ANALYTIC SIMULATION OF TRANSITION ZONE ASSOCIATED WITH UPCONING IN AQUIFER SYSTEMS

By

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WITH UPCONING IN AQUIFER SYSTEMS

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CHAPTER I

INTRODUCTION

Exploitation of groundwater resources for agricultural, municipal, and industrial uses is severely hampered in many regions of the world by the encroachment of saline water resulting from freshwater withdrawals. Examples of saltwater encroachment are numerous in coastal aquifers but a closely related problem sometimes occurs in inland aquifers as well.

When water is pumped by a discharge well, penetrating only the upper portion of a aquifer containing an underlying layer of saltwater, a local rise of the interface below the well occurs. This phenomenon is known as "upconing." In the early 1900s, researchers believed that a sharp interface existed between the freshwater and saltwater zones; but, from 1950 on, the field data on interface indicated that a transition zone of sufficient width can exist between these two. The two fluids are miscible and in reality, at their contact, they tend to mix by molecular diffusion and macroscopic dispersion. Therefore, they are not separated by a sharp interface. They do not constitute distinct fluid phases, and there is no pressure discontinuity where they are in contact. The salt water diffuses into the freshwater and accordingly, a brackish water band is formed, with decreasing salinity from its bottom to its top. This band, therefore, decreases in concentration from that of salt water to that of fresh water. The freshwater concentration is commonly taken as 250 mg/ℓ for drinking-water purposes.

In this study, the width of the transition zone has been calculated from analytic equations. Various positions of the transition zones at specific times are calculated, and figures have been presented to illustrate their positions above the initially sharp interface for the bottom of the well.

The study has been made to find the extent of the transition zone created by the operation of the discharge well, considering various conditions of aquifer properties, discharges, time effects, and well locations.

CHAPTER II

LITERATURE REVIEW

When an aquifer contains an underlying layer of saline water and is pumped by a well penetrating the upper portion of the aquifer, a local rise of the interface below the well occurs. This phenomenon is known as upconing. Upconing is a complex phenomenon and only in recent years has significant headway been made in research studies to enable criteria to be formulated for the design and operation of wells for skimming fresh water from above saline water (Todd, 1980). Most investigators of upconing have assumed an abrupt interface between the two fluids. This situation would obtain between immiscible fluids, but for miscible fluids such as fresh and saline groundwater, a mixing zone or transition zone having a finite thickness occurs.

Ghyben (1888) and Herzberg (1901) independently calculated the height of the cone below the well center, assuming a steady horizontal flow of fresh water to the well, no lateral movement of salt water, and a sharp interface.

Exact solutions for the shape of the saltwater front were obtained by Henry (1959) and an approximate equation for the steady-state interface between fresh water and salt water was developed by Rumer and Harleman (1963). Glover (1964) also developed an approximate equation for the shape of the freshwater-saltwater interface. Rumer and Shiau (1968) gave an analytic method to locate the position and to determine the shapes of the interface between the seaward flowing fresh water and the underlying saltwater in both isotropic and anisotropic, nonhomogeneous coastal aquifers.

Dagan and Bear (1968) gave mathematical solutions for interface upconing and checked their results by a physical model. Experimental studies were also conducted by Carlson (1968). Hantush (1968) also derived an approximate differential equations, the solution of which gave approximate expressions for the movement of the freshwater-saltwater interface in several flow systems.

Using existing theoretical equations Schmorak and Mercado (1969) described the mechanism of upconing of an abrupt interface. They found that the solutions of these equations are in agreement with field results up to some critical rise of the interface, which is approximately half the distance between the bottom of the well and the undisturbed initial interface. The salinity of the pumped water is probably caused by the intrusion of saline water above a certain critical depth. Schmorak and Mercado (1969) also found that the salinity increase of pumped water is about 5% to 8% of the average salinity of the saline water intruded above the critical depth.

Tyagi (1971) and Tyagi and Todd (1971) derived a dimensionless relationship between the dispersion coefficient and variables of flow and a porous medium. Hoque (1983) developed a numerical model using the block-centered finite difference method to solve two vertically integrated nonlinear partial differential equations. These equations describe the transient position of the freshwater-saltwater interface in an inland aquifer system.

Vacher (1974) published a report for the Bermuda Public Works Department that, among other findings, includes the documentation of the size and geometry of the transition zone surrounding and underlying the freshwater lenses. Ayer (1980) developed a numerical model treating the unsteady flow in Bermuda's groundwater system. He presented more accurate data and a better understanding of the transition zone of the Devonshire Lens. Ayer and Vacher (1980), in a report submitted to the Bermuda Public Works Department, formulated the equations for determining the width of the transition zone and the salinity profile. These two equations are utilized in carrying out this work.

CHAPTER III

MATHEMATICAL FORMULATION

The assumptions considered in this study are that the porous medium is homogeneous and nondeformable, that the two fluids are incompressible and separated by an abrupt interface, and that the flow obeys Darcy's law.

Upconing of the interface, below a partially penetrating pumping well, as a function of time and distance from the well, is described by:

$$Z(r,t) = \frac{Q}{2\pi(\frac{\Delta\gamma}{\gamma})k_r d}\left[\frac{1}{(1+\bar{R}^2)^{1/2}} - \frac{1}{\{(1+\bar{T})^2 + \bar{R}^2\}^{1/2}}\right] \qquad (1)$$

where $\bar{R}$ and $\bar{T}$ are the dimensionless distance and time parameters given by

$$\bar{R} = \frac{r}{d}\left(\frac{K_z}{K_r}\right)^{1/2} \qquad (2)$$

$$\bar{T} = \frac{(\Delta\gamma/\gamma)k_z t}{2nd} \qquad (3)$$

and

Z = distance of the interface rise above its initial position

Q = pumping rate of the well

$\Delta\gamma/\gamma$ = dimensionless density difference between the two fluids

d = distance between bottom of the well and the interface at $t = o$

r = distance from the well center

n = porosity of the aquifer

k_z, k_r = vertical and horizontal permeability, respectively

t = time elapsed since start of pumping

For r = o (i.e., at the pumping well), Equation (1) becomes

$$Z(O,t) = \frac{Q}{2\pi(\Delta\gamma/\gamma)k_r d}\left(1 - \frac{1}{1+\bar{T}}\right) \tag{4}$$

For $t \to \infty$ Equations (1) and (4) reduce to

$$Z(r,\infty) = \frac{Q}{2\pi(\Delta\gamma/\gamma)k_r d} \frac{1}{[(1 + \bar{R}^2)^{\frac{1}{2}}]} \tag{5}$$

For r = O and $t \to \infty$, Equation (1) yields

$$Z(O,\infty) = \frac{Q}{2\pi(\Delta\gamma/\gamma)k_r d} . \tag{6}$$

According to Equation (6), the ultimate rise of the interface at the new equilibrium is directly proporational to the pumping rate, Q.

This linear relationship between Z and Q in Equation (6) is limited to a certain critical rise Z_{cr}. Model experiments show that for values Z/d between 1/3 to 1/2, the rate of rise is accelerated, and that above a certain critical rise Z_{cr} = 0.5d there is instability, such that the interface reaches the bottom of the pumping well with a sudden jump.

The above relationship is derived considering that the upconing process involves a sharp interface between the two fluids. In the actual case, there is a transition zone between the two miscible fluids in which the concentration varies gradually from the concentration of salt water to the concentration of the fresh water. The hydrodynamic dispersion caused by fluctuations of the interface, results in the transition zone.

The salinity profile $\varepsilon(x)$ is a function of the depth, x, the depth of the transition zone center, $\bar{x}$, the equivalent of the total traveled distance, $|\bar{x}|$ (independent of direction), and the dispersivity Dm. It is given by:

$$\varepsilon(x) = \frac{1}{2}\left[1 - \operatorname{erf} \frac{x-\bar{x}}{2\sqrt{Dm|\bar{x}|}}\right] ; \qquad (7)$$

ε is the relative salinity and is defined by:

$$\varepsilon = \frac{C - C_b}{C_s - C_b} , \qquad (8)$$

where C is the measured concentration of chlorides at point x. C_b is the background concentration in the displaced water and C_s is the concentration of the invading fluid.

The width of the transition zone 2σ as a function of the total traveled distance, $|\bar{x}|$, and the dispersivity, Dm is:

$$2\sigma = 2(2Dm|\bar{x}|)^{\frac{1}{2}} \qquad (9)$$

or

$$2\sigma = 2(2D_L t)^{\frac{1}{2}} \qquad (10)$$

as $|\bar{x}| = u \cdot t$ and $D_L = Dm \cdot u_x$;

where u_x = fluid velocity, and

D_L = longitudinal dispersion coefficient.

The parameter, σ, is defined by

$$\sigma = \frac{1}{2}\left[(x)_{\varepsilon=15.9\%} - (x)_{\varepsilon=84.1\%}\right] \tag{11}$$

Thus, the $\varepsilon = \varepsilon(x)$ function can be expressed in terms of the transition zone parameter as:

$$\varepsilon(x) = \frac{1}{2}(1 - \text{erf}\,\frac{x-\bar{x}}{\sqrt{2}\sigma}) \quad . \tag{12}$$

Equation (11) and (12) can be used to superimpose the effect of dispersion on a sharp freshwater-saltwater interface. The assumption is that the sharp interface exists at the 84.13% concentration level of the transition zone.

The longitudinal dispersion coefficient (D_L) is computed by the product of dispersivity and velocity.

Velocity is computed from Darcy's law

$$u = ki$$

where

u = velocity

k = permeability

i = hydraulic gradient

The hydraulic gradient is defined as

$$\text{Hydraulic Gradient} = \frac{\text{Head Loss at Equal Intervals Away From the Well}}{\text{Distance of the Interval}} \tag{15}$$

The heads at various radii from the center of the well are computed, considering transient flow, for both confined and unconfined aquifers.

For the case of the confined aquifer, this solution was utilized. assuming that the well is fully penetrated in the aquifer. A simplified solution developed by Cooper and Jacop was adopted:

$$s = \frac{Q}{4\pi T} \ln \frac{2.25\ Tt}{r^2 S} \qquad (16)$$

where:

s = drawdown at a distance r from the well

Q = well discharge

T = transmissivity

S = storage coefficient

t = time since pumping started

For the unconfined aquifer, a solution presented by Boulton was adopted:

$$s = \frac{Q}{2\pi KH} (1 + C_k) V(t',r') \qquad (17)$$

where C_k is a correction factor and V(t',r') is Boulton's well function of t' and r' defined as

$$t' = \frac{Kt}{SH}$$

$$r' = \frac{r}{H} .$$

Values of V(t',r') are taken from Table 4.1, pp. 74 (Bouwer, 1978). C_k is obtained from a curve drawn from data points provided by Boulton.

Equation (1) was used to compute the rise of the initial interface at various radii from the well center at different time periods. Considering the interface as a sharp interface at the 84.13% concentration boundary, the width of the transition zone obtained from Equation (10) was added vertically upwards at the corresponding radii. The upper boundary of the transition zone thus obtained was considered at the 15.9% concentration boundary. The transition zone having a definite width, with distinct upper and lower demarcation boundaries, is thus shown.

CHAPTER IV

APPLICATION

Data from three aquifers were taken, and the width of the transition zone in each case was determined, in conducting this study. The first case consisted of an Island Aquifer System that represented a dense porous medium. The second case was that of an Inland Aquifer System that contained a highly porous medium. The third case used was the Garber-Wellington Aquifer System. Hydraulic parameters used for the computations were based on case studies (Tyagi 1982), and dispersion parameters were taken from Tyagi (1971) and Tyagi and Todd (1971).

The study determined the width of the transition zone at various times of pumping. The time of pumping in which the transition zone rises to the bottom of the well, is also determined.

Two separate calculations were performed to determine the upper and lower boundaries of the transition zone. The upper boundary has a value of 15.9% concentration and the lower one, a value of 84.1% concentration. First, the rise of the sharp interface was calculated at various distances from the well. Then the width of the transition zone at the corresponding distances was added vertically to the rise. The rise of the sharp interface is assumed to represent the 84.1% concentration boundary and the boundary obtained after adding the width represents an isochlore of 15.9% concentration.

