DYNAMIC ANALYSIS OF A MARINE RISER SYSTEM

By

VASAN LAXMINARASIMHAN

Bachelor of Engineering

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Hyderabad, India

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of the Oklahoma State University
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LIST OF SYMBOLS

z	- Distance measured along the riser from the bottom
T_O	- Axial tension at the top of the riser
ρ	- Mass density of the riser material
ρ_i	- Mass density of the fluid inside the riser
ρ_o	- Mass density of the surrounding fluid
g	- Gravitational force
ℓ	- Element length
L	- Riser total length
D_O	- Outer diameter of the riser
D_i	- Inner diameter of the riser
A_O	- Outer area of cross section
A_i	- Inner area of cross section
p_i	- Riser inside pressure $\{\rho_i g(L-z)\}$
p_O	- Riser outer pressure $\{\rho_o g(L-z)\}$
U_y, U_x	- Lateral displacement of riser
$\dot{U}_w$	- Wave particle velocity
$\dot{U}_c$	- Current velocity
$\ddot{U}_w$	- Wave particle acceleration
C_m	- Mass coefficient
D_D	- Drag coefficient
E	- Modulus of elasticity
H	- Wave height, crest to trough

L_w	-	Wave length
τ	-	Wave period
I	-	Second moment of cross sectional area
ξ	-	Local coordinate
$N_i(\xi)$	-	Shape functions, Hermitian
f_y, f_x	-	Lateral fluid loading on the riser

CHAPTER I

INTRODUCTION

The ever growing need for energy has taken mankind to very strange places in search of new energy sources. Almost all the land-based oil and gas sources have been discovered and, at the current rate of consumption, they would be depleted in not too long a future. So, it came as a pleasant surprise when it was discovered that the sea-beds hold vast quantities of oil and gas reserves. No time was lost in exploiting the newly discovered source. In future, the sea-beds may well be the only source of oil and gas.

The exploration and extraction of oil and gas from sea-beds follow a familiar pattern. Exploratory drilling ships are sent to places where the presence of oil is suspected. Drilling rods are lowered from the ship and the sea-bed is drilled. If oil is struck, big platforms are brought into that place and extraction begins. Initially, when the offshore drilling was in its infancy, the drilling was done very close to the shore in shallow waters not exceeding a few hundred feet deep. But now-a-days exploration and extraction are ventured far from shore at depths approaching 10,000 feet. The trend seems to be to go for greater depths in search of more oil and gas.

Offshore drilling has posed some of the greatest challenges to technology. The drilling and extraction have often to be carried out in extremely hostile environments. The structures and materials used must withstand severe loadings such as waves, currents and vessel displacements. One particular component that deserves special attention is the marine riser. It is the pipe that connects the well-head at the sea-bed to the drilling vessel (or platform) at the sea surface. During the drilling phase, the riser helps to guide the drill string and serves as a return path for the mud. During the extraction phase, the riser is the only transport path from the sea-bed to the ocean surface. This explains the great amount of importance placed on the proper analysis, design, construction and maintenance of marine risers. A single failure in a marine riser in operation can cost up to a million dollars per day.

A typical marine riser is shown schematically in Figure 1. The drilling vessel is held in position vertically above the well-head by moorings. The marine risers used during drilling phase are called drilling risers and the ones used during extraction phase are called production risers. The riser is attached to the well-head at the sea-bed through a blow-out preventer. The blow-out preventer provides control when well flows develop and it also provides a means of circulating, conditioning and returning the well-bore to a state of unpressured condition. At the top the riser is connected to the vessel (or platform) through slip joints (also called telescopic joints). The slip joint allows the riser to change its length as the vessel heaves and moves laterally. It is a common practice to apply axial tension at the top of the riser to reduce bending in the riser.

The marine riser must be structurally strong to withstand the unpredictable and varying forces exerted upon it under changing conditions. Moreover, current requirements to have risers with longer and more reliable service lives necessitates an analysis technique of acceptable

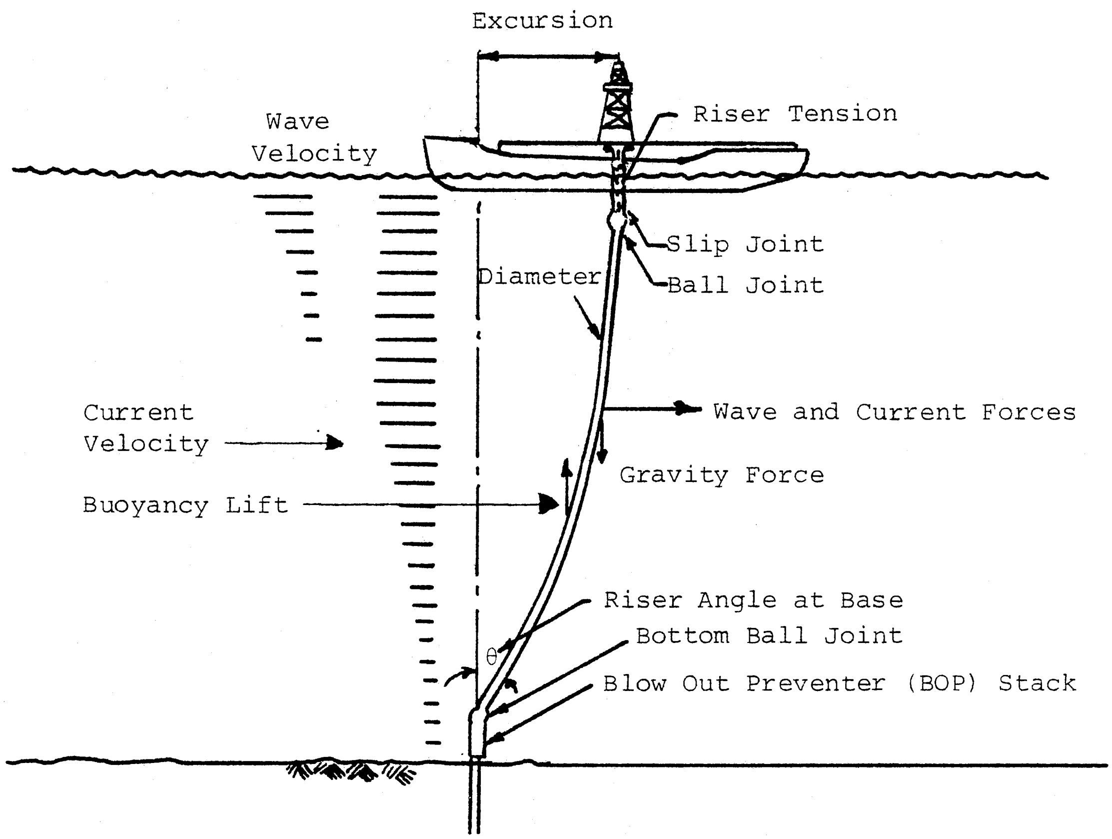

Figure 1. Riser Schematic Diagram

accuracy and low cost. The long slender marine riser configuration makes itself susceptible to a number of structural problems which complicate riser analysis and design.

In this thesis, the dynamic analysis of the marine riser is conducted by the Finite Element Method. Finite element method formulation provides an efficient and accurate solution. The analysis predicts riser deflections and bending moments due to wave and current loadings. It accounts for structural loads due to self weight, internal and external fluid pressures, applied top tension, hydrodynamic current and wave induced forces and surface vessel motion in waves. The hydrodynamic force acting on the riser is drag dominated and proportional to the square of the fluid velocity. The consequent non-linear term is also considered in the analysis.

CHAPTER II

LITERATURE REVIEW

Static Models

Attempts have been made to predict the behavior of marine risers using the principle of statics. Such analysis could be expected to be only approximate because the actual riser behavior is governed by the laws of dynamics. Both analytical and numerical studies have been reported under static models.

The equation of motion for a static case is derived by considering the riser as a simply supported beam with variable axial tension and current flow as the load after applying appropriate boundary conditions. The governing fourth-order differential equation is

$$EI \frac{d^4 y}{dz^4} + T(z) \frac{d^2 y}{dz^2} + w \frac{dy}{dx} = F_D \qquad (1)$$

where

E = modulus of elasticity

I = second moment of cross sectional area

T = tension varying along the length

w = weight of riser in sea water per unit length

$$F_D = \frac{1}{2} C_D \rho D_O |\dot{U}_c| \dot{U}_c$$

where

C_D = drag coefficient

ρ = mass density of water

D_O = riser outer diameter

Analytical Studies

The drill string has been analyzed by assuming the drill string as being made up of short beam sections having constant axial tension at the top and bottom, joined by a flexible cable with variable tension in the middle [1,2]. The papers conclude that the dynamic effects are negligible and that the bending stresses at the ocean surface are primarily due to the pitch and roll, while those at the ocean floor are due to lateral translations. The conclusion that the dynamic effects are insignificant has been proved to be wrong by later authors.

A simplified solution has been obtained by restricting the study to water depths less than 1,000 feet with moderate sea and vessel conditions [3]. Dimensionless variable has been introduced into the differential equation to facilitate the ease of solution by assuming infinite power series. The importance of tensioning the riser to prevent buckling and to control deflection and stresses has been demonstrated through design charts. Though this paper does not consider the various parameters in-depth, the paper has proved to be a useful guide in later works.

Numerical Studies

Sophisticated static analysis computer codes have been developed to study the relative importance of various parameters using the finite element method [4]. A fourth-order non-linear differential equation has been solved [5] using the finite difference technique by considering the riser as a simply supported beam with variable axial tension and variable current profile.

Dynamic Models

Three basic methods of solution are used for the dynamic response analysis: deterministic time-history analysis, a steady-state or frequency-domain analysis, and a non-deterministic random vibration analysis. The time-domain solution include the finite difference and finite element method. The finite difference method converts the equation of motion into a set of non-linear ordinary differential equations. The time-domain solution is quite flexible and can accommodate variation in riser dimension, boundary conditions, and external time-varying loads and/or motions. Dynamic analysis in the time domain is suited for the assessment of fatigue damage and also when a detailed knowledge of stress variation due to irregular seas is required.

The frequency domain solution is obtained by assuming steady-state wave loadings and vessel motions and reducing the equations of motion to an ordinary differential equation and numerically integrating it. The advantage of the frequency-domain analysis is that one can directly apply a frequency-domain definition of the environment or ship motion to the riser and generate, within a relatively short computer run, a response spectrum suitable for subsequent fatigue life estimation. The disadvantage include the unknown effect of drag linearization (if and when it is done) and the sensitivity of the method to minor changes in wave spectra.

The governing equation for a dynamic case, including varying top tension, internal and external fluid pressure, is

$$m \frac{\partial^2 y}{\partial t^2} + EI \frac{\partial^4 y}{\partial z^4} - \{T_O - \rho g(A_O - A_i)(L-z)\} \frac{\partial^2 y}{\partial z^2} - \{(\rho_O A_O - \rho_i A_i)(L-z)g\} \frac{\partial^2 y}{\partial z^2}$$

$$- \rho g(A_O - A_i) \frac{\partial y}{\partial z} + (\rho_O A_O - \rho_i A_i) g \frac{\partial y}{\partial z} = f_y \qquad (2)$$

where

$$m = \rho_i A_i + \rho(A_O - A_i)$$

$$f_y = \frac{\pi}{4} \rho_O C_m D_O^2 \frac{\partial^2 U_w}{\partial t^2} - \frac{\pi}{4} \rho_O (C_m - 1) D_O^2 \frac{\partial^2 y}{\partial t^2} + \frac{1}{2} \rho_O D_O C_D \left|\dot{U}_w + \dot{U}_c - \frac{\partial y}{\partial t}\right|$$

$$\times \left(\dot{U}_w + \dot{U}_c - \frac{\partial y}{\partial t}\right)$$

Deterministic Models

Analytical Studies

A frequency domain analysis by the normal mode approach has been carried out [6]. The steady state response has been obtained for two cases: (i) neglecting non-linear damping and (ii) including non-linear hydrodynamic damping. It has been shown that the non-linear hydrodynamic damping increases considerably as the amplitude of vessel motion increases and also that the force due to the vessel motion is the major cause for riser bending stress.

The drill string has been analyzed for maximum displacement and bending stress, depending on a given damping factor of the surrounding water [7]. A fourth-order homogeneous partial differential equation has been taken to be governed by non-homogeneous boundary conditions. The solution process involves transforming the problem as being governed by a non-homogeneous partial differential equation with homogeneous boundary conditions. Discussions have been presented based on practical values of damping factor (0.1 to 0.2). Resonance phenomena was also included. Plots of displacement due to drifting and rolling of a drilling vessel for different wave periods and damping factor of 0.1 is shown in Figures 2 and 3.

The dynamic response of marine risers has been obtained by Young et al. [8] in the frequency domain. The random nature of the waves has also been included in the analysis. The frequency domain approach was found to be very useful in fatigue calculations. The authors concluded that vessel motion is the primary factor influencing the dynamic response. In addition, short risers were found to be sensitive to wave period and range of operating tension, long risers were found to be sensitive to axial force variations. Young et al.'s computer programs allows for a choice of either the displacement or the force boundary conditions at either end of the riser.

A modal analysis procedure has been proposed by Dareing and Huang [9]. The eigenvalues and eigenfunctions developed earlier by the authors were used in obtaining the modal response [10].

Numerical Studies

Along with the static analysis a dynamic analysis has been carried out [11]. In the general fourth order linear differential equation the non-linear term has been substituted with an 'equivalent' linear term. The results of the response of eight riser configurations (bending stress, deflection, offset, sway and surge, bottom angle, water depth, wave forces, byoyed/unbyoyed) to one top tension, three wave heights, two vertical motion response functions and wave periods ranging from

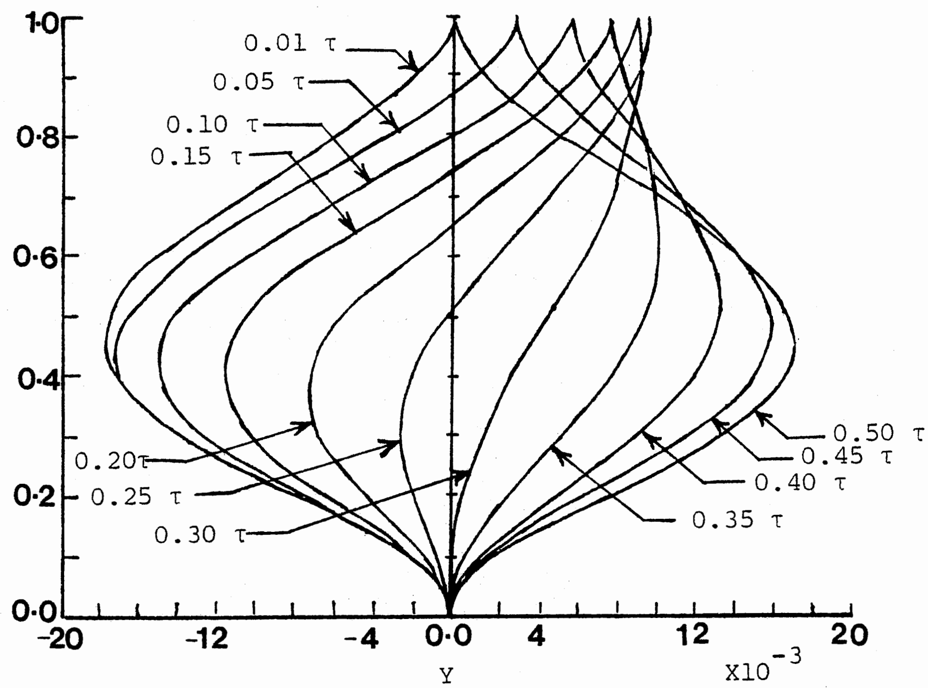

Figure 2. Displacement(Y) Due to Drifting of a Drilling Vessel [Ref. 7]

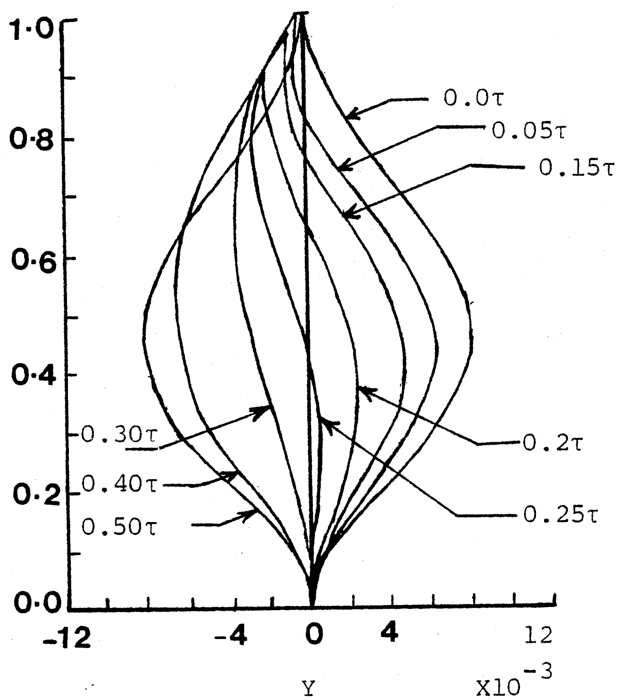

Figure 3. Dispacement(Y) Due to Rolling of a Drilling Vessel [Ref. 7]

6-20 seconds have been presented and discussed. The paper concludes that the hydrodynamic damping is a critical factor in limiting the riser dynamic deflections and stresses.

As with the static case, the dynamic case has been analyzed using sophisticated computer program [4]. The orderly importance of different parameters have been discussed briefly after solving the differential equation by the finite element method. The same method has been used to obtain the dynamic response [12]. The derivation of the governing equation includes intermediate ball joints with non-linear stiffness. The bending equation of motion has been transformed to discrete coordinate system to obtain the mass and stiffness matrix. The matrix equation of motion was numerically integrated by Newmark's method. The paper shows that the stress levels are quite sensitive to top tension, particularly just above the flexible joint and also the dynamic stress range reduces with increasing tension.

Coupled non-linear equations of motions for the axial (bar) and lateral (beam) response have been solved by direct time and spatial integration by Newmark-Beta method [13]. A brief descriptive analysis of the emergency disconnect maneuver has also been presented.

The hydrodynamic drag term has been linearized by a unique method and the results have been shown to agree well with the more accurate non-linear time-domain results [14]. The sinusoidal wave particle velocity has been represented by the real part of a complex variable. By substituting in the governing equation and using constants, the drag term was linearized. The constants in turn were evaluated by the describing function technique used in control theory. The variation of bending stress along the length of the riser is compared for two cases: (i) random waves with time and frequency domain compared with static

(i) random waves with time and frequency domain compared with static case (Figure 4) and (ii) regular wave in time and frequency domain (Figure 5).

The riser has been modeled as a discrete multi-degree-of-freedom dynamic system [15]. Various matrices such as mass, bending stiffness, geometric stiffness and damping matrices have been derived. A statistically equivalent load was determined to act at the nodal points. The non-linear force term was linearized by a scheme presented in Reference 21. The effect of variation of different parameters have been discussed. The results of the variation of maximum bending stress with wave period for different riser lengths is as shown in Figure 6. The plot obtained for the variation of bending stress along the riser length is as shown in Figure 7. Maximum bottom angle versus wave period for different wave heights is as shown in Figure 8.

Non-Deterministic Models

Results of the analytical studies of the effect of various problem parameters on the non-deterministic response of a marine riser to random wave forces have been presented [16].

A computer model for analyzing a marine riser has been developed [17]. The random wave model allows one to specify any wave spectrum, from which the model generates a synthetic wave by decomposing the spectrum. The model predicts a time history of riser stresses, deflection and lower ball joint angle. The method used was implicit finite difference solution to the tensioned beam column equation. Recommended top axial tension for various water depths and vessel offset for particular wave height of 15 feet and period 10.3 seconds is plotted as shown in Figure 9.

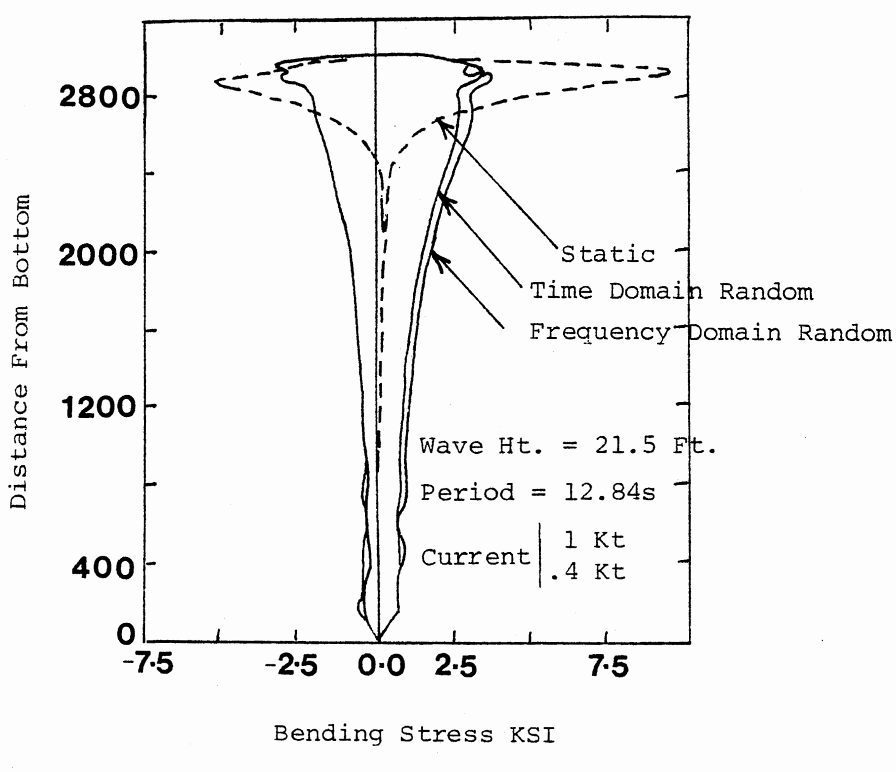

Figure 4. Bending Stress Along a 3000 Ft. Riser, for Random Wave [Ref. 14]

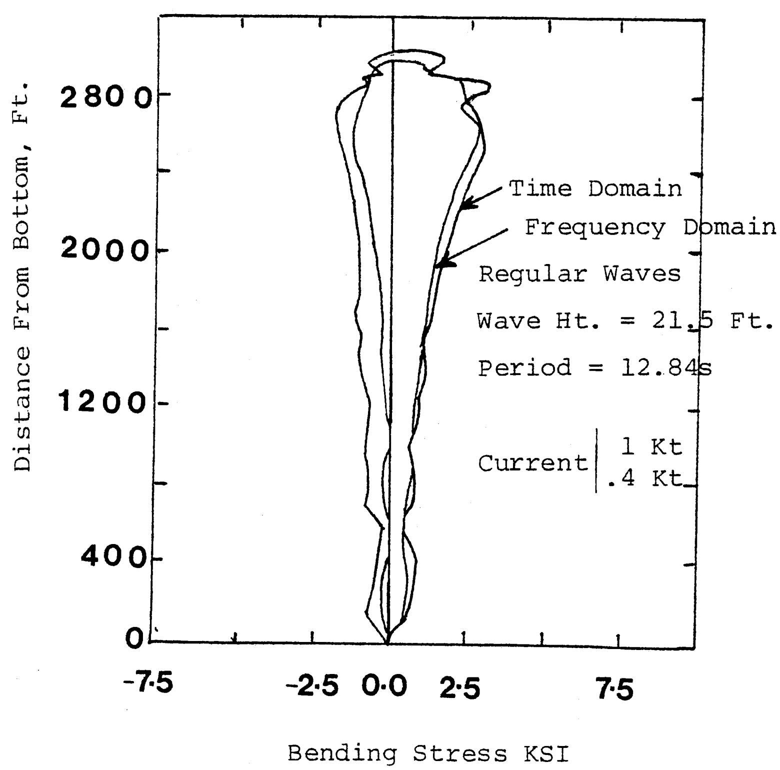

Figure 5. Bending Stress Along a 3000 Ft. Riser, for Regular Wave [Ref. 14]

