IMPACT OF PARAMETER UNCERTAINTY ON ONE-DIMENSIONAL VERTICAL UNSATURATED FLOW OF WATER

By

FETHI BEN JEMAA

Engineering Diploma

National Agronomical Institute of Tunis

Tunis, Tunisia

1986

Submitted to the Faculty of the
Graduate College of the
Oklahoma State University
in partial fulfillment of
the requirements for
the Degree of
MASTER OF SCIENCE
July, 1988

IMPACT OF PARAMETER UNCERTAINTY ON ONE-DIMENSIONAL VERTICAL UNSATURATED FLOW OF WATER

Thesis Approved:

C T Haan

Thesis Advisor

Bruce Wilson

D L Nofziger

Norman N. Durham

Dean of the Graduate College

ACKNOWLEDGMENTS

Although my name is the only one on the cover, this work is certainly not the result of my effort alone. Much of the credit should go to other people. Deserving special mention is Dr. Charles T. Haan, my major advisor, to whom I am especially indebted for his guidance, concern, and encouragement.

I wish also to express my sincere gratitude to my other committee members, Dr. Bruce N. Wilson and Dr. David L. Nofziger, for their advisement and invaluable help.

Special thanks are due to the University Mission of Tunisia for assistance, support, and positive attitude.

I would also like to thank all my friends here in the United States and back home in Tunisia. Without their friendship and help this master program would have been much less lively and enjoyable.

Last but not least, I would like to thank my father, my mother, my brothers and sisters, and the rest of all my extended family for all their love, understanding, and moral encouragements.

TABLE OF CONTENTS

LIST OF TABLES

Table Page

Table Page

Table Page

LIST OF FIGURES

Figure Page

CHAPTER I

INTRODUCTION

General

The unsaturated zone of the soil profile is a primary determinant of runoff formation and quantity as well as of subsurface water flow path and velocities (Warrick, 1983). Understanding unsaturated flow of water through porous media is of great importance toward an efficient and appropriate control of a number of phenomena occurring in the soil such as natural and artificial recharge of ground water, runoff and soil erosion control, evapotranspiration, and movement of pollutants and dissolved substances.

Unsaturated flow, which is governed by a nonlinear partial differential equation, typically involves random soil hydraulic parameters (Philip, 1980). An essential requirement to solve the flow equation would be an adequate determination of the soil's hydraulic conductivity-water content and pressure head-water content relationships and their distributions (Cosby et al., 1984). A wide range of empirical and quasi-analytical equations have been used. However, these equations are usually derived for some restricted conditions. As a result, more concern has recently been given to numerical models in which the soil hydraulic parameters are regarded as stochastic variables (Smith and Hebbert, 1979; Andersson, 1983; Dagan and Bressler, 1983; Morel-Seytoux and Billica, 1985).

Statement of Problem

Water movement through the unsaturated zone is largely affected by the spatial variability of the hydraulic characteristics of the soil. Soil-water properties may vary with depth and from one location to another within the field. In fact even in a soil uniform with respect to its texture, nonuniform soil-water parameters usually exist such as the water content-pressure head relationship.

Due to the extensive variability of soil-water parameters in the field, an estimate of the unsaturated hydraulic conductivity, K(h), is very difficult to obtain. Experimental field determinations are expensive and time consuming. A more convenient way of predicting K(h) has been to use the soil-water retention curve which is more easily measured (Van Genuchten, 1980). However this method involves empirical relationships incorporating parameters which are varying with respect to space and time within the field. Moreover, as the soil becomes less saturated, this variability becomes much more significant (Yeh et al., 1986). In saturated flow the variation of the hydraulic conductivity, K, with position results solely from the inhomogeneity of the porous medium. Whereas, in unsaturated flow, K(h) varies with position even in homogeneous soils, owing to the effect of the variation in hydraulic conductivity with moisture tension. (Freeze, 1969).

In most applications of flow theory, the problem of parameter variability has been handled by simply taking the mean value for a given number of samples and by making the assumption that the soil can be regarded as a homogeneous medium described by an average set of parameters determined from a number of locations over the field. Such an approach can be misleading and may generate flow predictions significantly different from those prevailing in the actual spatially variable field.

In this context many questions can be asked : How accurate can a model assuming a homogeneous field describe the flow in the actual spatially variable field? Is it satisfactory to determine the flow by an averaging concept? Are the average of flow parameters prevailing in the field equal to those that should be used in a model? If not, how can we extract a specific set of parameters which will best describe the water flow as it occurs in reality?

Objectives

The objectives of this study were:

1. To determine the random variability of the parameters incorporated in the functions relating the soil hydraulic properties: water content-matric potential and hydraulic conductivity-matric potential.

2. To develop a procedure for estimating average cumulative infiltration from knowledge of the random variability of the parameters describing the soil-water characteristic curve.

Procedure

To achieve the objectives of this study the partial differential equation of flow in unsaturated media has been repeatedly solved using a numerical procedure with the flow parameters considered as spatially random variables following a fixed probability density function. Based on each random set of flow parameters, the cumulative infiltration after 10 hours was computed. The average cumulative infiltration based on these computed infiltrations was also determined. A comparison of the resulting average infiltration from this procedure to that obtained by using a simple average of the flow parameters in the flow equation indicates how well such models describe the average flow of water when average parameter values are used.

This concept was considered because of the uncertainty of soil hydraulic parameters with respect to space in the field. The incorporation of random variability in the flow parameters will result in random variability in predicted infiltration.

For a given boundary condition, values of the flow parameters providing a good estimate of the spatially averaged infiltration will be sought. Because there are two parameters and one average infiltration, a unique set of parameters will not exist for a given boundary condition. Rather solutions will lie along a curve relating the parameters to each other. By examining several boundary conditions, a common region where the majority of solutions tend to converge was sought.

CHAPTER II

LITERATURE REVIEW

Governing Equations

The concept of a physical model of unsaturated flow through porous media was first developed by Buckingham (1907), when he suggested a modified form of Darcy's law to be used in unsaturated soils; a form in which the hydraulic conductivity is expressed as a function of the water content, and hence of the matric potential.

$$q = K(h)\ (\partial h/\partial z) + K(h) \tag{1}$$

where

q is the volumetric water flux.

h is the matric potential (or pressure head).

$K(h)$ is the unsaturated hydraulic conductivity as a function of the pressure head h.

z is the distance in depth.

In fact in unsaturated flow the hydraulic conductivity is related to the water content. As the water content of the porous medium decreases, the hydraulic conductivity decreases at a rate more than proportional (Hubert, 1978).

In unsaturated media the flow occurs as a result of the matric and gravitational gradient. As the soil drains under the effect of gravity, the hydraulic conductivity decreases rapidly. This dramatic reduction of the conductivity with the water content results from the fact that, when water content is reduced, the largest pores empty first, and small pores conduct water much less readily than large pores. In addition, the path for flow becomes much more tortuous as the soil desaturates (Campbell, 1985).

From the above concept, and by combining the continuity equation with Darcy's law, Richards (1931) developed the nonlinear partial differential equation of unsaturated flow in porous media as

$$\frac{\partial WC}{\partial t} = \frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) - \frac{\partial K_z}{\partial z} \qquad (2)$$

where

K_x, K_y, K_z are the unsaturated hydraulic conductivities in the x, y, and z directions respectively.

h is the soil matric potential.

WC is the volumetric water content.

t is the time.

For one-dimensional unsaturated flow in the z-direction taken positive downward the above equation becomes

$$\frac{\partial}{\partial z}\left[K_z\left(\frac{\partial h}{\partial z} - 1\right)\right] = \frac{\partial WC}{\partial t} \qquad (3)$$

Using the specific water capacity $C(h) = d(WC)/dh$ equation (3) becomes

$$C(h)\frac{\partial h}{\partial t} = \frac{\partial}{\partial z}\left[K(h)\left(\frac{\partial h}{\partial z} - 1\right)\right] \qquad (4)$$

Because of the strong nonlinearity of the unsaturated flow equation, there is no specific analytical solution. Attempts of linearization have been pursued by a number of researchers in order to reach an accurate and acceptable solution.

Philip (1957) developed an explicit algebraic equation as a general solution of Richards equation using an infinite power series in $t^{1/2}$ with coefficients as functions of the volumetric water content WC. Fok (1987) reported on a number of other empirical equations that have also been used.

Numerical Models

In the last few years, numerical models of flow through porous media using finite difference techniques have become more and more popular. Solving the flow partial differential equation requires the determination of soil hydraulic properties as functions of the moisture content and any solution would not be accurate unless these relationships have been expressed successfully.

Comparing solutions of the flow equation given by six numerical models to experimental results, Haverkamp et al. (1977), found excellent agreement between the observed and computed cumulative inflow, concluding that numerical models are a reliable tool for predicting water movement within the soil profile.

Using large number of experimental measurements (448 samples), El-Kadi (1985), examined the suitability of four models describing the soil-water characteristic function, h(WC), (Brooks and Corey, Brutsaert, Van Genuchten, and Vauclin et al. models). All these models were found to be successful. For sandy and silty samples, the Brooks and Corey model provided the highest accuracy. Whereas, for clayey samples, Van Genuchten produced the best results.

Khaleel and Yeh (1985), developed a Galarkin finite-element technique for solving the one-dimensional unsaturated flow equation. Excellent agreement was found when comparing the water content profiles obtained using this scheme to those obtained by Van Genuchten using a mass-lumped linear finite element method.

Parameter Uncertainty

Due to the uncertainty of soil hydraulic parameters, stochastic models are being used. Such a concept may utilize either univariate or multivariate parameter distributions (Russo and Bressler, 1982). "Using average values of uncertain parameters to estimate average infiltration may produce results that are significantly different from the estimate that would be obtained from averaging infiltrations calculated from each set of parameters" (Haan, 1987). As a matter of fact, any function of random variable is also a random variable (Haan, 1977). Since the soil hydraulic properties are random variables, the infiltration, a function of these properties, is also a random variable.

Haan (1987) demonstrated also that the flow solution is affected by the value assigned to the correlation coefficient between the parameters describing the soil hydraulic properties. Therefore, for an accurate solution of the flow equation, the correlation between the flow parameters should be considered.

Bresler and Dagan (1983) investigated two spatially variable soils and demonstrated that the traditional deterministic approach for solving the flow equation, which describes well the physics of water flow under uniform soil column conditions, fails to depict the average flow in a real spatially variable field. They also suggested a simplified solution of vertical flow which assumes the concept of a moving front, where the saturated hydraulic conductivity is regarded as a random variable following a lognormal distribution, hence, the matric potential and the moisture content are random variables.

Cosby et al. (1984) related the variability of soil moisture characteristics to soil physical properties, concluding that the variability of soil moisture parameters are most closely related to the variability in the texture (percent of sand, silt, and clay).

Parameter uncertainty may also be increased due either to errors made during experimental measurements, or to the type of function used to describe the soil water characteristic curve. Kool et al. (1985) showed that errors in the input data may contribute considerably to the variability in soil hydraulic parameters. Jones and Wagenet (1984) compared five methods of estimating the hydraulic conductivity, K, as a function of the water content, WC. Statistical comparison showed that the variation in the soil water flux is dependent upon the method used to characterize the K(WC) relationship.

CHAPTER III

HYDRAULIC MODEL

General

The vertical flow of water in an unsaturated porous medium can be described by the partial differential equation (3) known as Richards equation. The assumption that water percolates vertically through the soil profile was made, although it is recognized that water percolation within the soil is not strictly vertical, and lateral transfer is always observed (Vauchaud et al., 1987).

The flow equation is highly nonlinear and has no general analytical solution. It is generally required to know the relationships among the soil hydraulic properties of hydraulic conductivity K(h), water content WC(h), and pressure head potential h. Unless these relationships are determined, the flow equation cannot be solved.

