MIXED-LOOP ROBOT: KINEMATIC AND DYNAMIC ANALYSIS

By

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CHAPTER I

INTRODUCTION

Robots in Industry

When observed in operation on a production line, it is nearly impossible not to compare the industrial robot's motion with that of a human operator. The mystique that has been created about the industrial robot comes from the robot's ability to move an object from one point to another in any number of complex paths, while at the same time reorienting the direction of the object as it moves. The "manipulative" capability allows the robot to perform many complex operative tasks which otherwise could be only performed by humans. Because robots are typically installed as replacements for human operators rather than as replacement for other forms of automation, the tendency to view robots as automated human workers becomes even stronger.

There is some confusion over the exact definition of an industrial robot. In order to understand what a robot is, it is best to first review the various categories of manufacturing automation. Automation ranges in degrees from simply the use of powered or nonpowered tools to the complete control of a task by a computer aided manufacturing system involving mass storage memories, sensory devices, and periodic changes in programming. Between these extremes fall the categories of "hard automation" and "flexible automation."

In hard automation, a task is performed by a tool which has been set up using mechanical limits and adjustments so that no human control is required during operations [22]. Hard automation is typically dedicated to one application throughout the life of the tool. The primary disadvantage of hard automation is the difficulty of justifying the potentially high investment in dedicated equipment for use in a batch manufacturing operation in which changeovers may be required. An additional drawback is that human assistance is generally required in loading and unloading the tool.

Until recently, the alternative to hard automation was to increase the direct labor content of a manufacturing task. Flexible automation was developed as a means of increasing the range of tasks that can be performed and also to improve the changeover capability of manufacturing tools. In flexible automation, as in hard automation, a tool is preprogrammed by a human to perform a specific task [22]. In this case, however, the workpiece can be manipulated so that a greater number of different tasks can be performed in each cycle, such as machine loading and unloading as well as parts transfer. In addition, a changeover to another job can be typically accomplished by reprogramming rather than by reworking or replacing the equipment.

Industrial robots can be classified as a type of flexible automation. The Robot Institution of America (RIA) defines a robot as a "reprogrammable multifunctional manipulator designed to move materials, parts, tools or specialized devices through programmed motion for the performance of a variety of tasks" [20]. Joseph Engelberger of Unimation, the pioneering firm in the U.S. robotics field, defines an

The Japanese Industrial Robot Association, on the other hand, has defined four levels of industrial robot [22]:

- Manual manipulators that perform sequences of tasks which are fixed or preset.
- Playback robots that repeat fixed instructions.
- Numerically controlled robots that carry out tasks through numerically loaded information.
- Intelligent robots that perform through their own recognition capabilities.

The RIA definition of industrial robots is the best one to be presented to date. The first three words in the definition are essential to understand the basic concept of a robot:

Reprogrammable: An industrial robot is controlled by a programmable control device with memory, such as a microprocessor. The controller is programmed to command the robot arm and gripper to automatically repeat a specified series of movements, such as moving a workpiece through a drilling operation. If the robot is to be used in different operations, an entirely new sequence of movements can be created by reprogramming the controller. The requirement that a robot be programmable so that it can operate automatically prevents such devices as remote manipulators (e.g., those used to handle radioactive materials) or some types of locomotive devices from being classified as robots, since direct human control is required for them to operate.

Multifunctional: An industrial robot is much more flexible than hard automation in that it can perform a wide variety of tasks. During a single cycle of movement, for example, a robot can load a machine, unload the workpiece, transport it to another machining operation, deburr the part, and place it on a conveyor belt. It is therefore a general purpose device rather than a dedicated machine.

Manipulator: An industrial robot differs from other forms of automation in its ability to move an object through space while at the same time reorienting its position. It is this ability to manipulate objects that lead to the inevitable comparisons between robots and human arms and hands.

Robots can thus be thought of as machines that fill the gap between the specialized capabilities normally associated with hard automation and the extreme flexibility of human labor. Basically, a robot is an automatic device with a single arm for manipulating tools or parts through a programmed sequence of motions through space. What differentiates a robot from other types of automation is its ability to perform a sequence of operations quite different in nature, repetitive motions without the need for human involvement. Because of this unique capability to perform several different tasks, robots are used in a variety of industrial applications where the task can be performed in a more safe and effective manner than by human workers.

History of Robots

Robot development started about fifty years ago because of the speneed of handling radioactive materials. Argonne National Laboratory (ANL) (ANL) first initiated a project to develop a device which could duplicate the hand motions of a person at a remote control station. This work led to the development of a series of mechanically coupled "master-slave" systems. In the master-slave system the motion of the master was duplicated by the slave system, and forces on the slave system were to provide feed-back to the master system. The disadvantage with the master-slave system is that the master and the slave units have to be located fairly close to each other [17]. In order to overcome this disadvantage, ANL came up with an electronically controlled master-slave system, and Oak Ridge National Laboratory developed a hydraulic master-slave system. Several companies produced such robot systems as General Electric's "Man-Mate" and La Calhene's "MA-2 France."

But all these master-slave systems needed human operators to monitor, make decisions, and control the robot all the time. For these reasons, this kind of system is not good for tedious and repetitive tasks or remote control at a very long distance, which may cause time delay. Sheridan described "Supervisory Controlled Manipulators," which operated partially under human control, and Ernst described "Automatic Manipulator Systems," which carried out the tasks completely under computer control, involving sensory information about the environment in 1961. The first generation of automatic industrial robots was built by Illinois Institute of Research in 1971. Now there are thousands of first generation industrial robots in this country. "Unimate" and "Puma" by Unimation, Inc., and "T-3" by Cincinnati Milacron are the well known modern industrial robots. They are in use in the auto industry and other mass production industries for operations like welding, painting, and assembling.

First generation robots are controlled by minicomputers or microcomputers. One can either input the coordinates of working positions and some other data to the computer and let the computer calculate the working path, or teach the robot by leading it to do the work once while the computer memorizes the sequence of steps. Under computer control, the robot can then be made to repeat the operation [10]. These kinds of robots are good for tedious and repetitive tasks, but such robots cannot satisfy all situations on the production lines. For example, the robot cannot see, so it does not know where the machine part is, nor can it distinguish one part from another. Because it has no force feeling, the robot may crush the delicate workpieces which it may be wanting to transfer from one place to another. Problems such as these led the Robot Research and Design Group to incorporate several kinds of sensors on robots. The second generation of robots are equipped with vision sensors to see, proximate sensors to feel the distance from the object, and force and torque sensors to know the amount of force or torque applied on the object. Some of these second generation robots are already in use, though much needs to be accomplished in the vision sensor technology.

Now the industrial robots are designed to do more than human beings can do. Robots work three shifts a day without a break in noisy, hot, fumy, radioactive places, without any kind of problem. Also, their productivity and quality are quite stable. These are the very reasons an industry prefers to have robots installed on their production lines. Robots are not the ultimate solution to all the economic pressure that an industry is called upon to bear, but they do provide a partial answer to some of the productivity problems in U.S. industry.

Present Study

The objectives of the present study are to develop a computer program package for mixed-loop robots (Figure 1) and to make a comparative study with the conventional open-loop robot. In the present case a

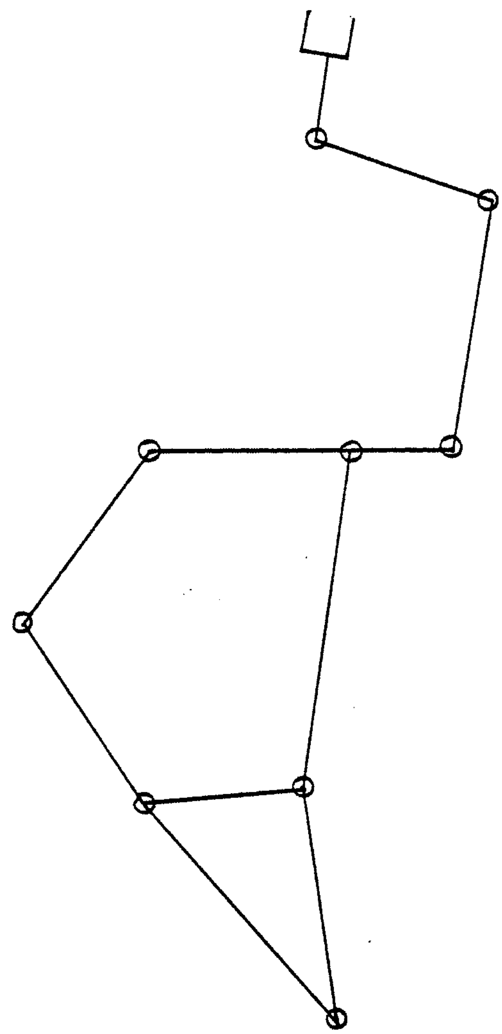

Figure 1. Mixed-Loop Robot

closed-loop is a planar-loop, but it can also be a spatial-loop. Any type of joint can be used: revolute, prismatic, or cylindrical.

The analysis problem is broken up into two sections: the analysis of closed-loop is performed first and then the results of this analysis are integrated with the open-loop part of the robot to complete the analysis of the entire robot. Kinematic analysis is done first, followed by dynamic analysis.

In kinematic analysis, the closed-loop is solved to get the values for the unknown variables of the mechanism from the known input values. Now all the joint variables are known including joint velocities and accelerations. With these variables and fixed parameters of the robot already known, the coordinate frame relationships between the links of the robot can be established. Once this is done the position, velocity, and acceleration of any point lying anywhere on the robot can be found by simple matrix multiplications. Points that are to be analyzed are defined with respect to the link coordinate frame of the link they belong to. After the point analysis is completed, link analysis is done to find out the link angular velocitie and angular acceleration. This also involves multiplication of already known matrices. Details of the procedure are described in the following chapter.

For the dynamic analysis, the open-loop part of the robot is analyzed to find the joint torques and forces using the Newton-Euler method. This is followed by the analysis of the closed-loop using the standard available method for dynamic analysis of planar mechanism. With this, the analysis of the mixed-loop robot is complete.

CHAPTER II

KINEMATIC ANALYSIS

The purpose of this chapter is to review the matrix method of kinematic analysis as applied to mechanisms and robots, to define the physical system to be analyzed, and to set down the notations and assumptions to be made subsequently.

A robot is made up of a number of physical links and all these links are considered to be rigid links. Also, these links are interconnected by joints or pairs. A pair between the links can be either revolute, prismatic, cylindrical, or spherical. In the present work only three kinds of joints are used: revolute, prismatic, and cylindric. A revolute pair is a connection allowing rotation about an axis between adjacent links as in journal bearing, while a prismatic pair allows rectilinear translation between connected links. Other connections such as a cylindrical pair are represented by the combination of revolute and prismatic pairs. All these pairs are considered to be geometrically perfect so that there is no "play" or "backlash."

There is a definite relationship between the various links of the robot (Figure 2). In order to describe this relationship between links, a coordinate frame is assigned to each link [2]. If the connection between the links is a revolute pair, then θ_i is the joint variable. The origin of the coordinate frame of link i is set to be at the intersection of the common normal between the axes of the joints i and joint i+1. In the

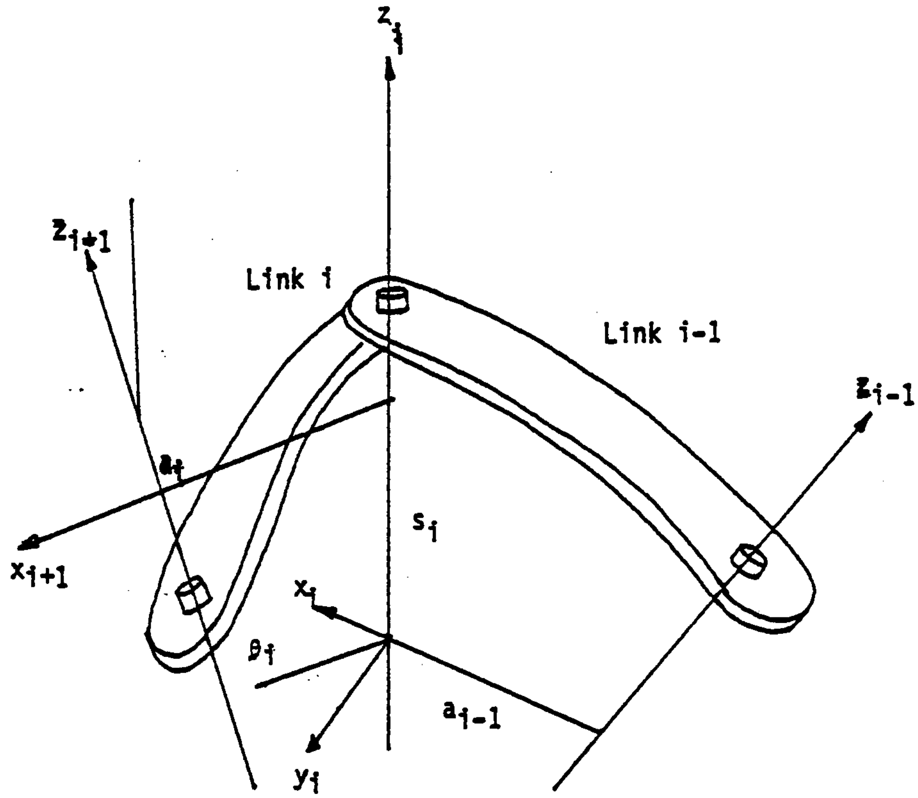

Figure 2. Link Parameters

case of intersecting joint axes, the origin is at the point of intersection of joint axes. If the axes are parallel, the origin is chosen to make the joint distance zero for the next link whose coordinate origin is defined. The z axis for link i will be aligned with the axis of joint i+1. The x axis is aligned with any common normal which exists and is directed along the normal from joint i and to joint i+1. In the case of intersecting joints, the direction of the x axis is parallel or antiparallel to the vector cross product $z_{i-1} \times z_i$ [12].

In the case of a prismatic joint, the distance s_i is the joint variable [12, 2]. The direction of the joint axis is the direction in which the joint moves. The direction of the axis is defined but, unlike a revolute joint, the position in space is not defined. The origin of the coordinate frame for a prismatic joint is coincident with the next defined link origin. The z axis of the prismatic joint is aligned with axis joint i+1. The x_i axis is parallel or antiparallel to the vector cross product of the direction of the prismatic joint and z_i. For a prismatic joint the zero position is defined when $s_i = 0$.

With the manipulator in zero position, the positive sense of rotation for revolute joints or displacement for prismatic joints can be decided and the sense of the direction of z axes determined. The reference coordinate frame is usually the base link coordinate frame, but if it is desired to define a different reference coordinate frame, then the relationship between the reference and the base coordinate systems can be described by a fixed homogeneous transformation. Normally, the origin of the last link to which the end-effector is attached is chosen to be coincident with the previous link coordinate frame [12]. If a tool (or end effector) is used whose origin and axes do not coincide with the coordinate system of the last link, then the tool can be related to this last link coordinate system by a fixed homogeneous transformation.

Now, having assigned the coordinate frame to each link according to the preceding scheme, we can establish the relationship between successive frames. The geometry and the relative position of link i relative to link i-1 is completely specified by four parameters, a_i, α_i, θ_i, and s_i [18]. These four parameters are measured for each joint or pair of the robot according to the following set of conventions:

i = number of a particular joint or pair.

z_i = Characteristic axis of motion of the pair involved.

x_i = Axis formed by common perpendicular directed from Z_{i-1} and Z_i. If these axes intersect, then orientation of x_i is arbitrary.

y_i = Axis implicitly defined to form a right handed coordinate system.

a_i = Length of common perpendicular from z_i to z_{i+1}, always positive.

α_i = Angle from positive z_i to positive z_{i+1}, measured counter clockwise about x_{i+1}.

θ_i = Angle from positive x_i to positive x_{i+1}, measured counter clockwise about z_i.

s_i = Distance along z_i from x_i to x_{i+1}, takes sign from orientation of z_i.

Once these four parameters are defined, the geometry of the mechanical assemblage is completely specified.

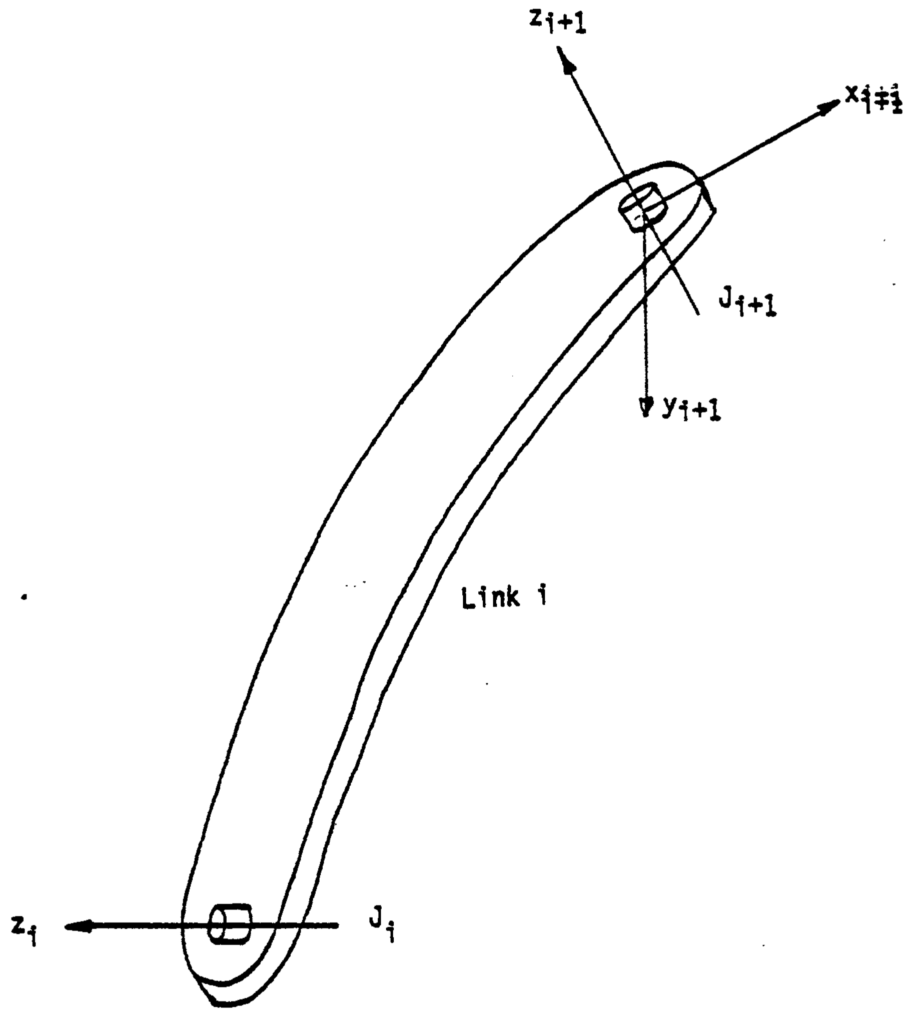

Figure 3. Link Coordinate Frame

The axes x_{i+1}, y_{i+1}, and z_{i+1} define a right-handed cartesian coordinate system associated with link i (Figure 3). The four parameters a_i, α_i, θ_i, and s_i fix the position of the coordinate system of link i relative to link i-1. The relative positions of these coordinate frames can be stated analytically in terms of a (4 x 4) homogeneous transformation matrix involving the four parameters a_i, α_i, s_i, and θ_i [18].

$$A_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & s_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

The above A matrix becomes a function of θ_i if the joint is revolute: if the joint is prismatic, then it becomes a function of s_i. This transformation A_i sets up the relationship between the links of the robot [12].

