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LONGITUDINAL DISPERSION OF THERMAL ENERGY IN UNCONSOLIDATED PACKED BEDS

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

degree of

DOCTOR OF PHILOSOPHY

BY

ROBERT EARL BABCOCK

Norman, Oklahoma

LONGITUDINAL DISPERSION OF THERMAL ENERGY

IN UNCONSOLIDATED PACKED BEDS

APPR AJOZ,

DISSERTATION COMMITTEE

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LONGITUDINAL DISPERSION OF THERMAL ENERGY

IN UNCONSOLIDATED PACKED BEDS

CHAPTER I

INTRODUCTION

Rate processes in porous media are of fundamental importance to a variety of commercial operations. Particular interest has been generated for a better understanding of longitudinal heat and mass transfer dispersion mechanisms involved in the displacement of one fluid from a porous medium by a second fluid which is physically similar but differs in either temperature or component concentration from the original fluid. Specific examples of such operations include fixed bed adsorbers (chromatography), regenerator heat exchangers, ion exchange columns, fuel cell electrodes, chemical reactors, and various petroleum recovery techniques involving fluid displacement. The velocity or rate of fluid displacement varies over a wide range depending on the particular operation of interest.

Longitudinal dispersion may be considered as the spreading of the potential involved (either temperature or concentration) as the displacement front moves longitudinally

through the porous medium. This type of process is an unsteady state process which uses a step function or unit pulse boundary condition at the bed entrance. Figure 1 compares the exit conditions with the input conditions for both a step function and a unit pulse input.

The longitudinal dispersion mechanisms involved are similar for both heat and mass transfer. These may be broadly classified as (1) bulk movement of the fluid, (2) molecular diffusion in the various phases present, (3) convective transfer between phases, which will be controlled by both a fluid film resistance and a resistance due to gradients existing within the solid particles, and (4) convective eddy mixing of the fluid phase within the flow channels of the medium. Figure 1 presents a schematic of these mechanisms.

In many mass transfer operations, such as miscible displacement of petroleum from underground reservoirs, the solid particles are not porous, and thus mechanism (3) is eliminated. This is never the case for heat transfer processes since all solids have a definite heat capacity.

The specific process of interest here is the longitudinal dispersion or spreading of a thermal step function when introduced into an unconsolidated packed bed. The dispersion is presented as an effective thermal conductivity which is a lumped parameter representing the sum of the individual mechanisms. Two mathematical models are discussed extensively. These are: (1) the previously used one







parameter diffusion model (4,39,50,67,69), which represents a one dimensional, semi-infinite porous medium, and (2)⁻the four parameter lumped model, which employs the solution of a pair of simultaneous differential equations which clearly shows the contribution of each mechanism.

CHAPTER II

REVIEW OF PREVIOUS WORK

Discussion of previous theoretical and experimental work applicable to this study may be broadly classified into three groups: (1) unsteady state heat transfer studies, mass transfer studies, and general packed bed studies. The last group was used previously for experimental design considerations and will be discussed at the end of this chapter as well as Chapter IV. There are many references available in the literature which are not directly applicable to this work but which are related to the general field of heat transfer in porous media. Included in these are steady state studies, studies in which heat radiation and radial transfer are considered, and solid-gaseous heat transfer studies. These studies are not discussed here. The reader is referred to Green (39) for a general review of them. It should be noted that the work of Green and this work are so closely related that any interested reader would undoubtedly benefit from the work of Green. The same experimental equipment was used in both works, and parts of this chapter and the following chapter were taken directly from Green's dissertation with permission.

Unsteady State Heat Transfer Studies

Square Front

The simplest case of heat transfer in the direction of fluid flow is exhibited by the "square front" model as discussed by several authors including Preston (67) and Churchill (16). In this highly idealized case only the heat transfer mechanisms (1) and (4) of the previous chapter are considered, i.e., bulk movement of the fluid and convective transfer between phases. The latter mechanism is restricted to the case of instantaneous equilibrium between phases, i.e., there is no solid or film resistance to transfer between phases. The differential equation describing this model is:

$$\frac{\partial T}{\partial t} = -V_F \frac{\partial T}{\partial z} \qquad \text{II-1}$$

The output response of equation II-1 to a step function input is a translated step function arriving at the bed exit at time $t = L/V_F$. For this reason V_F is designated as the "square" heat front velocity.

While this model is physically unrealistic it serves as an idealized reference case. Churchill points out that in some cases it is closely approached. Hadidi (44) discusses the application of this model to a system composed of two mobile phases and a stationary solid phase. In many practical cases it is desirous to hold thermal dispersion to a

minimum and thus attempts are made to obtain the square front. However, the square front is only rarely obtained.

Finite Convective Coefficient

When instantaneous equilibrium between the phases does not occur, i.e., ha is finite, the square front model may be modified to yield the following differential equations:

Fluid phase

$$\rho_{\mathbf{w}\mathbf{w}\mathbf{w}}^{\mathbf{c}\mathbf{\phi}}\frac{\partial \mathbf{T}_{\mathbf{w}}}{\partial \mathbf{t}} = -\mathbf{V}\rho_{\mathbf{w}\mathbf{w}\mathbf{w}}^{\mathbf{c}\mathbf{\phi}}\frac{\partial \mathbf{T}_{\mathbf{w}}}{\partial \mathbf{z}} - ha(\mathbf{T}_{\mathbf{w}} - \mathbf{T}_{\mathbf{s}}) \qquad \text{II-2}$$

Solid Phase

$$P_{s}c_{s}(1-\varphi)\frac{\partial T_{s}}{\partial t} = ha(T_{w} - T_{s}) \qquad \text{II-3}$$

This model does not allow molecular conduction in either phase, eddy mixing of the fluid phase, or solid resistance to transfer between the fluid and solid phases. Green (39) has discussed the work of Anzelieus (1), Schuman (76), Klinkenberg (51), Walter (81) and others (29,36,47,48, 49,68) as related to equations II-2 and II-3. The most useful solution obtained by these workers for a step function input is the approximation presented by Klinkenberg (51,52).

$$u = \frac{1}{2} \left[\operatorname{erfc} \left(\sqrt{Z} - \sqrt{Y} + \frac{1}{8\sqrt{Z}} + \frac{1}{8\sqrt{Y}} \right) \right] \qquad \text{II-4a}$$

where

$$Z = \frac{ha}{\rho_{s}c_{s}(1 - \varphi)} \left(t - \frac{z}{v} \right)$$
 II-4b

$$Y = \frac{haz}{\rho_w c_w \phi V} \qquad II-4c$$

Klinkenberg and Sjenitzer (53) have shown for the case of a pulse input of energy equations II-2 and II-3 give an output response which approaches a Gaussian distribution with a mean of Y and a variance of 2Y where Y is defined by equation II-4c.

Several investigators have conducted unsteady-state experiments where values of the heat transfer coefficient, ha, were determined by comparing effluent temperature response curves with equation II-4. Data were obtained by Greenstein and Preston (43) and Preston and Hazen (68) with liquid-solid systems at relatively low liquid velocities (3-24 ft/hr). Their work yielded heat transfer coefficients that correlated with fluid velocity as

$$ha = 0.196 G^{1.82}$$
 II-5

Green (39) has pointed out that the application of equation II-5 to systems in which piston flow is obtained would be questionable since the data were obtained using heating runs, and viscous fingering undoubtedly occurred. Preston and his co-workers measured a mixing cup temperature at the bed exit so no check of deviation from piston flow was possible.

Several workers have conducted experiments where steady state heat transfer coefficients have been measured. Unfortunately, all these workers used gases for the flowing fluid. A large number of investigations of mass transfer between phases in a packed bed have been conducted using both gases and liquids. Mass transfer data indicates a substantial difference between gaseous data and liquid data. For this reason the liquid mass transfer data were used by applying the Colburn "j" factor analogy rather than relying on gaseous heat transfer data.

One Parameter Diffusion Model

The alternate approach to the finite convective coefficient model is to consider longitudinal diffusion in both phases and to <u>mathematically</u> disregard any finite time required for communication between phases. This model was first proposed by Jenkins and Aronofsky (50) and is utilized by recent workers (4,39,69). The describing differential equation is:

$$\frac{\partial T}{\partial t} = -V_F \frac{\partial T}{\partial z} + k_e \frac{\partial^2 T}{\partial z^2} \qquad \text{II-6}$$

Notice that in this model $T = T_w = T_s$.

Preston (67) discusses the possibility that the effective conductivity, k_e , be composed of a static component, k_e^o , and a velocity dependent component, $k_e(v)$. Green extended this concept in proposing that $k_e(v)$ in turn be composed of an eddy mixing conductivity, k_{wm}^o , a heat transfer coefficient conductivity, k_{ha} , and possibly a third conductivity, $k_{s(ha)}$,

characterizing the contribution of the solid phase resistance to the transfer between phases. Green proposed that equation II-6 could adequately describe the physical situation if k_e was considered to be an overall dispersion coefficient composed of several more basic conductivities, each describing one of the previously mentioned mechanisms. Further, Green maintained that the individual conductivities were additive to yield k_e . Green's assumption of additivity was based on two results: 1) the numerical work of Green and Perry (39,41) and 2) a theoretical derivation showing the additivity of k_{ha} and $k_{wm}\varphi$ resulting from the work of Van Deemter et al. (39,79)

Static Conductivity

The diffusion model requires knowledge of k_e^0 , the conductivity of the medium, when a fluid is present but is not flowing. Static thermal conductivities of porous media have been investigated by several authors. In these studies the important criteria is not that the medium be porous, but that it consist of a continuous and discontinuous phase. In this and most other studies the fluid phase is usually taken as the continuous phase, and the solid phase designated as the discontinuous. This designation is not rigorous since the solid phase is somewhat continuous through point to point contact. However, designating the fluid as continuous results in good agreement between theoretically derived expression for $k_{\rm e}^{\rm O}$ and the existing data, thus indicating that point to point contact plays an insignificant role.

Euchen (26) modified the electrical conductivity

work of Maxuell and developed the following predictive equation applicable to heat transfer.

$$k_{e}^{O} = k_{w} \frac{[k_{s} + 2k_{w} - 2(1 - \varphi)(k_{w} - k_{s})]}{[k_{s} + 2k_{w} + (1 - \varphi)(k_{w} - k_{s})]} \qquad II-7$$

Hamilton (45) modified this expression so as to include non-spherical particles. Preston (67) made a rather extensive study of this parameter and his results generally agree with the Euchen equation. In this work k_e^0 was found by extrapolating available k_e data to zero velocity. Green and earlier work by the author (4) present comparisons of extrapolated static conductivities and Euchen static conductivities.

Mass Transfer Studies

Adsorption Studies

Several investigators have studied packed bed adsorption columns which have practical application in chromatography and fixed bed reactor work. (19,34,53,55,71,72,74,80) The mechanisms occurring in adsorption columns are identical to those occurring in heat transfer except that the mass concentration of a diffusing component is the potential involved instead of energy concentration, i.e., in adsorption columns the adsorbing component is dispersed axially by bulk movement of the fluid, by molecular and eddy diffusion within the fluid, and by adsorbing and desorbing on both the external and internal surfaces of the porous packing. The differential equations describing this type of an adsorption. process and the heat transfer process are identical. In particular, Rosen (71,72,73,74) and Deisler (19) have contributed considerably to the mathematical analysis of this model. Rosen has extensively considered the dispersion created by a definite time lag due to communication between fluid and the packing while neglecting any longitudinal molecular or eddy diffusion within the fluid phase. Deisler (19) considered all of the mechanisms mentioned but utilized only a cosine input instead of the more general step function used by Rosen.

Tracer Studies

Numerous workers have investigated dispersion in packed beds by injecting a non-adsorbing component into the flowing fluid. In this type of study the dispersion is due entirely to molecular diffusion and eddy mixing in the fluid phase. No communication between phases occurs. Eddy mixing in the fluid phase, k_{WM}^{ϕ} , cannot be measured directly in heat transfer work since it is impossible to prevent the solid packing from exchanging energy with the fluid phase. For this reason tracer studies are valuable to this study. Tracer dispersion coefficients can be compared with $k_{WM}^{\phi} \rho_{\mu} c_{\mu}^{\phi} \phi^{*}$ in an attempt to correlate eddy mixing coefficients. Since eddy mixing is a hydrodynamic mechanism pertaining to the fluid phase, one would expect heat transfer mixing and mass transfer mixing to be equivalent.

*Obtained in this work by difference, assuming additivity of various mechanisms.

The table on the following page summarizes the various experimental studies available in the literature for spherical beads.

Several theoretical studies have been presented concerning the eddy mixing of a fluid. Klinkenberg and Sjenitzer (53) have used a "similitude" approach in predicting that the eddy diffusion coefficient E be a linear function of Vd_p. This concept was extended by Klinkenberg and Sjenitzer and others to yield the mixing cell model. This model assumes a packed bed to consist of "n" perfect mixers in series, i.e., the fluid leaving a mixing cell has the same concentration as the bulk fluid in the cell. Equations have been derived expressing the effluent from the nth cell as a function of time for either an impulse or step function input. Carberry (12) has extended the model by introducing a cell mixing efficiency.

Aris and Amundson (3) compared the cell mixing and diffusion models and at distances equivalent to several mixing lengths down the bed, the two models gave approximately the same distribution expressions for injected tracer material. By equating these distributions, an expression for the Peclet number resulted.

$$Pe = \frac{Vd}{E + D} = \frac{2}{\gamma}$$
 II-7

Where y is the ratio of distance between successive mixing

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	Table 1	\$-	

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Table 1					
Author (Ref. No.)	Type of Bead	Type of Fluid	Tracer	D cm ² /sec	
Harleman & Rumer (46)	plastic (1 mm)	water	salt	1.35×10^{-5}	
Rumer (75)	glass (0.35 mm)	water	salt	1.35 x 10 ⁻⁵	
Liles & Geankoplis (59)	glass (0.47 & 6.13 mm)	water	2-napthal	1.0×10^{-5} assumed	
Ogata (63)	glass (3.45 & 1 mm)		methylene blue dye	1.0×10^{-5} assumed	
Ebach & White (23)	glass (1 mm)		pontamine blue dye	5 x 10 ⁻⁶	
Carberry & Bretton (13)	glass (1, 3, & 5 mm)		pontamine blue dye	5×10^{-6}	
Cairns (8)	glass (3.2 mm)		NaNO3	0.9 x 10 ⁻⁵	
Strang & Geankoplis (78)	glass (6 mm)	water	2-napthal	1.0 x 10 ⁻⁵ assumed	

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cells to the particle diameter of the pack. Notice that for the restrictions γ = constant and E >> 0 equation II-7 retains the linear dependence of E on Vd_p.

For spheres, and in certain velocity ranges, the length of the mixing cell is essentially equal to one particle diameter, i.e., $\gamma = 1$. For these conditions the mixing cell theory predicts a value of 2.0 for the Peclet number.

Giddings and Robinson (34) have attacked the validity of the mixing cell theory. Their model indicates a nonlinear dependence of E upon velocity at low velocities.

$$E = \frac{1}{\frac{1}{\lambda V d_p} + \frac{2}{c_g V^2}}$$
 II-8

Where λ is a proportionality constant and where C_g is a nonequilibrium term for diffusion in the interparticle spaces. Notice that at high velocities equation II-8 reduces to the linear dependence as presented by Klinkenberg and Sjentzer (53). Perkins and Johnson (64) have presented a similar but independent treatment of diffusion in the interparticle spaces. Their work will be discussed in more detail later.

Other mixing models have been proposed. Giddings (32) has proposed a random walk model. Cairns and Prausnitz (9, 10) have proposed a statistical model variation to the random walk model. Their work is based on the mathematics of Einstein (24). Taylor (79) has developed models for flow

through capillary tubes which have application to porous packs in some cases. Gottschlich (30) has discussed and presented a model predicting the influence of fluid dead spaces on eddy mixing.

In this work the mixing cell model is used.

Steady State Mass Transfer Studies

Several investigations have been conducted on steady state mass transfer between liquids and solid packings. (22, 30,82) The usual correlating procedure is to plot j_m versus the Reynolds number. In general, the analogy between heat and mass transfer holds, making it possible to use masstransfer results to predict heat transfer rates. This has been discussed by Gamson (31), Denton (20), and Colburn (17), among others.

The j_m correlation used in this work is the one presented by Dryden, Strang, and Withrow (22). It was chosen because the Dryden, et al. data was taken in the laminar flow region and required less extrapolation than other data to the velocity range of this work. Fulton (28) has presented a review of the various gas-solid j_m correlations.

General Packed Bed Studies

Flow Distributions in Packed Beds

Inherent in this work and most other packed bed studies is the assumption of piston-like fluid flow. Deviations from piston flow are known to occur. In the region near the wall the presence of the wall increases the void volume of the pack. The result is a higher fluid velocity in this region, and when the particle size becomes large compared to the tube diameter, significant deviations from piston flow can occur. Cairns and Prausnitz (11) and Schwartz and Smith (77) have reported this wall effect to be significant in cases in which the ratio of particle diameter to tube diameter is greater than 0.04-0.067.

Flow channeling may also cause deviations from pistonlike displacement. (6,15,27) Generally, flow channeling is caused by either an unfavorable viscosity ratio, i.e., the fluid being displaced has a higher viscosity than the displacing fluid, or natural convection due to density variations. (22,30) The latter mechanism is characterized by the Grashof number which involves the particle diameter raised to the third power. For this reason natural convection effects increase rapidly as the particle size is increased.

