# A MICROCOMPUTER APPROACH TO THE SYNTHESIS OF FOUR-BAR LINKAGES FOR FUNCTION GENERATION

By

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FOR FUNCTION GENERATION

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## NOMENCLATURE

| | |
|---|---|
| a | - length of the input link, AM |
| b | - length of the coupler link, AB |
| c | - length of the output link, BQ |
| d | - length of the ground link, MQ |
| $A_n$ | - the input link moving pivot in the $n^{\underline{th}}$ design position |
| $B_n$ | - the output link moving pivot in the $n^{\underline{th}}$ design position |
| M | - the input link fixed pivot |
| Q | - the output link fixed pivot |
| $(X_{A_n}, Y_{A_n})$ | - Coordinates of A in the $n^{\underline{th}}$ design position |
| $(X_{B_n}, Y_{B_n})$ | - coordinates of B in the $n^{\underline{th}}$ design position |
| $\Theta_1$ | - angle between MQ and the positive X-axis measured counter clockwise |
| $\Theta_2$ | - angle between AM and the positive X-axis measured counter clockwise |
| $\Theta_3$ | - angle between AB and the positive X-axis measured counter clockwise |
| $\Theta_4$ | - angle between BQ and the positive X-axis measured counter clockwise |
| $\Theta_{1n}$ | - angle between the first and $n^{\underline{th}}$ design position of the input link |

$\dot{\Theta}_n$ - angular velocity of the input link in the $n^{\underline{th}}$ design position

$\ddot{\Theta}_n$ - angular acceleration of the input link in the $n^{\underline{th}}$ design position

$\Phi_{1n}$ - angle between the first and $n^{\underline{th}}$ design position of the output link

$\dot{\Phi}_n$ - angular velocity of the output link in the $n^{\underline{th}}$ design position

$\ddot{\Phi}_n$ - angular acceleration of the output link in the $n^{\underline{th}}$ design position

CHAPTER I

# INTRODUCTION

Many analytical methods have been developed and implemented for the synthesis of planar mechanisms during the past three decades. A class of synthesis which has received considerable attention involves closed form solution for linkages with a finite number of prescribed precision positions. Synthesis for rigid body motion, path generation and function generation are typical of such synthesis problems.

The techniques used in this class of synthesis genrally yield all of the possible solutions. However, the conditions imposed are necessary but not sufficient to guarantee that all of the solutions will satisfy the design requirements. Hence, a rough analysis of the possible solutions is required just to discard those solutions which do not meet the design requirements. A detailed analysis is then necessary to select the best mechanism from the solutions which do meet the requirements. The final selection is generally based on the overall dynamic behavior of the mechanism.

The first analytical contributions to the synthesis of planar linkages were made in 1955 by Freudenstein (1) and his associates. This work was based on the graphical techniques of Burmester and has inspired most of the contributions that have been made in this area.

In 1965, Wilson (2) presented a method based on the classic rotation matrix operator. This led to the development of the general displacement matrix by Suh and Radcliffe (3). These contributions present methods for the synthesis of finite positions. Extensions to the basic theory is found in work by many others, including Coutant and Soni (4), for the synthesis of infinitesimal positions.

The basic theory has been implemented, using the digital computer, in many works. The first interactive computer package was the KINSYN package developed at M.I.T. by Kaufman (5) and his associates. The package was implemented on a small dedicated computer (IBM 1130) using special hardware also developed at M.I.T. Another package, LINCAGES, has been developed at the University of Minnesota which uses a mainframe computer system and graphics terminal (6). A similar package named RECSYN was developed at Ohio State University (7).

These and other packages presently available are extremely powerful aids to the linkage designer. The KINSYN package allows the user to specify complex linkages with an almost unlimted number of joints, sliders and links. Both synthesis and analysis are available to the designer. The synthesis is capable of rigid body, point path and function generation for three, four and five precision positions.

The analysis includes animation and calculation of other important design parameters.

However, the large computer system approach has many disadvantages. Perhaps the most obvious is implementation cost. A dedicated computer and graphics terminal are required for effective animation. Many prospective users simply cannot justify the cost of implementation.

The current uptrend in the availability and use of personal computers in engineering applications is apparent. Many personal computers presently available have the same capabilities that 20 years ago could only be found in large computer systems. The future promises even faster, more powerful personal computers than are presently available.

Although none of the personal computers presently available are capable of executing the type of synthesis package previously referred to, a series of programs developed for independent execution on a microcomputer is very feasible. A microcomputer system can be implemented with less cost than a single graphic terminal required in the main frame approach.

The objective of the present work is to illustrate a microcomputer approach for the synthesis of linkages. An interactive computer package for the synthesis of four-bar linkages for function generation was developed and unique problems solved to demonstrate its application.

The synthesis technique for finite and infinitesimal positions is presented in Chapter II. The displacement analysis required for both three and four position synthesis is presented in Appendix A. The closed form solution of the non-linear equations required in the synthesis of four positions is presented in Appendix B.

Two interactive programs, for three and four position synthesis, were developed on the Apple II Plus computer using the Apple Pascal Language System. Both programs are capable of finite and infinitesimal position synthesis. The concepts and use of these programs is presented in Chapter III.

A series of example problems are presented in Chapter IV to illustrate special applications for which the programs may be used.

# CHAPTER II

## SYNTHESIS TECHNIQUE

The objective of the synthesis for function generation is to determine linkages which have prescribed coordination between the input and output links. This coordination can be in the form of finite angular positions or infinitesimal positions (i.e. angular velocities or accelerations).

In this technique, the coordinates of the fixed pivots (M and Q) of a four-bar linkages are specified at zero and one on the X-axis as shown in Figure 1. The link given by AM is used to drive the linkage and is the input link. The pivot A is therefore referred to as the input link moving pivot. The output link is given by BQ and B referred to as the output link moving pivot.

The first design position is given by the coordinates of $A_1$ and $B_1$. The $n^{th}$ design position is specified relative to the first by $\Theta_{1n}$ and $\Phi_{1n}$ and therefore given by the coordinates of $A_n$ and $B_n$. Angular velocities and accelerations of the $n^{th}$ position are $\dot{\Theta}_n$, $\dot{\Phi}_n$ and $\ddot{\Theta}_n$, $\ddot{\Phi}_n$, respectively. This synthesis technique determines the coordinates of the moving pivots of the linkage in the first design position.

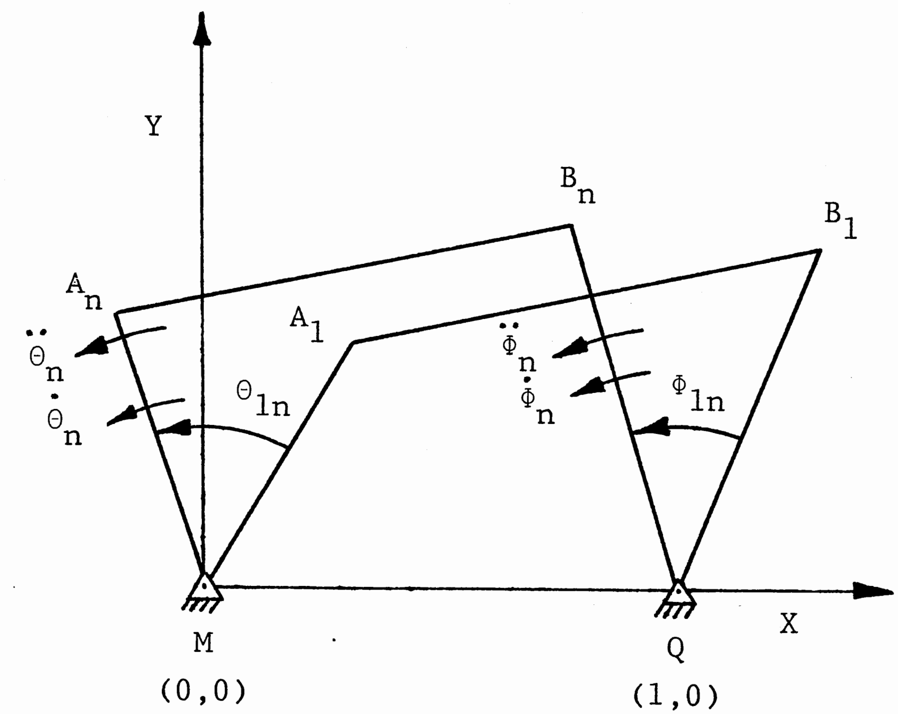



Figure 1. Synthesis Notation

## Relative Rotations

In order to develop the design equations for four-bar function generation, the $n^{\underline{th}}$ design position must be related to the first design position. This can be done using the displacement matrix developed by Suh and Radcliffe (3). Equation (2.1) can be used to perform rotations of a rigid body about the point (X,Y) through an angle $\Theta$. Equations (2.2) and (2.3) result for rotations of the links AM and BQ about M and Q through angles $\Theta_{1n}$ and $\Phi_{1n}$, respectively.

$$[R]_{(X,Y),\Theta} = \begin{bmatrix} \cos\Theta & -\sin\Theta & X(1-\cos\Theta) + Y\sin\Theta \\ \sin\Theta & \cos\Theta & Y(1-\cos\Theta) - X\sin\Theta \\ \Theta & \Theta & 1 \end{bmatrix} \tag{2.1}$$

$$\begin{Bmatrix} X_{A_n} \\ Y_{A_n} \\ 1 \end{Bmatrix} = [R]_{(0,0),\Theta_{1n}} \begin{Bmatrix} X_{A_1} \\ Y_{A_1} \\ 1 \end{Bmatrix} \tag{2.2}$$

$$\begin{Bmatrix} X_{B_n} \\ Y_{B_n} \\ 1 \end{Bmatrix} = [R]_{(1,0),\Phi_{1n}} \begin{Bmatrix} X_{B_1} \\ Y_{B_1} \\ 1 \end{Bmatrix} \tag{2.3}$$

Four algebraic equations (2.4 - 2.7) can be derived from Equations (2.2) and (2.3). The components of velocity for points $A_n$ and $B_n$ are expressed by Equations (2.8 - 2.11) and the acceleration components by Equations (2.12 - 2.15).

$$X_{A_n} = X_{A_1} \cos\Theta_{1n} - Y_{A_1} \sin\Theta_{1n} \tag{2.4}$$

$$Y_{A_n} = X_{A_1} \sin\Theta_{1n} + Y_{A_1} \cos\Theta_{1n} \tag{2.5}$$

$$X_{B_n} = X_{B_1} \cos\Phi_{1n} - Y_{B_1} \sin\Phi_{1n} - \cos\Phi_{1n} + 1 \tag{2.6}$$

$$Y_{B_n} = Y_{B_1} \sin\Phi_{1n} + Y_{B_1} \cos\Phi_{1n} - \sin\Phi_{1n} \tag{2.7}$$

$$\dot{X}_{A_n} = -Y_{A_n} \dot{\Theta}_n \tag{2.8}$$

$$\dot{Y}_{A_n} = X_{A_n} \dot{\Theta}_n \tag{2.9}$$

$$\dot{X}_{B_n} = -Y_{B_n} \dot{\Phi}_n \tag{2.10}$$

$$\dot{Y}_{B_n} = X_{B_n} \dot{\Phi}_n \tag{2.11}$$

$$\ddot{X}_{A_n} = -Y_{A_n} \ddot{\Theta}_n - X_{A_n} \dot{\Theta}_n^2 \tag{2.12}$$

$$\ddot{Y}_{A_n} = X_{A_n} \ddot{\Theta}_n - Y_{A_n} \dot{\Theta}_n^2 \tag{2.13}$$

$$\ddot{X}_{B_n} = -Y_{B_n} \ddot{\Phi}_n - (X_{B_n} - 1)\dot{\Phi}_n^2 \tag{2.14}$$

$$\ddot{Y}_{B_n} = (X_{B_n} - 1)\ddot{\Phi}_n - Y_{B_n} \dot{\Phi}_n^2 \tag{2.15}$$

## Design Equations

The design equations are derived by requiring the length of the coupler link, AB, to remain constant as the linkage moves (4). For a specified position, the length is expressed by Equation (2.16). When a velocity is specified, the first derivitive is expressed by Equation (2.17). For a specified acceleration, the second derivitive is expressed by Equation (2.18).

$$(AB)_n^2 = \text{constant}$$

$$(X_{B_n}-X_{A_n})^2 + (Y_{B_n}-Y_{A_n})^2 = (X_{B_1}-X_{A_1})^2 + (Y_{B_1}-Y_{A_1})^2 \qquad (2.16)$$

$$\frac{d}{dt}(AB)^2 = 0$$

$$(X_{B_n}-X_{A_n})(\dot{X}_{B_n}-\dot{X}_{A_n}) + (Y_{B_n}-Y_{A_n})(\dot{Y}_{B_n}-\dot{Y}_{A_n}) = 0 \qquad (2.17)$$

$$\frac{d^2}{dt^2}(AB)^2 = 0$$

$$(\dot{X}_{B_n}-\dot{X}_{A_n})(\dot{X}_{B_n}-\dot{X}_{A_n}) + (X_{B_n}-X_{A_n})(\ddot{X}_{B_n}-\ddot{X}_{A_n}) +$$
$$(\dot{Y}_{B_n}-\dot{Y}_{A_n})(\dot{Y}_{B_n}-\dot{Y}_{A_n}) + (Y_{B_n}-Y_{A_n})(\ddot{Y}_{B_n}-\ddot{Y}_{A_n}) = 0 \qquad (2.18)$$

## Position Equation

The design equation for position is derived in terms of the first design position and the parameters $\Theta_{1n}$ and $\Phi_{1n}$ by substituting Equations (2.4 - 2.7) into (2.16). After considerable manipulation, the position equation becomes:

$$A\,X_{B_1} + B\,X_{A_1}X_{B_1} + C\,Y_{A_1}X_{B_1} + D\,Y_{B_1} + E\,X_{A_1}Y_{B_1} + F\,Y_{A_1}Y_{B_1} + G + H\,X_{A_1} + I\,Y_{A_1} = 0 \qquad (2.19)$$

where:

$$A = \cos\Phi_{1n} - 1$$

$$B = 1 - \cos(\Theta_{1n} - \Phi_{1n})$$

$$C = \sin(\Theta_{1n} - \Phi_{1n})$$

$$D = -\sin\Phi_{1n}$$

$$E = -\sin(\Theta_{1n} - \Phi_{1n})$$

$$F = 1 - \cos(\Theta_{1n} - \Phi_{1n})$$

$$G = 1 - \cos\Phi_{1n}$$

$$H = \cos(\Theta_{1n} - \Phi_{1n}) - \cos\Theta_{1n}$$

$$I = -\sin(\Theta_{1n} - \Phi_{1n}) + \sin\Theta_{1n}$$

<u>Velocity Equation</u>

The design equation is derived for velocity in terms of the first design position and the parameters $\Theta_{1n}$, $\Phi_{1n}$, $\dot{\Theta}_n$ and $\dot{\Phi}_n$ by substituting Equations (2.4 - 2.11) into (2.17):

$$A\,X_{B_1} + B\,X_{A_1}X_{B_1} + C\,Y_{A_1}X_{B_1} + D\,Y_{B_1} + E\,X_{A_1}Y_{B_1} + F\,Y_{A_1}Y_{B_1} + G + H\,X_{A_1} + I\,Y_{A_1} = 0 \qquad (2.20)$$

where:

$$A = -\sin\Phi_{1n}\dot{\Phi}_n$$

$$B = \sin(\Theta_{1n} - \Phi_{1n})(\dot{\Theta}_n - \dot{\Phi}_n)$$

$$C = \cos(\Theta_{1n} - \Phi_{1n})(\dot{\Theta}_n - \dot{\Phi}_n)$$

$$D = -\cos\Phi_{1n}\dot{\Phi}_n$$

$$E = -\cos(\Theta_{1n} - \Phi_{1n})(\dot{\Theta}_n - \dot{\Phi}_n)$$

$$F = \sin(\Theta_{1n} - \Phi_{1n})(\dot{\Theta}_n - \dot{\Phi}_n)$$

$$G = \sin\Phi_{1n}\dot{\Phi}_n$$

$$H = (-\sin(\Theta_{1n} - \Phi_{1n}) + \sin\Theta_{1n})(\dot{\Theta}_n - \dot{\Phi}_n) + \sin\Theta_{1n}\dot{\Phi}_n$$

$$I = (-\cos(\Theta_{1n} - \Phi_{1n}) + \cos\Theta_{1n})(\dot{\Theta}_n - \dot{\Phi}_n) + \cos\Theta_{1n}\dot{\Phi}_n$$

