SIMULATION OF SALTWATER UPCONING

By

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CHAPTER I

INTRODUCTION

Almost thirty years after the first known publication on saltwater problems published in 1855 by Braithwaite, two investigators developed an approximate theory to find the boundaries of fresh water lenses in coastal aquifers (12). Their theory is now known as Ghyben-Herzberg theory. Under normal conditions, assuming fresh water and the denser salt water to be immiscible liquids, which is untrue in practice, a sharp interface is formed between the flowing fresh water and the underlying salt water. The steady position of the interface so formed is maintained by the hydrodynamic equilibrium between the fresh water and salt water potentials along the interface. Fresh water and salt water are actually miscible liquids and therefore the zone of contact between the two liquids will be a transition zone caused by molecular diffusion and hydrodynamic dispersion (18), rather than a sharp interface.

Statement of the Problem

The case of a confined, heterogeneous, and isotropic aquifer of infinite extent is considered in this study. Furthermore, it is considered that there is a static salt water body underlying the fresh water region. In the aquifer, the hydraulic head (Peizometric head) in the freshwater region is higher than that of the salt water region.

This study aims mainly at the simulation of the aquifer in order to predict the future peizometric level in the fresh water region with respect to some pumping schedule in specific areas of interest and then to estimate the location of the saltwater interface.

Objective and Scope of the Study

The objectives of the present study are; (a) to simulate for a confined aquifer which is not located in coast line but contains a saltwater body underneath the freshwater region, (b) to study the phenomenon of local upconing of the interface with respect to the above simulation.

A numerical model using the finite difference technique is developed in a two-dimensional physical plane. The Alternating Direction Implicit (ADI) Scheme is used as the solution technique. The solution scheme gives options to simulate a confined groundwater flow problem in homogeneous or heterogeneous, isotropic or anisotropic conditions for cases with or without a salt water body underneath the freshwater region. Furthermore, the model handles the equal or variable grid spacings in any direction. Variable pumping schedules and category map printout for the freshwater level and salt water interface levels have been incorporated into the simulation program. The model can also handle leaky artesian condition. An attempt is made to apply the model to the Yukon well field area of the Garber-Wellington formation in Oklahoma. However, the model application is limited to simple prediction of the factors in it, due to lack of essential well records and other pertinent data.

CHAPTER II

LITERATURE REVIEW

Muskat and Wyckoff (17), and Muskat (16) analyzed an idealized problem of upconing of the oil-water interface in a homogeneous formation of saturated oil zone overlying a water zone with the water being assumed in a static equilibrium condition. The interface in their analysis upcones towards a partially penetrating well pumping oil from the upper zone. They presented the fundamental physical principles of water-coning as referred to individual wells and then concluded that the critical nature of a water cone is marked by the accelerated rise of the cone as an oil-production rate is increased with the final attainment of instability when it has reached a point some 50 to 75 percent of the height to the bottom of the well; at the upper portion of the region of stability the cone is observed to be extremely sensitive to small changes in pressure differential, i.e., oil production rate. This leads to the fact that for a given average pressure differential or production rate a steadily flowing well will induce a lower cone height than one in which the flow is intermittent.

Meyer and Garder (15) derived an approximate solution for the maximum production of oil from a reservoir without having the affect of gas or water upcone in a partially penetrated well section. The assumptions they made in their study included a homogeneous reservoir that extends radially from the well a distance sufficiently larger than the well diameter; the fluids are initially in horizontal layers; and no flow takes place in the gas or water zone.

Bear and Dagan (2) studied the upconing of interfaces in aquifers, in detail, to obtain exact solutions for several simplified cases. Using a hodograph method, an exact solution for the shape of the interface was obtained for the case of a point sink withdrawing water above a horizontal interface in an infinite, two-dimensional flow field. The solution was obtained only for the critical situation when the interface upcones into the drain. With the hodograph method he also investigated the critical solution case of a sink located on the impermeable boundary above an initially horizontal interface in a semi-infinite aquifer. After these studies they concluded that Muskat's approach yields approximate qualitative results, but cannot be employed when a quantitative solution is required. To develop a theoretical equation for upconing of an interface in an unconfined aquifer they considered the general case based on Dupuit assumptions. They confirmed the agreement of their theoretical solution with the experimental results that they conducted on the Hele-Shaw model.

Wang (29) reported an approximate theoretical analysis for a partially penetrating well being used for skimming off fresh water overlying saltwater; and elucidated the interrelation between well spacing, well depth, rate of pumping, thickness of aquifer, and densities of fresh water and salt water. Wang assumed an unconfined aquifer with a sharp interface between fresh and saline water and concluded that there is a rapid fall-off in production above and below the optimal distance of penetration; there is a sharp upper limit, for a given set of aquifer constants, to the amount of water that can be pumped without entrainment of saltwater; there is an increase in maximum discharge, with other factors being held constant, as the difference of the densities of the two layers of fluid increases.

Schmorak and Mercado (23) presented an analytical expression (reported as being developed by Bear and Dagan) describing upconing of an interface as a function of time and distance from the pumping well partially penetrating a relatively thick coastal confined aquifer. The assumptions underlying their theoretical approach were that the porous medium is homogeneous and nondeformable, that the two fluids are incompressible and separated by an abrupt interface (a geometric surface), and that the flow obeys Darcy's law. Their theoretical solution was an approximate linearized development of the problem based on the method of small perturbations.

Pinder and Cooper (20) presented a numerical technique for determining the transient position of the saltwater front and the pattern of flow under the effects of dispersion involving irregular boundaries and nonuniform permeability. They used the method of characteristics in conjunction with the iterative alternating direction implicit procedure to solve the equation of groundwater flow and the equation governing the transport of the dissolved salt. Further assumptions made in their study include - release of water from storage has a negligible effect on the movement of the saltwater front; porosity and dynamic viscosity are constant in time and space; and the dispersion coefficient is constant in time and space and is a Scalar.

Strack (25) studied the influence of drains on the shape of the interface in coastal aquifers. He considered that freshwater is supplied from above (for instance infiltration water, supplied by canals, lakes, etc.) which is approximated by considering the upper boundary of the flow region as straight lines of constant head. He also assumed a homogeneous and isotropic medium with a very low flow rate in the saltwater region compared to the freshwater zone and hence neglected (i.e., the Static Saltwater zone). He solved for the interface in normal case and the case of upconing with drains by using the method of conformal mapping and hodograph. With the result of a test running in a parallel plate model he reported the satisfactory agreement of the formulae derived.

Sahni (22) evaluated the validity of the available steady state solutions of Wang (29) and Muskat and Wyckoff (17) to coning problems for axisymmetrical flow towards a well. He verified with the help of a physical model study that the highest stable cone always occurs with its apex at a lower elevation than the bottom of the well, which was in agreement with the findings of Bennet et al. (3) based on their electric analog studies. He also elucidated the physics of coning phenomenon beneath a freshwater skimming well in an unconfined, homogeneous and isotropic aquifer.

Streltsova and Kashef (27) and Kashef and Smith (14) reported some approaches and their validity while studying critical state of saltwater upconing beneath artesian discharge wells, and expansion of saltwater zone due to well discharge for coastal confined aquifers. Kashef and Safar (13) studied the fresh-saltwater interfaces for coastal artesian aquifer using finite element and hydraulic force techniques.

Chandler and McWhorter (5) investigated the upconing of saline water in response to pumping from an overlying layer of fresh water for a single well by numerical integration of the governing differential equation. They considered the transition zone between the fresh and saline water as an abrupt interface; there exists a steady flow toward partially penetrating pumping well in an isotropic and anisotropic, unconfined aquifer. They reported that an anisotropic medium with lower vertical permeability than an isotropic aquifer maximizes well discharge being free from salt contamination and also increases optimum depth of penetration of the pumping well. They compared their model results with the approximate analytic solution by Dupuit-Forchheimer assumptions and found that discharge computed by the numerical model was 1.7 percent higher than that by the latter.

Strack (26) discussed two three-dimensional interface flow problems in a shallow coastal aquifer with a fully penetrating well (having two zones, one adjacent to the coast and bounded by interface; and the other bounded by an impervious bottom) by using the single-potential technique. The first problem is one of unconfined interface flow where the upper boundary is a free water-table; the second one being of confined interface flow where the upper boundary is horizontal and impervious. He illustrated the use of single valued potential for an analytic technique and suggested its use in finite difference and finite element technique that may have some advantages over the analytic one. His discussion based on Dupuit-Forchheimer assumption was, of course, restricted to cases of steady state flow with homogeneous and isotropic medium.

Das Gupta (7) used finite element techniques to obtain a solution for the shape and location of two unknown boundaries (free surface and interface) simultaneously along with the freshwater discharge to the sea. He assumed a steady, two-dimensional flow in a homogeneous and isotropic phreatic aquifer with the freshwater zone overlies a static saltwater body; and the fresh and saltwater are immiscible. He also extended his numerical scheme to study the upconing of interface in the presence of an infiltration gallery system. He concluded that for the upconing case, the vertical component of flow is predominant in the vicinity of the gallery, and the effect of the upconing of the interface on lowering the water table head at the upstream boundary is negligible for the higher domain length-head ratio.

Rubin and Pinder (21) presented an estimate of the effect of salinity dispersion on the dynamics of flow as well as on salinity distribution in coastal confined aquifers with the help of a phenomenon that is described as migration of a sharp interface perturbed by small disturbances due to salinity dispersion. They considered both pumpage from an infinite strip of wells and from a single well with an assumption that a sharp interface between fresh and saline water initially exists within a finite distance from the pumpage location.

CHAPTER III

MATHEMATICAL FORMULATION

Formulation

The mathematical formulation of the problem considered in this study consists of two steps. The first step describes the development of the partial differential equations which represent the flow phenomenon of a fluid in an aquifer. The second step elaborates the theory of movement of an interface formed by two fluids.

A brief review of the development of equations which describe flow of single phase fluids in porous media is presented in this section.

Darcy's Law and Hubbert's Potential

Darcy's law may be expressed as

$$q = -K \frac{\partial h}{\partial L} = - \frac{k \rho g}{\mu} \cdot \frac{\partial h}{\partial L} \qquad (3.1)$$

where, q is the superficial velocity or specific discharge through the porous medium; K is the proportionality constant; k is specific permeability of the porous medium; g is acceleration due to gravity; ρ and μ are density and viscosity of the fluid respectively; h and L are hydraulic head and flow length respectively.

Hubbert (11) introduced the concept of the potential ϕ to relate Darcy's law to applications in petroleum reservoir engineering which is defined as

$$\phi = g \int_{z_0}^{z} dz + \int_{p_0}^{p} \frac{dp}{\rho p} \tag{3.2}$$

where, z_0 and p_0 are elevation and pressure at an arbitrary datum plane in the porous media, z and p are the elevation and pressure at a point where the potential is to be evaluated and the density, ρ, is regarded as a function of pressure only. If the hydraulic head (or, Peizometric head) is defined as: $h = \phi/g$, then equation (3.1) may be written in terms of

$$q = - \frac{k\rho}{\mu} \cdot \frac{\partial \phi}{\partial L} \tag{3.3}$$

Darcy's law extended to three dimensions in an anisotropic porous media is given by the following equations when the axes of the coordinate system coincide with the principal axes of the permeability tensor (1).

$$q_x = - \frac{k_x \rho g}{\mu} \cdot \frac{\partial h}{\partial x} \; ; \; q_y = - \frac{k_y \rho g}{\mu} \cdot \frac{\partial h}{\partial y}$$

$$q_z = - \frac{k_z \rho g}{\mu} \cdot \frac{\partial h}{\partial z} \tag{3.4}$$

Fluid Flow Equation by Conservation of Mass

Consider fluid flow through the differential element of porous media $\Delta x \Delta y \Delta z$ as shown in Figure 1. A fluid flowing through this media must satisfy the requirement of conservation of mass. The law of conservation of mass states that

$$(\text{Mass})_{in} - (\text{Mass})_{out} = (\text{change in storage or accumulation}) \tag{3.5}$$

$$(\text{Mass})_{in} = (\rho q_x)_x \, (\Delta y \Delta z \Delta t) + (\rho q_y)_y \, (\Delta x \Delta z \Delta t) + (\rho q_z)_z \, (\Delta x \Delta y \Delta t) + Q_{in} \Delta t \tag{3.6a}$$

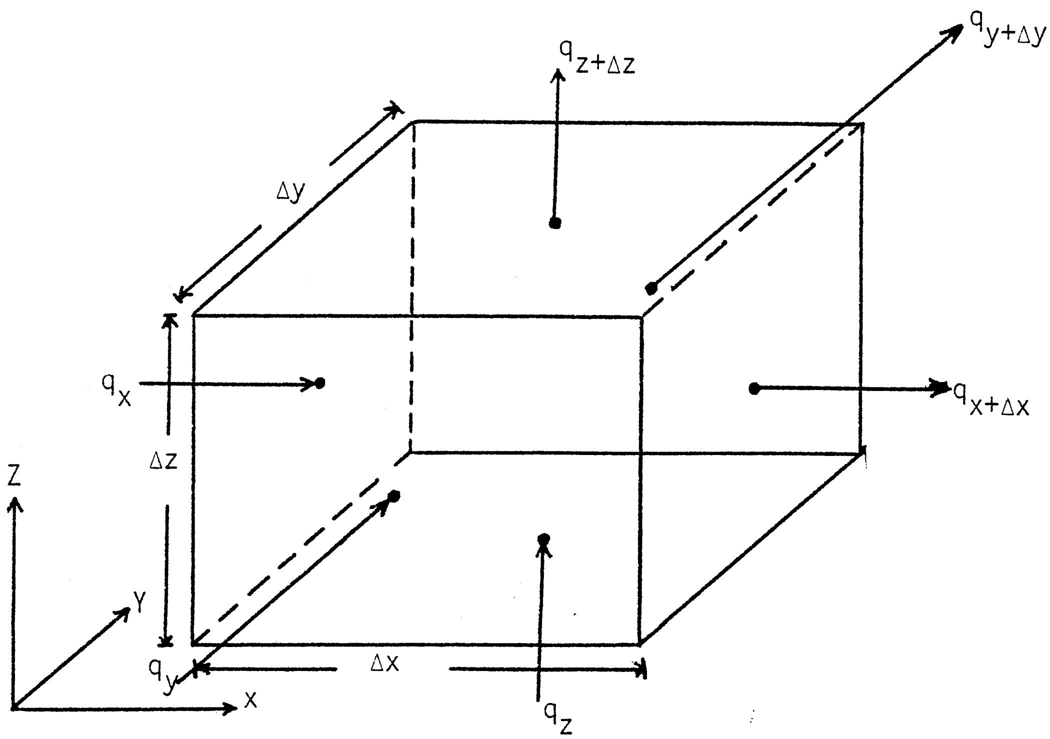


Figure 1. Control Volume for Derivation of 3-Dimensional Continuity Equation in Cartesian Coordinates

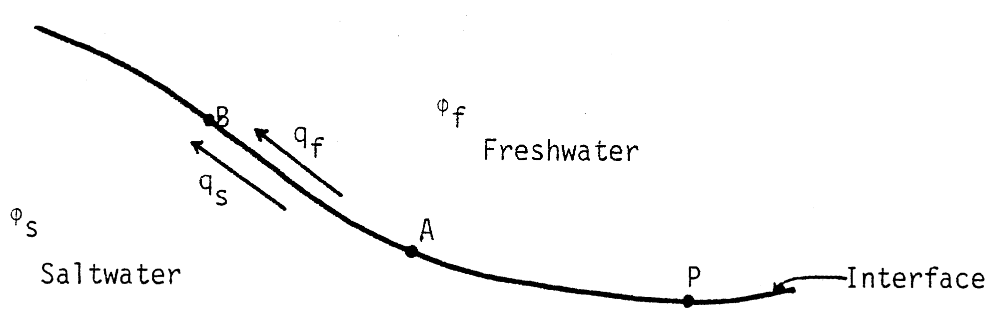


Figure 2. Dynamic Equilibrium at an Abrupt Interface

$$(\text{Mass})_{out} = (\rho q_x)_{x+\Delta x}\,(\Delta y \Delta z \Delta t) + (\rho q_y)_{y+\Delta y}\,(\Delta x \Delta z \Delta t) +$$

$$(\rho q_z)_{z+\Delta z}\,(\Delta x \Delta y \Delta t) + Q_{out}\Delta t \qquad (3.6b)$$

$$(\text{Change in Storage}) = [\,(\theta\rho)_{t+\Delta t} - (\theta\rho)_t\,]\ \Delta x \Delta y \Delta z \qquad (3.6c)$$

where, $(q_n)_{n+\Delta n}$ represents specific discharge in the direction of n at the location of $n+\Delta n$; Q_{in} and Q_{out} are the mass rate added to and withdrawn from the differential element considered per time step; and θ is the porosity of the media. Substituting equation (3.6) into equation (3.5), dividing throughout by $(\Delta x \Delta y \Delta z \Delta t)$, and taking limit that $\Delta x \Delta y \Delta z$ and Δt are infinitely small (that is, approaches zero); the partial differential equation describing the conservation of mass in porous media is obtained which is known as the continuity equation for fluid flow in porous media is given by

$$-\left[\frac{\partial}{\partial x}(\rho q_x) + \frac{\partial}{\partial y}(\rho q_y) + \frac{\partial}{\partial z}(\rho q_z)\right] + Q = \frac{\partial}{\partial t}(\rho\theta) \qquad (3.7)$$

where, Q is the net rate of mass added to the porous media per unit volume from an external source such as an injection well, recharge or discharge boundary, source or sink approximation.

Substituting Darcy's Law, equation (3.4), into the continuity equation (3.7), the governing fluid flow equation is derived as

$$\frac{\partial}{\partial x}\left(\frac{k_x\rho^2 g}{\mu}\cdot\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{k_y\rho^2 g}{\mu}\cdot\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{k_z\rho^2 g}{\mu}\cdot\frac{\partial h}{\partial z}\right)$$

$$+ Q = \frac{\partial}{\partial t}(\rho\theta) \qquad (3.8)$$

The right hand side of the above equation represents the rate of change of storage (rate of accumulation) of fluid per unit volume due to changes in the density of the fluid and the porosity of the porous media with time. Considering the compressibility of the grannular skeleton of the porous media to occur in the vertical direction, Davis and DeWiest (8) derived an expression as

$$\frac{\partial \theta}{\partial t} = \alpha \ (1 - \theta) \ \frac{\partial p}{\partial t}$$

where, α is the one dimensional vertical compressibility of the porous media and p is the internal fluid pressure. Therefore,

$$\frac{\partial}{\partial t} (\rho\theta) = [\rho\alpha \ (1 - \theta) + \rho\beta\theta] \ \frac{\partial p}{\partial t} \tag{3.9}$$

where, β is the fluid compressibility (given by $\partial\rho = \rho\beta\partial p$). Using Leibnitze's rule for the differentiation of integrals, the following can be expressed:

$$\frac{\partial h}{\partial t} = \frac{\partial}{\partial t} [z + \frac{1}{g} \cdot \int_0^p \frac{\partial p}{\rho}] = \frac{1}{\rho g} \cdot \frac{\partial p}{\partial t} \tag{3.10}$$

Specific storage factor of a porous medium can be given by:

$$S_s = \rho g \ [\alpha(1 - \theta) + \beta\theta] \tag{3.11}$$

Substitution of equations (3.9), (3.10) and (3.11) into equation (3.8), yields:

$$\frac{\partial}{\partial x} (\frac{k_x \rho^2 g}{\mu} \cdot \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y} (\frac{k_y \rho^2 g}{\mu} \cdot \frac{\partial h}{\partial y}) + \frac{\partial}{\partial z} (\frac{k_z \rho^2 g}{\mu} \cdot \frac{\partial h}{\partial z}) + Q$$
$$= \rho S_s \cdot \frac{\partial h}{\partial t} \tag{3.12}$$

Considering a fluid with constant density and a constant viscosity flowing through the porous media and that $k_n = \frac{k_n \rho g}{\mu}$ is the proportionality constant defined as the coefficient of permeability (or the hydraulic conductivity), equation (3.12) be rewritten, in a simpler form, as

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial h}{\partial z}\right) + Q = S_s\frac{\partial h}{\partial t} \qquad (3.13)$$

Phenomena of Fresh and Salt-Water Interface

Freshwater and saline water are miscible and at their contact (interface), they tend to mix by molecular diffusion and macroscopic dispersion. This leads to the fact that they are not separated by an oil-water type of interface, they do not constitute distinct fluid phases, and there is no pressure discontinuity where they are in contact. As a matter of fact, the boundary conditions on an interface take the form of two non-linear partial differential equations in h_f and h_s, which makes the direct derivation of the shape and position of an interface a practically impossible task. Even numerical methods fail here (1). Therefore, it is assumed that freshwater and saltwater are separated by an abrupt interface, as described in Figure 2; they have distinct and uniform densities; and that all fundamental idea and essential features of the interface are preserved. A potential ϕ can be defined for each of these fluids as follows:

$$\phi = z + (p_f/\rho_f g) \qquad (3.14)$$

$$\phi = z + (p_s/\rho_s g) \qquad (3.15)$$

where, p is the pressure, ρ is the density, z is the elevation of the point under consideration measured above some arbitrary datum, and the subscripts f and s denote fresh water and salt water respectively.

Since it is assumed that no pressure discontinuity exists across the interface, at any point P on the interface $p_f = p_s = p_i$, the pressure at the interface, denoting the elevation of p by z_i and eliminating p_i from equations (3.14) and (3.15) results in

$$(\phi_{if} - z_i)\ \rho_f g = (\phi_{is} - z_i)\ \rho_s g$$

where, the suffix i refers to the interface. Solving for z_i gives

$$z_i = \frac{\rho_s}{\rho_s - \rho_f}\ \phi_{is} - \frac{\rho_f}{\rho_s - \rho_f}\ \phi_{if}$$

or, $$z_i = (\rho_s/\Delta\rho)\ \phi_{is} - (\rho_f/\Delta\rho)\phi_{if} \qquad (3.16)$$

where, $\Delta\rho = \rho_s - \rho_f$

Equation (3.16) can be used to describe the interface, if the potentials ϕ_{is} and ϕ_{if} are known at a number of points along the interface. If flow exists in both fluids (that is, if both potentials vary along the interface), the shape and position of the interface depends on the velocity components along the interface in both fluids. If the Saltwater zone is assumed to be in static condition then the potential ϕ_{is} is the constant throughout this zone and hence the location of the interface is a function of freshwater potential only. Assuming the above condition is valid, the elevation difference between any two points A and B on the interface is given by

$$\Delta z_{AB} = z_A - z_B = (\rho_f/\Delta\rho)\ (\phi_{if_B} - \phi_{if_A}) \qquad (3.17)$$

Further, it may be of interest to determine the slope at a point on a stationary interface. Differentiating equation (3.16) with respect to s, distance measured along the interface, slope of the interface at any point along it is given by

$$\text{Sin } \theta = \frac{\partial z}{\partial s} = \frac{\rho_s}{\Delta\rho} \frac{\partial \phi_{is}}{\partial s} - \frac{\rho_f}{\Delta\rho} \frac{\partial \phi_{if}}{\partial s}$$

or, $$\text{Sin } \theta = \frac{1}{K} \left(\frac{\rho_f}{\Delta\rho} q_f - \frac{\rho_s}{\Delta\rho} q_s\right) \qquad (3.18)$$

where θ is the angle that the interface makes with the positive x-direction.

With the assumption of static saltwater zone, equation (3.18) reduces to

$$\text{Sin } \theta = (\rho_f/\Delta\rho)\ (q_f/K) \qquad (3.19)$$

Equations (3.17) and (3.19) represent the conditions usually being satisfied along the interface in the fresh water zone while obtaining the shape and position of the interface between the fresh and salt water.

Governing Equation

Equation (3.13) can be written for fresh water region as

$$\nabla q_f + S_{s_f} \frac{\partial h_f}{\partial t} = 0; \quad q_f = - K_f \nabla h_f \qquad (3.20)$$

where, ∇ is del-operator and is defined as $\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$

Assume that F (x, y, z, t) = 0 defines the shape and location of the interface. Denoting the elevation of points on interface by $\zeta = \zeta$ (x, y, t), the relationship for F becomes

$$z = \zeta\ (x,\ y,\ t);\ \text{or},\ F \equiv z - \zeta\ (x,\ y,\ t) = 0 \qquad (3.21)$$

The pressure at a point P (x, y, ζ) on the interface is the same when approached from both sides. Hence, from the definitions of h_f and h_s we have

$$\rho_f\ (h_f - \zeta) = \rho_s\ (h_s - \zeta)$$

or,

$$\zeta\ (x,\ y,\ t) = (1 + \delta)\ h_s - \delta h_f \qquad (3.22)$$

Once the distribution of $h_f = h_f\ (x, y, z, t)$ and $h_s = h_s\ (x, y, z, t)$ is known, equation (3.21) becomes the sought equation for F (x, y, z, t)

$$F \equiv z - h_s\ (1 + \delta) + h_f\ \delta = 0$$

The boundary conditions on the interface are as follows:

(a) Same specific discharge on both sides,

$$(q_n)_f = (q_n)_s \qquad \text{on } F$$

(b) Same pressure on both sides,

$$\rho_f\ (h_f - \zeta) = \rho_s\ (h_s - \zeta) \quad \text{on } F$$

Now, by integrating equation (3.20) along the vertical, making use of Leibinitz rule, the following is obtained for the fresh water region. (Refer to Figure 3) as

$$\int_{\zeta_1}^{\zeta_2} (\nabla q_f + S_{s_f} \frac{\partial h_f}{\partial t})\ dz = \nabla' . \int_{\zeta_1}^{\zeta_2} q_f'\ \ dz - q_f' \Big|_{\zeta_2} \nabla' \zeta_2$$

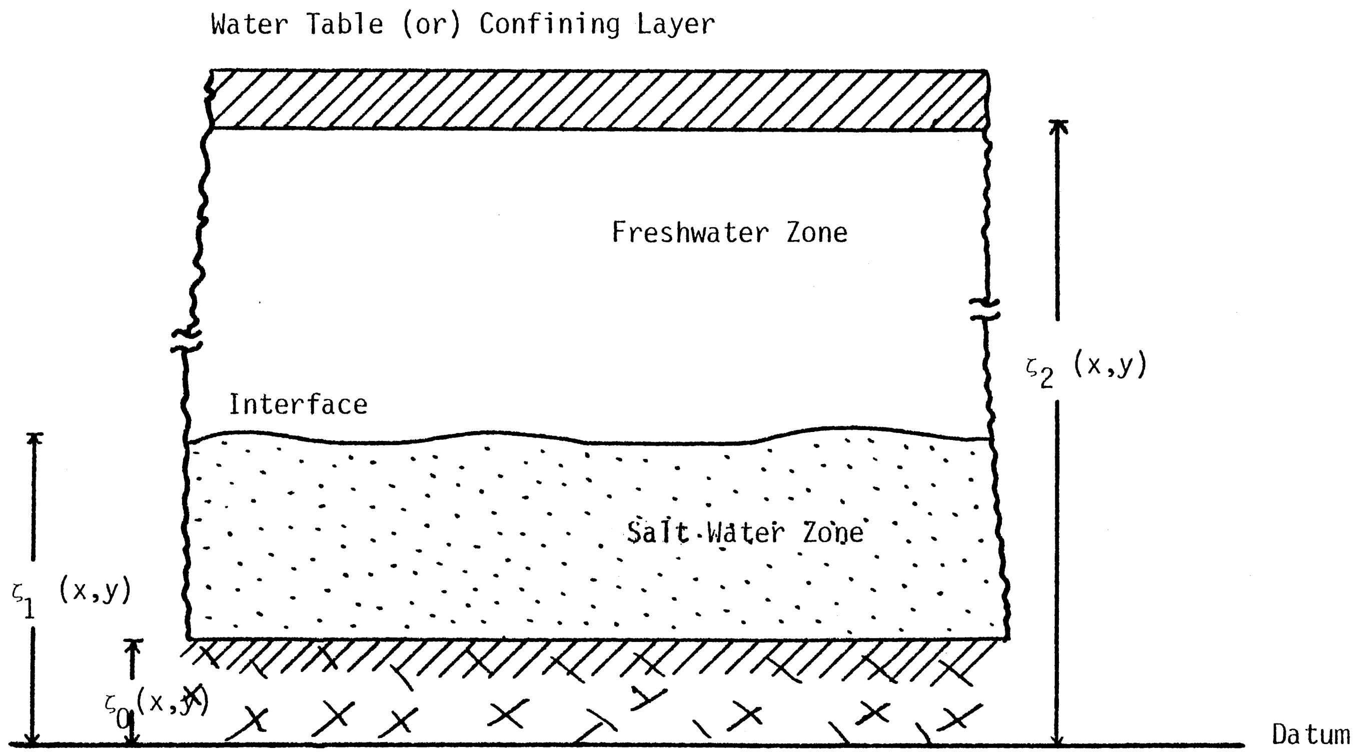


Figure 3. Nomenclature for an Interface in an Aquifer

$$+ q'_f\Big|_{\zeta_1} \cdot \nabla'\zeta_1 + \int_{\zeta_1}^{\zeta_2} \frac{\partial q_{fz}}{\partial z}\, dz \tag{3.23}$$

$$+ S_{s_f} \left(\frac{\partial}{\partial t} \int_{\zeta_1}^{\zeta_2} h_f dz - h_f\Big|_{\zeta_2} \frac{\partial \zeta_2}{\partial t}\right) = 0$$

where $\nabla'(\ ) \equiv [\partial(\ )/\partial x]\ \bar{x} + [\partial(\ )/\partial y]\ \bar{y}$; $q' = q_x\ \bar{x} + q_y\ \bar{y}$

Now,

$$\frac{\partial}{\partial t} \int_{\zeta_1}^{\zeta_2} h_f dz = \frac{\partial}{\partial t} (\tilde{h}_f\ b_f) = \tilde{h}_f \frac{\partial h_f}{\partial t} + b_f \frac{\partial \tilde{h}_f}{\partial t} = \tilde{h}_f \frac{\partial \zeta_2}{\partial t} + b_f \frac{\partial \tilde{h}_f}{\partial t}$$

where, $\simeq h_f\Big|_{\zeta_2}$ $\quad \tilde{h}_f = \frac{1}{b_f} \int_{\zeta_1}^{\zeta_2} h_f dz$; $b_f = \zeta_2 - \zeta_1$

Assuming $\tilde{h}_f \simeq h_f\Big|_{\zeta_2} = h_f\Big|_{\zeta_1}$, that is vertical equipotentials, which is equivalent to the Dupuit assumption, following is obtained from equation (3.23) as

$$\nabla' \cdot (b_f\ \tilde{q}'_f) - q'_f\Big|_{\zeta_2} \cdot (\nabla'\zeta_2) + q'_f\Big|_{\zeta_1} \cdot (\nabla'\zeta_1) + q_{fz}\Big|_{\zeta_2} -$$

$$q_{fz}\Big|_{\zeta_1} + S_{s_f}\ b_f \frac{\partial \tilde{h}_f}{\partial t} = 0 \tag{3.24}$$

where,

$$b_f\ \tilde{q}'_f = \int_{\zeta_1}^{\zeta_2} q'_f\ dz \equiv Q'_f$$

Along with these approximations, it also has from equation (3.22)

$$\zeta_1 = (1 + \delta)\ \tilde{h}_s - \tilde{h}_f$$

$$\text{and } F \equiv z - \zeta_1 = z - (1 + \delta)\ \tilde{h}_s + \delta\tilde{h}_f = 0 \tag{3.25}$$

as the equation describing the interface and satisfies

$$\theta \frac{\partial F}{\partial t} + q_f . \nabla F = 0$$

or,

$$\theta (1 + \delta) \frac{\partial \tilde{h}_s}{\partial t} - \theta\delta \frac{\partial \tilde{h}_f}{\partial t} = q_f \Big|_{\zeta_1} \nabla (z - \zeta_1)$$

$$= q_{fz}\Big|_{\zeta_1} - q_f'\Big|_{\zeta_1} \nabla' \zeta_1 \qquad (3.26)$$

in the fresh water region.

