TIMING AND MODE OF LOCOMOTION FOR HUMAN QUADRUPEDS

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TIMING AND MODE OF LOCOMOTION

FOR HUMAN QUADRUPEDS

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NOMENCLATURE

a	-	denotes the fixed coordinate axis x, y, or z
C_g	-	assumed center of gravity of body
C_{xi}	-	horizontal force constant for foot i
C_{zi}	-	vertical force constant for foot i
F_{ai}	-	force exerted on body by foot i in a direction
g	-	acceleration of gravity
H_x	-	measured horizontal diameter of hindleg workspace
I_{aa}	-	the body mass moment of inertia about the a axis
m	-	mass of the body
$\overline{P}_b$	-	normalized position vector of body C_g
$\overline{P}_i$	-	normalized position vector of foot i relative body C_g
R_i	-	force skewness constant for foot i
S_x	-	measured horizontal diameter of foreleg workspace
t	-	time measured in seconds
Q_i	-	force proportionality constant for foot i
V_x	-	normalized distance traveled per cycle by body C_g
α	-	foot position constant relating λ's
τ	-	cycle period in seconds
β_i	-	fraction of τ when leg i is supporting body
ζ_i	-	fraction of τ delay between leg 1 and leg i
γ	-	fraction of τ delay between diagonal leg pair
η	-	time normalized by τ

μ - ratio of β's of left or right leg pair

λ_i - maximum X_i position of foot i during cycle period

$\Delta\lambda$ - difference between λ's of left or right leg pair

ℓ - length of body in feet

ϕ - angular position of body about x-axis (roll)

θ - angular position of body about y-axis (pitch)

ψ - angular position of body about z-axis (yaw)

CHAPTER I

INTRODUCTION

Serious studies of the walking motions utilized by animals began over a century ago. The earliest attempts involved rigorous measurement of kinematic data from a walking animal and designing mechanisms which could duplicate this motion. These early studies were concerned primarily with the duplication of the motion, not the reasons why animals moved with a particular gait pattern. These early models were limited by the fact that their legs followed fixed patterns of motion and could not take advantage of isolated footholds or adapt to changing conditions.

Legged locomotion developments remained in this stage for nearly seven decades until the mid-sixties, when computer control and new, lightweight actuating systems made construction and control of multidegree of freedom mechanisms more feasible. This need for control algorithms and flexible mechanisms was the primary stumbling block in the advancement of legged locomotion research. To fully exploit the advantages of legged locomotion over wheeled locomotion, the legs of the mechanism must have a suitable number of controllable degrees of freedom. This requires a minimum of three degrees of freedom for each leg, a total of 12 for a quadruped mechanism and 18 for a hexapod.

The interest in developing legged locomotion systems was rekindled by published reports which stated that over half of the earth's surface area is inaccessible to conventional wheeled and tracked vehicles [23]. However, most of this inaccessable land mass can be traversed with little difficulty by animals employing legged locomotion. The problem of how to control mechanisms so that they can achieve successful legged locomotion comparable to their biological counterparts remained to be solved.

Typically, the control algorithms for legged locomotion mechanisms are constructed in several levels. The lowest levels control the details of locomotion, such as movements and monitoring of individual degrees of freedom, and sensor signal processing. The higher level tasks that must be performed are maintenance of balance, terrain sensing, determination of leg movement sequences, foothold selection, and path planning to name a few. It is these higher level tasks that require the most additional development before the goal of fully terrain adaptable systems can be realized.

Recent literature deals with attempts to explain the reasons why animals utilize unique gaits for locomotion. The underlying motivation for these investigations is to gain understanding of the higher level control strategies that animals employ during their locomotion cycles. The final result of this work will be realized when man can construct mechanisms and give them the decision-making capabilities that will allow them to move with the mobility and grace of their biological counterparts. The achievement of this goal will depend on man's ability to give these machines sensory capabilities, understanding, and adaptability of motion gait equivalent to that which is possessed by biological creatures.

Literature Review

Serious studies into the reasons why an animal utilizes a characteristic motion gait began in the mid-sixties. McGhee and Frank determined the criteria necessary for a idealized quadruped to maintain static equilibrium throughout a cycle period [18]. McGhee and Frank assumed that the body of the translating mechanism, or idealized animal, maintained a constant straight-line velocity. They further assumed that the dynamic effects caused by movement of the legs were negligible and that the quadruped's legs were capable of exerting any force required of them on the body. This reduced the problem of quadruped stability to a static form. The stability of the quadruped's body could be maintained by keeping the horizontal projection of the body C_g within the polygon formed by the horizontal projection of the positions of all feet that were in the support phase of their cycle and keeping at least three feet in a support phase at all times. This type of locomotion they defined as crawling. They then used an iteration technique to solve for the timing and dimensional parameters that would satisfy the static equilibrium criteria over an entire cycle period. They found three nonsingular gaits that would satisfy the equilibrium criteria with one being defined as optimum as it satisfied the equilibrium criteria over a greater range of leg duty factors (β).

Bessonov and Umnov extended this early work to include six-legged crawlers [8]. They used the same basic assumptions and static stability criteria that McGhee and Frank did in in their early study [18]. Bessonov and Umnov arrived at six nonsingular gaits that satisfied static equilibrium criteria for hexapods. McGhee and Iswandhi developed the algorithm further to include n-legged robots and investigated adaptive gaits that would allow locomotion in rough terrain environments while still maintaining static equilibrium for hexapods [19]. Another adaptive gait was developed by Hirose, Nose, Kikuchi, and Umetani based on these earlier works for crawling quadruped mechanisms or idealized animals [16].

Working mechanisms that utilize static equilibrium algorithms for stabilizing the crawler body have been constructed by various research groups. A quadruped crawling mechanism that negotiates stairways and other minor obstacles has been demonstrated [15,16]. Hexapods have also been developed and are in various stages of development [19,20]. The most ambitious of these projects is the ongoing development of the Adaptive Suspension Vehicle, a hexapod crawling vehicle that utilizes a human operator to determine gross motions with computer controls determining details of motion, such as leg placement and gait adaptations [23].

While the static stability algorithm was being developed, other hypotheses explaining the way animals moved were being forwarded. Alexander and Jayes found that the animal gaits they studied rarely satisfied static equilibrium criteria, even at low translational velocities [4]. Two explanations were forwarded to account for this phenomenon. The first resulted from gait analysis of quadrupedal and bipedal animals. Alexander and Jayes modeled force data taken during gait trials using an infinite trigonometric series [5]. Using dimensional parameters measured during gait trials and assuming that the legs were massless, mathematical models were developed that estimated the energy cost of locomotion [1,4]. Using this model, the timing parameters that minimized energy expenditures were determined and found to be comparable to measured gait data.

Jayes and Alexander used a different approach in their investigation of the slow speed locomotion of chelonians [17]. They expected the slow speed chelonians to use a gait that would maintain static equilibrium. What they found instead was that the chelonians moved in a trot-like walking gait with several periods in which only two legs were in a support phase. They calculated the vertical forces that need to be exerted by the feet for an idealized chelonian to maintain equilibrium in constant velocity locomotion. Plotting this information over the cycle period, they found that the gradient of the forces approached infinity at two points in time for each of the chelonian legs. The conclusion they reached was that the chelonians found such an abrupt change in exerted vertical force impossible for their slow speed muscles to deliver. Modeling the vertical forces exerted by the feet of the chelonian with a trigonometric series, they calculated the values of the force skewness constants R_i and the time delay factors ζ_i that would minimize unwanted vertical displacements and unwanted angular displacements of the body using an idealized chelonian model. The results were very close to measured gait data.

Significance of Research

Most of the research cited in the previous section concerns itself with developing a simplified mathematical model to describe legged locomotion. This model is then used to determine a limited set of parameters that will eliminate or minimize one or more undesirable characteristics. The results are then compared to data gathered from trial runs of a biological counterpart. For example, Jayes and Alexander gathered timing and force data for several species of chelonians. Then using the physical dimensions that were measured from both living and dead specimens, a model was developed for an ideal chelonian with massless legs and moving with a constant horizontal velocity. They then used this model to identify the optimal timing parameters ζ_i and force skewness factor R_i for a chelonian. Table I illustrates their findings and the comparison to measured data. The optimality in their findings comes from minimization of the calculated undesirable vertical and angular displacements of the chelonian body. However, this work proceeded under the assumption that the velocity of the body was constant and that the chelonians' gait was symetric, all β's are equal. Also, the effect of the duty factors β_i were not investigated and the physical dimensions used for calculations were measured from chelonian specimens. Therefore, this work will have applications limited to chelonians and animals with very similar physical characteristics.

Most of the work done by these research groups was concerned with the gaits of natural quadrupeds and the natural biped gait of humans. However, humans are also capable of quadruped locomotion. Quadruped locomotion is used by infant humans before their muscular structure and coordination are sufficiently developed to accommodate biped locomotion. Quadruped motion is also used by adults when negotiating certain obstacles where passage space is limited, footing is uncertain, or when the situation dictates, such as a soldier moving on a battlefield. It is

TABLE I

RANGES OF VERTICAL MOVEMENT, OF PITCH AND OF ROLL FOR A TYPICAL CHELONIAN WALKING AS SPECIFIED IN THE MODEL WITH SKEW FORCE PATTERNS AND DUTY FACTOR OF 0.83

Parameter	Gait 1	Gait 2	Gait 3
ζ_2	0.50	0.50	0.50
ζ_3	0.50	0.58	0.67
ζ_4	0.00	0.08	0.17
R_i	0.00	0.16	0.34
<u>Resulting displacement ranges:</u>			
vertical	0.12ℓ	0.06ℓ	0.12ℓ
roll angle	26°	16°	21°
pitch angle	0°	8°	14°

obvious that for a mechanism to achieve the mobility of a human, it should be capable of quadruped as well as biped locomotion. At this point in time, the motion strategies employed by humans in quadruped locomotion have not been investigated.

There are three basic types of quadruped locomotion gaits that have been identified by researchers [2]. The first, the crawl, refers to gaits in which the crawler maintains at least three legs in support positions at all times during the motion cycles [17]. With proper force and position control, it is possible for an animal to eliminate unwanted displacements of its body. However, as was explained above, this gait is used only in artificial mechanism locomotion and in special cases of very low velocity animal locomotion, such as grazing animals. Quadruped walking is the gait type in which at least two of the four walker legs support the body of the walker at all times in the cycle period [2]. It is no longer possible to eliminate unwanted displacements, as static equilibrium can no longer be maintained. This is the locomotion type used by most quadruped animals at the normal translational speed for that species. Quadruped running is defined as any gait in which fewer than two legs support the body and exert force on the ground at any time during the motion cycles. This type of gait is used most often by animals when startled, scared, or otherwise excited when a great need for maximum speed is needed. However, some species of animals, such as chelonians with their slow muscular speeds, are incapable of running. Since walking is the most common form of quadruped locomotion, this type of locomotion holds the greatest interest at this time.

Even in a simplified model of quadruped locomotion, there are an immense number of parameters that can be varied and the effects on the quadruped studied. To investigate all possibilities would be a very exhaustive procedure. Furthermore, there will be as many different optimum sets of parameters as there are individual body types. Therefore, a study describing gross motion characteristics for an average subject, using mean values for physical dimensions measured from a control group, would be more practical and useful. Even with this constraint, a study which includes all parametric variations is a formidable task.

Since studies of this mode of human locomotion have never been published, this study is introductory. With this in mind, the objectives were designed to provide a solid data base for future researchers to build on.

Objectives

The objectives of this study are the :

1) Development of a mathematical model that adequately describes unwanted displacements of the human body during quadruped locomotion.

2) Parametric study of human quadruped locomotion and identification of an optimum set of parameters that will

minimize undesirable displacements of the body for straightline translational locomotion.

3) Comparison of the information gained from the parametric study to data gathered from measurements of actual human performance.

CHAPTER II

HUMAN QUADRUPED LOCOMOTION: ACTUAL PERFORMANCE AND MODEL DEVELOPMENT

The development of a mathematical model describing any form of legged locomotion is nearly impossible without prior knowledge of the actual performance of the subject in question. The number of parameters involved in such a derivation is immense and threatens to overwhelm the researcher. However, if data is gathered and analyzed before beginning equation derivation, sound, simplifying assumptions can be made based on solid statistical data, which will reduce the task of equation development down to managable proportions. With this thought in mind, sets of videotapings were made of six adult human subjects, three male and three female, and sets of physical measurements were taken during the videotaping procedure. The subjects were each videotaped moving over a straightline 30 ft. course. The data taking procedures were designed to provide some insight into the timing and controlling mechanisms involved in human quadruped locomotion.

Measured Data

The data measured from the actual human performances can be divided into two subgroups. The first, dimensional data, is a set of distance measurements that is used to calculate or define the position of the support contact points (feet) of the quadruped, the position of the body C_g, or the reachable area of the subject's four legs. The second type of data group is termed the gait timing measurements. This group includes all duty factors (β_i) and time delay factors (ζ_i) that describe the order and timing of leg movements.

Physical Dimensions

Measurements of the approximate physical parameters of the six adult subjects were made during the videotaping procedure. Figures 1-3 define the coordinate axes, the physical dimensions, and the leg number assignment as they were used in the measurement procedure. Due to the diverse sizes and body configurations of the subjects, all distance parameters were normalized by the body length of the subject to give a more accurate standard of comparison. The body length (ℓ) was measured as the approximate distance from the hip joint of the subject to the shoulder joint of the subject.

Many of the dimensional values needed to describe the physical dimensions of the subjects' are defined as being measured relative to the C_g of the quadruped's body. This position is unknown for the subjects tested. In their study of chelonians, Jayes and Alexander solved this problem by

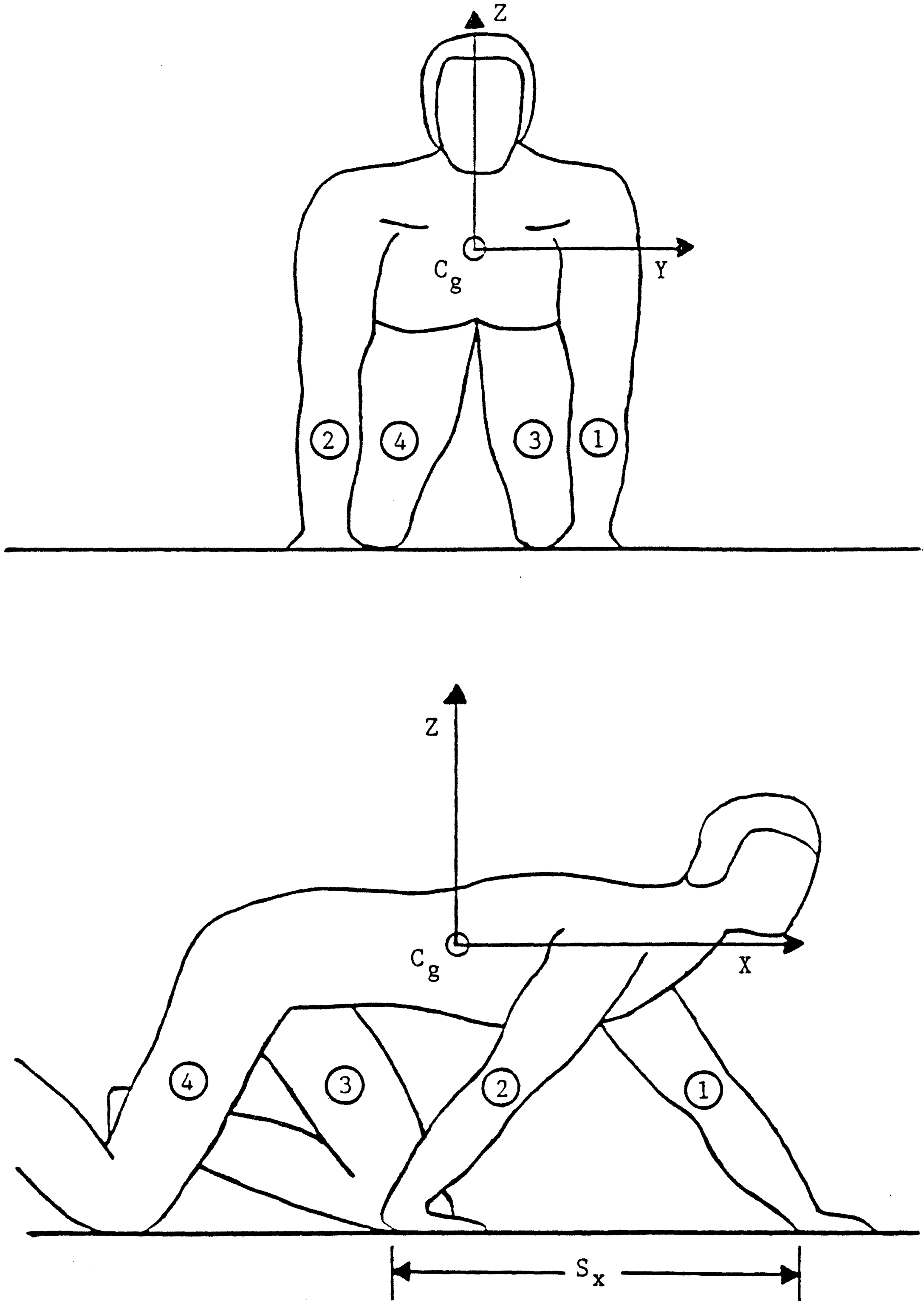


Figure 1. Measured Physical Dimensions

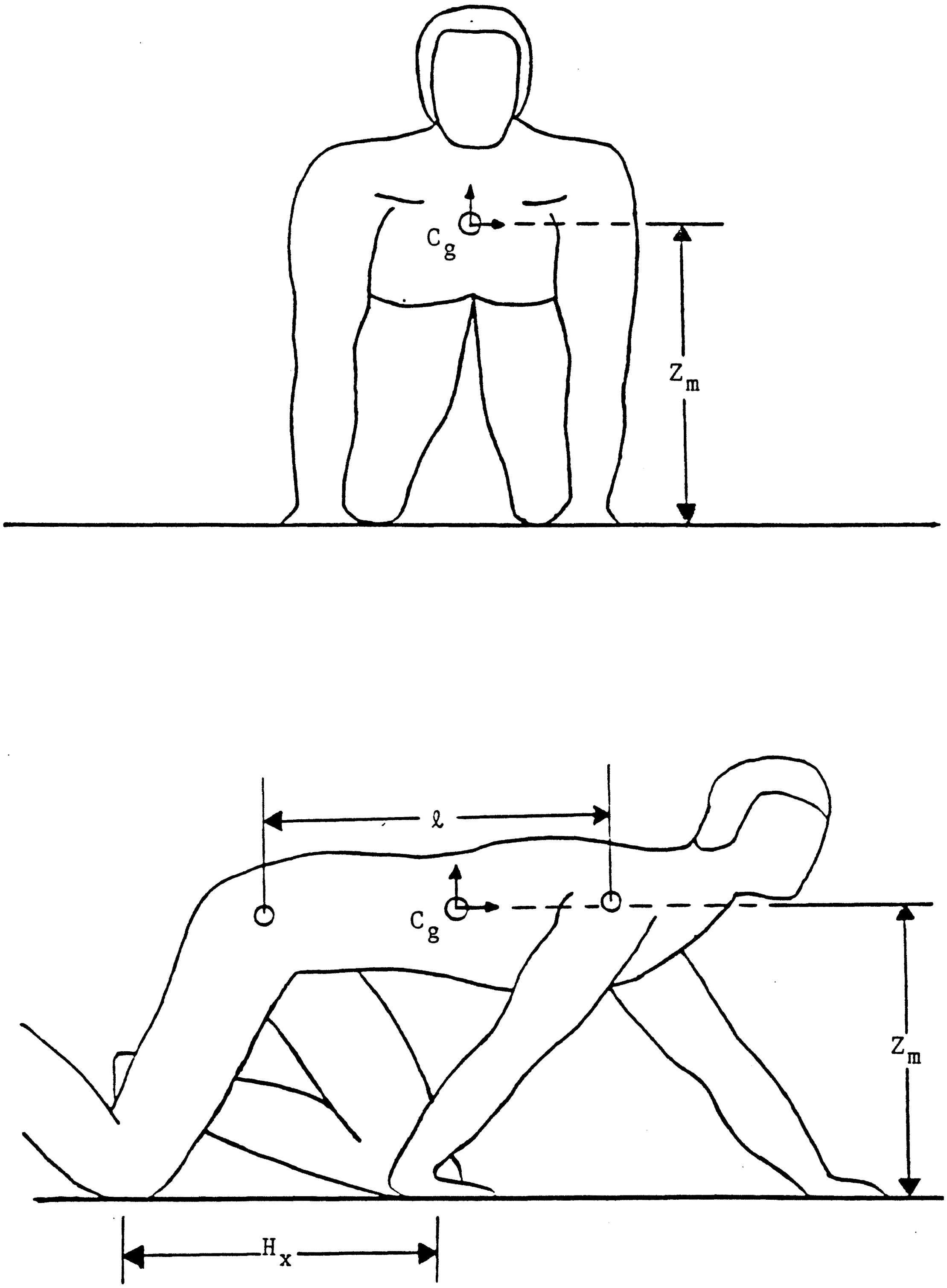
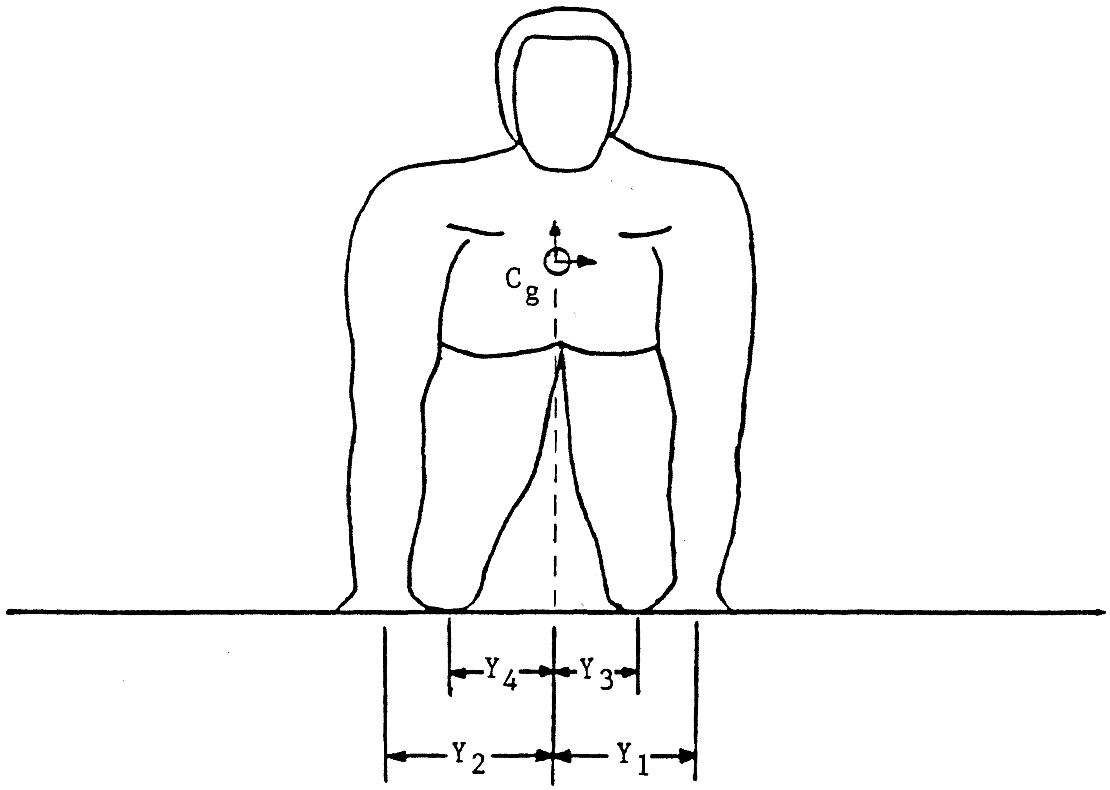


Figure 2. Measured Physical Dimensions



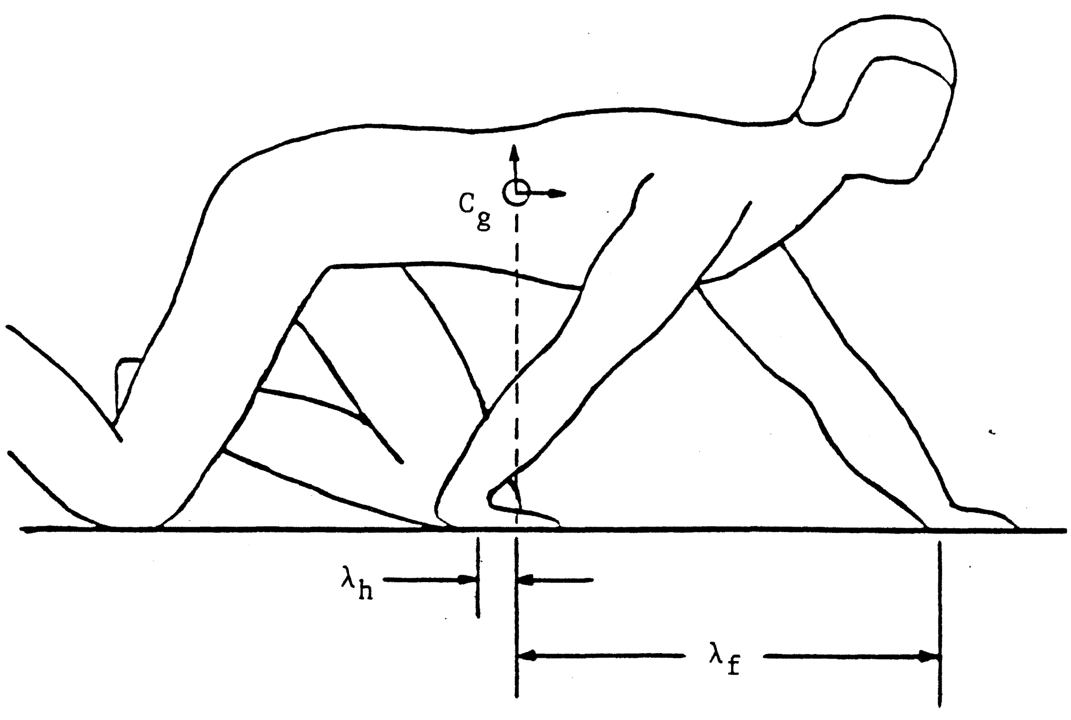


Figure 3. Measured Physical Dimensions

suspending animals as bifilar pendulums and determining the C_g for each individual [17]. As the human subjects expressed some displeasure with this this method, a different approach was used in this study. A C_g for the trunk, head, and neck link was assumed using mean values obtained from cadaver studies by Dempster and Roebuck [12]. It was found that the mean C_g for this link was located at a point approximately 0.541ℓ from the hip joint in the x direction and approximately at mass center in the y and z directions. This position was marked during the measurement procedures using an easily identifiable cloth strip. When a measurement relative to the C_g was required, it was taken relative to this assumed C_g.

The mean values and standard deviations of the subjects' normalized dimensional measurements were calculated using the six adult subjects as the statistical group and are contained in Table II.

Gait Timing Measurements

The gait timing values were determined after the videotaping procedure from reruns of the videotape records. The timings were for the subjects moving in quadrupedal locomotion over a 30 ft. straightline course. The raw data timings were made by hand using a stopwatch, since more sophisticated equipment was not available. The raw data was recorded in units of seconds. To account for the obvious errors that would result from such a crude timing method, a large number of timings was taken for each timing parameter. The mean values of the parameters were calculated for each subject. These mean values were used in all future calculations.

The timings that completely define the leg movement order are the four duty factors for the four legs, β_i, and the four time delay factors, ζ_i. Since ζ_1 by definition is equal to zero, there are actually three time delay factors to be determined. By definition, β_i and ζ_i are fractions of the cycle period τ. Therefore, the mean values of the raw data measurements were normalized by the mean value of τ for each subject. This produces dimensionless timing parameters that are compatible with the parameter definitions.

The mean values and standard deviations of the normalized timing constants were then calculated using the mean values calculated for each of the six subjects as the statistical group. The results are also contained in Table II.

Conclusions From Timings

The human subjects, like the chelonians studied by Jayes and Alexander, did not move with a gait that could produce static equilibrium. There are several times within the cycle period in which only two of the subjects' legs are supporting the weight of the body. This is illustrated by Figure 4. The human quadrupeds actually moved with a walking type gait as was defined earlier in this work.

TABLE II

RESULTS OF TIMING AND DIMENSION MEASUREMENTS OF HUMAN QUADRUPED LOCOMOTION

Parameter	Mean value	σ_n
Dimensional:		
$Y_1, -Y_2$	0.336	0.04
$Y_2, -Y_4$	0.286	0.05
λ_1, λ_2	1.053	0.15
λ_3, λ_4	-0.057	0.05
$\Delta\lambda$	1.109	0.11
S_x	1.594	0.21
H_x	1.121	0.13
Z_m	0.931	0.09
Timing:		
β_1	0.72	0.06
β_2	0.71	0.06
β_3	0.63	0.05
β_4	0.62	0.04
ζ_2	0.51	0.03
ζ_3	0.52	0.04
ζ_4	0.00	0.05
μ	1.143	0.03
τ	1.98	0.66
Mixed:		
V_x	1.596	0.33

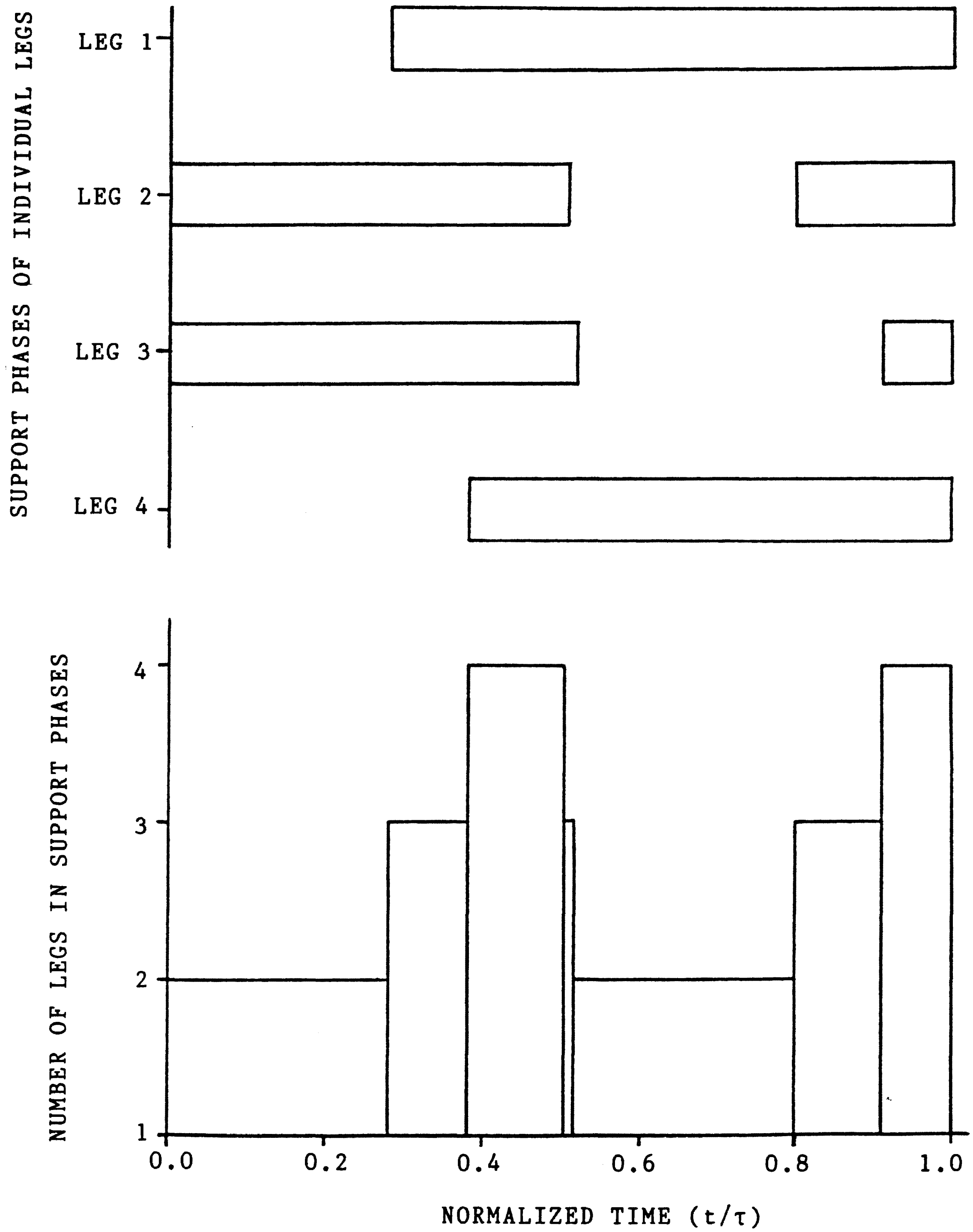


Figure 4. Measured Human Quadruped Support Phases

Mixed Parameters

There is one parameter which is a mixture of the timing and dimensional parameters that can be determined from the videotape records. This parameter, the mean horizontal velocity of the subject's body, was determined by dividing the test course length by the time required to travel the entire course. To make this measure compatible with the dimensionless constants defined above, the mean velocity of the subject was normalized by ℓ/τ with the result being a dimensionless velocity constant for each subject. The mean value and standard deviation was calculated as before and is included in Table II with the other performance data mean values.

Development of a Mathematical Model for Human Quadruped Locomotion

The development of the mathematical description of human quadruped walking began with a definition of the model characteristics. Dempster et al. calculated in their studies of cadavers that, on the average, 57.9% of the human body's total mass is located in the trunk, head, and neck [12]. The rest of the mass is distributed among the rest of the body parts in the proportions 5.1% in legs 1 and 2 and 15.9% in legs 3 and 4. Using this as justification, it was assumed that the dynamic effects due to movement of the human quadruped legs are negligible compared to the forces neccessary to support the body in low velocity walking.

The equations of motion describing the angular and linear displacements of the human body can then be derived using relatively simple dynamics principles. Using the coordinate axes as defined earlier, the forces exerted by the leg points of contact on the ground, and the position vector of the points of contact relative to the body C_g, the linear and angular acceleration of the body C_g can be calculated as follows

$$\sum_{i=1}^{4} \overline{F}_i = m(d^2\overline{P}_b/dt^2) \tag{2.1}$$

$$\sum_{i=1}^{4} (F_{zi}Y_i - F_{yi}Z_i) = I_{xx}(d^2\phi/dt^2) \tag{2.2}$$

$$\sum_{i=1}^{4} (F_{xi}Z_i - F_{zi}X_i) = I_{yy}(d^2\theta/dt^2) \tag{2.3}$$

$$\sum_{i=1}^{4} (F_{yi}X_i - F_{xi}Y_i) = I_{zz}(d^2\psi/dt^2) \tag{2.4}$$

where ϕ, θ, and ψ are the roll, pitch, and yaw angles of the body about the x, y, and z axes, respectively.

