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# AGGREGATE PRODUCTION AND MANPOWER PLANNING MODELS

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Scope of Study: This research incorporates the personnel transition rates, inherent in all industrial situations, into the aggregate planning problem, and introduces the definition of the aggregate production and manpower planning problem. Two models are developed. The first is a linear programming model in which the Orrbeck model is used for the purpose of comparison and as a point of departure from which the new model is developed. The second is an extension of the first model from a single objective to a multiple objectives decision making model, and the goal programming is used as a method of multiple objectives procedures. The analysis of these models indicate their capabilities in presenting more realistic situations than existing models. A nondominance algorithm is developed to test the dominance of the goal programming solution, and to generate a nondominated solution if the goal programming solution turns out to be dominated. Also, a solution methodology for linear goal programming to include all the goals in the optimization process is proposed.

Findings and Conclusions: A substantial improvement in the model's results can be obtained by integrating the personnel transition matrix with manpower requirements. For instance, the results of the first model indicate that the performance of the new model is better than that of the Orrbeck model in representing more realistic situations and providing substantial savings for the two cases that are considered. The solution methodology developed in this research is applied to the second model and all the goals are included in the optimization process. A preferred solution (goal programming solution) and a nondominated solution are also obtained. The new method enables the decision maker to be involved in the optimization process and to provide reasonable aspiration levels for the targets, particularly if the targets are not known. Some of the goal programming difficulties are discussed and solved by the nondominance algorithm developed in this research. The nondominance algorithm, as well as the solution methodology, can be used to evaluate the results of current goal programming applications.

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PLANNING MODELS

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## PREFACE

This research combines, for the first time, the aggregate production and manpower planning problem in one model. Two such models have been developed, both of which will allow the managers of production organizations to more easily and accurately project future manpower and production requirements. The first (an extension to the Orrbeck et al. model (68)) incorporates the effects of the personnel transition matrix of the organization on manpower and production decisions. The second is the development of a goal programming model for the aggregate production and manpower planning problem.

The research also led to the development of an algorithm to test the goal programming solution and to generate a nondominated solution if the goal programming solution turns out to be dominated. This algorithm is used to solve the second model, and furthermore, a solution methodology has been proposed to include all the goals in the optimization process.

I wish to express my sincere gratitude to all those who assisted me in this study and during my stay at Oklahoma State University. In particular, I am especially indebted to my adviser, Dr. Philip M. Wolfe, for his encouragement and invaluable guidance throughout this study.

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# CHAPTER I

# INTRODUCTION

## General

The aggregate planning problem has received a great deal of attention over the last three decades. The term "aggregate planning" is used for a broad planning of production and workforce to maintain an economical stability over time. Recognition of the widespread existence of this problem has led to the publication of a number of different approaches for solving the aggregate planning problem. A broad discussion of this problem and its proposed solution may be found in Buffa (16) or in Khoshnevis (47).

Unfortunately, even though there are several approaches available to managers of production organizations, aggregate planning methods are seldom used in practice. Taubert (1968, p. 343) states that "simplified aggregate scheduling models have not found widespread use in industry." It is apparent that there are several reasons which have prevented a dissemination of these approaches into management practice. The most important one is that the present approaches of aggregate planning assume an aggregate workforce without any classifications to manpower classes. However, reality would suggest a specification of the kinds and the number of workers an organization will need to accomplish its objectives.

The rapid growth and complexity of modern production organizations have increased the importance of manpower planning, and the managers of many such organizations now desire to include manpower as well as production planning models in their kits. However, although there are various papers and books describing myriad applications of manpower planning models, no model which combines the aggregate manpower and production planning model can be found in the literature. The increasing need of management to accurately project future manpower and production requirements has made the development of such a combined model potentially critical.

## Statement of the Problem

Aggregate planning is the problem of scheduling aggregate workforce, production and inventories. It has long interested businessmen and academicians, and hence a number of approaches to this problem have been proposed in the literature. All of these approaches address in common the same pr›lem in its general form, which is defined as: given $S_t$, the demand for each period t in the planning horizon which extends over T periods, determine the production level $X_t$, inventory level $I_t$, and workforce $N_t$ for periods $t = 1, 2, \ldots, T$ which minimize relevant costs over the planning horizon. Models and decision rules have been developed, and a variety of solution techniques can be found in the literature. Although some of these techniques can be proven to be optimal and have been widely promulgated in classrooms and textbooks, it is difficult to find real world solutions where these techniques are applied to aggregate production and workforce decisions.

It can be said that the main drawback of the existing approaches to aggregate production planning is not in the solution methodology, but in the assumptions of the model. All the present approaches assume that the workforce and productivity factors are scalars. However, because workforce and productivity factors are the most important controllable variables in this problem, treating them as scalars would get the problem far from reality. The assumptions of treating these variables as aggregate numbers is not an acceptable one to the management. This is probably one reason that managers are not willing to include the current aggregate production planning approaches in their kits.

There have been a variety of recent applications of stochastic models of the so-called Markov matrix type to manpower planning. These Markov models generally multiply a vector of personnel in various job categories by a matrix of transition rates. This allows one to obtain a projection of the current workforce based upon past trends. Many researchers suggest that Markov models contain an essential element for developing manpower projections. This turns out to be that the transition matrix allows the analyst to interconnect the internal and external manpower flows across time periods, which leads to dynamic models of the Markov decision variety.

No attempt has previously been made to represent aggregate planning in multidimensional space. The proposed research will not only represent this problem in multidimensional space, but will also develop appropriate aggregate production/manpower planning models. The concepts of mathematical manpower planning with embedded Markov processes will be used in this development. The research problem can be illustrated by introducing the definition given below.

## A General Definition of the Aggregate Production and Manpower Planning Problem

The problem of aggregate production and manpower planning can be defined as: given $S_t$, $I_o$, $\bar{N}_o$, $\bar{C}_p$ and M, determine $\bar{N}_t$, $\bar{X}_t$ and $I_t$; t = 1, 2,..., T to achieve organization goals.

Where:

$S_t$ = the demand for period t

$\bar{N}_t$ = the graded workforce in period t

$\bar{X}_t$ = the amount to be produced in period t

$I_t$ = the on-hand inventory

$\bar{C}_p$ = productivity factor

M = personnel transition matrix of the organization with dimension e x e (e is the number of graded workforce).

T = the number of periods in the planning horizon

$I_o$ and $\bar{N}_o$ are the initial values of the inventory and the graded workforce respectively.

The problem of aggregate production and manpower planning can be formulated by different methods. Such formulation will depend upon the desired details, solution technique, constraints, goals of organization, etc.

## Research Objectives

One major objective of this research is to develop appropriate aggregate production and manpower planning models which incorporate personnel transition rates. The development of the models will be based on the method of embedding Markov processes into mathematical programming decision models.

Another objective is to develop an algorithm to test the goal programming solution, to generate a nondominated solution if the goal programming solution turns out to be dominated, and to provide a solution methodology to include all the goals in the optimization process.

## Summary of Results

The objectives of this research have been met. The two new models developed in this study are evaluated using the Orrbeck data (68) for the first one and hypothetical data for the second. The evaluation results of these models have demonstrated their capabilities in representing more realistic situations. The results also show that these models are highly flexible and can easily incorporate additional constraints regarding manpower and production requirements. The major conclusions are:

1. A substantial improvement in the model's results can be made by integrating personnel transition rates with manpower requirements.

The fundamental change is that the model goals and constraints, expressed as manpower and production requirements, and as budgetary and other constraints, influence the final manpower and production decisions recommended by the model.

2. The results obtained from the first model have been compared with that of the Orrbeck model (68). The results indicate that the performance of the new model is much better than that of the Orrbeck model with respect to representing more realistic situations and yielding minimum cost. The results show a savings of 7.18% and 3.67% in

the total cost over the Orrbeck model for the two cases that have been investigated.

3. The models are formulated as mathematical programming models (linear and goal programming); therefore, they should be easy for managers to understand and use. Furthermore, they are capable of providing optimal decisions regarding:

a. The graded number of workers an organization needs to accomplish its objectives

b. The graded number of hiring and firing

c. Production and overtime decisions

d. Inventory decisions

4. Some of the goal programming difficulties have been solved by using the nondominance algorithm developed in this research.

5. The solution methodology, developed to include all the goals in the optimization process, has been accomplished. The results of this investigation indicate that it is possible to develop such a methodology and that the decision maker can be incorporated in the optimization process to provide reasonable aspiration levels for the goals.

## Contributions

This research has made several major contributions in the area of aggregate production and manpower planning. These include:

1. The introduction of a general definition to the aggregate production and manpower planning problem.

2. Incorporating the effect of the transition matrix on workforce and production decisions.

3. The development of a linear programming aggregate production and manpower planning model.

4. The development of a goal programming aggregate production and manpower planning model.

The developed models in this research have the following new characteristics:

a. They are considered as applications of large scale models for manpower and production planning in manufacturing firms.

b. The cases of quit, attrition, etc., are considered in the developed models by representing them in the personnel transition matrix of the firm.

c. The number of hiring or firing in each class of workforce for each period can be explicitly determined. For instance, the management may hire and fire in the same period (i.e., hiring for one class and firing from another).

d. The models achieve management goals such as stabilizing the graded workforce, minimizing cost, meeting the demand, etc., taking into consideration the dynamics of internal workforce that are represented in the personnel transition matrix.

Other major contributions are in the area of goal programming. These include:

5. The development of an algorithm to test the goal programming solution and to generate a nondominated solution if the goal programming solution turns out to be dominated.

6. The development of a solution methodology to include all the goals in the optimization process and to obtain a goal programming and a nondominated solution(s) to the model. This method allows the decision maker to be involved in the optimization process and to provide information regarding reasonable values of the targets.

# CHAPTER II

# BACKGROUND

## Introduction

The present research combines aggregate planning and manpower planning in one model. Upon reviewing the literature, no such model has been found and the areas of aggregate planning and manpower planning are treated as separate areas of research. Therefore, the background of each area will be independently reviewed in this chapter.

## Background of Aggregate Planning

The application of mathematical programming techniques to aggregate planning began during the great post-World War II management science movement. Mathematical programming is a recently developed branch of optimization theory. The older branches originated from minimization and maximization problems that arise in geometry and physical sciences. Mathematical programming originated during World War II from minimization and maximization problems that arose in the decision sciences; namely, management sciences, operations research, and engineering design Since then the work on application of mathematical techniques to aggregate planning has continued at an accelerated pace. This work has been motivated, in part, by the tremendous economic consequences of aggregated decisions and by the current development and improvement of research methodologies in the field of management science. The initial thrust of this work was to use mathematical optimizing techniques such as differential calculus and linear programming to solve necessarily simplified aggregate planning cost models. Solving a model yielded a set of decisions or decision rules, which produced mathematically optimum results with respect to the cost model.

More recently, perhaps following a newer wave of management science emphasis, new proposals for solving the aggregate planning problem have been taking the form of decision rules which are based on heuristic problem-solving approaches and computer search methods. The objective of this newer methodology is to enable the model builder and decision maker to introduce greater realism. This added realism should, hopefully, more than compensate for the fact that heuristic and computer search techniques do not guarantee mathematically optimum decision rules. Advocates of heuristic and search decision rule approaches argue that since the decisions produced by a model can be no better than the model itself, it follows that greater realism should produce better overall results. All of these approaches have one thing in common: they address the aggregate planning problem, which is one of the most important problems in industry today.

## Current Aggregate Planning Approaches

Apart from decisions which are made by managers or committees without any mathematical help, there is a group of approaches which uses more or less mathematical sophistication in order to better model or efficiently solve aggregate planning decision problems. In general, these approaches are divided into two classes: those which guarantee optimality of the solution for a given model, and those which do not guarantee optimality, but find a near optimal solution. Examples of the former include linear programming, differential calculus, dynamic programming, goal programming, and an application of the discrete and continuous maximum principle.

The decision rules which do not guarantee mathematically optimum solutions with respect to the model are of two general types. The first is heuristic in nature and hypothesizes that decision rules can be represented by heuristically derived equations. The numerical values assigned to the coefficients of the equations are obtained in two ways. Bowman (13), in his management coefficients approach, performs a regression analysis of historical management decisions to obtain coefficients. Jones (46), in his parametric production planning approach, builds a forward-looking multistage cost model and simulates the operation of the model by plugging in trial values of the coefficients. The simulation takes the form of a coarse grid search based on systematically evaluating certain combinations of coefficient values. At the conclusion of the coarse grid search, the best set of coefficients is selected for use in the heuristically postulated decision rules.

The second major solution methodology of this type does not postulate the form of decision rule equations, but rather obtains specific numerical values associated with various decisions by climbing or searching the mathematical response surface formed by the criteria function of the model. This approach combines the advantage of realistic model representation by means of a computational algorithm with newly developed computer routines which search for the optimal point, or points, on a mathematical response surface. This approach is termed the "Search Decision Rule," as devised by Taubert (79).

From the foregoing discussion, it is possible to use the solution methodology to classify aggregate planning models:

1. Mathematical Programming Optimal Decision Rules (MPODR).
2. Heuristic and Search Decision Rules.

The background of the studies relative to the above two areas will be presented by some details on the most successful approaches.

## Mathematical Programming Optimal Decision Rules (MPODR)

The area of aggregate planning has been the subject of intensive research and writing for more than two decades. Although under different titles (such as production smoothing or master production planning) it has been considered by some to be the major decision framework involved in production management. The best decisions, by using MPODR, are found in optimizing the model in each period. The simplest approach in this group is that of linear models with corresponding linear programming solutions. There are many models of this type in the literature with different assumptions about costs, capacities, and demand patterns. The models which will be mentioned here are related to the original aggregate planning problem. Bowman (12) proposed a transportation method formulation for aggregate planning in 1956. The Bowman approach required the specification of a restricted number of production levels for each period and neglected the costs of changing levels. Bowman did not consider the work force explicitly. The increased complexity of the simplex method of linear programming was proposed by Magee (51) to incorporate the workforce decision and the costs of changing levels. Additional linear programming formulations have been proposed for aggregate scheduling (McGarrah (57), Charnes, Cooper, and Farr (20), and Dzielinski and Gomory (26)).

Hanssman and Hess (36) describe a linear programming model similar to the model developed by Holt, Modigliani, and Simon (36), which will be described later. Their model is simple and easy to implement; therefore, it will be discussed in some detail.

Aggregate planning reached a significant point with the publication of Planning Production, Inventories, and Workforce by Holt, Modigliani, Muth, and Simon (37) in 1960. The orientation of this book was based on an intensive research study conducted by the authors in an empirical situation. Their formulation of the problem was based on the assumption that the costs involved in aggregate planning could be represented by linear or quadratic functions. The resultant cost model was then minimized by differentation with respect to the decision variables, production and workforce. This operation produced a set of linear equations which could be solved for the values of the two decision variables. The net result was a set of two linear decision rules which related the present state of the system and the forecasted sales for an infinite time horizon to give the minimum cost values for the production and workforce for the next time period. Their model (HMMS model) will be discussed in more detail.

Dynamic programming is another approach extensively used for this problem or related problems. Bellman (11) applied dynamic programming to aggregate planning in 1956. The most important related reference is the Wagner and Whitin (85) dynamic lot size model. Unfortunately, the so-called "curse of dimensionality" makes the solution of any real planning problem impractical.

Goodman (31) proposed a goal programming approach for solving nonlinear aggregate planning models. This approach was illustrated via two case applications. The first was applied to the HMMS model, and the second used a higher order of cost terms. The two case applications demonstrated that the effectiveness of such an approach is highly dependent upon the degree of nonlinearity which the goal programming models must approximate. The author suggested that for relatively low degree models, goal programming may provide an efficient and effective solution approach, while for higher degree models the approach may be inappropriate.

More recently, Masud and Hwang (56) describe a multiple objective formulation of the multi-product, multi-period aggregate production planning problem. A numerical example is solved by using three Multiple Objective Decision Making (MODM) methods. The methods used are: Goal Programming (GP), Step Method (STEM), and Sequential Multiple Objective Problem Solving (SEMOPS). Masud and Hwang indicate that if GP is used, the analyst can solve a set of problems using different goals and priority structures, and then let the Decision Maker (DM) make the final selection for implementation. In the case of interactive methods, such as STEM and SEMOPS used in their research, the analyst can provide the trade-off decision required in each iteration in lieu of the DM. The analyst can also generate a set of solutions by providing different trade-off information, and from these solutions, the DM can make the final selection. The authors conclude that there is at present no best MODM method for solving such problems and that all such possible options using MODM methods are highly flexible and adaptable to different circumstances.

The details of the HMMS model and the Hanssman and Hess model will be presented in turn.

HMMS Model or Linear Decision Rule (LDR)

The HMMS Model or Linear Decision Rule (LDR) is the basis of all the approaches that will be presented in detail. All the others have been compared with this one because it is based on a reasonable model and an optimal solution can be obtained. Other methods must lead to nearly the same costs as this one, for the same reality, in order to qualify for being useful.

Holt et al. (37) suggest that four cost terms should be considered. These costs are:

1. Regular Payroll Costs

The size of the workforce is adjusted once a month, and setting the workforce at a certain level implies a commitment to pay the employees at least their regular time wage for a month. This is a linear cost function as defined by

$$\text{Regular Payroll costs} = C_1 W_t$$

The assumption here is that the cost is linearly related to the size of the workforce $W_t$. An additional cost term can be added to the above equation, but that would not affect the solution.

2. Hiring and Layoff Costs

The cost of increasing or decreasing the workforce is assumed to take the form of the quadratic function:

$$\text{Hiring and Layoff Costs} = C_2(W_t - W_{t-1})^2$$

where $W_t - W_{t-1}$ is the change in the level of the workforce from period t-1 to t. Here the cost is assumed to be symmetrical, i.e., an increase or a decrease in the workforce by a given amount incurs the same cost. Asymmetry in the cost function can be introduced, for example, by $C_2(W_t - W_{t-1} - C_{/10})^2$, but Holt et al. (37, p. 53) state that "this additional constant proves to be irrelevant in obtaining optimal decisions."

3. Overtime and Undertime Costs

If the size of the workforce is held constant, changes in the production rate can be absorbed by overtime or undertime. Undertime is the cost of idle labor at regular payroll rates. The overtime cost depends on the size of the workforce, $W_t$, and the aggregate production rate, $P_t$. The overtime cost function is assumed to be

$$\text{Overtime Costs} = C_3(P_t - C_4W_t)^2 + C_5P_t - C_6W_t$$

where $C_3$, $C_4$, $C_5$, and $C_6$ are constants.

4. Inventory, Back Order, and Setup Costs

The minimum cost inventory level is assumed to be linearly related to the demand, taking the form $C_8 + C_9D_t$, where $D_t$ is the forecast demand for period t. In fact, it is known from inventory theory that the optimal inventory level is proportional not to demand, but to its square root. In the HMMS model it is assumed that the linear relationship is an adequate approximation.

The total cost of inventory, back order, and setup are then assumed to take the quadratic form:

Inventory, Back Order and Setup Costs $= C_7[I_t - (C_8 + C_9D_t)]^2$

Figure 1 summarizes the four basic cost equations. The data employed are from a paint factory which was used extensively in their study.

The HMMS model can be written as: the costs to be minimized are represented by the following function considering the workforce, $W_t$; aggregate production, $P_t$; net inventory, $I_t$; and demand $D_t$ (where the subscript t designates the time period):

$$C_T = \sum_{t=1}^{T} [C_1W_t + C_{13} + C_2(W_t - W_{t-1} - C_{11})^2$$

$$+ C_3(P_t - C_4W_t)^2 + C_5P_t - C_6W_t + C_{12}P_tW_t$$

$$+ C_7(I_t - C_8 - C_9D_t)^2] \qquad (2.1)$$

By definition, the excess of production over orders affects net inventory as:

$$I_t = I_{t-1} + P_t - D_t \qquad (2.2)$$

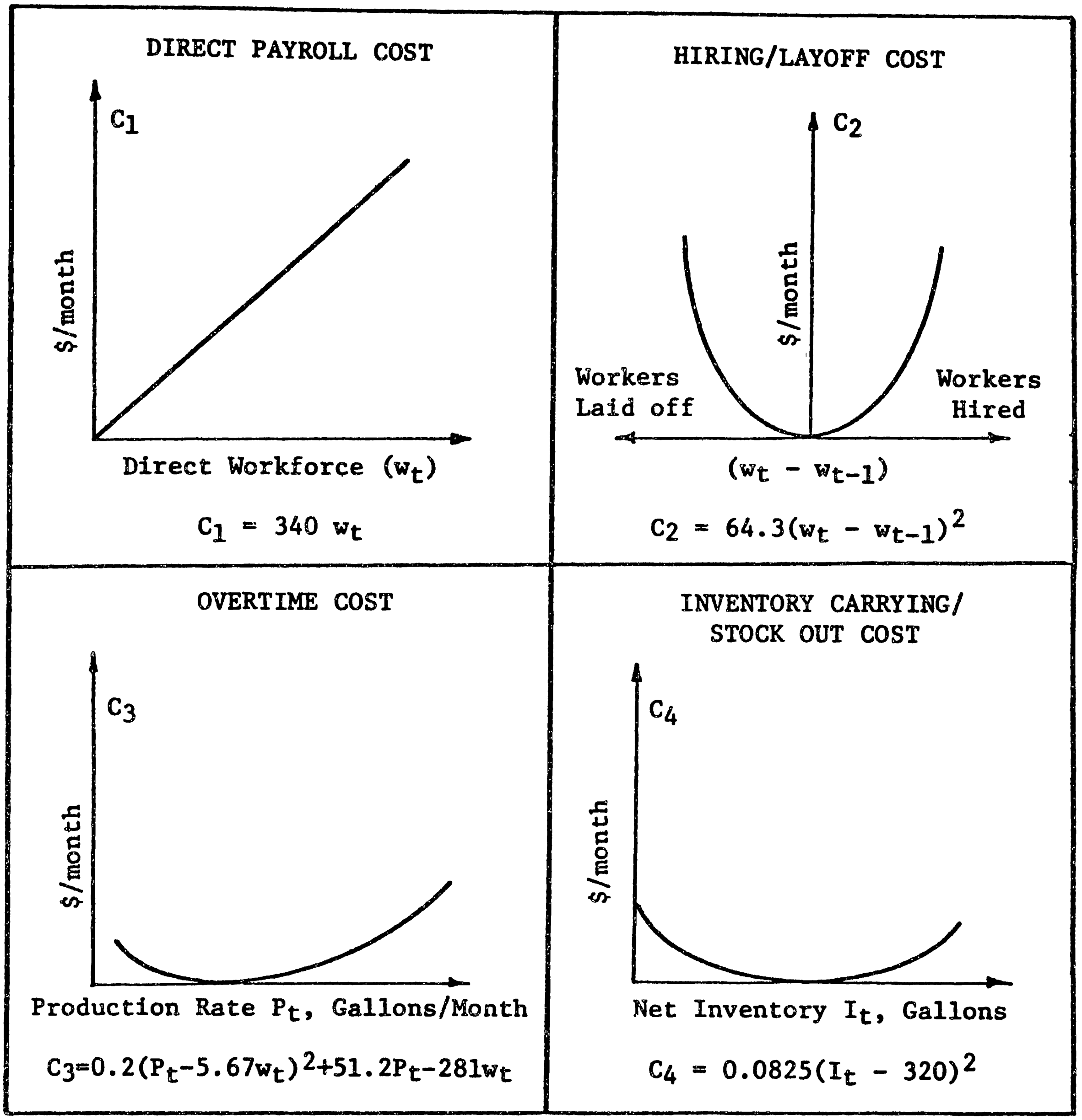
where t = 1, ..., T.

For a paint factory, which has been the example for comparisons, Holt et al. (37) determined the values of the $C_i$'s from statistical estimates based on accounting data and subjective estimates of intangibles. They found that the objective function could be stated as

$$C = \sum_{t=1}^{12} [340\ W_t + 64.3(W_t - W_{t-1})^2 + .2(P_t - 5.67\ W_t)^2$$

$$+ 51.2\ P_t - 281\ W_t + .0825(I_t - 320)^2] \qquad (2.3)$$

It is not a simple task to determine these coefficients, which is one difficulty in using this model.




Source: From Holt, C. C., Modigliani, F., Muth, J. F., and Simon, H. A., Planning Production, Inventories and Workforce, Prentice-Hall, 1960.

Figure 1. Cost Relationships of the Paint Factory Cost Model

By elimination of $P_t$ (or $I_t$) using the constraints, the model become quadratic with no constraints. Then by using differentiation a system of linear equations can be obtained. By inversion of the system matrix a set of linear decision rules (LDR) is found. As the interest is primarily in the first period decisions, only expressions for the optimal workforce ($W_1^*$) and the optimal production level for the first period ($P_1^*$) are needed.

$$W_1^* = a_1D_1 + \ldots + a_TD_T + bW_o + c - dI_o \tag{2.4}$$

$$P_1^* = e_1D_1 + \ldots + e_TD_T + fW_o + g - hI_o \tag{2.5}$$

The a's and e's decrease rapidly; therefore, the sensitivity to horizon increase is small.

The drawbacks of this approach are (besides the unusual quadratic cost expressions and the difficulty of finding the $C_i$'s) the possible occurrence of a negative $W_t$ or $P_t$, a negative component cost, and too high $I_t$, $W_t$, or $P_t$. Additional constraints have to be included in the model to control these variables.

Some advantages of this model are the ease of repetitive application of the linear decision rules and the guaranteed solution optimality (assuming that the optimum decision variables have a positive value).

## Hanssmann and Hess Model

The Hanssmann and Hess model (35) is based on the general assumptions of the HMMS model, but it uses linear functions. Their model is: minimize the function

$$C(P_1, \ldots, P_n; W_1, \ldots, W_n) = \sum_{i=1}^{n} \{ C_rW_i + C_k(W_i - W_{i-1})^+$$

$$+ C_f(W_i - W_{i-1})^- + C_o(KP_i - W_i)^+$$

$$+ C_1I^+ + C_2I^-\} \tag{2.6}$$

subject to the restrictions

$$P_i \geq 0 \tag{2.7}$$

$$W_i \geq 0 \tag{2.8}$$

$$I_i = I_{i-1} + P_i - D_i \quad (i = 1, \ldots, n) \tag{2.9}$$

where the $D_i$ and the initial conditions $(I_o, W_o)$ are given. If one defines (for any real number, a)

$$a^+ = \begin{cases} |a| & \text{for } a \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

and

$$a^- = \begin{cases} 0 & \text{for } a \geq 0 \\ |a| & \text{otherwise} \end{cases}$$

then,

$$a = a^+ - a^-$$

This definition may be thought of as an assumption rather than a restriction. Since it is generally known that an optimum solution of a linear programming problem will automatically yield pairs of numbers $(a^+, a^-)$ with the property that either $a^+ = 0$ or $a^- = 0$, the problem can be easily reformulated and solved by any linear programming algorithm.

Some advantages of this model are the possibility of establishing bounds for the variables, the ease of obtaining the cost coefficients, the possibility of obtaining more realistic cost functions using piecewise linear functions, and the possibility of performing sensitivity analysis using the dual solution. Some disadvantages are the linear assumptions and the computational work.

Orrbeck, Schwette, and Thompson in 1968 (68) developed a model in which the assumptions of constant wages and productivity in the production smoothing problem were dropped. Their model is an extension of the Hanssmann-Hess model; therefore, the necessary transformation to convert the model into linear programming format has been provided. The Orrbeck model classifies the workers into experience classes and can be used to illustrate the concept of incorporating personnel movement in aggregate planning models. For convenience, this model will be discussed in more detail in Chapter III.

## Heuristic and Search Decision Rules

The MPODR methods provide an optimum solution to a specific aggregate planning problem. The main drawback of these methods is that the assumptions are so restrictive that the models are unrealistic, or that realistic models are so complex that they are impossible to solve with current computational methods and equipment. The heuristic and search decision rule approaches are more free of the constraints of the mathematical forms. Thus, a trade-off must be made between the desirabilty of obtaining a known optimum solution to a relatively simplified model versus obtaining a near optimum solution to a more realistic model.

The most important approaches of heuristic and search decision rules are the management coefficients model, parametric production planning, and search decision rule. Therefore, these approaches will be discussed in turn.

## The Management Coefficients Model

Bowman (13) proposed a different approach to modeling managerial problems and tested his hypothesis on the aggregate planning problem. He said that on the average, managerial decisions are more accurate than those of any simplified model because managers have a more complex and complete mental model than can be expressed in mathematical terms. He showed that there was a high correlation between the actual decisions and those of the LDR, the decision rules corresponding to good regressions of the actual decisions. Then, by using the format of the LDR and regression, he tried to estimate decision rules for other cases. The best results were obtained using a feedback form similar to the original decision rules. For example, the versions of decision rules developed for regression were:

$$W_t = W_{t-1} + b_1(\bar{D}_{2-4} - \frac{\bar{D}}{\bar{W}})\, W_{t-1} + b_2(D_t \frac{\bar{I}}{\bar{D}} - I_{t-1}) + a_1 \quad (2.10)$$

$$P_t = b_3 W_t + b_4(\frac{\bar{W}}{\bar{D}}\, \bar{D}_{2-4} - W_t) + b_5(D_t \frac{\bar{I}}{\bar{D}} - I_{t-1}) + a_2 \quad (2.11)$$

where W, P, and I are as given before, $D_t$ represents actual sales in the current period, $\bar{D}_{2-4}$ represents average actual sales in the next three periods, and $\bar{D}$, $\bar{W}$, and $\bar{I}$ represent averages of these variables over the total period of investigation.

The theory behind Bowman's rules is that experienced managers are quite aware of and sensitive to the criteria of a system and the managerial decisions are basically sound. What is needed is to eliminate the "erratic" elements by making them more consistent. By averaging out the inconsistency, near optimal performance could be achieved.

Some of the advantages of this method are:

1. Easy implementation because it is not necessary to find costs and regression analysis is easy to perform.

2. More realism because implicitly a more realistic model is used.

Some of the criticisms of this approach are:

1. The form of the multiregression function is arbitrary and a particular regression of past decisions over a narrow range may lead to erroneous conclusions.

2. The regression model relies on decisions made by a particular manager or group of managers. Changes in personnel may render the model invalid.

3. The assumptions of unbiased managerial decisions and a nondynamic environment are not realistic.

## Parametric Production Planning (PPP)

Jones (46) developed a heuristic approach to aggregate planning which is called Parametric production Planning (PPP). PPP postulates the existence of two linear feedback rules. The first rule provides the number of workers and the second provides the production rate. Each rule contains two parameters. The rules are formulated to include the full range of possible decisions. The universe of possible parameters is searched to find the set of parameters that provides the lowest cost for a particular firm. Each set of parameters is evaluated by comparing the costs resulting from the application of rules bearing these parameter values to a likely sequence of sales forecasts and actual sales. The cost structure is not limited to linear functions or quadratic functions; therefore, it should be the best quantitative representation possible of the firm.

Jones (1967, p. 848) postulated the following rules:

1. Workforce rule.

$$W_1 = W_o + A\left(\sum_{i=1}^{E} b_i K(F_i) - W_o + b_1 K(I_i^* - I_o)\right) \qquad (2.12)$$

2. Production rule.

$$P_1 = K^{-1}(W_1) + C\left(\sum_{i=1}^{E} d_i F_i - K^{-1}(W_1) + d_1(I_1^* - I_o)\right) \qquad (2.13)$$

3. Weighting Function.

$$b_i = B^i / \sum_{i=1}^{E} B^i, \qquad d_i = D^i / \sum_{i=1}^{E} D^i$$

where:

A = Parameter between 0 and 1 indicating the portion of the desired workforce to be increased or decreased.

B = Parameter between 0 and 1 determining the relative weights to be placed on the forecasts for each of the E future periods.

$b_i$ = Weight applied to the sales forecast for the $i^{th}$ period in the future.

C = Parameter between 0 and 1 indicating the portion of the desired production to be increased or decreased.

D = Parameter between 0 and 1 determining the relative weights to be placed in the forecasts for each of the E future periods.

$d_i$ = Weight applied to the sales forecast for $i^{th}$ period in the future.

$E$ = Number of future periods to be included.

$F_i$ = The sales forecast for the $i^{th}$ period in the future.

$i$ = Number of the period where 0 is the period just completed and 1 is the immediate future period.

$I_o$ = Inventory of goods on hand.

$I_1^*$ = Optimal inventory at the end of the immediate future period.

$K(p)$ = Number of workers which can produce p units at the lowest total cost.

$K^{-1}(w)$ = Number of units which can be produced by w workers at the lowest cost unit.

$P_1$ = Production quantity determined by the production rule.

$W_o$ = Workforce on hand at the end of the zero period.

$W_1$ = Workforce determined by the workforce rule.

For the same paint factory used in the HMMS study, assuming that the HMMS cost model was realistic, Jones estimated his parameters, finding (with $I_1^* = C_8 = 320$) that

$$A = .2685, \quad B = .7745, \quad C = .9475, \quad D = .4692$$

In this comparison, PPP lost to the LDR by only .04 percent of the minimal cost, which shows a good approximation to the linear decision rule.

The great advantage of PPP is freedom from a given form of the reality model. The main disadvantage of this decision model is the limitation of four parameters; therefore, it has low flexibility for adaptation to complex situations.

## Search Decision Rule (SDR)

Taubert (79) used a general model and completely solved the optimization problem at each period. Therefore, he did not really provide any decision rule. As the problem formulation is general, techniques for general nonlinear programming must be used. Taubert used a pattern search routine (Hooke and Jeeves (38) and Weisman, Wood and Rivlin (87)) in order to find optimal decisions (W's and P's), given initial conditions and demands (see Buento-Neto (15)).

The Search Decision Rule (SDR) does not guarantee optimality, but it does offer a new way of breaking through the restrictive barrier imposed by the analytic model (the optimal solution methods discussed before). The SDR approach proposes building the most realistic cost or profit model possible and expressing it in the form of a computer subroutine which has the ability to compute the cost associated with any given set of decision variable values. Mathematically, the subroutine defines a multidimensional cost response surface with a dimensionality determined by the number of decision variables and the number of time periods included in the planning horizon. In short, the cost model forms a multistage decision system model in which the state represents the cost structure of the operation at the point in time when decisions are made, such as monthly, quarterly, etc. A computerized search routine is then used to systematically search the response surface of the cost model for the point (combination of decisions) producing the lowest total cost over the planning horizon. A mathematically optimum solution is not guaranteed, but the solutions found by the model cannot easily be improved.

For a practical application and comparison, Taubert used the same paint factory cost model as Holt et al. (37), but limited the planning horizon to 10 months in order to avoid too many dimensions in his search. Ten months means 20 dimensions for the search, as in each month there are two decision variables ($W_t$ and $P_t$). In comparison with LDR, SDR lost by only .1 percent. SDR cannot guarantee the exact optimum, but the difference will not be large.

The great advantage of SDR is its capability to handle any form of reality model, although for some functions we may have problems in the search. The disadvantage is the non-guarantee of optimality as the SDR may stop far from the optimum or at a local minimum or maximum. Also, if the cost function is complex the computation time and cost may offer some inconvenience, especially if a long horizon must be used.

There are other heuristic approaches to aggregate planning which can be found in the literature. Among them, Elmaleh and Eilon (28) suggested a switching procedure for use in industries in which production is limited to discrete levels. Millichamp and Love (58) proposed a simple modification to the production switching heuristic which renders the methodology appropriate for aggregate planning problems in general. They based their approach on the random walk approach to aggregate production planning proposed by Orr (67) and adapted by Elmaleh and Eilon (28).

More recently, Khoshnevis (47) incorporated the effects of the improvement curve productivity phenomena, present in most industrial situations, into the aggregate planning problem. He also described the effects of disruptions in productivity improvement, progress, and retrogression to the production and workforce planning area. Aggregate planning of both long cycle and short cycle production situations were considered and models peculiar to each case were developed in his work.

For more details of the aggregate planning problem and its extensions, refer to Khoshnevis (47). He presents a detailed discussion on the state-of-the-art of aggregate production models and analyzes the effects of a dynamic productivity factor throughout the planning horizon.

## Background of Manpower Planning
## (Human Resource Planning)

Manpower planning is a process intended to assure an organization that it will have the correct number of properly qualified and motivated employees in its workforce at some specified future time to carry on the work that will then have to be done. Manpower planning has been a function of management since the origin of modern industrial organization. The relatively sophisticated techniques available to management today are the outcome of a long period of evolution. A variety of approaches to manpower planning has been developed and proposed. These approaches are broadly termed "human resource planning models."

A review of human resource planning models by Milkovich and Mahoney (60) indicates that the general types of models observed in practice and in literature can be classified as:

1. Heuristic: to provide organization and direction

2. Theory-research based: for analysis and strategy development and determination

3. Technique oriented: for analytical models and their solutions

The general nature of each of these models and their applications is considered in turn. The heuristic and theory-research based models are taken from Milkovich and Mahoney (47). The current research is not concerned with these models; they are repeated here for illustration, not as a review of literature.

## Heuristic Models

These models are heuristic in the sense that they are designed to enable the users to organize their thoughts and to approach the issues in a systematic manner. Such models serve to provide aid or direction in the solution of the manpower planning problem. The literature has several illustrations of these conceptualizations of human resource planning (Burack and Walker (18). Generally, the common components of the models reported include:

1. Determining the human resource objectives;

2. Analyzing the internal labor supplies available and projecting into the future;

3. Matching the desired human resource position with the estimated actual position and identifying areas of surplus and/or shortages for each period;

4. Generating and analyzing alternative policies and strategies to achieve the human resource objectives, including alternative staffing, recruiting, job and organizational design, and training programs; and

5. Implementing the programs and reevaluating results against the human resource objectives.

## Theoretical-Research Based Models

Another major class of manpower models can be labeled as theoretical-research based models. These models are more concerned with the identification of the variables that influence an organization's human resource objectives. Some of the questions theoretical models are designed to answer include:

1. What are the specific determinants of unit productivity, employee performance, job satisfaction, or unit labor cost?

2. What relationships exist between budget expenditures on manpower programs such as training and unit productivity?

3. How does a policy of "promotion from within" impact unit productivity, labor costs, or legal compliances with EEO?

The focus is more on the specification of the substance or content of human resource objectives than on the issue to be considered or the analytical techniques to be used. For example, a heuristic model includes "determine human resource objectives," whereas a theoretical model may include "employee performance as a function of skills, motivation and technology."

These models are derived from economic and organizational research theories; therefore, they can provide critical input for human resource planning. Most managers currently operate with implicit models of the critical factors that will impact their human resources. Concepts and insights drawn from organization-related theory and research may also prove to be of value.

## Technique-Oriented Models

The third area of human resource modeling is the application of mathematical models to human resource issues. There is a wide variety of technique-oriented models that have been applied to various human resource planning elements with reasonable success. The most significant advances in human resource modeling techniques have occurred in the application of Markov chains, renewal, and goal programming models to the human resource stock and flow processes within the organization. The applications of these models include:

1. Forecasting the future human resources requirements that will be satisfied by the current inventory of personnel, and forecasting the future human resource budget commitments represented by the current stock of personnel

2. Analyzing the impact of proposed changes in policy and programs

3. Designing and structuring systems that will balance the flows of internal human resource supplies, requirements, and costs, and designing human resource information systems suitable for policy analysis and planning

It is an extremely difficult task to attempt to discover the first application of each concept in manpower planning. However, the most common mathematical models, as classified in the literature, are given below:

1. Markov Chain Models

2. Renewal Models

3. Normative or Optimization Models

These models are briefly reviewed in turn.