For a phreatic surface, it follows that

$$\theta \frac{\partial F}{\partial t} + (q - N).\nabla F = 0; \; N = - N \bar{z}$$

$$F = z - \zeta_2 = z - h_f \Big|_{\zeta_2} \approx z - \tilde{h}_f$$

$$\theta \frac{\partial \tilde{h}_f}{\partial t} = (q_f - N) \, . \, \nabla \, (z - \zeta_2) =$$

$$q_{fz}\Big|_{\zeta_2} + N - q_f'\Big|_{\zeta_2} . \nabla' \zeta_2 \qquad (3.27)$$

For $\tilde{q}_f$ we obtain

$$\tilde{q}_f = \frac{1}{b_f} \int_{\zeta_1}^{\zeta_2} q_f dz = - \frac{1}{b_f} \int_{\zeta_1}^{\zeta_2} K_f . \nabla h_f \, dz$$

$$= - \frac{K'_f}{b_f} . [\nabla' \int_{\zeta_e}^{\zeta_2} h_f \, dz - h_f\Big|_{\zeta_2} . \nabla' \zeta_2 + h_f\Big|_{\zeta_1} . \nabla' \zeta_1 +$$

$$h_f \Big|_{\zeta_2} - h_f\Big|_{\zeta_e}] \approx - K_f' . \nabla' \tilde{h}_f \qquad (3.28)$$

By combining equations (3.24), (3.26) and (3.27) the following is obtained for the fresh water region as

$$\nabla . \ (b_f K_f' \ . \ \nabla' \tilde{h}_f) + \theta \ (1 + \delta) \ \frac{\partial \tilde{h}_s}{\partial t}$$

$$- [\ \theta \ (1+\delta) + S_{s_f} \ b_f] \quad \frac{\partial \tilde{h}_f}{\partial t} + N = 0 \tag{3.29}$$

For a confined aquifer, $\zeta_2 = \zeta_2 \ (x, y)$ and the fresh water equation (3.29) reduces to

$$\nabla' . \ (b_f \ K_f' \ . \ \nabla \tilde{h}_f) + q_f' \Big|_{\zeta_2} \nabla' \zeta_2 - q_{fz} \Big|_{\zeta_2} + \theta \ (1 + \delta) \ \frac{\partial \tilde{h}_s}{\partial t}$$

$$- (\theta \delta + S_{s_f} \ b_f) \ \frac{\partial h_f}{\partial t} = 0 \tag{3.30}$$

Since at an impervious boundary

$$- q' \Big|_{\zeta_2} . \ \nabla' \zeta_2 + q_z \Big|_{\zeta_2} \equiv q \Big|_{\zeta_2} . \ \nabla \ (z - \zeta_2) = 0$$

Therefore, equation (3.30) reduces to

$$\nabla' . \ (b_f \ K_f' \ . \ \nabla' \tilde{h}_f) + \theta (1 + \delta) \ \frac{\partial \tilde{h}_s}{\partial t} - (\theta \delta + S_{s_f} \ b_f) \ \frac{\partial \tilde{h}_f}{\partial t} = 0 \tag{3.31}$$

Similarly for the salt water zone, it can be written as

$$\nabla' . \ (b_s K_s' \ . \ \nabla' \tilde{h}_s) - [S_{s_s} \ b_s + \theta \ (1 + \delta)] \ \frac{\partial \tilde{h}_s}{\partial t} + \theta \ \frac{\partial \tilde{h}_f}{\partial t} = 0 \tag{3.32}$$

Further, if it is assumed that the salt water zone is static, i.e., there is no change of salt water potential along the interface with respect to time, $\tilde{h}_s$ = a constant, then equation (3.32) does not exist and equation (3.31) becomes

$$\nabla' . (b_f \ K_f' \ . \ \nabla \tilde{h}_f) = (\theta \delta + S_{s_f} \ b_f) \ \frac{\partial \tilde{h}_f}{\partial t} \tag{3.33}$$

If sinks (e.g. wells) are located in the fresh water region, the term $-Q_f$ (x, y, t) or $-Q_f$ (x_i, y_i, t) δ $(x-x_i, y-y_i)$ is added on the left hand side of the above equation and it becomes in expanded form (after taking out the subscript f for fresh water), as:

$$\frac{\partial}{\partial x} (b\ K \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y} (b\ K \frac{\partial h}{\partial y}) = (\theta\delta + S_s b) \frac{\partial h}{\partial t} + Q\delta(x-x_i, y-y_i) \qquad (3.34)$$

If the upper confining layer is leaky, then the leakage term is added to the above equation and then it becomes

$$\frac{\partial}{\partial x} (bK \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y} (bK \frac{\partial h}{\partial y}) = (\theta\delta + S_s b) \frac{\partial h}{\partial t} + Q\delta\ (x-x_i, y-y_i) + \frac{K'}{b'} (h-h') \qquad (3.35)$$

where, super script (') means the values of the upper confining layer. This is the governing equation to be solved in this study.

Boundary Conditions

Boundary conditions for this type of problem can be defined as follows and are illustrated in Figure 4.

Impermeable Boundary

At impermeable boundaries, the specific flow rate is zero. Mathematically,

$$(q_n)_B = - K_n (\frac{\partial h}{\partial n})_B = 0 \qquad (3.36)$$

where, the subscript n refers to the coordinate direction normal to the impermeable boundary and B symbolizes the boundary of consideration. This condition can be achieved with either permeability or the gradient

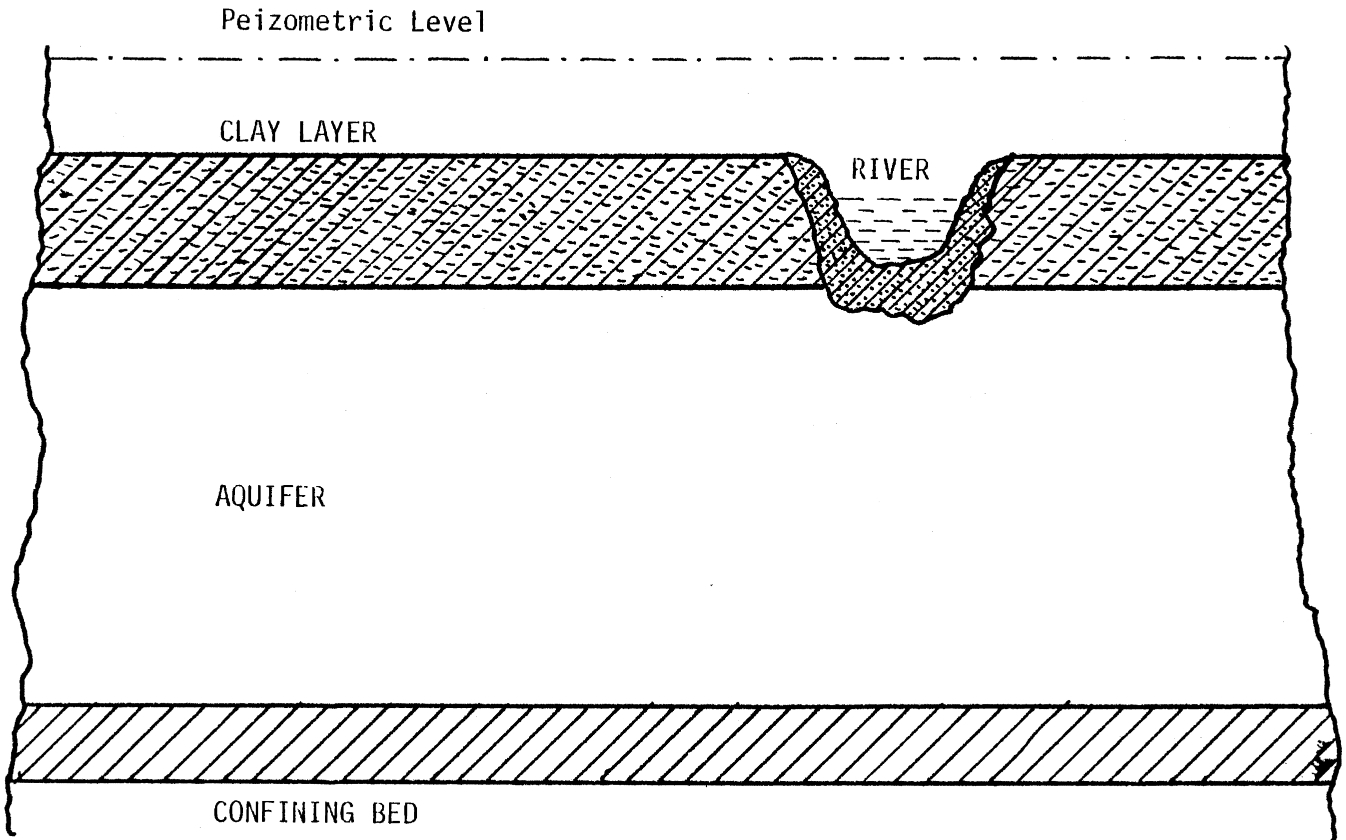


Figure 4. Boundary Conditions

set equal to zero. For this study, the no flow boundary conditions are maintained by specifying $K_n = 0$ at all exterior boundaries.

Specified Head Boundary

Specified head boundaries are simulated by approximating the normal derivative on the boundary, or, in other words, by specifying specific discharge across the boundary by:

$$(q_n)_B = - K_n \left(\frac{\partial h}{\partial n}\right)_B = - K_n \left(\frac{H_B - h_{B-\Delta n}}{\Delta n}\right) \tag{3.37}$$

where, H_b is the specified head at the boundary and $H_{B-\Delta n}$ is the head at a point $(B-\Delta n)$ from the boundary on the normal to the boundary. This boundary condition is handled by introducing as a source term Q_B at appropriate locations and then for all computational purposes, the permeability is set equal to zero along the boundaries.

Specified Flux Boundary

Specified flux boundaries, whether influx or withdrawal, are considered as source or sink terms, Q_B, at appropriate locations and the permeability is set equal to zero at all exterior boundaries.

Leaky Boundary

A leaky aquifer boundary is handled by introducing a source or sink term at appropriate positions which is approximated as the specific leakage factor in the normal direction and is estimated as

$$(q_n)'_B = - K'_n \left(\frac{H' - h_B}{\Delta n}\right) \tag{3.38}$$

where, K'_n is the permeability of the leaky region; H' is the specified head in the overlying region; h_B is the head along the leaky boundary; n is the thickness of the leaky region in the coordinate direction normal to the leaky boundary. This factor is already considered in the governing equation (3.35).

CHAPTER IV

NUMERICAL FORMULATION

Formulation

Let $f(x) = K_x\ (\frac{\partial h}{\partial x})$ at any location i, j in Figure 5 (4.1)

Then, by series expansion,

$$f\ (x + \frac{\Delta x}{2}) = f\ (x) + \frac{\Delta x}{2} \cdot f'\ (x) + \frac{(\Delta x)^2}{8} \cdot f''\ (x) + \ldots \quad (4.2a)$$

$$f\ (x - \frac{\Delta x}{2}) = f\ (x) - \frac{\Delta x}{2} \cdot f'\ (x) + \frac{(\Delta x)^2}{8} \cdot f''\ (x) - \ldots \quad (4.2b)$$

Subtracting equation (4.2b) from (4.2a) and solving for f' leads to:

$$f'\ (x) = \frac{f\ (x + \frac{\Delta x}{2}) - f\ (x - \frac{\Delta x}{2})}{\Delta x} \quad (4.3a)$$

Using nomenclature that $(x + \frac{\Delta x}{2})$ is located at $(i + \frac{1}{2})$ and $(x - \frac{\Delta x}{2})$ is located at $(i - \frac{1}{2})$ and then following similar procedure as (4.3a), it is obtained that

$$h'_{i+\frac{1}{2}} = \frac{1}{(\Delta x)_i} \cdot (h_{i+1} - h_i) \quad (4.3b)$$

and

$$h'_{i-\frac{1}{2}} = \frac{1}{(\Delta x)_i} \cdot (h_i - h_{i-1}) \quad (4.3c)$$

where, the subscript j is omitted temporarily for simplicity. With the

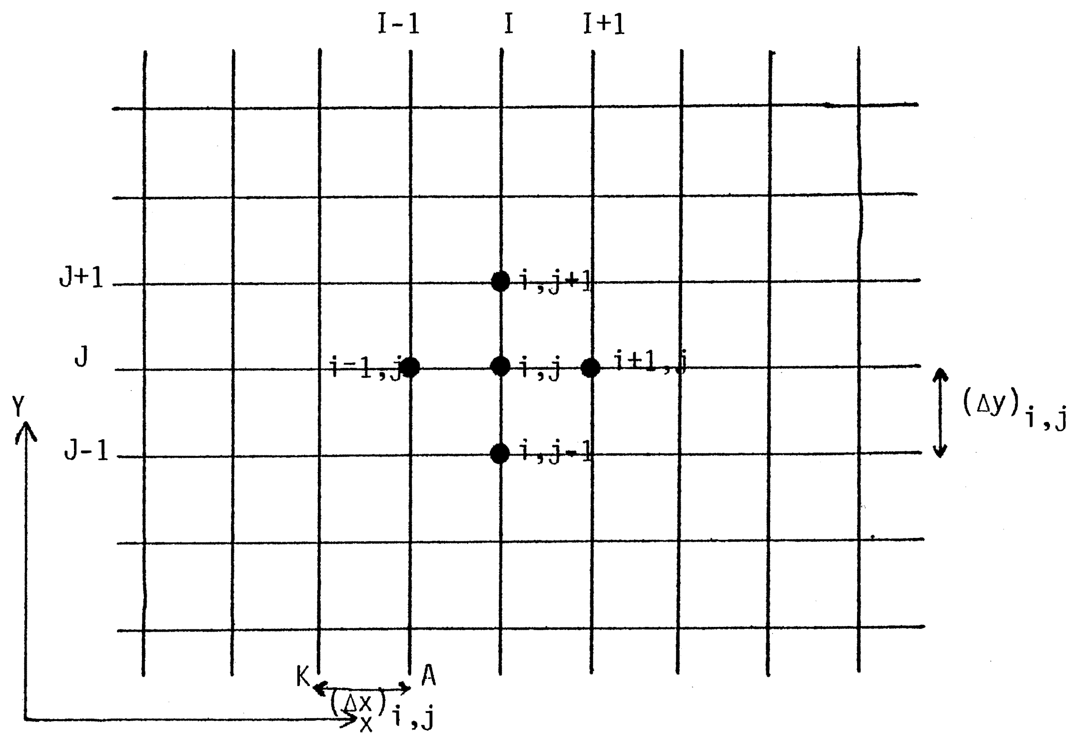


Figure 5. Grid Network for Finite-Difference Scheme

help of equations (4.1), (4.3a), (4.3b), and (4.3c), it can be formulated that

$$f'(x) = \frac{\partial}{\partial x}\left(bK_x \frac{\partial h}{\partial x}\right)_i$$

$$= \left(\frac{bK_x}{\Delta x}\right)_{i+\frac{1}{2}} \left(\frac{h_{i+1} - h_i}{\Delta x}\right) - \left(\frac{bKx}{\Delta x}\right)_{i-\frac{1}{2}} \left(\frac{h_i - h_{i-1}}{\Delta x}\right) \qquad (4.4)$$

K_x is essentially a mass conductivity and the effective conductivity term between nodes, $(K_x/\Delta x)_{i+\frac{1}{2}}$, can be evaluated as the two cell conductivities in series; in other words, by taking the harmonic mean between two consecutive nodal values. Thus,

$$\frac{(i+1) - i}{(K_x/\Delta x)_{i+\frac{1}{2}}} = \frac{(i+1) - (i+\frac{1}{2})}{(K_x/\Delta x)_{i+1}} + \frac{(i-\frac{1}{2}) - (i-1)}{(K_x/\Delta x)_i}$$

After rearranging the terms in the above equation, it gives

$$(K_x/\Delta x)_{i+\frac{1}{2}} = \frac{2(K_x)_{i+1}\,(K_x)_i}{(K_x)_i\,(\Delta x)_{i+1} + (K_x)_{i+1}\,(\Delta x)_i} \qquad (4.5a)$$

Similarly,

$$(K_x/\Delta x)_{i-\frac{1}{2}} = \frac{2(K_x)_i\,(K_x)_{i-1}}{(K_x)_{i-1}\,(\Delta x)_i\,(K_x)_i\,(\Delta x)_{i-1}} \qquad (4.5b)$$

The term b is a function of head and may be determined by a weighted arithmatic average, of the form:

$$(b)_{i+\frac{1}{2}} = \beta b_{i+1} + (1-\beta)\,b_i \qquad (4.5c)$$

Where β may be determined such that the head value at some upstream grid point is used by comparing the heads at nodes i and i + 1. It can also be obtained by arithmatic average method by setting $\beta = \frac{1}{2}$. However, since in the present study the interface is considered to be horizontal and the aquifer is confined, there the saturated thickness is expected to remain constant. Nevertheless, the numerical formulation, in this study, has been developed for a general case.

Substituting equations (4.5a), (4.5b), and (4.5c) into equation (4.4) results in:

$$\frac{\partial}{\partial x}\left(bK_x \frac{\partial h}{\partial x}\right)_{i,j} = (FX2)_{i,j}\,(h_{i+1,j} - h_{i,j}) - (FX1)_{i,j}\,(h_{i,j} - h_{i-1,j})$$

$$= (FX1)_{i,j}\,(h_{i-1,j}) - (FX1 + FX2)_{i,j}\,(h_{i,j}) + (FX2)_{i,j}\,(h_{i+1,j}) \qquad (4.6a)$$

Similarly, for y-direction, it can be obtained as

$$\frac{\partial}{\partial y}\left(bK_y \frac{\partial h}{\partial y}\right)_{i,j} = (FY1)_{i,j}(h_{i,j-1}) - (FY1 + FY2)_{i,j}\,(h_{i,j}) + (FY2)_{i,j}\,(h_{i,j+1}) \qquad (4.6b)$$

where,

$$(FX1)_{i,j} = [\beta(b)_{i,j} + (1-\beta)(b)_{i-1,j}]\cdot\frac{1}{(\Delta x)_{i,j}}\cdot\left[\frac{2(K_x)_{i,j}(K_x)_{i-1,j}}{(K_x)_{i,j}(\Delta x)_{i-1,j} + (K_x)_{i-1,j}(\Delta x)_{i,j}}\right] \qquad (4.7a)$$

$$(FX2)_{i,j} = [\beta(b)_{i+1,j} + (1-\beta)(b)_{i,j}] \cdot \frac{1}{(\Delta x)_{i,j}} \cdot$$

$$\left[\frac{2(K_x)_{i+1,j}(K_x)_{i,j}}{(K_x)_{i+1,j}(\Delta x)_{i,j} + (K_x)_{i,j}(\Delta x)_{i+1,j}}\right] \tag{4.7b}$$

$$(FY1)_{i,j} = [\beta(b)_{i,j} + (1-\beta)(b)_{i,j-1}] \cdot \frac{1}{(\Delta y)_{i,j}} \cdot$$

$$\left[\frac{2(K_y)_{i,j}(K_y)_{i,j-1}}{(K_y)_{i,j}(\Delta y)_{i,j-1} + (K_y)_{i,j-1}(\Delta y)_{i,j}}\right] \tag{4.7c}$$

$$(FY2)_{i,j} = [\beta(b)_{i,j+1} + (1-\beta)(b)_{i,j}] \cdot \frac{1}{(\Delta y)_{i,j}} \cdot$$

$$\left[\frac{2(K_y)_{i,j+1}(K_y)_{i,j}}{(K_y)_{i,j+1}(\Delta y)_{i,j} + (K_y)_{i,j}(\Delta y)_{i,j+1}}\right] \tag{4.7d}$$

The time derivative $\partial h/\partial t$ is approximated by a forward difference Taylor Series as

$$h^{t+\Delta t} = h^t + \Delta t\,(\partial h/\partial t)^t + \frac{(\Delta t)^t}{2}\left(\frac{\partial^2 h}{\partial t^2}\right)^t + \ldots$$

Neglecting higher orders terms, the above equation can be written as

$$(\partial h/\partial t)^t = \frac{h^{t+\Delta t} - h^t}{\Delta t}$$

By coding (n+1) for the time step ($t+\Delta t$) and n for t, the above condition is expressed as

$$\left(\frac{\partial h}{\partial t}\right)^t = \frac{h^{n+1} - h^n}{\Delta t} \tag{4.8}$$

Combining equations (4.6a), (4.6b) and (4.8) for the node (i,j), equation (3.35) is written in finite difference form as:

$$[FX1_{i,j}\,(h_{i-1,j}) - (FX1 + FX2)_{i,j}\,(h_{i,j}) + FX2_{i,j}\,(h_{i+1,j})]$$

$$+ [FY1_{i,j}\,(h_{i,j-1}) - (FY1 + FY2)_{i,j}\,(h_{i,j}) + FY2_{i,j}\,(h_{i,j+1})]$$

$$= (\theta\delta + S_s\, b_{i,j})\,\frac{1}{\Delta t}\,(h^{n+1}_{i,j} - h^{n}_{i,j}) + Q\delta(x-x_i,\ y-y_i) + \frac{K'}{b'}\,(h-h') \tag{4.9}$$

Boundary Conditions

The possible boundary conditions are specified in Figure 4. The methods of approximating these boundaries in the finite difference form are explained as follows.

Impermeable Boundary

An impermeable boundary between two nodes, say (i,j) and (i+1,j) in Figure 5, is equivalent to that $h_{i+1,j} = h_{i,j}$. This is achieved by setting

$$(K_x)_{i+1,j} = 0 \tag{4.10a}$$

The other no flow boundary conditions are specified in a similar manner.

Specified Head and/or Leaky Boundary

This condition is handled by assuming constant hydraulic heads at the aquifer boundaries and approximating the equivalent specific discharge across these boundaries. This is obtained, equivalent to equations (3.37) or (3.38), by computing

$$(q_B)_{i,j} = (-K_{Bn})_{i,j} \left[\frac{(H_B)_{i,j} - h_{i,j}^{t+\Delta t}}{\Delta L_n}\right]$$

or, in terms of a volumetric flow rate this can be given as

$$(Q_{Bn})_{i,j} = (-K_{Bn})_{i,j} \cdot \frac{(H_B)_{i,j} - h_{i,j}^{t+\Delta t}}{\Delta L_n} \tag{4.10b}$$

where, $(H_B)_{i,j}$ is the head in a hypothetical grid block a distance ΔL away from the center of the grid block (i,j) in a coordinate direction of n, and K_B is the effective permeability of the material in this interval. Introducing the practice followed by Pinder and Bredehoft (19) to represent volumetric flow rate across all boundaries in the study, the following expression is obtained as

$$(Q_B)_{i,j} = -(CFX + CFY)_{i,j} \left[(H_B)_{i,j} - h_{i,j}^{t+\Delta t}\right] \tag{4.11a}$$

where,

$$(CFX)_{i,j} = \left(\frac{K_{Bx}}{\Delta L_x}\right)_{i,j}$$

and (4.11b)

$$(CFY)_{i,j} = \left(\frac{K_{By}}{\Delta L_y}\right)_{i,j}$$

Specified Flux Boundary

The specified flux along the boundaries include volumetric rate of man made recharge or withdrawal to and from the aquifer system. This is incorporated in a way that

$$Q_{i,j} = \left(\frac{Q\ \delta\ (x-x_i\ ,\ y-y_i)}{\Delta x \Delta y}\right)_{i,j} = \left(\frac{Q\ (x,y,t)}{\Delta x \Delta y}\right)_{i,j} \qquad (4.11c)$$

Numerical Solution Scheme

Combining equations (4.11a), (4.11c) and (4.9) results in

$$[FX1_{i,j}\ (h_{i-1,j}) - (FX1+FX2)_{i,j}\ (h_{i,j}) + FX2_{i,j}\ (h_{i+1,j})]$$

$$+ [FY1_{i,j}\ (h_{i,j-1}) - (FY1+FY2)_{i,j}\ (h_{i,j}) + FY2_{i,j}\ (h_{i,j+1})]$$

$$= [\theta\delta+(S_s b)_{i,j}]\frac{1}{\Delta t}\ (h^{n+1}_{i,j} - h^{n}_{i,j}) + \left(\frac{Q}{\Delta x \Delta y}\right)_{i,j}$$

$$- (CFX+CFY)_{i,j}\ [(H_B)_{i,j} - h^{n+1}_{i,j}] \qquad (4.12)$$

Equation (4.12) governs the fluid flow for the present study in the finite difference form.

The Crank-Nicholson (6) form estimates $h_{i,j}$ as:

$$h_{i,j} \equiv \frac{1}{2}\ (h^{t+\Delta t} + h^{t}) = \frac{1}{2}\ .\ (h^{n+1}_{i,j} + h^{n}) \qquad (4.13)$$

The solution is seek for the (n+1) time step in implicit form. The Douglas method (9) gives the equation (4.12), with the help of equation (4.13), in the following forms for obtaining solution by the Alternating Direction Implicit (ADI) technique.