A formidable problem remained in defining the forces exerted by the subject during the leg support phases. It would be logical to assume that a slow speed quadruped would move in a manner that would satisfy the static equilibrium criteria. However as was pointed out earlier, this is not the case. Animals seem to avoid the abrupt change in force gradient needed to maintain static equilibrium and settle for a compromise, a gait with no abrupt changes in the force gradients but timed in a fashion that minimizes unwanted displacements [17]. Using a method published by Alexander and Jayes [5], the forces exerted by the legs during support phases of their cycle were modeled using a trigonometric series of the form

$$F_i = \sum_{n=1}^{\infty} A_n \sin(n\pi D_i) \qquad (2.5)$$

where A_n = arbitrary force constants

$D_i = (\eta-(\zeta_i-\beta_i))/\beta_i$

when $\zeta_i-\beta_i \leq \eta \leq \zeta_i$

At all other times, when leg i is in a movement phase, the forces exerted by leg i equal zero. Alexander and Jayes include no cosine terms in the forcing function because of the requirement that the force terms diminish to zero at the end points of the leg i support phase. Furthermore, if the force curves are smooth without abrupt changes in gradient, all higher-numbered terms in the infinite series will approach zero.

Vertical Forces

Alexander and Jayes state that the first three terms in the infinite series should be sufficient to model the forces present in any set of vertical force data. This leaves a series function for vertical forces of the form

$$F_{zi} = C_{zi}[\sin(\pi D_i) \pm R_i \sin(2\pi D_i) + Q_i \sin(3\pi D_i)] \qquad (2.6)$$

where $C_{zi} = A_1$ of leg i

$R_i = \pm A_2/A_1$

and $Q_i = A_3/A_1$

The ambiguous sign on the second series constant R_i comes from Jayes and Alexander's experiences with other quadruped animals in which the absolute value of the skewness factor R was negative for the forefeet (legs 1 and 2) and positive for hindfeet (legs 3 and 4) [17]. Alexander and Jayes showed that this three term representation of horizontal forces reasonably duplicated force records of all gaits followed by men, dogs, and sheep [4]. Therefore, this representation is used in all further calculations.

If the limitations placed on vertical forces exerted by a quadruped's legs are considered, some logical deductions can be made about the three constants C_{zi}, R_i, and Q_i. The vertical forces exerted by the ground on the leg contact points will always be positive, unless there is an adhesive attraction between the contact surface and the ground surface. By assuming no adhesive attraction is present, a limitation is placed on the values that R_i and Q_i can take. Recognizing that C_{zi} is positive, then for F_{zi} to be positive over the entire interval of the support phase, the inequalities

$$-0.5 \leq R_i \leq 0.5 \tag{2.7}$$

and

$$-B/3 \leq Q_i \leq B \tag{2.8}$$

where

$$B = 1-|2R_i|$$

must be satisfied.

The value of the constant C_{zi} also can be calculated to some degree of accuracy by careful observation. It is proper to assume that the mean value of the vertical acceleration is zero over the entire cycle period. This would maintain the relative position of the body with respect to the ground over subsequent cycles. For this to be true, the sum of the areas under the force vs. time curves of the terms in the vertical acceleration equation must be zero. Using Equation (2.1), the area under the vertical force curve for the entire cycle period would then be the integral equation

$$\sum_{i=1}^{4} \int_{0}^{\tau} F_{zi}\,dt - mg\tau = 0 \qquad (2.9)$$

Upon integration of this equation, the result is

$$\sum_{i=1}^{4} [2C_{zi}\beta_i(3+Q_i)]/3\pi = mg \qquad (2.10)$$

If each leg is assumed to support 0.25 of the body's weight during the movement cycle, then

$$C_{zi} = 0.375[mg\pi]/[(3+Q_i)\beta_i] \qquad (2.11)$$

This is the form of C_{zi} that was used in all subsequent calculations.

Horizontal Forces

The normal assumption made in the past by authors studying animal motion has been that the velocity of the animal is constant. It became obvious after reviewing the videotape records of the human crawling subjects that this was far from being an accurate assumption. Although some forward motion was maintained at all times, there appeared to be a cyclic acceleration and deceleration of the subjects' bodies during the movement cycles. There were obvious significant horizontal forces that could not be ignored in any model developed to describe the human quadruped motion sequence.

Alexander and Jayes developed equations describing the energy expenditures of quadrupeds and bipeds [1,4,6]. To describe the horizontal forces exerted by feet during locomotion, they developed an empirical relationship between the horizontal forces and vertical forces that closely approximates the results of force data records. The relationship is of the form

$$F_{xi} = C_{xi} T_c F_{zi} \tag{2.12}$$

where $$T_c = [\eta - (\zeta_i - \beta_i)/2]$$

when $$\zeta_i - \beta_i \leq \eta \leq \zeta_i$$

These horizontal forces will affect only the forward velocity, the yaw angle, and the pitch angle of the human model body.

The obvious reason for a subject to move with such an unsteady velocity is to use horizontal forces as a means to reduce pitching during the movement cycle. Yaw is effectively reduced to zero, if the horizontal forces were zero. Therefore, reduction of yaw angles cannot be the primary concern of the subject or the reason for horizontal force exertions of the feet.

The empirical function described above results in a negative value for the horizontal force in the first half of a leg support phase with a minimum in the first quarter.

Conversely, the second half of the support phase has a positive with a maximum in the final quarter of the support phase. This seems to support observations of videotape records in which the body of the crawler decelerates at the beginning of the support phases and accelerates near the end.

Lateral Forces

From the information gleened from previous papers regarding force data, the lateral forces F_{yi} exerted by the legs of walking animals were very small in magnitude compared with the vertical and horizontal forces [1,2,3,4,5,6,17]. Considering this evidence, the lateral forces were considered negligible for forward straightline locomotion.

Calculation of Displacements and Orientations From Acceleration Equations

Earlier in this chapter, the second derivatives of the position and orientation coordinates of the quadruped body were defined in Equations (2.1-2.4). The equations were defined in terms of the forces exerted on the quadruped feet and the position of the feet relative to the body C_g. Remember that the forces exerted by the ground on the legs, therefore also the moments generated due to the forces, are defined during the support phase of the cycle by a trigonometric series and are effectively zero during the movement phase. This presents a major problem when trying to integrate over an entire cycle period to determine the anti-derivatives and calculate the position and orientation vectors. Either a different set of equations must be derived for each combination of support conditions during the cycle or some numerical method must be used to integrate the acceleration functions.

Alexander and Jayes solve this problem by writing the acceleration equations in the form of a Fourier series [4,17]. They then integrate the sum of the series twice to obtain an infinite series for the displacement values. They can then find the linear and angular positions of the quadruped body at arbitrary times by summing the terms of the series at the specified times. They use the first 50 terms of the series at each time step. Although this method can produce results accurate to a large mumber of significant digits, it requires 50 evaluations of a complicated series function at each time step.

Rather than using this elegant, but also computationally intensive, method of solving the problem of the discontinuous force functions, methods of numerical integration were investigated. The simplest method that can be used is derived from the trapezoidal rule which states that

$$\int_{x_1}^{x_h} f(x)dx = h(f_1+2f_2+2f_3...+2f_n+f_{n+1})/2 \qquad (2.13)$$

where $h = (x_h-x_1)/n$

and $\qquad n$ = desired number of integration steps

with a estimated global error of

$$\text{Error} = -(x_h - x_1)h^2 f''(\varepsilon)/12 \qquad (2.14)$$

where $\qquad x_1 \leq \varepsilon \leq x_h$

over an integration interval [13]. This error estimation allowed the actual error to be bracketed which was an aid in choosing a suitable interval for the numerical integration. For the results that will be presented later, n was chosen to be 100. Since the integration interval is τ, this would result in an error estimation of

$$\text{Error} = -8.333 \cdot 10^{-6} \tau f''(\varepsilon) \qquad (2.15)$$

where $\qquad 0 \leq \varepsilon \leq \tau$

for each integration. This accuracy was considered sufficient for the purpose of describing gross motion characteristics of the quadruped.

The values for the accelerations at each time step were calculated and these accelerations were integrated twice using the trapezoidal rule. This yielded estimates of the displacement and orientation coordinates of the quadruped's body at each time step value.

Maintaining Function Continuity

Care had to be taken during displacement calculations to maintain proper mean values of the accelerations, velocities, and position coordinates over the cycle interval. These mean values are chosen such that there is a cyclic continuity of motion over multiple cycle periods.

The mean values that need to be maintained are

$$d^2Z_m/dt^2 = dZ_m/dt = 0 \tag{2.16}$$

$$Z_m = Z_b \tag{2.17}$$

$$d^2X_m/dt^2 = 0 \tag{2.18}$$

$$dX_m/dt = V_x\tau/\ell \tag{2.19}$$

$$d^2\phi_m/dt^2 = d\phi_m/dt = \phi_m = 0 \tag{2.20}$$

$$d^2\theta_m/dt^2 = d\theta_m/dt = \theta_m = 0 \tag{2.21}$$

$$d^2\psi_m/dt^2 = d\psi_m/dt = \psi_m = 0 \tag{2.22}$$

The mean values were preserved by first calculating the second derivatives of the displacements over the entire cycle and integrating these values twice. Then appropriate initial values were added to the values of the displacements, orientations, and their derivatives to produce the desired mean values.

CHAPTER III

PARAMETRIC STUDY OF HUMAN QUADRUPED WALKING

The last chapter dealt with development of a mathematical model that would describe quadruped walking. The parameters contained in this model that affect orientation and displacement of the quadruped's body can be split into four major classifications. The first three parameter groups are the same groups defined in the data gathering procedures. The final classification group is the set of force parameters which defines the shape and magnitude of the forces exerted by the ground on the quadruped's legs.

Using the previously defined mathematical model, the unwanted displacement and orientation values were calculated over an entire cycle period with a set of parameters which define a walking gait as the input to the procedure. After adjusting the mean values over the cycle period to the required values, the maximum and minimum deviations of the unwanted displacements and orientations from the mean values during one cycle period were determined. The basis for optimization was defined as a range of deviation from the required mean in an equation of the form

$$\text{Range} = (\text{Max}-\text{Min})/2 \qquad (3.1)$$

The optimality of one set of parameters defining a gait over another set results from a comparison of these range values. The gait producing the smaller range is designated as the better of the two.

Originally, an attempt was made to write a single optimization procedure, based on the complex search algorithm, which would converge on the optimum parameters that minimize the unwanted displacement ranges. There were several problems that thwarted this attempt. First, there is a mixture of units in the displacement and orientation values. The calculated displacements were in units of feet while the roll, pitch, and yaw angles were calculated in units of radians. Secondly, all parameters are not present in every set of displacement and orientation equations. Finally, there was very little difference in range values for several completely different sets of parameters. In other words, there is no clearcut minimum value on which the algorithm can converge. Due to these complications, the complex search algorithm did not converge to a solution at a rate that suited the needs of this study.

Since one all-encompassing optimization was not feasible, a different method of evaluation needed to be developed. Toward this end, the characteristics of each of the unwanted displacements were analyzed. The relative effects of each parameter on the displacements and orientations were classified and rated in the order of their relative importance. The parameters were then analyzed and assigned a value in the order resulting from this preliminary classification. In each case, the results of the previous parameter analysis procedures were used in subsequent parameter determinations.

Simplifying Observations

Even though the equations that described movement of the quadruped's body are in a simplified form, there are still 38 parameters in these equations that affect the magnitude and shape of the motion curves. Obviously, the optimization of such a large number of parameters would be an exhaustive procedure to undertake. The optimality being a minimization of unwanted displacements and orientation angles of the quadruped's body. Fortunately, analysis of the data resulting from measurements of actual human performance as well as the experiences of past researchers dealing with similar subjects leads to assumptions that reduce the complexity of parameter optimization to managable proportions.

Gait Timing Observations

There are nine timing parameters: β_i and ζ_i for each of the four quadruped legs and the cycle period τ. Optimization of these parameters alone would be a formidable task owing to the nonlinear relationship between these parameters and the unwanted displacements. There were two assumptions made that affected the timing parameters. The first was based on the results of the timing measurements contained in Table II. From these timings, a marked similarity between the forefeet duty factors (legs 1 and 2) and the hindfeet duty factors (legs 3 and 4) can be recognized. With Table II as evidence, it was assumed that

$$\beta_1 = \beta_2 = \beta_f \tag{3.2}$$

$$\beta_3 = \beta_4 = \beta_h \tag{3.3}$$

and

$$\mu = \beta_f/\beta_h \tag{3.4}$$

for the purposes of optimization.

Jayes and Alexander found in their studies of chelonians that the optimum time delay factors occurred when the cycle of forefeet and hindfeet pairs were a half cycle out of phase with each other. Although this reduces the possible leg movement order to two possibilities, 1423 and 1324, there are an infinite number of combinations of these two leg movement orders that could be investigated. An assumption of this type can be justified, if a careful look is taken at Equations (2.2) and (2.3) and the known information about the dimensional parameters are used.

The coordinate axes that were defined earlier in this study ensure that Y_1 and Y_3 will always be positive and Y_2 and Y_4 will always be negative values. Referring to Equation (2.2), there will be a net cancellation of the moment exerted on the crawler body about the x axis if either the fore and hind leg pairs (1 and 2, 3 and 4) or the diagonal pairs (1 and 4, 2 and 3) have similar time delay values. One of these pairings can be dismissed by considering Equation (2.3). The value of the position coordinates X_i will intuitively be positive for legs 1 and 2 and negative for legs 3 and 4 during most of the motion cycle. As a result, moments about the y axis due to the vertical forces F_{zi} will tend to cancel one another if either left and right leg pairs (1 and 3, 2 and 4) or the diagonal leg pairs have similar relative phases. Only the diagonal pairings will provide partial cancellation of moments about both the y and x axes. Referring to the definition of quadruped walking, the two diagonal pairs will have to move out of phase with each other a sufficient amount such that at least two legs maintain contact with the ground at all times. Thus, leg 2 is assumed to be out of phase relative to leg 1 and leg 3 is assumed to be out of phase relative to leg 4 the amount of a half cycle. The resulting relationships were of the form

$$\zeta_2 = 0.5 \tag{3.5}$$

$$\zeta_3 = \zeta_2+\gamma \tag{3.6}$$

and

$$\zeta_4 = \gamma \tag{3.7}$$

This is the form used in all future procedures.

With these two assumptions, the timing parameters chosen for optimization were the parameters β_f, μ, γ, and τ.

Quadruped Dimension Observations

There are 12 dimensional parameters that have an impact on the calculated roll, pitch, yaw, and vertical displacements of the quadruped body. These parameters are Y_i and λ_i for each of the four legs, the body mass, and the mass moments of inertia of the body about the major axes. Y_i, m, the mass moments of inertia, and $\Delta\lambda$ are constants that are related to the limb lengths and physical characteristics of the individual quadruped. For this reason, these parameters were not considered to be variable parameters that were available for optimization. The values of these parameters, used in displacement calculations, were the mean values determined in the measurement procedures or were determined by other means that will be discussed later.

This leaves only the relationship of the λ's to $\Delta\lambda$ left to determine. Since the limb lengths are of similar length for the fore and hind foot pairs a reasonable assumption is that

$$\lambda_f = \lambda_1 = \lambda_2 \tag{3.8}$$

and

$$\lambda_h = \lambda_3 = \lambda_4 \tag{3.9}$$

and can be related by defining a parameter α such that

$$\lambda_f = \alpha\Delta\lambda \tag{3.10}$$

and

$$\lambda_h = (\alpha-1)\Delta\lambda \tag{3.11}$$

reducing the number of variable dimensional parameters to one.

Force Constant Observations

Due to the lack of force records for quadruped human locomotion, assumptions that other authors have made concerning the 16 force parameters were used in this study.

Determination of C_{zi} has already been discussed at some length earlier. Using information from Alexander and Jayes' investigations of other quadruped gaits, the values for Q_i and C_{xi} are assumed to be equal for each of the four quadruped legs [4,17]. Similarly, the absolute values of R_i are assumed to be equal for all legs, but with a negative sign for the forelegs and a positive value for hindlegs. These assumptions leave three force parameters that can be varied independently to gage the effects on human quadruped motion.

Mixed Parameter Observations

The parameter V_x is a mixture of the timing and dimensional parameters as discussed earlier. No assumptions were made that affected the variable nature of this parameter.

After all assumptions were made, nine parameters were left that could be varied independently in the optimization procedures that follow.

Classification of Parameters

This procedure began with the identification and the comparison of the proportionality constant values contained in Equations (2.1-2.4). The first constants determined were the mass moments of inertia of the quadruped model's body about the major axes. In earlier studies of animal gaits, the moments of inertia of a subject's body were determined by suspending the subject as a bifilar pendulum and measuring the natural frequencies. Human subjects, however, expressed no wish to repeat this technique. Instead, approximate values for the mass moments of inertia were calculated using the results of cadaver studies by earlier groups [9,10,12]. These researchers presented their results in the form of constants N_x, N_y, and N_z such that the radius of gyration of a human body segment could be estimated as

$$K_a = N_a L \tag{3.12}$$

The moment of inertia of the body segment about the a axis could then be estimated as

$$I_{aa} = mK_a^2 \tag{3.13}$$

where m = body segment mass

and L = body segment length

This method was used to calculate assumed values for the moments of inertia of the torso, neck, and head link about the defined axes with the results

$$I_{xx} = 0.0424m\ell^2 \tag{3.14}$$

$$I_{yy} = 0.263m\ell^2 \tag{3.15}$$

and

$$I_{zz} = 0.205m\ell^2 \tag{3.16}$$

It was obvious that because of the body's small mass moment of inertia about the x axis that the roll angle would be the most sensitive unwanted displacement to parameter changes. Therefore, the roll angle range was the first unwanted displacement that was minimized. If this criteria is carried further, the yaw angle would be the next to be evaluated. However, a comparison of Equations (2.2) and

(2.4) indicates that the partial cancellation of net moment values due to the sign convention for Y_i would be similar for both roll and yaw calculations. Therefore, an optimum set of parameters that minimizes roll would also be close to an optimum for yaw.

Parameters Affecting Roll

First, all parameters that affected the roll angle range of the quadruped's body needed to be determined and classified as to their relative effect on the roll range. Starting with the moment equation about the x axis, Equation (2.2), and the function describing the vertical forces exerted by the crawler legs, Equation (2.6), the parameters β_i, μ, γ, R_i, and Q_i are obvious candidates for optimization.

Since all time parameters are normalized by the cycle period, then if the value of h used in the integration procedure is also normalized by the cycle period, a simple relationship between the roll range and τ is developed. The roll range will vary at a rate that is proportional to the square of the cycle period. Similarly, all other parameters that have a linear and therefore easily recognized relationship with the roll angle range were identified. The calculated roll range was actually a normalized value of the actual roll range by a factor such that

$$\phi_{act} = S_c \phi_{calc} \tag{3.17}$$

where

$$S_c = (m \ell \tau^2)/I_{xx} \tag{3.18}$$

This normalized roll range is the value that appears in the optimization procedures that follow.

The classification of the relative importance of each parameter to roll calculations was determined by trial and error simulations and experiences of past authors [17]. The results obtained from these trials placed β_f as the parameter most sensitive to small deviations from the optimum value. The duty factor was closely followed in order by γ, R_i, μ, and Q_i. The parameters were determined in the order indicated by the preliminary trials.

Parameters Affecting Vertical Displacement

Care had to be taken to keep the vertical displacement in check while the parameters for minimum roll were worked out. Some parameter sets that produce small roll ranges result in relatively large vertical displacement ranges. Therefore, the displacement was watched during the roll optimization procedure to ensure unacceptable displacements did not occur.

In a fashion similar to the roll range calculation, all parameters with linear relationships to the vertical displacement range were identified. The vertical displacement calculated was the actual displacement range normalized by these parameters such that

$$Z_{act} = Z_{calc}\tau^2 \qquad (3.19)$$

Since only the deviation of Z from the mean is important, the calculated Z is sufficient for analysis.

Remaining Parameters

The remaining four parameters are not contained in the equations governing roll angle or vertical displacement range. These must be determined by analysis of one of the other undesireable angular displacement ranges. Of these four parameters, only two appear in the equations governing yaw. The effects these two parameters have on yaw can easily be determined by referring to Equations (2.4) and (2.12). The two remaining parameters, C_{xi} and τ, have a simple linear relationship with the yaw range. Therefore, the yaw range is reduced to zero if either parameter approaches zero. Since a zero value of τ is physically impossible and videotape records reveal that C_{xi} does not equal zero, reduction of yaw ranges does not seem to be the primary concern of the quadruped.

Of all the undesirable displacements discussed thus far, only the equations describing the range of pitch angles contain all nine parameters earmarked for investigation. Referring to Equations (2.3), (2.6), and (2.12), it can be seen that the five parameters discussed earlier affect pitch ranges. However, C_{xi}, τ, α, and V_x are also present in the equations defining the horizontal force and horizontal displacement of the feet. The moment arm terms, X_i and Z_i, are not constants in Equation (2.3). Therefore, the actual values of X_i and Z_i, not normalized ranges, are needed for pitch calculations. For this reason, the cycle period cannot be normalized out of pitch calculations. The cycle period is needed to convert the normalized values of calculated X_i and Z_i into absolute distance values.

By a set of trial and error procedures, it was found that the pitch range was most sensitive to variances in the parameter α, followed by C_{xi}, V_x, and τ.

Determination of Parameter Relationships

As was stated earlier, the optimization of all nine parameters with one optimization procedure met with several insurmountable problems. Instead, a technique employing contour plotting was adopted. The parameters were varied and the resulting displacement ranges calculated for each set of parameters using a computer program written for a Harris 800 Super-mini computer. The listing of this program is contained in the Appendix A. The use of contour plotting allowed two parameters to be varied for each program run. The combinations of these two parameters that produced specified contour values of the displacement ranges could then be plotted (See Appendix B). By varying another parameter on each plot, the effects of a total of three parameters could be investigated at any one time. From these contour plots, relationships between parameters and equations describing the optimum combinations of parameters resulting in minimum displacement ranges were developed. The effects of the different parameters were analyzed in the order specified earlier.

Determination of Forefeet Duty Factors

The normalized roll angle ranges were calculated and contour plots were made for varying values of γ and μ with the assumption that $R_i = Q_i = 0$. A plot was made for different values of β_f. The ratio μ was confined to values that satisfy the criteria for walking type gaits. This limits the range of values that μ and β_f can take to the ranges

$$0.5 \leq \beta_f < 1.0$$

and

$$\beta_f < \mu \leq 2\beta_f$$

The γ values range from -0.5 to 0.5 which is adequate to cover all possible gaits that can be followed with the assumptions made.

Figures 5-8 illustrate the results of this plotting procedure. These plots revealed that the minimum roll ranges occurred along a curve such that

$$\gamma = (\beta_h - \beta_f)/2 = \beta_f(1/\mu - 1)/2 \tag{3.20}$$

for all values of β_f between 0.5 and 1.0.

Using Equation (3.20) to determine γ, the interval steps for varying β_f and μ were shortened and the minimum roll range calculated for each value of the duty factor along with the μ that the minimum occurred at. The results are contained in Table III. It is obvious from Table III that the roll ranges may be minimized when $\beta_f = 0.71$. Based on this observation, β_f is given a value of 0.71 for all future calculations.

The range of vertical displacement was plotted in a similar manner to check the effect that β_f, γ, and μ had on

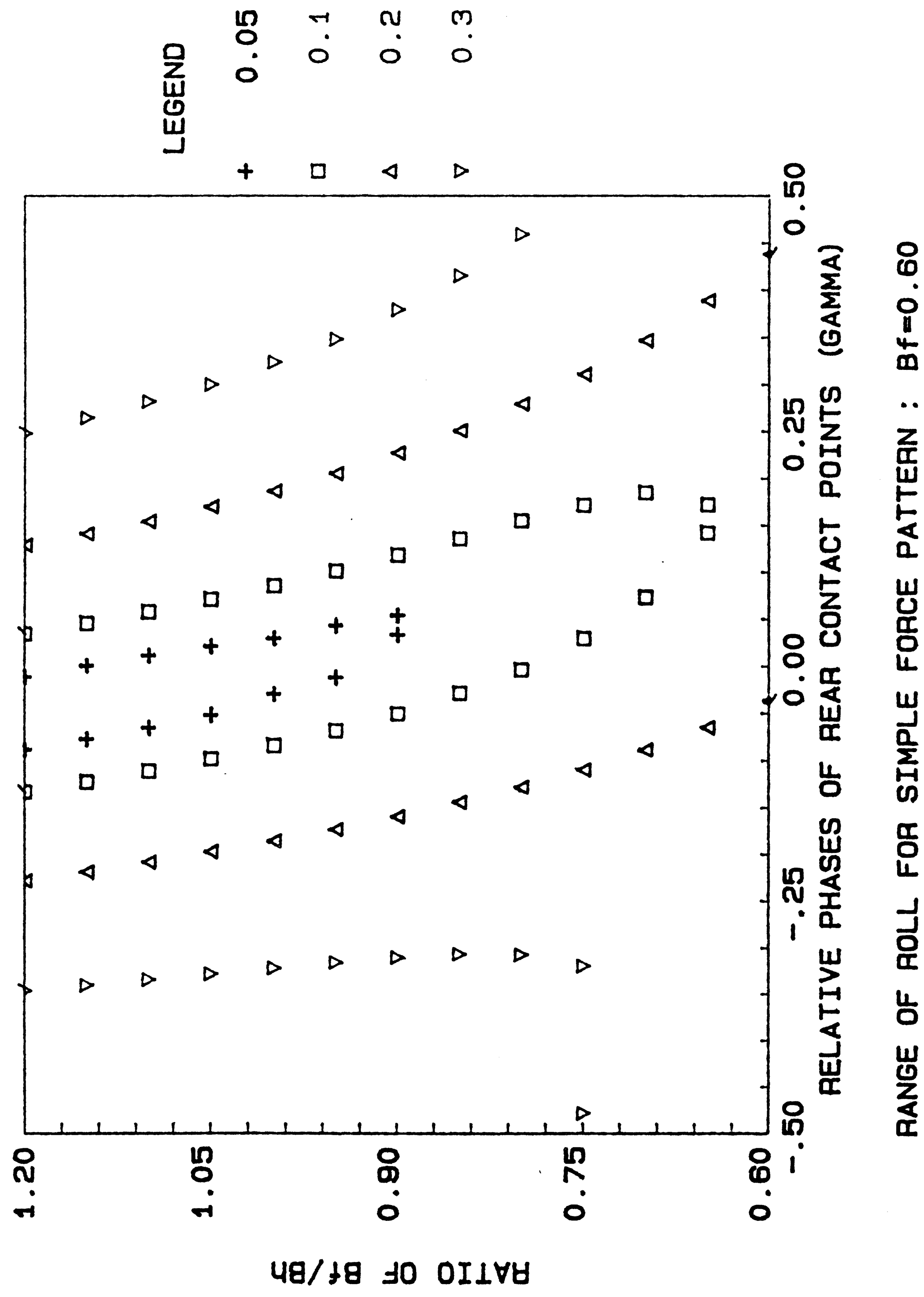


Figure 5. Roll Angle Range for β_f Study

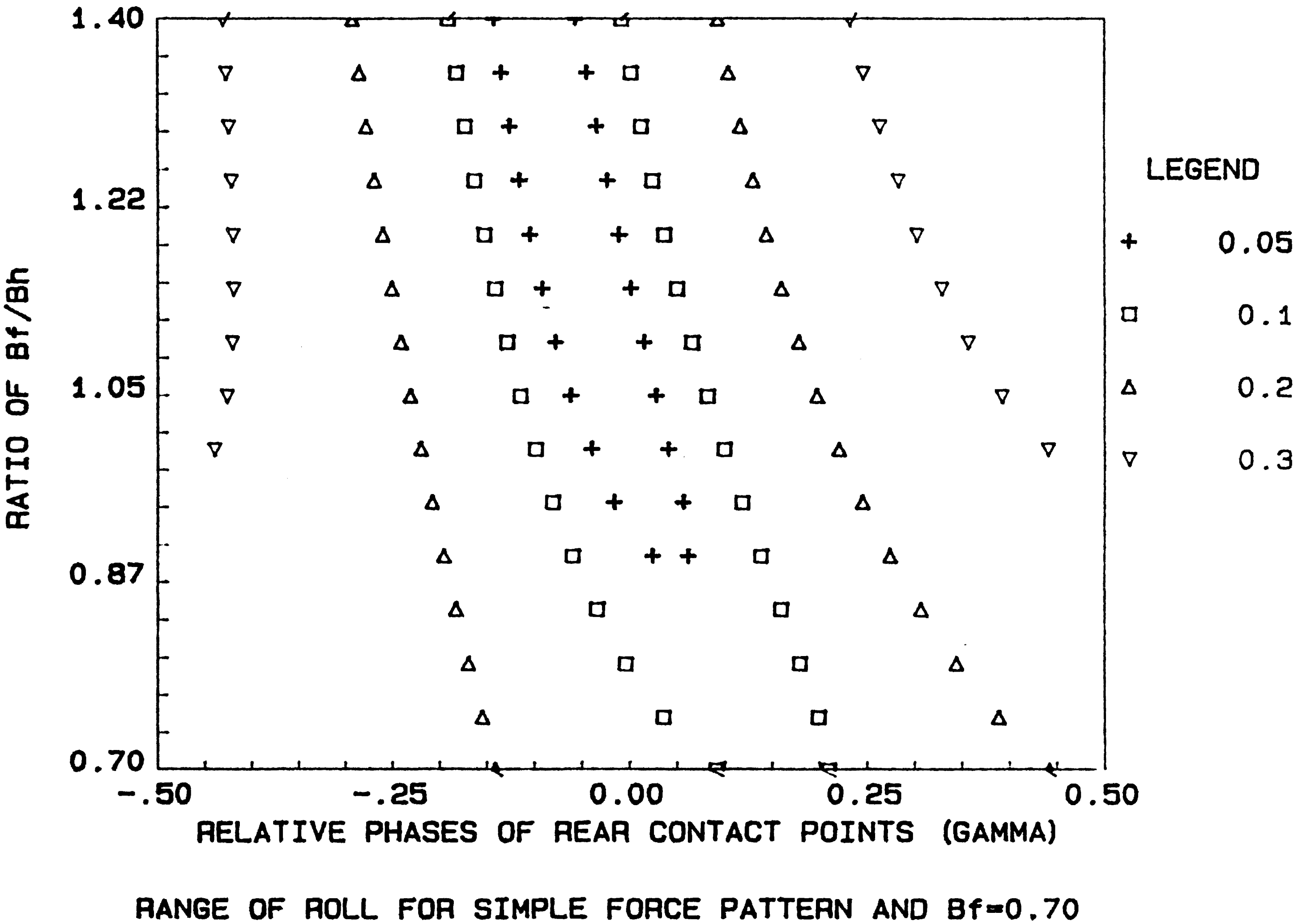


Figure 6. Roll Angle Range for β_f Study

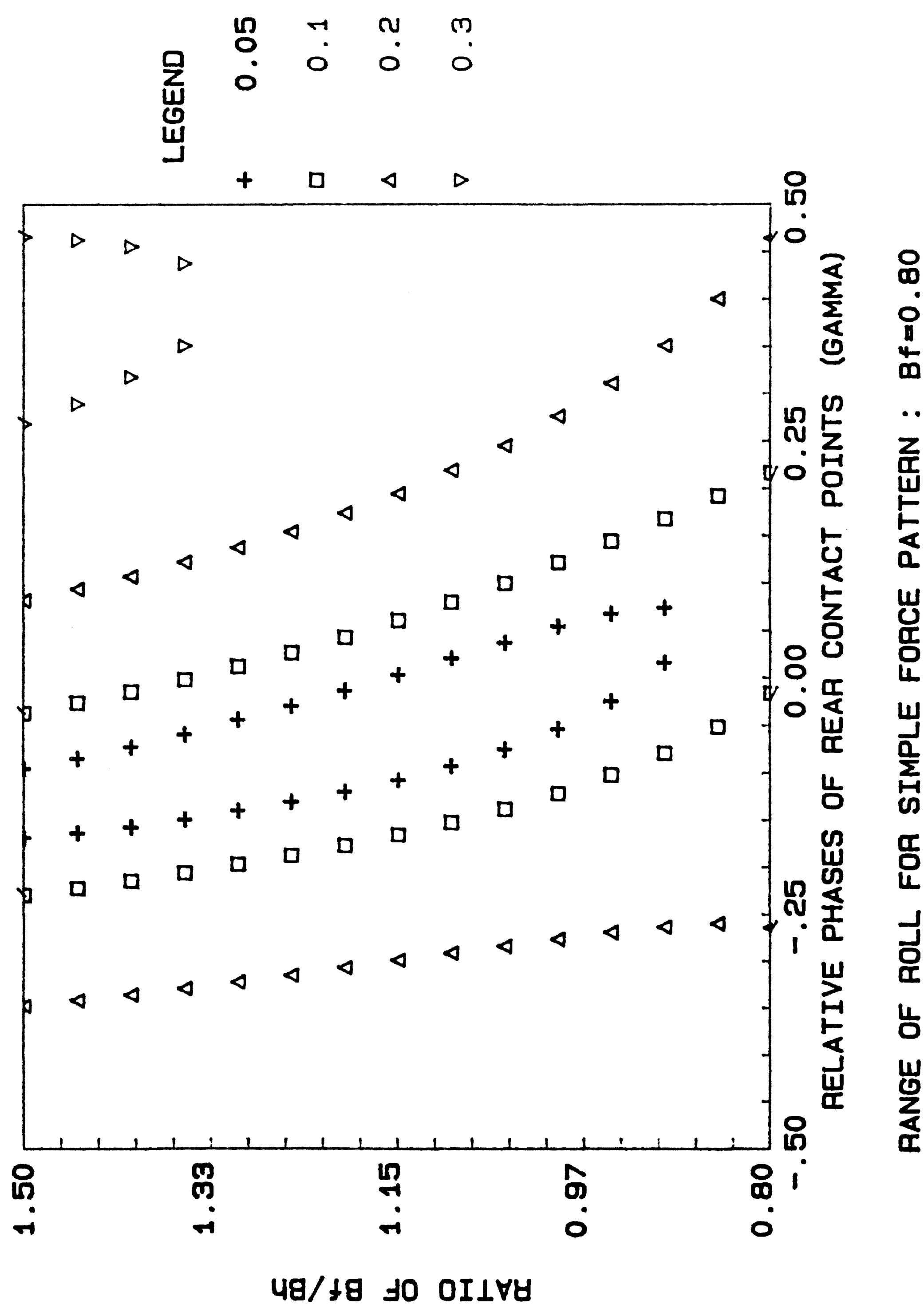


Figure 7. Roll Angle Range for β_f Study

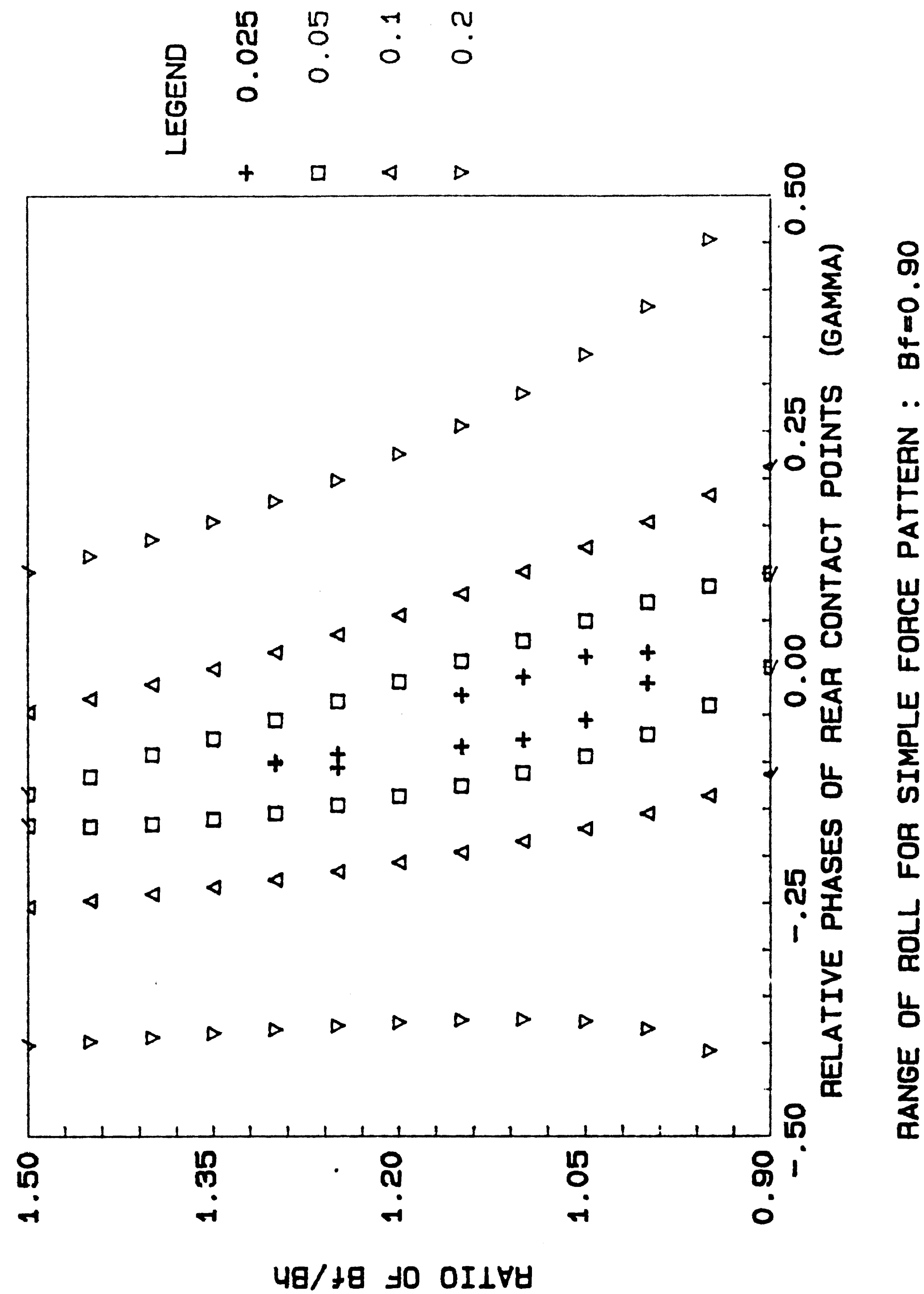


Figure 8. Roll Angle Range for β_f Study

TABLE III

MINIMUM NORMALIZED ROLL RANGES FOR VARYING VALUES OF β_f AND THE CORRESPONDING RATIO μ

β_f	μ	ϕ range($\times 10^{-3}$)
0.66	1.278	1.715
0.67	1.265	1.289
0.68	1.253	0.997
0.69	1.240	0.692
0.70	1.230	0.504
0.71	1.218	0.425
0.72	1.208	0.531
0.73	1.198	0.640
0.74	1.190	0.789
0.75	1.183	0.903
0.76	1.175	0.973
0.77	1.168	0.999
0.78	1.163	1.124
0.79	1.155	1.158
0.80	1.150	1.163

it. Figures 9-12 contain the results. If the plots of roll and vertical displacement are overlayed, it can be seen that parameters that result in minimum roll range correspond to maximum values of vertical displacement. However, the vertical displacement range is still comparatively small for all parameter sets plotted.

