COMPUTER SIMULATION OF
CRITICAL HEIGHT
SAMPLING

By

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PREFACE

In order to better understand the work presented in this study, it is necessary to understand several topics from mensuration and forest sampling. First, the problem of concern is improvement of the variance associated with the estimation of forest growth from remeasurement of permanently established locations in a forested area. When sample trees at these locations are chosen by an angle gauge the locations are often called permanent points. To simulate this on a computer data are required which give the position of each tree (using a Cartesian coordinate system) relative to every other tree and the entire border of the forest stand. Data of this type are known as mapped stand data. The data consist of the Diameter at Breast Height (DBH), (the diameter of the tree at a standardized height of 4.5 feet above the ground) and the total height of the tree at each specified age.

Second, two different sampling systems are compared in order to determine which one estimates growth with the least variance. The first method is known as Horizontal Point Sampling and shall also be written as HPS. It is a system of forest sampling that selects trees by using an angle whose vertex is centered at a point in the forest. The second method is called Critical Height Sampling and shall also be called CHS. Two characteristics of each system will be examined. The first, the volume estimator, refers to the estimate of forest volume obtained with the use of each sampling system. The second, the growth estimator, refers to the estimate of forest growth obtained with the use of each sampling system.

Third, two common topics in mensuration will be mentioned. In this study K is a constant which, when multiplied by the square of the Diameter at Breast Height (DBH) of a tree, gives the cross sectional area of the tree at Breast Height (4.5 feet above the ground) in square feet. This is called the Basal Area (BA) of the tree. Basal Area (BA) can also be expressed in terms of an entire stand (on a square feet per acre basis). Similarly, there is a constant K associated with metric units which when multiplied by the square of the DBH in centimeters (cm) yields Basal Area in square meters. In the metric system, the Breast Height is 1.3 meters (m) above the ground and Basal Area (BA) is in square meters (for a single tree) or square meters per hectare (for an entire stand). The Basal Area Factor (BAF) is the number of square feet of Basal Area per acre that is represented by each and every sample tree selected by an angle gauge in horizontal point sampling or critical height sampling. In HPS and CHS, the

BAF is the same for each tree tallied.

Finally, each sample tree will be classified into one of five different categories. The categories represent the different types of individual tree growth encountered in growth estimation through horizontal point sampling. The categories and their definitions are from Martin (1982).

1) Ingrowth trees are below the minimum dbh and "in" at the first measurement but grow enough to exceed the minimum dbh at the second measurement.
2) Survivor trees are above the minimum dbh and "in" at both measurements.
3) Mortality trees are above the minimum dbh and "in" at the first measurement but die prior to the second measurement.
4) Ongrowth trees are below the minimum dbh and "out" at the first measurement but are above the minimum dbh and "in" at the second measurement.
5) Nongrowth Trees are above the minimum dbh and "out" at the first measurement but are "in" at the second measurement.

I wish to express my gratitude to Dr. Thomas B. Lynch for his guidance and help during this project. I also wish to thank Dr. David W. Robinson and Dr. P. Larry Claypool for serving on my graduate committee. Many thanks are also extended to Mr. Joe Douglas of the University Computer Center for his help with several seemingly unsolvable computer problems.

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NOMENCLATURE

ALTCD	alternate critical diameter
BA	basal area
BAF	basal area factor
CA	critical area
CD	critical diameter
CH	critical height
ch_i	critical height of tree i
CHS	critical height sampling
d	top diameter in cm. outside bark at height h
DBH	diameter at breast height
h	height in meters above ground to top diameter d
H	total height of a single tree
HPS	horizontal point sampling
i	ingrowth estimate from final volumes and final basal areas
K	$[\pi/(144 \times 4)] = 0.00545415$, English units $[\pi/(10000 \times 4)] = 0.00007854$, metric units
m	mortality estimate from initial volumes and initial basal areas
n	nongrowth estimate from final volumes and final basal areas

N	sample size
N_c	number of samples required in critical height sampling
N_h	number of samples required in horizontal point sampling
o	ongrowth estimate from final volumes and final basal areas
π	Pi=3.14159265358979
r	shape factor
s_1	survivor estimate from initial volumes and initial basal areas
s_2	survivor estimate from final volumes and final basal areas
V	total cubic volume of a single tree
V_1	volume estimate at time 1
V_2	volume estimate at time 2
Var_c	estimated sample variance of critical height sampling
Var_h	estimated sample variance of horizontal point sampling
VBAR	volume to basal area ratio of a single tree
z	relative tree height from tip to d $= (H-h)/H$

CHAPTER I

INTRODUCTION

A "new" method of sampling for volume without the use of a volume table has existed for over twenty years. The method, known as critical height sampling, provides an unbiased estimate of stand volume by extending the use of horizontal point sampling to the third dimension in order to sample volume directly (Iles 1974, McTague and Bailey 1985).

The critical diameter of a tree is the point on the stem at which both sides of the sampling angle touch the stem when the vertex of the angle is at a randomly or systematically located sample point on the ground. The critical height is the distance from the groundline diameter to the critical diameter. An unbiased estimate of stand volume can be obtained by multiplying the sum of critical heights at a single sample point by the basal area factor.

$$\text{Volume per acre} = \text{BAF} * \left(\sum_{i=1}^{n} ch_i\right) \qquad (1)$$

An advantage of critical height sampling is that it provides a direct estimate of stand volume without the use of a volume table. Thus, any bias due to volume tables is eliminated (Iles 1979b). Since the method is sensitive to tree form and requires no taper assumptions, it works for any species, tree taper or utilization standard (Iles 1979b, Lynch 1986, Van Deusen and Meerschaert 1986). Another significant advantage is in the treatment of ongrowth and nongrowth trees in growth estimation (Iles 1979a and 1979b). The growth estimator in critical height sampling may allow the contribution to volume by ongrowth and nongrowth trees to be gradual, thus avoiding the big jump in the total volume estimate caused by ongrowth and nongrowth trees in permanent points. If the variance of the volume growth estimator of critical height sampling can be shown to be comparable to (numerically) or smaller than the variance of the volume growth estimator of horizontal point sampling, this would provide strong evidence in evaluating critical height sampling for practical use in growth estimation. Therefore, the main objective of this study is to compare the variances of the two growth estimators and the two volume estimators (numerically) to evaluate the practical use of critical height sampling in growth and volume estimation.

There are some disadvantages to CHS which should be considered in this analysis. It is not obvious whether mortality affects the CHS volume estimator more than the HPS volume estimator, since the critical height of a mortality tree can be larger or smaller than the tree VBAR (Iles 1979b). There are also several disadvantages invloving the field application of Critical Height sampling. Iles (1979b) lists the following problems.

1) The critical point may not be visible from the sample point, generally because of foilage.
2) The angle of measurement may be so steep that it makes measurement difficult.
3) The instrument (usually a relascope) simply may not be sufficiently accurate in locating the critical point even when it is clearly visible at a reasonable angle.

A detailed explanation of these disadvantages will be given in a treatment of the field application of critical height sampling.

Critical height sampling was discovered in 1962 in Japan by Masami Kitamura. In 1968 he published a paper concerning indirect methods of critical height measurement. Iles (1979b) documents that in 1971 Bitterlich reported on Kitamura's method and that a brief summary was included by Finalyson in 1969 in the manual for the wide scale relascope.

In 1974 Kim Iles independently rediscovered the method. He called it Penetration Sampling and represented his system as a series of random lines penetrating the volume of space in the forest. He then discovered a translated article by Kitamura (1968) in a literature review containing a diagram and formula which helped establish the similarity between his own work and that of Kitamura.

In 1976 the first English journal article on CH sampling appeared in Commonwealth Forestry Review. It was written by Bitterlich and had been translated from a German periodical. Bitterlich's textbook (Bitterlich 1984) provides a summary of the relationship between critical height sampling and the relascope as well as an explanation of similar methods presented in the Japanese literature.

Unbiasedness of critical height sampling for two specific published tree taper models is reported by McTague and Bailey (1985). Van Deusen and Meerschaert (1986) show critical height sampling to provide unbiased estimates of volume for any taper model. Unbiasedness is shown when tree selection is made by diameter at stump height. Selection by DBH assumes that the volume between DBH and stump height is a clyinder. Lynch (1986) also showed unbiasedness for any tree taper with tree selection made by diameter at stump height and describes a method for correcting the bias resulting from tree selection by DBH. This correction uses an appropriate taper equation to estimate the volume of the tree between DBH and stump height that is outside the cylinder defined by DBH. McTague and Bailey (1985) present a factor that corrects for bias due to selection at DBH that is correct when the taper function of Clutter (1980) is applicable. Kitamura (1962) shows unbiasedness for his system by using a different mathematical approach.

Kitamura's 1968 paper seems to have been an attempt to eliminate the direct measurement of critical height by using an indirect method such as a taper equation or a form factor to estimate the critical height. Ueno (1979) proposed a system which has been termed space point sampling. Tree selection is made using an angle gauge but at each sample tree a random height between 0 and an estimated maximum tree height is chosen and compared to either the ocularly estimated or directly measured critical height. If the random height falls between 0 and the critical height the tree is measured as "1" otherwise it is measured as "0". Volume per unit area is obtained by multiplying the BAF, the maximum tree height and the sum of tree measures at a single point. The advantage of this method is that very few critical heights need to be measured so the method will be faster in the field while still giving unbiased estimates of volume. Therefore, Uneo's method greatly simplifies the application of Kitamura's basic concept (Bitterlich 1984).

A 1982 computer simulation study by Sterba compared

Bitterlich's, Kitamura's, Ueno's and Minowa's (1979) methods of volume estimation. The study showed that horizontal point sampling estimates volume more precisely than critical height sampling. Ueno's method ranked below CHS and Minowa's method had the highest variance of the methods tested by Sterba (1982). Minowa's method (1979) is based on measurements of upper-stem diameters at a fixed vertical angle from a centerline on the ground of fixed length (see Bitterlich 1984). The most significant result of Sterba's study is the evaluation of the field procedure of each method. Ueno's method only requires the user to measure approximately one or two trees per sample point (in Sterba's simulation) while about 11 or 12 trees were measured per point or line with Bitterlich's, Kitamura's and Minowa's methods. Thus, Ueno's method allows one to put in about three times as many sample points in the same amount of time which helps to make up for the lack of precision in the method. Therefore, Uneo's method is the most cost efficent of the methods tested by Sterba (1982).

Iles (1979a) suggests that the variance of critical height sampling is "approximately" the same as the variance of horizontal point sampling in volume estimation. McTague and Bailey (1985) give a proof indicating that the variance of horizontal point sampling is less than the variance of the critical height sampling volume estimator when parabolic taper is assumed. Van Deusen and Meerschaert

(1986) discuss conditions under which the variance of volume estimation using VBAR sampling is less than would be obtained by using CHS.

Iles (1979a) first proposed the use of critical height sampling for growth estimation. His objective was to reduce the problems created by ongrowth and nongrowth trees. He also realized that mortality may cause problems because a tree that dies can cause the sum of critical heights to decrease more than the sum of VBAR's on a permanent point. McTague and Bailey (1985) showed that the variance of nongrowth is less with critical height sampling than with horizontal point sampling under the assumption of a random spatial distribution. They also showed that the critical height sampling growth estimate is equal to the difference in volume estimates at the measurement times defining the growth interval. McTague and Bailey (1985) also suggested that the variance of the growth estimate of critical height sampling might be less than that of horizontal point sampling, but did not give a proof.

CHAPTER II

METHODS AND MATERIALS

Remeasured Plot Data

Remeasured permanent plot data is often used to measure volume growth. Data consisting of remeasured diameters and heights at intervals for mapped stands were not available for this study. Therefore, it was necessary to simulate such data so that they could be used to compare the performance of the forest stand growth estimators. A forest growth simulator written in FORTRAN by Daniels and Burkhart (1975) was used to simulate remeasured mapped stand data for loblolly pine (Pinus taeda). The simulator was used to generate loblolly pine plantations with 600 surviving trees per acre at age seven. The stands were grown to age 60 and the DBH and total height were recorded at five year intervals from age 15 to age 60. A site index of 60 (base age = 25) was used and no cultural treatments were applied during the simulations. The program allowed simulation of thinning and fertilization but these factors were not included because test stands containing cultural treatments were not initially desired. Forty-nine blocks of 0.9878 acres each were created and stored on computer disk. Each block contained 676 cordinates or possible tree locations. The program adjusted for mortality so not all the trees were alive at age 15. Trees which died between age 15 and age 60 were assigned zero height and dbh at every age after the five year interval in which the tree died. The 49 blocks can be merged into one data set in order to create a mapped stand which is larger than one acre. Since the blocks are approximately one acre in size and are all square only square stands of approximately 1, 4, 9, 16, 25, 36 and 49 acres could be used.

Mirage Method for Correcting Boundary Overlap

Since the simulations in this study occur on a one acre stand, a correction for boundary overlap must be applied. When a sample point is located too close to the edge of the tract the sample estimate will be biased because the area to be sampled by that point does not lie completely within the tract (Beers 1976).

When the area to be sampled by a fixed radius plot extends beyond the tract boundary it is subject to boundary overlap. In horizontal point sampling no boundary overlap can occur when plot centers are required to be at least as far from a boundary as the radius of the variable plot associated with the largest tree.

On a large tract with minimal boundary (square or rectangular tract shape), the sample points can be located far enough away from the border so that no correction is necessary, and bias will be negligible if the nature of the forest in the boundary area is not greatly different from the forest as a whole. In critical height sampling the selection of trees is the same as in horizontal point sampling therefore, any correction method for point sampling will also be applicable in critical height sampling.

The mirage method developed by Schmid-Hass (1969) for correction of boundary overlap has been described by Beers (1976). To apply the correction one simply establishes another sample point on the opposite side of the boundary. The distance from the "Mirage point" to the boundary is the same as the distance from the original point to the boundary (Figure 1). Then the sample angle is projected back onto the tract and those trees selected by the angle gauge which are also located inside the original tract are tabulated again and included in the estimate of the sampling characteristic at that point. If the sample point is located in a corner three "Mirage points" are established (one in relation to each side and a third which is diagonally opposite of the original point in relationship to the two boundaries). Then the angle gauge is used to tabulate trees which fall into the sample angle

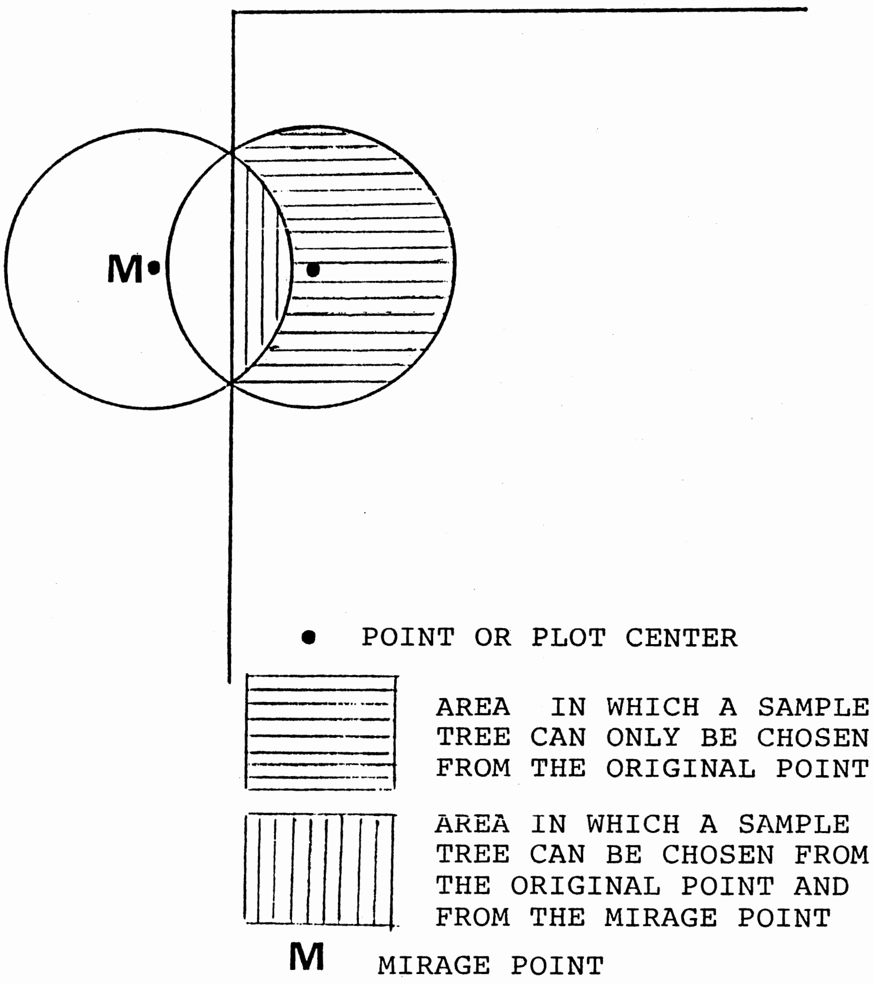

Figure 1. Mirage Method for the Correction of Boundary Overlap.

and are still located on the original tract. These trees are also added to the point estimate. Thus it is possible for a tree to be tabulated 1, 2, 3, or 4 times if the sample point is located in a corner. In variable plot sampling only the plots of trees located partially out of the tract in question need to be corrected. The Mirage method actually corrects boundary overlap on a per tree basis. For these reasons the Mirage method was used to correct for boundary overlap in all simulations of horizontal point sampling and critical height sampling in this study.

Taper Equations and Shape Assumptions

Since the data set used for computer simulation of critical height sampling consists only of tree coordiantes, DBH's and heights, a taper function must be used to obtain critical heights of sample trees. The consideration of different tree shape assumptions is important because different sets of assumptions may influence the variance of both volume and growth estimation. Three different shape assumptions were used.

