DEVELOPMENT AND EVALUATION OF CONTROL CHARTS USING EXPONENTIALLY WEIGHTED MOVING AVERAGES

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PREFACE

The study is concerned with the development and evaluation of control charts using the Exponentially Weighted Moving Average (EWMA) of appropriate sample statistics based upon data from a process. The primary objectives are to present methodology for constructing the control limits of control charts using the EWMA of sample statistics and to use computer simulation to determine the mean action time of these control charts. Then, the equivalent Type I error of the EWMA control charts is examined. A computer simulation program is used in the study to determine the mean action time for a particular scenario. Modification of the program is then done to facilitate the determination of mean action time for other scenarios. Comparisons of these mean action times with those of other popularly used control charts are also made.

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CHAPTER I

THE RESEARCH PROBLEM

Purpose

There are currently only a handful of articles mentioning control charts using the Exponentially Weighted Moving Average (EWMA) of certain sample statistics. Different articles have different ways of manipulating exponential smoothing methods to derive and construct control charts of the EWMA of sample statistics, such as the EWMA of the sample mean and the EWMA of individual data. All these control charts are meant to monitor the process mean only. Only three articles mention the derivation and construction of coupled control charts of the EWMA of sample statistics to monitor both the process mean and dispersion.

This thesis research presents a standard and comprehensive method of manipulating the exponential smoothing method to derive and construct control charts for the EWMA of sample statistics. Comprehensible and easily applied pairs of coupled control charts employing the EWMA are introduced to monitor the process mean and dispersion simultaneously.

The objectives of the research study are stated as follows :

(1) Illustrate the derivation of control limits of control charts using the EWMA,

(2) Illustrate the methodology of constructing pairs of coupled control charts using the EWMA of statistics from subgroups of size n,

(3) Illustrate the methodology of constructing pairs of coupled control charts using the EWMA of statistics from individual measurements,

(4) Determine the Mean Action Time of each proposed control chart under a variety of out-of-control scenarios,

(5) Compare the ability of control charts using the EWMA to detect process changes to that of $\overline{X}$ and R control charts using the AT&T run rules.

Introduction

In the industrial world, quality control charts are applicable for two general categories of processes. The first type is a process in which observed data can be collected in a subgroup of size 2 or more. The other is the process in which observed data is collected as individual samples, such as data collected from a continuous flow process.

A good quality control charting technique must be simple to construct, easy to use and easy to understand in any industrial setting. Moreover, the most important property is the ability of the control chart to detect process changes. Normally, coupled control charts such as $\overline{X}$, R charts and I, MR(2) charts, are used simultaneously to detect both changes of process mean and dispersion. The proposed control charting technique is aimed to satisfy all these properties.

The control charts developed in this study are :

(1)

(a) Control chart using the EWMA of the sample mean, EWMASM

(b) Control chart using the EWMA of the sample range, EWMASR

(2)

(a) Control chart using the EWMA of individual data, EWMAID

(b) Control chart using the EWMA of the moving range of subgroup size two of individual data, EWMAMR

The EWMA of sample statistics for control charts is evolved from the exponential smoothing method.

Exponential Smoothing

Exponential Smoothing is a forecasting technique developed by Brown (1959) [2] and is widely used in inventory control and other business problems. Exponential smoothing in forecasting is of the form :

$$\hat{Y}_{t+1} = \hat{Y}_t + \alpha e_t \qquad (1.1)$$

where

$\hat{Y}_{t+1}$ = Predicted value at time t+1

$\hat{Y}_t$ = Prior predicted value at time t

Y_t = Observed value at time t

e_t = Observed error at time t = $Y_t - \hat{Y}_t$

α = Smoothing constant ($0 \le \alpha \le 1$)

Substituting e_t in equation (1.1) results in

$$\hat{Y}_{t+1} = \hat{Y}_t + \alpha(Y_t - \hat{Y}_t) \qquad (1.2)$$

$$\hat{Y}_{t+1} = \alpha Y_t + (1 - \alpha)\hat{Y}_t \qquad (1.3)$$

Equation (1.3) is the normal expression for exponential smoothing in a forecasting application. This expression is well noted by its arithmetic simplicity. It requires only four arithmetic operations to determine an average, and in order to repeat this application only two values, α and the prior predicted value, $\hat{Y}_t$ are required to be remembered.

Exponentially Weighted Moving Averages in Quality Control Charts

Application of exponential smoothing techniques in quality control charting enables the most recent observed data to be given the greatest relative weight (α), with all the previous observed data having weights decreasing in a geometric progression from the most recent back to the first piece of data.

After modification, the exponential smoothing method used in quality control charting is:

$$\hat{Z}_t = \alpha Y_t + (1 - \alpha)\hat{Z}_{t-1} \qquad (1.4)$$

where

$\hat{Z}_t$ = EWMA of sample statistics at time t

$\hat{Z}_{t-1}$ = EWMA of sample statistics at time t-1

Y_t = Sample statistics at time t

α = A weighting factor ($0 \le \alpha \le 1$)

That is, every observed statistic Y_t is transformed into $\hat{Z}_t$. This transformed statistic $\hat{Z}_t$, is plotted on the appropriate EWMA chart. The expression in equation (1.4) enables data such as the sample mean, sample range, individual measurement and moving range of subgroup size two of individual measurements to be transformed in order to be plotted on the respective EWMA chart. The $\hat{Z}_t$ in equation (1.4) is known as the Exponentially Weighted Moving Average (EWMA) of statistic Y_t. Therefore, if the sample mean is transformed using equation (1.4), $\hat{Z}_t$ is known as the EWMA of the sample mean, and is plotted on a control chart of the EWMA of the sample mean.

The term α is known as a weighting factor since it gives relative weight to observed data. Selection of the weighting factor (α) determines the ability of the EWMA charts to detect a process shift. The smaller the value of α, the greater the influence of the past data in the decision making. The criterion to select the weighting factor can be based on the analysis of the sum of squares of errors associated with the EWMA [7].

It is noted that EWMA charts do not use each observed data values by themselves to make an independent test of the null hypothesis that a process is unchanged. Instead, information from the new data value is combined with past data value before any decision is made about a possible process change. This combination of the past and current data provides a high degree of sensitivity that will speed the detection of small process changes. Combining successive points brings these small changes to be noticed.

Therefore the EWMA chart provides a regular and formal use of historical data. Runs and all other data configurations are encompassed in the EWMA. Furthermore, once an EWMA of a certain statistic has been computed, it contains all the information provided by the historical record.

Average Age of the EWMA Data

The average age of data is the age of each data value used in the average, weighted as the data of that age would be weighted. In the exponential smoothing process, the weight of a given data k periods ago is $\alpha(1-\alpha)^k$. Hence, the average age of data in the exponential smoothing process or the average age of the EWMA data is

$$\bar{k} = 0\alpha + 1\alpha(1-\alpha) + 2\alpha(1-\alpha)^2 + \ldots$$

$$= \alpha \sum_{k=0}^{\infty} k(1-\alpha)^k$$

Simplify, we have

$$\bar{k} = \frac{(1 - \alpha)}{\alpha} \qquad (1.5)$$

Thus, it is clear that the average age of the EWMA data can be expressed in terms of the weighting factor, α.

Mean Action Time

The MAT is defined as the average number of subgroups that must be taken from a process before an "out-of-control" point will be found and corrective action can be implemented. The MAT is associated with the probability of a control chart to detect a process change. Normally, traditional Operating Characteristic Curves can be used to determine the ability of a control chart to detect a process change if the points plotted are independent of one another. Since EWMAs are used in the control chart, the probability that the current point falls beyond control limits is dependent upon the locality of the previously plotted point. Therefore, instead of talking about the risks of failing to detect a shift of a given magnitude, a better method might be an account for how much data it will take to detect the shift. Therefore MAT is better used as a measure to determine the ability of the EWMA charts to detect the process change.

Summary

This chapter presents the research objectives of this thesis. The foundation of the EWMA and exponential smoothing methods are discussed and linked to quality control charting techniques. Some other material related to the control charts using EWMAs is also briefly discussed in this chapter.

CHAPTER II

LITERATURE REVIEW

Introduction

Statistical quality control was introduced by Walter Shewhart in the 1920's and 1930's. One important tool of statistical quality control is the control chart. After Shewhart control charts were first introduced, numerous control chart techniques then developed. As no control charting technique is perfect, all the control charting techniques developed are meant to compensate for the weaknesses of one another. Isaac N. Gibra [5] and Lonnie C. Vance [11] have compiled a bibliography of statistical quality control charting techniques. This bibliography shows the width of scope in quality control charts.

Background

In 1959, Robert [9] developed a control chart using the exponential weighted moving average (EWMA) of the sample mean, EWMASM (there called the geometric moving average chart). He proposed that while making use of past data, primary consideration be given to more recently observed data. Therefore, he computes a weighted average using the exponential smoothing method with a progressively smaller weight being assigned to the older data as time passes. In his paper, he also compares the properties of control chart tests based on the EWMA with tests based on ordinary moving averages. He also includes several MAT curves (also known as Average Run Length (ARL) curves) for various weighting values (α) in his paper.

The use of the MAT to compare the ability of the Cumulative Sum Chart, the EWMA Chart and the Acceptance Control Chart to detect process shifts is considered by Freund [4]. He suggests the use of MAT rather than Operating Characteristic Curve plotting to determine the power of the EWMA chart to detect process changes. Wortham and Ringer [12] suggest the use of the EWMA to construct a control chart to monitor the variance of a process. Thus, simultaneous control charts using EWMA can be employed to monitor the process mean and variance. They and Hunter [7] separately suggest that the EWMA can be used to generate a control point for each data value.

Wortham and Heinrich [14] apply the EWMA to individual measurements. They point out that this approach may be justified when the cost of inspection is high or when expensive destructive testing is involved. Wortham [13] uses the EWMA on data from a continuous process. That is, he constructs control chart using EWMA on data from a flow process. Sweet [10] suggests two models to construct simultaneous control charts to monitor the mean and the standard deviation or variance of a process using the EWMA. These are modifications of the model proposed by Wortham and Ringer [12] to construct a control chart for the variance of a process.

Hunter [7] points out that the EWMA can be viewed as a compromise between Shewhart and CUSUM charting procedures. Perhaps more important, the EWMA can be used as a method for establishing real-time dynamic control of industrial processes. He suggests that the EWMA may be graphed simultaneously with data appearing on a Shewhart chart, and the EWMA is best plotted one time position ahead of the most recent observation. He also proposes that the weighting factor, $\alpha = 0.2$, is preferred since it provides the smaller error sum of squares. He further points out that the EWMA can be modified to enhance its ability to forecast.

Need for the Study

There are at least two good reasons why this research study should be done :

(1) Some studies, up to now, have been done to construct control charts using the EWMA. However, it has been

done implicitly for subgroups of size n and individual measurement. In this research study, methodology to construct control charts using EWMAs for subgroups of size n and individual measurements is discussed explicitly and separately.

(2) The emphasis of the control chart of the EWMA is only on the monitoring of the process mean. Even though there are articles discussing the derivation of control charts of the EWMA to monitor process dispersion, their derivations are rather difficult and not easily comprehended. Sweet [10] and Wortham [12] have two contradictory opinions on the derivation of an EWMA control chart to monitor process dispersion. Thus, in this research, a consistent and systematic approach of deriving control charts of the EWMA to monitor process dispersion is presented.

As Shewhart $\overline{X}$ and R charts are used to monitor the process mean and variance where observed data is collected in subgroups of size n; coupled control charts of the EWMA of the sample mean, EWMASM, and the sample range, EWMASR, are used for the same purpose. Similarly, control charts of individual measurements and moving range of subgroup size two are used to monitor the process mean and variance; control charts of the EWMA of individual measurements, EWMAID, and moving range of subgroup size two, EWMAMR, are used for the same purpose. This provides a systematic approach to derive the control limits as well as making EWMA control charting techniques more comprehensive and more easily applied in industrial processes.

Conclusion

Since the EWMA control charting technique is quite new to the industrial world, more studies need to be done to investigate its ability to detect process shifts. From the literature above, there is no uniformity present in the manipulation of exponential smoothing methods in the derivation of the EWMA control charts. Thus, it is also the effort of this study to present a systematic and consistent approach to apply exponential smoothing methods in the derivation of the EWMA control charts.

CHAPTER III

CONTROL CHART OF EWMA OF SAMPLE MEAN AND RANGE

Introduction

In this chapter, derivation of control limits for control charts of the EWMA of the sample mean and sample range is presented. Traditionally, in any control chart the control limits are three standard deviations from the expected value of the statistics being plotted. The proposed control charts also employ the same tradition.

Notation

$\overline{Y}_t$ = The mean of sample t

$\sigma_{\overline{Y}_t}$ = The standard deviation of sample mean

Z_t = The EWMA of t^{th} sample mean (EWMASM)

σ_{Z_t} = The standard deviation of EWMASM

α = A weighting factor, $0 \le \alpha \le 1$

$\hat{Z}_0$ = The starting point of the EWMASM = $\bar{\bar{Y}}$

$\bar{\bar{Y}}$ = The grand average of the sample mean

$\bar{Y}'$ = The true process mean

σ'_{Y_t} , σ'_{Y} = The true process standard deviation

$\bar{R}$ = The average of the sample range

n = Subgroup size

d_2 = A factor used in connection with sampling by variables that is a function of n and expresses the ratio between the expected value of R from a long series of samples from a normal universe and the σ' of that universe

CL_{SM} = The central line of EWMASM chart

UCL_{SM} = The upper control limit of EWMASM chart

LCL_{SM} = The lower control limit of EWMASM chart

F1 = Factor for constructing the LCL_{SM} and UCL_{SM}

R_t = The range of sample t

$\sigma_{\hat{R}_t}$ = The standard deviation of sample range

r_t = The EWMA of t^{th} sample range (EWMASR)

$\sigma_{\hat{r}_t}$ = The standard deviation of EWMASR

r_0 = The starting point of EWMASR = $\bar{R}$

d_3 = The ratio between the standard deviation of sample of subgroup size n and the standard deviation of the sample range

D_4 = A multiplier of $\bar{R}$ to determine the 3-sigma upper control limit on a R chart

CL_{SR} = The central line of EWMASR chart

UCL_{SR} = The upper control limit of EWMASR chart

LCL_{SR} = The lower control limit of EWMASR chart

F4 = Factor for constructing UCL_{SR}

F3 = Factor for constructing LCL_{SR}

Control Limits for EWMASM Chart

Every observed sample mean is transformed into a corresponding EWMA before it is plotted on the control chart. Let the mean of sample t be denoted as $\hat{Y}_t$. Thus, every $\hat{Y}_t$ should be transformed into a corresponding EWMA, $\hat{Z}_t$. The transformation is done by

$$\hat{Z}_t = \alpha\overline{Y}_t + (1-\alpha)\hat{Z}_{t-1}$$

To determine control limits for the EWMASM chart, the standard deviation of the transformed data, $\sigma_{\hat{Z}_t}$ is required. We know,

$$\hat{Z}_t = \alpha\overline{Y}_t + (1-\alpha)\hat{Z}_{t-1} \qquad (3.1)$$

Substituting

$$\hat{Z}_{t-1} = \alpha\overline{Y}_{t-1} + (1-\alpha)\hat{Z}_{t-2}$$

into Equation (3.1) results in

$$\hat{Z}_t = \alpha\overline{Y}_t + (1-\alpha)[\alpha\overline{Y}_{t-1} + (1-\alpha)\hat{Z}_{t-2}]$$

$$\hat{Z}_t = \alpha\overline{Y}_t + \alpha(1-\alpha)\overline{Y}_{t-1} + (1-\alpha)^2\hat{Z}_{t-2}$$