Markov Chain Models

There have been a variety of recent applications of Markov chain models to manpower planning. These Markov models generally multiply a vector of personnel in various job categories by a matrix of transition rate. This allows one to obtain a projection of the current workforce based upon past trends. Early work in this field dates back to the late 1940's, but it was only in the late 1960's that a coherent body of theory began to emerge. Probably the best known applications are those of Vroom and MacCrimmon (84), Bartholomew (6), Merch (59), and Mahoney and Milkovich (52). Among the others using Markov models for manpower planning are Forbes (29), Rowland and Sovereign (75), Marshall and Oliver (54), Stewman (78), and Nielsen and Young (66).

Markov chain models are most appropriate where the job classes and rates of flow between them are stable and the flows out of a class depends on the class occupied and the number of personnel in the class. The rates of movement depend upon the current class which has been defined in terms of organization level, salary grade, function, experience, age, sex or race. The Markov models contain an essential element for developing manpower projections. This turns out to be that the transition matrix allows the analyst to interconnect the external and internal flows across time periods.

The Markov chain model is capable of describing the changes in a graded manpower system. Given suitable assumptions about future loss and transition probabilities, the model can be used for forecasting the grade structure. It can also be used as a tool for exploring the consequences of different manpower policies and hence for controlling the structure. It is also possible to validate the model to the extent that recent history can be verified on the basis of prediction from the distant past.

## Renewal Models

Renewal models are most usefully applied to situations where grade size is closely controlled within the organization and where promotion and hiring decisions are made only to fill vacant positions. In many situations, this type of model can be used to examine various policies and to evaluate the results of their application on system parameters such as promotion rates, length of stay in grade, etc.

Bartholomew (7) developed the mathematical equations of the renewal models that permit the evaluation of variables in both discrete and continuous time. Bartholomew and Forbes (8) show how renewal models can be used to study career patterns and contrast these results with those that can be obtained from Markov models. Piskor and Dudding (70) describe the incorporation of a renewal model in a conversational program in use for the planning of grade sizes, hiring, firing and transfers in the Canadian Public Service. Stewman (78) compares the performance of the Markov chain, the Markov chain with duration of stay (Semi-Markov) and a vacancy model having both renewal and Markov properties. He finds that the renewal or opportunity model performs better in general than either the Markov chain or the Markov chain with duration of stay.

## Normative Models

The Markov chain and renewal models are descriptive in nature and are used to forecast future manpower requirements or to study the various policies on manpower systems. Normative models suggest a solution to the manpower planning model. This solution is optimal for a set of management goals or objectives.

One of the earliest applications of the normative techniques to the manpower planning models was the use of linear programming and its extensions. Kildebeck, Kipnis and Macky (48) developed a linear programming model for the pilot training cycle of the U.S. Air Force. The Marine Corps, as described by Marsh (53), used a linear programming model to assist in the planning of troop rotations. Purkiss (74) describes a linear programming model that was used to help drive training budgets for manpower in the British steel industry, while Morgan (62) and Clough, Dudding and Price (24) used this framework in studies of the Royal Air Force and the Canadian forces. Among the other works that one could cite are the industrial manpower models utilizing mathematical programming by Purkiss (73) for the British Iron and Steel Institute, and that of Alagizy (1) for IBM. Most of these models have experienced implementation difficulties. In addition to the problem of management communications, their implementation has been slowed by the model's construction. They have optimized a single objective function, and generally, have not handled the problem of multiple period planning very well. In personnel management, objectives are multiple and the appropriate solution technique is goal programming.

Charnes, Cooper and Niehaus (23) describe a goal programming model for guiding and controlling manpower planning at the level of the Office of Civilian Manpower Management (OCMM) of the U.S. Navy. The personnel requirements are accommodated by the goal programming aspect and the transition of recruits and job incumbents from one position to another are accommodated by the stochastic elements of a Markov chain. This model has been extended to consider more complex transitional effects, e.g., those due to retirement and to allow for interperiod Markov transition matrices which change over time. Most of these models have been developed and applied in military and government settings. Price and Piskor (72) describe a successful application of goal programming to the planning of hiring and promotions in the Canadian Armed Forces.

Zanakis and Maret (90, 92) presented a Markov chain application to model the manpower supply of over 1,000 engineers in a department of a large chemical company. They also suggested a Markov chain/preemptive goal programming sequential approach for solving manpower macro planning problems under various restrictions and conflicting goals.

More recently, Martel and Price (55) showed how state space models for human resource planning may be extended from linear and goal programming formulations to cover the case where manpower demands and available resources for future periods are not known for certain. However, they stated that the model can be treated as a multi-period stochastic program with simple recourse. They used normal and Beta probability distributions to fit the right hand sides and solved the equivalent deterministic program using convex separable programming. They also applied their methodology to a military human resource planning problem.

There have been a number of books published on the subject of manpower systems. The proceeding of NATO-sponsored meetings on human resources planning contains many illustrations of the use of diverse methods and models (14, 25, 77, 89). In addition, Bartholomew (7), Bartholomew and Smith (10), Bartholomew and Forbes (8), Bartholomew and

Morris (9) Charnes et al. (23), Grinold and Marshall (33), Moore and Charach (61), Niehaus (64), Vajda (81), Walker (86) and Verhoeven (82) have published books on specific areas of the mathematics and techniques of manpower and human resources planning.

# CHAPTER III

# INCORPORATING THE PERSONNEL TRANSITION MATRIX IN AGGREGATE PRODUCTION PLANNING MODELS

## Introduction

One major objective of this research is to study the effect of a personnel transition matrix on aggregate production planning models and to develop appropriate aggregate production and manpower planning models. The developed models will consider the fact that the workers must be treated as a graded workforce and hence differ in both productivity and wage. Such models incorporate the effect of a personnel transition matrix on workforce and production decisions.

To achieve this objective, two models will be developed. The first one will be developed in this chapter and be called Model I. The second will be developed in Chapter VI and will be called Model II.

Model I is a linear programming model of the aggregate production and manpower planning problem. The Orrbeck model (68) will be used as a point of departure from which this new model will be developed. This model will be considered as a starting point for developing aggregate production and manpower planning models and will also be used to verify the results of Model II.

Model II is an extension to Model I from a single objective to a multiple objectives model in which goal programming is selected as a multiple objectives solution procedure.

In this chapter, some of the definitions and concepts of manpower planning will be presented. The Orrbeck model will also be discussed in detail since it is the first model which incorporated the effect of worker productivity on production smoothing and classified the workers into different classes. These materials are appropriate for developing the new models.

## Manpower Planning

Vetter (1967), among others, defined manpower planning as:

> The process by which management determines how the organization should move from its current manpower position to its desired manpower position. Through planning, management strives to have the right number and the right kinds of people, at the right spaces, at the right time, doing things which result in both the organization and the individual receiving maximum long run benefits (p. 15).

## Manpower System

A manpower system is considered to be composed of mutually exclusive and exhaustive classes of states so that each member of the system may be in one and only one class at any given time. These classes may be defined in terms of any relevant variables. The manpower system is concerned with the numbers in each of these classes at discrete points in time, and with the numbers (or flows) moving between these classes from one point to the next. The system is open so that flows to and from the outside world are permitted. These flows correspond to wastage and recruitment respectively.

## Markov Chain Models

There have been a variety of recent applications of stochastic models of the so-called Markov matrix type to manpower planning. In the

Markov model the flows are assumed to be governed by transition probabilities, and each class is homogeneous and independent with respect to these probabilities. That is, each member of a class has the same probability of making a particular transition, and furthermore, these probabilities operate independently. The basis of the Markov assumption is that the transition probability depends only on the class of state occupied at present.

The Markov models generally multiply a vector of personnel in various job categories by a matrix of transition rates. This allows one to obtain a projection of the current workforce based upon past trends. Many researchers suggest that Markov models contain an essential element for developing manpower projections. This turns out to be that the transition matrix allows one to interconnect the internal and external manpower flows across time periods, which leads to dynamic models of the Markov decision variety.

The Markov chain model can be represented by the matrix equation:

$$X(t+1) = X(t)\ M + n(t+1)\ P$$

where:

$X(t+1)$ = the expected stocks vector at time t+1.

$X(t)$ = the stocks vector that is observed at time t.

$M$ = Personnel Transition Matrix (PTM) or transition probability matrix of the organization.

$n(t+1)$ = the number of entrants at time t+1.

$P$ = a vector showing how the entrants are distributed among the state of the system.

Repeated application of this equation allows forecasting of the stocks vector for later points in time. The Markov models contain an essential element for developing manpower projections since PTM allows the analyst to interconnect the internal and external manpower flows across time periods.

## Orrbeck Model

As previously mentioned, the first aggregate planning model which incorporated the effect of worker productivity and classified the workers into classes was developed by Orrbeck et al. (68). This model is an extension of the Hanssman-Hess model (35) which presents a linear programming formulation of the aggregate planning problem. The Hanssman-Hess model was discussed in Chapter II and will be repeated here for the purpose of clarity. The essential cost elements of the Hanssman-Hess model are regular payroll costs, overtime pay, costs of hiring and firing workers, and storage and shortage costs. The sum of these costs accounts for the total relevant cost in any period. The problem, then, is one of choosing production and workforce levels in order to minimize the sum of the total relevant costs over the planning horizon. The regular payroll costs in any period t are assumed to be proportional to the number of workers employed in that period. The cost of overtime is found by first establishing an upper limit on the production that can take place on regular time. Any production in excess of this amount must be done on overtime. To establish the upper limit to regular time production, Hanssmann and Hess (35) assume that each employee can produce exactly the same amount in a given period. The hiring or firing costs in any period t are assumed to be proportional to the number of workers hired or fired in that period. The inventory carrying costs and back order costs are assumed to be proportional to the amount of inventory or shortage at the end of the period. The production planning problem, then, is to determine $X_t$ and $N_t$ $(t = 1, \ldots, T)$ in order to minimize

$$C = \sum_{t=1}^{T} [C_r N_t \qquad \text{Payroll Costs}$$

$$+ C_o(\frac{1}{K} X_t - N_t)^+ \qquad \text{Overtime Pay}$$

$$+ C_h (N_t - N_{t-1})^+ \qquad \text{Hiring Costs}$$

$$+ C_f (N_t - N_{t-1})^- \qquad \text{Firing Costs}$$

$$+ C_I I_t^+ \qquad \text{Inventory Costs}$$

$$+ C_s I_t^- ] \qquad \text{Shortage Costs}$$

Subject to

$$X_t \geq 0, N_t \geq 0, I_t = I_{t-1} + X_t - S_t, t = 1, \ldots, T$$

where:

$T$ = number of periods in the planning horizon

$N_t$ = workforce level in period t

$X_t$ = production level in period t

$S_t$ = demand in period t

$C_r$ = wage rate per period

$C_o$ = overtime pay per worker per period

$\frac{1}{K}$ = number of units of output per employee per period

$C_h$ = hiring cost per employee per period

$C_f$ = firing cost per employee per period

$C_I$ = inventory cost per unit per period

$C_s$ = shortage cost per unit per period

$I_t$ = inventory level in period t

By using the proper transformations, the problem can be converted into a linear form and thus be solved by standard linear programming methods (refer to Hanssman and Hess (35) for details).

Orrbeck (68) made the following assumptions:

1. All employees fall into one of e experience classes, where class e represents the most experienced class of workers.

2. The number of workers in an experience class will be the number of workers in the next most experienced class in the preceding period, minus the number of workers released from the group. Exceptions are the first and last groups. The first group will consist of newly hired workers and the most experienced class will consist of employees in this group in the previous period plus those promoted into the class by the passage of time.

3. If workers are to be fired, the least experienced workers are fired first. Should the number of workers fired in a period exceed the number of employees in the first class of the previous period, some workers from the second experience class would have to be laid off.

4. Constraints governing the assignments of overtime are: (a) the unduly large amount of overtime not assigned to any class of employees and (b) the workers will be called upon in order of seniority. Thus the most experienced workers will work overtime first, subject to the limit of their capacity. If overtime work still remains, the next most experienced class will be called upon until all overtime work is assigned.

5. No shortages will be allowed and the inventory carrying cost will be assumed proportional to the average inventory.

As a result of the above assumptions, Orrbeck (68) added a set of new constraints to the original Hanssmann-Hess model, then transformed the model into a linear programming format. The Orrbeck model prior to transformation has the following structure:

$$\text{Min. } C = \sum_{t=1}^{T} [\sum_{i=1}^{e} N_t^i C^i + C_h N_t^1 + C_f N_t^f + a \sum_{i=1}^{e} \frac{C^1}{P^i} O_t^i + \frac{1}{2} C_I (I_t + I_{t-1})] \quad (3.1)$$

Subject to the following constraints:

$$I_t = I_{t-1} + X_t - S_t \quad (3.2)$$

$$O_t = [X_t - \sum_{i=1}^{e} P^i N_t^i]^+ \quad (3.3)$$

$$R_t^i = [O_t - \sum_{j=i+1}^{e} (\ell-1) P^j N_t^j]^+, \quad i = 1, 2, \ldots, e-1 \quad (3.4)$$

$$O_t^i = R_t^i - R_t^{i-1}, \quad i = 1, 2, \ldots, e \quad (3.5)$$

$$X_t \leq \sum_{i=1}^{e} \ell P^i N_t^i \quad (3.6)$$

$$N_t^i = [N_{t-1}^{i-1} - (\sum_{j=1}^{i-2} N_{t-1}^j - N_t^f)^-]^+, \quad i = 2, \ldots, e-1 \quad (3.7)$$

$$N_t^e = [N_{t-1}^e + N_{t-1}^{e-1} - (\sum_{j=1}^{e-2} N_{t-1}^j - N_t^f)^-]^+ \quad (3.8)$$

$$N_t^i \geq 0, \; O_t^i \geq 0, \; N_t^f \geq 0, \; X_t \geq 0, \; I_t \geq 0, \; t = 1, \ldots, T.$$

Where:

e = maximum number of experience classes.

$P^i$ = the number of units produced by each member of the $i^{th}$ experience class on regular time.

$O_t$ = total amount of overtime production in period t.

$O_t^i$ = amount of overtime production assigned to class i in period t.

$R_t^i$ = amount of overtime work remaining available in period t to the members of class i and the workers with less experience after overtime work has been assigned to the more experienced workers.

$N_t^i$ = number of workers in class i in period t.

$N_t^1$ = number of workers hired in period t.

$N_t^f$ = number of workers fired in period t.

$C^i$ = regular payroll cost per worker in class i per period.

$\ell$ = a constant such that $\ell p^i$ is the maximum production (in units) by one worker of experience class i on regular time and overtime.

a = a constant such that a $C^i$ is the overtime payment per worker in class i.

The remainder of the variables were defined previously.

## Personnel Transition Matrix and the Orrbeck Model

As previously stated, Orrbeck (68) assumed that the number of workers in an experience class would be the number of workers in the next most experienced class in the preceding period, minus the number of workers released from the group. This assumption does not represent the dynamics of the internal workforce which includes both movement (or lack of it) within the organization and external losses. The loss rates may be further subdivided into terminal losses from persons retiring and quitting. Considerable insight can be gained into the structure of the organization through analysis of movements and retirements. The internal movements are important if one is to obtain correct estimates of internal supplies and losses of personnel in the future. For example, the movement of a worker in class 1 to class 2 represents both a loss to class 1 and a gain to class 2. In planning aggregate skills, the basic source of this information is the transition matrix.

For the sake of clarity, consider a hypothetical example of the steps required to develop a transition matrix. For the purposes of this example, the following job categories will be used:

| Job Category | Code |
|---|---|
| Management | Mgt |
| General Administration | Gen |
| Skilled Worker | SW |
| Unskilled Worker | UW |

These categories can be used to go into the historical personnel files to obtain the data needed to build the transition matrix. What is needed are data on the job categories occupied by each individual in a sample (or complete count) at two relevant time periods. This allows a "snapshot" to be taken of personnel population between the two time periods. In this numerical example these data take the form shown in Figure 2.

| Employee Number | Job Category Time 1 | Job Category Time 2 |
|---|---|---|
| 3024 | Mgt | Mgt |
| 3025 | SW | |
| 3047 | Gen | Mgt |
| 3072 | | UW |

Figure 2. Transition Data File

The resulting file can now be used to develop the transition matrix by using a table, such as Table I. In this transition table, of the 50 employees in the management category at Time 1, only 40 remained in that job category by Time 2. Also, five of the 50 transferred to the general administrative category and five had left the organization. By adding the columns, one can obtain the population distribution at Time 2. The rates of movement can be obtained by dividing the number in each category in a given row of the row total. For example, in the row associated with the management, the 40 remaining in management are divided by the 50 at the start to give .8 (80%) and the five that moved to general administration results in a .1 (10%) movement rate, etc. The rates for the complete transition matrix are given in Table II.

Transition matrices can be established for a wide variety of job categories and time periods. In a planning model, the critical factor is that the transition rates be consistent with time periods used in the model. The transition matrix may also be modified to more accurately reflect the period being projected.

TABLE I

HISTORICAL TRANSITION STATISTICS

| Job Category | Totals Time 1 | Mgt | Gen | UW | SW | Exits |
|---|---|---|---|---|---|---|
| Mgt | 50 | 40 | 5 | | | 5 |
| Gen | 300 | 10 | 210 | | | 80 |
| UW | 600 | | | 360 | 60 | 180 |
| SW | 500 | | | | 450 | 50 |
| Entries | | 5 | 110 | 300 | | |
| Totals Time 2 | | 55 | 325 | 660 | 510 | |

TABLE II

TRANSITION RATES FROM TIME 1 TO TIME 2

| | MGT | GEN | UW | SW | Exits |
|---|---|---|---|---|---|
| Mgt | .80 | .10 | | | .10 |
| Gen | .03 | .70 | | | .27 |
| UW | | | .60 | .10 | .30 |
| SW | | | | .90 | .10 |

Source: From Niehaus, R. J., Computer Assisted Human Resources Planning, Wiley Interscience, New York, 1979.

From the foregoing discussion, it seems natural to drop assumptions 2 and 3 of the Orrbeck model in favor of a model in which the personnel transition matrix (PTM) of the organization will be used. The PTM governs the personnel movement during the time horizon and it will achieve the following characteristics which do not exist in the Orrbeck model. These characteristics are as follows:

1. The cases of quit, attrition, promotion, etc., will be considered in the new model by representing them in the PTM of the organization.

2. The number of workers hired or fired in each class for each period can be explicitly determined. For instance, management may hire and fire in the same period, i.e., hiring for one class and firing from another class.

The other assumptions of the Orrbeck model will not change in the new model since they are relevant assumptions.

## Workforce Constraints

A general formula of workforce is:

$$\text{Number Remaining in a Class} + \text{New Hires} - \text{Fires} = \text{Number in a Class}$$

To calculate the above equation for each period in the planning horizon, the initial number in each class (Job Category) is assumed to be constant and known. This initial number is then multiplied by the transition rates to project those staying in a particular class, those being promoted, and those leaving the organization. New hiring or firing is added to the number remaining in each class to get the number of workers in each class in the first period of the forecast. The process is repeated for the next period, multiplying the projected number at the end of the first period by the transition rates to obtain the number remaining and class changes in the second period. Again the new hiring and firing are added to obtain the number of workers in each class in the second time period. This process is then repeated for all the periods included in the model.

It is convenient to use a matrix notation to develop a mathematical expression of the above word equation. This can be done by introducing the following notation:

$M$ = Personnel Transition Matrix (PTM) of the organization with dimension e x e.

$\bar{N}_t$ = a column vector represents the number in each class in period t.

$\bar{N}_t^h$ = a column vector represents the number of hires in each class in period t.

$\bar{N}_t^f$ = a column vector represents the number of fires in each class in period t.

$\bar{N}_o$ = a column vector represents the number of workers in each class initially.

$\bar{N}_t$, $\bar{N}_t^h$, $\bar{N}_t^f$, $\bar{N}_o$ are nonnegative vectors each with dimension e.

By using the above notation one can write the following workforce constraints:

Period 1:

$$M\bar{N}_o + \bar{N}_1^h - \bar{N}_1^f = \bar{N}_1$$

Period 2:

$$M^2\bar{N}_o + M\bar{N}_1^h - M\bar{N}_1^f + \bar{N}_2^h - \bar{N}_2^f = \bar{N}_2$$

$$\vdots$$

Period t:

$$M^t\bar{N}_o + \sum_{i=1}^{t} M^{t-i}\, \bar{N}_i^h - \sum_{i=1}^{t} M^{t-i}\, \bar{N}_i^f = \bar{N}_t$$

Thus, the workforce constraints can be given by

$$\bar{N}_t - \sum_{i=1}^{t} M^{t-i}\, \bar{N}_i^h + \sum_{i=1}^{t} M^{t-i}\, \bar{N}_i^f = M^t\, \bar{N}_o \text{ and } t = 1, \ldots, T. \qquad (3.9)$$

Model Formulation

As mentioned before, the assumptions concerning the workforce constraints in the Orrbeck model are unrealistic because they do not represent the dynamics of the personnel movement in the firm. In the proposed model, these constraints will be replaced by those developed in the previous section. Therefore, the new model can now be formulated as:

$$\text{Min. } C = \sum_{t=1}^{T} [\bar{C}\, \bar{N}_t + \bar{C}_h\, \bar{N}_t^h + \bar{C}_f\, \bar{N}_t^f + a \sum_{i=1}^{e} \frac{c^i}{p^i}\, O_t^i$$

$$+ \frac{1}{2}\, C_I\, (I_t + I_{t-1})] \qquad (3.10)$$

Subject to the following constraints:

$$I_t = I_{t-1} + X_t - S_t \qquad (3.11)$$

$$O_t = (X_t - \bar{P}\, \bar{N}_t)^+ \qquad (3.12)$$

$$R^i = [O_t - \sum_{j=i+1}^{e} (\ell-1)\, P^j\, N^j]^+ \quad i = 1, 2, \ldots, e-1 \qquad (3.13)$$

$$O^i = R_t^i - R_t^{i-1} \qquad i = 1, \ldots, e \qquad (3.14)$$

$$X_t \leq \ell \bar{P} \bar{N}_t \qquad (3.15)$$

$$\bar{N}_t - \sum_{i=1}^{t} M^{t-i} \bar{N}_i^h + \sum_{i=1}^{t} M^{t-i} \bar{N}_i^f = M^t \bar{N}_o \qquad (3.16)$$

$$\bar{N}_t \geq 0,\ \bar{N}_t^h \geq 0,\ \bar{N}_t^f \geq 0,\ X_t \geq 0,\ I_t \geq 0 \text{ for } t = 1, 2, \ldots, T.$$

where:

$\bar{C}$ = a constant row vector represents the regular payroll cost with elements $C^i$, $i = i, \ldots, e$.

$\bar{C}_h$ = a constant row vector represents the hiring cost.

$\bar{C}_f$ = a constant row vector represents the firing cost.

$\bar{P}$ = productivity row vector with elements $P^i$, $i = 1, \ldots, e$.

The other variables were previously defined.

## Model Transformation

In order to solve the above model by linear programming methods, a set of variables must be determined in such a way that the cost functions and constraints are linear. In its present form, the overtime constraints are the only nonlinear constraints. To convert the overtime constraints to linear functions, define the variables

$$U_t^e = [X_t - \sum_{i=1}^{e} P^i N_t^i]^-$$

For the next most experienced class

$$U_t^{e-1} = [R_t^e - (\ell-1) P^e N_t^e]^-$$

The general relationship is

$$U_t^i = [R_t^e - \sum_{j=i+1}^{e} (\ell-1) P^j N_t^j]^-$$

Then, from the definition of $R_t^i$ one can write

$$R_t^e - \sum_{j=i+1}^{e} (\ell-1) P^j N_t^j = R_t^i - U_t^i$$

and also from the definition of $R_t^e$ and $U_t^e$ one can write

$$X_t - \sum_{i=1}^{e} P^i N_t^i = R_t^e - U_t^e$$

From the foregoing results the transformed model will be

$$\text{Min. } C = \sum_{t=1}^{T} [\bar{C} \bar{N}_t + \bar{C}_h \bar{N}_t^h + \bar{C}_f \bar{N}_t^f$$

$$+ a \sum_{i=1}^{e} \frac{C^i}{P^i} O_t^i + \frac{1}{2} C_I (I_t + I_{t-1})] \qquad (3.17)$$

Subject to the following constraints:

$$I_t = I_{t-1} + X_t - S_t \qquad (3.18)$$

$$R_t^e = O_t = X_t + U_t^e - \sum_{j=1}^{e} P^j N_t^j \tag{3.19}$$

$$R_t^i = R_t^e + U_t^i - \sum_{j=i+1}^{e} (\ell-1) P^j N_t^j, \; i = 1, 2, \ldots, e-1 \tag{3.20}$$

$$O_t^i = R_t^i - R_t^{i-1} \tag{3.21}$$

$$X_t \leq \ell \sum_{j=1}^{e} P^j N_t^j \tag{3.22}$$

$$\bar{N}_t - \sum_{i=1}^{t} M^{t-i} \bar{N}_i^h + \sum_{i=1}^{t} M^{t-i} \bar{N}_i^f = \bar{N}_o M^t \tag{3.23}$$

$$\bar{N}^t \geq 0, \bar{N}_t^h \geq 0, \bar{N}_t^f \geq 0, X_t \geq 0, I_t \geq 0, R_t^i \geq 0, U_t^i \geq 0,$$

$$i = 1, \ldots, e \text{ for } t = 1, 2, \ldots, T.$$

## Remarks

The model developed in this chapter is by no means the final production manpower planning model. It does, however, illustrate how the important aspects of a personnel transition matrix of the organization and Markov processes, heretofore neglected, can be incorporated into the aggregate production planning models. The model has been formulated as a linear programming model and its solution can be found by any available linear programming package. The model application, along with the comparison with the Orrbeck model, will be presented in Chapter VII.

A substantial improvement of the linear programming models can be made by the use of goal programming procedures. An early contribution is the work of Charnes, Cooper, and Ferguson [22]. In a model they designed for the General Electric Company to assist in setting executive compensation, they developed the concept called "goal programming." Here, the idea is to try to hit a number of management goals "as closely as possible," subject to a set of underlying constraints. However, goal programming models with embedded Markov processes have been developed and used in manpower planning [23]. A goal programming model (Model II) for aggregate production and manpower planning will be developed in Chapter VI after reviewing goal programming and some of its difficulties in Chapter IV, and developing a nondominance algorithm for goal programming in Chapter V.

CHAPTER IV

# NONDOMINANCE IN LINEAR GOAL PROGRAMMING

## Introduction

The area of multiple objective decision making has received a great deal of interest in recent years due to the realization that many real world decision making problems rarely involve only one objective. Among the many methods presented for solving the multiple objective problems, goal programming (GP) has received considerable attention. GP is a relatively new tool that has been used as a methodology for analyzing multiple objective decision making problems. It is an outgrowth of the early ideas of Charnes and Cooper (20) and has been extended by Ijiri (45), Lee (50), and Ignizio (41), among others. It has also been applied in many diverse areas such as manpower planning, energy/water resources, transportation problems, production planning, etc. For further applications and references, the reader is referred to Ignizio (42).

A goal programming solution can turn out to be dominated, that is, not the best one with respect to currently available alternatives. This suboptimizing feature of GP is implied by the fact that the goals are set a priori, as discussed in Zeleny (92). Hannan (34) gives a few numerical examples of GP difficulties. To overcome these difficulties Hannan suggests setting the goals a priori and then maximizing or minimizing the corresponding goal functions on a further constrainted set. Goicoechea, Hansen, and Ducksten (30) stated that it is possible for a

GP solution to be a dominated solution, in which case the targets would need to be adjusted and the model solved again. Ignizio (43, 44) suggested that by setting the objective aspiration levels high enough that they may not be attained for any solution, the GP solution cannot be dominated.

In this chapter some of the definitions and concepts of GP, its formulation, and its solution methods will be presented. The dominance in a GP solution will also be discussed through an example. These materials are appropriate for developing the nondominance test theorem to GP, and can be found in (2, 3, 5, 30, 34, 41, 42, 43, 44, 92, 93).

## Terminology and Concepts

Terminology and concepts, as always, play an important part in the understanding and appreciation of a methodology. GP has a number of special terms, concepts, and definitions that are appropriate for develop ing the GP model. Included among these are:

Objective: An objective is a relatively general statement (in narrative or quantative terms) that reflects the desires of the decision maker. For example, one may wish to "maximize profit" or "minimize labor turnover" or "wipe out poverty."

Aspiration level: An aspiration level is a specific value associated with a desired or acceptable level of achievement of an objective. Thus, an aspiration level is used to measure the achievement of an objective and generally serves to "anchor" the objective to reality.

Goal: An objective in conjunction with an aspiration level is termed a goal. For example, we may wish to "achieve at least X units of profit" or "reduce the rate of inflation by Y percent."

Goal Deviation: The difference between what one accomplishes and what one aspires to is the deviation from his goal. A deviation can represent overachievement as well as underachievement of a goal.

Deviation Variables: A deviation variable reflects either the underachievement (negative deviation) or overachievement (positive deviation) of an objective. All deviation variables are assumed to be nonnegative.

Achievement Function: The goal programming achievement function indicates the degree of achievement of the associated goals. Given a function that is to be lexicographically minimized, the achievement function is an ordered (i.e., ranked or prioritized, vector). This vector can be written as:

$$\bar{a} = (a_1, a_2, \ldots, a_k, \ldots, a_K),$$

where

$$a_k = g_k(\bar{d}-, \bar{d}+), \; k = 1, 2, \ldots, K$$

where

$a$ = achievement vector,

$k$ = ranking or priority,

$\bar{d}-$ = negative deviation vector,

$\bar{d}+$ = positive deviation vector, and

$g_k(\bar{d}-,\bar{d}+)$ = linear function of the goal or constraint deviation variables that are to be minimized at rank or priority k.

Lexicographic Minimum: Given an ordered array $\bar{a}$ of nonnegative elements $a_k$'s, the solution given by $\bar{a}^{(1)}$ is preferred to $\bar{a}^{(2)}$ if

$$a_k^{(1)} < a_k^{(2)}$$

and all higher order elements, i.e., a,.1, ..., $a_{k-1}$ are equal. If no other solution is preferred to $\bar{a}$, then $\bar{a}$ is the lexicographic minimum. Note that the lexicographic minimum is a nondominated solution (43).

## Goal Programming Formulation

Key aspects of the formulation for a goal programming model used here are the specification of the preemptive priorities, establishment of an aspiration level for each objective, and generation of the achievement function.

The concept of assigning a preemptive priority structure to goals is fundamental to the specific goal programming formulation discussed herein. It assumes that one can establish a preference relationship for the goal set comprising the problem. The prioritization of goals is preemptive in the sense that the assignment of goals, say $G_1$, to priority level 1, say $P_1$, will be satisfied before the goals at $P_2$ through $P_k$, assuming that there are K priority levels in the problem. One of the key features of the preemptive priority-based goal programming model is that it involves no weighting or quantative multiplier in relating one priority level to the next. As Ignizio (41) points out, the achievement of the set of objectives at "any one priority level is immeasurably preferred to the achievement of the objective set at any lower priority" for a large number of real world problems.

One of the more troublesome problems encountered in GP formulation is the establishment of an aspiration level for goals. In the typical linear programming model, there is a constraint set containing a column vector of right hand side values. This column is often referred to as the "resource level." In the context of GP, this is termed the aspiration level. This aspiration level must always be specified. For absolute goals (rigid or real constraints) this assignment is straight forward, but for goals which are actually objective functions from the single objective optimization domain, the process is less straight forward. The model builder must specify what he or she feels to be a reasonable aspiration level which should be exceeded, or conversely, not exceeded. Deviation from the aspiration level is measured via a pair of deviation variables; one negative, one positive. Every goal in the GP model carries a negative and positive deviation variable. Label these $d_i^-$ and $d_i^+$ respectively for the $i^{th}$ goal. Based upon the material presented so far, it should be obvious that the GP has the following assumptions and components:

Assumptions:

1. Aspiration levels may be associated with all objectives so as to transform them into goals.

2. Any real (rigid) constraints, i.e., absolute goals, are ranked at priority 1. All remaining goals may be ranked according to importance.

3. With the exception of priority 1, i.e., the set of real constraints, all goals within a given priority must either be commensurable, i.e., measured in the same units, or be made commensurable by means of weights.

Components:

1. A set of decision variables

$$\bar{x} = (x_1, x_2, \ldots, x_n)$$

2. A set of priority levels

$P_1, P_2, \ldots, P_K$ Where $P_1 >>> P_2 >>> \ldots >>> P_K$

3. A set of goals $G_1, G_2, \ldots, G_m$ which have a one-one correspondence with objectives $f_1(\bar{x}), f_2(\bar{x}), \ldots, f_m(\bar{x})$.

4. A set of aspiration levels $b_1, b_2, \ldots, b_m$; one for each goal.

5. A set of deviation variables $(d_1^-, d_1^+), (d_2^-, d_2^+), \ldots, (d_m^-, d_m^+)$ to measure the amount of deviation away from the aspiration level from goals.

6. An achievement function $\bar{a} = [g_1(\bar{d}-,\bar{d}+), g_2(\bar{d}-,\bar{d}+), \ldots, g_K(\bar{d}-,\bar{d}+)]$ to indicate the degree of achievement of the associated goals.

The appearance of deviation variables in the achievement function is based upon the nature of the goals. Achievement is measured as follows:

1. If the $i^{th}$ goal is of the "less than or equal to" type, $f_i(\bar{x}) \leq b_i$, "$d_i^+$" appears in the achievement function.

2. If the $i^{th}$ goal is of the "greater than or equal to" type, $f_1(\bar{x}) \geq b_1$, "$d_i^-$" appears in the achievement function.

3. If the $i^{th}$ goal is of the "equality" type, $f_i(\bar{x}) = b_i$, "$d_i^- + d_i^+$" appears in the achievement function.

A general model for the "n-variable, m-objective, and K-priority level" goal programming problem can now be stated as:

Find $\bar{x} = (x_1, x_2, \ldots, x_n)$,

so as to lexicographically minimize

$$\bar{a} = [g_1(\bar{d}-,\bar{d}+), g_2(\bar{d}-,\bar{d}+), \ldots, g_K(\bar{d}-,\bar{d}+)]$$

such that:

$$f_i(\bar{x}) + d_i^- - d_i^+ = b_i, \; i = 1, 2, \ldots, m$$

and

$\bar{x}, \bar{d}-, \bar{d}+ \geq 0.$

## Methods of Goal Programming Solution

The most commonly used algorithms for solving linear goal programming (LGP) problems are sequential linear goal programming [44], multiphase linear goal programming [44] and partitioning [5]. The first and second algorithms are taken from Ignizio [44] and each algorithm will be briefly discussed.

### Sequential Linear Goal Programming (SLGP) Algorithm

Step 1. Set k=1 (where k is used to represent the priority level under consideration and K is the total of these).

Step 2. Establish the mathematical formulation for priority level 1 only: that is, minimize $a_1 = g_1(\bar{d}-, \bar{d}+)$ subject to

$$\sum_{j=1}^{n} C_{i,j}x_j + d_i^- - d_i^+ = b_i \text{ for } i \in P_1$$

and

$$\bar{x}, \bar{d}-, \bar{d}+ \geq 0.$$

The resulting problem is simply a conventional (single-objective) linear programming problem and may be solved by the simplex method.

Step 3. Solve the single-objective problem associated with priority level k via any appropriate algorithm or code. Let the optimal solution to this problem be given as $a_k^*$, where $a_k^*$ is the optimal value of $g_k(\bar{d}-,\bar{d}+)$.

Step 4. Set k = k+1. If $k \geq K$, go to Step 7.

Step 5. Establish the equivalent, single-objective model for the next priority level (level k). This model is given by: minimize $a_k = g_k\ (\bar{d}-,\bar{d}+)$ subject to

$$f_t(\bar{x}) + d_t^- - d_t^+ = b_t$$

$$g_s(\bar{d}-,\bar{d}+) = a_s^*$$

$$\bar{x},\ \bar{d}-,\ \bar{d}+ \geq 0$$

where

$s = 1, \ldots, k-1.$

$t$ = set of subscripts associated with those goals or constraints included in priority levels 1, 2, ..., k.

Step 6. Go to Step 3.

Step 7. The solution vector $\bar{x}^*$, associated with the last single-objective model solved, is the optimal vector for the original goal programming model.

## The Multiphase Linear Goal Programming Algorithm

The multiphase (or modified simplex) algorithm is simply a refinement of the well-known two phase method. Before discussing the algorithm, the special tableau that is used in the procedure is presented in Table III. Table III differs somewhat from those employed for the single objective because it shows the general, initial multiphase tableau in its condensed form, i.e., only nonbasic columns are included.

The headings and elements within this tableau may be defined as follows:

## TABLE III

## THE INITIAL MULTIPHASE TABLEAU

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $P_K$ | $W_{K,1}$ | $\cdots$ | $W_{K,n}$ | $W_{K,n+1}$ | $\cdots$ | $W_{K,n+m}$ | | |
| | | | $\vdots$ | | $\vdots$ | | | $\vdots$ | | | |
| | | | $P_1$ | $W_{1,1}$ | $\cdots$ | $W_{1,n}$ | $W_{1,n+1}$ | $\cdots$ | $W_{1,n+m}$ | | |
| $P_K$ | $\cdots$ | $P_1$ | $V$ | $x_1$ | $\cdots$ | $x_n$ | $d_1^+$ | $\cdots$ | $d_m^+$ | $X_B$ | |
| $u_{1,K}$ | $\cdots$ | $u_{1,1}$ | $d_1^-$ | $y_{1,1}$ | $\cdots$ | $y_{1,n}$ | $y_{1,n+1}$ | $\cdots$ | $y_{1,n+m}$ | $b_1$ | |
| | $\vdots$ | | $\vdots$ | | $\vdots$ | | | $\vdots$ | | $\vdots$ | |
| $u_{m,K}$ | $\cdots$ | $u_{m,1}$ | $d_m^-$ | $y_{m,1}$ | $\cdots$ | $y_{m,n}$ | $y_{m,n+1}$ | $\cdots$ | $y_{m,n+m}$ | $b_m$ | |
| | | | $P_1$ | $R_{1,1}$ | $\cdots$ | $R_{1,n}$ | $R_{1,n+1}$ | $\cdots$ | $R_{1,n+m}$ | $a_1$ | |
| | | | $\vdots$ | | $\vdots$ | | | $\vdots$ | | $\vdots$ | |
| | | | $P_K$ | $R_{K,1}$ | $\cdots$ | $R_{K,n}$ | $R_{K,n+1}$ | $\cdots$ | $R_{K,n+m}$ | $a_K$ | |

Headings: $P_k$ = $k^{th}$ priority level, k = 1, ..., K.

V = problem variables--both decision and deviation. The variables to the right of V ($x_j$ and $d_j^+$) are the initial set of nonbasic variables; the variables below V ($d_i^-$) are the initial set of basic variables.

