COORDINATION OF INPUT-CRANK MOTION WITH THE ENVELOPING TANGENT-LINE VIA GEARED FIVE-BAR LINKAGE

By

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PREFACE

The purpose of this study is to develop a method for synthesizing a geared five-link mechanism by coordinating the finite and infinitesimal displacements of the input-crank with that of a tangent-line which draws a prescribed enveloping curve. The position, velocity, acceleration, jerk, and kerk of both the input-crank and the tangent-line are possible design parameters. The procedure developed would be incorporated into a computer program, for ease of synthesis.

I deeply wish to acknowledge the assistance and continuous encouragement received from my thesis adviser, Dr. A.H. Soni. Thanks are also due to my friends, Dave Sathyadev, Ram Gudavlli, and Mohan Sankaran for their stimulative discussions on the subject.

I also wish to express my appreciation to my parents for their continuous love and moral support. Special thanks go to my friend Mehrdad Tehrani for his valuable encouragement.

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CHAPTER I

INTRODUCTION

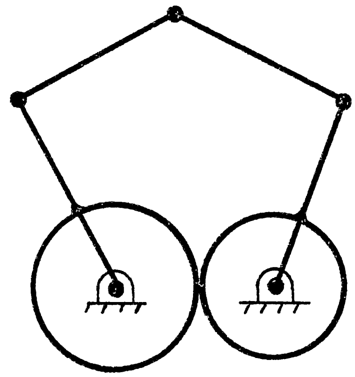
Developing fundamental theories for rigid body motion on a plane or in space has been a problem of significant interest kinematicians. With these developments, it then becomes possible to design mechanical devices or mechanisms to perform a variety of tasks for industrial applications. In developing such theories, kinematicians have contributed significantly to the problems on rigid-body guidance, point-path generation, and coordination of input and output links. Recently, there is an increased interest in studying the motion of a line and developing synthesis procedures to design mechanisms to generate an enveloping curve. This thesis extends this concept to the design of a geared five-link mechanism, in which, the motion of the input-crank is coordinated with the positions of a tangent-line drawing a prescribed enveloping curve.

Design of linkages with gears dates back to more than two centuries. According to Duffy [1]*, the first study of geared mechanisms performed in 1706 by De La Hires [2] showed that two distinct cycloidal systems generated the same point path. The geometric, algebraic, and harmonic properties of more complex gear curves were

*Numbers in brackets refer to numbered references in the Bibliography.

considered by Wunderlich [3,4] in detail with careful consideration for their synthesis to approximate a desired point-path. Detailed analytical as well as graphical descriptions of cycloidal point paths were given by Schmidt [5]. A detailed analysis of the algebraic properties of the knee curve for these mechanisms was suggested by Freudenstein and Primrose [6]. A synthesis procedure for designing geared five-bar mechanisms useing the displacement matrix was developed by Suh and Radcliffe [7]. Rooney and Jones [8] analyzed of various five-bar mechanisms including sliding pairs and supplemented their results using analog simulation. Kaufman and Sandor used cycloidal constraints to synthesize, a specified Co-planar motion [9]. Sandor and Kaufman [10] discussed a synthesis procedure for finitely separated positions, for the geared five-bar. Myklebust and Tesar [11], studied coplanar synthesis by algebraic methods for five finitely separated positions of these mechanisms. Mohan Rao and Sandor [12] studied the correlation of input and output crank positions of geared five-bars for four and five point approximation. Erdman and Sandor [13] discussed the kinematic synthesis of a geared five-bar function generator. Lee and Freudenstein [14] investigated the design of geared five-bar mechanisms for unlimited crank rotation and optimum transmission. Fitcher and Hunt [15] discussed the degree of the input-output equations of certain geared five-bar mechanisms. And, finally, Vadasz and Soni [16] studied the limit and dead center positions of geared five-link mechanisms. However, a clear and general method leading to the coordination of input-crank motion of geared five-bars with the enveloping tangent-line for a number of finitely and infinitesimally separated positions has not been addressed in any of the previous works. In this thesis, the loop-closure synthesis method is combined with the planar tangent-line concept, in order to develop an analytical closed-form synthesis procedure for five finitely separated positions. This method is preferred because the equations used for the finitely separated positions synthesis can be readily differentiated to obtain the equations for the infinitesimally separated position synthesis of any degree. The advantage of combining the loop-closure synthesis method with the planar tangent-line concept is that the synthesis equations are independent of the output angle. Consequently, the designer is not required to provide any initial information concerning the output link oscillation angles. Moreover, this method accommodates for any combination of finitely and infinitesimally separated positions, thus enhancing the capability of the designer to meet a wide range of position requirements. The increased mobility, versatility, and force transmission characteristics of the geared five-link mechanisms have been utilized in many practical applications, such as shaking machines for vibration testing, high-speed presses, textile machinery, automobile hood linkages, etc.

Geared five-link mechanisms have long been studied in a variety of forms and under a variety of names. Three types of geared five-link mechanisms consisting of two gears are illustrated in Figure (1). The geared five-link mechanism of type 1 consists of two geared cranks mounted on fixed revolute pairs. The motion of the mechanism can be controlled by attaching the input source to either of the two gears. Geared five-link mechanism of type 2 consists of one fixed gear at one



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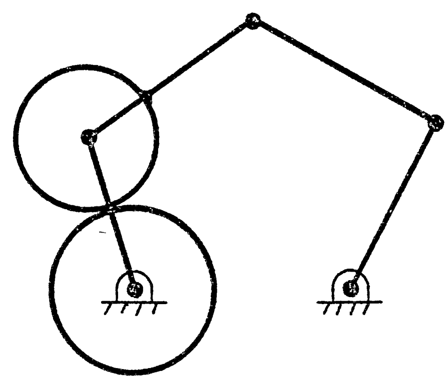

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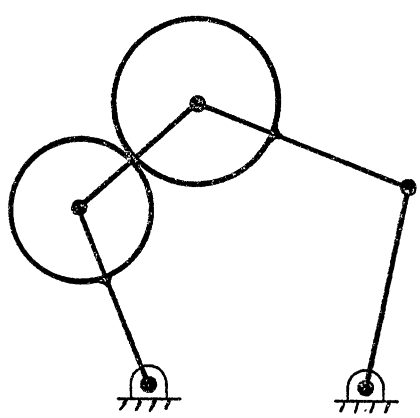

Type-3

Figure 1. Geared Five-Link Mechanisms

of the fixed revolute pairs and a second gear which is rigidly fixed to the second input link, pivots at the end of the first input link. In this case, the input source can be attached to the link which connects the gears. Geared five-link mechanism of type 3 consists of two gears which are fixed to two revolute pairs located at the ends of a floating link. The nearest gear to any of the fixed revolute pairs is fixed with the link which is pivoting about that fixed revolute pair. The gear farther from the fixed link is fixed onto the second floating link. Geared five-link mechanisms of type 1 and 2 are the two most commonly used in industry. This may be due to their superior dynamic characteristics and versatility over the type 3 mechanisms. The input source for type 1 and 2 of these mechanisms can be mounted on the fixed base. But for type 3, the input source should be carried on the floating link which connects the gears. Consequently, it creates a severe dynamic problem from the point of view of balancing. Geared five-link mechanisms of type 1 and 2 have two unattached links to the gears which provide increased mobility for these mechanisms. Geared five-link mechanism of type 3 has one of the two free links grounded, which greatly reduces the versatility of this type.

Although these three types of geared five-link mechanisms differ in their dynamic characteristics, the synthesis procedures developed in this thesis are quite similar. Therefore, this thesis will decument the synthesis procedure for the geared five-link mechanisms of type 2 only.

The significant contributions of this thesis are summarized below.

(1) Development of a generalized method for the synthesis of geared five-link mechanisms with an enveloping tangent-line for five

finitely separated positions.

(2) Development of a generalized method for the synthesis of geared five-link mechanisms with an enveloping tangent-line for first, second, third, and fourth infinitesimally separated positions.

CHAPTER II

DESCRIPTION OF THE GEARED FIVE-LINK MECHANISM IN STUDY

In general, a five-link mechanism has two degrees of freedom. In the case of geared five-link mechanisms, one degree of freedom is obtained by using a pair of gears. Figure (2) shows the geared five-link mechanism which will be studied in this thesis. In this figure, M, A, B, C, and Q are revolute pairs joining the rigid links a, b, c, and d, and the ground link MQ. The links a, b, c, and d are the first input link, second input link, coupler link, and output link respectively. The rigid link f is attached rigidly to the coupler link at the pin joint B and its orientation is measured by an angle α with respect to the coupler link. The link oo' is a rigid link pivoting about the origin of the fixed coordinate system (XY). The motion of the rigid tangent-line EE' is controlled by two sliders at points E and P. These two sliders are oriented at right angles to each other. The sliders at P and E are fixed perpendicular to the link f and the tangent-line EE' respectively. The gear G_1 is the fixed gear and the gear G_2 pivots on first input link and is rigidily fixed onto the second input link. The angles θ_2, θ_3, θ_4, and θ_5 measure the orientations of the links a, b, c, and d respectively, in counterclockwise direction with respect to X-axis. The angle θ measures the orientations of the links f and oo' in counterclockwise direction with respect to X-axis. The input angles θ_2

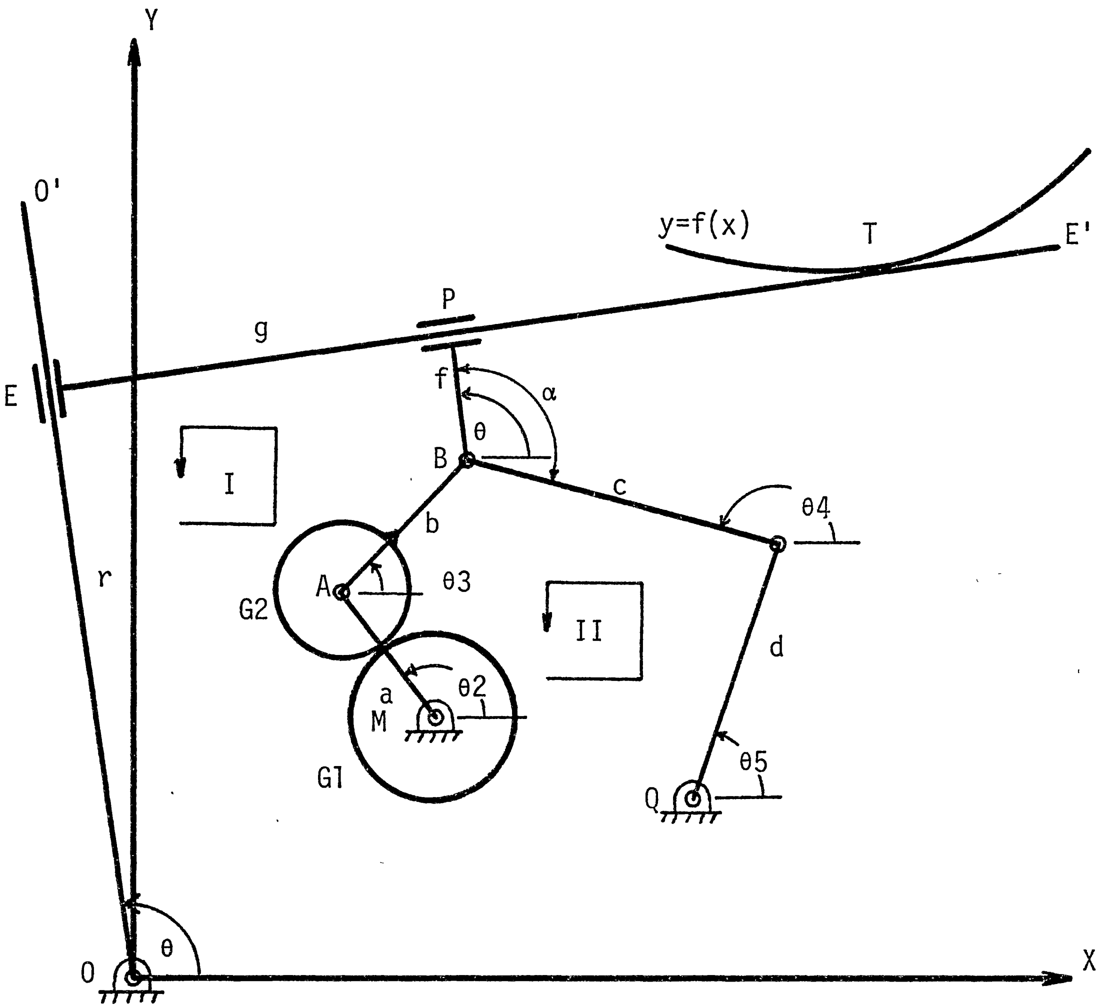

Figure 2. Configuration of the Geared
Five-Link Mechanism

and θ_3 are linearly related according to the following relationship.

$$\theta_3 = (1+GR)\theta_2 + \psi = N\theta_2 + \psi \tag{1}$$

where, GR is the gear ratio of the gears G_1 and G_2, and is equal to the ratio of pitch diameters of the gears G_1 and G_2. The angle ψ is called the phase angle and it represents the initial orientation of the second input link with respect to the first input link. N is called the gear train speed ratio and is equal to (1+GR).