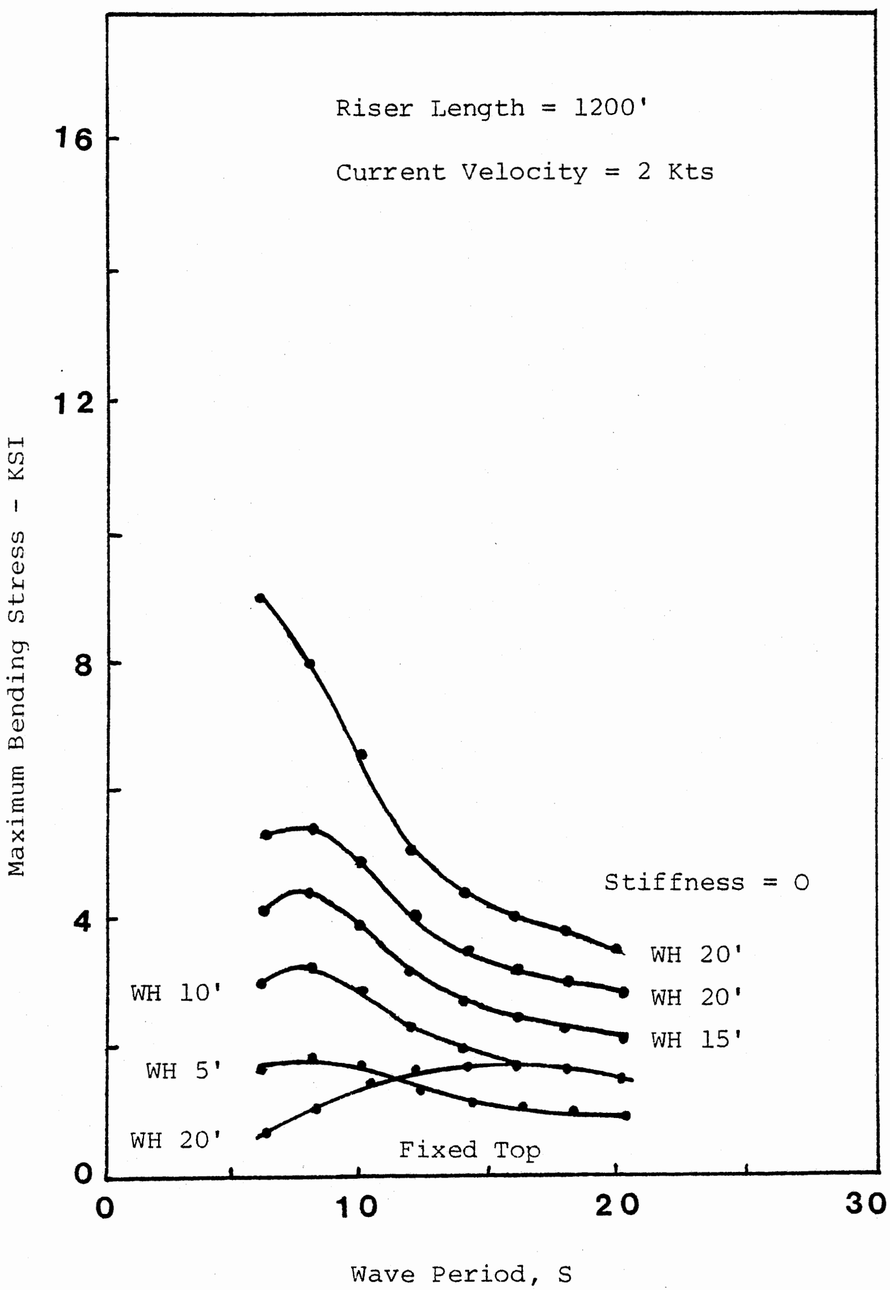

Figure 6. Maximum Bending Stress Versus Wave Period Wave Period for Different Heights [Ref. 15]

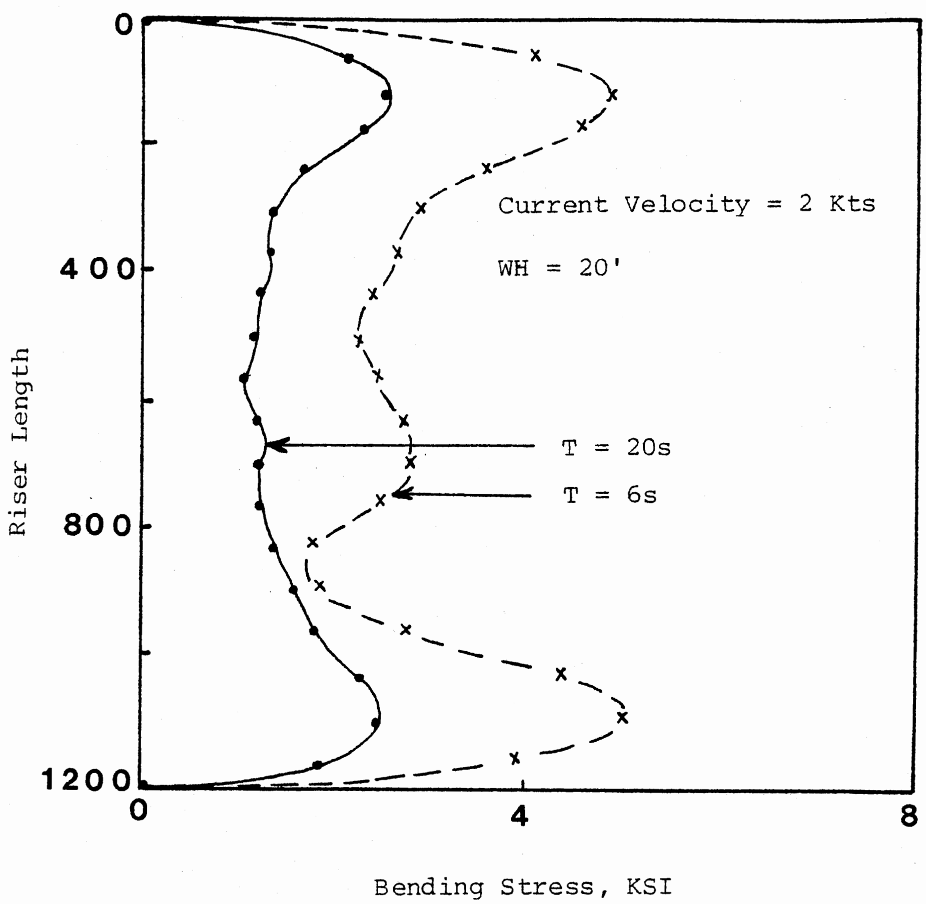

Figure 7. Bending Stress Amplitude Along Riser Length [Ref. 15]

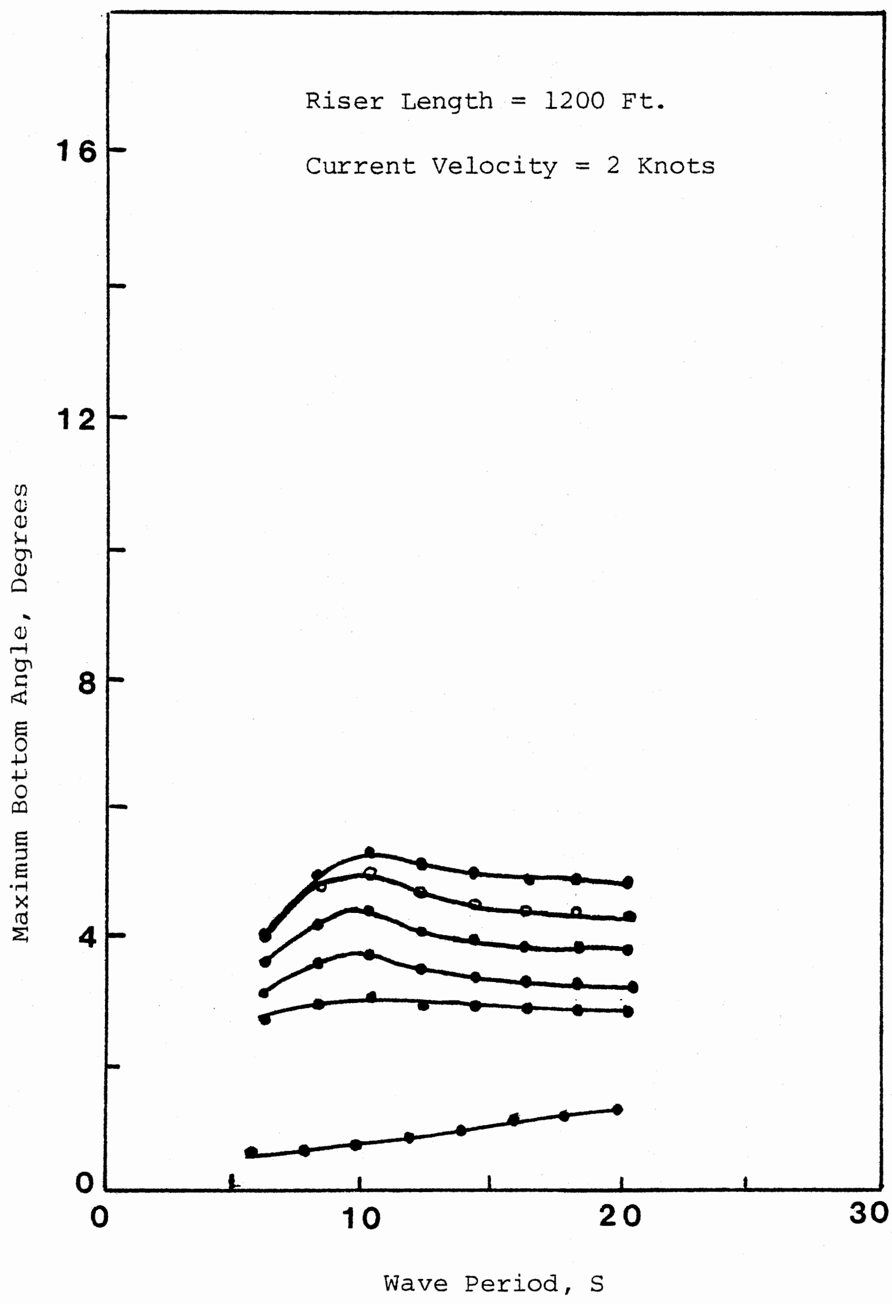

Figure 8. Maximum Bottom Angle Versus Wave Period [Ref. 15]

Current = 1.25 Kts to 0.25 Kts (500 Ft From Top)

Period = 10.3 Sec.

Wave Height = 15 Ft.

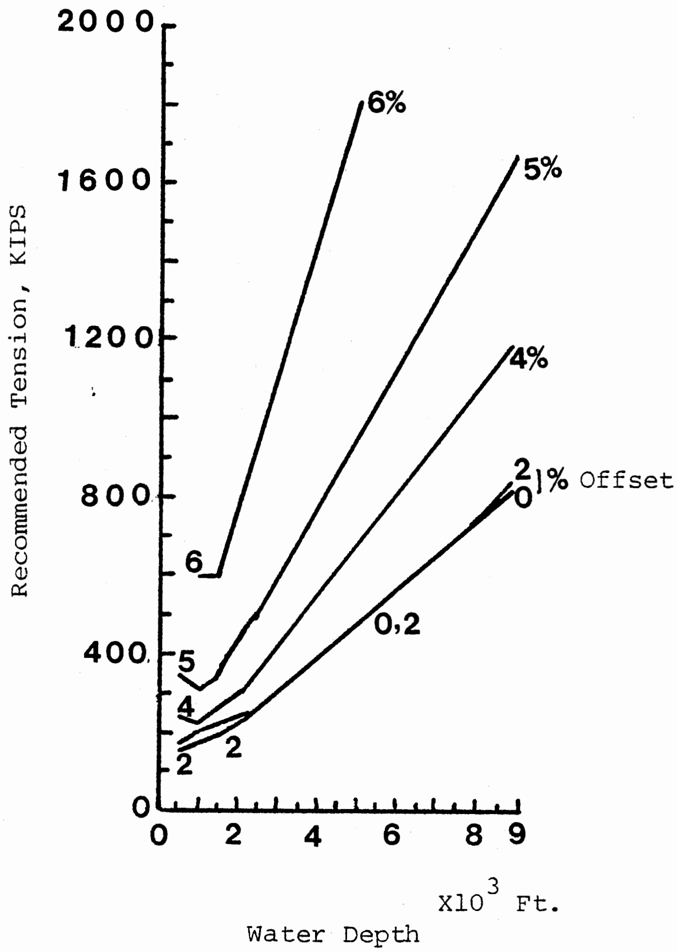

Figure 9. Recommended Top Tension
Versus Water Depth
[Ref. 17]

CHAPTER III

FORMULATION OF THE PROBLEM

In this chapter the governing equation of motion of the marine riser is derived by considering all the forces acting on a differential element as shown in Figure 10. The motion is assumed to be decoupled in the XZ and XY planes. Hence the same argument for the derivation of the governing equation applies in either of the planes.

The various parameters acting on the riser are

1. Top axial tension, varying along length
2. Internal pressure due to mud and fluid
3. External pressure due to sea water
4. Weight of riser acting downwards
5. Lateral fluid loading

Expression for Axial Tension T

The riser is assumed to have constant cross section throughout its length. The riser is supposed to be moving only laterally, i.e. its vertical motion is ignored as this is taken care of by the slip joint at the top.

For equilibrium, the sum of the forces in the Z direction should be equal to zero, therefore,

$$\frac{\partial}{\partial z}(T - p_i A_i) - \{\rho_i A_i + \rho(A_O - A_i)\}g = 0 \qquad (3)$$

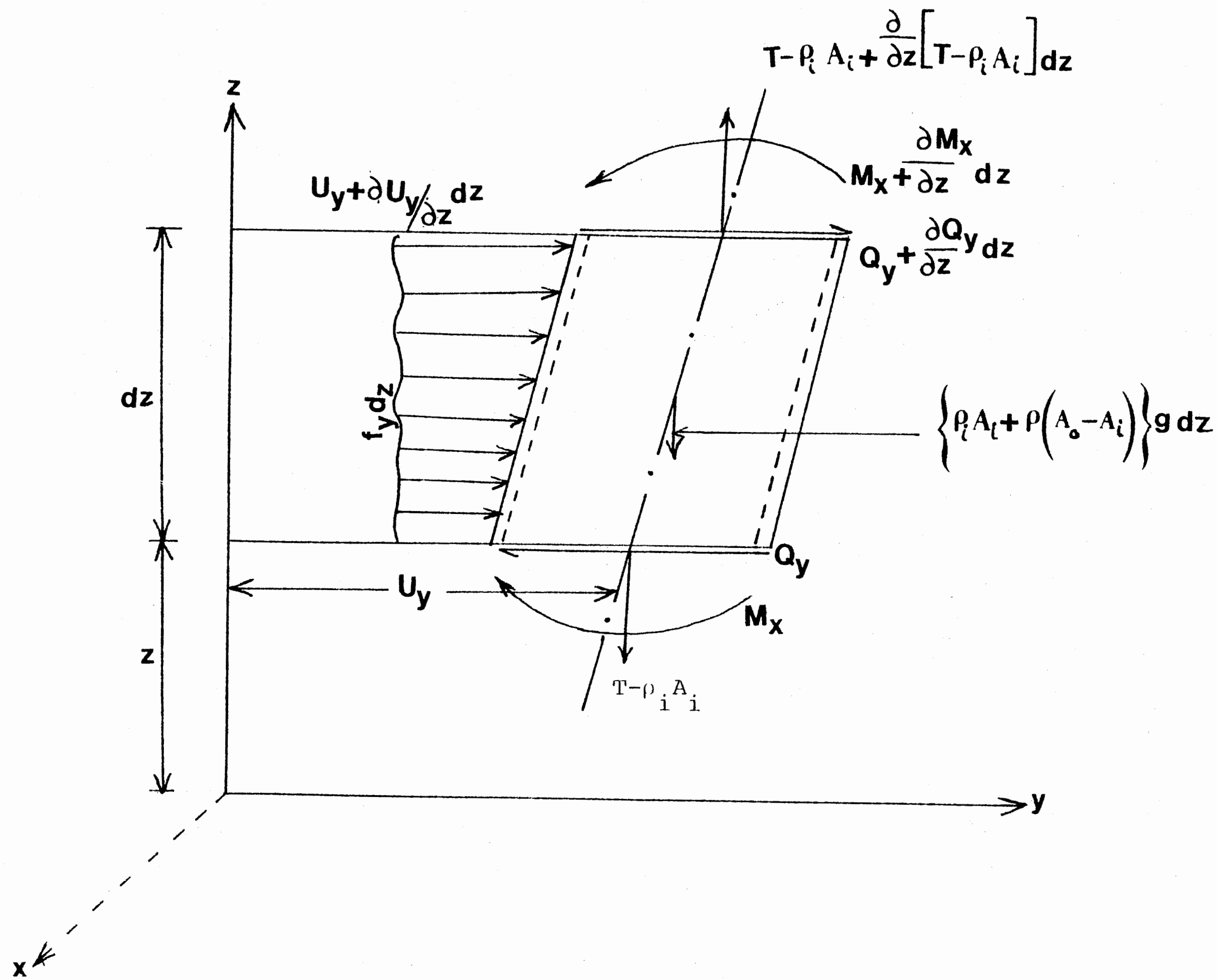

Figure 10. Free Body Diagram of a Differential Beam Element

Integrating with respect to z

$$T - p_i A_i = \{\rho_i A_i + \rho(A_o - A_i)\}gz + C \quad (4)$$

where C is the constant of integration.

Applying boundary conditions at $z = L$, $T = T_o$, $p_i = 0$, substituting in Equation (4) to find the value of C

$$C = T_o - \{\rho_i A_i + \rho(A_o - A_i)\}gL \quad (5)$$

Equation (4) becomes

$$T - p_i A_i = T_o - \rho_i g(L-z)A_i - \rho g(A_o - A_i)(L-z) \quad (6)$$

but $\rho_i g(L-z)A_i = p_i A_i$, therefore,

$$T = T_o - \rho g(A_o - A_i)(L-z) \quad (7)$$

Forces and Moments Due to External Pressure

From Figure 11,

ϕ = the angle between the normal to the surface (pressure direction and the horizontal plane

$= \frac{\partial U_y}{\partial z} \sin\theta$

Force on the elemental area,

$$\vec{dF} = -(p_o r_o d\theta dz)\cos\phi\, \hat{\varepsilon}_r + (p_o r_o d\theta dz)\sin\phi\, \hat{\varepsilon}_z \quad (8)$$

for small ϕ, Equation (8) becomes

$$\vec{dF} = -(p_o r_o d\theta dz)\, \hat{\varepsilon}_r + (p_o r_o d\theta dz)\phi\hat{\varepsilon}_z \quad (9)$$

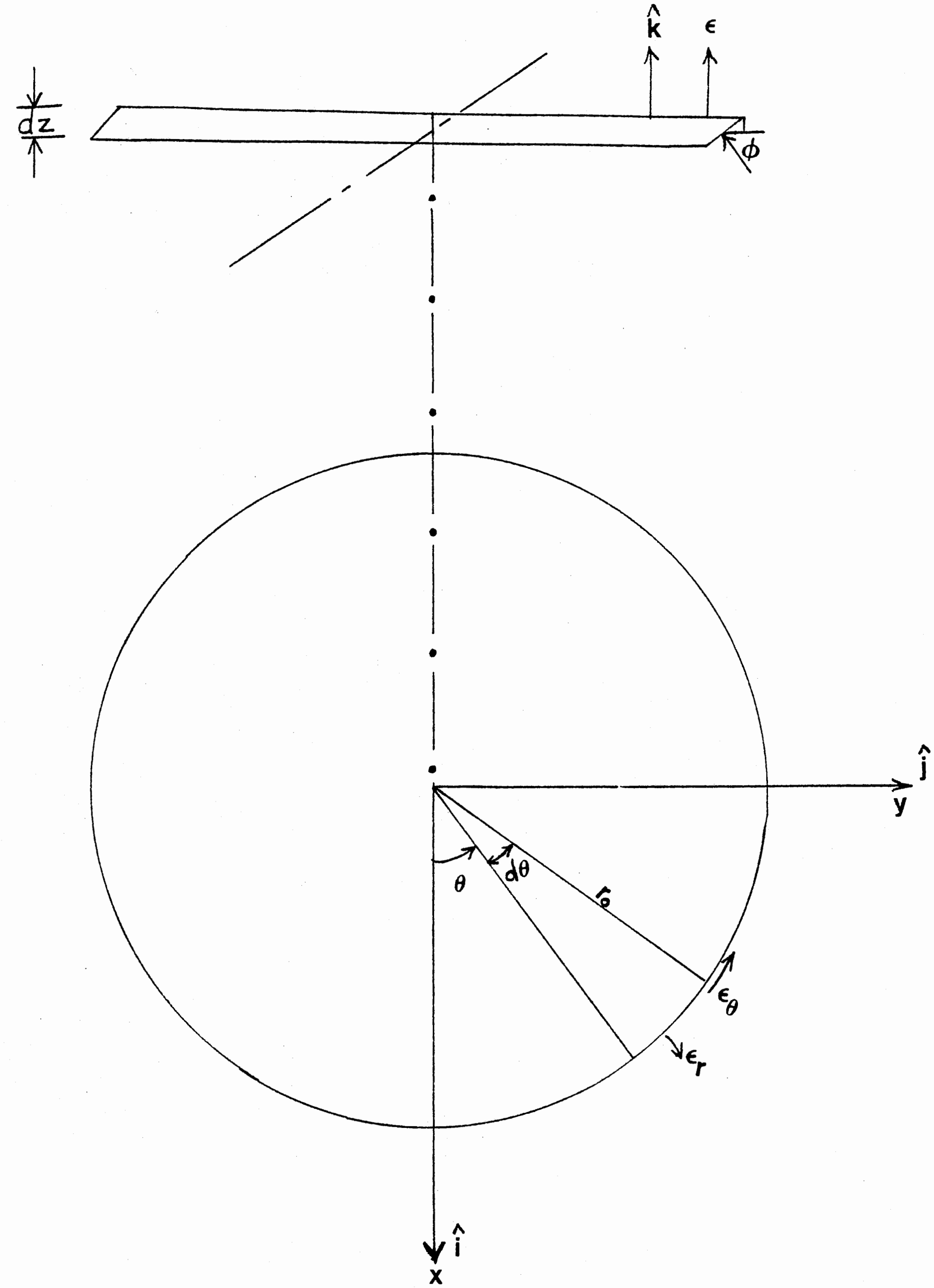

Figure 11. Coordinates for Forces and Moments Due to External Pressure

but $\hat{\varepsilon}_r = \cos\theta\hat{i} + \sin\theta\hat{j}$, $\hat{\varepsilon}_z = \hat{k}$, therefore,

$$d\vec{F} = -(p_o r_o d\theta dz)\cos\theta\hat{i} - (p_o r_o d\theta dz)\sin\theta\hat{j} + (p_o r_o d\theta dz)\frac{\partial U_y}{\partial z}\sin\theta\hat{k} \qquad (10)$$

Moment due to this force about the center of the mass

$$d\vec{m} = \vec{r} \times d\vec{F} \text{ and } \vec{r} = r_o\cos\theta\hat{i} + r_o\sin\theta\hat{j}$$

$$d\vec{m} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_o\cos\theta & r_o\sin\theta & o \\ -(p_o r_o\cos\theta) & -(p_o r_o\sin\theta) & (p_o r_o \frac{\partial U_y}{\partial z}\sin\theta) \end{vmatrix} d\theta dz \qquad (11)$$

$$d\vec{m} = p_o r_o^2 \frac{\partial U_y}{\partial z}\sin^2\theta d\theta dz\hat{i} - p_o r_o^2 \frac{\partial U_y}{\partial z}\sin\theta\cos\theta d\theta z\hat{j} + (-p_o r_o^2 \sin\theta\cos\theta$$

$$+ \; p_o r_o^2 \sin\theta\cos\theta) d\theta dz\hat{k} \qquad (12)$$

Integrating the above forces and moments with respect to θ, varying from o to 2π

$$\int_o^{2\pi} d\vec{F} = o \text{ from Equation (10)} \qquad (13)$$

from Equation (12)

$$\int_o^{2\pi} d\vec{m} = p_o A_o \frac{\partial U_y}{\partial z} dz\,\hat{i} \qquad (14)$$

Equation of Motion

The shear deformation and rotary inertia effects are ignored as the riser is a very slender beam. Then, according to classical beam theory, from Figure 10, summing forces in the Y-direction

$$\frac{\partial U_y}{\partial z} + f_y = \{\rho_i A_i + \rho(A_O - A_i)\}\ddot{U}_y \qquad (15)$$

Summing moments about the top edge

$$\frac{\partial M_x}{\partial z} - Q_y + (T - p_i A_i)\frac{\partial U_y}{\partial z} + P_O A_O \frac{\partial U_y}{\partial z} = 0 \qquad (16)$$

From moment-curvature relation

$$M_x = EI\frac{\partial \theta_x}{\partial z} = -EI\frac{\partial^2 U_y}{\partial z^2} \qquad (17)$$

Differentiating Equation (16) with respect to z

$$\frac{\partial^2 M_x}{\partial z^2} - \frac{\partial Q_y}{\partial z} + (T - p_i A_i)\frac{\partial^2 U_y}{\partial z^2} + p_O A_O \frac{\partial^2 U_y}{\partial z^2} + \{\rho g(A_O - A_i) + \rho_i g A_i\}\frac{\partial U_y}{\partial z}$$

$$- \rho_O g A_O \frac{\partial U_y}{\partial z} = 0 \qquad (18)$$

From Equations (15), (17) and (18)

$$\{\rho_i A_i + \rho(A_O - A_i)\}\ddot{U}_y + EI\frac{\partial^4 U_y}{\partial z^4} - \{T_O - \rho g(A_O - A_i)(L-z)\}\frac{\partial^2 U_y}{\partial z^2} - (p_O A_O - p_i A_i)$$

$$x\frac{\partial^2 U_y}{\partial z^2} - \rho g(A_O - A_i)\frac{\partial U_y}{\partial z} + (\rho_O A_O - \rho_i A_i)g\frac{\partial U_y}{\partial z} = f_y \qquad (19)$$

Rearranging, the governing equation is

$$\{\rho_i A_i + \rho(A_O - A_i)\}\ddot{U}_y + EI\,\frac{\partial^4 U_y}{\partial z^4} - \{T_O - \rho g(A_O - A_i)(L-Z)\}\,\frac{\partial^2 U_y}{\partial z^2} - (\rho_O A_O - \rho_i A_i)$$

$$\times\ (L-z)g\,\frac{\partial^2 U_y}{\partial z^2} - \rho g(A_O - A_i)\,\frac{\partial U_y}{\partial z} + (\rho_O A_O - \rho_i A_i)g\,\frac{\partial U_y}{\partial z} = f_y \qquad (20)$$

where

$$f_y = \frac{\pi}{4}\rho_O C_m D_O^2\,\frac{\partial^2 \ddot{U}_w}{\partial t^2} - \frac{\pi}{4}\rho_O (C_m - 1) D_O^2\,\frac{\partial^2 U_y}{\partial t^2} + \frac{1}{2}\rho_O D_O C_D\,\left|\dot{U}_w + \dot{U}_c - \frac{\partial U_y}{\partial t}\right|$$

$$\times\ (\dot{U}_w + \dot{U}_c - \frac{\partial U_y}{\partial t}) \qquad (21)$$

Similarly considering XZ plane we have, the governing equation as,

$$\{\rho_i A_i + \rho(A_O - A_i)\}\ddot{U}_x + EI\,\frac{\partial^4 U_x}{\partial z^4} - \{T_O - \rho g(A_O - A_i)(L-z)\}\,\frac{\partial^2 U_x}{\partial z^2} - (\rho_O A_O - \rho_i A_i)$$

$$\times\ (L-z)g\,\frac{\partial^2 U_x}{\partial z^2} - \rho g(A_O - A_i)\,\frac{\partial U_x}{\partial z} + (\rho_O A_O - \rho_i A_i)g\,\frac{\partial U_x}{\partial z} = f_x \qquad (22)$$

The above fourth order differential equation has been solved using finite element method after considering the drag term to be non-linear. This is described in later chapters.