Determination of the Relative Phase

Once the value of β_f was determined, the next parameter, γ, was treated similarly. The program source code was modified to calculate roll ranges for varying values of the relative phase and the vertical force skewness constant R_i while holding the force constant Q_i equal to zero. Contour plots were formed for various values of the ratio μ (See Figures 13-16). Comparison reveals that the relationship between γ and the minimum roll range was changed significantly by the addition of the skewness factor R_i, and Equation (3.20) no longer holds true. At first glance, it would appear that the relationship between γ, μ, and the minimum roll ranges is linear. However, in the analysis that follows, this last statement is proven false.

When a non-zero R_i is introduced into the vertical force functions, the force curves for each leg is skewed to the left if R_i is positive and to the right if R_i is negative. Obviously, it is this shift in the vertical forces that affects the optimum values of γ.

The key to developing the relationship between these

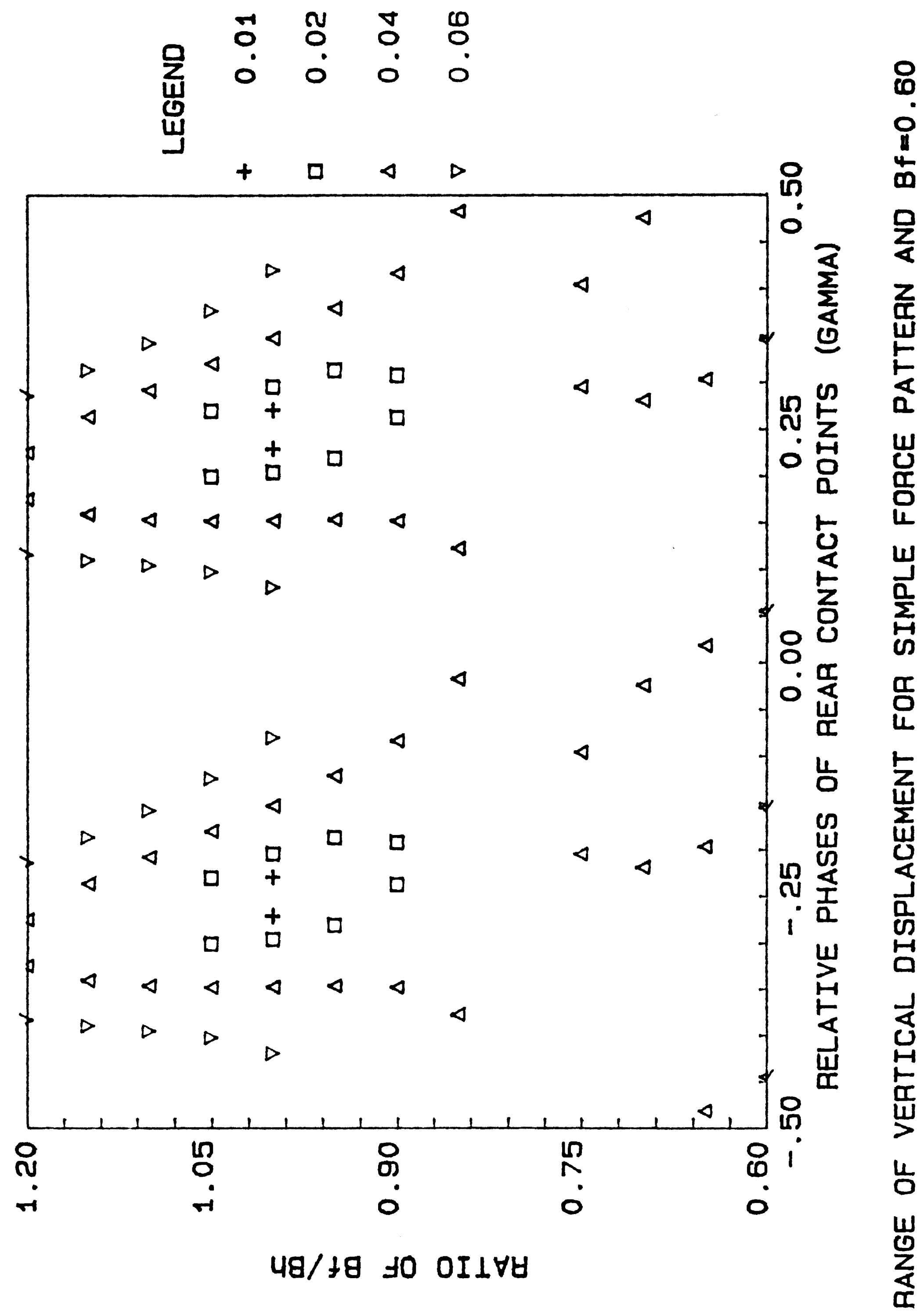


Figure 9. Z_b Displacement Range for β_f Study

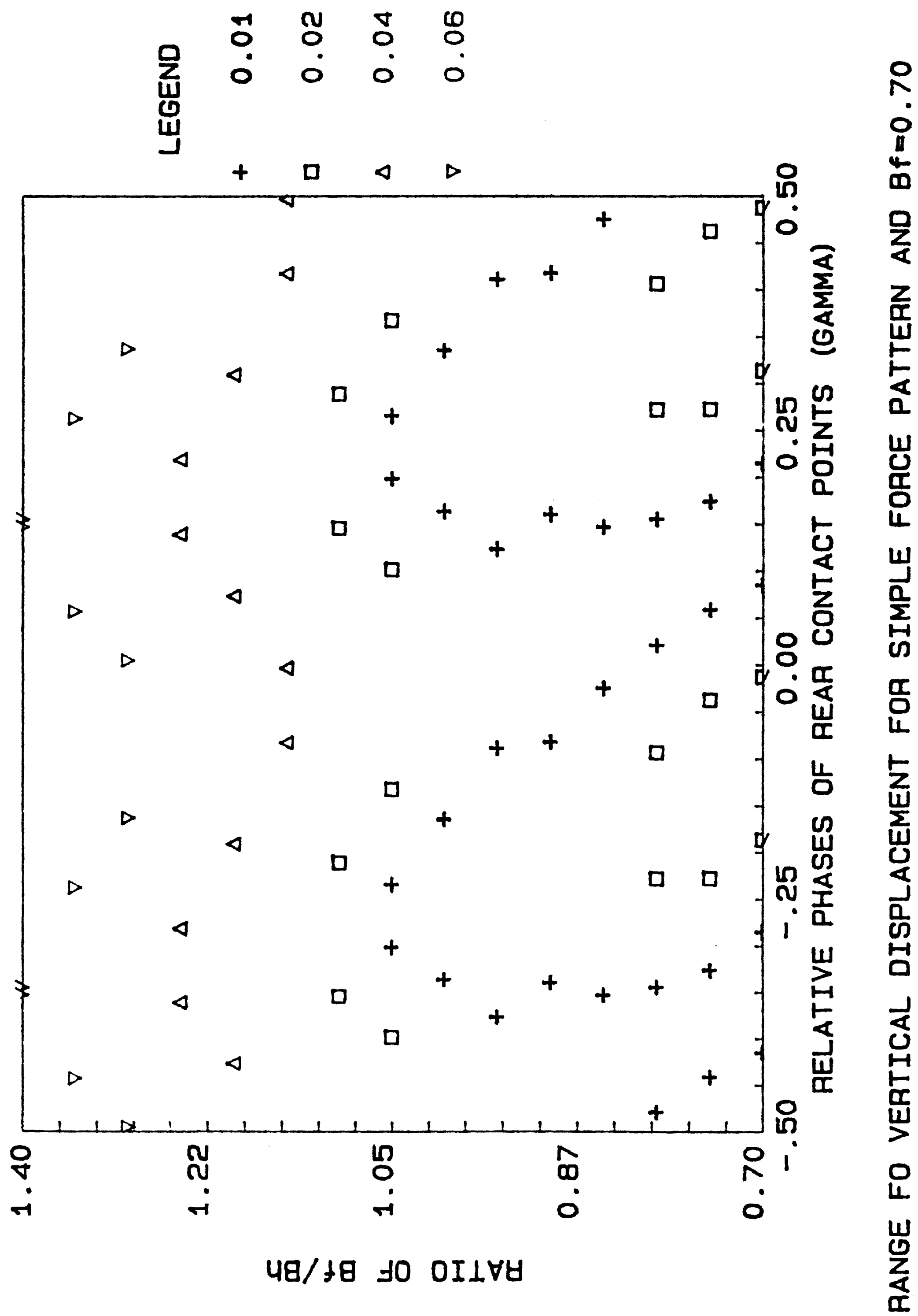


Figure 10. Z_b Displacement Range for β_f Study

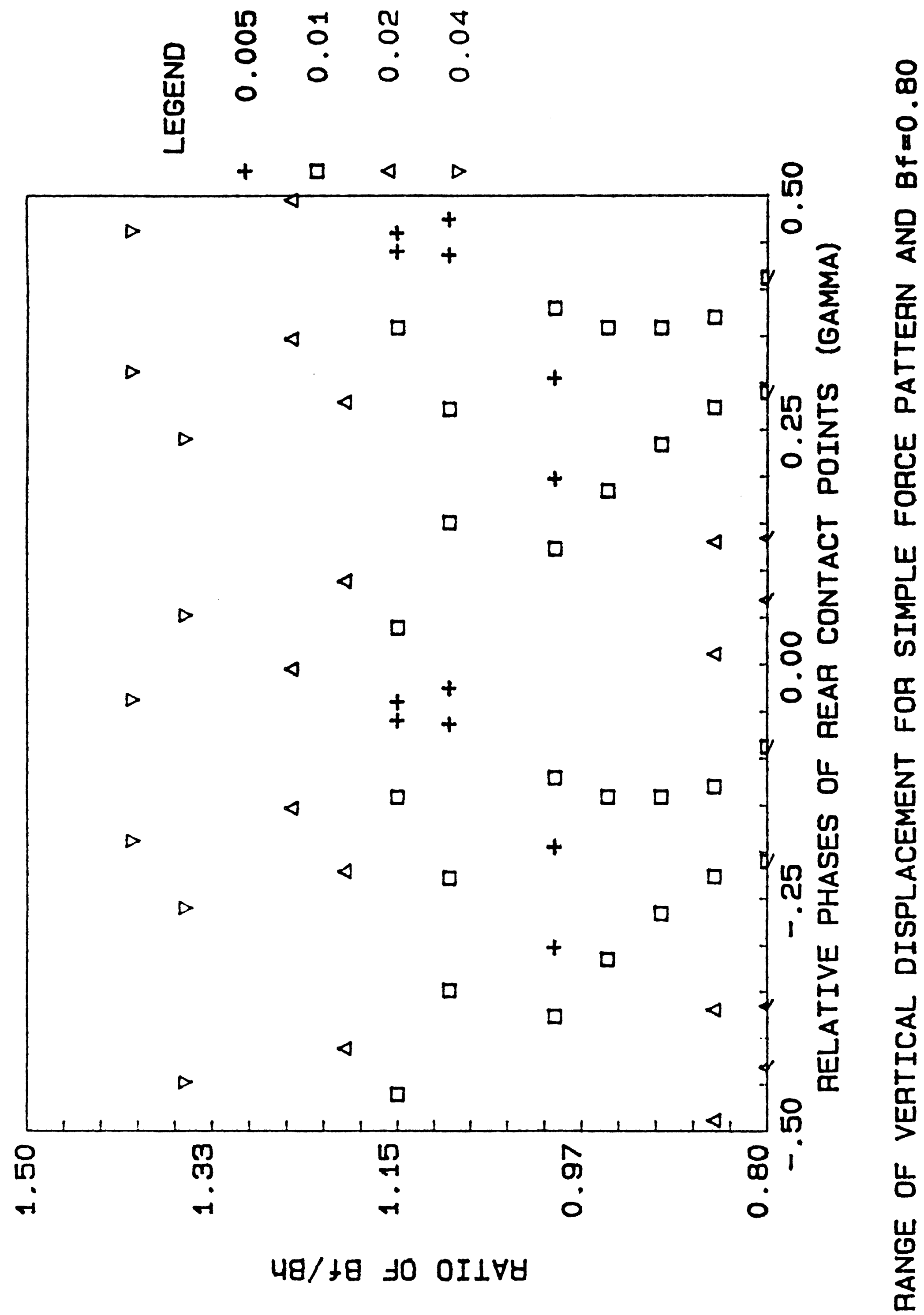


Figure 11. Z_b Displacement Range for β_f Study

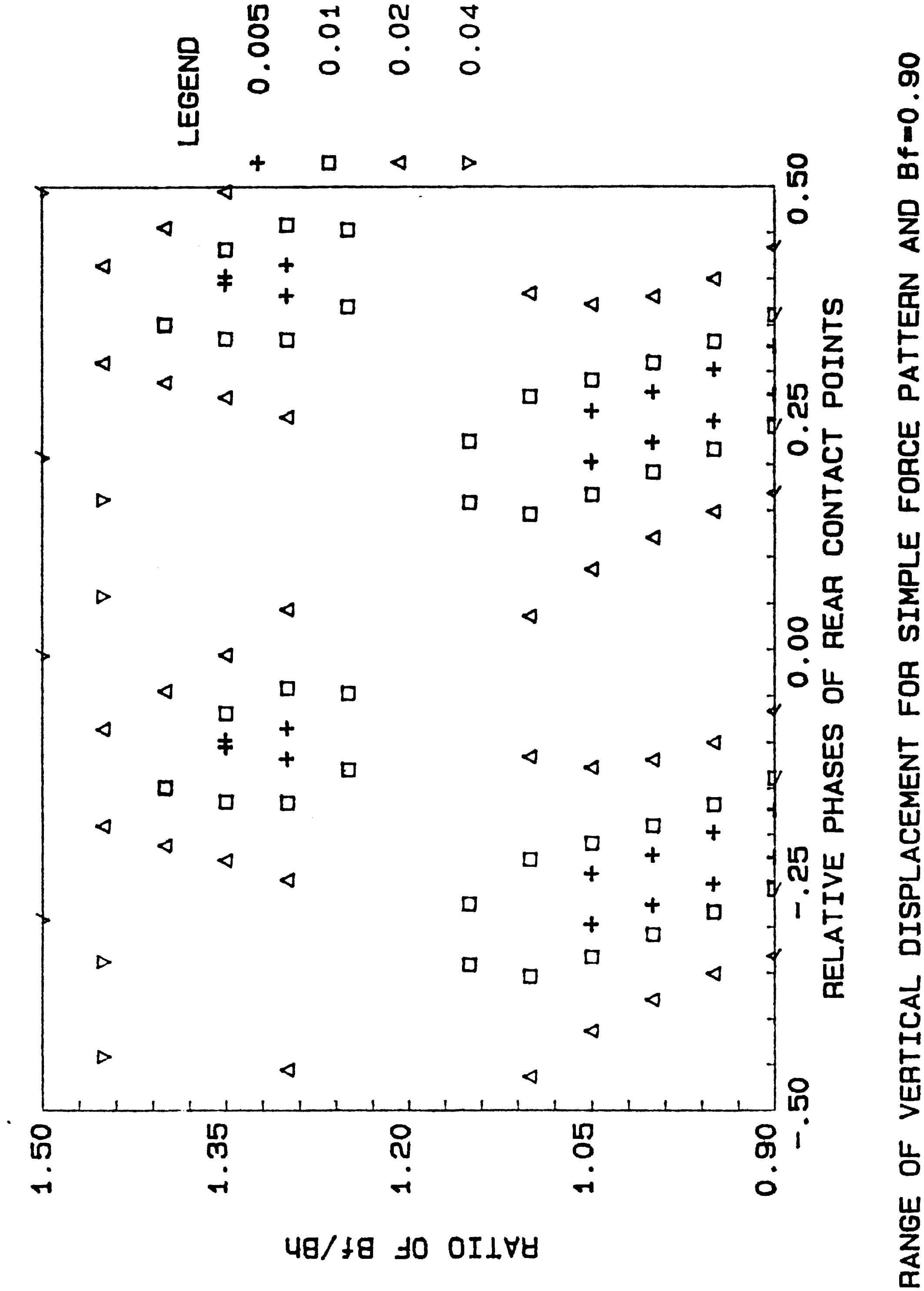


Figure 12. Z_b Displacement Range for β_f Study

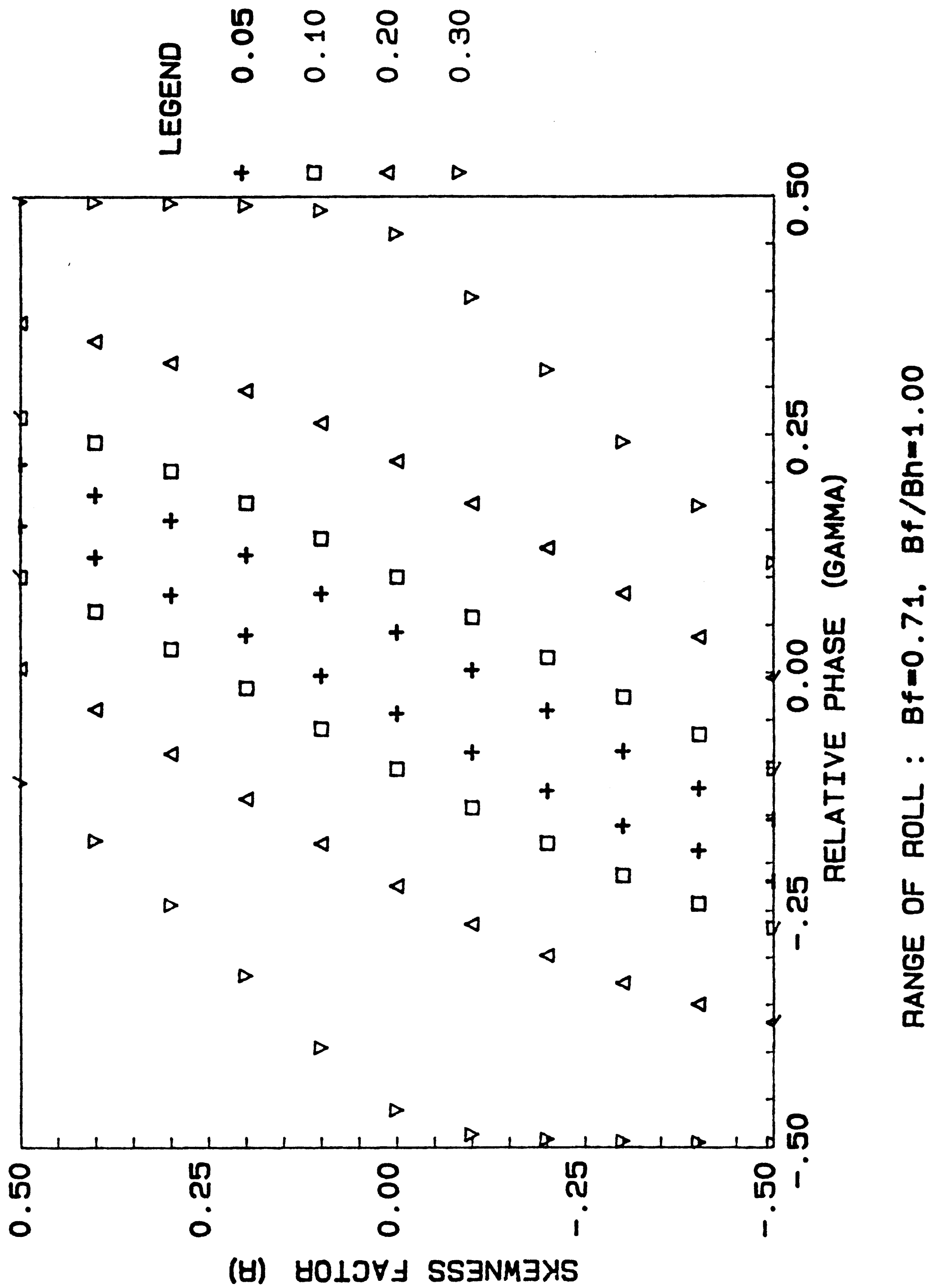


Figure 13. Roll Angle Range for γ Study

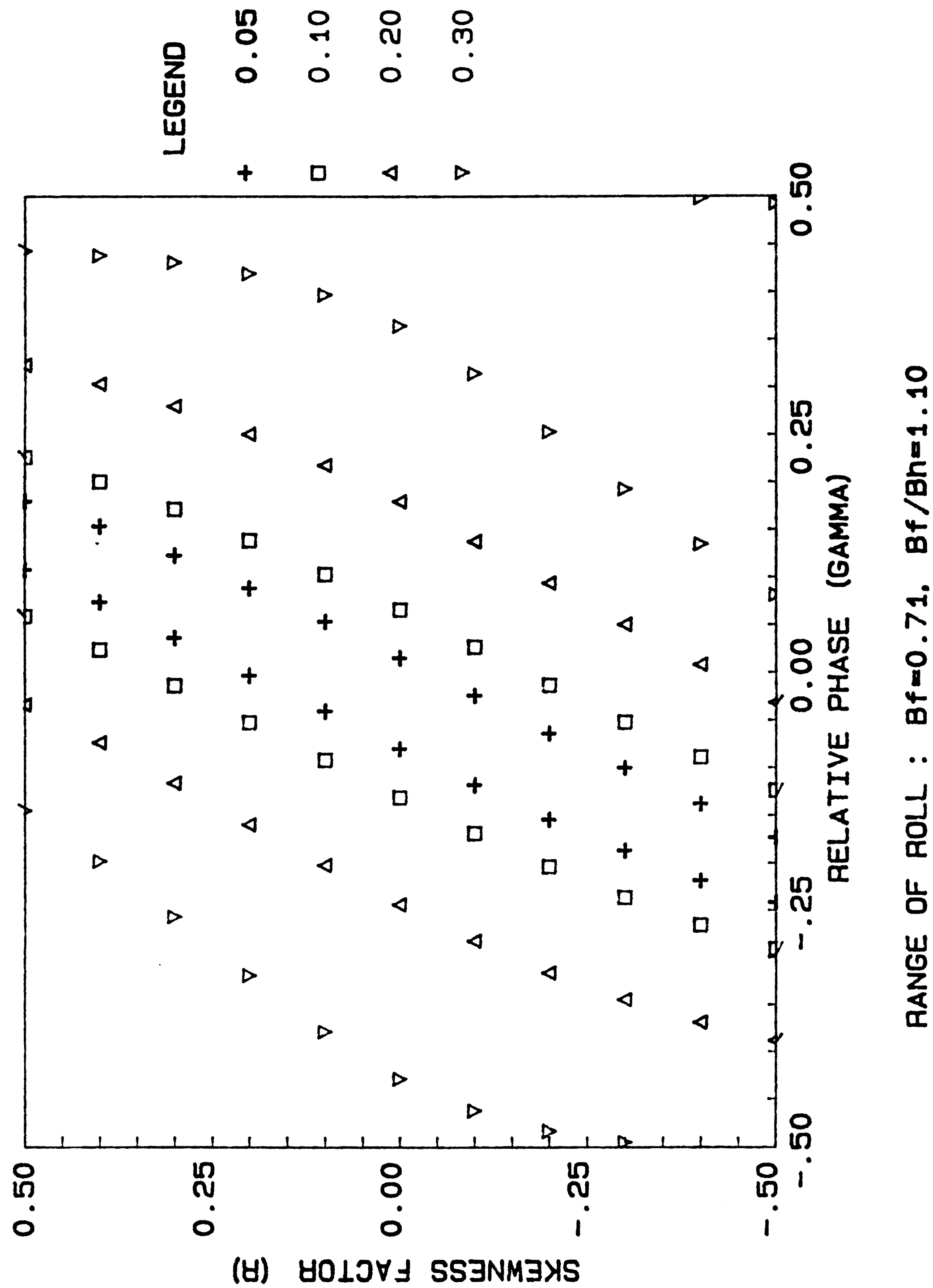


Figure 14. Roll Angle Range for γ Study

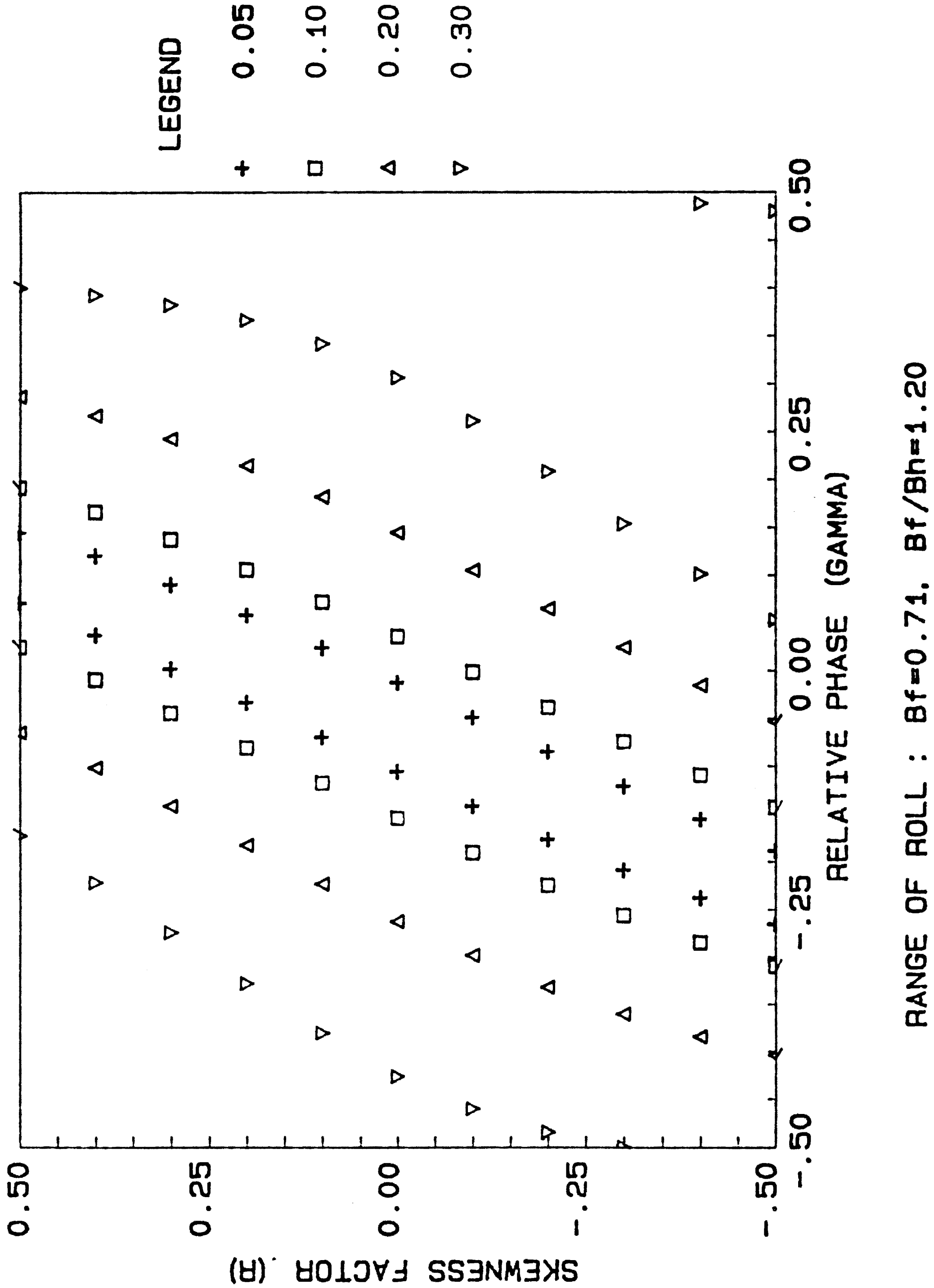


Figure 15. Roll Angle Range for γ Study

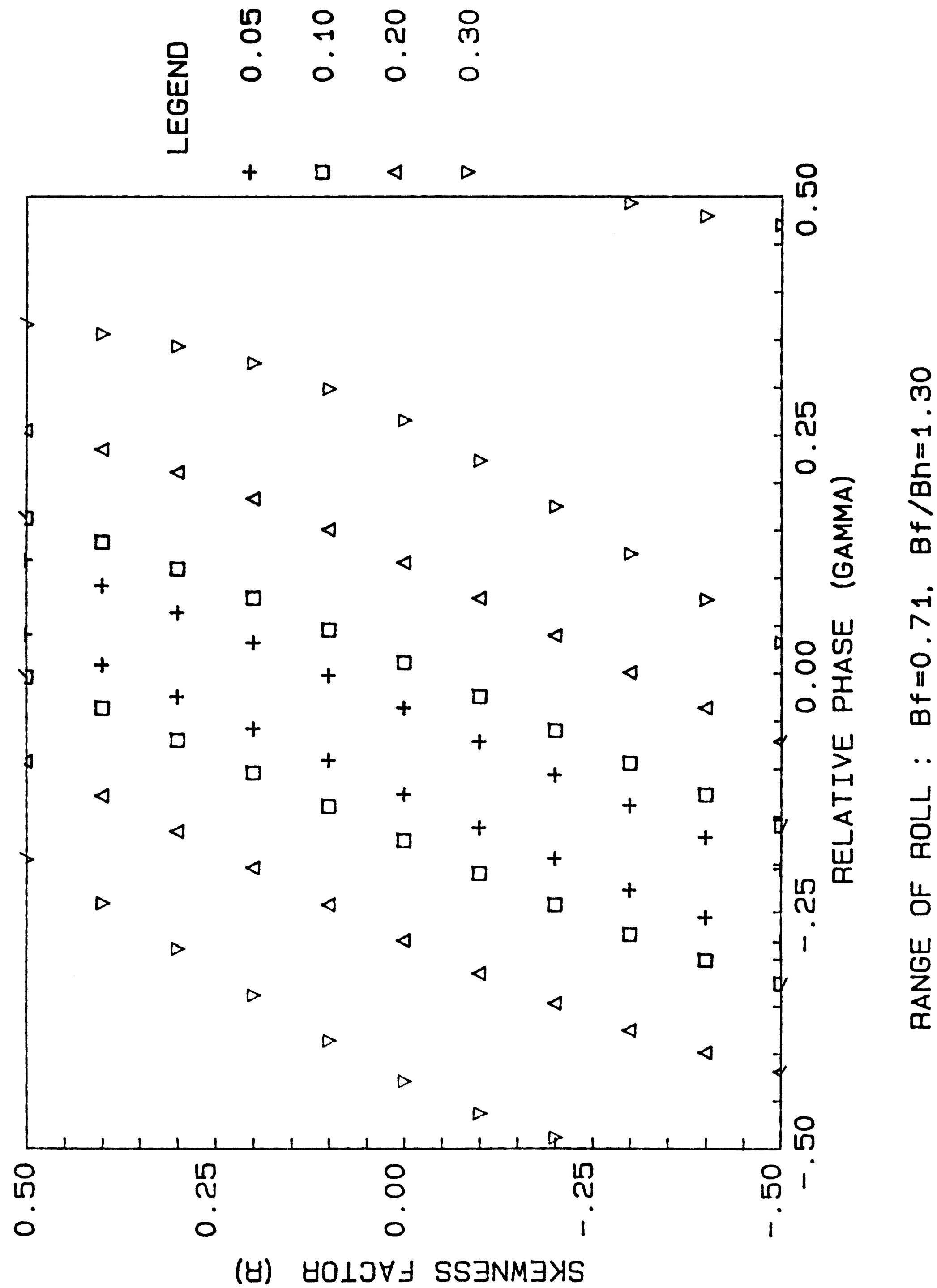


Figure 16. Roll Angle Range for γ Study

parameters is found by examining Equation (3.20) when $R_i=0$. This condition forces two events to occur at the same point in time. First, the diagonal leg pairs reach their point of maximum vertical force exertion at the same point in time. Secondly, the diagonal leg pairs reach the point when the moment impulse exerted on the body about the x axis equals one half of the total impulse exerted by the individual legs over the entire cycle at the same point in time. This second event is the event that holds and dictates the conditions for minimum roll range when R_i is non-zero.

