SYMMETRICAL COUPLER CURVE WITH A SINGLE CUSP OR TWO SYMMETRICAL CUSPS

By

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CHAPTER I

INTRODUCTION

A coupler curve is the path of a point on the connecting link of a four-bar linkage. The coupler point may be used directly as an output or to drive other links to achieve certain motions (Figure 1).

In 1784, Watt invented the "straight line motion" generated by the coupler of a four-bar linkage (1). His invention of this significant motion diverted the attention from the follower link as the output towards the coupler. The coupler curve was first studied analytically by Prony (2), who examined Watt's mechanism for deviations in 1796. Samuel Roberts (3) in 1876 found that the coupler curve of a four-bar linkage is of the sixth order. He called it a "Three-Bar Curve"; since then only the moving links were counted and called bars. Later, Cayley and others (4) tried to explore some linkages that are able to generate specific algebraic curves of any order. For more information about the four-bar coupler curve, the reader is referred to Chapter 6 of Reference (1).

Coupler curves have a variety of shapes. They may have symmetry about an axis as well as double points. A double point is a point where the curve intersects itself; that is, it has two tangents at that point. The double point might be a crunode where the tangents are distinct and intersect at an angle. It also might be a cusp where the tangents are coincident; that is, the curve is tangent to itself. Also, the four-bar

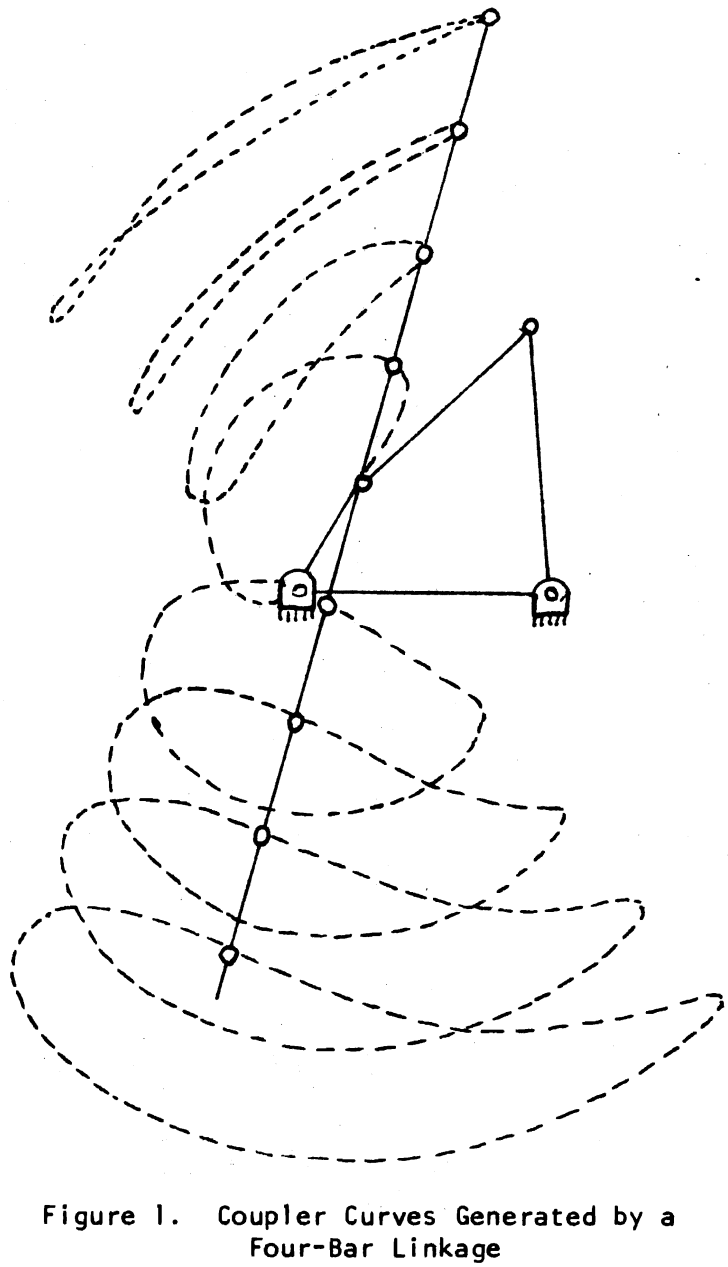

Figure 1. Coupler Curves Generated by a Four-Bar Linkage

linkage is able to generate an approximate straight line coupler curve.

Burmester (5) motivated by finding a general method for the synthesis of a four-bar linkage developed his famous theory which is an important part of the curvature theory. His graphical method guides a point through up to five finitely separated positions. Mueller (6) attacked the same problem but concentrated on the concept of a single-position design. His work was purely geometrical. Allievi (7, 8) directed his attention towards purely analytical methods in coplanar motion. Hain (9), Beyer (10), Hall (16), and Cowie (19) also presented some synthesis procedures in their books.

There is no general and easy procedure to synthesize a four-bar linkage that generates a desired coupler point motion. The curvature theory is very complicated and researchers usually attack special problems in special ways.

In this work the emphasis is oriented towards a single-position design (6). The higher-path curvature theory which was first introduced by Wolford (13) in 1960, and Freudenstein (14) in 1964 can serve the purpose. The basic characteristics of the motion are then studied in terms of the "instantaneous invariants" in which the motion is represented as a displacement from a reference position by means of a power series in the motion coordinates. This concept was first introduced by Bottema (15) in 1961, and elaborated by Veldkamp (16) later in 1963. For a better view about this method, see Appendix C.

The higher-order curvature theory is not going to be used in this work since the geometrical methods are more convenient for this particular problem. In other problems, such as the design of a linkage that generates an approximate straight-line motion or a circular arc, the higher-order curvature theory become more powerful. A brief introduction about the straight-line motion is presented in Appendix C.

This study deals mainly with the double point of a coupler curve. In particular its aim is to synthesize a four-bar linkage that generates a symmetrical coupler curve with a single cusp and another which generates two symmetrical cusps.

CHAPTER II

GENERAL BACKGROUND

2.1 Introduction

Before dealing with the synthesis problem, it is felt that a brief introduction to the theory behind it is necessary. The theoretical concepts that are going to be dealt with later on in Chapter III can be divided into four topics:

1. Instant Center
2. Fixed and Moving Centrodes
3. Symmetrical Coupler Curve
4. Double Point.

2.2 Instant Center

When two bodies move relative to one another, the instant center is the point common to both bodies and has the same velocity in each of the two bodies (1, 17, 18).

The instant center of a four-bar linkage, relative to which the coupler link moves, coincides with the intersection of the two straight lines colinear with the two grounded links, respectively. In Figure 2, I is the instant center of the moving link AB of the linkage A_0ABB_0.

2.3 Fixed and Moving Polodes

Fixed and moving polodes are the curves traced by the instant center

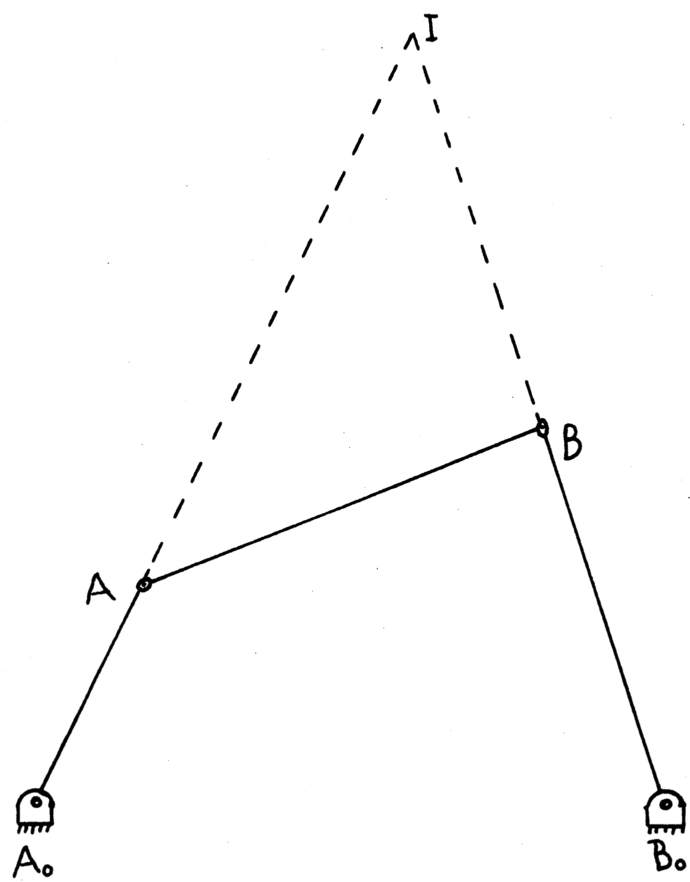

Figure 2. Instantaneous Center of the Coupler of a Four-Bar Linkage

of velocity on both fixed and moving planes. That is, a body moving with respect to another fixed body and two planes each is attached to one of them; the plane attached to the moving body is called the moving plane and the plane attached to the fixed plane is called the fixed plane. The curve traced by the instant pole on the moving plane is called the moving polode or centrode; it is also called the body polode. The curve traced on the fixed plane is called the fixed polode or centrode; it is also called the space polode (Figure 3).

This relative motion of the moving body with respect to the fixed body can be reproduced by having two bodies with profiles similar to the polodes moved against each other by pure rolling. The motion is pure rolling because the polodes are the locii of the instant center of velocity in two distinct planes. The relative velocity between the two bodies at the position of the instant center is zero.

As an example, a circular disk rolling along a straight path represents a circular body polode and a straight-line space polode (Figure 4) (1, 17, 18).

2.4 Symmetrical Coupler Curve

A four-bar linkage will generate a symmetrical coupler curve if the length of the coupler link AB is equal to the length of the follower link BB_0 and equal to the coupler arm BP (Figure 5) (1, 19, 20). The axis of symmetry of the coupler curve passes through the fixed point B_0 and through the point P^*; P^* is the position of P when the input link A_0A is in line with the fixed link A_0B_0. The angle made by the intersection of the axis of symmetry and the fixed link is equal to $[\pi/2 - \beta]$; β is the angle PAB.