Continuously substituting the last term, we have

$$\hat{Z}_t = \alpha\overline{Y}_t + \alpha(1-\alpha)\overline{Y}_{t-1} + \alpha(1-\alpha)^2\overline{Y}_{t-2} + \alpha(1-\alpha)^3\overline{Y}_{t-3} + \ldots + \alpha(1-\alpha)^{t-1}\overline{Y}_1 + (1-\alpha)^t\hat{Z}_0$$

The term $\hat{Z}_0$ is the starting point of the EWMA of the sample mean. Now, let the average of the sample mean be the starting point, thus $\hat{Z}_0 = \bar{Y}$. Then,

$$V(\hat{Z}_0) = V(\bar{Y}) = \frac{1}{t^2} \sum_{i=1}^{t} V(\bar{Y}_i) \qquad (3.2)$$

Assuming all the observed data are independent,

$$V(\bar{Y}_t) = V(\bar{Y}_{t-1}) = \ldots = V(\bar{Y}_2) = V(\bar{Y}_1)$$

Therefore,

$$V(\hat{Z}_0) = \frac{1}{t^2}\, t\, V(\bar{Y}_t) = \frac{V(\bar{Y}_t)}{t}$$

So,

$$V(\hat{Z}_t) = \alpha^2 V(\bar{Y}_t) + \alpha^2(1-\alpha)^2 V(\bar{Y}_t) + \alpha^2(1-\alpha)^4 V(\bar{Y}_t)$$

$$+ \alpha^2(1-\alpha)^6 V(\bar{Y}_t) + \ldots + \alpha^2(1-\alpha)^{2t-2} V(\bar{Y}_t)$$

$$+ (1-\alpha)^{2t} \frac{V(\bar{Y}_t)}{t}$$

$$V(\hat{Z}_t) = \alpha^2 V(\bar{Y}_t) [1 + (1-\alpha)^2 + (1-\alpha)^4 + (1-\alpha)^6 +$$

$$\ldots + (1-\alpha)^{2t-2} + \frac{(1-\alpha)^{2t}}{t\alpha^2}]$$

$$V(\hat{Z}_t) = \alpha^2 V(\bar{Y}_t) \{ \frac{1 - [(1-\alpha)^2]^t}{1 - (1-\alpha)^2} + \frac{(1-\alpha)^{2t}}{t\alpha^2} \} \qquad (3.3)$$

As t increases, $V(\hat{Z}_t)$ approaches a limit.

$$V(\hat{Z}_t) = \alpha^2 V(\bar{Y}_t) [\frac{1}{1-(1-\alpha)^2}]$$

$$= V(\bar{Y}_t) \frac{\alpha}{2 - \alpha} \qquad (3.4)$$

Therefore,

$$\sigma_{\tilde{Z}_t} = \sigma_{\bar{Y}_t} \sqrt{\alpha / (2-\alpha)} \qquad (3.5)$$

or

$$\sigma_{\tilde{Z}_t} = \sigma_{\tilde{Y}_t} / \sqrt{n} \; (\sqrt{\alpha / (2-\alpha)}) \qquad (3.6)$$

Thus, the standard deviation of EWMASM can be determined if the process standard deviation, σ'_Y, is known (or can be estimated) and the sample size and weighting factor are determined beforehand.

An estimate of process standard deviation is

$$\sigma'_Y = \frac{\bar{R}}{d_2}$$

Therefore,

$$\sigma_{\tilde{Z}_t} = \frac{\bar{R}}{d_2 \sqrt{n}} \sqrt{\alpha / (2-\alpha)} \qquad (3.7)$$

Then the control limits of EWMASM chart : -

$$\text{Central line} = \tilde{Z}_0 = \bar{Y} \qquad (3.8)$$

$$\text{Control limits} = \tilde{Z}_0 \pm 3\,\sigma_{\tilde{Z}_t}$$

$$= \bar{Y} \pm 3 \frac{\bar{R}}{d_2 \sqrt{n}} \sqrt{\alpha / (2-\alpha)} \qquad (3.9)$$

Let $$F1 = \frac{3}{d_2 \sqrt{n}} \sqrt{\alpha / (2-\alpha)} \qquad (3.10)$$

then the control limits may be calculated as :

$$UCL_{SM} = \bar{\bar{Y}} + F1\ \bar{R} \qquad (3.11)$$

$$LCL_{SM} = \bar{\bar{Y}} - F1\ \bar{R} \qquad (3.12)$$

Values of F1 for different sample sizes and weighting factors are tabulated in Appendix B.

Control Limits for EWMASR Chart

Denote the range of sample t as R_t. Every value of R_t must be transformed into a corresponding EWMA, $\hat{r}_t$. The transformation is done by the expression

$$\hat{r}_t = \alpha R_t + (1-\alpha)\hat{r}_{t-1}$$

To determine control limits for the EWMASR chart, the standard deviation of the transformed data, $\sigma_{\hat{r}_t}$ is required. As we know,

$$\hat{r}_t = \alpha\ R_t + (1-\alpha)\hat{r}_{t-1} \qquad (3.13)$$

Substituting

$$\hat{r}_{t-1} = \alpha R_{t-1} + (1-\alpha)\hat{r}_{t-2}$$

into Equation (3.13) results in

$$\hat{r}_t = \alpha R_t + (1-\alpha)[\alpha R_{t-1} + (1-\alpha)\hat{r}_{t-2}]$$

$$\hat{r}_t = \alpha R_t + \alpha(1-\alpha)R_{t-1} + (1-\alpha)^2\hat{r}_{t-2}$$

Continuously substituting the last term, we have

$$\hat{r}_t = \alpha R_t + \alpha(1-\alpha)R_{t-1} + \alpha(1-\alpha)^2R_{t-2} + \alpha(1-\alpha)^3R_{t-3} +$$

$$\ldots + \alpha(1-\alpha)^{t-1}R_1 + (1-\alpha)^t\hat{r}_0$$

The term $\bar{r}_0$ is the starting point of the EWMA of the sample range. Now, let the average of the sample range be the starting point, thus $\bar{r}_0 = \bar{R}$. Then,

$$\hat{V}(r_0) = V(\bar{R}) = \frac{1}{t^2} \sum_{i=1}^{t} V(R_i) \qquad (3.14)$$

Assuming all the observed data are independent,

$$V(R_t) = V(R_{t-1}) = \ldots = V(R_2) = V(R_1)$$

Therefore,

$$V(\bar{r}_0) = \frac{1}{t^2}\, t\, V(R_t) = \frac{V(R_t)}{t}$$

So,

$$V(\bar{r}_t) = \alpha^2 V(R_t) + \alpha^2(1-\alpha)^2 V(R_t) + \alpha^2(1-\alpha)^4 V(R_t)$$
$$+ \alpha^2(1-\alpha)^6 V(R_t) + \ldots + \alpha^2(1-\alpha)^{2t-2} V(R_t)$$
$$+ (1-\alpha)^{2t} \frac{V(R_t)}{t}$$

$$V(\bar{r}_t) = \alpha^2 V(R_t) \, [\, 1 + (1-\alpha)^2 + (1-\alpha)^4 + (1-\alpha)^6 +$$
$$\ldots + (1-\alpha)^{2t-2} + \frac{(1-\alpha)^{2t}}{t\alpha^2} \,]$$

$$\hat{V}(r_t) = \alpha^2 V(R_t) \left\{ \frac{1 - [(1-\alpha)^2]^t}{1 - (1-\alpha)^2} + \frac{(1-\alpha)^{2t}}{t\alpha^2} \right\} \qquad (3.15)$$

As t increases, $V(\bar{r}_t)$ approaches a limit.

$$\hat{V}(r_t) = \alpha^2 V(R_t) \left[\frac{1}{1-(1-\alpha)^2} \right]$$

$$= V(R_t) \frac{\alpha}{2 - \alpha} \qquad (3.16)$$

Therefore,

$$\sigma_{\tilde{r}_t} = \sigma_{R_t} \sqrt{\alpha / (2-\alpha)} \qquad (3.17)$$

From the R chart, it is known that

$$\text{Upper control limit} = D_4\bar{R}$$

$$= R + 3\,\sigma_{R_t}$$

where $D_4 = \left(1 + 3 \dfrac{d_3}{d_2} \right)$

Then,

$$D_4\bar{R} = \left(1 + 3 \frac{d_3}{d_2} \right) \bar{R} = \bar{R} + 3\,\sigma_{R_t}$$

Therefore,

$$\frac{d_3}{d_2} \bar{R} = \sigma_{R_t} \qquad (3.18)$$

Thus,

$$\hat{\sigma}_{r_t} = \frac{d_3}{d_2} \bar{R} \sqrt{\alpha / (2-\alpha)} \qquad (3.19)$$

The control limits of the EWMASR chart are

$$\text{Central line} = \hat{r}_0 = \bar{R} \qquad (3.20)$$

$$\text{Control limits} = \hat{r}_0 \pm 3\,\sigma_{\tilde{r}_t}$$

$$= \bar{R} \pm 3 \frac{d_3}{d_2} \bar{R} \sqrt{\alpha / (2-\alpha)} \qquad (3.21)$$

If we let

$$F4 = 1 + 3 \frac{d_3}{d_2} \sqrt{\alpha / (2-\alpha)} \qquad (3.22)$$

$$F3 = 1 - 3 \frac{d_3}{d_2} \sqrt{\alpha / (2-\alpha)} \qquad (3.23)$$

then the control limits may be calculated as :

$$UCL_{SR} = F4\ \bar{R} \qquad (3.24)$$

$$LCL_{SR} = F3\ \bar{R} \qquad (3.25)$$

Values of F4 and F3 for different sample sizes and weighting factors are tabulated in Appendix B.

Numerical Illustration

Suppose 30 subgroups of size 4 each are generated from a normal process with $\bar{Y}' = 50$ and $\sigma'_{Y} = 10$. The sample mean, $\bar{Y}$ and range, R for the 30 samples are calculated. Also, the averages of the sample mean, $\bar{\bar{Y}}$ and the sample range, $\bar{R}$ are determined. Using weighting factor $\alpha = 0.2$, the EWMAs of the sample mean and sample range are further computed. All these quantities are tabulated in TABLE 3.1.

In order to develop the control chart and plot the EWMA of the sample mean, Equations (3.8), (3.11) and (3.12) derived in this chapter are used. Thus, the central line and control limits of the EWMASM chart are

$$CL_{SM} = \bar{\bar{Y}}$$

$$UCL_{SM} = \bar{\bar{Y}} + F1\bar{R}$$

$$LCL_{SM} = \bar{\bar{Y}} - F1\bar{R}$$

In this numerical example, n = 4, F1 = 0.243 (from Appendix B), $\bar{\bar{Y}} = 48.91$ and $\bar{R} = 18.69$, resulting in

TABLE 3.1

SAMPLE MEAN, SAMPLE RANGE, EWMASM AND EWMASR OF 30 SUBGROUPS OF SIZE 4 (ALPHA=0.2)

Sample Number	Y1	Y2	Y3	Y4	Sample Mean	Sample Range	EWMASM	EWMASR
1	66.19	42.09	63.65	46.54	54.62	24.10	50.05	19.77
2	46.61	52.86	35.56	38.09	43.28	17.29	48.70	19.27
3	49.30	60.19	37.26	43.16	47.48	22.93	48.45	20.01
4	58.06	52.93	53.73	54.59	54.83	5.13	49.73	17.03
5	49.92	42.12	36.44	44.33	43.20	13.49	48.42	16.32
6	51.11	37.56	59.56	58.70	51.73	22.00	49.08	17.46
7	48.80	40.21	47.10	33.21	42.33	15.59	47.73	17.08
8	59.41	41.06	61.41	47.59	52.37	20.36	48.66	17.74
9	63.81	37.03	33.84	61.62	49.07	29.97	48.74	20.18
10	40.37	44.60	47.97	62.41	48.84	22.04	48.76	20.55
11	36.28	41.75	46.30	63.80	47.03	27.51	48.42	21.95
12	37.75	48.83	60.78	38.35	46.42	23.03	48.02	22.16
13	39.87	38.99	47.97	42.78	42.40	8.98	46.89	19.53
14	56.65	58.88	49.36	37.23	50.53	21.65	47.62	19.95
15	41.84	50.08	67.53	29.21	47.17	38.33	47.53	23.63
16	51.20	42.02	48.77	42.61	46.15	9.19	47.25	20.74
17	46.91	36.96	52.70	52.67	47.31	15.74	47.27	19.74
18	44.56	47.68	59.68	37.30	47.31	22.38	47.27	20.27
19	53.59	28.46	56.50	65.51	51.01	37.05	48.02	23.62
20	41.53	50.04	40.97	43.51	44.02	9.07	47.22	20.71
21	42.98	59.15	46.16	49.17	49.37	16.17	47.65	19.80
22	53.62	63.15	64.78	45.39	56.73	19.39	49.47	19.72
23	50.14	60.66	44.39	53.02	52.05	16.26	49.98	19.03
24	56.62	48.44	54.10	54.09	53.31	8.18	50.65	16.86
25	57.89	45.52	53.25	35.67	48.08	22.22	50.14	17.93
26	50.70	64.86	52.86	55.39	55.95	14.15	51.30	17.18
27	53.13	43.69	60.42	48.21	51.37	16.72	51.31	17.08
28	42.98	43.27	48.50	38.69	43.36	9.81	49.72	15.63
29	55.71	51.71	58.10	48.22	53.43	9.88	50.46	14.48
30	52.59	54.36	47.04	32.36	46.59	22.00	49.69	15.98
			AVERAGE		48.91	18.69		

$$CL_{SM} = 48.91$$

$$UCL_{SM} = 48.91 + 0.243 \times 18.69 = 53.54$$

$$LCL_{SM} = 48.91 - 0.243 \times 18.69 = 44.37$$

Similarly, using Equations (3.20), (3.24) and (3.25) the central line and control limits of the EWMASR chart are

$$CL_{SR} = \bar{R}$$

$$UCL_{SR} = F4\ \bar{R}$$

$$LCL_{SR} = F3\ \bar{R}$$

In this numerical example, F4 = 1.427 (from Appendix B), F3 = 0.573 (from Appendix B) and $\bar{R}$ = 18.69, resulting in

$$CL_{SR} = 18.96$$

$$UCL_{SR} = 1.427 \times 18.69 = 26.67$$

$$LCL_{SR} = 0.573 \times 18.69 = 10.71$$

By plotting the EWMAs of the sample mean on the control chart, the required EWMASM chart is obtained. In a similar way, the control chart for the EWMA of sample range is obtained. They are plotted in Figures 3.1 and 3.2, respectively.