Acceleration Equation

The design equation for acceleration is derived in terms of the first design position and the parameters $\Theta_{1n}$, $\Phi_{1n}$, $\dot{\Theta}_n$, $\dot{\Phi}_n$, $\ddot{\Theta}_n$ and $\ddot{\Phi}_n$ by substituting Equations (2.4 - 2.15) into (2.18):

$$A\,X_{B_1} + B\,X_{A_1}X_{B_1} + C\,Y_{A_1}X_{B_1} + D\,Y_{B_1} + E\,X_{A_1}Y_{B_1} + F\,Y_{A_1}Y_{B_1} + G + H\,X_{A_1} + Y_{A_1} = 0 \qquad (2.21)$$

where:

$$L_1 = -2\dot{\Phi}_n\dot{\Theta}_n + \dot{\Phi}_n^2 + \dot{\Theta}_n^2$$

$$L_2 = \ddot{\Phi}_n - \ddot{\Theta}_n$$

$$A = -\cos\Phi_{1n}\dot{\Phi}_n^2 - \sin\Phi_{1n}\ddot{\Phi}_n$$

$$B = \cos(\Theta_{1n} - \Phi_{1n})\,L_1 - \sin(\Theta_{1n} - \Phi_{1n})\,L_2$$

$$C = -\sin(\Theta_{1n} - \Phi_{1n})\,L_1 - \cos(\Theta_{1n} - \Phi_{1n})\,L_2$$

$$D = \sin\Phi_{1n}\dot{\Phi}_n^2 - \cos\Phi_{1n}\ddot{\Phi}_n$$

$$E = \sin(\Theta_{1n} - \Phi_n)\,L_1 + \cos(\Theta_{1n} - \Phi_{1n})\,L_2$$

$$F = \cos(\Theta_{1n} - \Phi_{1n})\,L_1 - \sin(\Theta_{1n} - \Phi_{1n})\,L_2$$

$$G = \cos\Phi_{1n}\dot{\Phi}_n^2 + \sin\Phi_{1n}\ddot{\Phi}_n$$

$$H = -\cos(\Theta_{1n} - \Phi_{1n})\,L_1 + \sin(\Theta_{1n} - \Phi_{1n})\,L_2 + \cos\Theta_{1n}\dot{\Theta}_n^2 + \sin\Theta_{1n}\ddot{\Theta}_n$$

$$I = \sin(\Theta_{1n} - \Phi_{1n})\,L_1 + \cos(\Theta_{1n} - \Phi_{1n})\,L_2 - \sin\Theta_{1n}\dot{\Theta}_n^2 + \cos\Theta_{1n}\ddot{\Theta}_n$$

Implementation

The three design equations may be combined in many ways to form a set of simultaneous equations. The first design position always results from the solution to this set. Therefore, the rest of the design positions are given relative to the first, e.g. $\Theta_{1n}$ and $\Phi_{1n}$. Due to this, the synthesis of n-positions requires only (n - 1) design equations.

In the case of a specified angular velocity or acceleration in the first position, the relative positions would be zero, e.g. $\Theta_{11} = \Phi_{11} = 0$. If an acceleration equation is to be used at a design position, the corresponding velocity equation must also be used for the same position. This is required since the angular acceleration of a link depends upon its angular velocity.

In the synthesis for three positions, two design equations are required. The location of the first moving pivot, $X_{A_1}$ and $Y_{A_1}$, is usually given and the second, $X_{B_1}$ and $Y_{B_1}$, is obtained from the solution. This yields two linear equations with two unknowns and is easily solved.

The four possible combinations of the design equations are: P-P-P, PP-P, P-PP and PPP.

where: P - position

PP - position and velocity

PPP - position, velocity and acceleration

The first specifies three separate positions and requires two position equations. The second specifies two positions and the angular velocity at the first position, requiring one position and one velocity equation. The third is similar to the second but specifies the velocity in the second position. The fourth specifies the angular velocity and acceleration in the first design position, requiring a velocity and acceleration equation.

In the synthesis of four positions, three design equations are required. One coordinate of the first moving pivot is selected and the remaining three coordinates result from the solution. This yields a set of three non-linear simultaneous equations. There are actually eight combinations but since jerk has been omitted only seven are given:

P-P-P-P

PP-P-P

P-PP-P

P-P-PP

PPP-P

P-PPP

PP-PP

Explanation of the four position cases follows the same rules as the three position cases, e.g. P-P-P-P, four separate positions, three position equations, etc. The case which is not presented, PPPP, represents one design position with angular velocity, acceleration and jerk specified.

## Verifying Solutions

The design equations provide necessary but insufficient conditions to guarantee the solution meets the required specifications. A defect in the position equation exists because a four-bar linkage may be assembled in one of two possible configurations. A mirror image is obtained by inverting the linkage about the link d and can assume either the crossed or uncrossed configuration (Figure 2). A linkage assembled in the half plane above d must be disassembled to reach a mirror image configuration.

A crank rocker linkage can not assume the crossed configuration. A drag link assumes both uncrossed and crossed configurations during a full cycle of rotation. A double rocker linkage must pass through a dead center position to change from the crossed to uncrossed configuration.

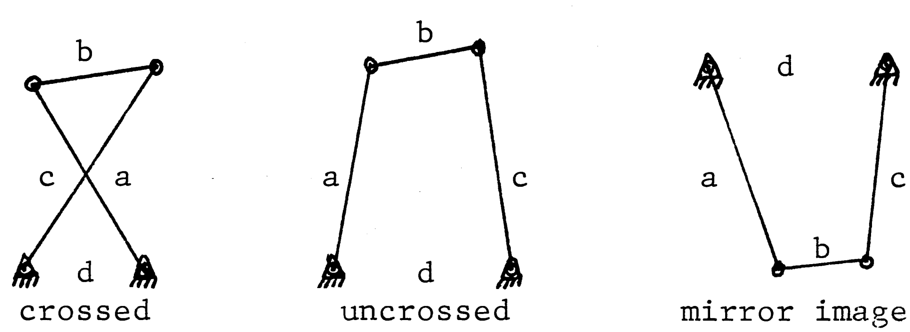



Figure 2. Configurations of the Four-Bar Linkage

The design equations may obtain a solution which has positions in any configuration. Hence, the linkage could require disassembly to reach all of the design positions. A solution of this type is 'not verified' and can only be determined by analyzing the solution after the synthesis has been performed.

The displacement analysis of a four-bar linkage is presented in Appendix A. The configuration of the linkage is determined by the sign in the quadratic Equation (A.12). For a crank rocker or drag link, the sign determines if a position is in the mirror image configuration. For a double rocker linkage the sign determines crossed or uncrossed configurations.

Crank rocker and drag link solutions can be verified by the following. The correct sign is determined for the first design position. Each remaining position is analyzed using this sign. If the specified output parameters match the analysis, a 'verified' solution has been obtained.

The defect is less prevalent in double rocker solutions and may be determined by animating the linkage. The defect is only present in the position equations. The specified velocity and/or acceleration at a design position is always correct.

# CHAPTER III

## MICROCOMPUTER IMPLEMENTATION

The solution space of a three position synthesis problem is quite different than that found in a four position problem. A three position problem has possible solutions in the entire plane, whereas solutions to the four position problem are limited to two third order curves. The number of calculated points required to represent the possible solutions of a three position problem is far greater than that required by a four position problem. Therefore, the synthesis of three and four position design problems on a microcomputer must be approached in two entirely different ways.

Two interactive programs were developed for three and four position synthesis on the Apple II Plus computer using the Apple Pascal Language system. Many similarities exist in the basic function of both programs. This chapter illustrates the use of each program and the concepts used in their development. The computer generated displays represent the case of finite positions. Similar displays for infinitesimal cases can be found in Appendix D.

## Pascal Language

There are many excellent references to the Pascal language. A few of these are listed in the bibliography. This section is included only to highlight the extensions of Apple Pascal as they apply to the present work.

Apple Pascal is a semi-compiled language. Source programs are compiled to P-code. The P-code is interpreted at execution time. Source programs of approximately 18 kilo-bytes can be edited. A maximum of 39 kilo-bytes are available for execution of compiled code.

The Pascal equivalent to a subroutine is referred to as a procedure. A function is a procedure which returns some numerical value. Apple Pascal provides extensions to these basic tools which allow maximum usage of the limited dynamic memory of the Apple II. The extensions are called segment procedures and units (10).

A segment procedure is similar to a regular procedure in that it remains in the main body of the program. However, segment procedures are given separate codefiles when the program is compiled.

A unit is similar to a library file on a large computer system. Units are developed and compiled separately from the main program. An interface part of the unit specifies variables which may pass between it and a calling main program. A main program which uses a particular unit simply declares that it is to be used. The unit is then linked to the main program before it is executed. This concept allows the programmer to develop and compile special units that may be used by any calling main program.

The programmer has complete control of both segment procedure and units. They may be specified to be resident in dynamic memory during the entire execution of the program or only when referenced. Both are loaded from the disk very quickly because the main program creates a dictionary which contains their exact location. This allows the programmer to create very large programs which can be executed in the limited memory of the Apple II.

The Apple Pascal language also supplies a library of units which may be used by the programmer. A set of turtlegraphics procedures and transcendental functions have been provided in this way and are used extensively in the present work.

Segment procedures and units have also been used extensively. The three position program contains seven segment procedures and uses seven units. The four position program contains nine segment procedures and uses nine units. A unit which animates four-bar linkages is used by both programs.

## Three Position Program

When program execution is initiated, the first display supplies the user with a variety of options (Figure 3). These options are used to propose a problem to be solved. The discussion of these options does not follow the order of listing in the display.

```
THREE POSITION INPUT/OUTPUT SYNTHESIS
----- -------- ------------ ---------

TYPE             COMMAND
----             -------

  A       VIEW INPUT DESCRIPTION

  B       REVIEW SETUP PARAMETERS

  D       ENTER THE DESIGN MODE

  P       RESET THE PROBLEM TYPE

  R       READ WORKFILE FROM DISK

  S       WRITE WORKFILE TO DISK

  Q       EXIT THE PROGRAM
```

Figure 3. Initial Options - Three Positions

## Reset the Problem Type

This option allows the user to select the type of problem desired (Figure 4). The possible finite and infinitesimal cases are displayed using the notation referred to in Chapter II.

```
        PRESENT SETTING IS 1

   TYPE                 PROBLEM
   ----                 -------

    1                    P-P-P

    2                    PP-P

    3                    P-PP

    4                    PPP

          TYPE X TO EXIT
```

Figure 4. Problem Type - Three Positions

Upon exit, variables are established which control the execution of the entire program. These control variables determine display formats, input variables and the design equations required for the solution. This concept allows four cases to be solved with very little duplication of program code.

View Input Description

The user may view a graphic illustration of the problem selected (Figure 5).

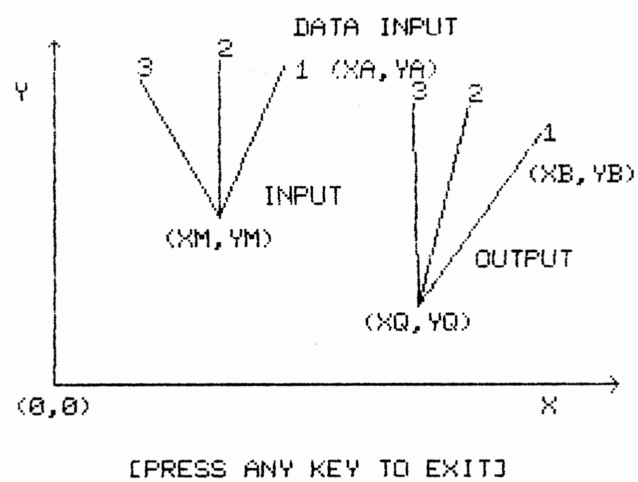



Figure 5. Description - Three Positions

Review Set Up Parameters

The user may enter or change the parameters of the problem in this mode (Figure 6). The finite positions are specified in degrees. If an infinitesimal case was selected, angular velocity and acceleration would be specified in rad/sec and rad/sec$^2$. The coordinates of the fixed pivots can be specified with any single system of units. Therefore, the solution produces the coordinates of the moving pivots and length of the links in the same units used to describe the location of the fixed pivots.

```
INPUT / OUTPUT PARAMETERS
-----   ------ ----------

INPUT ANGLE(1/2) ....  1.00000E2

OUTPUT ANGLE(1/2) ...  2.00000E1

INPUT ANGLE(1/3) ....  2.00000E2

OUTPUT ANGLE(1/3) ...  5.00000E1

XM ..................  0.00000

YM ..................  0.00000

XQ ..................  1.00000

YQ ..................  0.00000

TYPE [X] TO EXIT, [C] TO CHANGE
```

Figure 6. Parameters - Three Positions

## Enter the Design Mode

The solutions to a three position problem are easily obtained since the equations are linear. However, the number of solutions required to represent the solution space is too large to store in the limited dynamic memory of the Apple II. This suggests that only the type of solution should be stored, not the actual solution. A map can be generated to represent the solution space. Figure 7 is the principal display while in the design mode.

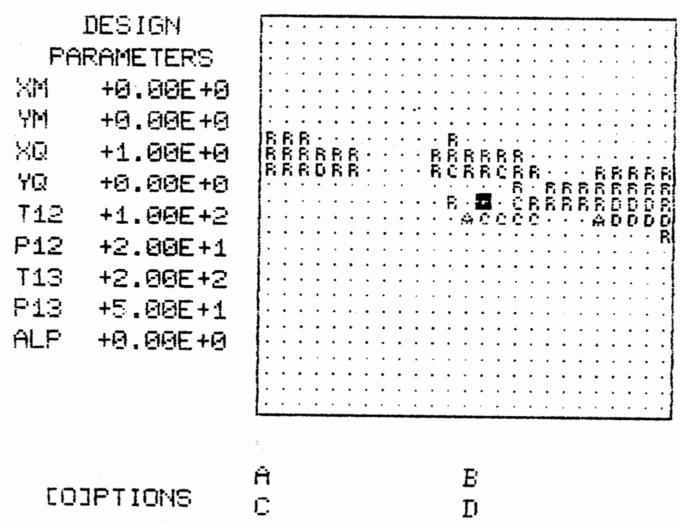



Figure 7. Design Mode Display - Three Positions

There are four possible types of the four-bar linkage: crank rocker, drag link, double rocker or not verified. The first three are determined by Grashoff's Criteria (12). The type "not verified" represents a case where the solution does not achieve the desired results and is determined by the method presented in Chapter II. These types are represented by 'C', 'D', 'R' and '.', respectively, at the coordinates of the input link moving pivot in the first design position.

In the design mode display, the parameters XM, YM, XQ and YQ are coordinates of the fixed pivots. The specified angles $\Theta_{12}$, $\Phi_{12}$, $\Theta_{13}$ and $\Phi_{13}$ are displayed as T12, P12, T12 and P13, respectively. The parameter ALP refers to the angle between MQ and the horizontal.

A list of options is available to the user at any time by pressing 'O' (Figure 8). The most important options are mapping, single solution and animation.

DESIGN MODE OPTIONS

| TYPE | COMMAND |
|---|---|
| A | ANIMATE A CHOSEN SOLUTION |
| D | DUMP MAP TO THE PRINTER |
| M | CREATE MAP OF DESIGN SOLUTIONS |
| R | READOUT OF CURSOR POSITION |
| N | NO READOUT OF CURSOR POSITION |
| S | SINGLE POSITION SOLUTION MODE |
| Q | EXIT THE DESIGN MODE |

Figure 8. Design Mode Options - Three Positions

The mapping option creates the display referred to previously. Pressing 'M' allows control of a cursor by using the 'O', 'K', 'L' and ',' keys to move up, left, right and down, respectively. Two screen positions are located and accepted by pressing 'Q'. The program maps all of the solutions between these two positions. As shown in Figure 9, the lower left corner of the mapped area was the first point located and the upper right corner the second.

