THE EFFECTS OF THE ECONOMIC TAX RECOVERY ACT OF 1981 ON OPTIMAL REPLACEMENT OF FARM MACHINERY

By

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TABLE OF CONTENTS

Chapter Page

LIST OF TABLES

LIST OF FIGURES

Figure	Page

CHAPTER I

INTRODUCTION

Investment in physical capital as an input to a production process is cyclical. The property is acquired, utilized, and replaced when it becomes uneconomical to maintain or is rendered obsolete by a more technically efficient asset. The purchase and disposition of farm chattel is an especially difficult decision because of uncertainty in agriculture. The acquisition and replacement of farm assets is influenced both by agents attributable to the machine (i.e. efficiency, maintenance, likelihood of breakdown, etc.) and exogenous factors (the firm financial status, economic conditions, agricultural, monetary, and fiscal policy, price and yield expectations, weather conditions, etc.). The Economic Tax Recovery Act of 1981 imposes some changes in the environment in which farm investment occurs.

Many aspects of the Economic Tax Recovery Act of 1981 are departures from previous statutes. The Accelerated Cost Recovery System (ACRS) governs depreciation and related tax issues for assets placed in service after 1980. The new depreciation methods classify assets according to expected useful life and after straight-line and accelerated cost recovery options. The major changes in tax issues relating to replacement of farm chattel are discussed in the following section.

Classification of Assets

Nonreal farm property is classed as a 3-year or 5-year asset. Some assets eligible for depreciation over three years are light trucks, automobiles, race horses over 2 years of age, working or breeding horses over 12 years of age, and breeding hogs. Property in the 5-year class includes machinery, other equipment, and cattle.

Depreciation of Property

An investor can elect to depreciate by either the ACRS method or the alternative ACRS straight-line approach. The respective ACRS depreciation schedules for 3-year and 5-year assets are given in Table 1. Additional first-year depreciation can no longer be taken . The Tax Equity and Fiscal Responsibility Act of 1982 (a revision of Economic Tax Recovery Act of 1981) stipulates that the basis for depreciation is to be reduced by one-half the amount of the investment credit taken. Prior to 1982 there was no basis adjustment.

If the alternative ACRS straight-line convention is elected, depreciation taken in the first year is one-half that of the yearly rate, regardless of the month in which the asset is placed in service. The remaining half-year's depreciation is taken in the year following the period of the elected life. Under the alternative ACRS straight-line method, 3-year property can be assigned a 3, 5, or 12 year life whereas a 5-year asset can be depreciated over 5, 12, or 25 years.

Prior to 1981, the investor had more flexibility in choosing both the method of depreciation and the elected life of the asset. For example, farm implements and tractors were frequently depreciated

Table 1. Depreciation Taken Under the Accelerated Cost Recovery System

Class	Year	% of Purchase Price
3-year Assets	1	25
	2	38
	3	37
5-year Assets	1	15
	2	22
	3-5	21

Table 2. Allowable Investment Tax Credit Expressed as a Percentage of Purchase Price

Elected Life	Old Law	Elected Life	New Law
3-4 years	3 1/3	3 years	6
5-6 years	6 2/3	5 years	10
7 years or more	10		

straight-line over 5 and 7 years, respectively, or the declining-balancing approach was used under the new legislation. Under the old law, a salvage value was used in computing depreciation (if the straight-line method is used), the first year's depreciation was prorated to the months in which the asset was in service, and a declining and/or double-declining balance schedule was employed.

Regardless of the election chosen, all property within a class purchased in the same year must be depreciated by the same method. A salvage value is disallowed. The recovery periods for the accelerated ACRS approach are considerably shorter than those under the old law.

Investment Tax Credit

For 3-year property, the investment credit equals 6 percent of the purchase price. For assets included in the 5-year class, the credit is computed as 10 percent of the investment.[1] The percentage bases for investment credit under the old and new legislation are given in Table 2. For instance, the currently defined 3-year asset could have had any elected life under the old law and the investment credits for the 3 or 4-year, 5 or 6-year, and 7-year or more elections were, respectively, 3 1/3, 6 2/3, and 10 percent of the purchase price. The 1981 legislation confined the same asset to either a 3 year ACRS or 5-year straight-line election with a 6 percent credit regardless.

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[1]The statute expresses the investment credit as 10-percent of the eligible portion of the investment where 60 percent of the 3-year asset price is considered eligible and 100 percent of the price of the 5-year asset is eligible.

The 1982 revision provides for an 8 percent and 4 percent investment tax credit deduction in lieu of the basis adjustment of depreciation for 5 and 3-year property, respectively. The limitation on investment credit taken is equal to the lesser of the entire tax liability or $25,000 plus eighty-percent of the tax that exceeds $25,000. The maximum amount of investment in used property qualifying for investment credit under the recent statutes is $125,000 (which exceeds the previous limit of $100,000). If the investment credit exceeds the entire tax liability, it can then be carried-forward to another tax year.

Disposition of Assets

There may be two additional tax considerations, both of which are contingent upon the sale of the asset. Recapture of investment tax credit applies only if the property is sold before it has reached its elected life. The recapture or repayment of unearned investment credit then becomes a tax liability in the year in which disposition occurs. Table 3 illustrates the recapture schedules of both tax legislations, where the values within the table express the tax liability as a percentage of purchase price. Under the old law, if the investor elected to retain the asset for 5-years and used the 6 2/3 percent investment credit but disposed of the property after 3 years, one-half of the credits unearned (3 1/3 percent of the purchase price) had to be repaid. Similarly, under the current rules a 5-year election (for a 5-year asset) allows a 10 percent investment tax credit, but sale of the asset after three years results in a 4 percent recapture since 5 percent of the purchase price is unearned.

Table 3. Recapture of Investment Tax Credit as a Percentage of Investment

Elected Life	Year of Disposal							
	Within Year 1	After Year 1	After Year 1	After Year 1	After Year 1	After Year 1	After Year 1	After Year 1
Old Law								
3	3 1/3	3 1/3	3 1/3	-0-	-0-	-0-	-0-	-0-
5	6 2/3	6 2/3	6 2/3	3 1/3	3 1/3	-0-	-0-	-0-
7	10	10	10	6 2/3	6 2/3	3 1/3	3 1/3	-0-
New Law								
3	6	4	2	-0-	-0-	-0-	-0-	-0-
5	10	8	6	4	2	-0-	-0-	-0-

Tax treatment of gains is pertinent regardless of the timing of the sale. The receipts from sale of the chattel can be taxed at the ordinary income tax rate and the capital gains tax rate. The portion of gain realized upon sale of an asset that is taxable at the ordinary rate is computed in the same manner as that prior to 1981: if the summed depreciation taken exceeds or equals the gain, the gain is taxed at the ordinary tax rate. If the gain exceeds the accumulated depreciation, the amount of the gain equalling the depreciation taken is taxed at the normal rate and the excess of gain over accumulated depreciation is taxed at the capital gains rate. The basis adjustment is considered to be purchase price minus depreciation taken. The new tax rate for capital gains is 40 percent (previously 50 percent) of the ordinary rate. The maximum capital gains tax has been reduced from 28 percent to 20 percent (after multiplication of the ordinary rate by the capital gains proportion). For example, if an investor whose marginal tax rate is 30 percent sells an asset, the portion of income equalling the accumulated depreciation taken is taxed at the ordinary or marginal tax rate (30 percent). The excess of the income over the accumulated depreciated is taxed at the capital gains rate which is 40 percent of the ordinary tax rate (12 percent).

The Problem

The Accelerated Cost Recovery System was designed to stimulate economic growth by increasing investment in capital assets. Will ACRS influence financial strategies, rates of replacement of depreciable assets, capital-labor mixes, and the adoption of new technology in agriculture? If so, will the influence be positive or negative?

Discovering the effects of ACRS in these areas would benefit certain groups: policymakers must understand the ramifications of ACRS on various facets of the agricultural sector to formulate effective policies and financial lenders must be cognizant of the implications of ACRS to adequately serve their clientele. Farm operators themselves can make informed decisions if they understand the economic changes induced by the current tax legislation. The following hypotheses describe the problem and provide a general direction for the research.

1. ACRS will increase the rate of replacement of depreciable assets.
2. The new recovery system will result in an increase in after-tax income.
3. The new set of statutes will augment production disparities between large capital-intensive farms and smaller units since the larger farm will realize greater relative tax savings.

The objectives of the research are:

1. To develop a model that will be useful in comparing the post-tax farm asset costs incurred from compliance with the current and former tax legislations, within an environment described by general farm conditions, tax rates, inflation rates, and farm sizes.
2. To analyze the impact of changes in the foregoing variables on optimal replacement of machinery.
3. To draw inferences about the small and large farm dichotomy and the associated farm income differences based on the results generated from the analysis.

The remaining portion of the study is a sequence of logical steps taken to achieve the objectives. In Chapter II, recent literature is reviewed related to investment analysis and alternative methods of considering income tax features in an analysis at the firm level. Chapter III contains a description of the model developed and the empirical framework for the analysis. Chapter IV evaluates the effects of the recent tax legislation on asset replacement, including the impact of variations in the rate of inflation, marginal income tax rates, hours of annual use, residual value equations and downtime costs. Chapter V contains a summary and concluding comments.

CHAPTER II

METHODOLOGY OF OPTIMAL REPLACEMENT

Theoretical Principles of Optimal Replacement

Replacement theory differs from static production theory in that the quantity of an input is not varied but rather is "lumpy" or fixed and a time dimension is introduced. Both are fundamentally alike as the dependent variable of concern (cost) is subjectively assigned a value (dollars) and the objective functions are quite similar. Within the cost-output plane of production theory, net revenues are maximized or costs are minimized while in replacement theory, the present-value stream of net revenues (addition to wealth of the firm or individual) is maximized or the present-value stream of costs is minimized. Furthermore, there are unifying themes in the optimization criteria in the frameworks.

Static vs. Intertemporal

Assumptions of Static Production Theory. In the following description of production theory, maximization of profit and utility is assumed as well as a) a large number of homogeneous, atomistic firms, b) perfect knowledge by both producers and consumers, c) no single firm can influence input or output prices, d) a timeless environment, and e) tastes and preferences, income, and technology are fixed. An approach occasionally employed when revenues and costs are examined is to consider each a function of quantity of output (Q). The resulting cost, C(Q), and revenue, R(Q), functions are illustrated in Figure 1. Since the time variable is held constant, the axes are also labelled "U.T.," to be read as "per unit of time." The revenue function is linear given the assumption that the producer is a "price taker", - i.e. faces one price and can sell all production at that price. Figure 2 illustrates the marginal revenue, marginal cost, and average cost functions denoted respectively by MR, MC, and AC. Marginal revenue equals $\frac{\partial R(Q)}{\partial Q}$, marginal cost equals $\frac{\partial C(Q)}{\partial Q}$, and average cost equals $\frac{C}{Q}$. At the quantity Q^1 average cost equals marginal cost and average cost is at a minimum. The high-profit level of output is that where the limit-slopes of both cost and revenue schedules are equal. Quantity Q' is such a level. Total net revenue is signified in Figure 1 by E. As the revenue function is linear and with no intercept, marginal revenue is constant and equals the average revenue AR (Q). Since revenue equals the product of price and quantity, (P:Q), average revenue equals the price received for the product, P(Q) = P. Thus, unit profit is the difference between price or average revenue and average cost. If price fell and remained below the minimum average cost, the producer would likely terminate production since continued production would incur a net loss. The net revenue or profit function $\Pi(Q)$ shown in Figure 3, equals the net of revenue R(Q) minus costs C(Q), or $\Pi(Q) = R(Q) - C(Q)$. Note that at levels of Q less that Q_0 or greater than Q_1, C(Q) exceeds R(Q) and thus $\Pi(Q)$ is negative.

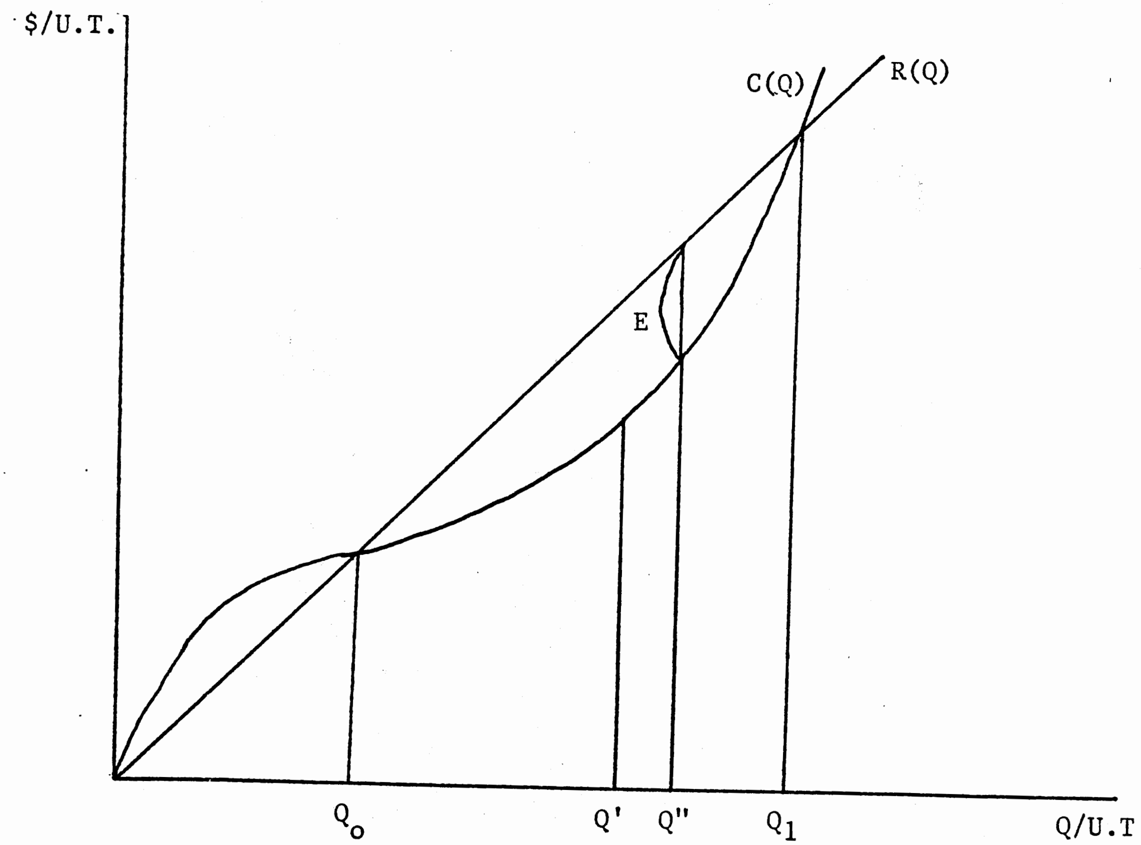

Figure 1. Cost and Revenue Functions

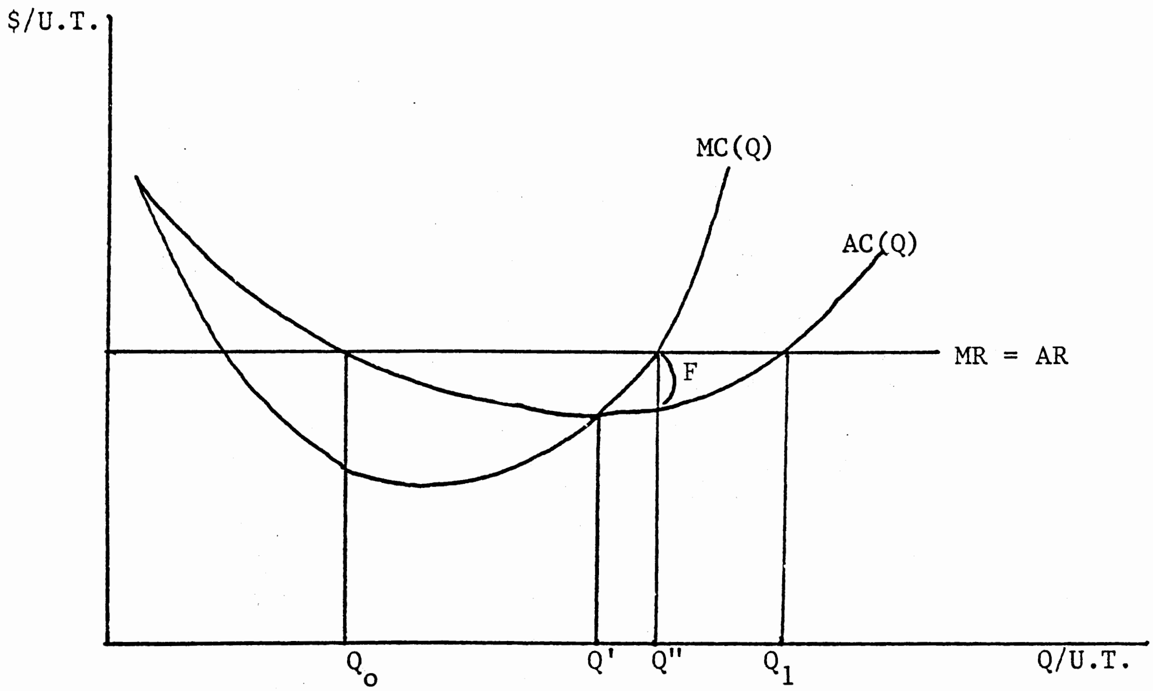

Figure 2. Marginal and Average Cost Curves

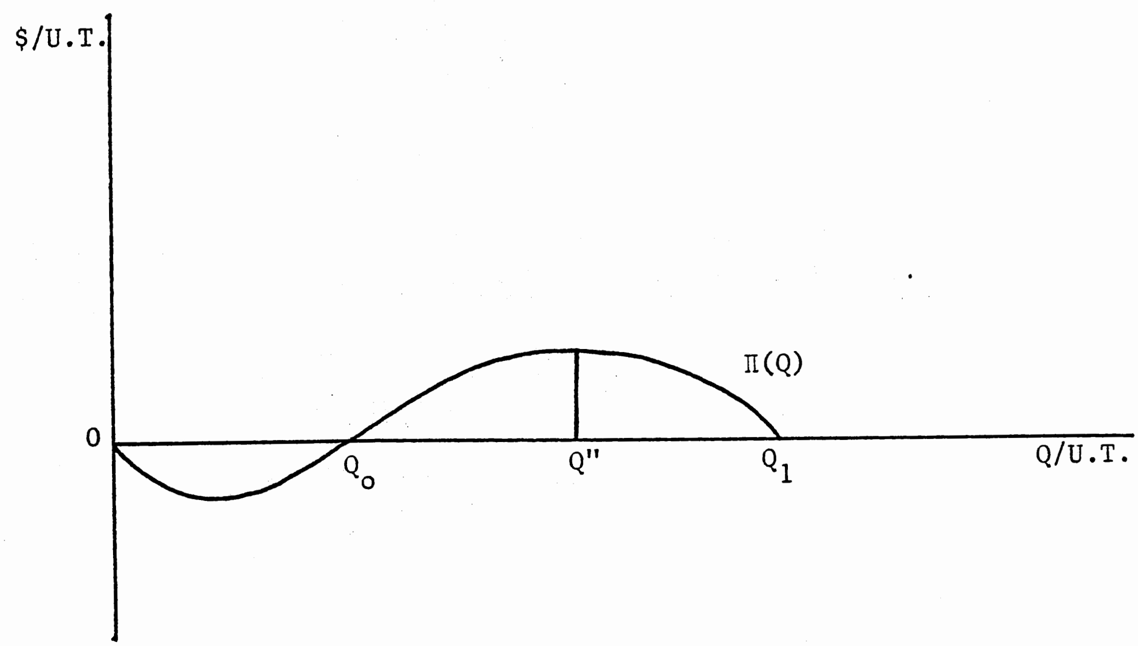

Figure 3. Profit Function

Assumptions of Replacement Theory. The primary assumptions of replacement theory are (a) decision making occurs in a risk-free environment, (b) acquisition and disposition costs are non-existent, (c) residual earnings can be imputed to the asset, (d) the producer's (investor's) objective is either to minimize the present-value stream of costs or to maximize the present value stream of net earnings and (e) the replacement is an identical asset.