The diagonal leg pair 1 and 4 is used for the derivation that follows. Leg pair 2 and 3 can also be used with equivalent results. Referring to Equations (2.2) and (2.6), the total moment impulse exerted by a single leg during one cycle period is $mgY_i/4$ when normalized by the cycle period. Since F_{zi} is zero unless the leg is in a support cycle, an equation of the form

$$Y_i\int_{l}^{\eta} F_{zi}\,dt = mgY_i/8 \tag{3.21}$$

where $l = \zeta_i - \beta_i$

can be written and evaluated to find the normalized point in time that the moment impulse about the x axis equals one half of the total for each leg during a cycle period. Integrating Equation (3.21) using parameters from legs 1 and 4, the result is for leg 1

$$(R_i)\cos(2\pi D_1) - \cos(\pi D_1) - R_i/2 = 0 \tag{3.22}$$

and for leg 4

$$(R_i)\cos(2\pi D_4)+\cos(\pi D_4)-R_i/2 = 0 \tag{3.23}$$

where $$D_i = [\eta-(\zeta_i-\beta_i)]/\beta_i$$

By substituting the trigonometric identity

$$\cos(2X) = 2\cos^2(X)-1$$

Equations (3.22) and (3.23) become for leg 1

$$\cos^2(\pi D_1)-(1/R_i)\cos(\pi D_1)-1 = 0 \tag{3.24}$$

and for leg 4

$$\cos^2(\pi D_4)+(1/R_i)\cos(\pi D_4)-1 = 0 \tag{3.25}$$

from which the cosine terms can be found using the quadratic equation. From experience, it was found that only one of the two roots found from the quadratic equation made physical sense. The value of D_i can then be found from the root as

$$D_i = (1/\pi)\cos^{-1}(\text{root}) \tag{3.26}$$

Recalling the definition of D_i and Equations (3.4) and (3.7), by substitution the relative phase becomes

$$\gamma = D_1\beta_h-D_4\beta_f \tag{3.27}$$

Checking the equations that resulted against the plotted information, this definition for γ agrees exactly with the optimum γ curve on the contour plots. Therefore, γ_{opt} is defined as the value which relates the moment impulse about the x axis of diagonal leg pairs as stated above.

Determination of the Force Skewness Constant

The skewness factor R_i was determined using contour plotting techniques similar to those used to define the relationship between γ and the roll angle. The contour plots were created by varying the skewness factor R_i and the vertical force proportionality constant Q_i. A separate plot was made for selected values of μ.

However, before these plots could be constructed the relationship between γ and the moment impulses of the legs needed to be rederived for non-zero values of Q_i. Solution of the integral Equation (3.21), Equations (3.22) and (3.23) became for leg 1

$$\cos(\pi D_1)-(R_i/2)\cos(2\pi D_1)+(Q_i/3)\cos(3\pi D_1) + R_i/2 = 0 \quad (3.28)$$

and for leg 4

$$\cos(\pi D_4)+(R_i/2)\cos(2\pi D_4)+(Q_i/3)\cos(3\pi D_4) + R_i/2 = 0 \quad (3.29)$$

Substitution of the trigonometric identities

$$\cos(2X) = 2\cos^2(X)-1$$

and $$\cos(3X) = 4\cos^3(X)-3\cos(X)$$

then these equations become for leg 1

$$W^3-(3R_i/4Q_i)W^2+[(3-3Q_i)/4Q_i]W+(3R_i/4Q_i) = 0 \quad (3.30)$$

and for leg 4

$$W^3+(3R_i/4Q_i)W^2+[(3-3Q_i)/4Q_i]W-(3R_i/4Q_i) = 0 \quad (3.31)$$

where $$W = \cos(\pi D_i)$$

The three roots to the cubic Equations (3.30) and (3.31) were found using an iterative method called Bairstow's Method [13]. This method was chosen because it is stable for all equations having at least one real root, is computationally efficient, and is mathematically simple. Using Bairstow's Method, a real root of the cubic equations is found very quickly. When this real root is factored out of the original cubic equation, the final two roots can be easily found from the remaining quadratic equation. From experience, it was found that only one of these roots has an absolute value of less than one, a requirement for W to make mathematical sense as a cosine function. The remaining roots were either complex or did not satisfy criteria. After the solutions were found, Equations (3.26) and (3.27) were used to calculate γ_{opt} as before.

The results of the plotting routines are contained in Figures 17-21. It is evident from these plots the the value of the roll angle range will be minimized at values of R_i that fall on the $R_i=0$ axis. Therefore, R_i was assigned the value of zero for all future calculations.

Determination of the Vertical Force Constant and the Duty Factor Ratio

Again, the analysis procedure began with the construction of contour plots. This time the remaining two parameters that affect roll angle range, Q_i and μ were varied, within previously defined limits. The resulting plot is shown in Figure 22. Obviously, there is a mathematical relationship describing the combination of these parameters that produces a minimized roll value. This relationship, however, was not readily apparent.

A similar plot was made by varying the same parameters, but with the vertical displacement range making up the contour lines, which resulted in Figure 23. The combination of parameters that produce minimum roll ranges and minimum

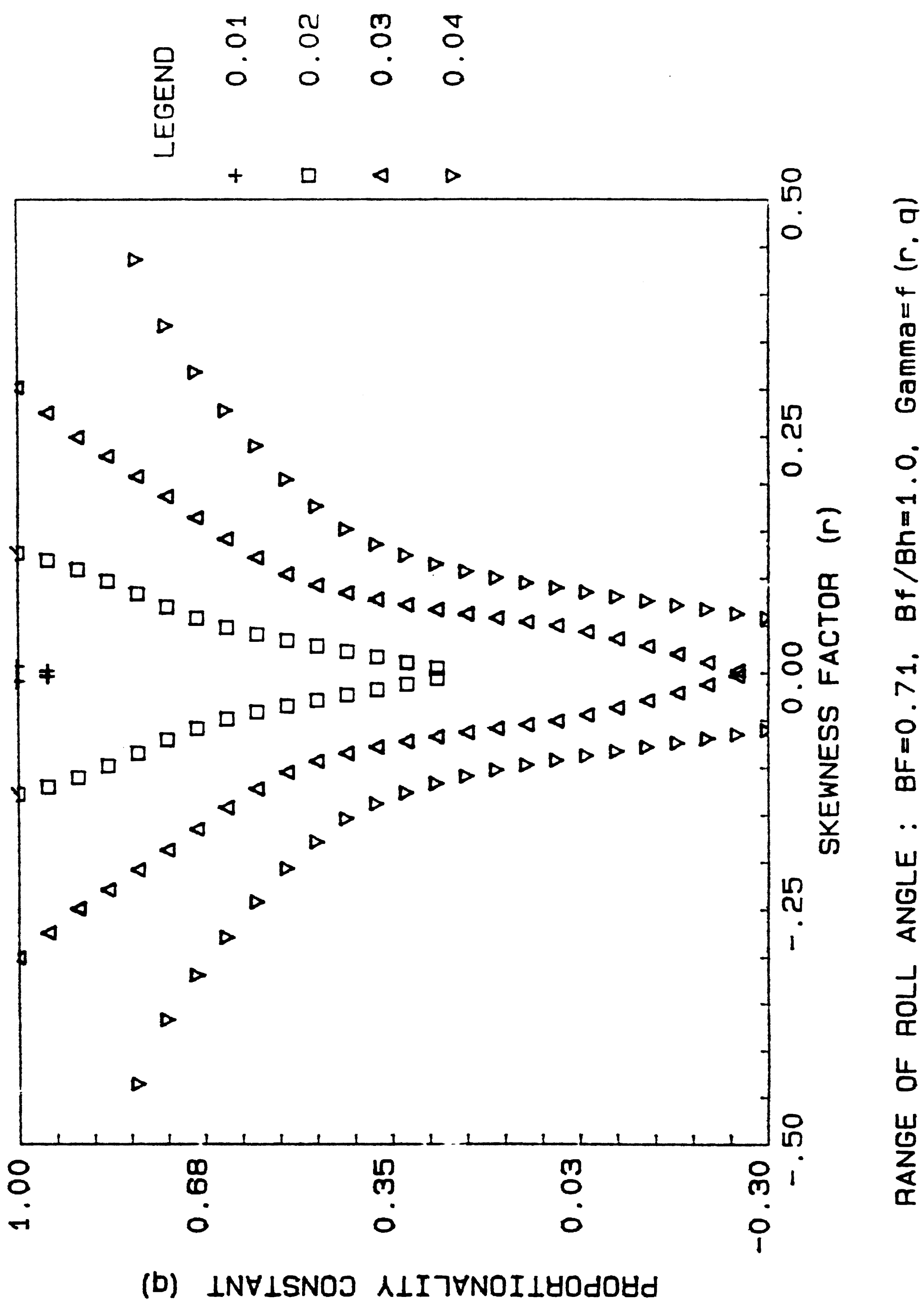


Figure 17. Roll Angle Range for R_i Study

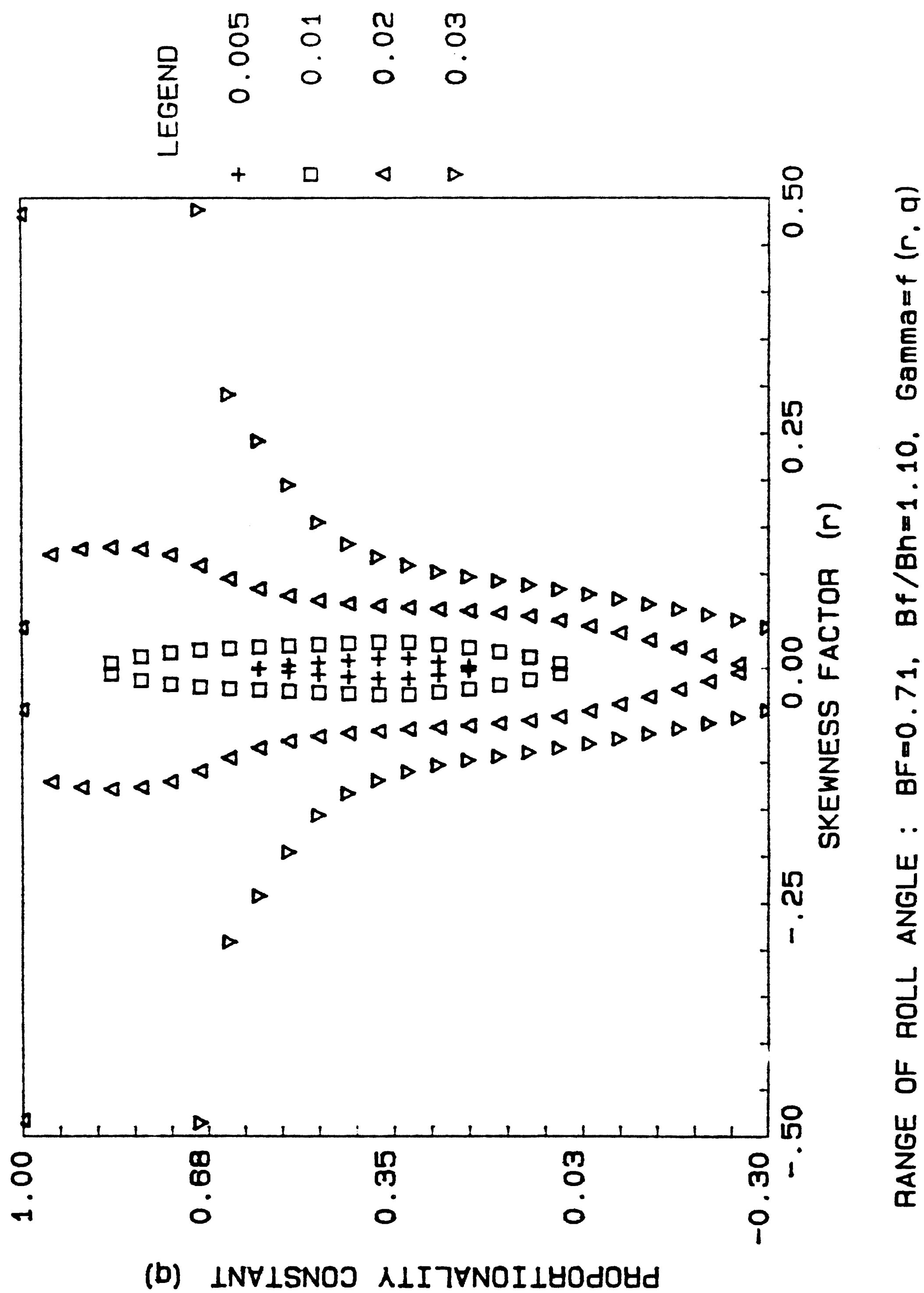


Figure 18. Roll Angle Range for R_i Study

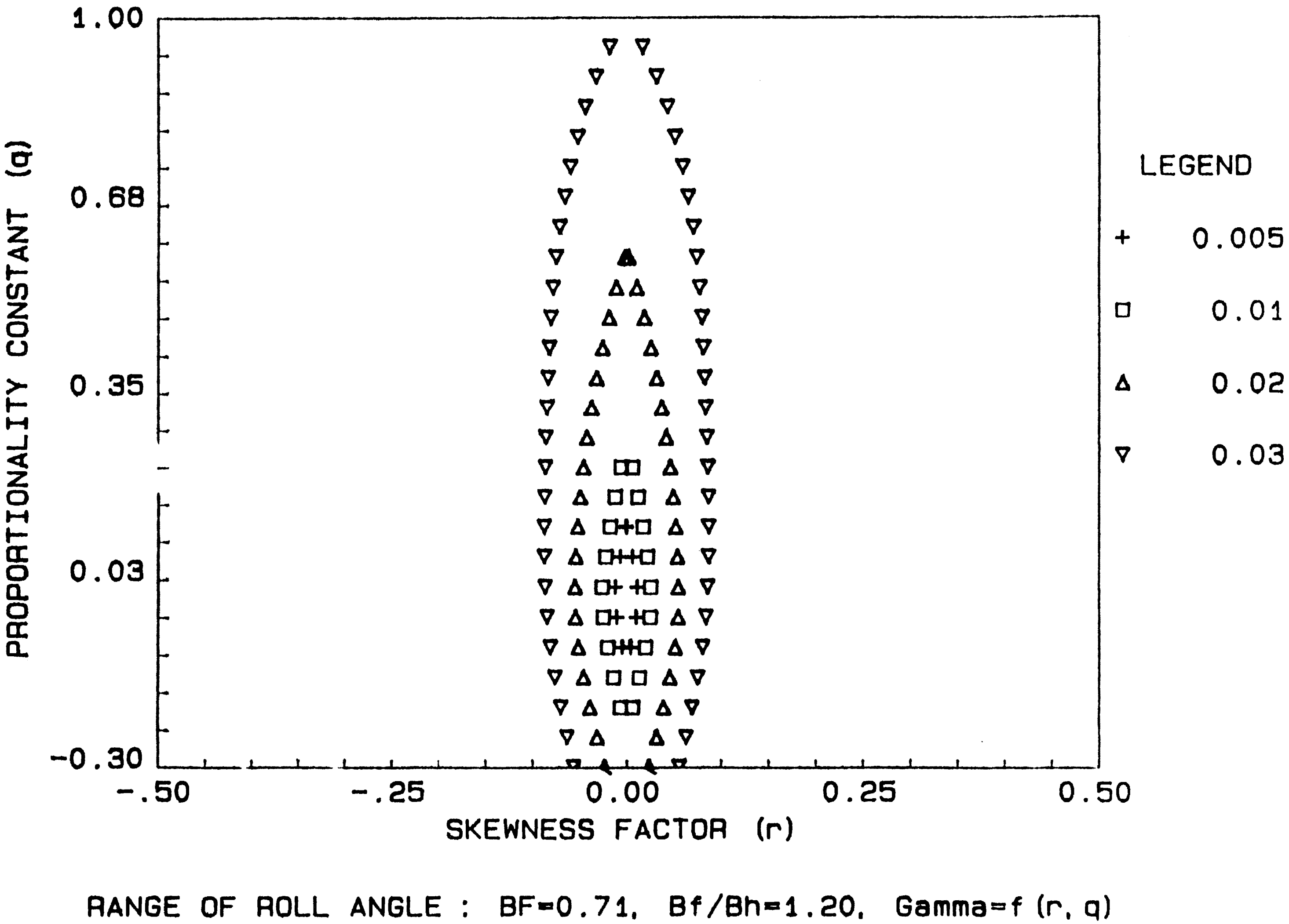


Figure 19. Roll Angle Range for R_i Study

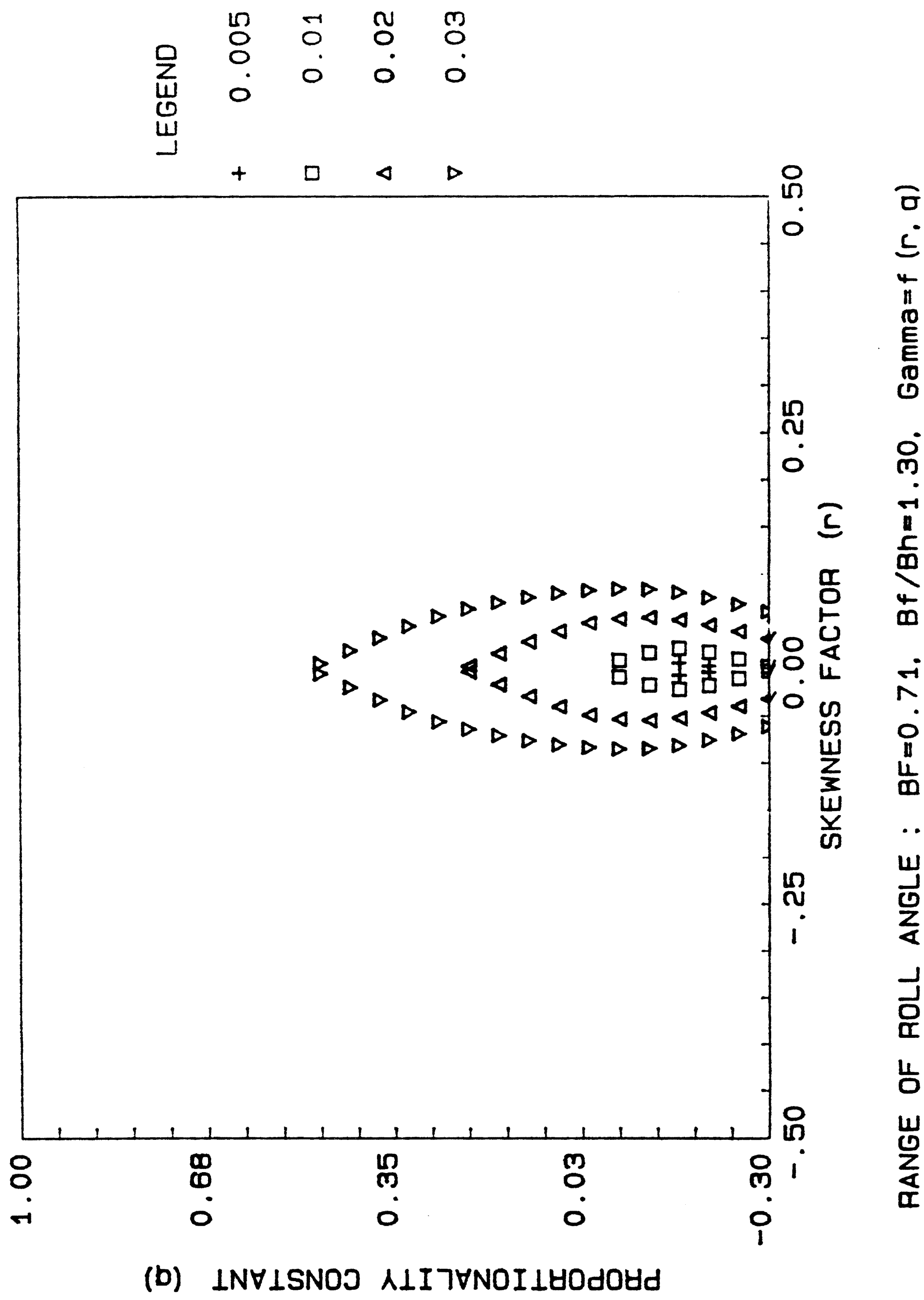


Figure 20. Roll Angle Range for R_i Study

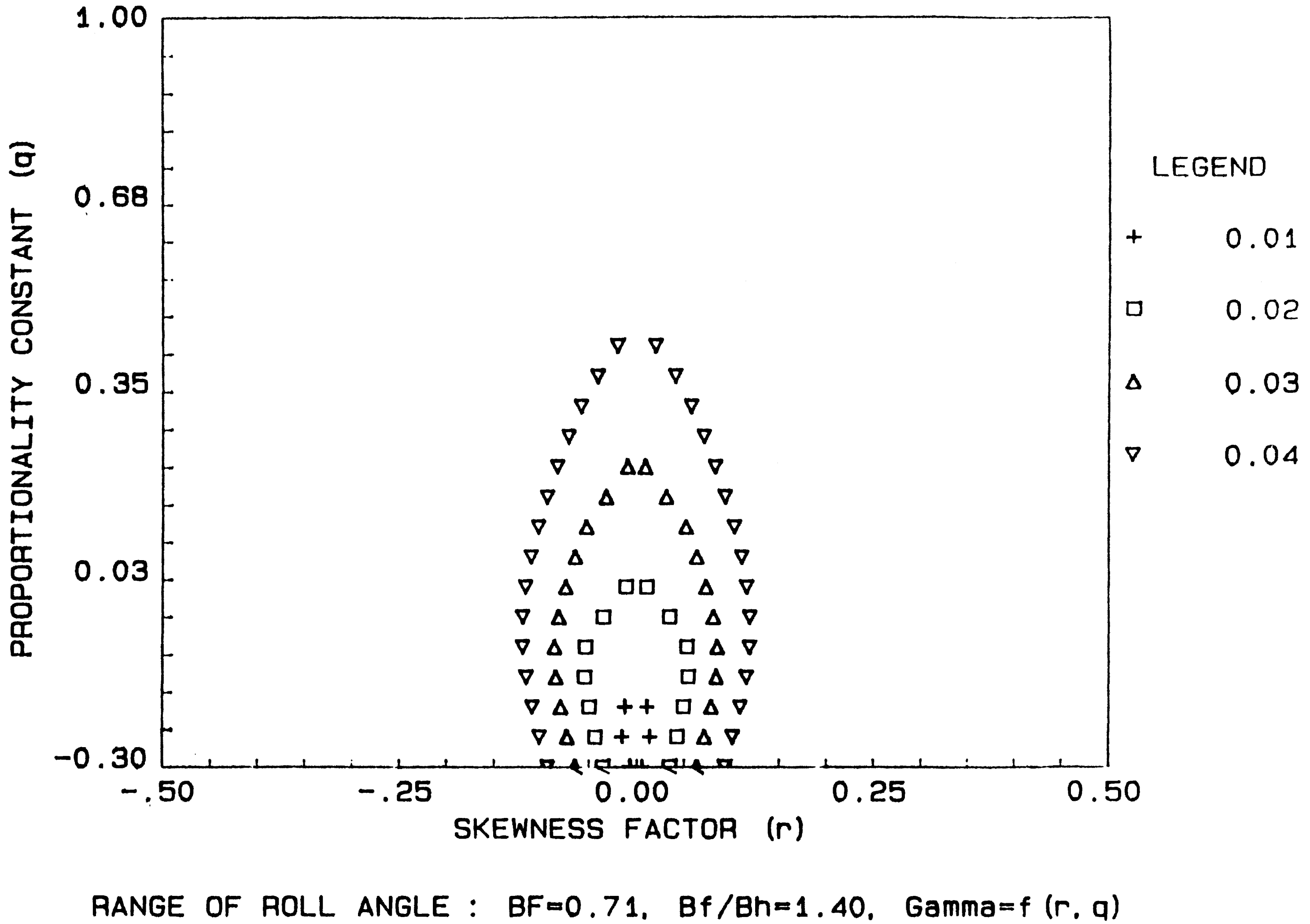


Figure 21. Roll Angle Range for R_i Study

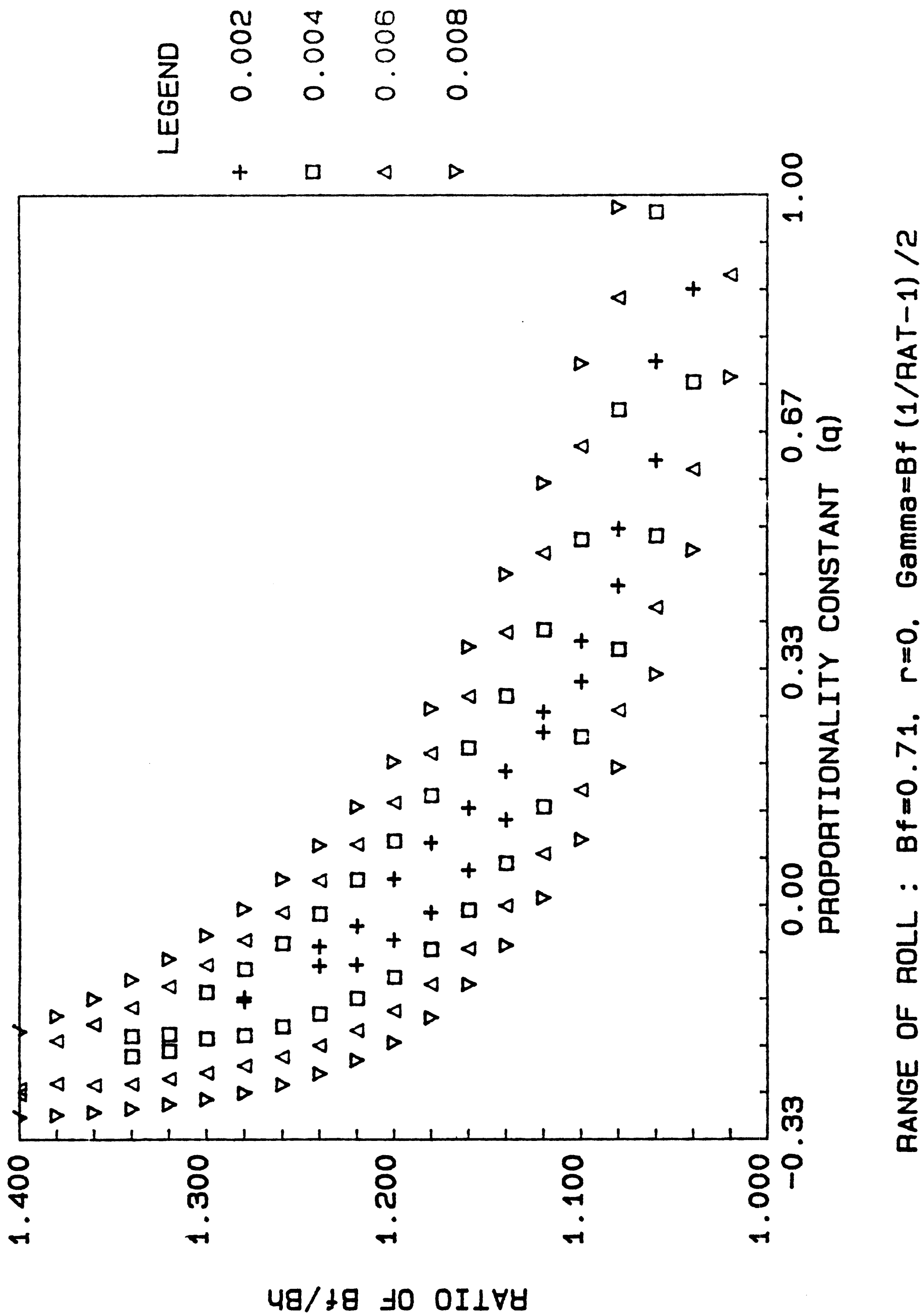


Figure 22. Roll Angle Range for Q_i and μ Study

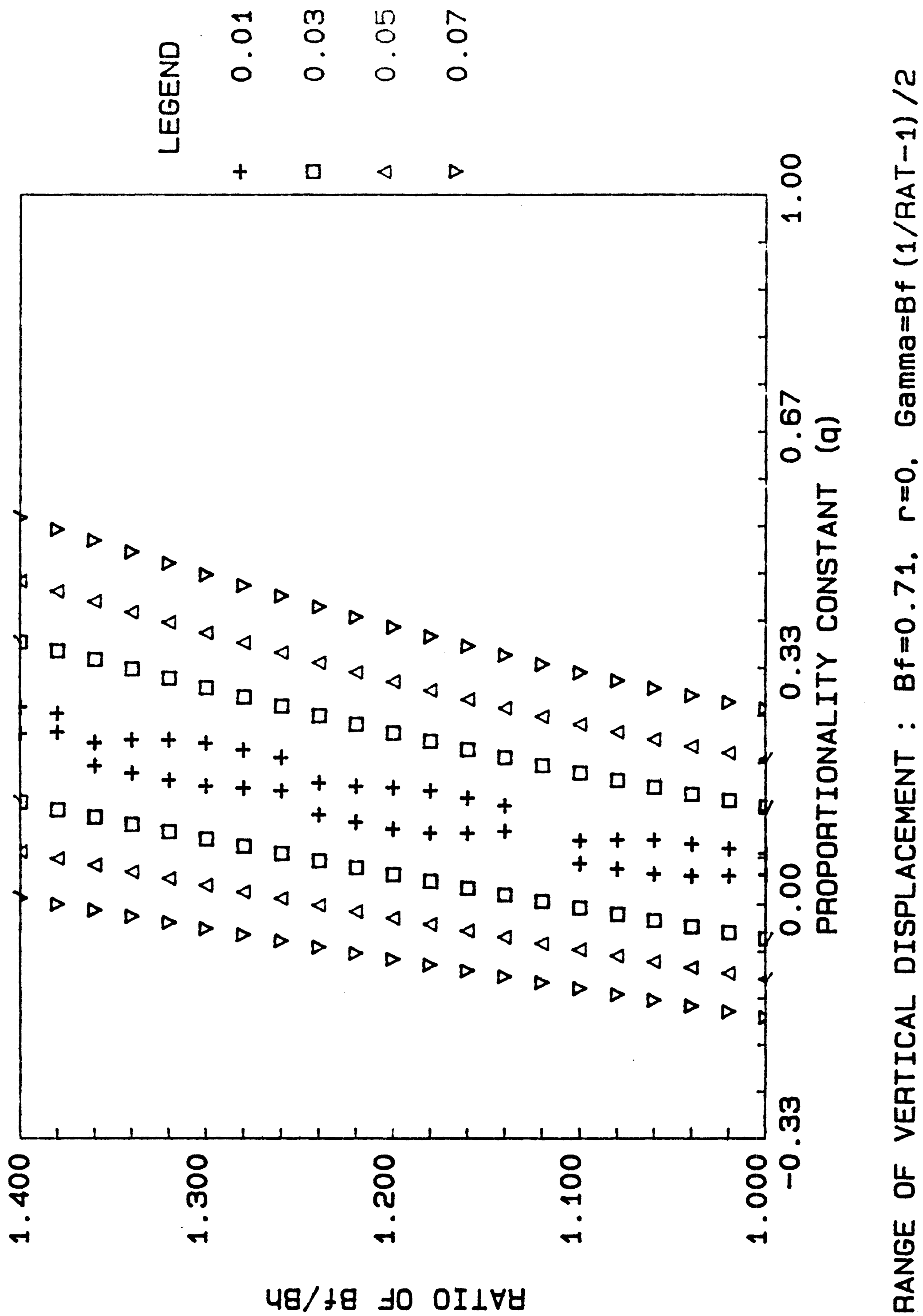


Figure 23. Z_b Displacement Range, Q_i and μ Study

vertical displacement ranges were then plotted on the same graph (See Figure 24). The point where these two curves intersect was then determined from the graph. It was assumed that this point of intersection defined the optimum values of Q_i and μ for the quadruped, since both displacements were at minimums. This gives a value of 0.113 and 1.152 to Q_i and μ, respectively. Evaluation of Equations (3.26-3.27) and (3.28-3.31) yielded a relative time delay (γ) of -0.0468. These were the values given to these parameters and carried over into subsequent calculations.

Determination of α

As was outlined earlier, to determine the last parameters, the pitch angle range was used as the quantity to be minimized. The parameters β_f, γ, R_i, μ, and Q_i remained unchanged from the optimums determined earlier. Following the order determined by the preliminary classification of parameters, α and V_x were varied and the normalized pitch range was calculated, while C_{xi} was held to zero. The results are presented in the form of a contour graph (See Figure 25).

