FUNDAMENTAL LIMITS OF FREQUENCY FLUCTUATIONS IN LASERS: 1/f NOISE PHENOMENA

By

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I would like to dedicate this thesis to my family and Ms. Laura B. Hall.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure | Page

CHAPTER I

INTRODUCTION

The term "laser" is an acronym for light amplification by stimulated emission of radiation. The advent of lasers was the starting point of a new era in the technology of mankind. Lasers have spread their applications into vast areas of science ranging from simple experiments in the laboratory to "high technology", medical surgery, and semiconductor manufacturing, to name a few. One of the applications of lasers is in the area of precision measurements because of their very stable frequency. Knowledge of the frequency stability of lasers can be advantageous in a given domain of applications.

A laser system, like any other system, is subjected to unwanted disturbances, so-called noises (either deterministic or nondeterministic). Fundamental noises (or fluctuations) are inherent to the laser system and it is impossible to reduce them unless the parameters of the system are altered. There are some extraneous noises by which the laser frequency stability is affected. These can be reduced by controlling environmental variations. Study of fundamental fluctuations becomes therefore very important to determine the ultimate sensitivity of laser systems.

After the advent of masers (microwave amplification by stimulated emission of radiation), Gordon et al. (1955) noted that these devices can be used for precision measurements (e.g. clocks) because of their very stable frequency. Gordon (1964) used an oscillator model to calculate the frequency linewidth of a maser. This linewidth was surprisingly small compared to the frequency at which the maser oscillates.

Schawlow and Townes (1958) had predicted the possibility of making masers in optical frequencies by using a Fabry-Perot resonator. They also calculated the frequency linewidth of such devices.

Javan (1959) published a paper showing a possibility of making masers in optical frequencies using gas discharges. In 1961, Javan and his coworkers were able to build the first optical maser (laser) which gives a continuous beam. They used He-Ne gas, and the laser was called He-Ne laser. The first measurement of frequency stability of He-Ne laser was achieved by Javan et al. (1962). They found a frequency stability of one part in 10^{14}. One year later, Jaseja et al. (1963) measured the frequency fluctuations with respect to time. They found the frequency linewidth due to spontaneous emission to be about 0.02 Hz for a power output of 1 mW.

One of the early measurements of the spectrum of frequency fluctuations was done by Siegman et al. (1967). Two linear lasers were heterodyned to measure the frequency fluctuations. They found white noise due to spontaneous emission and the $1/f^2$ noise due to extraneous disturbances. Manes et al. (1971) measured the frequency lineshape of a laser. They found that the lineshape is a mixture of Gaussian and Lorentzian lineshapes.

In heterodyning two independent linear laser scheme, the white noise is masked by $1/f^2$ noise which limits the observation of other types of noise. It is very difficult to stabilize linear lasers for reduction of $1/f^2$ noise.

Here we suggest that ring lasers can be used to study frequency fluctuations in lasers. Two counterpropagating beams share the same cavity in ring lasers. This produces very highly correlated noise which is due to extraneous disturbances. By combining the output of a ring laser, one can reduce the technical noises (common modes) to a substantial degree. We can therefore investigate noise phenomena in lasers.

One of the interesting types of noise is 1/f noise. 1/f noise was first noticed as a low-frequency noise in vacuum tubes by Johnson (1925). Since the mid fifties, 1/f noise has been observed as fluctuations in the parameters of many physical systems. In spite of its appearance in so many systems, its origins and its mechanism still remain a mystery.

Bilger (1981) speculated that there should exist 1/f noise in ring lasers. Since 1/f noise was reported in masers and in almost all physical systems, one is led to believe that 1/f noise exists in lasers. In this work, we analyze some data showing 1/f noise in frequency fluctuations, in four-frequency differential ring lasers, and two-frequency ring lasers. It is also shown that the 1/f noise level is proportional to Q^{-4}, where Q is the quality factor of the passive cavity.

A review of fluctuation phenomena and mathematical tools used to characterize the frequency fluctuations are given in Chapter II. Linear lasers and ring lasers are discussed in Chapter III. Fundamental frequency fluctuations in lasers are considered in Chapter IV. The experimental evidences showing 1/f noise in lasers are described in Section 4.2. Theoretical models for white noise and 1/f noise are given in Chapter V. We compare the experimental results and the theoretical models in Chapter VI. A summary and concluding remarks are given in Chapter VII.

CHAPTER II

FLUCTUATION PHENOMENA

Understanding of nature leads us to classify observed phenomena into various groups according to their common behavior. It was and is always desirable to comprehend nature so well that all phenomena can be explained by a single law (e.g. the quest for a unified theory of forces). But this is an endless journey.

Sir Isaac Newton, in the 18th century, applied mathematics to understand observed phenomena. Today, the broad field of science investigates the facts about nature by utilizing mathematics. Some complex natural phenomena cannot be explained in terms of an explicit mathematical relationship. These may be classified as nondeterministic (or random). Statistical methods, in this case, can be used to investigate fluctuating phenomena.

2.1 Classification of Random Processes

A random signal cannot be explained by an explicit mathematical equation because each observation of the signal will be unique. In other words, any given observation of the signal represents only one of many possible results which may have occurred.

A random process $\{y(t)\}$ is a collection of random vectors (functions) varying with a parameter t (e.g. t = time)(Melsa and Sage, 1973). A random process may be viewed as a stationary or a nonstationary random process.

If a random process y(t) has the same statistics for y(t) and y(t + τ) (for all τ), it is called a stationary random process. It is often difficult to realize a random process as a strictly stationary random process in practice. A weaker form of stationarity called wide-sense stationary random process is therefore defined. If the mean value and the correlation function of a random process stay unchanged in t(time), namely

$$E\{y(t)\} = \text{Constant in time} \tag{2.1}$$

$$E\{y(t)y(\tau)\} = \text{Function of } (t - \tau) \tag{2.2}$$

where $E\{\cdot\}$ is an ensemble average then it is called a wide-sense stationary random process. For many practical stationary processes, the ensemble (statistical) averages are equal to the time averages. In this case, the process is an ergodic process, namely

$$E\{y(t)\} = < y(t) > \tag{2.3}$$

where <y(t)> is given by

$$< y(t) > = \lim_{T \to \infty} \frac{1}{T} \int_0^T y(t)dt. \tag{2.4}$$

In this work, ergodicity is assumed for random processes. A nonstationary random process is defined as a random process whose statistical properties vary with t(time).

2.2 Mathematical Tools for Frequency Fluctuation Measurements

The concepts of power spectral density in the Fourier frequency domain and the Allan variance in the time domain are used to measure the frequency fluctuation of an oscillator (Barnes et al., 1971). However, it is customary to evaluate also the variance or autocorrelation function as a measure of frequency fluctuations.

2.2.1 Autocorrelation Function

The autocorrelation function of a random process describes the general dependence of the values of the observation at one time on the values at another time. If the autocorrelation function of a random process y(t) is represented by $R_y(\tau)$, the autocorrelation function is then defined as

$$R_y(\tau) = \langle y(t)\, y(t+\tau) \rangle \tag{2.5}$$

where <.> is the time average. The autocorrelation function can be used to find the power spectral density (Wiener-Khintchine relation) and the variance of a stationary process. This is shown in Sections 2.2.2, 2.2.3, and 2.2.4.

2.2.2 Variance

If we view the mean (or average) value as a measure of the static component of a random process, the variance can then be considered as a measure of the dynamic component of the random process. If the mean value is zero, the variance of a random process y(t) is given by

$$\sigma^2 = R_y(0) = \langle y^2(t) \rangle. \tag{2.6}$$

The variance in terms of τ is given by

$$\sigma^2(\tau) = \langle \bar{y}_k^2 \rangle$$

$$\bar{y}_k = \frac{1}{\tau} \int_{t_k}^{t_k+\tau} y(t)dt.$$

2.2.3 Allan Variance

For a random process with 1/f-type noises, the classical variance does not converge in the infinite time average (Allan, 1966). A more meaningful time-domain measure of frequency fluctuations which converges for most 1/f-type noises has been recommended by Barnes et al. (1971). This measure is called Allan variance, two-sample variance, pair variance or cluster variance. However, we refer to this as Allan variance in this work.

Allan variance $\sigma_A^2(\tau)$ of a random process y(t) is defined as a time average of two-sample variance of y(t), namely

$$\sigma_A^2(\tau) = \frac{1}{2} \langle (\bar{y}_{k+1} - \bar{y}_k)^2 \rangle \tag{2.7}$$

where $\bar{y}_k$ is given by

$$\bar{y}_k = \frac{1}{\tau} \int_{t_k}^{t_k+\tau} y(t)dt. \tag{2.8}$$

2.2.4 Power Spectral density function

The power spectral density function of a random process describes the general Fourier frequency composition of the observation in terms of the mean square variation in a unit bandwidth. For simplicity the power spectral density function is sometimes called the power spectrum or simply the spectrum.

We can define the one-sided spectral density function $S_y(f)$ of a random process y(t) as (Wiener-Khintchine relation)

$$S_y\ (f) = 4 \int_0^{\infty} R_y(\tau)\cos(2\pi f\tau)d\tau \tag{2.9}$$

where f is the Fourier frequency.

It can be shown that the Allan variance is related to the power spectral density function as (Barnes et al., 1971)

$$\sigma_A^{\ 2}(\tau) = \frac{2}{\pi\tau} \int_0^{\infty} du\ S(\frac{u}{\pi\tau})\ \frac{\mathrm{Sin}^4 u}{u^2} \tag{2.10}$$

Using Eq. (2.10) we can find the Allan variance of white noise, 1/f noise, and $1/f^2$ noise. These relations are summarized in Table I. It is advantageous to note that 1/f noise gives rise to a constant Allan variance.

In order to compute the power spectral density of a given data points, we can use the finite Fourier transform (Bendat and Piersol, 1971).

Consider the power spectral density $S_y(f)$ of an ergodic Gaussian random process {y(t)}. For a sample function y(t) of length T, the one-sided power spectral density is given by (Bendat and Piersol, 1971)

$$S_y(f) = \lim_{T\to\infty} \frac{2}{T}\, E\left\{|Y(f,T)|^2\right\} \tag{2.11}$$

where Y(f,T) is the finite Fourier transform of y(t), namely

$$Y(f,T) = \int_0^T y(t)e^{-j2\pi ft}dt. \tag{2.12}$$

TABLE I

CONVERSION TABLE FOR SPECTRUM AND ALLAN VARIANCE

NOISE	POWER SPECTRAL DENSITY S(f)	ALLAN VARIANCE $\sigma_A^2(\tau)$
White Noise	h_0	$h_0/(2\tau)$
1/f Noise	$h_{-1}f^{-1}$	$2\ln(2)h_{-1}$
$1/f^2$ Noise	$h_{-2}f^{-2}$	$(2\pi)^2 h_{-2}\,\tau/6$

Now an estimate of $S_y(f)$ can be obtained by simply omitting the limiting and expectation operators. This gives

$$\hat{S}_y(f) = \frac{2}{T} |Y(f,T)|^2. \tag{2.13}$$

Using the discrete Fourier transform (DFT) for equispaced data, we have

$$Y(f,T) = \Delta t \sum_{n=0}^{N-1} y_n \exp(-j2\pi nf\Delta t) \tag{2.14}$$

where $y_n = y(n\Delta t)$, N is number of data points, and Δt is the sampling interval. There exists a fast method to compute the DFT of the data, the FFT algorithm (Fast Fourier Transform) (Cooley and Tukey, 1965).

2.2.5 Frequency Linewidth

In early investigations of frequency fluctuations, (for example, Javan et al., 1962) the concept of frequency linewidth (or spectral purity) was used. It is advantageous to derive a relation in which the frequency linewidth $\Delta\nu$ is related to the frequency stability S(f).

Consider a sinusoidal oscillation x(t) with averaged frequency ν_0 and fluctuating phase $\phi(t)$ as

$$x(t) = x_0 \cos[2\pi\omega_0 t + \phi(t)] \tag{2.15}$$

where x_0 is a constant amplitude. Before speaking of an instantaneous frequency we need to realize that the following constraint exists

$$2\pi\nu_0 >> \frac{d\phi(t)}{dt} . \tag{2.16}$$

The instantaneous frequency fluctuation is then given as follows

$$\delta\nu(t) = \frac{d\phi(t)}{2\pi dt} \tag{2.17}$$

where the nominal frequency ν_o is taken out. The autocorrelation function of $\delta\nu(t)$ (stationary) is related to the power spectral density of $\delta\nu(t)$ as follows (Wiener-Khintchine relation)

$$R_{\delta\nu}(\tau) = \int_o^\infty S_{\delta\nu}(f) \cos(2\pi f\tau)df \tag{2.18}$$

The phase fluctuation for a time interval τ is given by

$$\phi(t,\tau) = 2\pi \int_t^{t+\tau} \delta\nu(y)dy \tag{2.19}$$

The mean square value of $\phi(t,\tau)$ can then be calculated as

$$<\phi(\tau)^2> = 8\pi^2\tau \int_o^\tau (1 - y/\tau)\, R_{\delta\nu}(y)dy \tag{2.20}$$

or, by using Eq. (5.4) we have

$$<\phi(\tau)^2> = (2\pi\tau)^2 \int_o^\infty \left(\frac{\mathrm{Sin}\pi f\tau}{\pi f\tau}\right)^2 S_{\delta\nu}(f)df. \tag{2.21}$$

The power spectral density of x(t) can then be calculated as

$$S_x(f) = x_0{}^2 \int_0^\infty \exp\left(-\frac{1}{2} <\phi(\tau)^2>\right)\cos[2\pi(f - f_0)\tau]d\tau. \tag{2.22}$$

For example, the power spectral density of $\delta\nu(t)$ for white noise in frequency is

$$S_{\delta\nu}(f) = h_0. \tag{2.23}$$

Let us calculate the power spectral density of x(t) as follows

$$<\phi(\tau)^2> = (2\pi\tau)^2 \int_o^\infty \left(\frac{\mathrm{Sin}\pi f\tau}{\pi f\tau}\right)^2 (h_0)\, df \tag{2.24}$$

The power spectral density of x(t) is then given by [using Eq. (2.22)]

$$S_x(f) = x_o^2/\{1 + [2(f - f_o)/(\pi h_o)]^2\}. \tag{2.25}$$

Equation (2.25) shows a power spectral density of x(t) which has a Lorentzian lineshape. The frequency linewidth $\Delta\nu$ is therefore defined as the full width of the lineshape at $S_x(f) = \frac{x_o^2}{2}$, namely

$$\Delta\nu = \pi h_o . \tag{2.26}$$

Investigation of frequency fluctuations is more fundamental than that of spectral purity. In other words, $S_x(f)$ can be calculated, given $S_{\delta\nu}(f)$. It is easy to show that the Markov noise $[S_{\delta\nu}(f) = 1/(f_o^2 + f^2)]$ has a Lorentzian lineshape. This means that it is impossible to distinguish the difference of a Markov noise from white noise based on spectral purity.

CHAPTER III

REVIEW OF LASERS

Many electronic devices, for example transistors, use moving electrons to obtain electronic amplification and oscillation. However, a new class of electronic devices use the internal resonances of atoms, or the transitions between quantum energy levels in atoms to gain amplification and oscillation. These devices are called masers and lasers. They usually operate at high frequencies (10^9 - 10^{15} Hz). The term maser is an acronym for microwave (or molecular) amplification by stimulated emission of radiation. The name laser is an acronym for light amplification by stimulated emission of radiation.

The ingredients to construct masers were known long before the invention of these devices. In the words of Townes (1984),

> ... there is no single component idea involved in the construction of masers or lasers which had not been known for at least 20 years before the advent of these devices... whatever unnecessary delay occurred was in part because quantum electronics lies between two fields, physics and electrical engineering. In spite of the closeness of these two fields, the necessary quantum mechanical ideas were generally not known or appreciated by electrical engineers, while physicists who understood well the needed aspects of quantum mechanics were often not acquainted with pertinent ideas of electrical engineering.

3.1 Linear Lasers

Figure 1 shows a basic structure of a linear laser. This consists of two reflectors (mirrors) and a gain medium. A laser system is similar to an RLC resonator with a feedback gain loop.

The beam amplification requires that the atomic population of the upper energy level is greater than that of the lower energy level. To achieve this the atoms are "pumped" to the upper energy level by some techniques. For example, in the He-Ne laser, the He atoms pump the Ne atoms to the upper energy level. The first He-Ne laser, which was the first continuous laser, was operated by Javan in 1961 (Javan et al., 1961).

The Laser beam can resonate in different modes. The most commonly occuring modes are Hermite-Gaussian modes. These modes may resonate also at different frequencies called transversal frequencies or longitudinal (or axial) frequencies.

Lasers are capable of producing very coherent beams. The spatial coherence is due to the geometry of the resonator (mirrors, etc.). The degree of frequency coherence is due to the atomic transitions (stimulated emission). The frequency coherence of lasers is used in precision measurements. Naturally, the knowledge of frequency fluctuations becomes important.