Analogous to the profit function in static economic theory is the net revenue function in replacement theory given in Figure 4 (Faris 1960, p. 757). This function has corresponding marginal and average schedules, such as those exhibited in Figure 5. Marginal net revenue over time, MNR, is the derivative of the intertemporal net revenue schedule $(\frac{\partial NR}{\partial time})$ and average net revenue, ANR, over time is as it implies $(\frac{NR}{time})$. Upon scrutinizing the schedules, one wonders what replacement decision would yield the maximum stream of net revenue. Only when returns are not discounted for time (i.e. the discount rate is zero), are ANR and MNR useful for an analysis of maximization of net revenue, since neither includes a discounting factor. Given this assumption, if there will not be a replacement, the optimal time to dispose of an asset is when MNR is maximized - i.e. at T_1 in Figure 5 (net income over time is maximized). If replacement of the property is assumed, the optimal replacement occurs at the maximum ANR across all replacement cycles. For example, assume the replacing asset is an identical one; replacement in T' would not be a maximizing decision since the marginal net revenue associated with use of the asset currently in service (N') is declining and below the maximum average net revenue resulting from use of the substituting asset (N_1).

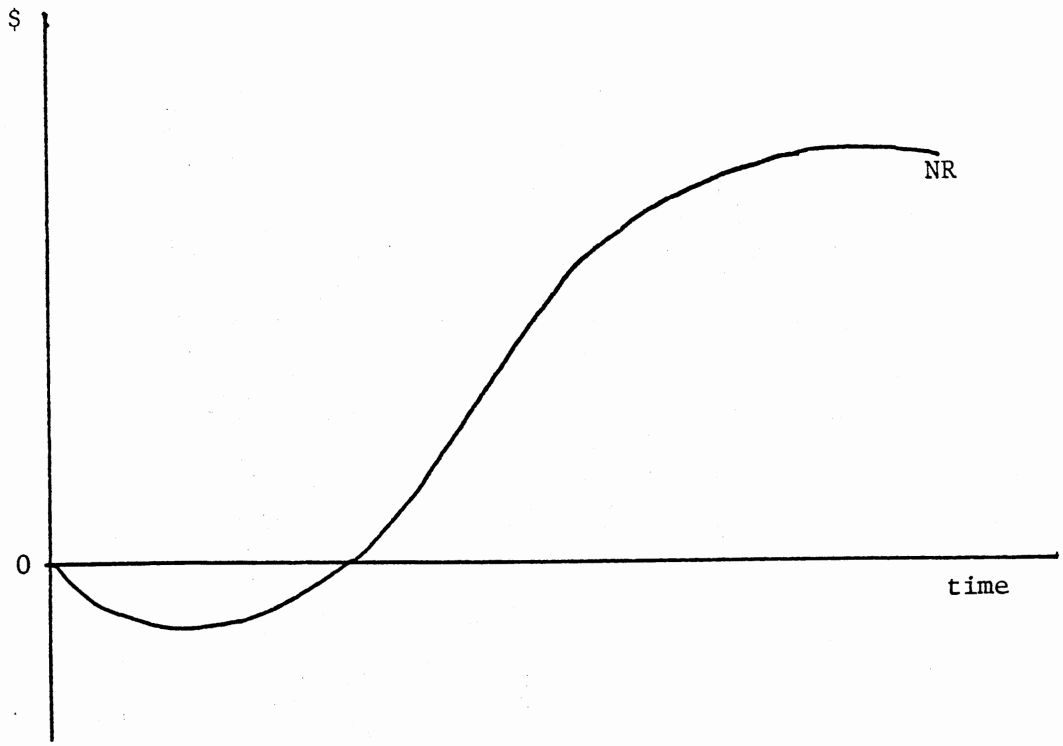

Figure 4. Net Revenue Over Time

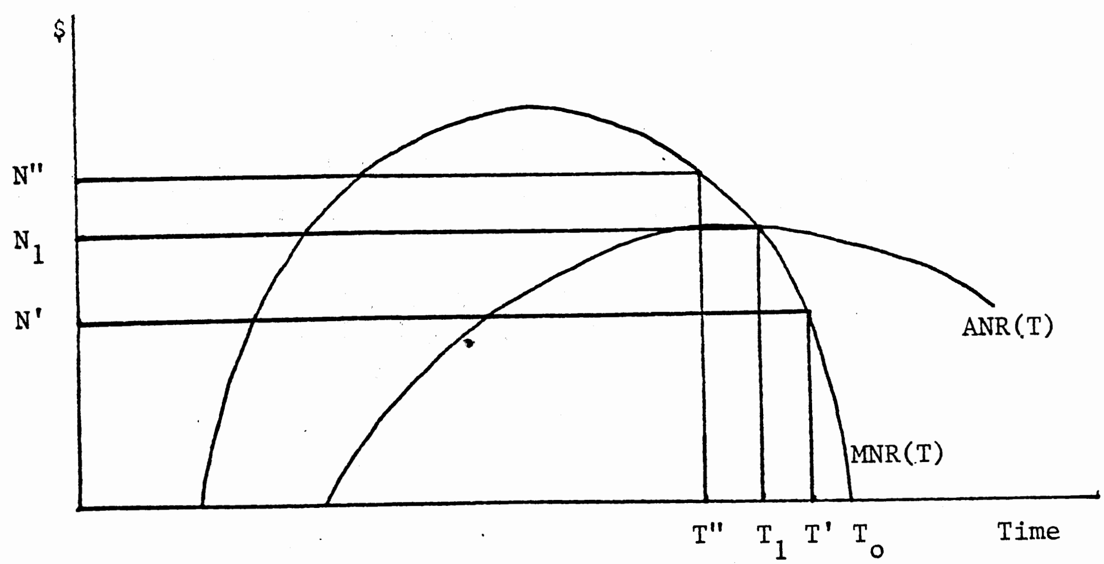

Figure 5. Marginal and Average Net Revenues from the Current Asset and Its Replacement, Respectively

Replacement in year T" would also be non-optimal as marginal net revenue (N") would exceed the maximum attainable average from use of the replacement (N_1). The decision yielding the greatest residual earnings is for disposition of the capital in year T_1. Since average net revenue in one use cycle equals the maximum average in the following cycle, opportunity costs are minimized. If the replacing asset effected greater imputed net revenue as in Figure 6, the decision rule would be to trade when the marginal net revenue from the asset currently utilized, MNR_0, equalled the maximum average net revenue from employing the substitute, ANR_1, (at T_1). This is both a necessary and sufficient condition that maximizes a stream of net revenue (recall that the discount rate was in these examples assumed to be zero). This approach is applicable when considering either identical or superior assets.

Continuous and Discrete Approaches

When a discount rate is used, average net revenue is not appropriate since earnings must be discounted for time. Given the need for discounting, the amortized present value of net revenue serves as an analogue of average net revenue over time (the latter no longer relevant since future returns are reduced by a time factor). The amortized present-value of net revenue is an average of discounted net revenue. If only one asset is considered and returns are discounted for time, the present value of net revenue is maximized by disposing of the asset when the amortized net revenue from use of the property is maximized. If the asset will be sold and another purchased, optimal replacement occurs when the marginal net revenue

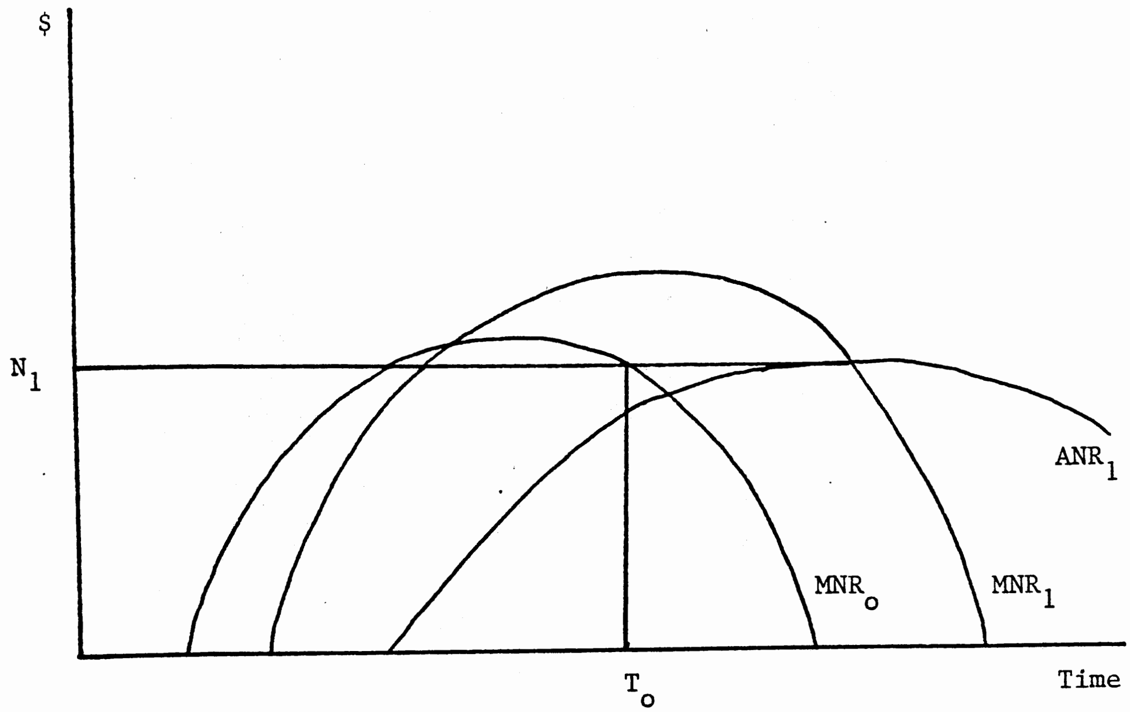

Figure 6. Different Marginal and Average Net Revenues

realized from use of the existing asset equals the greatest amortized present value of expected net returns accruing from utilizing the replacing asset (Faris 1960, p. 766). The amortized present value is used as a proxy for average net revenue. Net revenue as expressed by Faris is equal to total receipts net of interest charges, annual variable costs and fixed expenses or:

$$NR_k = Y_k - a_{k-1} i \; b_k - c_k$$

where

Y_k = total revenue in year k,

a_{k-1} = the unpaid balance of fixed or establishing costs at the end of year k-1,

i = the rate of interest paid,

b_k = annual operating costs in year k,

c_k = total fixed or establishing costs.

The amortized present value of an asset (ANR_n) yielding yearly returns, used until year n is given as:

$$(1) \quad A_{NR_n} = \left[\sum_{k=0}^{n} \frac{Y_k - a_{k-1} i - b_k - C_k}{(1+r)^k} \right] \left[\frac{r(1+r)^n}{(1+r)^n - 1} \right]$$

where $a_{0-1} = 0$

and r = the discount rate.

The present value of net revenue is maximized when the marginal net revenue in year n associated with use of asset a equals the maximum amortized present value of expected net revenues occurring when asset b replaces asset a in year n+1 and is retained until year 2n. This relationship is presented in Figure 7 where ANR is the amortized present value of net revenue and MNR is the marginal net

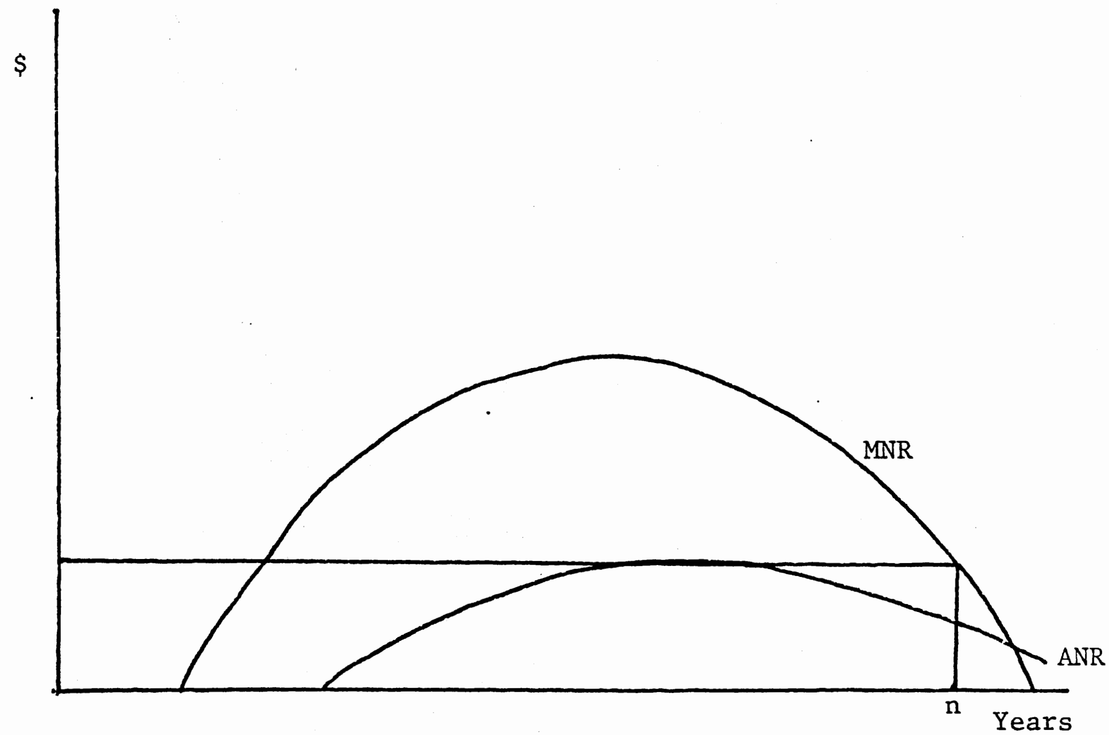

Figure 7. Marginal and Annualized Net Revenues from the Current Asset and its Replacement, Respectively

revenue. Since expected residual earnings from the replacing asset are used in computing the amortized values, anticipated earnings from future superior assets can also be entered.

Conversely, the maximization rule given by Faris can be transformed into a present value cost minimization principle. Suppose:

r = the real rate of time preference,

PV_n = the present-value of costs incurred from acquisition, use, and disposition in year n,

MC = the marginal (yearly) undiscounted cost,

AC_n = the anticipated amortized present-value cost associated with the replacing asset being utilized for n years, and

$$(2) \qquad AC_n = [PV_n] \left[\frac{r(1+r)^n}{(1+r)^n-1}\right].$$

The present value of costs from use of current capital and its replacement are minimized by trading when the marginal cost (added cost in the following year) equals the minimum amortized present-value cost associated with the replacement. In Figure 8, the optimal timing of disposition of the first asset occurs in year n.

Henderson and Quandt define the perpetualized present value of a stream of consequent quasi-rents, n, from incorporating a machine into a production process as:

$$(3) \qquad \Pi(T) = \left[\int_o^T z(t)e^{-it}dt - I_o + S(T)e^{-iT}\right] \frac{1}{1-e^{-iT}}$$

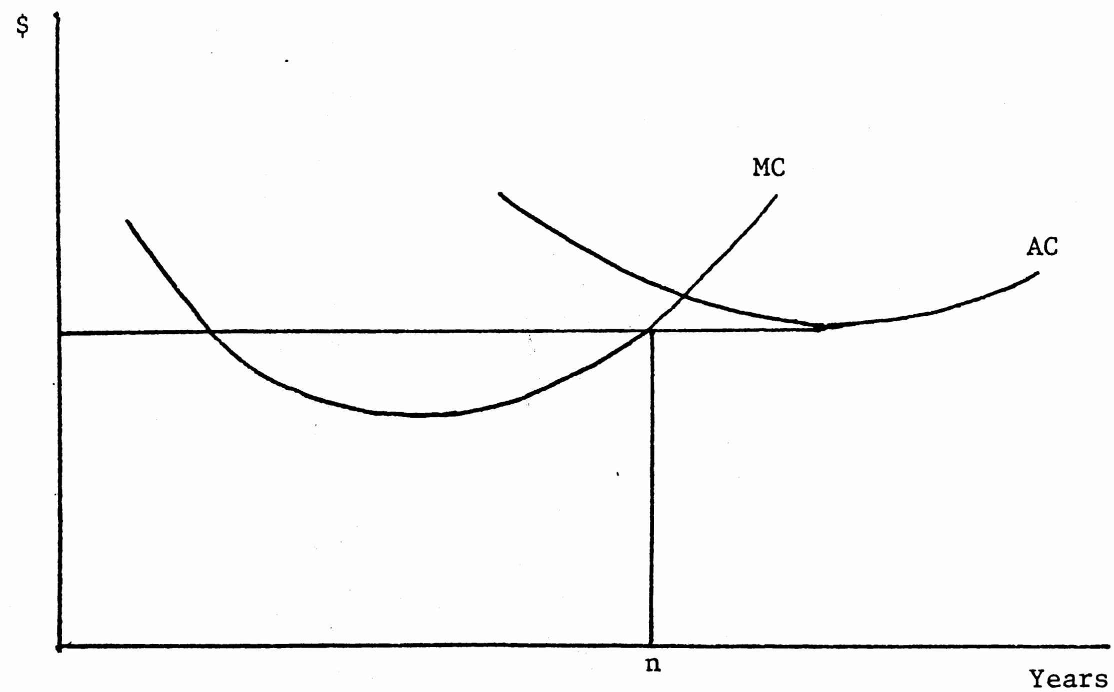

Figure 8. Marginal Cost (MC) of the Current Asset and Annualized Costs (AC) of its Replacement

where: i = the real discounted rate,

T = the time period in which the asset is sold,

$Z(t)$ = the residual earnings inputed to the machine,

I_o = the cost of the machine,

$S(T)$ = the scrap or salvage value in year T,

$$\frac{i}{1-e^{-iT}} = [\int_o^T e^{-it}dt] = [-\frac{1}{i}(e^{-iT} - 1)]^{-1} = [\frac{1-e^{-iT}}{i}]^{-1}$$

$$\frac{i}{1-e^{-iT}} = \text{Amortization Factor}$$

$$\frac{i}{1-e^{-iT}} = \frac{\text{Amortization Factor}}{i} = \text{Perpetuity Factor}$$

The accompanying assumptions are:

(a) an infinite time horizon,

(b) a competitive and constant price, p, and one level of output, Q,

(c) input costs are a function of output,

(d) repair costs vary with output and age of the machine,

(e) a profit-maximizing objective of the producer.

Present-value residual earnings are maximized by replacing when (3) is greatest. The authors differentiate the equation as follows:

$$\frac{d\Pi}{dT} = [\frac{ie^{-iT}}{(1-e^{-iT})^2}] [\int_o^T z(t)e^{-it}dt - I_o + S(T)e^{-iT}] + [\frac{1}{1-e^{-iT}}]$$
$$[z(t)e^{-it}dt - iS(T)e^{-iT} + S'(T)e^{-iT}] = 0$$

(4) $$z(t)dt - iS(T) + S'(T) = \frac{i}{(1-e^{-iT})} [\Pi_T]$$

(5) $$z(t)dt + S'(T) = \frac{i}{(1-e^{-iT})} [\int_o^T z(t)e^{-iT}dt - I_o + S(T)]$$

Henderson and Quandt depict (5) as being the marginal condition leading to the same solution as an iterative search to ascertain that level of time in which (3) is maximized. However, the approach given in (4) is a corollary to that in (1), since Π_T equals the present value of net revenue and $\frac{i}{1-e^{-iT}}$ is the amortization factor.[1] The terms on the left-hand-side of (4) comprise the marginal undiscounted net revenue where S'(T) is the change in market-value (which is the continuous analogue of the discrete (yearly) change in value) and S(T) is the opportunity cost of the periodic market rate of interest that could be earned from lending or investing an amount equal to the salvage value rather than repurchasing another machine. Therefore quasi-rent or net earnings contingent on the employment of an infinite chain of machines is maximized at that period of replacement, T, in which (3) is maximized. Maximization is also achieved by incrementally equating the marginal undiscounted net revenue occurring from use of the current asset to the maximum amortized anticipated net revenue forthcoming from use of the next asset. The latter decision must be made in every replacement cycle but may be more appropriate when superior machines are considered since reexamination of expectations is possible throughout the time horizon.

If the objective is to minimize the present value stream of costs over an infinite time horizon, then present-value net revenue in (3) and (4) is replaced with the present value of costs. Regardless of the objective, the optimization principles are fundamentally equivalent.

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[1] Faris expresses the amortization factor as $\frac{r(1+r)^n}{(1+r)^n-1}$.