$X_B$ = the initial values of the basic variables (elements below $X_B$). Since the initial basis (associated with $d_1^-$ ..., $d_m^-$) is an identity matrix, these initial values are simply the original right-hand side values ($b_i$'s) of the model.

Elements: j = 1, 2, ..., n

i = 1, 2, ..., m

s = 1, 2, ..., S

k = 1, 2, ..., K

$y_{i,s}$ = interior tableau element in the $i^{th}$ row under the $s^{th}$ nonbasic variable; initially, $y_{i,s}$ is simply the coefficient of the $s^{th}$ nonbasic variable in the $i^{th}$ goal.

$W_{k,s}$ = weighting factor for the nonbasic variable in column s at priority level k ($P_k$).

$u_{i,k}$ = weighting factor for the basic variable in row i at the $k^{th}$ priority level.

$R_{k,s}$ = indicator row element for priority level k under the $s^{th}$ nonbasic variable, that is, the "shadow price" or "marginal utility" for the $s^{th}$ nonbasic variable at the $k^{th}$ priority level.

$a_k$ = level of achievement of the goals in priority k,

where

$$\bar{a} = (a_1, \ldots a_k, \ldots a_K)$$

All the elements in the initial tableau, except for $R_{k,s}$ and $a_k$, are simply obtained from the mathematical model. However, $R_{k,s}$ and $a_k$, must be computed as follows:

$$R_{k,s} = U_k^T Y_s - W_{k,s} \qquad (4.1a)$$

or

$$R_{k,s} = \sum_{i=1}^{m} (y_{i,s} u_{i,k}) - W_{k,s} \qquad (4.1b)$$

and

$$a_k = U_k^T X_B \qquad (4.2a)$$

or

$$a_k = \sum_{i=1}^{m} (x_{B,i} u_{i,k}) \qquad (4.2b)$$

The Multiphase Simplex Algorithm: By following the steps given below, the optimal solution to the linear goal programming model may be derived.

Step 1. Initialization. Establish the initial multiphase tableau and the indicator row for priority level 1 only (the $R_{1,s}$ elements). Set k = 1 and proceed to Step 2.

Step 2. Check for optimality. Examine each positive-valued indicator row element ($R_{k,s}$) in indicator row k. Select the largest positive $R_{k,s}$ for which there are no negative-valued indicator numbers at a higher priority in the same column. Designate this column as s′. In the event of ties, the selection of $R_{k,s}$ may be made arbitrarily. If no such $R_{k,s}$ may be found in the $k^{th}$ row, go to Step 6. Otherwise, go to Step 3.

Step 3. Determining the entering variable. The nonbasic variable associated with column s′ is the new entering variable.

Step 4. Determining the departing variable. Determine the row associated with the minimum nonnegative value of

$$\frac{x_{B,i}}{y_{i,s'}}$$

In the event of ties, select the row having the basic variable with the higher-priority level. Designate this row as i′. The basic variable associated with row i′ is the departing variable.

Step 5. Establishment of the new tableau.

(a) Set up a new tableau with all $y_{i,s}$, $x_{B,i}$, $R_{k,s}$, and $a_k$ elements empty. Exchange the positions of the basic variable heading in row i′ (of the preceding tableau) with the nonbasic variable heading in column s′ (of the preceding tableau).

(b) Row i′ of the new tableau (except for $y_{i',s'}$) is obtained by dividing row i′ of the preceding tableau by $y_{i',s'}$.

(c) Column s′ of the new tableau (except for $y_{i',s'}$) is obtained by dividing column s′ of the preceding tableau by the negative of $y_{i',s'}$ (i.e., by $-y_{i',s'}$).

(d) The new element at position $y_{i',s'}$ is given by the reciprocal of $y_{i',s'}$ (from the preceding tableau). The remaining tableau elements are computed as follows: Let any element with a caret over it (i.e., $\hat{x}_{B,i}$, $\hat{y}_{i,s}$, etc.) represent the new set of elements, while those without the caret denote the values of these elements from the preceding tableau. Then, for those elements not in either row i´ or column s´:

$$\hat{x}_{B,i} = x_{B,i} - \frac{(x_{B,i'})(y_{i,s'})}{y_{i',s'}} \qquad (4.3)$$

$$\hat{y}_{i,s} = y_{i,s} - \frac{(y_{i',s})(y_{i,s'})}{y_{i',s'}} \qquad (4.4)$$

$$\hat{R}_{k,s} = R_{k,s} - \frac{(y_{i',s'})(R_{k,S'})}{y_{i',s'}} \qquad (4.5)$$

$$\hat{a}_{k} = a_{k} - \frac{(x_{B,i'})(R_{k,s'})}{y_{i',s'}} \qquad (4.6)$$

An alternative approach to computing the new $R_{k,s}$ and $a_k$ values is to employ (4.1) and (4.2). Note that (4.3) through (4.6) all have the following form:

$$\text{New Value} = \text{Old Value} - \frac{(\text{APRV})(\text{APCV})}{\text{PNC}}$$

where:

APRV = associated pivot row value

APCV = associated pivot column value

PNV = pivot number value, i.e., $y_{i',s'}$

i´ = pivot row

s´ = pivot column

(e) Return to Step 2.

Step 6. Convergence check. Examine each column vector of indicator elements ($R_s$) in the present tableau. At least one of these column vectors must consist solely of zeros if the present solution is to be improved. If so, go to Step 7. Otherwise, we have reached the optimal solution and may stop.

Step 7. Evaluate the next-lower priority level. Set $k = k + 1$. If k now exceeds K (the total number of priorities), then stop, as the present solution is optimal. If $k \leq K$, establish the indicator row for priority k ($P_k$) from (4.1) and (4.2) and go to Step 2.

## The Partitioning Algorithm

Arthur and Ravindran (5) have devised an efficient partitioning algorithm which consists of solving the series of linear programming subproblems, with the solution to the higher priority problem used as the initial solution to the lower priority problem. They also reported that the partitioning algorithm takes only between 12 and 60 percent of the computer time required by Lee's goal programming algorithm (50).

The partitioning algorithm begins by solving the smallest subproblem $S_1$, which is composed of those goal constraints assigned to the highest priority and the corresponding terms in the objective function. The optimal tableau for this subproblem is then examined for alternate optimal solutions. If none exist, then the present solution is optimal for the original problem with respect to all of the priorities. The algorithm then substitutes the values of the decision variables into the goal constraints of lower priorities in order to calculate their attainment levels, and the problem is solved. However, if alternate optimal solutions do exist, the next set of goal constraints (those assigned to the second highest priority) and their objective function terms are added to the problem. This brings the algorithm to the next largest subproblem in the series, and the optimization resumes. The algorithm continues in this manner until no alternate optimum exists for one of the subproblems, or until all priorities have been included in the optimization. The linear dependence between each pair of deviational variables simplifies the operation of adding the new goal constraints to the optimal tableau of the previous subproblem without the need for a dual-simplex iteration.

At the time when the optimal solution to the subproblem $S_{k-1}$ is obtained, a variable elimination step is performed prior to the addition of the goal constraints of priority k. The elimination step involves deleting from further consideration all nonbasic columns which have a positive relative cost ($C_j - Z_j \geq 0$) in the optimal tableau of $S_{k-1}$. This is based on the well known LP result that a nonbasic variable with a positive relative cost in an optimal tableau cannot enter the basis to form an alternate optimal solution.

## Dominance in Linear Goal Programming

As mentioned before, the GP solution can turn out to be dominated, i.e., not the best one with respect to other available solutions. The present approaches for solving the dominance in GP are not practical ones. They change the original problem to another by changing the goals and/or constraints. A proposed method will be developed later in this chapter to examine the nondominance of GP and determine the nondominated solution(s) if the GP solution turns out to be dominated.

To clarify the discussion of the dominance in GP and the proposed method, the following example is adopted from Zimmermann (93) and referred to throughout this chapter.

Example. A company manufactures two products, 1 and 2. Product 1 yields a profit of $2 per piece and product 2 of $1 per piece. While product 2 can be exported, yielding a revenue of $2 per piece in foreign countries, product 1 needs imported raw materials of $1 per piece. Two goals are established: (a) Profit maximization and (b) maximum improvement of the balance of trade, i.e., maximum difference of exports minus imports. This problem can be modeled as follows:

$$\max Z_1(x) = 2x_1 + x_2$$

$$\max Z_2(x) = -x_1 + 2x_2$$

such that

$$-x_1 + 3x_2 \leq 21$$

$$x_1 + 3x_2 \leq 27$$

$$4x_1 + 3x_2 \leq 45$$

$$3x_1 + x_2 \leq 30$$

$$x_1, x_2 \geq 0.$$

The aspiration level (target) for $Z_1(x)$ is 15 and for $Z_2(x)$ is 10; also that $Z_1(x)$ is ranked before $Z_2(x)$. The resultant linear goal programming formulation is:

$$\min \bar{a} = \{ (d_1^+ + d_2^+ + d_3^+ + d_4^+), d_5^-, d_6^- \}$$

such that:

$$-x_1 + 3x_2 + d_1^- - d_1^+ = 21$$

$$x_1 + 3x_2 + d_2^- - d_2^+ = 27$$

$$4x_1 + 3x_2 + d_3^- - d_3^+ = 45$$

$$3x_1 + x_2 + d_4^- - d_4^+ = 30$$

$$Z_1(x): 2x_1 + x_2 + d_5^- - d_5^+ = 15$$

$$Z_2(x): -x_1 + 2x_2 + d_6^- - d_6^+ = 10$$

$$\bar{x}, \bar{d}-, \bar{d}+ \geq 0$$

The solution of the above GP problem can be obtained by any of the above previous methods. For the purpose of clarity, the graphical solu tion will be used.

A graphical solution of the example can be pursued by plotting the six goals (4 constraints and 2 objectives) as straight lines (Figure 3). Note that only the decision variable $x_1$ and $x_2$ appear in the plot. The effect of increasing either $d_i^-$ or $d_i^+$ is reflected by arrows perpendicular to each goal line. The particular deviation variables to be minimized, i.e., those which appear within the achievement vector, have been circled.

The four goals with the highest priority are considered first. These goals may be satisfied by simultaneously minimizing $d_1^+$, $d_2^+$, $d_3^+$,

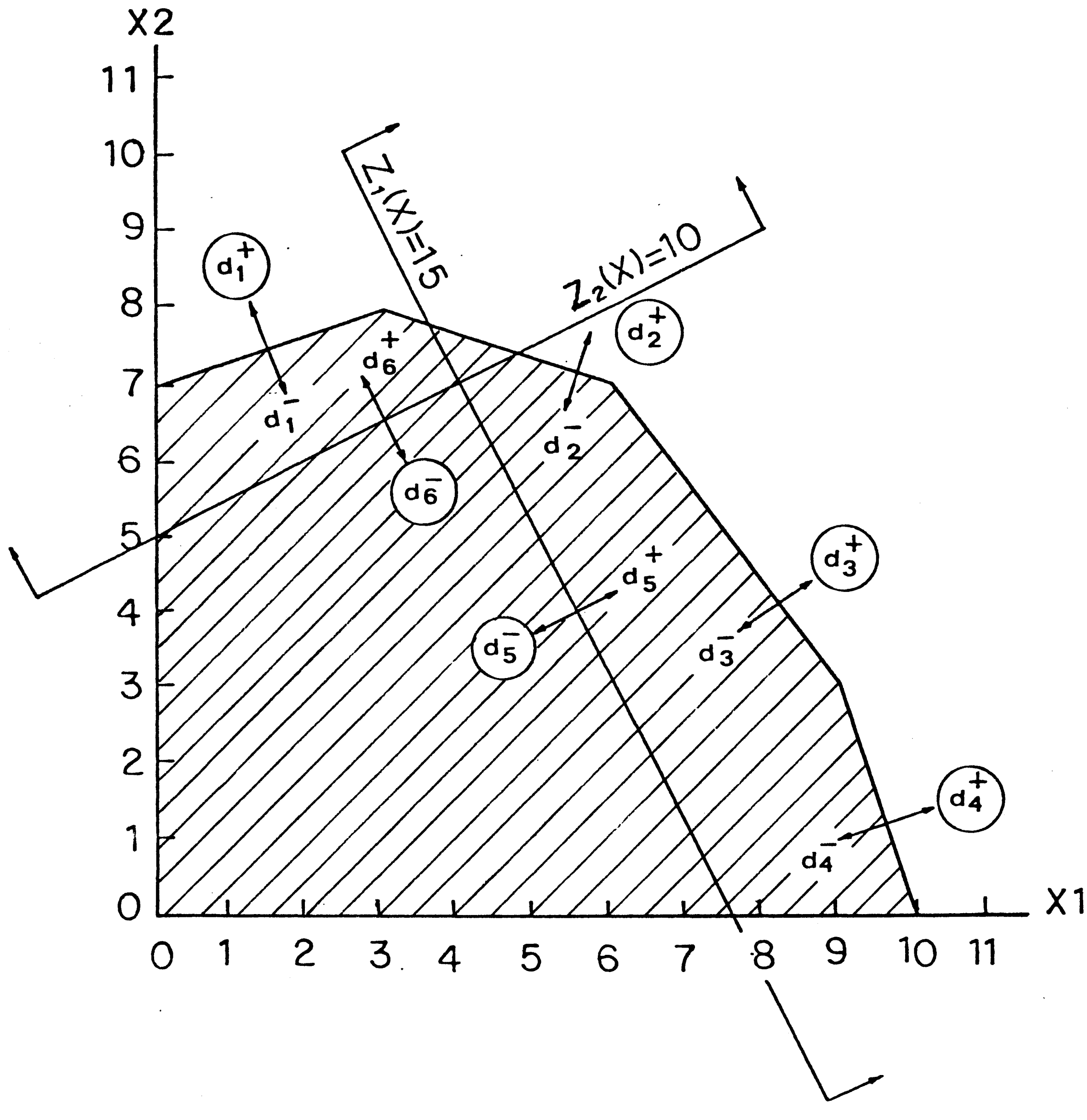



Figure 3. Graphical Solution to the First Priority

and $d_4^+$; in fact, they may be completely achieved by setting $d_1^+ = d_2^+ = d_3^+ = d_4^+ = 0$. The region for the first priority is shown in Figure 3 as the crosshatched area. Note that the crosshatched area is the feasible region of the original constraints.

Next, move to priority level 2, which is achieved through the minimization of $d_5^-$. Note that in Figure 3, $d_5^-$ may be set to zero without degrading the solution achieved for priority 1. The new reduced area is now indicated as the crosshatched region in Figure 4.

Last, move to priority level 3, the final priority level, and attempt to minimize $d_6^-$. Again $d_6^-$ may be set to zero without increasing either the value of $a_1$ or $a_2$ from the achievement function (p. 56). The reduced region is now indicated as the crosshatched area in Figure 5, which represents the final solution to the example.

From Figure 5, one obtains an optimal solution of $x_1 = 4$ and $x_2 = 7$ which should be obtained by any of the previous analytical methods. This implies that $Z_1(x)$ is satisfied fully, that is, $2(4) + 1(7) = 15$; $Z_2(x)$ is also satisfied fully, that is, $-4 + 2(7) = 10$.

This solution is dominated; it is inferior. For example, $x_1 = 4.8$ and $x_2 = 7.4$. At this feasible solution $Z_1(x)$ reaches $2(4.8) + 7.4 = 17$, that is $d_5^- = 0$, $d_5^+ = 2$; $Z_2(x)$ attains $-4.8 + 2(7.4) = 10$. A vector of values (4.8, 7.4) dominates the previous one (4,7). Also a vector of values (3.6, 7.8) dominates (4,7).

Dominated solutions are returned by any goal programming approach, regardless of the used method for obtaining the GP solution. The reason is that the goals are determined a priori without the true potentials of a feasible region being first explored (92).

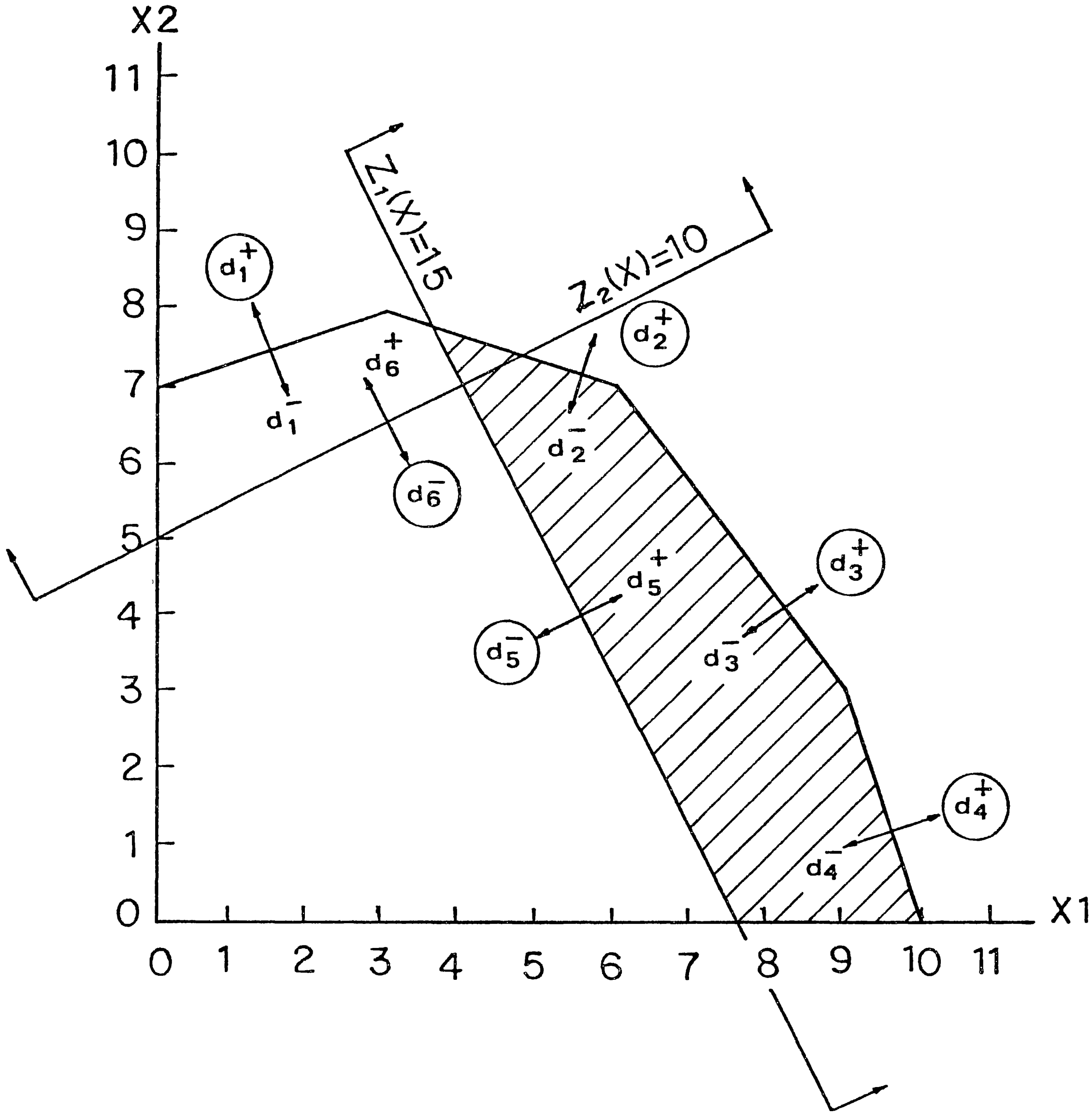



Figure 4. Graphical Solution to First and Second Priorities

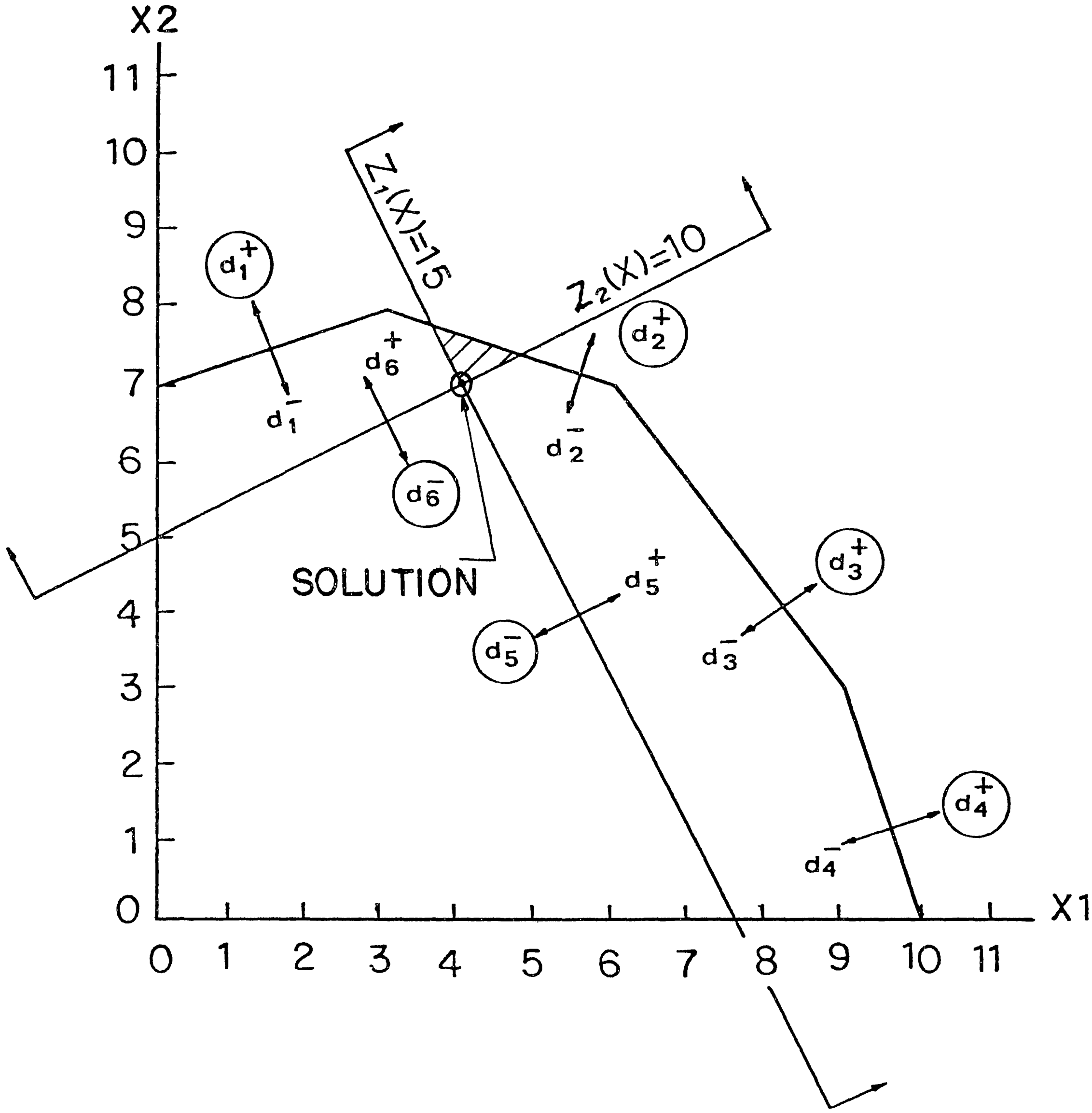



Figure 5. Solution to all Priority Levels

Hannan (34) gives an example of an unbounded solution which will also go undetected by the GP procedure:

$$\text{Minimize } d_1^- + d_2^-$$

subject to:

$$x_2 - x_3 \leq 6$$

$$x_1 \leq 4$$

$$2x_2 + x_3 + d_1^- - d_1^+ = 12$$

$$x_1 + x_2 + x_3 + d_2^- - d_2^+ = 10$$

Solving the above GP problem, one obtains $x_1 = 4$, $x_2 = 6$ and $x_3 = 0$; that is, $d_1^- = d_2^- = 0$. But note that both objectives can actually be raised beyond any bounds, because $x_3$ can be made arbitrarily large. Thus, setting the goals to 12 and 10 respectively is certainly suboptimal in this case.

It should be noted that the solution obtained by GP may be a preferred solution to the decision maker since it satisfies the set goals as closely as possible. However, it is desired to identify the nondominance of such solution and make available to the decision make a nondominated solution to his model. The following section is devoted to a theorem to test the nondominance of a GP solution and to obtain a nondominated solution if the GP solution turns out to be dominated.

## Nondominance Test for Linear Goal Programming

Suppose that x* is the solution of a goal programming problem. Consider what could be the results of solving the following linear programming problem.

$$\text{Maximize } w = \sum_i d_i + \sum_k d_k \tag{4.7}$$

Subject to:

$$\sum_{j=1}^{n} a_{rj}\, x_j = b_r \qquad r = 1, 2, \ldots, m \tag{4.8}$$

$$\sum_i c_{ij}\, x_j - d_i \geq f_i(x^*) \tag{4.9}$$

$$\sum_k c_{kj}\, x_j + d_k \leq f_k(x^*) \tag{4.10}$$

$$x_j,\ d_i,\ d_k \geq 0, \qquad j = 1, 2, \ldots, n$$

where:

Constraints (4.8) represent the rigid constraints.

Constraints (4.9) represent the objectives to be maximized; i.e., the goals of type greater than or equal to.

Constraints (4.10) represent the objectives to be minimized; i.e., the goals of type less than or equal to.

$i$ is the subscript for goal constraints of type greater than or equal to.

$k$ is the subscript for goal constraints of type less than or equal to.

$f_i(x^*)$ is the value of the goal $i$ at $x^*$.

$f_k(x^*)$ is the value of the goal $k$ at $x^*$.

$x^*$ is the goal programming solution.

Note that the objective function, $w$, has no dimension. It serves to obtain the maximum or the minimum of some or all goals.

The following is then true:

1. $x^*$ is a nondominated solution if and only if $w = 0$ (all $d_i$ and $d_k$ are zeros) and $\bar{f}(x^*)$ is a reasonable aspiration level vector.

2. $x^*$ is a dominated solution if and only if $w > 0$. The solution $x^o$ of the above linear programming problem is a nondominated solution to the original goal programming problem and the reasonable aspiration levels might be:

1. for maximization of goals

$$f_i(x) = f_i(x^*) + d_i$$

2. for minimization of goals

$$f_k(x) = f_k(x^*) - d_k$$

where

$$f_i(x) = \sum_i c_{ij} x_j$$
$$f_k(x) = \sum_k c_{kj} x_j$$

Proof:

1. $w = 0$ implies that for all i and k, $d_i$ and $d_k$ are equal to zero. If $d_i = 0$ and $d_k = 0$ for all i and k, then $f_i(x^*)$ cannot be decreased and $f_k(x^*)$ cannot be increased. Therefore, it is impossible to find a solution to the LP problem (4.7 to 4.10) that can dominate $x^*$, thus $x^*$ should be a nondominated solution to the GP problem. On the other hand, suppose that $x^*$ is a nondominated solution and $d_i$ or $d_k$ is greater than zero for some i or k. Then, there can be a solution that dominates $x^*$. This contradicts the nondominance of $x^*$. Accordingly, $d_i$ and $d_k$ must be equal to zero for all i and k, and therefore $w = 0$.

2. $w > 0$ implies that $d_i$ and/or $d_k$ are greater than zero for some i and/or k. Therefore, $f_i(x^*)$ could be decreased and/or $f_k(x^*)$ could be increased. Accordingly, there will be a solution $x^o$ to the LP problem that dominates $x^*$ and $x^*$ is a dominated solution. On the other hand, suppose that $x^*$ is a dominated solution, and $d_i$ and $d_k$ are equal to zero for all i and k. This implies that $f_i(x^*)$ cannot be decreased and $f_k(x^*)$

cannot be increased. Therefore, $x^*$ is a nondominated solution and this contradicts the dominance of $x^*$. Accordingly, $d_i$ and/or $d_k$ should be greater than zero and therefore $w > 0$.

Corollary. In the course of obtaining a GP solution, if the optimal tableau for subproblem $S_k$ has no alternate optimum solution, then the GP solution of the original problem is nondominated and there is no need to perform the nondominance test.

The corollary follows immediately from the preceeding test and the observation that obtaining a unique optimum of the subproblem $S_k$ means that at least one of the goals of $S_k$ attain its maximum or (minimum) and any trial to solve the LP test problem will lead to the same solution. To clarify the above nondominance test, the previous example will be considered. $x^* = (4,7)$ is a solution obtained by goal programming. To test the nondominance of $x^*$, the values of the objectives $Z_1(x)$ and $Z_2(x)$ at $x^*$ are calculated:

$$Z_1(x^*) = f_1(x^*) = 2(4) + 7 = 15$$

$$Z_2(x^*) = f_2(x^*) = -4 + 2(7) = 10$$

Then the linear programming test problem can now be formulated as:

$$\text{Maximize } w = d_1 + d_2$$

Subject to:

$$-x_1 + 3x_2 \leq 21$$

$$x_1 + 3x_2 \leq 27$$

$$4x_1 + 3x_2 \leq 45$$

$$3x_1 + x_2 \leq 30$$

$$2x_1 + x_2 + d_1 \geq 15$$

$$-x_1 + 2x_2 + d_2 \geq 10$$

$x_j \geq 0$, $d_i \geq 0$, $j = 1, 2$, and $i = 1, 2$.

A solution to the above linear programming problem is:

$$x_1 = 3.6$$

$$x_2 = 7.8$$

$$d_1 = 0.0$$

$$d_2 = 2.0$$

and

$$w = 2.0$$

Therefore, the solution $x^* = (4,7)$ is dominated and a nondominated solution to the given goal programming problem is $x^o = (3.6, 7.8)$.

The reasonable aspiration levels might be:

$$f_1(x) = 15$$

$$f_2(x) = 12$$

# CHAPTER V

# A NONDOMINANCE ALGORITHM FOR LINEAR GOAL PROGRAMMING

## Introduction

Having shown that the GP solution may be a dominated solution, the nondominance test has been proposed in the previous chapter to identify the GP solution and to generate a nondominated solution(s). The nondominance test is simply formulating and solving a linear programming problem. To implement this test in any goal programming algorithm, a subroutine(s) may be added to set up and solve the linear programming problem. In this research, PAGP, the partitioning algorithm for linear goal programming problems (5), is modified to include the nondominance test; however, similar modifications can be done for any other GP computer code. The partitioning algorithm is chosen for the following reasons:

1. It consists of solving the series of linear programming subproblems with the solution of the higher priority problem used as the initial solution to the lower priority problem.

2. It has been coded in FORTRAN. The program structure and notations are similar to those of Ignizio (41). Ignizio's code is one of the well-known codes in goal programming.

3. It finds the solution to the original problem in less time than the other methods. This is because the constraint partitioning and

variable elimination steps used in the partitioning algorithm decrease the basis size and number of columns.

In this chapter, the partitioning algorithm will be reviewed in some detail and notes on it will be addressed since they are appropriate for developing the new algorithm. General concepts and some features of the new algorithm are presented and algorithm limitations are provided.

## The Partitioning Algorithm

The GP methods presented by both Lee (50) and Ignizio (41) used the simplex algorithm (69) as their base and they added a modified decision rule for selecting the nonbasic variable to enter the basis at each iteration. However, both failed to take advantage of the reduction in the number of computations at each iteration offered by the definition of the preemptive priority factor which states that any lower priority level cannot be satisfied to the detriment of a higher priority level. Arthur and Ravindran (5) developed an efficient algorithm which consists of three procedures: partitioning, elimination, and termination. For convenience, each procedure will be briefly discussed (refer to Arthur (3) for further details).

### The Partitioning Procedure

The partitioning of the GP problem is accomplished by observing that for any goal constraint i, one and only one of three things may occur:

1. only $d_i^-$ appears in the objective function,

2. only $d_i^+$ appears in the objective function, and

3. both $d_i^-$ and $d_i^+$ appear in the objective function.

In case (1) the partition would assign goal constraint i to the priority factor associated with $d_i^-$, in case (2) constraint i would be assigned to the priority factor associated with $d_i^+$, while in case (3) the partition would determine the higher order priority factor (in terms of the ordinal ranking) associated with either $d_i^-$ or $d_i^+$ and constraint i would be assigned to that priority.

## The Elimination Procedure

The elimination procedure is based on the fact that in order to maintain the levels of achievement for the higher priority goals, a number of nonbasic variables (whose introduction into the basis can only destroy the level achieved for the higher order goals) can be eliminated. The motivation behind the elimination procedure comes from the theory of linear programming which states: "Let $\bar{z}$ be the optimal value of the LP problem; Min Z subject to Ax = b, $x \geq o$, and suppose that $\bar{c}_j = c_j - z_j \geq o$ for some nonbasic variable $x_j$. Then $x_j$ cannot enter the basis to form an alternate optimal solution."

It follows from the above theorem that in the course of obtaining a GP solution, if the optimal tableau for subproblems $S_k$ has been found, then any nonbasic variable $t_s$ (where $t_s$ can be a decision variable or a deviational variable) which has at least one positive relative cost, i.e., $\bar{c}_{js} \geq o$ where $\bar{c}_{js}$ is the relative change in priority $P_j$ per unit increase in $t_s$) can be eliminated from entering the basis in subproblems $S_{k+1}, \ldots, S_K$.

## The Termination Procedure

The termination procedure is based on the linear dependence between each pair of deviation variables $d_i^-$ and $d_i^+$. To clarify the discussion of the termination procedure, suppose a unique optimal solution has been found to subproblem $S_k$, then no nonbasic variable can enter the basis at priority $P_k$. Now suppose that goal constraint i is assigned to priority $P_{k+1}$ by the partitioning procedure. By adding this goal constraint to the optimal tableau of subproblem $S_k$ and performing the row reduction necessary to maintain a canonical form, i.e., eliminating the basic variables from constraint i through elementary row operations, either $d_i^-$ or $d_i^+$ will enter the basis as the basic variable corresponding to this new row in the new tableau. Hence, there will still be a unique optimal tableau at priority $P_{k+1}$ since no nonbasic variable can enter the basis. If more goal constraints were assigned to priority $P_{k+1}$, the same thing would happen as each constraint was added to the new tableau. Therefore, if a unique optimal solution has been found to subproblem $S_k$, there is no need to try to improve the lower priorities $P_{k+1}, \ldots, P_K$ and the algorithm terminates.

## The Algorithm

The partitioning algorithm can now be summarized in the following steps:

Step 1: Find a basic feasible solution to the real constraints by using a Phase I simplex method with a full artificial basis. If the real constraints have no feasible solution, the algorithm terminates. If the real constraints are feasible or the problem has no real constraints, move to Step 2.

Step 2: Solve the smallest subproblem containing only the goal constraints and associated variables belonging to the highest priority level.

Step 3: Examine the optimal tableau for alternate optimal solutions. If none exists, it is not possible to optimize the goals of lower priorities and the current solution is optimal for the original problem; move to Step 4. Otherwise go to Step 5.

Step 4: Substitute the values of the decision variables ($x_j$'s) in the goal constraints assigned to lower priority levels, calculate their levels of achievement, then terminate the algorithm.

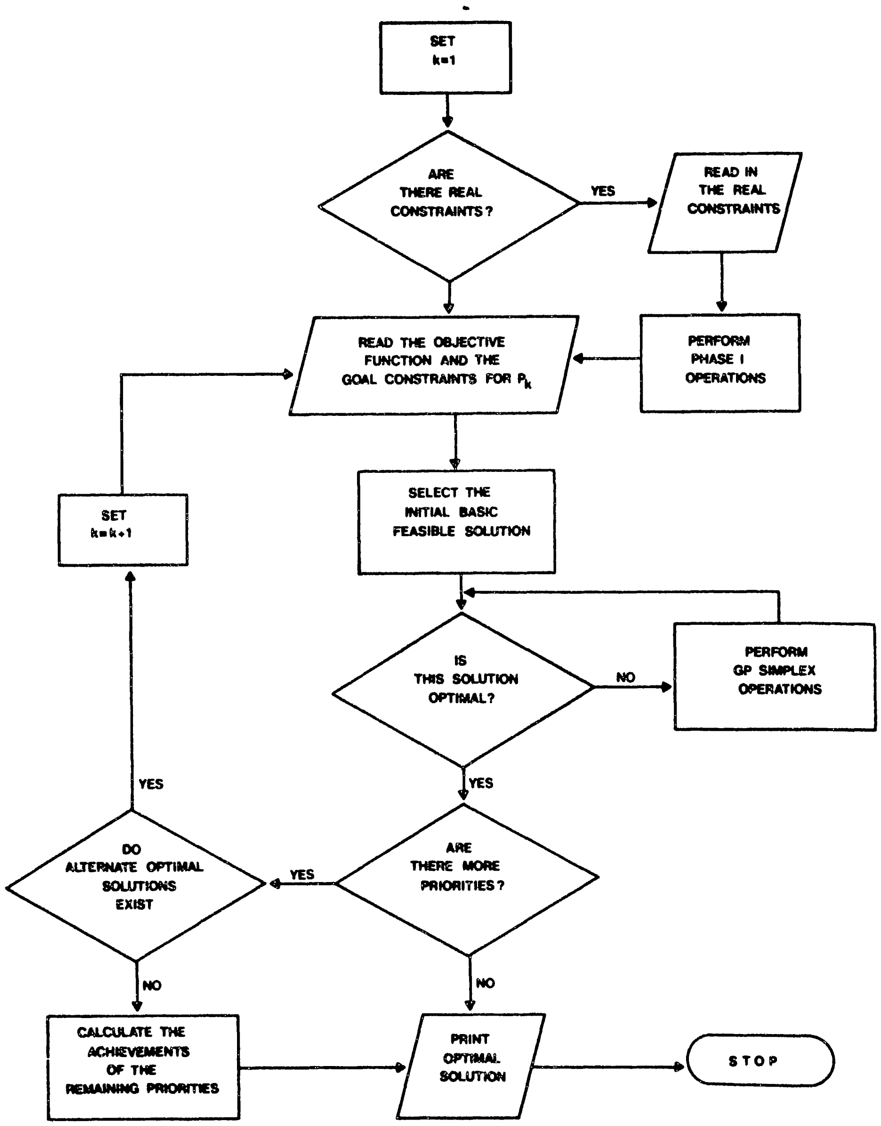
Step 5: If alternate optimal solutions do exist, goal constraints assigned to the next highest priority and the corresponding terms in the objective function are appended to the tableau while preserving its feasibility. The elimination procedure is then used to delete all the nonbasic columns with positive criteria coefficients and the optimization resumes.

Step 6: If all priority levels have been included in the optimization, the algorithm terminates. Otherwise go to Step 3.

The flowchart of the partitioning algorithm is given in Figure 6.

## Notes on the Partitioning Algorithm

The partitioning algorithm has been coded in FORTRAN and problems of various sizes and complexities have been solved to test its efficiency with the widely used goal programming algorithm by Lee (50). In all of the test problems, the partitioning algorithm did much better than the algorithm by Lee, taking as little as 12 percent of Lee's time and never more than 60 percent. The authors of PAGP attribute the efficiency of their algorithm to the constraint partitioning and variable elimination




Source: From Arthur, A. L. and Ravindran, A., "PAGP, A Partitioning Algorithm for (Linear) Goal Programming Problems," <u>ACM Transactions on Mathematical Software</u>, Vol. 6, No. 3, September 1980.