Now, after having established interrelationships between the manipulator links, kinematic analysis can be performed. As said earlier, the analysis is done in two stages (Figure 4). First, the unknown variables of the closed-loop or linkage are determined from the known input variables. Unknown joint variables are joint displacements, velocities, and accelerations. Determination of these variables makes further kinematic analysis on the robot as straight forward as open-loop robot analysis.

Kinematic Analysis of a Closed-Loop

In the closed-loop of the mixed-loop robot a five bar mechanism is considered (Figure 1). One can have any mechanism instead of the one

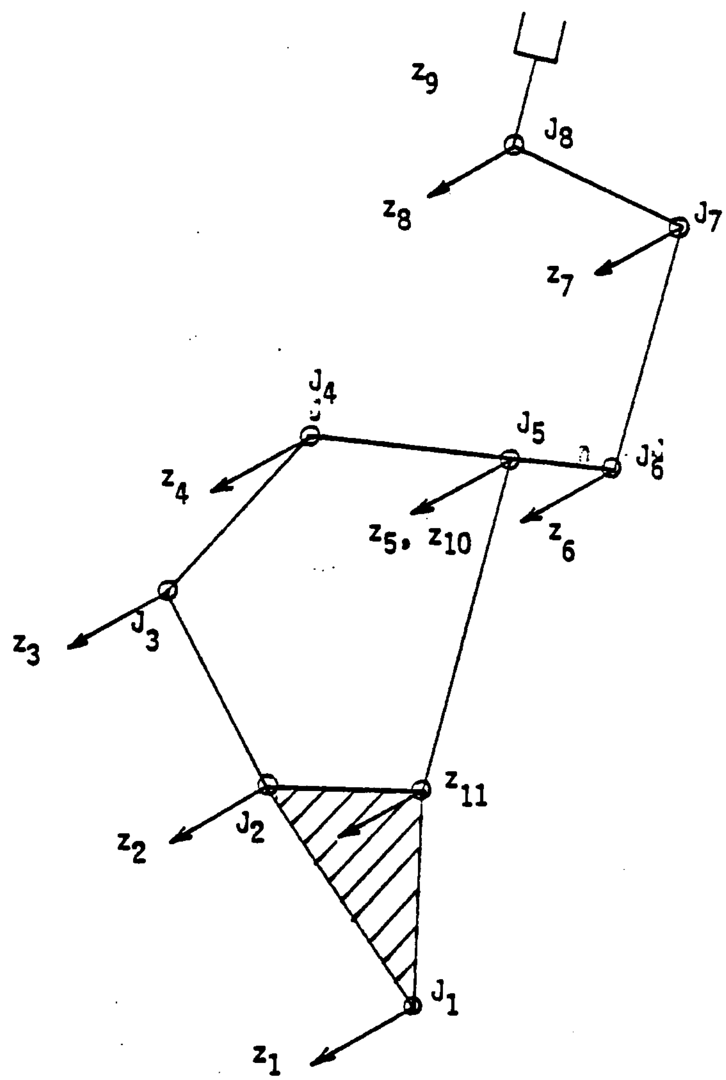

Figure 4. Link and Joint Numbering Convention

incorporated in the present case; however, the analysis procedure remains the same.

The kinematic analysis of a constrained system like in the present case involves determination of position, velocity, and acceleration of the part of the system when certain "input" positions, velocities, and accelerations are known [18]. A constrained mechanical system is essentially an assemblage of coupled links whose degrees of freedom is restricted after one of the links has been grounded. In the present case, however, the grounded link itself is a moving link. But still analysis methodology does not change. In a constrained system the number of independent motion variables is equal to the degrees of freedom of the mechanical system [18].

Displacement Analysis

The formulation for maintaining a closed-loop assembly in the constrained system for a given value of independent variables result in nonlinear simultaneous equations. Generally, such equations are solved with the aid of numerical techniques on computers. There are a variety of iterative schemes to solve such nonlinear equations numerically, but the most well known and often used is the Newton-Raphson method [18]. The technique used here, developed by Uicker, Denavit, and Hartenburg, is also iterative in nature and is similar to the N-R technique [18].

Simultaneous nonlinear equations are the result of the matrix-loop equation, which is of the form

$$A1 \cdot A2 \cdot A3 \text{———} A_i \text{———} A_n = I \qquad (2)$$

where [I] on the right hand side of the equation is an identity matrix indicating that it is a closed-loop. N is a number of links in a loop.

The number of transformations will be equal to the number of links in the loop since a coordinate frame is associated with each link and between two successive coordinate frames we have a transformation. This transformation is a relationship between the two coordinate frames.

In the case under consideration, a five bar mechanism is considered as a part of the manipulator and equations are developed for it (N = 5). In any mechanism in general one of the links is held stationary by fixing it to the ground and all the relationships are developed relative to this link. In here, since the ground link is attached to one of the moving links of the robot, all the relationships are developed relative to this moving link of the robot.

The present method is intended for computer use, so all the continuous variables like time are reduced to a series of tabulated values. For the same reason mechanism is analyzed at a series of instantaneous positions [3]. At any of these positions, input variables θ_i or S_i, and a_i, α_i are known for all the links. To determine the values of the remaining variables, an iterative scheme is used.

For the development of an iteration process, initial estimates of unknown variables of the mechanism are made. These are expressed as

$$\theta_i = \tilde{\theta}_i + d\theta_i \quad i = 2, 3, 4 \tag{3}$$

if the joint is prismatic θ_i is replaced by an S_i. θ_i here is a joint variable, $\bar{\theta}_i$ is an initial estimate of the variable, and $d\theta_i$ is the correction added to it to reduce the error. The loop equation can now be expressed as

$$A1(\theta_1) \; . \; A2(\theta_2 + d\theta_2) \text{———} A5 = 1 \tag{4}$$

Transformations are written in terms of initial estimates.

$$\begin{bmatrix} \mathrm{COS}(\bar{\theta}_i + d\theta_i) & -\mathrm{SIN}(\bar{\theta}_i + d\theta_i)\mathrm{COS}\theta_i & \mathrm{SIN}(\bar{\theta}_i + d\theta_i)\mathrm{SIN}\theta_i & a_i\mathrm{COS}(\bar{\theta}_i + d\theta_i) \\ \mathrm{SIN}(\bar{\theta}_i + d\theta_i) & \mathrm{COS}(\bar{\theta}_i + d\theta_i)\mathrm{COS}\theta_i & -\mathrm{COS}(\bar{\theta}_i + d\theta_i)\mathrm{SIN}\theta_i & a_i\mathrm{SIN}(\bar{\theta}_i + d\theta_i) \\ 0 & \mathrm{SIN}\alpha_i & \mathrm{COS}\alpha_i & S_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

Assuming that initial estimates are fairly accurate to make use of small angle approximations on $d\theta_i$ and using Trignometric identities for the sum of angles, the foregoing matrix can be expressed as the sum of two matrices [18].

$A_i(\theta_i)$

$$\begin{bmatrix} \mathrm{COS}\theta_i & -\mathrm{SIN}\theta_i\mathrm{COS}\alpha_i & \mathrm{SIN}\theta_i\mathrm{SIN}\alpha_i & a_i\mathrm{COS}\theta_i \\ \mathrm{SIN}\theta_i & \mathrm{COS}\theta_i\mathrm{COS}\alpha_i & -\mathrm{COS}\theta_i\mathrm{SIN}\ _i & a_i\mathrm{SIN}\theta_i \\ 0 & \mathrm{SIN}\alpha_i & \mathrm{COS}\alpha_i & S_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$+$$

$$\begin{bmatrix} \mathrm{SIN}\theta_i & -\mathrm{COS}\theta_i\mathrm{COS}\alpha_i & -\mathrm{COS}\theta_i\mathrm{COS}\alpha_i & -a_i\mathrm{SIN}\theta_i \\ \mathrm{COS}\theta_i & -\mathrm{SIN}\theta_i\mathrm{COS}\alpha_i & -\mathrm{SIN}\theta_i\mathrm{COS}\alpha_i & a_i\mathrm{COS}\theta_i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

The first of these two matrices is the original transformation matrix evaluated for $\bar{\theta}_i$ and the second matrix is the first partial derivative of the transformation matrix with respect to θ_i, also evaluated for θ_i. The first matrix is denoted by A_i.

$$A_i(\theta_i) \equiv \bar{A}_i + \frac{A_i(\theta_i)}{d\theta_i} \Big|_{\theta_i = \bar{\theta}_i} d\theta_i \tag{7}$$

Since the problem is to be adapted to the computer operation, a linear operator Q_θ is introduced to perform the differentiation

$$\frac{\partial A(\theta_i)}{\partial \theta_i} = \Big|_{\theta_i = \bar{\theta}_i} = Q \bar{A}_i \tag{8}$$

Under this definition there are two operator matrices Q_θ for the revolute pair and Q_s for the sliding pair.

$$Q_\theta = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{9}$$

$$Q_s = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{10}$$

With these definitions the above equation can be rewritten as [18]:

$$A_i(\theta_i \equiv A_i + [Q_\theta]\, A_i\, d\ _i$$

$$(I + Q_\theta\ d\theta_i)\ A_i \qquad (11)$$

Substituting in equation (4) yields

$$A1(I + Q\ d\theta_2)\ A2(I + Q\ d\theta_3)\ A3 \text{———} A5 = I \qquad (12)$$

In this multiplication all higher order terms of the form $(d\theta_i d\theta_j)$ are neglected. The expanded form of the equation becomes

$$\begin{aligned} & (A1 \,.\, Q \,.\, A2 \,.\, A3 \,.\, A4 \,.\, A5)\ d\theta_2 \\ + \quad & (A1 \,.\, A2 \,.\, Q \,.\, A3 \,.\, A4 \,.\, A5)\ d\theta_3 \\ + \quad & (A1 \,.\, A2 \,.\, A3 \,.\, Q \,.\, A4 \,.\, A5)\ d\theta_4 \\ = \quad & I - (A1.\ A2 \,.\, A3 \,.\, A4 \,.\, A5) \end{aligned} \qquad (13)$$

Transformations [A1] and [A5] are known since input variables are known. To write the above equation in the compact form, the following notation is used:

$$B1 = A1 \,.\, A2 \,.\, A3 \,.\, A4 \,.\, A5 \qquad (14)$$

$$B_i = (A1 \,.\, A2 \text{———} QA_i \text{———} A5)d\ _i \qquad (15a)$$

With these notations, equation (12) becomes

$$B2 + B3 + B4 = I - B1 \qquad (15b)$$

Single matrix can represent the entire left hand side.

$$E = I - B1 \tag{16}$$

In the closed-loop only the revolute and prismatic pairs are considered. Of the total of 16 equations generated by equation (12), only nine are required to solve the system completely. The left hand side of equation (16) can be written in the form

$$[M]\ \{D\} = \{V\} \tag{17}$$

Where

$$M = \begin{bmatrix} B2(1,4) & B3(1,4) & B4(1,4) \\ B2(2,4) & B3(2,4) & B4(2,4) \\ B2(3,4) & B3(3,4) & B4(3,4) \\ B2(1,1) & B3(1,1) & B4(1,1) \\ B2(2,1) & B3(2,1) & B4(2,1) \\ B2(3,1) & B3(3,1) & B4(3,1) \\ B2(2,2) & B3(2,2) & B4(2,2) \\ B2(3,2) & B3(3,2) & B4(3,2) \\ B2(3,3) & B3(3,3) & B4(3,3) \end{bmatrix} \tag{18}$$

$$D = \begin{bmatrix} d_2 \\ d_3 \\ d_4 \end{bmatrix} \qquad V = \begin{bmatrix} B1(1,4) \\ B1(2,4) \\ B1(3,4) \\ B1(1,1) \\ B1(2,1) \\ B1(3,1) \\ B1(2,2) \\ B1(3,2) \\ B1(3,3) \end{bmatrix} \tag{19}$$

In general this system of nine equations, MD = V, in six or less unknowns has no exact solution. However, since the entire method revolves around an iterative approach, no significant error is introduced if the system is solved for the closest approximation of a solution to all nine equations in the root-mean square sense [18].

$$D = (M^t M)^{-1} M^t V \tag{20}$$

where M^t is a transpose of matrix M. This [D] vector gives explicitly the unknown error terms. When the error terms $d\theta_i$ have been evaluated they are added to the initial estimates θ_i to give an improved approximation of the exact values of pair variables. If further accuracy is required, new approximations are used and the entire process is repeated. This iteration process may be continued until the required accuracy is achieved. At that point, the mechanism is said to be completely solved for the specified inputs [18].

To solve the mechanism for a new position, inputs are incremented by small amounts d_θ or d_S, and previously calculated values of θ_i or s_i are used as initial estimates. After a few iterations by the same method, these values converge to the proper values of θ_i for the new position. Increments in the input values should be sufficiently small.

Usually, the process converges if the estimates are within 15 to 20 degrees. If $(M^t M)$ matrix is singular, then it cannot be inverted and the mechanism is said to be in the locking position or in the dead point [18].

Velocity Analysis

Here the method of evaluating the relative velocities between adjacent links in a two degrees of freedom, five link mechanism. The relative velocities are the rates of change of the proper variable q_i, and will be denoted by q_i

$$q_i = \frac{dq_i}{dt}$$

Since this method is intended to follow the iterative method of displacement analysis using matrices, the velocities are found numerically at a specified point in the cycle of operation rather than as an analytical function of the parameters [3]. Link 1 is considered a fixed or ground link, q_1 and q_5 the input variables.

At any given point of the cycle, the matrix-loop equation (2) may be differentiated by means of derivative operator matrices Q_θ and Q_s to give

$$Q1A1 \cdot A2 — A5 + A1 \cdot Q2 \cdot A2 — A5 + — + A1 \cdot A2 \cdot A3 — Q5A5 = 0 \tag{21}$$

Each of the A_i- matrices is available from the displacement analysis as a numerical 4 x 4 matrix. In consequence, the matrix products denoted by B_i can be formed:

$$B_i = A1 \cdot A2 \cdot — A_{i-1}\, Q_i A_i — A5 \tag{22}$$

By reason of loop equation (2), it follows that

$$B1 = Q1A1 \cdot A2 — A_i — A5 = Q1I = Q1$$

Using these definitions and rearranging terms, equation (21) is reduced to

$$B2q_2 + B3q_3 + B4q_4 = 1Q1 - B5 \tag{23}$$

This is the matrix form of the velocity equation giving the relative velocities q_2——q_4 as functions of the input velocities q_1 and q_5.

Owing to the antisymmetric properties of the derivative operator matrices Q_i, and the orthogonality of the 3 x 3 submatrix of rotation in the transformation matrix A_i, it can be shown that the 16 linear equations implied in equation (23) and the equalities of the six elements below the major diagonal are sufficient to satisfy the entire matrix equation. Again, the above equation can be written in terms of

$$MD_v = C \tag{24}$$

where

$$M = \begin{bmatrix} B114 & B214 & B314 \\ B124 & B224 & B324 \\ B134 & B234 & B334 \\ B121 & B221 & B321 \\ B131 & B231 & B331 \\ B132 & B232 & B332 \end{bmatrix} \tag{25}$$

$$D_v = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \qquad C = \begin{bmatrix} C(1,4) \\ C(2,4) \\ C(3,4) \\ C(2,1) \\ C(3,1) \\ C(3,2) \end{bmatrix} \tag{26}$$

$$C_v \text{ matrix here is } = [Q1]q_1 - [Q5] \cdot A5 \cdot q_5 \tag{27}$$

In the present geometric configuration (five link mechanism) where a mechanism is movable with links $n < 7$, the system of equations (24) remains compatible because of the particular geometry, but some of the equations are redundant. Since the inverse of the rectangular matrix [M] does not exist in this case, the equation (24) cannot be directly solved for D_v, a vector of unknown velocities. However, the method of least squares may be used and, in this case, gives an exact solution since the system of equations is compatible.

$$D_v = (M^t M)^{-1} M^t C_v \tag{28}$$

The product $(M^t M)$ forms a square matrix that can be inverted assuming that it is nonsingular. If the determinant is zero, the matrix cannot be inverted and the velocities are undefined. By definition, this occurs if and only if the mechanism is at a dead point for the position considered.

Since the matrix M^t and the matrix $[M^t M]^{-1}$ have already been calculated in the displacement analysis, less computational effort is required. Velocity analysis, therefore, requires only two matrix multiplications of equation (28) and multiplication by the scaler input velocities.