The approach taken by Perrin is similar to that of Henderson and Quandt. When considering residual earnings and given a replacement decision of 5 years with an infinite planning period, the present-value stream of residuals earnings is maximized by continuous replacement in that years associated with the greatest perpetualized present value of net revenue or:

$$(6)\quad \frac{1}{1-e^{-PS}}\ [\int R(t)e^{-Pt}dt + M(S)e^{-PS} - M(0)] = \frac{1}{1-e^{-PS}}\ [C(0,S,1)]$$

$$= C(0,S,\infty)$$

where

$C(0,S,1)$ = the present value of residual earnings in the first use cycle ending in year s,

r = the real discount rate,

S = the year of asset disposal,

t = the year t ranging from 0 to S,

$R(t)$ = the flow of current residual earnings that can be ascribed to the asset,

$M(S)$ = the market value of the asset in year S,

$M(b)$ = the initial cost of the asset

Perrin likewise differentiates the expression to eliminate the need for repetitive searching. The solution is given as:

$$(7)\quad R(S) + M'(S) = PM(S) + \frac{1}{1-e^{-PS}}\ [C(0,S,1)]$$

$$= \frac{P}{1-e^{-PS}}\ [\int R(t)e^{-Pt}dt + M(S) - M(o)$$

where

$M^1(S)$ = the derivative of the market price in year S with respect to S. The present value stream of net revenues is maximized when replacement occurs at that age s. At s, the forthcoming net returns from use of the old asset plus the change in market value equals the amortized present value of both expected residual earnings and the difference between the salvage value of the original asset and the purchase price of its replacement. Subtracting the opportunity cost, PM(S), from both sides results in

$$(8)\quad R(S) + M'(S) - PM(S) = \frac{P}{1-e^{-PS}}\,[C(0,S,1)]$$

which bears semblance to (4). Figure 9 illustrates the equality conditions of (7) and (8) where optimal replacement occurs at S_o. The discrete equivalents of (6) and (7) are expressed by:

$$(9)\quad C(0,S,\infty) = \frac{1}{1-(1+r)^{-S}}\left[\sum_{t=1}^{S}(1+r)^{-t}R(t) + (1+R)^{-S}M(S) - M(o)\right]$$

and

$$(10)\quad R(S+1) + \Delta M(S+1) = \frac{r}{1-(1+r)^{-S}}\left[\sum_{t=1}^{S}(1+r)^{-t}R(t) + M(S) - M(o)\right]$$

where

r = the real discount rate.

Perrin reasons that returns occurring in the next period, (s+1), should be compared to those realized in the current period. Furthermore, it is recommended that if accuracy is desired, (9) should be implemented as the decision criterion since (10) is only an approximation to a continuous approach and the consequent error can result in a replacement decision being one period greater or less than that actually desired.

Again the present-value stream of costs can be minimized by substituting costs for net revenues in R(t) and R(S + 1). An

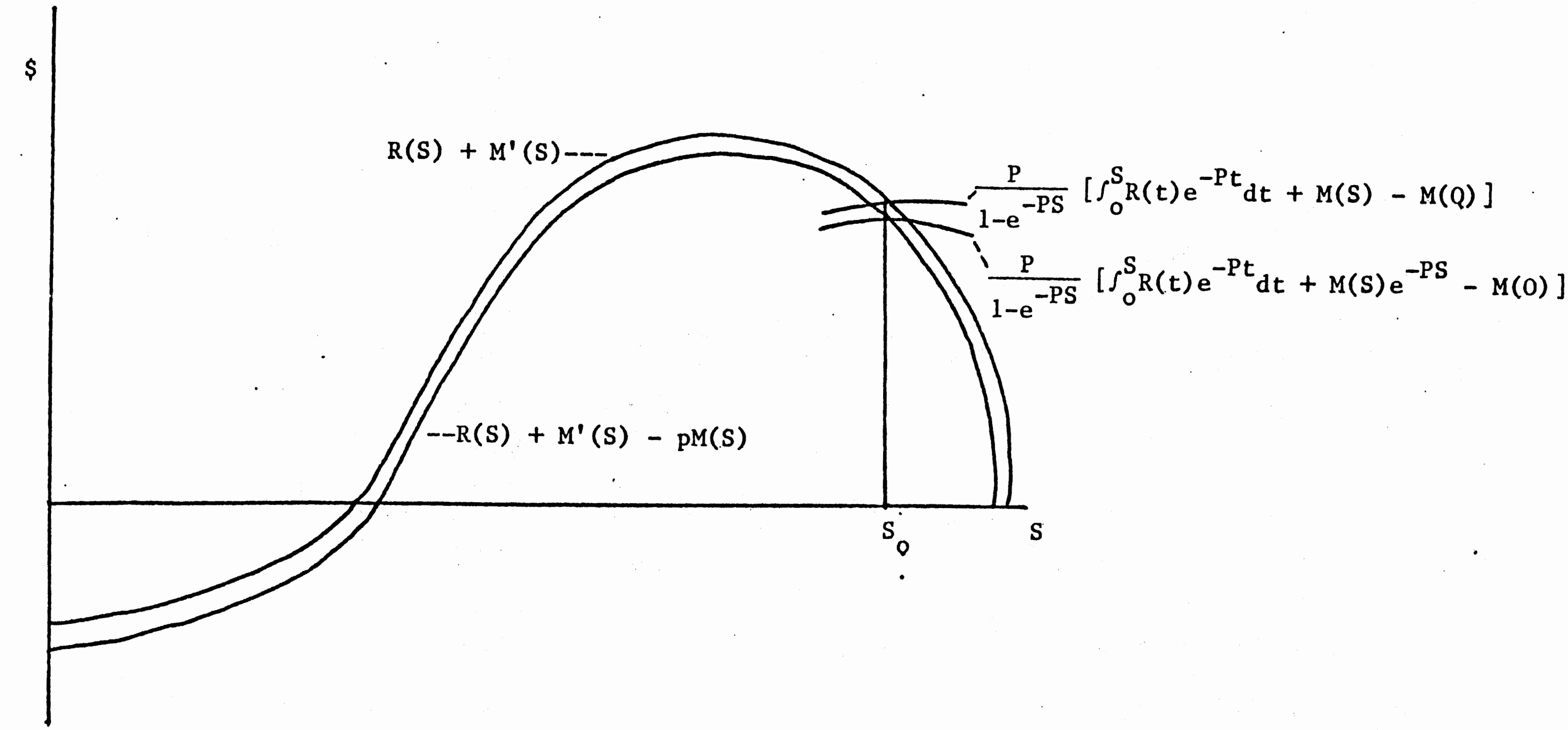

Figure 9. Equality Conditions for Maximization

illustration of the criterion cost curves in (7) is Figure 10, where $\int R(t)e^{-Pt}dt$ is the expected present value of ownership costs (maintenance, repair, etc.).

Review of Literature

The foregoing models are useful for theoretical purposes however tax aspects of asset purchase-disposition decisions need consideration. Chisholm (1974), in his study of Australian tax legislation, used a discrete time model to assimilate the conditions under which tractor replacement would occur. Respective of a single machine, the model is as follows:

$$(11)\quad Q_n = Q_n = (M_o - M_n[1+r]^{-n}) + (1-T)(\sum_{k=1}^{n} R_k[1+r]^{-k}) - T(I[1+r]^{-1} - T(\sum_{k=1}^{n} D_k\ [1+r]^{-k}) + T([\sum_{k=1}^{n} D_k - M_o + M_n]\ [1+r]^{-n})$$

where

n = the replacement age measured in years,

r = the firm's after tax discount rate (assumed constant),

M_o = the acquisition cost of a new machine,

M_n = the resale value of a machine aged n years,

R_k = the machine operating cost in years K,

D_k = the amount of depreciation allowance in year K,

I = the amount of an investment credit,

T = the firm's rate of income tax (assumed constant),

Q_n = the after-tax present value of the stream of costs for a single machine.

If V_n = the after-tax present value of the perpetualized stream of costs for an infinite chain of identical machines, each replaced at

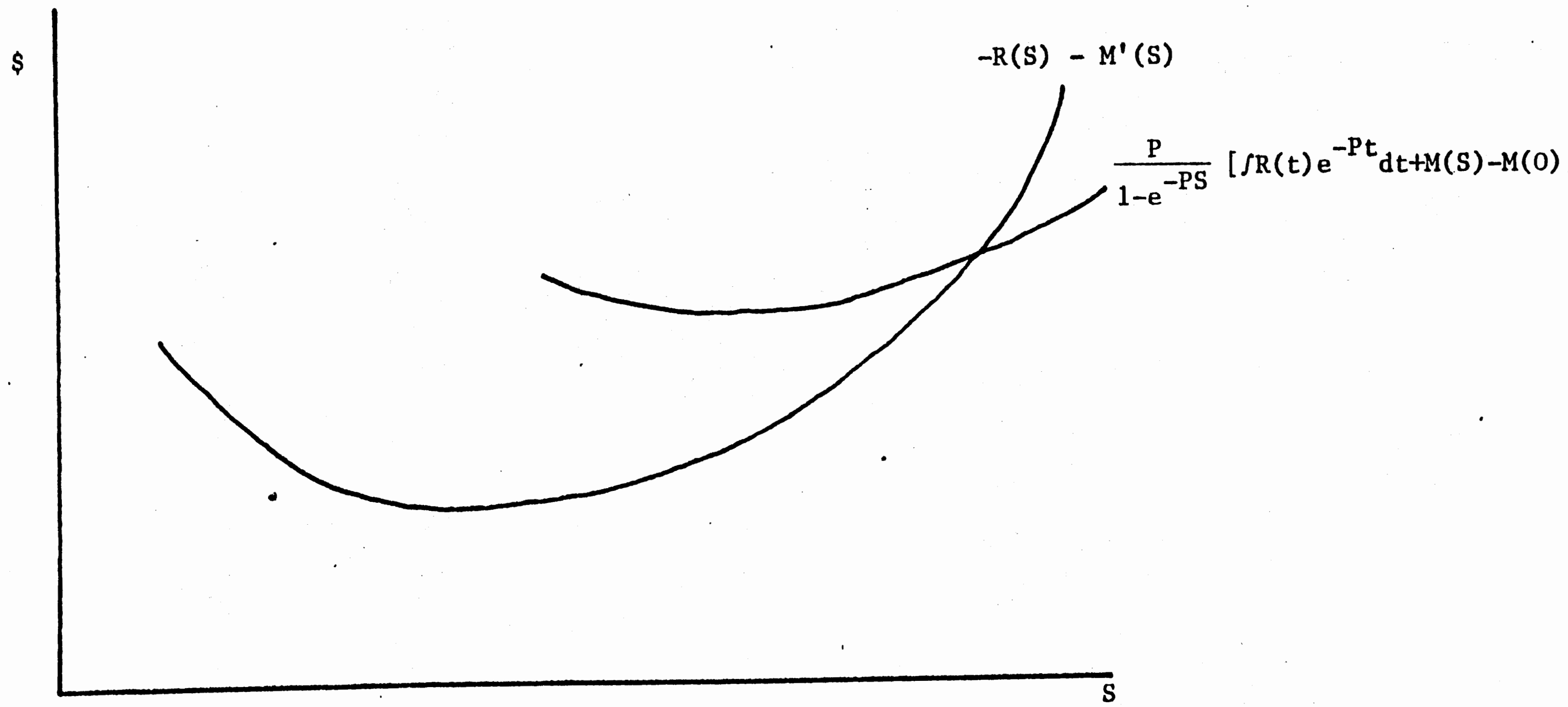

Figure 10. Criterion Cost Curves

age n years;

then

$$V_n = \frac{Q_n}{1-(1-r)^{-n}}$$

$$V_n \leq V_{n+1},$$

$$V_n \leq V_{n-1},$$

and

$$(12)\quad rV_n = \frac{r}{1-(1+r)^{-n}}\,[M_o - M_n[1+r]^{-n}) + (1-T)(\sum_{k=1}^{n} R_k[1+r]^{-k})$$
$$- T(I[1+r]^{-1} - T(\sum_{k=1}^{n} D_k[1+r]^{-k})$$
$$+ T([\sum_{k=1}^{n} D_, -M_o + M_n][1+r]^{-k})$$

Chisholm concluded that prior to the modification of the tax laws, the income tax and discount rates respectively were the influential factors affecting the replacement interval negatively and positively. Terms for recapture of depreciation and investment credit were not included as it was assumed that the farmer would not replace sooner than the asset's elected life. The model is useful since it is flexible for alternative methods of depreciating tax rates, and investment allowances; however, an inflationary element is needed in this approach.

The Kay and Rister optimal replacement model (1976) is distinguished from that of Chisholm. The former includes the additional first year depreciation permitted when the asset is replaced in n years (14). The perpetualized present-value cost of the machine is:

$$(13)\quad PV_n = \frac{1}{1-(1+r)^{-n}}\{(C_o - C_n[1+r]^{-n}) + (1+T)(\sum_{k=1}^{n} R_k[1+r]^{-k})$$
$$- T(A_n[1+r]^{-1}) - T(\sum_{k=1}^{n} D_k[1+r]^{-k}) - I_n(1+r)^{-1}\}$$

r = after-tax discount rate,

C_o = purchase price,

C_n = market value at end of year n,

T = income tax rate,

R_k = repair cost in year K,

A_n = additional first year depreciation,

D_k = regular depreciation in year K,

I_n = investment credit taken with a replacement in n years.

Kay and Rister also evaluated tractor replacement and concluded that the primary determinants of decisions regarding disposition and acquisition were the post-tax discount rate, additional first year depreciation, investment credit and repair costs, while depreciation method chosen did not effect replacement.

Bates, Rayner, and Custance (1979) suggest that repair costs for the $15,000 tractor in Kay and Rister's study (in constant dollars) are a continuous function of K years or;

$$R_k = \int 464.211\ K^{.5}\ (1+r)^{-h} dk \text{ (h expressed as an integer of } K - h = [K])$$

The authors' rationale for including an inflation element is 1) as depreciation is based on historical cost, 'real' costs are not recovered, 2) tax allowances frequently taken in the first year are 'depreciated' by that years actual inflation rate, 3) upon sale of the tractor, the inflated resale price may exceed the remaining depreciation allowance. With e^f being the rate of inflation, the 'real' present value cost is given as:

$$(14) \quad PV_n = \frac{1}{1-e^{-rn}} \{ (C_o - C_n e^{-rn}) + (1-T)R_n^* - T\,(A_n e^{-(r+f)}) - T(\int_o^n D_k e^{-(r+f)k} dk) - I_n e^{-(r+f)} \}$$

where R_n^* = the present value of the stream of repair costs. This framework is also based on the assumption that disposition will not occur before all depreciation deductions have been taken (given a straight-line election).

Bates, Rayner, and Custance found that when higher expected inflation rates were chosen, the optimal replacement age was extended.

Watts and Helmers (1979) compared traditional and capital budgeting methods for estimating annual machinery costs. Those costs were assumed to be either depreciation and opportunity costs or adjunct costs. Inflationary terms were introduced along with the assumptions:

(a) a constant discount rate is relevant,

(b) a constant inflation rate will prevail,

(c) inflation impacts all factors equally,

(d) the marginal tax rate is constant,

(e) adjunct costs are constant.

The annual costs could be expressed as:

$$(15) \quad MC = \overline{A}_c + \overline{O}_c + D_c$$

where

MC = real annual machinery costs,

$\overline{A}_c$ = the amortized after-tax adjust costs,

$\overline{O}_c$ = the amortized after-tax estimate of opportunity costs,

D_c = the amortized present value of depreciation.

Hence

$$\overline{A} = [\int_0^n \overline{A} e^{fi} d^{-\overline{r}i} di][\int_0^n e^{-\bar{\bar{r}}i} di]^{-1}$$

where

A = real after-tax adjunct costs,

$\overline{r}$ = r(1-T), T being the marginal tax rate

$\bar{\bar{r}}$ = real after-tax discount rate = r-f = (1-T)r-f,

f = rate of inflation,

$$\overline{O}_c = [\int_0^n \bar{\bar{O}}_c(i) e^{-\bar{\bar{r}}i} di][\int_0^n d^{-\bar{\bar{r}}i} fi^{\ -1}]$$

where

$\bar{\bar{r}}$ = real after-tax discount rate = (r-f)(1-T),

$\bar{\bar{O}}_c$ = the real after-tax opportunity cost at machine age i in a dollar value associated with machine age 0,

V(i) = value of machine age i in a dollar value associated with a machine age 0,

and

$$D_c = [\int_0^n D(i) e^{-ri} di][\int_0^n e^{-\bar{\bar{r}}i} di^{\ -1}]$$

where

$\overline{D}_i$ = machine depreciation at age I in dollar value associated with machine age c = $\frac{-\partial V(i)}{\partial i}$

Watts and Helmers conclude that inflationary conditions are significant and estimation of machinery costs should allow for inflationary consideration. In this regard, traditional budgeting was found to be deficient unless real opportunity costs and salvage values are initially given.

Reid and Bradford employed a model similar to that of Kay and Rister's discrete model but with the addition of terms for recapture and capital gains. The framework is as follows:

$$(16)\quad PV_n = \frac{1}{1-(1+r)^{-n}}\{C_o - C_n(1+r)^{-n} + (1-T)\sum_{k=1}^{n} R_k(1+r)^{-k}$$
$$-T\,A(1+r)^{-1} - T\sum_{k=1}^{n} D_k(1+r)^{-k} - I(1+r)^{-1} + IR_n(1+r)^{-n}$$
$$+ T\,OI(1+r)^{-n} + .4T\,CG(1+r)^{-n}\}$$

where

PV_n = the present value of perpetual costs from replacing every n years with an infinite time horizon,

r = the risk free, after-tax discount rate,

C_o = the purchase price of the asset,

C_n = the market or residual value of the asset when it is sold in year n,

T = the marginal tax rate,

R_k = the costs of repairs, insurance, and opportunity (i.e., when the asset is 'broken down' in year k,

A = the added allowable first year depreciation with a replacement decision of n years,

D_k = the depreciation of year k,

I = the investment tax credit taken,

IR_n = the recapture of investment credit resulting from sale of the asset in the n^{th} year,

OI = the portion of the gain realized upon sale of the asset that is taxable at the ordinary marginal income tax rate (equalling the accumulated depreciation),

CG = the remaining portion of the gain that is subject to the capital gains tax rate (the excess of the sale over the accumulated depreciation).

The capital gains tax rate under the recent tax statutes is 40 percent of the ordinary marginal income tax rate, hence the element T is multiplied by .4.

Leatham and Baker (1981) present a model as follows:

$$(17)\quad \left[\frac{r}{1-(1+r^*)^{-n}}\right]\left[\left[C_o - C_n(1+r)^{-n}\right] + (1-T)\sum_{k=1}^{n} R_k(1+r^*)^{-k} - T\left[\sum_{k=1}^{n} D_k(1+r)^{-k}\right] - \left[I_n(1+r)^{-1}\right]\right]$$

where

$$C_n = S_n - [S_n - (C_o - \sum_{k=1}^{n} D_k)]\,T, \text{ for } S_n \leq C_o,$$

$$C_n = S_n - \sum_{k=1}^{n} D_k T - (S_n - C_o)T^* \text{ for } S_n > C_o,$$

A_n = average yearly real after-tax cost of the asset retained n years,

r^* = the real post-tax cost of capital,

r = the nominal post-tax cost of capital,

C_o = the purchase price of the asset,

C_n = the nominal after-tax residual (market) value of the n-year old machine,

T = the marginal income tax rate,

T^* = the long term capital gains tax rate,

R_x = the real repair cost of the machine in year K,

D_k = the depreciation taken in year K,

I_r = the investment tax credit allowable with an elected life of n years, and

S_n = the nominal before tax market value of the asset following year n.

Leatham and Baker investigated the effects of inflation on the replacement decision of a \$15,000 tractor using the Kay and Rister data. Inflation was found to increase costs at low rates and decrease costs at high rates. Furthermore, high rates of inflation slightly reduced the optimal replacement age. This result conflicted with previous literature which maintained that the introduction of inflation increased optimal replacement age. The authors rationalized this as a consequence of an increased real salvage value, which lowers costs.

Reid and Bradford addressed the replacement question by including each of four different used tractor market value estimators. Under alternative scenarios, the resultant optimal replacements occurred mostly within 7-10 years (considerably earlier than the 11-14 year decisions of their colleagues). Reid and Bradford attributed this difference to the choice of residual value function.

