A THEORY OF PRICE-DEPENDENT DEMAND WITH APPLICATIONS TO THE MAJOR U.S. MEATS

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Bachelor of Science

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Lubbock, Texas

1981

Submitted to the Faculty of the Graduate College
of the Oklahoma State University
in partial fulfillment of the requirements
for the degree of
MASTER OF SCIENCE
December, 1983

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Thesis Approved:

Thesis Adviser

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PREFACE

The primary concern of this study was the theory of price-dependent demand. The major objectives of the study were to construct a general theory of price-dependent demand with emphasis given to the undeveloped areas, and to empirically test the validity of the theory for the major U.S. meats.

I wish to extend thanks to Dr. Bruce Bullock, who introduced me to this topic. His innovative ideas were a continual source of mental stimulation, and much of the conceptual seed for this thesis was sown by Dr. Bullock.

I extend abundant thanks to Dr. L.V. Blakley, whose numerous contributions significantly improved this work. His suggestions were invariably constructive, and few if any could have directed this project as effectively.

Dr. Dan Tilley was of considerable help in the refinement of the theoretical work. I thank Dr. Tilley for his continual willingness to assist, even though he was not obligated to do so.

I thank Dr. Glenn Knowles and Dr. Daryll Ray for their assistance, which was primarily through a teaching capacity. Their courses in econometrics and mathematical economics were taught with distinguished abilities.

I extend my appreciation to Dr. James Osborn for his administrative efforts, and for his patience with this project, which required more than an average length of time. Because this additional time was granted, many features were added to both the theoretical developments and the empirical analysis.

My greatest appreciation is for the pleasant working atmosphere provided by all of these men, who should not only be noted for their various abilities, but also for their upstanding characters. I thank many friends, particularly Mr. and Mrs. Tom Whaley, for their continual support, and my parents, who never cease to be a source of inspiration, and to whom this work is dedicated.

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CHAPTER I

INTRODUCTION

Throughout the development of modern consumer theory, demand functions have been almost consistently expressed in the quantity-dependent form. Price-dependent functions are occasionally observed in empirical work, but only few theoretical investigations of price-dependent demand can be found. Moreover, the few investigations into the properties of price-dependent functions have produced conflicting conclusions. Waugh (1964) derived a set of properties for the flexibilities from the first-order conditions of utility maximization. In a later study, Houck (1966) performed linear operations upon a set of quantity-dependent functions to obtain the implied price-dependent forms; however, the flexibilities of these functions did not follow the properties derived by Waugh. In the most recent of these studies, Heien (1982) derived yet a third set of properties with use of duality theory. Thus, the demand literature has not only failed to produce consistent conclusions concerning the properties of the price-dependent function, but has failed to develop an established methodology in the analysis thereof.

There are other respects in which price-dependent demand theory is incomplete. First, the properties of flexibilities under special forms of the utility function have not been thoroughly explored. For example, there have been only a few inquiries into the implications of separable utility for the price-dependent function. Second, little has been said of how that substitutability and complementarity are to be measured in terms of price-dependent parameters. Third, the investigation of relative prices has been insufficient; consequently, little has been said of the properties of the relative price function or of its analytical potentiality. Thus, there are several unexplored areas in the theories of both absolute and relative prices.

The incompleteness of the price-dependent theory has also prevented its application in empirical work. Few have proposed practical schemes whereby flexibilities can be properly estimated with the usual statistical techniques. Also, it is not certain as to how that elasticities and other quantity-dependent parameters are to be estimated when the associated data are generated in a price-dependent fashion.

The overall objective of this study is to construct a general theory of price-dependent demand, with emphasis given to the undeveloped areas. The specific objectives are:

1. to compare the price-dependent and quantity-dependent approaches, and to discuss the criteria that should govern the selection between the two,
2. to derive the properties of the flexibilities under general utility,
3. to determine how that the general properties are affected by homotheticity and separability in utility,
4. to determine how that substitutability and complementarity are manifested in terms of price-dependent parameters, and
5. to demonstrate the various methods by which elasticities and

other quantity-dependent measures are calculated from price-dependent parameters.

The development of the theoretical results is always directed with experimental applications in mind. The various price-dependent models are constructed with emphasis given to comformability with empirical limitations. Of these limitations, the problems that are often associated with large numbers of commodities are of particular concern. A theoretical justification is presented for certain schemes whereby partial sets of elasticities, flexibilities, and other parameters can be estimated by price-dependent methods with consideration given only to certain subsets of the commodity basket. Moreover, these schemes are empirically applied to the major U.S. meats for a demonstration of the methodology. Also, the quantity-dependent empirical techniques are applied to the meats for a comparison of the associated results with those obtained by the price-dependent methods. The results of statistical tests of the various theoretical implications from both the quantity-dependent and price-dependent approaches are presented as well.

CHAPTER II

A THEORY OF PRICE-DEPENDENT DEMAND

Price-Dependence Versus Quantity-Dependence In Demand

The proper selection between the price-dependent and quantity-dependent approaches to demand must be largely based upon empirical considerations. One will not commonly find markets in which there is a definite direction of causality from prices to quantities or vice-versa, but rather, both prices and quantities will usually be endogenously determined by the interactions of market supply and demand. The popularity of the quantity-dependent approach would seem to indicate that there must be some conceptual appeal or analytical superiority with this method of analysis. Indeed, if the theory were confined to the case of an individual consumer, then there is both conceptual appeal and analytical convenience with the quantity-dependent function. It is reasonable to assume that prices are generally fixed to the individual consumer; therefore, at this level, equilibrium adjustments must be accomplished entirely through quantity changes. Clearly, this is a case of quantity-dependence. Moreover, the various equilibria may be analyzed completely within a partial equilibrium framework if the demand functions are written in the quantity-dependent form. Since the larger portion of modern demand theory is constructed from the case of an individual consumer, the popularity of the quantity-dependent version is of no surprise. However, when aggregate markets are under consideration, it is no longer proper to think of quantities as being determined by prices, or of prices as being determined by quantities. Instead, both prices and quantities will usually be simultaneously and endogenously determined within a general equilibrium context. Nor is there advantage with either approach with respect to analytical convenience. The analysis of general equilibrium is equally inconvenient under either functional form.

There are two circumstances under which one of these approaches to demand will have analytical advantage over the other. These occur when the equilibria are located upon portions of the supply curve that are either perfectly elastic or perfectly inelastic. The former extreme occurs with an individual consumer in a perfectly competetive market. In such cases, prices become exogenous to the equilibrating process; subsequently, the quantity-dependent functions become more appropriate. First, such functions are consistent with the direction of causality. Second, the various equilibria may be analyzed within the relatively simplistic partial equilibrium framework if the demand functions are written with dependent quantities. Likewise, if supply is perfectly inelastic, then the price-dependent functions become both consistent with the direction of causality and the more amenable to the analysis of equilibria.

Though it is unlikely that either of these two extremes are to be observed among the various commodity markets, it is reasonable to suspect that perfectly inelastic supply is commonly approximated. It is certain that resource limitations determine an upper bound upon the supply of any commodity within a given length of run. Therefore, the mere existence of resource scarcity insures that quantities must become highly if not completely irresponsive to price changes at certain price levels. If the supply function is monotonic, then at extremely high prices, quantities must become highly if not completely invariant to price changes. There are situations in which supply could become extremely inelastic at low prices as well. For example, suppose a highly perishable product that is produced in a periodic fashion. Between production periods, an upper bound is determined upon the supply of this sort of good, for sales cannot exceed the output of the previous period. Moreover, if the product cannot be stored over a span of two production periods, then suppliers will maximize profits by selling all product at any price, as long as total revenues cover marketing costs. It is apparent that the supply for this product will be highly inelastic over nearly the entire range of the supply curve. Many examples of this sort of product can be found among the agricultural commodities. Products that are not perishable, but subject to obsolescence, provide examples as well.

It is important to observe that resource scarcity does not insure that equilibrium will occur at points of highly inelastic supply. With the former example, it was necessary to assume the additional condition of product perishability to guarantee that supply would be highly inelastic at low prices. There is a tendency to think that supply must be extremely inelastic for commodities such as antiques or precious stones, since there is limited availability of the items or of their resource components. However, this will not be the case if suppliers become increasingly unwilling to sell with reduced prices. Nor is it necessarily true that supply must be highly inelastic in extremely short lengths of run. Obviously, the extent to which suppliers can respond to price increases is diminished with shorter periods of time; however, they are not prevented from withdrawing product in the case of price reductions. For highly inelastic supply, there must exist factors that not only constrain the ability to sell, but also the willingness. Since there are several markets in which both sorts of factors do exist, highly inelastic supply should not be an uncommon phenomenon. All commodities are known to have extremely inelastic supply in certain price ranges; moreover for some commodities, these ranges probably embrace nearly the entire supply curve.

Nevertheless, it yet remains that perfectly inelastic supply is apt to be extremely uncommon. In usual situations, both quantities and prices will be endogenously determined, and the choice between quantity-dependent and price-dependent functions will be a matter of indifference insofar as theory is concerned. However, four empirical considerations may make one of these two approaches advantageous. First, the objectives of the experiment may require parameters for a particular form. For example, if elasticities were required, then quantity-dependent functions may be preferred. Second, when a demand function is not simultaneously estimated with the associated supply function, there are, nonetheless, implicit assumptions made of supply. For example, with the estimation of a price-dependent function, supply is implicitly assumed to be perfectly inelastic. Naturally, the form that implies the most approximate assumptions should be selected. Third, there may be fewer statistical problems associated with the estimation of one form than with the other. Price data tend to be more highly correlated than quantity data, so that the multicollinearity problem might be partially circumvented with price-dependent functions. On the other hand, price data are often estimated with greater efficiency than quantity data; subsequently, prices could be better regressors in this respect. Fourth, data limitations may be such that one form is more easily estimated than the other. In general, these empirical considerations should dictate the choice of form.

Properties Of Quantity-Dependent Functions

The properties of quantity-dependent functions have been derived on numerous occasions in the demand literature. Generally, these properties are presented in terms of the associated price and income elasticities. The properties of the elasticities are derived here by the usual methods for an introduction to notation, and for a demonstration of the consistency between the elasticities and the corresponding flexibilities.

The derivation of the elasticities generally proceeds from the case of an individual consumer, who is assumed to purchase according to the maximization of a utility function, $F(x_1,x_2,,,x_n)$, where $(x_1,x_2,,,x_n)$ are the various items within the commodity bundle. The utility function is taken here to have the usual properties, which include quasi-concavity and monotonicity in every argument. Additionally, cardinal significance is not required of the measure of utility, which implies that the function, $F(x_1,x_2,,,x_n)$, and all order preserving (monotonic increasing) transformations thereof are regarded as equally acceptable measures of utility. In interest of maintaining the ordinal character, the utility function is subjected to the arbitrary transformation, T, and subsequent calculations deal with the transformed function, $U(x_1,x_2,,,x_n)$, where U = T(F). All parameters that fail to demonstrate invariance with respect to T are treated as being indeterminate. Also, the consumer is assumed to exhaust his or her income in the pursuit of maximum utility; therefore, the optimal commodity combination must satisfy:

$$m = \sum_{t=1}^{n} x_t p_t$$

where p_t is the per-unit price of x_t, and m is the consumer's income for the period of concern. The objective function is generally written in terms of the Lagrange function, L, and the behavioral postulations are briefly comprehended with:

$$\text{maximize: } L = U(x_1,x_2,,,x_n) + \lambda(m - \Sigma\, p_t x_t)$$

Supply is assumed to be perfectly elastic; consequently, prices are treated as constants in the maximization. The resulting first-order conditions are:

$$\partial L/\partial x_i = U_i - \lambda p_i = 0; \qquad i = 1,2,,,n \tag{1}$$

$$\partial L/\partial m = m - \Sigma\, p_t x_t = 0 \tag{2}$$

The first condition is often written in terms of the proportionality rule:

$$U_i/U_j = p_i/p_j$$

which is perhaps the most general of theoretical results concerning the conduct of exchange. Since the utility function is assumed to be quasi-concave, the implicit function theorem guarantees that the quantities and λ can be written as functions of prices and income in a neighborhood about the optimum; subsequently:

$$x_i = x_i(p_1,p_2,,,p_n,m); \qquad i = 1,2,,,n$$

which are the quantity-dependent demand functions; moreover:

$$\lambda = \lambda(p_1,p_2,,,p_n,m)$$

The underlying postulations of the theory determine that certain properties must follow in the resulting demand functions. Most of these properties can be derived from the first-order relations. In most cases, the quantities in equations one and two are replaced with the associated demand functions. These equations are then differentiated with respect to prices or income to obtain the implied properties for the demand functions.

The first property, which is generally known as the "Engel aggregation condition", is derived by differentiating the budget constraint with respect to income. The differentiation yields:

$$\Sigma\ p_t(\partial x_t/\partial m) = 1$$

which implies that:

$$\Sigma\ (x_t m/x_t m)p_t(\partial x_t/\partial m) = 1$$

$$\Sigma \frac{\partial x_t}{\partial x}\frac{m}{x_t}\frac{x_t p_t}{m}$$

or:

$$\Sigma\ w_t \varepsilon_{tm} = 1 \tag{3}$$

where ε_{tm} is the elasticity of x_t with respect to m, and w_t is the expenditure proportion for x_t. Engel aggregation is useful toward the development of an interpretation for λ. First, the quantities under $U(x_1,x_2,,,x_n)$ are replaced with the demand functions. Second, U is differentiated by m to obtain:

$$\partial U/\partial m = \Sigma\ U_t(\partial x_t/\partial m)$$

Equation one is substituted here with the result that:

$$\partial U/\partial m = \lambda\Sigma\ p_t(\partial x_t/\partial m)$$

or:

$$\partial U/\partial m = \lambda\Sigma\ w_t \varepsilon_{tm}$$

and from Engel aggregation, the latter relation becomes:

$$\partial U/\partial m = \lambda$$

Therefore, λ is the marginal utility of income when prices are held constant.