The first method was to calculate the volumes and critical heights assuming that each tree had the shape of a cone with the given DBH and total height. The total cubic foot volume of the tree was calculated using the formula for the volume of a cone. To calculate the volume in ft^3

from a diameter in inches and a height in feet the following equation was used:

$$V= (\pi/3)(DBH/24)^2(H) \qquad (2)$$

To calculate the critical height of an individual tree the following taper equation was used which gives heights to specific upper stem diameters for solids generated by rotating a power curve about an axis (Van Deusen and Lynch 1987):

$$h(d)= H - H\ (d/DBH)^{(r/2)} \qquad (3)$$

If r=2, then equation (3) generates a cone. The second type of shape assumption used was that of a paraboloid. The formula for the volume of a paraboloid was used to calculate individual tree volumes. Since DBH was given in inches and total height in feet, the equation was converted to

$$V=(\pi/2)(DBH/24)^2(H) \qquad (4)$$

To obtain the critical height using the assumption of paraboloic shape, taper equation (3) was used with r=4. For simplicity the difference between DBH and stump diameter was ignored in equation (2), (3), and (4).

The third shape assumption consisted of a segmented taper equation presented by Cao, Burkhart and Max (1980). Their study compared two different methods used to estimate volumes to specific upper-stem diameter limits. One of the methods involved using taper equations which can be integrated to give volume to any top diameter limit. There were twelve different functions evaluated in the study using loblolly pine (Pinus taeda) data from plantations and natural stands. Equation (5) ranked first in predicting volumes to top diameters, third in estimating diameters and fourth in predicting volumes to various heights. Equation (5) was also judged to be a reasonably good multipurpose taper equation. It was for these reasons that equation (5) was used as the main shape assumption in this study. Since parameters in the equation were given in metric units, tree dimensions were converted to metric units when the equation was used to calculate critical heights. The critical height was then converted to English units. Cao, Burkhart, and Max (1980) used the following equation to calculate upper-stem diameters corresponding to given heights:

$$\begin{aligned}(d^2KH/V - 2z) = {} & b_1(3z^2-2z) \\ & + b_2(z-a_1)^2I_1 \\ & + b_3(z-a_2)^2I_2 \qquad (5)\end{aligned}$$

where

b_1, b_2, b_3, a_1, a_2 = regression coefficients

$I_i = 1, z > a_i ; i=1, 2$

$= 0, z < a_i$

I_1 and I_2 are indicator variables that determine which part of the equation is used, dependent upon the segment of the tree in which the critical diameter is located.

The critical height was determined in the following manner (Cao 1978). Since a_1 and a_2 represent the two points joining portions of the equation on the bole of the tree, the equation was solved for a_1 and a_2 and their corresponding diameters d_1 and d_2. Then the critical diameter was compared with d_1 and d_2 in order to decide which section of the tree contained the critical height. The equation was then solved as a quadratic for z. The critical height was then calculated by rearranging

$$z = (H - h)/H \quad (6)$$

to

$$h = H(1 - z) \quad (7)$$

This resulted in a critical height in meters which was then converted to feet.

In order to obtain single tree volumes a compatible volume equation was used. A volume equation is said to be compatible with a taper equation if it gives volumes equal to those obtained by intergrating the taper equation over the length of the tree. The equation had the following form:

$$V = b_0 + b_1 D^2 H \quad (8)$$

where

b_0, b_1 = regression coefficients

As reported by Cao, Burkhart and Max (1980) equation (5) is not completely compatible with volume equation (8), therefore, the volume given by equation (8) must be multiplied by a correction factor of 0.9896 to obtain compatible total tree volumes. The regression coefficients used were reported by Burkhart (1977). Equation (8) was used to calculate single tree cubic foot volumes used in point sampling. Volumes required in the computation of the critical height were calculated in cubic feet and then converted to cubic meters in taper equation (5).

The Measurement of Forest Growth

Forest growth can be measured by simply subtracting two separate estimates of volume at the same location taken at different times. A common interval might be 5 or 10 years. Growth can also be measured by classifying each sample tree into different categories of growth and then determining net change in volume from time 1 to time 2 arithmetically. In addition, the contribution to variance by each component of growth can be examined in order to analyze differences in the performance of the HPS and CHS growth estimators. Classifications of sample trees on permanent points used in this section were taken from Martin (1982) and Van Deusen, Dell and Thomas (1986), and are defined in the preface.

Combinations of the six classes can be used to estimate volume and change in volume in permanent point sampling (Van Deusen, Dell and Thomas 1986). If no cutting occurs in the interval, the volume at time 1 and time 2 of a sample point can be estimated by

$$V_1 = s_1 + m \tag{9}$$

and

$$V_2 = i + o + s_2 + n \qquad (10)$$

The volume change can then be estimated at a single point by

$$V_2 - V_1 = i + o + s_2 - s_1 + n - m \qquad (11)$$

The estimation process begins by assigning each sample tree to its proper component. The appropriate estimator is then incremented by either the VBAR (for horizontal point sampling) or the critical height (for critical height sampling). When the simulation is complete estimates of each component of growth at each point can be used to compute an estimate of the volume and the net change in volume. Additionally, the estimate of the variance of each growth component estimator can be examined to determine how each sampling system treats each component of growth.

It has been suggested that critical height sampling decreases contribution to the variance by ongrowth and nongrowth trees in growth estimation, (McTague and Bailey 1985, Iles 1979a). At the first measurement, time 1, an ongrowth tree is submerchantable and "out", while a nongrowth tree is merchantable and "out". Both trees are merchantable and "in" at the second measurement at time 2. When using permanent points to measure growth, the measured growth between time 1 and time 2 on ongrowth and nongrowth trees also includes the growth that occurred between age zero and time 1. Growth estimation using critical height sampling allows ongrowth and nongrowth trees to creep into a permanent point estimate gradually. Since a tree is only in up to a certain point on the stem (the critical diameter) only a portion of the volume of that tree is included in the volume per acre estimate. Thus, the critical height measurement of the growth may more closely reflect the true increment between time 1 and time 2. However, the effects of mortality trees may cause the variance of critical height sampling to increase because a tree which dies may decrease the sum of critical heights proportionally more than it would decrease the sum of VBAR's in a ordinary permanent growth point (Iles 1979a). A mortality tree is one that is merchantable and "in" at time 1 but dies prior to time 2. An ingrowth tree is one which was "in" at time 1 but was too small to be measured. Ingrowth trees may create an additional problem because they may have a relatively high critical height due to the fact that they are close to the sample point.

Criteria For Comparing Estimators

An objective of this study is to evaluate the accuracy of CHS estimators relative to HPS estimators. There are two components of accuracy, bias and precision. If the estimators are mathematically unbiased, the variance can be used to evaluate the accuracy of the estimators. If the true variance can be calculated, the estimator with the lowest variance would be declared superior. If the true variance cannot be calculated, simulation can sometimes be used to closely approximate the true variances for variance comparisons.

Horizontal point sampling has been shown to be mathematically unbiased (Palley and Horwitz 1961). Critical height sampling has also been proven to be mathematically unbiased (Kitamura 1962, McTague and Bailey 1985, Lynch 1986, and Van Deusen and Meerschaert 1986). It is difficult to calculate the true variance of HPS and CHS due to the problems of computing the area of overlap between plots of two or more trees. Therefore, simulation was used to closely approximate the variances of the volume and growth estimators of HPS and CHS in order determine which system provides the most accurate volume and growth estimators.

CHAPTER III

RESULTS AND DISCUSSION

The results of this study indicate that the variance of CHS in both volume and growth estimation is higher than that of HPS in most of the situations investigated in which the Cao, Burkhart, and Max (1980) taper equation is used, and sample trees are selected by groundline diameter. However, a detailed analysis of a complete simulation is necessary in order to understand all of the factors relevant to the comparison between the growth estimators of critical height sampling and horizontal point sampling. Therefore, a detailed description of a comparison will be given between CHS and HPS for both volume estimation and growth estimation, in which the segmented taper model presented by Cao, Burkhart, and Max (1980) was used to obtain volumes and critical heights. This simulation was chosen for a more detailed examination because the taper function of Cao, Burkhart, and Max (1980) more closely resembles the shape of a real tree than does a cone or a paraboloid. The mensurational characteristics of the test stand used in this study are given in Tables IX and X in appendix E. The summary in these tables includes the number of trees per acre, the basal area per acre, and the volume per acre (according to equation 8) for each age. Table X includes the actual volume of all ingrowth trees, all mortality trees, and all the survivor trees, as well as the net change in volume for each growth interval. In this simulation a one acre stand was used with 1000 sample points and a BAF of 10. Sample trees were selected by groundline diameter rather than by DBH in order to obtain exact unbiasedness. Selection of trees by DBH is the same as assuming that the tree is a cylinder below breast height (Van Deusen and Meerschaert 1986).

Volume Estimation

The graph of the volume estimator of CHS and HPS, (Figure 2), demonstrates the unbiasedness of CHS but gives no indication of the precision of the CHS volume estimator. The quality of the volume estimators can be evaluated by comparing their variances. Figure 3 shows the variance of the volume estimators of CHS and HPS at nine ages. The variance of the CHS volume estimator is larger than that of HPS in volume estimation for the conditions of this simulation. These results support the theoretical work of McTague and Bailey (1985) who give a proof indicating that the variance of the CHS volume estimator is greater than that of HPS when parabolic taper is assumed. The largest difference occurs at older ages where there are

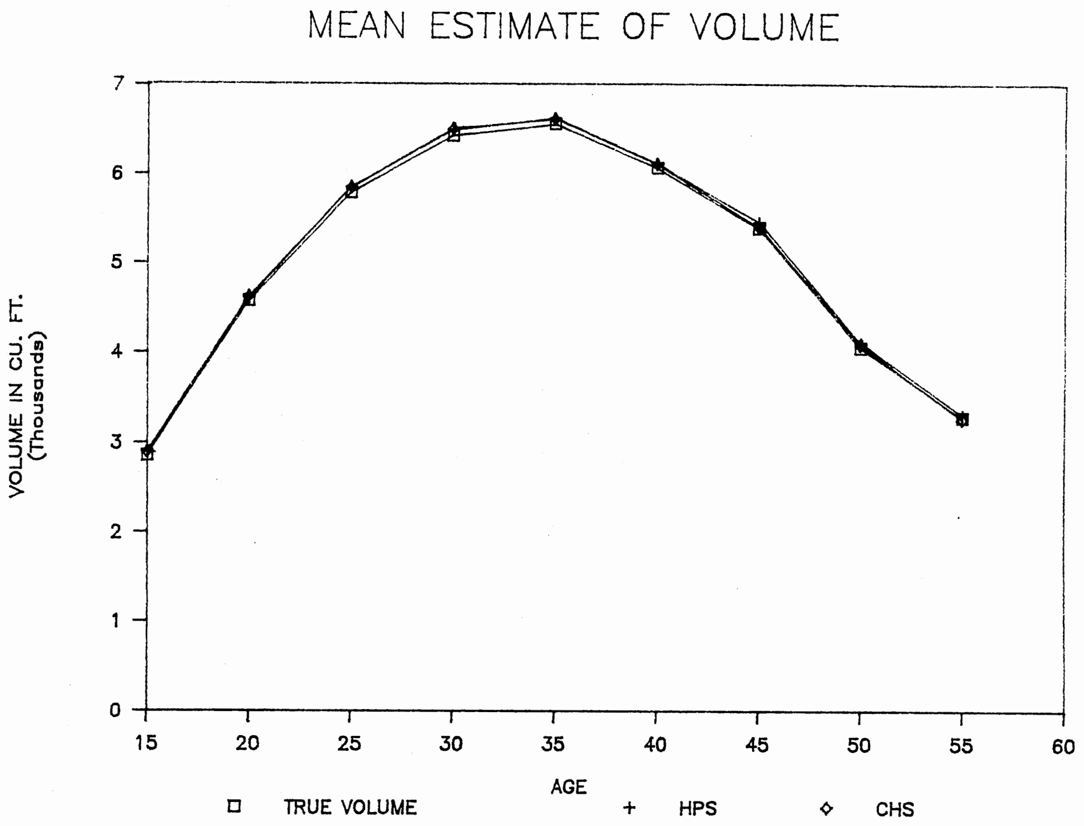

Figure 2. Mean Estimate of Volume for Critical Height Sampling and Horizontal Point Sampling. The Taper Equation used was from Cao, Burkhart, and Max (1980).

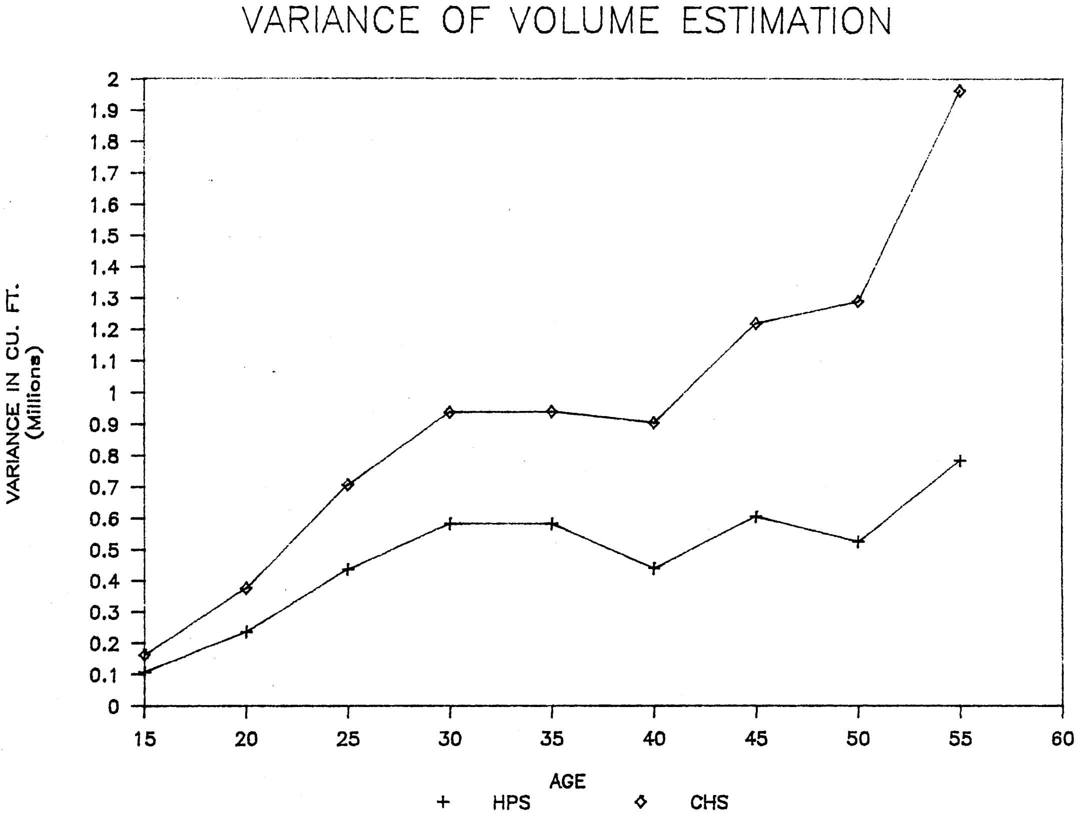

Figure 3. Variance of Volume Estimation for Critical Height Sampling and Horizontal Point Sampling. The Taper Equation used was from Cao, Burkhart, and Max (1980).

fewer but larger trees. The variance of HPS is much higher when tree selection is by DBH, and at several ages is even higher than that of CHS (Figure 4). The cases in which the variance of HPS is higher than CHS may be due to the fact that CHS underestimates volume by about 3 to 4 percent in this stand when tree selection is by DBH (Figure 5). However, the variance of the CHS volume estimator remains virtually unchanged in these two situations.

The coefficient of variation is a unitless measure which expresses variance on a relative basis (Freese, 1962), therefore, an additional comparison and evaluation of the variation of both volume estimators is possible using the coefficient of variation. The coefficient of variation of CHS is higher than that of HPS because CHS has a larger variance at every age (Figure 6). A study by Iles (1979b) using 200 sample points and varying BAF's showed no more than a five percent margin between the CHS and HPS coefficients of variation. The coefficent of variation of CHS was not always greater than that of HPS. The study assumed a conical shape for all trees. The comparisons made by Iles (1979b) were with respect to the average number of sample trees selected at the sample points and not by age of stand. Coefficents of variation in Figure 6 are much lower than those obtained by Iles (1979b), which ranged from about 65 to 140 percent. This can probably be attributed to the homogeniety of the loblolly pine stand

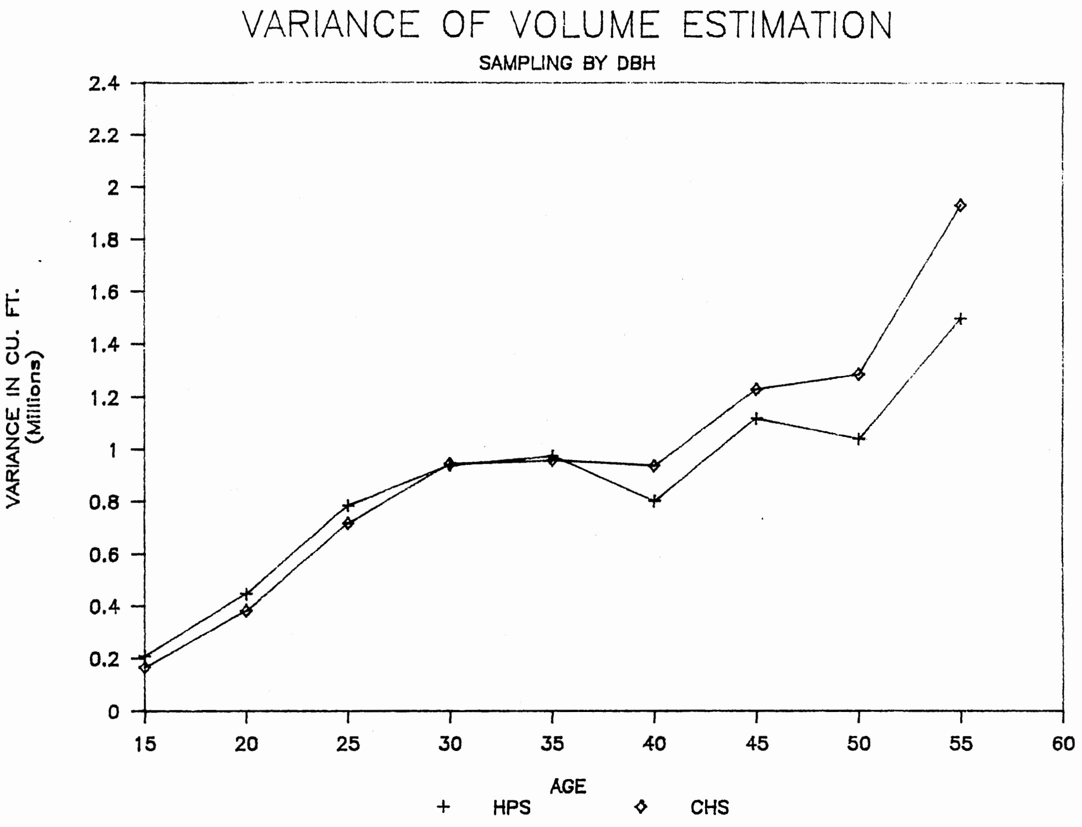

Figure 4. Variance of Volume Estimation for Critical Height Sampling and Horizontal Point Sampling When Tree Selection is Made by DBH. The Taper Equation used was from Cao, Burkhart, and Max (1980).