CHAPTER IV

FINITE ELEMENT FORMULATION

The mathematical model considered in the analysis treats the riser pipe as an assembly of beam elements of the form shown in Figure 12. Each element possesses four degrees of freedom (see Figure 13) with one translation and one rotation at each end.

The axial load on the riser is due to the applied tension at the top which prevents the pipe from buckling under its own weight and enables its deflections and stresses to be controlled. The magnitude of this axial tension varies along the length due to the counteracting effect of the riser self weight. The effective tension acting on each beam element is therefore estimated by subtracting the weight of the riser pipe above the element from the applied top tension. The weight of the inner fluid is neglected in these calculations as they do not contribute to the net axial forces.

The lateral load intensity and consequent riser deflection and stresses are primarily influenced by top vessel offset, current and wave velocities. The non-linear hydrodynamic exciting force is taken to be a modified form of Morrison's equation including mass and drag coefficients.

The bottom end, i.e. the first node that coincides with the riser system, has zero translational displacement. However, it is free to rotate. At the top, the nth node which coincides with the vessel bottom is assumed to have a sinusoidal displacement.

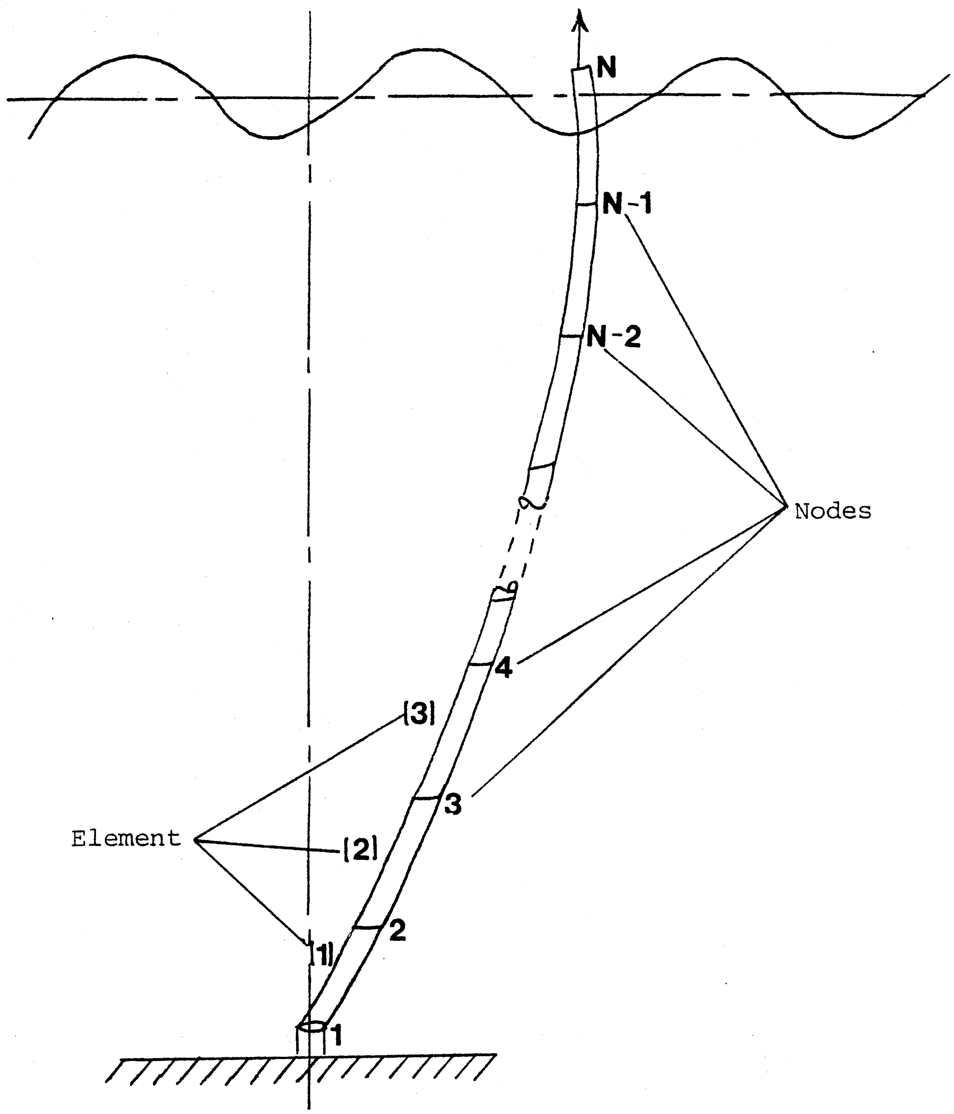

Figure 12. Element and Global Node Description

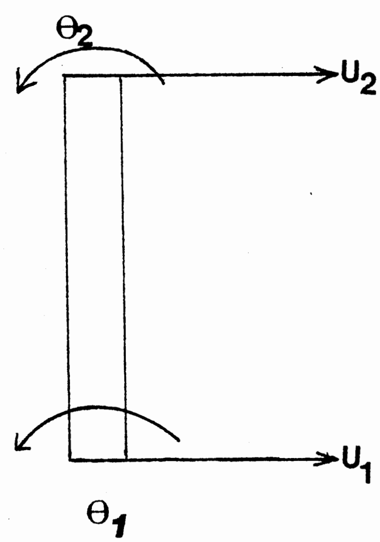

Figure 13. Beam Element Showing DOF

Galerkin's Technique

Before applying finite element method, the differential equation is rendered in an integral form. Since the non-linear forcing term cannot be represented by a potential term, the variational method cannot be used. Hence Galerkin's technique is adopted. According to this method the differential equation is successively multiplied by the shape functions and integrated over the domain. The residue is then set to zero. The various terms in the differential equation give rise to the mass, conventional stiffness, geometric stiffness, damping matrices and force vector.

Element Property Formulation

The element mass, damping and stiffness matrices along with the force vector are derived, considering Hermitian interpolation function. The shape functions are derived in the local coordinate (ξ) which is related to the global coordinate (z) as

$$z = (\ell_2 - \ell_1)\xi + \ell_1$$

The cubic polynomial is

$$U = a_o + a_1\xi + a_2\xi^2 + a_3\xi^3 \qquad (26)$$

Evaluating the constants a_o, a_1, a_2 and a_3 for $o < \xi < 1$, the shape functions arrived at, in local coordinates are

$$N_1(\xi) = 1 - 3\xi^2 + 2\xi^3$$

$$N_2(\xi) = 1(\xi - 2\xi^2 + \xi^3)$$

$$N_3(\xi) = 3\xi^2 - 2\xi^3$$

$$N_4(\xi) = l(-\xi^2+\xi^3)$$

Deflection $U_y(\xi,t) = U_i N_1(\xi) + \Theta_i N_2(\xi) + U_{i+1} N_3(\xi) + \Theta_{i+1} N_4(\xi)$ (27)

where U_i and Θ_i are ith node displacement and rotation.

Mass Matrix

In the formulation of the beam element mass matrix, the consistent mass approach is used. This leads to greater accuracy compared to lumped mass approach. From Equation (20), multiplying the inertial term $\{\rho_i A_i + \rho(A_O - A_i)\}\ddot{U}_y$ with N_i, i = 1,2,3,4 and integrating for ξ varying from 0 to 1, the mass matrix is formed. Differentiating Equation (27) twice with respect to time we get $\ddot{U}_y$.

$$\ddot{U}_y(\xi,t) = \ddot{U}_i N_1(\xi) + \ddot{\Theta}_i N_2(\xi) + \ddot{U}_{i+1} N_3(\xi) + \ddot{\Theta}_{i+1} N_4(\xi) \qquad (28)$$

substituting in the inertial term and integrating, let $m = \rho_i A_i + \rho(A_O - A_i)$, then, with $N_1(\xi)$

$$m \int_0^1 \{\ddot{U}_1(1-3\xi^2+2\xi^2) + \ddot{\Theta}_1 \ell(\xi-2\xi^2+\xi^3) + \ddot{U}_2(3\xi^2-2\xi^3) + \ddot{\Theta}_2 \ell(-\xi^2+\xi^3)\}$$

$$\times \{1-3\xi^2+2\xi^3\}\ell \, d\xi = m\{\frac{13}{35} \ell\ddot{U}_1 + \frac{22}{420} \ell^2\ddot{\Theta}_1 + \frac{9}{70} \ell\ddot{U}_2 - \frac{13}{420} \ell^2\ddot{\Theta}_2\} \qquad (29)$$

with $N_2(\xi)$

$$m \int_0^1 \{\ddot{U}_1(1-3\xi^2+2\xi^3) + \ddot{\Theta}_1 \ell(\xi-2\xi^2+\xi^3) + \ddot{U}_2(3\xi^2-2\xi^3) + \ddot{\Theta}_2 \ell(-\xi^2+\xi^3)\}$$

$$\times \{\xi - 2\xi^2+\xi^3\} \ell d\xi = m\ell^2\{\frac{22}{420} \ddot{U}_1 + \frac{4}{420} \ell\ddot{\Theta}_1 + \frac{13}{420} \ddot{U}_2 - \frac{3}{420} \ell\ddot{\Theta}_2\} \qquad (30)$$

with $N_3(\xi)$

$$m \int_0^1 \{\ddot{U}_1(1-3\xi^2+2\xi^3) + \ddot{\Theta}_1 \ell(\xi-2\xi^2+\xi^3) + \ddot{U}_2(3\xi^2-2\xi^3) + \ddot{\Theta}_2 \ell(-\xi^2+\xi^3)\}$$

$$\times \{3\xi^2-2\xi^3\}\ell \, d\xi = m\ell\{\frac{54}{420} \ddot{U}_1 + \frac{13}{420} \ell\ddot{\Theta}_1 + \frac{156}{420} \ddot{U}_2 - \frac{22}{420} \ell\ddot{\Theta}_2\} \quad (31)$$

with $N_4(\xi)$

$$m \int_0^1 \{\ddot{U}_1(1-3\xi^2+2\xi^3) + \ddot{\Theta}_1 \ell(\xi-2\xi^2+\xi^3) + \ddot{U}_2(3\xi^2-2\xi^3) + \ddot{\Theta}_2 \ell(-\xi^2+\xi^3)\}$$

$$\times \{-\xi^2+\xi^3\}\ell \, d\xi = m\ell^2\{- \frac{13}{420} \ddot{U}_1 - \frac{3}{420} \ell\ddot{\Theta}_1 - \frac{22}{420} \ddot{U}_2 + \frac{4}{420} \ell\ddot{\Theta}_2\} \quad (32)$$

From Equations (29), (30), (31) and (32), the mass matrix is

$$\frac{ml}{420} \begin{bmatrix} 156 & 22\ell & 54 & -13\ell \\ 22\ell & 4\ell^2 & 13\ell & -3\ell^2 \\ 54 & 13\ell & 156 & -22\ell \\ -13\ell & -3\ell^2 & -22\ell & 4\ell^2 \end{bmatrix}$$

<u>Conventional Stiffness Matrix</u>

From Equation (21) consider the term, $EI \dfrac{\partial^4 U_y}{\partial z^4}$. Multiplying with $N_i(\xi)$ and integrating from 0 to 1

$$\frac{EI}{\ell^3} \int_0^1 \frac{\partial}{\partial\xi} \left(\frac{\partial^3 U_y}{\partial\xi^3}\right) N_i(\xi)\, d\xi = \frac{EI}{\ell^3} \left[\{\frac{\partial^3 U_y}{\partial\xi^3} N_i(\xi)\}\right]_0^1 - \frac{EI}{\ell^3} \int_0^1 \frac{\partial^3 U_y}{\partial\xi^3} \frac{\partial N_i(\xi)}{\partial\xi}\, d\xi$$

$$= \frac{EI}{\ell^3} \left[\{\frac{\partial^3 U_y}{\partial\xi^3} N_i(\xi)\}\right]_0^1 - \frac{EI}{\ell^3} \left[\{\frac{\partial^2 U_y}{\partial\xi^2} \frac{\partial N_i}{\partial\xi}\}\right]_0^1 + \frac{EI}{\ell^3} \int_0^1 \frac{\partial^2 U_y}{\partial\xi^2} \frac{\partial^2 N_i(\xi)}{\partial\xi^2}\, d\xi$$

The first two terms of Equation (33) cancel out when added for all the elements. Hence only the 3rd term needs to be evaluated,

with $N_1(\xi)$

$$\frac{EI}{\ell^3}\int_0^1 \{U_1(-6+12\xi) + \Theta_1\ell(-4+6\xi) + U_2(6-12\xi) + \Theta_2\ell(-2+6\xi)\}\{-6+12\xi\}$$

$$\times\ d\xi = \frac{EI}{\ell^3}\{12U_1+6\ell\Theta_1 - 12U_2+6\ell\Theta_2\} \qquad (34)$$

with $N_2(\xi)$

$$\frac{EI}{\ell^3}\int_0^1 \{U_1(-6+12\xi) + \Theta_1\ell(-4+6\xi) + U_2(6-12\xi) + \Theta_2\ell(-2+6\xi)\}\{-4+6\xi\}$$

$$\times\ \ell d\xi = \frac{EI}{\ell^3}\{6\ell U_1+4\ell^2\Theta_1-6\ell U_2+2\ell^2\Theta_2\} \qquad (35)$$

with $N_3(\xi)$

$$\frac{EI}{\ell^3}\int_0^1 \{U_1(-6+12\xi) + \Theta_1\ell(-4+6\xi) + U_2(6-12\xi) + \Theta_2\ell(-2+6\xi)\}\{6-12\xi\}$$

$$\times\ d\xi = \frac{EI}{\ell^3}\{12U_1-6\ell\Theta_1+12U_2-6\ell\Theta_2\} \qquad (36)$$

with $N_4(\theta)$

$$\frac{EI}{\ell^3}\int_0^1 \{U_1(-6+12\xi) + \Theta_1\ell(-4+6\xi) + U_2(6-12\xi) + \Theta_2\ell(-2+6\xi)\}\{-2+6\xi\}$$

$$\times\ \ell d\xi = \frac{EI}{\ell^3}\{6\ell U_1+2\ell^2\Theta_1-6\ell U_2+4\ell^2\Theta_2\} \qquad (37)$$

From Equations (34), (35), (36) and (37), the conventional stiffness matrix is expressed as

$$\frac{EI}{\ell^3}\begin{bmatrix} 12 & 6\ell & -12 & 6\ell \\ 6\ell & 4\ell^2 & -6\ell & 2\ell^2 \\ -12 & -6\ell & 12 & -6\ell \\ 6\ell & 2\ell^2 & -6\ell & 4\ell^2 \end{bmatrix}$$

Geometric Stiffness Matrix

From Equation (20) consider terms

$$-\{T_O-\rho g(A_O-A_i)(L-z)\}\frac{\partial^2 U_y}{\partial z^2} - \rho g \frac{\partial U_y}{\partial z}$$

Multiplying with $N_i(\xi)$ and integrating over O to 1

$$-\frac{1}{\ell} - \int_O^1 \{T_O-\rho g(A_O-A_i)(L-\ell\xi-\ell_1)\}\, N_i(\xi)\, \frac{\partial^2 U_y}{\partial \xi^2}\, d\xi - \int_O^1 \rho g(A_O-A_i)N_i(\xi)$$

$$x\, \frac{\partial U_y}{\partial \xi}\, d\xi = -\frac{1}{\ell}\{T_O-\rho g(A_O-A_i)(L-\ell\xi-\ell_1)\}\{\frac{\partial U_y}{\partial \xi}\} + \frac{1}{\ell}\int_O^1 \frac{\partial U_y}{\partial \xi}$$

$$x\, \frac{\partial}{\partial \xi}\{T_O-\rho g(A_O-A_i)(L-\ell\xi-\ell_1)\}\{N_i(\xi)\}\, d\xi - \int_O^1 \rho g(A_O-A_i)N_i(\xi)\, \frac{\partial U_y}{\partial \xi}\, d\xi \qquad (38)$$

First term of Equation (38) when added for all elements, cancels out. From 2nd and 3rd terms we have

$$\frac{1}{\ell}\int_O^1 \{T_O-\rho g(A_O-A_i)(L-\ell\xi-\ell_1)\}\frac{\partial U_y}{\partial \xi}\; \frac{\partial N_i(\xi)}{\partial \xi} + \frac{\partial}{\partial \xi}\{T_O-\rho g(A_O-A_i)(L-\ell\xi-\ell_1)\}$$

$$x - \int_O^1 \rho g(A_O-A_i)N_i(\xi)\, \frac{\partial U_y}{\partial \xi}\, d\xi = \frac{1}{\ell}\int_O^1 \{T_O-\rho g(A_O-A_i)(L-\ell\xi-\ell_1)\}$$

$$x \frac{\partial U_y}{\partial \xi} \frac{\partial N_i(\xi)}{\partial \xi} d\xi + \int_0^1 \rho g(A_o-A_i)N_i(\xi) \frac{\partial U_y}{\partial \xi} - \int_0^1 \rho g(A_o-A_i)N_i(\xi)$$

$$x \frac{\partial U_y}{\partial \xi} d\xi = \frac{1}{\ell} \int_0^1 \{T_o-\rho g(A_o-A_i)(L-\ell\xi-\ell_1)\} \frac{\partial U_y}{\partial \xi} \frac{\partial N_i(\xi)}{\partial \xi} d\xi \qquad (39)$$

Similarly considering the terms

$$-(\rho_o A_o-\rho_i A_i)(L-z)g \frac{\partial^2 U_y}{\partial z^2} + (\rho_o A_o-\rho_i A_i)g \frac{\partial U_y}{\partial z}$$

we will be only left with the term

$$\frac{1}{\ell} \int_0^1 (\rho_o A_o-\rho_i A_i)(L-\ell\xi-\ell_1)g \frac{\partial U_y}{\partial \xi} \frac{\partial N_i(\xi)}{\partial \xi} d\xi \qquad (40)$$

Multiplying with $N_i(\xi)$, $i = 1,2,3,4$ and integrating over 0 to 1, both Equations (39) and (40). Let us first consider the Equation (39) with $N_1(\xi)$

$$\frac{1}{\ell} \int_0^1 \{T_o-\rho g(A_o-A_i)(L-\ell\xi-\ell_1)\}\{U_1(-6\xi+6\xi^2)+\Theta_1(1-4\xi+3\xi^2)\ell+U_2(6\xi-6\xi^2)$$

$$+ \Theta_2(-2\xi+3\xi^2)\ell\}\{-6\xi+6\xi^2\} d\xi = \frac{1}{\ell} A(\frac{6}{5} U_1 + \frac{\ell}{10} \Theta_1 - \frac{6}{5} U_2 + \frac{\ell}{10} \Theta_2)$$

$$+ B(\frac{3}{5} U_1 - \frac{9}{10} 1\Theta_1 - \frac{3}{5} U_2) \qquad (41)$$

where $A = T_o - \rho g(A_o-A_i)(L-1)$ and $B = \rho g\ell(A_o-A_i)$

with $N_2(\xi)$

$$\int_0^1 (A+B\xi)\{U_1(-6\xi+6\xi^2)+\Theta_1\ell(1-4\xi+3\xi^2) + U_2(6\xi-6\xi^2)+\Theta_2\ell(-2\xi+3\xi^2)\}$$

$$x \{1-4\xi+3\xi^2\} d\xi = A(\frac{1}{10} U_1 + \frac{2}{15} \ell\Theta_1 - \frac{1}{10} U_2 - \frac{1}{30} \ell\Theta_2)$$

$$+ B(\frac{1}{10} U_1 + \frac{1}{30} \ell\Theta_1 - \frac{1}{10} U_2 - \frac{1}{60} \ell\Theta_2) \qquad (42)$$

with $N_3(\xi)$

$$\frac{1}{\ell} \int_0^1 (A+B\xi)\{U_1(-6\xi+6\xi^2)+\Theta_1(1-4\xi+3\xi^2)\ell+U_2(6\xi-6\xi^2)+\Theta_2(-2\xi+3\xi^2)\ell\}$$

$$\times \{6\xi-6\xi^2\}\, d\xi = \frac{A}{\ell} (- \frac{6}{5} U_1 - \frac{1}{10} \Theta_1 + \frac{6}{5} U_2 - \frac{1}{10} \Theta_2)$$

$$+ \frac{B}{\ell}(- \frac{3}{5} U_1 + \frac{9}{10} \ell\Theta_1 + \frac{3}{5} U_2+0) \qquad (43)$$

with $N_4(\xi)$

$$\frac{1}{\ell} \int_0^1 (A+B\xi)\{U_1(-6\xi+6\xi^2)+\Theta_1(1-4\xi+3\xi^2)\ell+U_2(6\xi-6\xi^2)+\Theta_2(-2\xi+3\xi^2)\ell\}$$

$$\times \{-2\xi+3\xi^2)\ \ell d\xi = A(\frac{1}{10} U_1 - \frac{1}{10} \ell\Theta_1 - \frac{1}{10} U_2 + \frac{2}{15} \ell\Theta_2)$$

$$+ B(0+0 - \frac{1}{60} \Theta_1 + \frac{1}{10} \ell\Theta_2) \qquad (44)$$

From Equations (41), (42), (43) and (44), we arrive with the matrices

$$\frac{A}{\ell}\begin{bmatrix} \frac{6}{5} & \frac{1}{10}\ell & -\frac{6}{5} & \frac{1}{10}\ell \\ \frac{1}{10}\ell & \frac{2}{15}\ell^2 & -\frac{1}{10}\ell & -\frac{1}{30}\ell^2 \\ -\frac{6}{5} & -\frac{1}{10}\ell & \frac{6}{5} & -\frac{1}{10}\ell \\ \frac{1}{10}\ell & -\frac{1}{30}\ell^2 & -\frac{1}{10}\ell & \frac{2}{15}\ell^2 \end{bmatrix} + \frac{B}{\ell}\begin{bmatrix} \frac{3}{5} & \frac{1}{10}\ell & -\frac{3}{5} & 0 \\ \frac{1}{10}\ell & \frac{1}{30}\ell^2 & -\frac{1}{10}\ell & -\frac{1}{60}\ell^2 \\ -\frac{3}{5} & -\frac{1}{10}\ell & \frac{3}{5} & 0 \\ 0 & -\frac{1}{60}\ell^2 & 0 & \frac{1}{10}\ell^2 \end{bmatrix}$$

From Equation (40)

Let $C = (\rho_o A_o - \rho_i A_i) g (L - \ell_1)$ and $D = -(\rho_o A_o - \rho_i A_i) g \ell$

Except for the coefficients A, B, C and D the rest is the same as in Equation (39), hence the matrices can be written as above. The coefficients $\frac{A}{\ell}$ and $\frac{B}{\ell}$ are replaced by $\frac{C}{\ell}$ and $-\frac{D}{\ell}$ respectively. Thereafter, the four matrices can be combined to form two geometric stiffness matrices. Now the new coefficient $\frac{A + C}{\ell}$ takes into account the varying top tension and the other coefficient $\frac{B - D}{\ell}$ accounts for the effect of internal and external fluid pressure.

$$\frac{A+B}{\ell}\begin{bmatrix} \frac{6}{5} & \frac{1}{10}\ell & -\frac{6}{5} & \frac{1}{10}\ell \\ \frac{1}{10}\ell & \frac{2}{15}\ell^2 & -\frac{1}{10}\ell & -\frac{1}{30}\ell^2 \\ -\frac{6}{5} & -\frac{1}{10}\ell & \frac{6}{5} & -\frac{1}{10}\ell \\ \frac{1}{10}\ell & -\frac{1}{30}\ell^2 & -\frac{1}{10}\ell & \frac{2}{15}\ell^2 \end{bmatrix} + \frac{B-C}{\ell}\begin{bmatrix} \frac{3}{5} & \frac{1}{10}\ell & -\frac{3}{5} & 0 \\ \frac{1}{10}\ell & \frac{1}{30}\ell^2 & -\frac{1}{10}\ell & -\frac{1}{60}\ell^2 \\ -\frac{3}{5} & -\frac{1}{10}\ell & \frac{3}{5} & 0 \\ 0 & -\frac{1}{60}\ell^2 & 0 & \frac{1}{10}\ell \end{bmatrix}$$

Damping Matrix

The damping matrix is derived from the non-linear drag term. The relative velocity squared term is successively multiplied with $N_i(\xi)$, $i = 1, 2, 3, 4$ and integrated to form the matrix. The absolute value of the relative velocity within the matrix necessitates the computation of the matrix at every time step. All these have been included in the computer program which is described later.