The "Cournot aggregation condition" is derived by differentiating the budget constraint with respect to p_j. This yields:

$$\sum_t p_t(\partial x_t/\partial p_j) + x_j = 0$$

which implies that:

$$\sum_t w_t \varepsilon_{tj} = -w_j; \qquad j = 1,2,,,n \tag{4}$$

where ε_{tj} is the elasticity of x_t with respect to p_j.

A third condition becomes apparent upon the observation that neither the budget constraint nor the proportionality rules are altered by proportional changes in all prices and income. Consequently, the optimal commodity combination under $(p_1,p_2,,,p_n,m)$ must be the same as that obtained with $(\alpha p_1,\alpha p_2,,,\alpha p_n,\alpha m)$, where α is the factor of proportionality. In terms of the demand functions, this condition becomes:

$$x_i(\alpha p_1,\alpha p_2,,,\alpha p_n,\alpha m) = x_i(p_1,p_2,,,p_n,m)$$

From Euler's theorem, the associated elasticities must follow:

$$\sum_t \varepsilon_{it} + \varepsilon_{im} = 0; \qquad i = 1,2,,,n \tag{5}$$

which is the "homogeneity condition" for elasticities.

The fourth condition is generally known as "Slutsky's equation." The condition is that:

$$w_i\varepsilon_{ij} = w_j\varepsilon_{ji} + w_iw_j(\varepsilon_{jm} - \varepsilon_{im}) \tag{6}$$

Slutsky's equation can be derived in several ways; however, the methodology of Frisch (1959) is used here, since this approach yields useful intermediate results. To begin, an important concept used in Frisch's calculations is the utility accelerator, Φ_{ij}, which is defined as:

$$\Phi_{ij} = U_{ij}(x_j/U_i)$$

Subsequently, Φ_{ij} is the flexibility of U_i with respect to x_j. It can be easily confirmed that the utility accelerator satisfies:

$$w_i\Phi_{ij} = w_j\Phi_{ji} \tag{7}$$

Likewise, if Φ^{ij} denotes the inverse element of Φ_{ij}, then:

$$w_i\Phi^{ij} = w_j\Phi^{ji} \tag{8}$$

To demonstrate the latter property, suppose that Φ represents the n x n matrix of utility accelerators. Also, let W be a diagonal matrix with the expenditure proportions being situated on the diagonal. Now, the matrix version of equation seven is:

$$W\Phi = \Phi'W' = \Phi'W$$

If Φ is invertible, then this result can be premultiplied by $(\Phi')^{-1}$ to obtain:

$$(\Phi')^{-1}W = W\Phi^{-1}$$

Hence, the inverse of Φ follows the same symmetry as Φ. Frisch refers to the Φ^{ij} as "want elasticities", and this nomenclature is adopted here. Now, utility maximization requires that $U_i = \lambda p_i$ for every i. If the quantities and λ are regarded as functions in this relation, then differentiation by p_i yields:

$$\sum_t U_{it}(\partial x_t/\partial p_i) = \lambda + p_i(\partial\lambda/\partial p_i)$$

which implies that:

$$\sum_t \Phi_{it}\varepsilon_{ti} = 1 + \Delta_i$$

where Δ_i is the elasticity of λ with respect to p_i. In similar fashion, it may be confirmed that:

$$\sum_t \Phi_{it}\varepsilon_{tj} = \Delta_j$$

and that:

$$\sum_t \Phi_{it}\varepsilon_{tm} = \Delta_m$$

Of the last three equations, the first two are summarized in the matrix expression:

$$\Phi\varepsilon = I + \Delta$$

where ε is the n x n matrix of direct and cross price elasticities. I is an identity matrix, and Δ is an n x n matrix with Δ_i being the element for every row in column i. The matrix variant of the third equation is:

$$\Phi\varepsilon_m = \Delta_m c$$

where ε_m is the column vector of income elasticities, and c is a column vector of ones. The two matrix expressions imply that:

$$\varepsilon = \Phi^{-1} + \Phi^{-1}\Delta$$

and:

$$\varepsilon_m = \Delta_m \Phi^{-1} c$$

The respective scaler representations of these are:

$$\varepsilon_{ij} = \Phi^{ij} + \Delta_j \sum_t \Phi^{it}$$

and:

$$\varepsilon_{im} = \Delta_m \sum_t \Phi^{it} \tag{9}$$

The latter is substituted into the former with the result that:

$$\varepsilon_{ij} = \Phi^{ij} + \Delta_j \varepsilon_{im}/\Delta_m \tag{10}$$

The next step is to solve for Δ_j. This is accomplished by multiplying the latter relation by w_i and summing over i. This produces:

$$\sum_i w_i \varepsilon_{ij} = \sum_i w_i \Phi^{ij} + (\Delta_j/\Delta_m) \sum_i w_i \varepsilon_{im}$$

Cournot aggregation and Engel aggregation are substituted here to obtain:

$$-w_j = \sum_i w_i \Phi^{ij} + \Delta_j/\Delta_m$$

Since $w_i \Phi^{ij} = w_j \Phi^{ji}$, this result may be written:

$$-w_j = w_j \sum_i \Phi^{ji} + \Delta_j/\Delta_m$$

Equation nine is substituted here to obtain:

$$-w_j = w_j \varepsilon_{jm}/\Delta_m + \Delta_j/\Delta_m$$

which implies that:

$$\Delta_j = -w_j \varepsilon_{jm} - \Delta_m w_j$$

This equation is substituted back into equation 10 with the result that:

$$\varepsilon_{ij} = \Phi^{ij} - w_j \varepsilon_{im} \varepsilon_{jm}/\Delta_m - w_j \varepsilon_{im} \tag{11}$$

Slutsky's equation is obtained by evaluating $w_i\varepsilon_{ij} - w_j\varepsilon_{ji}$. In terms of the last result, this becomes:

$$w_i\varepsilon_{ij} - w_j\varepsilon_{ji} = w_i\Phi^{ij} - w_j\Phi^{ji} + w_iw_j(\varepsilon_{jm} - \varepsilon_{im})$$

The symmetry relation in equation eight is substituted here to obtain the final result:

$$w_i\varepsilon_{ij} = w_j\varepsilon_{ji} + w_iw_j(\varepsilon_{jm} - \varepsilon_{im})$$

This derivation of Slutsky's equation employs the assumption that Φ is nonsingular, which is not necessarily the case. However, it can be shown that Slutsky's equation will hold for all quasi-concave utility functions.

Other conditions will follow from the quasi-concavity of the utility function. Each of these would be inequality relations, and would vary in number according to the dimension of the consumption vector. A discussion of these properties can be found in Silberberg (1978) or Phlips (1974).

Properties Of Price-Dependent Demand Functions

While the quantity-dependent function is derived from the case of an individual consumer, the price-dependent function is derived within the context of an aggregate market. First, it is assumed that each individual purchases in a competetive fashion; however, the supply to the aggregate market is assumed to be perfectly inelastic. Therefore, the condition for market equilibrium is that:

$$X_i = \sum_t^q x_{it}(p_1,p_2,,,p_n,m_t); \qquad i = 1,2,,,n$$

where X_i is the fixed supply of commodity i. $x_{it}(p_1,p_2,,,p_n,m_t)$ is the demand function of individual t for commodity i. q is the total number of consumers, and m_t is the income of individual t. At this point, it is assumed that there exists a consumer whose purchasing decisions are identical with market averages. Specifically, the representative consumer is defined as one who will purchase average per-capita quantities when provided with average per-capita income. With such a consumer, the equilibrium conditions may be rewritten with:

$$x_i = x_i(p_1,p_2,,,p_n,m); \qquad i = 1,2,,,n$$

where $x_i(p_1,p_2,,,p_n,m)$ is the demand function of the representative consumer. x_i is the average per-capita availability of commodity i, which is also fixed, and m is now interpreted as average per-capita income. It should be observed that the assumption of a representative consumer is necessary to the quantity-dependent approach as well when dealing with the aggregate market. The theory pertaining to the quantity-dependent function is built upon the assumption of an individual consumer; therefore, if the theory is to extend to the aggregate market, then the behavior of the market must be represented in an individual. Moreover, the same definition of the representative consumer is employed under either approach; therefore, the only difference between the two approaches is in the representation of the final product. The underlying assumptions and behavioral postulations are the same. Now, if the latter system is solved for the reduced form, then the first-order conditions for utility maximization are obtained. That is:

$U_i = \lambda p_i; \qquad i = 1,2,,,n$

$\Sigma\, x_t p_t = m$

where U is now interpreted as being the utility function for the representative consumer. Prices are solved in terms of quantities and income by substituting the latter into the former with the result that:

$(1/\lambda)\Sigma\, x_t U_t = m$

which implies that:

$$\lambda = (1/m)\Sigma\, x_t U_t \tag{12}$$

Next, the last relation is substituted back into $U_i = \lambda p_i$ to produce:

$$p_i = U_i m/\Sigma\, U_t x_t; \qquad i = 1,2,,,n \tag{13}$$

which is the general form for the price-dependent function.

From equation 13, it is apparent that prices are proportional to income; subsequently, any percentage change in income will generate equivalent percentage changes in all prices. Therefore, the first property of the price-dependent function is that the associated income flexibilities are unitary. That is:

$$\gamma_{im} = 1; \qquad i = 1,2,,,n \tag{14}$$

where γ_{im} is the flexibility of p_i with respect to m. Since quantities are constant in the differentiation, the proportionality rules are unaffected; consequently, the relative prices must be constant as well. Therefore, if the budget is to be exhausted, then proportional changes in income must be absorbed through equivalent proportional changes in the prices.

The flexibilities also follow a property similar to the Cournot aggregation condition for elasticities. This is demonstrated by differentiating the budget constrain with respect to x_j to obtain:

$$\sum_t x_t(\partial p_t/\partial x_j) + p_j = 0$$

Here, and in subsequent calculation, prices are regarded as functions of quantities and income. The latter relation implies that:

$$(x_j/m)\sum_t (p_t/p_t)x_t(\partial p_t/\partial x_j) = -p_j x_j/m$$

or:

$$\sum_t w_t \gamma_{tj} = -w_j; \qquad j = 1,2,,,n \tag{15}$$

where γ_{tj} is the flexibility of p_t with respect to x_j.

A third property is analagous to Slutsky's equation inasmuch as it demonstrates a form of symmetry. To begin, the first-order conditions require that $U_i = \lambda p_i$, which implies that:

$$\gamma_{ij} = U_{ij}(x_j/U_i) - \partial\lambda/\partial x_j(x_j/\lambda)$$

or:

$$\gamma_{ij} = \Phi_{ij} - \Theta_j \tag{16}$$

where Φ_{ij} is the utility accelerator, and Θ_j is the flexibility of λ with respect to x_j. As with the prices, λ is treated as a function of quantities and income. Θ_j is solved in terms of the utility accelerators with use of equation 12:

$$\lambda = (1/m)\Sigma\, U_t x_t$$

which implies that:

$$\partial\lambda/\partial x_j = (1/m)(\sum_t U_{tj} x_t + U_j)$$

By the symmetry of cross derivatives, the last result can be written:

$$\partial\lambda/\partial x_j = (1/m)(\sum_t U_{jt} x_t + U_j)$$

which implies that:

$$\partial\lambda/\partial x_j (x_j/\lambda) = (U_j x_j/\lambda m)\{\sum_t U_{jt}(x_t/U_j) + 1\}$$

or:

$$\Theta_j = w_j(\sum_t \Phi_{jt} + 1) \tag{17}$$

Similarly:

$$\Theta_i = w_i(\sum_t \Phi_{it} + 1)$$

and:

$$w_j\Theta_i - w_i\Theta_j = w_i w_j \sum_t (\Phi_{it} - \Phi_{jt})$$

Now, equation 16 implies that:

$$\gamma_{it} - \gamma_{jt} = \Phi_{it} - \Phi_{jt} \tag{18}$$

subsequently, the previous result may be written:

$$w_j\Theta_i - w_i\Theta_j = w_i w_j \sum_t (\gamma_{it} - \gamma_{jt})$$

Equation 16 also implies that:

$$w_i\gamma_{ij} - w_j\gamma_{ji} = (w_i\Phi_{ij} - w_j\Phi_{ji}) + (w_j\Theta_i - w_i\Theta_j)$$

From equation seven, the first term on the right is zero, and substitution of the previous result for the second term yields:

$$w_i\gamma_{ij} = w_j\gamma_{ji} + w_i w_j (\sum_t \gamma_{it} - \sum_t \gamma_{jt}) \tag{19}$$

which is the symmetry relation for flexibilities.

A more general symmetry relation can be derived if the assumption of the budget constraint is abandoned. That is, it is only assumed that the consumer purchases according to the proportionality rule. First, the flexibility of a ratio of prices with respect to a quantity is equal to the difference of the flexibilities for the individual prices. That is:

$$\partial(p_i/p_k)/\partial x_j\,(x_j p_k/p_i) = \gamma_{ij} - \gamma_{kj}$$

and from equation 18:

$$\gamma_{ij} - \gamma_{kj} = \Phi_{ij} - \Phi_{kj}$$

Similarly:

$$\gamma_{ji} - \gamma_{ki} = \Phi_{ji} - \Phi_{ki}$$

With use of the symmetry of the utility accelerator, which is shown in equation seven, it may be confirmed that the last two equations can be combined in:

$$w_i(\gamma_{ij} - \gamma_{kj}) - w_j(\gamma_{ji} - \gamma_{ki}) = (w_i w_j / w_k)(\Phi_{ik} - \Phi_{jk})$$

Finally, equation 18 is substituted here for the right-hand term to obtain:

$$w_i(\gamma_{ij} - \gamma_{kj}) - w_j(\gamma_{ji} - \gamma_{ki}) = (w_i w_j / w_k)(\gamma_{ik} - \gamma_{jk}) \qquad (20)$$

which is the generalized symmetry relation for flexibilities. It is difficult to imagine a theory of exchange under which the consumer would not buy according to the proportionality rule; therefore, the latter result should hold within almost any theoretical context.