Figure 6. Flowchart of the Partitioning Algorithm

steps. They used the Phase I simplex procedure with a full artificial basis to find a basic feasible solution to the real constraints before optimizing the goals.

As previously stated, this research uses the PAGP as a GP algorithm. As this research progressed, some interesting observations were made regarding PAGP. These observations are presented below:

## Phase I with a Full Artificial Basis

A major requirement of the simplex method is the availability of an initial basis solution in canonical form. Without it the initial simplex tableau cannot be formed. An approach to finding an initial basis involves using artificial variables. A Phase I algorithm may then be to find an initial basic solution to the original problem by removing the artificial variables.

It should be noted that many linear programming problems do not need the use of Phase I with a full artificial basis. For instance, if one of the real constraints is in the form of "less than or equal to," then that constraint will have a basis and there is no need to use the full artificial basis. However, since PAGP uses a full artificial basis, the following cases may arise from the PAGP output:

1. If the original GP program has an alternate optimum solution and at least one of the constraints is in the form "$\leq$" or "=", then the solution obtained by PAGP may be different than the solution obtained by the other methods.

2. If the original program has a unique optimal solution, then the solution obtained by any method must be the same.

Another requirement in the structure of PAGP is that the real constraints must be in the form Ax = b, hence the slack (surplus) variables should be treated as decision variables in the original problem.

With regard to the structure of PAGP, if the real constraints have no feasible solution, the algorithm terminates. While Ignizio (41) pointed out that if the problem has no solution that satisfies the real constraints, then the final results derived will indicate the solution that is nearest to being implementable.

Based upon the previous discussion, Phase I has been deleted in the development of the nodominance algorithm.

## Missing Statements in PAGP

There are some missing statements in the output subroutine of PAGP (as published by Arthur and Ravindran (4)). These statements are necessary to correct the output when the number of decision variables in the GP program are greater than the number of goals. To illustrate the discussion, consider the following example which is taken from Murty (63) and formulated as a goal programming model:

$$\text{Min } \bar{a} = \{(d_1^- + d_1^+ + d_2^- + d_2^+),\ d_3^-,\ d_4^-\}$$

Subject to:

$$8x_1 + x_2 + 3x_3 + 2x_4 + 3x_5 - 3x_6 + d_1^- - d_1^+ = 17$$

$$3x_1 + 2x_3 + x_4 + x_5 - x_6 + d_2^- - d_2^+ = 5$$

$$5x_1 + x_3 + 2x_4 + x_5 - 4x_6 + d_3^- - d_3^+ = 8$$

$$12x_1 + x_2 + 2x_3 + 5x_4 + 4x_5 - 6x_6 + d_4^- - d_4^+ = 30$$

and all the variables are nonnegative.

Note that the number of the decision variables (x's) are 6 and the number of the deviation variables (d's) are 4.

Table IV shows the output of PAGP for the above example before the correction, while Table V shows the output after the correction. It is clear that the example does not have the variables $d_5^-$, $d_5^+$, $d_6^-$ and $d_6^+$ which appear as zeros in the output summary of Table IV. These variables should not have any values since they are not in the problem. The output summary of Table V does not have values for these variables.

A complete list of the output subroutine (Subroutine POUT) after the correction is given in Appendix A as a subprogram of the nondominance algorithm.

## The Nondominance Algorithm

Having shown that the nondominance test is the formulation and solution of a LP problem, the development of a nondominance algorithm can now be summarized in the following steps:

Step 1. Solve the GP problem.

Step 2. Formulate a LP problem from the information of the original problem and the GP solution obtained from Step 1.

Step 3. Solve the LP problem formulated in Step 2.

Step 4. Identify the nondominance of the GP solution from the LP solution. These steps can be done by two methods. In the first method, the solution of the GP problem and the formulated LP problem may be obtained by a GP code and a LP code respectively. This method is straight-forward, but it may

# TABLE IV

## COMPUTER OUTPUT OF PAGP BEFORE THE CORRECTIONS

THE OPTIMIZATION ENDED ON SUBPROBLEM 3
THERE WERE 4 CONSTRAINTS IN THE FINAL OPTIMAL TABLEAU.

*******************************************************************************

THE OPTIMAL SOLUTION FOR THE DECISION VARIABLES X(J)

X( 1)= 0.4000

X( 2)= 6.2000

X( 3)= 0.0000

X( 4)= 3.8000

X( 5)= 0.0000

X( 6)= 0.0000

*******************************************************************************

THE GOAL ACHIEVEMENTS ARE

| PRIORITY | GOAL NUMBER | OVER-ACHIEVEMENT | UNDER-ACHIEVEMENT |
|---|---|---|---|
| 1 | 1 | 0.0000 | 0.0000 |
| 1 | 2 | 0.0000 | 0.0000 |
| 2 | 3 | 1.6000 | 0.0000 |
| 3 | 4 | 0.0000 | 0.0000 |

*******************************************************************************

THE PRIORITY ACHIEVEMENTS ARE

| PRIORITY | ACHIEVEMENT |
|---|---|
| 1 | 0.0000 |
| 2 | 0.0000 |
| 3 | 0.0000 |

*******************************************************************************

OUTPUT SUMMARY

| SUBSCRIPT | A OPT | X OPT | POS DEV | NEG DEV |
|---|---|---|---|---|
| 1 | 0.0000 | 0.4000 | 0.0000 | 0.0000 |
| 2 | 0.0000 | 6.2000 | 0.0000 | 0.0000 |
| 3 | 0.0000 | 0.0000 | 1.6000 | 0.0000 |
| 4 | | 3.8000 | 0.0000 | 0.0000 |
| 5 | | 0.0000 | 0.0000 | 0.0000 |
| 6 | | 0.0000 | 0.0000 | 0.0000 |

TABLE V

COMPUTER OUTPUT OF PAGP AFTER THE CORRECTIONS

THE OPTIMIZATION ENDED ON SUBPROBLEM 3
THERE WERE 4 CONSTRAINTS IN THE FINAL OPTIMAL TABLEAU.

**************************************************************************

THE OPTIMAL SOLUTION FOR THE DECISION VARIABLES X(J)

X( 1)= 0.4000

X( 2)= 6.2000

X( 3)= 0.0000

X( 4)= 3.8000

X( 5)= 0.0000

X( 6)= 0.0000

**************************************************************************

THE GOAL ACHIEVEMENTS ARE

| PRIORITY | GOAL NUMBER | OVER-ACHIEVEMENT | UNDER-ACHIEVEMENT |
|---|---|---|---|
| 1 | 1 | 0.0000 | 0.0000 |
| 1 | 2 | 0.0000 | 0.0000 |
| 2 | 3 | 1.6000 | 0.0000 |
| 3 | 4 | 0.0000 | 0.0000 |

**************************************************************************

THE PRIORITY ACHIEVEMENTS ARE

| PRIORITY | ACHIEVEMENT |
|---|---|
| 1 | 0.0000 |
| 2 | 0.0000 |
| 3 | 0.0000 |

**************************************************************************

OUTPUT SUMMARY

| SUBSCRIPT | A OPT | X OPT | POS DEV | NEG DEV |
|---|---|---|---|---|
| 1 | 0.0000 | 0.4000 | 0.0000 | 0.0000 |
| 2 | 0.0000 | 6.2000 | 0.0000 | 0.0000 |
| 3 | 0.0000 | 0.0000 | 1.6000 | 0.0000 |
| 4 | | 3.8000 | 0.0000 | 0.0000 |
| 5 | | 0.0000 | | |
| 6 | | 0.0000 | | |

take a considerable amount of time to formulate and solve the LP test problem.

In the second method, a modification which includes the formulation and the solution of the LP test problem should be made as part of the GP code. This method is more attractive than the first one since there is no need to use two different codes (one for GP and the other for LP). Also the analyst does not need to formulate the LP test problem. The only limitation of this method is that the developed algorithm may occupy a large amount of computer memory. Therefore, the first method would likely be used when computer storage is a limiting factor, while the second method is the preferred one when computer memory is not limited.

As previously mentioned, the main purpose of this chapter is to develop a nondominance algorithm for an existing GP code. Therefore, the second method will be presented in some detail.

## General Concepts of the Algorithm

The concepts of a nondominance algorithm can actually be generalized so as to be utilized in any GP computer code. These concepts can be described in the following steps:

Step 1. Perform minor modifications in the GP code to facilitate the embedding of a nondominance algorithm into the code. Such modifications may include definitions of new variables, dimension statements, adding or deleting subprograms, etc.

Step 2. Save the original information to be used in the nondominance test after obtaining the GP solution.

Step 3. If the GP program has no alternate optimal solutions, then the GP solution is nondominated and the algorithm terminates. Otherwise, move to Step 4.

Step 4. Set up the LP problem:

$$\text{Max } w = \sum_i D_i + \sum_k D_k$$

Subject to:

$$\sum_{j=1}^{n} a_{rj} X_j \pm S_r = b_r, \qquad r = 1, 2, \ldots, m$$

$$\sum_i c_{ij} X_j - D_i - G_i = f_i(X^*)$$

$$\sum_k c_{kj} X_j + D_k + G_k = f_k(X^*)$$

$$j = 1, 2, \ldots, n$$

and all variables are nonnegative.

Where:

X = decision variables

D = variables to be maximized

S = slack or surplus variables in real constraints

G = slack or surplus variables in goal constraints

Step 5. Solve the above LP problem.

Step 6. Check for alternate (nondominated) solutions.

Step 7. Print out the results of the nondominance test.

The key to the effectiveness of the algorithm is the choice of the appropriate GP computer code and implementation of the above steps into the code. The partitioning algorithm has been chosen as an appropriate GP computer code and the above steps have been successfully implemented. Only one more input card is added to the current input cards of PAGP. This card is for the type of real and goal constraints.

The nondominance algorithm for linear goal programming (NAGP) has been coded in FORTRAN. The program structure and notations are similar to those of Arthur and Ravindran (4) and Ignizio (41). Appendix A gives a complete listing and documentation of the nondominance algorithm along with the partitioning algorithm.

## Some Special Features of the Algorithm

The program for the nondominance algorithm given in Appendix A has the following features:

1. Phase I is deleted from PAGP.

2. The first priority is associated with the set of real constraints and the remaining priorities are associated with the ranking of the original objective set.

3. The case of the infeasibility of real constraints is treated by assuming that the real constraints are considered as goals in the nondominance test, i.e., the values of the R.H.S. of real constraints are calculated from the final GP solution.

4. The traditional simplex method is used for solving the LP problem.

Some GP problems of varying difficulty were selected in order to test the efficiency and validation of the new algorithm. Appendix B has four problems and their solutions by the new algorithm.

Problem 1 shows that the GP solution can turn out to be dominated. A nondominated solution is obtained by the algorithm.

Problem 2 indicates the case of an unbounded solution which also can go undetected by the GP procedure. In such case, the algorithm is designed to show that the solution is unbounded.

Problems 3 and 4 illustrate the effect of establishing high values for the aspiration levels (targets) on the GP solution. The GP solution of Problem 3, as presented by Ignizio (43), may be obtained by solving a LP problem. In such case, the GP problem can be reduced to a LP problem. Problem 4 is taken from Zanakis and Maret (92) to show that setting a low value (in case of minimization) for an aspiration level may have a direct impact on the other goals, i.e., not all the goals would be included in the optimization process. An attempt to include all the goals in the optimization process will be presented in the next chapter as a solution methodology to GP for aggregate production and manpower planning models.

## Algorithm Limitations

Unfortunately, the algorithm is not totally general. For example, it does not generate all nondominated solutions, but rather it is designed to investigate a GP solution and determine a nondominated solution if the GP solution turns out to be dominated. However, this limitation is common in most of the efficient (nondominated) solution techniques which have difficulty in solving problems of other than small to moderate size.

# CHAPTER VI

# A GOAL PROGRAMMING MODEL FOR AGGREGATE PRODUCTION AND MANPOWER PLANNING

## Introduction

In recent years there has been an increased awareness of the need to identify and consider simultaneously several objectives in the solution and optimization of some problems, in particular those derived from the study of large scale systems. For instance, the problem of manpower planning is to determine the number of people by grade to best meet future manpower needs of an organization in the light of multiple objectives, e.g., economic conditions, production/sales trends, people skills, inventory, government regulations, organization history and policies regarding personnel hiring, training, promotion, firing and retirement. The problem of aggregate production planning, like many other real life problems, involves multiple objectives which are often conflicting. For example, a decrease in inventory levels may necessitate either increasing overtime or decreasing customer service. Increased overtime results in higher costs and hence less profit. On the other hand, an unbalanced workforce generates increased back orders and shortages, unfavorable customer relations, lost sales, and again, less profit.

In Chapter III, an aggregate production/manpower planning model (Model I) was presented. The Orrbeck model was used as a point of departure from which the model was developed, and the linear programming technique was used as a solution methodology. A substantial improvement can be made in the use of personnel transition rates when they are integrated in a model with manpower and production requirements. The fundamental change is that management decisions, expressed as manpower and production requirements or goals and as budgetary and other capacity constraints, influence the final manpower flows and production decisions recommended by the model. It seems, therefore, natural to model and optimize the problem of aggregate production and manpower planning by a method of multiple-objectives procedures. Goal programming as a method of these procedures has been used in manpower planning and aggregate production planning (42).

This chapter is devoted to the development of a goal programming model (Model II) for aggregate production and manpower planning. The new model is an extension of Model I and incorporates some basic concepts of the Charnes et al. models (23) which were developed for managing and controlling the Navy's civilian labor force. Furthermore, a solution methodology is proposed to include all the goals in the optimization process.

## Assumptions of the Model

The assumptions of the model developed in this chapter are the same as those of the model developed in Chapter III (Model I). A summary of these assumptions are:

1. The objective functions and constraints are linear functions.

2. Demand is deterministic and production is constrainted to meet the forecasting demand.
3. No shortages will be allowed and the inventory carrying cost is based upon the average of the beginning and ending inventory for each period.
4. The personnel transition matrix will be used for projecting manpower transition and it is assumed to be constant.
5. The cost of retirement and quit are not considered in the model.
6. The most experienced workers will work overtime first, subject to the limit of their capacity. If overtime work still remains, the next most experienced class will be called upon, and similarly for the remaining experience classes until all overtime work is assigned.

## Objectives of the Model

Among the objectives mentioned separately in the literature of manpower planning and aggregate production planning, the objectives listed below have been selected to be used in the model. These objectives will be presented in the order in which they are ranked by management, where $P_1$ is the highest priority and associated with the real constraints. Of course, a different set of objectives can be used to represent the actual situation.

$P_1$: Operate within the manpower and production requirements ($P_1$ is assigned to the real constraints).

$P_2$: Minimize the total number of hiring and firing.

$P_3$: Minimize the total production cost.

$P_4$: Minimize the total inventory carrying cost.

$P_5$: Hold the total overtime production down to a minimum.

### Formulation of the Goal Programming Model

In this section the GP model will be presented by first defining additional variables which are not defined in the previous chapters, followed by formulating the constraints of manpower and production requirements. Next, management objectives will be presented according to their priorities. Lastly, the complete goal programming model will be formulated by including the constraint and goal deviational variables.

#### Notation

In addition to the variables defined in previous chapters, the following have been employed to facilitate the goal programming formulation:

$B_t$ = total dollar budget of workers in period t.

$L_t$ = total manpower ceiling in period t.

$\bar{s}$ = a row vector represents the maximum payroll (regular and overtime) for each class of workers.

$\bar{u}$ = (1, 1, ..., 1); unity row vector with dimension e.

$\bar{d}^-_{mt}$, $\bar{d}^+_{mt}$, $d^-_{ct}$, $d^+_{ct}$, $d^-_{bt}$, $d^+_{bt}$, $d^-_{vt}$, $d^+_{vt}$, $d^-_{et}$, $d^+_{et}$, $d^-_{irt}$, $d^+_{irt}$, $d^-_{iot}$, $d^+_{iot}$, $d^-_{xt}$, $d^+_{xt}$, $d^-_{g_1}$, $d^+_{g_1}$, $d^-_{g_2}$, $d^+_{g_2}$, $d^-_{g_3}$, $d^+_{g_3}$, $d^-_{g_4}$, and $d^+_{g_4}$ are the appropriate deviational terms of the real and goal constraints.

Constraints

For each period, there are two sets of constraints: manpower requirements and production requirements. Constraints can be added (modified) to suit the actual problem.

1. Manpower constraints:

$$\bar{N}_t = M \bar{N}_{t-1} + \bar{N}_t^h - \bar{N}_t^f, \quad t = 1, \ldots, T \tag{6.1}$$

$$\bar{u} \bar{N}_t \leq L_t, \quad t = 1, \ldots, T \tag{6.2}$$

$$\bar{s} \bar{N}_t + \bar{C}_h \bar{N}_t^h + \bar{C}_f \bar{N}_t^f \leq B_t, \quad t = 1, \ldots, T \tag{6.3}$$

Equations (6.1) ensure that the manpower requirement of each class of workers in any period must equal the projected manpower (remaining from the previous period) plus the new hires minus the fires in the current period. Equations (6.2) ensure that the sum of the manpower for all classes in any period must be less than or equal to the total manpower ceiling. Equations (6.3) are the budget constraints. They are used to guarantee that the regular and overtime payroll, hiring and firing costs must be "less than or equal to" the total dollar budget that is stipulated for each period.

2. Production Constraints:

$$I_t = I_{t-1} + X_t - S_t \tag{6.4}$$

$$R_t^e = O_t = X_t + U_t^e - \sum_{j=1}^{e} P^j N_t^j \tag{6.5}$$

$$R_t^i = R_t^e + U_t^i - \sum_{j=i+1}^{e} (\ell-1) P^j N_t^j, \quad i = 1, \ldots, e-1 \tag{6.6}$$

$$O_t^i = R_t^i - R_t^{i-1} \tag{6.7}$$

$$X_t \leq \ell \bar{P} \bar{N}_t \tag{6.8}$$

Equations (6.4) through (6.8) are the equations (3.18) through (3.22) of Chapter III.

Objective Functions

As mentioned before, the following objectives are considered in the model according to their priorities:

$$\text{Min} \sum_{t=1}^{T} (\bar{u} \ \bar{N}_t^h + \bar{u} \ \bar{N}_t^f) \tag{6.9}$$

This objective is to minimize the total number of hiring and firing.

$$\text{Min} \sum_{t=1}^{T} [\bar{C} \ \bar{N}_t + \bar{C}_h \ \bar{N}_t^h + \bar{C}_f \ \bar{N}_t^f + a \sum_{i=1}^{e} \frac{C^i}{P^i} O_t^i] \tag{6.10}$$

This is to minimize the production cost.

$$\text{Min} \sum_{t=1}^{T} [\frac{1}{2} C_I (I_t + I_{t-1})] \tag{6.11}$$

This is to minimize the total inventory carrying cost.

$$\text{Min} \sum_{t=1}^{T} O_t \tag{6.12}$$

This is to keep the total overtime production down to a minimum.

The Model

The complete goal programming model can now be formulated by introducing the deviational variables in the real and goal constraints and by specifying the appropriate deviational variables in the objective function of the model.

$$\text{Min} \quad \bar{a} = \{ \sum_{t=1}^{T} [\bar{u}\, \bar{d}^-_{mt} + \bar{u}\, \bar{d}^+_{mt} + d^+_{ct} + d^+_{bt}$$

$$+ d^-_{vt} + d^+_{vt} + d^-_{et} + d^+_{et}$$

$$+ \sum_{i=1}^{e-1} (d^-_{irt} + d^+_{irt}) \qquad \text{1st priority}$$

$$+ \sum_{i=1}^{e} (d^-_{iot} + d^+_{iot}) + d^+_{xt}],$$

$$d^+_{g_1}, \qquad \text{2nd priority}$$

$$d^+_{g_2}, \qquad \text{3rd priority}$$

$$d^+_{g_3}, \qquad \text{4th priority}$$

$$d^+_{g_4} \quad \} \qquad \text{5th priority}$$

Real constraints:

$$\bar{N}_t - M\, \bar{N}_{t-1} - \bar{N}^h_t + \bar{N}^f_t + \bar{d}^-_{mt} - \bar{d}^+_{mt} = 0$$

$$\bar{u}\, \bar{N}_t + d^-_{ct} - d^+_{ct} = L_t$$

$$\bar{s}\ \bar{N}_t + \bar{C}_h\ \bar{N}_t^h + \bar{C}_f\ N_t^f + d_{bt}^- - d_{bt}^+ = B_t$$

$$X_t + I_{t-1} - I_t + d_{vt}^- - d_{vt}^+ = S_t$$

$$R_t^e - X_t - U_t^e + \sum_{j=1}^{e} P^j\ N_t^j + d_{et}^- - d_{et}^+ = 0$$

$$R_t^e - R_t^i + U_t^i - \sum_{j=i+1}^{e} (\ell-1)\ P^j\ N_t^j$$

$$+ d_{irt}^- - d_{irt}^+ = 0,\ i=1,\ \ldots,\ e-1$$

$$O_t^i - R_t^i + R_t^{i-1} + d_{iot}^- - d_{iot}^+ = 0,\ i=1,\ \ldots,\ e$$

$$X_t - \ell\ \bar{P}\ \bar{N}_t + d_{xt}^- - d_{xt}^+ = 0$$

for t = 1, 2, ..., T.

Goal constraints:

$$\sum_{t=1}^{T} (\bar{u}\ \bar{N}_t^h + \bar{u}\ \bar{N}_t^f) + d_{g_1}^- - d_{g_1}^+ = 0$$

$$\sum_{t=1}^{T} [\bar{C}\ \bar{N}_t + \bar{C}_h\ \bar{N}_t^h + \bar{C}_f\ \bar{N}_t^f + a \sum_{i=1}^{e} \frac{C^i}{P^i}\ O_t^i]$$

$$+ d_{g_2}^- - d_{g_2}^+ = 0$$

$$\sum_{t=1}^{T} \frac{1}{2} C_I (I_t + I_{t-1}) + d_{g_3}^- - d_{g_3}^+ = 0$$

$$\sum_{t=1}^{T} O_t + d_{g_4}^- - d_{g_4}^+ = 0$$

for $t = 1, 2, \ldots, T$,

where all the variables are nonnegative.

## Solution Methodology

Most of the published papers on the applications of preemptive goal programming used the same procedure as presented in the previous section for establishing the aspiration levels of the goals. For instance, if management wants to minimize cost, the aspiration level for this goal might be zero. The method of establishing low values (in case of minimization) or high values (in case of maximization) for the aspiration levels may cause some of the goals to have no impact on the model. In other words, not all the goals would be included in the optimization process and some of these goals may be eliminated from the model without any effect on the model solution.

For the purpose of illustration, consider the industrial case study (in a department of a large chemical company) presented by Zanakis and Maret (91). In their study they used a preemptive GP model to determine the manpower mix that best satisfies several conflicting socio-econo-organizational objectives. Their model has five structural variables (x's), 15 deviational variables (d's), and six priorities which are formulated as follows:

| Goal | Priority | Formulation | Objective Function |
|---|---|---|---|
| (1) | 1 | <u>Impose lower limit on new hires</u><br>$X_1 + d_1^- - d_1^+ = 40$ | minimize $d_1^-$ |
| (2) | 1 | <u>Maintain at least 100 contract employees</u><br>$X_5 + d_2^- - d_2^+ = 100$ | minimize $d_2^-$ |
| (3) | 1 | <u>Number of contract people not to exceed department's people</u><br>$-X_1 - X_2 - X_3 - X_4 + X_5$<br>$+ d_3^- - d_3^+ = 1063$ | minimize $d_3^+$ |
| (4) | 2 | <u>Number of re-hires to be 5 or a few more</u><br>$X_2 + d_4^- - d_4^+ = 5$ | minimize $d_4^-$ |
| (5) | 2 | <u>Keep transfers-in at 20 or a few more</u><br>$X_3 + d_5^- - d_5^+ = 20$ | minimize $d_5^-$ |
| (6) | 2 | <u>Non-exempt promotions to be 30 or less</u><br>$X_4 + d_6^- - d_6^+ = 30$ | minimize $d_6^+$ |
| (7) | 3 | <u>Satisfy the forecast future year people required</u><br>$X_1 + X_2 + X_3 + X_4 + 2X_5/3$<br>$+ d_7^- - d_7^+ = 787$ | minimize $d_7^- + d_7^+$ |
| (8) | 4 | <u>Minimize labor costs</u><br>$13.358X_1 + 14.846X_2 + 18.073X_3$<br>$+ 7.024X_4 + 26X_5 + d_8^- - d_8^+ = 0$ | minimize $d_8^+$ |

(9) 5 <u>Limit department growth rate</u>

$X_1 + X_2 + X_3 + X_4$
$+ d_9^- - d_9^+ = 219$ minimize $d_9^+$

(10) 6 <u>Satisfy position level ratio goals</u>

$0.0048X_1 + 0.0513X_2 - 0.1659X_3$
$+ d_{10}^- - d_{10}^+ = 32.273$ minimize $d_{10}^+$

(11) 6 $- 0.0048X_1 - 0.0513X_2 + 0.0711X_3$
$+ d_{11}^- - d_{11}^+ = 28.789$ minimize $d_{11}^+$

(12) 6 $0.9568X_1 + 0.5383X_2 + 0.9670X_3$
$+ X_4 + d_{12}^- - d_{12}^+ = 875.715$ minimize $d_{12}^+$

(13) 6 $0.9712X_1 + 0.6922X_2 - 0.3271X_3$
$+ X_4 + d_{13}^- - d_{13}^+ = 239.78$ minimize $d_{13}^-$

(14) 6 $0.7627X_1 - o.0512X_2 - 0.4834X_3$
$+ 0.9643X_4 + d_{14}^- - d_{14}^+ = 338.926$ minimize $d_{14}^+$

(15) 6 $- 0.8402 - 0.2821X_2 + 0.0758X_3$
$- 0.9762X_4 + d_{15}^- - d_{15}^+ = 47.411$ minimize $d_{15}^+$

where:

$X_1$ = number of new hires

$X_2$ = number of re-hires

$X_3$ = number of transfers-in from other departments

$X_4$ = number of promotions from non-exempt

$X_5$ = number of contract engineers

As can be shown from the above formulation, goal number 8 (priority 4) has an aspiration level of zero. The solution of the above GP model is the same solution which may be obtained by eliminating goals 9 through 15 (all the goals of priorities 5 and 6). Appendix B has the solution of the above GP model and the solution after eliminating goals 9 through 15 from the model. The results obtained in both cases are:

$$X_1 = 655.333$$

$$X_2 = 5.0$$

$$X_3 = 20.0$$

$$X_4 = 30.0$$

$$X_5 = 100.0$$

It is clear from the above example that a procedure to include all the goals in the optimization process is critically essential. Such a procedure is described in the steps given below:

Step 1. Formulate the model with hypothetical aspiration levels.

Step 2. Solve the model by using NAGP. If one or more of the priority achievement values are greater than zero, move to Step 3. Otherwise go to Step 4.

Step 3. Establish reasonable aspiration levels for the goals of priority k (where k is the highest priority which has a non-zero achievement value) by using the following relations:

$$AL'(\cdot) \geq AL(\cdot) + AT(\cdot);$$

for minimization of goals,

$$AL'(\cdot) \leq AL(\cdot) - AT(\cdot);$$

for maximization of goals,

$$AL'(\cdot) = AL(\cdot) \pm AT(\cdot);$$

for the goals of equalty type,

where:

$AL'(\cdot)$ = new aspiration level

$AL\ (\cdot)$ = hypothetical aspiration level

$AT\ (\cdot)$ = the value of the deviational variable in the achievement function.

Then, go to Step 2.

Step 4. All the goals are included in the optimization process. Check the nondominance of the final solution and the procedure terminates.

A numerical example of hypothetical data will be presented in Chapter VII to illustrate the above procedure to a GP model for aggregate production and manpower planning.

# CHAPTER VII

# ANALYSIS OF RESULTS

## Introduction

The objective of this chapter is to evaluate the new models developed in this research and illustrate the solution methodology proposed for linear goal programming models.

Model I (developed in Chapter III) is an extension to the single objective Orrbeck model and may serve as a general case for the linear programming models of aggregate production and manpower planning. To evaluate the performance of this model, a comparison with the Orrbeck model will be presented.

Model II (developed in Chapter VI) is a completely new model in the sense that a multiple objectives model has been developed which incorporates the personnel transition matrix into aggregate production planning models. Also, the solution methodoloy which aims to include all the goals in the optimization process is a new procedure for optimizing linear goal programming models, especially if the targets are not known. Because no model can be found in the literature for the purpose of comparison, hypothetical data will be furnished to demonstrate Model II and to illustrate the solution methodology, and Model I will be used to verify the results.

## Evaluation of Model I

As previously mentioned, Model I is an extension of the Orrbeck model and incorporates the PTM of the organization in manpower constraints. Therefore, to test the performance of Model I, hypothetical data are furnished by introducing personnel movement data (Table VI).

TABLE VI

PERSONNEL MOVEMENT DATA

| From \ To | Class 1 | Class 2 |
|---|---|---|
| Class 1 | 0.2 | 0.75 |
| Class 2 | 0.0 | 0.95 |

Reading across, .2 or 20% of workers in class 1 remain in class 1 and .75 or 75% transfer from class 1 to class 2. Similarly, 0.0 or 0.0% of class 2 are projected to transfer to class 1 with .95 or 95% remaining in class 2.

For the purpose of comparison, the data of Table VI are very close to the data described by Orrbeck (68). The personnel movement data as described by Orrbeck is shown in Table VII. Note that in Table VI the cases of quit and retirement (which are only 5%), are not considered in the Orrbeck model.

TABLE VII

PERSONNEL MOVEMENT DATA FOR

THE ORRBECK MODEL

| From \ To | Class 1 | Class 2 |
|---|---|---|
| Class 1 | 0 | 1.0 |
| Class 2 | 0 | 1.0 |

The other data used by Orrbeck are given below:

The set of demands over the six period planning horizon will be:

$S_1 = 11,000$

$S_2 = 11,500$

$S_3 = 9,000$

$S_4 = 12,300$

$S_5 = 8,400$

$S_6 = 9,200$

The common cost and productivity parameters will be:

$\bar{C} = (400, 450)$

$\bar{C}_h = (200, -)$:- means no hiring for class 2 is permitted.

$$\bar{C}_f = (100,\ 100)$$

$$P^2 = 30$$

$$\ell = 1.5$$

$$a = 1.5$$

$$C_I = 1.0$$

The initial conditions are:

$$\bar{N}_o = \begin{pmatrix} 50 \\ 200 \end{pmatrix}, \quad I_o = 1000$$

Two cases for productivity of class 1 workers will be considered:

$P^1 = 25$ for case 1 and $P^1 = 10$ for case 2.

The linear programming problem to be solved contains 72 variables and 42 constraints (not including non-negative constraints). For the general case of a T period horizon and e experience classes, the number of variables is T(5e+3) and the number of constraints is T(2e+3). These numbers can be reduced by using equality constraints to eliminate variables from the problem. They may also be reduced by model restrictions. For instance, no hiring for class 2 is permitted in the above example, and consequently, the number of variables is reduced to T(5e+2).

The resulting linear programming problem can be solved by using one of the linear programming packages available on most large computers. The Mathematical Programming System Extended (MPSX) is used and the details of the results of Model I and the Orrbeck model for $P^1 = 25$ and $P^1 = 10$ are given in Tables VIII, IX, X and XI. All the numbers of these tables have been rounded off to the first decimal point.

TABLE VIII

OPTIMUM EMPLOYMENT AND PRODUCTION
SCHEDULES OF MODEL I

($P^1$ = 25)

| $t$ | $S_t$ | $N_t^1$ | $N_t^2$ | $H_t^1$ | $F_t^1$ | $F_t^2$ | $X_t$ | $O_t^1$ | $O_t^2$ | $I_t$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 11000 | 166.4 | 227.5 | 156.4 | 0.0 | 0.0 | 10984.7 | 0.0 | 0.0 | 984.7 |
| 2 | 11500 | 33.3 | 340.9 | 0.0 | 0.0 | 0.0 | 11059.4 | 0.0 | 0.0 | 544.1 |
| 3 | 9000 | 6.7 | 348.8 | 0.0 | 0.0 | 0.0 | 10631.2 | 0.0 | 0.0 | 2175.4 |
| 4 | 12300 | 1.3 | 336.4 | 0.0 | 0.0 | 0.0 | 10124.6 | 0.0 | 0.0 | 0.0 |
| 5 | 8400 | 0.0 | 300.9 | 0.0 | 0.3 | 19.7 | 9025.6 | 0.0 | 0.0 | 625.6 |
| 6 | 9200 | 0.0 | 285.8 | 0.0 | 0.0 | 0.0 | 8574.4 | 0.0 | 0.0 | 0.0 |

TOTAL COST = $949,295.7

TABLE IX

OPTIMUM EMPLOYMENT AND PRODUCTION SCHEDULES OF MODEL I

($P^1$ = 10)

| t | $S_t$ | $N_t^1$ | $N_t^2$ | $H_t^1$ | $F_t^2$ | $F_t^1$ | $X_t$ | $O_t^1$ | $O_t^2$ | $I_t$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 11000 | 184.1 | 227.5 | 174.1 | 0.0 | 0.0 | 10000.0 | 0.0 | 1334.0 | 0.0 |
| 2 | 11500 | 36.8 | 354.2 | 0.0 | 0.0 | 0.0 | 11500.0 | 0.0 | 505.9 | 0.0 |
| 3 | 9000 | 0.0 | 364.1 | 0.0 | 7.4 | 0.0 | 10923.1 | 0.0 | 0.0 | 1923.1 |
| 4 | 12000 | 0.0 | 345.9 | 0.0 | 0.0 | 0.0 | 10376.9 | 0.0 | 0.0 | 0.0 |
| 5 | 8400 | 0.0 | 300.9 | 0.0 | 0.0 | 27.7 | 9025.6 | 0.0 | 0.0 | 625.6 |
| 6 | 9200 | 0.0 | 285.8 | 0.0 | 0.0 | 0.0 | 8574.4 | 0.0 | 0.0 | 0.0 |

TOTAL COST = $1,016,407.70

TABLE X

OPTIMUM EMPLOYMENT AND PRODUCTION
SCHEDULES OF THE ORRBECK MODEL
($P^1 = 25$)

| $t$ | $S_t$ | $N_t^1$ | $N_t^2$ | $H_t^1$ | $F_t^1$ | $F_t^2$ | $X_t$ | $O_t^1$ | $O_t^2$ | $I_t$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 11000 | 118.2 | 250.0 | 118.2 | 0.0 | 0.0 | 10454.5 | 0.0 | 0.0 | 454.5 |
| 2 | 11500 | 0.0 | 368.2 | 0.0 | 0.0 | 0.0 | 11045.5 | 0.0 | 0.0 | 0.0 |
| 3 | 9000 | 0.0 | 355.0 | 0.0 | 0.0 | 13.2 | 10650.0 | 0.0 | 0.0 | 1650.0 |
| 4 | 12300 | 0.0 | 355.0 | 0.0 | 0.0 | 0.0 | 10650.0 | 0.0 | 0.0 | 0.0 |
| 5 | 8400 | 0.0 | 293.3 | 0.0 | 0.0 | 61.7 | 8800.0 | 0.0 | 0.0 | 400.0 |
| 6 | 9200 | 0.0 | 293.3 | 0.0 | 0.0 | 0.0 | 8800.0 | 0.0 | 0.0 | 0.0 |

TOTAL COST = \$943,080.3

TABLE XI

OPTIMUM EMPLOYMENT AND PRODUCTION SCHEDULES OF THE ORRBECK MODEL

$(P^1 = 10)$

| t | $S_t$ | $N_t^1$ | $N_t^2$ | $H_t^1$ | $F_t^1$ | $F_t^2$ | $X_t$ | $O_t^1$ | $O_t^2$ | $I_t$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 11000 | 105.0 | 250.0 | 105.0 | 0.0 | 0.0 | 10000.0 | 0.0 | 1450.0 | 0.0 |
| 2 | 11500 | 0.0 | 355.0 | 0.0 | 0.0 | 0.0 | 11500.0 | 0.0 | 850.0 | 0.0 |
| 3 | 9000 | 0.0 | 355.0 | 0.0 | 0.0 | 0.0 | 10650.0 | 0.0 | 0.0 | 1650.0 |
| 4 | 12000 | 0.0 | 355.0 | 0.0 | 0.0 | 0.0 | 10650.0 | 0.0 | 0.0 | 0.0 |
| 5 | 8400 | 0.0 | 293.3 | 0.0 | 0.0 | 61.7 | 8800.0 | 0.0 | 0.0 | 400.0 |
| 6 | 9200 | 0.0 | 293.3 | 0.0 | 0.0 | 0.0 | 8800.0 | 0.0 | 0.0 | 0.0 |

TOTAL COST = \$979,217.7

Where:

$S_t$ = demand in period t.

$N_t^1$ = number of workers in class 1 in period t.

$N_t^2$ = number of workers in class 2 in period t.

$H_t^1$ = number of workers hired in class 1 in period t.

$F_t^1$ = number of workers fired from class 1 in period t.

$F_t^2$ = number of workers fired from class 2 in period t.

$X_t$ = production level in period t.

$O_t^1$ = amount of overtime production assigned to class 1 in period t.

$O_t^2$ = amount of overtime production assigned to class 2 in period t.

## Comparison with the Orrbeck Model

The solutions obtained by using Model I and the Orrbeck model are optimal ones based on the assumptions of each model. The comparison of these kind of models is somewhat irrelevant because each model has its assumption and its optimal solution. However, a traditional comparison for these kind of models may be done as described below:

Suppose that the actual PTM in the data described by Orrbeck is the matrix

$$M = \begin{pmatrix} 0.20 & 0.0 \\ 0.75 & 0.95 \end{pmatrix}$$

as presented in Table VI. What will be the actual system cost if:

1. Model I is used.

2. The Orrbeck Model is used.

Clearly, if Model I is used, the actual system cost is $949,295.70 for $P^1 = 25$ and $1,016,407.70 for $P^1 = 10$. The corresponding workforce decisions and the production schedules are given in Tables VIII and IX respectively.

If Orrbeck's decisions are employed, calculations for the workforce decisions and related cost components should be made to obtain the actual system cost. These calculations can be computed by using the following steps:

Step 1. Calculate the actual number of workers in each class from the relation:

$$M\,\bar{N}_{t-1} + \bar{N}_t^h - \bar{N}_t^f = \bar{N}_t, \quad t = 1, \ldots, 6$$

and workers of class 1 are fired first if workers are to be fired.

Step 2. Calculate the regular payroll cost based on the actual number of the workers obtained from Step 1.

Step 3. Calculate the overtime by using the relations:

$$X_t = P^1 N_t^1 + P^2 N_t^2 + O_t^1 + O_t^2$$

$$O_t^2 \leq 0.5\, P^2 N_t^2$$

and overtime is assigned first to class 2 workers.

Step 4. Calculate the overtime cost based on the actual overtime obtained from Step 3.

