MONTE CARLO SIMULATIONS OF
THE ANALYSIS OF VARIANCE
FOR DISCRETE DATA

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CHAPTER I

INTRODUCTION

Although probability is a relatively new discipline, counting problems associated with games of chance have been studied for much longer. The binomial expansion and the binomial coefficients have been discovered and rediscovered many times. According to Folks (20), in 1303 the Chinese writer Chu Shih-chieh published the arithmetical triangle of binomial coefficients and described it as an ancient method. Pascal rediscovered the triangle 351 years later, and it is now commonly known as Pascal's triangle. In addition to giving a triangular array of coefficients in his 1713 *Ars Conjectandi*, Bernoulli used the binomial expansion to solve probability problems. Thus, the binomial probability distribution is often attributed to Bernoulli. The binomial distribution arises when n independent trials are performed with two possible outcomes on each trial. If the probability is p that an outcome will occur on a trial and the probability is $q = 1 - p$ of its not occurring, then the probability of x such outcomes in the n trials is

$$P\ (X = x) = \binom{n}{x}\ p^x\ q^{n-x} \qquad x = 0,1,2,\ldots,n$$

where $p > 0$, $q > 0$, and $p + q = 1$. The binomial distrition is widely used. Extensive applications are found in genetics, acceptance sampling and reliability theory.

De Moivre was the first to publish the Poisson distribution as a limiting form of the binomial in the first edition of The Doctrine of Chance in 1718. Folks (20) states that Poisson's exponential limit of the binomial was presented in 1837 and has since been known as the Poisson distribution even though his work was predated by that of De Moivre. In both cases, the Poisson distribution was derived from the binomial distribution by allowing n to approach ∞ and p to approach 0, while requiring np to remain constant, i.e.,

$$P(X = x) = \lim_{\substack{n \to \infty \\ p \to 0}} \binom{n}{x} p^x q^{n-x} = \frac{\lambda^x e^{-\lambda}}{x!} \quad x = 0,1,2,\ldots$$

where $\lambda = np > 0$.

One of the classic examples of the use of the Poisson distribution was the number of cavalrymen killed from the kick of a horse which was presented by Bortkiewicz (8) in 1898. Bortkiewicz argued that a cavalryman will either be killed or not be killed by a horsekick during a year's time.

Furthermore, if it can be supposed that the chance of this rare event is the same for all cavalrymen, and that cavalrymen have independent chances of being killed; then, the number of cavalrymen killed during a year by a horsekick is a binomial variable. The probability of being killed is very small however, and the number of cavalrymen is very large. The Poisson limit of the binomial should therefore give a good description of the number of cavalrymen killed by a horsekick in a year. These data, collected from the records of the Prussian Army Corps, fitted the Poisson distribution. This classic example is not very practical.

More realistic uses of the Poisson distribution include the number of cells in an area when counting with a hemacytometer, the number of radioactive particles emitted in a given time period, the number of telephone calls received in a given time period, and the number of equipment failures in a given time period (33).

Another distribution derived from the binomial is the negative binomial. Although discussed by Pascal and Fermat, the negative binomial distribution was first formulated by Montmort (39) in 1714.

One of the earliest uses of the negative binomial distribution was by Student (43) in 1907. He studied the distribution of yeast cells counted with a hemocytometer. He assumed that if the liquid in which the cells were suspended was properly mixed, each cell had an equal chance of falling on any unit area of the hemocytometer. Thus he expected the binomial distribution. Student estimated p, q and n from the first and second sample moments, but in two of his four series he obtained negative estimates of p and n. He concluded that the negative estimates may have occurred because of a tendency of the yeast cells to stick together even though the liquid was vigorously shaken.

Throughout the early 1900's, several other cases of negative estimates of the binomial parameters were reported. Whitaker (44) investigated the claim that for small p and large n the variability of the estimators would cause some negative estimates to be obtained. In addition to Student's 1907 work, she reviewed studies of Mortara (40) who dealt with deaths due to chronic alcoholism, and Bortkiewicz, who studied suicides of children in Prussia and of women in German states, accidental deaths in trade societies, and as mentioned earlier, the deaths in the Prussian army corps from the kicks of horses. She found that it was highly unlikely that all negative estimates of p and n could be explained by variability alone. This led her to a new interpretation of the negative binomial

$$P(X = x) = \binom{n + x - 1}{n - 1} p^x q^{-(x+n)} \quad , \; x = 0,1,2,\ldots.$$

The negative binomial distribution was also studied by Greenwood and Yates in 1920, Eggenberger and Polya in 1923, and Anscombe in the 1940's (46). Willson (46) gives Anscombe's reparameterization of the distribution which uses $\mu = np$ and $k = -n$. A random variable X is then distributed as a negative binomial random variable if the probability mass function is given by

$$P(X = x) = \binom{k + x - 1}{k - 1} \left(\frac{\mu}{\mu + k}\right)^x \left(\frac{k}{\mu + k}\right)^k \quad , \; x = 0,1,2,\ldots.$$

The Poisson distribution is also a limiting form of the negative binomial distribution, arising when $k \to \infty$.

The negative binomial distribution is used in the fields accident statistics, population counts, psychological data and communications.

The logarithmic series distribution is new compared to the three previous distributions mentioned. This distribution was first used in a paper by Fisher, Corbet and Williams (19) in 1942. Here again the distribution was first applied in biological settings: the results of sampling butterflies (Corbet's data) and the collection of moths using light traps (Williams' data). If the number of species represented by only one individual in the sample is n_1, then the number of species represented by $2,3,\ldots,k,\ldots$ individuals is approximately

$$\left(\frac{n_1}{\theta}\right)\left(\frac{\theta^2}{2}\right),\ \left(\frac{n_1}{\theta}\right)\left(\frac{\theta^3}{3}\right),\ldots,\ \left(\frac{n_1}{\theta}\right)\left(\frac{\theta^k}{k}\right),\ldots$$

respectively, where θ is a positive number less than 1. The total number of species is approximately

$$S = \sum_{k=1}^{\infty}\left(\frac{n_1}{\theta}\right)\left(\frac{\theta^k}{k}\right) = -\left(\frac{n_1}{\theta}\right)\ln(1-\theta)$$

and the total number of individuals collected is approximately

$$M = \sum_{k=1}^{\infty}\left(\frac{n_1}{\theta}\right)\theta^k = \frac{n_1}{1-\theta}$$

Then the random variable X which equals the number of species observed has the following logarithmic distribution

$$P(X = x) = \left(\frac{1}{S}\right)\left(\frac{n_1}{\theta}\right)\left(\frac{\theta^x}{x}\right) = \frac{-\theta^x}{x\ln(1-\theta)},\quad x = 1,2,\ldots$$

where $0 < \theta < 1$.

The logarithmic distribution has been widely used in biological studies, including studies of the number of different plant species found on quadrats of various sizes, the number of mosquitos caught in light traps in several locations, and the number of species observed in an area (33). It has also been applied in economics as the distribution of numbers of items of a product purchased by a buyer in a specified period of time (9). These four discrete distributions were developed in biological situations along with many other statistical methods. One of the most widely used statistical methods ever developed is the analysis of variance published in 1923 by R. A. Fisher (20). This method, like most early statistical procedures, was explicitly developed to analyze crop yield data.

The analysis of variance may imply by its name that it is the study of variance. Actually, it is the partitioning of the total variation in the data into specific sources of variation. This procedure arranges and presents the results of an experiment in a single compact table. It not only shows the structure of the experiment, but also the relevant results in such a way that the necessary tests of significance are clear. For example, the analysis of variance for the randomized block design, which was developed by Fisher in the early 1930's, partitions the total variation in the data into the variation due to blocks, treatments, and experimental error.

Although the analysis of variance is useful in analyzing data after it has been collected, the interpretation of results is governed by the manner in which the experiment was conducted. The main interest in the methods used to conduct the experiment arises from the need for valid estimates of error. If an experiment allows computation of a valid estimate of error, its structure completely determines the statistical procedure by which this estimate is to be calculated (18).

The analysis of variance procedure, as presented by Fisher, assumes that the experimental error is normally distributed. If this assumption is not true, then any inferences made from tests of significance in the analysis of variance may not be valid (18). With this in mind, many transformations have been suggested for use on data that is known to be from a distribution other than the normal. One of the earliest such transformations was suggested by Fisher to stabilize the variance of a binomial variate. Since then, literally hundreds of transformations have been developed for use on data which are known to have come from a discrete distribution.

The purpose of this paper is to compare three methods for analyzing discrete data using the analysis of variance procedure. The first two methods have been mentioned previously: analysis of the raw data and analysis of a standard transformation of the data. The third method will analyze the ranks of the discrete data. The three methods will be compared by looking at the power and robustness of each when applied to discrete data. Many factors influence data collected in an experiment; therefore, simulated data must try to include as many of these factors as possible. Only one design, the randomized complete block, is studied and the generated data are from four discrete distributions: binomial, Poisson, negative binomial, and logarithmic. Many different circumstances are studied by varying the number of treatments, the number of blocks, and the number of observations per plot as well as the cell means.

Chapter II gives a brief review of some of the literature on the underlying assumptions of the analysis of variance, the effects of not meeting these assumptions, and various transformations that have been proposed. In Chapter III, the method of generating and collecting the data will be discussed. The methods of analyzing the results are described in Chapter IV. Some of the problems encountered in this study along with possibilities for further research are discussed in Chapter V.

CHAPTER II

PREVIOUS RESEARCH ON THE ANALYSIS OF VARIANCE AND DATA TRANSFORMATIONS

Assumptions of the Analysis of Variance

As mentioned in the first chapter, the analysis of variance was developed by Fisher in the context of biological data analysis. Cochran (10) presented four assumptions that he felt must be met in order for any inferences made from an analysis of variance to be valid. These assumptions are:

1. The treatment effects and the environmental effects must be additive. The treatment effects are the effects of procedures deliberately introduced by the experimenter. The environmental effects are the effects of features of the environment. It is further assumed that the experimental errors, which are all elements of variation that are not accounted for by treatment or environmental effects, have a mean of zero.

2. The experimental errors must all be independent.

3. The experimental errors must have a common variance.

4. The experimental errors should be normally distributed (p. 23).

Investigations Into the Effects of Violations of These Assumptions

Most experimental data never meet all of these assumptions, but the tests of significance that are usually performed as a result of the analysis of variance are assumed to be robust enough to permit these departures.

Violation of Normality Assumption

Investigations into the effects of the experimental data not meeting one or more of these assumptions have mainly been conducted with respect to continous data. Most studies have attempted to determine what effect, if any, non-normal data have on the validity of the inferences and tests of significance. The conclusion of these investigations is that no serious error in the significance levels of the F-test or of the two-tailed t-test is introduced by non-normality.

Pearson (42), studying continuous non-normal distributions, indicates that F-tests from an analysis of variance have approximately the stated Type I error rate, but in extreme cases of non-normal variation the probability of a Type II error is high because of the lack of a more efficient test. In another study, Eden and Yates (14) concluded that Fisher's z-test can be used on data from a skewed continuous distribution. They state that the test behaves similarly to the z-test based on data from a normal distribution. Bartlett (2) discusses the effects of moderate departures from normality on the t-test using theoretical results. He stressed that although the results are incomplete they tended to agree with the results of the two previous investigations in showing that for moderate departures from normality the t-test can still be used with confidence.

A study by Kanji (34) indicates that for inferences concerning means, the power calculated under normal theory is only slightly affected by the non-normality of the errors. Cochran (10) states that non-normality is likely to be accompanied by a loss of efficiency in the estimation of treatment effects and a corresponding loss of power in the F- and t-tests, but the loss of efficiency is not often great. These investigations seem to indicate that for minor deviations from normality data from non-normal continuous distributions can still be analyzed as if the data were from a normal distribution.

Violation of Other Assumptions

Cochran also discusses the effects of not having a common error variance on estimates and tests of significance. When the true error variance differs from one observation to another there will be a loss of efficiency in estimates of treatment effects, and a loss of sensitivity in tests of significance. Nevertheless, the F-test for equality of treatment means is probably the least affected by the heterogeneity of the error variances. Also, correlations among the errors can cause substantial biases in t-tests.

Transformations of Discrete Data

Discrete data obviously do not meet the last three assumptions. The usual method of handling discrete data is to transform the data in order to make the analysis more valid. The transformed variate will, hopefully, have a variance that is constant from observation to observation as well as be asymptotically normally distributed. Many transformations have been developed, but only a few are used widely. It is these common ones that this paper will investigate.

Transformations of Data from the Binomial Distribution

One of the first transformations developed for use on binomial data was $y = \sin^{-1}(x)$ suggested by Fisher (16). This transformation was later dropped in favor of the more appropriate angular transformations.

Given a sample of size n, Bartlett (4) proposed the transformation, $y = \sin^{-1}\sqrt{x}$, which has an approximately constant variance of $\frac{821}{n}$ on the new scale provided that the inverse sine is measured in degrees. The variance is $\frac{1}{4n}$ when measured in radians. Anscombe (1) said that the transformation

$$y = \sin^{-1}\sqrt{\frac{x + \frac{3}{8}}{n + \frac{3}{4}}}$$

has a variance approximately $\frac{1}{4\,(n + 1/2)}$ when the transformation is performed in radians. He also presented a modification of this transformation

$$y = \sqrt{n + \frac{1}{2}} \quad \sin^{-1}\sqrt{\frac{x + \frac{3}{8}}{n + \frac{3}{4}}}$$

Freeman and Tukey (22) suggested an averaged angular transformation of

$$y = \sin^{-1}\sqrt{\frac{x}{n + 1}} \; + \; \sin^{-1}\sqrt{\frac{x + 1}{n + 1}}$$

Since problems could be encountered if $x = 0$ or $x = n$, Bartlett (4) and Eisenhart et al. (15) and Ghurye (24) suggested using

$$y = \begin{cases} \sin^{-1}\sqrt{\frac{1}{4n}} & \text{when } x = 0 \\ 90^{\circ} - \sin^{-1}\sqrt{\frac{1}{4n}} & \text{when } x = n. \end{cases}$$

In this work, we shall use Bartlett's transformation, $y = \sin^{-1}\sqrt{x}$, since it is the most widely used.