3.2 Ring Lasers

A ring laser gyroscope (RLG) consists of a ring interferometer formed by three or more mirrors and a gain medium inside the cavity as shown in Fig. 2 (Aronowitz, 1971). In a RLG configuration the two oppositely directed running waves may have different phases and frequencies. In particular, a clockwise rotation of the laser about an axis perpendicular to the plane of the beam causes a frequency difference between the two counterpropagating waves due to Sagnac effect. Heterodyning the two waves and measuring the beat frequency as

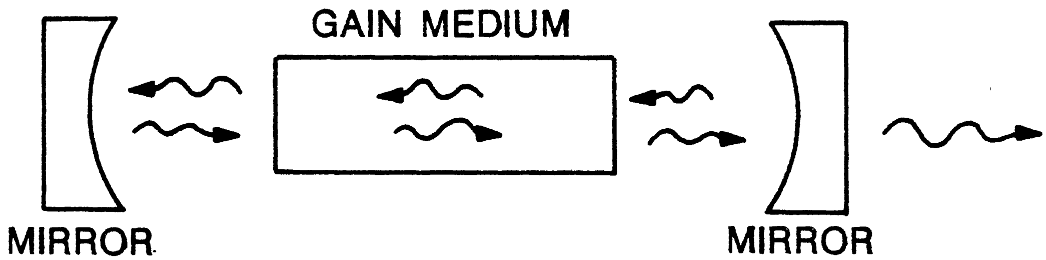


Figure 1. Structure of a Laser with Gain Medium and Two Mirrors. Laser Beam is Amplified, when passing through the gain medium due to the atomic population inversion.

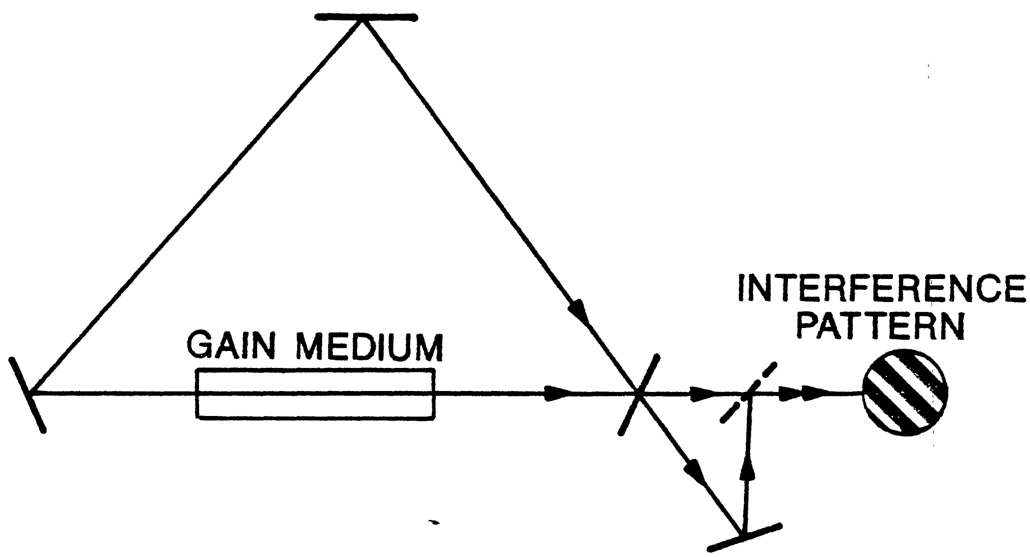


Figure 2. Structure of a Ring Laser with (at least) Three Mirrors and a Gain Medium. The two output beams at the right are combined to produce an interference pattern.

shown in Fig. 2, therefore, provides information about the rotation of the system with respect to an inertial frame. This leads to the use of RLG's as rotation sensors. We now give a simple explanation of the Sagnac effect.

For simplicity, we consider a circular ring interferometer of radius R rotating with a rate Ω around an axis perpendicular to its plane. Such an interferometer may be constructed using a glass fiber (see Fig. 3). Consider two counterpropagating beams starting from point A at time t=0. Since the interferometer rotates, the two beams have to traverse different path lengths L_{cw}, L_{ccw} in order to reach point A again. The beam which is co-directional with the direction of rotation has to catch up with A and has thus to travel a distance slightly larger than $2\pi R$, namely

$$L_{cw} = vt_{cw} = 2\pi R + R\Omega t_{cw} \tag{3.1}$$

where v is speed of light in the fiber and t_{cw} is the round-trip time for the beam travelling in the clockwise direction. Similarly, we note from Fig. 3 that the beam propagating against the direction of rotation obviously has to traverse a distance L_{ccw} slightly smaller than $2\pi R$, namely

$$L_{ccw} = vt_{ccw} = 2\pi R - R\Omega t_{ccw} \, . \tag{3.2}$$

Since the laser medium is inside the resonator, only oscillations with wavelength λ satisfying the resonance condition, where

$$\lambda n = L \tag{3.3}$$

can be sustained in the cavity; here n is an integer and L denotes the

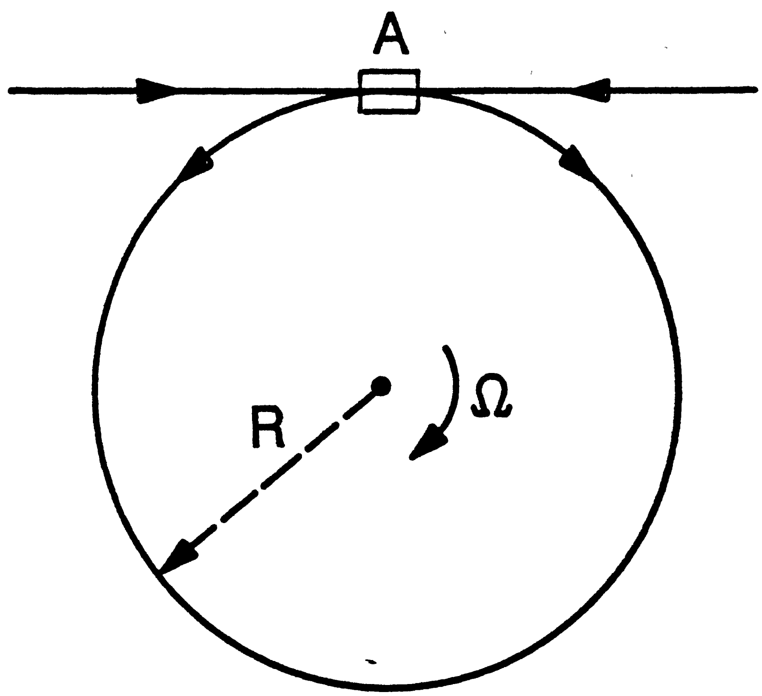


Figure 3. Sagnac Effect in a Passive Ring Interferometer with an Idealized Circular Beam Path. The two beams are entered into the interferometer at point A.

effective optical length of the resonator. Eq. (3.3) can be rewritten in terms of resonant frequencies ν as

$$\nu = \frac{nc}{L}. \tag{3.4}$$

Thus the different path lengths L_{cw}, and L_{ccw}, as seen by the two counterpropagating beams, cause a frequency difference

$$\Delta \nu = \nu_{cw} - \nu_{ccw} \tag{3.5}$$

between two modes. From Eqs. (3.1), (3.2), (3.3), (3.4), and (3.5) we find

$$\Delta \nu = nv(L_{cw} - L_{ccw})/(L_{cw}L_{ccw}) \simeq 2nvR\Omega t/p^2 \tag{3.6}$$

where we have approximated the sum of the traveling times t_{cw}, t_{ccw} by twice the travel time $t = 2\pi R/v$ in the absence of rotation; p is the perimeter of the interferometer.

The final expression for the frequency difference between the two counterpropagating waves is thus given by

$$\Delta \nu = 4A\,\Omega\ /(\lambda p) \tag{3.7}$$

where A is the area enclosed by the light beam (i.e. $A = \pi R^2$). In fact, it can be shown that Eq. (3.7) can be generalized to (Post, 1967)

$$\Delta \nu = 4\vec{A}\cdot\vec{\Omega}\ /(\lambda p) \tag{3.8}$$

where $\vec{A}$ denotes $A\vec{a}$ (the area A enclosed by the light times a unit vector $\vec{a}$ perpendicular to the plane of the beams) and $\vec{\Omega}$ is the rotation rate vector.

From Eq. (3.8) we note that the frequency difference $\Delta\nu$ induced by the rotation of the interferometer is proportional to the rotation rate Ω. Therefore, measuring the frequency difference between the two counterpropagating beams in a RLG by heterodyning the two beams provides information about the rotation of the system with respect to an inertial frame. The obvious advantage of such an optical gyroscope lies in the fact that no moving mechanical parts, in particular no spinning masses, are involved. A comprehensive review of ring lasers is given by Chow et al. (1985).

CHAPTER IV

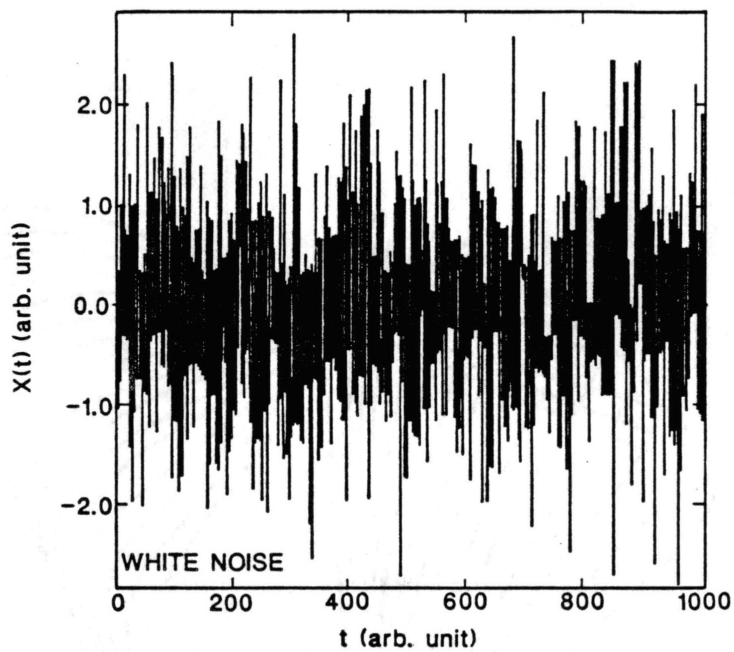
FUNDAMENTAL FREQUENCY FLUCTUATIONS IN LASERS

Why are we interested in frequency fluctuations of a laser? One of the applications of lasers and masers is in the area of precision measurements and defining standards. For example, timekeeping is one of the applications of masers (H maser, Cesium maser, etc.): definition of the second as of 1967 is the elapsed time of 9,192,631,770 oscillations of the "undisturbed" cesium atom (Jespersen and Fitz-Randolph, 1982). This links time directly to the knowledge of the frequency stability of a maser system. How would someone measure this frequency stability? We need to build several maser systems and take the relative frequency stability of each of them. It is, however, important to realize that the measurement of the absolute frequency stability of a given maser, or in fact any oscillators, is impossible. This idea is very close to the idea of measuring a speed of an object. All we can measure is the relative speed of an object with respect to another object.

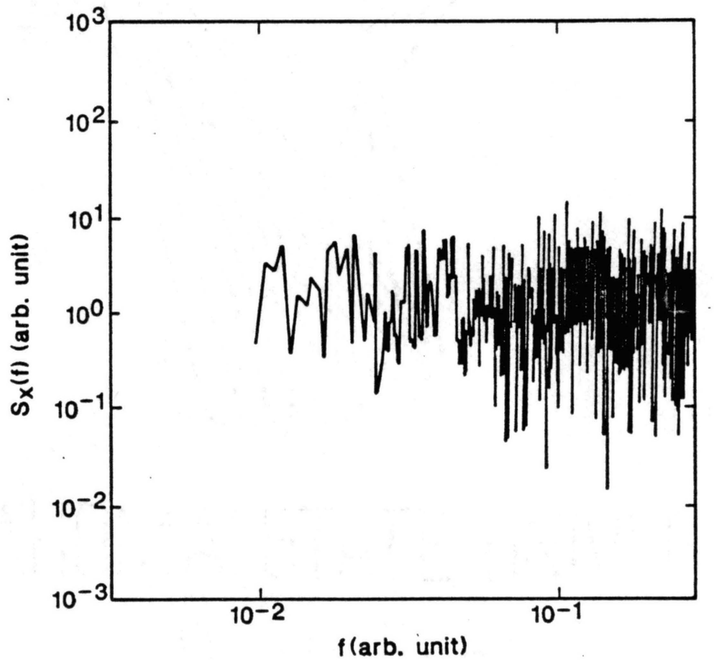
The performance limit of a ring laser gyroscope is indeed defined by the frequency stability of its radiations. Two independent laser beam countercirculating in a closed path with a finite area are combined at the output of the ring laser gyroscope to detect a rotation of the ring laser with respect to an inertial frame. Frequency fluctuations of the counterpropagating beams give rise to the false detection of the rotation which sets the performance limits of the ring laser gyroscope.

A definition of stability of an oscillator is given by the one-sided power spectral density of frequency fluctuations in the Fourier frequency domain and by the Allan variance of the frequency fluctuations in the time domain, as was proposed by Barnes et al. (1971)(ergodic random processes are assumed). The Fourier frequency analysis is preferred for stability measurements. It is relatively simple to find the time domain stability $\sigma_A^2(\tau)$ after that of Fourier frequency domain S(f) is obtained. This relation is given in Eq. (2.10). However, there is no simple relation which gives the power spectral density from the Allan variance.

Frequency fluctuations of an oscillator generally can be defined as a mixture of different noises. Among them, white noise, 1/f noise, and $1/f^2$ noise are most commonly occurring. For illustration purposes we simulate these three noises before we introduce any experimental evidences. The algorithms to generate these simulated noises are given in the appendix. The white noise and its power spectral density is depicted in Figs. 4a, 4b. This noise has very uncorrelated fluctuations. Besides the randomness apparent in Fig. 4b, the power spectral density appears to be frequency-independent. The 1/f noise and its power spectral density is shown in Figs. 5a, 5b. This noise is smoother than the white noise. The $1/f^2$ noise and its power spectral density is shown in Figs. 6a, 6b respectively. This noise is much smoother and slow-varying compared to the other two noises. $1/f^2$ noise seems to "wander" around its average.

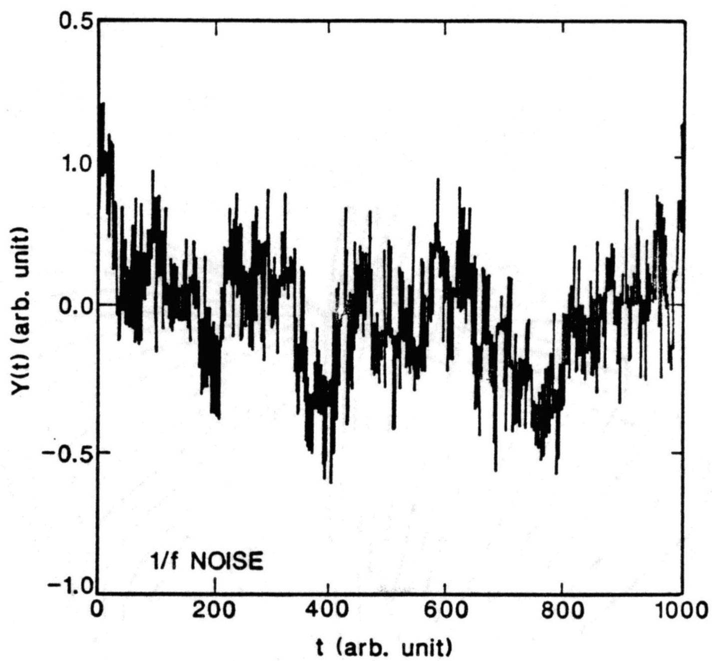


a. Fluctuations in Time Domain

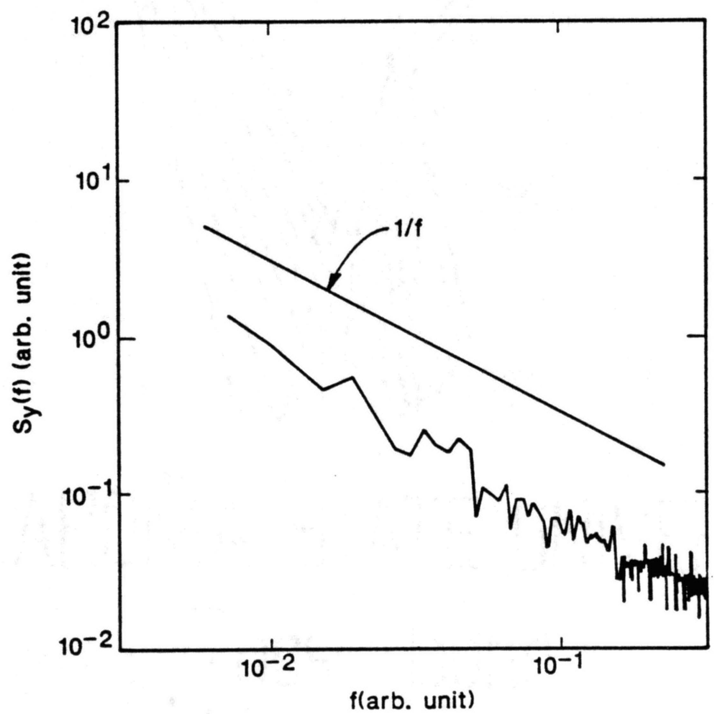


b. Power Spectral Density

Figure 4. Simulated White Noise with Mean Value 0 and Variance 1. The power spectral density shows a flat spectrum.