Force Vector

The non-linear drag term is

$$\frac{\pi}{4}\rho_O C_m D_O^2 \ddot{U}_w - \frac{\pi}{4}\rho_O (C_m - 1)\, D_O^2 \frac{\partial^2 U_y}{\partial t^2} + \frac{1}{2}\rho_O D_O C_D \left| \dot{U}_w + \dot{U}_c - \frac{\partial U_y}{\partial t} \right| (\dot{U}_w + \dot{U}_c - \frac{\partial U_y}{\partial t})$$

The force vector comprises a part of the relative velocity squared term and also the wave particle acceleration term. These are described further in later chapter dealing with computer program.

CHAPTER V

COMPUTER PROGRAM

The computer program used in the dynamic analysis is written in standard FORTRAN IV. The program has been written with the intention that every aspect of riser geometry, structure, top and bottom constraints, wave profile, current conditions be specified as input data. This leads to a program which requires comprehensive input data on the riser to be analyzed and at the same time allows a wide variety of configurations to be calculated without program changes.

The element property matrices such as mass, conventional stiffness, geometric stiffness matrices are time independent and are formulated and stored in separate subroutines. The subroutine used are MASS, STIFN and GESTFN respectively. A fifth order Gauss-Legendre integration scheme is used to formulate the damping matrix. The shape functions are computed in subroutine SHAPE. These element matrices are assembled for all the elements over the riser length in the subroutine GLOBAL. All the matrices except the damping matrix are assembled just once. The damping matrix is assembled at every time step. To reduce the storage, the global matrix is assembled and stored in a banded upper triangular form. The 4x4 element matrices are symmetrical and hence the band-width (MBAND) of the global matrix is four. The matrix is of the order NEQ by MBAND, where NEQ is the total degrees of freedom in the entire riser. Subroutine INPUT is formulated to read and echo check all the input parameters used for the analysis.

Boundary Conditions

The main program allows various combinations of boundary conditions to be specified at the highest and lowest riser nodes. This is done by including an ID array which defines the number of restraints, restrained degrees of freedom and so on for the riser configuration under consideration. A free degree of freedom is assigned a zero value and a restrained degree of freedom is specified as one. The lowest riser node has free rotational degree of freedom with the translational degree of freedom restrained. This condition is satisfied by assigning an added rotational stiffness value to the corresponding stiffness term. The highest node is assumed to be acted upon by a forcing function resulting due to vessel motion which in turn is dependent on the wave condition.

Time Integration

Newmark's time integration scheme is considered to be suitable for an analysis of this type. Table I shows briefly the implementation procedure. The non-linear drag term creates, by the finite element formulation, a non-linear damping matrix and a non-linear force vector. Since these two are time dependent, they have been separately computed and stored in subroutine DAMP. Subroutine NEWMAR which consists of Newmark's integration scheme, calls subroutine DAMP at every time step such that damping matrix and force vector are computed at that instant.

Subroutine NEWMAR computes the displacement and rotation at every time step. The system of equation stored in the banded form is solved by calling subprograms DECOM and SOLBAN. Subroutine DECOM decomposes

TABLE I

NEWMARK'S INTEGRATION SCHEME

1. Initialize $\{X\}_o$, $\{\dot{X}\}_o$ and $\{\ddot{X}\}_o$ to zero.

2. Set $\delta = 1/2$, $\alpha = 1/4$

$a_o = 1/(\alpha \cdot \Delta t^2)$, $a_1 = \delta/(\alpha \cdot \Delta t)$, $a_2 = 1/(\alpha \cdot \Delta t)$

$a_3 = 1/(2\alpha)-1$, $a_4 = (\delta/\alpha - 1)$, $a_5 = (\delta/\alpha - 2)\cdot \Delta t/2$

$a_6 = (1-\delta)\cdot \Delta t$, $a_7 = \delta \cdot \Delta t$

3. Calculate $\{\hat{F}\} = \{F\}_t + [M]_t\ (a_o\{X\}_{t-\Delta t} + a_2\{X\}_{t-\Delta t} + a_3\{X\}_{t-\Delta t})$

$+ [C]_t\ (a_1\{X\}_{t-\Delta t} + a_4\{X\}_{t-\Delta t} + a_5\{X\}_{t-\Delta t})$

4. Solve $([K]_t + a_o[M]_t + a_1[C]_t)\{X\}_t = \{\hat{F}\}_t$

5. Compute $\{X\}_t = a_o(\{X\}_t - \{X\}_{t-\Delta t}) - a_2\{X\}_{t-\Delta t} - a_3\{X\}_{t-\Delta t}$

$\{X\}_t = \{X\}_{t-\Delta t} + a_6\{X\}_{t-\Delta t} + a_7\{X\}_t$

6. Repeat from Step 3 for all intervals

the band matrix into an upper triangular matrix using the Gaussian elimination procedure. SOLBAN first decomposes the global force vector and then solves for the displacements and rotations using the method of backward substitution.

Check Problems

The program is checked thoroughly for proper formulation. The various checks for the individual element property matrix and the assembled global form are the following:

Mass and Stiffness Matrix

The mass and the stiffness matrices are checked by solving an eigenvalue problem. The free vibration of a simply supported beam is analyzed by suppressing the end displacements. The values and the subsequent mode shapes are compared with the theoretically calculated values and are as shown in Figure 14 and Figure 15. The end conditions are changed and the riser is considered as fixed at the bottom and sliding at the top. The eigenvalues and eigenvectors are solved by a simultaneous iteration scheme. The comparison of the results with the theoretical values show a good agreement. The above checks sufficiently validate the authenticity of the mass and stiffness matrix formulation.

Geometric Stiffness Matrix

The geometric stiffness matrix is checked by solving a buckling problem. The values of the buckling load at every mode is compared with the theoretical values and were found to agree well. The mode shape comparison are shown in Figure 16 and Figure 17.

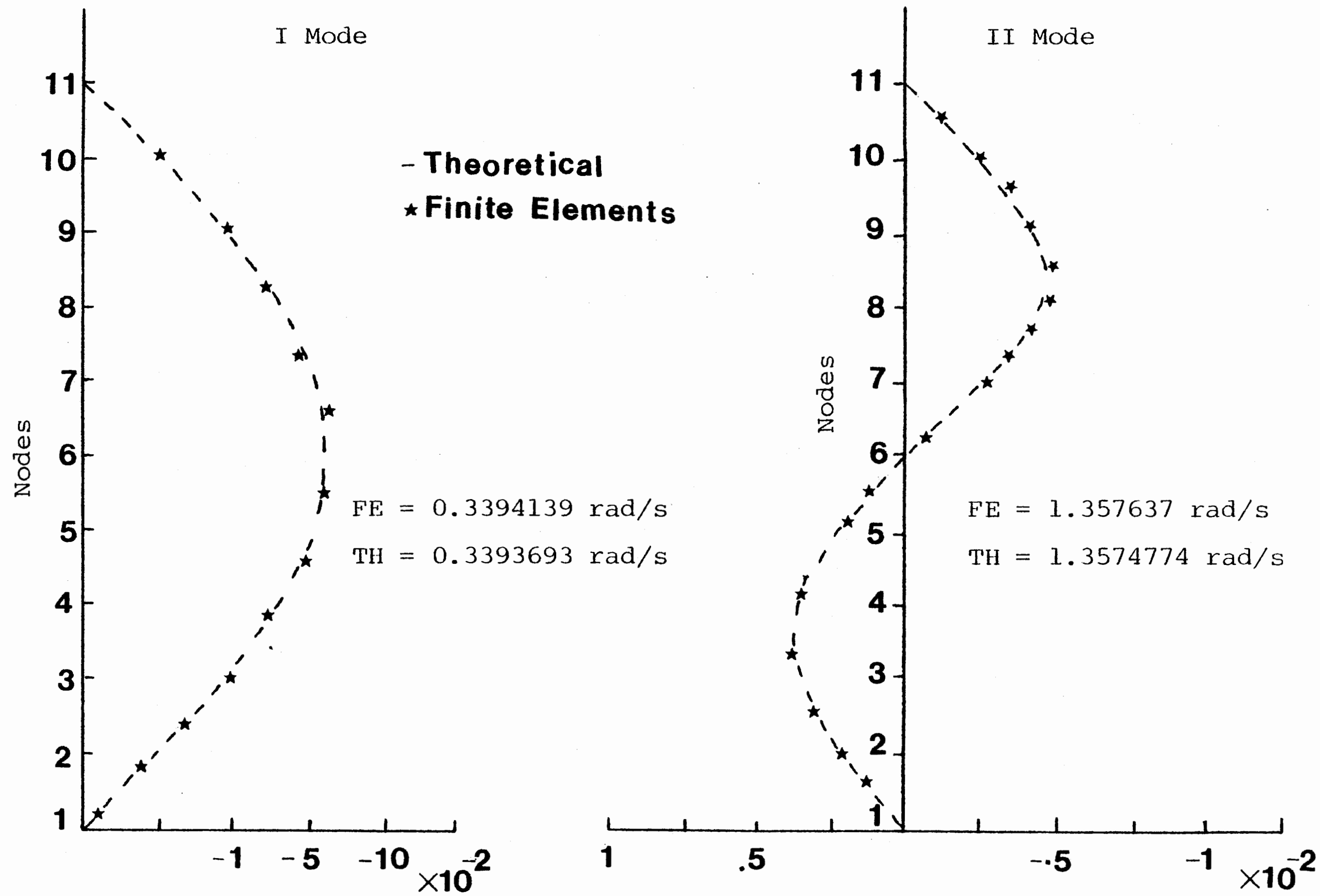

Figure 14. Free Vibration Mode Shapes

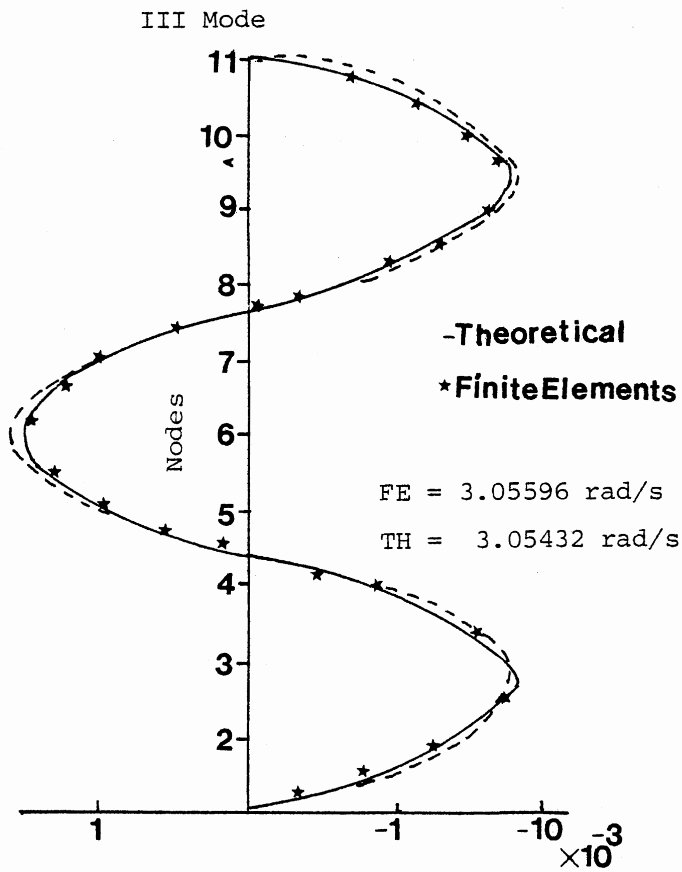

Figure 15. Free Vibration Mode Shape

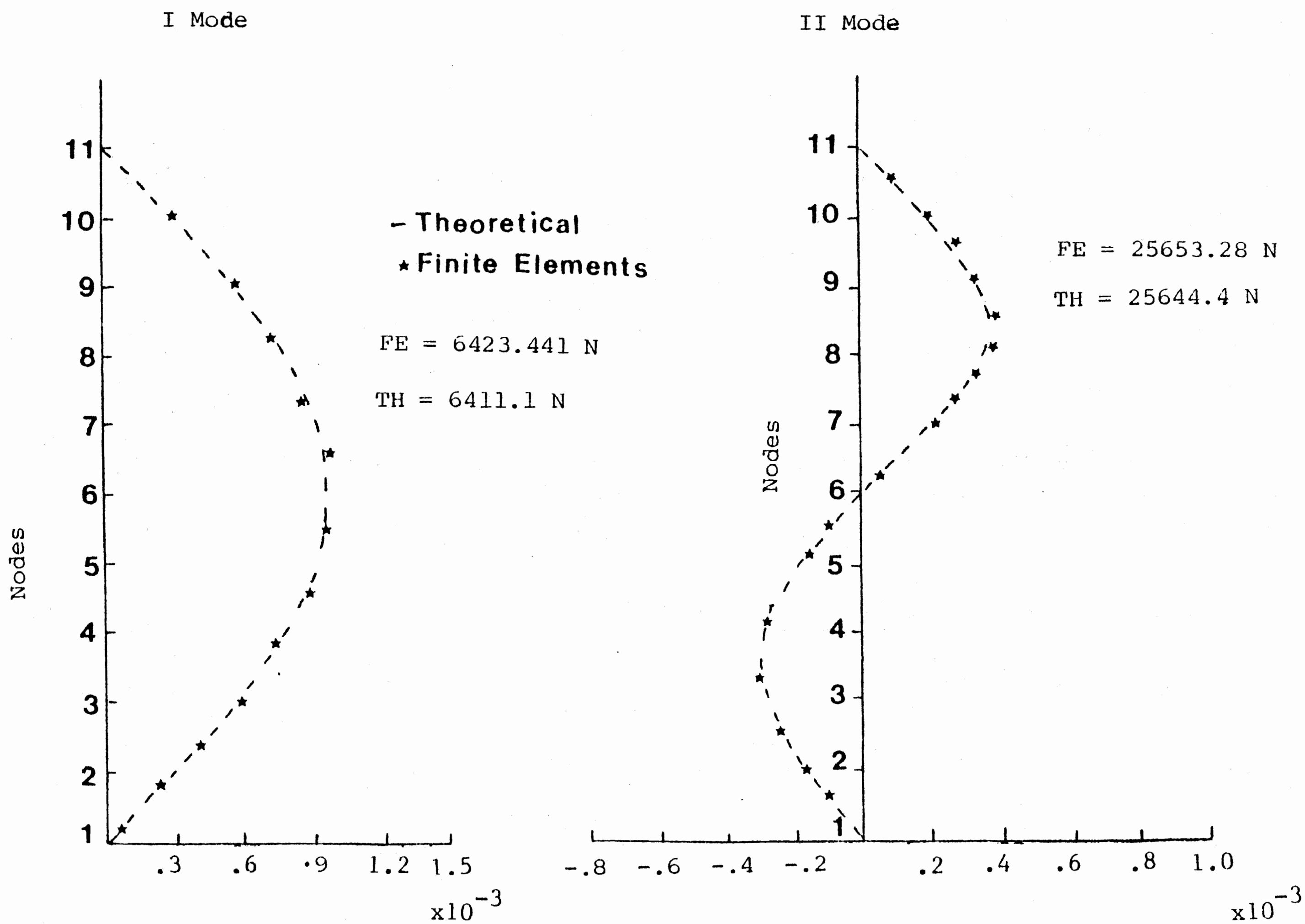

Figure 16. Buckling Mode Shapes

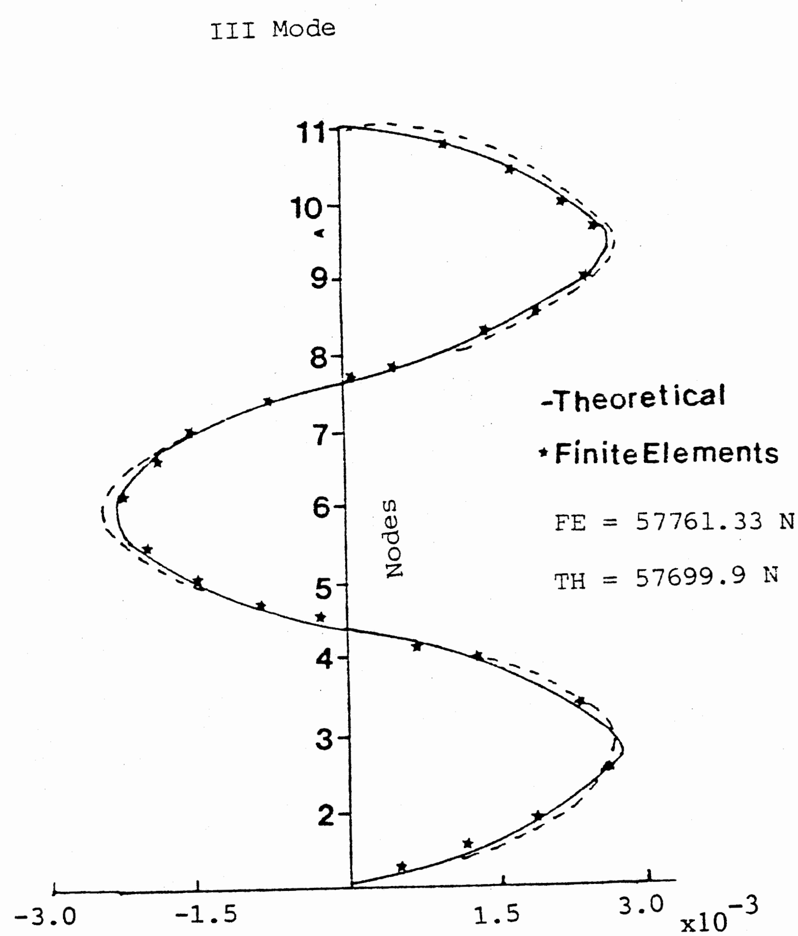

Figure 17. Buckling Mode Shape

Subroutines DECOM and SOLBAN

Subroutines DECOM and SOLBAN are checked by solving a static problem. For a 100 meter length riser the end displacement is assumed to be 0.03m and the force calculated from the formula

$$\text{Displacement} = \frac{\text{Force x (Length)}^3}{\text{12 x Young's Modulus x Inertia}}$$

The parameters taken are DIAI = 0.114m, DIAO = 0.1297m, DENS = 8000 Kg/m^3, $E = 20 \times 10^{10}$ N/m^2, $I = 0.0003047$ m^4. The force calculated is 21.65 N. The result obtained is a displacement of 0.0296554 m which verifies the validity of DECOM and SOLBAN.

Subroutine NEWMAR

The Newmark's integration scheme as formulated in the subroutine NEWMAR is checked by solving a forced boundary condition problem numerically. A sinusoidally time dependent displacement boundary condition is specified at the top. The theoretical verification is made by applying a method due to Mindlin and Goodman (18). The results are presented in Figure 18.

The input paramters considered for the various check problem are as follows:

Eigenvalue Problem - Simply Supported Beam:

To check mass and conventional stiffness matrices

DENSO	=	0.0 Kg/m^3
DENS	=	8000 Kg/m^3
DIAI	=	0.4172 m
DIAO	=	0.4572 m

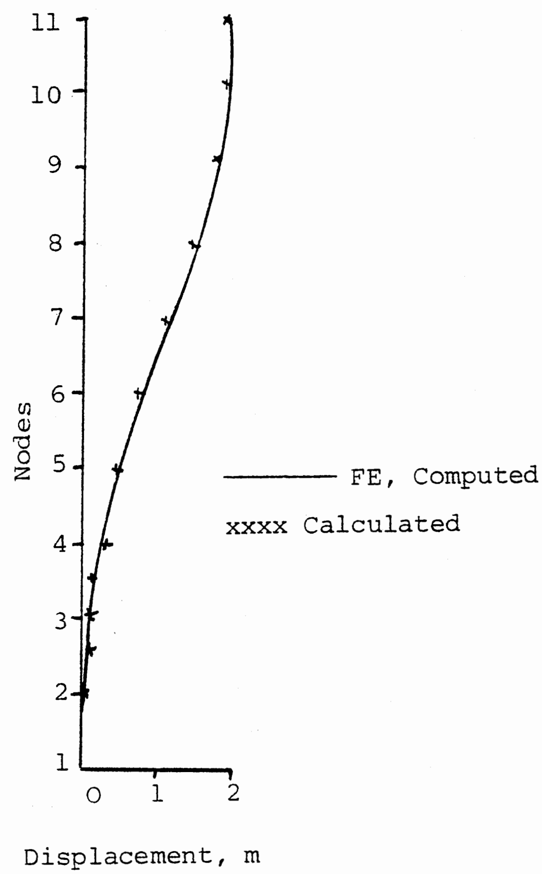

Figure 18. Displacement Comparison, Forced Vibration, Time = 9 Sec

E	=	20×10^{10} N/m^2
I	=	0.0006577 m^4
Length	=	150 m
Elements	=	10

Eigenvalue Problem - Fixed and Sliding Beam:

To check mass and conventional stiffness matrices

DENSO	=	0.0 Kg/m^3
DENS	=	8000 Kg/m^3
DIAI	=	0.4172 m
DIAO	=	0.4572 m
E	=	20×10^{10} N/m^2
I	=	0.0006577 m^4
Length	=	450 m
Elements	=	10

Buckling Problem:

To check geometric stiffness matrix

DENSO	=	0.0 Kg/m^3
DENS	=	8000 Kg/m^3
DIAI	=	0.4172 m
DIAO	=	0.4572 m
E	=	20×10^{10} N/m^2
I	=	0.0006577 m^4
Length	=	450 m
Elements	=	10
TENS	=	1.0 N

Static Problem:

To check subroutines DECOM and SOLBAN

DENSO	=	0.0 Kg/m^3
DENS	=	8000 Kg/m^3
DIAI	=	0.381 m
DIAO	=	0.4064 m
E	=	20×10^{10} N/m^2
I	=	0.0003047 m^4
Length	=	100 m
Elements	=	10
End Force	=	21.65 N
End Deflection	=	0.03 m

Time Dependent Boundary Condition Problem:

To check subroutine NEWMAR

DENSO	=	0.0 Kg/m^3
DENS	=	8000 Kg/m^3
DIAI	=	0.381 m
DIAO	=	0.4064 m
E	=	20×10^{10} N/m^2
I	=	0.0003047 m^4
Length	=	100 m
Elements	=	10
Amplitude	=	2.0 m
Wave Period	=	20 sec

CHAPTER VI

RESULTS, DISCUSSION AND CONCLUSIONS

The data for the example problems has been chosen carefully such as to include as many variations in sensitive parameters such as tension, wave period, etc. in as less number of problems as possible. The data chosen corresponds closely to API 500-20-ID test case as specified in (19).