CHAPTER III

GENERALIZED PROCEDURES FOR SYNTHESIS OF GEARED FIVE-LINK MECHANISMS

In this thesis, closed form synthesis equations are developed for the synthesis of a geared five-link mechanism function generator with an enveloping tangent-line for five finitely separated precision positions. Loop-closure method and tangent-line concepts are applied in order to derive the synthesis equations. Five precision positions were chosen, since five is the maximum number of precision positions for which the synthesis equations involving the loop (I) as shown in figure (2) can be made linear. The complete synthesis of the mechanism is performed in two steps. In the first step, a set of simultaneous linear synthesis equations will be solved for the unknowns loop (I). In the second step, a set of nonlinear synthesis equations will be solved using Newton-Raphson method [17] for the unknowns in loop (II) of the mechanism. For five precision positions synthesis, the designer must supply the input angles corresponding to each precision position as well as the gear ratio of the gears and the phase angle between the first and second input links. The selection of the gear ratio and the phase angle allows the designer to come up with a good practical or optimum design for a set of specified input angles. However, the optimization of the final design is beyond the scope of this thesis.

Once the synthesis equations for finitely separate precision positions were obtained, these equations are differentiated successively in order to obtain the synthesis equations for first, second, third, and fourth infinitesimally separated precision positions. A set of nomenclature for describing the possible combinations of finitely and infinitesimally separated displacements for five separate positions has been suggested by Tesar [18]. The possible combinations are:

P-P-P-P-P			
PP-P-P-P	P-PP-P-P	P-P-PP-P	P-P-P-PP
PP-PP-P	PP-P-PP	P-PP-PP	
PPP-P-P	P-PPP-P	P-P-PPP	
PPP-PP	PP-PPP		
PPPP-P	P-PPPP		
PPPPP			

The symbols P-P and PP represent two finitely and two infinitesimally separated positions respectively. In this thesis, we present an analytical method of synthesizing for all of the above motions. These combinations can be applied to obtain a wide variety of function generation motions. These procedures can be readily adapted to any specified function in the form of $y = f(x)$. Graphical configurations of the arbitrarily designed mechanisms will also be presented for some of the above combinations of motions, in order to demonstrate the capabilities of the synthesis procedures.

CHAPTER IV

LOOP-CLOSURE SYNTHESIS METHOD FOR FINITELY SEPARATED PRECISION POSITIONS

The basic constraint on the function which needs to be generated is that it is continuous and, differentiable in the interval between the first and the last precision position. We let the function be expressed as $y = f(x)$ and the coordinates of each specified precision position (SPP) on this function as X_{Ti} and Y_{Ti} for $i = 1,2,..,5$. We also let the input angles be expressed as θ_{2i}, and their corresponding second input angles are obtained from equation (1) ($\theta_{3j} = N\theta_{2j} + \psi$, $J = 2,..,5$). At the first precision position, the orientation of the second input link is specified by the phase angle measured with respect to the first input link.

Since, the tangent-line EE' must remain tangent to the function at all specified separated precision positions, we can express the slope of the tangent-line at the i^{th} precision position by differentiating the function with respect to x for each (SPP) as

$$S_i = \frac{d}{dx} f(x) \bigg|_{x_{Ti}} \qquad i = 1,2,..,5$$

where S_i is the slope of the tangent-line at the i^{th} (SPP). Having defined the slope of the tangent-line at the i^{th} (SPP), we can express the magnitude and orientation of an oscillating vector $\bar{r}$, which corresponds to the position and orientation of the slider E, as

$$|r_i| = (X^2_{Ei} + Y^2_{Ei})^{1/2}$$

$$\theta_i = \text{ATAN }(Y_{Ei}/X_{Ei})$$

where

$$X_{Ei} = (S_i\ X_{Ti} - Y_{Ti})/(S_i + 1/S_i)$$

and

$$Y_{Ei} = (-X_{Ei})/(S_i) \qquad \text{for all } i = 1,2,..,5$$

All the above equations are valid for all values of S_i; except for the case $S_i = 0$. However, in most practical cases, a null slope can be replaced by a very small nonzero value, in order to avoid division by zero errors. Having described the vector $\bar{r}$, we can now write the synthesis equations for the loop (I) in figure (2) using vector loop-closure method.

$$\overline{OM} + \overline{MA} + \overline{AB} + \overline{BP} + \overline{PE} = \overline{OE} \qquad (2)$$

where each vector in equation (2) can be expressed in terms of its real and imaginary components as

$$\overline{OM} = X_m - jY_m$$

$$\overline{MA} = a \cos \theta_2 - ja \sin \theta_2$$

$$\overline{AB} = b \cos \theta_3 - jb \sin \theta_3$$

$$\overline{BP} = f \cos \theta - jf \sin\theta$$

$$\overline{PE} = -g \sin \theta - jg \cos\theta$$

$$\overline{OE} = r \cos \theta - jr \sin\theta$$

where X_m, Y_m are the x and y coordinates of the pin-joint M in the fixed coordinate system XY. And "g" is the distance between the sliders at P and E. Now substituting the above expressions into equation (2) and separating the real and imaginary terms, we have

$$X_m + a \cos \theta_2 + b \cos \theta_3 + f \cos \theta - g \sin \theta = r \cos \theta \qquad (3\text{-a})$$

$$Y_m + a \sin \theta_2 + b \sin \theta_3 + f \sin \theta + g \cos \theta = r \sin \theta \qquad (3\text{-b})$$

Multiplying equation (3-a) by ($\cos\theta$), and equation (3-b) by ($\sin\theta$) and adding both equations together in order to eliminate the term "g", it yields:

$$X_m \cos \theta + Y_m \sin\theta + a \cos (\theta - \theta_2) + b \cos (\theta - \theta_3) + f = r \qquad (4)$$

Such an equation can be written for each (SPP). So, it is more convenient to express equation (4) in its general, simplified form as

$$K_{1i} X_m + K_{2i} Y_m + K_{3i}a + K_{4i}b + f = r_i \qquad (5)$$

where

$$K_{1i} = \cos \theta_i$$
$$K_{2i} = \sin \theta_i$$
$$K_{3i} = \cos (\theta_i - \theta_{2i})$$
$$K_{4i} = \text{Cos} (\theta_i - \theta_{3i}) \qquad i = 1,2,..,5$$

Equation (5) yields the synthesis equations loop (I) of the mechanism from which five linear equations corresponding to the five (SPP) are obtained. Then, this set of linear equations is to be solved simultaneously, in order to find the unknown values X_m, Y_m, a, b, and f.

The equations (3-a) and (3-b) can also be manipulated in a different manner to yield an expression for g_i (the distance between the sliders P and E at the i^{th} (SPP)). The procedures would still be the same, excepting that we multiply equation (3-a) by (sinθ) and equations (3-b) by (-cosθ) in order to eliminate the unwanted term "f" from these two equations. This yields:

$$k_{2i} X_m - K_{1i} Y_m + K_{5i} a + K_{6i} b = g_i \tag{6}$$

where

$$K_{5i} = \sin(\theta_i - \theta_{2i})$$
$$K_{6i} = \sin(\theta_i - \theta_{3i}) \qquad i = 1,2,..,5$$

After solving equation (5) for the unknown terms, we can evaluate equation (6) to check if there is any crossing-over of the sliders at E and P. If the sign of g_i changes as the mechanism moves from the i^{th} (SPP) to the $(i+1)^{th}$ (SPP), the slides will cross over. From a practical design point of view, such a phenomenon cannot be tolerated. However, selection of the gear ratio and the phase angle will give the designer enough freedom to come up with a practical design and avoid such a problem in most cases.

Similarly, another set of synthesis equations can be written for the loop (II) of the mechanism by following the same procedures explained above. We first write the vector loop-closure expression for the loop (II) from figure (2) as

$$\overline{OM} + \overline{MA} + \overline{AB} + \overline{BC} = \overline{OQ} + \overline{QC} \tag{7}$$

the above vectors can be expressed in terms of their real and imaginary components as:

$$\overline{OM} = X_m - jY_m$$
$$\overline{MA} = a \cos\theta_2 - ja \sin\theta_2$$
$$\overline{AB} = b \cos\theta_3 - jb \sin\theta_3$$
$$\overline{BC} = c \cos(\theta - \alpha) - jc \sin(\theta - \alpha)$$
$$\overline{OQ} = X_Q - j\,Y_Q$$
$$\overline{QC} = d \cos\theta_5 - jd \sin\theta_5$$

where X_Q and Y_Q are the x and y coordinates of the pin-joint Q in the fixed coordinate system XY. Substituting the above expressions into equation (7) and separating the real and imaginary terms, it yields

$$X_m + a \cos\theta_2 + b \cos\theta_3 + c \cos(\theta - \alpha) - X_Q = d \cos\theta_5 \tag{8-a}$$

$$Y_m + a \sin\theta_2 + b \sin\theta_3 + c \sin(\theta - \alpha) - Y_Q = d \sin\theta_5 \tag{8-b}$$

Now, we can eliminate the unwanted output angle (θ_5) from these equations by squaring and adding these two equations and expressing the resulting equation for the i^{th} (SPP) in its general form, as

$$X_Q^2 + Y_Q^2 + c^2 - d^2 + W_{1i}X_Q + W_{2i}Y_Q + W_{3i}c + w_{4i} = 0 \tag{9}$$

$$i = 1,2,..,5$$

where

$$W_{1i} = (K_{7i} + K_{8i}c \cos\alpha + K_{9i}c \sin\alpha)$$
$$W_{2i} = (K_{10i} + K_{9i}c \cos\alpha - K_{8i}c \sin\alpha)$$

$W_{3i} = (K_{11i} + K_{12i} + K_{13i} + K_{14i})\cos\alpha + (K_{15i} + K_{16i} + K_{17i} + K_{18i})\sin\alpha]$

$W_{4i} = K_{19i}$

$K_{7i} = -2(X_m + \cos\theta_{2i} + b\cos\theta_{3i})$

$K_{8i} = -2K_{1i}$

$K_{9i} = -2k_{2i}$

$K_{10i} = -2(Y_m + a\sin\theta_{2i} + b\sin\theta_{3i})$

$K_{11i} = 2X_m K_{1i}$

$K_{12i} = 2Y_m K_{2i}$

$K_{13i} = 2a\cos(\theta_i - \theta_{2i})$

$K_{14i} = 2b\cos(\theta_i - \theta_{3i})$

$K_{15i} = 2X_m K_{2i}$

$K_{16i} = -2Y_m K_{1i}$

$K_{17i} = -2a\sin(\theta_{2i} - \theta_i)$

$K_{18i} = -2b\sin(\theta_{3i} - \theta_i)$

$$K_{19i}=X_m^2+Y_m^2+a^2+b^2+2a(X_m\cos\theta_{2i}+Y_m\sin\theta_{2i})+2b(X_m\cos\theta_{3i}+Y_m\sin\theta_{3i})$$
$$+2ab\cos(\theta_{3i}-\theta_{2i}) \qquad i=1,2,..,5$$

Equation (9) consists of five independent nonlinear equations corresponding to the five (SPP)s which can be solved by Newton-Raphson method to yield the solution for the unknown parameters X_Q, Y_Q, c, d, and α. Having solved equations (5) and (9), we can construct our designed geared five link mechanism. While constructing the designed mechanism, one needs to know the output angle at the i^{th} (SPP) which can be readily determined from dividing equation (8-a) by (8-b) which yields:

$$\theta_{5i} = ATAN(Z_{1i}/Z_{2i}) \qquad (10)$$
$$i=1,2,..,5$$

where

$$Z_{1i} = (X_m + a\ \cos\theta_{2i} + b\ \cos\theta_{3i} + c\ \cos(\theta_i - \alpha) - X_Q)$$

$$Z_{2i} = (Y_m + a\ \sin\theta_{2i} + b\ \sin\theta_{3i} + c\ \sin(\theta_i - \alpha) - Y_Q)$$

An example of the above sample synthesis problem for five finitely separated precision positions is provided in Table I. The mechanism that was designed for this input data is drawn to full scale in figure 3.

TABLE I

SYNTHESIS OF A GEARED FIVE-LINK MECHANISM
FOR FIVE FINITELY (SPP)

INPUT DATA

Function $y = -0.3x^3 + 0.2x^2 - 0.3x + 3.0$

XT(1) = -1.000	YT(1) = 3.800	S(1) = -1.600
XT(2) = -0.500	YT(2) = 3.237	S(2) = -0.725
XT(3) = 0.000	YT(3) = 3.000	S(3) = -0.300
XT(4) = 0.500	YT(4) = 2.862	S(4) = -0.325
XT(5) = 1.000	YT(5) = 2.600	S(5) = -0.800
θ(1) = 32.005	θ2(1) = 160.000	θ3(1)=220.000
θ(2) = 54.058	θ2(2) = 150.000	θ3(2)=200.000
θ(3) = 73.301	θ2(3) = 140.000	θ3(3)=180.000
θ(4) = 71.996	θ2(4) = 130.000	θ3(4)=160.000
θ(5) = 41.340	θ2(5) = 120.000	θ3(5)=140.000

GR = 1.000
ψ = 60.000

RESULTS

X_m = 1.002	g(1) = -2.291
Y_m = 0.289	g(2) = -1.642
a = 3.038	g(3) = -0.651
b = 1.317	g(4) = -0.396
f = 0.729	g(5) = -0.910
X_Q = -0.741	θ5(1) = 95.738
Y_Q = -1.101	θ5(2) = 86.581
c = 0.241	θ5(3) = 78.381
d = 3.185	θ5(4) = 72.997
α = 174.258	θ5(5) = 70.471

All angles are in degree.