It would appear that the values of α and V_x which produce a small pitch range are related in a linear fashion. It was found that the optimum α for a value of V_x is such that the average values of X_i for diagonal leg pairs have equal absolute values but opposite signs during the legs'

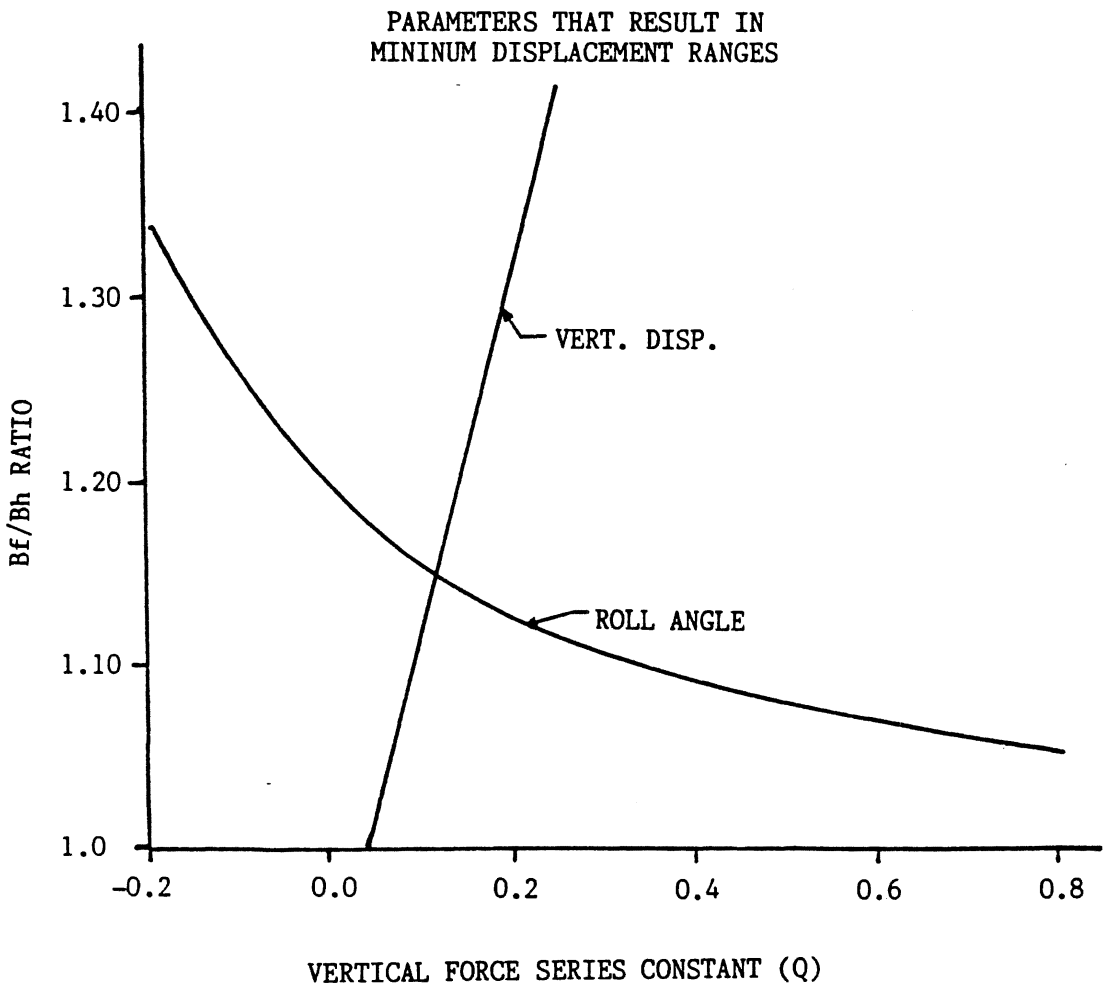


Figure 24. Optimum Combinations of Q_i and μ

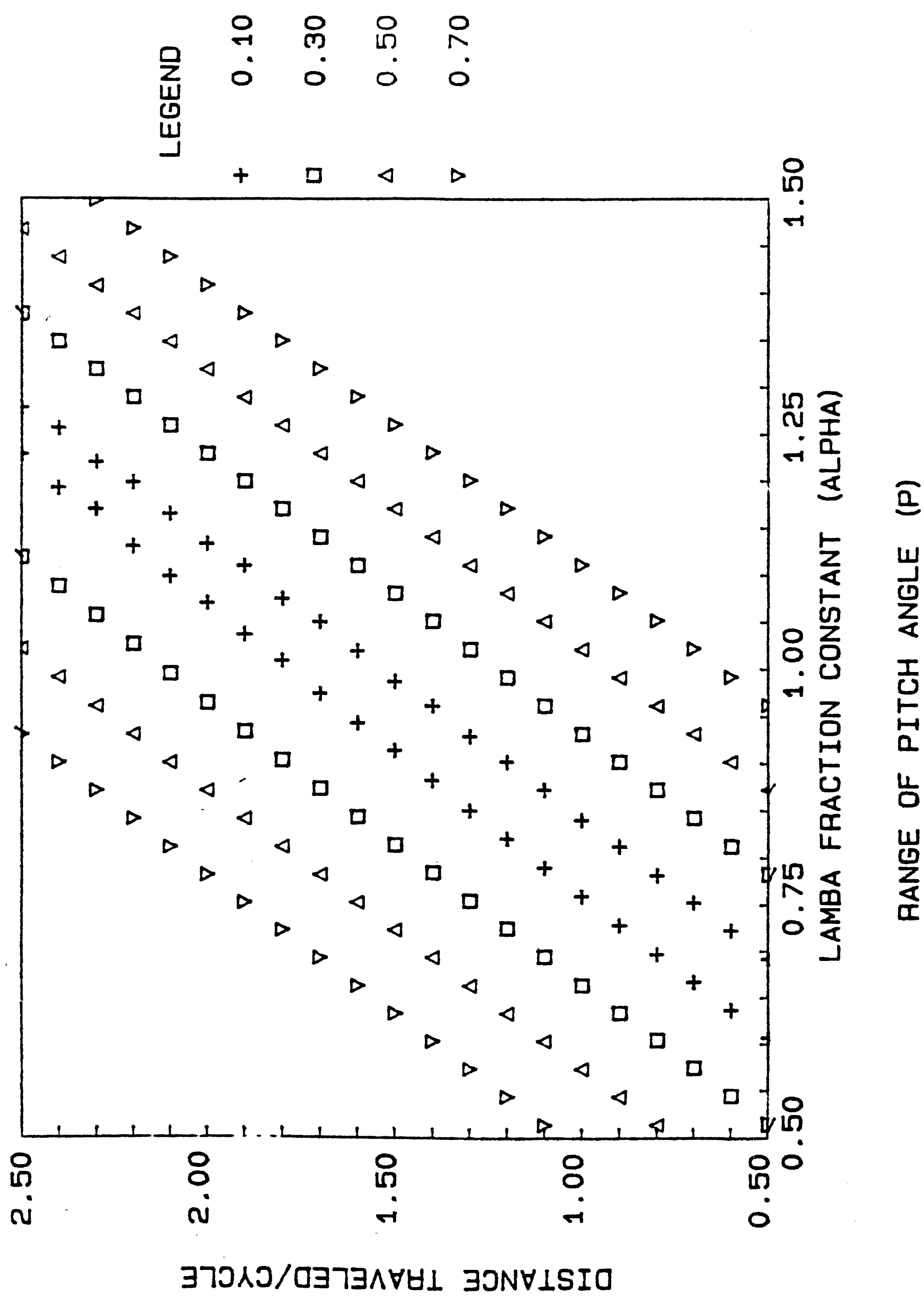


Figure 25. Pitch Angle Range for α Study

support phases. Since the horizontal position of the legs at any time η, can be described by the equation

$$X_i(\eta) = \lambda_i-[X_b(\eta)-X_b(\zeta_i-\beta_i)] \qquad (3.32)$$

during the leg support phase. And since when $C_{xi}=0$ the V_x is constant, the body position is

$$X_b(\eta) = V_x\eta \qquad (3.33)$$

Recalling Equations (3.10) and (3.11), the absolute values of the mean value of X_i will be equal for diagonal leg pairs when

$$\alpha = [V_x/(4\Delta\lambda)][\beta_f(1+1/\mu)]+0.5 \qquad (3.34)$$

which upon testing, agrees with the contour plot data.

Determination of the Horizontal Force Constant

The horizontal force constant C_{xi} was the next parameter that required evaluation. Allowing C_{xi} to take other values besides zero, allowed the cycle period τ to enter into the equations governing pitch angle range, since V_x was no longer constant and F_{xi} was no longer equal to zero at all times. To aquire information, V_x and C_{xi} were varied and the affect these parameters had on pitch range was recorded in the form of contour plots. One plot was constructed for each of several selected values of τ (See Figures 26-30).

From this information, there appeared to be a linear relationship between the values of V_x and C_{xi} that resulted in minimum pitch ranges. However, this linear relationship

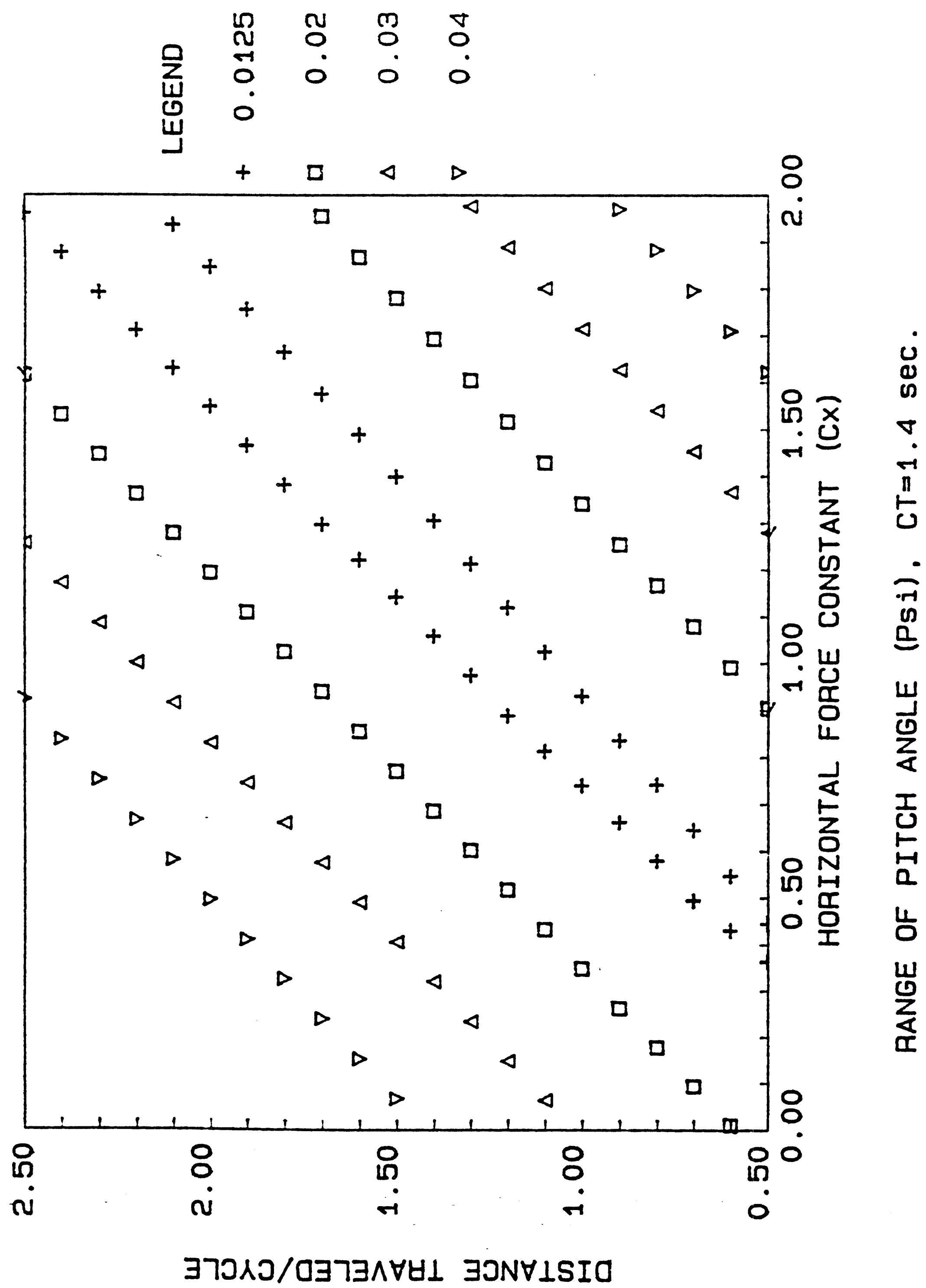


Figure 26. Pitch Angle Range for C_{xi} Study

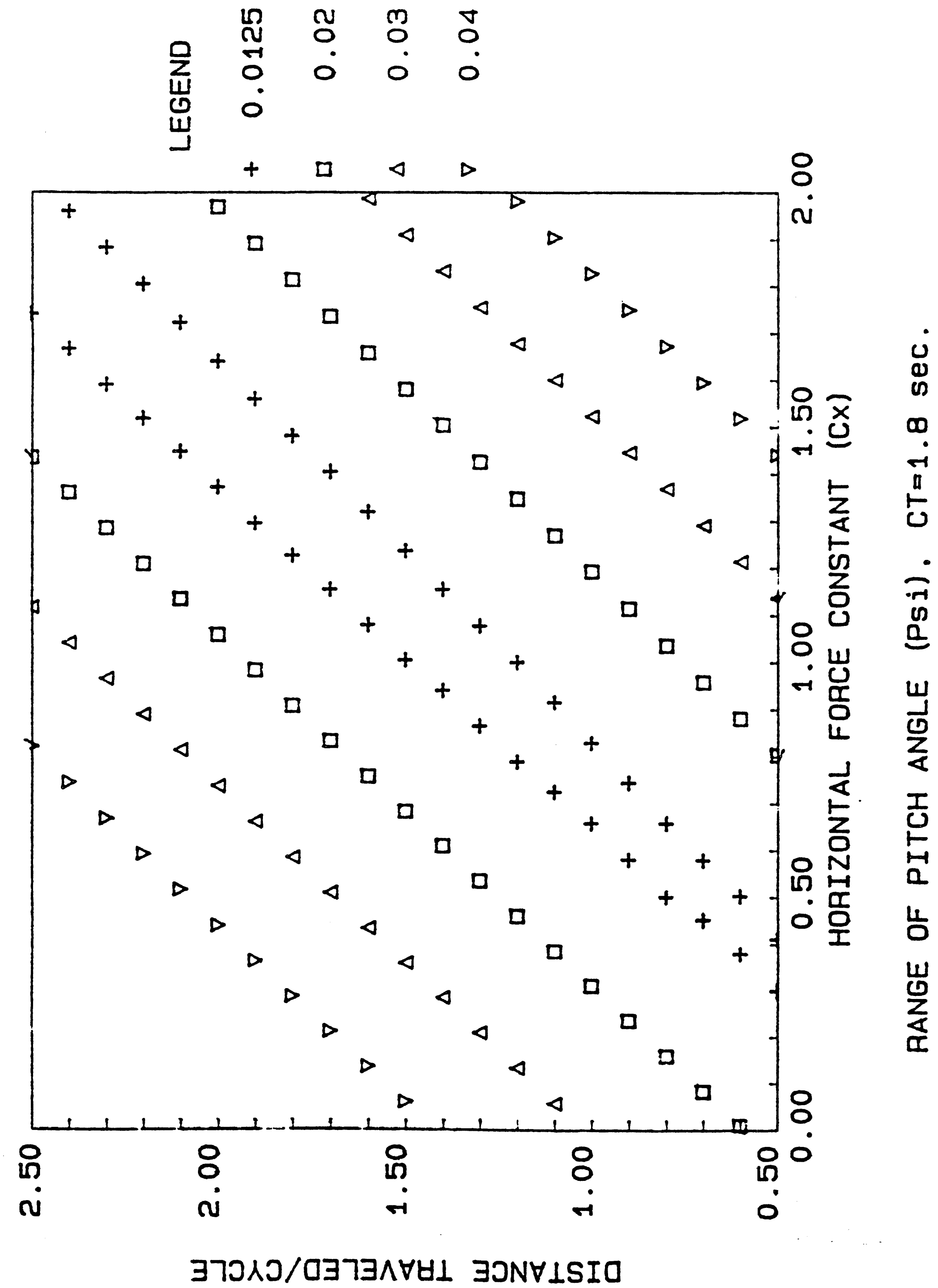


Figure 27. Pitch Angle Range for C_{xi} Study

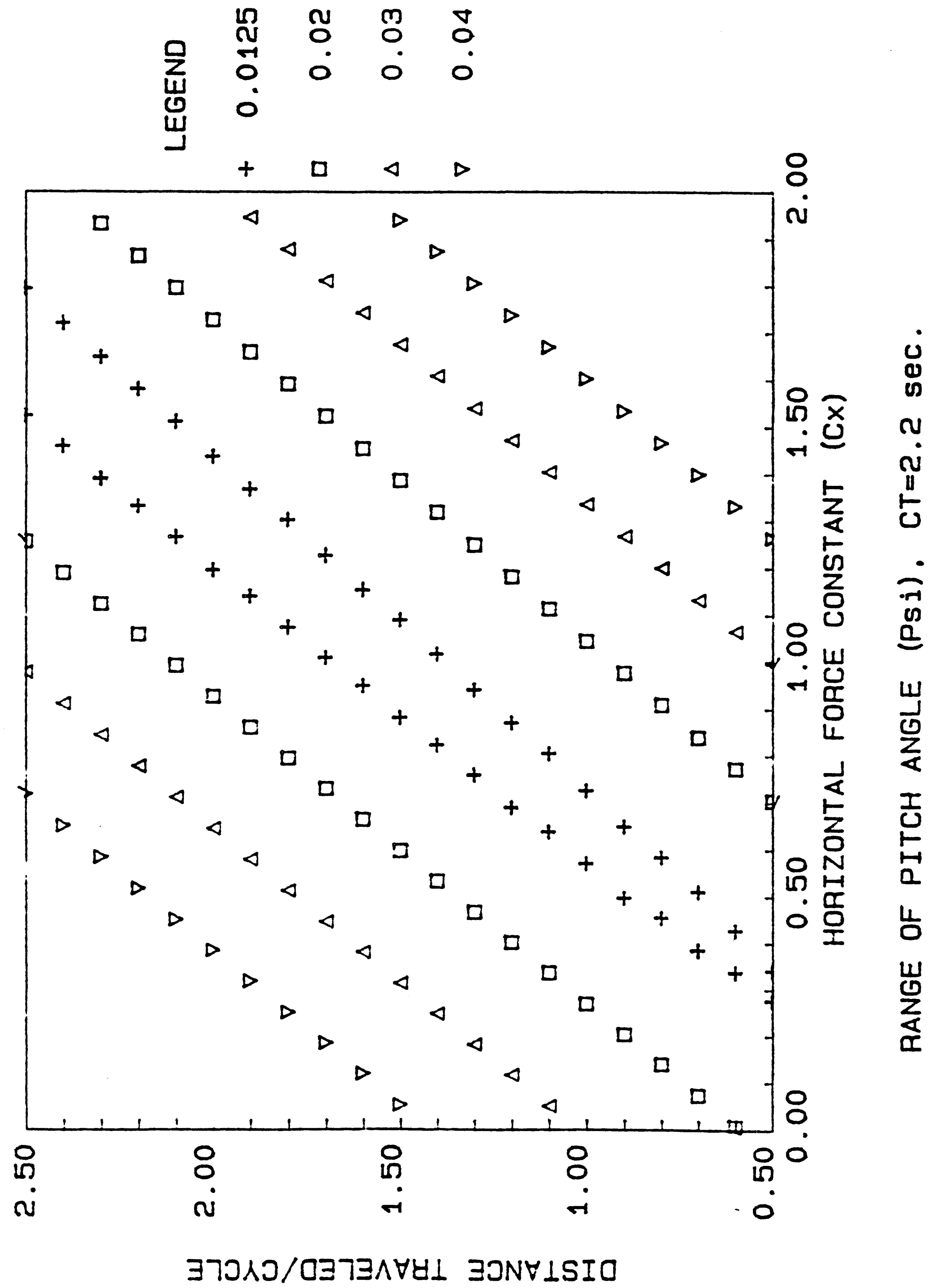


Figure 28. Pitch Angle Range for C_{xi} Study

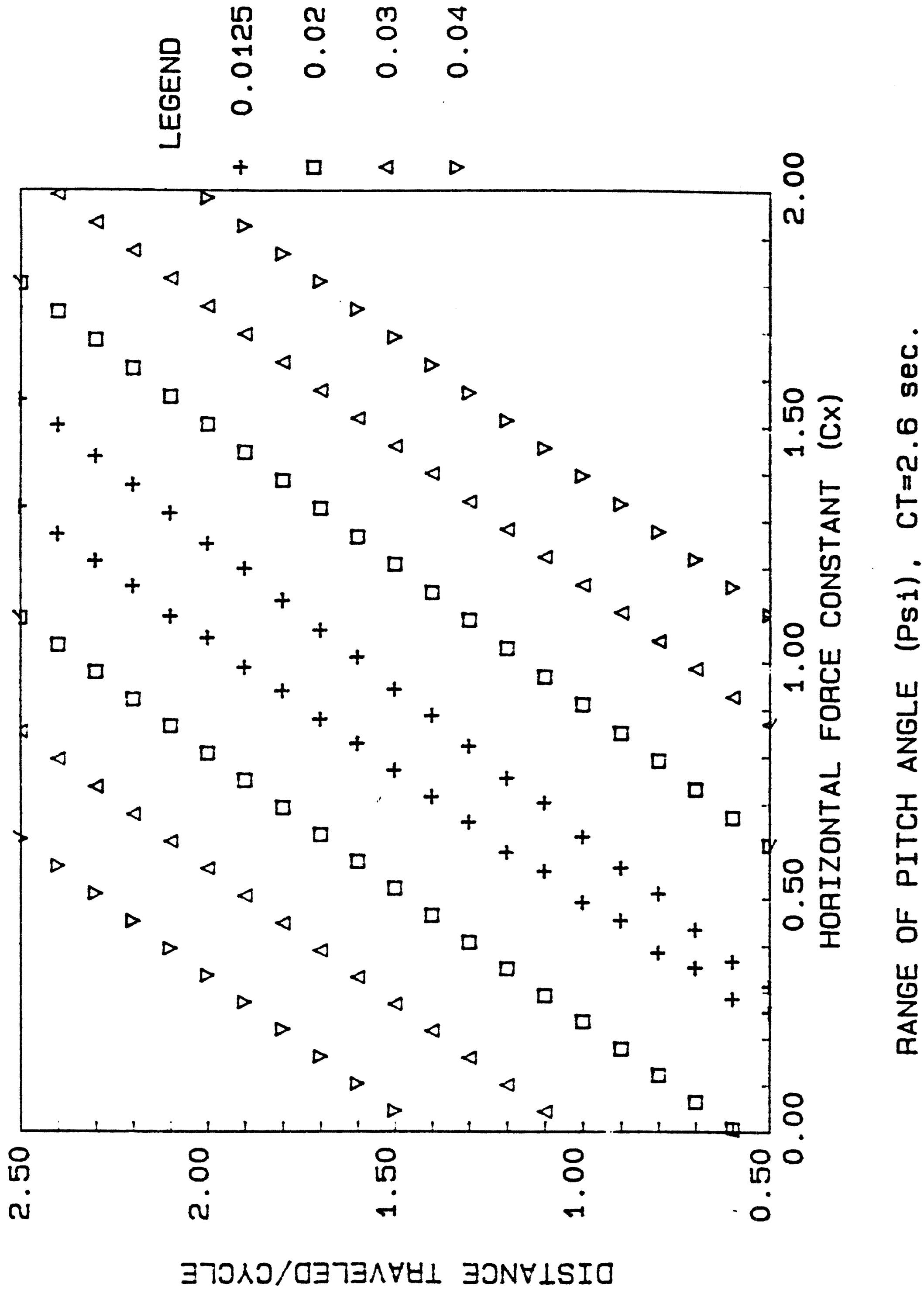


Figure 29. Pitch Angle Range for C_{xi} Study

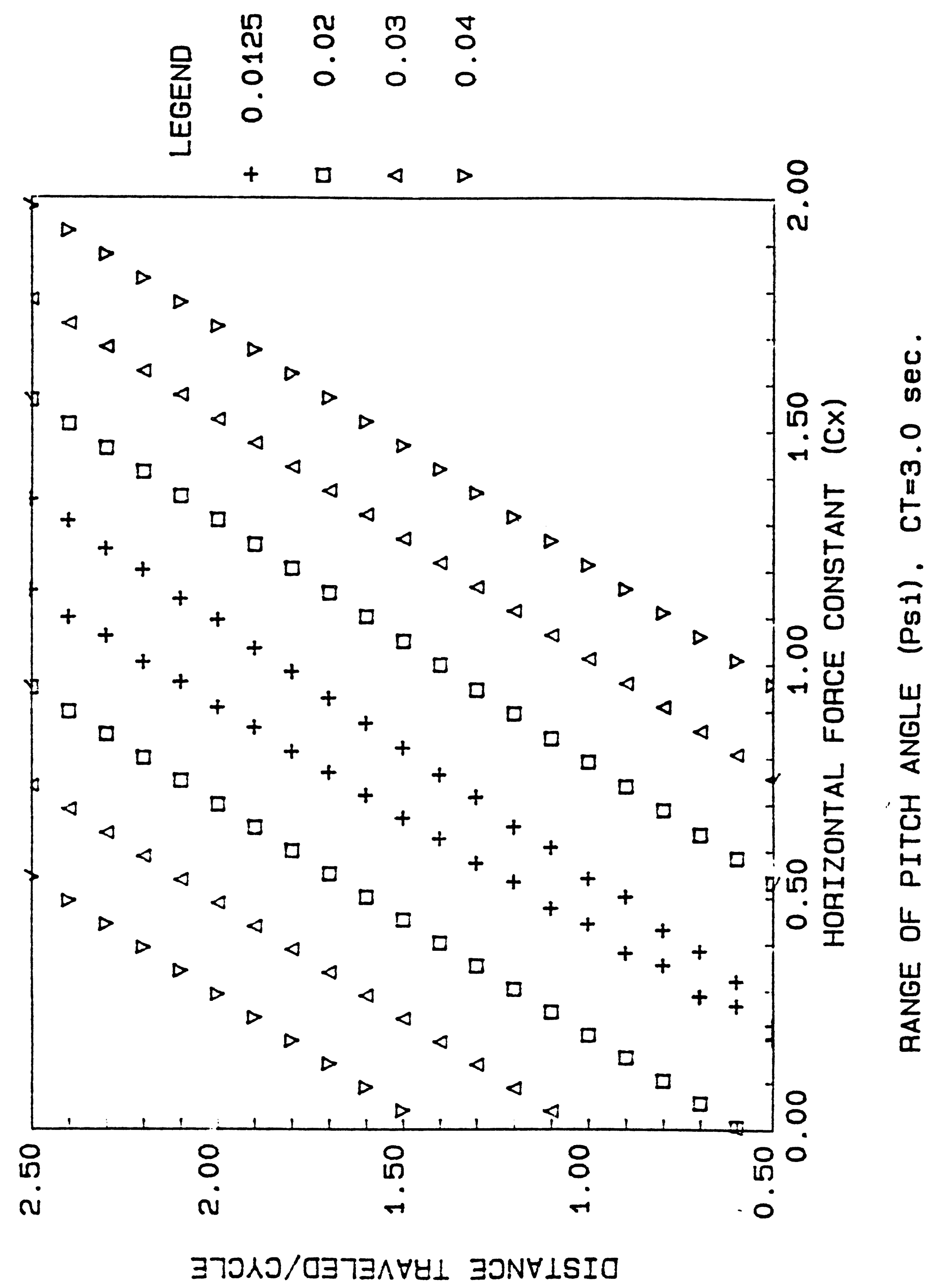


Figure 30. Pitch Angle Range for C_{xi} Study

was not constant for all values of τ. An exact relationship could not be determined with any confidence in the results. Instead, the parameters that produced minimum pitch ranges seemed to fit an equation of the form

$$[C_1 - C_2\tau]V_x = C_{xi} \tag{3.35}$$

Using a trial and error technique to determine the two constants, Equation (3.35) became

$$[4.7 - 0.9\tau]V_x/4 = C_{xi} \tag{3.36}$$

which agreed with the plotted data precisely. This equation predicted optimum C_{xi} to within $\pm 0.001\%$ of the actual parameter value found from the graphed data. Therefore, C_{xi} was defined as in Equation (3.36) for all future uses.

Determination of the Velocity

Using Equation (3.36), the pitch angle ranges were calculated for varied values of V_x and τ. The results were compiled in a electronic file in tabular form. This set of calculations revealed that for all cycle period values the range of normalized pitch was approximately $1.235 \cdot 10^{-2}$ when $V_x=0.5$ and steadily decreased in value to $1.111 \cdot 10^{-2}$ when $V_x=2.5$. A sample of the results is contained in Table IV.

This suggests that the pitch range can actually be reduced if the distance travelled per cycle (V_x) is increased. Obviously, V_x cannot be increased indefinitely. V_x will have a limit, determined by the reachable workspace of the limbs of the quadruped. The maximum and minimum values of X_i during a leg's support phase cannot extend

TABLE IV

NORMALIZED PITCH RANGES AND CALCULATED HORIZONTAL FORCE CONSTANT FOR VARIED VALUES OF τ and V_x

V_x	C_x	θ range($\times 10^{-2}$)
Cycle Period = 1.2 sec.		
0.50	0.453	1.236
1.00	0.905	1.206
1.50	1.358	1.175
2.00	1.810	1.144
2.50	2.263	1.114
Cycle Period = 2.0 sec.		
0.50	0.363	1.232
1.00	0.725	1.200
1.50	1.088	1.169
2.00	1.450	1.140
2.50	1.813	1.113
Cycle Period = 2.8 sec.		
0.50	0.273	1.228
1.00	0.545	1.192
1.50	0.818	1.157
2.00	1.090	1.124
2.50	1.363	1.108

beyond the boundaries of the reachable area. This leads to the following conclusion: the quadruped moves at the maximum mean velocity allowed by the workspace restrictions of the limbs so as to minimize the pitch angle range experienced by the quadruped's body.

This assumption was tested by calculating the maximum and minimum X_i of each leg during the leg's support cycle, using various values of V_x and τ. These values were compared to the reachable area of the limbs that were measured during the videotaping session (See Table II). From this procedure, some estimate of the allowable V_x could be made. Recall that the assumed C_g of the quadruped body is located a distance of 0.541ℓ from the hip joint. Using S_x and H_x mean values from Table II and the assumption that the workspace is roughly symetric about the shoulder and hip joints, the following estimates of the maximum and minimum values of X_i can be made for the fore and hind legs

$$X_{fmax} = 0.459+S_x/2 = 1.256 \tag{3.37}$$

$$X_{fmin} = 0.459-S_x/2 = -0.339 \tag{3.38}$$

$$X_{hmax} = -0.541+H_x/2 = 0.020 \tag{3.39}$$

$$X_{hmin} = -0.541-H_x/2 = -1.102 \tag{3.40}$$

Recognizing that the minimum and maximum positions X_i of the legs for each gait will occur at the beginning and the end of the legs' support phases, the maximum and minimum values were calculated for varied combinations of τ and V_x. A shortened sample of the tabular output is contained in Table V. Defining the following variables

TABLE V

REQUIRED WORKSPACE BOUNDARY VALUES

V_x	X_{fmax}	X_{fmin}	X_{hmax}	X_{hmin}
Cycle Period = 1.2 sec				
1.000	0.886	0.138	-0.223	-0.866
1.500	1.052	-0.070	-0.057	-1.023
2.000	1.218	-0.278	0.109	-1.179
2.500	1.383	-0.486	0.241	-1.335
Cycle Period = 1.6 sec				
1.000	0.886	0.115	-0.223	-0.883
1.500	1.052	-0.104	-0.057	-1.047
2.000	1.218	-0.324	0.109	-1.211
2.500	1.383	-0.543	0.274	-1.375
Cycle Period = 2.0 sec				
1.000	0.886	0.092	-0.223	-0.900
1.500	1.052	-0.140	-0.057	-1.073
2.000	1.218	-0.371	0.109	-1.245
2.500	1.383	-0.602	0.274	-1.418
Cycle Period = 2.4 sec				
1.000	0.886	0.070	-0.223	-0.916
1.500	1.052	-0.173	-0.057	-1.096
2.000	1.218	-0.415	0.109	-1.277
2.500	1.383	-0.657	0.274	-1.458

V_{f1} = the allowable V_x before X_{fmax} is exceeded

V_{f2} = the allowable V_x before X_{fmin} is exceeded

V_{h1} = the allowable V_x before X_{hmax} is exceeded

V_{h2} = the allowable V_x before X_{hmin} is exceeded

Table VI was constructed. It is obvious from Table VI that the mean velocity of the quadruped is most severely limited by X_{hmin}. Using the mean value of τ for the six subjects tested of 1.98 seconds, then V_{xmax} is approximately 1.589 for the modeled quadruped.

Comparison of the Model to Actual Performance

The predicted parameter values, found during the optimization process, were compared to the measured mean values from the videotape records in Table VII. The predicted values of these parameters were very close to the actual values used by the tested subjects. The largest deviations occurred in the time delay factors ζ_i. However, even these deviations are relatively small.

The force constants cannot be compared to mean values because of the lack of data records available for evaluation. However, some observations can be made from the videotape record and conclusions drawn from the results of past works. Alexander found in his studies of the gaits of other quadruped species that the skewness factor (R_i) was negligible for most of the individuals he studied [1]. In another published report, Jayes and Alexander observed values of the constant Q_i in the range 0.1 to 0.2 for

TABLE VI

ALLOWABLE VELOCITIES OF QUADRUPED BODY DUE TO WORKSPACE LIMITATIONS OF LIMBS

τ	V_{f1}	V_{f2}	V_{h1}	V_{h2}
1.20	2.115	2.145	1.730	1.755
1.60	2.115	2.034	1.730	1.667
2.00	2.115	1.930	1.730	1.585
2.40	2.115	1.842	1.730	1.515
2.80	2.115	1.777	1.730	1.463

TABLE VII

COMPARISON OF MEASURED AND PREDICTED PARAMETER VALUES FOR HUMAN QUADRUPEDS

Parameter	Measured Value	Predicted Value
β_f	0.714	0.710
β_h	0.625	0.616
μ	1.143	1.152
ζ_2	0.51	0.500
ζ_3	0.52	0.453
ζ_4	0.00	-0.047
α	0.949	0.975
V_x	1.596	1.589
R_i	?	0.000
Q_i	?	0.113
C_{xi}	?	1.157

several species of quadrupeds [4]. Both of these findings are consistent with the predicted values presented here. As for the parameter C_{xi}, no proof of the validity of the predicted value can be offered except indirectly through the results of the prediction of V_x.

The differences between the predicted and actual parameters used by human quadrupeds can be explained by any number of possible error sources. First, due to the lack of more sophisticated and specialized instruments, measurements of parameters were taken using relatively crude equipment which may have introduced error into the measured values. Assumptions were made in the development of the model equations that could have resulted in a divergence between calculated and actual values of the displacement values. Primarily, the assumption of massless limbs and the ignoring of the dynamic effects due to limb movement is suspected of introducing error. The values of the mass moments of inertia and the location of the C_g of the body were values assumed from cadaver studies [9]. If the actual moments of inertia for the subjects differed, then this is another source causing deviations.