Acceleration Analysis

The relative accelerations between adjacent links, that is, second time derivatives of the pair variable will be denoted by the symbol q_i, that is

$$q_i = \frac{d^2 q_i}{dt^2} = \frac{dq_i}{d^t} \tag{29}$$

Again, a similar procedure is followed as done in the velocity analysis and again after the equation (2) has been differentiated. The second time we get expression on the left hand side the same as the one we get in the velocity analysis [3].

$$B2q_2 + B3q_3 + B4q_4 = C_a \tag{30a}$$

Detailed derivation is given in the Appendix. Again, this equation can be written in the form

$$D_a = [M^tM]^{-1} M^t C_a \tag{30b}$$

Note that the solution is defined except when the matrix $[M^tM]$ is singular; i.e., except at dead points of the cycle. With the analysis of closed-loop done, all the joint displacements, velocities, and accelerations are known. These matrices can now be used to find the position, velocity, and acceleration of any point on the robot. The same matrices are later used for evaluating link angular velocities as well as angular accelerations

Kinematics of Points of Interest

In the development of the matrix method of analysis, a right handed cartesian coordinate $o_{i+1}x_{i+1}y_{i+1}z_{i+1}$ has been defined to be associated with each link i. Any point on link i can therefore be described by its coordinates x_{i+1}, y_{i+1}, z_{i+1} with respect to that system.

The figure on the next page gives the details as to how the interrelationships between the link coordinate frames are derived and the path chosen from the base frame to form the products of matrices. To find the position of a point relative to the base frame, numerical

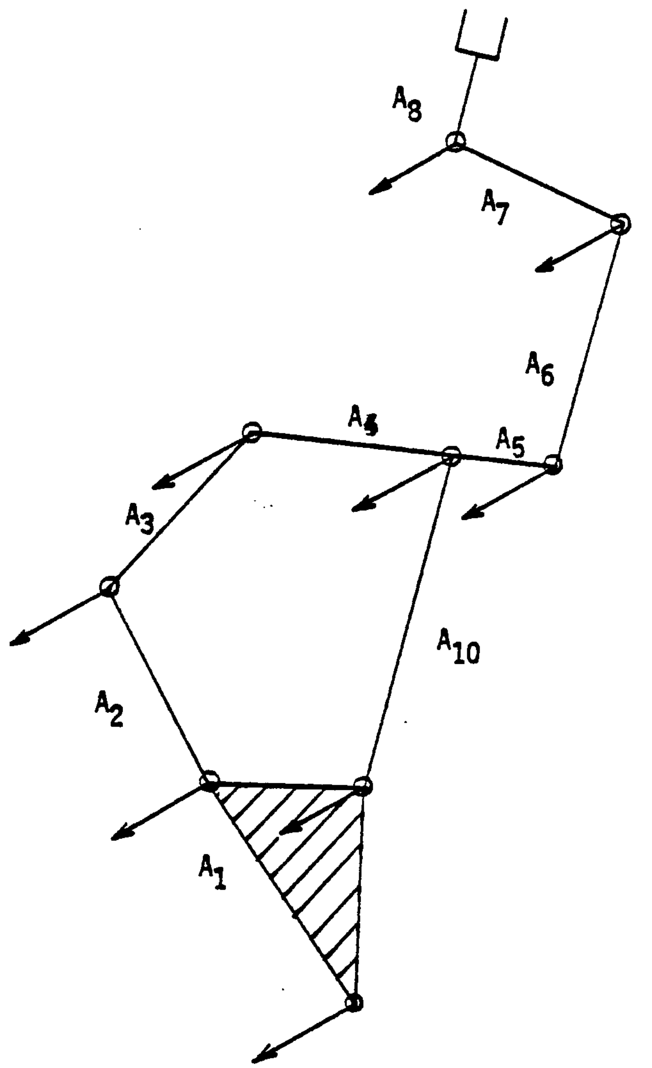

Figure 5. Formation of Matrix Products

matrices available from the displacement analysis are used. Let a vector x_i be the vector of a point position with respect to the link's local coordinate frame and X_i be the vector representing point in the base frame.

$$x_i = \begin{bmatrix} x_{i+1} \\ y_{i+1} \\ z_{i+1} \\ 1 \end{bmatrix} \quad (31a) \qquad X_i = \begin{bmatrix} X_{i+1} \\ Y_{i+1} \\ Z_{i+1} \\ 1 \end{bmatrix} \quad (31b)$$

Note that the coordinate frame $o_{i+1}X_{i+1}y_{i+1}z_{i+1}$ is associated with link i and hence point coordinates x_{i+1}, y_{i+1}, z_{i+1}.

The relationship between these two vectors is

$$(X_i) = T_i\ (x_i) \quad (32)$$

where T_i is a numerical matrix available from the displacement analysis.

$$T_i = A1\ .\ A2\ .\text{———}A_i \quad (33)$$

The instantaneous position of any point on the moving links may be found from these matrix products. They are the displacement matrices for the The absolute velocity components of a point

$$\dot{x}_{i+1} = dx_{i+1}/dt$$

$$\dot{y}_{i+1} = dy_{i+1}/dt$$

$$\dot{z}_{i+1} = dz_{i+1}/dt \quad (34)$$

on moving link i may be found by differentiating equation (32).

$$\frac{dT_i}{dt} = \sum_{j=1}^{i} A1 \cdot A2 \text{———} Q_j A_j \text{———} A_i a_j \tag{35}$$

$$x_i = \left(\frac{dT_i}{dt}\right) x_i \tag{36}$$

Similarly, acceleration components can also be found by differenting $[T_i]$ matrix twice with respect to time.

$$\frac{d^2T_i}{dt^2} = \sum_{k=1}^{i} \sum_{j=1}^{i} A1 \cdot A2 \text{———} Q_j A_j \text{———} Q_k A_k \text{———} A_i q_j a_k \tag{37}$$

$$x_i = \left(\frac{d^2T_i}{dt^2}\right) x_i \tag{38}$$

Kinematics of Links

In this section a procedure is developed to evaluate an angular velocity and an angular acceleration of a link i with respect to a reference frame. The figure on the following page is used to develop the relationships of angular velocity and acceleration.

A link i is considered with x_{i+1}, y_{i+1}, and z_{i+1} coordinate frame attached to it. This coordinate frame has a pure rotation in the reference frame. In Figure 6 vector d locates the origin o_{i+1} of the frame attached to link i. Vector R and r locate the point p with respect to the reference and the rotating frame respectively. These three vectors are related by a relationship

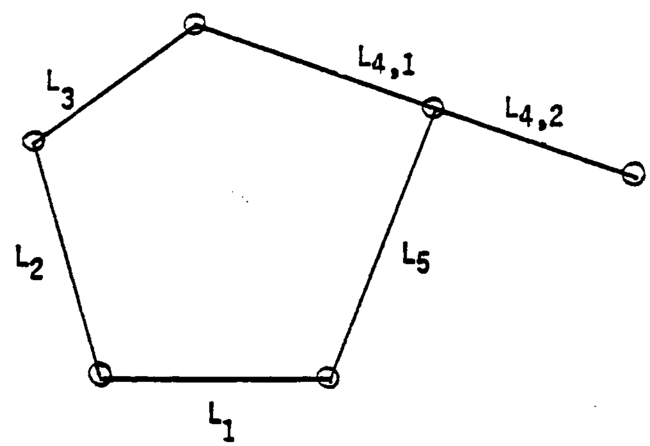

Figure 6. Closed-Loop of Mixed-Loop Robot

$$R = d + v \tag{39a}$$

where

$$R = (X_1)\ u_1 + (Y_1)\ v_1 + (Z_1)\ w_1$$

and $$v = (x_{i+1})\ u_{i+1} + (y_{i+1})\ v_{i+1} + (z_{i+1})\ w_{i+1}$$

u, v, and w are all unit vectors along three directions of mutually perpendicular axes of the frames under consideration. To get the angular quantities, equation (38) is differentiated with respect to time

$$R' = d' + r' \tag{39b}$$

$v' = w \times \dot{r}'$ since pure rotation is considered.

$$R' = d' + w \times \dot{r} \tag{39c}$$

This equation can be written in the matrix form to extract the angular velocity components. The equation written in matrix form appears below:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{y}_i \\ \dot{z}_i \end{bmatrix} = \begin{bmatrix} \dot{d}_x \\ \dot{d}_y \\ \dot{d}_z \end{bmatrix} + \begin{bmatrix} o & -\omega_{z_1} & \omega_{y_1} \\ \omega_{z_1} & o & -\omega_{x_1} \\ -\omega_{y_1} & \omega_{x_1} & o \end{bmatrix} \begin{bmatrix} T_{i_{11}} & T_{i_{12}} & T_{i_{13}} \\ T_{i_{21}} & T_{i_{22}} & T_{i_{23}} \\ T_{i_{31}} & T_{i_{32}} & T_{i_{33}} \end{bmatrix} \begin{bmatrix} x_{i+1} \\ y_{i+1} \\ z_{i+1} \end{bmatrix} \tag{39d}$$

$$[\dot{T}_i^{*}] = \begin{bmatrix} o & -\omega_{z_1} & \omega_{y_1} \\ \omega_{z_1} & o & -\omega_{x_1} \\ -\omega_{y_1} & \omega_{x_1} & o \end{bmatrix} \begin{bmatrix} T_{i_{11}} & T_{i_{12}} & T_{i_{13}} \\ T_{i_{21}} & T_{i_{22}} & T_{i_{23}} \\ T_{i_{31}} & T_{i_{32}} & T_{i_{33}} \end{bmatrix} \tag{39e}$$

Recollecting that only pure rotation is considered to develop the required relationships.

Matrices $[T_i]$ and $[T_i]$ are both available in the numerical form from the previous analysis. Now we can write

$$[\omega] = \begin{bmatrix} o & -\omega_{z_1} & \omega_{y_1} \\ \omega_{z_1} & \omega o & -\omega_{x_1} \\ -\omega_{y_1} & \omega_{x_1} & + o \end{bmatrix} = [T_i]' [T_i]^{-1} = [T_i]' [T_i]^t \tag{40}$$

where $[T_i]^t$ is a transpose of a matrix $[T_i]$ and since only 3 x 3 part of $[T_i]$ is considered which is orthogonal, the inverse of it is given by its transpose.

For angular acceleration, equation (39a) can be differentiated once more with respect to time.

$$R'' = d'' + \bar{\omega} \times (\omega \times v) + \bar{\alpha} + v \tag{41a}$$

The above equation can again be written in the matrix form to separate the components of angular acceleration

$$\bar{R}'' = [T_i]_{3 \times 3} \cdot \begin{bmatrix} x_{i+1} \\ y_{i+1} \\ z_{i+1} \end{bmatrix} \tag{41b}$$

$$x\ (\omega \times r) = [\omega]\ [\omega]\ [T_i] \cdot \begin{bmatrix} x_{i+1} \\ y_{I+1} \\ z_{i+1} \end{bmatrix} \tag{41c}$$

$$\alpha \times r = [\alpha]\ [T_i] \cdot \begin{bmatrix} x_{i+1} \\ y_{i+1} \\ z_{i+1} \end{bmatrix} \tag{41d}$$

From all these, expression $(x_{i+1}, y_{i+1}, z_{i+1})^t$ can be eliminated, which leads to

$$[T_i''] = [\omega]\ [\omega]\ [T_i] + [\alpha]\ [T_i] \tag{41e}$$

Note that the d" translation part is ignored as only rotation need be considerd. From equation (41e) accelertion components can be extracted.

$$[\alpha] = \quad [[T_i''] - [\omega]\ [\omega]\ [T_i]]\ [T_i]^t$$

Recalling that all the matrices involved are (3 x 3) matrices with orthogonal properties.

CHAPTER III

DYNAMIC ANALYSIS

Open-Loop Dynamic Analysis

Calculations for the open-loop dynamic analysis are initiated from the farthest link from the ground, which is the end-effector. The quantities required for the dynamic analysis are already available from the kinematic analysis. The mass of the link, mass moment of inertia for each link, location of mass center of each link, linear velocity and linear acceleration, and angular velocity and angular acceleration are either known or calculated quantities. From all these quantities, weights of the links, inertia forces, and moment about the mass center are determined.

D'Alembert's principle is applied to each link of the open-loop of the robot [19]. To develop dynamic equations, let

m_i = the total mass of link i, a scalar

$\hat{r}_i$ = the position of the center of mass of link i with reference to base coordinates

v_i = dr/dt the linear velocity of the center of mass of link i with reference to base coordinates

F_i = the total external vector force exerted on link i

N_i = the total external vector moment exerted on like i

I_i = the inertia matrix of link i about its center of mass

To derive dynamic relationships for the open-loop part of the manipulator, a link i is considered. All the equations are developed for this link and these same equations are applied to the other, constituting an open-loop in successive manner. As opposed to the kinematic analysis, in dynamic analysis the analysis is started from the end-effector of a manipulator.

In Figure 7, showing link i, there are three kinds of forces that act upon the link. These three forces are:

1. Inertia force.
2. Weight of the link itself.
3. The reaction forces due to attached links at the two ends of the link.

The direction of the inertia force is along the acceleration direction of the mass center of the link. The weight of the link always acts vertically down. The directions of the reaction forces depends upon what kind of connection exists between the link under consideration and the links attached to it at its two ends.

The following vector definitions are needed to calculate the magnitude of the forces acting on link i.

Let

$$\{r_G\}_i \qquad \begin{Bmatrix} r_{x_{G_i}} \\ r_{y_{G_i}} \\ r_{z_{G_i}} \end{Bmatrix}_1 \qquad (42)$$

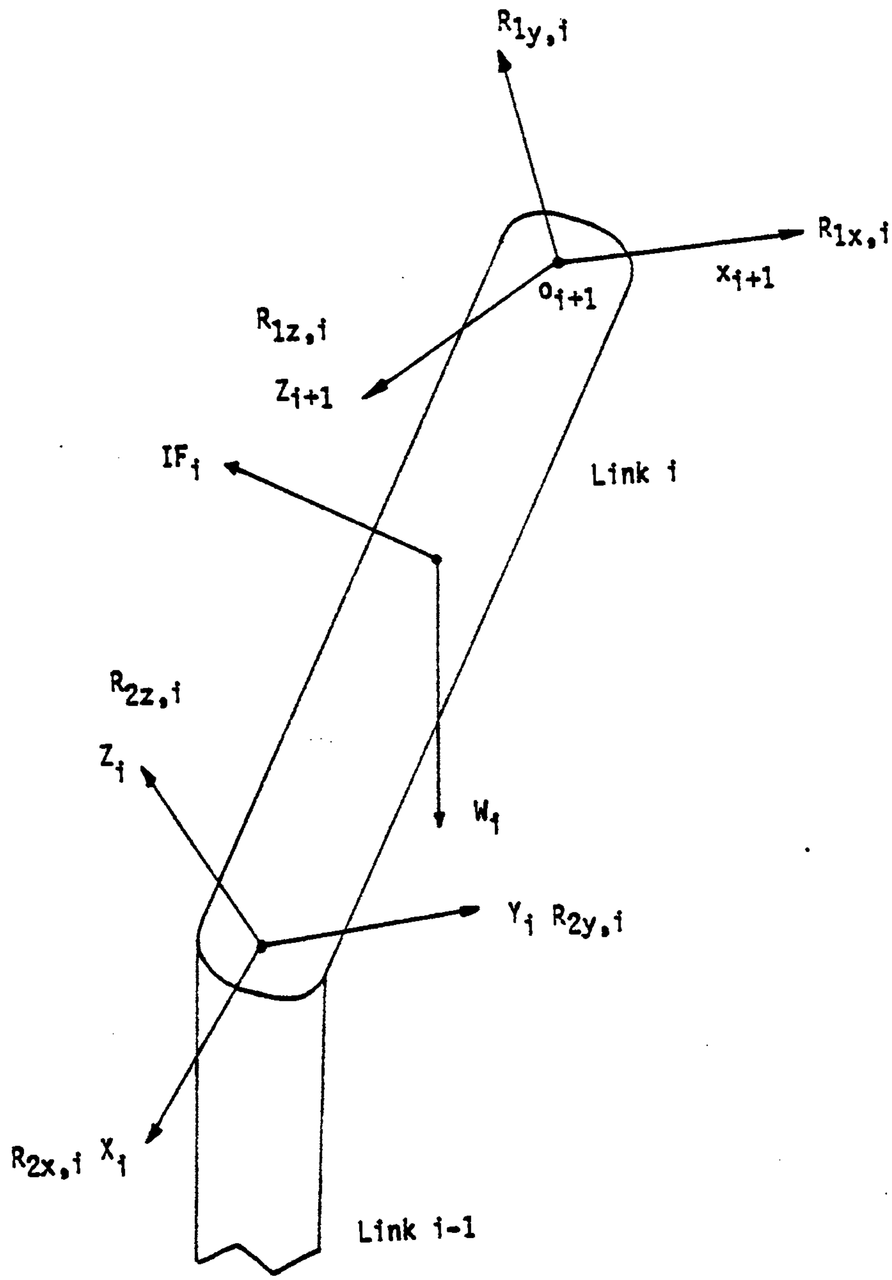

Figure 7. Forces Acting on the Link

be the vector representing the location of mass center of the link i in its local coordinate frame. The local coordinate frame associated with the link i is $x_{i+1}y_{i+1}z_{i+1}$. This frame moves with link i.

$$\{g\}_B = \begin{Bmatrix} g_x \\ g_y \\ g_z \end{Bmatrix} \tag{43}$$

This vector represents the components of the acceleration due to gravity in the base frame of the manipulator. Only one of the three will have a non-zero value as one of the axis of the base frame is normally aligned with the direction of gravity [19]. With these two vectors defined, the inertia force and the weight of the link can be calculated.

Let $\{W\}_B$ be the vector representing the weight vector for the link in the base frame. Its magnitude is given by

$$\{W_i\}_B = \begin{Bmatrix} W_{x_i} \\ W_{y_i} \\ W_{z_i} \end{Bmatrix}_B = m_i \begin{Bmatrix} g_x \\ g_y \\ g_z \end{Bmatrix} \tag{44}$$

Subscript B means that the quantities are expressed in the base coordinate frame. Inertia force is given by the product of mass of the link and the linear acceleration of mass center of the link.