Length Effects

Green (39) applied a statistical "t" test to k_e measurements made at 6.77 inches and 11.04 inches and found the data at the two locations to be in agreement at a 95% confidence level. Other literature data on longitudinal heat and mass transfer in packed beds indicate that length effects should not be significant in beds of the length used here. (13,23,44,77)

Shape Factors

Aris (2) has presented a theoretical discussion concerning the role packing shape plays in connection with longitudinal dispersion of heat and mass. It is shown that taking sphericity, the surface area of a sphere having the same volume as the particle divided by the surface area of the particle, as the characteristic dimension does not reconcile the results for different particle shapes. A new shape factor is introduced from which the contribution of internal diffusion to the longitudinal dispersion of heat or matter may be estimated for various shapes.

Reymond (69) conducted unsteady state longitudinal dispersion experiments, similar to the experiments conducted in this study, in which helixes, Raschig rings, and perforated spheres were used as packing. Reymond also concluded that the particle volume to surface area parameter was insufficient. Reymond's work was somewhat inconclusive since the effect of matices trapped fluid within the individual was not determined.

Research Objectives

This research is the conclusion of a long-range program supported by the National Science Foundation designed to study the longitudinal dispersion of thermal energy as a cold fluid displaces the original fluid present in a previously heated packed bed. The goal is a better understanding of the mentioned mechanisms, their interactions, if any, and the conditions under which each is significant. A

secondary goal is to compare the eddy mixing mechanism for heat transfer to the extensive mixing data available in the literature for mass transfer.

Specifically a mathematical derivation is desired that will show the additivity of the mechanisms as proposed by Green (39), and to show under what range of system parameters this additivity is valid. Also, it is desirable to show that the solid thermal diffusivity exerts an influence on fluid mixing that is not present in mass transfer mixing.

CHAPTER III

THEORETICAL INVESTIGATION

One Parameter Lumped Model

The following is a detailed review of the literature concerning the one parameter diffusion model. It is used to initially reduce the experimental data, and is presented here in order to allow the reader to compare it to the four parameter lumped model to be discussed.

It is known that when a fluid is displaced from a packed section by a displacing fluid of different temperature a characteristic "s" shaped erfc* curve response will result at any point downstream (1,39,41,44,50,66). The simplest mathematical model that yields an erfc solution is that resulting from an energy balance applied to a one dimensional system of differential length dz and composed of both solid and fluid phases. The major assumptions involved are:

- 1.) Piston-like displacement occurs.
- 2.) Constant physical properties.
- 3.) Fluid and solid temperatures become equal

*The erfc function is discussed by Carslow and Jaeger (14) page 482.

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instantaneously, i.e., $ha = \infty$.

- 4.) No temperature gradients exist in the individual solid particles normal to the direction of fluid flow. This allows the solid phase to act as an evenly distributed heat sink having the property of allowing conduction in the direction of fluid flow.
- 5.) No radial temperature gradients exist in either phase, i.e., the bed is perfectly insulated radially.



The differential equation which results has been solved by Jenkins and Aronofsky (50).

$$\frac{\partial T}{\partial t} = -V_F \frac{\partial T}{\partial z} + D_e \frac{\partial^2 T}{\partial z^2}$$
 III-1

where
$$V_F = \frac{\rho_w c_w \phi V}{\rho_w c_w \phi + \rho_s c_s (1 - \phi)}$$

and

$$D_{e} = \frac{k_{e}}{\rho_{w} c_{w} \phi + \rho_{s} c_{s} (1 - \phi)}$$

Equation III-1 is the usual one dimensional heat conduction equation with the additional bulk flow term $\left(-V_{\rm F} \frac{\partial T}{\partial z}\right)$. The k_e contained in the effective thermal diffusivity, D_e, represents an effective thermal conductivity dependent upon flow conditions as well as static properties of the system.

If the following dimensionless variables are defined:

$$u = \frac{T - T_o}{T_T - T_o}, \quad \tau = \frac{V_F t}{L}, \quad \text{and} \quad X = \frac{z}{L}$$

Equation III-1 becomes

$$\frac{\partial u}{\partial \tau} = -\frac{\partial u}{\partial u} + \frac{\nabla_{e}}{D_{e}} \frac{\partial^{2} u}{\partial^{2} u}$$
 III-2

Using the following boundary conditions

$$u(X,0) = 0$$

 $u(0,\tau) = 1$
 $u(\infty,\tau) = 0$

the solution given by Jenkins and Aronofsky (50) at the bed exit where z = L, (X = 1) is:

$$u(1,\tau) = \frac{1}{2} \left[\operatorname{erfc} \left(\frac{1-\tau}{2\sqrt{(D_e/V_F L)\tau}} \right) + R \right] \qquad \text{III-3}$$

where R =
$$e^{(V_F L/D_e)}$$
 erfc $\left(\frac{1+\tau}{\sqrt{(D_e/V_F L)\tau}}\right)$

Preston (66) has suggested that R be approximated by

$$R = \frac{\exp \left[z! - t_p^2\right]}{2\sqrt{\pi} t_p}$$

where $z' = V_F L/D_e$ and

$$t_{p} = \frac{1+\tau}{2\sqrt{(D_{e}/V_{F}L)\tau}}; \quad t_{p}^{2} = z'\left(\frac{1}{2}+\frac{\tau}{4}+\frac{1}{4\tau}\right)$$
 III-5

From equation III-5 it may be shown that t_p^2 is always greater than z' thus rendering R small for small values of (D_e/V_FL) , and becoming increasingly important as (D_e/V_FL) increases, to the point that the characteristic "s" shaped curve no longer exists at $(D_e/V_FL) = \infty$. Levenspiel (58) has constructed curves of u vs. τ with values of (D_e/V_FL) as a third parameter.



Figure 2

The maximum value of R may be approximated by differentiating the approximation of R with respect to τ and setting the derivative equal to zero. The results obtained by Preston (67) are

1.) R occurs at $\tau = 1.0$, i.e., the arrival time of the heat front.

2.)
$$R_{max} = (\frac{1}{2}\sqrt{\pi})\sqrt{(D_e/V_FL)} = .282\sqrt{(D_e/V_FL)}$$
 III-6

Specific values of (D_e/V_FL) for the systems studied in this work are tabulated in Appendix B. The values range from 0.006 to 0.100. This range of values is in the general region Levenspiel (58) has designated as "intermediate." Because of the relatively low values of (D_e/V_FL) the second term in the solution, R, has been neglected here. Green (39) has concluded that R is negligible if $(2D_e/V_FL) \ll 1$.

According to the above discussion one would expect to characterize the system by a single parameter (D_e/V_FL) . This is true only for a particular system at a particular velocity. That is, at a particular velocity there exists a value of the parameter (D_e/V_FL) which will cause the experimental and theoretical "s" shaped temperature distributions to coincide. However, attempts to correlate (D_e/V_FL) as a function of velocity, particle diameter, and system physical properties have been unsuccessful, indicating the need for a more advanced mathematical model.

The work of Green (39) and others (4,44,67) in heat transfer studies has shown that assumptions (3) and (4) are

invalid, i.e., there exists a finite time lag for communication between phases due to temperature gradients within the individual solid particles and a film resistance to heat transfer in the fluid phase. Due to the inadequacy of the one parameter diffusion model to contribute information concerning these two mechanisms, the four parameter lumped model has been presented. The lumped model shows that these mechanisms may be characterized by expressions which contain system parameters that are known under a given set of experimental conditions.

Four Parameter Lumped Model

It is proposed in this work that the system be characterized by three velocity dependent parameters and one velocity independent parameter. These are:

- k which characterizes the finite time required for heat to be transported across a stagnant fluid film surrounding the solid phase.
- 2. k which characterizes the finite time res(ha) quired to eliminate temperature gradients within the solid phase normal to the direction of fluid flow.
- 3. $k \phi$ which characterizes the eddy mixing of the fluid due to stagnant and trapped pockets of fluid.
- k_wφ which characterizes molecular conduction in the fluid phase.

The mathematical solution which represents the four parameter system results from the simultaneous solution of differential equations describing both the fluid temperature and the temperature profile existing within the solid particles. These equations are:

1.) The fluid equation

1

$$\rho_{W}c_{W}\frac{\partial T_{W}}{\partial t} + \rho_{W}c_{W}V\frac{\partial T_{W}}{\partial z} - (E+D)_{W}\rho_{W}c_{W}\frac{\partial^{2}T_{W}}{\partial z^{2}} = -\frac{\rho_{S}c_{S}}{m}\frac{\partial^{2}T_{S}}{\partial t} \qquad III-7$$

where $\rho_{s}c_{s}\frac{\partial T_{s}}{\partial t}$ represents the rate of change of the average heat content of the solid phase with respect to time. The boundary conditions which apply to equation III-7 are:

$$T_{w}(z,0) = T_{0}$$
$$T_{w}(0,t) = T_{I}$$
$$T_{w}(\infty,t) = T_{0}$$

2.) The solid equation

$$\rho_{s}c_{s}\frac{\partial T_{si}}{\partial t} = \rho_{s}c_{s}D_{s}\left[\frac{\partial^{2}T_{si}}{\partial n^{2}} + \frac{2}{r}\frac{\partial T_{si}}{\partial r}\right] \qquad \text{III-8}$$

The boundary conditions for equation III-8 are

 $T_{si}(r,z,0) = T_{si_0}$ $T_{si}(0,z,t) = finite (i.e., the temperature of the center of the particle is not infinite.)$ $T_{si}(b,z,t)$ is given by the expression

$$\rho_{s}c_{s}\frac{\partial \tilde{T}_{s}}{\partial t} = \frac{3h}{b\rho_{w}c_{w}}\left(\rho_{w}c_{w}T_{w} - \frac{\rho_{s}c_{s}T_{si}(b,z,t)}{K}\right) \qquad \text{III-9}$$

where $K = \frac{\rho_{s}c_{s}}{\rho_{w}c_{w}}$

While the basic ideas for the simultaneous solution of equations III-7 and III-8 follow from the work of Rosen (71-73) and Deisler (18,19), the two equations have not been solved to this author's knowledge for the boundary conditions indicated. Rosen solved the case in which the second order term $(E + D)_w \rho_w c_w \frac{\partial^2 T_w}{\partial z^2}$ was neglected, and much of his analysis is applicable to the work presented here. Deisler solved equations III-7 and III-8 for the steady state response to a cosine input. The step function solution presented here is mathematically more general and has wider practical application than the steady state cosine response.

The major assumptions involved in the advanced model are:

- (1) Piston like displacement occurs.
- (2) No radial heat losses from the packed bed.
- (3) Radial symmetry exists within the individual spherical particles, i.e., $\left(\frac{\partial T_W}{\partial z}\right) dp \ll T_0 T_T$
- (4) No heat is conducted downstream by exchange between particles through point to point

contact or longitudinally within the individual particles.

(5) Physical properties of both phases are temperature independent.

A more detailed discussion of assumptions (3) and (4) is presented in Appendix H. An approximation for nonspherical packing would replace equation III-8 by a parallel slab model in which conduction in the direction of fluid flow is not allowed. Aris (2) has proposed other treatments for the cases in which the solid particles are non-spherical.

The simultaneous solution for equations III-7 and III-8 with the boundary conditions indicated involves the following steps, the details of which are presented in Appendix A.

- The solid and fluid dependent variables are redefined in order to yield energy concentration variables.
- 2.) A solution to the equation describing the solid phase (Equation A-2) is then obtained yielding a concentration distribution within the solid phase in terms of a variable surface concentration. (72)
- 3.) This expression is first integrated over the entire volume of the particle, and then differentiated with respect to time, to obtain an expression for the rate of change of the average
solid concentration with respect to time. (72) (Equation A-4)

- 4.) Equation A-4 is then introduced into the equation describing the fluid phase, and the Laplace transform with respect to time taken.
 (Equation A-5)
- 5.) By utilizing boundary condition III-9 (or Equation A-3) the surface concentration is expressed in terms of the average solid concentration which may be expressed as the left hand side of the equation describing the fluid phase. (Equation A-1). When this expression is introduced into Equation A-5 the result is an ordinary, second order, linear differential equation in the Laplace transform domain. (Equation A-6)
- 6.) After Equation A-6 is solved, the work of Rosen (72) is used to obtain the inverse Laplace transform. The resulting solution is the following infinite integral involving a variable of integration, λ .

$$\begin{split} u(z,\theta) &= \frac{1}{2} + \frac{2}{\pi} \int_{0}^{\infty} \exp \left\{ \gamma x H_{1} + \frac{z D_{L}}{v^{3}} \left(\sigma^{2} \lambda^{4} + \frac{2 \sigma \lambda^{2} \gamma H_{2}}{m} + \frac{\gamma^{2} (H_{2}^{2} - H_{1}^{2})}{m^{2}} \right) \right\} &. \sin \left\{ \sigma_{\theta} \lambda^{2} - \gamma x H_{2} + \frac{z D_{L}}{v^{3}} \left(2 \sigma \lambda^{2} \gamma H_{1} + \frac{2 H_{1} H_{2} \gamma^{2}}{m} \right) \right\} \frac{d\lambda}{\lambda} \end{split}$$

where ${\rm H}_1$ and ${\rm H}_2$ are complicated hyperbolic functions of λ and ν

$$H_{1}(\lambda,\nu) = \frac{H_{D_{1}} + \nu(H_{D_{1}}^{2} + H_{D_{2}}^{2})}{(1 + \nu H_{D_{1}})^{2} + (\nu H_{D_{2}})^{2}}$$
 A-18

$$H_{2}(\lambda,\nu) = \frac{H_{D_{2}}}{(1 + \nu H_{D_{1}})^{2} + (\nu H_{D_{2}})^{2}}$$
 A-19

and where H_{D_1} and H_{D_2} are defined as:

$$H_{D_{1}} = \left[\lambda \left(\frac{\sinh 2\lambda + \sin 2\lambda}{\cosh 2\lambda - \cos 2\lambda}\right) - 1\right] \qquad A-14$$

$$H_{D_2} = \lambda \left(\frac{\sinh 2\lambda - \sin 2\lambda}{\cosh 2\lambda - \cos 2\lambda} \right)$$
 A-15

Rosen (72) has tabulated exact values of H_{D_1} and H_{D_2} as a function of λ . If equations A-14 and A-15 are expanded in a Maclaurin series the following limiting expressions may be obtained

$$H_{D_1} = \frac{4\lambda^4}{45} \qquad \text{III-10}$$

$$H_{D_2} = \frac{2\lambda^2}{3}$$
 III-11

Limiting values obtained from these expressions compare within 4% of the exact values obtained by Rosen (72)

if λ < 1. If these limiting expressions for ${\rm H}_{D_1}$ and ${\rm H}_{D_2}$ are substituted into equations A-18 and A-19, the following approximations are obtained for H_1 and H_2 if the product $\lambda^{4}\nu < 0.5.$

$$H_{1}(\lambda,\nu) = \frac{4\lambda^{4}(1+5\nu)}{45}$$
 A-20

$$H_2(\lambda,\nu) = \frac{2\lambda^2}{3} \qquad A-21$$

If equations A-20 and A-21 are introduced into equation A-24, the resulting infinite integral reduces to the following simplified form.

$$u(z,\theta) = \frac{1}{2} + \frac{2}{\pi} \int_{0}^{\infty} e^{-\lambda^{4} \left\{ (4/45)\gamma x (1+5\nu) + \frac{zD_{L}}{\nu 3} \left(\sigma + \frac{2\gamma}{3m}\right)^{2} \right\}}$$

$$\sin \left\{ \sigma \theta \lambda^{2} - (2/3)\gamma x \lambda^{2} \right\} \frac{d\lambda}{\lambda} \qquad A-25$$

Equation A-25 in turn reduces to the following erfc form. The details are presented in Appendix A.

$$u = \frac{1}{2} \left[erfc \left(\frac{1 - \tau}{2 \sqrt{(D_e'/V_FL)}} \right) \right]$$
 A-31

where

ł

$$D_{e}' = \frac{k_{wm} \varphi + k_{w} \varphi + k_{s}(ha) + k(ha)}{\rho_{w} c_{w} \varphi + \rho_{s} c_{s}(1 - \varphi)}$$
$$k_{s}(ha) = \frac{[\rho_{s} c_{s}(1 - \varphi) V_{F} d_{p}]^{2}}{60 k_{s}(1 - \varphi)} \qquad \text{III}$$

and

III-12

$$k_{(ha)} = \frac{\left[\rho_{s}c_{s}(1-\varphi)\nabla_{F}\right]^{2}}{ha} \qquad \text{III-13}$$

The conclusion drawn from the four parameter model is that the total dispersion of the system may be approximated by the summation of four independent conductivities each of which represents one of the previously mentioned mechanisms. In practice $k_w \varphi$ is replaced by the static conductivity, k_e^{o} , so as to include downstream conduction through the solid phase. The other three parameters, $k_{wm} \varphi$, $k_{s}(ha)$ and $k_{(ha)}$ are all velocity dependent, and their sum is designated as $k_e(v)$.

Three of the conductivities are known or may be calculated for a given set of experimental conditions. The exception is the term $k \varphi$. This term is obtained by difwm ference, and the mixing cell model is used to correlate it.