Model Components

A model describing the soil hydraulic properties relationships that has been proved to be successful was proposed by Van Genuchten (1980). This model relates both the hydraulic conductivity, K(h), and the water content, WC(h), to the hydraulic head, h, as

$$\begin{aligned} WC(h) &= WC_r + (WC_s - WC_r)[1+|Ah|^B]^{-m} && \text{for } h<0 \\ WC(h) &= WC_s && \text{for } h>=0 \end{aligned} \tag{5}$$

$$\begin{aligned} K(h) &= K_s \frac{\left[1-|Ah|^{B-1}\{1+|Ah|^B\}^{-m}\right]^2}{(1+|Ah|^B)^{m/2}} && \text{for } h<0 \\ K(h) &= K_s && \text{for } h>=0 \end{aligned} \tag{6}$$

Where

$WC(h)$ is the water content at a matric potential h.

WC_r is the residual (or irreducible) water content.

WC_s is the saturated water content (WC at $h = 0$).

h is the pressure head (or matric potential).

$K(h)$ is the hydraulic conductivity at a matric potential h.

K_s is the saturated hydraulic conductivity (K at $h = 0$).

A and B are parameters.

$m = 1 - 1/B$

$B > 1$

Of several models investigated, it was found that the above model of Van Genuchten has the greatest flexibility in describing WC(h) data (Greminger et al., 1985). Other advantages of this model are that it has a simple inverse function, and it provides a closed-form analytical equation of the hydraulic conductivity.

Because of its popularity the Van Genuchten model will be used in this study. Therefore the soil hydraulic properties will be described by equations (5) and (6) and the flow parameters to be estimated are A and B of these equations.

CHAPTER IV

DATA ANALYSIS

Introduction

The data used in the current study were from Nofziger et al. (1983) contributing to the regional project S-124, entitled "Movement and Retention of Water and Solutes in Selected Southern Region Field Soils". The data were collected for three soil series (Bethany, Konawa, and Tipton series) at 13 sites within the State of Oklahoma.

The representative sites of each soil were selected by soil classifiers in Oklahoma State University and in the Soil Conservation Service. Further description of the soils properties are given in the appendix A.

Volumetric water content measurements at selected pressure heads were obtained from desorption curves determined using a standard pressure plate apparatus. The soil samples used were 7.6 cm in diameter and 7.6 cm long. Selecting two given depths (15 and 30 cm), a total of 168 sets of water content-pressure head data were considered in this study (72, 60, and 36 sets for Bethany, Konawa, and Tipton series, respectively).

Estimation of the Hydraulic Model Parameters

Observed soil-water retention data were analyzed in order to estimate the parameters of Van Genuchten model WC_s, WC_r, A, and B. For this purpose a large number of soil hydraulic data have to be considered for a trustworthy estimate of a statistical treatment of flow.

While the parameters A and B can only be sought by a nonlinear least squares curve-fitting procedure, the saturated and the residual water contents may be available experimentally. Even though the residual water content is not always available, it can be estimated by extrapolation from the available soil-water data simultaneously with the estimation of A and B. However, due to the limited number of observations within each single set of data, it would not be accurate in the present study to fit a three parameter model by estimating A, B, and WC_r. The residual water content, WC_r, was therefore taken equal to zero. As a matter of fact, WC_r is defined nominally as the water content at which the matric potential approaches negative infinity. Such a condition is only met when $WC_r = 0$ (Kool et al, 1985). Furthermore, the choice of attributing a zero value to the residual water content was supported by the fact that it has no great influence on the estimation of the other parameters because the data used have a matric potential range not reaching very low values.

The saturated water content could be easily estimated from experimental measurements of the matric potential near zero or from the bulk density (d_b) using the relation

$$WC_s = 1 - d_b/d_s$$

where d_s is the particle density taken as 2.65.

The latter method, called "gravimetric", estimates a theoretical saturated water content which assumes that the soil is perfectly saturated; a condition which is not always true in practice. Complete saturation is seldom attained since some air is nearly always present and may become trapped in a very wet soil (Hillel, 1971). For the above reason, values of the saturated water content were taken as the volumetric water content at a matric potential approaching zero (-8 cm for Bethany and Tipton series and -4 cm for Konawa series).

Starting with known values of the saturated and residual water contents, a computer program using a least squares fitting procedure (Nofziger, 1988) was used to estimate the remaining parameters, A and B. The fitting procedure was repeated for all cores within each site. Tables 1 through 3 contain the regression coefficients A and B for Bethany, Konawa, and Tipton series respectively. In almost all the regressions made for the total 168 samples, the hydraulic model of Van Genuchten was found to fit the soil hydraulic data very well. Large values of R^2 and low sum of squares of residuals are observed in all cases. R^2 as used here refers to the difference in the total sum of squares corrected for the mean and the

TABLE 1

SOLUTIONS OF THE HYDRAULIC MODEL PARAMETERS FOR BETHANY SOIL

Soil Name: Bethany

Site	Depth	Core	A (1/cm)	B	R^2	WCs
1	1	1	.0044	1.9356	.9821	.3610
1	1	2	.0050	2.0253	.9847	.3860
1	1	3	.0046	1.8121	.9869	.3720
1	1	4	.0049	1.9165	.9705	.3770
1	1	5	.0091	1.2486	.9134	.2820
1	1	6	.0051	1.8579	.9899	.3700
1	2	1	.0735	1.1177	.9517	.4520
1	2	2	.0399	1.0819	.9756	.3840
1	2	3	.0263	1.1084	.9815	.4430
1	2	4	.0405	1.0629	.9587	.4020
1	2	5	.0132	1.2437	.9784	.3240
1	2	6	.0186	1.1869	.9808	.2810
2	1	1	.0049	1.5106	.9855	.3670
2	1	2	.0027	1.2935	.8496	.2840
2	1	3	.0045	1.6453	.9760	.3490
2	1	4	.0076	1.4419	.9915	.3740
2	1	5	.0094	1.4359	.9882	.3770
2	1	6	.0065	1.6770	.9844	.3850
2	2	1	.0279	1.0782	.9863	.4130
2	2	2	.0311	1.0905	.9808	.4120
2	2	3	.0429	1.0743	.9654	.4180
2	2	4	.0175	1.1111	.9877	.3910
2	2	5	.0156	1.1981	.9678	.2190
2	2	6	.0141	1.1513	.9726	.2850
3	1	1	.0079	1.4406	.9955	.3720
3	1	2	.0049	1.6930	.9818	.3630
3	1	3	.0061	1.5069	.9886	.3690
3	1	4	.0045	1.5840	.9856	.3430
3	1	5	.0054	1.7223	.9935	.3780
3	1	6	.0063	1.5262	.9824	.3640
3	2	1	.0328	1.0685	.9818	.3990
3	2	2	.0105	1.0801	.9895	.4030
3	2	3	.0243	1.0575	.9910	.3960

TABLE 1 (Continued)

Soil Name: Bethany

Site	Depth	Core	A (1/cm)	B	R^2	WCs
3	2	4	.0122	1.0934	.9953	.4050
3	2	5	.0245	1.2149	.9809	.2360
3	2	6	.0190	1.1712	.9857	.1960
4	1	1	.0046	1.3401	.9956	.3780
4	1	2	.0040	1.3467	.9908	.3750
4	1	3	.0041	1.3124	.9759	.3810
4	1	4	.0027	1.4179	.9905	.3850
4	1	5	.0041	1.4990	.9940	.3930
4	1	6	.0037	1.7679	.9924	.4010
4	2	1	.0132	1.1232	.9737	.4390
4	2	2	.0182	1.1245	.9928	.4680
4	2	3	.0100	1.0972	.9877	.4380
4	2	4	.0273	1.0913	.9872	.4730
4	2	5	.0140	1.1150	.9888	.4720
4	2	6	.0046	1.2424	.9696	.4420
5	1	1	.0147	1.2613	.9945	.4430
5	1	2	.0030	1.5267	.9815	.3670
5	1	3	.0089	1.1980	.9417	.3940
5	1	4	.0074	1.2215	.9481	.3990
5	1	5	.0035	1.8242	.9977	.3890
5	1	6	.0037	1.7920	.9982	.3880
5	2	1	.0164	1.1180	.9952	.4400
5	2	2	.0188	1.1033	.9623	.4330
5	2	3	.0259	1.0945	.9859	.4520
5	2	4	.0238	1.0869	.9838	.4440
5	2	5	.0042	1.2968	.9788	.4360
5	2	6	.0067	1.2174	.9788	.4480
6	1	1	.0097	1.2983	.9840	.4380
6	1	2	.0068	1.4021	.9684	.4530
6	1	3	.0130	1.2277	.9841	.4230
6	1	4	.0052	1.7799	.9958	.4230
6	1	5	.0038	1.6379	.9836	.4100
6	1	6	.0088	1.4133	.9858	.4480

TABLE 1 (Continued)

Soil Name: Bethany

Site	Depth	Core	A (1/cm)	B	R^2	WCs
6	2	1	.0260	1.0690	.9620	.4020
6	2	2	.0035	1.2281	.9821	.3940
6	2	3	.0034	1.3573	.9853	.4320
6	2	4	.0065	1.2927	.9880	.4530
6	2	5	.0097	1.1986	.9859	.4280
6	2	6	.0129	1.1899	.9943	.4360

TABLE 2

SOLUTIONS OF THE HYDRAULIC MODEL PARAMETERS FOR KONAWA SOIL

Soil Name: Konawa

Site	Depth	Core	A (1/cm)	B	R^2	WCs
1	1	1	.0085	1.5719	.9938	.3310
1	1	2	.0067	1.7425	.9855	.3170
1	1	3	.0079	1.6598	.9937	.3310
1	1	4	.0100	1.5672	.9746	.3270
1	1	5	.0072	1.9953	.8888	.3820
1	1	6	.0050	1.9049	.9679	.2940
1	2	1	.0058	1.5667	.9056	.3190
1	2	2	.0266	1.1616	.9859	.3510
1	2	3	.0075	1.6885	.9665	.3260
1	2	4	.0092	1.6064	.9898	.3380
1	2	5	.0069	1.4857	.9434	.3250
1	2	6	.0109	1.1693	.9964	.3340
2	1	1	.0121	1.7195	.9969	.3160
2	1	2	.0099	1.8207	.9954	.3400
2	1	3	.0160	1.6787	.9994	.3350
2	1	4	.0147	1.7135	.9905	.3420
2	1	5	.0112	1.8450	.9986	.3570
2	1	6	.0145	1.6303	.9974	.3380
2	2	1	.0251	1.2168	.9805	.3440
2	2	2	.0225	1.2790	.9970	.3320
2	2	3	.0167	1.3326	.9811	.3260
2	2	4	.0478	1.1667	.9948	.3390
2	2	5	.0322	1.3365	.9926	.3300
2	2	6	.0376	1.3760	.9894	.3690
3	1	1	.0363	1.8552	.9539	.3400
3	1	2	.0405	1.5906	.9707	.3260
3	1	3	.0331	1.8787	.9494	.3480
3	1	4	.0529	1.5381	.9570	.2850
3	1	5	.0327	1.6740	.9684	.2610
3	1	6	.0315	1.4760	.9712	.3020
3	1	7	.0330	1.6685	.9331	.3610
3	1	8	.0333	1.6233	.9518	.3730
3	1	9	.0320	1.9185	.9305	.3750

TABLE 2 (Continued)

