

ANALYSIS OF INDUCTION GENERATORS  
ON UNBALANCED POWER SYSTEMS

By

MARCUS OLIVER DURHAM

Bachelor of Science  
Louisiana Tech University  
Ruston, Louisiana  
1967

Master of Engineering  
The University of Tulsa  
Tulsa, Oklahoma  
1978

Submitted to the Faculty of the Graduate College  
of the Oklahoma State University  
in partial fulfillment of the requirements  
for the Degree of  
DOCTOR OF PHILOSOPHY  
December, 1985

Thesis  
1885D  
D961a  
cop.2



ANALYSIS OF INDUCTION GENERATORS  
ON UNBALANCED POWER SYSTEMS

Thesis Adviser:

*R. Ramakumar.*

Thesis Adviser

*W. P. Fitzpatrick*

*Wm. L. Hughes*

*Jerald D. Parker*

*Norman N. Klenham*

Dean of the Graduate College

## PREFACE

A model for determining the effect of imbalance on an induction generator is demonstrated to be the equivalent circuit of an induction motor. A line balancer was developed for the power system. This works equally well for balanced or unbalanced systems and for single- or three-phase conditions. Experimental results using a three-horsepower machine demonstrate the applicability of these models.

I wish to express my sincere thanks to the people at Oklahoma State University who assisted me in this project. Special thanks goes to Dr. R. Ramakumar, my adviser, for the direction, concern, support, and encouragement he provided. Dr. William Hughes is commended for his practical suggestions, advice, and provision of equipment. I am especially grateful to Dr. James Rowland, who encouraged me and provided the direction to get me started on my doctoral program at Oklahoma State University. I also appreciate the other committee members, Dr. Dan Lingelbach and Dr. Jerald Parker, for their support of my program.

Another individual who deserves special recognition is Dr. Robert Stratten at the University of Tulsa, who encouraged me in the early days of my graduate work and who provided assistance when I needed it.

My wife, Rosemary, and my children, Robert, Christopher, Karen, and Sarah, deserve a medal for the toleration and support provided over a long period in graduate school while working and supporting a family. My wife gets special thanks for typing the many drafts of this document.

## TABLE OF CONTENTS

| Chapter  | Page |
|--|------|
| I. INTRODUCTION . . . . .  | 1    |
| 1.1 General Literature Survey . . . . .                                      | 1    |
| 1.2 Statement of Requirements . . . . .                                      | 4    |
| 1.3 Objectives . . . . .   | 6    |
| II. MACHINE ANALYSIS . . . . .   | 9    |
| 2.1 Introduction . . . . .   | 9    |
| 2.2 Induction Generator Model . . . . .                                      | 11   |
| 2.3 Asymmetrical Models . . . . .  | 15   |
| 2.4 Unbalanced Analysis of the Induction<br>Machine . . . . .                | 17   |
| III. POWER SYSTEM BALANCER . . . . .   | 23   |
| 3.1 Introduction . . . . .   | 23   |
| 3.2 Assumptions . . . . .  | 33   |
| 3.3 General Line Balancer Model . . . . .                                    | 36   |
| 3.4 Single-Phase Balancer Model . . . . .                                    | 41   |
| 3.5 Balancer Model Terminal Characteristics . . . . .                        | 41   |
| 3.6 Balancer Model Internal Design . . . . .                                 | 51   |
| 3.7 Example of Line Balancer Calculation . . . . .                           | 60   |
| 3.8 Example of Single-Phase to Balanced<br>Three-Phase Calculation . . . . . | 64   |
| IV. EXPERIMENTAL DEVELOPMENT . . . . .                                       | 69   |
| 4.1 Introduction . . . . .   | 69   |
| 4.2 Balanced System Experiments . . . . .                                    | 77   |
| 4.3 Balancing Currents in Three-Phase Lines . . . . .                        | 81   |
| 4.4 Balancing Reactive Power Flow . . . . .                                  | 83   |
| 4.5 Balancing Single-Phase to Three-Phase . . . . .                          | 85   |
| V. SUMMARY AND CONCLUSIONS . . . . .   | 94   |
| 5.1 Summary of Results and Conclusions . . . . .                             | 94   |
| 5.2 Suggestions for Further Work . . . . .                                   | 97   |
| BIBLIOGRAPHY . . . . .   | 100  |
| APPENDIX . . . . .   | 102  |

## LIST OF TABLES

| Table  | Page |
|--|------|
| I. Experimental Results . . . . .  | 73   |
| II. Three-Phase Line Balancer Verification . . . . .   | 83   |
| III. Comparison of Symmetrical Components: Calculated<br>Torque and Measured Torque . . . . .  | 86   |
| IV. Single-Phase to Three-Phase Balancer for Resis-<br>tance Load . . . . .  | 89   |
| V. Currents for One-Phase to Three-Phase Balancer<br>Used With a Three-Phase Induction Generator<br>Operating at 1247 RPM on a Single-Phase Line . . . . . | 91   |

## LIST OF FIGURES

| Figure  | Page |
|---|------|
| 1. Minimum Required Protection for Induction Machines                                     | 3    |
| 2. Induction Machine Equivalent Circuit . . . . .   | 3    |
| 3. Negative Sequence Equivalent Circuit . . . . .   | 19   |
| 4. Balancer Representation of One Phase of a Three-<br>Phase Power Line Segment . . . . . | 25   |
| 5. Balancer for One Line . . . . .  | 26   |
| 6. Balanced Three Phase From Single Phase . . . . .                                       | 28   |
| 7. Determination of Current Angles . . . . .  | 31   |
| 8. Power System Schematic . . . . .   | 34   |
| 9. Power System as Two Port Network . . . . .   | 35   |
| 10. Two Port Network Model . . . . .  | 37   |
| 11. Illustrating the Concept of Power Factor . . . . .                                    | 43   |
| 12. Three-Phase Line Balancing by Impedance Example . . . . .                             | 61   |
| 13. Single-Phase to Three-Phase Balancer Example . . . . .                                | 66   |
| 14. Autotransformers and Resistance Load Used in<br>the Experiment . . . . .              | 74   |
| 15. Capacitors Used in the Experiment . . . . .   | 74   |
| 16. Variable Speed Drive Controller . . . . .   | 75   |
| 17. Meters and Instrumentation . . . . .  | 75   |
| 18. Motor Driver Coupled to the Induction Generator . . . . .                             | 76   |
| 19. Instrumentation and Inductive Reactance Used . . . . .                                | 76   |
| 20. Experimentally Obtained Machine Performance Curves . . . . .                          | 78   |

| Figure   | Page |
|--|------|
| 21. Machine Model Verification Under Balanced<br>Conditions . . . . .                      | 79   |
| 22. One-Phase to Three-Phase Balancer Line Current<br>Variation as Speed Changes . . . . . | 93   |



## NOMENCLATURE

|                 |   |
|-----------------|---|
| $C_n, C_o, C_p$ | Capacitance of balancer   |
| $f$             | frequency   |
| $i$             | Subscript for sequence: 0 = zero, 1 = positive, 2 = negative, 3 = composite |
| $I_n, I_o, I_p$ | Current magnitude line to line for balancer                                 |
| $I_r$           | Current magnitude of rotor  |
| $I_x, I_y, I_z$ | Current magnitude line to line  |
| $M_n, M_o, M_p$ | Magnetizing inductance of balancer  |
| $M_s$           | Magnetizing inductance of stator  |
| $P_f$           | Power transferred to balancer   |
| $P_{hi}$        | Machine rotor loss for sequence, $i = 0, 1, 2$                              |
| $\pi$           | Constant 3.14159  |
| $P_m$           | Machine core losses   |
| $P_n, P_o, P_p$ | Power of balancer   |
| $P_{ri}$        | Mechanical power of rotor for sequence $i$                                  |
| $P_{si}$        | Machine copper losses for sequence $i$                                      |
| $P_{ti}$        | Power of machine terminal for sequence                                      |
| $Q_f$           | Reactive power transferred to balancer                                      |
| $Q_m$           | Reactive power of magnetizing inductance                                    |
| $Q_r$           | Reactive power of rotor leakage reactance                                   |
| $Q_s$           | Reactive power of stator winding leakage                                    |
| $Q_{ti}$        | Reactive power of machine for sequence $i$                                  |

|                                |  |
|--------------------------------|--|
| $R_{ha}, R_{hb}$               | Resistance at temperature a,b                  |
| $R_n, R_o, R_p$                | Resistance of balancer                         |
| $R_r$                          | Resistance of rotor                            |
| $R_1, R_{xy}$                  | Temporary value of resistance in balancer      |
| $S_f$                          | Apparent power at balancer terminals           |
| $S_g$                          | Apparent power within balancer                 |
| $S_n, S_o, S_p$                | Apparent power of balancer                     |
| $S_{ti}$                       | Apparent power of machine for sequence i       |
| $S_x, S_y, S_z$                | Apparent power line to line                    |
| $T_{ha}, T_{hb}$               | Temperature variable at point a,b              |
| $T_{ri}$                       | Torque of machine rotor for sequence i         |
| $V_n, V_o, V_p$                | Voltage magnitude for balancer                 |
| $V_r$                          | Voltage magnitude of rotor                     |
| $V_s$                          | Voltage magnitude of stator                    |
| $V_t$                          | Voltage magnitude of machine terminal          |
| $V_5, V_6$                     | Product of $V_n$ and $V_o$ with angles         |
| $V_7, V_8$                     | Product of $V_n$ and $V_o$ with angles         |
| $\omega_n$                     | Synchronous speed                              |
| $\omega_r$                     | Rotor speed                                    |
| $\omega_s$                     | Slip   |
| $X_n, X_o, X_p$                | Reactance of balancer                          |
| $X_1, X_4$                     | Temporary value of reactance in balancer       |
| $Y_1, Y_4$                     | Temporary value of admittance in balancer      |
| $Y_5$                          | Admittance for SLFE                            |
| $Z_n, Z_o, Z_p$                | Impedance magnitude of balancer                |
| $\delta$                       | Voltage angle difference $\delta_n - \delta_o$ |
| $\delta_n, \delta_o, \delta_p$ | Voltage angle for balancer                     |

|                                |                                |
|--------------------------------|--------------------------------|
| $\theta_n, \theta_o, \theta_p$ | Current angle for balancer     |
| $\theta_x, \theta_y, \theta_z$ | Current angle line to line     |
| $\phi_n, \phi_o, \phi_p$       | Power angle for balancer       |
| $\phi_x, \phi_y, \phi_z$       | Power angle line to line       |
| $\phi_f$                       | Angle of power into balancer   |
| $\phi_g$                       | Angle of power within balancer |

## CHAPTER I

### INTRODUCTION

#### 1.1 General Literature Survey

Historically, the term "generator" has implied a synchronous machine because of the overwhelming preponderance of this type of equipment. The National Electric Manufacturers Association has developed standards for synchronous machines but has not refined documents for induction generators (1). Synchronous generators cause ideas of complex equipment with tricky controls but yielding a quality power source. Current interest for non-utility generation has tended toward induction machines because of their operating flexibility.

Griffith and Owen (2) present an argument for use of large induction machines. They cover many of the systems problems associated with using induction generators, but do not address small machine use. Induction machines as motors are very familiar to electrical energy users. The applicability of these machines as generators has been known since the late 1800's when Tesla (3) and Danielson (4) published their early research. Use as generators has been restricted because of their inability to generate and regulate voltage. Nailen (5) addresses many of the operational problems that have restricted use of induction generators.

The interest in induction generators is keen because of the widespread familiarity with induction motors. There are large numbers of these machines available. They are easy to install and operate, and

they are very price competitive because of their commonness and utility. Large scale government and private studies (6) have demonstrated the cost effectiveness of these machines. Nailen (7) has illustrated the applicability of large scale induction generators where stable power systems exist. Durham and Ramakumar (8) proposed the use of small scale machines for cogeneration in remote areas.

The installation of an induction generator is the same as the installation of the machine as a motor. The device should be called a motor/generator since the only difference is the speed at which the shaft is driven. The application and installation of induction motors is very standardized. The National Electrical Code (NEC) provides guidance for proper procedures (9). The latest NEC, however, provides little guidance for installation of generators. The induction machine can be used as both a motor and generator in the same installation. The procedures outlined for a motor should be used. The one-line diagram in Figure 1 illustrates the application, while the equivalent circuit of the machine is shown in Figure 2.

Each of these components has a unique function. Line overcurrent protection is accomplished by a fuse or circuit breaker. The purpose is to protect the line from a short circuit in the motor/generator circuit. A disconnect means such as a mechanical switch provides manual isolation for safety purposes. The controller includes circuitry operated by sensors such as timers and remote switches. Running current protection is accomplished by overload relays. The purpose is to stop the machine if excessive current flows in either direction in the power circuit. Breedlove and Harbough provide an analysis of protection required for motor/generators (10).

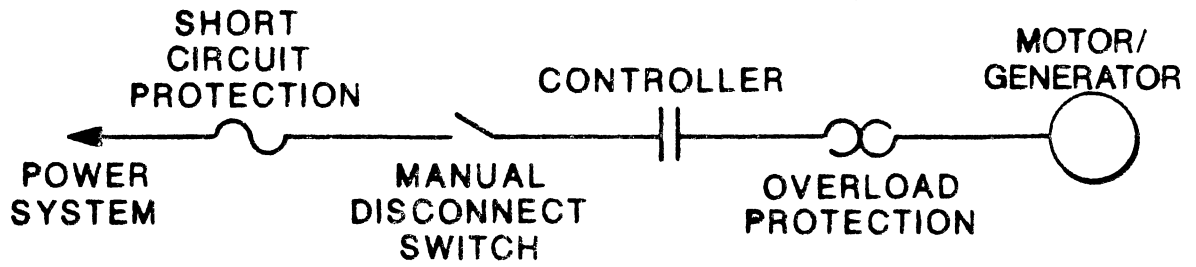


Figure 1. Minimum Required Protection for Induction Machines

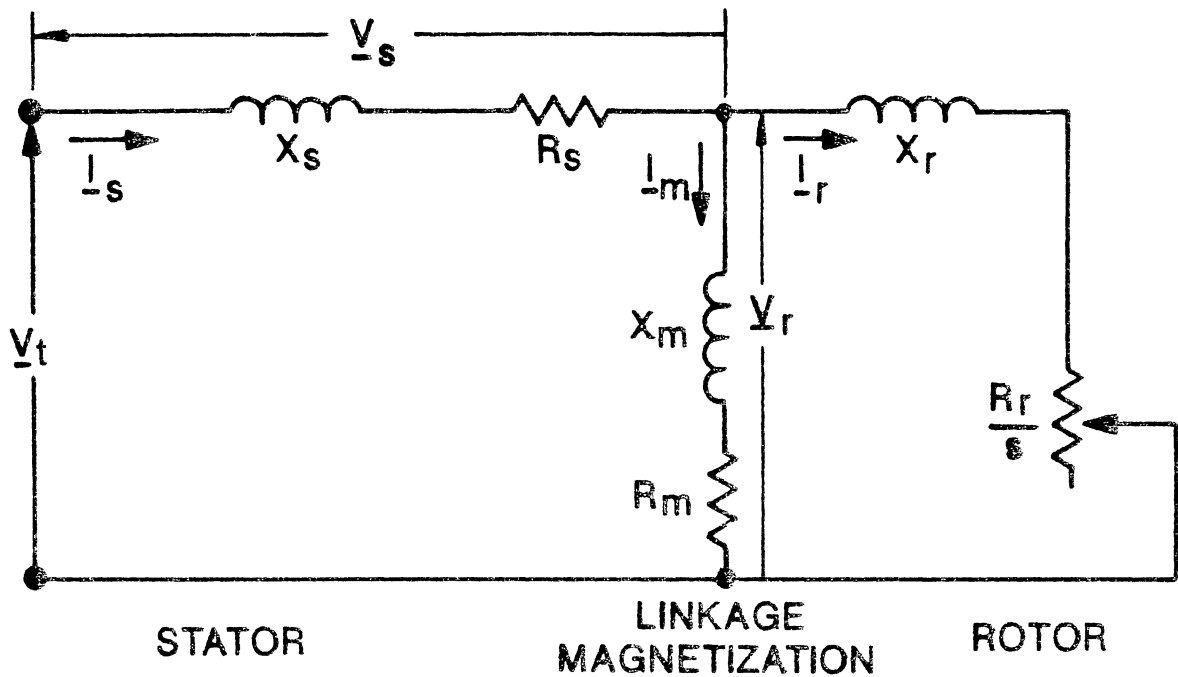


Figure 2. Induction Machine Equivalent Circuit

One additional control device is required to protect a generator--a shaft overspeed control. Excessive excitation can cause the machine to "run away" or to have excessive no load voltage (5).

## 1.2 Statement of Requirements

Induction generators are being applied in a variety of industrial applications. The size of induction generators in use ranges from small integral horsepower units to 15,000 HP machines. Operating voltage ranges are from common distribution system voltages of 480V to medium voltage applications of 13,000 volts. The American National Standards Institute has classified these voltages (11).

The source drivers for the machines vary widely--from waste heat recovery in petroleum plants, paper mills, and cement plants to prime movers such as gas engines, aeroturbines, and hydroturbines. With any drive used, the machine must be connected to a power grid for reactive excitation and for a receptor of power. Ideally, a power system is a perfectly balanced, symmetrical infinite bus. All studies found in the literature on application of induction generators make this assumption.

There are vast areas for the use of induction generators where this is not true. In many areas, balanced three-phase power is not available, particularly in remote, rural areas. These are often the areas where inexpensive or otherwise desirable sources of energy are available. The high plains of western Oklahoma provide one of the richest sources of wind energy in the country. This area is also rich in natural gas. This is often in relatively small, unmarketable quantities as a byproduct of oil production.

It is desirable to use these energy sources to generate electric power since it is one of the easiest energy forms to transport. The power grid in this area is as sparse as the population. In many cases, the only power available is single-phase residential service. In a few areas, two-wire three-phase power systems exist; however, these systems are totally unbalanced. A third type of unbalanced power system is a three-phase system created from two or three independent single-phase sources.

The induction generators in this service would be relatively small, less than 100 horsepower, compared to most induction generator applications. This size is significantly larger than available single-phase motors. The cost of three-phase motors over five horsepower is substantially less than an equivalent size single-phase motor. For these reasons, it would be desirable to use a three-phase motor as an induction generator on an unbalanced system, if possible. This research will evaluate that proposition. To accomplish this goal, the three-phase machine performance must be evaluated under unbalanced conditions.

Because the induction generator is the same machine as a motor, many things are known about the equipment and its performance. Some of the more readily available information are the performance characteristics, including the impact of speed current, power, and torque.

The equivalent circuit of the induction machine shown in Figure 2 has been used extensively. Howell and Hogwood (12) present the circuit in a very straightforward usable manner. The need for external excitation of an induction machine operating as motor or generator is apparent. Because of the need for protection of the machine, overload characteristics of the machine have been studied extensively also (10).



It is obvious the generator will continue to operate if one phase is open on the power system since the machine has an external driver. From observation, it is apparent the generator will not stall if lightly loaded and will be protected by overload relays if it is heavily loaded. Although these observations can be made, it is desirable to have a quantitative measure of the level of unbalance on the machine. This will require symmetrical component analysis of the line currents and machine power.

This study must examine the following areas:

1. The effect of unbalanced power systems on the machine.
2. The real power available from the machine.
3. A method of reducing the imbalance so reasonable performance can be obtained.

Before these analyses can be performed, several of the known parameters must be discussed. These include equivalent circuits for the machine, power models, and balancer models.

A review of the available literature on industrial cogeneration, induction generators, and unbalanced power system analysis illustrates the sparseness of information on operation of three-phase induction machines in very small installations. Literature on balancing systems is virtually nonexistent. Hughes (13) has presented some of the recent work that demonstrates power flow can be controlled between busses.

### 1.3 Objectives

There are several potential problem areas that must be solved in these systems. Ultimately on practical systems, an engine will be used to drive a three-phase induction generator. It is desirable to use the

induction machine as a motor to start the engine so that the system will be as simple as possible and investment in starting equipment will be eliminated. Starting and, for that matter, running a three-phase motor on a single-phase system cannot be done without additional equipment. Typically for motors with ratings less than 50 horsepower, phase converters have been used. These are either static devices or modified rotating machines. If the machine is not used as a starting motor, a different design technique may be used.

The machine will generate real power, but magnetizing current must be supplied from an external source. The power system itself or capacitors at machine terminals can be used to supply this. Too much capacitance will overexcite the machine and may cause it to speed up or generate excessive voltage.

Another area that must be investigated is the amount of loading that can be placed on an unbalanced three-phase machine. For a constant load, as the current is reduced in one phase, the currents in the other two phases increase. These currents must be limited to reduce the copper losses and overheating of the machine. The current levels are used to calculate the horsepower loading that the machine will experience under unbalanced power conditions.

Besides the induction generator performance, a parallel study that must be undertaken is the analysis of the power system. To accomplish this, a suitable model must be developed that is acceptable for balanced or unbalanced operation.

Fortescue (14) proposed a method for solution of unbalanced systems that has been widely used. This technique only demonstrates the problem without providing a method to correct the imbalance. Ramakumar (15)

discussed the merits of this system. Elgerd (16) has a detailed discussion of this technique with other methods of analyzing balanced system performance. Since users will be conducting this analysis, much of the information about the power company will not be available. The model has to be developed from simple parameters that are readily measurable, such as voltages and currents.

It is assumed the machine performance on an unbalanced system is probably not acceptable. A method of balancing the system is desirable. There are several constraints that should be placed on such a balancer. The major one is resistive components should not be added since they consume real power. A secondary consideration is passive devices are preferred since the equipment would be simple to install and operate. To satisfy these critical needs, an acceptable model for a power system balancer is required.

## CHAPTER II

### MACHINE ANALYSIS

#### 2.1 Introduction

The equivalent circuit of an induction motor is well known. The Institute of Electrical and Electronics Engineers (17) has developed a standard model for use in testing motors. Relatively less has been written and discussed about the induction generator.

An induction generator is an induction motor that has its shaft driven above synchronous speed. It is reasonable to expect the model of the motor to be acceptable for a generator. This is the case, but with some restrictions. The reactance values are different for performance as a motor and as a generator. Typically, motor reactance values are approximately ten percent greater than for the same machine operating as a generator. Torque values also change in the same percentage range. The other parameter that obviously changes is the shaft speed.

Since much has been written about the motor equivalent circuit, only a brief overview of the shaft speed and power performance will be given. More detailed information is available from many sources. One of the more readable is Howell and Hogwood (12).

Balanced three-phase machines can be analyzed on a per-phase basis. The equivalent circuit model is shown in Figure 2. The rotor is electrically isolated from the stator and the power source. The rotor acts as

the secondary winding of a transformer. For an induction machine, the rotor winding is shorted. This would cause very large currents and excessive heat on a stationary transformer, but a motor permits the secondary winding to move or rotate. The induction machine, acting as a transformer with rotating secondary, converts the electrical energy on the primary side to mechanical energy on the secondary side.

Slip is a dimensionless or per-unit parameter. It is equal to the difference between synchronous speed and rotor speed divided by synchronous speed:

$$s = \frac{\omega_n - \omega_r}{\omega_n} \quad (2.1)$$

The synchronous speed of an induction machine is determined by the design of the machine and the line frequency:

$$\omega_n = \frac{120 f}{\text{Number of Poles}} \quad (2.2)$$

Under steady-state operating conditions, the resistance and reactance of windings are constant with a constant voltage and frequency supply. The only parameter in the model that can change is slip. The mechanical power developed in the rotor shaft is a result of the rotor slip.

The total effective rotor resistance can be separated into two parts--one corresponding to rotor copper losses and the other corresponding to mechanical power developed:

$$\frac{R_r}{s} = \frac{R_r (1 - s)}{s} + R_r \quad (2.3)$$

The mechanical power developed is dependent on slip, rotor resistance, and rotor current. Rotor resistance is constant and current is dependent on the impedance and slip. Thus mechanical power developed varies with slip (rotor speed).

The rotor power loss per-phase due to heating can be determined:

$$P_h = I_r^2 R_r \quad (2.4)$$

The mechanical power developed is:

$$P_r = I_r^2 R_r \frac{(1-s)}{s} \quad (2.5)$$

Machine torque is directly related to the mechanical power developed:

$$T_r = \frac{7.04 P_r}{W_r} \text{ lb-ft} \quad \text{or} \quad \frac{9.54 P_r}{W_r} \text{ N-m per-phase} \quad (2.6)$$

Although slip is calculated as a per-unit value, it is generally given as a percentage (100 X per-unit) by manufacturers. Typical values of slip for general purpose motors range from one to five percent. Special high-slip, high-torque motors may have slips as high as 25 percent.

The equivalent circuit model is typically developed on a per-phase basis. By superposition, the model can represent a three-phase machine. If the input voltage to the model is assumed to be line to neutral, then the model for a three-phase machine will be the sum of three separate identical models. The power and torque for a complete three-phase machine is three times that calculated from the per-phase model.

## 2.2 Induction Generator Model

The motor equivalent circuit demonstrates that mechanical power

developed is dependent on rotor speed. For steady-state operation, the machine rotor resistance does not change. Mechanical power developed then is dependent only on rotor current and slip (rotor speed):

$$P_r = I_r^2 R_r \frac{(1-s)}{s} \quad (2.7)$$

Rotor current is an internal value that cannot be readily monitored on induction machines with squirrel cage rotors. Stator current at the terminals is the current that is easily measured. From an analysis of the equivalent circuit, it is obvious that rotor current and, in turn, stator current change with slip/speed.

The rotor resistance is essentially a constant over normal operating speed ranges. The rotor reactance is constant at line frequency and a constant internal voltage. Then, the rotor current changes only with speed. With these variables established, the shaft power of the machine is dependent only on slip. However, slip is calculated from synchronous speed--a constant--and rotor speed. Therefore, by changing only rotor speed, the machine power is changed.

The single-phase, and by superposition three-phase, equivalent circuit of an induction motor is a reasonably acceptable equivalent circuit for an induction machine regardless of its mode of operation. It can be used for a motor or within limits for a generator. The fundamental difference is that the shaft is a "driver" when operating as a motor and the shaft is "driven" when operating as a generator.