Conditions for Convergence

The convergence of equation A-24 to the desired erfc form is dependent upon the validity of the limiting expressions for H_1 and H_2 (A-20, A-21). The critical point in both limiting expressions is the convergence of the denominator of equations A-18 and A-19 to a value of 1.0. Using the approximate values of H_{D_1} and H_{D_2} , this requires that

$$\left(1 + \frac{4\lambda^{4}\nu}{45}\right)^{2} + \frac{4\lambda^{4}\nu^{2}}{9} \rightarrow 1.0$$
 III-14

For small values of λ the λ^4 term insures this convergence; however, for values of λ approaching 1.0, the restriction of small values of ν must be added. The following table presents values of equation III-14 for various conditions of λ and ν .

Table 2

λ	ν	$\left(1 + \frac{4\lambda^{4}\nu}{45}\right)^{2} + \frac{4\lambda^{4}\nu^{2}}{9}$	% error
1	.1	1.02	2.0
1	•5	1.20	20
1	l	1.63	63
•5	.1	1.002	0.2
•5	•5	1.01	1.0
•5	1	1.04	4.0

A detailed study of the exponential term in equation A-25 reveals an apparent built-in safety factor that assures the convergence to the erfc form. For example, if \vee becomes approximately 1.0 the term 1 + 5 \vee increases the value of the exponential and thus the infinite integral converges for a smaller value of λ . Table 2 indicates a \vee value of 1.0 may be tolerated if $\lambda_{\chi}^{\checkmark} < 0.5$. For the case of small values of ν , the λ^{4} term assures convergence at values slightly greater than 1.0. Table 2 indicates that values of $\lambda > 1.0$ may be tolerated at low values of ν . The only time convergence to the erfc form would result in appreciable error would be a case in which $\nu > 1.0$ and the second term in the exponential, $\frac{zD_L}{v3} \left(\sigma + \frac{2\gamma}{3m}\right)^2$, was very small. For the systems studied here, σ is of the order of magnitude of 10^2 causing the second term in the exponential to be comparable in size to $\frac{4\gamma x (1 + 5\nu)}{45}$.

CHAPTER IV

EXPERIMENTAL INVESTIGATION

The experimental model was designed to simulate the basic requirements of the real physical system. The main requirements were:

(a) Piston flow of a liquid in one direction(axial) through a packed bed.

(b) The establishment of a known initial constant temperature throughout the bed.

(c) The introduction of a step function in temperature into one end of the bed.

(d) The measurement of the response temperature profile at a known position down the bed.

(e) Negligible heat losses in the direction perpendicular (radial) to the fluid flow.

Early considerations indicated that a design similar to that used by Preston (6%) and Hadidi (44) would be satisfactory. The heat-transfer section in these investigations consisted of a packed bed of particles contained in a thinwalled, insulated, cylindrical tube. An open-volume section immediately above the bed face served to distribute the flow evenly across the pack. A step-function temperature input

was approximated by first bringing the bed to a desired temperature, T_c , using the test liquid as the heating media. Then, the entrance-face temperature was quickly changed by "flushing out" the open volume above the bed with liquid at the different input temperature, T_i . These design concepts were followed in this work.

Experimental Apparatus

Flow System

A schematic diagram of the flow system is shown in Figure 3. Fluid storage was in an 8 gallon closed tank. Flow rates through the system were controlled by regulated air pressure on the liquid in the storage tank. Liquid leaving the storage passed through a rotameter (C). To maintain constant volumetric flow rates during an experimental run, a constant differential type flow controller (not shown) was installed between the fluid storage tank and the rotameter (C). The flow controller was necessary due to the changing upstream pressure caused by the sudden change in fluid viscosity at the time of application of the step function.

To preheat the liquid, it was passed through the heat-transfer coils in two heating baths in series. Water was used as the heating media. In the first bath, a crude temperature control was maintained with the test liquid being heated to within a few degrees of the final





temperature. The heat source was a 1000 watt immersion heater (D) which contained its own thermostat. The second bath maintained a fine temperature control $(\pm 0.1^{\circ} F)$ and brought the liquid entering the packed tube to a set temperature level. The control in this second bath consisted of a mercury temperature regulator (E) and an electronic relay in conjunction with electric immersion heaters (D).

The test liquid went directly from the heating baths to the packed-bed test section. To reduce heat losses between the temperature baths and the test cell, a double-pipe heat exchanger insulated the flow line (A). Hot water from the second constant-temperature bath was circulated through the annulus of this exchanger.

When the test liquid was not preheated, it went immediately from the flow meter (C) to the test cell, bypassing the heating baths. With a constant flow rate, the temperature at the entrance to the cell was found to hold steady $(\pm 0.1^{\circ} F)$ over the time of an experimental run.

A small resistance heater (G), manually controlled by means of a rheostat, was wound around the liquid flow line just above the cell entrance. The need for this heater is discussed later. When desired, the packed bed could be by-passed by closing the exit flow line and opening the flush-out line (B) from the entrance cap. Provision was made to catch timed samples of the packed-bed effluent in order to determine volumetric flow rates.

Temperatures were measured using iron-constantan thermocouples and a Minneapolis-Honeywell multipoint temperature recorder (Y 153 x 87-C-11-111-106-8-B-V). Up to 24 separate points could be recorded during a given run, however, a maximum of six were used in this work. Print speed was two seconds per point, with a recording chart speed of one inch per minute. The temperature range was 60 to 220°F. The chart was graduated at 1.0°F and temperature recordings could easily be read to 0.2°F. Chart speed was 1 inch per minute. Both 30 and 24 gauge, spun-glass insulated thermocouples were used. In order to obtain small thermocouple beads, an electric arc was used to form the wire junctions.

Test Cell

More detailed sketches of the test cell are shown in Figures (4), (5), (6), and (7). The packed bed, consisting of solid spheres, was held in a cylindrical metal container which was 3.66 ± 0.1 inches ID and 13.67 inches in length (including threaded end pieces). The spheres were held in place between two end retaining screens, each of which was composed of a 100 mesh over a 6 mesh screen. The outlet retaining screen was soldered permanently in place while the entrance screen was fixed using Armstrong A-1 adhesive (Armstrong Company, Warsaw, Indiana).

The wall of the packed bed was made of 0.010 inch stainless steel sheet formed into a cylindrical shape and soldered at the seam. Threaded end pieces were soldered to



Figure 4 - Test Cell

the tube. The thinness of the tube resulted both in a low wall heat capacity and small heat conduction down the tube in the direction of fluid flow. The heat capacity of the tube wall was approximately 1.4% of the total packed-bed heat capacity, with water as the test liquid.

Fluid entrance and exit caps were threaded to match the tube end pieces. The use of "O" rings at this point prevented fluid leak (Figure 4). The thin-walled retaining tube, plus end caps, were inserted into a heavy-walled, steel casing. "O" rings were also used here to hold the inner tube in place and to seal off the annular space. This arrangement allowed a vacuum to be pulled around the packed bed, providing insulation. Vacuums on the order of 2 mm total pressure were used. An alumina-foil radiation shield around the inner tube, at a distance of about 0.25 inches from the tube, provided further against heat losses from the packed bed.

The entrance cap contained a flow sparger which served to distribute the incoming fluid over the face of the packed bed (Figure 5). There was a void space of 0.4 inches above the bed in which mixing of the feed liquid occurred. Two flush-out exit lines were fitted into the entrance cap allowing the bed to be by-passed. The use of this flush-out will be discussed in the Procedure Section. Thermocouples were fixed at different radial positions just above the entrance retaining screen (Figures 6 and 7). The



ENTRANCE CAP FLUSH OUT LINES AND THERMOCOUPLE OPENING



Figure 5 - Test-Cell Entrance Cap; Flow Sparger and Flush-Out Lines



Figure 6 - Test-Cell Entrance and Exit Caps; Thermocouple Openings



Figure 7 - Test Cell; Thermocouple Positions

thermocouples were sealed by taking the leads in through 1/4 inch copper tubing and applying Armstrong adhesive at the outlets.

The exit cap contained seven openings for thermocouple leads (Figures 6 and 7). Thermocouples (30 gauge) located in the packed bed itself were run into the cap openings, through small holes in the bottom retaining screen, and up into the bed. These were fixed in place with Armstrong adhesive prior to packing the bed. Lead wires were sufficiently rigid to hold the thermocouples in upright positions. Thermocouples could thus be placed at any selected depth in the bed, or at the packed-section exit, just below the retaining screen. The thermocouple openings were designed to serve as fluid flow outlets, however, it was found satisfactory to use only the center exit line.

Experimental Materials

Two liquids were used in the experimental program. These were distilled water and Soltrol.* Sources and physical properties of these liquids are given in Appendix F.

Four different solid spheres were used to formulate the packed section. These were glass, lucite, lead, and stainless steel. The sizes used were 3 mm and 6 mm in diameter for the glass, lucite and lead systems, and 6 mm only for the stainless steel. Sources and descriptions of the

*See Appendix F for a discussion concerning the Soltrol used with the lead packing.

beads are also presented in Appendix F. The 3 mm beads were screened between U. S. Sieve sizes 6 and 7. The diameter was then taken as the mean of the two screen openings which resulted in a diameter of 0.118 inches. The 6 mm beads were measured with a micrometer and the average of twenty-five measurements was taken as the diameter of the bead. The densities of the beads were determined by measuring the liquid volume displaced by a bead sample of known weight. Care was taken to eliminate the effect of air bubbles clinging to the surface of the spheres.

Experimental Procedure

Preliminary Procedures

The Minneapolis-Honeywell Temperature Recorder was calibrated using a Leeds and Northrup Potentiometer #8662, as prescribed in the Leeds and Northrup manual.

Thermocouples were checked against the best available thermometers over the experimental temperature range. The precision of all thermocouples used was judged to be better than $\pm 0.2^{\circ}$ F between 70° F and 175° F. Accuracy was within 0.5° F.

Prior to packing the spheres, thermocouples which were to be located within the bed were fixed in position and their locations measured. The thermocouples generally were placed approximately 2.0 inches from the outlet end of the bead pack and approximately 11.0 inches from the entrance

face. Also, prior to packing, the empty volume of the tube between the retaining screens was measured to allow a calculation of bed porosity.

The beads were wet packed using a mechanical shaker, with two to four inches of water maintained above the beads during packing. When the final bed height was reached, the top was smoothed, and the retaining screen fixed in position using Armstrong adhesive. Once the top screen was fixed and the entrance cap threaded on, the complete tube could be inverted with no shifting of the bed.

Bed porosity was calculated, knowing the empty tube volume, bead density, and total weight of beads in the bed. Packing as described gave porosities reproducible to within two per cent.

The packed heat-transfer tube, with the radiation shield in place, was next inserted into the heavy-walled outside cylinder which was fixed in a tri-pod metal stand. The "O" rings were clamped into place. The assembled tube was placed in its operating location, leveled, and a vacuum was pulled on the annular space. The tube was now ready for operation.

Experimental Run Procedure

The experimental procedure consisted of bringing the bed to a constant temperature, injecting a step function in temperature at one end, and measuring the response curve at fixed positions.

The packed section was heated to a constant specified temperature throughout by preheating the test liquid in the constant-temperature baths and flowing it through the section until thermocouple readings at the inlet and outlet agreed. Initial bed temperatures between 135°F and 150°F were used, with about 145°F being the usual value. The hot flow through the bed was stopped. The entrance cap (space above packing) was flushed out at a high flow rate with cool (room temperature) test fluid. This was done by closing in the tube exit line and opening the entrance-cap, flush-out line. Flush-out was continued until the entrance thermocouples indicated a constant temperature. The approximate flow rate for the run was set using the flow meter. After allowing a few seconds for further temperature adjustment at the entrance, flow was started through the test section by simultaneously opening the tube exit line and closing the flush-out line. The temperature recorder was started at this same instant. Total time of flush-out was held to one minute or less and, as discussed in the next section, this procedure resulted in a satisfactory approximation to a temperature step function.

Even with flush-out, the entrance temperature tended to drift downward $1-3^{\circ}F$ during a run. This was apparently due to a combination of initial incomplete cooling of the entrance cap and back diffusion of heat out of the bed. To offset this drift, a small resistance heater was wound around the feed line, just above the entrance cap. By manually

controlling the input from this heater, a constant temperature of $\pm 0.5^{\circ}$ F was maintained at the packed-section face except during approximately the first 20 seconds of a run. A drift of 1-2° F sometimes occurred in this short initial period. These tolerances were exceeded very slightly in a few of the runs, with no noticeable effect on the measured temperature profile. At interstitial velocities of approximately 5 ft/hr or less, back diffusion of heat out of the bed prevented complete control of the input temperature.

During a run the flow rate was held constant within one percent by the flow controller. Flow was measured by catching timed samples of the effluent. It was necessary to control the flow rate during a run because of changing pressure drop across the packed bed as the heat front progressed down the bed.

The temperatures at six pre-selected points in the system were recorded during the run. The two center positions, at the bed face and within the packed section (positions 1 and 8, see Figure 7) were always recorded. These points were all that were really necessary to the data calculations. The other thermocouples provided auxiliary information on piston flow and length effects. No deviations from piston flow were noticed. Wall effects were controlled by keeping $d_p/d_t < 0.068$ as suggested by previous workers. (11, 77) Viscous fingering and natural convection effects were eliminated by flowing vertically upward while displacing a

warm fluid by a cooler one. An experimental run concluded when all thermocouples in the bed reached the temperature of the input fluid. A typical data sheet and temperature recorder chart are presented in Appendix E.

In changing from one fluid system to another, the bed was unpacked, the bead dried, and then the bed repacked. This is necessary when the two fluids are immiscible as was the case here. (4)

Exploratory Data

A few preliminary runs were made to explore the possibility of using a pulse input instead of a step function. Although the results were not completely discouraging it was decided that the step function would be more practical. A recorder chart for a pulse run is presented in Figure 8.

Also, preliminary data was taken on an empty tube hoping to be able to obtain information on the contribution of molecular axial diffusion only. The results were useless due to the fact that the heat front arrived much earlier than would be expected (i.e. $V_F >> V$). No logical reason for this phenomenon has occurred to this author.

Step Function Injection

One of the weaker points of the procedure was the injecting a temperature step function into the bed. A preliminary calculation was made to estimate the amount of heat conduction into or out of the bed during a two-minute



Pulse Time 30 Seconds

flush-out. This indicated that the very maximum heat conduction would be about 2% of the total heat input. An experimental check was made by conducting entrance-cap, flushout tests with thermocouples at known positions just under the packed-bed face. These tests showed the calculated maximum conduction rate to be high, and that a negligible amount of mixing between the bed liquid and entrance-cap liquid occurred. A one-minute flush-out period would therefore not cause adverse effects, and actually flush-out times up to two minutes resulted in no appreciable effects on the temperature response curves of the bed.

The temperature of the injected fluid was constant across the inlet face of the packed section to $\pm 0.5^{\circ}$ F in most runs. In some cases, there was as much as a 3° F temperature difference between the center and outside radius.

It is expected that the results were not affected by using a static injection, i.e., fluid flow through the bed stopped during flush-out. This has been checked in a study on the longitudinal dispersion of mass in porous media (13).

Bed Heat Losses

Calculated heat losses from the glass bead pack were on the order of 1.5-3% of the heat input at a water flow velocity of 4.5 ft/hr, decreasing with increasing flow rate. For the metal systems heat losses were slightly higher due to the increased time required for completion of the run.

The use of a vacuum did not significantly reduce the air thermal conductivity, but convective transfer was essentially eliminated. An estimation of radiation indicated that the radiation shield reduced these losses to a negligible value.

Temperature Measurement

Thermocouples were made of 30 gauge wire allowing small junction beads to be formed (on the order of .02 inches). However, a slight time lag would occur in the heating of the thermocouple bead. Recorded temperatures therefore represent some intermediate value between the true liquid temperature and the solid-sphere temperature. Estimations of this time lag, based on literature heat-transfer coefficient correlations indicated that it had negligible effect on the data, with the temperature difference between fluid and thermocouple being less than .01.

Experimental Data Sets

The experimental systems investigated are summarized in Table 3. In all cases the cooling runs were made with flow being upward. Green (39) has presented recorder charts in which severe channeling was present for heating runs. The velocity range was between 5 ft/hr and 150 ft/hr. For the metal systems, runs below 10 ft/hr were discarded due to heat losses. The data is tabulated in Appendix B. Table 3

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Run No.	Solid	Liquid	d _p	φ
D-BB-24C-42C	Glass	Water	0.232"	0.362
D-BB-118C-135C	Glass	Water	0.232"	0 .36 2
D-BB-46C-64C	Glass	Soltrol	0.232"	0.370
D-BB-136C-147C	Glass	Soltrol	0.118"	0.363
D-BB-88C-117C	Lucite	Water	0.232"	0 .36 8
PM-157C-170C	Lucite	Water	0.118"	0.350
D-BB-68C-84C	Lucite	Soltrol	0.232"	0.370
D-BB-148C-165C	Lead	Water	0.250"	0.375
D-BB-166C-187C	Lead	Soltrol	0.250"	0.346
D-BB-188C-199C	Lead	Soltrol	0.118"	0.382
D-BB-2000-2200	Stainless Steel	Water	0.250"	0.380
D-BB-221C-240C	Stainless Steel	Soltrol	0.250"	0.380

CHAPTER V

DATA REDUCTION AND ANALYSIS

Data Reduction

The data were initially reduced by means of the solution to the one parameter conduction equation in which R was neglected.

$$u = \frac{T - T_{I}}{T_{o} - T_{I}} = \frac{1}{2} \operatorname{erfc} \left(\frac{1 - \tau}{2\sqrt{(D_{e}/V_{F}L)\tau}} \right) \qquad \text{III-3}$$

Equation III-3 may be rearranged to yield

$$u = \frac{1}{2} \operatorname{erfc} \frac{F}{2\sqrt{D_e}} \qquad V-1$$

where

$$F = \frac{L}{\sqrt{t}} - V_F \sqrt{t} \quad \text{and } D_e = \frac{k_e}{\rho_w c_w \phi + \rho_s c_s (1 - \phi)}$$

The procedure for determining k_e from the raw data utilizing the above equation has been presented other places (4,39,67), and it will only briefly be described here. A detailed calculation of run D-BB-99C including a temperature chart is presented in Appendix E. The procedure consists of:

(1) t vs. F is plotted on arithmetic - probability

paper which produces a straight line.