The results of Steps 1 and 2 are shown in Tables XII and XIII for

TABLE XII

ACTUAL WORKFORCE DECISIONS AND THE REGULAR PAYROLL COST FOR THE ORRBECK MODEL

$(P^1 = 25)$

| t | Orrbeck's Decisions | | | Actual Decisions | | |
|---|---|---|---|---|---|---|
| | $N_t^1$ | $N_t^2$ | Payroll Cost ($) | $N_t^1$ | $N_t^2$ | Payroll Cost ($) |
| 1 | 118.2 | 250.0 | 159780.0 | 128.2 | 227.5 | 153655.0 |
| 2 | 0.0 | 368.2 | 165690.0 | 25.6 | 312.3 | 150775.0 |
| 3 | 0.0 | 355.0 | 159750.0 | 0.0 | 307.1 | 138195.0 |
| 4 | 0.0 | 355.0 | 159750.0 | 0.0 | 291.7 | 131265.0 |
| 5 | 0.0 | 293.3 | 131985.0 | 0.0 | 215.4 | 96930.0 |
| 6 | 0.0 | 293.3 | 131985.0 | 0.0 | 204.7 | 92115.0 |
| TOTAL | | | 908940.0 | | | 762935.0 |

TABLE XIII

ACTUAL WORKFORCE DECISIONS AND THE REGULAR PAYROLL COST FOR THE ORRBECK MODEL

($P^1 = 10$)

| t | Orrbeck's Decisions | | | Actual Decisions | | |
|---|---|---|---|---|---|---|
| | $N_t^1$ | $N_t^2$ | Payroll Cost ($) | $N_t^1$ | $N_t^2$ | Payroll Cost ($) |
| 1 | 105.0 | 250.0 | 154500.0 | 115.0 | 227.5 | 148375.0 |
| 2 | 0.0 | 355.0 | 159750.0 | 23.0 | 302.4 | 145280.0 |
| 3 | 0.0 | 355.0 | 159750.0 | 4.6 | 304.5 | 138865.0 |
| 4 | 0.0 | 355.0 | 159750.0 | 0.9 | 292.7 | 132075.0 |
| 5 | 0.0 | 293.3 | 131985.0 | 0.0 | 217.3 | 97785.0 |
| 6 | 0.0 | 293.3 | 131985.0 | 0.0 | 206.4 | 92880.0 |
| TOTAL | | | 897720.0 | | | 755260.0 |

$P^1 = 25$ and $P^1 = 10$ respectively, while the results of Steps 3 and 4 are shown in Tables XIV and XV.

The actual total cost incurred (when Orrbeck's decisions are used) can now be calculated in the following way:

1. $P^1 = 25$

| | | |
|---|---|---|
| Regular Payroll Cost | | = $ 762,935.0 |
| Hiring Cost | = $200 (118.2) | = 23,640.0 |
| Firing Cost | = 100 (13.2 + 61.7) | = 7,490.0 |
| Overtime Cost | | = 220,365.0 |
| Inventory Cost | = (500 + 454.5 + 1650 + 400) | = 3,004.5 |
| Total Cost | | = $1,017,434.5 |

2. $P^1 = 10$

| | | |
|---|---|---|
| Regular Payroll Cost | | = $ 755,260.0 |
| Hiring Cost | = $200 (105) | = 21,000.0 |
| Firing Cost | = 100 (61.7) | = 6,170.0 |
| Overtime Cost | | = 268,672.5 |
| Inventory Cost | = (500 + 1650 + 400) | = 2,550.0 |
| Total Cost | | = $1,053,652.5 |

Table XVI summarizes the cost analysis for Model I and the actual cost incurred when Orrbeck's decisions are employed. It can be seen from Table XVI that Model I yields a total cost of $949,295.70 and $1,016,407.70 for $P^1 = 25$ and $P^1 = 10$ respectively, while the Orrbeck model yields $1,017,434.50 and $1,016,407.70. These results show a saving of $68,138.80 (7.18%) for $P^1 = 25$ and $37,244.80 (3.67%) for $P^1 = 10$ when Model I is used.

TABLE XIV

ACTUAL OVERTIME COST FOR THE ORRBECK MODEL

$(P^1 = 25)$

| t | Orrbeck's Overtime | | | Actual Overtime | | |
|---|---|---|---|---|---|---|
| | $O_t^1$ | $O_t^2$ | Overtime Cost ($) | $O_t^1$ | $O_t^2$ | Overtime Cost ($) |
| 1 | 0 | 0 | 0 | 0 | 424.5 | 9551.3 |
| 2 | 0 | 0 | 0 | 0 | 1036.5 | 23321.3 |
| 3 | 0 | 0 | 0 | 0 | 1437.0 | 32332.5 |
| 4 | 0 | 0 | 0 | 0 | 1899.0 | 42727.5 |
| 5 | 0 | 0 | 0 | 0 | 2338.0 | 52605.0 |
| 6 | 0 | 0 | 0 | 0 | 2659.0 | 59827.5 |
| TOTAL | | | 0 | | | 220365.0 |

TABLE XV

ACTUAL OVERTIME COST FOR THE ORRBECK MODEL

$(P^1 = 10)$

| t | Orrbeck's Overtime | | | Actual Overtime | | |
|---|---|---|---|---|---|---|
| | $O_t^1$ | $O_t^2$ | Overtime Cost ($) | $O_t^1$ | $O_t^2$ | Overtime Cost ($) |
| 1 | 0 | 1450 | 32625 | 0 | 2025.0 | 45562.5 |
| 2 | 0 | 850 | 19125 | 0 | 1698.0 | 38205.0 |
| 3 | 0 | 0 | 0 | 0 | 1469.0 | 33052.5 |
| 4 | 0 | 0 | 0 | 0 | 1860.0 | 41850.0 |
| 5 | 0 | 0 | 0 | 0 | 2281.0 | 51322.5 |
| 6 | 0 | 0 | 0 | 0 | 2608.0 | 58680.0 |
| TOTAL | | | 51750 | | | 268672.5 |

TABLE XVI

COST ANALYSIS OF MODEL I AND
THE ORRBECK MODEL ($)

| Cost Components | Model I | | The Orrbeck Model | |
|---|---|---|---|---|
| | $P^1 = 25$ | $P^1 = 10$ | $P^1 = 25$ | $P^1 = 10$ |
| Regular Payroll | 911191.3 | 933630.7 | 762935.0 | 755260.0 |
| Hiring | 31277.6 | 34819.4 | 23640.0 | 21000.0 |
| Firing | 1997.0 | 3511.2 | 7490.0 | 6170.0 |
| Overtime | 0.0 | 41397.7 | 220365.0 | 268672.5 |
| Inventory | 4829.8 | 3048.7 | 3004.5 | 2550.0 |
| TOTAL | 949295.7 | 1016407.7 | 1017434.5 | 1053652.5 |

## Remarks

The foregoing analyses have indicated that the important aspect of the personnel transition matrix is that it can be successfully incorporated into the aggregate production problem. The results show that the performance of the new model is much better than that of the Orrbeck model with respect to representing more realistic situations and yielding minimum cost. For instance, if

$$M = \begin{pmatrix} .2 & .0 \\ .7 & .8 \end{pmatrix}$$

in the above example, Orrbeck's decisions will not be able to meet the demand.

As shown in Tables VIII through XI, the new model can provide detailed information regarding workforce, production and inventory decisions; consequently, the model might be considered as the first step toward establishing (building) integrated manpower and production policies for manufacturing firms. Also, the new model has some characteristics which are non-existent in the present aggregate planning models. These characteristics are summarized below.

1. It is a large-scale model for manpower and production planning.
2. The cases of quit, attrition, etc., are considered in the model by representing them in the personnel transition matrix of the firm.
3. The number of hiring or firing in each class of workforce for each period is explicitly determined.

The new model is formulated as a LP model. The widespread and successful use of LP techniques in management operations makes it feasible to employ the model in practical applications.

## Evaluation of Model II

As described in detail in Chapter VI, Model II is a GP model for aggregate production and manpower planning. The concepts of PTM, as previously discussed, are not incorporated in the existing aggregate production planning models and hence no such model can be found for the purpose of evaluation and comparison. For this reason, hypothetical data are furnished to illustrate the application of this model and to clarify the solution methodology for the linear goal programming models proposed in Chapter VI.

### Numerical Example

Tables XVII through XXI give purely hypothetical data of a four periods-two experience classes aggregate production and manpower planning model. Some of the data in this example was used in the analysis of Model I.

Pertinent data not in the tables are:

| | | |
|---|---|---|
| Initial inventory | = | 1000 units |
| Hiring cost | = | $ 200/man for each class of workers. |
| Firing cost | = | $ 100/man for each class of workers. |
| Inventory carrying cost | = | $ 1./period/unit. |
| Overtime pay | = | 1.5 times the regular pay. |
| Maximum overtime duration | = | .5 times regular time. |

TABLE XVII

BUDGET, CEILING, AND REGULAR PAYROLL DATA

| Period t | Budget B(t) ($) | Ceiling L(t) (men) | Regular Payroll ($/man/period) Class 1 | Class 2 |
|---|---|---|---|---|
| 1 | 250000.0 | 350 | 400.0 | 450.0 |
| 2 | 250000.0 | 350 | 400.0 | 450.0 |
| 3 | 250000.0 | 350 | 400.0 | 450.0 |
| 4 | 250000.0 | 350 | 400.0 | 450.0 |

TABLE XVIII

PERSONNEL TRANSITION DATA

| From \ To | Class 1 | Class 2 |
|---|---|---|
| Class 1 | 0.2 | 0.7 |
| Class 2 | 0.0 | 0.8 |

TABLE XIX

INITIAL POPULATION DATA

| | Initial Population |
|---|---|
| Class 1 | 50 |
| Class 2 | 200 |

TABLE XX

DEMAND DATA

| Period t | Demand S(t) |
|---|---|
| 1 | 11000 |
| 2 | 11500 |
| 3 | 9000 |
| 4 | 12300 |

TABLE XXI

PRODUCTIVITY DATA

| | Productivity (units/man/period) |
|---|---|
| Class 1 | 20 |
| Class 2 | 30 |

Hypothetical aspiration levels for goals:

| Goal | Aspiration Level |
|---|---|
| 1 | 0 |
| 2 | 0 |
| 3 | 0 |
| 4 | 0 |

The goal programming problem to be solved contains five priorities (the first priority is assigned to the real constraints and the others to the goals), 36 real constraints, 4 goal constraints, 52 decision variables, and 80 deviational variables. The linear programming problem of the nondominance test contains 40 constraints and 96 variables.

The above example is solved by the nondominance algorithm (NAGP) developed in Chapter V. Five runs have been made to include the goals in the optimization process and, at the end of each run, the aspiration levels are calculated according to the method described in Chapter VI. For example, the first run has been made with the hypothetical aspiration level values of goals, i.e., 0, 0, 0, 0 for the goals of priorities $P_2$, $P_3$, $P_4$ and $P_5$ respectively. From the output computer results of run 1, the aspiration levels are changed to 130, 0, 0, 0 for the goals of the above priorities, which then become the input of run 2. The procedure continues until all the goals are included in the optimization process.

Table XXII shows the values of aspiration levels and achievement functions of the five runs. In the last run (run 5), the values of the

TABLE XXII

THE ASPIRATION LEVELS AND VALUES OF ACHIEVEMENT FUNCTIONS FOR THE FIVE RUNS

| | $P_1$ Real Constraints | $P_2$ Goal 1 | $P_3$ Goal 2 | $P_4$ Goal 3 | $P_5$ Goal 4 |
|---|---|---|---|---|---|
| RUN 1 | | | | | |
| Aspiration Level | RHS | 0.0 | 0.0 | 0.0 | 0.0 |
| Achievement Value | 0 | 117.6 | 774722.2 | 13695.4 | 14266.7 |
| RUN 2 | | | | | |
| Aspiration Level | RHS | 130.0 | 0.0 | 0.0 | 0.0 |
| Achievement Value | 0 | 0.0 | 768226.7 | 11650.6 | 13195.8 |
| RUN 3 | | | | | |
| Aspiration Level | RHS | 130.0 | 775000.0 | 0.0 | 0.0 |
| Achievement Value | 0 | 0.0 | 0.0 | 9256.1 | 14097.8 |
| RUN 4 | | | | | |
| Aspiration Level | RHS | 130.0 | 775000.0 | 10200.0 | 0.0 |
| Achievement Value | 0 | 0.0 | 0.0 | 0.0 | 13647.3 |
| RUN 5 | | | | | |
| Aspiration Level | RHS | 130.0 | 775000.0 | 10200.0 | 14000.0 |
| Achievement Value | 0 | 0.0 | 0.0 | 0.0 | 0.0 |

achievement function for all goals are equal to zero, which indicate that all the goals have been included in the optimization process. Two solutions are obtained from this run: a GP and a nondominated solution. Tables XXIII and XXIX show the workforce and production decisions of the GP and the nondominated solution respectively.

Where:

$H_t^2$ = the number of workers hired in class 2 in period t,

and all the other variables are previously defined. Appendix C has the computer output of run 5.

The solution results for the five runs in terms of goal attainment and goal value are given in Table XXV. Note that $P_1$ is for the real constraints and is achieved in each run. It should also be noted that the solutions of runs 1 through 4 are nondominated, but in each case some of the goals are not included in the optimization process and may be elimated without effecting the model solution. For instance, the solution obtained from run 4 is nondominated, but the goal of $P_5$ is not involved in the optimization process.

## Solution Difficulties

The PAGP code, as well as the nondominance algorithm, use the pivoting operation to obtain a new basis. The major drawback in this method is that round off errors accumulate as the algorithm moves from step to step. After several steps, the basis obtained by using the pivoting operation may be quite different from the basis which would be obtained if round off errors did not occur. Consequently, the commonly available goal programming codes are unable to solve large scale

TABLE XXIII

WORKFORCE AND PRODUCTION DECISIONS
OF THE GP SOLUTION

| t | $S_t$ | $N_t^1$ | $N_t^2$ | $H_t^1$ | $H_t^2$ * | $F_t^1$ | $F_t^2$ | $X_t$ | $O_t^1$ | $O_t^2$ | $I_t$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 11000 | 10.9 | 280.0 | 0.9 | 85.0 | 0.0 | 0.0 | 12615.6 | 0.0 | 3998.4 | 2615.6 |
| 2 | 11500 | 2.2 | 270.5 | 0.0 | 38.9 | 0.0 | 0.0 | 12216.5 | 0.0 | 4057.6 | 3332.1 |
| 3 | 9000 | 5.7 | 217.9 | 5.2 | 0.0 | 0.0 | 0.0 | 9920.3 | 0.0 | 3269.0 | 4252.4 |
| 4 | 12300 | 1.1 | 178.3 | 0.0 | 0.0 | 0.0 | 0.0 | 8047.6 | 0.0 | 2674.9 | 0.0 |

* $H_t^2$ is the number of workers hired in class 2 in period t, and all other variables are previously defined.

## TABLE XXIV

### WORKFORCE AND PRODUCTION DECISIONS OF A NONDOMINATED SOLUTION

| $t$ | $S_t$ | $N^1_t$ | $N^2_t$ | $H^1_t$ | $H^2_t$ * | $F^1_t$ | $F^2_t$ | $X_t$ | $O^1_t$ | $O^2_t$ | $I_t$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 11000 | 10.0 | 287.9 | 0.0 | 92.9 | 0.0 | 0.0 | 12490.4 | 0.0 | 3654.1 | 2490.4 |
| 2 | 11500 | 2.0 | 274.4 | 0.0 | 37.1 | 0.0 | 0.0 | 12389.2 | 0.0 | 4116.4 | 3379.6 |
| 3 | 9000 | 0.4 | 220.8 | 0.0 | 0.0 | 0.0 | 0.0 | 9950.6 | 0.0 | 3314.1 | 4330.0 |
| 4 | 12000 | 0.1 | 177.0 | 0.0 | 0.0 | 0.0 | 0.0 | 7970.0 | 1.2 | 2655.5 | 0.0 |

* $H^2_t$ is the number of workers hired in class 2 in period t, and all other variables are previously defined.

TABLE XXV

GOAL ATTAINMENT AND GOAL VALUE
FOR THE FIVE RUNS

| | | Minimization of hiring and firing (man) | Minimization of production cost ($) | Minimization of inventory Cost ($) | Minimization of overtime production (units) |
|---|---|---|---|---|---|
| Run 1 | Goal attain. | Not achieved | Not achieved | Not achieved | Not achieved |
| | Goal value | 117.6 | 774722.2 | 14195.4 | 14266.7 |
| Run 2 | Goal attain. | Achieved | Not achieved | Not achieved | Not achieved |
| | Goal value | 130.0 | 768226.7 | 12150.6 | 13195.8 |
| Run 3 | Goal attain. | Achieved | Achieved | Not achieved | Not achieved |
| | Goal value | 130.0 | 775000.0 | 9756.1 | 14097.8 |
| Run 4 | Goal attain. | Achieved | Achieved | Achieved | Not achieved |
| | Goal value | 130.0 | 775000.0 | 10700.0 | 13647.3 |
| Run 5 GP solution | Goal attain. | Achieved | Achieved | Achieved | Achieved |
| | Goal value | 130.0 | 775000.4 | 10700.0 | 14000.0 |
| Run 5 Nondom. solution | Goal attain. | Achieved | Achieved | Achieved | Achieved |
| | Goal value | 130.0 | 772247.9 | 10700.0 | 13741.3 |

problems (91). Thus, the initial results of the above example were unsatisfactory because, due to round off error, some of the real (goal) constraints were not satisfied.

To reduce the effect of these round off errors, four functions have been investigated. The first one is used in Ignizio's code (41). This function brings the floating point values that are either + or - 0.0001 from an integer to that integer. The second brings the floating points that are either + or - 0.000001 from an integer to that integer. The third and fourth functions are double precision functions which delete the floating point values whose absolute values are less than or equal to 0.001 and 0.0001 respectively. The four functions are listed in Appendix D.

To test the efficiency of these functions, the final run (run 5) has been made by using each function. Run 5 is used because the LP problem in the NAGP code is supposed to be solved in this run. The absolute value of the errors in the real constraints is calculated for the GP and nondominated solutions. Table XXVI shows the performance of each function in terms of the absolute error and CPU time. As shown from the above table, function 2 has the lowest absolute error but longest computer time.

It should be noted that function 4 has been used in runs 1, 2, 3 and 4 and function 2 was used in run 5 of the previous numerical example.

## Verification of Results

In this section Model I will be used to verify the results of Model II. To perform this analysis, the previous numerical example will

TABLE XXVI

ABSOLUTE ERRORS AND CPU TIME
FOR FIXING ERROR FUNCTIONS

| Function | Absolute Error | | Total absolute error in NAGP | CPU * (sec) |
| --- | --- | --- | --- | --- |
| | GP Solution | LP Solution | | |
| 1 | 2579.0477 | ** | 2579.0477 | 7.02 |
| 2 | 1.8043 | 15.0088 | 16.8031 | 13.93 |
| 3 | 70.4732 | ** | 70.4732 | 6.77 |
| 4 | 1.6583 | 30.4169 | 32.0752 | 9.18 |

* IBM 3081 (FORTVCL 77 compiler) has been used in this analysis.

** The LP test problem is not performed.

be considered and formulated as a LP model similar to that of Model I. The constraints are the same and the objective function is to minimize the cost of hiring and firing, payroll, inventory, and overtime. The size of the resultant LP problem can be reduced to 32 constraints and 48 decision variables instead of 36 constraints and 52 decision variables. One set of the overtime constraints and $O_t$ (total overtime production in period t) are eliminated by using the relation

$$O_t = O_t^1 + O_t^2$$

The resultant LP problem is then solved by MPSX.

Two goal programming problems are then considered. The first problem is equivalent to the linear programming formulation which consists of two priorities and one goal. The first priority is for the real constraints and the goal is for the objective function. The aspiration level for the goal is set equal to zero. The solution obtained from this GP formulation should not be dissimilar from the LP solution.

The second problem is a goal programming problem with five priorities and four goals. The first priority is for the real constraints and the other four priorities are for the goals. The goals are considered according to their priorities as follows:

Goal 1: minimize hiring and firing cost

Goal 2: minimize payroll cost

Goal 3: minimize inventory cost

Goal 4: minimize overtime cost

The goals are constructed from the components of the LP objective function and the aspiration levels (R.H.S.) of the goals are calculated from the LP solution as given below:

| Goal | Aspiration Level |
|---|---|
| 1 | 69862.0 |
| 2 | 620092.0 |
| 3 | 1474.0 |
| 4 | 34734.0 |

The reason for the above structure of the GP problems is that, in order to verify the results of Model II, the results of the LP problem and GP problems should agree.

The two GP problems are then solved by the NAGP. Table XXVII shows the results of the cost components for the LP problem and the two GP problems. Although the results of the second GP problem do not agree exactly with that of the LP solution, the results are close enough for the purpose of verification. However, better accuracy of the NAGP can be obtained by using the LU decomposition or Cholesky factorization method to alleviate the round off errors which result from the pivoting operation in the current code (for further details about these methods, refer to Murty (63)).

Appendix E has has the computer output results of:

1. MPSX for the LP problem
2. NAGP output for the equivalent GP problem of the LP problem
3. NAGP output for the GP problem constructed from the solution of the LP problem

The decision variables used in the computer outputs are also defined in Appendix E.

TABLE XXVII

COMPARISON OF MODEL I AND MODEL II RESULTS

| Cost Components | LP Problem (Model I) | GP Problem 1 * (Model II) | GP Problem 2 ** (Model II) |
|---|---|---|---|
| Hiring and Firing | 69862.0 | 69865.0 | 69862.0 |
| Payroll | 620092.0 | 620122.0 | 620187.0 |
| Inventory | 1474.0 | 1474.0 | 1470.0 |
| Overtime | 34734.0 | 34707.0 | 34734.0 |
| Total | 726162.0 | 726168.0 | 726253.0 |

* GP problem 1 is the equivalent GP problem for the LP problem

** GP problem 2 is the GP problem constructed from the solution of the LP problem

## Remarks

The previous analyses have demonstrated that a GP model can be developed and applied to the aggregate production and manpower planning problem. The solution methodology has been applied successfully, and all the goals are included in the optimization process. A preferred solution (GP solution) and a nondominated solution are also obtained. The new method can provide a set of solutions by providing different trade-off information. It also allows the decision maker to be involved in the optimization process and to provide reasonable aspiration levels for the targets, especially if the targets are not known.

It should also be noted that Model II has the same new characteristics as Model I. Furthermore, it is a multiple objectives decision making model in which a GP procedure is used, and accordingly, the resultant model will have the flexibility of choosing priorities. For instance, in one application the decision maker might assign the highest priorities to the manpower goals, while in another application the first priorities might be reserved for production costs--or even a combination of the two.

It should be remembered that the models presented in this research were developed to show the applicability of incorporating the personnel transition matrix in aggregate planning models, rather than the sophistication of the models themselves. With this is mind, it is felt that the goals of this research have been achieved.

# CHAPTER VIII

# CONCLUSIONS AND RECOMMENDATIONS

## Conclusions

The advances in information processing technology and quantative methodology during the past two decades have had a major impact on the design of production planning and control systems for manufacturing and distribution industries. Production planning and control systems, in a broad sense, are concerned with planning the use of productive resources to satisfy projected demand and then controlling the production process so that the plan is effectively carried out. The two essential elements in production planning are materials (equipment, raw materials, or semifinished products) and manpower.

The material requirements have been extensively studied and myriad applications of computer based techniques have been developed (such as material requirement planning systems (MRP)) and are in use in many manufacturing firms. However, the concept of the personnel transition matrix in manpower planning has not previously been considered in aggregate production planning. One of the major contributions of this research is the incorporation of the PTM into production planning; thus the present research can be considered as a first step toward building integrated computer based aggregate production and manpower planning systems for manufacturing firms. This research should also be helpful to theoreticians and practitioners who are involved in the design, development and operation of production planning and control systems.

The models developed in this research are by no means the final production and aggregate planning models. They do, however, illustrate how the important aspects of the personnel transition matrix and Markov processes can be properly incorporated into the aggregate production planning models. The research has originated a definition of the aggregate production and manpower planning problem.

Two models have been developed. The first is a linear programming model in which the Orrbeck model (68) has been used for the purpose of comparison and as a point of departure from which the new model (Model I) was developed. The second (Model II) is an extension of the first model from a single objective to a multiple objectives decision making model, and the goal programming technique has been used as a method of multiple objectives procedures. The analysis of these models indicated their flexibilities in presenting more realistic situations and accommodating budgetary and manpower ceilings as twin aspects of a simultaneous decision process.

The second major contribution of this research has been in the area of goal programming. The dominance in linear goal programming has been discussed and a nondominance test has been proposed. Furthermore, an algorithm has been developed to test the goal programming solution and to generate a nondominated solution if the goal programming solution turns out to be dominated. A solution methodology has also been proposed to include all the goals in the optimization process. This new method allows the decision maker to be involved in the optimization stages and to provide reasonable aspiration levels for the targets, especially if the targets are not known.

Finally, some of the goal programming difficulties have been discussed and solved by the nondominance algorithm developed in this research. The algorithm and solution methodology can also be used to evaluate the results of current goal programming applications.

## Recommendations

Because the problem of aggregate production and manpower planning is new, there are many possible areas of future research. These include:

1. The development of aggregate production and manpower planning models for multiproduct, multiplant firms.

2. The possibility of applying the effect of incorporating the personnel transition matrix to the current aggregate production planning models.

3. Extension of the models developed in this dissertation to include the effect of training and recruiting decisions.

4. It is assumed in the models developed in this research that the productivity factors are known. In most situations the product is produced by a varying number of workers, and consequently, the productivity factors are difficult to estimate. Further research should be done to accurately determine these factors. The work of Koshnevis (47) may be a good starting point for such research since he considered dynamic factors such as learning, design changes, etc., on worker productivity. Furthermore, the statistical methods and/or simulation analysis may be

used to get either a good estimation or approximated formulas to the productivity factors of the firm being considered.

The research on the goal programming technique presented in this dissertation has raised many new areas of further study. These include:

5. The application of the nondominance test to nonlinear goal programming models.

6. The development of a nondominance algorithm for nonlinear goal programming.

7. The application of the nondominance test to integer goal programming.

8. The development of a nondominance algorithm to integer goal programming.

9. Application of the solution methodology proposed in Chapter VI of this dissertation to nonlinear goal programming models.

10. Further research should be devoted to the development of a GP code to solve large scale problems, recognizing the fact that commonly available GP codes are unable to solve them. The inclusion of the goal programming technique as an option of MPSX (40), if possible, would be helpful for solving large scale GP problems.

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# APPENDIX A

# COMPUTER PROGRAM OF THE NONDOMINACE ALGORITHM FOR LINEAR GOAL PROGRAMMING (NAGP)

The nondominance algorithm developed in Chapter V has been coded in FORTRAN to test the nondominance of the GP solution obtained by PAGP and to generate a nondominated solution to the linear goal programming problems. The algorithm can solve and test problems with up to 60 constraints and 140 variables (decision and deviational) and 10 priority levels. These restrictions can be increased by changing the appropriate dimension statements. The notations used in the computer code are the same as in Arthur and Ravindran (4) and Ignizio (41). The program uses PAGP to obtain the GP solution (for an explanation of the notation in the computer code, see Arthur and Ravindran (4)).

To clarify the input to the NAGP algorithm, consider the example given in Chapter IV. The resultant linear goal programming formulation for this example is:

$$\text{Min } \bar{a} = \{ (d_1^+ + d_2^+ + d_3^+ + d_4^+),\ d_5^-,\ d_6^- \}$$

Real Constraints:

$$-X_1 + 3X_2 + d_1^- - d_1^+ = 21$$

$$X_1 + 3X_2 + d_2^- - d_2^+ = 27$$

$$4X_1 + 3X_2 + d_3^- - d_3^+ = 45$$

$$3X_1 + X_2 + d_4^- - d_4^+ = 30$$

Goal Constraints:

$$2X_1 + X_2 + d_5^- - d_6^+ = 15$$

$$-X_1 + 3X_2 + d_6^- - d_6^+ = 10$$

$$\bar{X},\ \bar{d}-,\ \bar{d}+ \geq 0$$

The following list gives the order and input format of the data cards for the above example. Note that card types 1, 3, 4 and 5 of NAGP are card types 1, 3, 4 and 6 of PAGP respectively.

| Card Type | | Description | Format |
|---|---|---|---|
| 1 | | Total number of priorities - NPRIT<br>Number of decision variables - NVAR<br>Number of real constraints - NRCON<br>Number of goal constraints - NGCON | 4I5 |
| | Example: | 3 2 4 2 | |
| 2 | | How many constraints are assigned to each priority. | 10I5 |
| | Example: | 4 1 1 | |
| 3 | | For each priority, one card is needed which gives the subscript of the constraint(s) assigned to priority $P_k$. | 16I5 |
| | Note: | If there are no constraints assigned to $P_k$, no card type 3 is necessary. | |
| | Example: | 1 2 3 4<br>5<br>6 | |
| 4 | | The number of terms (deviational variables) assigned to each priority in the objective function. | 10I5 |
| | Example: | 4 1 1 | |

| Card Type | | Description | Format |
|---|---|---|---|
| 5 | | For each goal constraint (real constraint in case of NRCON $\neq$ 0) assigned to priority $P_1$, read in the right hand side and the coefficients of the decision variables ($X_j$). If NVAR $\geq$ 7, go to another card. Enter as many type 5 cards as there are goal constraints assigned to priority $P_1$. The sequence of constraints must be in the order specified in card type 3 for $P_1$. | 8F10.0 |
| | Note: | The first priority $P_1$ is assigned to the real constraints unless there are no real constraints. | |
| | Example: | 21.0 -1.0 3.0<br>27.0 1.0 3.0<br>45.0 4.0 3.0<br>30.0 3.0 1.0 | |
| 6 | | For each deviational variable assigned to priority $P_1$, enter the following:<br><br>ISUB - the variable subscript<br><br>ITYPE = 3, if positive deviational variable<br>4, if negative deviational variable<br><br>WGHT - the cardinal weight assigned to the deviational variable | (2I5,F10.0) |
| | Example: | 1 3 1.0<br>2 3 1.0<br>3 3 1.0<br>4 3 1.0 | |
| | | Repeat card types 5 and 6 for priorities $P_2$, $P_3$,..., until all priorities are exhausted. Note that if for some priority $P_k$ there are no goal constraints assigned, then no type 5 card is required for $P_k$. However, for every priority $P_k$, there will be at least one type 6 card. | |
| | Example: | The type 5 and type 6 cards for priorities $P_2$ and $P_3$ are as follows: | |

| Card Type | Description | Format |
|---|---|---|
| | $P_2$: 15.00 2.0 1.0<br>5.0 4.0 1.0<br>$P_3$: 10.0 -1.0 2.0<br>6.0 4.0 1.0 | |
| 7 | Enter the type of real or goal constraints as follows:<br><br>NRGT = { 8, if the constraint of type "$\leq$"<br>9, if the constraint of type "$\geq$"<br>10, if the constraint of equality type.<br><br>Example: 8 8 8 8 9 9 | 12I5 |

The example problem has been solved using the computer program of the nondominance algorithm (see pages 156 to 158 for the computer output). The first part of the computer output is the solution of the GP problem and the second part is the summary of the nondominance test including a nondominated GP solution, which is the solution of the LP test problem. In this example two nondominated solutions are obtained with the same value of the objective function, i.e., the LP problem has an alternate solution. The computer output is self-explanatory.

The FORTRAN program listing of the NAGP algorithm is given on pages 159 to 177. The program was written to be performed on the Oklahoma State University IBM 3081 computer using a FORTVCL 77 complier. Slight modifications may be necessary for other systems.

THE OPTIMIZATION ENDED ON SUBPROBLEM 3
THERE WERE 6 CONSTRAINTS IN THE FINAL OPTIMAL TABLEAU.

**************************************************************************

THE OPTIMAL SOLUTION FOR THE DECISION VARIABLES X(J)

X( 1)= 4.0000

X( 2)= 7.0000

**************************************************************************

THE GOAL ACHIEVEMENTS ARE

| PRIORITY | GOAL NUMBER | OVER-ACHIEVEMENT | UNDER-ACHIEVEMENT |
|---|---|---|---|
| 1 | 1 | 0.0000 | 4.0000 |
| 1 | 2 | 0.0000 | 2.0000 |
| 1 | 3 | 0.0000 | 8.0000 |
| 1 | 4 | 0.0000 | 11.0000 |
| 2 | 5 | 0.0000 | 0.0000 |
| 3 | 6 | 0.0000 | 0.0000 |

**************************************************************************

THE PRIORITY ACHIEVEMENTS ARE

| PRIORITY | ACHIEVEMENT |
|---|---|
| 1 | 0.0000 |
| 2 | 0.0000 |
| 3 | 0.0000 |

**************************************************************************

OUTPUT SUMMARY

| SUBSCRIPT | A OPT | X OPT | POS DEV | NEG DEV |
|---|---|---|---|---|
| 1 | 0.0000 | 4.0000 | 0.0000 | 4.0000 |
| 2 | 0.0000 | 7.0000 | 0.0000 | 2.0000 |
| 3 | 0.0000 | | 0.0000 | 8.0000 |
| 4 | | | 0.0000 | 11.0000 |
| 5 | | | 0.0000 | 0.0000 |
| 6 | | | 0.0000 | 0.0000 |

**************************************************************************

***********************************************************************************

OUTPUT SUMMARY OF THE NONDOMINANCE TEST

***********************************************************************************

THE GOAL PROGRAMMING SOLUTION IS DOMINATED .

***********************************************************************************

THE OBJECTIVE FUNCTION IN THE NONDOMINATED SOLUTION = 2.0000

***********************************************************************************

OUTPUT SUMMARY OF A NONDOMINATED SOLUTION

| SUBSCRIPT | X NONDOMINATED | D | G | S |
|---|---|---|---|---|
| 1 | 4.8000 | | | 3.6000 |
| 2 | 7.4000 | | | 0.0000 |
| 3 | | | | 3.6000 |
| 4 | | | | 8.2000 |
| 5 | | 2.0000 | 0.0000 | |
| 6 | | 0.0000 | 0.0000 | |

***********************************************************************************

WHERE :

X = DECISION VARIABLES
D = VARIABLES TO BE MAXIMIZED IN THE LP PROBLEM OF THE NONDOMINANCE TEST
G = SLACK OR SURPLUS VARIABLES IN GOAL CONSTRAINTS
S = SLACK OR SURPLUS VARIABLES IN REAL CONSTRAINTS

***********************************************************************************

THE LINEAR PROGRAMMING PROBLEM OF THE NONDOMINANCE TEST HAS ALTERNATE OPTIMUM SOLUTION(S) .