Soil Name: Konawa

Site	Depth	Core	A (1/cm)	B	R^2	WCs
3	2	1	.0391	1.7211	.9493	.3190
3	2	2	.0319	1.6036	.9522	.2720
3	2	3	.0271	1.7471	.9461	.3060
3	2	4	.0486	1.4962	.9935	.2760
3	2	5	.0339	1.5122	.9786	.2450
3	2	6	.0293	1.4384	.9504	.2370
3	2	7	.0463	1.6738	.9579	.3040
3	2	8	.0399	1.6600	.9494	.2930
3	2	9	.0364	1.6057	.9387	.2750
4	1	1	.0341	1.5825	.9665	.2650
4	1	2	.0275	1.6359	.9649	.3190
4	1	3	.0312	1.5793	.9699	.2770
4	1	4	.0519	1.5304	.9824	.3360
4	1	5	.1147	1.4501	.9766	.3990
4	1	6	.0560	1.3798	.9724	.3470
4	1	7	.0253	1.5609	.9343	.3180
4	1	8	.0346	1.8068	.9660	.3840
4	1	9	.0383	1.5343	.9580	.3840
4	2	1	.0393	1.5014	.9641	.2710
4	2	2	.0536	1.3007	.9486	.2560
4	2	3	.0336	1.4906	.9840	.2450
4	2	4	.0490	1.4471	.9718	.2300
4	2	5	.0570	1.4457	.9541	.2460
4	2	6	.0865	1.4639	.9624	.2940
4	2	7	.0349	1.6463	.9699	.2730
4	2	8	.0342	1.8012	.9589	.2910
4	2	9	.0454	1.4798	.9747	.2770

TABLE 3

SOLUTIONS OF THE HYDRAULIC MODEL PARAMETERS FOR TIPTON SOIL

Soil Name: Tipton

Site	Depth	Core	A (1/cm)	B	R^2	WCs
1	1	1	.0266	1.1473	.9667	.3590
1	1	2	.0115	1.1606	.9200	.3190
1	1	3	.0044	1.1844	.9181	.2930
1	1	4	.0046	1.2257	.9601	.3050
1	1	5	.0033	2.3826	.9899	.2970
1	1	6	.0037	2.5770	.9950	.3090
1	2	1	.0066	1.2961	.9971	.3600
1	2	2	.0032	1.4314	.9933	.3490
1	2	3	.0061	1.2465	.9788	.3420
1	2	4	.0041	1.4126	.9937	.3610
1	2	5	.0033	1.7947	.9927	.3410
1	2	6	.0046	1.6069	.9889	.3530
2	1	1	.0069	1.3489	.9713	.3480
2	1	2	.0061	1.3617	.9473	.3370
2	1	3	.0037	1.5455	.9741	.3520
2	1	4	.0103	1.2603	.9222	.3580
2	1	5	.0042	2.5267	.9923	.3260
2	1	6	.0046	1.8016	.9759	.3340
2	2	1	.0066	1.3639	.9956	.4010
2	2	2	.0042	1.4691	.9952	.3670
2	2	3	.0066	1.3589	.9951	.3680
2	2	4	.0038	1.5425	.9921	.3540
2	2	5	.0067	1.4326	.9882	.3810
2	2	6	.0067	1.3958	.9545	.3810
3	1	1	.0031	1.4475	.9870	.3000
3	1	2	.0133	1.2326	.9674	.3400
3	1	3	.0059	1.3230	.9495	.3170
3	1	4	.0072	1.2511	.9628	.3180
3	1	5	.0114	1.3709	.9842	.3580
3	1	6	.0048	1.8675	.9884	.3250
3	2	1	.0100	1.2822	.9938	.3610
3	2	2	.0067	1.2978	.9965	.3610
3	2	3	.0107	1.3111	.9977	.3760

TABLE 3 (Continued)

Soil Name: Tipton

Site	Depth	Core	A (1/cm)	B	R^2	WCs
3	2	4	.0032	1.5028	.9959	.3290
3	2	5	.0048	1.7559	.9915	.3480
3	2	6	.0037	1.9177	.9954	.3370

residual sum of squares divided by the total sum of squares corrected for the mean. The fitting process indicated also that the values given to the parameters as initial estimates, a required input for the nonlinear least squares fitting procedure, have no significant effect on the final solution obtained. Figure 19 in Appendix C shows a typical example of the plot of equation (5).

From all the parameter estimations it was found that A ranges from 0.003 to 0.115 cm^{-1} while B ranges from 1.058 to 2.577. The range of variability of the above parameters is in reasonable agreement with those in the literature. Kool et al. (1985) wrote: "A generally ranges from 0.5 to 5.0 m^{-1}, while B usually varies from 1.1 to 3.5". Tables 4 through 7 show detailed information about the ranges of A and B and their logarithms for the three considered soil series.

Depth Considerations

As mentioned earlier two different depths were considered (15 and 30 cm). The previously estimated parameters of Van Genuchten model were divided into two groups according to the sample's depth. A statistical t-test for differences in means was conducted to decide whether or not the two groups of parameters can be considered as being from the same population. In other words the hypothesis, Ho, was that the population means are equal for the two depths. Table 8 shows the results of the

TABLE 4

STATISTICAL SUMMARY OF THE ESTIMATED PARAMETERS OF VAN GENUCHTEN MODEL FOR BETHANY SOIL (15 cm)

	Original Data		Logs of Data	
	A (1/cm)	B	A (1/cm)	B
Number of Obs.	36	36	36	36
Minimum	.003	1.198	-5.915	.181
Maximum	.015	2.025	-4.220	.706
Mean	.006	1.543	-5.206	.423
Standard Dev.	.003	.231	.414	.149

TABLE 5

STATISTICAL SUMMARY OF THE ESTIMATED PARAMETERS OF VAN GENUCHTEN MODEL FOR BETHANY SOIL (30 cm)

	Original Data		Logs of Data	
	A (1/cm)	B	A (1/cm)	B
Number of Obs.	36	36	36	36
Minimum	.003	1.058	-5.684	.056
Maximum	.074	1.357	-2.610	.305
Mean	.020	1.145	-4.127	.134
Standard Dev.	.014	.076	.725	.065

TABLE 6

STATISTICAL SUMMARY OF THE ESTIMATED PARAMETERS OF VAN GENUCHTEN MODEL FOR KONAWA SOIL

	Original Data		Logs of Data	
	A (1/cm)	B	A (1/cm)	B
Number of Obs.	60	60	60	60
Minimum	.005	1.162	-5.298	.150
Maximum	.115	1.995	-2.165	.691
Mean	.031	1.576	-3.684	.447
Standard Dev.	.020	.190	.722	.124

TABLE 7

STATISTICAL SUMMARY OF THE ESTIMATED PARAMETERS OF VAN GENUCHTEN FOR TIPTON SOIL

	Original Data		Logs of Data	
	A (1/cm)	B	A (1/cm)	B
Number of Obs.	36	36	36	36
Minimum	.003	1.147	-5.776	.137
Maximum	.027	2.577	-3.627	.947
Mean	.007	1.512	-5.158	.391
Standard Dev.	.004	.361	.487	.208

TABLE 8

t-TEST FOR DIFFERENCES IN MEANS OF THE REGRESSION PARAMETERS BETWEEN DEPTHS 15 AND 30 cm

Soil-Depth	A Means	B Means	Calculated t-Test For A	Calculated t-Test For B	t-TEST (Table) (*)
Bethany-15 cm	.006	1.543	-2.530	9.800	2.030
Bethany-30 cm	.020	1.145			
Tipton-15 cm	.008	1.556	1.340	.726	2.110
Tipton-30 cm	.006	1.468			
Konawa-15 cm	.029	1.671	.225	.008	2.040
Konawa-30 cm	.034	1.481			

(*) The hypothesis that the population means are equal is being tested (5% level of significance) t-test for differences in means. The hypothesis was rejected for Bethany soil but not for Konawa and Tipton soils. Therefore the regression coefficients estimated for the two considered depths were combined, assuming that they are from the same population, for Konawa and Tipton soil series but not for Bethany soil where each depth was analyzed separately.

Parameter Distributions

Normal and lognormal distributions were tested for the parameters A and B. Table 9 summarizes the results found, and gives the values of the maximum deviations, D1 and D2, between the fitted and empirical normal and lognormal distributions, respectively, as well as the critical values of the Kolmogorov-Smirnov test statistic (Haan, 1977). The mentioned test was used as a criterion of acceptance or rejection of the proposed distribution (acceptance when the maximum deviation is less than the critical value of the Kolmogorov-Smirnov test, and visa-versa). The cumulative normal distribution was approximated using a relationship given by Abramowitz and Stegun (1972).

Although the normal distribution was not rejected and could be assigned to both parameters, the lognormal distribution was found to more accurately describe the parameters. Figures 1 to 8 show the probability distributions and the lognormal fit for both A and B for each considered soil.

TABLE 9

KOLMOGOROV-SMIRNOV TEST STATISTIC FOR NORMALITY AND LOGNORMALITY OF A AND B

Soil	A		B		K
	D1	D2	D1	D2	
Bethany-15cm	.196	.134	.113	.090	.220
Bethany-30cm	.147	.073	.219	.211	.220
Tipton	.250	.145	.214	.179	.220
Konawa	.121	.185	.052	.067	.170

D1 maximum deviation from the normal distribution
D2 maximum deviation from the lognormal distribution
K the critical value of the Kolmogorov-Smirnov Test

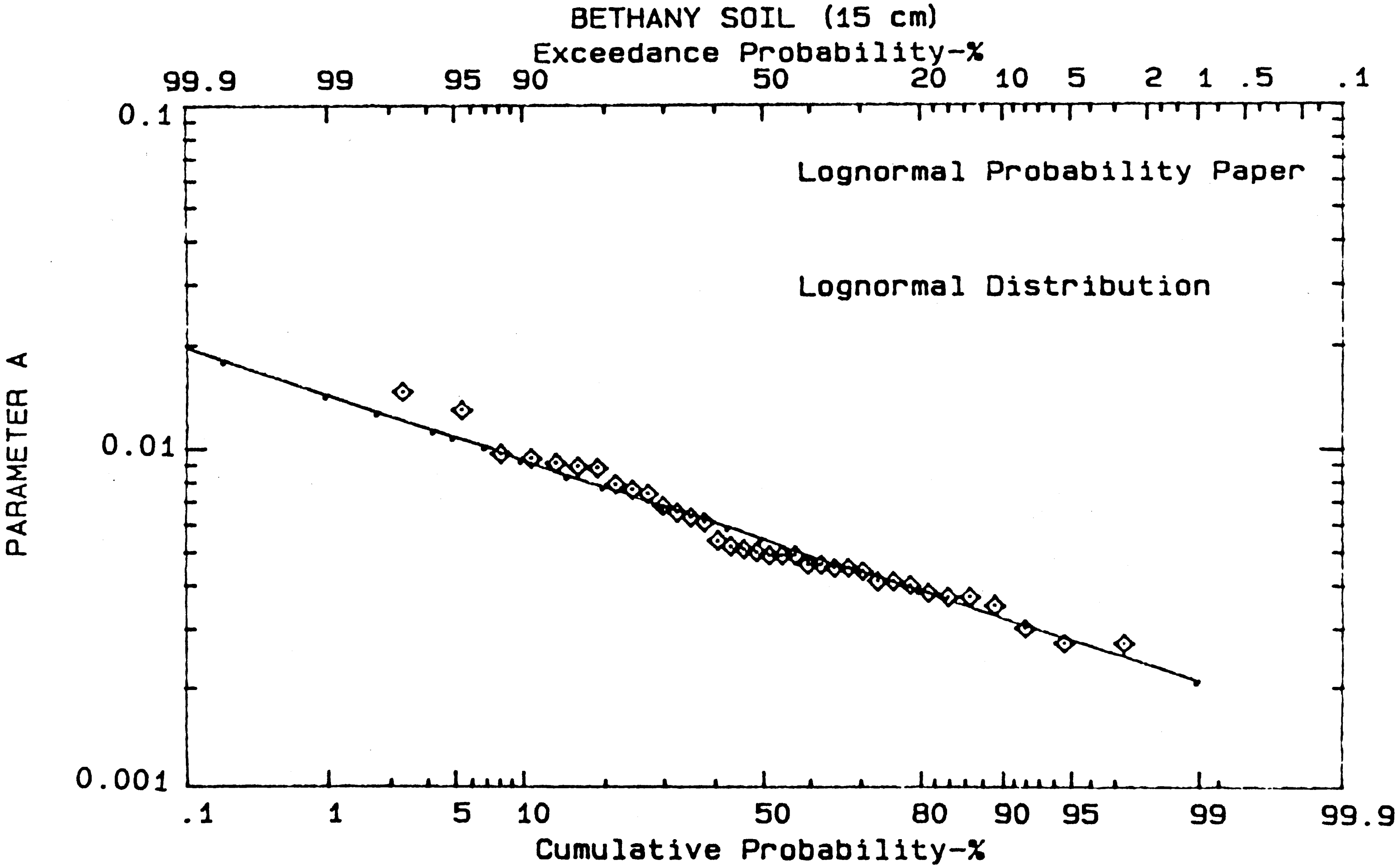

Figure 1. Probability Distribution of Parameter A for Bethany Soil (15 cm)