Reid and Bradford also developed a framework for optimal replacement including inflationary elements and terms reflecting technological change. Their alternative model was born out of need for modelling the effect of technological change on the cost-efficiency of a tractor. The conventional approach also failed to examine the different impacts of inflation on new and used market values and costs. The following equation is a combination of estimates of future present values and a perpetualized present value.

$$PV(S^*) = [-M(1) - \sum_{t=1}^{S1} (R_{t1} \prod_{i=1}^{t} \frac{(1+g_{i1})}{(1+r_{i1})(1+f_{i1})}) + M(S1) \prod_{t=1}^{S1}$$

$$\frac{(1+h_{t1})}{(1+r_{t1})(1+f_{t1})}] + \frac{1}{\prod_{t=1}^{S1} (1+r_{t1})(1+f_{t1})} [-M(2) \prod_{t=1}^{S1} (1+k_{t1})$$

$$- \prod_{t=1}^{S1} (1+g_{t1}) \sum_{t=1}^{S2} (R_{t2} \prod_{i=1}^{t} \frac{(1+g_{i2})}{(1+r_{i2})(;+f_{i2})} + \prod_{t=1}^{S1} (1+h_{t1}) M(S2)$$

$$\prod_{t=1}^{S2} \frac{(1+h_{t2})}{(1+r_{t2})(1+f_{t2})}] + \frac{[1-(1+r)^{-S3}]^{-1}}{\prod_{t=1}^{S1} (1+r_{t1})(1+f_{t1}) \prod_{t=1}^{S2} (1+r_{t2})(1+f_{t2})}$$

$$[-M(3) \prod_{t=1}^{S1} (1+k_{t1}) \prod_{t=1}^{S2} (1+k_{t2} - \prod_{t=1}^{S1} (1+g_{t1}) \prod_{t=1}^{S2} (1+g_{t2}) \sum_{t=1}^{S3} \frac{R_{t3}}{(1+r)^t}$$

$$+ \prod_{t=1}^{S1} (1+h_{t1}) \prod_{t=1}^{S2} (1+h_{t2}) \frac{M(S3)}{(1+r)^{S3}}]$$

where

$M(i)$ = purchase price of the new machine i,

$M(S_i)$ = remaining market value of used machine i,

$R(t_i)$ = the insurance, maintenance, and opportunity costs of breakdown, net of tax deductions for depreciation in year t of period i,

$S(i)$ = optimal replacement age of machine i,

r_{ti} = real discount rates during ownership of machine i,

f_{ti} = general inflation rate predominating in year t for machine i,

K_{ti} = annual rate of change in purchase price of machine i during year t,

g_{ti} = annual rate of change in operating costs of machine i in year t,

h_{ti} = annual rate of change in market value of used machine i.

Reid and Bradford addressed the issue of improved technology's effect on replacement. Their results supported the theoretical foundations presented by Perrin - a more efficient challenging asset will induce earlier replacement of the defender.

Reid and Bradford's general conclusion is that the most significant determinants of optimal replacement age for a machine are relative annual cash flow changes in a given machine-cycle and relative changes in the present value of the cash flows between machines.

The objectives of the review of literature were a) to examine various models of optimal replacement of depreciable assets, b) to peruse the methodology for including alternative depreciation schedules in asset purchase decisions, and c) to provide additional information for constructing a framework. The model developed allows for the incorporation of an inflationary assumption. Though formulated prior to the publication of the latest Reid and Bradford paper, the latter was nonetheless conceptually valuable for its treatment of inflation. Furthermore, the model to be discussed in Chapter III is essentially a variation on the theme presented by others (i.e. Chisholm, Kay and Rister, Bates, et al., etc.). The literature review was successful in achieving the objectives.

CHAPTER III

MODEL DEVELOPMENT

This chapter contains a description of the model and its development into a computer algorithm. Estimates for specific elements within the equation are then discussed.

As previously indicated, the decision rule for optimal asset replacement is either net revenue maximization or cost minimization. Since the former rule involves the use of projections of gross margins that are at best uncertain and are prone to concurrent complication by the introduction of cost differences among farms of alternative sizes, the analysis will be based on a generalized fixed cost minimization objective.

Empirical Framework

The fixed or ownership costs incurred in farming include depreciation, interest, repairs, property taxes, insurance, and shelter. The interest expense incurred to acquire the asset through a financial arrangement and property taxes were excluded from consideration. Shelter will be excluded from consideration as an expense attached to chattel (shelter is intrinsically a separate asset). The relevant components of the model are a) estimates of insurance and repair costs for deduction from taxable income, b) depreciation deductions, and c) after tax considerations including purchase price, investment tax credit, additional first-year depreciation deductions, and salvage or market value.

The empirical framework utilized will be:

(18) $$PV_n = \frac{1}{1-(1+r)^{-n}} \{ C_o - \frac{C_n}{(1+r)^n} + (1-T) \sum_{k=1}^{n} \frac{R_k}{(1+r)^k} - \frac{T \sum_{k=1}^{n} D_k}{[(1+r)(1+z)]^{-k}} - \frac{T\ AD}{(1+r)(1+z)} - \frac{I}{(1+r)(1+z)} + \frac{RC_n}{[(1+r)(1+z)]^n} + \frac{T\ OI_n}{[(1+r)(1+z)]^n} + \frac{pT\ CG_n}{[(1+r)(1+z)]^n} \}$$

where

PV_n = the annualized present-value costs incurred from acquisition, use, and disposition of one asset every n years for an infinite time horizon,

C_o = the purchase price of the asset,

C_n = the real market or residual value at the end of year n,

r = the risk-free rate of time preference,

z = the rate of inflation,

T = the marginal income tax rate,

R_k = the repair and maintenance, insurance, and downtime costs in year K,

D_k = the regular depreciation taken in year K,

AD = the additional depreciation taken in the first year,

I = the investment credit permitted with the asset life (not necessarily equalling n) elected,

RC_n = the recapture of investment credit due following the n^{th} year,

OI_n = the portion of the gain taxable at the marginal or ordinary tax rate,

pT = the capital gains tax rate where p represents the factor by which the capital gains rate is computed,

CG_n = the portion of the gain that is subject to taxation at the capital gains rate.

The difference $(C_0 - \frac{C_n}{(1+r)^n})$ can be interpreted as the net cost of the asset (in present-value terms). The term $(1-T) \sum_{k=1}^{n} \frac{R_k}{(1+r)^k}$ represents the present value of maintenance, downtime, and insurance costs net of the present value of reductions in tax liabilities imputed to these agents. The term is positive because consideration of net maintenance, downtime and insurance costs increases the present value of total fixed costs. Depreciation reduces the present value of fixed costs by the proportion of income tax (marginal income tax rate) multiplied times the present value of depreciation (annual plus additional first-year). Hence $(\frac{T \sum_{k=1}^{n} D_k}{[(1+r)(1+z)]^k} + \frac{T\ AD}{(1+r)(1+z)})$ reduces the present value of fixed costs and is therefore subtracted from costs. Investment tax credit reduces income tax liability by the amount of the credit. Thus, the present value of investment credit $(\frac{I}{(1+r)(1+z)})$ is subtracted in determining ownership costs. Alternatively, recapture of investment credit increases the present value of overhead costs by $(\frac{RC_n}{[(1+r)(1+z)]})$. Lastly, the receipts from sale of the asset are taxable and augment the present value of fixed costs. The portion of income from the sale of the asset equalling total depreciation taken is taxed at the ordinary marginal tax rate. The excess of the sale price over the total depreciation elected (if any excess is realized) is taxed at the capital gains rate. Including these future additional tax liabilities increases the present-value ownership costs by the nominal term $(\frac{RC_n}{[(1+r)(1+z)]^n})$.

Machinery values have traditionally increased during times of inflation. To account for inflation, an asset's nominal market value in year n, C_n, is inflated by the factor $(1+z)^n$ where z represents the rate of inflation. The nominal value is then discounted by the factor $[(1+r)(1+z)]^n$, to account for time and inflation. The terms $(1+z)$ in the numerator and denominator cancel leaving a discount factor of $(1+r)^n$. The component for insurance, repair, and downtime costs in year k, R_k, is likewise compounded by the multiple of $(1+z)^k$, to account for inflation and discounted by $[(1+r)(1+z)]^k$. Once again, the terms $(1+z)^k$ cancel an R_k is discounted by $(1+r)^k$. Tax authorities do not permit depreciation taken in any year, K, to be adjusted for inflation. Thus, depreciation recovery is based on the initial purchase price. AD, I, RC, OI, and CG, are also computed from historical cost. If inflation exists, the reduction in tax liability from depreciation is further reduced in present-value terms, and division by $[(1+r)(1+z)]^k$ reflects this reduction. Additional depreciation, AD, and investment tax credit, I, taken at the end of year 1 are also reduced by a multiple of $[(1+r)(1+z)]^n$. The perpetuity factor $1/1-(1+r)^{-n}$ converts the present value of real costs (within the brackets) incurred in the n-year period to a constant payment or real cost incurred every n years for an infinite time horizon. The inflationary component is excluded since amortized costs are expressed in real terms. To reflect the opportunity cost of capital, the term (1+r) can be substituted by (1+i) everywhere in the equation where i is the rental rate on capital.

Optimal Replacement Cycle

Two approaches can identify the optimal replacement cycle, n. The first method involves solving the model for PV_n (perpetualized present-value of costs) as the replacement year is increased from zero to a number specified as an upper limit to cycle length. The resulting PV_n costs may be plotted, as in Figure 11. The optimal replacement year occurs where the amortized costs are lowest. The alternative method involves comparing incremental (yearly) undiscounted costs from using the current asset with amortized (not perpetualized) costs associated with replacement in that year and employing the new asset (see Appendix A). Cost minimization over an infinite chain of cycles occurs from replacement at the end of year n when those marginal or incremental costs most nearly equal the minimum amortized costs. Although this approach is more flexible since more efficient replacements can be considered, the former method is empirically more useful.

The optimal replacement model employed in this analysis is similar to that used by Reid and Bradford, except that the present-value costs are adjusted for inflation. In addition to the assumptions enumerated in Chapter II, the following assumptions are necessary:

a. The rate of inflation will be constant and all future transactions will be nominally augmented by that magnitude,

b. the future market values of the asset are known with certainty,

c. the replacements are identical assets,

d. the yearly downtime, repair, and insurance costs can be determined without error,

e. the statutes prescribed by the current tax legislation will not change, and

f. an otherwise deterministic environment prevails.

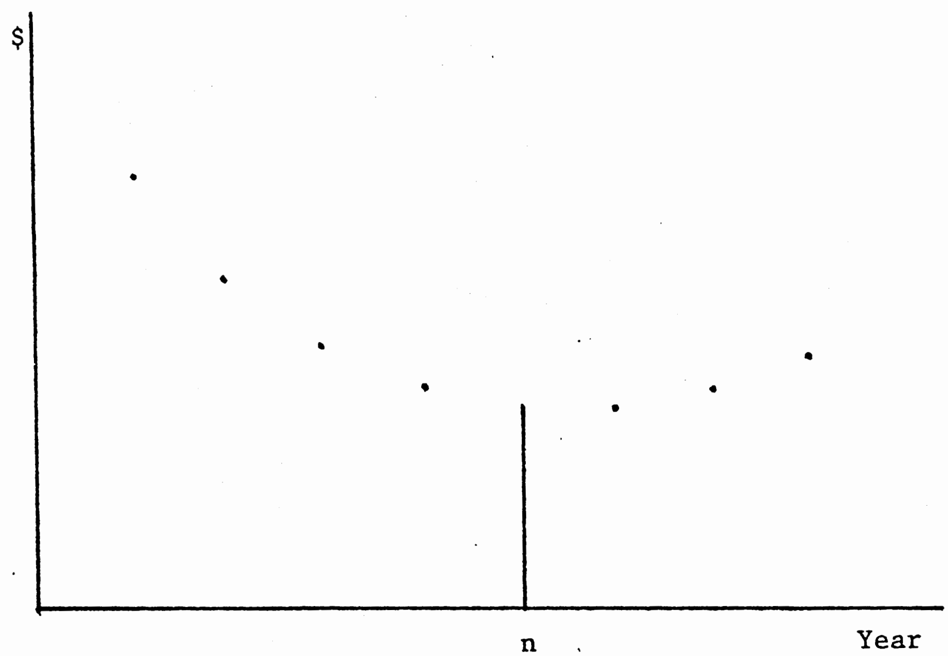

Figure 11. Long Run Annual Ownership Costs for Different Lengths of Machine Ownership

Program Development

A computer program was developed to enter values for exogenous variables and minimize equation 18. Figure 12 is a flow diagram of the computer program. A printout of the program is given in Appendix B. The hardware utilized was a Radio Shack (TRS-80, Model II) "micro-computer." To find the optimal replacement cycle, a large replacement period, n, was chosen along with either cost equation (if a tractor is considered) and values for the elements of the model. Upon initialization, the perpetualized present value of costs associated with an n-year replacement cycle was computed and printed. Since all tractor residual value (RV) equations examined four solutions resulted at every replacement year, n. The initial value for n was then decremented, all other decision variables were retained, and the second iteration began. The present value of net costs using each RV equation was again computed, converted into a perpetuity, and printed. This looping continued until the replacement period equaled zero. If a sufficiently large replacement period is entered, a bowl-shaped discrete cost curve is traced out. Figure 13 is an example of the printout where the numbers 1 through 15 are the disposition ages and the large values are the perpetualized costs. Costs are minimized with a replacement cycle of 8, 8, 7, and 5 years using the respective RV equations of Leatham and Baker, Reid and Bradford, Agricultural Engineers, and McNeill.

Specific Research

After specification of the empirical framework, the subsequent research involved obtaining proxy equations for market values and costs of alternative farm chattel.

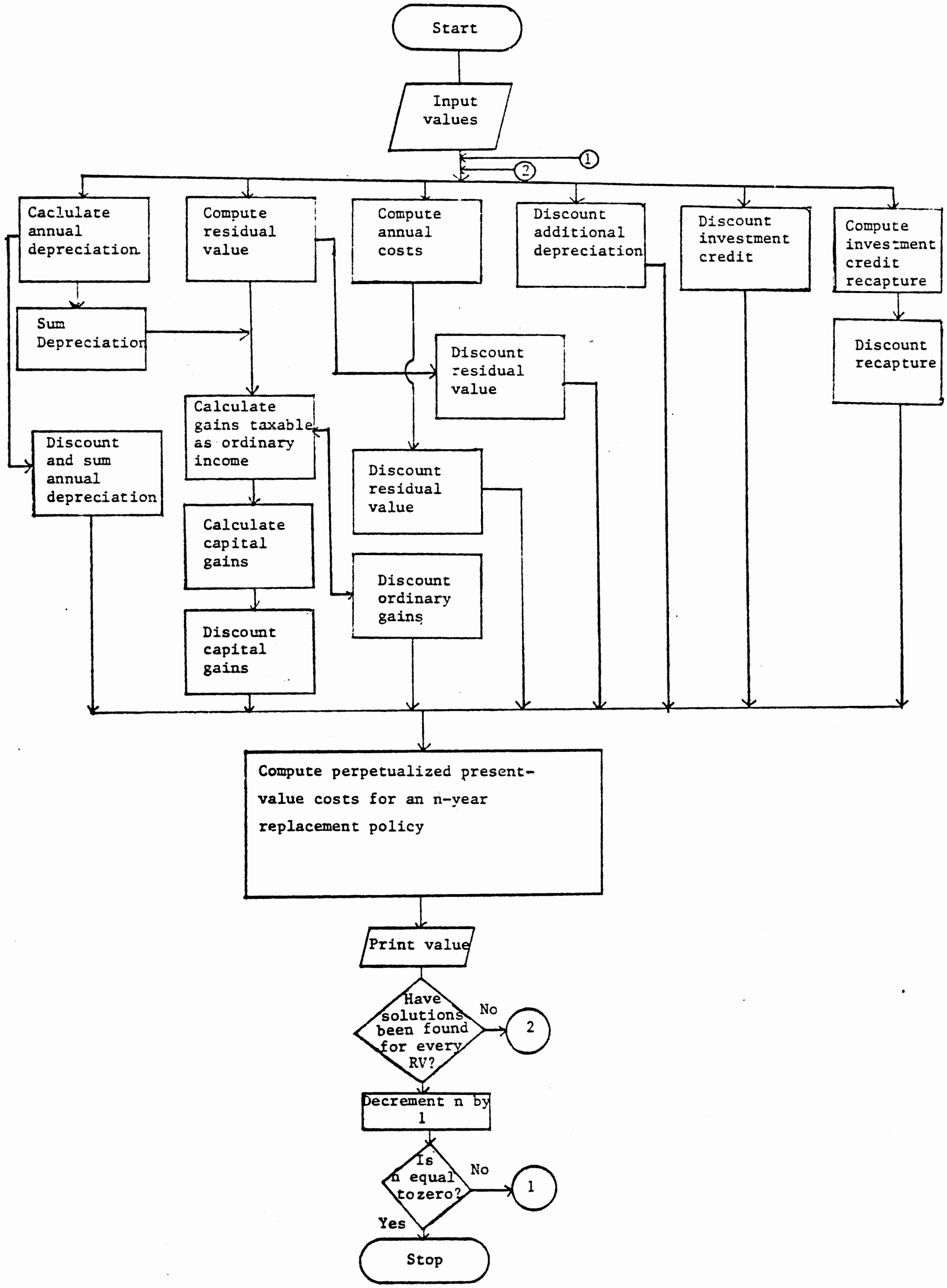

Figure 12. Flow Diagram of Computer Program

15	LEATHAM AND BAKER		127467	
AGRICULTURAL ENGINEERS		126076	MCNEILL	40710.4
	REID AND BRADFORD		127025	
14	LEATHAM AND BAKER		125068	
AGRICULTURAL ENGINEERS		123477	MCNEILL	39251.8
	REID AND BRADFORD		125093	
13	LEATHAM AND BAKER		122813	
AGRICULTURAL ENGINEERS		120989	MCNEILL	37758.9
	REID AND BRADFORD		123455	
12	LEATHAM AND BAKER		120741	
AGRICULTURAL ENGINEERS		118643	MCNEILL	36266.3
	REID AND BRADFORD		122196	
11	LEATHAM AND BAKER		118903	
AGRICULTURAL ENGINEERS		116479	MCNEILL	34818.8
	REID AND BRADFORD		112338	
10	LEATHAM AND BAKER		117368	
AGRICULTURAL ENGINEERS		114552	MCNEILL	33477.6
	REID AND BRADFORD		110834	
9	LEATHAM AND BAKER		116232	
AGRICULTURAL ENGINEERS		112939	MCNEILL	29414.8
	REID AND BRADFORD		109840	
8	LEATHAM AND BAKER		115637	
AGRICULTURAL ENGINEERS		111752	MCNEILL	25279.7
	REID AND BRADFORD		109533	
7	LEATHAM AND BAKER		115801	
AGRICULTURAL ENGINEERS		111164	MCNEILL	21071.9
	REID AND BRADFORD		110174.	
6	LEATHAM AND BAKER		117082	
AGRICULTURAL ENGINEERS		111457	MCNEILL	16789.8
	REID AND BRADFORD		112164	
5	LEATHAM AND BAKER		120114	
AGRICULTURAL ENGINEERS		113134	MCNEILL	12428.9
	REID AND BRADFORD		116149	
4	LEATHAM AND BAKER		144829	
AGRICULTURAL ENGINEERS		137694	MCNEILL	30180
	REID AND BRADFORD		142507	
3	LEATHAM AND BAKER		169915	
AGRICULTURAL ENGINEERS		160207	MCNEILL	41621.9
	REID AND BRADFORD		167790	
2	LEATHAM AND BAKER		225202	
AGRICULTURAL ENGINEERS		210468	MCNEILL	71879.5
	REID AND BRADFORD		218645	
1	LEATHAM AND BAKER		401917	
AGRICULTURAL ENGINEERS		372405	MCNEILL	178806
	REID AND BRADFORD		349779	

Figure 13. Perpetualized Yearly Costs (in real dollars)

Recall that the examination concerns 3-year and 5-year property. From those assets included as 5-year property, tractors were selected for analysis since they are typically employed by all farmers. Also, a chisel plow, grain drill, and a row cultivator were chosen primarily because of availability of cost information. Light trucks were not considered since their replacement is fairly regular and probably invariant to tax legislation.