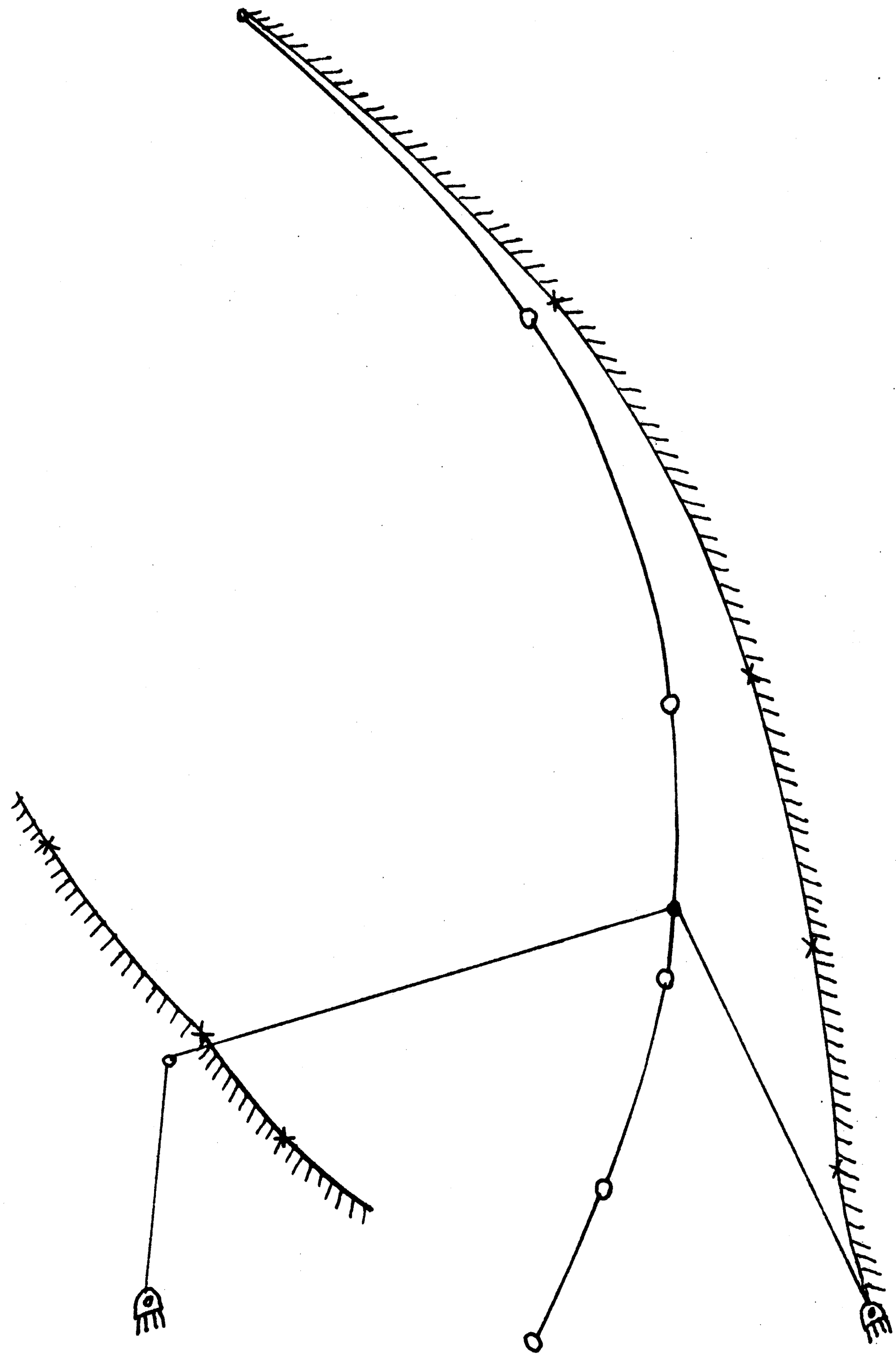

Figure 3. Fixed and Moving Polodes

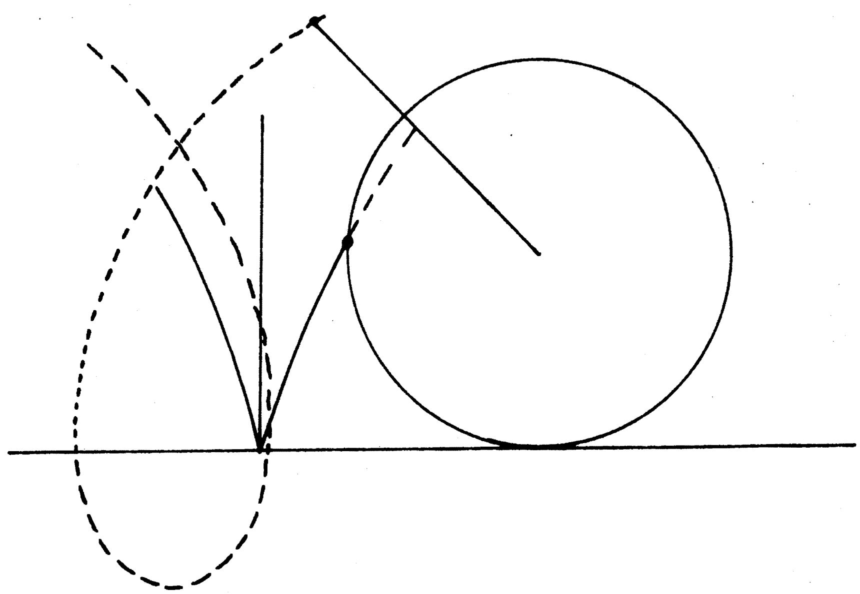

Figure 4. A Straight Line Space and a Circular Body Centrode

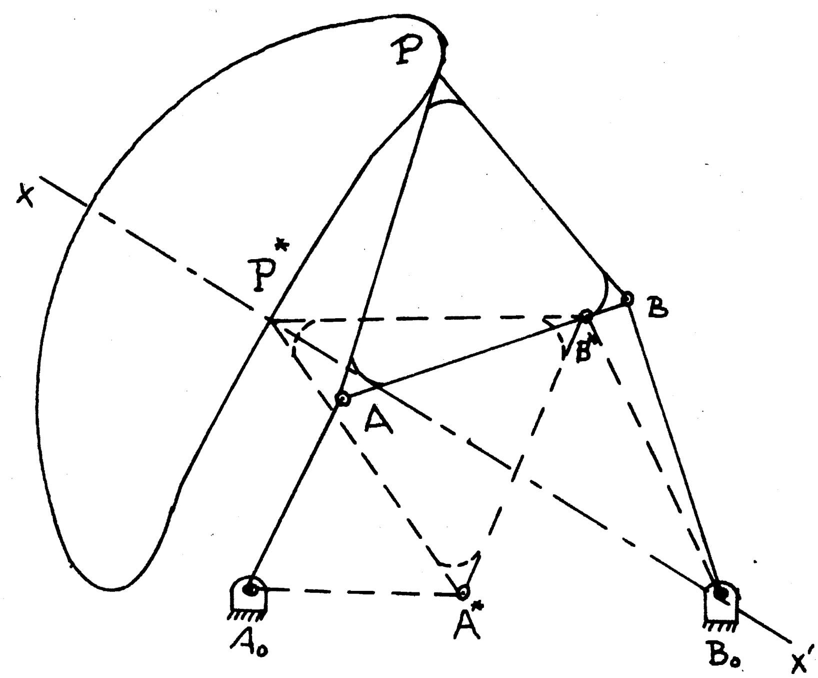

Figure 5. Symmetrical Coupler Curve

2.5 Double Point

A double point is the point where the coupler curve intersects itself. The double point is always located on a circle that circumscribes the triangle A_0B_0C, which is similar to the coupler triangle ABP (Figure 6) (1, 19, 20). It is also called the circle of singular foci. A double point can be a crunode or a cusp.

2.5.1 Crunode

If the tangents at the double point intersect at an angle, the double point is called a crunode. An eight shape (∞) is an example of a crunode. It is possible that a coupler curve intersects itself in more than one point (1, 9, 20).

2.5.2 Cusp

A cusp is a double point where the tangents are colinear or the angle between the two tangents is zero. A very common example of a cusp is the curve traced by a point on the periphery of a rolling wheel. In Figure 7, the curve is the cycloid. The wheel is the body polode and the straight line is the space polode. A point P on the periphery belongs to the moving or body polode. Let I be a point on the fixed or space polode. When P coincides with I, I becomes the instantaneous center of velocity. P comes down towards I, stops as it coincides with I, and moves back in an opposite approach. Although the velocity of P is zero at I, it experiences no discontinuity.

A cusp, then, can be generated if a point belongs to the body polode and coincides with the instantaneous center of velocity. For a

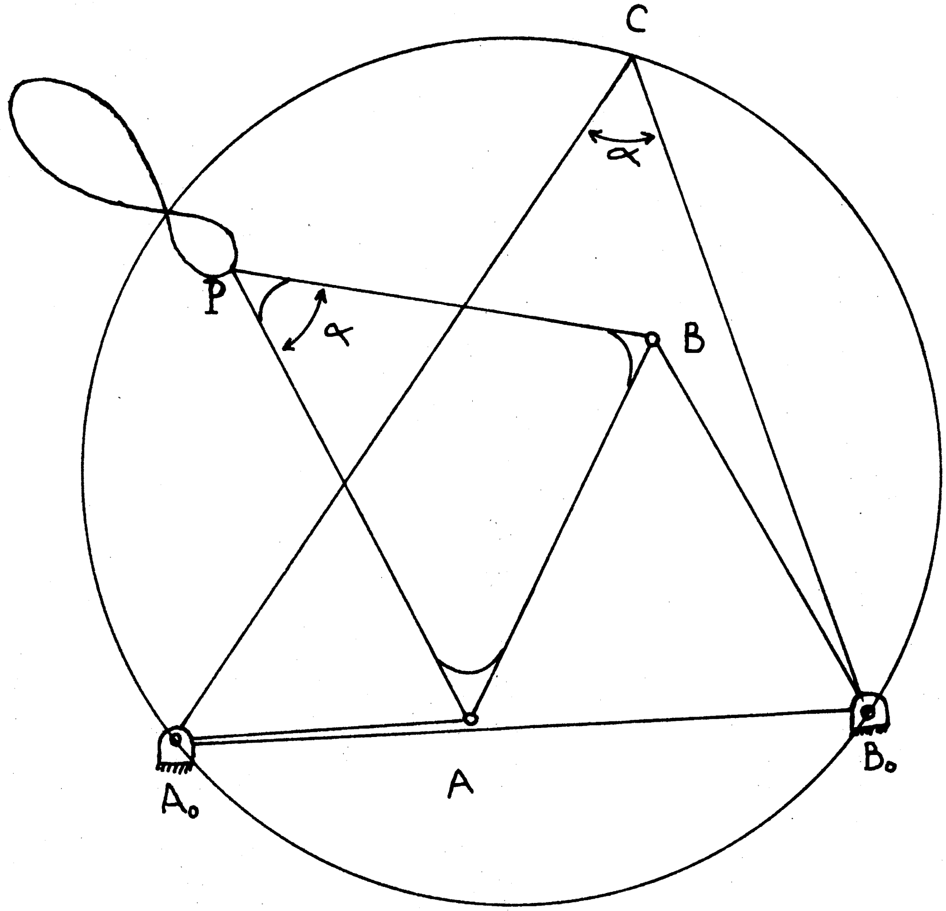

Figure 6. Coupler Curve With a Double Point

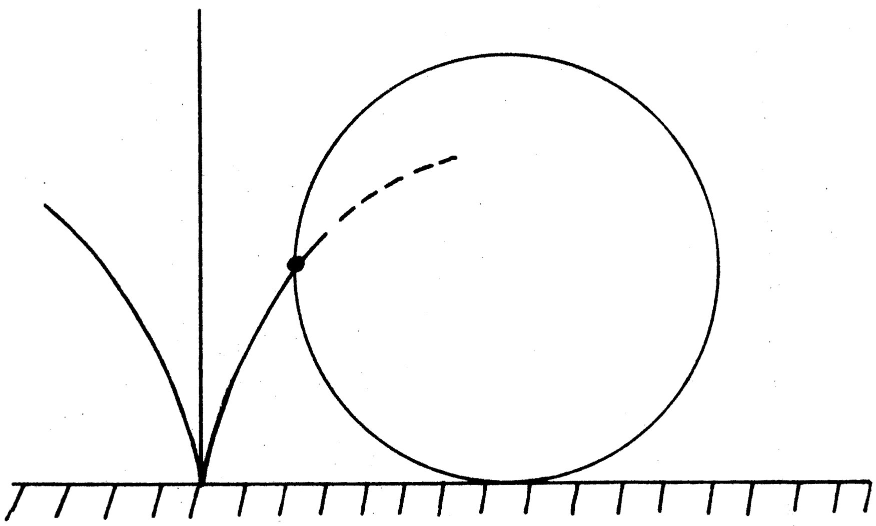

Figure 7. A Cusp Generated by a Point on the Periphery of a Rolling Wheel

four-bar linkage, a coupler point that is lying on the moving centrode will develop a cusp if it is coinciding with the instant center at that instant.

The tangent to the coupler curve at the cusp is normal to the fixed polode (1, 9).

CHAPTER III

SYNTHESIS

3.1 Synthesis of a Four-Bar Linkage That Generates a Symmetrical Coupler Curve With Two Cusps

A coupler point which happens to lie on the moving polode will generate a cusp as it coincides with the instantaneous center of velocity (1). The concept of single-position design is followed here. Let the design position be taken at the instant the coupler point is coinciding with the instantaneous center of velocity of the coupler link (Figure 8). This position of the four-bar will satisfy the condition for a cusp. A point P (Figure 8) coinciding with I, the instant center of the coupler link belongs to the moving centrode which at that instant is rolling over the fixed centrode.

Introducing symmetry to the problem, another cusp will be generated. The two cusps are symmetrical about an axis that passes through B_0 and makes an angle α with the fixed link of the four-bar (Figure 9).

$$\alpha = \left(\frac{\pi}{2} - \beta\right)$$

where β is the angle PAB.

Symmetry will add the condition that requires the length of the follower link BB_0 to be equal to the length of the coupler link and to the length of the coupler arm BP.

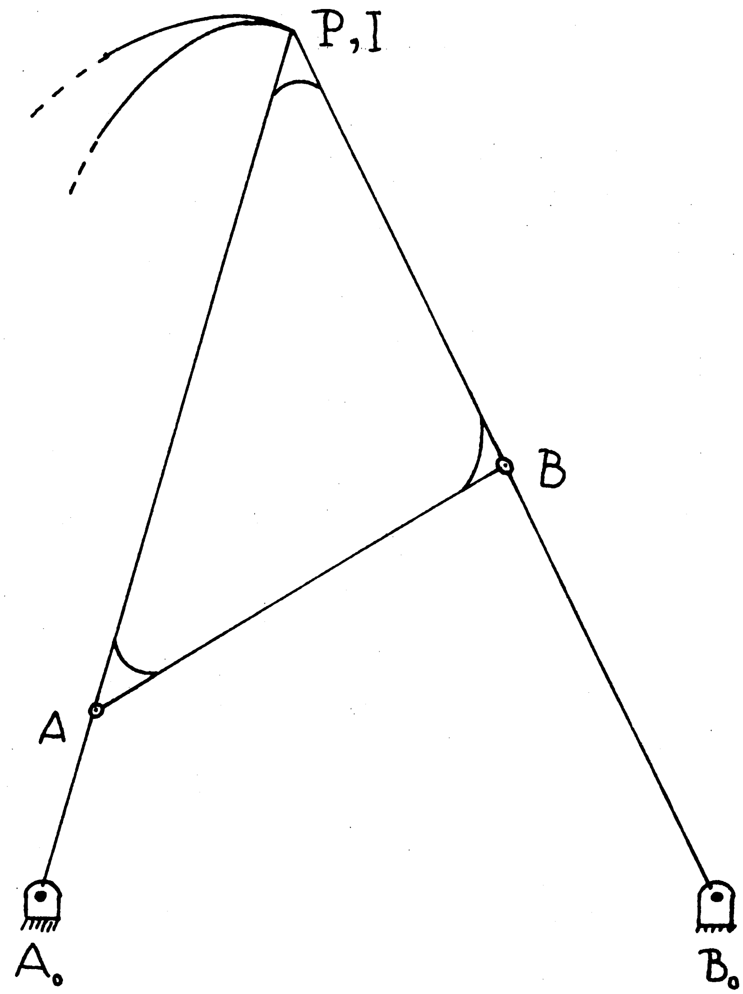

Figure 8. Design Position; the Coupler Point is Coinciding With the Instant Center

