

A STUDY OF THREE-PHASE BRIDGE RECTIFIER  
LINE CURRENT HARMONICS

By

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## PREFACE

The purpose of this study was to develop a mathematical model for the full-wave rectifier line current harmonics using a combination of analytical and numerical methods. The validity of the theoretical investigation was verified experimentally. The results of this study are useful in predicting the line current harmonic content.

I wish to express my sincere gratitude and appreciation to my thesis advisor, Dr. William L. Hughes, for his continuous assistance and encouragement. I am also grateful to Dr. R. Ramakumar, my committee member, for his assistance in finding some of the references, and for his encouragement and teaching excellence.

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My deepest appreciation is extended to the Tunisian government which provided me through the scientific Mission of Tunisia the financial support during my undergraduate and graduate studies. Also, I deeply express my thanks and appreciation to the School of Electrical Engineering, Oklahoma State University, for providing me an excellent education and support.

Finally, I would like to thank my friends in the USA and in Tunisia to whom I owe all the joy and happiness that I had during my stay in Stillwater.

I would like to dedicate this work to my lovely mother, Fatma, and caring father, Salem. Also, I dedicate this study to my loving and beautiful fiancée, Raja, my dear brother Moncef, and my caring sister, Mahbouba.

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## CHAPTER I

### INTRODUCTION

#### Rectifier Description

About thirty years ago, silicon solid-state rectifiers came on the market (1). The early silicon rectifier applications and operations were limited to the electromechanical and electrometallurgical industries. These rectifiers were economical, simple to operate and had high efficiency. Later, when the silicon controlled rectifiers became available, their use became wide-spread. They can be applied for hundreds of applications such as pumping machines, weaving machines, printing presses, paper making machines, and any other function where controllable and continuously variable speed is important. Other vital applications include AC-DC-AC inversion and frequency changers where substantial amounts of power are involved (2).

The basic function of a three-phase full wave rectifier is to convert alternating current into direct current. Such a system consists of six power diodes or SCR's (silicon controlled rectifiers), and is connected to a three-phase power source, which consist of three voltages having the same magnitude and a phase difference of 120 electrical degrees between each other. Unlike inverters, which are similar to rectifiers but they have opposite function (convert DC into AC), rectifiers can operate in an uncontrolled mode. That is, the rectifier valves are not controlled by a pulse generator. A basic three-phase

rectifier is shown in Figure 1. The output of the rectifier is connected to a load which can be a dc machine, a battery bank, or any other kinds of loads depending on the needs.

### Harmonics In Three-Phase Bridge Rectifiers

The nonlinearity of rectifier devices makes them a major source of harmonics in the line current. Stratford (2) has reported two incidents that illustrated the problems that arose from the operation of rectifiers at early installations.

The first event was the installation of rectifiers at a copper refinery west of Salt Lake City, Utah. When the installation was energized, the transcontinental telephone conversations occurring at that time were interrupted. The reason was that the ac power system feeding the rectifiers at the plant was parallel to the open-wire transcontinental telephone lines that passed between the mountain range and the great salt-lake. The harmonics caused by these rectifiers induced voltages in the telephone lines large enough to create noise on the telephone circuits, thus interrupting the conversations.

A second event happened at a mine site in Eastern Canada where a rectifier installation was energized. The communication lines, sharing the same right of way as the power system feeding the mine location, had noise induced into them which made communication impossible.

These two incidents of rectifier folklore had made manufacturers, electric utilities, and telephone companies study the problem of harmonics and establish standards for measurements of noise. Between 1930 and 1960 most of the rectifier uses were in electrochemical plants and were of the mercury arc variety. One of the means developed to

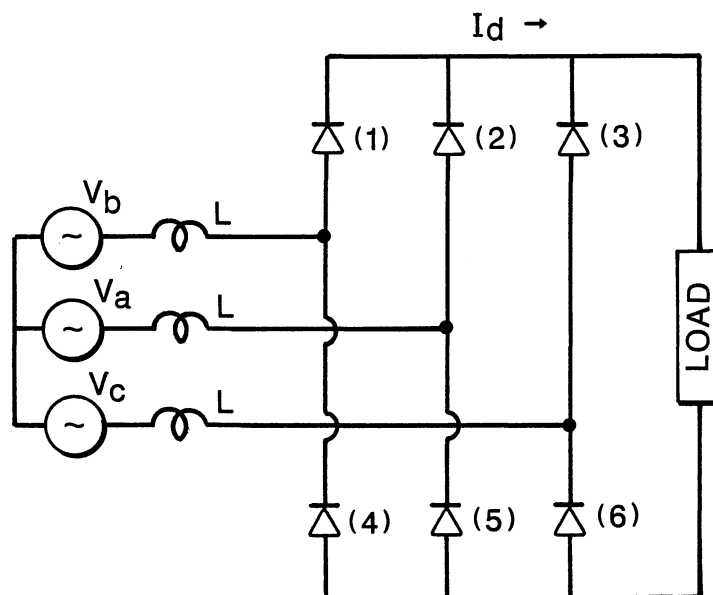


Figure 1. Typical Three-phase Bridge Rectifier Configuration.

limit the harmonic currents that were reflected into the utility systems was called multiphasing. Using this technique, a major portion of the harmonic currents that were causing the trouble in communication circuits was eliminated.

Nowadays, because of the rapid advances in solid-state electronics, rectifier industrial applications have increased, and the problem of harmonics has again become very important. In fact, large AC line current harmonics can cause serious problems in the transmission or distribution of electric power.

Harmonic currents generally flow through the least impedance paths. Since the power source is normally the lowest impedance path, most of these harmonics flow back to the source. This flow reduces the efficiency of the power source and raises many problems such as fuse-blowing, static condenser (SC) burning, errors in underfrequency detection relays, computer trouble, and abnormal motion of elevators due to mis-operation of the voltage regulator. Moreover, these harmonics cause machine heating due to increased copper and iron losses, and changes in the electromagnetically induced torque which affects machine efficiency and torsional oscillation. A comprehensive study of the harmonic effects on induction motors has been done, and the results are reported in references 4 and 5.

Finally, metering and instrumentation are affected by harmonic currents, especially if resonant conditions occur. Devices such as watt-hour meters and overcurrent relays are subject to erroneous operation. It was shown in references 6 and 7 that errors due to harmonics may be either positive or negative with third harmonics. The error depends upon the type of meter under consideration. Solid-state

meters usually measure power currents and voltages on an assumed wave shape which is the fundamental wave. In general, significant errors cannot be detected unless the distortion is perhaps greater than 20% or more.

### Problem Statement and Objective

It is clear from the previous section that the control of harmonics generated by power rectifiers is very important because of their effects on rectifier costs and efficiency, and other connected loads such as computers, motors, communication equipment, etc.

Most of the work done thus far is an attempt to design filters that minimize these harmonics. There are many novel approaches dealing with filtering or reduction of harmonics. Some of this work is reported in reference 8 and 9. The problem of harmonics thus continues to cause electric utilities to be very concerned. Therefore, further testing and analysis is needed to predict the harmonic content of ac rectifier line currents under various rectified load situations.

The objective of this thesis is to use an accurate method, described in the following section, to analyze and measure the harmonic content of rectifier AC line currents.

### Method of Study

Most of the studies (if not all) performed on a 3-phase rectifier used line-to-line waveforms. These analyses involve working with a set of six waves which creates many difficulties. It was shown by Lubdook and Murray that this technique can be simplified by analyzing the rectifier current waveforms using line-to-neutral voltage waveforms

(10). Hence the bridge can be considered as being made up of two 3-phase one-way circuits. This circuit, which is known in Europe as Gaetz circuit, is shown in Figure 2.

The action of each one-way circuit can be analyzed separately using phase-to-neutral waveforms. However, each group of three-diodes have voltages and currents of opposite sign and a different phase compared to the other group of the remaining diodes.

The analysis of this rectifier circuit will be a numerical analysis using a computer program to solve the differential equations that describe the line currents. The current waveforms thus obtained will be analyzed to find Fourier coefficients using the fast Fourier transform which is described in Appendix A. Also, the analysis will focus on the investigations of a one-way 3-phase circuit (half-wave rectifier) and will be extrapolated to the 3-phase bridge rectifier.

The measurements will be performed on a 100 watt rectifier which is a model for a large rectifier. A small resistor was inserted in series with each line to observe and record the current waveforms. This resistor affects the line current. However, this effect is neglected because its voltage drop is small compared to the output voltage. An oscilloscope and a spectrum analyzer were used to study the shape of the waveforms and the harmonic content of the line currents.

The model used includes inductors designed for rectifier short circuit protection. The effects of these inductors on harmonics were investigated. The analysis and measurement are based on certain assumptions which are described in the next section.

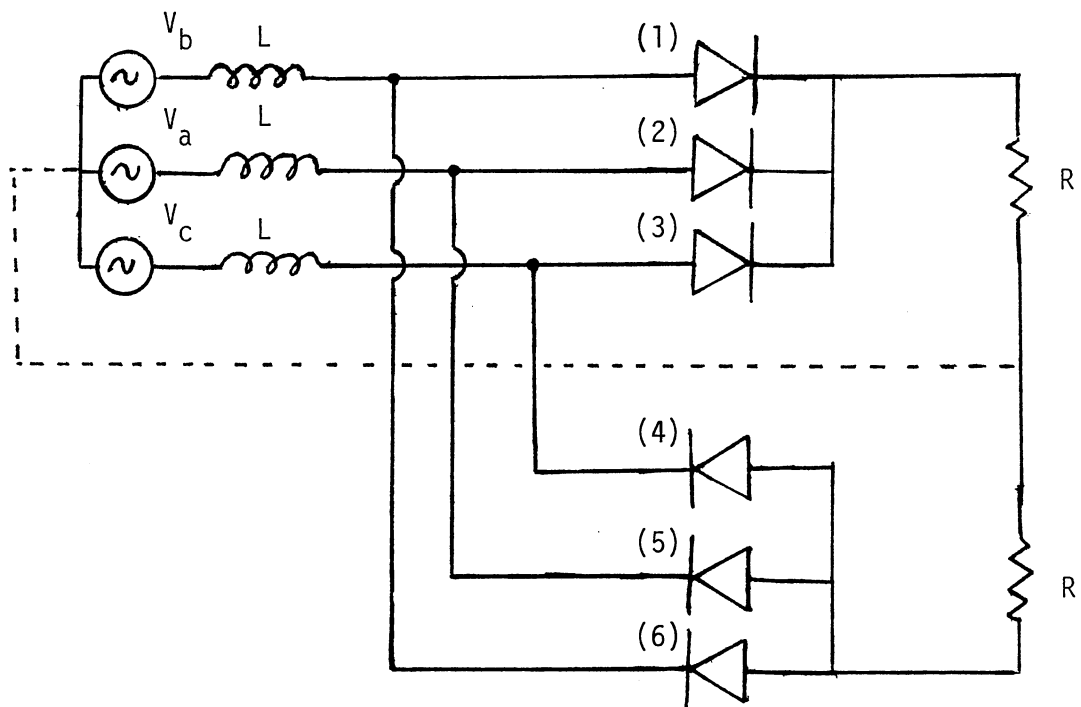


Figure 2. A Three-Phase Rectifier, consisting of Two Three-Phase One-Way Circuits.



### Assumptions

First, the diodes are assumed to be operating as ideal switches which have zero voltage drop when they are conducting. No recovered charge effects are considered. Second, the ac source is assumed to have no internal impedance, deliver constant-voltage sinusoidal waveforms, and to have constant frequency.

## CHAPTER II

### CHARACTERISTICS OF THE THREE-PHASE RECTIFIER

#### Introduction

The rectifier which is studied in this thesis is a line-commutated AC/DC power converter. The ac supply voltage also plays the role of commutating voltage. The line-to-neutral voltage is used to provide the bias across the diodes which are turned off and on.

Commutation between diodes is characterized by a commutation angle which is a measure of the amount of time taken to transfer current from one conducting diode to another. This commutation can never be instantaneous because of the inductances in series with the power source and the diodes' recovered charge.

The remainder of this chapter describes the operation of the rectifier. In addition, the analysis of this rectifier system will include commutation effects and rectifier protection.

#### Rectifier Operation

The basic operation of the three-phase bridge is illustrated in Figures 1 and 3. The rectifier is connected to an ohmic resistance and an ideal supply voltage. Also, commutation angle is neglected that is, the rectifiers are assumed to switch from on to off and vice versa instantaneously. When the rectifier is energized, the voltage  $V_{Ca}$  attains its maximum value during the first period of conduction which

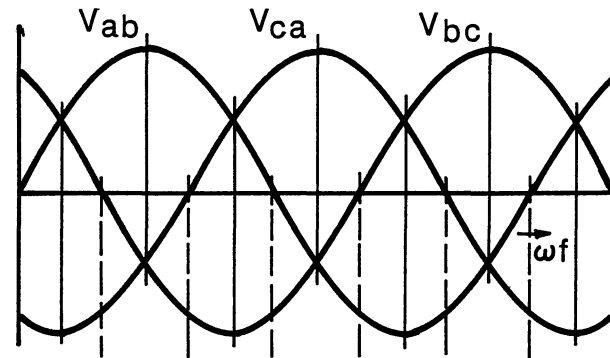
lasts 60 electrical degrees. As shown in Figure 3, the voltage  $V_{Ca}$  is negative which makes the voltage  $V_a$  greater than the voltage  $V_c$ . Hence, diode 2 starts conducting and allows diode 6 to conduct. During the second period of conduction the voltage  $V_{ab}$  reaches its maximum with  $V_a$  larger than  $V_b$  which allows diode 2 to remain conducting and diode 4 starting to conduct. The condition changes are shown in Figure 3.c which lists the conducting diodes during each of the six periods of 60 electrical degrees each.