Since both resistance and reactances are in the equivalent circuit, the power (input or output) at the terminals will be complex:

$$\underline{S}_t = P_t + jQ_t \quad (2.8)$$

The real part of the terminal power of the machine is the sum of the power consumed by stator copper losses, core losses, rotor copper losses, and the equivalent of mechanical power developed. The imaginary part of the terminal power is the sum of the reactive power absorbed by stator winding leakage reactance, mutual magnetizing reactance, and rotor leakage reactance:

$$P_t = P_s + P_m + P_h + P_r \quad (2.9)$$

$$Q_t = Q_s + Q_m + Q_r \quad (2.10)$$

When operating as a motor, the shaft power is less than the real power input to the terminals because of the losses in the machine. The reactive power must be supplied at the terminals. This can be from synchronous machines that are running in parallel with the induction machine or from static capacitors that are connected across motor terminals.

When the machine is run as the induction generator by driving the shaft, more power is fed to the shaft than is delivered at the terminals. The shaft power has a negative sign since the shaft is being driven rather than being a driver:

$$P_r = P_t - (P_s + P_m + P_h) \quad (2.11)$$

The first major weakness of an induction generator is that it delivers only real power. Reactive power for the machine must be supplied from the power system at the machine terminals just as is required for operation as an induction motor. Because of this limitation the induction generator cannot readily operate in the stand-alone mode but generally operates in parallel with synchronous generators (a power system).



Nevertheless, the simplicity, cost, availability, and ease of operation dictate the induction generator for an increasing number of applications.

The second major weakness of an induction generator is the fact that voltage control must be supplied by the power terminal. Because of the voltage drop in the stator impedance, the voltage across the magnetizing impedance is greater when the machine is operating as a generator than when operating as a motor. Overvoltage could result in some extreme conditions:

$$\underline{V}_r = \underline{V}_t + \underline{V}_s \quad (2.12)$$

The motor model has a slight inaccuracy when used as a generator. Much of the performance information is contained in the slip parameter. Operation on either side of locked rotor ( $s=1$ ) conditions changes all of the machine equivalent reactances slightly.

Because of the difference in voltages across the reactances and the difference in slip/frequency, the inductive reactances have different values for operation as a motor and a generator. The reactances are approximately 10 percent larger when operating as a motor than when operating as a generator. This also appears to cause torque values to be in error by the same percentage.

In summary, the performance of an induction machine is influenced primarily by shaft speed which is contained in the slip parameter. As the speed shifts to either side of synchronous speed, the direction of real power flow changes. The greater the slip, whether positive or negative, the greater the power becomes. Reactive power direction does not change, but the complex power changes and causes proportional voltage

changes in the circuit. The slip also contains the line frequency effect on reactances because slip is dependent on line frequency.

Because of the model assumptions, the analysis has been for balanced operation on a stable, balanced power system. This is not always the case. Only slight modifications to the model will permit analysis of unbalanced operations.

### 2.3 Asymmetrical Models

The development of the equivalent circuit assumed a balanced power system with complete phase symmetry. This results in decoupling between the models of individual phases. A per-phase representation can be developed and the representation for phases is identical. The three-phase system model is developed by superposition of three identical single-phase models. A system that meets these criteria is an ideal system.

All physical realizations of systems have some element of asymmetry between phases. The limit of this asymmetry is reached when one phase is opened, often referred to as "single phasing," because only two current carrying conductors are available.

Any asymmetry in the system forces a coupling between phases. The resulting imbalance precludes representing the system by a per-phase equivalent circuit. The unbalanced current in each phase must be calculated interdependently.

There are a variety of methods that have been developed to analyze the asymmetry and to determine the effect of imbalances on the system. All the procedures for calculating imbalances map the currents into a different domain. Although there are significant differences in techniques, all the procedures have the process of mapping in common.

Depending on the problem, some techniques are more beneficial for one type of system than another.

The procedures have a one-to-one correspondence and a map can be made from one system to another. Two broad categories can be established to define most of the unbalanced analysis procedures.

The first category can be called a generalized machine approach. The theory uses two axes--direct and quadrature--to redefine the system. The direct/quadrature system analysis works very well for direct current machines, for synchronous machines, and for equipment with brushes.

The second category separates each of the three currents into three components--positive, negative, and zero; or A, B, and zero; or forward, backward, and zero. One of the most commonly used theories is the symmetrical components proposed by Fortescue in 1918 (14).

Symmetrical components is the most common technique for unbalanced system analysis used in industrial applications. It can be used directly with the equivalent circuit model of the induction machine. Because so much practical information is available, this theory will be used to determine the effect of unbalanced systems on the induction machine.

The model for the positive sequence of the symmetrical components is the same as the model under balanced conditions and analysis can be carried out on a per-phase basis. The model for negative sequence components can be obtained by a suitable modification of the slip "s" as discussed in the next section. Once again, analysis on a per-phase basis is possible. For these reasons, the model voltage should be line to neutral, the current becomes line current, and the machine is represented by a wye connected impedance. This provides a direct correspondence

between the sequence model and the system model, since both are relative to neutral.

The benefit of symmetrical components can be summarized. It enables the representation of a set of unbalanced three-phase parameters--voltage, current, impedance, power--in terms of three sets of balanced quantities--positive, negative, and zero sequence components. Since each of the set of components is balanced, matrix manipulation can be used to aid in calculations and analysis can be conducted on a per-phase basis. These advantages have made symmetrical component analysis a very important and powerful tool in the analysis of unbalanced operation of power systems (15).

#### 2.4 Unbalanced Analysis of the Induction Machine

The equivalent circuit developed to represent the induction machine was for operation on a symmetrical three-phase balanced system. With phases unbalanced, an equivalent circuit must be developed for each of the symmetrical components--positive, negative, and zero sequence.

The positive sequence equivalent circuit is the same as the conventional equivalent circuit developed for balanced three-phase operation, since the positive sequence is the only component that exists in a balanced system.

Negative sequence analysis implies the rotating field in the stator (and in the air-gap) is in the opposite direction of the positive sequence rotation. However, the rotor continues to turn in the same direction. Alternately it can be assumed the stator remains connected to a positive sequence source, but the rotor is turning forward with a speed equal to the actual rotor speed plus twice the synchronous speed.

Comparing this to the positive sequence equivalent circuit, it appears the only elements that change are the rotor components of reactance and resistance.

If the rotor is turning opposite the stator field, the slip component of the rotor resistance will be different. Slip is the per-unit difference between stator speed and rotor speed for a positive sequence rotation. If the positive sequence and the general equivalent circuit have a one-to-one correspondence, then Equation (2.1) can be rewritten as a positive sequence relationship:

$$s_1 = \frac{\omega_n - \omega_r}{\omega_n} \quad (2.13)$$

With negative sequence, the rotor and the air-gap magnetic field rotate in opposite directions. The effective slip,  $s_2$ , is given by

$$s_2 = 2 - s_1 \quad (2.14)$$

To establish a negative sequence equivalent circuit, the positive sequence equivalent circuit can be modified by substituting  $s_2$  for  $s_1$ . Thus the effective rotor resistance is the physical resistance divided by  $(2 - s_1)$  rather than  $s_1$ .

The conditions in the stator have not changed; hence the stator impedances are the same in the positive and negative sequence equivalent circuits. Only the rotor circuit representation is changed as shown in Figure 3.

The equivalent rotor resistance can be separated into a rotor copper loss component and a negative torque component:

$$\frac{R_{r2}}{2-s} = R_{r2} + R_{r2} \frac{s-1}{2-s} \quad (2.15)$$

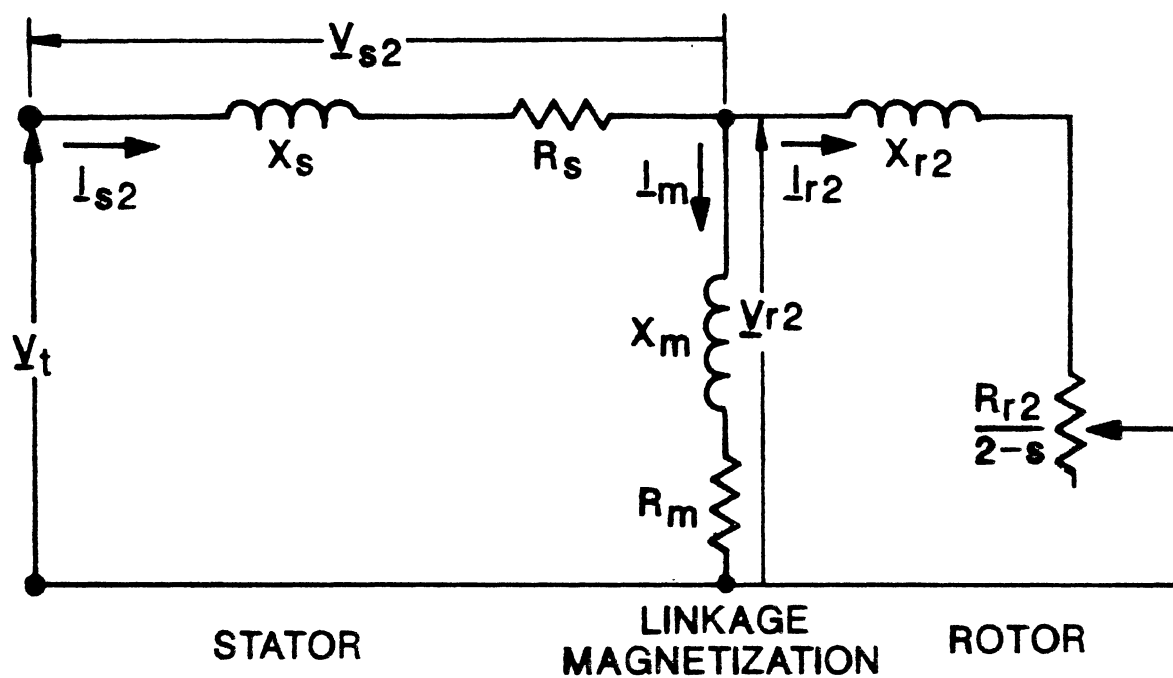


Figure 3. Negative Sequence Equivalent Circuit

The negative sequence power loss relationship for the rotor is determined by the rotor resistance component and the negative sequence rotor current:

$$P_{h2} = I_{r2}^2 R_{r2} \quad (2.16)$$

The negative sequence mechanical power developed is

$$P_{r2} = I_{r2}^2 R_{r2} \frac{s-1}{2-s} \quad (2.17)$$

$P_{r2}$  is negative, indicating that the mechanical power developed is negative or mechanical power must be supplied by the prime mover.

The torque is found using unit conversion and angular velocity as was done for the positive sequence case:

$$T_{r2} = 7.04 \frac{P_{r2}}{\omega_r} \text{ lb-ft} \quad \text{or} \quad 9.54 \frac{P_{r2}}{\omega_r} \text{ N-M} \quad (2.18)$$

Observing the negative sequence equivalent circuit, the rotor impedance changes with slip while the other circuit impedances are constant with speed. All equivalent circuit parameters that depend on slip will have different values in the positive sequence and negative sequence equivalent circuits. The magnetizing impedance does not change with speed and as such it has the same value in both equivalent circuits.

The presence or absence of zero sequence components depends solely on the electrical connections of the motor stator winding. Zero sequence component is one-third of the unbalance that flows in the neutral of the circuit. If the machine stator winding is connected in a wye configuration with the neutral connection coming out, then zero sequence currents can exist since a path is available for them to flow. If the machine is connected in a delta configuration or in a wye with a floating

neutral, then zero sequence cannot exist even though the circuit models use line to neutral equivalence. Without a physical connection between the machine windings and neutral, zero sequence is not realizable. During ground faults, a connection to ground may arise permitting zero sequence to be realized; but this condition is not expected in normal unbalanced operation.

Induction machines are designed and constructed to operate with a restricted temperature rise to prevent destruction of insulation and to preserve its performance characteristics. As a machine becomes more unbalanced, its temperature begins to rise. Because of additional losses generated, all the resistance elements in the motor equivalent circuit change. This is best demonstrated by the temperature relationship:

$$\frac{R_{ha}}{R_{hb}} = \frac{(A + T_{ha})}{(A + T_{hb})} \quad (2.19)$$

All calculations have assumed fixed values of impedance for many conditions. One of these is frequency. This is particularly important in reactance calculations. For example,

$$X_s = 2 \pi f M_s \quad (2.20)$$

Changes in frequency will cause proportional changes in the inductive reactances of the circuit. The sensitivity of the machine parameters to frequency changes must be considered in some systems with soft power sources and the resulting wide changes in system frequency.

The equivalent circuit for operation of the induction machine has been discussed. Since symmetrical component analysis is used, the equivalent circuit consists of three parts--the positive, negative, and zero sequence equivalent circuits. The positive sequence circuit is the one



derived for operation under balanced conditions. The negative sequence circuit is similar except that the rotor impedances have changed. These impedances change since coupling to the rotor is through the slip--transformer turns ratio. The slip value in the positive-sequence equivalent circuit must be replaced by the negative sequence value. The presence of zero sequence components depends on the machine connection. A four wire wye connection has a path to neutral and results in zero sequence values whereas a delta connection or a three wire wye does not have a path to neutral, so no zero sequence component can exist.

## CHAPTER III

### POWER SYSTEM BALANCER

#### 3.1 Introduction

An analysis of the induction machine operating on unbalanced power systems can be performed using the equivalent circuit model and symmetrical components as discussed in Chapter II. A study of this analysis indicates there is a limit on the amount of imbalance to which the machine can be exposed and still have reasonable operating characteristics. Typically, manufacturers express concern if the current imbalance is greater than 10 percent.

On many remote power systems away from large industrial loads, the three line-currents of an induction motor may differ by greater than 10 percent. For this reason methods of analyzing the power system with the objective of making corrections (balancing) to the imbalance are required.

One of the situations that causes imbalance is attributable to the type or method of delivering three-phase power. In sparsely populated areas, three-phase imbalance can be caused by:

1. three-phase system with single-phase loads,
2. two-current carrying conductors (v-phase), and
3. three-phase supply created from more than one single-phase source.

The limiting case of imbalance comes with attempting to operate three-phase machines on a single-phase line.

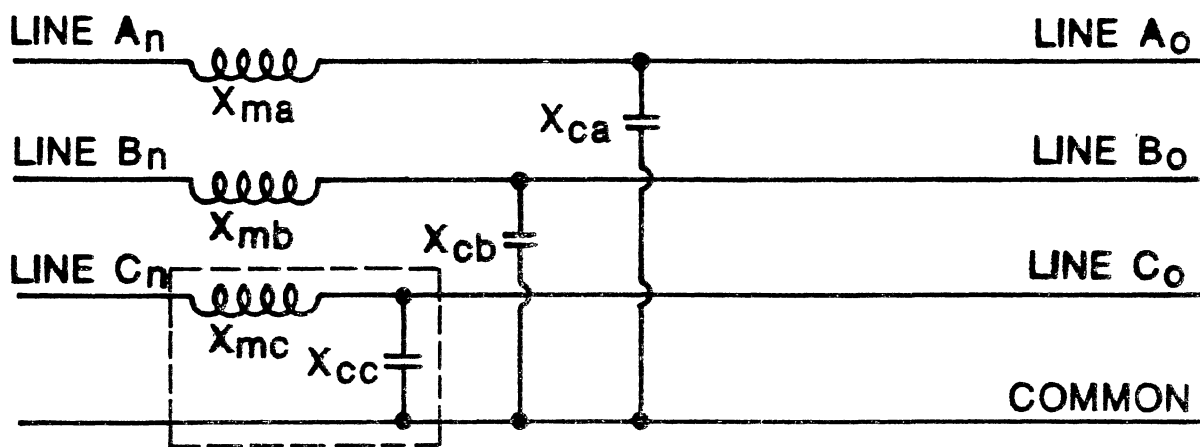
This chapter presents an idealized model of a power system balancer for steady-state analysis and discusses how it can be used to resolve unbalanced conditions in a power system.

The balancer model presented is generalized to permit it to be used to define any segment of the power system. Figure 4 demonstrates its applicability to represent one phase of a three-phase power line segment. Each phase of the power system is represented individually by one two-port network model.

Since each phase is modeled individually, unbalanced conditions can be resolved on a per-phase basis. By adding appropriate impedances in two lines, the currents in those lines can be forced to become equal to the current in the third phase. For this discussion balanced conditions are defined as equal current magnitudes with 120 degrees phase difference between any two.

The circuit elements of a line balancer can be calculated from the same balancer model, redrawn as shown in Figure 5. The model is defined by setting the output side to the existing line conditions and the input side to the line condition to be achieved after balancing. If the internal impedances calculated from the model are added to the line, the line should be balanced as desired.

The impedances added to each line not only affect the current but also the apparent power flow. Power factor correction is commonly required for power systems with large inductive (motor) loads. This correction is typically accomplished by adding shunt capacitors to the power line.



THREE PHASE LINE LUMPED PARAMETERS

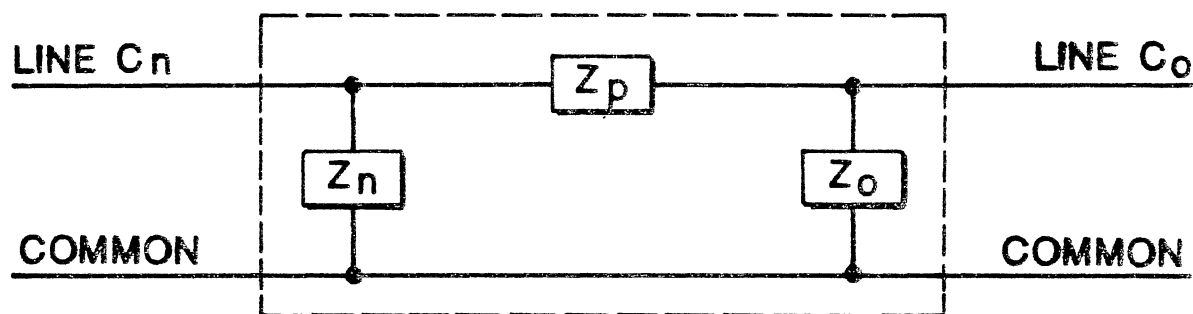
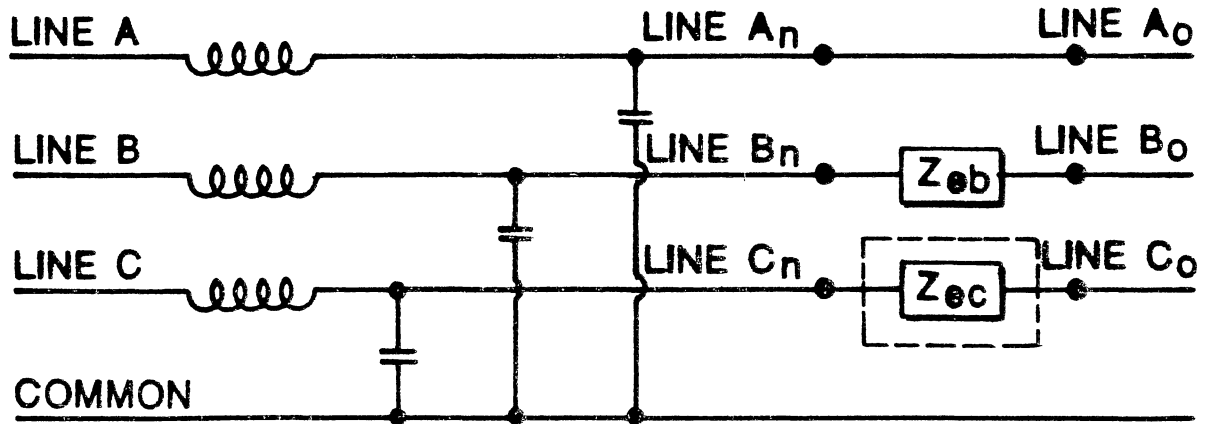


Figure 4. Balancer Representation of One Phase of a Three-Phase Power Line Segment



THREE PHASE LINE BALANCING BY IMPEDANCE

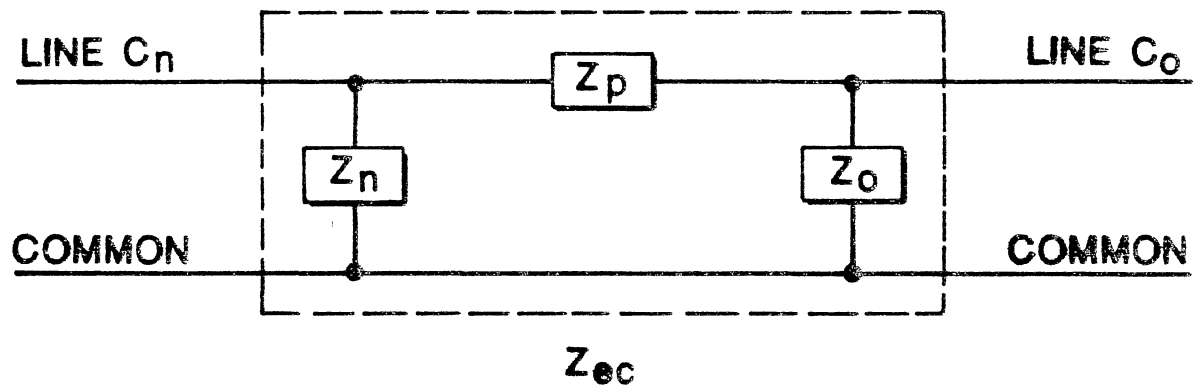


Figure 5. Balancer for One Line

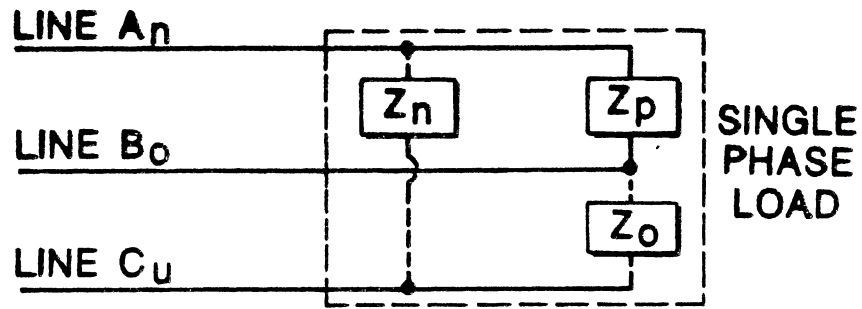
The balancer model will also determine the capacitors necessary for controlling this reactive power flow in the same manner that current was controlled.

An approach to control real power flow between busses has been discussed by Hughes (13). The balancer model resolves this problem similar to the current balancer problem.

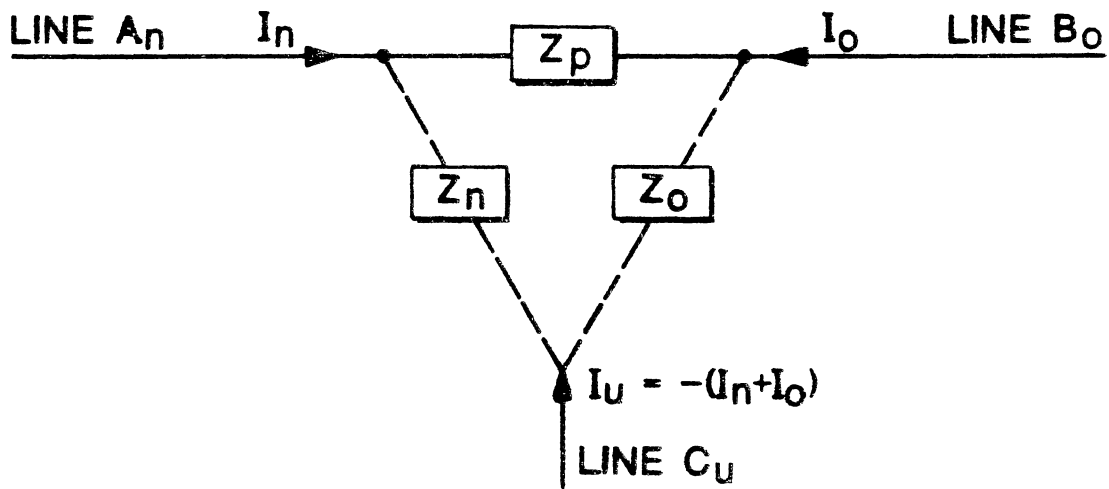
Obtaining single-phase power from a three-phase source is commonly done by connecting the single-phase load across two lines, as shown in Figure 6. Although this is adequate for the load, it places an imbalance on the source since one phase has no load. The balancer model can be used to calculate the circuit elements needed to create the third phase as shown by the dashed line in Figure 6.

The previous paragraph describes a balancer for a single-phase load on a three-phase source. The same analogy applies for obtaining three-phase power from a single-phase power line. By the use of the balancer model, the elements needed to create a third line necessary for a three-phase system from a single-phase line can be designed. The currents on all three lines will be equal in magnitude and separated by 120 degrees in phase.

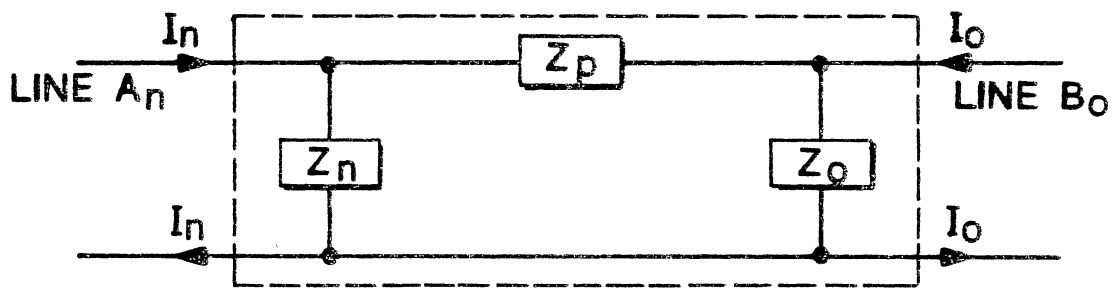
For operation of a three-phase induction generator on a power system, at least two basic requirements exist. The machine stator currents must be reasonably balanced or the machine will not operate effectively. The second, but more critical item, is that the induction generator supplies only real power to the system, but the machine requires reactive (lagging) power for operation. This must be supplied by parallel synchronous generators and/or by capacitors on the line.



