermoswitch.

<u>Flow Systems</u>: The oil was circulated by a California bronze gear pump (1/4 inch pipe connections) driven

by a Goulds Number 2 electric motor (3/4 horsepower). The pump discharge pressure was set at 50 psi by a valve on the bypass line. The oil passed through a Kates, Model MF, flow controller rated at 0.1 to 1.5 gallons per minute. The glycol solution was circulated by a 1/4 horsepower centrifugal pump which had a bypass line.

Flow Controllers and Transducers: The reactor inlet flows of both oil and coolant were controlled by a flow splitter arrangement. In each stream the flow was divided so that the fluid passed through an air-to-open control valve into the reactor and through an air-to-close control valve to the constant temperature feed tanks. All four valves were 1/4 inch, Type 75, Research control valves with 3 to 15-pound range springs. The valves each had a C_v of 0.2 for the oil and 0.08 for the coolant.

The pneumatic signal to the valves originated in the Taylor Transet electro-pneumatic transducers 701T which had a range of 3 to 15 psi. The input signals to the transducers were generated at the analog computer usually from a DC amplifier output. The anti-hysteresis controller, which was described in the second section of this chapter, proved to be so successful for the oil flow rate control system that a similar controller was built for the coolant flow system. Although hysteresis was not so serious a problem with the coolant flow system, a sluggish response was noted. The anti-hysteresis controller decreased the response time considerably.

Flow Measurements and Transducers: The flow measuring devices operated on two different principles, enabling the dynamic effects of two flow sensor types to be evaluated. The oil flow rate sensor was a Waugh turbine flowmeter, Model FL-6SB-1, rated at 0.15 to 1.0 gallons per minute. The pulses from this pickup were converted to a continuous voltage by a Waugh, Model FR-111, pulse rate converter. The output of the pulse rate converter was a voltage of 250 millivolts, maximum, which was available at the analog computer where it could be amplified.

The flow of coolant caused a pressure drop through a Hoke metering valve, Model 4RB281, with a C_v of 0.30. The valve stem had twenty turns and, for accurate positioning, was fitted with a micrometer vernier-indicating handwheel. The pressure drop created by the metering valve was transduced to electrical current by a Minneapolis-Honeywell, differential pressure to current transmitter. The output of this transmitter was 4 to 20 milliamps DC which created a 2 to 10 DC voltage drop across a precision 500 ohm resistor. This voltage was telemetered to the analog computer.

Temperature Measurements and Preamplifiers: All thermocouples were 24 gauge copper-constantan wire. An average wall temperature was measured with thermocouples placed in parallel inside the reactor wall. The inlet oil temperature was measured by a thermopile consisting of three thermocouples arranged in series. This thermopile was placed inside a Poly-Flo tee fitting which was installed at the reactor inlet and sealed with a teflon plug. The bulk temperature of the oil was measured by a thermocouple placed in a thermowell which extended into the reactor from the top. The coolant inlet and outlet temperatures were each measured by thermocouples inserted through the walls of Poly-Flo tubing and sealed. The locations of these thermocouples were approximately two inches from the inlet and four inches from the outlet.

All thermocouples had a reference junction at 32°F (ice bath). The thermocouple voltages for all the temperatures mentioned, and two constant temperature feed tank values, could be selected on a two-section, rotary switch. The common junctions of this switch were connected to a Leeds and Northrup potentiometer, Model 8662. The potentiometer readings were used in the determination of the steady-state operating temperatures and to check temperatures during dynamic testing.

The three temperatures, wall, bulk and coolant outlet, which were considered as process outputs could be telemetered in the form of thermocouple voltages to the analog computer. These voltages went through the preamplifiers before telemetry. The preamplifier for the controlled variable in each case was a Sanborn, Model 350-1500, low-level DC preamplifier with a Model 350-2 plug-in unit. This instrument allowed an adjustable gain up to 50,000 and an input suppression of $\pm$ 100 millivolts. A Hewlett-Packard, DC vacuum tube voltmeter, Model 412A, which contained an amplifier was used as a preamplifier for the measured temperatures, when it was needed. A second stage of amplification was performed on the analog computer in this case. The total amplifier gain for both thermocouple voltage amplifications was 2500.

The instrumentation discussed above resulted in all process inputs and outputs appearing at the analog computer. From the analog computer the physical process and associated transducers appeared to constitute a large complex function generator. From this standpoint the experimentation could be performed like any purely analog computer study. An instrumentation diagram is given in Figure 30.

The analog computer used for this study was the Donner, Model 3100D, expanded to 60 amplifiers and modified for use in the Process Dynamics Laboratory. A complete description of this facility has been given by Bishop and Sims (7).

The disturbances for this process were arbitrarily chosen as sinusoidal and square waves. The waves were electronically generated by an Exact, Type 240, function generator, which has a frequency range of 0.001 to 10,000 cycles per second. The disturbance variable chosen was the coolant flow rate and, therefore, the function generator output was connected to the input of the Taylor transducer which

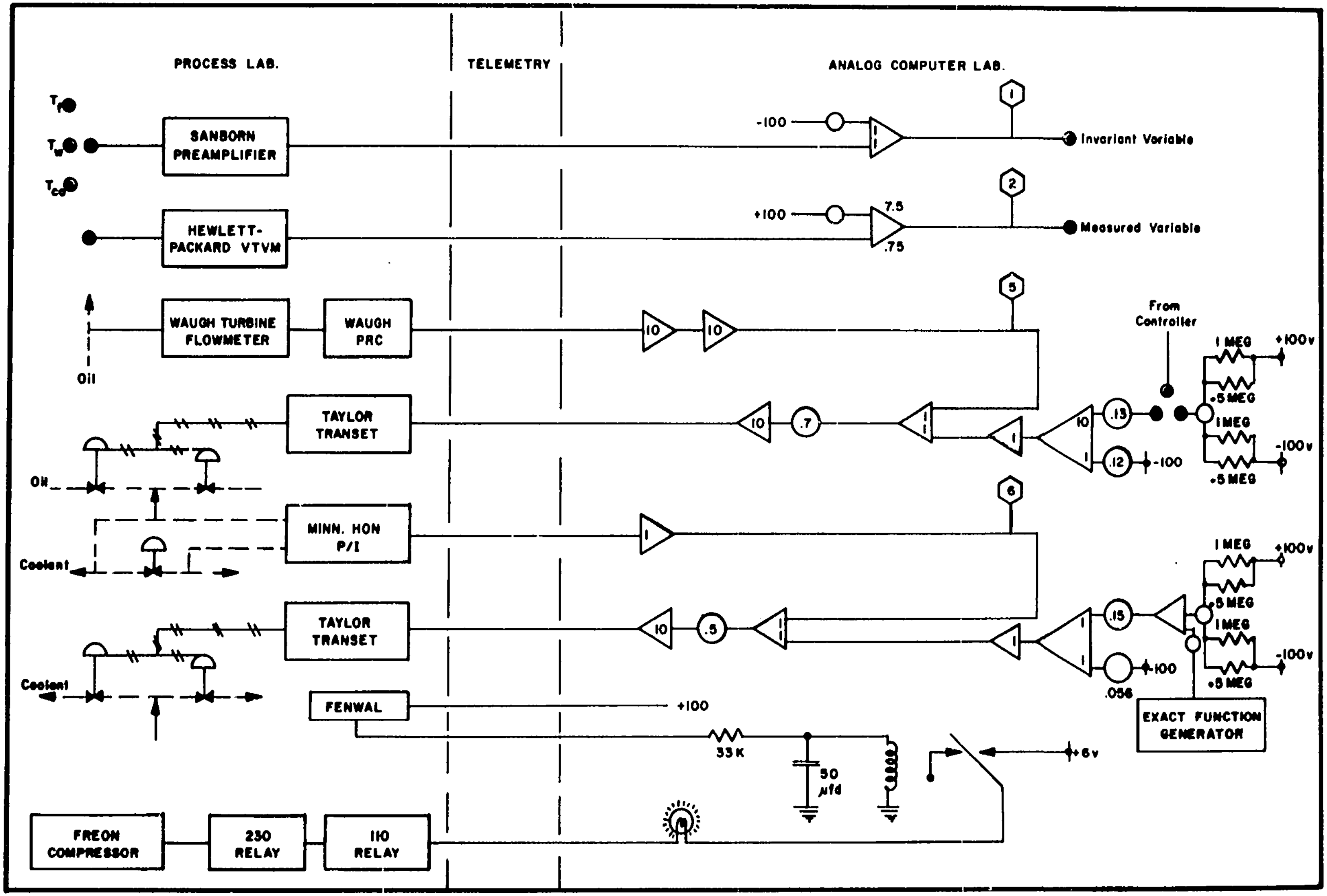


Figure 30. Instrumentation Diagram.

supplied the pneumatic signal to the coolant flow controllers. The measured variables could be chosen to be any of the three output temperatures or the coolant flow rate. These signals could be operated upon by the analog-computer-programmed controllers according to the control laws specified by the invariance theory. The analog controller output signal could be sent to the other Taylor transducer which supplied the pneumatic signal to the oil flow rate control valves. The oil flow rate was the manipulated input in each case. All the process variables were available for recording on a Sanborn, Model 156-1100C, six-channel recorder, if desired.

Experimental Procedure

The experimental testing was conducted in a manner similar to the analog computer simulation studies. The efficiency of the experimental controllers was defined by the same equation as before, Equation (98). Each experimental controller was programmed in turn according to Figures 21 through 23. The proper electrical connections were made between the points marked (⊙) on the instrumentation diagram, Figure 30. Once the system had reached steady-state operating conditions, the disturbance input was introduced and the testing begun.

The integral absolute value circuit which was used in the simulation studies was not used because the voltage levels were too low. The bias voltage of the absolute value circuit prohibited the automatic calculation of the integral absolute value of the controlled variable. If the voltage levels were increased to a useable magnitude, noise became a serious problem. Furthermore, slight variations in the steady-state conditions during a controller test period could have caused major changes in the integral absolute value even though the disturbance was compensated by manipulated input corrections. The area under the time-voltage curves was measured directly from recordings by making straight line approximations to the curves and subsequently calculating the areas of squares and triangles thus formed. The steady-state level of the controlled variable was determined before and after each run with the disturbance input disconnected. This procedure was accurate enough for efficiency calculations since extremely precise values of the efficiencies were not necessary to indicate the quality of control.

For each controller configuration the following tests were made:

1) No controller
2) Linear model controller
3) Nonlinear model controller.

Testing was done at three frequencies: 0.0032 cps, 0.0064 cps, and 0.01 cps. Above 0.01 cycles per second the frequency response characteristics of the process reduced the output signals to levels that were too low for dynamic analysis. Testing at lower frequencies could have been done, but the time for each test would be excessive in proportion to the amount of additional information that would be obtained.

The disturbance magnitude was $\pm$ 20 pounds per hour of coolant about a steady state value of 54 pounds per hour.

After the controlled system was operating on line, the coefficient pots in the controllers were varied to determine if any improvement in control would result from using constants different from those derived by the model equations. During optimization of the controllers the pots could not be varied independently because the steady-state output signal of the controllers had to be held constant. Furthermore, the optimization was time consuming because the controllers had just as slow a response to tuning as the process had to a disturbance. The only alternative to tuning would be to obtain the constants empirically from experimental data obtained by dynamic testing.

The simplest controller (configuration B) was used to study the problem associated with a set-point change. The results of this study could be extended to all other controller configurations.

CHAPTER VII

DISCUSSION OF RESULTS

The contents of this chapter will include a discussion of the results of the analog computer simulation studies and the experimental investigations. The implications of the specific results to general synthesis procedures will be made.

Analog Computer Simulation Studies

A sample of the Sanborn recorder output is shown in Figure 31. The runs shown are D103-5, and the pertinent information is given in Table 8. Figure 32 shows the response of all the process variables for the nonlinear controller D. Although this type of recording was not used for any simulation study, it is illustrative of the quality of control possible with invariance theory. The recording shows the uncontrolled response for sine, triangular, and square wave disturbances of a frequency of 0.05 cycles per second. This response is followed by the controlled response to the same disturbances in reverse order. The last part of Figure 32 shows the system response to various wave shapes and frequencies imposed in a somewhat random manner with and without control.