The market value of many depreciable assets frequently deviates from the accountants' "book value" (undepreciated portion of the original purchase price). Hence, a better estimate of residual value is necessary if more accurate predictions of asset disposition prices are to be made. Equations of residual values for tractors will first be discussed, after which estimates of various implements will be identified.

Tractor Market Values

Peacock and Brake examined an 11-year data series (1953-1963) of tractor values and fit the equation

(19) $RV = 64.3 - 3.11X$

RV = the nominal sale price expressed as a percentage of purchase price,

X = the age of the tractor at disposition (years).

McNeill reported the market value to be described by

(20) $RV = (100)e^{-.4299 - .0436X + .0691C}$

where

RV = the real market value of the tractor as a of percentage of replacement price,

X = the age of the tractor,

C = a dummy variable assigned a value between 0 and 4 on the basis of the assets working condition, (0 being poor, 4 being excellent) with the value being reduced yearly by .4 to reflect actual depreciation.

A statistical description of the equation was not available.

The equation estimated by the Agricultural Engineers is

(21) $RV = 68(.92)^X$

where

RV = the constant dollar residual value as a percentage of initial price. The R^2 value for the equation exceeded .9.

Reid and Bradford estimate market value as

(22) $RV = 368.7\ (X)^{-.273}\ (HP)^{.242}\ (NF)^{-.305}\ (T1)^{-.621}\ (T2)^{-.205}\ (MX)^{-.121}\ (MY)^{-.263}$

where

RV = the real-dollar remaining value as a percentage of purchase price,

X = the age (years) of the tractor at disposition.

HP = the tractor PTO horsepower rating,

NF = mean net farm income, weighted by a three year moving average and based on 1967 dollars,

T1,T2 = Trend variables to reflect technological improvement of tractors, T1 being 1 for the first 11 years, afterword becoming equal to the constant, e. T2 equals 1 until the tractor is age 20; and thereafter equals e. The use of e is necessary since the least-squares estimation involves a log-linear transformation.

Mx,MY = Dummy variables collectively indicating the brand of the tractor. Each is valued at either 1 or e. If MX = e and MY = 1, the tractor was manufactured by firm X and vice-versa for company Y. When MX = MY = 1, the tractor was produced by company X. The e's however are naturally exclusive.

The statistical specifications of Reid and Bradford's equation are (a) significance of all parameter estimates at the 1 percent level of type I error, (b) an R^2 value of .87, and (c) a Durbin-Watson statistic of 1.77 (deemed sufficiently large to reject the existence of either negative or positive serial correlation).

Leatham and Baker estimated remaining market value as

$$\frac{C_t}{C_0} = I^{1.4358}H^{-.0543}(1.054)^{D_1}(1.087)^{D_2}(.9930)^{D_3}(.7282)^{D_5}(.7582)^{D_6}(.7534)^{D_7}(.7414)^{D_8}(.9982)^{D_3A_t}(.9933)^{D_3A_t}(.9048)^{D_5A_t}(.8963)^{D_6A_t} \times (.9171)^{D_7A_t}(.9001)^{D_8A_t}e^{E_t} \tag{23}$$

where

C_t = the market value of the tractor in year t,

C_0 = the purchase price of the tractor in year 0,

I = a price index quotient where the numerator is the price index for new tractors purchased in year t, and the denominator is the index for tractors purchased in year 0,

H = the drawbar horsepower rating of the tractor,

D_1 = 1 if the the year, t, is 1974 or 1975, otherwise it is 0,

D_2 = 1 if the tractor is diesel-powered, 0 otherwise

D_3 = 1 if the tractor is a 4-wheel drive, 0 otherwise

D_5 = 1 if the manufacturer is Allis - Chalmers, 0 otherwise

D_6 = 1 if the manufacturer is International Harvester, 0 otherwise

D_7 = 1 if the manufacturer is John Deere, 0 otherwise

D_8 = 1 if the manufacturer is Massey-Ferguson, 0 otherwise

Given the test of statistical significance (H_0:B=0, H_A: =0), all but the b-value for D_3 were significant at the 5 percent level of type I error. The R^2 value was .733. The data series was taken from years 1963 to 1975.

The foregoing equations are illustrated in Figure 14 where horsepower is fixed at 115 PTO and the values for net farm income and tractor purchase prices are \$10,000 and \$35,000, respectively. The letters E, L, M, and R respectively denote the Agricultural Engineer, Leatham and Baker, McNeill, and Reid and Bradford estimates. The brand of tractor is John Deere (the most expensive). The ratio of prices, I_t, has been fixed at 1. The tractor is powered by diesel fuel.

None of the equations evidenced any functional form that was clearly preferred to the others. The replacement decision was sensitive to choice of residual value estimator, however, and hence the equations of McNeill, Agricultural Engineers, Reid and Bradford, and Leatham and Baker were employed in the analysis.

Implement Residual Values

The equation used for estimating the market values of the implements was taken from Agricultural Engineer estimates. It is as follows:

(24) $RV = .60(.885)^X$

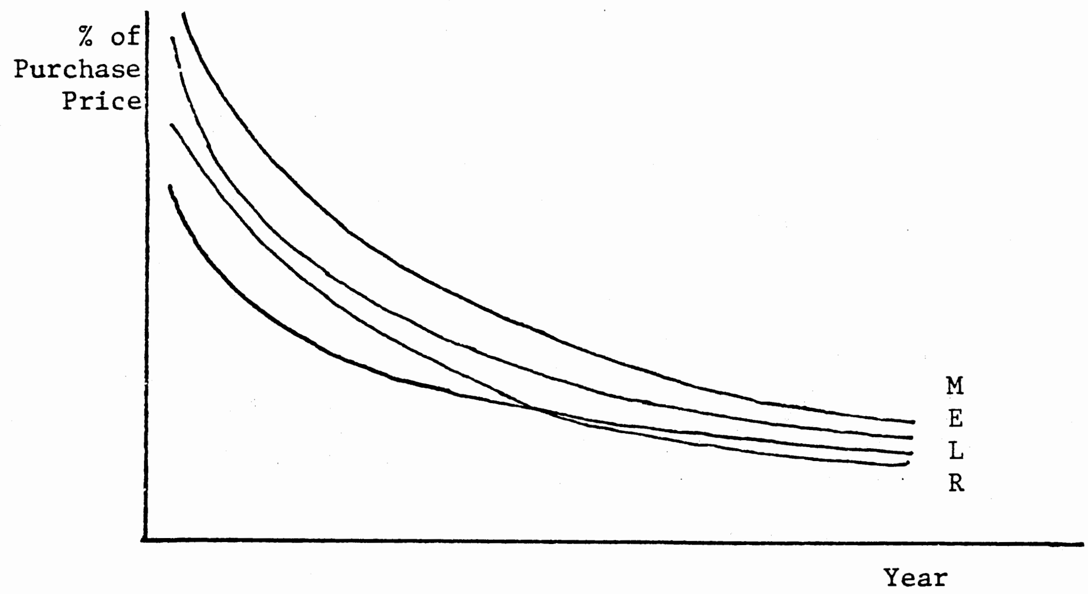

Figure 14. Residual Tractor Values using McNeill (M), Agricultural Engineers (E), Leatham and Baker (L), and Reid and Bradford (R) Equations.

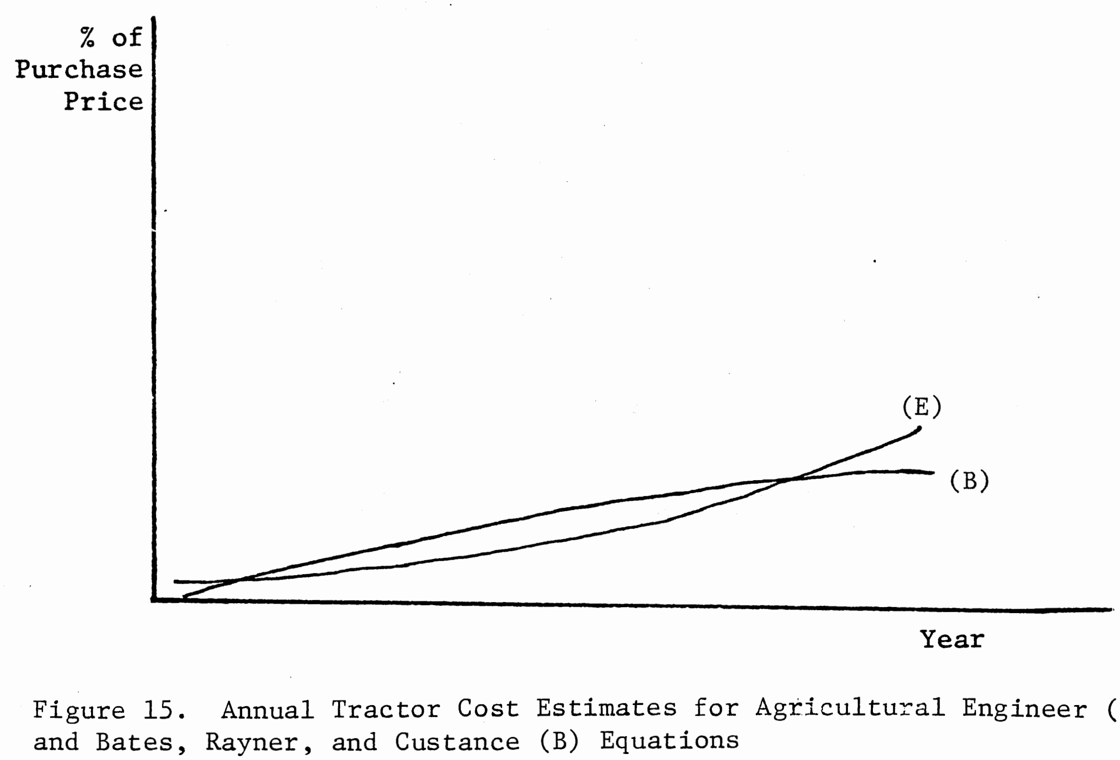

Figure 15. Annual Tractor Cost Estimates for Agricultural Engineer (E) and Bates, Rayner, and Custance (B) Equations

where

RV = the real residual or market value of the implement as a percentage of purchase price

X = age of implement (years).

<u>Tractor Costs-Repair, Insurance, and Downtime</u>

Since the objective is to minimize annualized present-value of so-called 'fixed' expenses only ownership costs were considered. For purposes of this study, they included repair and maintenance costs, insurance costs, and downtime costs. The repair costs estimated by the Agricultural Engineers for a diesel tractor are given by

(25) $C = 01.20(XK)^{2.033}$

where

C = the accumulated real dollar costs as a percentage of purchase price,

X = the yearly tractor use (hours) divided by 1000.

K = the age of the tractor (years)

Since the yearly costs are of concern when using a replacement model, was differentiated with respect to years, K. The resulting equation was

(26) $R_k = 02.44(X)^{2033}(K)^{1.033}$

where

R_k = the real dollar repair costs as a percentage of purchase price in year K.

Bates, Rayner, and Custance express tractor repair costs as

(27) $C = 464.211K^{.5}$

where

C = the constant dollar repair costs of a \$15,000 tractor, incurred in year K

K = the age of the tractor (years)

From division of C_k by the tractor price (\$15,000) a cost equation based on percentage of purchase price is obtained. The yearly cost is then

(28) $R_k = 3.0947K^{.5}$

where

R_k = the real repair cost in year K expressed as a percentage of purchase price.

Assuming 800 hours of use per year, the cost equations of the Agricultural Engineers and Bates, Rayner, and Custance are illustrated in Figure 15 (denoted respectively A and B). The former equation was employed for computing repair costs as it was consistent with the a-priori expectation of real yearly costs increasing at a positive rate as the tractor ages.

Insurance costs are computed as being a fixed proportion of the purchase price. Those costs are entered as a decision variable in the framework.

Opportunity costs of breakdown or downtime costs were considered. The relationship between downtime and age of a diesel tractor, taken from estimates given in the Agricultural Engineers yearbook is

(29) $DT = .0003234(XK)^{1.4173}$

where

DT = the accumulated hours of downtime,

X = the yearly use (hours),

K = the age of the tractor, in years

The differential of the equation is a proxy for the hours of downtime in year K, or

(30) $\frac{d}{dk} DT_k = .0004584(X)^{1.4173}(K)^{.4173}$

Each hour of downtime is then assigned a cost conditioned by output prices and labor costs.

Implement Repair Costs

The cumulative repair cost associated with each implement as estimated by the Agricultural Engineers is as follows:

(31) $C = 1.03(XK)^{1.400}$ (chisel plow or field cultivator),

(32) $C = 3.59(XK)^{2.626}$ (grain drill),

(33) $C = .94(XK)^{2.207}$ (row cultivator),

where

C = the real dollar accumulated repair costs as a percent of purchase price,

X = the yearly use (acres) divided by 1000,

K = the age of the implement (years).

The yearly repair costs are then the respective derivatives of the cumulative equations, or:

(34) $R_k = 1.442(X)^{1.4}(K)^{.4}$ (chisel plow or field cultivator),

(34) $R_k = 9.4273(X)^{2.626}(K)^{1.626}$ (grain drill),

(36) $R_k = 2.0746(X)^{2.207}(K)^{1.207}$ (row cultivator),

where

R_k = the real yearly repair costs given as a percentage of purchase price.

The foregoing cost and residual value equations were used in the model. The estimates for all the terms in the model have been identified in this chapter. After obtaining those equations, construction of the algorithm followed. Chapter IV contains a description of the subsequent research.

CHAPTER IV

APPLICATION OF MODEL AND RESULTS

This chapter is composed of two sections. The first portion is devoted to identifying the variables pertinent to asset replacement decisions and their alternative values. Secondly, an analysis of the scenarios is presented.

Variables

The following exogenous variables comprise the input for obtaining model solutions. Values assigned are provided, with explanations.

Asset Price

The tractors considered were the John Deere model 8440 (179.83PTO) and the John Deere model 2940 (81.46PTO); their costs are $63,518 and $25,117. Tractors of alternative sizes were examined since horsepower (PTO rating) was a significant statitistic in both the Reid and Bradford and the Leatham and Baker residual value equations.

The implements chosen for analysis are (all John Deere); a) a 19 ft. rigid chisel plan model 1610 priced at $8,246, b) a 20 x 8 double-run end-wheel grain drill model 8000 (plain) listing for $4,353.00, and c) a 12 row John Deere cultivator selling for $4,899.

Average National Net Farm Income For The Previous Year

Average net farm income was deemed by the Reid and Bradford to be a significant statistic in estimating the future market value of a tractor. The alternative values assigned that variable were $10,000 and $20,000. Although in Bradford and Reid's study income was weighted by a three-year moving average based on 1967 dollars, it was appropriate to assume only current income levels to insure consistency with asset price (expressed in nominal terms).

Marginal Income Tax Rate

The income tax rates selected were 20 percent and 40 percent. Assuming that the farmer's income is relatively constant throughout the time horizon (at either the 20 percent or 40 percent taxable level), the tax rates chosen were a proxy for farm size and intensity.

Additional Depreciation

Prior to 1981, first-year capital expensing was permitted (not to exceed 20 percent of the asset price). when the asset considered was treated as dictated by the former tax legislation, 20 percent capital expensing was claimed in the first year.

Rate of Inflation

Inflation rates of 0 percent and 10 percent were alternatively assumed. Inflation has a two-fold influence on the replacement model- it augments repair costs and market values and increases the discounting factor. Higher rates of inflation are expected to defer replacement of the existing asset.

Real Discount Rate

The real discount rate or risk-free rate of time preference was assigned a value of 4 percent. As was previously stated, the only effect of the discount rate is to reduce net asset costs by adjusting the flow of future costs and returns for time.

Downtime Costs Per Hour

The cost of downtime consists of the actual non-repair costs incurred (i.e. labor) and opportunity costs of delayed harvest and marketing. Since opportunity cost is quite volatile in a short period (being based on commodity prices) and cannot be precisely estimated, downtime costs were \$20 and \$40, depending on price expectations and labor costs.

Yearly Use of Asset

The Agricultural Engineers' equations of repair costs for tractors and specific implements were based on annual use in hours and acres, respectively. Yearly use of a tractor is primarily dependent on the crop(s) produced and annual average rainfall. Yearly tractor employment was assumed to be 600 hours. This is the expected use of a tractor in an operation employing just one machine or a larger farm utilizing several tractors. The farm sizes considered were 400 and 800 acres. The estimated use in terms of 'times-over' per year was one for the implements.

Annual Rate of Inflation of Tractor Costs

Nominal tractor prices were assumed to increase at a rate equal to that of inflation. Hence, the rate of inflation of tractor purchase prices was assigned values of 0 percent and 10 percent.

Insurance Rate

Insurance costs for a tractor were .75 percent of market price annually (Reid and Bradford, 1981).

Investment Tax Credit

Both the current and former tax legislations permit 10 percent of purchase price to be taken as investment credit. Prior to 1981, the '5-year' ACRS asset was required to have an elected life of 7 years, and 6 percent for 3-year property previously for 5-year elections (3 years under ACRS). Hence investment tax credit taken with the tractors was 10 percent of purchase price.

Elected Life and Depreciation Method

Three depreciation methods were evaluated: the previously common 10-year straight-line (with tax treatments being subject to the previous statutes), the ACRS schedules, and the alternative straight-line ACRS approach. Each implement was depreciated in accordance with the 7-year straight line approach, the 3-year ACRS method, and the alternative ACRS schedules of 3 years and 5 years. A 10 percent salvage value of both tractors and implements was assumed with use of the 10-year and 7-year straight line methods.

Replacement year

The initial replacement cycle was arbitrarily assigned a length of 15 years. A solution for the perpetuity value was obtained, and the length of the replacement cycle was decremented to 14-years. This processing continued until a discrete approximation such as that in Figure 11 was obtained. The perpetualized costs were at a minimum with a year replacement cycle.

Scenarios

The four variables selected for describing a specific situation or scenario are marginal tax rate, rate of inflation, the previous year's average net farm income, and downtime cost. Table 4 illustrates the alternative scenarios under which the 81 P.T.O. tractor was purchased, utilized and replaced, perpetually. The results under the assumptions of Baseline 1 were a basis for comparing the outcomes under the alternative scenarios. In Baseline 1 the marginal income tax rate is .20 or 20 percent, the rate of inflation is .10 or 10 percent, the previous year's average net farm income is $20,000, and the cost of downtime is $40. Scenario 1-a differs from Baseline 1 in that the former marginal tax rate is zero. The effect of a reduction in the tax rate on the optimal replacement decision was partitioned. In Scenario 1-b, the rate of inflation is zero but the remaining variables have the same values as those in Baseline 1. In Scenario 1-c the cost of downtime was $20. Two factors were altered in Scenarios 1-d and 1-e. In the first, the marginal income tax rate is 10 percent and the rate of inflation is 5 percent. In Scenario

Table 4. The Variables Describing the Environment for Use of the 82 P.T.O. Tractor

	Baseline 1	Scenario 1-a	Scenario 1-b	Scenario 1-c	Scenario 1-d	Scenario 1-e
Marginal Tax Rate (%)	20	0	20	20	10	0
Rate of Inflation (%)	10	10	0	10	5	0
Last Year's Average Farm Income	$20,000	20,000	20,000	20,000	20,000	20,000
Downtime Cost	$40	40	40	20	40	40

1-e, both the tax rate and inflation rate are zero. The concurrent changes in those variables were made to guage their interactive effect on optimal replacement.

Table 5 exhibits results for the 129 P.T.O. tractor scenarios. In Baseline 2 the marginal tax rate is 40 percent, the inflation rate is 10 percent, the average net farm income for the past year is \$20,000, and downtime cost \$40. Scenario 2-a is identical to Baseline 2 and in contrast to Baseline 2 insofar as the tax rate being 20 percent instead of 40 percent. Scenario 2-b illustrates a further reduction in the tax rate to zero. In Scenario 2-c, only the inflation rate differs from that in Baseline 2-c it is zero. Likewise, in Scenario 2-c the cost of downtime is reduced to \$20 with all other factors valued as in Baseline 2. The Scenarios 2-e and 2-f contain simultaneous changes in the marginal tax rate and rate of inflation. Scenario 2-e exhibits reduced tax and inflation rate to 20 percent and 5 percent, respectively. Scenario 2-f is identical to Scenario 1-e where both tax and inflation rates are zero. The conditions surrounding the use of the 179 P.T.O. machine were intended to simulate those of a large farm while the situation for the 81 P.T.O. tractor described a small operation. The 8440 model tractor was assigned 600 hours of annual use and the model 2940 was assumed to be used 400 hours annually.