- (2) This line is shifted so as to pass through the point F = 0, U = 0.5 as required by equation V-1.
- (3) An arbitrary point is picked (usually u = 0.10) which will yield F at this point, (F 10).
- (4) From equation III-3; 0.20 = erfc $\frac{F.10}{2\sqrt{D_e}}$ which upon taking the inverse erfc yields a value of

D_e. It in turn yields a value of k_e when multiplied by the system parameters $\rho_w c_w \phi + \rho_s c_s (1 - \phi)$.

The k_e so obtained is broken into two components, a static component and a velocity dependent component

$$k_{e} = k_{e}^{o} + k_{e}(v)$$

The static component is the effective conductivity of the system at zero velocity. Several authors have studied this parameter (26,45,67), and they have developed methods for predicting the magnitude of k_e^0 for a wide variety of systems. k_e^0 in this work is obtained by extrapolating k_e data to zero velocity, and the results agree well with the various predictive models. The static conductivity serves as a corrected fluid molecular conductivity, $k_w \varphi$, because the four parameter mathematical model does not allow conduction downstream in the solid phase. Static conductivity, k_e^0 , is subtracted from the raw k_e data to yield the velocity dependent component.

The velocity component, $k_e(v)$, is the sum of three

independent velocity dependent mechanisms.

$$k_{e}(\mathbf{v}) = k_{wm} \phi + \frac{[\rho_{s} c_{s} (1 - \phi) \mathbf{v}]^{2}}{ha} + \frac{[\rho_{s} c_{s} (1 - \phi) \mathbf{v} d_{p}]^{2}}{60 k_{s} (1 - \phi)}$$
$$= k_{wm} \phi + k_{ha} + k_{s} (ha)$$

 k_{ha} and $k_{s(ha)}$ are functions which may be predicted so that it is possible to further reduce the data so that dispersion caused only by fluid mixing may be studied. The parameter which can be predicted with least accuracy is the term ha. The correlating curve of Dryden et al. (22) ($j_m \phi$ vs. Re) was used to calculate ha by assuming the analogy between heat and mass transfer to be valid. For heat transfer $j_h \phi$ is defined as

$$J_{h}^{\phi} = \frac{h\phi}{c_{w}G} \left(\frac{c_{w}^{\mu}}{k_{w}}\right)^{2/3} \qquad V-2$$

Dryden (21) has suggested that the data of Williamson et al. (82) may be more applicable for the lower Reynolds numbers than his data. Figure 9 presents a graphical comparison of these. (22,82) The difference between the correlations is primarily due to definition of $j_h \varphi$. Williamson et al. have correlated their data using the Prandtl number to the 0.58 power instead of the more commonly used 2/3 power.

$$J_{h}^{\phi} = \frac{h\phi}{c_{w}G} \left(\frac{c_{w}\mu}{k_{w}}\right)^{0.58}$$
 V-3



It is the opinion of this author that insufficient evidence has been presented to justify the 0.58 power. Also, using the Williamson et al. correlation results in values of $k_{(ha)}$ so large that k_{wm}^{ϕ} must become negative in order to preserve the theory of additivity of the various conductivities. A negative k_{wm}^{ϕ} has no meaning.

Data Analysis

Mixing Cell Theory

Several workers have studied the eddy mixing of a fluid flowing through a porous medium. The most comprehensive review of this type of dispersion is the work of Perkins and Johnson (64) who use the mixing cell theory. This theory has been independently proposed by several authors. (3,23, 53,55) It suggests that a packed section be thought of as a large number of small mixing chambers within each of which complete mixing occurs.



 $C_{Bulkn} = C_{o_n} = C_{I_{n+1}}$

L' =length of a mixing cell.

Differential equations for each cell are linked by having the output of one cell be the input of the next, i.e., assume $C_0 = C_{bulk}$ for each mixing cell. When the number of cells approaches infinity, it may be shown that (3,53,55)

$$Pe = \frac{Vd_p}{E + D} = \frac{2}{\gamma} \qquad \qquad V-4$$

Where $Y = L'/d_p$, the number of mixing cell lengths per particle diameter.

The critical assumption of the mixing cell theory, for y defined as it is above, is that complete backmixing must occur in each of the individual cells. In other words, L' is the length required for complete backmixing. This makes y a variable dependent upon flow conditions. An alternate treatment has been proposed by Carberry (12) which sets $\gamma = 1$ for all conditions and then defines a mixing cell efficiency. A cell having an efficiency of 100% would yield a constant value of the Peclet number, Pe = 2.0, indicating that E, the eddy mixing diffusion coefficient, is a linear function of Vd $_{\rm D}$ under the restriction of E >> D. Whether γ is treated as a variable or as a mixing cell efficiency is immaterial. Both treatments serve the same purpose and the difference is merely one of presentation. The variable y is used in this study.

The mass transfer studies discussed in Chapter II indicate that the Peclet number is a function of flow

conditions and that only at certain ranges of the variable Vd_p/D is it equal to approximately 2.0. Perkins and Johnson (64) and others (33,34) have developed a reasonable explanation for this. When a fluid flows through a pack at a very low rate there will be time for molecular diffusion to eliminate concentration gradients within each mixing cell and thus complete mixing will occur. If the velocity is increased, yet still maintaining laminar flow, a point will be reached at which there is insufficient time for molecular diffusion to act within a particle diameter, causing the mixing cell length, L', to increase, (or the mixing cell efficiency to decrease) resulting in a lower Peclet number, i.e., an increase in the mixing coefficient E. The ability of the molecular diffusion of the fluid phase to eliminate these pockets within a certain specified range of $Vd_{\rm D}^{\rm }/D$ does not mean that the eddy dispersion has been eliminate i. It does mean that if the pockets are eliminated within the specified times, the mathematics of the mixing cell model predict a linear dependence of the eddy mixing coefficient, E, on Vd_p.

The dimensionless group, Vd_p/D , may be considered as the ratio of the time required for molecular diffusion to damp out concentration gradients to the residence time of the fluid in a mixing cell of length d_p . Thus, it is a measure of the efficiency of the mixing within the cells. Although the region is not sharply defined, mass transfer studies indicate that the cell mixing length is approximately



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one particle diameter in the region of $10 < Vd_p/D < 50$. For $Vd_p/D < 10$ molecular diffusion is predominant; for $Vd_p/D > 0$ 50 insufficient time is available for complete mixing in one particle diameter length. When $Vd_p > 50$ eddy dispersion, E, increases as Vd_p to some power greater than one up to the point where transition from laminar to turbulent flow begins. This transition zone is characterized best by the Reynolds number rather than Vd_p/D , but since the group $\mu/\rho D \approx 1000$ for most mass transfer processes, Vd_p/D again may be used as the correlating variable. The transition zone begins at approximately $Vd_p/D = 60,000$, and complete turbulence occurs roughly at $Vd_p/D = 10 \times 10^6$. The reader is referred to Figure 10 compiled from literature liquid mass transfer studies using spherical beads. Note that when the velocity is decreased to the point where $E \approx 0$, that the Peclet number then becomes a linear function of Vd_p .

The correlating variable should be γ instead of the Peclet number. This choice is somewhat arbitrary since $\gamma =$ 2/Pe according to the mixing cell theory. However, using γ as the correlating variable helps visualize the physical situation. The combined data of this work and the work of Green (39) cover a range of $3 < Vd_p/D < 800$. As in mass transfer mixing data, one would expect complete mixing within one particle diameter length to occur in the general region of $10 < Vd_p/D < 50$. In heat transfer, however, another variable is influential, the thermal diffusivity of the

solid packing. Eddy mixing is assumed to occur because of the difference in velocity of the fluid around the solid particles. It is usually assumed that stagnant pockets form, or there are slugs of fluid traveling at very low relative velocity. In the case of mass transfer studies using nonporous packing, the only mechanism that can eliminate these stagnant pockets is molecular diffusion in the fluid phase. In the case of heat transfer, the heat contained in the stagnant pockets can also be eliminated by heat transfer into the solid phase, followed by conduction through the solid to a region of high fluid velocity. Also, heat may be conducted down the bed in the solid phase through point to point contact of the solid particles. However, Masamune and Smith (61) indicate that point to point contact thermal conductivities are of the order of 10^{-2} Btu/hr-ft-°F. The net result of this additional mechanism would be a decrease in mixing cell length, i.e., $\gamma < 1.0$ at $10 < Vd_p/D < 50$.

Since this added mechanism does occur for heat transfer, it is necessary to define a normalization factor in order to compare mass transfer and heat transfer mixing data. For this reason the factor γ_n , which is a function of the physical properties of the solid phase, has been introduced. γ_n is defined as the factor required to force the mixing cell length to equal one particle diameter $(\gamma/\gamma_n = 1)$ in the region of 10 < Vd_p/D < 50. When this is done, the heat transfer mixing data correlate with the mass transfer mixing data over the range of 3 < Vd_p/D < 800.
The normalization factor γ_n is a sole function of the thermal properties of the solid phase. The thermal diffusivity was chosen since the efficiency of the added mechanism is dependent upon how quickly the heat contained within the stagnant pockets can be conducted to a region of higher fluid velocity. The mechanism being an unsteady state process, the diffusivity was chosen as the correlating parameter. Also, a base of $\gamma_n = 1$ was chosen for the mass transfer case where $D_g = 0$.

A detailed mathematical model would undoubtedly yield insight into the mixing process, but it is felt that the analysis is unwarranted at this time. It would be unwise to attempt to prove such a model with the data presented since it is so far removed from the measured parameter, k_e , when comparing mixing data. For this reason γ_n is presented as a function of thermal diffusivity only, and the more advanced models deferred to future work where the experimental program may be designed accordingly.

Figures 21 to 24 (Appendix C) present γ as a function of Vd_p/D for the various solid systems. Figure 11 presents γ_n as a function of the solid thermal diffusivity D_s. The data points of Figure 11 indicate the maximum variation in γ_n for the various solid systems. Figure 12 presents all of the heat transfer mixing data available (this work, 4,39) in a form comparable to the mass transfer mixing data of the previously mentioned authors.





The following design procedure is outlined in order to predict a value of k_e . The following system properties must be known: ρ_w , μ , c_w , ϕ , ρ_s , c_s , k_w , k_s , V, and d_p .

1) The static thermal conductivity is predicted by the Euchen or Hamilton equation.

$$k_{e}^{O} = \frac{k_{W} \left[k_{s} + 2k_{W} - 2(1 - \phi)(k_{W} - k_{s})\right]}{k_{s} + 2k_{W} + (1 - \phi)(k_{W} - k_{s})}$$
 II-7

2) The solid resistance contribution is estimated by means of equation III-12.

$$k_{s(ha)} = \frac{\left[\rho_{s}c_{s}(1-\varphi)\nabla_{F}d_{p}\right]^{2}}{60k_{s}(1-\varphi)} \qquad \text{III-12}$$

3) Upon calculation of the Reynolds number, Dryden's $j_m \varphi$ factor may be obtained from Figure 9. Under the assumption that $j_m \varphi = j_h \varphi$, a value of h is calculated from the definition of $j_h \varphi$

$$J_{h} \varphi = \frac{h\varphi}{c_{w}G} \left(\frac{c_{w} \mu}{k_{w}}\right)^{2/3} \qquad V-2$$

$$ha = \frac{J_{h}c_{w}G \left[\frac{6(1-\varphi)}{d_{p}}\right]}{(c_{w}\mu/k_{w})^{2/3}} \qquad V-5$$

where $a = 6(1 - \phi)/d_p$ for spherical particles. Equation III-13 is then used to obtain the contribution of the film resistance

$$k_{ha} = \frac{(\rho_{s}c_{s}(1 - \varphi)V_{F})^{2}}{ha}$$
 III-13

4) Calculation of Vd_p/D yields a value of γ/γ_n from Figure 12. Knowing D_s allows, a value of γ_n to be taken from Figure 11. The eddy mixing contribution is then obtained by the following calculation.

$$\mathbf{k}_{\mathbf{W}\mathbf{M}} \varphi = [(\gamma/\gamma_n)(\gamma_n)(\nabla d_p) - 2D][\rho_{\mathbf{W}} c_{\mathbf{W}} \varphi/2] \qquad \nabla -6$$

5) The overall longitudinal dispersion coefficient k_e is then the algebraic sum of the individual contributions.

$$k_{e} = k_{e}^{o} + k_{s(ha)} + k_{(ha)} + k_{wm} \phi \qquad V-7$$

CHAPTER VI

SUMMARY AND CONCLUSIONS

A fluid was considered in one-dimensional, steady, piston flow through a porous medium. At the entrance face, a step function in temperature was imposed on the injected fluid. The resulting heat front moved through the porous medium, with thermal energy being dispersed in the direction of fluid flow and away from the mean heat-front position by a combination of heat transfer mechanisms. The particular dispersion mechanisms of molecular conduction, eddy mixing, and a finite fluid-solid heat transfer rate were considered.

General differential equations describing the above mentioned mechanisms were solved analytically, and the resulting solution approximated to yield a convenient erfc form. The mathematical model indicated that the overall dispersion coefficient, k_e , could be represented by the sum of the following conductivities.

- k^O_e characterizing the static conductivity of the
 medium
- k_{wm}^{ϕ} characterizing eddy mixing within the fluid phase

Predictive equations for all of the above conductivitles except $k \quad \varphi$ were presented. The contribution of eddy mixing $(k_{wm} \varphi)$ was obtained from the experimental values of the total dispersion (k_e) by difference. The resulting eddy mixing coefficient was compared to mass transfer mixing data. The heat and mass transfer mixing data were correlated by introducing a heat transfer normalization factor to account for the added mechanism of internal conduction within the solid particles. The mixing cell theory was found to be adequate in correlating the mixing data for a variety of systems.

The application of the mathematical work to a similar mass transfer process (chromatography) resulted in an extension of the commonly used H.E.T.P. design equation originally presented by Van Deemter et al. (80)

Conclusions

The following conclusions may be drawn from the results of this investigation.

1) The solution to the general differential equations, equations III-7 and III-8, may be approximated by a simple conduction-equation solution (equation III-3) using an effective thermal conductivity which is equal to the sum

of four contributing conductivities each of which represents a separate mechanism. A qualitative discussion indicates that this approximation is valid for an extensive range of variables.

2) The mixing cell theory adequately correlates the heat transfer mixing data if γ , the dimensionless mixing cell length, is treated as a variable.

3) The added heat transfer mechanism of internal conduction within the solid phase, which does not occur in mass transfer mixing systems, tends to eliminate stagnant pockets, and thus decrease the eddy mixing coefficient E. This effect is presented in terms of a normalization factor γ_n , which is correlated as a smooth function of the solid phase diffusivity.