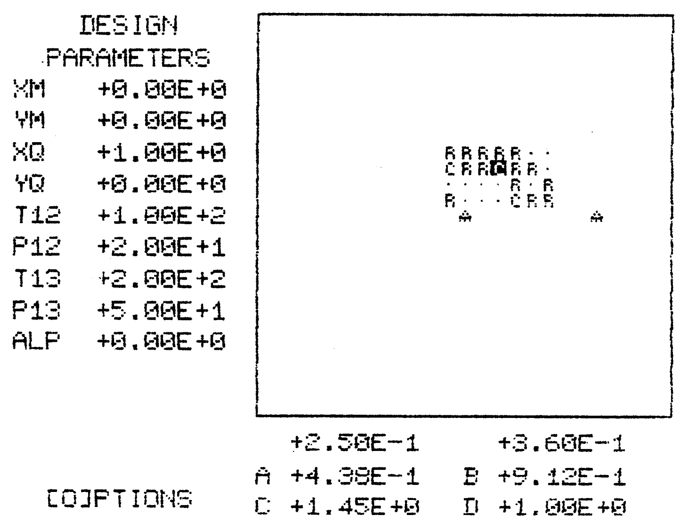



Figure 9. Options Illustration - Three Positions

The computer can generate and display 625 possible solutions in less than fifteen minutes. The input link fixed pivot is always displayed in the center of the screen and the output link fixed pivot at a relative angle given by the coordinates of both. The range of the display is governed by the distance between the two fixed pivots. The top, bottom, left and right edge of the display are located one and one half times this distance from the center of the screen.

The single solution option allows the user to display link lengths of a particular solution. 'S' is pressed and the cursor used to locate the desired coordinates. A, B, C and D represent the input, coupler, output and ground link, respectively, for the cursor position shown in Figure 9.

The animation option is used in the same manner as the single solution option. 'A' is pressed and the coordinates of the solution to be animated are located with the cursor. The display is cleared and the particular solution is animated. The animation is common to both three and four position programs and is discussed later.

When the readout option is in effect, the cursor position is displayed as the user locates positions for the other options. In Figure 9, the cursor position is displayed above the link lengths. The no readout option cancels the readout option and allows points to be located very quickly. The dump option allows hardcopy of the screen display to be produced on the printer.

## Write Workfile to the Disk

This option allows the user to save the parameters and map of a particular problem on the disk.

## Read Workfile from the Disk

This option allows the user to continue work on a previously saved problem at a later time.

## Considerations

The solutions in this program are presented in a user supplied reference frame. This reference frame is introduced by the selection of the fixed pivots. However, the actual solution of the design equations occurs in the fixed reference frame presented in Chapter II. Therefore, scale factors and coordinate transformations are used in the program to interface these two reference frames. A third reference frame, invisible to the user, is used in the graphic display.

# Four Position Program

The sequence of execution of the four position program differs from that of the three position program. The main difference is that solutions must be generated before the design mode is entered. This is required because of the large codefiles required to solve the non-linear design equations. Figure 10 is the initial display generated when the program is executed. Again, the order of discussion does not follow the order represented in the display.

```
FOUR POSITION INPUT/OUTPUT SYNTHESIS
---- -------- ------------ ---------

TYPE              COMMAND
----              -------

  A         VIEW INPUT DESCRIPTION

  B         REVIEW INPUT/OUTPUT PARAMETERS

  C         REVIEW ENVIRONMENT PARAMETERS

  D         ENTER THE DESIGN MODE

  G         GENERATE CIRCLEPOINT CURVE

  P         RESET THE PROBLEM TYPE

  Q         EXIT THE PROGRAM
```

Figure 10. Initial Options - Four Positions

## Reset the Problem Type

This option serves the same purpose as the corresponding option in the three position program (Figure 11). However, the display presents the type of problems solved by the four position program. The notation conforms with that of Chapter II. The last option, PPPP, has been included for future implementation.

PRESENT SETTING IS 1

| TYPE | PROBLEM |
|---|---|
| 1 | P-P-P-P |
| 2 | PP-P-P |
| 3 | P-PP-P |
| 4 | P-P-PP |
| 5 | PP-PP |
| 6 | PPP-P |
| 7 | P-PPP |
| 8 | PPPP |

TYPE X TO EXIT

Figure 11. Problem Type - Four Positions

## View Input Description

This option also duplicates the function of the three position option. The user may view a description of the four position problem selected (Figure 12).

## Review the Input/Output Parameters

The user may specify the parameters of the problem in this mode (Figure 13). The units of the parameters are identical to those of the three position problem.

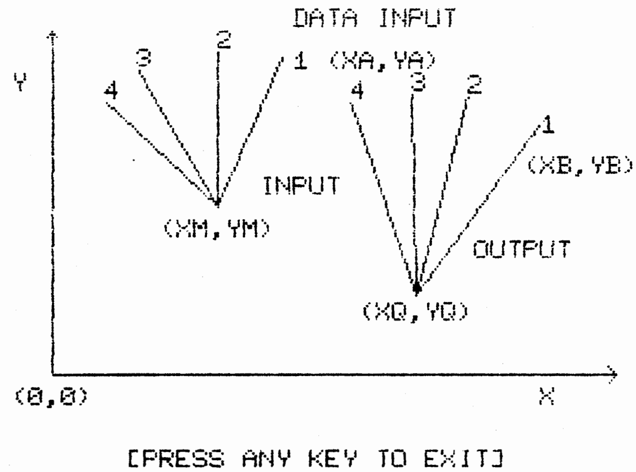



Figure 12. Description - Four Positions

INPUT / OUTPUT PARAMETERS

| | |
|---|---|
| INPUT ANGLE(1/2) .... | 3.00000E1 |
| OUTPUT ANGLE(1/2) ... | 1.00000E1 |
| INPUT ANGLE(1/3) .... | 6.00000E1 |
| OUTPUT ANGLE(1/3) ... | 2.00000E1 |
| INPUT ANGLE(1/4) .... | 9.00000E1 |
| OUTPUT ANGLE(1/4) ... | 2.50000E1 |
| XM .................. | 0.00000 |
| YM .................. | 0.00000 |
| XQ .................. | 1.00000 |
| YQ .................. | 0.00000 |

TYPE [X] TO EXIT, [C] TO CHANGE

Figure 13. Parameters - Four Positions

Review Environment Parameters

The four position program requires the user to specify parameters of the solution and screen environment (Figure 14). The screen parameters determine the scope of the display in the design mode. The solution parameters define the area where solutions are to be generated.

```
      ENVIRONMENT PARAMETERS
      ----------- ----------

XMIN [SOLUTION] ..... -5.00000E-1

YMIN [SOLUTION] ..... -5.00000E-1

XMAX [SOLUTION] .....  1.50000

YMAX [SOLUTION] .....  1.50000

XMIN [ SCREEN ] ..... -5.00000E-1

YMIN [ SCREEN ] ..... -5.00000E-1

XMAX [ SCREEN ] .....  1.50000

YMAX [ SCREEN ] .....  1.50000

NO. DATA POINTS .....  3.10000E1

TYPE [X] TO EXIT, [C] TO CHANGE
```

Figure 14. Environment - Four Positions

The solution parameters and fixed pivots must be specified within the limits of the screen parameters. The number of data points determines how many increments are used between the maximum and minimum solution parameters in generating the circlepoint curves.

<u>Generate Circlepoint Curve</u>

After all of the parameters are specified, this option must be used to generate solutions before entering the design mode. Solutions are generated using the design equations presented in Chapter II. These equations require one coordinate of a moving pivot to be specified. The other three coordinates are obtained by the solution. In this program, solutions are obtained in two ways.

The first specifies the X coordinate of the input moving pivot at increments between the maximum X and minimum X solution parameters. The second specifies the Y coordinate of the input moving pivot at increments between the maximum Y and minimum Y solution parameters. This is done in an attempt to produce smooth curves with equally spaced data points (Figure 15 and 16). Region 1 of the curve could be totally missed if only the first solution technique were used. The second technique could produce similar results in region 2.

Both techniques result in non-linear design equations of the form solved by the linear superposition method presented in Appendix B. This presents another problem.

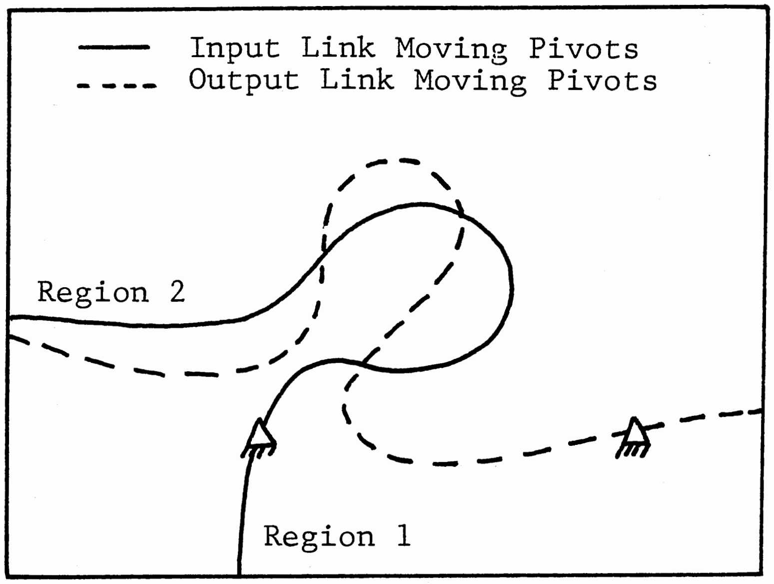



Figure 15. Circlepoint Curve, Continuous

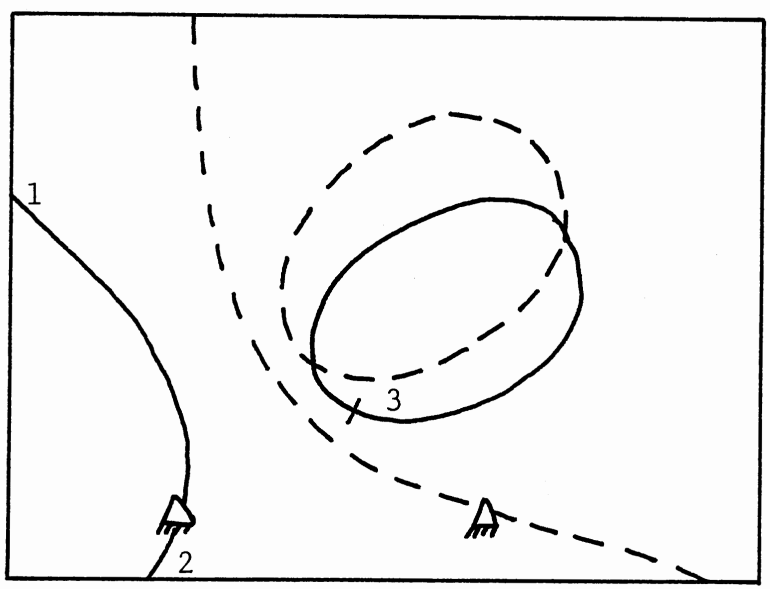
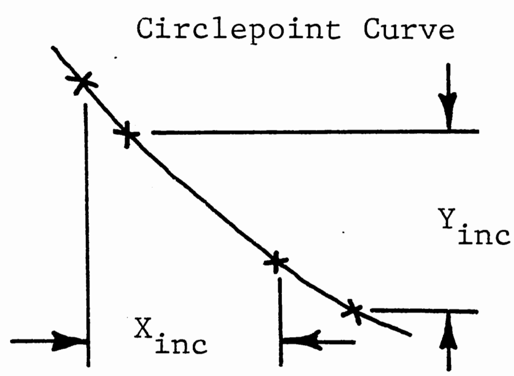



Figure 16. Circlepoint Curve, Discontinuous

The solutions are not in order after they have been generated. The solutions must be sorted to allow the computer to generate the curves on a graphic display. The curves may be continuous, shown in Figure 15, or form closed loops as shown in Figure 16.

The sorting is achieved by first finding a solution near the boundary (Figure 16, point 1). The remaining solutions are sorted for minimum distance between two consecutive points. If the distance between a point and the nearest point which has not been sorted (Figure 16, points 2 and 3) becomes greater than the distance shown in Figure 17, a new loop is formed and sorting continues until all points have been included.




$$\text{Distance} \le X_{inc}^2 + Y_{inc}^2$$

Figure 17. Maximum Distance between Data Points

Solutions are then removed, based on minimum distance, until only 60 data points remain unless there were less than 60 at the beginning of the sort. This is required because of variable limitations in the design mode. Note that all sorting is performed on the input link moving pivots. The order of the output link moving pivots always corresponds to that of the input.

## Enter the Design Mode

After the solutions are generated, the user may enter the design mode. The possible solutions are displayed by a pair of circlepoint curves as in Figure 18.

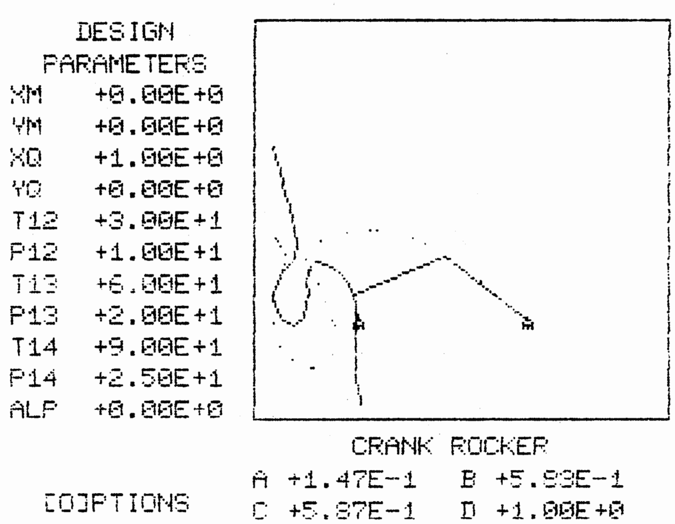



Figure 18. Design Mode Display - Four Positions

As in the three position program, a list of options is available to the user by pressing 'O' (Figure 19). Two of these options, animate and graphics dump, are identical to those of the three position program.

```
                DESIGN MODE OPTIONS
                ------ ---- -------

TYPE            COMMAND
----            -------

A        ANIMATE THE PRESENT SOLUTION

B        DISPLAY OF CIRCLEPOINT CURVES

C        CLEAR THE SCREEN

G        GRAPHICS DUMP TO THE PRINTER

R        READOUT OF PRESENT SOLUTION

N        ERASE READOUT OF SOLUTION

P        PRINTOUT WORKING SOLUTIONS

Q        EXIT THE DESIGN MODE

  LEFT/RIGHT ARROWS DISPLAY SOLUTIONS
```

Figure 19. Design Mode Options - Four Positions

The left and right arrows allow the user to display the possible solutions on the curve. Option C erases the display and allows the solutions to be displayed without the curves. The curves may be displayed again by using option B.