Calculated Angular and Linear Displacements

The actual unwanted displacements of the human quadruped's body were predicted using the optimum values of the parameters as defined in Table VII. The actual displacement predictions were found by first finding the normalized displacement ranges, then calculating the normalizing factors for each individual subject, and then calculating the actual displacements using Equations (3.14-3.17). Table VIII contains the resulting values. Figures 31-43 (See Appendix C) illustrate the assumed forces exerted by each of the legs over the cycle period and the resulting normalized displacements when the quadruped is using the parameter values as previously determined in the optimization procedures.

TABLE VIII

CALCULATED DISPLACEMENTS RANGES
OF HUMAN QUADRUPED BODY

Subject #	Z_b(ft.)	ϕ	θ	ψ
1	0.024	7.2°	12.8°	15.6°
2	0.018	6.1°	10.8°	12.1°
3	0.004	1.4°	2.4°	2.9°
4	0.006	2.0°	3.5°	4.3°
5	0.007	2.3°	4.1°	5.0°
6	0.008	2.5°	4.4°	5.4°
Mean	0.010	3.3°	5.8°	7.1°

CHAPTER IV

SUMMARY AND CONCLUSIONS

This work describes a mathematical model developed for human quadruped locomotion studies. The model describes the unwanted displacements of the human body as it is moving with a specified gait pattern during straightline quadruped walking. The model has been simplified by the assumptions of massless legs, a rigid body link, and use of data measured during studies of actual human performance.

This model was used to determine the optimum parameters defining the motion gait. Optimality was defined as the combination of gait parameters that resulted in a minimum unwanted displacement range of the quadruped body over one cycle period. The parameters that required optimization were first identified and isolated from the equations of the mathematical model. Beginning with a relatively large number of parameters, the number of parameters affecting displacements were reduced to a more managable amount. This was done by using assumptions based on measurement data and by identification of those parameters that are related to body shape and limb lengths and therefore not variable.

A single optimization procedure did not yield satisfactory results. Instead, an alternative method was developed and used. The displacements were classified as to their relative sensitivity to deviations from an optimum set of parameters. It was found that the roll angle range was by far the most sensitive. The parameters that affected the roll angle range were identified and classified as to their relative effect on the roll angle. Building on this, the roll was studied and the defined variable parameters that affected roll determined in such a way that would minimize the roll angle range and also keep the vertical displacement range within acceptable levels.

The parameters that remained undetermined were found by minimization of the pitching angle range of the body. The parameters determined in the roll angle studies remained unchanged from the previous values. It was found that the idealized model which used these predicted parameters for locomotion had small values for the unwanted displacement ranges of the body.

Comparison of the predicted parameters from the minimization process revealed that these values provided a close approximation of the actual parameters measured from recordings of actual human performance. There were small deviations between actual and predicted values that can be explained by individual differences in the subject's C_g, body shape and type differences, errors in measurement, errors in assumptions for deriving the mathematical model, or other unidentified error sources. However, the differences were small and the predictions sufficiently accurate to describe gross motion tendencies of humans that are employing a quadruped walking type gait for locomotion. This indicates that humans do indeed move in such a way as to minimize the unwanted vertical and angular displacement ranges of their body and also to avoid rapid changes in the forces exerted by their limbs. Whether the large force gradients are avoided because the human is unable to exert such rapidly changing forces or the exertion is merely physically uncomfortable, is unknown.

This study lays the groundwork for future studies into the nature of locomotion. The methods employed here can also be used to study the gaits of other species of quadrupeds. The data measurements provide information that can be used in the development of more detailed models that do not include the assumption of massless limbs.

It is suggested that a future follow-up study be performed which includes a more detailed model development that accounts for the dynamic effects produced by limb motion. A complimentary study should also be undertaken that will extend this work to include the motion of human quadrupeds traveling on a curvilinear path. The motions of a quadruped that has a net acceleration is another area to be discussed. Adaptive strategies for the quadruped walking gait when the terrain is no longer smooth or obstacles are present is another subject that should be addressed in a future study.

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APPENDIXES

APPENDIX A

DISPLACEMENT CALCULATION PROGRAM LISTING

```
C***********************************************************
C**              MAIN PROGRAM WLKDSP: VERSION 1            **
C**            FOR STUDY OF FOREFEET DUTY FACTOR           **
C***********************************************************
      REAL VXLST,ZBLST,Y(4),BETA(4),RATIO,XLAMBA(4),XH,CT
      REAL CL,ALPHA,Q,R,RANGE,PHI(4),ZB(401),TIME(401)
      INTEGER BRNCH,I,M
      LOGICAL ABORT
      COMMON/ROBDATA/VXLST,ZBLST,Y,BETA,RATIO,XLAMBA,XH
13    CALL PARADEF(CL)
      DO 26 I=1,5
         WRITE(3,'(1H,///)')
26    CONTINUE
15    WRITE(3,*)'ENTER THE DESIRED PLOTTING OPTION'
      WRITE(3,*)'1= Zb        2=ROLL       3=PITCH'
      WRITE(3,*)'4=YAW        5=RESTART    6=END PROGRAM'
      READ(3,*)BRNCH
      IF (BRNCH.EQ.5.OR.BRNCH.EQ.6) GOTO 23
      CALL GETWFN(201,'ENTER NAME OF DATA FILE$',ABORT)
      IF (BRNCH.EQ.3) THEN
          WRITE(3,*)'ENTER THE CYCLE VELOCITY Vx'
          READ(3,*)VXLST
          WRITE(3,*)'ENTER THE LAMBA RELATIONAL CONSTANT'
          READ(3,*)ALPHA
          XLAMBA(1)=ALPHA*CL
          XLAMBA(2)=XLAMBA(1)
          XLAMBA(3)=(ALPHA-1)*CL
          XLAMBA(4)=XLAMBA(3)
          WRITE(3,*)'ENTER THE CYCLE TIME(SEC.)'
          READ(3,*)CT
      ENDIF
      IF(BRNCH.EQ.4.OR.BRNCH.EQ.3) THEN
          WRITE(3,*)'ENTER THE HOR. FORCE CONSTANT, Cx'
          READ(3,*)CFX
      ENDIF
      WRITE(3,*)'ENTER THE VALUE FOR BF'
      READ(3,*) BETA(1)
      BETA(2)=BETA(1)
      WRITE(3,*)'ENTER THE SKEWNESS FORCE CONSTANT Ri'
      READ(3,*)R
      WRITE(3,*)'ENTER THE VERTICAL FORCE CONSTANT Qi'
      READ(3,*)Q
      DO 14 RATIO=BETA(1),2*BETA(1),0.05
         BETA(3)=BETA(1)/RATIO
         BETA(4)=BETA(3)
         DO 86 PHDIF=-0.5,0.5.0.05
            CALL PHICAL(PHI,PHDIF)
            IF (BETA(3).GT.1.OR.BETA(3).LT.0.5) GOTO 86
            IF (BRNCH.EQ.1) THEN
               CALL ZBCALC(ZB,PHI,TIME,M,R,Q,RANGE)
            ELSEIF(BRNCH.EQ.2) THEN
               CALL RLCALC(PHI,R,Q,RANGE)
            ELSEIF (BRNCH.EQ.3) THEN
               CALL PTCALC(PHI,CYCTIM,CFX,R,Q,RANGE)
```

```
            ELSEIF (BRNCH.EQ.4) THEN
               CALL YWCALC(PHI,R,Q,CFX,RANGE)
            ENDIF
            WRITE(201,*)RATIO,PHDIF,RANGE
86        CONTINUE
14      CONTINUE
        CLOSE(201)
23      IF (BRNCH.EQ.5) GOTO 13
        IF (BRNCH.NE.6) GOTO 15
        END
C
C**********************************************************
C**            MAIN PROGRAM WLKDSP: VERSION 2            **
C**          FOR STUDY OF RELATIVE PHASE, GAMMA          **
C**********************************************************
        REAL VXLST,ZBLST,Y(4),BETA(4),RATIO,XLAMBA(4),XH,CT
        REAL CL,ALPHA,Q,R,RANGE,PHI(4),ZB(401),TIME(401)
        INTEGER BRNCH,I,M
        LOGICAL ABORT
        COMMON/ROBDATA/VXLST,ZBLST,Y,BETA,RATIO,XLAMBA,XH
13      CALL PARADEF(CL)
        DO 26 I=1,5
          WRITE(3,'(1H,///)')
26      CONTINUE
15      WRITE(3,*)'ENTER THE DESIRED PLOTTING OPTION'
        WRITE(3,*)'1= Zb        2=ROLL        3=PITCH'
        WRITE(3,*)'4=YAW        5=RESTART     6=END PROGRAM'
        READ(3,*)BRNCH
        IF (BRNCH.EQ.5.OR.BRNCH.EQ.6) GOTO 23
        CALL GETWFN(201,'ENTER NAME OF DATA FILE$',ABORT)
        IF (BRNCH.EQ.3) THEN
            WRITE(3,*)'ENTER THE CYCLE VELOCITY Vx'
            READ(3,*)VXLST
            WRITE(3,*)'ENTER THE LAMBA RELATIONAL CONSTANT'
            READ(3,*)ALPHA
            XLAMBA(1)=ALPHA*CL
            XLAMBA(2)=XLAMBA(1)
            XLAMBA(3)=(ALPHA-1)*CL
            XLAMBA(4)=XLAMBA(3)
            WRITE(3,*)'ENTER THE CYCLE TIME(SEC.)'
            READ(3,*)CT
        ENDIF
        IF(BRNCH.EQ.4.OR.BRNCH.EQ.3) THEN
            WRITE(3,*)'ENTER THE HOR. FORCE CONSTANT, Cx'
            READ(3,*)CFX
        ENDIF
        WRITE(3,*)'ENTER THE VALUE FOR BF'
        READ(3,*) BETA(1)
        BETA(2)=BETA(1)
        WRITE(3,*)'ENTER THE RATIO Bf/Bh'
        READ(3,*)RATIO
        BETA(3)=BETA(1)/RATIO
        BETA(4)=BETA(3)
        WRITE(3,*)'ENTER THE VERTICAL FORCE CONSTANT Qi'
```

```
      READ(3,*)Q
      DO 14 R=-0.5,0.5,0.05
         DO 86 PHDIF=-0.5,0.5.0.05
            CALL PHICAL(PHI,PHDIF)
            IF (BETA(3).GT.1.OR.BETA(3).LT.0.5) GOTO 86
            IF (BRNCH.EQ.1) THEN
               CALL ZBCALC(ZB,PHI,TIME,M,R,Q,RANGE)
            ELSEIF(BRNCH.EQ.2) THEN
               CALL RLCALC(PHI,R,Q,RANGE)
            ELSEIF (BRNCH.EQ.3) THEN
               CALL PTCALC(PHI,CYCTIM,CFX,R,Q,RANGE)
            ELSEIF (BRNCH.EQ.4) THEN
               CALL YWCALC(PHI,R,Q,CFX,RANGE)
            ENDIF
            WRITE(201,*)R,PHDIF,RANGE
86       CONTINUE
14    CONTINUE
      CLOSE(201)
23    IF (BRNCH.EQ.5) GOTO 13
      IF (BRNCH.NE.6) GOTO 15
      END
C
C**********************************************************
C**             MAIN PROGRAM WLKDSP: VERSION 3          **
C**    FOR STUDY OF VERTICAL FORCE SKEWNESS CONSTANT Ri   **
C**********************************************************
      REAL VXLST,ZBLST,Y(4),BETA(4),RATIO,XLAMBA(4),XH,CT
      REAL CL,ALPHA,Q,R,RANGE,PHI(4),ZB(401),TIME(401)
      INTEGER BRNCH,I,M
      LOGICAL ABORT
      COMMON/ROBDATA/VXLST,ZBLST,Y,BETA,RATIO,XLAMBA,XH
13    CALL PARADEF(CL)
      DO 26 I=1,5
         WRITE(3,'(1H,///)')
26    CONTINUE
15    WRITE(3,*)'ENTER THE DESIRED PLOTTING OPTION'
      WRITE(3,*)'1= Zb        2=ROLL       3=PITCH'
      WRITE(3,*)'4=YAW       5=RESTART    6=END PROGRAM'
      READ(3,*)BRNCH
      IF (BRNCH.EQ.5.OR.BRNCH.EQ.6) GOTO 23
      CALL GETWFN(201,'ENTER NAME OF DATA FILE$',ABORT)
      IF (BRNCH.EQ.3) THEN
          WRITE(3,*)'ENTER THE CYCLE VELOCITY Vx'
          READ(3,*)VXLST
          WRITE(3,*)'ENTER THE LAMBA RELATIONAL CONSTANT'
          READ(3,*)ALPHA
          XLAMBA(1)=ALPHA*CL
          XLAMBA(2)=XLAMBA(1)
          XLAMBA(3)=(ALPHA-1)*CL
          XLAMBA(4)=XLAMBA(3)
          WRITE(3,*)'ENTER THE CYCLE TIME(SEC.)'
          READ(3,*)CT
      ENDIF
      IF(BRNCH.EQ.4.OR.BRNCH.EQ.3) THEN
```

```
            WRITE(3,*)'ENTER THE HOR. FORCE CONSTANT, Cx'
            READ(3,*)CFX
         ENDIF
         WRITE(3,*)'ENTER  THE  VALUE  FOR  BF'
         READ(3,*) BETA(1)
         BETA(2)=BETA(1)
         WRITE(3,*)'ENTER THE RATIO Bf/Bh'
         READ(3,*)RATIO
         BETA(3)=BETA(1)/RATIO
         BETA(4)=BETA(3)
         DO 14 Q=-0.33,1.0,0.05
            DO 86 R=-0.5,0.5,0.05
               PHDIF=GAMMAQ(BETA,R,Q)
               CALL PHICAL(PHI,PHDIF)
               IF (BETA(3).GT.1.OR.BETA(3).LT.0.5) GOTO 86
               IF (BRNCH.EQ.1) THEN
                  CALL ZBCALC(ZB,PHI,TIME,M,R,Q,RANGE)
               ELSEIF(BRNCH.EQ.2) THEN
                  CALL RLCALC(PHI,R,Q,RANGE)
               ELSEIF (BRNCH.EQ.3) THEN
                  CALL PTCALC(PHI,CYCTIM,CFX,R,Q,RANGE)
               ELSEIF (BRNCH.EQ.4) THEN
                  CALL YWCALC(PHI,R,Q,CFX,RANGE)
               ENDIF
               WRITE(201,*)Q,R,RANGE
86          CONTINUE
14       CONTINUE
         CLOSE(201)
23       IF (BRNCH.EQ.5) GOTO 13
         IF (BRNCH.NE.6) GOTO 15
         END
C
C**********************************************************************
C**              MAIN PROGRAM WLKDSP: VERSION 4                      **
C**     FOR STUDY OF FORCE CONSTANT Qi AND RATIO                     **
C**********************************************************************
         REAL VXLST,ZBLST,Y(4),BETA(4),RATIO,XLAMBA(4),XH,CT
         REAL CL,ALPHA,Q,R,RANGE,PHI(4),ZB(401),TIME(401)
         INTEGER BRNCH,I,M
         LOGICAL ABORT
         COMMON/ROBDATA/VXLST,ZBLST,Y,BETA,RATIO,XLAMBA,XH
13       CALL PARADEF(CL)
         DO 26 I=1,5
            WRITE(3,'(1H,///)')
26       CONTINUE
15       WRITE(3,*)'ENTER  THE  DESIRED  PLOTTING  OPTION'
         WRITE(3,*)'1=  Zb        2=ROLL        3=PITCH'
         WRITE(3,*)'4=YAW         5=RESTART     6=END  PROGRAM'
         READ(3,*)BRNCH
         IF (BRNCH.EQ.5.OR.BRNCH.EQ.6) GOTO 23
         CALL GETWFN(201,'ENTER  NAME  OF  DATA  FILE$',ABORT)
         IF (BRNCH.EQ.3) THEN
            WRITE(3,*)'ENTER THE CYCLE VELOCITY Vx'
            READ(3,*)VXLST
```

```
          WRITE(3,*)'ENTER THE LAMBA RELATIONAL CONSTANT'
          READ(3,*)ALPHA
          XLAMBA(1)=ALPHA*CL
          XLAMBA(2)=XLAMBA(1)
          XLAMBA(3)=(ALPHA-1)*CL
          XLAMBA(4)=XLAMBA(3)
          WRITE(3,*)'ENTER THE CYCLE TIME(SEC.)'
          READ(3,*)CT
      ENDIF
      IF(BRNCH.EQ.4.OR.BRNCH.EQ.3) THEN
          WRITE(3,*)'ENTER THE HOR. FORCE CONSTANT, Cx'
          READ(3,*)CFX
      ENDIF
      WRITE(3,*)'ENTER THE VALUE FOR BF'
      READ(3,*) BETA(1)
      BETA(2)=BETA(1)
      R=0
      DO 14 RATIO=BETA(1),2*BETA(1),BETA(1)/20
         BETA(3)=BETA(1)/RATIO
         BETA(4)=BETA(3)
         DO 86 Q=-0.33,1.0,1.33/20
            PHDIF=GAMMAQ(BETA,R,Q)
            CALL PHICAL(PHI,PHDIF)
            IF (BETA(3).GT.1.OR.BETA(3).LT.0.5) GOTO 86
            IF (BRNCH.EQ.1) THEN
               CALL ZBCALC(ZB,PHI,TIME,M,R,Q,RANGE)
            ELSEIF(BRNCH.EQ.2) THEN
               CALL RLCALC(PHI,R,Q,RANGE)
            ELSEIF (BRNCH.EQ.3) THEN
               CALL PTCALC(PHI,CYCTIM,CFX,R,Q,RANGE)
            ELSEIF (BRNCH.EQ.4) THEN
               CALL YWCALC(PHI,R,Q,CFX,RANGE)
            ENDIF
            WRITE(201,*)RATIO,Q,RANGE
86       CONTINUE
14    CONTINUE
      CLOSE(201)
23    IF (BRNCH.EQ.5) GOTO 13
      IF (BRNCH.NE.6) GOTO 15
      END
C
C*********************************************************
C**            MAIN PROGRAM WLKDSP: VERSION 5          **
C**           FOR STUDY OF LAMBA CONSTANT ALPHA        **
C*********************************************************
      REAL VXLST,ZBLST,Y(4),BETA(4),RATIO,XLAMBA(4),XH,CT
      REAL CL,ALPHA,Q,R,RANGE,PHI(4),ZB(401),TIME(401)
      INTEGER BRNCH,I,M
      LOGICAL ABORT
      COMMON/ROBDATA/VXLST,ZBLST,Y,BETA,RATIO,XLAMBA,XH
13    CALL PARADEF(CL)
      DO 26 I=1,5
         WRITE(3,'(1H,///)')
26    CONTINUE
```

```
15    WRITE(3,*)'ENTER THE DESIRED PLOTTING OPTION'
      WRITE(3,*)'1= Zb         2=ROLL         3=PITCH'
      WRITE(3,*)'4=YAW         5=RESTART      6=END PROGRAM'
      READ(3,*)BRNCH
      IF (BRNCH.EQ.5.OR.BRNCH.EQ.6) GOTO 23
      CALL GETWFN(201,'ENTER NAME OF DATA FILE$',ABORT)
      IF (BRNCH.EQ.3) THEN
          WRITE(3,*)'ENTER THE CYCLE TIME(SEC.)'
          READ(3,*)CT
      ENDIF
      IF(BRNCH.EQ.4.OR.BRNCH.EQ.3) THEN
          WRITE(3,*)'ENTER THE HOR. FORCE CONSTANT, Cx'
          READ(3,*)CFX
      ENDIF
      WRITE(3,*)'ENTER THE VALUE FOR BF'
      READ(3,*) BETA(1)
      BETA(2)=BETA(1)
      R=0
      Q=0.113
      RATIO=1.152
      BETA(3)=BETA(1)/RATIO
      BETA(4)=BETA(3)
      PHDIF=GAMMAQ(BETA,R,Q)
      CALL PHICAL(PHI,PHDIF)
      IF (BETA(3).GT.1.OR.BETA(3).LT.0.5) GOTO 86
      DO 14 VXLST=0.5,2.5,0.1
         DO 86 ALPHA=0,2.0,0.1
            XLAMBA(1)=ALPHA*CL
            XLAMBA(2)=XLAMBA(1)
            XLAMBA(3)=(ALPHA-1)*CL
            XLAMBA(4)=XLAMBA(3)
            IF (BRNCH.EQ.1) THEN
               CALL ZBCALC(ZB,PHI,TIME,M,R,Q,RANGE)
            ELSEIF(BRNCH.EQ.2) THEN
               CALL RLCALC(PHI,R,Q,RANGE)
            ELSEIF (BRNCH.EQ.3) THEN
               CALL PTCALC(PHI,CYCTIM,CFX,R,Q,RANGE)
            ELSEIF (BRNCH.EQ.4) THEN
               CALL YWCALC(PHI,R,Q,CFX,RANGE)
            ENDIF
            WRITE(201,*)VXLST,ALPHA,RANGE
86       CONTINUE
14    CONTINUE
      CLOSE(201)
23    IF (BRNCH.EQ.5) GOTO 13
      IF (BRNCH.NE.6) GOTO 15
      END
C
C***********************************************************
C**            MAIN PROGRAM WLKDSP: VERSION 6             **
C**           FOR STUDY OF LAMBA CONSTANT ALPHA           **
C***********************************************************
      REAL VXLST,ZBLST,Y(4),BETA(4),RATIO,XLAMBA(4),XH,CT
      REAL CL,ALPHA,Q,R,RANGE,PHI(4),ZB(401),TIME(401)
```

```
      INTEGER BRNCH,I,M
      LOGICAL ABORT
      COMMON/ROBDATA/VXLST,ZBLST,Y,BETA,RATIO,XLAMBA,XH
13    CALL PARADEF(CL)
      DO 26 I=1,5
         WRITE(3,'(1H,///)')
26    CONTINUE
15    WRITE(3,*)'ENTER THE DESIRED PLOTTING OPTION'
      WRITE(3,*)'1= Zb        2=ROLL        3=PITCH'
      WRITE(3,*)'4=YAW        5=RESTART     6=END PROGRAM'
      READ(3,*)BRNCH
      IF (BRNCH.EQ.5.OR.BRNCH.EQ.6) GOTO 23
      CALL GETWFN(201,'ENTER NAME OF DATA FILE$',ABORT)
      IF (BRNCH.EQ.3) THEN
          WRITE(3,*)'ENTER THE CYCLE TIME(SEC.)'
          READ(3,*)CT
      ENDIF
      WRITE(3,*)'ENTER THE VALUE FOR BF'
      READ(3,*) BETA(1)
      BETA(2)=BETA(1)
      R=0
      Q=0.113
      RATIO=1.152
      BETA(3)=BETA(1)/RATIO
      BETA(4)=BETA(3)
      PHDIF=GAMMAQ(BETA,R,Q)
      CALL PHICAL(PHI,PHDIF)
      IF (BETA(3).GT.1.OR.BETA(3).LT.0.5) GOTO 86
      DO 14 VXLST=0.5,2.5,0.1
         ALPHA=(VXLST/(4*CL))*(BETA(1)+BETA(2))+0.5
         XLAMBA(1)=ALPHA*CL
         XLAMBA(2)=XLAMBA(1)
         XLAMBA(3)=(ALPHA-1)*CL
         XLAMBA(4)=XLAMBA(3)
         DO 86 CFX=0,2.0,0.1
            IF (BRNCH.EQ.1) THEN
               CALL ZBCALC(ZB,PHI,TIME,M,R,Q,RANGE)
            ELSEIF(BRNCH.EQ.2) THEN
               CALL RLCALC(PHI,R,Q,RANGE)
            ELSEIF (BRNCH.EQ.3) THEN
               CALL PTCALC(PHI,CYCTIM,CFX,R,Q,RANGE)
            ELSEIF (BRNCH.EQ.4) THEN
               CALL YWCALC(PHI,R,Q,CFX,RANGE)
            ENDIF
            WRITE(201,*)VXLST,CFX,RANGE
86       CONTINUE
14    CONTINUE
      CLOSE(201)
23    IF (BRNCH.EQ.5) GOTO 13
      IF (BRNCH.NE.6) GOTO 15
      END
C
C**********************************************************
C**                 SUBROUTINE PHICAL                   **
```

```
C***********************************************************
      SUBROUTINE PHICAL(PHI,PHDIF)
      REAL PHI(4),PHDIF
      PHI(1)=0
      PHI(2)=0.5
C
C USE RELATIVE PHASE GAMMA TO CALCULATE INDIVIDUAL LEG
C PHASE RELATIONSHIPS
C
      PHI(4)=PHDIF
      PHI(3)=0.5+PHDIF
      IF (PHI(4).GT.1) THEN
         PHI(4)=PHI(4)-1.0
      ELSEIF(PHI(4).LT.0) THEN
         PHI(4)=PHI(4)+1.0
      ENDIF
      IF (PHI(3).LT.0) THEN
         PHI(3)=PHI(3)+1.0
      ELSEIF(PHI(3).GT.1) THEN
         PHI(3)=PHI(3)-1.0
      ENDIF
      END
C
C***********************************************************
C**  SUBROUTINE RLCALC => CALCULATE NORMALIZED ROLL RANGE **
C***********************************************************
      SUBROUTINE RLCALC(PHI,R,Q,RSPR)
      REAL PHI(4),TIME(401),VX,RBLST,Y(4),BETA(4),RATIO
      REAL XH,RLST,TM,PH,BT,VR(401),ARLST,VRLST,RHI,RS
      REAL RLO,RSPR,ZBLST,RL(401),AR(401),R,Q,XLAMBA(4)
      INTEGER HI,M,N
      COMMON/ROBDATA/VX,ZBLST,Y,BETA,RATIO,XLAMBA,XH
      VRLST=0
      HI=INT(1.0/XH)+1
      RHI=-1000
      RLO=1000
      ARLST=0
      RLST=0
      DO 32 M=1,HI
C
C CALC. TIME STEP, RESET ACC. TO ZERO FOR STEP
C
          TIME(M)=REAL(M-1)*XH
          AR(M)=0
          TM=TIME(M)
          DO 33 N=1,4
C
C SET SKEWNESS FACTOR TO REQUIRED VALUE FOR FORELEGS
C
             RS=R
             IF(N.EQ.1.OR.N.EQ.2) THEN
                RS=-R
             ENDIF
             PH=PHI(N)
```

```
              BT=BETA(N)
C
C CALCULATE ANGULAR ACCELERATION AT TIME
C
              AR(M)=FORCEZ(TM,PH,BT,RS,Q)  *  Y(N)  +  AR(M)
33          CONTINUE
C
C INTEGRATE ACCELERATION VALUES
C
           CALL DOUBLE(AR(M),VR(M),RL(M),ARLST,VRLST,RLST,XH)
32    CONTINUE
      VRLST=-RL(HI)
C
C ADJUST ROLL TO VALUES REQUIRED FOR CONTINUITY
C
      DO 88 M=1,HI
            RL(M)=RL(M) + VRLST * TIME(M)
            VR(M)=VR(M)+VRLST
C
C   FIND MAX AND MIN OF ROLL ANGLE OVER CYCLE
C
            IF (RL(M).GT.RHI) THEN
                RHI=RL(M)
            ELSEIF (RL(M).LT.RLO) THEN
                RLO=RL(M)
            ENDIF
88    CONTINUE
      DO 89 M=1,HI
         RL(M)=RL(M)-(RHI+RLO)/2
89    CONTINUE
C
C RANGE OF ROLL
C
      RSPR=(RHI-RLO)/2
      END
C
C************************************************************
C** FUNCTION FORCEZ => CALC VERT FORCE OF LEG AT TIME STEP**
C************************************************************
      FUNCTION FORCEZ(TME,ZPHI,ZBTA,RS,Q)
      REAL FORCEZ,PI,C,GRAV,TME,ZPHI,ZBTA,RS,D,Q
      PI=3.1415926
      GRAV=32.174
      C=(3*GRAV*PI)/(8*ZBTA*(3+Q))
      IF (ZPHI.GT.ZBTA) THEN
         IF (TME.LT.(ZPHI-ZBTA).OR.TME.GT.ZPHI) THEN
            FORCEZ=0.0
         ELSE
            D=(TME+ZBTA-ZPHI) * PI / ZBTA
            FORCEZ=C*(SIN(D)-RS*SIN(2*D)+Q*SIN(3*D))
         ENDIF
      ELSEIF(TME.LT.ZPHI) THEN
         D=(TME + ZBTA - ZPHI) * PI / ZBTA
         FORCEZ=C*(SIN(D)-RS*SIN(2*D)+Q*SIN(3*D))
```

```
      ELSEIF(TME.GT.(ZPHI+1-ZBTA)) THEN
         D=(TME + ZBTA - ZPHI -1) * PI / ZBTA
         FORCEZ=C * (SIN(D) - RS * SIN (2*D) + Q*SIN(3*D))
      ELSE
            FORCEZ=0.0
      ENDIF
      END
C
C************************************************************
C** FUNCTION GAMMA4 => CALCULATES RELATIVE PHASE WHEN Q=0 **
C************************************************************
      FUNCTION GAMMA4(R,BTA,RAT)
      REAL GAMMA4,R,BTA,RAT,PI,COSA,C4,A
      PI=3.1415926
      IF (R.EQ.0) THEN
         GAMMA4=BTA*(1/RAT-1)/2
      ELSE
C
C FIND ROOTS OF QUADRATIC FROM OBSERATIONS OF MOMENT IMPULSE
C
         IF (R.LT.0) THEN
            COSA=(-1/R - SQRT( 1/(R**2) + 4.0)) / 2.0
         ELSE
            COSA=(-1/R + SQRT( 1/(R**2) + 4.0)) / 2.0
         ENDIF
         IF (COSA**2.EQ.0) THEN
            GAMMA4=BTA*(1/RAT-1)/2
         ELSE
            A = ATAN (SQRT(( 1-COSA**2 )/( COSA**2 )))
            C4= A/PI
            IF (R.LT.0) THEN
               C4=1-C4
            ENDIF
            GAMMA4= BTA * ( 1/RAT - C4*( 1/RAT+1 ))
         ENDIF
      ENDIF
      END
C
C************************************************************
C** FUNCTION GAMMAQ => FINDS THE RELATIVE PHASE FOR NON-   **
C**     ZERO VALUES OF Q.  USES BAIRSTOW'S METHOD TO FIND **
C**     THE ROOTS OF THE CUBIC EQUATION FROM MOMENT        **
C**     IMPULSE OBSERVATION                                **
C************************************************************
      FUNCTION GAMMAQ(BETA,RFC,QFC)
      REAL BETA(4),RFC,QFC,GAMMAQ,EQR,EQS,DELR,DELS,PI
      REAL B4,C2,C3,DEL,COSA(3),COSD,SQCK,A2,A3,A4,B2,B3
      PI=3.1415926
C
C  ITERATE TO FIND FIRST ROOT
C
      IF (ABS(RFC).LT.0.001) THEN
         GAMMAQ=(BETA(3)-BETA(1))/2
      ELSE
```

```
         DEL=1000
         A2=-(3*RFC/(4*QFC))
         A3=(3-3*QFC)/(4*QFC)
         A4=(3*RFC)/(4*QFC)
         EQR=0
         EQS=0
         DEL=1000
161      IF (DEL.LT.1E-5) GOTO 162
            B2=A2+EQR
            B3=A3+EQR*B2+EQS
            B4=A4+EQR*B3+EQS*B2
            C2=B2+EQR
            C3=B3+EQR*C2+EQS
            DELR=(B4-B3*C2)/(C2**2-C3)
            DELS=(C3*B3-C2*B4)/(C2**2-C3)
            DEL=ABS(B3)+ABS(B4)
            EQR=EQR+DELR
            EQS=EQS+DELS
         GOTO 161
162      CONTINUE
C
C  COSA VARIABLES ARE ROOTS OF CUBIC
C
         COSA(1)=-(A2+EQR)
         SQCK=EQR**2+4*EQS
C
C  CHECK FOR COMPLEX ROOTS IN QUADRATIC REMAINDER
C
         IF (SQCK.LT.0) THEN
            COSA(2)=1000
            COSA(3)=1000
         ELSE
C
C FIND ROOTS OF QUADRATIC REMAINDER
C
            COSA(2)=(EQR+SQRT(SQCK))/2
            COSA(3)=(EQR-SQRT(SQCK))/2
         ENDIF
C
C FIND MATHEMATICALLY POSSIBLE ROOT
C
         DO 163 I=1,3
            IF (ABS(COSA(I)).LT.1) THEN
              COSD=COSA(I)
            ENDIF
163      CONTINUE
C
C SOLVE FOR RELATIVE PHASE FROM ROOT OF CUBIC
C
         B2=ATAN((1-COSD**2)/COSD**2)
         IF (COSD.LT.0) THEN
            B2=PI-B2
         ENDIF
         B2=(B2/PI)
```

```
         B4=1-B2
         GAMMAQ=(BETA(3)*B2-BETA(1)*B4)
      ENDIF
      END
C
C*************************************************************
C** SUBROUTINE PTCALC => FIND PITCH ANGLE RANGE FOR CYCLE **
C*************************************************************
      SUBROUTINE PTCALC(PHI,CYCTIM,CFX,R,Q,PTSPR)
      REAL PHI(4),TIME(401),VX,Y(4),BETA(4),RATIO,XLAMBA(4)
      REAL XH,TIM,PPH,BET,VPT(401),PTCH,APLST,VPLST,APT
      REAL PTLO,PTSPR,ZBLST,CYCTIM,XLMBA,XFT(4),XB(401)
      REAL PITCH(401),CFX,ZB(401),BODY,ASAVE,ACCP(401)
      REAL PHIHI,PTHI,R,Q,DUMMY,RS
      INTEGER II,IJ,IHI
      COMMON/ROBDATA/VX,ZBLST,Y,BETA,RATIO,XLAMBA,XH
      ASAVE=0
      PTLST=0
      BODY=1.7083
      PTHI=-1000
      PTLO=1000
      APLST=0
      VPLST=0
C
C FIND VARIABLE MOMENT ARMS FOR ANGULAR ACCELERATION CALC.
C
      CALL ZBCALC(ZB,PHI,TIME,IHI,R,Q,DUMMY)
      CALL XBCALC(XB,PHI,IHI,BETA,XH,CFX)
      DO 103 II=1,IHI
          VB(II)=VB(II)+VX
          ZB(II)=ZB(II) * (CYCTIM**2) / BODY + ZBLST
          XB(II)= (CYCTIM**2) * XB(II) / BODY+ TIME(II) * VX
103   CONTINUE
      DO 122 II=1,IHI
C
C RESET ACCELERATION VARIABLE FOR NEXT TIMESTEP
C
          APT=0
          TM=TIME(II)
          DO 123 IJ=1,4
              PPH=PHI(IJ)
              BET=BETA(IJ)
              XLMBA=XLAMBA(IJ)
C
C CALCULATE THE FOOT POSITION RELATIVE TO THE BODY Cg
C
              XFT(IJ)=XOFFT(PPH,BET,XB,XLMBA,IHI,TM)-XB(II)
              RS=R
              IF (IJ.EQ.1.OR.IJ.EQ.2) THEN
                 RS=-R
              ENDIF
C
C CALCULATE THE ANGULAR ACCELERATION
C
```

```
                APT=FORCEZ(TM,PPH,BET,RS,Q)*XFT(IJ) + APT
                APT=APT  +  FORCEX(TM,PPH,BET,CFX,RS,Q) * ZB(II)
123         CONTINUE
            ACCP(II)=-APT
            ASAVE=ASAVE+ACCP(II)
122     CONTINUE
        ASAVE=-ASAVE/IHI
C
C ADJUST TO REQUIRED MEAN VALUES
C
        DO 124 II=1,IHI
          ACCP(II)=ACCP(II)+ASAVE
          CALL DOUBLE(ACCP(II),VPT(II),PITCH(II),APLST,VPLST
       *,PTLST,XH)
124     CONTINUE
        VPLST=-PITCH(IHI)
        DO 178 II=1,IHI
            PITCH(II)=PITCH(II)+VPLST*TIME(II)
            VPT(II)=VPT(II)+VPLST
C
C FIND MAX AND MIN DEVIATION FROM MEAN DURING CYCLE
C
            IF (PITCH(II).GT.PTHI) THEN
                 PTHI=PITCH(II)
            ELSEIF (PITCH(II).LT.PTLO) THEN
                 PTLO=PITCH(II)
            ENDIF
178     CONTINUE
        PTSPR=(PTHI-PTLO)/2
        DO 179 II=1,IHI
           PITCH(II)=-(PHI+PLO)/2
178     CONTINUE
        END
C
C***********************************************************
C** SUBROUTINE XBCALC => CALCULATE POSITION DEVIATION     **
C**      OF Cg AWAY FROM THAT DUE TO Vx                   **
C***********************************************************
        SUBROUTINE XBCALC(XB,PHI,HI,BETA,XH,CFX)
        REAL XB(401),PHI(4),XLST,TM,PH,BT,VELX,X,AXLST,VLST
        REAL BETA(4),XH,CFX,VB(401),AB(401),R,Q,AX,XHI,XLO,RS
        INTEGER HI
        VLST=0
        XHI=-1000
        XLO=1000
        AXLST=0
        XLST=0
        DO 26 I=1,HI
            TM=REAL(I-1)*XH
            AX=0
            DO 27 J=1,4
                IF (J.EQ.1.OR.J.EQ.2) THEN
                   RS=-R
                ELSE
```

```
                  RS=R
               ENDIF
               PH=PHI(J)
               BT=BETA(J)
C
C ACCELERATION OF Cg
C
               AX=FORCEX(TM,PH,BT,CFX,RS,Q) + AX
27          CONTINUE
            CALL DOUBLE(AX,VELX,X,AXLST,VLST,XLST,XH)
            XB(I)=X
            VB(I)=VELX
            AB(I)=AX
26     CONTINUE
       VLST=-XB(HI)
C
C ADJUSTMENTS
C
       DO 89 I=1,HI
            TM=REAL(I-1)*XH
            VB(I)=VB(I)+VLST
            XB(I)=XB(I)+VLST*TM
            IF (XB(I).GT.XHI) THEN
                 XHI=XB(I)
            ELSEIF (XB(I).LT.XLO) THEN
                 XLO=XB(I)
            ENDIF
89     CONTINUE
       END
C
C************************************************************
C** FUNCTION XOFFT => CALCULATE POSITION OF FOOT RELATIVE **
C**      TO BODY Cg GIVEN BODY POSITION AND PRESENT TIME  **
C************************************************************
       FUNCTION XOFFT(PH,BT,XB,XLMBA,IHI,TM)
       REAL XOFFT,PH,BT,XB(401),XLMBA,TINTRP,XBASE
       INTEGER IHI,IBASE,IBSHI
       TINTRP=(PH-BT+1)*(IHI-1)
C
C VARIABLE USED IN AN INTERPOLATION PROCESS
C
       IF (PH.GT.BT) THEN
           TINTRP=(PH-BT)*(IHI-1)
       ENDIF
       IBASE=INT(TINTRP)+1
       IBSHI=IBASE+1
C
C INTERPOLATE TO FIND BODY POSITION AT BEGINNING OF SUPPORT
C      PHASE
C
       XBASE=(XB(IBSHI)-XB(IBASE))*(TINTRP-(IBASE-1))
      *+XB(IBASE)
       IF ( PH.GT.BT ) THEN
           XOFFT=XLMBA + XBASE
```

```
      ELSEIF ( TM.LT.PH) THEN
          XOFFT=XLMBA + XBASE - XB(IHI) + XB(1)
      ELSE
          XOFFT=XLMBA+XBASE
      ENDIF
      END
C
C***********************************************************
C** SUBROUTINE YWCALC => CALCULATE NORM. YAW ANGLE RANGES **
C***********************************************************
      SUBROUTINE YWCALC(PHI,R,Q,CFX,YSPR)
      REAL PHI(4),TIME(401),VX,YBLST,Y(4),BETA(4),RATIO,R,Q
      REAL XH,YLST,TM,PH,BT,VY(401),YAW,AYLST,VYLST,YHI
      REAL YLO,YSPR,ZBLST,YW(401),CFX,AY(401),XLAMBA(4)
      INTEGER HI,M,N
      COMMON/ROBDATA/VX,ZBLST,Y,BETA,RATIO,XLAMBA,XH
      VYLST=0
      HI=INT(1.0/XH)+1
      YHI=-1000
      YLO=1000
      AYLST=0
      YLST=0
      DO 32 M=1,HI
          TIME(M)=REAL(M-1)*XH
          AY(M)=0
          TM=TIME(M)
          DO 33 N=1,4
              PH=PHI(N)
              BT=BETA(N)
              IF(N.EQ.1.OR.N.EQ.2)THEN
                 RS=-R
              ELSE
                 RS=R
              ENDIF
C
C ANGULAR ACCELERATION CALCULATION
C
              AY(M)=-FORCEX(TM,PH,BT,CFX,RS,Q) * Y(N) + AY(M)
33        CONTINUE
C
C INTEGRATION OF ACCELERATION FUNCTION
C
          CALL DOUBLE(AY(M),VY(M),YAW,AYLST,VYLST,YLST,XH)
          YW(M)=YAW
32    CONTINUE
      VYLST=-YW(HI)
C
C MEAN VALUE ADJUSTMENT
C
      DO 88 M=1,HI
           YW(M)=YW(M) + VYLST * TIME(M)
           VY(M)=VY(M)+VYLST
C
C DETERMINATION OF MAX AND MIN
```

```
C
            IF (YW(M).GT.YHI) THEN
                YHI=YW(M)
            ELSEIF (YW(M).LT.YLO) THEN
                YLO=YW(M)
            ENDIF
88    CONTINUE
      YSPR=(YHI-YLO)/2
      END
C
C***********************************************************
C** FUNCTION FORCEX => HORIZONTAL FORCE OF A LEG CALC.    **
C***********************************************************
      FUNCTION FORCEX(T,PH,BT,CFX,RS,Q)
      REAL FORCEX,T,PH,BT,CX,PI,BETA(4),VX,CFX,RS,Q
      PI=3.1415926
      IF (PH.GT.BT) THEN
          IF (T.LT.(PH-BT).OR.T.GT.PH) THEN
             CX=0
          ELSE
             CX=T-(PH-BT/2)
          ENDIF
      ELSEIF (T.LT.PH)  THEN
          CX=T-(PH-BT/2)
      ELSEIF (T.GT.(PH+1-BT)) THEN
          CX=T-(PH+1-BT/2)
      ELSE
          CX=0.0
      ENDIF
      FORCEX=CFX*CX*FORCEZ(T,PH,BT,RS,Q)
      END
C
C***********************************************************
C** SUBROUTINE ZBCALC => HORIZONTAL DISP RANGE OF BODY Cg **
C***********************************************************
      SUBROUTINE ZBCALC(ZB,PHI,TIME,HI,R,Q,ZSPR)
      REAL ZB(401),VX,ZBLST,Y(4),BETA(4),RATIO,XLAMBA(4),XH
      REAL PHI(4),ZLST,TM,PH,BT,VZ(401),Z,AZLST,VZLST,RS
      REAL ZHI,ZLO,ZSPR,AZ(401),R,Q,TIME(401),GRAV
      INTEGER HI,I,J
      COMMON/ROBDATA/VX,ZBLST,Y,BETA,RATIO,XLAMBA,XH
C
C RESET VZLST,AZLST:  HI=>SET ITERATION NUMBER: SET ZLST TO
C INITIAL POSITION
C
      VZLST=0.0
      HI=INT(1.0/XH)+1
      ZHI=-100
      ZLO=100
      GRAV=32.174
      AZLST=0.0
      ZLST=0.0
      DO 22 I=1,HI
C
```

```
C   SET TIME CONSTANT FOR PLOTTING, RESET ACCELERATION, SET
C   TEMP TIME VAR.
C
         TIME(I)=REAL(I-1)*XH
         AZ(I)=0.0
         TM=TIME(I)
         DO 23 J=1,4
C
C   SET TEMPORY VARIABLES FOR CALL TO FORCEZ, CALCULATE AZ
C   FOR THIS TS
C
             PH=PHI(J)
             BT=BETA(J)
             RS=R
             IF (J.EQ.1.OR.J.EQ.2) THEN
                RS=-R
             ENDIF
             AZ(I)=FORCEZ(TM,PH,BT,RS,Q) + AZ(I)
23       CONTINUE
C
C  CALL TRAP. DOUBLE INTEGRATION ROUTINE TO CALCULATE THE
C  POS. Z
C
         AZ(I)=AZ(I)-GRAV
         CALL DOUBLE(AZ(I),VZ(I),Z,AZLST,VZLST,ZLST,XH)
         ZB(I)=Z
22     CONTINUE
      VZLST=-ZB(HI)
      DO 78 I=1,HI
         ZB(I)=ZB(I)+VZLST*TIME(I)
         VZ(I)=VZ(I)+VZLST
         IF (ZB(I).GT.ZHI) THEN
                ZHI=ZB(I)
         ELSEIF(ZB(I).LT.ZLO) THEN
                ZLO=ZB(I)
         ENDIF
78    CONTINUE
      ZLST=-(ZHI+ZLO)/2
      DO 77 I=1,HI
          ZB(I)=ZB(I)+ZLST
77    CONTINUE
      ZSPR=(ZHI-ZLO)/2
      END
C
C************************************************************
C**              SUBROUTINE WRTPARA                        **
C************************************************************
      SUBROUTINE WRTPARA(CL)
      REAL VX,ZLST,YF(4),BET(4),RATIO,XLAM(4),XH,CL
      COMMON/ROBDATA/VX,ZLST,YF,BET,RATIO,XLAM,XH
      LOGICAL ABORT
      CALL GETWFN(32,'ENTER FILENAME TO BE SAVED$',ABORT)
      IF (ABORT) RETURN
      WRITE(32,*) VX,ZLST,RATIO,XH,CL
```

```
      DO 25 I=1,4
        WRITE(32,*)YF(I)
25    CONTINUE
      CLOSE(32)
      END
C
C***********************************************************
C**                    SUBROUTINE INKEYP                  **
C***********************************************************
      SUBROUTINE INKEYP(CL)
      REAL VX,ZLST,YF(4),BET(4),RATIO,XLAM(4),XH,CL
      COMMON/ROBDATA/VX,ZLST,YF,BET,RATIO,XLAM,XH
      WRITE(3,'(1H,///)')
      WRITE(3,*)'ENTER THE VALUE FOR VELOCITY OF BODY'
      READ(3,*) VX
      WRITE(3,*)'ENTER THE VALUE FOR Z MEAN OF BODY'
      READ(3,*) ZLST
      WRITE(3,*)'ENTER THE VALUE FOR Y OF FEET'
      DO 27 I=1,4
          WRITE(3,*)'Y',I,'='
          READ(3,*) YF(I)
27    CONTINUE
      WRITE(3,*)'ENTER VALUE OF DELTA LAMBA'
      READ(3,*)CL
      WRITE(3,*)'ENTER DESIRED TIMESTEP FOR ITERATION'
      READ(3,*) XH
      END
C
C***********************************************************
C**                 SUBROUTINE RDPARAM                    **
C***********************************************************
      SUBROUTINE RDPARAM(CL)
      REAL VX,ZLST,YF(4),BET(4),RATIO,XLAM(4),XH,CL
      COMMON/ROBDATA/VX,ZLST,YF,BET,RATIO,XLAM,XH
      LOGICAL ABORT
      CALL GETRFN(32,'ENTER NAME OF FILE TO BE READ$',ABORT)
      IF (ABORT) RETURN
      READ(32,*) VX,ZLST,RATIO,XH,CL
      DO 24 I=1,4
          READ(32,*) YF(I)
24    CONTINUE
      CLOSE(32)
      END
C
C***********************************************************
C**                   SUBROUTINE PARADEF                  **
C***********************************************************
      SUBROUTINE PARADEF(CL)
      REAL CL,VX,ZLST,YF(4),BET(4),RATIO,XLAM(4),XH
      COMMON/ROBDATA/VX,ZLST,YF,BET,RATIO,XLAM,XH
      INTEGER IBRNCH
11    WRITE(3,'(1H,///)')
      WRITE(3,*)'ENTER CHOICE FOR PARAMETER DEFINITION'
      WRITE(3,*)'1=READFILE     2=WRITEFILE       3=KEYBOARD'
```

```
      WRITE(3,*)'                        4=END  INPUT  ROUTINE'
      READ(3,*) IBRNCH
      IF (IBRNCH.EQ.1) THEN
           CALL  RDPARAM(CL)
      ELSEIF (IBRNCH.EQ.2) THEN
           CALL  WRTPARA(CL)
      ELSEIF (IBRNCH.EQ.3) THEN
           CALL  INKEYP(CL)
      ENDIF
      IF (IBRNCH.NE.4) GOTO 11
      END
C
C************************************************************
C**  SUBROUTINE DOUBLE => DOUBLE INTEGRATION SUBROUTINE   **
C************************************************************
        SUBROUTINE DOUBLE(ACC,VEL,POS,ALST,VLST,POSLST,STP)
        REAL ACC,VEL,POS,ALST,VLST,POSLST,STP
        VEL=VLST+((ACC+ALST)/2)*STP
        POS=POSLST+((VEL+VLST)/2)*STP
        ALST=ACC
        VLST=VEL
        POSLST=POS
        END
```