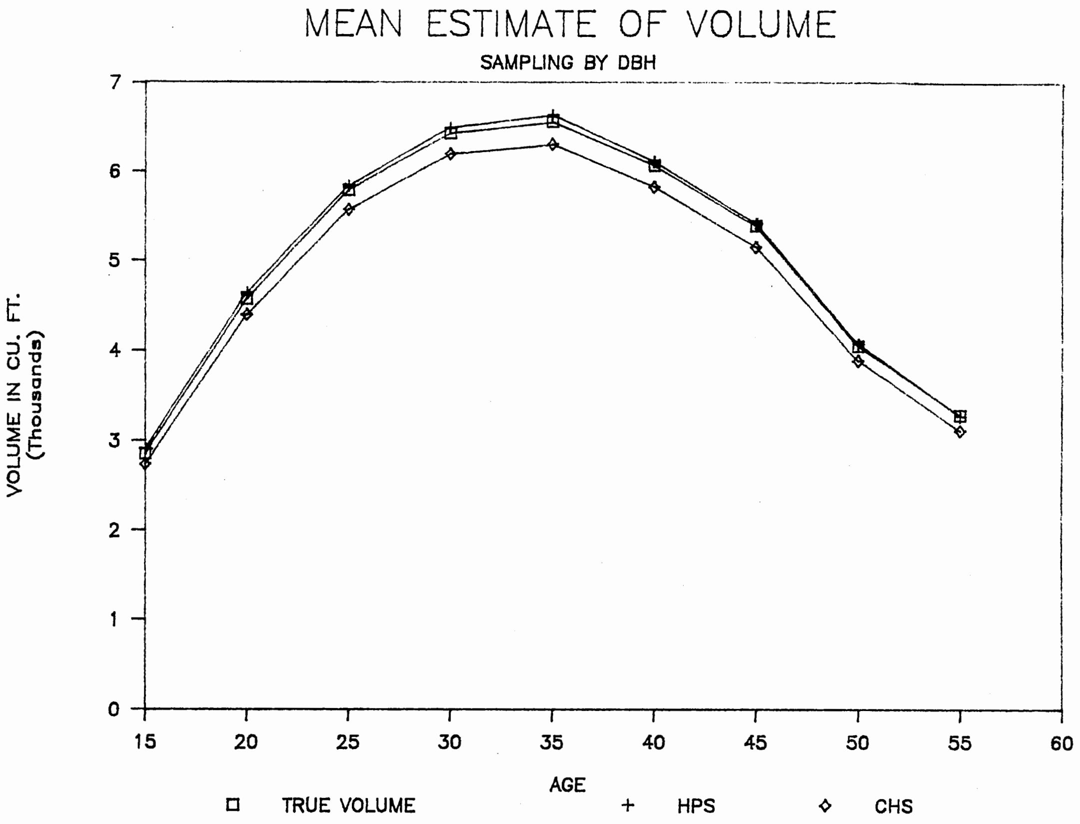

Figure 5. Mean Estimate of Volume When Tree Selection is made by DBH. The Taper Equation used was from Cao, Burkhart, and Max (1980).

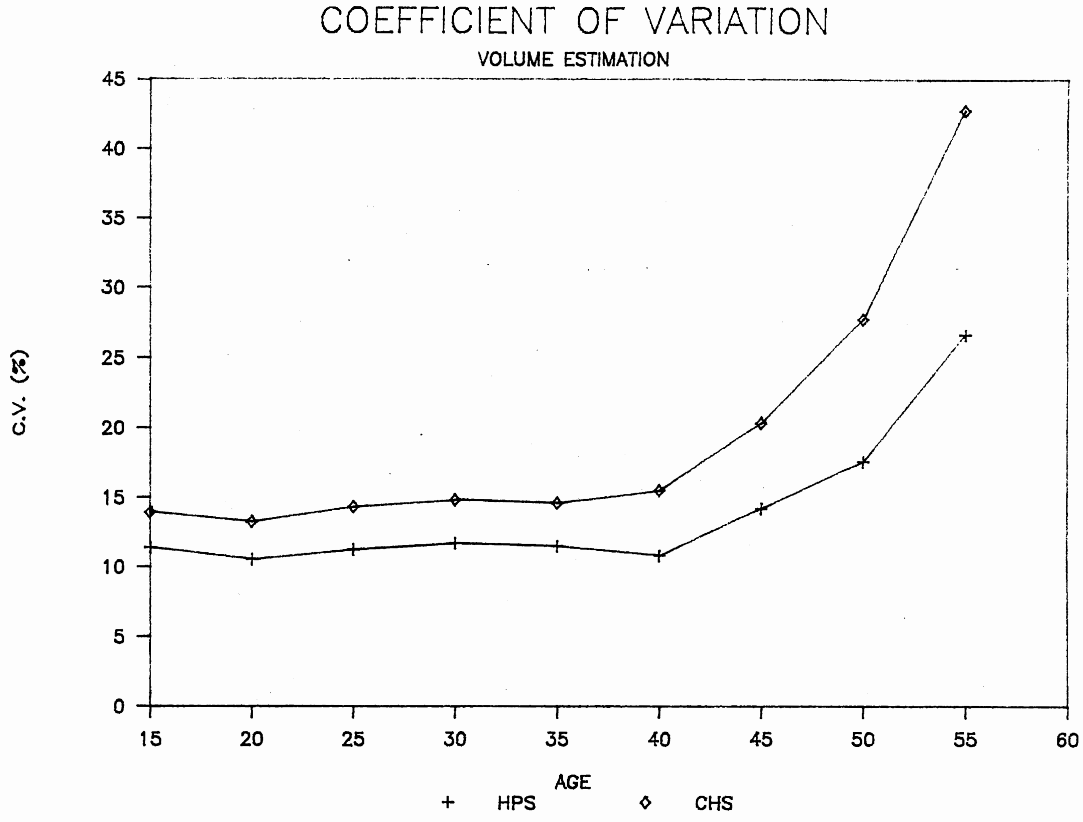

Figure 6. Coefficient of Variation for Critical Height Sampling and Horizontal Point Sampling in Volume Estimation. The Taper Equation used was from Cao, Burkhart, and Max (1980).

used in this study as compared to the Douglas fir stand used in the simulations of Iles (1979b). The fact that coefficients of variation for CHS and HPS were closer in the study of Iles (1979b) than in the current study may be due to differences in stand structures and taper functions used in the two studies.

Growth Estimation

The graph of the HPS and CHS growth estimators in Figure 7 shows that the estimates of growth from both sampling systems are essentially equal. This demonstrates the unbiasedness of CHS but does give any indication of the precision of the CHS growth estimator. Therefore, it is necessary to examine the variance of the growth estimators of CHS and HPS. In Figure 8 the variances of the growth estimators of HPS and CHS are compared. These results show that the variance of the HPS growth estimator is less than that of CHS at all ages execpt the first two. The lower CHS variance at the first growth interval probably results from a combination of a large HPS ongrowth variance, a low CHS mortality variance, and a high HPS nongrowth variance at the first growth interval (Figures 9, 10, and 11).

The gradual increase in the variance of CHS over HPS is probably due to an increase in the CHS mortality variance during the period from age 27.5 years to 42.5 years (Figure 10), and a decrease in HPS nongrowth variance at all

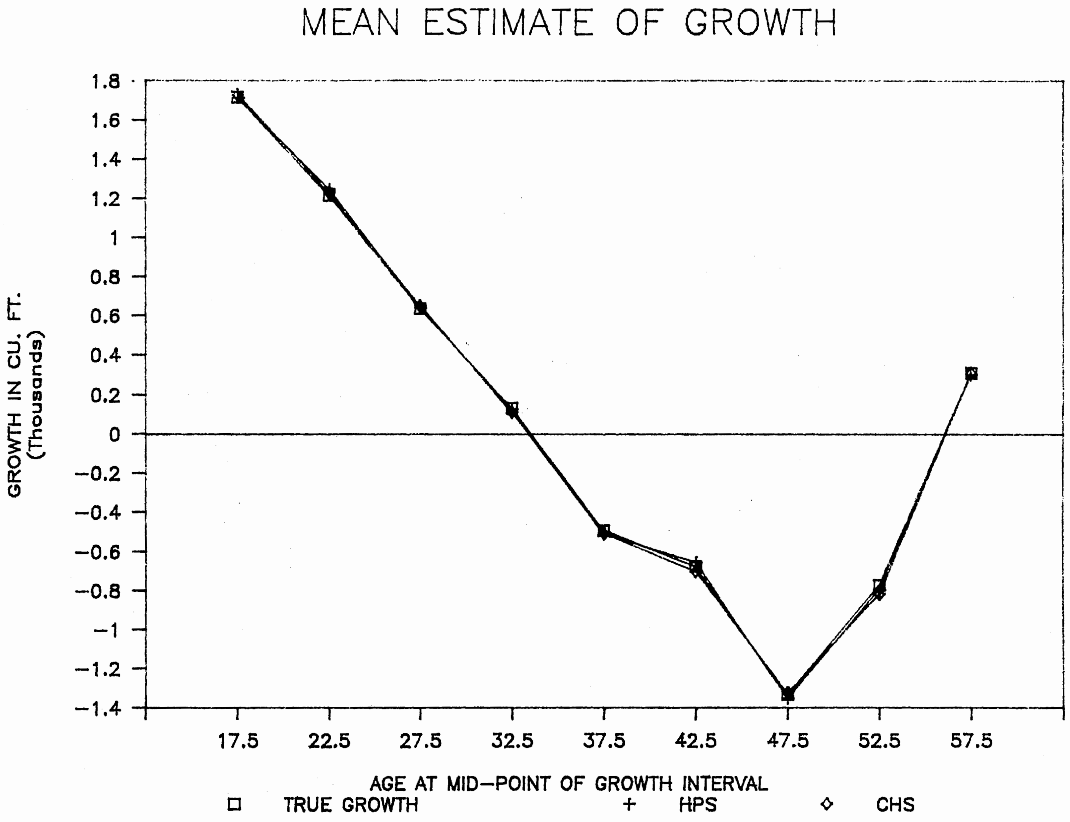

Figure 7. Mean Estimate of Growth by Critical Height Sampling and Horizontal Point Sampling. The Taper Equation used was from Cao, Burkhart, and Max (1980).

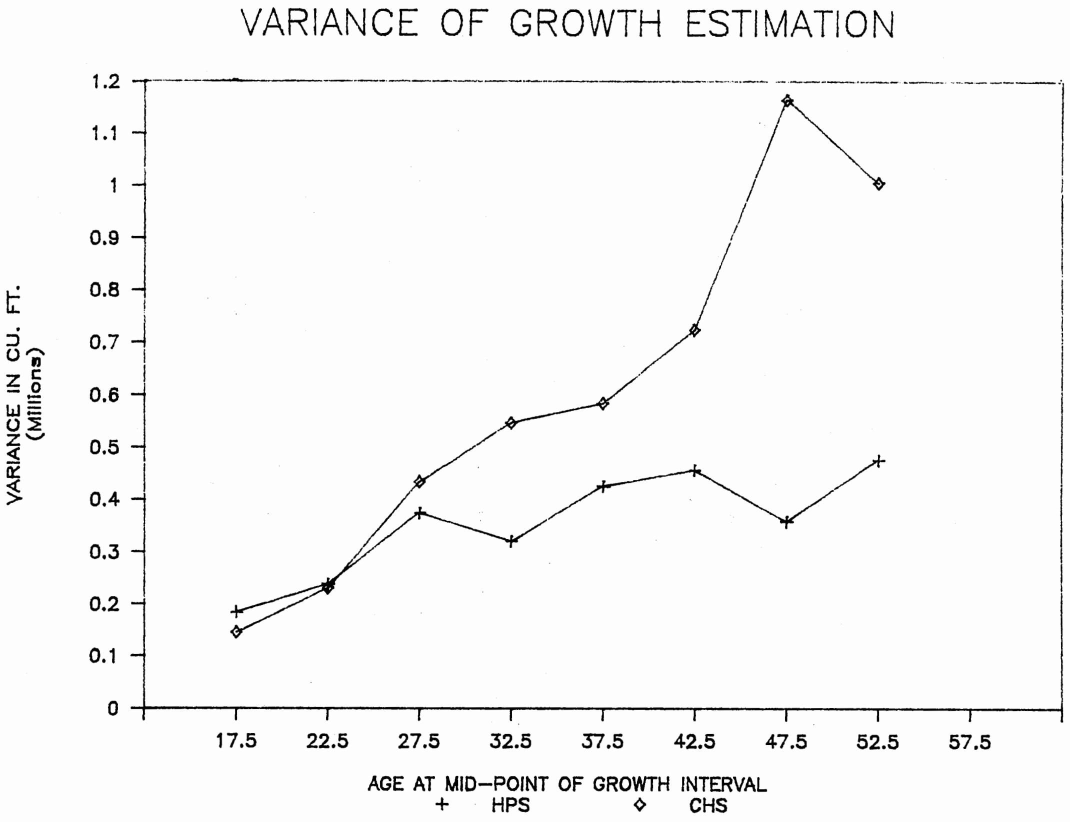

Figure 8. Variance of Growth Estimation by Critical Height Sampling and Horizontal Point Sampling The Taper Equation used was from Cao, Burkhart, and Max (1980).

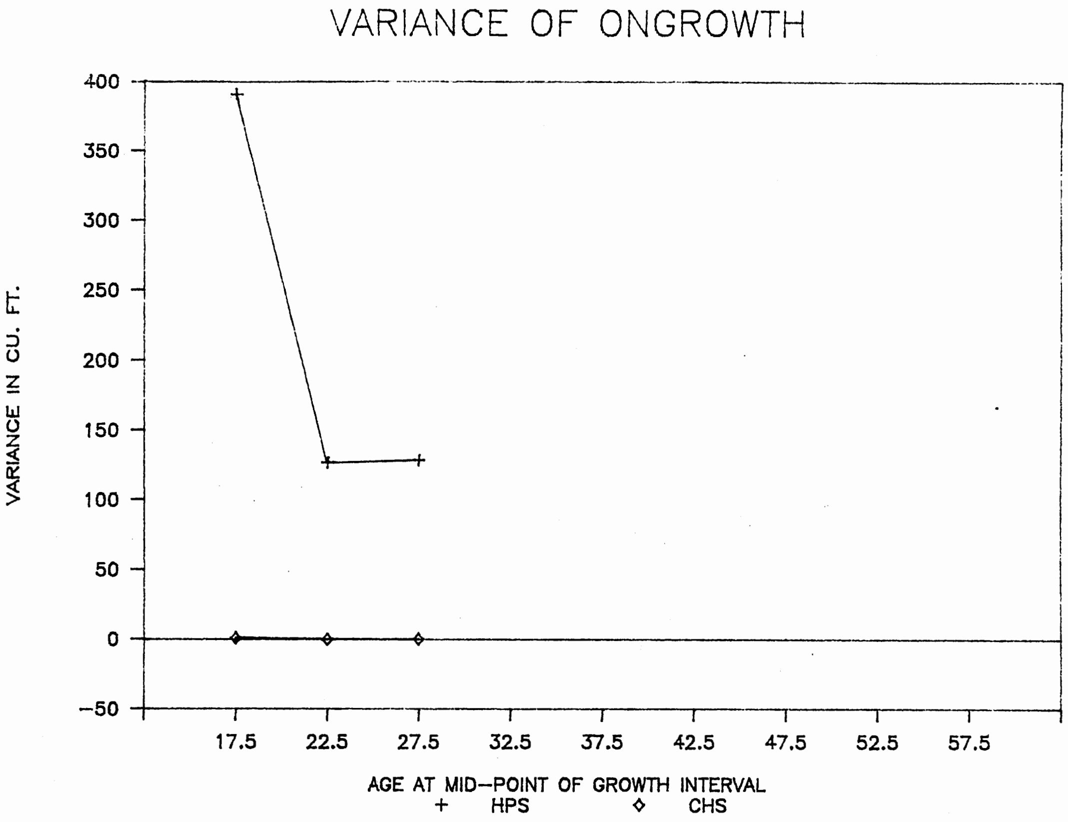

Figure 9. Variance of Ongrowth Trees in Critical Height Sampling and Horizontal Point Sampling. The Taper Equation used was from Cao, Burkhart, and Max (1980).