Let $\{IF\}$ be the vector representing the inertia force.

$$\{IF_i\}_i = \begin{bmatrix} IF_{x_i} \\ IF_{y_i} \\ IF_{z_i} \end{bmatrix}_i = m_i \cdot \begin{bmatrix} r''_{x_{G_i}} \\ r''_{y_{G_i}} \\ r''_{z_{G_i}} \end{bmatrix} \tag{45}$$

where the ($r''_{x_{G_i}}$ $r''_{y_{G_i}}$ $r''_{z_{G_i}}$) represents an acceleration (linear) vector for the link i expressed in the link's own coordinate frame. Absolute acceleration is calculated first and then this acceleration is resolved in the link's own coordinate frame. The steps involved are outlined below. $\{r_{G_i}\}_B$ is an absolute acceleration vector for the link i.

$$\{r_{G_i}\}_B = \begin{bmatrix} r_{G_{x_i}} \\ r_{G_{y_i}} \\ r_{G_{z_i}} \\ 1 \end{bmatrix}_B \tag{46}$$

$$= \frac{d^2}{d^2}[T_i] \begin{bmatrix} r_{G_{x_i}} \\ r_{G_{y_i}} \\ r_{G_{z_i}} \\ 1 \end{bmatrix}_B \tag{47}$$

where $[T_i] = A1 \cdot A2 \cdot A3 \cdots\cdots A_i$.

The second derivative of this transformation product gives the linear and angular acceleration of the frame attached to link i with respect to the base frame of the manipulator. When this transformation derivative is muliplied by a vector representing a point on the link, it gives linear acceleration.

$$F_i = m_i \{r''_{G_i}\}_i \tag{48}$$

where F_i represents the resultant of all the external forces acting on link i. To develop analytical relationships, it is necessary to write the above equation in the expanded form. Let the vectors R_{1_i} and R_{2_i} represent two reactions acting on the link to counter the external forces and thus keep the link in equilibrium.

$$\{R_{2_i}\}_i + \{R_{1_i}\}_i + \{W_i\} = m_i \{r''_{G_i}\}_i \tag{49}$$

As the calculations are begun from the end-effector, i.e., i = N last link, where the vector $\{R_{1_i}\}_i$ is nothing but weight or the load carried by the manipulator [ ]. More often this is a known quantity and from this known quantity, reaction at the other end of the end-effector $\{R_{2_i}\}_i$ can be calculated. It should be noted that the subscript i represents the linear acceleration or total acceleration of that point. The vector multiplied in (7) represents a mass center so it gives the linear acceleration of the mass center. Now this linear acceleration has to be resolved along the three axes of the link's own coordinate frame.

$$\{r_{G_i}\}_i = [T_i]^{-1}_{3 \times 3} \{r''_{G_i}\}_B \tag{50}$$

Recollecting that 3 x 3 part of a transformation represents an orientation of the coordinate frame of link i in the base frame.

Since the inertia force and the weight vectors are now known, D'Alembert's principle can be applied to determine the unknown reaction forces. The principle, when stated mathematically, has the form

$$F_i = m_i \{a_{G_i}\}_i$$

Subscript i means that the vector is expressed or calculated in the link i coordinate frame. To calculate the reaction of the link attached to the end-effector on the end-effector i, the direction of the vector $\{R_{2_i}\}$ is reversed.

$\{R_{2_i}\}_i$ = Reaction force acting on the link i (end-effector i = n) due to link i-1 in $o_{i+1}\ y_{i+1}\ Z_{i+1}$

$-\{R_{2_i}\}_i$ = Reaction force acting on the link i-1 due to link i, which happens to be the end-effector

Now apply the equation(s) to the succeeding link i-1. The vector $(-\{R_{2_i}\}_i)$ has to be transformed to the link i-1, the coordinate frame. The new vector now becomes the vector $\{R_{1_{i-1}}\}_{i-1}$ for the link i-1.

$$\{R_{1_{i-1}}\}_{i-1} = [A_i]\ \{-R_{2_i}\}_i \qquad (51)$$

where $[A_i]$ represents a transformation between the coordinate frame attached to link i, i.e., $o_{i+1}\ x_{i+1}\ z_{i+1}$ and the coordinate frame attached to link i-1, i.e., $o_i\ x_i\ y_i\ z_i$. With application equation (49), $\{R_{w_{i-1}}\}_{i-1}$ is evaluated and the same procedure as outlined above is adapted to transform to this force to the frame of the succeeding link.

Successive application of the above procedure yields all the reaction forces acting on the open-loop part of the manipulator. Now, since the forces on the links are known, the equations can be developed for the unknown moments or torques. The procedure to calculate these torques is explained below. The quantities required for the torque calculation are mass moment of inertia of the link i, angular acceleration, and angular velocity. As done in the case of linear acceleration, the absolute angular velocity and the absolute angular acceleration are resolved along the link's own coordinate frame axes.

Both the absolute velocity and the absolute angular acceleration are available from the kinematic analysis already done. These quantities are transformed to the link i frame by

$$\{\omega_i\}_i = [T_i]_{3 \times 3} \{\ _i\}_B \tag{52}$$

$$\{\alpha_i\}_i = [T_i]_{3 \times 3} \{\ _i\}_B \tag{53}$$

where

$\{\omega_i\}_i$ = angular velocity of link i along the local coordinate frame

$\{\alpha_i\}_i$ = angular acceleration of link i along the local coordinate frame

$\{\omega_i\}_B$ = angular velocity of link i along the base frame

$\{\alpha_i\}_B$ = angular acceleration link i along the base frame

$[T_i]^{-1}$ = the same transformation as defined above.

Moment about the mass center is calculated from the angular velocity, angular acceleration, and mass moment inertia matrix. [uI] is a mass moment of inertia matrix as defined below:

Using the above definition the resultant moment about the mass center can be calculated. Let $\{RM_i\}_i$ be the vector representing the resultant moment.

$$\{RM_i\}_i = [MI] \cdot \{\alpha_i\}_i + \{\omega_i\}_i \times ([MI] \cdot (\omega_i)_i) \qquad (54)$$

Moments due to weight of the link and also moment due to inertia force vanish since the moment arm in both cases is zero. Let a be a vector between the mass center and the origin of the coordinate frame o_{i+1}, $x_{i+1}y_{i+1}z_{i+1}$. And let b be a vector connecting the point G_i with the origin of the coordinate frame $x_iy_iz_i$, i.e., o_i. c vector connects o_i with o_{i+1}. These three vectors are related by the relationship (Figure 8)

$$c = a + b \qquad (55)$$

{a} is a defined vector since it is a location of mass center with respect to the link i coordinate frame. {c} is a vector describing the location of o_{i+1}, the origin of the link i coordinate frame in the coordinate frame associated with link i-1, i.e., $o_ix_iy_iz_i$. To make it compatible with {a} it also should be expressed in the link i coordinate frame.

$$\{c\}_{i-1} = \begin{Bmatrix} A_i(1,4) \\ A_i(2,4) \\ A_i(3,4) \end{Bmatrix}_{i-1} \qquad (56)$$

To convert it to a vector in the link i coordinate frame, this vector has to be premultiplied by $[A_i]^{-1}$.

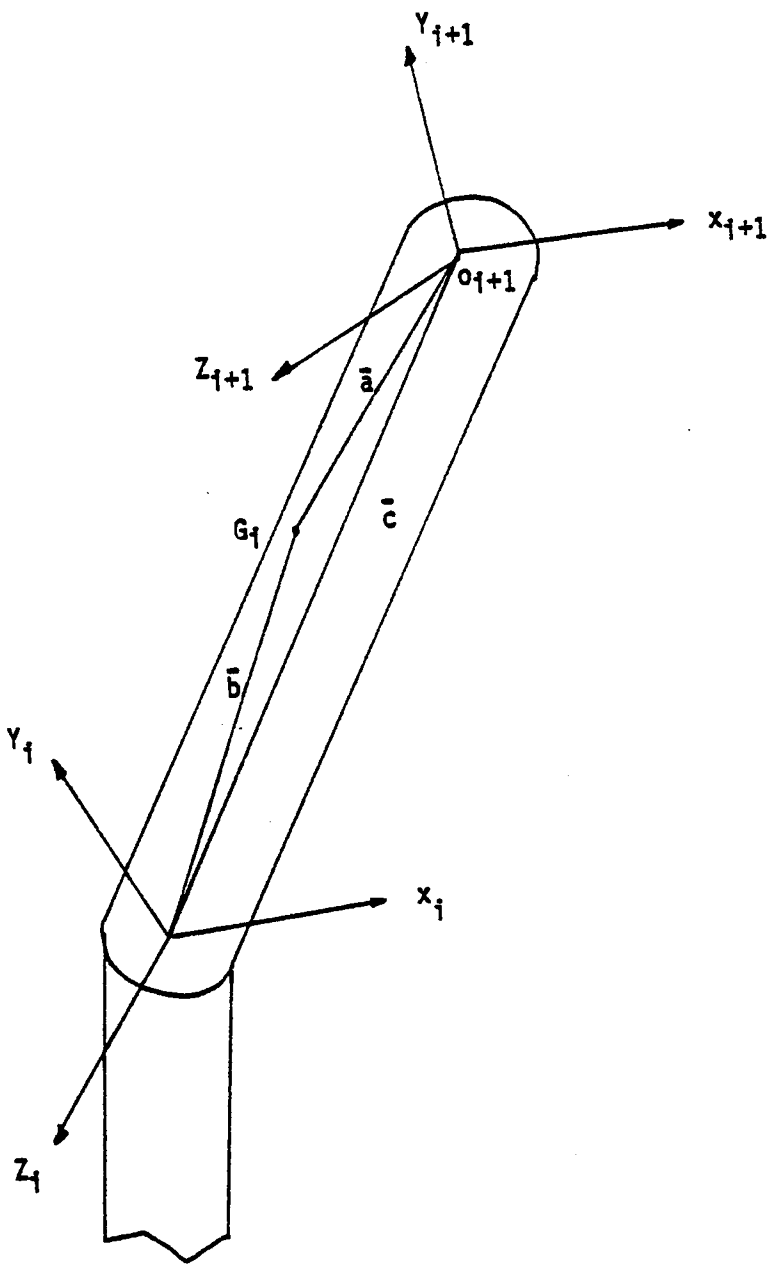

Figure 8. Relation Between Coordinate Frames and C.G. of Link

$$\{c\}_i = [A_i]^{-1} \{c\}_{i-1} \tag{57}$$

Moments due to reaction forces at joint i and joint i+1 can be calculated. Reaction at joint i+1 $\{R_{1_i}\}_i$ and that at joint i is $\{R_{2i}\}_i$. Moment arm for the $\{R_{1_i}\}_i$ is $\{a\}$ and one for the $\{R_{2_i}\}_i$ is $\{b\}$. Moment due to reaction force at joint i+1 $\{MR_{1_i}\}_i$

$$\{MR_{1_i}\}_i = \{a\}_i \times \{RF_{1_i}\}_i \tag{58}$$

where for vector cross product vector $\{a\}_i$ is expressed in the 3 x 3 matrix form.

$$\{MR_{1_i}\}_i = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}_i \cdot \begin{bmatrix} RF_{1_i}(1) \\ RF_{1_i}(2) \\ RF_{1_i}(3) \end{bmatrix}_i . \tag{59}$$

Similarly, moment due to reaction at joint i can be found. Moment arm for it is found from equation (55)

$$b = c - a \tag{60}$$

Moment due to $\{RF_{2_i}\}_i$ at joint i $\{MR_{2_i}$

$$\{MR_{2_i}\}_i = \{b\}_i \times \{RF_{2_i}\}_i \tag{61}$$

Again vector $\{b\}_i$ is expressed in the form of (3 x 3) matrix for multiplication.

$$\{MR_{2_i}\}_i = \begin{bmatrix} 0 & -b_z & b_y \\ b_z & 0 & -b_x \\ -b_y & b_x & 0 \end{bmatrix}_i \cdot \begin{bmatrix} RF_{2_i}(1) \\ RF_{2_i}(2) \\ RF_{2_i}(3) \end{bmatrix}_i \tag{62}$$

Again D'Alembert's principle is applied to the link i to determine unknown joint torques. Mathematically the principle is expressed as

$$N_i = I_i \cdot \alpha_i + \omega_i \times (I_i \cdot \omega_i) \tag{63}$$

where N_i is a vector sum of all the external moments acting on the link. The right hand expression is moment about mass center of the link. Here also, like in unknown force calculations, computations are started from the end-effector. At the free end of the end-effector there is no reaction moment acting, so for it $\{MR_i\}$ is set to zero. From the known resultant moment about mass center and moment due to reaction force $\{RF_{2_i}\}_i$ at the other end of the end-effector, reaction moment components $\{MR_{2_i}\}_i$ are calculated. It is to be noted again that the vector $\{MR_{2_i}\}_i$ is expressed link i coordinate frame. Now the reaction moment on the link i-1 due to link i is

$$\{MR_{1_{i-1}}\}_i = 1\{MR_{2_i}\}_i \tag{64}$$

This vector should be expressed in link i-1 coordinate frame before reaction moment at the other end of the link can be evaluated. It is done by premultiplying the vector $\{RM_{1_i}\}_i$ by a transformation $[A_i]^{-1}$. Only the orientation part of the $[A_i]^{-1}$ is considered.

$$\{RM_{1_i}\}_{i-1} = [A_i]_{3 \times 3} \{RM_{1_{i-1}}\}_i \tag{65}$$

$$\{RM_{1_i}\}_i + \{RM_{2_i}\}_i + \{MR_{1_i}\}_i + \{MR_{2_i}\}_i = \{RM_i\}_i \tag{66}$$

This equation (65) is now applied repeatedly to calculate all the unknown torques at each of the joints of the open-loop.

Since the loop considered herein is a planar loop, it cannot resist any forces perpendicular to its plane and also cannot resist moments about the axes lying in the plane.

At point A there are three force components and three moment components, but for the evaluation of forces and moments in the planar mechanism, not all of them are considered. Forces lying in the plane of the mechanism and moments about the axis perpendicular to the plane are only to be considered. The remaining components are transferred to the base joint. Moments MA_x, MA_y, and FA_z are all transferred to the base. This is a single matrix multiplication operation. The required transformation to transfer vector quantities at point A to the base joint is already known.

Dynamic Analysis of a Closed-Loop

From the open-loop part dynamic analysis by the Newton-Euler method, we have forces and torques acting at point A. The point is actually a joint connecting this closed-loop with the open-loop part of the robot. Forces and torques acting at point A are regarded as external forces for the analysis of the closed-loop. For the closed-loop joint 1 and joint 5 are the inputs. Input information for the procedure consists of all the parameters which are either known or determined on the position solution (θ_i, s_i). The parameters which are either known or determined in the velocity solution (θ_i and s_i'), the parameters which are either known or determined in the acceleration analysis (θ_i'' and s_i), the location of each mass center, the value of mass and mass moment of inertia about the mass center of each part, and known parameters of the known reaction force/torque problem are indicated in Figure 9. The symbol has been used to indicate the mass center of each moving part of the mechanism. These mass centers are located in the local coordinate frame of the link. These coordinates are termed local since they move with the part to which they are attached. To determine unknown reaction forces, D'Alembert's principle is applied. Mathematically, the principle can be stated as [11, 16]

$$F - ma = 0 \tag{67}$$

$$T - I = 0 \tag{68}$$

In other words, if the linear and angular acceleration of each part of the mechanism are known, the products ma and "Iα can be treated as a force and torque respectively. This reduces the dynamics problem to an equivalent statics problem. The symbols "m" and "I" used are mass and mass moment of inertia of a part. The symbol a is linear acceleration of the mass center of the part and "α" is an angular acceleration of the part. F and T represent the resultant of all applied and reaction forces acting on the part and vector sum of all applied and reaction torques respectively. In order to use this principle, it is necessary to have all inertial forces and torques associated with each mass center. The information necessary to determine the inertial forces is available from already developed position velocity and acceleration analysis and the knowledge of mass, center of mass location, and the mass moment of inertia of each of the parts. The unknown reactions are indicated in Figure 9. The notation $F_{i,j}$ means the force exerted on part i by part j. Thus $F_{2,1}$ means force exerted on part 2 by part 1, and $T_{3,2}$ is a torque exerted on part 3 by part 2.

Procedure

As an intermediate step in the determination of reaction forces, inertial forces associated with mass center of each part must be evaluated. The information necessary for doing this is available from the position, velocity, and acceleration analysis results. In order to develop the -ma term for each of the parts, acceleration with respect to the reference frame must be calculated. For this closed-loop mechanism, the reference frame at joint 1 is chosen. It can be chosen anywhere, but choosing at joint 1 greatly facilitates the analysis. Normally one of the parts in the mechanism is ground, but in the present case, as has been pointed out earlier, the ground link itself is a moving link. However, all the kinematic quantities involved like angular velocities, angular accelerations, and linear velocities and linear accelerations are referenced to a coordinate frame attached to it. Similarly, terms -ma and $-I\alpha$ can be developed.

The complete set of equations that result when D'Alembert's principle is applied to each of the five link mechanisms are listed below:

Link 2

$$F_{2,1x} + F_{2,3x} + IF_{2x} + W_{2x} = 0$$

$$F_{2,1y} + F_{2,3y} + IF_{2y} + W_{2y} = 0$$

$$T_{2,1} + T_{2,3} + r_2 \times F_{2,3}$$
$$+ r_2 \times (IF_2 + W_2)$$
$$+ I_2 \alpha_2 = 0$$

Link 3

$$F_{3,2x} + F_{3,4x} + IF_{3x} + W_{3x} = 0$$

$$F_{3,2y} + F_{3,4y} + IF_{3y} + W_{3y} = 0$$

In each of the equations above there are more than two unknowns: therefore, it will not be possible to solve any single equation. The number of unknowns can be reduced from the system of equations above by introducing the following simplifications:

$$F_{3,2} = -F_{2,3}$$

$$F_{4,3} = -F_{3,4}$$

$$F_{5,4} = -F_{4,5}$$

$$T_{3,2} = -T_{2,3}$$

$$T_{4,3} = -T_{3,4}$$

$$T_{5,4} = -T_{4,5}$$

Even after these simplifications, there are fifteen unknowns with only twelve equations [16]. Three additional equations need to be generated to solve the system of equations. These equations are generated by imposing appropriate conditions of nonactive joints like joint 2, 3, and 4.

$$T_{3,2} + T_{3,4} + r_3 \times F_{3,4}$$

$$+ r_3 \times (IF_3 + W_3)$$

$$+ I_3 \alpha_3 = 0$$

Link 4

$$F_{4,3x} + F_{4,5x} + (IF_{4,1} + IF_{4,2})x$$

$$+ (W_{4,1} + W_{4,2})y = 0$$

$$F_{4,3y} + F_{4,5y} + F_y + (IF_{4,1} + IF_{4,2})y$$

$$+ (W_{4,1} + W_{4,2})y = 0$$

$$T_{4,3} + T_{4,5} + M + r_{4,1} \times F_{4,5} + r_{4,2} \times F$$

$$+ r_{4,1} \times (IF_{4,1} + W_{4,1})$$

$$+ r_{4,2} \times (IF_{4,2} + W_{4,2})$$

$$+ I_{4,1} \alpha_{4,1} + I_{4,2} \alpha_{4,2} = 0$$

Link 5

$$F_{5,4x} + F_{5,1x} + IF_{5x} + W_{5x} = 0$$

$$F_{5,4y} + F_{5,1y} + IF_{5y} + W_{5y} = 0$$

$$T_{5,1} + T_{5,4} + r_5 \times F_{5,1}$$

$$+ r_5 \times (IF_5 + W_5)$$

$$+ I_5 \ \alpha_5 = 0$$

Three equations are written for each part. The first represents the sum of all applied inertial and reaction forces along the x-direction of the reference frame; the second represents the sum of all applied inertial and reaction forces along the y-direction of the reference frame, and the third equation represents a torque equation where all applied inertial and reaction torques are summed up. Quantity like r_i represents lengths of the part identified by its subscripts i. For instance, r_2 is a length of part 2 and r_i represents the mass center locations of the part.