Island Aquifer System

The first case is the consideration of an island aquifer in which the confined and unconfined conditions of flow are included separately.

Confined Aquifer

The following hydrologic data were obtained from a case study at an island:

Discharge, Q = 20 gpm

Permeability, k = 1333.33 gpd/ft^2

Transmissivity, T = 40,000 gpd/ft

Initial distance between well bottom and interface, d = 25 ft

Saturated fresh water thickness, b = 30 ft

Piezometric surface above the initial interface, H = 45 ft

Storage coefficient, S = 0.005

Dispersivity, Dm = $\frac{1}{200}$ m

Ratio of vertical and horizontal permeability, $\frac{k_z}{k_r} = \frac{1}{20}$

Dimensionless density difference between fresh and salt water

$$\frac{\Delta\gamma}{\gamma} = 0.025$$

Porosity, n = 0.30.

The rise of the sharp interface was computed from Equation (1). After the values of Q, $\frac{\Delta\gamma}{\gamma}$, d, n and $\frac{k_z}{k_r}$ were substituted in the equation. The following relationship resulted:

$$Z(r,t) = 5.5[\{(1 + (0.0088xr)^2\}^{-\frac{1}{2}} - \{(1 + 108.42xt)^2 + (0.0088xr)^2\}^{-\frac{1}{2}}] \quad (18)$$

The variables--radius r, and period of pumping t--were used to compute the rise of the interface at different distances from the well and at different periods of pumping. The rise was computed up to a distance of 1000 feet and the periods of pumping considered were one year, five years, and the year the well becomes contaminated. The results of the calculations are shown in Table I.

TABLE I

RISE OF SHARP INTERFACE IN ISLAND AQUIFER

Distance From Well Center	Rise in Feet			
	Year			
	1	5	11	12
1	4.62	5.30	5.39	5.41
500	0.49	1.01	1.12	1.13
1000	0.14	0.49	0.59	0.60

The width of the transition zone was computed from Equation (9) that, using the above value of Dispersivity, yielded:

$$2\sigma = 6.32(u \cdot t)^{\frac{1}{2}} \tag{19}$$

The velocity u, was computed from Darcy's law ($u = ki$). The hydraulic gradient i, was computed by calculating the drawdowns of the piezometric surface at different distances from the well. The drawdown was computed from Equation (16), which substituting the values of Q, T and s, had the following relationship:

$$s = 0.05 \ln(8.78 \times 10^8 \times \frac{t}{r^2}) \qquad (20)$$

Radius r varied from 1 to 1000 feet, and pumping periods included 1 year, 5 years and 12 years (the time of well contamination). The results of the computations of the width are presented in Table II.

TABLE II

WIDTH OF TRANSITION ZONE IN CONFINED ISLAND AQUIFER

Distance From Well Center	Width in Feet			Height of Upper Boundary (ft.)		
	Year			Year		
	1	5	12	1	5	12
1	5.59	12.51	19.38	10.21	17.81	24.79
500	0.38	0.67	1.04	0.87	1.68	2.17
1000	0.20	0.49	0.72	0.34	0.98	1.32

The Island Aquifer System considered here has low values of discharge and permeability. The width of the transition zone decreases rapidly within 200 feet of the well, and beyond 700 feet, the decrease is gradual. In the first five years, the width of the transition zone is 12.5 feet. The rise then decreases with time. In 12 years it becomes 19.38 feet. Thus, the distance between the initially sharp interface and the upper boundary of the transition zone is 24.79 feet, compared to the fresh water thickness of 45 feet. Because the bottom of the well is 25 feet above the initial interface, the well is contaminated in 12 years. The locations of the transition zone in 1, 5, and 12 years are shown in Figures 1, 2, and 3 respectively.

Unconfined Aquifer

For an unconfined island aquifer, the following hydrologic data were obtained:

Discharge, $Q = 20$ gpm

Permeability, $k = 1333.33$ gpd/ft^2

Initial distance between well bottom and interface, $d = 25$ ft

Saturated fresh water thickness, $H = 30$ ft

Specific Yield, $S = 0.01$

Dispersivity, $Dm = \frac{1}{200}$ m

Ratio of vertical and horizontal permeability, $\frac{k_z}{k_r} = \frac{1}{20}$

Dimensionless density difference between fresh and salt water,

$$\frac{\Delta\gamma}{\gamma} = 0.025$$

Porosity, $n = 0.30$.

Equation (1), was used to compute the rise of the sharp interface when substitutions are made for the values of Q, $\frac{\Delta\gamma}{\gamma}$, d, w and $\frac{k_z}{k_r}$, the equation, results as follows:

$$Z(r,t) = 5.5[\{(1 + (0.0088xr)^2\}^{-\frac{1}{2}} - \{(1 + 108.42xt)^2 + (0.0088xt)^2\}^{-\frac{1}{2}}] \quad (21)$$

The radius r, and period of pumping t, were varied to compute the rise, at different distances from the well and at different pumping