First Sweep: in the x-direction

$$\frac{1}{2}(FX1)_{i,j}(h^{n+\frac{1}{2}}_{i-1,j}+h^{n}_{i-1,j})-\frac{1}{2}(FX1+FX2)_{i,j}(h^{n+\frac{1}{2}}_{i,j}+h^{n}_{i,j})$$

$$+\frac{1}{2}(FX2)_{i,j}(h^{n+\frac{1}{2}}_{i+1,j}+h^{n}_{i+1,j})=$$

$$-[(FY1)_{i,j}(h^{n}_{i,j-1})-(FY1+FY2)_{i,j}(h^{n}_{i,j})+(FY2)_{i,j}(h^{n}_{i,j+1})]$$

$$+\frac{1}{\Delta t}[\theta\delta+(S_s b)_{i,j}](h^{n+\frac{1}{2}}_{i,j}-h^{n}_{i,j})+(\frac{Q}{\Delta x\Delta y})_{i,j}-(CFX+CFY)_{i,j}\cdot$$

$$(H_{B_{i,j}}-h^{n+\frac{1}{2}}_{i,j})$$

or,

$$[\frac{1}{2}(FX1)_{i,j}(h^{n+\frac{1}{2}}_{i-1,j})]+[-\frac{1}{2}(FX1+FX2)_{i,j}-\frac{1}{\Delta t}(\theta\delta+S_s b)_{i,j}$$

$$-(CFX+CFY)_{i,j}]\ h^{n+\frac{1}{2}}_{i,j}+[\frac{1}{2}(FX2)_{i,j}(h^{n+\frac{1}{2}}_{i+1,j})]=$$

$$[\frac{1}{2}(FX1+FX2)_{i,j}+(FY1+FY2)_{i,j}-\frac{1}{\Delta t}(\theta\delta+S_s b)_{i,j}]\ h^{n}_{i,j}$$

$$-[\frac{1}{2}[(FX1)_{i,j}(h^{n}_{i-1,j})+(FX2)_{i,j}(h^{n}_{i+1,j})]$$

$$+[(FY1)_{i,j}(h^{n}_{i,j-1})+(FY2)_{i,j}(h^{n}_{i,j+1})]]$$

$$+[(\frac{Q}{\Delta x\Delta y})_{i,j}-(CFX+CFY)_{i,j}H_{B_{i,j}}] \tag{4.14a}$$

and in the y direction,

2nd Sweep: in the y-direction

$$\frac{1}{2}[(FY1)_{i,j}(h^{n+1}_{i,j-1}+h^{n}_{i,j-1})-(FY1+FY2)_{i,j}(h^{n+1}_{i,j}+h^{n}_{i,j})$$

$$+ (FY2)_{i,j} \quad (h^{n+1}_{i,j+1} + h^{n}_{i,j+1})] =$$

$$- \frac{1}{2} [(FX1)_{i,j} \; (h^{n+\frac{1}{2}}_{i-1,j} + \; h^{n}_{i-1,j}) - (FX1+FX2)_{i,j} \; (h^{n+\frac{1}{2}}_{i,j} + h^{n}_{i,j})$$

$$+ (FX2)_{i,j} \; (h^{n+\frac{1}{2}}_{i+1,j} \quad h^{n}_{i+1,j})] + [\frac{1}{\Delta t} \; (\theta\delta + S_s b)_{i,j} \; (h^{n+1}_{i,j} - h^{n}_{i,j})$$

$$+(\frac{Q}{\Delta x \Delta y})_{i,j} - (CFX+CFY)_{i,j} \; (H_{B_{i,j}} - h^{n+1}_{i,j})]$$

or,

$$[\frac{1}{2} (FY1)_{i,j} \; (h^{n+1}_{i,j-1})] + [- \frac{1}{2} (FY1+FY2)_{i,j} - \frac{1}{\Delta t} \; (\theta\delta + S_s b)_{i,j}$$

$$- (CFX+CFY)_{i,j}] \; (h^{n+1}_{i,j}) + [\frac{1}{2} (FY2)_{i,j} \; (h^{n+1}_{i,j+1})]$$

$$= [\frac{1}{2} (FX1+FX2)_{i,j} + \frac{1}{2} (FY1+FY2)_{i,j} - \frac{1}{\Delta t} \; (\theta\delta + S_s b)_{i,j}] \; h^{n}_{i,j}$$

$$- [\frac{1}{2} [(FX1)_{i,j} \; (h^{n}_{i-1,j}) + (FX2)_{i,j} \; (h^{n}_{i+1,j})] + \frac{1}{2} [(FY1)_{i,j} \; (h^{n}_{i,j-1})$$

$$+ (FY2)_{i,j} \; (h^{n}_{i,j+1})]]$$

$$+ [- \frac{1}{2} (FX1)_{i,j} \; (h^{n+\frac{1}{2}}_{i-1,j}) + \frac{1}{2} (FX1+FX2)_{i,j} \; h^{n+\frac{1}{2}}_{i,j} - \frac{1}{2} (FX2)_{i,j} \; (h^{n+\frac{1}{2}}_{i+1,j})]$$

$$+ \quad (\frac{Q}{\Delta x \Delta y})_{i,j} - (CFX+CFY)_{i,j} \; H_{B_{i,j}} \qquad (4.14b)$$

If m number of iterations are considered for each time incremental step, and defining a counter k such that k = 1,2,...m, equation (4.14a) may be written as:

$$[\frac{1}{2} (FX1)_{i,j} \; (h^{k+\frac{1}{2}}_{i-1,j})] + [- \frac{1}{2} (FX1+FX2)_{i,j} - \frac{1}{\Delta t} \; (\theta\delta + S_s b)_{i,j}$$

$$- (CFX+CFY)_{i,j} - H^{k}] \; h^{k+\frac{1}{2}}_{i,j} + [\frac{1}{2} (FX2)_{i,j} \; (h^{k+\frac{1}{2}}_{i+1,j})] =$$

$$[\frac{1}{2}(FX1+FX2)_{i,j} + (FY1+FY2)_{i,j} - \frac{1}{\Delta t}(\theta\delta+S_s b)_{i,j}]\ h^n_{i,j}$$

$$- [\frac{1}{2}[(FX1)_{i,j}(h^n_{i-1,j}) + (FX2)_{i,j}(h^n_{i+1,j})] + [(FY1)_{i,j}$$

$$(h^n_{i,j-1}) + (FY2)_{i,j}\ (h^n_{i,j+1})]]$$

$$+ [(\frac{Q}{\Delta x \Delta y})_{i,j} - (CFX+CFY)_{i,j}\ H_{B_{i,j}} - H^k\ (h^k_{i,j})] \qquad (4.15a)$$

where, $k = 1,2,3,\ldots m$ and H^k is a normalized iteration parameter determined for each node at the beginning of each iteration. Similarly, equation (4.14b) may be written as:

$$[\frac{1}{2}(FY1)_{i,j}(h^{k+1}_{i,j-1})] + [-\frac{1}{2}(FY1+FY2)_{i,j} - \frac{1}{\Delta t}(\theta\delta+S_s b)_{i,j}$$

$$- (CFX+CFY)_{i,j} - H^k]\ h^{k+1}_{i,j} + [\frac{1}{2}(FY2)_{i,j}(h^{k+1}_{i,j+1})] =$$

$$[\frac{1}{2}(FX1+FX2)_{i,j} + \frac{1}{2}(FY1+FY2)_{i,j} - \frac{1}{\Delta t}(\theta\delta+S_s b)_{i,j}]\ h^n_{i,j}$$

$$- [\frac{1}{2}[(FX1)_{i,j}(h^n_{i-1,j}) + (FX2)_{i,j}(h^n_{i+1,j})] +$$

$$\frac{1}{2}[(FY1)_{i,j}(h^n_{i,j-1}) + (FY2)_{i,j}(h^n_{i,j+1})]]$$

$$+ [-\frac{1}{2}(FX1)_{i,j}(h^{k+\frac{1}{2}}_{i-1,j}) + \frac{1}{2}(FX1+FX2)_{i,j}(h^{k+\frac{1}{2}}_{i,j}) - \frac{1}{2}(FX2)_{i,j}$$

$$(h^{k+\frac{1}{2}}_{i+1,j})] + [(\frac{Q}{\Delta x \Delta y})_{i,j} - (CFX+CFY)_{i,j}\ H_{B_{i,j}} - H^k\ h^{k+\frac{1}{2}}_{i,j}] \qquad (4.15b)$$

Equations (4.15a) and (4.15b) may be written for each time step, in a compact form as:

$$(CL1)\ h^{k+u1}_{i+v1,j+w1} + (CL2)\ h^{k+u1}_{i,j} + (CL3)\ h^{k+u1}_{i+v2,j+w2}$$

$$= (CR1) + (u2)\ (CR2) + (u3)\ (CR3) + (CR4)\ (h^{k+u4}_{i,j}) \qquad (4.16)$$

where, the different coefficients, subscripts and superscripts are defined below.

While working in x-direction, i.e., in the 1st sweep:

$$CL1 = \frac{1}{2}\,(FX1)_{i,j}$$

$$CL2 = -\frac{1}{2}\,(FX1+FX2)_{i,j} - \frac{1}{\Delta t}\,(\theta\delta+S_s b)_{i,j} - (CFX+CFY)_{i,j} - H^k$$

$$CL3 = \frac{1}{2}\,(FX2)_{i,j}$$

$$u1 = 1/2\ ;\ v1 = -1;\ v2 = +1;\ w1 = w2 = 0;\ u2 = 1;\ u3 = 0;\ u4 = 0 \quad (4.17a)$$

While working in y-direction, i.e., in the 2nd sweep:

$$CL1 = \frac{1}{2}\,(FY1)_{i,j}$$

$$CL2 = -\frac{1}{2}\,(FY1+FY2)_{i,j} - \frac{1}{\Delta t}\,(\theta\delta+S_s b)_{i,j} - (CFX+CFY)_{i,j} - H^k$$

$$CL3 = \frac{1}{2}\,(FY2)_{i,j}$$

$$u1 = 1;\ v1 = v2 = 0;\ w1 = -1;\ w2 = +1;\ u2 = 1/2;\ u3 = 1;\ u4 = 1/2 \quad (4.17b)$$

The right hand side coefficients are:

$$CR1 = -\frac{1}{2}\left[(FX1)_{i,j}\,(h^n_{i-1,j}) + (FX2)_{i,j}\,(h^n_{i+1,j})\right] - (CFX+CFY)_{i,j}\,H_{B_{i,j}}$$

$$+\left[\frac{1}{2}\,(FX1+FX2)_{i,j} - \frac{1}{\Delta t}\,(\theta\delta + S_s b)_{i,j}\right] h^n_{i,j} + \left(\frac{Q}{\Delta x\Delta y}\right)_{i,j}$$

$$CR2 = -\left[(FY1)_{i,j}\,(h^n_{i,j-1}) - (FY1+FY2)_{i,j}\,h^n_{i,j} + (FY2)_{i,j}\,(h^n_{i,j+1})\right]$$

$$CR3 = -\frac{1}{2}\left[(FX1)_{i.j}\,(h^{n+\frac{1}{2}}_{i-1,j}) - (FX1+FX2)_{i,j}\,h^{n+\frac{1}{2}}_{i,j} + (FX2)_{i,j}\,(h^{n+\frac{1}{2}}_{i+1,j})\right]$$

$$CR4 = -H^k \quad (4.17c)$$

Estimation of Iteration Parameter

Spillette and Nielsen (24) presented a method to estimate the normalized iteration parameter as follows. Iteration parameters used in the solution of the fluid flow equations are applied in a cyclic fashion. To assign the value of H^{k+1}, it is convenient to consider the $(k+1)^{st}$ iteration as the m^{th} iteration of the r^{th} cycle (k=0 for the first iteration, r = 0 for the first cycle; and there are N iteration per cycle); hence

$$m = k+1 - r.N \tag{4.18}$$

The iteration paremeters are obtained from the relation

$$H^{k+1} = \begin{cases} W_{max} \, [W_{min}/W_{max}]^{\left(\frac{m-1}{N-2}\right)} & \text{for } m = 1,2,3,\ldots(N-1) \\ 0 & \text{for } m = N \end{cases}$$

In order to achieve a faster convergence of the solution scheme, the above equation is replaced (28) by

$$H^k = W^k \, (FX1+FX2+FY1+FY2) \tag{4.19}$$

where, W ranges (over grid) between W_{min} and W_{max} defined by

$$W_{max} = \text{Max} \quad \frac{2}{\left(\frac{K_y(\Delta x)^2}{K_x(\Delta y)^2} + 1\right)} \cdot \left(1 - \frac{\pi^2}{4I^2}\right);$$

$$\frac{2}{\left(\frac{K_x(\Delta y)^2}{K_y(\Delta x)^2} + 1\right)} \cdot \left(1 - \frac{\pi^2}{4J^2}\right) \tag{4.20}$$

$$W_{min} = \text{Min}\left[\frac{1}{\left(\frac{K_y(\Delta x)^2}{K_x(\Delta y)^2}+1\right)} \cdot \left(\frac{\pi^2}{2I^2}\right) ;\right.$$

$$\left.\frac{1}{\left(\frac{K_x(\Delta y)^2}{K_y(\Delta x)^2}+1\right)} \cdot \left(\frac{\pi^2}{2J^2}\right)\right] \tag{4.21}$$

where, I and J are column and row numbers of the nodal point under consideration; and Δx and Δy are the grid spacings in the respective direcrions.

The set of parameters are spaced in geometric sequence given by

$$W^{k+1} = r\, W^k \tag{4.22}$$

in which

$$\ln r = [\ln (W_{max}/W_{min}]/(LNT-1) \tag{4.23}$$

LNT = the number of iteration parameters used. The iteration parameters starting with W_{min} are cycled until convergence is achieved.

Solution Procedure

As stated earlier, solution to the present problem is achieved by following the iteration procedure in a cyclic fashion. The solution procedure may be explained in two steps. The first step deals with the problem definition or job setup in which all the input parameters, initial and boundary values are being read. For a better follow up of the problem, all the input values are printed with a standard format as a part of the first step. First of all, in the job setup, the aquifer system properties are discretized by superimposing a certain mesh pattern of finite difference grids over maps of the aquifer properties. Thus, the total dimensions of the grid are defined in x-direction as the number of columns of the model and in y-direction as the number of rows in the model. The parameter and default value cards are defined next which provides data for the whole aquifer system assuming it as if having homogeneous-isotropic properties with identical initial heads and net withdrawal rates at all nodes. A node card deck is then provided which contains the model parameters for each node that has any aquifer system properties differing from those defined on the default value card. The input formats and input parameter cards are assembled in order of occurrence in Appendix (D) and (E) respectively. The aquifer boundaries and grid spacings are described in Appendix (A). The computer flow chart is presented in a subroutine form in Appendix (B) and (C) respectively.

The second step contains the program operational sequences and consists of three cycles of operations. The first and the outermost cycle proceeds with the time incremental steps. The model simulation starts with time equals to zero and increases in a certain pre-specified fashion with the increase of time steps. At the end of every time step, the model results are printed out and then proceeds for the next time step operations and continues until all the time steps have been considered. Each time step undergoes a set of iterations for the model simulation which is termed as the second or the inner cycle of operation. The inner cycle of iterations continues until the solution scheme converges. The convergence test accounts the total system by controlling the greatest change in heads during iterations over the entire model. If the greatest difference between head values calculated (for each node) in a particular iteration from that of the previous iteration lies within a pre-determined or pre-defined tolerance limit then the iteration procedure is terminated, the upconed surface is located, the estimated results are printed out and then the operational sequence goes back to the outer cycle for simulation of the next time step. Mathematically, the convergence criterion and conditions may be given for each node as:

$$h_i^k - h_i^{k+1} \leqslant E \qquad \text{for NP nodes} \tag{4.24}$$

where, NP is the total number of grid points considered in the model, and E is some predefined limit of tolerance or error check. If the equation (4.24) is satisfied, then

$$h_i^{n+1} = h_i^{k+1} \tag{4.25}$$

where, n is the indication for time step. Otherwise, the next iteration is carried out which constitutes the inner most cycle of operation to simulate the head values of the model. This proceeds in two directions; first, it estimates head values in x-direction by solving equation (4.15a) which is denoted as of $(k+\frac{1}{2})^{th}$ iteration and then that in the y-direction by solving equation (4.15b) and being known as $(k+1)^{th}$ iteration; thus completing one iteration.

CHAPTER V

HYDROGEOLOGY

Garber-Wellington Aquifer

The aquifer consists of two geologic formations - the Garber Sandstone and the Wellington Formation. Because these two formations are similar, they are commonly referred to as the Garber-Wellington Aquifer or, simply, the Garber.

The formation consists of about 900 ft. of alternating layers of fine sandstone, shale, and siltstone. That portion of the formation which underlies most of Cleveland and Oklahoma counties contains a large volume of freshwater that can be produced quite economically with wells. Efficient wells can produce 200 to 400 gpm and sometimes more in areas where the sand is thickest and most permeable.

Saltwater underlies the freshwater in Cleveland, Oklahoma, and Logan counties. West of these counties the water in the formation is considered too salty to be useful. The Garber-Wellington formation is exposed at the land surface in the eastern part of the three counties and it slants downward to the west and at the western edge of the county the top of the formation is several hundred feet down below the land surface (see Figure 6). This slant or dip is why wells in the eastern part of the formation would be fairly shallow and while those in the western part would be much deeper. The general details of the aquifer

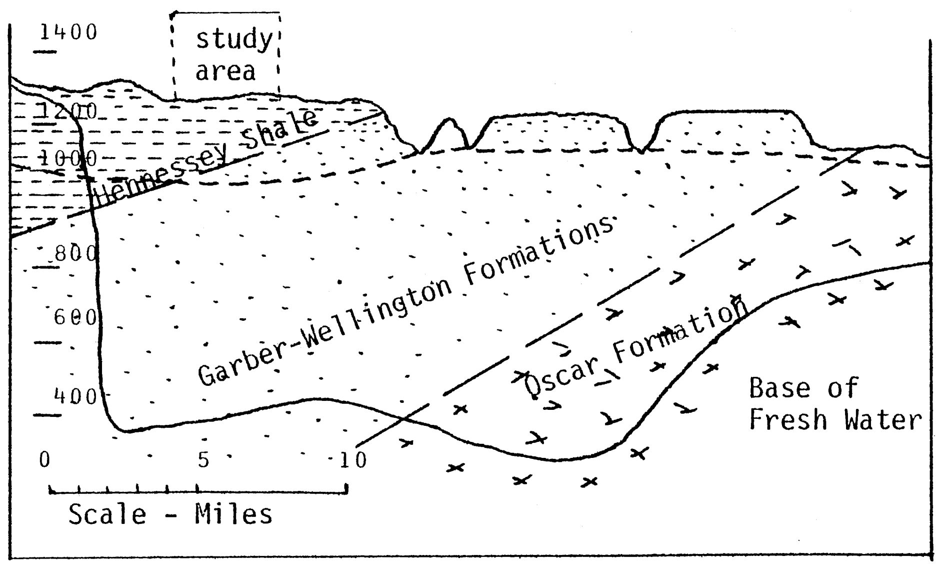


Figure 6. Diagramatic Sketch of Garber-Wellington Aquifer

are given by Carr and Marcher (4), and Wickersham (30).

Water from the Garber-Wellington formation is not widely used for irrigation because the tightness of the formation makes wells fairly expensive and they have a fairly low yield per well, which combined with the cost for deep pumping makes irrigation use prohibitive. The water is however, economical for municipalities. Several cities in central Oklahoma derive part or all of their supply from the Garber-Wellington. Norman, Moore, Midwest City, Edmond, Nichols Hills, and others use Garber-Wellington water (10).

Geologic Framework

The geologic framework of the Garber-Wellington aquifer largely controls the occurrence and movement of ground water. Principal components of the geological framework are (1) geologic structure, including regional and local dip and faulting, (2) lateral and vertical distribution of sandstone and shale units, and (3) characteristics of the rock units, particularly permeability of the sandstone beds.

Sediments that now comprise these rocks represent deltaic deposits laid down by streams flowing from the east into a broad basin that extended into western Oklahoma and Texas. The main part of the delta apparently was in the latitude of central Oklahoma county where sandstone comprises about 75 percent of the aquifer. North and South of this part of the area, the proportion of sandstone to shale decreases, but from near central Oklahoma county, the sandstone grades into siltstone and shale toward the west.

Because of its origin as part of a delta system, the Garber-Wellington aquifer is a complex of interfingering beds of sandstone, siltstone, and shale. The thickness of individual beds changes over short distances and beds may end abruptly, so that correlation for more than short distances is virtually impossible. The maximum thickness of individual beds of sandstone is about 40 ft. but beds 5 to 10 ft. thick are more common. Sandstone comprises 35 to 75 percent of the aquifer and averages about 50 percent. Rocks exposed at the surface are Permian and Quaternary in age. The Permian rocks include the Wellington formation, Garber sandstone, Hennessey shale, and Duncan sandstone.

Because of its low hydraulic conductivity, the Hennessey group acts as a confining layer for the Garber-Wellington aquifer in localities where the aquifer is fully saturated.

Hydraulic and Hydrologic Characteristics

Occurrence of Groundwater

Vertical and lateral variations in hydraulic characteristics of the Garber-Wellington aquifer caused by variations in lithology result in groundwater occurring under unconfined, semi-artesian, and artesian conditions. Unconfined conditions generally exist at depths of less than 200 ft. where the aquifer is exposed at the surface. Artesian conditions exist below 200 ft. and in most of the area where the aquifer is overlain by the Hennessey group. Vertical variations in the hydraulic characteristics of the aquifer present a particularly significant problem in defining the hydrologic system. The average transmissibility for the Garber-Wellington formation is estimated (31) to be around 3,000 gallons per day per square foot.

Movement of Ground Water

Water in the upper part of the aquifer has two components of movement. The principal component is essentially lateral from areas of recharge to points of discharge. A secondary component of movement is usually vertical downward. Movement of water in the lower part of the aquifer is difficult to define because of the lack of water-level data in most of the area. The regional direction of movement is southwest in the same general direction as the regional structural dip. The locations of points or areas of discharge of water from lower parts of the aquifer are unknown.

Freshwater in Storage

The total volume of water available from storage in the freshwater zone may be estimated by multiplying the area, one-half the thickness of the fresh-water zone, and the porosity of the sandstone. One-half the thickness of the freshwater zone is used because the aquifer consists of about equal amounts of sandstone and shale as previously indicated (4). Even though the shale contains large amounts of water it is available only by very slow drainage over a long period of time. Although porosity determines the amount of water the aquifer can hold, the amount of water that the rocks will yield is less because some of the water is retained in the pore spaces. Thus, a better estimate of water available from storage is based on specific yield rather than porosity. Using a specific yield of 0.2 and an average saturated sandstone thickness of 200 feet for the total of about 2,000 square mile area, Wickersham (30) estimated that more than 50 million acre-foot (1 ac. ft. = 325,850 gallons) of water are stored in the Garber-Wellington ground water basin. Additional amounts of water are also supplied to the basin annually by recharge.

Recharge and Discharge

Recharge to the Garber-Wellington aquifer is derived primarily from rainfall on the outcrop area in the northern and eastern portions of the basin. Wood and Burton (31) simply estimated recharge to the Garber-Wellington aquifer to be five percent of the average annual precipitation. However, using pumpage and water-level data, they calculated that recharge in the area south of the North Canadian river was greater than five percent for the years 1957-61. Five percent of the average annual rainfall would provide a recharge of 90 acre feet per square mile as reported by them with an outcrop of 800 square miles as recharge area and thus a total aquifer recharge of 72,000 acre feet annually.

Carr and Marcher (4) used actual field data (streamflow measurements) to determine recharge for the Garber-Wellington aquifer and found it was at least 10 percent of the average annual rainfall. These estimates were based on measurements of Wildhorse Creek, a drainage basin that is typical of most of the outcrop area of the aquifer. Such estimates presume that as the aquifer is essentially in a state of equilibrium in most of the area, the volume of water discharged to the streams is nearly equal to the amount of recharge. According to them the recharge rate in 1975 was 190 acre feet per square mile, however, the rainfall was 10 percent above normal for that year, so they reduced the recharge rate to 170 acre feet per square mile as the average annual rate.

Carr and Marcher (4) added approximately 400 square miles to the area studied by Wood and Burton (31) but neither study considers any recharge from the outcrop area in Lincoln and Pottawatomie counties. The outcrop area of the two published studies is 1,200 square miles with an average recharge of 171 acre feet per square mile. This gives an annual recharge of 205,000 acre feet (186 mgd) occuring in Cleveland, Oklahoma, and southern Logan counties. Directly east of this area is approximately 500 square miles of additional outcrop area in Pottawatomie and Lincoln counties which contributes recharge to the deeper sands at the Garber-Wellington formation.

Peizometric Surface and Well Yield

Generally the good quality water in the Garber-Wellington aquifer has a higher peizometric level (greater head) than both the Garber Salt-water and the transitional salty water. Well yield varies widely from a higher value in the east to a lower value on the west. This is due to the reason that well yield is a function of specific capacity of a properly constructed well and the available drawdown; and that the specific capacity is usually less in the west due to less availability of usable sand bodies.

Study Area

The City of Yukon had its well field being located within the alluvium of the North Canadian River. Considerable work was done in 1974 to renovate and increase the well field yield. Additional studies were made by different agencies to determine the feasibility of expanding the existing well field within the alluvium. The study pointed out several limitations such as naturally poor water quality, competition with irrigators, and limited long-term potential, but still recommended proceeding with some development within the area and looking to the Garber-Wellington aquifer for a longer term supply.

While the ground-water reservoir formed by the Garber-Wellington aquifer underlies much of central Oklahoma, the part that is logical to supply Yukon's water requirements underlies T11N, R4W Oklahoma county and is bounded by I-40 on the north, Portland Avenue on the east, the Cleveland county line on the south and the Canadian county line on the west. The study area and the municipal production deep well locations are shown on Figures 7, 8 and 9 respectively.

Geologic framework in the study area is very similar to the general trend of the Garber-Wellington formation, that is, the Hennessey group above the aquifer thickens towards west and south. This can be observed by taking a cross-sectional view of the area along A-A' as shown in Figure 9 and Figure 10. Thickness of the Hennessey group in the study area is in the range of 300 to 450 ft. as can be seen from Figure 6. Figure 11 presents the water level elevation map (Peizometric map) from mean sea level (MSL) in the study area. From the Figures 11 and 12, it can be concluded that within the study area the Garber-Wellington formation is slightly artesian. The base of freshwater region are plotted in Figure 13. Distribution of freshwater zone in the study area of the Garber-Wellington aquifer may be obtained from Figures 12 and 13 by subtracting the elevation of base of freshwater from that of the contact zone between Garber-Hennessey group (in the present study). Figures

