EFFECTS OF INITIAL NUMBERS AND INTRINSIC RATES OF INCREASE ON THE OUTCOME OF INTERSPECIFIC COMPETITION AS PREDICTED BY THE LOTKA-VOLTERRA COMPETITION EQUATIONS

Ву

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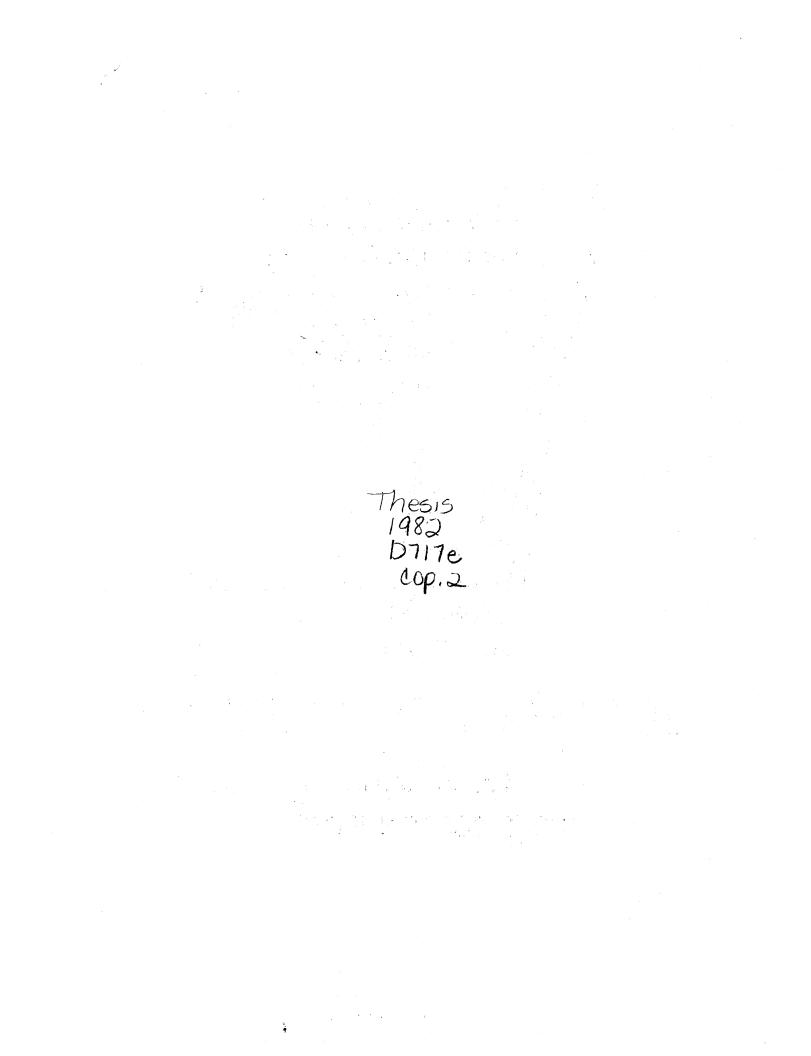
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Thesis Approved: Thesis Adviser Ban hald m au 0 Graduate College

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CHAPTER I

INTRODUCTION

The well known competition equations of Lotka-Volterra can be written as a set of differential equations, namely,

1)
$$\frac{dN_1}{dt} = r_1 N_1 \left(\frac{K_1 - N_1 - \alpha_{12}N_2}{K_1} \right)$$

2)
$$\frac{dN_2}{dt} = r_2 N_2 \left(\frac{K_2 - N_2 - \alpha_{21}N_1}{K_2} \right)$$

where N_1 and N_2 are the population sizes of the two competing species with K_1 and K_2 their respective carrying capacities and r_1 and r_2 their instantaneous per capita rates of growth (biotic potentials). The two alphas are competition coefficients: α_{12} is the per capita inhibitory influence of species 2 on species 1 and α_{21} is the reciprocal effect of species 1 on species 2. When these equations are coupled, they describe the simultaneous growth of two competing species in a homogenous environment over ecological time.

Ecologists have utilized these competition equations, and continue to do so, despite recognized drawbacks. These include linear approximations of the competition coefficients (Gilpin and Justice 1973, Smith-

Gill and Gill 1978, Abrams 1980), the assumption that all individuals are competitive equivalents, and that biotic potentials and carrying capacities are constant (Pinaka 1978). Lacking from these equations are such considerations as stochastic fluctuations (Rao 1979) and time lags (Wangersky and Cunningham 1957, Hassel and Comins 1976). Nonlinear alternatives are discussed in Vandermeer (1973), Rosenzweig and MacArthur (1963) and Gilpin (1973). Due to these deficiences, the equations are generally considered to be poor representatives of reality (Wilbur 1972), but they still model competition adequately for some species in nature (Istock 1977). Even Gause (1934) concluded that in all probability no population ever grows exactly according to the Lotka-Volterra predictions but that the true equations are likely to yield solutions very similar to those of Lotka-Volterra.

Simple algebraic manipulations of the equations can define the conditions where the growth of either population is zero in the presence of its competitor for any given values of K_1 , K_2 , α_{12} , and α_{21} . Four possible outcomes are realized depending upon the geometric configurations formed when the resulting isoclines are plotted (Gause and Witt 1934, Vandermeer 1970, Pianka 1978).

The outcomes, or cases, may be described by a set of inequalities formed by the competition coefficients and carrying capacities (Reiners, et al. 1973, Pinka 1978). These are:

1)
$$\alpha_{12} < \frac{K_1}{K_2}$$
 and $\alpha_{21} > \frac{K_2}{K_1}$ 2) $\alpha_{12} > \frac{K_1}{K_2}$ and $\alpha_{21} < \frac{K_2}{K_1}$
3) $\alpha_{12} > \frac{K_1}{K_2}$ and $\alpha_{21} > \frac{K_2}{K_1}$ 4) $\alpha_{12} < \frac{K_1}{K_2}$ and $\alpha_{21} < \frac{K_2}{K_1}$

Species 1 is the sole survivor, or winner, in case 1 whereas species 2 wins in case 2. A stable two species equilbrium is realized in case 4; but of greatest interest is case 3, because its outcome is not determined by these inequalities alone.

The superimposed isoclines of species 1 and species 2 have the same general appearance in all competitions of case 3 (Fig. 1). For each species, population size will decrease for points above its isocline and increase for points below its isocline. The point of isocline intersection is regarded as an unstable equilbrium condition; arrows pointing toward it indicate the areas where both species will grow or decrease, arrows pointing away from the intersection denote regions in which only one species is able to grow. An arrow at either carrying capacity, K_1 or K_2 , indicates a stable condition where the competitor has become extinct.

Under case 3, each species inhibits the other's growth more than its own and the outcome of competition has been generally thought to depend solely upon the initial densities of species 1 and species 2 (Vandermeer 1970, Pianka 1978).

Park (1962) was able to correlate environmental conditions with success or failure in interspecific competition including a zone of indeterminate outcome where slight changes in initial numbers could alter the usual competitive outcome. In other experiments, Park (1957) found starting densities to make little difference. However, in each of these papers it is unclear as to which case the competitions represent. Using initial numbers, Coste et al. (1978) delinated the stability domains of each species under case 3 and showed that r values can influence the outcome of competition. Strobeck (1973) has determined r values to be

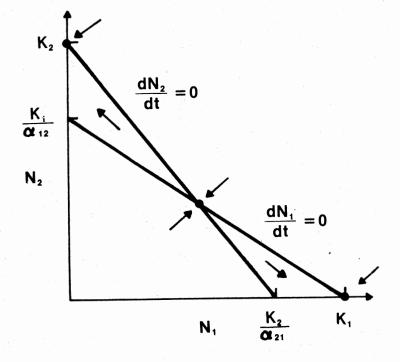


Figure 1. Generalized Graphical Appearance of Case 3 of the Lotka-Volterra competition equations.

important in the coexistance of n > 2 species only, but MacArthur (1969) did not consider them important even in this. Gill (1972) suggested that competitive ability is unrelated to either r or K, but later (Gill 1974) showed graphically that r values can alter the outcome of case 3 competition.

The objective of this study was to determine the effects of initial population sizes and biotic potentials on the outcome of case 3 of the Lotka-Volterra equations. For more detailed information on these and other related equations, the reader is advised to turn to Wangersky (1978) and May (1976).

CHAPTER II

MATERIALS AND METHODS

The objectives of this study were met through the utilization of "compet-l", a computer program modeling the Lotka-Volterra competition equations (Reiners et al.1973). "Compet-l" was altered to allow the winning population to reach its exact K while the numbers of its competitor fell to zero and to output values for dN/Ndt and dN/dt. Further trivial changes facilitated input.