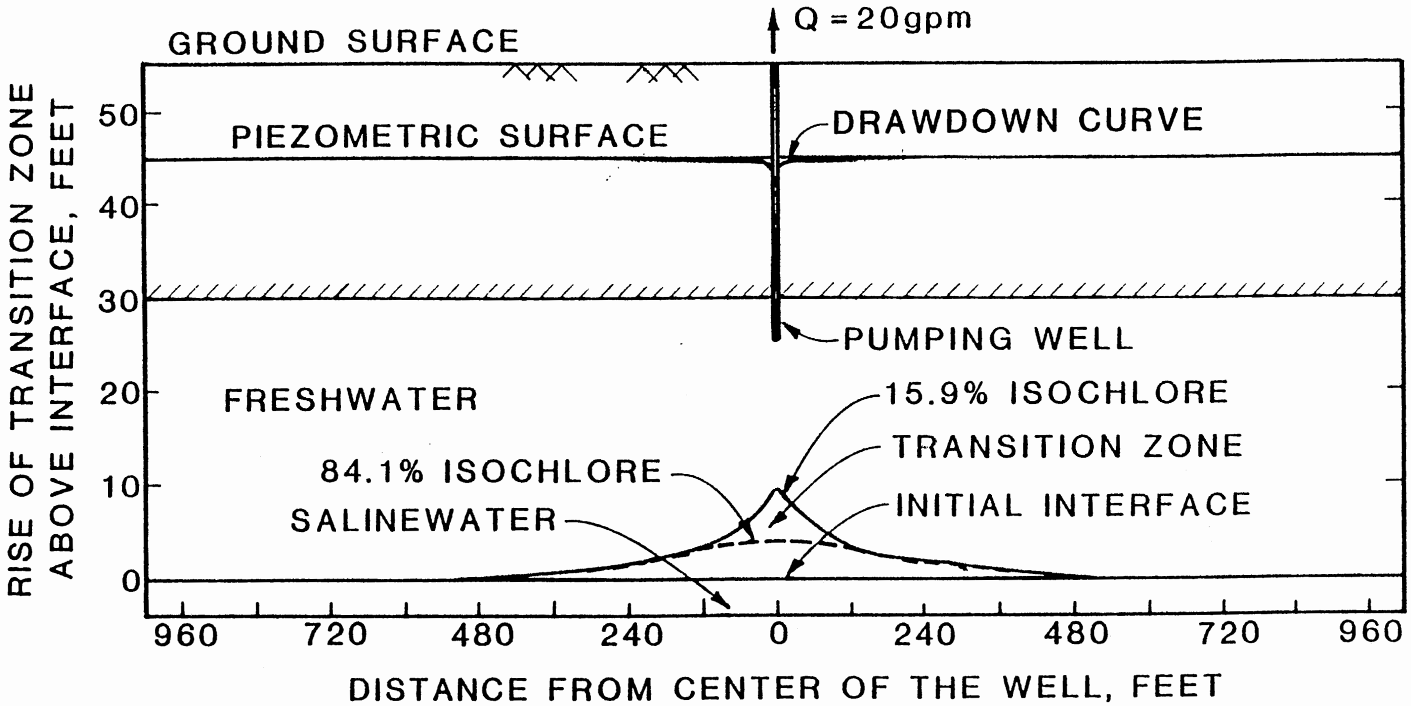

Figure 1. Transition Zone in a Confined Island Aquifer in 1 Year

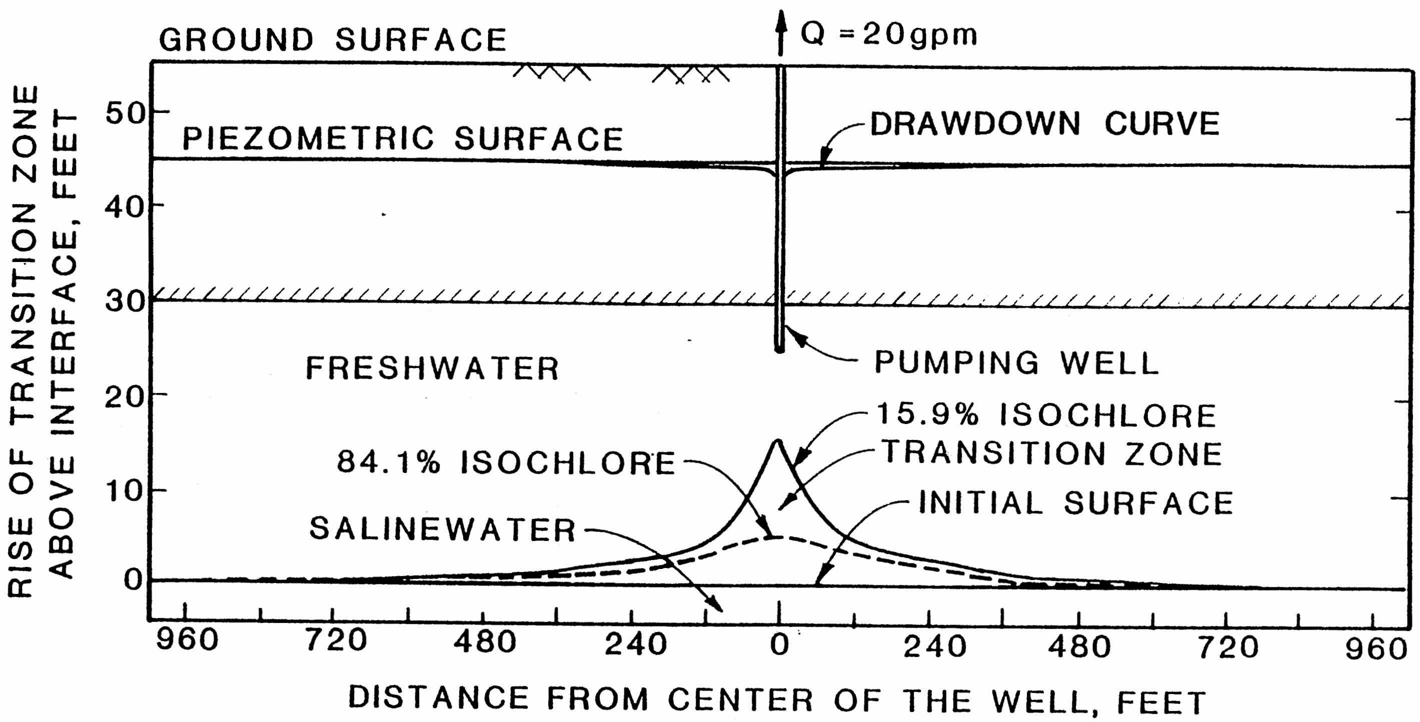

Figure 2. Transition Zone in a Confined Island Aquifer in 5 Years

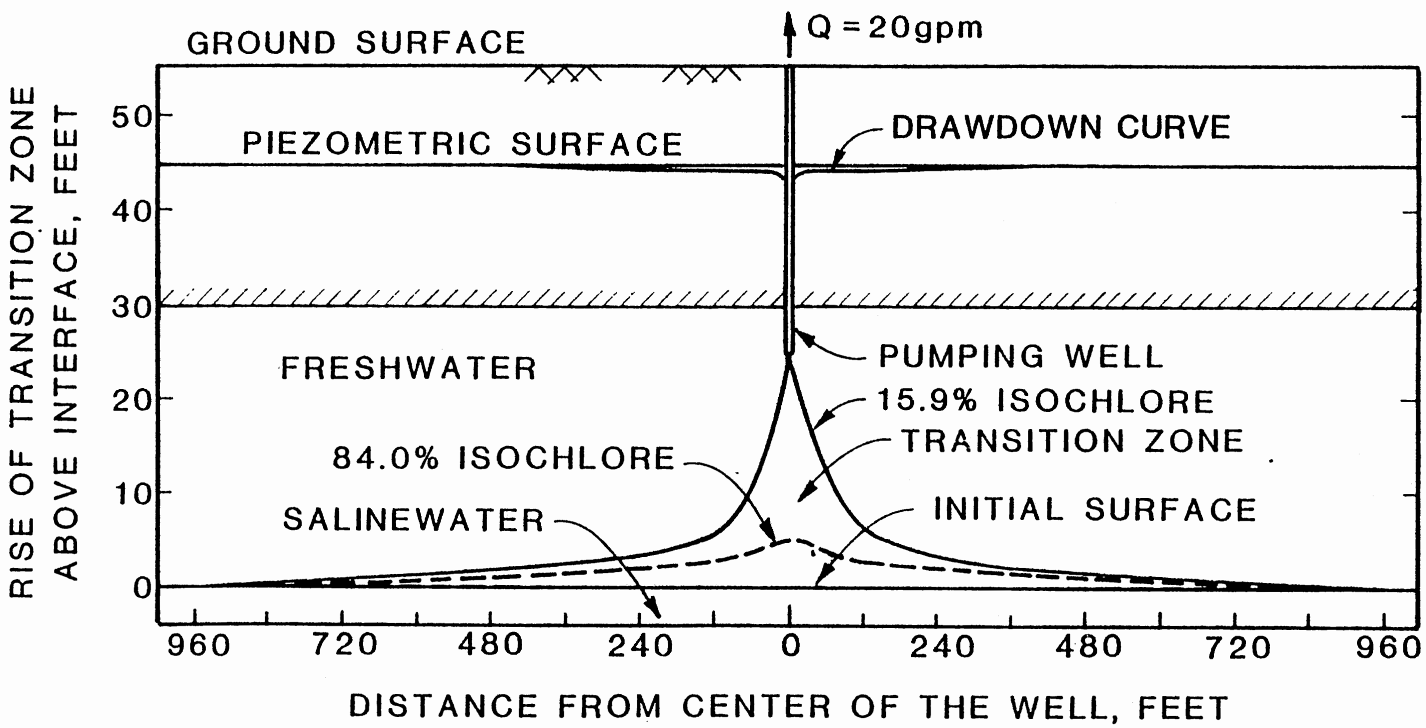

Figure 3. Transition Zone in a Confined Island Aquifer in 12 Years

periods. Results of the computations are shown in Table I.

The width of the transition zone was computed from Equation (9). With the value of dispersivity being substituted, the equation results:

$$2\sigma = 6.32(u \times t)^{\frac{1}{2}} \qquad (22)$$

Darcy's law was used to compute the velocity of flow, which included the hydraulic gradient. Hydraulic gradient was computed, from calculating the drawdowns of the water table with the help of Equation (17). Substituting the values of Q, K, and H, the following relationship results:

$$s = 0.11\ (1 + C_k)v(t',r') \qquad (23)$$

Boultan's well function $v(t',r')$ and correction factor C_k were dependent upon the radius r, and the period of pumping t. Radius r was varied from 1 to 1000 feet and the pumping periods included 1 year, 5 years, and 11 years. The results of the computations of the width of the transition zone are shown in Table III.

TABLE III

WIDTH OF THE TRANSITION ZONE IN UNCONFINED ISLAND AQUIFER

Distance From Well Center	Width in Feet			Height of Upper Boundary (ft.)		
	Year			Year		
	1	5	11	1	5	11
1	5.72	12.87	19.82	10.34	18.17	25.21
500	0.41	0.72	1.12	0.09	1.73	2.24
1000	0.23	0.52	0.77	0.37	1.01	1.36

Within 150 feet of the well, the width of the transition zone decreases rapidly, and beyond 400 feet the decrease is gradual. After 5 years of continuous pumping, the width of the transition zone is 12.87 feet. After 11 years of pumping, the width is 19.82 feet, making the distance from the initial interface to the upper boundary 25 feet, to reach to the bottom of the well. Thus, the well is contaminated in 11 years. Figures 4, 5, and 6 show the position of the transition zone in 1 year, 5 years, and 11 years respectively.

Inland Aquifer System

In the second case of an inland aquifer, the confined and unconfined flow conditions are considered separately.

Confined Aquifer

For the confined inland aquifer system, the data obtained from a case study were as follows:

Discharge, Q = 1000 gpm

Permeability, k = 2000 gpd/ft^2

Transmissivity, T = 400,000 gpd/ft

Initial distance between well bottom and interface, d = 100 ft

Saturated fresh water thickness, b = 200 ft

Piezometric surface above the initial interface, H = 300 ft

Storage coefficient, S = 0.005

Dispersivity, $Dm = \frac{1}{10}$ m

Ratio of vertical and horizontal permeability, $\frac{k_z}{k_r} = \frac{1}{20}$

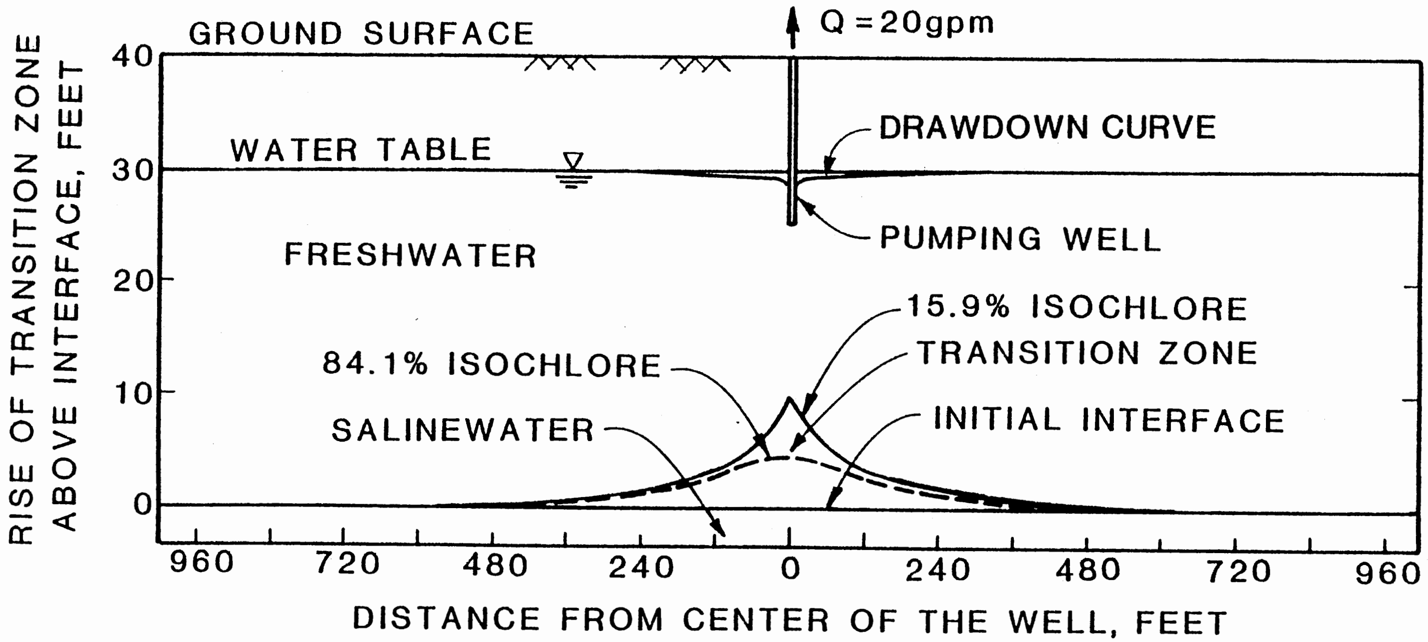

Figure 4. Transition Zone in an Unconfined Island Aquifer in 1 Year

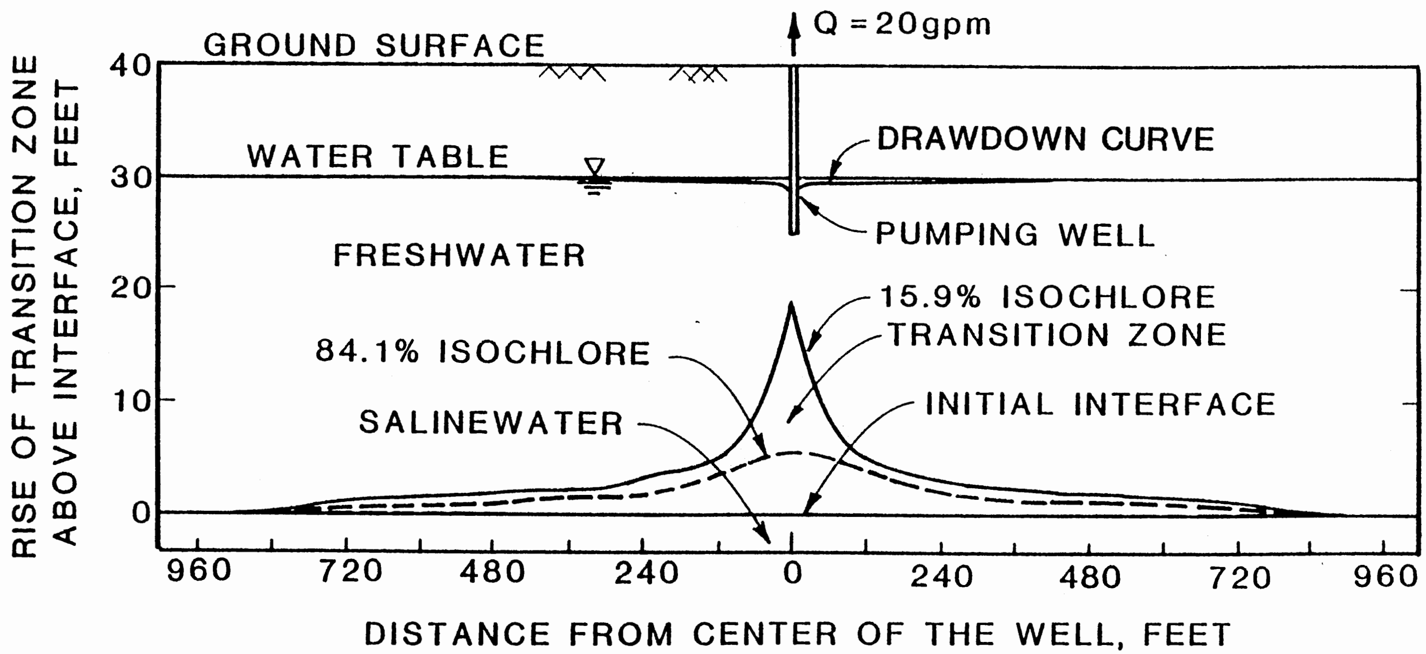

Figure 5. Transition Zone in an Unconfined Island Aquifer in 5 Years

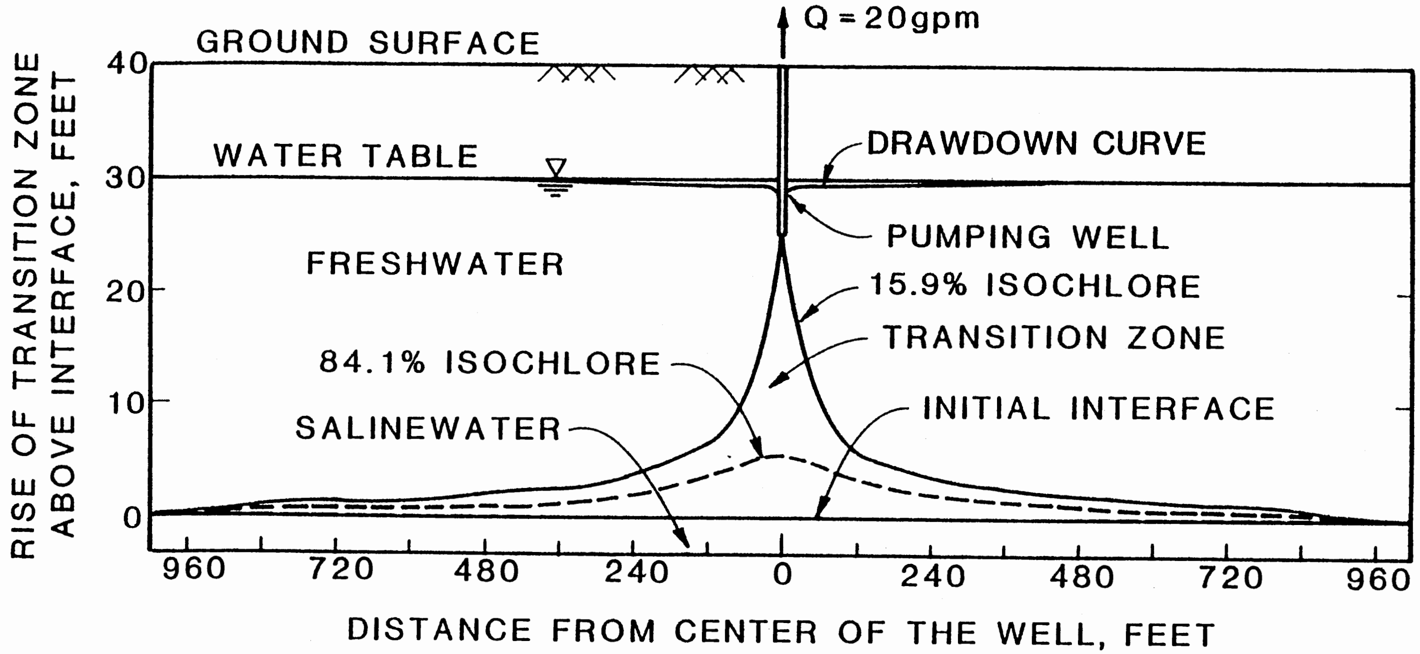

Figure 6. Transition Zone in an Unconfined Island Aquifer in 11 Years

Dimensionless density difference between fresh and salt water,

$$\frac{\Delta\gamma}{\gamma} = 0.025$$

Porosity, n = 0.30.

To compute the rise of the sharp interface, Equation (1) was utilized, which with the substitutions of Q, $\frac{\Delta\gamma}{\gamma}$, d, n and $\frac{k_z}{k_r}$ was as follows:

$$Z(r,t) = 45.85[\{1 + (0.02xr)^2\}^{-\frac{1}{2}} - \{(1 + 40.65xt)^2 + (0.02xr)^2\}^{-\frac{1}{2}}] \quad (24)$$

In this case also, radius r was varied from 1 to 1000 feet, but pumping times considered were 1 year, 5 years, 10 years, and 16 years (the sixteenth is the year the well gets contaminated). The results of the computations are shown in Table IV.

TABLE IV

RISE OF SHARP INTERFACE IN INLAND AQUIFER

Distance From Well Center	Rise in Feet				
	Year				
	1	5	10	14	16
1	30.40	41.62	42.56	43.82	44.44
500	16.32	26.60	28.84	29.20	29.40
1000	7.80	16.49	18.16	18.99	19.24

When dispersivity was substituted in Equation (9) for computing the width of the transition zone, the following expression was obtained:

$$2\sigma = 9.342 \, (u \times t)^{\frac{1}{2}} \qquad (25)$$

Velocity u was computed from Darcy's law. To compute the hydraulic gradient, the heads of the piecometric surface at different distances from the well were computed from Equation (16). When the values of Q, T and S were substituted in the equation, it yielded

$$s = 0.28 \, \ln(8.78 \times 10^9 \times \frac{t}{r^2} \qquad (26)$$

The results of the computations with r varying from 1 to 1000 feet and pumping time 1 year, 5 years, 10 years, and 16 years, are shown in Table V.

TABLE V

WIDTH OF TRANSITION ZONE IN CONFINED INLAND AQUIFER

Distance From Well Center	Width in Feet				Height of Upper Boundary (ft.)			
	Year				Year			
	1	5	10	16	1	5	10	16
3	13.96	31.22	44.14	55.85	44.36	72.84	86.70	100.2
500	1.16	2.60	3.67	4.65	17.48	29.20	32.51	34.05
1000	0.81	1.88	2.56	3.26	8.61	18.37	20.72	22.50

In the first 5 years, the increment in height of the upper boundary is 28.48, whereas in the later 11 years the increment is 27.36, compared to 300 feet of freshwater thickness. After 16 years of continuous pumping, the height of the upper boundary is 100 ft above the initial interface, thus reaching the bottom of the well and contaminating it. Figures 7, 8, 9, and 10 represent the transition zone in 1, 5, 10, and 16 years respectively.

Unconfined Aquifer

The hydrologic data used for the unconfined inland aquifer case, were as follows:

Discharge, Q = 1000 gpm

Permeability, k = 2000 gpd/ft^2

Initial distance between bottom and interface, d = 100 ft

Saturated fresh water thickness, H = 200 ft

Specific Yield, S = 0.01

Dispersivity, Dm = $\frac{1}{10}$ m

Ratio of vertical and horizontal permeability, $\frac{k_z}{k_r} = \frac{1}{20}$.

Dimensionless density difference between fresh and salt water,

$$\frac{\Delta\gamma}{\gamma} = 0.025$$

Porosity, n = 0.30.

Substituting the values of Q, $\frac{\Delta\gamma}{\gamma}$, d, n and $\frac{k_z}{k_r}$ in Equation (1), the rise of the sharp interface was calculated, which has the following expression:

$$Z(r,t) = 45.85[\{1 + (0.02xr)^2\}^{-\frac{1}{2}} - \{(1 + 40.65xt)^2 + (0.02xr)^2\}^{-\frac{1}{2}}] \quad (27)$$