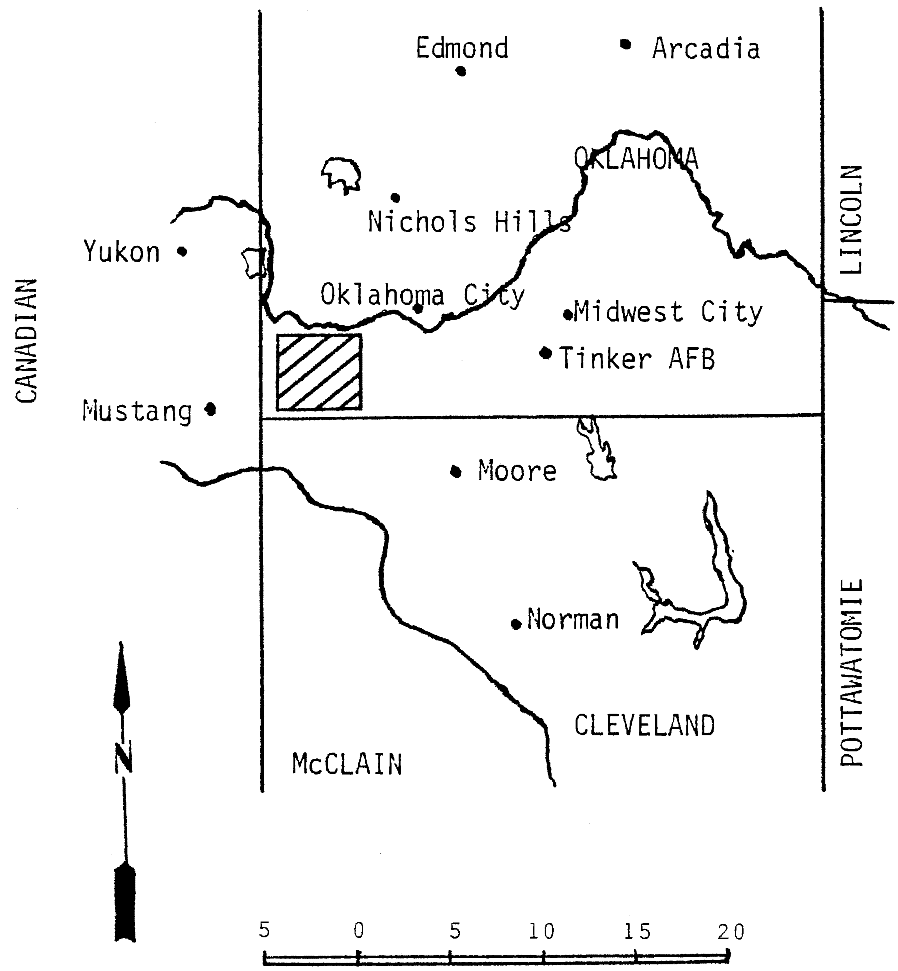


Figure 7. Location of Study Area

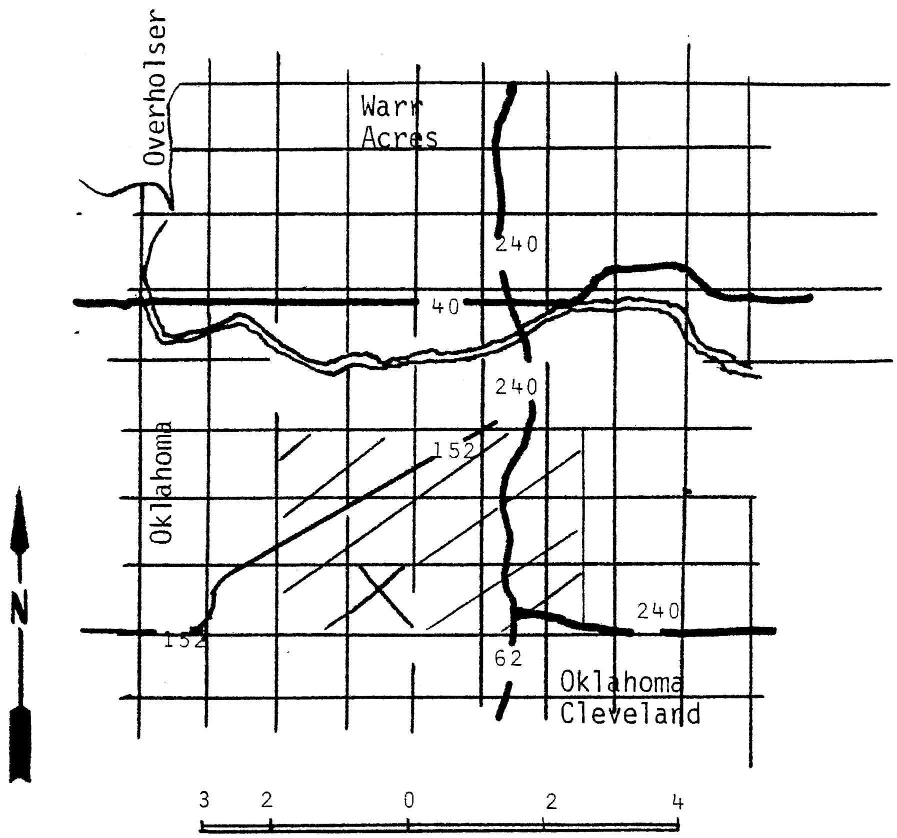


Figure 8. Location of Yukon Well Field

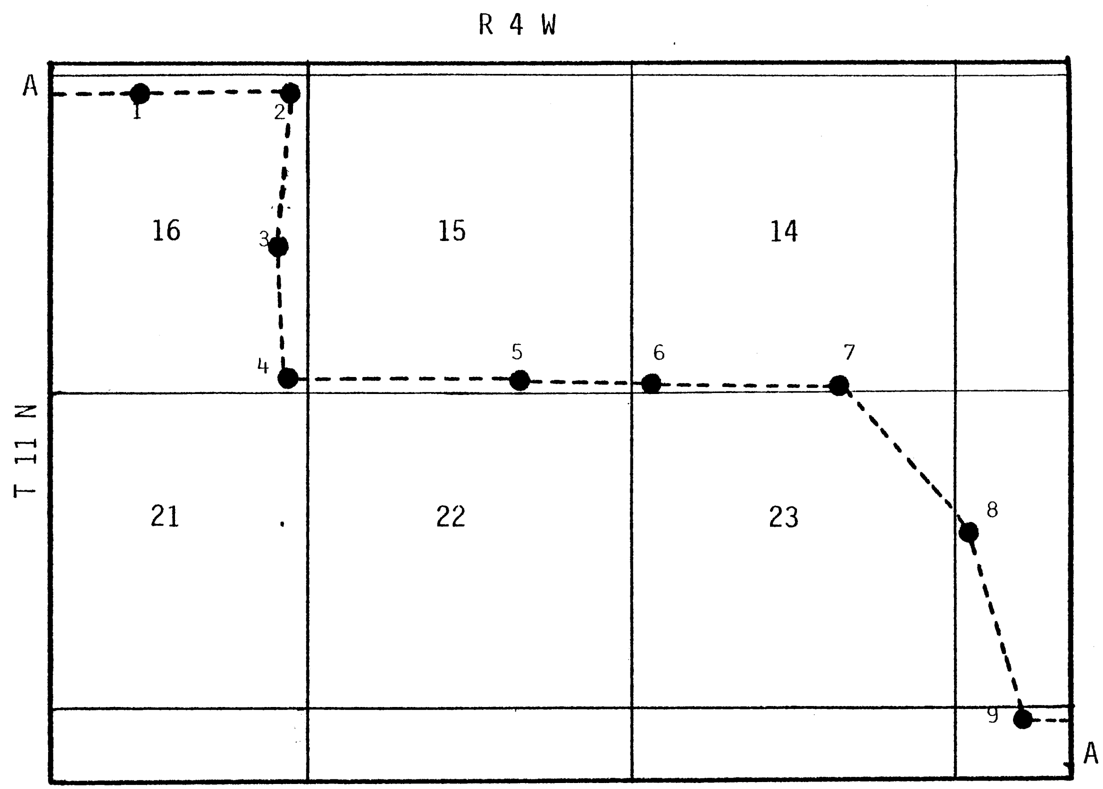


Figure 9. Well Location Map

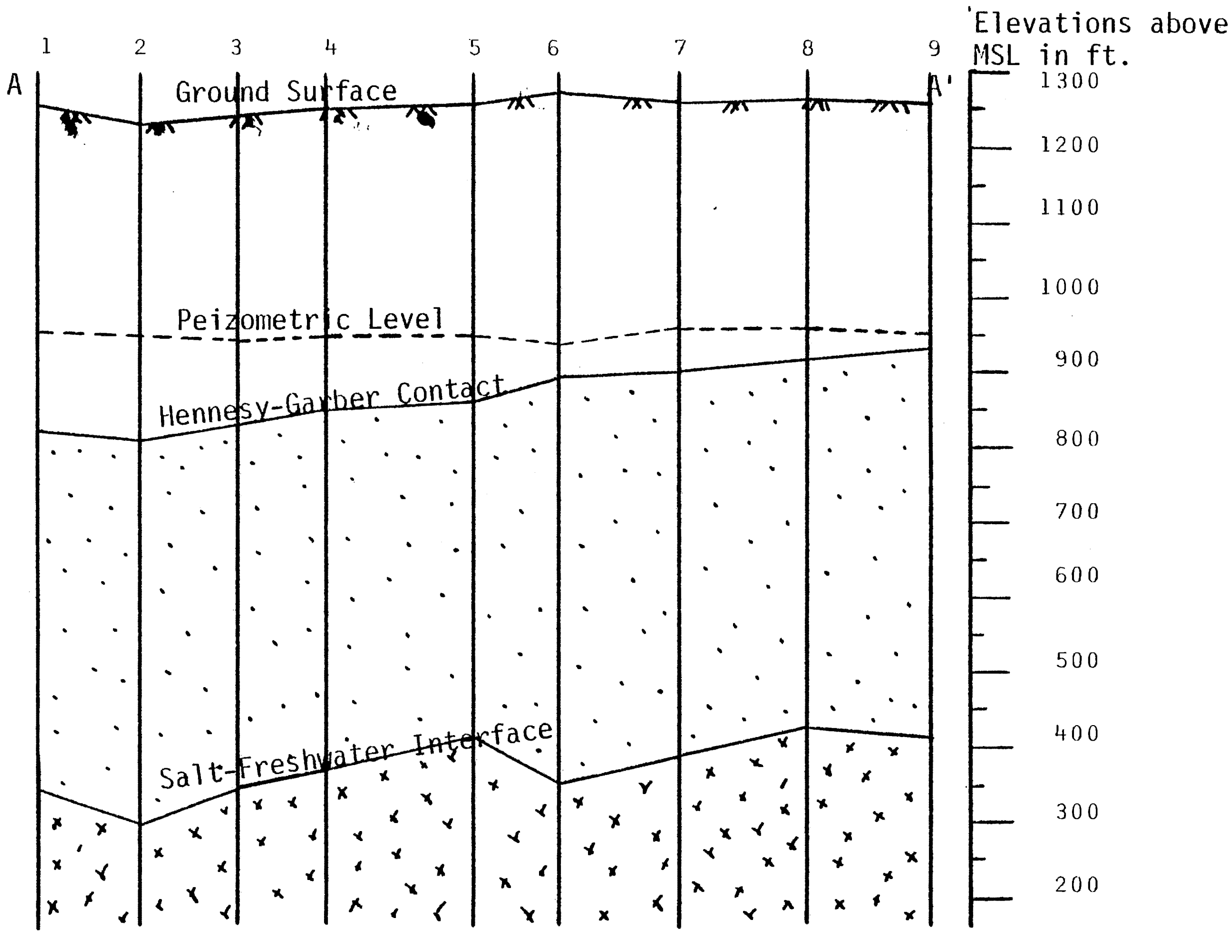
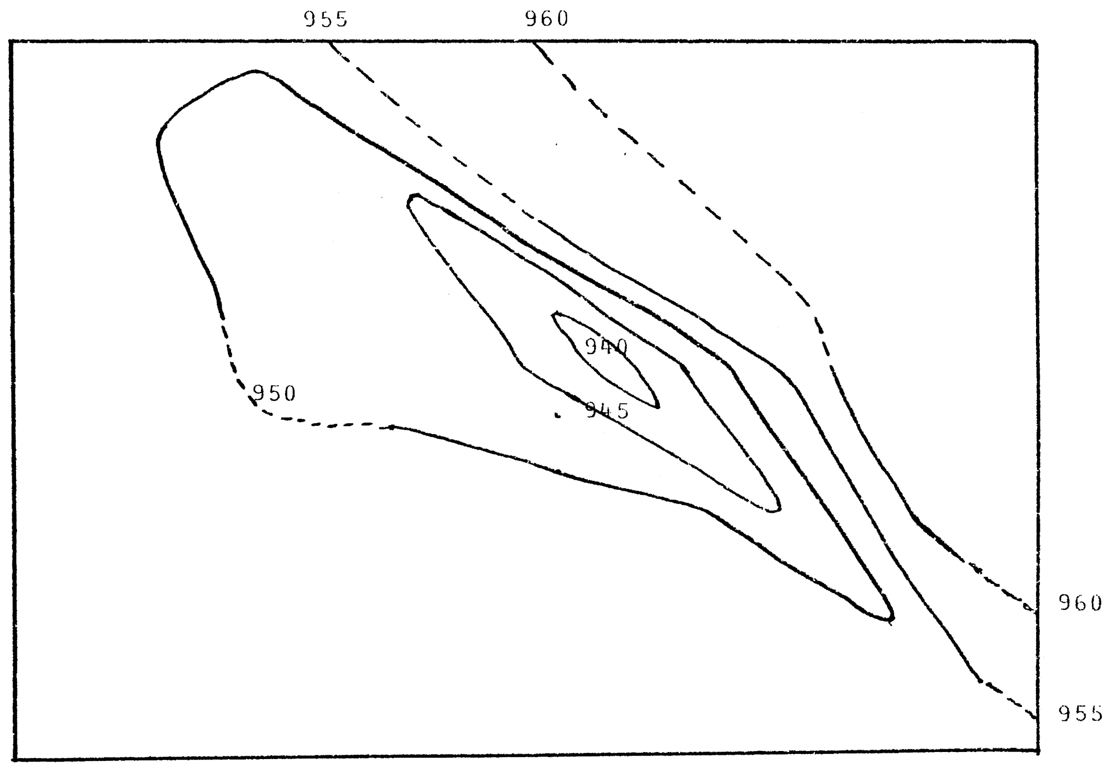


Figure 10. Diagramatic Sketch of Cross-Section along A-A'

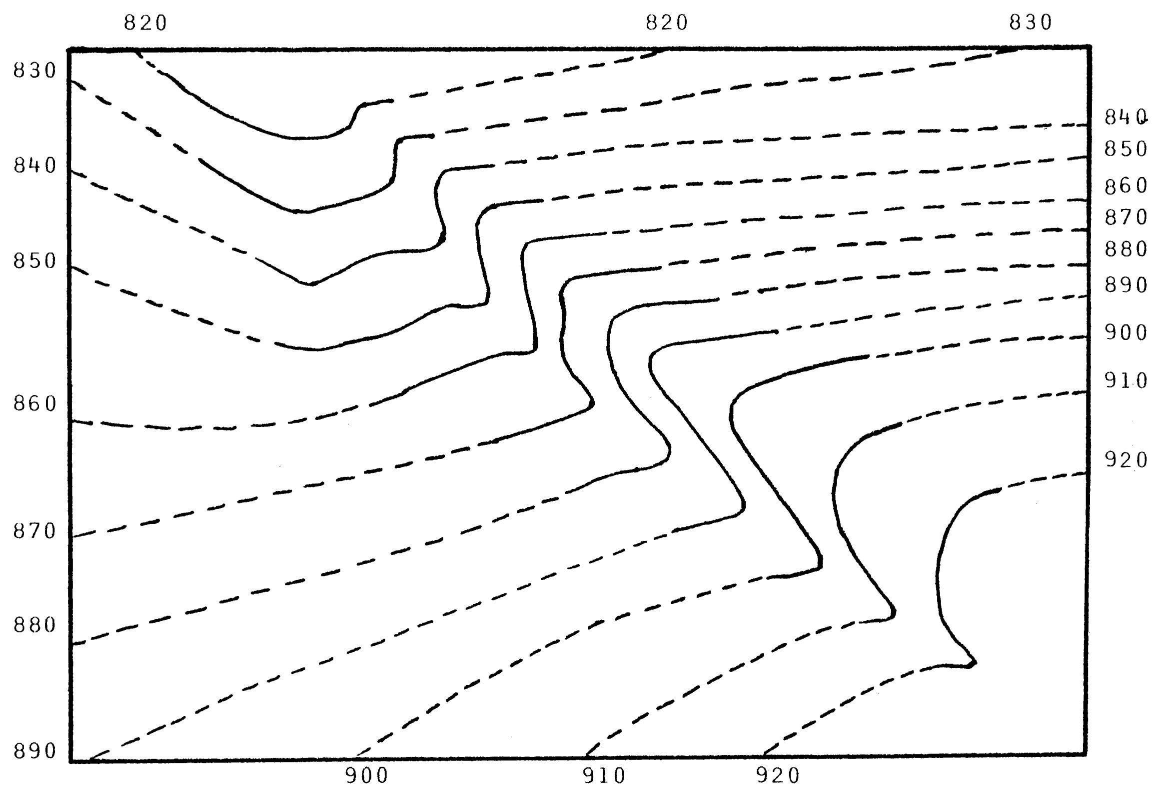


—— 960 —— Contour interval of 5 ft.

---- 960 ---- Inferred

(Elevations above Mean Sea Level)

Figure 11. Peizometric Head Distribution Map

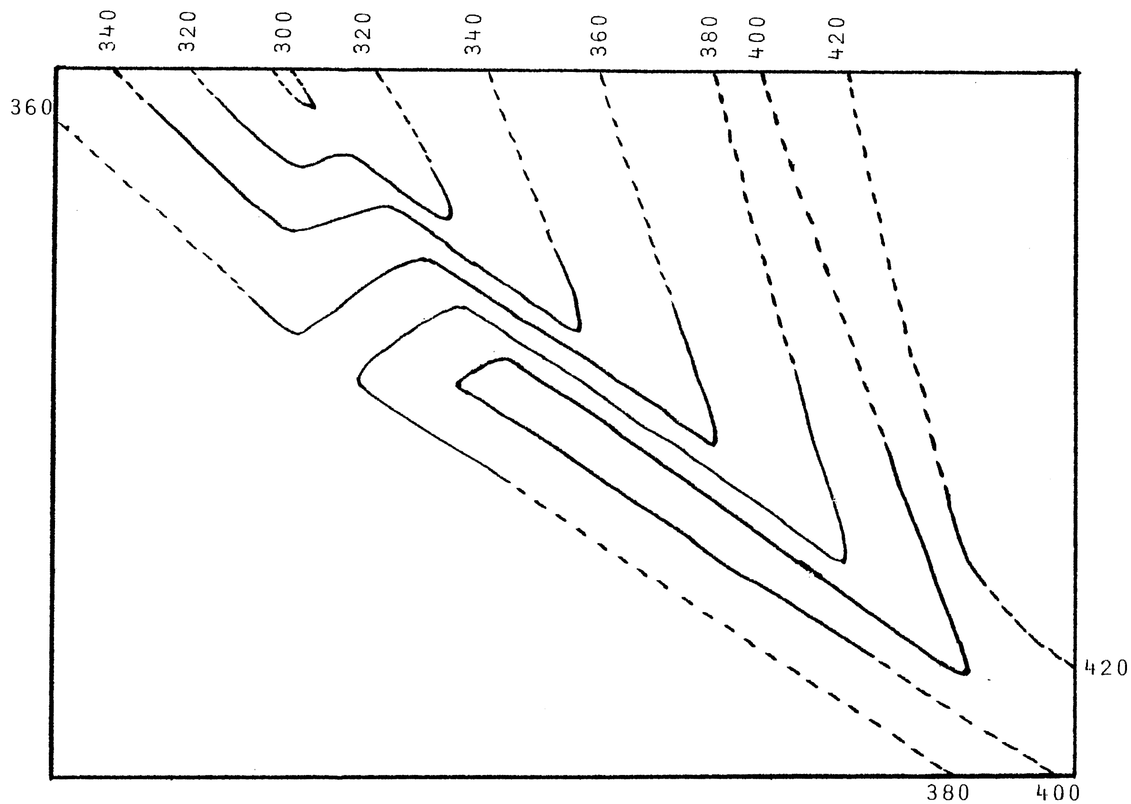


—— 900 —— Contour interval of 10 ft.

---- 900 ---- Inferred

(Elevations above Mean Sea Level)

Figure 12. Hennessey and Garber-Wellington Formation Contact Elevation Distribution Map

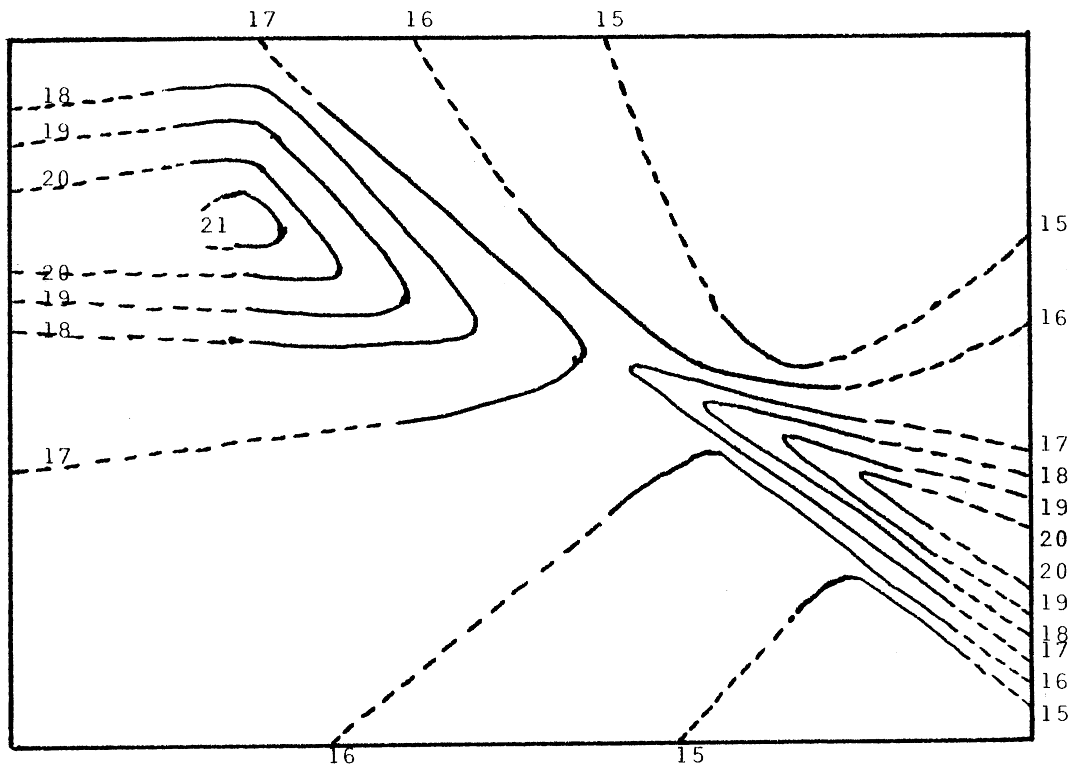


—— 380 —— Contour interval of 20 ft.

---- 380 ---- Inferred

(Elevations above Mean Sea Level)

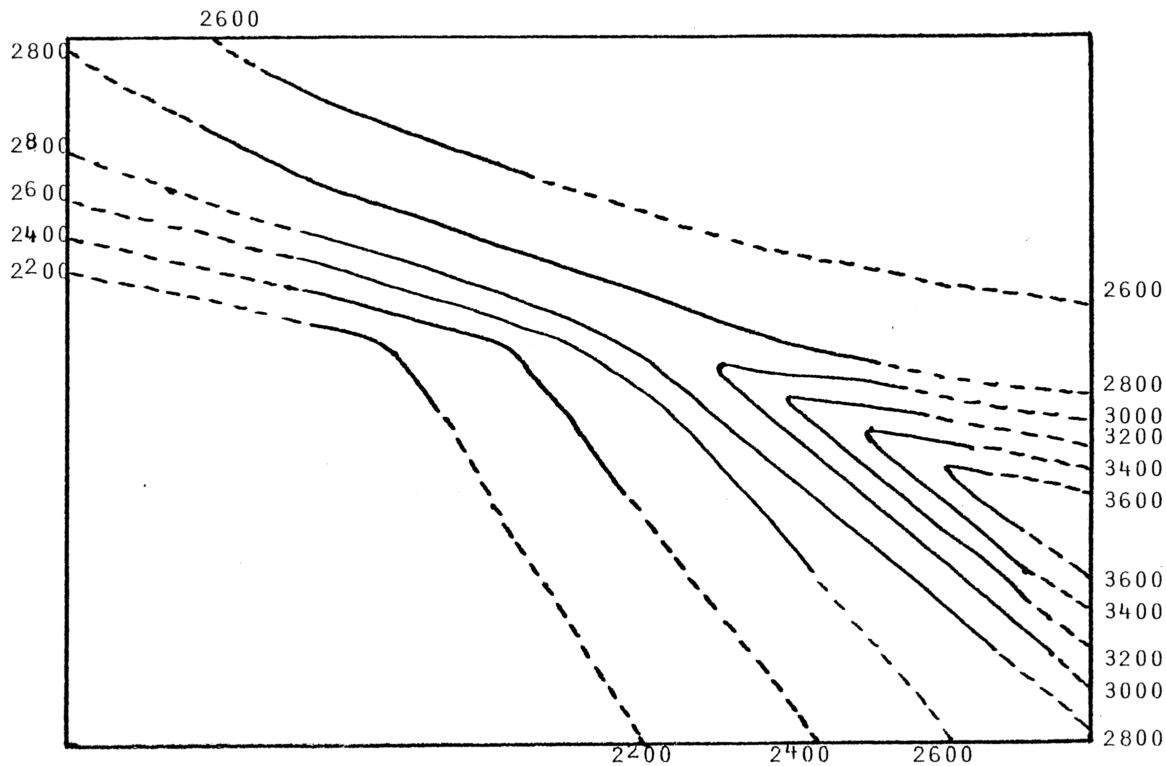
Figure 13. Salt-Fresh-Water Interface Elevation Distribution Map



——— 17 ——— Contour interval of 1 gpd/ft.^{2}

- - - - 17 - - - - Inferred

Figure 14. Permeability Distribution Map



—— 2000 —— Contour interval of 200 gpd/ft.

---- 2000 ---- Inferred

Figure 15. Transmissibility Distribution Map

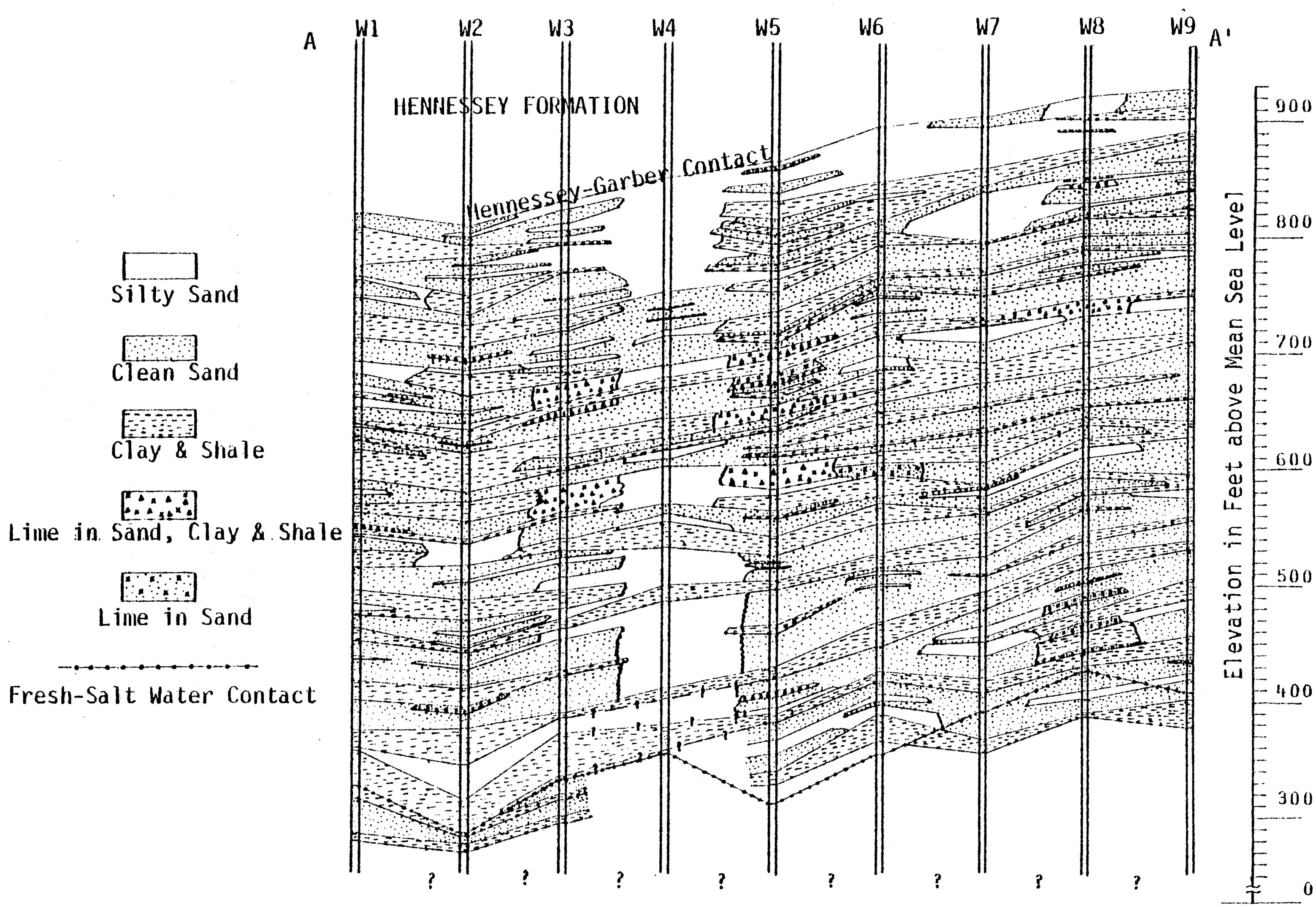


Figure 16. Geologic Cross-Sectional View Along A-A'

14 and 15 present maps showing aerial distribution of average permeability and transmissibility of the Garber-Wellington aquifer in the study area, respectively. The electric logs recorded by Engineering Enterprises, Inc. were used to establish a correlation with the measured water quality, determine the distribution of sands in the Garber-Wellington formation, and locate the Garber-Hennessey boundary in the study area. The interpreted results of driller's and electric logs are presented in Figure 16 as the Geologic cross-section of the study area.

The Yukon study area is part of the Prairie Plains Homocline. The surface is a gently eastward sloping plain with westward dipping rocks. Within the study area, surface elevation varies from about 1,250 to 1,300 feet. Drainage consists of eastward-flowing streams. Tributary streams generally flow northward or southward. The area has a subhumid climate with pronounced day-to-day changes and mild seasonal variations. The total average annual precipitation is about 32 inches. May is commonly the wettest month. The average annual temperature is about 61 degrees. Within the study area (T11N, R4W) recharge from directly above the aquifer is negligible. The study area receives a resonable amount of recharge from the North Canadian river along its northern portion.

On the west edge, beyond the study area, the high chloride (saltwater) and high sulfate (gypsum water) water zones nearly intersect or mix together. The saltwater-freshwater interface is not a sharp line in the study area. Therefore the upper boundary of the transition zone with total dissolved solids content of the water less than 1000 mg/l is considered as the interface location for modeling purpose. The average specific capacity for the study area may be considered as 2 gpm per foot of available drawdown.

CHAPTER VI

MODEL APPLICATION

This chapter is exclusively devoted to application of the model developed in the previous chapters.

Job Setup

As the first step in the computer job setup, the aquifer system properties are discretized by superimposing a square/rectangular mesh finite difference grid over maps of the aquifer properties. This is illustrated in Appendix A. The total dimensions of the grid are defined by NX, the number of grids in the longitudinal direction ($\leqslant$ NC, the number of columns of the model) and by NY, the number of grids in the transverse direction ($\leqslant$ NR, the number of rows in the model). Next, the check card and control card are prepared according to the formats illustrated in Appendix D. The check card provides a check in correct dimension specification of the arrays and assigns the other parametric values that are essential for the model run. Each discrete portion of the aquifer associated with numbered node of the grid is assigned an average value of transmissivity, storage factor, initial head, and net withdrawal rate. The above nodal values are limited by the control card which specifies the exact formats required for the node cards and accordingly directs the user. The node cards include all the nodes information of the model for a particular property of the aquifer. The boundary of the aquifer is approximated in a stepwise fashion, however, for the present study, the boundaries are straight lines. Therefore, they don't need such approximation (see Appendix A). All the boundaries after approximation are extended to incorporate one external grid system which serves as the outermost boundary for the model and this boundary is assigned zero transmissivity/permeability at all times. The aquifer simulation and upcone prediction deck are presented in Appendix C, while the input card deck for a sample program is included in Appendix E. The program deck is in the form of a Subroutine so that it can be called by any calling program. A sample calling program titled as Main Program has also been included in Appendix C. The next step in job setup is to include the appropriate computer installation job control cards and then the program is ready to run. For easy understanding, sample job control language cards are included in Appendix C. The computer output is in the form of printed numerical values of heads for all nodes and in the form of a category map printout at the end of every time increment.

Program Logic and Operational Sequence

The step-by-step operation of the program is explained in the following discussion according to the program Flow Chart and the Algorithm given in Appendices B and C respectively. The program for the model has five main sections. The first section includes the variable descriptions which is important for a user to read before proceeding deeper into the program. The next section prepares the computer for the problem and handle the data input and output of the input data. The third section includes the simulation steps, while following to it the section estimates upconing of Saltwater from the baseline. The last section covers the printout statements for the results output.

Computer Preparation and Data Input

The Arguments into the Subroutine reserves core storage sufficient for models. For the present problem it is limited to 40 columns and 40 rows. However, the core storage utilized is quite less than this, such as 37 columns and 28 rows. These limits can be changed according to available computer storage and user's need. The Subroutine ERRSET (for IBM-S360/370) is called at the beginning of the calling program in order to avoid the termination of computation due to possible underflow errors, especially in the first time interval. This is due to the reason that drawdowns, on occasion, are extremely small at distant places from the pumping locations; and if they are so small to fall beyond the limit of the computer to handle the underflow occurs; however, with an error set, (ERRSET) it automatically sets these small numbers equal to zero and allows the processing to continue. The input-output storage unit numbers are sent as arguments into the subroutine MODEL. These data are used for all READ and WRITE statements in the program.