<u>Transformations of Data from the Poisson Distribution</u>

Since the Poisson distribution has variance equal to the mean, the square root scale is generally used to stabilize the variance. Bartlett (3) showed that if a Poisson random variable X with mean λ is transformed by $y = \sqrt{x}$, then Y is distributed more nearly normally than X. Later Bartlett (4) recommended the transformation $y = \sqrt{x + 1/2}$, while Anscombe (1) believed that $y = \sqrt{x + 3/8}$ was optimum for practical purposes. Freeman and Tukey (22) presented the transformation $y = \sqrt{x} + \sqrt{x + 1}$ which they believed more nearly stabilizes the variance of the transformed variates. Hoyle (27) mentions the work of Kihlberg, Herson and Scholtz, who used computer studies to show that $y = \sqrt{x + .386}$ is the optimal transformation for data from a Poisson distribution. The transformation $y = \sqrt{x + 3/8}$ will be used in this study.

Transformations of Data from the Negative Binomial Distribution

In the negative binomial distribution the variance is greater than the mean and hyperbolic functions are usually used to stabilize the variance of the transformed variate Y. Beall (5) suggested $y = \sinh^{-1} \sqrt{\frac{x}{k}}$ while Anscombe (1) recommended

$$y = \sinh^{-1}\sqrt{\frac{x + \frac{3}{8}}{k - \frac{3}{4}}}$$

as a good transformation. Anscombe also mentioned

$$y = \sqrt{k - \frac{1}{2}} \ \sinh^{-1}\sqrt{\frac{x + \frac{3}{8}}{k - \frac{3}{4}}}$$

as an even better transformation. He also gives a much simpler transformation that is in more common usage and the one used in this paper, $y = \ln(x + .5\,k)$.

The Rank Transformation

Another way to analyze a set of data is to analyze the ranks of the data. The ranks instead of the true values were used by Hotelling and Pabst (26) in the calculation of a correlation coefficient. They stated that the greatest asset of this method was that no assumption of normality was needed. Friedman (23) developed a widely used ranking procedure in which the data in each row of a two-way table was ranked, then the column means were tested as to whether they came from the same population.

Using ranks in the analysis of variance was first employed by Wilcoxon (45) to obtain a rapid approximation of the significance of the differences in means. Kruskal and Wallis (37) also used ranks in detecting differences among population means but used the H-statistic instead of the analysis of variance. Kruskal (36) previously had developed a use for the H-statistic in a one-way analysis of variance. Hodges and Lehman (25) provided a method for constructing rank tests for two-way analysis using an alignment of the observations in each block by removing treatment effects before ranking the observations. Mehra and Sen (38) extended Hodges' and Lehmann's work by removing both block and treatment effects and then tested for interaction. The techniques discussed so far have a theoretical basis behind them, but a technique developed by Iman (28) and later extended by both Iman and Conover (29), (11), (30), (31), (12), (13) used what they call the rank transform of the data in the usual analysis of variance. They stated that although no theory supports the method at this time, it still produces impressive results in simulation. Through simulation of data from several continuous distributions, the power and robustness of this transformation is demonstrated in various experimental designs.

A great advantage of this transformation is its ease of use. Basically all the transformation does is rank in ascending order all the data, regardless of any classification or level (block, subtreatment, etc.).

Example: For a randomized complete block design with three blocks and five treatments the data would be transformed as follows:

		Block 1	Block 2	Block 3
Treatment	1	3.5	3.9	3.1
	2	7.2	2.7	1.3
	3	.1	.6	.9
	4	8.1	5.4	.5
	5	6.9	3.7	4.2

becomes,

		Block 1	Block 2	Block 3
Treatment	1	8	10	7
	2	14	6	5
	3	1	3	4
	4	15	12	2
	5	13	9	11

Since most package programs have a ranking procedure this transformation can be used with only a few extra programming steps.

Iman (28) simulated data from three continuous distributions in a two-way model with interaction: normal, contaminated normal and exponential. He states that the null distribution of the F-statistic computed on ranks behaves similarly to the F-statistic computed on raw data. The power of the F-statistic computed on ranks when the data is not normal can also increase considerably over the power of the F-statistic computed on raw data. When the data is normally distributed, the loss of power for the rank transform method is little, if any.

In an extensive simulation study, Iman and Conover (29) show the rank transform method to be preferable to the usual analysis of variance procedures, except where the usual analysis of variance assumptions are met. When the data is from contaminated normal, lognormal or exponential distributions the analysis of rank transformed data appeared to be more robust and more powerful than the analysis of the raw data.

CHAPTER III

METHODS AND PROCEDURES

The Simulations

The Designs Used

Simulation methods were employed to investigate the robustness (Type I error rate) and the power (1 - Type II error rate) of the F-test for equality of treatment means in a randomized complete block design. The model used in this study was

$$Y_{ijk} = \mu + \alpha_i + \beta_j + e_{ijk} \quad , \quad \begin{aligned} i &= 1,2,\ldots,t \\ j &= 1,2,\ldots,b \\ k &= 1,2,\ldots,m \end{aligned} \qquad (1)$$

where μ is the initial mean, α_i is the non-negative treatment effect, β_j is the non-negative block effect, e_{ijk} is the random error, t is the number of treatments, b is the number of blocks, and m is the number of observations in each treatment - block combination (cell). The cell mean is $E(Y_{ijk}) = \mu + \alpha_i + \beta_j$, and the interaction $(\alpha\beta)_{ij}$ is zero for all i and j.

Several factors were studied in this investigation. Four discrete distributions were examined: the binomial (n = 100), Poisson, negative binomial (k = 1), and logarithmic. For each distribution, initial means (μ) of 1, 5, and

10 were studied except for the logarithmic distribution which cannot have a mean of 1. The number of treatments (t) was 5, 10, or 20, the number of blocks (b) was 3, 4, or 10, and the number of observations per cell (m) was 1, 5, or 10. All possible combinations of these factors were simulated.

Uniform (0,1) random variables were generated using GGUW from the IMSL library (32). To transform the uniform variates into variates from the desired discrete distribution, an algorithm developed by J. E. Norman and L. E. Cannon (41) was used.

During the study of the Type I error rate or robustness of the F-tests for equality of treatment means, α_i in (1) was set equal to 0 for i = 1, 2, ..., t. The block effects were equally spaced fractions of the standard deviation; thus the mean of cell (i,j) was

$$\mu + \frac{j-1}{b-1}\sigma , \quad j = 1,2,\ldots,b$$

where σ is the standard deviation when the mean of the distribution is μ.

In the power study of the test for equality of treatment means, cell (i,j) had mean

$$\mu + \frac{i-1}{t-1}\sigma + \frac{j-1}{b-1}\sigma , \quad \begin{array}{l} i = 1,2,\ldots,t \\ j = 1,2,\ldots,b. \end{array}$$

Tables I, II, III, and IV (Appendix A) present the block effects for the binomial (n = 100), Poisson, negative binomial (k = 1) and logarithmic distributions, respectively. Tables V, VI, VII, and VIII (Appendix B) present the treatment effects used in the generation of data to study the power when the data were from the binomial ($n = 100$), Poisson, negative binomial ($k = 1$) and logarithmic distributions, respectively. In each table μ is the mean of the cell before the addition of block or treatment effects, t is the treatment number, and b is the block number. The power of the F-test for the equality of block means was also studied.

Methods of Analysis

Three methods of analyzing discrete data were investigated in this study. The first method uses the standard analysis of variance to analyze discrete data. The other two methods first transform the data and then apply the standard analysis of variance to the transformed data.

The first transformation was a standard one developed so that the assumptions of the analysis of variance are more nearly satisfied. The proper choice of a transformation depends on the distribution of the data. Suppose X is the value of the original observation, and Y is the transformed value; then, data from a binomial distribution were transformed using $y = \sin^{-1}\sqrt{x}$. The transformation $y = \sqrt{x + .375}$ was used when data arose from the Poisson distribution. Data generated from a negative binomial distribution were transformed using $y = \ln(x + .5\,k)$. A fourth distribution, the logarithmic distribution, has no commonly suggested transformation; therefore, none was used.

Example: Consider data taken from a negative binomial distribution with a k of 1

in a randomized complete block design
with four treatments and three blocks.

Original Data

Treatments

Block	1	2	3	4
1	1	1	0	2
2	1	0	4	3
3	4	17	2	7

Applying the transformation

$y = \ln(x + .5\ k)$ gives:

Transformed Data

Treatments

Block	1	2	3	4
1	.40547	.40547	-.69315	.91629
2	.40547	-.69315	1.50408	1.25276
3	1.50408	2.86220	.91629	2.01490

The other transformation is the rank transformation. The original data were ranked in ascending order using a modified version of the ranking subroutine RANK in the IMSL library (32). The ranks were then analyzed. In the event of ties the average of the associated ranks was used.

Example: Consider the following set of discrete data:

0,2,0,3,0,6,1,5,7.

Ranking the data in ascending order gives:

2,5,2,6,2,8,4,7,9.

The three 0's should have had the ranks 1,2 and 3. The average is

$$\frac{1 + 2 + 3}{3} = 2$$

which is used as the rank of a 0.

When more than one observation per cell occurred, the cell totals were used in the analysis. The standard transformation was applied to the data before computing the cell total. The rank transformation was applied after the raw data within cells had been summed.

Results of the Simulations

For each combination of the factors, 1000 sets of simulated data were generated. Each set was analyzed separately by all three methods. In each case, the analysis of variance was computed using a modified version of RANBLK (32). F-statistics for testing for the equality of treatment means and for the equality of block means were computed and compared to the tabled F-values at the 1%, 5% and 10% levels. A record was kept of the number of times in the 1000 sets of data the hypothesis of equal block means was rejected at each of the three significance levels for each of the methods.

More extensive records were kept of the rejection rates of the hypothesis of equal treatment means. For a particular set of data, there are eight possible outcomes. The outcomes range from all three F-tests indicating that the hypothesis should be rejected, to all three suggesting that the hypothesis should not be rejected. In between these two extremes are six cases in which one F-test implies something different from the other two. For each set of 1000 simulations the number of times each of the eight outcomes occurred was tabulated.

Tables IX through XIX (Appendix C) present the results of the investigation of the robustness of the methods for the binomial (n = 100), Poisson, negative binomial (k = 1), and logarithmic distributions. Each value in the table is the result of 1000 simulations. A 1% Type I error rate means that, on the average, the F-test's in 10 out of the 1000 sets of simulated data will imply that there are treatment differences when in fact there are none. Similarily at the 5% and 10% levels, 50 and 100 F-tests should, on the average, reject the hypothesis.

Tables X through XXX (Appendix D) present the results of the simulations used in the power study of the methods. Each value in the table is the number of times in 1000 sets of simulated data that the hypothesis of equal treatment means was rejected when the hypothesis was false. The method with the greatest power is the one with the largest table values. For example, in Table XXVII (Appendix D) for 5 treatments, 3 blocks and 10 observations per cell the rank transformation method is the most powerful of the three methods at all three significance levels.

Tables XXXI through XLI (Appendix E) contain the counts of the number of times the hypothesis of equal block means was rejected when only block effects were present in the simulations. Tables XLII through LII (Appendix F) present the number of times the hypothesis was rejected when both block and treatment effects were present.

CHAPTER IV

POWER AND ROBUSTNESS OF THE F-TESTS

In Chapter III the results of the simulations were presented. The task now is to decide which of the three methods is "best" in a particuar situation. The "best" method will be the method that is robust, as well as the most powerful, in as many situations as possible.

K-Statistic as a Measure of Agreement

To measure the extent of the agreement between two of the methods, the K-statistic was employed. The result of each method's F-test for the hypothesis of equal treatment means was previously categorized in one of two ways - reject or fail to reject the hypothesis of equal treatment means. As an example, a 2 x 2 table can be formed of these results as follows:

		Rank Transformation: Reject	Rank Transformation: Fail to Reject
Original Data	Reject	x_{11}	x_{12}
	Fail to Reject	x_{21}	x_{22}

where x_{11} is the number of times in the 1000 simulations both the rank transformation method and original data method rejected the hypothesis, x_{22} is the number of times both methods fail to reject the hypothesis, x_{12} is the number of times analysis of the raw data leads to rejection of the hypothesis but analysis of the ranked data does not indicate rejection, and x_{21} is the number of times the hypothesis is rejected under the rank transformation but not under analysis of the raw data. Three 2 x 2 tables can be constructed to compare the methods for each set of 1000 simulations.

The measure of agreement K is defined by Bishop, Fienberg and Holland (2) to be

$$K = \frac{N \sum_{i=1}^{2} x_{ii} - \sum_{i=1}^{2} x_{i.}x_{.i}}{N^2 - \sum_{i=1}^{2} x_{i.}x_{.i}}$$

where N (= 1000) is the total number of observations (simulations), $x_{i.}$ is the total number of observations in the ith row and $x_{.i}$ is the total number of observations in the ith column. A K of 1 indicates that the two methods agree perfectly. Values less than 1 mean that the agreement between the methods is not perfect, but there is still agreement, unless the value is close to zero which means that there is little agreement. Perfect agreement cannot be expected in all cases, but the larger the K value the more often the two methods both reject or fail to reject the hypothesis on the same set of data. Taking the methods two at a time, the results of the simulations were compared using the K-statistic.

Robustness of the F-Test for All Treatment Means Equal

Each of the F-tests in this study was conducted at three error rates: 1%, 5% and 10%. A 5% error rate means that we expect to make an average of 5 Type I errors in 100 sets of data. A test is considered robust if the Type I error rate of the test is at the stated level even though the assumptions are not fully satisfied.