The option R, displays the link lengths and type of linkage for the solution shown in the display (Figure 18). Option N erases the link lengths and type of solution. The option P produces a printout of all of the working solutions (Figure 20).

| INPUT LINK | COUPLER LINK | OUTPUT LINK | GROUND LINK | *FIRST POSITION | MECHANISM TYPE |
|---|---|---|---|---|---|
| 4.35995E-1 | 4.49240 | 5.05702 | 1.00000 | 2.72776E2 | DOUBLE ROCKER |
| 4.11042E-1 | 5.20105 | 5.78980 | 1.00000 | 2.72249E2 | CRANK ROCKER |
| 3.71074E-1 | 7.41781 | 8.04505 | 1.00000 | 2.71370E2 | CRANK ROCKER |
| 3.06452E-1 | 6.82593E1 | 6.89484E1 | 1.00000 | 2.69910E2 | CRANK ROCKER |
| 2.42009E-1 | 6.95360 | 6.20317 | 1.00000 | 2.68589E2 | CRANK ROCKER |
| 1.77554E-1 | 2.93541 | 2.12290 | 1.00000 | 2.67770E2 | CRANK ROCKER |
| 1.12978E-1 | 1.76466 | 8.87518E-1 | 1.00000 | 2.67911E2 | CRANK ROCKER |
| 4.83906E-2 | 1.23570 | 2.89888E-1 | 1.00000 | 2.69311E2 | CRANK ROCKER |
| 1.61385E-2 | 9.37995E-1 | 8.04856E-2 | 1.00000 | 9.19623E1 | CRANK ROCKER |
| 8.10466E-2 | 7.38718E-1 | 3.55211E-1 | 1.00000 | 9.57048E1 | CRANK ROCKER |
| 1.47627E-1 | 5.83042E-1 | 5.87966E-1 | 1.00000 | 1.00486E2 | CRANK ROCKER |
| 1.94665E-1 | 4.88009E-1 | 7.35551E-1 | 1.00000 | 1.04393E2 | CRANK ROCKER |
| 2.18759E-1 | 4.41977E-1 | 8.07443E-1 | 1.00000 | 1.06567E2 | CRANK ROCKER |
| 2.85493E-1 | 3.18641E-1 | 9.96831E-1 | 1.00000 | 1.13295E2 | CRANK ROCKER |
| 3.03297E-1 | 2.85889E-1 | 1.04540 | 1.00000 | 1.15305E2 | DOUBLE ROCKER |
| 3.46908E-1 | 2.04127E-1 | 1.16152 | 1.00000 | 1.20759E2 | DOUBLE ROCKER |
| 3.91456E-1 | 1.11700E-1 | 1.28053 | 1.00000 | 1.28173E2 | DOUBLE ROCKER |
| 3.07875E-1 | 1.67593E-1 | 1.14018 | 1.00000 | 1.74489E2 | DOUBLE ROCKER |
| 3.15005E-1 | 1.86845E-1 | 1.13266 | 1.00000 | 1.77065E2 | DOUBLE ROCKER |
| 3.71270E-1 | 2.69171E-1 | 1.16292 | 1.00000 | 1.82312E2 | DOUBLE ROCKER |
| 4.35604E-1 | 3.10702E-1 | 1.27317 | 1.00000 | 1.79654E2 | DOUBLE ROCKER |
| 4.42094E-1 | 3.12269E-1 | 1.28730 | 1.00000 | 1.77909E2 | DOUBLE ROCKER |
| 4.85261E-1 | 3.03386E-1 | 1.39553 | 1.00000 | 1.70434E2 | DOUBLE ROCKER |
| 5.05281E-1 | 2.71356E-1 | 1.45934 | 1.00000 | 1.63304E2 | DOUBLE ROCKER |
| 5.11402E-1 | 2.22154E-1 | 1.49214 | 1.00000 | 1.55776E2 | DOUBLE ROCKER |
| 5.06786E-1 | 1.70195E-1 | 1.49412 | 1.00000 | 1.49239E2 | DOUBLE ROCKER |
| 5.03629E-1 | 1.51450E-1 | 1.48964 | 1.00000 | 1.47014E2 | DOUBLE ROCKER |

*First Position is given as the angle (degrees) that the input link is displaced, measured from MQ, in the first design position.

Figure 20. Printout of Working Solutions

Considerations

The majority of the work performed in this program is accomplished in generating the circlepoint curves. The design mode is used only to display and evaluate the alternative solutions. This approach is necessary because the solutions are non-linear and hence, harder to obtain.

## Animation

The animation option is available in both the three and four position synthesis programs. Animation can be a valuable aid to the user in selecting the most suitable linkage for his requirements. Although it is only a displacement analysis, this option gives the experienced user an idea of the kinematic and dynamic response of the linkage.

An initialization procedure is required to locate the screen coordinates of the fixed pivots and the scale factor for the link lengths. The positions of the linkage are then calculated and displayed sequentially. Using turtlegraphics, only the coupler displacement need be calculated while animating. This is accomplished using Equation (A.7) given in Appendix A.

Figure 21 describes the animation procedure. The turtle is moved to the location of the fixed pivot M and the pencolor set to white. An angle $\Theta_2$ is assumed and the turtle moved a distance (a) to position A. The turtle is then turned an angle $-(\Theta_2-\Theta_3)$ and moved a distance (b) to assume position B. The turtle then moves to the position of the fixed pivot Q. The above is repeated with the pencolor set to black in order to erase the screen. Angle $\Theta_2$ is incremented and the entire procedure repeated.

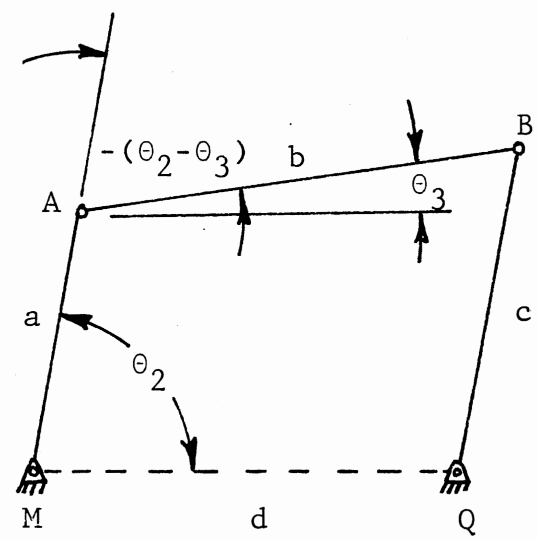



Figure 21. Animation Description

The sign of the term under the square root in Equation (A.7) must be observed to determine if the linkage has reached a limit position. If the sign becomes negative, a limit has been reached and the angle $\Theta_2$ incremented in the opposite direction.

As in the other modes, a list of options is available to the user at any time be pressing 'O' (Figure 22).

Option A allows the user to increase or decrease the increments of angle $\Theta_2$. The option B is used to display only the design positions (Figure 23). Option C instructs the program not to erase the positions as they are displayed (Figure 24). Normal animation may be resumed by using option D.

ANIMATION OPTIONS

| | |
|---|---|
| A | SELECT NEW INCREMENT |
| B | DISPLAY DESIGN POSITIONS |
| C | TURN ON TRACE MODE |
| D | TURN OFF TRACE MODE |
| E | INC OR DEC THE SCALE |
| F | SCREEN DUMP TO PRINTER |
| G | FLIP AT LIMIT POSITION |
| H | TURN OFF FLIP |
| Q | EXIT ANIMATION |

Figure 22. Animation Options

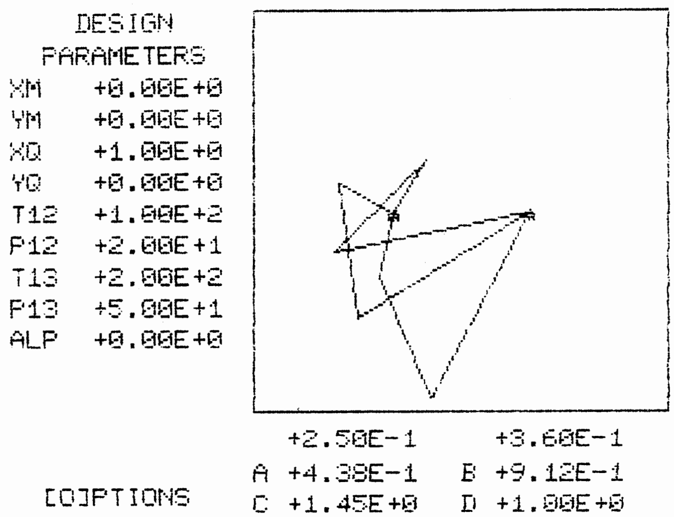



Figure 23. Display of Design Positions

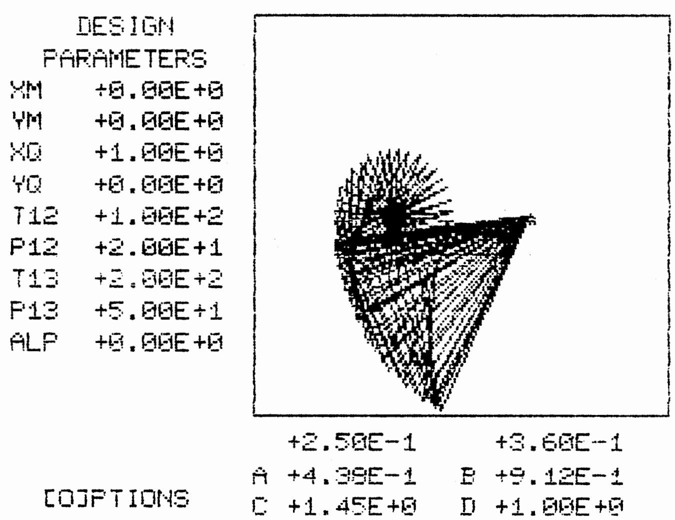



Figure 24. Trace Mode on Display

The scale of the display can be increased or decreased by using option E. Turtlegraphics automatically clips any move off the screen (Figure 25). Option F allows hardcopy to be obtained at any time. All of the displays presented were obtained with this option.

Options G and H may be used, in the case of a double rocker solution, to force the linkage to assume the opposite configuration when a limit position has been reached. Only double rocker solutions can be manipulated in this way. Option Q returns the user to the design mode.

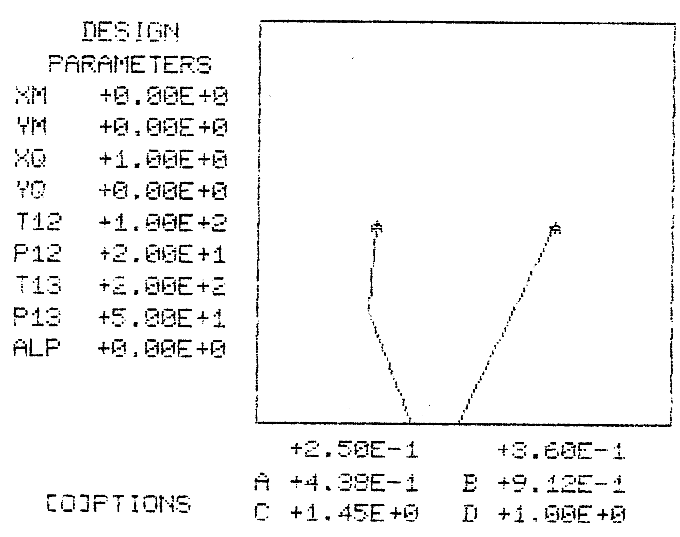



Figure 25. Clipping of Display

# CHAPTER IV

# APPLICATIONS

The three and four position synthesis programs presented in Chapter III may be applied to a wide variety of problems. If the user has data which exactly matches that required in one of the eleven possible cases, the application is straightforward. An example of each case is illustrated in Appendix D. However, many desirable characteristics may be obtained by the careful selection of data. These include prescribed limit positions, angle of oscillation and approximate dwell. The following examples illustrate how the programs may be used in achieving such characteristics.

## Limit Positions

Any of the infinitesimal cases may be used to define limit positions. The output link angular velocity of a four-bar linkage is only zero at a limit position. However, due to the nature of computer calculations, a very small angular velocity should be specified instead of zero.

Suppose a crank rocker linkage is desired where the output link rotates 50° for an input rotation of 100°. However, the second position of the output is required to be a limit position. The three position program may be used to meet this objective with the P-PP case. Figure 26 is a mapping of the solution space and Figure 27, a possible solution.

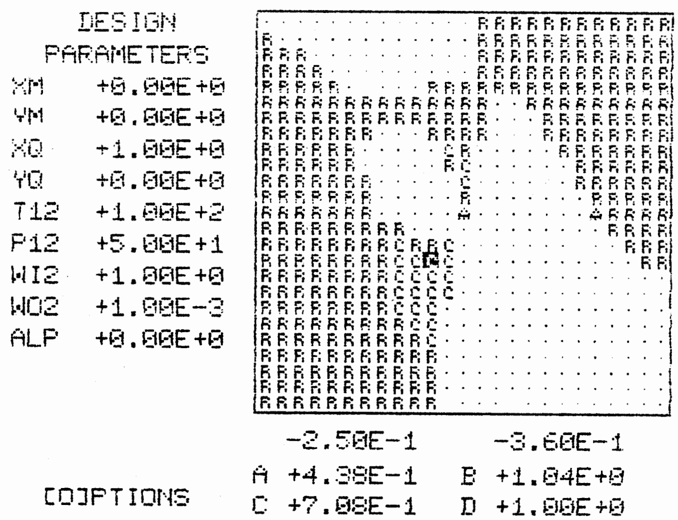



Figure 26. Limit Positions, Case P-PP

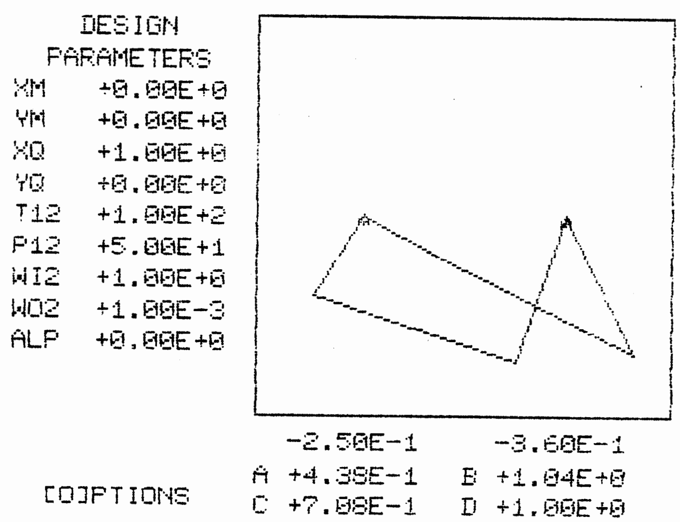



Figure 27. Limit Position Solution, Case P-PP

If the designer wished to control the velocity between the two positions, a third position could be specified. This would require the four position program and P-P-PP case to be executed. If the user desired the last 10° of output rotation to have low velocity, the added position $\Theta = 25°$ and $\Phi = 40°$ could be specified. This results in the circlepoint curves of Figure 28 and possible solutions like Figure 29.

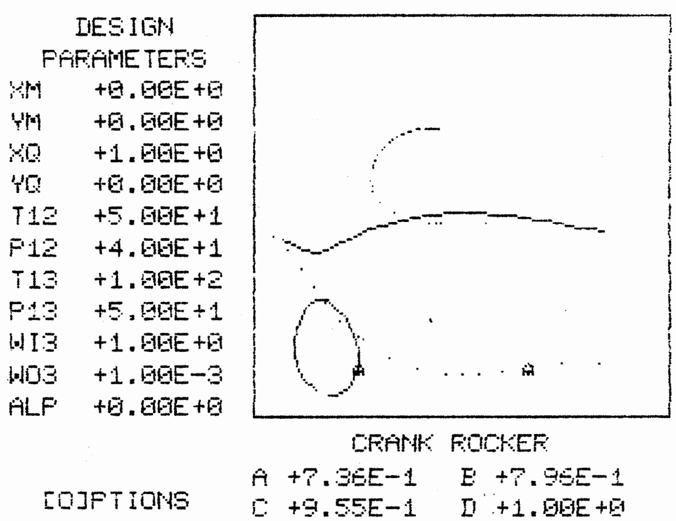



Figure 28. Limit Positions, Case P-P-PP

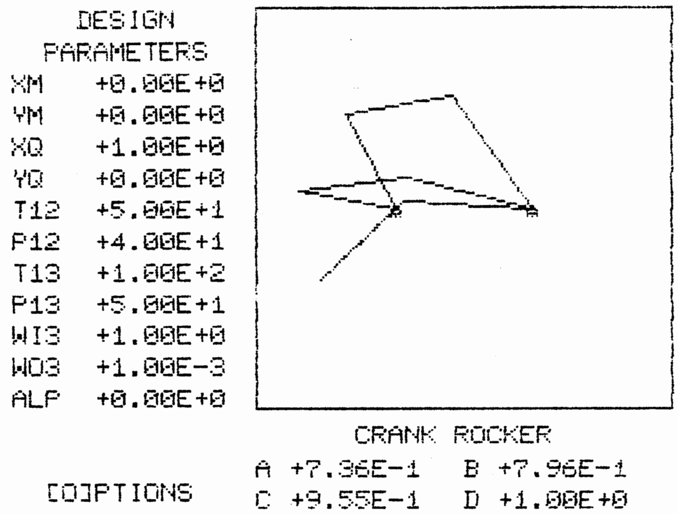



Figure 29. Limit Position Solution, Case P-P-PP

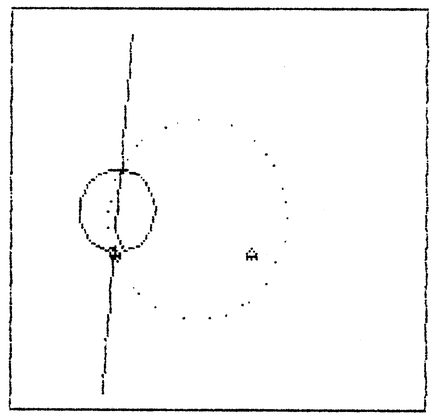
## Angle of Oscillation

In many applications, the linkage designer may desire a crank rocker solution with a prescribed angle of output oscillation. This objective can be accomplished using the PP-PP case in the four position program. The angle of oscillation is the angle between the two limit positions of a four-bar linkage. The output angular velocity of the two design positions is specified near zero. The relative angle between design positions (output link) is specified as the desired angle of oscillation. The increase the probability of crank rocker solutions, the input angle between design positions should be specified near 180°. The deviation from 180° can be used to govern the time ratio between forward and reverse oscillations.