APPENDIX B

CONTOUR PLOTTING PROGRAM LISTING

```
C  PROGRAM CONTOUR WILL PLOT A SIMPLE CONTOUR PLOT FROM A
C  FILE, DATA1, THAT IS PRODUCED FROM AN EXTERNAL PROGRAM IN
C  THE FORMAT: Y-AXIS, X-AXIS, CONTOUR FUNCTION.  A SEPARATE
C  FILE, TITLES, THAT CONTAINS THE AXES MARKINGS AND CURVE
C  LABELS IS ALSO REQUIRED
C
      REAL Y(4,100),X(4,100),TGT(4),YVARB,XVARB,CONTOR,YLST
     *,XLST
      REAL CONLST,YOUT(100),XOUT(100),YLOW,YHIGH,XHIGH,XLOW
      INTEGER NP(4),NPLT,NOUT,I,J,K
      LOGICAL MARK1,MARK2,MARK3,MARK4,ABORT
      CHARACTER CURVMK(4)*8,MARK*8,TITLE*80,XLAB*60,YLAB*60
      CALL GETRFN(34,'ENTER NAME OF DATA READ FILE$',ABORT)
      K=1
      OPEN(214,FILE='TITLES')
C
C READ THE CURVE LABELS AND AXES TITLES FROM FILE
C   TGT => THE VALUES DESIRED FOR THE CONTOUR LINES, 4 MAX.
C
      DO 39 I=1,4
         READ(214,*)TGT(I)
         READ(214,*)CURVMK(I)
39    CONTINUE
      NPLT=4
      READ(214,*)TITLE
      READ(214,*)XLAB
      READ(214,*)YLAB
      YLST=12
      XLST=0.5
      CLOSE(214)
      CONLST=0
C
C READ THE VALUES FOR THE TRI-DIMENSIONAL PLOT
C  YVARB IS THE Y-AXIS, XVARB IS THE X-AXIS, CONTOR IS THE
C  Z-AXIS FUNCTION.  READ UNTIL FILE END ENCOUNTERED
C
50     READ(34,*,END=100) YVARB,XVARB,CONTOR
C
C FIND THE LOW AND HIGH VALUES OF THE AXES VALUES FOR PLOT
C  SCALING
C
      IF (K.EQ.1) THEN
         YLOW=YVARB
         YHIGH=YVARB
      ELSEIF(YVARB.GT.YHIGH) THEN
         YHIGH=YVARB
      ELSEIF(YVARB.LT.YLOW) THEN
         YLOW=YVARB
      ENDIF
      IF (K.EQ.1) THEN
         XHIGH=XVARB
         XLOW=XVARB
      ELSEIF (XVARB.GT.XHIGH) THEN
         XHIGH=XVARB
```

```
      ELSEIF (XVARB.LT.XLOW) THEN
         XLOW=XVARB
      ENDIF
      K=K+1
C
C FIND IF PRESENT AND PAST Z-AXIS FUNCTION STRADDLES A
C   DESIRED CONTOUR LINE VALUE
C
      DO 14 I=1,4
         MARK1=(CONTOR.GT.TGT(I))
         MARK2=(CONLST.LT.TGT(I))
         MARK3=(MARK1.EQ.MARK2)
         MARK4=(YVARB.EQ.YLST)
C
C IF YES INTERPOLATE TO FIND APPROXIMATE CONTOUR LINE POINT
C   THAT THIS INDICATES AND SAVE TO PLOTING MATRIX
C
        IF (MARK3.AND.MARK4) THEN
            NP(I)=NP(I)+1
            X(I,NP(I))=((TGT(I)-CONLST)/(CONTOR-CONLST))
            X(I,NP(I))= X(I,NP(I))*(XVARB-XLST)+XLST
            Y(I,NP(I))=YVARB
         ENDIF
14    CONTINUE
C
C SAVE LAST READ VALUES FOR POSSIBLE FUTURE INTERPOLATION
C
      CONLST=CONTOR
      YLST=YVARB
      XLST=XVARB
      GOTO 50
100   CONTINUE
      CLOSE(34)
C
C DESIGNATE PLOTTING FILE AND SAVE MATRICES IN FORM THAT
C   IS COMPATABLE WITH QCKPLT LIBRARY SUBROUTINE
C
      CALL GETWFN(201,'ENTER THE PLOTFILE NAME$',ABORT)
      DO 16 I=1,4
         DO 18 J=1,NP(I)
            XOUT(J)=X(I,J)
            YOUT(J)=Y(I,J)
18       CONTINUE
         NOUT=NP(I)
         XOUT(NOUT+1)=XLOW
         XOUT(NOUT+2)=XHIGH
         YOUT(NOUT+1)=YLOW
         YOUT(NOUT+2)=YHIGH
         MARK=CURVMK(I)
C
C WRITE PLOT TO FILE USING LIBRARY SUBROUTINES QCKPLT AND
C                    ADDPLT
         IF (I.EQ.1) THEN
            CALL QCKPLT(XOUT,YOUT,NOUT,4,XLAB,YLAB,TITLE,
```

```
     *       MARK,1,43)
          ELSE
             CALL ADDPLT(XOUT,YOUT,NOUT,MARK,1)
          ENDIF
16     CONTINUE
       CLOSE(201)
       STOP
       END
```