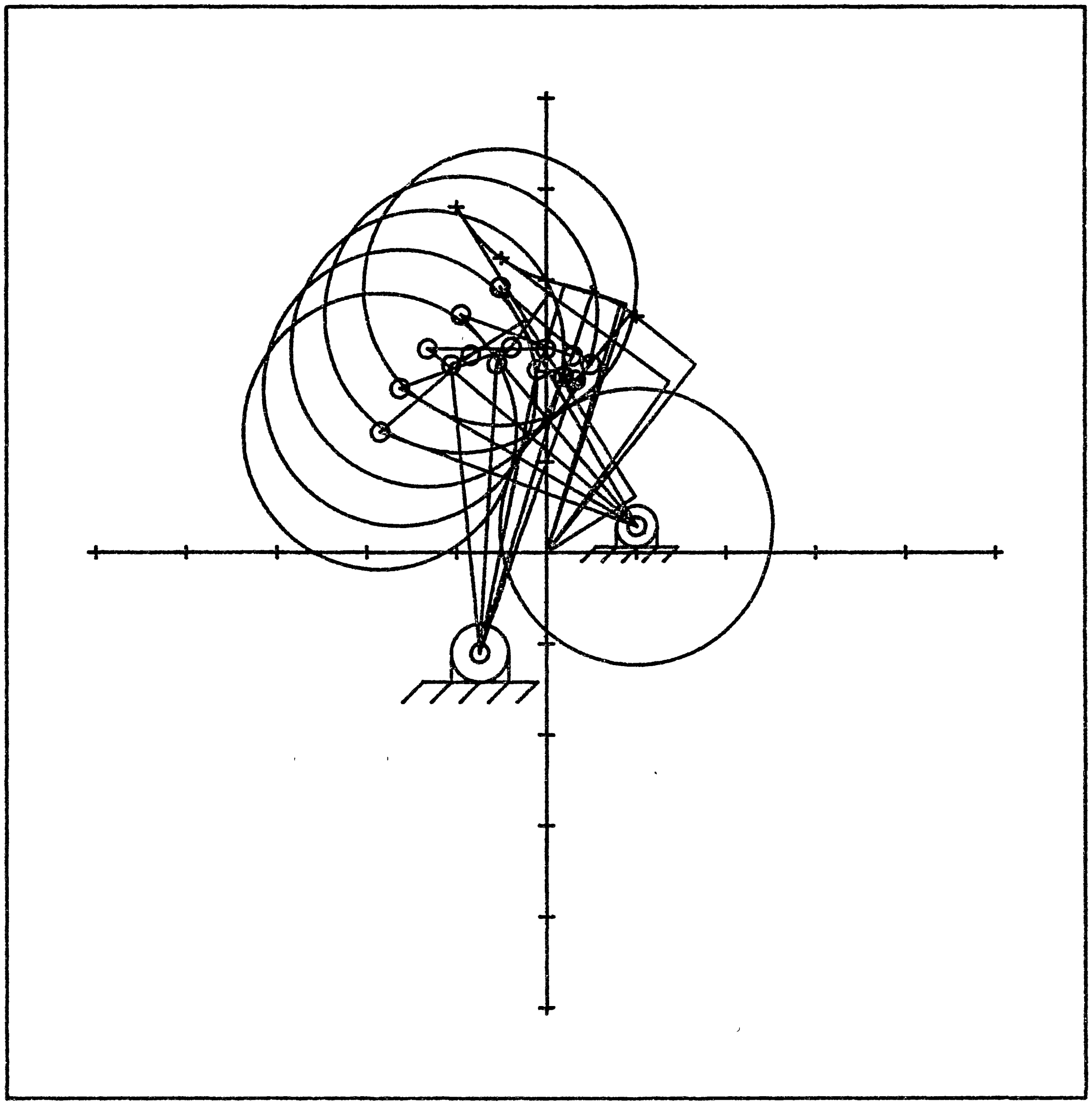

Figure 3. Configuration of the Designed
Mechanism For Type P-P-P-P-P

CHAPTER V

DESCRIPTION OF INFINITESIMALLY SEPARATED DISPLACEMENTS OF RIGID BODY

Generally, motion of a rigid body can be described in a number of ways. For instance, it can be described by a series of successive infinitesimal screw displacements or by the displacement of a point in the body and the rotation of the body. In either case, one can choose one of parameters as the reference parameter of the motion, and express the other parameter as a function of the reference parameter. In the same fashion, the instantaneous motion of a rigid body involving infinitesimal changes in angular displacement can be described by taking the derivative with respect to time successively upto any desired order, along with specifying the successive derivatives of the linear motion with respect to time upto any desired order.

The infinitesimally separated displacements of a rigid body tangential to any curve, is described by the properties of the rigid body as it approaches the curve at the tangential point. These properties may be the velocity, acceleration, jerk, time rate of change of jerk (kerk) etc. In this thesis the tangent-line EE' is considered as a rigid body whose properties are considered to be the same as that of the desired specified infinitesimal displacement at the i^{th} finitely separated precision position on the specified function. Hence, the instantaneous angular motion of the rigid body EE' involving infinitesimal changes in angular displacements can be described with respect to changes in time by specifying $d\theta/dt$, $d^2\theta/dt^2$, $d^3\theta/dt^3$, $d^4\theta/dt^4$, and so on, upto the required order.

In the previous chapter, we developed the synthesis equations for five finite precision positions of the geared five-link function generation mechanism. In the following chapters, we will develop the synthesis precedures for infinitesimal displacements of these function generation mechanisms.

CHAPTER VI

SYNTHESIS PROCEDURES FOR FIRST ORDER INFINITESIMALLY SEPARATED POSITIONS

The synthesis equations derived in the previous chapter will be used here derive the synthesis equations for any combination of first order infinitesimal displacements for five finite precision positions. In general, for the first order infinitesimal displacement of five finitely (SPP), two basic combinations exist.

PP-P-P-P	P-PP-PP
P-PP-P-P	PP-P-PP
P-P-PP-P	PP-PP-P
P-P-P-PP	

The first combination, describes four finite precision positions and one first order infinitesimal displacement at one of the four precision positions. The second combination describes three precision positions and one infinitesimal displacement at two of the finite positions. However, in either case, the first infinitesimal displacement synthesis equations at the precision position J is obtained by differentiating the synthesis equations (5) and (9) with respect to time, it yields

$$F_{1J}X_m + F_{2J}Y_m + F_{3J}a + F_{4J}b = \dot{r}_J \tag{11}$$

where

$F_{1J} = -\dot{\theta}_j \sin\theta_J$

$F_{2J} = \dot{\theta}_J \cos\theta_J$

$F_{3J} = -(\dot{\theta}_J - \dot{\theta}_{2J}) \sin(\theta_J - \theta_{2J})$

$F_{4J} = -(\dot{\theta}_J - \dot{\theta}_{3J}) \sin(\theta_J - \theta_{3J}) = -(\dot{\theta}_J - N\dot{\theta}_{2J}) \sin(\theta_J - \theta_{3J})$

$J = 1,..,4$

The term $\dot{r}_J$ is the first time rate of change of radial component of the vector $\bar{r}$ (procedures for evaluating the radial time derivatives of the vector $\bar{r}$ is provided in Appendix A). From equation (1), we have

$$\theta_3 = (1+GR)\theta_2 + \psi = N\theta_2 + \psi$$

Successive differentiation of equations (1) yields:

$$\dot{\theta}_3 = N\dot{\theta}_2 \quad (12)$$

$$\ddot{\theta}_3 = N\ddot{\theta}_2 \quad (13)$$

$$\dddot{\theta}_3 = N\dddot{\theta}_2 \quad (14)$$

$$\ddddot{\theta}_3 = N\ddddot{\theta}_2 \quad (15)$$

- - - - -

- - - - -

Now, differentiating equation (9) with respect to time,

$$F_{5J}X_Q+F_{6J}cX_Q+F_{7J}Y_Q+F_{8J}cY_Q+(F_{9J}+F_{10J}+F_{11J})c+(F_{12J}+F_{13J}+F_{14J})=0 \quad (16)$$

where

$F_{5J} = \dot{\theta}_{2J}(a \sin\theta_{2J} + Nb \sin\theta_{3J})$

$F_{6J} = \dot{\theta}_J(\sin(\theta_J - \alpha))$

$F_{7J} = -\dot{\theta}_{2J}(a \cos\theta_{2J} + Nb \cos\theta_{3J})$

$$F_{8J} = -\dot{\theta}_J(\cos(\theta_J - \alpha))$$

$$F_{9J} = \dot{\theta}_J(-X_m \sin(\theta_J - \alpha) + Y_m \cos(\theta_J - \alpha))$$

$$F_{10J} = -a(\dot{\theta}_{2J} - \dot{\theta}_J)(\sin(\theta_{2J} - \theta_J + \alpha))$$

$$F_{11J} = -b(N\dot{\theta}_{2J} - \dot{\theta}_J)(\sin(\theta_{3J} - \theta_J + \alpha))$$

$$F_{12J} = a\dot{\theta}_{2J}(-X_m \sin\theta_{2J} + Y_m \cos\theta_{2J})$$

$$F_{13J} = bN\dot{\theta}_{2J}(-X_m \sin\theta_{3J} + Y_m \cos\theta_{3J})$$

$$F_{14J} = ab(\dot{\theta}_{2J} - N\dot{\theta}_{2J}) \sin(\theta_{2J} - \theta_{3J})$$

$J=1,..,4$

Equation (11) is a linear equation in unknowns X_m, Y_m, a, and b and equation (16) is a nonlinear equation in unknowns X_Q, Y_Q, c, and α. Note that the differentiation of equation (5) and (9) causes the constant terms such as "f" and "d^2" to vanish. Again the subscript J corresponds to the J^{th} precision point, at which there are infinitesimal displacement. All the coefficients F_1........F_{15} are time dependent coefficients. The term $\dot{\theta}_{2J}$ is the known angular velocity of the first input link at the J^{th} finite precision point. $\dot{\theta}_j$ is the known angular velocity of the tangent-line as it approaches the specified function at the J^{th} finite precision position. The Tables (II) and (III) present two sample synthesis problems of the geared five-link mechanism for first order infinitesimal displacement of types PP-P-P-P and PP-PP-P respectively. The actual configurations of the arbitrarily designed mechanisms are shown in figures 4 and 5 respectively.

TABLE II

SYNTHESIS OF A GEARED FIVE-LINK MECHANISM FOR FIRST ORDER INFINITESIMAL DISPLACEMENT OF TYPE PP-P-P-P

INPUT DATA

Function $y = -0.3x^3 + 0.2x^2 - 0.3x + 3.0$

XT(1) = -1.000	YT(1) = 3.800	S(1) = -1.600
XT(2) = -0.500	YT(2) = 3.237	S(2) = -0.725
XT(3) = 0.500	YT(3) = 2.862	S(3) = -0.325
XT(4) = 1.500	YT(4) = 1.987	S(4) = -1.725
$\theta(1)$ = 32.005	$\theta 2(1)$ = 110.000	$\theta 3(1)$ = 55.000
$\theta(2)$ = 54.058	$\theta 2(2)$ = 100.000	$\theta 3(2)$ = -15.000
$\theta(3)$ = 71.996	$\theta 2(3)$ = 90.000	$\theta 3(3)$ = -85.000
$\theta(4)$ = 30.101	$\theta 2(4)$ = 70.000	$\theta 3(4)$ = -225.000
$\dot{\theta}(1)$ = 1.000	$\dot{\theta} 2(1)$ = 1.000	$\dot{r}(1)$ = -4.834

GR = 6.000
ψ = -55.000

RESULTS

X_m = -6.542	g(1) = -1.962
Y_m = -6.924	g(2) = -3.175
a = 4.050	g(3) = -4.928
b = 1.037	g(4) = -0.890
f = 8.587	
X_Q = -4.164	$\theta 5(1)$ = 88.394
Y_Q = -4.073	$\theta 5(2)$ = 97.854
c = 6.371	$\theta 5(3)$ = 116.156
d = 7.213	$\theta 5(4)$ = 75.367
α = 26.053	

All angles are in degree.