In Tables IX through XIX (Appendix C) the results of the simulations to check the Type I error are presented. If the F-test for each method is robust, then the columns with a 1% error rate should contain all 10's, the columns with a 5% error rate should contain all 50's and the columns with a 10% error rate should contain all 100's on the average.

The F-test computed by each method appears to be robust. The total number of Type I errors per 1000 simulations is approximately what it should be for every case except for the analysis of the raw data when the distribution is negative binomial ($k = 1$) or logarithmic.

The F-tests from the analysis of raw data do not reject at the expected level for the negative binomial ($k = 1$) or logarithmic distribution. The actual error rate of the test is below the stated level. This can be seen in Tables XV through XIX (Appendix C). The numbers in the seventh column of Table XV should fluctuate around 100 if the F-test is robust; but, the numbers are much less. Similar results are seen in the other columns for the original data method in Tables XV through XIX.

Using the K-statistic on these results, agreement in all cases was found to be at least moderate. In the cases of the Poisson and binomial distributions, the agreement between the decisions made by the three methods was strong. This moderate to strong agreement between the decisions means that all three methods tend to reject the hypothesis of equal treatment means on the same sets of data.

Power of the F-Test for All Treatment Means Equal

The power of the three methods depended on the distribution of the data, the mean of the cells, the size of the design, and the significance level of the test. Obviously, as the significance level of the test increases so does the power of the test. Nevertheless, the results of the power study were consistent for all significance levels; that is, the most powerful test at the 1% level tended to be the most powerful at the 5 and 10% levels as well. Therefore, the significance level will not be stated in the discussion of power.

The cell means greatly influence the power of the methods. The difference between the largest and smallest treatment effects was one standard deviation. With such a small difference in treatment effects, the power cannot be expected to be large when only a few observations are taken.

Tables XX through XXX (Appendix D) present the results of the power study by distribution and mean. Looking at these tables the first observation would be that the power increases as the number of total observations in the design increases. The power of all three F-tests is very small for designs with only one observation per cell, but the power increases dramatically as more observations are taken within each cell. Another big contributor to an increase in power is an increase in the number of treatments.

One surprising result was that none of the three standard transformations performed well. In general, they had less power than the F-tests based on the analysis of raw data or the ranks. Thus, the discussion will be mainly limited to a comparison of the other two methods.

Data generated from the Poisson and the binomial distributions exhibited similar behavior when analyzed by the three methods. Looking at Tables XX through XXV it can be seen that for small designs and small cell means, the rank transformation method had the greatest power. F-tests based on the analysis of raw data had the greatest power in designs which had at least 10 blocks, 10 treatments, 5 observations per cell and cell means of 5 or more. For both distributions, the standard transformation method at best had power similar to the original data method.

The cell means greatly influenced the power of the three methods when the data was from the binomial or Poisson distributions. One reason for this may be the shape of the probability mass functions of the distributions from which the data were generated. Plots of both the Poisson and binomial (n = 100) probability mass functions at a mean of 1 are skewed to the right, but for means of 5 and 10, the plots are nearly symmetric about the mean. Data taken from either of these distributions with the larger cell means would thus appear nearly normal and would be best analyzed with the original data method.

For both the negative binomial (k = 1) and logarithmic distributions, the method with the largest overall power was the rank transformation. This can be seen in Tables XXVI through XXX. As the size of the design increases, the difference in the power of the original data method and the rank transformation method diminishes. This is undoubtedly due to the fact that the conditions of the Central Limit Theorem are more nearly satisfied as the number of observations increases. For large designs with more than 500 total observations, F-tests based on the analysis of raw data were slightly more powerful than the tests based on the ranks.

Unlike the binomial (n = 100) and Poisson distibutions, the power is not as greatly affected by the cell means when the data is generated from the negative binomial (k = 1) or logarithmic distribution. Again, this is no doubt due to the probability mass functions for these distributions which are both highly skewed to the right and flatten out as the mean increases.

Another reason that the analysis of the raw data may have less power than the rank transformation method when the data is logarithmic or negative binomial ($k = 1$) is the fact that for these two distributions the actual Type I error rate of the test is below the stated level. With the actual error rate of the test being below the stated level, the power of the test would be affected in a similar manner.

Again, using the K-statistic, strong agreement was found between the methods for the binomial ($n = 100$) and Poisson distributions. This strong agreement between the power of each of the F-tests indicated that all three methods rejected the hypothesis on the same sets of data more often than by chance.

The agreement between the original data method and the rank transformation method is weak in the case of the logarithmic distribution. The agreement between the two methods is small when only 1 observation per cell is taken, but for 5 and 10 observations per cell, the level of agreement becomes moderate. Again, this is probably due to the fact that the actual significance level is below the stated one when analysis is performed on the raw data.

The degree of agreement between the rank transformation and the original data methods for the negative binomial was moderate, lying between the strength of agreement found for the logarithmic and for the binomial and Poisson.

Power of the F-Test for All Block Means Equal

Although the mean of each cell in all of the simulations included a known but variable block effect, the purpose of these simulations was not to study the F-test for equality of block means. However, the information accumulated from these F-tests was studied.

Since block effects were always present, the robustness of the F-test for equality of block means could not be verified, but the power of the F-tests could be examined. The F-tests for the hypothesis of equal block means reacted similarly to their counterparts for equal treatment means. The power of the rank transformation method's F-test is greatest for small design sizes and data from negative binomial or logarithmic series distributions just as it was for the test for treatment differences. Also, the analysis of the original data is more powerful in most other situations.

Looking at the Tables in Appendices E and F, the power of these F-tests can be directly observed. Looking at Tables XXXVII through XXXIX and Tables XLVIII through L for the negative binomial ($k = 1$) distribution, the F-test with the greatest power is the one associated with the rank transformation method. This method remains the most powerful until the design becomes large. In that case, both the original data method and the rank transform method perform similarly. The standard transformation method has less power than either of these methods.

Tables XXXI through XXXVI and Tables XLII through XLVII present the results of the power study of the binomial (n = 100) and Poisson distributions. For both these distributions, the standard transformation method produces an F-test of less power than either the original data method or the rank transformation method. The rank transformation method has greatest power only for small data sets. The most powerful method for both distributions is to use the original data F-tests to check for block effects unless the data set is very small.

Tables XL, XLI, LI, and LII present the results of the power study for the logarithmic series distribution. The F-test from the rank transformation method has more power than the F-test from the analysis of the original data when the design has less than 10 treatments. At about 10 treatments the F-tests have similar power.

Conclusions

The three standard transformations are not suggested as possible methods of analyzing discrete data due to their overall poor performance. These transformations are supposed to work best on large sets of data. From the results seen in this study, by the time the data sets are large enough for the standard transformations to be effective, the conditions of the Central Limit Theorem are more nearly met. Thus, the original data method makes the best approach to analysis.

The study shows that experiments with a large number of observations can be effectively analyzed without transforming the data. If the experiment has a small number of observations, then ranking the data before analyzing with the analysis of variance seems to be the most powerful method.

Since no one method proved to be best in all situations, we suggest that both the original data and the rank transformed data be analyzed. Using both procedures gives extra protection against making an unnecessary error. When the two methods imply different results, it could be that one of the F-statistics is only marginally significant or that the data clearly falls into one of the two broad categories.

CHAPTER V

PROBLEMS, RECOMMENDATIONS AND SUMMARY

Problems Encountered and Recommendations for Further Research

Due to the extensive nature of this study, several problems were encountered. One of the first problems came in deciding when to transform the data. The randomized block design usually has one observation per cell; but, when discrete populations are sampled, more than one observation is frequently taken. The sum of these observations is then used as a single entry in the computation of the analysis of variance.

In order to use a transformation, a decision must be made concerning when the data are to be summed. The standard tranformations were developed for use on individual observations. Thus, transformations of this type were applied first and then summed. The rank transform was a different matter. Following the same principle, the original data were first ranked and then summed. Although this method was robust, the method was not powerful due to the large number of ties. The alternative method summed the data and then ranked it. This method was much more powerful than the previous one; therefore, this approach was used in the study.

Another problem involved the use of the logarithmic distribution. The logarithmic distribution was chosen as an extreme case. Unlike the other three distributions, the mean is not one of the parameters of the probability mass function. By trial and error, the necessary parameters were found such that the required cell means were achieved. Controlling the cell means took away direct control of the treatment and block effects; thus causing them to vary a little more from simulation to simulation. The effects as listed on the tables previously are therefore, not exact due to rounding error.

The problem of what parameter values to use came next. We wanted to study the widest possible situations without getting outside the range of reality. Extreme values of each parameter were used in the simulations in the hope that one method would be "best" for all values of the parameter. This did not occur in most cases, so generalizations had to be made concerning when each method was best. Since no one method proved to be the "best" in all situations more work needs to be done on exactly when each method works well and when it does not.

The biggest problem encountered in this study was the lack of specific methods or procedures to analyze the simulated data. Although the K-statistic was useful, it could only measure the agreement between two methods at a time. The conclusions made from the simulations were based on how we viewed the results looking at each simulation case separately and then comparing it to the other cases.

Another area that could be investigated is multiple comparison tests in association with the three methods. Once powerful and robust methods of determining treatment differences have been found the next natural step is to find the treatments that are different.

Summary

Monte Carlo simulations were conducted to investigate the robustness and power associated with three methods of testing for the equality of treatment means when discrete data were collected from a randomized complete block design. The standard analysis of variance was applied to the raw data (the original data method), to data transformed by a standard method (the standard transformation method), and to the associated ranks of the data (the rank transformation method). The binomial (n = 100), Poisson, negative binomial (k = 1) and logarithmic distributions were studied by simulating designs with different combinations of cell means, numbers of treatments, numbers of blocks, and numbers of observations per cell.

Originally the aim was to determine which method was superior to the other two. Unfortunately, there was no method uniformly better than the others. The standard transformation method was shown to have less power than either of the other two methods. The rank transformation method seemed to have greatest power on small data sets, while the original data method worked best on large data sets.

The methods were robust in most cases. The original data method was not robust when this method was used to analyze data from negative binomial or logarithmic distributions.

Another trend discovered during the simulations was that the degree of skewness in the distribution was a major factor in choosing the best method. Data from Poisson and binomial (n = 100) distributions were analyzed with more power by the original data method. The rank transformation method worked best on data from logarithmic and negative binomial distributions both of which have highly skewed probability mass functions.

Lastly, in designs of this kind it appears that taking more than one observation per cell is strongly recommended. Even though the individual observations are summed together in each cell, the power of the F-tests increases greatly when more than one observation per cell is taken.

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APPENDIX A

BLOCK EFFECTS

TABLE I

BINOMIAL DISTRIBUTION n = 100

b	μ = 1	μ = 5	μ = 10
3	0.000000	0.000000	0.000000
	0.497494	1.089725	1.500000
	0.994987	2.179450	3.000000
4	0.000000	0.000000	0.000000
	0.331663	0.726483	1.000000
	0.663325	1.452966	2.000000
	0.994987	2.179450	3.000000
10	0.000000	0.000000	0.000000
	0.110554	0.242161	0.333333
	0.221108	0.484322	0.666667
	0.331663	0.726483	1.000000
	0.442217	0.968644	1.333333
	0.552771	1.210805	1.666667
	0.663325	1.452966	2.000000
	0.773879	1.695127	2.333333
	0.884433	1.937288	2.666667
	0.994987	2.179450	3.000000

TABLE II

POISSON DISTRIBUTION

b	μ = 1	μ = 5	μ = 10
3	0.000000	0.000000	0.000000
	0.500000	1.118034	1.581139
	1.000000	2.236068	3.162278
4	0.000000	0.000000	0.000000
	0.333333	0.745356	1.054093
	0.666667	1.490712	2.108185
	1.000000	2.236068	3.162278
10	0.000000	0.000000	0.000000
	0.111111	0.248452	0.351364
	0.222222	0.496904	0.702728
	0.333333	0.745356	1.054093
	0.444444	0.993808	1.405457
	0.555556	1.242260	1.756821
	0.666667	1.490712	2.108185
	0.777778	1.739164	2.459549
	0.888889	1.987616	2.810913
	1.000000	2.236068	3.162278

TABLE III

NEGATIVE BINOMIAL DISTRIBUTION k = 1

b	μ = 1	μ = 5	μ = 10
3	0.000000	0.000000	0.000000
	0.707107	2.738613	5.244044
	1.414214	5.477226	10.488088
4	0.000000	0.000000	0.000000
	0.471405	1.825742	3.496029
	0.942809	3.651484	6.992059
	1.414214	5.477226	10.488088
10	0.000000	0.000000	0.000000
	0.157135	0.608581	1.165343
	0.314270	1.217161	2.330686
	0.471405	1.825742	3.496030
	0.628539	2.434323	4.661373
	0.785674	3.042903	5.826716
	0.942809	3.651484	6.992059
	1.099944	4.260064	8.157402
	1.257079	4.868645	9.322745
	1.414214	5.477226	10.488088

TABLE IV

LOGARITHMIC DISTRIBUTION

b	μ = 5	μ = 10
3	0.000000	0.000000
	3.409808	8.238520
	6.819735	16.474197
4	0.000000	0.000000
	2.273174	5.492352
	4.546451	10.984720
	6.819735	16.477005
10	0.000000	0.000000
	0.757658	1.830777
	1.515416	3.661573
	2.273174	5.492303
	3.030934	7.323084
	3.788684	9.153925
	4.546451	10.984720
	5.304212	12.815500
	6.061975	14.646143
	6.819735	16.477005