The various sets of data are as follows:

(i)	Length	=	225 m
	Elements	=	10
	Wave Height	=	6.09 m
	Period	=	9 sec & 20 sec
	Surface Current Velocity	=	2 Knots
	Tension/Weight of Riser	=	1.22 & 2.0
	Maximum Top Rotation	=	0.05 rad & 0.15 rad
	Maximum Top Displacement	=	3 m & 2 m
(ii)	Length	=	300 m
	Elements	=	15 & 20
	Wave Height	=	6.09 m
	Period	=	20 sec
	Surface Current Velocity	=	2 Knots
	Tension/Weight of Riser	=	2.0
	Maximum Top Rotation	=	0.25 rad

	Maximum Top Displacement	=	3.0 m
(iii)	Length	=	400 m
	Elements	=	15 & 20
	Wave Height	=	6.09 m
	Period	=	20 sec
	Tension/Weight of Riser	=	2.0
	Maximum Top Rotation	=	0.1 rad
	Maximum Top Displacement	=	2.0 m
(iv)	Length	=	600 m
	Elements	=	20
	Wave Height	=	6.09 m
	Period	=	5, 9, 12, 15, 20 sec
	Tension/Weight of Riser	=	2.0 & 2.5
	Maximum Top Rotation	=	0.1 rad
	Maximum Top Displacement	=	2.0 m

The other parameters that are constant for all the examples are:

DENSI	=	$1438\ Kg/m^3$
DENSO	=	$1025\ Kg/m^3$
DENS	=	$8690\ Kg/m^3$
DIAI	=	0.381 m
DIAO	=	0.4064 m
E	=	$21 \times 10^{10}\ N/m^2$
g	=	$9.81\ m/sec^2$
C_D	=	1.138
C_m	=	1.5
Current Velocity	=	2 Knots to (at 2nd node)

Figure 19 to Figure 26 correspond to the first case. Figure 19 and Figure 20 shows the wave propagation of displacements in the first few seconds. The displacements only due to drifting and that only due to rolling of the drilling vessel agrees well with Kazuo Aso et al. [7]. The paper does not present any displacement or bending stress graph due to both drifting and rolling. The method of superposition to arrive at such result as they argue may not be convincing particularly for a non-linear problem. As such, both drifting and rolling are taken to act simultaneously and the result of the displacement and bending moments are as shown in Figure 21 to Figure 24. It can be seen from Figure 24 that the maximum bending moment at the bottom occurs after 12 seconds and that at the top occurs after 14 seconds. The bending moment due to drifting and rolling as taken separately are presented in Figure 25 and Figure 26. From the figures it can be seen that the maximum bending moment, maximum vessel displacement and maximum deflection of the riser do not occur at the same time over a period of vibration. This agrees well with Kazuo Aso et al. [7].

Figure 27 shows displacement comparison for a 15 and 20 element riser model for 300 m length. It can be seen that the displacement profile differ less for the upper half than for the lower half. A similar comparison is made for a 400 m length of riser as shown in Figure 28 to Figure 30. The result is the same. Here a comparison for the bending moment represented in Figures 29 and 30 shows that the maximum moment, either at the top or at the bottom occurs at the same time for both the models. However, the profile differs very much near the bottom. For a 600 m length of riser the analysis was performed for two different top tension to riser weight ratios. The period was varied

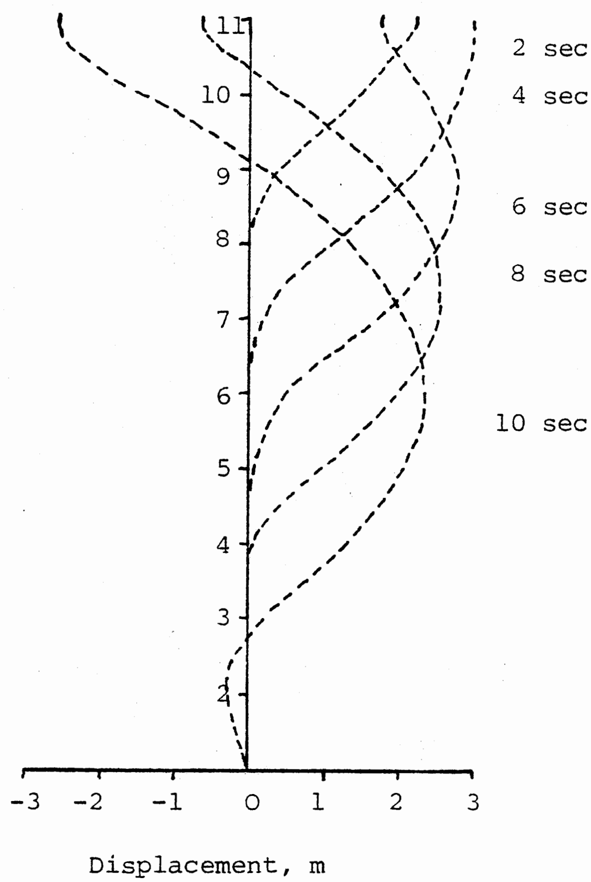

Figure 19. Wave Propagation Due to Drifting

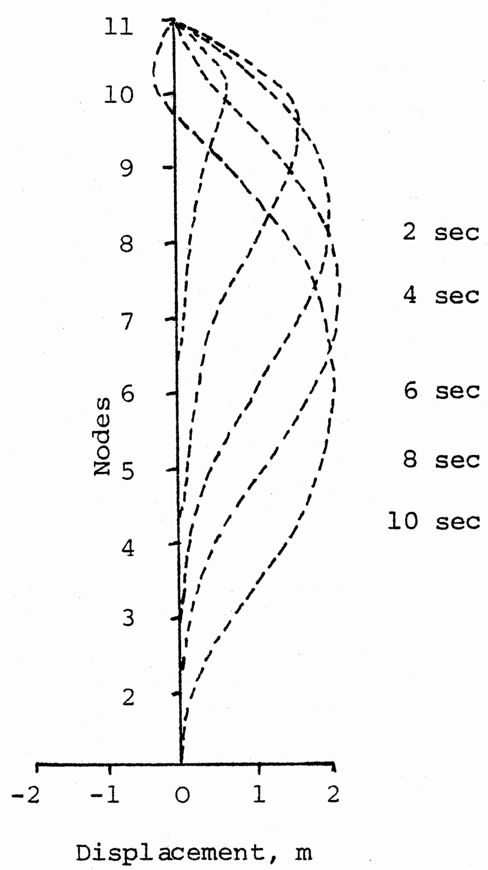

Figure 20. Wave Propagation Due to Rolling for 225 m Riser Length

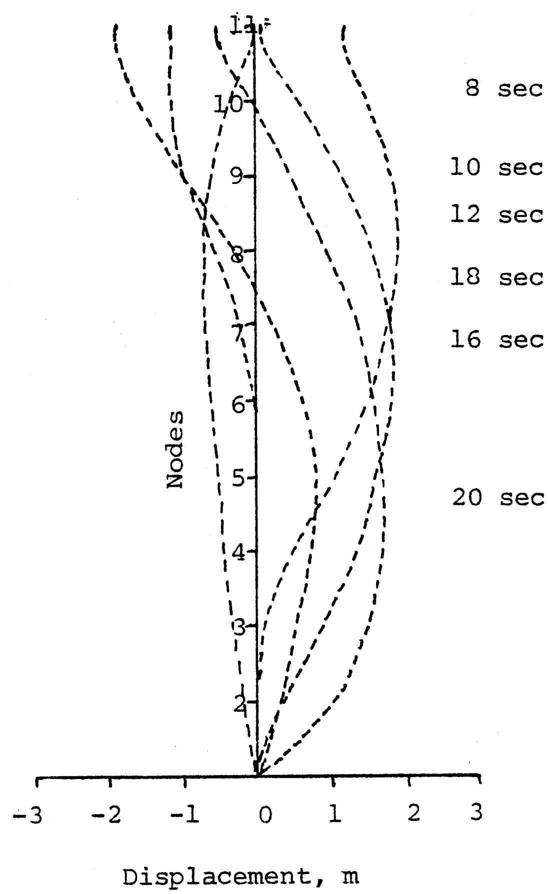

Figure 21. Displacements Due to Drifting for 225 m Riser Length

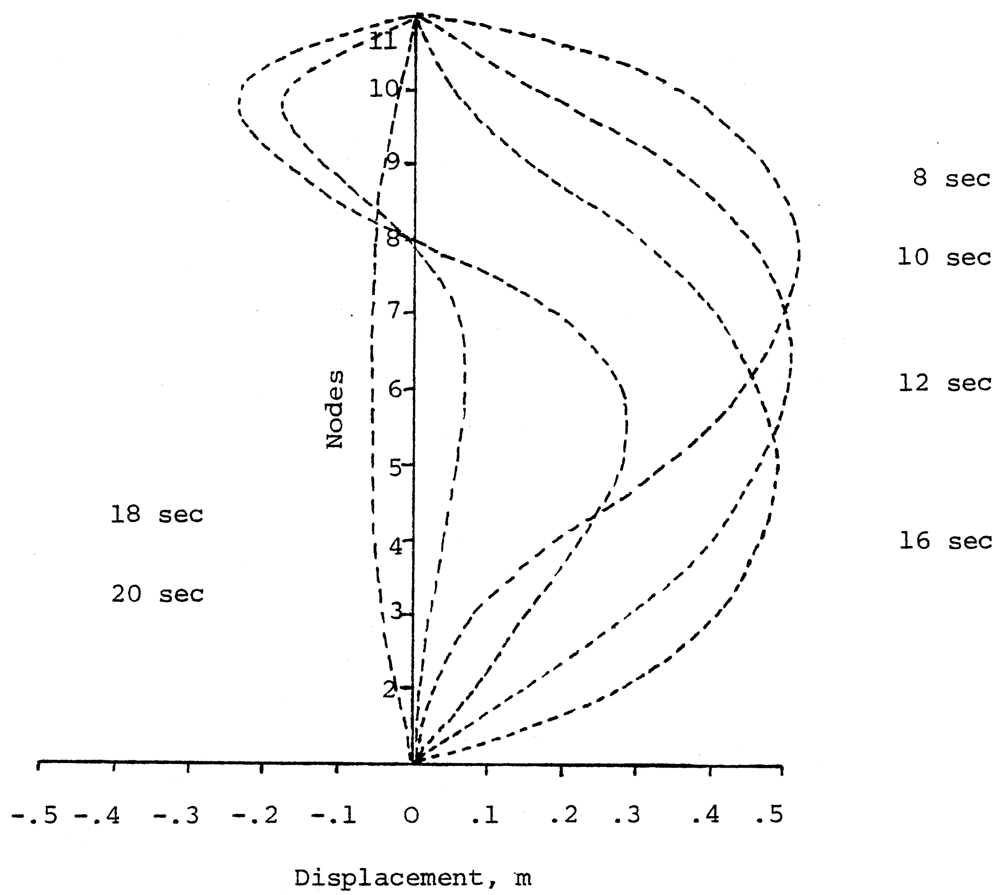

Figure 22. Displacements Due to Rolling for 225 m Riser Length

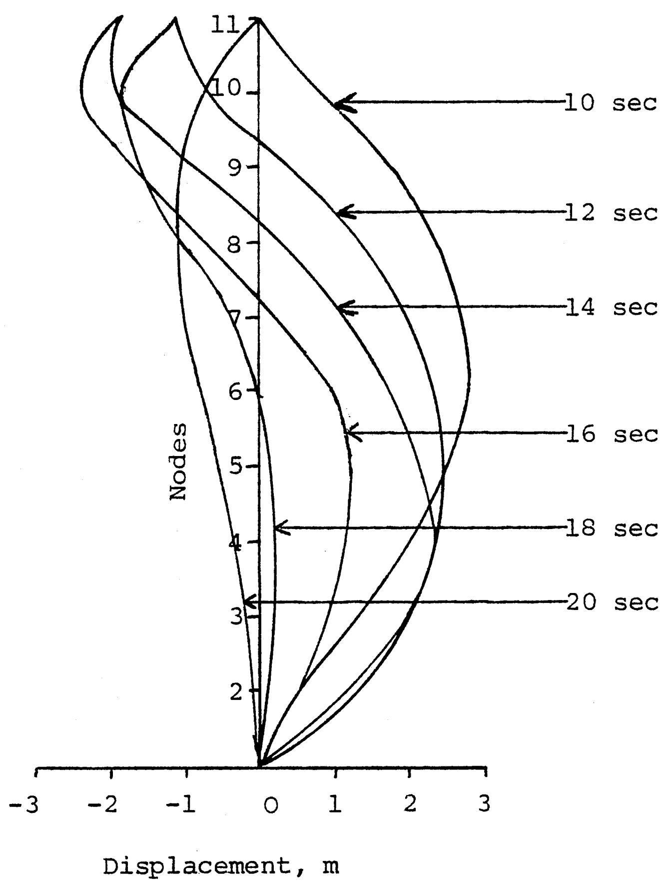

Figure 23. Displacements Due to Roll and Drift for 225 m Riser Length

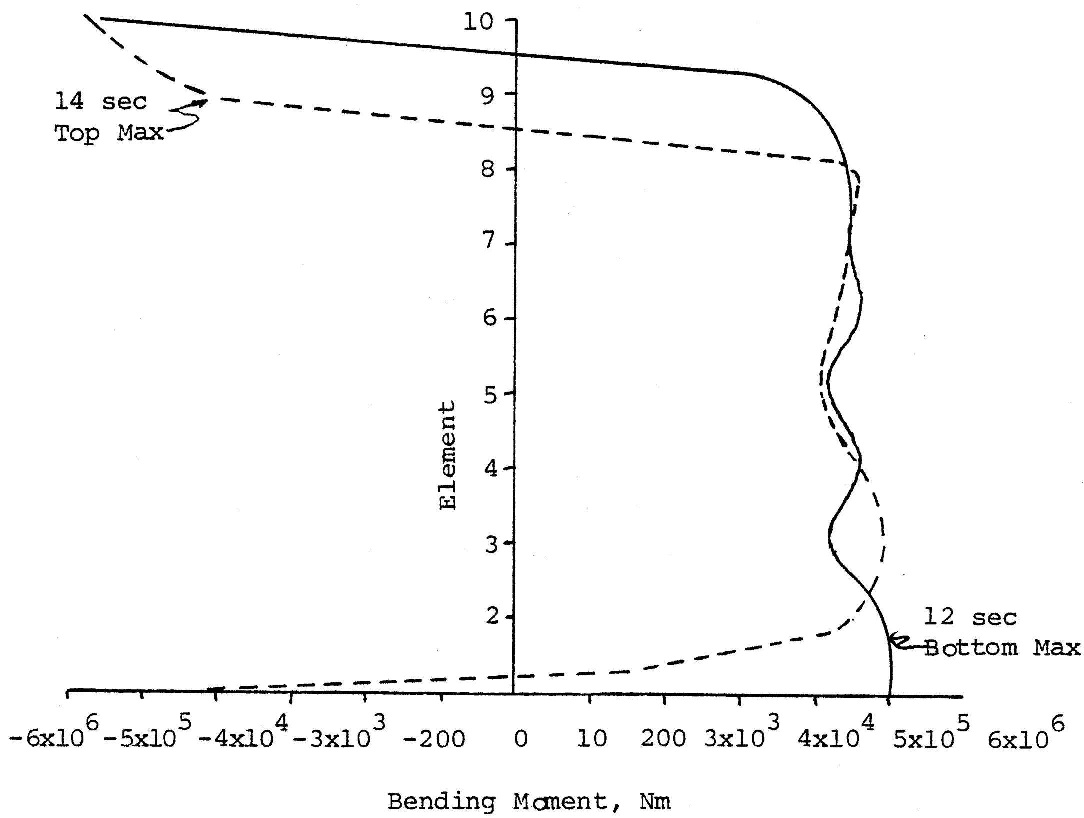

Figure 24. Bending Moment Variation Due to Drift and Roll for 225 m Riser Length

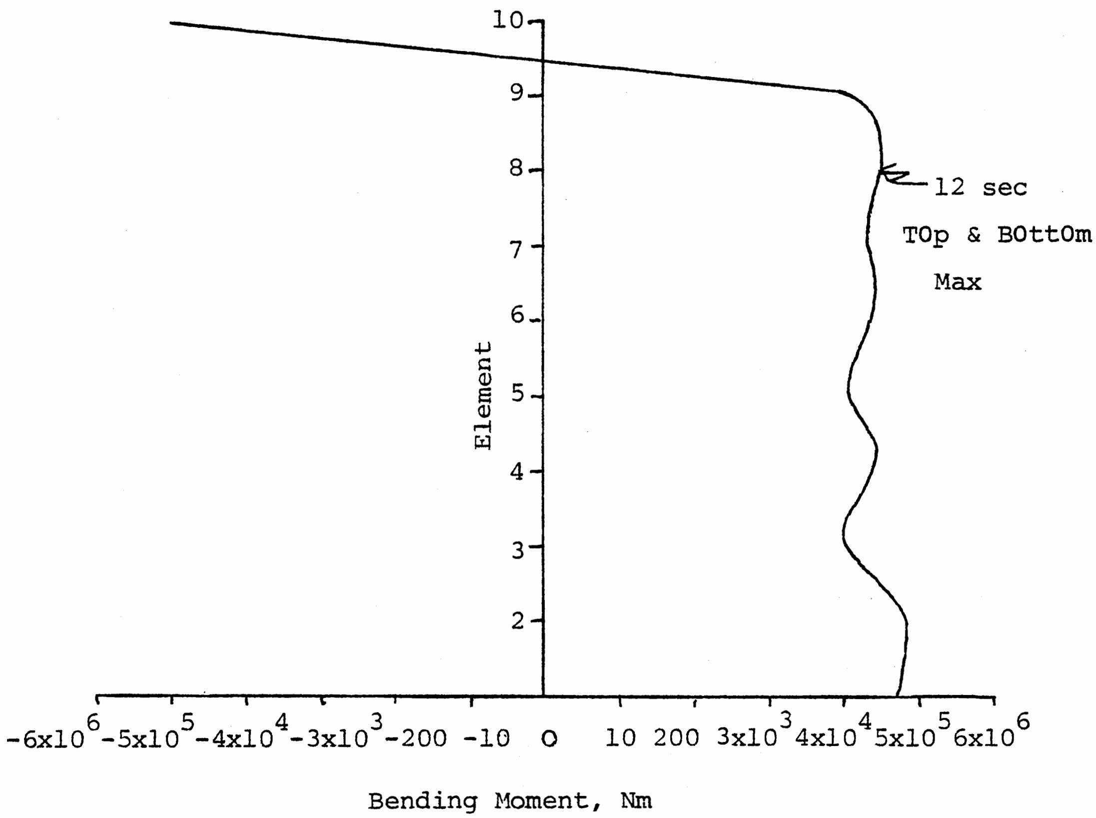

Figure 25. Bending Moment Variation Due to Drift for 225 m Riser Length

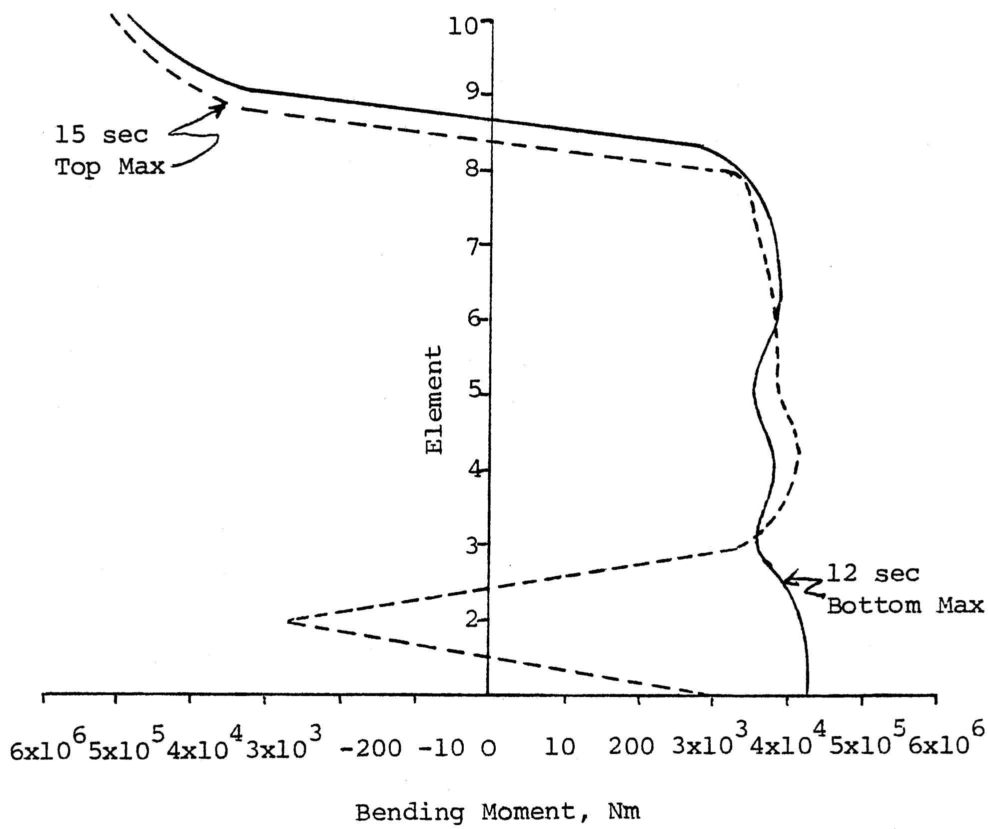

Figure 26. Bending Moment Variation Due to Rolling for 225 m Riser Length

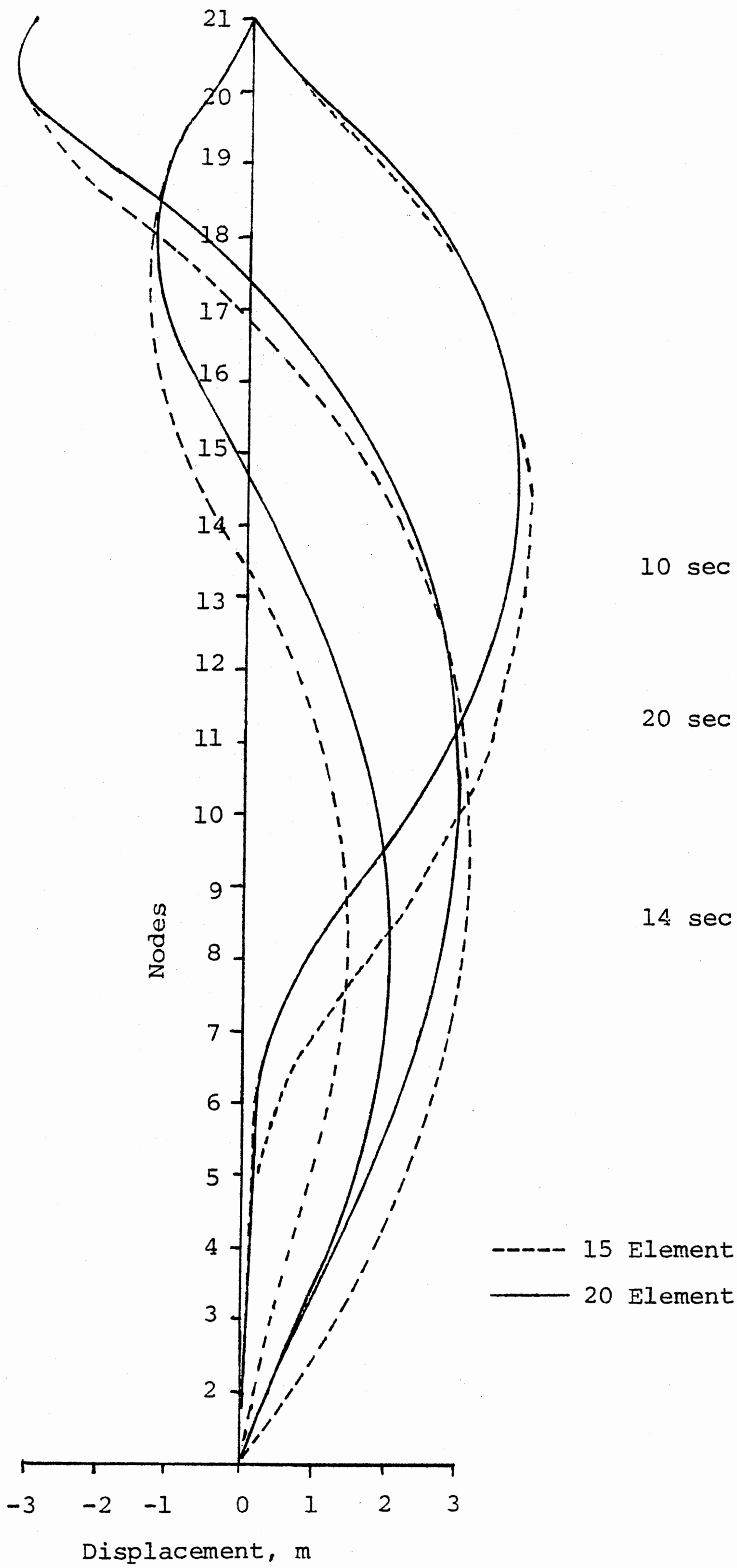

Figure 27. Displacement Comparison for 15 and 20 Element Model for 300 m Riser Length