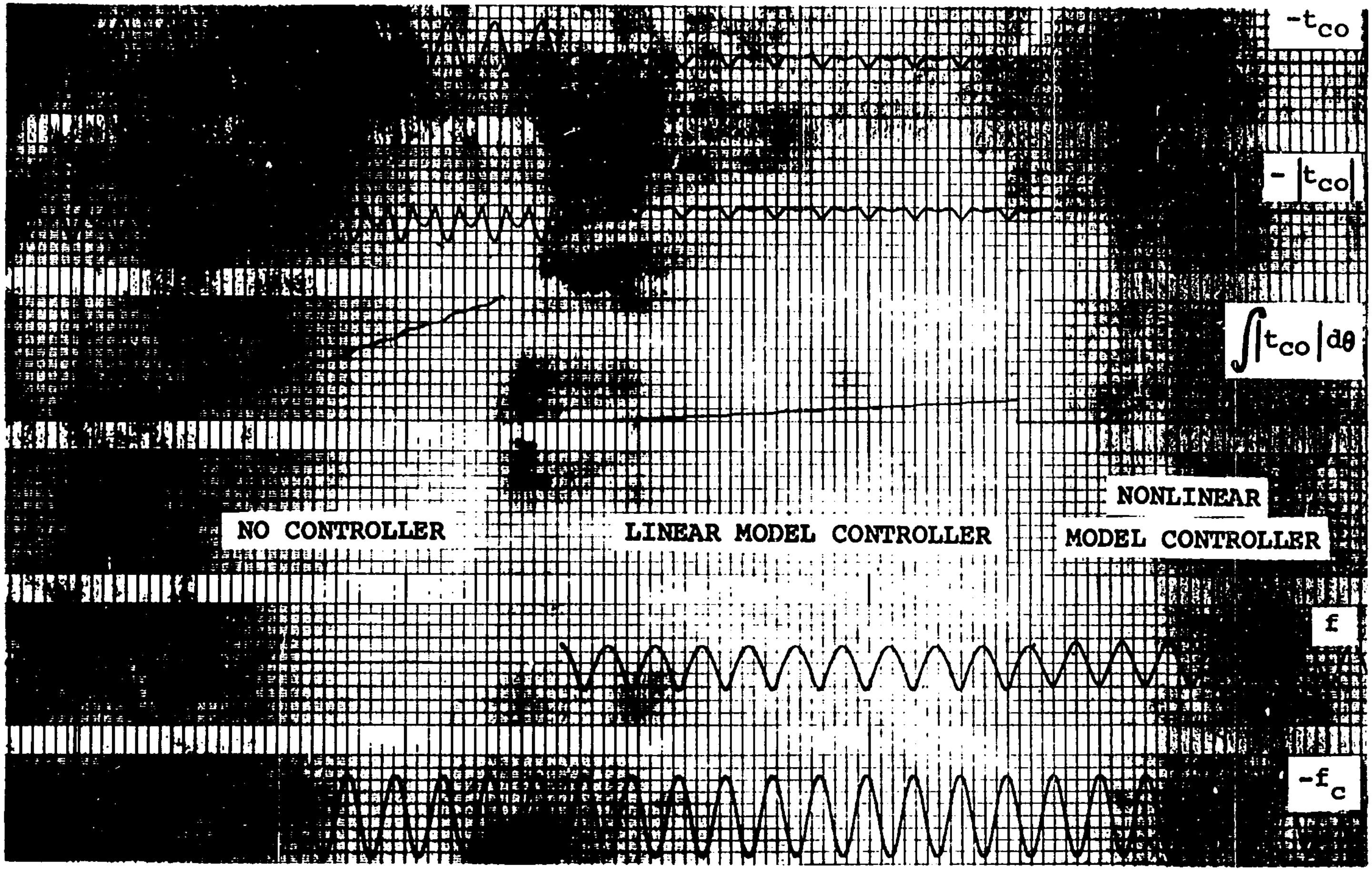


Figure 31. Sample Sanborn Recording for Simulation Runs D103-5.

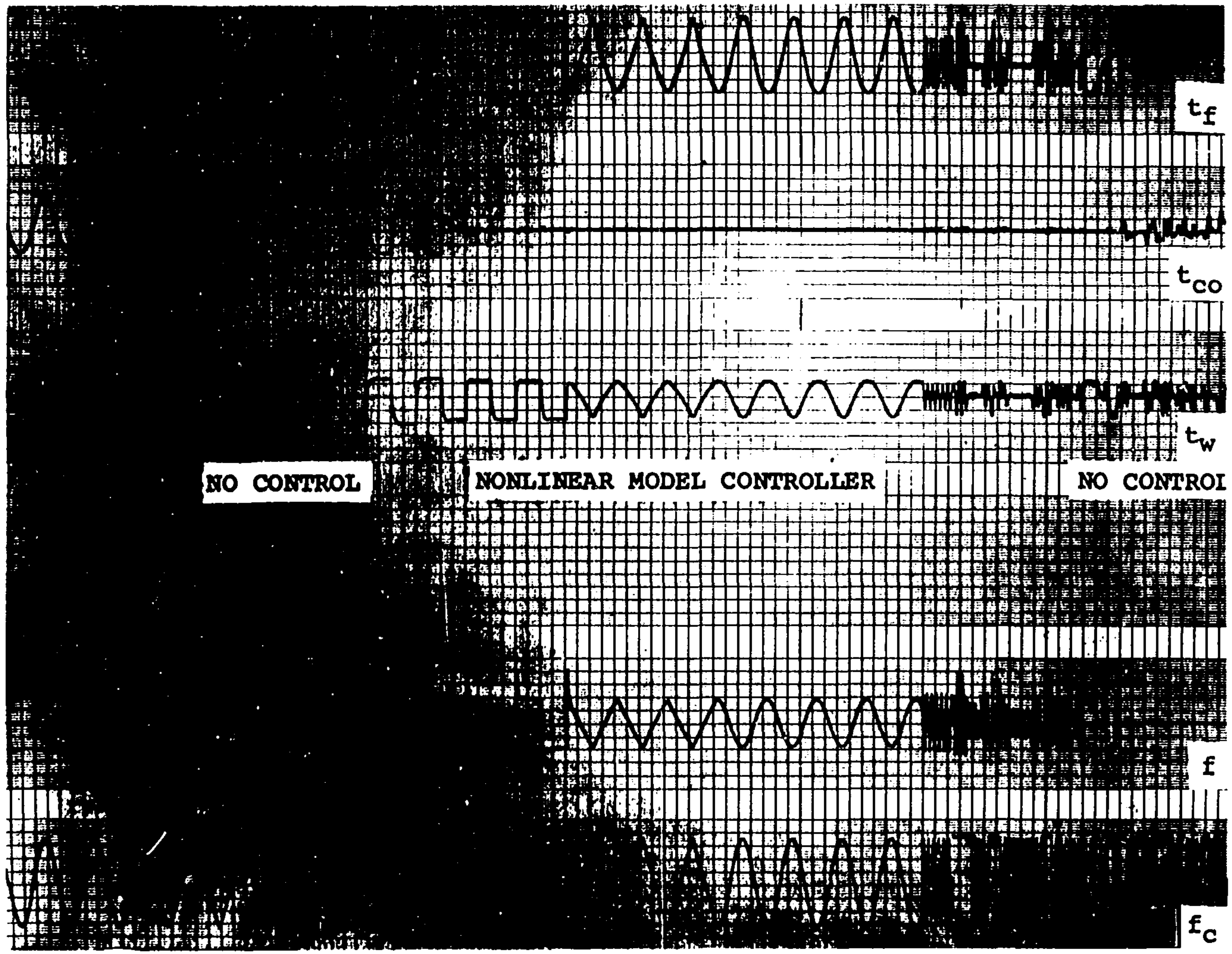


Figure 32. Simulated System Response for Various Types of Disturbances: Nonlinear Process Simulation, Nonlinear Model Controller D.

All controllers in the simulation study were tested exactly as they were programmed from the theory. No optimization of pot settings was necessary to obtain better control.

Controllers A and B: Both of these controllers were studied primarily with sine wave disturbances. There was only one controller type in each configuration because the linear and nonlinear models led to the same controller specifications. The results of these tests are given in Table 6. Both controllers were also tested for a variety of wave shapes and frequencies. No results are shown in the table for these runs, but it was found that the integral absolute value of the controlled variable never deviated from zero on the recorder during these tests. Thus, these controllers were considered to be 100 per cent efficient for all wave shapes and frequencies in the simulation study. No further testing was performed. These results represented the ideal behavior of an invariance control system since there was no way to vary the disturbance in order to cause a variation in the controlled variable.

Controller C: The nonlinear model and the linearized model each specified different control configurations in this case. The results of these simulation studies are given in Table 7. Runs C106, C107, C117, and C118 showed that the linear controller of a simulated linear process was 100 per cent efficient. The remaining runs compared the efficiencies of the linear and nonlinear controllers for

TABLE 6

EFFICIENCIES OF CONTROLLERS A AND B: ANALOG COMPUTER STUDIES

Run No.	Controller Configuration	Process Simulation	Disturbance Wave	Disturbance cps	No. of Cycles	Lines Start	Lines End	Δ Lines	Efficiency %
A1	none	linear	sine	0.05	2	13.1	42.7	29.6	-
A2	linear A	linear	sine	0.05	2	0	0	0	100
A3	none	nonlinear	sine	0.05	2	7.5	38.3	30.8	-
A4	linear A	nonlinear	sine	0.05	2	0	0	0	100
A5	none	nonlinear	sine	0.10	7	5.3	49.7	44.4	-
A6	linear A	nonlinear	sine	0.10	7	0	0	0	100
A7	none	nonlinear	sine	0.50	40	0.6	6.3	5.7	-
A8	linear A	nonlinear	sine	0.50	40	0	0	0	100
B1	none	linear	sine	0.05	3	4.7	48.9	44.2	-
B2	linear B	linear	sine	0.05	3	0	0	0	100
B3	none	nonlinear	sine	0.05	3	0.2	46.1	45.9	-
B4	linear B	nonlinear	sine	0.05	3	0	0	0	100
B5	none	nonlinear	sine	0.10	7	1.4	45.3	43.9	-
B6	linear B	nonlinear	sine	0.10	7	0	0	0	100
B7	none	nonlinear	sine	0.50	40	0.7	6.1	5.4	-
B8	linear B	nonlinear	sine	0.50	40	0	0	0	100

TABLE 7

EFFICIENCIES OF CONTROLLER C: ANALOG COMPUTER STUDIES

Run No.	Controller Configuration	Process Simulation	Disturbance Wave	Disturbance cps	No. of Cycles	Lines Start	Lines End	Δ Lines	Efficiency %
C106	none	linear	sine	0.05	3	0	44.0	44.0	-
C107	linear C	linear	sine	0.05	3	0	0	0	100
C108	none	nonlinear	sine	0.05	3	2.0	49.0	47.0	-
C109	linear C	nonlinear	sine	0.05	3	0	8.2	8.2	83
C110	nonlinear C	nonlinear	sine	0.05	3	0	0	0	100
C117	none	linear	sine	0.10	7	1.8	44.5	42.7	-
C118	linear C	linear	sine	0.10	7	0	0	0	100
C119	none	nonlinear	sine	0.10	7	1.0	45.9	44.9	-
C120	linear C	nonlinear	sine	0.10	7	0.1	8.5	8.4	81
C121	nonlinear C	nonlinear	sine	0.10	7	0	0	0	100
C111	none	nonlinear	triang.	0.05	3	5.0	42.6	37.6	-
C112	linear C	nonlinear	triang.	0.05	3	0.7	6.0	5.3	86
C113	nonlinear C	nonlinear	triang.	0.05	3	0	0	0	100
C114	none	nonlinear	square	0.05	2	0.4	48.4	48.0	-
C115	linear C	nonlinear	square	0.05	2	4.9	18.1	13.2	73
C116	nonlinear C	nonlinear	square	0.05	2	0	0	0	100

control of the simulated linear process. Runs C109, C120, C112, and C115 indicated that the controller designed, using the linear model, did not provide absolute invariance although the control efficiency was still quite good. Since perfect control was not attained, it was expected that the efficiency would be a function of wave shape and frequency. Frequent and severe disturbances should have lowered the efficiency, and the results of these runs support this belief. Changing to a higher frequency and from a sine to a square wave both lowered the efficiency. When the more complicated nonlinear controller was used, the invariance efficiency was 100 per cent, and it was not a function of wave shape or frequency.

Controller D: In these studies it was possible to compare linear and nonlinear controllers. These controllers also gave the first test of the analog differentiation circuit. The results are presented in Table 8. The linear controller used for control of the simulated linear process was 100 per cent efficient for all waves and frequencies except the highest frequency sine wave tested. These results tended to support the hypothesis that the differentiator gave a very good approximation to the time derivative, but it became less accurate as the frequency increased. Although it was a little sensitive to frequency increases, the differentiator could handle severe disturbances (step functions) very well. Runs D104, D140, and D145 demonstrated the decrease in efficiency with increase in frequency for the linear control of the

TABLE 8

EFFICIENCIES OF CONTROLLER D: ANALOG COMPUTER STUDIES

Run No.	Controller Configuration	Process Simulation	Disturbance		No. of Cycles	Lines		Δ Lines	Efficiency %
			Wave	cps		Start	End		
D101	none	linear	sine	0.05	3	0.4	39.3	38.9	-
D102	linear D	linear	sine	0.05	3	0	0	0	100
D103	none	nonlinear	sine	0.05	3	0.7	41.5	40.8	-
D104	linear D	nonlinear	sine	0.05	3	0.1	5.6	5.5	87
D105	nonlinear D	nonlinear	sine	0.05	3	0	0	0	100
D137	none	linear	sine	0.20	16	1.0	30.6	29.6	-
D138	linear D	linear	sine	0.20	16	0	0	0	100
D139	none	nonlinear	sine	0.20	16	0.5	30.6	30.1	-
D140	linear D	nonlinear	sine	0.20	16	0	5.8	5.8	81
D141	nonlinear D	nonlinear	sine	0.20	16	0	0.2	0.2	99
D142	none	linear	sine	0.50	40	0.6	10.4	9.8	-
D143	linear D	linear	sine	0.50	40	0	0.2	0.2	98
D144	none	nonlinear	sine	0.50	40	0.8	10.7	9.9	-
D145	linear D	nonlinear	sine	0.50	40	0.2	2.8	2.6	74
D146	nonlinear D	nonlinear	sine	0.50	40	0	0.4	0.4	96
D111	none	linear	triang.	0.05	4	2.5	45.0	42.5	-
D112	linear D	linear	triang.	0.05	4	0	0	0	100
D113	none	nonlinear	triang.	0.05	4	2.3	45.7	43.4	-
D114	linear D	nonlinear	triang.	0.05	4	0	5.2	5.2	88
D115	nonlinear D	nonlinear	triang.	0.05	4	0	0	0	100
D116	none	linear	square	0.05	2	7.8	46.7	38.9	-
D117	linear D	linear	square	0.05	2	0	0	0	100
D118	none	nonlinear	square	0.05	2	6.0	47.8	41.8	-
D119	linear D	nonlinear	square	0.05	2	1.0	10.8	9.8	77

TABLE 8--Continued

Run No.	Controller Configuration	Process Simulation	Disturbance Wave	Disturbance cps	No. of Cycles	Lines Start	Lines End	Δ Lines	Efficiency %
D120	nonlinear D	nonlinear	square	0.05	2	0	0.1	0.1	99.8
D147	none	nonlinear	sine	0.05	3	0.9	41.5	40.6	-
D148	linear D	nonlinear	sine	0.05	3	0.3	6.2	5.9	85
D149	none	nonlinear	sine	0.05	4	1.7	41.2	39.5	-
D150	linear D	nonlinear	sine	0.05	4	0.3	4.2	3.9	90
D151	none	nonlinear	sine	0.05	6	2.1	41.6	39.5	-
D152	linear D	nonlinear	sine	0.05	6	0.5	2.9	2.4	94
D153	none	nonlinear	sine	0.05	6	2.1	21.9	19.8	-
D154	linear D	nonlinear	sine	0.05	6	0	0.6	0.6	97

simulated nonlinear process. The linear control of the simulated nonlinear process was also less efficient for square waves than for sine waves, just as controller C was. The nonlinear controller efficiency for the simulated nonlinear process also was affected by the increase of frequency as runs D105, D141, and D146 demonstrated. Most of the loss of efficiency was attributed to the approximate differentiator, but some loss could have been caused by electronic multiplier inaccuracies.