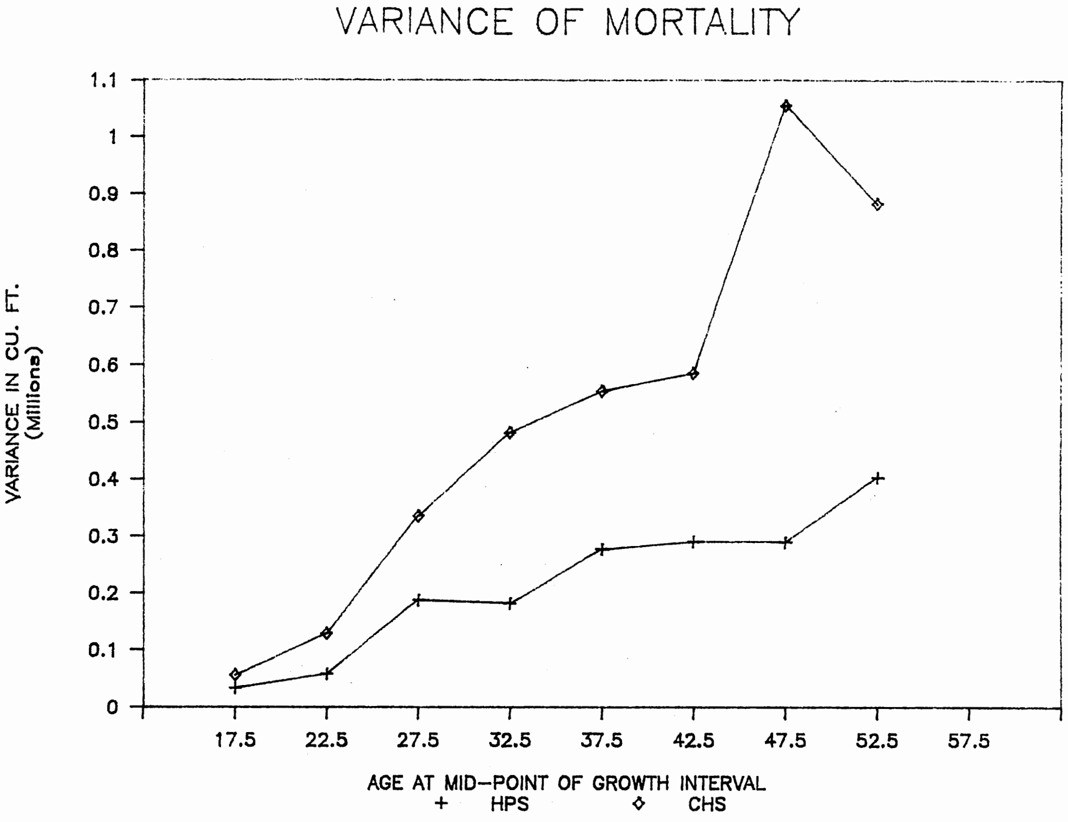

Figure 10. Variance of Mortality Trees in Critical Height Sampling and Horizontal Point Sampling. The Taper Equation used was from Cao, Burkhart, and Max (1980).

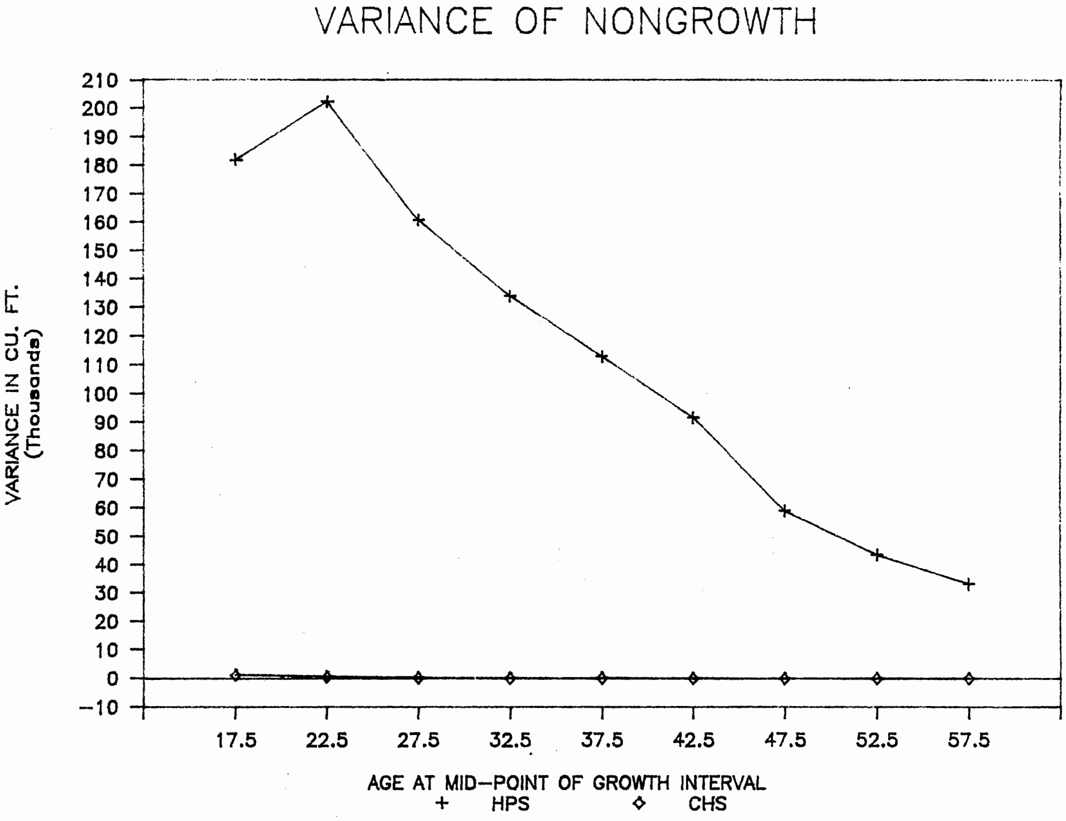

Figure 11. Variance of Nongrowth Trees in Critical Height Sampling and Horizontal Point Sampling. The Taper Equation used was from Cao, Burkhart, and Max (1980).

growth intervals, (Figure 11). The high increase in the variance of CHS at ages 47.5 and 52.5 is due to a large increase in the variance of the CHS mortality estimate at those ages. This indicates that the CHS growth estimator does not work well if mortality is high.

The results of the simulations which assume parabolic and conical tree shape differ from the comparisons which use the Cao, Burkhart, and Max (1980) taper equation. The mean estimate of growth for HPS and CHS using a parabolic shape assumption in Figure 12 shows that both the CHS and the HPS estimators produce unbiased estimates of growth. The variance of both growth estimates shown in Figure 13 show the variance of CHS to be lower than the variance of HPS at the first six measurement intervals. The distribution of mortality in the data set probably causes the variance of CHS to increase above the variance of HPS at the last two measurement intervals. The mean estimate of growth using a conical shape assumption shown in Figure 14 also verifies unbiasedness of CHS and HPS estimators. The variance of the growth estimators using a conical shape assumption presented in Figure 15 shows that the varaince of CHS is lower than the variance of HPS at only the first three measurement intervals. The variance of CHS is higher than that of HPS at the next three intervals but the difference is small. The CHS variance is much greater at the last two intervals but again that is probably due to a

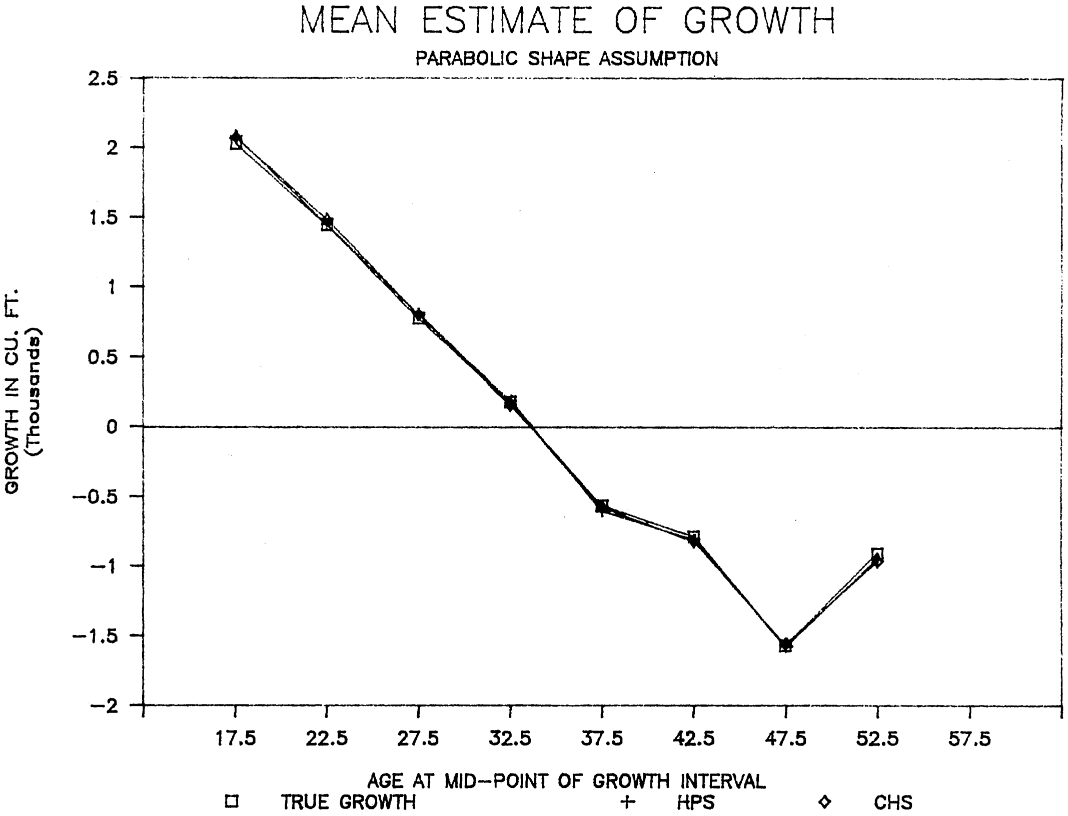

Figure 12. Mean Estimate of Growth Using A Parabolic Shape Assumption.

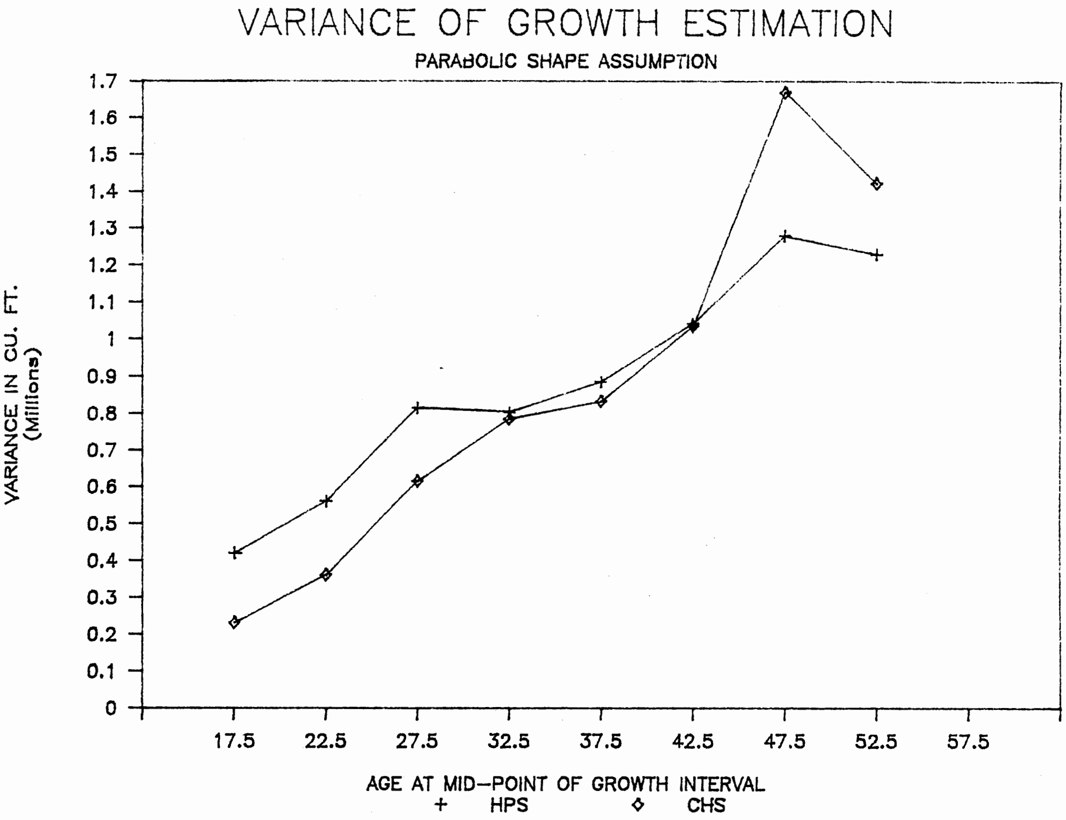

Figure 13. Variance of Growth Estimation Using A Parabolic Shape Assumption.

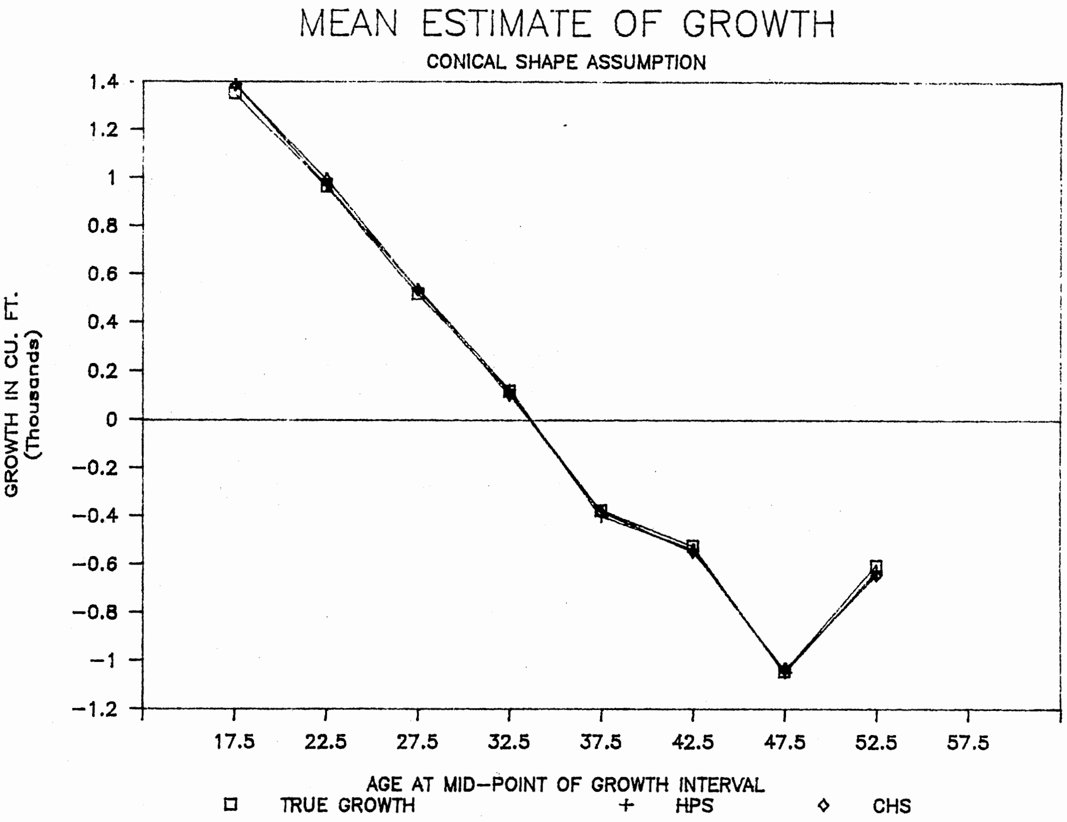

Figure 14. Mean Estimate of Growth Using A Conical Shape Assumption.

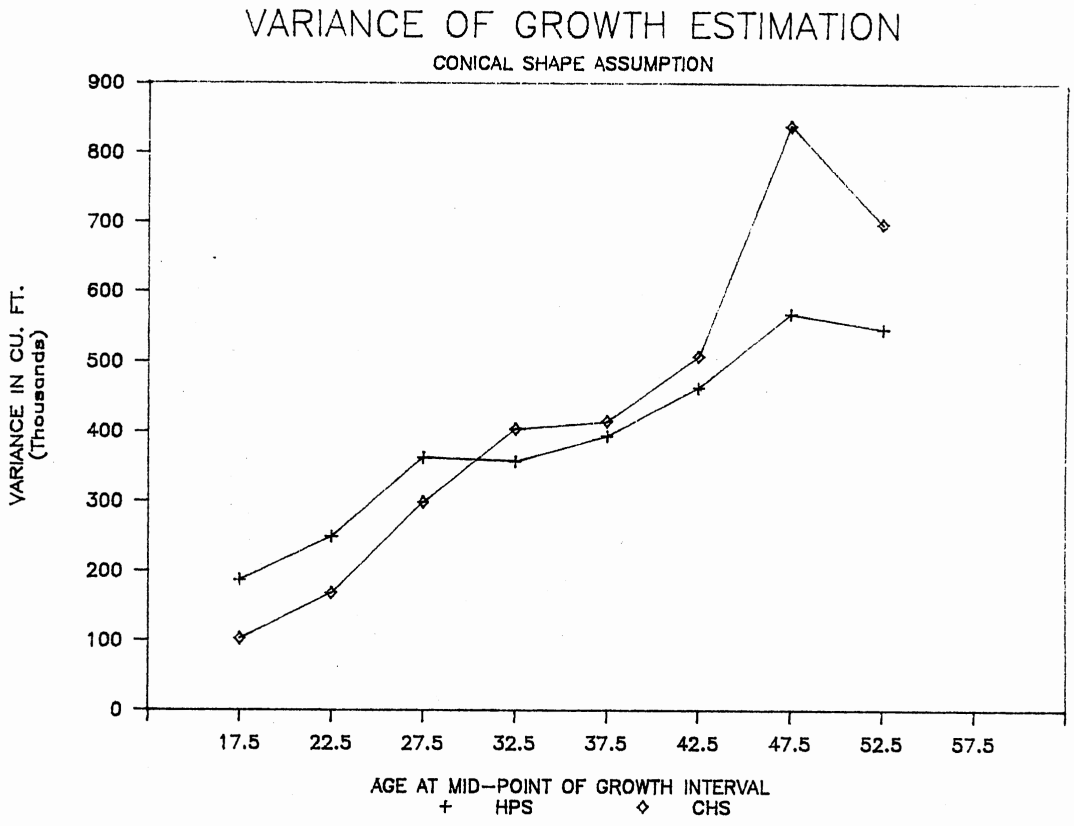

Figure 15. Variance of Growth Estimation Using a Conical Shape Assumption.

high level of mortality.