IF_i and W_i represent the inertia force and weight of part i. IF_2 and W_2 represent the inertia force and weight of part 2. Similarly, I_i represents a mass moment of inertia of part i. Link 4 has two sets of quantities because of the way the ternary link is represented in the kinematic analysis.

$IF_{4,1}$ = Inertia force of part 1 of link 4

$W_{4,1}$ = Weight of part 1 of link 4

$IF_{4,2}$ = Inertia force of part 2 of link 4

$W_{4,2}$ = Weight of part 2 of link 4

$r_{4,1}$, $r_{4,2}$ = Lengths of part 1 and part 2 of link 4

$r_{4,1}$, $r_{4,2}$ = Mass centers for two parts of link 4

If the joint is an ideal revolute joint, it can transmit a force from one part to another but it cannot transmit a torque. Thus at any nonactive revolute joint reaction force will occur, but no torque is transmitted.

Mathematically

$$T_i = 0.0$$

where i can take any value from 2 to 4 depending on the type of joints in the five-link mechanism. If a prismatic pair connects two links, in general, there will be a reaction force normal to the axis of sliding and a reaction torque. The reaction torque must exist because of the nature of the prismatic pair connection which permits no relative rotation of two parts. A torque is required to enforce this condition of no relative rotation.

Mathematically

$$F_i \cdot s = 0$$

With these additional conditions there are 15 equations for 15 unknowns. the system of scalar equations can be represented succinctly in matrix form as

$$[A]\ \{x\} = \{B\} \tag{69}$$

The coefficient matrix [A] contains the coefficient of the unknown torques and unknown reaction force components which are contained in {x}. Column vector {B} contains the known inertial terms.

As seen from the resulting matrix, it is found that the coefficient matrix is in upper triangular form. Using elementary row operations, the coefficient matrix may be transformed into upper triangular and with standard methods available, the system of equation can be solved for unknown quantities [16].

All the forces and torques evaluated are with respect to the reference frame chosen. These have to be transformed to the link's local coordinate frame. It is to be noted that in the analysis of a closed-loop an assumption has been made that all the forces normal to the plane of the mechanism are transferred to the base. There is a component of F perpendicular to the plane of the mechanism. Determining the reactions resulting due to this component becomes an indeterminate problem in the case of planar linkage. Similarly, the moment M components about the axes in the plane of the mechanism becomes an indeterminate problem. so the force F component perpendicular to the plane of the mechanism and the moment M components about axes in the plane of the paper are assumed to be resisted only by the base joint.

CHAPTER IV

SUMMARY AND CONCLUSIONS

The main objective of the present work was to develop a tool to do the analysis of the mixed-loop robot. These robots are relatively new but bound to receive attention in the near future. Since closed-loop is incorporated into the conventional open-loop structure, the analysis procedure differs to some extent from the open-loop robot.

Both kinematic and dynamic analysis are performed on the robot, kinematic analysis followed by dynamic analysis. In the kinematic analysis, the closed-loop part is analyzed first. Unknown joint displacements, velocities, and accelerations are determined using iterative techniques. If the mechanism is in locking position, the analysis is aborted. This situation is corrected by changing the link length ratios. This is followed by the kinematic analysis of the entire robot. Kinematic analysis for the entire robot consists of determining the links' angular velocities, angular acceleration, and point distance, velocity, and acceleration, all either in the reference frame or any other desired frame. Once all the kinematic parameters for the robot are available, dynamic analysis can be performed.

In the dynamic analysis the open-loop part of the robot is analyzed first and then the closed-loop. For the open-loop part the Newton-Euler approach is used. Analysis is begun from the end-effector. Forces and just before the closed-loop begins. From this point on a different approach is used for the closed-loop. It is very much like the Newton-Euler method used for the open-loop. The computer program has been coded to automate the analysis procedure. After the user has defined all the required parameters of the robot, both numerical and graphical results are presented. The program is interactive in nature.

In the present work a planar loop has been used, but a spatial loop also can be used. There will not be any change in the analysis procedure as far as the kinematic analysis is concerned. However, a different approach needs to be used for the dynamic analysis. Revolute and prismatic joints are allowed to be used. This restriction also can be removed to allow more generality as an extension to the present work. As a further addition to the generality, a unified approach for the analysis can be developed which will avoid the combination of two approaches, as it is in the present case.

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APPENDIX A

MATRIX RELATIONSHIPS

Symbols used in the following relationships are defined below:

$$B1 = A1 \cdot A2 \cdot A3 \cdot A4 \cdot A5$$

$$B2 = A1 \cdot [Q] \cdot A2 \cdot A3 \cdot A4 \cdot A5$$

$$B3 = A1 \cdot A2 \cdot [Q] \cdot A3 \cdot A4 \cdot A5$$

$$B4 = A1 \cdot A2 \cdot A3 \cdot [Q] \cdot A4 \cdot A5$$

Since A1 and A5 are defined by input variables, they are exact matrices. But A2, A3, and A4 are approximate matrices defined by initial guesses of the unknown variables. [Q] is an operator matrix. All the relationships are developed from the basic loop equation:

$$A1 \cdot A2 \cdot A3 \cdot A4 \cdot A5 = I$$

Displacement Relationships

$$A1 \cdot [Q] \cdot A2 \cdot A3 \cdot A4 \cdot A5$$

$$A1 \cdot A2 \cdot [Q] \cdot A3 \cdot A4 \cdot A5$$

$$A1 \cdot A2 \cdot A3 \cdot [Q] \cdot A4 \cdot A5$$

$$= I - A1 \cdot A2 \cdot A3 \cdot A4 \cdot A5 = I - B1$$

In developing the above relationship, all the joints in the closed-loop are assumed to be revolute joints only.

The relationship can be written in compact form using the definitions:

$$B2(d\theta_2) + B3(d\theta_3) + B4(d\theta_4) = I - B1$$

Velocity Relationships

Differenting the loop equation w . r. t . time

$$A1 \;.\; [Q] \;.\; A2 \;.\; A3 \;.\; A4 \;.\; A5 \;(\dot{\theta}_2)$$

$$A1 \;.\; A2 \;.\; [Q] \;.\; A3 \;.\; A4 \;.\; A5 \;(\dot{\theta}_3)$$

$$A1 \;.\; A2 \;.\; A3 \;.\; [Q] \;.\; A4 \;.\; A5 \;(\dot{\theta}_4)$$

$$= -\; [Q] \;.\; A1 \;.\; A2 \;.\; A3 \;.\; A4 \;.\; A5 \;(\dot{\theta}_1)$$

$$-\; A1 \;.\; A2 \;.\; A3 \;.\; A4 \;.\; [Q] \;.\; A5 \;(\dot{\theta}_5)$$

$$= -\; [Q] \;(\;_1) - A1 \;.\; A2 \;.\; A3 \;.\; A4 \;.\; [Q] \;.\; A5 \;(\dot{\theta}_5)$$

All the matrices are exact matrices here.

Acceleration Relationship

A = First differentiation of matrix A w.r.t. time

A = Second differentiation of matrix A w.r.t. time

The loop equation is differentiated twice w.r.t. time to develop acceleration relationships.

$$A1 \;.\; A2 \;.\; A3 \;.\; A4 \;.\; A5$$

$$+\; A1 \;.\; A2 \;.\; A3 \;.\; A4 \;.\; A5$$

$$+\; A1 \;.\; A2 \;.\; A3 \;.\; A4 \;.\; A5$$

$$+\; A1 \;.\; A2 \;.\; A3 \;.\; A4 \;.\; A5$$

$$+\; A1 \;.\; A2 \;.\; A3 \;.\; A4 \;.\; A5$$

$$= -\,2\,(A1 \;.\; A2 \;.\; A3 \;.\; A4 \;.\; A5 + A1 \;.\; A2 \;.\; A3 \;.\; A4 \;.\; A5$$

$$+\; A1 \;.\; A2 \;.\; A3 \;.\; A4 \;.\; A5 + A1 \;.\; A2 \;.\; A3 \;.\; A4 \;.\; A5$$

$$+\; A1 \;.\; A2 \;.\; A3 \;.\; A4 \;.\; A5 + A1 \;.\; A2 \;.\; A3 \;.\; A4 \;.\; A5$$

$$+\; A1 \;.\; A2 \;.\; A3 \;.\; A4 \;.\; A5 + A1 \;.\; A2 \;.\; A3 \;.\; A4 \;.\; A5)$$

$$= \dot{K}$$

A1 and A5 are known and all the velocity matrices A_i ($i = 1, 2, \ldots 5$) and displacement matrices A_i ($i = 1, 2, \ldots 5$) are known.

$$
\begin{aligned}
& A1 \cdot A2 \cdot A3 \cdot A4 \cdot A5 \\
+ \; & A1 \cdot A2 \cdot A3 \cdot A4 \cdot A5 \\
+ \; & A1 \cdot A2 \cdot A3 \cdot A4 \cdot A5 \\
= \; & K - A1 \cdot A2 \cdot A3 \cdot A4 \cdot A5 - A1 \cdot A2 \cdot A3 \cdot A4 \cdot A5
\end{aligned}
$$

APPENDIX B

EXAMPLE RESULTS

DATA FOR THE ROBOT

Number of joints = 10

Number of links = 8

Link	"a"	" "	"S"
1	3.00	0.0	0.0
2	2.00	0.0	0.0
3	4.00	0.0	0.0
4	4.00	0.0	0.0
5	2.75	0.0	0.0
6	3.45	0.0	0.0
7	2.00	0.0	0.0
8	2.50	0.0	0.0
10	4.00	0.0	0.0
11	2.00	0.0	0.0

****

KINEMATIC AND OYNAMIC ANALYSIS OF A MIXEO-LOOP ROBOT

****

STEP NO. = 1

RESULTS FOR THE CLOSEO LOOP

CONVERGENCE AFTER 6 ITERATIONS

DISPLACEMENT ANALYSIS RESULTS (DEG/IN)

JNT. 3 = -5.547709
JNT. 4 = 132.7041
JNT.10 = 64.84369

VELOCITY ANALYSIS RESULTS (RAD/S / IN/S)

JNT. 3 = -8.943070
JNT. 4 = 3.203922
JNT.10 = -0.2608112

ACCLERATION ANALYSIS RESULTS (RAD/S*S / IN/S*S)

JNT. 3 = -23.30211
JNT. 4 = -12.98520
JNT.10 = 36.28499

POINT ANALYSIS

TOTAL NO. OF POINTS = 5

POINT I.D.	LINK I.D.	POSITION (IN)	VELOCITY (IN/S)	ACCLERATION (IN/S*S)
1	8	-0.319E+00	-0.289E+02	0.284E+03
		0.114E+01	-0.656E+01	-0.594E+03
		0.000E+00	0.000E+00	0.000E+00
2	7	0.254E+01	-0.210E+02	-0.406E+03
		0.343E+00	0.367E+02	-0.624E+03
		0.000E+00	0.000E+00	0.000E+00
3	6	0.188E+01	0.583E+01	-0.284E+03
		-0.169E+01	0.250E+02	-0.236E+03
		0.000E+00	0.000E+00	0.000E+00
4	5	-0.439E+00	0.288E+02	0.531E+02
		-0.324E+01	-0.444E+01	0.192E+03
		0.000E+00	0.000E+00	0.000E+00
5	2	-0.307E+01	-0.377E+01	0.338E+03
		0.593E+00	-0.319E+02	-0.518E+01
		0.000E+00	0.000E+00	0.000E+00

LINK ANALYSIS

TOTAL NO. OF LINK = 10

LINK I.D.	ANGULAR VELOCITY (RAD/S)	ANGULAR' ACCLERATION (RAD/S*S)
1	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.100E+02	0.000E+00
2	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.150E+02	-0.153E-04
3	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.606E+01	-0.233E+02
4	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.926E+01	-0.363E+02
5	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.926E+01	-0.363E+02
6	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.153E+02	-0.363E+02
7	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.108E+02	-0.363E+02

DYNAMIC FORCE ANALYSIS RESULTS

JNT. ID	X - COMPONENT	Y - COMPONENT	Z - COMPONENT
1	0.943E+04	0.137E+05	0.000E+00
2	-0.428E+03	-0.285E+04	0.000E+00
3	0.243E+04	-0.953E+03	0.000E+00
4	0.198E+04	-0.124E+04	0.000E+00
5	-0.455E+04	0.147E+05	0.000E+00
6	0.108E+05	0.133E+05	0.000E+00
7	0.141E+05	0.369E+04	0.000E+00
8	0.602E+04	-0.456E+04	0.000E+00
10	-0.188E+05	0.527E+04	0.000E+00

DYNAMIC TORQUE ANALYSIS RESULTS

JNT. ID	X - COMPONENT	Y - COMPONENT	Z - COMPONENT
1	0.000E+00	0.000E+00	-0.195E+05
2	0.000E+00	0.000E+00	0.182E+03
3	0.000E+00	0.000E+00	-0.107E+02
4	0.000E+00	0.000E+00	0.646E+02
5	0.000E+00	0.000E+00	-0.265E+02
6	0.000E+00	0.000E+00	-0.665E+03
7	0.000E+00	0.000E+00	-0.127E+05
8	0.000E+00	0.000E+00	-0.357E+04
10	0.000E+00	0.000E+00	0.193E+05

STEP NO. = 2

RESULTS FOR THE CLOSEO LOOP

CONVERGENCE AFTER 3 ITERATIONS

DISPLACEMENT ANALYSIS RESULTS (DEG/IN)

JNT. 3 = -8.468669

JNT. 4 = 134.0745

JNT. 10 = 64.29425

VELOCITY ANALYSIS RESULTS (RAD/S / IN/S)

JNT. 3 = -9.083371

JNT. 4 = 3.132253

JNT. 10 = -0.4886782E-01

ACCLERATION ANALYSIS RESULTS (RAD/S*S / IN/S*S)

JNT. 3 = -24.41098

JNT. 4 = -13.80949

JNT. 10 = 38.21930

POINT ANALYSIS

TOTAL NO. OF POINTS = 5

POINT I.D.	LINK I.D.	POSITION (IN)	VELOCITY (IN/S)	ACCLERATION (IN/S*S)
1	8	-0.442E+00	-0.263E+02	0.321E+03
		0.104E+01	-0.865E+01	-0.542E+03
		0.000E+00	0.000E+00	0.000E+00
2	7	0.245E+01	-0.213E+02	-0.348E+03
		0.419E+00	0.342E+02	-0.622E+03
		0.000E+00	0.000E+00	0.000E+00
3	6	0.187E+01	0.541E+01	-0.249E+03
		-0.163E+01	0.237E+02	-0.241E+03
		0.000E+00	0.000E+00	0.000E+00
4	5	-0.397E+00	0.289E+02	0.587E+02
		-0.325E+01	-0.444E+01	0.191E+03
		0.000E+00	0.000E+00	0.000E+00
5	2	-0.308E+01	-0.259E+01	0.336E+03
		0.482E+00	-0.319E+02	0.753E+01
		0.000E+00	0.000E+00	0.000E+00

LINK ANALYSIS

TOTAL NO. OF LINK = 10

LINK I.D.	ANGULAR VELOCITY (RAD/S)	ANGULAR' ACCLERATION (RAD/S*S)
1	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.100E+02	0.381E-05
2	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.150E+02	-0.172E-04
3	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.592E+01	-0.244E+02
4	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.905E+01	-0.382E+02
5	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.905E+01	-0.382E+02
6	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.150E+02	-0.382E+02
7	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.105E+02	-0.382E+02

DYNAMIC FORCE ANALYSIS RESULTS

JNT. ID	X - COMPONENT	Y - COMPONENT	Z - COMPONENT
1	0.801E+04	0.121E+05	0.000E+00
2	-0.781E+03	-0.368E+04	0.000E+00
3	0.321E+04	-0.137E+04	0.000E+00
4	0.260E+04	-0.188E+04	0.000E+00
5	-0.470E+04	0.130E+05	0.000E+00
6	0.977E+04	0.132E+05	0.000E+00
7	0.134E+05	0.397E+04	0.000E+00
8	0.585E+04	-0.424E+04	0.000E+00
10	-0.175E+05	0.496E+04	0.000E+00

DYNAMIC TORQUE ANALYSIS RESULTS

JNT. ID	X - COMPONENT	Y - COMPONENT	Z - COMPONENT
1	0.000E+00	0.000E+00	-0.152E+05
2	0.000E+00	0.000E+00	0.760E+03
3	0.000E+00	0.000E+00	-0.622E+02
4	0.000E+00	0.000E+00	0.463E+02
5	0.000E+00	0.000E+00	-0.297E+02
6	0.000E+00	0.000E+00	0.131E+04
7	0.000E+00	0.000E+00	-0.119E+05
8	0.000E+00	0.000E+00	-0.342E+04
10	0.000E+00	0.000E+00	0.144E+05

STEP NO. = 3

RESULTS FOR THE CLOSED LOOP

CONVERGENCE AFTER 3 ITERATIONS

DISPLACEMENT ANALYSIS RESULTS (DEG/IN)

JNT. 3 = -11.43967

JNT. 4 = 135.4245

JNT.10 = 63.81519

VELOCITY ANALYSIS RESULTS (RAD/S / IN/S)