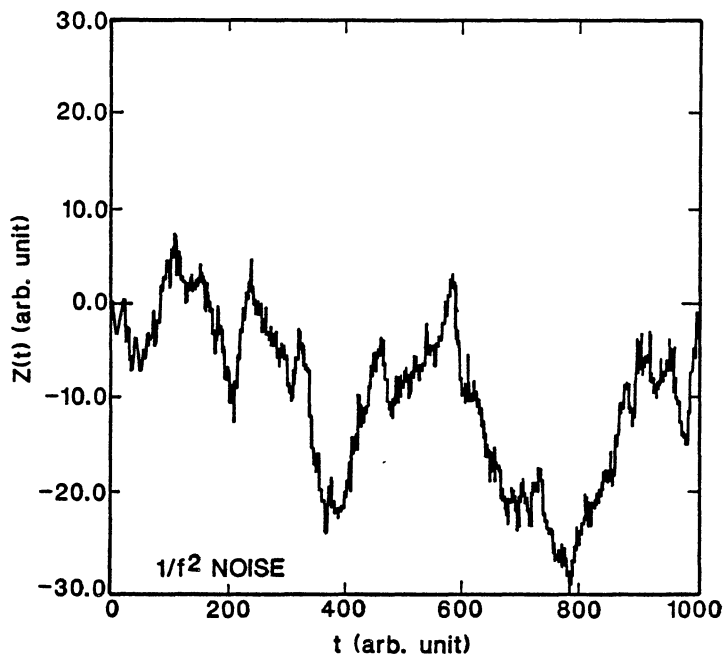


a. Fluctuations in Time Domain

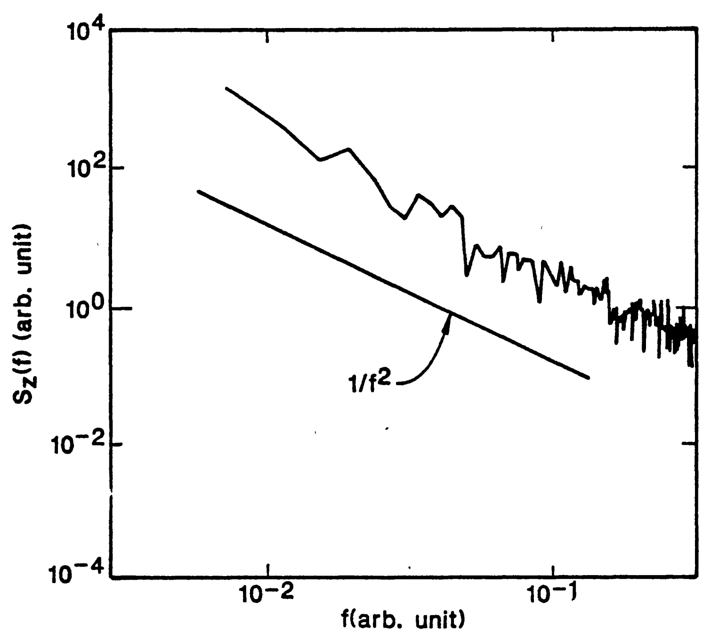


b. Power Spectral Density

Figure 5. Simulated 1/f Noise. The power spectral density shows a 1/f dependence of the spectrum.



a. Fluctuations in Time Domain



b. Power Spectral Density

Figure 6. Simulated $1/f^2$ Noise.

4.1 Heterodyning Two Linear Lasers

The first attempt to measure the frequency fluctuations of a laser was made by Javan et al. (1962). They set up two He-Ne lasers with no frequency stabilizing feedback. By combining the output of these two separate and independent oscillating lasers, they were able to measure the frequency linewidth which was better than one part in 10^{14}. The laser was from the 1.15 μm transition of the Ne atom. The observations were made by mixing the output of the detector with a crystal-controlled local oscillator to bring the beat signal into a suitable frequency range. A spectrum analyzer was used to measure the frequency spectrum of the signal. The result is shown in Fig. 7. The beat signal is about 5 MHz. They noted frequency jumps (this is not shown in Figure 7) for each laser of about 100 KHz around a central frequency. They speculated that the beat frequency jumps are due to microphonics. The temperature fluctuations resulted in a slow drift of the oscillation frequency by affecting the length of the cavity. The frequency shift due to this effect was 1 MHz per 100 s interval.

One year later, Jaseja et al. (1963) attempted to measure the frequency stability of a He-Ne laser again. The procedure was similar to that of Javan et al. (1962). The long-term frequency fluctuations are shown in Fig. 8. They were able to measure the frequency linewidth due to spontaneous emission to be about 0.02 Hz for a power output of 1 mW.

The first direct measurement of the power spectral density of frequency fluctuations was made by Siegman et al. (1967). Heterodyning the outputs from two stable He-Ne lasers (wavelength, λ = 0.633 μm), they measured the power spectral density of the frequency fluctuations. Both

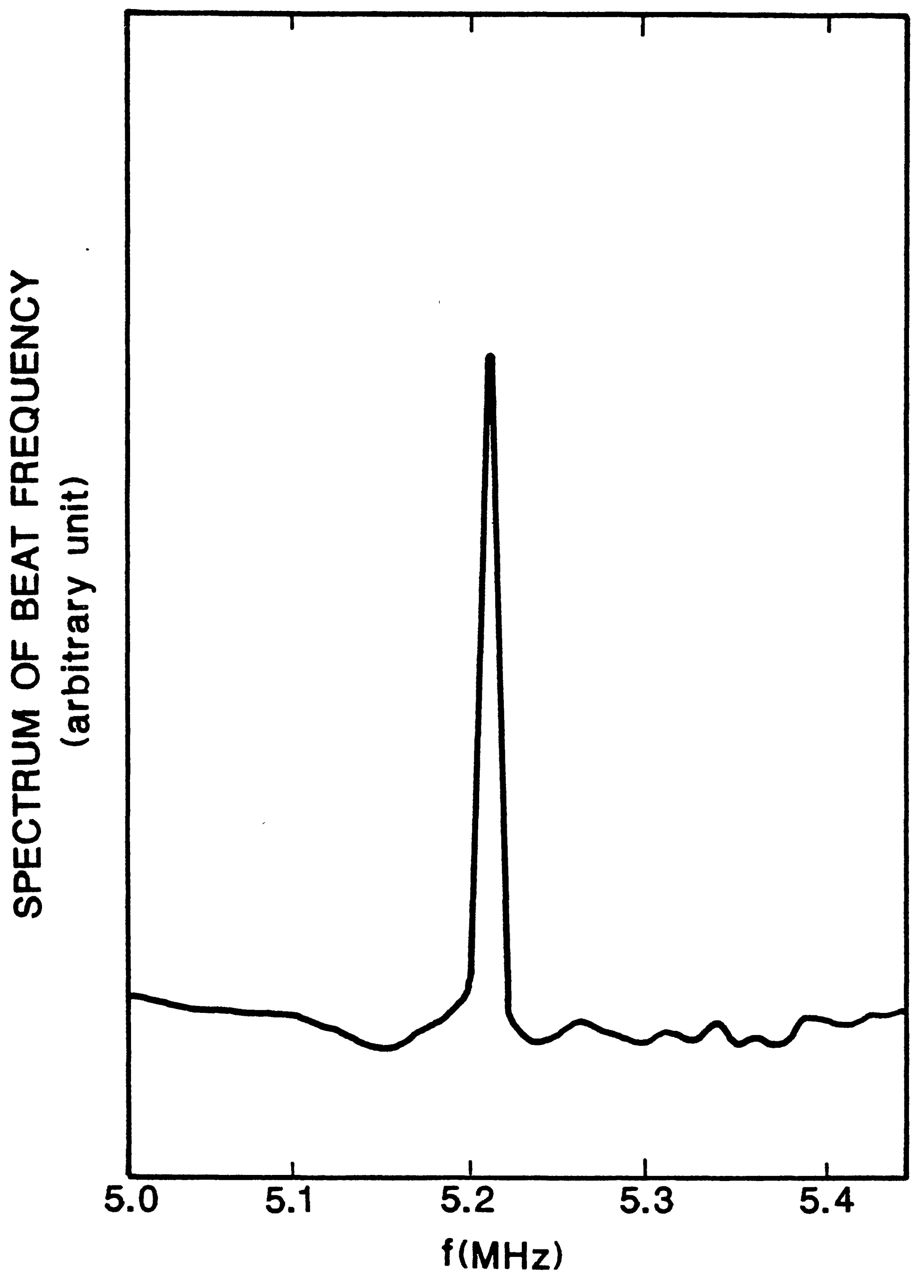


Figure 7. Power Spectral Density of Beat Frequency (Javan et al., 1962). This was originally taken with 0.05 s exposure time from a spectrum analyzer.

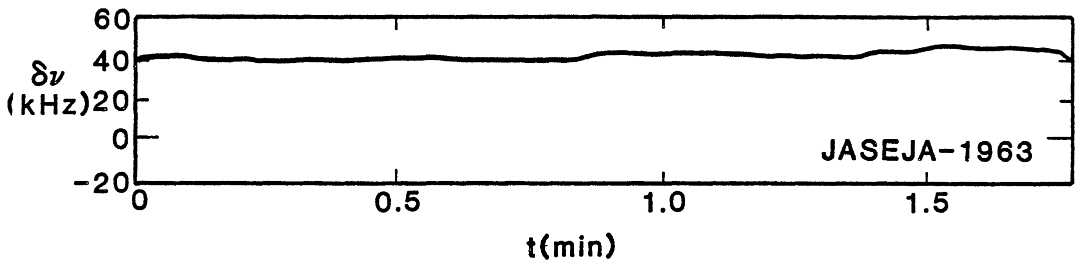


Figure 8. Beat Frequency Signal (Jaseja et al., 1963). This shows the beat frequency fluctuations between two lasers during two minutes. The frequency fluctuations is in the range of Kiloherz.

lasers were free-running with no frequency stabilizing feedback except for a slow automatic frequency control loop which kept one laser frequency at 30 MHz difference from the other one. The random frequency fluctuations of the beat frequency between two lasers were then measured. They found the power spectral density of the beat frequency to be a Gaussian lineshape with a standard deviation of 8.1 KHz. They measured the mean square value of the instantaneous phase fluctuations versus the sample time τ. It was found that the mean square value was proportional to τ^2 for $\tau \leq 100$ ns. They speculated that the τ^2 dependence of the mean square value is due to the mechanical instabilities. The beat frequency signal was fed into a frequency discriminator with the center frequency of 30 MHz in order to measure the instantaneous frequency fluctuations. The output of the discriminator was connected to a spectrum analyzer for power spectral density measurement of the frequency fluctuation. The measured power spectral density of instantaneous frequency fluctuation is shown in Fig. 9. The white quantum noise due to spontaneous emission is shown in the frequency range of 1 KHz to 10 KHz. Below 1 KHz, $1/f^2$ noise predominates, down to 50 Hz. In these preliminary measurements of laser short-term frequency fluctuations, they showed the feasibility of white quantum noise measurement by operating one laser at low power ($P < 10^{-9}$ W).

In 1971, Manes et al. observed the power spectral density of frequency fluctuations by combining the output of two stable He-Ne lasers at $\lambda = 3.39\ \mu$m. They successfully measured the frequency fluctuations due to spontaneous emission by increasing the losses in the cavity and lowering the power of the laser field. This increases the

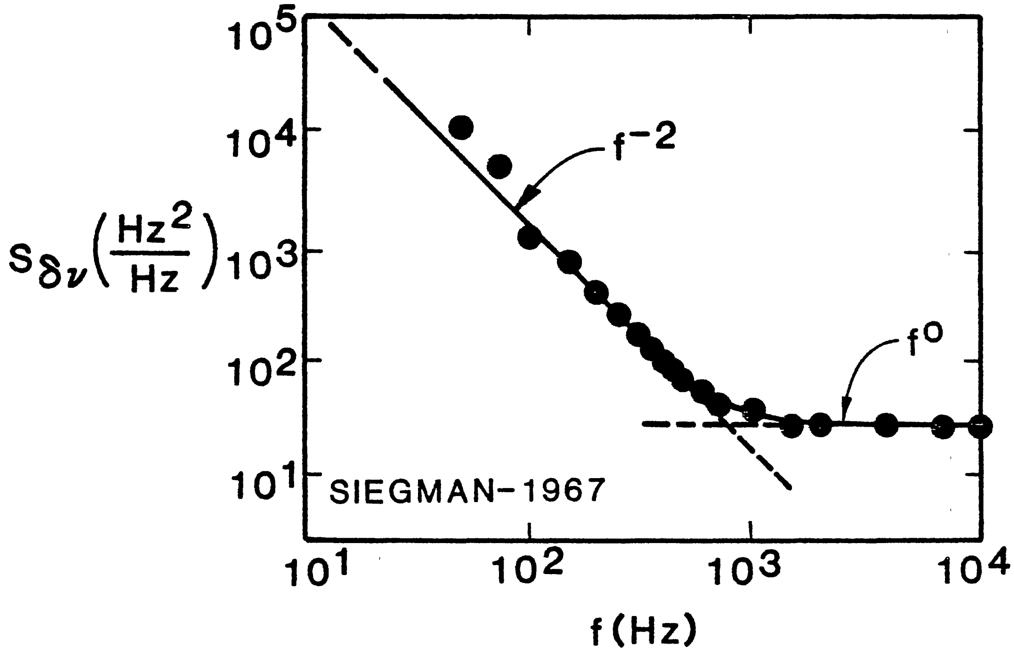


Figure 9. Power Spectral Density of Beat Frequency (Siegman et al., 1967). The transition frequency is at 800 Hz.

white quantum frequency noise contribution to the laser linewidth (see Chapter V). The laser field power spectral density was then found to be nearly Lorentzian with a linewidth inversely proportional to the output power. They showed that white noise and $1/f^2$ noise in frequency fluctuations would result in Lorentzian and Gaussian lineshapes in the laser field, respectively.

Direct observation of frequency fluctuations of the laser field was also achieved by heterodyning laser outputs. Essentially, the experimental set-up was similar to that of Siegman et al. (1967). The measured discriminator output power spectral density showed $1/f^2$ noise up to 500 Hz which indicates that the laser field has been disturbed by "technical" fluctuations. Above 500 Hz, the discriminator output power spectral density was constant (white) up to 50 KHz where the apparatus limits were reached.

4.2 Combining The Outputs Of A Ring Laser: Observation Of 1/f Noise

In all experimental cases we have discussed so far, there is not any observation of 1/f noise in frequency fluctuations of laser field. Heterodyning two independent linear lasers in the case of large "technical" noise, it is almost impossible to observe 1/f noise because of the presence of the dominant $1/f^2$ noise. Ring lasers can play a very important role in reducing the latter. Two counterpropagating beams essentially share the same cavity. This produces very high correlated "technical" noises in the laser field. By combining the output of a ring laser, one can reduce the technical noises (common modes) to a substantial degree.

Flicker (1/f) noise was first noticed as a low-frequency noise in vacuum tubes (Johnson, 1925) and much later in semiconductors. Since the midfifties, 1/f noise has been observed as fluctuations in the parameters of many systems. Many are completely unrelated to either vacuum tubes or semiconductors. The presence of 1/f noise in so many systems has led researchers to speculate that there exist some law of nature that applies to all systems which results in 1/f noise (the "aether" may fluctuate in 1/f noise). However, it is generally agreed that its origins and its mechanism are still remain not well understood.

1/f noise has been used to characterize, in part, the voltage across nerve membranes (Verveen and Derksen, 1965), fluctuations in temperature variation (Brophy, 1970), the rate of insulin intake by diabetics (Campbell and Jones, 1972), the amplitude and frequency of music (Voss and Clarke, 1975), traffic flow (Musha and Higuchi, 1976), economic data and the rate of computer errors (Mandelbrot, 1977), the frequency fluctuations of quartz crystal oscillators (Gagnepain et al., 1981), and so on.