The Subroutine first reads the check card and then checks if the dimensions defined are less than or equal to the assigned values. Next, it reads the control card to check if the above step is safe for computer usage, and sets up a system to handle the data input to the computer. All the input parameters are read completely and then it is tested if the user(s) desires printout of the input data. If so, then it is printed under the title of INPUT DATA, and otherwise the program proceeds for the simulation section.

Simulation and Upconing Estimation

Simulation starts at TIME equal to zero and is updated with the progress of simulation. The variable TIME is only used for labeling the printouts (results of the model), while the incremental time (DELTAT in the program) interval is used for all calculation purposes. All the calculations, which include either simulation only or both simulation and upconing estimation, are set under one DO LOOP which starts at the first time step and is carried out until the computation is completed for the last time step. However, the DO LOOP includes all the output formats too. In order to be more specific, this section is described below in six steps as in order of their occurrences in the program MODEL.

In the first step of the computational scheme, iteration parameters are estimated and are stored in the variable vector RHOP for further usages in the simulations. The second step estimates the parameters that are involved in the permeability and transmissibility factors, which serve as coefficients in the numerical governing equations; and above all, they are repeatedly used in the process of simulation. These are stored in the variable arrays named FPX and FPY. The next two steps involve calculation of head or drawdown in x and y directions respectively. The procedures for calculating heads/drawdowns in x and y directions (rows and columns respectively) are essentially the same, except that the subscripts and superscripts change in order to match the respective row or column. The first sweep of head/drawdown estimation is carried out in x-direction along the rows and then in the y-direction along the columns. At the end of each cycle of such operation, the absolute value of any changes in head/drawdown that have occurred since the column calculations in the preceding cycle is recorded and the maximum value of such deviation is stored in variable E. The fifth step, an important and controlling step in the simulation process, is the checking of error limit. At the end of each iteration a comparison is made between E and ERROR (the specified error criteria that is read in the check card). If E is greater than ERROR, it is interpreted that the solution has not converged and proceeds for the next iteration. The above iterative procedure is repeated until the convergence criteria is met and the solution results for that time step are ready for printing. If, however, the convergence criteria is not achieved within a specified number of iterations, then an error message is printed stating so and the program is terminated with complete printout of the results up to that time step. The last step is the option of estimating the upconed surface from its base line. It is estimated either by Muskat's approach of physical principles of brine upconing or by Jacob Bear's one-dimensional transport theory or by both the approaches. The results are printed exactly in the same format as that of the head/drawdown printouts.

Printout of Results

The last section in the program handles printing of the category map and the values of the heads/drawdowns for each time step. A category map and all heads are printed for the model after every time step. At the end of this section is the instruction DELTAT = DELTAT * TIMEIM, which indicates that the time increment DELTAT is made to increase in size as the simulation progresses. The reasons for this are related to increased accuracy during the early part of the simulation and increased efficiency as time progresses. After printing the heads/drawdowns and upconed values at the end of the time increment, control is provided to begin the computations for the next time increment.

A sample output of the simulation and upconing results is given at the end of this chapter, in the form of a computer printout. Contours of equal head/drawdowns and equal upconed values are drawn by joining the same values on the category map printout.

PRINTOUT OF ALL THE INPUT PARAMETERS.

```
NX      =  37
NY      =  28
NSTEP   =  15
NITER   =  20
ERROR   =   0.1000E-04
DELTAT=     0.5000E+01
TIMEIM=     0.1200E+01
THETA  =    0.3000E+00
DENSF  =    0.4000E+02
BETA   =    0.0
LNT     =   5


IEQX   = 1   (= 1,MEANS EQUAL GRID-SPACINGS IN X-DIRECTION,OTHERWISE UNEQUAL GRIDS ARE USED)
IEQY   = 1   (= 1,MEANS EQUAL GRID-SPACINGS IN Y-DIRECTION,OTHERWISE UNEQUAL GRIDS ARE USED)
IHOMXY= 0   (= 1,MEANS HOMOGENEOUS IN ALL DIRECTIONS,OTHERWISE HETEROGENEOUS CONDITION EXISTS)
IHOMX = 0   (= 1,MEANS HOMOGENEOUS CONDITION IN X-DIRECTION,OTHERWISE HETEROGENEOUS CONDITION EXISTS)
IHOMY = 0   (= 1,MEANS HOMOGENEOUS CONDITION IN Y-DIRECTION,OTHERWISE HETEROGENEOUS CONDITION EXISTS)
ISOT   = 1   (= 1,MEANS ISOTROPIC CONDITION EXISTS,OTHERWISE ANISOTROPICITY PREVAILS)
ICON   = 1   (= 1,MEANS CONFINED CASE CONSIDERED,OTHERWISE UNCONFINED CONDITION EXISTS)
IEQSS = 1   (= 1,MEANS EQUAL STORATIVITY IN WHOLE FLOW DOMAIN,OTHERWISE NOT EQUAL)
IHDRDN= 1   (= 1,MEANS EQUAL HEAD/DRAWDOWN THROUGHOUT THE FIELD,OTHERWISE EQUAL VALUES EXIST)
IPRINT= 1   (= 1,MEANS PRINTOUT OF ALL THE INPUTPARAMETERS IS DESIRED,OTHERWISE NOT)
IDISCH=  9  (INDICATES TOTAL NO. OF NODES HAVING NON-ZERO NET FLUX)
IUPCON= 1   (=1, IF UPCONING ESTIMATION IS DESIRED,OTHERWISE NOT)
ITRANS=-1   (=+1, IF UPCONING IS TO BE ESTIMATED BY TRANSPORT THEORY ONLY;
             = 0, IF BY MUSKAT APPROACH ONLY;
             =-1, IF BY BOTH APPROACHES)

SPACIAL  INCREMENTS :

IN X-DIRECTION...
  480.  480.  480.  480.  480.  480.  480.  480.  480.  480.  480.  480.  480.
  480.  480.  480.  480.  480.  480.  480.  480.  480.  480.  480.  480.  480.
  480.  480.  480.  480.  480.  480.  480.  480.  480.  480.  480.
IN Y-DIRECTION...
  480.  480.  480.  480.  480.  480.  480.  480.  480.  480.  480.  480.  480.
  480.  480.  480.  480.  480.  480.  480.  480.  480.  480.  480.  480.  480.
  480.  480.
NO. OF PUMPS=   1
NO. OF TIME INCREMENTS PER PUMPAGE CHANGE= 15
```

CATEGORY PRINTOUT LEGEND (HEAD/DRAWDOWN) :

	IF HEAD IS GREATER THAN OR EQUAL TO 0.0
0	IF HEAD IS GREATER THAN OR EQUAL TO 0.0001
A	IF HEAD IS GREATER THAN OR EQUAL TO 0.0005
B	IF HEAD IS GREATER THAN OR EQUAL TO 0.0010
C	IF HEAD IS GREATER THAN OR EQUAL TO 0.0050
D	IF HEAD IS GREATER THAN OR EQUAL TO 0.0100
E	IF HEAD IS GREATER THAN OR EQUAL TO 0.0200
F	IF HEAD IS GREATER THAN OR EQUAL TO 0.0500
G	IF HEAD IS GREATER THAN OR EQUAL TO 0.0700
H	IF HEAD IS GREATER THAN OR EQUAL TO 0.1000
J	IF HEAD IS GREATER THAN OR EQUAL TO 0.2000
K	IF HEAD IS GREATER THAN OR EQUAL TO 0.5000
1	IF HEAD IS GREATER THAN OR EQUAL TO 0.7500
2	IF HEAD IS GREATER THAN OR EQUAL TO 1.0000
3	IF HEAD IS GREATER THAN OR EQUAL TO 1.2000
4	IF HEAD IS GREATER THAN OR EQUAL TO 1.5000
5	IF HEAD IS GREATER THAN OR EQUAL TO 1.8000
6	IF HEAD IS GREATER THAN OR EQUAL TO 2.0000
7	IF HEAD IS GREATER THAN OR EQUAL TO 3.0000
8	IF HEAD IS GREATER THAN OR EQUAL TO 4.0000
9	IF HEAD IS GREATER THAN OR EQUAL TO 5.0000

CATEGORY PRINTOUT LEGEND (UPCONED SURFACE) :

	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 0.0
0	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 0.0001
A	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 0.0005
B	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 0.0010
C	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 0.0050
D	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 0.0100
E	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 0.0200
F	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 0.0500
G	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 0.0700
H	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 0.1000
J	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 0.2000
K	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 0.5000
1	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 0.7500
2	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 1.0000
3	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 1.2000
4	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 1.5000
5	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 1.8000
6	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 2.0000
7	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 3.0000
8	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 4.0000
9	IF UPCONE IS GREATER THAN OR EQUAL TO DENSF* 5.0000

TIME STEP= 1 DELTAT= 6.0000 DAYS TIME= 5.0000 DAYS

ERROR IN THE SOLUTION = 0.14901E-07 NO. OF ITERATIONS COVERED = 2

CATEGORY MAP PRINTOUT :

1																																			
2	0	0	0	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	C	0	0	0	0	0	0	0	0	0	0
3	0	0	F	0	0	0	0	F	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	C	0	C	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	C	0	0	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	C	0	C	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	C	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	E	0	0	0	0	0	0	0	F	0	0	0	F	0	0	C	0	C	0	F	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	C	0	0	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	C	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	C	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	F	0	0	0	0
19	0	0	0	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	C	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	C	C	C	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	C	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	F	0	0
25	0	0	0	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	C	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	C	0	0	0	0	0	0	0	0	0	
28																																			

CATEGORY MAP PRINTOUT FOR UPCONED SUEFACE :

```
 1
 2   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 3   0 0 F 0 0 0 0 F 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 4   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 5   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 6   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 7   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 8   0 0 0 0 0 0 0 E 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 9   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
10   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
11   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
12   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
13   0 0 0 0 0 0 0 E 0 0 0 0 0 0 0 F 0 0 0 F 0 0 0 0 0 0 F 0 0 0 0 0 0 0 0
14   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
15   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
16   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
17   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
18   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 F 0 0 0 0
19   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
20   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
21   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
22   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
23   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
24   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 F 0 0
25   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
26   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
27   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
28
```

```
TIME STEP=    12            DELTAT=    44.5803  DAYS            TIME=  197.9021   DAYS
ERROR IN THE SOLUTION =    0.64373E-05            NO. OF ITERATIONS COVERED =   2

CATEGORY MAP PRINTOUT :

  1
  2     0 A E A 0 0 A E A 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  3     0 E 2 E 0 0 E 2 E 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  4     0 A E A 0 0 A E A 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  5     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  6     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  7     0 0 0 0 0 0 A D A 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  8     0 0 0 0 0 0 D 1 D 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  9     0 0 0 0 0 0 A D A 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 10     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 11     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 A 0 0 0 0 0 0 A 0 0 0 0 0 0 0 0
 12     0 0 0 0 0 0 A D A 0 0 0 0 0 A E A 0 A E A 0 0 0 0 A E A 0 0 0 0 0 0 0
 13     0 0 0 0 0 0 D 1 D 0 0 0 0 0 D 2 D A E 2 E A 0 0 A E 3 E A 0 0 0 0 0 0
 14     0 0 0 0 0 0 0 D 0 0 0 0 0 0 A D A 0 A E A 0 0 0 0 A E A 0 0 0 0 0 0 0
 15     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 A 0 0 0 0 0 0 A 0 0 0 0 0 0 0 0
 16     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 A 0 0 0 0
 17     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 A E A 0 0 0
 18     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 A E 2 E A 0 0
 19     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 A E A 0 0 0
 20     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 A 0 0 0 0
 21     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 22     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 A 0 0
 23     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 A E A 0
 24     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 E 2 E A
 25     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 A E A 0
 26     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 27     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 28
```

CATEGORY MAP PRINTOUT FOR UPCONED SUEFACE :

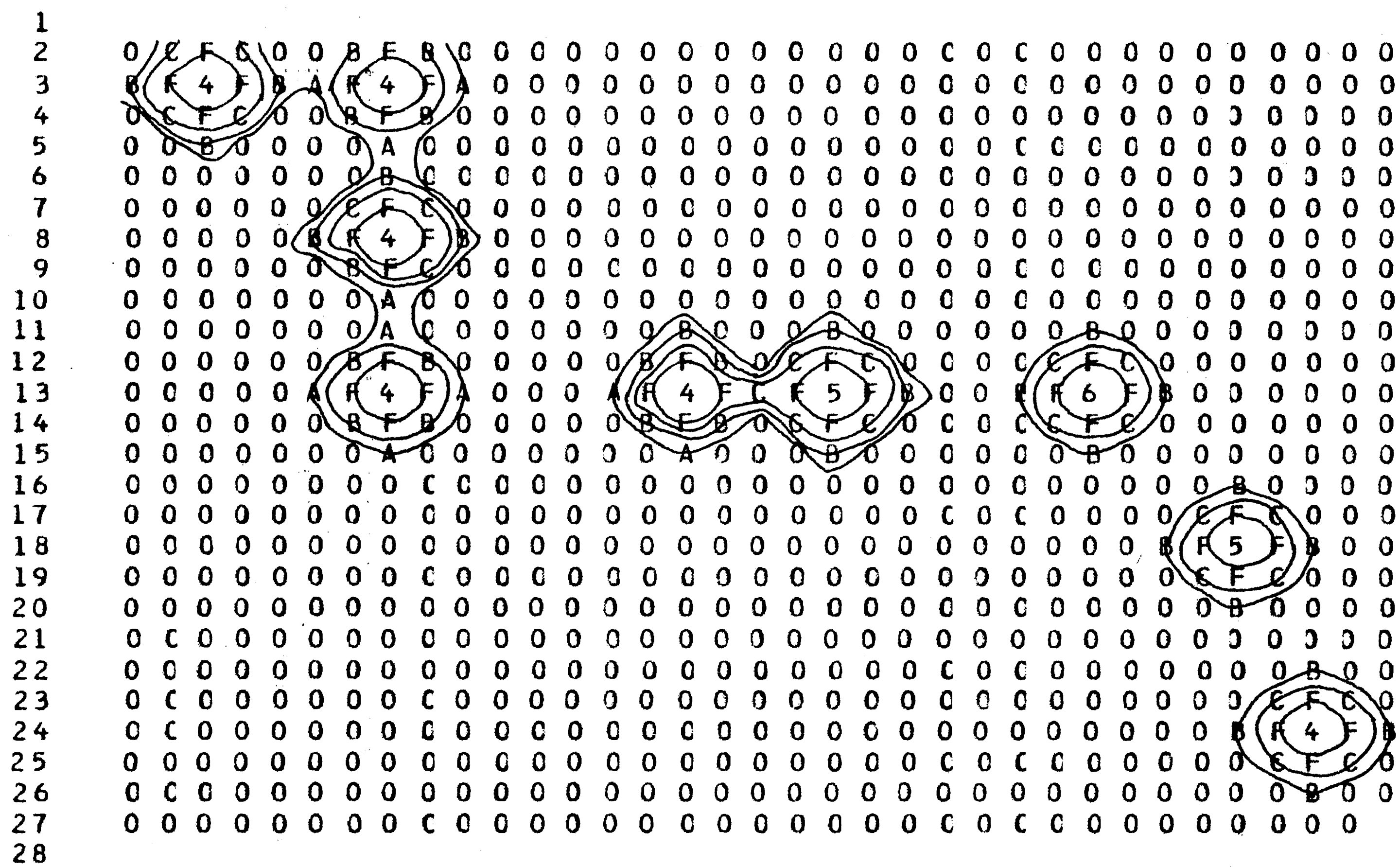
1																																			
2	0	A	E	A	0	0	A	E	A	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	C	0	0	0	0	0	0	0	0	0	0
3	0	E	2	E	0	0	E	2	E	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0	0	0
4	0	A	E	A	0	0	A	E	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	C	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	A	D	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	D	1	D	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	C	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	A	D	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	A	0	0	C	0	C	0	A	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	A	D	A	0	0	0	0	0	A	E	A	0	A	E	A	0	0	0	0	A	E	A	0	0	0	0	0	0	0
13	0	0	0	0	0	0	D	1	D	0	0	0	0	0	D	2	D	A	E	2	E	A	0	0	A	E	3	E	A	0	0	0	0	0	0
14	0	0	0	0	0	0	0	D	0	0	0	0	0	0	A	D	A	0	A	E	A	0	0	0	0	A	E	A	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0	A	0	0	0	0	0	0	A	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	C	0	0	0	0	0	A	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	A	E	A	0	0	0
18	0	0	0	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	0	0	A	E	2	E	A	0	0
19	0	C	C	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	C	0	0	0	0	A	E	A	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	A	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	C	C	C	C	0	0	0	0	0	0	0
22	0	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	A	0	0
23	0	0	0	0	0	0	0	0	C	0	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	A	E	A	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	0	0	0	0	0	E	2	E	A
25	0	0	0	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	A	E	A	0
26	0	C	0	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	0	C	0	0	0	0	0	0	0	0	0	
28																																			

```
TIME STEP=    15            DELTAT=    77.0348   DAYS             TIME=  360.1743   DAYS
ERROR IN THE SOLUTION =    0.11176E-07          NO. OF ITERATIONS COVERED =    3

CATEGORY MAP PRINTOUT :

  1
  2     0 C F C 0 0 B F B C 0 0 0 0 0 0 0 0 0 0 0 0 C 0 0 0 C 0 0 0 0 0 0 0 0
  3     B F 4 F B A F 4 F A 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C 0 0 0 0 0 0 0 0 0 0
  4     0 C F C 0 0 B F B 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
  5     0 0 B 0 0 0 0 A C 0 0 0 0 0 0 0 0 0 0 0 0 0 C 0 C C C 0 0 0 0 0 0 0 0
  6     0 0 0 0 0 0 0 B 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C 0 0 0 0 0 0 0 0 0 0
  7     0 0 0 0 0 0 C F C 0 C 0 0 0 0 0 0 0 0 0 0 0 C 0 C 0 0 0 0 0 0 0 0 0 0
  8     0 0 0 0 0 B F 4 F B 0 0 0 0 0 0 0 0 0 0 0 0 C 0 0 0 0 0 0 0 0 0 0 0 0
  9     0 0 0 0 0 0 B F C 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 10     0 0 0 0 0 0 0 A C 0 0 0 0 0 0 C 0 0 0 0 0 0 0 0 0 0 C 0 0 0 0 0 0 0 0
 11     0 0 0 0 0 0 0 A 0 0 0 0 0 0 0 B 0 0 0 B 0 0 0 0 0 0 B 0 0 0 0 0 0 0 0
 12     0 0 0 0 0 0 B F B 0 0 0 0 0 B F B 0 C F C 0 0 0 0 C F C 0 0 0 0 0 0 0
 13     0 0 0 0 0 A F 4 F A 0 0 0 A F 4 F C F 5 F B 0 0 B F 6 F B 0 0 0 0 0 0
 14     0 0 0 0 0 0 B F B 0 0 0 0 0 B F B B C F C 0 0 0 0 C F C 0 0 0 0 0 0 0
 15     0 0 0 0 0 0 0 A C 0 0 0 0 0 0 A 0 0 0 B 0 0 0 0 0 0 B 0 0 0 0 0 0 0 0
 16     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C 0 C 0 0 0 0 0 B 0 0 0 0
 17     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C F C 0 0 0
 18     0 0 0 0 0 0 0 0 C 0 0 C 0 C C 0 0 0 0 0 0 0 C 0 C 0 0 0 B F 5 F B 0 0
 19     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C 0 0 0 0 0 0 C F C 0 0 0
 20     0 0 0 0 0 0 0 0 C 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 B 0 0 0 0
 21     0 0 0 0 0 0 0 0 C 0 0 0 0 0 0 0 0 0 0 0 0 0 C 0 C C C 0 0 0 0 0 0 0 0
 22     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 B 0 0
 23     0 0 0 0 0 0 0 0 C C 0 0 0 0 0 0 0 0 0 0 0 0 C 0 0 0 0 0 0 0 0 C F C 0
 24     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C 0 0 0 0 0 B F 4 F B
 25     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C F C 0
 26     0 0 0 0 0 0 0 0 0 0 0 C 0 0 0 0 0 0 0 0 0 0 0 0 C 0 0 0 0 0 0 0 B 0 0
 27     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C 0 0 0 0 0 0 0 0 0 0 0
 28
```

CATEGORY MAP PRINTOUT FOR UPCONED SUEFACE :



HEAD VALUES AT THE END OF THIS TIME STEP :

	1	2	3	4	5	6	7	8	9	10
1		0.C	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	-0.1995E-04	-C.1062E-02	-0.3304E-01	-C.1026E-02	-0.1847E-04	-0.1631E-04	-0.9562E-03	-0.3196E-01	-0.9563E-03
3	0.0	-0.5373E-03	-0.3307E-01	-0.1429E+01	-0.3254E-01	-C.5101E-03	-0.4844E-03	-0.3148E-01	-0.1435E+01	-0.3149E-01
4	0.0	-0.2012E-04	-0.1061E-02	-0.3307E-01	-0.1026E-02	-0.1833E-04	-0.1720E-04	-0.9564E-03	-0.3147E-01	-0.9401E-03
5	0.0	-0.4201E-06	-(.20C7E-04	-0.5395E-03	-0.190EE-04	-0.3741E-06	-0.3383E-06	-0.1721E-04	-0.4844E-03	-0.1650E-04
6	0.0	-0.6243E-08	-0.2793E-06	-0.6849E-05	-0.2655E-06	-0.1136E-07	-0.3923E-06	-0.1922E-04	-0.5102E-03	-0.1846E-04
7	0.0	-0.7144E-10	-0.3035E-08	-0.6994E-07	-0.5726E-08	-0.2636E-06	-0.1931E-04	-0.1025E-02	-0.3083E-01	-0.1008E-02
8	0.0	-0.6636E-12	-0.2971E-10	-0.1075E-C8	-0.6111E-C7	-0.6194E-05	-0.5043E-03	-0.3083E-01	-0.1264E+01	-0.3083E-01
9	0.0	-0.5516E-14	-0.3233E-12	-0.2360E-10	-C.2344E-08	-0.2316E-06	-0.1806E-04	-0.9922E-03	-0.3034E-01	-0.1008E-02
10	0.0	-0.4512E-16	-C.3821E-14	-0.3964E-12	-0.4325E-10	-0.4202E-08	-0.3214E-06	-0.1692E-04	-0.4747E-03	-0.1764E-04
11	0.0	-0.4413E-17	-0.7169E-15	-0.1C79E-12	-0.1441E-10	-0.1642E-08	-0.1485E-06	-0.9339E-05	-0.3159E-03	-0.9605E-05
12	0.0	-0.1893E-15	-0.3322E-13	-0.5293E-11	-0.7418E-09	-0.8816E-07	-0.8406E-05	-0.5842E-03	-0.2359E-01	-0.5963E-03
13	0.0	-0.4742E-14	-C.8762E-12	-0.1471E-C9	-0.2181E-C7	-0.2762E-05	-0.2858E-03	-0.2310E-01	-0.1296E+01	-0.2310E-01
14	0.0	-0.1830E-15	-0.3197E-13	-0.5061E-11	-0.703CE-09	-0.8242E-07	-0.7862E-05	-0.5593E-03	-0.2312E-01	-0.5591E-03
15	0.0	-0.3688E-17	-C.6080E-15	-0.9055E-13	-0.1176E-10	-0.1302E-08	-0.1177E-05	-0.7747E-05	-0.2798E-03	-0.7746E-05
16	0.0	-0.5150E-19	-0.7986E-17	-0.1113E-14	-0.136EE-12	-0.1457E-10	-0.1260E-08	-0.7790E-07	-0.2584E-05	-0.7789E-07
17	0.0	-C.5579E-21	-0.8083E-19	-0.1066F-16	-0.1265E-14	-0.1302E-12	-0.1074E-10	-0.6275E-09	-0.1949E-07	-0.6275E-09
18	0.0	-0.4960E-23	-C.6788E-21	-0.8655E-19	-0.996EE-17	-0.9830E-15	-0.7724E-13	-0.4283E-11	-0.1256E-09	-0.4282E-11
19	0.0	-0.3802E-25	-0.5038E-23	-0.6247E-21	-0.6920E-19	-0.6521E-17	-0.4879E-15	-0.2570E-13	-0.7143E-12	-0.2570E-13
2C	0.0	-0.2647E-27	-0.3421E-25	-0.4092E-23	-0.4341E-21	-0.3903E-19	-0.2780E-17	-0.1392E-15	-0.3674E-14	-0.1392E-15
21	0.0	-0.1722E-29	-C.2152E-27	-0.2473E-25	-0.2509E-23	-0.2150E-21	-0.1458E-19	-0.6943E-18	-0.1742E-16	-0.6942E-18
22	0.0	-C.1054E-31	-0.1269E-29	-0.1398E-27	-0.1355E-25	-0.1107E-23	-0.7147E-22	-0.3238E-20	-0.7722E-19	-0.3237E-20
23	0.0	-0.6122E-34	-0.7083E-32	-0.7470E-30	-0.6916E-28	-0.5390E-26	-0.3314E-24	-0.1429E-22	-0.3241E-21	-0.1461E-22
24	0.0	-0.3394E-36	-0.3770E-34	-0.38C7E-32	-0.336EE-30	-0.2504E-28	-0.1467E-26	-0.6024E-25	-0.1300E-23	-0.6424E-25
25	0.0	-0.1807E-38	-0.1926E-36	-0.1863E-34	-0.1576E-32	-0.1119E-30	-0.6252E-29	-0.2445E-27	-0.5025E-26	-0.2713E-27
26	0.0	-0.9285E-41	-C.9503E-39	-0.8810E-37	-0.7130E-35	-0.4836E-33	-0.2579E-31	-0.9618E-30	-0.1883E-28	-0.1108E-29
27	0.0	-0.4647E-43	-0.4568E-41	-0.4061E-39	-0.3147E-37	-0.2041E-35	-0.1039E-33	-0.3698E-32	-0.6903E-31	-0.4415E-32
28	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

HEAD VALUES (CONTINUED) :

	11	12	13	14	15	16	17	18	19	20
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	-0.1638E-04	-0.2038E-06	-0.2031E-08	-0.1715E-10	-0.1274E-12	-0.8535E-15	-0.5267E-17	-0.3039E-19	-0.1660E-21	-0.2193E-23
3	-0.4703E-03	-0.5365E-05	-0.5000E-07	-0.3982E-09	-0.2799E-11	-0.1780E-13	-0.1043E-15	-0.5715E-18	-0.2977E-20	-0.2691E-21
4	-0.1608E-04	-0.1998E-06	-0.1989E-08	-0.1678E-10	-0.1244E-12	-0.8330E-15	-0.5882E-17	-0.6982E-19	-0.2622E-20	-0.4686E-19
5	-0.3062E-06	-0.4061E-08	-0.4283E-10	-0.3814E-12	-0.2982E-14	-0.2779E-16	-0.1366E-15	-0.7010E-17	-0.4093E-18	-0.8171E-17
6	-0.3639E-06	-0.5127E-08	-0.5572E-10	-0.5048E-12	-0.4055E-14	-0.1126E-14	-0.2329E-13	-0.1142E-14	-0.6375E-16	-0.1334E-14
7	-0.1856E-04	-0.2483E-06	-0.2615E-08	-0.2291E-10	-0.1806E-12	-0.1652E-12	-0.3662E-11	-0.1717E-12	-0.9159E-14	-0.2007E-12
8	-0.5042E-03	-0.6190E-05	-0.6212E-07	-0.5234E-09	-0.4330E-11	-0.2175E-10	-0.5110E-09	-0.2290E-10	-0.1191E-11	-0.2734E-10
9	-0.1910E-04	-0.2614E-06	-0.2812E-08	-0.2559E-10	-0.5463E-10	-0.2467E-08	-0.6178E-07	-0.2626E-08	-0.1341E-09	-0.3225E-08
10	-0.3602E-06	-0.5323E-08	-0.6343E-10	-0.6682E-10	-0.4697E-08	-0.2325E-06	-0.6272E-05	-0.2513E-06	-0.1229E-07	-0.3105E-06
11	-0.1624E-06	-0.2040E-08	-0.5576E-10	-0.3908E-08	-0.3071E-06	-0.1694E-04	-0.5028E-03	-0.1848E-04	-0.8580E-06	-0.2347E-04
12	-0.8832E-05	-0.9699E-07	-0.2379E-08	-0.1670E-06	-0.1455E-04	-0.9050E-03	-0.3143E-01	-0.9719E-03	-0.4118E-04	-0.1260E-02
13	-0.2856E-03	-0.2760E-05	-0.5741E-07	-0.4256E-05	-0.4121E-03	-0.2982E-01	-0.1439E+01	-0.3093E-01	-0.1069E-02	-0.3795E-01
14	-0.8038E-05	-0.8522E-07	-0.2027E-08	-0.1466E-06	-0.1321E-04	-0.8406E-03	-0.3042E-01	-0.8902E-03	-0.3633E-04	-0.1199E-02
15	-0.1204E-06	-0.1368E-08	-0.3624E-10	-0.2586E-08	-0.2196E-06	-0.1311E-04	-0.4290E-03	-0.1410E-04	-0.6384E-06	-0.2029E-04
16	-0.1278E-08	-0.1558E-10	-0.4450E-12	-0.3077E-10	-0.2518E-08	-0.1450E-06	-0.4433E-05	-0.1556E-06	-0.7734E-08	-0.2415E-06
17	-0.1084E-10	-0.1377E-12	-0.4339E-14	-0.2943E-12	-0.2332E-10	-0.1281E-08	-0.3662E-07	-0.1385E-08	-0.7454E-10	-0.2307E-08
18	-0.7775E-13	-0.1021E-14	-0.3594E-16	-0.2402E-14	-0.1853E-12	-0.9588E-11	-0.2584E-09	-0.1064E-10	-0.6124E-12	-0.1869E-10
19	-0.4902E-15	-0.6853E-17	-0.2629E-18	-0.1701E-16	-0.1261E-14	-0.6306E-13	-0.1610E-11	-0.7024E-13	-0.4395E-14	-0.1305E-12
20	-0.2853E-17	-0.4265E-19	-0.1737E-20	-0.1080E-18	-0.7557E-17	-0.3661E-15	-0.9067E-14	-0.4116E-15	-0.2778E-16	-0.8069E-15
21	-0.1552E-19	-0.2443E-21	-0.1057E-22	-0.6306E-21	-0.4205E-19	-0.1966E-17	-0.4706E-16	-0.2226E-17	-0.1629E-18	-0.4613E-17
22	-0.7844E-22	-0.1306E-23	-0.6008E-25	-0.3435E-23	-0.2180E-21	-0.9817E-20	-0.2234E-18	-0.1123E-19	-0.8856E-21	-0.2413E-19
23	-0.3764E-24	-0.6600E-26	-0.3225E-27	-0.1766E-25	-0.1066E-23	-0.4538E-22	-0.9825E-21	-0.5355E-22	-0.4467E-23	-0.1168E-21
24	-0.1729E-26	-0.3185E-28	-0.1651E-29	-0.8656E-28	-0.4969E-26	-0.2004E-24	-0.4130E-23	-0.2383E-24	-0.2149E-25	-0.5404E-24
25	-0.7647E-29	-0.1479E-30	-0.8115E-32	-0.4075E-30	-0.2227E-28	-0.8520E-27	-0.1672E-25	-0.1001E-26	-0.9944E-28	-0.2405E-26
26	-0.3277E-31	-0.6644E-33	-0.3654E-34	-0.1854E-32	-0.9651E-31	-0.3510E-29	-0.6563E-28	-0.4081E-29	-0.4356E-30	-0.1037E-28
27	-0.1373E-33	-0.2916E-35	-0.1785E-36	-0.3231E-35	-0.4085E-33	-0.1414E-31	-0.2521E-30	-0.1629E-31	-0.1834E-32	-0.4377E-31
28	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