Under normal operation there are always two diodes conducting. Each diode conducts for two periods. Hence, the conduction period of each diode is therefore 120 electrical degrees. Also, for each cycle there are six conduction states of the rectifier. Consequently, the rectifier operates in a six-pulse mode as shown in Figure 3.b.

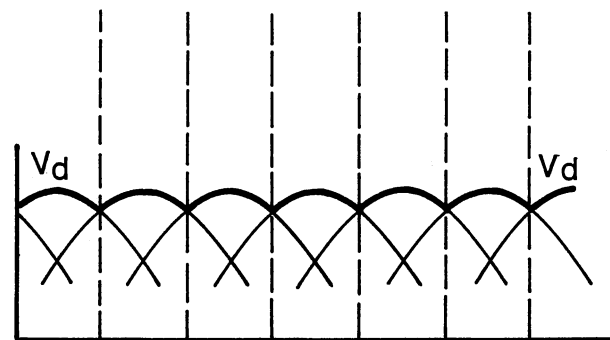
### Commutation Effects

In the previous section, the operation of the rectifier was based on the assumption that the supply is ideal. However, to obtain a realistic insight into the characteristics of the rectifier and to understand its behavior, it is essential to take into account the inductances of the system.

Usually the three-phase supply for the rectifier is provided through a transformer. Thus, the equivalent circuit of the rectifier always includes a leakage inductance in series with the supply source lines. This leakage reactance is calculated from the reactance percentage of the ac circuit. Also, sometimes another inductance is purposely added to the lines to limit fault currents.



(a) VOLTAGE WAVEFORMS



(b) OUTPUT VOLTAGE

	60	120	180	240	300	360	420
1					C	C	
2	C	C					C
3			C	C			
4		C	C				
5				C	C		
6	C					C	C

(c) CONDUCTING DIODES DURING EACH SIXTY ELECTRICAL DEGREES

Figure 3. Basic Operation of a Three-phase Bridge.

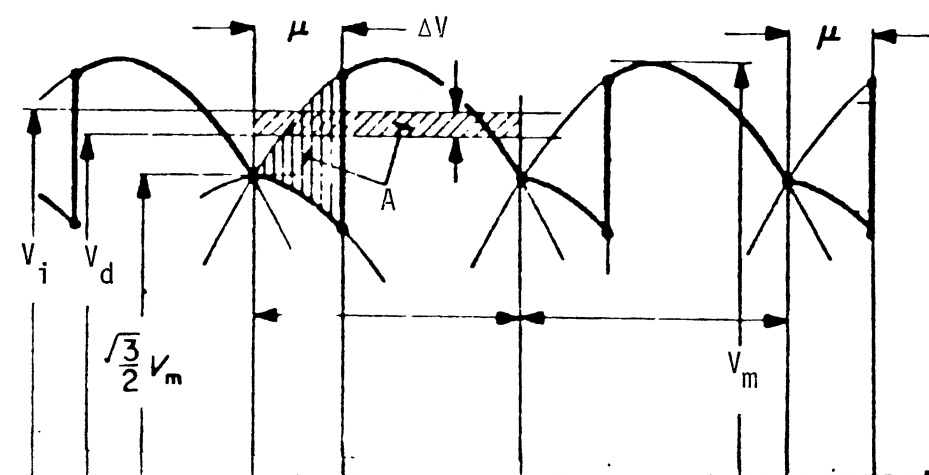
Adding these reactances to the system causes the commutation to be not instantaneous. As a result, during commutation all three diodes are momentarily conducting, for example, diode 1 and 2 are commutating and diode 6 is conducting. During this mode of operation, the mean direct voltage  $V_d$  (output voltage) is reduced compared to its normal value when two diodes are conducting. The effect of commutation on the output voltage is shown in Figure 4.

Figure 5 illustrates the two normal modes of operation, conduction mode and commutation mode. From this figure it can be deduced that six commutations occur in one cycle and each commutation is followed by a state without commutation. Thus, in total the rectifier exhibits twelve distinct switching states.

### Three-Phase Bridge Protection

Three-phase bridge rectifiers are generally protected from short circuit conditions by external means. During a fault (eg. a load short circuit), the diodes on the positive side of the dc circuit (1,2,3) commute alternatively with those on the negative side (4,5,6), see Figure 1. Hence, the three-phase transformer connected to the supply is short-circuited.

Fuses and reactors inserted in series with the power source lines are used for protection against overcurrent. Also, circuit breakers and protective relays are used for protection. In many industrial applications, reactors are used for protection in coordination with a fast acting breaker because of the following reasons:



$V_d$ : mean direct voltage, actual case

$V_i$ : mean direct voltage, ideal case

$A$ : voltage-time integral

$\Delta V$ : voltage drop =  $V_d - V_i$

Figure 4. Effect of Commutation on the Output Voltage.

- 1) They prevent commutation failure in the rectifier by limiting the rate of increase of direct current during commutation in the three-phase bridge.
- 2) They decrease harmonic voltages and currents in the ac line. This effect will be discussed in Chapter V where an investigation of the rectifier with and without an inductor is performed.
- 3) They smooth the ripple in the direct output voltage. This also is discussed in Chapter V.

As a conclusion a reactor/breaker combination is sometimes preferred to fuses which are sometimes almost as expensive as the protected device. A more detailed study of the effect of such an inductor is investigated experimentally and the results are discussed in Chapter V.

## CHAPTER III

### CONVENTIONAL THREE-PHASE FULL WAVE RECTIFIER THEORY

#### Introduction

The theory of a three-phase bridge is given in references 11 and 12. Also, reference 13 has an outline for the rectifier analysis which takes into consideration the triggering angle for the SCR's and the commutation angle. This method of analysis treats two modes of operation: the normal conduction mode and the normal commutation mode. Also, each period of conduction is studied separately. It is shown that the analysis for the following periods can be deduced from the first period by shifting the current expressions by an angle of  $\frac{\pi}{3}$  for the second period,  $\frac{2\pi}{3}$  for the third period and so on. As a result, analytical expressions for currents can be obtained. Then, Fourier series (see appendix A) of these expressions can be found analytically.

The following section describes three modes of operation which are: normal conduction mode, normal commutation mode and fault mode.

#### Modes of Operation

Referring to the three-phase bridge shown in Figure 1 (Chapter I), the modes of operation can be classified as follows.



1) Normal conduction mode: Two diodes on different sides of the bridge are on. That is, one of the diodes numbered 1, 2 or 3, and one of the diodes numbered 4, 5 or 6 are conducting. Two diodes in the same arm such as 1 and 4 can never be conducting at the same time during this mode.

2) Normal commutation mode: three diodes are on; one on each arm, two commutating and one is conducting.

3) Fault mode (eg. short-circuit mode):

Three failure cases are of interest. First case happens when both diodes on one arm are conducting, example 1 and 4, with no other diodes conducting. Second case occurs when two diodes on the same arm are conducting and one or both diodes on an other arm are conducting. Finally, a fault occurs when four or more diodes are conducting involving all three arms of the three-phase bridge.

#### A Brief Mathematical Description

Assuming that the diodes operate as switches and that no fault condition is present, the procedure outlined in reference 13 can be applied to the configuration of the three-phase bridge shown in Figure 1. (The triggering angle is not considered because the rectifier has diodes and not SCR's)

It was mentioned in the previous chapter that there are twelve switching states and six periods of conduction. The first period  $0 < \theta < \frac{\pi}{3}$  is divided into a commutation interval  $0 < \theta < u$  and a conduction interval  $u < \theta < \frac{\pi}{3}$ . Kirchhoff voltage law equations can be written for these time intervals.

Consider the first commutation period, which is illustrated in Figure 5.a. During this period (see Figure 3.a) the voltage  $V_C$  is dropping; however, the voltage  $V_{ac}$  is increasing to reach its peak value. Diodes 1 and 2 are then commutating while diode 6 is conducting. Using the circuit shown in Figure 5.a, we have the following equations.

$$L p i_a^{(1)}(t) - L p i_b^{(1)}(t) = V_{ab}(t) \quad (3.1)$$

$$R i_a^{(1)}(t) + 2 L p i_a^{(1)}(t) + R i_b^{(1)}(t) + L p i_b^{(1)}(t) = V_{ac}(t) \quad (3.2)$$

where  $p = \frac{d}{dt}$  and (1) is a superscript indicating the first switching state. The general solution for  $i_a^{(1)}(t)$  and  $i_b^{(1)}(t)$  can be shown to have the form:

$$i_a^{(1)}(t) = k_1^{(1)} + k_2^{(1)} e^{P_1 t} + f_a^{(1)}(t) \quad (3.3)$$

$$i_b^{(1)}(t) = -k_1^{(1)} + k_2^{(1)} e^{P_1 t} + f_b^{(1)}(t) \quad (3.4)$$

$$i_c^{(1)}(t) = -i_a^{(1)}(t) - i_b^{(1)}(t)$$

where  $P_1 = \frac{-2R}{3L}$  and  $f_a^{(1)}(t)$  and  $f_b^{(1)}(t)$  are functions of  $L, R, V_{ac}(t)$  and  $V_{ab}(t)$ . The constants  $k_1$  and  $k_2$  are integration constants that are found using the initial conditions.

The commutation angle  $u$  is found from  $i_b^{(1)}\left(\frac{u}{\omega_0}\right) = 0$  in equation (3.4).

During the second interval  $u < \theta < \frac{\pi}{3}$  expressions for  $i_a^{(2)}(t)$ ,  $i_b^{(2)}(t)$  and  $i_c^{(2)}(t)$  are derived using Figure 5.c, which shows that only diodes 2 and 6 are conducting.

$$i_a^{(2)}(t) R + 2 L p i_a^{(2)}(t) = V_{ac}(t) \quad (3.5)$$

General solution of equation 3.5 is given below.

$$i_a^{(2)}(t) = k_2^{(2)} e^{P_2 t} + f_a^{(2)}(t) \quad (3.6)$$

$$i_c^{(2)}(t) = -i_a^{(2)}(t) \quad (3.7)$$

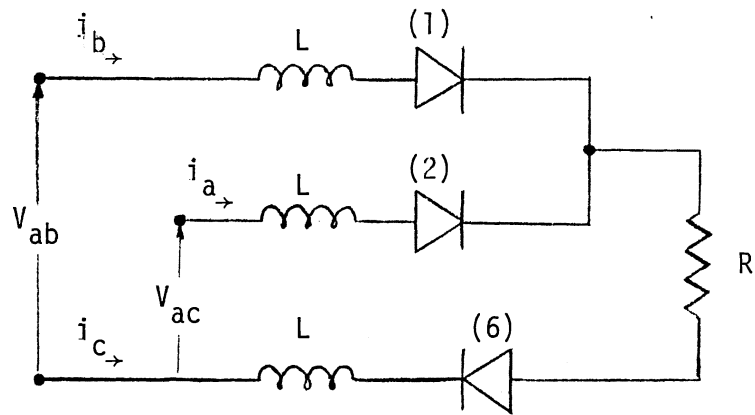
$$i_b^{(2)}(t) = 0 \quad (3.8)$$

where  $P_2 = -\frac{R}{2L}$  and  $f_a^{(2)}(t)$  is a function of  $R$ ,  $L$  and  $V_{ac}(t)$ .

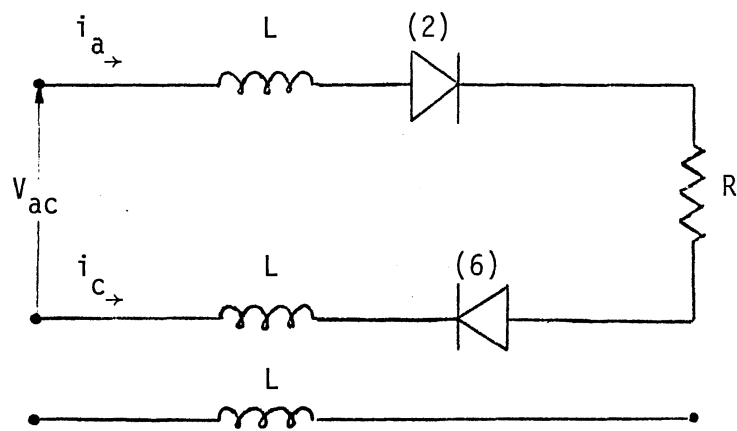
The next period of interest is  $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$ . Figure 5.d shows the commutation interval. Using the same procedure, expressions for the line currents can be derived. The results obtained show that  $i_a^{(k)}(t)$ ,  $i_b^{(k)}(t)$ ,  $i_c^{(k)}(t)$  are shifted by a multiple of  $\frac{\pi}{3}$  ( $\frac{\pi}{3}$  for second period,  $\frac{2\pi}{3}$  for 3 period and so on). For example,  $i_a^{(3)}(t) =$

$2k_2^{(1)} e^{P_1(t - \frac{\pi}{3\omega_0})} + f_a^{(1)}(t - \frac{\pi}{3\omega_0})$ . Table I shows a summary of the expression for current  $i_a(t)$  during the first conduction period of diode 2, which is  $\frac{2\pi}{3\omega_0}$  seconds.

