A UNITARITY BOUND CONSTRAINT ON W-PAIR PRODUCTION IN TWO-PHOTON COLLISIONS

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CHAPTER I

INTRODUCTION

A primary measure of the validity of any theory is the degree to which it is capable of providing predictions which can be put to the experimental test. The agreement between theory and experiment in the case of spinor electrodynamics is almost legendary; however, no such statement can be made regarding the electrodynamics of vector particles.

Much of the difficulty lies in the properties of the massive charged vector particles available for study; all are short lived members of the hadron family so that in production processes final state interactions due to the strong force obscure the more delicate radiative corrections which constitute the nontrivial predictions of the theory. In addition these vector bosons are thought to consist of quark-antiquark bound states, a fundamental theory involving point-like interactions therefore being inapplicable except as an approximation.

If current optimism is borne out, the above cited problems will be overcome when the next generation of colliding beam accelerators begin to produce the $W^{\pm}$, long hypothesized as the intermediaries of the weak interaction, which have the dual attributes of being both disposessed of strong interactions, and, it is expected, lacking in structure. Before the question of detailed comparison may be seriously addressed, however, it is necessary that one theoretical ambiguity be resolved by answering convincingly, if not conclusively, the question: what is the value of the magnetic moment parameter K for a pointlike particle? Although playing a role analogous to the g of spinor electrodynamics in the magnetic moment:

$$\vec{\mu} = \frac{e}{2M_W}(1 + K)\vec{S} \tag{1-1}$$

by the very nature of its introduction K cannot be calculated from the theory; rather the mathematics must be coupled with some compelling argument (1). An example in this regard is T. D. Lee's (1) result for the radiative corrections to the basic quadrupole moment

$$Q = -\frac{e}{M_W^2} K \tag{1-2}$$

obtained as an infinite sum of Feynman graphs using the renormalizable model he and C. N. Yang (2) developed:

$$Q = -\frac{e}{M_W^2}\left[K + \frac{1}{2} a_o \alpha \ln(\alpha K^2)\right] \tag{1-3}$$

It is readily observed from this form that all coefficients in the usual perturbation expansion are infinite, and so the theory is unrenormalizable, unless

$$K = \alpha^{-\frac{1}{2}} \tag{1-4}$$

or

$$a_o = 0 \tag{1-5}$$

Moreover, the coefficient of the log is given by

$$a_o = -\frac{(K+3)(K-1)^2}{2\pi} \tag{1-6}$$

thus three possibilities are provided.

In the face of the aforementioned uncertainty it becomes necessary to ask if there might not exist an alternative argument, one which allows a single value of K. It is exactly to that question that this thesis is addressed. In Chapter II an approach based upon the unitarity of the scattering matrix is described, and pair production in two-photon collisions is singled out for study in this regard. In Chapter III the concept of a transversality mapping is introduced as a simplification tool and its implications for this study discussed. Chapter IV concerns the comparison of asymptotic behavior with the prediction of unitarity while Chapter V checks the result with the literature. Appendixes are provided which detail vector electrodynamics, relevant kinematics, the application of the transversality mapping, and the computer programs used.

CHAPTER II

THE UNITARITY BOUND

Aside from the more obvious defect of providing several candidates for K, the renormalization argument cited in Chapter I has the additional deficit that it is rooted not in some profound physical principle but rather in the mathematics needed to deal with the recurrent infinities that plague quantum field theory. While renormalizability appears to be a common feature of those theories which correspond to nature it must be kept in mind that they are all limited in scope and the possibility of a completely finite unified theory cannot be excluded. An alternative approach should therefore have the merit of resting on much firmer physical grounds.

Just such an approach is provided by the consequences of the unitarity of the scattering matrix, which in essence is the generalization of the conservation of probability principle familiar to ordinary quantum mechanics, and as such may be taken to be a fundamental requirement of any physically meaningful theory. Taken together with an austere set of parallel assumptions, namely that the amplitude is an analytic function of the mandelstam variables S and T, and polynomially bounded in S, unitarity restricts the asymptotic behavior in S of a cross-section to be

$$\sigma(S) \underset{S \to \infty}{\leq} C(\ln S)^2 \qquad \forall\, S > S_a \tag{2-1}$$

here S_a denotes the beginning of the asymptotic region and C is some finite constant (3). It is therefore possible to make the following hypothesis: if there exists some set of K_is such that for a given process

$$\sigma(S,K) \underset{S\to\infty}{\leq} C(\ln S)^2 \qquad K = K_i \tag{2-2}$$

$$\sigma(S,K) \underset{S\to\infty}{>} C(\ln S)^2 \qquad K \neq K_i \tag{2-3}$$

then $\{K_i\}$ constitute the possible physical values of K. This is the line of attack that will be used.

In selecting a process for examination it is useful to keep in mind the advantage of excluding virtual photons and thereby remove questions of γ-Z^o mixing as predicted in the Weinberg-Salam (4) model. This essentially limits the possibilities to Compton scattering and two-photon production, the latter of which will be chosen, albeit somewhat arbitrarily. The three lowest order diagrams contributing to this process are illustrated in Figure 1 and using the rules of Table II, Appendix A, the amplitude is found to be given by

$$M_{fi} = -ie^2 M^{\alpha\beta\mu\nu} \varepsilon_\alpha(P_1)\varepsilon_\beta(P_2)\varepsilon_\mu(k_1)\varepsilon_\nu(k_2) \tag{2-4}$$

where

$$M^{\alpha\beta\mu\nu} = M^{\alpha\beta\mu\nu}_{(a)} + M^{\alpha\beta\mu\nu}_{(b)} + M^{\alpha\beta\mu\nu}_{(c)} \tag{2-5}$$

$$M^{\alpha\beta\mu\nu}_{(a)} \equiv V^{\alpha\sigma\mu}(-P_1,q_1,K)\, S_{\sigma\rho}(q_1)\, V^{\rho\beta\nu}(q_1,P_2,K) \tag{2-6}$$

$$M^{\alpha\beta\mu\nu}_{(b)} \equiv V^{\alpha\sigma\nu}(-P_1,q_2,K)\, S_{\sigma\rho}(q_2)\, V^{\rho\beta\mu}(q_2,P_2,K) \tag{2-7}$$

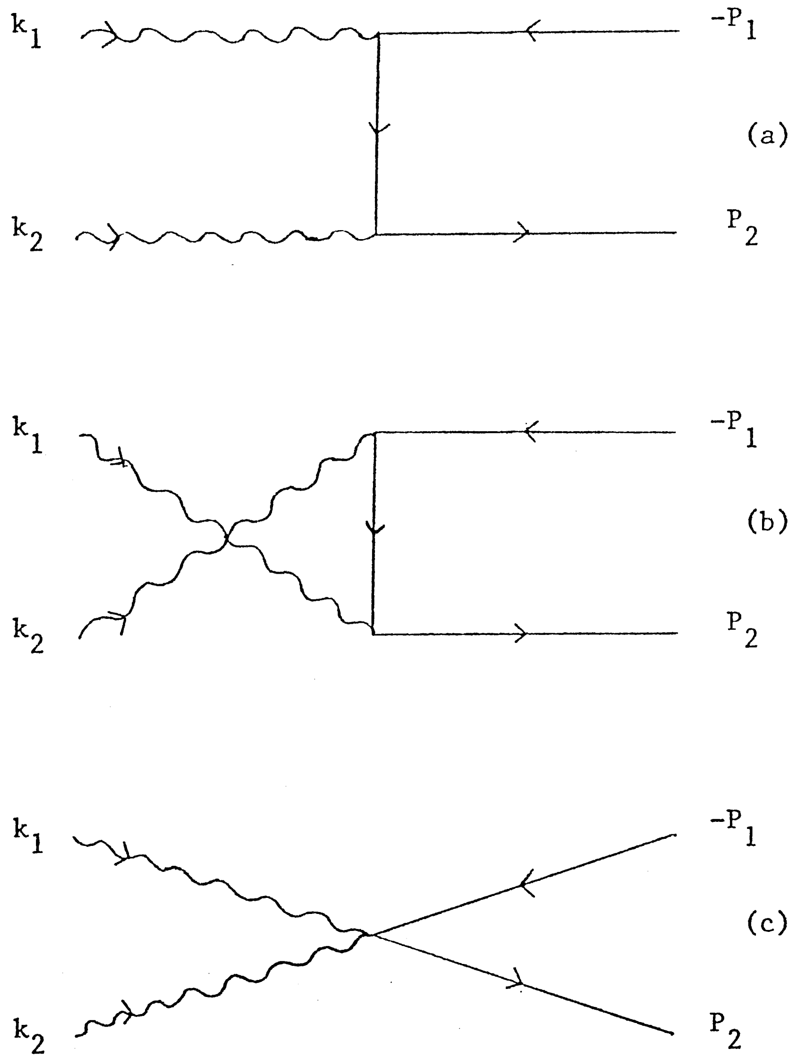

Figure 1. Lowest order graphs contributing to $\gamma\gamma \to W^+W^-$

$$M_{(c)}^{\alpha\beta\mu\nu} \equiv U^{\alpha\beta\mu\nu} \tag{2-8}$$

Applying Equations (A-10) and (A-11), the squared amplitude, summed over final and averaged over initial spins, is given by

$$\begin{aligned}
\overline{|M_{fi}|^2} &= \frac{e^4}{4} M^{\alpha\beta\mu\nu} M^{\alpha'\beta'\mu'\nu'} (-g_{\alpha\alpha'} + M^{-2} P_{1\alpha} P_{1\alpha'}) \\
&\quad \text{x} \ (-g_{\beta\beta'} + M^{-2} P_{2\beta} P_{2\beta'}) (-g_{\mu\mu'}) (-g_{\nu\nu'}) \\
&= \frac{e^4}{4} M^{\alpha\beta\mu\nu} M^{\alpha'\beta'}_{\mu\nu} (-g_{\alpha\alpha'} + M^{-2} P_{1\alpha} P_{1\alpha'}) \\
&\quad \text{x} \ (-g_{\beta\beta'} + M^{-2} P_{2\beta} P_{2\beta'})
\end{aligned} \tag{2-9}$$