One of the unusual properties of 1/f noise is the so-called "infrared catastrophe". If the power spectral density of a random process continues down to zero Fourier frequency while it remains proportional to $f^{-\alpha}$ for $\alpha \geq 1$ then the integral of this diverges and the variance is infinite. Infinite variance indeed represents infinite power which cannot be a property of a finite stationary physical system that exhibits 1/f noise.

One attempt to avoid this difficulty is to assume that at some low-frequency limit the power spectral density becomes flat like white noise, and a high-frequency limit is also assumed, above which the power spectral density decreases at least as rapidly as $1/f^2$ (as is assumed for white noise) to assume integrability of the power spectral density at high frequencies. With low and high frequency limits, the integral of the power spectral density would be finite, which results in a finite variance.

Even after intensive experimental efforts, low-frequency limits have not been detected. For example, Brophy (1970) has observed 1/f noise in seasonal temperature fluctuations down to Fourier frequency 10^{-10} Hz, or about one cycle in 300 years. In this case and almost all others, no change in slope of the spectrum was observed at low frequencies. In only two cases, as far as the author knows, a low frequency flattening has been observed: in nerve membranes (Verveen and Derksen, 1965) and in thin films of tin at the temperature of the superconducting transition (Clarke and Hsiang, 1975, 1976).

Let us return to the measurement of frequency fluctuations by means of ring lasers. Two counterpropagating beams are essentialy in the same cavity. If the ring laser does not rotate with respect to an inertial frame, the countercirculating beams will have the same frequency. In the presence of rotation the beam has, in principle, different frequencies proportional to the rotation rate. However, in practice, there exist some effects that cause the counterpropagating beams to be locked at the same frequency (for example, via back scattering of the beam off mirrors). The minimum rotation rate required to stay out of this "dead-band" is called lock-in rotation rate. There are ways to prevent this lock-in phenomenon. One way is biasing the ring laser by mechanical rocking (dithering)(Killpatric, 1967). One may also use four mirrors which are not in the same plane (Sanders and Anderson, 1981) and non-reciprocal bias elements in the cavity (for example, a Faraday cell, Zeeman bias, or magnetic mirrors) to prevent lock-in phenomenon. In the cases where one gets four different frequencies, the ring laser is called a four-frequency differential ring laser.

Fig. 10 shows the experimental set-up to obtain the beat frequency between two counterpropagating beams. Mounting a small detector on a stationary place relative to the prism converts the intensity of the fringes into an electrical signal. This signal may be translated to digital numbers by making use of A/D converters. These numbers may be stored in the memory of a digital computer for later analysis.

The long-term beat frequency fluctuations can be measured by evaluating the "average instantaneous" frequency of the signal. This is done by counting the zero crossings of the signal in a given time interval (i.e. a sample time). To minimize count errors, one may use the powerful method of a two-threshold crossing technique (Bilger and Sayeh, 1983).

4.2.1 Data Analysis Techniques

Figure 11 depicts the beat frequency fluctuations of a four-frequency differential ring laser. The analysis of beat frequency fluctuations can be summarized as follows: (a) detecting and replacing the outliers in the data set, (b) removing the linear trend (note that an average is a linear trend with zero slope), (c) applying a proper time window to the data, (d) computing the power spectral density estimate using DFT methods, (e) averaging the power spectral density estimate to reduce the uncertainty involved in the estimate due to DFT, and to define a confidence interval for the estimate, (f) fitting power

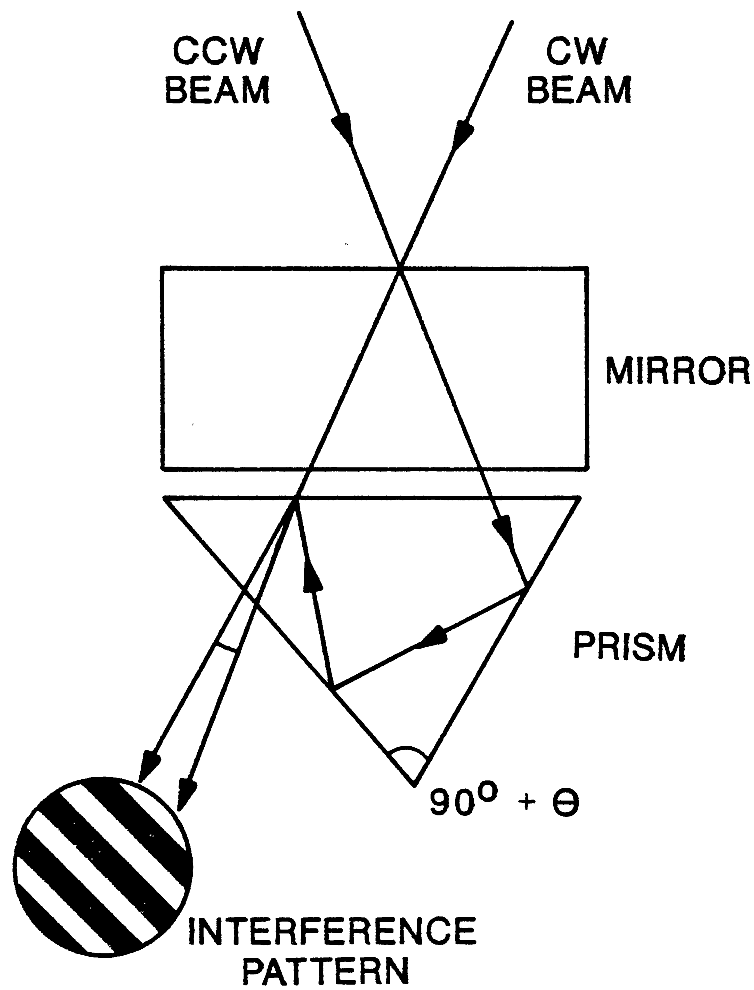


Figure 10. Set-up of Beat Frequency Read-Out (Aronowitz, 1971). The prism angle has small Deviation from 90^{o} to produce the Fringe pattern at the left.

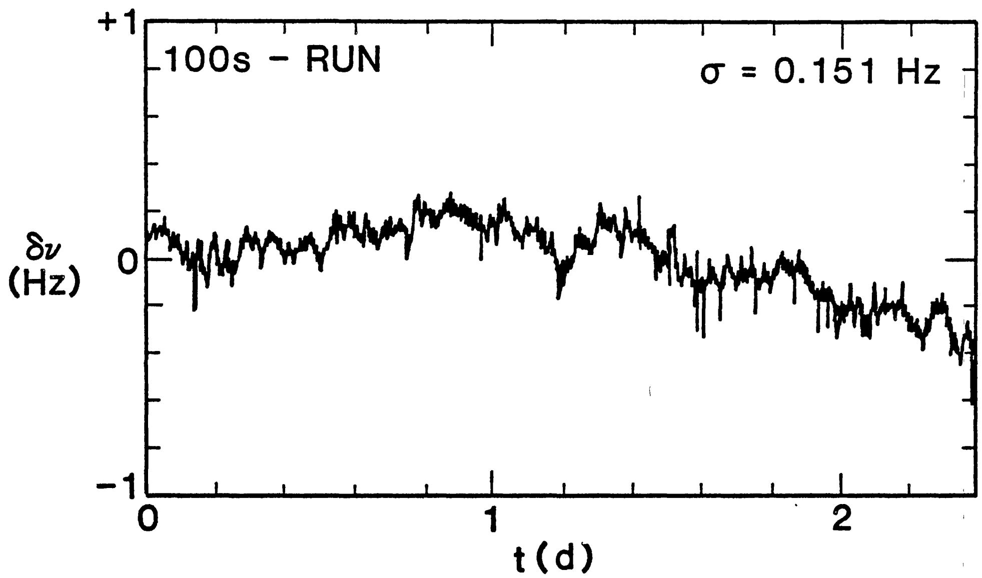


Figure 11. Record A, Beat Frequency (Bilger and Sayeh, 1983). The frequency deviations stay within a fraction of a Hertz. This is very small compared to what has been reported by Jaseja et al. (1963) and Siegman et al. (1967).

laws, if applicable, to the spectrum by means of non-linear least-squares fit methods, and (g) removing the induced quantization error due to the fact that the instantaneous frequency is represented always by an integral number of counts (zero crossings).

a. Outliers are data points whose values are very large compared to the local data point values. The value of outliers are several orders of magnitude larger than the sandard deviation of the data points. These points may enter into the set by faulty signals occurring during the data acquisition. In order to detect these outliers, we compare every point with its local average within a prescribed time window. A threshold is then set to decide whether a given data point is an outlier. The detected outlier is replaced by the average of the points in the window. This technique is applied to the data set starting at the begining of the data and proceeding by shifting the window by one point until all data are examined. It is assumed that there are no outliers within the first window.

b. The deterministic part of the data, namely a systematic trend, can be removed without loss of significant knowledge about the power spectral density of the frequency fluctuations. In general, a systematic trend is perfectly deterministic while the noise is nondeterministic. Consider a function x(t), which may be written in the form

$$x(t) = x_d(t) + x_n(t) \tag{4.1}$$

where $x_d(t)$ is some deterministic function of time and $x_n(t)$ is a nondeterministic function of time. For the special case of a systematic trend to be a linear trend, $x_d(t)$ is given by

$$x_d(t) = x_0 + x_1 t. \tag{4.2}$$

A linear least square fit program (Habib, 1984) is used to estimate x_0 and x_1. The deterministic part of the data, $x_d(t)$, is then removed from the set. $x_n(t)$ therefore remains for further analysis.

c. It is usually advantageous to window the data points to enhance certain characteristics of the power spectral density estimates. Windowing is simply multiplying the data by a prescribed function, a so called "window function". The purpose of windowing is to suppress large side lobes due to discontinuities at each end of the finite segment of the data being analyzed.

There are several window functions which can be used depending on nature of the problem. In this work the Hamming window is used. This window can be written as (Otnes and Enochson, 1972).

$$W_h(t) = 0.54 + 0.46 \cos 2\pi t/T \text{ for } t < T/2, \text{ 0 otherwise} \tag{4.3}$$

d. We can use the discrete Fourier transform concept to evaluate the power spectral density of a given set of data points. This was discussed in Chapter II.

e. Each Fourier frequency component of the estimate S(f) has a sampling distribution given by (Bendat and Piersol, 1971)

$$\frac{\hat{S}(f)}{S(f)} = \chi_2^{\,2}/2 \tag{4.4}$$

where χ_2^2 is the chi-square distribution with 2 degrees of freedom. The result in Eq. (4.4) is independent of the record length T. The normalized standard error is

$$\epsilon = \frac{\sigma[\hat{S}\ (f)]}{S(f)} = \sqrt{\frac{2}{n}} \tag{4.5}$$

In our case, n = 2, thus $\epsilon = 1$. This implies that the standard deviation of the estimate is as large as the quantity being estimated. This is an unacceptable error for most practical applications. One way to reduce this error is to average the power spectral density components. This can be done by averaging the power spectral density components for m adjacent components. The normalized standard error is then

$$\epsilon = \frac{1}{\sqrt{m}} \tag{4.6}$$

The sampling distribution of an averaged estimate is approximately chi-square with 2m degrees of freedom. We can then define a $(1 - \alpha)$ confidence interval for a power spectral density S(f) based on an averaged estimate $\hat{S}(f)$ as

$$2m/\chi^2_{2m;\alpha/2} \leq S(f)/\hat{S}(f) < 2m/\chi^2_{2m;(1-\alpha/2)} \tag{4.7}$$

This confidence interval is applied to each power spectral density component. The error bars on each data can also be defined by Eq. (4.6).

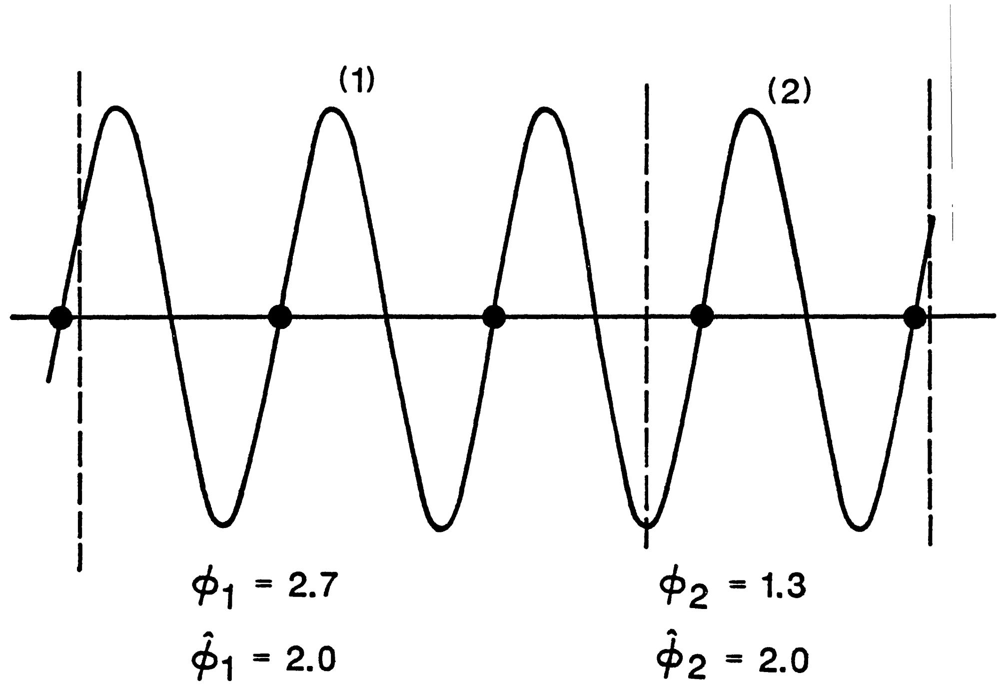
f. A noise model that has been found advantageous in oscillators (Barnes et al., 1971) consists of a set of five independent noises, with power spectral density

$$S_{\delta\nu}(f) = h_{-2}f^{-2} + h_{-1}f^{-1} + h_o + h_1 f + h_2 f^2 \tag{4.8}$$

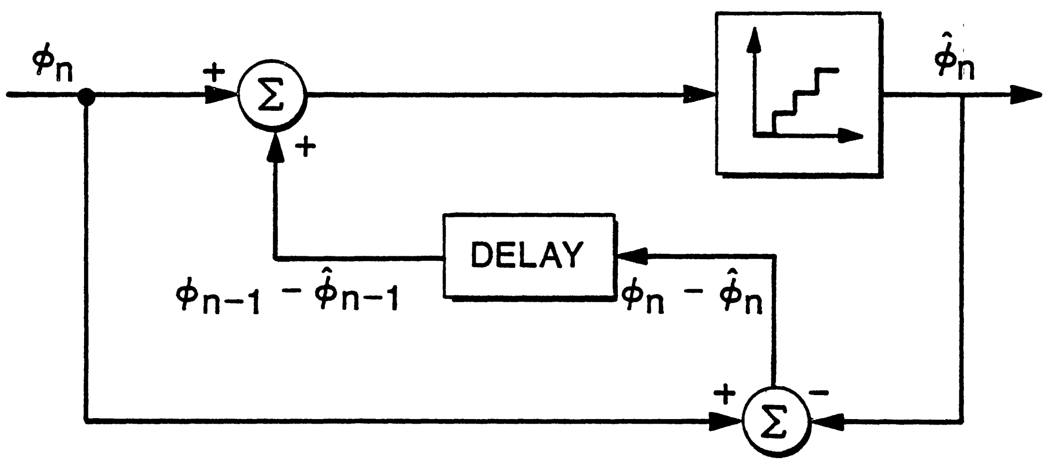
A laser system can be considered as an oscillator, the power spectral density in Eq. (4.8) can therefore be useful to characterize the laser system noises. These power laws are presented by sections of straight lines in a log-log plot of $S_{\delta\nu}(f)$ versus f. A nonlinear least square fit program (Habib, 1984) is used to estimate the coefficients h_{-2}, h_{-1}, h_0, h_1, and h_2. Some of these five independent noises may predominate the others. This can easily be seen by the relative errors from the least square fit.

e. If a zero crossing technique is used to record the average instantaneous phases, the quantization noise would be added to the data due to the integral nature of the counts.

Consider a pure sinusoidal signal shown in Fig. 12a. Without loss of generality, we assume the signal is free of noise, but the sampling intervals have fluctuations. Using a zero crossing technique, we count the number of zero-crossing ($\hat{\phi}_n$) where the signal is going from negative value to positive value in a given time interval (n). In interval (1), we have 2 counts ($\hat{\phi}_1 = 2$) whereas the true phase value (ϕ_1) is 2.7 ($\phi_1 = 2.7$). The difference between the number of counts and the true phase value (i.e. $\phi_n - \hat{\phi}_n$) is added to the next interval. In interval (2), we then have 2 counts ($\hat{\phi}_2 = 2$) whereas the true phase is 1.3 ($\phi_2 = 1.3$). This quantization of ϕ_n is shown by a block diagram in Fig. 12b. Calculating the power spectral density of ϕ_n given the power spectral density of $\hat{\phi}_n$ is complicated due to the nonlinearity of the truncating function. We therefore simulate the model on the computer. The quantization noise is then identified and is suppressed from the signal. It is shown by several simulation runs that the power spectral density of the counts is near-f noise, (i.e. $S \propto f$.) given a white spectrum for the true phase values.



a. Sinusoidal Signal



b. Block Diagram of Quantization Error

Figure 12. Quantization Error with Zero-Crossing Technique. In a., a sinusoidal signal is assumed with no phase fluctuations; however the sampling intervals have fluctuations.