THREE PHASE FROM ONE PHASE LINE



IMPEDANCES TO CREATE THIRD LINE



BALANCER MODEL TO CREATE THIRD PHASE

Figure 6. Balanced Three Phase From Single Phase

The solution to the problem of finding the network elements needed to satisfy both of these conditions has traditionally been tedious or nonexistent. The use of the balancer model discussed here has reduced this to a very practical problem.

For balancing three-phase lines, one model is required for each line. The model represents the line relative to a common or ground. This is a typical representation of electrical energy systems whether a three-wire ungrounded or a four-wire grounded system is considered.

The current in one phase affects the currents in other phases. Isolation of phases for analysis is done using superposition. This is a common practice but it is usually assumed that the phases are identical and balanced. The balancer model looks at a segment of line while the input and output terminals are isolated. Superposition can still be used. A two-port balancer is ideal for imbalance analysis because the output is defined by a black box network operating on the input. Even with imbalance caused by an open phase, the model will still describe the system.

The same two-port network used for the balancer model is often used for filter description. A filter is employed to pass or eliminate certain signals. It is this characteristic that is most appropriate for adjusting the power line to provide a desired performance. Real power may be passed while shunting reactive power through such a network.

In general, the balancer model is described in terms of the relationships between the input voltage and current, and the output voltage and current. Both magnitude and angle will be required for the input and output parameters. The power system performance required determines both real and reactive elements. Many power system conditions are



better described in terms of real power and power factor. These different operating criteria can be expressed in terms of voltages and currents.

Most phase angle measurements in power systems are made in the form of power factor. This is the cosine of the angle associated with real and apparent power. It is also the cosine of the phase angle between voltage and current phasors. As such it is the impedance angle. It is also the delay in time, expressed in electrical degrees, between when the voltage crosses the zero axis and the current crosses the same axis.

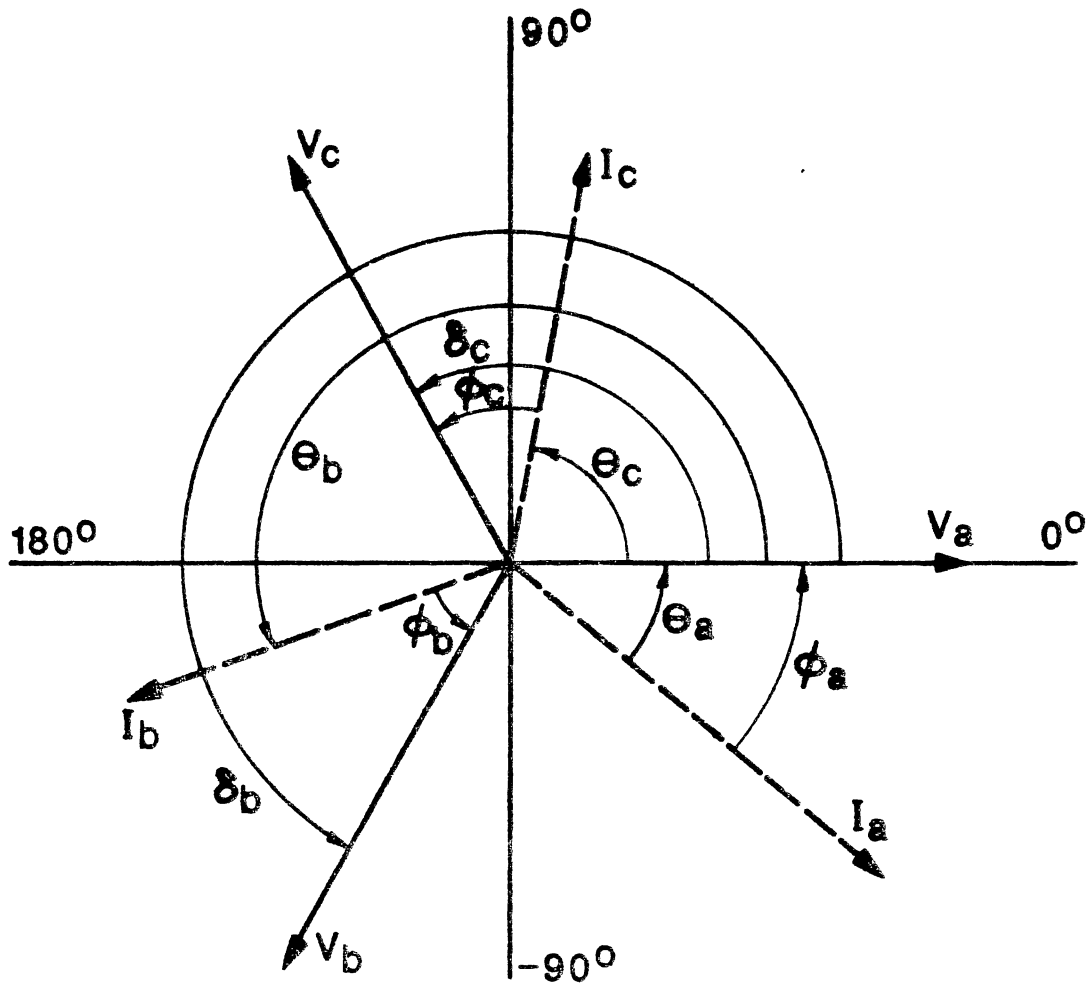
Instrumentation is available to measure the phase angle between a reference voltage and an unknown voltage. Use of these voltage angles with power factor angles provides all the required angle data for the balancer model. The reference can be any one of the phases of a three-phase system. The other angles are then expressed relative to this reference which is assumed to have an angular value of zero:

$$\phi_p = \arccos(\text{pf}_p) \quad p = a, b, \text{ or } c \quad (3.1)$$

$$\phi_p = \delta_p - \theta_p \quad (3.2)$$

Figure 7 is an illustration of the angles for a three-phase system. For balanced systems the phase quantities (voltages, currents) are separated by 120 degrees. For unbalanced systems and for three-phase systems that are created from a single-phase using a balancer, this is not necessarily true. Physical devices used to create the third phase will not be pure elements and may not have exact values. The results are phases that may be very close to but not exactly 120 degrees apart.

A further note is necessary concerning power factor. In a three-phase system, this is a per-phase quantity. One phase must be selected



$V_{ref} = V_a$

$\delta_a = 0 = \text{reference}$        $\phi_a = \text{pf angle relative to } \delta_a$

$\delta_b = \text{relative to } \delta_a$        $\phi_b = \text{pf angle relative to } \delta_b$

$\delta_c = \text{relative to } \delta_a$        $\phi_c = \text{pf angle relative to } \delta_c$

$\theta_a = \delta_a - \phi_a$

$\theta_b = \delta_b - \phi_b$

$\theta_c = \delta_c - \phi_c$

Figure 7. Determination of Current Angles

as the reference and other phases rotated relative to it depending on the phase-sequence.

Magnitude and phase are used to completely define any vector quantity. However, it is not always absolutely necessary to have both. For example, in a polyphase system without a neutral, if current magnitudes can be forced to be equal, then the currents must be balanced (separated by 120 degrees in phase from each other).

Line balancers can be used to correct a variety of different circuit conditions. Selection of shunt capacitance for power factor correction is extremely common. This permits the line to operate closer to unity power factor, thus delivering only real power and reducing current levels and resulting line losses. Balancing filters can be used to achieve equal current magnitudes by adding suitable impedances in the lines. The direction of real or reactive power flow in a line can be controlled with a phase balancer by adjusting voltage and current phase angle relationships. A standard balancer can be used to model and/or design a balancer for a three-phase line as well as a balancer to interface single-phase and three-phase systems.

All these different functions use the same balancer model. The only difference in the models is the form of the input and output data available. Without defining the internal configuration of the network, classes of models can be defined by their input, output, or control function.

Section 3.2 presents the assumptions made in the development of the model. Since the balancer model is generic, a general class of equations will be developed in section 3.3. Section 3.4 introduces the special requirements for a balancer on single-phase lines. The terminal characteristics of the model for a variety of conditions are discussed

in section 3.5. Section 3.6 provides the internal specifications and the solution of the model for balancing three-phase lines and for creating three-phase lines from a single-phase line.

### 3.2 Assumptions

The balancer employs only one model but can be used for two distinct balancing situations--three-phase lines and one-phase to three-phase lines. The three-phase line balancer may be used to control current magnitudes as well as real and/or reactive power flow. A schematic of the system under study is shown in Figure 8. On initial inspection, this appears to be a very simple circuit. However, it adequately represents the major elements of the system and reduces them into a minimum system consisting of a load, a source, and the connection between them. Figure 9 segregates the system into elements that can be represented by the generic balancer model discussed earlier.

Changing any component in the system will dramatically affect the operation of the entire system. A direct method of calculating these changes is desirable. If all three elements of the load are not identical, then unbalanced operation results in drastically impaired performance of the induction machine.

The following assumptions are made in the model development:

1. Only passive devices may be used.
2. The model must be acceptable for any unbalanced condition.
3. The same model must be capable of representing the line, load, source, or system behavior.
4. The model must permit decoupling between the input and output conditions.

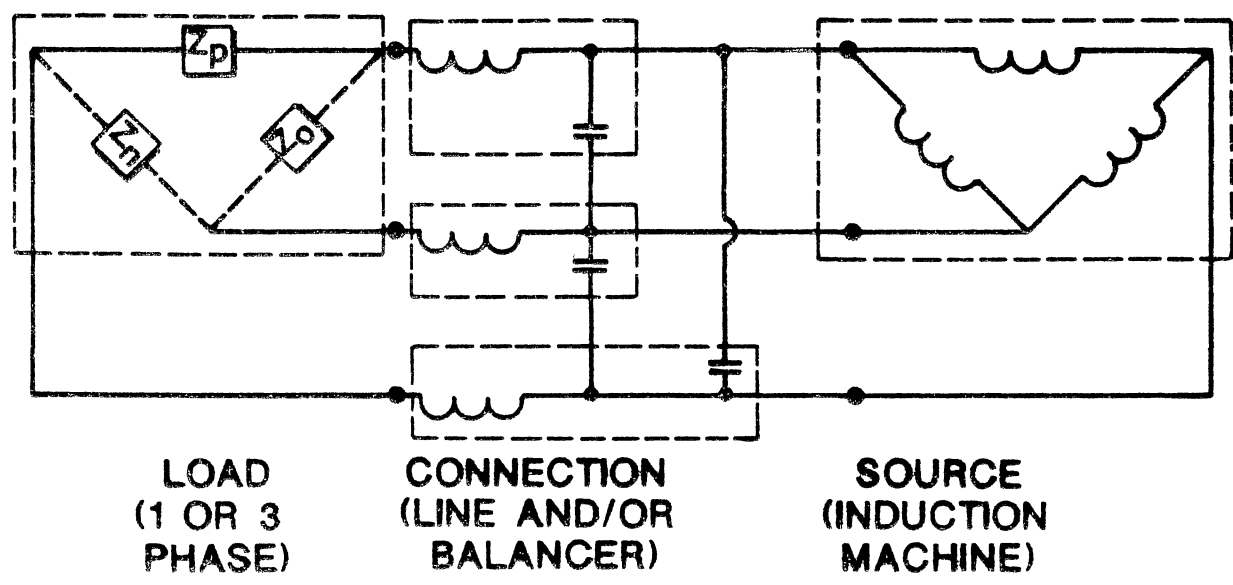


Figure 8. Power System Schematic

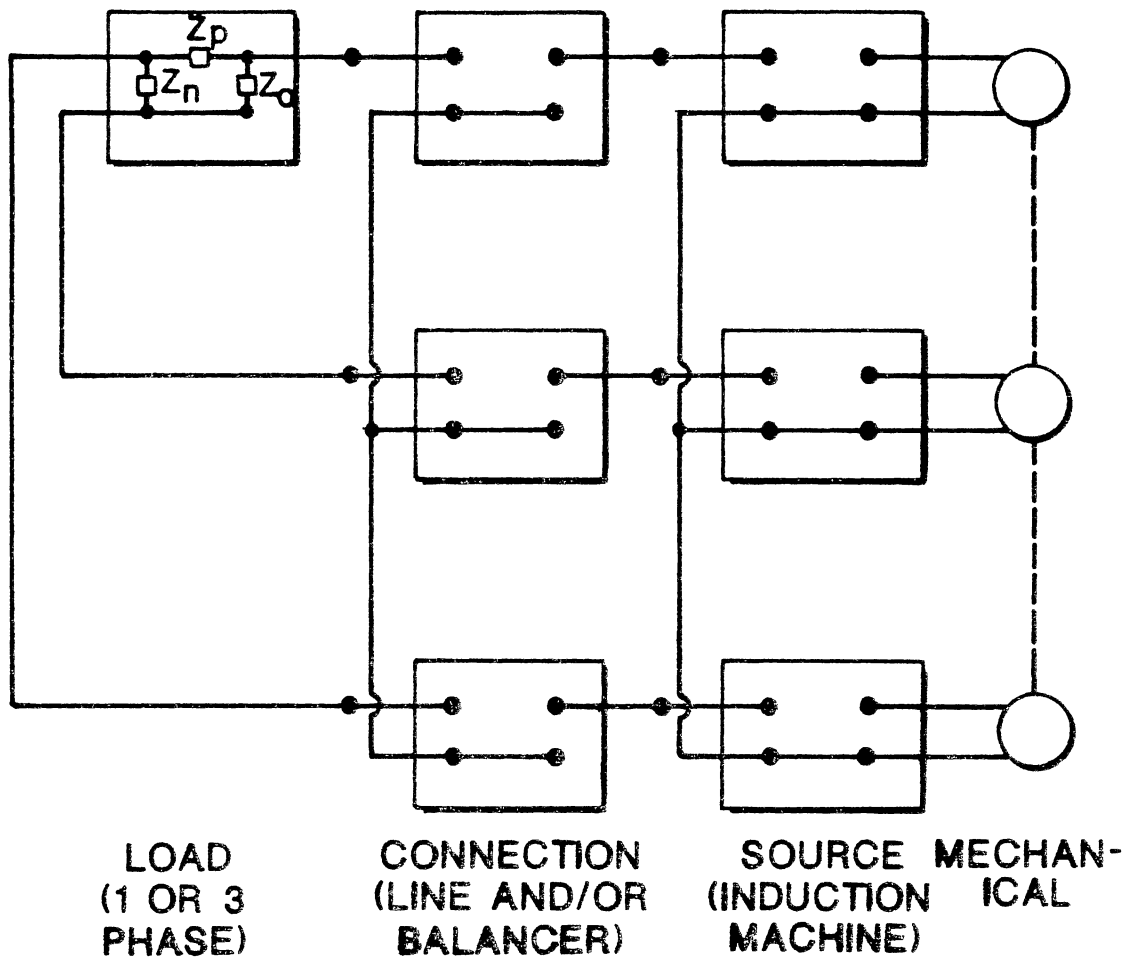


Figure 9. Power System as Two Port Network

5. The model must be suitable for single-phase and three-phase conditions.
6. The model must permit decoupling between phases of a polyphase system.
7. Elements used in the model are ideal.
8. Only steady-state conditions are considered.

### 3.3 General Line Balancer Model

Only the input and output parameters are required to define the balancer model. The internal connections of the network between the terminals can be found from these external parameters.

Figure 10 illustrates the parameters of the model and the sign conventions used. The direction conventions are made for a three-phase system. The general equations will be developed from Ohm's Law and the power equation. Both have three parameters--voltage, current, and impedance or power. Any two of these parameters will completely define an element in the model. All parameters are complex variables, which can be represented by their real and imaginary components or by their magnitude and phase.

With this basis, equations can be developed to define the terminal characteristics and the internal impedances of the elements in the model. For convenience of manipulation, the magnitude and angles are separated into two different variables. When the variables are used in the same equation, they are separated by a colon.

The total apparent power transfer into the model network is equal to the algebraic sum of the apparent power input into the two ports:

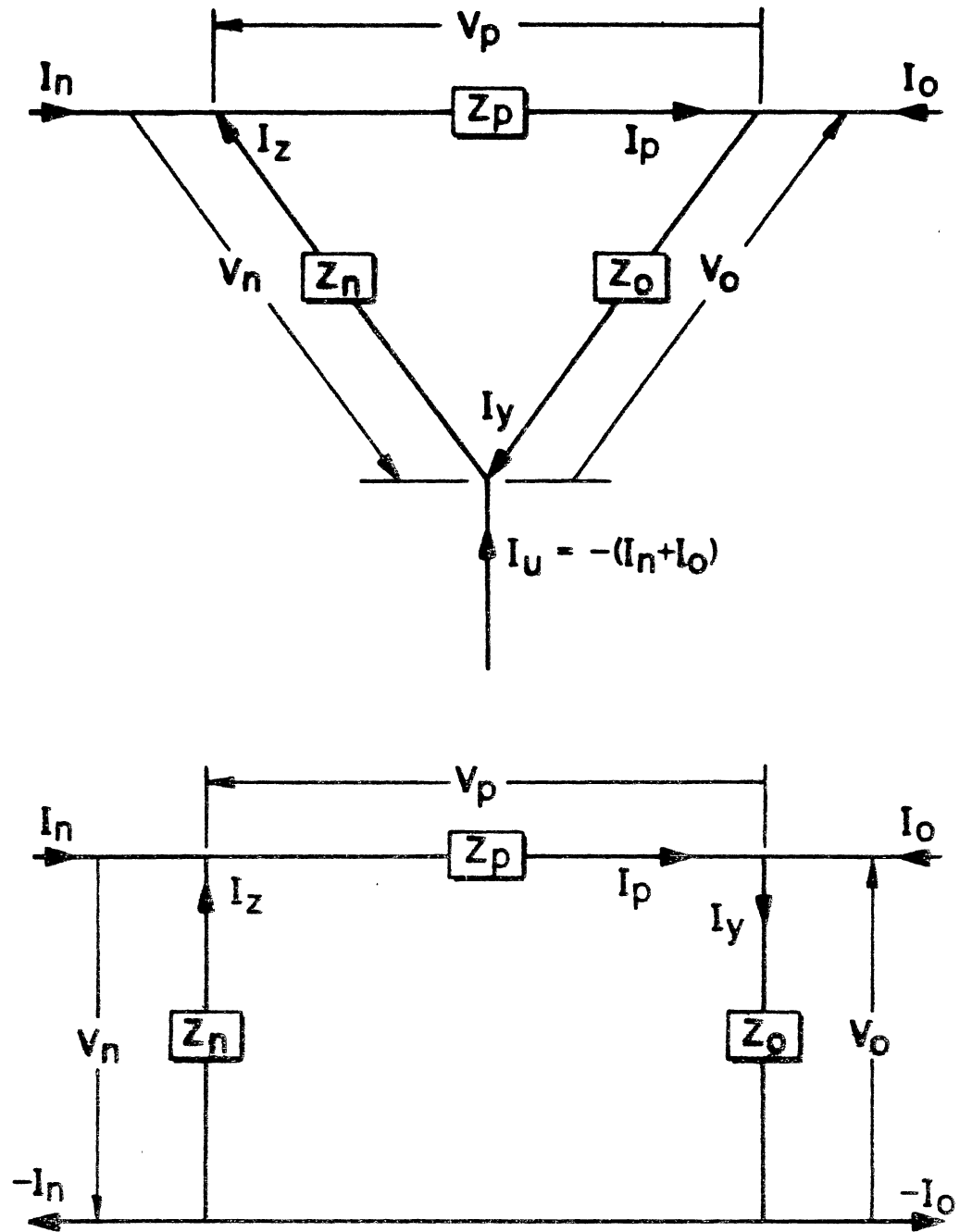


Figure 10. Two Port Network Model



$$S_f : \phi_f = S_n : \phi_n + S_o : \phi_o \quad (3.3)$$

This equation can be expressed in terms of real and reactive powers:

$$P_f = P_n + P_o \quad (3.4)$$

$$P_f = S_n \cos(\phi_n) + S_o \cos(\phi_o) \quad (3.5)$$

$$Q_f = Q_n + Q_o \quad (3.6)$$

$$Q_f = S_n \sin(\phi_n) + S_o \sin(\phi_o) \quad (3.7)$$

The complex power transfer can also be calculated from voltage and current. Expressing the magnitude and phase in separate equations, we have:

$$S_n = V_n I_n \quad (3.8)$$

$$\phi_n = \delta_n - \theta_n + 180^\circ \quad (3.9)$$

The presence of the  $180^\circ$  term in Equation (3.9) is due to the convention assumed for the input voltage (see Figure 10):

$$S_o = V_o I_o \quad (3.10)$$

$$\phi_o = \delta_o - \theta_o \quad (3.11)$$

The apparent power within the network is the algebraic sum of the apparent power in each element:

$$S_g : \phi_g = S_z : \phi_z + S_y : \phi_y + S_p : \phi_p \quad (3.12)$$

Each of the elements has apparent power calculated from a complex voltage and current:

$$S_z = V_n I_z \quad (3.13)$$

$$\phi_z = \delta_n - \theta_z \quad (3.14)$$

$$S_y = V_o I_y \quad (3.15)$$

$$\phi_y = \delta_o - \theta_y \quad (3.16)$$

$$S_p = V_p I_p \quad (3.17)$$

$$\phi_p = \delta_p - \theta_p \quad (3.18)$$

One of the assumptions made in model development is that no real power will be consumed within the balancer. For this condition a simplified set of equations result. From Equation (3.4):

$$P_f = 0 \quad (3.19)$$

$$P_n = -P_o \quad (3.20)$$

All power within the balancer is reactive. Therefore, the output power is equal in magnitude but opposite in direction to the input power.

The currents through the three elements of the balancer model are:

$$I_p : \theta_p = I_n : \theta_n + I_z : \theta_z \quad (3.21)$$

$$I_y : \theta_y = I_o : \theta_o + I_p : \theta_p \quad (3.22)$$

$$I_z : \theta_z = I_y : \theta_y + I_u : \theta_u \quad (3.23)$$

The voltage drop in each element is obtained from Ohm's Law:

$$V_n = I_z Z_n \quad (3.24)$$

$$\delta_n = \theta_z + \phi_n \quad (3.25)$$

$$V_o = I_y Z_o \quad (3.26)$$

$$\delta_o = \theta_y + \phi_o \quad (3.27)$$

$$V_p = I_p Z_p \quad (3.28)$$

$$\delta_p = \theta_p + \phi_p \quad (3.29)$$

Kirchhoff's laws can be used to sum the voltage drops around the loop and to sum the currents into the network:

$$V_n : \delta_n + V_o : \delta_o + V_p : \delta_p = 0 \quad (3.30)$$

$$I_n : \theta_n + I_o : \theta_o + I_u : \theta_u = 0 \quad (3.31)$$

These equations completely describe the balancer model. By defining any two complex input parameters--volts, amps, or power--and any two output parameters, all terminal characteristics of the model can be determined.

These same variables define the internal impedances of the balancer. Each special case of the network involves three equations and three variables. The three impedances each have two parts--real (resistance) and imaginary (reactance). If internal impedances are to be calculated from the input and output parameters, then six unknowns exist for three equations. Therefore, additional requirements must be imposed or found to define the internal impedances.

From the assumption that the balancers are designed not to consume real power, all impedances have zero real parts. Therefore,

$$R_z = R_y = R_p = 0 \quad (3.32)$$

Only three reactance components are to be found for the three unknown impedances from three equations. An exact solution can therefore be found.

### 3.4 Single-Phase Balancer Model

A single-phase load consuming (generating) real power (source) can be made to appear as a balanced load (source) on a three-phase system. As in the previous section, the shunt impedances of the balancer will be assumed to be purely reactive.

Unlike the three-phase balancer, if real power is to be transferred from a source to a load, the model must permit the transfer or pass impedance,  $Z_p$ , to be resistive. This impedance represents the single-phase load and is not an element that is added to the system. Although real power will be transferred to this impedance representing the load, the balancing impedances will not consume power. The same two-port balancer model can be used if the series or pass impedance element is a resistor with reactance equal to zero:

$$R_z = R_y = 0 \quad (3.33)$$

$$X_p = 0 \quad (3.34)$$

These equations assume ideal elements of pure resistance or reactance. For modeling purposes this is adequate. Physical realizations of circuit elements always have both resistance and reactance. After the model predicts the desired ideal element, a physical device must be built to satisfy the design values as closely as possible.

### 3.5 Balancer Model Terminal Characteristics

The balancer model is general and flexible so that a variety of problems can be solved. Five classifications will encompass most situations of interest. They are:

1. Balancing real and reactive power flow between busses.
2. Balancing current magnitude and angle on three-phase lines.
3. Balancing single-phase loads on three-phase lines.
4. Balancing real power flow with fixed line voltages.
5. Balancing real power flow into unbalanced three-phase voltages and currents.

These different conditions will be developed in five stages. These are vastly different problems. Each classification involves a variation of the available input and output parameters of the balancer model. If the input and output parameters can be converted to voltage and current, then the internal impedance elements of the balancer can be calculated. The input side of the model is the desired balanced condition, while the output side of the model is the existing line condition.

An unusual but useful concept of power factor values should be noted. Lagging power factor is defined positive while leading power factor is defined negative. This convention assumes real and reactive power flow in the same direction. While this is accurate for most problems, it is not correct for the induction generator where real power is transferred from the machine to the power system and reactive power is drawn from the power system by the machine. For induction generators, the supplement power factor is required.