Conclusion

In this chapter, the derivation of control limits for the control charts of the EWMA of the sample mean and sample range are systematically presented. An effort is made to ensure that the step-by-step derivation of the EWMA is

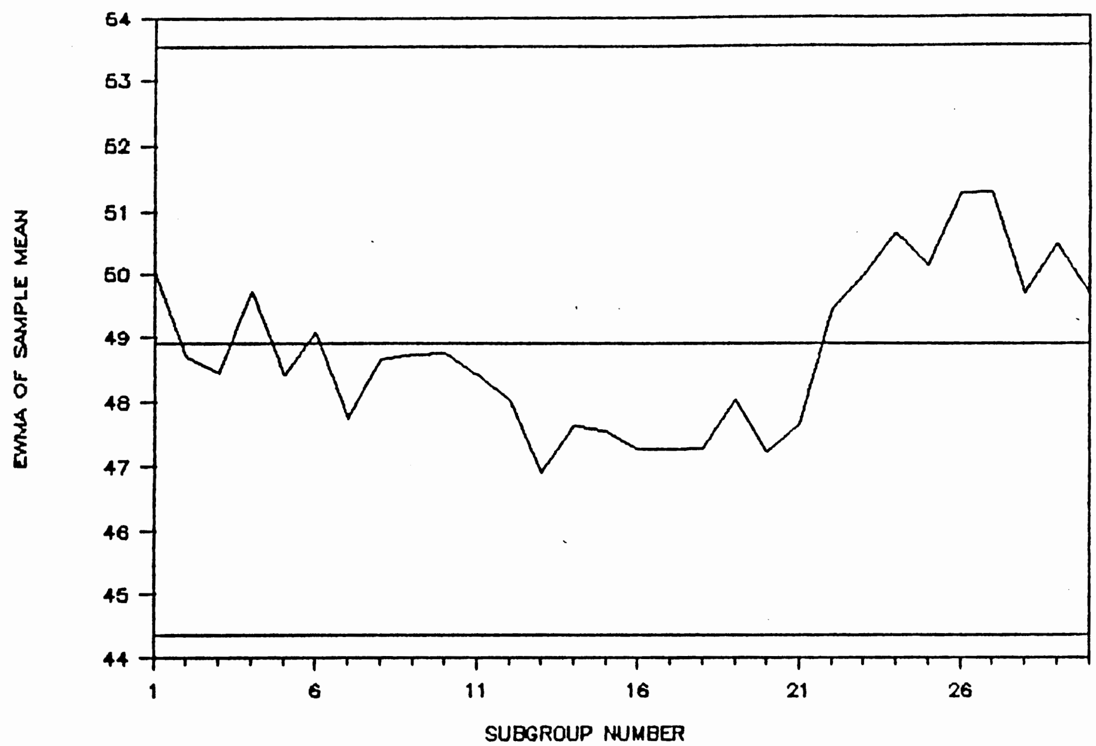

Figure 3.1 The EWMASM Chart, α = Ø.2.

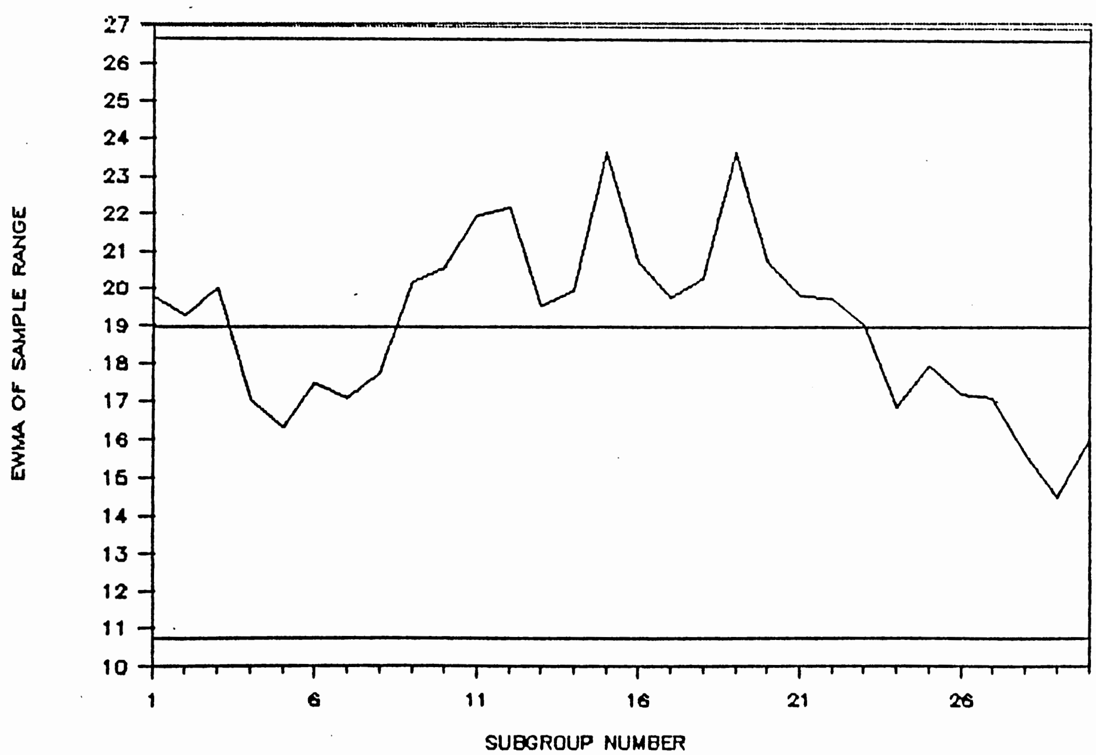

Figure 3.2. The EWMASR Chart, α = Ø.2.

consistent in both cases of the sample mean and sample range. Then, a numerical example is used to illustrate the construction of the proposed coupled EWMA charts to monitor the process mean and dispersion.

CHAPTER IV

CONTROL CHART OF EWMA OF INDIVIDUAL DATA AND MR(2) OF INDIVIDUAL DATA

Introduction

In this chapter, derivation of control limits for control charts of the EWMA of individual data and moving range of subgroup size two is presented. These are similar to traditional control charts, where the control limits are three standard deviations from the expected value of the statistics being plotted. The proposed control charts also employ the same tradition.

Notation

I_t = The t^{th} individual data value

σ_{I_t}, σ_I = The true process standard deviation

$\hat{i}_t$ = The EWMA of I_t (EWMAID)

$\sigma_{\hat{i}_t}$ = The standard deviation of EWMAID

α = A weighting factor, $0 \le \alpha \le 1$

$\hat{i}_0$ = The starting point of the EWMAID = $\bar{I}$

$\overline{I}$ = The average of individual data.

$\overline{MR}$ = The average of the moving range of subgroups of size two

d'_2 = A ratio between the expected value of the average of range of subgroup size of 2 from a long series of samples from a normal universe and the σ' of that universe

CL_{ID} = The central line of EWMAID chart

UCL_{ID} = The upper control limit of EWMAID chart

LCL_{ID} = The lower control limit of EWMAID chart

F2 = Factor for constructing the LCL_{ID} and UCL_{ID}

MR_t = The moving range of subgroup of size two at t^{th} individual data

σ_{MR_t} = The standard deviation of MR_t

m_t = The EWMA of t^{th} MR_t (EWMAMR)

σ_{m_t} = The standard deviation of EWMAMR

m_1 = The starting point of EWMAMR = $\overline{MR}$

d'_3 = The ratio between the standard deviation of sample of subgroup size two and the standard deviation of the sample range

D'_4 = A multiplier of $\overline{R}$ of subgroup size two to determine the 3-sigma upper control limit on a R chart

CL_{MR} = The central line of EWMAMR chart

UCL_{MR} = The upper control limit of EWMAMR chart

LCL_{MR} = The lower control limit of EWMAMR chart

F6 = Factor for constructing UCL_{MR}

F5 = Factor for constructing LCL_{MR}

Control Limits for EWMAID Chart

Let the individual data at t be denoted as I_t. Every value of I_t should be transformed into a corresponding EWMA, $\hat{i}_t$. The transformation is done by

$$\hat{i}_t = \alpha I_t + (1-\alpha)\hat{i}_{t-1}$$

To determine control limits for the EWMAID chart, the standard deviation of the transformed data, $\sigma_{\hat{i}_t}$ is required. As we know,

$$\hat{i}_t = \alpha I_t + (1-\alpha)\hat{i}_{t-1} \qquad (4.1)$$

Substituting

$$\hat{i}_{t-1} = \alpha I_{t-1} + (1-\alpha)\hat{i}_{t-2}$$

into Equation (4.1) results in

$$\hat{i}_t = \alpha I_t + (1-\alpha)[\alpha I_{t-1} + (1-\alpha)\hat{i}_{t-2}]$$

$$\hat{i}_t = \alpha I_t + \alpha(1-\alpha)I_{t-1} + (1-\alpha)^2\hat{i}_{t-2}$$

Continuously substituting the last term, we have

$$\hat{i}_t = \alpha I_t + \alpha(1-\alpha)I_{t-1} + \alpha(1-\alpha)^2 I_{t-2} + \alpha(1-\alpha)^3 I_{t-3} + \ldots + \alpha(1-\alpha)^{t-1}I_1 + (1-\alpha)^t\hat{i}_0$$

The term $\hat{i}_0$ is the starting point of EWMA of individual data. Now, let the average of the individual data be the starting point, thus $\hat{i}_0 = \bar{I}$. Then,

$$V(\hat{i}_0) = V(\bar{I}) = \frac{1}{t^2}\sum_{j=1}^{t} V(I_j) \qquad (4.2)$$

Assuming all the observed data are independent,

$$V(I_t) = V(I_{t-1}) = \ldots = V(I_2) = V(I_1)$$

Therefore,

$$V(\hat{i}_0) = \frac{1}{t^2}\, t\, V(I_t) = \frac{V(I_t)}{t}$$

So,

$$V(\hat{i}_t) = \alpha^2 V(I_t) + \alpha^2(1-\alpha)^2 V(I_t) + \alpha^2(1-\alpha)^4 V(I_t)$$
$$+ \alpha^2(1-\alpha)^6 V(I_t) + \ldots + \alpha^2(1-\alpha)^{2t-2} V(I_t)$$
$$+ (1-\alpha)^{2t} \frac{V(I_t)}{t}$$

$$V(\hat{i}_t) = \alpha^2\, V(I_t)\, [\, 1 + (1-\alpha)^2 + (1-\alpha)^4 + (1-\alpha)^6 +$$
$$\ldots + (1-\alpha)^{2t-2} + \frac{(1-\alpha)^{2t}}{t\alpha^2} \,]$$

$$V(\hat{i}_t) = \alpha^2\, V(I_t)\, \{ \frac{1 - [(1-\alpha)^2]^t}{1 - (1-\alpha)^2} + \frac{(1-\alpha)^{2t}}{t\alpha^2} \} \qquad (4.3)$$

As t increases, $V(\hat{i}_t)$ approaches a limit.

$$V(\hat{i}_t) = \alpha^2\, V(I_t)\, [\frac{1}{1-(1-\alpha)^2}]$$

$$= V(I_t) \frac{\alpha}{2 - \alpha} \qquad (4.4)$$

Therefore,

$$\sigma_{\hat{i}_t} = \sigma_{\hat{I}_t} \sqrt{\alpha / (2-\alpha)} \qquad (4.5)$$

Thus, the standard deviation of the EWMAID can be determined if the process standard deviation, σ_I, is known (or can be estimated) and the weighting factor is determined beforehand.

An estimate of the process standard deviation is

$$\sigma_I = \frac{\overline{MR}}{d_2'}$$

Therefore,

$$\sigma_{\tilde{I}_t} = \frac{\overline{MR}}{d_2'} \sqrt{\alpha / (2-\alpha)} \qquad (4.6)$$

Then the control limits of the EWMAID chart are

$$\text{Central line} = \tilde{i}_0 = \bar{I} \qquad (4.7)$$

$$\text{Control limits} = \tilde{i}_0 \pm 3\,\sigma_{\tilde{i}_t}$$

$$= \bar{I} \pm 3 \frac{\overline{MR}}{d_2'} \sqrt{\alpha / (2-\alpha)} \qquad (4.8)$$

Let

$$F2 = \frac{3}{d_2'} \sqrt{\alpha / (2-\alpha)} \qquad (4.9)$$

Then the control limits may be calculated as :

$$UCL_{ID} = \bar{I} + F2\,\overline{MR} \qquad (4.10)$$

$$LCL_{ID} = \bar{I} - F2\,\overline{MR} \qquad (4.11)$$

Values of F2 for different sample sizes and weighting factors are tabulated in Appendix B.

Control Limits for EWMAMR Chart

Denote the moving range of subgroup size two at sample t as MR_t. It is noted that t starts from 2. Every MR_t value must be transformed into a corresponding EWMA, $\hat{m}_t$. The transformation is done by the expression

$$\hat{m}_t = \alpha MR_t + (1-\alpha)\hat{m}_{t-1}$$

To determine control limits for the EWMAMR chart, the standard deviation of the transformed data, $\sigma_{\hat{m}_t}$ is required. As we know,

$$\hat{m}_t = \alpha MR_t + (1-\alpha)\hat{m}_{t-1} \tag{4.12}$$

Substituting

$$\hat{m}_{t-1} = \alpha MR_{t-1} + (1-\alpha)\hat{m}_{t-2}$$

into Equation (4.12) results in

$$\hat{m}_t = \alpha MR_t + (1-\alpha)[\alpha MR_{t-1} + (1-\alpha)\hat{m}_{t-2}]$$

$$\hat{m}_t = \alpha MR_t + \alpha(1-\alpha)MR_{t-1} + (1-\alpha)^2\hat{m}_{t-2}$$

Continuously substituting the last term, we have

$$\hat{m}_t = \alpha MR_t + \alpha(1-\alpha)MR_{t-1} + \alpha(1-\alpha)^2MR_{t-2} + \alpha(1-\alpha)^3MR_{t-3} +$$

$$+\ldots+ \alpha(1-\alpha)^{t-2}MR_2 + (1-\alpha)^{t-1}\hat{m}_1$$

Note that t for MR_t starts from 2. This is due to the fact that when t = 2, the second observed data value is collected and this is the time the first moving range of 2 can be determined. The term $\hat{m}_1$ is the starting point of the EWMA of MR(2) of individual data. Now, let the average of the moving range, $\overline{MR}$ be the starting point, thus $\hat{m}_1 = \overline{MR}$. Then,

$$V(\hat{m}_1) = V(\overline{MR}) = \frac{1}{(t-1)^2} \sum_{j=2}^{t} V(MR_j) \tag{4.13}$$