Various inequality conditions could also be derived from the quasi-concavity of the utility function. These would vary in number according to the dimension of the consumption vector. The concavity conditions are often described in terms of the indifference curves and surfaces, which are required to be convex to the origin. The indifference curve for any two goods, say x_i and x_j, will be convex to the origin if:

$$\begin{vmatrix} U_{ii} & U_{ij} & -p_i \\ U_{ji} & U_{jj} & -p_j \\ -p_i & -p_j & 0 \end{vmatrix} > 0$$

Linear operations may be performed upon this matrix to obtain the equivalent condition:

$$\begin{vmatrix} \Phi_{ii} - \Phi_{ji} & \Phi_{ij} - \Phi_{jj} & 0 \\ \Phi_{ji} & \Phi_{jj} & 1 \\ w_i & w_j & 0 \end{vmatrix} > 0$$

which implies that:

$$w_j(\Phi_{ji} - \Phi_{ii}) + w_i(\Phi_{ij} - \Phi_{jj}) > 0$$

Equation 18 is substituted here to obtain:

$$w_j(\gamma_{ji} - \gamma_{ii}) + w_i(\gamma_{ij} - \gamma_{jj}) > 0 \tag{21}$$

which is the condition for two-dimensional convexity. Similar conditions could be derived for three or more goods; however, these would be considerably more complex.

Association Between Elasticities And Flexibilities

The implicit association between elasticities and flexibilities is clearly shown in the work of Houck (1966). He demonstrates that a set of flexibilities and the associated properties can be derived from a set of elasticities that follows Slutsky's equation and the homogeneity, Cournot aggregation, and Engel aggregation conditions. Though the conditions for the flexibilities have already been derived, Houck's methodology is reproduced here to demonstrate the correspondence between the two approaches to demand, and to dispel some common misconceptions concerning the relationships between elasticities and flexibilities. As before, the analysis begins with a set of quantity-dependent functions, which are assumed to conform to the usual properties. The total derivatives of the functions are:

$$dx_i = \sum_t (\partial x_i/\partial p_t)dp_t + (\partial x_i/\partial m)dm$$

and if the differentials are replaced with differentials in logs, then these are written:

$$dlnx_i = \sum_t \varepsilon_{it} dlnp_t + \varepsilon_{im} dlnm; \qquad i = 1,2,,,n$$

Second, the latter set of equations are collected into matrices to obtain:

$$dlnX = \varepsilon dlnP + \varepsilon_m dlnm$$

where ε and ε_m are defined as before. dlnX and dlnP are column vectors containing the log differentials in quantities and prices, respectively, and dlnm is a scaler for the log differential in income. If this equation is premultiplied by ε^{-1}, then the price-dependent form is obtained:

$$dlnP = \varepsilon^{-1} dlnX - \varepsilon^{-1}\varepsilon_m dlnm$$

or:

$$dlnP = \gamma dlnX + \gamma_m dlnm$$

where γ is the matrix of direct and cross flexibilities, and γ_m is the column vector of income flexibilities.

It is not difficult to show that the inverse of ε does exist, so that the previous derivation of the flexibilities is general. The proof may be presented in a variety of ways; however, the approach presented here has an advantage in that it yields the specific value for the determinant of ε. First it may be confirmed that the elasticities must follow:

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & -1 \\ \Phi_{21} & \Phi_{22} & -1 \\ -w_i & -w_2 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{11} & \varepsilon_{21} & 0 \\ \varepsilon_{21} & \varepsilon_{22} & 0 \\ \Delta_1 & \Delta_2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ w_1 & w_2 & 0 \end{bmatrix}$$

Here, two goods are assumed; however, the proof readily extends to the n good case. Now, the determinant of the left-hand matrix is necessarily nonzero by the quasi-concavity of the utility function, and it may be confirmed that the determinant of the right-hand matrix is unitary. Since the determinant of the product is equal to the product of the determinants, it follows that the determinant of the center matrix must be nonzero, but it may be easily confirmed that the determinant of this matrix is the same with the determinant of ε. Specifically, if ψ denotes the determinant of the left-hand matrix, then $\det(\varepsilon) = 1/\psi$, and $\det(\gamma) = \psi$.

If the elasticities conform to properties such as homogeneity and Engel aggregation, then it is apparent from the latter calculations that these properties must be imputed to the flexibilities. First, it is known that the elasticities satisfy:

$$\Sigma\, w_t \varepsilon_{tm} = 1$$

$$\sum_t w_t \varepsilon_{tj} = -w_j; \qquad j = 1,2,,,n$$

$$\sum_t \varepsilon_{it} + \varepsilon_{im} = 0; \qquad i = 1,2,,,n$$

$$w_i \varepsilon_{ij} = w_j \varepsilon_{ji} + w_i w_j (\varepsilon_{jm} - \varepsilon_{im})$$

In matrix notation, the respective representations of these are:

$$w'\varepsilon_m = 1 \tag{3'}$$

$$w'\varepsilon = -w' \tag{4'}$$

$$\varepsilon c + \varepsilon_m = 0 \tag{5'}$$

and:

$$W\varepsilon = \varepsilon'W + W(c\varepsilon_m' - \varepsilon_m c')W \tag{6'}$$

where w is the column vector of expenditure proportions. c is a column vector of ones, and W is a diagonal matrix with the expenditure proportions being situated on the diagonal. These four conditions may be manipulated with linear operations to determine the properties that are imputed to the flexibilities.

The condition for unitary income flexibilities is a consequence of the homogeneity condition for the elasticities. This is shown in equation five prime, which implies that:

$$-\varepsilon^{-1}\varepsilon_m = c \tag{22}$$

but it has been shown that $\gamma_m = -\varepsilon^{-1}\varepsilon_m$; therefore:

$$\gamma_m = c$$

In scalers, this expression becomes equation 14:

$$\gamma_{im} = 1; \qquad i = 1,2,,,n$$

The columns in both the elasticity and flexibility matrices must satisfy certain weighted sums. For the elasticities, this is the Cournot aggregation condition. For the flexibilities, the condition is shown in equation 15, which is virtually identical with Cournot aggregation; moreover, equation four prime implies that either condition is imputed by the other. Postmultiplication of four prime by the inverse of ε yields:

$$w' = -w'\varepsilon^{-1}$$

However, $\gamma = \varepsilon^{-1}$; subsequently:

$$w'\gamma = -w'$$

In scaler notation, the latter becomes equation 15:

$$\sum_t w_t\gamma_{tj} = -w_j; \qquad j = 1,2,,,n$$

The symmetry relation for the flexibilities can be derived from equation six prime. To begin, six prime is premultiplied and postmultiplied by W^{-1} to obtain:

$$\varepsilon W^{-1} = W^{-1}\varepsilon' + c\varepsilon_m' - \varepsilon_m c'$$

This result is then premultiplied by ε^{-1} and postmultiplied by $(\varepsilon')^{-1}$ to produce:

$$W^{-1}(\varepsilon')^{-1} = \varepsilon^{-1}W^{-1} + \varepsilon^{-1}c\varepsilon_m'(\varepsilon')^{-1} - \varepsilon^{-1}\varepsilon_m c'(\varepsilon')^{-1}$$

At this point, ε^{-1} is replaced with γ, and equation 22 is substituted for the two right-hand terms with the result that:

$$W^{-1}\gamma' = \gamma W^{-1} + cc'\gamma' - \gamma cc'$$

This equation is premultiplied and postmultiplied by W, and terms are rearranged to obtain the final result:

$$W\gamma = \gamma'W + W(\gamma cc' - cc'\gamma')W$$

This is represented in scalers with equation 19:

$$w_i \gamma_{ij} = w_j \gamma_{ji} + w_i w_j (\sum_t \gamma_{it} - \sum_t \gamma_{jt})$$

which is the symmetry relation for flexibilities. Hence, symmetry in the flexibilities is imputed by Slutsky's equation and homogeneity in the elasticities.

Perhaps the most important implication of these results is that flexibilities are not generally equal to the reciprocals of the associated elasticities, as commonly supposed. Inasmuch as the elements in a matrix are not usually equal to the reciprocals of the elements in the corresponding inverse, the reciprocals of the elasticities will not generally be equal to the associated flexibilities, nor are they necessarily good approximations. This relationship can exist under only two conditions. The first case occurs when the commodity bundle is composed of only one good. The second case occurs when all cross elasticities are equal to zero. Neither case is apt to be common; moreover, the latter case is not implied by additive utility, as commonly imagined. From equation 11, it is evident that when the cross utility accelerators are equal to zero, the cross elasticities are, nonetheless, nontrivial through income effects.

Price-Dependence Under Special Forms Of The Utility Function

Relevance Of Special Forms

It is often expedient to adopt particular functional specifications or restricted functional forms of utility, either for the analytical or empirical convenience that such forms may render. In these cases, the previously derived properties for the flexibilities will still pertain, since in each derivation, a most general form of the utility function has been employed. However, the particular specifications of utility will determine that either the previously derived properties can be simplified, or that additional properties can be obtained.

Homothetic functions and functions that follow some form of separability are of particular interest to practical analysis. The appeal of the homothetic function is the ease with which it is examined. On the other hand, if it is known that the various commodities posses some sort of separability in the preference structure, then this information may be exploited to obtain theoretical results that are more conformable with empirical limitations. A problem with the general form of utility is that the associated demand functions show prices to be functions of every quantity, or quantities to be functions of every price. Unfortunately, in an actual market situation, the number of commodities is apt to range up to the hundreds if not thousands; consequently, the efficient estimation of demand parameters becomes not only numerically impractical, but statistically impossible. The problem can be partially circumvented either by aggregating the data, or by simply ignoring remote variables; however, there will generally be adverse statistical consequences with both methods. Therefore, the theoretical advantages of generality are overwhelmed by the empirical impracticalities, and the adoption of separable forms of utility becomes one of the few feasible alternatives. The advantage of the separable function is that the associated theory often allows for the estimation of most demand parameters with consideration given only to certain subsets of the commodities.

Within the price-dependent framework, the implications of both separable and homothetic functions are easily examined; moreover, the advantages of these functions can be fully exploited to expedite the empirical analysis.

Symmetry Under Homotheticity

The effect of homotheticity in utility is to simplify the symmetry relation. The homothetic function is of the form, $U = T\{F(x_1,x_2,,,x_n)\}$, where F is homogeneous of degree one. From equation 13, the implied price-dependent demand functions are:

$$p_i = T'F_i m/T'\Sigma\ F_t x_t; \qquad i = 1,2,,,n$$

or:

$$p_i = F_i m/\Sigma\ F_t x_t$$

By Euler's equation, the denominator of this expression is simply equal to $F(x_1,x_2,,,x_n)$; therefore:

$$p_i = F_i m/F$$

which implies that:

$$\partial p_i/\partial x_j = F_{ij} m/F - F_i F_j m/F^2$$

which is symmetrical in i and j; moreover:

$$1/m(p_i/p_i)x_i x_j(\partial p_i/\partial x_j) = 1/m(p_j/p_j)x_i x_j(\partial p_j/\partial x_i)$$

or:

$$w_i \gamma_{ij} = w_j \gamma_{ji}$$

which is the symmetry relation for homothetic functions.

Another implication of homotheticity is that the rows in the flexibility matrix must sum to negative one. This is demonstrated by summing the symmetry relation over j. The summation produces:

$$w_i \sum_j \gamma_{ij} = \sum_j w_j \gamma_{ji}$$

The weighted column sum condition in equation 15 is substituted here, and terms are rearranged to yield:

$$\sum_j \gamma_{ij} = -1; \qquad i = 1,2,,,n$$

Since the income flexibilities are unitary, the latter result indicates that the price-dependent functions are homogeneous of degree zero under homothetic utility.

It is not difficult to confirm that the elasticities must follow an identical symmetry relation; moreover, if the rows in the flexibility matrix sum to negative one, then the rows in the elasticity matrix must do the same. Also, it reasonably follows that the income elasticities must be unitary under homothetic utility, for if the rows in the elasticity matrix sum to negative one, then by the homogeneity condition for elasticities, the income elasticities are necessarily equal to one.

Additional Properties Under Separability

With separable utility, the various items within the commodity basket can be segregated into subsets where the commodities of differing subsets posses some degree of independence. The most general case of separability occurs with the weakly separable function, which is of the form, $U = F\{G^1(X^1), G^2(X^2),,, G^g(X^g)\}$, where X^i denotes subset i. That is:

$$X^i = (x_{n_1+n_2+\ldots+n_{i-1}+1}, x_{n_1+n_2+\ldots+n_{i-1}+2},,, x_{n_1+n_2+\ldots+n_i})$$

where n_i is the number of commodities in group i, and:

$$\sum_i^g n_i = n$$

Other types of separability will be special cases of this general form. For example, there is the case of strong separability under which the subfunctions are block-additive. Here, the general form is $U = F\{G^1(X^1) + G^2(X^2) +\ldots+ G^g(X^g)\}$. The most extreme case of separability occurs with the pointwise separable function, which is the special case of strong separability where the subsets are composed of but one commodity. The general form of this function is $U = F\{G_1(x_1) + G_2(x_2) +\ldots+ G_n(x_n)\}$.