4.2.2 Experimental Evidence

At the 11th Winter Colloquium on Quantum Electronics, 1981, Bilger speculated that there should exist 1/f noise in ring lasers (Bilger, 1981). This speculation has had its own strong point since 1/f noise was reported in atomic clocks and in almost all physical systems. At the time, there was not any published work that would show 1/f noise in lasers. In 1983, we published a paper in which the measured power spectral density of a four-frequency differential ring laser clearly showed this 1/f noise. We shall discuss these results in upcoming sections.

Investigation of frequency fluctuations in lasers can be done by making use of ring lasers. We will discuss the experimental results which were achieved in Four-frequency differential ring lasers and conventional (two-frequency) ring lasers in the following sections.

4.2.2.1 Four-Frequency Differential Ring Laser

A four-frequency differential ring laser, as it was discussed in previous chapters, has, as its name reveals, four different resonance frequencies in the same cavity. The resonance modes (frequencies) of a ring laser are four-fold degenerate in the absence of polarization-dependent elements. Each longitudinal cavity mode can oscillate as any of four distinct modes, clockwise and counterclockwise traveling, having either of two arbitrary orthogonal polarizations (i.e. left-circularly polarized and right-circularly polarized). In four-frequency differential ring laser, this mode degeneracy is removed by introducing, for example, out-of-plane geometry and polarization-dependent elements. Four distinct modes, for each longitudinal mode, can therefore oscillate in the cavity. For convenience, let us number the modes as follows:

ν_1 = clockwise, left-circularly polarized

ν_2 = clockwise, right-circularly polarized

ν_3 = counterclockwise, left-circularly polarized

ν_4 = counterclockwise, right-circularly polarized

Such a ring laser can be considered as two two-frequency ring lasers sharing the same cavity; one oscillates with a left-circularly polarized beam; another oscillates with the right-circularly polarized beam. The final beat frequency of the four-frequency differential ring laser is achieved by combining ν_1 and ν_3 then ν_2 and ν_4, namely

$$\Delta\nu_L = \nu_1 - \nu_3 \tag{4.9}$$

$$\Delta\nu_R = \nu_2 - \nu_4 \tag{4.10}$$

The beat frequencies $\Delta\nu_L$ and $\Delta\nu_R$ are detected by small light detectors. The final beat frequency is obtained by combining these two beat frequencies (i.e. output of the detectors), namely

$$\Delta\nu = \Delta\nu_L - \Delta\nu_R \tag{4.11}$$

The beat frequency fluctuations $\Delta\nu$ of a four-frequency differential ring laser is shown in Fig. 11 (after removing the average). This was sampled at the rate of one sample per 100 s for 2.5 days duration. The zero-crossing technique was used to count the number of crossings from negative to positive value.

78 outliers were detected and replaced by their local averages in the original data. The value of the outliers were at least 100 times

larger than the standard deviation of the data points. The number of data points in the analysis was 2048 which is suitable for the Fast Fourier Transform (FFT) algorithm ($N = 2^{11}$). The standard deviation of this run is 0.13 Hz. It is advantageous to note that the peak-to-peak frequency excursion is 0.74 Hz over 2.5 days which gives the frequency stability of 2 parts in 10^{13}. This is a relatively stable oscillator compared to the H-masers used for timekeeping purposes (the frequency stability of a H-maser is one part in 10^{9}). However, one can argue that major frequency fluctuations due to mechanical or other correlated noises are suppressed to some degree which could otherwise give rise to a worse frequency stability.

The ring laser was placed in a thermostat with 3×10^{-7} short-term temperature stability over minutes, and with 10^{-6} long-term stability over one week. The laser cavity had 4 mirrors with an out-of-plane geometry to produce left-circularly polarized and a right-circularly polarized waves. The cavity itself was made of Cervit with a thermal expansion coefficient $\leq 10^{-7}$ K^{-1}. The passive cavity quality factor Q was 3×10^{8}, the power loss per mode P was 80 μW and the wavelength λ was 633 nm. The cavity was filled with He-Ne gas with very low pressure.

The power spectral density of these data (see Fig. 11) is illustrated in Fig. 13. The 1/f noise is clearly shown over one decade of the spectrum. This 1/f noise cannot be due to technical fluctuations (temperature variation, etc.) because the same gyro, when operated without any temperature stabilization, gave the same 1/f noise level as shown in Fig. 13. White noise has not been achieved in this run because of predomination of 1/f noise down to the Nyquist frequency (with the given sampling interval Δt = 100 s the Nyquist frequency 5 mHz). 1/f

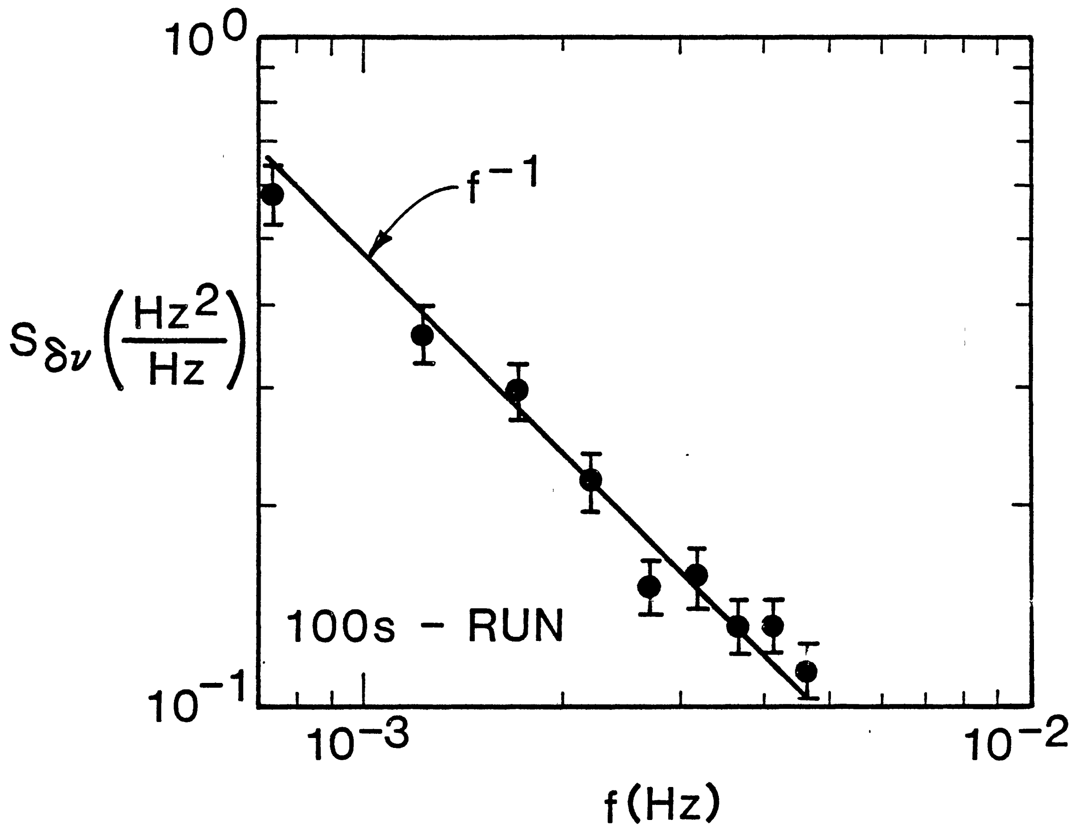


Figure 13. Power Spectral Density of Record A (Bilger and Sayeh, 1983). This shows 1/f noise at very low fourier frequencies ($<10^{-2}$ Hz); compare this to that of Siegman et al. (1967).

noise in frequency fluctuations is shown from $5x10^{-4}$ Hz down to $5x10^{-3}$ Hz. Only 100 averaged points are depicted in Fig. 13.

The series of measurement runs presented in this work are listed in Table II. Record A has already been discussed. Record B was achieved with the same four-frequency differential ring laser. However, this record lasted for 9 days. The sampling rate is again one sample per 100 s. The number of detected outliers in this record was only 12. The standard deviation of Record B is 1.36 Hz which is higher than that of Record A. This implies the presence of low-frequency noises with larger magnitude in Record B compared to Record A. This record is shown in Fig. 14.

The computed power spectral density of Record B is depicted in Fig. 15. This clearly shows the presence of two dominate noises, namely, 1/f noise and $1/f^2$ noise. $1/f^2$ noise predominates over five decades of the spectrum from $5x10^{-4}$ Hz down to the lower Fourier frequency $1.2x10^{6}$Hz. 1/f noise prevails over one decade of the spectrum from the Nyquist $5x10^{-3}$ Hz down to $5x10^{-4}$ Hz here $1/f^2$ noise predominates. The estimated white noise is drawn from the analysis on Record C (this run has a higher Nyquist frequency compared to Record A and Record B). The $1/f^2$ noise prevails at Fourier frequency $5x10^{-4}$ Hz. The noise magnitudes are several orders smaller compared to that of Siegman et al. (1967, 1971). Here the standard deviation of the beat frequency is 1.36 Hz over 9 days whereas that of Siegman et al. (1967) is of the order of Kilohertzes. This evidence show the reduction of technical noises by many orders of magnitude in this series of measurements. This shows the feasibility of frequency fluctuations investigation by means of ring lasers.

TABLE II

RECORDS OF FREQUENCY FLUCTUATIONS IN RING LASERS

RECORD	SAMPLING TIME	TOTAL DURATION	NOISES IN RECORD
Record A	100 s	22.5 d	1/f
Record B	100 s	9 d	$1/f^2$ and 1/f
Record C	10 s	11.5 h	1/f, white, and f
Record D	100 s	5 d	$1/f^2$, 1/f, white, and f
Record E	500.5 s	54 d	1/f, white, and f

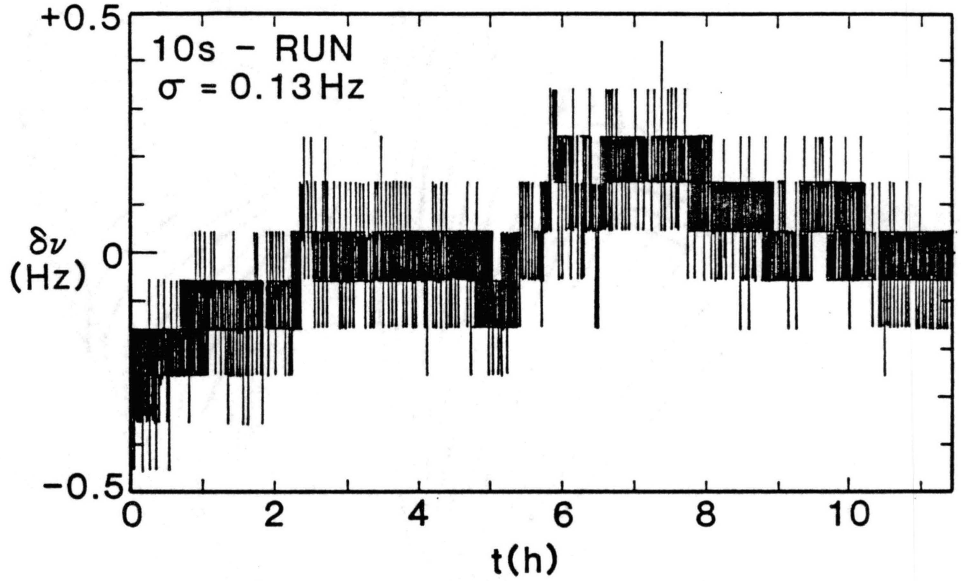


Figure 14. Record B, Beat Frequency. The signal was heavily quantized, which gives rise to high frequency quantization noise. However, the low frequency fluctuations are not affected significantly.

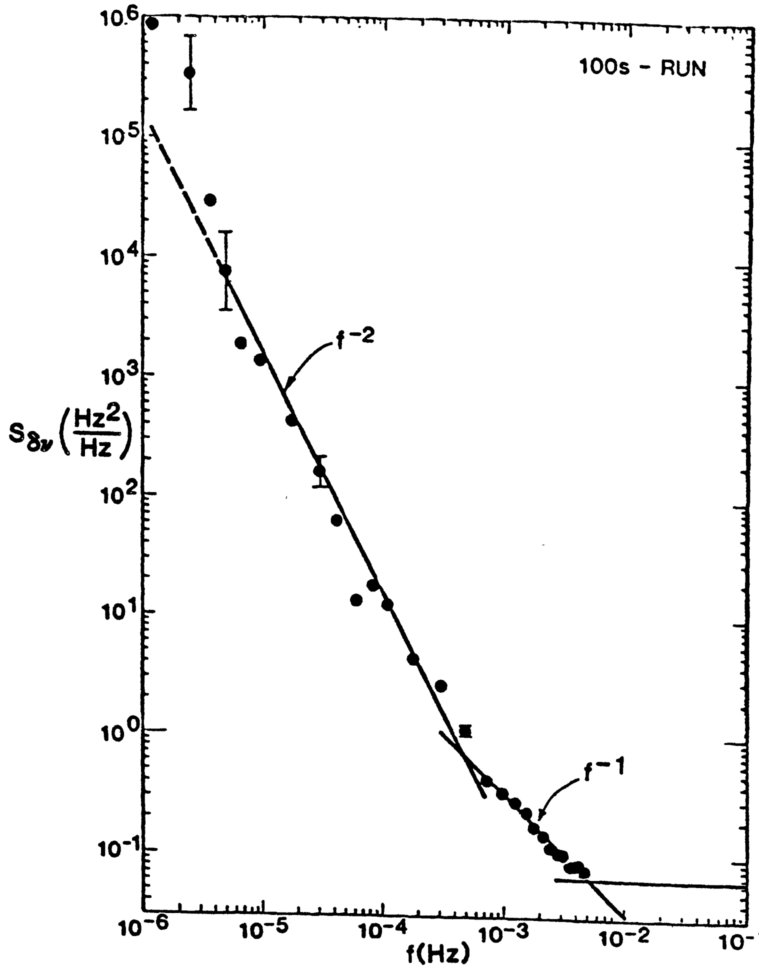


Figure 15. Power Spectral Density of Record B (Bilger and Sayeh, 1983). $1/f^2$ noise predominates over 5 decades of the spectrum and 1/f noise prevails in the higher fourier frequencies over 1 decade of the spectrum.

Record C was achieved on the same ring laser with the same specifications as were Record A and Record B. However, the sampling time was shortened to reach the white noise level. The sampling rate was then one sample per 10 s for a duration of 11.5 hours. The standard deviation is 0.13 Hz which is somewhat smaller than that of the other two records. Fig. 16 shows the corresponding power spectral density of Record C. This represents 1/f noise and f noise. The 1/f noise predominates at low frequencies from $5x10^{-3}$ Hz to higher frequencies. As was mentioned, f-noise is due to quantization of the phase to interger numbers. This is shown by open circles on Fig. 16. The quantization noise may rise from the fact that 0.1 Hz resolution of Record C was larger than the white noise level. By simulation runs on a computer, we decided that the white noise level is 0.063 Hz^2/Hz which is plotted by a solid line in Fig. 16.

In all these three Records (A, B, and C), we observe that the 1/f noise levels are the same (i.e. $h_{-1} = 3.3x10^{-4}$ Hz). Another four-frequency differential ring laser was investigated with a higher passive cavity quality factor over a long period of time to observe the 1/f noise phenomenon.

Record D was achieved in a four-frequency differential ring laser with four out-of-plane mirrors. The passive cavity quality factor Q was $5x10^8$, the power loss per mode was 80 μW. This ring laser was not in a temperature-controlled thermostant.