Supplement power factor is not a term encountered in the literature. It is a very legitimate concept that arises from the angle associated with power factor. Figure 11 illustrates the need this concept:

$\phi_1$  = lagging power factor angle

$\phi_2$  = leading power factor angle

$\phi_3$  = supplement power factor angle

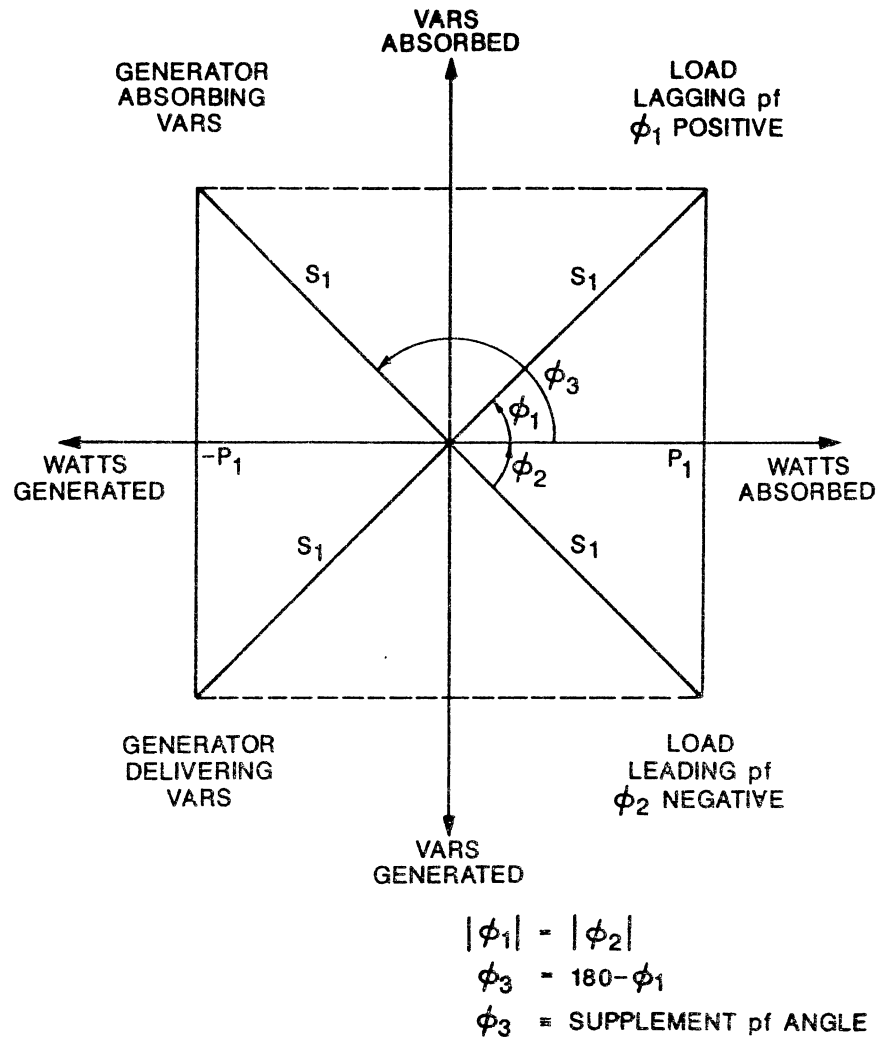


Figure 11. Illustrating the Concept of Power Factor

and

$$\text{pf}_1 = P_1/S_1 \quad (3.35)$$

$$\text{pf}_2 = P_1/(-S_1) \quad (3.36)$$

$$\text{pf}_3 = (-P_1)/S_1 \quad (3.37)$$

The magnitudes of  $\text{pf}_2$  and  $\text{pf}_3$  are equal, but the actual angles differ by 180 degrees. When power factor is used to calculate phase angles, then the direction of var flow must be noted. Otherwise an erroneous angle will be calculated. This is particularly the case when real power flows from a machine and reactive power flows to that machine.

### 3.5.1 Power Balancing Between Busses

The first classification of problems is balancing the real and reactive power flow between busses. Under ideal conditions with lossless components, there is no loss in the balancer.

Real power flow balancing can be accomplished by control of the voltage on the input and/or the output of the balancer. On power systems the voltage magnitudes are held relatively constant. However, the phase of the voltage can be controlled by a phase shift in the network. For real power flow control, the power factor on the input and output of the balancer is assumed to be unity. By controlling the phase difference between the input and output voltages, power flow can be controlled.

Reactive power control is a very common requirement in power systems. This is commonly referred to as power factor correction. Reactive power control is equivalent to controlling the power factor angle. Unlike real power balancing, in this case the phase difference between

the input and output voltages of the balancer must be zero. With this requirement and with a fixed quantity of real power, reactive power balancing is accomplished by controlling the magnitude and phase of current.

The general solution to balancing real and reactive power is derived from the known or required parameters. The solution of the balancer model for internal impedances requires input and output voltages and currents. The corresponding equations constitute a special case of the general equations discussed in section 3.3.

The known parameters are  $P_n$ ,  $\phi_n$ ,  $V_n$ ,  $\phi_o$ , and  $\delta_o$ .

$$S_n = P_n / \cos(\phi_n) \quad (3.38)$$

$$I_n = S_n / -V_n \quad (3.39)$$

Assuming the input voltage as the reference,

$$S_n = 0 \quad (3.40)$$

The current angle becomes

$$\theta_n = -\phi_n \quad (3.41)$$

For a lossless system,

$$P_o = -P_n \quad (3.42)$$

$$S_o = -P_n / \cos(\phi_o) \quad (3.43)$$

Power grid requirements dictate that voltage magnitudes are equal:

$$V_o = V_n \quad (3.44)$$

The remaining parameter required for the model is the output current:



$$I_o = S_o / V_n \quad (3.45)$$

$$\theta_o = \delta_o - \phi_o \quad (3.46)$$

### 3.5.2 Current Balancing in Three-Phase Lines

The second classification is balancing the current flow in three-phase lines. This is the traditional technique for balancing a system. Balancing implies the adjustment of both magnitude and phase of currents. The current magnitudes must be equal and their phases must be separated by 120 degrees. Because of the inherent physical restrictions on three-phase systems without a neutral return, if the current magnitudes are equal, then the phase angles of the three currents will automatically have a phase difference of 120 degrees between any two.

As in the previous derivation, line currents can be balanced by placing impedances in series in the line or across the line and the common. For an ideal lossless balancer, the power into and from it must be equal. With the balancing impedance added in series with the line, voltage will then be dropped across the impedance, resulting in a different output voltage. If the original imbalance is very large and very large impedances are placed in the line, it is possible that the voltage drop would be so great as to make an induction machine unable to start on the system. This is a limiting constraint on current balancers. With a fixed power load and a constant supply voltage, the line current will increase.

The derivation of the input and output voltage and current is made from the known parameters. The actual current angles are not required.

Only the phase shift or difference in input and output current angles is necessary for complete accuracy.

The known parameters are  $P_n$ ,  $pf_n$ ,  $I_n$ ,  $pf_o$ ,  $I_o$ , and  $\phi_o$ . The power factor is converted to angles for manipulation:

$$\phi_n = \arccos (pf_n) \quad (3.47)$$

$$\phi_o = \arccos (pf_o) \quad (3.48)$$

The apparent power into the network is

$$S_n = P_n / \cos (\phi_n) \quad (3.49)$$

$$V_n = -S_n / I_n \quad (3.50)$$

Assuming the input current as the reference,

$$\theta_n = 0 \quad (3.51)$$

Then the input voltage angle becomes

$$\delta_n = \phi_n \quad (3.52)$$

For a lossless system

$$P_o = -P_n \quad (3.53)$$

$$S_o = -P_n / \cos (\phi_o) \quad (3.54)$$

The remaining parameter is output voltage:

$$V_o = S_o / I_o \quad (3.55)$$

$$\delta_o = \phi_o - \theta_o \quad (3.56)$$

### 3.5.3 Balancing a Single-Phase Load on a Three-Phase Line

The third classification of balancing involves a general single-phase load and it is required to make it look like a balanced load on a three-phase line.

A three-phase line can be created from a single-phase line using this technique. By defining the existing voltages and currents for the output of the model and establishing the desired voltage as the input to the model, the balancer will define the impedance elements necessary to arrive at a balanced three-phase condition. Balancing single-phase lines requires the line-to-line voltage and also the line current magnitudes be equal while the angles are separated by 120 degrees.

This classification is also very useful for determining the equivalent model of a line. If the input to and output from the balancer model are given as the conditions on either end of a power line, the steady-state model of the line is determined by the impedance elements of the model.

The input and output voltage angles and current angles are relative to each other; therefore, the input angles can be assumed to be zero. Power factor is a commonly measured value. Relative voltage angles can be measured rather easily with available instrumentation. Absolute current angle measurements are not easily obtained. The current angle can be ascertained from the voltage angle and the power factor angle.

The supplement power factor angle should be used for balancing a single-phase load on a three-phase system. Using the convention assumed

for the network, the voltage direction and current direction cause a 180 degree phase shift in the power factor angle.

The only calculation required in this derivation is the determination of the current angle from the power factor. The known parameters are the terminal values for current and voltage required for calculation of the internal impedances of the balancer network. The known parameters are  $V_n$ ,  $I_n$ ,  $I_o$ ,  $\delta_o$ , and  $\phi_o$ .

Assuming the input voltage as the reference angle

$$\theta_n = -\phi_n \quad (3.57)$$

#### 3.5.4 Power Balancing With Fixed Voltage

For induction machines operating as generators on power systems, a combination of the above procedures is required. As indicated in Chapter II, the machine generates real power but consumes reactive power. The machine does not control its terminal voltage, but must operate at the existing line voltage. Machine overload is determined by the current flowing in the stator winding of the machine. With this set of constraints, a balancer classification can be established that is particularly suitable for balancing a three-phase induction generator on a single-phase line.

In this case, the power into the line, the line voltage, the machine power factor, the machine stator current, and the difference between the output and input voltage angles are available. The known parameters are  $P_n$ ,  $V_n$ ,  $pf_o$ ,  $\delta_o$ , and  $I_o$ .

Use the input voltage angle as the reference angle:

$$\delta_n = 0 \quad (3.58)$$

The power factor of the line must be corrected to unity:

$$\phi_n = 0 \quad (3.59)$$

Then

$$\theta_n = 0 \quad (3.60)$$

$$S_n = P_n / \cos(\phi_n) = P_n \quad (3.61)$$

$$I_n = S_n / V_n \quad (3.62)$$

Assuming a lossless balancer circuit:

$$P_o = -P_n \quad (3.63)$$

$$\phi_o = \arccos(\text{pf}_o) \quad (3.64)$$

$$S_o = -P_n / \cos(\phi_o) \quad (3.65)$$

The remaining parameter is output voltage:

$$V_o = S_o / I_o \quad (3.66)$$

The output current is required:

$$\theta_o = \delta_o - \phi_o \quad (3.67)$$

### 3.5.5 Real Power Into an Unbalanced

#### Three-Phase System

Balancing the flow of power may be required into an unbalanced system where voltage and currents are specified. Balancing the reactive power of the system can be accomplished by setting the power factor to the required value. For a fixed voltage and current, the real power flow can be controlled by adjusting the power factor. In essence, line

power factor can be used to balance the flow of real or reactive power when the output or load voltage and current are fixed.

The required line conditions are real power, power factor, input current, output current magnitude and angle, load voltage, and the difference between output and input voltage angles. The known parameters are  $P_n$ ,  $pf_n$ ,  $I_n$ ,  $V_o$ ,  $\delta_o$ ,  $I_o$ , and  $\theta_o$ .

Assuming the input voltage as the reference,

$$\phi_n = \arccos (pf_n) \quad (3.68)$$

$$\theta_n = -\phi_n \quad (3.69)$$

The apparent power input to the balancer is

$$S_n = P_n / \cos (\phi_n) \quad (3.70)$$

The remaining parameter is the input voltage:

$$V_n = -S_n / I_n \quad (3.71)$$

In summary, many more classifications for balancing power system performance could be defined. Each of these could be used for establishing the terminal voltage and current at the input and output of the balancer model. Any of these potential problems can then be resolved by using one of the above derivations. Two items are worth emphasizing--all parameters are complex (require both magnitude and angle) and the phase angles of output quantities may be defined as a phase shift or as phase difference relative to the input quantities.

### 3.6 Balancer Model Internal Design

The previous section established the system performance requirements

and the input/output parameters to the balancer model for different cases of interest. The parameters are independent of the internal components in the balancer. Regardless of what is used within, the network continues to appear the same to the power system outside the terminals of the balancer, if the input and output conditions are not violated.

This section will consider the internal design of the balancer and derive the relationships to calculate the impedances necessary to satisfy the terminal conditions. Since the balancer is similar to a filter, the number of alternatives available for the circuitry within the balancer is almost infinite. These can be mathematically reduced to a few practical designs even though a particular physical element (for example, an ideal inductance) may not be realizable. Another example is an "n" section filter which can be reduced to a "pi" or "t" section filter. It is possible that one of the elements of the reduced model would require a negative real value. This would imply negative resistance which cannot be physically realized under ordinary conditions without a source.

However, for the balancer problems under study, it is not necessary to have complicated networks within the model. The "pi" or "delta" design is sufficient for most conditions. Since delta-wye impedance conversions are well known, the delta is used as the general configuration for the balancer.

One alternative design that is convenient in some cases is the "L" section. This consists of forcing the input shunt impedance of a delta circuit to infinity and transferring all the reactive power to the output shunt impedance. For systems without series elements the input and output impedances are in parallel, so this is a totally accurate assumption. The assumption used for the "L" section is acceptable for designs

with series impedance if the series impedance is small compared to the shunt impedance.

The equations in this section are derived for a single-phase line. For a balanced three-phase system, the results for each line would be identical with the single-phase results. However, to design a balancer for unbalanced systems, the equations must be solved independently for the conditions on each phase. The results derived for each phase will be the impedance elements necessary to balance that line.

The internal impedance design of the balancer can be defined by the currents entering the input and output nodes, as illustrated in Figure 10. For a single-phase design, the following relationships are valid (16):

$$\underline{I}_n = -\underline{I}_z + \underline{I}_p \quad (3.72)$$

$$\underline{I}_o = +\underline{I}_y - \underline{I}_p \quad (3.73)$$

These can be redefined into a new set of equations using voltages rather than currents. The currents on the left-hand side are stated in terms of complex power relationships. The currents on the right-hand side are stated in terms of impedance relationships:

$$\frac{\underline{S}_n^*}{-\underline{V}_n^*} = \frac{-\underline{V}_n}{\underline{Z}_n} + \frac{(-\underline{V}_n - \underline{V}_o)}{\underline{Z}_p} \quad (3.74)$$

$$\frac{\underline{S}_o^*}{\underline{V}_o^*} = \frac{\underline{V}_o}{\underline{Z}_o} + \frac{(\underline{V}_o + \underline{V}_n)}{\underline{Z}_p} \quad (3.75)$$

Both sides of each equation are multiplied through by the denominator voltage. The left side is separated into real power and imaginary power components:



$$P_n + jQ_n^* = \frac{V_n^2}{Z_n} - \frac{(-V_n - V_o) V_n^*}{Z_p} \quad (3.76)$$

$$P_o + jQ_o^* = \frac{V_o^2}{Z_o} + \frac{(V_o + V_n) V_o^*}{Z_p} \quad (3.77)$$

Each of the equations can be separated into two equations, one for the real part and the other for the imaginary part. Equation (3.76) will be manipulated first. Define

$$\delta = \delta_n - \delta_o \quad (3.78)$$

Combine terms

$$P_n + jQ_n = \frac{V_n^2}{Z_n^*} + \frac{(V_n^2 + V_n V_o e^{j\delta})}{Z_p^*} \quad (3.79)$$

Use a common denominator

$$P_n + jQ_n = \frac{Z_p^* V_n^2 + Z_n^* (V_n^2 + V_n V_o e^{j\delta})}{Z_n^* Z_p^*} \quad (3.80)$$

Define the denominator

$$Z_n^* Z_p^* = R_l - jX_l \quad (3.81)$$

where

$$R_l = R_n R_p - X_n X_p \quad (3.82)$$

$$X_l = X_n R_p + R_n X_p \quad (3.83)$$

Then define

$$Y_{-1} = \frac{Z_n Z_p}{Z_n^2 Z_p^2} \quad (3.84)$$

$$Y_{-1} = \frac{R_1 + jX_1}{R_1^2 + X_1^2} \quad (3.85)$$

Restate

$$P_n + jQ_n = Y_{-1} (Z_p^* V_n^2 + Z_n^* V_n^2 + Z_n^* V_o V_n e^{j\delta}) \quad (3.86)$$

Expand into real and imaginary parts using Euler's rule

$$e^{j\delta} = \cos(\delta) + j\sin(\delta) \quad (3.87)$$

$$\begin{aligned} P_n &= (R_1^2 + X_1^2)^{-1} (R_1 R_p V_n^2 + R_1 R_n V_n^2 \\ &\quad + X_1 X_p V_n^2 + R_1 R_n V_n V_o \cos(\delta) \\ &\quad + R_1 X_n V_n V_o \sin(\delta) \\ &\quad - X_1 R_n V_n V_o \sin(\delta) \\ &\quad + X_1 X_n V_n V_o \cos(\delta)) \end{aligned} \quad (3.88)$$

$$\begin{aligned} Q_n &= (R_1^2 + X_1^2)^{-1} (-R_1 X_p V_n^2 - R_1 X_n V_n^2 \\ &\quad + X_1 R_p V_n^2 + X_1 R_n V_n^2 \\ &\quad + R_1 R_n V_n V_o \sin(\delta) \\ &\quad - R_1 X_n V_n V_o \cos(\delta) \\ &\quad + X_1 R_n V_n V_o \cos(\delta) \\ &\quad + X_1 X_n V_n V_o \sin(\delta)) \end{aligned} \quad (3.89)$$

A similar derivation can be made for Equation (3.77):

$$P_o + jQ_o = \frac{\underline{z}_p^* V_o^2 + \underline{z}_i^* V_o^2 + \underline{z}_o^* V_o V_n e^{-j\delta}}{\underline{z}_o^* \underline{z}_p^*} \quad (3.90)$$

Define the denominator

$$\underline{z}_o^* \underline{z}_p^* = R_{L_4} - jX_{L_4} \quad (3.91)$$

where

$$R_{L_4} = R_o R_p - X_o X_p \quad (3.92)$$

$$X_{L_4} = X_o R_p + R_o X_p \quad (3.93)$$

Then define

$$\underline{Y}_{L_4} = \frac{\underline{z}_o \underline{z}_p}{\underline{z}_o^2 \underline{z}_p^2} \quad (3.94)$$

$$\underline{Y}_{L_4} = \frac{R_{L_4} + jX_{L_4}}{R_{L_4}^2 + X_{L_4}^2} \quad (3.95)$$

Restate

$$P_o + jQ_o = \underline{Y}_{L_4} (\underline{z}_p^* V_o^2 + \underline{z}_c^* V_o^2 + \underline{z}_o^* V_o V_n e^{-j\delta}) \quad (3.96)$$

The equations are expanded into real and imaginary parts using Euler's rule similar to the derivations for the input power.

Now assume no real power is consumed in the shunt impedances:

$$R_n = R_o = 0 \quad (3.97)$$

Redefine Equation (3.81):

$$\underline{z}_1 = R_1 - jX_1 \quad (3.98)$$

$$\underline{z}_1 = (-X_n X_p) - j(X_n R_p) \quad (3.99)$$

Redefine Equation (3.91):

$$\underline{Z}_4 = R_4 - jX_4 \quad (3.100)$$

$$\underline{Z}_4 = (-X_o \ X_p) - j(X_o \ R_p) \quad (3.101)$$

Substitution of these values into the real and reactive power equations yields a set of four power equations referred to as static load flow equations (SLFE):

$$P_n = (R_p V_n^2 + R_p V_5 - X_p V_6) Y_5 \quad (3.102)$$

$$P_o = (R_p V_o^2 + R_p V_7 - X_p V_8) Y_5 \quad (3.103)$$

$$Q_n Y_5 = \left( \frac{R_p^2 + X_p^2}{X_n X_n} + X_p \right) V_n^2 + X_p V_5 + R_p V_6 \quad (3.104)$$

$$Q_o Y_5 = \left( \frac{R_p^2 + X_p^2}{X_o X_o} + X_p \right) V_o^2 + X_p V_7 + R_p V_8 \quad (3.105)$$

where

$$V_5 = V_n V_o \cos(\delta) \quad (3.106)$$

$$V_6 = V_n V_o \sin(\delta) \quad (3.107)$$

$$V_7 = V_n V_o \cos(-\delta) \quad (3.108)$$

$$V_8 = V_n V_o \sin(-\delta) \quad (3.109)$$

$$Y_5 = (X_p^2 + R_p^2)^{-1} \quad (3.110)$$

The four equations are calculated in terms of four impedances-- shunt input reactance, shunt output reactance, series resistance, and series reactance. These are nonlinear equations. Solution to determine the four impedances would be a tedious chore. Two simplifying assump-

tions will reduce the solution to exact linear impedances. The assumptions are determined by the type of problem under consideration. Two cases of interest will be considered in detail.

### 3.6.1 Derivation of Line Balancer Model

The line balancing model assumes a lossless, nonresistive design. The series resistance is driven to zero. Series reactance can be determined from the real power equations:

$$R_p = 0 \quad (3.111)$$

$$P_n = -V_6/X_p \quad (3.112)$$

$$X_p = -V_6/P_n \quad (3.113)$$

The reactive power equations yield the shunt reactances:

$$Q_n = \frac{V_n^2}{X_n} + \frac{V_n^2}{X_p} + \frac{V_6}{X_p} \quad (3.114)$$

$$Q_o = \frac{V_o^2}{X_n} + \frac{V_o^2}{X_p} + \frac{V_7}{X_p} \quad (3.115)$$

Then

$$X_n = \frac{X_p V_n^2}{X_p Q_n - V_n^2 - V_5} \quad (3.116)$$

$$X_o = \frac{X_p V_o^2}{X_p Q_o - V_o^2 - V_7} \quad (3.117)$$

The three impedances that solve the line balancer are calculated. All the impedances are reactive. No power is consumed in the balancer network.

### 3.6.2 Derivation of Single-Phase to Balanced Three-Phase Balancer

The creation of a balanced three-phase system from a single-phase line requires real power to be transferred. This implies that the series or pass impedance must be real. For this condition the series reactance must be zero.

In a physical realization any reactive component of the series impedance can be eliminated by adding a suitable parallel (or series) reactance. For example, capacitors can be used to counter the effect of inductive reactances of machines.

With the series reactance eliminated, a linear solution to the static load flow equation can be used to determine the impedance elements necessary to balance the single-phase line onto a three-phase system:

$$X_p = 0 \quad (3.118)$$

The real power equation yields the series resistance:

$$P_n = \frac{V_n^2 + V_7}{R_p} \quad (3.119)$$

$$R_p = \frac{V_n^2 + V_7}{P_n} \quad (3.120)$$

The reactive power equations yield the shunt reactances:

$$Q_n = \frac{V_n^2}{X_n} + \frac{V_6}{R_p} \quad (3.121)$$

$$X_n = \frac{R_p V_n^2}{R_p Q_n - V_6} \quad (3.122)$$

$$Q_o = \frac{V_o^2}{X_o} + \frac{V_\delta}{R_p} \quad (3.123)$$

$$X_o = \frac{R_p V_o^2}{R_p Q_o - V_\delta} \quad (3.124)$$

The solution of the equations, with some of the impedance elements driven to zero, will yield theoretically pure impedances. However, physical devices are never pure. Capacitors have bleeder resistors and the wire of an inductor has some resistance, but the ideal devices should be realized as close as possible. The actual values are then used as elements for a correlation between theoretical and experimental (physical) voltages and currents. The effect of using actual elements on the system can thus be ascertained.

The comparison requires that the three internal impedances be known as well as one other parameter--either voltage or current. Three combinations can be used to observe most systems. These require voltage line-to-line, line current, or phase current.