The values for K_1 , K_2 , α_{12} , and α_{21} were chosen in such a manner as to satisfy the inequalities appropriate for case 3. Species isoclines were plotted and initial population sizes were chosen which represented appropriate areas of the plot. The effects of different r values for populations at given starting densities were determined by holding the r of one species constant and altering the other. The data collected over 445 trials included the survivor (or winner) of the struggle, the amount of time required for the survivor to reach K (dubbed saturation time), and the exact numerical path in $N_1 - N_2$ space taken by the populations as the winner reached its K over time.

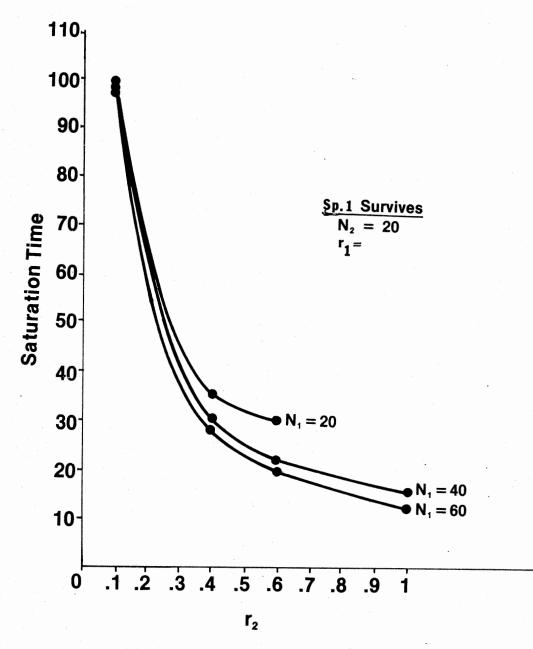
CHAPTER III

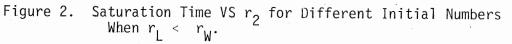
RESULTS

A difference in the r values of two competing species affects events leading to the outcome of competition often without changing the outcome itself. A difference in r values elicits some general effects, no matter what the initial densities. The greater the disparity between r_L and r_W (either $r_L < r_W$ or $r_L > r_W$; $r = r_L$ of loser, $r_W = r$ of winner) the longer the saturation time (Figs. 2 & 3). More growth is obtained by the losing species before its eventual demise if r_L is increased relative to r_W (r_L/r_W increases) for either $r_L < r_W$ (Fig. 4) or $r_L > r_W$ (Fig. 5) providing competition is initiated below both isoclines. The disparity between r_L and r_W also affects the precision with which the two species follow the isocline of the winner in N_1 - N_2 space. The precision increases as r_W/r_L increases for either $r_L < r_W$ (Fig. 4) or $r_L > r_W$ (Fig. 5).

Initial population size also affects saturation time when r values are held constant. The greater the initial size of the eventual winner, the faster it reaches K (Fig. 2). Conversely, the greater the initial numbers of the eventual loser, the more slowly the winner reaches K (Fig. 3).

The intersection of the two species' isoclines in case 3 competition 'is generally regarded as an unstable equilibrium; the point that falls directly on this intersection will remain stable at that value of N_1 and N_2 (coexistence), but points that fall away from this intersection should





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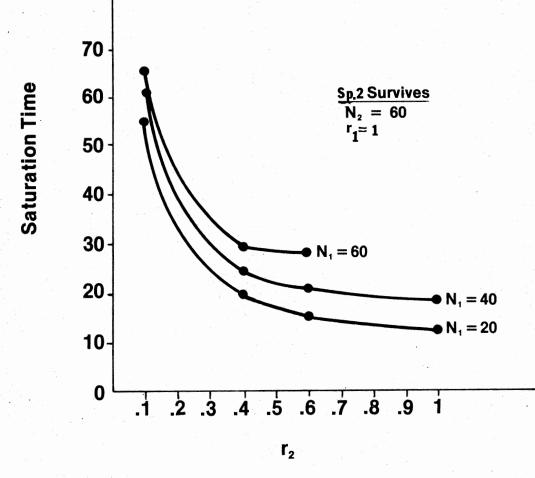


Figure 3. Saturation Time VS r_2 for Different Initial Numbers when $r_L > r_W^{}.$

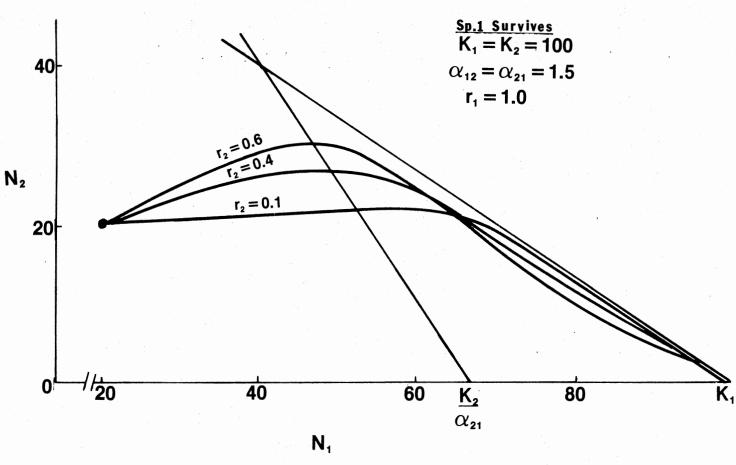
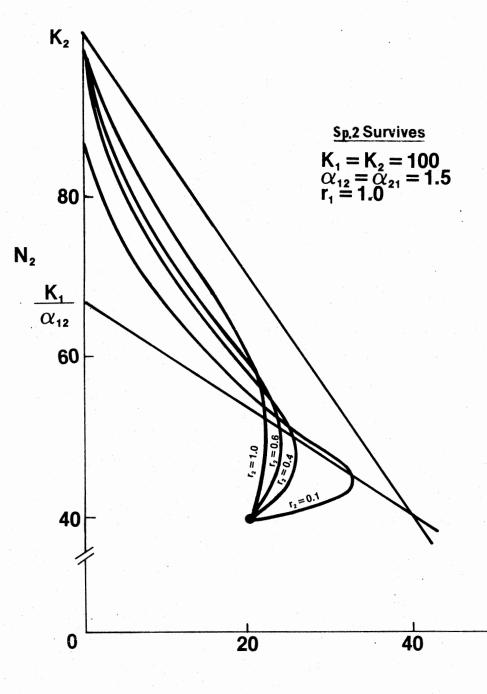


Figure 4. Paths in N₁ - N₂ Space for Two Competing Species With Three Values of r_2^{P} Where $r_L^{P} < r_W^{P}$.



 N_1

Figure 5. Paths in N₁ - N₂ Space for Two Competing Species With Four Values of r_2 Where $r_L \ge r_W$.

move toward the K of either species as the other goes extinct. There is, however, a specialized condition wherein points off the intersection are attracted to the intersection and, thus, coexistence. If initial densities fall directly on an extended line connecting the orgin and the isocline intersection (coexistence line), and r_1 equals r_2 , the two species approach the intersection from either above or below (Fig. 6). Coexistence of the two species is realized. In various computer runs within this specialized condition, one or the other species often won, but only after considerable "generations" (Tables I, II, Appendix B). Lack of coexistence in some cases is thought to be an artefact of the approximation technique of the computer model and the mathematical impossibility of obtaining an exact slope of the line from division of two endless decimals. In these cases, the initial densities could not be placed directly onto the coexistence lines, so true coexistence was not obtained.

For initial densities near but not on the coexistence line, saturation time increases as the distance from the line decreases (Table III, Appendix B, Fig. 7). This increase is most pronounced when initial numbers fall very close to the coexistence line and reaches infinity (coexistence) as the line is reached.

Given r_1 equal to r_2 , the outcome of competition for all initial densities is deterministic. Initial densities that fall below the coexistence line grow to K_1 , leaving species two extinct; initial densities that fall above the line grow to K_2 , leaving species one extinct (Figs. 8-11).

When r values are unequal, the outcome of competition is still deterministic, but is no longer dependent on initial densities relative to the extended straight line connecting the origin and the isocline

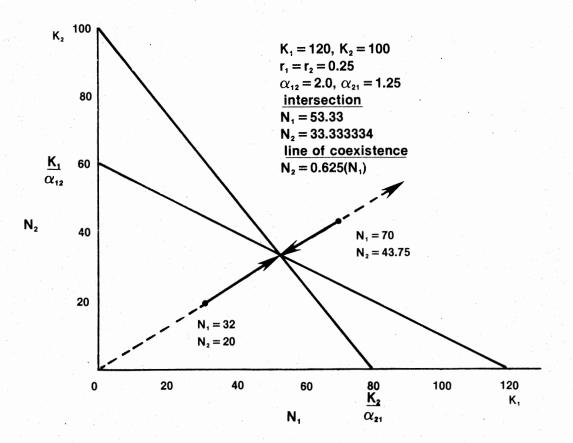


Figure 6. Convergence on the Isocline Intersection When Initial Numbers fall on the Coexistence Line and $r_1 = r_2$.