Under any form of separability, the flexibilities will satisfy the property that if x_i and x_k are of the same group, and if x_j is taken from a second group, then $\gamma_{ij} = \gamma_{kj}$. That is:

$$\gamma_{ij} = \gamma_{kj}; \qquad i,k \in a;\ j \notin a \tag{23}$$

where the subset is denoted by a, and the commodities are denoted by i, j, and k. This result is demonstrated by observing that the accelerators for the weakly separable function are of the general form:

$$\Phi_{ij} = F_{ab}G^b_j x_j/F_a + G^a_{ij}x_j/G^a_i; \qquad i \in a,\ j \in b$$

Now, if $a \neq b$, then the right-hand term vanishes; consequently:

$$\Phi_{ij} = F_{ab}G_j^b x_j/F_a; \qquad i \in a,\ j \in b,\ a \neq b$$

From this relation, it is evident that if x_k is also taken form group a, then $\Phi_{ij} = \Phi_{kj}$; therefore:

$$\Phi_{ij} = \Phi_{kj}; \qquad i,k \in a;\ j \notin a$$

From equation 18:

$$\gamma_{ij} - \gamma_{kj} = \Phi_{ij} - \Phi_{kj}$$

which is combined with the previous result to produce equation 23.

Equation 23 is a general result; however, it may be extended when the utility function is either strongly or pointwise separable. First, it should be observed that with a function of the form, $U = F\{G^1(X^1) + G^2(X^2) + \ldots + G^g(X^g)\}$, F effectively becomes a monotonic transformation. In previous calculations, it has not been necessary to attatch special significance to the ordinal property. Since elasticities and flexibilities are invariant under alternative transformations of utility, the properties that are derived for these parameters under any one function must also pertain to all of its ordinal equivalents. Consequently, in deriving the flexibilities under strong separability, F may be safely ignored, and the utility accelerators may be defined in terms of the subfunctions with:

$$\phi_{ij} = G_{ij}^a(x_j/G_i^a); \qquad i \in a \tag{24}$$

Likewise, the flexibility of λ with respect to x_j is also defined in terms of the subfunction accelerators with:

$$\theta_j = w_j(\sum_t \phi_{jt} + 1) \tag{25}$$

which is similar to equation 17, but with Φ_{jt} being replaced with ϕ_{jt}. Henceforth, when dealing with separable functions, the parameters of the subfunctions will be denoted in the lower case to distinguish them from the parameters of U. With these new definitions, equation 16 may be rewritten as:

$$\gamma_{ij} = \phi_{ij} - \theta_j \tag{26}$$

Now, it is obvious from equation 24 that if x_i and x_j are from different strongly separable groups, then ϕ_{ij} is equal to zero; consequently:

$$\gamma_{ij} = -\theta_j; \qquad j \in a,\ i \notin a$$

Also, if x_k is not from group a, then:

$$\gamma_{kj} = -\theta_j; \qquad j \in a,\ k \notin a$$

but the last two results imply that:

$$\gamma_{ij} = \gamma_{kj}; \qquad j \in a;\ i,k \notin a \tag{27}$$

which is simply an extension of equation 23. In both cases, it is required that x_j be of a different group than x_i and x_k; however, x_i and x_k are required to be of the same group in the previous equation, but not in the latter. Since pointwise separability is the special case of strong separability where each commodity comprises a group, the rule here is obtained with a slight modification of equation 27; namely:

$$\gamma_{ij} = \gamma_{kj}; \qquad i,k \neq j$$

Substitutability And Complementarity

The price-dependent approach to demand can yield simple and appealing measures of substitutability and complementarity. The traditional quantity-dependent measures all seem to fall short in that they are not entirely consistent with the concepts under consideration. The general idea embracing substitutability and complementarity is that many pairs of goods are related in such a way that the consumption of one will tend to affect the consumer's valuation of the other. Since the concern of substitutability and complementarity is with the impact of consumption upon valuation, the measurement of these concepts in terms of quantity-dependent parameters is necessarily awkward. With the quantity-dependent function, there is a description of how the consumer will react to a set of values that are prescribed by the market; however, nothing is said of how those values are determined, or of how they are affected by changes in consumption. Consequently, quantity-dependent parameters can measure substitutability and complementarity only in an indirect fashion. With price-dependent functions, quantities are given, and valuation is then determined; therefore, it reasonably follows that the two concepts are best measured in terms of price-dependent parameters.

The most common of the quantity-dependent measures have been the elasticities and some of the substitution effects contained therein. These substitution effects are divided in the partial derivative form of equation 11:

$$\varepsilon_{ij} = \Phi^{ij} - w_j \varepsilon_{im} \varepsilon_{jm} / \Delta_m - w_j \varepsilon_{im}$$

The derivative form is derived by multiplying this relation by x_i/p_j. The multiplication yields:

$$\partial x_i/\partial p_j = \lambda U^{ij} - m(\partial x_i/\partial m)(\partial x_j/\partial m)/\Delta_m - x_j(\partial x_i/\partial m)$$

where U^{ij} is an element in the inverse of the Hessian matrix for U. The "income effect" is $-x_j(\partial x_i/\partial m)$, and is always negative for normal goods. The "total substitution effect" is:

$$\partial x_i^u/\partial p_j = \lambda U^{ij} - m(\partial x_i/\partial m)(\partial x_j/\partial m)/\Delta_m$$

This expression is further divided into the "specific substitution effect", λU^{ij}, and the "general substitution effect", which is $-m(\partial x_i/\partial m)(\partial x_j/\partial m)/\Delta_m$. As measures of substitutability and complementarity, the most popular of these have been the specific and total substitution effects. Also, substitutes and complements are often identified by the elasticity variants of these effects, which are the same in sign with the partial derivative forms. The elasticity version of the specific effect is simply the want elasticity, Φ^{ij}. The elasticity form of the total effect is denoted by ε_{ij}^u, and is defined with:

$$\varepsilon_{ij}^u = \Phi^{ij} - w_j \varepsilon_{im} \varepsilon_{jm}/\Delta_m$$

The ε_{ij}^u are commonly called "constant utility elasticities." It can be demonstrated that these will result from the dual problem to utility maximization. The objective under the dual is:

$$\text{minimize: } \Sigma\, p_t x_t + \mu\{U - U(x_1, x_2,,, x_n)\}$$

Therefore, any given level of utility is attained at minimum cost. The resulting demand functions are:

$$x_i = x_i^u(p_1, p_2, , , p_n, U); \qquad i = 1, 2, , , n$$

which have the ε_{ij}^u for their elasticities with respect to the prices.

Although these measures have enjoyed considerable popularity, they have also been justly criticized on numerous occasions. For example, Phlips (1974) notes that the specific substitution effects are not invariant under various transformations of utility, and that the total substitution effects are generally biased towards substitutability. Pearce (1964) also objects on several points. However, the one objection here is that in every case, value is taken as given. The concern of substitutability and complementarity is with changes in value that are generated by changes in consumption, and not vice-versa. Yet, each of these parameters are measures of various consumption responses that are induced by predetermined changes in value.

If a cardinal significance were required of the utility function, then there would be little problem with the measurement of substitutability and complementarity. In such cases, the marginal utilities become absolute measures of the consumer's valuation, and the impact of any one good upon the value of another could be measured simply by examining the derivatives of the marginal utility. However, if F is a utility function, and if all monotonic increasing transformations of F are regarded as acceptable representations of utility, then the marginal utilities of the ordinal function are of the form:

$$U_i = T'F_i$$

Since T is an arbitrary transformation, it becomes apparent from the latter relation that the marginal utilities are indeterminate with respect to absolute magnitude, and are relevant only with respect to their various proportionalities. Consequently, the assumption of ordinal utility has deprived the theory of a primitive rule of value. Instead, any one good must be valued in terms of another according to the relative sizes of the respective marginal utilities. For example, if U_i is two times larger than U_j, then x_i has twice the value of x_j; however, there is no absolute rule whereby the two commodities can be valuated independently.

When value is reduced to a totally relative concept, it necessarily follows that substitutability and complementarity must be somewhat relative in nature as well. Therefore, it is not strictly proper to speak of one good as being a substitute or complement to another without making reference to the assumed unit of account. For example, if $\gamma_{ij} > 0$, then x_j is a complement to x_i relative to income, since the value of x_i is increased by the consumption of x_j when income is the unit of account. The reference to the unit of account is obviously important, seeing that the classification of the two goods could be changed with a different basis of value. For example, suppose that x_k is selected as the unit of account, and let the value of x_i be denoted by p_i^k where:

$$p_i^k = p_i/p_k$$

Furthermore, let γ_{ij}^k denote the flexibility of p_i^k with respect to x_j. Henceforth, γ_{ij}^k is called the "relative flexibility" to distinguish it from γ_{ij}. Now, if $\gamma_{ij}^k > 0$, then x_j is a complement to x_i relative to x_k.

However, it is quite possible that $\gamma_{ij} > 0$, but $\gamma_{ij}^{k} < 0$. Therefore, when goods are classified as being substitutes or complements, reference should be made to the basis of value under which the classifications are made.

The intuitive appeal of the relative flexibility, as a measure of substitutability and complementarity, is greatest when an independent good is selected for the numeraire. For example, suppose that x_k is regarded as being independent of x_j. If $\gamma_{ij}^{k} < 0$, then x_j is a stronger substitute for x_i than for an independent good. With this interpretation, the relative flexibility seems to be more consistent with the intuitive understanding of the concepts. However, there is some degree of subjectivity with this approach, for independence is interpreted here in the absolute sense, and not the relative. If x_j and x_k are independent, and if $\gamma_{ij}^{k} = 0$, then x_i must also be independent of x_j, but this relative method obviously cannot be used to confirm the independence between x_j and x_k. Therefore, the selection of the numeraire must be arbitrary to some extent. However, the designation of an independent numeraire should be limited by at least one objective rule; namely, if x_h and x_k are both independent of x_j, then $\gamma_{ij}^{h} = \gamma_{ij}^{k}$. That is, the extent of substitutability or complementarity relative to one independent good should be no different than when measured in terms of another. This condition is the same with the condition that $\gamma_{kj}^{h} = 0$, which is a reasonable result, for if $\gamma_{kj}^{h} \neq 0$, then x_j would have to be either a relative substitute or complement to x_k. This condition is obviously necessary if identical conclusions concerning substitutability and complementarity are to be obtained under different choices of the numeraire. Barten (1971) has also suggested a similar requirement of independent goods, and has proposed a similar measure of substitutability and complementarity. Barten's methods can also be found in Phlips (1974).

If the utility function is of the form, $U = F\{G^1(X^1) + G^2(X^2) + \ldots + G^g(X^g)\}$, then strongly separable goods are obvious candidates for the numeraire. If x_k and x_h are both strongly separable from x_j, then by definition, $\gamma^h_{kj} = 0$; therefore, strongly separable goods are consistent with the independence criterion. Of course, this does not prove that strongly separable goods are independent, since the condition that $\gamma^h_{kj} = 0$ is a necessary consequence of independence, and not a sufficient condition thereof. On the other hand, weak separability cannot constitute independence, since γ^h_{kj} is not necessarily equal to zero if x_h and x_k are from differing weakly separable groups.

When the numeraire good in p^k_i is strongly separable from x_i, there are certain properties that follow in the associated relative flexibilities. The first of these is that:

$$\gamma^k_{ij} = 0; \qquad i \in a;\ k \in b;\ j \notin a,b \tag{28}$$

which is consistent with the proposition that strongly separable goods are independent. Moreover, if x_j is taken from the same group as x_i, then:

$$\gamma^k_{ij} = G^a_{ij}(x_j/G^a_i); \qquad i,j \in a;\ k \notin a \tag{29}$$

or from equation 24:

$\gamma^k_{ij} = \phi_{ij}; \qquad i,j \in a;\ k \notin a$

Therefore, the relative flexibilities become the accelerators on the strongly separable subfunctions. These are known to satisfy:

$w_i\phi_{ij} = w_j\phi_{ji}$

Therefore:

$$w_i\gamma^k_{ij} = w_j\gamma^k_{ji}; \qquad i,j \in a;\ k \notin a \tag{30}$$

A third property results if x_j is taken from the same group as the numeraire. In this situation:

$\gamma^k_{ij} = -\phi_{kj}; \qquad j,k \in b;\ i \notin b$

Since the latter relation is invariant with respect to x_i, as long as x_i is not a component of the group containing the numeraire, it follows that:

$$\gamma^k_{ij} = \gamma^k_{hj}; \qquad j,k \in b;\ i,h \notin b \tag{31}$$

Therefore, the flexibilities of p^k_i and p^k_h with repect to x_j are equal if x_j and x_k are from the same group.

The Derivation Of Elasticities And Flexibilities From Relative Flexibilities Under Strong Separability

If the utility function is of the form, $U = F\{G^1(X^1) + G^2(X^2) + \ldots + G^g(X^g)\}$, then the flexibilities of certain relative prices will be equivalent to the accelerators corresponding to the block-additive representation of utility. It has been demonstrated in equation 29 that if x_i and x_j are of the same strongly separable group, and if x_k is of a different group, then γ^k_{ij} is equal to the accelerator on the subfunction containing x_i and x_j. That is:

$$\gamma^k_{ij} = G^a_{ij}(x_j/G^a_i); \qquad i,j \in a;\ k \notin a$$

Therefore, the relative flexibilities provide a means of obtaining the accelerators for one of the infinite representations of utility. Moreover, the particular representation produced by the relative flexibilities invariably corresponds to the block-additive transformation of utility. Because of these properties, the relative flexibilities provide a convenient means of estimating elasticities and flexibilities when the utility function is strongly separable. Although this is an indirect approach, there are at least two empirical considerations that could make this method advantageous. First, there are many experimental situations where elasticities or flexibilities are needed only for a particular commodity group. Under these circumstances, the usage of either quantity-dependent or price-dependent functions becomes unnecessarily burdensome, since such functions have the entire commodity set for their domain. However, it has been demonstrated that relative prices under strong separability are functions only of those commodities contained in certain subsets of the commodity basket. Consequently, the estimation of relative price functions requires less data, and can be accomplished with fewer degrees of freedom. Moreover, if estimates are available for all the relative flexibilities between the items of a particular group, then the flexibilities for that group can be calculated with use of the expenditure proportions, and the elasticities can be derived with the expenditure proportions and an a-priori estimate of Δ_m. This assumes that the numeraire commodity is of a different group than the group of concern. Therefore, if the elasticities and flexibilities are needed for a particular group, say group a, then these can be estimated with: 1) the expenditure proportions for the items in group a, 2) an a-priori estimate of Δ_m, 3) data for the commodities in group a, and 4) data for the group containing the numeraire. Thus, the data requirements can be reduced considerably with the relative flexibility approach. A second advantage with this approach occurs when the experiment calls for the measurement of substitutability and complementarity. As shown before, the relative flexibilities provide excellent measures of these concepts if strongly separable goods are interpreted as being independent.