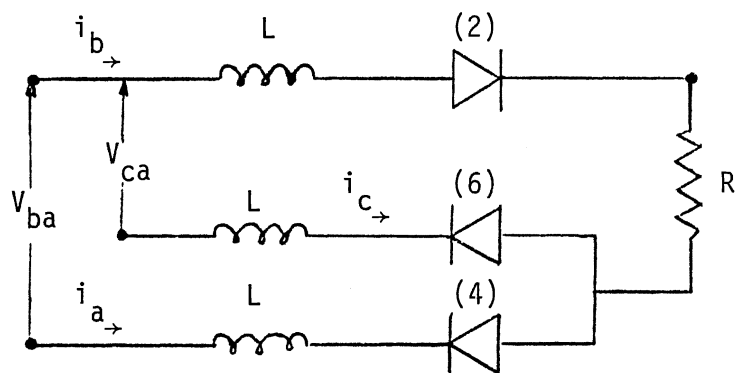
In summary, it is clear that the conventional method that could be used to analyze the system involves a great amount of effort to derive current expressions for each conducting and switching periods. A simple technique to analyze the system is presented in the next chapter.



5.a First commutation period.



5.b First conduction period.



5.c Second commutation period

Figure 5. Rectifier equivalent circuits.

TABLE I  
CURRENT  $i_a(t)$  DURING EACH SWITCHING PERIOD

Diodes Condition	Angle	Current $i_a(t)$
Diode 1 and 2 are commutating and diode 6 is conducting	$0 < \theta < u$	$k_1 + k_2 \exp(P_1 t) + f_a^{(1)}(t)$
Diodes 2 and 6 are conducting	$u < \theta < \frac{\pi}{3}$	$k_3 \exp(P_2 t) + f_a^{(2)}(t)$
Diode 2 is conducting and diodes 6 and 4 are commutating	$\frac{\pi}{3} < \theta < \frac{\pi}{3} + u$	$2 k_2 \exp[P_1(t - \frac{2\pi}{3\omega_0})] + f_a^{(2)}(t - \frac{2\pi}{3\omega_0})$
Diodes 2 and 4 are conducting	$\frac{\pi}{3} + u < \theta < \frac{2\pi}{3}$	$k_3 \exp[P_2(t - \frac{2\pi}{3\omega_0})] + f_a^{(2)}(t - \frac{2\pi}{3\omega_0})$
Diodes 2 and 3 are commutating and 4 is conducting	$\frac{2\pi}{3} < \theta < \frac{2\pi}{3} + u$	$k_2 \exp[P_1(t - \frac{2\pi}{3\omega_0})] + f_a^{(1)}(t - \frac{2\pi}{3\omega_0})$
Diodes 3 and 4 are conducting	$\frac{2\pi}{3} + u < \theta < 2$	0

## Harmonic Analysis

Most of the rectifier/inverter harmonic currents analysis are performed based on the assumptions that the diodes' currents are constant during conduction periods, and commutation periods are negligibly small (This type of rectifier/inverter model is called linearized model). Under these assumptions the diode current wave shapes take on the square waveform shown in Figure 6.

Through Fourier analysis, it can be shown that the harmonic components are related to the fundamental component of current as follows:

$$I_n = I_1/n \quad (3.9)$$

where  $I_n$  is the amplitude of the  $n^{\text{th}}$  harmonic current,  $I_1$  is the amplitude of the fundamental current and  $n$  is the harmonic number which has the expression:

$$n = 6K \pm 1 \quad (3.10)$$

where  $K$  is an integer (1, 2, 3, 4.....). The  $6K-1$  harmonics are negative sequence and the  $6K + 1$  harmonics are positive sequence.

The common three-phase bridge will have these values of harmonic currents:

Harmonic	5	7	11	13	17	19	23	25
Current	.200	.143	.091	.077	.059	.053	.043	.040
(per unit)								

Practical rectifier circuits have inductances which have an effect on these currents (See commutation effect in Chapter II). Realistic

values for medium voltage systems as reported by Stratford (14) are given below:

Harmonic	5	7	11	13	17	19	23	25
Current (per unit)	.175	.111	.045	.029	.015	.010	.009	.008

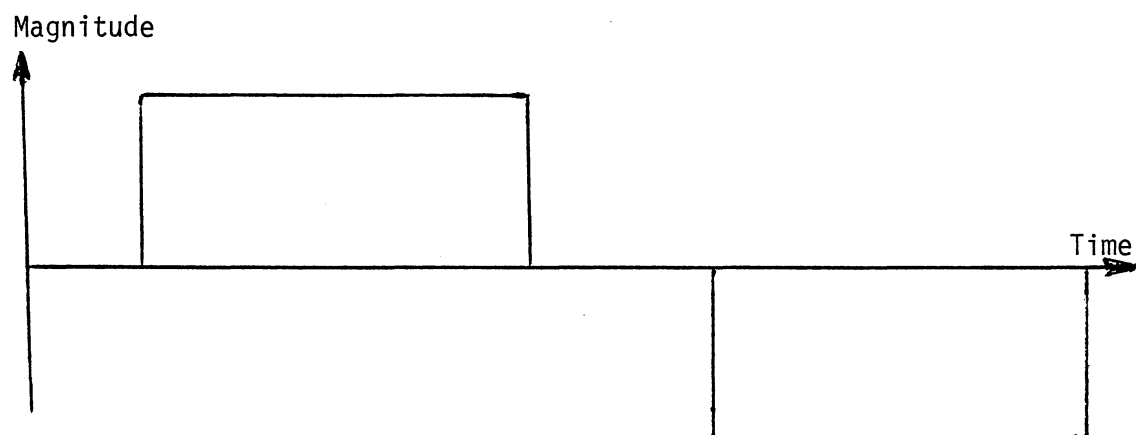


Figure 6. Idealized three-phase rectifier current.

## CHAPTER IV

### THREE-PHASE BRIDGE RECTIFIER STUDIES

#### INTRODUCTION

In the previous chapter, it was shown that the mathematical descriptions of the line currents can be obtained using line-to-line voltages. However, these current formulations involve a great amount of effort to find closed form expressions for the Fourier series coefficients of these currents.

In this chapter, a simpler technique is used to analyze the system and find Fourier coefficients of the line currents. Based on certain assumptions described below, the differential equations that describe the system were used and a computer program was written to solve these equations numerically. Using fast Fourier transform algorithm (a method for computing the finite Fourier transform of a series of N data points, see Appendix A), the second part of the program computes the Fourier coefficients of the line currents. These coefficients are the amplitudes of the harmonic components of the line currents.

The assumptions made for this analysis are enumerated below.

- 1) All diodes are assumed to be ideal switches with zero forward voltage drop and zero reverse currents.
- 2) The voltage source is ideal (no internal impedance).



3) The three lines are symmetric. That is, all lines have the same current magnitude and shape but are delayed successively by 0 period,  $\frac{1}{3}$  period, or  $\frac{2}{3}$  period corresponding to line frequency.

### Half-Wave Rectifier Analysis

Applying Kirchhoff voltage law to the circuit shown in Figure 7, the following equations can be derived

$$\frac{dI_1}{dt} = \frac{1}{L} (E \sin t - V_1) \quad (4.1)$$

$$\frac{dI_2}{dt} = \frac{1}{L} [E \sin (t + 120^\circ) - V_1] \quad (4.2)$$

$$\frac{dI_3}{dt} = \frac{1}{L} [E \sin (t - 120^\circ) - V_1] \quad (4.3)$$

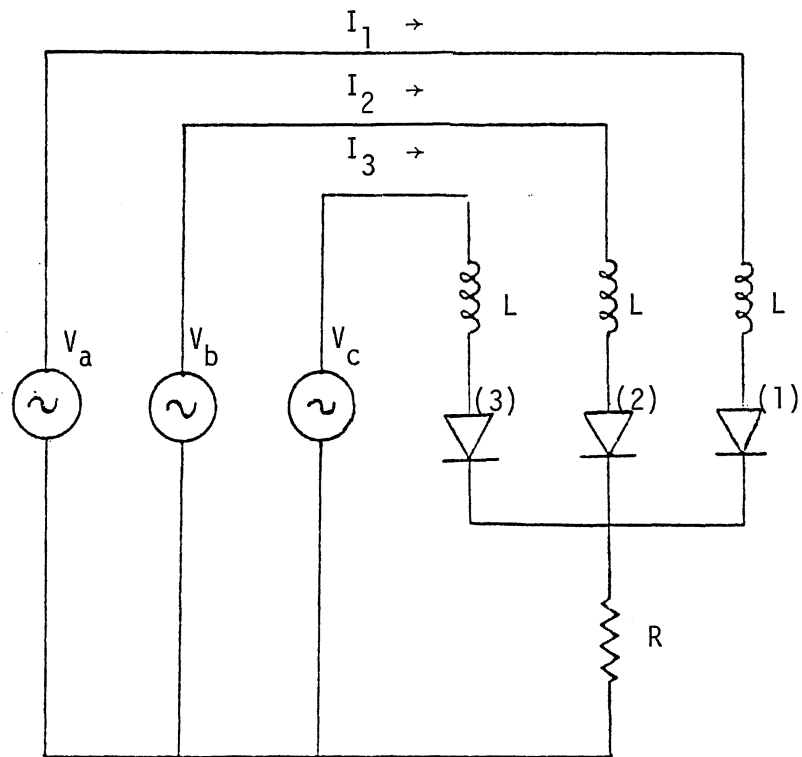
where  $V_1 = (I_1 + I_2 + I_3) R$ .

The method used to solve these equations was a numerical method named Euler method which is described in any numerical analysis text book. The algorithm of this method can be described as follows.

Given the following equations  $\frac{dI}{dt} = f(t, I(t))$  and  $I(t_0) = I_0$ , then for any time  $t$  an approximation of  $I(t)$  can be found. Assume that we have an interval  $[t_0, t]$ , if we choose an increment of  $dt$  which is equal to the difference between  $t$  and  $t_0$  divided by  $N$  which is a number of points. Then, the current recurrence expression is:

$$I_{n+1} = I_n + dt f(t_n, I_n) \quad (4.4)$$

The code of the computer program that was used to predict the line currents is shown in Appendix B. Figure 8 shows a flowchart for this program. The first part of this program has a loop that computes the currents after each time increment  $dt$ . Then, it stores their values.



$$V_a = E \sin \omega t$$

$$V_b = E \sin (\omega t + 120^\circ)$$

$$V_c = E \sin (\omega t - 120^\circ)$$

$$\omega = 2\pi f_0; f_0 = 60 \text{ Hz}$$

$$L = .36 \text{ mH}; R = 375 \text{ ohms}$$

Figure 7. Half-wave rectifier circuit.