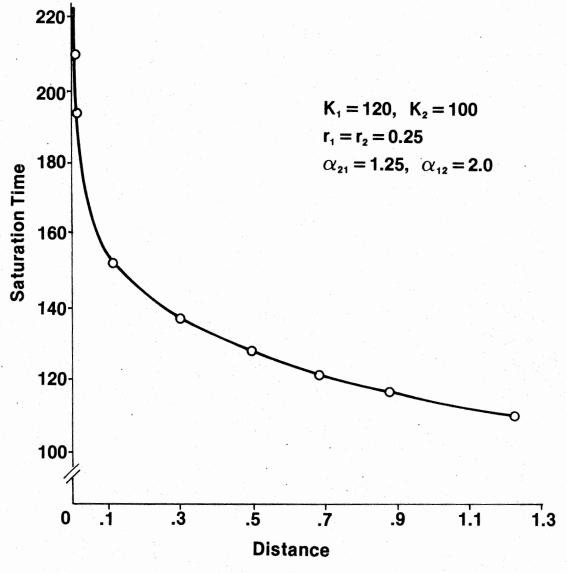


Figure 7. The Effect of Distance From the Coexistence Line on the Saturation Time of a Competitive Interaction when $r_1 = r_2$.

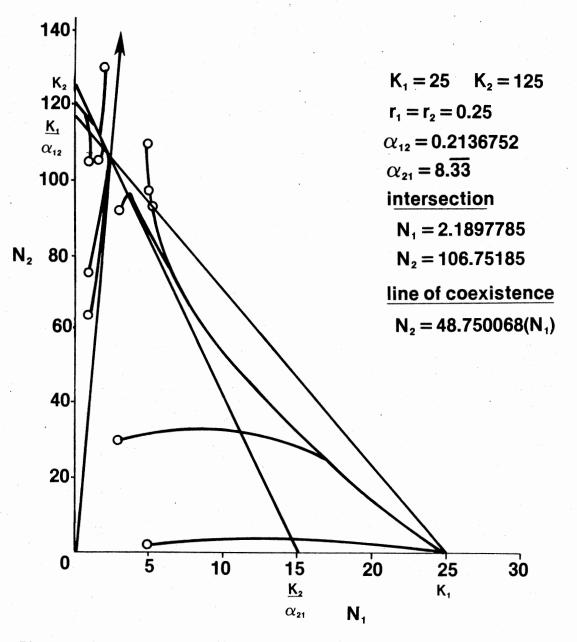
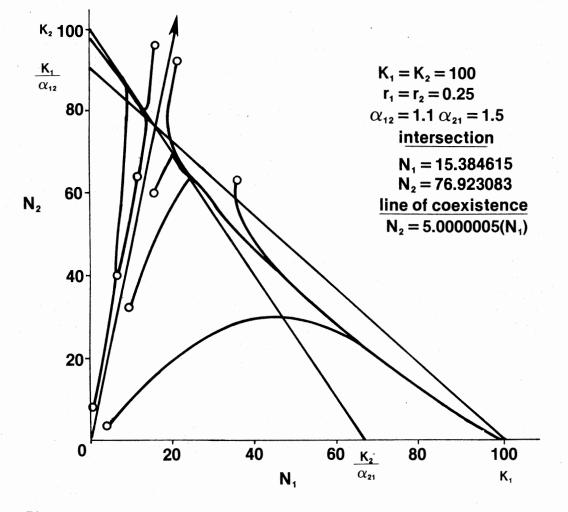
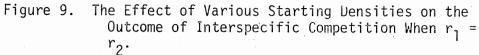


Figure 8. The Effect of Various Starting Densities on the Outcome of Interspecific Competition when $r_1 = r_2$.





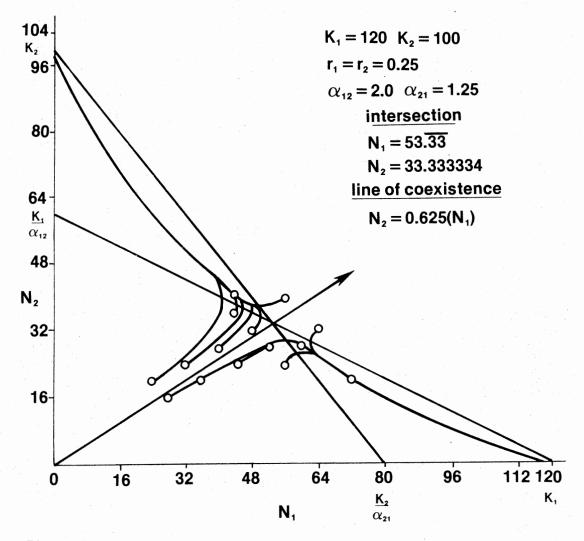


Figure 10. The Effect of Various Starting Densities on the Outcome of Interspecific Competition When $r_1 = r_2$.

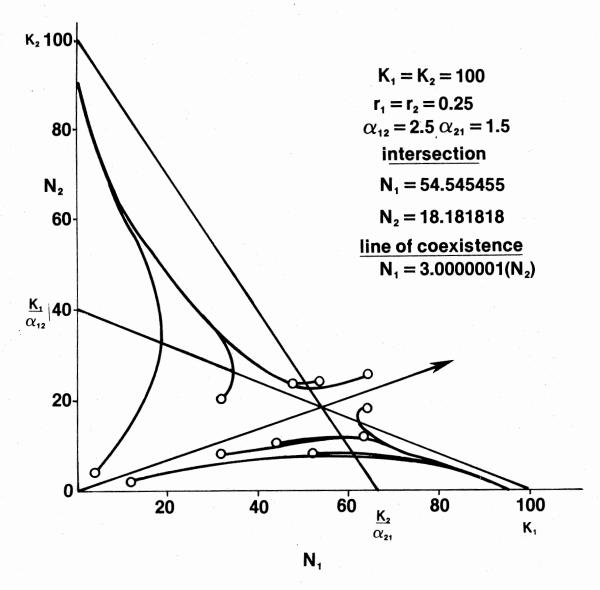


Figure 11. The Effect of Various Starting Densities on the Outcome of Interspecific Competition When $r_1 = r_2$.

intersection (coexistence line). Instead, the boundary that predicts the outcome is an extended curve from the origin, through the isocline intersection and beyond in N_1-N_2 space. The curve is concave for $r_2 > r_1$, convex for $r_1 > r_2$, and steeper for more disparate r values (Fig. 12). Initial densities that fall above this curved coexistence line grow until species 2 wins (Fig. 13, points A, B); initial densities that fall below this curved line grow until species 1 wins (Fig. 13, points C, D). This is true regardless where the initial points fall with respect to the original straight coexistence line (Fig. 13, Points B, C). In other words, at any perpendicular to the original straight coexistence line, the further away initial numbers are from the line, the more different r values must become in order to distort the curved coexistence line enough such that these points then lay to the opposite sides of this curve and thus reverse the outcome of competition predicted from equal r values (Fig. 14).

When r values are different, their effect on the competitive outcome is dependent upon where competition is initiated in relation to the isocline intersection. To reverse the outcome of competition predicted from equal r values, initial points below the intersection require a sufficient inequality as $r_1 > r_2$ (Fig. 15). Initial densities above the intersection require a sufficient inequality as $r_2 > r_1$ (Fig. 16).

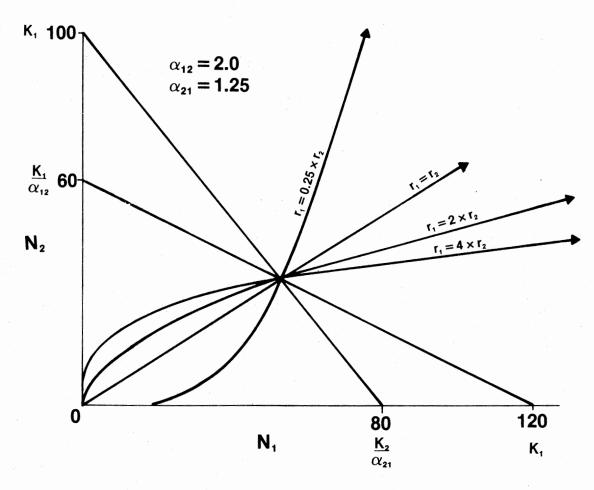


Figure 12. Effect of Diverging r Values on the Coexistence Line in $N_1 - N_2$ Space.