Flexibilities for a particular commodity group can be easily calculated if all of the direct and cross relative flexibilities between the items within that group are available. From equation 26, the flexibilities are determined with:

$$\gamma_{ij} = \phi_{ij} - \theta_j$$

where ϕ_{ij} and θ_j correspond to the block-additive representation of utility. Moreover, from equation 25:

$$\theta_j = w_j(\sum_t \phi_{jt} + 1)$$

However, under the block-additive transformation, ϕ_{jt} is equal to zero if x_j and x_t are of differing separable groups. Therefore, if x_j is from group a, then:

$$\theta_j = w_j(\sum_{t\varepsilon a} \phi_{jt} + 1); \qquad j \; \varepsilon \; a$$

This relation is substituted into equation 26 to obtain:

$$\gamma_{ij} = \phi_{ij} - w_j(\sum_{t\varepsilon a} \phi_{jt} + 1); \qquad j \; \varepsilon \; a$$

Next, the relative flexibilities are substituted for the accelerators to produce:

$$\gamma_{ij} = \gamma^k_{ij} - w_j(\sum_{t\varepsilon a} \gamma^k_{jt} + 1); \qquad j \; \varepsilon \; a, \; k \; \varepsilon \; b, \; i \not\varepsilon \; b, \; a \neq b \tag{32}$$

which is the relationship between flexibilities and relative flexibilities under strong separability. Similarly, if the utility function is pointwise separable, then the conversion is accomplished with:

$$\gamma_{ij} = \gamma^k_{ij} - w_j(\gamma^k_{jj} + 1); \qquad i,j \neq k$$

The elasticities for a particular commodity group can be calculated from the relative flexibilities if estimates are available for Δ_m and the expenditure proportions for the commodities in the group. First, from equation nine:

$$\varepsilon_{im} = \Delta_m \sum_t \Phi^{it}$$

which is replaced with:

$$\varepsilon_{im} = \delta_m \sum_t \phi^{it} \tag{33}$$

where δ_m is the flexibility of money under the block-additive transformation of utility. Likewise, the want elasticity is replaced with ϕ^{it}. Since the ϕ^{it} are calculated from the block-additive transformation, they shall henceforth be referenced as "block-additive want elasticities." Now, the matrix of utility accelerators is block-diagonal under the additive transformation of utility; subsequently, the matrix of block-additive want elasticities must be block-diagonal as well, since the latter is simply the inverse of the former. Therefore, in the latter relation, it is only necessary to sum over the group containing x_i. If x_i is an element in group a, then:

$$\varepsilon_{im} = \delta_m \sum_{t \varepsilon a} \phi^{it}; \qquad i \varepsilon a$$

In terms of the relative flexibilities, this equation becomes:

$$\varepsilon_{im} = \delta_m \sum_{t \varepsilon a} \gamma_k^{it}; \qquad i \varepsilon a,\ k \not\varepsilon a \tag{34}$$

where the γ_k^{it} are obtained by inverting the matrix of direct and cross relative flexibilities between the commodities in group a. Once the income elasticities are calculated, the direct and cross elasticities are derived with use of equation 11:

$$\varepsilon_{ij} = \Phi^{ij} - w_j \varepsilon_{im} \varepsilon_{jm} / \Delta_m - w_j \varepsilon_{im}$$

Here, Φ^{it} is replaced with the γ_k^{ij}, and δ_m is substituted for Δ_m to produce:

$$\varepsilon_{ij} = \gamma_k^{ij} - w_j \varepsilon_{im} \varepsilon_{jm} / \delta_m - w_j \varepsilon_{im}; \qquad i \varepsilon a,\ k \varepsilon b,\ j \not\varepsilon b,\ a \neq b \tag{35}$$

which is the relation from which the direct and cross elasticities are derived. In the case of pointwise separability, the income elasticities are calculated with:

$$\varepsilon_{im} = \delta_m / \gamma_{ii}^{k}; \qquad i \neq k$$

The direct and cross elasticities are then derived from:

$$\varepsilon_{ii} = 1/\gamma_{ii}^{k} - w_i \varepsilon_{im} \varepsilon_{im} / \delta_m - w_i \varepsilon_{im}; \qquad i \neq k$$

and:

$$\varepsilon_{ij} = -w_j \varepsilon_{im} \varepsilon_{jm} / \delta_m - w_j \varepsilon_{im}; \qquad i \neq j$$

In all the derivations above, δ_m was ultimately needed to calculate both the income and price elasticities. Unfortunately, the calculation of δ_m will require knowledge of the utility accelerators for every commodity group. This can be demonstrated from equation 33:

$$\varepsilon_{im} = \delta_m \sum_t \phi^{it}$$

This relation is multiplied by w_i and summed over i with the result that:

$$\sum_i w_i \varepsilon_{im} = \delta_m \sum_i \sum_t w_i \phi^{it}$$

Engel aggregation is substituted here, and terms are rearranged to yield:

$$\delta_m = 1/(\sum_i \sum_t w_i \phi^{it})$$

Hence, the calculation of δ_m will require the entire set of utility accelerators for the block-additive representation of utility. This poses a severe hindrance to those studies that are concerned only with the elasticities for a particular commodity group. Under such situations, the most expedient approach will probably be to aggregate the commodity set into pointwise separable quantities. If the commodities are pointwise separable, then the latter relation becomes:

$$\delta_m = 1/\sum_t w_t/\phi_{tt}$$

where the ϕ_{tt} are now interpreted as being the accelerators for the aggregate commodities. The cross accelerators are deleted, since these are known to be equal to zero under the additive transformation of the pointwise separable function. If there are k commodities, and if x_k is the numeraire, then:

$$d\ln p_i^k = \gamma_{ii}^k d\ln x_i + \gamma_{ik}^k d\ln x_k; \qquad i = 1,2,,,k-1$$

where:

$$\gamma_{ii}^k = \phi_{ii}$$

and:

$$\gamma_{ik}^k = -\phi_{kk}$$

Subsequently, δ_m can be calculated with:

$$\delta_m = 1/\{(\sum_t^{k-1} w_t/\gamma_{tt}^k) - w_k/\gamma_{ik}^k\}; \qquad i \neq k \tag{36}$$

x_i can be any good other than x_k, since from the previous relation, it is implied that:

$$\gamma^k_{ik} = \gamma^k_{jk}; \qquad i,j \neq k$$

This condition could be imposed as a restriction in the estimation, so that the k - 1 estimates of $-\phi_{kk}$ would all be equal. After having obtained an estimate of δ_m with this method, one could then proceed to calculate the elasticities for the disaggregated commodities in the group of concern.

Properties Of Relative Flexibilities Under General Utility

The relative flexibilities are of greatest interest when the utility function is strongly separable; however, there are some applications where they could be useful under other forms of the utility function as well. These applications include structural stability tests, and the tests of certain theoretical propositions. Also, the relative flexibilities could provide useful measures of relative substitutability and complementarity.

The assumptions of the theory imply at least two conditions for the relative flexibilities under general utility. The first of these is derived by observing that:

$$\gamma^k_{ij} = \gamma_{ij} - \gamma_{kj} \tag{37}$$

Also, from equation 20:

$$w_i(\gamma_{ij} - \gamma_{kj}) - w_j(\gamma_{ji} - \gamma_{ki}) = (w_i w_j / w_k)(\gamma_{ik} - \gamma_{jk})$$

The former is substituted into the latter to yield:

$$w_i\gamma_{ij}^k - w_j\gamma_{ji}^k = (w_iw_j/w_k)(\gamma_{ik}^k - \gamma_{jk}^k) \tag{38}$$

which is the symmetry relation for relative flexibilities. Since this equation assumes nothing other than the proportionality rule, the test of this condition is the test of proportionality. The second condition is derived from equation 21:

$$w_j(\gamma_{ji} - \gamma_{ii}) + w_i(\gamma_{ij} - \gamma_{jj}) > 0$$

Equation 37 is substituted here with the result that:

$$w_j(\gamma_{ji}^k - \gamma_{ii}^k) + w_i(\gamma_{ij}^k - \gamma_{jj}^k) > 0 \tag{39}$$

This condition is a consequence of the quasi-concavity of the utility function; therefore, the test of this condition is the same with the test of two-dimensional convexity in the indifference curves. Other conditions could be derived for convexity in greater dimensions.

Of course, there is nothing to prevent the validity of these tests under separability; however, the implications of the particular form of separability should be considered. For example, if the utility function is weakly separable, then equation 23 indicates that:

$$\gamma_{ik} = \gamma_{jk}; \qquad i,j \in a;\ k \notin a$$

Equation 37 is substituted here with the result that:

$$\gamma_{ik}^k = \gamma_{jk}^k; \qquad i,j \in a;\ k \notin a$$

This equation implies that the symmetry relation can reduce to:

$$w_i\gamma_{ij}^k = w_j\gamma_{ji}^k; \qquad i,j \in a;\ k \notin a$$

Since all forms of separability are special cases of weak separability, this is a general result for separable functions. Also, if the utility function is strongly separable, then from equation 28:

$$\gamma_{ij}^{k} = 0; \qquad i \in a;\ k \in b;\ j \notin a,b$$

Subsequently, the two-dimensional convexity relation can reduce to:

$$w_j\gamma_{ii}^{k} + w_i\gamma_{jj}^{k} < 0; \qquad i \in a;\ k \in b;\ j \notin a,b$$

Perhaps the most useful application of the relative flexibility is in tests of structural stability. Since the relative price, p_i^k, is equal to the slope of the indifference curve between x_i and x_k, the flexibilities of p_i^k determine the shape of the indifference curve. Therefore, if nonlinearities are ignored, then shifts in the relative flexibilities are synonymous with structural shifts in utility. This sort of structural test could be particularly useful if x_i and x_k were of the same weakly separable group, for in such cases, p_i^k can be analyzed with use of only those commodities in the group containing x_i and x_k.

CHAPTER III

AN APPLICATION OF PRICE-DEPENDENT METHODS TO MEATS

General Procedures And Description Of Data

Elasticities, flexibilities, and relative flexibilities were estimated by various methods for the major U.S. meats. The estimates are presented in this chapter, and comparisons are made between the results of the various methods. Also, the results of statistical tests for several of the theoretical propostions are presented

The analysis was conducted for the consumption of beef, pork, and chicken. All data are quarterly, beginning with the first quarter of 1965 and ending with the last quarter of 1980. The meats were selected for analysis, because the supply of each is probably extremely inelastic within any one quarter; subsequently, the price-dependent methods can be examined within the partial equilibrium framework.

All quantities for the meats include both fresh and processed consumption on the per-capita basis. All were calculated in terms of retail pound equivalents. The beef and pork data were taken from Meat And Livestock Situation. No distinction is made between beef and veal in the analysis; subsequently, the beef quantity variable is the sum of reported beef and veal consumption. The chicken consumption data were taken from Poultry And Egg Situation, and include the consumption of both broilers and other chicken.

All meat price variables are expressed in cents per retail pound. The beef price variable is a weighted average of the prices for veal and choice grade beef. The weights were .034 for veal and .966 for beef. These were based upon quantities consumed in the years 1967 through 1969, which was the period from which weights were calculated for the various cuts included within the individual prices. All quarterly prices were calculated as simple averages of the monthly prices within the quarter. Monthly prices for beef, veal, and pork were obtained from Meat And Livestock Situation. The price for grade A broilers was used as the chicken price variable. Broiler prices were obtained from Poultry And Egg Situation.

Other variables used in the analysis included per-capita income, and prices and quantities for aggregate food and nonfood consumption. Income was measured by per-capita nominal disposable personal income, as measured by Survey Of Current Business. Prices indices for food and nonfood consumption were taken from the Handbook Of Labor Statistics. Quantities for food and nonfood consumption were calculated by dividing the per-capita personal consumption expenditures for each by the respective price indices. The personal consumption expenditures for food and nonfood were obtained from the Survey Of Current Business. All data used in the study were unadjusted for seasonality except for disposable personal income and the personal consumption expenditures. The usage of unadjusted data would have been preferred for these, but was prevented by lack of availability. Aside from these, the only other variables used in the analysis were relative prices for the meats and aggregate food.

In most cases, nonfood was used as the numeraire. In all cases, relative prices were calculated as the averages of monthly ratios, rather than the ratios of monthly averages.

Expenditure proportions were frequently required throughout the analysis. These were obtained from the 1972 - 1973 consumer expenditure survey, which is summarized in the Handbook Of Labor Statistics. The survey results included the expenditures for poultry, but not for the individual components thereof. To obtain expenditures for chicken and turkey, price and quantity data from Poultry And Egg Situation were used to calculate total expenditures for chicken and turkey in the period, 1972 through 1973. The respective proportions of this measure represented by chicken and turkey were then used to disaggregate the poultry expenditure measure reported in the survey. These proportions were calculated at .7885 for chicken and .2115 for turkey.