APPENDIX B

TREATMENT EFFECTS

TABLE V

BINOMIAL DISTRIBUTION n = 100

t	μ = 1	μ = 5	μ = 10
5	0.000000	0.000000	0.000000
	0.248747	0.544862	0.750000
	0.497494	1.089725	1.500000
	0.746241	1.634587	2.250000
	0.994987	2.179450	3.000000
10	0.000000	0.000000	0.000000
	0.110554	0.242161	0.333333
	0.221108	0.484322	0.666667
	0.331663	0.726483	1.000000
	0.442217	0.968644	1.333333
	0.552771	1.210805	1.666667
	0.663325	1.452966	2.000000
	0.773879	1.695127	2.333333
	0.884433	1.937288	2.666667
	0.994987	2.179450	3.000000
20	0.000000	0.000000	0.000000
	0.052368	0.114708	0.157895
	0.104736	0.229416	0.315790
	0.157103	0.344124	0.473684
	0.209471	0.458832	0.631579
	0.261839	0.573539	0.789474
	0.314207	0.688247	0.947368
	0.366574	0.802955	1.105263
	0.418942	0.917663	1.263158
	0.471310	1.032371	1.421053
	0.523678	1.147079	1.578947
	0.576045	1.261787	1.736842
	0.628413	1.376494	1.894737
	0.680781	1.491202	2.052632
	0.733149	1.605910	2.210526
	0.785516	1.720618	2.368421
	0.837884	1.835326	2.526316
	0.890252	1.950034	2.684211
	0.942620	2.064742	2.842105
	0.994987	2.179450	3.000000

TABLE VI

POISSON DISTRIBUTION

t	μ: 1	μ: 5	μ: 10
5	0.000000	0.000000	0.000000
	0.250000	0.559017	0.790569
	0.500000	1.118034	1.581139
	0.750000	1.677051	2.371708
	1.000000	2.236068	3.162278
10	0.000000	0.000000	0.000000
	0.111111	0.248452	0.351364
	0.222222	0.496904	0.702728
	0.333333	0.745356	1.054093
	0.444444	0.993808	1.405457
	0.555556	1.242260	1.756821
	0.666667	1.490712	2.108185
	0.777778	1.739164	2.459549
	0.888889	1.987616	2.810913
	1.000000	2.236068	3.162278
20	0.000000	0.000000	0.000000
	0.052632	0.117688	0.166436
	0.105263	0.235376	0.332871
	0.157895	0.353063	0.499307
	0.210526	0.470751	0.665743
	0.263158	0.588439	0.832178
	0.315789	0.706127	0.998614
	0.368421	0.823815	1.165050
	0.421053	0.941502	1.331485
	0.473684	1.059190	1.497921
	0.526316	1.176878	1.664357
	0.578947	1.294566	1.830792
	0.631579	1.412253	1.997228
	0.684211	1.529941	2.163664
	0.736842	1.647629	2.330099
	0.789474	1.765317	2.496535
	0.842105	1.883005	2.662971
	0.894737	2.000692	2.829406
	0.947368	2.118380	2.995842
	1.000000	2.236068	3.162278

TABLE VII

NEGATIVE BINOMIAL DISTRIBUTION k = 1

t	μ 1	5	10
5	0.000000	0.000000	0.000000
	0.353553	1.369306	2.622022
	0.707107	2.738613	5.244044
	1.060660	4.107919	7.866066
	1.414214	5.477266	10.488088
10	0.000000	0.000000	0.000000
	0.157135	0.608581	1.165343
	0.314270	1.217161	2.330686
	0.471405	1.825742	3.496029
	0.628539	2.434322	4.661373
	0.785674	3.042903	5.826716
	0.942809	3.651484	6.992059
	1.099944	4.260064	8.157402
	1.257079	4.868645	9.322745
	1.414214	5.477266	10.488088
20	0.000000	0.000000	0.000000
	0.074432	0.288275	0.552005
	0.148865	0.576550	1.104009
	0.223297	0.864625	1.656014
	0.297729	1.153100	2.208019
	0.372161	1.441375	2.760023
	0.446594	1.729650	3.312028
	0.521026	2.017925	3.864033
	0.595458	2.306200	4.416037
	0.669891	2.594475	4.968042
	0.744323	2.882750	5.520047
	0.818755	3.171025	6.072051
	0.893188	3.459300	6.624056
	0.967620	3.747575	7.176061
	1.042052	4.035850	7.728065
	1.116484	4.314125	8.280070
	1.190917	4.612400	8.832075
	1.265349	4.900676	9.384079
	1.339781	5.188951	9.936084
	1.414214	5.477226	10.488088

TABLE VIII

LOGARITHMIC DISTRIBUTION

	μ	
t	5	10
5	0.000000	0.000000
	1.704850	4.119249
	3.409808	8.238525
	5.114759	12.357773
	6.819735	16.474213
10	0.000000	0.000000
	0.757658	1.830783
	1.515416	3.661579
	2.273174	5.492358
	3.030934	7.323090
	3.788684	9.153931
	4.546451	10.984726
	5.304212	12.815384
	6.061975	14.646149
	6.819735	16.477020
20	0.000000	0.000000
	0.358836	0.867210
	0.717777	1.734425
	1.076712	2.601641
	1.435648	3.468852
	1.794582	4.336034
	2.153523	5.203239
	2.512464	6.070450
	2.871394	6.937653
	3.230329	7.804886
	3.589272	8.672089
	3.948213	9.539291
	4.307144	10.406464
	4.666088	11.273636
	5.025026	12.140915
	5.383949	13.008057
	5.742889	13.875168
	6.101827	14.742340
	6.460765	15.609528
	6.819735	16.476852

APPENDIX C

TYPE I ERROR RATE OF THE F-TEST FOR THE EQUALITY OF TREATMENT MEANS

TABLE IX

BINOMIAL DISTRIBUTION $\mu = 1$, n = 100

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	10	18	16	48	51	41	98	117	103
		5	9	14	12	45	51	45	97	117	104
		10	9	9	11	59	56	57	124	114	122
	4	1	10	7	9	39	56	48	90	104	98
		5	12	13	7	43	59	48	92	92	96
		10	10	13	13	52	52	47	103	102	94
	10	1	8	7	10	57	60	58	106	112	113
		5	4	5	7	39	39	43	88	85	101
		10	9	13	9	52	57	52	99	92	100
10	3	1	7	15	13	38	50	60	88	92	99
		5	9	8	9	58	54	57	111	117	111
		10	14	11	16	40	41	52	86	96	115
	4	1	14	12	8	38	50	52	94	110	108
		5	9	13	10	43	58	50	105	116	115
		10	8	8	8	39	44	49	97	102	102
	10	1	7	4	7	45	46	46	108	96	116
		5	14	12	11	46	54	58	91	98	96
		10	10	10	10	45	46	50	99	95	88
20	3	1	11	20	17	53	63	61	99	110	107
		5	6	9	14	47	53	46	104	109	83
		10	12	11	15	54	64	60	106	107	119
	4	1	7	7	6	40	48	48	87	103	105
		5	8	8	6	40	49	43	90	90	83
		10	10	9	9	60	56	60	114	105	115
	10	1	11	7	9	39	48	52	94	95	98
		5	5	9	5	49	54	40	95	99	89
		10	6	10	4	43	35	43	92	89	93

*Method of Analysis:

I Original Data

II Rank Transformation

III Standard Transformation

TABLE X

BINOMIAL DISTRIBUTION μ = 5, n = 100

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	9	12	10	53	52	55	101	103	102
		5	12	11	8	56	51	67	103	103	99
		10	9	14	8	48	49	50	97	108	101
	4	1	4	8	5	40	50	42	93	99	92
		5	8	7	8	55	51	55	101	104	106
		10	12	10	11	54	51	48	99	112	103
	10	1	8	6	4	44	43	48	82	89	86
		5	10	12	10	45	46	46	85	85	88
		10	8	11	11	55	51	49	105	96	109
10	3	1	8	10	11	48	53	49	101	101	98
		5	7	8	6	42	45	46	87	89	92
		10	13	15	14	62	68	65	111	109	111
	4	1	9	10	11	46	54	47	114	111	105
		5	8	7	9	50	46	49	97	105	105
		10	9	10	13	52	51	54	101	101	102
	10	1	9	10	5	42	45	39	97	93	91
		5	15	14	14	47	50	47	104	100	106
		10	7	8	6	46	42	48	96	97	96
20	3	1	6	9	9	56	57	57	107	118	102
		5	5	9	9	52	57	51	99	97	110
		10	6	8	6	52	46	57	102	101	106
	4	1	6	9	8	49	49	46	97	96	92
		5	8	8	10	49	45	53	89	89	94
		10	13	12	10	50	52	54	103	99	111
	10	1	9	10	9	47	58	54	103	103	99
		5	8	7	5	43	39	39	88	87	90
		10	10	10	7	37	48	35	79	85	83

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE XI

BINOMIAL DISTRIBUTION $\mu = 10$, $n = 100$

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	6	7	4	64	64	53	103	121	107
		5	13	16	10	59	62	62	107	118	108
		10	8	7	11	46	50	45	95	114	92
	4	1	10	14	10	42	52	47	98	109	108
		5	8	18	11	61	59	64	102	102	98
		10	5	12	7	54	50	51	91	98	92
	10	1	14	14	15	51	55	52	95	109	94
		5	14	11	16	55	53	56	101	93	100
		10	2	7	2	39	40	40	86	88	88
10	3	1	12	13	9	55	52	55	95	93	93
		5	9	7	8	39	44	38	81	90	85
		10	13	17	11	56	60	65	112	111	112
	4	1	16	19	16	63	66	62	115	117	117
		5	10	8	7	42	44	47	88	87	91
		10	6	8	7	40	42	43	86	80	86
	10	1	10	8	9	46	52	41	92	99	93
		5	4	4	6	54	54	56	117	109	109
		10	11	10	11	49	54	48	93	100	86
20	3	1	13	13	12	56	48	51	109	116	107
		5	10	10	13	63	54	64	100	111	103
		10	8	6	9	47	48	42	91	98	99
	4	1	11	9	12	53	59	51	104	107	110
		5	6	6	4	52	53	48	114	116	111
		10	11	13	12	47	40	46	97	83	104
	10	1	7	10	9	55	51	49	107	98	102
		5	11	10	11	49	41	46	89	83	79
		10	11	9	11	41	40	38	87	86	87

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE XII

POISSON DISTRIBUTION μ = 1

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	13	11	14	45	54	52	99	113	101
		5	12	11	12	52	53	61	107	109	112
		10	6	10	11	41	49	49	81	96	87
	4	1	5	5	7	43	49	44	83	94	99
		5	16	20	14	48	57	55	101	104	102
		10	2	5	4	51	52	57	95	108	101
	10	1	11	10	12	52	44	49	98	103	98
		5	9	9	11	36	43	48	97	93	104
		10	9	6	9	41	39	41	105	100	100
10	3	1	7	11	10	51	44	47	91	92	99
		5	5	9	6	42	46	45	93	104	94
		10	9	8	9	61	58	53	111	108	110
	4	1	8	12	8	48	52	47	88	100	100
		5	7	4	6	53	54	55	102	106	112
		10	10	11	12	46	53	51	92	95	94
	10	1	8	12	9	48	49	48	97	100	97
		5	7	10	9	38	48	47	89	94	104
		10	9	9	11	61	53	53	108	112	109
20	3	1	6	10	7	41	47	41	87	87	91
		5	6	7	10	38	52	40	94	105	107
		10	5	6	5	34	36	37	79	92	82
	4	1	8	13	9	47	63	62	96	110	109
		5	7	6	8	49	47	46	90	99	103
		10	11	12	9	43	48	46	93	89	89
	10	1	7	7	6	38	40	45	88	99	92
		5	9	9	11	38	42	42	80	82	87
		10	6	8	7	46	56	50	109	95	99

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE XIII

POISSON DISTRIBUTION $\mu = 5$

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	8	12	12	42	50	36	90	108	92
		5	12	13	12	51	54	50	93	108	101
		10	9	13	9	55	55	57	97	116	99
	4	1	9	10	10	36	40	39	89	96	94
		5	12	17	13	57	52	58	109	109	109
		10	3	5	7	36	37	38	86	77	82
	10	1	18	19	22	66	72	67	106	113	112
		5	8	9	8	40	40	37	106	99	98
		10	12	11	11	52	47	48	100	95	95
10	3	1	12	13	14	56	59	52	107	113	109
		5	9	9	10	60	50	57	117	117	120
		10	8	11	8	56	52	51	104	113	112
	4	1	5	6	5	61	53	57	113	100	109
		5	5	7	3	32	32	44	90	96	86
		10	11	10	11	49	54	53	88	92	101
	10	1	7	9	8	49	55	45	92	92	100
		5	8	8	6	48	42	40	110	117	119
		10	10	12	11	49	57	51	95	90	97
20	3	1	8	5	9	44	49	39	99	106	93
		5	9	9	9	58	53	54	107	111	106
		10	9	9	14	47	42	52	111	108	117
	4	1	11	12	10	50	51	51	102	106	104
		5	13	12	11	62	61	61	108	105	104
		10	8	8	9	45	43	41	90	93	89
	10	1	10	9	12	56	48	48	91	100	99
		5	12	8	10	44	39	45	87	91	89
		10	4	4	6	42	43	44	91	90	94

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE XIV

POISSON DISTRIBUTION $\mu = 10$

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	10	17	12	51	64	53	114	114	106
		5	13	15	14	59	68	61	118	135	115
		10	11	13	5	52	63	51	101	110	105
	4	1	15	19	14	56	56	59	111	110	105
		5	6	7	7	34	47	40	84	102	86
		10	12	13	11	53	49	49	106	106	109
	10	1	8	11	10	45	55	49	101	109	104
		5	12	12	12	51	56	57	98	100	97
		10	4	5	3	41	40	39	81	86	84
10	3	1	7	6	10	57	55	58	104	108	98
		5	12	14	9	43	42	42	95	88	89
		10	9	12	14	54	48	51	107	106	107
	4	1	4	6	5	45	42	45	95	97	97
		5	9	8	8	54	57	54	117	121	112
		10	13	14	10	49	49	52	104	107	108
	10	1	7	11	5	47	44	41	103	102	101
		5	15	14	15	59	57	65	117	112	107
		10	7	10	9	43	47	37	94	93	86
20	3	1	11	12	10	53	54	48	102	101	103
		5	16	11	14	56	53	49	114	118	109
		10	9	10	8	47	51	51	103	108	98
	4	1	11	15	12	44	50	50	104	100	98
		5	9	8	11	50	47	56	112	106	112
		10	15	14	12	59	60	56	104	111	106
	10	1	13	16	12	45	46	46	94	96	94
		5	10	11	9	46	53	50	99	96	99
		10	6	6	7	47	48	40	87	85	87