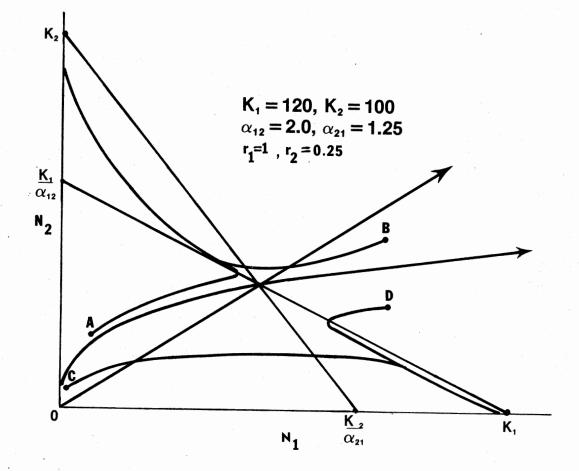


Figure 13. Paths in N₁ - N₂ Space for Four Different Initial Densities With Various Orientations to the Straight and Curved Coexistence Lines When r₁ ≠ r₂.

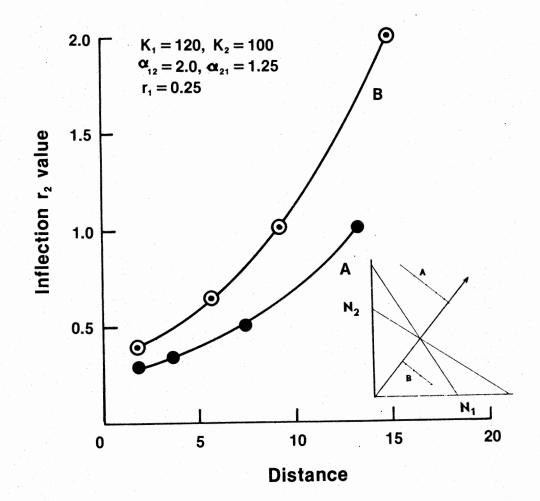


Figure 14. Relationship Between Distance From the Coexistence Line and the Influection r Value. Inflection r Value is that Value Which Results in Population Numbers Crossing the Straight Coexistence Line. Curve A Represents Initial Densities Above the Intersection and Coexistence Line; Curve B Contains Initial Numbers Below the Intersection and Coexistence Line.

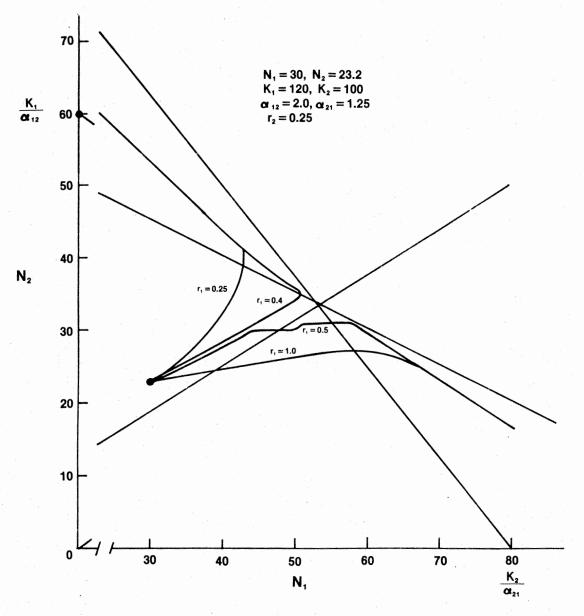


Figure 15. Paths in $N_1 - N_2$ space for Four Values of r_1 Showing a Reversed Competitive Outcome as r_1 Becomes Sufficiently Larger Than r_2 .

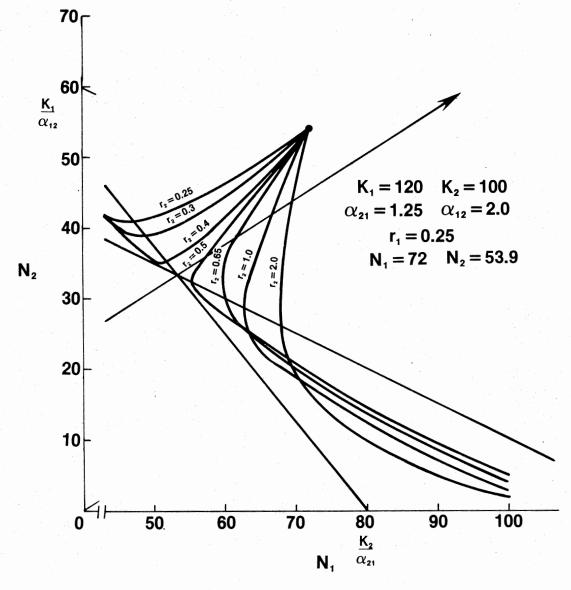


Figure 16. Paths in N₁ - N₂ Space for Seven Values of r₂ Showing a Reversed Competitive Outcome as r₂ Becomes²Sufficiently Larger Than r₁.

CHAPTER IV

DISCUSSION

Some effects due to unequal r values of two competitors are intuitive. One would expect, if there is room for both species to increase initially, that the eventual loser would grow at a faster rate with increasing r values until its isocline is reached. One would expect that the winning species would reach its K faster as its initial numbers are increased and that the greater the initial numbers of the loser, the more time it would take for the winner to reach its K. It is also reasonable that the eventual winner of a competition would require more time to reach its K with progressively lower r values. It seems counter intuitive, however, to expect a longer competitive interaction as observed when r_W became progressively larger than r_1 .

Perhaps the most interesting effects of unequal r values were those dealing with the coexistence line. Coste et al. (1978) correctly delineated the stability domains of a 2-species case 3 competitive interaction but failed to elucidate the effects of unequal r values in shaping these areas. They correctly stated that it is possible for the competitive process to select a particular species (say species one) whose growth rate, r_1 , is smaller than that of its competitor. This is possible, however, only when a certain inequality is met, namely:

$$r_1 > \frac{r_2}{\alpha_{12}}$$

But this inequality does not include initial numbers and therefore cannot account for their effect in determining competitive outcomes. There are initial densities which always favor the species with the lower biotic potential regardless of the magnitude of the competitor's r.

Another effect of unequal r values may be observed by invoking the concept of r-selection. When individuals are not in competition (density independence by definition) natural selection is thought to favor the genotype with the highest Malthusian parameter, resulting in a larger r for the population as a whole. This is the process of r-selection which can be graphically depicted by the addition of a third axis, the per capita logarithmic growth rate (dN/Ndt), to the already familiar plot of competitive isoclines in N_1 - N_2 space. At the point of density independence the per capita logarithmic growth rates are maximal and equal to r_1 and r_2 . As populations grow each dN/Ndt decreases and reaches zero at each respective carrying capacity. Growth rates then become negative for densities beyond these points.

Using this concept, the coexistence line is seen as the two dimensional projection of the line where $dN_1/N_1dt = dN_2/N_2dt$ when $r_1 = r_2$ (Fig. 17). However, r-selection warps this straight line into a curve which is convex for $r_1 > r_2$ and concave for $r_2 > r_1$.

Gill (1974) showed determinancy of outcome in a case 3 interaction dependent on disparity of r values but failed to appreciate some of the complexities of the relationship. In the case of $r_1 > r_2$, Gill maintained that species two must attain at least $N_2 > K_1 / \alpha_{12}$ because at any density lower than this the vector field favors species 1. This statement is clearly fallacious as species 2 may initiate competition at densities far less than this and still emerge victorious (Fig. 18). Conversely,

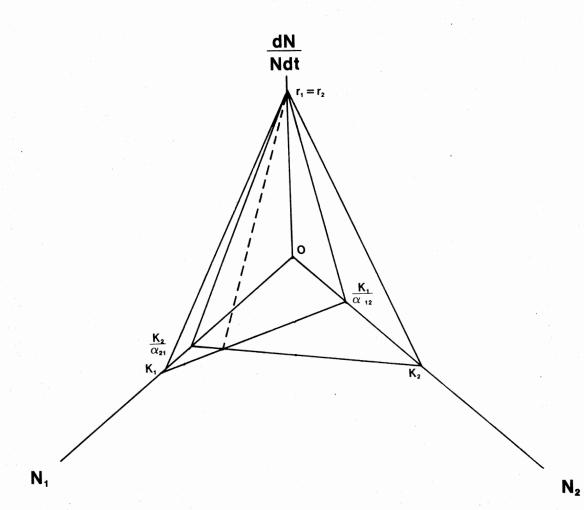


Figure 17. Vector Fields When Equal Growth Rates are Plotted as a Function of N₁ and N₂. For Any Set of Initial Densities Competition Will Favor the Species With the Largest Value of dN/Ndt.