An example of this technique is illustrated by Figures 30 and 31.

DESIGN
PARAMETERS

| | |
|---|---|
| XM | +0.00E+0 |
| YM | +0.00E+0 |
| XQ | +1.00E+0 |
| YQ | +0.00E+0 |
| T12 | +1.90E+2 |
| P12 | +6.00E+1 |
| WI1 | +1.00E+0 |
| WO1 | +1.00E-3 |
| WI2 | +1.00E+0 |
| WO2 | -1.00E-3 |
| ALP | +0.00E+0 |




CRANK ROCKER

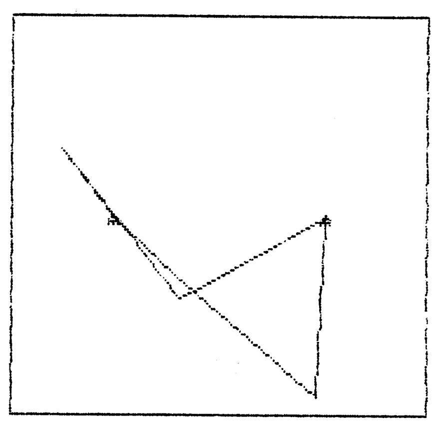
[O]PTIONS

A +3.84E-1  B +8.50E-1
C +7.80E-1  D +1.00E+0

Figure 30. Angle of Oscillation, Case PP-PP

DESIGN
PARAMETERS

| | |
|---|---|
| XM | +0.00E+0 |
| YM | +0.00E+0 |
| XQ | +1.00E+0 |
| YQ | +0.00E+0 |
| T12 | +1.90E+2 |
| P12 | +6.00E+1 |
| WI1 | +1.00E+0 |
| WO1 | +1.00E-3 |
| WI2 | +1.00E+0 |
| WO2 | -1.00E-3 |
| ALP | +0.00E+0 |



CRANK ROCKER

[O]PTIONS

A +3.84E-1  B +8.50E-1
C +7.80E-1  D +1.00E+0

Figure 31. Angle of Oscillation Solution - Case PP-PP

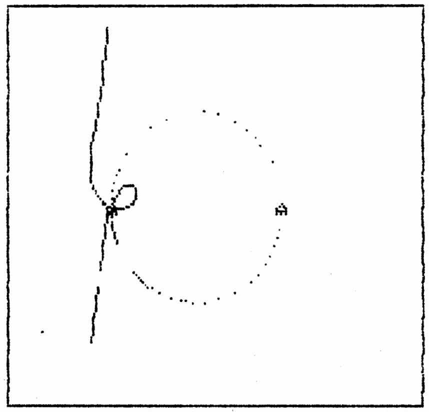
## Approximate Dwell

Approximate dwell is another characteristic often desired by the linkage designer. A four-bar linkage is incapable of absolute dwell. However, in many applications, an approximation is often all that is necessary. Again, the PP-PP case is required. This is actually an extension of the procedure that produces a specified angle of oscillation. The same methodology is used with a zero rotation between the two output link design positions.

The angular velocity of the output link at the two design positions is specified near zero. The deviation from zero governs the accuracy of the approximation. The specified angle of input motion governs the period of the dwell. Figures 32 and 33 illustrate this concept for a dwell corresponding to 40° of input rotation.

DESIGN
PARAMETERS

XM +0.00E+0
YM +0.00E+0
XQ +1.00E+0
YQ +0.00E+0
T12 +4.00E+1
P12 +0.00E+0
WI1 +1.00E+0
WO1 -1.00E-1
WI2 +1.00E+0
WO2 +1.00E-1
ALP +0.00E+0

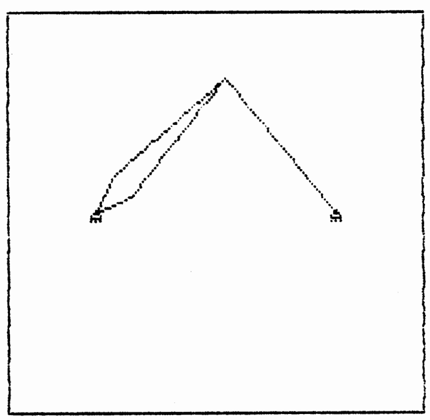


CRANK ROCKER
A +1.58E-1 B +5.89E-1
[O]PTIONS
C +6.77E-1 D +1.00E+0

Figure 32. Approximate Dwell, Case PP-PP

DESIGN
PARAMETERS

XM +0.00E+0
YM +0.00E+0
XQ +1.00E+0
YQ +0.00E+0
T12 +4.00E+1
P12 +0.00E+0
WI1 +1.00E+0
WO1 -1.00E-1
WI2 +1.00E+0
WO2 +1.00E-1
ALP +0.00E+0



CRANK ROCKER
A +1.58E-1 B +5.89E-1
[O]PTIONS
C +6.77E-1 D +1.00E+0

Figure 33. Approximate Dwell Solution, Case PP-PP

# CHAPTER V

## SUMMARY

The present work has demonstrated how a microcomputer can be utilized in the synthesis of four-bar mechanisms for function generation. Design equations for position, velocity and acceleration have been developed in the same general form to allow the eleven possible cases. The solution of the non-linear equations of the four-position cases has been developed in closed form using linear superposition.

The interactive programs allow the user to proceed from general to specific objectives. The core of each program is required to meet a minimum of objectives, resulting in very fast response from the computer. As options are selected, more calculations are required by the computer, hence, slowing the design process.

If, however, all of the options were required initially, the process would be much slower. This concept stimulates a high degree of interaction between the linkage designer and computer. As the designer gains experience in using the programs, good solutions are obtained very quickly.

The present work could be extended to synthesize linkages for rigid body motion and path generation. This would require a minimum of effort since both result in design equations similar to those of function generation. Synthesis for five positions could also be added. However, the feasibility should be studied since it would require a closed form solution of four non-linear equations in four unknowns and results in a maximum of four possible solutions.

Detailed kinematic and dynamic force analysis programs could also be included to aid the linkage designer in selecting the best linkage from the possible solutions. These could be separate or called by the synthesis programs as options.

The present work has been implemented on the Apple II Plus microcomputer using the Apple Pascal language. The programs could be modified to allow execution on any microcomputer with the Pascal language and similar capabilities.

The Apple II, compared with other microcomputers presently available, is limited in speed and dynamic memory. Implementation on a more advanced microcomputer would almost certainly allow synthesis of higher order linkages to be explored. This could include six and eight link mechanisms.

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APPENDIX A

DISPLACEMENT ANALYSIS

The displacement analysis of four-bar linkages is a well documented subject. The approach used in this work is based on Freudenstein's equation (12). This allows the coupler or output link angular displacement to be determined when the angular displacement of the input link and the length of all of the links are given. Refer to Figure (34).

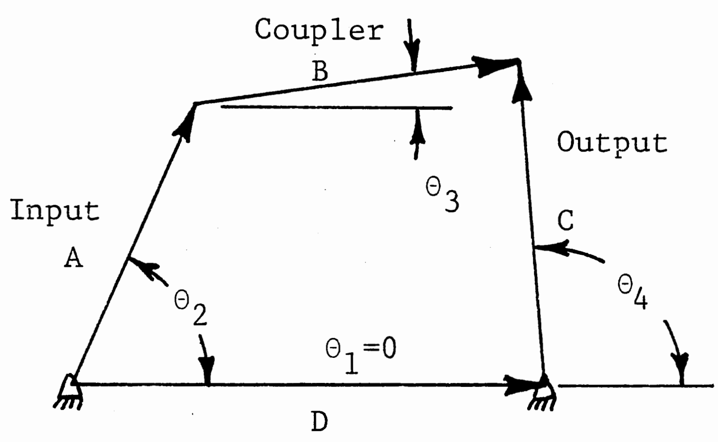


Figure 34. Displacement Notation

Coupler Displacement

The vectors A, B, C and D form a closed loop which can be expressed by: A + B = C + D. Allowing $\hat{i}$ and $\hat{j}$ to be unit vectors in the X and Y directions and a, b, c and d to be the magnitudes of A, B, C and D, respectively, this may be expressed as:

$$a\ (\cos\Theta_2\hat{i} + \sin\Theta_2\hat{j}) + b\ (\cos\Theta_3\hat{i} + \sin\Theta_3\hat{j})$$
$$= c\ (\cos\Theta_4\hat{i} + \sin\Theta_4\hat{j}) + d\ (\cos 0\hat{i} + \sin 0\hat{j}) \qquad (A.1)$$

Separating the $\hat{i}$ and $\hat{j}$ components and placing only the $\Theta_4$ terms on the right hand side:

$$a\cos\Theta_2 + b\cos\Theta_3 - d = c\cos\Theta_4 \qquad (A.2)$$

$$a\sin\Theta_2 + b\sin\Theta_3 = c\sin\Theta_4 \qquad (A.3)$$

Squaring both sides of each equation and adding gives

$$a^2 + b^2 = d^2 + 2ab\cos\Theta_2\cos\Theta_3 - 2ad\cos\Theta_2$$
$$-2bd\cos\Theta_3 + 2ab\sin\Theta_2\sin\Theta_3 = c^2 \qquad (A.4)$$

Simplifying equation (A.4) results in:

$$A_1\cos\Theta_3 + B_1\sin\Theta_3 + C_1 = 0 \qquad (A.5)$$

where: $A_1 = \cos\Theta_2 - \frac{d}{a}$

$B_1 = \sin\Theta_2$

$$C_1 = \frac{a^2 + b^2 + d^2 - c^2 - 2ad\cos\Theta_2}{2ab}$$

Substituting the trigonometric identities for

$$\sin \Theta = \frac{2 \tan (\Theta/2)}{1 + \tan (\Theta/2)} \quad \text{and} \quad \cos \Theta = \frac{1 - \tan^2 (\Theta/2)}{1 + \tan (\Theta/2)} \tag{A.6}$$

Equation (A.5) can be solved for $\Theta_3$:

$$\Theta_3 = 2 \tan^{-1}\left[\frac{Q_1 \pm \sqrt{Q_1^2 - 4 P_1 R_1}}{2 P_1}\right] \tag{A.7}$$

where: $P_1 = (C_a - A_1)$

$Q_1 = 2B_1$

$R_1 = (C_1 + A_1)$

## Output Displacement

If the output displacement is required, the $\hat{i}$ and $\hat{j}$ components of Equation (A.1) are separated placing only the $\Theta_3$ terms on the right hand side:

$$a \cos \Theta_2 - c \cos \Theta_4 - d = -b \cos \Theta_3 \tag{A.8}$$

$$a \sin \Theta_2 - c \sin \Theta_4 = -b \sin \Theta_3 \tag{A.9}$$

Squaring both sides of each equation and adding gives:

$$a^2 + c^2 + d^2 - b^2 - 2ac \cos \Theta_2 \cos \Theta_4 - 2ac \sin \Theta_2 \sin \Theta_4 - 2ad \cos \Theta_2 + 2cd \cos \Theta_4 = 0 \tag{A.10}$$

This equation can be simplified to:

$$A_1 \cos \Theta_4 + B_1 \sin \Theta_4 + C_1 = 0 \tag{A.11}$$

where: $A_1 = d/a - \cos \Theta_2$

$B_1 = -\sin \Theta_2$

$$C_1 = \frac{a^2 + \overset{c}{\cancel{b}}{}^2 + d^2 - b^2 - 2ad \cos\Theta_2}{2ac}$$

Substituting the trigonometric identities (A.6) into Equation (A.11) results in:

$$\Theta_4 = 2 \tan^{-1} \left[ \frac{Q_1 \pm \sqrt{Q_1{}^2 - 4\, P_1\, R_1}}{2\, P_1} \right] \qquad (A.12)$$

where: $P_1 = (C_1 - A_1)$

$Q_1 = \overset{2}{\cancel{\$}}B_1$

$R_1 = (C_1 + A_1)$

## APPENDIX B

## LINEAR SUPERPOSITION

The principles of linear superposition can be used to obtain a closed form solution to a set of n non-linear equations with n unknown parameters. This method is used to obtain the solution of the following set of equations:

$$A_iX_1 + B_iX_2 + C_iX_3 + D_iX_1X_3 + E_iX_2X_3 + F_i = 0 \qquad \text{(B.1)}$$

for i = 1 to 3

If we let $X_1 = (l_1 + \lambda_1 m_1 + \lambda_2 n_1)$

$$X_2 = (l_2 + \lambda_1 m_2 + \lambda_2 n_2)$$

$$X_3 = (l_3 + \lambda_1 m_3 + \lambda_2 n_3)$$

$$X_1X_3 = \lambda_1$$

$$X_2X_3 = \lambda_2$$

then we can express Equation (B.1) as:

$$A_i\ (l_1 + \lambda_1 m_1 + \lambda_2 n_1) + B_i\ (l_2 + \lambda_1 m_2 + \lambda_2 n_2)$$
$$+ C_i\ (l_3 + \lambda_1 m_3 + \lambda_2 n_3) + D_i\lambda_1 + E_i\lambda_2 + F_i = 0 \qquad \text{(B.2)}$$

for i = 1 to 3

Collecting the constant, $\lambda_1$ and $\lambda_2$ terms of (B.2) yields

$$l_1 A_i + l_2 B_i + l_3 C_i = -F_i \tag{B.3}$$

$$m_1 A_i + m_2 B_i + m_3 C_i = -D_i \tag{B.4}$$

$$n_1 A_i + n_2 B_i + n_3 C_i = -E_i \tag{B.5}$$

Each of these equations give three linear equations in three unknowns which are solved for the coefficients $l_i$, $m_i$ and $n_i$.