### 3.7 Example of Line Balancer Calculation

Through a series of measurements, the voltage magnitude and angles with current magnitude and angles can be determined for an unbalanced three-phase system. To balance the system, impedances can be placed in the lines. The values of these impedances are determined from the balancer model (Figure 12). The values are phase voltages and line currents:

|                 |                            |                   |                             |
|-----------------|----------------------------|-------------------|-----------------------------|
| $V_{ao} = 100V$ | $\delta_{ao} = 0^\circ$    | $I_{ao} = 5A$     | $\theta_{ao} = 60^\circ$    |
| $V_{bo} = 100V$ | $\delta_{bo} = -125^\circ$ | $I_{bo} = 5.156A$ | $\theta_{bo} = -47.6^\circ$ |
| $V_{co} = 100V$ | $\delta_{co} = 115^\circ$  | $I_{co} = 6A$     | $\theta_{co} = 185^\circ$   |

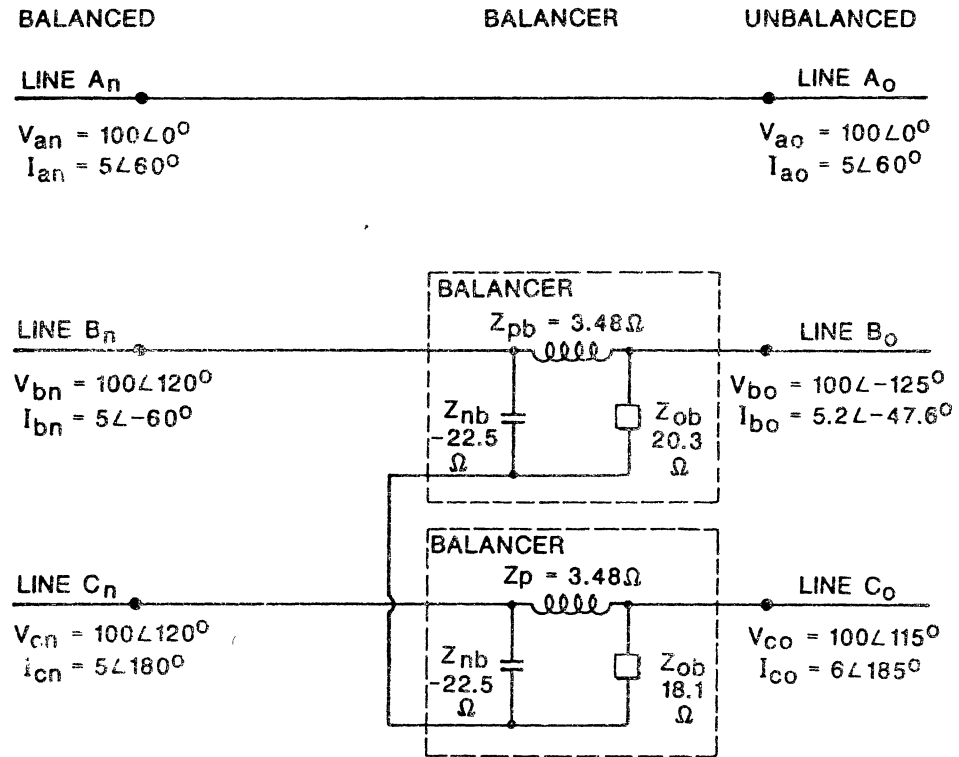


Figure 12. Three-Phase Line Balancing by Impedance Example



Determine the power at each node:

$$S_o = V_o I_o \quad (3.10)$$

$$\phi_o = \delta_o - \theta_o$$

$$P_o = S_o \cos (\phi_o)$$

$$Q_o = S_o \sin (\phi_o)$$

$$P_{ao} = (100)(-5.00) \cos (-60.0) = -250 \text{ watts}$$

$$Q_{ao} = (100)(-5.00) \sin (-60.0) = 433 \text{ vars}$$

$$P_{bo} = (100)(-5.15) \cos (-77.4) = -112 \text{ watts}$$

$$Q_{bo} = (100)(-5.15) \sin (-77.4) = 503 \text{ vars}$$

$$P_{co} = (100)(-6.00) \cos (-70.0) = -205 \text{ watts}$$

$$Q_{co} = (100)(-6.00) \sin (-70.0) = 564 \text{ vars}$$

To balance this system, lines "b" and "c" must have a voltage phase shift of five degrees assuming that line "a" is the reference:

$$\delta_b = \delta_{bn} - \delta_{bo} \quad (3.78)$$

$$= -120 + 125 = 5^\circ$$

$$\delta_c = \delta_{cn} - \delta_{co} \quad (3.78)$$

$$= 120 - 115 = 5^\circ$$

The current magnitudes must be equal:

$$V_{an} = 100V \quad \delta_{an} = 0^\circ \quad I_{an} = 5A \quad \theta_{an} = 60^\circ$$

$$V_{bn} = 100V \quad \delta_{bn} = -120^\circ \quad I_{bn} = 5A \quad \theta_{bn} = -60^\circ$$

$$V_{cn} = 100V \quad \delta_{cn} = 120^\circ \quad I_{cn} = 5A \quad \theta_{cn} = 180^\circ$$

Determine the power for balanced operation:

$$S_{an} = V_{an} I_{an} \quad (3.8)$$

$$= (-100)(5) = -500 \text{ VA}$$

$$\begin{aligned}\phi_{an} &= \delta_{an} - \theta_{an} \\ &= 0 - 60 + 180 = 120^\circ\end{aligned}$$

$$\begin{aligned}P_{an} &= S_{an} \cos(\phi_{an}) \\ &= (-500) \cos(120) = 250 \text{ watts}\end{aligned}$$

$$\begin{aligned}Q_{an} &= S_{an} \sin(\phi_{an}) \\ &= (-500) \sin(120) = -433 \text{ vars}\end{aligned}$$

$$P_{an} = P_{bn} = P_{cn}$$

$$Q_{an} = Q_{bn} = Q_{cn}$$

The input and output voltages are related by their phase angle differences:

$$V_5 = V_n V_o \cos(\delta) \quad (3.106)$$

$$V_{b5} = (-100)(100) \cos(5) = -9962$$

$$V_{c5} = (-100)(100) \cos(5) = -9962$$

$$V_6 = V_n V_o \sin(\delta) \quad (3.107)$$

$$V_{b6} = (-100)(100) \sin(5) = -872$$

$$V_{c6} = (-100)(100) \sin(5) = -872$$

$$V_7 = V_n V_o \cos(-\delta) \quad (3.108)$$

$$V_{b7} = (-100)(100) \cos(-5) = -9962$$

$$V_{c7} = (-100)(100) \cos(-5) = -9962$$

The impedance that is placed in series with the line can be determined. This is dependent on the real power to be transferred. For a lossless design:

$$R_p = 0 \quad (3.111)$$

$$X_p = -V_6/P_n \quad (3.113)$$

$$X_{bp} = 872/250 = 3.488 \text{ ohms}$$

$$X_{cp} = 872/250 = 3.488 \text{ ohms}$$

The shunt impedances for the balancers are dependent on the reactive power of the system. The input shunt impedance is calculated:

$$X_n = \frac{X_p V_n^2}{X_p Q_n - V_n^2 - V_5} \quad (3.116)$$

$$X_{bn} = \frac{(3.488)(100)(100)}{(3.488)(-433) - (100)(100) + 9962} = -22.5 \text{ ohms}$$

$$X_{cn} = \frac{(3.488)(100)(100)}{(3.488)(-433) - (100)(100) + 9962} = -22.5 \text{ ohms}$$

The output shunt impedances match to the existing unbalanced conditions:

$$X_o = \frac{X_p V_o^2}{X_p Q_o - V_o^2 - V_7} \quad (3.117)$$

$$X_{bo} = \frac{(3.488)(100)(100)}{(3.488)(503) - (100)(100) + 9962} = 20.3 \text{ ohms}$$

$$X_{co} = \frac{(3.488)(100)(100)}{(3.488)(564) - (100)(100) + 9962} = 18.1 \text{ ohms}$$

All the impedances are available to balance a three-phase system.

### 3.8 Example of Single-Phase to Balanced

#### Three-Phase Calculation

For a balanced system the line-to-line voltages must be the same magnitude. This is determined by the power system. For the single-phase load, the voltage across the load and current through the load can

be measured. This provides all the information necessary to balance the load on a three-phase system:

$$\begin{aligned} V_p &= 100V & \delta_p &= 0^\circ \\ I_p &= 17.3V & \theta_p &= 0^\circ \end{aligned}$$

The load must be converted to a resistance for real power flow. Even if the load is inductive, capacitance can be shunted across it to achieve an equivalent resistance.

For a balanced system, the other two line voltages are required to be equal in magnitude to the load voltage. However, they must be separated by 120 degrees:

$$\begin{aligned} V_n &= 100V & \delta_n &= 120^\circ \\ V_o &= 100V & \delta_o &= -120^\circ \end{aligned}$$

The real single-phase load will become a three-phase load under balanced conditions (see Figure 13):

$$\begin{aligned} 1.732 V_L I_L &= V_p I_p \\ V_L &= V_p \\ I_L &= I_p / 1.732 \end{aligned}$$

but

$$\begin{aligned} I_n &= 10A & \theta_n &= -30^\circ \\ I_o &= 10A & \theta_n &= 90^\circ \\ I_u &= 10A & \theta_n &= -150^\circ \end{aligned}$$

The power at the balancer terminals can be calculated:

$$\begin{aligned} S_n &= V_n I_n \\ &= (100)(10) = 1000 \text{ VA} \end{aligned}$$

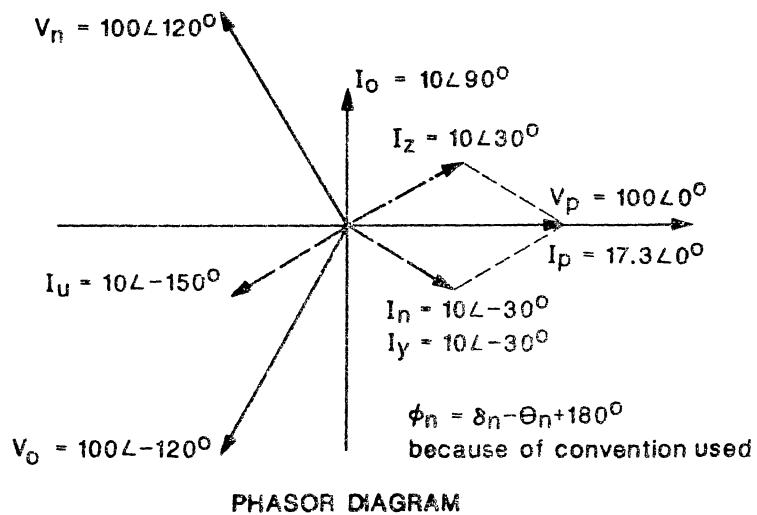
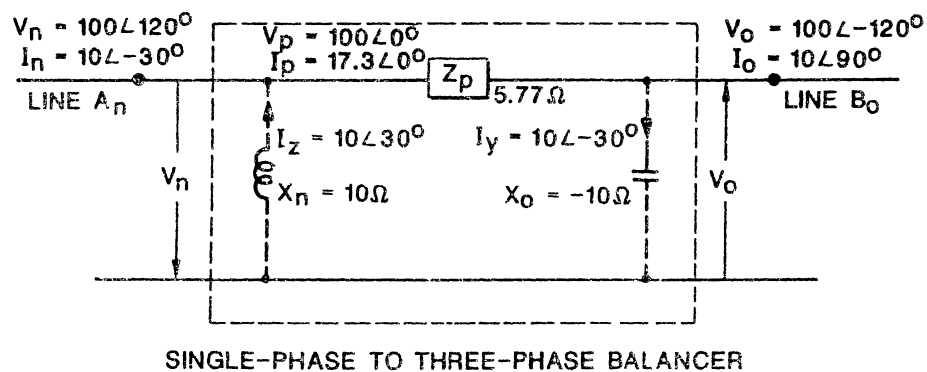
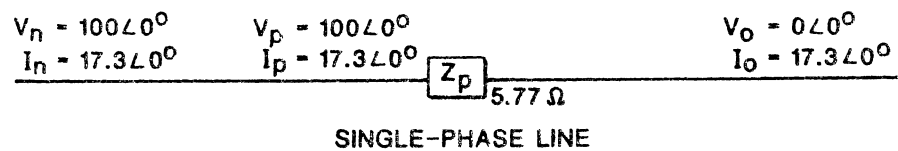


Figure 13. Single-Phase to Three-Phase Balancer Example

$$\begin{aligned}\phi_n &= \delta_n - \theta_n + 180 \\ &= 120 + 30 + 180 - 330^\circ\end{aligned}$$

$$\begin{aligned}P_n &= S_n \cos(\phi_n) \\ &= 1000 \cos(330) = 866 \text{ watts}\end{aligned}$$

$$\begin{aligned}Q_n &= S_n \sin(\phi_n) \\ &= 1000 \sin(330) = -500 \text{ vars}\end{aligned}$$

$$\begin{aligned}P_o &= V_o I_o \cos(\delta_o - \theta_o) \\ &= (100)(10) \cos(-120 - 90) = -866 \text{ watts}\end{aligned}$$

$$\begin{aligned}Q_o &= V_o I_o \sin(\delta_o - \theta_o) \\ &= (100)(10) \sin(-120 - 90) = 500 \text{ vars}\end{aligned}$$

The balancer input and output voltage and phase angles have the following relationship:

$$\delta = \delta_n - \delta_o$$

$$\delta = 120 + 120 = 240^\circ$$

$$V_6 = V_n V_o \sin(\delta)$$

$$V_6 = (100)(100) \sin(240) = -8666$$

$$V_7 = V_n V_o \cos(-\delta)$$

$$V_7 = (100)(100) \cos(-240) = -5000$$

$$V_8 = V_n V_o \sin(-\delta)$$

$$V_8 = (100)(100) \sin(-240) = +8666$$

With all the terminal values of power, current, and voltage established, the impedance necessary to balance the load can be calculated. Although it is not an added impedance, the equivalent load impedance

value can be found from the load current and voltage. Since these were forced to real values, the load impedance does not have a reactive component:

$$X_p = 0 \quad (3.118)$$

Then the equivalent resistance is calculated:

$$R_p = \frac{V_n^2 + V_7^2}{P_n} \quad (3.120)$$

$$R_p = \frac{(100)(100) - 5000}{866} = 5.77 \text{ ohms}$$

The shunt reactances are the impedance elements necessary to balance the load. Their values can be calculated:

$$X_n = \frac{R_p V_n^2}{Q_n R_p - V_6^2} \quad (3.122)$$

$$X_n = \frac{5.77 (100)(100)}{(-500)(5.77) + 8666} = 10 \text{ ohms}$$

$$X_o = \frac{R_p V_o^2}{Q_o R_p - V_8^2} \quad (3.123)$$

$$X_o = \frac{(5.77)(100)(100)}{(500)(5.77) - 8666} = -10 \text{ ohms}$$

This provides all the impedance values necessary to balance the single-phase load. The shunt impedances are equal in magnitude but are opposite reactances.

## CHAPTER IV

### EXPERIMENTAL DEVELOPMENT

#### 4.1 Introduction

Mathematical models are desirable for simulation of engineering problems, because they can be performed on paper or in computers without investing in expensive equipment, operating in complex environments or manipulating large quantities of power. Since the model is mathematical, some conditions can be projected that are not strictly realizable in the physical environment. There is also the potential for limitations or errors that will not permit the model to work in all real situations. For these and other reasons, it is necessary to ultimately build a physical device and experimentally verify the key results of simulation.

The purpose of this chapter is to present the results of experimental investigations undertaken and to discuss their implications. A series of experiments were conducted to verify the models developed. Most of the tests were conducted at multiple shaft speeds since operating speed controls the power output of an induction generator.

Experiments were conducted for the following conditions:

1. Balanced operation to verify machine model.
2. Balancing currents in three-phase lines:
  - a. Unbalanced line resulting from unknown impedance in line A.



- b. Unbalanced line balanced by inserting impedances in lines B and C.
3. Balancing reactive power flow:
  - a. Driver running, generator off, capacitors across terminals (overexcitation).
  - b. Driver running, generator on, capacitors across terminals (power factor correction).
4. Balancing single-phase to three-phase:
  - a. Three-phase generator operating on a single-phase line (line C disconnected) without a balancer.
  - b. Single-phase resistive load balanced on three-phase line.
  - c. Three-phase generator balanced on single-phase line.

To verify the balancer model, a laboratory setup was fabricated.

The hardware used for the experiments is listed below:

Generator-induction motor, 3Hp, 1130 RPM, NEMA D, three-phase, 230/460/796 volt, 9.4/4.7/2.7A, max. cap. 2.7 KVAR, Eff. 0.84, PF 0.67, mfg. Marathon Electric.

Driver-induction motor, 5Hp, 1160 RPM, NEMA B, three-phase, 230/460 volt, 14.7/7.35 A, max. cap. 3.2 KVAR, Eff. 0.85, PF 0.75, mfg. Marathon Electric.

Speed control-variable speed drive, 5Hp, 230/460 volt, frequency 0-90 Hz, mfg. Louis Allis, Model: Lancer, Jr.

Capacitor contactor--3 pole, 480 volt, 10 amp, Westinghouse Industrial control relay.

Fuse-disconnect switch, 3 pole, fused 30 amp, 240 volt, General Electric.

Capacitors, 480 volt, 20 microfarad and 30 microfarad series, parallel connected in 3 KVAR, 3-phase assembly, Elliott Industries.

Inductor, 10 amp, 100 millihenry, 4-100 turn windings, hand wound.

Inductors-autotransformer, 2 KVA, 120/240 16/32 volt, dry type, Square D.

Resistors-heat strips and lamps connected through 120/240, 2 KVA transformer.

The instrumentation used for the experiments is listed below:

Torque transducer-strain gage, 1000 lb-in., max. 15000 RPM, with 60 teeth per revolution speed pick-up, Lebow Products, Model 1104.

Torque display, 3 digit, calibrated, 120 volt, DJ Instruments, Model 415.

Speed display, 4 digit, 120 volt, 0-1999 Hz LFE Process Control, Model 4424.

Phase angle meter, 4 digit, reference and signal input, 120 volt, Keltronics Lab Model.

Power factor meter, 3 digit, 1- or 3-phase, transformer coupled current, direct coupled potential, 0-600 volt, 0-100 amp displays, 120 volt, Amprobe.

Analyzers, three-phase power system displays for volts, amps, kilowatts, and power factor, Westinghouse Power Analyzer.

Volt ohm meter, analog, Triplett.

Ammeter, clamp-on type, 0-100 amps, Amprobe RS-3.

Computer, 48K, disk drive, printer, Apple II, with BASIC.

The machinery was connected to a 120/208 volt, four-wire, three-phase power system through the fused disconnect switch. This allowed safety

protection and isolation. The generator was connected back to the power system through the same circuit.

Section 4.2 summarizes the experimental results and compares them with model data for balanced system operation. Section 4.3 illustrates balancing currents in three lines of the three-phase system. Section 4.4 introduces balancing reactive power flow. Section 4.5 presents balancing a single-phase line to operate as a three-phase line.

Table I is a tabulation of results from the experiments. The Appendix contains samples of theoretical results calculated using the model equations. The experimental setup is shown in Figures 14 through 19.

Figure 14 is a photograph of the autotransformers used in series with the lines to balance current flow as described in section 4.3. The heaters and lamps in the foreground were used to devise a single-phase resistance load for the second part of the experiment described in section 4.5. The capacitors shown in Figure 15 were used to correct power factor and for the capacitive reactance in the third part of the experimental setup and model described in section 4.5. The variable speed drive controller shown in Figure 16 was used in all the experiments involving the induction generator. It was used to control the speed of the motor used to drive the induction generator. The meters and instrumentation used for the various experiments are shown in Figure 17. A photograph of the machines is shown in Figure 18. The driving motor is on the left. Coupling the two machines is the torque transducer and speed pick-up. The induction generator is shown on the right. The inductive reactance used in the third experiment discussed in section 4.5 is shown in Figure 19. The Westinghouse Analyzers and Triplett volt-ohm meter are also pictured.

TABLE I  
EXPERIMENTAL RESULTS

| Case<br>RPM | Torque<br>In.-Lb | Volts L-L       |                 |                 | Angle (V = ref)    |                    | Line Amps      |                |                | PF x 100        |                 |
|-------------|------------------|-----------------|-----------------|-----------------|--------------------|--------------------|----------------|----------------|----------------|-----------------|-----------------|
|             |                  | V <sub>ab</sub> | V <sub>bc</sub> | V <sub>ca</sub> | V <sub>b</sub> deg | V <sub>c</sub> deg | I <sub>a</sub> | I <sub>b</sub> | I <sub>c</sub> | PF <sub>a</sub> | PF <sub>c</sub> |
| 1 1186      | 66               | 215             | 215             | 215             | -119.9             | 119.8              | 6.3            | 6.3            | 6.4            | 50              | 50              |
| 1220        | -73              | 215             | 215             | 215             | -119.8             | 119.5              | 6.4            | 6.5            | 6.6            | 47              | 47              |
| 1243        | -161             | 215             | 215             | 215             | -119.8             | 119.6              | 9.4            | 9.4            | 9.7            | 64              | 64              |
| 2 1230      | -97              | 218             | 216             | 210             | -123.6             | 115.1              | 6.1            | 8.4            | 7.8            | 55              | 65              |
| 1240        | -134             | 220             | 216             | 210             | -124.3             | 114.6              | 7.3            | 9.9            | 9.1            | 65              | 72              |
| 3 1230      | -101             | 214             | 216             | 214             | -119.8             | 118.7              | 7.4            | 7.4            | 8.0            | 60              | 54              |
| 1240        | -144             | 216             | 219             | 216             | -119.9             | 118.3              | 9.0            | 9.0            | 9.6            | 69              | 63              |
| 4 1220      | -19              | 248             | ---             | ---             | ---                | ---                | ---            | ---            | ---            | ---             | ---             |
| 1240        | -21              | 260             | ---             | ---             | ---                | ---                | ---            | ---            | ---            | ---             | ---             |
| 5 1220      | -69              | 216             | 216             | 216             | -119.8             | 119.5              | 2.7            | 2.7            | 2.8            | 85              | 85              |
| 1240        | -52              | 216             | 216             | 216             | -119.8             | 119.5              | 6.1            | 6.1            | 6.3            | 99              | 99              |
| 6 1220      | -50              | 214             | 175             | 206             | -120.9             | 0                  | 10.0           | 10.0           | 0              | -28             | -28             |
| 1230        | -85              | 214             | 171             | 214             | -120.4             | 0                  | 11.9           | 11.9           | 0              | ---             | ---             |
| 6A1220      | -63              | 215             | 221             | 231             | -119.6             | 0                  | 4.1            | 4.1            | 0              | 95              | 95              |
| 1238        | -139             | 215             | 212             | 246             | -119.6             | 0                  | 9.4            | 9.4            | 0              | -99             | -99             |
| 7 ---       | ---              | 216             | 218             | 216             | -121.2             | 120.8              | 4.9            | 4.1            | 4.7            | 99              | 99              |
| ---         | ---              | 216             | 218             | 215             | -119.5             | 119.5              | 7.7            | 6.5            | 7.4            | 99              | 99              |
| 8 1220      | -51              | 219             | 234             | 192             | -119.8             | 100.4              | 6.7            | 3.3            | 7.3            | -99             | -95             |
| 1247        | -153             | 219             | 219             | 215             | -119.3             | 116.8              | 6.6            | 6.6            | 7.3            | -99             | -99             |



Figure 14. Autotransformers and Resistance Load Used in the Experiment

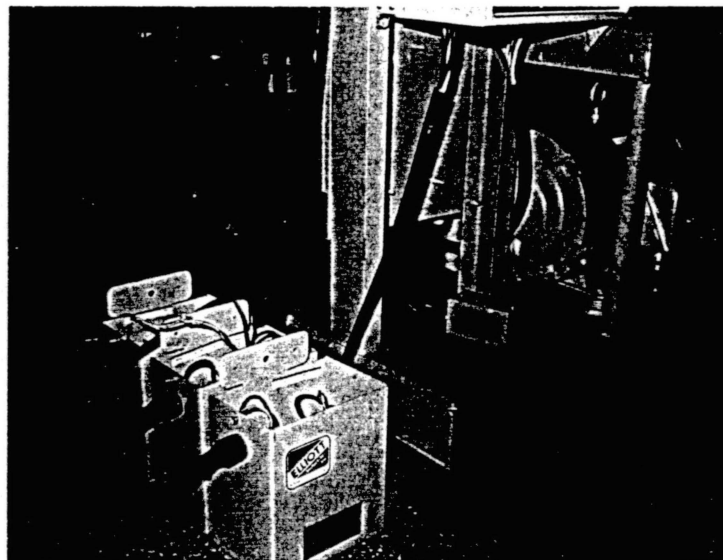


Figure 15. Capacitors Used in the Experiment



Figure 16. Variable Speed Drive Controller

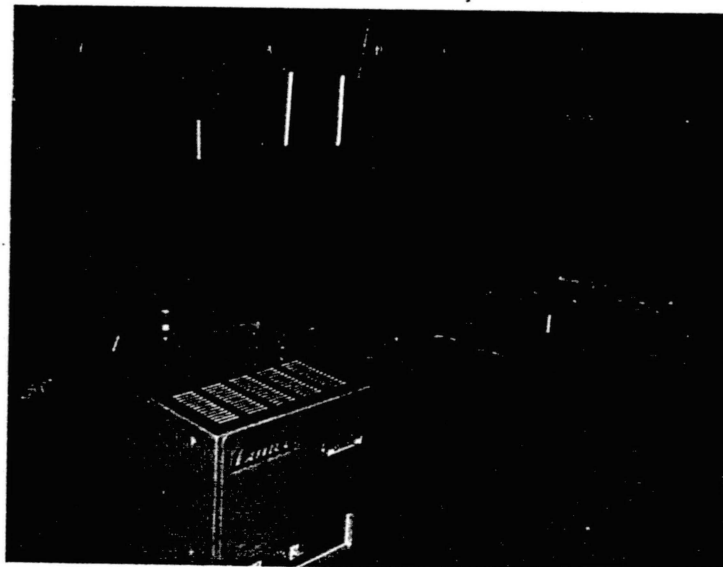


Figure 17. Meters and Instrumentation

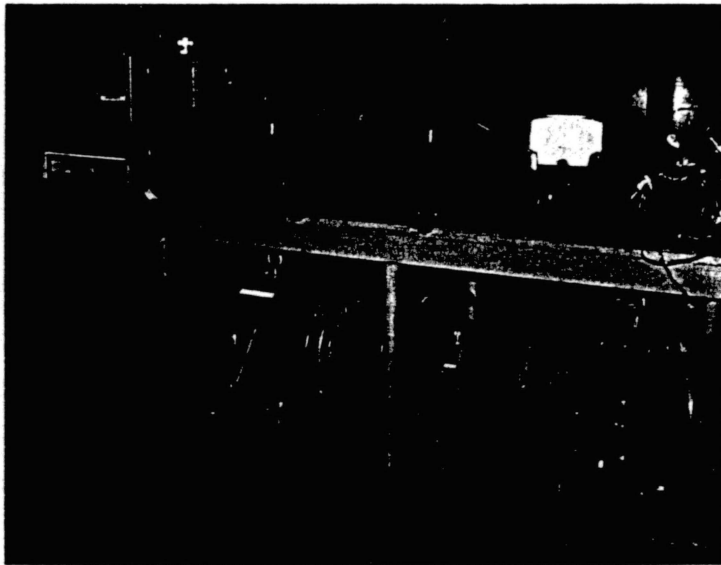


Figure 18. Motor Driver Coupled to the Induction Generator



Figure 19. Instrumentation and Inductive Reactance Used

## 4.2 Balanced System Experiments

From Chapter 11, the model for the induction machine requires parameters for stator and rotor resistances and leakage reactances. Typically, motor reactances are 5 to 10 percent greater than the reactances for the same machine operating as a generator. Analysis of the equivalent circuit shown in Figure 2 indicates that this is contributed to by the differences in internal voltages. For a fixed terminal voltage, since the internal voltages change, the same terminal current would represent different internal impedances and power. For generator operation, the increased speed causes increased generated voltages. This in turn changes the flux density and saturation of the magnetic circuit, resulting in changes in inductance.