NOMENCLATURE

A	feed volume of adsorbate per unit area of	bed, ft
a	particle area per unit volume of bed, ft	
Ъ	particle radius, in. or ft	
c	heat capacity, Btu/lb _m -°F	
C		
cg	non-equilibrium term for diffusion in the	inter-
	particle spaces	
D	molecular diffusivity (mass or heat trans	sfer),
	ft^2/hr . $D = D_w$ unless otherwise stated.	
De	effective thermal diffusivity of porous m	media for
	the one parameter model, ft ² /hr	•
D _e !	effective thermal diffusivity of porous m	edia for
	the four parameter model, ft ² /hr	
$\mathtt{D}_{\mathtt{L}}$	total diffusivity of the fluid phase, E +	D, ft ² /hr
D _v	velocity component of effective thermal d	liffusivity,
	ft ² /hr	
d _p	solid particle diameter, in. or ft	
d _t	inside diameter of heat transfer tube, ft	;
E	eddy mixing diffusivity (mass or heat tra	unsfer),
	ft ² /hr	
erf	denotes error function	
erfc	denotes co-error function, 1 - erf	

F	-	z∕√t – V _F √t
G	-	mass flux, lbm/ft ² -hr
ĥ	-	heat transfer coefficient, Btu/hr-ft ² -°F
Hl	-	defined in Appendix A, equation A-18
H ₂	-	defined in Appendix A, equation A-19
$^{\rm H}D_1$	-	defined in Appendix A, equation A-14
HD2	-	defined in Appendix A, equation A-15
H or H.E.T.I	-	height equivalent to a theoretical plate, ft
H	-	distance between theoretical plates, ft
j	-	empirical "j" factor
k	-	molecular thermal conductivity, Btu/hr-ft-°F
ke	-	effective thermal conductivity of porous media,
		Btu/hr-ft-°F
<u>⊮</u> e	-	static thermal conductivity of porous media, i.e.,
		thermal conductivity with fluid in-place but not
		moving, Btu/hr-ft-°F
$k_{e}(v)$	-	velocity dependent component of effective thermal
		conductivity, Btu/hr-ft-°F
^k (ha)	-	effective coefficient characterizing a finite heat
		transfer rate between the solid and fluid phases
		controlled by a fluid "film" around the particles,
		Equation (III-13), Btu/hr-ft-°F
k _n	-	thermal conductivity obtained by numerical integra-
		tion, Btu/hr-ft-°F
k _p	-	thermal conductivity through point to point contact
		of the packing, Btu/hr-ft-°F

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^k s(ha)	-	effective coefficient characterizing a finite time
		required to damp out temperature gradients within
		the solid particles, Equation (III-12), Btu/hr-ft-°F
k _{wm}	-	eddy mixing dispersion coefficient, Btu/hr-ft-°F
К	-	thermal equilibrium constant, ρ _s c _s /ρ _w c _w
L	-	length of packed section, ft
L'	-	length of a mixing cell, in. or ft
m	-	ratio of solid bed fraction to void fraction, $\frac{1-\varphi}{\omega}$
n	-	number of theoretical plates
Pe	-	Peclet number, Vd _p /(E + D)
q	-	$\rho_{s}c_{s}T_{s} - \rho_{s}c_{s}T_{s}$
R	-	real part as opposed to imaginary part
Re	-	Reynolds' number, $\frac{va_p^p}{\mu}$
R _f	-	film resistance, $\frac{b \rho_w c_w}{3h}$, hr
S	-	Laplace transform variable, α + i β
S	-	fluid injection volume per unit area of bed, ft
т	-	temperature, °F
t	-	time, min or hr
to	-	time of injection for pulse input
u	-	temperature fraction, $\frac{T - T_0}{T_T - T_0}$
V	-	interstitial fluid velocity, ft/hr
V!	-	effective plate volume, ft ³
VF	-	velocity of square heat front, $V_F = \frac{\rho_W c_W \phi_V}{\rho_w c_w \phi + \rho_c c_s (1 - \phi)}$
W	-	$ib\left(\frac{s}{D_{s}}\right)^{\frac{1}{2}}$
x	-	z/mV, hr
X	-	z/L

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Y - equation II-4

Y_D - defined in Appendix A

 $Y_{\rm T}$ - defined in Appendix A

y - defined in Appendix A

- z distance, ft
- Z equation II-4

 $z' - V_F L/D_e$

Greek Letters

α	- real part of complex Laplace transform variable s
β	- imaginary part of complex Laplace transform variable
	s or as defined in Appendix D, equation D-1
Ŷ	- $3D_{s}K/b^{2}$, $1/hr$
λ	- variable of integration or proportionality constant
	in the treatment of eddy diffusivity
ξ	- defined in Appendix A
μ	- viscosity, lb _m /ft-hr
ρ	- density, lb _m /ft ³
φ	- porosity, i.e., void fraction
θ	-t - z/V
т	- V _F t/L
σ	- 2D _s /b ² , 1/hr
ν	$-\gamma R_{f} = k_{g}/bh$

Subscripts

h	- denotes heat transfer variable
I	- denotes inlet condition

m	- denotes mass transfer variable
n	- denotes n th cell
0	- denotes initial condition or injection time for
	pulse input
s	- denotes solid phase
W	- denotes fluid phase

- denotes inside solid particles

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- denotes fluid mixing wm

Superscripts

o'A

- denotes average temperature with respect to solid A particle radius
 - denotes Laplace transform domain

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APPENDIX A

MATHEMATICAL TREATMENT

Four Parameter Model Solution

The mathematical model proposed in Chapter III requires the simultaneous solution of differential equations describing the temperature of both phases. These equations are:

$$\rho_{W}c_{W}\frac{\partial T_{W}}{\partial t} + V\rho_{W}c_{W}\frac{\partial T_{W}}{\partial z} - (E+D)_{W}\rho_{W}c_{W}\frac{\partial^{2}T_{W}}{\partial z^{2}} = -\frac{\rho_{S}c_{S}}{m}\frac{\partial^{2}T_{S}}{\partial t} \quad \text{III-7}$$

and

Ξ

$$\rho_{s}c_{s}\frac{\partial T_{si}}{\partial t} = \rho_{s}c_{s}D_{s}\left[\frac{\partial^{2}T_{si}}{\partial r^{2}} + \frac{2}{r}\frac{\partial T_{si}}{\partial r}\right] \qquad \text{III-8}$$

The following definitions are used:

$$C = \rho_{w}c_{w}T_{w} - \rho_{w}c_{w}T_{w_{0}}$$

$$D_{L} = E + D$$

$$a(z,t) = \rho_{s}c_{s}T_{s} - \rho_{s}c_{s}T_{s_{0}}$$

$$q_{i}(r,z,t) = \rho_{s}c_{s}T_{si} - \rho_{s}c_{s}T_{si_{0}}$$

$$q_{s}(z,t) = q_{i}(b,z,t), \text{ i.e., } q_{i} \text{ evaluated at the solid particle}$$

$$85$$

surface where subscript o denotes initial conditions, subscript i denotes inside solid particle, and superscript A denotes average value with respect to solid particle radius. Equations III-7 and III-8 then become

$$\frac{\partial C}{\partial t} + V \frac{\partial C}{\partial z} - D_L \frac{\partial^2 C}{\partial z^2} = - \frac{\partial q}{\partial t} m$$
 A-1

and

$$\frac{\partial q_1}{\partial t} = D_8 \left[\frac{\partial^2 q_1}{\partial r^2} + \frac{2}{r} \frac{\partial q_1}{\partial r} \right] \qquad A-2$$

The boundary conditions in this nomenclature become:

$$C(z,0) = 0$$

$$C(0,t) = C_{I} \equiv \rho_{W} c_{W} (T_{WI} - T_{WO})$$

$$C(\infty,t) = 0$$

and

$$q_{i}(r,z,0) = 0; z \ge 0$$

$$q_{i}(0,z,t) = \text{finite}; z \ge 0$$

$$q_{s} = q_{i}(b,z,t) \text{ given by } z \ge 0$$

$$\frac{\partial \dot{q}}{\partial t} = \frac{3h}{b\rho_W c_W} \left(c - \frac{q_B}{K} \right)$$
 A-3

The various C and q terms will be referred to as concentrations since they represent the heat concentration in Btu/ft^3 .

The last boundary condition (A-3) is the linking

equation between equations A-1 and A-2. Carslaw and Jaeger (14) have solved equation A-2 for the special case of a constant value of q_s . By applying Duhamel's theorem (14, p. 30) to the solution of Carslaw and Jaeger, Rosen was able to obtain an expression for $q_1(r,z,t)$ in terms of the surface concentration $q_s(b,z,t)$.

 $q_{i}(r,z,t) =$

$$2D_{s}\sum_{h=1}^{\infty}(-1)^{n+1}\sigma_{n}\frac{\sin(\sigma_{n}r)}{r}\int_{0}^{t}q_{s}(b,z,t)\exp\left[-D_{s}\sigma_{n}^{2}(t-\lambda)\right]d\lambda$$

If this expression is in turn integrated over the entire volume of the particle the following expression is obtained for the average solid concentration in terms of the surface concentration $q_s(b,z,t)$

$$\overset{A}{q}(z,t) = \frac{\frac{4\pi}{3} \int_{0}^{b} q_{1}(r,z,t)r^{2}dr}{\frac{4\pi}{3} \int_{0}^{b} r^{2}dr} = \frac{3}{b} \int_{0}^{b} q_{1}(r,z,t) r^{2}dr ;$$

$$\overset{\mathbf{A}}{\mathbf{q}}(\mathbf{z}, \mathbf{t}) = \frac{6\mathbf{D}_{\mathbf{s}}}{\mathbf{b}^2} \sum_{n=1}^{\infty} \int_{0}^{\mathbf{t}} \mathbf{q}_{\mathbf{s}}(\mathbf{b}, \mathbf{z}, \lambda) \exp \left[-\mathbf{D}_{\mathbf{s}}\sigma_{\mathbf{n}}^2(\mathbf{t} - \lambda)\right] d\lambda$$

Exchanging the order of differentiation and summation, applying Leibnitz's Rule, integrating the resulting integral by parts, and making use of the fact that $q_s(b,z,0) = 0$, the

following expression for the rate of change of the average solid concentration with respect to time is obtained

$$\frac{\partial \hat{\mathbf{q}}}{\partial t} = \frac{\partial \mathbf{D}_{\mathbf{s}}}{\mathbf{b}^2} \sum_{n=1}^{\infty} \int_{0}^{t} \frac{\partial \mathbf{q}_{\mathbf{s}}(z,\lambda)}{\partial \lambda} \exp\left[-\mathbf{D}_{\mathbf{s}}\sigma_{\mathbf{n}}^2(t-\lambda)\right] d\lambda \qquad A-4$$

Substituting equation A-4 into equation A-1 and taking the Laplace transform with respect to time we have

$$s\overline{C} + V \frac{\partial \overline{C}}{\partial z} - D_{L} \frac{\partial^{2}\overline{C}}{\partial z^{2}} = -\frac{6D_{g}}{b^{2}m} \sum_{n=1}^{\infty} \mathcal{L} \int_{c}^{t} \frac{\partial q_{g}}{\partial \lambda} \exp \left[-D_{g}\sigma_{n}^{2}(t-\lambda)\right] d\lambda$$
A-5

The Laplace transform of the integral on the righthand side of equation A-5 may be recognized as the Faltung integral theorem and therefore can be replaced by $\pounds f_1 \pounds f_2$ where $f_1 = \frac{\partial q_s}{\partial t}$ and $f_2 = \exp[-D_s \sigma_n^2 t]$. Using the boundary condition A-3 and equation A-1, $\pounds \frac{\partial q_s}{\partial t} = s\bar{q}_s$ can be expressed as

$$s\overline{q}_{s} = \frac{bKs}{3h} \left[ms\overline{c} + Vm \frac{\partial\overline{c}}{\partial z} - mD_{L} \frac{\partial^{2}\overline{c}}{\partial z^{2}} + \frac{3h}{b}c \right]$$

and $\mathcal{L} \exp \left[-D_s \sigma_n^2 t\right] = 1/(s + D_s \sigma_n^2)$

With these substitutions and the following notation

$$Y = (3D_{g}K)/(b^{2})$$

$$R_{f} = b\rho_{W}c_{W}/3h$$

$$Y_{D}(s) = 2Y \sum_{n=1}^{\infty} (s)/(s + D_{g}\sigma_{n}^{2})$$

$$Y_{T}(s) = \frac{Y_{D}(s)}{[1 + R_{f}Y_{D}(s)]}$$

equation A-5 becomes

$$\frac{\partial^2 \overline{c}}{\partial z^2} - \frac{V}{D_L} \frac{\partial \overline{c}}{\partial z} - \overline{c} \left(\frac{s}{D_L} + \frac{Y_T(s)}{D_L m} \right) = 0 \qquad A-6$$

Equation A-6 may be treated as an ordinary, second order, linear differential equation whose solution after applying the boundary conditions is

$$\overline{u} = \frac{\overline{C}}{C_{I}} = \frac{1}{s} e^{-\sqrt{\frac{v^{2}}{4D_{L}^{2}}} + \left(\frac{s}{D_{L}} + \frac{Y_{T}(s)}{mD_{L}}\right)} z$$
A-7

Equation A-7 may be rearranged to yield

$$\overline{u} = \frac{1}{s} \exp \left[\frac{Vz}{2D_L} \left(1 - \sqrt{1 + \frac{4D_L}{V^2} \left\{ s + \frac{Y_T(s)}{m} \right\}} \right) \right]$$
 A-8

Provided that $\left\{ \begin{array}{c} \frac{4D_{L}}{V^{2}} \mid s + \frac{Y_{T}(s)}{m} \mid \right\}^{2} < 1$ the radical of equation A-8 may be expanded in a binomial expansion and discarding all but the first three terms yields

$$\overline{u} = \frac{1}{s} e^{\frac{-zs}{V}} exp \left[-\frac{zY_{T}(s)}{V_{m}} + \frac{zD_{L}}{\sqrt{3}} \left(s + \frac{Y_{T}(s)}{m} \right)^{2} \right] \qquad A-9$$

$$-zs$$

The factor e^{V} in equation A-9 is a time translation factor and may be ignored if in the final result t is replaced by θ where $\theta = t - \frac{z}{v}$.

Making use of contour integrals Rosen (71) has been able to show that the inverse Laplace transform of equation A-9 is given by

$$u(z,\theta) = \frac{1}{2} + \frac{1}{\pi} \int_{0}^{\infty} R\left[e^{i\beta\theta}\overline{u}(z,i\beta)\right] d\beta$$
 A-10

thus necessitating the evaluation of $Y_T(i\beta)$. Again following the work of Rosen (71) a trigonometric expression for $Y_D(s)$ is obtained.

$$Y_{D}(s) = \gamma(w \text{ cot } w - 1)$$
 A-11

where $w = ib(s/D_s)^{\frac{1}{2}}$. From equation A-11

$$Y_{D}(i\beta) = \gamma ((i-1)\lambda \cot [(i-1)\lambda] - 1)$$
 A-12

where $w(i\beta) = (i - 1)\lambda$, $\lambda = (\beta/\sigma)^{\frac{1}{2}}$, $\sigma = (2D_g/b^2)$.

By rather tedious manipulations $Y_D(i\beta)$ reduces to

$$Y_{D}(i\beta) = \gamma[H_{D_{1}}(\lambda) + iH_{D_{2}})] \qquad A-13$$

where

$$H_{D_{1}}(\lambda) = \left[\lambda \left(\frac{\sinh 2\lambda + \sin 2\lambda}{\cosh 2\lambda - \cos 2\lambda}\right) - 1\right] \qquad A-14$$

$$H_{D_{2}}(\lambda) = \lambda \left(\frac{\sinh 2\lambda - \sin 2\lambda}{\cosh 2\lambda - \cos 2\lambda} \right)$$
 A-15

$$Y_{T}(i\beta) = \frac{\gamma(H_{D_{1}} + iH_{D_{2}})}{1 + \nu(H_{D_{1}} + iH_{D_{2}})} \quad \text{where } \nu = \gamma R_{f} \quad A-16$$

By multiplying numerator and denominator by $1 + v(H_{D_1} - iH_{D_2})$ equation A-16 reduces to

$$Y_{T}(i\beta) = \gamma(H_{1}(\lambda,\nu) + iH_{2}(\lambda,\nu))$$
 where A-17

$$H_{1}(\lambda,\nu) = \frac{H_{D_{1}} + \nu(H_{D_{1}}^{2} + H_{D_{2}}^{2})}{(1 + \nu H_{D_{1}})^{2} + (\nu H_{D_{2}})^{2}}$$
 A-18

$$H_{2}(\lambda,\nu) = \frac{H_{D_{2}}}{(1 + \nu H_{D_{1}})^{2} + (\nu H_{D_{2}})^{2}}$$
 A-19

If the hyperbolic and circular functions of equations A-14 and A-15 are expanded in a Maclaurin series and all but the first few terms discarded the following limiting values of H_{D_1} and H_{D_2} may be obtained

$$H_{D_1} = (4/45)\lambda^4$$

 $H_{D_2} = (2/3)\lambda^2$

which are accurate to within 4% if $\lambda < 1$. If the limiting values of H_{D_1} and H_{D_2} are in turn substituted into equations A-18 and A-19, with $\nu\lambda^4$ assumed less than 0.5, the following expressions for $H_1(\lambda,\nu)$ and $H_2(\lambda,\nu)$ are valid within 5%.

$$H_{1} = (4/45)\lambda^{4}(1 + 5\nu) \qquad A-20$$

$$H_2 = (2/3)\lambda^2$$
 A-21

Therefore
$$Y_{T}(1\beta) = \gamma[(4/45)\lambda^{4}(1+5\nu) + 1(2/3)\lambda^{2}]$$
 A-22

Substituting A-17 into equation A-9 and equation A-9 in turn introduced into equation A-10 the following results:

$$\dot{u}(z,\theta) = \frac{1}{2} + \frac{1}{\pi} \int_{0}^{\infty} R \left[e^{i\beta\theta} e^{-\gamma x \left[H_{1}(\lambda,\nu) + iH_{2}(\lambda,\nu) \right]} \right]$$
A-23

$$\cdot \exp \frac{zD_{\rm L}}{v3} \left(1\beta + \frac{\gamma}{m} (H_{\rm l} + 1H_{\rm 2}) \right)^2 \right] \frac{d\beta}{1\beta}$$

where x = (z/Vm). Remembering $\beta = \sigma \lambda^2$ and considering the real part as indicated in equation A-23 we have

$$u(z,\theta) = \frac{1}{2} + \frac{2}{\pi} \int_{0}^{\infty} e^{-\left\{\gamma x H_{1} + \frac{z D_{L}}{\sqrt{3}} \left(\sigma^{2} \lambda^{4} + \frac{2\sigma \lambda^{2} H_{2} \gamma}{m} + \frac{z \sigma \lambda$$

$$\frac{\frac{\gamma^{2}}{m^{2}}(H_{2}^{2}-H_{1}^{2}))}{\sin\left\{\sigma_{\theta\lambda}^{2}-\gamma_{x}H_{2}+\frac{zD_{L}}{v3}\left(2\sigma\lambda^{2}H_{1}\gamma+\frac{2H_{1}H_{2}\gamma^{2}}{m^{2}}\right)\right\}\frac{d\lambda}{\lambda}}$$
A-24

where $e^{i\omega} = \cos \omega + i \sin \omega$ has been used. Substituting the limiting values of H₁ and H₂ (A-20 and A-21) and throwing away all terms involving λ^6 or higher order we have

$$u(z,\theta) = \frac{1}{2} + \frac{2}{\pi} \int_{0}^{\infty} e^{-\lambda^{4} \left\{ (4/45)\gamma x (1+5\nu) + \frac{zD_{L}}{\sqrt{3}} \left(\sigma + \frac{2\gamma}{3m}\right)^{2} \right\}}$$

sin $\left\{\sigma\theta\lambda^{2} - (2/3)\gamma x\lambda^{2}\right\} \frac{d\lambda}{\lambda}$ A-25