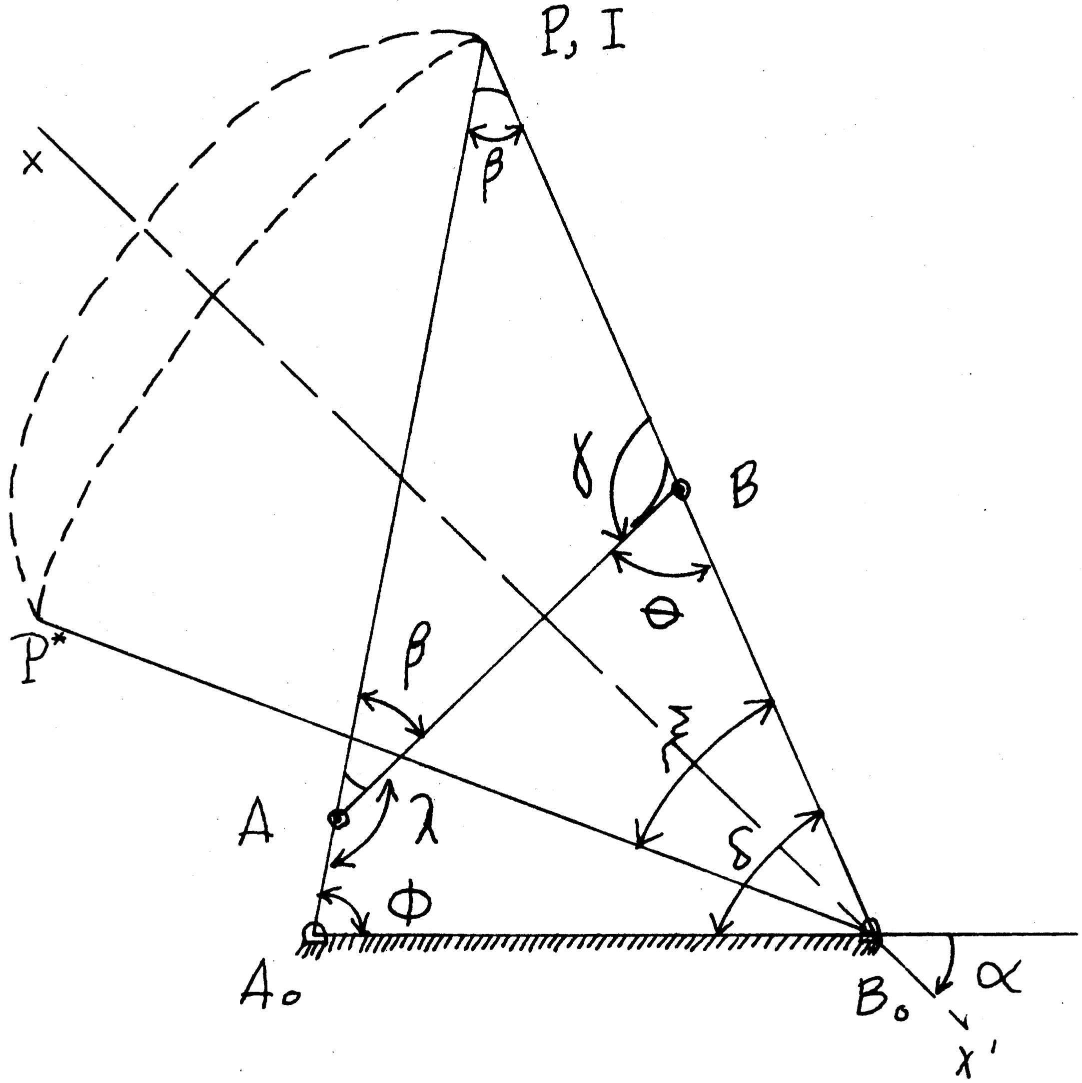

Figure 9. Detailed Figure of the Design Position of Two Symmetrical Cusps

To synthesize a four-bar, the lengths of its four links should be found. The coupler link's dimensions should also be found. The constraints on this design are introduced by the symmetry and the position of the four-bar linkage.

From symmetry,

$$BB_o = AB = BP$$

$$\alpha = \left(\frac{\pi}{2} - \beta \right)$$

Some assumptions have to be made. If the length of one of the links of the four-bar linkage is assumed, the generality of the problem is preserved. The lengths of the other three links will be found in terms of the assumed length. The angle α can be treated as a parameter. For every α there is a different design (Figure 9).

$$\alpha = \left(\frac{\pi}{2} - \beta\right)$$

Assume α is given and then find β.

$$\beta = \left(\frac{\pi}{2} - \alpha\right)$$

Angles APB and BAP are equal since the coupler triangle ABP is isosceles. Angle ABP is equal to γ.

$$\gamma = \pi = 2\beta$$

or

$$\gamma = 2\alpha$$

By assumption let the length of the link BB_0 be equal to unity. From symmetry then,

$$AB = BP = BB_0 = 1.0$$

and

$$(AP)^2 = (AB)^2 + (BP)^2 - 2(AB)(BP)\cos(\gamma)$$

or

$$(AP)^2 = 2(1 - \cos\gamma)$$

Let θ be the angle ABB_0.

$$\theta = \pi - \gamma$$

At this stage one more parameter is needed. Let the angle of span between the two cusps be equal to ξ: it is the angle at B_0 between PB_0 and P^*B_0 (Figure 9). Therefore, the angle between PB_0 and the axis of symmetry is equal to half of ξ. Since α is assumed before (α is the angle between the fixed link A_0B_0 and the axis of symmetry), the angle between PB_0 and A_0B_0 can be found. Let this angle be called δ.

$$\delta = \frac{\xi}{2} + \alpha$$

Let the angle A_0AB be called λ. Then,

$$\lambda = \pi - \beta$$

For the quadrilateral A_0ABB_0, the sum of the internal angles should be equal to 360°. Therefore,

$$\phi = 360° - \delta - \theta - \lambda$$

So far the lengths of the links A_0A and A_0B_0 are not yet found.

Applying the Sine law in triangle A_0PB_0:

$$\frac{B_0P}{\sin\phi} = \frac{A_0B_0}{\sin\beta} = \frac{A_0P}{\sin\delta}$$

B_0P is known and all the angles ϕ, β, and δ are also known; it is easy then to determine the length of the link A_0B_0.

$$A_0B_0 = \frac{B_0P \sin\beta}{\sin\phi}$$

and

$$A_0P = \frac{B_0P \sin\delta}{\sin\phi}$$

and finally,

$$A_0A = A_0P - AP.$$

3.2 Synthesis of a Four-Bar Linkage That Generates a Symmetrical Coupler Curve With a Single Cusp

It has been mentioned before that a cusp is generated when a coupler point lying on the moving centrode passes through the instantaneous center of velocity. In the first part of this chapter, a four-bar that generates a symmetrical coupler curve with two cusps is synthesized. The design position is chosen at a point that does not belong to the axis of symmetry. In that case, another cusp is generated symmetrical to the first with respect to the axis of symmetry. It is clear then that a point lying on the axis of symmetry chosen as the design position will develop a symmetrical coupler curve but it will have only one cusp.

A very convenient design position is B_0, the fixed end of the follower link (Figure 10). P should coincide with B_0. The instantaneous center of velocity "I" should also coincide with B_0. The points "P" and "I" will then coincide and P will belong to the moving centrode which at

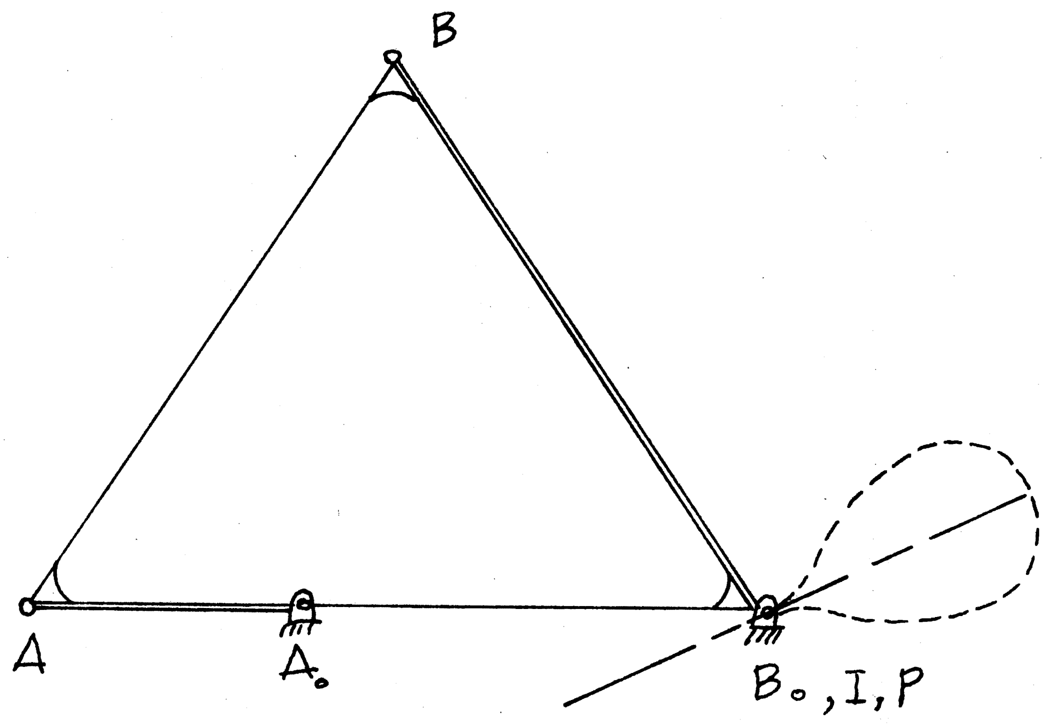

Figure 10. Symmetrical Coupler Curve With a Single Cusp

that instant is rolling over the fixed centrode at the position of the instant center I. This position will allow a cusp to be developed. The orientation of the four-bar will be such that the input link AB is in line with the fixed link A_0B_0. The coupler point P will coincide with B_0. B_0 belongs to the axis of symmetry of the coupler curve. The cusp will lie on the axis of symmetry and thus symmetry conditions will not help to generate another one. The curve itself is symmetrical and the axis of symmetry is the tangent at the cusp.

To synthesize the linkage two parameters are needed as in the previous case. Let the angle of inclination α of the axis of symmetry with respect to the fixed link A_0B_0 be one parameter. The second parameter is the length of the segment along the axis of symmetry limited by the two extreme positions of the coupler point. These two extreme positions are coordinated by the position of the linkage when the input link AB is in line with the fixed link A_0B_0 (Figure 11).

In Figure 12, α is known; then

$$\beta = \frac{\pi}{2} - \alpha$$

$$\gamma = \pi - 2\beta = 2\alpha$$

In triangle B_0B_2P

$$(B_0P)^2 = (B_0B_2)^2 + (B_2P)^2 - 2(B_2P)(B_0B_2)\cos\theta$$

$$\cos\theta = \frac{(B_0B_2)^2 + (B_2P)^2 - (B_0P)^2}{2(B_2P)(B_0B_2)}$$

If the follower link B_0B is assumed unity:

$$B_0B = BP = AB = 1.0$$

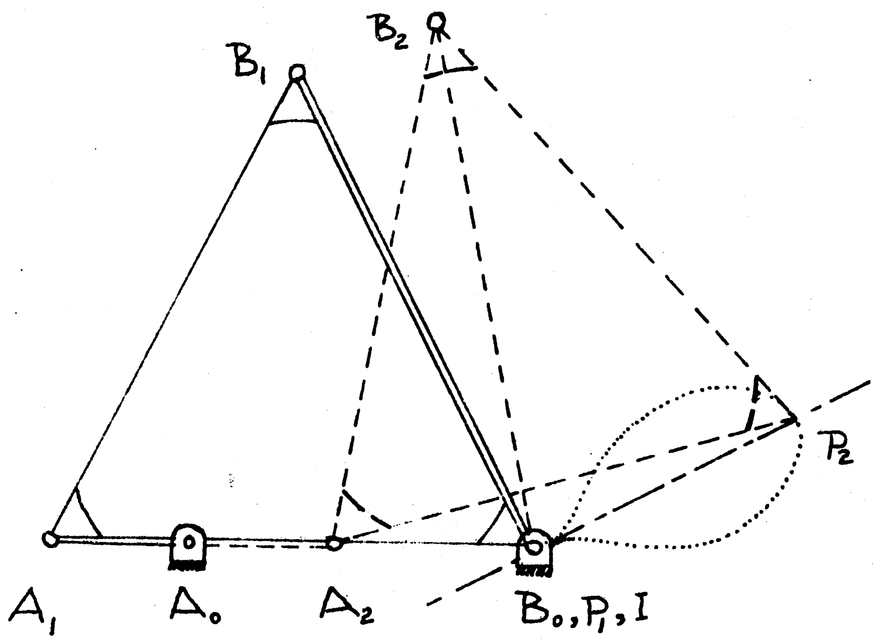

Figure 11. The Two Extreme Positions of the Coupler Point of a Four-Bar Linkage Generating a Coupler Curve With a Single Cusp

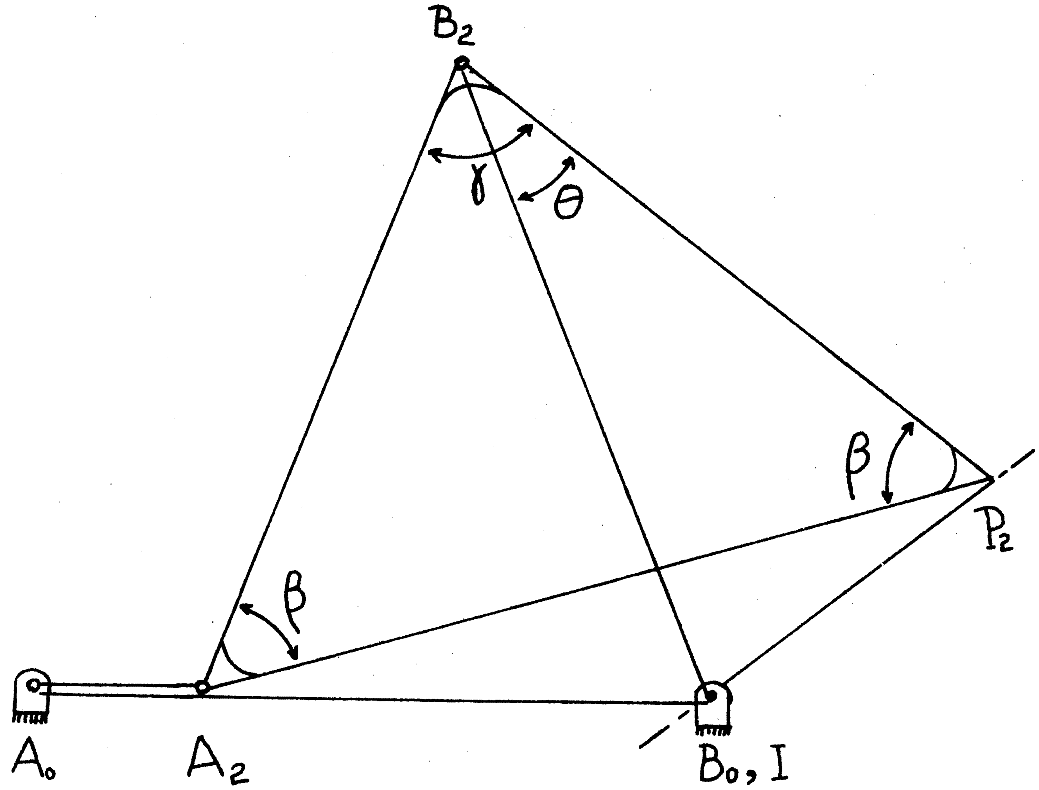

Figure 12. Detailed Figure for a Single Cusp Design
(Position 2)

therefore,

$$\cos\theta = \frac{2 - (B_0P)^2}{2}$$

Once θ is known, λ can be found and thus

$$(A_2B_0)^2 = 2(1 - \cos\lambda)$$

From Figure 13,

$$(A_1B_0)^2 = 2(1 - \cos\gamma)$$

but

$$A_1B_0 = A_2B_0 = 2AA_0$$

therefore,

$$AA_0 = \frac{A_1B_0 - A_2B_0}{2}$$

and finally,

$$A_0B_0 = A_2A_0 + A_2B_0.$$

3.3 Results and Discussion

To complete the design procedures, two computer programs were written. One solves for two symmetrical cusps and the other solves for a single cusp. Both programs calculate the lengths of the four links following the procedure presented in sections 3.1 and 3.2, respectively. The type of linkage is checked by Grashof's rule for a four-bar mechanism. A subroutine was written for this purpose. The crank-rocker mechanism is the one of significant importance to this work. In the