Most parameter estimates reported in this study were calculated from ordinary least squares estimators; however, statistical test results are often reported for both ordinary and generalized least squares estimates. In most cases, ordinary least squares estimates were more agreeable with expectation; subsequently, these were used in all calculations and comparisons other than statistical tests. The theoretical propositions were tested in terms of both ordinary and generalized estimates in order to add rigor to the testing process. In every case, generalized least squares estimation was conducted with the assumption of a homoskedastistic but autoregressive error of the first order, and the estimates were calculated with the Cochrane-Orcutt two-step method. Also, the correlations of errors in different equations were always assumed to be zero. Log-linear functions were uniformly applied throughout the analysis. This functional form was selected because of the convenience with which it is analyzed, and was uniformly applied for consistency in methodology.

Quantity-Dependent And Price-Dependent Estimates

Direct Estimation Of Elasticities

Since the supply for each of the three meats is almost completely predetermined to the quarterly equilibria, the legitimacy of the quantity-dependent approach for market data is questionable; nevertheless, quantity-dependent functions were estimated for a comparison of the resulting elasticity estimates with those obtained under the other approaches. Henceforth, x_b, x_p, and x_c shall denote the logs of the quantities for beef, pork, and chicken, respectively. p_b, p_p, and p_c shall denote the logs of the corresponding prices, and m shall represent the log of income. Each of the quantities was fitted to the three prices and income to obtain the elasticity estimates that are reported in Table I. The direct elasticities are consistent with the general notion that foods are inelastic goods. As for the cross elasticities, one would normally expect all of these to be positive, since the three meats are usually thought to substitute for one another. However, it shall be demonstrated shortly that it is reasonable for substitute goods to have negative cross elasticities. The extent to which the elasticity estimates conform to the theoretical propositions is of particular interest. Unfortunately, those properties that require the entire elasticity

TABLE I

OLS ELASTICITY ESTIMATES FOR
MEATS; U.S., 1965-80

Dependents	Independents					Misc. Stats.	
	constant	p_b	p_p	p_c	m	R^2	DW
x_b (beef)	1.465 (6.0)[a]	-.625 (-8.1)	.425 (6.4)	-.268 (-3.0)	.449 (6.5)	.75	.71
x_p (pork)	1.650 (5.5)	.382 (4.0)	-.662 (-8.0)	-.208 (-1.9)	.364 (4.2)	.81	1.51
x_c (chicken)	.143 (.5)	.277 (2.7)	-.053 (-.6)	.228 (-2.0)	.238 (2.6)	.86	1.44

[a] t value in parenthesis.

matrix cannot be tested, but Slutsky's equation is subject to testing. The elasticity estimates in Table II were constrained to conform to Slutsky's equation by linear restrictions upon the estimators. The condition was tested with estimates from both ordinary and generalized least squares. The calculations for these tests are presented in the lower portion of the table. At the five percent level of significance, both tests indicate statistical inconsistency with Slutsky's equation (null hypothesis is rejected); however, the generalized estimates are consistent with Slutsky's equation at the one percent level. Neither of the tests are perfectly valid, since the data were probably generated in a price-dependent fashion, and since the exclusion of nonmeat commodities may have exerted some degree of bias upon the estimators. The expenditure proportions used in these tests and in all subsequent tests are presented in the Appendix along with other descriptive statistics.

Direct Estimation Of Flexibilities

Each of the prices was fitted to the three quantities and income to obtain estimates of the flexibilities. This approach is more appropriate than the quantity-dependent methods for the particular commodities at hand, since the statistical models are consistent with highly inelastic supplies. The estimates, which are presented in Table III, are consistent in sign with expectation except for the flexibility of beef price with respect to chicken; however, this estimate is not significantly different from zero in the t test. The symmetry relation and the weighted column sum condition cannot be tested because of the incompleteness of the model; however, the unitary income flexibility condition was

TABLE II

STATISTICAL TESTS OF SLUTSKY'S EQUATION
FOR MEATS; U.S., 1965-80

Dependents	Independents				
	constant	p_b	p_p	p_c	m
Restricted OLS Elasticity Estimates					
x_b (beef)	1.031	-.734	.215	.044	.532
	(4.8)[a]	(-10.6)	(6.7)	(2.0)	(8.2)
x_p (pork)	1.602	.386	-.763	-.050	.348
	(7.2)	(6.7)	(-14.1)	(-1.5)	(5.4)
x_c (chicken)	-.129	.192	-.119	-.139	.313
	(-.4)	(2.0)	(-1.4)	(-1.2)	(3.6)

F Statistics

Method	DF	Numerator	Denominator	F
OLS	3/177	5.901	1.085	5.44
GLS	3/174	3.264	1.086	3.00

[a] t value in parenthesis.

TABLE III

OLS FLEXIBILITY ESTIMATES FOR MEATS; U.S., 1965-80

Dependents	Independents					Misc. Stats.	
	constant	x_b	x_p	x_c	m	R^2	DW
p_b (beef)	-.240	-.803	-.184	.016	.960	.98	.56
	(-.6)[a]	(-7.6)	(-2.7)	(.1)	(22.2)		
p_p (pork)	.280	-.122	-.982	-.463	1.005	.97	.82
	(.7)	(-1.1)	(-13.1)	(-3.5)	(21.2)		
p_c (chicken)	2.242	-.673	-.757	-.546	.846	.95	1.18
	(5.5)	(-5.9)	(-10.3)	(-4.2)	(18.1)		

[a] t value in parenthesis.

imposed to obtain the estimates in Table IV. The F tests under both the ordinary and generalized least squares estimates indicate that the income flexibilities are significantly different from unity when tested simultaneously; however, with a one percent significance level, the failure of both tests is by only a fraction. Also, when tested individually with ordinary estimators, the beef and pork income flexibilities are not significantly different from one at the five percent level. Under generalized estimators, the individual tests for unity in the income flexibilities yielded t statistics of -1.2, 0.0, and -3.2 for beef, pork, and chicken, respectively. These render the same conclusions as the ordinary estimators. Again, these tests are invalid to whatever extent that bias has been imposed upon the estimators by the exclusion of nonmeat commodities.

Comparison Of Elasticity And Flexibility Estimates

The estimates from the quantity-dependent and price-dependent functions were compared to determine the empirical consistencies or differences between the two approaches. In order to make the comparisons, the price-dependent functions in Table III were inverted to obtain the implied quantity-dependent forms. The resulting elasticity estimates are reported in Table V. Now, this method will yield consistent estimators for the elasticities if the entire flexibility matrix is involved in the inversion, and if the flexibility estimators are consistent. The first of these conditions is obviously not fulfilled for the problem at hand; moreover, because of the excluded variables in the estimation, the latter condition probably fails as well. Consequently, a great amount of

TABLE IV

STATISTICAL TESTS OF UNITARY INCOME FLEXIBILITIES FOR MEATS; U.S., 1965-80

Dependents	Independents				
	constant	x_b	x_p	x_c	m
Restricted OLS Flexibility Estimates					
p_b (beef)	-.210 (-.6)[a]	-.841 (-8.6)	-.189 (-2.8)	-.084 (-1.5)	1.00
p_p (pork)	.276 (.7)	-.117 (-1.1)	-.981 (-13.2)	-.450 (-7.6)	1.00
p_c (chicken)	2.356 (5.8)	-.822 (-7.8)	-.775 (-10.5)	-.931 (-15.9)	1.00

F Statistics

Method	DF	Numerator	Denominator	F
OLS	3/177	4.240	1.085	3.91
GLS	3/174	4.348	1.086	4.00

[a] t value in parenthesis.

TABLE V

IMPLIED ELASTICITIES FROM UNRESTRICTED FLEXIBILITY ESTIMATES

Dependents	Independents			
	p_b	p_p	p_c	m
x_b (beef)	-.913	.554	-.497	.740
x_p (pork)	-1.209	-2.212	1.843	1.826
x_c (chicken)	2.804	2.384	-3.776	-1.895

confidence cannot be placed in the indirect estimates; however, a comparison of the results in Table V with the quantity-dependent estimates in Table I reveals that the conclusions of the two approaches must be contradictory, even after the allowance for a considerable degree of error. The differences are particularly pronounced in the direct elasticities, which are more elastic under the price-dependent approach than under quantity-dependent estimation. Also, the income elasticities differ to the extent that chicken appears to be a normal good in Table I, but as an inferior good in Table V, and the income elasticities for beef and pork are considerably smaller under direct estimation than when calculated from the flexibilities.

The differences between the two sets of estimates were expected, for the two models cannot both be proper statistical specifications for either ordinary or generalized least squares. Now, suppose that the supply for each commodity is perfectly inelastic, so that the quantities become exogenous to the equilibria. If nonlinearities are ignored, then the equilibria may be statistically represented with:

$$P = \alpha' + X\gamma' + m\gamma_m' + E$$

If there are T observations and n commodities, then P is the T x n matrix of prices in logs with the observational vector for p_i being situated in column i. X is the T x n matrix of quantities in logs, and is similarly constructed. α' is tha 1 x n vector of intercept terms. γ' is the tranpose of the n x n flexibility matrix. γ_m' is the 1 x n vector of income flexibilities. m is the T x 1 observational vector on income, and is in logs. E is the T x n matrix of stochastic errors. With this sort of model, the generalized least squares estimator is known to render unbiased and efficient estimates for all parameters if X and m are nonstochastic, and if E has zero expectation. If this system is inverted, then the quantity-dependent form becomes:

$$X = -\alpha'\varepsilon' + P\varepsilon' - m\gamma_m'\varepsilon' + E\varepsilon'$$

where $\varepsilon = \gamma^{-1}$. Now, the latter model is severely afflicted with at least three statistical problems. First, the dependent variables are nonstochastic. Second, the price variables are stochastic regressors, and are correlated with the error terms; consequently, the generalized least squares estimator is necessarily inconsistent. Third, the error terms are heteroskedastistic and contemporaneously correlated, and the proper statistical correction will require knowledge of the elasticity matrix, which is the object of estimation. Similar conclusions are reached for the price-dependent form when supply is perfectly elastic. It is apparent that if one model is a proper statistical specification for generalized least squares, then the other model is necessarily precluded. Therefore, similarities are not to be expected between the quantity-dependent and indirect estimates for the elasticities.

In cases where supply is neither perfectly elastic nor perfectly inelastic, the proper statistical techniques are those that estimate the demand functions simultaneously with the supply functions. Since both quantities and prices are endogenous, either functional form is acceptable, and the quantity-dependent and indirect estimates should converge towards equality with increasing sample sizes. This assumes that the direct estimators for both the elasticities and the flexibilities are consistent, and that the inverted matrices involve the entire commodity basket.

Relative Price Estimates

Estimation Of Relative Flexibilities

Relative flexibilities were estimated in order to determine the extent of substitutability among the meats, and to derive the implied elasticities and flexibilities. Also, statistical tests were conducted to determine the validity of the various theoretical propositions for the actual parameters. Aggregate nonfood was selected for the numeraire, and was assumed to be strongly separable from the meats. The three meats were also assumed to comprise a strongly separable group. x_n is used to denote the quantity variable for nonfood consumption. The relative prices for beef, pork, and chicken are denoted by p_b^n, p_p^n, and p_c^n, respectively. Both x_n and the relative prices are in logs. Each relative price was fitted to the three meat quantities and nonfood consumption to obtain the estimates reported in Table VI. The negative signs on all the relative flexibilities between the meats are consistent with the expectation that the three goods are substitutes; moreover, the t statistics indicate that all but two of the estimates are highly significant. If the theory and the assumptions concerning the utility structure are correct, then the cross relative flexibilities between the meats should follow equation 30:

$$w_i\gamma_{ij}^k = w_j\gamma_{ji}^k; \qquad i,j \in a;\ k \notin a$$

TABLE VI

OLS RELATIVE FLEXIBILITY ESTIMATES
FOR MEATS; U.S., 1965-80

Dependents[a]	Independents					Misc. Stats.	
	constant	x_b	x_p	x_c	x_n	R^2	DW
p_b^n (beef)	-3.922	-.869	-.187	-.192	.982	.61	.48
	(-4.4)[b]	(-6.6)	(-2.4)	(-1.5)	(5.9)		
p_p^n (pork)	-4.051	-.230	-.980	-.644	1.112	.80	.72
	(-4.3)	(-1.6)	(-11.8)	(-4.9)	(6.4)		
p_c^n (chicken)	.207	-.728	-.775	-.821	.654	.75	.93
	(.2)	(-4.8)	(-8.8)	(-5.8)	(3.6)		

[a] All dependent variables are relative prices with nonfood as the numeraire.

[b] t value in parenthesis.

where the meats are assumed to be contained in group a. Also, the relative flexibilities on nonfood should all be equal, as indicated by equation 31:

$$\gamma_{ij}^{k} = \gamma_{hj}^{k}; \qquad j,k \in b;\ i,h \notin b$$

These conditions were imposed as restrictions upon the estimators to produce the estimates in Table VII. The F statistics at the bottom of the table indicate that there is no significant difference between the restricted and unrestricted estimates under ordinary least squares. The generalized estimates are inconsistent with the restrictions; however, the failure of the test is by only a fraction of the F statistic when the significance level is at one percent.

Since restrictions are known to increase the efficiency of the estimators when the restrictions are true, the restricted estimates were chosen for the subsequent calculations and for the analysis of substitutability. If the separability assumptions are correct, then there are at least three reasons to suspect that the restrictions are valid. First, their theoretical validity depends upon nothing other than the proportionality rule. Second, these functions should be complete statistical specifications, since commodities other than the meats and nonfood do not enter into the determination of the relative prices. Third, the statistical tests give little reason to reject the hypothesis that the restrictions are true.