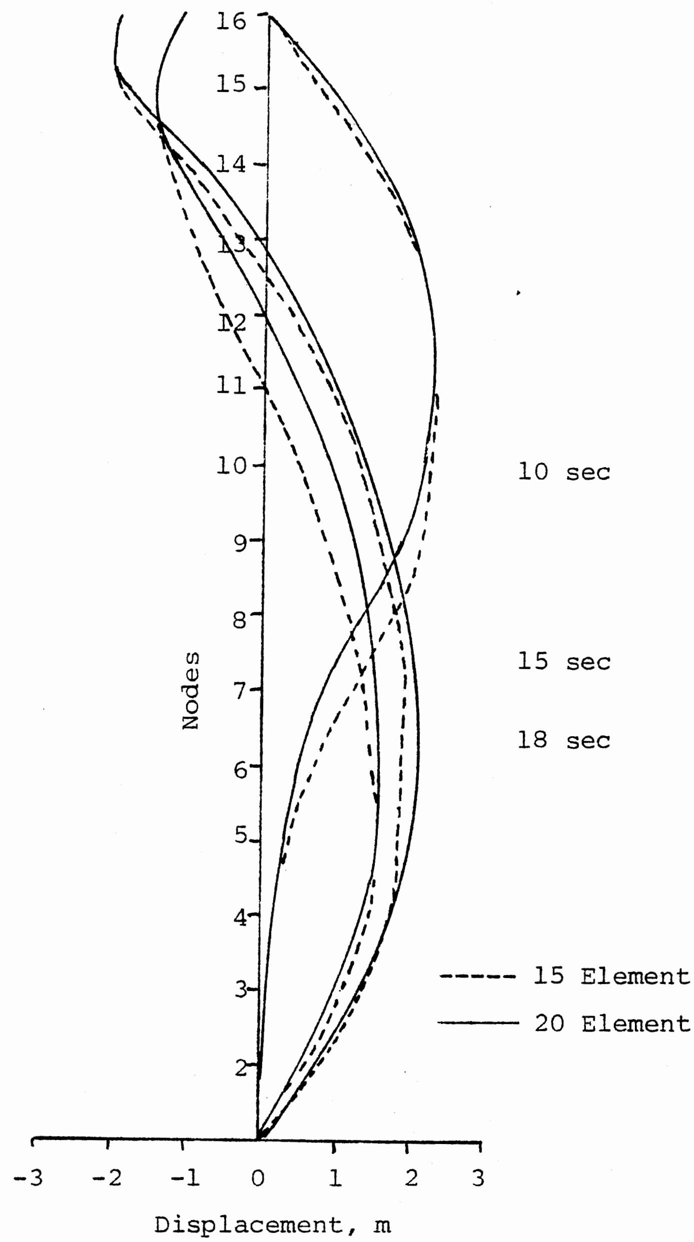

Figure 28. Displacement Comparison for 15 and 20 Element Model for 400 m Riser Length

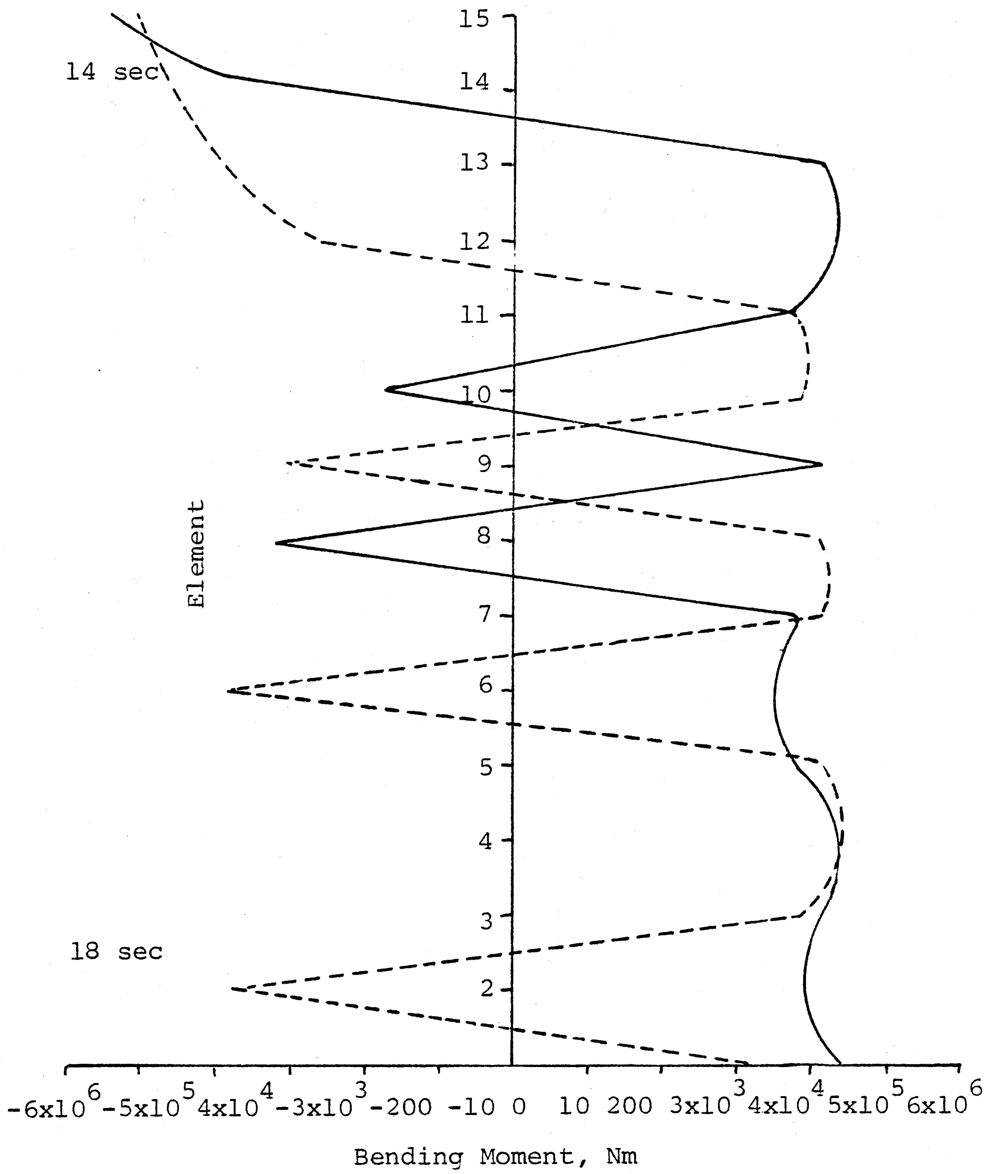

Figure 29. Bending Moment Variation for 15 Element Model for 400 m Riser Length

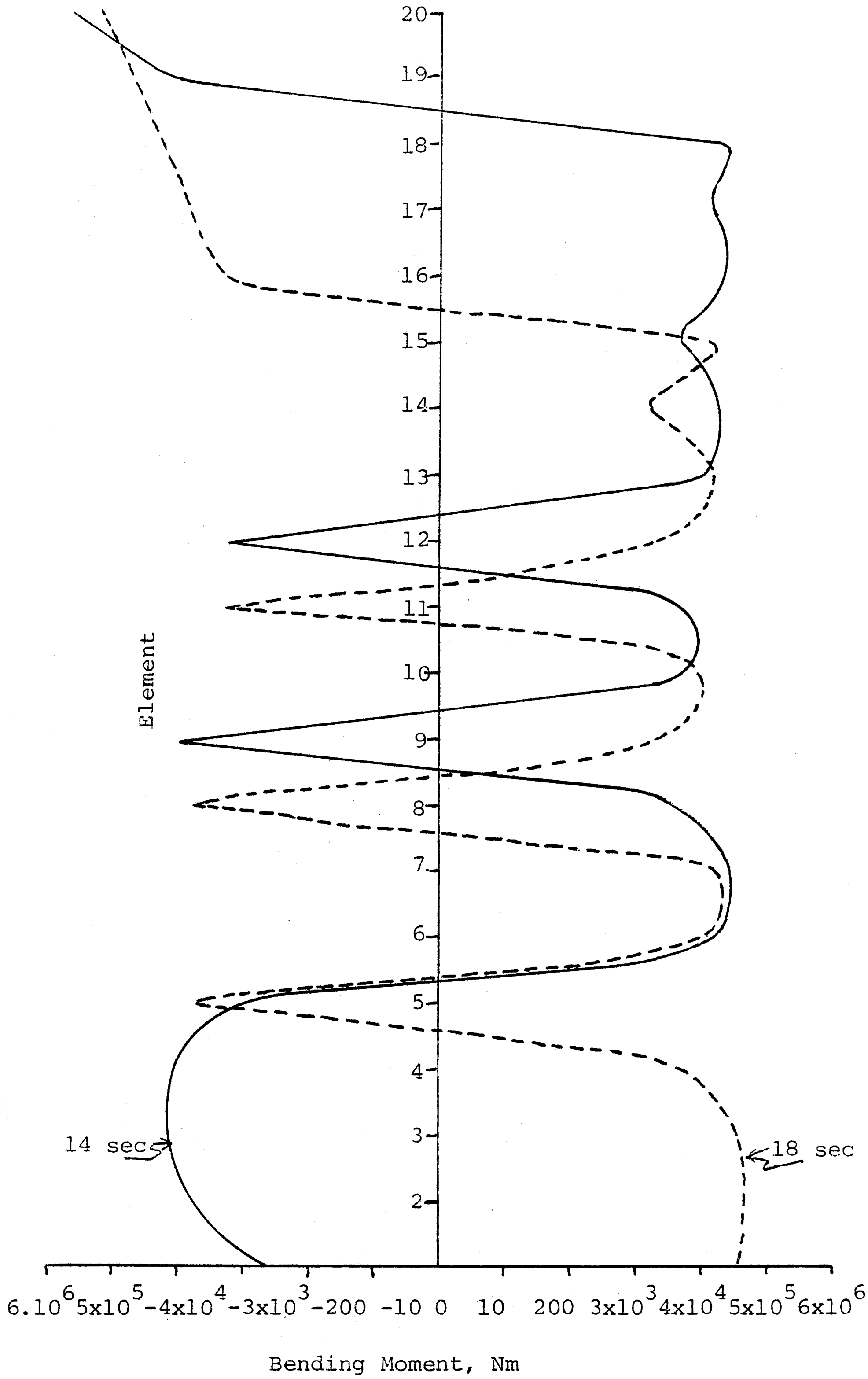

Figure 30. Bending Moment Variation for 20 Element Model for 400 m Riser Length

from 5 to 20 seconds to study the behavior of the bottom angle. Figures 31 and 32 show the displacement profile when the ratio of top tension to riser weight is 2.0 and 2.5 respectively. It can be seen that the displacement behavior is more orderly when the ratio is higher. This agrees with the general conclusion that the top tension ratio should be higher as the length of the riser is increased. Also in Figures 33 and 34 can be seen the bottom rotational behavior due to changing wave periods. It can be seen that the maximum bottom angle occurs when the wave period is between 10 and 15 seconds. This does not perfectly agree with Spanos and Chen [15]. They have arrived at the results by considering the top to be fixed which is a hypothetical approach. Here it may be argued that the results presented are more realistic than most of the previous work done in this area.

Conclusions

The general conclusion that the time domain analysis is very expensive and time consuming is proved wrong. The maximum CPU time taken in this analysis is about 6 seconds which costs not more than a couple of dollars. All the previous authors agree that the time domain analysis without any linearization technique is the most appropriate method of analysis. Hence there is no reason why a thorough analysis such as this one should not be carried out, especially when the cost of the whole project runs into millions of dollars. As in the case of all off-shore structures, the fluid loading seems to be the weakest link in the analysis. Morrison's equation still forms the basic equation for fluid loading. The sophistication in the analysis is, however, yet to be matched by experimental verification.

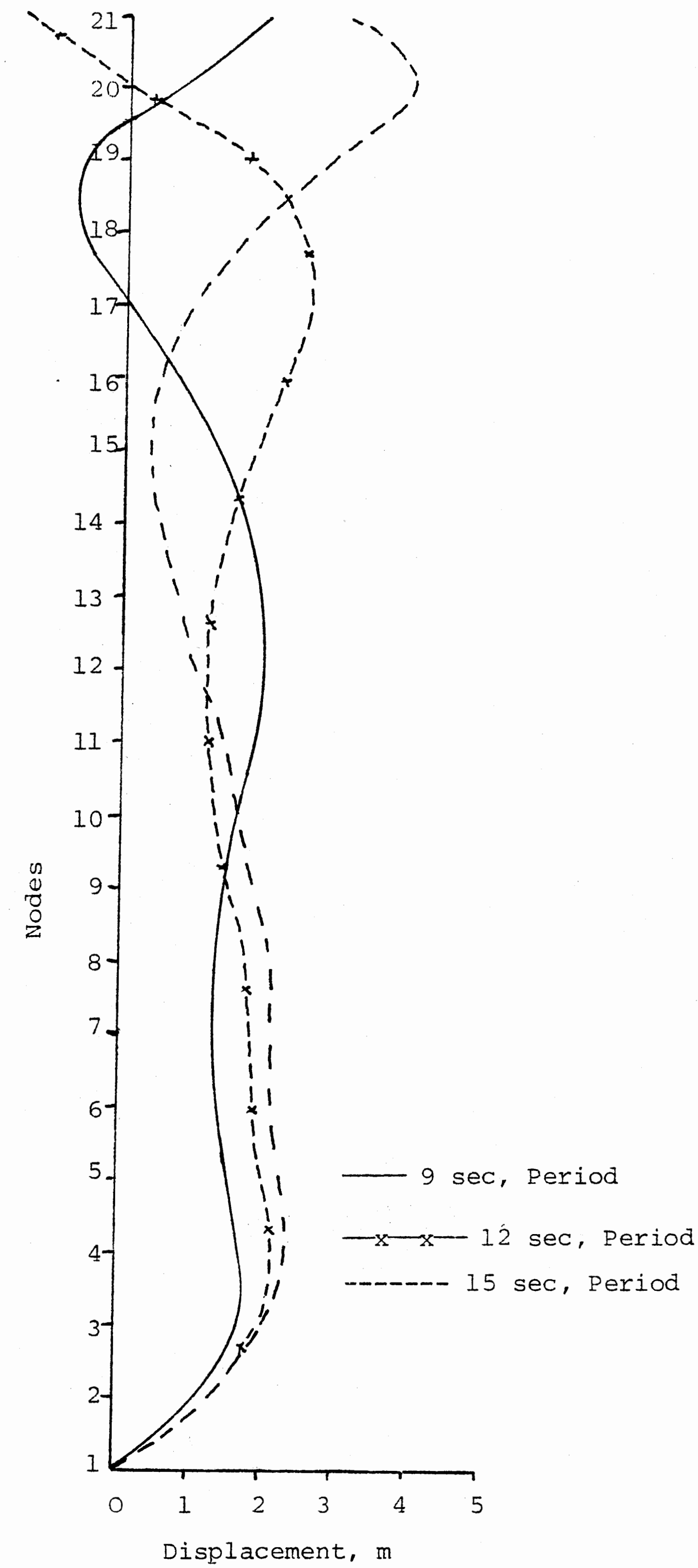

Figure 31. Deflection Comparison for Different Wave Periods

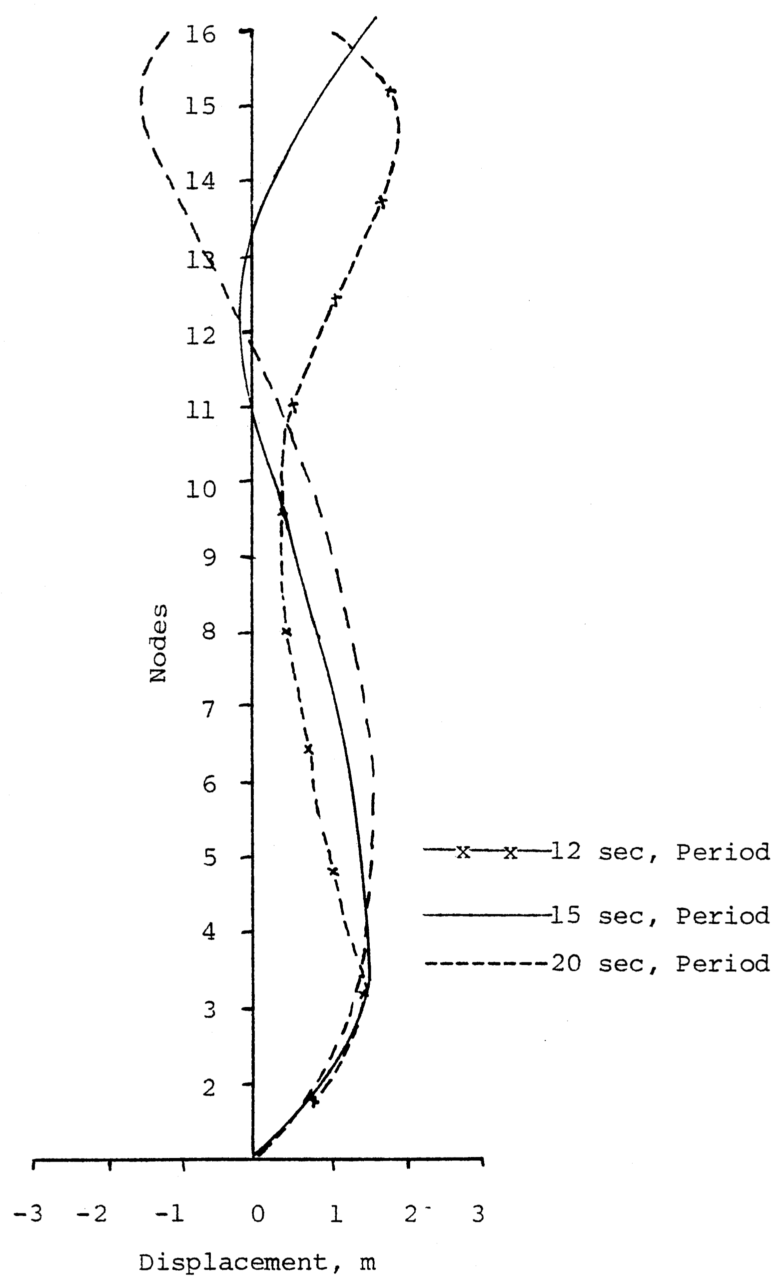

Figure 32. Deflection Comparison for Different Wave Periods for T/W = 2.5 and Riser Length = 600 m

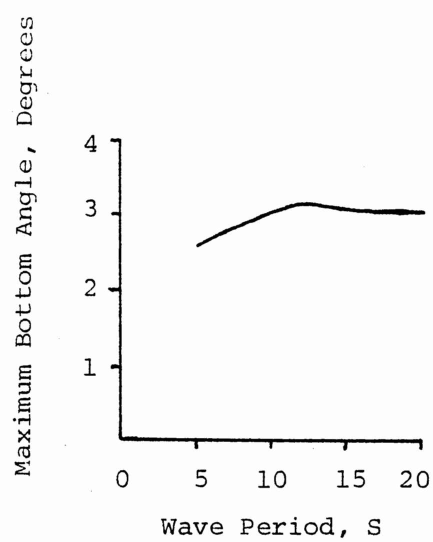

Figure 33. Maximum Bottom Angle Versus Wave Period for T/W = 2.0 and Riser Length = 600 m

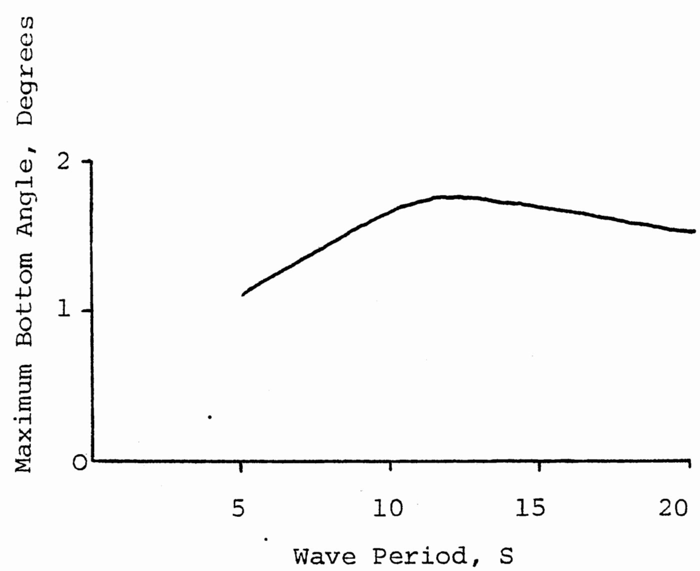

Figure 34. Maximum Bottom Angle Versus Wave Period for T/W = 2.5 and Riser Length = 600 m

The finite element formulation leads to an efficient and accurate solution for the dynamic analysis of risers. Herein a consistent mass matrix is derived which represents riser system inertia more accurately than lumped mass matrix. This along with the boundary conditions considered for calculating bending moment differs from that of Spanos and Chen [15]. This analysis also does not give rise to any loss of accuracy due to any linearization technique as is the case for most of the previous papers. Moreover, since the analysis is in time domain it can be concluded that this represents an accurate dynamic analysis of the marine riser.

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APPENDICES

APPENDIX A

MARIAN USER'S MANUAL

MARIAN is a Computer Program written in FORTRAN to carry out the dynamic analysis of a marine riser in time domain. This user's manual describes the way in which the data is supplied to the program.

CARD #	DATA AND DESCRIPTION
1	DENSI, DENS, DENSO, DIAI, DIAO, YMOD (5F10.3, E13.7)
	DENSI - Mass density of the fluid inside the riser, in Kg/m^3
	DENS - Mass density of the riser material, in Kg/m^3
	DENSO - Mass density of the surrounding fluid, in Kg/m^3
	DIAI - Riser inner diameter, in m
	DIAO - Riser outer diameter, in m
	YMOD - Young's Modulus of elasticity, in N/m^2
2	NNPE, NUMEL, NDOF, MBAND, NUMNP(5I5)
	NNPE - Number of nodes per element(=2)
	NUMEL - Number of elements
	NDOF - Number of degrees of freedom(=4)
	MBAND - Band width(=4)
	NUMNP - Number of nodal points
3	TENS, GRAV, DELT, AMP, THETA, DPER, ITER (6F10.4, IS)
	TENS - Top tension, in N
	GRAV - Gravitational acceleration, m/sec^2
	DELT - Incremental time step, in sec

CARD #	DATA AND DESCRIPTION
	AMP - Amplitude of drift, in m
	THETA - Amplitude of roll, in rad
	DPER - Period of drift and role, in sec
	ITER - Maximum number of time steps
4	DRA, AMASSC, SURFV, WAVEL, PERIOD, WHGT, IDEPTH (6F10.5, I5)
	DRA - Drag coefficient
	AMASSC - Mass coefficient
	SURFV - Surface current velocity, in m/sec
	WAVEL - Wave length, in m
	PERIOD - Wave period, in sec
	WHGT - Wave height, in m
	IDEPTH - Node number where the current velocity tapers to zero
5	I, LENGTH(I) (I5, F10.5)
	NOTE: I = NUMNP, LENGTH(I) = Distance of the ith nodal point from bottom. As many cards as NUMNP with one value of I and LENGTH(I) on each card
6	I, NP(J,I), J = 1, NNPE, I = 1, NUMEL (3I3)
	NOTE: I is the element number followed by the node numbers of that element on each card. As many cards as elements.
7	ID(I,J), I = 1, NNPE, J = 1, NUMNP (XI2)

CARD # DATA AND DESCRIPTION

NOTE: This contains two cards. Each card has as many values as NU NP. Hence the Format has to be changed accordingly. Give one if the DOF is suppressed, otherwise zero. First card all displacements, 2nd card all rotations.