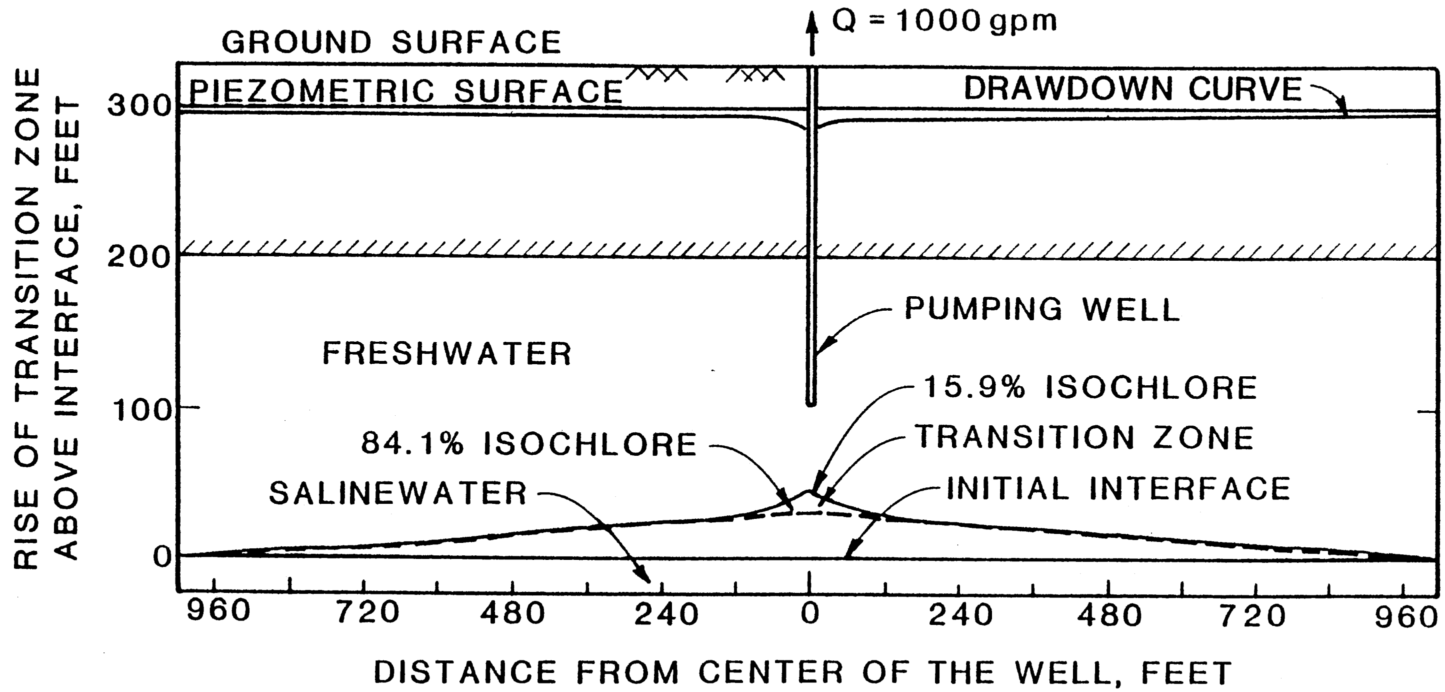

Figure 7. Transition Zone in a Confined Inland Aquifer in 1 Year

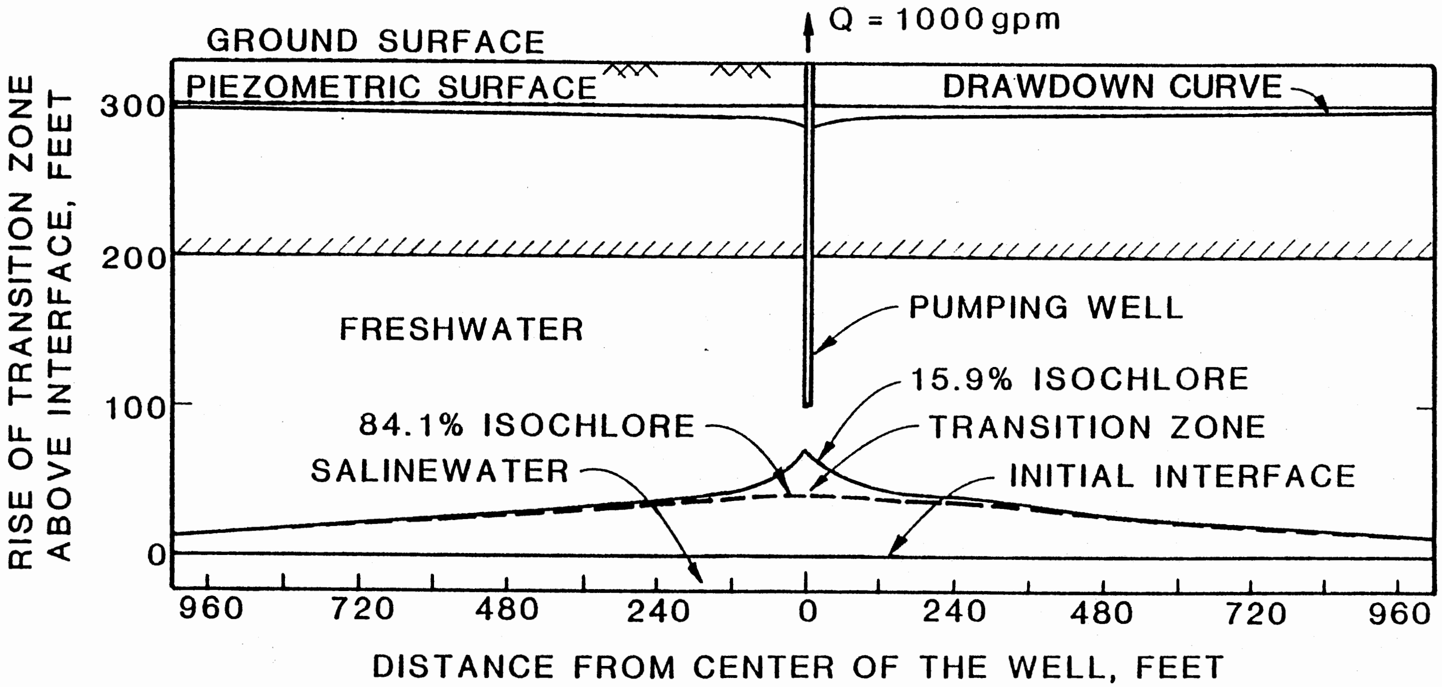

Figure 8, Transition Zone in a Confined Inland Aquifer in 5 Years

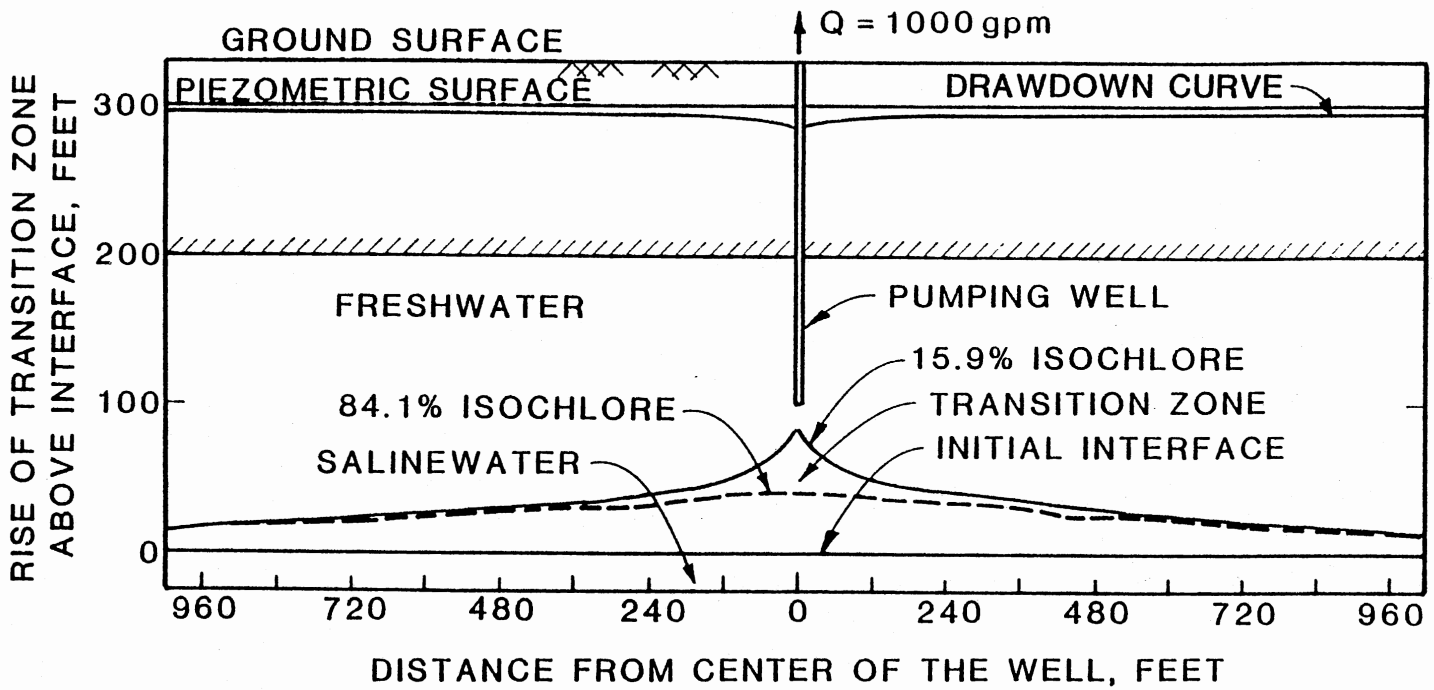

Figure 9. Transition Zone in a Confined Inland Aquifer in 10 Years

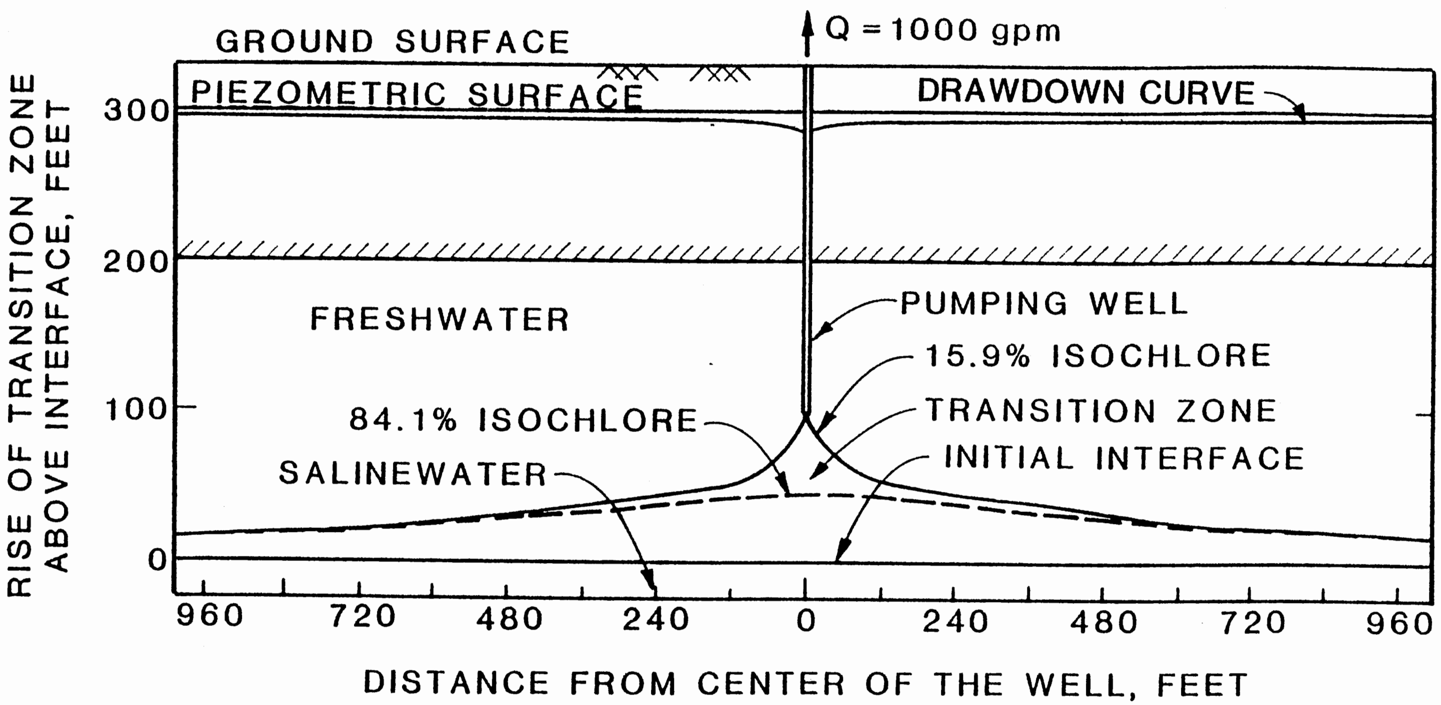

Figure 10. Transition Zone in a Confined Inland Aquifer in 16 Years

Radius r was varied from 1 to 1000 feet, and pumping times t considered were 1 year, 5 years, 10 years, and 14 years. The results of the computations are shown in Table IV.

After substituting dispersivity in Equation (9), the expression for width of the transition zone was as follows:

$$2\sigma = 9.342\ (u \times t)^{\frac{1}{2}} \tag{28}$$

To compute velocity u, the drawdown of the water table was computed from Equation (17), with substituting the values of Q, K and H, had the following relationship:

$$s = 0.57\ (1 + C_k)v(t',r') \tag{29}$$

Radius r and pumping period t' were the variables on which Boulton's well function v(t',r') and correction factor C_k depended. Radius r was varied from 1 to 1000 feet and pumping periods considered were 1 year, 5 years, 10 years, and 14 years (in the fourteenth year the well gets contaminated). The width of the transition zone and height of upper boundary are shown in Table VI.

The Inland Aquifer system has high discharge and permeability. The width of the transition zone decreases sharply within 150 feet from the well and beyond 300 feet the decrease is gradual. Compared to the fresh water thickness of 200 feet, the width of the transition zone is 40.87 in 5 years and 56.18 in 14 years. The height of the upper boundary of the transition zone in the first 5 years is 82.49 feet, and in the later 9 years is 100 feet from the initial interface. The well thus gets contaminated in 14 years of continuous pumping. The locations of the transition zone in 1, 5, 10, and 14 years are shown in Figures 11, 12, 13, and 14 respectively.

TABLE VI

WIDTH OF TRANSITION ZONE IN UNCONFINED INLAND AQUIFER

Distance From Well Center	Width in Feet, Year 1	Width in Feet, Year 5	Width in Feet, Year 10	Width in Feet, Year 14	Height of Upper Boundary (ft.), Year 1	Height of Upper Boundary (ft.), Year 5	Height of Upper Boundary (ft.), Year 10	Height of Upper Boundary (ft.), Year 14
1	16.08	40.87	48.29	56.18	46.48	82.49	90.85	100.00
500	2.04	3.48	4.08	4.68	18.36	30.08	32.92	33.88
1000	1.21	2.28	3.16	3.29	9.01	18.77	21.32	22.28

Garber-Wellington (Confined) Aquifer System

Based on the hydrologic data from the field, the Garber-Wellington Aquifer system in Oklahoma represents, on the average, confined flow conditions. This aquifer contains alternate layers of sand, clay, and shale. The following hydrologic data were considered:

Discharge, $Q = 200$ gpm

Permeability, $k = 13$ gpd/ft^2

Transmissivity, $T = 4550$ gpd/ft

Initial distance between well bottom and interface, $d = 175$ ft

Saturated fresh water thickness, $b = 350$ ft

Piezometric surface above the initial interface, $H = 450$ ft

Storage coefficient, $S = 0.005$

Dispersivity, $Dm = \frac{1}{5}$ m

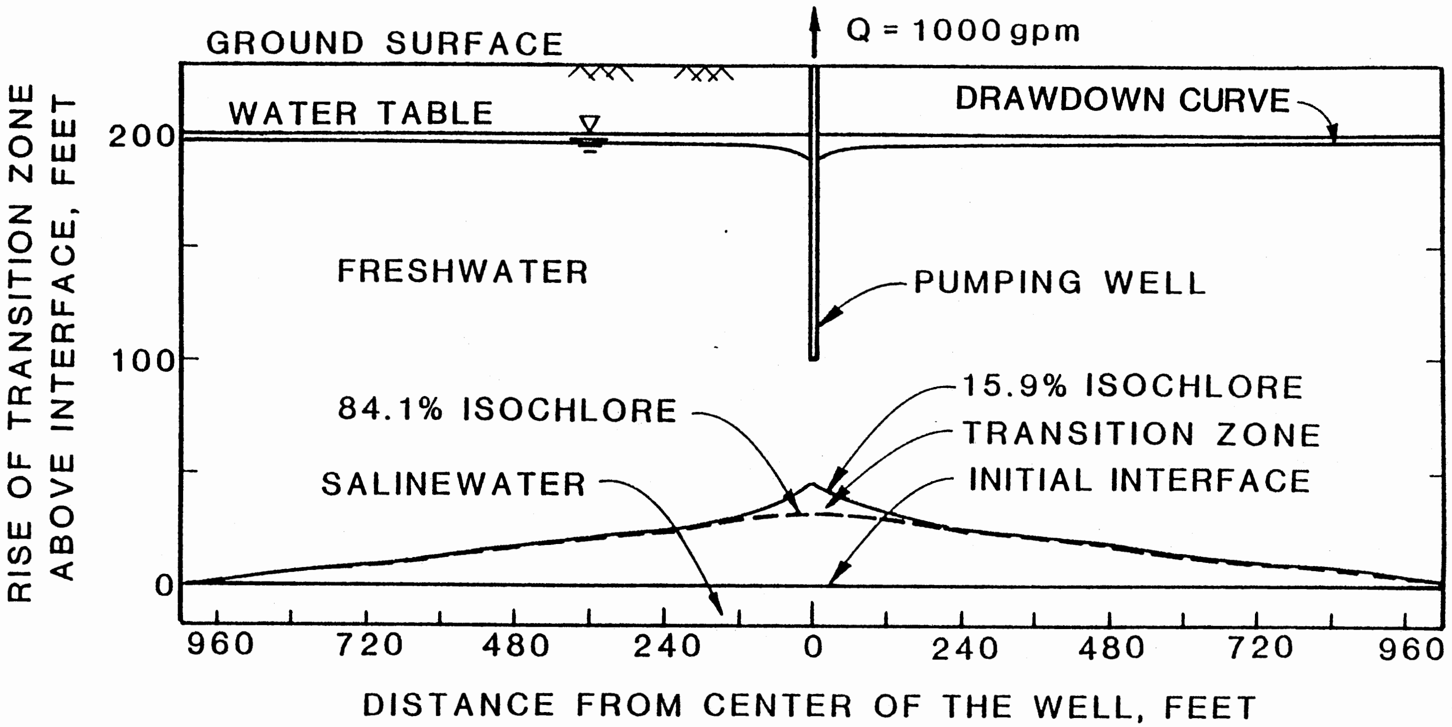

Figure 11. Transition Zone in an Unconfined Inland Aquifer in 1 Year

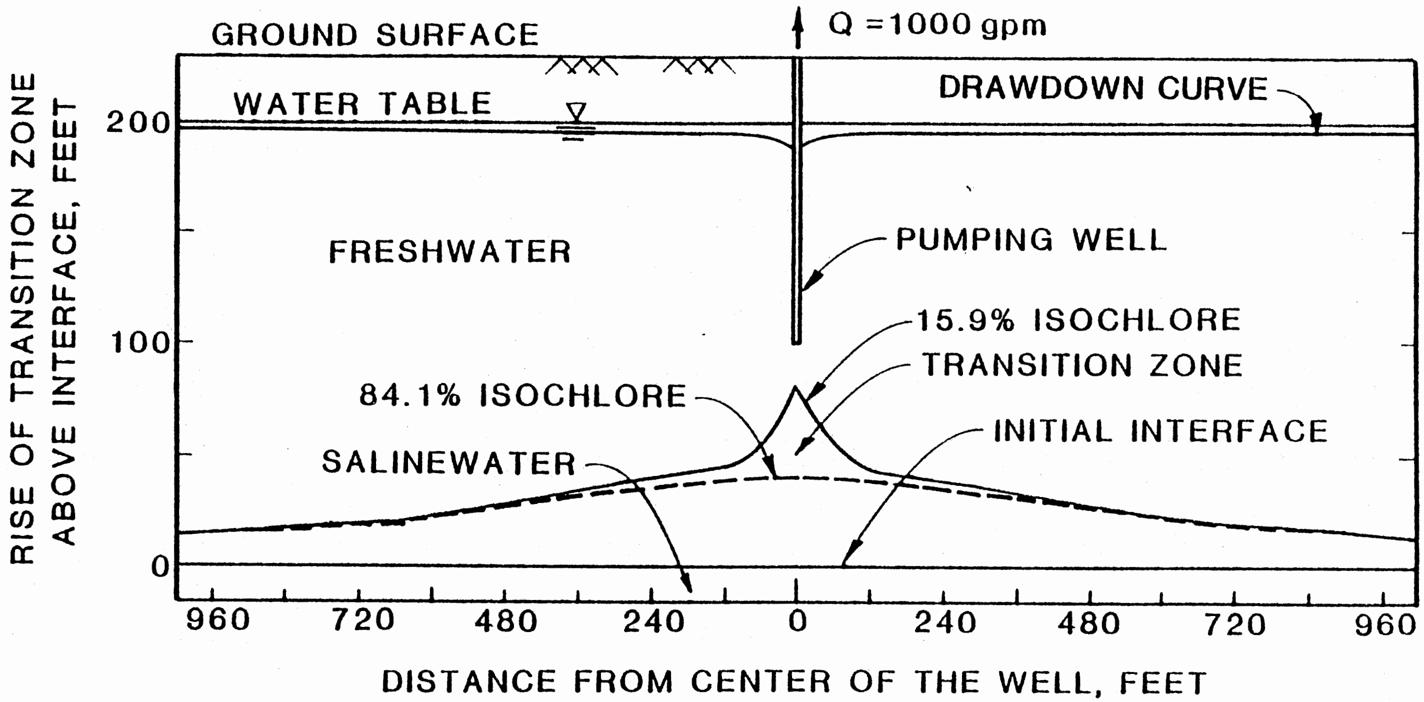

Figure 12. Transition Zone in an Unconfined Inland Aquifer in 5 Years

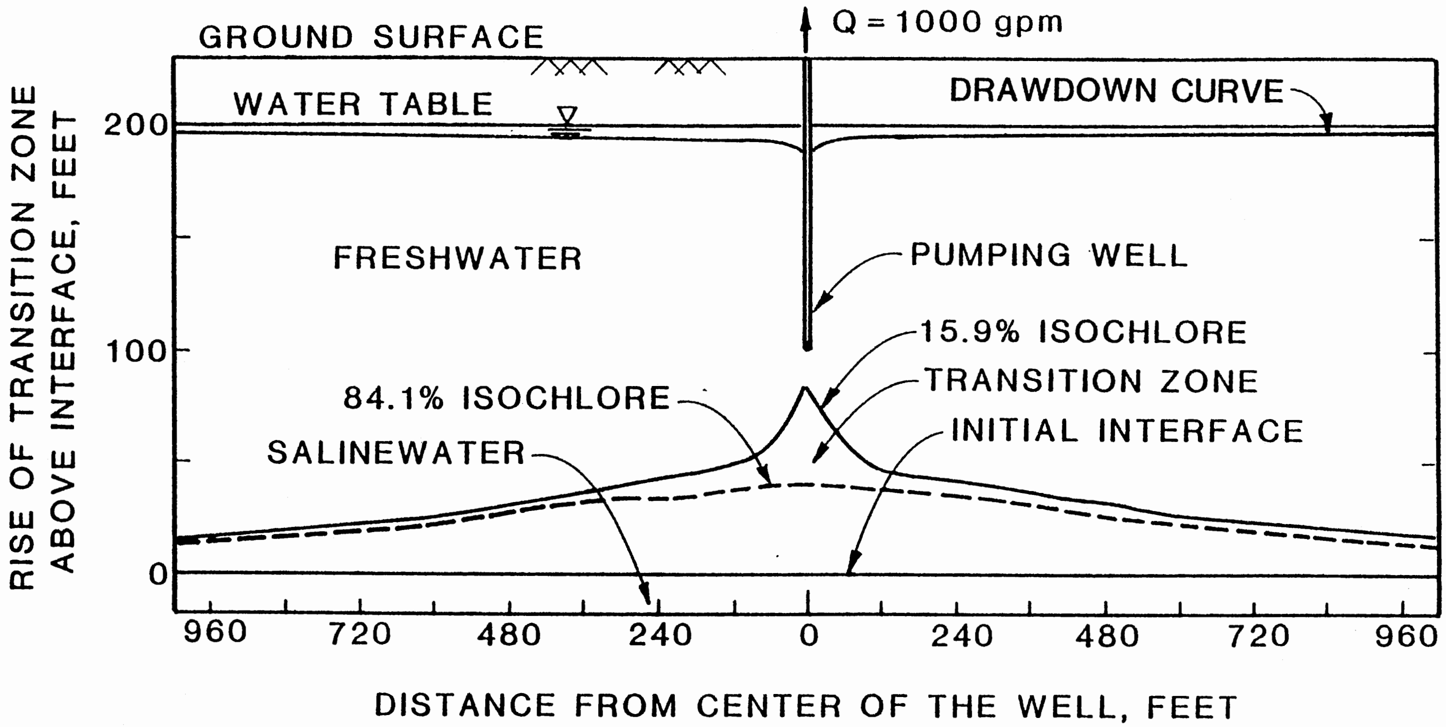

Figure 13. Transition Zone in an Unconfined Inland Aquifer in 10 Years

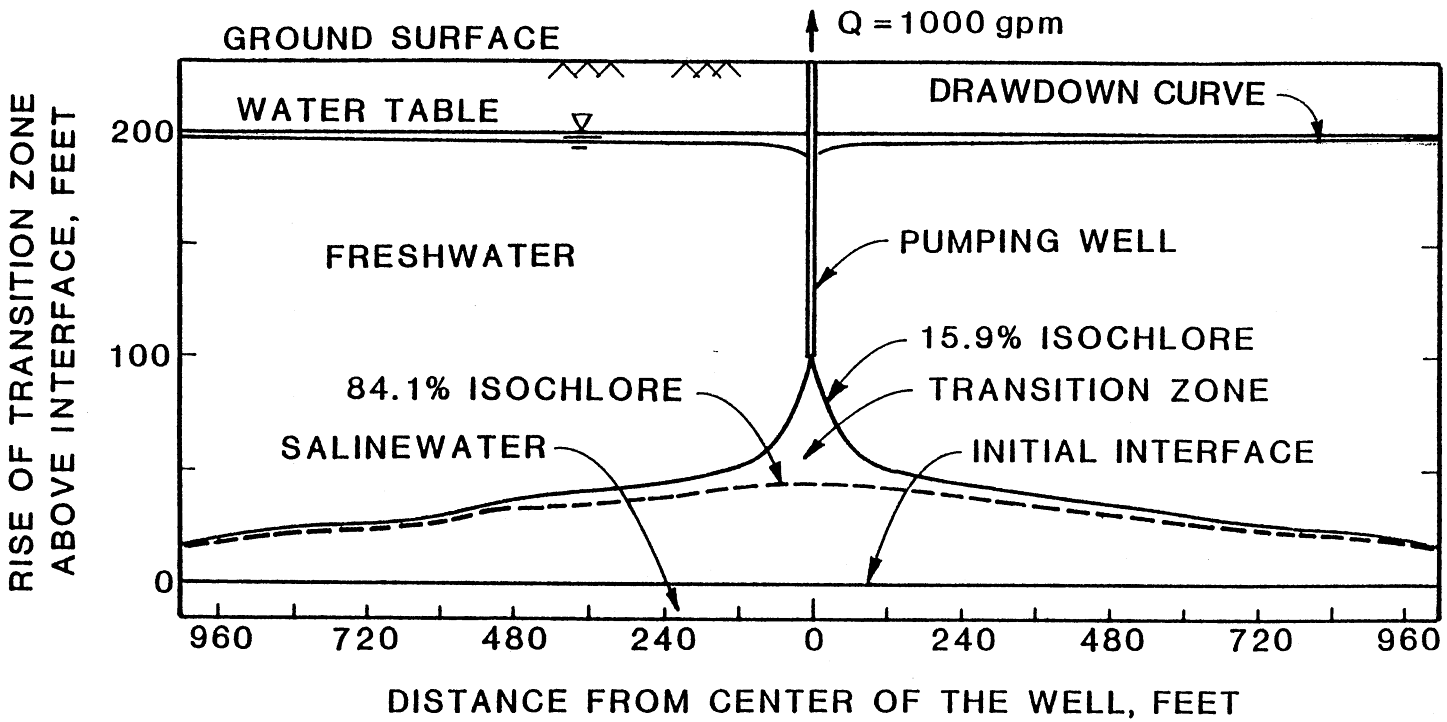

Figure 14. Transition Zone in an Unconfined Inland Aquifer in 14 Years

Ratio of vertical and horizontal permeability, $\frac{k_z}{k_r}$, $= \frac{1}{20}$

Dimensionless density difference between fresh and salt water,

$$\frac{\Delta\gamma}{\gamma} = 0.025.$$

Porosity, n = 0.30.