HEAD VALLES (CONTINLED) :

	21	22	23	24	25	26	27	28	29	30
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	-0.2227E-22	-0.1286E-23	-0.3912E-25	-0.8478E-27	-0.8139E-27	-0.3782E-25	-0.1246E-23	-0.2157E-22	-0.1246E-23	-0.3782E-25
3	-0.4470E-20	-0.2458E-21	-0.7128E-23	-0.1473E-24	-0.1416E-24	-0.6889E-23	-0.2382E-21	-0.4330E-20	-0.2382E-21	-0.6889E-23
4	-0.8615E-18	-0.4511E-19	-0.1247E-20	-0.2457E-22	-0.2366E-22	-0.1205E-20	-0.4373E-19	-0.8348E-18	-0.4372E-19	-0.1205E-20
5	-0.1572E-15	-0.7835E-17	-0.2065E-18	-0.3877E-20	-0.3740E-20	-0.1994E-18	-0.7597E-17	-0.1523E-15	-0.7597E-17	-0.1994E-18
6	-0.2681E-13	-0.1273E-14	-0.3197E-16	-0.5725E-18	-0.5530E-18	-0.3087E-16	-0.1234E-14	-0.2600E-13	-0.1234E-14	-0.3085E-16
7	-0.4214E-11	-0.1903E-12	-0.4561E-14	-0.7793E-16	-0.7541E-16	-0.4400E-14	-0.1847E-12	-0.4089E-11	-0.1847E-12	-0.4397E-14
8	-0.5989E-09	-0.2571E-10	-0.5880E-12	-0.9594E-14	-0.9308E-14	-0.5666E-12	-0.2496E-10	-0.5814E-09	-0.2496E-10	-0.5658E-12
9	-0.7372E-07	-0.3056E-08	-0.6672E-10	-0.1041E-11	-0.1015E-11	-0.6417E-10	-0.2969E-08	-0.7296E-07	-0.2969E-08	-0.6404E-10
10	-0.7610E-05	-0.3025E-06	-0.6310E-08	-0.9436E-10	-0.9293E-10	-0.6157E-08	-0.2997E-06	-0.7824E-05	-0.2998E-06	-0.6138E-08
11	-0.6306E-03	-0.2321E-04	-0.4601E-06	-0.6539E-08	-0.6617E-08	-0.4645E-06	-0.2387E-04	-0.6727E-03	-0.2388E-04	-0.4620E-06
12	-0.3855E-01	-0.1260E-02	-0.2347E-04	-0.3192E-06	-0.3249E-06	-0.2414E-04	-0.1344E-02	-0.4339E-01	-0.1344E-02	-0.2387E-04
13	-0.1581E+01	-0.3854E-01	-0.6408E-03	-0.8088E-05	-0.8458E-05	-0.6831E-03	-0.4334E-01	-0.1906E+01	-0.4338E-01	-0.6723E-03
14	-0.3795E-01	-0.1260E-02	-0.2452E-04	-0.3503E-06	-0.3882E-06	-0.2775E-04	-0.1457E-02	-0.4470E-01	-0.1412E-02	-0.2540E-04
15	-0.5891E-03	-0.2231E-04	-0.4741E-06	-0.7365E-08	-0.9985E-08	-0.6731E-06	-0.3158E-04	-0.7961E-03	-0.3066E-04	-0.6574E-06
16	-0.6605E-05	-0.2725E-06	-0.6204E-08	-0.1038E-09	-0.1687E-09	-0.1112E-07	-0.5011E-05	-0.1162E-04	-0.5280E-06	-0.1052E-05
17	-0.5930E-07	-0.2635E-08	-0.6416E-10	-0.1154E-11	-0.2233E-11	-0.1431E-09	-0.6345E-08	-0.1457E-06	-0.7112E-06	-0.4333E-04
18	-0.4548E-09	-0.2126E-10	-0.5483E-12	-0.1107E-13	-0.1088E-12	-0.1365E-10	-0.1587E-08	-0.1642E-06	-0.1420E-04	-0.9535E-03
19	-0.3078E-11	-0.1489E-12	-0.3992E-14	-0.1077E-15	-0.4395E-14	-0.6018E-12	-0.7277E-10	-0.7534E-08	-0.6416E-06	-0.4177E-04
20	-0.1846E-13	-0.9415E-15	-0.2601E-16	-0.1242E-17	-0.1063E-15	-0.1463E-13	-0.1758E-11	-0.1792E-09	-0.1491E-07	-0.9531E-06
21	-0.1020E-15	-0.5502E-17	-0.1573E-18	-0.1635E-19	-0.1900E-17	-0.2547E-15	-0.2955E-13	-0.2920E-11	-0.2394E-09	-0.1506E-07
22	-0.5271E-18	-0.2943E-19	-0.8710E-21	-0.2018E-21	-0.2619E-19	-0.3445E-17	-0.3973E-15	-0.3990E-13	-0.3406E-11	-0.2401E-09
23	-0.2475E-20	-0.1459E-21	-0.5403E-23	-0.1913E-21	-0.3723E-19	-0.6704E-17	-0.1130E-14	-0.1720E-12	-0.2321E-10	-0.2726E-08
24	-0.1087E-22	-0.7637E-24	-0.1648E-22	-0.3592E-20	-0.7340E-18	-0.1391E-15	-0.2420E-13	-0.3733E-11	-0.5208E-09	-0.6412E-07
25	-0.4603E-25	-0.6889E-26	-0.8360E-24	-0.1729E-21	-0.3415E-19	-0.6248E-17	-0.1046E-14	-0.1608E-12	-0.2197E-10	-0.2601E-08
26	-0.1893E-27	-0.1181E-27	-0.2245E-25	-0.4524E-23	-0.8521E-21	-0.1518E-18	-0.2479E-16	-0.3655E-14	-0.4874E-12	-0.5615E-10
27	-0.7677E-30	-0.2129E-29	-0.4233E-27	-0.8303E-25	-0.1522E-22	-0.2584E-20	-0.4099E-18	-0.5865E-16	-0.7417E-14	-0.8217E-12
28	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

HEAD VALUES (CONTINUED) :

	31	32	33	34	35	36	37
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	-0.8029E-27	-0.1339E-28	-0.1872E-30	-0.2280E-32	-0.2490E-34	-0.2509E-36	0.0
3	-0.1394E-24	-0.2217E-26	-0.2956E-28	-0.3440E-30	-0.3592E-32	-0.3463E-34	0.0
4	-0.2322E-22	-0.3521E-24	-0.4478E-26	-0.4974E-28	-0.4962E-30	-0.4575E-32	0.0
5	-0.3660E-20	-0.5285E-22	-0.6405E-24	-0.6786E-26	-0.6464E-28	-0.5695E-30	0.0
6	-0.5390E-18	-0.7410E-20	-0.8553E-22	-0.8639E-24	-0.7853E-26	-0.6609E-28	0.0
7	-0.7309E-16	-0.9560E-18	-0.1051E-19	-0.1011E-21	-0.8769E-24	-0.7051E-26	0.0
8	-0.8944E-14	-0.1113E-15	-0.1164E-17	-0.1067E-19	-0.8836E-22	-0.6794E-24	0.0
9	-0.9609E-12	-0.1136E-13	-0.1132E-15	-0.9900E-18	-0.7837E-20	-0.5782E-22	0.0
10	-0.8710E-10	-0.9784E-12	-0.9291E-14	-0.7777E-16	-0.5916E-18	-0.4227E-20	0.0
11	-0.6249E-08	-0.6679E-10	-0.6077E-12	-0.4907E-14	-0.3641E-16	-0.2585E-18	0.0
12	-0.3079E-06	-0.3190E-08	-0.2830E-10	-0.2265E-12	-0.1722E-14	-0.1317E-16	0.0
13	-0.7960E-05	-0.7814E-07	-0.6942E-09	-0.6178E-11	-0.5931E-13	-0.6165E-15	0.0
14	-0.3400E-06	-0.1392E-06	-0.6745E-08	-0.1741E-09	-0.3170E-11	-0.4627E-13	0.0
15	-0.6582E-06	-0.1288E-04	-0.6207E-06	-0.1570E-07	-0.2812E-09	-0.4040E-11	0.0
16	-0.4158E-04	-0.9029E-03	-0.4061E-04	-0.9816E-06	-0.1682E-07	-0.2321E-09	0.0
17	-0.1879E-02	-0.4637E-01	-0.1878E-02	-0.4310E-04	-0.7138E-06	-0.9553E-08	0.0
18	-0.4697E-01	-0.1542E+01	-0.4754E-01	-0.9909E-03	-0.1538E-04	-0.1944E-06	0.0
19	-0.1879E-02	-0.4697E-01	-0.1953E-02	-0.4626E-04	-0.7996E-06	-0.1126E-07	0.0
20	-0.4137E-04	-0.9400E-03	-0.4507E-04	-0.1287E-05	-0.2760E-07	-0.4882E-09	0.0
21	-0.6474E-06	-0.1401E-04	-0.1112E-05	-0.8877E-05	-0.4172E-06	-0.1097E-07	0.0
22	-0.1341E-07	-0.6003E-06	-0.2225E-04	-0.6013E-03	-0.2478E-04	-0.5931E-06	0.0
23	-0.2653E-06	-0.2041E-04	-0.1114E-02	-0.3464E-01	-0.1188E-02	-0.2486E-04	0.0
24	-0.6621E-05	-0.5487E-03	-0.3413E-01	-0.1422E+01	-0.3520E-01	-0.6220E-03	0.0
25	-0.2562E-06	-0.1985E-04	-0.1097E-02	-0.3414E-01	-0.1133E-02	-0.2251E-04	0.0
26	-0.5280E-08	-0.3820E-06	-0.1962E-04	-0.5393E-03	-0.2041E-04	-0.4376E-06	0.0
27	-0.7487E-10	-0.5202E-08	-0.2544E-06	-0.6453E-05	-0.2656E-06	0.0	0.0
28	0.0	0.0	0.0	0.0	0.0	0.0	0.0

UPCONED VALUES AT THE END OF THIS TIME STEP :

	1	2	3	4	5	6	7	8	9	10
1		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	-0.7980E-03	-0.4248E-01	-0.1321E+01	-0.4105E-01	-0.7387E-03	-0.6723E-03	-0.3825E-01	-0.1278E+01	-0.3825E-01
3	0.0	-0.2149E-01	-0.1323E+01	-0.5718E+02	-0.1302E+01	-0.2040E-01	-0.1938E-01	-0.1259E+01	-0.5741E+02	-0.1260E+01
4	0.0	-0.8046E-03	-0.4243E-01	-0.1323E+01	-0.4103E-01	-0.7331E-03	-0.6881E-03	-0.3826E-01	-0.1259E+01	-0.3760E-01
5	0.0	-0.1680E-04	-0.8028E-03	-0.2158E-01	-0.7631E-03	-0.1496E-04	-0.1353E-04	-0.6885E-03	-0.1938E-01	-0.6602E-03
6	0.0	-0.2497E-06	-0.1117E-04	-0.2740E-03	-0.1062E-04	-0.4543E-06	-0.1569E-04	-0.7687E-03	-0.2041E-01	-0.7384E-03
7	0.0	-0.2857E-08	-0.1214E-06	-0.2798E-05	-0.2290E-06	-0.1055E-04	-0.7723E-03	-0.4099E-01	-0.1233E+01	-0.4032E-01
8	0.0	-0.2655E-10	-0.1188E-08	-0.4298E-07	-0.2444E-05	-0.2478E-03	-0.2017E-01	-0.1233E+01	-0.5056E+02	-0.1233E+01
9	0.0	-0.2206E-12	-0.1293E-10	-0.9439E-09	-0.9375E-07	-0.9266E-05	-0.7224E-03	-0.3969E-01	-0.1214E+01	-0.4033E-01
10	0.0	-0.1805E-14	-0.1529E-12	-0.1586E-10	-0.1730E-08	-0.1681E-06	-0.1236E-04	-0.6769E-03	-0.1899E-01	-0.7058E-03
11	0.0	-0.1765E-15	-0.2867E-13	-0.4316E-11	-0.5762E-09	-0.6567E-07	-0.5941E-05	-0.3736E-03	-0.1264E-01	-0.3842E-03
12	0.0	-0.7572E-14	-0.1329E-11	-0.2117E-09	-0.2967E-07	-0.3526E-05	-0.3362E-03	-0.2337E-01	-0.9436E+00	-0.2385E-01
13	0.0	-0.1897E-12	-0.3505E-10	-0.5885E-08	-0.8722E-06	-0.1105E-03	-0.1143E-01	-0.9242E+00	-0.5186E+02	-0.9240E+00
14	0.0	-0.7321E-14	-0.1279E-11	-0.2025E-09	-0.2812E-07	-0.3297E-05	-0.3145E-03	-0.2237E-01	-0.9247E+00	-0.2237E-01
15	0.0	-0.1475E-15	-0.2432E-13	-0.3622E-11	-0.4703E-09	-0.5207E-07	-0.4709E-05	-0.3099E-03	-0.1119E-01	-0.3098E-03
16	0.0	-0.2060E-17	-0.3194E-15	-0.4450E-13	-0.5472E-11	-0.5829E-09	-0.5042E-07	-0.3116E-05	-0.1033E-03	-0.3116E-05
17	0.0	-0.2232E-19	-0.3233E-17	-0.4262E-15	-0.5061E-13	-0.5207E-11	-0.4295E-09	-0.2510E-07	-0.7795E-06	-0.2510E-07
18	0.0	-0.1984E-21	-0.2715E-19	-0.3462E-17	-0.3987E-15	-0.3932E-13	-0.3099E-11	-0.1713E-09	-0.5023E-08	-0.1713E-09
19	0.0	-0.1521E-23	-0.2015E-21	-0.2499E-19	-0.2768E-17	-0.2608E-15	-0.1952E-13	-0.1028E-11	-0.2857E-10	-0.1028E-11
20	0.0	-0.1059E-25	-0.1366E-23	-0.1637E-21	-0.1737E-19	-0.1561E-17	-0.1112E-15	-0.5568E-14	-0.1470E-12	-0.5568E-14
21	0.0	-0.6887E-28	-0.8609E-26	-0.9892E-24	-0.1003E-21	-0.8602E-20	-0.5832E-18	-0.2777E-16	-0.6966E-15	-0.2777E-16
22	0.0	-0.4217E-30	-0.5077E-28	-0.5591E-26	-0.5419E-24	-0.4429E-22	-0.2859E-20	-0.1295E-18	-0.3089E-17	-0.1295E-18
23	0.0	-0.2449E-32	-0.2833E-30	-0.2988E-28	-0.2766E-26	-0.2156E-24	-0.1326E-22	-0.5715E-21	-0.1296E-19	-0.5843E-21
24	0.0	-0.1357E-34	-0.1508E-32	-0.1523E-30	-0.1347E-28	-0.1002E-26	-0.5869E-25	-0.2409E-23	-0.5201E-22	-0.2570E-23
25	0.0	-0.7227E-37	-0.7705E-35	-0.7454E-33	-0.6304E-31	-0.4475E-29	-0.2501E-27	-0.9781E-26	-0.2010E-24	-0.1085E-25
26	0.0	-0.3714E-39	-0.3801E-37	-0.3524E-35	-0.2852E-33	-0.1934E-31	-0.1032E-29	-0.3847E-28	-0.7532E-27	-0.4430E-28
27	0.0	-0.1859E-41	-0.1827E-39	-0.1624E-37	-0.1259E-35	-0.8163E-34	-0.4158E-32	-0.1479E-30	-0.2761E-29	-0.1766E-30
28	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

UPCONED VALUES (CONTINUED):

	11	12	13	14	15	16	17	18	19	20
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	-0.6554E-03	-0.8153E-05	-0.8125E-07	-0.6861E-09	-0.5094E-11	-0.3414E-13	-0.2107E-15	-0.1216E-17	-0.6639E-20	-0.8772E-22
3	-0.1881E-01	-0.2146E-03	-0.2000E-05	-0.1593E-07	-0.1120E-09	-0.7119E-12	-0.4171E-14	-0.2286E-16	-0.1191E-18	-0.1077E-19
4	-0.6433E-03	-0.7993E-05	-0.7957E-07	-0.6712E-09	-0.4977E-11	-0.3332E-13	-0.2353E-15	-0.2793E-17	-0.1049E-18	-0.1874E-17
5	-0.1225E-04	-0.1624E-06	-0.1713E-08	-0.1525E-10	-0.1193E-12	-0.1111E-14	-0.5465E-14	-0.2804E-15	-0.1637E-16	-0.3268E-15
6	-0.1456E-04	-0.2051E-06	-0.2229E-08	-0.2019E-10	-0.1622E-12	-0.4505E-13	-0.9317E-12	-0.4570E-13	-0.2550E-14	-0.5334E-13
7	-0.7426E-03	-0.9934E-05	-0.1046E-06	-0.9166E-09	-0.7224E-11	-0.6609E-11	-0.1455E-09	-0.6866E-11	-0.3663E-12	-0.8029E-11
8	-0.2017E-01	-0.2476E-03	-0.2485E-05	-0.2093E-07	-0.1732E-09	-0.8701E-09	-0.2044E-07	-0.9160E-09	-0.4763E-10	-0.1093E-08
9	-0.7639E-03	-0.1045E-04	-0.1125E-06	-0.1040E-08	-0.2185E-08	-0.9867E-07	-0.2471E-05	-0.1050E-06	-0.5362E-08	-0.1290E-06
10	-0.1441E-04	-0.2129E-06	-0.2537E-08	-0.2673E-08	-0.1879E-06	-0.9300E-05	-0.2509E-03	-0.1005E-04	-0.4917E-06	-0.1242E-04
11	-0.6498E-05	-0.8160E-07	-0.2391E-08	-0.1563E-06	-0.1228E-04	-0.6777E-03	-0.2011E-01	-0.7393E-03	-0.3432E-04	-0.9390E-03
12	-0.3533E-03	-0.3880E-05	-0.5516E-07	-0.6679E-05	-0.5819E-03	-0.3620E-01	-0.1257E+01	-0.3888E-01	-0.1647E-02	-0.5041E-01
13	-0.1143E-01	-0.1104E-03	-0.2296E-05	-0.1703E-03	-0.1648E-01	-0.1193E+01	-0.5756E+02	-0.1237E+01	-0.4276E-01	-0.1518E+01
14	-0.3215E-03	-0.3409E-05	-0.8107E-07	-0.5863E-05	-0.5284E-03	-0.3363E-01	-0.1217E+01	-0.3561E-01	-0.1453E-02	-0.4797E-01
15	-0.4817E-05	-0.5473E-07	-0.1450E-08	-0.1034E-06	-0.8791E-05	-0.5245E-03	-0.1716E-01	-0.5641E-03	-0.2554E-04	-0.8117E-03
16	-0.5111E-07	-0.6232E-09	-0.1780E-10	-0.1231E-08	-0.1007E-06	-0.5798E-05	-0.1773E-03	-0.6224E-05	-0.3094E-06	-0.9661E-05
17	-0.4334E-09	-0.5509E-11	-0.1736E-12	-0.1177E-10	-0.9327E-09	-0.5125E-07	-0.1455E-05	-0.5539E-07	-0.2982E-08	-0.9228E-07
18	-0.3110E-11	-0.4085E-13	-0.1438E-14	-0.9609E-13	-0.7412E-11	-0.3835E-09	-0.1034E-07	-0.4255E-09	-0.2450E-10	-0.7476E-09
19	-0.1961E-13	-0.2741E-15	-0.1051E-16	-0.6806E-15	-0.5043E-13	-0.2523E-11	-0.6439E-10	-0.2810E-11	-0.1758E-12	-0.5221E-11
20	-0.1141E-15	-0.1706E-17	-0.6949E-19	-0.4321E-17	-0.3023E-15	-0.1464E-13	-0.3627E-12	-0.1646E-13	-0.1111E-14	-0.3228E-13
21	-0.6207E-18	-0.9771E-20	-0.4228E-21	-0.2523E-19	-0.1682E-17	-0.7864E-16	-0.1882E-14	-0.8904E-16	-0.6515E-17	-0.1845E-15
22	-0.3138E-20	-0.5223E-22	-0.2403E-23	-0.1374E-21	-0.8718E-20	-0.3927E-18	-0.8938E-17	-0.4493E-18	-0.3542E-19	-0.9650E-18
23	-0.1506E-22	-0.2640E-24	-0.1290E-25	-0.7065E-24	-0.4264E-22	-0.1815E-20	-0.3930E-19	-0.2142E-20	-0.1787E-21	-0.4674E-20
24	-0.6914E-25	-0.1274E-26	-0.6603E-28	-0.3462E-26	-0.1988E-24	-0.8016E-23	-0.1652E-21	-0.9532E-23	-0.8597E-24	-0.2162E-22
25	-0.3059E-27	-0.5914E-29	-0.3246E-30	-0.1630E-28	-0.8906E-27	-0.3408E-25	-0.6688E-24	-0.4004E-25	-0.3978E-26	-0.9621E-25
26	-0.1311E-29	-0.2658E-31	-0.1542E-32	-0.7416E-31	-0.3860E-29	-0.1404E-27	-0.2625E-26	-0.1632E-27	-0.1742E-28	-0.4149E-27
27	-0.5490E-32	-0.1167E-33	-0.7141E-35	-0.3292E-33	-0.1634E-31	-0.5655E-30	-0.1008E-28	-0.6516E-30	-0.7335E-31	-0.1751E-29
28	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

LFCONED VALUES (CONTINUED):

	21	22	23	24	25	26	27	28	29	30
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	-0.8909E-21	-0.5143E-22	-0.1565E-23	-0.3391E-25	-0.3256E-25	-0.1513E-23	-0.4983E-22	-0.8628E-21	-0.4983E-22	-0.1513E-23
3	-0.1788E-18	-0.9830E-20	-0.2851E-21	-0.5891E-23	-0.5565E-23	-0.2756E-21	-0.9527E-20	-0.1732E-18	-0.9526E-20	-0.2756E-21
4	-0.3446E-16	-0.1804E-17	-0.4988E-19	-0.9826E-21	-0.9464E-21	-0.4820E-19	-0.1749E-17	-0.3339E-16	-0.1749E-17	-0.4819E-19
5	-0.6286E-14	-0.3134E-15	-0.8259E-17	-0.1551E-18	-0.1496E-18	-0.7977E-17	-0.3039E-15	-0.6093E-14	-0.3039E-15	-0.7975E-17
6	-0.1072E-11	-0.5090E-13	-0.1279E-14	-0.2290E-16	-0.2212E-16	-0.1235E-14	-0.4937E-13	-0.1040E-11	-0.4937E-13	-0.1234E-14
7	-0.1686E-09	-0.7614E-11	-0.1824E-12	-0.3117E-14	-0.3016E-14	-0.1760E-12	-0.7388E-11	-0.1635E-09	-0.7388E-11	-0.1759E-12
8	-0.2396E-07	-0.1028E-08	-0.2352E-10	-0.3838E-12	-0.3723E-12	-0.2266E-10	-0.9986E-09	-0.2326E-07	-0.9986E-09	-0.2263E-10
9	-0.2949E-05	-0.1222E-06	-0.2669E-08	-0.4165E-10	-0.4060E-10	-0.2567E-08	-0.1188E-06	-0.2919E-05	-0.1188E-06	-0.2561E-08
10	-0.3044E-03	-0.1210E-04	-0.2524E-06	-0.3775E-08	-0.3717E-08	-0.2463E-06	-0.1199E-04	-0.3130E-03	-0.1199E-04	-0.2455E-06
11	-0.2522E-01	-0.9285E-03	-0.1840E-04	-0.2616E-06	-0.2647E-06	-0.1858E-04	-0.9550E-03	-0.2691E-01	-0.9551E-03	-0.1848E-04
12	-0.1542E+01	-0.5040E-01	-0.9388E-03	-0.1277E-04	-0.1300E-04	-0.9654E-03	-0.5376E-01	-0.1735E+01	-0.5377E-01	-0.9548E-03
13	-0.6322E+02	-0.1542E+01	-0.2563E-01	-0.3235E-03	-0.3383E-03	-0.2732E-01	-0.1734E+01	-0.7624E+02	-0.1735E+01	-0.2689E-01
14	-0.1518E+01	-0.5040E-01	-0.9810E-03	-0.1401E-04	-0.1553E-04	-0.1110E-02	-0.5830E-01	-0.1788E+01	-0.5648E-01	-0.1016E-02
15	-0.2356E-01	-0.8924E-03	-0.1896E-04	-0.2946E-06	-0.3994E-06	-0.2693E-04	-0.1263E-02	-0.3184E-01	-0.1226E-02	-0.2630E-04
16	-0.2642E-03	-0.1090E-04	-0.2481E-06	-0.4154E-08	-0.6749E-08	-0.4446E-06	-0.2004E-04	-0.4650E-03	-0.2112E-04	-0.4206E-04
17	-0.2372E-05	-0.1054E-06	-0.2566E-08	-0.4618E-10	-0.8933E-10	-0.5724E-08	-0.2538E-05	-0.5829E-05	-0.2845E-04	-0.1733E-02
18	-0.1819E-07	-0.8504E-09	-0.2193E-10	-0.4430E-12	-0.4352E-11	-0.5461E-09	-0.6348E-07	-0.6569E-05	-0.5681E-03	-0.3814E-01
19	-0.1231E-09	-0.5957E-11	-0.1597E-12	-0.4308E-14	-0.1758E-12	-0.2407E-10	-0.2911E-08	-0.3014E-06	-0.2567E-04	-0.1671E-02
20	-0.7385E-12	-0.3766E-13	-0.1040E-14	-0.4970E-16	-0.4253E-14	-0.5853E-12	-0.7031E-10	-0.7169E-08	-0.5965E-06	-0.3812E-04
21	-0.4079E-14	-0.2201E-15	-0.6293E-17	-0.6541E-18	-0.7599E-15	-0.1019E-13	-0.1182E-11	-0.1168E-09	-0.9575E-08	-0.6025E-06
22	-0.2109E-16	-0.1177E-17	-0.3484E-19	-0.8073E-20	-0.1048E-17	-0.1378E-15	-0.1599E-13	-0.1596E-11	-0.1363E-09	-0.9602E-08
23	-0.9899E-19	-0.5838E-20	-0.2161E-21	-0.7652E-20	-0.1489E-17	-0.2682E-15	-0.4518E-13	-0.6880E-11	-0.9285E-09	-0.1090E-06
24	-0.4349E-21	-0.3055E-22	-0.6593E-21	-0.1437E-18	-0.2936E-16	-0.5566E-14	-0.9679E-12	-0.1493E-09	-0.2083E-07	-0.2565E-05
25	-0.1841E-23	-0.2756E-24	-0.3344E-22	-0.6917E-20	-0.1366E-17	-0.2499E-15	-0.4182E-13	-0.6431E-11	-0.8787E-09	-0.1041E-06
26	-0.7570E-26	-0.4723E-26	-0.8982E-24	-0.1810E-21	-0.3408E-19	-0.6073E-17	-0.9916E-15	-0.1462E-12	-0.1950E-10	-0.2246E-08
27	-0.3071E-28	-0.8517E-28	-0.1693E-25	-0.3321E-23	-0.6089E-21	-0.1033E-18	-0.1639E-16	-0.2346E-14	-0.2967E-12	-0.3287E-10
28	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

UPCONED VALUES (CONTINUED):