The sampling rate was one sample per 100 s for a duration of 5 days. The power spectral density of this record is depicted in Fig. 17. The three dominant noises in this experiment are $1/f^2$ at low frequencies, 1/f noise at intermediate frequencies, and f noise at high

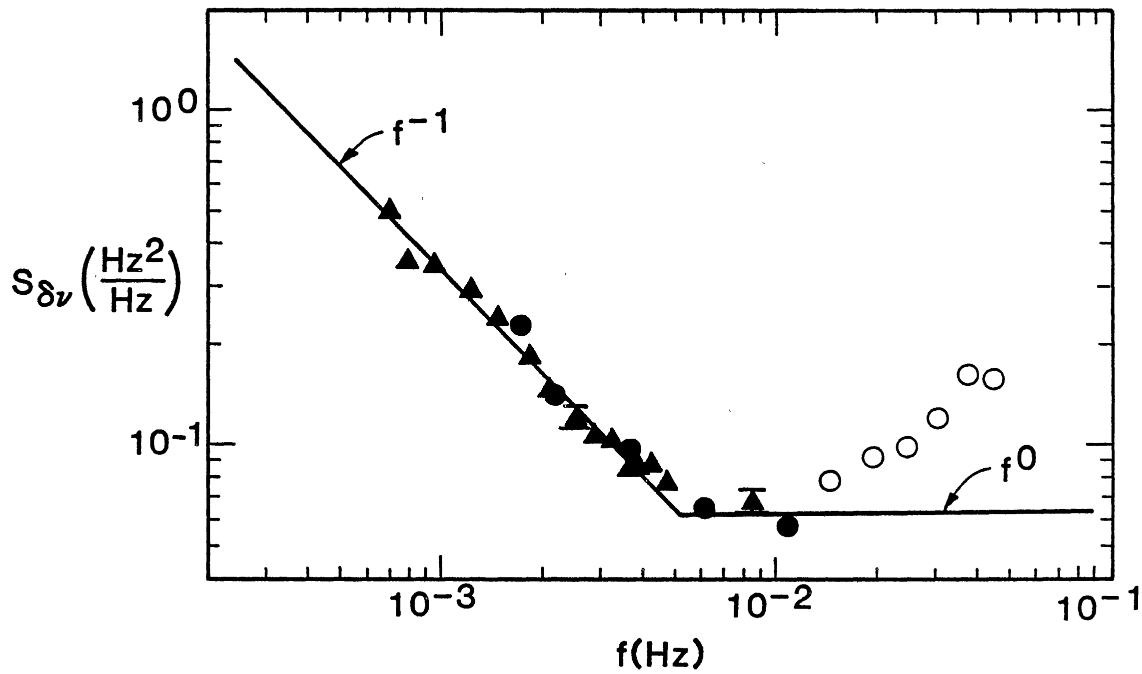


Figure 16. Power Spectral Density of Record C (Bilger, 1984). 1/f noise is shown over 1 decade of the spectrum. The open circles are due to quantization noise in the zero-crossing techniques.

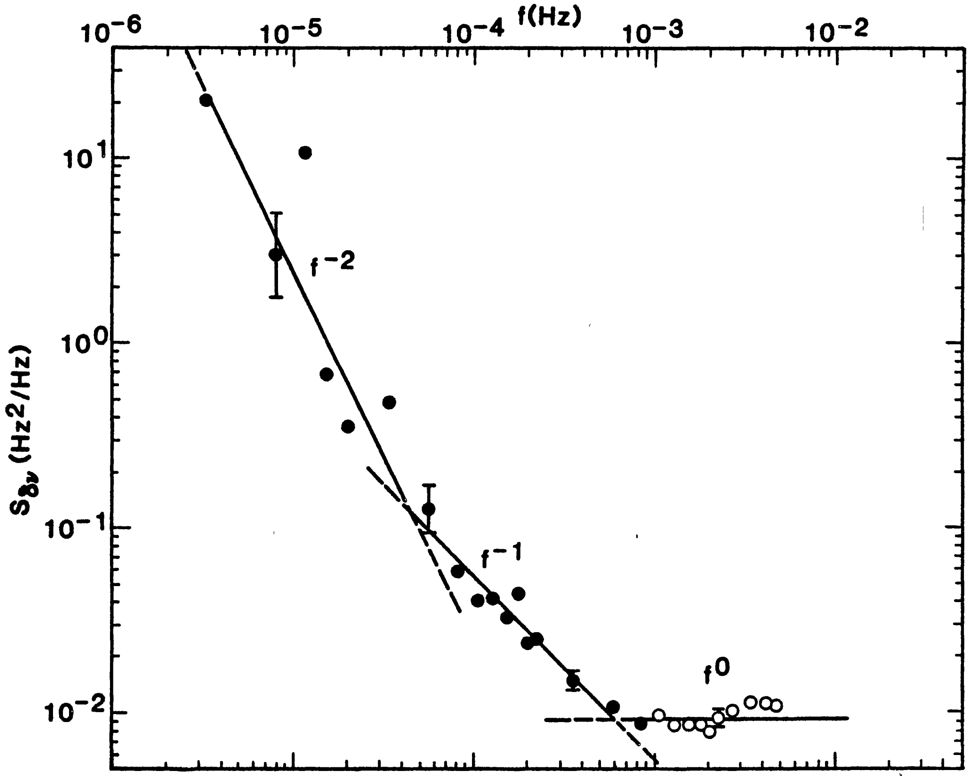


Figure 17. Power Spectral Density of Record D (Sayeh and Bilger, 1985). The white noise level is shown by open circles. The transition frequency for 1/f noise and white noise is at very low fourier frequency ($<6x10^{-4}$ Hz).

frequencies. The f noise is not shown, however, the corrected white noise is presented by open circles. The white noise level is substantially lower than that of Records A, B, or C (i.e. $h_0 = 10^{-2}$ Hz^2/Hz for Run D). The 1/f noise level is also lower than that of Records A, B, or C (i.e., $h_{-1} = 5x10^{-6}$ Hz^2 for Record D). The $1/f^2$ noise level predominates from the Fourier frequency $4x10^{-5}$ Hz down to $3x10^{-6}$ Hz over two decades of the spectrum. Let us take a close look at the transition frequencies of the white noise to the 1/f noise in these records. We note that the transition frequency (for Record D) is at a lower Fourier frequency (i.e. $6x10^{-4}$ Hz) than that of Records A, B, or C (i.e. $5x10^{-3}$ Hz). This implies that the 1/f noise contribution decreases with a faster rate than the white noise contribution when the quality factor is increased.

Let us consider the transition frequencies of 1/f noise into $1/f^2$ noise in Record D compared to Records A, B, or C. The transition frequency for Record D is at a lower frequency (i.e. $4x10^{-5}$ Hz) than that of Records A, B, and C (i.e. $5x10^{-4}$ Hz). This implies that the $1/f^2$ noise contribution decreases with a faster rate than that of 1/f noise.

In a review of the pertinent literature, we encounter Dorschner et al. (1980) on beat frequency fluctuation measurements of a four-frequency differential ring laser. They obtained the standard deviation of the beat frequency fluctuations as a stability measurement of their ring laser. They examined the beat frequency fluctuations for three different laser power levels, namely, 2.7 μW, 13 μW, and 52 μW. These were achieved in He-Ne ring laser with the passive quality factor Q = $4.6x10^8$ and the wavelength $\lambda = 633$ nm.

The variance of the beat frequency fluctuations is shown in Fig. 18. The white noise shows $1/\tau$ dependence in the variance plot. 1/f noise level, in general, diverges as the number of points in the cluster increases. The white noise predominates up to the sample time τ = 10 s. The slopes level off, however after the sample time τ = 10 s which may suggest the presence of 1/f noise. With this assumption at hand, we would like to estimate the 1/f noise level in this plot (Fig. 18). The variance σ^2 relates to the 1/f noise level, h_{-1} as

$$\sigma^2 = (2h_{-1} \; N \ln)/(N - 1) \tag{4.12}$$

where N is the number of points in the cluster. We estimated the number of points in each cluster, in Fig. 18, to be

$$N_n = N_o/\Delta \tag{4.13}$$

where Δ depends on the nature of clustering, in this case $\Delta = 2$, N_0 is the number of points in the first cluster, and n is the cluster number (i.e. n = 0, 1, 2, ...). In Fig. 18, we have $N_0 = \frac{200}{0.25} = 800$, thus

$$\sigma^2 \simeq 2h_{-1}[\ln 800 - (\ln 10) \log \tau] \tag{4.14}$$

where $\frac{N}{N - 1} = 1$. In order to estimate the 1/f noise level h_{-1}, we use the nonlinear least-square fit program to fit the following function through the data points shown in Fig. 18.

$$\eta = \log \left[\frac{h_o}{2} 10^{-\xi} + 2h_{-1} \; (\ln 800 - (\ln 10)\xi)\right] \tag{4.15}$$

where $\eta = \log \sigma^2$ and $\xi = \log \tau$.

The estimated noise levels are shown by solid lines in Fig. 18. The error of estimating 1/f noise level was large for the 2.7 μW data.

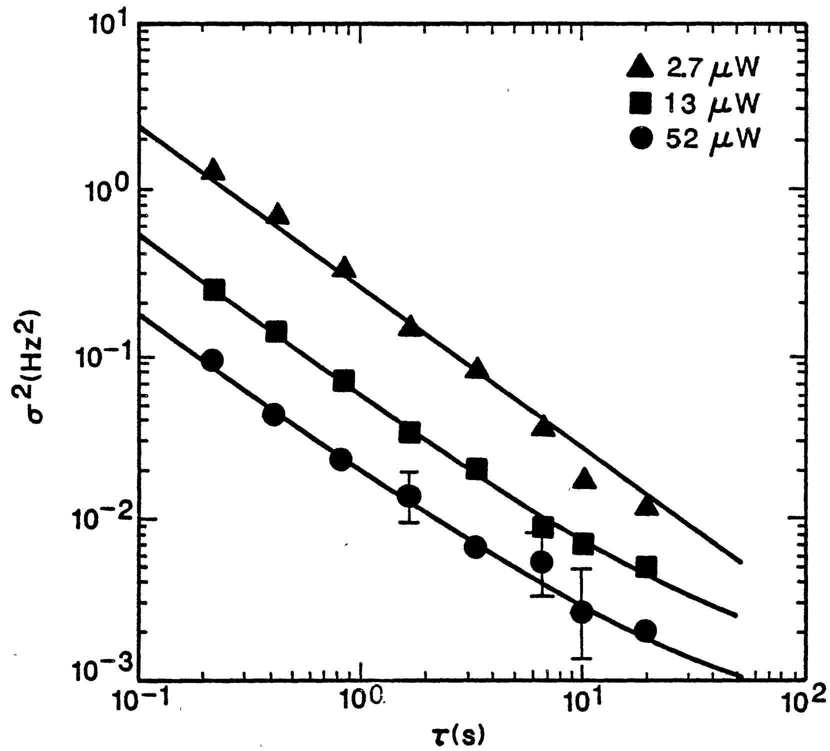


Figure 18. Variance of Beat Frequency for Different Power Loss Levels (Dorschner et al., 1980). The solid lines are least square fit through the data.

We decided that the 1/f noise is masked by white noise for this power level. The 1/f noise levels, in Fig. 18, show very small dependence on power. The transition corner of white noise into 1/f noise is moved toward higher sampling time for lower powers. This implies again (see above) that the power dependence of 1/f noise is weaker than that of the white noise.

4.2.2.2 Two-Frequency Ring Lasers

Let us examine the frequency fluctuations in two-frequency ring lasers. A two-frequency ring laser is similar to a four-frequency differential ring laser, except that there are no polarization-dependent elements inside the cavity. This reduces losses in the cavity, therefore the higher quality can be reached.

The clockwise beam is combined with the counterclockwise one to produce moving fringes at the detector. The output of the detector then gives the beat frequency signal. The zero-crossing technique is used to count the number of crossings from a negative to a positive value. The ring laser temperature was not controlled by any means of control loops. The passive quality factor was about 10^{10}, the power loss per mode was 100 μW, and the beam wavelength was 633 nm of He-Ne laser. Record E, in Table II, is achieved in this two-frequency ring laser.

The power spectral density of the beat frequency fluctuations is depected in Fig. 19. 1/f noise predominates from the Fourier frequency 10^{-5} Hz down to the lowest Fourier frequency over one decade of the spectrum. White noise prevails from Fourier frequency 10^{-5} Hz to the highest frequency.

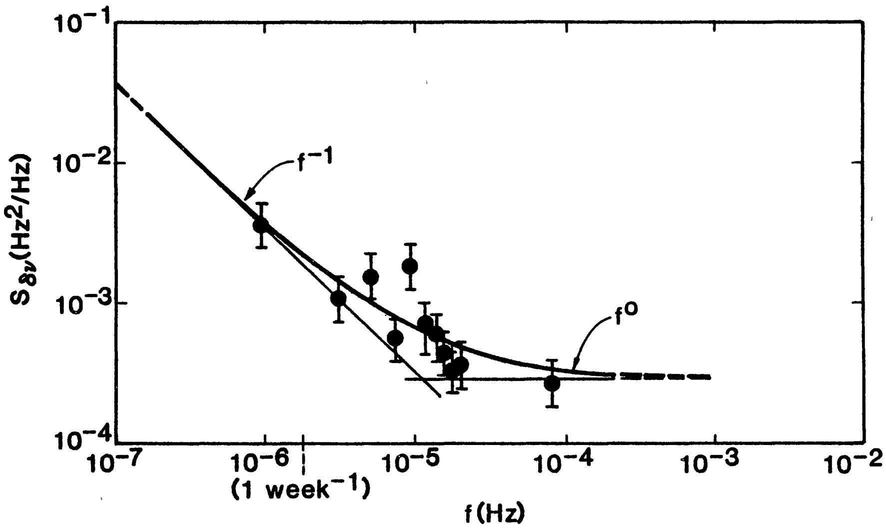


Figure 19. Power Spectral Density of Record E (Bilger and Sayeh, 1985). These points are fitted by white noise and 1/f noise. The 1/f noise and white noise are not very well documented. However, This shows an upper limit for 1/f noise.

Let us compare the transition corner frequencies of 1/f noise to white noise in Record E to Record D. The transition corner frequency for Record E is at substantially lower Fourier frequency (i.e. 10^{-5} Hz) than that of Record D (i.e., $6x10^{-4}$ Hz). This incredibly low 1/f noise level ($h_{-1} = 2.5x10^{-9}$ Hz^2) is a very strong support of the idea that the 1/f noise level is Q-dependent.

4.2.3 Summary

In Sections 4.2.2.1 and 4.2.2.2 we presented various experimental evidences showing 1/f noise in frequency fluctuation of ring lasers. Records A, B, and C were in a thermostat with $3X10^{-7}$ short-term temperature stability. Record D was not placed in a temperature-controlled thermostat. However, Record D shows a lower 1/f noise level which means the 1/f noise level cannot be affected by the environmental temperature variations.

1/f noise predominates (in presence of white noise and $1/f^2$ noise) over one decade of the power spectral density in most experimental evidences. Presence of white noise and $1/f^2$ noise in the spectrum makes us to think about the idea that the 1/f noise in the spectrum is actually the transition of white noise to $1/f^2$ noise.

In order to decline that, the spectrum is fitted by two different sets of noises. One set contains $1/f^2$ noise and white noise only. The other set contains $1/f^2$ noise, 1/f noise and white noise. The RMS residual error in latter is found to be smaller in all cases. This therefore shows the presence of 1/f noise in the spectrum.

The 1/f noise levels are summarized in Fig. 20. This shows the measured values of h_{-1} per mode, in Eq. (4.12), versus the passive

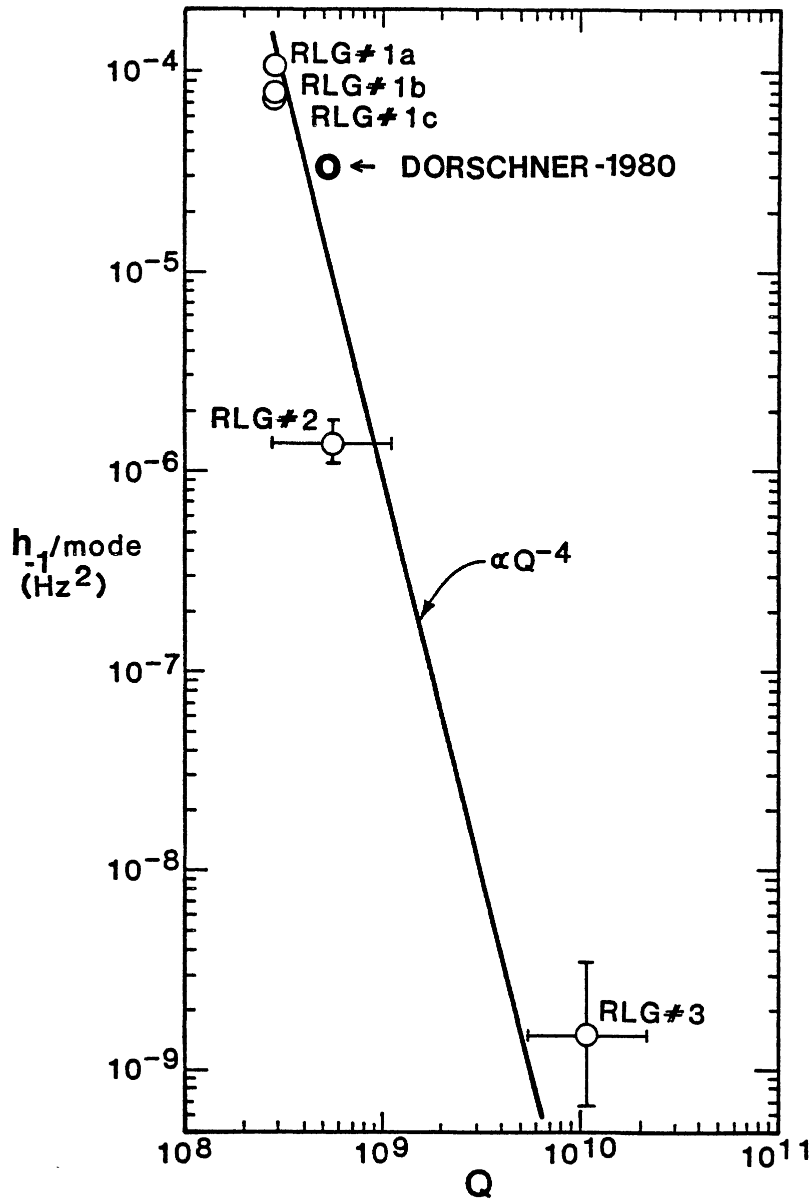


Figure 20. 1/f Noise Level vs. Passive Quality Factor (Sayeh and Bilger, 1985). The 1/f noise level (in Dorschner et al., 1980) is well within the fitted line.

quality factor Q for different types of ring lasers, namely, four-frequency differential ring lasers and two-frequency ring lasers. The 1/f noise level which was estimated from the variance of frequency fluctuations given by Dorschner et al. (1980), is shown in Fig. 20. These measured values of h_{-1}, excluding that of Dorschner, are fitted by a line in log-log plot. This shows a Q^{-4} dependence of the 1/f noise level with $h_{-1} = 9\times10^{29}\ Q^{-4}\ Hz^2$. The point which was estimated from the variance is well within the fitted line.