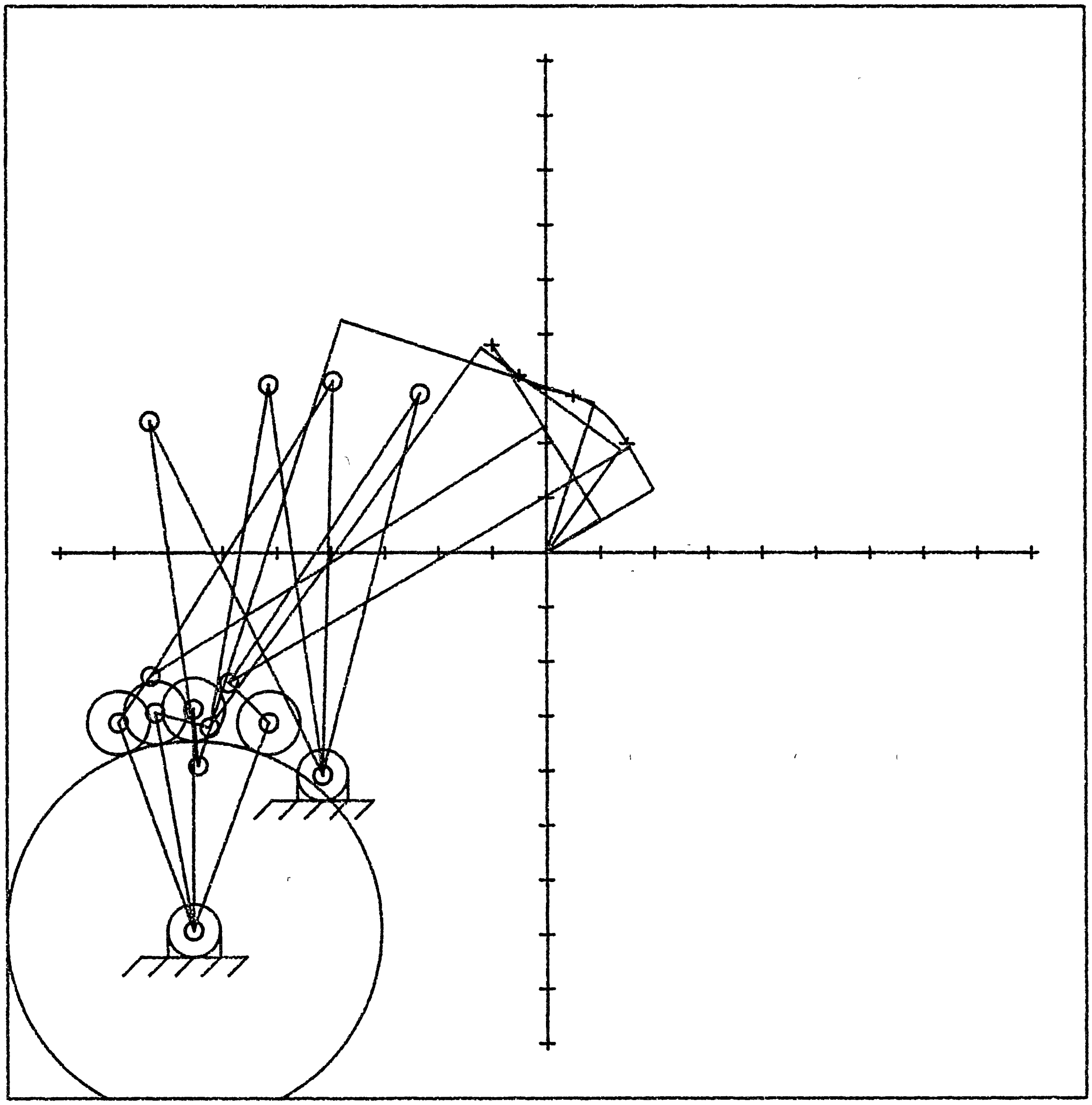

Figure 4. Configuration of the Designed
Mechanism For Type PP-P-P-P

TABLE III

SYNTHESIS OF A GEARED FIVE-LINK MECHANISM FOR FIRST ORDER INFINITESIMAL DISPLACEMENT OF TYPE PP-PP-P

INPUT DATA

Function $y = -0.3x^3 + 0.2x^2 - 0.2x + 3.0$

XT(1) = -1.000	YT(1) = 3.700	S(1) = -1.500
XT(2) = 0.500	YT(2) = 2.912	S(2) = -0.225
XT(3) = 1.500	YT(3) = 2.138	S(3) = -1.625
θ(1) = 33.690	θ2(1) = 95.000	θ3(1) = 105.000
θ(2) = 77.320	θ2(2) = 75.000	θ3(2) = -35.000
θ(3) = 31.608	θ2(3) = 55.000	θ3(3) = -175.000
$\dot{\theta}(1)$ = 1.000	$\dot{\theta}2(1)$ = 1.000	$\dot{r}(1)$ = -4.457
$\dot{\theta}(2)$ = 1.000	$\dot{\theta}2(2)$ = 1.000	$\dot{r}(2)$ = 4.181
GR = 6.000		
ψ = 10.000		

RESULTS

X_m = -1.229	g(1) = -3.407
Y_m = -1.974	g(2) = -0.022
a = 4.314	g(3) = -0.952
b = 0.615	
f = 1.070	
X_Q = 0.741	θ5(1) = 77.211
Y_Q = 3.992	θ5(2) = 111.496
c = 6.904	θ5(3) = 41.468
d = 4.940	
α = 24.885	

All angles are in degrees.

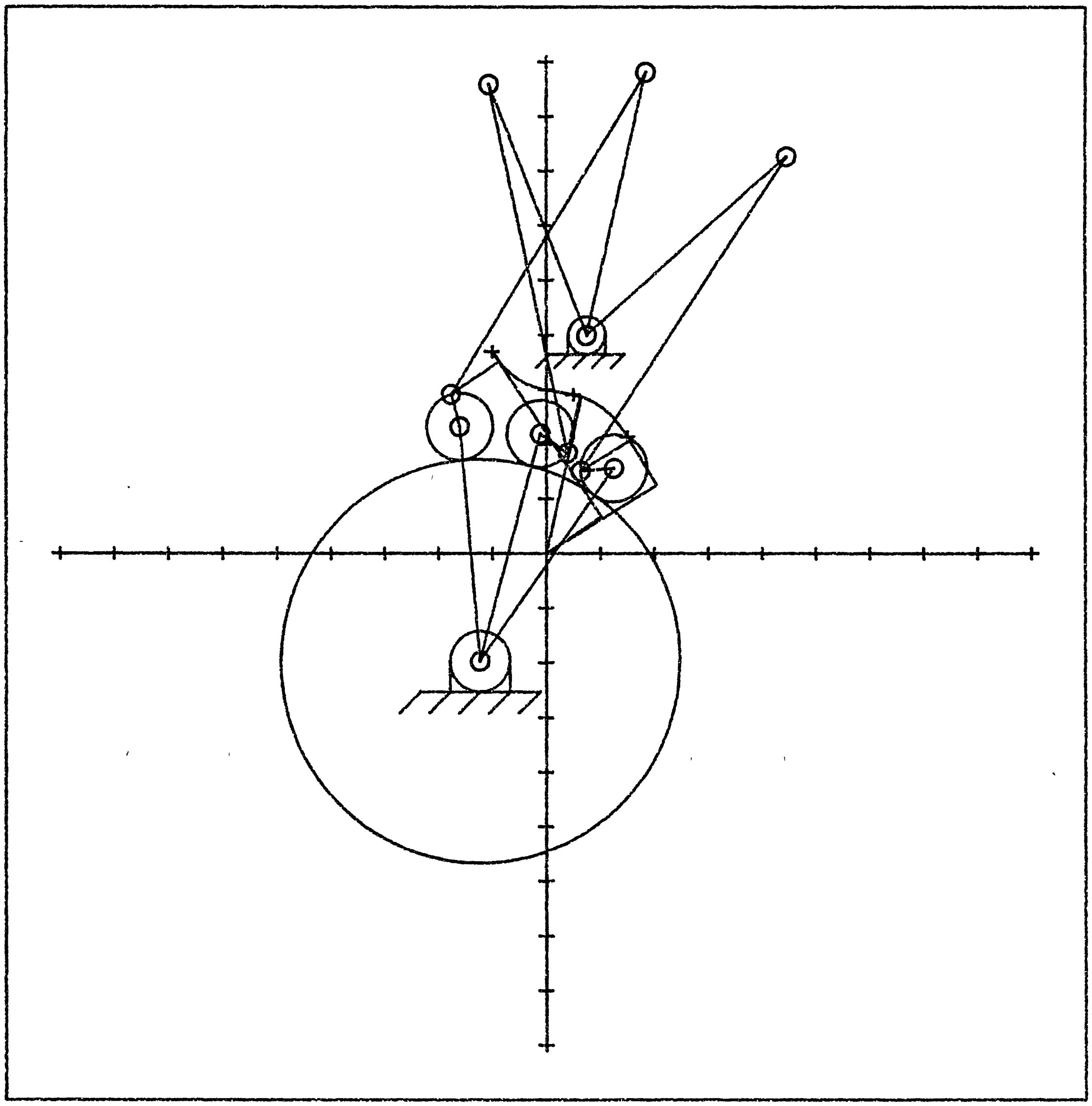

Figure 5. Configuration of the Designed Mechanism For Type PP-PP-P

CHAPTER VII

SYNTHESIS PROCEDURES FOR SECOND ORDER INFINITESIMALLY SEPARATED POSITIONS

An additional design criterion might be the specification of the acceleration characteristics of the function at the J^{th} precision position. Generally, for the second order infinitesimal displacement of five precision positions, there are two basic combinations as shown below.

PPP-P-P	PPP-PP
P-PPP-P	PP-PPP
P-P-PPP	

The first combination, describes three finite precision positions and one second infinitesimal displacement at one of the three finite precision positions. The second combination describes two finite precision positions with first and second order infinitesimal displacements at the finite precision positions.

In order to develop the synthesis equations for the second infinitesimal displacement, we differentiate equations (11) and (16), with respect to time. Differentiating equation (11), we obtain,

$$L_{1J}X_m + L_{2J}Y_m + L_{3J}a + L_{4J}b = \ddot{r}_J \qquad (17)$$

where

$$L_{1J} = -(\ddot{\theta}_J \sin\theta_J + \dot{\theta}_J^2 \cos\theta_J)$$

$$L_{2J} = (\ddot{\theta}_J \cos\theta_J - \dot{\theta}_J^2 \sin\theta_J)$$

$$L_{3J} = -(\ddot{\theta}_J - \ddot{\theta}_{2J}) \sin(\theta_J - \theta_{2J}) - (\dot{\theta}_J - \dot{\theta}_{2J})^2 \cos(\theta_J - \theta_{2J})$$

$$L_{4J} = -(\ddot{\theta}_J - N\ddot{\theta}_{2J}) \sin(\theta_J - \theta_{3J}) - (\dot{\theta}_J - N\dot{\theta}_{2J})^2 \cos(\theta_J - \theta_{3J})$$

For all J=1,2,3

The terms $\ddot{r}_J$, $\ddot{\theta}_J$, and $\ddot{\theta}_{2J}$ are known second time rate of change of radial component of the vector $\bar{r}$, angle θ, and the first input angle θ_2 at the J^{th} precision position respectively.

Similarly, differentiation of equation (16) with respect to time results in

$$L_{5J}X_Q+L_{6J}cX_Q+L_{7J}Y_Q+L_{8J}cY_Q+(L_{9J}+L_{10J}+L_{11J})c+(L_{12J}+L_{13J}+L_{14J})=0 \qquad (18)$$

where

$$L_{5J} = -[\dot{\theta}_{2J}^2(a \cos\theta_{2J} + bN^2 \cos\theta_{3J}) + \ddot{\theta}_{2J}(a \sin\theta_{2J} + bN \sin\theta_{3J})]$$

$$L_{6J} = [\ddot{\theta}_J \sin(\theta_J - \alpha) + \dot{\theta}_J^2 \cos(\theta_J - \alpha)]$$

$$L_{7J} = [\dot{\theta}_{2J}^2(a \sin\theta_{2J} + bN^2 \sin\theta_{3J}) - \ddot{\theta}_{2J}(a \cos\theta_{2J} + bN \cos\theta_{3J})]$$

$$L_{8J} = (-\ddot{\theta}_J \cos(\theta_J - \alpha) + \dot{\theta}_J^2 \sin(\theta_J - \alpha)]$$

$$L_{9J} = [\ddot{\theta}_J(-X_m\sin(\theta_J-\alpha)+Y_m\cos(\theta_J-\alpha))+\dot{\theta}_J^2(-X_m\cos(\theta_J-\alpha)-Y_m\sin(\theta_J-\alpha))]$$

$$L_{10J} = -a[(\ddot{\theta}_{2J}-\ddot{\theta}_J)\sin(\theta_{2J} - \theta_J + \alpha) + (\dot{\theta}_{2J} - \dot{\theta}_J)^2 \cos(\theta_{2J} - \theta_J + \alpha)]$$

$$L_{11J} = -b[(N\ddot{\theta}_{2J} - \ddot{\theta}_J)\sin(\theta_{3J} - \theta_J + \alpha)+(N\dot{\theta}_{2J} - \dot{\theta}_J)^2\cos(\theta_{3J} - \theta_J+\alpha)]$$

$$L_{12J} = a[-X_m(\ddot{\theta}_J \sin\theta_{2J} + \dot{\theta}_{2J}^2\cos\theta_{2J}) + Y_m(\ddot{\theta}_J \cos\theta_{2J} - \dot{\theta}_{2J}^2\sin\theta_{2J})]$$

$$L_{13J} = bN[-X_m(\ddot{\theta}_{2J}\sin\theta_{3J} + N\dot{\theta}^2_{2J}\cos\theta_{3J}) + Y_m(\ddot{\theta}_{2J}\cos\theta_{3J} - N\dot{\theta}^2_{2J}\sin\theta_{3J})]$$

$$L_{14J} = ab[(\ddot{\theta}_{2J} - N\ddot{\theta}_{2J})\sin(\theta_{2J} - \theta_{3J})+(\dot{\theta}_{2J} - N\dot{\theta}_{2J})^2\cos(\theta_{2J} - \theta_{3J})]$$

$$j=1,..,3$$

Equations (17) and (18) are similar to equations (11) and (16) respectively, with the same unknown parameters. Again, in these two equations, all the terms with subscript J correspond to the J^{th} precision position at which a second order infinitesimal displacement is under consideration. Tables IV and V show two of the arbitarily designed mechanisms for types PPP-P-P and PP-PPP and their corresponding configurations are shown in figures (6) and (7) respectively.