Runs D147 through D154 illustrated the effect of disturbance magnitude upon efficiency. Because the magnitude of the disturbance affected the nonlinear terms in the model, the simulated process behavior should have become more linear as the disturbance magnitude decreased. Figure 33, which was taken from the results of runs D147 through D154, illustrates this point.

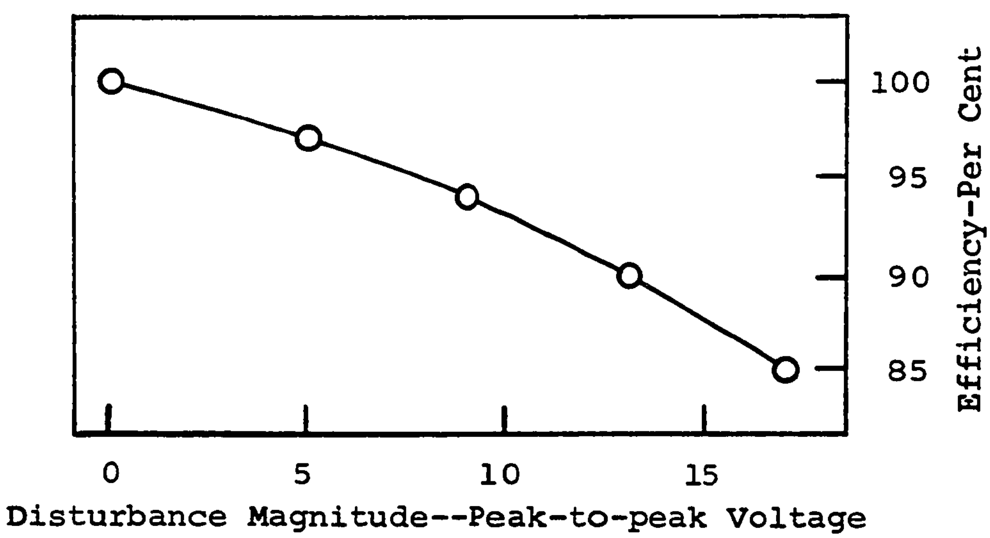


Figure 33. Efficiency of Controller D as a Function of Disturbance Magnitude.

The effect of disturbance magnitude upon the efficiency of a linear controller for a simulated nonlinear process could have been demonstrated by any of the configurations whose linear and nonlinear controllers were different.

Controller E: Table 9 lists the results of the simulation studies for controller E. The linear and nonlinear controller efficiencies were not strongly affected by disturbance frequency or wave shape. These controller configurations had one interesting feature: the efficiencies of the linear model controller were remarkably high. Most of the control quality improvement came from using the invariance device which was designed on a linear model basis. Very little further improvement came from applying the more complicated nonlinear controller. The reason for the high efficiencies was that the simulated nonlinear process and the nonlinear controller each contained two nonlinearities which effectively cancelled one another dynamically. Runs E22 through E25 reinforced this belief. After the base case run E22 was evaluated, the efficiencies were determined for the nonlinear controller operating first with one of the controller nonlinearities and then with the other. Run E23 was for the multiplier nonlinearity only, and run E24 was for the divider nonlinearity alone. Run E25 was made with both nonlinear control elements functioning. Calculation of the efficiencies showed that nonlinear control with either half of the nonlinear portion of the controller alone was less

TABLE 9

EFFICIENCIES OF CONTROLLER E: ANALOG COMPUTER STUDIES

Run No.	Controller Configuration	Process Simulation	Disturbance		No. of Cycles	Lines		Δ Lines	Efficiency %
			Wave	cps		Start	End		
E101	none	linear	sine	0.05	3	0.9	40.0	39.1	-
E102	linear E	linear	sine	0.05	3	0	0	0	100
E103	none	nonlinear	sine	0.05	3	0.7	41.5	40.8	-
E104	linear E	nonlinear	sine	0.05	3	0.2	0.7	0.5	99
E105	nonlinear E	nonlinear	sine	0.05	3	0	0	0	100
E106	none	linear	sine	0.10	7	7.9	47.0	39.1	-
E107	linear E	linear	sine	0.10	7	0	0.1	0.1	99.7
E108	none	nonlinear	sine	0.10	7	5.0	45.6	40.6	-
E109	linear E	nonlinear	sine	0.10	7	0.3	1.1	0.8	98
E110	nonlinear E	nonlinear	sine	0.10	7	0.6	0.7	0.1	99.8
E111	none	linear	triang.	0.05	4	3.2	46.9	43.7	-
E112	linear E	linear	triang.	0.05	4	0	0	0	100
E113	none	nonlinear	triang.	0.05	4	4.8	49.8	45.0	-
E114	linear E	nonlinear	triang.	0.05	4	0.3	0.8	0.5	99
E115	nonlinear E	nonlinear	triang.	0.05	4	0	0	0	100
E116	none	linear	square	0.05	2	8.3	48.0	39.7	-
E117	linear E	linear	square	0.05	2	0	0	0	100
E118	none	nonlinear	square	0.05	2	6.2	49.1	42.9	-
E119	linear E	nonlinear	square	0.05	2	0.2	1.9	1.7	96
E120	nonlinear E	nonlinear	square	0.05	2	-0.2	0	0.2	99.5
E22	none	nonlinear	sine	0.05	3	6.1	47.1	41.0	-
E23	nonlinear E*	nonlinear	sine	0.05	3	1.4	9.3	7.9	81
E24	nonlinear E*	nonlinear	sine	0.05	3	0.9	8.1	7.2	82
E25	nonlinear E	nonlinear	sine	0.05	3	0	0	0	100

*Part of nonlinear controller only.

efficient than pure linear control. Furthermore, nonlinear control with all the nonlinearities is only a little more efficient than linear control. The conclusion drawn was that linear control gave practically all the quality gain, and if the last little bit of efficiency improvement was desired, the entire nonlinear controller was needed.

<u>Controller F</u>: This configuration presented the most serious problems of all the simulation studies. The results are given in Table 10. The existence of the double differentiator caused some doubts about the successful operation of this controller. For the sine and triangular wave disturbances these controllers operated very well and no particular effect of increasing the frequency was noted. This behavior was surprising, since the single differentiator had this effect. The biggest problem was encountered when square waves were introduced. It was not surprising that square waves were hard to cope with because the controller in that case was required to accurately compute the second derivatives of step functions. Controllers D and E had demonstrated that first derivatives of step functions, which are impulse functions, could be approximated to a sufficient degree that very good control was obtained. However, the derivatives of impulses in controller F were not very accurate. As a result, the efficiencies of the linear controller for the simulated linear process and the nonlinear controller for the simulated nonlinear process were not close to the usual 100 per cent.

TABLE 10

EFFICIENCIES OF CONTROLLER F: ANALOG COMPUTER STUDIES

Run No.	Controller Configuration	Process Simulation	Disturbance		No. of Cycles	Lines		Δ Lines	Efficiency %
			Wave	cps		Start	End		
F101	none	linear	sine	0.01	1.5	9.7	40.0	30.3	-
F102	linear F	linear	sine	0.01	1.5	0	0	0	100
F109	none	nonlinear	sine	0.01	1	6.4	26.3	19.9	-
F110	linear F	nonlinear	sine	0.01	1	2.7	7.8	5.1	74
F111	nonlinear F	nonlinear	sine	0.01	1	0	0	0	100
F103	none	linear	sine	0.05	6	2.0	25.3	23.3	-
F104	linear F	linear	sine	0.05	6	0	0	0	100
F112	none	nonlinear	sine	0.05	6	2.1	25.4	23.3	-
F113	linear F	nonlinear	sine	0.05	6	1.2	6.6	5.4	77
F114	nonlinear F	nonlinear	sine	0.05	6	0	0	0	100
F105	none	linear	sine	0.10	15	1.0	28.4	27.4	-
F106	linear F	linear	sine	0.10	15	0.2	0.5	0.3	99
F115	none	nonlinear	sine	0.10	15	1.7	29.0	27.3	-
F116	linear F	nonlinear	sine	0.10	15	0.2	6.7	6.5	76
F117	nonlinear F	nonlinear	sine	0.10	15	0	0.2	0.2	99
F33	none	linear	sine	0.15	22	0.2	24.8	24.6	-
F34	linear F	linear	sine	0.15	22	0	1.4	1.4	94
F121	none	linear	triang.	0.05	5	1.0	16.9	15.9	-
F122	linear F	linear	triang.	0.05	5	0	0	0	100
F123	none	nonlinear	triang.	0.05	5	1.0	16.4	15.4	-
F124	linear F	nonlinear	triang.	0.05	5	-0.2	1.9	2.1	86
F125	nonlinear F	nonlinear	triang.	0.05	5	0	0	0	100
F126	none	linear	square	0.05	4	2.5	28.0	25.5	-
F127	linear F	linear	square	0.05	4	1.6	8.1	6.5	75

TABLE 10--Continued

Run No.	Controller Configuration	Process Simulation	Disturbance		No. of Cycles	Lines		Δ Lines	Efficiency %
			Wave	cps		Start	End		
F128	none	nonlinear	square	0.05	4	5.7	31.0	25.3	-
F129	linear F	nonlinear	square	0.05	4	5.7	27.5	21.8	14
F130	nonlinear F	nonlinear	square	0.05	4	1.0	7.0	6.0	76
F150	none	nonlinear	sine	0.05	6	2.7	25.8	23.1	-
F151	nonlinear F*	nonlinear	sine	0.05	6	0	0.1	0.1	99.6
F152	nonlinear F*	nonlinear	sine	0.05	6	1.1	6.0	4.9	79
F153	nonlinear F	nonlinear	sine	0.05	6	0	0	0	100

*Part of nonlinear controller only.

The linear controller for the simulated nonlinear process, run 129, suffered badly and gave by far the lowest efficiency of all the simulation studies.

This controller exhibited characteristics which caused further doubts about its ultimate successful application to a real chemical process. Even for slowly varying sine waves, the magnitude of the disturbance had to be kept very low to keep the controller from saturating. Moreover, the entire operation of these controllers could be described as "touchy." The double differentiator tended to operate on all signals, including switching transients. Although noise caused no apparent problem in this well-behaved system, the noise in a real process would be expected to cause problems.

Runs F150 through F153 demonstrated a type of simplifying approximation that might be applicable to the nonlinear controllers. The nonlinearity in controller F consisted of a function of the measured variable divided by a linear combination of the measured variable and its derivative. Run F151 used a nonlinear controller which omitted the derivative term in the divisor, and run F152 omitted the variable itself. The controller used in run F151 would be a suitable simplifying approximation to that used in run F153 because the efficiencies were practically identical. The controller used in run F152 was not as efficient. The conclusion was that the derivative term in the divisor was not significant compared to the variable itself.

Summary of Simulation Study Results

This study has fulfilled the purposes that were outlined in Chapter V. The application of invariance theory using linear and nonlinear perturbation equations has been demonstrated. The use of the analog computer as a tool for studying the practical application of the theory has been illustrated. Linear and nonlinear controller qualities, measured by efficiency, have been compared for the simulated process. Several other important observations which are pertinent in the application of invariance theory to a well-behaved and accurately-modeled system were:

a) Whenever invariance was very close to absolute, or 100 per cent efficient, the quality of control was independent of the disturbance magnitude and frequency. If invariance was not absolute, the quality of control was progressively worse for increasing frequency and magnitude of disturbance.

b) Although analog differentiation is a seldom recommended operation, no particular problem resulted from its use in these controllers. The use of the second derivative, which to the author's knowledge has never been advocated in the literature, has been demonstrated. Although it must be used with care, a second derivative controller should not be ruled-out before it is attempted.

c) A successful minor simplification of one controller (F) has led to the consideration of possible approximation techniques in search of less complicated controllers.

d) The most important result has been an indication that linear systems theory still has many applications. Even though these studies have been concerned with a nonlinear system and nonlinear controllers, several results have shown that the majority of the control quality improvement has been obtained by strictly linear techniques. Specifically, four instances have been given:

1) In some cases linear theory gave the same controller equations as nonlinear theory.
2) In some cases the linear part of the nonlinear controller, which could have been designed from linear theory, provided practically all the control improvement.
3) In all cases linear theory alone gave improved control and was usually better than 75 per cent efficient even for very severe disturbances to the simulated process.
4) In those cases in which linear control was not absolute, the control quality improved as the disturbance magnitude was lowered. Since these disturbance magnitudes were severe to begin with, it followed that the usually less severe disturbances in chemical processes could be compensated by linear controllers.