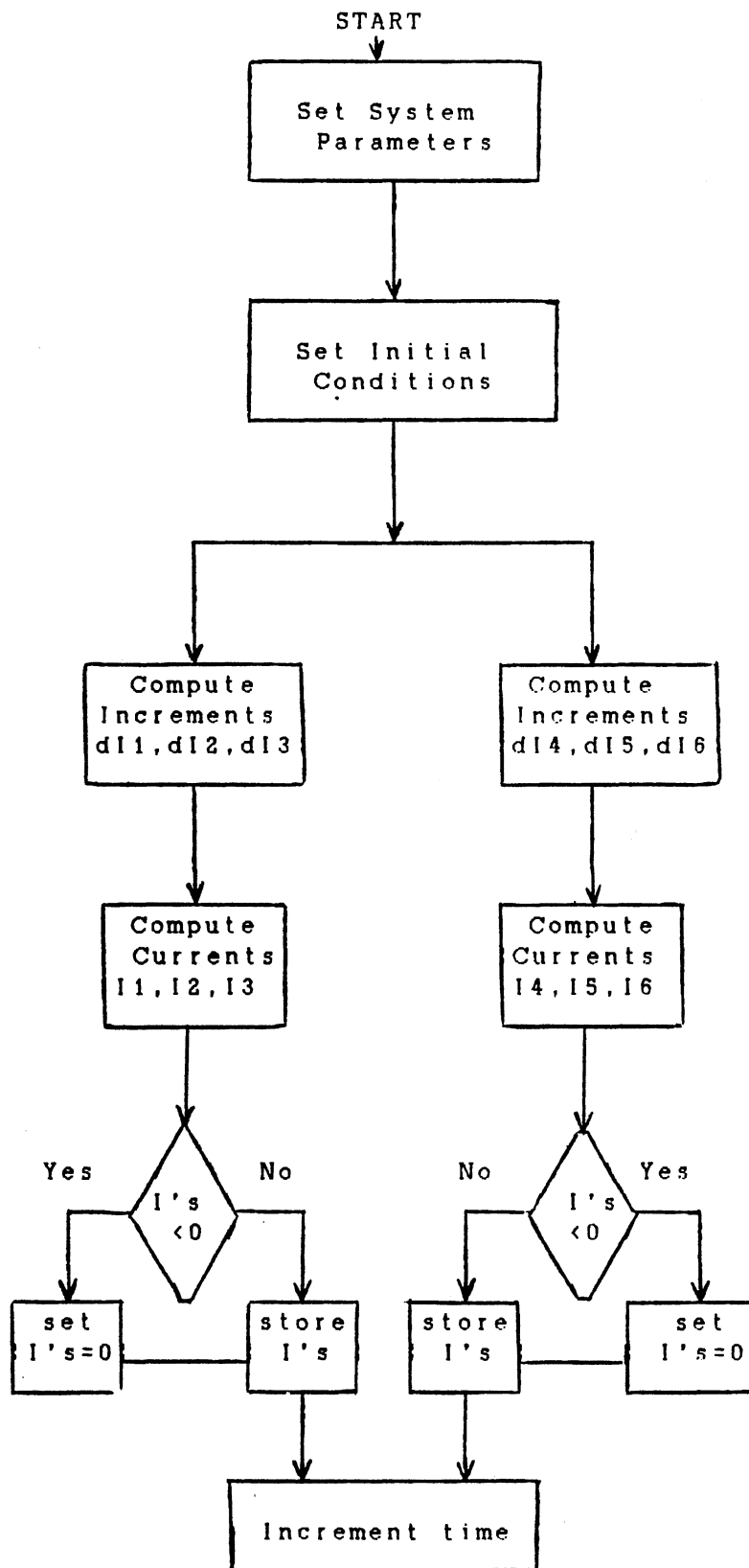


Figure 8. Flow chart of the computer program used.

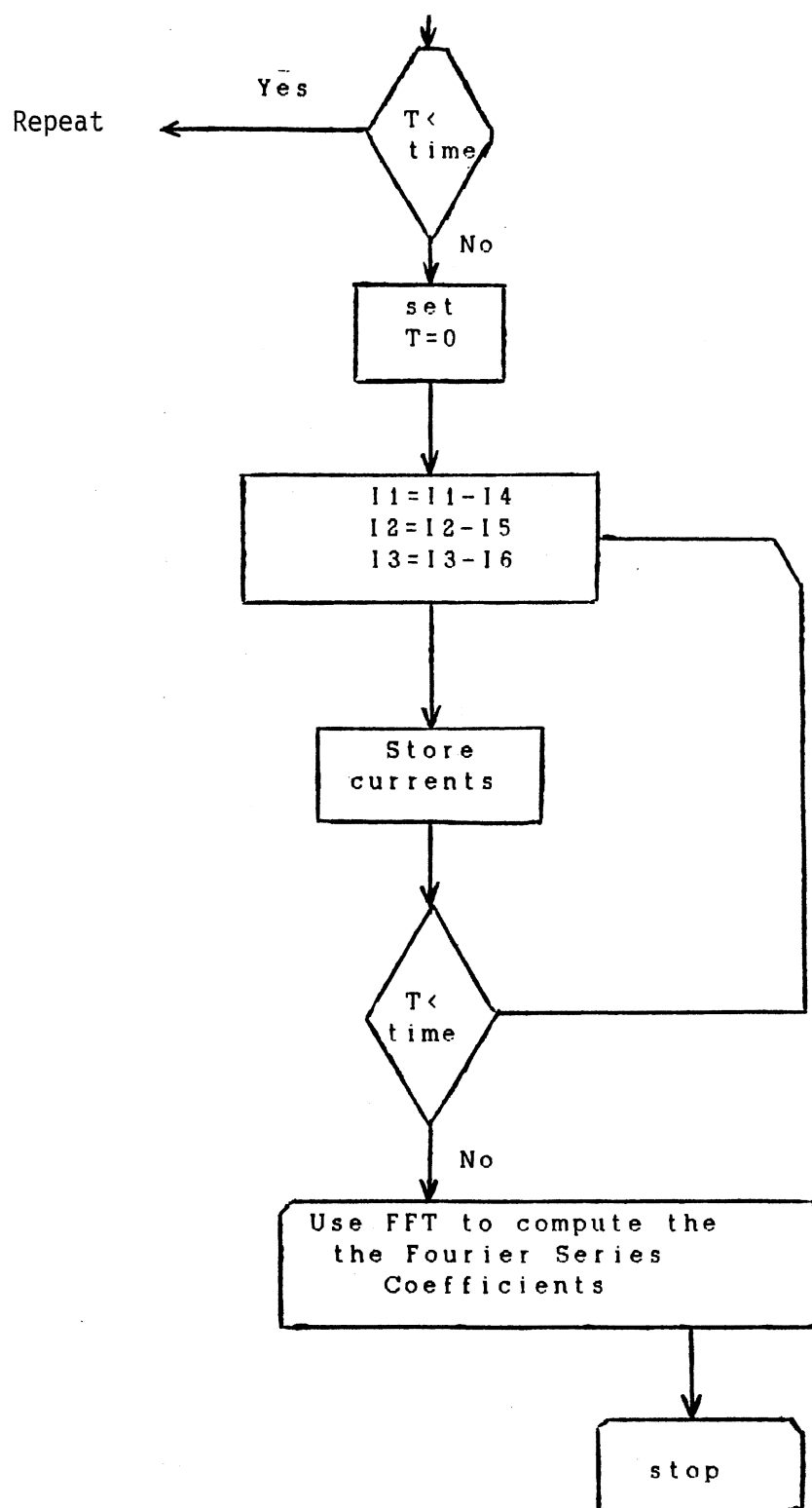


Figure 8. (continued)

Since the diodes are assumed to be ideal they were turned on when conducting and off when they are reverse biased or nonconducting.

The initial conditions chosen for these currents were equal to zero at  $t = 0$ . The choice turns out to be a good one. At  $t = 0$ , both currents  $I_1$  and  $I_3$  are equal to zero. The current  $I_2$  reaches its actual value after a few milliseconds. A plot of these currents is shown in Figure 9. This plot shows that each diode is conducting for approximately one third of a period (5.83 milliseconds). Also, commutation intervals are shown. That is, conduction of two diodes at the same time is shown for a fraction of a millisecond (.78 milliseconds).

#### Half-Wave Rectifier Harmonics Study

The second part of the computer program mentioned earlier takes the stored line current values and uses Fast Fourier Transform (see Appendix A) to find the magnitudes of the line current harmonics. Table II shows the normalized magnitudes of the harmonics, expressed in per unit of the fundamental. All harmonics, odd and even, are present.

#### Full-Wave Rectifier Analysis

The full-wave rectifier shown in Figure 2 (Chapter I) can be considered to be made up of two half-wave rectifiers (each one being one-way). Each one of these rectifiers can be analyzed separately. Then, based on some observations, the line currents of the full-wave rectifier can be deduced.

Consider Figure 2, the equations that describe the currents in the remaining half-wave rectifier are:

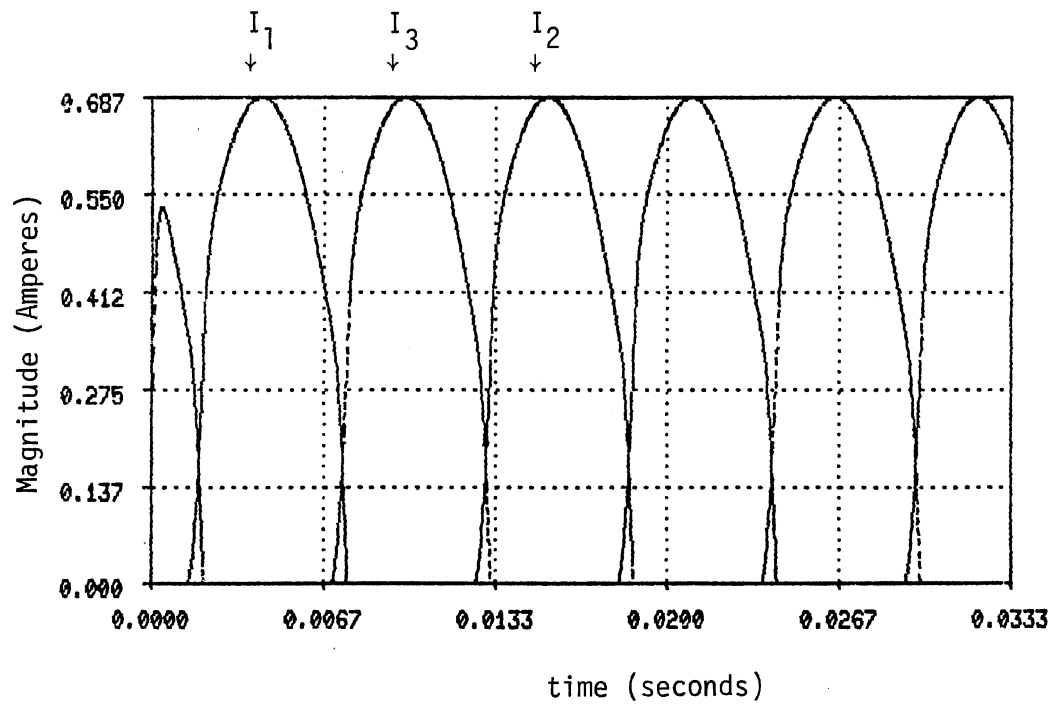
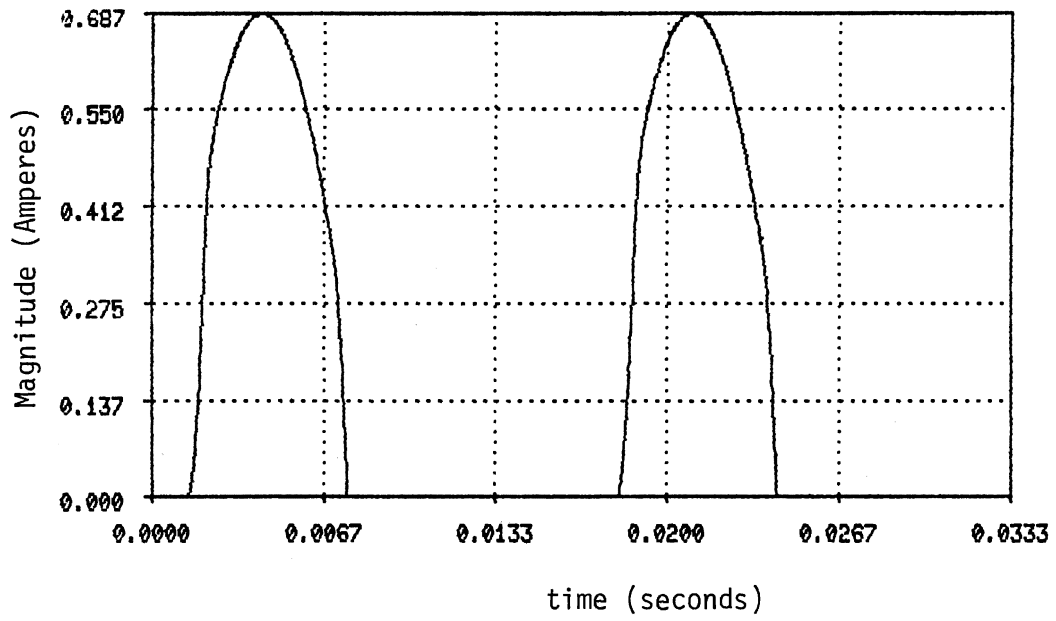


Figure 9. Half-wave rectifier line currents.

$$\frac{dI_4}{dt} = \frac{1}{L} [-E \sin(377t) - V_2] \quad (4.5)$$

$$\frac{dI_5}{dt} = \frac{1}{L} [-E \sin(377t + \frac{2\pi}{3}) - V_2] \quad (4.6)$$

$$\frac{dI_6}{dt} = \frac{1}{L} [-E \sin(377t - \frac{2\pi}{3}) - V_2] \quad (4.7)$$

$$\text{where } V_2 = (I_4 + I_5 + I_6)R \quad (4.8)$$

Using these equations and the procedure used to find the currents of the half-wave rectifier analyzed previously, values of the currents  $I_4$ ,  $I_5$ , and  $I_6$  are obtained and stored. The flowchart shown in Figure 8 shows the addition to the program that computes these currents.