Assuming all the observed data are independent,

$$V(MR_t) = V(MR_{t-1}) = \ldots = V(MR_2)$$

Therefore,

$$V(\hat{m}_1) = \frac{t-1}{(t-1)^2} V(MR_t) = \frac{V(MR_t)}{t-1}$$

So,

$$V(\hat{m}_t) = \alpha^2 V(MR_t) + \alpha^2(1-\alpha)^2 V(MR_t) + \alpha^2(1-\alpha)^4 V(MR_t)$$

$$+ \alpha^2(1-\alpha)^6 V(MR_t) + \ldots + \alpha^2(1-\alpha)^{2t-4} V(MR_t)$$

$$+ (1-\alpha)^{2t-2} \frac{V(MR_t)}{t-1}$$

$$V(\hat{m}_t) = \alpha^2 V(MR_t) [1 + (1-\alpha)^2 + (1-\alpha)^4 + (1-\alpha)^6 +$$

$$\ldots + (1-\alpha)^{2t-4} + \frac{(1-\alpha)^{2t-2}}{\alpha^2 (t-1)}]$$

$$V(\hat{m}_t) = \alpha^2 V(MR_t) \{ \frac{1 - [(1-\alpha)^2]^{t-1}}{1 - (1-\alpha)^2} + \frac{(1-\alpha)^{2t-2}}{\alpha^2(t-1)} \} \tag{4.14}$$

As t increases, $V(\hat{m}_t)$ approaches a limit.

$$V(\hat{m}_t) = \alpha^2 V(MR_t) \left[\frac{1}{1-(1-\alpha)^2}\right]$$

$$= V(MR_t) \frac{\alpha}{2-\alpha} \tag{4.15}$$

Therefore,

$$\sigma_{\hat{m}_t} = \sigma_{MR_t} \sqrt{\alpha / (2-\alpha)} \tag{4.16}$$

From MR(2) chart, it is known that

Upper control limit $= D_4\overline{MR}$

$= \overline{MR} + 3\,\sigma_{MR_t}$

where $D_4 = \left(1 + 3\frac{d_3}{d_2}\right)$

Then,

$$D_4\overline{MR} = \left(1 + 3\frac{d_3}{d_2}\right)\overline{MR} = \overline{MR} + 3\,\sigma_{MR_t}$$

Therefore,

$$\frac{d_3}{d_2}\overline{MR} = \sigma_{MR_t} \tag{4.17}$$

Thus,

$$\hat{\sigma}_{m_t} = \frac{d_3}{d_2}\overline{MR}\sqrt{\alpha / (2-\alpha)} \tag{4.18}$$

The control limits of EWMAMR chart are

$$\text{Central line} = \hat{m}_1 = \overline{MR} \tag{4.19}$$

$$\text{Control limits} = \hat{m}_1 \pm 3\,\sigma_{\hat{m}_t}$$

$$= \overline{MR} \pm 3 \frac{d_3}{d_2} \overline{MR} \sqrt{\alpha / (2-\alpha)} \qquad (4.20)$$

If we let

$$F6 = 1 + 3 \frac{d_3}{d_2} \sqrt{\alpha / (2-\alpha)} \qquad (4.21)$$

$$F5 = 1 - 3 \frac{d_3}{d_2} \sqrt{\alpha / (2-\alpha)} \qquad (4.22)$$

then the control limits may be calculated as :

$$UCL_{MR} = F6\ \overline{MR} \qquad (4.23)$$

$$LCL_{MR} = F5\ \overline{MR} \qquad (4.24)$$

Values of F6 and F5 for different weighting factors are tabulated in Appendix B.

Numerical Illustration

Suppose 30 individual data values are generated from a normal process with $\overline{Y}' = 50$ and $\sigma'_Y = 10$. The moving range of subgroup size 2 for the 30 data values are calculated. And the mean of these individual data, $\overline{I}$, and the average of those moving ranges are determined. Using weighting factor, $\alpha = 0.2$, the EWMAs of individual data and moving average of subgroup size two are further computed. All these quantities are tabulated in TABLE 4.1.

In order to develop the control chart and plot the EWMA of the individual data, Equation (4.7), (4.10) and (4.11) derived in this chapter are used. Thus, the central line and control limits of the EWMAID chart are

TABLE 4.1

INDIVIDUAL DATA, MOVING RANGE OF 2, EWMAID AND EWMAMR OF 30 SUBGROUPS (ALPHA=0.2)

Sample Number	Y	Moving Range of 2	EWMAID	EWMAMR
1	42.09		46.53	8.68
2	52.86	10.77	47.79	9.10
3	60.19	7.33	50.27	8.74
4	52.93	7.26	50.80	8.45
5	42.12	10.81	49.07	8.92
6	37.56	4.56	46.77	8.05
7	40.21	2.65	45.46	6.97
8	41.06	0.85	44.58	5.74
9	37.03	4.03	43.07	5.40
10	44.60	7.58	43.37	5.84
11	41.75	2.85	43.05	5.24
12	48.83	7.07	44.20	5.61
13	38.99	9.84	43.16	6.45
14	58.88	19.89	46.31	9.14
15	50.08	8.80	47.06	9.07
16	42.02	8.06	46.05	8.87
17	36.96	5.06	44.23	8.11
18	47.68	10.72	44.92	8.63
19	28.46	19.22	41.63	10.75
20	50.04	21.59	43.31	12.92
21	59.15	9.10	46.48	12.15
22	63.15	4.01	49.81	10.52
23	60.66	2.49	51.98	8.92
24	48.44	12.21	51.27	9.58
25	45.52	2.93	50.12	8.25
26	64.86	19.34	53.07	10.47
27	43.69	21.16	51.19	12.60
28	43.27	0.42	49.61	10.17
29	51.71	8.43	50.03	9.82
30	54.36	2.65	50.90	8.39
Average	47.64	8.68		

$$CL_{ID} = \overline{I}$$

$$UCL_{ID} = \overline{I} + F2\ \overline{MR}$$

$$LCL_{ID} = \overline{I} + F2\ \overline{MR}$$

In this numerical example, F2 = 0.887 (from Appendix B), $\overline{I}$ = 47.64 and $\overline{MR}$ = 8.68, resulting in

$$CL_{ID} = 47.64$$

$$UCL_{ID} = 47.64 + 0.887x8.68 = 55.34$$

$$LCL_{ID} = 47.64 - 0.887x8.68 = 39.94$$

Similarly, using Equations (4.19), (4.23) and (4.24) the central line and control limits of the EWMAMR chart are

$$CL_{MR} = \overline{MR}$$

$$UCL_{MR} = F6\ \overline{MR}$$

$$LCL_{MR} = F5\ \overline{MR}$$

In this numerical example, F6 = 1.756 (from Appendix B), F5 = 0.244 (from appendix B), $\overline{MR}$ = 8.68, resulting in

$$CL_{MR} = 8.68$$

$$UCL_{MR} = 1.756x8.68 = 15.24$$

$$LCL_{MR} = 0.244x8.68 = 2.12$$

By plotting the EWMAs of individual data on the control chart, the required EWMAID chart is obtained. In a similar way, the EWMAMR control chart is obtained. They are plotted in Figures 4.1 and 4.2, respectively.

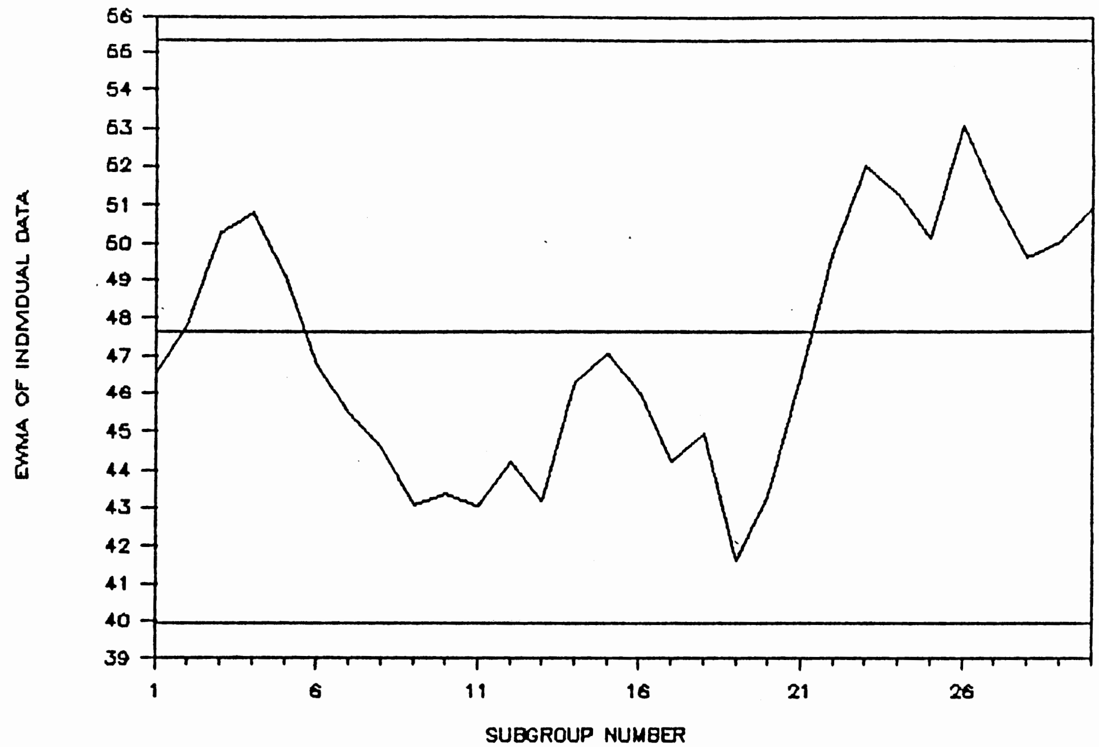

Figure 4.1 The EWMAID Chart, α = Ø.2.

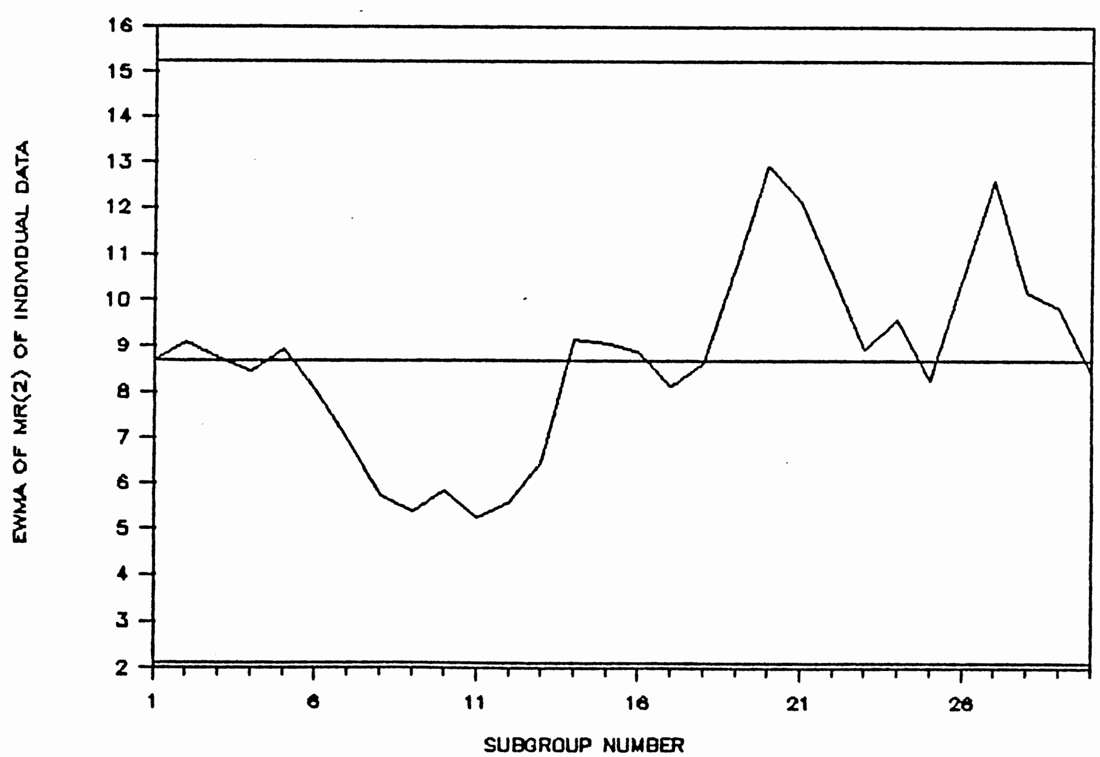

Figure 4.2. The EWMAMR Chart, α = Ø.2.

Conclusion

In this chapter, the derivation of control limits for the control charts of the EWMA of individual data and moving range of subgroup size 2 of individual data are systematically presented. An effort is made to ensure that the step-to-step derivation of the EWMA is consistent. Then, a numerical example is used to illustrate the construction of the proposed coupled EWMA charts to monitor process mean and dispersion.

CHAPTER V

MAT SIMULATION

Introduction

It has been pointed out by Freund [4] that probability is not suitable for measuring the ability of an EWMA chart of sample statistic to detect process changes. He recommends the use of the Mean Action Time (MAT) to achieve this purpose. Thus, a computer simulation is designed here to determine the MAT of each proposed control chart under a variety of conditions. An out-of-control condition will be indicated only when the EWMA plotted falls outside control limits.

A normal distribution is used to generate the process for computer simulation. For each proposed control chart, a computer simulation program is designed to determine the MAT for only one particular scenario. Modification of that program is done so that the MAT of the control chart under different scenarios can also be determined.

Notation

$\sigma'_{Y_t\ old}$, σ_{old} = The original process standard deviation, STD

$\sigma'_{Y_t\ new}$, σ_{new} = The new process standard deviation, NWSTD

$\sigma_{\bar{Y}}$ = The sample standard deviation

Simulation of MAT of EWMASM Chart

A simulation program is designed to determine the MAT of the EWMASM chart. The simulation study consists of 45 different scenarios broken down in the following manner :

1 subgroup size : n = 4

5 weighting factors : α = 0.2, 0.4, 0.6, 0.8, 1.0

9 shifts in process mean (multiples of $\sigma/\sqrt{n}$) : 0.00, 0.25, 0.50, 1.00, 1.50, 2.00, 2.50, 3.00, 4.00

In order to generalize the simulation, individual data are generated through the intrinsic normal random variate generator GGNQF. The individual data generated are standard normal variates. These data are then transformed into normal variates with mean, $\bar{Y}' = 50$ and standard deviation, $\sigma' = 10$. To embed the effect of a shift in mean (in multiples of sample standard deviation, $\sigma_{\bar{Y}}$) into the (50,10) normal variates, the sample standard deviation is multiplied by the magnitude of shift and added to the process mean. Thus, the normal variates generated for the simulation are

$$Z = GGNQF(dseed) * \sigma' + (Y' + shift * \sigma_{\bar{Y}})$$

As numbers are generated, they are then combined into subgroups of size four and the mean of sample, $\bar{Y}$ is determined. As the process mean and standard deviation are known, the control limits of the EWMASM chart can be determined. Using Equations (3.6) and (3.9), we have,

$$UCL_{SM} = \bar{Y}' + 3/\sqrt{n} \times \sigma' \times \sqrt{\alpha/(2-\alpha)}$$
$$= 50 + 3/2 \times 10 \times \sqrt{\alpha/2-\alpha)}$$
$$= 50 + 15 \times \sqrt{\alpha/(2-\alpha)}$$

$$LCL_{SM} = \bar{Y}' - 3/\sqrt{n} \times \sigma' \times \sqrt{\alpha/(2-\alpha)}$$
$$= 50 - 3/2 \times 10 \times \sqrt{\alpha/2-\alpha)}$$
$$= 50 - 15 \times \sqrt{\alpha/(2-\alpha)}$$

The exact values of the control limits depend on the value of α used.