Results

Effects of Market Value Estimators

81 P.T.O. Tractor. In Table 6, when the 81 P.T.O. tractor was depreciated with the ACRS schedule goven conditions of Baseline 1,

Table 5. Variables Describing the Environment for Use of the 179 P.T.O Tractor

	Baseline 2	Scenario 2-a	Scenario 2-b	Scenario 2-c	Scenario 2-d	Scenario 2-e	Scenario 2-f
Marginal Tax Rate (%)	40	20	0	40	40	20	0
Rate of Inflation (%)	10	10	10	0	10	5	0
Last Year's Average Farm Income	$20,000	20,000	20,000	20,000	20,000	20,000	20,000
Downtime Cost	$40	40	40	40	20	40	40

Table 6. Optimal Replacement Solutions for the 81 P.T.O. Tractor

Residual Value Equation	Baseline 1	Scenario 1-a	Scenario 1-b	Scenario 1-c	Scenario 1-d	Scenario 1-e
			ACRS			
Leatham & Baker	15[a] (56,565)	15 (66,798)	15 (53,740)	15 (56,537)	15 (60,927)	15 (66,304)
Reid & Bradford	15 (60,026)	15 (67,444)	15 (54,257)	15 (59,998)	15 (61,787)	15 (66,950)
McNeill	14 (23,358)	5 (28,178)	15 (50,009)	14 (23,332)	13 (46,538)	15 (61,640)
Agricultural Engineers	15 (55,879)	15 (55,879)	15 (53,055)	15 (55,852)	15 (60,156)	15 (65,447)
			ALTERNATIVE ACRS			
Leatham & Baker	15 (59,085)	15 (66,798)	15 (55,022)	15 (59,058)	15 (62,001)	15 (66,324)
Reid & Bradford	15 (62,546)	15 (67,444)	15 (55,538)	15 (62,520)	15 (62,861)	15 (66,950)
McNeill	15 (25,905)	5 (28,178)	15 (51,290)	15 (25,878)	15 (67,734)	15 (61,640)
Agricultural Engineers	15 (58,400)	15 (65,941)	15 (54,336)	15 (58,372)	15 (61,230)	15 (65,447)

Table 6. (Continued)

Residual Value Equation	Baseline 1	Scenario 1-a	Scenario 1-b	Scenario 1-c	Scenario 1-d	Scenario 1-e
			7-YEAR STRAIGHT LINE			
Leatham & Baker	15 (56,986)	15 (66,798)	15 (54,301)	15 (56,959)	15 (61,179)	15 (66,304)
Reid & Bradford	15 (57,503)	15 (67,444)	15 (54,818)	15 (57,476)	15 (60,408)	15 (66,950)
McNeill	14 (24,271)	7 (28,861)	15 (50,569)	14 (24,245)	13 (46,537)	15 (61,640)
Agricultural Engineers	15 (55,879)	15 (55,879)	15 (53,055)	15 (55,852)	15 (60,156)	15 (65,447)
			10-YEAR STRAIGHT LINE			
Leatham & Baker	15 (57,711)	15 (66,798)	15 (54,667)	15 (57,684)	15 (61,488)	15 (66,304)
Reid & Bradford	15 (58,228)	15 (67,444)	15 (55,184)	15 (58,200)	15 (62,069)	15 (66,956)
McNeill	14 (25,034)	7 (28,861)	15 (50,936)	14 (25,007)	13 (46,424)	15 (61,640)
Agricultural Engineers	15 (56,940)	15 (65,941)	15 (53,982)	15 (56,913)	15 (60,716)	15 (66,447)

[a] optimal replacement year
[b] yearly perpetual cost

perpetualized costs (yearly costs into perpetuity) were minimized at a 15-year replacement pattern based on three of the market-value estimates. A 14-year interval resulted using McNeill's equation. These results also occurred when the 7-year and 10-year depreciation methods were used. The alternative ACRS approach effected a 15-year cycle regardless of the residual value equation.

In Table 6, under Scenario 1-a with every depreciation method, only the McNeill solution changed relative to that of the Baseline. The solution with that equation was reduced to the year in which the elected life terminated. The primary agents for the more rapid replacement patterns are 1) no tax savings realized from costs, and 2) the high level of the McNeill market value vis-a-vis maintenance costs at those optimal replacement ages. Alternatively, the solutions accompanying the other residual value equations are explained by the lower market values compared to costs in the early and middle years of the cycle. The removal of a tax rate increased the perpetualized cost regardless of the estimating equation.

The rate of inflation of the general price level, purchase and sale price of tractors, and maintenance costs (all assumed equal) were eliminated in Scenario 1-b. The replacement cycles for the 81 P.T.O. tractor under Scenario 1-b were nearly identical to those with Baseline 1.

In Scenario 1-c, the opportunity and cash costs associated with breakdown were reduced from the Baseline value of \$40 to \$20 per hour. The solutions for all market value estimators were unchanged, however.

Inflation and tax rate were simultaneously reduced in Scenarios 1-d and 1-e (to 5 percent and 10 percent, respectively in 1-d and 0 percent for both in 1-e). This was done to address interaction between the two variables and their positive effect on length of asset replacement cycle. The cyclical length under both scenarios was relatively unchanged under all depreciation methods.

The differences in replacement patterns among residual value equations for the 81 P.T.O. tractor are explained in Figure 16 which describes the real market value of the tractor (John Deere model 2940) as a fraction of purchase price, given the assumptions of Baseline 1. The equations with the higher values in the 6-10 year range generally had shorter replacement solutions. The letters M, B, E, and L respectively denote the residual value estimates of McNeill, Reid and Bradford, Agricultural Engineers, and Leatham and Baker. Note in Table 5 that under the Alternative ACRS approach the McNeill solution generally prescribed a shorter replacement period. This was perhaps due to the yearly decline in the McNeill market value being large relative to the increase in undiscounted costs leading to earlier replacement.

179 P.T.O. Tractor. Consideration of the 179 P.T.O. tractor produced trends dissimilar to those with the smaller machine. In Table 7, under the conditions depicted by Baseline 2 (40 percent tax rate, 10 percent inflation rate, $20,000 average net farm income for previous year, $40 cost per hour of downtime) and assuming the market value relationship given by Leatham and Baker, the tractor replacement cycle was 15 years in length with all schedules other than the 7-year straight line, under which the tractor was kept until it was 14 years old. When the Reid and Bradford estimator was used, replacement occurred at years 10, 11, 11, and 15 based on depreciation methods

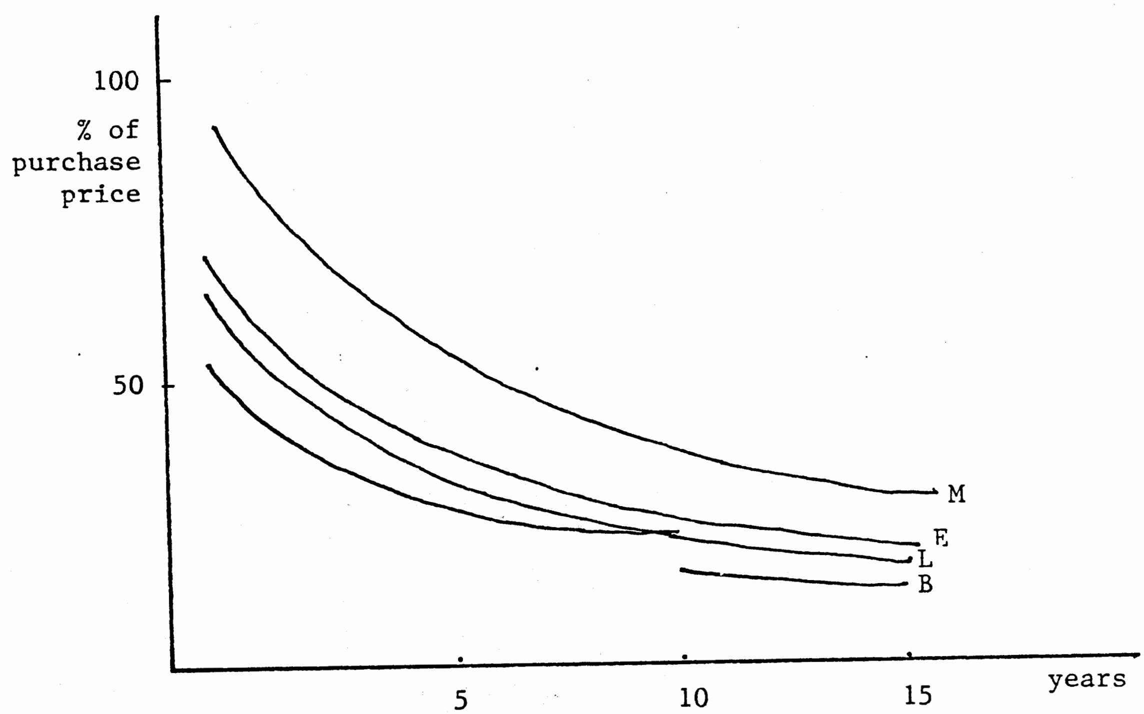

Figure 16. Residual Value Equations for an 81 P.T.O. Tractor Estimated by McNeill (M), Agricultural Engineers (E), Leatham and Baker (L), and Reid and Bradford (B)

Table 7. Optimal Replacement solutions Employing a 179 P.T.O. Tractor

Residual Value Equation	Baseline 2	Scenario 2-a	Scenario 2-b	Scenario 2-c	Scenario 2-d	Scenario 2-e	Scenerio 2-f
			ACRS				
Leatham & Baker	15 (153,562)	15 (191,572)	11 (225,803)	12 (138,821)	15 (153,470)	12 (187,605)	11 (224,218)
Reid & Bradford	10 (165,595)	11 (188,809)	11 (214,066)	11 (131,965)	11 (163,469)	11 (178,180)	11 (212,482)
McNeill	7 (67,580)	5 (83,363)	5 (94,758)	7 (125,246)	7 (67,509)	5 (133,558)	5 (196,522)
Agricultural Engineers	15 (152,045)	15 (189,550)	10 (220,652)	11 (136,028)	15 (151,953)	11 (183,845)	10 (218,941)
			ALTERNATIVE ACRS				
Leatham & Baker	15 (166,311)	15 (197,947)	11 (225,803)	13 (146,282)	15 (166,219)	13 (193,994)	11 (213,482)
Reid & Bradford	11 (179,732)	11 (196,899)	11 (214,066)	11 (140,188)	11 (179,649)	11 (185,074)	11 (212,482)
McNeill	12 (87,476)	6 (97,300)	5 (94,759)	9 (135,983)	12 (87,390)	7 (145,456)	5 (196,527)
Agricultural Engineers	15 (164,794)	15 (195,924)	10 (220,652)	12 (143,987)	15 (164,702)	12 (190,693)	10 (218,941)

Table 7. (Continued)

Residual Value Equation	Baseline 2	Scenario 2-a	Scenario 2-b	Scenario 2-c	Scenario 2-d	Scenario 2-e	Scenerio 2-f
			7-YEAR STRAIGHT LINE				
Leatham & Baker	14 (155,624)	13 (191,291)	11 (225,803)	12 (142,178)	14 (155,534)	15 (196,217)	11 (224,218)
Reid & Baker	11 (149,271)	11 (181,669)	11 (214,066)	11 (135,561)	11 (149,188)	11 (178,819)	11 (212,482)
McNeill	7 (69,377)	7 (86,779)	5 (100,766)	8 (130,240)	7 (69,306)	7 (133,560)	7 (197,855)
Agricultural Engineers	13 (153,315)	12 (187,705)	10 (220,652)	11 (139,625)	13 (153,227)	11 (184,237)	10 (218,941)
			10-YEAR STRAIGHT LINE				
Leatham & Baker	15 (159,359)	13 (193,331)	11 (225,803)	13 (144,286)	15 (159,267)	15 (197,777)	11 (224,218)
Reid & Bradford	15 (159,242)	11 (183,994)	11 (214,066)	11 (137,914)	11 (153,839)	11 (180,799)	11 (212,482)
McNeill	7 (76,165)	7 (90,173)	5 (100,756)	8 (133,301)	7 (76,094)	7 (136,451)	7 (197,855)
Agricultural Engineers	14 (157,213)	12 (189,876)	10 (220,652)	12 (141,863)	14 (157,123)	11 (186,217)	10 (218,941)

ACRS, Alternative ACRS, 7-year straight-line, 10-year straight-line, respectively. The respective solutions with the ACRS, Alternative ACRS, 7, and 10-year straight-line methods employing the Agricultural Engineer equation were 15, 15, 13, 14. Use of the McNeill estimate resulted in a 7-year replacement decision for all depreciation methods other than Alternative ACRS, with which replacement occurred in 12 years.

Introduction of the Scenario 2-a assumptions resulted in earlier patterns with the McNeill equation and the ACRS or alternative ACRS combinations while the Reid and Bradford solution prescribed a higher replacement age under ACRS and the same age using the alternative approach. The remaining two equations had the same solutions when the depreciation schedule was either ACRS or the Alternative ACRS. Depreciation of the tractor by the 7 and 10-year straight-line methods produced a 1 to 3-year reduction in the replacement pattern for the Leatham and Baker and Agricultural Engineer decisions. The McNeill solution was unaffected and that Reid and Bradford was shortened by 4 years.

In Scenario 2-b, the tax rate was eliminated and the Leatham and Baker, McNeill, and Agricultural Engineer patterns were essentially shortened. The Reid and Bradford solutions however, were unchanged.

Given the conditions of Scenario 2-c (inflation rate at zero), disposition occurred at a lower age than with baseline 2 for the Leatham and Baker and Agricultural Engineer solutions, and changed but little for the Reid and Bradford values (excepting the result attached to the 10-year straight-line approach). For the McNeill equation, there was no consistent trend among depreciation methods. The

Leatham and Baker and Agricultural Engineer solutions are consonant with previous literature in which a positive relationship between inflation and replacement age of a machine was reported (Bates, et al.). As the rate of inflation is reduced, given the sloping nature of the residual value equations, the level of real market value is increased relative to repair costs and therefore disposition occurs at an earlier age. It is not clear why the Reid and Bradford and McNeill solutions did not exhibit this movement.

The solutions for scenario 2-d (downtime cost reduced to $20) were identical to those of the Baseline.

Scenarios 2-e and 2-f included a 20 percent tax rate-5 percent inflation rate and a zero percent tax rate-zero percent inflation rate combination, respectively. The optimal replacement solutions prescribed shorter cycles under the Scenario 2-e environment. Given the assumptions of Scenario 2-f, the replacement periods were further curtailed.

With the exception of the 10-year straight-line method and assuming the conditions of Baseline 2 (40 percent tax rate, 20 percent inflation rate, $20,000 average net farm income, $40 cost of downtime), the Reid and Bradford equation induced replacement at age 10 or 11. This result is explained in Figure 17 where the decline in market value as estimated by Reid and Bradford is greatest between 10 and 12 years. Using the McNeill equation, the change in market value is greater between years 7 and 8 than any other yearly difference in which the tractor has been fully depreciated. The McNeill solutions reflect this relationship. The solution accompanying the Agricultural Engineer and Leatham and Baker estimates indicated that replacement

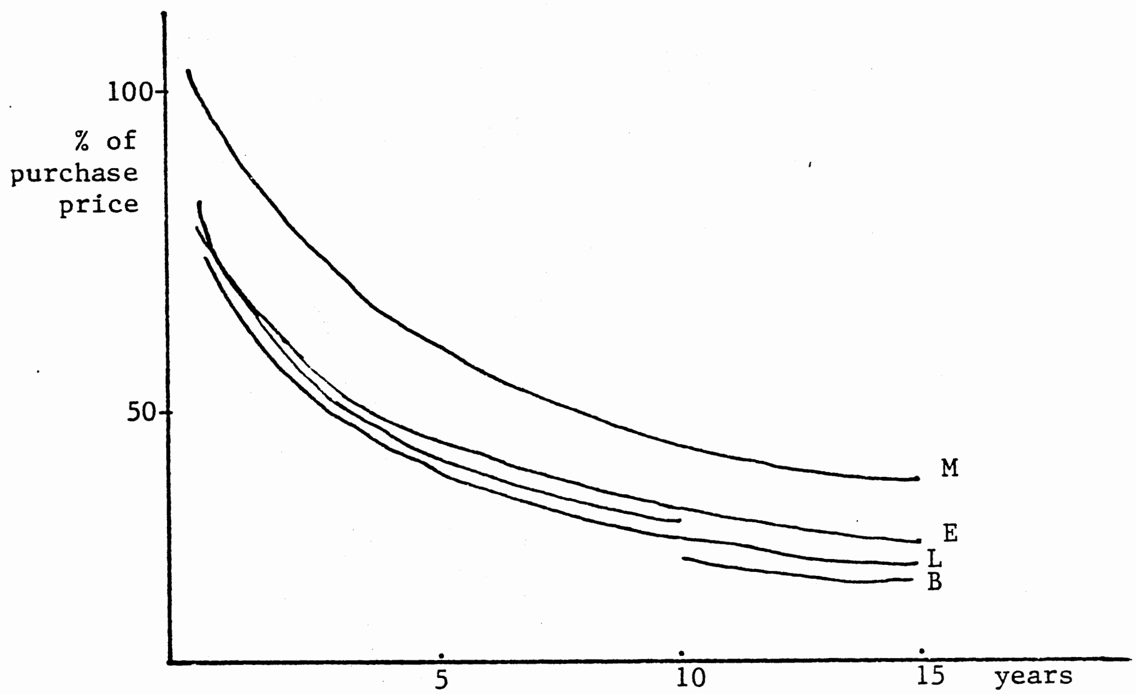

Figure 17. Residual Value Equations for an 179 P.T.O. Tractor Estimated by McNeill (M), Agricultural Engineers (E), Leatham and Baker (L), and Reid and Bradford (B).

should not occur before the tractor is 13 years of age. This was probably due to the equations being generally more asymptotic than the other two.

Implements. In the environment described by Baseline 2 (marginal tax rate at 40 percent, rate of inflation at 10 percent, previous year's average net farm income being $20,000, cost of downtime $40), the optimal replacement periods for the chisel plow and row cultivator where consistently found to be 15 years, while the cycles for the grain drill were shorter with the solution using the 20-year straight-line market value resulting in a more rapid replacement cycle than that with use of the Agricultural Engineer residual value estimate.

The chisel plows and row cultivator replacement periods were unaffected by the changes in tax rate while the grain drill solutions evidenced earlier disposition patterns (Tables 8, 9, and 10). The differences in repair cost equations explain this contrast.

The Scenario 2-c conditions (zero rate of inflation) effected shorter replacement periods with use of either market value estimator for the grain drill and row cultivator. Replacement length of the chisel plow was unchanged.