CHAPTER III

THE TRANSVERSALITY MAPPING

Although Equations (2-5) through (2-9) together with the definitions of Table II, Appendix A, constitute all the information needed to calculate $\overline{|M_{fi}|^2}$ directly, it should be noted that such a straight forward evaluation involves some one hundred and sixty thousand terms; thus it is only prudent to consider means by which the tensor structure of $M^{\alpha\beta\mu\nu}$ can be simplified, so as to make the problem more manageable. Towards this end the concept of a transversality mapping is introduced as follows: consider an arbitrary vector electrodynamic process involving N external lines, with amplitude

$$M_{fi} = M^{\lambda} \prod_{i=1}^{N} \varepsilon_{\lambda_i}(q_i) \qquad (3\text{-}1)$$

$$M^{\lambda} \equiv M^{\lambda_1 \lambda_2 \lambda_3 \cdots \lambda_N} \qquad (3\text{-}2)$$

Defining

$$M^{\lambda} \equiv M'^{\lambda} + q_i^{\lambda_i} \Omega_i^{\tilde{\lambda}} \qquad i \leq n \leq N \qquad (3\text{-}3)$$

$$\Omega_i^{\tilde{\lambda}} \equiv \Omega_i^{\lambda_1 \cdots \lambda_{i-1} \lambda_{i+1} \cdots \lambda_n} \qquad (3\text{-}4)$$

if n of the N transversality conditions

$$q_j^{\lambda_j} \varepsilon_{\lambda_j}(q_j) = 0 \qquad (3\text{-}5)$$

are used, Equations (3-1) and (3-3) imply

$$M_{fi} \rightarrow M'_{fi} = M'^{\lambda} \prod_{i=1}^{N} \varepsilon_{\lambda_i}(q_i) \tag{3-6}$$

or equivalently

$$M^{\lambda} \rightarrow M'^{\lambda} \tag{3-7}$$

so that the operation maps the tensor M^{λ} onto M'^{λ}; now let

$$\rho_{\lambda_i \lambda'_i} = \sum_{\sigma} \varepsilon^{(\sigma)}_{\lambda_i}(q_i)\, \varepsilon^{(\sigma)}_{\lambda'_i}(q_i) \tag{3-8}$$

then according to Equation (3-5)

$$q_i^{\lambda_i} \rho_{\lambda_i \lambda'_i} = \sum_{\sigma} \varepsilon^{(\sigma)}_{\lambda'_i}(q_i) \left[q_i^{\lambda_i} \varepsilon^{(\sigma)}_{\lambda_i}(q_i) \right]$$

$$= 0 \tag{3-9}$$

and similarly

$$\rho_{\lambda_i \lambda'_i}\, q_i^{\lambda'_i} = 0 \tag{3-10}$$

so that with N_o the number of lines in the initial state

$$\begin{aligned} \overline{|M_{fi}|^2} &= 2^{-N_o} M^{\lambda} M^{\lambda'} \prod_{i=1}^{N} \rho_{\lambda_i \lambda'_i} \\ &= 2^{-N_o} [M'^{\lambda} + q_i^{\lambda_i} \Omega_i^{\tilde{\lambda}}][M'^{\lambda'} + q_j^{\lambda'_j} \Omega_j^{\tilde{\lambda}'}] \prod_{i=1}^{N} \rho_{\lambda_i \lambda'_i} \\ &= 2^{-N_o} M'^{\lambda} M'^{\lambda'} \prod_{i=1}^{N} \rho_{\lambda_i \lambda'_i} \\ &= \overline{|M'_{fi}|^2} \end{aligned} \tag{3-11}$$

thus the mapping leaves $\overline{|M_{fi}|^2}$ invariant.

As it stands, the transversality mapping is sufficient to allow all terms proportional to $P_{1\alpha}$ and $P_{2\beta}$ to be dropped from $M^{\alpha\beta\mu\nu}$ since Equation (A-11) is unique. In the case of $k_{1\mu}$ and $k_{2\nu}$, however, Equations (3-9) and (3-10) are unnecessarily restrictive as the following theorem demonstrates: let $\{k_j\} \subset \{q_i\}$ such that

$$k_j^{\,2} = 0 \tag{3-12}$$

$$k_{j\lambda_j} M^{\lambda} = 0 \tag{3-13}$$

and choose

$$\rho_{\lambda_j\lambda_j'} \equiv - g_{\lambda_j\lambda_j'} \tag{3-14}$$

then

$$\overline{|M_{fi}|^2} = \overline{|M'_{fi}|^2} - \delta \tag{3-15}$$

where

$$\delta = 2^{-N_o} \omega_{j\ell}\, \omega'_{\ell j} \prod_{i\neq j\neq \ell} \rho_{\lambda_i\lambda_i'} \tag{3-16}$$

with $\omega_{j\ell}$ the negative of the coefficient of $k_\ell^{\lambda_\ell}$ in $k_{j\lambda_j} M'^{\lambda}$. The proof is as follows: writing in analogy to equation (3-3)

$$M'^{\lambda} = M^{\lambda} - k_j^{\lambda_j} \Omega_j^{\tilde{\lambda}} \qquad j \leq N_k \tag{3-17}$$

it follows from Equations (3-12) and (3-13) that

$$\overline{|M'_{fi}|^2} = 2^{-N_o}[M^{\lambda} - k_j^{\lambda_j}\,\Omega_j^{\tilde{\lambda}}][M^{\lambda'} - k_\ell^{\lambda'_\ell}\,\Omega_\ell^{\tilde{\lambda}'}] \prod_{i=1}^{N} \rho_{\lambda_i \lambda'_i}$$

$$= 2^{-N_o}[M^{\lambda}M^{\lambda'} + k_j^{\lambda_j}\,k_\ell^{\lambda'_\ell}\,\Omega_j^{\tilde{\lambda}}\,\Omega_\ell^{\tilde{\lambda}'}] \prod_{i=1}^{N} \rho_{\lambda_i \lambda'_i}$$

$$= \overline{|M_{fi}|^2} + 2^{-N_o}\,(-1)^2 (k_j \cdot \Omega_\ell^{\tilde{\lambda}'})(k_\ell \cdot \Omega_j^{\tilde{\lambda}}) \prod_{i \neq j \neq \ell}^{N} \rho_{\lambda_i \lambda'_i}$$

$$\equiv \overline{|M_{fi}|^2} + \delta \qquad (3\text{-}18)$$

but also

$$k_{j\lambda_j}\,M'^{\lambda} = 0 - k_\ell^{\lambda_\ell}\,(k_j \cdot \Omega_\ell^{\tilde{\lambda}}) \qquad \ell \neq j \qquad (3\text{-}19)$$

since the k_ℓ's are arbitrary it must be that

$$k_{j\lambda_j}\,M'^{\lambda} = k_\ell^{\lambda_\ell}\,\omega_{j\ell}$$

$$= -\,k_\ell^{\lambda_\ell}\,(k_j \cdot \Omega_\ell^{\tilde{\lambda}}) \qquad (3\text{-}20)$$

and Equation (3-16) follow, Q.E.D. It may be noted that if there is only one external line belonging to $\{k_i\}$, or if only one of the conditions (3-5) is applied to $\{k_i\}$, then equations (3-12), (3-13) and (3-18) have the immediate consequence that

$$\delta = 0 \qquad (3\text{-}21)$$

Identical arguments apply in the case of scalar electrodynamics, the sole difference being that $\{\varepsilon_{\lambda_i}(q_i)\} \in \{k_i\}$. This extention allows a simple example to illustrate the correctness of Equations (3-15) and (3-16); apart from an overall factor of ie^2 the amplitude for pair production of scalar particles in two-photon collisions is $M^{\mu\nu}\varepsilon_\mu(k_1)\varepsilon_\nu(k_2)$, where

$$M^{\mu\nu} = \frac{(2P_1-k_1)^\mu (k_2-2P_2)^\nu}{2k_1\cdot P_1} + \frac{(2P_1-k_2)^\nu (k_1-2P_2)^\mu}{2k_2\cdot P_1} + 2g^{\mu\nu} \quad (3\text{-}22)$$

is easily seen to satisfy Equation (3-13). Proceeding as in Equations (3-17) and (3-18):

$$M'^{\mu\nu} = -2\left[\frac{P_1{}^\mu P_2{}^\nu}{k_1\cdot P_1} + \frac{P_1{}^\nu P_2{}^\mu}{k_2\cdot P_1} - g^{\mu\nu}\right]$$

$$= M^{\mu\nu} - k_1{}^\mu\left[\frac{P_2{}^\nu}{k_1\cdot P_1} + \frac{P_1{}^\nu}{k_2\cdot P_1}\right] - k_2{}^\nu\left[\frac{P_1{}^\mu}{k_1\cdot P_1} + \frac{P_2{}^\nu}{k_2\cdot P_1}\right]$$

$$+ k_1{}^\mu k_2{}^\nu\left[\frac{1}{2k_1\cdot P_1} + \frac{1}{2k_2\cdot P_1}\right] \quad (3\text{-}23)$$

$$\overline{|M'_{fi}|^2} = \overline{|M_{fi}|^2} + \frac{1}{4}(2)\, k_{1\mu}\left[\frac{P_1{}^\mu}{k_1\cdot P_1} + \frac{P_2{}^\mu}{k_2\cdot P_1}\right] k_2{}^\nu\left[\frac{P_{2\nu}}{k_1\cdot P_1} + \frac{P_{1\nu}}{k_2\cdot P_1}\right]$$

$$= \overline{|M_{fi}|^2} + \frac{1}{2}[2][2]$$

$$= \overline{|M_{fi}|^2} + 2$$

$$\equiv \overline{|M_{fi}|^2} + \delta \quad (3\text{-}24)$$

Now applying the transversality mapping theorem

$$k_{1\mu} M'^{\mu\nu} = -2k_2{}^\nu$$

$$= -\omega_{12} k_2{}^\nu \quad (3\text{-}25)$$

$$k_{2\nu}\, M'^{\mu\nu} = -\, 2k_1^{\ \mu}$$

$$= -\, \omega_{21} k_1^{\ \mu} \qquad (3\text{-}26)$$

and

$$\delta = (2)^{-2}\left[\omega_{12}\omega_{21} + \omega_{21}\omega_{12}\right]$$

$$= \frac{1}{4}\left[(2)(2) + (2)(2)\right]$$

$$= 2 \qquad (3\text{-}27)$$

so the equivalence is demonstrated; if, for example, (3-5) is applied to $k_2^{\ \nu}$ only

$$M'^{\mu\nu} = -\frac{P_2^{\ \nu}(2P_1 - k_1)^{\mu}}{k_1 \cdot P_1} + \frac{P_1^{\ \nu}(k_1 - 2P_2)^{\nu}}{k_2 \cdot P_1} + 2g^{\mu\nu} \qquad (3\text{-}28)$$

$$\omega_{12} = 2 \qquad (3\text{-}29)$$

$$\omega_{21} = 0 \qquad (3\text{-}30)$$

thus δ vanishes and the note preceeding Equation (3-21) is also verified.