Let
$$\xi^2 = \lambda^4 \left\{ (4/45)\gamma x (1 + 5\nu) + \frac{zD_L}{v^3} \left(\sigma + \frac{2\gamma}{3m}\right)^2 \right\}$$
 A-26

and
$$y = \frac{\sigma_{\theta} - (2/3)\gamma x}{2\left\{(4/45)\gamma x(1+5\nu) + \frac{zD_{L}}{\sqrt{3}}\left(\sigma + \frac{2\gamma}{3m}\right)^{2}\right\}^{\frac{1}{2}}}$$
 A-27

Then

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$$u(z,\theta) = \frac{1}{2} + \frac{1}{\pi} \int_{0}^{\infty} e^{-5^{2}} \sin 25y \frac{d5}{5}$$

which reduces to

$$u(z,\theta) = \frac{1}{2} \operatorname{erfc} \left(\frac{(2/3)\gamma x - \sigma \theta}{2(zD_{L}/V^{3})^{\frac{1}{2}}} \right) \qquad A-28$$

where reference is made to the fact that

$$\frac{1}{\pi}\int_{0}^{\infty} e^{-\xi^{2}} \sin 2\xi y \frac{d\xi}{\xi} = \frac{1}{2} \operatorname{erf} y$$

Carslaw and Jaeger (14) p. 483. Thus

$$u(z,\theta) = \frac{1}{2} \operatorname{erfc} \left(\frac{(2/3)\gamma x - \sigma \theta}{2 \left((4/45)\gamma x (1+5\nu) + (zD_{L}/\sqrt{3}) \left[\sigma + \frac{2\gamma}{3m} \right]^{2} \right)^{\frac{1}{2}}} \right)$$

A-29

If the argument of the erfc in Equation A-29 is multiplied top and bottom by $(b^2 V_F / 2D_S / t)$ and substituting x = (z/mV), $\theta = t - \frac{z}{v}$ and further replacing z by $V_F t$ everywhere but in the numerator, equation A-29 becomes

$$u(z,t) = \frac{1}{2} \operatorname{erfc} \left[\frac{(z/\sqrt{t}) - V_{F}\sqrt{t}}{2\sqrt{D_{e}}'} \right] \qquad A-30$$

where
$$D_{e_{i}}^{\dagger} = \frac{\frac{\left[\rho_{s}c_{s}(1-\varphi)\nabla_{F}d_{p}\right]^{2}}{60k_{s}(1-\varphi)} + \frac{\left[\rho_{s}c_{s}(1-\varphi)\nabla_{F}\right]^{2}}{ha}}{\frac{\rho_{w}c_{w}\varphi + \rho_{s}c_{s}(1-\varphi)}{ha}}$$

If $k_{s(ha)}$ and k_{ha} are defined as

$$k_{s(ha)} = \frac{\left[\rho_{s}c_{g}(1-\varphi)\nabla_{F}d_{p}\right]^{2}}{60k_{s}(1-\varphi)}$$
$$k_{ha} = \frac{\left[\rho_{s}c_{s}(1-\varphi)\nabla_{F}\right]^{2}}{ha}$$

ha

then
$$D_e'$$
 becomes

$$D_e' = \frac{k_{wm}\phi + k_w\phi + k_{ha} + k_s(ha)}{\rho_w c_w \phi + \rho_s c_s(1 - \phi)}$$

$$k_{wm}\phi + k_w\phi$$

where
$$D_{L} = \frac{R_{WM} \phi + R_{W} \phi}{\rho_{w} c_{w} \phi + \rho_{s} c_{s} (1 - \phi)}$$
 has been used.

Equation A-30 may be further reduced to the form used in Chapter III.

$$u(l,\tau) = \frac{1}{2} \operatorname{erfc} \left(\frac{1-\tau}{2\sqrt{(D_{e'}/V_{F}L)\tau}} \right) \qquad A-31$$

where X has been evaluated at z = L.

APPENDIX B

Raw Data

In the following tables all conductivities are in Btu/hr-ft-°F, Vd_p is in ft²/hr, and temperature is in °F.

Glass-Water		d _p = (0.0193'	φ = 0.362		
		-	c = 0.5	1		
Run #	Vd _p	Re	ke	$\frac{k_{e}(v)}{v}$	D _e /V _F L	Tavg
D-BB-24C 25C 26C 27C 28C 29C 30C 31C 32C 33C 34C 35C	0.29 0.42 0.80 0.47 1.12 0.19 0.61 0.38 Run Ti 0.52 0.25 0.34 0.40	13.18 19.09 36.36 21.36 50.90 8.64 27.73 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27 17.27	3.32 5.96 15.83 6.96 24.34 1.94 10.12 4.68 2.70 4.72 5.60	2.81 5.45 15.32 6.45 23.83 1.43 9.61 4.17 7.56 2.19 4.20	0.010 0.013 0.018 0.019 0.009 0.019 0.019 0.015 0.011 0.014 0.010 0.011	121.6 119.5 125.5 117.3 115.5 119.3 116.4 116.4 117.5 121.8 122.1
37C 38C 39C 40C 41C	0.49 0.14 0.56 0.25 Bun Ti	22.27 6.36 25.45 11.36	7.78 1.65 8.86 3.26	7.27 1.14 8.35 2.75	0.014 0.011 0.014 0.012	115.4 120.5 117.1 118.5
420 1180 1190 1200 1210	0.30 Run Tl 0.62 0.62 0.82	13.64 170wn Out 28.18 28.18 37.27	4.00 ; 9.54 9.41 14.12	3.49 9.03 8.90 13.61	0.012 0.014 0.014 0.015	118.1 116.3 116.0 114.9
1220 1230	0.80 0.81	36.36 36.81	14.00 14.39	13.49 13.88	0.015 0.015	114.4 114.2

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*Run thrown out denotes a run that is in disagreement with one or more other runs at the same Vd_p . Run discarded denotes a run that is known to be defective.

Run #	Vdp	Re	ke	k _e (v)	$\frac{D_e / V_F L}{E}$	Tavg
1240 1250 1260 1270 1280 1290 1300 1310 1310 1320 1330 1340 1350	0.94 0.99 1.78 1.78 1.72 0.68 1.11 2.27 2.24 2.17	42.72 45.00 80.90 80.90 78.17 30.91 39.09 50.45 103.17 101.81 98.63	20.46 20.72 22.89 56.64 53.18 50.86 12.09 17.58 24.95 84.83 80.27 73.36	19.95 20.21 22.38 56.13 52.67 50.35 11.58 17.07 24.44 84.32 79.76 72.85	0.020 0.019 0.021 0.028 0.027 0.026 0.016 0.018 0.020 0.033 0.032 0.030	114.0 111.7 110.2 108.6 112.8 113.1 113.5 113.6 113.4 110.1 115.3 112.7
Glass	-Soltro	l d _p =	= 0.0193	(6 mm)	φ = 0.	370
			$k_e^0 = 0.2$	25		
D-BB-46C 47C 48C 50C 51C 52C 53C 54C 55C 56C 57C 58C 59C 60C 61C 62C 63C 64C	0.71 0.73 0.85 0.46 Run D: 0.50 1.04 Run D: 1.16 0.54 0.59 0.70 0.76 Run D: 0.47 0.83 1.86 1.48 1.66	14.41 14.82 17.25 9.34 iscarded 10.15 21.11 iscarded 23.55 10.96 11.98 14.21 15.43 iscarded 9.54 16.85 37.76 30.05 33.70	9.46 10.22 13.01 5.23 (Poor en 5.52 17.11 (Poor en 20.00 5.97 6.74 8.46 10.69 (Poor en 5.23 11.57 43.16 30.08 34.41	9.21 9.97 12.76 4.98 fc curve 5.27 16.86 fc curve 19.75 5.72 6.49 8.21 10.44 fc curve 4.98 11.32 42.91 29.83 34.16	0.031 0.032 0.035 0.026 fit) 0.026 0.038 fit) 0.040 0.025 0.026 0.028 0.032 fit) 0.026 0.032 fit) 0.026 0.032 fit) 0.026 0.032 fit) 0.026 0.032 fit) 0.026	113.8 115.3 114.7 116.0 115.5 115.6 116.5 120.8 118.5 118.8 116.0 122.0 120.9 120.4 119.8 121.8
Glas	s-Soltro	ol* d _r	= 0.009	983'(3 m	m) φ =	• 0.363
			$k_{e}^{o} = 0.2$	25		
D-BB-136C 137C	0.35 0.36	7.10 7.40	4.53 4.25	4.28 4.00	0.015 0.014	114.8 120.5

*The 3 mm glass-Soltrol data was not used in the correlation of γ_n for glass since it gives a value of γ_n approximately 100% higher than would be expected from the other glass data.

_Run #	Vd _p	Re	k _e	k _e (v)	$\frac{D_e / V_F L}{2}$	Tavg
138C 139C 140C 141C 142C 143C 143C 144C 145C 146C 147C	0.36 0.58 0.57 1.30 1.28 1.31 0.13 0.13 Run Di	7.40 11.78 11.78 11.57 26.39 25.99 26.60 2.64 2.64 scarded	4.69 9.15 7.88 7.36 23.19 22.50 24.10 1.06 1.12 (Heat L	4.44 8.90 7.63 7.11 22.94 22.25 23.85 0.81 0.87 osses Due	0.015 0.019 0.016 0.015 0.021 0.021 0.021 0.010 0.010 to Low	123.4 116.0 115.5 114.1 113.3 112.2 112.5 116.2 115.9 Velocity)
Lucite	-Water	đp	= 0.019	3' (6 mm)	φ	= 0 .36 8
		, –	$k_e^0 = 0.3$	20		
D-BB-88C 89C 90C 91C 92C 93C 94C 95C 96C 97C 98C 99C 100C 101C 102C 103C 104C 105C 106C 107C 108C 109C 110C 111C 112C 113C	1.26 0.86 1.35 1.26 1.05 1.42 0.21 1.42 0.47 0.57 0.57 0.65 0.65 0.71 0.82 0.65 0.71 0.82 0.65 0.71 0.82 0.70 0.87 0.70 0.87 0.70 0.87 0.70 0.87 0.70 0.87 0.70 0.85 0.71 0.86 0.71 0.93 0.75 0.75 0.75 0.75 0.75 0.75 0.75 0.75	57.26 39.36 487.72 64.27 91.36 64.27 91.20 64.27 91.20 21.04 16.737 21.04 16.737 21.04 16.737 21.04 16.737 21.04 16.737 19.31 25.31 39.00 20 35.31 39.00 20 30 35.31 39.00 20 30 35.31 39.00 20 30 35.31 39.00 30 30 30 30 30 30 30 30 30 30 30 30 3	74.72 37.07 92.26 47.025 44.92.64 361.17 162.88 92.64 19.49 12.97 12.97 12.97 12.97 12.97 12.97 12.97 12.20 23.52 17.22 23.52 23.13 12.97 12.23 23.55 1.17 12.97 22.52 23.52 23.52 23.52 23.52 23.52 23.52 23.52 23.52 23.52 23.52 23.52 23.52 23.52 23.52 23.52 23.52 23.52 23.52 23.52 23.52 23.52 23.52 23.52 23.52 23.52 23.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.55 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 25.52 2	74.52 36.87 92.06 46.05 44.32 92.72 82.72 82.72 82.72 87.97 162.69 12.29 17.97 12.77 12.77 12.77 12.77 17.03 20.63 23.705 27.35 33.01 15 16 2.15 15	0.051 0.037 0.059 0.042 0.042 0.038 0.055 0.050 0.035 0.043 0.069 0.028 0.023 0.023 0.021 0.026 0.026 0.026 0.029 0.029 0.030 0.036 0.033 0.027 0.036 0.033 0.027	120.0 123.1 126.5 126.5 120.1 120.8 120.8 120.8 120.5 120.6 117.1 118.0 117.9 117.1 116.8 119.8 119.8 119.8 119.5 121.0 120.2 118.6
1140 1150 1160 1170	2.09 1.87 1.69 2.11	94.99 84.99 76.81 95.90	187.35 168.40 129.70 195.94	187.15 168.20 129.50 195.74	0.077 0.077 0.066 0.079	118.5 118.5 118.4 118.5

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•	Run #	Vdp	Re	ke	$\frac{k_e(v)}{v}$	D _e /V _F L	Tavg
	Lucit	e-Water	• đ _p	= 0.009	83' (3 mm)) φ	= 0.350
			-	$k_e^C = 0.$	20		
	PM-157C 158C 159C 160C 161C 162C 163C 164C 165C 166C 168C 169C 170C	0.21 0.16 0.14 0.12 0.20 0.33 0.36 0.31 0.48 0.28 0.36 0.48 0.48 0.57	9.73 7.34 5.29 9.04 15.04 13.91 21.54 16.54 22.00 26.00	3.16 2.64 2.25 1.60 3.78 9.03 8.71 9.03 14.59 5.86 13.08 13.08	2.96 2.44 2.05 1.40 3.58 8.51 8.13 14.39 5.66 12.88 19.80	0.006 0.007 0.007 0.006 0.008 0.012 0.011 0.012 0.013 0.009 0.012 0.012 0.012	116.5 117.8 112.8 118.0 116.5 113.0 114.8 115.5 115.1 118.5 112.0 113.5 113.8
	Lucite	-Soltro	l d _r	, = 0.01	93' (6 mm) ç ı	- 0.370
				$k_e^0 = 0.$	38		
	D-BB-68C 69C 70C 71C 72C 73C 74C 75C 75C 76C 77C 79C 80C 81C 82C 83C 84C	1.41 1.59 1.31 1.75 2.28 1.39 0.84 1.18 0.62 1.58 0.59 0.94 Run Di 0.36	28.63 32.28 26.60 35.53 46.29 28.22 17.05 23.96 12.59 37.56 42.63 32.08 11.98 19.08 scarded 7.31	47.86 59.09 37.78 62.08 103.19 43.33 17.86 34.94 10.47 70.11 91.85 54.94 10.14 20.64 (Heat L 5.45	47.48 58.71 37.40 61.70 102.81 42.95 17.48 34.56 10.09 69.73 91.47 54.56 20.26 0sses Due 5.07	0.076 0.083 0.064 0.079 0.100 0.070 0.048 0.048 0.048 0.038 0.098 0.098 0.078 0.038 0.049 to Low 0.033	115.8 118.5 119.0 125.0 123.2 123.1 122.1 122.1 120.4 122.8 120.8 123.7 123.6 129.0 119.0 Velocity) 121.8
	Lead-	Water	₫ _p =	= 0.0208	' (6 mm)	φ:	= 0.375
				$k_e^0 = 3.$	50		
D	-BB-148C 149C 150C 151C	0.61 0.61 0.75 0.96	27.68 27.54 34.18 43.68	12.41 12.99 17.38 24.49	8.91 9.49 13.88 20.99	0.017 0.018 0.019 0.021	114.8 114.7 114.5 113.9