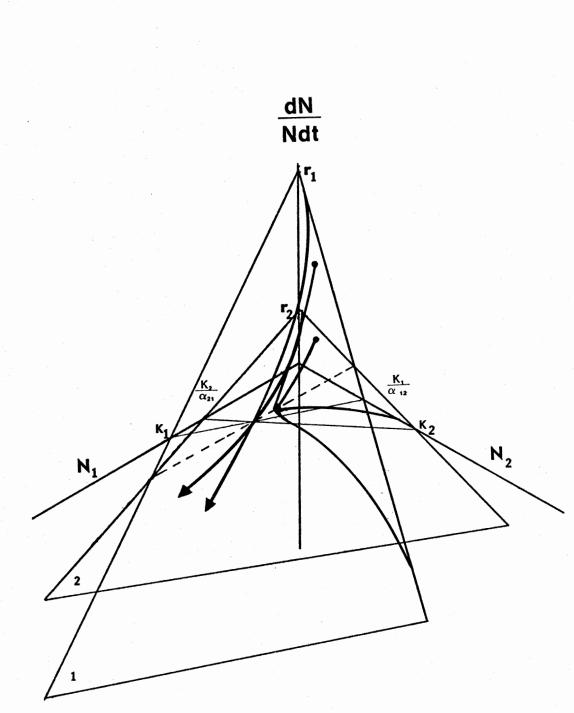


Figure 18. Vector Fields When Unequal Growth Rates $(r_1 > r_2)$ Are Plotted as a Function of N₁ and N₂. Dotted Line Represents all Points Where $dN_1/N_1^2dt = dN_2/N_2dt$ and Curved Lines With Arrows Represent the Curved Coexistence Line. A Competitive Interaction is Depicted in Which Species 2 Survives Despite Initial Numbers $< K_1/\alpha_{12}$. species 1 is not doomed to extinction if introduced at densities greater than those separated by the line where $dN_1/N_1 dt = dN_2/N_2 dt$ (Fig. 19). A species may reach such high densities during periods of mass movement instigated by migration or ecological disaster, as in a fire.

One applied area where this work is of potential use is in biological control. It is generally agreed upon that the larger the number of beneficial species in a complex, the greater the chances that the complex will regulate a host population. For this reason, researchers in biological control may strive for the establishment of exotic beneficials but at the same time do not want their potential competitors (the indigenous beneficials) to become extinct. If the inequalities formed by the exotic and indigenous species match those of case 3, competitive exclusion of one of the species may not necessarily be the end result. These findings suggest that there are initial population numbers that may result in coexistence or very long saturation times which would make extinction in a short time very unlikely. Workers should introduce the appropriate number of exotics that will result in a point on or near the line of coexistence, or the isocline intersection.

Another benefit may be visualized utilizing the process of competitive exclusion. Certain desirable species may be useful as competitors of an undesirable species. Numbers should be introduced resulting in a point that lies to the proper side of the curved coexistence line. For example, if there exists a pest of a desirable species which is detrimental due to a certain physiological factor (ie. a toxin) an ecological equivalent may be found that does not exhibit this trait. The harmless competitor could then be introduced in the appropriate numbers and the extinction of the harmful species would be the predicated outcome of the struggle.

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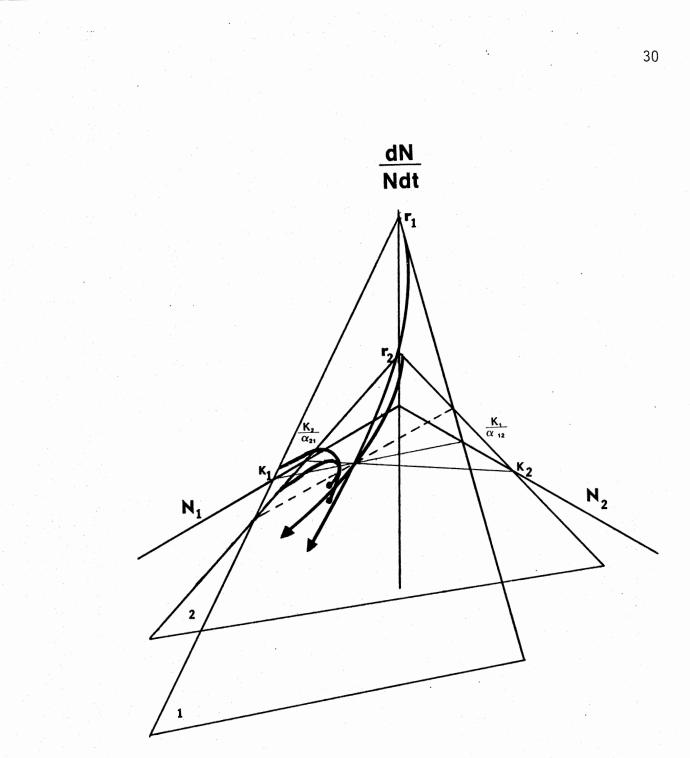


Figure 19. Vector Fields When Unequal Growth Rates $(r_1 > r_2)$ are Plotted as a Function of N_1 and N_2 . A Competitive Interaction is Shown in Which Species 1 Survives Despite Initial Numbers Greater Than Those Separated by $dN_1/N_1dt=dN_2/N_2dt$.

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APPENDIX A

EDITED VERSION OF COMPET-1

- 10 REM FOR FURTHER INFORMATION ABOUT THIS PROGRAM CONTACT
- 20 REM CONDUIT P. O. BOX 388, IOWA CITY, IOWA 52240 (319) 353-3170
- 30 REM REGISTRY NUMBER: B10083
- 40 REM LAST REVISION 7/26/77 BY NEIL S. FERGUSON FOR CONDUIT
- 50 REM THIS PROGRAM IS DISTRIBUTED WITH THE SUPPORT OF THE NATIONAL
- 60 REM SCIENCE ROUNDATION GRANT NO. SED75-06596. ANY OPINION, FINDINGS,
- 70 REM CONCLUSIONS, OR RECOMMENDATIONS EXPRESSED OR IMPLIED ARE THOSE
- 80 REM OF THE AUTHORS AND DO NOT NECESSARILY REFLECT THE VIEWS OF THE
- 90 REM NATIONAL SCIENCE FOUNDATION.
- 100 REM "COMPET-1" -- (BASIC PROGRAM BEGINS AT LINE 360)
- 110 REM
- 120 REM VERSION 1: WRITTEN BY WILLIAM E. GLANZ '70, AND WILLIAM A.
- 130 REM REINERS. DEPT. OF BIOLOGY. DARTMOUTH COLLEGE. WRITTEN IN
- 140 REM BASIC ON THE DARTMOUTH TIME-SHARING SYSTEM.
- 150 REM
- 160 REM COPYRIGHT 1974 BY THE TRUSTEES OF DARTMOUTH COLLEGE
- 170 REM
- 180 REM DESCRIPTION:
- 190 REM
- 200 REM THIS PROGRAM IS AN INTERSPECIFIC COMPETITION MODEL BASED
- 210 REM ON THE CLASSICAL LOTKA-VOLTERRA EQUATION. IT CALCULATES THE
- 220 REM POPULATION SIZE OF TWO COMPETING SPECIES IN SUCCESSIVE TIME
- 230 REM STEPS USING DATA FROM WITHIN THE PROGRAM. THE CALCULATIONS
- 240 REM ARE DONE USING INCREMENTAL ADDITION.
- 250 REM
- 260 REM
- 270 REM

280	REM THIS PROGRAM IS TAKEN FROM THE TEXT ECOLOGICAL MODELING BY
290	REM WILLIAM E. GLANZ AND WILLIAM A. REINERS.
300	REM
310	REM
320	REM
330	REM * * * * * * * * * * * * * * * * * * *
340	REM READ DATA FOR POPULATION ONE! INITIAL POPULATION SIZE, GROWTH
350	REM RATE, ENVIRONMENTAL CAPACITY, AND FRACTION OVERLAP BY POP. #2.
360	DIM Z\$(3)
370	PRINT " VALUES FOR POPULATION ONE"
380	PRINT
390	PRINT "ENTER INITIAL POPULATION SIZE"
400	INPUT N1
410	PRINT "ENTER GROWTH RATE"
420	INPUT R1
430	PRINT "ENTER ENVIRONMENTAL CAPACITY"
440	INPUT K1
450	PRING "ENTER FRACTION OF OVERLAP BY POPULATION #2"
460	INPUT J1
470	PRINT
480	PRINT "
490	PRINT
500	PRINT "ENTER INITIAL POPULATION SIZE"
510	INPUT N2
520	PRINT "ENTER GROWTH RATE"
530	INPUT R2
540	PRINT "ENTER ENVIRONMENTAL CAPACITY"