The application of the transversality mapping and theorem are detailed in Appendix B. Rather more has been done than simply dropping $P_{1\alpha}$, $P_{2\beta}$, $k_{1\mu}$ and $k_{2\nu}$; instead a program of recombination has been carried out which allows $M'^{\alpha\beta\mu\nu}$ to be written compactly in terms of the constants

$$C_1 \equiv 1 - K \qquad (3\text{-}31)$$

$$C_2 \equiv 1 + K \tag{3-32}$$

and tensors

$$I_o^{\alpha\beta\mu\nu}(P_1,P_2,k_1) \equiv 2g^{\alpha\beta}\left[\frac{P_1^{\ \mu}P_2^{\ \nu}}{k_1\cdot P_1} + \frac{P_1^{\ \nu}P_2^{\ \mu}}{k_1\cdot P_2} - g^{\mu\nu}\right] \tag{3-33}$$

$$I_1^{\alpha\mu}(k_1,P_1) \equiv (k_1\cdot P_1)\, g^{\alpha\mu} - k_1^{\ \alpha}P_1^{\ \mu} \tag{3-34}$$

$$I_2^{\mu\alpha\sigma}(k_1) \equiv g^{\sigma\mu}\, k_1^{\ \alpha} - g^{\alpha\mu}\, k_1^{\ \sigma} \tag{3-35}$$

as

$$\begin{aligned}
-M'^{\alpha\beta\mu\nu} &= I_o^{\ \alpha\beta\mu\nu}(P_1,P_2,k_1) \\
&+ \frac{1}{2k_1\cdot P_1}\{(\frac{C_1}{M})^2\, I_1^{\alpha\mu}(k_1,P_1)I_1^{\beta\nu}(k_2,P_2) + C_2^2 g_{\sigma\rho}I_2^{\mu\alpha\sigma}(k_1)I_2^{\nu\beta\rho}(k_2) \\
&+ 2C_2[P_1^{\ \mu}I_2^{\nu\beta\alpha}(k_2) + P_2^{\ \nu}I_2^{\mu\alpha\beta}(k_1)]\} \\
&+ \frac{1}{2k_2\cdot P_1}\{(\frac{C_1}{M})^2 I_1^{\alpha\nu}(k_2,P_1)\, I_1^{\beta\mu}(k_1,P_2) + C_2^2 g_{\sigma\rho}I_2^{\nu\alpha\sigma}(k_2)I_2^{\mu\beta\rho}(k_1) \\
&+ 2C_2[P_1^{\ \nu}\, I_2^{\mu\beta\alpha}(k_1) + P_2^{\mu}\, I_2^{\nu\alpha\beta}(k_2)]\}
\end{aligned} \tag{3-36}$$

The initial objective has been achieved for these are now less than three thousand terms to be considered, and Equation (2-9) becomes

$$\begin{aligned}
\overline{|M_{fi}|^2} &= \frac{e^4}{4}\{M'^{\alpha\beta\mu\nu}\, M'^{\alpha'\beta'}_{\mu\nu}\, (-g_{\alpha\alpha'} + \frac{P_{1\alpha}P_{1\alpha'}}{M^2})(-g_{\beta\beta'} + \frac{P_{2\beta}P_{2\beta'}}{M^2}) \\
&- 8[3 + \frac{(k_1\cdot k_2)^2 - 2M^2(k_1\cdot k_2)}{M^4}]\}
\end{aligned}$$

One final manipulation that will prove useful is to eliminate C_2 in favor of C_1 via

$$C_1 + C_2 = 2 \tag{3-38}$$

so that

$$\begin{aligned}
- M'^{\alpha\beta\mu\nu} &= N^{\alpha\beta\mu\nu} \\
&+ C_1{}^2 \{ \frac{I_1^{\alpha\mu}(k_1,P_1) I_1^{\beta\nu}(k_2,P_2)}{2M^2 (k_1 \cdot P_1)} + \frac{I_1^{\alpha\nu}(k_2,P_1) I_1^{\beta\mu}(k_1,P_2)}{2M^2 (k_2 \cdot P_1)} \\
&+ C_1 (C_1 - 4) g_{\sigma\rho} \{ \frac{I_2^{\mu\alpha\sigma}(k_1) I_2^{\nu\beta\rho}(k_2)}{2(k_1 \cdot P_1)} + \frac{I_2^{\nu\alpha\sigma}(k_2) I_2^{\mu\beta\rho}(k_1)}{2(k_2 \cdot P_1)} \} \\
&- 2C_1 \{ \frac{[P_1^{\mu} I_2^{\nu\alpha\beta}(k_2) + P_2^{\nu} I_2^{\mu\alpha\beta}(k_1)]}{2(k_1 \cdot P_1)} \\
&+ \frac{[P_1^{\nu} I_2^{\mu\beta\alpha}(k_1) + P_2^{\mu} I_2^{\nu\alpha\beta}(k_2)]}{2(k_2 \cdot P_1)} \}
\end{aligned} \tag{3-39}$$

$$\begin{aligned}
N^{\alpha\beta\mu\nu} &= - 2g^{\mu\nu} g^{\alpha\beta} \\
&+ 2g_{\sigma\rho} \{ \frac{[I_2^{\mu\alpha\sigma}(k_1) + P_1^{\mu} g^{\alpha\sigma}] [I_2^{\nu\beta\rho}(k_2) + P_2^{\nu} g^{\beta\rho}]}{(k_1 \cdot P_1)} \\
&+ \frac{[I_2^{\nu\alpha\sigma}(k_2) + P_1^{\nu} g^{\alpha\sigma}] [I_2^{\mu\beta\rho}(k_1) + P_2^{\mu} g^{\beta\rho}]}{(k_2 \cdot P_1)} \}
\end{aligned} \tag{3-40}$$

CHAPTER IV

ASYMPTOTIC BEHAVIOR AND THE UNITARITY BOUND

Owing to the fact that

$$q_1^{\,2} = T \tag{4-1}$$

$$q_2^{\,2} = U \tag{4-2}$$

the general form of the function A(S,T) defined in Appendix C is, by Equations (3-27) through (3-30):

$$A(S,T) = \frac{1}{(M^2)^2} \frac{1}{\left[(T-M^2)(T+S-M^2) \right]^2} \sum_{i+j+k=6} a_{ijk}(K)\, T^i (M^2)^j S^k \tag{4-3}$$

where the constraint i+j+k = 6 follows from dimensionality and note has been taken of the property of I_1 that

$$I_1^{\sigma_i \rho}(k,P_i)\, \rho_{i\sigma_i} = 0 \tag{4-4}$$

Of the two hundred and sixteen coefficients appearing in Equation (4-3), only a few may be expected to appear in the limit $S \to \infty$. This is readily demonstrated; let

$$Z = \frac{T}{M^2} \tag{4-5}$$

$$Y = \frac{M^2}{S} \tag{4-6}$$

then

$$A(Y,Z) = \left[(Z-1)(Z + \frac{1}{Y} - 1) \right]^{-2} \sum_{i+k \leq 6} a_{i,6-(i+k),k} \frac{Z^i}{Y^k} \tag{4-7}$$

and Equation (C-15) becomes

$$M^2 \sigma(S,M^2,K) = \frac{\pi\alpha^2}{4} Y^2 \int_{1 - \frac{(1+\xi)}{2Y}}^{1 - \frac{(1-\xi)}{2Y}} dZ \, A(Y,Z)$$

$$\frac{\pi\alpha^2}{4} h(y,K) \tag{4-8}$$

where

$$\xi = \sqrt{1-4y} \tag{4-9}$$

A change of variables

$$Z = W + 1 - \frac{1}{2y} \tag{4-10}$$

gives

$$h(y,K) = \int_{-\frac{\xi}{2y}}^{\frac{\xi}{2y}} dW \left[(yW - \frac{1}{2})(yW + \frac{1}{2}) \right]^2 \sum_{i+j \neq k=6}$$

$$x \quad a_{ijk} Y^j \left[(yW - \frac{1}{2}) + Y \right]^i \tag{4-11}$$

a second change of variables

$$r = YW - \frac{1}{2} \tag{4-12}$$

allows this to be reexpressed as

$$h(y,K) = \frac{1}{y} \int_{-\frac{1}{2}(\xi+1)}^{\frac{1}{2}(\xi-1)} dr \left[r(r+1) \right]^{2} \sum_{i+j+k=6} a_{ijk}\, y^{j} (r+y)^{i}$$

$$\equiv \frac{1}{y} H(y,K) \tag{4-13}$$

As H(y,K) is, by Equation (4-6), $\lim_{y \to o} H(y,K)$ in the limit $s \to \infty$, Equation (4-13) may be expanded about $y = o$; to leading order in y:

$$h(y,K) \to \frac{1}{y} \int_{-1}^{o} dr \left[r(r+1) \right]^{-2} \sum_{i} b_{i}\, r^{i} \tag{4-14}$$

$$b_{i} = a_{i,o,6-i} \tag{4-15}$$

By one more change of variables:

$$r = \frac{T}{S} \tag{4-16}$$

$$h_{asymp}(Y,K) = \frac{1}{S^{2} M^{2}} \int_{-s}^{o} dT \left[T(T+Y) \right]^{-2} \sum_{i} b_{i}\, T^{i} S^{6-i} \tag{4-17}$$