Once a subgroup has been created, its statistic Y_t is transformed into its corresponding EWMA value. The EWMA is checked to see if it falls beyond or within the control limits. If it falls beyond the control limits, an action signal results. Then, the number of EWMAs created since the previous action signal is recorded. This number is an "action time". If the EWMA falls within the control limits, another sample is generated and transformed into its EWMA, and so on. This procedure is repeated until the 10,000th action signal results. The average of these 10,000 action times is the MAT for that particular scenario.

Since the execution of the simulation of each scenario can be quite long, only simulation for one scenario is done at a time. After obtaining the MAT for one scenario, parameters are changed to form a new scenario. It is important to point out that each scenario has the same random number seed. Thus, each scenario is subject to the same string of random numbers. This is to show that all the studies are based on the same "process".

The MAT values for the EWMASM chart are tabulated in TABLE 5.1.

TABLE 5.1

THE MAT VALUES FOR THE EWMASM CHART

	SHIFT IN MEAN (IN MULTIPLE OF SIGMA XBAR)								
ALPHA	0.00	0.25	0.50	1.00	1.50	2.00	2.50	3.00	4.00
0.2	555.54	165.42	45.01	10.85	5.62	3.80	2.92	2.40	1.88
0.4	421.52	197.78	64.10	13.48	5.67	3.44	2.48	1.97	1.41
0.6	388.21	225.23	89.32	19.25	6.88	3.65	2.41	1.79	1.25
0.8	374.43	256.94	122.24	28.57	9.58	4.42	2.61	1.79	1.19
1.0	370.47	284.41	159.56	43.66	15.15	6.28	3.26	2.00	1.19

Simulation of MAT of EAMASR Chart

This simulation study consists of the 30 different scenarios broken down in the following manner:-

1 subgroup size : n = 4

5 weighting factors : α = 0.2, 0.4, 0.6, 0.8, 1.0

6 shifts in process standard deviation ($\sigma'_{Y_t new} / \sigma'_{Y_t old}$) : 1.00, 1.20, 1.50, 2.00, 3.00, 4.00

The individual standard normal variates are generated as in (a). These data are then transformed into normal variates with mean, $\bar{Y}' = 50$ and standard deviation, $\sigma' = 10$. To embed the effect of dispersion change (in ratio of $\sigma'_{Y_t new} / \sigma'_{Y_t old}$) into the (50,10) normal variates, the original process standard deviation is multiplied by the ratio of new process standard deviation to original standard deviation. Thus, the normal variate generated for simulation is

Z = GGNQF(dseed) * (ratio * σ') + $\bar{Y}'$

As numbers are generated, they are combined into subgroups of size four, and the range of the subgroup, R, is determined. As the process mean and standard deviation are known, the control limits of the EWMASR chart can be determined. Using Equations (3.21) and $\bar{\sigma}' = \bar{R} / d_2$, we have

$$UCL_{SR} = \bar{R} + 3 \frac{d_3}{d_2} \bar{R} \sqrt{\alpha/(2-\alpha)}$$

$$= d_2 x \sigma' + 3 x \frac{d_3}{d_2} x\ d_2 x \sigma' x \sqrt{\alpha/(2-\alpha)}$$

$$= d_2 x 10 + 3 x \frac{d_3}{d_2} x\ d_2 x 10\ x \sqrt{\alpha/(2-\alpha)}$$

$$LCL_{SR} = \bar{R} - 3 \frac{d_3}{d_2} \bar{R} \sqrt{\alpha/(2-\alpha)}$$

$$= d_2 x \sigma' - 3 x \frac{d_3}{d_2} x\ d_2 x \sigma' x \sqrt{\alpha/(2-\alpha)}$$

$$= d_2 x 10 - 3 x \frac{d_3}{d_2} x\ d_2 x 10\ x \sqrt{\alpha/(2-\alpha)}$$

The exact values of the control limits depend on the value of α used.

Once a sample range has been created, it is transformed into its corresponding EWMA. Then, it follows the procedure as in (a) to determine the MAT of the EWMASR chart for different scenarios. The MAT values for the EWMASR chart are tabulated in TABLE 5.2.

TABLE 5.2

THE MAT VALUES FOR THE EWMASR CHART

	$\sigma_{new} / \sigma_{old}$					
ALPHA	1.00	1.20	1.50	2.00	3.00	4.00
0.2	512.96	32.35	7.74	3.35	1.81	1.39
0.4	324.85	30.97	7.11	2.86	1.55	1.25
0.6	250.32	31.25	7.19	2.71	1.47	1.21
0.8	215.56	32.40	7.60	2.74	1.45	1.19
1.0	202.72	34.79	8.46	2.90	1.45	1.19

Simulation of MAT of EWMAID Chart

This simulation study consists of the 45 different scenarios broken down in the following manner :

1 subgroup size : n = 4

5 weighting factors : α = 0.2, 0.4, 0.6, 0.8, 1.0

9 shifts in process mean (in multiples of σ) : 0.00, 0.25, 0.50, 1.00, 1.50, 2.00, 2.50, 3.00, 4.00

The individual standard normal variates are generated as in (a). These data are then transformed into normal variates with mean, $\overline{Y}' = 50$ and standard deviation, $\sigma' = 10$. To embed the effect of shift in mean (in multiples of σ') into the (50,10) normal variates, the σ' is multiplied by the magnitude of shift and added to the process mean. Thus, the individual normal variates generated for the simulation are

$$Z = GGNQF(dseed) * \sigma' + (\overline{Y}' + shift * \sigma')$$

As the process mean and standard deviation are known, the control limits of EWMAID chart can be determined. Using Equations (4.6) and (4.8), we have

$$\begin{aligned}
UCL_{ID} &= \overline{Y}' + 3 \times \sigma' \times \sqrt{\alpha/(2-\alpha)} \\
&= 50 + 3 \times 10 \times \sqrt{\alpha/2-\alpha)} \\
&= 50 + 30 \times \sqrt{\alpha/(2-\alpha)} \\
LCL_{ID} &= \overline{Y}' - 3 \times \sigma' \times \sqrt{\alpha/(2-\alpha)} \\
&= 50 - 3 \times 10 \times \sqrt{\alpha/2-\alpha)} \\
&= 50 - 30 \times \sqrt{\alpha/(2-\alpha)}
\end{aligned}$$

The exact values of the control limits depend on the value of α used.

Once an individual number has been created, it is transformed into its corresponding EWMA. Then it follows the procedure as in (a) to determine the MAT of the EWMAID chart for different scenarios. The MAT values for the EWMAID chart are tabulated in TABLE 5.3.

TABLE 5.3

THE MAT VALUES FOR THE EWMAID CHART

	SHIFT IN MEAN (IN MULTIPLE OF STD DEV)								
ALPHA	0.00	0.25	0.50	1.00	1.50	2.00	2.50	3.00	4.00
0.2	567.37	164.65	44.47	10.81	5.59	3.80	2.92	2.40	1.88
0.4	422.37	195.70	64.33	13.47	5.62	3.43	2.47	1.98	1.41
0.6	385.47	225.12	88.58	18.97	6.81	3.65	2.41	1.81	1.24
0.8	370.45	253.63	119.28	28.61	9.60	4.43	2.60	1.90	1.18
1.0	364.94	276.75	155.28	43.43	14.87	6.33	3.27	2.00	1.18

Simulation of MAT of EWMAMR Chart

This simulation study consists of the 30 different scenarios broken down in the following manner:-

1 subgroup size : n = 4

5 weighting factors : α = 0.2, 0.4, 0.6, 0.8, 1.0

6 shifts in process standard deviation ($\sigma'_{Y_t new} / \sigma'_{Y_t old}$) : 1.00, 1.20, 1.50, 2.00, 3.00, 4.00

The individual standard normal variates are generated as in (a). These data are then transformed into normal variates with mean, $\overline{Y}'$ = 50 and standard deviation, σ' = 10. To embed the effect of dispersion change (in ratio of $\sigma'_{Y_t new} / \sigma'_{Y_t old}$) into the (50,10) normal variates, the original process standard deviation is multiplied by the ratio of the new process standard deviation to the original standard deviation. Thus, the normal variate generated for simulation is

$$Z = GGNQF(dseed) * (ratio * \sigma') + Y'$$

As numbers are generated, they are grouped into a moving subgroup size of two, and the moving ranges of subgroup size 2, MR are determined. As the process mean and standard deviation are known, the control limits of the EWMAMR chart can be determined. Using Equations (4.20) and $\hat{\sigma}' = \overline{MR} / d_2'$

$$UCL_{MR} = \overline{MR} - 3 \frac{d_3'}{d_2'} \overline{MR} \sqrt{\alpha/(2-\alpha)}$$

$$= d_2' x \sigma' - 3 x \frac{0.880}{2.059} x\ d_2' x \sigma' x \sqrt{\alpha/(2-\alpha)}$$

$$= 2.056x10 - 3 x \frac{0.880}{2.059} x\ 2.056x10\ x \sqrt{\alpha/(2-\alpha)}$$

$$= 20.56 - 26.40 \sqrt{\alpha/(2-\alpha)}$$

$$LCL_{MR} = \overline{MR} + 3 \frac{d_3'}{d_2'} \overline{MR} \sqrt{\alpha/(2-\alpha)}$$

$$= d_2' x \sigma' + 3 x \frac{0.880}{2.059} x\ d_2' x \sigma' x \sqrt{\alpha/(2-\alpha)}$$

$$= 2.056x10 + 3 x \frac{0.880}{2.059} x\ 2.056x10\ x \sqrt{\alpha/(2-\alpha)}$$

$$= 20.56 + 26.40 \sqrt{\alpha/(2-\alpha)}$$

The exact values of the control limits depend on the value of α used.

Once a moving range has been created, it is transformed into its corresponding EWMA. Then, it follows the procedure as in (a) to determine the MAT of EWMAMR for different scenarios. The MAT values for the EWMAMR chart are tabulated in TABLE 5.4.

TABLE 5.4

THE MAT VALUES FOR THE EWMAMR CHART

	$\sigma_{new} / \sigma_{old}$					
ALPHA	1.00	1.20	1.50	2.00	3.00	4.00
0.2	157.01	42.14	15.51	7.02	3.54	2.53
0.4	101.99	33.48	13.30	6.09	3.05	2.19
0.6	92.41	31.64	12.76	5.79	2.90	2.09
0.8	98.58	33.20	13.08	5.79	2.83	2.04
1.0	120.81	38.17	14.16	6.05	2.86	2.06

Conclusion

In this MAT evaluation, substantial computer simulation has been carried out. The MAT for each control chart of the EWMA of each form of sample statistic has been determined and tabulated. These MAT values indicate the ability of the EWMA control charts to detect process changes.

For each EWMA control chart, it is easy to modify the simulation program. A complete listing of the program used for each of the four EWMA control charts is included in Appendices C through F.

CHAPTER VI

ANALYSIS OF RESULTS

Introduction

In this chapter, the results obtained from the MAT simulation are analyzed and compared with the MAT of other popularly used control charts such as $\bar{X}$ and R charts. The MAT values for the EWMA control charts are listed in TABLES 5.1 through 5.4. in the previous chapter. These control charts demonstrate a desirably high ability to detect process changes.

Tables of Results

The MAT table for each control chart consists of 5 rows and 6/9 columns. The rows show the different values of the weighting factors used, α = 0.2, 0.4, 0.6, 0.8 and 1.0. The columns of each table show the level of shift in either the process mean or standard deviation. All the other entries in the tables are the MAT of the proposed control chart for a particular combination of weighting factor and process shift.

So, in TABLE 5.1, for a scenario in which the weighting factor is 0.6 and the shift in process mean is $0.50\sigma/\sqrt{n}$, the MAT is found to be 89.32 -- that is, the mean number of samples that will be taken before an out-of-control condition is signaled is 89.32.

Due to the nature of the EWMA expression, it is found that for weighting factor, $\alpha = 1.0$, the following observations hold:

(1) The EWMASM chart with subgroup size n is equivalent to the $\bar{X}$ chart of subgroup size n.

(2) The EWMASR chart with subgroup size n is equivalent to R chart with subgroup size n.

(3) The EWMAID chart is equivalent to the I chart.

(4) The EWMAMR chart is equivalent to the MR(2) chart.

It should be noted that the I chart is a particular case of the $\bar{X}$ chart where subgroup size is unity. Thus, from (1) and (3) above, together with TABLE 5.1 and 5.3, it may be observed that the MAT of the $\bar{X}$ chart is the same regardless of subgroup size.

For $\bar{X}$ and R charts, the four run rules proposed by AT&T [1] are used to signal any process changes. In EWMA control charts, due to the interdependence of the points plotted, none of the AT&T run rules except rule 1 can be used. Thus, all the MAT values found in these tables are only associated with a plotted point falling outside control limits. This means that the MAT for $\alpha = 1.0$ in TABLE 5.1 and 5.3 are also the MAT values for the $\overline{X}$ chart when only AT&T run rule 1 is used. Likewise, the MAT for α = 1.0 in TABLE 5.2 is also the MAT value for the R chart when only AT&T rule 1 is used. Graham [6] and Raiman [8] have determined the MAT of $\overline{X}$ and R charts for different combinations of the four AT&T run rules. Their results will be used to compare against the MAT of the EWMA control charts.

From these tables, it is clear that the MAT is a very valuable tool. It is certainly not restricted as is a probability measure since it can take a value as large as is desired and a minimum value no smaller than one, because a MAT = 1 means that every point is a signal for action.

EWMA Distribution

In this research study, the parent population used to generate the process is a normal distribution with mean 50 and standard deviation 10. Even though every plotted number has gone through several "transformations", normality of the EWMA can be correctly assumed. Brown [3] has clearly shown that the output of the exponential smoothing methods with input which is normally distributed is also normally distributed. This proof is further extended to prove the normality of the EWMA of normal sample statistics. This proof is included in Appendix A.

Since the EWMA has been shown to be the sum of independent sample statistics weighted exponentially by values of α, we should expect from the central limit theorem that, for a large number of sample statistics which are not essentially normally distributed, the EWMA would be approximately normally distributed. It is also shown in Appendix A that the mean of the EWMA is the same as the mean of the sample statistic regardless of the number of subgroups. However, the standard deviation of the EWMA converges to a limit as the number of subgroups becomes larger. Thus, the distribution of the EWMA will experience a transient period before its standard deviation approaches the limit. The number of subgroups required for the standard deviation of the EWMA to achieve a certain degree of accuracy (in terms of number of decimal places in the numerical value when compared to the limit) depends on the weighting factor used to compute the EWMA.