- 550 INPUT K2
- 560 PRINT "ENTER FRACTION OF OVERLAP BY POPULATION #1"
- 570 INPUT J2
- 580 REM READ NUMBER OF TIME PERIODS TO SIMULATE AND CALCULATION
- 590 PRINT INVERAL
- 620 PRINT "INTERVAL OF COMPUTATION IS 0.1"
- 630 LET I=0.1
- 640 PRINT
- 650 PRINT "TIME", "POP, #1", "POP, # 2"
- 660 PRINT
- 661 LET E=1
- 670 FOR T=1 TO 2000
- 672 LET P1=0
- 674 LET P2=0
- 676 LET D1=0
- 678 LET D2=0
- 680 REM I IS COMPUTATION INTERVAL, SO 1/I IS NUMBER OF INTERVALS
- 690 FOR A=1 TO INT (1/I+.5)
- 700 LET G1=1*R1*N1*(K1-N1-J1*N2)/K1
- 710 LET G2=I*R2*N2*(K2-N2-J2*N1)/K2
- 712 LET P1=G1/N1+P1
- 714 LET P2=G2/N2+P2
- 720 LET N1=N1+G1
- 730 LET N2-N2+G2
- 732 LET D1=D1+G1
- 734 LET D2=D2+G2
- 740 NEXT A

LET Q1=INT (N1+.5)

LET Q2=INT (N2+.5)

- 780 PRINT T, Q1, Q2, P1, P2, D1, D2
 781 IF Q1=0, OR, Q1=K1 GOTO 783
 782 GOTO 784
 783 IF Q2=0, OR, Q2=K2 GOTO 791
 784 LET E=E+1
 - 785 NEXT T
 - 786 PRINT

750

760

770

- 791 PRINT "EXTINCTION TIME WAS", E
- 792 PRINT
- 793 PRINT
- 800 PRINT "DO YOU WISH TO RERUN THIS PROGRAM" (YES/NO)"
- 810 INPUT Z\$
- 820 IF Z\$="YES" THEN 370
- 830 IF Z\$="NO" THEN 860
- 840 PRINT "PLEASE ENTER A YES OR A NO"
- 850 GOTO 800
- 860 END

APPENDIX B

TABLES

TABLE I

THE EFFECT OF VARIOUS r VALUES ON THE OUTCOME OF INTERSPECIFIC COMPETITION WHEN INITIAL SPECIES NUMBERS FALL ON THE COEXISTENCE LINE ABOVE, BELOW, AND AT THE POINT OF ISOCLINE INTERSECTION

For:	$K_1 = K_2 = 1$	00 ^α 12	^{= α} 21 ⁼	1.5			
		(belc N ₁ = N ₂ =	ow) 20 20	$(intersec N_1 = 40 N_2 = 40)$	tion)	(abov N1 = N2 =	
r]	r ₂	sat. time	winner	sat. time	winner	sat. time	winner
1	0.01	966	N	> 2,000	tie	517	N ₂
1	0.1	106	N	> 2,000	tie	65	N ₂
1	0.2	60	Ν ₁	> 2,000	tie	40	N ₂
1	0.4	32	N٦	> 2,000	tie	29	N ₂
1	0.5	30	Nl	> 2,000	tie	28	N ₂
1	0.6	30	N	> 2,000	tie	28	N ₂
1	0.9	31	N ₁	> 2,000	tie	32	N ₂
1	0.99	41	N	> 2,000	tie	42	N ₂
1	0.999	64	N	> 2,000	tie	66	N ₂
1	1,	> 2,000	tie	> 2,000	tie >	2,000	tie
0.01	1	966	N ₂	> 2,000	tie	517	N _T
0.1	1	106	N ₂	> 2,000	tie	65	N ₁
0.6	1	30	N ₂	> 2,000	tie	28	N ₁
0.99	1	41	N2	> 2,000	tie	42	N

TABLE I (Continued)

	FOR K	1 = 120 ,	K ₂ = 1	00 , ∝12 =	= 2.0, α2	1 = 1.25	
		N ₁ = 32		N ₁ = 53	.33	N ₁ = 76	
		N ₂ = 20		N ₂ = 33	. 333334	N ₂ = 47	.5
r	r ₂	sat. time	winner	sat. time	winner	sat. time	winner
2	0.25	48	N	> 2,000	tie	30	N ₂
0.25	2	37	N ₂	> 2,000	tie	27	N ₁
0.25	0.25	253	NJ	>2,000	tie	257	NJ
	FOR K ₁	= K ₂ =	100	α]2 =].]	α2] =	1.5	· · ·
		N ₁ = 8		N ₁ = 1	5.384615	N ₁ = 20	
		N ₂ = 40	.000004	N ₂ = 76	.923083	N ₂ = 10	0.00001
r _l	r ₂						
0.25	0.25	615	NJ	> 2,000	tie	> 2,000	tie
1	0.25	61	N	>2,000	tie	63	N ₂
0.25	1	179	N ₂	>2,000	tie	97	N ₁
	FOR K ₁	= 25 , K	2 = 125	α 12 =	0.2136752	α 21 = 8.	33
		N ₁ = 1		N ₁ = 2.	1897785	N ₁ = 3	
		N ₂ = 4	8.750068	$N_2 = 10$	6.75185	$N_2 = 14$	6.2502
rı	r ₂						
0.25	0.25	850	N ₂	> 2,000	tie	201	N
1	0.25	50	N	> 2,000	tie	41	N ₂
0.25	1	230	N ₂	> 2,000	tie	30	N

TABLE I (Continued)

FOR: $K_1 = K_2 = 100$, $\alpha_{12} = 2.5$, $\alpha_{21} = 1.5$							
		N ₁ = 3 N ₂ = 1		N ₁ = 54. N ₂ = 18.		N ₁ = 7 N ₂ = 2	72.000002 24
r	r ₂						· .
0.25	0.25	202	N	>2,000	tie	201	N ₁
1	0.25	51	N	>2,000	tie	41	N ₂
0.25	1	26	N ₂	>2,000	tie	30	N

THE EFFECT OF V	ARIOUS	STARTING	DENSITIES	С
THE OUTCOME OF	INTERS	SPECIFIC	COMPETITIO	N

TABLE II

	N ₁	N ₂	Winner	Saturation Tim
	16	60	NJ	139
	12	64	N ₂	311
	12	81	N ₂	226
	16	96	N ₂	253
	22	92	N	161
	36	64	NJ	80
	10	32	N	121
	7	40	N ₂	274
	4	20.000002	. N [.] l	600
	15.384615	76.923083	tie	> 2,000
	20	100.00001	tie	> 2,000
<1 =	$K_2 = 100, \alpha_2$	$\alpha_{1} = 1.5, \alpha_{12} = 3$	2.5, r ₁ = r ₂	2 = 0.25
	52	8	N	50
	12	2	N	60
	4	4	N ₂	44
	48	24	N ₂	49
	54	24	N ₂	53
	64	26	N ₂	

42

ON

		TABLE II (CO	rt muea j	
	64	18	N ₁	56
	64	12	N ₁	56
	36.000001	12	N	207
	54.545455	18.181818	tie	>2,000
	70.00002	24	N ₁	201
= 2	25, $K_2 = 125, \alpha_{12}$	= 0.2136752, α	₂₁ = 8. <u>33</u> ,	$r_1 = r_2 = 0.25$
	2	105	N ₂	399
	5	94	N	81
	1	115	N ₂	210
	4	97.5	N	98
	5	110	N ₁	90
	5	2	N	25
	3	30	N	62
	1	64	N ₂	307
	1	75	N ₂	271
	3	92	N	121
	1	48.750068	N ₂	850
	2.1897785	106.75185	tie	> 2,000
	3	146.2502	N ₂	850

K٦

TABLE II (Continued)

$= K_2 = 100,$	$\alpha_{12} = \alpha_{21} = 1.5$	$r_1 = r_2 = 1.0$	
40	20	N ₁	16
60	20	N	13
20	40	N ₂	16
60	40	NŢ	18
20	60	N ₂	13
40	60	N ₂	18
20	20	tie	> 2,000
40	40	tie	> 2,000
60	60	tie	> 2,000
₁ = 120, K ₂ =	100, α ₁₂ = 2.0,	α ₂₁ = 1.25, r ₁	= r ₂ = 0.25
	100, $\alpha_{12} = 2.0$,		
44	100, α ₁₂ = 2.0, 40	N ₂	65
	100, $\alpha_{12} = 2.0$,	N ₂ N ₂	
44 56	100, $\alpha_{12} = 2.0$, 40 40	N ₂ N ₂ N ₂	65 83
44 56 44	100, $\alpha_{12} = 2.0$, 40 40 36	N ₂ N ₂ N ₂ N ₂ N ₂	65 83 71
44 56 44 48	100, $\alpha_{12} = 2.0$, 40 40 36 32	N ₂ N ₂ N ₂	65 83 71 96
44 56 44 48 40	100, $\alpha_{12} = 2.0$, 40 40 36 32 28	N ₂ N ₂ N ₂ N ₂ N ₂ N ₂	65 83 71 96 87
44 56 44 48 40 32	100, $\alpha_{12} = 2.0$, 40 40 36 32 28 24	N ₂ N ₂ N ₂ N ₂ N ₂ N ₂ N ₂	65 83 71 96 87 79
44 56 44 48 40 32 24	100, $\alpha_{12} = 2.0$, 40 40 36 32 28 24 20	$\begin{array}{c} N_2 \\ N_2 \end{array}$	65 83 71 96 87 79 72
44 56 44 48 40 32 24 64	100, $\alpha_{12} = 2.0$, 40 40 36 32 28 24 20 32	$ N_{2} \\ N_{1} \\ \end{pmatrix} $	65 83 71 96 87 79 72 81