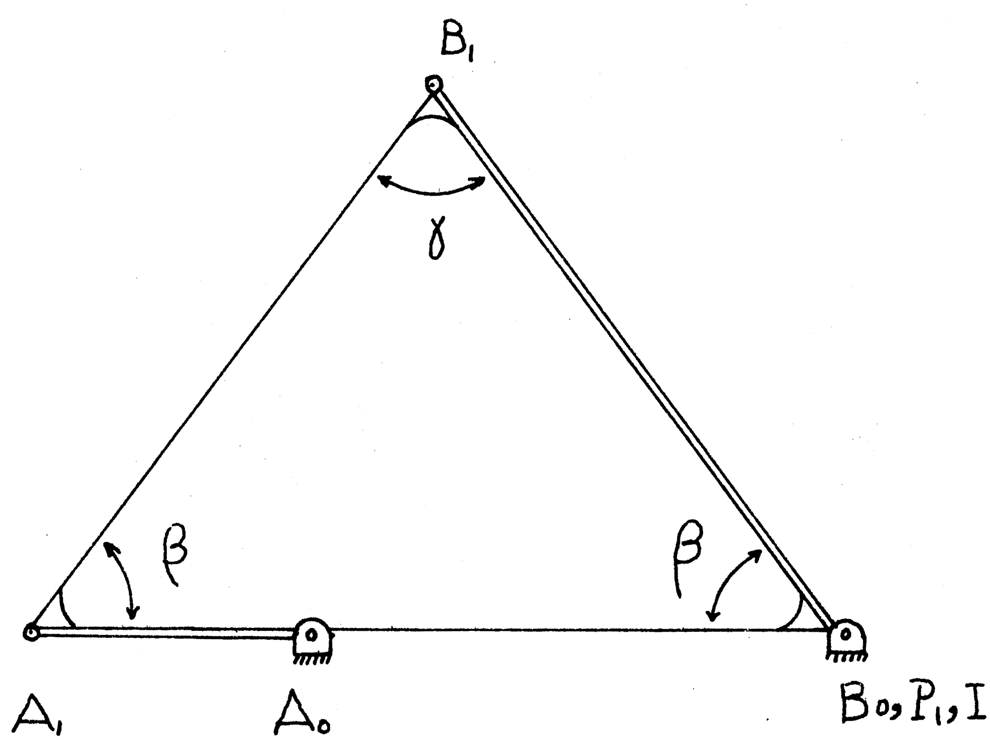

Figure 13. Detailed Figure for a Single Cusp Design (Position 1)

case of a crank-rocker, the minimum and maximum transmission angles are calculated.

The results are tabulated in Tables I through XVI and some design charts for a crank-rocker mechanism are prepared. The tabulated results include the length of each link, the coupler arm AP and the angle β between AP and the coupler link AB (see Figure 14). For two symmetrical cusps, β is measured counterclockwise from AB. In the case of a single cusp design, β is measured in a clockwise direction from AB. Also, the design tables contain the minimum and maximum transmission angles as well as the type of the mechanism. Each table lists all of the above mentioned quantities for different values of the angle α and one value of the angle ξ for the two cusps design or one value of the length B_0P for a single cusp design. For ξ equal to 10.0 and B_0P equal to 0.5, see Tables I and II, respectively. Tables III through IX (Appendix A) are prepared for two symmetrical cusps design, while Tables X through XVI (Appendix B) are for a single cusp design.

The tabulated results are plotted in the form of design charts. The plotted curves are those of the link proportions of the input, the coupler and the ground links normalized with respect to the follower link length. The input plot is a dotted line, the coupler plot is a solid line, and the ground plot is a hyphenated line. The abscissa in these graphs is α, the angle of inclination of the axis of symmetry with respect to the fixed link. The ordinates are the normalized lengths. Due to the symmetry, the coupler link should have the same length as the follower, and hence the normalized curve of the coupler is a straight line parallel to the α axis at a value of 1.

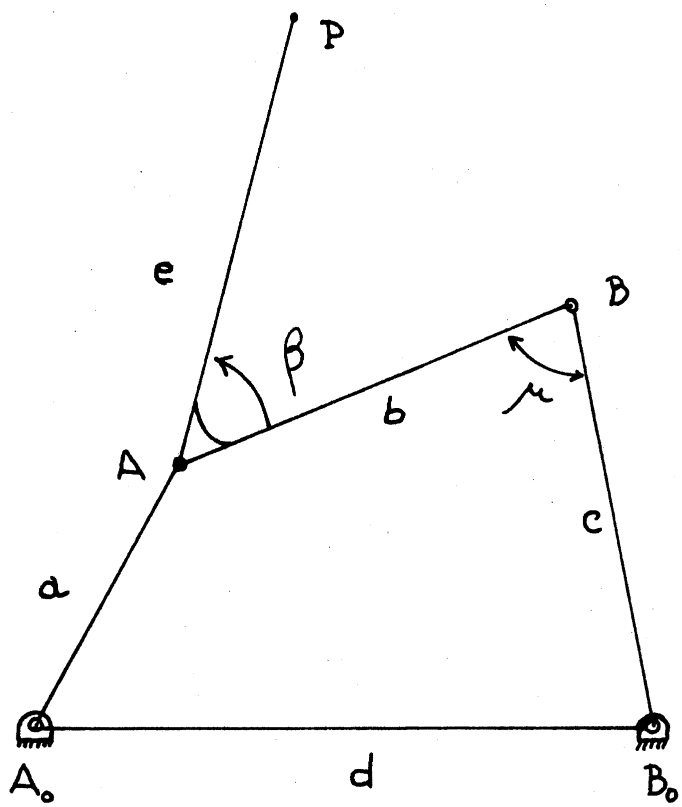

Figure 14. Detailed Figure of a Synthesized Four-Bar Linkage

TABLE I

DESIGN TABLE FOR TWO SYMMETRICAL CUSPS WITH ξ = 10 DEGREES

ξ	α	a	b	c	d	e	β	μ Min	μ Max	Type
10.0	15.0	0.4132	1.0	1.0	2.6288	0.5176	75.0	---	---	Double Rocker
10.0	20.0	0.5720	1.0	1.0	2.7927	0.6840	70.0	---	---	Double Rocker
10.0	25.0	0.1580	1.0	1.0	1.8184	0.8452	65.0	112.25	162.39	Crank-Rocker
10.0	30.0	0.1472	1.0	1.0	1.7321	1.0000	60.0	104.83	140.00	Crank-Rocker
10.0	35.0	0.1442	1.0	1.0	1.6456	1.1472	55.0	97.30	127.00	Crank-Rocker
10.0	40.0	0.1523	1.0	1.0	1.5577	1.2856	50.0	89.30	117.50	Crank-Rocker
10.0	45.0	0.1750	1.0	1.0	1.4669	1.4142	45.0	80.50	110.40	Crank-Rocker
10.0	50.0	0.2159	1.0	1.0	1.3717	1.5321	40.0	70.60	105.10	Crank-Rocker
10.0	55.0	0.2792	1.0	1.0	1.2700	1.6383	35.0	59.40	101.50	Crank-Rocker
10.0	60.0	0.3697	1.0	1.0	1.1595	1.7320	30.0	46.50	99.70	Crank-Rocker
10.0	65.0	0.4933	1.0	1.0	1.0370	1.8126	25.0	31.55	99.84	Crank-Rocker
10.0	70.0	0.6577	1.0	1.0	0.8983	1.8794	20.0	13.80	102.20	Crank-Rocker
10.0	75.0	0.8737	1.0	1.0	0.7373	1.9319	15.0	---	---	Drag-Link

TABLE II

DESIGN TABLE FOR A SINGLE CUSP WITH $B_0P = 0.5$

B_0P	α	a	b	c	d	e	β	Min	Max	Type
0.5	30.0	0.2324	1.0	1.0	0.7676	1.0000	60.00	31.00	60.00	Crank-Rocker
0.5	35.0	0.2230	1.0	1.0	0.9241	1.1472	55.00	41.00	70.00	Crank-Rocker
0.5	40.0	0.2119	1.0	1.0	1.0737	1.2856	50.00	51.00	80.00	Crank-Rocker
0.5	45.0	0.1992	1.0	1.0	1.2150	1.4142	45.00	61.00	90.00	Crank-Rocker
0.5	50.0	0.1850	1.0	1.0	1.3471	1.5321	40.00	71.00	100.00	Crank-Rocker
0.5	55.0	0.1694	1.0	1.0	1.4689	1.6383	35.00	81.00	110.00	Crank-Rocker
0.5	60.0	0.1525	1.0	1.0	1.5795	1.7320	30.00	91.0	120.00	Crank-Rocker
0.5	65.0	0.1344	1.0	1.0	1.6732	1.8126	25.00	101.00	130.00	Crank-Rocker
0.5	70.0	0.1153	1.0	1.0	1.7640	1.8794	20.00	111.00	140.00	Crank-Rocker

Another parameter governs the results. For two symmetrical cusps the parameter is the angle of span ξ between the two cusps. For a single cusp it is the segment B_0P between the two extreme positions of the coupler point along the axis of symmetry. In Figure 15, the curves are those of the normalized lengths of the input, coupler, and ground links for a span angle of 10 degrees. In the case of a single cusp (Figure 16), the normalized lengths are plotted against α for B_0P equal to 0.5. The rest of the design data are plotted as design charts for both kinds. For two cusps (Figure 17), the curves are continuous within a range. For smaller values of α, the type of mechanism is a double rocker, while for larger angles the mechanism becomes a crank-rocker. When α approaches 90 degrees, the mechanism will more likely become a drag-link, depending on ξ. The design procedure for the single cusp gives a crank-rocker if the solution exists (Figure 18). The charts include only the designs whose maximum and minimum transmission angles are not larger than 140 and not less than 30 degrees. The best transmission angle is 90 degrees. Transmission angles smaller than 40 or larger than 140 are not recommended, especially for high speed designs of mechanisms.

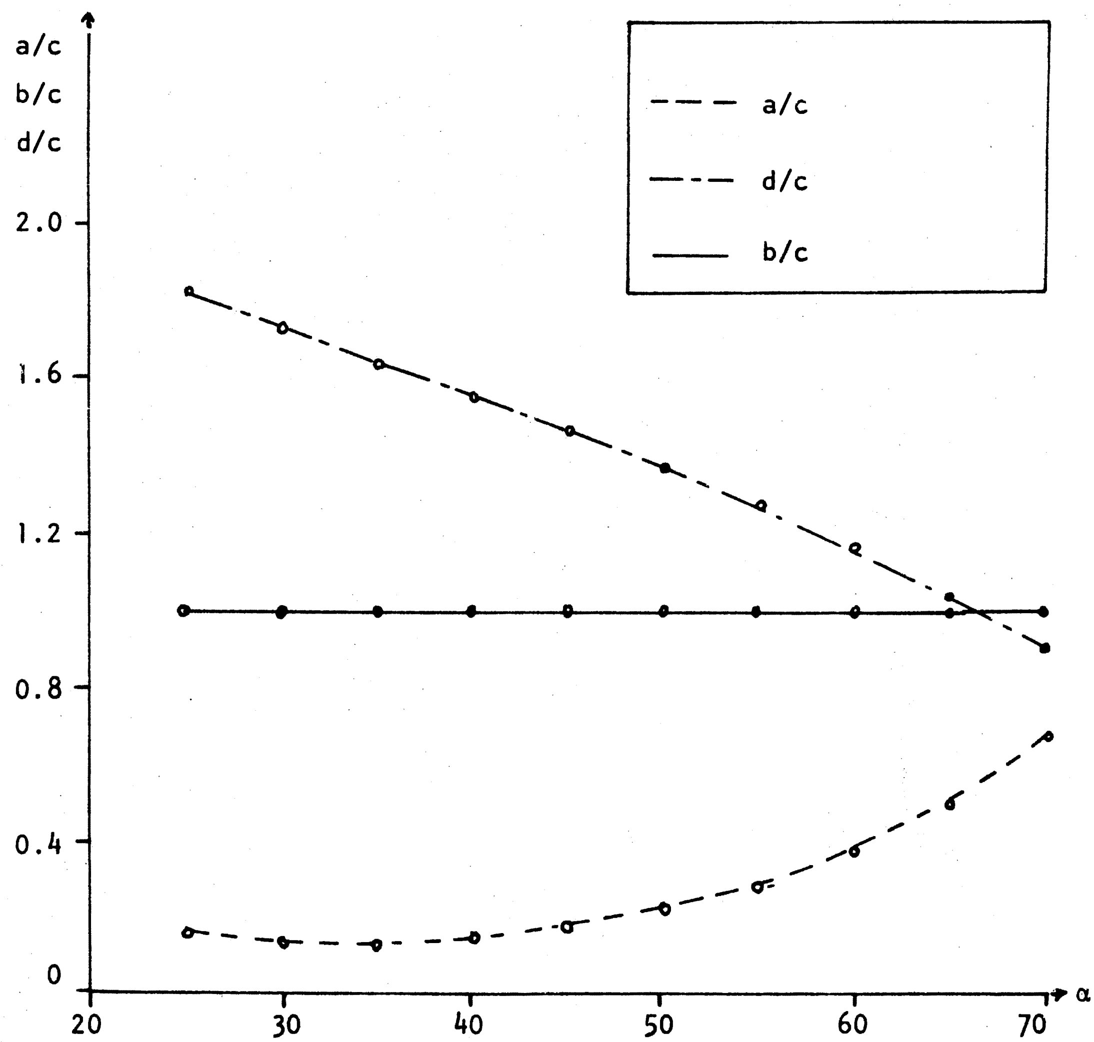

Figure 15. Sample Design for Two Symmetrical Cusps; Length Proportions Versus α for $\xi = 10$