JNT. 3 = -9.232052

JNT. 4 = 3.056132

JNT.10 = 0.1759908

ACCLERATION ANALYSIS RESULTS (RAD/S*S / IN/S*S)

JNT. 3 = -25.53426

JNT. 4 = -14.69156

JNT.10 = 40.22499

POINT ANALYSIS

TOTAL NO. OF POINTS = 5

POINT I.D.	LINK I.D.	POSITION (IN)	VELOCITY (IN/S)	ACCLERATION (IN/S*S)
1	8	-0.551E+00	-0.237E+02	0.352E+03
		0.930E+00	-0.105E+02	-0.489E+03
		0.000E+00	0.000E+00	0.000E+00
2	7	0.236E+01	-0.214E+02	-0.291E+03
		0.486E+00	0.318E+02	-0.616E+03
		0.000E+00	0.000E+00	0.000E+00
3	6	0.186E+01	0.515E+01	-0.214E+03
		-0.157E+01	0.224E+02	-0.245E+03
		0.000E+00	0.000E+00	0.000E+00
4	5	-0.354E+00	0.291E+02	0.645E+02
		-0.326E+01	-0.445E+01	0.191E+03
		0.000E+00	0.000E+00	0.000E+00
5	2	-0.309E+01	-0.142E+01	0.334E+03
		0.370E+00	-0.319E+02	0.201E+02
		0.000E+00	0.000E+00	0.000E+00

LINK ANALYSIS

TOTAL NO. OF LINK = 10

LINK I.D.	ANGULAR VELOCITY (RAD/S)	ANGULAR' ACCLERATION (RAD/S*S)
1	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.100E+02	-0.153E-04
2	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.150E+02	-0.191E-05
3	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.577E+01	-0.255E+02
4	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.882E+01	-0.402E+02
5	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.882E+01	-0.402E+02
6	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.148E+02	-0.402E+02
7	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.103E+02	-0.402E+02

DYNAMIC FORCE ANALYSIS RESULTS

JNT. ID	X - COMPONENT	Y - COMPONENT	Z - COMPONENT
1	0.653E+04	0.106E+05	0.000E+00
2	-0.119E+04	-0.443E+04	0.000E+00
3	0.391E+04	-0.182E+04	0.000E+00
4	0.313E+04	-0.258E+04	0.000E+00
5	-0.487E+04	0.113E+05	0.000E+00
6	0.870E+04	0.130E+05	0.000E+00
7	0.127E+05	0.425E+04	0.000E+00
8	0.570E+04	-0.392E+04	0.000E+00
10	-0.164E+05	0.449E+04	0.000E+00

DYNAMIC TORQUE ANALYSIS RESULTS

JNT. ID	X - COMPONENT	Y - COMPONENT	Z - COMPONENT
1	0.000E+00	0.000E+00	-0.110E+05
2	0.000E+00	0.000E+00	0.144E+04
3	0.000E+00	0.000E+00	0.109E+03
4	0.000E+00	0.000E+00	0.366E+02
5	0.000E+00	0.000E+00	-0.424E+02
6	0.000E+00	0.000E+00	0.330E+04
7	0.000E+00	0.000E+00	-0.112E+05
8	0.000E+00	0.000E+00	-0.328E+04
10	0.000E+00	0.000E+00	0.959E+04

STEP NO. = 4

RESULTS FOR THE CLOSED LOOP

CONVERGENCE AFTER 3 ITERATIONS

DISPLACEMENT ANALYSIS RESULTS (DEG/IN)

JNT. 3 = -14.46423

JNT. 4 = 136.7528

JNT.10 = 63.41145

VELOCITY ANALYSIS RESULTS (RAD/S / IN/S)

JNT. 3 = -9.389841

JNT. 4 = 2.975307

JNT.10 = 0.4145532

ACCLERATION ANALYSIS RESULTS (RAD/S*S / IN/S*S)

JNT. 3 = -26.66052

JNT. 4 = -15.63498

JNT.10 = 42.29482

POINT ANALYSIS

TOTAL NO. OF POINTS = 5

POINT I.D.	LINK I.D.	POSITION (IN)	VELOCITY (IN/S)	ACCLERATION (IN/S*S)
1	8	-0.645E+00	-0.209E+02	0.379E+03
		0.816E+00	-0.121E+02	-0.435E+03
		0.000E+00	0.000E+00	0.000E+00
2	7	0.227E+01	-0.212E+02	-0.235E+03
		0.543E+00	0.295E+02	-0.606E+03
		0.000E+00	0.000E+00	0.000E+00
3	6	0.185E+01	0.503E+01	-0.179E+03
		-0.152E+01	0.212E+02	-0.246E+03
		0.000E+00	0.000E+00	0.000E+00
4	5	-0.310E+00	0.294E+02	0.705E+02
		-0.327E+01	-0.447E+01	0.190E+03
		0.000E+00	0.000E+00	0.000E+00
5	2	-0.309E+01	-0.256E+00	0.331E+03
		0.259E+00	-0.318E+02	0.325E+02
		0.000E+00	0.000E+00	0.000E+00

LINK ANALYSIS

TOTAL NO. OF LINK = 10

LINK I.D.	ANGULAR VELOCITY (RAD/S)	ANGULAR' ACCLERATION (RAD/S*S)
1	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.100E+02	0.000E+00
2	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.150E+02	-0.153E-04
3	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.561E+01	-0.267E+02
4	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.859E+01	-0.423E+02
5	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.859E+01	-0.423E+02
6	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.146E+02	-0.423E+02
7	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.101E+02	-0.423E+02

DYNAMIC TORQUE ANALYSIS RESULTS

JNT. ID	X - COMPONENT	Y - COMPONENT	Z - COMPONENT
1	0.000E+00	0.000E+00	-0.708E+04
2	0.000E+00	0.000E+00	0.231E+04
3	0.000E+00	0.000E+00	0.400E+02
4	0.000E+00	0.000E+00	0.537E+02
5	0.000E+00	0.000E+00	-0.500E+02
6	0.000E+00	0.000E+00	0.531E+04
7	0.000E+00	0.000E+00	-0.104E+05
8	0.000E+00	0.000E+00	-0.315E+04
10	0.000E+00	0.000E+00	0.477E+04

STEP NO. = 5

RESULTS FOR THE CLOSED LOOP

CONVERGENCE AFTER 3 ITERATIONS

DISPLACEMENT ANALYSIS RESULTS (DEG/IN)

JNT. 3 = -17.54626

JNT. 4 = 138.0578

JNT.10 = 63.08850

VELOCITY ANALYSIS RESULTS (RAD/S / IN/S)

JNT. 3 = -9.556497

JNT. 4 = 2.889145

JNT.10 = 0.6674049

ACCLERATION ANALYSIS RESULTS (RAD/S*S / IN/S*S)

JNT. 3 = -27.76442

JNT. 4 = -16.64032

JNT.10 = 44.40331

POINT ANALYSIS

TOTAL NO. OF POINTS =   5

POINT I.D.	LINK I.D.	POSITION (IN)	VELOCITY (IN/S)	ACCLERATION (IN/S*S)
1	8	-0.723E+00	-0.181E+02	0.401E+03
		0.696E+00	-0.134E+02	-0.382E+03
		0.000E+00	0.000E+00	0.000E+00
2	7	0.219E+01	-0.208E+02	-0.180E+03
		0.589E+00	0.272E+02	-0.592E+03
		0.000E+00	0.000E+00	0.000E+00
3	6	0.184E+01	0.507E+01	-0.144E+03
		-0.148E+01	0.199E+02	-0.244E+03
		0.000E+00	0.000E+00	0.000E+00
4	5	-0.264E+00	0.296E+02	0.766E+02
		-0.329E+01	-0.450E+01	0.191E+03
		0.000E+00	0.000E+00	0.000E+00
5	2	-0.309E+01	0.896E+00	0.328E+03
		0.148E+00	-0.317E+02	0.446E+02
		0.000E+00	0.000E+00	0.000E+00

LINK ANALYSIS

TOTAL NO. OF LINK = 10

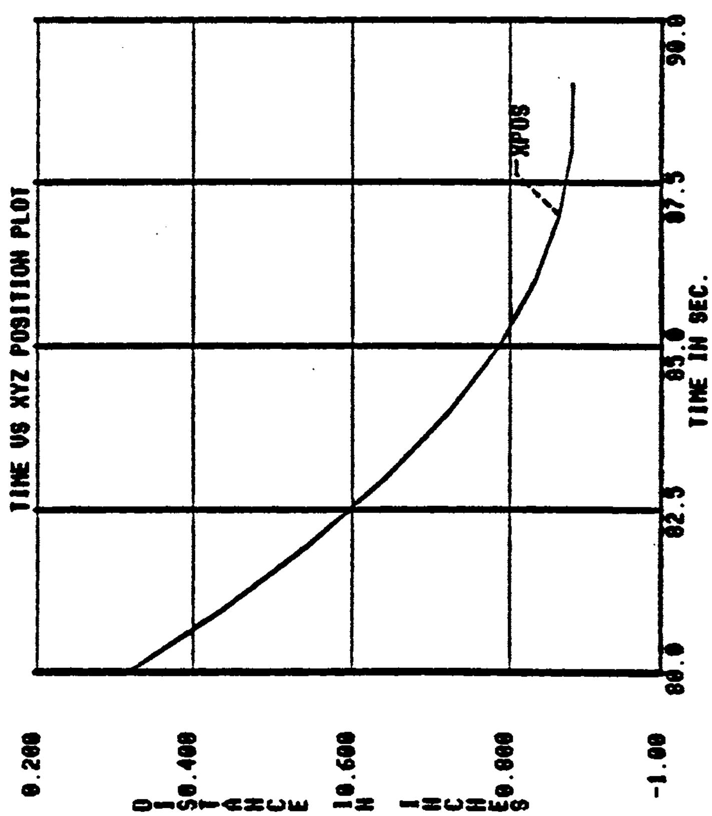
LINK I.D.	ANGULAR VELOCITY (RAD/S)	ANGULAR' ACCLERATION (RAD/S*S)
1	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.100E+02	-0.153E-04
2	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.150E+02	-0.610E-04
3	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.544E+01	-0.278E+02
4	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.833E+01	-0.444E+02
5	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.833E+01	-0.444E+02
6	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.143E+02	-0.444E+02
7	0.000E+00	0.000E+00
	0.000E+00	0.000E+00
	0.983E+01	-0.444E+02

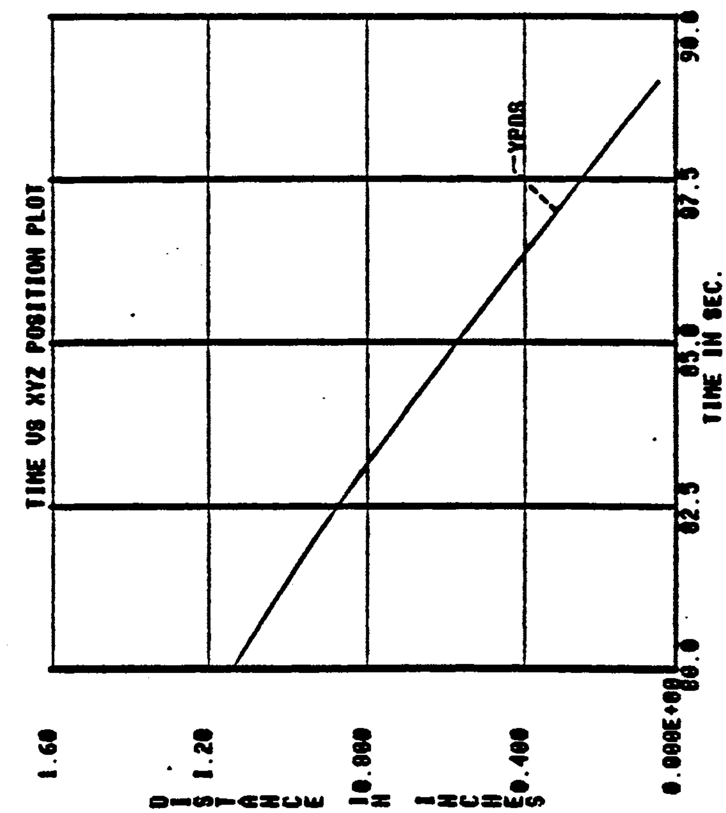
DYNAMIC FORCE ANALYSIS RESULTS

JNT. ID	X - COMPONENT	Y - COMPONENT	Z - COMPONENT
1	0.364E+04	0.774E+04	0.000E+00
2	-0.210E+04	-0.572E+04	0.000E+00
3	0.516E+04	-0.270E+04	0.000E+00
4	0.378E+04	-0.407E+04	0.000E+00
5	-0.560E+04	0.817E+04	0.000E+00
6	0.652E+04	0.126E+05	0.000E+00
7	0.112E+05	0.481E+04	0.000E+00
8	0.539E+04	-0.325E+04	0.000E+00
10	-0.142E+05	0.346E+04	0.000E+00

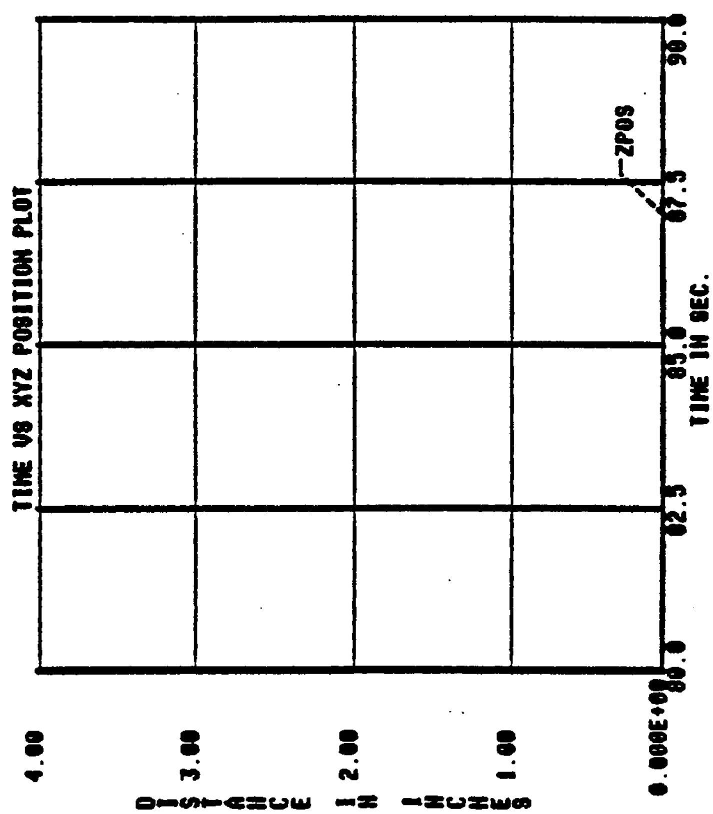
DYNAMIC TORQUE ANALYSIS RESULTS

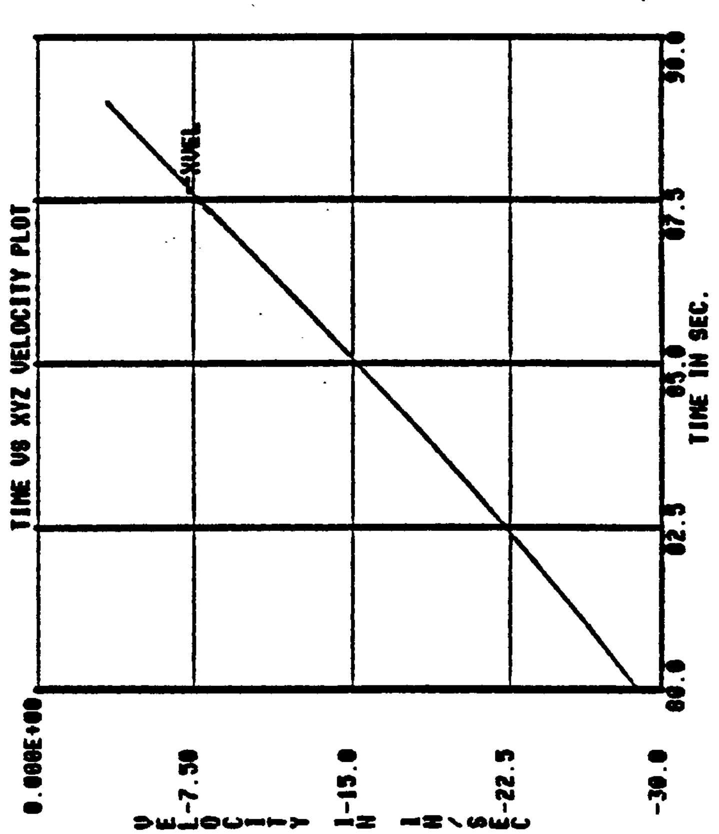
JNT. ID	X - COMPONENT	Y - COMPONENT	Z - COMPONENT
1	0.000E+00	0.000E+00	-0.308E+04
2	0.000E+00	0.000E+00	0.312E+04
3	0.000E+00	0.000E+00	0.513E+01
4	0.000E+00	0.000E+00	0.121E+03
5	0.000E+00	0.000E+00	-0.338E+02
6	0.000E+00	0.000E+00	0.732E+04
7	0.000E+00	0.000E+00	-0.955E+04
8	0.000E+00	0.000E+00	-0.302E+04
10	0.000E+00	0.000E+00	-0.445E+02