There are two important observations that allow the analysis of the full-wave rectifier to be deduced from the combined analysis of the two half-wave rectifiers. The first one is two diodes on the same arm such as diodes 1 and 4 cannot conduct at the same time. The other observation is that the current in diode 1, for example, is the same as the current through diode 4, except that the two currents are of opposite sign and have a delay angle between them. As a result, the line current of the full-wave rectifier is equal to the current through diode 1 when it is conducting, and it is equal to the negative of the current through diode 4 when the latter is conducting.

The resulting line current is shown in Figure 10 which is a plot of the line current versus time. This plot is for the current  $I_1$ . Since the system is symmetric the currents  $I_2$  and  $I_3$  can be obtained using the same reasoning.

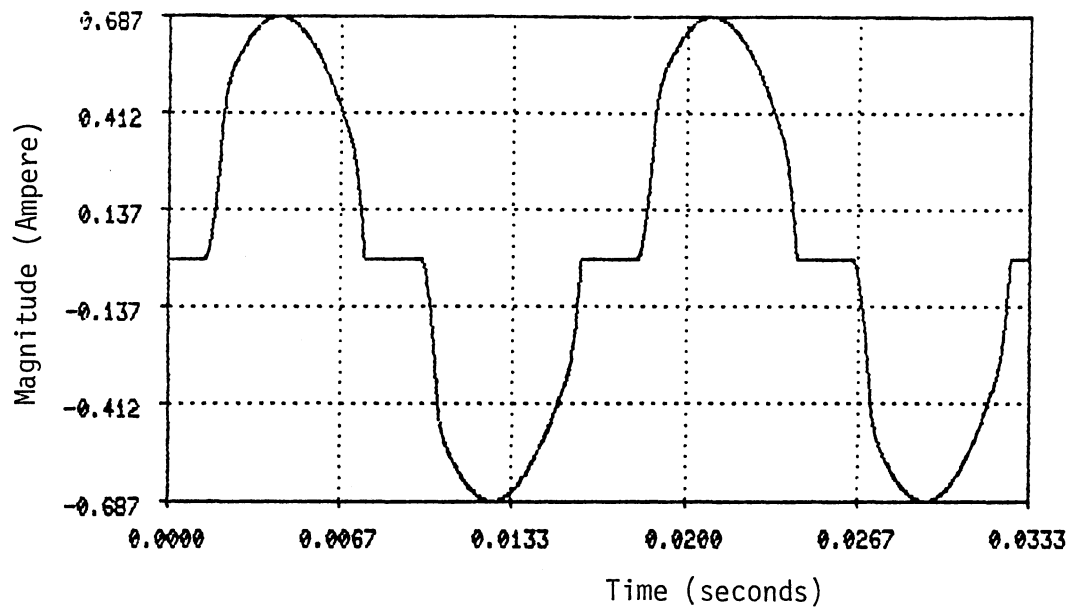
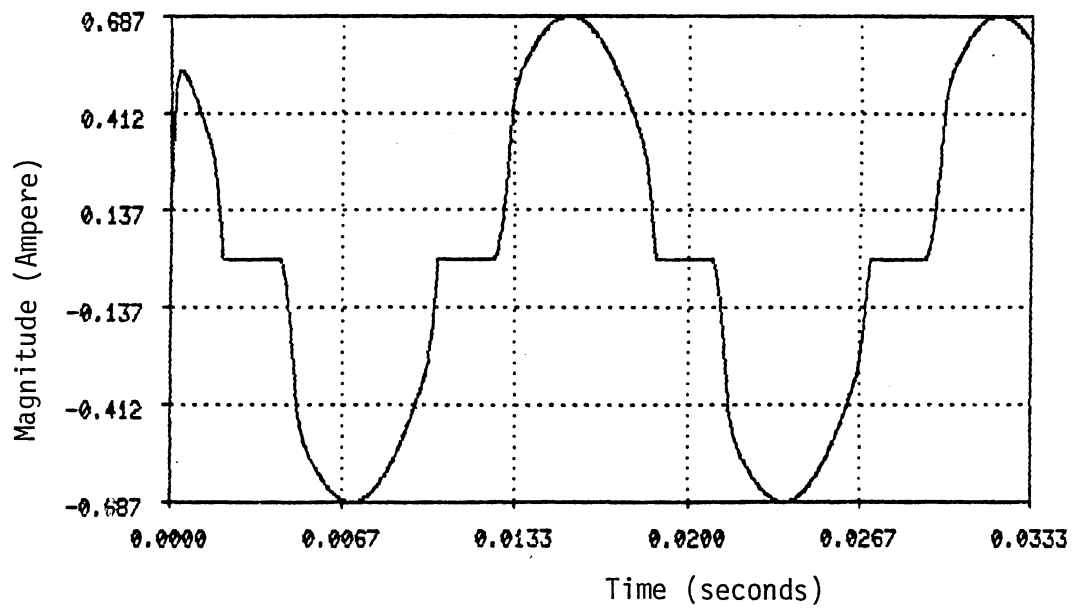
(a) Current  $I_1$ (b) Current  $I_2$ 

Figure 10. Full-wave rectifier line currents.



## Full-Wave Rectifier Harmonic Study

Using a procedure similar to the one used for the half-wave rectifier, the amplitudes of the line current harmonics are computed and Table II shows these values. Compared to the half-wave rectifier harmonics, the full-wave line currents do not have even harmonics. The per unit values of the remaining harmonic components are very close to the corresponding values of the half-wave rectifier. The next section presents a more detailed discussion of the results.

## Discussions and Conclusions

From Figure 3.c (Chapter II), which showed an ideal case, it can be observed that each diode on one arm conducts for  $\frac{2}{3}$  period, and the two diodes have a current through them in opposite directions and a delay angle of  $\frac{1}{6}$  period between them. For example, diode 2 conducts for  $\frac{1}{3}$  period in the positive direction, then after  $\frac{1}{6}$  period diode 5 conducts in the opposite direction for  $\frac{1}{3}$  period, and after  $\frac{1}{6}$  period diode 2 conducts again for  $\frac{1}{3}$  period.

This observation holds for the current  $I_1$ , which had the plot shown in Figure 10. This plot shows that diode 1 is conducting for approximately  $\frac{1}{3}$  period, then after approximately  $\frac{1}{6}$  period diode 4 starts to conduct in the opposite direction. Thus, the method used for the full-wave analysis provided an accurate approximation of the line currents which were used to find the magnitudes of the line current harmonics.

Most of the references consider the line current to have a square waveform. Which have a Fourier series that is composed only of harmonics of order 5, 7, 11, 13, 17, 19, 23, 25 and so on. The previous

TABLE II  
COMPUTED LINE CURRENT HARMONICS

Harmonic number	Half-wave rectifier current (per unit)	Full-wave rectifier current (per unit)
1	1.0000	1.0000
2	.5921	.0000
3	.1641	.1642
4	.1106	.0000
5	.1406	.1428
6	.0448	.0000
7	.0625	.0624
8	.0754	.0000
9	.0213	.0213
10	.0395	.0000
11	.0467	.0465
12	.0116	.0000
13	.0259	.0258
14	.0306	.0000
15	.0069	.0069
16	.0173	.0000
17	.0203	.0222
18	.0048	.0000
19	.0155	.0154
20	.0074	.0000
21	.0043	.0043
22	.0075	.0000
23	.0093	.0092
24	.0041	.0000
25	.0050	.0047
26	.0064	.0000
27	.0038	.0030
28	.0029	.0000
29	.0050	.0050
30	.0034	.0000
31	.0017	.0020
32	.0012	.0000
33	.0031	.0030
34	.0032	.0000
35	.0040	.0031
36	.0022	.0000
37	.0031	.0010
38	.0033	.0000
39	.0018	.0020
40		.0000

full-wave analysis, however, showed that the line current is not a square wave because of the presence of reactances in the system. As a result some harmonics of different order showed up. These harmonics were of order 3, 9, 15, 21, and soon. All the even harmonics disappeared.

As a conclusion, in practice, full-wave rectifier circuits have inductors designed for protection and other reactances such as the voltage source reactances. These inductors introduce some additional harmonics in the systems besides the expected harmonics of order 5, 7, 11, 13, 17, 19 ... . Also, the imbalance of the system causes these harmonics to be present.

The validity of the analysis of the full-wave rectifier presented in this chapter was tested experimentally. The next chapter provides a description of the complete experimental investigations undertaken.

## CHAPTER V

### EXPERIMENTAL INVESTIGATIONS

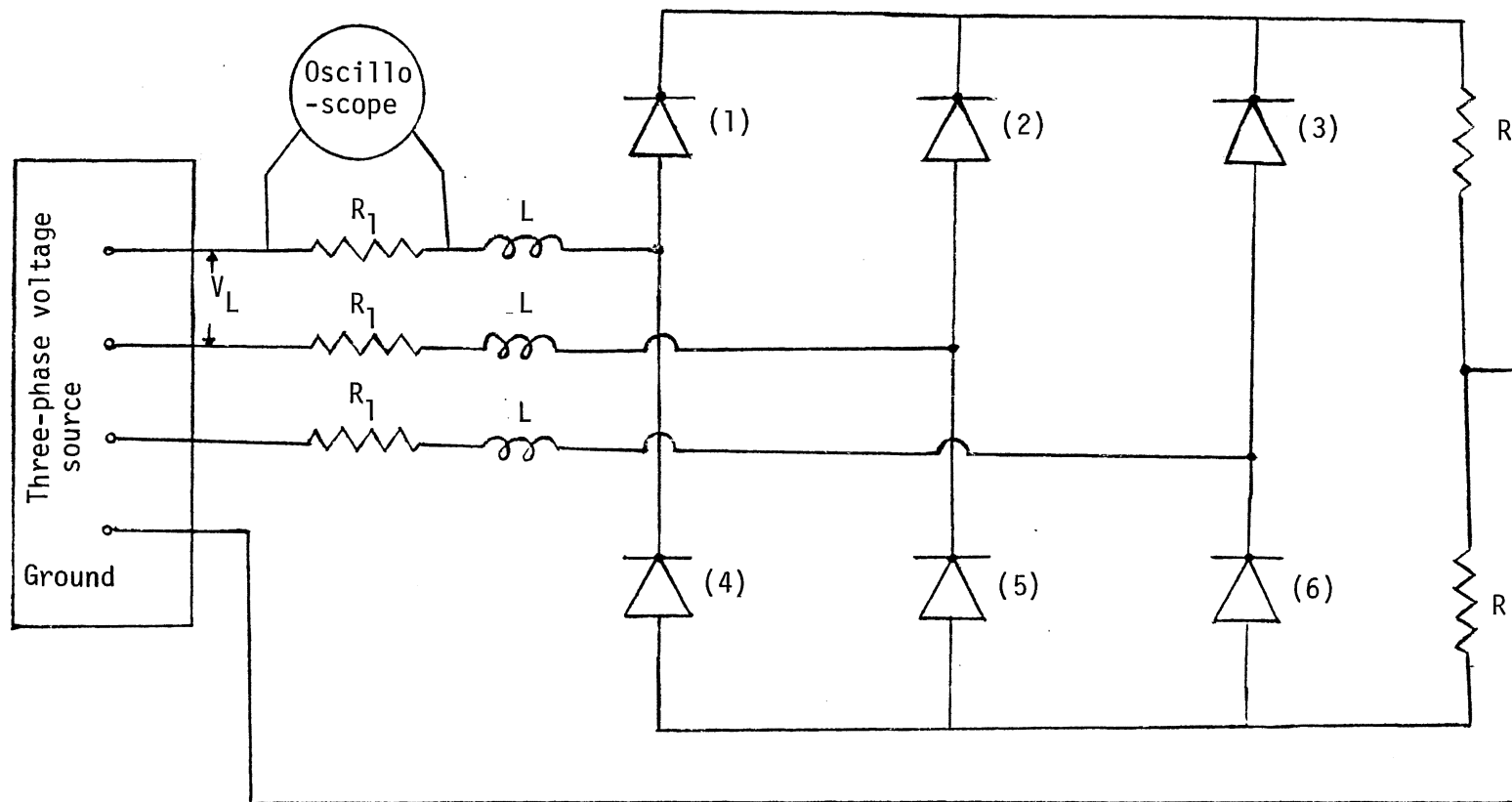
#### INTRODUCTION

The method used in Chapter IV to analyze the rectifier current waveforms and harmonics has been verified experimentally to determine the validity of the approach used for this analysis. The experimental investigations were performed for both cases with and without protective inductors. The results are discussed and compared with the corresponding theoretical values.

The experiment was performed on a 100 watt system which is a model for a large system. The first section of this chapter describes the experimental setup and the instrumentation used. The remaining sections present a summary of the investigation of the half-wave and full-wave rectifiers. Oscillograms of specific currents and output voltages were obtained. Also, spectrograms of the line currents were obtained. Finally, the data recorded were used for comparing the measured values with the theoretical results.