This successful application of linear systems theory to simulated nonlinear processes clearly demonstrates that the current emphasis on nonlinear theory, and its penalizing complications, is not completely justified. Much fruitful work remains to be done in linear systems theory as applied to nonlinear systems.

Results of Experimental Work

Figures 34 and 35 are sample recordings which show several of the process variables. The temperature, T_w, was the variable which was to be made invariant in this example. The recordings show the quality of control provided by an actual experimental invariance device.

During the experimental data collection the controller settings were changed by small amounts to determine the possible existence of better controller constants than theory predicted. In most cases the theoretically calculated constants were about as good as any values found. In a few instances slightly better control was provided with different settings. However, it was noted that these optimum settings were generally different for each frequency and wave shape of disturbance. The optimization of controller constants was difficult for the following reasons:

1) The controller response was generally slow to changes in the constants because the speed of response of the controller was essentially that of the slow process. Since a constant, steady-state controller output was demanded, much time was consumed in waiting for the controller to "line-out" during optimization.

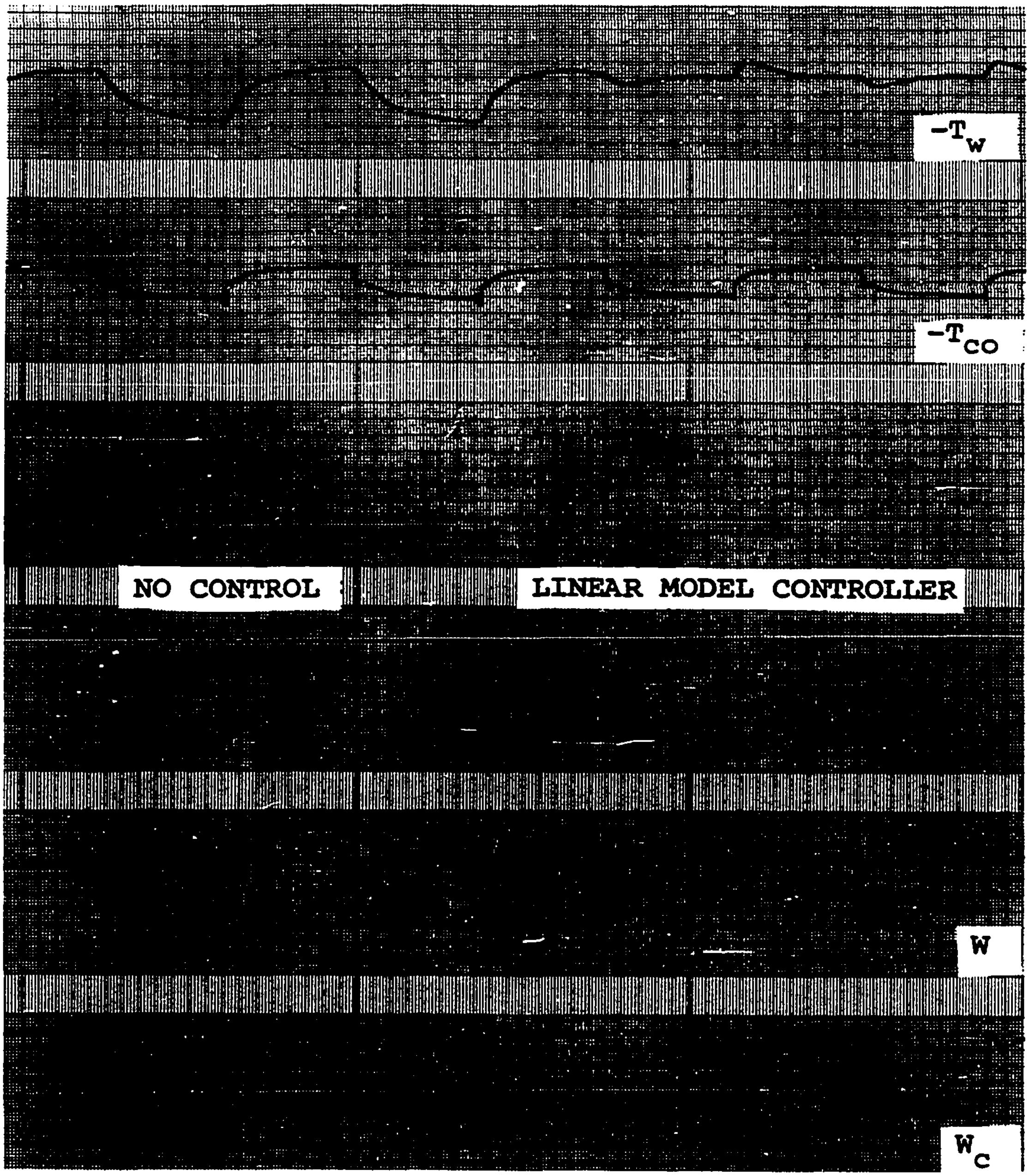


Figure 34. Experimental Data Recording for Controller E: Square Wave Disturbance, 0.0032 cps.

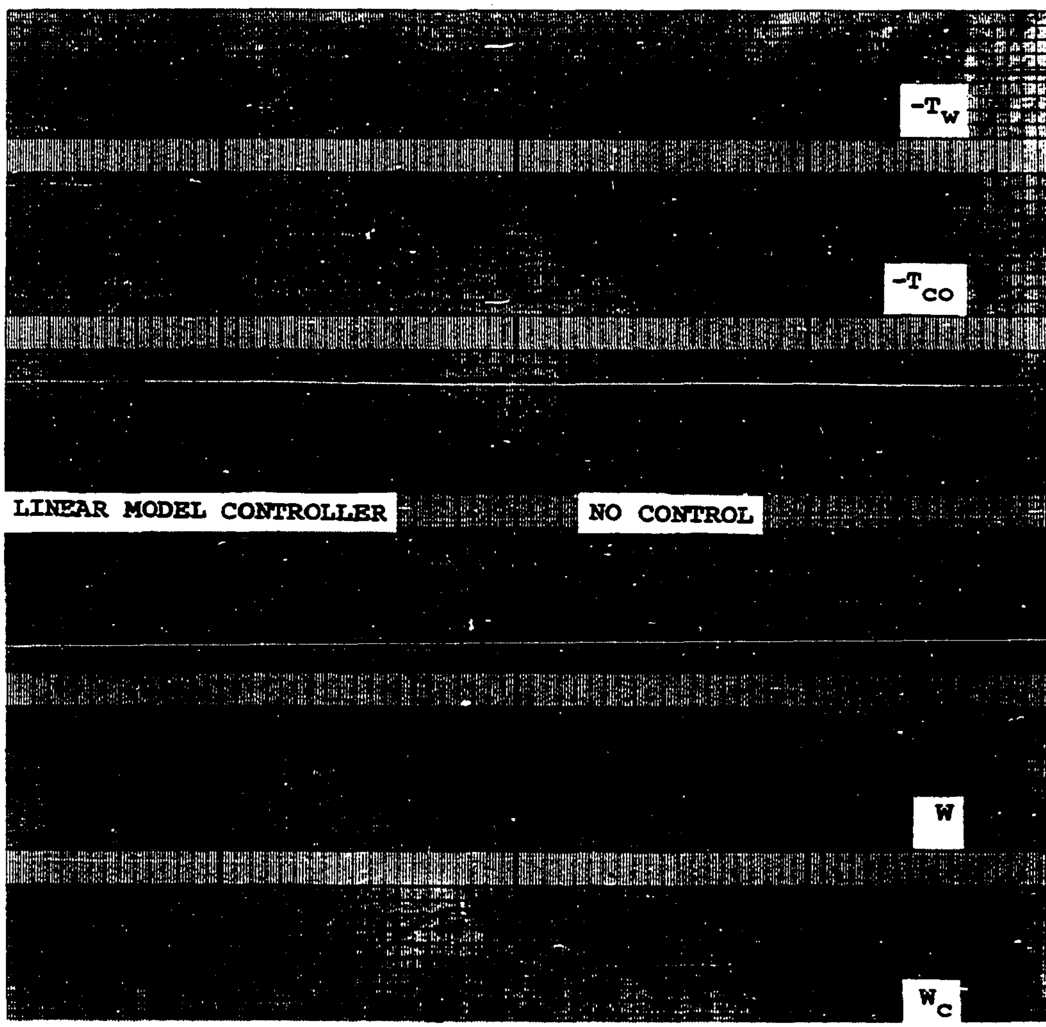


Figure 35. Experimental Data Recording for Controller E: Sine Wave Disturbance, 0.0032 cps.

2) The testing itself was slow because one had to observe several low frequency cycles of response to check controller efficiency. This testing time added to the controller optimization time.

3) Some controllers had as many as ten coefficient pots. A tenfold optimization procedure is a formidable obstacle even though all settings are not entirely independent.

4) The optimum efficiencies were not sharp functions of controller settings. Therefore, convergence to the optimum was a rather slow procedure.

In Tables 11 through 15, in which the controller efficiencies are listed, the values listed are for the best controller constants found. If they are other than the calculated values, this fact will be mentioned during the discussion of each controller which follows. Included in the tables are data on the range of the controlled variable. This number is the spread in degrees Fahrenheit of the controlled variable measured over a cycle of disturbance. This number is another measure of control quality.

<u>Controller A</u>: It was found that controller A operated slightly more efficiently if the controller gain and speed of response were increased slightly. The needed increase in speed of response could be explained as an attempt to overcome a lag in the instrumentation that was not included in the model. The efficiencies are given in Table 11, runs X1

TABLE 11

EFFICIENCIES OF CONTROLLERS A AND B: EXPERIMENTAL DATA

Run No.	Controller Configuration	Disturbance		Integral Absolute Value	Efficiency %	Range °F
		Wave	cps			
X1	none	square	0.0032	216	-	2.00
X2	linear A	square	0.0032	25	88	0.50
X3	none	square	0.0064	102	-	1.00
X4	linear A	square	0.0064	12	88	0.33
X5	none	sine	0.0032	88	-	1.17
X6	linear A	sine	0.0032	10	89	0.33
X7	none	sine	0.0064	65	-	0.67
X8	linear A	sine	0.0064	8	88	0.33
X9	none	square	0.0032	230	-	2.83
X10	linear B	square	0.0032	10	96	0.33
X11	none	square	0.0064	180	-	1.67
X12	linear B	square	0.0064	30	83	0.67
X13	none	sine	0.0032	366	-	2.00
X14	linear B	sine	0.0032	50	86	0.83
X15	none	sine	0.0064	50	-	0.67
X16	linear B	sine	0.0064	7	86	0.33

through X16. These runs were made at one set-point. Therefore, only the linear model controller needed to be tested. The square wave data indicated that efficiency was a function of the frequency of disturbance, which the sine wave did not. The high efficiency for the square wave was caused by the controller being tuned for a square wave disturbance. Although the controller settings were optimum for square wave disturbances, they were not necessarily so for sine waves.