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Run #	Vdp	Re	ke	$\frac{k_{e}(v)}{v}$	D _e /V _F L	Tavg
152C 153C 154C 155C 156C 157C 158C 159C 160C 161C-10	1.05 1.54 2.09 2.13 2.16 0.44 0.33 0.27 0.20 55C Run	47.50 69.99 94.85 96.63 98.22 19.86 15.18 12.45 9.23 s Discar	28.36 45.20 68.39 70.07 10.42 8.32 7.21 5.54 cded (Hea	24.86 41.70 64.89 66.57 66.57 6.92 4.82 3.71 2.04 t Losses	0.022 0.024 0.027 0.027 0.027 0.020 0.021 0.022 0.023 Due to Ve	113.5 113.3 116.4 116.4 116.5 117.1 119.2 119.7 119.0 Low locity)
Lead-Sc	oltrol	đp	= 0.0208	3' (6 mm)	φ	- 0.346
:			$k_{e}^{o} = 1.5$	5 <u>0</u>		
D-BB-166 167 168 169 170 171 172 173 174 175 178 179 180 181 182 183 184 185 186 187 188 189 190 191 192 193 194 195 196 197 198	0.68 0.54 0.39 0.322 Run Di Run 0.28 0.29 0.29 0.29 0.29 0.29 0.29 0.29 0.29	13.74 10.92 7.96 6.23 4.53 scarded rown Out 6.09 5.70 5.79 8.06 10.60 17.99 22.87 42.02 57.42 57.05 55.42 57.05 55.42 57.05 55.42 57.05 55.42 57.05 55.42 57.05 55.42 57.05 55.42 57.05 55.42 57.05 55.42 57.05 55.42 57.32 25.32 25.18 & & & & & & & & & & & & & & & & & & & &	7.44 5.70 4.14 3.39 3.05 (Heat Lo 3.31 3.39 3.56 4.74 6.08 11.14 16.81 26.10 36.90 55.70 51.10 56.90 51.10 56.90 54.10 55.70 54.10 20.24 (Poor er 22.05 21.80 4.16 3.16 2.42 2.07 (Produce (Heat Lo	5.94 4.20 2.64 1.89 1.55 5.5 5.5 5.5 5.5 5.0 5.3 5.3 5.40 5.40 5.40 5.40 5.40 5.40 5.40 5.40	0.027 0.026 0.027 0.033 to Low 0.027 0.029 0.029 0.029 0.029 0.029 0.029 0.029 0.029 0.030 0.029 0.029 0.030 0.030 0.039 0.030 0.039 0.036 0.039 0.043 0.043 0.045 0.045 0.045 0.045 0.045 0.017 0.019 0.019 0.019	115.1 115.5 118.8 119.8 119.5 Velocity) 119.0 119.1 118.3 117.6 115.2 114.2 114.0 113.1 113.7 116.4 115.8 113.1 113.5 112.4 114.6 115.8 120.2 Velocity)

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	T avg	= 0.38	114.4	118.3 118.3 117.3	Velocity Velocity 1778		111 111 113 10 113 10 10 10 10 10 10 10 10 10 10 10 10 10	114.0 115.1 115.6	- 0.38	9 11 17 17	115.6 115.6 118.6 118.6	Velocity Velocity 116.0 115.8	116.5 117.1 114.8	110.9 109.6 1103.4	118.1
	D _e /V _F L	8	710.0	0.016 0.018 0.020	to Low	0.017	0.020 0.021 0.026 0.026 0.027	0.027 0.027 0.025 0.023	8		0.044 0.040 0.036 0.036	fit) to Low to Low 0.029 0.033	f1t) 0.047 0.050 0.061 0.062	0.089 0.089 0.089 0.065	0.051
	k _e (v)	3' (6 mm) .50	9.06	3.62 2.71 2.06	DESES Due DESES Due 1 60	39.10 3.10 3.10 3.10 3.10 3.10 3.10 3.10 3	12.69 337.29 48.10 56.10	113.20 58.00 45.20 29.60	08' (6 mm)		9.75 9.75 4. 6 3	ric curve osses Due osses Due 2.15 3.79	rfc curve 14.12 19.30 34.10	75.60 59.90 36.40 36.40	19.60
100	к ө	= 0.0200 ko = 2	11.56	6.12 5.21 4.56	(Heat Lo (Heat Lo 4 10	95.66 2661	15.19 35.70 50.60 50.70 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 58.60 59.60 50 50 50 50 50 50 50 50 50 50 50 50 50	115.70 47.70 32.10	= 0.02	Ke = 1.	13.40 10.75 6.71 6.71	(Foor e. (Heat L (Heat L 3.15 4.79	(Poor 15.12 25.30 35.50	37.40 37.40	20.60
į	Re	đ	27.86	hrown Out 15.54 12.04 9.41	iscarded iscarded o 50	15.23	30.91 38.77 79.54 90.45	hrown Out 125.44 90.90 79.08 57.22	р ^{сс} т	L L T	22.00 22.00 22.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.00 20.000	iscarded iscarded iscarded 5.56 7.39	1scarded 16.54 21.00 29.64 29.44	38.77 65.57 65.16 39.18 29.64	20.91
	Vd	L-Water	0.61	Run 1 0.34 0.27 0.21		0.26	0.68 1.26 1.25 0.85	Run 7 2.76 1.74 1.26	-Soltro		0.40 0.40 0.40	Run D Run D Run D 0.27 0.36	Run D 0.82 1.04 1.46		1.03
	Run #	Stee]	D-BB-2000	2010 2020 2030 2040	2020 2000 20070	2080 2090 2100	2110 2120 2130 2140 2140 2150	2160 2190 2190 2190	Steel.		D-BB-221C 222C 223C	5250 5260 5280 5580 5580 5580 5580 5580 5580 558	5300 53300 53300 53300 53300 53300 53300 53300 53300 53300 53300 53300 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 53500 535000 53500 53500 53500 53500 500000000	2350 2360 2380 2380 2380 2380 2380	240 C

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Reduced Data

Glass-Water

Run #	k _{ha}	$\frac{k_{s(ha)}}{k_{s(ha)}}$	k _{wm} φ	E/D	Pe	<u>Y</u>	$\frac{\gamma/\gamma_n}{\gamma_n}$	Vdp/D
D-BB-24C 25C 26C 27C 28C 29C 30C 33C 35C 36C 37C 38C 37C 120C 122C 123C 124C 125C 126C 125C 126C 125C 126C 125C 126C 125C 125C 125C 125C 125C 125C 125C 125	97803479602000855990000999400545862483 1428 312 112 2 334446777776358543 222	$\begin{array}{c} .43 \\ .98 \\ .139 \\ .143 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197 \\ .197$	1.2739 4.131223 4.124355588 12395368413 1212395368413 1212395368413 12115 12395368413 12115 12395368413 12115 12395368413	$\begin{array}{c} 10.90\\ 213.863\\ 3149\\ 534.77\\ 7060\\ 9112\\ 912\\ 912\\ 912\\ 912\\ 912\\ 912\\ 91$	4 3 2 3 2 4 2 4 2 4 2 3 2 3 3 3 3 3 3 3	.5643643900788338766519024865478311080 1.9978833160 1.111	9752867093667531195606510946733072	48.30 1178.6.77 186.777 186.777 186.777 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.6.77 103.777 105.777 103.777 103.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.777 105.7777 105.7777 105.7777 105.7777 105.7777 105.7777 105.77777 105.77777 105.77777777 105.777777777777777777777777777777777777
			Glass-	Soltrol				
D-BB-46C 47C 48C 49C 51C 52C	5.17 5.46 7.00 2.52 2.90 9.75	.86 .91 1.24 .36 .43 1.85	3.17 3.60 4.52 2.10 1.94 5.51	107.33 121.62 152.70 70.95 65.61 186.14	1.92 1.74 1.62 1.87 2.20 1.64	1.04 1.15 1.23 1.07 .91 1.22	2.08 2.30 2.46 2.14 1.82 2.44	208.8 214.7 250.0 135.3 147.1 305.9

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Run #	^k ha	$\frac{k_{s(ha)}}{k}$	k _{wm} φ	E/D	Pe	Y	<u>Y/Y</u> n	Vd _p /D
540 550 560 570 620 630 1370 13890 1410 1420 1430 1440 1450 1460	11.74 3.883 5.913 5.963 64.270 1.770 5.963 247.270 1.7705 5.020 1.3.35 1.3.5020 1.3.550 1.3.5020 1.3.33 1.3.14 1.333	2.30 .604 .98 .19771 .222776 .9994 .03 .03	5.71 1.847 2.3597 3.5970 2.35970 2.3835 2.2531 2.632 3.835 2.2531 2.632 3.255 2.2531 9.026 6.26 6.26 6.26 6.26 6.26 6.26 6.26	192.91 62.16 69.78 77.77 119.66 66.62 121.69 416.35 295.54 82.77 69.93 415.20 101.05 211.49 230.07 15.54 17.23	1.750304480934542828533666006 1.222.212111111111111222	1.14 .882 .7798 1.5324 1.905 1.5324 1.6354 5833384 1.287 .93 .93	211121232272806666804480804668044808644808044123227774128478	341.28 153.58 1735.52 1344.7.132 2228.447.53 2447.538 1055.06 1050.06 1700.66 1707.64 1682.6 3782.8 38 38 38 38 38 38 38 38 38 38 38 38 38
			Lucit	e-Water				
D-BB-88C 89C 90C 91C 92C 93C 93C 95C 95C 97C 98C 99C 100C 101C 102C 103C 104C 105C 106C 107C 108C 109C 110C	8.41 9.63 9.68 6.35 105 77 2.42 117 8.32 9.88 9.29 20 20 20 20 20 20 20 20 20 20 20 20 20	31.64 14.72 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 32.31.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.94 34.	34.47 17.51 17.51 17.06 15.94 15.98 15.89 15.89 10.99 10.26 9.26 9.26 9.26 9.26 9.26 9.26 9.26 9	253.83 129.02 128.92 162.44 117.38 238.42 175.71 238.42 175.71 74.39 678.44 51.128 67.45 61.128 67.45 61.128 67.45 80.43 121.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.02 129.020	.82 1.65 1.247 .9353 1.702 1.307 58 1.5584 1.12 1.5584 1.12 1.5584 1.11 1.5584 1.11	2.43 3.1.576 3.4.04967 1.1.4.97 3.6.775 3.6.775 3.6.775 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 3.6.70 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y/y_n k_{ha} Run # ks(h<u>a)</u> E/D $k_{wm}^{\phi}\phi$ Pe Va_D/D Y 14.91 9.88 13.44 4.66 98.97 1.80 2.25 144.2 1110 1,11 9.01 3.32 66.35 1.17 2.39 117.3 1120 1.91 3.47 3.95 3.13 3.72 18.53 15.48 87.06 600.59 .58 4.34 348.3 114C 81.56 .51 .64 69.69 611.41 4.94 **311.**7 115C 83.03 13.23 18.79 56.92 88.73 59.35 88.22 437.04 3.91 116C 281.7 •54 4.65 1170 649.63 351.7 .72 .90 2.78 .45 1.60 35.7 26.9 PM-157C .91 11.76 .52 158c .97 1.03 .83 1.21 .29 1.63 12.03 2.06 1.45 1.95 .22 10.71 1.29 1590 22.9 ···95 2.397 5.16 5.41 5.41 160C .17 7.03 1.03 19.4 1.77 1.38 1.53 1.24 17.56 39.55 38.00 .79 2.18 1.41 .41 1.13 161C 33.2 .96 1.11 .85 1.45 1.31 1.81 162C 55.2 2.57 59.8 163C 1.64 164C 39.87 2.01 51.0 1.87 1.61 1.73 .72 1.10 4.50 60.00 1.29 1.55 79.2 46.0 165C 1.94 3.42 1.52 1.38 1.85 25.20 166C ī.48 5.92 6.41 1.35 168C 60.7 47.21 169C 1.80 4.67 1.20 1.50 80.7 1.66 2.41 80.85 170C 2.30 6.52 10.97 1.16 1.93 95.3 Lucite-Soltrol 19.72 23.78 12.87 .62 4.63 3.24 414.7 D-BB-68C 14.64 13.12 666.22 18.34 -58 -88 786.49 434.80 467.7 69C 16.69 3.26 2.28 11.33 700 13.20 385.3 3.86 690.88 1215.88 538.39 2.70 3.65 2.65 21.04 20.45 710 20.21 .74 514.7 5.21 5.29 2.57 3.69 1.9 32.51 14.30 6.35 35.99 15.94 6.47 34.31 12.75 .55 .75 720 670.6 73C 408.8 1.80 74C 4.66 218.67 1.11 247.1 750 760 9.19 2.54 14.26 481.75 .78 1.47 2.58 11.11 347.1 182.4 3.92 3.63 .66 3.03 3.72 2.99 1.54 4.33 5.31 4.27 22.59 29.11 77C 820.27 544.1 28.55 .54 .67 33.81 1142.20 790 800 617.6 20.45 16.48 689.69 464.7 2.30 3.55 7.69 3.91 6.74 2.20 81C 173.5 132.09 1.30 2.37 227.70 82C 1.66 1.20 276.5 1.58 84C .86 2.63 1.17 1.71 105.9 Lead-Water 2.23 .96 1.05 1.91 2.09 D-BB-148C 6.68 50.00 54.57 2.09 .04 101.5 1490 .04 7.29 1.91 101.0 3.15 1500 10.67 79.87 1.23 .06 1.63 2.45 125.3 1.46 1510 .09 16.24 121.56 1.37 2.91 160.2

5.41 9.87

15.64

1520

1530 1540 .11

.23

.43

19.34

31.60

48.82

144.76

236.53

365.42

1.59 1.75

2.00

1.26

1.14

1.00

3.16 3.50

4.00

174.2

256.7

347.8

Run #	k _{ha}	^k s(ha)	k_φ wm ^φ	E/D	Pe	Y	<u>Y/Y</u> n	Vd _p /D
155C 156C 157C 158C 158C 159C 160C	16.17 16.68 1.30 .85 .61 .37	.44 .46 .02 .01 .01	49.96 49.43 5.60 3.96 3.09 1.67	373.95 369.99 41.92 29.64 23.13 12.50	.99 1.02 1.78 1.91 1.99 2.67	2.02 1.96 1.12 1.05 1.01 .75	4.04 3.91 2.23 2.09 2.02 1.5	354.3 360.2 72.8 55.7 45.7 33.8
			Lead-	Soltrol				
D-BB-166C 167C 168C 169C 170C 170C 173C 174C 175C 178C 179C 180C 181C 182C 183C 184C 185C 184C 185C 186C 187C 186C 187C 188C 189C 190C 192C 193C 194C 195C 197C	3.60 2.48 1.46 .57 .88 1.258 .5358 1.258 .5354 .54 .258 .5354 .258 .5354 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .258 .268 .268 .268 .268 .268 .268 .268 .268 .268 .275 .268 .268 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .275 .27	.02 .01 .01 .03 .05 .09 .15 .28 .27 .28 .26 .26 .06 .06 .06	2.718 9.98760 0.272298 9.557367 9.18.10 9.8886 9.18.10 9.8886 9.18.10 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882.3 882.3 816.56 626.59 637.6 8803.6 815.3 600.6 2803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 803.6 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803.6 800
			St ee 1	-Water				
D-BB-200C 202C 203C 204C 207C 208C 209C 210C	5.63 2.20 1.44 0.95 0.96 1.36 2.12 3.18	0.13 0.04 0.03 0.02 0.02 0.02 0.02 0.04 0.07	3.30 1.38 1.24 1.09 0.71 0.93 1.00 4.03	23.57 9.86 8.86 7.79 5.07 6.64 .7.14 28.79	4.17 5.27 4.494 5.63 5.63 6.84 2.42	0.48 0.38 0.45 0.51 0.34 0.36 0.29 0.83	1.17 0.95 1.12 1.27 0.85 0.90 0.73 2.07	102.17 57.00 44.17 34.50 34.83 43.17 55.83 71.67

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Run #	k _{ha} k _{s(ha}	a) <u>k</u> wm ^φ	E/D	Pe	Y	<u>Y/Yn</u>	Vd _p /D
211C 212C 213C 214C 215C 215C 217C 218C 219C 220C	6.71 0.1 9.69 0.20 18.03 0.5 30.12 1.0 36.52 1.4 60.47 2.7 36.71 1.4 29.67 1.0 17.70 0.5	5.81 7.44 14.60 16.89 18.17 50.00 19.86 14.45 11.33	41.50 53.14 104.29 120.65 129.79 357.00 141.86 103.22 80.93	2.68 2.63 2.00 2.41 2.59 2.34 2.79 2.57	0.75 0.76 1.00 0.83 0.79 1.55 0.72 0.78	1.87 1.90 2.50 2.07 1.98 3.88 2.12 1.80 1.95	113.34 142.17 210.34 291.67 331.67 460.00 333.34 290.00 209.83
		Steel-	Soltrol				
D-BB-221C 222C 223C 224C 228C 229C 231C 232C 232C 233C 234C 235C 235C 236C 237C 238C 239C 240C	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	3.84 2.85 1.72 1.72 1.72 1.72 1.33 4.67 5.359 10.27 26.31 21.259 11.89 5.71	125 93 58 20 43 152 174 3124 3380 5855 20 43 174 3124 3380 5855 6986 186	1.76 2.09 2.37 3.81 2.38 1.75 1.29 1.09 0.80 0.80 0.80 0.80 0.80 0.80 0.80 0	$1.14 \\ 0.94 \\ 0.89 \\ 0.53 \\ 1.14 \\ 0.14 \\ 0.93 \\ 0.53 \\ 1.14 \\ 0.13 \\ 1.63 \\ 0.13 \\ 1.48 \\ 0.13 \\ 0.13 \\ 1.48 \\ 0.13 \\ 0.14 \\ 0.13 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ 0.14 \\ $	2.40 2.10 2.17 2.22 2.13 2.12 3.10 3.23 3.10 5.15 5.15 5.15 5.15 5.15 5.15 5.15 5	225.3 199.4 142.0 118.0 107.1 240.2 426.1 562.0 9543.7 429.2 567.0 303.2

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APPENDIX C

GRAPHICAL PRESENTATION OF DATA





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APPENDIX D

APPLICATION OF MATHEMATICAL TREATMENT TO CHROMATOGRAPHIC WORK

Longitudinal dispersion of an adsorbate in a fixed bed system may be described using the same mechanisms as with longitudinal energy dispersion. That is, the adsorbate may be dispersed by 1) longitudinal molecular and eddy diffusion within the fluid phase, 2) fluid film resistance to transfer between phases, 3) solid resistance to transfer between phases due to time required for the adsorbate to diffuse into the porous solid phase, and 4) bulk movement of the fluid. Several authors have discussed this problem. Rosen (72) has developed an exact and an approximate mathematical solution for a model which neglects the effect of molecular and eddy diffusion in the fluid phase. Lapidus and Aximundson (56) have treated the case in which solid resistance is ignored. Deisler (19) treated all of the above mentioned mechanisms, but conducted a frequency response analysis utilizing a cosine wave input instead of the step function input utilized by Rosen and Lapidus and The value of the step function input is that Ammundson. its response may be differentiated to yield the response to

a pulse input. The pulse input is extensively used in chromatographic work (54). Van Deemter et al. (80) have studied the case for a pulse input in which the system is assumed to consist of a series of theoretical plates each of which accomplishes complete mixing. The result is:

$$\frac{C_{n}}{C_{I}} = \frac{A}{V' \sqrt{2\pi n}} \exp \left\{ -\frac{(S/V' - n)^{2}}{2n} \right\} \qquad D-1$$

where

 $V' = \phi H' / \beta, \text{ the effective plate volume}$ H' = distance between theoretical plates $\beta = \frac{1}{1 + \frac{K(1 - \phi)}{\phi}}$

K = equilibrium constant between phases: It is analogous $to <math>\rho_s c_s / \rho_w c_w$ in heat transfer variables $A = V \phi t_o$, feed volume of adsorbate per unit area of bed n = number of plates $S = V \phi t$, fluid injection volume per unit area of bed $C_n = concentration exiting from n^{th} plate$ $C_I = concentration of feed$

Notice that the degree of spreading accomplished by the injected pulse is a strong function of n, the number of theoretical plates.