The coefficients of variation of both CHS and HPS in growth estimation with Cao, Burkhart, and Max (1980) taper and selection by groundline diameter are shown in Figure 16. Since the net change in volume is negative at some age intervals, some of the coefficients of variation will be negative. Thus, if the points located at ages 32.5 and 37.5 were connected they would incorrectly represent the change in the coefficent of variation between the fourth and fifth growth intervals, therefore, the graph is constructed of four distinct lines to more accurately represent transition between the positive coefficients of variation and the negative coefficients of variation for both sampling systems.

As mentioned previously there is a bias in total cubic volume when trees are selected by groundline diameter rather than by DBH. The difference in total cubic volume encountered in this study was between 3 and 4 percent. Sighting trees to groundline diameter causes the tree factor in HPS to decrease which causes the HPS variance to decrease. The CHS variance is not reduced correspondingly. Therefore, the variance of volume and growth estimation in CHS looks better relative to HPS when compared to the other results presented in this chapter. The mean estimate of growth for HPS and CHS when tree selection is by dbh is graphed in Figure 17. The estimates

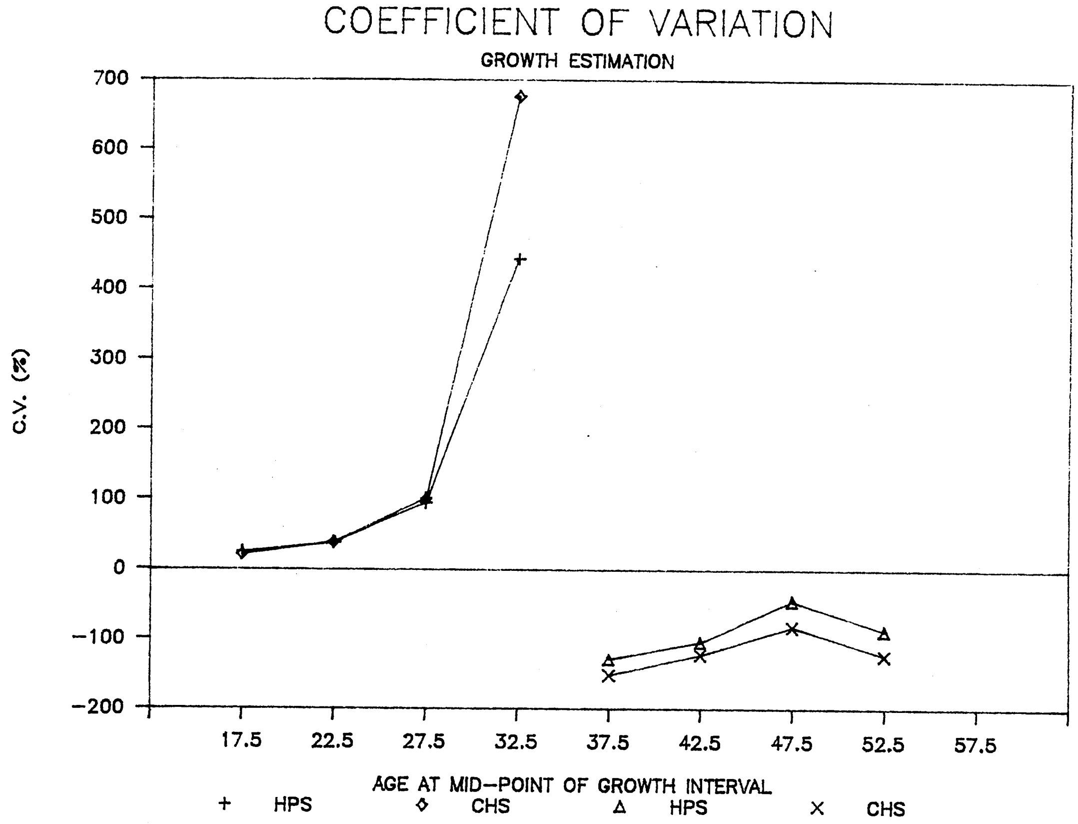

Figure 16. Coefficient of Variation for Critical Height Sampling and Horizontal Point Sampling in Growth Estimation. The Taper Equation used was from Cao, Burkhart, and Max (1980). There are Four Different Lines on the Graph to Enable Differentiation Between Positive and Negative Coefficients of Varaiation for HPS and CHS.

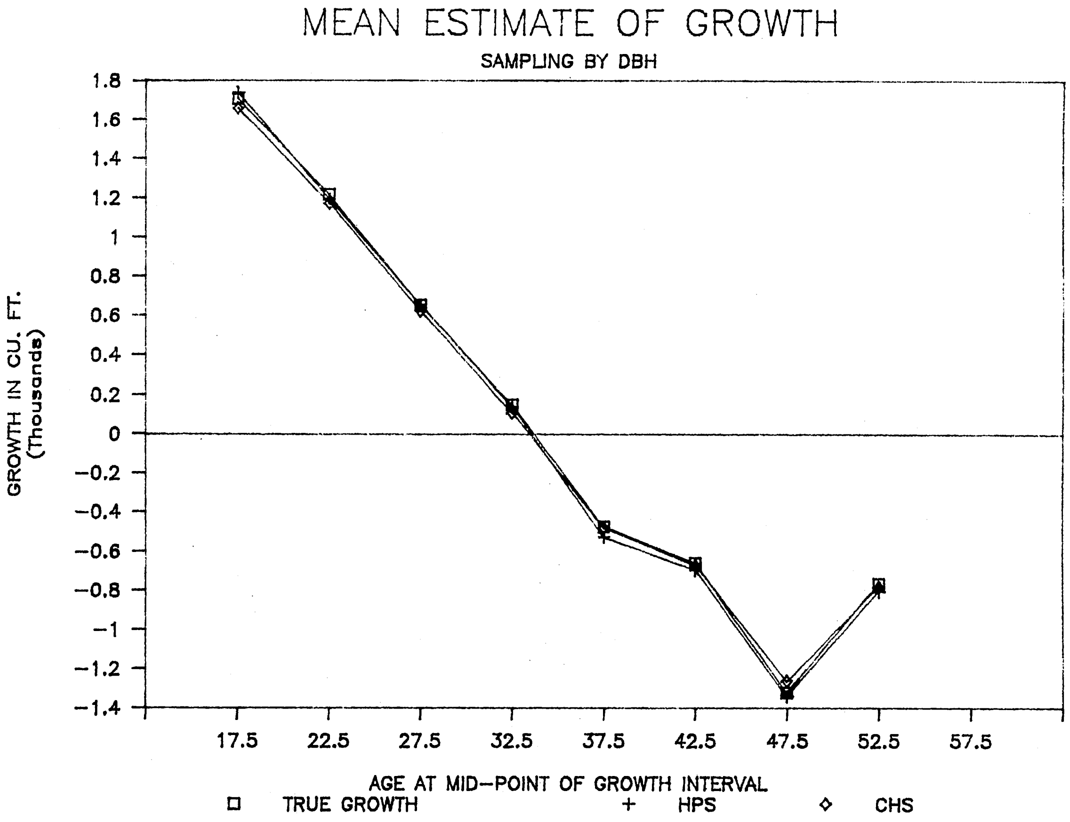

Figure 17. Mean Estimate of Growth When Tree Selection is Made by DBH. The Taper Equation used was from Cao, Burkhart, and Max (1980)

appear to be nearly unbiased indicating that the 3 to 4 percent bias that occurs in volume estimation does not affect the growth estimate greatly. The variance of the growth estimates when tree selection is by DBH is shown in Figure 18. The graph shows the variance of CHS to be less than that of HPS at six growth intervals. The variance of the CHS growth estimate is larger than that of the HPS growth estimate at the last two growth intervals, probably due to large CHS mortality variances in these intervals. McTague and Bailey (1985) and Lynch (1986) all proposed the use of taper equations to correct for the bias. The taper equations would be used to estimate the volume below dbh that is not contained by the cylinder between DBH and groundline diameter.

It should be noted that the estimation bias that results from selecting sample trees on the basis of DBH is not purely additive. Therefore, it can be expected to affect the variance of the CHS estimator. For example, it is well known that a percentage bias in the estimation of a mean will affect the corresponding variance according to the square of the percentage. However, this is not the only factor affecting the relationships of the variances of CHS estimators based on selection by DBH to those based selection by groundline diameter. The shape of the lower bole implied by the Cao, Burkhart, and Max (1980) taper function could be an important factor here. Critical

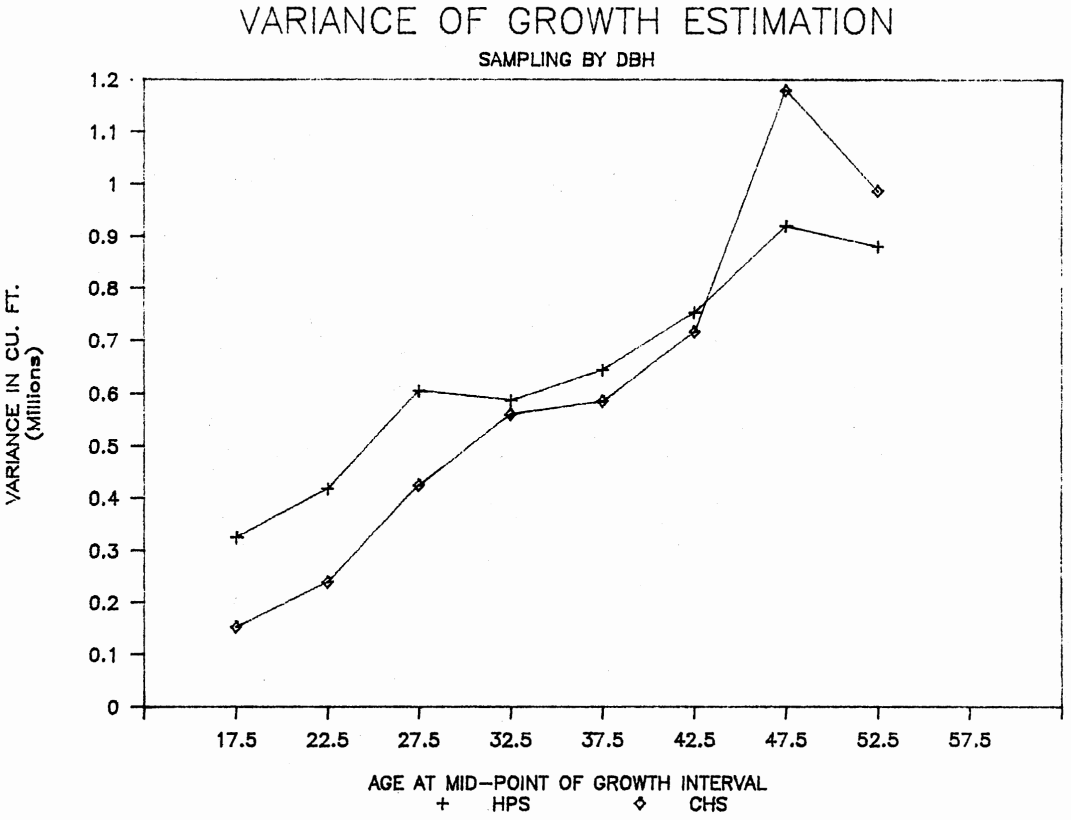

Figure 18. Variance of Growth Estimation When Tree Selection is Made by DBH. The Taper Equation used was from Cao, Burkhart, and Max (1980).

height sampling performed better relative to horizontal point sampling under both parabolic and conical shape assumptions (Figures 12, 13, 14, and 15). The results presented for cones and paraboloids cannot be directly compared to the results based on the Cao, Burkhart, and Max, (1980) taper equation because DBH was assumed to be groundline diameter in these simulations. However, they do suggest that the relationship between HPS and CHS can be different under different shape assumptions.

An additional comparison between HPS and CHS can be made by using ratios of variances to determine the number of points required to obtain the same standard error of volume growth estimate in each system. Here the standard error of volume growth estimate refers to the standard error of an average over several points. These ratios could be used to obtain sample sizes needed for equal standard errors in the stand that was used in this study. Given a sample size (number of points) N_h for horizontal point sampling, the sample size N_c required to obtain an equal standard error in critical height sampling is found by rearranging

$$\frac{VAR_c}{N_c} = \frac{VAR_h}{N_h} \qquad (12)$$

to

$$N_c = (VAR_c/VAR_h)N_h \qquad (13)$$

Tables XI and XII in appendix F list the variance ratios for each age for both volume and growth estimation for the main simulation (that is, Cao, Burkhart, and Max (1980) taper and sample tree selection by groundline diameter) examined in this study. This table applies only to the stand used in this study and not necessarily to other stands.

Components of Growth

When comparing variances of growth components one should note that the expected value of s_2, o, n, i, and s_2-s_1 are not the same in CHS as in HPS. To see why this occurs consider n (nongrowth). Since nongrowth trees were not in at time 1, they are far away from the point and have a small critical height at time 2. Consequently, the expected value of n under CHS is always less than under HPS. The expected values of m and s_1, however, are the same in either CHS or HPS. Tables I, II, III, and IV in appendix A give the means and variances of all components of growth for CHS and HPS. Figure 19 and Figure 20 show the variance of the s_1 and the s_2 volumes respectively. The variance associated with CHS is higher than that of HPS at all ages for both s_1 and s_2 volumes. The graph of the variance of s_2-s_1 of CHS and HPS in Figure 21 shows that the variance of s_2-s_1 for CHS is consistently less than for HPS. The variance of

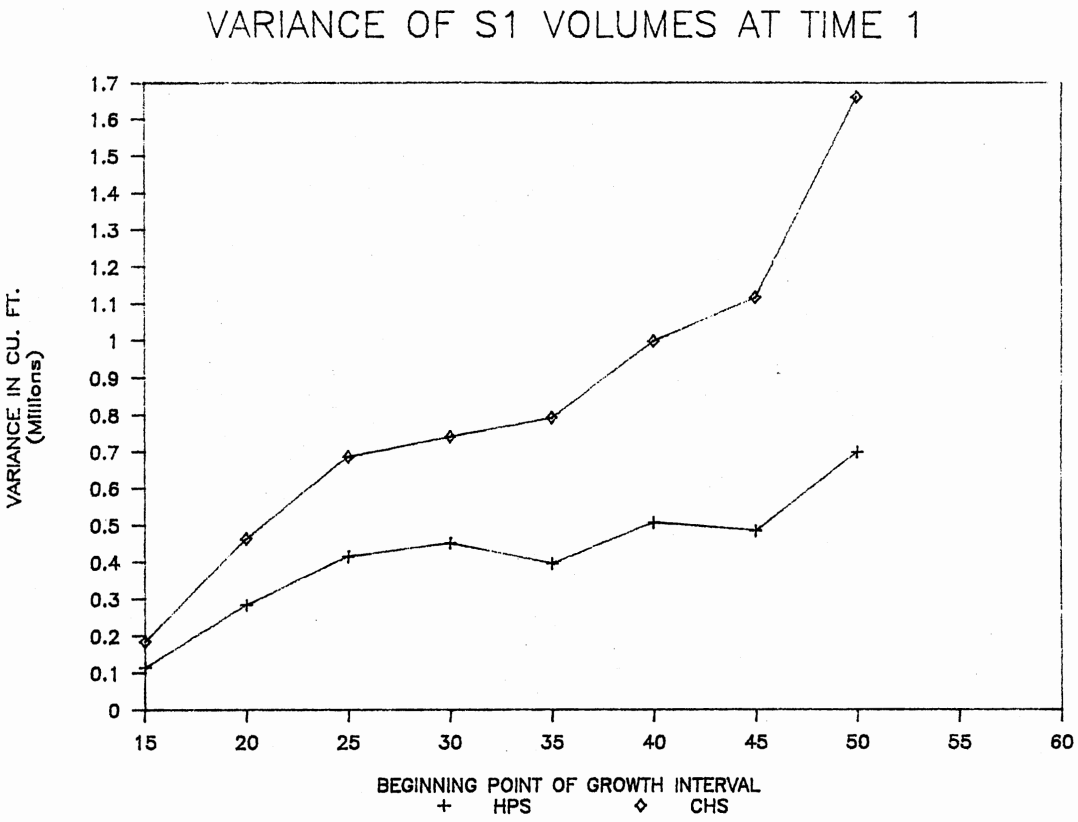

Figure 19. Variance of the Volumes of S_1 Trees at Time 1. Since S_1 Trees are Measured and Tabulated Only at Time 1, They are Plotted Against the Beginning of the Growth Interval.