It may readily be seen by comparison to

$$h(S,K,M^{2}) = \frac{1}{SM^{2}} \int_{M^{2} - \frac{s}{2}(1+\xi)}^{M^{2} - \frac{s}{2}(1-\xi)} dT \left[(T-M^{2})(T+S-M^{2}) \right]^{-2} \sum_{i+j+k=6}$$

$$x\; a_{ijk}\, T^{i} (M^{2})^{j} S^{k} \tag{4-18}$$

that the asymptotic form of the cross-section is given by

$$\sigma \to \frac{\pi\alpha^{2}}{4s^{2}} \frac{1}{M^{4}} \left[M^{4} \int_{M^{2} - \frac{S}{2}(1+\xi)}^{M^{2} - \frac{S}{2}(1+\xi)} dT\, A(S,T) \right] \Bigg|_{M^{2}=O}$$

$$= \frac{\pi\alpha^2}{4S^2} \frac{1}{M^4} \int_{-S}^{0} dT \, A'(S,T) \tag{4-19}$$

$$A'(S,T) = [M^4 A(S,T)]\Big|_{M^2=0} \tag{4-20}$$

provided that

$$\sum_i b_i \, T^i S^{6-i} = f(S,T) \, [T(T+S)]^2 \tag{4-21}$$

It is next noted that Equations (3-27) and (3-28) imply that A(S,T) may be written as

$$A(S,T) = A_1(S,T,C_1) + A_2(S,T,\delta) \tag{4-22}$$

where $A_2(S,T,\delta)$ is given by the substitution

$$M'^{\alpha\beta\mu\nu} = N^{\alpha\beta\mu\nu} \tag{4-23}$$

in Equation (3-27), except for an overall factor of $\frac{e^4}{4}$. Contracting $P_{1\alpha}$ and $P_{2\beta}$:

$$N^{\alpha\beta\mu\nu} P_{1\alpha} P_{2\beta} = 2g^{\mu\nu}(k_1 \cdot P_1 + k_2 \cdot P_1 - P_1 \cdot P_2)$$

$$+ 2\{P_2^\nu (P_2 - k_2)^\mu + P_1^\mu (P_1 - k_1)^\nu - \frac{P_1^\mu P_2^\nu (M^2 - 2P_1 \cdot k_1)}{(P_1 \cdot k_1)}\}$$

$$+ \{P_2^\mu (P_2 - k_1)^\nu + P_1^\nu (P_1 - k_2)^\mu - \frac{P_1^\nu P_2^\mu (M^2 - 2P_1 \cdot k_2)}{(P_1 \cdot k_2)}\}$$

$$= 2M^2 \{g^{\mu\nu} - \frac{P_1^\mu P_2^\nu}{k_1 \cdot P_1} - \frac{P_1^\nu P_2^\mu}{P_1 \cdot k_2}\} + 2\{2(k_1+k_2)^\mu (k_1+k_2)^\nu - k_2^\mu (k_1+k_2)^\nu$$

$$- k_1^\nu (k_1+k_2)^\mu\}$$

$$\equiv 2M^2 N^{\mu\nu} + 2\{k_1^\mu (k_1+k_2)^\nu + (k_1+k_2)^\mu k_2^\nu\} \tag{4-24}$$

so

$$(N^{\alpha\beta\mu\nu} P_{1\alpha} P_{2\beta}) (N_{\mu\nu}^{\alpha'\beta'} P_{1\alpha'} P_{2\beta'}) = 4M^4 N^{\mu\nu} N_{\mu\nu}$$

$$+ 8\{(k_1 \cdot k_2)^2 - 2M^2 (k_1 \cdot k_2)\} \tag{4-25}$$

Also

$$N^{\alpha\beta\mu\nu} P_{1\alpha} = 2g^{\mu\nu}(k_1+k_2-P_1)^\beta + 2 [g^{\beta\mu}(P_2-k_1)^\nu + g^{\beta\nu}(P_2-k_2)^\mu]$$

$$+ 2\{\frac{[-P_1^\mu P_2^\nu k_2^\beta + P_1^\mu g^{\beta\nu}(k_2 \cdot P_2) + P_1^\mu P_2^\nu (P_1-k_1)^\beta]}{(k_1 \cdot P_1)}$$

$$+ \frac{[-P_1^\nu P_2^\mu k_1^\beta + P_1^\nu g^{\beta\mu}(k_1 \cdot P_2) + P_1^\nu P_2^\mu (P_1-k_2)^\beta]}{(k_2 \cdot P_1)}\}$$

$$= 2P_2^\beta N^{\mu\nu} + 2 [g^{\beta\mu} k_2^\nu + g^{\beta\nu} k_1^\mu] \tag{4-26}$$

then

$$(N^{\alpha\beta\mu\nu} P_{1\alpha}) (N_{\beta\mu\nu}^{\alpha'} P_{1\alpha'}) = 4M^2 N^{\mu\nu} N_{\mu\nu} \tag{4-27}$$

and similarly

$$(N^{\alpha\beta\mu\nu} P_{2\beta}) (N_{\alpha\mu\nu}^{\beta'} P_{2\beta'}) = 4M^2 N^{\mu\nu} N_{\mu\nu} \tag{4-28}$$

The function $A_2(S,T,\delta)$ is thus

$$A_2(S,T,\delta) \quad = \quad N^{\alpha\beta\mu\nu} N_{\alpha\beta\mu\nu} - 4 N^{\mu\nu} N_{\mu\nu} - 24 \qquad (4\text{-}29)$$

so that lacking any factors of M^{-4}, it does not contribute to the leading asymptotic behavior of Equation (4-19).

With the foregoing developments in mind, the function $A_1'(S,T,C_1)$ has been calculated using the Reduce II program listed in Appendix D (5). The result is expressed in terms of the variable

$$T' \quad = \quad T + \frac{1}{2} S \qquad (4\text{-}30)$$

for which

$$\underset{S\to\infty}{\sigma(K)} \quad = \quad \frac{\pi\alpha^2}{4S^2} \int_{-\frac{1}{2}S}^{\frac{1}{2}S} dT' \; A_1'(S,T',C_1) \qquad (4\text{-}31)$$

Equation (4-21) is precisely satisfied, and

$$A_1'(S,T',C_1) \quad = \quad \frac{C_1^2}{M^4} \{C_1^2 \frac{T'^2}{4} + S^2 [\frac{13}{16} C_1^2 - 2C_1 + 2]\} \qquad (4\text{-}32)$$

Substituting Equation (4-32) into (4-31) and integrating

$$\underset{s\to\infty}{\sigma(S,M)} \quad = \quad \frac{\pi\alpha^2 C_1^2}{24M^4} \{5C_1^2 - 12 C_1 + 12\}S \qquad (4\text{-}33)$$

In utilizing Equation (2-1), it is useful to differentiate both sides with respect to S and then take the limit $s \to \infty$, so that

$$\frac{\pi\alpha^2 C_1^2}{24M^4} \{5C_1^2 - 12 C_1 + 12\} \leq C \lim_{s\to\infty} \frac{\ln s}{s}$$

$$= 0 \qquad (4\text{-}34)$$

Since the cross section must be non-negative, the unitarity bound requires

$$C_1^2\{5C_1^2 - 12\,C_1 + 12\} = 0 \tag{4-35}$$

The polynomial in brackets has two roots which give

$$K = \frac{-1 \mp 2i\sqrt{6}}{5} \tag{4-36}$$

$$\vec{\mu} = \mp\, i\,\frac{\sqrt{6}}{5}\,\frac{e}{M_W}\,\vec{S} \tag{4-37}$$

however as $\vec{\mu}$ is a physically measureable quantity these must be excluded. Thus, there remains only

$$C_1 = 0 \tag{4-38}$$

$$K = 1 \tag{4-39}$$

$$\vec{\mu} = \frac{e}{M_W}\,\vec{S} \tag{4-40}$$

and K is uniquely fixed.

CHAPTER V

IMPLICATIONS OF K = 1

One immediate consequence of Equation (4-39) is that

$$a_o = 0 \tag{5-1}$$

and so the theory is renormalizable. This argument between the first order unitarity approach and the infinite order quadrupole calculation may be easily understood by noting that in the Lee and Young model, K is a renormalized quantity. It should further be noted that K = 1 is the value assigned in the Weinberg-Salam (4) theory of electro-weak interactions, the preceeding argument thus lending support to that model.