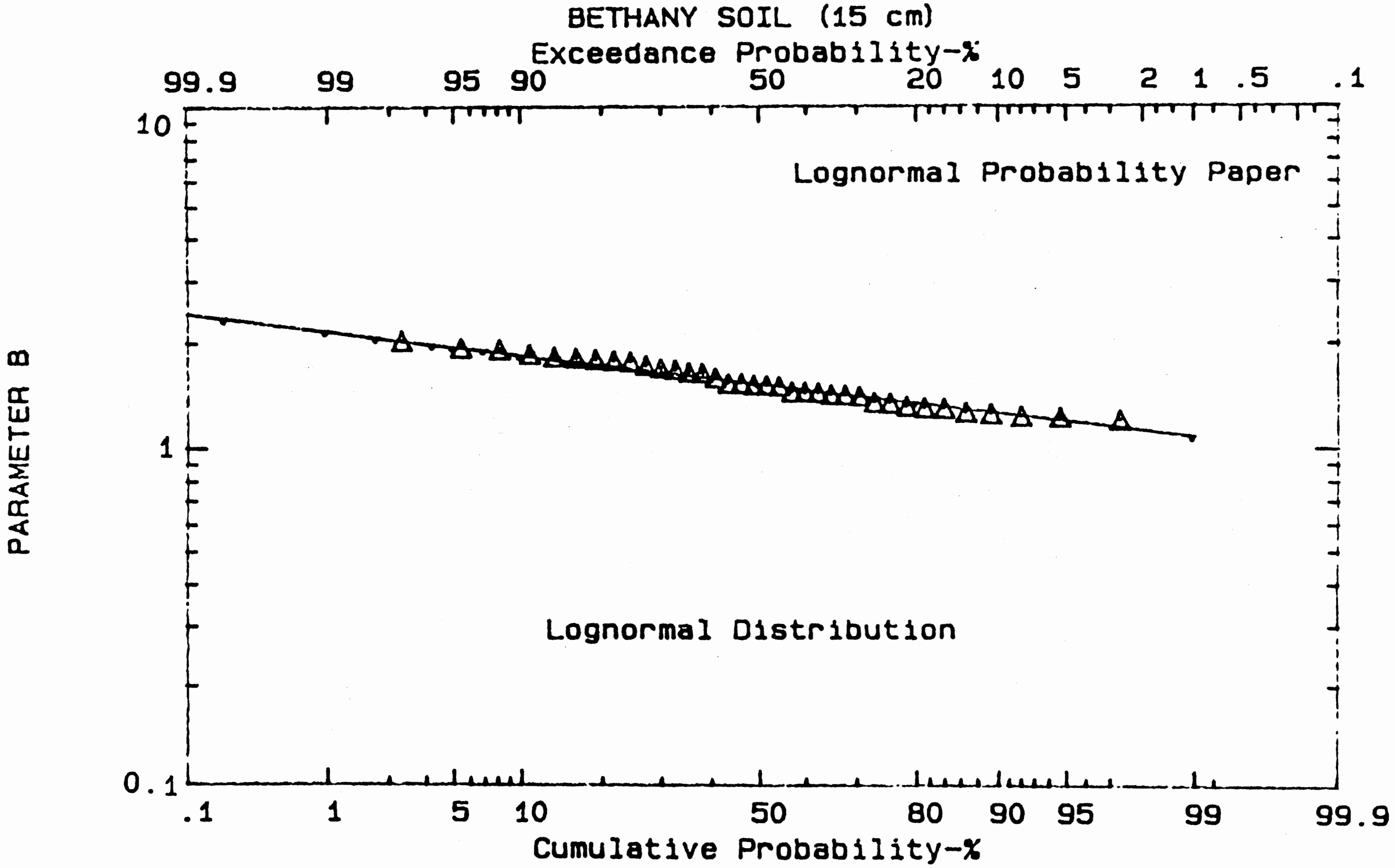

Figure 2. Probability Distribution of Parameter B for Bethany Soil (15 cm)

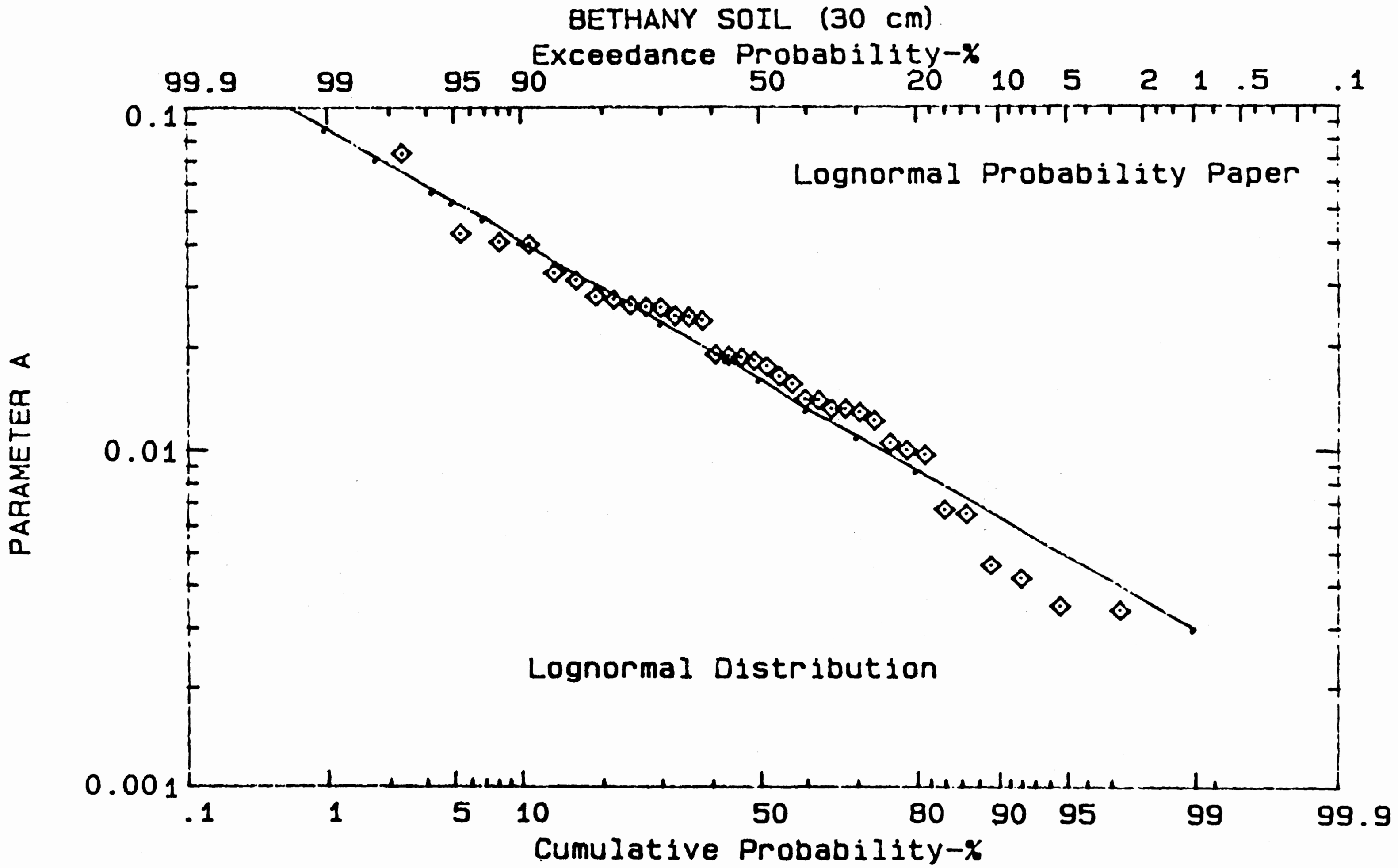

Figure 3. Probability Distribution of Parameter A for Bethany Soil (30 cm)

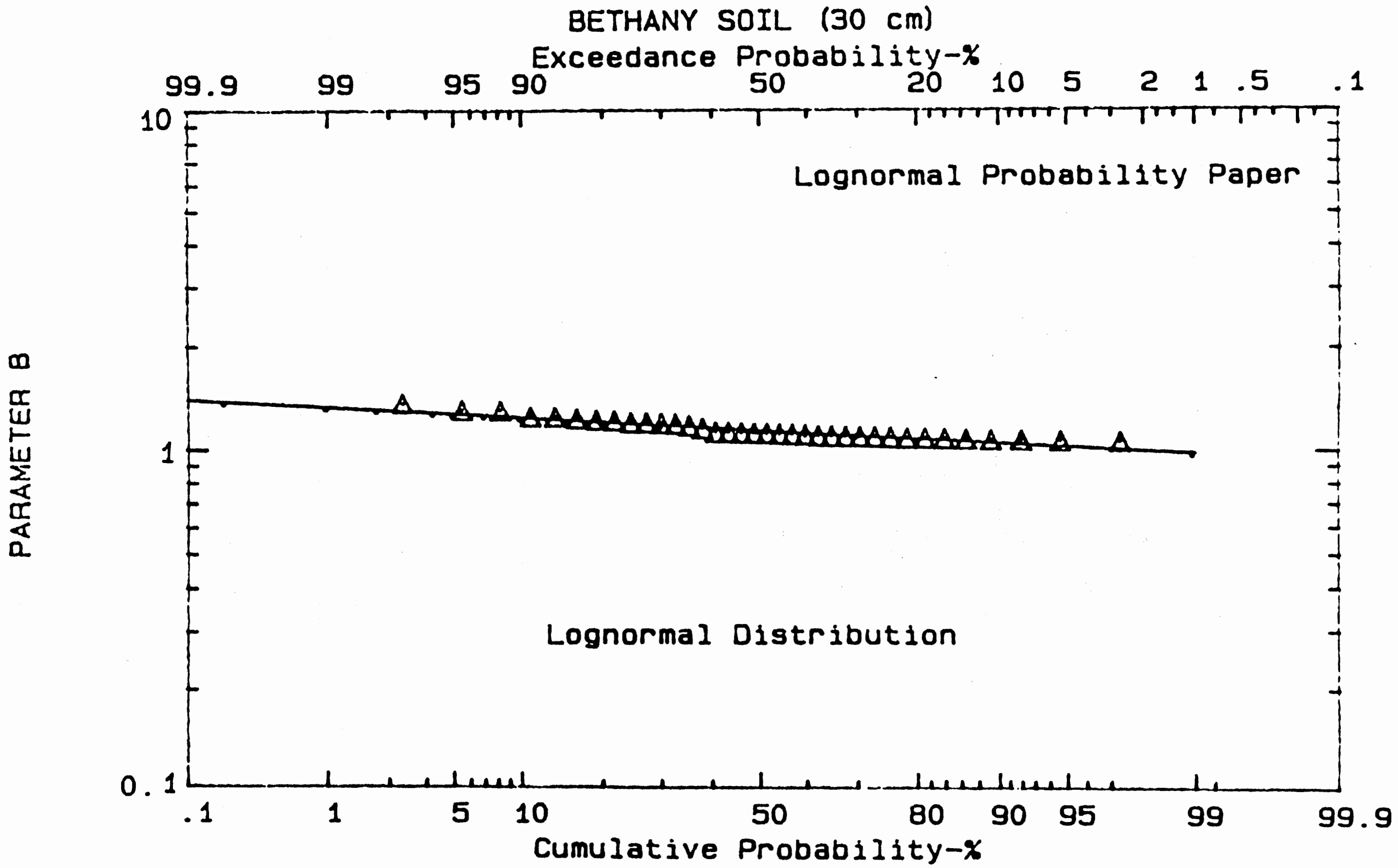

Figure 4. Probability Distribution of Parameter B for Bethany Soil (30 cm)