When the above values of Q, $\frac{\Delta\gamma}{\gamma}$, d, n and $\frac{k_z}{k_r}$ are substituted in Equation (1) to compute the rise of the sharp interface, the equation has the following expression:

$$Z(r,t) = 806.32[\{1 + (0.00125xr)^2\}^{-\frac{1}{2}} - \{(1 + 0.003xt)^2 + (0.00125xr)^2\}^{-\frac{1}{2}}] \quad (30)$$

Variables of radius r and pumping time t and used to compute the rise to 1000 feet from the well, for 1 year, 5 years, 10 years, and 17 years of pumping. The results of the computations are shown in Table VII.

TABLE VII

RISE OF SHARP INTERFACE IN CONFINED GARBER-WILLINGTON AQUIFER

Distance From Well Center	Rise in Feet, Year 1	5	10	17
3	21.38	47.81	67.60	87.16
500	1.78	3.98	5.63	7.34
1000	1.32	2.79	4.04	5.14

The expression for Equation 9, along with the substitution for Dispersivity is as follows:

$$2\sigma = 43.76\ (u \times t)^{\frac{1}{2}} \qquad (31)$$

Velocity u is computed from the calculations of the drawdowns of the piezometric surface at unit intervals from the well. When the values of Q, T and S are substituted in Equation (16), it yields:

$$s = 5.04\ \ln(9.9 \times 10^{7} \times \frac{t}{r^{2}}) \qquad (32)$$

The width is thus computed up to a distance of 1000 feet from the well and pumping times considered were 1 year, 5 years, 10 years, and 17 years (in the seventeenth year the well gets contaminated). The results of the computations of the width of the transition zone are shown in Table VIII.

TABLE VIII

WIDTH OF TRANSITION ZONE IN CONFINED GARBER-WELLINGTON AQUIFER

Distance From Well Center	Width in Feet				Height of Upper Boundary (ft.)			
	1	5	10	17	1	5	10	17
3	5.85	28.42	53.51	88.11	27.23	76.23	121.11	175.20
500	3.55	17.48	30.53	56.56	5.33	21.46	36.16	63.90
1000	1.70	8.49	12.06	28.40	3.02	11.28	16.10	33.54

Within the first 5 years of continuous pumping, the maximum rise of the transition zone is 76.23 feet, whereas in 17 years it rises to 17.50 feet, compared to 450 feet of fresh water thickness. So within the first 5 years it rises more quickly than in the later years. Within a 120 feet radius of the well, the rise of the transition zone is seen to be more prominent than beyond 200 feet. After 17 years of continuous pumping, the well gets contaminated because the upper boundary of the transition zone reaches a height of 175 feet, which is the distance from the well bottom and initial interface. Figures 15, 16, 17, and 18 show the transition zones in 1, 5, 10, and 17 years respectively.