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE XV

NEGATIVE BINOMIAL DISTRIBUTION k = 1, μ = 1

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	11	20	19	41	58	56	84	119	116
		5	5	11	9	40	59	47	90	111	93
		10	5	12	6	29	53	38	73	96	89
	4	1	9	15	13	39	56	53	71	101	97
		5	4	8	7	35	54	44	89	108	93
		10	5	8	7	33	37	45	82	83	96
	10	1	7	8	10	42	51	49	87	99	101
		5	11	13	11	52	58	50	105	108	106
		10	5	10	14	54	55	60	112	113	106
10	3	1	9	9	12	35	47	51	77	101	111
		5	7	11	8	42	54	52	97	104	102
		10	5	11	4	29	43	36	89	99	83
	4	1	9	15	14	49	49	48	97	109	106
		5	5	5	5	34	49	40	91	104	104
		10	2	9	7	38	52	38	93	110	92
	10	1	7	6	5	40	46	46	95	97	99
		5	8	10	11	55	53	56	93	102	106
		10	10	14	5	46	50	39	104	94	112
20	3	1	11	7	8	43	51	54	88	95	99
		5	9	13	8	40	53	52	97	98	99
		10	5	9	8	34	46	44	88	101	88
	4	1	10	14	16	61	71	71	110	120	128
		5	4	10	10	55	53	57	88	108	97
		10	8	8	4	43	46	48	95	100	85
	10	1	5	10	8	39	44	48	82	93	93
		5	7	15	9	45	48	48	100	99	97
		10	8	8	10	38	43	42	87	89	93

*Method of Analysis:

I Original Data

II Rank Transformation

III Standard Transformation

TABLE XVI

NEGATIVE BINOMIAL DISTRIBUTION k = 1, μ = 5

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	6	15	12	34	53	55	70	105	97
		5	6	13	7	36	54	45	73	95	99
		10	5	11	11	37	50	53	84	99	94
	4	1	7	12	9	31	53	51	65	106	99
		5	9	14	10	48	64	56	98	123	112
		10	7	11	10	37	50	57	91	97	102
	10	1	5	9	11	42	63	51	92	103	105
		5	8	6	9	54	52	44	96	101	100
		10	11	13	8	44	53	46	101	97	101
10	3	1	8	11	7	32	48	51	77	98	95
		5	2	4	9	28	49	50	81	97	113
		10	8	8	9	25	40	42	64	88	80
	4	1	7	7	11	42	56	52	90	116	109
		5	10	16	15	56	67	58	111	115	118
		10	5	9	11	41	49	55	89	102	100
	10	1	11	12	15	45	47	56	95	90	94
		5	6	7	11	50	53	67	95	105	117
		10	13	13	11	49	52	39	92	95	92
20	3	1	9	12	15	41	57	61	78	115	115
		5	5	12	8	36	44	45	76	88	83
		10	4	6	7	47	53	55	100	102	118
	4	1	6	6	7	42	36	42	69	81	83
		5	5	13	8	45	54	49	97	104	100
		10	4	7	9	41	42	47	78	98	98
	10	1	11	8	12	53	50	53	84	109	98
		5	11	13	11	53	51	56	104	101	86
		10	10	14	10	43	45	49	100	106	104

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE XVII

NEGATIVE BINOMIAL DISTRIBUTION $k = 1, \mu = 10$

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	10	14	14	34	59	45	79	108	96
		5	9	10	10	46	64	54	95	120	106
		10	8	11	18	37	57	55	88	111	103
	4	1	5	8	10	34	47	51	78	100	92
		5	7	5	10	37	42	49	73	84	104
		10	7	10	8	36	48	45	78	92	93
	10	1	4	9	10	40	45	47	89	94	91
		5	7	10	11	45	50	48	91	90	104
		10	9	12	14	50	51	48	97	95	97
10	3	1	12	18	16	50	60	56	86	103	98
		5	6	9	5	37	51	52	82	99	94
		10	8	15	10	45	59	58	100	125	117
	4	1	10	13	9	49	55	50	89	111	107
		5	9	6	11	45	53	52	102	101	106
		10	6	7	10	40	42	51	82	93	98
	10	1	10	8	9	39	51	60	92	107	116
		5	10	11	10	42	53	47	96	102	103
		10	5	11	8	54	54	39	114	101	90
20	3	1	8	7	9	35	43	39	83	88	81
		5	12	11	17	43	60	67	94	120	113
		10	5	6	7	34	40	50	80	102	121
	4	1	8	10	8	46	61	56	88	107	93
		5	13	13	8	53	58	46	104	119	110
		10	11	10	11	36	54	47	79	105	95
	10	1	12	12	13	45	62	56	90	96	95
		5	7	8	10	57	55	45	100	103	98
		10	12	13	11	51	51	45	91	84	106

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE XVIII

LOGARITHMIC DISTRIBUTION $\mu = 5$

			Significance Level					
			1%		5%		10%	
t	b	m	I*	II	I	II	I	II
5	3	1	3	11	18	50	53	99
		5	9	18	31	57	67	112
		10	3	9	27	44	57	92
	4	1	4	5	20	45	44	82
		5	2	8	18	44	70	103
		10	13	17	43	54	89	105
	10	1	2	8	32	39	62	103
		5	9	8	46	55	84	104
		10	5	5	42	46	89	98
10	3	1	13	14	37	52	65	104
		5	7	7	25	46	74	94
		10	4	11	31	57	68	104
	4	1	14	13	41	52	78	100
		5	10	7	36	52	83	109
		10	7	11	40	45	90	113
	10	1	10	17	39	56	77	115
		5	5	12	56	51	108	119
		10	8	5	36	46	86	108
20	3	1	9	5	35	57	62	113
		5	6	5	29	44	72	95
		10	4	10	38	49	70	98
	4	1	10	11	34	59	78	107
		5	4	9	36	54	88	102
		10	10	12	34	51	89	95
	10	1	7	12	31	46	62	87
		5	8	8	49	43	86	82
		10	8	3	45	45	91	84

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE XIX

LOGARITHMIC DISTRIBUTION $\mu = 10$

			Significance Level					
			1%		5%		10%	
t	b	m	I*	II	I	II	I	II
5	3	1	7	8	26	62	53	112
		5	6	10	25	61	59	107
		10	5	12	32	59	74	98
	4	1	4	9	21	69	47	129
		5	7	10	34	51	67	118
		10	7	10	36	69	88	122
	10	1	4	13	27	54	82	119
		5	15	10	52	60	105	105
		10	7	8	46	47	82	97
10	3	1	8	4	26	42	49	102
		5	3	10	28	51	79	94
		10	6	9	35	58	89	113
	4	1	10	12	25	49	60	92
		5	7	9	40	51	92	100
		10	8	9	38	72	86	123
	10	1	3	9	37	49	88	100
		5	11	10	50	70	103	125
		10	10	13	43	51	88	98
20	3	1	9	12	43	54	86	120
		5	6	8	35	38	87	99
		10	2	6	24	41	76	99
	4	1	11	7	33	39	83	95
		5	6	10	43	50	90	103
		10	5	9	34	52	61	96
	10	1	14	14	54	57	101	97
		5	10	14	62	56	109	100
		10	10	4	38	49	87	94

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

APPENDIX D

POWER OF THE F-TEST FOR THE EQUALITY OF TREATMENT MEANS

TABLE XX

BINOMIAL DISTRIBUTION μ = 1, n = 100

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	21	24	20	75	93	76	145	160	144
		5	78	85	70	250	269	224	382	383	353
		10	165	193	143	427	426	387	598	596	576
	4	1	26	27	29	88	106	104	179	198	182
		5	118	134	106	349	346	314	498	499	454
		10	354	354	307	650	627	600	770	760	744
	10	1	85	100	80	254	252	252	371	360	353
		5	678	657	621	879	860	847	929	920	917
		10	971	967	955	998	996	994	999	999	997
10	3	1	16	16	12	69	89	79	164	165	160
		5	107	109	98	296	299	268	442	431	389
		10	314	301	274	589	563	552	742	704	693
	4	1	23	23	17	82	95	87	158	163	170
		5	205	210	185	422	417	411	562	557	535
		10	519	511	463	796	783	755	895	881	871
	10	1	107	111	98	264	268	251	393	397	371
		5	839	830	774	939	928	915	970	961	957
		10	1000	998	993	1000	1000	1000	1000	1000	1000
20	3	1	22	20	19	89	109	87	162	172	158
		5	174	174	146	397	386	371	558	550	517
		10	530	519	465	798	785	747	880	884	852
	4	1	36	34	30	127	137	121	217	226	213
		5	341	337	292	584	593	558	724	699	682
		10	789	787	734	936	939	915	976	965	964
	10	1	139	130	128	320	318	318	449	444	427
		5	968	959	947	994	994	985	998	998	995
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000

*Method of Analysis:
- I Original Data
- II Rank Transformation
- III Standard Transformation

TABLE XXI

BINOMIAL DISTRIBUTION μ = 5, n = 100

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	17	19	18	90	102	80	178	196	180
		5	108	105	104	307	301	289	448	441	441
		10	264	256	246	585	576	582	731	702	729
	4	1	37	41	35	145	156	139	239	248	244
		5	223	216	213	494	471	476	635	607	633
		10	535	497	531	818	801	809	912	895	917
	10	1	120	110	110	313	295	314	438	425	438
		5	847	837	839	952	941	946	976	968	971
		10	995	991	995	1000	1000	1000	1000	1000	1000
10	3	1	36	29	30	95	93	98	186	175	186
		5	144	148	146	379	380	387	549	523	541
		10	443	396	444	745	721	754	863	838	861
	4	1	39	37	33	138	136	139	244	236	231
		5	332	333	327	614	602	614	737	716	733
		10	763	736	754	933	912	924	973	961	966
	10	1	169	167	174	355	358	352	500	496	501
		5	963	950	958	995	988	993	998	996	999
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
20	3	1	34	34	33	147	130	145	239	238	234
		5	299	280	299	580	563	575	725	708	726
		10	780	745	774	935	915	933	971	960	964
	4	1	56	57	60	173	184	177	283	285	275
		5	571	547	556	811	798	805	902	884	896
		10	962	949	960	996	992	996	999	997	998
	10	1	267	247	269	512	508	509	634	617	623
		5	997	997	998	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE XXII

BINOMIAL DISTRIBUTION $\mu = 10$, $n = 100$

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	17	24	20	99	106	104	179	171	177
		5	104	111	103	336	322	343	484	481	489
		10	288	285	294	638	605	640	776	747	774
	4	1	38	40	41	126	122	124	226	231	223
		5	238	226	236	521	507	522	668	647	672
		10	584	552	579	859	803	861	927	902	925
	10	1	155	151	151	362	356	365	488	480	480
		5	902	885	897	972	958	973	992	985	992
		10	999	998	999	1000	999	999	1000	1000	1000
10	3	1	32	28	28	108	98	107	190	193	185
		5	204	195	203	449	423	449	596	584	601
		10	525	484	518	801	740	805	903	882	897
	4	1	32	37	31	155	155	148	246	257	247
		5	417	396	417	696	669	687	811	796	802
		10	840	789	836	950	937	950	976	966	974
	10	1	182	187	184	421	407	431	564	531	551
		5	983	976	986	999	997	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
20	3	1	35	42	39	149	148	145	250	247	255
		5	350	335	356	626	601	615	761	738	757
		10	826	774	822	952	936	953	988	977	985
	4	1	58	54	59	172	171	172	278	275	275
		5	657	632	663	863	846	859	930	914	930
		10	979	967	977	997	994	997	1000	999	999
	10	1	285	262	288	555	516	539	660	649	662
		5	1000	999	1000	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000

*Method of Analysis:
 I Original Data
 II Rank Transformation
 III Standard Transformation

TABLE XXIII

POISSON DISTRIBUTION $\mu = 1$

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	16	26	21	77	100	83	152	178	163
		5	67	82	57	244	261	251	372	389	384
		10	154	171	148	419	421	413	599	580	584
	4	1	23	30	32	104	100	101	174	180	184
		5	129	137	130	341	352	341	496	499	491
		10	330	321	328	624	598	617	787	742	767
	10	1	69	79	83	232	233	237	345	337	340
		5	670	663	664	879	865	876	945	930	935
		10	971	963	966	993	993	993	997	996	997
10	3	1	26	24	32	81	88	89	159	156	157
		5	101	111	107	287	295	293	444	427	433
		10	267	252	256	581	557	565	724	725	724
	4	1	27	27	31	116	119	110	189	211	195
		5	210	201	200	441	449	427	598	596	591
		10	545	514	529	780	756	772	876	851	868
	10	1	110	100	105	270	259	270	392	395	403
		5	816	784	813	947	943	950	979	970	981
		10	998	996	998	1000	1000	1000	1000	1000	1000
20	3	1	15	18	20	87	79	82	161	160	163
		5	161	169	172	387	401	393	543	526	538
		10	520	501	514	772	749	762	874	870	864
	4	1	45	42	39	135	137	143	230	227	232
		5	328	322	306	607	572	599	732	714	731
		10	783	760	781	931	923	929	969	965	966
	10	1	143	125	138	355	344	344	473	459	476
		5	971	970	969	993	992	995	998	996	998
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE XXIV