Generally we can summarize the power spectral density of freqnecy fluctuations in lasers as follows

$$S_{\delta\nu}(f) = h_{-2}f^{-2} + h_{-1}f^{-1} + h_o \qquad (4.20)$$

which includes $1/f^2$ noise, 1/f noise, and white noise. The magnitude of these coefficients will be examined in Chapter V and VI. $1/f^2$ noise is not discussed in this work.

CHAPTER V

THEORETICAL CONSIDERATIONS

In this chapter we consider theoretical models to explain the fundamental frequency fluctuations (white noise and 1/f noise in frequency) in lasers. Here a laser system is regarded as an oscillator with well-stabilized amplitude, but with frequency (or phase) fluctuations.

The frequency fluctuations in the beat frequency represent the difference between the instantaneous frequency fluctuations of the two (or more) laser beams. Hence, if the fluctuations of the two (or more) beams are statistically independent, the mean-square fluctuations (in time or in Fourier frequency domain) of the beat frequency are essentially twice (or more times) as large as the mean-square fluctuations in each beam independently. We therefore divide the power spectral density of the beat frequency by the number of beams involved to estimate the spectrum per mode of the laser frequency fluctuations.

The concept of frequency linewidth was introduced in Section 2.2.5. In early investigations of frequency fluctuations, the frequency linewidth was used (for example, Javan et al., 1962). We will use the relations between the frequency linewidth and the power spectral density of frequency fluctuation, which were derived in Section 2.2.5, later in this chapter.

Generally it has been observed that the power spectral density of frequency fluctuations of lasers obeys the following relation

$$S_{\delta\nu}(f) = h_{-2}f^{-2} + h_{-1}f^{-1} + h_0 \tag{5.1}$$

where h_{-2}, h_{-1}, and h_0 are constants corresponding to $1/f^2$ noise, flicker noise, and white noise levels, respectively. The collected data, as have seen in Chapter IV, from different types of ring laser show that 1/f noise is occuring in most cases. Now, in this chapter, we review the existing theories for white noise level h_0 and discuss models of 1/f noise level h_{-1}.

5.1 Quantum White Noise: Spontaneous Emission

Most electro-optic phenomena associated with laser theory such as stimulated emission, reaction of the emitted field on atoms, and so on, do not require the quantization of the field for their explaination (Sargent et al., 1974). These processes can all be quantitatively explained and physically understood in terms of the semiclassical theory of the matter-field interaction in which the field is treated classically while the atoms obey the laws of quantum mechanics. However, the quantized field is fundamentally required for accurate descriptions of certain processes involving fluctuations in the electromagnetic field, e.g. spontaneous emission.

In 1927 Dirac (1958) quantized the radiation field. This is one of the most fundamental consequences of quantum theory since it allows a unification of the particle and wave properties of light.

Consider a one-dimensional cavity of length L that has perfectly reflecting mirrors. We take the electric field E and magnetic field H to be polarized in the $\hat{x}$ and $\hat{y}$ directions, respectively, namely

$$E = \alpha q(t)\ \sin(kz)\hat{x} \tag{5.2}$$

$$H = \beta p(t)\ \cos(kz)\hat{y} \tag{5.3}$$

where q(t) and p(t) are time-varying quantities, α and β are constants, and k is the wavenumber. We see that the electric and magnetic field act as position and momentum coordinates. The corresponding energy in the cavity is given by the integral over volume of the electric and magnetic field densities as follows

$$W = \frac{1}{2}\int(\epsilon_0 E^2 + \mu_0 H^2)dvol = \frac{1}{2}\ (p^2 + \omega_0^2 q^2) \tag{5.4}$$

which is just the energy of a simple harmonic oscillator for a particle oscillating with frequency ω_o . To quantize the field we treat the electric field as position operator q and the magnetic field as momentum operator p according to the laws of quantum mechanics. We request the commutation relation

$$[p,q] = i\hbar \tag{5.5}$$

where $\hbar$ = h/2 (h is Planck's constant). Our single-mode field is then described by the quantum-mechanical wave function

$$\Psi(q,t) = \sum_{n=0}^{\infty} c_n(t)\varphi_n(q) \tag{5.6}$$

where $|c_n|^2$ is the probability that the radiation oscillator is excited to the nth energy state characterized by the eigenfunction $\varphi_n(q)$ and having energy $\hbar\omega_o(n + 1/2)$. This nth quantum state is called n-photon state. The first thing to note about the quantized field is that it has fluctuation even in the absence of photons. In fact, denoting the vacuum state (zero photon) by $|0>$, we find the Hamiltonian in eq. (5.4) has (1/2) $\hbar\omega_o$ expectation value, namely

$$< 0|W|0 > = \frac{1}{2}\hbar\omega. \qquad (5.7)$$

and the electric field has zero expectation, namely

$$< 0|E|0 > = 0 \qquad (5.8)$$

However, the vacuum average of the field squared is

$$< 0|E^2|0 > = E_0^{\;2} \sin(kz). \qquad (5.9)$$

Thus the field has fluctuations about a zero mean in the vacuum. These vacuum fluctuations stimulate the atom to emit photons spontaneously, i.e. we have spontaneous emission. Therefore, the frequency of the laser is affected by these fluctuations.

5.1.1 Linear Oscillator

A laser oscillator can be represented by an RLC circuit (Yariv, 1975) as shown in Fig. 21. The presence of a laser medium with negative loss (i.e., gain) is explained by including a negative conductance $-G_s$ while ordinary loss mechanisms are represented by the positive conductance, G_t. The noise associated with the loss G_t is

$$<i_s^{\;2}> = 4h\nu\Delta\nu N_2 \qquad (5.10)$$

where h is Planck's constant, k is Boltzmann constant, and T is the

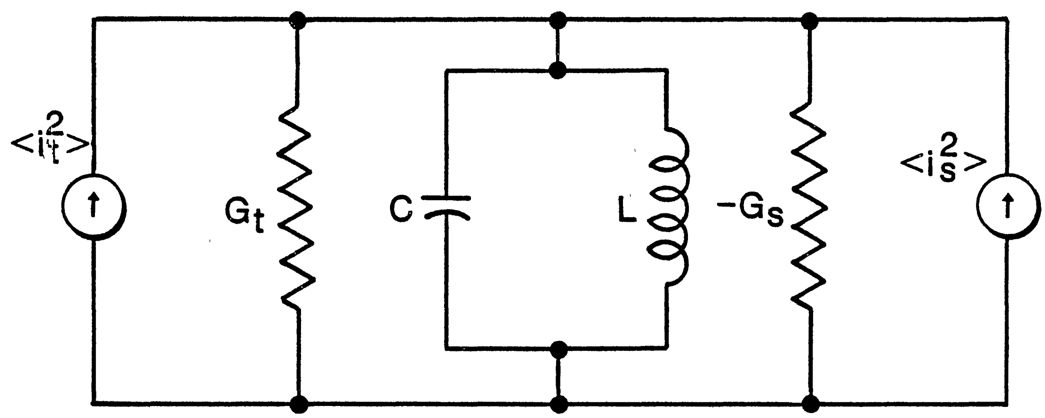


Figure 21. RLC Circuit of Laser Oscillator. The two noise sources are due to thermal noise and spontaneous emission noise. L and C define the nominal laser frequency.

temperature in Kelvin. Spontaneous emission is represented by a noise source as

$$\Delta\nu = 2\pi h\nu_0{}^3/(Q^2P) \tag{5.11}$$

where N_2 is the population of atoms in upper energy level. This shows that the spontaneous emission noise relates directly to the population of atoms in upper energy level.

It is easy to show that the frequency linewidth of this linear oscillator model is given by

$$\Delta\nu = 2\pi h\nu_0{}^3/(Q^2P). \tag{5.12}$$

where Q is the passive quality factor and P is the power loss. Using Eq. (2.26) we can find the white noise level as

$$h_0 = 2h\ \nu_0{}^3/(Q^2P). \tag{5.13}$$

This was originally obtained by Schawlow and Townes (1958).

5.1.2 Driven Van der Pol Oscillator

It was originally proposed by Lamb (1964) that the laser oscillation can be modelled by using a Van der Pol oscillator.

Let us consider a noise-driven Van der Pol oscillator as a model for a laser oscillator (Yariv, 1975):

$$\ddot{x} + [r - (g - \gamma x^2)]\dot{x} + \omega_0^2 x = N(t) \tag{5.14}$$

where x is the mode amplitude, r is the energy decay rate, g is the unsaturated gain, γ is the saturation parameter, and N(t) is the noise source due to the spontaneous emission processes. The mode amplitude x is chosen so that x^2 is the mode energy. We write x(t) as

$$x(t) = x_0 \cos \omega_0 t + C(t) \cos \omega_0 t + S(t) \sin \omega_0 t \tag{5.15}$$

where x_0 denotes the magnitude of the coherent term ($x_0{}^2 = \frac{4}{\gamma}(g-r)$), and C(t) and S(t) are slow varying fluctuation amplitudes. The random force N(t) can be written as

$$N(t) = N_c(t) \cos \omega_0 t + N_s(t) \sin \omega_0 t \tag{5.16}$$

where $N_c(t)$ and $N_s(t)$ are slow varying fluctuation amplitudes. If the laser is sufficiently above threshold, we can make the assumption that (r, g, and γx^2) $\ll \omega_0$ and also $x_0{}^2 \gg$ (C^2 and S^2). Using Eqs. (5.15) and (5.16) and the mentioned assumptions, we find

$$\frac{dS(t)}{dt} = \frac{N_c(t)}{2\omega_0} \tag{5.17}$$

We then use Eq. (5.17) to obtain the one-sided power spectral density of the instantaneous frequency fluctuations of the laser radiation. Under conditions $x_0{}^2 \gg$ (C^2 and S^2), Eq. (5.15) can be written as

$$x(t) \simeq [x_0 + C(t)] \cos [\omega_0 t + \phi(t)] \tag{5.18}$$

where $\phi(t) \simeq -\frac{S(t)}{x_0}$. Therefore, the instantaneous frequency $\nu(t)$ is

$$\nu(t) = \frac{1}{2\pi} [\omega_0 - \frac{ds}{dt} \quad \frac{1}{x_0}]. \tag{5.19}$$

Using Eq. (5.17) the frequency fluctuation $\delta\nu$ is

$$\delta\nu = \nu(t) - \nu_0 = \frac{N_c(t)}{2\pi\omega_0 x_0} \tag{5.20}$$

where $\nu_0 = \frac{\omega_0}{2\pi}$. It is easy to show that PSD of $N_c(t)$ is

$$S_{N_c}(f) = \frac{4r\omega_0^3 h}{\pi}\left[\frac{N_2 g(\nu + \nu_0)}{(N_2 - N_1 g_2/g_1)} + n_{th}\right] \qquad (5.21)$$

where f is the Fourier frequency, N_1 and N_2 are the populations of the lower and upper laser levels respectively, g_1 and g_2 are the level degeneracies, g(.) is the transition lineshape and n_{th} is the number of thermally emitted photons. The PSD of the frequency fluctuations $\delta\nu$, $S_{\delta\nu}(f)$ is obtained by using Eqs. (5.20) and (5.21) as

$$S_{\delta\nu}(f) = \frac{\gamma r h f_0}{8\pi^2(g - r)}\left[\frac{N_2 g(\nu + \nu_0)}{(N_2 - N_1 g_2/g_1)} + n_{th}\right] \qquad (5.22)$$

Some of the parameters in Eq. (5.22) are almost impossible to measure accurately, for example quantity (g-r). However, we can avoid this difficulty by introducing the passive quality factor Q and the power loss per mode. Under limiting conditions ($N_2 >> N_1$, $f << \nu_0/Q$, $h\nu_0/kT >> 1$), Eq. (5.22) converges to

$$S_{\delta\nu}(f) = \frac{h\nu_0\gamma r}{8\pi^2(g - r)} \qquad (5.23)$$

The power loss per mode P is given by

$$P = r x_0^2/2$$

where x_0 is derived by using Poincare's method (Andronov et al., 1966) as

$$x_0^2 = 4(g - r)/\gamma$$

thus,

$$P = 2r(g - r)/\gamma.$$

The passive quality factor is

$$Q = \omega_o/r \ .$$

Equation (5.23) can be written (by using above equations) as

$$S_{\delta\nu}(f) = \frac{h\nu_o^2}{Q^2 P}$$

Here the frequency fluctuations $\delta\nu(t)$ due to quantum noise gives an approximate white-noise level h_o which has been verified by experiment (Manes et al., 1971). The quantum noise level is also verified in this work. This is discussed in Chapter VI.

5.2 Quantum 1/f Noise: Loss Fluctuations

In Chapter IV, we discussed the experimental aspects of the frequency fluctuations in lasers. The Q^{-4} dependence of the 1/f noise level was summarized in Fig. 20. Here we will give a model which describes this dependence. To this end, the Van der Pol model, which was used in Sec. 5.1.2, will be examined with a different noise source, namely loss fluctuations. Then we will consider a driven (forced) linear oscillator as a mode of laser oscillation. In this model, we assume that the loss fluctuations manifest themselves in a fluctuation of the damping coefficient (energy decay rate r). These models are then compared with the experimental results (in Chapter VI).

5.2.1 Van der Pol Oscillator

Let us assume that there exist fluctuations in the loss r, in Eq. (5.14), independent of the existence of white noise, i.e. for the purpose of this derivation N(t) in Eq. (5.14) is set to zero. Now we like to find the power spectral density of the frequency fluctuations due to loss fluctuations.

We first establish an approximate solution to Eq. (5.14) ($N(t) = 0$) for small $(g-r)/\omega_o$ using Poincare's method (Andronov et al., 1966):

$$x(t) \simeq 2 \left(\frac{g - r}{\gamma}\right)^{1/2} \cos (\xi \omega_0 t) \tag{5.24}$$

where $\xi = 1 - (g-r)^2/16\,\omega^2$. The resonance frequency $\omega = \xi \omega_o$ therefore depends on the loss r. g is assumed to be a constant with respect to time. The loss fluctuation δr can be related to the frequency fluctuation $\delta \nu$ as follows

$$\delta\nu/\nu_0 = \frac{g - r}{8\omega_0 Q} \frac{\delta r}{r} \tag{5.25}$$

The power spectral density of frequency fluctuations can be related to that of fractional loss fluctuations by

$$S_{\delta\nu}(f) = \left[\frac{\nu_0 (g - r)}{8\omega_0 Q}\right]^2 S_{\delta r/r}(f). \tag{5.26}$$

This model will give the Q^{-4} dependency of the frequency fluctuation spectrum if we assume $g = 2r$, namely

$$S_{\delta\nu}(f) = \frac{1}{16} \nu_0^2 Q^{-4} S_{\delta r/r}(f) \tag{5.27}$$

This means that the gain of the system has to be twice of the loss, In Van der Pol oscillator, the gain g is always larger than the loss r. The assumption $g = 2r$ can therefore be viable in laser systems.

The power spectral density of fractional loss fluctuations is discussed in Section 5.3.