#### Experimental Setup and Procedure

The experimental setup used in this study is shown in Figure 11. The system is connected to a three-phase power source which has inductors, designed for protection, inserted in series with its lines. Also, a small resistor, instead of a shunt or a current transformer, was



$L = .036$  Henry;  $R_1 = 5.6$  ohms  
 $R = 375$  ohms,  $V_L = 208$  Vrms

Figure 11. Experimental setup.

installed in each line to observe and record the current waveforms and harmonics. This resistor has a negligible effect on the line current. The results were normalized with respect to the fundamental component to be used for comparison.

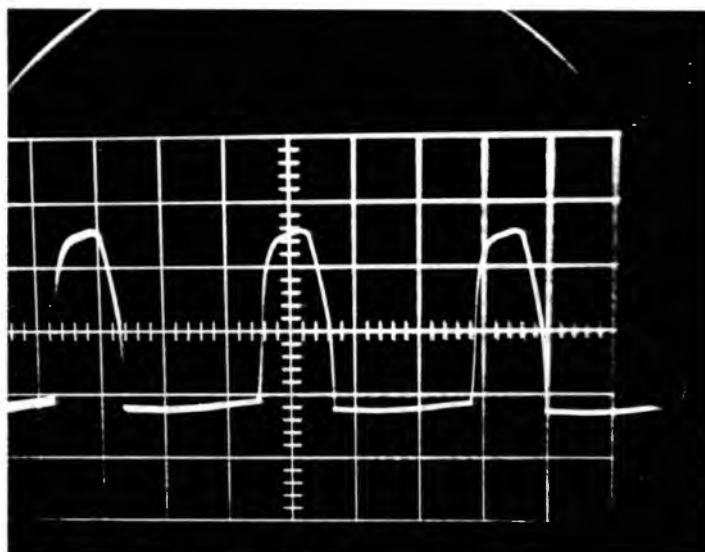
The spectrum analyzer used in the measurement of the current harmonic amplitudes was a 3585A spectrum analyzer. It is controlled by three internal microprocessors, one central processor and two smaller, control-oriented processors, all products of HP's advanced microcomputer/LSI technology.

The microprocessor control provides a number of features such as automatic calibration. The computational power of the microprocessor has made it possible to have features such as conversion from dB's reading to rms values. It has a frequency range of 20 Hz to 40 MHz. The instrument has a manual mode which allows the operator to manually tune the analyzer's frequency with a frequency dial. This mode was used to obtain the oscillograms.

The experiment was conducted as follows. First, three of the six diodes were disconnected from the system which then became a half-wave rectifier. Once the data were taken, the inductors were shorted and the measurements were repeated. Second step, the diodes are reconnected and data were collected with and without shorted inductors.

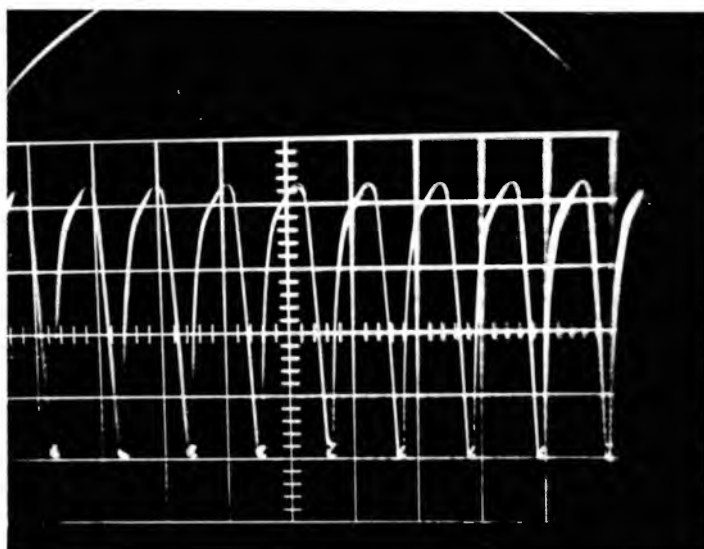
#### Half-Wave Rectifier Experimental Study

The output voltage and the line current of the half-wave rectifier with and without inductors are shown in Figures 12 and 13, respectively. The waveforms obtained with the protective inductor inserted in series with the power source lines are shown to be smoother



(a) Line current

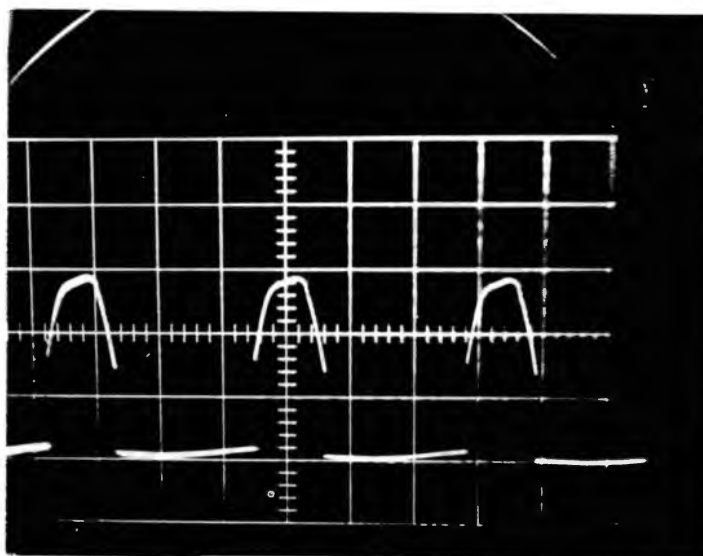
Scales X: 5ms/cm; y = .2A/cm



(b) Output voltage ripple

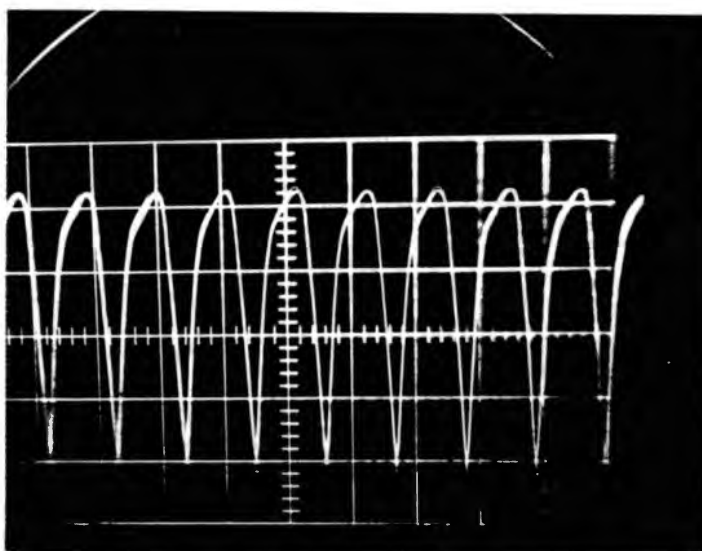
Scales X: 2ms/cm; y = 20V/cm

Figure 12. Half-wave rectifier waveforms,  
with inductors.



(a) Line current

Scales X: 5ms/cm; y = .2A/cm



(b) Output voltage ripple

Scales X: 2ms/cm; y = 20V/cm

Figure 13. Half-wave rectifier waveforms,  
with shorted inductors.

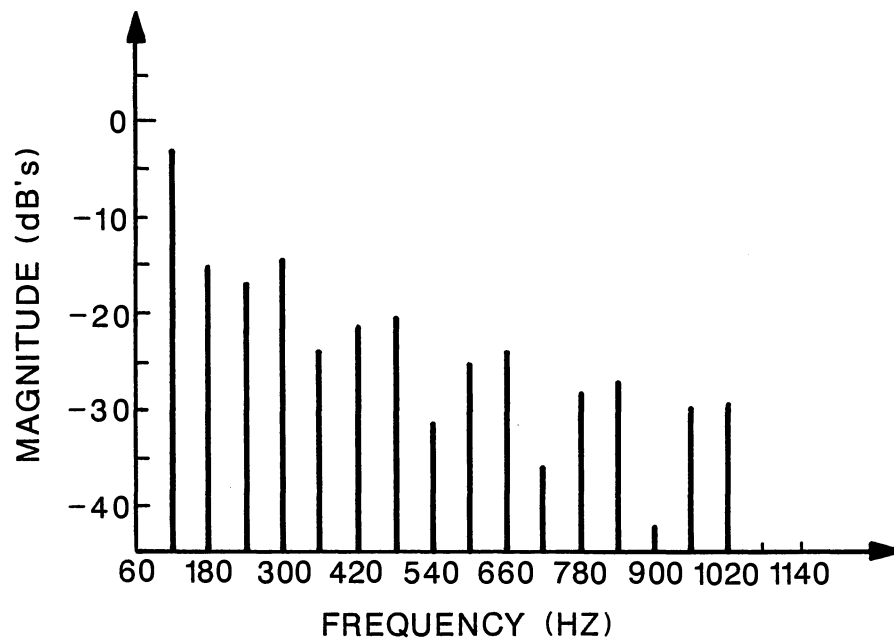


than the other waveforms. With reference to the oscillograms, the current has a peak value of approximately .56 A. The peak value of the ripple voltage is 85 V. The DC level of the output voltage was measured to be 85 V. This results agrees with the expected theoretical value 85 volts. The DC level is found by multiplying the line-to-neutral voltage ( $120 \sqrt{2}$ ) by .5). Also, from these oscillograms it can be observed that each diode is conducting for one third of a period, or for 5.5 milliseconds. Hence, the waveforms obtained experimentally and theoretically agree well.

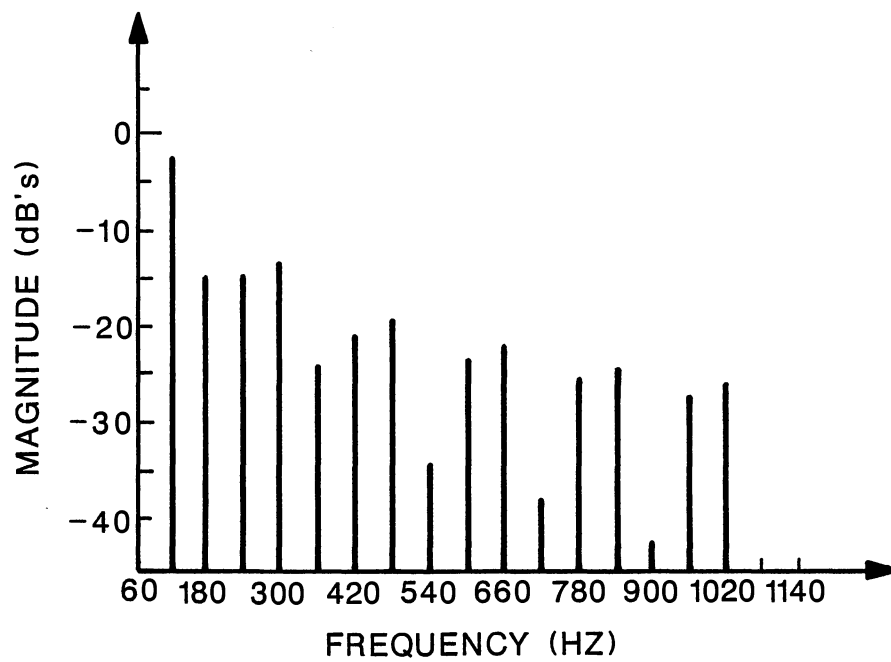
The second part of the half-wave rectifier experimental analysis was an investigation of the line current harmonics. After connecting the spectrum analyzer across one of the resistors installed in series with the power source lines, the spectrograms shown in Figure 14 were obtained. Then, the peaks of the first forty harmonics were measured and recorded.

Figure 14 shows the harmonics of order 2 up to the harmonic of order 17. The oscillograms obtained with and without inductors indicate a slight difference between them. For example, the peak of the 9th harmonic obtained with inductors present is greater than the corresponding one obtained without inductors. However, the harmonics of order 13 and 14 are shown to have higher peaks when the inductors are shorted. The difference between these peaks was so small that it could be neglected. Hence, the inductors inserted in the lines for system protection do not have a significant effect on the harmonics of low order, harmonics of order less than 20.

The recorded data were normalized with respect to the fundamental and Table III presents a summary of the results. This Table shows both



(a) With inductors



(b) Without inductors

Figure 14. Half-wave rectifier spectrograms.

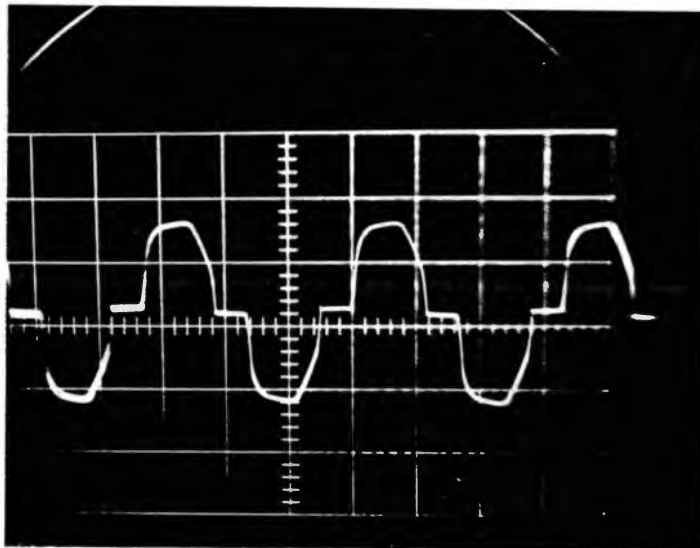
sets of data collected, with and without inductors. It can be observed that the presence of the inductors in the circuit has a large effect on the harmonics of high order, order greater than 20. In fact, the peaks of the harmonics measured when the inductors were shorted are significantly higher than the corresponding ones measured with inductors included in the system. For instance, the 31-st harmonic has a magnitude four times higher when the inductor is shorted.