TABLE II (Continued)

ų,

44	24	N	91
36	20	N	95
28	16	N	101
53.33	33.333334	tie	> 2,000
32	20	N	253
76	47.5	N	257
36	48	N ₂	52
			•

TABLE II (Continued

TABLE III

$K_1 = 120, K_2 = 100, \alpha_{12} = 2.0, \alpha_{21} = 1.25, r_1 = r_2 = 0.25$							
۲N	N ₂	Winner	Sat. Time	Distance			
76	47.5	N	257	0.000			
77	45.9	N ₁	108	1.887			
79	42.7	Nl	88	5.660			
81	39.5	N	78	9.434			
85	33.1	N	66	16.98			
93	20.3	N	52	32.076			
75	49.1	N2	101	1.887			
74	50.7	N ₂	88	3.774			
72	53.9	N ₂	76	7.547			
69	58.7	N ₂	66	13.208			
61	71.5	N ₂	53	28.302			
32	20	N	253	0.000			
33	18.4	N	96	1.887			
35	15.2	Nl	75	5.66			
37	12	N ₁	64	9.434			
40	7.2	N ₁	52	15.094			
31	21.6	N ₂	89	1.887			
30	23.2	N ₂	77	3.774			
27	28	N ₂	61	9.434			
21	37.6	N ₂	50	20.755			

INFLUENCE OF DISTANCE FROM THE COEXISTENCE LINE ON THE SATURATION TIME OF A COMPETITIVE INTERACTION

N ₁	N2	Winner	Sat. Time	Distance
53.333	33.333334	tie	2,000	0.000
53.3335	33.33307	N	220	0.0003124
53.335	33.33067	N	216	0.0031426
53.336	33.3291	N	209	0.005004
53.338	33.3259	N	200	0.0087775
53.34	33.32267	N	193	0.0125766
53.4	33.22667	N ₁	152	0.1257843
53.5	33.06667	N ₁	136	0.314464
53.6	32.90667	N	127	0.5031436
53.7	32.74667	N	121	0.6918232
53.8	32.58667	N ₁	117	0.8805028
54	32.26667	N ₁	110	1.2578621
54.5	31.46667	N _T	100	2.2012618
55	30.66667	N ₁	94	3.1446583
57	27.46667	N	79	6.9182507
61	21.06667	N ₁	64	14.465436
68	9.86667	N ₁	47	27.673
53	33.86667	N ₂	115	0.6289342
51	37.06667	N ₂	81	4.4025
49	40.26667	N ₂	70	8.1761
44	48.26667	N ₂	57	17.6101
37	59.46667	N ₂	47	30.8177
76.01	47.484	N	191	0.018868

TABLE III (Continued)

	NJ	N ₂	Winner	Sat. Time	Distance
. '	76.05	47.42	N	162	0.0943398
	76.09	47.356	N	152	0.1698
	76.14	47.276	N _T	144	0.2642
	76.2	47.18	N	137	0.3774
	76.27	47.068	N ₁	132	0.5094
	76.32	46.988	N ₁	129	0.6038
•	76.45	46.78	N ₁	123	0.8491
	76.55	46.62	N ₁	119	1.0377
	76.65	46.46	N ₁	116	1.2264

TABLE III (Continued)

TABLE IV

-					
ĸ _l	= K ₂ = 100	$\alpha_{12} = \alpha_{21}$	= 1.5		
r _l	r ₂	N ₁	N ₂	Winner	Sat. Time
]	0.1	40	20	N ₁	99
1	0.4	40	20	N ₁	30
1	0.6	40	20	N ₁	22
1	0.1	60	20	N ₁	99
]	0.4	60	20	N ₁	28
1	0.6	60	20	N ₁	20
]	0.1	20	40	N ₂	97
1	0.4	20	40	N2	27
]	0.1	60	40	N	167
1	0.4	60	40	N ₁	41
1	0.6	60	40	N ₁	28
ן י י	0.1	20	60	N ₂	57
1	0.4	20	60	N ₂	20
	0.6	20	60	N ₂	16
1	0.1	40	60	N ₂	62
1	0.4	40	60	N ₂	24
1	0.6	40	60	N ₂	21
0.1	1	40	20	N ₁	96
0. 4	1	40	20	N ₁	27

THE EFFECT OF VARIOUS r VALUES AND INITIAL DENSITIES ON THE OUTCOME OF INTERSPECIFIC COMPETITION

r	r ₂	N	N ₂	Winner	Sat. Time
0.1	1	20	60	N ₂	99
0.4	···] ···	20	60	N ₂	28
0.1	1	40	60	N ₂	167
0.6]	40	60	N ₂	18
0.1	1	21	20	N ₂	109
1	0.1	21	20	NJ	106
<u> </u>	20, 12 - 10	$\alpha_{12} = 2.0$, α 21 - 1.25		-
1	0.1	24	24	Na	135
	0.1 1	24 24	24 24	N ₁ N2	135 78
1 0.1 1				N ₂	
0.1	1	24	24	· · · · ·	78
0.1 1 0.1	1 0.1 1	24 40	24 8 8	N ₂ N ₁ N ₂	78 80
0.1 1 0.1	1 0.1 1	24 40 40	24 8 8	N ₂ N ₁ N ₂	78 80
0.1 1 0.1 K ₁ = 2	1 0.1 1 5, K ₂ = 125	24 40 40 5, $\alpha_{12} = 0.213$	24 8 8 6752, $\alpha_{21} = 8$	N ₂ N ₁ N ₂ 3.33	78 80 123
0.1 1 0.1 $K_1 = 2$ 1	1 0.1 1 5, $K_2 = 125$ 0.1	24 40 40 5, $\alpha_{12} = 0.213$ 3	24 8 8 6752, $\alpha_{21} = 8$ 30	N ₂ N ₁ N ₂ 3.33 N ₁	78 80 123 74

TABLE IV (Continued)

K ₁ = K ₂	= 100, a]	$2 = 2.5, \alpha 21$	= 1.5		
1	0.1	44	12	NJ	111
0.1	1	44	12	N ₂	59
1	0.1	44	20	N ₂	99
0.1	1	44	20	N ₂	53
κ ₁ = κ ₂	= 100, α ₂	$_{21} = 1.5, \alpha_{12}$, = 1.1		
]	0.1	10	32	NJ	109
0.1	1	10	32	N ₂	482
1	0.1	12	64	N	164
0.1	1	12	64	N ₂	526

TABLE IV (Continued)

TABLE V	ŤA	BL	E.	۷
---------	----	----	----	---

	FFECT						
THE	COEXI	ISTEN	ICE L	INE	IN	$N_1 - N_2$,
		SF	PACE			1 2	

For: K	₁ = 120, K ₂ = 1	00, $\alpha_{12} = 2$,	α ₂₁ = 1.	25, r ₁ = 1.(), r ₂ = 0.25
NJ	N ₂	Winner	NJ	N ₂	Winner
]	5	N ₁	68	38	N ₂
1	12	N ₂	78	36	N
3	12	N _l	78	40	N ₂
6	12	N 2	86	36	N
4	16	N ₂	86	38	N ₁
6	16	N ₁	86	39	N ₂
5	20	N ₂	92	38	N
8	20	N ₂	92	40	N ₂
9	20	N ₂	98	40	N ₁
10	20	N	98	42	N ₂
12	23	N2	106	40	N ₁
14	23	N ₁	106	42	N ₂
16	23	N ₁	114	42	N ₁
18	25	N ₂	114	44	N ₂
20	25	N ₂	120	42	N
22	25	N ₁	120	44	N ₂
24	26	N ₁	46	34	N ₂
36	28	N	54	31	N ₁
36	32	N ₂	54	36	N ₂
46	31	N _T	68	35	N ₁

N	N ₂	Winner	N	N2	Winner
r ₁ = 0	.5, r ₂ = 0.25				
2	4	NJ	32	26	N ₂
2	6	N ₂	44	28	N ₁
4	8	N ₁	44	30	N ₂
8	10	Nl	62	36	N
8	12	N ₂	70	38	N ₁
12	14	N ₁	70	40	N ₂
12	16	N ₂	86	42	N _T
16	16	N	86	44	N ₂
16	18	N ₂	140	56	N ₁
22	20	N	140	58	N ₂
22	22	N ₂	140	62	N ₂
32	24	N			
r ₁ = 0	.25, r ₂ = 1.0				
16]	N ₂	48	22	N ₁
20	1	N ₂	48	24	N ₂
24	1	Nl	56	38	N ₁
18	.5	N	56	40	N ₂
24	3	N ₂	59	44	N ₁
28	4	N	59	46	N ₂