APPENDIX B

COMPUTER PROGRAM

```
C*********************************************************************
C*                                                                   *
C***** MARIAN - PROGRAM TO COMPUTE THE DYNAMIC RESPONSE OF A MARINE   *
C*                                                                   *
C***** RISER SYSTEM IN THE TIME DOMAIN.                               *
C*                                                                   *
C***** WRITTEN BY L.VASAN                                             *
C*                                                                   *
C*********************************************************************
C
        COMMON /C1/ DENSI,DENS,AREAI,AREAO,ELEN(50),NDOF,GRAV,YMOD,
     1 INERT,TENS,LENGTH(50),DENSO,NUMEL,NP(2,50),LL(4),NUMNP,
     2STMAT(4,4),EMAT(4,4),GSMAT(4,4),GLMASS(50,6),GLSTIF(50,6),TIME(30)
     3,GLGEST(50,6),NUMB,MBAND,ID(2,50),NEQ,NNPE,AMASSC,GLTOST(50,6)
       COMMON /C2/SH1(10),SH2(10),SH3(10),SH4(10),ZHI(5),WGT(5),AMP,DPER
       COMMON /C3/ NINT,ITER,DIS(50,30),SURFV,WAVEL,WHGT,PERIOD,VCUR(50)
     1,VCW(20),DELT,GLDAMP(50,6),CMAT(4,4),Z(20),IDEPTH,FUNC(50,30),DIAI
     2,DIAO,VEL(50),DRA,PMAT(4,4),GLGEOS(50,6),FORCE(50),VWAVE(50),THETA
C
C
       DIMENSION AMOM(50), SHEAR(50)
C
C
       REAL LENGTH,INERT
C
       DATA NINT,JJ,LP / 5,0,6 /
       DATA ZHI / 0.0,0.538469,-0.538469,0.90618,-0.90618 /
       DATA WGT / 0.568889,0.478629,0.478629,0.236927,0.236927 /
C
C
       CALL INPUT
C
C
       DO 10 I = 1, NUMEL
  10   ELEN(I) = LENGTH(I+1) - LENGTH(I)
C
C
      INERT = (3.1416/64.0)*((DIAO**2) + (DIAI**2))*(DIAO +DIAI)
     .        *(DIAO - DIAI)
C
      AREAO = (3.1416/4.0) *(DIAO**2)
      AREAI = (3.1416/4.0) *(DIAI**2)
C
C
C
       NEQ = 0
       TIME(1) = 0.0
C
       DO 77 I = 2, ITER
        TIME(I) = TIME(I-1) + DELT
 77   CONTINUE
```

```
C
C
      DO 13 N = 1, NUMNP
      DO 12 J = 1, NNPE
      IF (ID(J,N).GT.0) GO TO 150
      NEQ = NEQ + 1
      ID(J,N) = NEQ
      GO TO 12
 150  ID(J,N) = 0
  12  CONTINUE
  13  CONTINUE
C
C
      DO 41 K = 1,NEQ
      DO 41 KJ = 1,ITER
41    DIS(K,KJ) = 0.0
C
C INITIALIZE BANDED GLOBAL MATRICES.
C
      NUMB = NUMNP * 2
      DO 40 I = 1, NEQ
      DO 50 J = 1, MBAND
      GLMASS(I,J) = 0.0
      GLGEOS(I,J) = 0.0
      GLDAMP(I,J) = 0.0
      GLSTIF(I,J) = 0.0
      GLGEST(I,J) = 0.0
      GLTOST(I,J) = 0.0
 50   CONTINUE
 40   CONTINUE
C
C
        DO 554 K = 1,4
        DO 554 J = 1,4
        CMAT(K,J) = 0.0
 554    CONTINUE
C
       DO 498 K = 1,NUMB
       VEL (K) = 0.0
 498   CONTINUE
      CALL SHAPE
      DO 60 N = 1, NUMEL
      I = NP(1,N)
      J = NP(2,N)
      CALL STIFN(N)
      CALL MASS(N)
      CALL GESTFN(N)
      CALL PRESS(N)
      CALL GLOBAL(I,J,JJ)
  60  CONTINUE
C     GLSTIF(1,1) = GLSTIF(1,1) + 3250134.0
```

```
      DO 63 KL = 1,NEQ
      DO 53 ML = 1,MBAND
      GLTOST(KL,ML) = GLSTIF(KL,ML)+GLGEST(KL,ML)+GLGEOS(KL,ML)
53    CONTINUE
63    CONTINUE
C
C
      NEQ = NEQ - 2
C
C
C
     CALL NEWMAR
C
      DO 543 JJ = 1, ITER
C
       M = 0
       DO 200 I = 1,NUMNP
       DO 220 J = 1,NNPE
       N = ID(J,I)
       M = M + 1
       FUNC(M,JJ) = 0.0
       IF (N.EQ.0) GO TO 220
       FUNC(M,JJ) = DIS(N,JJ)
 220   CONTINUE
 200   CONTINUE
C
C
          MN = 0
          KLM = 2
        DO 342 JL = 1,NUMEL
        MN = MN + 2
        KLM = KLM + 2
        MPQ = MN - 1
        NQR = KLM - 1
C
       AMOM(JL)= -YMOD*INERT*(-FUNC(MN,JJ) + FUNC(KLM,JJ))/ELEN(JL)
       SHEAR(JL) = (-YMOD*INERT /(ELEN(JL) ** 3))*(12.0*FUNC(MPQ,JJ)
     1 + 6.0*ELEN(JL)*FUNC(MN,JJ) - 12.0*FUNC(NQR,JJ) + 6.0*
     2 ELEN(JL)*FUNC(KLM,JJ))
C
C
 342   CONTINUE
     WRITE (4,445) TIME(JJ)
445  FORMAT (///10X,'TIME = ',F5.2,' SECS.'//)
C
     WRITE (4,333)
333  FORMAT (/10X,4HNODE,8X,12HDISPLACEMENT,10X,8HROTATION,8X,
     1 7HELEMENT,10X,6HMOMENT,15X,5HSHEAR//)
C
        J = 0
       DO 239 I = 1,NUMB,2
```

```
        J = I - J
      WRITE (4,444) J, FUNC(I,JJ), FUNC(I+1,JJ)
444   FORMAT (11X,I2,8X,E13.6,8X,E13.6/)
C
       IF (J.GT.NUMEL) GO TO 239
C
      WRITE (4,456) J, AMOM(J), SHEAR(J)
456   FORMAT(61X,I2,8X,E13.6,8X,E13.6/)
 239    CONTINUE
C
 543    CONTINUE
       STOP
       END
C
C
                    SUBROUTINE SHAPE
C
       COMMON /C2/SH1(10),SH2(10),SH3(10),SH4(10),ZHI(5),WGT(5),AMP,DPER
       COMMON /C3/ NINT,ITER,DIS(50,30),SURFV,WAVEL,WHGT,PERIOD,VCUR(50)
     1,VCW(20),DELT,GLDAMP(50,6),CMAT(4,4),Z(20),IDEPTH,FUNC(50,30),DIAI
     2,DIAO,VEL(50),DRA,PMAT(4,4),GLGEOS(50,6),FORCE(50),VWAVE(50),THETA
C
C
        DO 5 J = 1,NINT
        SH1(J) = 0.5 - 0.75 * ZHI(J) + 0.25 * (ZHI(J) ** 3)
        SH2(J) = 0.25 * (1.0-ZHI(J)-ZHI(J) ** 2 + ZHI(J)**3)
        SH3(J) = 0.5 + 0.75 * ZHI(J) - 0.25 * (ZHI(J) ** 3)
        SH4(J) = 0.25 * (-1.0 - ZHI(J) + ZHI(J) ** 2 + ZHI(J) **3)
 5      CONTINUE
C
C
        RETURN
        END
C
                      SUBROUTINE GLOBAL(I,J,JJ)
C
         COMMON /C1/ DENSI,DENS,AREAI,AREAO,ELEN(50),NDOF,GRAV,YMOD,
     1 INERT,TENS,LENGTH(50),DENSO,NUMEL,NP(2,50),LL(4),NUMNP,
     2STMAT(4,4),EMAT(4,4),GSMAT(4,4),GLMASS(50,6),GLSTIF(50,6),TIME(30)
     3,GLGEST(50,6),NUMB,MBAND,ID(2,50),NEQ,NNPE,AMASSC,GLTOST(50,6)
       COMMON /C3/ NINT,ITER,DIS(50,30),SURFV,WAVEL,WHGT,PERIOD,VCUR(50)
     1,VCW(20),DELT,GLDAMP(50,6),CMAT(4,4),Z(20),IDEPTH,FUNC(50,30),DIAI
     2,DIAO,VEL(50),DRA,PMAT(4,4),GLGEOS(50,6),FORCE(50),VWAVE(50),THETA
C
       LL(1) = ID(1,I)
       LL(2) = ID(2,I)
       LL(3) = ID(1,J)
       LL(4) = ID(2,J)
C
       DO 400 K = 1, NDOF
       IF (LL(K).LE.0) GO TO 400
```

```
      KI = LL(K)
      DO 300 M = 1, NDOF
      IF (LL(M).LT.KI) GO TO 300
      LK = LL(M) - KI + 1
      IF (JJ.GT.0) GO TO 200
      GLSTIF(KI,LK) = GLSTIF(KI,LK) + STMAT(K,M)
      GLGEST(KI,LK) = GLGEST(KI,LK) + GSMAT(K,M)
      GLGEOS(KI,LK) = GLGEOS(KI,LK) + PMAT(K,M)
      GLMASS(KI,LK) = GLMASS(KI,LK) + EMAT(K,M)
  200 GLDAMP(KI,LK) = GLDAMP(KI,LK) + CMAT(K,M)
  300 CONTINUE
  400 CONTINUE
C
      RETURN
      END
C
C
                         SUBROUTINE INPUT
C
C
        COMMON /C1/ DENSI,DENS,AREAI,AREAO,ELEN(50),NDOF,GRAV,YMOD,
     1 INERT,TENS,LENGTH(50),DENSO,NUMEL,NP(2,50),LL(4),NUMNP,
     2STMAT(4,4),EMAT(4,4),GSMAT(4,4),GLMASS(50,6),GLSTIF(50,6),TIME(30)
     3,GLGEST(50,6),NUMB,MBAND,ID(2,50),NEQ,NNPE,AMASSC,GLTOST(50,6)
       COMMON /C2/SH1(10),SH2(10),SH3(10),SH4(10),ZHI(5),WGT(5),AMP,DPER
       COMMON /C3/ NINT,ITER,DIS(50,30),SURFV,WAVEL,WHGT,PERIOD,VCUR(50)
     1,VCW(20),DELT,GLDAMP(50,6),CMAT(4,4),Z(20),IDEPTH,FUNC(50,30),DIAI
     2,DIAO,VEL(50),DRA,PMAT(4,4),GLGEOS(50,6),FORCE(50),VWAVE(50),THETA
C
C
       REAL INERT, LENGTH
       IN = 5
       LP = 6
C
C
       READ(IN,10)   DENSI,DENS,DENSO,DIAI,DIAO,YMOD
  10  FORMAT(5F10.3,E13.7)
C
       READ(IN,20) NNPE,NUMEL,NDOF,MBAND,NUMNP
  20  FORMAT(5I5)
C
       READ(IN,30) TENS,GRAV,DELT,AMP,THETA,DPER,ITER
  30  FORMAT(6F10.4,I5)
       READ(IN,140) DRA,AMASSC,SURFV,WAVEL,PERIOD,WHGT,IDEPTH
 140  FORMAT (6F10.5,I5)
C
       READ(IN,70) (I,LENGTH(I),IO=1,NUMNP)
       READ(IN,80) (I,(NP(J,I),J=1,NNPE),IO=1,NUMEL)
  70  FORMAT(I5,F10.5)
  80  FORMAT(3I3)
C
```

```
C
       DO 130 I = 1, NNPE
       READ (IN,110) (ID(I,J),J=1,NUMNP)
 110  FORMAT(16I2)
 130   CONTINUE
C
C
      WRITE (4,900) DENSI,NNPE,DRA,WHGT,YMOD
      WRITE (4,910) DENS.NUMEL,AMASSC,AMP,TENS
      WRITE (4,920) DENSO,MBAND,SURFV,THETA,GRAV
      WRITE (4,930) DIAI,NDOF,WAVEL,DPER,DELT
      WRITE (4,940) DIAO,NUMNP,PERIOD,LENGTH(NUMNP),ITER
C
 900  FORMAT(5X,'DENSI =',E13.5,8X,'NNPE  =',I5,8X,'DRA    ='
     . ,F10.3,8X,'WHGT    =',F10.3,8X,'YMOD =',E13.7/)
 910  FORMAT(5X,'DENS  =',E13.5,8X,'NUMEL =',I5,8X,'AMASSC =',
     . F10.3,8X,'AMP     =',F10.3,8X,'TENS =',E13.7/)
 920  FORMAT(5X,'DENSO =',E13.5,8X,'MBAND =',I5,8X,'SURFV  =',
     . F10.3,8X,'THETA   =',F10.3,8X,'GRAV    =',E13.7/)
 930  FORMAT(5X,'DIAI  =',E13.5,8X,'NDOF  =',I5,8X,'WAVEL  =',
     . F10.3,8X,'DPER    =',F10.3,8X,'DELT =',F10.3/)
 940  FORMAT(5X,'DIAO  =',E13.5,8X,'NUMNP =',I5,8X,'PERIOD =',
     . F10.3,8X,'LENGTH =',F10.3,8X,'ITER =',I10/)
C
C
       RETURN
       END
C
C
                       SUBROUTINE MASS(N)
C
C
         COMMON /C1/ DENSI,DENS,AREAI,AREAO,ELEN(50),NDOF,GRAV,YMOD,
     1 INERT,TENS,LENGTH(50),DENSO,NUMEL,NP(2,50),LL(4),NUMNP,
     2STMAT(4,4),EMAT(4,4),GSMAT(4,4),GLMASS(50,6),GLSTIF(50,6),TIME(30)
     3,GLGEST(50,6),NUMB,MBAND,ID(2,50),NEQ,NNPE,AMASSC,GLTOST(50,6)
       COMMON /C3/ NINT,ITER,DIS(50,30),SURFV,WAVEL,WHGT,PERIOD,VCUR(50)
     1,VCW(20),DELT,GLDAMP(50,6),CMAT(4,4),Z(20),IDEPTH,FUNC(50,30),DIAI
     2,DIAO,VEL(50),DRA,PMAT(4,4),GLGEOS(50,6),FORCE(50),VWAVE(50),THETA
C
C
       AMASS = DENSI * AREAI + DENS * (AREAO-AREAI)
       COEFF = AMASS * (ELEN(N)/420.0) + 3.14/4.0 * (DENSO * (AMASSC-1.0
     1  ) * DIAO ** 2)
C
C
       EMAT(1,1) = 156.0 * COEFF
       EMAT(1,2) = 22.0 * ELEN(N) * COEFF
       EMAT(1,3) = 54.0 * COEFF
       EMAT(1,4) = -13.0 * ELEN(N) * COEFF
       EMAT(2,2) = 4.0 * (ELEN(N) ** 2) * COEFF
```

```
      EMAT(2,3) = 13.0 * ELEN(N) * COEFF
      EMAT(2,4) = -3.0 * (ELEN(N) ** 2) * COEFF
      EMAT(3,3) = 156.0 * COEFF
      EMAT(3,4) = -22.0 * ELEN(N) * COEFF
      EMAT(4,4) = 4.0 * (ELEN(N) ** 2) * COEFF
C
      DO 10 J = 1, NDOF
      DO 20 K = 1, J
      EMAT(J,K) = EMAT(K,J)
   20 CONTINUE
   10 CONTINUE
C
       IF(N.GT.1) RETURN
C
      RETURN
      END
C
C
C
                       SUBROUTINE STIFN(N)
C
C
        COMMON /C1/ DENSI,DENS,AREAI,AREAO,ELEN(50),NDOF,GRAV,YMOD,
     1 INERT,TENS,LENGTH(50),DENSO,NUMEL,NP(2,50),LL(4),NUMNP,
     2STMAT(4,4),EMAT(4,4),GSMAT(4,4),GLMASS(50,6),GLSTIF(50,6),TIME(30)
     3,GLGEST(50,6),NUMB,MBAND,ID(2,50),NEQ,NNPE,AMASSC,GLTOST(50,6)
C
       REAL INERT
C
      COEFF = YMOD * INERT/ELEN(N) ** 3
C
C
      STMAT(1,1) = 12.0 * COEFF
      STMAT(1,2) = 6.0 * ELEN(N) * COEFF
      STMAT(1,3) = -12.0 * COEFF
      STMAT(1,4) = 6.0 * ELEN(N) * COEFF
      STMAT(2,2) = 4.0 * (ELEN(N) ** 2) * COEFF
      STMAT(2,3) = -6.0 * ELEN(N) * COEFF
      STMAT(2,4) = 2.0 * (ELEN(N) ** 2) * COEFF
      STMAT(3,3) = 12.0 * COEFF
      STMAT(3,4) = -6.0 * ELEN(N) * COEFF
      STMAT(4,4) = 4.0 * (ELEN(N) ** 2) * COEFF
C
      DO 10 J = 1, NDOF
      DO 20 K = 1, J
      STMAT(J,K) = STMAT(K,J)
   20 CONTINUE
   10 CONTINUE
C
C
       IF(N.GT.1) RETURN
```

```
C
C
      RETURN
      END
C
                         SUBROUTINE GESTFN(N)
C
C
        COMMON /C1/ DENSI,DENS,AREAI,AREAO,ELEN(50),NDOF,GRAV,YMOD,
     1 INERT,TENS,LENGTH(50),DENSO,NUMEL,NP(2,50),LL(4).NUMNP,
     2STMAT(4,4),EMAT(4,4),GSMAT(4,4),GLMASS(50,6),GLSTIF(50,6),TIME(30)
     3,GLGEST(50,6),NUMB,MBAND,ID(2,50),NEQ,NNPE,AMASSC,GLTOST(50,6)
C
       REAL LENGTH,INERT
C
      COEFF = ((TENS - DENS*GRAV*(AREAO-AREAI) *(LENGTH(NUMNP)-
     .    LENGTH(N+1))) + (DENSO*AREAO - DENSI*AREAI)*(LENGTH(NUMNP)
     .    - LENGTH(N+1)) * GRAV)/ELEN(N)
C
C
      GSMAT(1,1) = 1.2 * COEFF
      GSMAT(1,2) = 0.1 * ELEN(N) * COEFF
      GSMAT(1,3) = -1.2 * COEFF
      GSMAT(1,4) = 0.1 * ELEN(N) * COEFF
      GSMAT(2,2) = (2.0/15.0)* (ELEN(N) ** 2) * COEFF
      GSMAT(2,3) = -0.1 * ELEN(N) * COEFF
      GSMAT(2,4) = (-1.0/30.0)* (ELEN(N) ** 2) * COEFF
      GSMAT(3,3) = 1.2 * COEFF
      GSMAT(3,4) = -0.1 * ELEN(N) * COEFF
      GSMAT(4,4) = (2.0/15.0)* (ELEN(N) ** 2) * COEFF
C
      DO 10 J = 1, NDOF
      DO 20 K = 1, J
      GSMAT(J,K) = GSMAT(K,J)
   20 CONTINUE
   10 CONTINUE
C
C
       IF(N.GT.1) RETURN
C
C
      RETURN
      END
C
C
                         SUBROUTINE PRESS(N)
C
C
        COMMON /C1/ DENSI,DENS,AREAI.AREAO,ELEN(50),NDOF,GRAV,YMOD,
     1 INERT,TENS,LENGTH(50),DENSO,NUMEL,NP(2,50),LL(4),NUMNP,
     2STMAT(4,4),EMAT(4,4),GSMAT(4,4),GLMASS(50,6),GLSTIF(50,6),TIME(30)
```

```
     3,GLGEST(50,6),NUMB,MBAND,ID(2,50),NEQ,NNPE,AMASSC,GLTOST(50,6)
C
       COMMON /C3/ NINT,ITER,DIS(50,30),SURFV,WAVEL,WHGT,PERIOD,VCUR(50)
     1,VCW(20),DELT,GLDAMP(50,6),CMAT(4,4),Z(20),IDEPTH,FUNC(50,30),DIAI
     2,DIAO,VEL(50),DRA,PMAT(4,4),GLGEOS(50,6),FORCE(50),VWAVE(50),THETA
        REAL LENGTH,INERT
C
      COEFF = ((DENS*GRAV*LENGTH(NUMNP)*(AREAO-AREAI)) - (DENSO*
     .    AREAO - DENSI*AREAI)*LENGTH(NUMNP)*GRAV)/ELEN(N)
C
C
       PMAT(1,1) = 0.6 * COEFF
       PMAT(1,2) = 0.1 * ELEN(N) * COEFF
       PMAT(1,3) = -0.6 * COEFF
       PMAT(1,4) = 0.0 * ELEN(N) * COEFF
       PMAT(2,2) = (1.0/30.0)* (ELEN(N) ** 2) * COEFF
       PMAT(2,3) = -0.1 * ELEN(N) * COEFF
       PMAT(2,4) = (-1.0/60.0)* (ELEN(N) ** 2) * COEFF
       PMAT(3,3) = 0.6 * COEFF
       PMAT(3,4) = -0.0 * ELEN(N) * COEFF
       PMAT(4,4) = 0.1 * (ELEN(N) ** 2) * COEFF
C
       DO 10 J = 1, NDOF
       DO 20 K = 1, J
       PMAT(J,K) = PMAT(K,J)
  20   CONTINUE
  10   CONTINUE
C
C
        IF(N.GT.1) RETURN
C
C
       RETURN
       END
C
C
                        SUBROUTINE DAMP (JJ,TEMP)
C
C
         COMMON /C1/ DENSI,DENS,AREAI,AREAO,ELEN(50),NDOF,GRAV,YMOD,
     1 INERT,TENS,LENGTH(50),DENSO,NUMEL,NP(2,50),LL(4),NUMNP,
     2STMAT(4,4),EMAT(4,4),GSMAT(4,4),GLMASS(50,6),GLSTIF(50,6),TIME(30)
     3,GLGEST(50,6),NUMB,MBAND,ID(2,50),NEQ,NNPE,AMASSC,GLTOST(50,6)
       COMMON /C2/SH1(10),SH2(10),SH3(10),SH4(10),ZHI(5),WGT(5),AMP,DPER
       COMMON /C3/ NINT,ITER,DIS(50,30),SURFV,WAVEL,WHGT,PERIOD,VCUR(50)
     1,VCW(20),DELT,GLDAMP(50,6),CMAT(4,4),Z(20),IDEPTH,FUNC(50,30),DIAI
     2,DIAO,VEL(50),DRA,PMAT(4,4),GLGEOS(50,6),FORCE(50),VWAVE(50),THETA
C
        DIMENSION TEMP(50)
C
C
```

```
        REAL LENGTH,INERT
         DIMENSION AWAVE(50),ACCL(50)
C
C
        NDUM = NEQ + 1
C
          DO 122 I = 2, NDUM
          VEL(I) = TEMP(I-1)
 122      CONTINUE
C
C
          DO 250 I = 1, NUMNP
          VCUR(I) = 0.0
 250      CONTINUE
C
          VCUR(NUMNP) = SURFV
          DUMM = 0.0
          TOTLEN = LENGTH(NUMNP) - LENGTH(IDEPTH)
C
           DO 350 I = 1, NUMNP
            IF((I).EQ.NUMNP) GO TO 350
            IF(I.LE.IDEPTH) GO TO 350
            DUMM = DUMM + ELEN(I-1)
            VCUR(I) = DUMM * SURFV / TOTLEN
 350       CONTINUE
C
C
C
           DO 450 I = 1, NUMNP
           TOTL = -(LENGTH(NUMNP) - LENGTH(I))
           VWAVE(I) = (3.1416 * WHGT/ PERIOD)* (EXP(6.283 * TOTL / WAVEL
      1 )) * COS(6.283 * TIME(JJ) / PERIOD)
        AWAVE(I) = -  (19.74*WHGT/(PERIOD**2))*(EXP(6.283*TOTL/WAVEL))
      . *SIN(6.283*TIME(JJ)/PERIOD)
 450       CONTINUE
C
       DO 62 NL = 1,NUMB
 62    FORCE(NL) = 0.0
C
         COEFF = 0.5 * DENSO + DIAO * DRA
         DO 35 NN = 1,NUMEL
         IF (NN.EQ.1) MN = 3
         IF (NN.GT.1) MN = MN + NN
C
C
          DO 888 III = 1,4
          DO 888 JJJ = 1,4
 888      CMAT(III,JJJ) = 0.0
C
          VONE = VWAVE(NN) + VCUR(NN)
          VTWO = VWAVE(NN+1) + VCUR(NN+1)
```

```
C
      DO 30 K = 1,NINT
      VCW(NN) = VONE * 0.5 * (1.0- ZHI(K)) + VTWO*0.5*(1.0+ZHI(K))
      Z(NN) = ABS(VCW(NN) - (VEL(MN-2) * SH1(K) + VEL(MN-1)
     1          * SH2(K) + VEL(MN) * SH3(K) + VEL(MN+1) * SH4(K)))
      CMAT(1,1) = CMAT(1,1) + Z(NN) * SH1(K) **2.0 * WGT(K) * COEFF
      CMAT(1,2) = CMAT(1,2) + Z(NN) * SH1(K) * SH2(K) * WGT(K) * COEFF
      CMAT(1,3) = CMAT(1,3) + Z(NN) * SH1(K) * SH3(K) * WGT(K) * COEFF
      CMAT(1,4) = CMAT(1,4) + Z(NN) * SH1(K) * SH4(K) * WGT(K) * COEFF
      CMAT(2,2) = CMAT(2,2) + Z(NN) * SH2(K) **2.0 * WGT(K) * COEFF
      CMAT(2,3) = CMAT(2,3) + Z(NN) * SH2(K) * SH3(K) * WGT(K) * COEFF
      CMAT(2,4) = CMAT(2,4) + Z(NN) * SH2(K) * SH4(K) * WGT(K) * COEFF
      CMAT(3,3) = CMAT(3,3) + Z(NN) * SH3(K) **2.0    * WGT(K) * COEFF
      CMAT(3,4) = CMAT(3,4) + Z(NN) * SH3(K) * SH4(K) * WGT(K) * COEFF
      CMAT(4,4) = CMAT(4,4) + Z(NN) * SH4(K) * SH4(K) * WGT(K) * COEFF
 30   CONTINUE
C
C
       DO 60 I = 1,4
       DO 60 J = 1,4
 60    CMAT(J,I) = CMAT(I,J)
C
       IJ = NP(1,NN)
       JL = NP(2,NN)
       CALL GLOBAL(IJ,JL,JJ)
       MN = MN - NN + 1
 35    CONTINUE
C
     COEF1 = 3.1416/4.0 *DENSO*AMASSC*(DIAO**2)
    IF(ID(1,1).EQ.O.AND.ID(2,1).GT.O) JML = 2
    IF(ID(1,1).EQ.O.AND.ID(2,1).EQ.O) JML = 1
    DO 84 MM = 1,NUMEL
        KML = JML+2
    DO 94 K = 1,NINT
     ACCL(MM) = AWAVE(MM)*0.5*(1.0 - ZHI(K)) + AWAVE(MM+1)*0.5*
   .  (1.0 + ZHI(K))
     FORCE(JML) = (COEF1*ACCL(MM) + COEFF*Z(MM)*VCW(MM))*SH1(K)
   .             +FORCE(JML)
     FORCE(KML) = (COEF1*ACCL(MM) + COEFF*Z(MM)*VCW(MM))*SH3(K)
   .             + FORCE(KML)
94  CONTINUE
     JML = JML+2
84  CONTINUE
C
    PHI = 0.0
C
    FORCE(NEQ)=FORCE(NEQ)-GLTOST(NEQ,2)*AMP*SIN(6.283*TIME(JJ)/
   .DPER+PHI)+GLTOST(NEQ,3)*THETA*SIN(6.283*TIME(JJ)/DPER+PHI)
   .-(6.283/DPER)*GLDAMP(NEQ,2)*AMP*COS(6.283*TIME(JJ)/DPER
   .+PHI)+(6.283/DPER)*GLDAMP(NEQ,3)*THETA*COS(6.283*TIME(JJ)
   ./DPER+PHI)+((6.283/DPER)**2)*GLMASS(NEQ,2)*AMP*SIN(6.283*
```

```
     .TIME(JJ)/DPER+PHI)-((6.283/DPER)**2)*GLMASS(NEQ,3)*THETA*
     .SIN(6.283*TIME(JJ)/DPER+PHI)
C
C
      FORCE(NEQ-1)=FORCE(NEQ-1)-GLTOST(NEQ-1,3)*AMP*SIN(6.283*TIME(JJ)/
     .DPER+PHI)+GLTOST(NEQ-1,4)*THETA*SIN(6.283*TIME(JJ)/DPER+PHI)
     .-(6.283/DPER)*GLDAMP(NEQ-1,3)*AMP*COS(6.283*TIME(JJ)/DPER
     .+PHI)+(6.283/DPER)*GLDAMP(NEQ-1,4)*THETA*COS(6.283*TIME(JJ)
     ./DPER+PHI)+((6.283/DPER)**2)*GLMASS(NEQ-1,3)*AMP*SIN(6.283*
     .TIME(JJ)/DPER+PHI)-((6.283/DPER)**2)*GLMASS(NEQ-1,4)*THETA*
     .SIN(6.283*TIME(JJ)/DPER+PHI)
C
C
      RETURN
      END
C
C
                              SUBROUTINE NEWMAR
C
          COMMON /C1/ DENSI,DENS,AREAI,AREAO,ELEN(50),NDOF,GRAV,YMOD,
     1 INERT,TENS,LENGTH(50),DENSO,NUMEL,NP(2,50),LL(4),NUMNP,
     2STMAT(4,4),EMAT(4,4),GSMAT(4,4),GLMASS(50,6),GLSTIF(50,6),TIME(30)
     3,GLGEST(50,6),NUMB,MBAND,ID(2,50),NEQ,NNPE,AMASSC,GLTOST(50,6)
        COMMON /C2/SH1(10),SH2(10),SH3(10),SH4(10),ZHI(5),WGT(5),AMP,DPER
        COMMON /C3/ NINT,ITER,DIS(50,30),SURFV,WAVEL,WHGT,PERIOD,VCUR(50)
     1,VCW(20),DELT,GLDAMP(50,6),CMAT(4,4),Z(20),IDEPTH,FUNC(50,30),DIAI
     2,DIAO,VEL(50),DRA,PMAT(4,4),GLGEOS(50,6),FORCE(50),VWAVE(50),THETA
C
         DIMENSION A(50,6),B(50,1),XOLD(50),XNEW(50),VXOLD(50),VXNEW(50)
     1 ,AXNEW(50),AXOLD(50),TEMP(50)
C
       NC = 3
C
C
          DO 10 I = 1,NEQ
            XOLD(I) = 0.0
            VXOLD(I) = 0.0
            AXOLD(I) = 0.0
            XNEW(I) = 0.0
            VXNEW(I) = 0.0
            AXNEW(I) = 0.0
          B(I,1) = 0.0
 10       CONTINUE
C
C
          DO 200 JJ = 2, ITER
         DO 123 LK = 1,NEQ
123      TEMP(LK) = VXOLD(LK) + DELT*AXOLD(LK)
C
C
C
```

```
C ****** NEWMARK'S ALGORITHM
C
           ALFA = 0.25
           DELTA = 0.5
           AO = 1.0 / (ALFA * DELT ** 2)
           A1 = DELTA / (ALFA * DELT)
      A2=1.0/(ALFA*DELT)
      A3=(0.5/ALFA) - 1.0
      A4=(DELTA/ALFA) - 1.0
      A5=DELT*((DELTA/ALFA)-2.0)*0.5
      A6=DELT*(1.0-DELTA)
      A7=DELTA*DELT
C
      CALL DAMP(JJ,TEMP)
C
      DO 420 I=1,NEQ
      B(I,1)=FORCE(I)
      KBAND = 2 * NC + 1
      DO 420 J=1,KBAND
      JN=J+I-NC-1
      IF (JN.LE.O.OR.JN.GT.NEQ) GO TO 420
      IF (J.LE.NC) GO TO 421
      JM = J - NC
      B(I,1)=B(I,1)+GLMASS(I,JM)*(AO*XOLD(JN)+A2*VXOLD(JN)+A3*AXOLD(JN))
     *+ GLDAMP(I,JM)*(A1*XOLD(JN) + A4*VXOLD(JN) + A5*AXOLD(JN))
      GO TO 420
 421   JM = I - JN + 1
      B(I,1)=B(I,1)+GLMASS(JN,JM)*(AO*XOLD(JN)+A2*VXOLD(JN)+A3*AXOLD(JN)
     *)+ GLDAMP(JN,JM)*(A1*XOLD(JN) + A4*VXOLD(JN) + A5*AXOLD(JN))
420    CONTINUE
        DO 425 I = 1,NEQ
        DO 435 J = 1,MBAND
       A(I,J) = GLTOST(I,J) + AO*GLMASS(I,J) + A1*GLDAMP(I,J)
435    CONTINUE
425    CONTINUE
       CALL DECOM(A,B,JJ)
       CALL SOLBAN(A,B,JJ)
      DO 430 I=1,NEQ
      XNEW(I)=DIS(I,JJ)
      AXNEW(I)=AO*(XNEW(I)-XOLD(I)) - A2*VXOLD(I) - A3*AXOLD(I)
      VXNEW(I)=VXOLD(I) + A6*AXOLD(I) + A7*AXNEW(I)
430    TEMP(I) = VXNEW(I)
C
C
      CALL DAMP(JJ,TEMP)
C
C
      DO 422 I=1,NEQ
      B(I,1)=FORCE(I)
      KBAND = 2 * NC + 1
      DO 422 J=1,KBAND
```

```
      JN=J+I-NC-1
      IF (JN.LE.0.OR.JN.GT.NEQ) GO TO 422
      IF (J.LE.NC) GO TO 423
      JM = J - NC
      B(I,1)=B(I,1)+GLMASS(I,JM)*(AO*XOLD(JN)+A2*VXOLD(JN)+A3*AXOLD(JN))
     *+ GLDAMP(I,JM)*(A1*XOLD(JN) + A4*VXOLD(JN) + A5*AXOLD(JN))
      GO TO 422
 423   JM = I - JN + 1
      B(I,1)=B(I,1)+GLMASS(JN,JM)*(AO*XOLD(JN)+A2*VXOLD(JN)+A3*AXOLD(JN)
     *)+ GLDAMP(JN,JM)*(A1*XOLD(JN) + A4*VXOLD(JN) + A5*AXOLD(JN))
422    CONTINUE
        DO 445 I = 1,NEQ
        DO 455 J = 1,MBAND
       A(I,J) = GLTOST(I,J) + AO*GLMASS(I,J) + A1*GLDAMP(I,J)
455    CONTINUE
445    CONTINUE
       CALL DECOM(A,B,JJ)
       CALL SOLBAN(A,B,JJ)
      DO 432 I=1,NEQ
      XNEW(I)=DIS(I,JJ)
      AXNEW(I)=AO*(XNEW(I)-XOLD(I)) - A2*VXOLD(I) - A3*AXOLD(I)
432   VXNEW(I)=VXOLD(I) + A6*AXOLD(I) + A7*AXNEW(I)
      DO 470 I=1,NEQ
      XOLD(I)=XNEW(I)
      DIS(I,JJ) = XOLD(I)
      VXOLD(I)=VXNEW(I)
  470 AXOLD(I)=AXNEW(I)
C
      PHI = 0.0
C
      DIS(NEQ+1,JJ) = AMP * SIN(6.283*TIME(JJ)/DPER+PHI)
      DIS(NEQ+2,JJ) = -THETA * SIN(6.283*TIME(JJ)/DPER+PHI)
 200   CONTINUE
C
C
       RETURN
       END
C
C
C
                     SUBROUTINE DECOM(A,B,JJ)
C
C
         COMMON /C1/ DENSI,DENS,AREAI,AREAO,ELEN(50),NDOF,GRAV,YMOD,
     1 INERT,TENS,LENGTH(50),DENSO,NUMEL,NP(2,50),LL(4),NUMNP,
     2STMAT(4,4),EMAT(4,4),GSMAT(4,4),GLMASS(50,6),GLSTIF(50,6),TIME(30)
     3,GLGEST(50,6),NUMB,MBAND,ID(2,50),NEQ,NNPE,AMASSC,GLTOST(50,6)
       COMMON /C2/SH1(10),SH2(10),SH3(10),SH4(10),ZHI(5),WGT(5),AMP,DPER
       COMMON /C3/ NINT,ITER,DIS(50,30),SURFV,WAVEL,WHGT,PERIOD,VCUR(50)
     1,VCW(20),DELT,GLDAMP(50,6),CMAT(4,4),Z(20),IDEPTH,FUNC(50,30),DIAI
     2,DIAO,VEL(50),DRA,PMAT(4,4),GLGEOS(50,6),FORCE(50),VWAVE(50),THETA
```

```
C
        DIMENSION A(50,6),B(50,1),XOLD(50),XNEW(50),VXOLD(50),VXNEW(50)
     1 ,AXNEW(50),AXOLD(50)
C
      NP1 = NEQ - 1
      DO 226 I = 1,NP1
      MJ = I + MBAND - 1
      IF(MJ.GT.NEQ) MJ = NEQ
      NJ = I + 1
      MK = MBAND
      IF((NEQ-I+1).LT.MBAND) MK = NEQ-I+1
      ND = 0
      DO 224 J = NJ,MJ
      MK = MK - 1
      ND = ND + 1
      NL = ND + 1
      DO 224 K = 1,MK
      NK = ND + K
224   A(J,K) = A(J,K)-A(I,NL)*A(I,NK)/A(I,1)
226   CONTINUE
C
C
      RETURN
      END
C
C
                  SUBROUTINE SOLBAN(A,B,JJ)
C
C
        COMMON /C1/ DENSI,DENS,AREAI,AREAO,ELEN(50),NDOF,GRAV,YMOD,
     1 INERT,TENS,LENGTH(50),DENSO,NUMEL,NP(2,50),LL(4),NUMNP,
     2STMAT(4,4),EMAT(4,4),GSMAT(4,4),GLMASS(50,6),GLSTIF(50,6),TIME(30)
     3,GLGEST(50,6),NUMB,MBAND,ID(2,50),NEQ,NNPE,AMASSC,GLTOST(50,6)
       COMMON /C2/SH1(10),SH2(10),SH3(10),SH4(10),ZHI(5),WGT(5),AMP,DPER
       COMMON /C3/ NINT,ITER,DIS(50,30),SURFV,WAVEL,WHGT,PERIOD,VCUR(50)
     1,VCW(20),DELT,GLDAMP(50,6),CMAT(4,4),Z(20),IDEPTH,FUNC(50,30),DIAI
     2,DIAO,VEL(50),DRA,PMAT(4,4),GLGEOS(50,6),FORCE(50),VWAVE(50),THETA
C
        DIMENSION A(50,6),B(50,1),XOLD(50),XNEW(50),VXOLD(50),VXNEW(50)
     1 ,AXNEW(50),AXOLD(50)
C
      NP1 = NEQ - 1
C
C****   DECOMPOSITION OF THE COLUMN VECTOR FORCE
C
      DO 250 I = 1,NP1
      MJ = I + MBAND - 1
      IF(MJ.GT.NEQ) MJ = NEQ
      NJ = I+1
      L= 1
      DO 250 J = NJ,MJ
```

```
      L = L + 1
250   B(J,1) = B(J,1) - A(I,L)*B(I,1)/A(I,1)
C
C****  BACKWARD SUBSTITUTION FOR DETERMINATION OF DIS
C
      DIS(NEQ,JJ) = B(NEQ,1)/A(NEQ,1)
      DO 253 K = 1,NP1
      I = NEQ - K
      MJ = MBAND
      IF((I+MBAND-1).GT.NEQ) MJ = NEQ-I+1
      SUM = 0.0
      DO 251 J = 2,MJ
      N = I+J-1
251   SUM = SUM + A(I,J)*DIS(N,JJ)
253   DIS(I,JJ) = (B(I,1) - SUM)/A(I,1)
C
C
      RETURN
      END
```