Tables 8, 9, and 10 identify the optimal replacement ages of a chisel plow, grain drill, and a row cultivator, respectively under some of the scenarios associated with the large tractor. Two market value estimates were used -- a 20-year straight-line schedule and the Agricultural Engineer equation for implements. They are respectively identified as S and E in Figure 18, which implicitly assumes the conditions of Baseline 2. The optimal replacement ages of the chisel

Table 8. Optimal Replacement Solutions Utilizing a Chisel Plow

	BASELINE 2	SCENARIO 2-a	SCENARIO 2-b	SCENARIO 2-c
		ACRS		
Agricultural Engineer	15 (13,005)	15 (15,629)	15 (18,254)	15 (11,989)
20-Year Straight Line	15 (12,719)	15 (15,249)	15 (17,779)	15 (10,425)
		ALTERNATIVE ACRS		
Agricultural Engineer	15 (14,660)	15 (16,457)	15 (18,254)	15 (12,830)
20-Year Straight Line	15 (14,375)	15 (16,077)	15 (17,779)	15 (11,639)
		7-YEAR STRAIGHT LINE		
Agricultural Engineer	15 (13,282)	15 (15,768)	15 (18,254)	15 (12,357)
20-Year Straight Line	15 (12,997)	15 (15,388)	15 (17,779)	15 (11,166)
		10-YEAR STRAIGHT LINE		
Agricultural Engineer	15 (13,758)	15 (16,006)	15 (18,254)	15 (12,597)
20-Year Straight Line	15 (13,473)	15 (15,626)	15 (17,779)	15 (11,406)

Table 9. Optimal Replacement Solutions Utilizing a Grain Drill

	BASELINE 2	SCENARIO 2-a	SCENARIO 2-b	SCENARIO 2-c
		ACRS		
Agricultural Engineer	6 (20,956)	5 (25,994)	5 (30,801)	4 (20,236)
20-Year Straight Line	7 (17,350)	3 (19,620)	1 (19,653)	2 (10,365)
		ALTERNATIVE ACRS		
Agricultural Engineer	7 (22,683)	6 (26,952)	5 (30,801)	4 (21,596)
20-Year Straight Line	5 (19,807)	3 (21,370)	1 (19,653)	3 (12,919)
		7-YEAR STRAIGHT LINE		
Agricultural Engineer	7 (21,336)	6 (26,486)	5 (31,212)	5 (22,014)
20-Year Straight Line	3 (18,197)	3 (20,227)	1 (21,632)	1 (11,438)
		10-YEAR STRAIGHT LINE		
Agricultural Engineer	7 (21,801)	6 (26,753)	5 (31,212)	5 (22,332)
20-Year Straight Line	5 (18,904)	3 (20,730)	1 (21,632)	2 (12,259)

Table 10. Optimal Replacement Solutions Utilizing a Row Cultivator

	BASELINE 2	SCENARIO 2-a	SCENARIO 2-b	SCENARIO 2-c
		ACRS		
Agricultural Engineer	15 (9,480)	15 (11,624)	15 (13,769)	12 (11,447)
20-Year Straight Line	15 (9,310)	15 (11,398)	15 (13,489)	5 (7,272)
		ALTERNATIVE ACRS		
Agricultural Engineer	15 (10,462)	15 (12,115)	15 (13,769)	12 (12,039)
20-Year Straight Line	15 (10,293)	15 (11,890)	15 (13,487)	5 (8,519)
		7-YEAR STRAIGHT LINE		
Agricultural Engineer	15 (9,644)	15 (11,706)	15 (13,769)	12 (11,706)
20-Year Straight Line	15 (9,475)	15 (11,481)	15 (13,487)	5 (8,337)
		10-Year Straight Line		
Agricultural Engineer	15 (9,926)	15 (11,847)	15 (13,769)	12 (11,875)
20-Year Straight Line	15 (9,757)	15 (11,622)	15 (13,487)	5 (8,693)

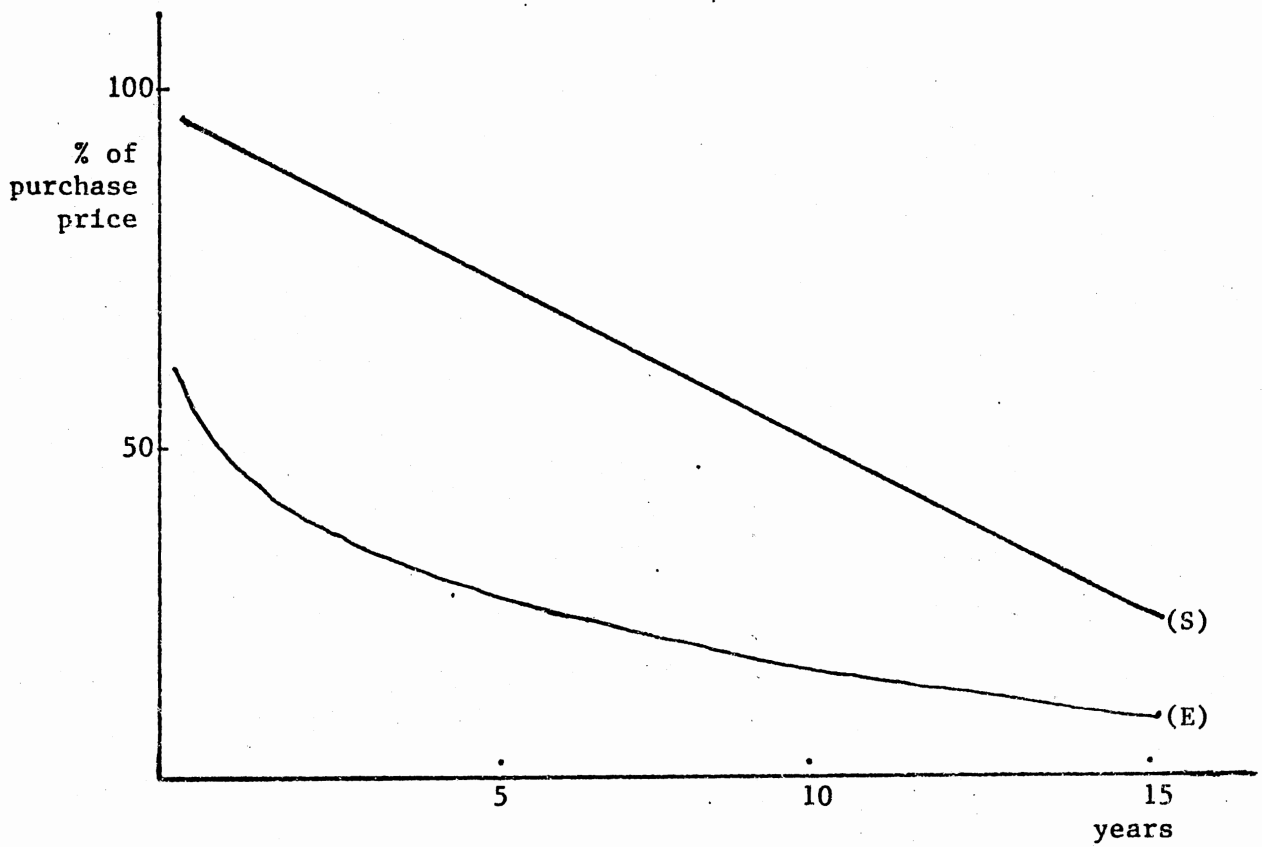

Figure 18. Residual Value Estimates for Implements Using the 20-Year Straight-Line (S) and Agricultural Engineer (E) Equations

plow under Baseline 2 were 15 years, regardless of depreciation method; these results were identical to those of the purchase-replacement decision for the row cultivator. The solutions for the grain drill consistently recommended replacement at considerably earlier ages. The grain drill cost equation is parabolically upturning whereas those equations for chisel plow and row cultivator costs are parabolically downturning. Hence, the shorter replacement pattern associated with the grain drill is due to the explosive nature of maintenance costs relative to those costs of the other implements. The differences in solutions between market values of the grain drill are explained by the excess of the straight-line market value over the Agricultural Engineer estimate in the earlier years. The selling price (as estimated by the 20-year straight-line) is at the level that induces disposition of the drill before the expiration of its elected life.

Effects of Alternative Depreciation Methods. For the 81 P.T.O. tractor, altering the method of depreciation had no effect on optimal replacement pattern. The solutions stabilized at 15 years.

For the larger tractor, the choice of depreciation method was more influential on optimal replacement. The Leatham and Baker solutions differed between the ACRS and the alternative methods only by 1 year under Scenarios 2-c (zero inflation rate) and 2-e (20 percent tax, 5 percent inflation rate). Upon comparing the ACRS approach with the 7-year straight, a trend of shorter replacement is exhibited excepting the solutions under Scenarios 2-c, 2-b (20 percent tax rate), and 2-f(zero tax and inflation rate). Given the assumption of Scenario 2-e replacement actually was deferred considerably. In general, the 10-year straight-line method resulted in longer replacement patterns than those with the 7-year approach, some of which equalled the ACRS patterns.

The Reid and Bradford solutions were invariant to the depreciation method selected. They continually prescribed an 11-year replacement cycle.

The McNeill patterns were found to generally increase when cost recovery was changed from the conventional ACRS to its alternative. Similarly to the Leatham and Baker values, many of the 7-year straight-line replacement ages were less than those under both the ACRS and alternative ACRS schedules. The 10-year straight-line solutions were identical to the 7-year values.

Under most conditions, depreciaton methods ACRS and the alternative ACRS effected equivalent replacement ages for the large tractor having a market value described by the Agricultural Engineer equation. Given the assumptions of 3 Scenarios, the 7-year straight-line solutions were lower than the ACRS patterns, and consideration of the remaining four scenarios induced replacement at the same ages as those of the ACRS method. Conversely, upon comparison of the two straight-line schedules, under 3 scenarios the pattern increased with the latter approach. The other sets of assumptions the optimal ages were unchanged.

It is notable that with the elimination of inflation, most solutions prescribed a longer replacement pattern when the method of cost recovery (pertinent to the large tractor) was varied from ACRS to the alternative.

As the 7-year straight-line method was likely the most common depreciation method employed under the previous tax legislation, the solutions were contrasted with the ACRS solutions for the 179 P.T.O. tractor. This was done to address the question concerning the effectiveness of the new tax legislation.

If one assumes the tractor residual value to be closely approximated by the Leatham and Baker equation, ACRS would effect earlier tractor replacement only under the Scenario 2-c conditions where the marginal tax rate is 20 percent and the rate of inflation is 5 percent. This is also true with the Reid and Bradford estimate. Given the McNeill residual values, ACRS curtails the replacement cycle under the assumptions of Scenarios 2-a (20 percent tax rate), 2-c, 2-e, and 2-f, with the tax rate being the most significant factor. Use of the Agricultural Engineer equation resulted in a longer replacement pattern under 3 Scenarios (the remaining scenario solutions being unchanged).

The solutions for the grain drill (Figure 14) having a residual value described by the Agricultural Engineer equation had shorter patterns with the ACRS method than the Alternatative ACRS, 7 and 10-year straight-line for 2, 3 and 3 scenarios respectively. The 20-year straight-line market value estimate solutions resulted in earlier replacement with use of ACRS than with the alternative ACRS and 10-year straight-line for 2 and 1 scenario respectively. Regardless of scenario or market value estimate, depreciation of either the row cultivator or chisel plow resulted in an ACRS replacement cycle was indistinguishable from the solutions of the other methods of cost recovery.

Effects of Specific Variables

Tax Rate. With exception of the Reid and Bradford equation, as the tax rate was reduced, replacement occurred at an earlier age, ceteris parabus. This result is consistent with a-priori expectations since a reduction in tax benefits increases net costs. Therefore with a parabolically increasing cost function, the replacement pattern will be shorter. The Reid and Bradford market value is somewhat flatter in the neighborhood of the baseline optimal age interacting with the incremental repair costs were probably the significant factors influencing the invariance of the solution. Those implements whose cost functions are upturning also had shorter replacement cycles when tax rate is reduced.

Inflation Rate. At the 40 percent tax rate, removal of the inflation rate typically hastened replacement. Alternatively at the lower rate the purchase-disposition decision was unchanged by inflation rate.

Downtime Cost. The opportunity and cash costs were inconsequential. Reducing the hourly cost of downtime did not effect any replacement cycles though perpetualized costs were lessened slightly.

Summary

The implications of the Accelerated Costs Recovery System and the Tax Equity and Fiscal Responsibility Act for the following issues will be discussed.

Rate of Asset Replacement

It was hypothesized that employment of the ACRS depreciation schedule would hasten the rate of asset replacement relative to the rate under the former depreciation rules. The replacement patterns for the 81 P.T.O. tractor remained unchanged after use of the ACRS approach. The 179 P.T.O. tractor replacement cycles generally lengthened or did not vary when ACRS was introduced. Most of the McNeill replacement ages declined in response to the ACRS statutes, but principally there was no reduction is cyclical length with the other residual solutions.

Among the implements, the solutions resulting from utilizing the Agricultural Engineer market value estimates for the grain drill exhibited a shorter replacement cycle under the conditions of Baseline 2, Scenario 2-a(marginal tax rate reduced to 20 percent), and Scenario 2-c(inflationary situation eliminated).

Given the model and the assumptions in Chapter II and III, ACRS would not effect replacement of a small tractor. Unless the McNeill market value estimated relationship was assumed, the replacement pattern for a larger tractor would either increase or not change under the ACRS statutes vis-a-vis those of the previous tax legislation. The purchase-disposition decision of an implement would be shortened, under recovery only if its yearly maintenance costs increased at an increasing rate with age (i.e. if the function was parabolically upturning).

Post-Tax Net Income

The impact of ACRS on the gross receipts of the farm operator is indeterminant since th analytical tools in this study were not capable of generating any projections of future farm receipts. However, assuming that real gross sales are perpetually constant, the minimization of ownership costs of machinery can lead to the optimization of wealth. In this context the relevant decision rule is to minimize the perpetualized present-value of costs. Disregarding the Reid and Bradford solutions, intertemporal net farm income increased for the operator using either tractor when the ACRS schedules were utilized. This result likewise occurred by using ACRS schedules with the implements.

Size-Related Production Disparaties

The Baseline 1 and 2 scenarios were intended for modelling a small and large farm situation, respectively. Tractor size and use were proxy variables for physical farm size and intensity while the marginal tax rate was a surrogate for income level (assuming that size of the farm operation and net farm income vary positively). The hypothesis suggested that size-related cost differences would be further amplified by the introduction of the recent tax legislation. The results indicate that if the Reid and Bradford residual value equation is not used, the absolute cost differences accruing from the ACRS method are minimal for either size tractor (operation).

CHAPTER V

SUMMARY AND CONCLUSIONS

A General Summary

The Problem

The Accelerated Costs Recovery System (ACRS) implemented under the Economic Tax Recovery Act of 1981 is significantly different from previous tax legislation. The Tax Equity and Fiscal Responsibility Act of 1982 is a revision of the Economic Tax Recovery Act. The most notable feature of the 1982 Act is a reduction in the depreciable basis to 95 percent if an investment tax credit is taken. ACRS has ramifications for financial strategies, investment decisions related to farm assets, the rate of adoption of new technology, and capital-labor mixes. The specific implications of ACRS for the rate of asset replacement, post-tax net farm income, and size-related production disparities were addressed in this study.

The Model

The empirical framework employed was a model that simulated the purchase-replacement decisions for farm tractors and implements chattel over an infinite time horizon. The objective was minimization of a perpetual stream of ownership costs.

Scenarios Examined

Replacement was evaluated under 2 sets of scenarios which included variation of three factors that described the economic and tax situations. There were two sets of scenarios - Baseline 1 with its alternative scenarios and Baseline 2, also having alternative assumptions. The components were marginal income tax rate, rate of inflation, the average net farm income for the previous year, and downtime cost. Baseline 1 assumed a 20 percent tax rate, a 10 percent rate of inflation, $20,000 average net farm income, and $40 downtime cost. Scenarios 1-a through 1-c differed from the Baseline in that the value for one variable was altered. Scenarios 1-a, 1-b, and 1-c were characterized by a zero tax rate, a zero rate of inflation, and $20 downtime cost respectively (the variables not mentioned having the same values as those in the Baseline). Scenarios 1-d and 1-e contained changes in two variables, tax and inflation rates having 10 percent-5 percent and zero percent-zero percent combinations, respectively.

Baseline 2 assumed a 40 percent marginal income tax rate, a 10 percent inflation rate, $20,000 average net farm income for the previous year, and $40 downtime cost. Scenarios 2-a through 2-d contained only one assumption that differed from Baseline 2. In Scenario 2-a the tax rate was reduced to 20 percent, and further reduced to zero percent in Scenario 2-b. Scenario 2-c and a zero inflation rate and 2-d assumed $20 cost of downtime. Scenarios 2-e and 2-f assumed concurrent changes in tax and inflation rates. In Scenario 2-e the tax rate was 20 percent and the inflation rate was 5 percent while in 2-f both were zero percent.

The sets of scenarios were each associated with a tractor of particular size-Baseline 1 with an 81 P.T.O. tractor and Baseline 2 with a 179 P.T.O. machine. A chisel plow, a row cultivator, and a grain drill were considered under the second set of scenarios.

Issues Addressed

Rate of Asset Replacement. Introduction of either the ACRS or alternative ACRS depreciation schedules and associated tax statutes did not effect the replacement decision of the 81 P.T.O. tractor. The ACRS schedule generally lengthened the replacement cycle for the 179 P.T.O. tractor. The purchase-disposition decision occurred earlier under the new tax legislation for implements with a parabolically increasing cost function.

The choice of residual value estimator was an influential agent for the replacement period of the tractors. However, the imposition of the ACRS rules did not result in earlier replacement.

Post-Tax Net Income. The costs associated with acquisition, utilization, and disposition of either tractor were reduced when either the ACRS or alternative ACRS schedules were employed. Therefore, given the assumption of a constant real income stream, after-tax net farm income is increased under the ACRS legislation vis-a-vis the previous tax environment.

Size-Related Production Disparities. Differences in ownership costs between small and large farmers were hypothesized to be further augmented by compliance with the ACRS statutes. The Baselines 1 and 2 modelled a small and large farm situation, respectively (tax rate serving as a proxy for income level and tractor size representing physical size). The relative cost differences with asset consideration under the previous legislation and in the ACRS environment for the smaller and larger operation were negligable.

Future Research Needs

In application of the empirical framework its fundamental limitations were revealed. The following areas need further study so that the rather simple yet useful concept of asset replacement theory may be employed with a greater degree accuracy.

Estimation of Machine and Implement Residual Values

The market value equations used in this study differed significantly from one another and hence the solutions were not compatable. This variation between estimates may instill doubt concerning the integrity of any single estimate (though the Leatham and Baker equation was 'well-behaved' and most statistically sound as it was based on considerably more observations than the other equations). Hence more comprehensive estimates are needed.

Estimation of Cost Equations

The Agricultural Engineer cost equations served adequately the purposes of this study. However, newer cost estimates are needed since consideration must be given to technological change and obsolescence.

Incorporation of Statistical Methods

A statistical procedure is needed to reduce the uncertainty of income, repair and downtime costs, marginal tax rates, etc., making the model more amenable to utilization by an investor or even a policy maker. A discrete systems simulation model might lend itself well to use in optimal replacement theory.

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APPENDIXES

APPENDIX A

PRESENT VALUE, ANNUITIES, AND PERPETUITIES

Present Value

The future value of a_o dollars in one year, compounded at the yearly rate i, $FV = a_o(1+i)$. Therefore a_o equals the present value of FV or $a_o = \frac{FV}{(1+i)}$. If a_o was compounded yearly at interest rate i for 2 years, then $FV = a_o(1+i)(1+i) = a_o(1+i)^2$ (the present value of FV, $a_o = \frac{FV}{(1+i)^2}$). Compounding the initial amount a_o yearly over n years, $FV = a_o(1+i)^n$ or the present value of FV, $PV(FV) = a_o = \frac{FV}{(1+i)^n}$. The present value of a_o received yearly for n years hence equals $a_o + \frac{a_o}{1+i} + \frac{a_o}{(1+i)^n} + \ldots\ldots + \frac{a_o}{(1+i)^n}$ or $PV = \sum_{j=1}^{n} \frac{a_o}{(1+i)^j} = a_o \sum_{j=1}^{n} (1+i)^{-j}$. The real rate of time preference is that rate at which individual is indifferent between receiving a_o dollars (or consuming \$$a_o$ worth of goods) immediately or receiving $a_o(1+i)$ dollars (or consuming an amount equalling \$$a_o(1+i)$) one period in the future (i.e. one year).

Continuous compounding of an initial amount occurs only in theory. Letting a_o again equal that amount, the future value at the end of year n will be $a_o[(1+\frac{i}{k})^k]^n$ where the compounding occurs k times yearly. This can also be expressed as $FV = a_o[(1+\frac{i}{k})^{\frac{k}{i}}]^{in}$ and in the limit, a_o is compounded infinitely in each year thus $FV = \lim_{k\to\infty} a_o[(1+\frac{i}{k})^{\frac{k}{i}}]^{in}$. Letting w equal $\frac{k}{i}$ the equation becomes $FV = \lim_{w\to\infty} a_o[(1+\frac{1}{w})^w]^{in}$, hence $FV = a_o e^{in}$ since $\lim_{w\to\infty} (1+\frac{1}{w})^w = e$ where e is the natural constant 2.71828183.... As a_o is the present value $a_o = FVe^{-in}$.

Annuities

An annuity is constant payment or receipt due or forthcoming respectively every period (typically one month) for n periods (years). the following is a derivation of the amortization factor used for computing a payment schedule for repaying a loan over n years at 100(i) percent yearly interest.