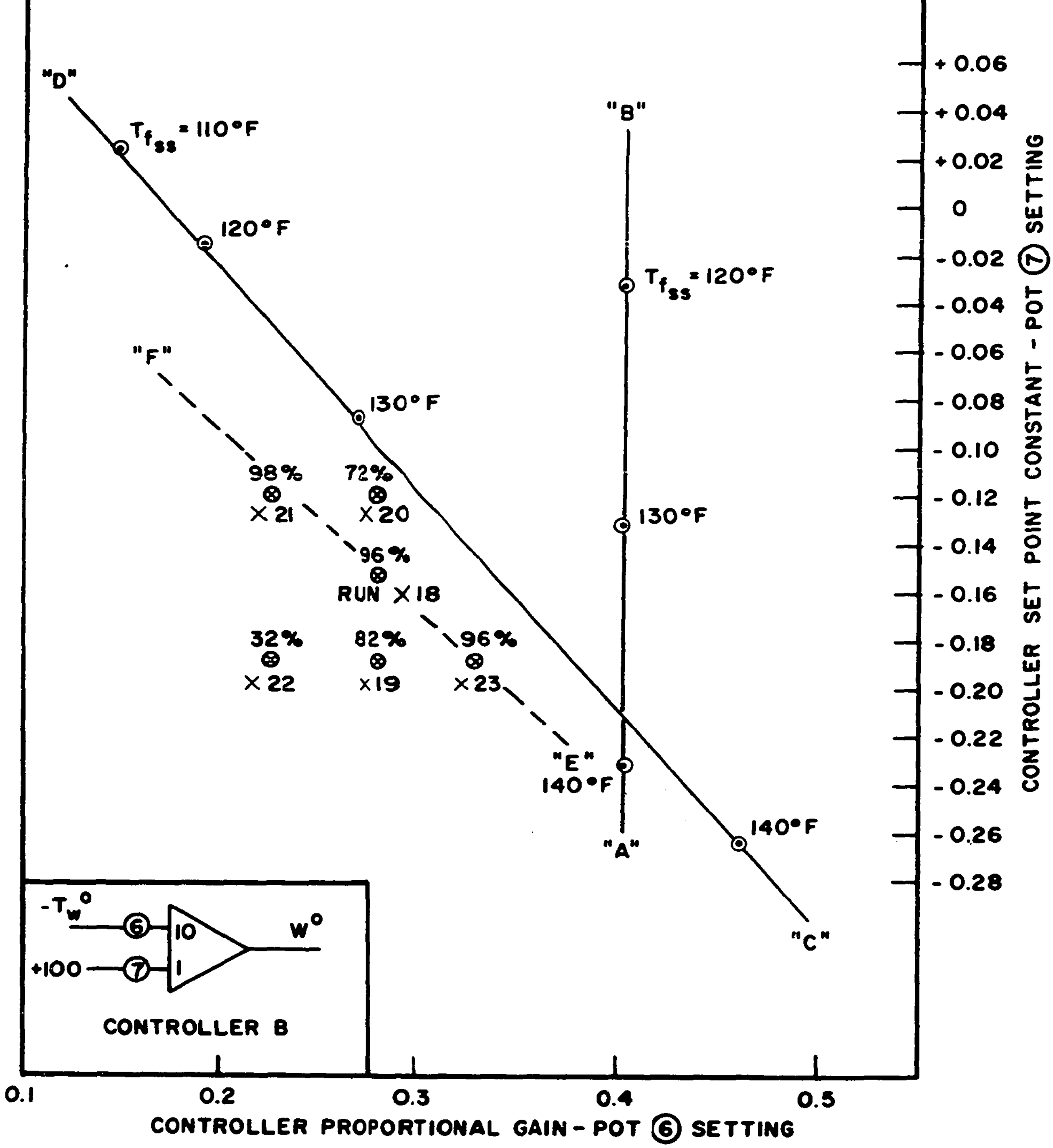
Set-Point Variation Study: Table 12 presents the data for the variation in efficiency of controller B due to changes in the controller settings. Figure 36 illustrates the results. If the linear model was used for controller synthesis, the proportional gain was found to be independent of the set-point value, $T_{f_{ss}}$. Thus, the linear model dictated that the set-point be changed by leaving pot 6 fixed and by changing pot 7. This procedure is shown as line "A"-"B" in Figure 36. On this line three values of pot 7, for three values of set-point temperatures, are shown. If the nonlinear model was used for controller synthesis, the proportional gain was found to be dependent on the steady-state temperature. Thus, the nonlinear model dictated that the set-point be changed by varying both pots 6 and 7. This procedure is shown as line "C"-"D" and values of four steady-state set-point temperatures are shown as points along this line. In order to verify this specified procedure for set-point changes, several runs were made and efficiencies calculated. The run

TABLE 12

EFFICIENCIES OF CONTROLLER B FOR THE SET-POINT VARIATION STUDY: EXPERIMENTAL DATA

Run No.	Controller Configuration	Disturbance Wave	Disturbance cps	Integral Absolute Value	Efficiency %	Range °F
X17	none	square	0.0032	230	-	2.83
X18	nonlinear B	square	0.0032	10	96	0.33
X19	nonlinear B	square	0.0032	41	82	0.50
X20	nonlinear B	square	0.0032	65	72	1.17
X21	nonlinear B	square	0.0032	5	98	0.17
X22	nonlinear B	square	0.0032	156	32	1.83
X23	nonlinear B	square	0.0032	10	96	0.33

Figure 36. Set-Point Variation for Controller B.



numbers and efficiencies are shown in Figure 36 with the plotted values of pots 6 and 7. An examination of the efficiencies indicated that (just as predicted) the controller for the nonlinear process required changes in both pot settings to maintain high efficiency. The optimum curve which was found is approximately line "E"-"F." The difference between lines "C"-"D" and "E"-"F" was thought to be due to dynamics that were not included in the model of the system. Although the linear and nonlinear models had predicted the same dynamic controller, the two models dictated different methods of changing the set-point.

<u>Controller C</u>: The efficiencies for controller C are presented in Table 13. Because of the number of settings to optimize, the theoretically calculated values of the constants were used. Some of the efficiencies could have been improved for specific wave shapes and frequencies of the disturbance. In this case it was decided to determine the quality of control provided by the theoretically derived controllers. The efficiencies that were realized were not as high as those for controllers A and B, but the quality of control was still good. For the square wave, the efficiencies did not appear to give a trend with frequency of disturbance. The sine wave data gave the expected result: nonabsolute invariance controller efficiency that decreased with increasing frequencies. The more complicated nonlinear controller was not significantly better than the linear one, and in one case (run X35) was much worse.

TABLE 13

EFFICIENCIES OF CONTROLLER C: EXPERIMENTAL DATA

Run No.	Controller Configuration	Disturbance		Integral Absolute Value	Efficiency %	Range °F
		Wave	cps			
X24	none	square	0.0032	152	-	0.67
X25	linear C	square	0.0032	50	67	0.42
X26	none	square	0.0032	88	-	0.84
X27	nonlinear C	square	0.0032	50	43	0.67
X28	none	square	0.0064	84	-	0.75
X29	linear C	square	0.0064	40	52	0.33
X30	none	square	0.0064	88	-	0.50
X31	nonlinear C	square	0.0064	38	57	0.50
X32	none	square	0.01	65	-	0.50
X33	linear C	square	0.01	25	62	0.17
X34	none	square	0.01	45	-	0.33
X35	nonlinear C	square	0.01	36	20	0.50
X36	none	sine	0.0032	140	-	0.67
X37	linear C	sine	0.0032	37	74	0.58
X38	none	sine	0.0032	110	-	0.50
X39	nonlinear C	sine	0.0032	20	82	0.33
X40	none	sine	0.0064	87	-	0.33
X41	linear C	sine	0.0064	43	51	0.33
X42	none	sine	0.0064	60	-	0.33
X43	nonlinear C	sine	0.0064	25	58	0.33
X44	none	sine	0.01	50	-	0.25
X45	linear C	sine	0.01	28	44	0.33
X46	none	sine	0.01	37	-	0.33
X47	nonlinear C	sine	0.01	20	46	0.33

Controller D: The efficiencies for controller D are given in Table 14. These data were obtained with no optimization of pot settings. Increased frequency of disturbance for both wave shapes resulted in decreased efficiency as noted previously. The additional complication of the nonlinear controller gave an increase in efficiency in only one case. One unexplained phenomenon noted was the tendency for the linear controller to saturate more easily and frequently than the nonlinear one.

Controller E: Data for controller E are given in Table 15. No optimization was performed. The efficiency decreased with frequency of disturbance for both wave shapes. The simulation studies had predicted that the nonlinear controller would offer very little improvement over the linear controller. However, in general the data did not agree with the prediction. The simulation studies were based on the ultimate attainment of 100 per cent efficiency. In the experimental work this efficiency was not attained. It was concluded from the data and the simulation studies for controllers C, D, and E that if the model was very accurate, then the improvement provided by using the nonlinear controller could be predicted. On the other hand, if there was something slightly wrong with the model, which will be the usual case, and if absolute invariance is not attained, then the improvement provided by the nonlinear model can only be determined by experimental testing.

TABLE 14

EFFICIENCIES OF CONTROLLER D: EXPERIMENTAL DATA

Run No.	Controller Configuration	Disturbance Wave	Disturbance cps	Integral Absolute Value	Efficiency %	Range °F
X48	none	square	0.0032	647	-	9.05
X49	linear D	square	0.0032	222	66	4.70
X50	nonlinear D	square	0.0032	188	71	4.35
X51	none	square	0.0064	521	-	6.61
X52	linear D	square	0.0064	221	58	3.65
X53	nonlinear D	square	0.0064	246	53	3.83
X54	none	square	0.01	271	-	4.87
X55	linear D	square	0.01	155	43	2.96
X56	nonlinear D	square	0.01	161	41	3.65
X57	none	sine	0.0032	406	-	6.26
X58	linear D	sine	0.0032	88	78	1.57
X59	nonlinear D	sine	0.0032	129	68	1.91
X60	none	sine	0.0064	319	-	4.52
X61	linear D	sine	0.0064	117	63	2.44
X62	nonlinear D	sine	0.0064	178	44	2.96
X63	none	sine	0.01	180	-	3.13
X64	linear D	sine	0.01	92	49	2.09
X65	nonlinear D	sine	0.01	119	34	2.96

TABLE 15

EFFICIENCIES OF CONTROLLER E: EXPERIMENTAL DATA

Run No.	Controller Configuration	Disturbance Wave	Disturbance cps	Integral Absolute Value	Efficiency %	Range °F
X66	none	square	0.0032	617	-	8.00
X67	linear E	square	0.0032	152	75	3.48
X68	none	square	0.0032	472	-	6.26
X69	nonlinear E	square	0.0032	60	87	1.22
X70	none	square	0.0064	451	-	6.09
X71	linear E	square	0.0064	196	57	3.31
X72	none	square	0.0064	329	-	4.87
X73	nonlinear E	square	0.0064	119	64	1.91
X74	none	square	0.01	350	-	4.87
X75	linear E	square	0.01	187	47	2.78
X76	none	square	0.01	160	-	3.65
X77	nonlinear E	square	0.01	95	41	1.74
X78	none	sine	0.0032	351	-	5.74
X79	linear E	sine	0.0032	101	71	1.74
X80	none	sine	0.0032	321	-	4.87
X81	nonlinear E	sine	0.0032	112	65	1.74
X82	none	sine	0.0064	289	-	4.18
X83	linear E	sine	0.0064	156	46	2.09
X84	none	sine	0.0064	128	-	3.65
X85	nonlinear E	sine	0.0064	59	54	1.74
X86	none	sine	0.01	147	-	3.13
X87	linear E	sine	0.01	100	32	2.09
X88	none	sine	0.01	130	-	2.61
X89	nonlinear E	sine	0.01	54	58	1.39

The differentiator performed well in both controllers D and E; these devices were found to be useable for invariance type controllers. Both controllers D and E gave better quality control when the signals were low-pass filtered.

Controller F: The simulation studies for configuration F gave indication that this controller could cause troubles in an experimental system. This difficulty indeed was the case. The noise present in the system was amplified by the double differentiator to a level which was unacceptable for control purposes. The result was that the controller had large alternating positive and negative output signals which caused the pneumatically operated valve to cycle from one saturation limit to the other. This cycling ultimately would have caused the control system to fail from excessive wear. In addition the control quality was poor. Three abortive attempts to get useful data were made. First of all, the signals in the controller were low-pass filtered to reduce the noise, but when the filtering began to be adequate, the signal information was destroyed. The second attempt consisted of making the disturbance very low in magnitude and very slow to change. Finally, the troublesome second derivative term was omitted from the controller in the hope that it was not significant. A similar maneuver had good results in one of the simulation study controllers. However, although the controller operation was less troublesome, the quality of control was bad. Thus, the second derivative was assumed to be important. None of these attempts were fruitful, and control in each case was unacceptable. No results are given for this controller configuration because in no case was the controlled system better than the uncontrolled. It was concluded from these experiences that the use of a double differentiator is questionable unless some successful smoothing of the controller operation can be obtained. Another point which should be made is: in feedforward control and in other types of invariance control a great deal of information has to be known about the system. Unlike the ordinary feedback control, these systems must be applied with some care, or the controlled system will turn out to be worse than the uncontrolled.

In general, the biggest problem encountered in the realization of practical controllers was the construction of the controllers using analog computing elements. These elements have been designed for analog computation with relatively high speed and high voltage signals. The kind of controllers actually needed are ones designed for low voltage and dynamically slow signals. Some devices with these characteristics are available commercially; however, caveat emptor.

An example of the problems encountered can be given by the equation for the linear controller, F, from Table 5:

$$W^{o} = -(12{,}929\ D^{2}+788.7\ D+10.02)\ W_{c}^{o}-3.173\ T_{co_{ss}}+255$$

This equation could not be time scaled, which is the usual procedure for differential equation programming. The coefficient of the second derivative of W_c^o was approximately 1300 times the coefficient of W_c^o. Magnitude scaling proved to be no help in reducing the order of magnitude difference in the coefficients. The size of these coefficients demonstrates the noise amplification feature of differentiators. This problem is assumed to become more difficult as the system size and time lags increase.

The practical attainment of efficiencies that were somewhat less than 100 per cent could not be entirely attributed to specific causes. Primarily, two reasons existed for nonabsolute invariance.

1) Assumptions were made in the derivation of the model which were not necessarily correct. The most dangerous assumption was probably that the coolant temperature for heat transfer purposes was the mean of the inlet and outlet values. The controller constants were not considered to be the cause of inefficiency because little significant improvement resulted from changing them. Thus, it was concluded that most of the error in the model is fundamentally in its topology.
2) The dynamics of the measuring, transducing, and controlling devices were ignored, although in

every case attempts were made to improve the speed and accuracy of responses. For example, it was necessary to install the anti-hysteresis controllers on the flow systems. In the experimental process the relative time constants of the auxiliary devices were larger in relation to process time constants than for most chemical processes. For larger and slower processes the auxiliary equipment dynamics will become progressively less important.