Although the EWMA can be assumed normally distributed under all circumstances except α = 0.0 and 1.0, the 0.0027 probability value used to describe the Type I error of a normally distributed in-control process with 3 sigma control limits cannot be applied to the EWMA control charts. This is because the points plotted on the EWMA charts are not independent. Only when the weighting factor, α = 1.0, the points plotted on the EWMASM, EWMASR and EWMAID charts are independent. Therefore, the probability of a point falling outside the 3 sigma control limits of the EWMA control charts must be determined by other methods. Using simulation to determine the MAT of the EWMA control charts helps to solve this problem. The MAT when the process is in-control can be used to determine the equivalent Type I error of these control charts.

The MAT and Equivalent Type I Error

When a process is in-control, a high value of the MAT is desirable since it corresponds to a small value of Type I error. However, when a process is out-of-control, a small value of the MAT is desired. The relation of the MAT and the probability of not detecting a process change, P, is of the form

$$MAT = \frac{1}{1 - P}$$

Thus, when a process is in-control, the probability of rejecting the null hypothesis will be small if the MAT is a large value.

Comparison of Results

EWMASM Chart and $\overline{X}$ Chart

Since the EWMASM and EWMAID charts have the same MAT table, only a comparison of the EWMASM and $\overline{X}$ chart is made. From the work done by Graham [6], the MAT for 3 different combinations of AT&T run rules on an $\overline{X}$ chart are used for comparison. The combinations of AT&T rules are :

(1) Only rule 1 is used

(2) Only rules 1 and 2 are used

(3) All 4 rules are used

These 3 combinations are selected for comparison because they give good MAT results and/or they are popular in application. To ease comparison, the MAT values concerned are tabulated in TABLE 6.1. The MAT values for the EWMASM chart with $\alpha = 1.0$ are omitted since they are identical to the MAT values for the $\bar{X}$ chart when rule 1 is used. The research by Graham [6] is up to only 3.00 $\sigma/\sqrt{n}$. Therefore, the MAT for an $\bar{X}$ chart beyond this shift is not available. However, the MAT for an $\bar{X}$ chart when rule 1 is used at a shift of 4.00 $\sigma/\sqrt{n}$ is available from the EWMASM chart for α = 1.0.

TABLE 6.1

THE MAT VALUES FOR THE EWMASM AND SOME OF THE $\bar{X}$ CHARTS

		Shift in Mean (Multiples of Sigma Xbar)								
		0.00	0.25	0.50	1.00	1.50	2.00	2.50	3.00	4.00
EWMASM Chart,	= 0.2	555.54	165.42	45.01	10.85	5.62	3.80	2.92	2.40	1.88
	0.4	421.52	197.78	64.10	13.48	5.67	3.44	2.48	1.97	1.41
	0.6	388.21	225.23	89.32	19.25	6.88	3.65	2.41	1.79	1.25
	0.8	374.43	256.94	122.24	28.57	9.58	4.42	2.61	1.79	1.19
Xbar Chart with										
rule 1		369.22	284.24	155.64	44.55	15.00	6.35	3.22	2.00	1.19
rule 1,2		222.78	157.45	77.04	20.18	7.26	3.65	2.29	1.68	N/A
rule 1,2,3,4		92.59	57.83	27.24	9.28	4.89	3.14	2.20	1.66	N/A

From this table, the MAT of the EWMASM chart shows that α = 0.2 is the most powerful among other control charts under comparison when the process is in-control. That is, the EWMASM chart with α = 0.2 has a desirably high MAT value of 555.54 when the process is in-control (that is, a shift in mean in multiples of $\sigma/\sqrt{n}$ = 0.00). The $\bar{X}$ chart with all 4 rules used has the smallest MAT value of 92.59. This shows that the EWMASM chart with α = 0.2 gives an excellent protection when the process is in-control, but the $\bar{X}$ chart with all 4 rules used signals a false alarm every 92.59 samples. This is very costly and may have negative psychological effects on the person who uses this $\bar{X}$ chart to monitor any process.

For a shift of 0.25 $\sigma/\sqrt{n}$ in the process mean, the $\bar{X}$ chart with all 4 rules used has the desired smallest MAT value of 57.83 which indicates that this chart is the most powerful tool to detect this magnitude of shift. At this magnitude of shift, the performances of the EWMASM charts are not comparable to the $\bar{X}$ chart with all 4 rules used. From TABLE 6.1, it is clear that the $\bar{X}$ chart with all 4 rules used is the most powerful chart to detect any magnitude of shift in process mean. Ironically, it has the largest Type I error. Thus, for a steady process, the user of this chart has to prepare to bear this unfavorable Type I risks.

Excluding the $\bar{X}$ chart with all 4 rules used from the comparison, it is found that for a shift of 0.5 $\sigma/\sqrt{n}$ in the process mean, the EWMASM charts with α = 0.2 becomes the most favorable as its MAT value is the smallest. This EWMASM chart with α = 0.2 attains the highest level of performance until the magnitude of shift in mean increases to 2.0 $\sigma/\sqrt{n}$. For a mean shift of 2.0 $\sigma/\sqrt{n}$, the EWMASM chart with α = 0.4 has the best performance. For further increase in the shift in mean, the $\overline{X}$ chart with rules 1 and 2 used becomes the most powerful chart.

Therefore, it is clear that for a user who is interested in detecting a shift of 2.00 $\sigma/\sqrt{n}$ in the process mean and at the same time maintaining a small Type I error, the EWMASM chart with α = 0.4 is the most suitable control chart to be used. Thus, the usefulness of the EWMASM chart depends on the objective of the user.

EWMASR Chart and R Chart

From the research by Raiman [8], the R charts of three different combinations of AT&T run rules are selected for comparison. The 3 combinations of AT&T run rules are : -

(1) Only rule 1 is used

(2) Only rules 1 and 4 are used

(3) All 4 rules are used

These 3 combinations are selected because they give good MAT results and/or they are popular in application. To ease comparison, the MAT values concerned are tabulated in TABLE 6.2. The MAT values for the EWMASR chart with α = 1.0 are

TABLE 6.2

THE MAT VALUES FOR THE EWMASR AND SOME OF THE R CHARTS

		$\sigma_{new} / \sigma_{old}$					
		1.00	1.20	1.50	2.00	3.00	4.00
EWMASR Chart, =	0.2	512.96	32.35	7.74	3.35	1.81	1.39
	0.4	324.85	30.97	7.11	2.86	1.55	1.25
	0.6	250.32	31.25	7.19	2.71	1.47	1.21
	0.8	215.56	32.40	7.60	2.74	1.45	1.19
R Chart with							
rule 1		199.19	34.42	8.46	2.92	1.46	1.19
rule 1,4		108.17	28.17	7.69	2.89	1.46	N/A
rule 1,2,3,4		54.37	18.86	5.95	2.62	1.44	N/A

omitted since they are identical to the MAT for a R chart when rule 1 is used. The research by Raiman [8] is up to only 3 times the ratio of new standard deviation to original standard deviation, the MAT values for a R chart beyond this shift are not available. However, the MAT for a R chart when rule 1 is used at a shift of $4x\frac{\sigma_{new}}{\sigma_{old}}$ is obtained from the EWMASR chart for $\alpha = 1.0$.

From this table, the MAT of the EWMASR chart shows that $\alpha = 0.2$ is the most powerful among other control charts under comparison when the process is in-control. That is, the EWMASR chart with $\alpha = 0.2$ has a desirably high MAT value of 512.96 when the process is in-control (that is, a 0 % shift in dispersion). The R chart with all 4 rules used has the smallest MAT value of 54.37. This shows that the EWMASR chart with $\alpha = 0.2$ gives excellent protection when the process is in-control, but the R chart with all 4 rules used signals a false alarm every 54.37 subgroups.

For a 20 % shift (or $\sigma_{new} / \sigma_{old}$ = 1.20) in the process dispersion, the R chart with all 4 rules used has the desired smallest MAT value of 18.86 which indicates that this chart is the most powerful tool to detect this magnitude of shift. At this magnitude of shift, the performances of the EWMASR charts are not comparable to the R chart with all 4 rules used. From TABLE 6.2, it is clear that the R chart with all 4 rules used is the most powerful chart to detect any magnitude of shift in process dispersion. Similar to the $\bar{X}$ chart with all 4 rules used, this R chart has the largest Type I error. Thus, for a steady process, the user of this chart has to prepare to bear this unfavorable Type I risks.

Excluding the R chart with all 4 rules used from the comparison, it is found that for a 20 % shift in the process dispersion, the EWMASR chart with α = 0.4 becomes the most favorable as its MAT value of 30.97 is the smallest. This EWMASR chart with α = 0.4 attains the highest level of performance until the magnitude of shift in the process dispersion increase to 200 %. For a 200 % shift in the process dispersion the EWMASR chart with α = 0.6 has the best MAT value of 2.71. For further increase in the shift in process dispersion, the EWMASR chart with α = 0.8 becomes the most powerful chart.

It is clear that for a user who is interested in detecting a 150 % shift in the process dispersion and at the same time maintaining a small Type I error, the EWMASR chart with α = 0.4 is the most suitable control chart to be used. Thus, similarly, the usefulness of the EWMASR chart depends on the objective of the user.

<u>EWMAMR Chart and MR(2) Chart</u>

Before this study, the MAT of a MR(2) chart had not been determined by anybody. It is known that the MAT of the EWMAMR chart with α = 1.0 is equivalent to the MAT of a MR(2) chart. Hence, in this comparison, the MAT values of EWMAMR charts with α = 0.2, 0.4, 0.6 and 0.8 are compared with the MAT of an EWMAMR with α = 1.0 which is also the MAT value of a MR(2) chart. To ease comparison, TABLE 5.4 is reproduced here as TABLE 6.3, but with some modification.

TABLE 6.3

THE MAT VALUES FOR THE EWMAMR
AND THE MR(2) CHARTS

		$\sigma_{new} / \sigma_{old}$					
		1.00	1.20	1.50	2.00	3.00	4.00
EWMAMR Chart, =	0.2	157.01	42.14	15.51	7.02	3.54	2.53
	0.4	101.99	33.48	13.30	6.09	3.05	2.19
	0.6	92.41	31.64	12.76	5.79	2.90	2.09
	0.8	98.58	33.20	13.08	5.79	2.83	2.04
MR(2) Chart		120.81	38.17	14.16	6.05	2.86	2.06

From this table, the MAT of the EWMAMR chart shows that $\alpha = 0.2$ is the most powerful among other control charts under comparison when the process is in-control. That is, the EWMAMR chart with $\alpha = 0.2$ has a desirably high MAT value of 157.01 when the process is in-control (that is, a 0 % shift in dispersion). The EWMAMR chart with $\alpha = 0.6$ has the smallest MAT value of 92.41. This shows that the EWMAMR chart with $\alpha = 0.2$ gives an excellent protection when the process is in-control, but the EWMAMR chart with $\alpha = 0.6$ signals false alarm every 92.41 subgroup.

For a 20 % shift in the process dispersion, the EWMAMR chart with $\alpha = 0.6$ has the desired smallest MAT value of 31.64 which indicates that this chart is the most powerful tool to detect this magnitude of shift. This EWMAMR chart with $\alpha = 0.6$ attains the highest level of performance until the magnitude of shift in process dispersion increases to 200 %. For a 200 % shift in the process dispersion the EWMAMR charts with $\alpha = 0.6$ and 0.8 equally have the best MAT value of 5.79. For further increase in the shift in the process dispersion, the EWMAMR chart with $\alpha = 0.8$ becomes the most powerful chart.

Thus, it is clear that for a range of 20-200 % shift in process dispersion, the EWMAMR with $\alpha = 0.6$ is the best control chart to detect this magnitude of shift. However, for a range of a 200-400 % shift in the process dispersion, the EWMAMR with $\alpha = 0.8$ is the most powerful chart to signal the shift of dispersion. For an in-control process, both EWMAMR charts have largest Type I error. Therefore the usefulness of the EWMAMR chart to detect a shift in the process dispersion depends on the objective of the user.

Conclusion

This chapter explains the results obtained in the MAT simulation. The results are analyzed and compared with the MAT of the $\overline{X}$, R and MR(2) charts. Analysis and comparisons are done in terms of MAT values. It is clear that the usefulness of the EWMA charts to user depends on the objectives of the user in the detection of level of process changes.

CHAPTER VII

CONCLUSIONS

This research study shows that a systematic and consistent derivation of the EWMA of variables is possible and may be more understandable than that derived by Wortham [12] and Sweet [10]. This study also introduces the factors used in constructing control charts for the EWMA of a sample statistic. These factors are F1, F3 and F4 for subgroups of size n and F2, F5 and F6 for individual data.

The MAT simulation carried out in this study serves as a mean to compare the ability of an EWMA control chart with $\bar{X}$ and R control charts using the AT&T run rules. The parent population of the process generated in the simulation is from a normal distribution. This research also illustrates that for a normal process, its EWMA is also normally distributed. For a process which is not normally distributed, by using the central limit theorem, its EWMA can be considered normally distributed.

Since no chart is able to perform at superior levels in all situations, other factors need to be taken into consideration to determine the usefulness of the control chart under certain circumstances. Further study can be geared to determine the optimal weighting factor used in those proposed control charts and how each proposed control chart responds to instantaneous or other pattern shifts of the process mean or dispersion. The economic design of the proposed control chart should be done in order to enhance its ability as an appreciable control chart.

Generally speaking, control charts of the EWMA of sample statistics, especially the sample mean, sample range, individual data, and moving range of individual data, provide the best protection against a false indication of an out-of-control process. These charts do respond quite well to process changes. Effort should be made to let people be aware and appreciate the ability of the EWMA control charts. These control charts should be used whenever possible, particularly when there is a need to compensate the weaknesses of other control charts in detecting desired level of shift in process mean or/and dispersion.

REFERENCE

(1) AT&T - Statistical Quality Control Handbook, AT&T, Indianapolis, 1956,

(2) Brown, Robert G.- Statistical Forecasting for Inventory Control, Mcgraw-Hill Book Co., New York, 1959,

(3) Brown, Robert G.- Smoothing, Forecasting and Prediction of Discrete Time Series, Prentice-Hall, Inc., N.J., 1962,

(4) Freund, Richard A. - "Graphical Process Control", Industrial Quality Control, Vol. 18, No. 7, 1962,

(5) Gibra, Isaac N. - "Recent Developments in Control Chart Techniques", Journal of Quality Control, Vol. 7-8, 1975-76,

(6) Graham, J.Mark - Mean Action Times for Xbar Charts and Western Electric Rules, Master's Degree Project, OSU, 1986,

(7) Hunter, J. Stuart - "The Exponentially Weighted Moving Averages", Journal of Quality Control, Vol. 18,

(8) Raiman, Laura - A Simulation of Mean Action Times on The R Chart Using The Western Electric Run Rules, Master's Degree Project, OSU, 1986,

(9) Robert, S.W. - "Control Chart Based on Geometric Moving Averages", Technometrics, Vol. 1, No 3, 1959,

(10) Sweet, Arnold L. - "Control Chart Using Coupled EWMA", IIE Transactions, Vol. 18, March 1986,

(11) Vance, Lonnie C. - "A Bibliography of Statistical Quality Control Chart Techniques, 1970-1980", Journal of Quality Control, Vol. 15, No. 2, 1983,

(12) Wortham, A.W. and L.J. Ringer - "Control via Exponential Smoothing", The Transportation and Logistic Review, 7, 32, 1971,

(13) Wortham, A.W. - "The Use of Exponentially Smoothing Data in Continuous Process Control", Int. J. Prod. Res., Vol. 10, No. 4, 1972,

(14) Wortham, A.W. and G.F. Heinrich - "Control Chart Using Exponential Smoothing Techniques", Annual Technical Conference Transactions of ASQC, 26, 1972.