The equivalent circuit impedances for the machine are listed below. These were obtained from calibration tests performed by the manufacturer to IEEE Standard 112 (17). The same machine is used but it is operating in different modes:

|                          | <u>Motor</u> | <u>Generator</u> |
|--------------------------|--------------|------------------|
| Stator resistance        | 0.022 pu     | 0.022 pu         |
| Rotor resistance         | 0.025 pu     | 0.025 pu         |
| Stator leakage reactance | 0.073 pu     | 0.070 pu         |
| Rotor leakage reactance  | 0.059 pu     | 0.056 pu         |
| Magnetizing reactance    | 0.801 pu     | 0.730 pu         |
| Base impedance           | 70.939 ohm   | 70.939 ohm       |

Figure 20 shows the performance characteristics of the three-phase induction machine operating under balanced conditions as a motor obtained from calibration data. Figure 21 illustrates the correlation between theoretical (model) and experimental results for balanced operation of



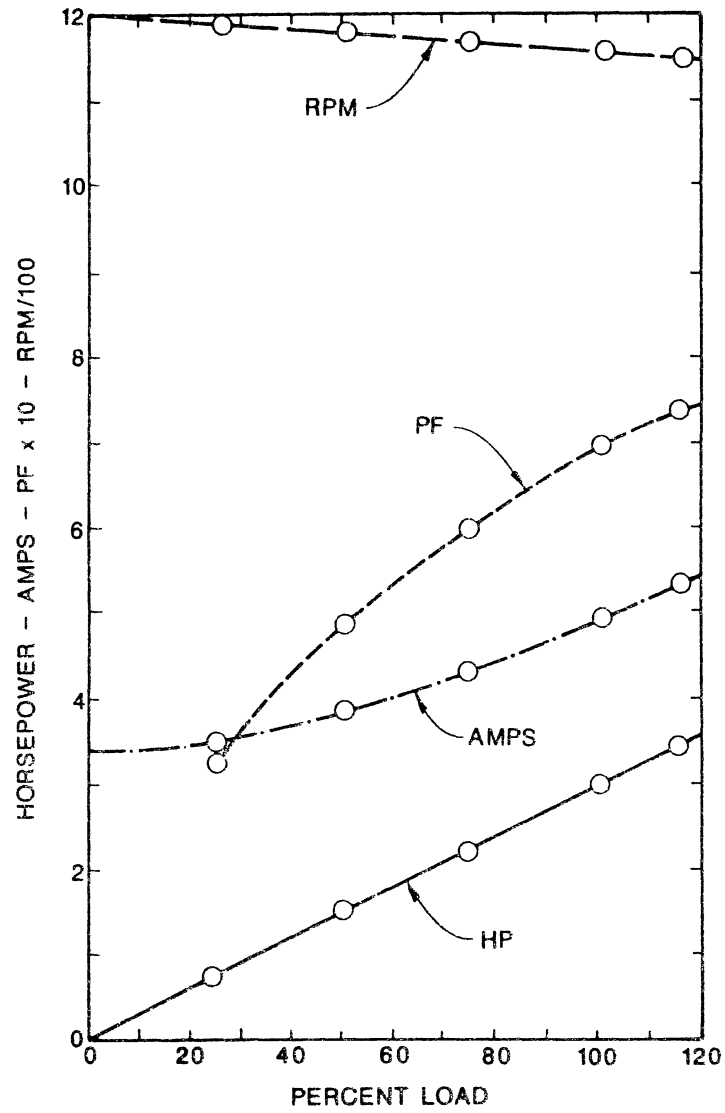


Figure 20. Experimentally Obtained Machine Performance Curves

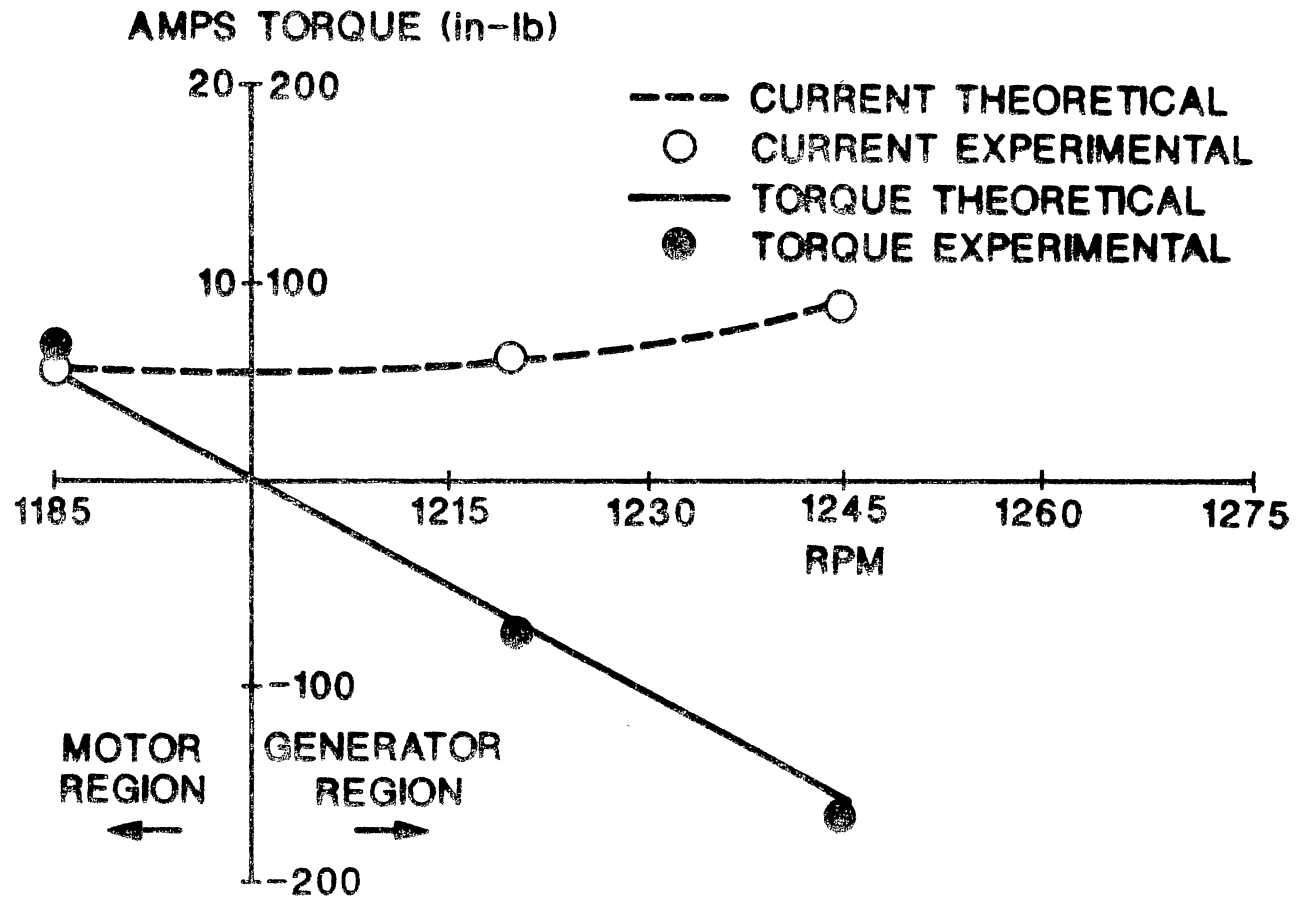


Figure 21. Machine Model Verification Under Balanced Conditions

the machine as a motor and generator. For balanced operation, it is apparent the equivalent circuit model is quite accurate. Experimental and theoretical values of line currents agree very closely for both generator and motor operation.

The transfer of power from the driver to the generator was determined from the torque transmitted through the shaft. Measured torque values vary consistently from theoretically calculated values. The model results are about 8 percent lower than the measured values for generator operation. The torque measurements for motor operation have a similar response. The model values could be made virtually identical by increasing speed 7 rpm.

This variation is apparently due to the same conditions of internal voltage increase, current change, and flux density change that cause a difference in reactance values between the generator and motor operation. It may also be due to a calibration error between the resistance bridge of the torque transducer and the electronic readout that displays the values. However, the display was calibrated against a known resistance. Nevertheless, the constant offset does have characteristics of calibration problems.

The measured line currents for the machine under balanced operation on a three-phase system were not exactly the same for all three phases. By rotating the machine connections between lines, the imbalance could be moved from one phase to the other. It was greater with some of the connections than with others. This indicates both the machine and the power system had some inherent imbalance.

### 4.3 Balancing Currents in Three-Phase Lines

This experiment was conducted in two parts. The first segment was conducted with an "unknown" inductance inserted in series with line A of the generator. The inductance was obtained by using the winding of an autotransformer. The addition of this impedance caused unbalanced currents to exist in the three-phase line.

#### 4.3.1 Experimental Setup and Model

The unbalanced currents, voltages, and angles were measured and used as input to the symmetrical component model. The currents in the two undisturbed lines were not identical. This was apparently due to the inherent imbalance noted earlier. This relative imbalance did not appreciably affect the machine performance.

The current imbalance amounted to a 35 percent difference between the lowest and the highest values of line currents (7.3 amps to 9.9 amps). This great an imbalance would be expected to have a very dramatic impact on the machine through reduced torque and negative sequence heating. The model data are shown in the Appendix as Case 2 Generator 1240 rpm.

The torques from the symmetrical component model and experimental measurements compared very favorably. The model predicted a value of 129.5 inch-pounds while the measured results from Table 1, Case 2 at 1240 rpm, was 134 inch-pounds.

The second segment of the experiment was to determine a balancer to be placed in the lines to cause equal currents (magnitudes) to flow in all three phases. For 1240 rpm operation, the desired line current was selected to be 8.7 amps. This is the line current on the input side of the balancer.

The impedance of the model must be calculated independently for each line of the machine. The output current for line B was selected as 9.1 amps, the current of the machine when unbalanced. The output current for line C was 9.9 amps.

An L section network was used for the balancer. The same value of series impedance was required to balance each line. This was because the voltage phase shift required to balance the phases was the same on both lines B and C. From Table 1, Case 2 at 1240 rpm, it is noted that both lines B and C have shifted approximately 4.5 degrees from the nominal values of Case 1. The value of inductive reactance required was 1.72 ohms. This is shown in the Appendix as Case 3, Line Balancer.

The shunt impedances required for balanced operation on each line were different. Although the voltage magnitudes were identical, the power factor on lines B and C were different. The shunt impedance primarily reflects the reactive power necessary to balance the reactive component of current. Line A was selected as the reference phase. There was no phase shift in line A. It was not necessary to add a series impedance in line A.

#### 4.3.2 Experimental Results

The L section balancers were placed in lines B and C. Identical auto transformers were used to obtain the balancing impedance as were used to cause the imbalance. These should have created a perfectly balanced system with the impedances matched. However, from the voltage measured across and the current through each autotransformer, the impedance was calculated as different values. Line B impedance was calculated to be 1.38 ohms and line C was calculated to be 1.04 ohms.

Line B balanced almost perfectly with line A. The current magnitudes were the same and the angle deviation from the theoretical 120 degrees was less than 0.2 degrees. The magnitude of 9 amps was slightly higher than the projected magnitude of 8.7 amps. This is within the expected accuracy of instrumentation and device impedances.

Line C did not balance as closely. The current was slightly higher in magnitude than in lines A and B. The residual imbalance is attributed to the inherent imbalance observed in the first experiment and the fact that the balancing impedance value was not as close as expected. Table II illustrates the actual line currents under the different operating conditions.

TABLE II  
THREE-PHASE LINE BALANCER VERIFICATION

| Quantity (Amps)        | Line A | Line B | Line C |
|------------------------|--------|--------|--------|
| Normal Running         | 8.7    | 8.7    | 8.9    |
| Unbalanced Z in Line A | 7.3    | 9.9    | 9.1    |
| Theoretical            | 8.7    | 8.7    | 8.7    |
| Experimental           | 9.0    | 9.0    | 9.6    |

#### 4.4 Balancing Reactive Power Flow

This experiment was conducted in two parts--the generator isolated from the line and the generated connected to the line. The first segment was

the observation of the machine performance when the generator was isolated from the line. The machine was driven by the shaft and three one-KVAR capacitors were connected across the terminals. The machine became self-excited under these conditions.

There was no load. Line current did not exist. However, the capacitor/machine combination became self-excited. Terminal voltages increased from 215 volts to 260 volts as the machine speed was increased. The overexcitation was so great that the generator could not be loaded with the capacitors across the line. The overvoltage protection would de-energize the controller. It was necessary to de-energize the terminal capacitors before loading the induction generator. When the capacitors were removed, the unloaded speed increased by five rpm.

The equivalent circuit model assumes generation of real power and the voltage is supplied by the line. It was not used to predict overexcitation from a capacitive load. This should be one of the subjects of future study.

The second part of the experiment was conducted with the generator connected to the line. Again the machine was driven by the shaft and capacitors were connected to the terminals. The power factor was to be corrected to unity. The actual capacitance used was 0.81 KVAR. This is a very good correlation. The differences can be attributed to the accuracy of the measurements.

The model results using real power requirements and power factor with no change in voltage angles exactly match published power factor tables (17). For a 100 KW load, an original power factor of 0.60, and a desired unity power factor, the computed reactive power is 133 KVAR.

The sequence for starting the generator must be as follows: bring the driver to near synchronous speed, place the generator on line, then connect the capacitors. The shutdown sequence must remove the capacitors from the terminals before the generator is allowed to rotate unloaded.

## 4.5 Balancing Single-Phase to Three-Phase

### 4.5.1 Experimental Setup and Model

This experiment was conducted in three parts--measurement of conditions on the generator with one line disconnected, balancing single-phase to three-phase for a purely resistive load, and balancing single-phase to three-phase for the induction generator.

The first part is to measure the performance parameters of the machine with line C disconnected. This is the worst case of imbalance that can exist. The two remaining lines constitute a single-phase condition. Even with one line removed, line-to-line voltage continues to exist on that terminal although they are grossly unbalanced, as shown in Table I, Case 6.

Experimental data were taken at only two speeds. Since line currents were excessive, the machine was overheating and vibration/noise problems were encountered. The symmetrical component equivalent circuit model of Chapter II was used to determine the impact of this operation on the machine. Negative sequence heating is the best model measurement for detrimental effects on the generator. The model indicates the machine suffers from significant negative sequence heating. The results are shown in the Appendix as Case 6.



A comparison of theoretical and experimental torques is shown in Table III. Although there is some variation between values, the results compare adequately. The experimental values were slightly more negative throughout the experiments as noted previously.

TABLE III  
COMPARISON OF SYMMETRICAL COMPONENTS: CALCULATED  
TORQUE AND MEASURED TORQUE

| Quantity<br>(In.-Lb) | Torque<br>(1220 RPM) | Torque<br>(1230 RPM) |
|----------------------|----------------------|----------------------|
| Theoretical          | -45                  | -75                  |
| Experimental         | -50                  | -85                  |

Single-phase operation of a three-phase machine can be very detrimental to the machine. It is desirable to try to correct this problem. However, so many potential problems exist with accurately modeling rotating machines that the tests were first conducted on a purely resistive load.

The second part of the experiment was to determine if a purely resistive single-phase load could be made to appear as a balanced three-phase load. The single-phase load was a group of heaters and lamps connected across lines A and B of the three-phase system. A potential transformer was used to reduce the line voltage of 208 volts to the heater rated voltage of 120 volts. Because only fixed values of balancer

impedances were available, tests were conducted at only two different line current values.

The balancer model was used to calculate the impedances necessary to make the single-phase load appear as a balanced three-phase load on the power system. Criteria listed in the assumptions of Chapter III are re-emphasized. Only passive devices are allowed and the line currents must be balanced.

The model indicates the voltages are precisely equal and 120 degrees apart. The currents in all three lines are identical and 120 degrees apart. Appendix Case 7 presents the model results. For the first experiment, line currents were selected at 4.3 amps. The second experiment was at 6.9 amps. The model predicts the current through the shunt impedances is equal to the line currents and the current through the load impedance is 1.732 times the load impedance. The impedances from Case 7 at 6.9 amps are compared. These are 31.3 ohms for both the shunt reactances.

The actual capacitive reactance was 31 ohms and the inductive reactance was 31 when calculated from the voltage and currents. The values were so close since series/parallel combinations of capacitors were made and the inductor winding turns were changed until the proper values were obtained.

The results of balancing the resistive load were good enough that the third part of the experiment was conducted on the three-phase induction generator. As in the previous analysis, line C was removed from the generator to cause a single-phasing condition. The third line was created by a balancer. The balancer model predicted results for the

machine to be almost identical to the results for the resistive load, as shown in Appendix Case 8.

The line currents were equal. A nominal value of 7.3 amps was used to represent near full load on the generator. The currents through the shunt impedances were equal to the line currents. The current through the single-phase line equivalent was 1.732 times the line currents. The impedance magnitudes necessary to create the three-phase balancer was 1.732 times the single-phase line equivalent impedance. The shunt impedance magnitudes were 29.7 ohms. The same actual values were used as for the resistance load. These were 31 ohms.

#### 4.5.2 Experimental Results

The first experiment was conducted with the resistor load connected across two lines of a three-phase system. The results of using the balancing impedances were very good, as shown in Table IV. The calculations were based on line currents of 4.3 amps. The created third phase, line C, had a measured current value of 4.7 amps. Lines A and B had measured current values of 4.9 and 4.1 amps, respectively. The power factor in all lines was 0.99.

The second test was conducted at a nominal line current of 6.9 amps. The current in the created third phase had a value of 7.4 amps. Lines A and B had measured current values of 7.7 and 6.5 amps, respectively. The phase difference between the line voltages were within one-half percent of the desired 120 degrees.

The experimental data indicate that single-phase resistive loads can be made to look like a balanced three-phase load very adequately. The phase currents were very close to predicted values. It should be

TABLE IV  
SINGLE-PHASE TO THREE-PHASE BALANCER  
FOR RESISTANCE LOAD

| Quantity<br>(Amps) | Line<br>A | Line<br>B | Line<br>C | Load<br>A-B | Imped<br>A-C | Imped<br>B-C |
|--------------------|-----------|-----------|-----------|-------------|--------------|--------------|
| Theoretical LA     | 4.3       | 4.3       | 4.3       | 7.45        | 4.3          | 4.3          |
| Experimental LA    | 4.9       | 4.1       | 4.7       | 7.35        | 4.3          | 4.3          |
| Theoretical HA     | 6.9       | 6.9       | 6.9       | 11.95       | 6.9          | 6.9          |
| Experimental HA1   | 7.7       | 6.5       | 7.4       | 11.90       | 6.9          | 7.0          |
| Experimental HA2   | 7.4       | 6.9       | 7.9       | 12.10       | 6.9          | 6.9          |

Notes: Phase A-B = load.

Line C = created line.

LA = low current test.

HA1 = high current test.

HA2 = high current test with increased load.

pointed out that for balancing conditions, the phase current in the load is 1.732 times the line current while the phase current in the balancing impedance equals the line current. The line current deviations were in the range of 10 percent. These differences in line currents can be attributed to several factors. The elements used in the balancer are not pure impedances. Inductors notably have significant resistances. The second major problem is that the shunt impedances must be equal in magnitude with one being a capacitor and one being an inductor. Furthermore, these impedances must be 1.732 times the load impedance. It is not possible to achieve all of these conditions precisely. Moreover, the currents through both impedances must be perfect sine waves to achieve perfect balance. The iron core inductor is prone to distort the wave shape and introduce harmonics.

Even with these shortcomings, sufficiently accurate balanced lines were obtained to justify attempting to balance a three-phase induction generator on a single-phase line. This was the third part of the experiment.

A nominal value of 7.3 amps was used for the model to balance single-phase to three-phase on the induction generator. As noted above, the model (theoretical or calculated) values were 29.7 ohms and the actual values were 31 ohms. The impedances were added to the terminals of the induction machine to recreate the line C. The results are shown in Table V.

For the three-phase generator, a current of 7.3 amps was maintained on the created line C regardless of load. This was exactly as predicted. The speed was then adjusted to change the output of the induction generator. Line A held a relatively constant value near 6.7 amps even as speed

TABLE V  
 CURRENTS FOR ONE-PHASE TO THREE-PHASE BALANCER  
 USED WITH A THREE-PHASE INDUCTION GENERATOR  
 OPERATING AT 1247 RPM ON A  
 SINGLE-PHASE LINE

| Quantity<br>(Amps)  | Line<br>A | Line<br>B | Line<br>C | Load<br>A-B | Imped<br>A-C | Imped<br>B-C |
|---------------------|-----------|-----------|-----------|-------------|--------------|--------------|
| Normal Running      | 7.1       | 7.1       | 7.3       | ---         | ---          | ---          |
| Line C Disconnected | 11.9      | 11.9      | 0         | ---         | ---          | ---          |
| Theoretical         | 7.3       | 7.3       | 7.3       | 12.6        | 7.3          | 7.3          |
| Experimental        | 6.6       | 6.6       | 7.3       | 10.7        | 6.6          | 7.0          |

was changed. Line B changed dramatically from 1.8 amps to 7.3 amps as the speed increased, as indicated in Figure 22.

Shaft speed for the induction generator controls the real power output. The adjustment of the speed and the resulting load can be used to achieve near complete balance of line currents for selected values of balancer circuit impedances.

Several observations are appropriate. The created line current is constant and is independent of speed or load. If the generator is operated at speeds near or slightly above where the output current matches the created line current, then balance is virtually complete. If the balancer is designed for full load conditions, the line currents to the two existing power lines will decrease and then increase from no load to full load.

Two precautions should be noted and appropriate protection provided. All capacitors connected to the machine cause overexcitation leading to high voltage if the load is removed while the generator is being driven. A more serious problem to be avoided was the removal of load with the created line C connected. Without the load, a series resonant network is present (the inductance and capacitance used to create the third phase) in which the voltages across the capacitor or inductor could reach dangerously high values. To avoid damage to the balancer components, some means must be provided to either de-tune the circuit or provide overvoltage protection if the voltage exceeds a preset value. When line C between the generator and the balancer was disconnected, excessive voltages and currents resulted as expected, causing a breakdown in the inductor insulation.

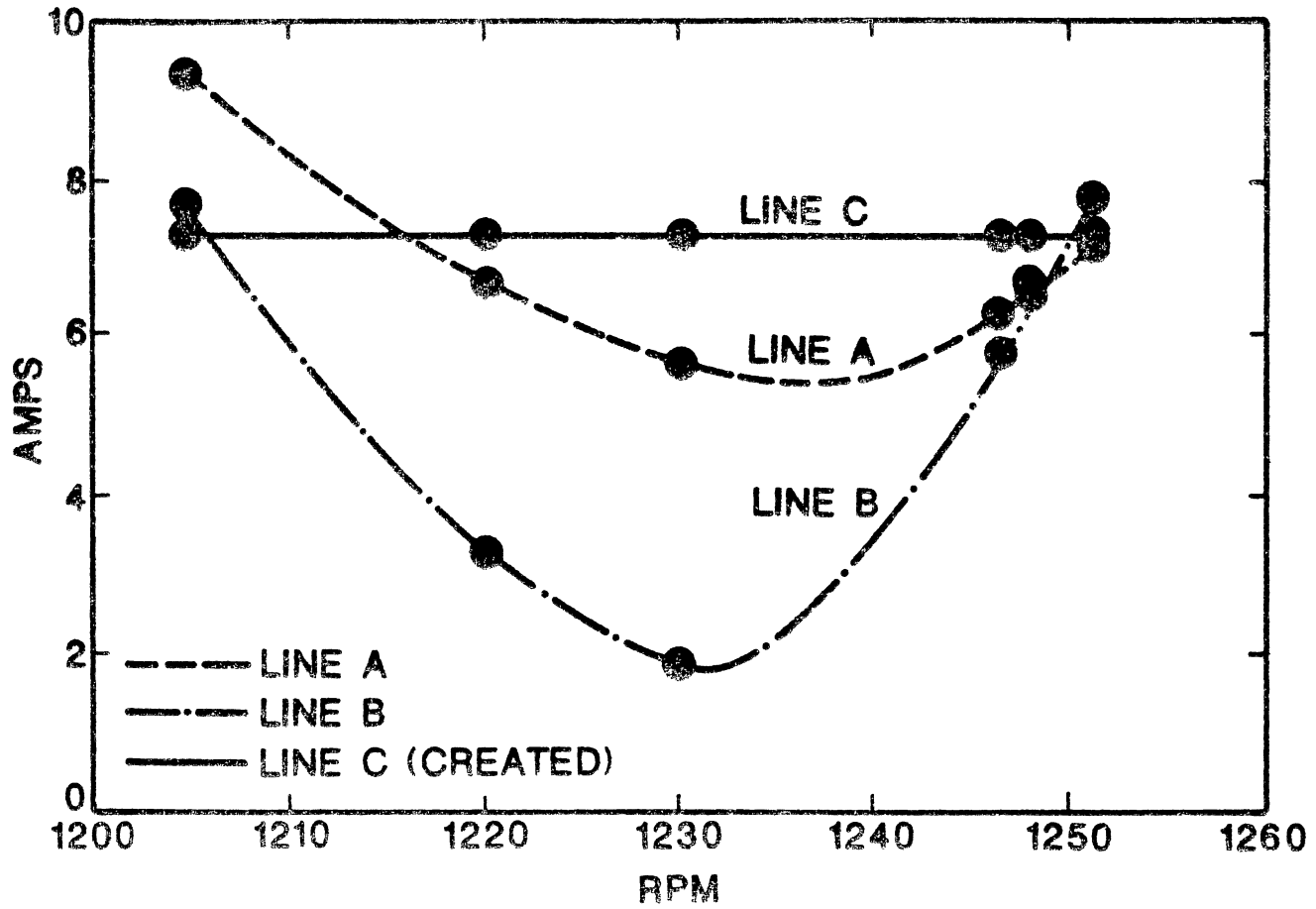


Figure 22. One-Phase to Three-Phase Balance Line Current Variation as Speed Changes



## CHAPTER V

### SUMMARY AND CONCLUSIONS

#### 5.1 Summary of Results and Conclusions

Operation of three-phase induction generators on unbalanced power systems has been studied. The imbalance can vary from slight differences in line impedances to the absence of a line (single-phasing).