A further consequence of Equation (4-39) is

$$A(S,T) = A_2(S,T,\delta) \tag{5-2}$$

since $A_1(S,T,C_1)$ is proportional to C_1. The second term of Equation (4-29) is readily calculated:

$$N^{\mu\nu}N_{\mu\nu} = 4 - 2\,\frac{M^2(k_1\cdot k_2)}{(k_1\cdot P_1)(k_2\cdot P_1)} + \left[\frac{M^2(k_1\cdot k_2)}{(k_1\cdot P_1)(k_2\cdot P_1)}\right]^2 \tag{5-3}$$

while the first term has been found by the Reduce II program of Appendix E (5):

$$N^{\alpha\beta\mu\nu} N_{\alpha\beta\mu\nu} = 16\{M^4(\frac{1}{(k_1 \cdot P_1)^2} + \frac{2}{(k_1 \cdot P_1)(k_2 \cdot P_1)} + \frac{1}{(k_2 \cdot P_1)^2})$$

$$- 4M^2 \frac{(k_1 \cdot k_2)}{(k_1 \cdot P_1)(k_2 \cdot P_1)} + 2M^2 (\frac{1}{k_1 P_1} + \frac{1}{k_2 \cdot P_1})$$

$$- \frac{(P_1 \cdot k_1)}{(P_1 \cdot k_2)} - \frac{(P_1 \cdot k_2)}{(P_1 \cdot k_1)} + 6 \frac{(k_1 \cdot k_2)^2}{(k_1 \cdot P_1)(k_2 \cdot P_1)}$$

$$- 5 \frac{(k_1 \cdot k_2)}{(k_1 \cdot P_1)} + (\frac{k_1 \cdot k_2}{P_1 \cdot k_1})^2 - 5 \frac{(k_1 \cdot k_2)}{(k_2 \cdot P_1)} + (\frac{k_1 \cdot k_2}{k_2 \cdot P_1})^2 + 2\} \tag{5-4}$$

Combining these results

$$N^{\alpha\beta\mu\nu} N_{\alpha\beta\mu\nu} - 4 N^{\mu\nu} N_{\mu\nu} - 24 = 12\{2 - 2 \frac{M^2 (k_1 \cdot k_2)}{(k_1 \cdot P_1)(k_2 \cdot P_1)} + [\frac{M^2 (k_1 \cdot k_2)}{(k_1 \cdot P_1)(k_2 \cdot P_1)}]^2 \}$$

$$+ 16\{ [\frac{(k_1 \cdot k_2)^2}{(k_1 \cdot P_1)(k_2 \cdot P_1)}]^2 - 2 \frac{(k_1 \cdot k_2)^2}{(k_1 \cdot P_1)(k_2 \cdot P_1)}\} \tag{5-5}$$

so

$$A_2(S,T,\delta) = 48\{\frac{1}{2} + \frac{M^2 S}{X} + \frac{M^4 S^2}{X^2}\} + 16\{\frac{S^4}{X^2} + 2 \frac{S^2}{X}\} \tag{5-6}$$

where

$$X \equiv (T - M^2)(T + S - M^2)$$

$$= T^2 + T(S-2M^2) - M^2(S-M^2) \quad (5\text{-}7)$$

Integrating over the azimuthal angle ϕ, the differential cross-section is given by

$$\frac{d\sigma}{dT} = \frac{4\pi\alpha^2}{S^2}\left\{\frac{3}{2} + \frac{S(3M^2+2S)}{X} + \frac{S^2(S^2+3M^4)}{X^2}\right\} \quad (5\text{-}8)$$

and then

$$\sigma = \frac{4\pi\alpha^2}{S^2}\int_{M^2-\frac{S}{2}(1+\xi)}^{M^2-\frac{S}{2}(1-\xi)} dT\left\{\frac{3}{2} + \frac{S(3M^2+2S)}{X} + \frac{S^2(S^2+3M^4)}{X^2}\right\}$$

$$= \frac{4\pi\alpha^2}{S^2}\left[\frac{3}{2}\,\xi S + \frac{6M^2}{S}\,(S-2M^2)\ln\left(\frac{1-\xi}{1+\xi}\right) + \frac{2}{M^2}\,\xi(S^2+3M^4)\right]$$

$$= \frac{4\pi\alpha^2\xi}{M^2}\left[\frac{3}{2}\frac{M^2}{S} + 2\left(1 + \frac{3M^4}{S^2}\right) + \frac{6M^2}{S}\left(\frac{M^2}{S} - \frac{2M^4}{S^2}\right)\frac{1}{\xi}\ln\left(\frac{1-\xi}{1+\xi}\right)\right] \quad (5\text{-}9)$$

Expressed in terms of Y:

$$\sigma = \frac{2\pi\alpha^2\xi}{M^2}\left[4 + 3Y + 12Y^2 - 12\,\frac{Y^2}{\xi}\,(1-2Y)\ln\left(\frac{1+\xi}{1-\xi}\right)\right] \quad (5\text{-}10)$$

with ξ given by Equation (4-9). This result agrees in detail with that given by Sushkov, Flambaum, and Khriplovich (6). In the asymptotic limit

$$\sigma(K=1)_{Y\to o} = \frac{8\pi\alpha^2}{M^2} \quad (5\text{-}11)$$

thus the unitarity bound is respected; this behavior, and that of Equation (4-33)

$$\sigma(K\neq 1)_{S\to\infty} \sim \frac{\alpha^2 S}{M^4} \qquad (5\text{-}12)$$

also agree with the qualitative forms given by Pesic (7).

Reactions in which $\gamma\gamma \to W^+W^-$ appears as a subprocess have been considered in several papers, their asymptotic results for K = 1 being of particular interest. With the diagram of Figure 2a dominating, Sushkov, Flambaum and Khriplovich (6) find

$$\sigma_{e^-e^+ \to e^-e^+ W^-W^+} \sim (\ln S)^4 \qquad (5\text{-}13)$$

Kompaniets (8), as well as Ter-Isaakyan and Khoze (9) find

$$\sigma_{\gamma e^- \to e^-W^+W^-} \sim S(\ln S) \qquad (5\text{-}14)$$

for the dominant diagrams of Figure 2b. While Equations (5-13) and (5-14) seem to imply a unitarity violation for K=1, it must be noted that these calculations ignore the mixing of the Z^o with the virtual photon. Given the fact that this mixing has controlled bad behavior in other processes (4), they cannot, therefore, be considered as conflicting with this papers results; more realistic investigations would be desirable.

Summary and Conclusions

In this paper the process $\gamma\gamma \to W^+W^-$ has been examined in comparison to the unitarity bound

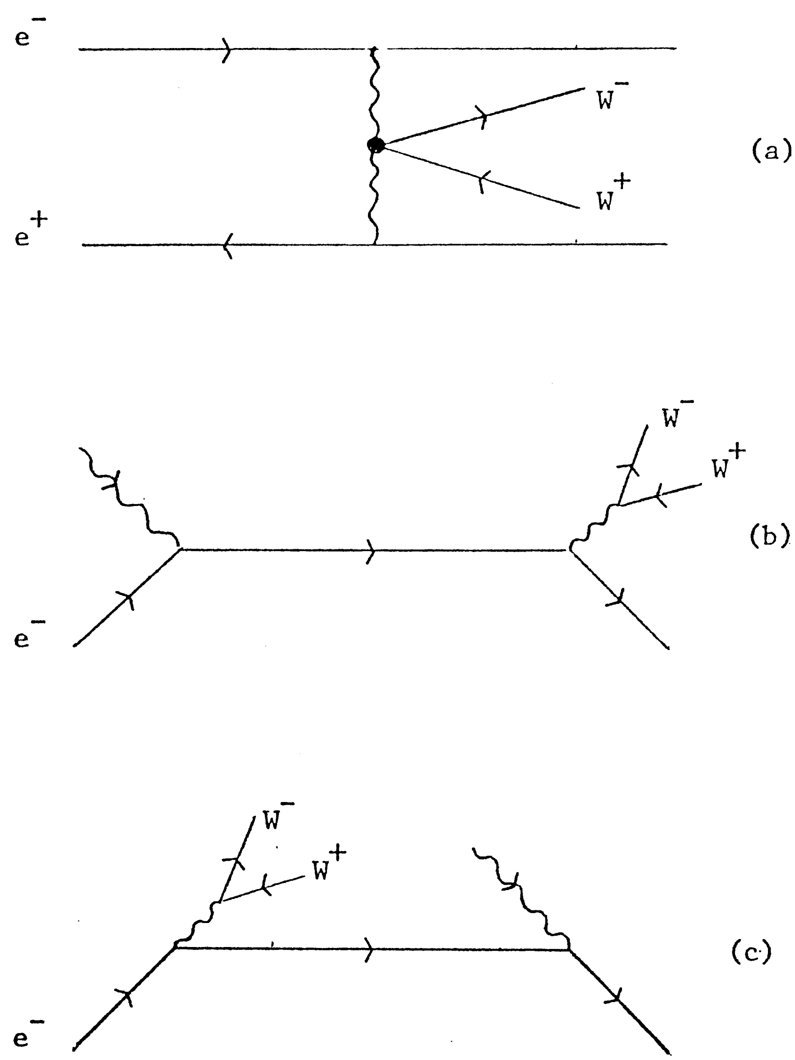

Figure 2. Dominant diagrams for $e^-e^+ \to e^-e^+W^-W^+$ and $\gamma e^- \to e^-W^+W^-$

$$\sigma_{(s\to\infty)} \leq C(\ln s)^2$$

leading to the algebraic equation

$$C_1^2 [5C_1^2 - 12C_1 + 12] = 0$$

In conjunction with the requirement that measurable quantities be purely real, this has been shown to give K=1 as the sole physical value of the anomolous magnetic moment parameter. It should be noted, however, that this result only constrains K_o in

$$K \equiv \sum_{i=o}^{\infty} K_i (\alpha)^i$$

to be unity. The higher order terms could be found by the unitarity approach, or by direct calculation with K_o so fixed (10).

It remains to be seen if the W boson has a K value consistent with $K_o = 1$, although a deviation would imply structure, with serious consequences for the Weinberg-Salam model. The results of Pesic (7) for $e^- e^+ \to e^- e^+ w^+ w^-$ and general K are based upon an unrealistically low W mass of less than 10 GeV; revision and extention to differential cross sections is needed here. In addition, as remarked at the end of Chapter V, this process as well as photoproduction want for more careful study in the context of the Weinberg-Salam model and K_o=1.

Finally, while the transversality mapping and theorem of Chapter III have resulted as a byproduct of this investigation their power should not be underestimated. As demonstrated by Appendix B, they expedite manipulations while allowing the metric form of the photon polarization sum to be retained. This resulted in a factor of fifty in simplification here; even more dramatic reductions may be expected in processes having a larger number of external legs.