POISSON DISTRIBUTION $\mu = 5$

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	24	31	21	99	119	105	170	178	173
		5	94	87	94	286	293	281	437	437	436
		10	247	249	238	570	547	551	733	705	734
	4	1	19	25	25	119	110	07	204	195	195
		5	199	205	195	460	439	459	612	585	605
		10	509	481	508	799	770	801	911	871	903
	10	1	116	123	119	314	305	306	430	432	433
		5	842	814	834	957	949	960	980	968	981
		10	993	991	992	999	997	999	999	999	999
10	3	1	26	22	23	103	106	103	182	195	188
		5	162	146	148	371	356	360	529	495	519
		10	425	407	420	759	711	752	868	837	864
	4	1	37	40	40	145	153	148	236	228	237
		5	323	284	314	584	559	594	726	704	717
		10	750	718	740	917	898	910	970	945	958
	10	1	154	154	158	371	371	369	495	492	505
		5	960	949	955	990	987	989	998	994	997
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
20	3	1	32	31	27	119	113	115	207	219	219
		5	272	269	267	551	527	535	691	655	694
		10	749	714	735	917	894	914	962	949	962
	4	1	40	42	44	168	159	164	281	268	278
		5	553	534	556	790	756	781	874	857	874
		10	960	946	956	990	986	998	997	993	995
	10	1	242	222	234	490	476	483	614	601	616
		5	998	996	997	1000	999	1000	1000	999	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE XXV

POISSON DISTRIBUTION $\mu = 10$

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	26	29	24	104	98	102	185	187	183
		5	110	125	108	333	338	343	493	489	497
		10	292	278	282	595	581	589	770	740	764
	4	1	36	43	38	124	141	130	232	225	228
		5	220	198	223	502	485	496	643	618	655
		10	545	522	538	827	788	821	915	896	909
	10	1	132	133	132	320	323	314	443	434	441
		5	871	856	873	979	966	977	990	988	989
		10	997	997	997	1000	999	1000	1000	1000	1000
10	3	1	29	26	32	124	115	118	211	200	208
		5	204	193	204	475	444	468	614	598	619
		10	513	475	506	797	763	787	882	858	885
	4	1	39	35	37	125	127	125	214	224	212
		5	329	332	327	614	606	626	748	725	748
		10	795	753	798	938	918	934	973	961	973
	10	1	197	182	193	427	419	428	560	549	554
		5	959	947	957	986	987	986	997	992	995
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
20	3	1	36	28	36	135	128	131	225	226	228
		5	349	325	355	642	622	633	758	751	765
		10	787	757	788	942	930	951	975	964	976
	4	1	62	54	56	172	174	161	263	278	259
		5	612	582	606	846	816	846	913	904	914
		10	969	954	967	993	991	992	997	996	997
	10	1	261	252	258	513	490	508	635	621	638
		5	1000	1000	1000	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000

*Method of Analysis:
- I Original Data
- II Rank Transformation
- III Standard Transformation

TABLE XXVI

NEGATIVE BINOMIAL DISTRIBUTION k = 1, μ = 1

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	9	12	10	45	65	57	81	113	116
		5	32	34	38	123	151	132	210	251	224
		10	50	74	49	198	236	197	335	378	304
	4	1	15	15	17	46	67	67	106	133	129
		5	36	53	47	163	198	168	276	316	252
		10	111	144	109	317	342	293	450	481	412
	10	1	32	29	34	99	108	109	182	194	196
		5	249	280	194	513	531	475	665	673	591
		10	616	656	565	845	859	770	911	923	860
10	3	1	14	16	14	69	69	68	124	143	139
		5	44	46	52	141	167	150	244	274	247
		10	81	95	82	258	281	239	404	412	379
	4	1	20	12	12	58	72	74	119	136	137
		5	60	78	58	209	233	186	325	349	300
		10	154	176	138	379	388	354	541	545	489
	10	1	31	41	41	135	134	136	227	219	222
		5	337	335	278	603	618	547	742	749	678
		10	804	827	696	941	941	891	969	965	941
20	3	1	15	12	12	59	55	57	109	129	124
		5	48	59	50	176	197	169	299	326	282
		10	159	187	129	386	430	336	534	593	492
	4	1	14	17	18	65	79	80	129	153	159
		5	91	103	77	259	288	248	398	407	376
		10	309	340	246	583	606	498	718	714	633
	10	1	38	41	37	151	144	144	249	220	228
		5	537	530	456	806	794	717	879	887	811
		10	960	955	892	995	993	972	999	997	988

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE XXVII

NEGATIVE BINOMIAL DISTRIBUTION $k = 1, \mu = 5$

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	12	13	13	42	69	59	88	127	114
		5	27	27	30	103	139	106	193	233	194
		10	56	79	44	202	244	181	334	385	307
	4	1	10	17	17	51	60	56	112	124	102
		5	48	65	39	164	184	154	272	299	261
		10	121	144	70	315	321	245	458	462	378
	10	1	19	28	28	112	105	101	191	187	171
		5	228	260	167	513	509	394	651	658	536
		10	638	654	466	856	853	734	917	923	821
10	3	1	19	15	11	69	68	58	116	138	117
		5	23	42	35	136	150	130	231	251	216
		10	76	87	61	250	270	181	386	396	318
	4	1	25	16	15	62	76	75	124	136	142
		5	66	83	56	205	223	155	325	334	269
		10	180	193	127	409	431	322	555	571	462
	10	1	34	40	34	105	115	113	185	195	180
		5	356	381	236	617	643	486	748	747	621
		10	824	834	636	940	940	849	973	971	921
20	3	1	21	15	13	57	51	48	97	112	101
		5	60	64	44	188	196	145	292	328	254
		10	135	163	107	379	399	285	532	568	429
	4	1	19	20	16	76	93	89	132	151	147
		5	119	122	76	289	307	199	407	423	329
		10	299	318	206	564	586	415	708	722	557
	10	1	49	39	32	159	138	139	257	242	216
		5	568	584	384	807	796	662	887	891	754
		10	964	957	819	989	991	947	997	996	977

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE XXVIII

NEGATIVE BINOMIAL DISTRIBUTION k = 1, μ = 10

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	5	16	14	47	66	55	100	131	101
		5	24	39	22	100	120	95	168	208	156
		10	45	56	37	179	207	146	299	341	262
	4	1	16	14	11	61	77	64	105	151	134
		5	47	63	41	167	183	156	275	299	257
		10	135	137	73	318	342	244	475	484	389
	10	1	33	38	35	109	115	106	186	191	189
		5	260	285	174	505	521	371	651	635	507
		10	637	655	451	854	867	736	931	932	837
10	3	1	9	12	13	50	67	55	98	120	113
		5	39	52	31	165	161	111	259	273	209
		10	101	98	54	292	291	199	433	453	333
	4	1	25	18	18	78	72	68	130	137	126
		5	61	68	51	211	222	170	339	357	279
		10	183	199	133	441	450	327	599	610	473
	10	1	42	37	32	116	120	101	213	210	177
		5	363	376	222	617	625	462	745	745	619
		10	816	827	631	940	942	840	973	980	904
20	3	1	17	13	20	60	68	59	111	132	121
		5	71	77	34	201	209	151	319	326	249
		10	136	143	96	376	381	284	538	550	433
	4	1	17	20	18	77	77	72	149	144	129
		5	113	119	75	289	293	208	411	408	324
		10	296	333	177	564	595	413	700	736	551
	10	1	47	37	32	163	147	131	253	245	223
		5	520	529	319	784	764	581	850	857	705
		10	953	959	809	993	994	934	996	998	964

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE XXIX

LOGARITHMIC DISTRIBUTION μ = 5

			Significance Level					
			1%		5%		10%	
t	b	m	I*	II	I	II	I	II
5	3	1	7	18	20	73	67	114
		5	13	16	59	84	121	160
		10	26	30	91	135	185	230
	4	1	6	11	20	54	54	111
		5	32	34	96	115	154	189
		10	37	67	150	185	255	276
	10	1	12	23	63	84	120	132
		5	68	83	225	228	347	362
		10	217	252	483	504	625	637
10	3	1	9	15	54	63	97	118
		5	21	22	87	93	144	173
		10	36	44	130	160	233	261
	4	1	12	16	56	60	91	100
		5	28	25	106	102	178	178
		10	64	70	178	196	305	315
	10	1	24	13	80	75	145	126
		5	109	109	271	279	426	393
		10	285	332	536	570	686	698
20	3	1	22	15	58	50	114	108
		5	30	24	90	91	164	171
		10	53	49	166	181	276	310
	4	1	15	11	55	53	109	106
		5	36	34	110	132	197	223
		10	93	98	237	254	354	394
		1	19	11	77	59	151	134
		5	173	155	374	346	509	468
		10	491	507	765	761	857	865

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE XXX

LOGARITHMIC DISTRIBUTION $\mu = 10$

			Significance Level					
			1%		5%		10%	
t	b	m	I*	II	I	II	I	II
5	3	1	4	10	19	57	58	106
		5	17	27	65	87	110	150
		10	30	39	97	110	160	193
	4	1	7	20	34	76	72	131
		5	14	17	84	97	148	173
		10	37	41	124	157	227	249
	10	1	10	12	54	57	103	124
		5	64	83	207	232	354	338
		10	181	175	424	426	571	551
10	3	1	10	9	49	60	89	110
		5	22	25	72	96	130	153
		10	34	34	107	125	200	218
	4	1	12	16	43	55	79	108
		5	34	31	125	122	193	203
		10	46	57	154	188	266	295
	10	1	21	20	67	80	124	135
		5	93	73	238	227	367	347
		10	268	274	532	506	656	651
20	3	1	20	16	53	50	81	99
		5	24	21	81	79	153	147
		10	33	29	117	115	221	215
	4	1	16	17	47	69	96	120
		5	39	37	130	121	212	211
		10	71	83	220	225	333	325
	10	1	17	12	74	68	149	119
		5	127	109	324	278	448	422
		10	450	434	686	684	809	802

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

APPENDIX E

POWER OF THE F-TEST FOR THE EQUALITY OF BLOCK MEANS WHEN ALL TREATMENT MEANS ARE EQUAL

TABLE XXXI

BINOMIAL DISTRIBUTION n = 100, μ = 1

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	41	53	36	136	150	137	243	242	236
		5	241	297	236	571	597	550	728	727	701
		10	567	608	533	862	876	821	947	943	925
	4	1	26	39	33	142	152	135	226	231	214
		5	274	310	262	567	593	540	718	728	686
		10	596	625	528	860	848	828	935	928	902
	10	1	48	50	41	152	156	158	247	249	252
		5	476	485	416	751	753	691	851	849	814
		10	888	892	828	970	970	953	993	989	982
10	3	1	94	120	103	326	314	301	441	446	436
		5	789	808	734	945	937	920	969	971	959
		10	991	988	983	999	999	997	999	999	999
	4	1	122	140	112	306	318	311	432	428	415
		5	823	814	768	954	945	928	981	975	970
		10	985	990	978	999	997	999	1000	1000	999
	10	1	144	134	139	364	356	327	478	467	439
		5	951	947	920	986	986	978	994	993	991
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
20	3	1	355	345	329	626	603	580	752	745	706
		5	996	996	993	999	999	998	999	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	4	1	339	343	325	595	597	572	728	717	705
		5	997	997	994	1000	1000	999	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	10	1	470	447	417	720	703	678	826	802	763
		5	1000	1000	1000	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000

*Method of Analysis:

I Original Data

II Rank Transformation

III Standard Transformation

TABLE XXXII

BINOMIAL DISTRIBUTION n = 100, μ = 5

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	47	66	50	188	199	174	288	290	283
		5	336	373	333	676	689	672	817	813	813
		10	702	723	683	934	927	933	976	972	973
	4	1	38	38	36	165	175	173	292	285	288
		5	361	379	351	664	675	655	808	790	798
		10	770	789	751	952	941	945	977	974	976
	10	1	67	68	61	184	177	187	292	303	303
		5	597	614	587	823	818	818	906	893	899
		10	963	961	959	994	994	993	998	997	998
10	3	1	157	147	157	368	375	367	509	491	500
		5	869	856	844	970	964	971	986	984	986
		10	995	995	995	999	1000	998	1000	1000	1000
	4	1	151	156	147	358	356	355	496	487	496
		5	901	891	887	981	970	987	994	986	991
		10	999	999	999	1000	1000	1000	1000	1000	1000
	10	1	201	196	197	428	422	416	564	563	563
		5	983	978	979	997	997	996	1000	1000	998
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
20	3	1	436	438	428	692	684	684	803	790	795
		5	1000	1000	1000	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	4	1	438	416	438	697	684	690	806	799	801
		5	999	998	999	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	10	1	619	584	602	823	798	814	881	867	876
		5	1000	1000	1000	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000

*Method of Analysis:
- I Original Data
- II Rank Transformation
- III Standard Transformation

TABLE XXXIII

BINOMIAL DISTRIBUTION n = 100, μ = 10

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	44	48	42	158	174	152	272	282	271
		5	395	425	389	699	698	692	830	816	826
		10	702	727	705	927	914	924	974	962	968
	4	1	45	60	47	180	188	184	297	320	299
		5	395	407	392	700	696	707	822	814	817
		10	781	785	777	948	944	949	988	976	989
	10	1	70	64	71	214	211	215	337	324	332
		5	650	658	656	858	851	859	924	909	919
		10	967	963	965	996	994	995	1000	998	999
10	3	1	165	170	161	380	361	388	535	511	526
		5	896	886	890	973	970	974	992	993	993
		10	998	998	997	999	999	999	1000	1000	1000
	4	1	140	144	139	363	358	366	503	494	496
		5	920	907	924	980	975	977	993	990	993
		10	998	999	998	1000	1000	1000	1000	1000	1000
	10	1	229	225	219	461	442	464	596	586	603
		5	995	992	996	1000	999	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
20	3	1	479	471	476	728	705	719	831	813	827
		5	1000	999	999	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	4	1	465	456	471	713	689	713	816	794	815
		5	1000	1000	1000	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	10	1	657	632	649	846	838	845	908	903	906
		5	1000	1000	1000	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000