The relative flexibility estimates indicate that there is significant substitutability between the meats. The substitutability of beef for chicken and of pork for chicken is so intensive that a percentage

TABLE VII

STATISTICAL TESTS OF RELATIVE FLEXIBILITY RESTRICTIONS FOR MEATS; U.S., 1965-80

Dependents[a]	Independents				
	constant	x_b	x_p	x_c	x_n
Restricted OLS Relative Flexibility Estimates					
p_b^n (beef)	-3.490 (-7.8)[b]	-.781 (-6.7)	-.127 (-2.4)	-.189 (-6.2)	.858 (13.2)
p_p^n (pork)	-2.637 (-5.3)	-.223 (-2.4)	-1.043 (-13.4)	-.351 (-10.3)	.858 (13.2)
p_c^n (chicken)	-.662 (-1.5)	-.801 (-6.2)	-.844 (-10.3)	-.944 (-12.5)	.858 (13.2)
F Statistics					
Method	DF	Numerator	Denominator	F	
OLS	5/177	2.205	1.085	2.03	
GLS	5/174	4.121	1.086	3.79	

[a] All dependent variables are relative prices with nonfood as the numeraire.

[b] t value in parenthesis.

increase in the quantity of either tends to reduce the relative price of chicken by almost the same percentage as an increase in chicken itself. Chicken is also the best substitute for both beef and pork.

The difference between any cross flexibilitiy and its transpose is explained by the disparity in the expenditure proportions. For example, the relative flexibility of pork price with respect to beef is larger than for beef with respect to pork, because the two goods represent differing proportions of the total budget. Since beef is consumed in greater quantities than pork, a percentage change in the former implies a greater absolute change than with the latter; therefore, a percentage change in the quantity of beef will induce a greater absolute change in the price of pork than vice-versa. This assertion rests upon the assumption that nonfood is strongly separable from beef and pork, so that the cross derivatives for the relative prices of the meats are symmetrical. Also, since pork price is generally less than beef price, the cross effects become even more disproportionate when expressed in percentage terms. Therefore, if substitutability and complementarity are to be measured in terms of percentage changes, then account must be made of the respective proportions represented by each good within the budget. In some applications, substitutability and complementarity may be better measured in terms of absolute changes. In such cases, the relative flexibility can be converted to the absolute measure, ω_{ij}^{k}, which is defined with:

$$\omega_{ij}^{k} = p_i p_j \gamma_{ij}^{k} / w_j; \qquad i,j \in a; \; k \notin a$$

The ω_{ij}^{k} are not equal to the relative price derivatives, but are easily shown to be proportional to them. To begin, from equation 29:

$$\gamma^{k}_{ij} = G^{a}_{ij}(x_j/G^{a}_{i}); \qquad i,j \in a;\ k \notin a$$

This relation implies that:

$$\omega^{k}_{ij} = G^{a}_{ij}m/\lambda; \qquad i,j \in a;\ k \notin a$$

The partial derivatives are:

$$\partial p^{k}_{i}/\partial x_j = G^{a}_{ij}/G^{b}_{k}; \qquad i,j \in a;\ k \in b;\ a \neq b$$

which are clearly proportional to the ω^{k}_{ij}. These coefficients were calculated for the meats, and are reported in Table VIII. Mean values were used for the prices in the calculations. The conclusions from these coefficients concerning the relative importance of the three goods in substitution are the same as from the relative flexibilities. Beef and pork still appear to be strong substitutes for chicken with pork being the stronger of the two. Also, chicken appears to be the best substitute for both beef and pork, as before.

Calculation Of Implied Elasticities

Elasticities were calculated from the restricted relative flexibility estimates in Table VII. Incomes elasticities were calculated with equation 34:

$$\varepsilon_{im} = \delta_m \sum_{t \in a} \gamma^{it}_{k}; \qquad i \in a,\ k \notin a$$

Once the income elasticities were calculated, the direct and cross elasticities were derived with equation 35:

$$\varepsilon_{ij} = \gamma^{ij}_{k} - w_j\varepsilon_{im}\varepsilon_{jm}/\delta_m - w_j\varepsilon_{im}; \qquad i \in a,\ k \in b,\ j \notin b,\ a \neq b$$

TABLE VIII

ω SUBSTITUTABILITY COEFFICIENTS

Dependents[a]	Independents		
	x_b	x_p	x_c
p_b^n (beef)	-42.80	-9.11	-16.78
p_p^n (pork)	-9.11	-55.83	-23.19
p_c^n (chicken)	-16.78	-23.19	-32.02

[a] All dependent variables are relative prices with nonfood as the numeraire.

Each of these formulae require knowledge of δ_m. Unfortunately, the estimation of this parameter requires consideration of the entire commodity basket. As a feasible alternative, it was previously suggested that the commodity bundle be aggregated into pointwise separable commodities. The estimation of δ_m could then be accomplished with the relative flexibilities of the aggregate variables in equation 36:

$$\delta_m = 1/\{(\sum_{t\neq k} w_t/\gamma^k_{tt}) - w_k/\gamma^k_{ik}\}; \qquad i\neq k$$

In this study, the commodities were aggregated into food and nonfood. The food to nonfood price ratio was then regressed upon the two aggregate quantities to estimate the relative flexibilities needed for equation 36. The estimated equation was:

$$p^n_f = \alpha + \gamma^n_{ff}x_f + \gamma^n_{fn}x_n + e$$

where p^n_f is the price ratio, and x_f and x_n are the quantity variables for food and nonfood, respectively. All three of the variables are in logs. δ_m was then calculated with:

$$\delta_m = 1/(w_f/\gamma^n_{ff} - w_n/\gamma^n_{fn})$$

The estimated relative flexibilities are reported in Table IX. These estimates have δ_m at -.627, and this value was consistently used in the calculation of the elasticities.

The resulting elasticity estimates are reported in Table X, and the associated constant utility elasticities and block-additive want elasticities are reported in Table XI. A comparison of these elasticity

TABLE IX

OLS RELATIVE FLEXIBILITY ESTIMATES FOR AGGREGATE FOOD; U.S., 1965-80

Dependent	Independents			Misc. Stats.	
	constant	x_f	x_n	R^2	DW
p_f^n (food/nonfood)	4.165 (7.9)[a]	-1.330 (-13.9)	.560 (21.3)	.88	.46

[a] t value in parenthesis.

TABLE X

IMPLIED ELASTICITIES FROM RESTRICTED RELATIVE FLEXIBILITY ESTIMATES

Dependents	Independents			
	p_b	p_p	p_c	m
x_b (beef)	-1.612	-.094	.346	.851
x_p (pork)	-.158	-1.381	.537	.627
x_c (chicken)	1.518	1.314	-1.843	-.619

TABLE XI

IMPLIED ELASTICITY COMPONENTS FROM RESTRICTED RELATIVE FLEXIBILITY ESTIMATES

Dependents	Independents					
	Constant Utility Elasticities			Block-Additive Want Elasticities		
	p_b	p_p	p_c	p_b	p_p	p_c
x_b (beef)	-1.584	-.078	.353	-1.622	-.094	.360
x_p (pork)	-.137	-1.369	.542	-.166	-1.381	.546
x_c (chicken)	1.497	1.302	-1.848	1.526	1.314	-1.853

estimates with the quantity-dependent estimates in Table I reveals that there are considerable differences between the two. However, since the quantity-dependent estimators are inconsistent under fixed supply, there is little reason to expect similar results from the two methods. The only apparent consistency in the differences between the two sets is that the quantity-dependent estimates are smaller in absolute size in 10 out of 12 cases. The contrast is most conspicuous with the direct elasticities, for in every case, the imputed estimates are the more elastic of the two. These results give some reason to suspect that the direction of bias for the quantity-dependent estimates is towards zero, and particularly for the direct elasticities.

Since food is a necessity in consumption, it is often reasoned that food quantities cannot significantly respond to changes in prices. The direct elasticities under the quantity-dependent estimators are consistent with expectation in this respect. However, this reasoning ignores the possibility of substitution between food items. If a particular food item can be easily replaced with a substitute good, then it is reasonable to expect the consumption of that item to significantly respond to price changes, regardless of how necessary the two items may be to the consumer. Since the relative flexibilities indicate that the meats are highly substitutable, it is of no surprise that the implied elasticities show all three items to be elastic goods. The comparatively large direct elasticity for chicken is consistent with this reasoning, for it is evident from the relative flexibilities that substitution is most intensive with chicken; therefore, chicken is the most elastic of the three.

It is commonly imagined that the cross elasticities between substitute goods must be positive; however, the cross elasticities between beef and pork demonstrate that this is not necessarily the case. First, some substitute goods may have cross elasticites with positive substitution effects, but these may be dominated by the income effects, which are generally negative. Moreover, the constant utility elasticities between beef and pork indicate that there are cases in which even the substitution effects can be negative for substitute goods. It is commonly reasoned that if beef and pork are substitutes, then an increase in pork price will reduce the consumption of pork, which will increase the consumer's valuation of beef, and subsequently, the consumption of beef will increase. This logic would probably hold if pork were the only substitute for beef; however, if there are two or more substitutes, then there can be secondary cross effects that can reverse the process. For example, suppose that pork is a good substitute for both beef and chicken. Moreover, assume that pork is a better substitute for chicken than for beef, as measured by the relative flexibility. Now, the first round affects of an increase in pork price will be to reduce the consumption of pork, which will increase the consumer's valuation of beef and chicken, since pork is a substitute for both. However, the consumer's valuation of chicken will be increased by a greater percentage than for beef, since pork was assumed to be a better substitute for the former than the latter. Since the prices of beef and chicken are assumed to be constant in the differentiation, an effect of the first round is a disproportionality between the marginal utilities of beef and chicken with their respective prices. In the second round, proportionality must be restored, which will be accomplished with a combination of an increase in the consumption of chicken and a decrease in the consumption of beef. The second round reduction in beef will be the greatest when chicken is a good substitute for beef. In such cases, the upward adjustments in chicken will further reduce the consumer's valuation of beef, and subsequently, its consumption.

To demonstrate the process mathematically, observe that the constant utility elasticity between beef and pork is given by:

$$\varepsilon^{u}_{bp} = \gamma^{bp}_{n} - w_p \varepsilon_{pm} \varepsilon_{bm} / \delta_m$$

Since the second term is generally positive, negative total substitution effects must usually be explained by negative block-additive want elasticities. Now, let ψ denote the determinant of the relative flexibility matrix for beef, pork, and chicken, and observe that ψ is negative for this particular set of estimates. The inverse element, γ^{bp}_{n}, is calculated with:

$$\gamma^{bp}_{n} = -(\gamma^{n}_{bp}\gamma^{n}_{cc} - \gamma^{n}_{cp}\gamma^{n}_{bc})/\psi$$

which is less than zero if:

$$\gamma^{n}_{bp}\gamma^{n}_{cc} - \gamma^{n}_{cp}\gamma^{n}_{bc} < 0$$

Since all of the elements above are less than zero, the latter relation is the same with:

$$\gamma^{n}_{bp}/\gamma^{n}_{cp} - \gamma^{n}_{bc}/\gamma^{n}_{cc} < 0$$

This relation demonstrates how that the total substitution effects between beef and pork can be negative if pork is a better substitute for chicken than for beef, and if chicken is a good substitute for beef. Therefore, it is reasonable for substitute goods to have negative cross elasticities; moreover, there is no reason to expect such occurrences to be improbable. In general, if there are three substitute goods, say x_i, x_j, and x_k, then ε_{ij} can be negative if x_j is a better substitute for x_k, which is also a substitute for x_i. Of course, similar reasoning will lead to the conclusion that complementary goods can have positive cross elasticities and total substitution effects.

The negative income elasticity for chicken is difficult to reconcile with expectation, since most food items are generally thought to be normal goods. However, the negative income elasticity is consistent with the extreme substitutability of beef and pork for chicken. To explain the process in terms of relative flexibilities, suppose that increases in income are allocated to the quantities in two steps, and observe that the allocation must be such that proportionality with the prices is preserved. In the first step, assume that the increase in income is proportionately allocated to all quantities. Now, it is apparent from the relative flexibilities that such an allocation will reduce the consumer's valuation of all three meats in terms of nonfoods. However, since beef and pork are strong substitutes for chicken, the proportional increases in the meats will reduce the relative price of chicken by a greater percentage than for beef and pork. Therefore, a consequence of the first step will be a disproportionality between the marginal utilities of the three meats with their respective prices. Therefore, in the second step, proportionality must be restored with a combination of increases in the consumption of beef and pork, and a decrease in the consumption of chicken. If the decrease in the consumption of chicken in the second step is large, then the income elasticity could be negative. The income elasticity of chicken will be lowest when substitution between beef and pork is weak, for in such cases, the disproportionality created in the first step will be the greatest. Therefore, the negative estimate for the income elasticity of chicken is a result of the intensive substitutability of beef and pork for chicken, and the comparatively low degree of substitutability between beef and pork.

A second point in the defense of the peculiar estimate for the income elasticity of chicken is the degree to which these estimates fit the data. In a simulation study, the indirect elasticity estimates were used as parameters in log-linear models. These functions were then used to generate predicted values of the quantities over the entire range of the data. The degree of fit was measured by the correlation of the predicted values with the actual quantities. The correlation coefficient for chicken was calculated at .859, which is reasonably high considering the roundabout nature of this approach. However, the coefficients for beef and pork were only -.004 and .470, respectively. The poor performance of the beef and pork estimates is probably due to the exclusion of nonmeat commodities in these calculations. This suspicion tends to be confirmed upon an examination of the errors in the simulation. The errors for beef and pork tended to increase in positive increments when moving from the earlies to the latest points in the period, but the errors for chicken tended to move in the opposite direction. Correlation coefficients were calculated between the errors and a trend variable in order to measure the consistency of the movements in the errors. The coefficients were .940 for the beef errors, .880 for pork, and -.779 for chicken. Thus, there are definite trends in the errors.