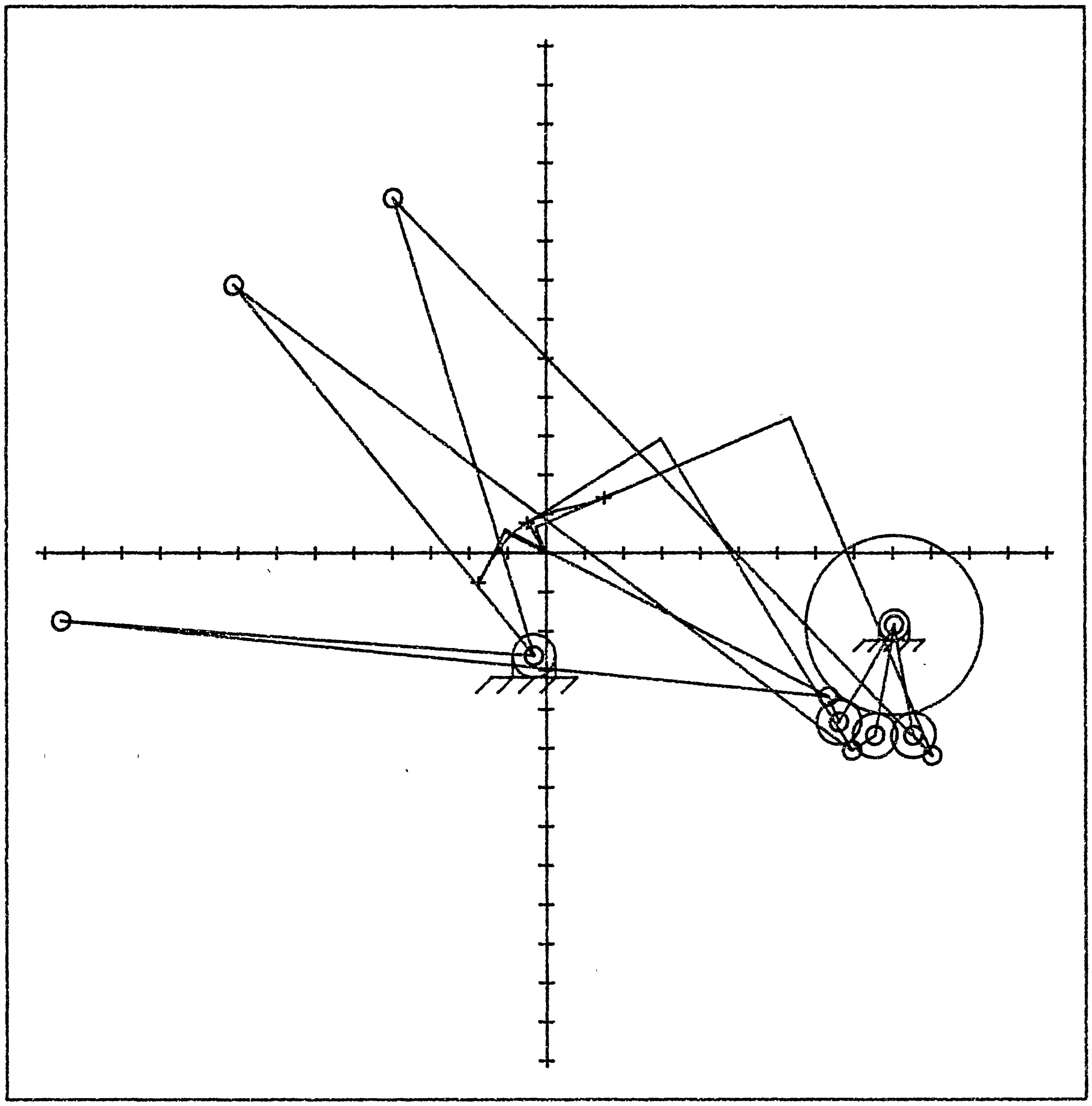

Figure 6. Configuration of the Designed Mechanism For Type PPP-P-P

TABLE V

SYNTHESIS OF A GEARED FIVE-LINK MECHANISM
FOR SECOND ORDER INFINITESIMAL
DISPLACEMENTOF TYPE PP-PPP

INPUT INFORMATION

Function $y = 1.25e^{x} - 0.75$

XT(1) = -1.500	YT(1) = -0.471	S(1) = 0.279
XT(2) = -0.250	YT(2) = 0.224	S(2) = 0.974
$\theta(1)$ = -74.416	$\theta 2(1)$ = 130.000	$\theta 3(1)$ = 75.000
$\theta(2)$ = 134.231	$\theta 2(2)$ = 60.000	$\theta 3(2)$ =-205.000
$\dot{\theta}(1)$ = 1.000	$\dot{\theta}2(1)$ = 1.000	$\dot{r}(1)$ = -0.816
$\dot{\theta}(2)$ = 1.000	$\dot{\theta}2(2)$ = 1.000	$\dot{r}(2)$ = -0.031
$\ddot{\theta}(2)$ = 0.000	$\ddot{\theta}2(2)$ = 0.000	$\ddot{r}(2)$ = -5.587

GR = 3.000
ψ = -55.000

RESULTS

X_m = 2.080	g(1) = 0.786
Y_m = -3.870	g(2) = 3.831
a = 5.666	
b = 1.166	
f = 1.928	
X_Q = -0.855	$\theta 5(1)$ = 137.792
Y_Q = -0.914	$\theta 5(2)$ = 15.464
c = 4.021	
d = 7.252	
α = 141.531	

All angles are in degrees

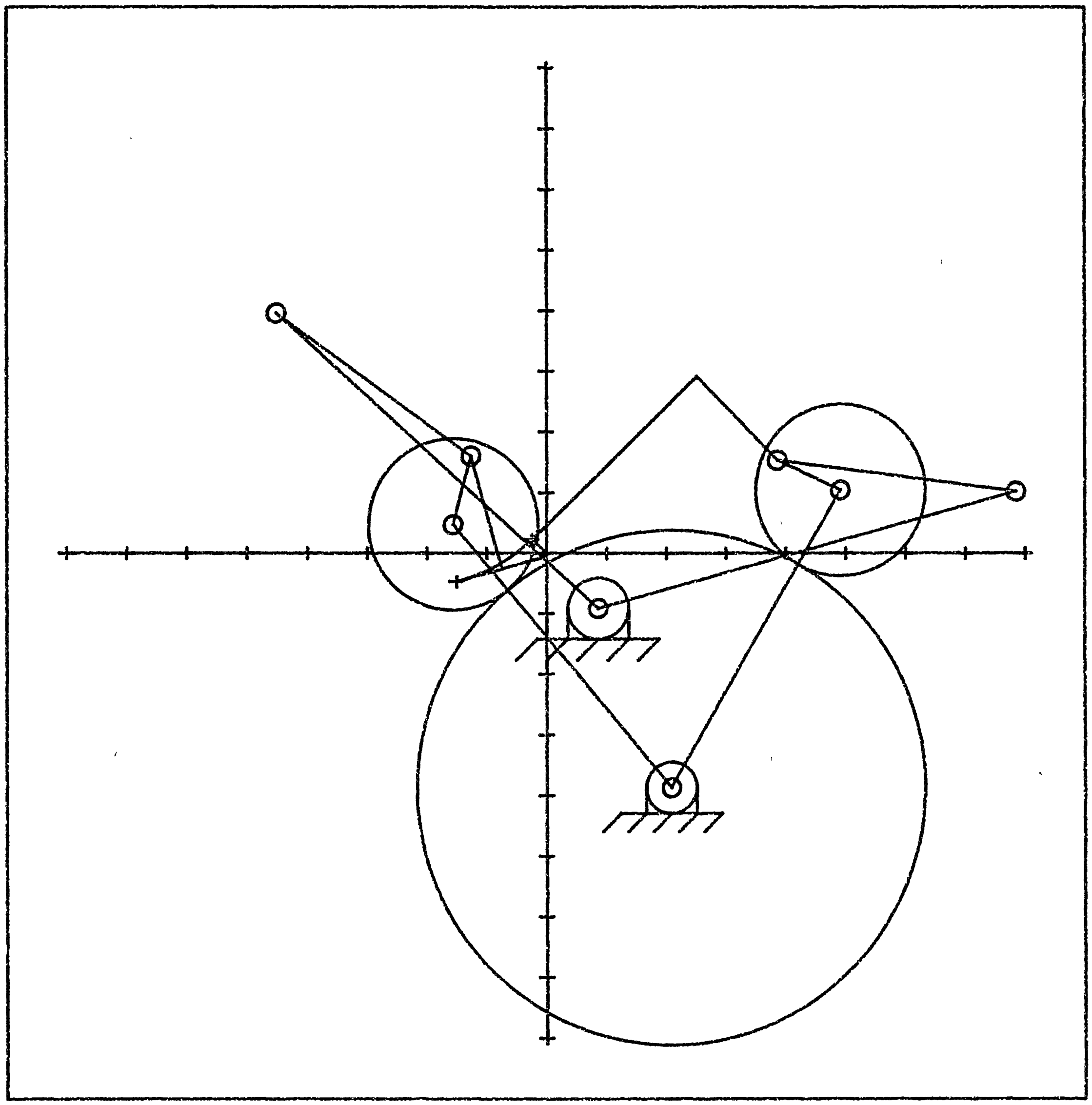

Figure 7. Configuration of the Designed Mechanism For Type PP-PPP

CHAPTER VIII

SYNTHESIS PROCEDURES FOR THIRD ORDER INFINITESIMALLY SEPARATED POSITIONS

An additional design criterion might be the specification of the jerk characteristics of the function at the J^{th} precision position. In such cases, the synthesis equations derived in the previous chapter will be used to derive the synthesis equations for third order infinitesimally separated positions. Generally, for the third infinitesimal displacement of five finite precision positions, there is only one basic combination as shown below:

PPPP-P

P-PPPP

which specifies two finite precision positions and a third order infinitesimal displacement at either of the finite precision positions. Again, in order to develop the synthesis equations for the third order infinitesimal displacement, we differentiate equations (17) and (18) with respect to time. Differentiating equation (17), we have

$$q_{1J}X_m + q_{2J}Y_m + q_{3J}a + q_{4J}b = \dddot{r}_J \qquad (19)$$

where

$$q_{1J} = -[(\dddot{\theta}_J - \dot{\theta}_J^3)\ \sin\theta_J + 3\ddot{\theta}_J\dot{\theta}_J\ \cos\theta_J]$$

$$q_{2J} = [(\dddot{\theta}_J - \dot{\theta}_J^3)\cos\theta_J - 3\ddot{\theta}_J\dot{\theta}_J \sin\theta_J]$$

$$q_{3J}=-[((\dddot{\theta}_J-\dddot{\theta}_{2J})-(\dot{\theta}_J-\dot{\theta}_{2J})^3)\sin(\theta_J-\theta_{2J})+3(\ddot{\theta}_J-\ddot{\theta}_{2J})(\dot{\theta}_J-\dot{\theta}_{2J})\cos(\theta_J-\theta_{2J})]$$

$$q_{4J}=-[((\dddot{\theta}_J-N\dddot{\theta}_{2J})-(\dot{\theta}_J-N\dot{\theta}_{2J})^3)\sin(\theta_J-\theta_{3J})+3(\ddot{\theta}_J-N\ddot{\theta}_{2J})(\dot{\theta}_J-N\dot{\theta}_{2J})\cos(\theta_J-\theta_{3J})]$$

and similarly differentiation of equation (18) with respect to time results in

$$q_{5J}X_Q+q_{6J}cX_Q+q_{7J}Y_Q+q_{8J}cY_Q+(q_{9J}+q_{10J}+q_{11J})c+(q_{12J}+q_{13J}+q_{14J})=0 \qquad (20)$$

where

$$q_{5J}=-[a(\dddot{\theta}_{2J}+\dot{\theta}_{2J}^3)\sin\theta_{2J}-3\ddot{\theta}_{2J}\dot{\theta}_{2J}(a\cos\theta_{2J}+bN\cos\theta_{3J})-b(\dddot{\theta}_{2J}-N^2\dot{\theta}_{2J}^3)\sin\theta_{3J}]$$

$$q_{6J}=-[(-\dddot{\theta}_J+\dot{\theta}_J^3)\sin(\theta_J-\alpha)-3\ddot{\theta}_J\dot{\theta}_J \cos(\theta_J-\alpha)]$$

$$q_{7J}=-[a(-\dddot{\theta}_{2J}-\dot{\theta}_{2J}^3)\cos\theta_{2J}-3\ddot{\theta}_{2J}\dot{\theta}_{2J}(a\sin\theta_{2J}+Nb\sin\theta_{3J})+b(\dddot{\theta}_{2J}-N^2\dot{\theta}_{2J}^3)\cos\theta_{3J}]$$

$$q_{8J}=-[(\dddot{\theta}_J-\dot{\theta}_J^3)\cos(\theta_J-\alpha)-3\ddot{\theta}_J\dot{\theta}_J \sin(\theta_J-\alpha)]$$

$$q_{9J}=[(-X_m\dddot{\theta}_J-3\ddot{\theta}_J\dot{\theta}_JY_m+X_m\dot{\theta}_J^3)\sin(\theta_J-\alpha)+(\dddot{\theta}_JY_m-3\ddot{\theta}_J\dot{\theta}_JX_m-\dot{\theta}_J^3Y_m)\cos(\theta_J-\alpha)]$$

$$q_{10J}=-a[((\dddot{\theta}_{2J}-\dddot{\theta}_J)-(\dot{\theta}_{2J}-\dot{\theta}_J)^3)\sin(\theta_{2J}-\theta_J+\alpha)+3(\ddot{\theta}_{2J}-\ddot{\theta}_J)(\dot{\theta}_{2J}-\dot{\theta}_J) \cos(\theta_{2J}-\theta_J+\alpha)]$$

$$q_{11J}=-b[((N\dddot{\theta}_{2J}-\dddot{\theta}_J)-(N\dot{\theta}_{2J}-\dot{\theta}_J)^3)\sin(\theta_{3J}-\theta_J+\alpha)+3(N\ddot{\theta}_{2J}-\ddot{\theta}_J)(N\dot{\theta}_{2J}-\dot{\theta}_J) \cos(\theta_{3J}-\theta_J+\alpha)]$$

$$q_{12J}=a[(-\dddot{\theta}_JX_m-3\ddot{\theta}_J\dot{\theta}_{2J}Y_m+\dot{\theta}_{2J}^3X_m)\sin\theta_{2J}+(\dddot{\theta}_JY_m-3\ddot{\theta}_J\dot{\theta}_{2J}X_m-\dot{\theta}_{2J}^3Y_m)\cos\theta_{2J}]$$

$$q_{13J}=bN[(-X_m\dddot{\theta}_{2J}-3N\ddot{\theta}_{2J}\dot{\theta}_{2J}Y_m+N^2\dot{\theta}_{2J}^3X_m)\sin\theta_{3J}+(\dddot{\theta}_{2J}Y_m-3N\ddot{\theta}_{2J}\dot{\theta}_{2J} X_m-N^2\dot{\theta}_{2J}^3 Y_m)\cos\theta_{3J}]$$

$$q_{14J}=ab[((\dddot{\theta}_{2J}-N\dddot{\theta}_{2J})-(\dot{\theta}_{2J}-N\dot{\theta}_{2J})^3)\sin(\theta_{2J}-\theta_{3J})+3(\ddot{\theta}_{2J}-N\ddot{\theta}_{2J})(\dot{\theta}_{2J}-N\dot{\theta}_{2J})$$

$\cos(\theta_{2J}-\theta_{3J})]$

Equations (19) and (20) are similar to the previous infinitesimal synthesis equations. Again, all the terms with subscript J in these two synthesis equations correspond to the J^{th} precision position at which a third order infinitesimal displacement is under consideration. The term $\dddot{r}_J$ is the known third time rate of change of the radial component of vector $\bar{r}$ at the J^{th} precision position. $\dddot{\theta}_J$, and $\dddot{\theta}_{2J}$ represent the known third time rate of change of the angle θ and the first input angle θ_2 at the J^{th} precision position respectively. A sample synthesis of geared five-link mechanism for third order infinitesimal displacement is included in Table VI.The corresponding design is shown in figure 8.