The geometric series $(1+r+r^2+...+R^n)(1+r) = 1-r^{n+1}$ where $r \neq 1$ can be described as $(1+r+r^2+...+r^n) = \frac{1-r^{n+1}}{1-r}$. Letting $r = \frac{1}{(1+i)}$ where i = the interest rate, the series becomes $a_o(1+r+r^2+...+r^n) = a_o(\frac{1-r^{n+1}}{1-r})$. Hence the present value of a stream of constant payments, a_o for j=n periods beginning immediately (j=0) equals $a_o(\frac{1-r^{n+1}}{1-r})$. However, the first payment is frequently not due until the end of the first period (j=1) and thus the present value becomes $a_o(\frac{1-r^{n+1}}{1-r} -1) = a_o(\frac{1-r^{n+1}-1+r}{1-r}) = a_o(\frac{r(1-r^n)}{1-r})$. Dividing all terms by r $PV = a_o\frac{(1-r^n)}{1/r-1} = a_o\frac{(1-(1+i)^{-n})}{1+i-1} = a_o(\frac{1-(1+i)^{-n}}{i})$. The last term is the inverse of the familiar amortization factor. If the present value of the amount of the loan was known, the periodic payment a_o is $PV(\frac{i}{1-(1+i)^{-n}})$ where the amortization factor is multiplied by the amount to be financed (PV).

Alternatively by continuously compounding the interest, the present value is $\int_0^n a_o e^{-it}dt = \frac{-a_o e^{-it}}{i} \Big|_0^n = a_o(\frac{1-e^{-in}}{i})$

Again $a_o = PV(\frac{i}{1-e^{in}})$.

Perpetuities

The present value of a perpetual stream of constant payments is $PV = \lim_{n\to\infty} a_o\left(\frac{1-(1+r)^{-n}}{i}\right) = \frac{a_o}{i}$. Therefore a perpetuity can be computed from the present value as $\frac{a_o}{i} = \frac{PV}{i}\left(\frac{i}{1-(1+i)^{-n}}\right) = PV\left(\frac{1}{1-(1+i)^{-n}}\right)$.

Using continuous terms the present value is as follows; $\int_o^{\infty} a_o e^{-it}dt = \frac{-a_o e^{-it}}{i}\Big|_o^{\infty} = \frac{a_o}{i}$, which is analogous to the discrete approach.

APPENDIX B

LIST OF COMPUTER PROGRAM

Table 11. Illustration of Program in Basic

```
190 CLS
195 '     ASSIGNMENT OF VALUES TO VARIABLES'
200 INPUT "ASSET PRICE";E1
205 INPUT"MARGINAL TAX RATE (%/100)";G6
210 INPUT "LAST YEAR'S AVERAGE NET FARM INCOME ";G1
220 INPUT "ADDITIONAL DEPRECIATION IN DOLLARS"; E2
240 INPUT "RATE OF INFLATION"; E3
250 CLS
260 INPUT "REAL DISCOUNT RATE";E4
280 INPUT "DOWN TIME COSTS PER HOUR"; E5
300 INPUT"ENTER IMPLEMENT CONSIDERED(IF TRACTOR, ENTER 0), (1)CORN PICKER,(2) MO
     LDBOARD PLOW, (3) DISK HARROW, (4) CHISEL PLOW OR FIELD CULTIVATOR,(5) ROW
     PLANTER, (6) GRAIN DRILL, (7) ROW CULTIVATOR, (8) ROTARY HOE";N9
305 INPUT"YEARLY USE(IN HOURS IF TRACTOR, OTHERWISE IN ACRES)";  O1
315 CLS
320 INPUT"HORSEPOWER"; E7
323 INPUT"IF DIESEL-POWERED TRACTOR, ENTER 1";PA
325 INPUT"IF FOUR WHEEL DRIVE TRACTOR, ENTER 1";PB
326 INPUT"OF THE THREE MOST POPULAR TRACTOR MANUFACTURERS, IS YOUR'S THE LEAST E
     XPENSIVE? IF SO, ENTER EXP(1), OTHERWISE ENTER 1";B5
327 CLS
328 INPUT"THE NEXT LEAST EXPENSIVE? IF SO, ENTER EXP(1), OTHERWISE ENTER 1";B4
330 INPUT"IF MANUFACTURER IS ALLIS-CHALMERS, ENTER 1";PC
332 INPUT"IF MANUFACTURER IS INTERNATIONAL HARVESTER, ENTER 1";PH
334 INPUT"IF MANUFACTURER IS JOHN DEERE, ENTER 1";PD
336 INPUT"IF MANUFACTURER IS MASSEY-FURGESON, ENTER 1";PM
337 INPUT"RATE OF INFLATION OF TRACTOR PRICE(%/100)";PI
339 CLS
340 INPUT"INVESTMENT CREDIT"; E8
345 INPUT"INSURANCE COSTS AS A DECIMAL PERCENTAGE OF MARKET VALUE ";B8
350 INPUT"ENTER TYPE OF COST EQUATION DESIRED (0) BATES, RAYNER, AND CUSTANCE (1
     ) AGRICULTURAL ENGINEERS"; B9
360 INPUT "SALVAGE VALUE(APPLICABLE FOR ITEMS PLACED INTO SERVICE BEFORE 1981)";
      E9
410 INPUT "ELECTED LIFE";F3
411 G8=E1
419 INPUT"REPLACEMENT YEAR"; F1
420 O2=F1
425 IF F1>=(F3 + 1) THEN GOTO  440
430 DIM DEP(F3+1), DDP(F3 + 1), F6(F3 + 1), CST(F1), DCS(F1), RVV(F1), DRV(F1)
     :GOTO 600
440 DIM DEP(F1), DDP(F1), F6(F1), CST(F1), DCS(F1), RVV(F1), DRV(F1)
600 CLS:PRINT E1,G6,G1,E2,E3,E4,E5,E7,E8,E9,F1,F2,F3
700 PRINT:INPUT"METHOD OF DEPRECIATION DESIRED (1) STRAIGHT-LINE,NEW METHOD(2)ST
     RAIGHT-LINE, OLD METHOD (3) 5-YEAR ACRS (4) 3-YEAR ACRS (5) 3-YEARSTRAIGHT-
     LINE, NEW METHOD (6) 5-YEAR STRAIGHT-LINE, NEW METHOD "; D1
702 LPRINT"          Leatham    Agri                 Reid &"
703 LPRINT"Year      & Baker    Engineer   McNeil    Bradford"
710 FOR PP=O2 TO 1 STEP -1
```

Table 11. (Continued)

```
711 F1=PP
714 LPRINTF1;
716 IF N9>0 THEN GOTO 718
717 FOR B7=1 TO 4 STEP 1:GOTO 720
718 FOR S7=1 TO 2
719 '     COMPUTATION OF YEARLY DEPRECIATION'
720 ON D1 GOSUB 820, 920, 1020, 1205, 1255, 1300
799 GOTO 1560
820 FOR I=1 TO F3+1
840 IF I=1  THEN DEP(I)=(E1-((E2/E1)*.5*G8)-((E8/E1)*.5*G8))*(1/(2*F3)): ELSE GO
    TO 860
845 GOTO 885
860 IF I=(F3+1) THEN DEP(I)=(E1-((E2/E1)*.5*G8)-((E8/E1)*.5*G8))*(1/(2*F3))ELSE
    GOTO 880
865 GOTO 885
880 DEP(I)=(E1-((E2/E1)*.5*G8)-((E8/E1)*.5*G8))*(1/F3):
885 NEXT
890 FOR I=1 TO F3 + 1 :NEXT
899 RETURN
920 FOR I=1 TO F3
940 DEP(I)=(E1-((E2/E1)*G8)-((E9/E1)*G8))*(1/F3)
960  NEXT
999 RETURN
1020 IF F3= 3 THEN GOTO 1200
1040 FOR I=1 TO F3
1060 IF I=1 THEN DEP(I)= .15*(E1-((E2/E1)*G8)-((E8/E1)*.5*G8)): GOTO 1120
1080 IF I=2 THEN  DEP(I)= .22*(E1-((E2/E1)*G8)-((E8/E1)*.5*G8)): GOTO 1120
1100 DEP(I)= .21*(E1-((E2/E1)*G8)-((E8/E1)*.5*G8))
1120 NEXT
1140 RETURN
1200 STOP
1205 FOR I=1 TO F3
1210 IF I=1 THEN DEP(I)=.25*(G8-((E2/E1)*G8)-((E8/E1)*.5*G8)): GOTO 1225
1215 IF I=2 THEN DEP(I)=.38*(G8-((E2/E1)*G8)-((E8/E1)*.5*G8)): GOTO 1225
1220 DEP(I)=.37*(G8-((E2/E1)*G8)-((E8/E1)*.5*G8)): GOTO 1225
1225 NEXT
1230 RETURN
1255 FOR I=1 TO F3+1
1260 IF I=1 THEN DEP(I)=(G8-((E2/E1)*G8)-((E8/E1)*.5*G8))/(2*F3):
GOTO 1275
1265 IF I=(F3+1) THEN DEP(I)=(G8-((E2/E1)*G8)-((E8/E1)*.5*G8))/(2*F3): GOTO 1275
1270 DEP(I)=(G8-((E2/E1)*G8)-((E8/E1)*.5*G8))/(F3)
1275 NEXT
1280 RETURN
1300 FOR I=1 TO F3+1
1305 IF I=1 THEN DEP(I)=(G8-((E2/E1)*G8)-((E8/E1)*.5*G8))/(2*F3):GOTO 1320
1310 IF I=(F3+1) THEN DEP(I)=(G8-((E2/E1)*G8)-((E8/E1)*.5*G8))/(2*F3): GOTO 1320
1315 DEP(I)=(G8-((E2/E1)*G8)-((E8/E1)*.5*G8))/(F3)
1320 NEXT
1325 RETURN
```

Table 11. (Continued)

```
1510 IF D1=1 THEN GOTO 1560
1514 IF D1>4 THEN GOTO 1560
1520 FOR J=1 TO F3:GOTO 1562
1560 FOR J=1 TO F3+1
1562 M4=0
1563 G2=G2 + 1:F5=(R1*F1) + G2
1585 IF D1=1 THEN GOTO 1592
1586 IF D1>4 THEN GOTO 1592
1590 IF J>F3 THEN DEP(J)=0
1591 GOTO 1595
1592 IF J>F3+1 THEN DEP(J)=0
1593 '    DISCOUNTING DEPRECIATION'
1595 DDP(J)=DEP(J)/((1+E3)*(1+E4))^F5
1600 NEXT
1620 GOTO 1800 : NEXT
1800 FOR K=1 TO G2
1805 M4=0
1820 G4=DEP(K)+G4: G3=DDP(K)+G3
1840 NEXT:
1850 FOR J1=1 TO F1
1860 G7=J1
1880 IF N9<>0 THEN GOTO 1900
1890 IF F1<12 THEN T=1 ELSE T=EXP(1):GOTO 1922
1895 '    RESIDUAL VALUE CALCULATION'
1900 IF S7=2 THEN GOTO 1906
1902 '    IMPLEMENT RESIDUAL VALUE-ESTIMATION USING THE AGRICULTURAL ENGINEER
              EQUATION FOLLOWED BY THAT USING THE 20-YEAR STRAIGHT-LINE ESTIMA
     TE;          THE VARIABLE S7 IS THE COUNTER'
1904 RVV(J1)=E1*(.6)*(.855^G7):GOTO 1910
1906 RVV(J1)=E1*(1-(.05*G7))
1910 ON N9 GOTO 1941, 1942, 1943, 1944, 1945, 1946, 1947, 1948
1915 '    TRACTOR RESIDUAL VALUE ESTIMATION USING THE LEATHAM AND BAKER, AGRI-
              CULTURAL ENGINEER, MCNEILL, AND REID AND BAKER EQUATIONS; B7 IS
     THE          COUNTER'
1922 IF B7>1 THEN GOTO 1925
1923 RVV(J1)=(E1)*(((E7)^-.0543)*((1.0876)^(PA))*((.9930)^(PB))*((.7282)^(PC))*(
     (.7582)^(PH))*((.7534)^(PD))*((.7414)^(PM))*((.9942)^((PA)*(J1)))*((.9933)^
     (PB*J1))*((.8948)^(PC*J1))*((.8968)^(PH*J1))*((.9171)^(PD*J1))*((.9001)^(PM
     M*J1))*((1+PI)^G7))
1924 GOTO 1938
1925 IF B7>2 THEN GOTO 1930
1927 RVV(J1)=(.68)*((.92)^(J1))*(((1+PI)^(G7)))*(E1)
1928 GOTO 1938
1930 IF B7>3 THEN GOTO 1935
1931 P3=4-(.4*(J1)):IF P3<0 THEN P3=0
1932 RVV(J1)=(E1)*(EXP(-.4299-(.0436*(J1))+(.0691*(P3))))*((1+PI)^(2*(G7)))
1933 GOTO 1938
1935 IF J1>11 THEN T1=EXP(1) ELSE T1=1:IF J1>19 THEN T2=EXP(1) ELSE T2=1
1936 RVV(J1)=(E1)*(((1+PI)^G7)*(3.687)*((E7)^.242)/(((J1)^.273)*((G1)^.305)*((T1
     )^.621)*((T2)^.205)*((B4)^.121)*((B5)^.263)))
```

Table 11. (Continued)

```
1937 '    TRACTOR COST ESTIMATION USING THE BATES, RAYNER, AND CUSTANCE COST
            EQUATION OR THE AGRICULTURAL ENGINEER COST EQUATION, DEPENDING O
     N          THE VALUE OF B9(0 OR 1)'
1938 IF B9=0 THEN GOTO 1940
1939 CST(J1)=((G8)*(.024396*((O1/1000)^2.033)*((J1)^1.033)*((1+E3)^G7))+((.00000
     4584*((O1/1000)^1.4173))*((J1)^.4173)*(E5*((1+E3)^G7))*G8)+(RVV(J1)*B8)):GO
     TO 1950
1940 CST(J1)=((G8*(.0309474*((J1)^.5))*((1+E3)^G7)) + ((.0004584*((O1/1000)^1.41
     73)*((J1)^.4173))*(E5*((1+E3)^G7))*G8) + (RVV(J1)*B8)): GOTO 1950
1941 CST(J1)=(E1)*(.192536*((O1/1000)^2.348)*((J1)^1.348)):GOTO 1950
1942 CST(J1)=(E1)*(.1267*((O1/1000)^1.81)*((J1)^.81)):GOTO 1950
1943 CST(J1)=(E1)*(.004285*((O1/1000)^1.714)*((J1)^.714)):GOTO 1950
1944 CST(J1)=(E1)*(.01442*((O1/1000)^1.4)*((J1)^.4)):GOTO 1950
1945 CST(J1)=(E1)*(.028636*((O1/1000)^2.137)*((J1)^1.137)):GOTO 1950
1946 CST(J1)=(E1)*(.094273*((O1/1000)^2.626)*((J1)^1.626)):GOTO 1950
1947 CST(J1)=(E1)*(.020746*((O1/1000)^2.207)*((J1)^1.207)):GOTO 1950
1948 CST(J1)=(E1)*(.0049*((O1/1000)^1.369)*((J1)^.369)):
1949 '    DISCOUNTED COSTS AND RESIDUAL VALUES'
1950 DCS(J1)=(CST(J1)/(((1+E4)*(1+E3))^G7))
1955 DRV(J1)=(RVV(J1)/(((1+E4)*(1+E3))^G7))
1956 '    ADDITION OF DISCOUNTED MAINTANANCE COSTS'
1962 M3=RVV(J1)
1965  NEXT
2050 H1= (R1*F1)+1
2060 DA=(((E2/E1)*G8)/(((1+E4)*(1+E3))^H1))
2062 DI=(((E8/E1)*G8)/(((1+E4)*(1+E3))^H1))
2070 J1=J1 - 1
2071 H2=R1 + 1
2080 '    COMPUTATION AND DISCOUNTING RECAPTURE OF INVESTMENT TAX CREDIT'
2100 IF D1<>2 THEN GOTO 2199
2110 IF J1>=3 THEN GOTO 2120
2115 H4=(.1*G8)/(((1+E4)*(1+E3))^(H2*F1)): GOTO 2280
2120 IFJ1>=5 THEN GOTO 2130
2125 H4=(.067*G8)/(((1+E4)*(1+E3))^(H2*F1)): GOTO 2280
2130 IFJ1>=7 THEN   GOTO 2140
2135 H4=(.033*G8)/(((1+E4)*(1+E3))^(H2*F1)): GOTO 2280
2140 H4=0: GOTO 2280
2199 IF D1>3 THEN GOTO 2210
2200 H4=((.1-(.02*F1))*G8)/(((1+E4)*(1+E3))^((H2*F1) )):GOTO
2280
2210 H4=((.06-(.02*F1))*G8/(((1+E3)*(1+E4))^(H2*F1)))
2280 IF H4<0 THEN H4=0
2289 G9=E1-E9
2290 '    PARTITIONING AND DISCOUNTING THE DISPOSITION RECEIPTS TAXABLE AS
            ORDINARY INCOME AND CAPITAL GAINS'
2292 IF D1=2 THEN GOTO 2300
2294 IF M3>G4+.5*E8 THEN GOTO 2297
2295 M1=M3/(((1+E4)*(1+E3))^G7):GOTO 2351
2297 M1=(G4+.5*E8)/(((1+E4)*(1+E3))^G7)
2298 M2=(M3-(G4+.5*E8))/(((1+E4)*(1+E3))^G7):GOTO 2351
```

Table 11. (Continued)

```
2300 IF M3>(G4) THEN GOTO 2320
2310 M1=M3/(((1+E4)*(1+E3))^G7)
2315 M2=0: GOTO 2351
2320 M1=((G4)/(((1+E4)*(1+E3))^G7))
2325 M2=(M3-G4)/(((1+E4)*(1+E3))^G7)
2351 IF D1=2 THEN GOTO 2360
2353 '     CALCULATION OF TOTAL PRESENT VALUE COSTS'
2355 N1=(E1-DRV(J1)+ (1-G6)*(SD) - (G6*DA) - (G6*G3)-(DI)        + (H4) + (G6*M1) + (G
     6*(.4)*M2)) : GOTO 2375
2360 N1=(E1-DRV(J1)+(1-G6)*(SD)-(G6*DA)-(G6*G3)-(DI)
+(H4)+(G6*M1)+(G6*(.5)*M2)
     )
2370 '     CONVERSION OF PRESENT VALUE INTO A PERPETUAL STREAM OF COSTS'
2375 N2=((N1/(1-(1/(((1+E4))^(G7)))))*(E4))
2378 N3=((N1/(1-(1/(((1+E4))^(G7)))))):IF D1=2 THEN O7=.5 ELSE O7=.4:IF M1=0 THE
     N M1=1:IF M2=0 THEN M2=1
2379 IF B7=1 THEN LPRINTTAB(10);N3;
2380 IF S7<>0 THEN GOTO 2390
2382 IF B7=2 THEN LPRINTTAB(20)N3; ELSE  GOTO 2384:GOTO 2400
2384 IF B7=3 THEN LPRINTTAB(30)N3;  ELSE GOTO 2385
2385 IF B7=4 THEN LPRINT TAB(40);N3:GOTO 2400
2387 GOTO 2400
2390 IF S7=1 THEN LPRINT F1,"BUDGET GENERATOR MARKET VALUE", N3 ,ELSE GOTO 2395
2392 GOTO 2400
2395 LPRINT"
          20-YEAR STRAIGHT-LINE MARKET VALUE",N3
2400 G2=0: F5=0: G4=0: G3=0: SD=0:J1=0:K=0:G7=0
2414 NEXT
2415 S7=0
2416 B7=0
2420 NEXT
2430 '     PRINTOUT OF VALUES ASCRIBED TO THE VARIABLES'
2440 LPRINT  "PRICE",E1,  "INCME",G1,  "TXRT",G6  "ADDEP",E2,  "INRTE",E3,  "DSR
     TE",E4,  "DMCST",E5,  "IMPMNT",N9,  "YRLYUSE",O1, "HRSPWR",E7,  "DSL",PA,
     "FWD",PB, "LSTEXPEN",B5,  "NXTLST",B4,  "AC",PC,  "TH",PH,  "JD",PD,  "MF",
     ,PM, "RTRACTORINFL",PI
2450 LPRINT  "INCRDT",E8,  "INSURCST",B8,  "CSTEQUTN",B9,  "SLVGVLUE",E9,  "ELCT
     DLF",F3,  "RPMTYR",F1,  "DEPMTHD",D1
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2

VITA

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