These trends were expected, since the prices of nonmeat commodities also followed consistent upward trends throughout the period. If the indirect elasticity estimates were perfectly accurate, then the simulated models could not account for the variation that has been induced by the upward trends in nonmeat prices. Moreover, the directions of the trends in the errors were consistent with expectation as well. By the homogeneity condition, the sum of the cross elasticities of the meats with repect to the nonmeat items can be estimated by the negative of the row sums of the elasticities in Table X. These indicate that the cross elasticities of beef and pork with respect to the nonmeat goods are generally positive. In similar fashion, the estimates for chicken imply that the cross elasticities of chicken with respect to the nonmeats are generally negative. Since the nonmeat prices have followed inflationary trends, these items have tended to induce upward trends in the consumption of beef and pork, but a downward trend in the consumption of chicken. Since the simulated models cannot account for this variation, they should tend to underestimate the consumption of beef and pork with increasing error when moving toward the end of the data, and should tend to overestimate the consumption of chicken in similar fashion. Subsequently, the trends in the errors of the simulated models are consistent with expectation.

If the variation that is due to the nonmeat prices is removed from the actual quantities, then the resulting variables should be more closely approximated by the simulated models. If it is assumed that the affects of the nonmeat prices are approximated by a linear trend, and if the indirect elasticity estimates for the meats are reasonably accurate, then the variation that has been induced by the nonmeat prices should be represented by the trend components in the simulation errors. These trend components were estimated by regressing the errors on a trend variable. The predicted values for these regressions were then subtracted from the actual quantities, and correlation coefficients were calculated between the resulting variables and the simulated meat consumption estimates. The correlations were .932 for beef, .906 for pork, and .910 for chicken. The significant increases in the coefficients for beef and pork probably indicate that a large percentage of the variation in the consumption of these commodities has been generated by changes in the nonmeat prices. Also, when the nonmeat affects are removed, the simulated models account for a considerable portion of the remaining variation. Therefore, if degree of fit is regarded as a criterion of the estimator efficiency, then the simulated models tend to confirm the accuracy of the indirect elasticity estimates.

Although remote variables are often ignored in the estimation of demand functions, this practice can hardly be justified by the theory. Moreover, the previous results indicate that the exclusion of nonmeat commodities in the estimation of meat demand functions could have severe statistical consequences. In such cases, the parameter estimates will be forced to account for the variation generated by both the meats and the excluded variables. Since meat prices and nonmeat prices have been highly correlated, and since the nonmeats probably account for a large portion of the variation in meat consumption, the failure to include these variables could severely bias the estimators for the elasticities among the meats.

It is commonly assumed that the affects of remote variables are approximated by a linear trend, and a trend variable is inserted into the regression equation to account for the excluded regressors. However, an alternative method is to simply first-difference the data. If the excluded variables were exactly represented by a linear trend, then first-differencing would completely remove the variation generated by these variables. However, an unfortunate consequence of this method is that the included variables are similarly affected, so that the explanatory power of the estimated model could be considerably reduced. This is demonstrated in the results of a second simulation. The indirect elasticity estimates were used as parameters in first-differenced log-linear models. The correlations between the predicted and actual values were .436 for beef, .495 for pork, but only .059 for chicken. Thus, the removal of nonmeat variation by the first-differencing increases the correlations for beef and pork, relative to the correlations that were previously produced by the log-linear models. The significant reduction in the coefficient for chicken is probably due to a strong upward trend that occurred in chicken consumption over the entire range of the data. The changes in income and in the meat prices have probably served to produce an upward trend in chicken consumption that has more than offset the downward trend induced by the increases in the nonmeat prices; however, the explanatory powers of both the included and the excluded variables are destroyed in the first-differencing. Therefore, it is possible that the affects of excluded variables could be removed by first-differencing; however, the explanatory powers of the included variables could be reduced.

Calculation Of Implied Flexibilities

With strongly separable utility, better estimators for the flexibilities could possibly be obtained from the relative flexibilities than from direct estimation. The superiority of the indirect estimators would be entirely due to the missing variable problem that is often associated with the estimation of absolute price functions. There is reason to suspect that the exclusion of remote variables could significantly bias the estimators for flexibilities. From equation 26:

$$\gamma_{ij} = \phi_{ij} - \theta_j$$

Since ϕ_{ij} is trivial for remotely related commodities, the latter expression will reduce to $-\theta_j$; however, it is not certain that this term will be trivial. From equation 25:

$$\theta_j = w_j(\sum_t \phi_{jt} + 1)$$

which does not necessarily imply that θ_j will be small.

In order to compare the two methods, flexibilities were calculated from the restricted relative flexibility estimates. The conversion was accomplished with equation 32:

$$\gamma_{ij} = \gamma^k_{ij} - w_j(\sum_{t \varepsilon a} \gamma^k_{jt} + 1); \qquad j \varepsilon a,\ k \varepsilon b,\ i \notin b,\ a \neq b$$

The results of the calculations are presented in Table XII. A comparison

TABLE XII

IMPLIED FLEXIBILITIES FROM RESTRICTED RELATIVE FLEXIBILITY ESTIMATES

Dependents	Independents			
	x_b	x_p	x_c	m
p_b (beef)	-.778	-.115	-.176	1.000
p_p (pork)	-.220	-1.031	-.338	1.000
p_c (chicken)	-.798	-.832	-.931	1.000

of these with the direct estimates in Tables III and IV reveals that the differences between the two sets are generally trivial; moreover, there are no apparent consistencies in the differences. Therefore, these estimates indicate that the two methods yield similar conclusions, even though there is some theoretical appeal to the relative price approach.

Convexity And Stability In The Indifference Curves

The relative flexibility estimates are consistent with two-dimensional and three-dimensional convexity in the indifference curves for the meats. Two-dimensional convexity may be established by observing that all of the relative flexibilities satisfy equation 39. However, an easier approach is to simply observe that the estimated relative flexibility matrices for both the restricted and the unrestricted estimators are negative definite. This indicates that the strongly separable subfunction for the meats is concave, which implies that the indifference curves between the meats must be convex to the origin.

The stability of the utility structure for these three commodities is of particular interest. The period of analysis was characterized by persistent and significant increases in the consumption of chicken. For example, in the first half of the period, average quarterly chicken consumption was 9.5 pounds, but increased by 19% to 11.3 pounds in the second half. On the other hand, beef consumption increased by 6%, and pork changed by only -2%. The significant increase in chicken consumption leads to the suspicion that structural shifts in favor of chicken have occurred within the period. However, there is the possibility that the consumption changes are mostly due to general movements in prices and income, and not to structural shifts in preferences. For example, the beef price to chicken price ratio increased from a quarterly average of 2.38 in the first half to 2.76 in the second, and the price of pork relative to chicken increased from 1.78 to 2.06. Also, quarterly income increased form 3,102 to 5,925 dollars per-capita.

To test the structural stability for the three goods, beef price relative to chicken and pork price relative to chicken were regressed upon the quantities and shift variables for both the slope and intercept terms. The shifts were located at the center of the period, which occurred between 1972 and 1973. The relative prices for beef and pork are now denoted by p_c^b and p_p^c, since the numeraire has been changed from nonfood to chicken. This change was made for two reasons. First, it is known that these relative prices are completely determined by the three meat quantities if these commodities comprise a weakly separable group. Therefore, the assumption of strong separability can be replaced with considerably less restrictive assumptions. Second, it is more reasonable to assume perfectly inelastic supply for chicken than for nonfood. Both of these relative prices are in logs. The tests were conducted with both ordinary and generalized least squares estimators. The results are presented in Tables XIII and XIV. The t statistics on the shift parameters indicate that the utility structure for the meats has been reasonably stable. None of the estimates on the shift parameters show to be significantly different from zero under either set of estimates. Therefore, it is best to conclude that the increases in chicken consumption have been mostly due to the decreases in the price of chicken relative to the prices of beef and pork.

The elasticity estimates in Table X indicate that chicken is apt to

TABLE XIII

OLS STABILITY TESTS FOR MEATS; U.S., 1965-80

Dependents[a]	Independents								Misc. Stats.	
	constant	x_b	x_p	x_c	d[b]	$d*x_b$	$d*x_p$	$d*x_c$	R^2	DW
p_b^c (beef)	1.833 (2.1)[c]	.122 (.3)	.558 (4.3)	.778 (4.1)	.953 (.8)	-.276 (-.6)	.045 (.3)	-.084 (-.4)	.87	.91
p_p^c (pork)	2.130 (2.4)	1.234 (2.9)	-.328 (-2.5)	.082 (.4)	-.070 (-.1)	-.460 (-1.0)	.282 (1.6)	.328 (1.5)	.75	1.32

[a] All dependent variables are relative prices with chicken as the numeraire.

[b] The variable, d, is equal to zero for 1965-72, and equal to one for 1973-80.

[c] t value in parenthesis.

TABLE XIV

GLS STABILITY TESTS FOR MEATS; U.S., 1965-80

Dependents[a]	Independents								Misc. Stats.	
	constant	x_b	x_p	x_c	d[b]	$d*x_b$	$d*x_p$	$d*x_c$	R^2	DW
p_b^c (beef)	1.081	.082	.581	.591	.028	-.139	.131	.039	.71	1.7
	(2.6)[c]	(.3)	(4.5)	(3.4)	(0.0)	(-.4)	(.8)	(.2)		
p_p^c (pork)	2.085	.845	-.146	-.050	-1.472	.013	.216	.391	.59	1.5
	(3.6)	(2.3)	(-1.1)	(-.3)	(-1.2)	(0.0)	(1.2)	(1.7)		

[a] All dependent variables are relative prices with chicken as the numeraire.

[b] The variable, d, is equal to zero for 1965-72, and equal to one for 1973-80.

[c] t value in parenthesis.

sell in large quantities when purchasing conditions are unfavorable for beef and pork. The large cross elasticities indicate that increases in the prices of beef and pork will cause the consumer to abandon these commodities and retreat to chicken. Likewise, with unfavorable income conditions, chicken consumption will increase, but at the expense of beef and pork. However, the elasticity estimates also indicate that the recent trends in chicken consumption could be abruptly reversed with increases in income and a discontinuation in the downward trend in the relative price of chicken.

CHAPTER IV

SUMMARY AND CONCLUSIONS

Although demand analysis is usually conducted under the quantity-dependent approach, it has been demonstrated that there are some situations in which the price-dependent approach is equally if not more appropriate. Moreover, these situations should not be uncommon.

It has been shown that the first-order conditions for utility maximization and the quasi-concavity of the utility function impute several properties to the associated flexibilities. Also, these properties posses a reflective consistency with the properties of the corresponding elasticities. In addition to the general properties, other properties will sometimes result when the utility function is of a restricted form, and in some cases, the general properties can be simplified. If there are any conceptual problems with the proposed derivation of the price-dependent function, then the same problems must also be associated with the quantity-dependent function when applied to the aggregate market. The assumptions and behavioral postulations employed in the derivation of the price-dependent function are the same with those used under the traditional quantity-dependent approach. Perhaps the most critical of these assumptions is the existence of a representative consumer. Without such a consumer, none of the presented theoretical results for the price-dependent function will pertain; however, the results for the quantity-dependent function will still be relevant when dealing with a single consumer.

Since substitutability and complementarity are concerned with the impact of consumption upon consumer valuation, the measurement of the concepts is best accomplished with price-dependent parameters. The relative flexibility is an intuitively appealing measure when a strongly separable good is selected for the numeraire. However, an obvious problem with this method occurs when there are several strongly separable goods. By utility theory, all strongly separable goods are equally acceptable candidates for the numeraire; however, it is almost certain that identical conclusions will not be obtained in applied work under different choices of the numeraire. Since the theory does not discriminate between the strongly separable goods, the proper choice of the numeraire must be based upon empirical considerations. Perhaps the best method is to choose that numeraire commodity under which the resulting relative price functions are most nearly linear. This assumes that the estimation techniques pertain to linear functions, as is usually the case. Linearity could be measured by R^2 or some other criterion of fit.

The estimation of elasticities and flexibilities from relative flexibilities is theoretically appealing; however, the estimates produced by these methods will also depend upon the selection for the numeraire. The linearity criterion should also be an acceptable rule of numeraire selection here.

The price-dependent methods should be appropriate when examining the quarterly consumption of the meats, since the supply for each of these commodities is almost completely predetermined to the quarterly equilibria. The empirical applications of the price-dependent methods to the meats tend to confirm the relevance of this approach. The resulting estimates are reasonably consistent with introspection; moreover, the various statistical tests indicate that these estimates are not inconsistent with the price-dependent theory. Perhaps better estimates could be obtained under the price-dependent methods with a different choice for the numeraire. The usage of nonfood has probably introduced aggregation errors into the statistical models. Also, commodities with more highly inelastic supplies should provide better choices for the numeraire good.

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APPENDIX

DESCRIPTIVE STATISTICS

Variable	mean	std. dev.	exp. prop.
x_b	21.573	1.560	.0335
x_p	14.964	1.534	.0190
x_c	10.377	1.331	.0079
x_f	567.160	16.436	.1835
x_n	2190.414	229.376	.8165
p_b	135.468	48.277	-
p_p	100.859	31.783	-
p_c	51.767	12.431	-
p_f	150.944	50.767	-
p_n	143.490	43.774	-
p_b^n	.933	.086	-
p_p^n	.704	.094	-
p_c^n	.369	.048	-
p_f^n	1.042	.056	-
p_b^c	2.569	.390	-
p_p^c	1.919	.203	-
m	4513.766	1706.146	-

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VITA

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