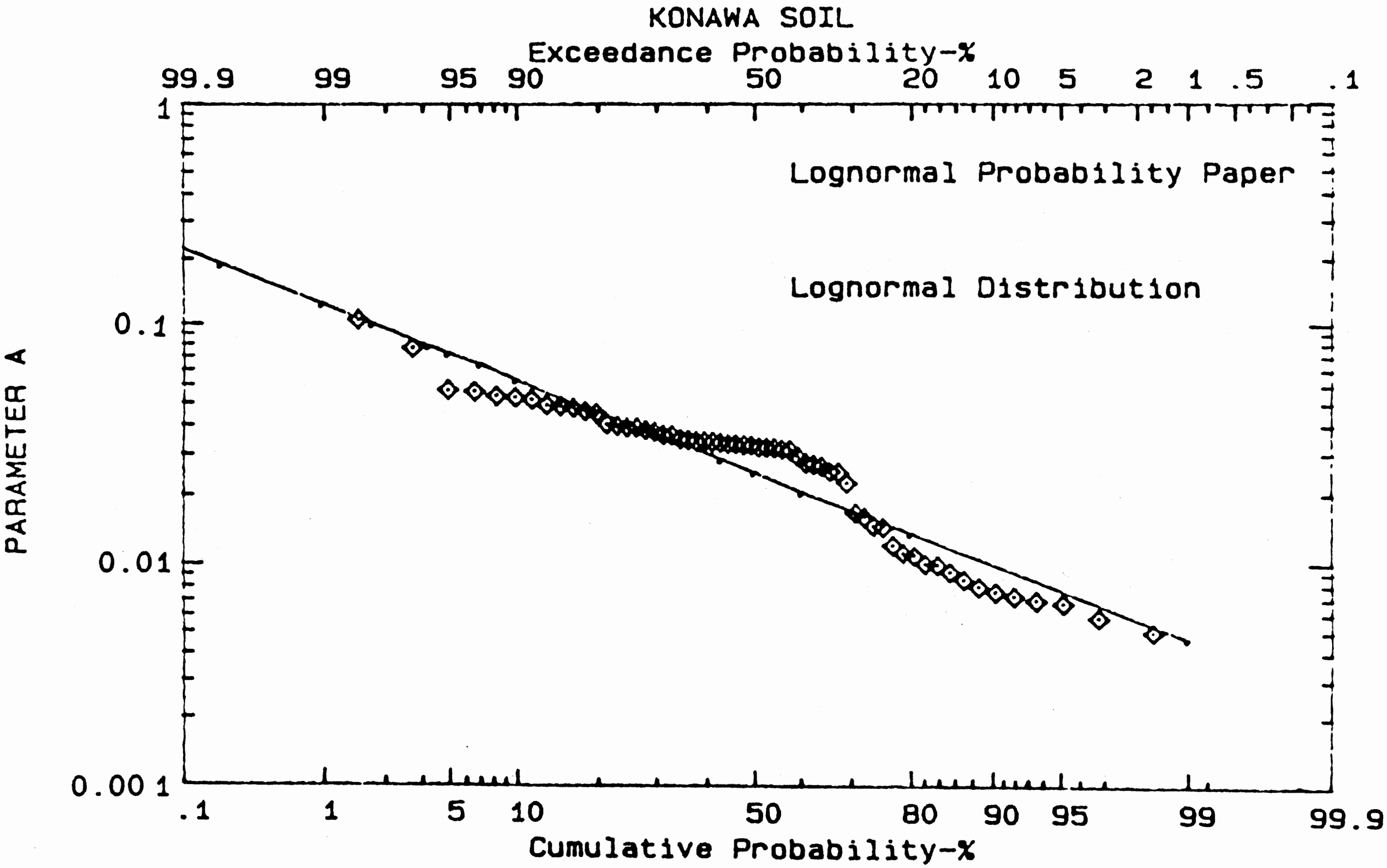

Figure 5. Probability Distribution of Parameter A for Konawa Soil

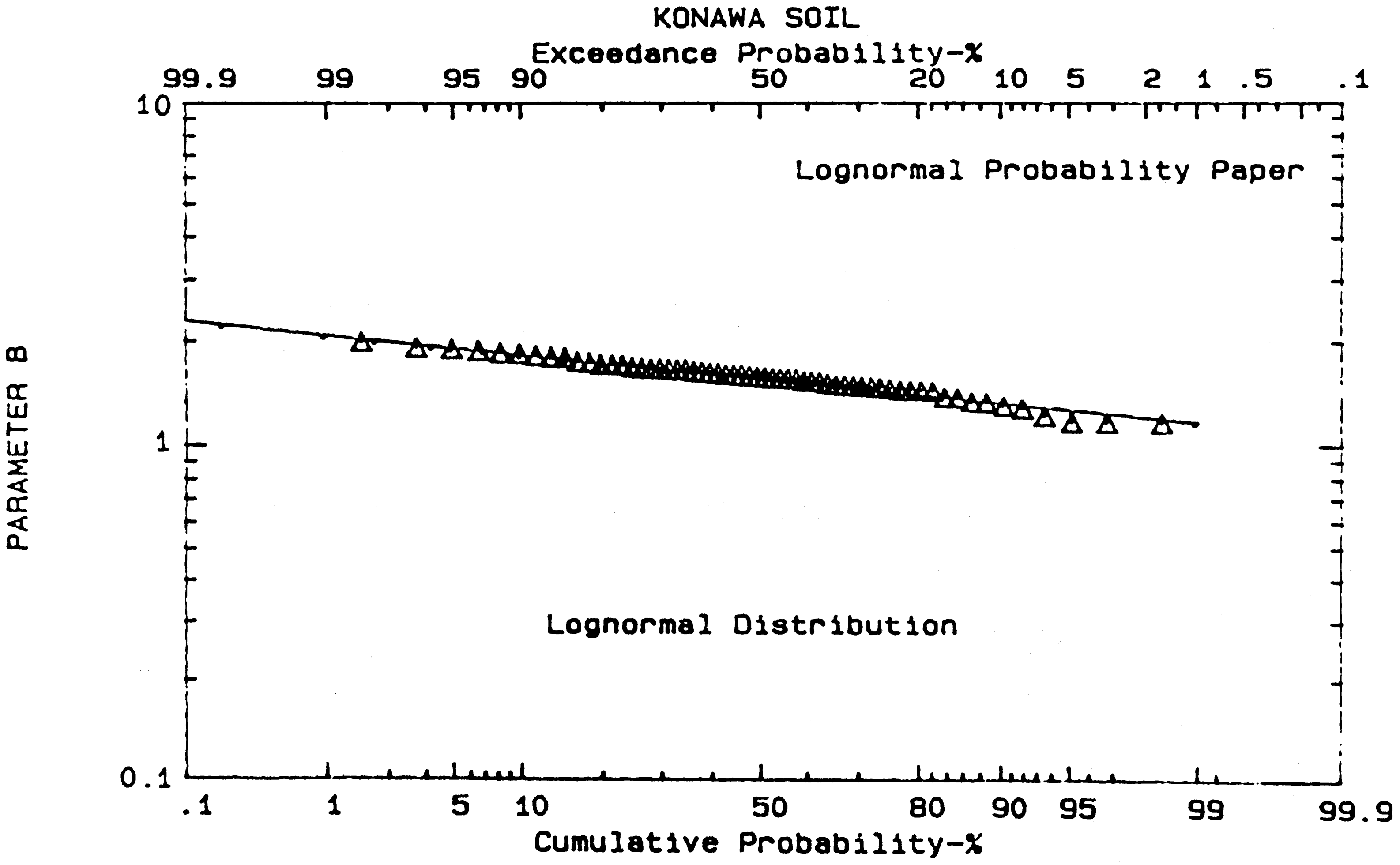

Figure 6. Probability Distribution of Parameter B for Konawa Soil

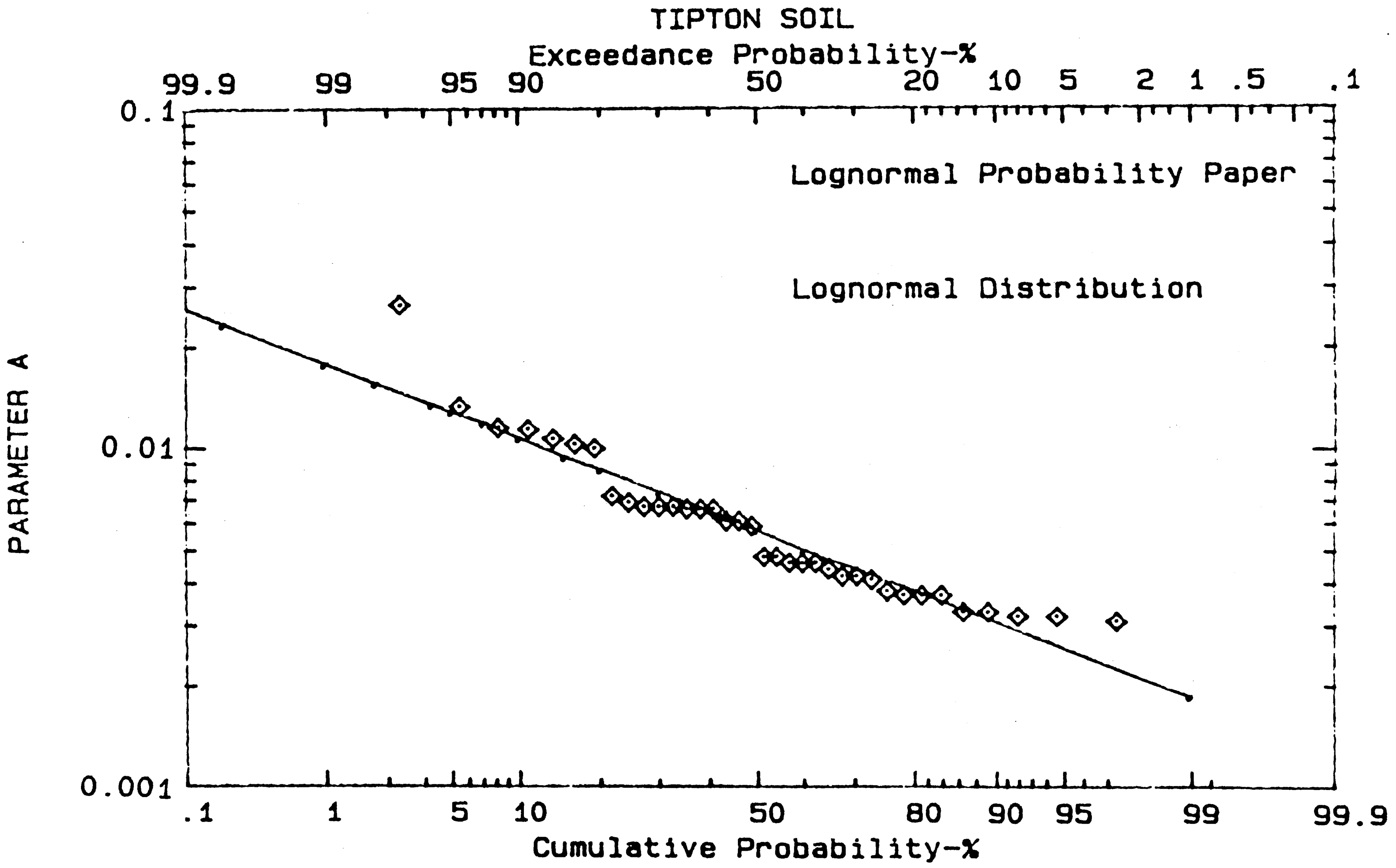

Figure 7. Probability Distribution of Parameter A for Tipton Soil

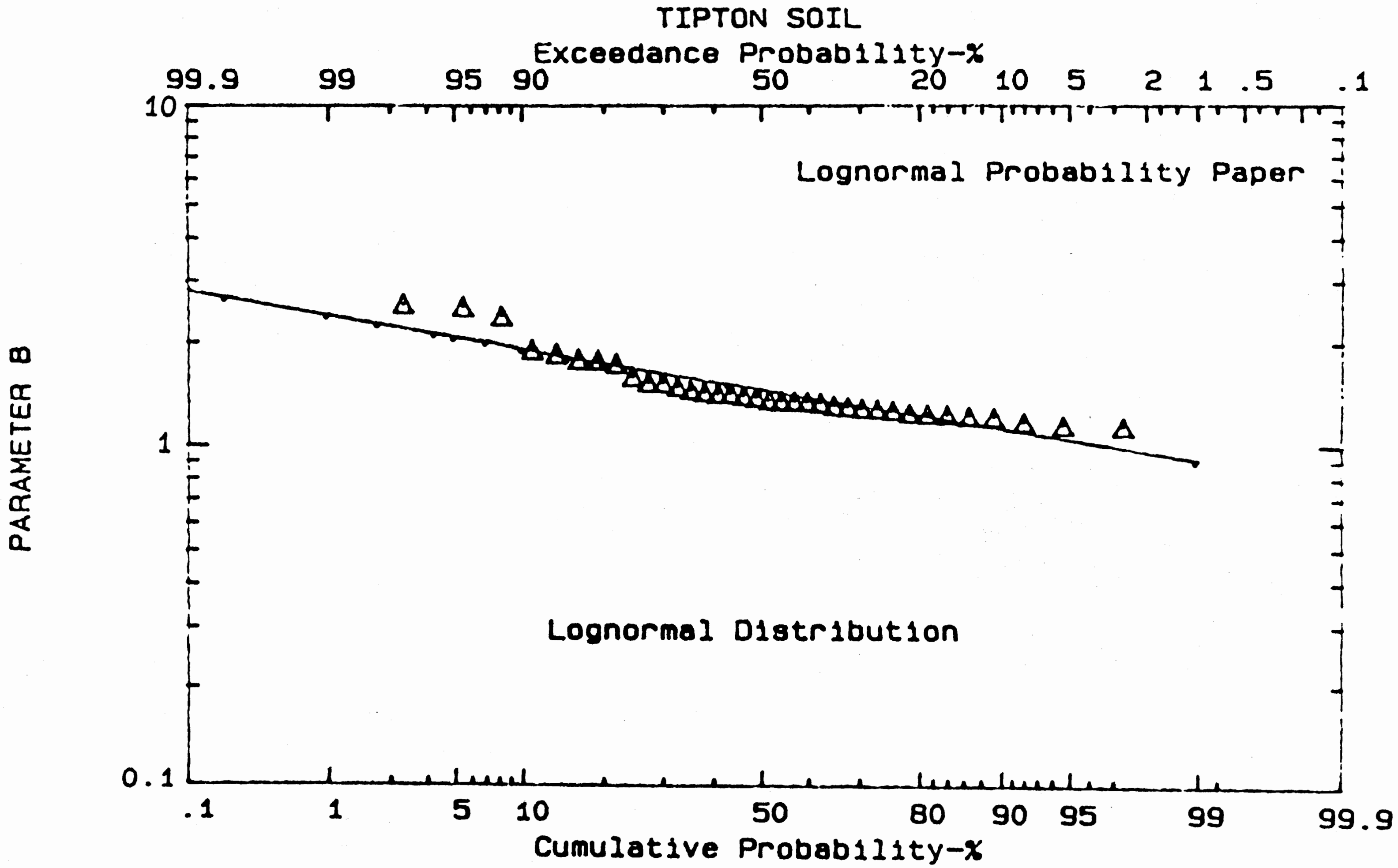

Figure 8. Probability Distribution of Parameter B for Tipton Soil

Parameter Correlation

The parameters A and B are inversely related to the air entry tension and variance of the pore size distribution (Kool et al., 1985). Since the air entry tension is affected in part by pore size, it is expected to find a significant correlation between the above parameters. The correlation coefficients were calculated and are shown on table 10. A t-test was conducted to affirm the significance of this correlation. For this purpose the hypothesis that the parameter populations are uncorrelated was tested. This hypothesis was rejected for all cases using a level of significance of 5%. Thus correlation must be maintained in the generation process for random A and B pairs for use in the flow simulation.

TABLE 10

t-TEST STATISTIC FOR SIGNIFICANCE OF CORRELATION BETWEEN A AND B

Soil	Correlation Coeff.	t-Test (*)	
		Calculated	Table
Bethany-15 cm	-.465	-3.063	2.020
Bethany-30 cm	-.760	-6.818	2.020
Tipton	-.590	-4.260	2.020
Konawa	-.295	-2.350	2.000

(*) The hypothesis that populations are uncorrelated is being tested (5% level of significance)

CHAPTER V

WATER INFLOW SIMULATION

Bivariate Generation of Parameters

A bivariate generation of random and lognormally distributed pairs of A and B was used to maintain the correlation between A and B. The procedure given in Haan (1977) was used.

Knowing the correlation matrix $\underline{R}$ between A and B, the equation

$$\underline{X}_{(n\times 2)} = \underline{Z}_{(n\times 2)} * \underline{A}'_{(2\times 2)}$$

was used to generate random values of A and B where

$\underline{A}'$ is the transpose of a 2x2 orthogonal matrix of characteristic vectors ($\underline{L}_1$) of the correlation matrix,

$\underline{Z}$ is an nx2 matrix of independent standard normal deviates

$\underline{X}$ is an nx2 matrix of n generated standardized logarithms of observations on A and B, and

n is the number of random observations to be generated.

The characteristic vectors $\underline{L}_1$ or eigenvectors of the correlation matrix $\underline{R}$ are obtained from the equation

$$\mathrm{Det}(\underline{R} - \underline{L}*\underline{I}) = \underline{0}$$

where

$\underline{I}$ is the identity matrix.

$\underline{0}$ is the zero matrix.

The $\underline{X}_{(n \times 2)}$ matrix of n generated standardized logarithmic observations are converted to A and B from

$$A_i = \mathrm{EXP}(\mathrm{SD}(A) * X_{(i,1)} + \mathrm{M}(A))$$

$$B_i = \mathrm{EXP}(\mathrm{SD}(B) * X_{(i,2)} + \mathrm{M}(B))$$

where

Ai and Bi are the ith generated values for A and B,

SD(A) and SD(B) are the standard deviations of A and B,