TABLE VI

SYNTHESIS OF A GEARED FIVE-LINK MECHANISM
FOR THIRD ORDER INFINITESIMAL
DISPLACEMENT OF TYPE PPPP-P

INPUT INFORMATION

Function $y = 0.25e^x - 0.15$

XT(1) = -1.000	YT(1) = 0.426	S(1) = 0.276
XT(2) = 1.000	YT(2) = 2.189	S(2) = 2.039
θ(1) = 105.425	θ2(1) = 110.000	θ3(1) = 87.500
θ(2) = 153.872	θ2(2) = 100.000	θ3(2) = 17.500
$\dot{\theta}(1)$ = 1.000	$\dot{\theta}2(1)$ = 1.000	$\dot{r}(1)$ = 0.636
$\ddot{\theta}(1)$ = 0.000	$\ddot{\theta}2(1)$ = 0.000	$\ddot{r}(1)$ = -5.287
$\dddot{\theta}(1)$ = 0.000	$\dddot{\theta}2(1)$ = 0.000	$\dddot{r}(1)$ = 1.710

GR = 6.000
ψ = -22.500

RESULTS

X_m = 3.181	g(1) = 0.264
Y_m = -10.606	g(2) = 1.140
a = 8.235	
b = 0.478	
f = -3.083	
X_Q = 9.648	θ5(1) = -140.402
Y_Q = -1.603	θ5(2) = 123.382
c = 6.628	
d = 3.655	
α = 61.118	

All angles are in degrees

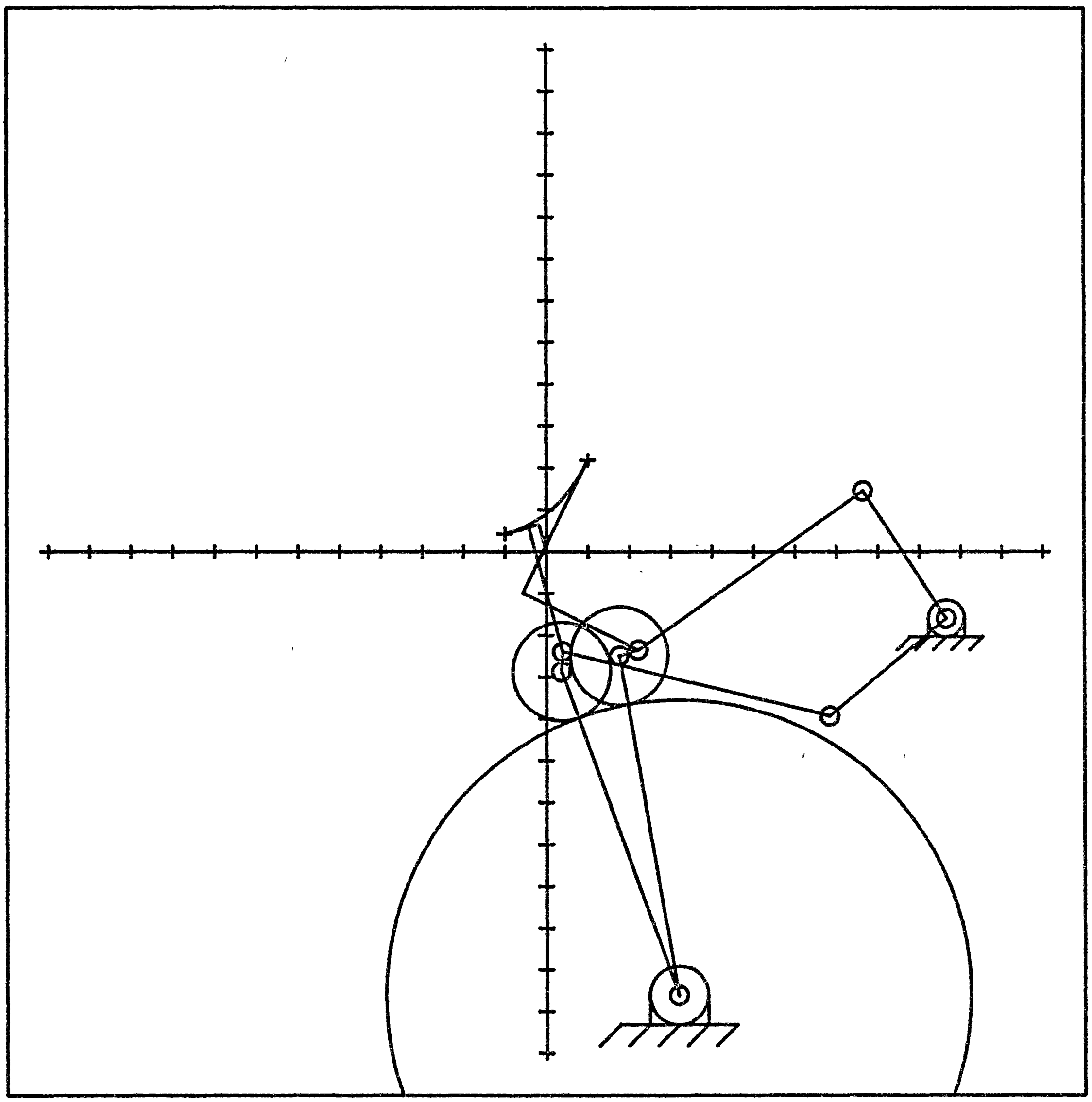

Figure 8. Configuration of the Designed
Mechanism For Type PPPP-P

CHAPTER IX

SYNTHESIS PROCEDURES FOR FOURTH ORDER INFINITESIMALLY SEPARATED POSITIONS

In case our objective is to design a mechanism for fourth order infinitesimal displacement of the function, the synthesis equations developed in the previous chapter have to be differentiated with respect to time. The resulting synthesis equations will grant this design criterion. Basically, the only possible combination for the fourth order infinitesimal displacement of five finitely specified precision positions is

PPPPP

which specifies one finite precision position and a fourth order infinitesimal displacement at that finite precision position. Following the same precedure as in the previous chapter, we start with differentiating equation (19) with respect to time, for synthesis of loop(I). We obtain,

$$S_{11}X_m + S_{21}Y_m + S_{31}a + S_{41}b = \ddddot{r}_1 \qquad (22)$$

The coefficients S_{11}, S_{21}, S_{31}, and S_{41} have been included in Appendix B.

Similarily, the synthesis equations for loop (II) are obtained by differentiating equation (20) with respect to time which results in

$$S_{51}X_Q+S_{61}CX_Q+S_{71}Y_Q+S_{81}CY_Q+(S_{91}+S_{101}+S_{111})C+(S_{121}+S_{131}+S_{141})=0 \qquad (23)$$

The coefficients S_{51}, $S_{51}, \ldots, S_{141}$ have been provided in Appendix C. The term $\ddddot{r}_J$ describes the known fourth time rate of change of the radial component of the vector $\bar{r}$ at the J^{th} precision position. An example of the above synthesis for for fourth other infinitesimal displacement is provided in Table VII. The corresponding mechanism is shown in figure 9.

TABLE VII

SYNTHESIS OF A GEARED FIVE-LINK MECHANISM
FOR FOURTH ORDER INFINITESIMAL
DISPLACEMENT OF TYPE PPPPP

INPUT DATA

Function $y = 0.4e^{x} + 4.0$

XT(1) = -1.000	YT(1) = 4.147	S(1) = 0.147
$\theta(1)$ = 98.371	$\theta 2(1)$ = 40.000	$\theta 3(1)$= 160.000
$\dot{\theta}(1)$ = 1.000	$\dot{\theta}2(1)$ = 1.000	$\dot{r}(1)$ = -0.631
$\ddot{\theta}(1)$ = 0.000	$\ddot{\theta}2(1)$ = 0.000	$\ddot{r}(1)$ = 2.118
$\dddot{\theta}(1)$ = 0.000	$\dddot{\theta}2(1)$ = 0.000	$\dddot{r}(1)$ = -1.926
$\ddddot{\theta}(1)$ = 0.000	$\ddddot{\theta}2(1)$ = 0.000	$\ddddot{r}(1)$ = 2.216

GR = 1.000
ψ = 120.000

RESULTS

X_m = -1.551	g(1) = 2.386
Y_m = -0.565	
a = 5.426	
b = 0.701	
f = 1.403	
X_Q = -2.303	$\theta 5(1)$ = 85.075
Y_Q = -4.280	
c = 4.033	
d = 5.952	
α = 103.654	

All angles are in degrees

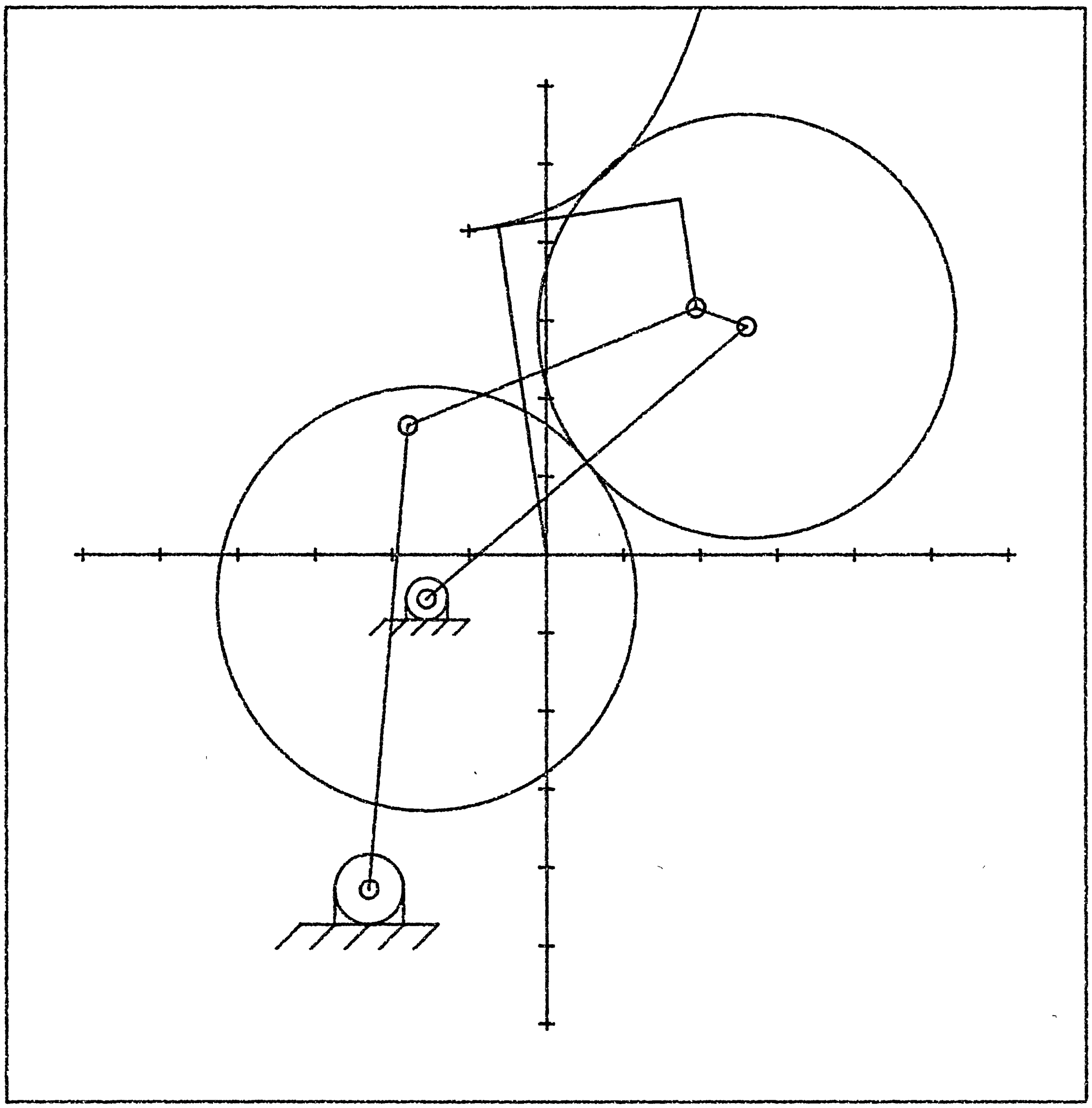

Figure 9. Configuration of the Designed
Mechanism For Type PPPPP

CHAPTER X

SUMMARY AND CONCLUSIONS

This thesis develops an approach for synthesizing the geared five-link mechanisms to coordinate the input-crank with an enveloping tangent-line for five positions. These five positions consist of both finitely and infinitesimally separated positions. There exist sixteen possible mixed combinations of finite and infinitesimally separated positions for which function generation motion can be obtained. The existence of the enveloping tangent-line facilitates the development of a set of synthesis equations, for both finitely and infinitesimally separated positions, independent of the output angle.