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APPENDIXES

APPENDIX A

VECTOR BOSON ELECTRODYNAMICS

The total Lagrangian for vector electrodynamics is

$$L = - \xi (\partial^*_\lambda \phi^*_\lambda)(\partial_\omega \phi_\omega) - \frac{1}{2}\left(\frac{\partial A_\lambda}{\partial X_\omega}\right)\left(\frac{\partial A_\lambda}{\partial X_\omega}\right) - \frac{1}{2} G_{\lambda\omega} G^*_{\lambda\omega} - m^2 \phi_\lambda \phi^*_\lambda$$

$$- ieK\, F_{\lambda\omega}\, \phi^*_\lambda\, \phi_\omega \qquad \text{(A-1)}$$

$$F_{\lambda\omega} = \frac{\partial A_\omega}{\partial X_\lambda} - \frac{\partial A_\lambda}{\partial X_\omega} \qquad \text{(A-2)}$$

$$G_{\lambda\omega} = \partial_\lambda \phi_\omega - \partial_\omega \phi_\lambda \qquad \text{(A-3)}$$

where ξ is a regularization parameter that will be put equal to zero as lowest order processes are being considered, A_ω is the electromagnetic field, and ϕ_ω is the boson field (2). The resulting Feynman rules are given in Table I. To convert these rules, which use imaginary fourth component, scalar product

$$A_\lambda B_\lambda = \vec{A}\cdot\vec{B} - A_4 B_4$$

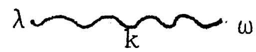
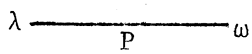
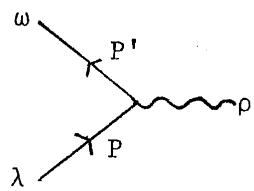
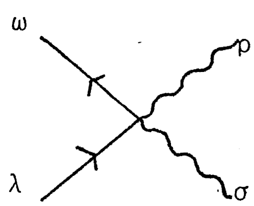
and metric

$$g_{\lambda\omega} = \delta_{\lambda\omega}$$

to standard notation involving real zeroth component, scalar product

TABLE I

FEYNMAN RULES FOR VECTOR BOSON ELECTRODYNAMICS

Element	Graph	Value
Internal Photon Line		$- i\delta_{\lambda\omega}(k^2)^{-1}$
Internal Boson Line		$- i(P^2-m^2)^{-1}(\delta_{\lambda\omega} + m^{-2}P_\lambda P_\omega)$
Three-Vertex		$- ie[\delta_{\lambda\omega}(P+P')_\rho - \delta_{\lambda\rho}(-KP'+P+KP)_\omega$ $-\delta_{\omega\rho}(-KP+P'+KP')_\lambda]$
Four-Vertex		$- ie^2[\partial\delta_{\rho\sigma}\,\delta_{\lambda\omega} - \delta_{\omega\rho}\,\delta_{\lambda\sigma} - \delta_{\omega\sigma}\delta_{\lambda\rho}]$

$$A_\lambda B^\lambda = A_o B^o - \vec{A}\cdot\vec{B}$$

and matric

$$g_{\lambda\omega} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{A-7}$$

the transformation

$$q^2 \rightarrow - q^2 \tag{A-8}$$

$$\delta_{\lambda\omega} \rightarrow - g_{\lambda\omega} \tag{A-9}$$

is used to obtain the resultant rules of Table II.

Polarization sums are given by

$$\sum_\lambda \varepsilon_\omega^{(\lambda)}(k)\ \varepsilon_\rho^{(\lambda)}(k) = - g_{\omega\rho} \tag{A-10}$$

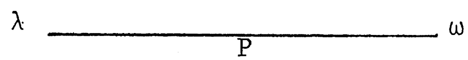
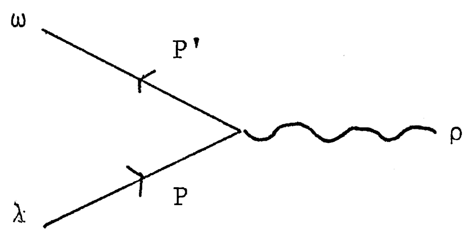
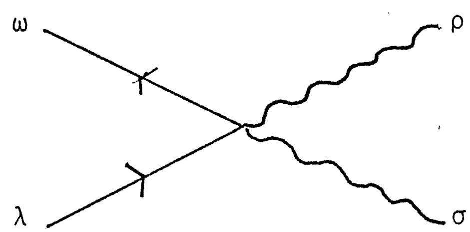
for massless vector particles and

$$\sum_\lambda \varepsilon_\omega^{(\lambda)}(P)\ \varepsilon_\rho^{(\lambda)}(P) = - g_{\omega\rho} + m^{-2} P_\omega P_\rho \tag{A-11}$$

for massive vector particles.

TABLE II

VECTOR BOSON ELECTRODYNAMICS IN STANDARD NOTATION

Element	Graph	Value
Internal Photon Line		$-i\ D_{\lambda\omega}(k^2) = -\ ig_{\lambda\omega}(k^2)^{-1}$
Internal Meson Line		$iS_{\lambda\omega}(P) = i(P^2-m^2)^{-1}(-g_{\lambda\omega}+m^{-2}P_\lambda P_\omega)$
Three-Vertex		$-i\ eV_{\lambda\omega\rho}(P,P',K) = -ie\,[g_{\lambda\omega}(P+P')_\rho$ $-g_{\lambda\rho}(P+P')_\rho$ $-\ g_{\lambda\rho}(-KP'+P+KP)_\omega$ $-\ g_{\omega\rho}(-KP+P'+KP')_\lambda]$
Four-Vertex		$-i\ e^2U_{\lambda\omega\rho\sigma} = -ie^2\,[2g_{\lambda\omega}g_{\rho\sigma} - g_{\lambda\rho}g_{\omega\sigma}$ $-\ g_{\lambda\sigma}g_{\omega\rho}]$

APPENDIX B

APPLICATION OF THE TRANSVERSALITY MAPPING

In applying the transversality mapping to $M^{\alpha\beta\mu\nu}$ it is useful to note that this tensor is symmetric under exchange of the two photons; then according to Equations (2-5) through (2-8) and the definition of $U^{\alpha\beta\mu\nu}$

$$M^{\alpha\beta\mu\nu}_{(a)} \rightarrow M^{\alpha\beta\mu\nu}_{(b)} \tag{B-1}$$

under the operation

$$k_1 \leftrightarrow k_2 \tag{B-2}$$

$$\mu \leftrightarrow \nu \tag{B-3}$$

so that it is sufficient to work with $M^{\alpha\beta\mu\nu}_{(a)}$:

$$M^{\alpha\beta\mu\nu}_{(a)} \equiv V^{\alpha\sigma\mu}(-P_1,q_1,K)\ S_{\sigma\rho}(q_1)\ V^{\rho\beta\nu}(q_1,P_2,K)$$

$$= [\,g^{\alpha\sigma}(q_1-P_1)^{\mu} + g^{\alpha\mu}(P_1+Kk_1)^{\sigma} - g^{\sigma\mu}(q_1+Kk_1)^{\alpha}]$$

$$\times\ [\ \frac{-g_{\sigma\rho} + \dfrac{q_{1\sigma}q_{1\rho}}{M^2}}{q_1^{\ 2} - M^2}]$$

$$\times\ [\,g^{\rho\beta}(q_1+P_2)^{\nu} - g^{\rho\nu}(q_1-Kk_2)^{\beta} - g^{\beta\nu}(P_2+Kk_2)^{\rho}]$$

$$= [\,-2g^{\alpha\sigma}P_1^{\mu} + g^{\alpha\mu}(P_1+Kk_1)^{\sigma} - (1+K)g^{\sigma\mu}k_1^{\alpha}]$$

$$\left[\frac{-g_{\sigma\rho} + \frac{q_{1\sigma} q_{1\rho}}{M^2}}{q_1^{\ 2} - M^2} \right] x$$

$$[\, 2g^{\rho\beta} P_2^{\nu} + (1+K) g^{\rho\nu} k_2^{\beta} - g^{\beta\nu} (P_2 + Kk_2)^{\rho}] \qquad \text{(B-4)}$$

Taking first the terms involving $q_{1\sigma}\ q_{1\rho}$:

$$M^{-2} [-2q_1^{\alpha} P_1^{\mu} + g^{\alpha\mu} (k_1 \cdot P_1 - M^2 - Kk_1 \cdot P_1) - (1+K) q_1^{\mu} k_1^{\alpha}] \ x$$

$$[\, 2q_1^{\ \beta} P_2^{\ \nu} + (1+K) q_1^{\ \nu} k_2^{\ \beta} - g^{\beta\nu} \ (M^2 - P_2 \cdot k_2 + KP_2 \cdot k_2)] =$$

$$M^{-2} [-2k_1^{\alpha} P_1^{\mu} + (1-K)(k_1 \cdot P_1) g^{\alpha\mu} + (1+K) P_1^{\mu} k_1^{\alpha} - M^2 g^{\alpha\mu}] \ x$$

$$[-2k_2^{\beta} P_2^{\nu} + (1+K) P_2^{\nu} k_2^{\beta} + (1-K)(k_2 \cdot P_2) - M^2 \ g^{\beta\mu}] =$$

$$M^{-2} [\, (1-K)[\, (k_1 \cdot P_1) g^{\alpha\mu} - k_1^{\alpha} P_1^{\mu}] - M^2 g^{\alpha\mu}] \ x$$

$$[\, (1-K)[\, (k_2 \cdot P_2) g^{\beta\nu} - k_2^{\beta} P_2^{\nu}] - M^2 g^{\beta\nu}] \qquad \text{(B-5)}$$

then

$$M_{(a)}^{\alpha\beta\mu\nu} = \frac{1}{q_1^2 - M^2} \{ \frac{(1-K)^2}{M^2} [\, (k_1 \cdot P_1) g^{\alpha\mu} - k_1^{\alpha} P_1^{\mu}] [\, (k_2 \cdot P_2) g^{\beta\nu} - k_2^{\beta} P_2^{\nu}]$$

$$- (1-K) g^{\alpha\mu} [\, (k_2 \cdot P_2) g^{\beta\nu} - k_2^{\beta} P_2^{\nu}] - (1-K) g^{\beta\nu} [\, (k_1 \cdot P_1) g^{\alpha\mu} - k_1^{\alpha} P_1^{\mu}]$$

$$+ M^2 g^{\alpha\mu} g^{\beta\nu} + g_{\sigma\rho} [2g^{\alpha\sigma} P_1^{\mu} + (1+K) g^{\sigma\mu} k_1^{\alpha} - g^{\alpha\mu} (P_1 + Kk_1)^{\sigma}]$$

$$\times \left[2g^{\rho\beta}P_2^{\nu} + (1+K)g^{\rho\nu}k_2^{\beta} - g^{\beta\nu}(P_2+Kk_2)^{\rho} \right] \} \qquad \text{(B-6)}$$