	31	32	33	34	35	36	37
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	-0.3211E-25	-0.5357E-27	-0.7486E-29	-0.9121E-31	-0.9961E-33	-0.1004E-34	0.0
3	-0.5575E-23	-0.8868E-25	-0.1183E-26	-0.1376E-28	-0.1437E-30	-0.1385E-32	0.0
4	-0.9290E-21	-0.1408E-22	-0.1791E-24	-0.1990E-26	-0.1985E-28	-0.1830E-30	0.0
5	-0.1464E-18	-0.2114E-20	-0.2562E-22	-0.2714E-24	-0.2586E-26	-0.2278E-28	0.0
6	-0.2156E-16	-0.2964E-18	-0.3421E-20	-0.3455E-22	-0.3141E-24	-0.2644E-26	0.0
7	-0.2924E-14	-0.3824E-16	-0.4202E-18	-0.4044E-20	-0.3508E-22	-0.2820E-24	0.0
8	-0.3578E-12	-0.4450E-14	-0.4655E-16	-0.4270E-18	-0.3534E-20	-0.2718E-22	0.0
9	-0.3844E-10	-0.4544E-12	-0.4526E-14	-0.3960E-16	-0.3135E-18	-0.2313E-20	0.0
10	-0.3484E-08	-0.3914E-10	-0.3717E-12	-0.3111E-14	-0.2367E-16	-0.1691E-18	0.0
11	-0.2499E-06	-0.2671E-08	-0.2431E-10	-0.1963E-12	-0.1456E-14	-0.1034E-16	0.0
12	-0.1232E-04	-0.1276E-06	-0.1132E-08	-0.9060E-11	-0.6889E-13	-0.5269E-15	0.0
13	-0.3184E-03	-0.3126E-05	-0.2777E-07	-0.2471E-09	-0.2372E-11	-0.2466E-13	0.0
14	-0.1360E-04	-0.5570E-05	-0.2698E-06	-0.6963E-08	-0.1268E-09	-0.1851E-11	0.0
15	-0.2633E-04	-0.5151E-03	-0.2483E-04	-0.6282E-06	-0.1125E-07	-0.1616E-09	0.0
16	-0.1663E-02	-0.3612E-01	-0.1624E-02	-0.3927E-04	-0.6728E-06	-0.9285E-08	0.0
17	-0.7516E-01	-0.1855E+01	-0.7512E-01	-0.1724E-02	-0.2855E-04	-0.3821E-06	0.0
18	-0.1879E+01	-0.6166E+02	-0.1902E+01	-0.3963E-01	-0.6151E-03	-0.7778E-05	0.0
19	-0.7516E-01	-0.1879E+01	-0.7814E-01	-0.1850E-02	-0.3199E-04	-0.4506E-06	0.0
20	-0.1655E-02	-0.3760E-01	-0.1803E-02	-0.5147E-04	-0.1104E-05	-0.1953E-07	0.0
21	-0.2590E-04	-0.5605E-03	-0.4448E-04	-0.3551E-03	-0.1669E-04	-0.4389E-06	0.0
22	-0.5364E-06	-0.2401E-04	-0.8901E-03	-0.2405E-01	-0.9913E-03	-0.2372E-04	0.0
23	-0.1061E-04	-0.8164E-03	-0.4456E-01	-0.1386E+01	-0.4751E-01	-0.9944E-03	0.0
24	-0.2648E-03	-0.2195E-01	-0.1365E+01	-0.5688E+02	-0.1408E+01	-0.2488E-01	0.0
25	-0.1025E-04	-0.7939E-03	-0.4388E-01	-0.1365E+01	-0.4532E-01	-0.9004E-03	0.0
26	-0.2112E-06	-0.1528E-04	-0.7848E-03	-0.2157E-01	-0.8165E-03	-0.1751E-04	0.0
27	-0.2995E-08	-0.2081E-06	-0.1017E-04	-0.2581E-03	-0.1063E-04	0.0	0.0
28	0.0	0.0	0.0	0.0	0.0	0.0	0.0

CHAPTER VII

SUMMARY AND CONCLUSION

The present study developed a computer program in FORTRAN IV for modeling two-dimensional unsteady flow of freshwater in a confined, isotropic/anisotropic, homogeneous/heterogeneous aquifer which is underlained by a lens of saltwater. A one-dimensional transport theory is coupled into it for predicting the location of the interface. Finite-Difference techniques are used to approximate the partial differential equations, and the resulting matrix problem is solved using Alternating Directional Implicit Procedure. The model developed is reasonably compact, efficient and economic to use for simulation purposes. The program is very flexible in nature and is written in the form of a subroutine which is believed to be easier to call from an external source. The program can be used for either simulation only or for both the simulation and upconing of saltwater underlying the freshwater. The program has many options in it and is programmed in such a fashion that it leads the user to a sure way of better conceptualizing the problem and its area of occurrence. The model was applied to the Yukon well field of Oklahoma which is the main source of municipal water supply to the city of Yukon.

Suggestions for Further Work

A basic lack in the present study is the right way of model application. Generally speaking, a model has to be calibrated with at least

5 to 10 years of past records. Due to unavailability of such data for the Yukon well field, calibration of the model for the area was unsuccessful. For this reason and many more, the following are some of the suggestions for future work in the same area.

a. Calibration of the model to verify its validity.

b. Experimental verification of the model for upconing of saltwater; this can be achieved with the help of a viscous-flow model study in laboratory.

c. Examining other available methods (such as line-successive overrelaxation, Strongly Implicit Procedure, etc.) for solving the matrix generated by Finite-Difference Technique in order to find the best method of solution for the groundwater flow equation(s).

d. Examining other available numerical techniques (such as Finite Element and Boundary Element) applicable to this type of solution to determine the most convenient technique of solution.

e. Incorporating a two-dimensional transport equation into the present model to achieve a better prediction of saltwater interface.

f. A three-dimensional case may be considered for further detailed study.

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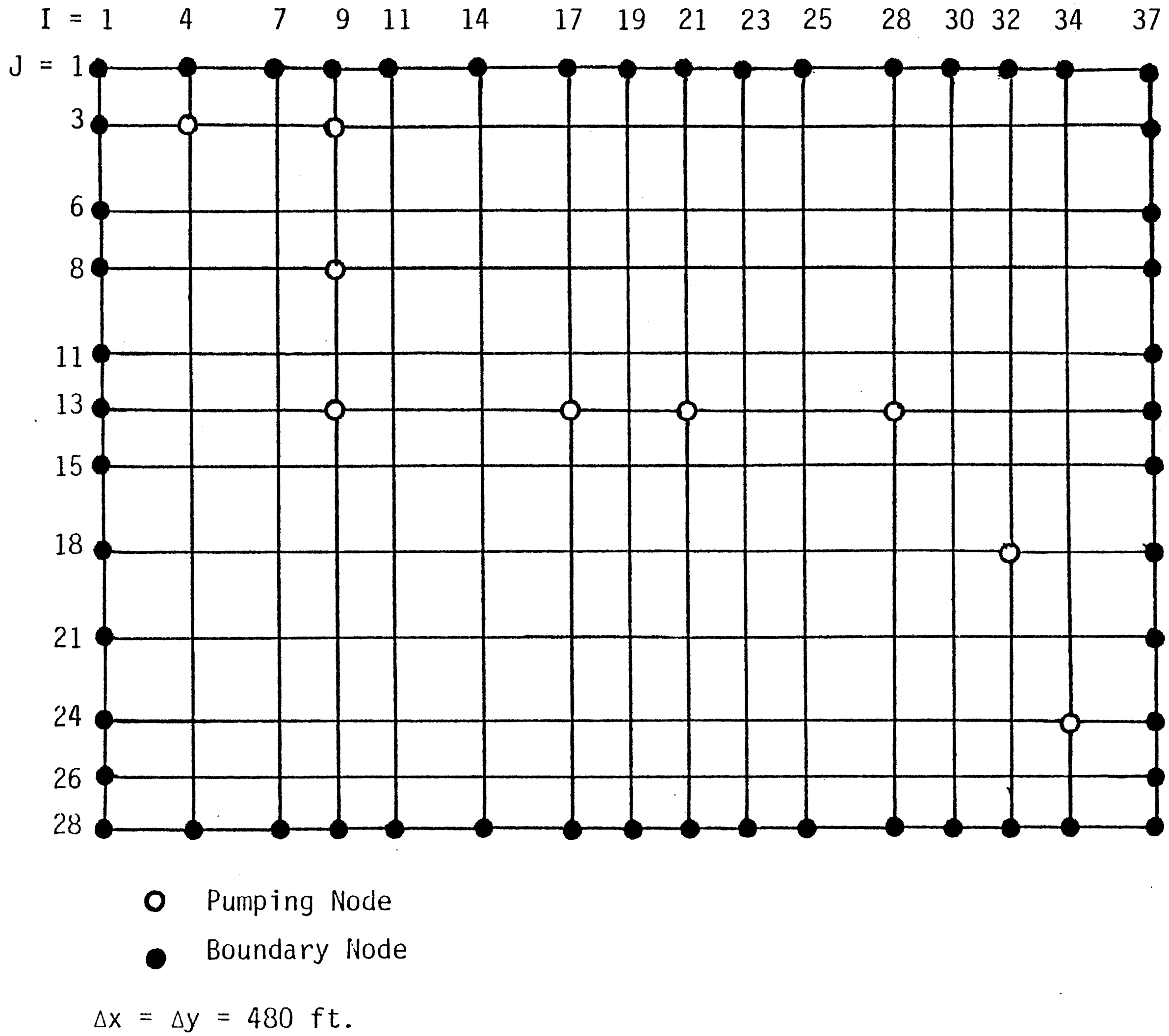
APPENDICES

APPENDIX A

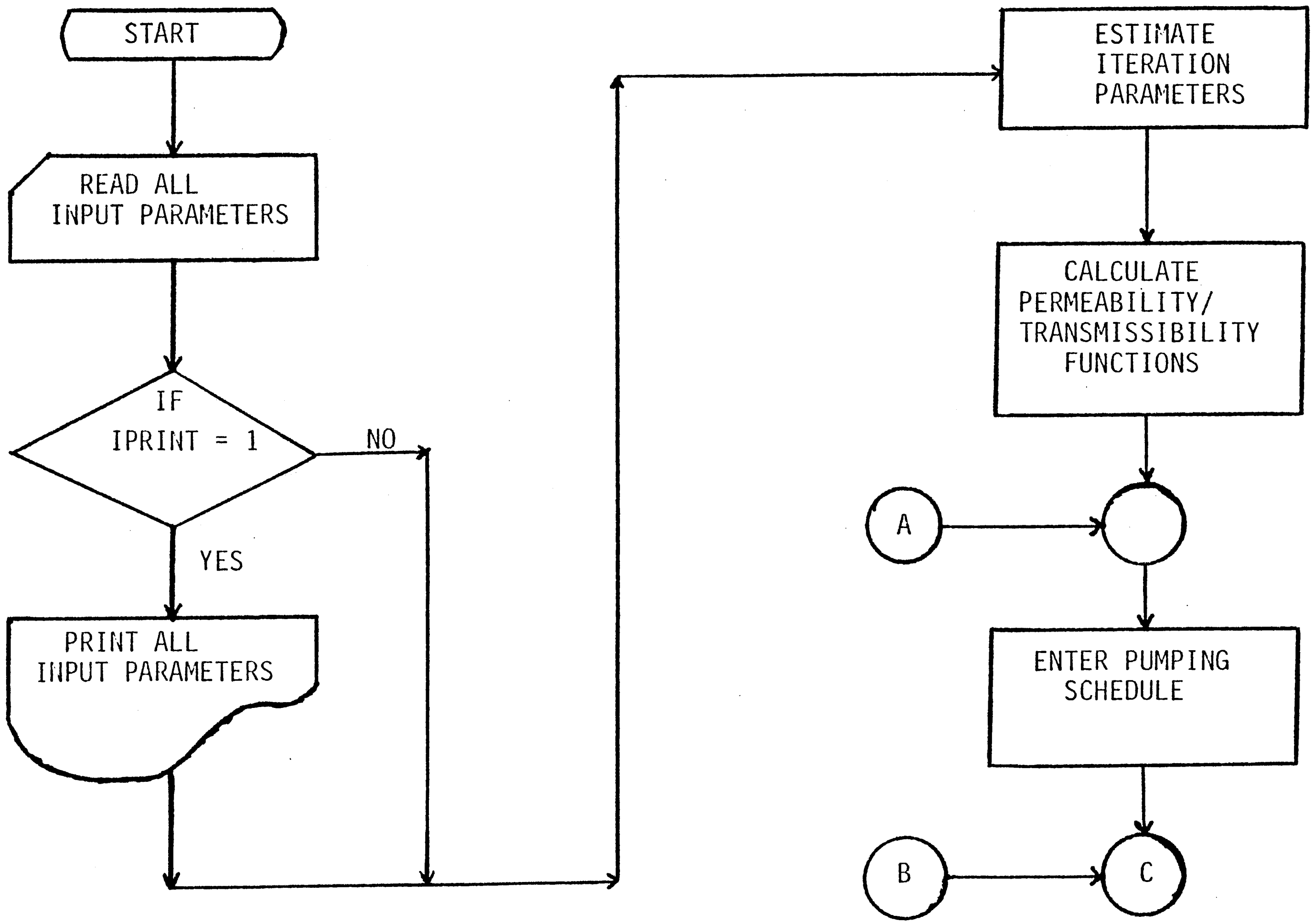
DISCRETIZATION OF STUDY AREA

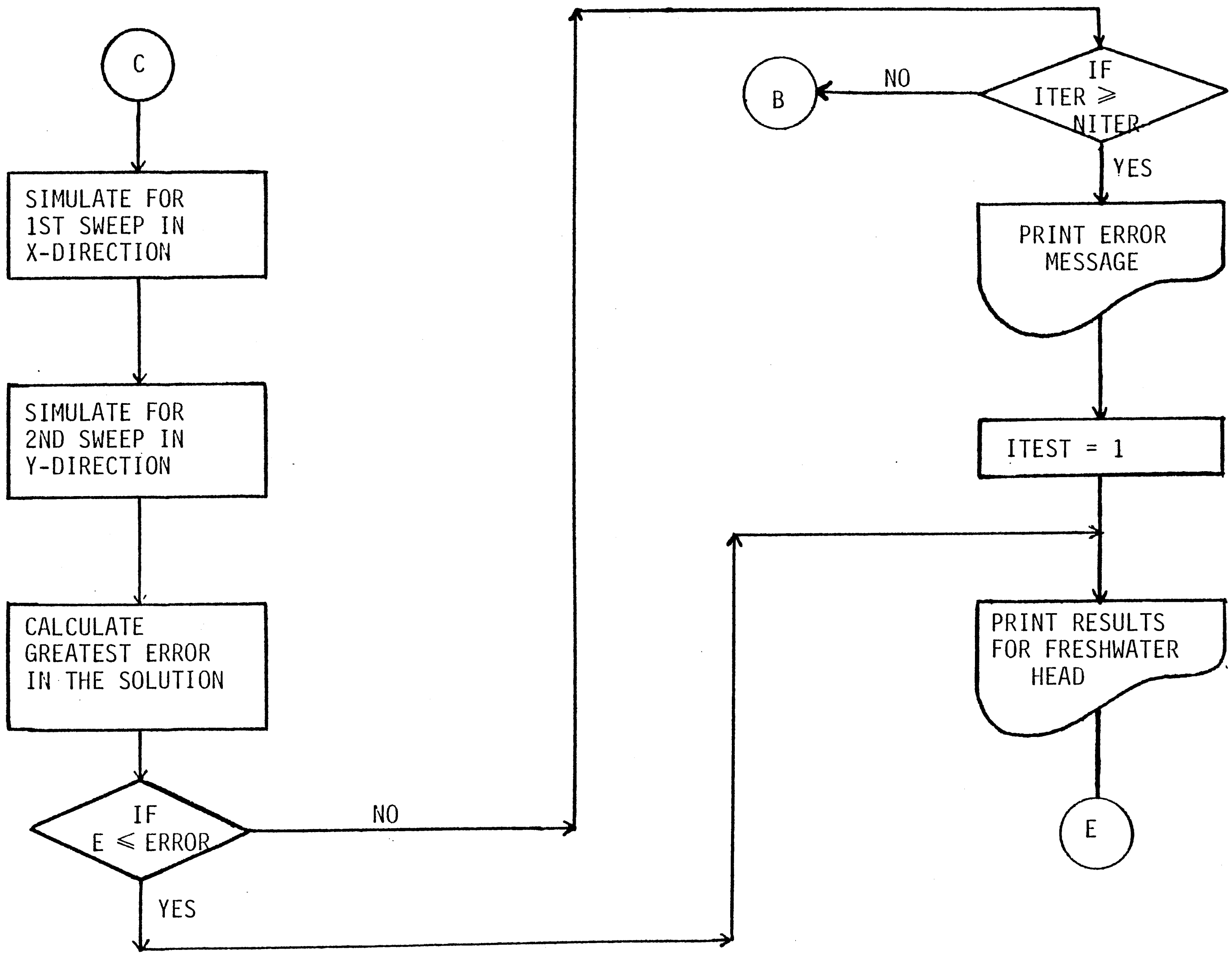


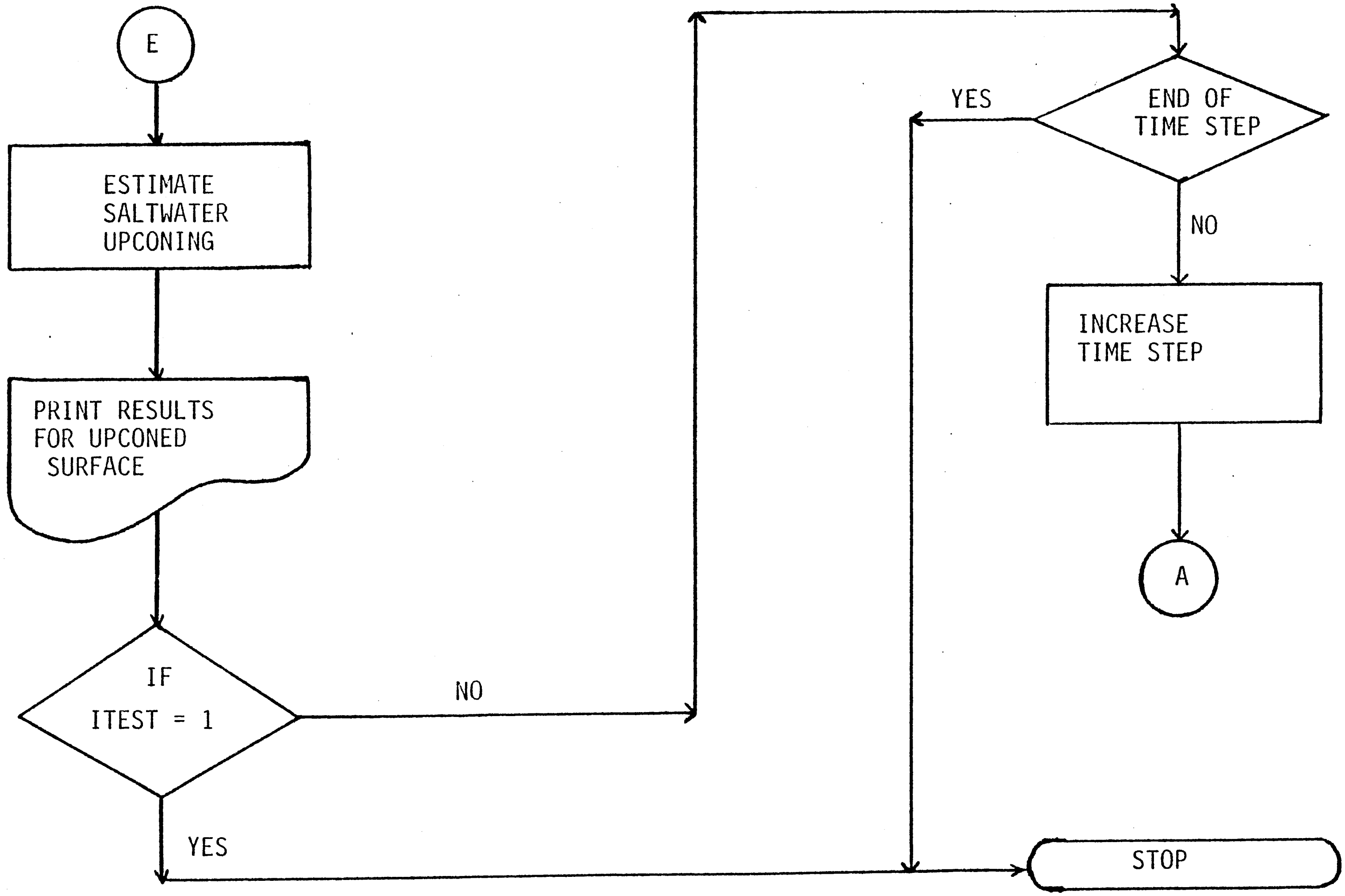
DISCRETIZATION OF THE STUDY AREA

APPENDIX B

FLOW CHART


START
READ ALL INPUT PARAMETERS
IF IPRINT = 1
NO
YES
PRINT ALL INPUT PARAMETERS
ESTIMATE ITERATION PARAMETERS
CALCULATE PERMEABILITY/ TRANSMISSIBILITY FUNCTIONS
A
ENTER PUMPING SCHEDULE
B
C


C
SIMULATE FOR 1ST SWEEP IN X-DIRECTION
SIMULATE FOR 2ND SWEEP IN Y-DIRECTION
CALCULATE GREATEST ERROR IN THE SOLUTION
IF E ≤ ERROR
NO
YES
IF ITER ≥ NITER
NO
B
YES
PRINT ERROR MESSAGE
ITEST = 1
PRINT RESULTS FOR FRESHWATER HEAD
E


E
ESTIMATE SALTWATER UPCONING
PRINT RESULTS FOR UPCONED SURFACE
IF ITEST = 1
NO
YES
END OF TIME STEP
YES
NO
INCREASE TIME STEP
A
STOP