APPENDICES

APPENDIX A

USING THE CHARACTERISTIC FUNCTION TO PROVE THE NORMALITY OF THE EWMA

USING THE CHARACTERISTIC FUNCTION TO PROVE THE NORMALITY OF THE EWMA

Let Y_t = The t^{th} sample statistic with mean m and standard deviation σ

$\overline{Y}_t$ = Average of sample statistics with mean m and standard deviation $\sigma/\sqrt{t}$

Z_t = EWMA of the t^{th} sample statistic

α = A weighting factor, $0 < \alpha < 1$

Then

$$Z_t = \alpha Y_t + \alpha(1-\alpha)Y_{t-1} + \alpha(1-\alpha)^2 Y_{t-2} + \ldots + \alpha(1-\alpha)^{t-2}Y_2 + \alpha(1-\alpha)^{t-1}Y_1 + (1-\alpha)^t Y_0 \quad (1)$$

$$= \sum_{k=0}^{t-1} \alpha(1-\alpha)^k Y_{t-k} + (1-\alpha)^t Y_0$$

Let $g(k) = \alpha(1-\alpha)^k$, $h(t) = (1-\alpha)^t$ and $Y_0 = \overline{Y}_t$

Therefore,

$$Z_t = \sum_{k=0}^{t-1} g(k)Y_{t-k} + h(t)\overline{Y}_t \quad (2)$$

Let the characteristic function of the sample statistic be $F_y(jw)$, thus

$$F_y(jw) = E(e^{-jwY}) \quad (3)$$

Let the characteristic function of EWMA of sample statistic to be $F_z(jw)$, thus

$$F_z(jw) = E(e^{-jwZ}) \quad (4)$$

Substituting Equation (2) into (4) results in

$$F_z(jw) = E\{\exp-[\, jw\, (\sum_{k=0}^{t-1} g(k)Y_{t-k} + h(t)\overline{Y}_t)\,]\}$$

$$= E\{\exp-[\ jw\ (\sum_{k=0}^{t-1} g(k)Y_{t-k}\ ]\} \times E\{\ \exp-[\ jw\ h(t)\bar{Y}_t\ ]\}$$

Since the successive values of Y_t are independent, therefore

$$F_x(jw) = \prod_{k=0}^{t-1} F_y[jwg(k)]\ \bar{F_y}[jwh(t)] \tag{5}$$

If Y_t is normally distributed with mean m and standard deviation σ, then,

$$F_y(jw) = \exp - (\frac{\sigma^2 w^2}{2} + jwm) \tag{6}$$

$$\bar{F_y}(jw) = \exp - (\frac{\sigma^2 w^2}{2t} + jwm\ ) \tag{7}$$

Subtituting Equations (6) and (7) into (5) results in

$$F_x(jw) = \prod_{k=0}^{t-1} \{\ \exp-[\ \frac{\sigma^2 w^2}{2} g^2(k) + jwmg(k)]\ \} \times$$

$$\{\ \exp-[\ \frac{\sigma^2 w^2}{2t} h^2(t) + jwmh(t)]\ \}$$

$$F_x(jw) = \{\ \exp-[\ \frac{\sigma^2 w^2}{2} \sum_{k=0}^{t-1} g^2(k) + jwm \sum_{k=0}^{t-1} g(k)]\ \} \times$$

$$\{\ \exp-[\ \frac{\sigma^2 w^2}{2t} h^2(t) + jwmh(t)]\ \}$$

$$F_x(jw) = \exp-\{[\ \frac{\sigma^2 w^2}{2} (\ \sum_{k=0}^{t-1} g^2(k) + \frac{h^2(t)}{t}\ )] +$$

$$[\ jwm(\ \sum_{k=0}^{t-1} g(k) + h(t)\ ]\} \tag{8}$$

Therefore, by comparison of the characteristic functions, we have proved that the EWMA of sample statistic Y is normally distributed with mean,

$$\bar{Z} = m \left[\sum_{k=0}^{t-1} g(k) + h(t) \right] \tag{9}$$

and

$$\sigma^2_z = \sigma^2 \left[\sum_{k=0}^{t-1} g^2(k) + \frac{h^2(t)}{t} \right] \tag{10}$$

It is realized that the term $[\sum_{k=0}^{t-1} g(k) + h(t)]$ is equal to 1 for any value of t. Thus, the expected value of an EWMA is the expected value of its sample statistic. However, the standard deviation of the EWMA depends on the number of sample collected. This is because the term

$$\left[\sum_{k=0}^{t-1} g^2(k) + \frac{h^2(t)}{t} \right] = \sum_{k=0}^{t-1} \alpha^2(1-\alpha)^{2k} + \frac{(1-\alpha)^{2t}}{t}$$

$$= \alpha^2 \left[\frac{1-(1-\alpha)^{2t}}{1-(1-\alpha)^2} + \frac{(1-\alpha)^{2t}}{t} \right] \tag{11}$$

The right hand side of Equation (11) converges to a limit as t becomes larger.

$$\lim_{t \to \infty} \alpha^2 \left[\frac{1-(1-\alpha)^{2t}}{1-(1-\alpha)^2} + \frac{(1-\alpha)^{2t}}{t} \right] = \frac{\alpha^2}{2\alpha - \alpha^2} = \frac{\alpha}{2 - \alpha}$$

Thus, for a large t, the standard deviation of EWMA is equal to the standard deviation of its sample statistic multiplied the term $\sqrt{\alpha / (2-\alpha)}$.

APPENDIX B

TABLES OF FACTORS

TABLE B.1

FACTORS OF CONTROL LIMITS FOR THE EWMASM, F1

NUMBER OF OBSERVATIONS IN SAMPLE, N	VALUE OF ALPHA										
	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
2	0.000	0.431	0.627	0.790	0.940	1.085	1.231	1.380	1.535	1.701	1.880
3	0.000	0.235	0.341	0.430	0.512	0.591	0.670	0.751	0.835	0.925	1.023
4	0.000	0.167	0.243	0.306	0.365	0.421	0.477	0.535	0.595	0.659	0.729
5	0.000	0.132	0.192	0.242	0.289	0.333	0.378	0.423	0.471	0.522	0.577
6	0.000	0.111	0.161	0.203	0.242	0.279	0.316	0.354	0.394	0.437	0.483
7	0.000	0.096	0.140	0.176	0.210	0.242	0.274	0.307	0.342	0.379	0.419
8	0.000	0.086	0.124	0.157	0.137	0.215	0.244	0.274	0.305	0.337	0.373
9	0.000	0.077	0.112	0.142	0.169	0.195	0.221	0.247	0.275	0.305	0.337
10	0.000	0.071	0.103	0.129	0.154	0.178	0.202	0.226	0.251	0.279	0.308
11	0.000	0.065	0.095	0.120	0.143	0.165	0.187	0.209	0.233	0.258	0.285
12	0.000	0.061	0.089	0.112	0.133	0.154	0.174	0.195	0.217	0.241	0.266
13	0.000	0.057	0.083	0.105	0.125	0.144	0.163	0.183	0.203	0.225	0.249
14	0.000	0.054	0.078	0.099	0.118	0.136	0.154	0.172	0.192	0.213	0.235
15	0.000	0.051	0.074	0.094	0.112	0.129	0.146	0.164	0.182	0.202	0.223
16	0.000	0.049	0.071	0.089	0.106	0.122	0.139	0.156	0.173	0.192	0.212
17	0.000	0.047	0.068	0.085	0.102	0.117	0.133	0.149	0.166	0.184	0.203
18	0.000	0.045	0.065	0.081	0.097	0.112	0.127	0.142	0.158	0.175	0.194
19	0.000	0.043	0.062	0.079	0.094	0.108	0.122	0.137	0.153	0.169	0.187
20	0.000	0.041	0.060	0.076	0.090	0.104	0.118	0.132	0.147	0.163	0.180
21	0.000	0.040	0.058	0.073	0.087	0.100	0.113	0.127	0.141	0.156	0.173
22	0.000	0.038	0.056	0.070	0.084	0.096	0.109	0.123	0.136	0.151	0.167
23	0.000	0.037	0.054	0.068	0.081	0.094	0.106	0.119	0.132	0.147	0.162
24	0.000	0.036	0.052	0.066	0.079	0.091	0.103	0.115	0.128	0.142	0.157
25	0.000	0.035	0.051	0.064	0.077	0.088	0.100	0.112	0.125	0.138	0.153

TABLE B.2

FACTORS OF UPPER CONTROL LIMIT FOR THE EWMASR, F4

NUMBER OF OBSERVATIONS IN SAMPLE, N	VALUE OF ALPHA										
	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
2	1.000	1.520	1.756	1.953	2.134	2.310	2.485	2.665	2.852	3.052	3.269
3	1.000	1.361	1.525	1.661	1.787	1.909	2.030	2.155	2.285	2.423	2.574
4	1.000	1.294	1.427	1.539	1.641	1.740	1.839	1.941	2.047	2.160	2.292
5	1.000	1.256	1.371	1.468	1.557	1.643	1.729	1.818	1.910	2.000	2.114
6	1.000	1.230	1.335	1.422	1.502	1.580	1.657	1.737	1.820	1.908	2.004
7	1.000	1.212	1.300	1.388	1.462	1.534	1.605	1.678	1.755	1.836	1.924
8	1.000	1.198	1.288	1.363	1.432	1.499	1.566	1.634	1.705	1.781	1.864
9	1.000	1.187	1.272	1.343	1.408	1.471	1.534	1.599	1.666	1.738	1.816
10	1.000	1.178	1.259	1.326	1.388	1.449	1.509	1.570	1.634	1.703	1.777
11	1.000	1.171	1.248	1.313	1.372	1.430	1.487	1.546	1.608	1.673	1.744
12	1.000	1.164	1.239	1.301	1.358	1.414	1.469	1.526	1.585	1.648	1.716
13	1.000	1.159	1.231	1.291	1.346	1.400	1.453	1.508	1.565	1.626	1.693
14	1.000	1.154	1.224	1.282	1.335	1.387	1.439	1.492	1.548	1.607	1.671
15	1.000	1.150	1.217	1.274	1.326	1.377	1.427	1.479	1.533	1.590	1.652
16	1.000	1.146	1.212	1.267	1.318	1.367	1.416	1.467	1.519	1.575	1.636
17	1.000	1.143	1.207	1.261	1.311	1.359	1.407	1.456	1.507	1.562	1.621
18	1.000	1.140	1.203	1.255	1.304	1.351	1.398	1.446	1.497	1.550	1.608
19	1.000	1.137	1.199	1.250	1.298	1.344	1.390	1.437	1.487	1.539	1.596
20	1.000	1.134	1.195	1.246	1.293	1.338	1.383	1.430	1.478	1.530	1.585
21	1.000	1.132	1.192	1.242	1.287	1.332	1.376	1.422	1.469	1.520	1.575
22	1.000	1.130	1.188	1.238	1.283	1.326	1.370	1.415	1.462	1.512	1.565
23	1.000	1.128	1.186	1.234	1.278	1.321	1.364	1.409	1.455	1.504	1.557
24	1.000	1.126	1.183	1.230	1.274	1.317	1.359	1.402	1.448	1.496	1.548
25	1.000	1.124	1.180	1.227	1.271	1.312	1.354	1.397	1.442	1.489	1.541

TABLE B.3

FACTORS OF LOWER CONTROL LIMIT FOR THE EWMASR, F3

NUMBER OF OBSERVATIONS IN SAMPLE, N	VALUE OF ALPHA										
	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
2	1.000	0.480	0.244	0.047	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	1.000	0.639	0.244	0.339	0.213	0.091	0.000	0.006	0.000	0.000	0.000
4	1.000	0.706	0.573	0.461	0.359	0.260	0.161	0.059	0.000	0.000	0.000
5	1.000	0.744	0.629	0.532	0.443	0.357	0.271	0.182	0.090	0.000	0.000
6	1.000	0.770	0.665	0.578	0.498	0.420	0.343	0.263	0.180	0.092	0.000
7	1.000	0.788	0.692	0.612	0.538	0.466	0.395	0.322	0.245	0.164	0.076
8	1.000	0.802	0.712	0.637	0.568	0.501	0.434	0.366	0.295	0.219	0.136
9	1.000	0.813	0.728	0.657	0.592	0.529	0.466	0.401	0.334	0.262	0.184
10	1.000	0.822	0.741	0.674	0.612	0.551	0.491	0.430	0.366	0.297	0.223
11	1.000	0.829	0.752	0.687	0.628	0.570	0.513	0.454	0.392	0.327	0.256
12	1.000	0.836	0.761	0.699	0.642	0.586	0.531	0.474	0.415	0.352	0.284
13	1.000	0.841	0.769	0.709	0.654	0.600	0.547	0.492	0.435	0.374	0.307
14	1.000	0.846	0.776	0.718	0.665	0.613	0.561	0.508	0.452	0.393	0.329
15	1.000	0.850	0.783	0.726	0.674	0.623	0.573	0.521	0.467	0.410	0.348
16	1.000	0.854	0.789	0.733	0.682	0.633	0.584	0.533	0.481	0.425	0.364
17	1.000	0.857	0.793	0.739	0.689	0.641	0.593	0.544	0.493	0.438	0.379
18	1.000	0.860	0.797	0.745	0.696	0.649	0.602	0.554	0.503	0.450	0.392
19	1.000	0.863	0.801	0.750	0.702	0.656	0.610	0.563	0.513	0.461	0.404
20	1.000	0.866	0.805	0.754	0.707	0.662	0.617	0.570	0.522	0.470	0.415
21	1.000	0.868	0.008	0.758	0.713	0.668	0.624	0.578	0.531	0.480	0.425
22	1.000	0.870	0.812	0.762	0.717	0.674	0.630	0.585	0.539	0.488	0.435
23	1.000	0.872	0.814	0.766	0.722	0.679	0.636	0.591	0.545	0.496	0.443
24	1.000	0.874	0.817	0.770	0.726	0.683	0.641	0.598	0.552	0.504	0.452
25	1.000	0.876	0.820	0.773	0.729	0.688	0.646	0.603	0.558	0.511	0.459

TABLE B.4

FACTORS OF CONTROL LIMITS FOR THE EWMAID, F2

VALUE OF ALPHA	F2
0.00	0.000
0.10	0.160
0.20	0.887
0.30	1.117
0.40	1.330
0.50	1.536
0.60	1.741
0.70	1.952
0.80	2.172
0.90	2.406
1.00	2.660