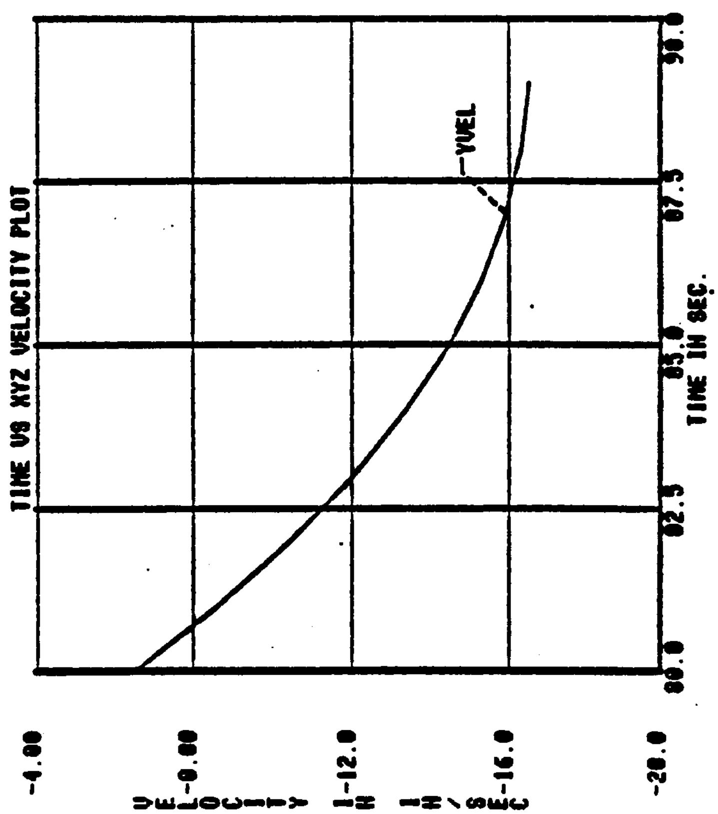

TIME VS XYZ POSITION PLOT
TIME IN SEC.
XPOS
80.0
82.5
85.0
87.5
90.0
0.200
0.400
0.600
0.800
-1.00
DISTANCE IN INCHES

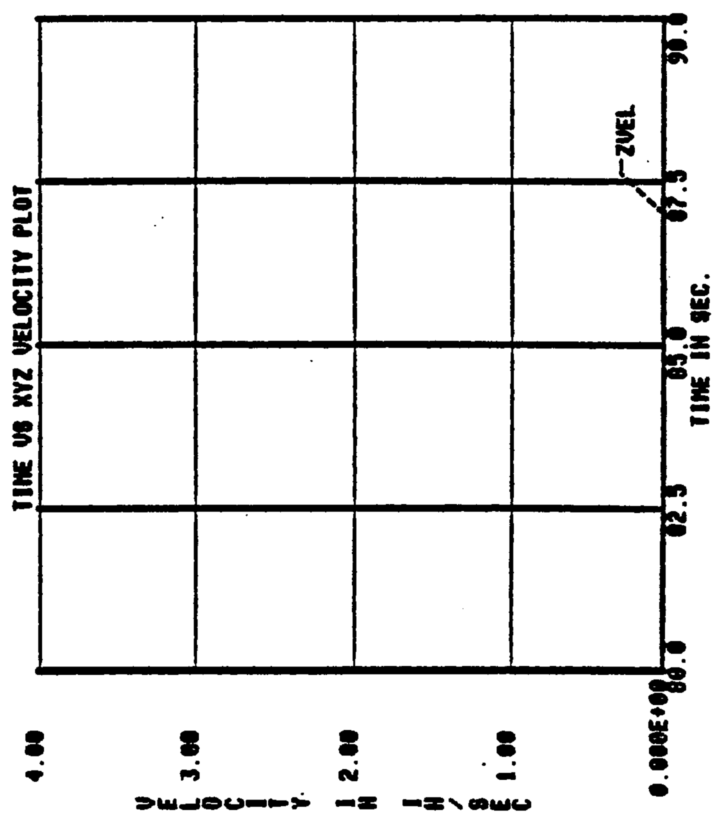


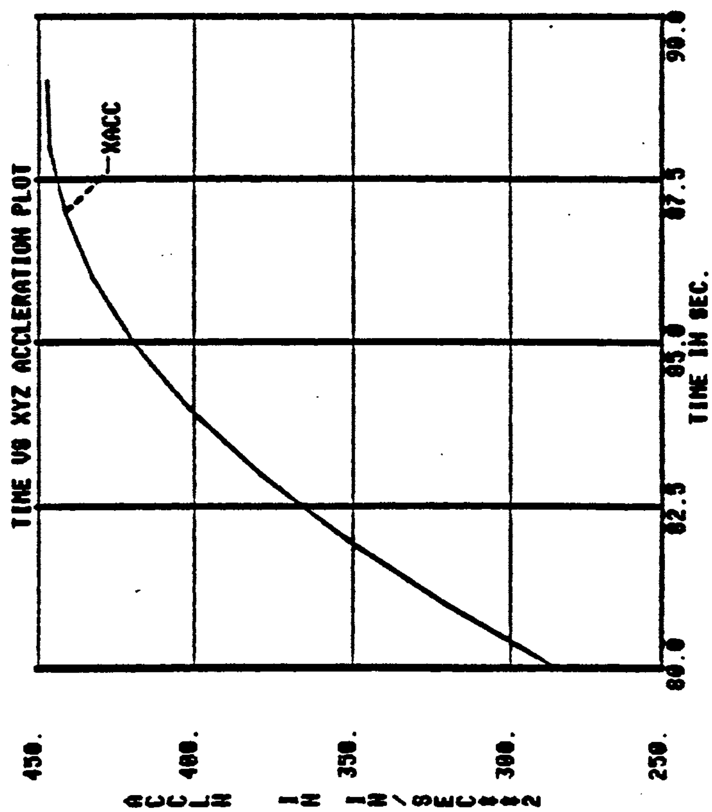
TIME VS XYZ POSITION PLOT
YPOS
TIME IN SEC.
DISTANCE IN INCHES
1.60
1.20
0.400
0.000E+00
00.0
02.5
05.0
07.5

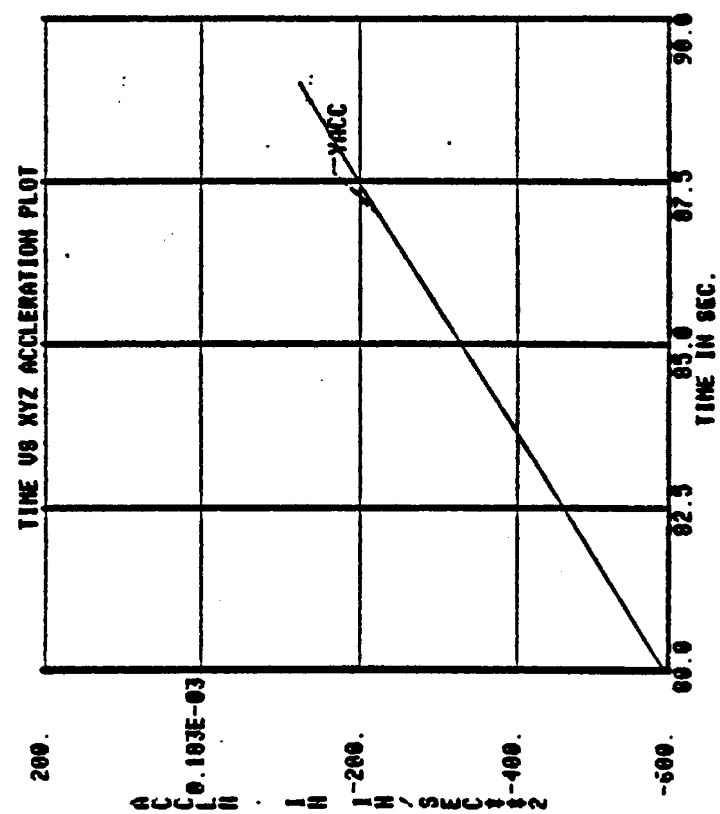

TIME VS XYZ POSITION PLOT
ZPOS
80.0
82.5
85.0
87.5
90.0
TIME IN SEC.
0.000E+00
1.00
2.00
3.00
4.00
DISTANCE IN INCHES

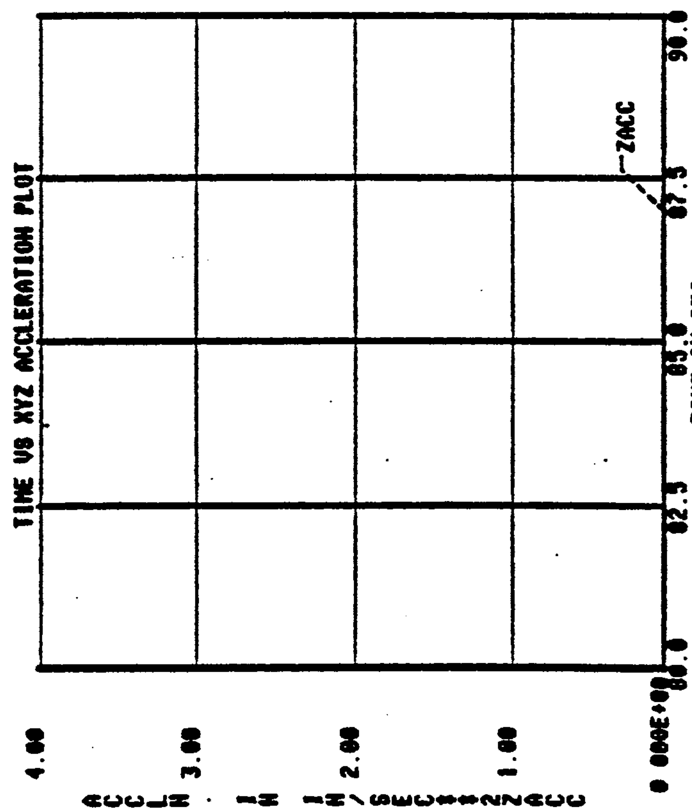

TIME VS XYZ VELOCITY PLOT
XVEL
0.000E+00
-7.50
-15.0
-22.5
-30.0
VELOCITY IN M/SEC
80.0
82.5
85.0
87.5
90.0
TIME IN SEC.


TIME VS XYZ VELOCITY PLOT
VVEL
TIME IN SEC.
VELOCITY IN M/SEC
-4.00
-8.00
-12.0
-16.0
-20.0
80.0
82.5
85.0
87.5
90.0

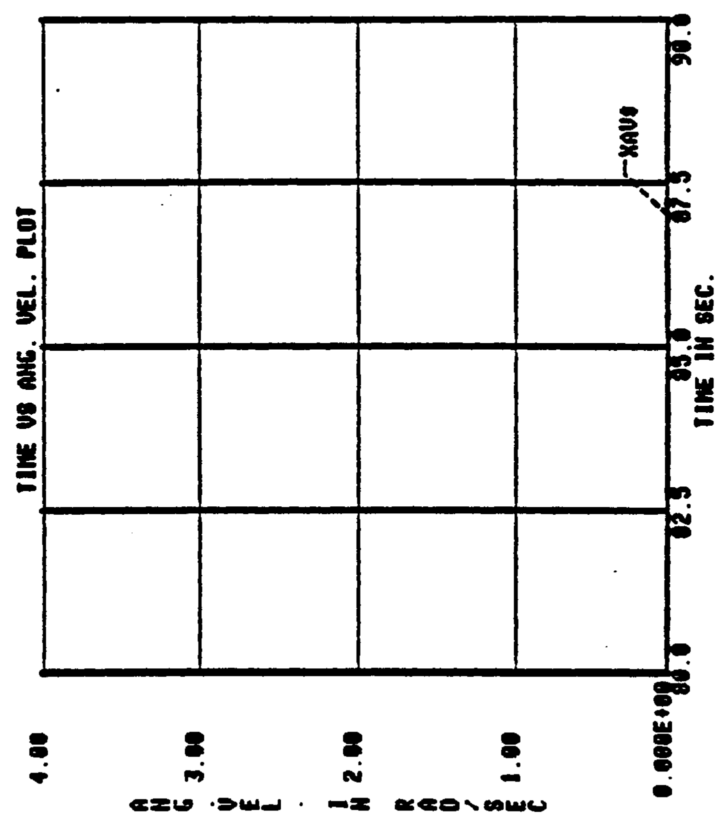

TIME VS XYZ VELOCITY PLOT
ZVEL
80.0
82.5
85.0
87.5
90.0
TIME IN SEC.
0.000E+00
1.00
2.00
3.00
4.00
VELOCITY IN IN/SEC

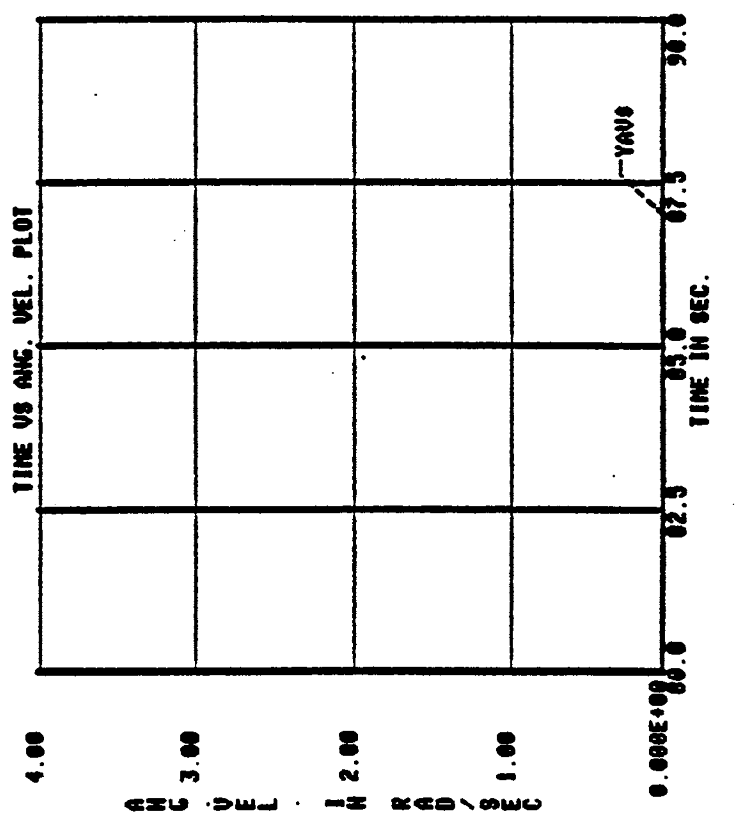

TIME VS XYZ ACCLERATION PLOT
XACC
80.0
82.5
85.0
87.5
90.0
TIME IN SEC.
450.
400.
350.
300.
250.
ACCLN IN IN/SEC**2

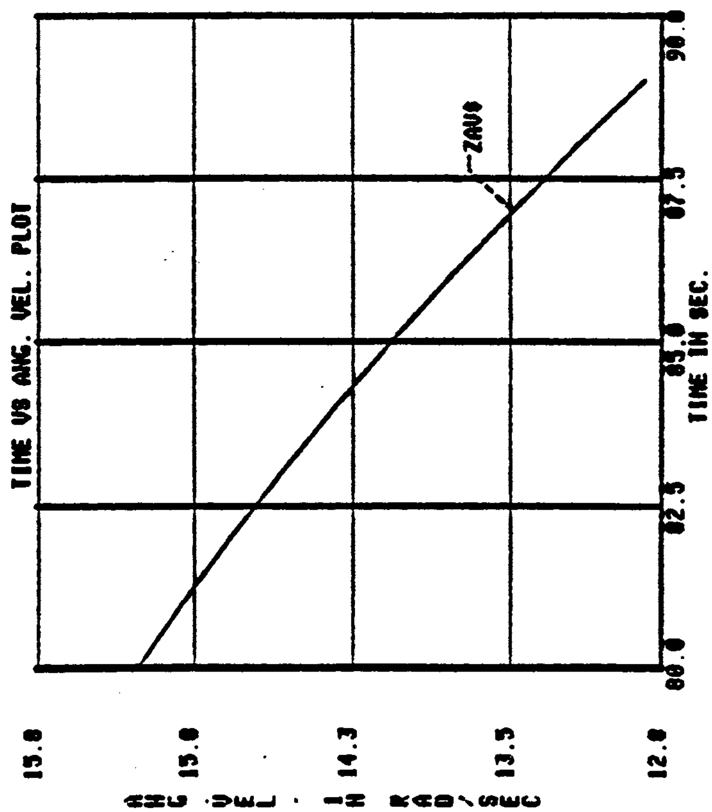

TIME VS XYZ ACCLERATION PLOT
-YACC
200.
0.183E-03
-200.
-400.
-600.
ACCEL. IN IN/SEC**2
80.0
82.5
85.0
87.5
90.0
TIME IN SEC.