Next considering terms proportional to $g_{\sigma\rho}$:

$$g_{\sigma\rho} \left[2g^{\alpha\sigma}P_1^{\mu} + (1+K)g^{\sigma\mu}k_1^{\alpha} - g^{\alpha\mu}(P_1+Kk_1)^{\sigma} \right] \times$$

$$\left[2g^{\rho\beta}P_2^{\nu} + (1+K)g^{\rho\nu}k_2^{\beta} - g^{\beta\nu}(P_2+Kk_2)^{\rho} \right] =$$

$$g_{\sigma\rho} \left[2g^{\alpha\sigma}P_1^{\mu} - g^{\alpha\mu}(P_1-k_1)^{\sigma} + (1+K) \left[g^{\sigma\mu}k_1^{\alpha} - g^{\alpha\mu}k_1^{\sigma} \right] \right] \times$$

$$\left[2g^{\beta\rho}P_2^{\nu} - g^{\beta\nu}(P_2-k_2)^{\rho} + (1+K) \left[g^{\rho\nu}k_2^{\beta} - g^{\beta\nu}k_2^{\rho} \right] \right] =$$

$$(1+K)^2 \left[g^{\sigma\mu}k_1^{\alpha} - g^{\alpha\mu}k_1^{\sigma} \right] \left[g^{\rho\nu}k_2^{\beta} - g^{\beta\nu}k_2^{\rho} \right] g_{\sigma\rho}$$

$$+ (1+K) \left[k_2^{\beta} \left[2g^{\alpha\nu}P_1^{\mu} - g^{\alpha\mu}(P_1-k_1)^{\nu} \right] - g^{\beta\nu} \left[2k_2^{\alpha}P_1^{\mu} - g^{\alpha\mu}(-k_2 \cdot P_2) \right] \right]$$

$$+ (1+K) \left[k_1^{\alpha} \left[2g^{\beta\mu}P_2^{\nu} - g^{\beta\nu}(P_2-k_2)^{\mu} \right] - g^{\alpha\mu} \left[2k_1^{\beta}P_2^{\nu} - g^{\beta\nu}(-P_1 \cdot k_2) \right] \right]$$

$$+ 4g^{\alpha\beta}P_1^{\mu}P_2^{\nu} - 2P_1^{\mu}(P_2-k_2)^{\alpha}g^{\beta\nu} - 2g^{\alpha\mu}P_2^{\nu}(P_1-k_1)^{\beta} + g^{\alpha\mu}g^{\beta\nu}(2k_1 \cdot P_1 - M^2) =$$

$$(1+K)^2 \left[g^{\sigma\mu}k_1^{\alpha} - g^{\alpha\mu}k_1^{\sigma} \right] \left[g^{\rho\nu}k_2^{\beta} - g^{\beta\nu}k_2^{\rho} \right] g_{\sigma\rho}$$

$$- (1+K)g^{\alpha\mu} \left[(k_2 \cdot P_2)g^{\beta\nu} - k_2^{\beta}P_2^{\nu} \right] + 2(1+K)P_1^{\mu} \left[g^{\alpha\nu}k_2^{\beta} - g^{\beta\nu}k_2^{\alpha} \right]$$

$$- (1+K)g^{\beta\nu} \left[(P_1 \cdot k_1)g^{\alpha\mu} - k_1^{\alpha}P_1^{\mu} \right] + 2(1+K)P_2^{\nu} \left[g^{\beta\mu}k_1^{\alpha} - g^{\alpha\mu}k_1^{\beta} \right]$$

$$+ 4g^{\alpha\beta}P_1^{\mu}P_2^{\nu} - 2P_1^{\mu}k_1^{\alpha}g^{\beta\nu} - 2g^{\alpha\mu}P_2^{\nu}k_2^{\beta} + g^{\alpha\mu}g^{\beta\nu}(2k_1 \cdot P_1 - M^2) \qquad \text{(B-7)}$$

Combining these expressions

$$M'^{\alpha\beta\mu\nu}_{(a)} = (q_1^2-M^2)^{-1} \{\frac{(1-K)^2}{M^2}[(k_1\cdot P_1)g^{\alpha\mu}-k_1^{\alpha}P_1^{\mu}][(k_2\cdot P_2)g^{\beta\nu}-k_2^{\beta}P_2^{\nu}]$$

$$+ (1+K)^2 [g^{\sigma\mu}k_1^{\alpha} - g^{\alpha\mu}k_1^{\sigma}][g^{\rho\nu}k_2^{\beta} - g^{\beta\nu}k_2^{\rho}]g_{\sigma\rho}$$

$$- 2g^{\alpha\mu}[(k_2\cdot P_2)g^{\beta\nu}-k_2^{\beta}P_2^{\nu}] + 2(1+K)\ P_1^{\mu}[g^{\alpha\nu}k_2^{\beta} - g^{\beta\nu}k_2^{\alpha}]$$

$$- 2g^{\beta\nu}[(k_1\cdot P_1)g^{\alpha\mu} - k_1^{\alpha}P_1^{\mu}] + 2(1+K)P_2^{\nu}[g^{\beta\mu}k_1^{\alpha} - g^{\alpha\mu}k_1^{\beta}]$$

$$+ 4g^{\alpha\beta}P_1^{\mu}P_2^{\nu} - 2P_1^{\mu}k_1^{\alpha}g^{\beta\nu} - 2g^{\alpha\mu}P_2^{\nu}k_2^{\beta} + g^{\alpha\mu}g^{\beta\nu}(2k_1\cdot P_1)\}$$

$$= (q_1{}^2-M^2)^{-1} \{\frac{(1-K)^2}{M^2}[(k_1\cdot P_1)g^{\alpha\mu} - k_1^{\alpha}P_1^{\mu}][(k_2\cdot P_2)g^{\beta\nu}-k_2^{\beta}P_2^{\nu}]$$

$$+ (1+K)^2 [g^{\sigma\mu}k_1^{\alpha} - g^{\alpha\mu}k_1^{\sigma}][g^{\rho\nu}k_2^{\beta} - g^{\beta\nu}k_2^{\rho}]g_{\sigma\rho}$$

$$+ 2(1+K)[P_1^{\mu}(g^{\alpha\nu}k_2^{\beta} - g^{\beta\nu}k_2^{\alpha}) + P_2^{\nu}(g^{\beta\mu}k_1^{\alpha} - g^{\alpha\mu}k_1^{\beta})]$$

$$+ 4g^{\alpha\beta}P_1^{\mu}P_2^{\nu} - 2g^{\alpha\mu}g^{\beta\nu}(k_1\cdot P_1)\} \qquad \text{(B-8)}$$

where

$$q_1^2 - M^2 = -2k_1\cdot P_1 \qquad \text{(B-9)}$$

Applying the transformation of Equations (B-2) and (B-3), $M'^{\alpha\beta\mu\nu}_{(b)}$ is obtained from $M'^{\alpha\beta\mu\nu}_{(a)}$ and

$$M'^{\alpha\beta\mu\nu} = U^{\alpha\beta\mu\nu} + M'^{\alpha\beta\mu\nu}_{(a)} + M'^{\alpha\beta\mu\nu}_{(b)} \qquad \text{(B-10)}$$

then

$$- M'^{\alpha\beta\mu\nu} = 2g^{\alpha\beta}\left[\frac{P_1^\mu P_2^\nu}{k_1\cdot P_1} + \frac{P_1^\nu P_2^\mu}{k_2\cdot P_1} - g^{\mu\nu}\right]$$

$$+ \frac{1}{2k_1\cdot P_1}\{\frac{(1-K)^2}{M^2}[(k_1\cdot P_1)g^{\alpha\mu} - k_1^\alpha P_1^\mu][(k_2\cdot P_2)g^{\beta\nu} - k_2^\beta P_2^\nu]$$

$$+ (1+K)^2[g^{\sigma\mu}k_1^\alpha - g^{\alpha\mu}k_1^\sigma][g^{\rho\nu}k_2^\beta - g^{\beta\nu}k_2^\rho]g_{\sigma\rho}$$

$$+ 2(1+K)[P_1^\mu(g^{\alpha\nu}k_2^\beta - g^{\beta\nu}k_2^\alpha) + P_2^\nu(g^{\beta\mu}k_1^\alpha - g^{\alpha\mu}k_1^\beta)]\}$$

$$+ \frac{1}{2k_2\cdot P_1}\{\frac{(1-K)^2}{M^2}[(k_2\cdot P_1)g^{\alpha\nu} - k_2^\alpha P_1^\nu][(k_1\cdot P_2)g^{\beta\mu} - k_1^\beta P_2^\mu]$$

$$+ (1+K)^2[g^{\sigma\nu}k_2^\alpha - g^{\alpha\nu}K_2^\sigma][g^{\rho\mu}k_1^\beta - g^{\beta\mu}k_1^\rho]g_{\sigma\rho}$$

$$+ 2(1+K)[P_1^\nu(g^{\alpha\mu}k_1^\beta - g^{\beta\mu}k_1^\alpha) + P_2^\mu(g^{\beta\nu}k_2^\alpha - g^{\alpha\nu}k_2^\beta)]\} \qquad \text{(B-11)}$$