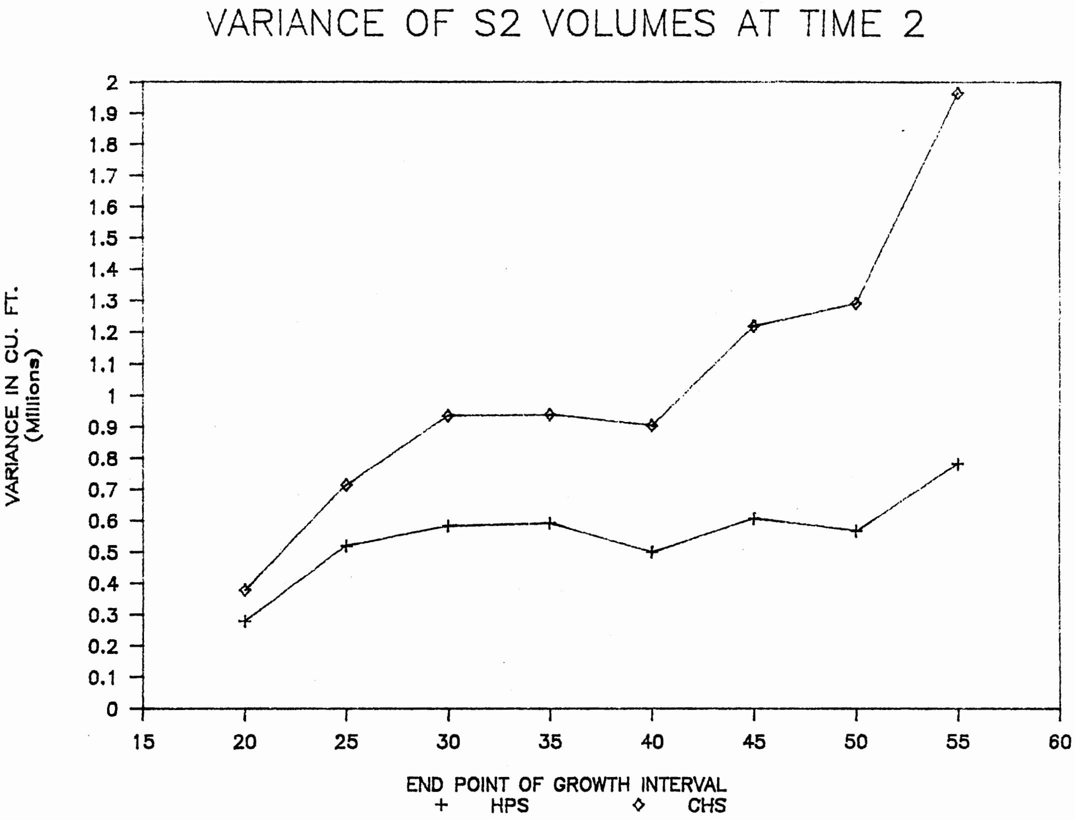

Figure 20. Variance of the Volumes of S_2 Trees at Time 2. Since S_2 trees are Measured and Tabulated Only at Time 2, They are Plotted Against the End Point of the Growth Interval.

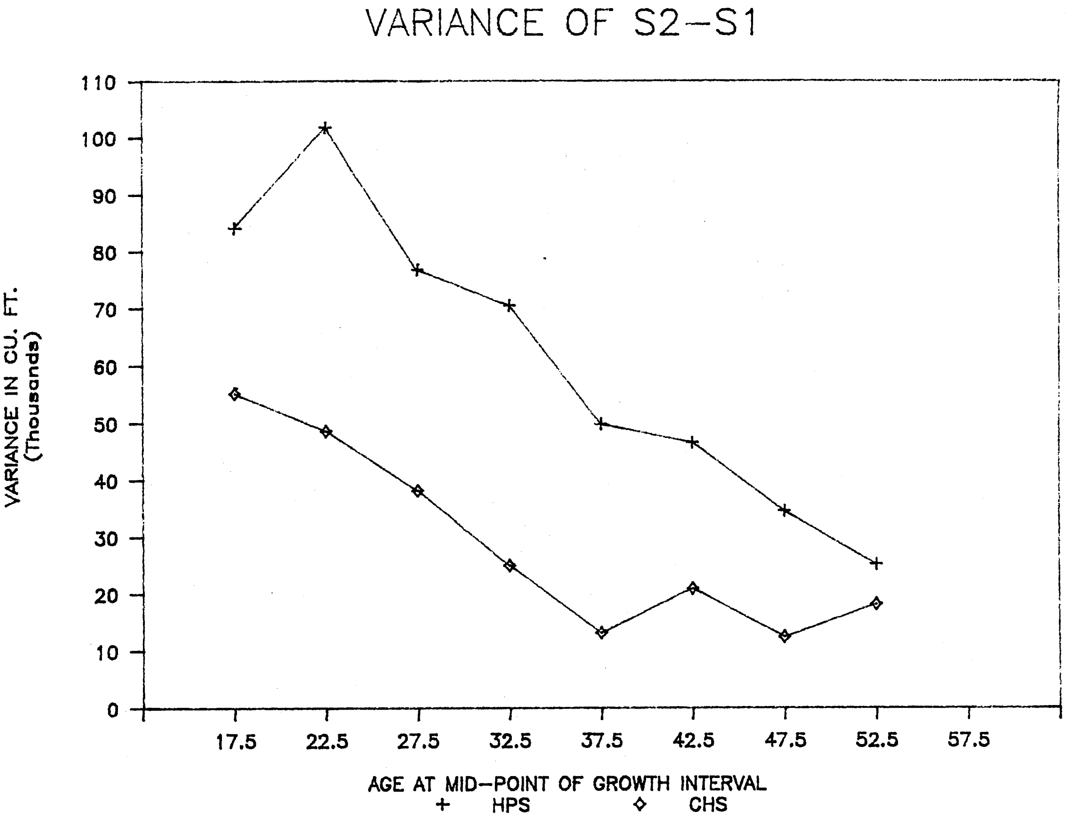

Figure 21. Variance of S_2-S_1. The Taper Equation used was from Cao, Burkhart, and Max (1980).

mortality by CHS and HPS shown previously in Figure 10 showed the variance of CHS when estimating mortality to be much higher than that of HPS. A graph of the actual mortality of the stand shown in appendix B (Figure 27) shows that CHS was more sensitive to large changes in mortality as the shape of the CHS mortality variance curve follows that of the actual stand mortality very closely. The large value of the CHS mortality variance at age 47.5 indicates that CHS may not estimate growth very efficiently if mortality is high. This could be due to the large increase in mortality causing a proportionally more variable decrease in the sum of critical heights than in the sum of VBAR's in a conventionally computed plot as was suggested by Iles (1979b).

Ingrowth and ongrowth trees were only encountered in the intervals between ages 20, 25 and 30. The magnitude of the ingrowth and ongrowth estimates as well as their variances are small, thus it is difficult to interpret their effect on the variance of the growth estimators of CHS and HPS. Figure 9 indicated that the variance of the contribution to growth of the CHS ongrowth trees was much smaller than that for the HPS ongrowth trees. An ongrowth tree is one that is submerchantable and out at time 1 but merchantable and in at time 2. Ongrowth trees are submerchantable, out and thus relatively distant from the point at time 1 therfore, the CH is low when they are in at the end of the growth interval. Thus, CHS under-estimates the value of ongrowth trees relative to HPS, (that is, the expected value of "ongrowth" in CHS does not equal the expected value of "ongrowth" in HPS).

The variance of the contribution to growth of ingrowth trees in CHS and HPS graphed in Figure 22 indicates that the CHS ingrowth trees contribute more to the variance of growth than the HPS ingrowth trees. An ingrowth tree is one which is submerchantable and in at time 1 but is merchantable and in at time 2. Since an ingrowth tree has to be close to the sample point it is possible that it could have a relatively high critical height. Thus, the expected value of the contribution of ingrowth trees to volume growth is higher in CHS than it is in HPS.

The variance of the nongrowth trees in CHS and HPS was shown in Figure 11. McTague and Bailey (1985) reported that the variance of the contribution of nongrowth trees to volume growth is higher in CHS than in HPS. An examination of the relationships presented in Figure 11 confirms this and shows that the variance of nongrowth in CHS is considerably less than that for HPS. A nongrowth tree is one which is merchantable and out at time 1 but is merchantable and in at time 2. Trees that are out and merchantable at the intial measurement period will have a small CH when they are in at time 2, thus, the nongrowth estimate by CHS should be smaller than that of HPS.

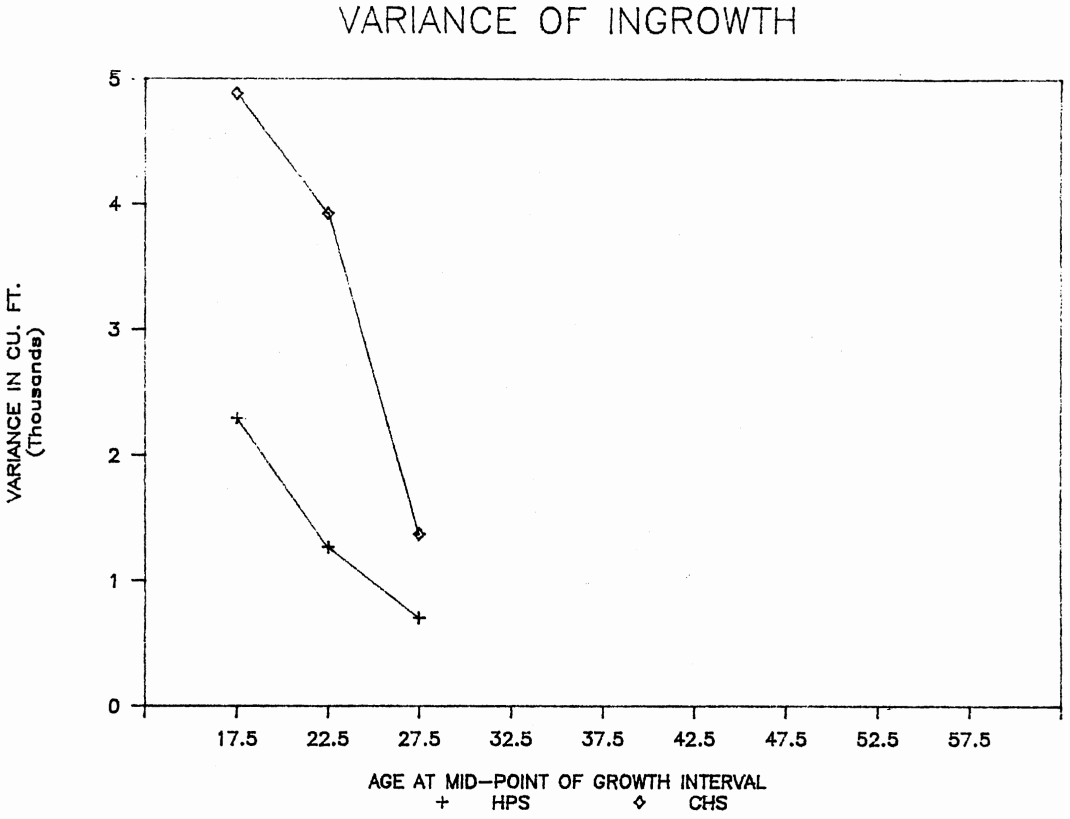

Figure 22. Variance of Ingrowth Trees in Critical Height Sampling and Horizontal Point Sampling. The Taper Equation used was from Cao, Burkhart, and Max (1980).

Thinned Stand Data Set

To investigate the influence of mortality trees in volume and growth estimation a data set representing a thinned stand of trees was used in some simulations. The stand used the same parameters described previously except that a low thinning was conducted at ages 10, 20, 30, 40, and 50 to a residual basal area of 80 square feet per acre. The simulations again used 1000 sample points, BAF of 10 and a one acre stand. Cao, Burkhart, and Max (1980) taper was assumed and sample trees were selected according to their groundline diameters. The graph of the volume estimators is given in Figure 23. Both CHS and HPS provided unbiased estimates of volume at all ages. Estimators of net volume change in Figure 24 show unbiased estimates by CHS and HPS. The variance of the HPS and CHS net volume change estimators on the thinned stand shown in Figure 25 demonstrates some interesting points. The variance of CHS was lower than that of HPS at three net change intervals. These were intervals during which no cutting occured. Overall, the variances of both net change estimators were much closer in the intervals where no thinning occured than the previous results (Figure 8). This means that if the effects of high mortality can be removed, CHS could possibly have a lower variance than HPS in the estimation of net volume change.

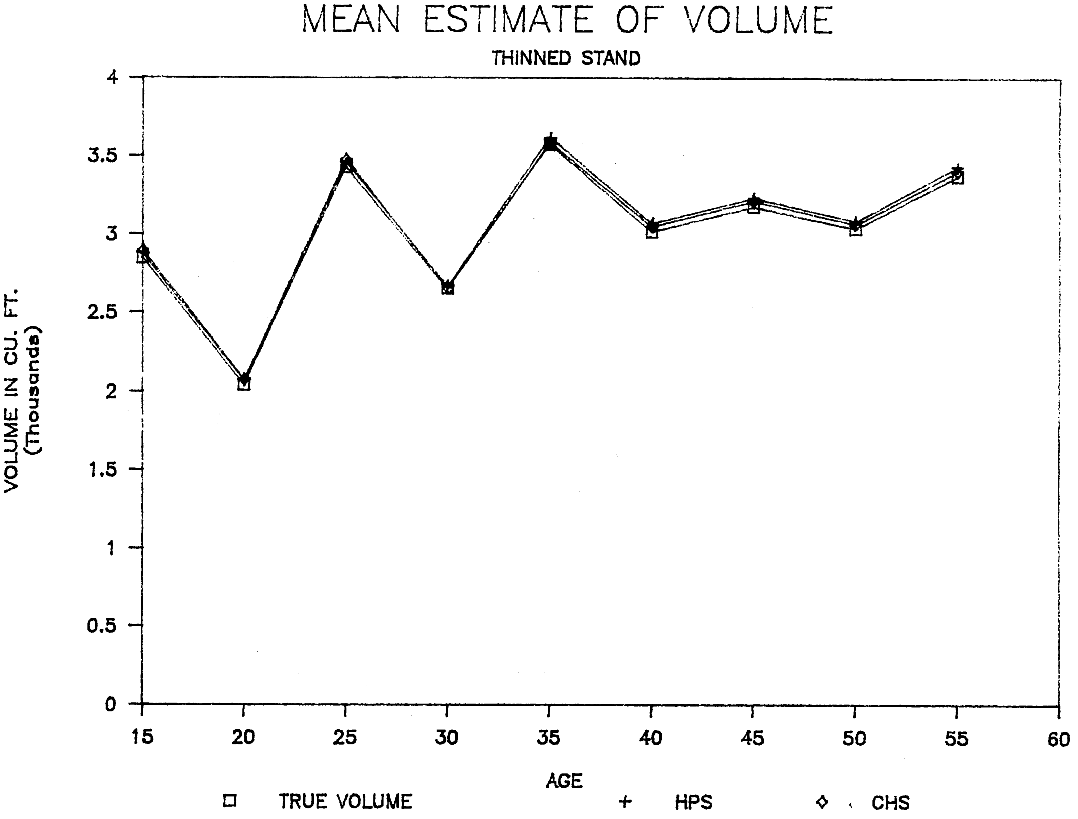

Figure 23. Mean Estimate of Volume Using a Thinned Stand. The Data Represents a Stand Which Had Undergone a Low Thinning to 80 Ft^2/Acre Residual BA at Ages 10,20,30,40, and 50 Years. The Taper Equation used was from Cao, Burkhart, and Max (1980).

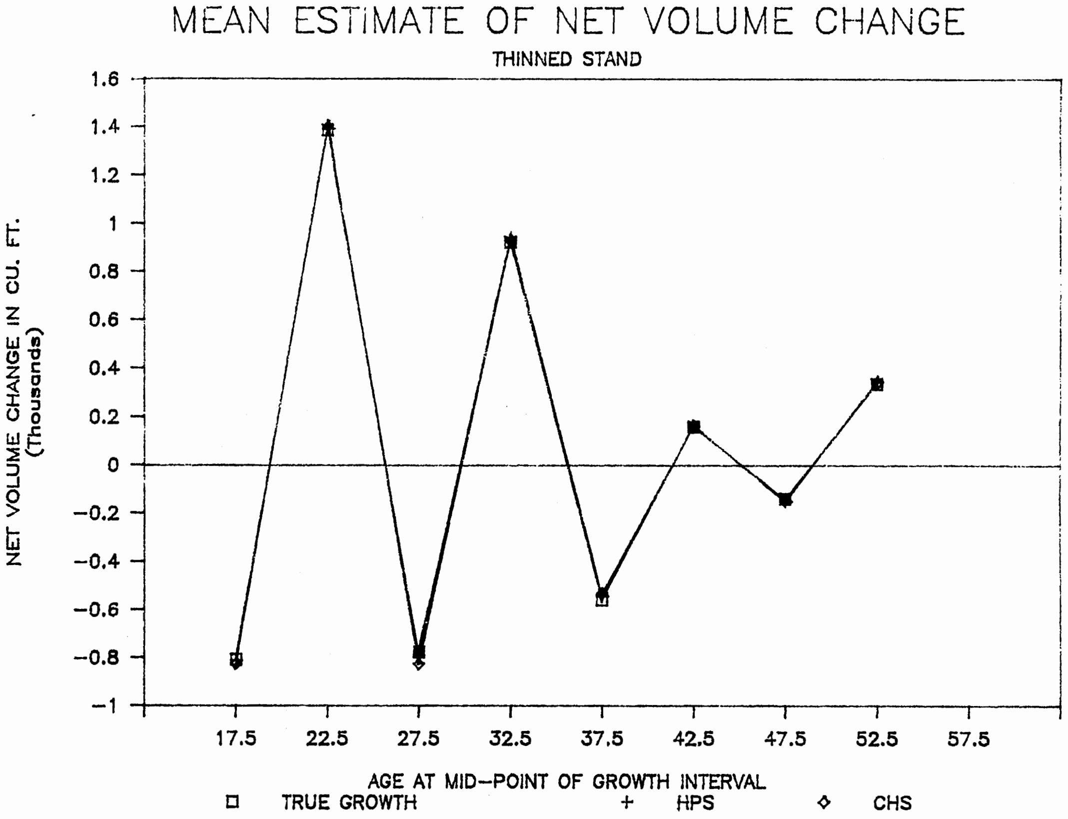

Figure 24. Mean Estimate of Net Volume Change. The Data Used in This Simulation Represents a Thinned Stand. The Taper Equation used was from Cao, Burkhart, and Max (1980).