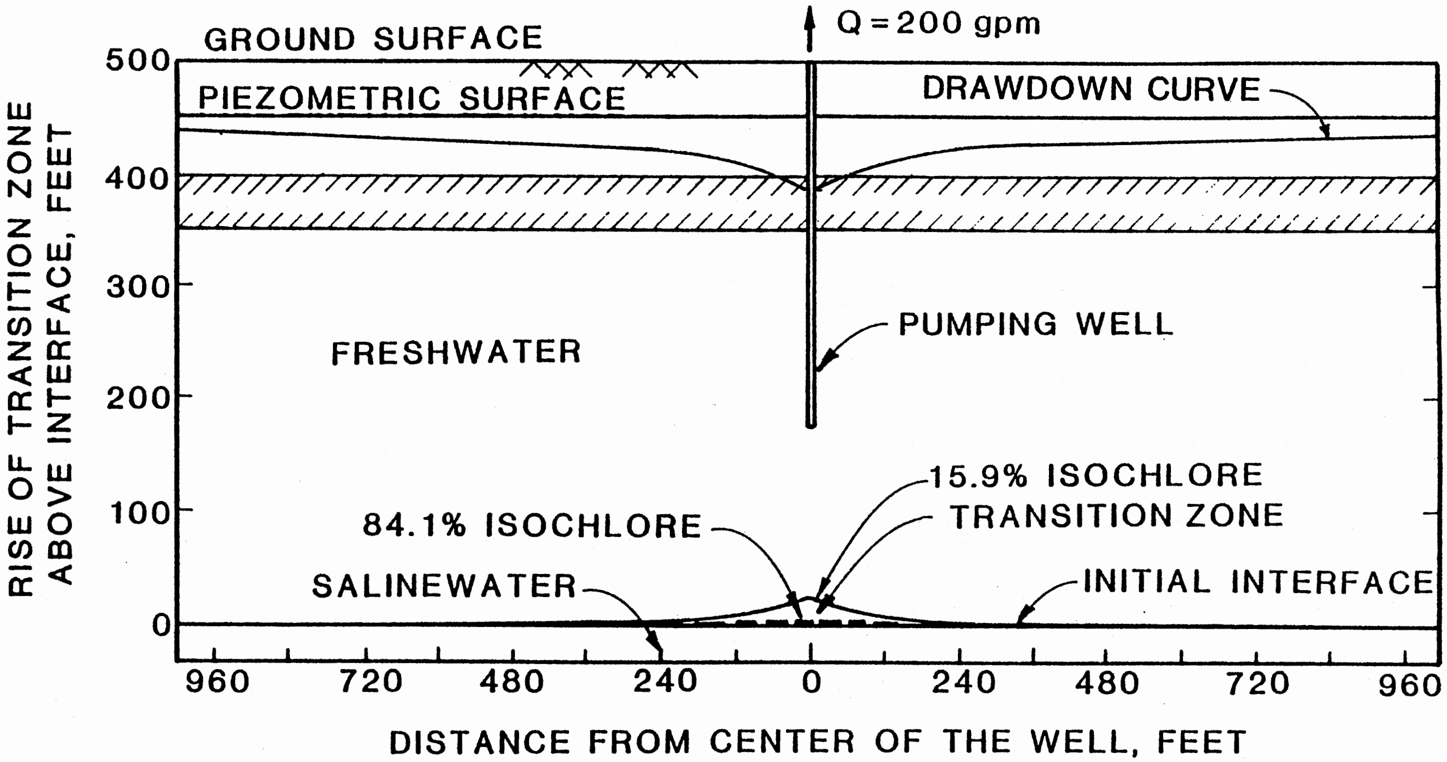

Figure 15. Transition Zone in the Confined Garber-Wellington Aquifer in 1 Year

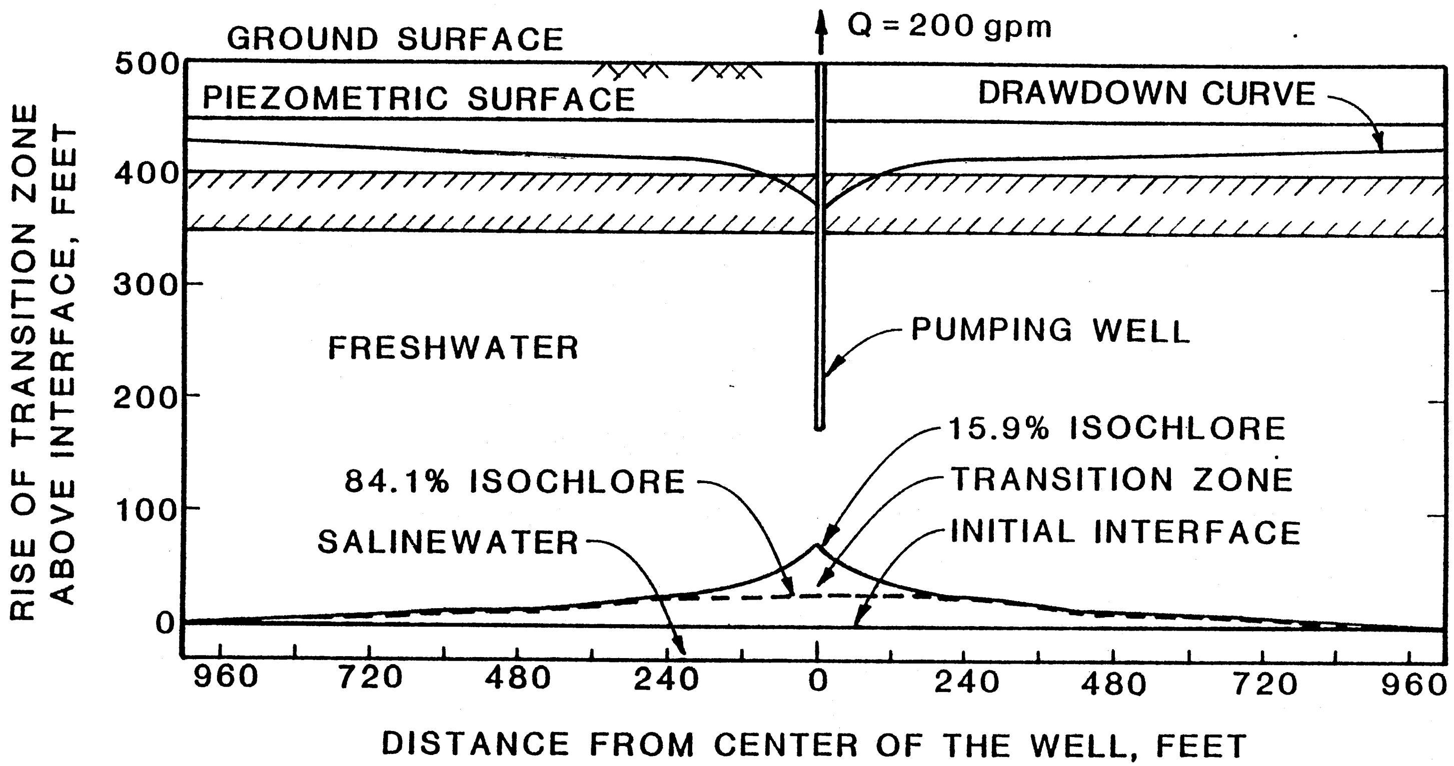

Figure 16. Transition Zone in the Confined Garber-Wellington Aquifer in 5 Years

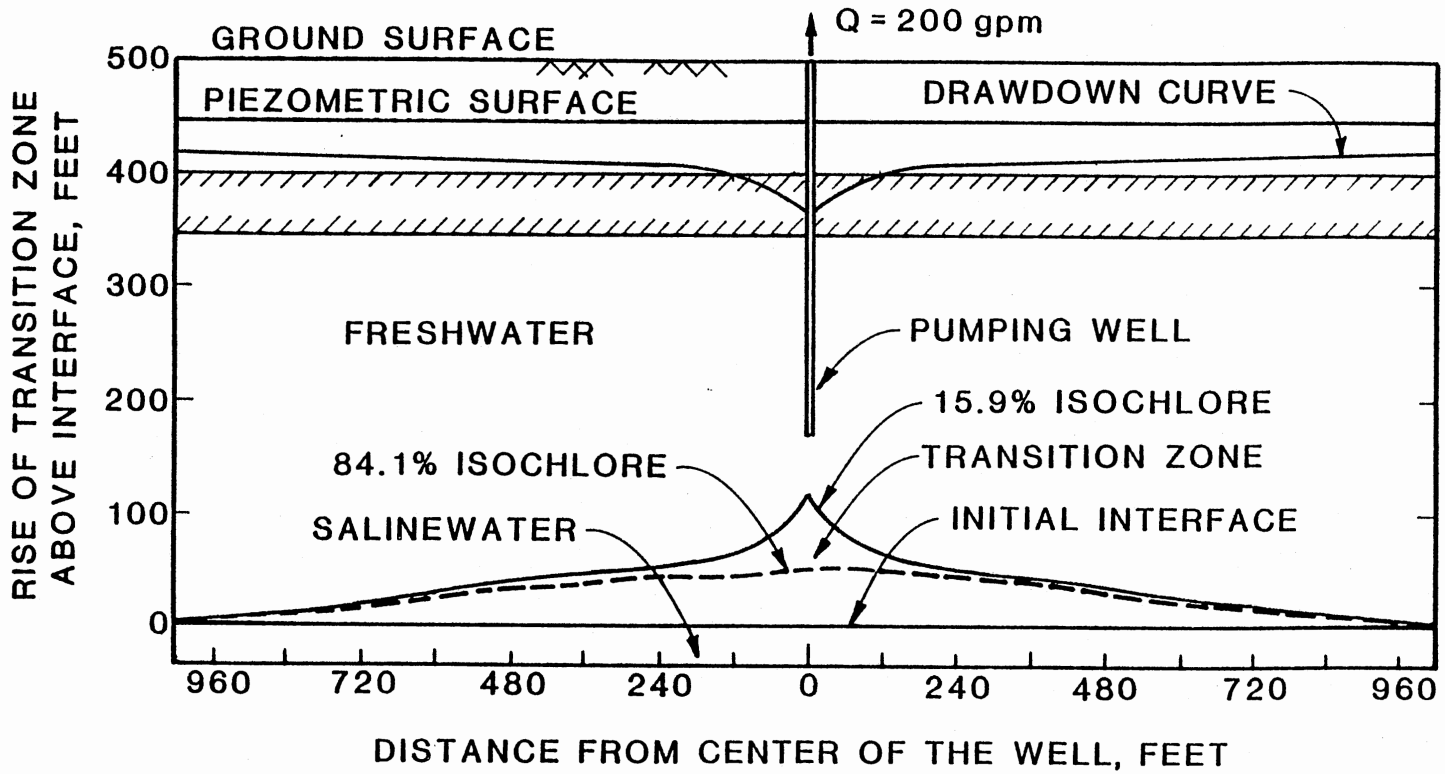

Figure 17. Transition Zone in the Confined Garber-Wellington Aquifer in 10 Years

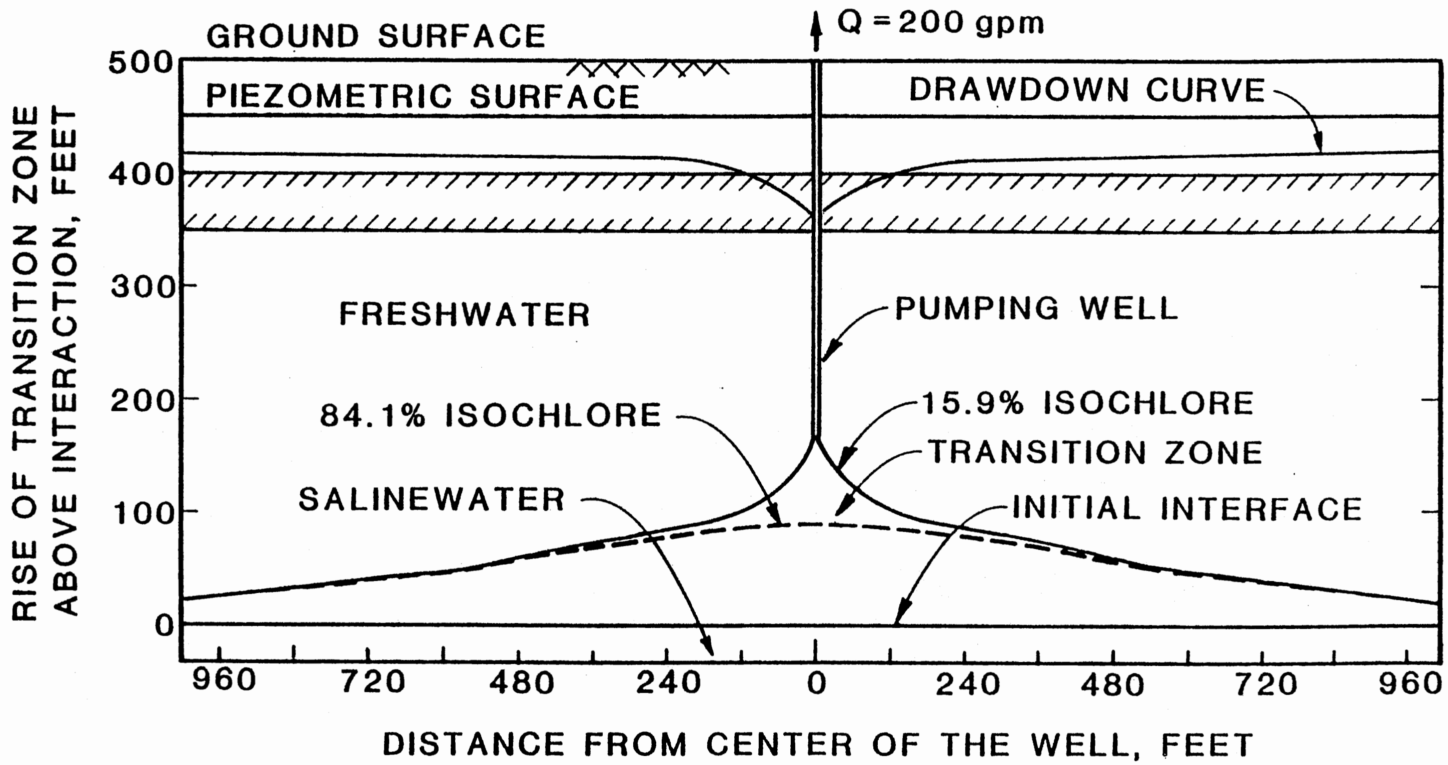

Figure 18. Transition Zone in the Confined Garber-Wellington Aquifer in 17 Years

CHAPTER V

CONCLUSION

The assumptions made in this study are that the well fully penetrates the aquifer and that the velocity of flow is computed for an homogeneous aquifer. The following conclusions are drawn from the results of this study:

1. In the case of the confined Island Aquifer, in 12 years, the upper boundary of the transition zone rises 25 feet, in an overall freshwater thickness of 45 feet, and contaminates the well.

2. In case of the unconfined Island Aquifer, in 11 years, the upper boundary of the transition zone rises 25 feet, in an overall freshwater thickness of 30 feet, and contaminates the well.

3. When the well is contaminated, the transition zone in the Island Aquifer exhibits an average slope of 0.178 within 100 feet radius of the well, after which the slope becomes 0.006 up to 1000 feet.

4. In case of the confined Inland Aquifer, in 16 years, the upper boundary of the transition zone rises 100 feet, in an overall freshwater thickness of 200 feet, and contaminates the well.

5. In case of the unconfined Inland Aquifer, in 14 years, the upper boundary of the transition zone, rises 100 feet, in all overall fresh water thickness of 200 feet, and contaminates the well.

6. When the well is contaminated, the transition zone in the Inland Aquifer exhibits an average slope of 0.450 within 100 feet radius of the

well. Beyond 100 feet, from the well, the average slope is 0.036.

7. In the case of the Garber-Wellington Aquifer, in 17 years the upper boundary of the transition zone, rises 175 feet in an overall fresh water thickness of 350 feet and contaminates the well.

8. In the Garber-Wellington Aquifer, when the well is contaminated the transition zone exhibits an average slope of 0.685 within 100 feet radius from the well and beyond 100 feet the average slope becomes 0.081.

9. In all the cases, it has been seen that within the first 5 years the rate of rise of the transition zone is rapid, and beyond 5 years the rate of rise tends to decrease.

10. A maximum value of average slope within 100 feet of a well in the transition zone is 0.685 based on the data used in this study. Between 100 and 1000 feet from a well, the maximum value of average slope is 0.081.

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```
$JOB             ,TIME=(0,3)
C
C
C      THIS PROGRAM COMPUTES THE RISE OF THE SHARP INTERFACE
C
C
       DATA Q,PI,COND,DEPTH,POROS/1440000.,3.14,2000.,100.,0.30/
       R=.22
       WRITE(6,300)
       WRITE(6,400)
       K=16
       A=FLOAT(K)
       WRITE(6,500) A
       DO 100 M=1,1000,100
       WRITE(6,600) M
       B=FLOAT(M)
       RISE=Q/(2.*PI*.025*COND*DEPTH)*(1./SQRT(1.+((B/DEPTH)*R)**2)-1./
      1SQRT((1.+.025*COND*(R)**2*48.79*A/(2.*POROS*DEPTH))**2+((B/DEPTH)*R)
      1R)**2))
       WRITE(6,700) RISE
   100 CONTINUE
   200 CONTINUE
   300 FORMAT('1',3X,'PUMPING TIME',5X,'RADIUS',5X,'RISE')
   400 FORMAT(7X,'(YEAR)',9X,'(FT)',7X,'(FT)')
   500 FORMAT(7X,F4.1)
   600 FORMAT(22X,I4)
   700 FORMAT(30X,F15.9)
       STOP
       END

$ENTRY
```

```
$JOB            ,TIME=(0,3)
C
C
C      THIS PROGRAM COMPUTES THE MAGNITUDE OF THE WIDTH OF THE
C      TRANSITION ZONE CONSIDERING TRANSIENT FLOW CONDITION
C
C
       DIMENSION H(950)
       READ(5,40) T,Q,S,DIST,PERM,LP
       READ(5,50) DISPV,PHI
       WRITE(LP,260) T,Q,S,DIST,PERM
       WRITE(LP,300) DISPV,PHI
       WRITE(LP,124)
       WRITE(LP,125)
       M=1
       WRITE(LP,126) M
       DO 200 K=3,900
       WRITE(LP,127) K
       D=FLOAT(K)
C      COMPUTATION OF DRAWDOWNS(THEIS METHOD)
       H(K)=(Q/(4.*PHI*T))*(ALOG((2.25*T*365.*FLOAT(M))/(D**2*S)))
       H(K+1)=(Q/(4.*PHI*T))*(ALOG((2.25*T*365.*FLOAT(M))/((D+1)**2*S)))
C      COMPUTATION OF HEADLOSS
       HLOSS=H(K)-H(K+1)
C      COMPUTATION OF HYDRAULIC GRADIENT
       HGRAD=HLOSS/DIST
C      COMPUTATION OF VELOCITY OF FRESH WATER (DARCY'S LAW)
       VF=PERM*HGRAD
C      COMPUTATION OF VELOCITY OF SALINE WATER
       VS=VF*.9524
C      COMPUTATION OF DISPERSION COEFFICIENT
       DL=VS*DISPV
C      TIME OF TRAVEL CONVERSION FACTOR TO YEARS
       TIME=60.*60.*24.*365.
C      COMPUTATION OF WIDTH OF TRANSITION ZONE
       WIDTH=2.*SQRT(2.*DL*TIME*(FLOAT(M)))
       WRITE(LP,100) M,WIDTH
   40  FORMAT(5F14.6,1X,I1)
   50  FORMAT(5F15.9,I1)
  100  FORMAT('+',38X,I2,6X,F15.9)
  124  FORMAT('1',8X,'TIME STEP',5X,'RADIUS',5X,'PUMPING TIME',5X,'WIDTH'
      1)
  125  FORMAT(/,9X,'(YEAR)',8X,'(FT)',10X,'(YEAR)',8X,'(FT)')
  126  FORMAT(/,10X,I2)
  127  FORMAT(23X,I4)
  200  CONTINUE
  250  CONTINUE
  260  FORMAT(2X,5F14.6)
  300  FORMAT(2X,5F15.9)
       STOP
       END
```

VITA

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