In conclusion, the inductors used for protection do not have a significant effect on the harmonics of low order. However, they have a significant effect on the higher harmonics, the ones at greater than 1200 Hz. Hence, an inductor not only protects the system from short circuit conditions but also eliminates some of the harmonics that occur at high frequencies.

#### Full-Wave Rectifier Experimental Study

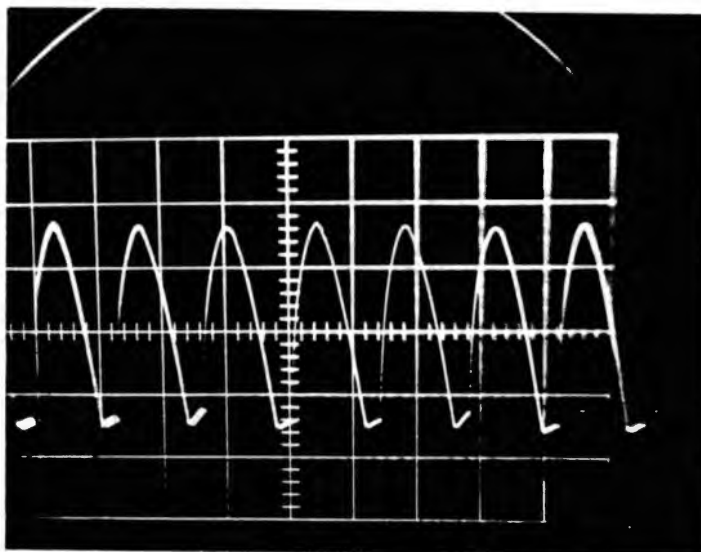
Oscillograms of the output voltage and the line current for the full-wave rectifier configuration are shown in Figure 15. The same waveforms obtained by shorting the inductors inserted in the system are presented in Figure 16. Similar to the half-wave rectifier case, the full-wave rectifier has smoother waveforms when the inductors are included in the system. The measured dc level and the output voltage ripple were respectively 260 volts and 60 volts. It can be seen from the oscillograms that each diode conducts for 5.5 milliseconds. Also, the measured peak-to-peak current was 1.12 A. Later in this chapter, these results will be compared with the theoretical results.

The harmonic analysis of the full-wave rectifier was conducted by analyzing each harmonic individually. This was the most accurate method



(a) Line current

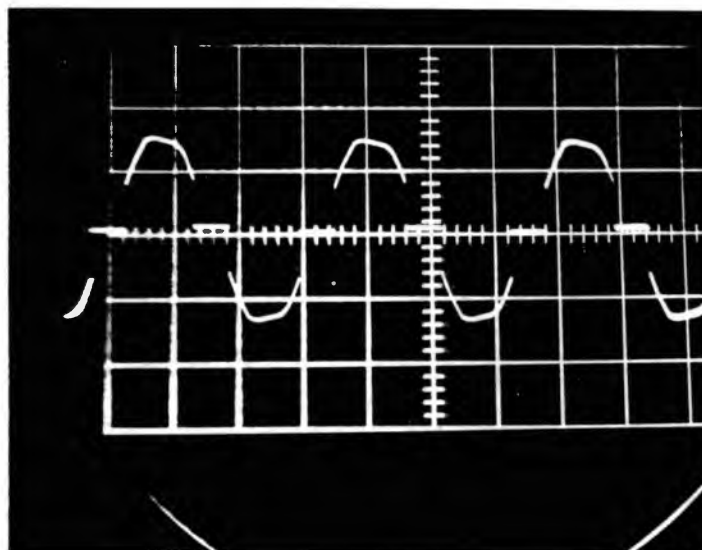
Scales X: 5ms/cm; y: .36A/cm



(b) Output ripple

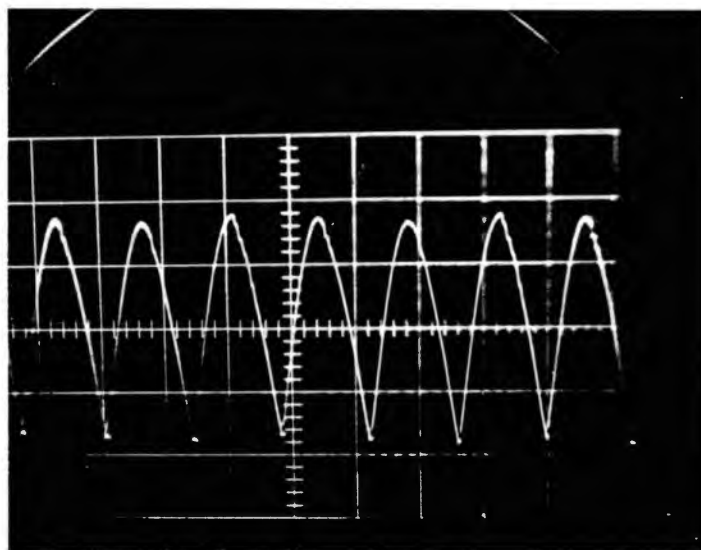
Scales X: 2ms/cm; y: 20V/cm

Figure 15. Full-wave rectifier waveforms,  
with inductors.



(a) Line current

Scales X: 5ms/cm; y: .36A/cm



(b) Output ripple

Scales X: 2ms/cm; y: 20V/cm

Figure 16. Full-wave rectifier waveforms,  
with shorted inductors.

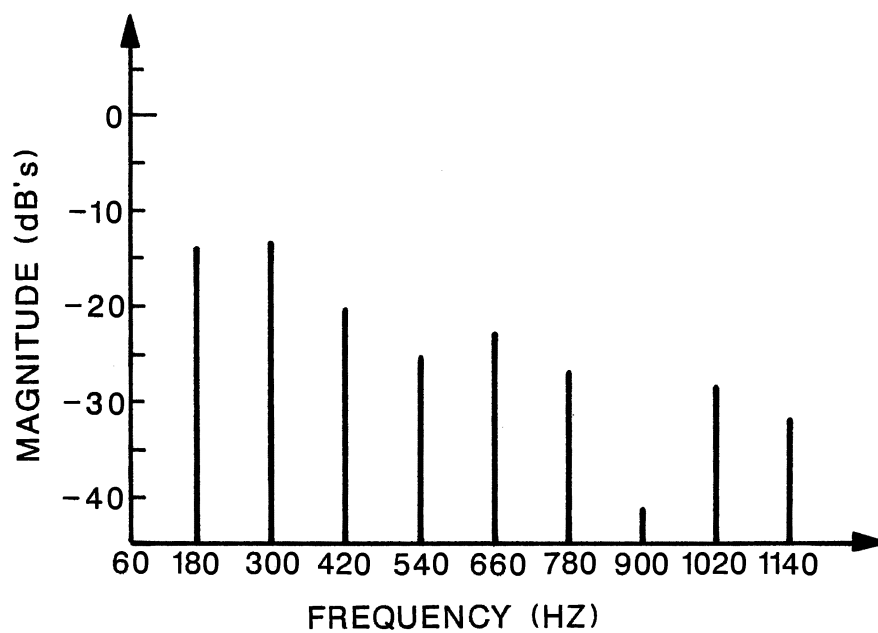
that could be used to measure the harmonic magnitudes. The recorded data was normalized with respect to the fundamental amplitude and presented in Table III. The spectrograms presented in Figure 17 reveal that all the even harmonics are absent. All the odd harmonics are present; however, the ones which have an order that is a multiple of three (triplen harmonics) such as 9 and 15 are smaller than the harmonics of order 5, 7 and 11. For instance, the 7th harmonic magnitude is two times the 9th harmonic magnitude. The presence of the unexpected harmonics 9, 15, 21, etc. was mainly due to the imbalance in the system.

In addition to the conclusion stated in the previous section, namely the inductors used for protection have an effect on the harmonics that have high frequencies, the full-wave rectifier line current included unexpected harmonics which are odd and multiples of three. These harmonics are zero sequence. Their presence was mainly due to the imbalance present in the system.

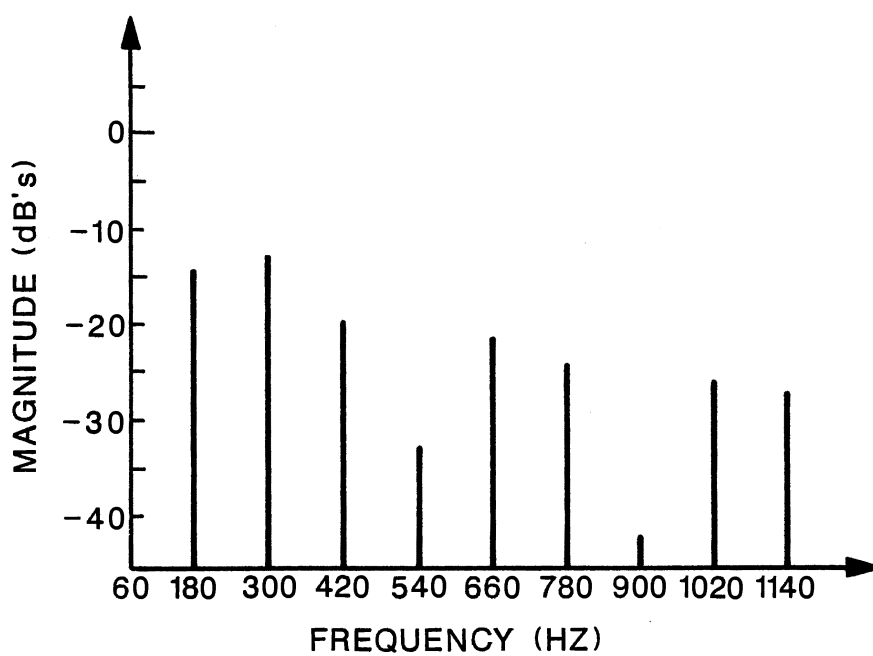
#### Comparison of Experimental and Theoretical Results

The comparison of the experimental and theoretical waveforms shows a reasonable agreement. The assumption that the voltage source was ideal and the presence of the resistor in series with the line currents influence the magnitudes of the line current and voltage ripple. However, this effect is not considered because the results were normalized.

Table IV presents a comparison between the predicted and the experimental harmonics magnitude. This table indicates that the error is within 10% for the harmonics that have frequencies less than 1200



(a) With inductors



(b) Without inductors

Figure 17. Full-wave rectifier spectrograms.

TABLE III  
EXPERIMENTAL ANALYSIS

Harmonic	Half-wave Rectifier		Full-wave Rectifier	
	With inductor	Without inductor	With inductor	Without inductor
1	1.0000	1.0000	.0000	1.0000
2	.5792	.5535	.0000	.0000
3	.1469	.1253	.1423	.1296
4	.1177	.1263	.0000	.0000
5	.1510	.1596	.1528	.1479
6	.0504	.0509	.0000	.0000
7	.0667	.0594	.0659	.0637
8	.0800	.0767	.0000	.0000
9	.0215	.0180	.0283	.0192
10	.0437	.0458	.0000	.0000
11	.0523	.0549	.0517	.0569
12	.0137	.0107	.0000	.0000
13	.0283	.0369	.0290	.0373
14	.0344	.0432	.0000	.0000
15	.0076	.0084	.0079	.0080
16	.0218	.0294	.0000	.0000
17	.0240	.0345	.0247	.0351
18	.0046	.0060	.0000	.0000
19	.0167	.0250	.0167	.0258
20	.0087	.0292	.0000	.0000
21	.0030	.0049	.0032	.0046
22	.0122	.0214	.0000	.0000
23	.0135	.0249	.0136	.0252
24	.0026	.0394	.0000	.0000
25	.0087	.0190	.0088	.0196
26	.0097	.0219	.0000	.0000
27	.0026	.0031	.0026	.0030
28	.0061	.0175	.0000	.0000
29	.0068	.0198	.0070	.0198
30	.0024	.0029	.0000	.0000
31	.0046	.0156	.0046	.0161
32	.0051	.0781	.0000	.0000



Hz. The error becomes high for the high order harmonics and it goes up to about 40%. The increased discrepancy could be due to the noise introduced into the system. Filtering this noise may lead to better results. However, it was not necessary to include a filter in the system because the experimental and theoretical results are in good agreement for low-order harmonics.