Contracting with $k_{1\mu}$ and $k_{2\nu}$:

$$k_{1\mu} M'^{\alpha\beta\mu\nu} = 2g^{\alpha\beta}k_2^\nu$$

$$= - \omega_{12}k_2^\nu \qquad \text{(B-12)}$$

$$k_{2\nu} M'^{\alpha\beta\mu\nu} = 2g^{\alpha\beta}k_1^\mu$$

$$= \omega_{21}\, k_1^\mu \qquad \text{(B-13)}$$

so except for a factor of e^4

$$\delta = 2^{-2}\left[\,8g^{\alpha\beta}g^{\alpha'\beta'}\left(-g_{\alpha\alpha'} + \frac{P_{1\alpha}P_{1\alpha'}}{M^2}\right)\left(-g_{\beta\beta'} + \frac{P_{2\beta}P_{2\beta'}}{M^2}\right)\right]$$

$$= \frac{1}{4}\left\{8\left[\,3 + \frac{(k_1\cdot k_2)^2 - 2M^2(k_1\cdot k_2)}{M^4}\right]\right\} \qquad \text{(B-14)}$$

APPENDIX C

MANDELSTAM VARIABLES AND PHASE SPACE

The Mandelstam variables for the process $\gamma\gamma \rightarrow \;^{+\;-}$ are:

$$S = (k_1 + k_2)^2$$

$$= (P_1 + P_2)^2 \qquad \text{(C-1)}$$

$$T = (k_1 - P_1)^2$$

$$= (P_2 - k_1)^2 \qquad \text{(C-2)}$$

$$U = (k_1 - P_2)^2$$

$$= (P_1 - k_2)^2 \qquad \text{(C-3)}$$

In the center of mass frame

$$S = 2k_1 \cdot k_2 \qquad \text{(C-4)}$$

$$T = M^2 - \frac{s}{2}(1-\xi \cos\theta) \qquad \text{(C-5)}$$

$$U = M^2 - \frac{s}{2}(1+\xi \cos\theta) \qquad \text{(C-6)}$$

where

$$\xi \equiv \sqrt{1 - \frac{4M^2}{S}} \qquad \text{(C-7)}$$

and in this frame

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}\frac{1}{(4\pi)^2 S}\,\xi\,\overline{|M_{fi}|^2} \tag{C-8}$$

$$\sigma = \frac{1}{4}\frac{1}{(4\pi^2)S}(2\pi)\int_{-1}^{1}\overline{|M_{fi}|^2}\,d(\cos\theta) \tag{C-9}$$

Noting the constraints

$$k_1 + k_2 = P_1 + P_2 \tag{C-10}$$

$$S + T + U = 2M^2 \tag{C-11}$$

A(S,T) is defined as

$$\frac{e^4}{4}A(S,T) = \overline{|M_{fi}|^2}\,\Big|_{k_1\cdot k_2 = \frac{S}{2},\; k_1\cdot P_1 = \frac{M^2-T}{2},\; k_2\cdot P_1 = \frac{S+T-M^2}{2}} \tag{C-12}$$

then since by Equation (C-5)

$$\frac{dT}{d\cos\theta} = \frac{S\xi}{2} \tag{C-13}$$

Equation (C-8) becomes

$$\frac{d\sigma}{d\phi dT} = \frac{1}{2}\frac{1}{(4\pi)^2 S^2}\frac{e^4}{4}A(S,T) \tag{C-14}$$

and the cross-section is given by

$$\sigma = \frac{\pi\alpha^2}{4S^2}\int_{M^2 - \frac{S}{2}(1+\xi)}^{M^2 - \frac{S}{2}(1-\xi)} A(S,T)\,dT \tag{C-15}$$

APPENDIX D

REDUCE II PROGRAM FOR $A_1'(S,T,C_1)$

```
VECTORƀ K1,K2,P1,P2;
MASS ƀ K1=Θ,K2=Θ,P1=M,P2=M;
MSHELL ƀ K1,K2,P1,P2;
OPERATOR ƀ I1,I2,A,A1,A2,A3,A4,F;
FURƀALLƀ B1,K1,J3,P1,J1,J5 ƀ LET
I1(B1,K1,J3,P1,J1) = B1*(J1.J3)-(K1.J3)*(P1.J1),
I2(J1,K1,J5,J3) = (J1.J3)*(K1.J5)-(J1.J5)*(K1.J3);
LET ƀ C2 = 2 - C1;
INDEX ƀ J5,J6;
2*(J3,J4)*((P1.J1)*(P2.J2)/B1 + (P1.J2)*(P2.J1)/B2 - (J1.J2))
+(I1(B1,K1,J3,P1,J1)*I1(B2,K2,J4,P2,J2)*(C1/M)**2
+(J5.J6)*I2(J1,K1,J5,J3)*I2(J2,K2,J6,J4)*C2**1
+2*C2*((P1.J1)*I2(J2,K2,J4,J3) + (P2.J2)*I2(J1,K1,J3,J4)))/(2*B1)
+(I1(B2,K2,J3,P1,J2)*I1(B2,K1,J4,P2,J1)*(C1/M)**2
+(J5.J6)*I2(J2,K2,J5,J3)*I2(J1,K1,J6,J4)*C2**2
+2*C2*((P1.J2)*I2(J1,K1,J4,J3) + (P2.J1)*I2(J2,K2,J3,J4)))/(2*B2) $
SAVE AS ƀ A(J3,J4) $
SUB ƀ(J3=J7,J4=J8,A(J3,J4))$
SAVEAS ƀ A(J7,J8) $
INDEX ƀ J1,J2 $
A(J3,J4)*A(J7,J8) $
SAVE AS ƀ A(J3,J4,J7,J8) $
INDEX ƀ J3,J4,J7,J8;
A(J3,J4,J7,J8)*(J3,J7)*(J4.J8) $
SAVE AS ƀ A1;
A(J3,J4,J7,J8)*(J3.J7)*(P2.J4)*(P2.J8) $
SAVEAS ƀ A2;
A(J3,J4,J7,J8)*(P1.J3)*(P1.J7)*(J4.J8) $
SAVEAS ƀ A3;
A(J3,J4,J7,J8)*(P1.J3)*(P1.J7)*(P2.J4)*(P2.J8) $
SAVEAS ƀ A4;
A1*(M**4) - A2*(M**2) - A3*(M**2) + A4;
SUB ƀ (B1 = K1.P1,B2 = K2.P1,!*ANS) $
SUB ƀ (P1.P2=K1.K2-M**2, P2.K2=P1.K1,P2.K1=P1.K2,!*ANS);
SUB ƀ (K1.K2=S/2,K1.P1 = S/4-T1/2,K2.P1 = S/4 + T1/2,!*ANS);
SUB ƀ (M=Θ,!*ANS);
FORALL ƀ C1 ƀ SAVEAS ƀ F(C1);
F(C1) - F(Θ);
SAVEAS ƀ F(S,T1);
ARRAY ƀ X(6);
COEFF(F(S,T1),T1,X);
WRITE ƀ "AΘ ƀ 1S",X(Θ);
```

```
WRITE b̸ "A2 b̸ 1S",X(2);
WRITE b̸ "A4 b̸ 1S",X(4);
WRITE b̸ "A6 b̸ 1S",X(6);
```

APPENDIX E

REDUCE II PROGRAM FOR $N^{\alpha\beta\mu\nu} N_{\alpha\beta\mu\nu}$

```
VECTOR b̸ K1,K2,P1,P2;
MASS b̸ K1=0,K2=0,P1=M,P2=M;
MSHELL b̸ K1,K2,P1,P2;
OFFb̸MCD;
OPERATORb̸I1,I2,A,A1,A2,A3,A4,F;
FORALLb̸B1,K1,J3,P1,J1,J5 b̸ LET
I1(B1,K1,J3,P1,J1) = B1*(J1.J30-(K1.J3)*(P1.J1),
I2(J1,K1,J5,J3) = (J1.J3)*(K1.J5)-(J1.J5)*(K1.J3);
LET b̸ C1=0,C2=2;
INDEX b̸ J5,J6;
2*(J3,J4)*((P1.J1)*(P2.J2)/B1 + (P1.J2)*(P2.J1)/B2 - (J1.J2))
+(I1(B1,K1,J3,P1,J1)*I1(B2,K2,J4,P2,J2)*(C1/M)**2
+(J5.J6)*I2(J1,K1,J5,J3)*I2(J2,K2,J6,J4)*C2**2
+2*C2*((P1.J1)*I2(J2,K2,J4,J3) + (P2.J2)*I2(J1,K1,J3,J4)))/(2*B1)
+(I1(B2,K2,J3,P1,J2)*I1(B1,K1,J4,P2,J1)*(C1/M)**2
+(J5.J6)*I2(J2,K2,J5,J3)*I2(J1,K1,J6,J4)*C2**2
+2*C2*((P1.J2)*I2(J1,K1,J4,J3) + (P2.J1)*I2(J2,K2,J3,J4)))/(2*B2) $
SAVE AS b̸ A(J3,J4) $
SUB b̸(J3=J7,J4=J8,A(J3,J4)) $
SAVEAS b̸ A(J7,J8) $
INDEX b̸ J1,J2;
A(J3,J4)*A(J7,J8) $
SAVEAS b̸ A(J3,J4,J7,J8) $
INDEX b̸ J3,J4,J7,J8;
A(J3,J4,J7,J8)*(J3.J7)*(J4.J8) $
SUB b̸(B1 = K1.P1, B2 = K2.P1,!*ANS) $
SUB b̸(P1.P2=K1.K2-M**2, P2.K2=P1.K.,P2.K1=P1.K2,!*ANS);
```

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