The synthesis procedures start by separating the mechanism structure into two loops and writing the vector loop-closure equations for each loop. The vector loop-closure method is preferred since the synthesis equations obtained for the finitely separate positions can readily differentiated successively to obtain any the equations for any desired order of infinitesimal displacement. The synthesis procedure developed in this thesis can be applied to synthesize different mechanisms with an enveloping tangent-line, not only for finite positions, but also for infinitesimally separated positions. It can be applied to both function generation synthesis problems as well as rigid body guidance synthesis problems. For instance, such a geared five-link mechanism designed in this thesis, can be used for filing the contour of a precision cam if particular or locations on the contour of the cam must be filed with a set of prescribed characteristics such as velocity, acceleration, jerk, kerk, etc. These mechanisms can also be used for picking up an object with a certain velocity, acceleration, or jerk and locating it with a certain velocity, acceleration, or jerk at another location. A computer program has been developed on H P-9000 minicomputer to synthesize and draw the configuration of the mechanism for any type of finitely or infinitesimally separated position motions.

It should be emphasized here that all of the above synthesis procedures are in closed-form. Hence, they yield efficient and accurate routines independent of motion increment size or any other implied limitations characteristic of numerical, iterative procedures. A closed form solution the added advantage of disclosing all branches of closure. This is important to a designer who has synthesized a satisfactory solution by all other measures, but who has no idea if the closure branches intersect. The synthesis procedures developed in this thesis is well suited for both rational and non-rational gear-ratios. This others the designer more flexibility to design the mechanism with any available gear pair. It should also be mentioned that although all the synthesis equations developed for the first loop of the mechanism are linear, care must be taken to check if the scalar length of the first and second input links are positive. When a design yields a negative value for the first input link or the second input link, this means that the specified input angles or the phase angle should be modified by a 180 degree offset. In such cases, it is possible to obtain a satisfactory design by varying the gears or the phase angle.

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APPENDICES

APPENDIX A

PROCEDURES FOR FINDING THE RADIAL TIME DERIVATIVE COMPONENTS OF VECTOR $\bar{r}$

APPENDIX A

PROCEDURES FOR FINDING THE RADIAL TIME DERIVATIVE COMPONENTS OF VECTOR $\bar{r}$.

Consider a moving tangent-line ll' in the canonical system XOY (Figure 10). The equation of the tangent-line ll' at the i^{th} position can be expressed [19] as

$$r_i = X_i \cos\theta_i + Y_i \sin\theta_i \tag{A-1}$$

Equation (A-1) gives the family of straight-lines with a single parameter θ_i and a dependant variable r_i. In Chapter IV, we showed that the slope of such a tangent-line, tangential to a prescribed function, in the form of y=f(x), at the i^{th} (SPP) can be obtained from the following relationship

$$S_i' = \frac{d}{dx} f(x) \Big|_{x=x_{Ti}} \tag{A-2}$$

Letting ϕ_i be the angle that the tangent-line makes with X-axis at the i^{th} (SPP). Then

$$\phi_i = \tan^{-1} (S_i) = \tan^{-1} (Y_{Ei}/X_{Ei}) \tag{A-3}$$

Since, the vector $\bar{r}$ is always considered normal to the tangent-line ll', the relationship between the angles ϕ_i and θ_i at the i^{th} (SPP) can be shown as

$$\phi = \theta_i \pm \frac{\pi}{2} \tag{A-4}$$

The angles ϕ_i and θ_i are the angular displacement of tangent-line ll' and vector $\bar{r}$ at the i^{th} (SPP) respectively. Differentiating equation (A-4) with respect to

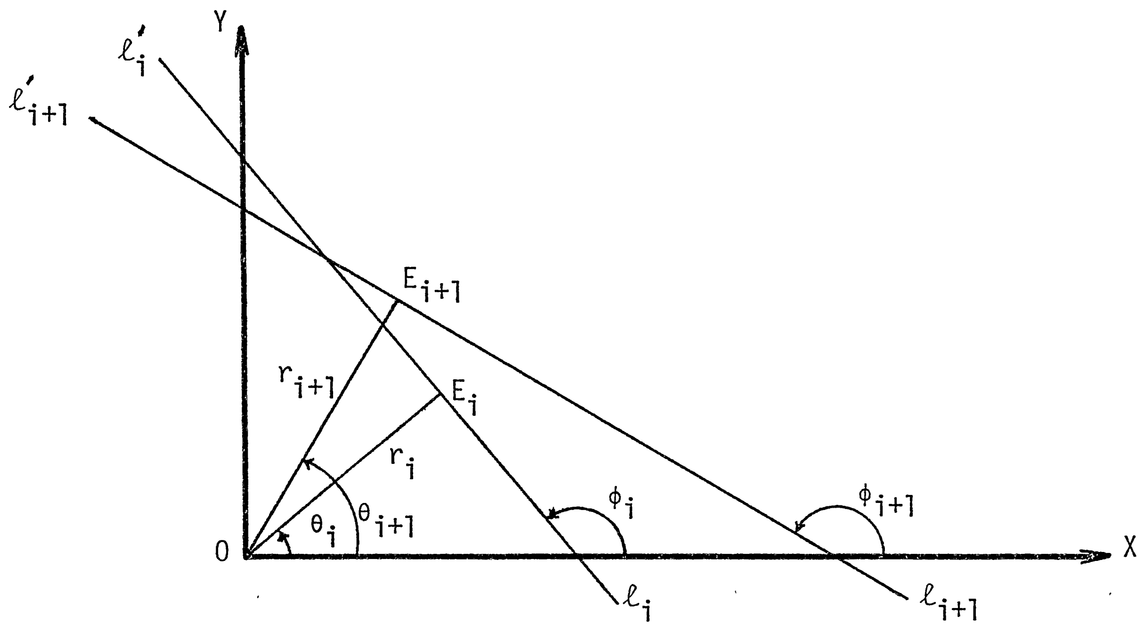

Figure 10. Tangent-Line in a Canonical System

time successively,

$$\dot{\phi} = \dot{\theta}_i \tag{A-5}$$

$$\ddot{\phi} = \ddot{\theta}_i \tag{A-6}$$

$$\dddot{\phi} = \dddot{\theta}_i \tag{A-7}$$

$$\ddddot{\phi} = \ddddot{\theta}_i \tag{A-8}$$

where $\dot{\phi}_i$, $\ddot{\phi}_i$, $\dddot{\phi}_i$, and $\ddddot{\phi}_i$ are the angular velocity, acceleration, jerk and kerk of the tangent-line ll' at the i^{th} (SPP). Similarily, $\dot{\theta}_i$, $\ddot{\theta}_i$, $\dddot{\theta}_i$, and $\ddddot{\theta}_i$ are the angular velocity, acceleration, jerk, and kerk of vector $\bar{r}$ at the i^{th} (SPP). Now, considering equation (A-1), we can express the first, second, third, and fourth time derivatives of radial displacement of the vector $\bar{r}$ as

$$\dot{r}_i = A_{1i} \cos\theta_i + B_{1i} \sin\theta_i \tag{A-9}$$

$$\ddot{r}_i = A_{2i} \cos\theta_i + B_{2i} \sin\theta_i \tag{A-10}$$

$$\dddot{r}_i = A_{3i} \cos\theta_i + B_{3i} \sin\theta_i \tag{A-11}$$

$$\ddddot{r}_i = A_{4i} \cos\theta_i + B_{4i} \sin\theta_i \tag{A-12}$$

where

$$A_{1i} = (\dot{X} + Y\dot{\theta})_i$$

$$A_{2i} = (\ddot{X} + 2\dot{Y}\dot{\theta} + Y\ddot{\theta} - X\dot{\theta}^2)_i$$

$$A_{3i} = (\dddot{X} + 3\ddot{Y}\dot{\theta} + 3\dot{Y}\ddot{\theta} + Y\dddot{\theta} - 3\dot{X}\dot{\theta} - 3X\ddot{\theta}\dot{\theta} - Y\dot{\theta}^3)_i$$

$$A_{4i} = (\ddddot{X}+4\dddot{Y}\dot{\theta}+6\ddot{Y}\ddot{\theta}+4\dot{Y}\dddot{\theta}+Y\ddddot{\theta}-6\ddot{X}\dot{\theta}^2-12\dot{X}\ddot{\theta}\dot{\theta}-4X\dddot{\theta}\dot{\theta}-3X\ddot{\theta}^2-4\dot{Y}\dot{\theta}^3-6Y\ddot{\theta}\dot{\theta}^2+X\dot{\theta}^4)_i$$

$$B_{1i} = (\dot{Y} - X\dot{\theta})_i$$

$$B_{2i} = (\ddot{Y} - 2\dot{X}\dot{\theta} - X\ddot{\theta} - Y\dot{\theta}^2)_i$$

$$B_{3i} = (\dddot{Y} - 3\ddot{X}\dot{\theta} - 3\dot{X}\ddot{\theta} - X\dddot{\theta} - 3\dot{Y}\dot{\theta}^2 - 3Y\ddot{\theta}\dot{\theta} + X\dot{\theta}^3)_i$$

$$B_{4i}=(\ddddot{Y}-4\dddot{X}\dot{\theta}-6\ddot{X}\ddot{\theta}-4\dot{X}\dddot{\theta}-X\ddddot{\theta}-6\ddot{Y}\dot{\theta}^2-12\dot{Y}\ddot{\theta}\dot{\theta}-4Y\dddot{\theta}\dot{\theta}-3Y\ddot{\theta}^2+4\dot{X}\dot{\theta}^3+6X\ddot{\theta}\dot{\theta}^2-Y\dot{\theta}^4)_i$$

$$i = 1,..,5$$

Now, if we express θ as a function of time, $\theta = t$, then

$$\dot{\theta}_i = 1$$

$$\ddot{\theta}_i = \dddot{\theta}_i = \ddddot{\theta}_i = 0 \tag{A-13}$$

Substituting equations (A-13) into the above coefficients and expressing the resulting coefficients for point E on the tangent-link

$$A_{1i} = (\dot{X}_E + Y_E)_i$$

$$A_{2i} = (\ddot{X}_E + 2\dot{Y}_E - X_E)_i \tag{A-14}$$

$$A_{3i} = (\dddot{X}_E + 3\ddot{Y}_E - 3\dot{X}_E - Y_E)_i$$

$$A_{4i} = (\ddddot{X}_E + 4\dddot{Y}_E - 6\ddot{X}_E - 4\dot{Y}_E + X_E)_i$$

$$B_{1i} = (\dot{Y}_E - X_E)_i$$

$$B_{2i} = (\ddot{Y}_E - 2\dot{X}_E - Y_E)_i \qquad \text{(A-15)}$$

$$B_{3i} = (\dddot{Y}_E - 3\ddot{X}_E - 3\dot{Y}_E + X_E)_i$$

$$B_{4i} = (\ddddot{Y}_E - 4\dddot{X}_E - 6\ddot{Y}_E + 4\dot{X}_E - Y_E)_i$$

Now, the left-hand side components of the above equations can be evaluated by considering equations (A-2) and (A-3). From these two equations, we can write

$$\tan \phi_i = \frac{dy}{dx}\Big|_{x = x_{Ei}} = g(x)\Big|_{x = x_{Ei}} \qquad \text{(A-16)}$$

Rearranging equation (A-16) to find an expression for $\frac{dx}{dt}$, we have

$$\frac{d}{d\phi} \tan \phi \cdot \frac{d\phi}{dt}\Big|_i = \frac{dg(x)}{dx} \cdot \frac{dx}{dt}\Big|_i \qquad \text{(A-17)}$$

From above equation, we can result that

$$\frac{dx}{dt}\Big|_i = \left[\frac{d}{d\phi} \tan \phi \cdot \frac{d\phi}{dt} \Big/ \frac{dg(x)}{dx}\right]_i \qquad \text{(A-18)}$$

By considering, equation (A-5), and assumption that $\theta = t$, then the above equation can be simplified as

$$\frac{dx}{dt}\Big|_i = \left[(1/\cos^2\phi) \Big/ \frac{dg(x)}{dx}\right]_i \qquad \text{(A-19)}$$

Any terms in left-hand side of equation (A-19) are known. So, we can easily compute $\frac{dx}{dt}$ at any i^{th} (SPP). Please note that the term $\frac{dg(x)}{dx}$ must be evaluated at $x = x_{Ei}$.