***********************************************************************************

**************************************************************************************

ALTERNATE NONDOMINATED SOLUTION NUMBER 1

OUTPUT SUMMARY OF A NONDOMINATED SOLUTION

| SUBSCRIPT | X NONDOMINATED | D | G | S |
|---|---|---|---|---|
| 1 | 3.6000 | | | 1.2000 |
| 2 | 7.8000 | | | 0.0000 |
| 3 | | | | 7.2000 |
| 4 | | | | 11.4000 |
| 5 | | 0.0000 | 0.0000 | |
| 6 | | 2.0000 | 0.0000 | |

**************************************************************************************

WHERE :

X = DECISION VARIABLES
D = VARIABLES TO BE MAXIMIZED IN THE LP PROBLEM OF THE NONDOMINANCE TEST
G = SLACK OR SURPLUS VARIABLES IN GOAL CONSTRAINTS
S = SLACK OR SURPLUS VARIABLES IN REAL CONSTRAINTS

**************************************************************************************

```
C **********************************************************************
C ****                                                                 *
C ****                                                                 *
C **** THE FORTRAN PROGRAM LISTING OF THE NAGP, NONDOMINANCE ALGORITHM *
C **** FOR (LINEAR) GOAL PROGRAMMING                                   *
C ****                                                                 *
C ****                                                                 *
C **********************************************************************
C ****
C ****
C **** NAGP (THE NONDOMINACE ALGORITHM FOR LINEAR GOAL PROGRAMMING) IS
C **** DESIGNED TO TEST THE GP SOLUTION AND TO GENERATED A NONDOMINATED
C **** SOLUTION IF THE GP SOLUTION TURNS OUT TO BE DOMINATED .
C **** PAGP (THE PARTITIONING ALGORITHM FOR GOAL PROGRAMMING) IS
C **** MODIFIED TO INCLUDE THE NONDOMINANCE ALGORITHM, AND ACCORDINGLY,
C **** MUCH OF THE NOTATION AND STRUCTION OF THE NAGP ARE TAKEN FROM
C **** ARTHUR AND RAVINDRAN (4,5) AND IGNIZIO (41) .
C ****
C ****
C **********************************************************************
C ****
C ****
C **** THE CODE FOR NAGP USES THE FOLLOWING ARRAYS :
C ****
C ****          TE(NROW,NCOL) = THE COEFFICIENT OF THE VARIABLE IN
C ****                          COLUMN NCOL IN THE CONSTRAINT IN ROW NROW
C ****
C ****          TED(NROW,NCOL)= THE COEFFICIENT OF THE VARIABLE IN
C ****                          COLUMN NCOL IN THE CONSTRAINT IN ROW NROW
C ****                          IN THE LP PROBLEM OF NONDOMINANCE TEST
C ****
C ****          TT(NP,NCOL)   = THE WEIGHT OF THE VARIABLE IN COLUMN NCOL
C ****                          AT PRIORITY NP
C ****
C ****          TB(NROW)      = THE RIGHT HAND SIDE CONSTANT OF THE
C ****                          CONSTRAINT IN ROW NROW
C ****
C ****          TBD(NROW)     = THE RIGHT HAND SIDE CONSTANT OF THE
C ****                          CONSTRAINT IN ROW NROW IN THE LP PROBLEM
C ****                          OF NONDOMINANCE TEST
C ****
C ****          TL(NROW,NP)   = THE WEIGHT ASSIGNED TO THE BASIC VARIABLE
C ****                          IN ROW NROW AT PRIORITY NP
C ****
C ****          TI(NP,NCOL)   = THE RELATIVE WEIGHT OF THE VARIABLE IN
C ****                          COLUMN NCOL AT PRIORITY NP
C ****
C ****          TA(NP)        = THE TOTAL DEVIATION FROM THE GOALS AT
C ****                          PRIORITY NP
C ****
C ****          NC(NP)        = THE NUMBER OF GOAL CONSTRAINTS ASSIGNED
C ****                          TO PRIORITY NP BY THE PARTITION
C ****
C ****          NCON(I,NP)    = THE SUBSCRIPT OF THE I-TH CONSTRAINT
C ****                          ASSIGNED TO PRIORITY NP (I=1,...,NC(NP))
C ****
C ****          NTOF(NP)      = THE NUMBER OF TERMS IN THE OBJECTIVE
C ****                          FUNCTION AT PRIORITY NP
C ****
C ****          IND(NCOL)     = 1, IF THE VARIABLE IN COLUMN NCOL IS
C ****                          ELIGIBLE TO ENTER THE BASIS
C ****                        = 0, OTHERWISE
C ****
C ****          JROW(NROW,1)  = THE TYPE OF BASIC VARIABLE IN ROW NROW,
C ****                          WHERE TYPE IS GIVEN BELOW
C ****
C ****                                    TYPE      JROW( . ,1)
C ****                                    ****      ***********
C ****                                     X            2
C ****                                     D+           3
C ****                                     D-           4
C ****                                     D            5
```

```
C ****                                  G             6
C ****                                  S             7
C ****
C ****         JROW(NROW,2)  = THE SUBSCRIPT OF THE BASIC VARIABLE IN
C ****                         ROW NROW
C ****
C ****         JCOL(NCOL,1)  = THE TYPE OF VARIABLE IN COLUMN NCOL
C ****                         (TYPE IS DEFINED AS ABOVE)
C ****
C ****         JCOL(NCOL,2)  = THE SUBSCRIPT OF THE VARIABLE IN
C ****                         COLUMN NCOL
C ****
C ****         NRGT(.)       = TYPE OF REAL OR GOAL CONSTRAINT,
C ****                         WHERE TYPE IS GIVEN BELOW
C ****
C ****                                 TYPE          NRGT(.)
C ****                                 ****          *******
C ****                                  LE             8
C ****                                  GE             9
C ****                                  EQ            10
C ****
C ****
C ***********************************************************************
C ****
C ****
C **** THE MAIN PROGRAM
C ****
C ****
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON TT(10,140),TB(60),TE(60,140),TL(60,10),TA(10),TI(10,140),
     1TED(60,180),TBD(60),JCOL(180,2),NCOLI,NROWI,NPRIC,NC(10),
     1JROW(60,2),NVAR,NPRIT,NRCON,IND(140)
      COMMON /CHNG/ NCON(60,10),NTOF(10)
      COMMON /OUTPT/ WOUT(140,4)
      COMMON /DOMNC/ NGCON,NRAG,IOBJ,NCOLR,NCOLG,NCOL,NPHS1,IALT,
     1NRGT(60)
      COMMON /OBJDM/ W,WART,C(180),CR(180),CB(60)
      COMMON /ENTDPR/ NEVC,NDVR
      INTEGER ALTST
C
C **** READ IN PROBLEM DATA
C ****
C **** NPRIT=THE TOTAL NUMBER OF PRIORITIES
C ****
C **** NVAR=THE NUMBER OF DECISION VARIABLES
C ****
C **** NRCON=THE NUMBER OF REAL CONSTRAINTS
C ****
C **** NGCON=THE NUMBER OF GOALS
C
      READ (5,130) NPRIT,NVAR,NRCON,NGCON
      READ (5,131) (NC(NP),NP=1,NPRIT)
      DO 101 NP=1,NPRIT
         IF (NC(NP).EQ.0) GO TO 101
         NCTMP=NC(NP)
         READ (5,132) (NCON(N,NP),N=1,NCTMP)
  101 CONTINUE
      READ (5,131) (NTOF(NP),NP=1,NPRIT)
C
C **** INITIALIZE SUBPROBLEM DIMENSIONS AND COLUMN INDICATORS.
C ****
C **** NCOLI=THE NUMBER OF COLUMNS IN THE CURRENT WORKING TABLEAU
C ****
C **** NROWI=THE NUMBER OF ROWS IN THE CURRENT WORKING TABLEAU
C ****
C **** NPRIC=THE PRIORITY CURRENTLY BEING OPTIMIZED
C ****
C **** ZERO THE TE, TL, TT, AND TI ARRAYS.
C
      NCOLI=0
      NROWI=0
      NPRIC=0
```

```
      DO 104 NCR=1,140
         IND(NCR)=1
         DO 102 NR=1,60
  102    TE(NR,NCR)=0.
         DO 103 NP=1,10
            TI(NP,NCR)=0.
  103    TT(NP,NCR)=0.
  104 CONTINUE
      DO 105 NR=1,60
      DO 105 NP=1,10
  105 TL(NR,NP)=0.
C
C **** THE PARTITIONING ALGORITHM BEGINS.
C
  106 NPRIC=NPRIC+1
      IF (NPRIC.EQ.1) GO TO 107
      GO TO 108
  107 CALL READ1
      GO TO 109
  108 CALL READ2
  109 CALL CINDX
      CALL TEST (NEVC,NDVR)
C
C **** IF NEVC IS LESS THAN ZERO, THE SUBPROBLEM IS OPTIMIZED.
C
      IF (NEVC.LE.0) GO TO 110
C
C **** IF NDVR IS LESS THAN ZERO, NO MINIMUM POSITIVE RATIO WAS FOUND.
C
      IF (NDVR.LE.0) GO TO 116
      CALL PERM (NEVC,NDVR)
      GO TO 109
C
C **** IF THERE ARE NO MORE PRIORITIES, TOTAL PROBLEM IS OPTIMIZED.
C **** PRINT THE OPTIMAL SOLUTION.
C
  110 IF (NPRIC.EQ.NPRIT) GO TO 115
C
C **** SINCE THERE ARE MORE PRIORITIES, MOVE ON TO THE NEXT SUBPROBLEM
C **** IF THERE ARE ALTERNATE SOLUTIONS.  FIRST, ELIMINATE THOSE
C **** COLUMNS WHICH CAN NOT ENTER THE BASIS.  IF THERE ARE NO
C **** ALTERNATE SOLUTIONS, PRINT THE UNIQUE OPTIMAL SOLUTION.
C
      INOND=0
  120 ALTST=0
      DO 112 NCR=1,NCOLI
         IF (IND(NCR).EQ.0) GO TO 112
         IF (TI(NPRIC,NCR).GT.0.) GO TO 112
         DO 111 NR=1,NROWI
            IF (JROW(NR,1).EQ.JCOL(NCR,1).AND.JROW(NR,2).EQ.JCOL(NCR,2))
     1      GO TO 112
  111    CONTINUE
         ALTST=1
  112 CONTINUE
C
C **** IF ALTST=1, THERE ARE ALTERNATE SOLUTIONS.
C
      IF (INOND.EQ.0.AND.ALTST.EQ.1) GO TO 113
      IF (INOND.EQ.1.AND.ALTST.EQ.0) GO TO 118
      IF (INOND.EQ.1.AND.ALTST.EQ.1) GO TO 119
      GO TO 115
C
C **** ELIMINATE THOSE COLUMNS WITH A POSITIVE RELATIVE COST AT
C **** PRIORITY NPRIC.
C
  113 DO 114 NCR=1,NCOLI
  114 IF (TI(NPRIC,NCR).GT.0.) IND(NCR)=0
      GO TO 106
C
C **** THE OPTIMIZATION IS OVER. PRINT OUT THE FINAL SOLUTION.
C
  115 CALL POUT
```

```
C
      GO TO 117
  116 WRITE (6,133) NPRIC
  117 CONTINUE
C
C **** THE NONDOMINANCE TEST BEGINS
C
      WRITE (6,135)
      IF (NPRIC.LT.NPRIT) GO TO 118
      INOND=1
      GO TO 120
  118 WRITE (6,143)
      GO TO 125
  119 NRAG=NRCON+NGCON
C
C **** READ IN THE CONSTRAINT OR GOAL TYPE
C
      READ (5,134) (NRGT(NG),NG=1,NRAG)
      CALL SETUP
      CALL PHSE1
C
C **** PHASE 1 IS NOT USED IF THE REAL AND GOAL CONSTRAINTS ARE OF
C **** TYPE '<' .
C
      IF (NPHS1.EQ.0) WRITE (6,136)
C
      IF (DABS(WART).GT.0.0 ) GO TO 126
C
      CALL PHSE2
C
C **** IF NDVR=0 , THE PROBLEM HAS UNBOUNDED SOLUTION .
C
      IF (NDVR.EQ.0) GO TO 124
C
C **** IF W=0 , THE GOAL PROGRAMMING SOLUTION IS NONDOMINATED .
C
      IF (DABS(W).LE.0.0005) GO TO 121
      WRITE (6,137)
C
       W=-W
      WRITE (6,138) W
      CALL DOUT
      GO TO 122
  121 WRITE (6,140)
      CALL DOUT
C
C **** CHECK FOR ALTERNATE OPTIMUM
C
  122 CALL ALTOP
      IF (IALT.EQ.0) GO TO 123
      GO TO 125
  123 WRITE (6,141)
      GO TO 125
  124 WRITE (6,142)
      GO TO 125
C
  126 WRITE (6,143)
      WRITE (6,144)
      WRITE (6,145) WART
C
  125 STOP
C
  130 FORMAT (4I5)
  131 FORMAT (10I5)
  132 FORMAT (16I5)
  133 FORMAT (/ 40H   THE PROGRAM TERMINATED ON SUBPROBLEM ,I4, 42H  NO
     1 MINIMUM POSITIVE RATIO COULD BE FOUND)
  134 FORMAT (12I5)
  135 FORMAT (1H1,//120(1H*)///20X,  41H OUTPUT SUMMARY OF THE NONDOMINA
     1NCE TEST ,///120(1H*))
  136 FORMAT (//  21H  PHASE 1 IS NOT USED,////120(1H*))
  137 FORMAT (//  46H  THE GOAL PROGRAMMING SOLUTION IS DOMINATED .,
```

```
     1////120(1H*))
  138 FORMAT (//  55H  THE OBJECTIVE FUNCTION IN THE NONDOMINATED SOLUTI
     1ON =,F15.4,////120(1H*))
  140 FORMAT (//   48H THE GOAL PROGRAMMING SOLUTION IS NONDOMINATED .,
     1////120(1H*))
  141 FORMAT (//  93H  THE LINEAR PROGRAMMING PROBLEM OF THE NONDOMINANC
     1E TEST HAS NO ALTERNATE OPTIMUM SOLUTION .,////120(1H*))
  142 FORMAT (//  114H  THE ORIGINAL PROBLEM HAS UNBOUNDED SOLUTION AND
     1THE GOAL PROGRAMMING SOLUTION IS CERTAINLY SUBOPTIMAL SOLUTION .,
     1////120(1H*))
  143 FORMAT (////  55H  THE ABOVE GOAL PROGRAMMING SOLUTION IS NONDOMIN
     1ATED .,////120(1H*))
  144 FORMAT (//  43H  THE LP PROBLEM TERMINATES AT PHASE 1 AND )
  145 FORMAT (//  44H  THE VALUE OF PHASE 1 OBJECTIVE FUNCTION  =,F15.4,
     1////120(1H*))
C
      END
C ****
C ******************************************************************
C ****
      SUBROUTINE READ1
C
C **** SUBROUTINE READ1 READS IN THE GOAL CONSTRAINTS AND OBJECTIVE
C **** FUNCTION TERMS ASSIGNED TO PRIORITY ONE.
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON TT(10,140),TB(60),TE(60,140),TL(60,10),TA(10),TI(10,140),
     1TED(60,180),TBD(60),JCOL(180,2),NCOLI,NROWI,NPRIC,NC(10),
     1JROW(60,2),NVAR,NPRIT,NRCON,IND(140)
      COMMON /CHNG/ NCON(60,10),NTOF(10)
C
C **** SET COLUMN AND ROW HEADINGS.
C
      DO 100 NV=1,NVAR
         JCOL(NV,1)=2
  100 JCOL(NV,2)=NV
      NC11=NC(1)
      DO 101 NCR=1,NC11
         NC1=NVAR+2*NCR-1
         NC2=NVAR+2*NCR
         JCOL(NC1,1)=4
         JCOL(NC1,2)=NCON(NCR,1)
         JCOL(NC2,1)=3
         JCOL(NC2,2)=NCON(NCR,1)
         JROW(NCR,1)=4
  101 JROW(NCR,2)=NCON(NCR,1)
C
C **** READ IN THE GOAL CONSTRAINTS ASSIGNED TO PRIORITY 1.
C
      NC1=NC(1)
      DO 103 NCR=1,NC1
         NV1=NVAR+2*NCR-1
         NV2=NVAR+2*NCR
         READ (5,105) TB(NCR),(TE(NCR,NV),NV=1,NVAR)
C
C **** SAVE THE INFORMATION FOR THE NONDOMINACE TEST.
C
         IGSUB=NCON(NCR,1)
         DO 102 NV=1,NVAR
            TED(IGSUB,NV)=TE(NCR,NV)
  102    CONTINUE
         IF (NRCON.NE.0) TBD(IGSUB)=TB(NCR)
C
C **** PUT +1 IN FOR D- AND -1 IN FOR D+.
C
         TE(NCR,NV1)=1.
         TE(NCR,NV2)=-1.
  103 CONTINUE
      NCOLI=NV2
      NROWI=NC(1)
C
C **** READ IN THE OBJECTIVE FUNCTION TERMS FOR PRIORITY 1.
```

```
C
      NT1=NTOF(1)
      DO 104 NT=1,NT1
         READ (5,106) ISUB,ITYPE,WGHT
         CALL PLACE (ISUB,ITYPE,WGHT)
  104 CONTINUE
      RETURN
C
  105 FORMAT (8F10.0)
  106 FORMAT (2I5,F10.0)
C
      END
C ****
C ************************************************************************
C ****
      SUBROUTINE READ2
C
C **** SUBROUTINE READ2 READS IN THE GOAL CONSTRAINTS AND OBJECTIVE
C **** FUNCTION TERMS ASSIGNED TO PRIORITY NPRIC.
C **** SUBROUTINE READ2 IS ALSO USED TO READ IN THE FIRST PRIORITY GOAL
C **** CONSTRAINTS AND OBJECTIVE FUNCTION TERMS IF REAL CONSTRAINTS ARE
C **** PRESENT.
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON TT(10,140),TB(60),TE(60,140),TL(60,10),TA(10),TI(10,140),
     1TED(60,180),TBD(60),JCOL(180,2),NCOLI,NROWI,NPRIC,NC(10),
     1JROW(60,2),NVAR,NPRIT,NRCON,IND(140)
      COMMON /CHNG/ NCON(60,10),NTOF(10)
      IF (NC(NPRIC).EQ.0) GO TO 107
C
C **** READ IN THE COEFFICIENTS OF THE X'S.
C
      NCTMP=NC(NPRIC)
      DO 106 NRI=1,NCTMP
         NR=NRI+NROWI
         NC1=NCOLI+2*NRI-1
         NC2=NCOLI+2*NRI
         JCOL(NC1,1)=4
         JCOL(NC1,2)=NCON(NRI,NPRIC)
         JCOL(NC2,1)=3
         JCOL(NC2,2)=NCON(NRI,NPRIC)
         READ (5,109) TB(NR),(TE(NR,NV),NV=1,NVAR)
C
C **** SAVE THE INFORMATION FOR THE NONDOMINACE TEST.
C
         IGSUB=NCON(NRI,NPRIC)
         DO 100 NV=1,NVAR
            TED(IGSUB,NV)=TE(NR,NV)
  100    CONTINUE
C
         TE(NR,NC1)=1.
         TE(NR,NC2)=-1.
C
C **** PERFORM THE ROW REDUCTION.
C
         DO 102 NRC=1,NROWI
            IF (JROW(NRC,1).NE.2) GO TO 102
            J=JROW(NRC,2)
            TB(NR)=TB(NR)-TE(NR,J)*TB(NRC)
            DO 101 NCR=1,NC2
               IF (NCR.EQ.J) GO TO 101
               TE(NR,NCR)=TE(NR,NCR)-TE(NR,J)*TE(NRC,NCR)
  101       CONTINUE
            TE(NR,J)=0.
  102    CONTINUE
C
C **** DETERMINE THE DEVIATIONAL VARIABLE TO ENTER THE BASIS.
C
         IF (TB(NR)) 103,105,105
C
C **** SINCE TB IS LESS THAN ZERO, MULTIPLY THE ROW BY -1 AND ENTER D+
C **** IN THE BASIS.
```

```
C
  103     DO 104 NCR=1,NC2
  104     TE(NR,NCR)=-TE(NR,NCR)
          TB(NR)=-TB(NR)
          JROW(NR,1)=3
          JROW(NR,2)=NCON(NRI,NPRIC)
          GO TO 106
C
C **** SINCE TB IS GREATER THAN OR EQUAL TO ZERO ENTER D- IN THE BASIS.
C
  105     JROW(NR,1)=4
          JROW(NR,2)=NCON(NRI,NPRIC)
  106 CONTINUE
C
C **** INCREASE THE PARAMETERS NCOLI AND NROWI.
C
      NCOLI=NC2
      NROWI=NR
C
C **** READ IN THE OBJECTIVE FUNCTION TERMS FOR PRIORITY NPRIC.
C
  107 NTTMP=NTOF(NPRIC)
      DO 108 NT=1,NTTMP
         READ (5,110) ISUB,ITYPE,WGHT
         CALL PLACE (ISUB,ITYPE,WGHT)
  108 CONTINUE
      RETURN
C
  109 FORMAT (8F10.0)
  110 FORMAT (2I5,F10.0)
      END
C ****
C *********************************************************************
C ****
      SUBROUTINE PLACE (ISUB,ITYPE,WGHT)
C
C **** SUBROUTINE PLACE PUTS THE OBJECTIVE FUNCTION WEIGHTS FOR THE
C **** DEVIATION VARIABLES AT THE CURRENT PRIORITY LEVEL (NPRIC) IN THE
C **** CORRECT POSITIONS IN THE AUGMENTED TABLEAU.
C ****
C **** ISUB=THE SUBSCRIPT OF THE DEVIATIONAL VARIABLE
C ****
C **** ITYPE=3, IF POSITIVE DEVIATIONAL VARIABLE (D+)
C ****       4, IF NEGATIVE DEVIATIONAL VARIABLE (D-)
C ****
C **** WGHT=THE CARDINAL WEIGHT OF THIS DEVIATIONAL VARIABLE AT THE
C ****      CURRENT PRIORITY LEVEL
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON TT(10,140),TB(60),TE(60,140),TL(60,10),TA(10),TI(10,140),
     1TED(60,180),TBD(60),JCOL(180,2),NCOLI,NROWI,NPRIC,NC(10),
     1JROW(60,2),NVAR,NPRIT,NRCON,IND(140)
      COMMON /CHNG/ NCON(60,10),NTOF(10)
C
C **** PLACE THE WEIGHT IN THE PROPER COLUMN IN THE TOP STUB.
C
      NC1=NVAR+1
      DO 101 NCR=NC1,NCOLI
         IF (JCOL(NCR,1).EQ.ITYPE.AND.JCOL(NCR,2).EQ.ISUB) GO TO 102
  101 CONTINUE
  102 TT(NPRIC,NCR)=WGHT
C
C **** PLACE THE WEIGHT IN THE PROPER ROW IN THE LEFT STUB.
C
      DO 103 NR=1,NROWI
         IF (JROW(NR,1).EQ.ITYPE.AND.JROW(NR,2).EQ.ISUB) GO TO 104
  103 CONTINUE
      GO TO 105
  104 TL(NR,NPRIC)=WGHT
  105 CONTINUE
      RETURN
      END
```

```
C ****
C ***********************************************************************
C ****
      SUBROUTINE CINDX
C
C **** SUBROUTINE CINDX COMPUTES THE RELATIVE COST COEFFICIENTS FOR EACH
C **** VARIABLE IN THE CURRENT TABLEAU(THE TI( . , . ) ARRAY) AND THE
C **** OBJECTIVE FUNCTION VALUE(THE TA(.) ARRAY) AT THE CURRENT
C **** PRIORITY(NPRIC)
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON TT(10,140),TB(60),TE(60,140),TL(60,10),TA(10),TI(10,140),
     1TED(60,180),TBD(60),JCOL(180,2),NCOLI,NROWI,NPRIC,NC(10),
     1JROW(60,2),NVAR,NPRIT,NRCON,IND(140)
C
C **** COMPUTE TA(NPRIC) AND TI(NPRIC,NC)   NC=1,....,NCOLI
C
      TA(NPRIC)=0.
      DO 101 NR=1,NROWI
  101 TA(NPRIC)=TA(NPRIC)+TB(NR)*TL(NR,NPRIC)
      DO 102 NCR=1,NCOLI
         TI(NPRIC,NCR)=TT(NPRIC,NCR)
      DO 102 NR=1,NROWI
  102 TI(NPRIC,NCR)=TI(NPRIC,NCR)-TE(NR,NCR)*TL(NR,NPRIC)
      RETURN
      END
C ****
C ***********************************************************************
C ****
      SUBROUTINE TEST (NEVC,NDVR)
C
C **** SUBROUTINE TEST DETERMINES THE NEXT ENTERING VARIABLE'S COLUMN
C **** (NEVC) AND THE NEXT DEPARTING VARIABLE'S ROW(NDVR).  IF NO
C **** FURTHER OPTIMIZATION IS POSSIBLE, THE VALUE NEVC=O IS RETURNED.
C **** IF NDVR=O IS RETURNED, NO MINIMUM POSITIVE RATIO COULD BE FOUND
C **** IN THE CURRENT PIVOT OPERATION,I.E., ALL OF THE COEFFICIENTS
C **** TE( . ,NEVC) ARE NONPOSITIVE.
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON TT(10,140),TB(60),TE(60,140),TL(60,10),TA(10),TI(10,140),
     1TED(60,180),TBD(60),JCOL(180,2),NCOLI,NROWI,NPRIC,NC(10),
     1JROW(60,2),NVAR,NPRIT,NRCON,IND(140)
      NDVR=0
      NEVC=0
      VEVC=0.
      VDVR=10.0E+20
C
C **** DETERMINE ENTERING VARIABLE'S COLUMN.
C
      DO 101 NCR=1,NCOLI
         IF (TI(NPRIC,NCR).GE.O.) GO TO 101
         IF (IND(NCR).EQ.0) GO TO 101
         IF (TI(NPRIC,NCR).GE.VEVC) GO TO 101
         NEVC=NCR
         VEVC=TI(NPRIC,NCR)
  101 CONTINUE
C
C **** IF NEVC=O, SUBPROBLEM NPRIC IS OPTIMIZED. RETURN.
C
      IF (NEVC.EQ.0) RETURN
C
C **** DETERMINE DEPARTING VARIABLE'S ROW.
C
      DO 105 NR=1,NROWI
         IF (TE(NR,NEVC).LE.O.) GO TO 105
         V=TB(NR)/TE(NR,NEVC)
         IF (NDVR.EQ.0) GO TO 104
         IF (V-VDVR) 104,102,105
  102    DO 103 NP=1,NPRIC
            IF (TL(NR,NP)-TL(NDVR,NP)) 105,103,104
  103    CONTINUE
  104    VDVR=V
```

```
         NDVR=NR
  105 CONTINUE
      RETURN
      END
C ****
C ***********************************************************************
C ****
      SUBROUTINE PERM (NEVC,NDVR)
C
C **** SUBROUTINE PERM PERFORMS THE PIVOT OPERATION USING THE PIVOT
C **** ELEMENT IN COLUMN NEVC AND ROW NDVR AND COMPUTES THE NEW TABLEAU.
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON TT(10,140),TB(60),TE(60,140),TL(60,10),TA(10),TI(10,140),
     1TED(60,180),TBD(60),JCOL(180,2),NCOLI,NROWI,NPRIC,NC(10),
     1JROW(60,2),NVAR,NPRIT,NRCON,IND(140)
C
C **** REPLACE HEADING FOR ROW NDVR.
C
      JROW(NDVR,1)=JCOL(NEVC,1)
      JROW(NDVR,2)=JCOL(NEVC,2)
C
C **** REPLACE TL VECTOR FOR ROW NDVR
C
      DO 101 NP=1,NPRIC
  101 TL(NDVR,NP)=TT(NP,NEVC)
C
C **** COMPUTE NEW TE ARRAY.
C
      PIV=TE(NDVR,NEVC)
      PIB=TB(NDVR)
      DO 103 NR=1,NROWI
         IF (NR.EQ.NDVR) GO TO 103
         IF (DABS(TE(NR,NEVC)).LE.0.000050) GO TO 103
         PIX=TE(NR,NEVC)/PIV
         TB(NR)=FIX(TB(NR)-PIX*PIB)
         DO 102 NCR=1,NCOLI
  102    TE(NR,NCR)=FIX(TE(NR,NCR)-TE(NDVR,NCR)*PIX)
  103 CONTINUE
      TB(NDVR)=FIX(PIB/PIV)
      DO 104 NCR=1,NCOLI
  104 TE(NDVR,NCR)=FIX(TE(NDVR,NCR)/PIV)
      RETURN
      END
C ****
C ***********************************************************************
C ****
      DOUBLE PRECISION FUNCTION FIX(Z)
C
C **** FUNCTION FIX DELETES FLOATING POINT VALUES WHOSE ABSOLUTE
C**** VALUES ARE LESS THAN OR EQUAL TO 0.0001 .
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      FIX=DINT(Z+DSIGN(.5D+0,Z))
      IF (DABS(FIX-Z).GT. 1.D-4) FIX=Z
      RETURN
      END
C ****
C ***********************************************************************
C ****
      SUBROUTINE POUT
C
C **** SUBROUTINE POUT PREPARES AND PRINTS THE SOLUTION INFORMATION OF
C **** THE GOAL PROGRAMMING PROBLEM .
C
      IMPLICIT REAL*8(A-H,O-Z)
C
      COMMON TT(10,140),TB(60),TE(60,140),TL(60,10),TA(10),TI(10,140),
     1TED(60,180),TBD(60),JCOL(180,2),NCOLI,NROWI,NPRIC,NC(10),
     1JROW(60,2),NVAR,NPRIT,NRCON,IND(140)
      COMMON /CHNG/ NCON(60,10),NTOF(10)
```

```
      COMMON /OUTPT/ WOUT(140,4)
      DIMENSION DIFF(60),RLHS(60,10)
C
      WRITE (6,122)
      WRITE (6,123) NPRIC,NROWI
C
C **** OUTPUT ARRAY IS ZEROED.
C
      DO 100 I=1,140
      DO 100 J=1,4
  100 WOUT(I,J)=0.
C
C **** OUTPUT ARRAY IS FILLED.
C
      DO 101 NP=1,NPRIC
  101 WOUT(NP,1)=FIX(TA(NP))
      DO 102 NR=1,NROWI
         I1=JROW(NR,1)
         I2=JROW(NR,2)
  102 WOUT(I2,I1)=FIX(TB(NR))
C
C **** IF ALL PRIORITIES HAVE BEEN INCLUDED, PRINT OPTIMAL SOLUTION.
C **** IF NOT, WE MUST CALCULATE VALUES FOR REMAINING TA'S AND D- AND D+
C
      IF (NPRIC.GE.NPRIT) GO TO 114
      NP1=NPRIC+1
      DO 113 NP=NP1,NPRIT
         TA(NP)=0.
         IF (NC(NP).EQ.0) GO TO 106
C
C **** READ IN THE GOAL CONSTRAINTS ASSIGNED TO PRIORITY NP.
C
         NCTMP=NC(NP)
         DO 105 NCI=1,NCTMP
            NR=NROWI+NCI
            READ (5,124) TB(NR),(TE(NR,NV),NV=1,NVAR)
C
C **** SAVE THE INFORMATION FOR THE NONDOMINACE TEST.
C
            IGSUB=NCON(NCI,NP)
            DO 103 NV=1,NVAR
               TED(IGSUB,NV)=TE(NR,NV)
  103       CONTINUE
C
            RLHS(NCI,NP)=0.
            DO 104 NV=1,NVAR
  104       RLHS(NCI,NP)=RLHS(NCI,NP)+TE(NR,NV)*WOUT(NV,2)
            DIFF(NCI)=TB(NR)-RLHS(NCI,NP)
  105    CONTINUE
C
C **** READ THE OBJECTIVE FUNCTION TERMS FOR PRIORITY NP.
C
  106    NTTMP=NTOF(NP)
         DO 112 NT=1,NTTMP
            READ (5,125) ISUB,ITYPE,WGHT
            IF (NC(NP).EQ.0) GO TO 111
            NCTMP=NC(NP)
            DO 110 NCI=1,NCTMP
               IF (ISUB.NE.NCON(NCI,NP)) GO TO 110
               IF (DIFF(NCI)) 107,108,109
  107          IF (ITYPE.NE.3) GO TO 110
               WOUT(ISUB,3)=-DIFF(NCI)
  108          GO TO 110
  109          IF (ITYPE.NE.4) GO TO 110
               WOUT(ISUB,4)=DIFF(NCI)
  110       CONTINUE
  111       TA(NP)=TA(NP)+WGHT*WOUT(ISUB,ITYPE)
  112    CONTINUE
         NROWI=NROWI+NC(NP)
C
C **** FILL IN THE OUTPUT VALUE FOR ATTAINMENT OF PRIORITY NP.
C
```

```
         WOUT(NP,1)=FIX(TA(NP))
  113 CONTINUE
C
C **** PRINT OPTIMAL SOLUTION
C
  114 WRITE (6,126)
      WRITE (6,127)
      DO 115 NV=1,NVAR
         WRITE (6,128) NV,WOUT(NV,2)
  115 CONTINUE
      WRITE (6,126)
      WRITE (6,129)
      DO 116 NP=1,NPRIT
         IF (NC(NP).EQ.0) GO TO 116
         NCTMP=NC(NP)
      DO 139 NCO=1,NCTMP
         N=NCON(NCO,NP)
         WRITE (6,130) NP,N,WOUT(N,3),WOUT(N,4)
  139 CONTINUE
  116 CONTINUE
      WRITE (6,126)
      WRITE (6,131)
      DO 117 NP=1,NPRIT
         WRITE (6,132) NP,WOUT(NP,1)
  117 CONTINUE
      WRITE (6,126)
      WRITE (6,133)
      WRITE (6,134)
      I=MAX0(NPRIT,NVAR,NROWI)
      DO 121 K=1,I
         IF (K.GT.NPRIT) GO TO 119
         IF (K.GT.NVAR) GO TO 118
         WRITE (6,135) K,(WOUT(K,J),J=1,4)
         GO TO 121
  118    WRITE (6,136) K,WOUT(K,1),(WOUT(K,J),J=3,4)
         GO TO 121
  119    IF (K.GT.NVAR) GO TO 120
         IF (K.GT.NROWI) GO TO 140
         WRITE (6,137) K,(WOUT(K,J),J=2,4)
         GO TO 121
  140    WRITE (6,141) K,WOUT(K,2)
         GO TO 121
  120    WRITE (6,138) K,(WOUT(K,J),J=3,4)
  121 CONTINUE
      WRITE (6,126)
C
      RETURN
C
  122 FORMAT (1H1)
  123 FORMAT (/  39H  THE OPTIMIZATION ENDED ON SUBPROBLEM ,I5 /  13H  T
     1HERE WERE ,I5, 42H CONSTRAINTS IN THE FINAL OPTIMAL TABLEAU.)
  124 FORMAT (8F10.0)
  125 FORMAT (2I5,F10.0)
  126 FORMAT (//120(1H*))
  127 FORMAT (1H0, 52HTHE OPTIMAL SOLUTION FOR THE DECISION VARIABLES X(
     1J))
  128 FORMAT (1H0,  2HX(,I3,  2H)=,F15.4)
  129 FORMAT (1H0, 25HTHE GOAL ACHIEVEMENTS ARE //   9H PRIORITY,2X, 11H
     1GOAL NUMBER,8X, 16HOVER-ACHIEVEMENT,4X, 17HUNDER-ACHIEVEMENT)
  130 FORMAT (4X,I2,10X,I2,10X,F15.4,10X,F15.4)
  131 FORMAT (1H0, 29HTHE PRIORITY ACHIEVEMENTS ARE //  9H PRIORITY,13X,
     1 11HACHIEVEMENT)
  132 FORMAT (4X,I2,10X,F15.4)
  133 FORMAT (1H0, 15H OUTPUT SUMMARY)
  134 FORMAT (1H0,  9HSUBSCRIPT,11X,  8H   A OPT,7X,  8H   X OPT,7X,  9H
     1  POS DEV,6X,  9H  NEG DEV /)
  135 FORMAT (I8,7X,4F15.4)
  136 FORMAT (I8,7X,F15.4,15X,2F15.4)
  137 FORMAT (I8,22X,3F15.4)
  138 FORMAT (I8,37X,2F15.4)
  141 FORMAT (I8,22X,F15.4)
C
```

```
      END
C ****
C **********************************************************************
C ****
      SUBROUTINE SETUP
C
C **** SUBROUTINE SETUP ESTABLISHES THE INITIAL TABLEAU OF
C **** THE LINEAR PROGRAMMING PROBLEM FOR THE NONDOMINANCE TEST .
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON TT(10,140),TB(60),TE(60,140),TL(60,10),TA(10),TI(10,140),
     1TED(60,180),TBD(60),JCOL(180,2),NCOLI,NROWI,NPRIC,NC(10),
     1JROW(60,2),NVAR,NPRIT,NRCON,IND(140)
      COMMON /OUTPT/ WOUT(140,4)
      COMMON /DOMNC/ NGCON,NRAG,IOBJ,NCOLR,NCOLG,NCOL,NPHS1,IALT,
     1NRGT(60)
C
C **** INITIALIZE JCOL(.,.) AND JROW(.,.)
C
      NVD=NVAR+1
      DO 97 NV=NVD,180
         JCOL(NV,1)=0
         JCOL(NV,2)=0
   97 CONTINUE
C
      DO 98 NV=1,60
         JROW(NV,1)=0
         JROW(NV,2)=0
   98 CONTINUE
C
      IOBJ=NRCON+1
C
C **** IF THE REAL CONSTRAINTS HAVE NO FEASIBLE SOLUTION (TA(1)>0) ,
C **** THEN THE REAL CONSTRAINTS WILL BE TREATED AS GOALS .
C
      IF (TA(1).GT.0.0000500) IOBJ=1
C
C **** INITIALIZE TBD(.) ARRAY
C
      DO 99 IB=IOBJ,NRAG
         TBD(IB)=0.0
   99 CONTINUE
C
C **** CALCULATE THE RHS OF GOALS FROM THE GP SOLUTION
C
      DO 100 ID=IOBJ,NRAG
      DO 100 JD=1,NVAR
         TBD(ID)=TBD(ID)+WOUT(JD,2)*TED(ID,JD)
  100 CONTINUE
C
C **** CASE OF NEGATIVE GOAL VALUES
C
      DO 104 NEG=IOBJ,NRAG
         IF (TBD(NEG).GE.0.) GO TO 104
         IF (NRGT(NEG).EQ.8) GO TO 101
         IF (NRGT(NEG).EQ.9) GO TO 102
  101    NRGT(NEG)=9
         GO TO 103
  102    NRGT(NEG)=8
  103    TBD(NEG)=-TBD(NEG)
      DO 104 NV=1,NVAR
         TED(NEG,NV)=-TED(NEG,NV)
  104 CONTINUE
C
C **** SET COLUMN AND ROW HEADINGS FOR SLACK OR SURPLUS OF REAL
C **** CONSTRAINTS
C **** SET COLUMN AND ROW HEADINGS FOR SLACK OR SURPLUS OF GOAL
C **** CONSTRAINTS
C
      IF (NRCON.EQ.0) GO TO 108
         IF (TA(1).GT.0.0000500) GO TO 108
      JR=NVAR
```

```
      DO 107 NR=1,NRCON
         IF (NRGT(NR).EQ.10) GO TO 107
         JR=JR+1
         JCOL(JR,1)=7
         JCOL(JR,2)=NR
         IF (NRGT(NR).EQ.8) GO TO 105
         IF (NRGT(NR).EQ.9) GO TO 106
  105    TED(NR,JR)=1.
         JROW(NR,1)=7
         JROW(NR,2)=NR
         GO TO 107
  106    TED(NR,JR)=-1.
  107 CONTINUE
C
      NCOLR=JR
      GO TO 109
  108 NCOLR=NVAR
  109 KGL=0
      DO 112 IG=IOBJ,NRAG
         IF (NRGT(NG).EQ.10) GO TO 112
         KGL=KGL+1
         JG1=NCOLR+2*KGL-1
         JG2=NCOLR+2*KGL
         JCOL(JG1,1)=5
         JCOL(JG1,2)=IG
         JCOL(JG2,1)=6
         JCOL(JG2,2)=IG
         IF (NRGT(IG).EQ.8) GO TO 110
         IF (NRGT(IG).EQ.9) GO TO 111
  110    TED(IG,JG1)=1.
         JROW(IG,1)=5
         JROW(IG,2)=IG
         TED(IG,JG2)=1.
         GO TO 112
  111    TED(IG,JG1)=-1.
         TED(IG,JG2)=-1.
  112 CONTINUE
C
      NCOLG=JG2
      RETURN
      END
C ****
C ***********************************************************************
C ****
      SUBROUTINE PHSE1
C
C **** SUBROUTINE PHSE1 PERFORMS A PHASE 1 SIMPLEX PROCEDURE IN ORDER TO
C **** FIND AN INITIAL BASIC FEASIBLE SOLUTION TO
C **** THE LINEAR PROGRAMMING PROBLEM FOR THE NONDOMINANCE TEST .
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON TT(10,140),TB(60),TE(60,140),TL(60,10),TA(10),TI(10,140),
     1TED(60,180),TBD(60),JCOL(180,2),NCOLI,NROWI,NPRIC,NC(10),
     1JROW(60,2),NVAR,NPRIT,NRCON,IND(140)
      COMMON /DOMNC/ NGCON,NRAG,IOBJ,NCOLR,NCOLG,NCOL,NPHS1,IALT,
     1NRGT(60)
      COMMON /OBJDM/ W,WART,C(180),CR(180),CB(60)
      COMMON /ENTDPR/ NEVC,NDVR
C
      DO 100 NV=1,180
         C(NV)=0.0
  100 CONTINUE
C
C **** SUBROUTINE PHASE 1 IS NOT USED IF THE REAL AND GOAL CONSTRAINTS
C **** ARE OF TYPE " < " .
C
      NPHS1=0
      DO 101 NR=1,NRAG
         IF (NRGT(NR).EQ.8) GO TO 101
         NPHS1=NPHS1+1
         GO TO 102
  101 CONTINUE
```

```
      IF (NPHS1.EQ.0) RETURN
  102 CONTINUE
C
      DO 103 NV=1,NRAG
         CB(NV)=0.0
  103 CONTINUE
C
C **** SET COLUMN AND ROW HEADINGS FOR ARTIFICIAL VARIABLES
C **** SET 1. IN TED(.,.) FOR EACH ARTIFICIAL VARIABLE
C **** SET C(J)=0.0 FOR ALL DECISION VARIABLES AND C(J)=1. FOR THE
C **** ARTIFICIAL VARIABLES
C
      IAR=NCOLG
      DO 104 NR=1,NRAG
         IF (NRGT(NR).EQ.8) GO TO 104
         IAR=IAR+1
         JROW(NR,1)=1
         JROW(NR,2)=NR
         JCOL(IAR,1)=1
         JCOL(IAR,2)=NR
         TED(NR,IAR)=1.
         C(IAR)=1.
         CB(NR)=1.
  104 CONTINUE
C
      NCOL=IAR
  105 CALL CHKOP
      IF (NEVC.EQ.0) GO TO 106
      NENT=NEVC
      CALL DPRT
      NDPR=NDVR
      CALL PIVOT (NENT,NDPR)
      GO TO 105
C
  106 WART=0.0
      DO 107 NR=1,NRAG
         WART=WART+TBD(NR)*CB(NR)
  107 CONTINUE
       IF (WART.GT.0.0) RETURN
C
      DO 108 NR=1,NRAG
      DO 108 NV=1,NCOL
         IF (NV.LE.NCOLG) GO TO 108
         TED(NR,NV)=0.0
  108 CONTINUE
C
      RETURN
      END
C ****
C **********************************************************************
C ****
      SUBROUTINE PHSE2
C
C **** SUBROUTINE PHSE2 PERFORMS A PHASE 2 SIMPLEX PROCEDURE IN ORDER TO
C **** FIND AN OPTIMAL SOLUTION TO THE LP PROBLEM OF THE NONDOMINANCE
C **** TEST .
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON TT(10,140),TB(60),TE(60,140),TL(60,10),TA(10),TI(10,140),
     1TED(60,180),TBD(60),JCOL(180,2),NCOLI,NROWI,NPRIC,NC(10),
     1JROW(60,2),NVAR,NPRIT,NRCON,IND(140)
      COMMON /DOMNC/ NGCON,NRAG,IOBJ,NCOLR,NCOLG,NCOL,NPHS1,IALT,
     1NRGT(60)
      COMMON /OBJDM/ W,WART,C(180),CR(180),CB(60)
      COMMON /ENTDPR/ NEVC,NDVR
C
      NCOL=NCOLG
      DO 101 NV=1,NCOL
         C(NV)=0.0
  101 CONTINUE
      DO 102 NV=1,NRAG
         CB(NV)=0.0
```

```
  102 CONTINUE
      NG=NCOLR+1
      DO 103 NV=NG,NCOL
         KSUB=JCOL(NV,2)
         IF (JCOL(NV,1).EQ.5.AND.NRGT(KSUB).EQ.8) C(NV)=-1.
         IF (JCOL(NV,1).EQ.5.AND.NRGT(KSUB).EQ.9) C(NV)=-1.
  103 CONTINUE
C
      NRB=1
      IF (NPHS1.EQ.0) NRB=IOBJ
      DO 104 NR=NRB,NRAG
         KSUB=JROW(NR,2)
         IF (JROW(NR,1).EQ.5.AND.NRGT(KSUB).EQ.8) CB(NR)=-1.
         IF (JROW(NR,1).EQ.5.AND.NRGT(KSUB).EQ.9) CB(NR)=-1.
  104 CONTINUE
C
  105 CALL CHKOP
      IF (NEVC.EQ.0) GO TO 106
      NENT=NEVC
      CALL DPRT
C
C **** IF NDVR=0 , THE PROBLEM HAS UNBOUNDED SOLUTION .
C
      IF (NDVR.EQ.0) RETURN
      NDPR=NDVR
      CALL PIVOT (NENT,NDPR)
      GO TO 105
  106 W=0.0
      DO 107 NR=1,NRAG
         W=W+TBD(NR)*CB(NR)
  107 CONTINUE
      RETURN
      END
C ****
C ***********************************************************************
C ****
      SUBROUTINE CHKOP
C
C **** SUBROUTINE CHKOP CALCULATES RELATIVE COST COEFFICIENTS,
C **** PERFORMS A CHECK FOR OPTIMALITY AND
C **** DETERMINES THE ENTERING VARIABLE'S COLUMN .
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON TT(10,140),TB(60),TE(60,140),TL(60,10),TA(10),TI(10,140),
     1TED(60,180),TBD(60),JCOL(180,2),NCOLI,NROWI,NPRIC,NC(10),
     1JROW(60,2),NVAR,NPRIT,NRCON,IND(140)
      COMMON /DOMNC/ NGCON,NRAG,IOBJ,NCOLR,NCOLG,NCOL,NPHS1,IALT,
     1NRGT(60)
      COMMON /OBJDM/ W,WART,C(180),CR(180),CB(60)
      COMMON /ENTDPR/ NEVC,NDVR
C
C **** COMPUTE RELATIVE COST COEFFICIENTS .
C
      DO 101 NV=1,NCOL
         CR(NV)=C(NV)
      DO 101 NR=1,NRAG
         CR(NV)=CR(NV)-CB(NR)*TED(NR,NV)
  101 CONTINUE
C
C **** CHECK FOR OPTIMALITY
C
      VEVC=0.
      NEVC=0
      DO 102 NCO=1,NCOL
         NV=NCO
         IF (CR(NV).GE.0.0) GO TO 102
         IF (CR(NV).GE.VEVC) GO TO 102
         VEVC=CR(NV)
         NEVC=NV
  102 CONTINUE
      RETURN
      END
```

```
C ****
C ***********************************************************************
C ****
      SUBROUTINE DPRT
C
C **** SUBROUTINE DPRT DETERMINES DEPARTING VARIABLE'S ROW .
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON TT(10,140),TB(60),TE(60,140),TL(60,10),TA(10),TI(10,140),
     1TED(60,180),TBD(60),JCOL(180,2),NCOLI,NROWI,NPRIC,NC(10),
     1JROW(60,2),NVAR,NPRIT,NRCON,IND(140)
      COMMON /DOMNC/ NGCON,NRAG,IOBJ,NCOLR,NCOLG,NCOL,NPHS1,IALT,
     1NRGT(60)
      COMMON /ENTDPR/ NEVC,NDVR
C
      NDVR=0
      VDVR=10.0E+20
      DO 102 ND=1,NRAG
         IF (TED(ND,NEVC).LE.0.0) GO TO 102
         V=TBD(ND)/TED(ND,NEVC)
         IF (NDVR.EQ.0) GO TO 101
         IF (V-VDVR) 101,101,102
 101     VDVR=V
         NDVR=ND
 102  CONTINUE
      RETURN
      END
C ****
C ***********************************************************************
C ****
      SUBROUTINE PIVOT (NEVC,NDVR)
C
C **** SUBROUTINE PIVOT COMPUTES THE NEW TABLEAU : GIVEN A VALUE OF THE
C **** ENTERING VARIABLE'S COLUMN (NEVC) AND THE DEPARTING VARIABLE'S
C **** ROW (NDVR) .
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON TT(10,140),TB(60),TE(60,140),TL(60,10),TA(10),TI(10,140),
     1TED(60,180),TBD(60),JCOL(180,2),NCOLI,NROWI,NPRIC,NC(10),
     1JROW(60,2),NVAR,NPRIT,NRCON,IND(140)
      COMMON /DOMNC/ NGCON,NRAG,IOBJ,NCOLR,NCOLG,NCOL,NPHS1,IALT,
     1NRGT(60)
      COMMON /OBJDM/ W,WART,C(180),CR(180),CB(60)
C
      JROW(NDVR,1)=JCOL(NEVC,1)
      JROW(NDVR,2)=JCOL(NEVC,2)
      CB(NDVR)=C(NEVC)
C
      PIV=TED(NDVR,NEVC)
      PIB=TBD(NDVR)
      DO 102 NR=1,NRAG
         IF (NR.EQ.NDVR) GO TO 102
         IF (DABS(TED(NR,NEVC)).LE.0.000050) GO TO 102
         PIX=TED(NR,NEVC)/PIV
         TBD(NR)=FIX(TBD(NR)-PIX*PIB)
C
         DO 101 NV=1,NCOL
 101     TED(NR,NV)=FIX(TED(NR,NV)-TED(NDVR,NV)*PIX)
 102  CONTINUE
C
      TBD(NDVR)=FIX(PIB/PIV)
C
      DO 103 NV=1,NCOL
         TED(NDVR,NV)=FIX(TED(NDVR,NV)/PIV)
 103  CONTINUE
C
      RETURN
      END
C ****
C ***********************************************************************
C ****
      SUBROUTINE ALTOP
```

```
C
C **** SUBROUTINE ALTOP CHECKS THE LINEAR PROGRAMMING PROBLEM
C **** OF THE NONDOMINANCE TEST FOR ALTERNATE OPTIMUM SOLUTIONS.
C **** IF ONE OR MORE VALID SOLUTIONS, ALTOP GENERATES AND OUTPUTS
C **** THEMS.
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON TT(10,140),TB(60),TE(60,140),TL(60,10),TA(10),TI(10,140),
     1TED(60,180),TBD(60),JCOL(180,2),NCOLI,NROWI,NPRIC,NC(10),
     1JROW(60,2),NVAR,NPRIT,NRCON,IND(140)
      COMMON /OUTPT/ WOUT(140,4)
      COMMON /DOMNC/ NGCON,NRAG,IOBJ,NCOLR,NCOLG,NCOL,NPHS1,IALT,
     1NRGT(60)
      COMMON /OBJDM/ W,WART,C(180),CR(180),CB(60)
      COMMON /ENTDPR/ NEVC,NDVR
C
      IALT=0
      NEVC=0
      DO 104 NCR=1,NCOL
         IF (CR(NCR).GT.0.0) GO TO 104
         DO 101 NR=1,NRAG
            IF (JROW(NR,1).EQ.JCOL(NCR,1).AND.JROW(NR,2).EQ.JCOL(NCR,2))
     1         GO TO 104
  101    CONTINUE
            IALT=IALT+1
            IF (IALT.EQ.1) WRITE (6,105)
            WRITE (6,106)
            WRITE (6,107) IALT
            NEVC=NCR
            CALL DPRT
            NDPR=NDVR
            NENT=NEVC
C
C **** PIVOT IS CALLED AGAIN TO RETURN THE TABLEAU TO ITS ORIGINAL
C **** FORM FOR FURTHER ALTERNATE SOLUTION SEARCH.
C
            ND=NDVR
            DO 102 NE=1,NCOL
               IF (JROW(ND,1).EQ.JCOL(NE,1).AND.JROW(ND,2).EQ.
     1         JCOL(NE,2)) GO TO 103
  102       CONTINUE
  103       NEAG=NE
            CALL PIVOT (NENT,NDPR)
            CALL DOUT
            CALL PIVOT (NEAG,NDPR)
  104 CONTINUE
      RETURN
C
  105 FORMAT (//  93H  THE LINEAR PROGRAMMING PROBLEM OF THE NONDOMINANC
     1E TEST HAS ALTERNATE OPTIMUM SOLUTION(S) .,////120(1H*))
  106 FORMAT (1H1,//120(1H*))
  107 FORMAT (//  41H  ALTERNATE NONDOMINATED SOLUTION NUMBER ,I3)
C
      END
C ****
C *********************************************************************
C ****
      SUBROUTINE DOUT
C
C **** SUBROUTINE DOUT PREPARES AND PRINTS THE SOLUTION INFORMATION OF
C **** THE NONDOMINANCE TEST .
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON TT(10,140),TB(60),TE(60,140),TL(60,10),TA(10),TI(10,140),
     1TED(60,180),TBD(60),JCOL(180,2),NCOLI,NROWI,NPRIC,NC(10),
     1JROW(60,2),NVAR,NPRIT,NRCON,IND(140)
      COMMON /DOMNC/ NGCON,NRAG,IOBJ,NCOLR,NCOLG,NCOL,NPHS1,IALT,
     1NRGT(60)
      COMMON /OUTPT/ WOUT(140,4)
C
C **** OUTPUT ARRAY IS ZEROED
C
```

```
      DO 101 I=1,140
      DO 101 J=1,4
         WOUT(I,J)=0.0
  101 CONTINUE
C
C **** OUTPUT ARRAY IS FILED
C
      DO 104 NR=1,NRAG
         I1=JROW(NR,1)
         I2=JROW(NR,2)
         IF (I1.EQ.2) GO TO 102
         I1=I1-3
         GO TO 103
  102    I1=1
  103    WOUT(I2,I1)=FIX(TBD(NR))
  104 CONTINUE
C
C **** PRINT NONDOMINATED SOLTION
C
      WRITE (6,121)
      WRITE (6,122)
      I=MAXO(NVAR,NRAG)
C
C **** THE REAL CONSTRAINTS WILL BE TREATED AS GOALS IF THEY HAVE NO
C **** FEASIBLE SOLUTION .
C
      IF (IOBJ.EQ.1) GO TO 112
C
      DO 111 K=1,I
         IF (NRGT(K).EQ.10) GO TO 105
         IF (K.GT.NVAR) GO TO 108
         IF (K.GT.NRAG) GO TO 106
         IF (K.GT.NRCON) GO TO 110
         WRITE (6,123) K,WOUT(K,1),WOUT(K,4)
         GO TO 111
  105    IF (K.GT.NVAR) GO TO 107
  106    WRITE (6,124) K,WOUT(K,1)
         GO TO 111
  107    WRITE (6,125) K
         GO TO 111
  108    IF (K.GT.NRCON) GO TO 109
         WRITE (6,126) K,WOUT(K,4)
         GO TO 111
  109    WRITE (6,127) K,WOUT(K,2),WOUT(K,3)
         GO TO 111
  110    WRITE (6,128) K,(WOUT(K,J),J=1,3)
  111 CONTINUE
C
      GO TO 118
C
  112 DO 117 K=1,I
         IF (NRGT(K).EQ.10) GO TO 113
         IF (K.GT.NVAR) GO TO 116
         IF (K.GT.NRAG) GO TO 114
         WRITE (6,128) K,(WOUT(K,J),J=1,3)
         GO TO 117
  113    IF (K.GT.NVAR) GO TO 115
  114    WRITE (6,124) K,WOUT(K,1)
         GO TO 117
  115    WRITE (6,125) K
         GO TO 117
  116    WRITE (6,127) K,WOUT(K,2),WOUT(K,3)
  117 CONTINUE
C
  118 WRITE (6,129)
      WRITE (6,130)
      RETURN
C
  121 FORMAT (1HO,  42H OUTPUT SUMMARY OF A NONDOMINATED SOLUTION)
  122 FORMAT (1HO,  9HSUBSCRIPT,9X,14HX NONDOMINATED,9X,1HD,14X,1HG,
     114X,1HS/)
  123 FORMAT (I8,7X,F15.4,30X,F15.4)
```

```
  124 FORMAT (I8,7X,F15.4)
  125 FORMAT (I8)
  126 FORMAT (I8,52X,F15.4)
  127 FORMAT (I8,22X,2F15.4)
  128 FORMAT (I8,7X,3F15.4)
  129 FORMAT (//120(1H*)///,10X, 8H WHERE :)
  130 FORMAT (//20X,   24H  X = DECISION VARIABLES,/20X,
     1   74H  D = VARIABLES TO BE MAXIMIZED IN THE LP PROBLEM OF THE NON
     1DOMINANCE TEST,/20X,  52H  G = SLACK OR SURPLUS VARIABLES IN GOAL
     1CONSTRAINTS,/20X,  53H  S = SLACK OR SURPLUS VARIABLES IN REAL CON
     1STRAINTS ,///120(1H*)//)
C
      END
```