*Method of Analysis:
- I Original Data
- II Rank Transformation
- III Standard Transformation

TABLE XXXIV

POISSON DISTRIBUTION μ = 1

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	42	51	51	137	160	152	236	261	261
		5	265	310	277	587	592	568	743	734	718
		10	551	603	551	857	856	852	941	935	932
	4	1	29	36	28	122	134	121	218	222	218
		5	271	277	262	570	554	554	714	704	711
		10	630	675	629	883	891	887	958	951	951
	10	1	57	52	55	159	171	167	266	274	275
		5	467	467	441	705	702	697	817	811	804
		10	876	884	869	975	973	966	993	989	989
10	3	1	110	122	117	294	308	313	450	441	447
		5	741	751	743	950	941	937	983	976	977
		10	985	988	983	999	999	999	1000	999	999
	4	1	115	126	119	303	306	314	408	412	420
		5	789	790	773	935	935	933	976	968	970
		10	991	993	985	999	998	999	1000	1000	999
	10	1	147	145	153	362	348	359	495	488	490
		5	946	940	937	984	982	982	992	992	991
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
20	3	1	334	333	337	578	554	564	716	687	704
		5	996	996	996	999	999	999	999	999	999
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	4	1	314	307	315	570	556	565	696	678	693
		5	998	993	995	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	10	1	462	454	463	705	684	693	789	784	787
		5	1000	1000	1000	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000

*Method of Analysis:
- I Original Data
- II Rank Transformation
- III Standard Transformation

TABLE XXXV

POISSON DISTRIBUTION $\mu = 5$

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	55	61	54	186	200	185	294	297	291
		5	289	340	284	634	647	630	791	771	785
		10	676	717	671	922	914	920	974	969	970
	4	1	39	38	36	144	158	141	248	253	244
		5	364	396	360	670	661	660	802	791	802
		10	751	759	740	932	927	928	977	973	973
	10	1	67	64	60	204	190	201	308	306	311
		5	602	601	590	844	831	835	902	902	898
		10	966	960	963	996	996	995	1000	998	1000
10	3	1	150	163	154	362	358	358	505	472	492
		5	856	861	845	969	965	965	988	985	985
		10	994	995	995	1000	999	1000	1000	1000	1000
	4	1	147	149	148	370	348	362	504	493	502
		5	897	899	892	973	975	973	992	987	991
		10	999	999	998	1000	1000	1000	1000	1000	1000
	10	1	204	193	196	454	438	446	596	561	576
		5	983	980	984	996	997	996	998	997	999
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
20	3	1	415	405	419	658	652	648	785	759	772
		5	999	997	998	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	4	1	421	419	406	659	640	663	780	764	781
		5	999	999	999	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	10	1	579	549	570	813	780	804	878	863	877
		5	1000	1000	1000	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000

*Method of Analysis:
- I Original Data
- II Rank Transformation
- III Standard Transformation

TABLE XXXVI

POISSON DISTRIBUTION $\mu = 10$

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	38	57	44	180	205	180	315	295	314
		5	364	428	356	704	707	694	843	822	835
		10	690	700	688	927	921	929	976	966	975
	4	1	39	47	37	157	168	161	263	273	257
		5	369	382	364	675	679	676	824	801	814
		10	766	757	767	943	941	942	983	980	983
	10	1	69	73	73	215	214	211	326	328	325
		5	647	634	636	854	844	854	910	912	909
		10	967	965	968	994	992	993	997	996	998
10	3	1	175	185	181	394	388	396	517	505	526
		5	872	867	869	975	963	974	993	987	992
		10	997	999	999	1000	1000	1000	1000	1000	1000
	4	1	167	166	158	380	371	371	517	500	509
		5	907	907	904	971	972	970	985	983	983
		10	999	999	999	1000	1000	1000	1000	1000	1000
	10	1	236	215	226	460	440	453	567	552	574
		5	987	985	986	994	994	994	998	997	998
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
20	3	1	459	433	454	742	730	748	847	828	842
		5	999	999	999	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	4	1	498	465	496	719	705	711	823	812	822
		5	1000	1000	1000	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	10	1	614	582	613	830	810	830	906	892	909
		5	1000	1000	1000	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000

*Method of Analysis:
- I Original Data
- II Rank Transformation
- III Standard Transformation

TABLE XXXVII

NEGATIVE BINOMIAL DISTRIBUTION k = 1, μ = 1

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	18	44	39	70	117	115	158	204	205
		5	110	169	113	351	391	354	537	553	512
		10	303	402	299	638	696	606	801	811	749
	4	1	22	28	28	80	111	107	154	180	180
		5	125	158	121	328	383	320	490	531	445
		10	359	418	330	671	712	637	822	825	771
	10	1	32	29	27	103	115	120	179	202	215
		5	210	230	185	466	482	423	599	611	556
		10	632	675	552	864	872	789	930	933	876
10	3	1	47	64	63	160	188	188	270	281	286
		5	468	512	417	765	776	710	872	861	827
		10	879	912	824	979	977	953	995	990	984
	4	1	39	51	56	168	164	173	282	272	271
		5	483	538	437	767	792	736	877	871	830
		10	880	909	814	985	982	957	993	992	990
	10	1	67	65	74	197	196	208	319	281	290
		5	735	733	666	915	903	862	955	952	916
		10	988	990	964	998	997	995	1000	999	999
20	3	1	133	161	171	369	354	372	553	482	495
		5	938	936	898	992	985	974	996	996	991
		10	1000	1000	997	1000	1000	1000	1000	1000	1000
	4	1	150	147	154	353	336	344	497	473	482
		5	954	944	915	993	988	972	997	995	989
		10	1000	1000	999	1000	1000	1000	1000	1000	1000
	10	1	199	160	172	437	361	375	582	491	516
		5	996	993	987	999	999	997	1000	999	998
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000

*Method of Analysis:
- I Original Data
- II Rank Transformation
- III Standard Transformation

TABLE XXXVIII

NEGATIVE BINOMIAL DISTRIBUTION $k = 1, \mu = 5$

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	21	33	33	87	105	96	155	177	165
		5	128	180	121	384	425	303	555	576	461
		10	299	378	221	651	703	520	820	829	695
	4	1	13	31	26	72	98	86	147	173	162
		5	134	169	112	364	395	299	507	529	439
		10	365	428	289	671	716	569	825	835	714
	10	1	31	32	28	95	109	106	173	196	184
		5	243	264	159	500	499	381	644	637	530
		10	637	667	451	863	857	721	932	934	816
10	3	1	51	65	55	165	195	170	275	286	264
		5	504	554	401	781	792	669	885	864	774
		10	883	891	712	978	978	908	992	988	958
	4	1	41	55	45	172	168	161	277	269	265
		5	506	541	377	785	786	663	880	866	786
		10	910	918	753	987	985	940	997	994	976
	10	1	68	60	59	201	175	162	302	287	279
		5	728	728	531	904	891	766	954	945	866
		10	991	988	942	1000	997	988	1000	1000	996
20	3	1	131	152	128	388	353	327	541	487	468
		5	939	930	844	995	988	960	1000	995	983
		10	1000	1000	995	1000	1000	998	1000	1000	1000
	4	1	170	146	135	372	327	309	510	467	442
		5	943	937	846	991	989	949	999	999	980
		10	1000	1000	996	1000	1000	999	1000	1000	1000
	10	1	223	181	162	461	392	357	594	524	483
		5	995	993	955	998	998	990	1000	999	995
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000

*Method of Analysis:
- I Original Data
- II Rank Transformation
- III Standard Transformation

TABLE XXXIX

NEGATIVE BINOMIAL DISTRIBUTION k = 1, μ = 10

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	14	35	23	89	128	111	160	198	176
		5	125	177	101	375	431	279	529	553	441
		10	352	409	237	688	734	556	834	841	711
	4	1	16	22	13	88	93	90	163	185	168
		5	158	185	126	369	406	305	525	543	445
		10	368	425	248	684	719	535	827	831	687
	10	1	23	19	23	83	99	87	173	171	177
		5	242	271	162	504	513	348	643	650	493
		10	642	669	420	858	868	699	932	926	813
10	3	1	47	71	60	158	173	164	277	283	254
		5	485	525	353	775	761	626	884	859	743
		10	895	910	718	981	976	901	990	993	958
	4	1	45	43	43	161	165	151	261	269	253
		5	525	548	383	803	789	642	899	872	756
		10	903	915	762	984	985	919	995	992	965
	10	1	77	68	58	205	179	157	341	300	267
		5	735	737	513	898	893	762	951	942	859
		10	994	995	934	1000	1000	985	1000	1000	996
20	3	1	151	153	125	381	349	316	540	465	438
		5	944	937	810	992	986	946	1000	994	975
		10	999	1000	990	1000	1000	998	1000	1000	1000
	4	1	153	143	118	381	334	302	548	458	429
		5	950	947	823	993	991	947	999	998	976
		10	1000	1000	998	1000	1000	1000	1000	1000	1000
	10	1	228	174	139	466	398	344	585	503	468
		5	999	996	946	1000	1000	981	1000	1000	990
		10	1000	1000	999	1000	1000	1000	1000	1000	1000

*Method of Analysis:
- I Original Data
- II Rank Transformation
- III Standard Transformation

TABLE XL

LOGARITHMIC DISTRIBUTION $\mu = 5$

			Significance Level					
			1%		5%		10%	
t	b	m	I*	II	I	II	I	II
5	3	1	3	20	41	83	93	144
		5	50	110	201	265	335	383
		10	107	197	396	475	557	621
	4	1	6	14	33	62	76	123
		5	67	99	187	249	322	365
		10	152	214	395	469	578	642
	10	1	21	17	70	73	124	132
		5	116	126	278	315	405	446
		10	274	338	564	598	700	726
10	3	1	17	25	73	97	146	165
		5	203	302	524	547	683	687
		10	550	663	858	886	945	947
	4	1	13	32	66	101	151	184
		5	238	292	500	531	671	649
		10	578	646	850	853	926	925
	10	1	37	22	111	106	196	179
		5	366	351	635	622	767	759
		10	846	866	961	957	981	977
20	3	1	37	57	181	191	326	287
		5	673	681	903	891	964	935
		10	977	972	998	997	1000	999
	4	1	44	36	155	142	279	251
		5	658	670	893	870	947	923
		10	978	975	996	995	998	996
	10	1	72	49	223	159	325	253
		5	881	842	972	946	987	974
		10	1000	1000	1000	1000	1000	1000

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE XLI

LOGARITHMIC DISTRIBUTION $\mu = 10$

			Significance Level					
			1%		5%		10%	
t	b	m	I*	II	I	II	I	II
5	3	1	5	21	37	75	81	128
		5	39	86	167	251	298	366
		10	122	191	378	443	552	592
	4	1	8	17	39	78	93	138
		5	52	78	178	246	331	360
		10	158	217	406	477	556	609
	10	1	21	4	54	54	90	116
		5	84	88	239	248	369	374
		10	269	318	538	572	676	692
10	3	1	3	20	56	75	131	146
		5	151	225	438	467	623	590
		10	517	619	831	836	916	909
	4	1	11	23	74	101	176	166
		5	194	210	480	458	616	597
		10	587	625	845	851	935	914
	10	1	29	22	97	92	187	166
		5	328	324	610	572	728	694
		10	802	795	936	922	972	962
20	3	1	37	52	167	141	289	239
		5	613	634	874	816	938	886
		10	970	965	996	993	999	997
	4	1	37	46	168	144	300	226
		5	675	625	885	834	936	891
		10	977	974	997	996	999	998
	10	1	74	42	198	146	334	240
		5	858	778	958	922	978	955
		10	998	998	1000	999	1000	1000

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

APPENDIX F

POWER OF THE F-TEST FOR THE EQUALITY OF BLOCK MEANS WHEN THE TREATMENT MEANS ARE NOT EQUAL

TABLE XLII

BINOMIAL DISTRIBUTION n = 100, μ = 1

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	39	45	34	124	151	127	216	235	226
		5	193	209	178	452	457	431	610	586	590
		10	412	427	350	769	751	713	882	859	848
	4	1	25	39	35	133	137	120	220	229	206
		5	181	193	161	442	445	401	586	574	550
		10	467	463	420	771	749	711	858	850	835
	10	1	30	41	34	127	135	131	219	225	204
		5	298	299	280	588	569	538	710	699	667
		10	743	711	682	910	904	885	957	943	944
10	3	1	71	82	62	227	217	214	346	345	338
		5	604	610	559	839	829	806	913	898	891
		10	931	922	904	987	984	984	997	995	997
	4	1	78	82	69	223	233	222	339	335	321
		5	620	621	582	832	814	799	899	895	870
		10	958	940	926	995	989	990	999	997	997
	10	1	109	99	98	262	250	245	385	388	360
		5	847	830	780	945	943	920	977	974	956
		10	998	995	997	1000	1000	1000	1000	1000	1000
20	3	1	233	229	208	495	483	454	630	625	608
		5	975	963	950	993	998	993	1000	998	998
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	4	1	228	227	207	460	457	416	615	595	561
		5	979	968	968	998	995	994	1000	998	999
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	10	1	307	285	260	525	510	502	652	635	616
		5	1000	997	994	1000	1000	999	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE XLIII