APPENDIX C

FORCES EXERTED BY LEGS AND BODY DISPLACEMENTS

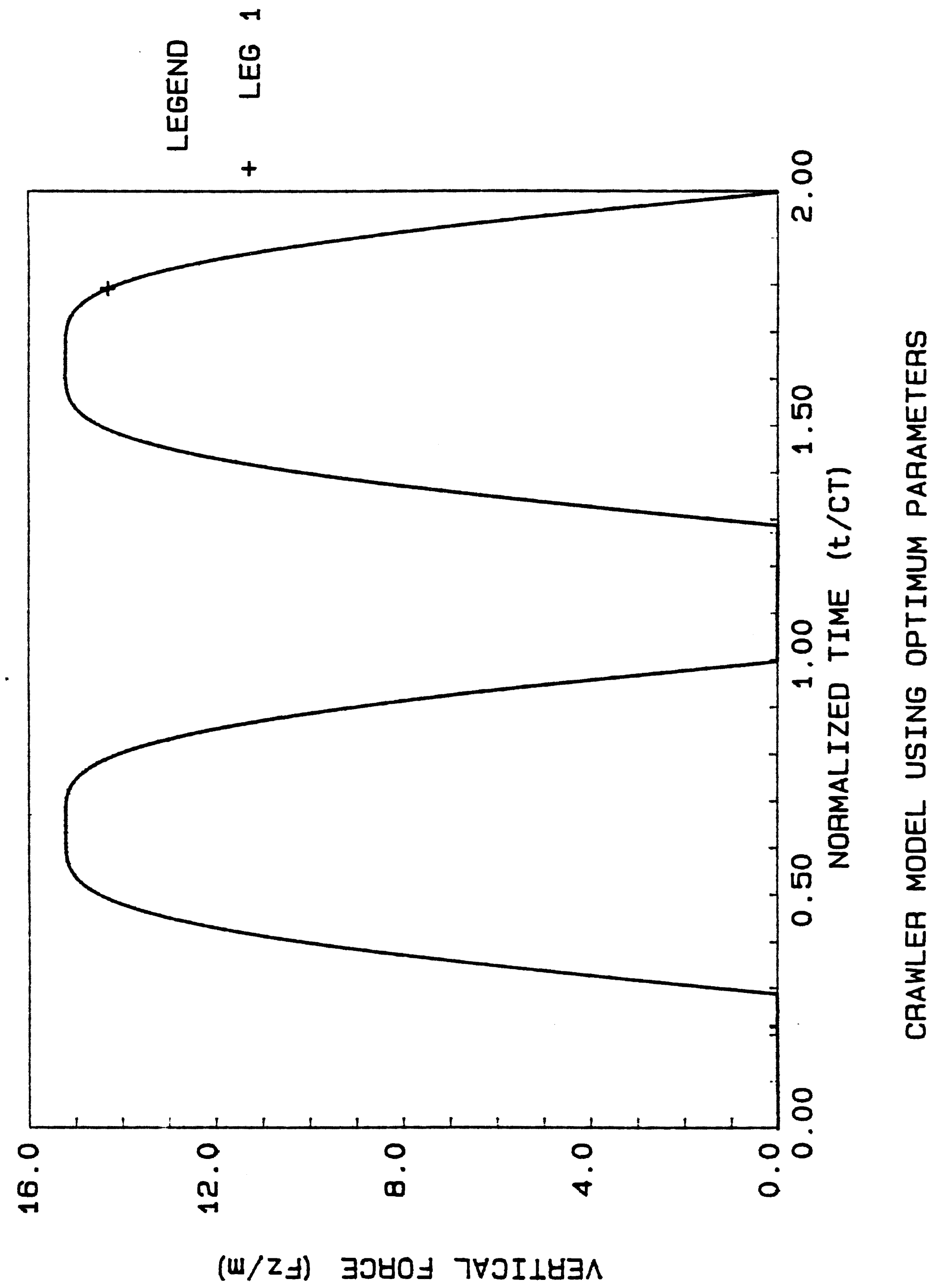


Figure 31. Vertical Force of Leg 1

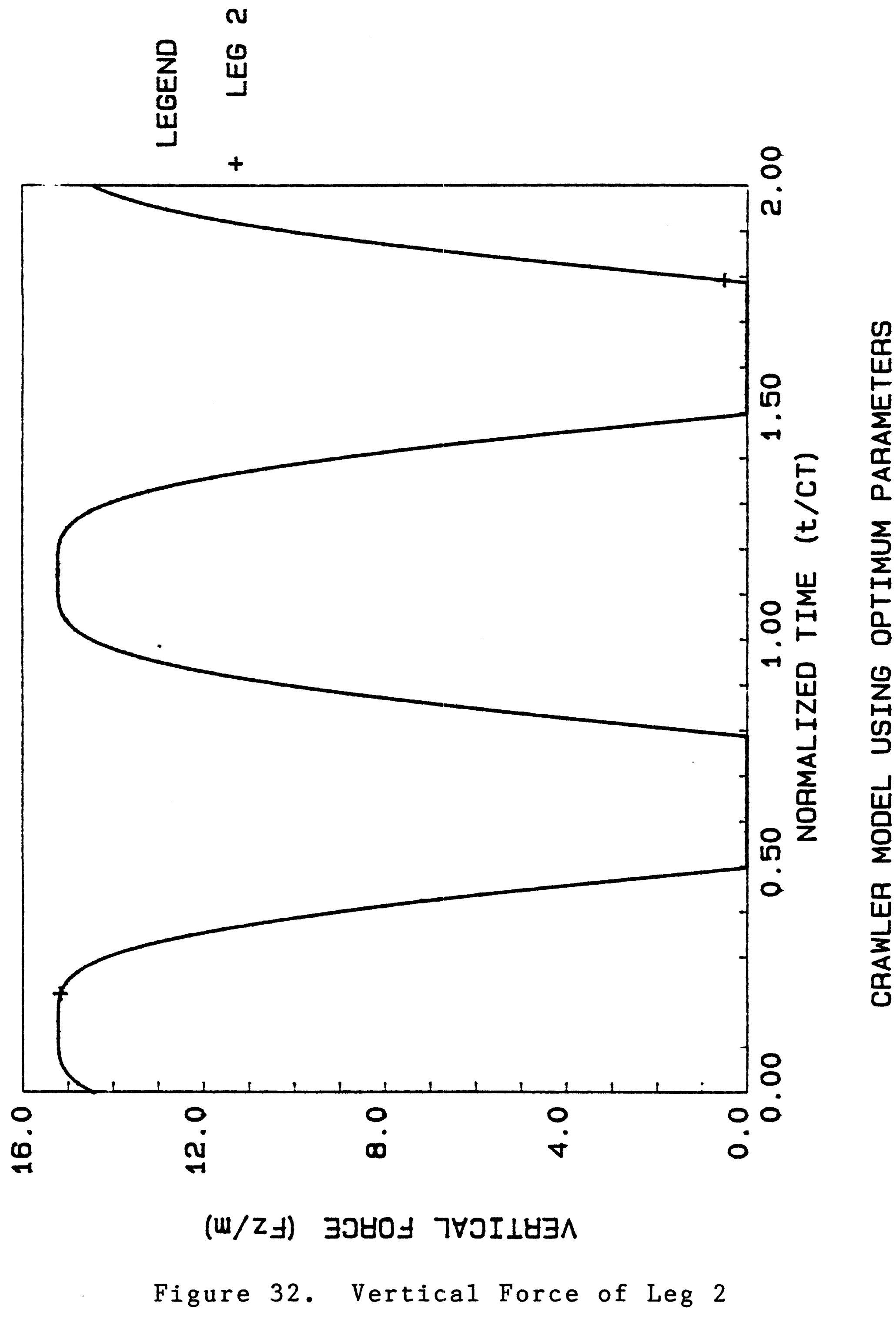


Figure 32. Vertical Force of Leg 2

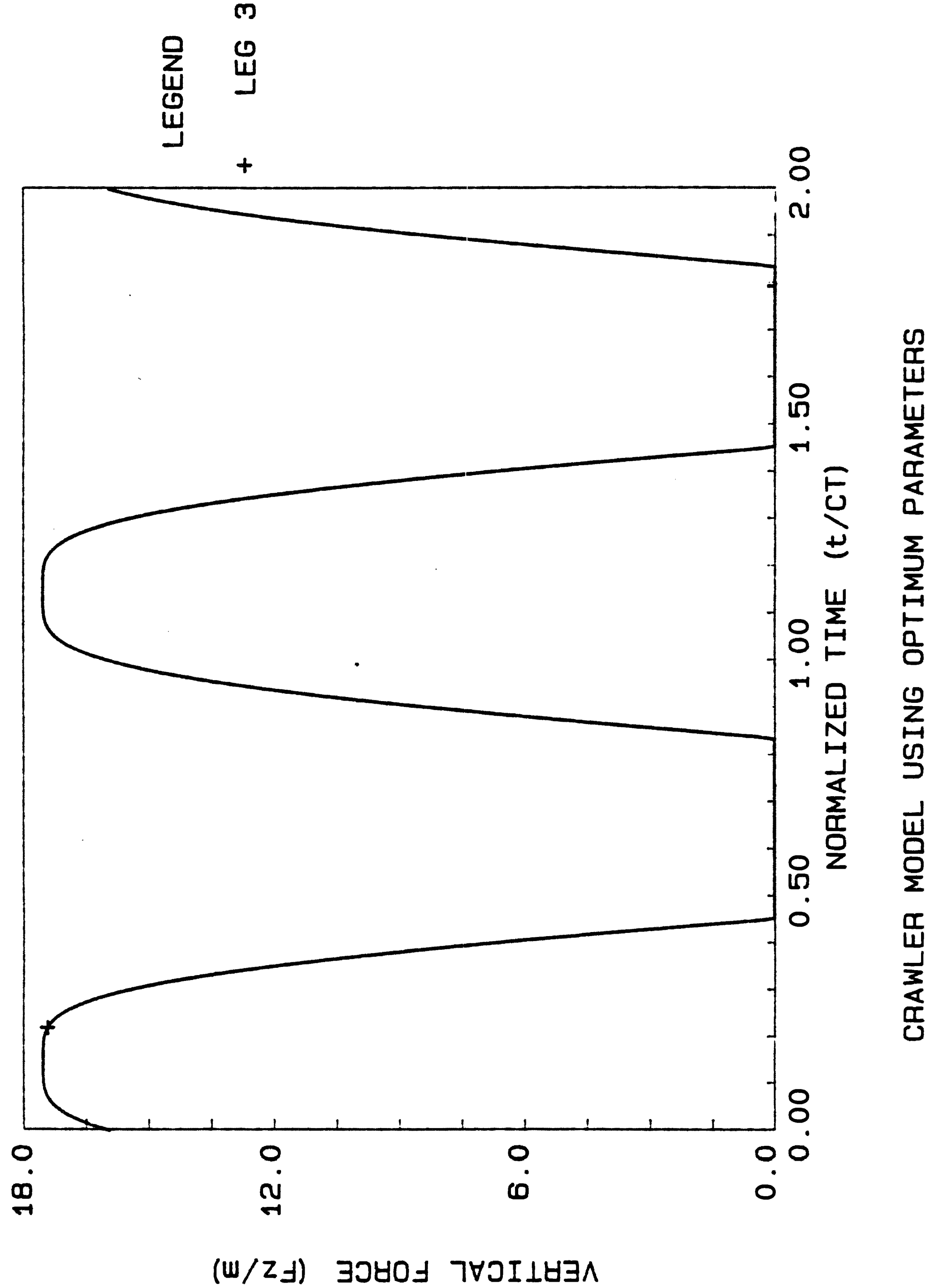


Figure 33. Vertical Force of Leg 3

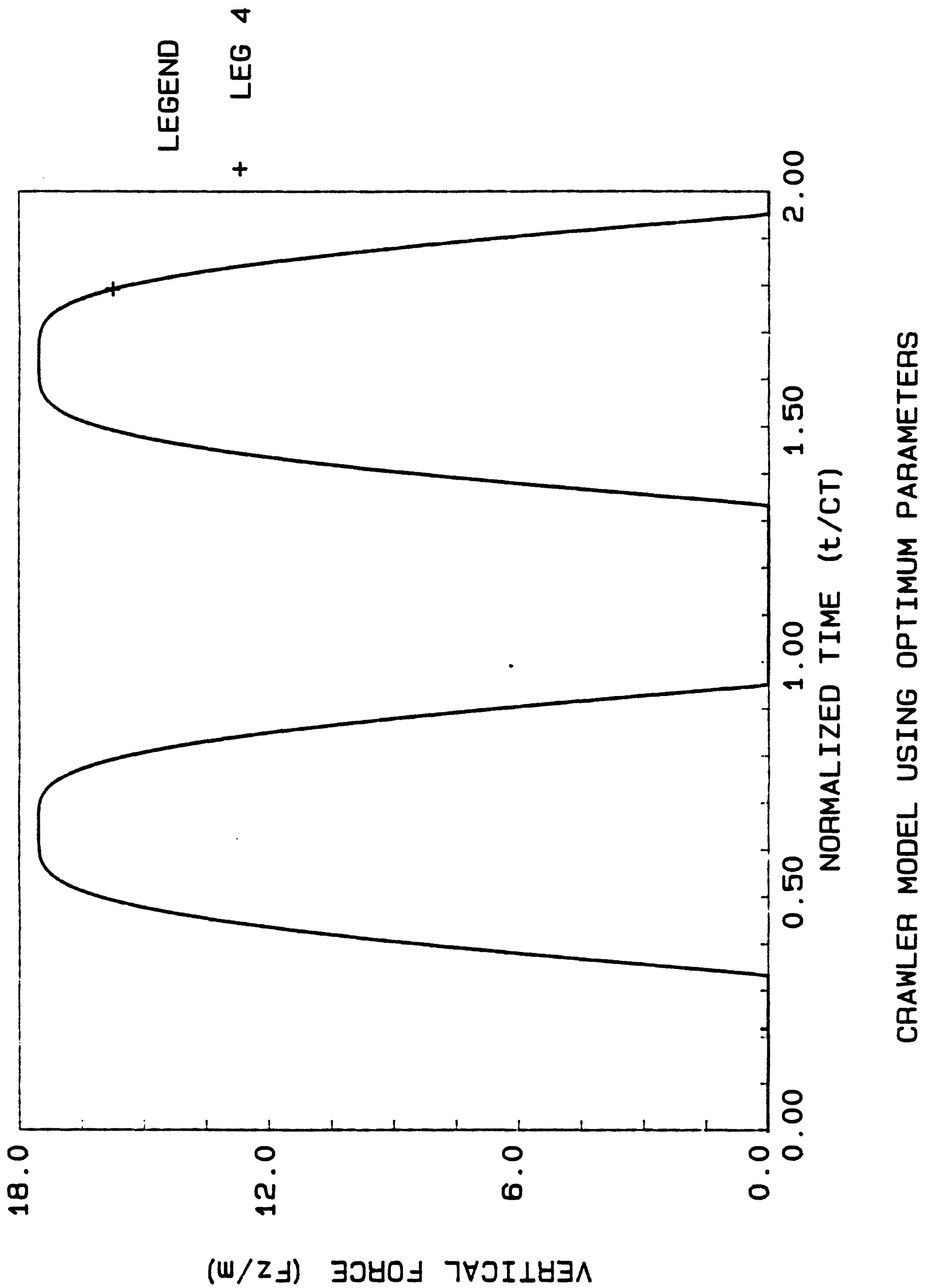


Figure 34. Vertical Force of Leg 4

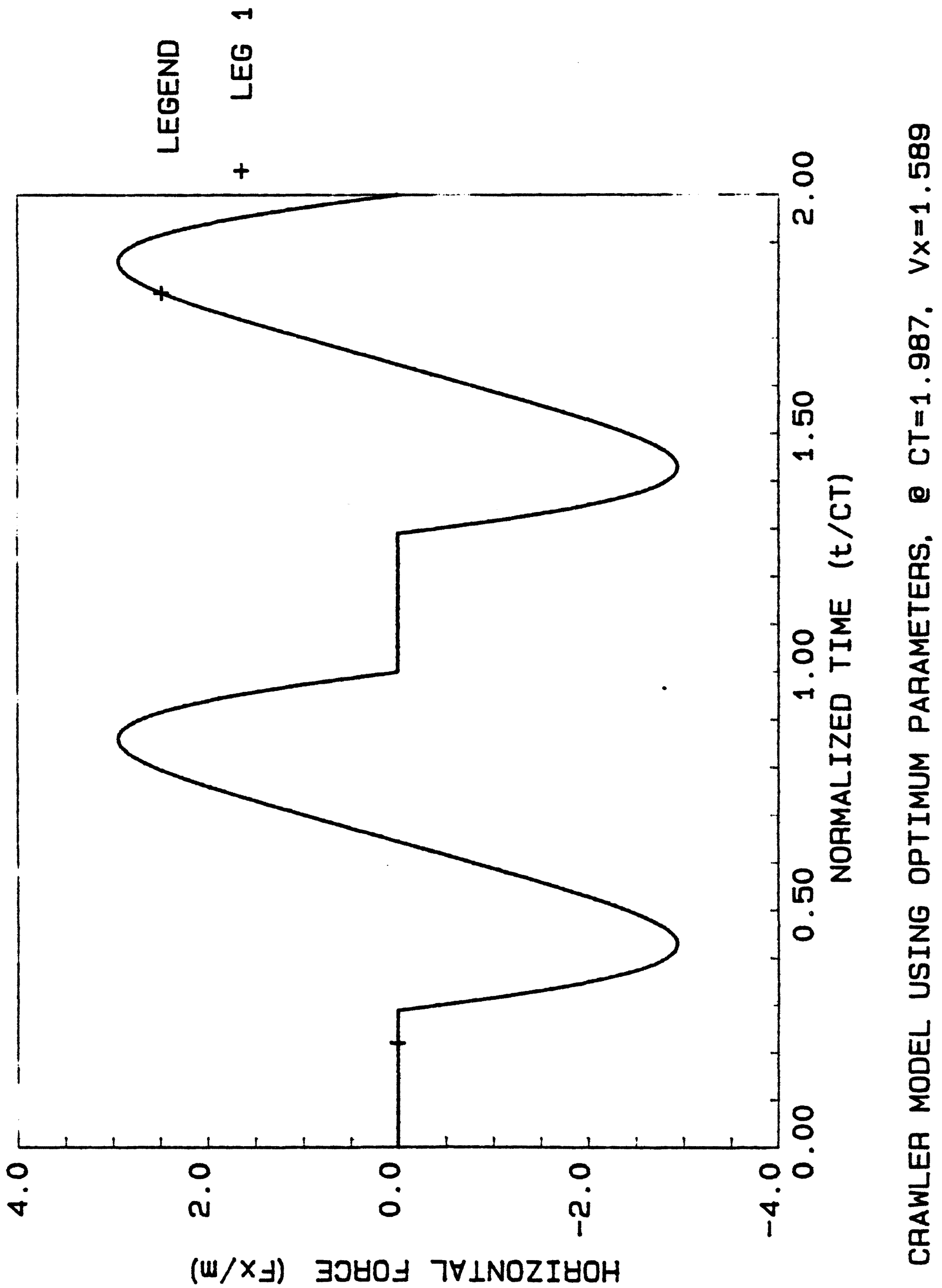


Figure 35. Horizontal Force of Leg 1

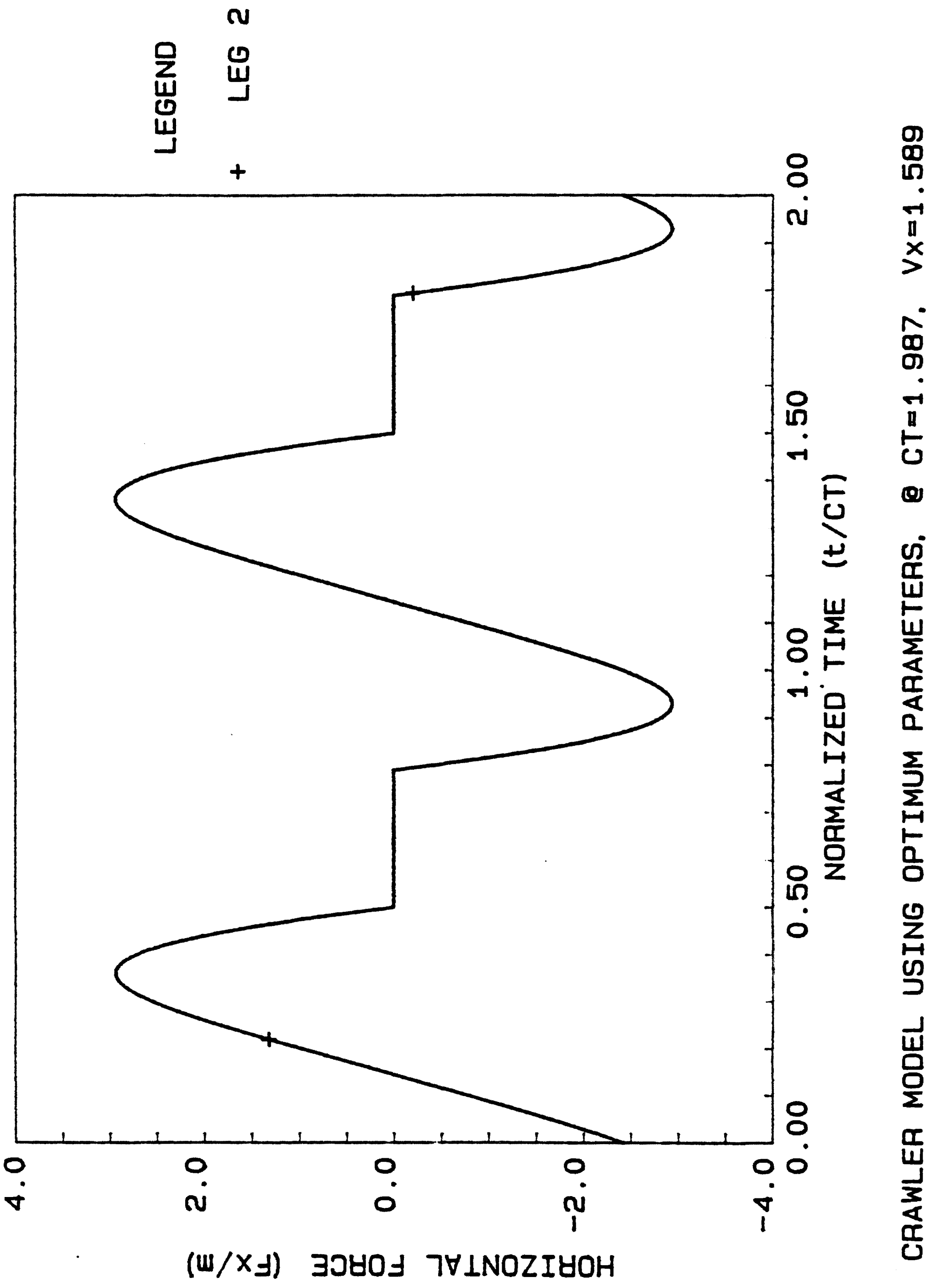


Figure 36. Horizontal Force of Leg 2

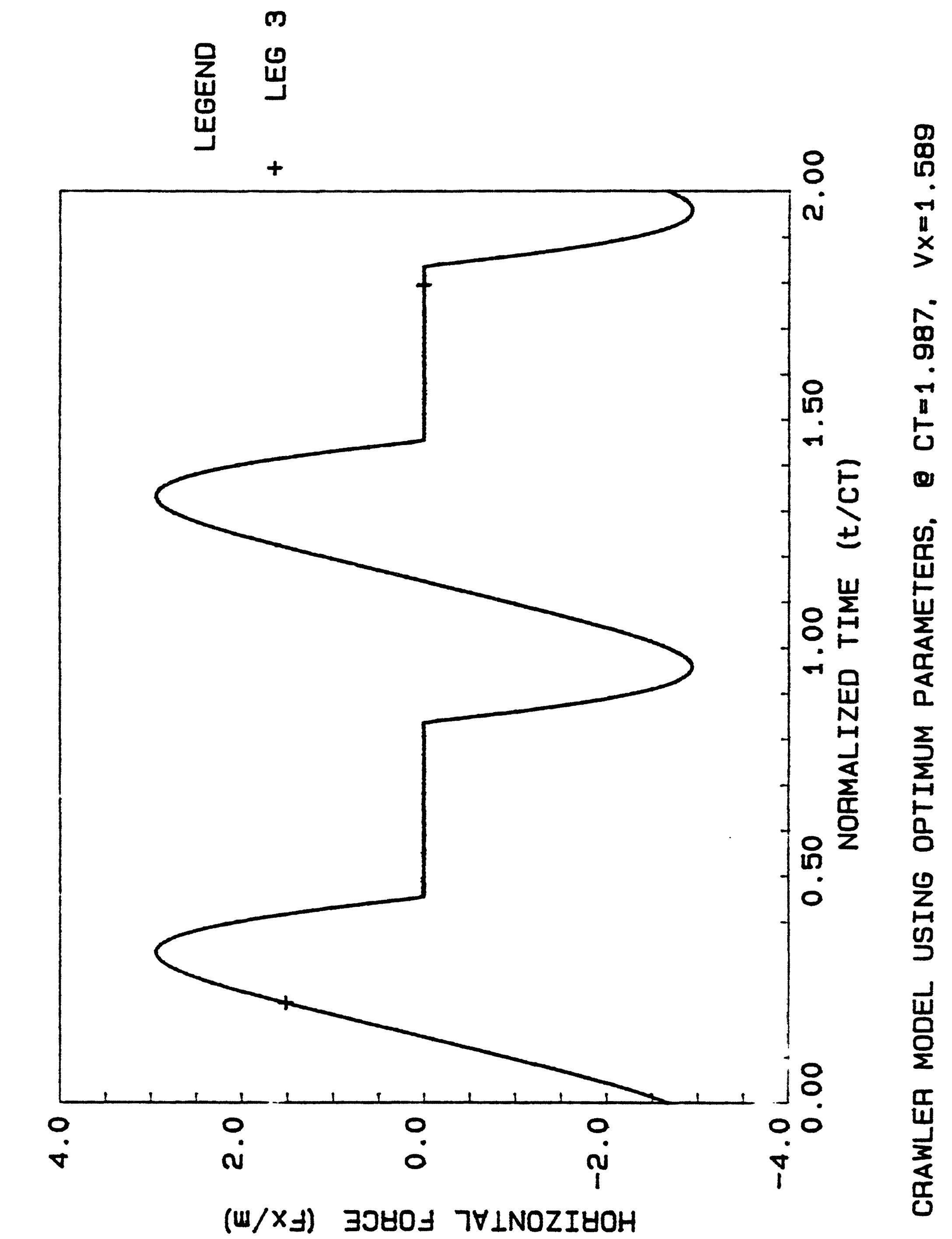


Figure 37. Horizontal Force of Leg 3

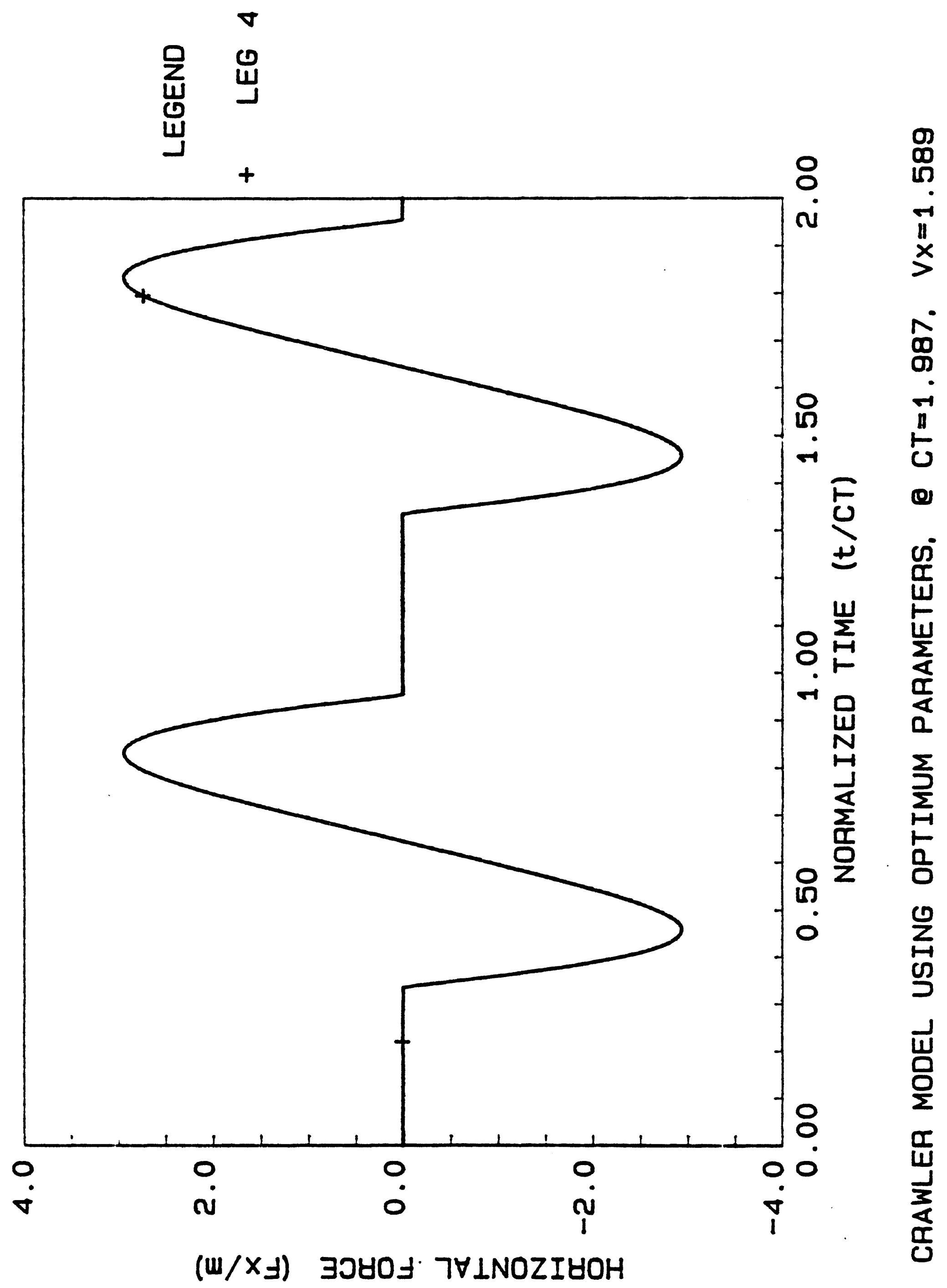
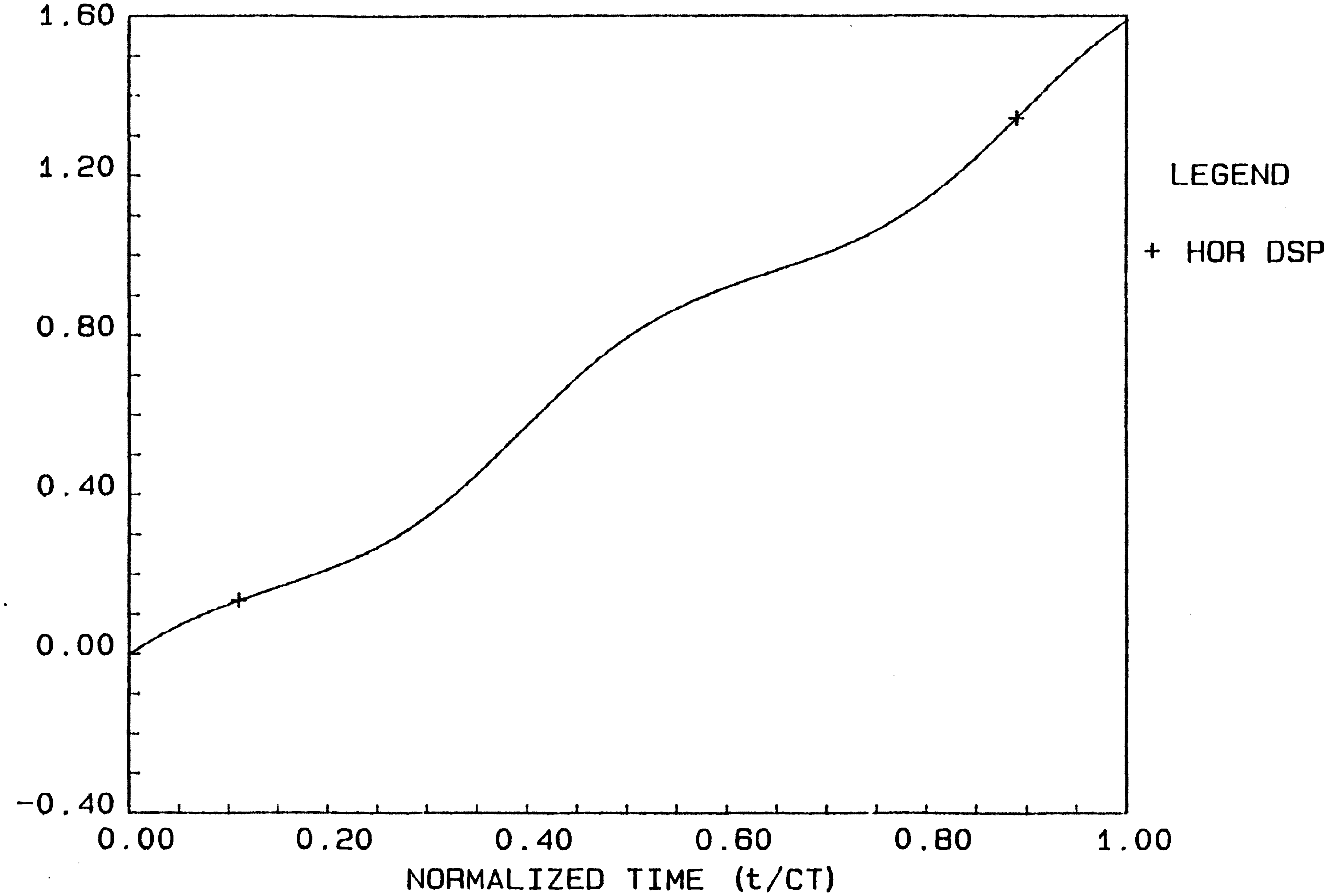


Figure 38. Horizontal Force of Leg 4



CALCULATED HORIZONTAL DISP. OF BODY CG (Vxm=1.589, CT=1.987s.)

Figure 39. Normalized X_b of Body C_g

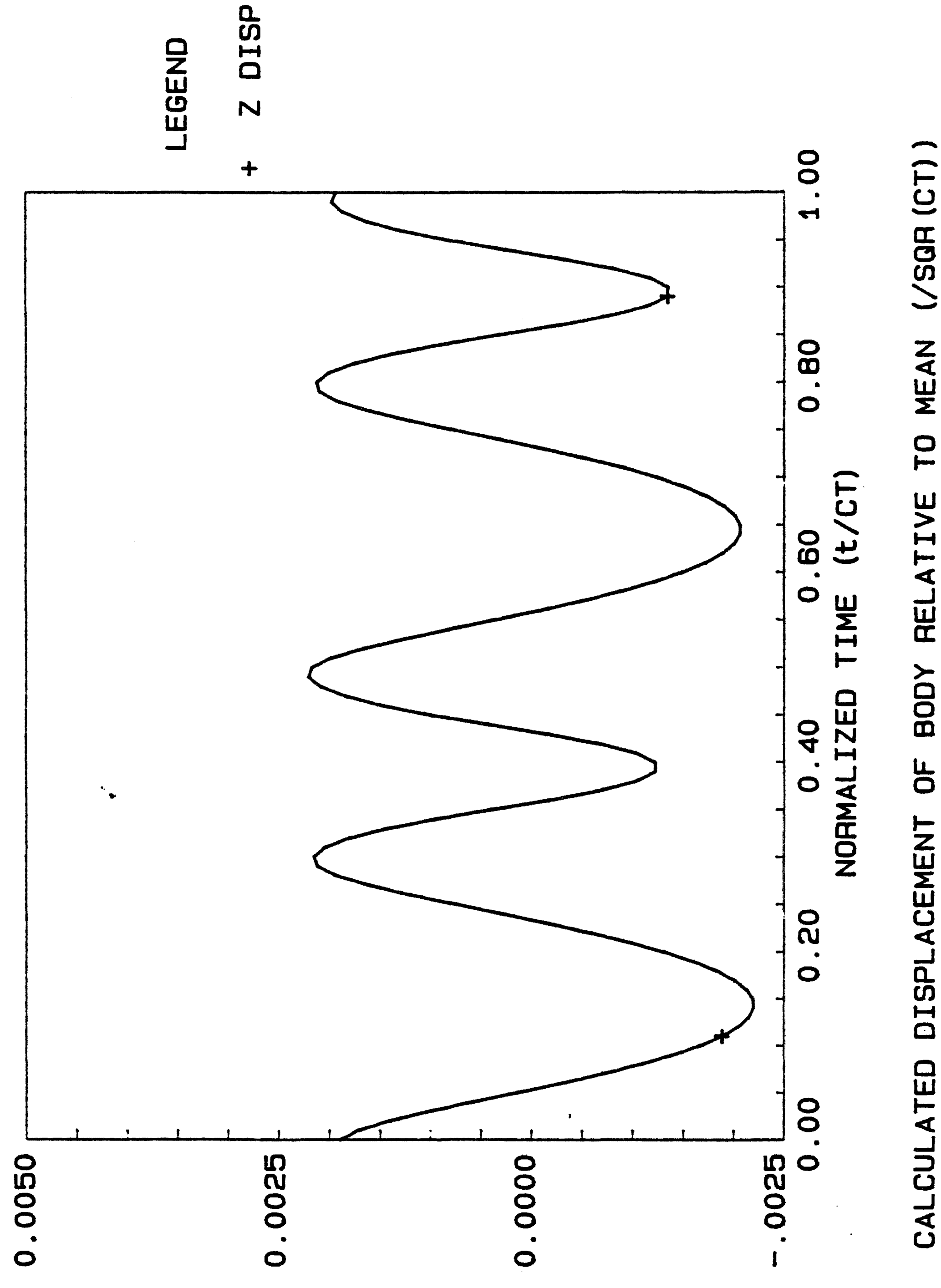


Figure 40. Normalized Z_b of Body C_g

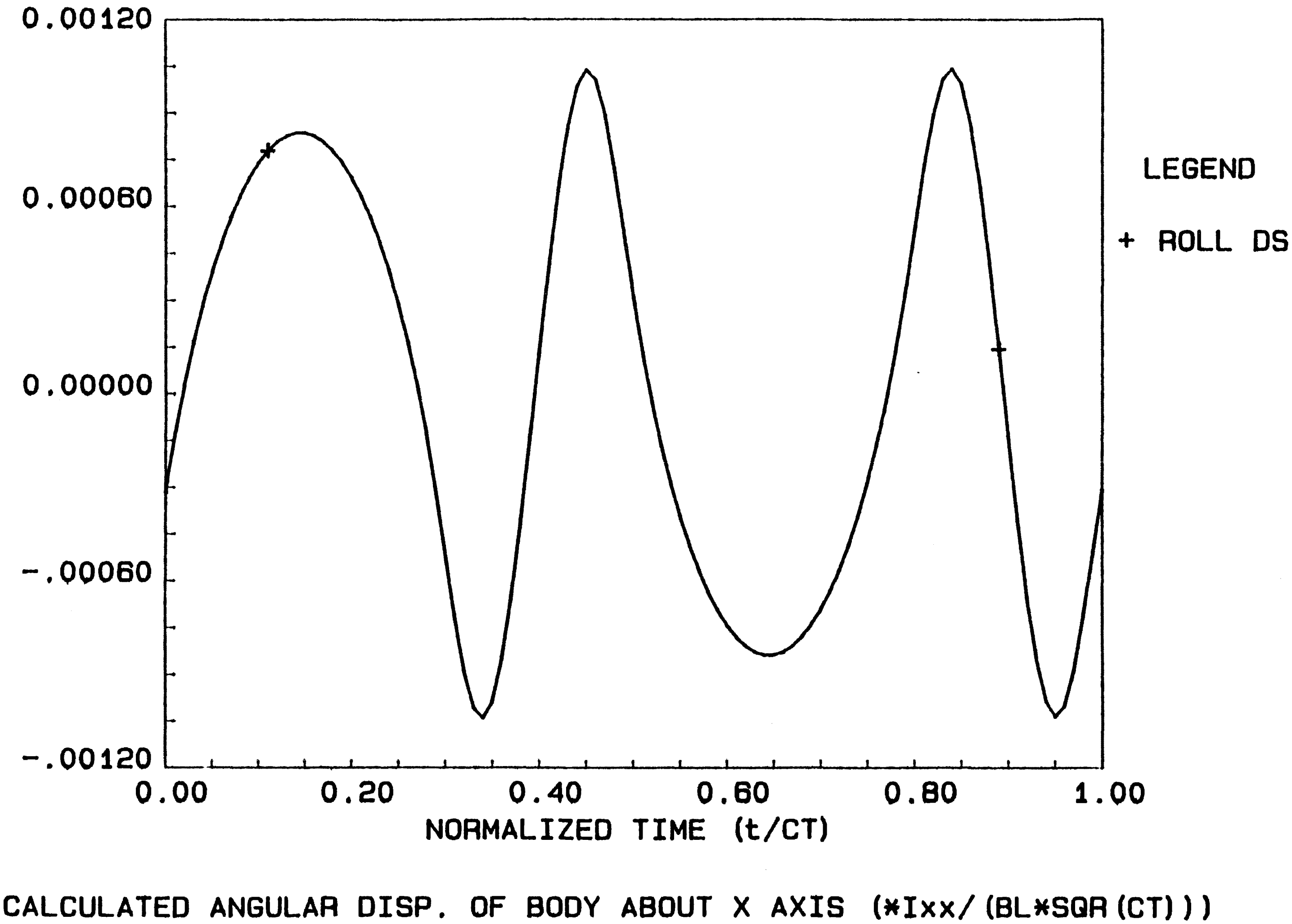


Figure 41. Normalized Body Roll Angle

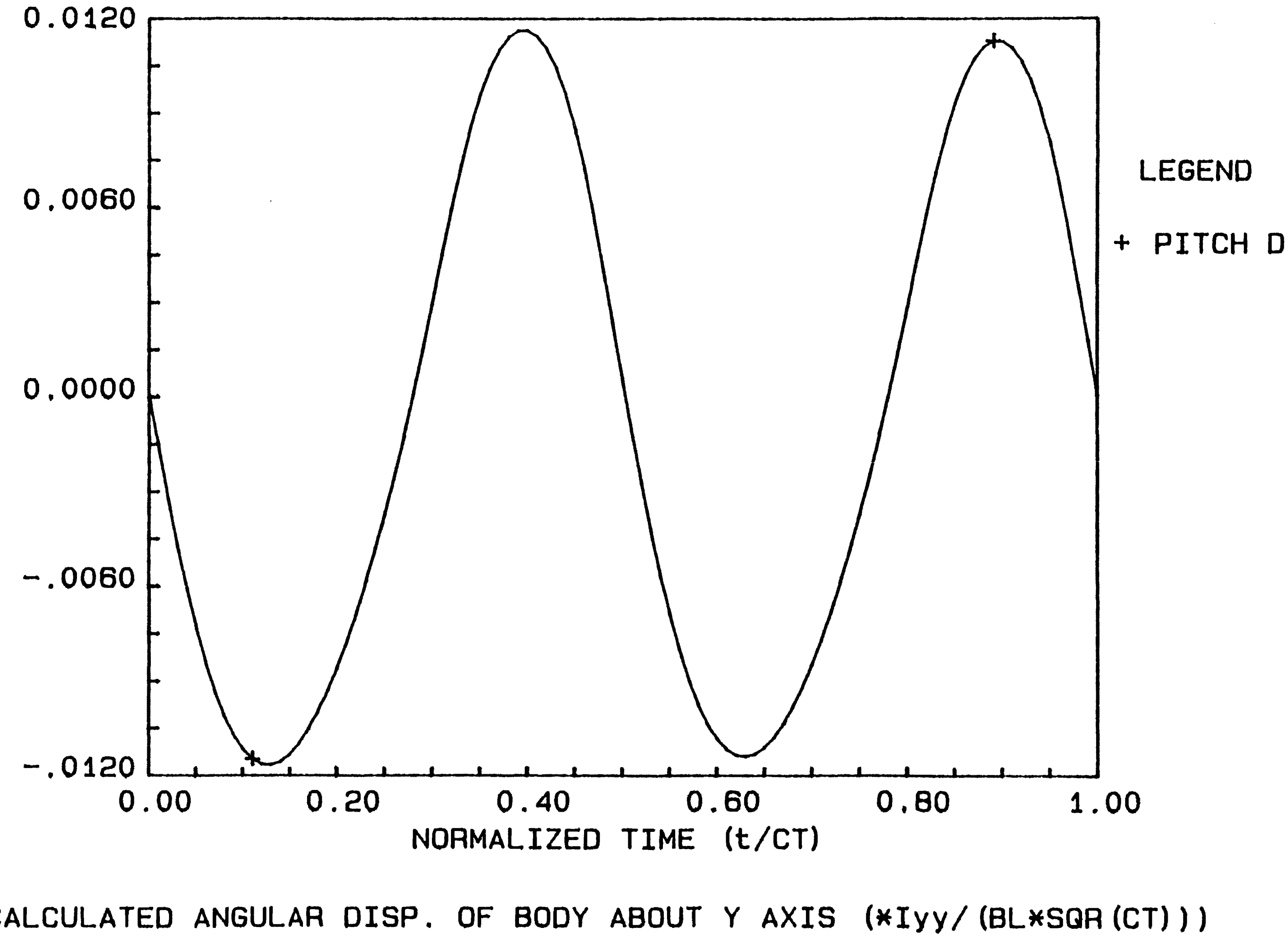


Figure 42. Normalized Body Pitch Angle

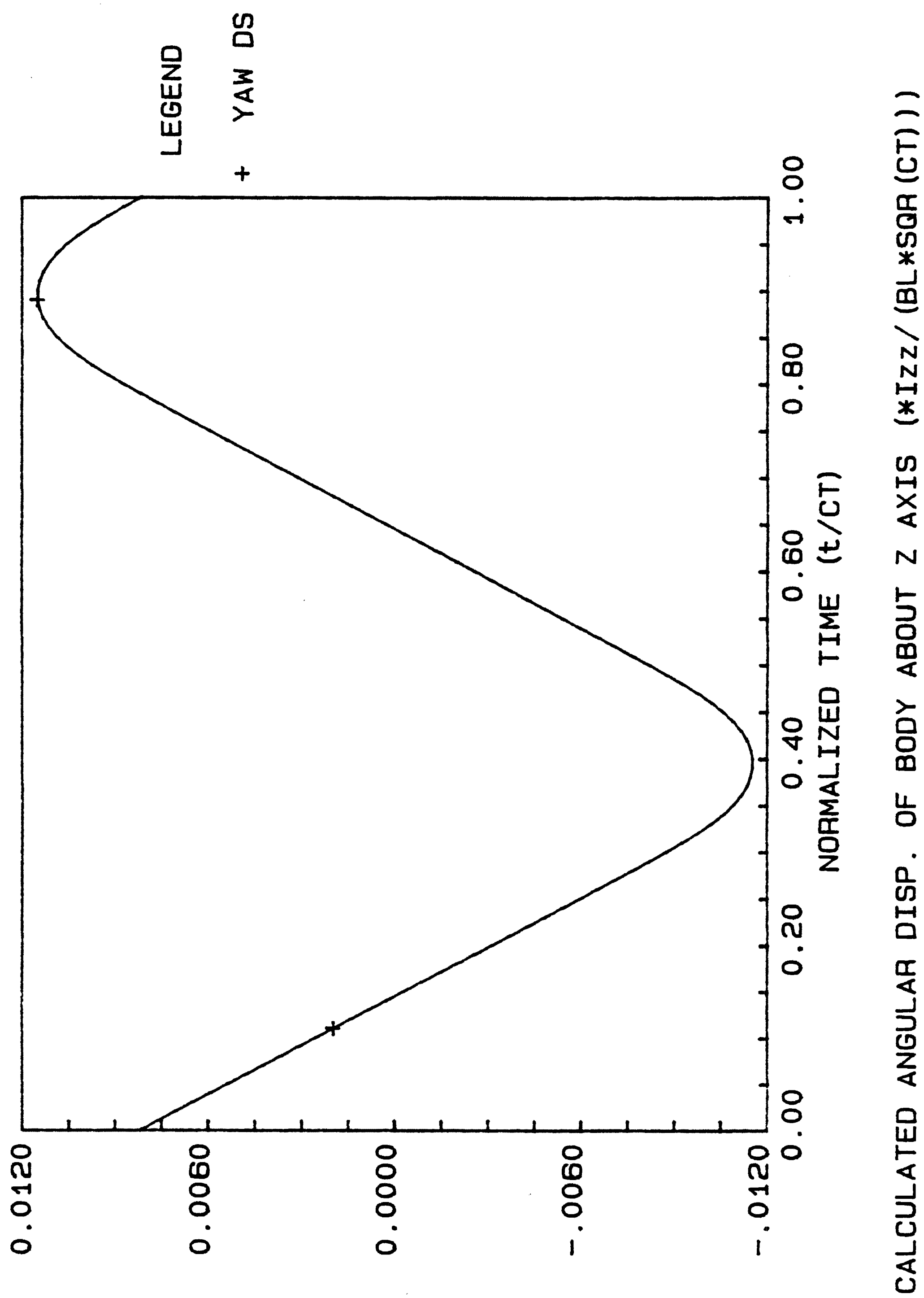


Figure 43. Normalized Body Yaw Angle

VITA

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