Summary of the Results of the Experimental Work

In general, the purposes of this part of the investigation, which were outlined in Chapter VI, have been fulfilled. The use of linear and nonlinear total variable models for the derivation of control laws has been demonstrated. The experimental results were promising, considering the fact that the experimental system proved to be a severe test of the practical application of the theory. The laboratory system was somewhat faster and a great deal smaller than the usual chemical process. Both factors presumably worked against successful invariance attainment. Nevertheless, the theory was considered to be entirely feasible for practical applications. The quality of control, which was easily attainable by straightforward practical application of the theory, was measured on a physical system. The problems of practical application of the theory have been delineated, and the usual results of any experimental work were noted:

1) Some problems that <u>were</u> anticipated before the experimentation <u>did not</u> materialize.

2) Some problems that <u>were not</u> considered before experimentation <u>did</u> materialize.

3) Some problems that were anticipated before experimentation did materialize but in a different way than originally thought.

Specifically, several conclusions from the results can be made.

a) In relation to a physical system, whose model must be an approximation of its behavior because of various assumptions, the invariance principle represented a formulation of the optimum system behavior. The degree to which invariance was obtained depended on the accuracy of the model and upon the ability of the control system to execute the prescribed control law.

b) The use of nonlinear model equations for the synthesis of an efficient invariance controller was not necessary for the experimental system used. If the model had been more accurate or the nonlinearities more severe, then the nonlinear model could have been used with good results. This fact added emphasis to a conclusion drawn from the simulation studies: <u>linear systems theory is still important</u>. However, this investigation has also provided the ability of the theory to treat nonlinear systems when necessary.

c) If the ability to change the set-point and to retain the invariance quality was required, the nonlinear model theory became important. In practice it was preferable to change the set-point and to retune the controller for optimum control. Until an optimization technique is established, the tuning for invariance may be a formidable task for some controllers.

d) The use of differentiators was shown to be possible as long as the second (or higher) derivative was not required.

e) The most severe problem was the assembling of components to execute the control law at the proper time. These components required a high degree of stability and accuracy even when operating as slowly as the physical process itself.

CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS

The complete procedure for the synthesis of invariance principle control systems for chemical processes has been discussed. This investigation started with the mathematical formulation of the theory in which it has been shown that two conditions are necessary for the mathematical attainment of invariance. The use of matrix techniques and different model types has been demonstrated. This proves to be useful for the first condition of invariance: that the controlled variable solution is a constant and its derivative zero. Three methods of determining the second condition of invariance--the existence of dual channels of information--have been shown to be equivalent: mathematical, topological, and physical. Using energy and material balances for the mathematical model construction and the analog computer for simulation studies, examples of invariance principle control systems were presented. The problems involved in synthesizing real controllers have been discussed. Finally, experimental data have been obtained on a physical system; these results suggest the feasibility of the invariance control synthesis procedures.

From the experience gained during the experimentation, several recommendations can be made concerning each phase of the synthesis procedure.

Model Construction: If material or energy balances are to be used for the mathematical model, there is usually a great deal of information about the system parameters which must be obtained. Often some additional experimental work may be required for the determination of physical properties, rate operation coefficients and system dimensions. Alternately, it may be preferable to undertake a dynamic testing experiment to get an approximate model for the system behavior. This procedure is common in systems engineering because many parameters are unknown and because of the good system identification techniques which are available. The mathematical model can be obtained in either the perturbation variable or the total variable form. There is also a choice of linear or nonlinear models.

Control Law Determination: For linear equations, the techniques that are most useful are linear operators and matrix techniques. The Laplacian operator is considered to be the simplest and safest, but the choice is primarily one of preference. Time domain methods are applicable for all models and necessary for nonlinear ones.

Analog Computer Simulation: Any of the models can be programmed for study on the analog computer, but the perturbation models are recommended for simplicity. The simulation studies are useful for studying the overall system dynamics and the feasibility of specified controller configurations. However, it must be remembered that successful analog simulation studies do not guarantee the complete success of the real control system. The ultimate purpose of the synthesis procedure is a successful practical control system.

Experimental Verification of Control Quality: Total variable models are recommended for determining control laws for real systems because of the more useful set-point specifications. Controllers should be built according to the control laws, keeping in mind the need for precise, high quality instrumentation. Although the theory is applicable to nonlinear systems, its more valuable contributions will probably be based primarily on linear models and controllers. Linearized models should always be examined because the resulting, less-complicated controllers may give about all the possible gain.

The attainment of invariance is fundamentally based on three requirements:

1) The mathematical model adequately describes the system behavior.
2) The control system is permitted by satisfaction of the dual channel theorem.
3) The execution of the theoretically derived control laws can be accurately performed.

If these conditions can be met, invariance theory is a methodical design procedure for control systems and represents a definitive statement of the optimum control system. The quality of control which is experienced in practice depends on how well these conditions are met.

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APPENDIX A

NOMENCLATURE

$a_{1,2,\ldots5}$ = proportional constants used to convert physical variables to analog computer controller variables

A = a controller configuration

$\underline{A}$ = constant coefficient n x n matrix

$\underline{A}$ (D) = characteristic matrix of the uncontrolled process, differential operator form

$\underline{A}$ (s) = characteristic matrix of the uncontrolled process, Laplacian operator form

A_i = reactor wall inside area, $ft.^2$

A_o = reactor wall outside area, $ft.^2$

$b_{1,2,\ldots5}$ = constants used to convert physical variables to analog computer controller variables

B = a controller configuration

$\underline{B}$ = constant coefficient n x m matrix

C = a controller configuration

$\underline{C}$ = constant coefficient n x p matrix

$\underline{C}$ (D) = expanded form of matrix $\underline{C}$ for a controlled system, differential operator form

$\underline{C}$ (s) = expanded form of matrix $\underline{C}$ for a controlled system, Laplacian operator form

Cp_c = coolant heat capacity, BTU/lb.-°F

Cp_f = oil heat capacity, BTU/lb.-°F

Cp_w = wall metal heat capacity, BTU/lb.-°F

det	= determinant
D	= a controller configuration
D	= the differential operator
e	= the difference between W' and its set point value (W'_{sp}), volts
E	= a controller configuration
ε	= controller efficiency
f	= analog variable corresponding to W*, volts
f_c	= analog variable corresponding to W_c*, volts
F	= a controller configuration
F (s)	= Laplace transformation of f
$F_c(s)$	= Laplace transformation of f_c
$\underline{G}$	= constant coefficient n x 1 matrix
h_i	= reactor wall inside heat transfer coefficient, BTU/hr.-ft.^{2}-°F
h_o	= reactor wall outside heat transfer coefficient, BTU/hr.-ft.^{2}-°F
$\underline{H}$ (D)	= characteristic matrix for a controlled system, differential operator form
$\underline{H}$ (s)	= characteristic matrix for a controlled system, Laplacian operator form
$\underline{H}_c(s)$	= one-column-modified form of $\underline{H}$ (s)
$\underline{H}_{cg}(D)$	= one-column-modified form of $\underline{H}$ (D)
i	= index for measured, process output variables
$\underline{I}$	= the identity matrix
I_c	= integral absolute value for the controlled system variable
I_u	= integral absolute value for the uncontrolled system variable
j	= index for controllable, process input variables

k = index for uncontrollable, process input variables

$k_{1,2,\ldots 11}$ = numerical constants in analog computation model

$k_c(s)$ = transfer function of the feedforward controller on variable W_c

$k_{co}(s)$ = transfer function of the feedback controller on variable T_{co}

$K_f(s)$ = transfer function of the feedback controller on variable T_f

$\underline{K}_u(D)$ = a matrix of transfer functions for feedforward controllers, differential operator form

$\underline{K}_u(s)$ = a matrix of transfer functions for feedforward controllers, Laplacian operator form

$K_{u1}(s)$ = transfer function for the feedforward controller on disturbance u_1

$K_w(s)$ = transfer function of the feedback controller on variable T_w

$\underline{K}_x(D)$ = a matrix of transfer functions for system input devices, differential operator form

$\underline{K}_x(s)$ = a matrix of transfer functions for system input devices, Laplacian operator form

$K_{x1}(s)$ = transfer function of a system input device

$\underline{K}_y(D)$ = a matrix of transfer functions for feedback controllers, differential operator form

$\underline{K}_y(s)$ = a matrix of transfer functions for feedback controllers, Laplacian operator form

$K_{y1}(s)$ = transfer function for the feedback controller on the variable y_1

$K_{y2}(s)$ = transfer function for the feedback controller on the variable y_2

$K_{y3}(s)$ = transfer function for the feedback controller on the variable y_3

m = total number of process controllable inputs

n = total number of process outputs, or the order of the system

p	= total number of process uncontrollable inputs
q	= index for control law equation
Q_L	= heat loss term, BTU/hr.
s	= Laplacian operator
S1...S3	= summing junctions
t	= process (real) time
t_{co}	= analog variable corresponding to $T_{co}*$, volts
t_f	= analog variable corresponding to T_f*, volts
t_w	= analog variable corresponding to T_w*, volts
T_{ci}	= coolant inlet temperature, °F
T_{cm}	= coolant mean temperature, °F
T_{co}	= coolant outlet temperature, °F
T_f	= oil bulk temperature, °F
T_{in}	= oil inlet temperature, °F
T_w	= wall temperature, °F
$\underline{u}$ (t)	= column p matrix of process uncontrollable inputs
u_k	= a measurable and uncontrollable, process input
$\underline{U}*(s)$	= Laplace transformation of $\underline{u}*(t)$
V_c	= reactor coolant volume, ft.^{3}
V_f	= reactor oil volume, ft.^{3}
V_w	= reactor wall volume, ft.^{3}
W	= oil flow rate, lb./hr.
W'	= output signal of flowmeter (proportional to W), volts
W_c	= coolant flow rate, lb./hr.
W'_{sp}	= set-point value of W'
$\underline{x}$ (t)	= column m matrix of process controllable inputs

x_j = a controllable, process input

$\underline{X}^*(s)$ = Laplace transformation of $\underline{x}^*(t)$

$\underline{y}$ (t) = column n matrix of process outputs

y_i = a measurable process output

$\underline{Y}^*(s)$ = Laplace transformation of $\underline{y}$ (t)

$\underline{z}$ (t) = combined $\underline{x}$ (t) and $\underline{y}$ (t) column matrix

$\underline{Z}^*(s)$ = Laplace transformation of $\underline{z}^*(t)$

Greek Symbols:

$|\underline{\Delta}|$ = a characteristic matrix for a specific controlled system

θ = analog computer time (one second equals one minute of real time)

ρ_c = density of coolant, lb./ft.^{3}

ρ_f = density of oil, lb./ft.^{3}

ρ_w = density of wall metal, lb./ft.^{3}

$\underline{\Phi}$ = column n matrix of nonlinear functions of $\underline{x}$, $\underline{y}$, and $\underline{u}$

$\underline{\Psi}$ = column m matrix of functions (control laws)

Subscripts:

c = coolant variable

f = oil variable

w = wall variable

ss = steady-state variable

$\underline{\quad}$ = matrix notation

Superscripts

. = first time derivative

..	= second time derivative
*	= perturbation (transient) variable
o	= analog computer variable, volts

APPENDIX B

GLOSSARY

This glossary is included to assist the reader in the use of this work. In the text a few terms have specific meanings which differ slightly from those normally encountered in systems engineering; however, the majority of definitions agree with those found in the following references:

1) AIEE Subcommittee on Terminology and Nomenclature of the Feedback Control Systems Committee.

2) D'Azzo, J. J., and Houpis, C. H., <u>Feedback Control System Analysis and Synthesis</u>, McGraw-Hill Book Company, Inc., New York, 1960.

3) Seifert, W. W., and Steeg, C. W., <u>Control Systems Engineering</u>, McGraw-Hill Book Company, Inc. New York, 1960.

4) <u>Webster's New International Dictionary of the English Language</u>, unabridged, 2d ed., G. and C. Merriam Company, Springfield, Massachusetts, 1955.

<u>Backlash</u>. A form of hysteresis in a mechanical device. For machinery, it can be defined as the distance which one part of connected machinery, such as a gear, can be moved without moving the connected parts.

<u>Block diagram</u>. A pictorial representation of the flow of information and the functions performed by each component in the system.

Controllable Variable. A variable whose magnitude at any time can be prescribed.

Control law. The rule which governs a controller's actions. In this work it is used as the time-domain equivalent of a transfer function.

Controller. An instrument which executes a control law to bring about the regulation of a variable.

Control system. A group of elements which performs the functions of measuring the error in the controlled variable, obtaining the actuating signal, and changing the manipulated variable.

Dead time. The elapsed time after the initiation of a disturbance and before any measureable response is detected.

Device. An instrument for effecting a purpose. In this work it refers to actual hardware and usually is synonymous with controller.

Distributed-parameter. Referring to a variable which is a function of both time and space (cf. lumped-parameter).

Disturbance. An unwanted signal that tends to affect the controlled variable.

Domain. A region of values to which a variable is confined; also called a space in this work. The time domain is real-valued and the Laplace domain is complex-valued.

Dynamics. The qualities of a system which are changing during a period of time; opposed to statics.

Feedback. Referring to a flow of information from a system output toward a system manipulated input.

Feedforward. Referring to a flow of information from a system disturbance input toward a manipulated input.

First order. Implying some relationship to a differential equation whose highest derivative of the dependent variable is the first.

Hysteresis. A double-valued phenomenon in which the magnitude of the dependent variable depends on whether the independent variable is increasing or decreasing. It is caused by a lagging effect.

Identification. In systems engineering, the procedure for determining a mathematical model.

Linear. Characterized by the principle of superposition which is the following: the total response of a system is the sum of the responses due to all applied forces individually. Mathematically a system is linear if the expression relating input and output involves only the first powers of the input and output and their derivatives.