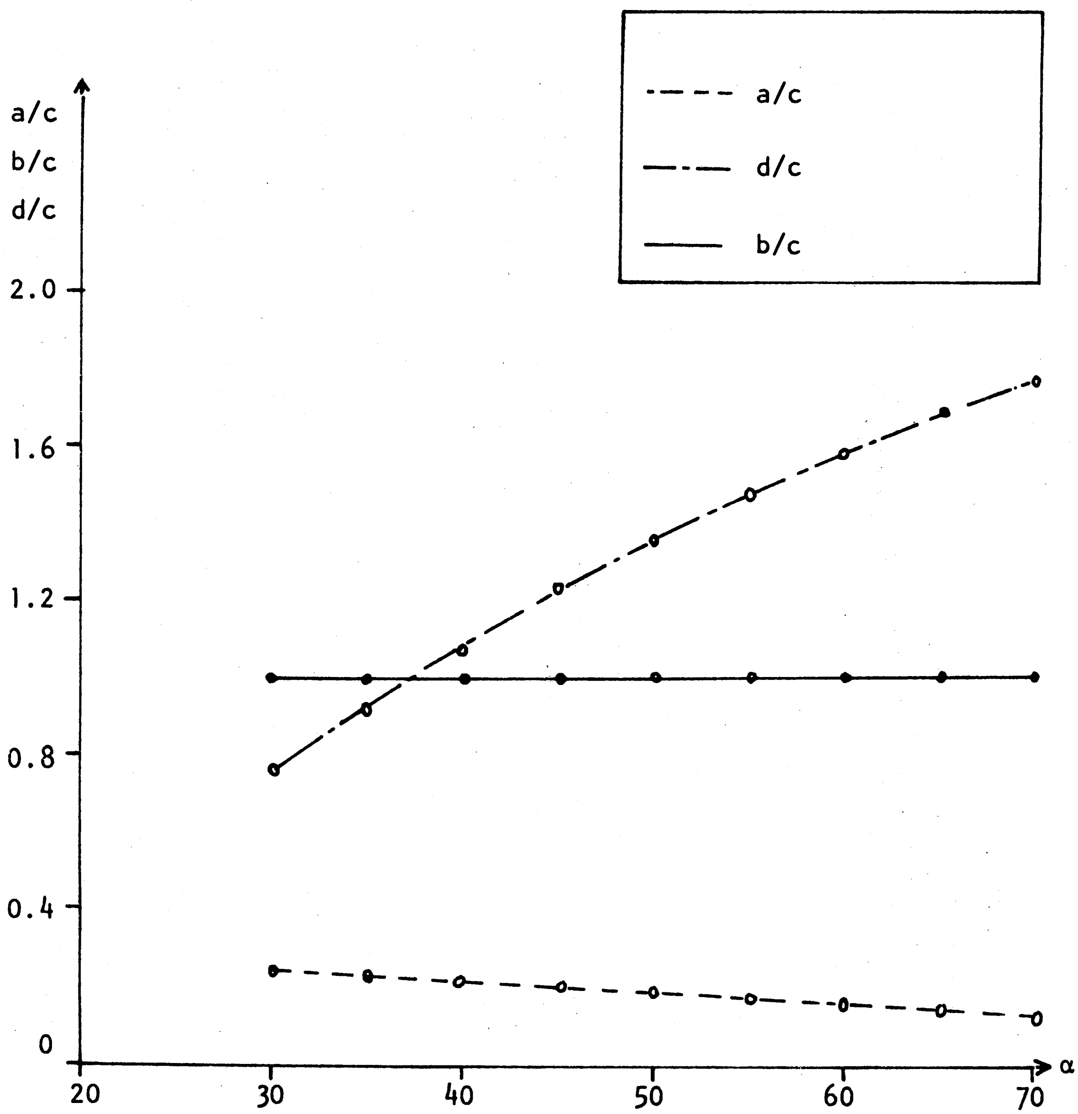

Figure 16. Sample Design for a Single Cusp; Length Proportions Versus α for $B_0P = 0.5$

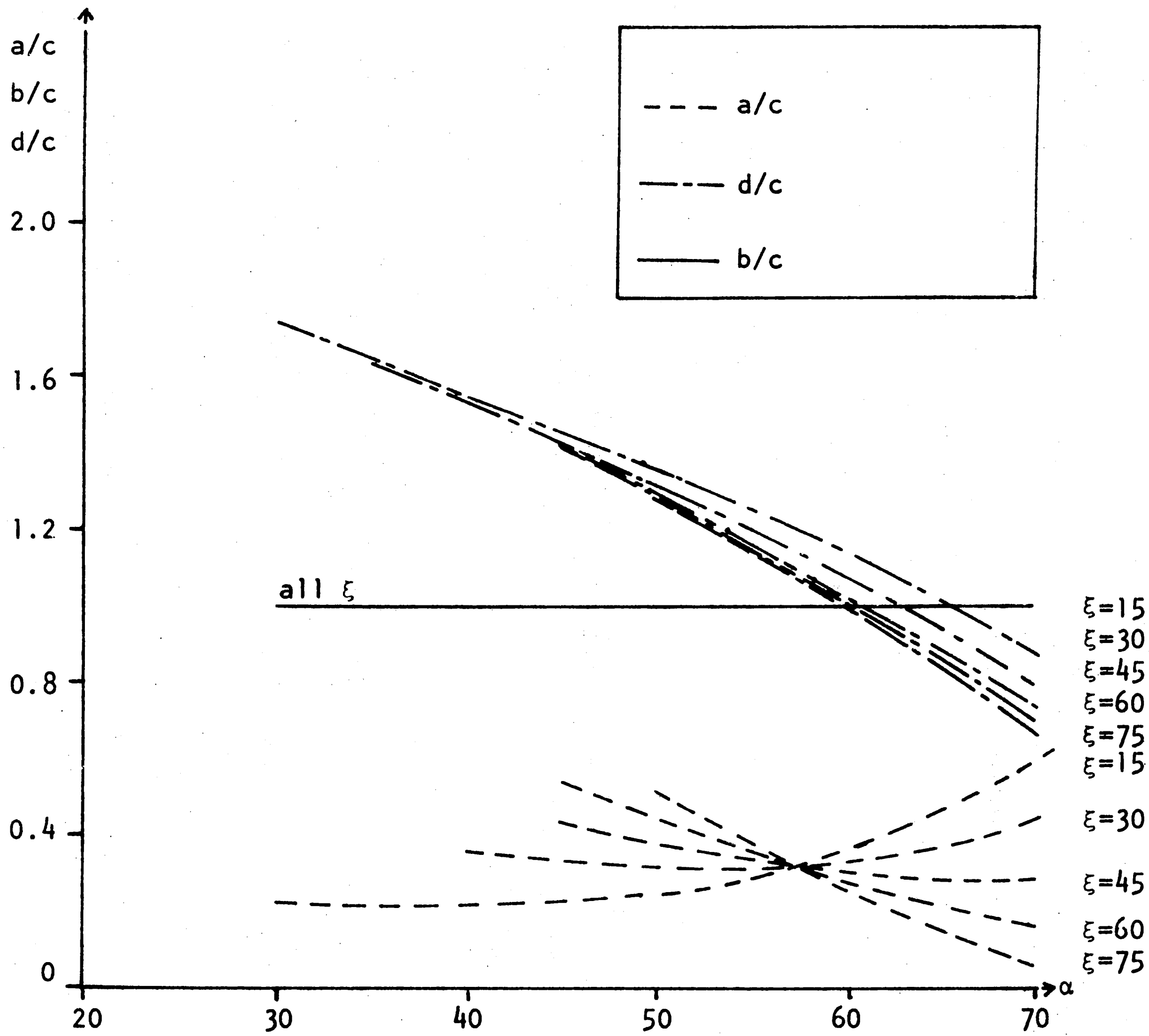

Figure 17. Design Chart for a Symmetrical Coupler Curve With Two Symmetrical Cusps

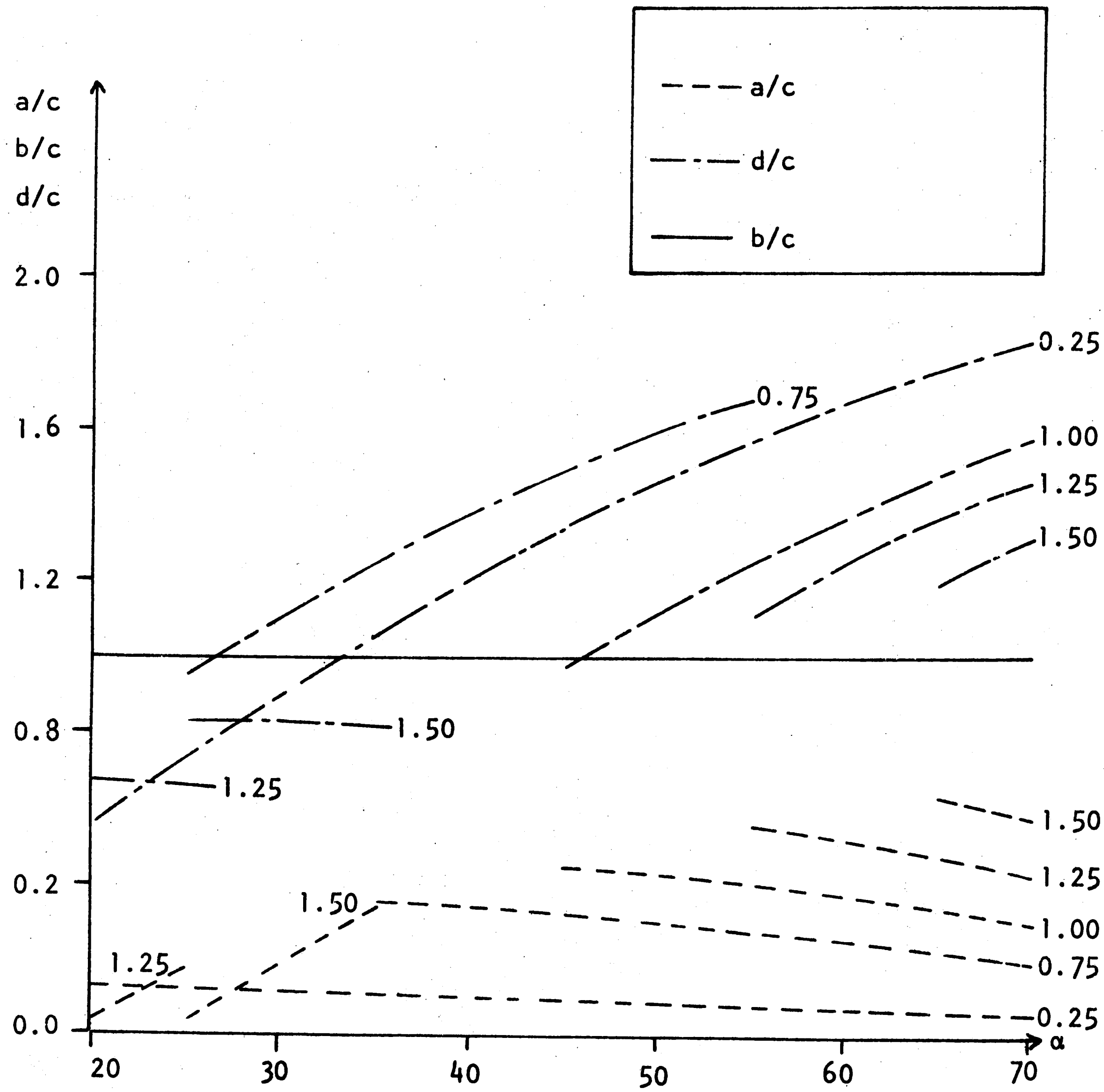

Figure 18. Design Chart for a Symmetrical Coupler Curve With a Single Cusp

CHAPTER IV

SUMMARY AND CONCLUSION

4.1 Analysis and Summary

The design procedure followed in the two problems presented before depends on the single-position method. In the first problem the design position does not belong to the symmetry axis; a case that will produce another cusp symmetrical to the first one. The design position belongs to the axis of symmetry in the second problem. The double point will not have a symmetrical image. The key point in both design is that a coupler point on the moving polode of the coupler link of a four-bar mechanism will produce a cusp if it coincides with the instantaneous center. As a start the third vertex of the coupler triangle is assumed to coincide with the instant center. This point will develop a cusp.