TABLE V (Continued)

N ₁		N ₂		Winner	N	• • •	^N 2	Winner
28	<u> </u>	6		N ₂	62		50	NJ
36		8		N	62		54	N ₂
36		10		N ₂	66		64	NJ
42		14		N٦	66		66	N ₂
42		15		N ₂				
70		76		N				•
70		78		N ₂				
73		86	•	N				
73		88		N ₂				
76		98		N				
76	1	00		N ₂				

TABLE V (Continued)

TABLE VI

For: K _l	= 120, K ₂ =	= 100, ^a 2	1 = 1.25,	$\alpha_{12} = 2.0$		
NJ	N ₂	۳	r ₂	Winner	Sat.Time	Distance
77	45.9	0.25	0.25	NJ	108	1.887
77	45.9	0.3	0.25	N ₂	136	1.887
77	45.9	0.4	0.25	N ₂	72	1.887
77	45.9	0.5	0.25	N ₂	58	1.887
77	45.9	0.65	0.25	N ₂	48	1.887
77	45.9	1.0	0.25	N ₂	39	1.887
77	45.9	2.0	0.25	N ₂	31	1.887
79	42.7	0.25	0.25	N	88	5.66
79	42.7	0.3	0.25	N	90	5.66
79	42.7	0.4	0.25	N	104	5.66
79	42.7	0.5	0.25	N ₂	79	5.66
79	42.7	0.65	0.25	N ₂	58	5.66
79	42.7	1.0	0.25	N ₂	44	5.66
79	42.7	2.0	0.25	N ₂	34	5.66
81	39.5	0.25	0.25	N	78	9.434
81	39.5	0.4	0.25	N ₁	79	9.434
81	39.5	0.5	0.25	۲N	82	9.434
81	39.5	0.65	0.25	N	92	9.434
81	39.5	1.0	0.25	N ₂	57	9.434
81	39.5	2.0	0.25	N ₂	40	9.434

THE EFFECTS OF BIOTIC POTENTIAL AND DISTANCE FROM THE COEXISTENCE LINE ON THE OUTCOME OF INTERSPECIFIC COMPETITION

TABLE VI (Continued)

N	N ₂	r	r ₂	Winner	Sat. Time	Distance
85	33.1	0.25	0.25	NJ	66	16.98
85	33.1	0.3	0.25	N	65	16.98
85	33.1	0.4	0.25	N	63	16.98
85	33.1	0.5	0.25	N	63	16.98
85	33.1	0.65	0.25	N	62	16.98
85	33.1	1.0	0.25	N	62	16.98
85	33.1	2.0	0.25	N	64	16.98
75	49.1	0.25	0.25	N ₂	101	1.887
75	49.1	0.25	0.3	N	128	1.887
75	49.1	0.25	0.4	N	72	1.887
75	49.1	0.25	0.5	N	57	1.887
75	49.1	0.25	0.65	NJ	46	1.887
75	49.1	0.25	1.0	N	36	1.887
75	49.1	0.25	2.0	۲ ^N	28	1.887
74	50.7	0.25	0.25	N ₂	88	3.774
74	50.7	0.25	0.3	N2	97	3.774
74	50.7	0.25	0.4	N	81	3.774
74	50.7	0.25	0.5	N	61	3.774
74	50.7	0.25	0.65	N	48	3.774
74	50.7	0.25	1.0	NŢ	37	3.774
74	50.7	0.25	2.0	N	28	3.774

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	N			11	C-+ Time	Distance
N	N ₂	۲ _ا	r ₂	Winner	Sat.Time	Distance
72	53.9	0.25	0.25	N ₂	76	7.547
72	53.9	0.25	0.3	N ₂	77	7.547
72	53.9	0.25	0.4	N ₂	88	7.547
72	53.9	0.25	0.5	N	77	7.547
72	53.9	0.25	0.65	N	55	7.547
72	53.9	0.25	1.0	N	40	7.547
72	53.9	0.25	2.0	N	30	7.547
69	58.7	0.25	0.25	N ₂	66	13.208
69	58.7	0.25	0.3	N ₂	66	13.208
69	58.7	0.25	0.4	N ₂	66	13.208
69	58.7	0.25	0.5	N ₂	70	13.208
69	58.7	0.25	0.65	N ₂	96	13.208
69	58.7	0.25	1.0	N ₁	46	13.208
69	58.7	0.25	2.0	N	32	13.208
61	71.5	0.25	0.25	N ₂	53	28.302
61	71.5	0.25	0.3	N ₂	52	28.302
61	71.5	0.25	0.4	N ₂	51	28.302
61	71.5	0.25	0.5	N ₂	51	28.302
61	71.5	0.25	0.65	N ₂	51	28.302
61	71.5	0.25	1.0	N ₂	53	28.302
61	71.5	0.25	2.0	N2	99	28.302

TABLE VI (Continued)

NJ	N ₂	r _l	r ₂	Winner	Sat.Time	Distance
33	18.4	0.25	0.25	Nl	96	1.887
33	18.4	0.25	0.3	NJ	106	1.887
33	18.4	0.25	0.4	N2	77	1.887
33	18.4	0.25	0.65	N ₂	53	1.887
33	18.4	0.25	1.0	N ₂	45	1.887
33	18.4	0.25	2.0	N ₂	38	1.887
35	15.2	0.25	0.25	N	75	5.66
35	15.2	0.25	0.3	N	71	5.66
35	15.2	0.25	0.4	N	70	5.66
35	15.2	0.25	0.5	N	84	5.66
35	15.2	0.25	0.65	N ₂	67	5.66
35	15.2	0.25	1.0	N ₂	50	5.66
35	15.2	0.25	2.0	N ₂	41	5.66
37	12	0.25	0.25	N _l	64	9.434
37	12	0.25	0.3	N _l	59	9.434
37	12	0.25	0.4	۲N	54	9.434
37	12	0.25	0.5	N	53	9.434
37	12	0.25	0.65	N ₁	55	9.434
37	12	0.25	1.0	N ₂	63	9.434
37	12	0.25	2.0	N ₂	44	9.434
40	7.2	0.25	0.25	NJ	52	15.094
40	7.2	0.25	0.3	N	48	15.094
40	7.2	0.25	0.4	N	43	15.094

TABLE VI (Continued)

TABLE VI (Continued)

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NJ	N ₂	r	r ₂	Winner	Sat.Time	Distance
40	7.2	0.25	0.5	N	40	15.094
40	7.2	0.25	0.65	N	39	15.094
40	7.2	0.25	1.0	N ₁	39	15.094
40	7.2	0.25	2.0	N ₂	62	15.094
31	21.6	0.25	0.25	N ₂	89	1.887
31	21.6	0.3	0.25	N ₂	100	1.887
31	21.6	0.4	0.25	N ₁	89	1.887
31	21.6	0.5	0.25	Р	75	1.887
31	21.6	0.65	0.25	NJ	65	1.887
31	21.6	1.0	0.25	NJ	57	1.887
31	21.6	2.0	0.25	N	50	1.887
30	23.2	0.25	0.25	N ₂	77	3.774
30	23.2	0.3	0.25	N ₂	76	3.774
30	23.2	0.4	0.25	N ₂	89	3.774
30	23.2	0.5	0.25	N ₁	91	3.774
30	23.2	0.65	0.25	N	73	3.774
30	23.2	1.0	0.25	N ₁	61	3.774
30	23.2	2.0	0.25	N	53	3.774
27	28	0.25	0.25	N ₂	61	9.434
27	28	0.3	0.25	N ₂	57	9.434
27	28	0.4	0.25	N2	53	9.434
27	28	0.5	0.25	N ₂	52	9.434
27	28	0.65	0.25	N2	52	9.434

NJ	N ₂	rl	r ₂	Winner	Sat.Time	Distance
27	28	1.0	0.25	N ₂	59	9.434
27	28	2.0	0.25	N	68.	9.434
21	37.6	0.25	0.25	N ₂	48	20.755
21	37.6	0.3	0.25	N2	44	20.755
21	37.6	0.4	0.25	N ₂	39	20.755
21	37.6	0.5	0.25	N2	36	20.755
21	37.6	0.65	0.25	N ₂	34	20.755
21	37.6	1.0	0.25	N2	32	20.755
21	37.6	2.0	0.25	N ₂	31	20.755

TABLE VI (Continued)

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Candidate for the Degree of

Master of Science

Thesis: EFFECTS OF INITIAL NUMBERS AND INTRINSIC RATES OF INCREASE ON THE OUTCOME OF INTERSPECIFIC COMPETITION AS PREDICATED BY THE LOTKA-VOLTERRA COMPETITION EQUATIONS

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