In summary, the method used to predict the line current harmonics was accurate enough, inspite of the assumption made. Considering the internal impedance of the power source, using a current transformer instead of the resistors inserted in the lines, and filtering some of the noise introduced into the system may improve the agreement between theoretical and experimental results.

TABLE IV  
COMPARISON OF RESULTS

Harmonics	3	5	7	9	11	13	15	17	19
% error	15	6	5	6	10	11	11	11	8
Harmonic	21	23	25	27	29	31	33	35	37
% error	34	32	46	15	30	56	30	17	54

## CHAPTER VI

### CONCLUSIONS AND RECOMMENDATIONS

#### Summary of Results

The primary objective of this work was to develop a mathematical model for analyzing the full-wave rectifier line current harmonics and testing this model experimentally. The need for further analysis and testing of the line current harmonics necessitated the finding of a simple technique that predicts these line current harmonic contents. The study of the rectifier system was performed as follows.

- 1) The system was split into two one-way circuits, and a computer program was written to compute the line current harmonic contents of a one-way circuit. Then, this program was extended by including the other one-way circuit, differential equations. Finally, the results of the two one-way circuits were combined to obtain the line current harmonics of the full-wave rectifier.

- 2) The validity of the mathematical model developed was verified experimentally.

Based on a set of well-defined assumptions, Chapter IV presented a theoretical study of the half-wave and full-wave rectifiers. The half-wave rectifier results showed that all harmonics are present, even and odd. However, the full-wave rectifier results indicated that the even harmonics are absent.

Experimental study of the half-wave rectifier was presented in Chapter V. This study had two goals. The first one was to verify the validity of the method used to compute the harmonic contents, and the second one was to investigate the effects of the inductors introduced for the protection of the system.

The experimental results were in good agreement with the computed values. Both results, experimental and theoretical, revealed the presence of unexpected harmonics which were odd and multiple of three. These harmonics were due to an imbalance in the system.

The error was within 10% for low order harmonics, which have frequencies less than 1200 Hz. The differences become significant, as high as 40%, as the frequency increases to values greater than 1200 Hz. Filtering the undesirable noise introduced into the system would improve the agreement at higher frequencies.

The effect of the inductors inserted in series with the lines of the power source was also studied. It was concluded that the presence of these reactances make the system current waveforms smoother. Moreover, they reduce the magnitudes of the harmonic contents that have high frequencies, 1200 Hz and above.

This work should be considered only as a first step in the full-wave rectifier line current harmonic investigation. Further work is left for future investigations.

#### Scope of Further Work

The need of an accurate computer program that computes the line current harmonics of the three-phase full-wave rectifier makes the extension of this study necessary. The method described briefly in

Chapter III could be used to model the system, and a comparison between this model and the model developed in this thesis could be made. Also the system model has to include the recovered charge and should allow the study of the system under various load conditions (resistive, inductive, and capacitive).

Finally, this work could be extended to include the modeling of silicon controlled rectifiers in which gate triggering is delayed to provide variable dc voltage. Nowadays such systems are in common use, and are causing increasingly severe problems for the power companies. An accurate analytical prediction method would find extensive use in the utility industry.

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APPENDIX A

FOURIER'S SERIES ANALYSIS

The harmonic analysis of nonsinusoidal voltages and currents is usually performed with the use of Fourier's series representation of these waveforms. In general, Fourier's series expressions of a periodic function can be defined as follows.

Assume that  $f(x)$  has a period of  $2L$ , then

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right) \quad (1)$$

where the coefficients  $a_n$  and  $b_n$  are described by:

$$a_0 = \frac{1}{L} \int_{-L}^L f(x) dx \quad (2)$$

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx \quad (3)$$

$$b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx \quad (4)$$

Since most of the electrical networks are subject to waveforms which have a period of  $2\pi$ , these expressions become:

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x) dx \quad (5)$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx dx \quad (6)$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx dx \quad (7)$$

where  $n = 1, 2, 3, 4, \dots$ . Also  $f(x)$  can be defined by:

$$f(x) = C_0 + C_1 \sin(x + \alpha_1) + C_2 \sin(2x + \alpha_2) + C_n \sin(nx + \alpha_n) \quad (8)$$

$$\text{where } C_n = \sqrt{a_n^2 + b_n^2} \quad (9)$$

As a conclusion, the analysis of the harmonic contents of a signal is basically the study of the coefficients  $C_n$  where  $n$  represents the



ratio between the n-th harmonic frequency and the frequency of the waveform. These coefficients could be determined numerically by a method named FFT which is presented in the next section.

### Fast Fourier Transform

The application of Fourier's series is limited to waveforms that are easy to analyze analatically such as square waveforms and triangular waveforms. In most of the practical cases however, experimentally or numerically derived waveforms are not easy to investigate by the use of Fourier's series. Thus, the application of Fast Fourier Transform (FFT) represents the most efficient numerical method available for the harmonic study of these type of waveforms.

The FFT is a method for computing the Discrete Fourier Transform (DFT) of a discrete data samples. Basically, the algorithm of this method can be described as follows.

Given a set of data  $\{x_j\}$  where  $j = 0, 1, 2, 3, \dots n-1$ , and  $n$  is an even positive integer. Then, the Fourier coefficients are described by

$$A_n = \frac{2}{K} \sum_{j=0}^{K-1} x_j \cos \left( 2\pi n \frac{j}{K} \right)$$

$$B_n = \frac{2}{K} \sum_{j=0}^{K-1} x_j \sin \left( 2\pi n \frac{j}{K} \right)$$

where  $n = 0, 1, 2, \dots \frac{K}{2}$

In order to avoid the problems such as alising that may be encountered by the use of this method, the sampling rate frequency of a waveform has to be large enough, at least two times the signal frequency. The selection of a reasonable time step yield results within less than 5% error.

APPENDIX B

COMPUTER PROGRAM LISTING

```

C =====
C   THE FOLLOWING PROGRAM IS USED TO COMPUTE THE LINE  "
C   CURRENTS OF THE FULL_WAVE RECTIFIER,CONSISTING OF TWO  "
C   ONE-WAY CIRCUITS. EACH ONE-WAY IS ANALYSED SEPARATELY.  "
C   THE RESULTS WERE COMBINED TO DESCRIBE THE LINE CURRENT  "
C   WAVEFORMS. THE DATA OBTAINED WERE USED TO FIND THE  "
C   HARMONIC CONTENTS OF THESE CURRENTS.  "
C   =====
C
C
C
C
C   REAL*8 I1(10000),I2(10000),I3(10000),V1(10000),T
C   DT,L REAL*8 I1,DI2,DI3,A(50),B(50),C(50),R(50),E
C   REAL*8 PI,I5(10000),I6(10000),V2(10000),I4(10000)
C   REAL*8 DI4,DI5,DI6
C   INTEGER I,J,K
C
C   I1,I2,I3 ARE THE CURRENTS THROUG DIODES 1,2 AND
C   3,RESPECTIVALLY. THESE 3 DIODES CONSIST ONE-WAY.
C   THE OTHER ONE-WAY HAS THE CURRENTS I4,I5 AND I6.
C   THEIR VALUES ARE COMPUTED USING DOUBLE PRECISION.
C
C   L IS THE VALUE OF THE INDUCTOR USED FOR
C   RECTIFIER PROTECTION.ITS VALUE IS .036 HENRY.
C
C   V1 IS THE RESISTIVE LOAD VOLTAGE.ITS VALUE
C   VALUE EQUALS THE VALUE OF THE RESISTOR R (CHOSEN
C   375 OHMS) MULTIPLIED BY THE SUM OF THE CURRENTS I1,I2
C   AND I3 .
C
C   V2 IS THE RESISTIVE LOAD VOLTAGE THAT HAS A VALUE
C   EQUALS THE VALUE OF THE RESISTOR R MULTIPLIED BY THE
C   SUM OF THE CURRENTS I4,I5 AND I6.
C
C   T AND DT ARE TIME AND TIME INCREMENT,RESPECTIVALLY.
C
C   E IS THE MAGNITUDE OF THE VOLTAGE SOURCE.
C
C   DI1,DI2, DI3 DI4 DI5 AND DI6 ARE THE LINE
C   CURRENTS INCREMENTS.
C
C   A AND B ARE THE FOURIER SERIES COEFFICIENTS .C
C   IS THEIR SQUAREROOT.R
C
C   *
C   * *****
C   * SYSTEM PARAMETERS *
C   * *****
C   *

```

```

L = .036
E = 169.7056
PI = 3.141594
DT = 166.67E-7

```

```

      *
      * *****
      *   SYSTEM INITIAL CONDITIONS   *
      * *****
      *

```

```

      I1(1)=0.0
      I2(1)=0.0
      I3(1)=0.0
      I4(1)=0.0
      I5(1)=0.0
      I6(1)=0.0
      V1(1)=0.0
      V2(1)=0.0

```

```

      *****
      * START THE COMPUTATION OF THE LINE CURRENTS *
      * I1,I2,I3,I4,I5 AND I6.....*
      *****

```

THE LINE CURRENTS ARE COMPUTED USING THE INCREMENT DT.  
AT EACH TIME T THESE CURRENTS ARE COMPUTED AND STORED  
TO BE PLOTTED AND USED FOR THE COMPUTATION OF THE HARMONIC  
MAGNITUDES.

```

      DO 20 I=2,2000
      T=T+DT
      DI1=(E*SIN(377*T)-V1(I-1))*DT/L
      DI2=(E*SIN(377*T+(2*PI)/3)-V1(I-1))*DT/L
      DI3=(E*SIN(377*T-(2*PI)/3)-V1(I-1))*DT/L
      DI4=(-E*SIN(377*T)-V2(I-1))*DT/L
      DI5=(-E*SIN(377*T+(2*PI)/3)-V2(I-1))*DT/L
      DI6=(-E*SIN(377*T-(2*PI)/3)-V2(I-1))*DT/L
      I1(I)=I1(I-1)+DI1
      I2(I)=I2(I-1)+DI2
      I3(I)=I3(I-1)+DI3
      I4(I)=I4(I-1)+DI4
      I5(I)=I5(I-1)+DI5
      I6(I)=I6(I-1)+DI6
      V1(I)=(I1(I)+I2(I)+I3(I))*375
      V2(I)=(I4(I)+I5(I)+I6(I))*375

```

C  
C  
C  
C  
C  
C  
C  
C  
C

```

      *
      *
      * *****
      * CONDITIONS FOR DIODES' CONDUCTION *
      * *****
      *
      *

```

```

      IF(I1(I).LT.0) THEN

```

```

        I1(I)=0

```

```

      END IF

```

```

        IF(I2(I).LT.0) THEN

```

```

          I2(I)=0

```

```

        END IF

```

```

          IF(I3(I).LT.0) THEN

```

```

            I3(I)=0

```

```

          END IF

```

```

            IF(I4(I).LT.0) THEN

```

```

              I4(I)=0

```

```

            END IF

```

```

              IF(I5(I).LT.0) THEN

```

```

                I5(I)=0

```

```

              END IF

```

```

                IF(I6(I).LT.0) THEN

```

```

                  I6(I)=0

```

```

                END IF

```

```

      WRITE(1,*)T,I1(I)

```

```

      WRITE(8,*)T,I2(I)

```

```

      WRITE(9,*)T,I3(I)

```

```

20  CONTINUE

```

C  
C  
C  
C  
C  
C  
C  
C

THE FOLLOWING LOOP IS USED TO COMBINE THE COMPUTED  
VALUES OF THE TWO HALF\_ WAVE RECTIFIERS(TWO ONE-WAY  
CIRCUITS).

```

      T=0

```

```

      DO 30 J=1,2000

```

```

        T=T+DT

```

```

          I1(J)=I1(J)-I4(J)

```

```

          I2(J)=I2(J)-I5(J)

```

```

          I3(J)=I3(J)-I6(J)

```

```

      WRITE(2,*)T,I1(J)

```

```

      WRITE(3,*)T,I2(J)

```

```

      WRITE(4,*)T,I3(J)

```

```

30  CONTINUE

```

C  
C  
C

```

C
C      THE FOLLOWING PART OF THE PROGRAM COMPUTES THE
C      FOURIER COEFFICIENTS WHICH REPRESENT THE MAGNITUDES
C      OF THE HARMONIC CONTENTS OF THE CURRENT I1.
C      THE RESULTS ARE THE SAME FOR THE CURRENTS I2,
C      I3 BECAUSE THE THREE CURRENTS ARE SIMILAR EXCEPT
C      THEY DO NOT OCCUR AT THE SAME TIME. THE METHOD USED
C      IS CALLED "FAST FOURIER TRANSFORM"
      DO 50 K=1,45
      A(K)=0
      B(K)=0
      DO 40 J=1,999
      A(K)=A(K)+(.002*I1(J)*COS(2*PI*J*K/1000))
      B(K)=B(K)+(.002*I1(J)*SIN(2*PI*J*K/1000))
40      CONTINUE
      C(K)=(A(K)**2+B(K)**2)**.5
      R(K)=C(K)/C(1)
      WRITE(11,*)K,C(K),R(K)
50      CONTINUE
      STOP

```

2  
VITA

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