To verify it is necessary that the design position satisfies the double point conditions. In order that a coupler curve contains a double point, this point should belong to the circle of singular foci. In Chapter II, a brief introduction about this circle is presented. It circumscribes the triangle A_0B_0C such that this triangle is similar to the coupler triangle ABP (Figure 6). In the design position considered in the synthesis, the link A_0A is in line with the coupler arm AP and the follower link BB_0 is also in line with the arm BP. Any triangle A_0B_0C which is similar to the coupler triangle ABP will have its angle A_0CB_0 equal to the angle APB. This means that the angle A_0PB_0 is equal to the angle A_0CB_0. If a circle circumscribes the triangle A_0B_0C, the angle A_0CB_0 will subtend the arc A_0B_0 and will be equal to half of it (Figure 19). But the angle A_0PB_0 is equal to the angle A_0CB_0 and thus it will be equal to half of the arc A_0B_0. This implies that the point P must lie on the circumference of the same circle that circumscribes the triangle A_0B_0C. From definition that the circle is the circle of singular foci, and hence point P at that instant satisfies the condition for a double point. Furthermore, this point coincides with the instant center of the four-bar linkage. Kurt Hain (9) mentioned that a cusp will be developed at the point of intersection between the circle of singular foci and the fixed polode. The instantaneous center belongs to the fixed centrode.

It is interesting to notice that if the coupler link AB is parallel to the fixed link A_0B_0 and the coupler point P coincides with the instantaneous center I (Figure 20), then the triangle A_0B_0C will be identical to the triangle A_0B_0P and C will coincide at P. P will still produce a cusp since it lies on the circle of the foci.

In the case of a single cusp, double point conditions are more obvious. The instantaneous center and the coupler point P both coincide at B_0, which is always on the circle of the foci. It also belongs to the fixed centrode.

Then the necessary and sufficient conditions for a coupler point to generate a cusp are satisfied by the design presented in Chapter III.

4.2 Conclusion

In conclusion, the design of a four-bar linkage that generates a symmetrical coupler curve with two cusps or with a single cusp is

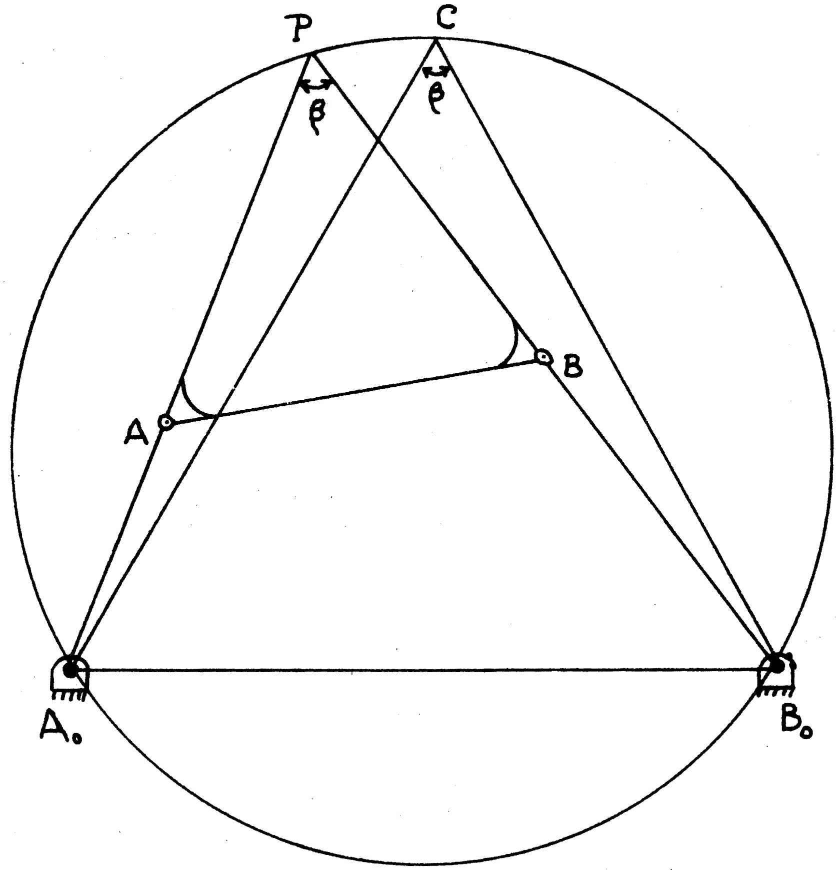

Figure 19. The Circle of Singular Foci Passing Through the Design Position

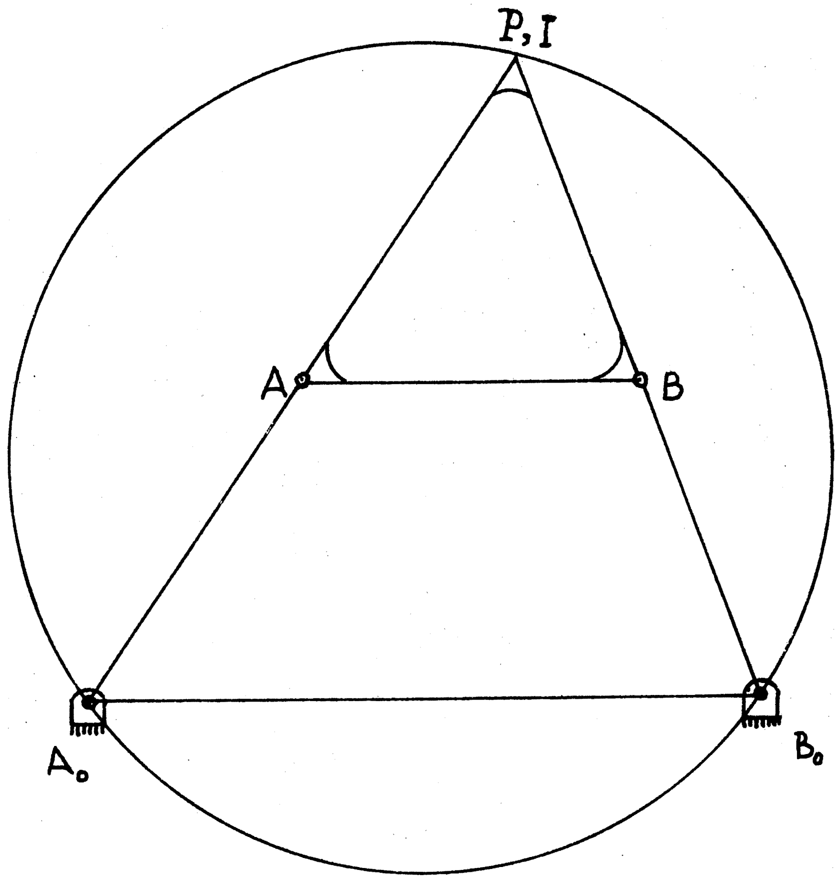

Figure 20. Special Case of the Design Position Coinciding With the Vertex of the Triangle Similar to the Coupler Triangle

completed. The method followed is the single-position design. Once the position is defined, the solution of the problem is relatively easy to obtain. It is a geometrical solution depending on the position of the four-bar linkage. Two parameters are needed for each of the two designs. In the case of two symmetrical cusps, the inclination angle of the symmetry axis with the fixed link is one and the other is the span angle between the two cusps. In the second design, the angle of inclination and the length of the segment between the two extreme positions of the coupler point along the axis of symmetry are needed. For practical purposes the two parameters in each case are suitable. A knowledge of the size of the path and its orientation will provide the needed parameters. In addition to the two parameters, an assumption of the magnitude of the length of one of the links is helpful. This assumption will not affect the generality of the problem. The results are all normalized in terms of the assumed length. The available space and the size of the mechanism will dictate the choice of that length.

All the design specifications are presented in tables so that it can be used by designers. A set of design charts is also prepared for the crank-rocker design.

This design is of practical importance to designers. Its uses can be divided into two main categories. The coupler curve can be used as an output of a four-bar linkage or as a guiding motion to drive other linkages; most important is the six-bar linkage.

As an output of a four-bar, the design can be used in mechanisms that require a state of dwell enduring over a finite period of time. The cusp will cause an infinitesimal dwell at the double point and the coupler point will come to a complete stop. This means that the coupler point is forced to approach this position very slowly. That is also true for the departure. All of that will generate a state of approximate dwell, and it is sufficient for most practical purposes.

As a guiding motion, the coupler curve can be used to synthesize a six-bar mechanism. This six-bar is called a Coupler Driver Six-Bar Mechanism. In Figure 21, a six-bar driven by a coupler point of a four-bar is shown. Any of the revolute pairs could be substituted by a prismatic pair since both have one degree of freedom. The output of the six-bar can be through a folower link or through a slider. There are 21 types of six-bar linkages that have one degree of freedom and are driven by the coupler point of a four-bar linkage. These types can be classified by the two basic types of motion they execute: rotary to rotary, where the input and output cranks are both performing rotary motion; and rotary to linear, where the input crank performs rotary motion while the output performs sliding motion. For more information, the reader is referred to unit 15 of Reference (17).

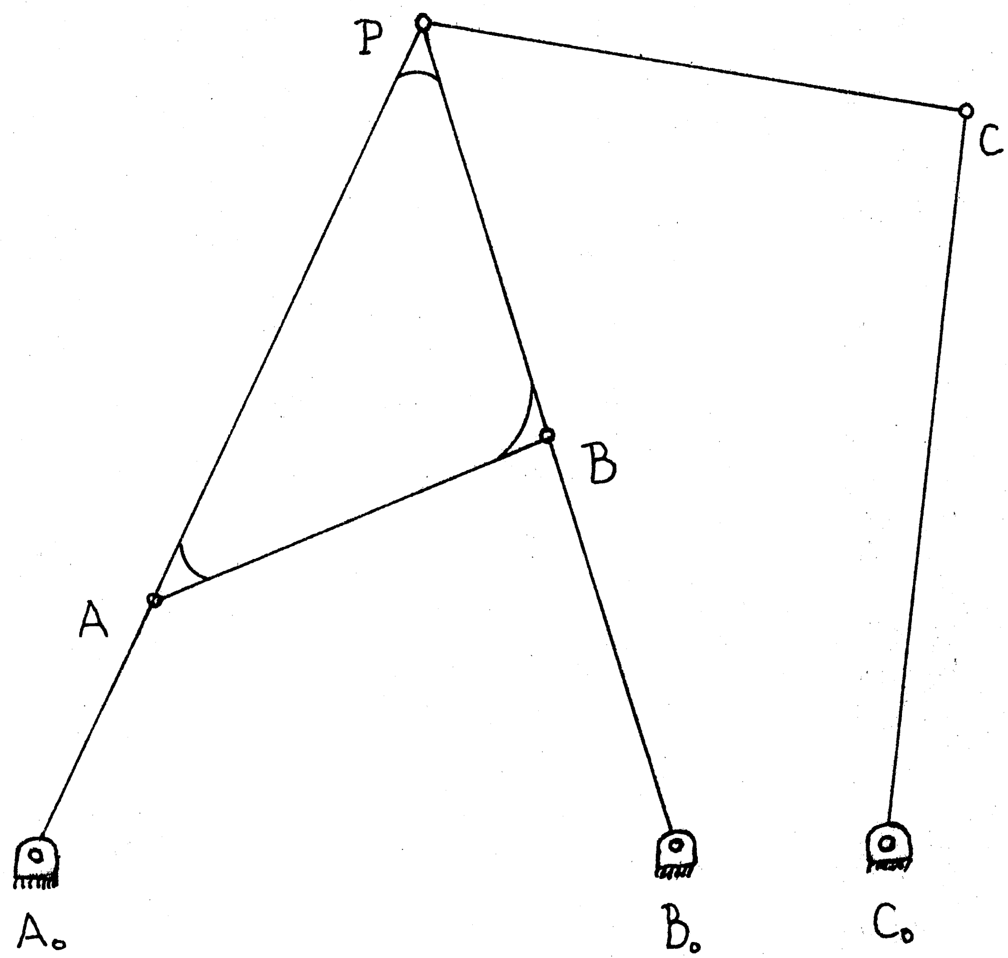

Figure 21. Coupler-Driven Six-Bar Mechanism

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APPENDIX A

TABULATED RESULTS OF A SYMMETRICAL COUPLER CURVE WITH TWO SYMMETRICAL CUSPS DESIGN

TABLE III

DESIGN TABLE FOR TWO SYMMETRICAL CUSPS WITH ξ = 15 DEGREES

ξ	α	a	b	c	d	e	β	ξ Min	ξ Max	Type
15.0	10.0	0.3887	1.0	1.0	2.4105	0.3473	80.00	---	---	Double Rocker
15.0	25.0	0.2376	1.0	1.0	1.8266	0.8452	65.00	---	---	Double Rocker
15.0	30.0	0.2183	1.0	1.0	1.7332	1.0000	60.00	98.50	154.70	Crank-Rocker
15.0	35.0	0.2058	1.0	1.0	1.6404	1.1472	55.00	91.70	134.80	Crank-Rocker
15.0	40.0	0.2031	1.0	1.0	1.5468	1.2856	50.00	84.40	122.00	Crank-Rocker
15.0	45.0	0.2136	1.0	1.0	1.4509	1.4142	45.00	76.40	112.70	Crank-Rocker
15.0	50.0	0.2406	1.0	1.0	1.3510	1.5321	40.00	67.50	105.50	Crank-Rocker
15.0	55.0	0.2876	1.0	1.0	1.2454	1.6383	35.00	57.20	100.10	Crank-Rocker
15.0	60.0	0.3589	1.0	1.0	1.1316	1.7320	30.00	45.50	96.40	Crank-Rocker
15.0	65.0	0.4594	1.0	1.0	1.0068	1.8126	25.00	31.80	94.30	Crank-Rocker
15.0	70.0	0.5954	1.0	1.0	0.8670	1.8794	20.00	15.60	94.00	Crank-Rocker
15.0	75.0	0.7754	1.0	1.0	0.7067	1.9319	15.00	---	---	Drag-Link