Interaction. The effect which one variable has upon another; also called coupling.

Lumped-parameter. Referring to a variable which is only a function of time, not space (cf. distributed-parameter).

Manipulated. Referring to a system input variable whose value can be changed at will.

Model. Forms, patterns, or equations appropriate for the representation of individual ideas and for the interrelationships existing within a group of ideas.

Multivariable. Referring to the existence of more than one variable. Multivariable systems are also called multipole, multiloop, multidimensional, and complex.

Noise. In general any unwanted message in the presence of a wanted message; usually an interference of higher frequency than the wanted message.

Nonlinear. Not characterized by the superposition principle (cf. linear).

Optimization. The procedure of looking for the best result which is determined by a specified criterion.

Performance criterion. A standard for determining how a system should function.

Perturbation variable. A variable whose normal value is zero; also referred to as a transient variable because it usually has non-zero values for only short periods of time.

Process. In this work a collection of equipment items which performs chemical engineering unit operations.

Programming. The procedure for converting mathematical relationships into forms that are usable in an analog or digital computer.

Real. Referring to an actual thing; opposed to simulated. In this work real process refers to an actual chemical process, not to a model, and real time refers to solar time and not to artificial, accelerated, analog computer time.

Response time. The approximate length of time for which a given disturbance affects the system output.

Saturation. A condition whereby further increase in the system input can no longer increase the output.

Scaling. The process of changing variables proportionally.

Servosystem. A mechanical control system.

Set-point. A reference input to a control system which specifies the value which is desired for the controlled variable.

Simulation. An assumed imitation of reality; a system which operates in a similar manner to the real process.

Stability. A characteristic of a system whereby all variables tend to return in time to finite steady-state values.

Steady state. The state of a process when no disturbances are introduced.

Synthesis. The process of combining elements to form a whole system.

System. A combination of components that act together.

Systems engineering. An interdisciplinary approach to optimum processes involving a necessary ability to treat a whole system--design, operation, economics, instrumentation, and control. It involves the use of advanced mathematical techniques, analog and digital computers, as well as the most advanced aspects of basic engineering and science.

Time lag. An elapsed amount of time after a disturbance before the system output variable reaches a new steady-state value. It is usually characterized by the time-constant; e.g., for a system output which is dynamically characterized by an exponential decay, e^{-at}, the time constant is that value of t which makes the exponent of e equal to minus one.

Time-varying. Referring to a parameter whose value is only a function of the independent variable, time. A linear, time-varying system has the property of superposition (cf. linear).

Topology. Referring to the descriptive and analytical study of a place; in this work the place is the block diagram of the model.

Total variable. The sum of the steady-state variable and the perturbation variable.

Transfer function. The ratio of the operational form of the output of the operational form of the input.

APPENDIX C

STEADY-STATE DATA

This appendix outlines the procedure that was used to determine the numerical values for the constant parameters in the model which was used for all simulation studies and experimental controller calculations. The model which was used for determining three of the constants was given by a steady-state form of Equations (3).

$$0 = h_i \; A_i \; (T_w - T_f) + W \; Cp_f (T_{in} - T_f) - Q_L \qquad \text{(C-1)}$$

$$0 = h_i \; A_i \; (T_f - T_w) + h_o \; A_o \left[\frac{(T_{co} + T_{ci})}{2} - T_w \right] \qquad \text{(C-2)}$$

$$0 = h_o \; A_o \left[T_w - \frac{(T_{co} + T_{ci})}{2} \right] + W_c \; Cp_c \; (T_{ci} - T_{co}) \qquad \text{(C-3)}$$

Since the process had been assembled in a manner which allowed the direct measurement of W, W_c, T_w, T_f, T_{in}, T_{co}, and T_{ci}, steady-state operation could yield data which allowed calculation of the three unknowns--h_oA_o, h_iA_i, Q_L. These calculations depended on a knowledge of the values of Cp_f and Cp_c at operating conditions. Also, it was necessary for the dynamic model to know the values of ρ, V, and Cp for the oil, coolant, and wall metal. Thus, attention was first directed toward the gathering of data on physical properties and system dimensions.

Reactor Volumes

The volume which the oil occupied in the reactor was taken from the dissertation of Stewart (81) who used the same reactor. The volumes of the wall and coolant space were calculated from simple geometrical relationships using measurements made on the reactor during its assemblage.

Densities

The density of the oil was measured in the laboratory because the oil was not specifically identified by its name, "Super Service hydraulic lift oil." The density was determined simply by weighing a known volume of oil at a given temperature. The equipment for this determination consisted of a triple-beam balance, 100 ml. graduated cylinder, thermometer, beaker, and bunsen burner. The data for oil density were reduced and the results are shown in Figure 37.

The density of the ethylene glycol-water solution was obtained from a data book published by Carbide and Carbon Chemicals Company (now Union Carbide Chemicals Company).* The solution was 50 per cent (by weight) water. The density

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*Carbide and Carbon's Glycols, Carbide and Carbon Chemicals Company, A Division of Union Carbide and Carbon Corporation, New York, 1955, p. 42.

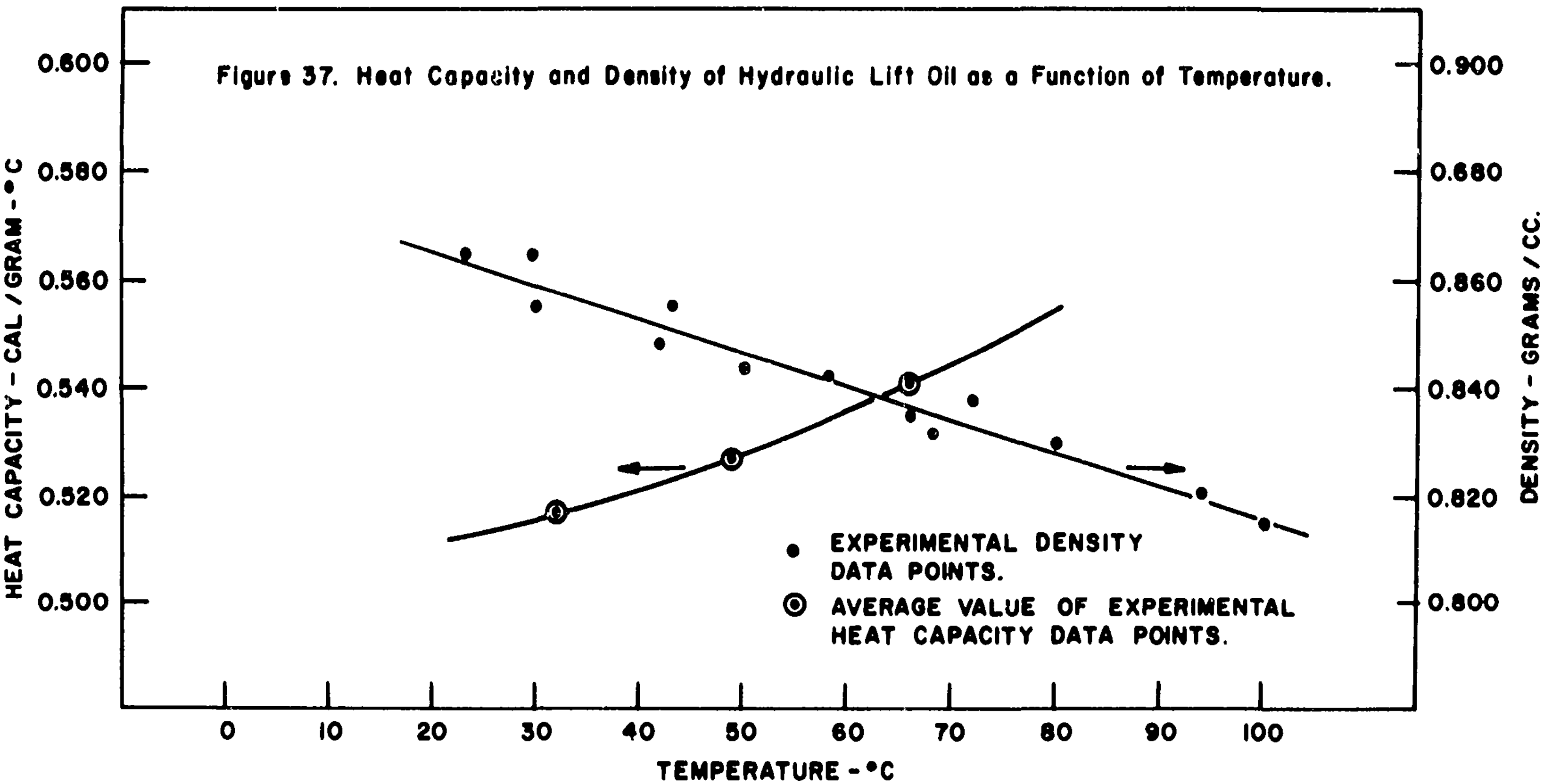

Figure 37. Heat Capacity and Density of Hydraulic Lift Oil as a Function of Temperature.

was checked by the same method used for the oil and the results were in good agreement.

The density of the wall metal (type metal) was obtained directly from a handbook. A quick check of this value was made by measuring the volume of water displaced by a sample of the metal which had been weighed.

Heat Capacities

The heat capacity of the oil was determined in the laboratory by means of a simple calorimetric experiment. The calorimeter consisted of a Dewar flask containing an insulated aluminum can with a thermally insulated cover. The sample was stirred by a plastic agitator powered by a small electric motor. The heating element was a nichrome strip wound in a coil; its resistance was approximately 1 ohm. Direct electric current was passed through the heating element and the voltage drop and current were measured. The temperature was measured by a Beckmann thermometer. The temperature was measured every 30 seconds and plotted versus time for each run. The straight-line temperature curves before and after the heating period were extrapolated in the usual manner to yield a corrected temperature rise. The heat capacity was calculated from the corrected temperature rise, the known weight of sample, and the measured electrical energy which was supplied. The calorimeter was calibrated with n-heptane at each temperature to yield a calorimeter heat loss constant. A total of 15 runs with n-heptane and

16 runs with the oil were made. The temperature levels were 32°, 49°, and 66°C. The averaged results for each temperature are shown in Figure 37.

The heat capacity of the glycol solution was obtained from the Carbide and Carbon data book.* The 50 per cent water (by weight) data were used.

The heat capacity of the wall metal was estimated from the molar heat capacities of the elements constituting type metal. The accuracy of this method was estimated from similar calculations for alloys for which heat capacity data were available. The agreement was found to be acceptable.

Heat Transfer Coefficients and Heat Loss Term

After the physical properties and the system dimensions were determined, the process was operated at steady-state conditions to provide data for the calculation of h_iA_i, h_oA_o, and Q_L. The steady-state runs were made over the full range of both coolant and oil flow rates to determine the effect of flow rates on heat transfer coefficients. The temperatures were measured by a Leeds and Northrup, Model 8662, potentiometer using the thermocouples described in Chapter VI. The flow measurements were made with the flowmeters described in Chapter VI. Equations (C-1) through (C-3) were used to calculate h_iA_i, h_oA_o, and Q_L. The data and calculated results for 16 runs are given in Table 16, and the heat transfer coefficients are plotted in Figure 38.

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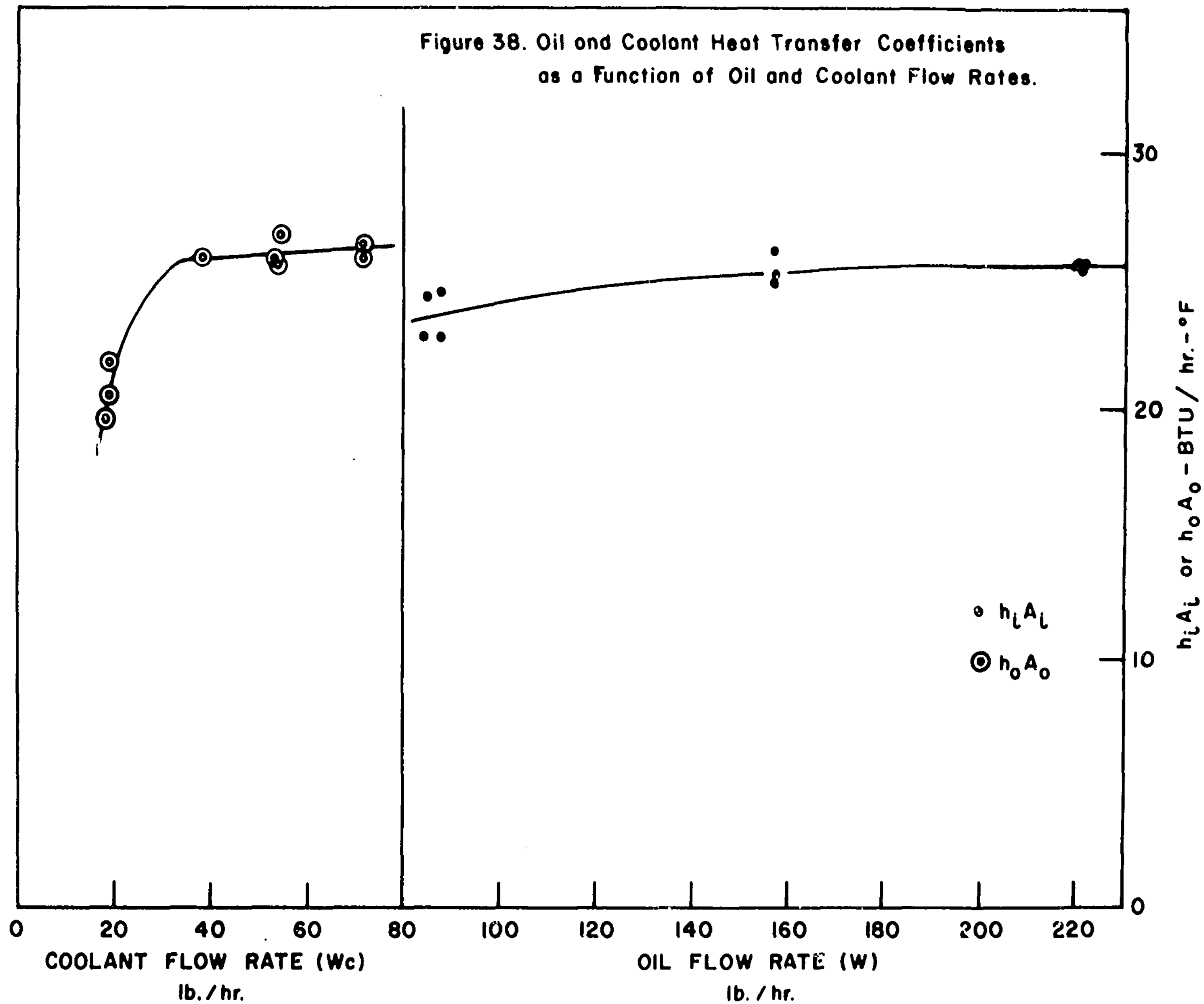
*Ibid., p. 47.