APPENDIX C

SAMPLE RESULTS

DENSI =	0.14380E 04	NNPE =	2	DRA =	1.138	WHGT =	6.090	YMOD =	0.2100000E 12
DENS =	0.86900E 04	NUMEL =	15	AMASSC =	1.500	AMP =	2.000	TENS =	0.1072000E 07
DENSO =	0.10250E 04	MBAND =	4	SURFV =	1.028	THETA =	0.100	GRAV =	0.9810000E 01
DIAI =	0.38100E 00	NDOF =	4	WAVEL =	100.000	DPER =	20.000	DELT =	1.000
DIAO =	0.40640E 00	NUMNP =	16	PERIOD =	20.000	LENGTH =	400.000	ITER =	21

TIME = 0.00 SECS.

NODE	DISPLACEMENT	ROTATION
1	0.000000E 00	0.000000E 00
2	0.000000E 00	0.000000E 00
3	0.000000E 00	0.000000E 00
4	0.000000E 00	0.000000E 00
5	0.000000E 00	0.000000E 00
6	0.000000E 00	0.000000E 00
7	0.000000E 00	0.000000E 00
8	0.000000E 00	0.000000E 00
9	0.000000E 00	0.000000E 00
10	0.000000E 00	0.000000E 00
11	0.000000E 00	0.000000E 00
12	0.000000E 00	0.000000E 00
13	0.000000E 00	0.000000E 00
14	0.000000E 00	0.000000E 00
15	0.000000E 00	0.000000E 00
16	0.000000E 00	0.000000E 00

ELEMENT	MOMENT	SHEAR
1	-0.000000E 00	0.000000E 00
2	-0.000000E 00	0.000000E 00
3	-0.000000E 00	0.000000E 00
4	-0.000000E 00	0.000000E 00
5	-0.000000E 00	0.000000E 00
6	-0.000000E 00	0.000000E 00
7	-0.000000E 00	0.000000E 00
8	-0.000000E 00	0.000000E 00
9	-0.000000E 00	0.000000E 00
10	-0.000000E 00	0.000000E 00
11	-0.000000E 00	0.000000E 00
12	-0.000000E 00	0.000000E 00
13	-0.000000E 00	0.000000E 00
14	-0.000000E 00	0.000000E 00
15	-0.000000E 00	0.000000E 00

TIME = 1.00 SECS.

NODE	DISPLACEMENT	ROTATION
1	0.000000E 00	0.352095E-08
2	0.742309E-06	0.104050E-06
3	0.123598E-04	0.113609E-05
4	0.860253E-04	0.426838E-05
5	0.298746E-03	0.924749E-05
6	0.692263E-03	0.148714E-04
7	0.126916E-02	0.202862E-04
8	0.203216E-02	0.259960E-04
9	0.297873E-02	0.317307E-04
10	0.410945E-02	0.376876E-04
11	0.550599E-02	0.520190E-04
12	0.781260E-02	0.127070E-03
13	0.157827E-01	0.628887E-03
14	0.602336E-01	0.366621E-02
15	0.291817E 00	0.158080E-01
16	0.618016E 00	-0.309008E-01

ELEMENT	MOMENT	SHEAR
1	-0.241249E 00	-0.280220E-01
2	-0.247854E 01	-0.199030E 00
3	-0.750555E 01	0.611397E-01
4	-0.119532E 02	0.132322E 01
5	-0.135013E 02	0.292584E 01
6	-0.130236E 02	0.445843E 01
7	-0.136663E 02	0.580626E 01
8	-0.137569E 02	0.715495E 01
9	-0.143276E 02	0.846245E 01
10	-0.343404E 02	0.802422E 01
11	-0.180038E 03	-0.329953E 01
12	-0.120562E 04	-0.851863E 02
13	-0.727795E 04	-0.519867E 03
14	-0.290940E 05	-0.114539E 04
15	0.111923E 06	0.212843E 05

TIME = 2.00 SECS.

NODE	DISPLACEMENT	ROTATION
1	0.000000E 00	0.264075E-06
2	0.154380E-04	0.145846E-05
3	0.123754E-03	0.861816E-05
4	0.569336E-03	0.271148E-04
5	0.167619E-02	0.555778E-04
6	0.363982E-02	0.877955E-04
7	0.649239E-02	0.119843E-03
8	0.102510E-01	0.152348E-03
9	0.149152E-01	0.187286E-03
10	0.206972E-01	0.243534E-03
11	0.293198E-01	0.447267E-03
12	0.505773E-01	0.143406E-02
13	0.130191E 00	0.566918E-02
14	0.419382E 00	0.181221E-01
15	0.106976E 01	0.254454E-01
16	0.117554E 01	-0.587770E-01

ELEMENT	MOMENT	SHEAR
1	-0.286626E 01	-0.304821E 00
2	-0.171946E 02	-0.105193E 01
3	-0.443214E 02	-0.126865E 01
4	-0.683307E 02	0.201752E 00
5	-0.773445E 02	0.215715E 01
6	-0.770800E 02	0.371137E 01
7	-0.778003E 02	0.485608E 01
8	-0.838137E 02	0.547169E 01
9	-0.135288E 03	0.212394E 01
10	-0.488179E 03	-0.241882E 02
11	-0.236721E 04	-0.155000E 03
12	-0.101748E 05	-0.608402E 03
13	-0.298393E 05	-0.114651E 04
14	-0.175481E 05	0.277313E 04
15	0.201813E 06	0.222148E 05

TIME = 3.00 SECS.

NODE	DISPLACEMENT	ROTATION	ELEMENT	MOMENT	SHEAR
1	0.000000E 00	0.314652E-05	1	-0.135033E 02	-0.143440E 01
2	0.123489E-03	0.877338E-05	2	-0.563335E 02	-0.264287E 01
3	0.604574E-03	0.322302E-04	3	-0.116516E 03	-0.331770E 01
4	0.203201E-02	0.808556E-04	4	-0.169154E 03	-0.206746E 01
5	0.507473E-02	0.151316E-03	5	-0.193547E 03	-0.108973E 00
6	0.101789E-01	0.231938E-03	6	-0.198942E 03	0.118859E 01
7	0.174777E-01	0.314651E-03	7	-0.209835E 03	0.827836E 00
8	0.270806E-01	0.402320E-03	8	-0.307677E 03	-0.646567E 01
9	0.393610E-01	0.530578E-03	9	-0.873867E 03	-0.469953E 02
10	0.571545E-01	0.893903E-03	10	-0.342009E 04	-0.208436E 03
11	0.949088E-01	0.232121E-02	11	-0.119494E 05	-0.634894E 03
12	0.207552E 00	0.730245E-02	12	-0.284899E 05	-0.943483E 03
13	0.536703E 00	0.191609E-01	13	-0.242537E 05	0.103977E 04
14	0.120921E 01	0.292828E-01	14	0.408498E 05	0.268029E 04
15	0.182992E 01	0.122350E-01	15	0.223169E 06	0.284263E 05
16	0.161800E 01	-0.809001E-01	-		

TIME = 4.00 SECS.

NODE	DISPLACEMENT	ROTATION
1	0.000000E 00	0.182462E-04
2	0.573295E-03	0.324992E-04
3	0.201951E-02	0.852153E-04
4	0.537418E-02	0.176576E-03
5	0.116202E-01	0.300360E-03
6	0.214700E-01	0.443602E-03
7	0.353035E-01	0.601589E-03
8	0.539821E-01	0.820495E-03
9	0.813958E-01	0.135586E-02
10	0.135377E 00	0.315290E-02
11	0.276517E 00	0.859354E-02
12	0.636666E 00	0.198629E-01
13	0.130676E 01	0.289725E-01
14	0.197497E 01	0.170012E-01
15	0.229586E 01	0.810966E-02
16	0.190209E 01	-0.951045E-01

ELEMENT	MOMENT	SHEAR
1	-0.342042E 02	-0.417893E 01
2	-0.126602E 03	-0.494376E 01
3	-0.218917E 03	-0.565654E 01
4	-0.297166E 03	-0.442857E 01
5	-0.343878E 03	-0.257162E 01
6	-0.379992E 03	-0.275797E 01
7	-0.523949E 03	-0.131661E 02
8	-0.128427E 04	-0.650746E 02
9	-0.432223E 04	-0.244159E 03
10	-0.130367E 05	-0.632237E 03
11	-0.270341E 05	-0.781786E 03
12	-0.218857E 05	0.807113E 03
13	0.286853E 05	0.219644E 04
14	0.213059E 05	-0.578282E 03
15	0.247321E 06	0.309613E 05

TIME = 5.00 SECS.

NODE	DISPLACEMENT	ROTATION
1	0.000000E 00	0.673302E-04
2	0.185901E-02	0.889563E-04
3	0.525550E-02	0.180298E-03
4	0.117849E-01	0.324033E-03
5	0.227069E-01	0.510029E-03
6	0.390650E-01	0.734166E-03
7	0.622432E-01	0.104943E-02
8	0.976190E-01	0.175013E-02
9	0.165612E 00	0.384693E-02
10	0.328434E 00	0.953898E-02
11	0.709210E 00	0.202253E-01
12	0.137400E 01	0.283165E-01
13	0.204437E 01	0.183384E-01
14	0.236899E 01	0.891735E-02
15	0.258432E 01	0.155316E-02
16	0.200000E 01	-0.100000E 00

ELEMENT	MOMENT	SHEAR
1	-0.518980E 02	-0.908747E 01
2	-0.219364E 03	-0.771445E 01
3	-0.344417E 03	-0.820295E 01
4	-0.446516E 03	-0.778519E 01
5	-0.538082E 03	-0.895474E 01
6	-0.758278E 03	-0.221776E 02
7	-0.167712E 04	-0.820206E 02
8	-0.502996E 04	-0.268860E 03
9	-0.136905E 05	-0.620486E 03
10	-0.256064E 05	-0.668702E 03
11	-0.194098E 05	0.707822E 03
12	0.239722E 05	0.199870E 04
13	0.225746E 05	-0.158312E 04
14	0.176460E 05	0.304714E 04
15	0.243341E 06	0.294424E 05

VITA

VASAN LAXMINARASIMHAN

Candidate for the Degree of

Master of Science

Thesis: DYNAMIC ANALYSIS OF A MARINE RISER SYSTEM

Major Field: Mechanical Engineering

Biographical:

Personal Data: Born in Pasur, Tamil Nadu, India, June 28, 1955, the son of Mr. and Mrs. V. L. Narasimhan.

Education: Graduated from St. John's Church High School, Secunderabad, India, in May, 1971; two year intermediate from Railway Jr. College, Secunderabad, India, in May, 1973; received Bachelor of Engineering degree in Mechanical Engineering from Osmania University, in 1978; completed requirements for the Master of Science degree at Oklahoma State University in July, 1983.

Professional Experience: Worked as Design Engineer for Unicorn Industries, Secunderabad, India, June 1978 to November 1980; Production Supervisor, Bharat Dynamics Ltd., Hyderabad, India, December 1980 to July 1980.