TABLE IV

DESIGN TABLE FOR TWO SYMMETRICAL CUSPS WITH ξ = 30 DEGREES

ξ	α	a	b	c	d	e	β	μ Min	μ Max	Type
30.0	10.0	0.6074	1.0	1.0	2.2246	0.3473	80.00	---	---	Double Rocker
30.0	35.0	0.3899	1.0	1.0	1.6436	1.1472	55.00	---	---	Double Rocker
30.0	40.0	0.3528	1.0	1.0	1.5321	1.2856	50.00	72.30	140.90	Crank-Rocker
30.0	45.0	0.3256	1.0	1.0	1.4205	1.4142	45.00	66.40	121.60	Crank-Rocker
30.0	50.0	0.3109	1.0	1.0	1.3071	1.5321	40.00	59.80	108.00	Crank-Rocker
30.0	55.0	0.3112	1.0	1.0	1.1899	1.6383	35.00	52.10	97.30	Crank-Rocker
30.0	60.0	0.3292	1.0	1.0	1.0670	1.7320	30.00	43.30	88.50	Crank-Rocker
30.0	65.0	0.3679	1.0	1.0	0.9357	1.18126	25.00	33.00	81.40	Crank-Rocker
30.0	70.0	0.4308	1.0	1.0	0.7931	1.8794	20.00	20.90	75.50	Crank-Rocker

TABLE V

DESIGN TABLE FOR TWO SYMMETRICAL CUSPS WITH ξ = 45 DEGREES

ξ	α	a	b	c	d	e	β	Min	Max	Type
45.0	10.0	0.7978	1.0	1.0	2.0988	0.3473	80.00	---	---	Double Rocker
45.0	40.0	0.5021	1.0	1.0	1.5439	1.2856	50.00	---	---	Double Rocker
45.0	45.0	0.4348	1.0	1.0	1.4152	1.4142	45.00	58.70	135.30	Crank-Rocker
45.0	50.0	0.3778	1.0	1.0	1.2872	1.5321	40.00	54.10	112.70	Crank-Rocker
45.0	55.0	0.3330	1.0	1.0	1.1582	1.6383	35.00	48.70	96.40	Crank-Rocker
45.0	60.0	0.3022	1.0	1.0	1.0259	1.7320	30.00	42.40	83.20	Crank-Rocker
45.0	65.0	0.2872	1.0	1.0	0.8883	1.8126	25.00	35.00	72.00	Crank-Rocker
45.0	70.0	0.2898	1.0	1.0	0.7426	1.8794	20.00	26.20	62.20	Crank-Rocker

TABLE VI

DESIGN TABLES FOR TWO SYMMETRICAL CUSPS WITH ξ = 60 DEGREES

ξ	α	a	b	c	d	e	β	Min	Max	Type
60.0	10.0	0.9705	1.0	1.0	2.0190	0.3473	80.00	---	---	Double Rocker
60.0	40.0	0.6564	1.0	1.0	1.5832	1.2856	50.00	---	---	Double Rocker
60.0	45.0	0.5450	1.0	1.0	1.4343	1.4142	45.00	52.80	163.50	Crank-Rocker
60.0	50.0	0.4439	1.0	1.0	1.2897	1.5321	40.00	50.00	120.20	Crank-Rocker
60.0	55.0	0.3541	1.0	1.0	1.1472	1.6383	35.00	46.72	97.30	Crank-Rocker
60.0	60.0	0.2769	1.0	1.0	1.0045	1.7320	30.00	42.70	79.70	Crank-Rocker
60.0	65.0	0.2131	1.0	1.0	0.8594	1.8126	25.00	37.70	64.90	Crank-Rocker
60.0	70.0	0.1637	1.0	1.0	0.7095	1.8794	20.00	31.70	51.80	Crank-Rocker
60.0	75.0	0.1294	1.0	1.0	0.5523	1.9319	15.00	24.40	39.90	Crank-Rocker

TABLE VII

DESIGN TABLE FOR TWO SYMMETRICAL CUSPS WITH ξ = 75 DEGREES

ξ	α	a	b	c	d	e	β	μ Min	μ Max	Type
75.0	10.0	1.1334	1.0	1.0	1.9778	0.3473	80.00	---	---	Double Rocker
75.0	45.0	0.6603	1.0	1.0	1.4796	1.4142	45.00	---	---	Double Rocker
75.0	50.0	0.5113	1.0	1.0	1.3147	1.5321	40.00	47.40	131.90	Crank-Rocker
75.0	55.0	0.3752	1.0	1.0	1.1560	1.6383	35.00	46.00	99.90	Crank-Rocker
75.0	60.0	0.2522	1.0	1.0	1.0007	1.7320	30.00	44.00	77.60	Crank-Rocker
75.0	65.0	0.1425	1.0	1.0	0.8463	1.8126	25.00	41.20	59.30	Crank-Rocker
75.0	70.0	0.0464	1.0	1.0	0.6906	1.8794	20.00	37.60	43.20	Crank-Rocker

TABLE VIII

DESIGN TABLE FOR TWO SYMMETRICAL CUSPS WITH ξ = 90 DEGREES

ξ	α	a	b	c	d	e	β	μ Min	μ Max	Type
90.0	10.0	1.2923	1.0	1.0	1.9712	0.3473	80.00	---	---	Double Rocker
90.0	45.0	0.7853	1.0	1.0	1.5553	1.4142	45.00	---	---	Double Rocker
90.0	50.0	0.5827	1.0	1.0	1.3646	1.5321	40.00	46.00	153.60	Crank-Rocker
90.0	55.0	0.3970	1.0	1.0	1.1854	1.6383	35.00	46.40	104.60	Crank-Rocker
90.0	60.0	0.2272	1.0	1.0	1.0142	1.7320	30.00	46.30	76.70	Crank-Rocker
90.0	65.0	0.0728	1.0	1.0	0.8480	1.8126	25.00	45.60	54.80	Crank-Rocker

TABLE IX

DESIGN TABLE FOR TWO SYMMETRICAL CUSPS WITH ξ = 120 DEGREES

ξ	α	a	b	c	d	e	β	μ Min	μ Max	Type
120.0	10.0	1.6211	1.0	1.0	2.0629	0.3473	80.0	---	---	Double Rocker
120.0	50.0	0.7509	1.0	1.0	1.5616	1.5321	40.0	---	---	Double Rocker
120.0	55.0	0.4462	1.0	1.0	1.3192	1.6383	35.0	51.80	123.90	Crank-Rocker
120.0	60.0	0.1728	1.0	1.0	1.0993	1.7320	30.0	55.20	79.00	Crank-Rocker

APPENDIX B

TABULATED RESULTS OF A SYMMETRICAL COUPLER CURVE WITH A SINGLE CUSP DESIGN

TABLE X

DESIGN TABLE FOR A SINGLE CUSP WITH B_0P = 0.25

B_0P	α	a	b	c	d	e	β	μ Min	μ Max	Type
0.25	20.0	0.1201	1.0	1.0	0.5639	0.6840	70.00	25.60	40.00	Crank-Rocker
0.25	25.0	0.1166	1.0	1.0	0.7286	0.8452	65.00	35.60	50.00	Crank-Rocker
0.25	30.0	0.1122	1.0	1.0	0.8878	1.0000	60.00	45.60	60.00	Crank-Rocker
0.25	35.0	0.1069	1.0	1.0	1.0403	1.1472	55.00	55.60	70.00	Crank-Rocker
0.25	40.0	0.1008	1.0	1.0	1.1848	1.2856	50.00	65.60	80.00	Crank-Rocker
0.25	45.0	0.0939	1.0	1.0	1.3203	1.4142	45.00	75.60	90.00	Crank-Rocker
0.25	50.0	0.0864	1.0	1.0	1.4457	1.5321	40.00	85.60	100.00	Crank-Rocker
0.25	55.0	0.0781	1.0	1.0	1.5602	1.6383	35.00	95.60	110.00	Crank-Rocker
0.25	60.0	0.0693	1.0	1.0	1.6628	1.7320	30.00	105.60	120.00	Crank-Rocker
0.25	65.0	0.0599	1.0	1.0	1.7527	1.8126	25.00	113.60	130.00	Crank-Rocker
0.25	70.0	0.0501	1.0	1.0	1.8293	1.8794	20.00	125.60	140.00	Crank-Rocker

TABLE XI

DESIGN TABLE FOR A SINGLE CUSP WITH $B_0P = 0.75$

B_0P	α	a	b	c	d	e	β	μ Min	μ Max	Type
0.75	35.00	0.3490	1.0	1.0	0.7981	1.1472	55.00	26.00	70.00	Crank-Rocker
0.75	40.00	0.3342	1.0	1.0	0.9514	1.2856	50.00	35.95	80.00	Crank-Rocker
0.75	45.00	0.3168	1.0	1.0	1.0974	1.4142	45.00	46.00	90.00	Crank-Rocker
0.75	50.00	0.2969	1.0	1.0	1.2351	1.5321	40.00	56.00	100.00	Crank-Rocker
0.75	55.00	0.2749	1.0	1.0	1.3634	1.6383	35.00	66.00	110.00	Crank-Rocker
0.75	60.00	0.2507	1.0	1.0	1.4813	1.7320	30.00	76.00	120.00	Crank-Rocker
0.75	65.00	0.2246	1.0	1.0	1.5880	1.8126	25.00	86.00	130.00	Crank-Rocker
0.75	70.00	0.1968	1.0	1.0	1.6826	1.8794	20.00	96.00	140.00	Crank-Rocker

TABLE XII

DESIGN TABLE FOR A SINGLE CUSP WITH B_0P = 1.00

B_0P	α	a	b	c	d	e	β	μ Min	μ Max	Type
1.00	45.00	0.4483	1.0	1.0	0.9659	1.4142	45.00	30.00	90.00	Crank-Rocker
1.00	50.00	0.4240	1.0	1.0	1.1081	1.5321	40.00	40.00	100.00	Crank-Rocker
1.00	55.00	0.3965	1.0	1.0	1.2418	1.6383	35.00	50.00	110.00	Crank-Rocker
1.00	60.00	0.3660	1.0	1.0	1.3660	1.7320	30.00	60.00	120.00	Crank-Rocker
1.00	65.00	0.3327	1.0	1.0	1.4799	1.8126	25.00	70.00	130.00	Crank-Rocker
1.00	70.00	0.2969	1.0	1.0	1.5825	1.8794	20.00	80.00	140.00	Crank-Rocker
1.00	75.00	0.2588	1.0	1.0	1.6730	1.9319	15.00	90.00	150.00	Crank-Rocker

TABLE XIII

DESIGN TABLE FOR A SINGLE CUSP WITH B_0P = 1.25

B_0P	α	a	b	c	d	e	β	μ Min	μ Max	Type
1.25	20.00	0.0217	1.0	1.0	0.6623	0.6840	70.00	37.40	40.00	Crank-Rocker
1.25	25.00	0.1861	1.0	1.0	0.6592	0.8452	65.00	27.40	50.00	Crank-Rocker
1.25	55.00	0.5382	1.0	1.0	1.1001	1.6383	35.00	32.60	110.00	Crank-Rocker
1.25	60.00	0.5025	1.0	1.0	1.2296	1.7320	30.00	42.60	120.00	Crank-Rocker
1.25	65.00	0.4630	1.0	1.0	1.3497	1.8126	25.00	52.60	130.00	Crank-Rocker
1.25	70.00	0.4199	1.0	1.0	1.4595	1.8794	20.00	62.60	140.00	Crank-Rocker

TABLE XIV

DESIGN TABLE FOR A SINGLE CUSP WITH B_0P = 1.50

B_0P	α	a	b	c	d	e	β	μ Min	μ Max	Type
1.50	25.00	0.0224	1.0	1.0	0.8228	0.8452	65.00	47.20	50.00	Crank-Rocker
1.50	30.00	0.1812	1.0	1.0	0.8186	1.0000	60.00	37.20	60.00	Crank-Rocker
1.50	35.00	0.3386	1.0	1.0	0.8086	1.1472	55.00	27.20	70.00	Crank-Rocker
1.50	65.00	0.6238	1.0	1.0	1.1888	1.8126	25.00	32.80	130.00	Crank-Rocker
1.50	70.00	0.5747	1.0	1.0	1.3047	1.8794	20.00	42.80	140.00	Crank-Rocker
1.50	75.00	0.5211	1.0	1.0	1.4107	1.9319	15.00	52.30	150.00	Crank-Rocker

TABLE XV

DESIGN TABLE FOR A SINGLE CUSP WITH B_0P = 1.75

B_0P	α	a	b	c	d	e	β	μ Min	μ Max	Type
1.75	35.00	0.1345	1.0	1.0	1.0127	1.1472	55.00	52.00	70.00	Crank-Rocker
1.75	40.00	0.2837	1.0	1.0	1.0019	1.2856	50.00	42.00	80.00	Crank-Rocker
1.75	45.00	0.4307	1.0	1.0	0.9835	1.4142	45.00	32.00	90.00	Crank-Rocker
1.75	50.00	0.5745	1.0	1.0	0.9576	1.5321	40.00	22.00	100.00	Crank-Rocker
1.75	75.00	0.7248	1.0	1.0	1.2071	1.9319	15.00	27.90	150.00	Crank-Rocker
1.75	80.00	0.6600	1.0	1.0	1.3096	1.9696	10.00	37.90	160.00	Crank-Rocker

TABLE XVI

DESIGN TABLE FOR A SINGLE CUSP WITH B_0P = 2.00

B_0P	α	a	b	c	d	e	β	Min	Max	Type
2.00	50.00	0.1233	1.0	1.0	1.4088	1.5321	40.00	80.00	100.00	Crank-Rocker
2.00	55.00	0.2456	1.0	1.0	1.3927	1.6383	35.00	70.00	110.00	Crank-Rocker
2.00	60.00	0.3660	1.0	1.0	1.3660	1.7320	30.00	60.00	112.00	Crank-Rocker
2.00	65.00	0.4837	1.0	1.0	1.3289	1.8126	25.00	50.00	130.00	Crank-Rocker
2.00	70.00	0.5977	1.0	1.0	1.2817	1.8794	20.00	40.00	140.00	Crank-Rocker
2.00	75.00	0.7071	1.0	1.0	1.2247	1.9319	15.00	30.00	150.00	Crank-Rocker

APPENDIX C

THEORETICAL BACKGROUND ABOUT STRAIGHT LINE MOTION, CANONICAL SYSTEMS, AND INSTANTANEOUS INVARIANTS

C.1 Straight Line Segment

At any instant there exist some points in the moving plane, part of which the coupler link is, that are going into inflection with respect to the fixed plane. The locus of these points is called the inflection circle. At each inflection point there is three-point contact between the curve and its tangent. Moreover, there is at least one real point among the inflection points that is going into four-point contact with its tangent. This happens when the inflection circle is intersecting the cubic of stationary curvature. The cubic of stationary curvature is the locus of all points that are going through a fixed curvature at a particular instant. The intersection of the inflection circle and the cubic of stationary curvature is called Ball's point (1, 8, 13, 14, 15, 21).