TABLE 16

STEADY-STATE DATA AND CALCULATED CONSTANTS

Run No.	Temperatures °F					Flow Rates lb/hr		Heat Capacities BTU/lb-°F		BTU/hr-°F		BTU/hr
	T_w	T_f	T_{in}	T_{co}	T_{ci}	W	W_c	Cp_f	Cp_c	h_oA_o	h_iA_i	Q_L
36	91.1	138.3	154.1	59.1	31.2	157.0	54.0	0.538	0.762	25.8	25.1	187
37	91.3	138.2	154.1	59.5	31.3	157.0	53.6	0.538	0.762	26.0	25.4	191
38	91.3	138.3	154.7	59.6	31.4	157.0	54.3	0.538	0.762	27.0	26.3	218
39	94.5	143.1	154.8	61.4	31.5	221.0	54.6	0.540	0.763	26.0	25.6	150
40	94.4	143.0	155.2	61.2	32.0	220.0	55.6	0.540	0.763	26.2	25.8	163
41	94.3	143.4	155.3	61.2	32.0	221.0	56.6	0.540	0.763	26.6	25.9	159
42	85.6	129.6	155.8	54.6	32.0	88.0	57.3	0.537	0.761	25.7	24.7	253
43	85.6	130.0	154.8	54.5	32.0	88.0	57.3	0.537	0.761	24.2	23.0	191
44	86.5	128.8	154.8	58.8	32.5	85.0	53.0	0.537	0.763	25.4	24.5	123
45	86.2	129.5	154.7	58.5	32.0	84.5	51.2	0.537	0.763	24.3	22.9	108
49	89.7	138.9	155.8	53.3	31.7	156.5	71.7	0.539	0.761	26.7	25.9	247
50	90.2	138.9	155.8	53.3	31.9	153.0	71.7	0.539	0.761	26.1	25.5	226
51	114.8	142.6	155.3	109.3	33.7	160.0	19.0	0.540	0.783	21.9	34.1	-28
52	117.5	143.4	155.6	112.4	33.0	161.5	19.0	0.540	0.784	20.4	35.3	-119
53	118.2	143.8	155.6	114.4	32.0	162.5	18.4	0.540	0.784	19.7	34.6	-154
56	97.0	140.5	155.8	74.0	31.1	159.0	38.0	0.539	0.768	26.1	26.7	59

Figure 38. Oil and Coolant Heat Transfer Coefficients as a Function of Oil and Coolant Flow Rates.



The points for h_oA_o were calculated from data taken at constant oil flow rate and for h_iA_i from data taken at constant coolant flow rate.

The calculated values of the heat transfer coefficients were reasonably constant over most of the ranges of flow rates. A rather serious drop in the coolant-side coefficient occurred below a coolant flow rate of 30 pounds per hour. Therefore, this low range of coolant flow rate was avoided in the subsequent experimentation as much as possible. The calculated values of h_iA_i for run numbers 51-53 also showed an increase for low coolant flow rates. This increase was not attributed to an actual change in the inside heat transfer coefficient, but it was thought to be a peculiarity of the model selected. The coefficients h_i and h_o were not true film coefficients, but functions of the resistance of the liquid film and the wall metal between the liquid-metal interface and the thermocouple location. For example,

$$h_i = \frac{1}{\frac{1}{h_f} + \frac{x}{k}\frac{D_i}{D_L}}$$

where h_f is the inside film coefficient, x equals $(D_t - D_i)/2$, D_t is the diameter of the thermocouple location, D_i is the inside diameter of the reactor, k is the metal thermal conductivity, and D_L is the logarithmic mean of D_i and D_t. The values of h_i and h_o also depend on the location of the heat loss term, Q_L, in the model. Thus the constants h_iA_i and h_oA_o were useable only in the model form given by Equations (3).

The values for all constants that were given in Table 1 except Q_L were obtained from the steady-state data represented by run numbers 36-38. The heat loss term Q_L was chosen to be slightly lower than run numbers 36-38 indicated, but this selection had very little effect on the model and the resulting controller calculations.

APPENDIX D

RECOMMENDATIONS FOR FUTURE WORK

One of the purposes of this research was to indicate the directions for future studies in the applications of invariance theory. Theoretical considerations and conclusions drawn from experimental experiences suggest a number of topics for future work. These topics can be broken down conveniently into three categories--short-range, medium-range, and long-range projects. The short-range projects are mostly concerned with immediate problems encountered in the practical application of the theory. These projects are not estimated to be of great magnitude, although any one of them could develop into a larger project. The medium-range projects are either logical extensions of short-range projects or else new topics that are expected to require more effort. The long-range projects are estimated to be major extensions of smaller projects or complex separate topics. The three categories are not entirely distinct; the amount of effort in each topic may vary according to the specific purposes of each investigation. Table 17 gives an outline of the three categories with the topics listed in each. The arrows indicate how the scope of each topic could be logically

TABLE 17

TOPICS FOR FUTURE WORK

Short-range Projects	Medium-range Projects	Long-range Projects
1) Arrhenius-type Nonlinearities	6) Nonlinearities Other Than Product or Arrhenius Types	
2) Better Set-point Methods	7) Pot-setting Optimization	11) Adaptive Control
3) Approximations for Difficult Devices	8) Identification Techniques	
4) Auxiliary Equipment Dynamics		
5) Slow Dynamic Devices		
	9) Stability Considerations	
	10) Signal Flow Diagrams	
		12) Distributed-parameter Systems
		13) Economic Models

extended. Each topic will be briefly discussed to indicate present thinking as to directions for future work.

Short-range Projects

1) Arrhenius-type Nonlinearities

The type of nonlinearity which has been treated in this work is the relatively simple, though frequently encountered, product-type. A convenient feature of this type of nonlinearity is that the time-domain models yield explicit equations for the control laws in most cases. Another common type of nonlinearity which occurs in chemical reaction systems is the Arrhenius (exponential) type. The immediate problem presented by these nonlinearities is that the time-domain model equations do not yield explicit equations for control laws. Although the conditions for invariance are still very specific, the existence of a dependent variable in exponential form may make practical application of these invariance controllers difficult. Techniques for synthesizing these controllers need to be determined and, if necessary, approximation methods evaluated.

2) Better Set-point Methods

The total variable models discussed in this work have been shown to be preferable for synthesizing real controllers; however, these controllers do not always incorporate the set-point of the controlled variable in a simple manner. Methods for introducing the set-points more conveniently should be examined in order to provide simpler controller configurations.

3) Approximations for Difficult Devices

One of the problems encountered in the experimental work was that the invariance conditions in one case specified a "double-differentiator." As a result of this work, it appears that devices for computing second and higher derivatives are not practical. It is possible that some better filtering method may be found to smooth the operation of such devices; if not, approximation techniques need to be determined. Similar techniques would also prove useful for simplification of complex controllers that would otherwise be economically unattractive. It appears that the best approximation methods would be found more easily from identifying the system by dynamic testing (frequency response, pulse testing, etc.) rather than attempting to simplify the exact control laws determined from the theoretical model equations.

4) Auxiliary Equipment Dynamics

The experimental work has led to the conclusion that one of the reasons for nonabsolute invariance was the omission of the dynamics of the auxiliary devices--transducers, final control elements, measuring devices, etc. At this point some study needs to be made to develop a criterion for determining when the auxiliary equipment dynamics need to be considered. This criterion would probably involve the relative values of the time constants for the process and the auxiliary devices.

5) Slow Dynamic Devices

The major obstacle encountered in the practical application of invariance theory was the necessity for assembling invariance controllers which are highly accurate and able to operate as slowly as the control law dictates. It is not clear that such devices, either electronic or pneumatic, are readily available; nor has it been determined what the practical upper limits on controller time constants are. This effort will require considerable knowledge of controller "hardware" and probably will entail liaison between the investigator and industry. It is not certain that manufacturers of control components will be entirely cooperative, but some effort needs to be made.

The continued use of the analog computer as a controlling device for research needs to be reevaluated because of the problems encountered in this work. It may be necessary to build or purchase special purpose controllers that are specifically designed for this type of process control application.

Medium-range Projects

6) Nonlinearities Other Than Product or Arrhenius Types

A more involved project in nonlinear system applications of invariance theory is concerned with general types of nonlinearities, besides the product and Arrhenius types.

These nonlinearities should be classified and studied in order to determine possible problem areas for invariance theory applications to chemical processes. One problem encountered in this work and treated successfully herein was the hysteresis phenomenon; however, the solution to hysteresis problems may not always be quite so simple. Other double-valued functions may also cause problems and, if they cannot be circumvented, new techniques may be required.

7) Pot-setting Optimization

If the complex controller types which are composed of many adjustable parameters are to be useful, optimization techniques need to be developed. Better methods of introducing the controlled-variable set-points may alleviate this problem; however, it still would exist even for simple controllers. The problem of optimization is aggravated by the use of uncertain, inaccurate models.

8) Identification Techniques

An alternative to optimization of complex controllers is the use of identification techniques in the hope of obtaining simple, but useable, models. It is not certain whether any of the several identification techniques is preferable for application of invariance theory. Conceivably any one of them is usable, but some further investigation should be performed to find which identification technique, if any, is preferable. Frequency response testing has been used most frequently in the past for identification of a model for the design of a control system such as invariance control. From a practical viewpoint the various identification methods should be considered in the way that each provides for convenient set-point inclusion in the controller.

9) Stability Considerations

To date no problems with stability have been encountered. It can be shown that feedforward devices add no instability to the total system. This fact is a result of the absence of feedforward dynamic terms in the system (controlled) characteristic matrix. On the other hand, any feedback controller, either primary or secondary, has the ability to create instability beyond that of the uncontrolled process. The experience with invariance theory applications thus far leads to the belief that in a process which is naturally heavily damped, such as the experimental process of this work, invariance application presents no problems of stability. However, for processes that are only marginally stable or unstable, invariance feedback controllers will have a great effect on stability. Invariance theory stability considerations are virtually unmentioned in the literature. For linear systems the standard techniques of linear stability theory will be applicable; for nonlinear systems the new techniques, such as Lyapunov's methods, will probably be required.

10) Signal Flow Diagrams

Signal flow diagrams appear to currently be in vogue, especially among electrical engineers. These diagrams are reputed to be particularly applicable to multivariable systems; however, the previous warnings about further unnecessary abstraction of the problem should be heeded. The problems of abstraction are particularly acute in the process of verifying the existence of dual channels of information. The concept of a signal flow is, however, close to a channel of information; therefore, signal flow diagrams should be investigated to determine their applicability to invariance theory.

Long-range Projects

11) Adaptive Control

A logical long-range project is the extension of the optimization and identification problems to adaptive control. The application of invariance theory to complex processes may require a "learning" controller. Techniques for synthesizing controllers that would improve themselves with time should be investigated. Another type of adaptive controller which needs study is the type of device which has the ability to change its parameters with changes in process operating conditions. Since the invariance controllers can be derived from the process models in terms of steady-state conditions, it should be relatively easy to synthesize controllers that would automatically change their constants with changes in operating conditions. Invariance theory is particularly suitable for systematic design procedures for this type of adaptive controller.

12) Distributed-parameter Systems

The work to date has been concerned mainly with lumped-parameter systems. Some work (at the Universities of Delaware and Texas) has been noted concerning feedforward control of distributed-parameter systems. Further work is necessary to develop better techniques for controller synthesis for these systems. The field of distributed-parameter modeling techniques is developing rapidly; much of the present theory could be incorporated in invariance theory for distributed-parameter systems.

13) Economic Models

A particularly interesting possibility for applications of invariance theory is the addition of economic criteria into the system model. There is no reason to believe that invariance theory should apply only to physically measurable system output variables. A broader view is to consider the system output to be dollars. If the system profit could be optimized under steady-state conditions, the invariance problem could be restated as the need for making profit invariant to system disturbances. Invariance theory offers the advantage of not requiring the controlled system output (dollars) to be measured. The development of nonlinear techniques herein allows the possibility of treating nonlinear criteria with little difficulty.