# APPENDIX B

# EXAMPLES OF GP DIFFICULTIES

Some GP problems of varying difficulty have been selected in order to test the correctness and efficiency of the new algorithm. These problems are:

Problem 1 (Hannan (34)):

$$\text{Min } P_1 d_1^- + P_2 d_2^- + P_3 d_3^-$$

Subject to:

$$X_2 + X_3 \leq 6$$

$$X_1 \leq 4$$

$$2X_2 + x_3 + d_1^- - d_1^+ = 10$$

$$X_1 + X_2 + X_3 + d_2^- - d_2^+ = 12$$

$$X_1 + 3X_2 + d_3^- - d_3^+ = 16$$

Where all the variables are nonnegative.

Pages 180, 181 show the computer output of Problem 1. $X_1 = 4$, $X_2 = 4$, $X_3 = 2$ is the GP solution and this solution is dominated. $X_1 = 4$, $X_2 = 6$, $X_3 = 0$ is a nondominated solution for this problem.

Problem 2. (Hannan (34)):

Another example of an unbounded solution which will also go undetected by the goal programming procedure is:

$$\text{Min } d_1^- + d_2^-$$

COMPUTER OUTPUT OF PROBLEM 1

THE OPTIMIZATION ENDED ON SUBPROBLEM 4
THERE WERE 5 CONSTRAINTS IN THE FINAL OPTIMAL TABLEAU.

**************************************************************************

THE OPTIMAL SOLUTION FOR THE DECISION VARIABLES X(J)

X( 1)= 4.0000

X( 2)= 4.0000

X( 3)= 2.0000

**************************************************************************

THE GOAL ACHIEVEMENTS ARE

| PRIORITY | GOAL NUMBER | OVER-ACHIEVEMENT | UNDER-ACHIEVEMENT |
|---|---|---|---|
| 1 | 1 | 0.0000 | 0.0000 |
| 1 | 2 | 0.0000 | 0.0000 |
| 2 | 3 | 0.0000 | 0.0000 |
| 3 | 4 | 0.0000 | 2.0000 |
| 4 | 5 | 0.0000 | 0.0000 |

**************************************************************************

THE PRIORITY ACHIEVEMENTS ARE

| PRIORITY | ACHIEVEMENT |
|---|---|
| 1 | 0.0000 |
| 2 | 0.0000 |
| 3 | 2.0000 |
| 4 | 0.0000 |

**************************************************************************

OUTPUT SUMMARY

| SUBSCRIPT | A OPT | X OPT | POS DEV | NEG DEV |
|---|---|---|---|---|
| 1 | 0.0000 | 4.0000 | 0.0000 | 0.0000 |
| 2 | 0.0000 | 4.0000 | 0.0000 | 0.0000 |
| 3 | 2.0000 | 2.0000 | 0.0000 | 0.0000 |
| 4 | 0.0000 | | 0.0000 | 2.0000 |
| 5 | | | 0.0000 | 0.0000 |

**************************************************************************

**************************************************************************************

OUTPUT SUMMARY OF THE NONDOMINANCE TEST

**************************************************************************************

THE GOAL PROGRAMMING SOLUTION IS DOMINATED .

**************************************************************************************

THE OBJECTIVE FUNCTION IN THE NONDOMINATED SOLUTION = -8.0000

**************************************************************************************

OUTPUT SUMMARY OF A NONDOMINATED SOLUTION

| SUBSCRIPT | X NONDOMINATED | D | G | S |
|---|---|---|---|---|
| 1 | 4.0000 | | | 0.0000 |
| 2 | 6.0000 | | | 0.0000 |
| 3 | 0.0000 | 2.0000 | 0.0000 | |
| 4 | | 0.0000 | 0.0000 | |
| 5 | | 6.0000 | 0.0000 | |

**************************************************************************************

WHERE :

X = DECISION VARIABLES
D = VARIABLES TO BE MAXIMIZED IN THE LP PROBLEM OF THE NONDOMINANCE TEST
G = SLACK OR SURPLUS VARIABLES IN GOAL CONSTRAINTS
S = SLACK OR SURPLUS VARIABLES IN REAL CONSTRAINTS

**************************************************************************************

THE LINEAR PROGRAMMING PROBLEM OF THE NONDOMINANCE TEST HAS NO ALTERNATE OPTIMUM SOLUTION .

**************************************************************************************

Subject to:

$$X_2 - X_3 \leq 6$$

$$X_1 \leq 4$$

$$2X_2 + X_3 + d_1^- - d_1^+ = 12$$

$$X_1 + X_2 + X_3 + d_2^- - d_2^+ = 10$$

Where all the variables are nonnegative.

Assign priority $P_1$ to the real constraints and perform the necessary changes in the subscripts of d's. The problem can be written as:

$$\text{Min } \bar{a} = \{ (d_1^+ + d_2^+),\ d_3^-,\ d_4^- \}$$

Real Constraints:

$$X_2 - X_3 + d_1^- - d_1^+ = 6$$

$$X_1 + d_2^- - d_2^+ = 4$$

Goal Constraints:

$$2X_2 + X_3 + d_3^- - d_3^+ = 12$$

$$X_1 + X_2 + X_3 + d_4^- - d_4^+ = 10$$

Where all the variables are nonnegative.

The computer output of Problem 2 is shown on pages 183, 184. $X_1 = 4$, $X_2 = 6$, $X_3 = 0$ is the GP solution. The nondominance algorithm indicates that the GP problem has unbounded solutions as shown in the computer output.

COMPUTER OUTPUT OF PROBLEM 2

THE OPTIMIZATION ENDED ON SUBPROBLEM 3
THERE WERE 4 CONSTRAINTS IN THE FINAL OPTIMAL TABLEAU.

*************************************************************************

THE OPTIMAL SOLUTION FOR THE DECISION VARIABLES X(J)

X( 1)= 4.0000

X( 2)= 6.0000

X( 3)= 0.0000

*************************************************************************

THE GOAL ACHIEVEMENTS ARE

| PRIORITY | GOAL NUMBER | OVER-ACHIEVEMENT | UNDER-ACHIEVEMENT |
|---|---|---|---|
| 1 | 1 | 0.0000 | 0.0000 |
| 1 | 2 | 0.0000 | 0.0000 |
| 2 | 3 | 0.0000 | 0.0000 |
| 3 | 4 | 0.0000 | 0.0000 |

*************************************************************************

THE PRIORITY ACHIEVEMENTS ARE

| PRIORITY | ACHIEVEMENT |
|---|---|
| 1 | 0.0000 |
| 2 | 0.0000 |
| 3 | 0.0000 |

*************************************************************************

OUTPUT SUMMARY

| SUBSCRIPT | A OPT | X OPT | POS DEV | NEG DEV |
|---|---|---|---|---|
| 1 | 0.0000 | 4.0000 | 0.0000 | 0.0000 |
| 2 | 0.0000 | 6.0000 | 0.0000 | 0.0000 |
| 3 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | | | 0.0000 | 0.0000 |

*************************************************************************

*********************************************************************

OUTPUT SUMMARY OF THE NONDOMINANCE TEST

*********************************************************************

THE ORIGINAL PROBLEM HAS UNBOUNDED SOLUTION AND THE GOAL PROGRAMMING SOLUTION IS CERTAINLY SUBOPTIMAL SOLUTION .

*********************************************************************

Problem 3. (Ignizio (43)):

$$\text{Min} \quad \bar{a} = \{ (d_1^+ + d_2^+ + d_3^+ + d_4^+),\ d_5^-,\ d_6^- \}$$

Real Constraints:

$$-X_1 + 3X_2 + d_1^- - d_1^+ = 21$$

$$X_1 + 3X_2 + d_2^- - d_2^+ = 27$$

$$4X_1 + 3X_2 + d_3^- - d_3^+ = 45$$

$$3X_1 + X_2 + d_4^- - d_4^+ = 30$$

Goal Constraints:

$$2X_1 + X_2 + d_5^- - d_5^+ = 40$$

$$-X_1 + 2X_2 + d_6^- - d_6^+ = 20$$

Where all the variables are nonnegative.

Page 186 shows the computer output of Problem 3 which demonstrates that the second goal has no impact on the solution as a result of setting a high value for the aspiration level of the first goal, and the GP problem may be reduced to a LP problem.

Problem 4 (Zanakis (91)):

This problem has been discussed in Chapter VI and the resultant GP formulation is:

$$\text{Min} \quad \bar{a} = \{ d_1^- + d_2^- + d_3^+,\ d_4^- + d_5^- + d_6^+,$$

$$d_7^- + d_7^+,\ d_8^+,\ d_9^+,$$

$$d_{10}^+ + d_{11}^+ + d_{12}^+ + d_{13}^- + d_{14}^+ + d_{15}^+ \}$$

COMPUTER OUTPUT OF PROBLEM 3

THE OPTIMIZATION ENDED ON SUBPROBLEM 2
THERE WERE 5 CONSTRAINTS IN THE FINAL OPTIMAL TABLEAU.

******************************************************************************

THE OPTIMAL SOLUTION FOR THE DECISION VARIABLES X(J)

X( 1)= 9.0000

X( 2)= 3.0000

******************************************************************************

THE GOAL ACHIEVEMENTS ARE

| PRIORITY | GOAL NUMBER | OVER-ACHIEVEMENT | UNDER-ACHIEVEMENT |
|---|---|---|---|
| 1 | 1 | 0.0000 | 21.0000 |
| 1 | 2 | 0.0000 | 9.0000 |
| 1 | 3 | 0.0000 | 0.0000 |
| 1 | 4 | 0.0000 | 0.0000 |
| 2 | 5 | 0.0000 | 19.0000 |
| 3 | 6 | 0.0000 | 23.0000 |

******************************************************************************

THE PRIORITY ACHIEVEMENTS ARE

| PRIORITY | ACHIEVEMENT |
|---|---|
| 1 | 0.0000 |
| 2 | 19.0000 |
| 3 | 23.0000 |

******************************************************************************

OUTPUT SUMMARY

| SUBSCRIPT | A OPT | X OPT | POS DEV | NEG DEV |
|---|---|---|---|---|
| 1 | 0.0000 | 9.0000 | 0.0000 | 21.0000 |
| 2 | 19.0000 | 3.0000 | 0.0000 | 9.0000 |
| 3 | 23.0000 | | 0.0000 | 0.0000 |
| 4 | | | 0.0000 | 0.0000 |
| 5 | | | 0.0000 | 19.0000 |
| 6 | | | 0.0000 | 23.0000 |

******************************************************************************

THE ABOVE GOAL PROGRAMMING SOLUTION IS NONDOMINATED.

******************************************************************************

Goal constraints:

$$X_1 + d_1^- - d_1^+ = 40$$

$$X_5 + d_2^- - d_2^+ = 100$$

$$-X_1 - X_2 - X_3 - X_4 + X_5$$

$$+ d_3^- - d_3^+ = 1063$$

$$X_2 + d_4^- - d_4^+ = 5$$

$$X_3 + d_5^- - d_5^+ = 20$$

$$X_4 + d_6^- - d_6^+ = 30$$

$$X_1 + X_2 + X_3 + X_4 + 2X_5/3$$

$$+ d_7^- - d_7^+ = 787$$

$$13.358X_1 + 14.846X_2 + 18.073X_3$$

$$+ 7.024X_4 + 26X_5 + d_8^- - d_8^+ = 0$$

$$X_1 + X_2 + X_3 + X_4$$

$$+ d_9^- - d_9^+ = 219$$

$$0.0048X_1 + 0.0513X_2 - 0.1659X_3$$

$$+ d_{10}^- - d_{10}^+ = 32.273$$

$$- 0.0048X_1 - 0.0513X_2 + 0.0711X_3$$

$$+ d_{11}^- - d_{11}^+ = 28.789$$

$$0.9568X_1 + 0.5383X_2 + 0.9670X_3$$

$$+ X_4 + d_{12}^- - d_{12}^+ = 875.715$$

$$0.9712X_1 + 0.6922X_2 - 0.3271X_3$$

$$+ X_4 + d_{13}^- - d_{13}^+ = 239.78$$

$$0.7627X_1 - o.0512X_2 - 0.4834X_3$$

$$+ 0.9643X_4 + d_{14}^- - d_{14}^+ = 338.926$$

$$- 0.8402 - 0.2821X_2 + 0.0758X_3$$

$$- 0.9762X_4 + d_{15}^- - d_{15}^+ = 47.411$$

Where all the variables are nonnegative.

Pages 189, 190 show the computer output of Problem 4 when all the goals are included, and page 191 shows the computer output when the goals of $P_5$ and $P_6$ are eliminated. The results demonstrate that if one or more of the higher priority goals has a lower aspiration level, some goals of the lower priorities may be eliminated from the GP model without changing the solution results. For instance, goals of $P_5$ and $P_6$ are eliminated and the solution is the same as shown in the computer outputs.

COMPUTER OUTPUT OF PROBLEM 4

( ALL THE GOALS ARE INCLUDED )

THE OPTIMIZATION ENDED ON SUBPROBLEM 4
THERE WERE 8 CONSTRAINTS IN THE FINAL OPTIMAL TABLEAU.

****************************************************************************

THE OPTIMAL SOLUTION FOR THE DECISION VARIABLES X(J)

X( 1)= 665.3330

X( 2)= 5.0000

X( 3)= 20.0000

X( 4)= 30.0000

X( 5)= 100.0000

****************************************************************************

THE GOAL ACHIEVEMENTS ARE

| PRIORITY | GOAL NUMBER | OVER-ACHIEVEMENT | UNDER-ACHIEVEMENT |
|---|---|---|---|
| 1 | 1 | 625.3330 | 0.0000 |
| 1 | 2 | 0.0000 | 0.0000 |
| 1 | 3 | 0.0000 | 1683.3330 |
| 2 | 4 | 0.0000 | 0.0000 |
| 2 | 5 | 0.0000 | 0.0000 |
| 2 | 6 | 0.0000 | 0.0000 |
| 3 | 7 | 0.0000 | 0.0000 |
| 4 | 8 | 12133.9282 | 0.0000 |
| 5 | 9 | 501.3330 | 0.0000 |
| 6 | 10 | 0.0000 | 0.0000 |
| 6 | 11 | 0.0000 | 0.0000 |
| 6 | 12 | 0.0000 | 0.0000 |
| 6 | 13 | 0.0000 | 0.0000 |
| 6 | 14 | 187.5285 | 0.0000 |
| 6 | 15 | 0.0000 | 0.0000 |

****************************************************************************

THE PRIORITY ACHIEVEMENTS ARE

| PRIORITY | ACHIEVEMENT |
|---|---|
| 1 | 0.0000 |
| 2 | 0.0000 |
| 3 | 0.0000 |
| 4 | 12133.9282 |
| 5 | 501.3330 |
| 6 | 187.5285 |

****************************************************************************

OUTPUT SUMMARY

| SUBSCRIPT | A OPT | X OPT | POS DEV | NEG DEV |
|---|---|---|---|---|
| 1 | 0.0000 | 665.3330 | 625.3330 | 0.0000 |

| | | | | |
|---|---|---|---|---|
| 2 | 0.0000 | 5.0000 | 0.0000 | 0.0000 |
| 3 | 0.0000 | 20.0000 | 0.0000 | 1683.3330 |
| 4 | 12133.9282 | 30.0000 | 0.0000 | 0.0000 |
| 5 | 501.3330 | 100.0000 | 0.0000 | 0.0000 |
| 6 | 187.5285 | | 0.0000 | 0.0000 |
| 7 | | | 0.0000 | 0.0000 |
| 8 | | | 12133.9282 | 0.0000 |
| 9 | | | 501.3330 | 0.0000 |
| 10 | | | 0.0000 | 0.0000 |
| 11 | | | 0.0000 | 0.0000 |
| 12 | | | 0.0000 | 0.0000 |
| 13 | | | 0.0000 | 0.0000 |
| 14 | | | 187.5285 | 0.0000 |
| 15 | | | 0.0000 | 0.0000 |

****************************************************************************

THE ABOVE GOAL PROGRAMMING SOLUTION IS NONDOMINATED.

****************************************************************************

COMPUTER OUTPUT OF PROBLEM 4
( ALL THE GOALS ARE NOT INCLUDED )

THE OPTIMIZATION ENDED ON SUBPROBLEM 4
THERE WERE 8 CONSTRAINTS IN THE FINAL OPTIMAL TABLEAU.

****************************************************************************

THE OPTIMAL SOLUTION FOR THE DECISION VARIABLES X(J)

X( 1)= 665.3330

X( 2)= 5.0000

X( 3)= 20.0000

X( 4)= 30.0000

X( 5)= 100.0000

****************************************************************************

THE GOAL ACHIEVEMENTS ARE

| PRIORITY | GOAL NUMBER | OVER-ACHIEVEMENT | UNDER-ACHIEVEMENT |
|---|---|---|---|
| 1 | 1 | 625.3330 | 0.0000 |
| 1 | 2 | 0.0000 | 0.0000 |
| 1 | 3 | 0.0000 | 1683.3330 |
| 2 | 4 | 0.0000 | 0.0000 |
| 2 | 5 | 0.0000 | 0.0000 |
| 2 | 6 | 0.0000 | 0.0000 |
| 3 | 7 | 0.0000 | 0.0000 |
| 4 | 8 | 12133.9282 | 0.0000 |

****************************************************************************

THE PRIORITY ACHIEVEMENTS ARE

| PRIORITY | ACHIEVEMENT |
|---|---|
| 1 | 0.0000 |
| 2 | 0.0000 |
| 3 | 0.0000 |
| 4 | 12133.9282 |

****************************************************************************

OUTPUT SUMMARY

| SUBSCRIPT | A OPT | X OPT | POS DEV | NEG DEV |
|---|---|---|---|---|
| 1 | 0.0000 | 665.3330 | 625.3330 | 0.0000 |
| 2 | 0.0000 | 5.0000 | 0.0000 | 0.0000 |
| 3 | 0.0000 | 20.0000 | 0.0000 | 1683.3330 |
| 4 | 12133.9282 | 30.0000 | 0.0000 | 0.0000 |
| 5 | | 100.0000 | 0.0000 | 0.0000 |
| 6 | | | 0.0000 | 0.0000 |
| 7 | | | 0.0000 | 0.0000 |
| 8 | | | 12133.9282 | 0.0000 |

****************************************************************************

THE ABOVE GOAL PROGRAMMING SOLUTION IS NONDOMINATED .

APPENDIX C

COMPUTER OUTPUT OF THE

NUMERICAL EXAMPLE

(RUN 5)

THE OPTIMIZATION ENDED ON SUBPROBLEM 5
THERE WERE 40 CONSTRAINTS IN THE FINAL OPTIMAL TABLEAU.

************************************************************

THE OPTIMAL SOLUTION FOR THE DECISION VARIABLES X(J)

| | |
|---|---|
| X( 1)= | 10.9039 |
| X( 2)= | 279.9687 |
| X( 3)= | 2.1808 |
| X( 4)= | 270.5083 |
| X( 5)= | 5.7156 |
| X( 6)= | 217.9106 |
| X( 7)= | 1.1431 |
| X( 8)= | 178.3294 |
| X( 9)= | 0.9039 |
| X( 10)= | 84.9687 |
| X( 11)= | 0.0000 |
| X( 12)= | 38.9006 |
| X( 13)= | 5.2285 |
| X( 14)= | 0.0000 |
| X( 15)= | 0.0000 |
| X( 16)= | 0.0000 |
| X( 17)= | 0.0000 |
| X( 18)= | 0.0000 |
| X( 19)= | 0.0000 |
| X( 20)= | 0.0000 |
| X( 21)= | 0.0000 |
| X( 22)= | 0.0000 |
| X( 23)= | 0.0000 |
| X( 24)= | 0.0000 |
| X( 25)= | 2615.5768 |
| X( 26)= | 3332.0676 |
| X( 27)= | 4252.3556 |

```
X( 28)=        0.0000
X( 29)=    12615.5768
X( 30)=    12216.4908
X( 31)=     9920.2880
X( 32)=     8047.6444
X( 33)=     3998.4380
X( 34)=     4057.6250
X( 35)=     3268.9983
X( 36)=     2674.9386
X( 37)=        0.0000
X( 38)=        0.0000
X( 39)=        0.0000
X( 40)=        0.0000
X( 41)=     3998.4380
X( 42)=     4057.6250
X( 43)=     3268.9983
X( 44)=     2674.9406
X( 45)=      201.0920
X( 46)=        0.0000
X( 47)=        0.0000
X( 48)=        0.0000
X( 49)=        0.0000
X( 50)=        0.0000
X( 51)=        0.0000
X( 52)=        0.0000
```

**********************************************************************

THE GOAL ACHIEVEMENTS ARE

| PRIORITY | GOAL NUMBER | OVER-ACHIEVEMENT | UNDER-ACHIEVEMENT |
|---|---|---|---|
| 1 | 1 | 0.0000 | 0.0000 |
| 1 | 2 | 0.0000 | 0.0000 |
| 1 | 3 | 0.0000 | 0.0000 |
| 1 | 4 | 0.0000 | 0.0000 |
| 1 | 5 | 0.0000 | 0.0000 |
| 1 | 6 | 0.0000 | 0.0000 |
| 1 | 7 | 0.0000 | 0.0000 |
| 1 | 8 | 0.0000 | 0.0000 |
| 1 | 9 | 0.0000 | 59.1274 |

| | | | |
|---|---|---|---|
| 1 | 10 | 0.0000 | 77.3109 |
| 1 | 11 | 0.0000 | 126.3738 |
| 1 | 12 | 0.0000 | 170.5288 |
| 1 | 13 | 0.0000 | 37.3043 |
| 1 | 14 | 0.0000 | 58.3183 |
| 1 | 15 | 0.0000 | 98.4506 |
| 1 | 16 | 0.0000 | 128.8359 |
| 1 | 17 | 0.0000 | 0.0000 |
| 1 | 18 | 0.0000 | 0.0000 |
| 1 | 19 | 0.0000 | 0.0000 |
| 1 | 20 | 0.0000 | 0.0000 |
| 1 | 21 | 0.0000 | 0.0000 |
| 1 | 22 | 0.0000 | 0.0000 |
| 1 | 23 | 0.0000 | 0.0000 |
| 1 | 24 | 0.0000 | 0.0000 |
| 1 | 25 | 0.0000 | 0.0000 |
| 1 | 26 | 0.0000 | 0.0000 |
| 1 | 27 | 0.0000 | 0.0000 |
| 1 | 28 | 0.0000 | 0.0000 |
| 1 | 29 | 0.0000 | 0.0000 |
| 1 | 30 | 0.0000 | 0.0000 |
| 1 | 31 | 0.0000 | 0.0000 |
| 1 | 32 | 0.0000 | 0.0000 |
| 1 | 33 | 0.0000 | 310.1314 |
| 1 | 34 | 0.0000 | 21.8079 |
| 1 | 35 | 0.0000 | 56.6466 |
| 1 | 36 | 0.0000 | 11.4314 |
| 2 | 37 | 0.0000 | 0.0000 |
| 3 | 38 | 0.0000 | 0.0000 |
| 4 | 39 | 0.0000 | 0.0000 |
| 5 | 40 | 0.0000 | 0.0000 |

******************************************************************

THE PRIORITY ACHIEVEMENTS ARE

| PRIORITY | ACHIEVEMENT |
|---|---|
| 1 | 0.0000 |
| 2 | 0.0000 |
| 3 | 0.0000 |
| 4 | 0.0000 |
| 5 | 0.0000 |

************************************************************************

OUTPUT SUMMARY

| SUBSCRIPT | A OPT | X OPT | POS DEV | NEG DEV |
|---|---|---|---|---|
| 1 | 0.0000 | 10.9039 | 0.0000 | 0.0000 |
| 2 | 0.0000 | 279.9687 | 0.0000 | 0.0000 |
| 3 | 0.0000 | 2.1808 | 0.0000 | 0.0000 |
| 4 | 0.0000 | 270.5083 | 0.0000 | 0.0000 |
| 5 | 0.0000 | 5.7156 | 0.0000 | 0.0000 |
| 6 | | 217.9106 | 0.0000 | 0.0000 |
| 7 | | 1.1431 | 0.0000 | 0.0000 |
| 8 | | 178.3294 | 0.0000 | 0.0000 |
| 9 | | 0.9039 | 0.0000 | 59.1274 |
| 10 | | 84.9687 | 0.0000 | 77.3109 |
| 11 | | 0.0000 | 0.0000 | 126.3738 |
| 12 | | 38.9006 | 0.0000 | 170.5288 |
| 13 | | 5.2285 | 0.0000 | 37.3043 |
| 14 | | 0.0000 | 0.0000 | 58.3183 |
| 15 | | 0.0000 | 0.0000 | 98.4506 |

| | | | |
|---|---|---|---|
| 16 | 0.0000 | 0.0000 | 128.8359 |
| 17 | 0.0000 | 0.0000 | 0.0000 |
| 18 | 0.0000 | 0.0000 | 0.0000 |
| 19 | 0.0000 | 0.0000 | 0.0000 |
| 20 | 0.0000 | 0.0000 | 0.0000 |
| 21 | 0.0000 | 0.0000 | 0.0000 |
| 22 | 0.0000 | 0.0000 | 0.0000 |
| 23 | 0.0000 | 0.0000 | 0.0000 |
| 24 | 0.0000 | 0.0000 | 0.0000 |
| 25 | 2615.5768 | 0.0000 | 0.0000 |
| 26 | 3332.0676 | 0.0000 | 0.0000 |
| 27 | 4252.3556 | 0.0000 | 0.0000 |
| 28 | 0.0000 | 0.0000 | 0.0000 |
| 29 | 12615.5768 | 0.0000 | 0.0000 |
| 30 | 12216.4908 | 0.0000 | 0.0000 |
| 31 | 9920.2880 | 0.0000 | 0.0000 |
| 32 | 8047.6444 | 0.0000 | 0.0000 |
| 33 | 3998.4380 | 0.0000 | 310.1314 |
| 34 | 4057.6250 | 0.0000 | 21.8079 |
| 35 | 3268.9983 | 0.0000 | 56.6466 |
| 36 | 2674.9386 | 0.0000 | 11.4314 |
| 37 | 0.0000 | 0.0000 | 0.0000 |
| 38 | 0.0000 | 0.0000 | 0.0000 |
| 39 | 0.0000 | 0.0000 | 0.0000 |
| 40 | 0.0000 | 0.0000 | 0.0000 |
| 41 | 3998.4380 | | |
| 42 | 4057.6250 | | |
| 43 | 3268.9983 | | |
| 44 | 2674.9406 | | |
| 45 | 201.0920 | | |
| 46 | 0.0000 | | |
| 47 | 0.0000 | | |
| 48 | 0.0000 | | |
| 49 | 0.0000 | | |
| 50 | 0.0000 | | |
| 51 | 0.0000 | | |
| 52 | 0.0000 | | |

****************************************************************************

**************************************************************************

OUTPUT SUMMARY OF THE NONDOMINANCE TEST

**************************************************************************

THE GOAL PROGRAMMING SOLUTION IS DOMINATED .