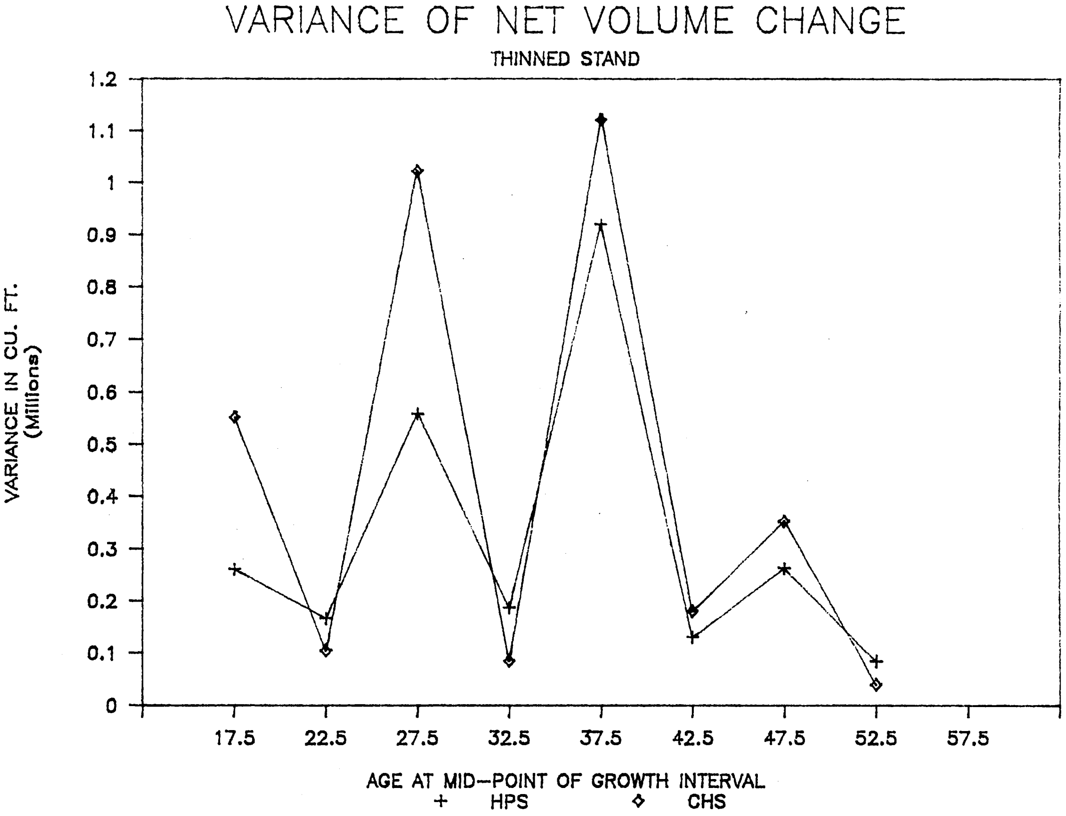

Figure 25. Variance of Net Volume Change. The Data Set Represents a Thinned Stand. CHS has the Lower Variance at The Second, Fourth, and Eigth Growth Intervals. The Difference Between CHS and HPS Decreases as The Stand Gets Older. The Taper Equation used was from Cao, Burkhart, and Max (1980) and Sample Trees were Selected by Groundline Diameter.

CHAPTER IV

FIELD APPLICATION

The overall goal of a field test as stated by Iles (1979b), is to "identify problems in application and possible solutions". Although it was not possible to conduct a field test of CHS in remeasured point sampling for this study, it was possible to observe some of the oddities which may occur in the field while estimating volume using critical height sampling.

There are several types of problems which occur in the field application of critical height sampling. Most of the problems fall into one of three categories. The first is that the critical point may not be visible due to foilage, other trees or some other factor which blocks the line of sight. The second type of problem is created by trees which are very close to the sample point. The resulting angle of measurement is too steep to allow successful measurement of the critical height of the tree. The third category of problem originates from the inability of the measuring instrument to locate the critical point.

In order to examine some of these problems more closely a very small scale field exercise was conducted in a natural stand of shortleaf pine (Pinus echinata) in Southeast Oklahoma. A single entry volume table was used to make comparable estimates of volume through horizontal point sampling. The table had been constructed the previous week from data measured on standing trees by undergraduates students of the OSU forestry program.

Seven sample points were located and measured using horizontal point sampling and critical height sampling. All measurements were made in metric units because only metric scale relascopes were available. A metric scale relascope mounted on a tripod was used to choose the sample trees and to measure the critical height of each "in" tree. In cases where the angle of measurement was too steep for the relascope, the critical diameter was calculated using the distance from the sample point to the tree and the plot radius factor. A Wheeler Pentaprism was then used to locate the critical point and the critical height was measured using a clinometer.

Foilage and other vegetation which blocked the view of a tree was a problem which might be correctable. Since remeasurement of permanent points is often done during winter in temperate climates, this problem could be either partially or completely eliminated. During other seasons, the only alternate solution may be to improve on the quality of the measuring device in an attempt to get around the obstructions.

However, there is a possible solution to the second type of problem. In situations where a tree is too close to the sample point, the critical diameter can be adjusted in order to allow the measurement of critical height to be made more easily. This alternate critical diameter can be calculated by the following equation.

$$ALTCD = [(BA - CA) / K]^{(1/2)} \qquad (14)$$

The effect of using the alternate critcal diameter is to lower the critical height of a tree which is too close to the sample point and to raise the critical height of a tree that is far away. Trees which are too close to the sample point usually are difficult to measure because the angle is too steep. Thus, by lowering the critical diameter it becomes easier to measure the critical height of the tree. If a tree is located far away from the sample point the critical diameter will be close to the DBH thus making it difficult to detect the difference between the critical point and the DBH or groundline diameter. This will make the measurement of the critical height difficult with the relascope. Use of the alternate critical diameter to obtain critical heights also produces an unbiased estimate of volume. Figure 26 illustrates the effects of using the alternate critical diameter.

There are several situations which should be discussed with respect to the field procedure. Magnification (at several different levels) may not help the problem of

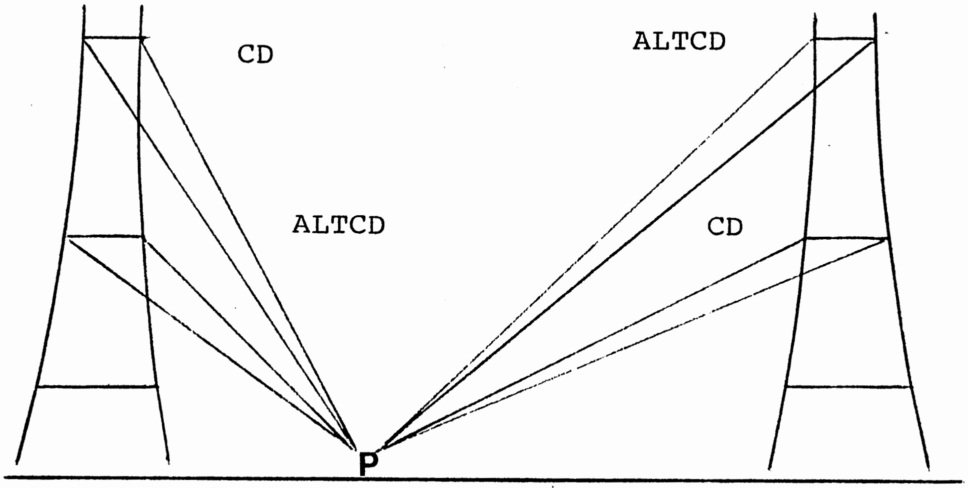

Figure 26. Effect of the Alternate Critical Diameter on the Critical Height of a Tree.

locating the critical diameter because foliage and other obstructions are also magnified (Iles 1979b). A relascope and a Wheeler pentaprism are difficult to use even if one has a fairly clear area. During the field trial, the problem of having more than one critical diameter was evident and on some trees the selection of the critical point seemed almost arbitrary.

The alternate critical diameter is not difficult to calculate with a programmable hand held calculator or portable computer. If one measures the distance and the DBH one can calculate the CD and the ALTCD easily. The program should be such that the instrument reading can be input to obtain the critical height. Thus, the advantage of the Alternate Critical Diameter is that only one measurement of distance to the sample tree must be made, even if the tree is close to the sample point. The "ordinary" CD requires two measurements to the tree when foilage blocks the view of the critical point or when the sample point is located too close to the tree. As a result, the distance from the point to the tree must be measured in order to calculate the critical diameter. Then the distance from the tree to some other location, chosen to facilitate the measurement of the critical height, must be made in order to measure the critical height.
Simulations using the ALTCD in the measurement of critical heights were run and the results are presented in appendix

C. The variance of the ALTCD is much higher than the variance of HPS and the variance of CHS using the CD. This indicates that the ALTCD is probably impractical for use in CHS.

Under the conditions encountered in this exercise it seems possible to put in a single point in a one half hour. With large trees it will take longer and the measurements will probably be less precise. Factors including stand density, terrain, season, weather, and proficiency of instrument use will affect the time requirement. What needs to be known is if the bias introduced by lack of exact measurements in CHS is equal to, less than, or greater than the bias introduced through the use of a volume table in ordinary point sampling.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

It has been shown that the variance of the growth estimator of CHS can be lower than that of HPS under some conditions. When tree selection is made by groundline diameter the variance of the CHS growth estimator is usually larger than that of HPS. However, when tree selection is made by DBH the variance of the HPS growth estimator is higher than that of CHS at some ages. Simulations performed using parabolic and conical taper assumptions indicate that the relationship between the HPS and CHS volume growth estimators is sensitive to taper assumptions. CHS performs better relative to HPS under parabolic and conical shape assumptions than when the Cao, Max, and Burkhart (1980) taper equation is used.

The results indicate that variances of the contributions to growth of ongrowth and nongrowth trees are smaller in CHS than HPS. The variance of the contribution to growth of ingrowth trees is higher with CHS than with HPS, thus the problem of ingrowth trees is increased through the use CHS rather than reduced. However, the magnitude of the ingrowth variance is small and the difference between the variance of the ingrowth trees in CHS and HPS probably does not greatly affect the variance of the net growth estimate. Growth estimation using CHS is highly sensitive to changes in mortality and the variance of growth estimation is increased by a large number of mortality trees.

Critical height sampling was not as efficient at estimating volumes of forest stands as HPS in the stand studied here. Widespread use of CHS in volume estimation is not likely until an improvement in the measuring instrument is made. At the present time the use of Ueno's (1979,1980a) space point sampling (see Bitterlich 1984) may be the most efficient way to apply CHS in volume estimation, according to the results of Sterba (1982). The advantage of the system (under conditions similar to those assumed by Sterba (1982)) lies in reduction of requirements for sample tree measurement to about one to two trees on each sample point. Thus, it is possible to establish three or four times as many sample locations using Ueno's method as would be required by HPS or ordinary CHS in the same amount of time.

The use of CHS in growth estimation would utilize CH measurements rather than counts as in Uneo's method. More care is inherently taken in measuring growth from permanent points than when measuring volume during temporary "one-time" inventories. Therefore, it is easier to justify the extra time and manpower required to measure each critical height individually using a relascope or other individual height measuring instruments when permanent points are being installed or measured.

There are a few areas where further research is possible. First, simulated comparisons of CHS to HPS should be run with data representing other forest types to see whether the results are similar to those reported here. This would include simulations using natural stand data constructed using a growth simulator or measured from existing stands. Stands having varying levels of mortality trees should be used to examine the effects of high or low levels of mortality trees on CHS. Age levels can be varied on test stands to study the effects of high or low ongrowth and ingrowth on CHS. Spacing can be varied to study the effects of nongrowth in greater detail. Also, factors such as BAF, size of stand, number of sample points and method of determining critical heights can be used to test the effects each has on the use of CHS in growth estimation.

Second, an extensive test of the field procedure should be made. It should include an investigation into technologically based improvements availble for enhancing the accuracy of the individual measurement of critical height. The possibility of providing an electronic measurement of the distance from the sample point to the tree could be considered, since distance measurements in the current CHS procedure are time consuming. Initially, an attempt might be made to evaluate an improvement in the magnification of the view through the instrument. This could include an adaptation of the Telarelascope (an instrument based on the relascope but providing a magnified image of the tree, see Bitterlich (1984)) to the field procedure associated with Critical Height Sampling.

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APPENDICES

APPENDIX A

MEANS AND VARIANCES OF ALL GROWTH CONPONENTS

TABLE I

HORIZONTAL POINT SAMPLING MEANS

class mid-point	s_1	m	i	o	s_2	n	s_2-s_1
	cubic feet/acre						
17.5	2647	211	19	3	--	1009	874
22.5	4036	517	8	1	3521	879	963
27.5	4955	833	5	1	4899	719	741
32.5	5388	1034			5696	557	603
37.5	5111	1437			5991	452	481
42.5	4650	1394			5592	354	392
47.5	3576	1821			5043	229	257
52.5	2940	1122			3833	150	188

TABLE II

CRITICAL HEIGHT SAMPLING MEANS

class mid-point	s_1	m	i	o	s_2	n	s_2-s_1
	cubic feet/acre						
17.5	2652	211	22	.1	--	70	1828
22.5	4053	519	10	.02	4481	43	1681
27.5	4940	848	5	.03	5735	26	1461
32.5	5380	1053			6402	16	1146
37.5	5113	1428			6526	11	916
42.5	4623	1418			6030	7	716
47.5	3553	1794			5340	4	481
52.5	2893	1145			4034	2	337

TABLE III

HORIZONTAL POINT SAMPLING VARIANCES

class mid-point	s_1	m	i	o	s_2	n	s_2-s_1
	------------------cubic feet/acre--------------------						
17.5	116375	34409	2302	391	--	182041	84288
22.5	285331	59158	1271	127	281387	202478	101948
27.5	417131	188444	705	129	521263	160796	76851
32.5	454016	182196			584900	134007	70502
37.5	398489	277896			594211	112944	49968
42.5	508571	291793			500816	91736	46721
47.5	495571	290710			608879	59081	34672
52.5	699993	404595			568230	43626	25283

TABLE IV

CRITICAL HEIGHT SAMPLING VARIANCES

class mid-point	s_1	m	i	o	s_2	n	s_2-s_1
	-----------------------cubic feet/acre---------------						
17.5	186445	56614	4884	1.0	--	1279	55311
22.5	465410	129565	3926	0.1	379401	685	48764
27.5	687618	336419	1375	0.2	715225	298	38224
32.5	742171	483443			937214	163	25085
37.5	793422	555347			940765	106	13244
42.5	1000014	587063			905186	59	21006
47.5	1118473	1055859			1221171	33	12505
52.5	1662534	883796			1293522	17	18240

TABLE V

VOLUME ESTIMATION USING CHS AND HPS

age	Volume			Variance		C.V.	
	true	hps	chs	hps	chs	hps	chs
	-----------	cubic	feet/acre	------		------%-------	
15	2859	2893	2899	109397	164300	11.4	13.9
20	4577	4609	4629	237103	377192	10.5	13.2
25	5795	5859	5860	438335	707526	11.2	14.3
30	6432	6501	6512	584622	937908	11.7	14.8
35	6564	6629	6623	583855	939997	11.5	14.6
40	6070	6120	6116	440310	904413	10.8	15.5
45	5395	5465	5413	606953	1220519	14.2	20.4
50	4060	4113	4089	526050	1291279	17.6	27.7
55	3290	3319	3273	785746	1964094	26.7	42.8

TABLE VI

GROWTH ESTIAMTION USING CHS AND HPS

class mid-point	Growth			Variance		C.V.	
	true	hps	chs	hps	chs	hps	chs
	cubic feet/acre					%	
17.5	1718	1716	1730	184724	146576	25	22
22.5	1218	1250	1230	239314	231307	39	39
27.5	636	641	652	375728	434588	95	101
32.5	131	128	109	321743	548082	444	677
37.5	-493	-509	-506	426852	585251	-128	-151
42.5	-675	-654	-702	457923	725576	-103	-121
47.5	-1334	-1351	-1324	359085	1164682	-44	-81
52.5	-770	-793	-815	477407	1006990	-87	-123

APPENDIX B

MORTALITY OF DATA SET

TABLE VII

MORTALITY OF DATA SET

Age	# Dead at age	% of 676 dead at age	# Dying in interval	% Dying in interval	% Dying based on new base number
15	143	21.15			
			83	12.20	15.57
20	226	33.43			
			90	13.31	20.00
25	316	46.75			
			82	12.13	22.78
30	398	58.88			
			56	8.28	20.14
35	454	67.16			
			61	9.02	27.48
40	515	76.18			
			42	6.21	26.09
45	557	82.40			
			43	6.36	36.13
50	600	88.76			
			20	2.96	26.32
55	620	91.72			
			0	0.00	0.00
60	620	91.72			

Table VII. Distribution of mortality for the test data set used in this study. The last column is the percentage of trees which died in the interval based upon the number of live trees at the beginning of that specific growth interval. The last column is also plotted on the graph in figure 27.