Compatability conditions are imposed by:

$$\lambda_1 = X_1 X_2 = (l_1 + m_1\lambda_1 + n_1\lambda_2)(l_3 + m_3\lambda_1 + n_3\lambda_2) \tag{B.6}$$

$$\lambda_2 = X_2 X_3 = (l_2 + m_2\lambda_1 + n_2\lambda_2)(l_3 + m_3\lambda_1 + n_3\lambda_2) \tag{B.7}$$

These equations can be simplified to:

$$F_1\lambda_2^2 + (F_2\lambda_1 + F_3) + (F_4\lambda_1^2 + F_5\lambda_1 + F_6) = 0 \tag{B.8}$$

$$G_1\lambda_2^2 + (G_2\lambda_1 + G_3) + (G_4\lambda_1^2 + G_5\lambda_1 + G_6) = 0 \tag{B.9}$$

where

$$\begin{aligned}
F_1 &= n_1 n_3 & G_1 &= n_2 n_3 \\
F_2 &= n_1 m_3 + m_1 n_3 & G_2 &= n_2 m_3 + m_2 n_3 \\
F_3 &= l_1 n_3 + n_1 l_3 & G_3 &= l_2 n_3 + n_2 l_3 - 1 \\
F_4 &= m_1 m_3 & G_4 &= m_2 m_3 \\
F_5 &= l_1 m_3 + m_1 l_3 - 1 & G_5 &= l_2 m_3 + m_2 l_3 \\
F_6 &= l_1 l_3 & G_6 &= l_2 l_3
\end{aligned}$$

For convenience, Equations (B.8) and (B.9) are expressed by:

$$H_1\lambda_2^2 + H_2\lambda_2 + H_3 = 0 \tag{B.10}$$

$$I_1\lambda_2^2 + I_2\lambda_2 + I_3 = 0 \tag{B.11}$$

where

$$H_1 = F_1$$

$$H_2 = (F_2\lambda_1 + F_3)$$

$$H_3 = (F_4\lambda_1^2 + F_5\lambda_1 + F_6)$$

$$I_1 = G_1$$

$$I_2 = (G_2\lambda_1 + G_3)$$

$$I_3 = (G_4\lambda_1^2 + G_5\lambda_1 + G_6)$$

A common root, $\lambda_2$, of the compatibility equations can be found using the Sylvester technique. This is achieved by:

$$\det \begin{vmatrix} H & H & H & 0 \\ 0 & H & H & H \\ I & I & I & 0 \\ 0 & I & I & I \end{vmatrix} = 0 \tag{B.12}$$

Expanding Equation (B.12) and collecting terms yields a third order polynomial in $\lambda_1$:

$$L_1\lambda_1^3 + L_2\lambda_1^2 + L_3\lambda_1 + L_4 = 0 \qquad \text{(B.13)}$$

where:

$$\begin{aligned} L_1 = {} & 2\ F_1^2G_4G_5 + 2\ F_1F_4G_2G_3 + F_1F_5G_2^2 - F_1F_2G_2G_5 - \\ & F_1F_2G_3G_4 - F_1F_3G_2G_4 - 2\ F_1F_4G_1G_5 - 2\ F_1F_5G_1G_4 + \\ & F_2^2G_1G_5 + 2\ F_2F_3G_1G_4 + 2\ F_4F_5G_1^2 - F_2F_5G_1G_5 - \\ & F_2F_5G_1G_2 - F_2F_4G_1G_3 \end{aligned}$$

$$\begin{aligned} L_2 = {} & 2\ F_1^2G_4G_6 + F_1^2G_5^2 + F_1F_4G_3^2 + 2\ F_1F_5G_2G_3 + \\ & F_1F_6G_2^2 - F_1F_2G_2G_6 - F_1G_2G_3G_5 - F_1F_3G_2G_5 - \\ & F_1F_3G_3G_4 - 2\ F_1F_4G_1G_6 - 2\ F_1F_5G_1G_5 - 2\ F_1F_6G_1G_4 + \\ & F_2^2G_1G_6 + 2\ F_2F_3G_1G_5 + F_3^2G_1G_4 + 2\ F_4F_6G_1^2 + \\ & F_5^2G_1^2 - F_2F_6G_1G_2 - F_2F_5G_1G_3 - \\ & F_3F_5G_1G_2 - F_3F_4G_1G_3 \end{aligned}$$

$$\begin{aligned} L_3 = {} & 2\ F_1^2G_5G_6 + F_1F_5G_3^2 + 2\ F_1F_6G_2G_3 - F_1F_2G_3G_6 - \\ & F_1F_3G_2G_6 - F_1F_3G_3G_5 - 2\ F_1F_5G_1G_6 - 2\ F_1F_6G_1G_5 + \\ & 2\ F_2F_3G_1G_6 + F_3^2G_1G_5 + 2\ F_5G_6G_1^2 - F_2F_6G_1G_3 - \\ & F_3F_6G_1G_2 - F_3F_5G_1G_3 \end{aligned}$$

$$\begin{aligned} L_4 = {} & F_1^2G_6^2 + F_1F_6G_3^2 - F_1F_3G_3G_6 - 2\ F_1F_6G_1G_6 + \\ & F_3^2G_1G_6 + F_6^2G_1^2 - F_3F_6G_1G_3 \end{aligned}$$

The solution of Equation (B.13) yields either one real root or three real roots in $\lambda_1$. Each of these roots must be substituted back into Equations (B.8) and (B.9). A common root, $\lambda_2$, of Equation (B.8) and (B.9) exists. This root, $\lambda_2$, and the matching $\lambda_1$ can be substituted back into:

$$X_1 = (l_1 + \lambda_1 m_1 + \lambda_2 n_1)$$

$$X_2 = (l_2 + \lambda_2 m_2 + \lambda_2 n_2)$$

$$X_3 = (l_3 + \lambda_1 m_3 + \lambda_2 n_3)$$

Hence, the solution is obtained. It must be noted that for each set of parameters, there is either one or three solutions to the set of equations. A closed form solution for Equation (B.13) is given in Appendix C.

## APPENDIX C

## SOLUTION OF A CUBIC

The solution of a cubic polynomial can be obtained in closed form by the procedure found in Mark's Standard Handbook for Mechanical Engineers (11). The polynomial: $X^3 + a X^2 + b X + c = 0$ can be expressed as:

$$X_1^3 = A X_1 + B$$

where: $X_1 = X + a/3$

$$A = 3(a/3) - b$$

$$B = -2(a/3) + b(a/3) - c$$

Allowing $p = A/3$ and $q = B/2$ the general solution is:

Case 1: $q^2 - p^3$ = positive, yields one real and two imaginary roots. The real root is given by:

$$X_1 = \sqrt[3]{q + \sqrt{q^2 - p^3}} + \sqrt[3]{q - \sqrt{q^2 - p^3}}$$

Case 2: $q^2 - p^3$ = zero, yields three real roots with two equal: $X_1 = 2\sqrt[3]{q}$, $X_1 = -\sqrt[3]{q}$, $X_1 = -\sqrt[3]{q}$.

Case 3: $q - p$ = negative, yields three real, distinct roots:

$$X_1 = 2\sqrt{p}\ \cos(u/3)$$

$$X_1 = 2\sqrt{p}\ \cos(u/3 + 120)$$

$$X_1 = 2\sqrt{p}\ \cos(u/3 + 240)$$

where: $u = \cos^{-1}(q/(p\sqrt{p}))$

The original X may be found using: $X = X_1 - a/3$.