TABLE B.5

FACTORS OF UPPER CONTROL LIMIT FOR THE EWMAMR, F6

VALUE OF ALPHA	F6
0.00	1.000
0.10	1.520
0.20	1.756
0.30	1.953
0.40	2.134
0.50	2.310
0.60	2.485
0.70	2.665
0.80	2.852
0.90	3.052
1.00	3.269

TABLE B.6

FACTORS OF LOWER CONTROL LIMIT FOR THE EWMAMR, F5

VALUE OF ALPHA	F5
0.00	1.000
0.10	0.480
0.20	0.244
0.30	0.047
0.40	0.000
0.50	0.000
0.60	0.000
0.70	0.000
0.80	0.000
0.90	0.000
1.00	0.000

APPENDIX C

LISTING OF COMPUTER PROGRAM USED TO DETERMINE THE MAT OF THE EWMASM CHART

```
**** TSO FOREGROUND HARDCOPY ****
DSNAME=U11563A.MODELA.CNTL

//U11563A JOB (11563,111-11-1111),'NG',CLASS=4,TIME=(15,0),
// MSGCLASS=X,NOTIFY=*
/*PASSWORD ????
/*JOBPARM ROOM=N
/*ROUTE PRINT LOCAL
// EXEC FORTVCLG
//FORT.SYSIN DD *
C
C           SIMULATION PROGRAM TO DETERMINE THE MEAN ACTION TIME
C                  OF CONTROL CHART FOR EWMA OF SAMPLE MEAN
C
C           SAMPLE SIZE : 4         WEIGHTING FACTOR : 0.2
C           SHIFT IN MEAN (IN MULTIPLE OF SIGMA XBAR) : 0.0
C           SIMULATION RUN : 10000
C
C           TO COMPUTE MAT FOR OTHER SITUATION, ALTER THE VALUE OF
C           VARIABLES  ALPHA, NSIZE AND SHIFT
C
      DOUBLE PRECISION DSEED,SUM,MAT
      REAL NUM,LCL,NMEAN,MEAN
C
      READ *,MEAN,STD
      PRINT *,'ORIGINAL PROCESS MEAN                    ',MEAN
      PRINT *,'ORIGINAL PROCESS STANDARD DEVIATION      ',STD
      PRINT *
C
C           WEIGHTING FACTOR , ALPHA
C           SAMPLE SIZE   , NSIZE
C
      ALPHA = 0.2
      NSIZE = 4
      A     = NSIZE
      SQR   = SQRT(A)
      B     = 3. / SQR
      C     = ALPHA / (2. - ALPHA)
      D     = SQRT(C)
      E     = B * STD * D
      UCL   = MEAN + E
      LCL   = MEAN - E
C
C           SHIFT IN MEAN (IN MULTIPLE OF SIGMA XBAR)
C           NEW MEAN , NMEAN
C
      SHIFT = 0.0
      NMEAN = MEAN + (SHIFT * STD / SQR)
C
      DSEED = 8064697.
      SUM   = 0
      DO 200 NTRIAL = 1, 10000
         EWMA = MEAN
         NUM  = 0
50       ZBAR = 0
         DO 100 I = 1, NSIZE
            Z     = GGNQF(DSEED) * STD + NMEAN
            ZBAR = Z + ZBAR
100      CONTINUE
         ZBAR = ZBAR / A
         EWMA = ALPHA * ZBAR + (1. - ALPHA) * EWMA
         IF ((EWMA .GE. LCL) .AND. (EWMA .LE. UCL)) THEN
```

```
                  NUM = NUM + 1
                  GO TO 50
               ELSE
                  NUM = NUM + 1
                  SUM = SUM + NUM
            ENDIF
200     CONTINUE
C
C             MEAN ACTION TIME , MAT
C
         MAT = SUM / (NTRIAL - 1.)
C
        WRITE (6,300) ALPHA,NSIZE,SHIFT,MAT
300     FORMAT (1X,'THE WEIGHTING FACTOR         = ',F10.2/
     $           1X,'THE SAMPLE SIZE             = ',I10/
     $           1X,'THE SHIFT IN MEAN           = ',F10.2,' SIGMA XBAR'//
     $           1X,'THE MEAN ACTION TIME OF    '/
     $           1X,'CONTROL CHART FOR EWMA     '/
     $           1X,'OF SAMPLE MEAN              = ',F10.2/)
C
        STOP
        END
//GO.SYSIN DD *
50,10
//
```

APPENDIX D

LISTING OF COMPUTER PROGRAM USED TO DETERMINE THE MAT OF THE EWMASR CHART

```
**** TSO FOREGROUND HARDCOPY ****
DSNAME=U11563A.MODELB.CNTL

//U11563A JOB (11563,111-11-1111),'NG',CLASS=4,TIME=(15,0),
// MSGCLASS=X,NOTIFY=*
/*PASSWORD ????
/*JOBPARM ROOM=N
/*ROUTE PRINT LOCAL
// EXEC FORTVCLG
//FORT.SYSIN DD *
C
C          SIMULATION PROGRAM TO DETERMINE THE MEAN ACTION TIME
C                OF CONTROL CHART FOR EWMA OF SAMPLE RANGE
C
C          SAMPLE SIZE : 4          WEIGHTING FACTOR : 0.2
C          RATIO OF NEW STD DEV / STD DEV : 1.0
C          SIMULATION RUN : 10000
C
C          TO COMPUTE MAT FOR OTHER SITUATION, ALTER THE VALUE OF
C          VARIABLES ALPHA,NSIZE,SD2 AND RATIO
C          IF NSIZE IS CHANGED, THE VALUE OF SD2 AND THE WAY TO
C          DETERMINE THE SAMPLE RANGE MUST BE CHANGED ACCORDINGLY
C
      DIMENSION Z(10)
      DOUBLE PRECISION DSEED,SUM,MAT
      REAL NUM,LCL,MEAN,NWSTD
C
      READ *,MEAN,STD
      PRINT *,'ORIGINAL PROCESS MEAN                       ',MEAN
      PRINT *,'ORIGINAL PROCESS STANDARD DEVIATION         ',STD
      PRINT *
C
C          WEIGHTING FACTOR , ALPHA
C          SAMPLE SIZE   , NSIZE
C          CONTROL CHART FACTOR , SD2 (DEPENDENT UPON NSIZE)
C
      ALPHA = 0.2
      NSIZE = 4
      SD3   = 0.880
      SD2   = 2.059
      B     = ALPHA / (2. - ALPHA)
      C     = SQRT(B)
      D     = SD3 / SD2
      E     = 3. * C * D
      F4    = 1. + E
      F3    = 1. - E
      IF (F3 .LE. 0) THEN  F3 = 0
      RBAR  = SD2 * STD
      UCL   = F4 * RBAR
      LCL   = F3 * RBAR
C
C          NEW PROCESS STANDARD DEVIATION, NWSTD
C          NWSTD / STD = RATIO
C
      RATIO = 1.0
      NWSTD = RATIO * STD
C
      DSEED = 8064697.
      SUM   = 0
      DO 200 NTRIAL = 1, 10000
         EWMA = RBAR
         NUM  = 0
```

```
50        DO 100 I = 1, NSIZE
             Z(I) = GGNQF(DSEED) * NWSTD + MEAN
100       CONTINUE
C
C            DETERMINE SAMPLE RANGE OF SUBGROUP SIZE 4
C
          BIG   = AMAX1(Z(1),Z(2),Z(3),Z(4))
          SMALL = AMIN1(Z(1),Z(2),Z(3),Z(4))
          RANGE = BIG - SMALL
C
          EWMA = ALPHA * RANGE + (1. - ALPHA) * EWMA
          IF ((EWMA .GE. LCL) .AND. (EWMA .LE. UCL)) THEN
                 NUM = NUM + 1
                 GO TO 50
              ELSE
                 NUM = NUM + 1
                 SUM = SUM + NUM
          ENDIF
200    CONTINUE
C
C            MEAN ACTION TIME , MAT
C
        MAT = SUM / (NTRIAL - 1.)
C
       WRITE (6,300) ALPHA,NSIZE,RATIO,MAT
300    FORMAT (1X,'THE WEIGHTING FACTOR             = ',F10.2/
     $         1X,'THE SAMPLE SIZE                  = ',I10/
     $         1X,'THE RATIO OF NWSTD/STD           = ',F10.2//
     $         1X,'THE MEAN ACTION TIME OF        '/
     $         1X,'CONTROL CHART FOR EWMA         '/
     $         1X,'OF SAMPLE RANGE                  = ',F10.2/)
C
       STOP
       END
//GO.SYSIN DD *
50,10
//
```

APPENDIX E

LISTING OF COMPUTER PROGRAM USED TO DETERMINE THE MAT OF THE EWMAID CHART

```
**** TSO FOREGROUND HARDCOPY ****
DSNAME=U11563A.MODELC.CNTL

//U11563A JOB (11563,111-11-1111),'NG',CLASS=4,TIME=(15,0),
// MSGCLASS=X,NOTIFY=*
/*PASSWORD ????
/*JOBPARM ROOM=N
/*ROUTE PRINT LOCAL
// EXEC FORTVCLG
//FORT.SYSIN DD *
C
C          SIMULATION PROGRAM TO DETERMINE THE MEAN ACTION TIME
C             OF CONTROL CHART FOR EWMA OF INDIVIDUAL DATA
C
C          WEIGHTING FACTOR : 0.2
C          SHIFT IN MEAN (IN MULTIPLE OF STD DEVIATION) : 0.0
C          SIMULATION RUN : 10000
C
C          TO COMPUTE MAT FOR OTHER SITUATION, ALTER THE VALUE OF
C          VARIABLES ALPHA AND SHIFT
C
      DOUBLE PRECISION DSEED,SUM,MAT
      REAL NUM,LCL,NMEAN,MEAN
C
      READ *,MEAN,STD
      PRINT *,'ORIGINAL PROCESS MEAN                  ',MEAN
      PRINT *,'ORIGINAL PROCESS STANDARD DEVIATION    ',STD
      PRINT *
C
C          WEIGHTING FACTOR , ALPHA
C
      ALPHA = 0.2
      C     = ALPHA / (2. - ALPHA)
      D     = SQRT(C)
      E     = 3 * STD * D
      UCL   = MEAN + E
      LCL   = MEAN - E
C
C          SHIFT IN MEAN (IN MULTIPLE OF STANDARD DEVIATION)
C          NEW MEAN , NMEAN
C
      SHIFT = 0.0
      NMEAN = MEAN + (SHIFT * STD)
C
      DSEED = 8064697.
      SUM   = 0
      DO 200 NTRIAL = 1, 10000
         EWMA = MEAN
         NUM  = 0
50       Z    = GGNQF(DSEED) * STD + NMEAN
         EWMA = ALPHA * Z + (1. - ALPHA) * EWMA
         IF ((EWMA .GE. LCL) .AND. (EWMA .LE. UCL)) THEN
              NUM = NUM + 1
              GO TO 50
           ELSE
              NUM = NUM + 1
              SUM = SUM + NUM
         ENDIF
200   CONTINUE
C
C          MEAN ACTION TIME , MAT
C
```

```
       MAT = SUM / (NTRIAL - 1.)
C
      WRITE (6,300) ALPHA,SHIFT,MAT
300   FORMAT (1X,'THE WEIGHTING FACTOR           = ',F10.2/
     $         1X,'THE SHIFT IN MEAN             = ',F10.2,' STD DEV'//
     $         1X,'THE MEAN ACTION TIME OF      '/
     $         1X,'CONTROL CHART FOR EWMA       '/
     $         1X,'OF INDIVIDUAL DATA            = ',F10.2/)
C
      STOP
      END
//GO.SYSIN DD *
50,10
//
```

APPENDIX F

LISTING OF COMPUTER PROGRAM USED TO DETERMINE THE MAT OF THE EWMAMR CHART

```
**** TSO FOREGROUND HARDCOPY ****
DSNAME=U11563A.MODELD.CNTL

//U11563A JOB (11563,111-11-1111),'NG',CLASS=4,TIME=(15,0),
// MSGCLASS=X,NOTIFY=*
/*PASSWORD ????
/*JOBPARM ROOM=N
/*ROUTE PRINT LOCAL
// EXEC FORTVCLG
//FORT.SYSIN DD *
C
C          SIMULATION PROGRAM TO DETERMINE THE MEAN ACTION TIME
C          OF CONTROL CHART FOR EWMA OF MR(2) OF INDIVIDUAL DATA
C
C          WEIGHTING FACTOR : 0.2
C          RATIO OF NEW STD DEV / STD DEV : 1.0
C          SIMULATION RUN : 10000
C
C          TO COMPUTE MAT FOR OTHER SITUATION, ALTER THE VALUE OF
C          VARIABLES ALPHA AND RATIO.
C
      DIMENSION Z(10)
      DOUBLE PRECISION DSEED,SUM,MAT
      REAL NUM,LCL,MEAN,MRBAR,MR2,NWSTD
C
      READ *,MEAN,STD
      PRINT *,'ORIGINAL PROCESS MEAN                 ',MEAN
      PRINT *,'ORIGINAL PROCESS STANDARD DEVIATION    ',STD
      PRINT *
C
C          WEIGHTING FACTOR , ALPHA
C          CONTROL CHART FACTORS ,SD2 AND SD3
C
      ALPHA = 0.2
      SD3   = 0.853
      SD2   = 1.128
      B     = ALPHA / (2. - ALPHA)
      C     = SQRT(B)
      D     = SD3 / SD2
      E     = 3. * C * D
      F6    = 1. + E
      F5    = 1. - E.
      IF (F5 .LE. 0) THEN  F5 = 0
      MRBAR = SD2 * STD
      UCL   = F6 * MRBAR
      LCL   = F5 * MRBAR
C
C          NEW PROCESS STANDARD DEVIATION, NWSTD
C          NWSTD / STD = RATIO
C
      RATIO = 1.0
      NWSTD = RATIO * STD
C
      DSEED = 8064697.
      SUM   = 0
      DO 200 NTRIAL = 1, 10000
         EWMA = MRBAR
         NUM  = 0
         Z(1) = GGNQF(DSEED) * NWSTD + MEAN
50       Z(2) = GGNQF(DSEED) * NWSTD + MEAN
         MR2  = ABS(Z(1) - Z(2))
         EWMA = ALPHA * MR2 + (1. - ALPHA) * EWMA
```

```
         IF ((EWMA .GE. LCL) .AND. (EWMA .LE. UCL)) THEN
               NUM  = NUM + 1
               Z(1) = Z(2)
               GO TO 50
            ELSE
               NUM = NUM + 1
               SUM = SUM + NUM
         ENDIF
200   CONTINUE
C
C           MEAN ACTION TIME . MAT
C
       MAT = SUM / (NTRIAL - 1.)
C
      WRITE (6,300) ALPHA,RATIO,MAT
300   FORMAT (1X,'THE WEIGHTING FACTOR         = ',F10.2/
     $         1X,'THE RATIO OF NWSTD/STD       = ',F10.2//
     $         1X,'THE MEAN ACTION TIME OF     '/
     $         1X,'CONTROL CHART FOR EWMA      '/
     $         1X,'OF MR(2) OF IND. DATA        = ',F10.2/)
C
      STOP
      END
//GO.SYSIN DD *
50,10
//
```

VITA 2

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