M(A) and M(B) are the means of A and B.

The correlation observed in the original data is maintained by using the following expressions derived from Matalas (1967):

$$\mathrm{SD}(A) = \{\mathrm{LOG}([\mathrm{SD}(A_o)/\mathrm{M}(A_o)]^2 + 1)\}^{1/2}$$

$$\mathrm{SD}(B) = \{\mathrm{LOG}([\mathrm{SD}(B_o)/\mathrm{M}(B_o)]^2 + 1)\}^{1/2}$$

$$\mathrm{M}(A) = \{\mathrm{LOG}(\mathrm{M}(A_o) - \mathrm{SD}(A_o)^2\}/2$$

$$\mathrm{M}(B) = \{\mathrm{LOG}(\mathrm{M}(B_o) - \mathrm{SD}(B_o)^2\}/2$$

$$\mathrm{R}(A,B) = \mathrm{LOG}(1 + \mathrm{R}(A_o,B_o)*\{\mathrm{EXP}[\mathrm{SD}(A_o)^2 - 1]* \mathrm{EXP}[\mathrm{SD}(B_o)^2 - 1]\}^{1/2})/\mathrm{SD}(A_o)*\mathrm{SD}(B_o)$$

where

A_o and B_o are the parameters A and B from the observed data.

SD(X) is the standard deviation of the variable X, M(X) is the mean of the variable X, and

R(X,Y) is the correlation coefficient between the variables X and Y.

Using the procedure shown above, random lognormally distributed observations of A and B were generated (a listing of the computer program, BIVAR.BAS, used for the generation is given in appendix B). The number of generated observations was increased until the variation in means and standard deviations of the simulated cumulative inflow values was found to be reasonably stable. As a result, 10 sets of 100 pairs of A an B were generated for each of the four considered soils. However, as mentioned earlier in Chapter III, the parameter B of the Van Genuchten model must be strictly greater than 1. Respecting this condition, some generated values of B had to be discarded. Tables 11 through 14 show the correlation coefficients between A and B for each generated sample compared to the original values, means and standard deviations are also shown. From these tables it can be seen that the initial correlation was maintained during the generation process.

Solving the Flow Equation

For the simulations described below values of K_s and WC_s as required in equations (5) and (6) were taken as:

TABLE 11

MEANS AND STANDARD DEVIATIONS OF GENERATED A AND B FOR BETHANY SOIL (15 cm)

Run #	A		B		Corr.
	Mean	SD	Mean	SD	
DATA	.0060	.0030	1.5430	.2310	-.5110
1	.0059	.0027	1.5205	.2305	-.5390
2	.0061	.0039	1.5395	.2538	-.5058
3	.0058	.0028	1.5438	.2400	-.5260
4	.0059	.0029	1.5409	.2295	-.5263
5	.0059	.0027	1.5449	.1988	-.5620
6	.0059	.0026	1.5584	.2077	-.4760
7	.0064	.0030	1.5415	.2341	-.6180
8	.0059	.0030	1.5392	.2321	-.5620
9	.0059	.0027	1.5620	.2391	-.4797
10	.0060	.0031	1.5275	.2330	-.5760

TABLE 12

MEANS AND STANDARD DEVIATIONS OF GENERATED A AND B FOR BETHANY SOIL (30 cm)

Run #	A		B		Corr.
	Mean	SD	Mean	SD	
DATA	.0200	.0140	1.1450	.0760	-.5740
1	.0209	.0151	1.1454	.0741	-.5390
2	.0177	.0116	1.1449	.0710	-.4940
3	.0199	.0131	1.1498	.0713	-.5550
4	.0198	.0110	1.1417	.0776	-.5950
5	.0192	.0132	1.1442	.0648	-.5200
6	.0216	.0146	1.1299	.0797	-.5697
7	.0225	.0142	1.1348	.0783	-.5250
8	.0200	.0140	1.1447	.0770	-.5360
9	.0209	.0146	1.1371	.0794	-.5440
10	.0184	.0112	1.1454	.0711	-.5840