# APPENDIX D

# ENUMERATED DISPLAYS

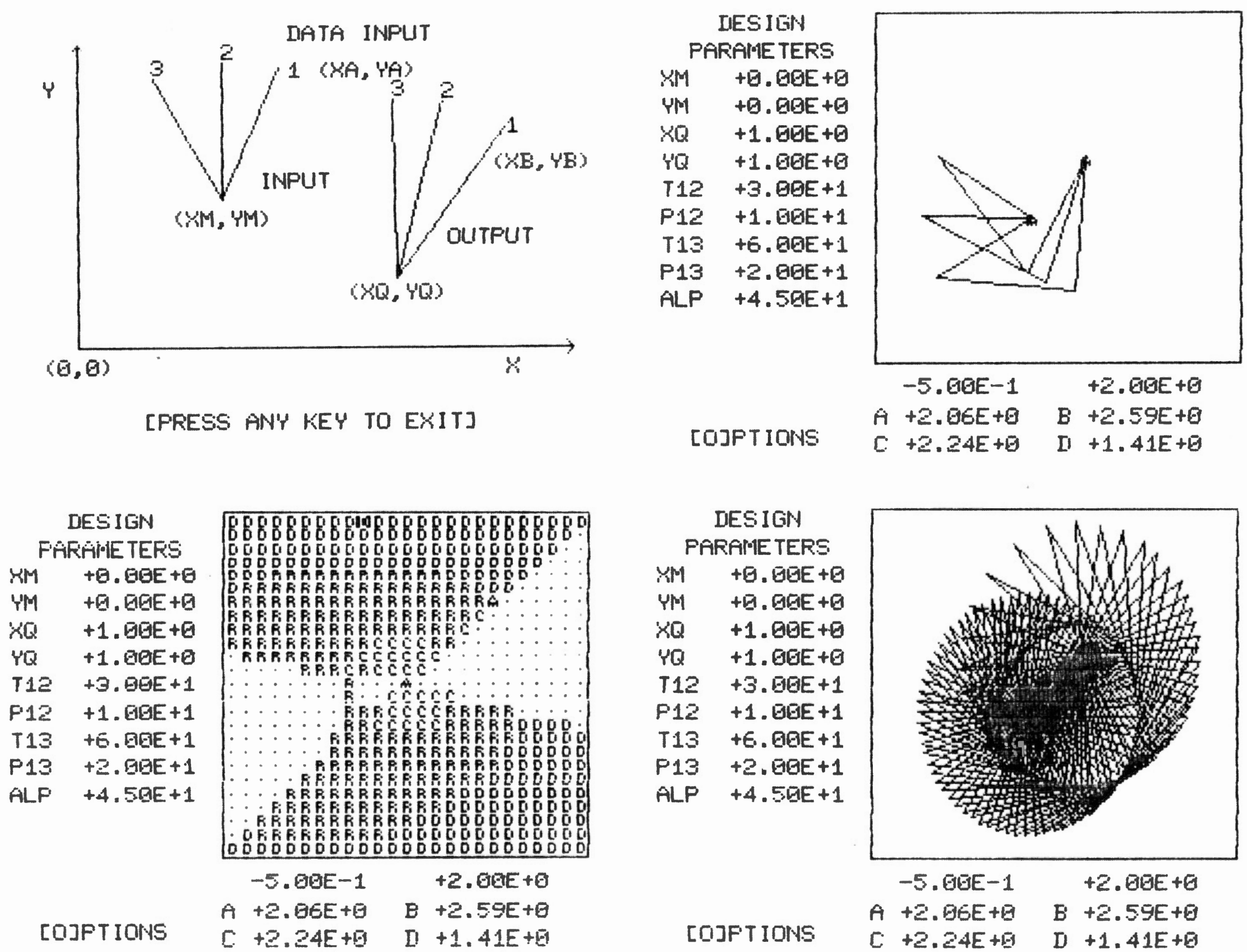


Figure 35. Example, Case P-P-P

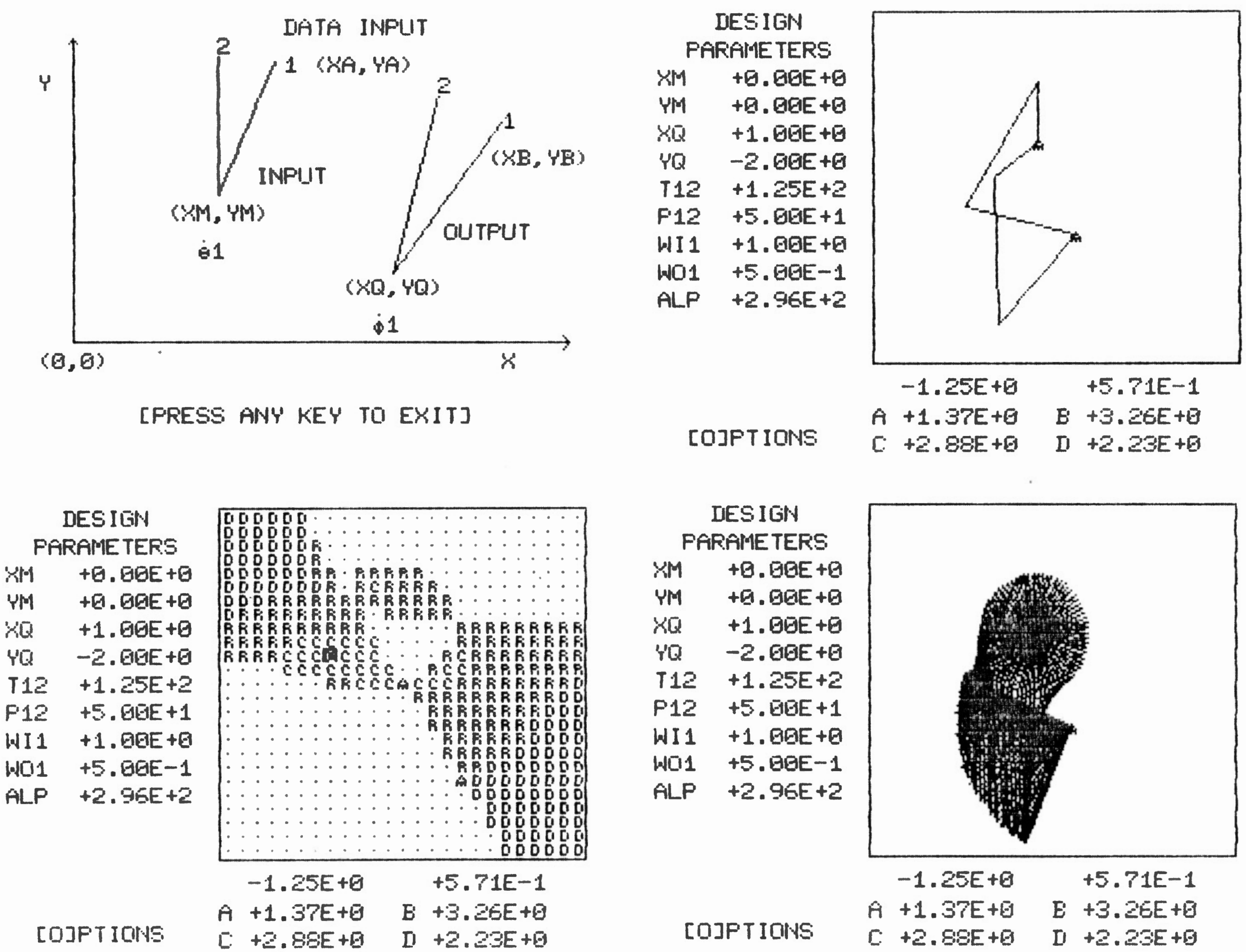


Figure 36. Example, Case PP-P

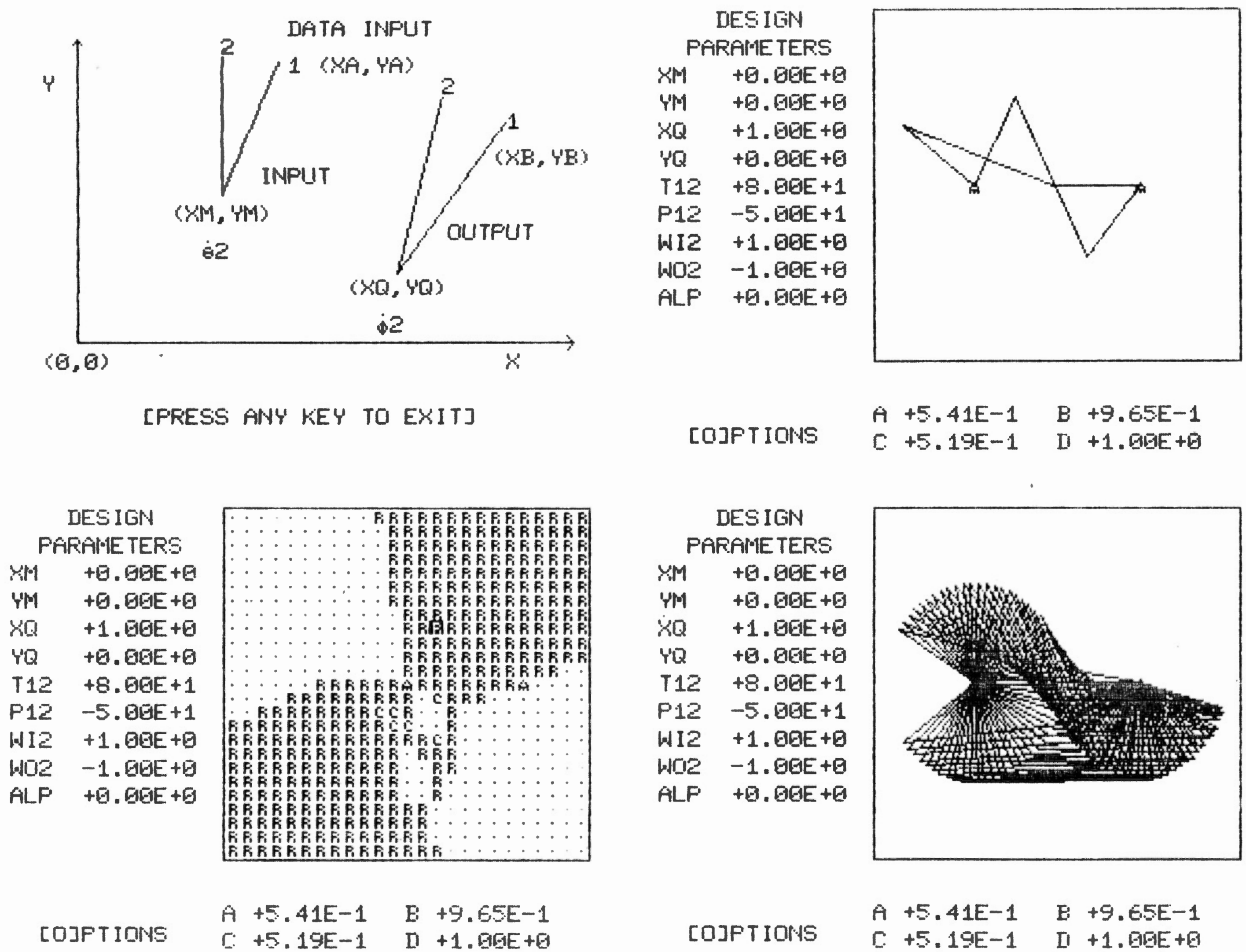


Figure 37. Example, Case P-PP

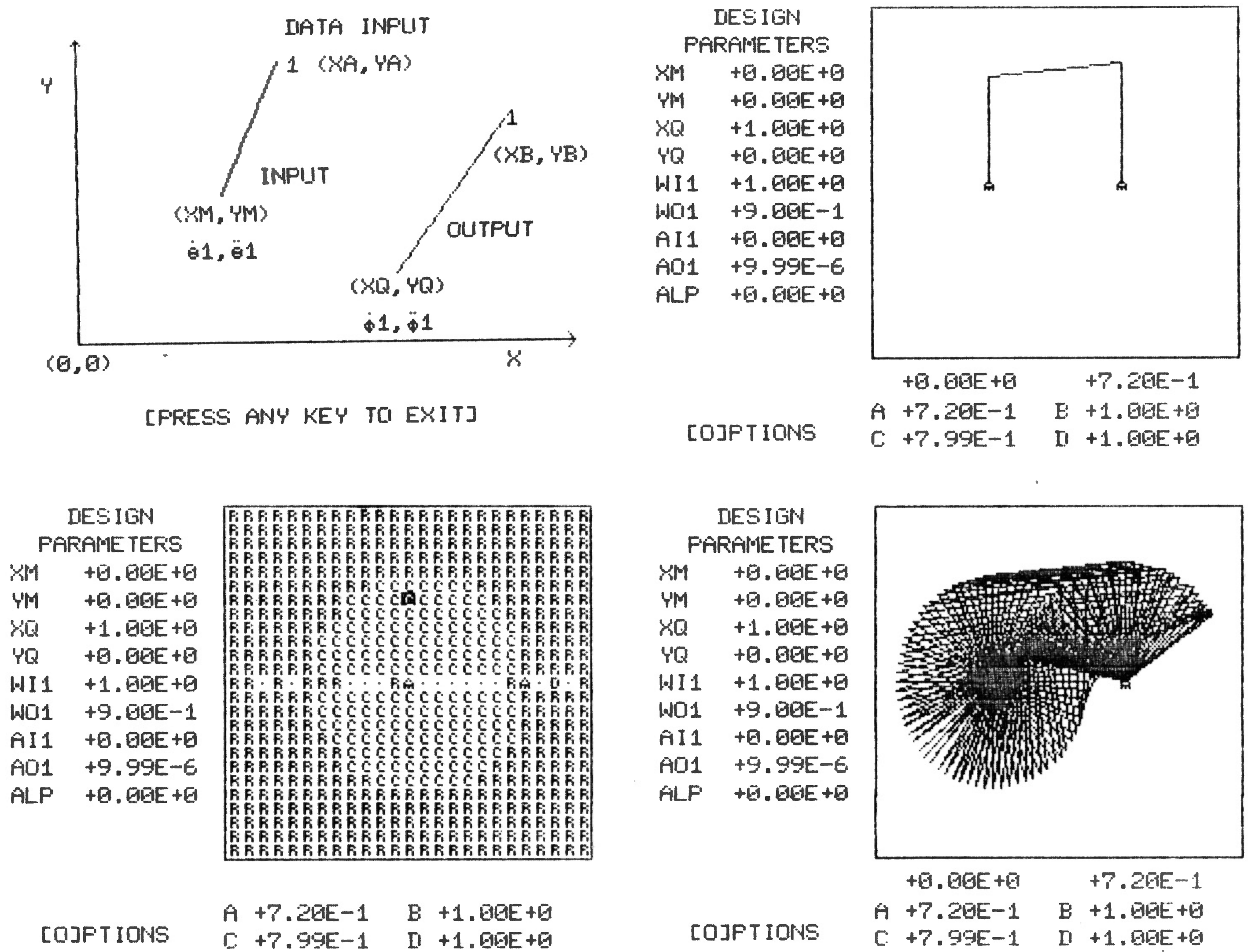


Figure 38. Example, Case PPP

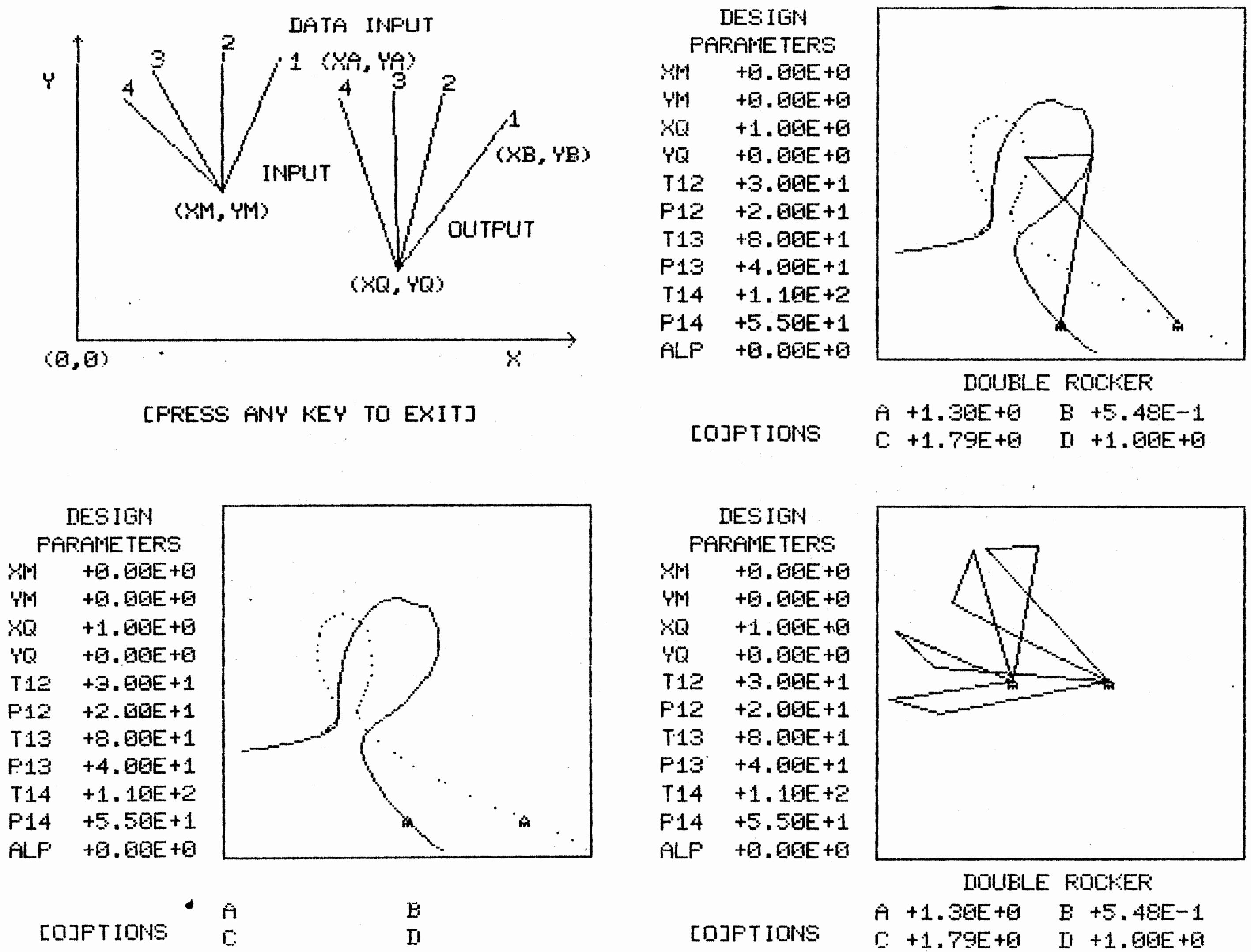


Figure 39. Example, Case P-P-P-P

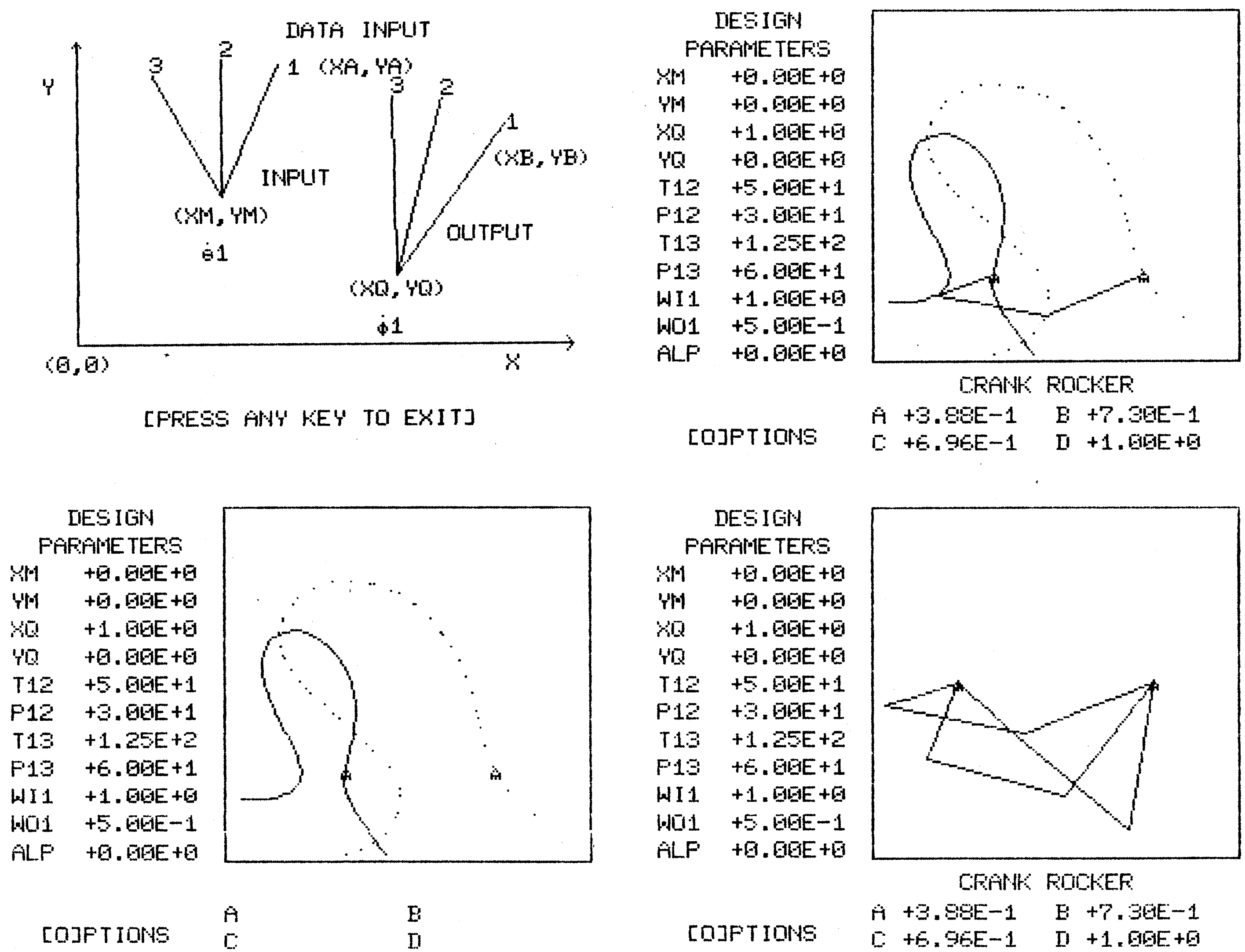