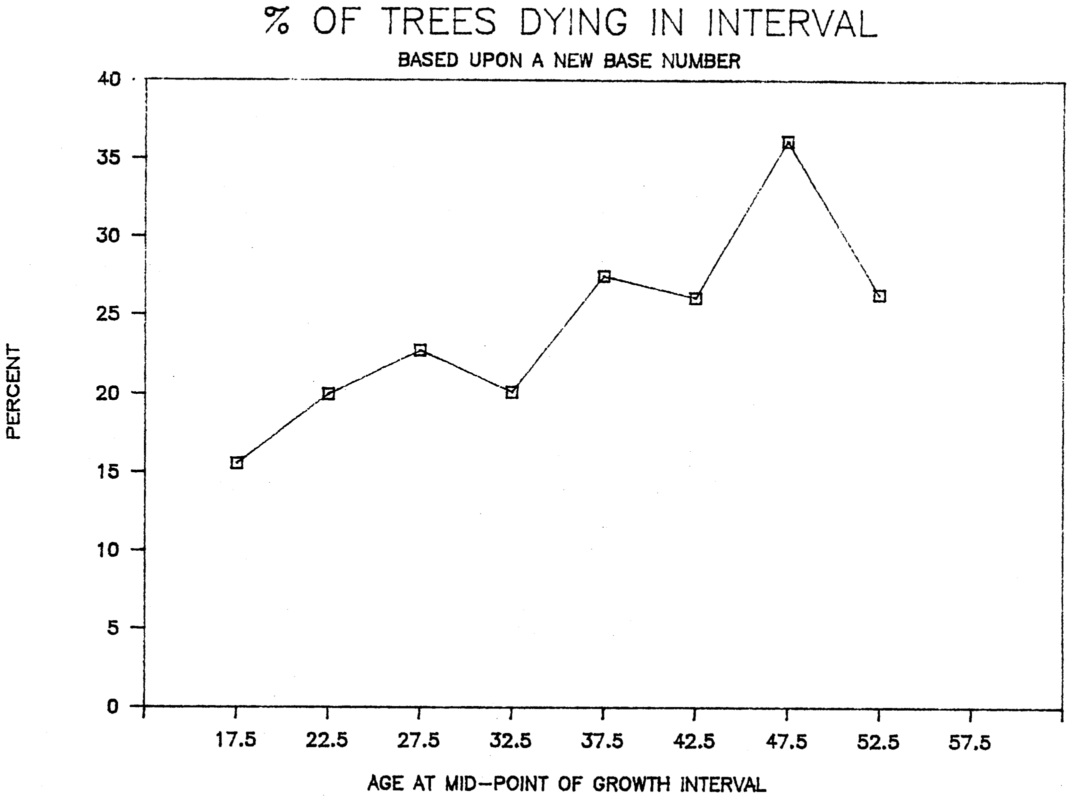

Figure 27. Mortality of the Data Set Used in All Simulations Involving an Unthinned Stand. The Curve Represents the Percentage of Trees That Died in the Measurement Interval Relative to the Number That Were Alive at the Beginning of That Interval.

APPENDIX C

RESULTS OF FIELD EXERCISE

TABLE VIII

RESULTS OF FIELD EXERCISE

Point No.	BAF	Volume (CHS)	Volume (ALTCD)	Volume (HPS)
	m^2/ha	m^3/ha	m^3/ha	m^3/ha
1	2.25	166.9883	112.5248	155.8087
2	2.25	186.694	186.335	163.8411
3	4.0	217.992	97.82	186.9165
4	4.0	254.92	165.76	194.4192
5	4.0	82.4	41.384	81.2715
6	4.0	128.68	226.84	190.6945
7	4.0	269.84	276.4	249.1650
points 3-7:	Avg.=	190.7664	161.6409	180.4933
	s^2 =	6676.8874	8993.0684	3275.0051

Table VIII. Results of field exercise using CHS and HPS to sample volume from the same sample points. Volume (CHS) means to estimate volume by ordinary critical height sampling. Volume (ALTCD) means to estimate volume by ctitical height sampling using the alternate critical diameter. Volume (HPS) means to estimate volume by ordinary horizontal point sampling. The averages and variances listed at the bottom are for points 3 through 7 only. The BAF's and the estimates are all in cubic meters per hectare.

APPENDIX D

RESULTS USING THE ALTERNATE CRITICAL DIAMETER

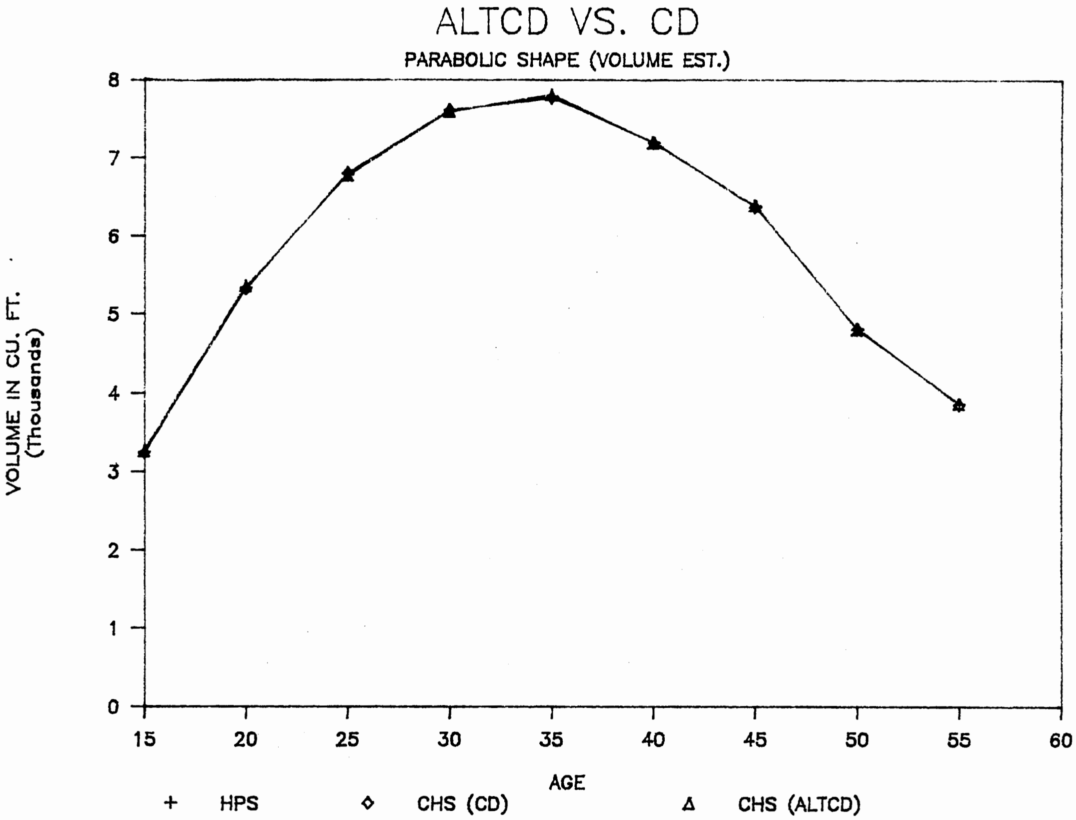

Figure 28. Mean Estimate of Volume Using The Critical Diameter and the Alternate Critical Diameter. This Demonstrates the Unbiasedness of The Alternate Critical Diameter. The Simulation Assumed a Parabolic Shape.

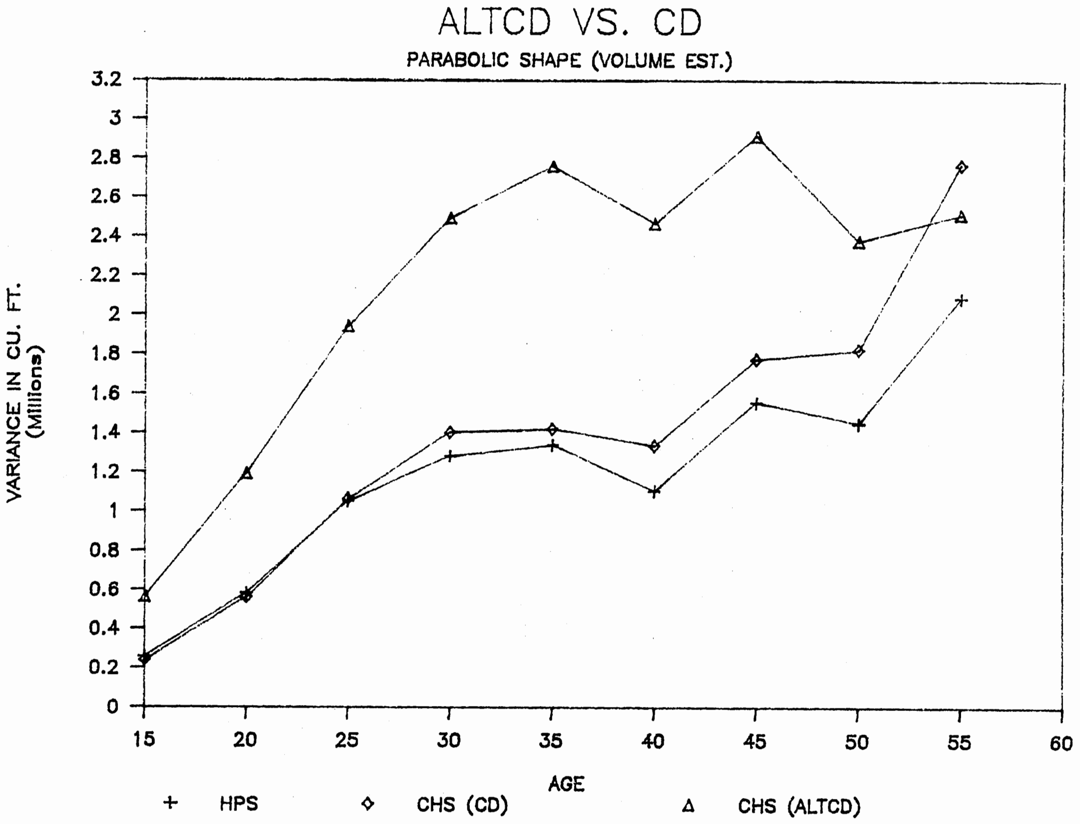

Figure 29. Variance of Volume Estimation Using the Critical Diameter and the Alternate Critical Diameter. The Simulation Assumed a Parabolic Shape.

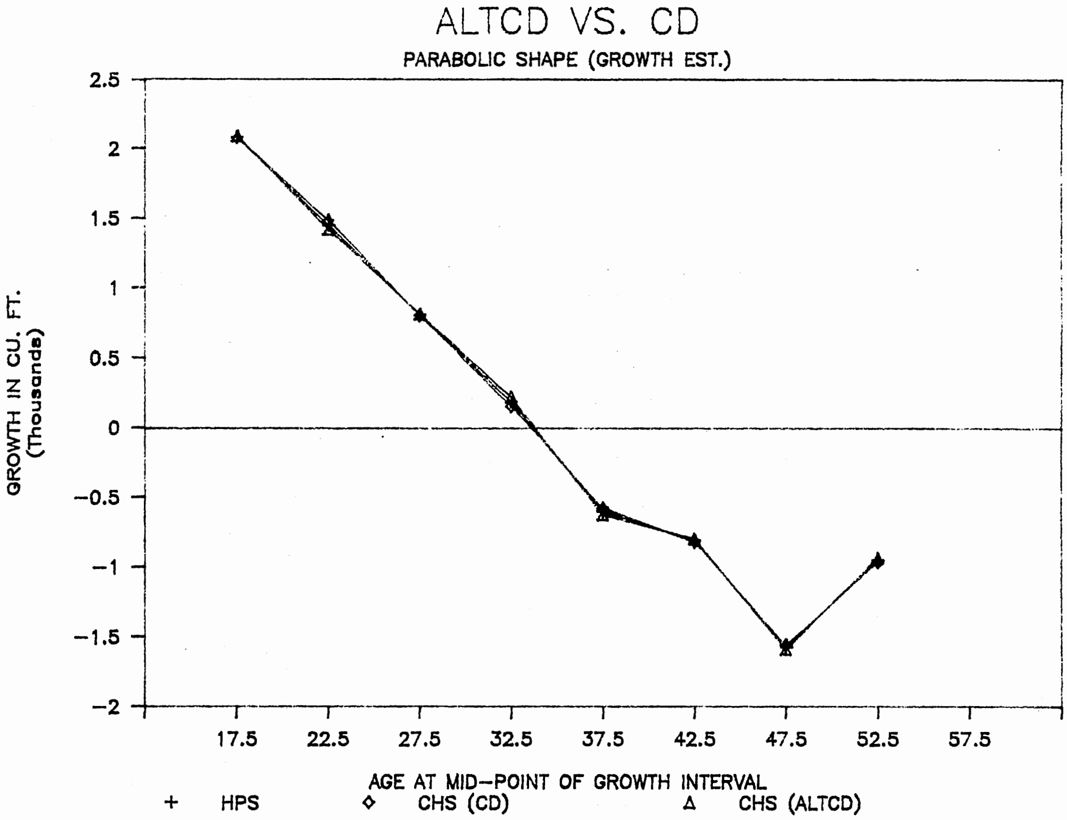

Figure 30. Mean Estimate of Growth Using The Critical Diameter and the Alternate Critical Diameter The Simulation Assumed a Parabolic Shape.

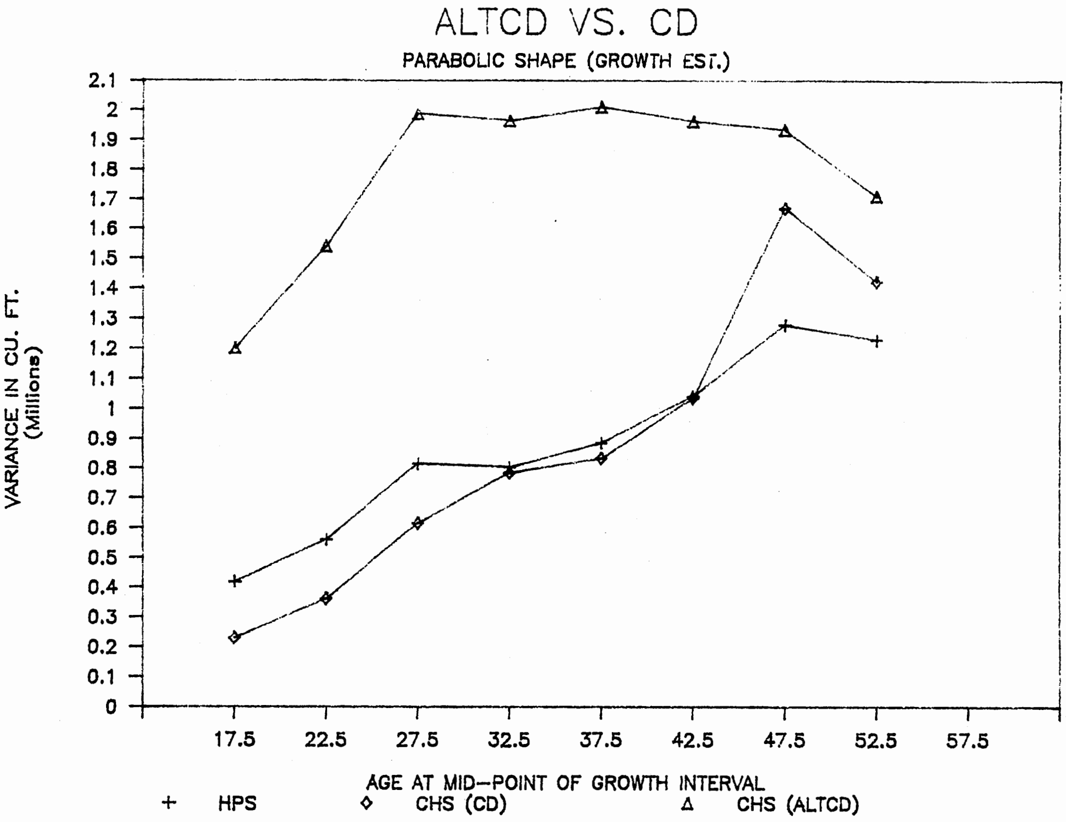

Figure 31. Variance of Growth Estimation Using The Critical Diameter and the Alternate Critical Diameter The Simulation Assumed a Parabolic Shape.

APPENDIX E

INITIAL CONDITIONS OF TEST STAND

TABLE IX

VOLUME SUMMARY

age	no. trees/ac.	BA/ac.	volume/ac.
		--ft^2--	---ft^3---
15	533	143	2859
20	450	183	4577
25	360	200	5795
30	278	200	6432
35	222	199	6564
40	161	189	6070
45	119	165	5395
50	76	140	4060
55	56	101	3290

TABLE X

GROWTH SUMMARY

age-class midpoint	ingrowth	mortality	s_2-s_1	V_2-V_1
	-----------------cubic feet-----------------			
17.5	0	281	0	1718
22.5	22	620	1912	1218
27.5	8	970	1731	637
32.5		1171	1488	131
37.5		1578	1164	-493
42.5		1583	926	-675
47.5		2012	730	-1334
52.5		1319	485	-770

APPENDIX F

RATIOS OF CRITICAL HEIGHT SAMPLING AND HORIZONTAL POINT SAMPLING VARIANCES

TABLE XI

RATIOS OF CRITICAL HEIGHT SAMPLING VARAINCE TO HORIZONTAL POINT SAMPLING VARIANCE IN VOLUME ESTIMATION

age	Variance of HPS	Variance of CHS	Ratio
15	109397	164300	1.5019
20	237103	377192	1.590
25	438335	707526	1.614
30	584622	937908	1.604
35	583855	939997	1.610
40	440310	904413	2.054
45	606953	1220519	2.010
50	526050	1291279	2.454
55	785746	1964094	2.499

Table XI. Ratios of critical height sampling and horizontal point sampling variances for volume estimation. Multiplication of the ratio by a desired sample size in horizontal point sampling will result in the number of sample points required to obtain an equal standard error in critical height sampling. These values apply only to the test stand used in this study and are intended to illustrate the differences between HPS and CHS.

TABLE XII

RATIOS OF CRITICAL HEIGHT SAMPLING VARIANCE TO HORIZONTAL POINT SAMPLING VARAINCE IN GROWTH ESTIMATION

age	Variance HPS	Variance CHS	Ratio
17.5	184724	146576	.7935
22.5	239314	231307	.967
27.5	375728	434588	1.157
32.5	321743	548082	1.704
37.5	426852	585251	1.372
42.5	457923	725576	1.584
47.5	359085	1164682	3.243
52.5	477407	1006990	2.110

Table XII. Ratios of critical height sampling and horizontal point sampling variances for growth estimation. Multiplication of the ratio by a desired sample size in horizontal point sampling will result in the number of sample points required to obtain an equal standard error in critical height sampling. These values apply only to the test stand used in this study and are intended to illustrate the differences between HPS and CHS.

2

VITA

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