APPENDIX C

ALGORITHM

```
// EXEC FORTGC,PARM=NOSOURCE
//SYSLIN  CD  DISP=OLD,
//   DCB=(RECFM=FB,LRECL=80,BLKSIZE=6160),
//   SPACE=(TRK,(10,10),RLSE),
//   UNIT=SYSTSD,
//   DSN=U11370F.FINAL.CBJ
//FORT.SYSIN DD *
C      MAIN PROGRAM............FINAL...........................
C            ***************************************************
C            *  PROGRAMMED  BY  ::     BIJAY  KUMAR  PANIGRAHI      *
C            *                 AT  ::     OKLAHOMA STATE UNIVERSITY,  *
C            *                           STILLWATER, OCTOBER 1980 .  *
C            ***************************************************
C
      DIMENSION DELX(40),DELY(40),B(40),G(40),CAT1(20),CAT2(20),
     $          H(40,40),SS(40,40),Q(40,40),HL(40,40),BT(40,40),
     $          FPX(40,40),FPY(40,40),ROW(40),UPC(40,40),
     $          PX(40,40),PY(40,40)
      INTEGER OUT
      CALL ERRSET(208,256,-1,1)
C     ALL THE CHECKING PARAMETERS ARE  SET IN SAME FORMAT AS FOLLOWS::
      NC=40
      NR=40
      NM=20
      IN=5
      OUT=6
      PI=3.1416
      CALL MODEL(IN,OUT,NC,NR,NM,
     $            DELX,DELY,CAT1,CAT2,PX,PY,SS,H,Q,BT,FPX,FPY,
     $            ROW,HL,UPC,B,G,PI)
      STOP
      END
C
C     FOLLOWINGS ARE THE JOB CONTROL CARDS FOR THIS PROGRAM
/*JOBPARM F=9001,N=1
//RUN EXEC PGM=LOADER,REGION=400K
//SYSLIB DD DISP=SHR,DSN=SYS1.FORTLIB
//SYSLOUT DD SYSOUT=A
//SYSLIN DD DISP=SHR,DSN=U11370F.FINAL.OBJ
//FT06F001 DD SYSOUT=A
//FT05F001 DD DISP=SHR,DSN=U11370F.FINAL.DATA
//
```

```
      SUBROUTINE MODEL( IN,OUT,NC,NR,NM,
     $                  DELX,DELY,CAT1,CAT2,PX,PY,SS,H,Q,BT,FPX,FPY,
     $                  ROW,HL,UPC,B,G,PI)
C          ***************************************************
C          *           M O D E L   S I M U L A T I O N       *
C          *-------------------------------------------------*
C          *  PROGRAMMED  BY  ::    BIJAY  KUMAR  PANIGRAHI  *
C          ***************************************************
      DIMENSION DELX(NC),DELY(NR),CAT1(NM),CAT2(NM),
     $          PX(NC,NR),PY(NC,NR),SS(NC,NR),H(NC,NR),Q(NC,NR),
     $          BT(NC,NR),FPX(NC,NR),FPY(NC,NR),ROW(NC),UPC(NC,NR),
     $          HL(NC,NR),B(NR),G(NR),CL(40,40),IP(20),JP(20),
     $          P(20,20),RC1(40),RC2(40),RHOP(20),DCM(40)
      INTEGER OUT
C     ************************************************************
C          VARIABLE  DESCRIPTIONS
C     ************************************************************
C     NX     =TOTAL NO. OF NODAL POINTS IN X-DIRECTION (=NC)
C     NY     =TOTAL NO. OF NODAL POINTS IN Y-DIRECTION (=NR)
C     NSTEP  =TOTAL NO. OF TIME INCREMENTAL STEPS
C     NITER  =MAXIMUM NO. OF ALLOWABLE ITERATIONS PER TIME STEP
C     ERROR  =MAXIMUM ALLOWABLE ERROR LIMIT FOR EACH NODAL VALUE
C     DELTAT=INITIAL TIME INTERVAL
C     TIMEIP=A MULTIPLIER FOR INCREASING EACH TIME STEP
C     THETA  =REPRESENTABLE POROSITY OF THE MEDIUM (IN FRACTION)
C     DENSF  =DENSITY RATIO (=FRESHWATER DENSITY/(1.-SALTWATER DENSITY))
C     BETA   =A FRACTIONAL VALUE USED FOR CALCULATING THICKNESS OF FRESH-
C             WATER ZONE
C     LNT    =NO. OF ITERATION PARAMETERS INCORPRATED IN THE PROGRAM
C ----------------------------------------------------------------------
C     IEQX   =INDICATOR TO SHOW IF EQUAL INCREMENT IN X-DIRECTION (=1)
C     IEQY   =INDICATOR TO SHOW IF EQUAL INCREMENT IN Y-DIRECTION (=1)
C     ISOT   =INDICATOR TO SHOW IF ISOTROPIC CONDITION EXISTS (=1)
C     IHOMXY=INDICATOR TO SHOW IF HOMOGENEOUS IS THE WHOLE DOMAIN (=1)
C     IHOMX =INDICATOR TO SHOW IF EQUAL PERMEABILITY IN X-DIRECTION (=1)
C     IHOMY =INDICATOR TO SHOW IF EQUAL PERMEABILITY IN Y-DIRECTION (=1)
C     ICON   =INDICATOR TO SHOW IF CONFINED CONDITION EXISTS (=1)
C     IECSS =INDICATOR TO SHOW IF EQUAL STORATIVITY CONDITION
C             EXISTS (=1)
C     IPRINT=INDICATOR TO SHOW IF PRINTOUT OF ALL THE INPUT PARAMETERS
C             ARE DESIRED (=1)
```

```
C     IHDRCN=INDICATOR TO SHOW IF EQUAL HEAD/DRAWDOWN EXISTS
C            THROUGH OUT THE FLOW-FIELD (=1)
C     IDISCH=TOTAL NO. OF GRID POINTS AT WHICH NET FLUX IS NOT ZERO
C     IUPCON=INDICATOR TO SHOW IF UPCONING IS TO BE ESTIMATED (=1)
C     ITRANS=INDICATOR TO SHOW IF UPCONING IS TO BE ESTIMATED BY
C            TRANSPORT THEORY ONLY (=+1) ,OR, BY MUSKAT APPROACH ONLY
C            (=0) ,OR, BY BOTH METHODS (=-1)
C ----------------------------------------------------------------------
C     DELX(I) =SPATIAL INCREMENTS IN X-DIRECTION (MAXM. = NC)
C     DELY(J) =SPATIAL INCREMENTS IN Y-DIRECTION (MAXM. = NR)
C     PX(I,J) =PERMEABILITY/TRANSMISSIBILITY OF POINT (I,J) IN X-DIRCN.
C     PY(I,J) =PERMEABILITY/TRANSMISSIBILITY OF POINT (I,J) IN Y-DIRCN.
C     BT(I,J) =SATURATED THICKNESS AT THE POINT (I,J)
C     SS(I,J) =SPECIFIC STORAGE/STORAGE COEFFICIENT OF POINT (I,J)
C     H(I,J)  =HYDRAULIC HEAD/DRAWDOWN OF POINT (I,J)AT THE START OF
C              TIME STEP
C     Q(I,J)  =NET FLUX AT POINT (I,J);+VE FOR OUTFLUX & -VE FOR INFLUX
C     NP      =NO. OF PUMPS
C     NSP     =NO. OF TIME INCREMENTS PER PUMPAGE CHANGE
C     NRT     =NO. OF RATES IN PUMPING SCHEDULE
C     IP(L)                      (MAXM. L=NP)
C     JP(L)
C     P(L,K)                      (MAXM. K=NRT)
C     MCM
C     CAT1(M)                      (MAXM. M=MCM)
C     CAT2(M)
C ----------------------------------------------------------------------
C     RHOP(N)= ITERATION PARAMETERS (MAXM. N=LNT)
C     HL(I,J)=HYDRAULIC HEAD/DRAWDOWN OF POINT (I,J) AT THE END OF
C              TIME STEP/ITERATION
C     DL(I,J)=HYDRAULIC HEAD/DRAWDOWN VALUE OF POINT (I,J) AT THE
C              BEGINING OF ITERATION
C     TIME    =TOTAL TIME COVERED IN DAYS/YEARS SINCE THE BEGINING
C              OF SIMULATION
C     ITER    =A COUNTER DENOTING THE NUMBER OF ITERATIONS COMPLETED
C     PARM    =ITERATION PARAMETER =RHOP(N)
C     E       =LARGEST ERROR ENCOUNTERED IN A SINGLE ITERATION
C     ******************************************************************
C     READ AND WRITE ALL THE PARAMETERS REQUIRED.
C     ******************************************************************
```

```
      READ(IN,901) NX,NY,NSTEP,NITER,ERROR,DELTAT,TIMEIM,THETA,DENSF,
     $             BETA,LNT
      IF(NX.GT.NC.OR.NY.GT.NR) GO TO 888
      NY1=NY-1
      NX1=NX-1
      READ(IN,902) IEQX,IEQY,ISOT,IHOMXY,IHOMX,IHOMY,ICON,IEQSS,
     $             IHORDN,IPRINT,IDISCH,IUPCON,ITRANS
      IF(IEQX) 10,10,20
10    READ(IN,903) (DELX(I),I=1,NX )
      GO TO 40
20    READ(IN,904) DUMMY
      DO 30 I=1,NX
30    DELX(I)=DUMMY
40    IF(IEQY) 50,50,60
50    READ(IN,903) (DELY(J),J=1,NY )
      GO TO 80
60    READ(IN,904) DUMMY
      DO 70 J=1,NY
70    DELY(J)=DUMMY
80    IF(ISOT) 86,86,81
81    IF(IHOMXY) 82,82,84
82    READ(IN,903) ((PX(I,J), I=1,NX ),J=1,NY )
      DO 83 J=1,NY
      DO 83 I=1,NX
83    PY(I,J)=PX(I,J)
      GO TO 160
84    READ(IN,904) DUMMY
      DO 85 J=1,NY
      DO 85 I=1,NX
      PX(I,J)=DUMMY
85    PY(I,J)=DUMMY
      GO TO 160
86    IF(IHOMX) 90,90,100
90    READ(IN,903) ((PX(I,J), I=1,NX ),J=1,NY )
      GO TO 120
100   READ(IN,904) DUMMY
      DO 110 J=1,NY
      DO 110 I=1,NX
110   PX(I,J)=DUMMY
120   IF(IHOMY) 130,130,140
130   READ(IN,903) ((PY(I,J),I=1,NX ),J=1,NY )
      GO TO 160
```

```
140   READ(IN,904) DUMMY
      DO 150 J=1,NY
      DO 150 I=1,NX
150   PY(I,J)=DUMMY
160   IF(ICON) 170,170,180
170   READ(IN,903) ((BT(I,J),I=1,NX ),J=1,NY )
      GO TO 200
180   DO 190 J=1,NY
      DO 190 I=1,NX
190   BT(I,J)=1.0
C     READ THE INITIAL HEAD & OTHER PARAMETER VALUES FOR EACH NODE.
200   IF(IEQSS) 201,201,202
201   READ(IN,903) ((SS(I,J),I=1,NX ),J=1,NY )
      GO TO 204
202   READ(IN,904) DUMMY
      DO 203 J=1,NY
      DO 203 I=1,NX
203   SS(I,J)=DUMMY
204   IF(IHORDN) 205,205,206
205   READ(IN,903) ((H(I,J),I=1,NX ),J=1,NY )
      GO TO 208
206   READ(IN,904) DUMMY
      DO 207 J=1,NY
      DO 207 I=1,NX
207   H(I,J)=DUMMY
208   DO 209 J=1,NY
      DO 209 I=1,NX
209   Q(I,J)=0.0
      READ(IN,905) (IP(K),JP(K),Q( IP(K),JP(K)),K=1,IDISCH)
C     READ PUMP PARAMETERS AND PUMPING SCHEDULE .
      READ(IN,906) NP,NSP,NRT
      DO 210 I=1,NP
210   READ(IN,907) IP(I),JP(I),(P(I,K),K=1,NRT)
C     READ CATEGORY MAP PRINTOUT PARAMETERS ::
      READ(IN,908) MCM,(CAT2(I),CAT1(I),I=1,MCM)
      IF(IPRINT.NE.1) GO TO 220
C
C     WRITE ALL THE INPUT PARAMETERS READ BEFORE.
C
      WRITE(OUT,951) NX,NY,NSTEP,NITER,ERROR,DELTAT,TIMEIM,THETA,DENSF,
     $               BETA,LNT
      WRITE(OUT,952) IEQX,IEQY,IHOMXY,IHOMX,IHOMY,ISOT,ICON,IEQSS
      WRITE(OUT,960) IHORDN,IPRINT,IDISCH,IUPCON,ITRANS
```

```
      WRITE(OUT,953)
      WRITE(OUT,954) (DELX(I),I=1,NX )
      WRITE(OUT,955) (DELY(J),J=1,NY )
      WRITE(OUT,956) NP,NSP,NRT
      DO 211 I=1,NP
211   WRITE(OUT,957) IP(I),JP(I),(P(I,K),K=1,NRT)
      WRITE(OUT,958) (CAT2(I),CAT1(I),CAT2(I),CAT1(I),I=1,MCM)
      WRITE(OUT,959)
220   IF(ICON.NE.1) GO TO 885
C     ****************************************************************
C     NUMERICAL SOLUTION SCHEME STARTS FROM HERE.
C     ****************************************************************
      TIME=0.0
C ---------------------------------------------------------------------
C     ESTIMATE THE ITERATION PARAMETERS
C ---------------------------------------------------------------------
      HMAX=0.0
      HMIN=2.
      XVAL=PI*PI/(2.*NX*NX)
      YVAL=PI*PI/(2.*NY*NY)
      XVM=2.-XVAL
      YVM=2.-YVAL
      DO 230 I=1,NX1
      DO 230 J=1,NY1
      IF(PX(I,J).EQ.0..OR.PY(I,J).EQ.0.) GO TO 230
      XPART=1./(1.+DELX(I)**2*PY(I,J)/(DELY(J)**2*PX(I,J)))
      YPART=1./(1.+DELY(J)**2*PX(I,J)/(DELX(I)**2*PY(I,J)))
      XF=XVAL*XPART
      YF=YVAL*YPART
      HMIN=AMIN1(HMIN,XF,YF)
      XF=XVM*XPART
      YF=YVM*YPART
      HMAX=AMAX1(HMAX,XF,YF)
230   CONTINUE
      ALPHA=EXP(ALOG(HMAX/HMIN)/(LNT-1))
      RHOP(1)=HMIN
      DO 240 NTIME=2,LNT
240   RHOP(NTIME)=RHOP(NTIME-1)*ALPHA
      WRITE(OUT,970) LNT,(RHOP(I),I=1,LNT)
C ---------------------------------------------------------------------
C     CALCULATE THE FLUX & PERMEABILITY FUNCTIONS THOSE WOULD BE
C     DIRECTLY CONTROLLING THE NUMERICAL SOLUTION SCHEME.
C ---------------------------------------------------------------------
```

```
      DO 270 I=1,NX1
      II=I+1
      DO 270 J=1,NY1
      FPX(I,J)=0.0
      FPY(I,J)=0.0
      IF(PX(I,J).EQ.0..AND.PX(II,J).EQ.0.) GO TO 250
      FPX(I,J)=2.*PX(I,J)*PX(II,J)/
     $          (PX(I,J)*DELX(II)+PX(II,J)*DELX(I))
250   JJ=J+1
      IF(PY(I,J).EQ.0..AND.PY(I,JJ).EQ.0.) GO TO 260
      FPY(I,J)=2.*PY(I,J)*PY(I,JJ)/
     $          (PY(I,J)*DELY(JJ)+PY(I,JJ)*DELY(J))
260   SS(I,J)=(SS(I,J)+THETA*DENSF)*7.48
      DUMMY=DELX(I)*DELY(J)
      FPX(I,J)=FPX(I,J)*DUMMY
      FPY(I,J)=FPY(I,J)*DUMMY
      SS(I,J)=SS(I,J)*DUMMY
270   HL(I,J)=H(I,J)
C
      DELT=1.0
      DELTA=DELTAT
      K1=1
C ---------------------------------------------------------------------
C     MODEL SIMULATION STARTS FROM HERE
C ---------------------------------------------------------------------
      DO 600 ISTEP=1,NSTEP
      IF(NSP.EQ.NSTEP) GO TO 300
C
C     ENTER PUMPAGE SCHEDULES.
C
      ZK=(ISTEP-1.0)/NSP+1.0
      IF(ZK-K1) 300,280,300
280   DO 290 K=1,NP
      I=IP(K)
      J=JP(K)
290   Q(I,J)=P(K,K1)
      DELTAT=DELTA
      K1=K1+1
300   TIME=TIME+DELTAT
      DO 310 J=2,NY1
      DO 310 I=2,NX1
310   SS(I,J)=SS(I,J)*(DELT/DELTAT)
```

```
      ITER=0
      NTH=0
C
C     ITERATIONS PER EACH TIME STEP BEGINS HERE.
C
320   ITER=ITER+1
      DO 330 I=2,NX1
      DO 330 J=2,NY1
330   DL(I,J)=HL(I,J)
      IF(MOD(ITER,LNT)) 340,340,350
340   NTH=0
350   NTH=NTH+1
      PARM=RHOP(NTH)
      E=0.0
C     ............................................................
C     X-DIRECTION SWEEP STARTS HERE (1ST. SWEEP) :
C     ............................................................
      DO 390 J=2,NY1
      JJ=J-1
      JK=J+1
      DO 360 I=2,NX1
      B(I)=0.0
      G(I)=0.0
      II=I-1
      IK=I+1
      IF((FPX(I,J)+FPY(I,J)).EQ.0..AND.SS(I,J).EQ.0.) GO TO 350
      A1=FPX(II,J)/DELX(I)
      A2=FPX(I,J)/DELX(I)
      C1=FPY(I,JJ)/DELY(J)
      C2=FPY(I,J)/DELY(J)
      HI=(A1+A2+C1+C2)*PARM
      AA=-A1
      BB=A1+A2+2.*SS(I,J)+HI*2.
      CC=-A2
      DD=A1*(HL(II,J)+H(II,J))+A2*(HL(IK,J)+H(IK,J))
      DD=DD+C1*(HL(I,JJ)+H(I,JJ))+C2*(HL(I,JK)+H(I,JK))
      DD=DD-(A1+A2+C1+C2)*(HL(I,J)+H(I,J))-2.*Q(I,J)
      DD=DD+2.*SS(I,J)*(H(I,J)-HL(I,J))
      WW=BB-AA*B(II)
      B(I)=CC/WW
      G(I)=(DD-AA*G(II))/WW
360   CONTINUE
```

```
      HBP=C.0
      DO 390 I=2,NX1
      N=NX-I+1
      IF((FPX(N,J)+FPY(N,J)).NE.0..AND.SS(N,J).GE.0.) GO TO 370
      HA=0.0
      GC TC 380
370   HA=G(N)-B(N)*HBP
      HL(N,J)=HL(N,J)+HA
380   HBP=HA
390   CONTINUE
C     ..........................................................
C     Y-DIRECTION SWEEP STARTS HERE (2ND. SWEEP) :
C     ..........................................................
      DO 430 I=2,NX1
      II=I-1
      IK=I+1
      DO 400 J=2,NY1
      B(J)=0.0
      G(J)=0.0
      JJ=J-1
      JK=J+1
      IF((FPX(I,J)+FPY(I,J)).EQ.0..AND.SS(I,J).EQ.0.) GO TO 400
      A1=FPX(II,J)/DELX(I)
      A2=FPX(I,J)/DELX(I)
      C1=FPY(I,JJ)/DELY(J)
      C2=FPY(I,J)/DELY(J)
      HI=(A1+A2+C1+C2)*PARM
      AA=-C1
      BB=C1+C2+2.*SS(I,J)+HI*2.
      CC=-C2
      DD=A1*(HL(II,J)+H(II,J))+A2*(HL(IK,J)+H(IK,J))
      DD=DD+C1*(HL(I,JJ)+H(I,JJ))+C2*(HL(I,JK)+H(I,JK))
      DD=DD-(A1+A2+C1+C2)*(HL(I,J)+H(I,J))-2.*Q(I,J)
      DD=DD+2.*SS(I,J)*(H(I,J)-HL(I,J))
      WW=BB-AA*B(JJ)
      B(J)=CC/WW
      G(J)=(DD-AA*G(JJ))/WW
400   CONTINUE
      HBP=0.0
      DO 430 J=2,NY1
      N=NY-J+1
      IF((FPX(I,N)+FPY(I,N)).NE.0..AND.SS(I,N).GE.0.) GO TO 410
      HA=0.0
```

```
      GO TO 420
410   HA=C(N)-B(N)*HBP
      HL(I,N)=HL(I,N)+HA
420   HBP=HA
      ECHK=ABS(HL(I,N)-DL(I,N))
      IF(ECHK.GT.E) E=ECHK
430   CONTINUE
C     ..............................................................
C     CHECK THE ERROR IN THIS ITERATION & ALSO CHECK THE NO. OF
C     ITERATIONS COMPLETED.
C     ..............................................................
      IF(E-ERROR) 460,460,440
440   IF(ITER-NITER) 320,450,450
450   WRITE(OUT,971) ITER
      ITEST=1
460   DO 470 I=2,NX1
      DO 470 J=2,NY1
470   H(I,J)=HL(I,J)
      WRITE(OUT,972) ISTEP,DELTAT,TIME,E,ITER
      DELT=DELTAT
      DELTAT=DELTAT*TIMEIM
C     PRINTOUT CATEGORY MAP ::
      WRITE(OUT,973)
      DO 500 J=1,NY
      DO 490 I=2,NX1
      DO 480 MK=1,MCM
      IF(H(I,J)+CAT1(MK)) 480,490,490
480   CONTINUE
490   ROW(I)=CAT2(MK)
500   WRITE(OUT,974) J,(ROW(I),I=1,NX )
      IF(ISTEP.NE.NSTEP) GO TO 511
C     PRINTOUT HEAD VALUES ::
      ISEQ=(NX+9)/10
      DO 510 M=1,ISEQ
      IFIRST=(M-1)*10+1
      ILAST=M*10
      IF(ILAST.GT.NX) ILAST=NX
      IF(M.EQ.1) WRITE(OUT,975)
      IF(M.GT.1) WRITE(OUT,976)
      WRITE(OUT,977) (I,I=IFIRST,ILAST)
      DO 510 J=1,NY
510   WRITE(OUT,978) J,(H(I,J),I=IFIRST,ILAST)
```

```
C     ************************************************************************
C     UPCONING ESTIMATION FOR THE TIME STEP.
C     ************************************************************************
511   IF(IUPCON) 595,595,515
515   IF(ITRANS) 520,520,540
520   DO 525 J=1,NY
      DO 525 I=1,NX
525   UPC(I,J)=H(I,J)*DENSF
      WRITE(OUT,983)
      WRITE(OUT,979)
C     PRINTOUT CATEGORY MAP FOR UPCONED SURFACE ::
      DO 528 J=1,NY
      DO 527 I=2,NX1
      DO 526 MK=1,MCM
      IF(UPC(I,J)+CAT1(MK)*DENSF) 526,527,527
526   CONTINUE
527   ROW(I)=CAT2(MK)
528   WRITE(OUT,974) J,(ROW(I),I=1,NX )
      IF(ISTEP.NE.NSTEP) GO TO 540
C     PRINTOUT UPCONED VALUES FOR THE TIME STEP.
      ISEQ=(NX+9)/10
      DO 530 M=1,ISEQ
      IFIRST=(M-1)*10+1
      ILAST=M*10
      IF(ILAST.GT.NX) ILAST=NX
      IF(M.EQ.1) WRITE(OUT,980)
      IF(M.GT.1) WRITE(OUT,981)
      WRITE(OUT,977) (I,I=IFIRST,ILAST)
      DO 530 J=1,NY
530   WRITE(OUT,978) J,(UPC(I,J),I=IFIRST,ILAST)
540   CONTINUE
C     WRITE(OUT,982)
C     INCLUDE  AT  THIS  LOCATION  THE  TRANSPORT  ESTIMATION...
595   IF(ITEST.EQ.1) GO TO 9999
600   CONTINUE
C ---------------------------------------------------------------------------
C     FORMAT STATEMENTS
C ---------------------------------------------------------------------------
C
C     ALL  INPUT FORMAT STATEMENTS ARE LISTED HERE
C
```

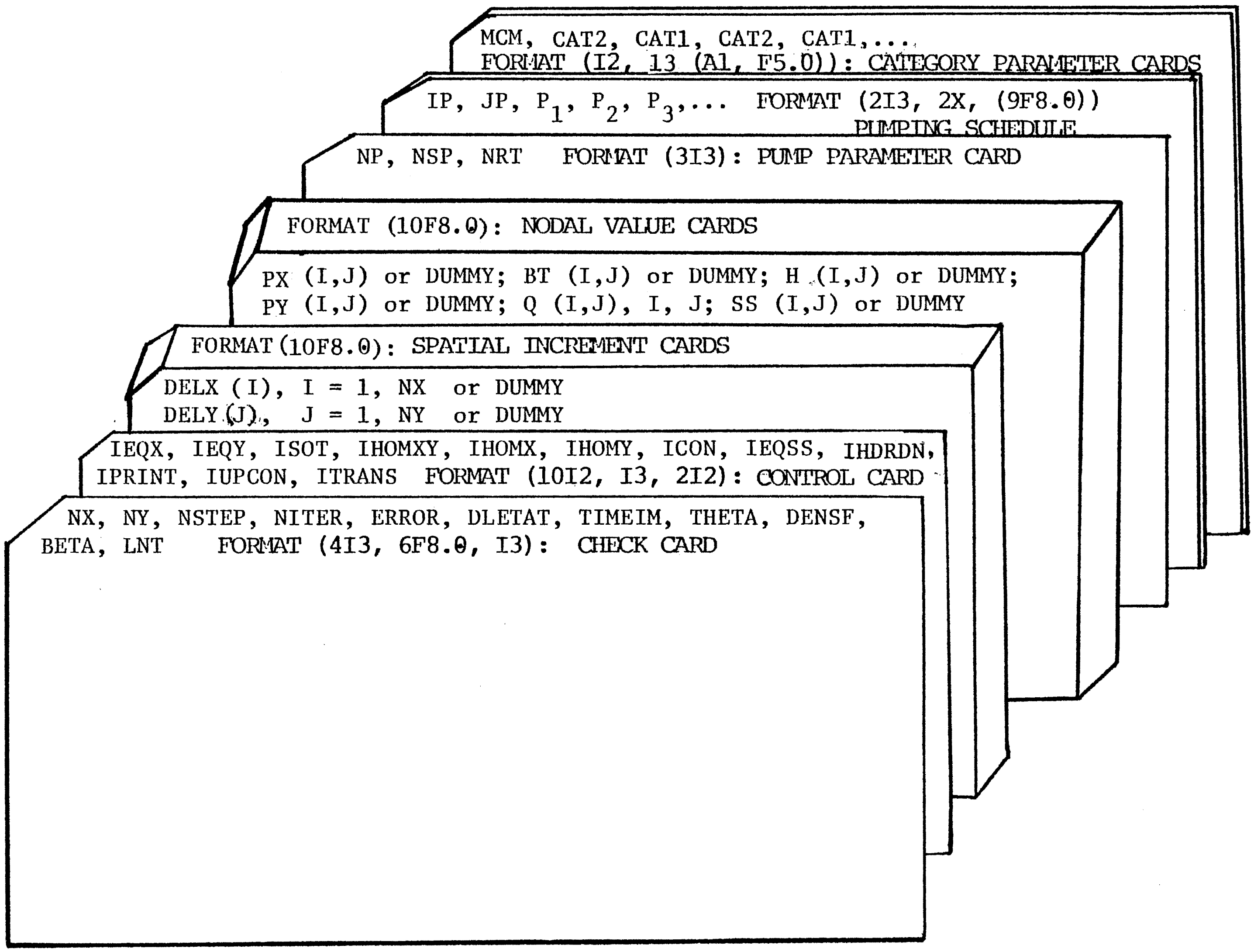
```
901   FORMAT(4I3,6F8.0,I3)
902   FORMAT(10I2,I3,2I2)
903   FORMAT(10F8.0)
904   FORMAT(F8.0)
905   FORMAT(5(I3,1X,I3,1X,F8.0))
906   FORMAT(3I3)
907   FORMAT(2I3,2X,(9F8.C))
908   FORMAT(I2,13(A1,F5.C),/(2X,13(A1,F5.0)))
C
C     ALL OUTPUT FORMAT STATEMENTS ARE LISTED HERE
C
951   FORMAT(1H1,////////////,
951   FORMAT(1H1,//,35X,'PRINTOUT OF ALL THE INPUT PARAMETERS.',/,35X,
     A        '------------------------------------',/,5X,
     B        'NX      = ',I3,/,5X,'NY      = ',I3,/,5X,'NSTEP = ',I3,/,
     C        5X,'NITER = ',I3,/,5X,'ERROR = ',E12.4,/,5X,'DELTAT= ',
     D        E12.4,/,5X,'TIMEIM= ',E12.4,/,5X,'THETA = ',E12.4,/,5X,
     E        'DENSF = ',E12.4,/,5X,'BETA  = ',E12.4,/,5X,'LNT    = ',
     F        I3,//)
952   FORMAT(5X,'IEQX  =',I2,'  (= 1,MEANS EQUAL GRID-SPACINGS IN ',
     A        'X-DIRECTION,OTHERWISE UNEQUAL GRIDS ARE USED)',/,
     B        5X,'IEQY  =',I2,'  (= 1,MEANS EQUAL GRID-SPACINGS IN ',
     C        'Y-DIRECTION,OTHERWISE UNEQUAL GRIDS ARE USED)',/,
     D        5X,'IHOMXY=',I2,'  (= 1,MEANS HOMOGENEOUS IN ALL ',
     E        'DIRECTIONS,OTHERWISE HETEROGENEOUS CONDITION EXISTS)',/,
     F        5X,'IHOMX =',I2,'  (= 1,MEANS HOMOGENEOUS CONDITION IN ',
     G        'X-DIRECTION,OTHERWISE HETEROGENEOUS CONDITION EXISTS)',/,
     H        5X,'IHOMY =',I2,'  (= 1,MEANS HOMOGENEOUS CONDITION IN ',
     I        'Y-DIRECTION,OTHERWISE HETEROGENEOUS CONDITION EXISTS)',/,
     J        5X,'ISOT  =',I2,'  (= 1,MEANS ISOTROPIC CONDITION ',
     K        'EXISTS,OTHERWISE ANISOTROPICITY PREVAILS)',/,
     L        5X,'ICON  =',I2,'  (= 1,MEANS CONFINED CASE CONSIDERED',
     M        ',OTHERWISE UNCONFINED CONDITION EXISTS)',/,
     N        5X,'IEQSS =',I2,'  (= 1,MEANS EQUAL STORATIVITY IN ',
     O        'WHOLE FLOW DOMAIN,OTHERWISE NOT EQUAL)')
960   FORMAT (5X,'IHDRDN=',I2,'  (= 1,MEANS EQUAL HEAD/DRAWDOWN ',
     Q        'THROUGHOUT THE FIELD,OTHERWISE EQUAL VALUES EXIST)',/,
     R        5X,'IPRINT=',I2,'  (= 1,MEANS PRINTOUT OF ALL THE INPUT',
     S        'PARAMETERS IS DESIRED,OTHERWISE NOT)',/,
     T        5X,'IDISCH=',I3,'  (INDICATES TOTAL NO. OF NODES HAVING ',
     U        'NON-ZERO NET FLUX)',/,
     V        5X,'IUPCON=',I2,'  (=1,IF UPCONING ESTIMATION IS ',
     W        'DESIRED,OTHERWISE NOT)',/,
```

```
     X       5X,'ITRANS=',I2,'  (=+1,IF UPCONING IS TO BE ESTIMATED ',
     Y       'BY TRANSPORT THEORY ONLY;',/,17X,'= 0,IF BY MUSKAT ',
     Z       'APPROACH ONLY;',/,17X,'=-1,IF BY BOTH APPROACHES)',/)
953   FORMAT (5X,'SPACIAL  INCREMENTS :',/)
954   FORMAT (5X,'IN X-DIRECTION...',/,(2X,13F6.0))
955   FORMAT (5X,'IN Y-DIRECTION...',/,(2X,13F6.0))
956   FORMAT(5X,'NO. OF PUMPS=',I3,/,5X,'NO. OF TIME INCREMENTS PER ',
     $       'PUMPAGE CHANGE=',I3,/,5X,'NUMBER OF RATES IN PUMPING ',
     $       'SCHEDULE=',I3,/,5X,'PUMPING SCHEDULES GIVEN BELOW :::',/)
957   FORMAT(5X,2I3,10(1X,E10.4))
958   FORMAT(5X,'CATEGORY PRINTOUT LEGEND : ',//,
     $       (2X,A2,' IF HEAD IS GREATER THAN OR EQUAL TO',F8.2))
958   FORMAT(1H1,//,5X,'CATEGORY PRINTOUT LEGEND (HEAD/DRAWDOWN) :',10X,
     $       'CATEGORY PRINTOUT LEGEND (UPCONED SURFACE) :',//,
     $       (5X,A2,' IF HEAD IS GREATER THAN OR EQUAL TO',F8.4,8X,
     $       A2,' IF UPCONE IS GREATER THAN OR EQUAL TO DENSF*',F8.4))
959   FORMAT(1H1,//,35X,'PRINTOUT OF THE RESULTS.',/,35X,
     $       '------------------------',//)
970   FORMAT(5X,I3,2X,'NO. OF ITERATION PARAMETERS AS FOLLOWS :',//,
     $       5X,10(1X,E11.5))
971   FORMAT(/,5X,'NO. OF ITERATIONS EXCEEDED THE LIMIT SPECIFIED',/)
972   FORMAT (5X,'TIME STEP=',I5,10X,'DELTAT=',F10.4,'  DAYS',10X,
     $       'TIME=',F10.4,'  DAYS',/,
     $        5X,'ERROR IN THE SOLUTION = ',E13.5,10X,
     $        'NO. OF ITERATIONS COVERED = ',I3,/)
973   FORMAT (5X,'CATEGORY MAP PRINTOUT ( HEAD/DRAWDOWN ) :',/)
974   FORMAT (5X,I4,2X,50A2)
975   FORMAT(/////,5X,'HEAD VALUES AT THE END OF THIS TIME STEP :',/)
976   FORMAT(5X,'HEAD VALUES(CONTINUED): ')
977   FORMAT(14X,I3,9(8X,I3))
978   FORMAT(5X,I3,2X,10E11.4)
979   FORMAT(//,5X,'CATEGORY MAP PRINTOUT ( UPCONED SURFACE ): ',/)
980   FORMAT(//,5X,'UPCONED VALUES AT THE END OF THIS TIME STEP :',/)
981   FORMAT(5X,'UPCONED VALUES (CONTINUED):')
982   FORMAT(/,5X,'UPCONING HERE IS ESTIMATED BY ONE-DIMENSIONAL ',
     $            'TRANSPORT PHENOMENON DEVELOPED BY JACOB BEAR.',/)
983   FORMAT(1H1,///)
9999  RETURN
```

```
888   WRITE(OUT,991) NC,NR,NX,NY
991   FORMAT(//,5X,'WARNING :: ERROR OCCURED AT THE STEP OF DIMENSION',
     $        ' CHECKING',/,5X,'DIMENSION DEFINED FOR THE ARRAYS IS  ',
     $        I3,',',I3,/,5X,'DIMENSION READ FOR THE ARRAYS IS  ',I3,
     $        ',',I3,//)
      RETURN
885   WRITE(OUT,992)
992   FORMAT(//,5X,'SORRY SIR/MADAM    THIS PROGRAM CAN NOT HANDLE ',
     $         'UNCONFINED CASE FOR SIMULATION.',/,5X,'IF YOU DESIRE TO '
     $         ,'HAVE CONFINED CASE SOLUTION THEN DEFINE THE VALUE OF ',
     $         'ICON = 1',//)
      RETURN
      END
```

APPENDIX D

INPUT CARD FORMAT



GENERAL FORMAT OF INPUT CARD SETUP FOR THE MODEL

APPENDIX E

INPUT CARD DECK

```
37 28 15 20    1E-5     5E0   12E-1    3E-1     4E1     0E0  5
1 1 1 0 0 0 1 1 1 1  9 1-1
  48E1
  48E1
   0E0    0E0    0E0    0E0    0E0    0E0    0E0    0E0    0E0    0E0
   0E0    0E0    0E0    0E0    0E0    0E0    0E0    0E0    0E0    0E0
   0E0    0E0    0E0    0E0    0E0    0E0    0E0    0E0    0E0    0E0
   0E0    0E0    0E0    0E0    0E0    0E0    0E0    0E0   28E2   28E2
  27E2   27E2   27E2   26E2   26E2   26E2   26E2   26E2   26E2   26E2
  26E2   26E2   26E2   26E2   26E2   26E2   26E2   26E2   26E2   26E2
  26E2   26E2   26E2   26E2   26E2   26E2   26E2   26E2   26E2   26E2
  26E2   26E2   26E2    0E0    0E0   28E2   28E2   27E2   27E2   27E2
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   0E0
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21  13        1E5  28   13      12E4  32   18        1E5  34   24        9E4
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 1   1        0E0
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VITA 2

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Master of Science

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Major Field: Civil Engineering Minor Field: Geology

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