It is necessary to have two independent models for analysis of the problem. These are a model for the generator and a model for the line or power system. The power system model can also be used as a tool to resolve the problem of balancing the power system.

The model used for machine analysis is the conventional equivalent circuit model. Operation under unbalanced conditions can be considered by using symmetrical component analysis. This technique is well known and significant literature exists to document its use with motors. Not much literature was found on the application of induction generators on unbalanced systems.

Once it was verified that a machine model exists that is reasonably accurate, a model was sought that would permit the analysis of the problem of balancing an unbalanced power system. Typically, symmetrical components and fault analysis are used to determine the magnitude of problems in unbalanced power systems. However, these do not provide a technique for correcting the imbalance.

A model to study the balancing of all parts of the power system is

presented. The balancer, power system, line, loads, and sources can be represented by the same model. Although two-port network use is very common in filter studies, its use in power system analysis is rare. The two-port network permits description of the output conditions and the input conditions independent of the definition of the internal network. This makes it uniquely suited for analysis of unbalanced systems and for arriving at the elements necessary to balance the system. A significant advantage of this model is that the system can be analyzed using relatively simple mathematics.

The input and output of the balancer model can be designated as the desired and existing line conditions, respectively. These conditions can be used to calculate internal circuit impedances to match the external conditions. These impedance parameters provide a balancer for the power system. Any level of imbalance, up to single-phasing, can be resolved using the balancer analysis presented.

The physical realization of the model results has a number of minor problems. The actual impedance values of inductors and capacitors must match the values calculated using the model. These also must change for different load conditions. Devices available generally have fixed values of impedances, with the possibility of change in discrete steps. Therefore, the impedance values of the elements used may not be exactly equal to the required values nor can the magnitudes be precisely changed for varying loads. Continuously variable capacitors and inductors would be desirable.

If the capacitor-inductor circuit is fixed, the balancer will yield a fixed current output in the created third phase. This is adequate for a constant load. However, the current available on that phase will be

insufficient to start the induction machine since starting currents are typically six times running currents.

The combination of impedances also creates a resonant circuit that has very high stored energy. If the load is removed from that circuit, the remaining high-Q series resonant circuit will lead to excessive voltages across the inductance (or capacitance). This will generally cause catastrophic failure of the inductors and/or capacitors.

The output of an induction generator is determined by its shaft speed. Since no-load to full-load corresponds to a speed change of less than 50 rpm, any small change in speed causes a significant change in the power output. These power changes are accompanied by corresponding current changes which in turn contribute to the balancing problem discussed above. Establishing an exact speed to maintain a certain output while matching the balancer currents can be tedious.

When balancing the lines, a fourth line is required to satisfy the model. For four-wire systems with a neutral, this is no problem. However, three-wire systems can present a problem. The common side of all balancers must be tied together. Under balanced conditions, the potential of this line will be at the neutral point. If the lines are not exactly balanced, this floating line may have some potential different from neutral.

A phase angle meter was used to measure the phase angles of voltages relative to each other. Power factor on each phase must be measured and used with the voltage phase angles to find the current phase angles. Since power factor seldom is less than 0.3, the instruments often do not measure less than that value. However, in unbalanced systems, power factors can be near zero. Furthermore, when used with an induction generator,

the reactive power enters the machine when real power leaves the machine. This requires the calculations to use supplement power factor angles.

Experimental investigations were undertaken to verify the models developed. The results indicate the induction generator models and the line balancer models are acceptable.

The measured value of generator torque was approximately eight percent greater than the theoretical calculated values of torque. Although this is more than desirable, it is adequate for most practical purposes. The balancer model for balancing lines yielded currents approximately three percent less than the measured values. This is very good. The comparison of results for balancers to interface single-phase to three-phase was not as good. One line was measured with a ten percent deviation above and one line was measured with a ten percent deviation below the nominal value. However, this was easily changed with proper motor speed and the resulting loading. Although these currents were significantly different than desired values, they were dramatically better than operating with no current on one line. Neither significant temperature rise nor vibration problems were noted on the three-phase generator supplying power to a single-phase line through a balancer.

Induction machines can operate adequately on somewhat unbalanced lines if they are not fully loaded. A line balancer can be used if the line is significantly unbalanced and/or to permit balanced operation of a three-phase induction generator on single-phase lines.

## 5.2 Suggestions for Further Work

This research has uncovered several areas where additional study is warranted. The first area for further study would be the development of

a balancer for starting large three-phase machines on a single-phase line. The one-phase to three-phase balancer developed in this study is suitable for running conditions but is not acceptable for starting. The balancer delivers a constant current on the created line. However, starting typically requires the current to be five to six times the running current. The second area for further study would be the development of a more general solution for the internal impedances of the balancer without the constraint of ideal components assumed in this analysis.

In a similar vein, a generalized solution could be developed permitting any impedance for the elements of the balancer. The analysis presented here assumed that all shunt impedances would be inductances and capacitances only. Further study is necessary to quantify overexcitation and underexcitation phenomena in induction generators. This is critical to successful operation with the load suddenly removed.

The development of a control system for the induction generator operating on an unbalanced power system merits consideration. With static reactance values, the level of imbalance changes with speed and load. Speed control or variable reactances could be used to maintain acceptable balance over a range of operations.

A refined model of the induction machine should be evaluated for the generator. The motor model results in reactances that are approximately 8 to 10 percent less when the machine operates as a generator rather than as a motor. These variations obviously will affect current values and resulting power. The measured torque was approximately 8 percent greater than the theoretical value. The model possibly would require additional impedances so that the current through the rotor resistance

would remain the same during generator operation even though the voltages have increased.

These are some areas that would permit further refinements in the modeling and analysis of three-phase induction machines operating on unbalanced power systems. Resolution of some of these issues will enable the design of hardware required to make the operation of such machines more efficient and reliable.

## BIBLIOGRAPHY

- (1) NEMA Standards MG1-1978: Motors and Generators. Washington, D.C.: National Electrical Manufacturers Association, 1978.
- (2) Owen, Edward L., and Glenn R. Griffith. "Induction Generator Applications for Petroleum and Chemical Plants." IEEE-PCIC Conference Paper PCI-82-14. New York, NY, 1982.
- (3) Tesla, N. "A New System of Alternate-Current Motors and Transformers." AIEE Transactions, Vol. 5, 1888, pp. 308-327.
- (4) Danielson, E. "Reversibility of the Three Phase Motors With Inductive Windings." Electrical World, Vol. 21, No. 3 (January 21, 1893), p. 44.
- (5) Nailen, Richard L. "Spooks on the Power Line? Induction Generators and the Public Utility." IEEE Transactions on Industry Applications, Vol. IA-18, No. 6 (November/December, 1982), pp. 608-615.
- (6) Little, A. D. A Technical, Economical and Policy Analysis of Electric Motor Equipment. Contract No. Co-04-50127-00. Cambridge, MA, 1976.
- (7) Nailen, Richard L. "Watts From Waste Heat--Induction Generators for the Process Industries." IEEE Transactions on Industry Applications, Vol. IA-19, No. 3 (May/June, 1983), pp. 470-475.
- (8) Durham, Marcus O., and R. Ramakumar. Small Power Production Using Waste Gas. Stillwater, OK: Engineering Energy Laboratory, Oklahoma State University, July, 1985.
- (9) National Electrical Code. Boston, MA: National Fire Protection Association, 1978.
- (10) Breedlove, H., and J. R. Harbough. "Protection of the Motor/Generator." IEEE-PCIC Conference Paper PCI-81-3. New York, NY, 1981.
- (11) "Voltage Ratings for Electric Power Systems Equipment (60 HZ)." ANSI C84.1. New York: American National Standards Institute, 1984.

- (12) Howell, J. K., and E. E. Hogwood, Jr. Electrified Oil Production --An Enginnering Text. Tulsa, OK: Petroleum Publishing Co., 1962.
- (13) Hughes, William L. "Control of Power Flow on the Interconnected Grid." Energy Information Dissemination Program. Stillwater, OK: Oklahoma State University, June 30, 1983, pp. 137-155.
- (14) Fortescue, C. L. "Method of Symmetrical Co-Ordinates Applied to the Solution of Polyphase Networks." AIEE Transactions, Vol. 37, Pt. 2 (1918), pp. 1027-1115.
- (15) Ramakumar, R. "Notes on Symmetrical Components." Oklahoma State University, Stillwater, OK, April, 1985.
- (16) Elgerd, Olle I. Electrical Energy Systems Theory. New York: McGraw-Hill Book Co., 1971.
- (17) IEEE Standard Test Procedure for Polyphase Induction Motors and Generators. Standard 112. New York: Institute of Electrical and Electronic Engineers, 1978.
- (18) Electrical Engineering Pocket Handbook. St. Louis, MO: EASA, 1982.



## Computer Nomenclature

Machine--Sequence Parameters

|              |   |
|--------------|---|
| FT (0,1,2,3) | Angle of complex terminal power                   |
| HA, HB, HC   | Angle of current in lines A, B, C                 |
| HM (0,1,2)   | Angle of magnetizing current                      |
| HR (0,1,2)   | Angle of rotor current                            |
| HS (0,1,2)   | Angle of stator current                           |
| IA, IR, IC   | Magnitude of current in lines A, B, C             |
| IM (0,1,2)   | Magnitude of magnetizing current                  |
| IR (0,1,2)   | Magnitude of rotor current                        |
| IS (0,1,2)   | Magnitude of stator current                       |
| LE (0,1,2)   | Angle of equivalent impedance                     |
| LF (0,1,2)   | Angle of rotor impedance divider                  |
| LG (0,1,2)   | Angle of magnetizing impedance driver             |
| PL (0,1,2)   | Rotor copper losses                               |
| PR (0,1,2,3) | Mechanical power developed                        |
| PS (0,1,2,3) | Stator copper losses                              |
| PT (0,1,2,3) | Terminal power                                    |
| QT (0,1,2,3) | Terminal reactive power                           |
| ST (0,1,2,3) | Terminal apparent power                           |
| TR (0,1,2,3) | Rotor torque developed                            |
| UR (0,1,2,3) | Angle of voltage across magnetizing impedance     |
| VR (0,1,2,3) | Magnitude of voltage across magnetizing impedance |
| WN           | Synchronous speed                                 |
| WR           | Rotor speed                                       |
| WS           | Slip  |

(            )    Sequence values:  
 0 = zero, 1 = positive, 2 = negative, 3 = total,  
 4 = alternate calculation

Balancer

|            |  |
|------------|--|
| CN, CO, CP | Capacitance of balancer impedances               |
| FN, FO     | Angle of complex power into balancer             |
| FP         | Angle of complex power in load                   |
| FY, FZ     | Angle of complex power in shunt impedance        |
| HN, HO     | Angle of current into balancer                   |
| HP         | Angle of current through load (series) impedance |
| HU         | Angle of unbalanced current (-IN-IO)             |
| HY, HZ     | Angle of current through shunt impedances        |
| IN, IO     | Magnitude of current into balance                |
| IP         | Magnitude of current through load                |
| IU         | Magnitude of current imbalance (-IN-IO)          |
| IY, IZ     | Magnitude of current through shunt impedances    |
| LN, LO, LP | Angle of balancer impedances                     |
| MN, MO, MP | Inductance of balancer impedance                 |
| PN, PO     | Power into balancer                              |
| PP         | Power in load (series impedance)                 |
| QP         | Reactive power in load (series impedance)        |
| RN, RO, RP | Resistance of balancer impedances                |
| SN, SO     | Complex power into balancer                      |
| SP         | Complex power into load (series impedance)       |
| SY, SZ     | Complex power in shunt impedances                |
| UN, UO     | Angle of voltage on balancer input               |

UP                    Angle of voltage across load  
 VN, VO               Magnitude of voltage on balancer input  
 VP                    Magnitude of voltage across load  
 XN, XO, XP          Reactance of balancer impedance  
 ZN, ZO, ZP          Balancer impedance

### Case 1 Motor 1186 RPM Impedances

#### Machine Parameters: Print

Stator R = 1.561  
 Stator X = 5.170  
 Magnet R = 0  
 Magnet X = 56.82  
 Rotor+ R = 1.773  
 Rotor- R = 1.773  
 Rotor0 R = 0  
 Rotor+ X = 4.185  
 Rotor- X = 4.185  
 Rotor0 X = 0  
 Speed FL = 1157  
 HPWR FL = 3

#### Machine Impedance: Print

|       |                    |       |                |
|-------|--------------------|-------|----------------|
| WN    | = 1200             | WR    | = 1186         |
| WS    | = 0.0116666667     |       |                |
| ZG    | = Divider Rotor Z  |       |                |
| ZG(1) | = 0.347            | LG(1) | = 1.18906284   |
| ZG(2) | = 0.931            | LG(2) | = 0.0146158179 |
| ZG(0) | = 1                | LG(0) | = 0            |
| ZF    | = Divider Magnet Z |       |                |
| ZF(1) | = 0.928            | LF(1) | = -0.354202367 |
| ZF(2) | = 0.070            | LF(2) | = -0.195315716 |
| ZF(0) | = 0                | LF(0) | = 0            |
| ZE    | = Equiv. Machine Z |       |                |
| ZE(1) | = 58.142           | LE(1) | = 1.22225602   |
| ZE(2) | = 9.374            | LE(2) | = 1.31916834   |
| ZE(0) | = 0                | LE(0) | = 0            |

## Case 1 Motor 1186 RPM

Line Current: Radians

|    |       |    |           |
|----|-------|----|-----------|
| IA | = 6.3 | HA | = 0       |
| IB | = 6.3 | HB | = -2.0926 |
| IC | = 6.4 | HC | = 2.0909  |

Machine Currents

|       |                   |       |                   |
|-------|-------------------|-------|-------------------|
| IS(1) | = 6.33333334      | HS(1) | = -5.82079034E-04 |
| IS(2) | = -0.0376666667   | HS(2) | = 0.79774441      |
| IS(0) | = -0.0313333333   | HS(0) | = -1.34925771     |
| IR(1) | = 2.1976667       | HR(1) | = 1.18848077      |
| IR(2) | = -0.0350676667   | HR(2) | = 0.812360228     |
| IR(0) | = -0.0313333333   | HR(0) | = -1.34925771     |
| IM(1) | = 5.87733334      | HM(1) | = -0.354784447    |
| IM(2) | = -2.63666667E-03 | HM(2) | = 0.602428694     |
| IM(0) | = 0               | HM(0) | = -1.34925771     |
| WR    | = 1186            |       |                   |

Machine Power

|       |                  |       |                   |
|-------|------------------|-------|-------------------|
| ST(1) | = 2332.14022     | FT(1) | = 1.22225602      |
| ST(2) | = 0.0132996229   | FT(2) | = 1.31916834      |
| ST(0) | = 0              | FT(0) | = 0               |
| ST(3) | = 2332.153       | FT(3) | = 1.22225658      |
| PT(1) | = 796.487088     | QT(1) | = 2191.91385      |
| PT(2) | = 3.31135346E-03 | QT(2) | = 0.0128807961    |
| PT(0) | = 0              | QT(0) | = 0               |
| PT(3) | = 796.490399     | QT(3) | = 2191.92673      |
| PR(1) | = 725.419175     | TR(1) | = 51.5989558      |
| PR(2) | = -1.083769E-03  | TR(2) | = -7.70883246E-05 |
| PR(0) | = 0              | TR(0) | = 0               |
| PR(3) | = 725.418091     | TR(3) | = 51.5988787      |
| VR(1) | = 9.988395       | UR(1) | = 2.35854545      |
| VR(2) | = -0.159382545   | UR(2) | = 1.98242491      |
| VR(0) | = 0              | UR(0) | = 0.221538618     |
| VR(3) | = -9.84          | UR(3) | = -0.777097816    |

Machine Power Loss

|                        |                        |
|------------------------|------------------------|
| PL(1) = 8.56312686     | PL(2) = 2.18033123E-03 |
| PL(0) = 0              | PL(3) = 8.56530719     |
| PS(1) = 62.6134445     | PS(2) = 2.21471211E-03 |
| PS(0) = 1.53255511E-03 | PS(3) = 62.6171918     |
| PT(4) = 796.60059      | PR(4) = 0.97241031     |

## Case 1 Generator 1220 RPM

Machine Parameters: Print

Stator R = 1.561  
 Stator X = 4.966  
 Magnet R = 0  
 Magnet X = 51.785  
 Rotor+ R = 1.773  
 Rotor- R = 1.773  
 Rotor0 R = 0  
 Rotor+ X = 3.972  
 Rotor- X = 3.972  
 Rotor0 X = 0  
 Speed FL = 1157  
 HPWR FL = 3

Machine Impedance: Print

|                       |                      |
|-----------------------|----------------------|
| WN = 1200             | WR = 1220            |
| WS = -0.0166666667    |                      |
| ZG = Divider Rotor Z  |                      |
| ZG(1) = -0.431        | LG(1) = 2.05356144   |
| ZG(2) = 0.929         | LG(2) = 0.0157666411 |
| ZG(0) = 1             | LG(0) = 0            |
| ZF = Divider Magnet Z |                      |
| ZF(1) = 0.886         | LF(1) = 0.445444603  |
| ZF(2) = 0.073         | LF(2) = -0.202064098 |
| ZF(0) = 0             | LF(0) = 0            |
| ZE = Equiv. Machine Z |                      |
| ZE(1) = -49.834       | LE(1) = -1.19661346  |
| ZE(2) = 8.972         | LE(2) = 1.30935391   |
| ZE(0) = 0             | LE(0) = 0            |

Line Current: Radians

|    |       |    |            |
|----|-------|----|------------|
| IA | = 6.4 | HA | = 0        |
| IB | = 6.5 | HB | = -2.09439 |
| IC | = 6.6 | HC | = 2.09439  |

Machine Currents

|       |                   |       |                   |
|-------|-------------------|-------|-------------------|
| IS(1) | = 6.5             | HS(1) | = -2.61482984E-08 |
| IS(2) | = -0.057666667    | HS(2) | = 0.522106784     |
| IS(0) | = -0.057666667    | HS(0) | = -0.522612642    |
| IR(1) | = -2.8015         | HR(1) | = 2.05356141      |
| IR(2) | = -0.0535723333   | HR(2) | = 0.537873425     |
| IR(0) | = -0.057666667    | HR(0) | = -0.522612642    |
| IM(1) | = 5.759           | HM(1) | = 0.445444577     |
| IM(2) | = -4.20966667E-03 | HM(2) | = 0.320042686     |
| IM(0) | = 0               | HM(0) | = -0.522612642    |
| WR    | = 1220            |       |                   |

Machine Power

|       |                   |       |                  |
|-------|-------------------|-------|------------------|
| ST(1) | = -2105.4865      | FT(1) | = -1.19661346    |
| ST(2) | = 0.0298358876    | FT(2) | = 1.30935391     |
| ST(0) | = 0               | FT(0) | = 0              |
| ST(3) | = -2105.511       | FT(3) | = -1.19662187    |
| PT(1) | = -769.580704     | QT(1) | = 1959.80074     |
| PT(2) | = 7.71180789E     | QT(2) | = 0.028822009    |
| PT(0) | = 0               | QT(0) | = 0              |
| PT(3) | = -769.572992     | QT(3) | = 1959.82956     |
| PR(1) | = -848.828248     | TR(1) | = -58.6943861    |
| PR(2) | = -2.56527734E-03 | TR(2) | = -1.7738262E-04 |
| PR(0) | = 0               | TR(0) | = 0              |
| PR(3) | = -848.830814     | TR(3) | = -58.6945635    |
| VR(1) | = -12.186525      | UR(1) | = 3.20452276     |
| VR(2) | = -0.23303965     | UR(2) | = 1.68883478     |
| VR(0) | = 0               | UR(0) | = 1.04818368     |
| VR(3) | = 12.202          | UR(3) | = 0.043858812    |

Machine Power Loss

|       |              |       |                  |
|-------|--------------|-------|------------------|
| PL(1) | = 13.9152172 | PL(2) | = 5.08850096E-03 |
| PL(0) | = 0          | PL(3) | = 13.9203057     |

Machine Power Loss (Cont.)

|                        |                        |
|------------------------|------------------------|
| PS(1) = 65.95225       | PS(2) = 5.19101878E-03 |
| PS(0) = 5.19101878E-03 | PS(3) = 65.9626321     |
| PT(4) = -768.947877    | PR(4) = -1.13784291    |

## Case 1 Generator 1243 RPM

Line Current: Radians

|          |              |
|----------|--------------|
| IA = 9.4 | HA = 0       |
| IB = 9.4 | HB = -2.0909 |
| IC = 9.7 | HC = 2.0874  |

Machine Currents

|                         |                         |
|-------------------------|-------------------------|
| IS(1) = 9.5             | HS(1) = -1.22801049E-03 |
| IS(2) = -0.112666667    | HS(2) = 0.795666419     |
| IS(0) = -0.095          | HS(0) = -1.34806153     |
| IR(1) = -6.6025         | HR(1) = 2.41455155      |
| IR(2) = -0.104667333    | HR(2) = 0.811284646     |
| IR(0) = -0.095          | HR(0) = -1.34806153     |
| IM(1) = 6.327           | HM(1) = 0.763650635     |
| IM(2) = -8.22466667E-03 | HM(2) = 0.595441319     |
| IM(0) = 0               | HM(0) = -1.34806153     |
| WR = 1243               |                         |

Machine Power

|                         |                         |
|-------------------------|-------------------------|
| ST(1) = -3363.16625     | FT(1) = -0.928733571    |
| ST(2) = 0.113863187     | FT(2) = 1.31010776      |
| ST(0) = 0               | FT(0) = 0               |
| ST(3) = -3363.237       | FT(3) = -0.928760149    |
| PT(1) = -2014.02773     | QT(1) = 2693.43267      |
| PT(2) = 0.0293477711    | QT(2) = 0.110016061     |
| PT(0) = 0               | QT(0) = 0               |
| PT(3) = -2013.99838     | QT(3) = 2693.54269      |
| PR(1) = -2234.2318      | TR(1) = -151.63298      |
| PR(2) = -9.88277064E-03 | TR(2) = -6.70724482E-04 |
| PR(0) = 0               | TR(0) = 0               |
| PR(3) = -2234.24168     | TR(3) = -151.633651     |

Machine Power (Cont.)

|                    |                     |
|--------------------|---------------------|
| VR(1) = -28.720875 | UR(1) = 3.56551291  |
| VR(2) = -0.4553029 | UR(2) = 1.962246    |
| VR(0) = 0          | UR(0) = 0.222734793 |
| VR(3) = 28.71      | UR(3) = 0.408069094 |

Machine Power Loss

|                     |                      |
|---------------------|----------------------|
| PL(1) = 77.2904002  | PL(2) = 0.0194236594 |
| PL(0) = 0           | PL(3) = 77.3098238   |
| PS(1) = 140.88025   | PS(2) = 0.0198149871 |
| PS(0) = 0.014088025 | PS(3) = 140.914153   |
| PT(4) = -2016.01771 | PR(4) = -2.99496204  |

Case 2 Generator 1230 RPM

Line Current: Radians

|          |               |
|----------|---------------|
| IA = 6.1 | HA = 0        |
| IB = 8.4 | HB = -2.15722 |
| IC = 7.8 | HC = 2.00887  |

Machine Currents

|                       |                       |
|-----------------------|-----------------------|
| IS(1) = 7.429         | HS(1) = -0.0535846786 |
| IS(2) = -0.793666667  | HS(2) = -0.492958653  |
| IS(0) = -0.619333333  | HS(0) = -0.03-7657774 |
| IR(1) = -4.264246     | HR(1) = 2.18347848    |
| IR(2) = -0.737316333  | HR(2) = -0.477256884  |
| IR(0) = -0.619333333  | HR(0) = -0.0357657774 |
| IM(1) = 5.846623      | HM(1) = 0.556733836   |
| IM(2) = -0.0579376667 | HM(2) = -0.694219123  |
| IM(0) = 0             | HM(0) = -0.0357657774 |
| WR = 1230             |                       |

Machine Power

|                     |                     |
|---------------------|---------------------|
| ST(1) = -2436.19879 | FT(1) = -1.05406672 |
| ST(2) = 5.6508937   | FT(2) = 1.30967695  |



Machine Power (Cont.)

|                      |                      |
|----------------------|----------------------|
| ST(0) = 0            | FT(0) = 0            |
| ST(3) = -2440.228    | FT(3) = -1.05569178  |
| PT(1) = -1203.57813  | QT(1) = 2118.12753   |
| PT(2) = 1.45884684   | QT(2) = 5.45933746   |
| PT(0) = 0            | QT(0) = 0            |
| PT(3) = -1202.11929  | QT(3) = 2123.58686   |
| PR(1) = -1321.83453  | TR(1) = -90.6585051  |
| PR(2) = -0.487882547 | TR(2) = -0.033461603 |
| PR(0) = 0            | TR(0) = 0            |
| PR(3) = -1322.32242  | TR(3) = -90.6919667  |
| VR(1) = -18.5494701  | UR(1) = 3.33443983   |
| VR(2) = -3.20732605  | UR(2) = 0.67370447   |
| VR(0) = 0            | UR(0) = 1.53503055   |
| VR(3) = 15.776       | UR(3) = 0.0986704631 |

Machine Power Loss

|                     |                     |
|---------------------|---------------------|
| PL(1) = 32.2398667  | PL(2) = 0.963865521 |
| PL(0) = 0           | PL(3) = 33.2037322  |
| PS(1) = 86.151654   | PS(2) = 0.98328448  |
| PS(0) = 0.598758667 | PS(3) = 87.7336972  |
| PT(4) = -1201.38499 | PR(4) = -1.77255016 |

Case 2 Generator 1240 RPM

Line Current: Radians

|          |               |
|----------|---------------|
| IA = 7.3 | HA = 0        |
| IB = 9.9 | HB = -2.16944 |
| IC = 9.1 | HC = 2.000147 |