BINOMIAL DISTRIBUTION n = 100, $\mu = 5$

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	38	52	38	155	168	157	259	256	258
		5	269	273	262	562	547	557	722	694	712
		10	579	569	511	879	849	870	946	920	945
	4	1	41	45	40	160	172	162	266	273	261
		5	327	305	319	631	614	622	770	754	756
		10	668	633	662	910	876	909	967	948	963
	10	1	62	63	58	187	180	176	283	291	277
		5	518	482	506	777	759	763	861	854	865
		10	909	886	905	981	975	978	999	989	996
10	3	1	108	126	114	314	313	317	442	425	448
		5	770	752	769	935	916	938	977	965	978
		10	991	988	991	1000	999	1000	1000	1000	1000
	4	1	139	144	139	334	318	334	454	444	453
		5	843	817	843	958	947	957	982	975	983
		10	996	993	996	1000	999	1000	1000	1000	1000
	10	1	160	159	163	363	362	354	510	485	502
		5	963	954	965	994	990	993	998	999	998
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
20	3	1	406	397	403	647	635	637	760	745	759
		5	996	993	994	1000	999	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	4	1	396	366	390	618	607	619	745	734	738
		5	998	995	999	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	10	1	492	497	506	729	710	733	827	812	810
		5	1000	1000	1000	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE XLIV

BINOMIAL DISTRIBUTION n = 100, μ = 10

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	57	64	51	178	177	183	315	298	304
		5	293	315	296	644	614	643	789	756	789
		10	672	644	670	915	891	914	976	947	973
	4	1	36	46	39	166	169	161	267	264	269
		5	332	326	329	630	613	637	786	762	776
		10	727	686	733	930	911	927	970	958	967
	10	1	55	61	58	186	181	186	307	296	304
		5	583	559	586	810	783	816	889	870	883
		10	954	929	951	989	987	992	995	994	995
10	3	1	137	144	142	369	357	374	506	512	509
		5	832	819	837	963	950	961	985	979	983
		10	994	994	993	999	999	999	1000	1000	1000
	4	1	142	132	139	362	347	354	504	487	504
		5	869	842	866	967	954	964	987	979	988
		10	997	996	997	999	998	1000	1000	999	1000
	10	1	190	189	190	393	385	398	545	526	543
		5	985	974	983	999	997	998	999	998	999
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
20	3	1	419	420	413	680	671	683	806	777	800
		5	995	994	994	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	4	1	399	394	398	638	641	646	774	738	776
		5	1000	1000	1000	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	10	1	576	559	578	796	761	795	868	847	866
		5	1000	1000	1000	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000

*Method of Analysis:
- I Original Data
- II Rank Transformation
- III Standard Transformation

TABLE XLV

POISSON DISTRIBUTION $\mu = 1$

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	34	48	38	124	137	132	200	208	211
		5	178	203	179	435	439	447	600	591	608
		10	390	419	403	726	724	730	845	846	840
	4	1	38	42	40	123	135	127	212	224	211
		5	213	216	196	478	464	481	631	627	632
		10	478	473	467	781	759	773	872	864	876
	10	1	44	44	44	134	138	145	224	239	236
		5	319	315	306	594	591	596	736	721	722
		10	723	710	722	915	904	910	949	946	943
10	3	1	72	95	83	228	240	226	349	347	356
		5	577	578	585	837	815	818	914	894	904
		10	949	933	929	993	987	989	998	996	996
	4	1	70	70	65	224	217	216	341	345	331
		5	634	614	612	866	850	849	923	923	923
		10	943	938	937	988	981	987	995	993	997
	10	1	96	109	106	260	269	262	385	377	378
		5	825	816	821	949	940	944	974	967	974
		10	999	998	999	1000	1000	1000	1000	1000	1000
20	3	1	246	236	249	462	460	462	588	589	600
		5	963	962	960	995	993	993	997	998	997
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	4	1	219	215	213	431	432	428	582	559	576
		5	971	969	971	997	997	997	1000	998	999
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	10	1	308	296	294	536	523	529	679	645	661
		5	1000	1000	1000	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000

*Method of Analysis:
- I Original Data
- II Rank Transformation
- III Standard Transformation

TABLE XLVI

POISSON DISTRIBUTION $\mu = 5$

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	44	66	46	138	153	141	235	232	231
		5	249	273	248	587	563	591	725	709	731
		10	559	545	576	875	846	876	954	928	950
	4	1	33	39	30	141	150	142	247	237	242
		5	285	279	288	580	561	582	733	714	728
		10	660	626	651	888	873	884	945	935	940
	10	1	59	59	55	183	188	183	281	283	280
		5	495	467	480	751	729	749	844	830	844
		10	911	883	902	987	975	983	995	992	994
10	3	1	123	133	125	288	296	288	413	413	420
		5	772	758	759	940	933	942	979	961	976
		10	988	973	988	1000	994	1000	1000	1000	1000
	4	1	110	135	124	291	296	302	432	432	433
		5	794	755	787	942	922	938	977	966	978
		10	990	986	989	997	995	999	1000	1000	1000
	10	1	160	155	157	354	347	354	489	476	476
		5	955	943	951	989	985	989	996	994	997
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
20	3	1	376	371	377	643	629	634	762	743	756
		5	994	992	994	999	999	999	1000	999	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	4	1	360	343	349	582	592	592	714	704	711
		5	993	995	994	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	10	1	445	437	442	705	690	708	807	781	799
		5	1000	1000	1000	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000

*Method of Analysis:
- I Original Data
- II Rank Transformation
- III Standard Transformation

TABLE XLVII

POISSON DISTRIBUTION μ = 10

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	50	67	50	177	173	176	289	271	288
		5	307	322	310	631	604	633	772	746	770
		10	646	632	643	909	892	910	963	951	965
	4	1	39	49	40	141	159	144	261	263	262
		5	338	344	334	652	626	658	792	764	787
		10	677	654	668	909	880	910	964	943	960
	10	1	66	64	63	197	184	193	291	284	289
		5	539	513	540	782	770	787	881	860	878
		10	952	936	951	994	985	995	998	996	998
10	3	1	146	148	147	333	330	332	464	449	468
		5	852	820	851	961	948	965	983	972	979
		10	990	987	991	999	999	999	1000	999	1000
	4	1	128	140	124	315	308	317	452	441	455
		5	848	829	853	962	948	963	979	976	981
		10	995	990	995	1000	1000	1000	1000	1000	1000
	10	1	173	170	171	388	378	393	529	506	534
		5	977	970	977	993	994	994	998	998	999
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
20	3	1	393	374	390	680	656	680	765	755	767
		5	997	994	998	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	4	1	388	390	382	634	626	629	750	735	750
		5	998	998	998	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000
	10	1	553	518	551	765	753	761	840	833	838
		5	1000	1000	1000	1000	1000	1000	1000	1000	1000
		10	1000	1000	1000	1000	1000	1000	1000	1000	1000

*Method of Analysis:
- I Original Data
- II Rank Transformation
- III Standard Transformation

TABLE XLVIII

NEGATIVE BINOMIAL DISTRIBUTION k = 1, μ = 1

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	15	24	19	64	102	90	125	150	150
		5	65	83	68	203	235	203	319	338	317
		10	149	185	139	402	442	354	570	584	516
	4	1	14	18	17	62	80	74	124	148	140
		5	60	79	56	185	229	185	319	334	315
		10	189	219	161	431	474	389	592	608	529
	10	1	15	18	16	72	69	70	126	151	145
		5	98	117	90	279	296	263	406	423	380
		10	263	277	218	513	532	458	655	680	603
10	3	1	27	29	30	96	118	121	189	190	183
		5	228	261	203	502	512	448	656	641	586
		10	543	568	479	816	813	741	898	892	850
	4	1	24	29	33	110	124	127	197	210	203
		5	233	262	194	493	505	426	643	635	555
		10	565	598	506	798	807	751	891	892	850
	10	1	20	24	25	109	118	117	203	199	189
		5	327	346	276	584	599	506	721	719	654
		10	760	793	676	927	934	872	968	970	925
20	3	1	64	66	63	205	202	196	334	311	309
		5	652	673	575	855	859	791	927	921	863
		10	963	963	920	991	992	986	999	1000	995
	4	1	69	85	85	215	214	216	349	327	329
		5	632	642	546	847	839	774	916	910	864
		10	963	968	915	996	996	982	999	999	993
	10	1	98	84	83	227	229	232	341	337	331
		5	829	835	723	934	938	887	966	964	939
		10	997	995	989	1000	999	1000	1000	1000	1000

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE XLIX

NEGATIVE BINOMIAL DISTRIBUTION $k = 1$, $\mu = 5$

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	13	27	22	52	86	82	115	150	148
		5	53	73	58	203	236	174	338	351	296
		10	164	209	128	432	444	326	589	599	493
	4	1	16	21	15	61	91	72	121	151	137
		5	81	95	61	214	247	182	344	364	306
		10	174	198	101	411	431	309	569	566	476
	10	1	20	22	15	73	86	82	141	150	147
		5	102	114	79	286	297	217	413	419	352
		10	295	304	198	579	587	421	716	713	576
10	3	1	25	27	25	106	142	122	205	218	212
		5	239	262	184	507	536	433	651	662	572
		10	548	579	407	820	820	699	903	906	800
	4	1	23	40	36	112	115	118	198	210	205
		5	224	254	182	495	513	394	637	648	531
		10	588	624	428	854	842	710	915	905	817
	10	1	36	34	30	117	125	116	209	199	195
		5	344	362	233	617	617	450	741	752	591
		10	794	809	635	937	931	834	965	959	913
20	3	1	74	81	66	240	229	212	357	338	322
		5	614	633	492	856	858	746	932	921	839
		10	958	963	890	993	991	963	997	997	984
	4	1	71	81	66	226	206	211	342	331	306
		5	647	661	500	856	860	731	929	923	823
		10	968	967	884	997	997	961	999	1000	986
	10	1	98	71	61	267	220	207	371	328	309
		5	835	837	652	953	945	854	977	975	905
		10	998	999	981	1000	1000	996	1000	1000	998

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE L

NEGATIVE BINOMIAL DISTRIBUTION $k = 1, \mu = 10$

			Significance Level								
			1%			5%			10%		
t	b	m	I*	II	III	I	II	III	I	II	III
5	3	1	17	30	20	55	92	70	120	158	132
		5	54	91	48	226	272	182	355	388	309
		10	146	189	103	419	447	314	581	584	472
	4	1	12	25	17	63	81	75	127	140	134
		5	60	84	54	208	238	169	332	347	279
		10	170	201	115	428	431	324	578	582	464
	10	1	19	11	13	71	66	64	135	126	131
		5	106	108	65	282	317	220	434	435	322
		10	287	289	175	557	564	419	695	709	549
10	3	1	20	34	26	125	135	118	222	210	202
		5	232	273	173	502	214	386	661	650	541
		10	587	619	403	845	830	697	915	897	806
	4	1	25	41	30	95	119	109	170	196	186
		5	220	254	183	476	489	391	619	615	519
		10	606	628	425	832	832	701	916	914	805
	10	1	35	32	27	127	114	102	213	203	187
		5	359	384	232	595	616	476	745	747	602
		10	845	853	623	953	957	821	967	977	890
20	3	1	69	73	71	232	218	196	358	338	316
		5	671	672	498	875	872	725	932	927	825
		10	961	954	848	991	989	962	998	997	989
	4	1	71	66	51	190	190	176	316	288	269
		5	663	659	483	866	858	723	936	920	823
		10	967	957	866	992	991	955	999	996	977
	10	1	97	76	68	249	209	191	350	323	285
		5	852	834	644	956	955	837	975	972	901
		10	999	999	973	1000	1000	997	1000	1000	999

*Method of Analysis:
- I Original Data
- II Rank Transformation
- III Standard Transformation

TABLE LI

LOGARITHMIC DISTRIBUTION $\mu = 5$

			Significance Level					
			1%		5%		10%	
t	b	m	I*	II	I	II	I	II
5	3	1	3	19	30	66	71	119
		5	16	30	87	131	191	230
		10	56	84	183	250	327	354
	4	1	2	15	27	51	53	98
		5	31	44	121	147	209	241
		10	58	81	188	234	307	350
	10	1	13	14	50	59	95	118
		5	31	37	120	128	197	234
		10	88	97	242	246	345	372
10	3	1	6	18	55	80	125	135
		5	68	78	216	237	359	371
		10	171	224	457	484	609	632
	4	1	13	22	67	79	119	137
		5	60	83	220	233	334	354
		10	207	251	433	459	582	615
	10	1	14	16	56	64	105	111
		5	130	122	271	282	399	408
		10	330	372	587	612	720	728
20	3	1	13	29	96	96	182	166
		5	236	257	501	472	657	626
		10	606	620	816	826	898	901
	4	1	25	24	86	81	156	157
		5	224	234	493	488	637	612
		10	609	620	838	840	916	915
	10	1	37	19	115	91	193	159
		5	359	344	624	583	750	686
		10	788	804	960	924	959	958

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

TABLE LII

LOGARITHMIC DISTRIBUTION $\mu = 10$

			Significance Level					
			1%		5%		10%	
t	b	m	I*	II	I	II	I	II
5	3	1	2	17	24	63	59	123
		5	27	43	104	133	182	213
		10	48	71	158	183	288	307
	4	1	3	14	28	59	66	106
		5	23	29	93	119	172	202
		10	50	67	172	205	294	315
	10	1	8	16	41	57	90	107
		5	36	31	123	128	212	208
		10	86	81	218	242	334	374
10	3	1	5	21	40	80	104	148
		5	64	92	211	231	348	346
		10	173	208	418	440	589	573
	4	1	11	15	55	78	111	145
		5	65	76	205	223	348	331
		10	156	186	396	409	533	548
	10	1	15	15	56	75	114	132
		5	83	67	226	216	344	342
		10	273	279	538	542	665	674
20	3	1	19	22	83	89	166	141
		5	192	181	443	410	590	536
		10	510	490	783	760	880	863
	4	1	13	17	88	82	162	153
		5	200	198	446	401	601	535
		10	517	536	787	763	880	863
	10	1	30	19	92	87	177	147
		5	288	226	550	464	673	602
		10	732	716	906	885	949	935

*Method of Analysis:
I Original Data
II Rank Transformation
III Standard Transformation

2

VITA

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