Van Deemter et al. compared equation D-1 to the Lapidus and Ammundson solution for a fixed bed system, and obtained the following expression for H(Height Equivalent to a Theoretical Plate).

$$H = \frac{2(E + D)}{V} + \frac{2V\varphi}{\alpha} / \left[1 + \frac{\varphi}{K(1 - \varphi)}\right]^2 \qquad D-2$$

where α is the mass transfer coefficient per unit volume of packing. It is analogous to the heat transfer term $\frac{ha}{\rho_w c_w}$. Note that the K used here is the reciprocal of the one used by Van Deemter et al.

Equation D-2 has been further simplified by Van Deemter by representing the eddy diffusion coefficient, E, by $2\lambda Vd_p$ as given by the mixing cell theory, and also by representing the mass transfer coefficient, α , by means of a correlation given by Ergun (25). Giddings (34) has suggested a more reasonable approximation for E. Perkins and Johnson (64) have also discussed a similar treatment of eddy diffusion.

The mathematical treatment of this work may be used to extend and modify the work of Van Deemter et al. If the approximate solution given in Appendix A is differentiated with respect to time the response to a pulse input is obtained. (39)

$$\frac{C}{C_{I}} = \frac{\beta t_{o}}{\sqrt{2\pi\sigma^{2}}} \exp\left[-\frac{(Z/V - \beta t)^{2}}{2\sigma^{2}}\right] \qquad D-3$$

where t $\approx \frac{z}{V\beta}$ has been introduced into the denominator of the

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 \mathcal{A}_{2}

exponential term. In terms of mass transfer variables σ^2 may be expressed as:

$$\sigma^{2} = \frac{2(E + D)Z}{V^{3}} + \frac{2K^{2}(1 - \varphi)^{2}\beta^{2}Z}{V\varphi\alpha} + \frac{2\beta^{2}d_{p}^{2}K(1 - \varphi)Z}{60 D_{s}V\varphi} \qquad D-4$$

If equation D-3 is compared to the theoretical plate solution (D-1) a more advanced expression for H is obtained that includes the effect of solid resistance to transfer between phases.

$$H = \frac{2(E + D)}{V} + (2V\varphi\alpha) / (1 + \frac{\varphi}{(1 - \varphi)K})^{2} + \frac{2(1 - \varphi)d_{p}^{2}VK}{60 D_{g}\varphi (1 + \frac{(1 - \varphi)K}{\varphi})^{2}} D-5$$

It should be noted that equation D-5 is only valid when the diffusion in the solid phase occurs in spherical particles. Aris (2) discusses how internal diffusion may be predicted for irregular shaped particles.

APPENDIX E

Sample Calculations Run Number D-BB-99C Lucite-Water System

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Data Chart

Run No. <u>D-BB-99C</u>	Date <u>9-3-63</u>
Barometric Pressure 73.3	Room Temp. 80°F
Solid Lucite (6 mm)	Fluid H ₂ 0
Heat Transfer Tube <u>B-1</u>	Insulation Press. < 1 mm
Porosity <u>0.368</u>	Rotameter <u>1.0</u>
Initial Temp. <u>153.9°F</u>	Inlet Temp. <u>82°F</u>

Measured Flow Rate

CC	SEC	CC/SEC	•
243	36.8	396.18	
250	38.0	394.80	×.
235	35.4	398.40	
252	38.0	397.80	

Avg. CC/SEC. <u>396.80</u> V = 31.27 ft/hr $Vd_p = 0.60$ ft²/hr Entrance Flush-out time <u>1 min.</u>

ke 19.89 Btu/hr-ft-°F

Remarks

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k_e Calculation Sheet*

Run # D-BB-990	$T_0 = 153.9$	$F = z / \sqrt{t} - V_F \sqrt{t}$
V = 31.27 ft/hr	T _i = 82	TR# 1 & 5
z = 0.958 ft	$T_{o} - T_{i} = 71.9$	$F_{.1C} = 1.30$
	, . .	$(F_{.10})^2 = 1.69$
		$k_{e} = 19.89$

°T F	t min	√t min ²	u	$z/\sqrt{t_1}$ ft/hr ²	V _F √t ft/hr ²	ft/hr [‡]
139.2	2.45	1.565	.205	4.742	3.678	1.064
122.7	2.83	1.682	.434	4.412	3.953	.4590
114.3	3.04	1.744	.551	4.255	4.098	.1570
101.0	3.43	1.852	.736	4.007	4.352	.3450
92 .6	3.83	1.957	.853	3.792	4.599	.8070

$$k_{e} = \frac{\rho_{w}c_{w}\phi + \rho_{s}c_{s}(1 - \phi)}{3.2848} (F_{.10})^{2}$$
$$= \frac{(61.6)(.99)(.368) + 73.0(.35)(.632)}{3.2848} (1.69)$$

= 19.89 Btu/hr-ft-°F

*Some of the k_e calculations were calculated using an IBM 1620 digital computer. In this case the computer output listed F vs. u.

Sample Calculation (Run D-BB-99C)

$$k_e = 18.89$$

 $V = 31.27$
 $k_e^0 = 0.20$ (Extrapolated Value)
 $Vd_p = 31.27$ (0.0193) = 0.60 ft²/hr
 $Re = \frac{Vd_p\rho}{\mu} = \frac{0.60(61.6)}{(0.560)(2.42)} = 27.27$
j\$\psi (From Dryden's Correlation) = 0.225
ha = $\frac{j\varphi \rho_w c_w \varphi 6(1 - \varphi)V}{\varphi (\frac{C_w \mu_w}{k_w})^{2/3} d_p}$
 $= \frac{(0.225)(61.6)(.999)(.368)(6)(.632)(31.27)}{(368)(3.669)^{2/3}(0.0193)}$
 $k_{ha} = \frac{V_p^2 [\rho_s c_s (1 - \varphi)]^2}{ha}$
 $= \frac{[(31.27)(61.6)(.999)(.368)]^2 [(73.0)(.35)(.632)]^2}{[(61.6)(.999)(.368) + (73.0)(.35)(.632)]^2(35.540)}$
 $= 2.43$
 $k_s(ha) = \frac{(\rho_w c_w \varphi)^2 [\rho_s c_s (1 - \varphi)]^2 (Vd_p)^2}{[\rho_w c_w \varphi + \rho_s c_s (1 - \varphi)]^2 60k_s (1 - \varphi)}$
 $= \frac{[(61.6)(.999)(.369)]^2 [(154)(.202)(.632)]^2 (.60)^2}{[(61.6)(.999)(.368) + 73.0(.35)(.632)]^2 60(.117)(.632)]}$
 $= 7.17$
 $k_e(V) = k_e - k_e^0 = 19.89 - 0.20 = 19.69$
 $k_{wm} \varphi = k_e(v) - k_s(ha) - k_{(ha)} = 19.69 - 7.17 - 2.43 = 10.09$

$$\frac{E}{D} = \frac{k_{wm}\phi}{k_{w}\phi} = \frac{10.09}{(.369)(.368)} = 74.30$$

$$\frac{Vd_{p}}{D} = \frac{0.060}{0.006} = 100$$

$$E + D = \frac{k_{wm}\phi}{\rho_{w}c_{w}\phi} + D = \frac{10.09}{(61.6)(.999)(.368)} + 0.006 = 0.451$$

$$Pe = \frac{Vd_{p}}{E + D} = \frac{0.60}{0.451} = 1.32$$

$$\gamma = 2/Pe = 1.51$$

$$\gamma/\gamma_{n} = 1.50/0.80 = 1.89$$
Calculation of Porosity

Volume of tube 2257 cc Weight of beads 1669.74 grmsVolume of beads = (1669.74)/(1.17) = 1427.12 cc $1 - \varphi = (1427.12)/(2257) = 0.632$ $\varphi = 0.368$

APPENDIX F

EXPERIMENTAL MATERIALS

Source and Physical Properties

Liquids*

1.) Distilled Water

Source: University of Oklahoma Power Plant, Norman, Oklahoma μ 0.560 cp ρ 61.6 lbm/ft³ c 0.999 Btu/lbm-°F k 0.369 Btu/hr-ft-°F D (Thermal) 0.006 ft²/hr

2.) Soltrol

Source: Phillips Petroleum Company, Bartlesville,

Oklahoma

<u>j</u> ä	0.92 cp
ρ	46.8 lb _m /ft ³
c	0.50 Btu/lbm-°F
k**	0.08 Btu/hr-ft-°F
	$\lambda = 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$

D (Thermal) $0.0034 \text{ ft}^2/\text{hr}$

*Physical properties were taken at 120°F **Measured by R. Prabhudesai at 80°F

		Solids	
)	Glass 3 mm	(0.118") and 6 mm (0.232	")
·	Source: W. H	. Curtin & Co., Houston,	Texas
	Composition:	Calculated from semi-quar	ntitative spec-
		trographic analysis made	by Shilstone
		Testing Laboratory, 1714	West Capitol,
		Houston, Texas	
		Compound	Approximate %
		Si0 ₂	69
		CaO	8
		Na ₂ 0	16
		к ₂ 0	1
		РЪО	3
		(Misc)	3
	ρ	154 1b _m /ft ³	
	c	0.202 Btu/lb _m -°F	
	k	0.588 Btu/hr-ft-°F	
	D (Thermal)	0.019 ft ² /hr	
)	Lucite 3 mm	(0.118") and 6 mma (0.231	")
	Source: Ace	Plastic Co., 91-30 Van Wy	ck Expressway,
	Jama	ica 35, New York	
	Composition:	Assumed 100% methyl-meth	acrylate
	ρ	73.0 lb _m /ft ³	
	с	0.35 Btu/1bm-°F	
	k	0.117 Btu/hr-ft- [°] F	
	D (Thermal)	0.005 ft ² /hr	

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3.) Lead 3 mm (0.118") and 6 mm (0.250")

Source: National Lead Company, Box 7109, St. Louis 77, Missouri

Approximate % Composition: Compound 0.002 Max. Silver . Copper 0.0025 Max. 0.005 Max. Arsenic, antimony, and tin 0.002 Max. Zinc Iron 0.002 Max. Bismuth 0.150 Max. Lead (by difference) 99.85 Min. 693 1bm/ft3 ρ 0.0316 Btu/1b_m-°F 19.35 Btu/hr-ft-°F k $0.884 \, \text{ft}^2/\text{hr}$ D (Thermal) 4.) Stainless Steel 6 mm (0.250") Source: Hoover Ball and Bearing Company, Box 381, Middletown, Ohio Approximate % Composition: Compound 0.95-1.2 Carbon 0.75 Max. Molybdenum 16-18 Chromium 1.0 Max. Manganese 1.0 Max. Silicon 0.030 Max. Sulfur Iron (Balance) 78 Min. (AISI type 440C Stainless Steel)

The soltrol used with the lead beads was inadvertently mixed with kerosene making the mixture approximately 15% kerosene. Soltrol is a light refinery cut of kerosene and the viscosity is the only physical property that could be affected by this mistake. The viscosity of the mixture was measured using an Ostawald viscometer and the results compared to pure soltrol. Two different samples of the mixture were taken to insure complete mixing had occurred.

Ostawald	Viscometer #	Kinematic Viscosity	Viscosity cp
K527	(mixture)	1.345	1.07
K786	(mixture)	1.344	1.07
S254	(pure)	1.335	1.01

In light of the above data the mixture was assumed to be sufficiently similar to pure soltrol to retain the leadsoltrol data.

APPENDIX G

ADDITIONAL WORK

The qualitative arguments presented concerning the convergence of the exact solution of the four parameter model to the simplified erfc form presents an interesting topic for additional work. With the aid of modern high speed digital computer the exact conditions for convergence could be obtained. This knowledge would be beneficial in both heat transfer and mass transfer studies. This type of an investigation has been instigated by Jim Mehl at the University of Oklahoma as a special problem under Professor O. K. Crosser.

Other additional studies would be of interest. It would be of interest to conduct high velocity experiments in an attempt to validate the Perkins and Johnston (64) discussion in the turbulent velocity range of $10^6 < Vd_p/D < 10^8$. Also of interest would be the extension of the analysis presented here to unconsolidated cores. A very complex topic in which little progress has been made recently is the problem of non-piston flow. An attempt to treat viscous fingering and channeling presents a challenging topic.

Many other complications may be introduced such as:

1) moving heat source

3) high temperature (radiation)

4) simultaneous chemical reactions.

All of the above topics would help create a better understanding of rate processes occurring in packed beds.

APPENDIX H

QUALIFICATION OF MATHEMATICAL ASSUMPTIONS

Temperature Gradient Across A Particle Diameter

In deriving an expression for the contribution of the solid phase resistance to the over-all dispersion, it was necessary to assume radial symmetry within the individual particles. This seems to contradict the presence of downstream conduction within the fluid phase, i.e., there must be a longitudinal gradient across the particles in order for downstream conduction to occur. This section shows that while this gradient exists it is small compared to the temperature difference between phases, and should not seriously affect the temperature distribution within the particles.

If we take the one parameter solution (Equation III-3) and differentiate it with respect to \overline{z} we have

$$\frac{dT}{dz} = \frac{(T_0 - T_I)e}{2\sqrt{\pi D_e t}}$$
H-1

The maximum gradient across a particle will occur when the front arrives at the particle. If for a particular
particle a distance z from the bed entrance t $\approx \frac{z}{v_F}$, and equation H-1 becomes

$$\frac{dT}{dz}_{max} = \frac{\frac{T_o - T_I}{2\sqrt{\pi D_e z/V_F}}}{4 \sqrt{\pi D_e z/V_F}}$$

If this maximum gradient is present over the entire length of the particle then equation H-2 may be rearranged to yield

$$\left(\frac{\Delta T}{T_{o} - T_{I}}\right)_{max} = \frac{d_{p}}{2\sqrt{\pi D_{e} z/V_{F}}}$$
 H-3

where ΔT is the temperature drop across a particle diameter. Using run number D-BB-99C as a typical run where $d_p = 0.01983$ ft $D_e = 19.89 [(61.6)(.99)(.368) + (73.0)(.35)(.632)]$ $= 0.51 \text{ ft}^2/\text{hr}$ and

$$V_{F} = \frac{(61.6)(.99)(.368)(31.27)}{[(61.6)(.99)(.368) + (73.0)(.35)(.632)]}$$

= 18.3 ft/hr.

We have

$$\left(\frac{\Delta T}{T_{o} - T_{I}}\right)_{max} = \frac{0.01983\sqrt{18.3}}{2\sqrt{3.14(0.51)z}} = \frac{0.0355}{\sqrt{z}}$$
 H-4

If z is taken as the bed exit, i.e., z = 1 ft., then $\left(\frac{\Delta T}{T_0 - T_I}\right)_{max} = 3.55\%$. However, if we look at the first particle, i.e., z = 0.01983 ft., $\left(\frac{\Delta T}{T_0 - T_I}\right)_{max} = 25\%$. The following table shows the temperature drop as a function of bed length.

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$\left(\frac{\Delta T}{T_{o} - T_{I}}\right)_{max}$	z (ft)			
0.25	0.01983 (one particle diameter)			
0.11	.1			
0.079	.2			
0.045	.6			
0.036	1.0 (Bed exit)			

It is concluded that while the temperature distribution within the first several layers of particles will be seriously affected, that approximately 80% of the particles composing the bed will not be affected by these intraparticle gradients.

Conduction Through Point to Point Contact

Masamune and Smith (61) have measured the contribution of conduction downstream through contact regions of adjacent solid particles. The data were obtained using spherical glass and steel beads ranging in diameter from 29 microns to 470 microns. Very low pressures were used $(10^{-2} \text{ mm of Hg})$ to assure free molecular conduction was negligible. The results were independent of particle diameter and indicate a value for the point to point contact conductivity of 0.01 - 0.03 Btu/hr-ft-°F, which is truly insignificant for the work presented here.

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