C.1.1 Canonical System and Instantaneous Invariants

The concept of instantaneous invariants was introduced by Bottema and Veldkamp (8, 9). It is a powerful method to study the kinematic geometry of infinitesimally separated positions of a moving plane.

A plane m in a continuous motion with respect to a fixed plane f is shown in Figure 22. A cartesian system (x,y) is attached to the moving plane and another (X,Y) is attached to the fixed plane. The origin of the moving system at a particular instant has the coordinates (a,b) with respect to the fixed system. a and b are functions of ϕ, the rotation angle of m relative to f. This means X and Y are functions of ϕ. The coordinates of a point on the moving plane m with respect to the

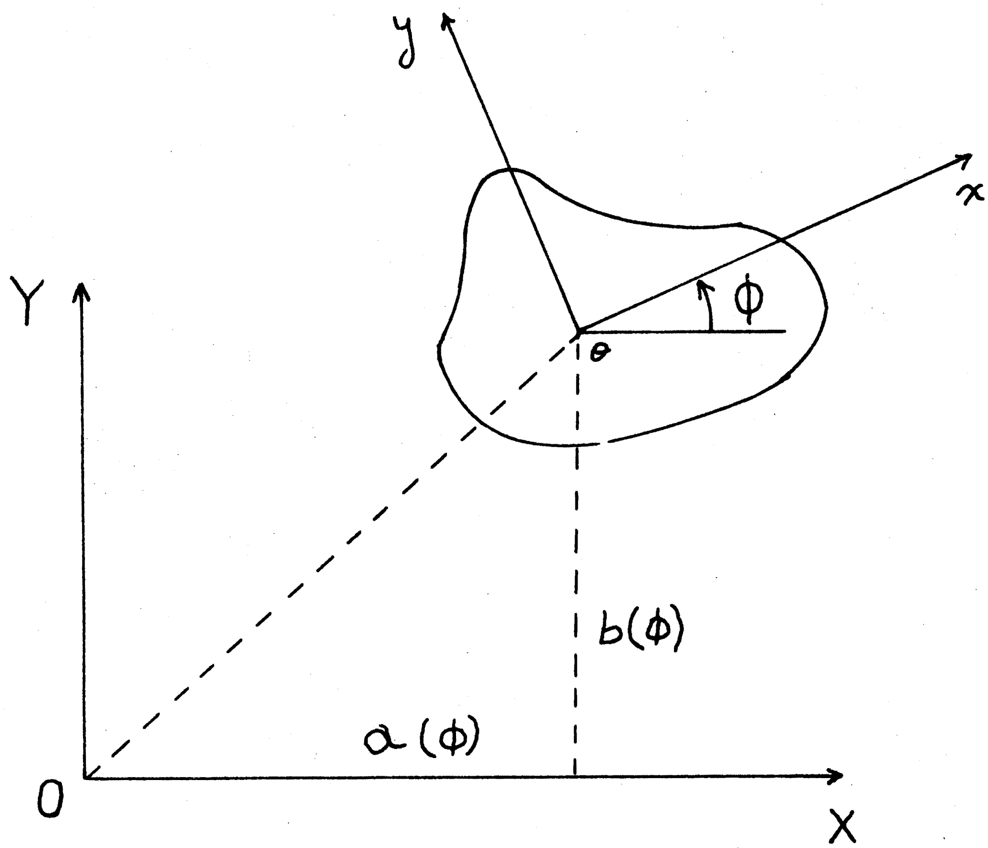

Figure 22. Relative Motion of Two Planes

fixed systems of axes can be expressed as follows (21, 22, 23):

$$\begin{bmatrix} X(\phi) \\ Y(\phi) \end{bmatrix} = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a(\phi) \\ b(\phi) \end{bmatrix} \tag{C.1}$$

Let the two origins of the two systems of axes m and f coincide and the x-axis of the moving system be in line with the X-axis of the fixed system. Let this instant be the reference or zero instant. At this instant ϕ is equal to zero. The set of coordinates consisting of the coordinates of the moving and stationary systems is called the canonical system.

Differentiate Equation (C.1) with respect to up to the ith order. Let the symbols X_i, Y_i, a_i, and b_i be the ith derivatives of X, Y, a, and b with respect to ϕ. The derivatives a_i and b_i for i varies from 1 to n are called the intantaneous invariants. The instantaneous invariants a_1, b_1, and a_2 are equal to zero because at the reference position ϕ is equal to zero and the origins of both axis systems are coinciding. To describe the instantaneous invariants kinematically, the chain rule of differentiation is applied and the derivatives with respect to time are obtained. The ith time derivative of Equation (C.1) is:

$$\frac{d^i[X]}{dt^i} = \left[\frac{d^i A}{dt^i}\right] [x] + \frac{d^i[a]}{dt^i} \tag{C.2}$$

where

$$[X] = \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$[x] = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$[a] = \begin{bmatrix} a \\ b \end{bmatrix}$$

If w, α, $\overset{o}{\alpha}$, and $\overset{oo}{\alpha}$ are the respective time derivatives of ϕ, and $\overset{o}{X}$, $\overset{oo}{X}$, and $\overset{\circ\circ\circ}{X}$, $\overset{\circ}{Y}$, $\overset{\circ\circ}{Y}$, and $\overset{\circ\circ\circ}{Y}$ are the velocity, acceleration, and jerk components of the coupler point, the final results will be listed as follows:

$$X = x \quad \text{and} \quad Y = y \tag{C.3.a}$$

$$\overset{\circ}{X} = -wy \quad \text{and} \quad \overset{\circ}{Y} = wx \tag{C.3.b}$$

$$\overset{\circ\circ}{X} = -(w^2x + \alpha y) \quad \text{and} \quad \overset{\circ\circ}{Y} = \alpha x - w^2y + w^2b_2 \tag{C.3.c}$$

$$\overset{\circ\circ\circ}{X} = -3w\;x + (w^3 - \overset{\circ}{\alpha})y + w^3a_3 \tag{C.3.d}$$

$$\overset{\circ\circ\circ}{Y} = -(w^3 - \overset{\circ}{\alpha})x + 3w\alpha(b_2 - y) + w^3b_3 \tag{C.3.e}$$

For more information about the instantaneous invariants, see References (21), (22), and (23).

APPENDIX D

LISTING OF COMPUTER PROGRAM

```
$JOB            ,TIME=(0,03)
      DATA NR,NP,PI/5,6,3.14159/
      BBO=1.0
      AB=BBO
      BP=BBO
      READ(NR,*) ZETA
      WRITE(NP,1050) ZETA
      ZETA=ZETA*PI/180
   10 READ(NR,*,END=999) ALPHA
      ALPHA=ALPHA*PI/180
      BETA=PI/2-ALPHA
      GAMA =2*ALPHA
      THETA= PI-GAMA
      DELTA=ZETA/2+ALPHA
      LAMBDA=PI-BETA
      PHI=2*PI-DELTA-THETA -LAMBDA
      AP=(2*(1.0-COS(GAMA)))**0.5
      BOP=BBO+BP
      AOBO=(BOP*SIN(BETA))/SIN(PHI)
      AOP=(BOP*SIN(DELTA))/SIN(PHI)
      AOA=AOP-AP
      WRITE(NP,1010) AOA,AB,BBO,AOBO
      BETA=BETA*180.0/PI
      WRITE(NP,1020) AP,BETA
      ALPHA=ALPHA*180.0/PI
      WRITE(NP,1040) ALPHA
      CALL GRASH(AOBO,AOA,AB,BBO,NP)
      GO TO 10
  999 WRITE(NP,1030)
 1010 FORMAT(//////////,5X,'INPUT = ',F10.4,5X,'COUPLER = ',F10.4,5X,
     $ 'FOLLOWER = ',F10.4,5X,'GROUND = ',F10.4,///)
 1020 FORMAT(5X,'ARM AP = ',F10.4,5X,'BETA = ',F10.4,///)
 1030 FORMAT('1')
 1040 FORMAT(5X,'ALPHA = ',F10.4,///)
 1050 FORMAT(5X,'ZETA = ',F10.4,/////)
      STOP
      END

      SUBROUTINE GRASH(AL1,AL2,AL3,AL4,NP)
C
C     SUBROUTINE GRASH CHECKS THE TYPE OF THE FOUR-BAR LINKAGE USING
C     GRASHOFF CRITERIA.
C
      ALMAX=AMAX1(AL1,AL2,AL3,AL4)
      ALMIN=AMIN1(AL1,AL2,AL3,AL4)
      ALTOT=AL1+AL2+AL3+AL4
      ALMAM=ALMAX+ALMIN
      ALREM=ALTOT-ALMAM
      IF(ALMAM.LE.ALREM) GO TO 100
      WRITE(NP,1010)
      RETURN
  100 IF(AL1.NE.ALMIN.AND.AL2.NE.ALMIN) GO TO 101
      IF (AL1.EQ.ALMIN) GO TO 102
      IF(AL2.EQ.ALMIN) WRITE(NP,1030)
      XMIN=AL1-AL2
      Y=(AL3**2+AL4**2-XMIN**2)/(2*AL3*AL4)
      THMIN=ARCOS(Y)
      XMAX=AL1+AL2
      Z=(AL3**2+AL4**2-XMAX**2)/(2*AL3*AL4)
      THMAX=ARCOS(Z)
```

```
      THMIN=THMIN*180./3.14159
      THMAX=THMAX*180./3.14159
      WRITE(NP,1040) THMIN,THMAX
      GO TO 103
  102 WRITE(NP,1020)
      GO TO 103
  101 WRITE(NP,1010)
 1010 FORMAT (5X,'DOUBLE ROCKER')
 1020 FORMAT(5X,'DRAG-LINK')
 1030 FORMAT(5X,'CRANK-ROCKER')
 1040 FORMAT(5X,'MIN PRES ANGLE = ',F10.4,5X,'MAX PRES ANGLE = ',F10.4)
  103 RETURN
      END
```

```fortran
      REAL LAMBDA
      DATA NR,NP,PI/5,6,3.14159/
      BBO=1.0
      AB=BBO
      BP=BBO
      READ(NR,*) BOP
   10 READ(NR,*,END=999) ALPHA
      ALPHA=ALPHA*PI/180
      BETA=PI/2-ALPHA
      GAMA=2*ALPHA
      X=(2.0-BOP**2)/2
      THETA=ARCOS(X)
      LAMBDA=GAMA-THETA
      A2BO=(2*(1.0-COS(LAMBDA)))**0.5
      A1BO=(2*(1.0-COS(GAMA)))**0.5
      AAO=(A1BO-A2BO)/2
      AOBO=AAO+A2BO
      WRITE(NP,1010) AAO,AB,BBO,AOBO
      AP=A1BO
      BETA=BETA*180.0/PI
      WRITE(NP,1020) AP,BETA
      ALPHA=ALPHA*180.0/PI
      WRITE(NP,1040) BOP,ALPHA
      CALL GRASH(AOBO,AAO,AB,BBO,NP)
      GO TO 10
  999 WRITE(NP,1030)
 1010 FORMAT(//////////,5X,'INPUT = ',F10.4,5X,'COUPLER = ',F10.4,5X,
     $ 'FOLLOWER = ',F10.4,5X,'GROUND = ',F10.4,///)
 1020 FORMAT(5X,'ARM AP = ',F10.4,5X,'BETA = ',F10.4,///)
 1030 FORMAT('1')
 1040 FORMAT(5X,'BOP = ',F10.4,5X,'ALPHA = ',F10.4,///)
      STOP
      END

      SUBROUTINE GRASH(AL1,AL2,AL3,AL4,NP)
C
C     SUBROUTINE GRASH CHECKS THE TYPE OF THE FOUR-BAR LINKAGE USING
C     GRASHOFF CRITERIA.
C
      ALMAX=AMAX1(AL1,AL2,AL3,AL4)
      ALMIN=AMIN1(AL1,AL2,AL3,AL4)
      ALTOT=AL1+AL2+AL3+AL4
      ALMAM=ALMAX+ALMIN
      ALREM=ALTOT-ALMAM
      IF(ALMAM.LE.ALREM) GO TO 100
      WRITE(NP,1010)
      RETURN
  100 IF(AL1.NE.ALMIN.AND.AL2.NE.ALMIN) GO TO 101
      IF (AL1.EQ.ALMIN) GO TO 102
      IF(AL2.EQ.ALMIN) WRITE(NP,1030)
      XMIN=AL1-AL2
      Y=(AL3**2+AL4**2-XMIN**2)/(2*AL3*AL4)
      THMIN=ARCOS(Y)
      XMAX=AL1+AL2
      Z=(AL3**2+AL4**2-XMAX**2)/(2*AL3*AL4)
      THMAX=ARCOS(Z)
      THMIN=THMIN*180./3.14159
      THMAX=THMAX*180./3.14159
      WRITE(NP,1040) THMIN,THMAX
      GO TO 103
  102 WRITE(NP,1020)
      GO TO 103
  101 WRITE(NP,1010)
 1010 FORMAT (5X,'DOUBLE ROCKER')
 1020 FORMAT(5X,'DRAG-LINK')
 1030 FORMAT(5X,'CRANK-ROCKER')
 1040 FORMAT(5X,'MIN PRES ANGLE = ',F10.4,5X,'MAX PRES ANGLE = ',F10.4)
  103 RETURN
      END
```

VITA

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