Now, in order to obtain an expression for $\frac{dy}{dt}$, we can write

$$\frac{dy}{dx}\Big|_i = [\frac{dy}{dt} / \frac{dx}{dt}]_i \qquad \text{(A-20)}$$

or

$$\frac{dy}{dt}\Big|_i = [\frac{dy}{dx} \cdot \frac{dx}{dt}]_i \qquad \text{(A-21)}$$

Please also note that the term $\frac{dy}{dx}$ must also be evaluated at $x=x_{Ei}$. Following the same procedure described above, we can obtain the higher time derivative of x and y terms as shown below.

$$\frac{dx^2}{dt^2}\Big|_i = [(2 \sin \phi / \cos^3\phi)/\frac{d^2g(x)}{dx^2}]_i \qquad \text{(A-22)}$$

$$\frac{dx^3}{dt^3}\Big|_i = [((2 + 4 \sin \phi)/\cos^4\phi)/ \frac{d^3g(x)}{dx^3}]_i \qquad \text{(A-23)}$$

$$\frac{dx^4}{dt^4}\Big|_i = [(4(1 + 2 \sin\phi + 3 \sin\phi)/\cos^5\phi)/\frac{d^4g(x)}{dx^4}]_i \qquad \text{(A-24)}$$

and

$$\frac{dy^2}{dt^2}\Big|_i = [\frac{d^2y}{dx^2} \cdot \frac{dx^2}{dt^2}]_i \qquad \text{(A-25)}$$

$$\frac{dy^3}{dt^3}\Big|_i = [\frac{d^3y}{dx^3} \cdot \frac{dx^3}{dt^3}]_i \qquad \text{(A-26)}$$

$$\frac{dy^4}{dt^4}\Big|_i = [\frac{d^4y}{dx^4} \cdot \frac{dx^4}{dt^4}]_i \qquad \text{(A-27)}$$

The equations (A-21) through (A-27) are obtained based on the assumption that $\theta=t$. Moveover, it is important to observe that, we can obtain the n^{th} order infinitesimal displacement of the enveloping tangent-line, only when the precribed function (y=f(x)) is differentiable with respect to x up to the $(n + 1)^{th}$ order.

APPENDIX B

COEFFICIENTS OF EQUATIONS (21)

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COEFFICIENTS OF EQUATION (21)

The coefficients of the equation (21) are as following:

$$S_{11}=-[(\ddddot{\theta}_J-3\ddot{\theta}_J\dot{\theta}_J^2)\sin\theta_J+\dot{\theta}_J(\dddot{\theta}_J-\dot{\theta}_J^3)\cos\theta_J+3\dddot{\theta}_J\dot{\theta}_J\cos\theta_J+3\ddot{\theta}_J^2\cos\theta_J-3\ddot{\theta}_J\dot{\theta}_J^2 \sin\theta_J]$$

$$S_{21}=[(\ddddot{\theta}_J-3\ddot{\theta}_J\dot{\theta}_J^2)\cos\theta_J-\dot{\theta}_J(\dddot{\theta}_J-\dot{\theta}_J{}^3)\sin\theta_J-3\dddot{\theta}_J\dot{\theta}_J\sin\theta_J-3\ddot{\theta}_J^2\sin\theta_J-3\ddot{\theta}_J\dot{\theta}_J^2 \cos\theta_J]$$

$$S_{31}=-[((\ddddot{\theta}_J-\ddddot{\theta}_{2J})-3(\ddot{\theta}_J-\ddot{\theta}_{2J})(\dot{\theta}_J-\dot{\theta}_{2J})^2)\sin(\theta_J-\theta_{2J})+((\dddot{\theta}_J-\dddot{\theta}_{2J})-(\dot{\theta}_J-\dot{\theta}_{2J})^3)(\dot{\theta}_J-\dot{\theta}_{2J})\cos(\theta_J-\theta_{2J})+3(\dddot{\theta}_J-N\dddot{\theta}_{2J})(\dot{\theta}_J-\dot{\theta}_{2J})\cos(\theta_J-\theta_{2J})+3(\ddot{\theta}_J-\ddot{\theta}_{2J})^2\cos(\theta_J-\theta_{2J})-3(\ddot{\theta}_J-\ddot{\theta}_{2J})(\dot{\theta}_J-\dot{\theta}_{2J})^2\sin(\theta_J-\theta_{2J})]$$

$$S_{41}=-[((\ddddot{\theta}_J-N\ddddot{\theta}_{2J})-3(\ddot{\theta}_J-N\ddot{\theta}_{2J})(\dot{\theta}_J-N\dot{\theta}_{2J})^2)\sin(\theta_J-\theta_{3J})+((\dddot{\theta}_J-N\dddot{\theta}_{2J})-(\dot{\theta}_J-N\dot{\theta}_{2J})^3)(\dot{\theta}_J-N\dot{\theta}_{2J})\cos(\theta-\theta_{3J})+3(\dddot{\theta}_J-N\dddot{\theta}_{2J})(\dot{\theta}_J-N\dot{\theta}_J)\mathrm{Cos}(\theta_J-\theta_{3J})+3(\ddot{\theta}_J-N\ddot{\theta}_{2J})^2\cos(\theta_J-\theta_{2J})-3(\ddot{\theta}_J-N\ddot{\theta}_{2J})(\dot{\theta}_J-N\dot{\theta}_{2J})^2\sin(\theta_J-\theta_{3J})]$$

APPENDIX C

COEFFICIENTS OF EQUATION (22)

APPENDIX C

COEFFICIENTS OF EQUATION (22)

The coefficients of the equations (22) are as followings:

$$S_{51}=[2a((\ddddot{\theta}_{2J}-3\ddot{\theta}_{2J}\dot{\theta}_{2J}^{2})\sin\theta_{2J}+\dot{\theta}_{2J}(\dddot{\theta}_{2J}+\dot{\theta}_{2J}^{3})\mathrm{Cos}\,\theta_{2J}+3(\dddot{\theta}_{2J}\dot{\theta}_{2J}+\ddot{\theta}_{2J}^{2})$$
$$\cos\theta_{2J}-3\dot{\theta}_{2J}^{2}\ddot{\theta}_{2J}\sin\theta_{2J})+2bN((\ddddot{\theta}_{2J}-3N^{2}\ddot{\theta}_{2J}\dot{\theta}_{2J})\sin\theta_{3J}+N\dot{\theta}_{2J}(\dddot{\theta}_{2J}-N^{2}$$
$$\dot{\theta}_{2J}^{3})\cos\theta_{3J}+3N(\dddot{\theta}_{2J}-\ddot{\theta}_{2J}^{2})\cos\theta_{3J}-3N^{2}\dot{\theta}_{2J}^{2}\ddot{\theta}_{2J}\sin\theta_{3J})]$$

$$S_{61}=[2((\ddddot{\theta}_{J}-6\ddot{\theta}_{J}\dot{\theta}_{J}^{2})\sin(\theta_{J}-\alpha)+((\dddot{\theta}_{J}-\dot{\theta}_{J}^{3})\theta_{J}+3(\dddot{\theta}_{J}\dot{\theta}_{J}+\ddot{\theta}_{J}^{2}))\cos(\theta_{J}-\alpha)]$$

$$S_{71}=[-2a(-3(\dddot{\theta}_{2J}\ddot{\theta}_{J}+\ddot{\theta}_{2J}\dddot{\theta}_{J})\sin\theta_{2J}-3\ddot{\theta}_{2J}\ddot{\theta}_{J}\dot{\theta}_{2J}\cos\theta_{2J}+(\ddddot{\theta}_{2J}-3\ddot{\theta}_{2J}\dot{\theta}_{2J}^{2})$$
$$\cos\theta_{2J}-(\dddot{\theta}_{2J}-\dot{\theta}_{2J}^{3})\dot{\theta}_{2J}\sin\theta_{2J})-2bN(-3N(\ddot{\theta}_{2J}^{2}+\dot{\theta}_{2J}\ddot{\theta}_{2J})\sin\theta_{3J}-3N^{2}\dot{\theta}_{2J}^{2}$$
$$\ddot{\theta}_{2J}\cos\theta_{3J}+(\ddddot{\theta}_{2J}-3N^{2}\ddot{\theta}_{2J}\dot{\theta}_{2J}^{2})\cos\theta_{3J}-N\dot{\theta}_{2J}(\dddot{\theta}_{2J}-N^{2}\dot{\theta}_{2J}^{3})\sin\theta_{3J})]$$

$$S_{81}=[2(-(\ddddot{\theta}_{J}-6\ddot{\theta}_{J}\dot{\theta}_{J}^{2})\cos(\theta_{J}-\alpha)+\dot{\theta}_{J}(\dddot{\theta}_{J}-\dot{\theta}_{J}^{3})+3(\dddot{\theta}_{J}\dot{\theta}+\ddot{\theta}_{J}^{2}))\sin(\theta_{J}-\alpha)]$$

$$S_{91}=2[((-X_{m}\ddddot{\theta}_{J}-3(\dddot{\theta}_{J}\dot{\theta}_{J}+\ddot{\theta}_{J}^{2})Y_{m}+3x_{m}\ddot{\theta}_{J}\dot{\theta}_{J}^{2})-(\dddot{\theta}_{J}Y_{m}-3\ddot{\theta}_{J}\dot{\theta}_{J}X_{m}-\dot{\theta}_{J}^{3}Y_{m}))\sin(\theta_{J}-\alpha)$$
$$+\ ((-X_{m}\dddot{\theta}_{J}-3\ddot{\theta}_{J}\dot{\theta}_{J}Y_{m}+X_{m}\dot{\theta}_{J}^{3})\dot{\theta}_{J}+(\ddddot{\theta}_{J}Y_{m}-3(\dddot{\theta}_{J}\dot{\theta}_{J}+\ddot{\theta}_{J}^{2})X_{m}-3\ddot{\theta}_{J}\dot{\theta}_{J}^{2}Y_{m}))\cos(\theta_{J}-\alpha)]$$

$$S_{101}=-2a[((\ddddot{\theta}_{2J}-\ddddot{\theta}_{J})-6(\ddot{\theta}_{6J}-\ddot{\theta}_{J})(\dot{\theta}_{2J}-\dot{\theta}_{J})^{2})\mathrm{Sin}(\theta_{2J}-\theta_{J}+\alpha)+(4(\dddot{\theta}_{2J}-\dddot{\theta}_{J})$$
$$(\dot{\theta}_{2J}-\dot{\theta}_{J})-(\dot{\theta}_{2J}-\theta_{J})^{4}+3(\ddot{\theta}_{2J}-\ddot{\theta}_{J})^{2})\cos(\theta_{2J}-\theta_{J}+\alpha)]$$

APPENDIX (C) CONTINUED

$$S_{111}=-2b[((N\ddddot{\theta}_{2J}-\ddddot{\theta}_J-6(N\ddot{\theta}_{2J}-\ddot{\theta}_J)(N\dot{\theta}_{2J}-\dot{\theta}_{2J})^2)\sin(\theta_{3J}-\theta_J+\alpha)+((4(N\dddot{\theta}_{2J}-$$
$$\dddot{\theta}_J)(N\dot{\theta}_{2J}-\dot{\theta}_J)-(N\dot{\theta}_{2J}-\dot{\theta}_J)^4+3(N\ddot{\theta}_{2J}-\ddot{\theta}_J)^2)\cos(\theta_{3J}-\theta_J+\alpha)]$$

$$S_{121}=2a[((-\ddddot{\theta}_J X_m-3(\dddot{\theta}_J\dot{\theta}_{2J}+3\ddot{\theta}_J\ddot{\theta}_{2J})Y_m+3\ddot{\theta}_{2J}\dot{\theta}^2_{2J}X_m)-(\dddot{\theta}_J Y_m-3\ddot{\theta}_J\dot{\theta}_{2J}X_m-\dot{\theta}^3_{2J}Y_m)$$
$$\text{Sin }\theta_{2J}+((-\dddot{\theta}_J X_m-3\ddot{\theta}_J\dot{\theta}_{2J}Y_m+\dot{\theta}^3_{2J}X_m)\dot{\theta}_{2J}+(\dddot{\theta}_J Y_m-3(\dddot{\theta}_J\dot{\theta}_{2J}+3\ddot{\theta}_J\ddot{\theta}_{2J})X_m-3\ddot{\theta}_{2J}$$
$$\dot{\theta}^2_{2J}Y_m))\text{Cos}\,\theta_{2J}]$$

$$S_{131}=2bN[((-X_m\ddddot{\theta}_{2J}-3N(\dddot{\theta}_{2J}\dot{\theta}_{2J}+\ddot{\theta}^2_{2J})Y_m+3N^2\ddot{\theta}_{2J}\dot{\theta}^2_{2J}x_m)-(\dddot{\theta}_{2J}Y_m-3N\ddot{\theta}_{2J}\dot{\theta}_{2J}x_m-$$
$$N^2\dot{\theta}^3_{2J}Y_m)N\dot{\theta}_{2J})\sin\theta_3+((-X_m\dddot{\theta}_{2J}-3N\ddot{\theta}_{2J}\dot{\theta}_J Y_m+N^2\dot{\theta}^3_{2J}X_m)N\dot{\theta}_{2J}+(\ddddot{\theta}_{2J}Y_m-$$
$$3N(\dddot{\theta}_{2J}\dot{\theta}_{2J}+\ddot{\theta}^2_{2J})X_m-3N^2\ddot{\theta}_{2J}\dot{\theta}^2_{2J}Y_m))\cos\theta_{3J}]$$

$$S_{141}=2ab[((1-N)\ddddot{\theta}_{2J}-3\ddot{\theta}_{2J}(\dot{\theta}_{2J}-N\dot{\theta}_{2J})^2)-3(1-N)^3\ddot{\theta}_{2J}\dot{\theta}^2_{2J})\sin(\theta_{2J}-$$
$$\theta_{3J})+((1-N)(\dddot{\theta}_{2J}-(\dot{\theta}_{2J}-N\dot{\theta}_{2J})^3\dot{\theta}_{2J})+3(1-N)^2(\dddot{\theta}_{2J}\dot{\theta}_{2J}+\ddot{\theta}_{2J}))$$
$$\cos(\theta_{2J}-\theta_{3J}]]$$

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VITA

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