5.2.2 Driven Linear Oscillator

A driven linear oscillator can be used to model the laser oscillator. The passive cavity, consisting of mirrors, can be viewed as a damped linear oscillator given by the following differential equation

$$\ddot{x}(t) + r\dot{x}(t) + \omega_0^2 x(t) = 0 \qquad (5.28)$$

where r represents losses. The driving force $F_s(t)$ originates from stimulated atoms giving rise to a sinusoidal signal given by

$$F_s(t) = A \cos \omega_s t \qquad (5.29)$$

where A is the driving amplitude and ω_s is the angular frequency of the driven oscillator. Consider x(t) as the amplitude of the laser oscillator with

$$\ddot{x}(t) + r\dot{x}(t) + \omega_0^2 x(t) = A \cos \omega_s t \qquad (5.30)$$

It is easy to show that x(t), in steady state, is given by

$$x(t) = x_0 \cos \omega_s t \qquad (5.31)$$

where x_0 depends on ω_s. In order to maximize the value for x_0, the angular frequency of the driving oscillator ω_m is given as

$$\omega_m^2 = \omega_0^2 - r^2/2 \qquad (5.32)$$

Now Eq. (5.32) can be used to derive the power spectral density of $\delta\nu$ given the power spectral density of δr.

The loss fluctuation δr can be related to the frequency fluctuation $\delta\nu$ as follows

$$\delta\nu/\nu_0 = -\frac{1}{2} Q^{-2} \delta r/r \qquad (5.33)$$

Equation (5.33) gives rise to the following power spectral density

$$S_{\delta\nu}(f) = \frac{1}{4}\ \nu_o^2 Q^{-4} \delta r/r \tag{5.34}$$

Equation (5.34) gives the power spectral density level which is larger than that of Eq. (5.27) by a factor of 4. The numerical factor can be identical assuming larger gain g in the Van der Pol oscillator. Equation (5.27) and (5.34) can be written as

$$S_{\delta\nu}(t) = (\tfrac{1}{4})^{\mu}\ \nu_o^2 Q^{-4}\ \delta r/r \tag{5.35}$$

where μ is 2 if the Van der Pol oscillator is used, and is 1 if the driven linear oscillator is used. In the following section we will discuss the spectrum of fractional loss fluctuations.

5.2.3 Spectrum of Fractional Loss Fluctuations

In Section 5.2.2 we showed that the loss fluctuation δr would give a Q^{-4} dependence of the power spectral density $S_{\delta\nu}(f)$ in Eq. (5.35). It is advantageous to note that the proportionality to Q^{-4} is independent of the specific assumptions on the types of loss fluctuations. Loss fluctuations may originate from losses through the mirrors, coupling between modes, or scattering at mirrors or at atoms.

Here we examine a new theory of 1/f noise given by Handel (1980). Using this theory, one would expect loss fluctuations to originate from loss processes inside the cavity whose elementary cross sections of interaction with the field fluctuate with a 1/f spectrum.

Briefly, Handel's quantum theory of 1/f noise states that the interference between the part of the particle's wave function which suffers losses due to bremsstrahlung scattering under the emission of infraquanta and the part of the wave function which does not suffer losses produce the quantum 1/f noise.

In light of the theory of Handel, the power spectral density of fractional scattering cross sections is given by

$$S_{\delta\sigma/\sigma}(f) = \frac{B}{f} \qquad (5.36)$$

where B is given by (Handel, 1980)

$$B = 4\alpha\Delta v^2/(3\pi c^2) \qquad (5.37)$$

for particles whose bremsstrahlung scattering is electromagnetic radiations. For those particles whose bremsstrahlung scattering is graviton, B is given by (Handel, 1985)

$$B = 16Gm^2v^4/(5hc^5) \qquad (5.38)$$

The definition of above quantities is as follows:

α = fine-structure constant

ΔV = velocity change of particles in scattering process

c = speed of ligh in vacuum

G = gravitational constant

m = reduced mass of two particles

v = relative velocity of two particles in scattering process

In the scattering processes, losses r are directly proportional to the scattering cross sections, namely

$$\delta\sigma/\sigma = \delta r/r \qquad (5.39)$$

The power spectral density of fractional loss fluctuations therefore can be written as

$$S_{\delta r/r}(f) = \frac{B}{f} \qquad (5.40)$$

Using Eq's. (5.40) and (5.35), the power spectral density of the frequency fluctuations can be written as

$$S_{\delta\nu}(f) = [(\frac{1}{4})^{\mu} B \nu_o^2 Q^{-4}]/f. \tag{5.41}$$

This gives the 1/f noise level h_{-1} as follows

$$h_{-1} = [(\frac{1}{4})^{\mu} B \nu_o^2 Q^{-4}] \tag{5.42}$$

Equation (5.42) is closely examined in the light of the experimental evidences in Chapter VI.

CHAPTER VI

COMPARISON WITH THE EXPERIMENT

The quantum white noise in a laser field was originally predicted by Schawlow and Townes (1958). The frequency linewidth of this noise discussed in Section 5.1.1. This linewidth shows very small frequency fluctuations compared to the frequency of the laser field (e.g., frequency stability of one part in 10^{14}). The frequency stability of lasers were then considered the lowest among the known oscillators. This led the researchers to pursue experimental results revealing this small frequency linewidth. We discussed some of the aspects of these experimental results in Chapter IV.

Here we will only compare the experimental results from ring lasers (in Section 4.2.1) with the theories given in Chapter V.

6.1 White Noise Level, h_0

The quantum white noise level h_0 was derived in Equation (5.13) using a linear oscillator and in Equation (5.23) using a Van der Pol oscillator. Equation (5.13) is refered to as Schawlow and Townes prediction which is a factor of 2 larger than that of Equation (5.23).

Dorschner et al., (1980) derived an equation for the frequency linewidth by using the uncertainty relation. Their relation for the frequency linewidth was equivalent to Equation (5.23). This suggests that Equation (5.23) is a better relation compared to Equation (5.13).

We therefore base our comparison between the experimental results and the Van der Pol model on Equation (5.23).

The measured white noise levels are given as

h_0 = masked, see Figure 13,

h_0 = masked, see Figure 15,

$h_0 \simeq 0.06$ H^2z/Hz, see Figure 16,

$h_0 \simeq 0.009$ Hz^2/Hz see Figure 17, and

$h_0 \simeq 0.0004$ Hz^2/Hz see Figure 19.

In Figures 13 and 15, the white noise is masked by 1/f noise up to the Nyquist frequency. It is therefore impossible to extract the white noise level in these records. However, in Figure 16, the sampling time is shorter which pushes the Nyquist frequency to higher frequensies to reveal the white noise. In this measurement the white noise level is h_0 = 0.063 Hz^2/Hz. The theoretical white noise level, in Equation (5.23), gives the value of 0.04 Hz^2/Hz. Here, the difference is explained by the fact that the atomic population of the upper level N_2 was not very much greater than that of the lower level N_1. This means the ratio $N_2/(N_2 = N_1 g_2/g_1)$ is not close to unity. This usually happens when the laser operates in low power.

In Figure 17, the white noise level is h_0 = 0.009 Hz^2/Hz. The theoretical value is 0.01 Hz^2/Hz which is slightly larger than that of measured value. This may be due to the quantization error correction.

In Figure 19, the white noise level is h_0 = 0.004 Hz^2/Hz whereas the theoretical value predicts a lower value of 0.00002 Hz^2/Hz. Since a two-frequency ring laser gyro was used, this relatively large value for the measured noise is due to the "dither" noise. The mechanical dither introduces white noise which depends upon the scale factor, lock-in rotation rate, and the peak amplitude of the dither. We can therefore argue that the quantum white noise limit was not reached by this two-frequency ring laser.

6.2 1/f Noise Level, h_{-1}

The 1/f noise level h_{-1} is given in Equation (5.42). Verifying Equation (5.42) can become very difficult taks. In order to change parameter in Equation (5.42), the ring laser, sometime, has to be redesigned. This may change the other parameters. For example, changing the frequency of a gas laser may demand using a totally different laser system (e.g. solid state laser). Changing the Q-parameter is less difficult compared to others. In experimental results, we examine different set of data using ring lasers with different quality factors (Q).

The measured 1/f noise level are given as

$h_{-1} \simeq 4\text{x}10^{-4}$ Hz, see Figure 13,

$h_{-1} \simeq 3\text{x}10^{-4}$ Hz, see Figure 15,

$h_{-1} \simeq 3\text{x}10^{-4}$ Hz, see Figure 16,

$h_{-1} \simeq 5\text{x}10^{-6}$ Hz, see Figure 17, and

$h_{-1} \simeq 2\text{x}10^{-9}$ Hz, see Figure 19.

The first four measured values were achieved in four-frequency differential ring lasers. The 1/f noise level per mode is therefore given by dividing those values by 4. The last measured value of the 1/f noise level was achieved in a two-frequency ring laser. In the case, h_{-1} per mode is given by dividing the value of h_{-1} by 2. Now we can plot the summary of those h_{-1} per mode versus the passive quality factor. This is shown in Figure 20. These points suggest Q^{-4}

dependence of the 1/f noise level. The experimental results shown in Figure 20 give the following relation for $\dot{h}_{-1}$, namely

$$h_{-1} \simeq \frac{4\,\nu_o^2}{Q^4} \tag{6.1}$$

Comparing Equation (6.1) to Equation (5.42), we have

$$\left(\frac{1}{4}\right)^{\mu} B \simeq 4 \tag{6.2}$$

The parameter B is related to loss fluctuations in Equations (5.37) and (5.38). If the losses produce electromagnetic radiations in the scattering process, B is given by Equation (5.37). The maximum value of B in vacuum is

$$B_m = \frac{4\alpha}{3\pi} \simeq 0.003 \tag{6.3}$$

when the velocity change is equal to the velocity of light in vacuum. This value of B is very small compared to what we expect from the experimental results. The value of B can be larger than B_m if we consider the scattering process in a medium. In this case, B is given by

$$B = \frac{4\alpha(\Delta V)^2}{3\pi v^2} \tag{6.4}$$

where v is the speed of light in medium. Here, the velocity change can indeed be larger than v which results in larger B. The velocity of light in medium, in which laser field is travelling, is very close to the speed of light in vacuum. B may therefore be very close to the value of B_m.

Now let us consider losses which produce gravitons in the scattering process. In this case, B is given by Equation (5.38). The value of B is examined with respect to the experimental results. The laser beam is considered as a beam of photons having the equivalent mass as

$$m = h\nu_o/c^2 \tag{6.5}$$

for each photon. Equation (5.28) is therefore written as

$$B = 16\ Gh\,\nu_o^2/(5c^5) \tag{6.6}$$

The calculted value of B is $13x10^{-57}$ for $\nu_o = 474x10^{12}$ Hz. This is an extremely small value for B.

These small values of B in both cases suggest that the observed 1/f noise is given by a superposition of many independent processes with 1/f noise spectrum.

Chapter VII

DISCUSSION AND CONCLUSION

All measurements in physical systems are ultimately limited by fluctuations in either the system being measured or the measuring device. Noise mechanisms limit the accuracy of precision measurements. Although many sources of noise are well understood, the origin of the 1/f noises remains, in general, not well known. There are some models for 1/f noise which describe the experimental results to some extend, however, in some cases these models give incorrect predictions of the 1/f noise level (Hooge, 1977), (Putterman, 1977), (Weissman, 1979), (Gruneis, 1984), (Larraza et al., 1985). Handel's theory of 1/f noise, which is used in this work, was originally developed to explain 1/f noise in the current fluctuation of conductors. The prediction of Handel (1980) generally gives smaller value of the 1/f noise level than what the experiments show.

In this end, we summarize the experimental results and the theoretical considerations which were discussed in Chapter IV, V, and VI. The long-term measurement of frequency fluctuations in ring lasers showed 1/f-type noises in the power spectral density. The spectrum contains three types of noise, namely, $1/f^2$ noise, 1/f noise, and white noise in most cases. Each noise has its range of Fourier frequency at which it dominates other noises. The white noise prevails at higher Fourier frequencies, the 1/f noise dominates at intermediate Fourier frequencies, and the $1/f^2$ noise shows up at lower Fourier frequencies. These noises define certain limits on frequency stability depending on whether short-term or long-term stability is considered. For example, in a ring laser which is designed to detect a rotation rate about 10^{-9} of earth rotation, we pay more attention to reducing the long-term noises (i.e., 1/f-type noises). According to the experimental results and the theoretical developements in the previous chapters, the quality factor Q is a very important factor for reducing the 1/f noise level of the frequency fluctuations. This noise level is indeed reduced quatricly in terms of the quality facotr. The Q^{-4} dependence of the 1/f noise level can be used to design some aspects of a ring laser (e.g. the necessary light reflection on mirrors, diffraction losses, etc.).

The 1/f noise level h_{-1} was formulated by Equation (5.42). The only parameter was verified, in some extent, was the quality factor Q. Figure 20 shows this Q^{-4} dependence of the 1/f noise level. This quality factor dependency is explained by loss fluctuations regardless of its type. The parameter B was evaluated in two different cases. The results are very small when the scattered beam is electromagnetic fields. The value of B is extremely small when the scattered beam is graviton. The experimental results, however, give a value of B which is orders of magnitude larger than that of the theories. This may be answered by assuming that there are several 1/f noise processes which are added to produce this large 1/f noise level.

The quantum white noise is considered as a fundamental frequency flucuations in lasers. The spontaneous emission is the origion of the white noise, therefore it cannot be due to extraneous fluctuations. The 1/f noise is also considered as a fundamental frequency fluctuation in lasers. The observed 1/f noise in series of experiments shows that the ambiant temperature is not cause of 1/f noise. According to Handel's theory of 1/f noise which based on electrodynamics and quantum mechanics, the flow of particles (changed or not changed) should present 1/f noise if this flow interacts with outside world (by loss mechanisms). The 1/f noise in laser beam is thus considered as a fundamental process which affects the output frequency of a laser.

The 1/f noise is of great practical importance, since averaging of data containing 1/f noise does not significantly reduce the noise level whereas averaging of data containing white noise over a time period reduces the noise level. The 1/f noise is thus a measure of the ulimate stability of a laser system.

In order to study frequency fluctuations in lasers, we considered ring lasers. Ring lasers are actually two lasers in the same cavity. This therefore reduces the correlated frequency fluctuations due to extaneous variations (e.g. mirror vibrations, temperature varations, etc.).

In this work, two types of ring lasers were under consideration, namely, the four-frequency differential ring lasers and the two-frequency ring lasers. The diversity of type of ring lasers lies in the problem of lock-in phenomenon. This lock-in threshold can be reduced by increasing the enclosed area where the two beams circulate and reducing the backscattering on mirrors. The formulation of the lock-in threshold is given by Chow et al. (1985). From this we can design a ring laser in which the lock-in threshold is smaller than the earth rotation rate. This ring laser would not therefore have frequency locking. In this case, the ring laser can operate without using any dither mechanisms or any polarization-dependent elements. The earth rotation rate is about $2x10^{-6}$ Hz. If the mirrors have backscattering coefficient of 0.1 Hz, a square ring laser with the perimeter of 40 m would give a lock-in threshold less than the earth rotation rate. This means that the ring laser would operate outside of the locking band. Now this ring laser may be used to study noise phenomena in lasers.

For further investigation toward modeling 1/f noise, chaos in nonlinear oscillators seems promising. Gwinn and Westervelt (1985) showed that the driven damped pendulum would give rise to chaos with the 1/f noise spectrum. They formulated the problem by a differential equation. This equation was simulated using a digital computer. For certain value of the paraters in the equations, they found the spectrum which exhibit 1/f noise.

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APPENDIX

White noise is relatively simple to simulate using an uncorrelated uniform distributed random generator. The resulting random numbers are converted to produce a Gaussian distributed random data.

$1/f^2$ noise can be generated by integrating simulated white noise. If X(k) is the white noise then simulate $1/f^2$ noise Z(k) is given as

$$Z(k) = Z(k - 1) + \Delta k \; x(x - 1) \tag{A.1}$$

where k is the time interval. Here $\Delta k = 1$ was used.

1/f noise may be simulated by taking the DFT of $1/f^2$ noise, then take the square root of the DFT components. Finally, the inverse DFT is applied. If Y(k) represents the simulated 1/f noise, then

$$Y(k) = \text{Inv. DFT} \; [\text{DFT } Z(k)]^{1/2} \; . \tag{A.2}$$

In this process we have to make sure that the inverse transform will give real numbers. Z(k) are real numbers for all k. The magnitude of DFT[Z(k)] is then an even function and the phase is an odd function. It can be shown that Y(k) are real numbers as what follows: Let us show DFT [Z(k)] by

$$\text{DFT}[Z(k)] = \text{mag DFT}[Z(k)]e^{j\theta} \tag{A.3}$$

where θ is the phase of the DFT [Z(k)]. Taking the square root of Eq. (A.3) gives

$$\mathrm{DFT}[Z(k)]^{1/2} = \mathrm{mag}\ \mathrm{DFT}[Z(k)]^{1/2}\ e^{j\Theta/2} \qquad (A.4)$$

The magnitude of Eq. (A.4) is an even function because the magnitude of DFT [Z(k)] is an even function. The phase in Eq. (A.4) is an odd function because θ is an odd function. Y(k) are therefore real numbers for all k.

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