Figure 40. Example, Case PP-P-P

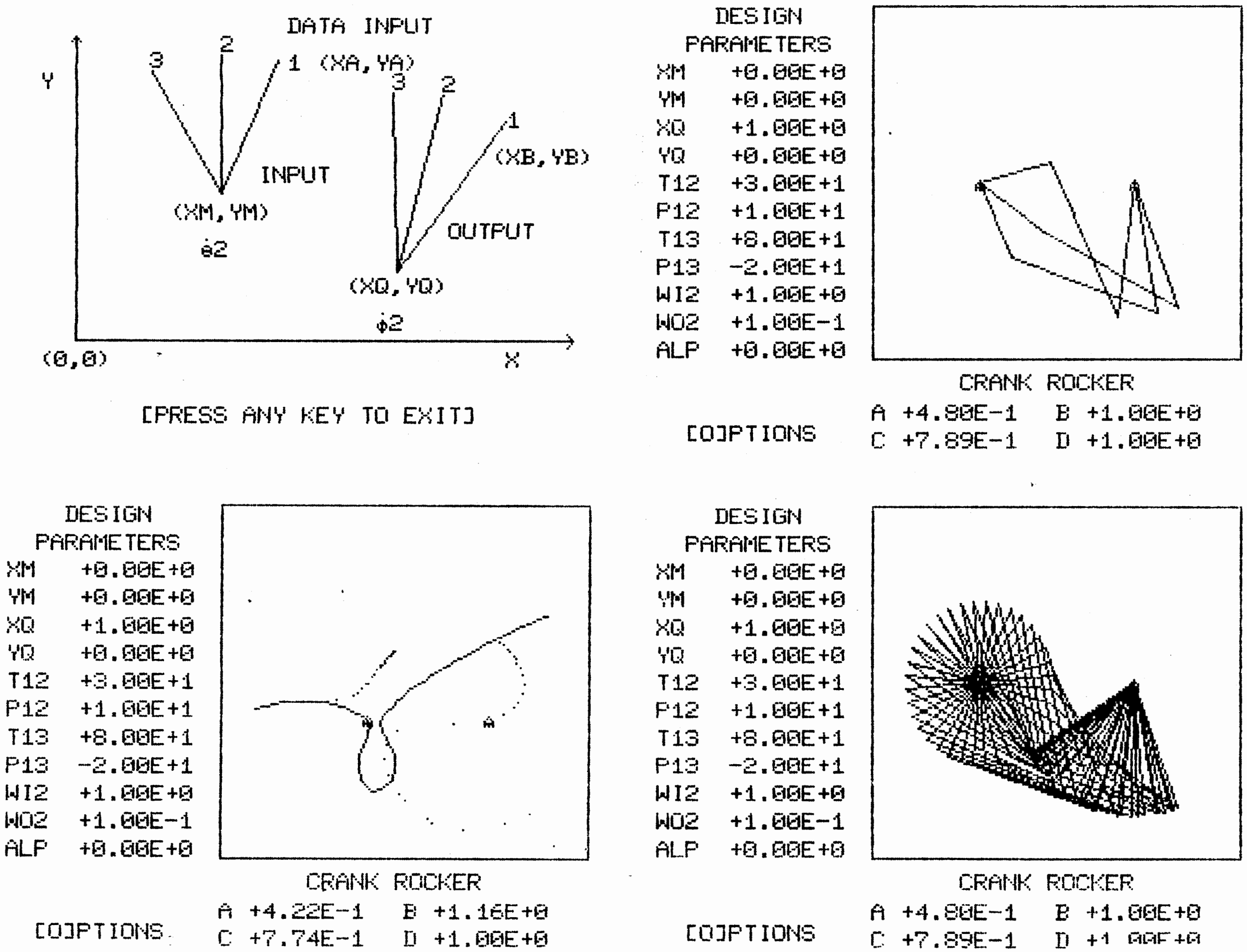


Figure 41. Example, Case P-PP-P

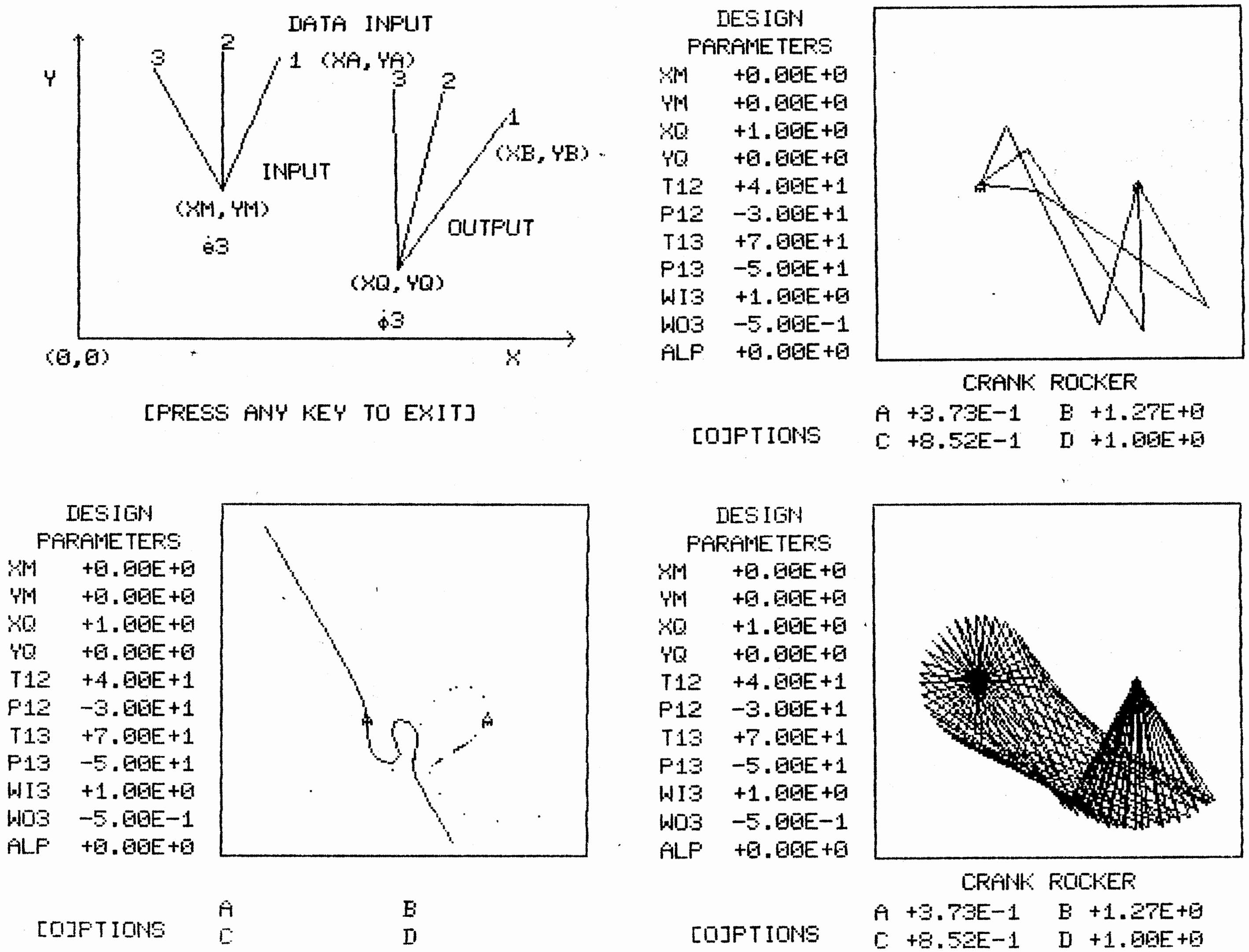


Figure 42. Example, Case P-P-PP

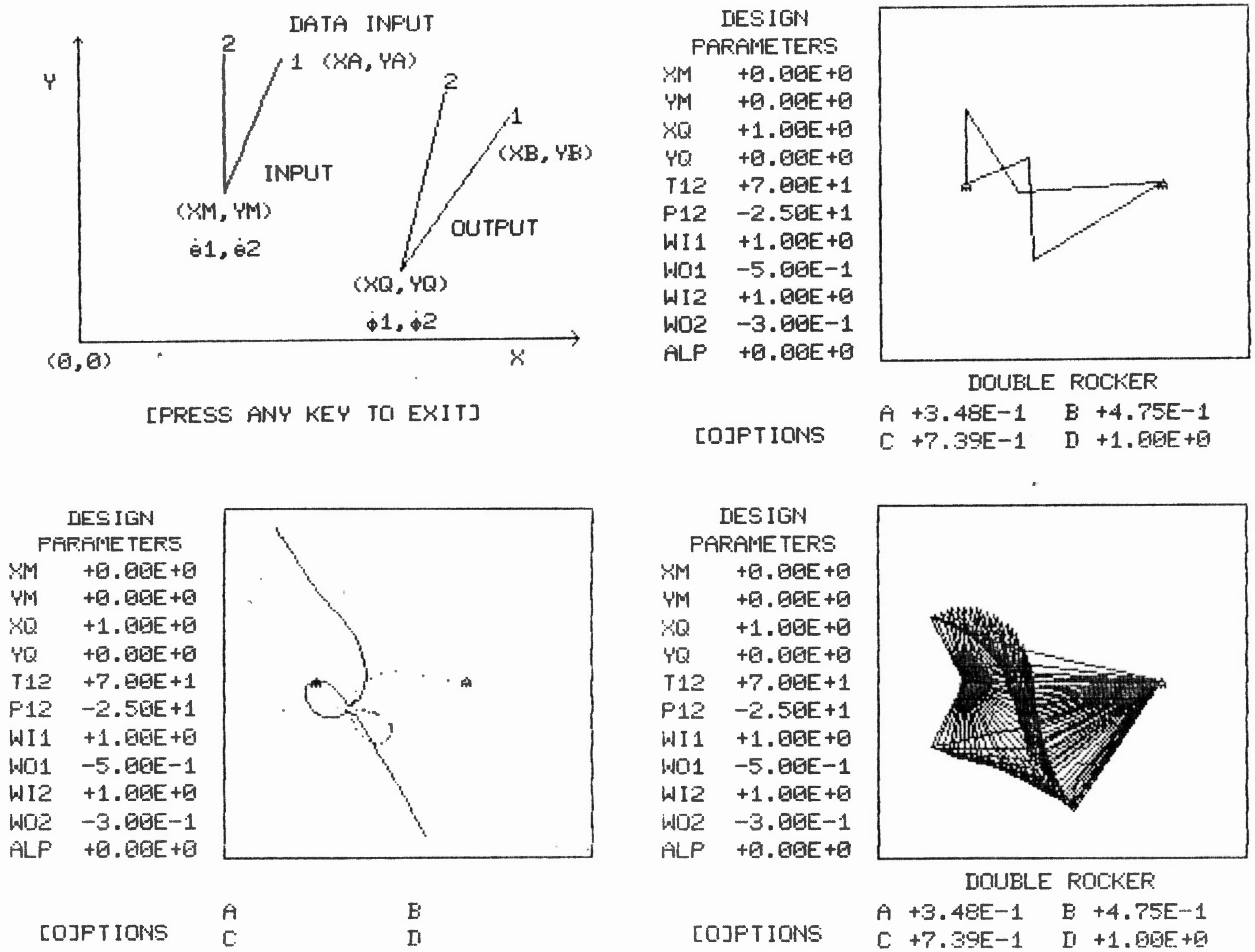


Figure 43. Example, Case PP-PP

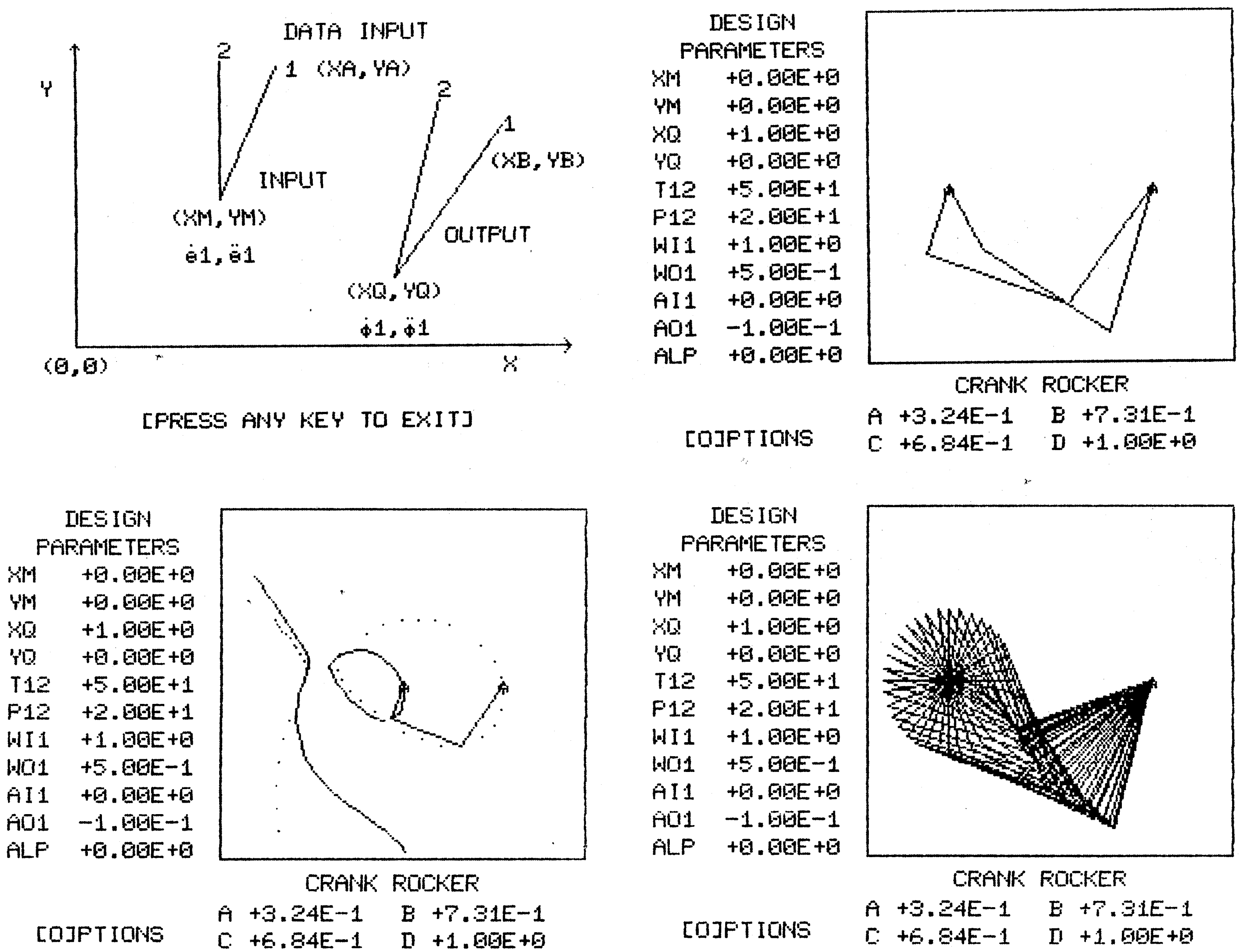


Figure 44. Example, Case PPP-P

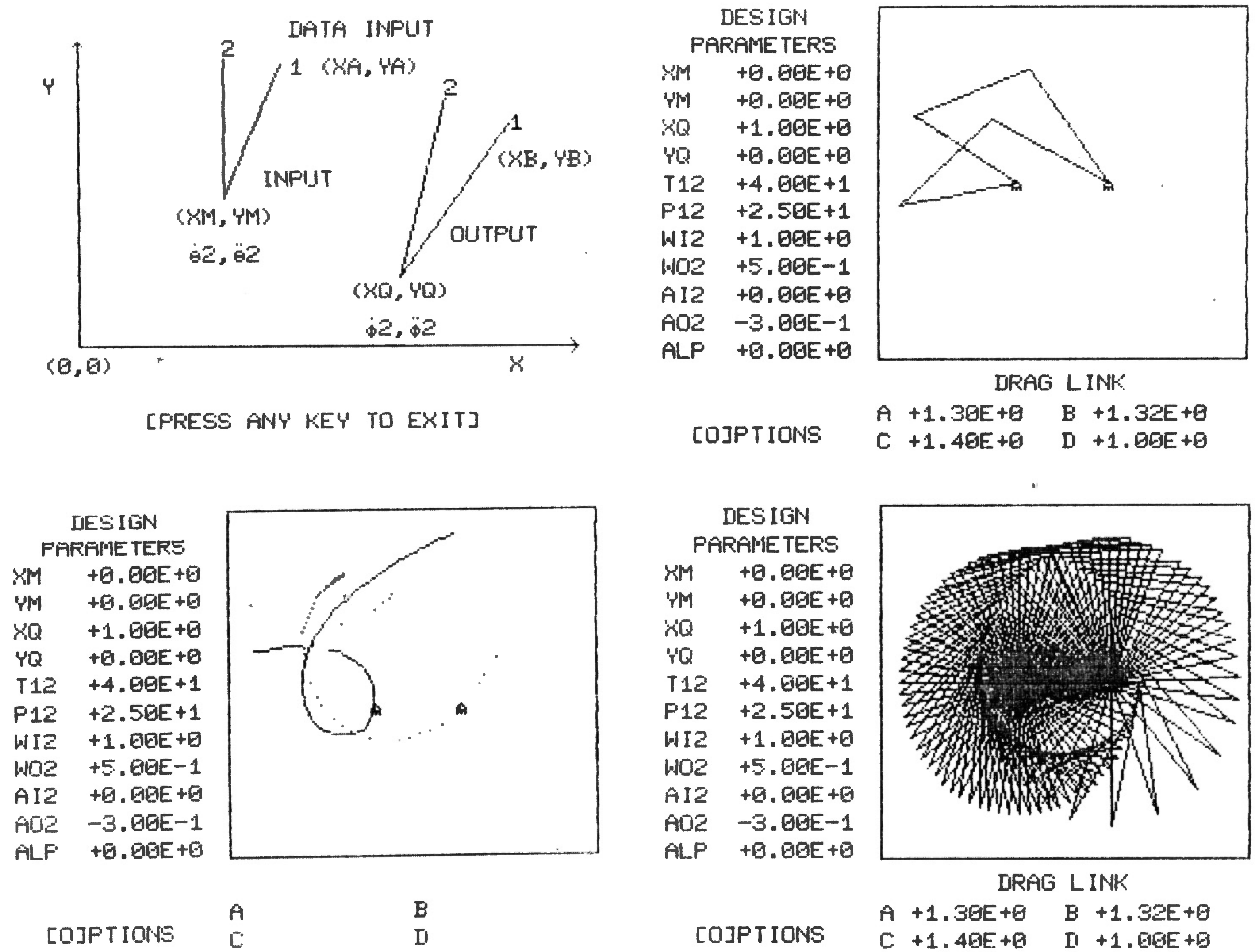


Figure 45. Example, Case P-PPP

VITA

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