TABLE 13

MEANS AND STANDARD DEVIATIONS OF GENERATED A AND B FOR KONAWA SOIL

Run #	A Mean	A SD	B Mean	B SD	Corr.
DATA	.0310	.0200	1.5760	.1900	-.2930
1	.0332	.0204	1.5784	.1932	-.2780
2	.0301	.0230	1.5684	.1893	-.2240
3	.0338	.0217	1.5428	.1827	-.3660
4	.0313	.0193	1.5495	.1889	-.1910
5	.0332	.0211	1.5828	.1946	-.2340
6	.0296	.0177	1.5770	.1883	-.2740
7	.0293	.0175	1.5865	.1790	-.3090
8	.0308	.0181	1.5953	.1856	-.2870
9	.0326	.0209	1.5521	.1925	-.3450
10	.0282	.0149	1.5806	.1981	-.3370

TABLE 14

MEANS AND STANDARD DEVIATIONS OF GENERATED A AND B FOR TIPTON SOIL

Run #	A		B		Corr.
	Mean	SD	Mean	SD	
DATA	.0070	.0040	1.5120	.3610	-.4420
1	.0069	.0036	1.5382	.3447	-.4070
2	.0068	.0038	1.5563	.4041	-.3940
3	.0070	.0042	1.4943	.3845	-.4980
4	.0074	.0040	1.4928	.3388	-.4330
5	.0074	.0041	1.4505	.3150	-.4500
6	.0069	.0037	1.5197	.3856	-.4410
7	.0062	.0033	1.5757	.3403	-.4990
8	.0070	.0039	1.4784	.3917	-.4860
9	.0068	.0037	1.4989	.3957	-.4520
10	.0074	.0042	1.4623	.3375	-.4900

Soil	K_s(cm/hr.)	WC_s
Bethany-15 cm	0.2	0.42
Bethany-30 cm	1.5	0.42
Konawa	0.3	0.40
Tipton	0.5	0.37

In all cases WC_r was taken as zero.

Once defined, the parameters of equation (5) serve as an input data for a computer program "Interactive Simulation of One-Dimentional Water Movement in Soils" (Nofziger, 1985) to solve the Richards partial differential equation for unsaturated flow. A finite difference method is used. The flow equation (4) is transformed as

$$C(i,j)\frac{h(i,j+1)-h(i,j)}{\Delta t} = \frac{1}{\Delta z}\left[K(i+1/2,j)\left(\frac{h(i+1,j+1)-h(i,j+1)}{\Delta z}-1\right) - K(i-1/2,j)\left(\frac{h(i,j+1)-h(i-1,j+1)}{\Delta z}-1\right)\right] \qquad (7)$$

Where

$h(i,j)$ is the pressure head at the i^{th} spacing step and j^{th} time step.

$C(i,j)$ is the specific water capacity at $h(i,j)$.

Δt is the mesh size in time.

Δz is the mesh size in depth.

$K(i+1/2,j) = [K(h(i,j))+K(h(i+1,j))]/2$

$$K(i-1/2,j) = [K(h(i-1,j))+K(h(i,j))]/2$$

Applying the finite difference equation above to each interior node, a system of linear equations results which can be solved by appropriate matrix equation solvers. Since each equation has only three unknowns, the augmented matrix of the system will be in a tridiagonal form which makes the computations easier and faster.

The descritization scheme of the flow domain used in the above model uses a grid system with respect to space and time. The space index (i) is defining a mesh size of Δz in depth. The time index (j) is defining a mesh size of Δt in time. The model offers the option of choosing the initial mesh sizes, then automatically adjusting it according to the mass balance and depth of wetting. The option of fixed mesh sizes is also available.

Boundary and Initial Conditions

The solution of the flow equation can be displayed for several boundary conditions at the upper boundary ($z = 0$) for a semi-infinite system and at both the upper and lower boundaries for a finite system.

In the present study a semi-infinite type of soil profile was chosen. Concerning the initial conditions, the simulation was done considering an initial matric potential

$$h(z,0) = -5000 \text{ cm}$$

Five different boundary conditions have been considered, consisting of imposing a constant matric potential values at the upper boundary of the soil profile:

BC#1: $h(0,t) = 0$ cm
BC#2: $h(0,t) = -50$ cm
BC#3: $h(0,t) = -100$ cm
BC#4: $h(0,t) = -150$ cm
BC#5: $h(0,t) = -200$ cm

Since a semi-infinite type of soil was considered, the length of the soil profile was supposed to be large enough so that variations occurring at the upper boundary does not affect the lower boundary.

Statistical Concept

The cumulative inflow was simulated for each set of parameters A and B generated. The average inflow was computed from all these simulations. The results obtained for 2.5, 5, 7.5, and 10 hours of simulation using the following boundary and initial conditions

BC. $h(0,t) = 0$ cm
IC. $h(z,0) = -5000$ cm

are displayed in tables 15 through 18. Similarly the simulation was done using average values of parameters A and B (A, B), the results are given in table 19.

Comparing the cumulative inflow obtained using average values of parameters, A_{avg} and B_{avg}, to those computed by

TABLE 15

MEANS AND STANDARD DEVIATIONS OF SIMULATED CUMULATIVE INFLOW FOR BETHANY SOIL (15 cm)

Run #	Cumulative Inflow (cm)							
	2.5 hrs.		5 hrs.		7.5 hrs.		10 hrs.	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
1	4.020	1.799	5.707	2.601	7.039	3.218	8.178	3.738
2	4.159	2.031	5.909	2.928	7.293	3.622	8.476	4.198
3	4.192	1.737	5.959	2.511	7.356	3.105	8.545	3.604
4	4.138	1.825	5.883	2.628	7.261	3.246	8.437	3.764
5	4.180	1.554	5.938	2.252	7.325	2.791	8.508	3.242
6	4.189	1.563	5.958	2.259	7.354	2.979	8.545	3.245
7	4.002	1.741	5.681	2.516	7.008	3.118	8.141	3.612
8	4.166	1.664	5.928	2.407	7.315	2.973	8.504	3.443
9	4.107	1.573	5.840	2.277	7.213	2.820	8.385	3.274
10	4.082	1.879	5.801	2.713	7.157	3.355	8.315	3.891
AVG	4.124		5.860		7.232		8.403	

TABLE 16

MEANS AND STANDARD DEVIATIONS OF SIMULATED CUMULATIVE INFLOW FOR BETHANY SOIL (30 cm)

Run #	Cumulative Inflow (cm)							
	2.5 hrs.		5 hrs.		7.5 hrs.		10 hrs.	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
1	2.420	1.908	3.534	2.829	4.487	3.604	5.362	4.304
2	2.453	1.658	3.588	2.469	4.560	3.157	5.455	3.786
3	2.414	1.626	3.536	2.426	4.502	3.109	5.394	3.736
4	2.399	1.815	3.511	2.697	4.463	3.433	5.340	4.100
5	2.362	1.543	3.453	2.302	4.388	2.946	5.247	3.535
6	2.211	1.760	3.232	2.621	4.105	3.353	4.908	4.024
7	2.250	1.608	3.293	2.397	4.189	3.069	5.018	3.687
8	2.383	1.800	3.491	2.674	4.440	3.414	5.317	4.085
9	2.983	1.545	3.366	2.310	4.284	2.965	5.133	3.574
10	2.421	1.659	3.545	2.478	4.503	3.171	5.386	3.809
AVG	2.430		3.455		4.392		5.256	

TABLE 17

MEANS AND STANDARD DEVIATIONS OF SIMULATED CUMULATIVE INFLOW FOR KONAWA SOIL

Run #	Cumulative Inflow (cm)							
	2.5 hrs.		5 hrs.		7.5 hrs.		10 hrs.	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
1	5.018	1.526	7.908	2.062	10.676	2.319	13.514	2.638
2	5.268	1.733	8.250	2.340	11.065	2.639	13.832	2.974
3	4.889	1.572	7.748	2.102	10.515	2.357	13.397	2.798
4	4.907	1.305	7.767	1.702	10.580	2.020	13.540	3.311
5	4.996	1.477	7.884	1.968	10.646	2.197	13.398	2.520
6	5.208	1.527	8.134	2.065	10.861	2.350	13.598	2.675
7	5.315	1.482	8.318	1.932	11.196	2.192	14.088	2.977
8	5.220	1.506	8.164	2.036	10.975	2.314	13.785	2.659
9	4.919	1.310	7.783	1.719	10.620	1.984	13.564	3.142
10	5.183	1.571	8.120	2.124	10.877	2.420	13.560	2.686
AVG	5.092		8.008		10.801		13.628	

TABLE 18

MEANS AND STANDARD DEVIATIONS OF SIMULATED CUMULATIVE INFLOW FOR TIPTON SOIL

Run #	Cumulative Inflow (cm)							
	2.5 hrs.		5 hrs.		7.5 hrs.		10 hrs.	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
1	5.456	2.935	8.009	4.190	10.121	5.101	11.947	5.887
2	5.534	2.937	8.121	4.181	10.240	5.111	12.068	5.914
3	4.969	2.826	7.312	4.001	9.275	4.849	10.943	5.603
4	5.017	2.543	7.354	3.633	9.279	4.449	10.968	5.131
5	4.984	2.945	7.314	4.186	9.305	5.047	10.987	5.824
6	5.561	3.073	8.157	4.389	10.318	5.332	12.195	6.124
7	5.961	2.890	8.688	4.140	10.924	5.056	12.847	5.846
8	4.952	2.985	7.284	4.219	9.273	5.075	10.937	5.861
9	5.503	2.813	8.054	4.014	10.151	4.900	11.963	5.660
10	5.031	2.710	7.375	3.868	9.358	4.685	11.050	5.408
AVG	5.297		7.767		9.824		11.591	

TABLE 19

CUMULATIVE INFLOW OBTAINED USING AVERAGE VALUES OF A AND B PARAMETERS

	Cumulative Inflow (cm)			
Soil	2.5 hrs.	5 hrs.	7.5 hrs.	10 hrs.
Bethany-15 cm	3.935	5.578	6.879	7.993
Bethany-30 cm	1.964	2.855	3.629	4.352
Tipton	5.277	7.671	9.619	11.320
Konawa	4.763	7.437	9.988	12.510

averaging the inflow values obtained from each set of parameters shows that they are considerably different. In all the cases studied the cumulative inflow obtained using the second method was found to be greater. It can also be seen that the longer the time of the simulation, the bigger is the difference in results between the two methods.

Inflow Distribution

The cumulative probability distribution of inflow was tested using the normal and lognormal distributions. Table 20 shows the maximum deviations of the cumulative inflow from both the above distributions. K is the critical value of the Kolmogorov-Smirnov test statistic. The cumulative inflow is best described by a lognormal distribution. Figures 9 through 12 show the probability distribution of the cumulative inflow after 10 hours of simulation.