**************************************************************************

THE OBJECTIVE FUNCTION IN THE NONDOMINATED SOLUTION = 2937.9943

**************************************************************************

OUTPUT SUMMARY OF A NONDOMINATED SOLUTION

| SUBSCRIPT | X NONDOMINATED | D | G | S |
|---|---|---|---|---|
| 1 | 10.0000 | | | |
| 2 | 287.8756 | | | |
| 3 | 2.0000 | | | |
| 4 | 274.4194 | | | |
| 5 | 0.4000 | | | |
| 6 | 220.8110 | | | |
| 7 | 0.1180 | | | |
| 8 | 177.0330 | | | |
| 9 | 0.0000 | | | 52.1244 |
| 10 | 92.8756 | | | 73.5734 |
| 11 | 0.0000 | | | 128.6587 |
| 12 | 37.0700 | | | 172.8489 |
| 13 | 0.0000 | | | 31.1088 |
| 14 | 0.0000 | | | 56.1368 |
| 15 | 0.0000 | | | 100.6246 |
| 16 | 0.0000 | | | 130.4341 |
| 17 | 0.0000 | | | |
| 18 | 0.0000 | | | |
| 19 | 0.0000 | | | |
| 20 | 0.0000 | | | |
| 21 | 0.0000 | | | |
| 22 | 0.0000 | | | |
| 23 | 0.0000 | | | |
| 24 | 0.0000 | | | |
| 25 | 2490.4149 | | | |
| 26 | 3379.6132 | | | |
| 27 | 4329.9719 | | | |
| 28 | 0.0000 | | | |
| 29 | 12490.4149 | | | |
| 30 | 12389.1984 | | | |
| 31 | 9950.3587 | | | |
| 32 | 7970.0281 | | | |
| 33 | 3654.1465 | | | 763.9877 |
| 34 | 4116.3995 | | | 20.0000 |
| 35 | 3314.1196 | | | 4.0000 |

| | | | | |
|---|---|---|---|---|
| 36 | 2656.6760 | | | 0.0000 |
| 37 | 0.0000 | 0.0000 | 0.0000 | |
| 38 | 0.0000 | 2679.3358 | 0.0000 | |
| 39 | 0.0000 | 0.0000 | 0.0000 | |
| 40 | 1.1804 | 258.6585 | 0.0000 | |
| 41 | 3654.1465 | | | |
| 42 | 4116.3995 | | | |
| 43 | 3314.1196 | | | |
| 44 | 2655.4956 | | | |
| 45 | 663.9877 | | | |
| 46 | 0.0000 | | | |
| 47 | 0.0000 | | | |
| 48 | 0.0000 | | | |
| 49 | 0.0000 | | | |
| 50 | 0.0000 | | | |
| 51 | 0.0000 | | | |
| 52 | 0.0000 | | | |

*****************************************************************************************

WHERE :

X = DECISION VARIABLES
D = VARIABLES TO BE MAXIMIZED IN THE LP PROBLEM OF THE NONDOMINANCE TEST
G = SLACK OR SURPLUS VARIABLES IN GOAL CONSTRAINTS
S = SLACK OR SURPLUS VARIABLES IN REAL CONSTRAINTS

*****************************************************************************************

THE LINEAR PROGRAMMING PROBLEM OF THE NONDOMINANCE TEST HAS NO ALTERNATE OPTIMUM •

*****************************************************************************************

# APPENDIX D

## FIXING ROUND-OFF ERROR FUNCTIONS

```
C ****               FUNCTION 1
C
C **** FUNCTION 1 BRINGS FLOATING POINT VALUES THAT ARE
C **** EITHER + OR - 0.0001 FROM AN INTEGER TO THAT INTEGER .
C
      FUNCTION FIX(Z)
      IMPLICIT REAL*8(A-H,O-Z)
C
      X=1.
      DO 101 N=1,3
         IF (N.NE.1) X=10.*X
         F=X*Z
         I=F
         J=I-2
      DO 101 K=1,3
         G=J+K
         IF (ABS(F-G)-.005) 102,102,101
  101 CONTINUE
      FIX=Z
      RETURN
  102 FIX=G/X
      RETURN
      END
C
C ****               FUNCTION 2
C
C **** FUNCTION 2 BRINGS FLOATING POINT VALUES THAT ARE
C **** EITHER + OR - 0.000001 FROM AN INTEGER TO THAT INTEGER .
C
      FUNCTION FIX(Z)
      IMPLICIT REAL*8(A-H,O-Z)
C
      A=1.
      DO 101 N=1,5
         IF (N.NE.1) A=10.*A
         F=A*Z
         I=F
         J=I-2
      DO 101 K=1,3
         G=J+K
         IF (ABS(F-G)-0.00005) 102,102,101
  101 CONTINUE
      FIX=Z
      RETURN
  102 FIX=G/A
      RETURN
      END
C
C ****               FUNCTION 3
C
C **** FUNCTION 3 DELETES FLOATING POINT VALUES WHOSE ABSOLUTE
C**** VALUES ARE LESS THAN OR EQUAL TO 0.001 .
C
      DOUBLE PRECISION FUNCTION FIX(Z)
      IMPLICIT REAL*8(A-H,O-Z)
      FIX=DINT(Z+DSIGN(.5D+0,Z))
      IF (DABS(FIX-Z).GT. 1.D-3) FIX=Z
      RETURN
      END
C
C ****               FUNCTION 4
C
C **** FUNCTION 4 DELETES FLOATING POINT VALUES WHOSE ABSOLUTE
C**** VALUES ARE LESS THAN OR EQUAL TO 0.0001 .
C
      DOUBLE PRECISION FUNCTION FIX(Z)
      IMPLICIT REAL*8(A-H,O-Z)
      FIX=DINT(Z+DSIGN(.5D+0,Z))
      IF (DABS(FIX-Z).GT. 1.D-4) FIX=Z
      RETURN
      END
```

# APPENDIX E

# COMPUTER OUTPUTS FOR VERIFICATION OF RESULTS

MPSX OUTPUT OF THE LP PROBLEM

```
MPSX/370 R1.6 PTF9    MPSCL EXECUTION

NAME          FILE1
ROWS
 N  OBJ
 E  CON1
 E  CON2
 E  CON3
 E  CON4
 E  CON5
 E  CON6
 E  CON7
 E  CON8
 L  CON9
 L  CON10
 L  CON11
 L  CON12
 L  CON13
 L  CON14
 L  CON15
 L  CON16
 E  CON17
 E  CON18
 E  CON19
 E  CON20
 E  CON21
 E  CON22
 E  CON23
 E  CON24
 E  CON25
 E  CON26
 E  CON27
 E  CON28
 L  CON29
 L  CON30
 L  CON31
 L  CON32
COLUMNS
    X1        OBJ          400.00000   CON1           1.00000
    X1        CON3     -      .20000   CON4     -      .70000
    X1        CON9           1.00000   CON13           .60000
    X1        CON21         20.00000   CON29    -    30.00000
    X2        OBJ          450.00000   CON2           1.00000
    X2        CON4     -      .80000   CON9           1.00000
    X2        CON13           .67500   CON21         30.00000
    X2        CON25         15.00000   CON29    -    45.00000
    X3        OBJ          400.00000   CON3           1.00000
    X3        CON5     -      .20000   CON6     -      .70000
    X3        CON10          1.00000   CON14           .60000
    X3        CON22         20.00000   CON30    -    30.00000
    X4        OBJ          450.00000   CON4           1.00000
    X4        CON6     -      .80000   CON10          1.00000
    X4        CON14           .67500   CON22         30.00000
    X4        CON26         15.00000   CON30    -    45.00000
    X5        OBJ          400.00000   CON5           1.00000
    X5        CON7     -      .20000   CON8     -      .70000
```

MPSX/370 R1.6 PTF9    MPSCL EXECUTION

| | | | | | | |
|---|---|---|---|---|---|---|
| X5 | CON11 | | 1.00000 | CON15 | | .60000 |
| X5 | CON23 | | 20.00000 | CON31 | - | 30.00000 |
| X6 | OBJ | | 450.00000 | CON6 | | 1.00000 |
| X6 | CON8 | - | .80000 | CON11 | | 1.00000 |
| X6 | CON15 | | .67500 | CON23 | | 30.00000 |
| X6 | CON27 | | 15.00000 | CON31 | - | 45.00000 |
| X7 | OBJ | | 400.00000 | CON7 | | 1.00000 |
| X7 | CON12 | | 1.00000 | CON16 | | .60000 |
| X7 | CON24 | | 20.00000 | CON32 | - | 30.00000 |
| X8 | OBJ | | 450.00000 | CON8 | | 1.00000 |
| X8 | CON12 | | 1.00000 | CON16 | | .67500 |
| X8 | CON24 | | 30.00000 | CON28 | | 15.00000 |
| X8 | CON32 | - | 45.00000 | | | |
| X9 | OBJ | | 200.00000 | CON1 | - | 1.00000 |
| X9 | CON13 | | .20000 | | | |
| X10 | OBJ | | 200.00000 | CON2 | - | 1.00000 |
| X10 | CON13 | | .20000 | | | |
| X11 | OBJ | | 200.00000 | CON3 | - | 1.00000 |
| X11 | CON14 | | .20000 | | | |
| X12 | OBJ | | 200.00000 | CON4 | - | 1.00000 |
| X12 | CON14 | | .20000 | | | |
| X13 | OBJ | | 200.00000 | CON5 | - | 1.00000 |
| X13 | CON15 | | .20000 | | | |
| X14 | OBJ | | 200.00000 | CON6 | - | 1.00000 |
| X14 | CON15 | | .20000 | | | |
| X15 | OBJ | | 200.00000 | CON7 | - | 1.00000 |
| X15 | CON16 | | .20000 | | | |
| X16 | OBJ | | 200.00000 | CON8 | - | 1.00000 |
| X16 | CON16 | | .20000 | | | |
| X17 | OBJ | | 100.00000 | CON1 | | 1.00000 |
| X17 | CON13 | | .10000 | | | |
| X18 | OBJ | | 100.00000 | CON2 | | 1.00000 |
| X18 | CON13 | | .10000 | | | |
| X19 | OBJ | | 100.00000 | CON3 | | 1.00000 |
| X19 | CON14 | | .10000 | | | |
| X20 | OBJ | | 100.00000 | CON4 | | 1.00000 |
| X20 | CON14 | | .10000 | | | |
| X21 | OBJ | | 100.00000 | CON5 | | 1.00000 |
| X21 | CON15 | | .10000 | | | |
| X22 | OBJ | | 100.00000 | CON6 | | 1.00000 |
| X22 | CON15 | | .10000 | | | |
| X23 | OBJ | | 100.00000 | CON7 | | 1.00000 |
| X23 | CON16 | | .10000 | | | |
| X24 | OBJ | | 100.00000 | CON8 | | 1.00000 |
| X24 | CON16 | | .10000 | | | |
| X25 | OBJ | | 1.00000 | CON17 | - | 1.00000 |
| X25 | CON18 | | 1.00000 | | | |
| X26 | OBJ | | 1.00000 | CON18 | - | 1.00000 |
| X26 | CON19 | | 1.00000 | | | |
| X27 | OBJ | | 1.00000 | CON19 | - | 1.00000 |
| X27 | CON20 | | 1.00000 | | | |
| X28 | OBJ | | .50000 | CON20 | - | 1.00000 |
| X29 | CON17 | | 1.00000 | CON21 | - | 1.00000 |
| X29 | CON29 | | 1.00000 | | | |

```
MPSX/370 R1.6 PTF9    MPSCL EXECUTION

     X30        CON18             1.00000   CON22      -      1.00000
     X30        CON30             1.00000
     X31        CON19             1.00000   CON23      -      1.00000
     X31        CON31             1.00000
     X32        CON20             1.00000   CON24      -      1.00000
     X32        CON32             1.00000
     X33        OBJ              30.00000   CON21             1.00000
     X34        OBJ              30.00000   CON22             1.00000
     X35        OBJ              30.00000   CON23             1.00000
     X36        OBJ              30.00000   CON24             1.00000
     X37        OBJ              22.50000   CON21             1.00000
     X37        CON25       -     1.00000
     X38        OBJ              22.50000   CON22             1.00000
     X38        CON26       -     1.00000
     X39        OBJ              22.50000   CON23             1.00000
     X39        CON27       -     1.00000
     X40        OBJ              22.50000   CON24             1.00000
     X40        CON28       -     1.00000
     X41        CON25       -     1.00000
     X42        CON26       -     1.00000
     X43        CON27       -     1.00000
     X44        CON28       -     1.00000
     X45        CON21       -     1.00000
     X46        CON22       -     1.00000
     X47        CON23       -     1.00000
     X48        CON24       -     1.00000
RHS
     RHS1       CON1             10.00000   CON2            195.00000
     RHS1       CON9            350.00000   CON10           350.00000
     RHS1       CON11           350.00000   CON12           350.00000
     RHS1       CON13           250.00000   CON14           250.00000
     RHS1       CON15           250.00000   CON16           250.00000
     RHS1       CON17         10000.00000   CON18         11500.00000
     RHS1       CON19          9000.00000   CON20         12300.00000
ENDATA
```

MPSX/370 R1.6 PTF9 MPSCL EXECUTION

SECTION 1 - ROWS

| | NUMBER | ...ROW.. | AT | ...ACTIVITY... | SLACK ACTIVITY | ..LOWER LIMIT. | ..UPPER LIMIT. | .DUAL ACTIVITY |
|---|---|---|---|---|---|---|---|---|
| | 1 | OBJ | BS | 726161.81109 | 726161.81109- | NONE | NONE | 1.00000 |
| | 2 | CON1 | EQ | 10.00000 | . | 10.00000 | 10.00000 | 87.13530 |
| | 3 | CON2 | EQ | 195.00000 | . | 195.00000 | 195.00000 | 251.23751 |
| | 4 | CON3 | EQ | . | . | . | . | 82.89619 |
| | 5 | CON4 | EQ | . | . | . | . | 248.95515 |
| | 6 | CON5 | EQ | . | . | . | . | 71.15102 |
| | 7 | CON6 | EQ | . | . | . | . | 236.47347 |
| | 8 | CON7 | EQ | . | . | . | . | 32.85714 |
| | 9 | CON8 | EQ | . | . | . | . | 205.71429 |
| | 10 | CON9 | BS | 333.42857 | 16.57143 | NONE | 350.00000 | . |
| | 11 | CON10 | BS | 347.08408 | 2.91592 | NONE | 350.00000 | . |
| | 12 | CON11 | BS | 349.26109 | .73891 | NONE | 350.00000 | . |
| | 13 | CON12 | BS | 349.59517 | .40483 | NONE | 350.00000 | . |
| | 14 | CON13 | UL | 250.00000 | . | NONE | 250.00000 | 256.18756 |
| | 15 | CON14 | UL | 250.00000 | . | NONE | 250.00000 | 244.77574 |
| | 16 | CON15 | UL | 250.00000 | . | NONE | 250.00000 | 182.36735 |
| | 17 | CON16 | UL | 250.00000 | . | NONE | 250.00000 | 28.57143 |
| | 18 | CON17 | EQ | 10000.00000 | . | 10000.00000 | 10000.00000 | 22.50000- |
| | 19 | CON18 | EQ | 11500.00000 | . | 11500.00000 | 11500.00000 | 22.50000- |
| | 20 | CON19 | EQ | 9000.00000 | . | 9000.00000 | 9000.00000 | 21.50000- |
| | 21 | CON20 | EQ | 12300.00000 | . | 12300.00000 | 12300.00000 | 22.50000- |
| | 22 | CON21 | EQ | . | . | . | . | 22.50000- |
| | 23 | CON22 | EQ | . | . | . | . | 22.50000- |
| | 24 | CON23 | EQ | . | . | . | . | 21.50000- |
| | 25 | CON24 | EQ | . | . | . | . | 22.50000- |
| A | 26 | CON25 | EQ | . | . | . | . | . |
| A | 27 | CON26 | EQ | . | . | . | . | . |
| A | 28 | CON27 | EQ | . | . | . | . | . |
| A | 29 | CON28 | EQ | . | . | . | . | . |
| | 30 | CON29 | BS | 4854.28571- | 4854.28571 | NONE | . | . |
| | 31 | CON30 | BS | 4088.78367- | 4088.78367 | NONE | . | . |
| | 32 | CON31 | BS | 5236.91634- | 5236.91634 | NONE | . | . |
| | 33 | CON32 | BS | 4904.41535- | 4904.41535 | NONE | . | . |

MPSX/370 R1.6 PTF9 MPSCL EXECUTION

SECTION 2 - COLUMNS

| NUMBER | .COLUMNS | AT | ...ACTIVITY... | ..INPUT COST.. | ..LOWER LIMIT. | ..UPPER LIMIT. | .REDUCED COST. |
|---|---|---|---|---|---|---|---|
| 34 | X1 | BS | 10.00000 | 400.00000 | . | NONE | . |
| 35 | X2 | BS | 323.42857 | 450.00000 | . | NONE | . |
| 36 | X3 | BS | 2.00000 | 400.00000 | . | NONE | . |
| 37 | X4 | BS | 345.08408 | 450.00000 | . | NONE | . |
| 38 | X5 | BS | .40000 | 400.00000 | . | NONE | . |
| 39 | X6 | BS | 348.86109 | 450.00000 | . | NONE | . |
| 40 | X7 | BS | .08000 | 400.00000 | . | NONE | . |
| 41 | X8 | BS | 349.51517 | 450.00000 | . | NONE | . |
| 42 | X9 | LL | . | 200.00000 | . | NONE | 164.10221 |
| 43 | X10 | BS | 128.42857 | 200.00000 | . | NONE | . |
| 44 | X11 | LL | . | 200.00000 | . | NONE | 166.05896 |
| 45 | X12 | BS | 79.34122 | 200.00000 | . | NONE | . |
| 46 | X13 | LL | . | 200.00000 | . | NONE | 165.32245 |
| 47 | X14 | BS | 71.39382 | 200.00000 | . | NONE | . |
| 48 | X15 | LL | . | 200.00000 | . | NONE | 172.85714 |
| 49 | X16 | BS | 70.14630 | 200.00000 | . | NONE | . |
| 50 | X17 | LL | . | 100.00000 | . | NONE | 212.75406 |
| 51 | X18 | LL | . | 100.00000 | . | NONE | 376.85627 |
| 52 | X19 | LL | . | 100.00000 | . | NONE | 207.37376 |
| 53 | X20 | LL | . | 100.00000 | . | NONE | 373.43272 |
| 54 | X21 | LL | . | 100.00000 | . | NONE | 189.38776 |
| 55 | X22 | LL | . | 100.00000 | . | NONE | 354.71020 |
| 56 | X23 | LL | . | 100.00000 | . | NONE | 135.71429 |
| 57 | X24 | LL | . | 100.00000 | . | NONE | 308.57143 |
| 58 | X25 | LL | . | 1.00000 | . | NONE | 1.00000 |
| 59 | X26 | LL | . | 1.00000 | . | NONE | 2.00000 |
| 60 | X27 | BS | 1473.83268 | 1.00000 | . | NONE | . |
| 61 | X28 | LL | . | .50000 | . | NONE | 23.00000 |
| 62 | X29 | BS | 10000.00000 | . | . | NONE | . |
| 63 | X30 | BS | 11500.00000 | . | . | NONE | . |
| 64 | X31 | BS | 10473.83268 | . | . | NONE | . |
| 65 | X32 | BS | 10826.16732 | . | . | NONE | . |
| 66 | X33 | LL | . | 30.00000 | . | NONE | 7.50000 |
| 67 | X34 | LL | . | 30.00000 | . | NONE | 7.50000 |
| 68 | X35 | LL | . | 30.00000 | . | NONE | 8.50000 |
| 69 | X36 | LL | . | 30.00000 | . | NONE | 7.50000 |
| 70 | X37 | BS | 97.14286 | 22.50000 | . | NONE | . |
| 71 | X38 | BS | 1107.47755 | 22.50000 | . | NONE | . |
| 72 | X39 | LL | . | 22.50000 | . | NONE | 1.00000 |
| 73 | X40 | BS | 339.11221 | 22.50000 | . | NONE | . |
| 74 | X41 | BS | 4754.28571 | . | . | NONE | . |
| 75 | X42 | BS | 4068.78367 | . | . | NONE | . |
| 76 | X43 | BS | 5232.91634 | . | . | NONE | . |
| 77 | X44 | BS | 4903.61535 | . | . | NONE | . |
| 78 | X45 | LL | . | . | . | NONE | 22.50000 |
| 79 | X46 | LL | . | . | . | NONE | 22.50000 |
| 80 | X47 | LL | . | . | . | NONE | 21.50000 |
| 81 | X48 | LL | . | . | . | NONE | 22.50000 |

NAGP OUTPUT OF THE EQUIVALENT

GP PROBLEM FOR THE LP PROBLEM

THE OPTIMIZATION ENDED ON SUBPROBLEM 2
THERE WERE 33 CONSTRAINTS IN THE FINAL OPTIMAL TABLEAU.

*************************************************************************

THE OPTIMAL SOLUTION FOR THE DECISION VARIABLES X(J)

X( 1)= 10.0000

X( 2)= 323.4286

X( 3)= 2.0000

X( 4)= 345.0841

X( 5)= 0.4510

X( 6)= 348.8384

X( 7)= 0.0902

X( 8)= 349.5490

X( 9)= 0.0000

X( 10)= 128.4286

X( 11)= 0.0000

X( 12)= 79.3412

X( 13)= 0.0000

X( 14)= 71.3938

X( 15)= 0.0000

X( 16)= 70.1625

X( 17)= 0.0000

X( 18)= 0.0000

X( 19)= 0.0000

X( 20)= 0.0000

X( 21)= 0.0000

X( 22)= 0.0000

X( 23)= 0.0000

X( 24)= 0.0000

X( 25)= 0.0000

X( 26)= 0.0000

X( 27)= 1473.8327

X( 28)= 0.0000

```
X( 29)=     10000.0000
X( 30)=     11500.0000
X( 31)=     10473.8327
X( 32)=     10826.1673
X( 33)=         0.0000
X( 34)=         0.0000
X( 35)=         0.0000
X( 36)=         0.0000
X( 37)=        97.1429
X( 38)=      1107.4776
X( 39)=         0.0000
X( 40)=       337.8935
X( 41)=      4754.2857
X( 42)=      4068.7837
X( 43)=      5232.9163
X( 44)=      4905.3413
X( 45)=         0.0000
X( 46)=         0.0000
X( 47)=         0.0000
X( 48)=         0.0000
```

*************************************************************************

THE GOAL ACHIEVEMENTS ARE

| PRIORITY | GOAL NUMBER | OVER-ACHIEVEMENT | UNDER-ACHIEVEMENT |
|---|---|---|---|
| 1 | 1 | 0.0000 | 0.0000 |
| 1 | 2 | 0.0000 | 0.0000 |
| 1 | 3 | 0.0000 | 0.0000 |
| 1 | 4 | 0.0000 | 0.0000 |
| 1 | 5 | 0.0000 | 0.0000 |
| 1 | 6 | 0.0000 | 0.0000 |
| 1 | 7 | 0.0000 | 0.0000 |
| 1 | 8 | 0.0000 | 0.0000 |
| 1 | 9 | 0.0000 | 16.5714 |
| 1 | 10 | 0.0000 | 2.9159 |
| 1 | 11 | 0.0000 | 0.7106 |
| 1 | 12 | 0.0000 | 0.3608 |
| 1 | 13 | 0.0000 | 0.0000 |
| 1 | 14 | 0.0000 | 0.0000 |
| 1 | 15 | 0.0000 | 0.0000 |
| 1 | 16 | 0.0000 | 0.0000 |
| 1 | 17 | 0.0000 | 0.0000 |
| 1 | 18 | 0.0000 | 0.0000 |
| 1 | 19 | 0.0000 | 0.0000 |
| 1 | 20 | 0.0000 | 0.0000 |

| | | | |
|---|---|---|---|
| 1 | 21 | 0.0000 | 0.0000 |
| 1 | 22 | 0.0000 | 0.0000 |
| 1 | 23 | 0.0000 | 0.0000 |
| 1 | 24 | 0.0000 | 0.0000 |
| 1 | 25 | 0.0000 | 0.0000 |
| 1 | 26 | 0.0000 | 0.0000 |
| 1 | 27 | 0.0000 | 0.0000 |
| 1 | 28 | 0.0000 | 0.0000 |
| 1 | 29 | 0.0000 | 4854.2857 |
| 1 | 30 | 0.0000 | 4088.7837 |
| 1 | 31 | 0.0000 | 5236.9163 |
| 1 | 32 | 0.0000 | 4906.2433 |
| 2 | 33 | 726191.2016 | 0.0000 |

***************************************************************************

THE PRIORITY ACHIEVEMENTS ARE

| PRIORITY | ACHIEVEMENT |
|---|---|
| 1 | 0.0000 |
| 2 | 726191.2016 |

***************************************************************************

OUTPUT SUMMARY

| SUBSCRIPT | A OPT | X OPT | POS DEV | NEG DEV |
|---|---|---|---|---|
| 1 | 0.0000 | 10.0000 | 0.0000 | 0.0000 |
| 2 | 726191.2016 | 323.4286 | 0.0000 | 0.0000 |
| 3 | | 2.0000 | 0.0000 | 0.0000 |
| 4 | | 345.0841 | 0.0000 | 0.0000 |
| 5 | | 0.4510 | 0.0000 | 0.0000 |
| 6 | | 348.8384 | 0.0000 | 0.0000 |
| 7 | | 0.0902 | 0.0000 | 0.0000 |
| 8 | | 349.5490 | 0.0000 | 0.0000 |
| 9 | | 0.0000 | 0.0000 | 16.5714 |
| 10 | | 128.4286 | 0.0000 | 2.9159 |
| 11 | | 0.0000 | 0.0000 | 0.7106 |
| 12 | | 79.3412 | 0.0000 | 0.3608 |
| 13 | | 0.0000 | 0.0000 | 0.0000 |
| 14 | | 71.3938 | 0.0000 | 0.0000 |
| 15 | | 0.0000 | 0.0000 | 0.0000 |
| 16 | | 70.1625 | 0.0000 | 0.0000 |
| 17 | | 0.0000 | 0.0000 | 0.0000 |
| 18 | | 0.0000 | 0.0000 | 0.0000 |
| 19 | | 0.0000 | 0.0000 | 0.0000 |
| 20 | | 0.0000 | 0.0000 | 0.0000 |
| 21 | | 0.0000 | 0.0000 | 0.0000 |
| 22 | | 0.0000 | 0.0000 | 0.0000 |
| 23 | | 0.0000 | 0.0000 | 0.0000 |
| 24 | | 0.0000 | 0.0000 | 0.0000 |
| 25 | | 0.0000 | 0.0000 | 0.0000 |
| 26 | | 0.0000 | 0.0000 | 0.0000 |
| 27 | | 1473.8327 | 0.0000 | 0.0000 |
| 28 | | 0.0000 | 0.0000 | 0.0000 |
| 29 | | 10000.0000 | 0.0000 | 4854.2857 |
| 30 | | 11500.0000 | 0.0000 | 4088.7837 |
| 31 | | 10473.8327 | 0.0000 | 5236.9163 |
| 32 | | 10826.1673 | 0.0000 | 4906.2433 |
| 33 | | 0.0000 | 726191.2016 | 0.0000 |
| 34 | | 0.0000 | | |
| 35 | | 0.0000 | | |
| 36 | | 0.0000 | | |

| | |
|---|---|
| 37 | 97.1429 |
| 38 | 1107.4776 |
| 39 | 0.0000 |
| 40 | 337.8935 |
| 41 | 4754.2857 |
| 42 | 4068.7837 |
| 43 | 5232.9163 |
| 44 | 4905.3413 |
| 45 | 0.0000 |
| 46 | 0.0000 |
| 47 | 0.0000 |
| 48 | 0.0000 |

****************************************************************************

****************************************************************************

OUTPUT SUMMARY OF THE NONDOMINANCE TEST

****************************************************************************

THE ABOVE GOAL PROGRAMMING SOLUTION IS NONDOMINATED .

****************************************************************************

NAGP OUTPUT OF THE GP PROBLEM CONSTRUCTED

FROM THE SOLUTION OF THE LP PROBLEM

THE OPTIMIZATION ENDED ON SUBPROBLEM 5
THERE WERE 36 CONSTRAINTS IN THE FINAL OPTIMAL TABLEAU.

************************************************************:

THE OPTIMAL SOLUTION FOR THE DECISION VARIABLES X(J)

| | |
|---|---|
| X( 1)= | 10.0000 |
| X( 2)= | 323.4286 |
| X( 3)= | 2.0000 |
| X( 4)= | 345.0841 |
| X( 5)= | 0.6094 |
| X( 6)= | 348.6428 |
| X( 7)= | 0.3488 |
| X( 8)= | 349.5188 |
| X( 9)= | 0.0000 |
| X( 10)= | 128.4286 |
| X( 11)= | 0.0000 |
| X( 12)= | 79.3412 |
| X( 13)= | 0.0000 |
| X( 14)= | 71.2686 |
| X( 15)= | 0.0000 |
| X( 16)= | 70.2716 |
| X( 17)= | 0.0000 |
| X( 18)= | 0.0000 |
| X( 19)= | 0.0000 |
| X( 20)= | 0.0000 |
| X( 21)= | 0.0000 |
| X( 22)= | 0.0000 |
| X( 23)= | 0.0000 |
| X( 24)= | 0.0000 |
| X( 25)= | 0.0000 |
| X( 26)= | 0.0000 |
| X( 27)= | 1470.0764 |
| X( 28)= | 0.0000 |

X( 29)= 10000.0000

X( 30)= 11500.0000

X( 31)= 10470.0764

X( 32)= 10829.9236

X( 33)= 0.0000

X( 34)= 0.0000

X( 35)= 0.0000

X( 36)= 0.0000

X( 37)= 97.1429

X( 38)= 1107.4776

X( 39)= 0.0000

X( 40)= 339.1129

X( 41)= 4754.2857

X( 42)= 4068.7837

X( 43)= 5231.0382

X( 44)= 4905.0735

X( 45)= 0.0000

X( 46)= 0.0000

X( 47)= 0.0000

X( 48)= 0.0000

*************************************************************************

THE GOAL ACHIEVEMENTS ARE

| PRIORITY | GOAL NUMBER | OVER-ACHIEVEMENT | UNDER-ACHIEVEMENT |
|---|---|---|---|
| 1 | 1 | 0.0000 | 0.0000 |
| 1 | 2 | 0.0000 | 0.0000 |
| 1 | 3 | 0.0000 | 0.0000 |
| 1 | 4 | 0.0000 | 0.0000 |
| 1 | 5 | 0.0000 | 0.0000 |
| 1 | 6 | 0.0000 | 0.0000 |
| 1 | 7 | 0.0000 | 0.0000 |
| 1 | 8 | 0.0000 | 0.0000 |
| 1 | 9 | 0.0000 | 16.5714 |
| 1 | 10 | 0.0000 | 2.9159 |
| 1 | 11 | 0.0000 | 0.4282 |
| 1 | 12 | 0.0000 | 0.1396 |
| 1 | 13 | 0.0000 | 0.0000 |
| 1 | 14 | 0.0000 | 0.0000 |
| 1 | 15 | 0.0000 | 0.1096 |
| 1 | 16 | 0.0000 | 0.3038 |
| 1 | 17 | 0.0000 | 0.0000 |
| 1 | 18 | 0.0000 | 0.0000 |
| 1 | 19 | 0.0000 | 0.0000 |
| 1 | 20 | 0.0000 | 0.0000 |

| | | | |
|---|---|---|---|
| 1 | 21 | 0.0000 | 0.0000 |
| 1 | 22 | 0.0000 | 0.0000 |
| 1 | 23 | 0.0000 | 0.0000 |
| 1 | 24 | 0.0000 | 0.0000 |
| 1 | 25 | 0.0000 | 0.0000 |
| 1 | 26 | 0.0000 | 0.0000 |
| 1 | 27 | 0.0000 | 0.0000 |
| 1 | 28 | 0.0000 | 0.0000 |
| 1 | 29 | 0.0000 | 4854.2857 |
| 1 | 30 | 0.0000 | 4088.7837 |
| 1 | 31 | 0.0000 | 5235.0382 |
| 1 | 32 | 0.0000 | 4906.2924 |
| 2 | 33 | 0.0000 | 0.0000 |
| 3 | 34 | 0.0000 | 101.3953 |
| 4 | 35 | 0.0000 | 3.9236 |
| 5 | 36 | 0.0000 | 0.0000 |

******************************************************************************

THE PRIORITY ACHIEVEMENTS ARE

| PRIORITY | ACHIEVEMENT |
|---|---|
| 1 | 0.0000 |
| 2 | 0.0000 |
| 3 | 0.0000 |
| 4 | 0.0000 |
| 5 | 0.0000 |

******************************************************************************

OUTPUT SUMMARY

| SUBSCRIPT | A OPT | X OPT | POS DEV | NEG DEV |
|---|---|---|---|---|
| 1 | 0.0000 | 10.0000 | 0.0000 | 0.0000 |
| 2 | 0.0000 | 323.4286 | 0.0000 | 0.0000 |
| 3 | 0.0000 | 2.0000 | 0.0000 | 0.0000 |
| 4 | 0.0000 | 345.0841 | 0.0000 | 0.0000 |
| 5 | 0.0000 | 0.6094 | 0.0000 | 0.0000 |
| 6 | | 348.6428 | 0.0000 | 0.0000 |
| 7 | | 0.3488 | 0.0000 | 0.0000 |
| 8 | | 349.5188 | 0.0000 | 0.0000 |
| 9 | | 0.0000 | 0.0000 | 16.5714 |
| 10 | | 128.4286 | 0.0000 | 2.9159 |
| 11 | | 0.0000 | 0.0000 | 0.4282 |
| 12 | | 79.3412 | 0.0000 | 0.1396 |
| 13 | | 0.0000 | 0.0000 | 0.0000 |
| 14 | | 71.2686 | 0.0000 | 0.0000 |
| 15 | | 0.0000 | 0.0000 | 0.1096 |
| 16 | | 70.2716 | 0.0000 | 0.3038 |
| 17 | | 0.0000 | 0.0000 | 0.0000 |
| 18 | | 0.0000 | 0.0000 | 0.0000 |
| 19 | | 0.0000 | 0.0000 | 0.0000 |
| 20 | | 0.0000 | 0.0000 | 0.0000 |
| 21 | | 0.0000 | 0.0000 | 0.0000 |
| 22 | | 0.0000 | 0.0000 | 0.0000 |
| 23 | | 0.0000 | 0.0000 | 0.0000 |
| 24 | | 0.0000 | 0.0000 | 0.0000 |
| 25 | | 0.0000 | 0.0000 | 0.0000 |
| 26 | | 0.0000 | 0.0000 | 0.0000 |
| 27 | | 1470.0764 | 0.0000 | 0.0000 |
| 28 | | 0.0000 | 0.0000 | 0.0000 |
| 29 | | 10000.0000 | 0.0000 | 4854.2857 |
| 30 | | 11500.0000 | 0.0000 | 4088.7837 |

| | | | |
|---|---|---|---|
| 31 | 10470.0764 | 0.0000 | 5235.0382 |
| 32 | 10829.9236 | 0.0000 | 4906.2924 |
| 33 | 0.0000 | 0.0000 | 0.0000 |
| 34 | 0.0000 | 0.0000 | 101.3953 |
| 35 | 0.0000 | 0.0000 | 3.9236 |
| 36 | 0.0000 | 0.0000 | 0.0000 |
| 37 | 97.1429 | | |
| 38 | 1107.4776 | | |
| 39 | 0.0000 | | |
| 40 | 339.1129 | | |
| 41 | 4754.2857 | | |
| 42 | 4068.7837 | | |
| 43 | 5231.0382 | | |
| 44 | 4905.0735 | | |
| 45 | 0.0000 | | |
| 46 | 0.0000 | | |
| 47 | 0.0000 | | |
| 48 | 0.0000 | | |

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OUTPUT SUMMARY OF THE NONDOMINANCE TEST

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THE ABOVE GOAL PROGRAMMING SOLUTION IS NONDOMINATED .

**************************************************************************

THE LP PROBLEM TERMINATES AT PHASE 1 AND

THE VALUE OF PHASE 1 OBJECTIVE FUNCTION = 3.7525

**************************************************************************

Definition of the Decision Variables

| i | X(i) |
|---|---|
| 1, 3, 5, 7 | Number of workers in class 1 in periods 1, 2, 3, and 4 respectively. |
| 2, 4, 6, 8 | Number of workers in class 2 in periods 1, 2, 3, and 4 respectively. |
| 9, 11, 13, 15 | Number of workers hired for class 1 in periods 1, 2, 3, and 4 respectively. |
| 10, 12, 14, 16 | Number of workers hired for class 2 in periods 1, 2, 3, and 4 respectively. |
| 17, 19, 21, 23 | Number of workers fired from class 1 in periods 1, 2, 3, and 4 respectively. |
| 18, 20, 22, 24 | Number of workers fired from class 2 in periods 1, 2, 3, and 4 respectively. |
| 25, 26, 27, 28 | Inventory level in periods 1, 2, 3, and 4 respectively. |
| 29, 30, 31, 32 | Production level in periods 1, 2, 3, and 4 respectively. |
| 33, 34, 35, 36 | Amount of overtime production assigned to class 1 workers in periods 1, 2, 3, and 4 respectively. |
| 37, 38, 39, 40 | Amount of overtime production assigned to class 2 workers in periods 1, 2, 3, and 4 respectively. |
| 41, 42, 43, 44, 45, 46, 47, 48 | Nonnegative variables used to transform the nonlinear overtime constraints to linear constraints. |

VITA 6

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