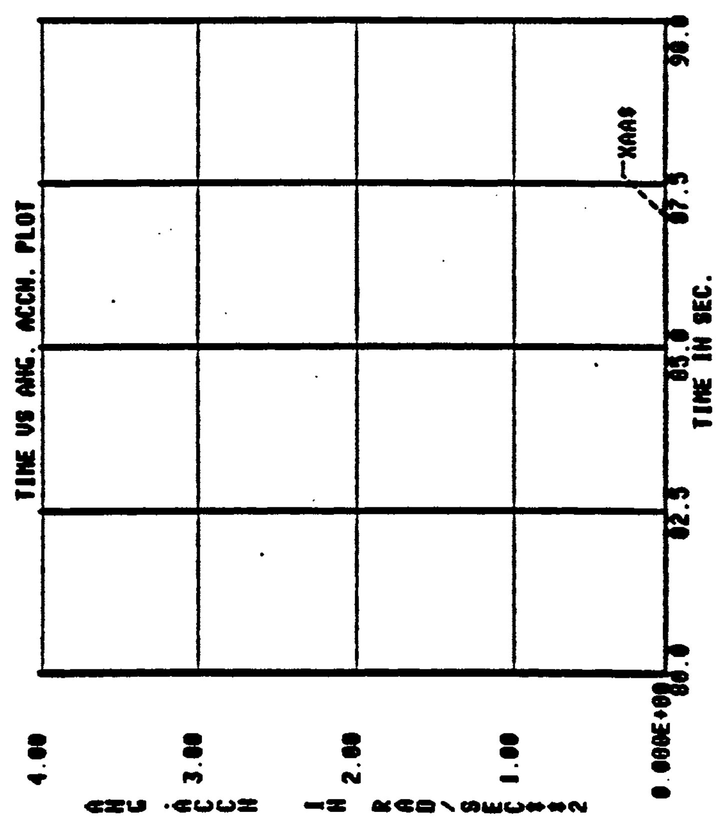


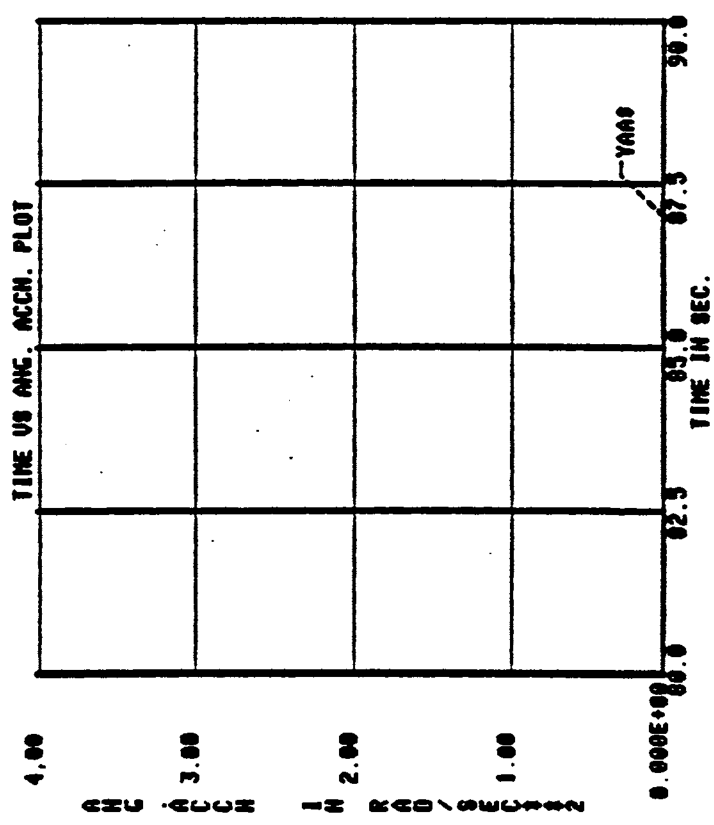
TIME VS XYZ ACCLERATION PLOT
ZACC
TIME IN SEC.
80.0
82.5
85.0
87.5
90.0
4.00
3.00
2.00
1.00
0.000E+00

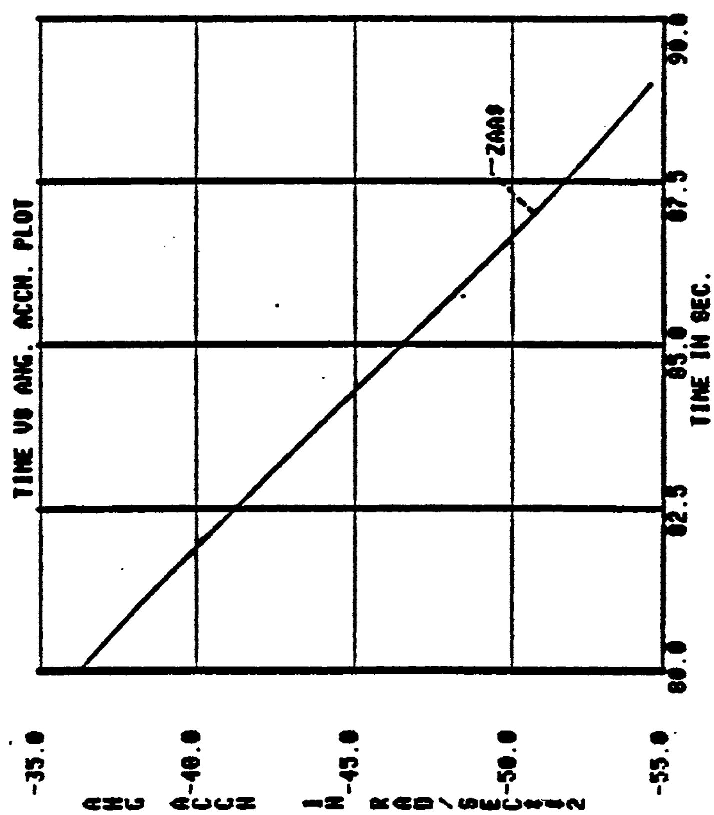

TIME VS ANG. VEL. PLOT
XAVS
TIME IN SEC.
80.0
82.5
85.0
87.5
90.0
ANG. VEL. IN RAD/SEC
0.000E+00
1.00
2.00
3.00
4.00

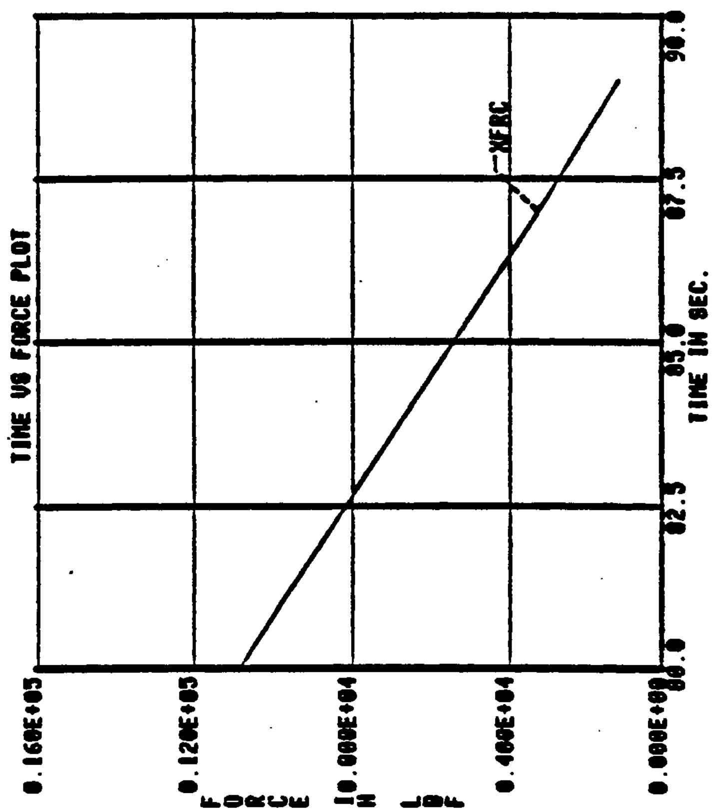

TIME VS ANG. VEL. PLOT
TIME IN SEC.
ANG. VEL. IN RAD/SEC
0.000E+00
1.00
2.00
3.00
4.00
80.0
82.5
85.0
87.5
90.0

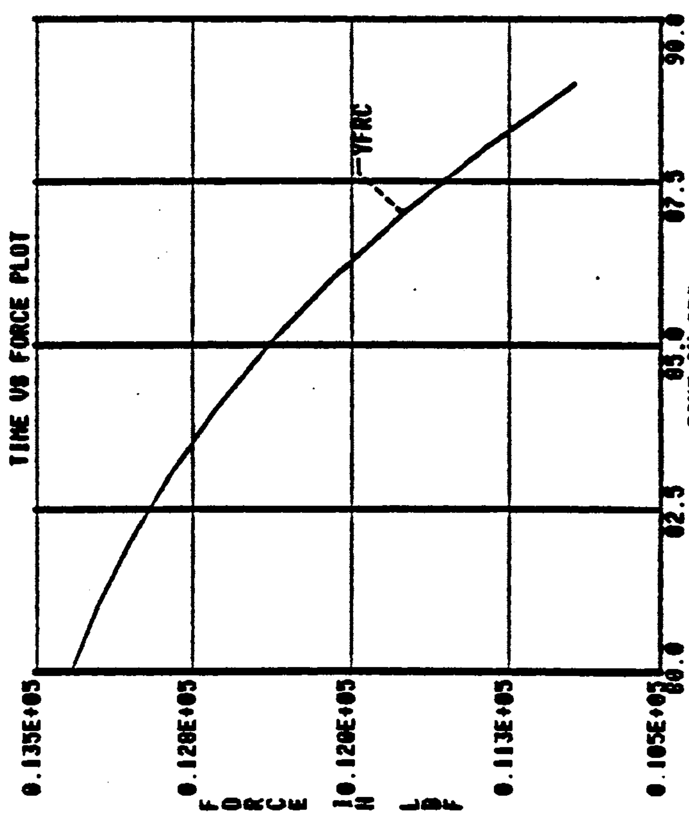

TIME VS ANG. VEL. PLOT
TIME IN SEC.
ANG. VEL. IN RAD/SEC
80.0
82.5
85.0
87.5
90.0
15.8
15.0
14.3
13.5
12.8

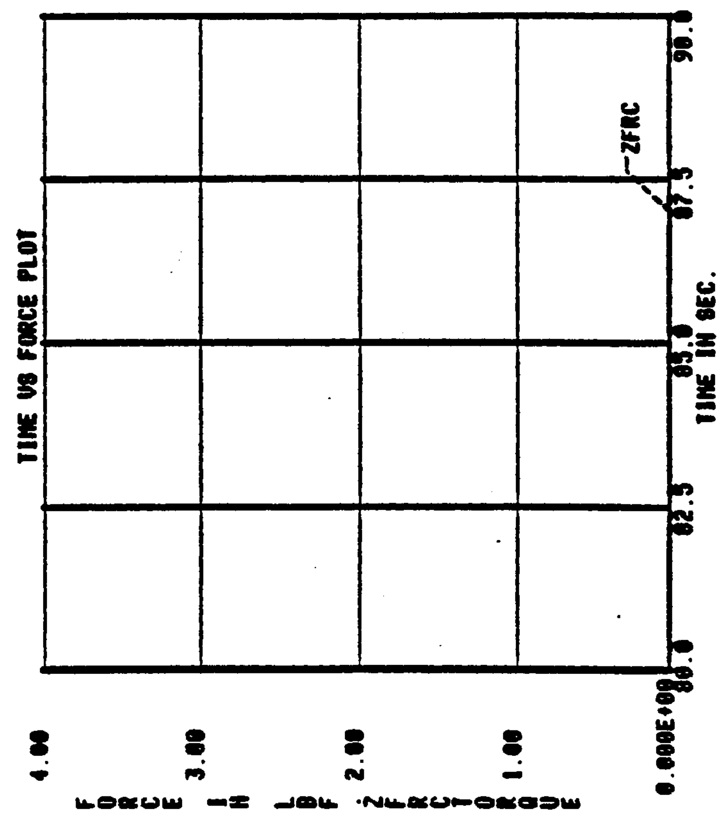

TIME VS ANG. ACCN. PLOT
TIME IN SEC.
ANG ACCN IN RAD/SEC**2
4.00
3.00
2.00
1.00
0.000E+00
80.0
82.5
85.0
87.5
90.0
XAA$

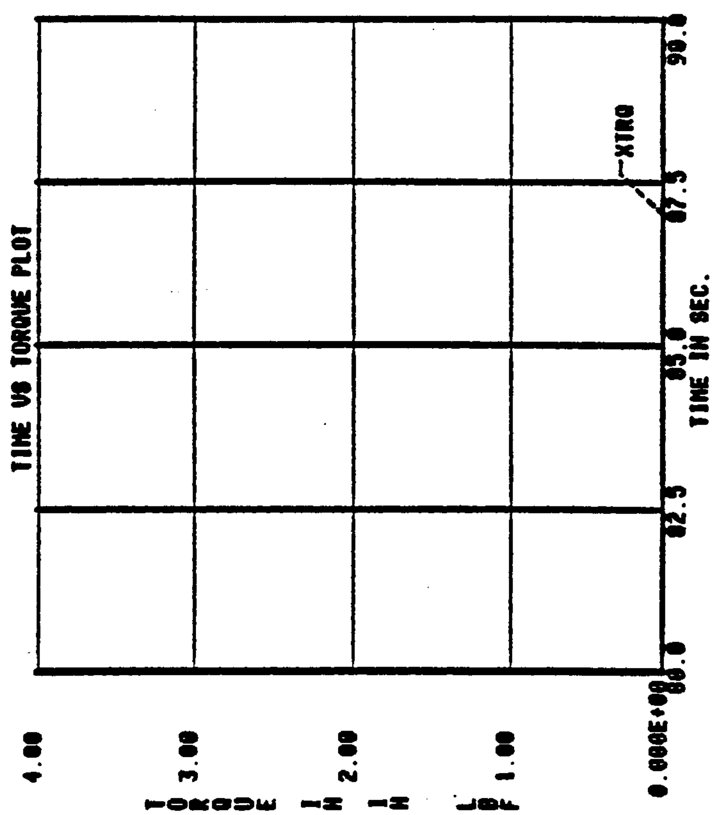

TIME VS ANG. ACCN. PLOT
YAAS
80.0
82.5
85.0
87.5
90.0
TIME IN SEC.
0.000E+00
1.00
2.00
3.00
4.00
ANG ACCN IN RAD/SEC**2

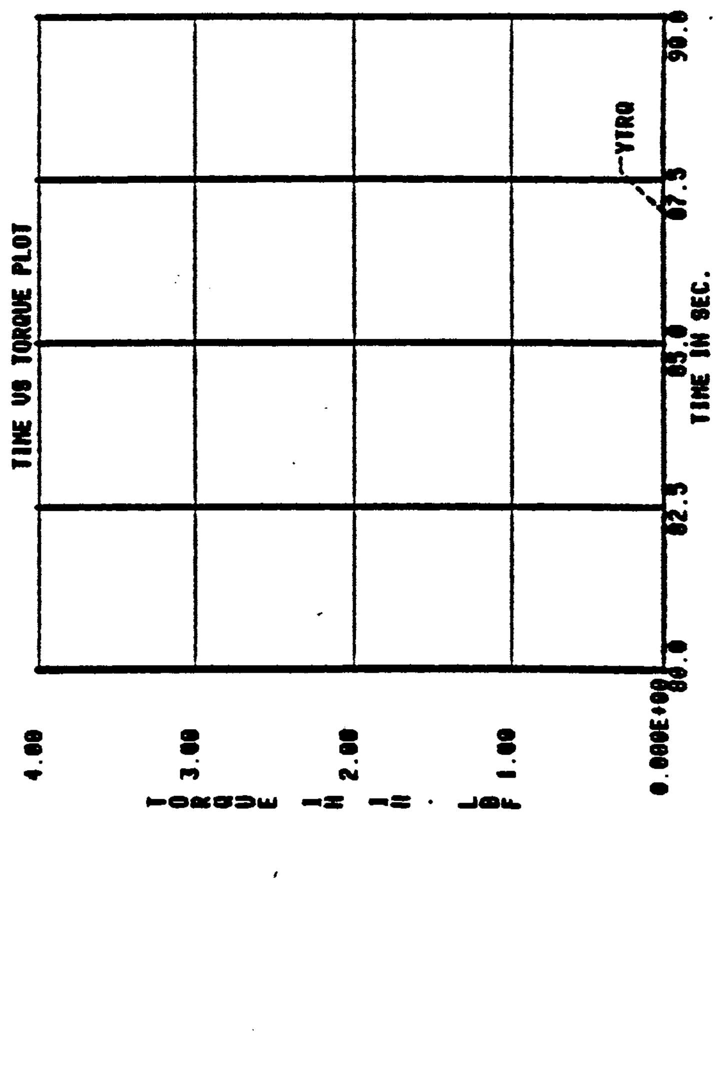

TIME VS ANG. ACCN. PLOT
TIME IN SEC.
80.0
82.5
85.0
87.5
90.0
-35.0
-40.0
-45.0
-50.0
-55.0

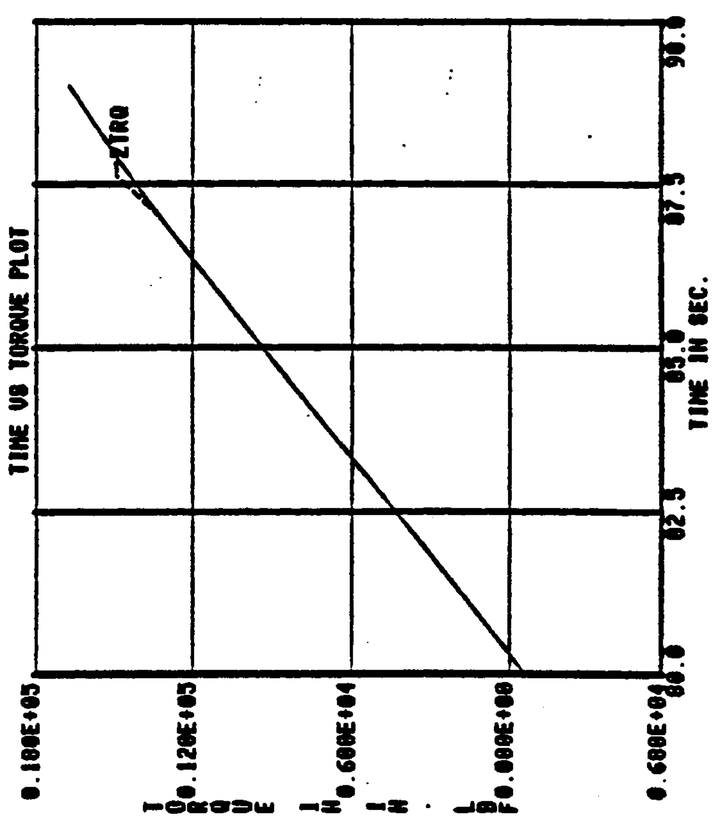

TIME VS FORCE PLOT
0.160E+05
0.120E+05
0.800E+04
0.400E+04
0.000E+00
FORCE IN LBF
80.0
82.5
85.0
87.5
90.0
TIME IN SEC.
XFRC


TIME VS FORCE PLOT
VFRC
0.135E+05
0.128E+05
0.120E+05
0.113E+05
0.105E+05
FORCE IN LBF
80.0
82.5
85.0
87.5
90.0
TIME IN SEC.


TIME VS FORCE PLOT
ZFRC
80.0
82.5
85.0
87.5
90.0
TIME IN SEC.
4.00
3.00
2.00
1.00
0.000E+00
FORCE IN LBF


TIME VS TORQUE PLOT
TIME IN SEC.
80.0
82.5
85.0
87.5
90.0
XTRQ
TORQUE IN IN LBF
0.000E+00
1.00
2.00
3.00
4.00


TIME VS TORQUE PLOT
TIME IN SEC.
TORQUE IN IN. LBF
YTRQ
0.000E+00
1.00
2.00
3.00
4.00
80.0
82.5
85.0
87.5
90.0


TIME VS TORQUE PLOT
TIME IN SEC.
TORQUE IN-LBF
ZTRQ
0.180E+05
0.120E+05
0.600E+04
0.000E+00

VITA

Ravindra Kulkarni

Candidate for the Degree of

Master of Science

Thesis: MIXED-LOOP ROBOT: KINEMATIC AND DYNAMIC ANALYSIS

Major Field: Mechanical Engineering

Biographical:

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