Machine Currents

|                      |                       |
|----------------------|-----------------------|
| IS(1) = 8.76         | HS(1) = -0.0608676206 |
| IS(2) = -0.906       | HS(2) = -0.58600595   |
| IS(0) = -0.689666667 | HS(0) = -0.0460268515 |
| IR(1) = -5.88672     | HR(1) = 2.31888443    |
| IR(2) = -0.841674    | HR(2) = -0.570368523  |
| IR(0) = -0.689666667 | HR(0) = -0.0460268515 |

Machine Currents (Cont.)

|                   |                       |
|-------------------|-----------------------|
| IM(1) = 6.06192   | HM(1) = 0.673550754   |
| IM(2) = -0.066138 | HM(2) = -0.786469051  |
| IM(0) = 0         | HM(0) = -0.0460268515 |
| WR = 1240         |                       |

Machine Power

|                      |                       |
|----------------------|-----------------------|
| ST(1) = -2973.12158  | FT(1) = -0.952437084  |
| ST(2) = 7.36289893   | FT(2) = 1.31000005    |
| ST(0) = 0            | FT(0) = 0             |
| ST(3) = -2977.823    | FT(3) = -0.954341466  |
| PT(1) = -1723.51561  | QT(1) = 2422.59073    |
| PT(2) = 1.89852338   | QT(2) = 7.11392224    |
| PT(0) = 0            | QT(0) = 0             |
| PT(3) = -1721.61709  | QT(3) = 2429.70465    |
| PR(1) = -1904.6588   | TR(1) = -129.578239   |
| PR(2) = -0.638305252 | TR(2) = -0.0434253476 |
| PR(0) = 0            | TR(0) = 0             |
| PR(3) = -1905.29711  | TR(3) = -129.621664   |
| VR(1) = -25.607232   | UR(1) = 3.46984578    |
| VR(2) = -3.6612819   | UR(2) = 0.580592831   |
| VR(0) = 0            | UR(0) = 1.52476947    |
| VR(3) = 22.081       | UR(3) = 0.286842793   |

Machine Power Loss

|                     |                     |
|---------------------|---------------------|
| PL(1) = 61.4406065  | PL(2) = 1.25602001  |
| PL(0) = 0           | PL(3) = 62.6966266  |
| PS(1) = 119.787394  | PS(2) = 1.281325    |
| PS(0) = 0.742474214 | PS(3) = 121.811193  |
| PT(4) = -1720.78929 | PR(4) = -2.55401757 |

Case 3 Generator 1230 RPM

Line Current: Radians

|          |              |
|----------|--------------|
| IA = 7.4 | HA = 0       |
| IB = 7.4 | HB = -2.0909 |
| IC = 8.0 | HC = 2.0717  |

Machine Currents

|                       |                         |
|-----------------------|-------------------------|
| IS(1) = 7.59933334    | HS(1) = -6.82865308E-03 |
| IS(2) = -0.216666667  | HS(2) = 0.743278492     |
| IS(0) = -0.202666667  | HS(0) = -1.37289921     |
| IR(1) = -4.36201733   | HR(1) = 2.23023451      |
| IR(2) = -0.201283333  | HR(2) = 0.75898026      |
| IR(0) = -0.202666667  | HR(0) = 01.37289921     |
| IM(1) = 5.98067533    | HM(1) = 0.603489862     |
| IM(2) = -0.0158166667 | HM(2) = 0.542018022     |
| IM(0) = 0             | HM(0) = -1.37289921     |
| WR = 1230             |                         |

Machine Power

|                      |                         |
|----------------------|-------------------------|
| ST(1) = -2549.19463  | FT(1) = -1.05406672     |
| ST(2) = 0.421138611  | FT(2) = 1.30967695      |
| ST(0) = 0            | FT(0) = 0               |
| ST(3) = -2549.495    | FT(3) = -1.05418264     |
| PT(1) = -1259.40253  | QT(1) = 2216.37058      |
| PT(2) = 0.108722047  | QT(2) = 0.406862687     |
| PT(0) = 0            | QT(0) = 0               |
| PT(3) = -1259.29381  | QT(3) = 2216.77744      |
| PR(1) = -1383.1439   | TR(1) = -94.8634305     |
| PR(2) = -0.036359944 | TR(2) = -2.49376006E-03 |
| PR(0) = 0            | TR(0) = 0               |
| PR(3) = -1383.18026  | TR(3) = -94.8659243     |
| VR(1) = -18.9747754  | UR(1) = 3.38119586      |
| VR(2) = -0.8755825   | UR(2) = 1.90994162      |
| VR(0) = 0            | UR(0) = 0.197897118     |
| VR(3) = 19.082       | UR(3) = 0.193928468     |

Machine Power Loss

|                      |                      |
|----------------------|----------------------|
| PL(1) = 33.7352171   | PL(2) = 0.07183306   |
| PL(0) = 0            | PL(3) = 33.8070502   |
| PS(1) = 90.1475425   | PS(2) = 0.0732802778 |
| PS(0) = 0.0641161671 | PS(3) = 90.284939    |
| PT(4) = -1259.08827  | PR(4) = -1.85412904  |

## Case 3 Generator 1240 RPM

Line Current: Radians

|    |       |    |            |
|----|-------|----|------------|
| IA | = 9.0 | HA | = 0        |
| IB | = 9.0 | HB | = -2.09264 |
| IC | = 9.6 | HC | = 2.0647   |

Machine Currents

|       |                 |       |                  |
|-------|-----------------|-------|------------------|
| iS(1) | = 9.1999        | HS(1) | = -9.7560865E-03 |
| IS(2) | = -0.225333333  | HS(2) | = 0.599375524    |
| IS(0) | = -0.217        | HS(0) | = -1.51366805    |
| IR(1) | = -6.181728     | HR(1) | = 2.36999596     |
| IR(2) | = -0.209334667  | HR(2) | = 0.615012951    |
| IR(0) | = -0.217        | HR(0) | = -1.51366805    |
| IM(1) | = 6.365708      | HM(1) | = 0.724662289    |
| IM(2) | = -0.0164493333 | HM(2) | = 0.398912423    |
| IM(0) | = 0             | HM(0) | = -1.51366805    |
| WR    | = 1240          |       |                  |

Machine Power

|       |                 |       |                   |
|-------|-----------------|-------|-------------------|
| ST(1) | = -3278.57931   | FT(1) | = -0.952437084    |
| ST(2) | = 0.455452746   | FT(2) | = 1.31000005      |
| ST(0) | = 0             | FT(0) | = 0               |
| ST(3) | = -3278.87      | FT(3) | = -0.952544069    |
| PT(1) | = -1900.58915   | QT(1) | = 2671.48707      |
| PT(2) | = 0.117438484   | QT(2) | = 0.440051596     |
| PT(0) | = 0             | QT(0) | = 0               |
| PT(3) | = -1900.47172   | QT(3) | = 2671.92712      |
| PR(1) | = -2100.34295   | TR(1) | = -142.891074     |
| PR(2) | = -0.0394841602 | TR(2) | = -2.68619657E-03 |
| PR(0) | = 0             | TR(0) | = 0               |
| PR(3) | = -2100.38243   | TR(3) | = -142.89376      |
| VR(1) | = -26.8905168   | UR(1) | = 3.52095731      |
| VR(2) | = -0.9106058    | UR(2) | = 1.76597431      |
| VR(0) | = 0             | UR(0) | = 0.0571282743    |
| VR(3) | = 26.739        | UR(3) | = 0.345878752     |

Machine Power Loss

PL(1) = 67.7529984  
 PL(0) = 0

PS(1) = 132.094319  
 PS(0) = 0.073505929

PT(4) = -1900.30466

PL(2) = 0.0776946377  
 PL(3) = 67.8306931

PS(2) = 0.0792599484  
 PS(3) = 132.274085

PR(4) = -2.81552605

## Case 3 Line B Balancer 1240 RPM

Filter: In/Out Print

VN = -124  
 IN = -8.7  
 SN = -1078.8  
 PN = -699.19175

VO = 124  
 IO = 9.9  
 SO = 1227.6  
 PO = 650.52889

VP = 9.736  
 IU = 2.406

UN = 0  
 HN = 0.865683308  
 FN = -0.865683308  
 QN = 821.547525

UO = 0.0785398163  
 HO = 1.09083078  
 FO = -1.01229097  
 QO = -1041.06384

UP = -1.53152642  
 HU = -1.11111946

Filter: PI(Delta)/ZP = XP

VP = 9.736  
 IP = 5.64274198  
 SP = 54.9377359  
 PP = 1.20554111E-07

IZ = -6.40384036  
 SZ = 794.076205  
 IY = -8.61721906  
 SY = -1068.53516

ZN = 19.3633809  
 ZO = -14.3897932  
 ZP = 1.7254023

UP = -1.53152642  
 HP = -3.10232275  
 FP = 1.57079633  
 QP = 54.9377359

HZ = -1.57079633  
 FZ = 1.57079633  
 HY = -1.49225651  
 FY = 1.57079633

LN = 1.57079633  
 LO = 1.57079633  
 LP = 1.57079633

Filter: L Section

VP = 9.736  
 IP = 5.64274198  
 SP = 54.9377359  
 PP = 1.20554111E-07

UP = -1.53152642  
 HP = -3.10232275  
 FP = 1.57079633  
 QP = 54.9377359

Filter: L Section (Cont.)

|                  |                  |
|------------------|------------------|
| IZ = 0           | HZ = 0           |
| SZ = 0           | FZ = 0           |
| IY = -2.21333914 | HY = -1.49225651 |
| SY = -274.454053 | FY = 1.57079633  |
| ZN = 1E+19       | LN = 0           |
| ZO = -56.0239495 | LO = 1.57079633  |
| ZP = 1.7254023   | LP = 1.57079633  |

Case 3 Line C Balancer 1240 RPM

Filter: Standard In/Out

|                  |                   |
|------------------|-------------------|
| VN = -124        | UN = 0            |
| IN = -8.7        | HN = 0.863937979  |
| SN = -1078.8     | FN = -0.863937979 |
| PN = -700.624556 | QN = 820.325955   |
| VO = 124         | UO = 0.0785398163 |
| IO = 9.1         | HO = 0.844739357  |
| SO = 1128.4      | FO = -0.766199541 |
| PO = 813.069875  | QO = -782.434623  |
| VP = 9.736       | UP = -1.53152642  |
| IU = -0.435      | HU = 0.450620918  |

Filter: PI(Delta)/ZP = XP

|                     |                  |
|---------------------|------------------|
| VP = 9.736          | UP = -1.53152642 |
| IP = 5.65430526     | HP = -3.10232275 |
| SP = 55.050316      | FP = 1.57079633  |
| PP = 1.20801154E-07 | QP = 55.050316   |
| IZ = -6.39353501    | HZ = -1.57079633 |
| SZ = 792.798341     | FZ = 1.57079633  |
| IY = -6.53195351    | HY = -1.49225651 |
| SY = -809.962235    | FY = 1.57079633  |
| ZN = 19.3945916     | LN = 1.57079633  |
| ZO = -18.9836011    | LO = 1.57079633  |
| ZP = 1.72187378     | LP = 1.57079633  |

Filter: L Section

|                     |                  |
|---------------------|------------------|
| VP = 9.736          | UP = -1.53152642 |
| IP = 5.65430526     | HP = -3.10232275 |
| SP = 55.050316      | FP = 1.57079633  |
| PP = 1.20801154E-07 | QP = 55.050316   |
| IZ = 0              | HZ = 0           |
| SZ = 0              | FZ = 0           |
| IY = -0.138378898   | HY = -1.49225651 |
| SY = -17.1589834    | FY = 1.57079633  |
| ZN = 1E+19          | LN = 0           |
| ZO = -896.090385    | LO = 1.57079633  |
| ZP = 1.72187378     | LP = 1.57079633  |

Case 5 PF Correct 1240 RPM

Filter: Standard In/Out

|                 |                   |
|-----------------|-------------------|
| VN = -124       | UN = 0            |
| IN = -6.05      | HN = 0            |
| SN = -750.2     | FN = 0            |
| PN = -750.2     | QN = 0            |
| VO = 124        | UO = 0            |
| IO = 9.305      | HO = 0.863239847  |
| SO = 1153.82    | FO = -0.863239847 |
| PO = 749.958486 | QO = -876.848254  |
| VP = 0          | UP = 1.57079633   |
| IU = 7.071      | HU = -1.57052089  |

Filter: Pi(Delta)/ZP = XP

|                  |                  |
|------------------|------------------|
| VP = 0           | UP = 1.57079633  |
| IP = 0           | HP = 0           |
| SP = 0           | FP = 1.57079633  |
| PP = 0           | QP = 0           |
| IZ = -1.24E-17   | HZ = 0           |
| SZ = 1.5376E-15  | FZ = 0           |
| IY = -7.07135689 | HY = -1.57079633 |
| SY = -876.848254 | FY = 1.57079633  |
| ZN = 1E+19       | LN = 0           |
| ZO = -17.5355313 | LO = 1.57079633  |
| ZP = 0           | LP = 0           |

Filter: L Section

|                  |                  |
|------------------|------------------|
| VP = 0           | UP = 1.57079633  |
| IP = 0           | HP = 0           |
| SP = 0           | FP = 1.57079633  |
| PP = 0           | QP = 0           |
| IZ = 0           | HZ = 0           |
| SZ = 0           | FZ = 0           |
| IY = -7.07135689 | HY = -1.57079633 |
| SY = -876.848254 | FY = 1.57079633  |
| ZN = 1E+19       | LN = 0           |
| ZO = -17.5355313 | LO = 1.57079633  |
| ZP = 0           | LP = 1.57079633  |

## Case 6 Generator 1220 RPM Line C Open

Line Current: Degrees Input

|         |                 |
|---------|-----------------|
| IA = 10 | HA = 0          |
| IB = 10 | HB = 3.14159265 |
| IC = 0  | HC = 0          |

Machine Currents

|                     |                       |
|---------------------|-----------------------|
| IS(1) = 5.77366667  | HS(1) = -0.523598778  |
| IS(2) = 5.77366667  | HS(2) = 0.523598776   |
| IS(0) = 0           | HS(0) = 1.57079633    |
| IR(1) = -2.48845033 | HR(1) = 1.52996266    |
| IR(2) = 5.36373634  | HR(2) = 0.539365417   |
| IR(0) = 0           | HR(0) = 1.57079633    |
| IM(1) = 5.11546867  | HM(1) = -0.0781541745 |
| IM(2) = 0.421477667 | HM(2) = 0.321534678   |
| IM(0) = 0           | HM(0) = 1.57079633    |
| WR = 1220           |                       |

Machine Power

|                     |                     |
|---------------------|---------------------|
| ST(1) = -1661.22769 | FT(1) = -1.19661346 |
| ST(2) = 299.083655  | FT(2) = 1.30935391  |
| ST(0) = 0           | FT(0) = 0           |
| ST(3) = -1910.171   | FT(3) = -1.289703   |



Machine Power (Cont.)

|                     |                     |
|---------------------|---------------------|
| PT(1) = -607.198752 | QT(1) = 1546.28171  |
| PT(2) = 77.3054157  | QT(2) = 288.92024   |
| PT(0) = 0           | QT(0) = 0           |
| PT(3) = -529.893337 | QT(3) = 1835.20195  |
| PR(1) = -699.725022 | TR(1) = -46.3098384 |
| PR(2) = 025.7150896 | TR(2) = -1.77813521 |
| PR(0) = 0           | TR(0) = 0           |
| PR(3) = -695.440111 | TR(3) = -48.0879736 |
| VR(1) = -10.8247589 | UR(1) = 2.68092401  |
| VR(2) = 23.332253   | UR(2) = 1.69032677  |
| VR(0) = 0           | UR(0) = 3.14159265  |
| VR(3) = 19.613      | UR(3) = 1.21052479  |

Machine Power Loss

|                     |                     |
|---------------------|---------------------|
| PL(1) = 10.9790987  | PL(2) = 51.0086204  |
| PL(0) = 0           | PL(3) = 61.9877191  |
| PS(1) = 52.036289   | PS(2) = 52.036289   |
| PS(0) = 0           | PS(3) = 104.072578  |
| PT(4) = -529.379814 | PR(4) = -0.93222535 |

Case 6 Generator 1230 RPM Line C Open

Line Current: Degrees Input

|           |                 |
|-----------|-----------------|
| IA = 11.9 | HA = 0          |
| IB = 11.9 | HB = 3.14159265 |
| IC = 0    | HC = 0          |

Machine Currents

|                     |                      |
|---------------------|----------------------|
| IS(1) = 6.87033334  | HS(1) = -0.523598778 |
| IS(2) = 6.87033334  | HS(2) = 0.523598776  |
| IS(0) = 0           | HS(0) = 1.57079633   |
| IR(1) = -3.94357133 | HR(1) = 1.71346438   |
| IR(2) = 6.38253967  | HR(2) = 0.539300544  |
| IR(0) = 0           | HR(0) = 1.57079633   |
| IM(1) = 5.40695234  | HM(1) = 0.0867197372 |
| IM(2) = 0.501534333 | HM(2) = 0.322338305  |
| IM(0) = 0           | HM(0) = 1.57079633   |

Machine Currents (Cont.)

WR = 1230

Machine Power

|                     |                     |
|---------------------|---------------------|
| ST(1) = -2083.56774 | FT(1) = -1.05406672 |
| ST(2) = 423.444478  | FT(2) = 1.30967695  |
| ST(0) = 0           | FT(0) = 0           |
| ST(3) = -2403.678   | FT(3) = -1.17800748 |
| PT(1) = -1029.36451 | QT(1) = 1811.53615  |
| PT(2) = 109.317335  | QT(2) = 409.090389  |
| PT(0) = 0           | QT(0) = 0           |
| PT(3) = -920.047171 | QT(3) = 2220.62654  |
| PR(1) = -1130.50372 | TR(1) = -77.536011  |
| PR(2) = -36.5590262 | TR(2) = -2.50741419 |
| PR(0) = 0           | TR(0) = 0           |
| PR(3) = -1167.06274 | TR(3) = -80.0434252 |
| VR(1) = -17.1545353 | UR(1) = 2.86442574  |
| VR(2) = 27.7640476  | UR(2) = 1.6902619   |
| VR(0) = 0           | UR(0) = 3.14159265  |
| VR(3) = 26.403      | UR(3) = 1.04766893  |

Machine Power Loss

|                     |                     |
|---------------------|---------------------|
| PL(1) = 27.5732614  | PL(2) = 72.2263688  |
| PL(0) = 0           | PL(3) = 99.7996302  |
| PS(1) = 73.6815105  | PS(2) = 73.6815105  |
| PS(0) = 0           | PS(3) = 147.363021  |
| PT(4) = -919.900092 | PR(4) = -1.56442727 |

Case 7 Resistor Load 1PH to 3PH Low Amp

Filter: In/Out Print

|                 |                   |
|-----------------|-------------------|
| VN = 216        | UN = 0            |
| IN = -4.3       | HN = 0.523598775  |
| SN = 928.8      | FN = -0.523598775 |
| PN = 804.364396 | QN = -464.4       |

Filter: In/Out Print (Cont.)

|                  |                   |
|------------------|-------------------|
| VO = 216         | UO = 2.0943951    |
| IO = -4.3        | HO = 2.61799388   |
| SO = -928.8      | FO = -0.523598774 |
| PO = -804.364396 | QO = 464.399999   |
| VP = -216        | UP = 1.04719755   |
| IU = 4.3         | HU = 1.57079633   |

Filter: Pi(Delta)/ZP = RP

|                  |                  |
|------------------|------------------|
| VP = -216        | UP = 1.04719755  |
| IP = -7.44781845 | HP = 1.04719755  |
| SP = 1608.72878  | FP = 0           |
| PP = 1608.72878  | QP = 0           |
| IZ = 4.29999999  | HZ = -1.57079633 |
| SZ = 928.799998  | FZ = 1.57079633  |
| IY = -4.29999999 | HY = 0.523598775 |
| SY = -928.799998 | FY = 1.57079633  |
| ZN = 50.2325583  | LN = 1.57079633  |
| ZO = -50.2325582 | LO = 1.57079633  |
| ZP = 29.0017811  | LP = 0           |

Balancer Impedance: Elements

|                      |                  |
|----------------------|------------------|
| RN = 1.10229176E-07  | XN = 50.2325582  |
| CN = 0               | MN = 0.133245999 |
| RO = -1.10229176E-07 | XO = -50.2325582 |
| CO = -52.8060382     | MO = 0           |
| RP = 29.001781       | XP = 0           |
| CP = 0               | MP = 0           |

Case 7 Resistor Load 1PH to 3PH High Amp

Filter: Standard In/Out

|                 |                   |
|-----------------|-------------------|
| VN = 216        | UN = 0            |
| IN = -6.9       | HN = 0.523598775  |
| SN = 1490.4     | FN = -0.523598775 |
| PN = 1290.72426 | QN = -745.2       |

Filter: Standard In/Out (Cont.)

|                  |                   |
|------------------|-------------------|
| VO = 216         | UO = 2.0943951    |
| IO = -6.9        | HO = 2.61799388   |
| SO = -1490.4     | FO = -0.523598774 |
| PO = -1290.72426 | QO = 745.199999   |
| VP = -216        | UP = 1.04719755   |
| IU = 6.9         | HU = 1.57079633   |

Filter: Pi(Delta)/ZP = RP

|                  |                  |
|------------------|------------------|
| VP = -216        | UP = 1.04719755  |
| IP = -11.9511505 | HP = 1.04719755  |
| SP = 2581.44851  | FP = 0           |
| PP = 2581.44851  | QP = 0           |
| IZ = 6.89999998  | HZ = -1.57079633 |
| SZ = 1490.4      | FZ = 1.57079633  |
| IY = -6.89999998 | HY = 0.523598775 |
| SY = -1490.4     | FY = 1.57079633  |
| ZN = 31.3043479  | LN = 1.57079633  |
| ZO = -31.3043479 | LO = 1.57079633  |
| ZP = 18.0735737  | LP = 0           |

Balancer Impedance: Elements

|                      |                   |
|----------------------|-------------------|
| RN = 6.86935449E-08  | XN = 31.3043479   |
| CN = 0               | MN = 0.0830373619 |
| RO = -6.86935448E-08 | XO = -31.3043479  |
| CO = 084.7352706     | MO = 0            |
| RP = 18.0735737      | XP = 0            |
| CP = 0               | MP = 0            |

Case 8 Generator 3PH From 1PH 1250 RPM

Filter: Standard In/Out

|                 |                   |
|-----------------|-------------------|
| VN = 216        | UN = 0            |
| IN = -7.3       | HN = 0.523598775  |
| SN = 1576.8     | FN = -0.523598775 |
| PN = 1365.54886 | QN = -788.4       |

Filter: Standard In/Out (Cont.)

|                  |                   |
|------------------|-------------------|
| VO = 217         | UO = 2.0943951    |
| IO = -7.3        | HO = 2.61799388   |
| SO = -1584.1     | FO = -0.523598774 |
| PO = -1371.87084 | QO = 792.049998   |
| VP = -216.502    | UP = 1.05119765   |
| IU = 7.3         | HU = 1.57079633   |

Filter: Pi(Delta)/ZP = RP

|                  |                  |
|------------------|------------------|
| VP = -216.502    | UP = 1.05119765  |
| IP = -12.7323022 | HP = 1.05119765  |
| SP = 2756.56889  | FP = 0           |
| PP = 2756.56889  | QP = 0           |
| IZ = 7.40186045  | HZ = -1.57079633 |
| SZ = 1598.80186  | FZ = 1.57079633  |
| IY = -7.35093022 | HY = 0.523598775 |
| SY = -1595.15186 | FY = 1.57079633  |
| ZN = 29.1818525  | LN = 1.57079633  |
| ZO = -29.5200735 | LO = 1.57079633  |
| ZP = 17.0041518  | LP = 0           |

Balancer Impedance: Elements

|                      |                   |
|----------------------|-------------------|
| RN = 6.40359895E-08  | XN = 29.1818525   |
| CN = 0               | MN = 0.0774072681 |
| RO = -6.47781738E-08 | XO = -29.5200735  |
| CO = -89.8569034     | MO = 0            |
| RP = 17.0041518      | XP = 0            |
| CP = 0               | MP = 0            |

Rec'd of #2

VITA

Marcus Oliver Durham

Candidate for the Degree of

Doctor of Philosophy

Thesis: ANALYSIS OF INDUCTION GENERATORS ON UNBALANCED POWER SYSTEMS

Major Field: Electrical Engineering

Biographical:

Personal Data: Born in Orange, Texas, September 19, 1945, the son of William O. and Pettie Durham. Married to Rosemary Garlington on December 9, 1967. Father of four children: Robert Allan, 15; Christopher Marcus, 13; Karen Diane, 10; and Sarah Jane, 7.

Education: Graduated from DeRidder High School, DeRidder, Louisiana, in May, 1963; received the Bachelor of Science degree in Electrical Engineering, cum laude, from Louisiana Tech University, Ruston, Louisiana, in May, 1967; received the Master of Engineering degree in Engineering Systems from The University of Tulsa, Tulsa, Oklahoma, in May, 1978; completed requirements for the Doctor of Philosophy degree at Oklahoma State University in December, 1985.

Professional Experience: Engineer, Atlantic Richfield Company, Dallas, Texas, 1967; Data Processing Officer, USAF, El Centro, California, 1968-1972; Design Engineer, ARCO Oil and Gas, Dallas, Texas, 1972-1974; Senior Electrical Engineer, ARCO Oil and Gas, Tulsa, Oklahoma, 1975-1980; District Engineer, ARCO Oil and Gas, Tulsa, Oklahoma, 1981-1985; Instructor, School of Electrical and Computer Engineering, Oklahoma State University, August, 1985-present; Principal Engineer and Operations Director, THEWAY Corporation, Tulsa, Oklahoma, August, 1985-present.

Professional Organizations: Senior Member, Institute of Electrical and Electronic Engineers; Member, Society of Petroleum Engineers; Standards Member, American Petroleum Institute.

Honor Societies: Eta Kappa Nu, Electrical Engineering; Tau Beta Pi, National Engineering; Phi Kappa Phi, National Academic.