Convergence in the Solution

In an attempt to find a set of A and B values that would produce infiltration estimates equal to the average infiltration, 100 simulations of the cumulative inflow were computed using each of the five boundary conditions considered. From these simulations sets of parameters A and B , giving cumulative inflow values close to the mean, were selected. A minimum of 7 points were selected. Plots of the best fitting curves through these selected sets of A and B are shown in figures 13 through 16. The curves were

TABLE 20

KOLMOGOROV-SMIRNOV TEST STATISTIC FOR NORMALITY AND LOGNORMALITY OF THE CUMULATIVE INFLOW

Soil	D1	D2	K
Bethany-15cm	.089	.062	.140
Bethany-30cm	.166	.090	.140
Tipton	.115	.136	.140
Konawa	.167	.140	.140

D1 maximum deviation from the normal distr.
D2 maximum deviation from the lognormal dist.
K the critical value of the K-S test

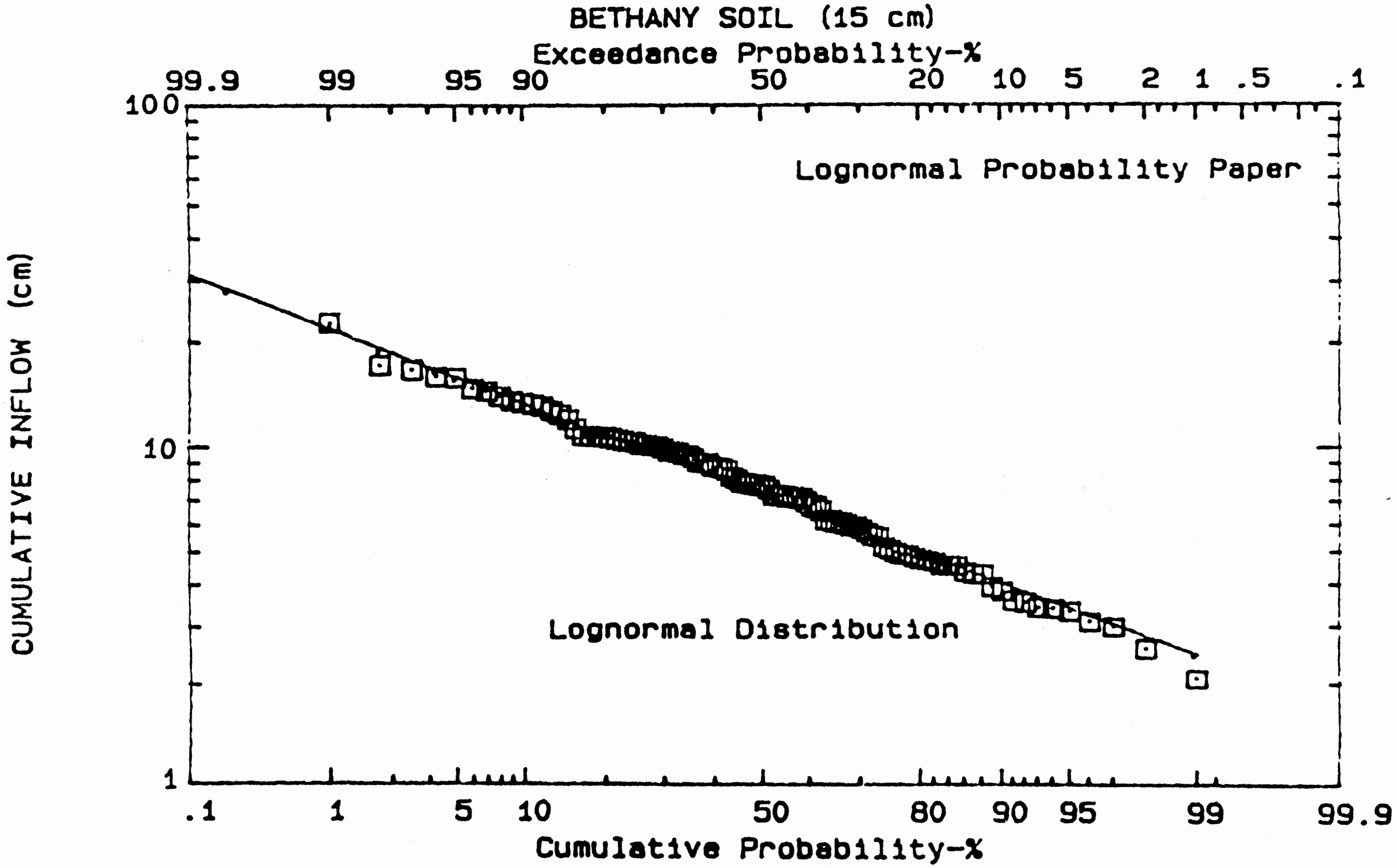

Figure 9. Probability Distribution of Cumulative Inflow After 10 hrs. of Simulation for Bethany Soil (15 cm)

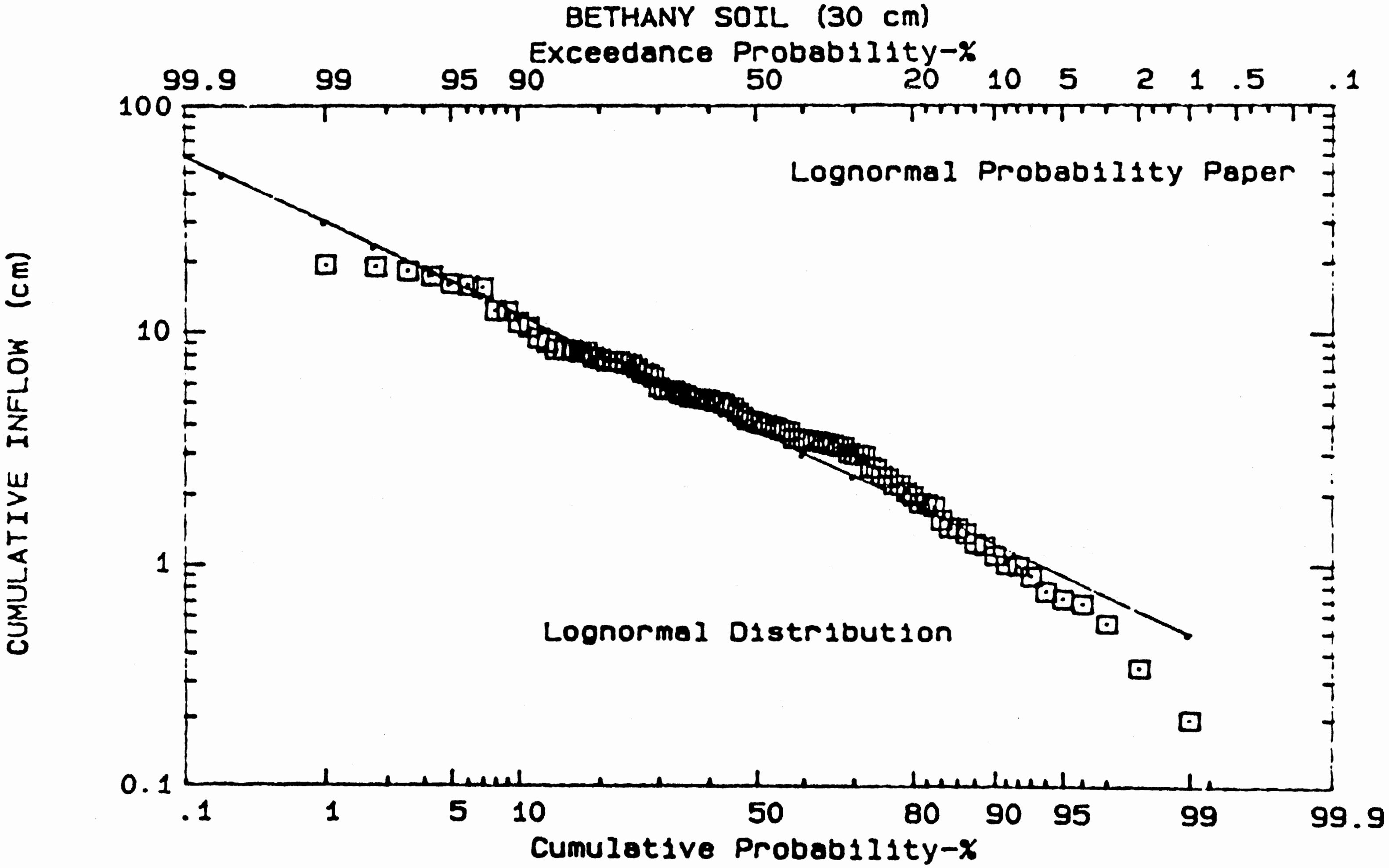

Figure 10. Probability Distribution of Cumulative Inflow After 10 hrs. of Simulation for Bethany Soil (30 cm)

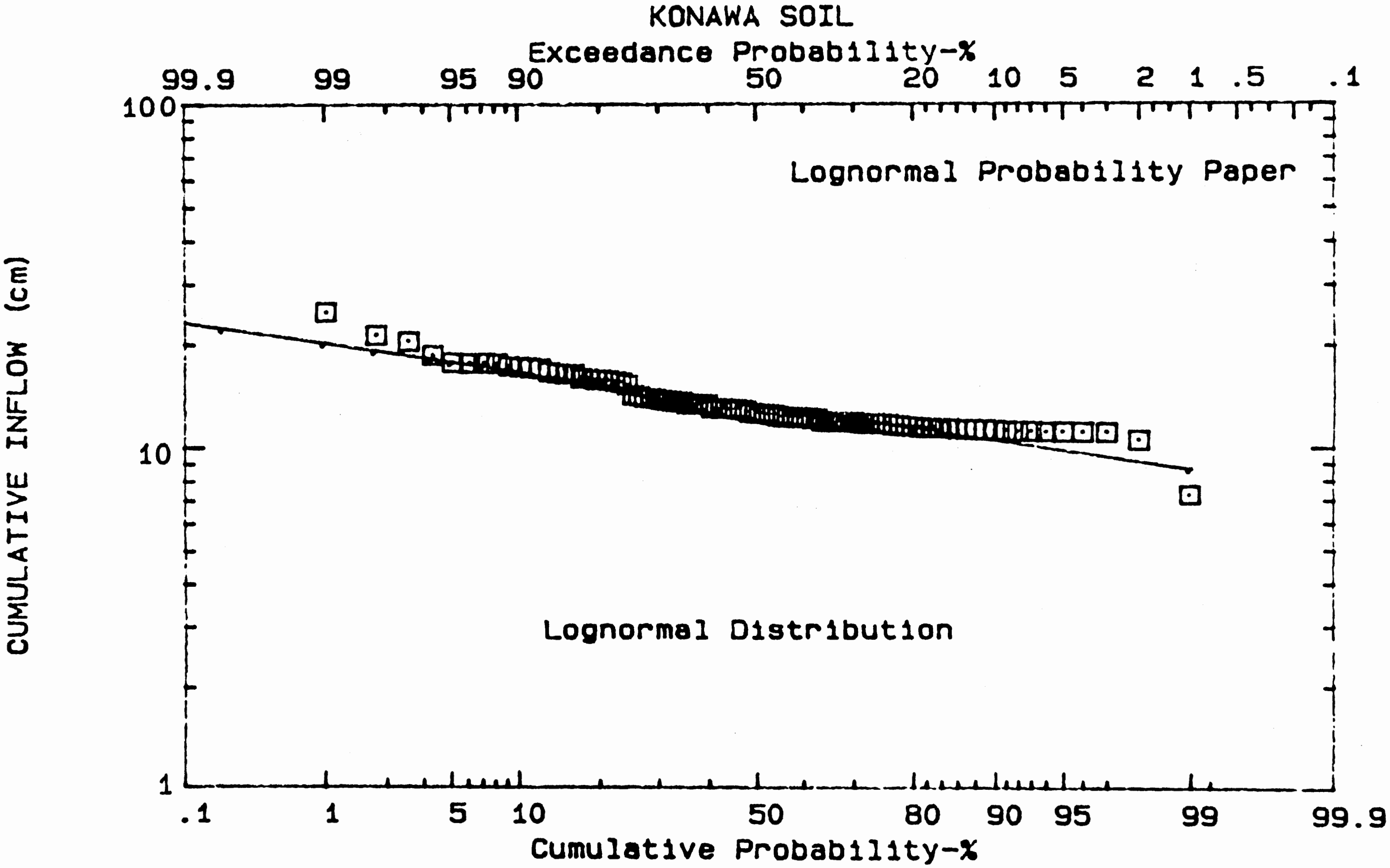

Figure 11. Probability Distribution of Cumulative Inflow After 10 hrs. of Simulation for Konawa Soil

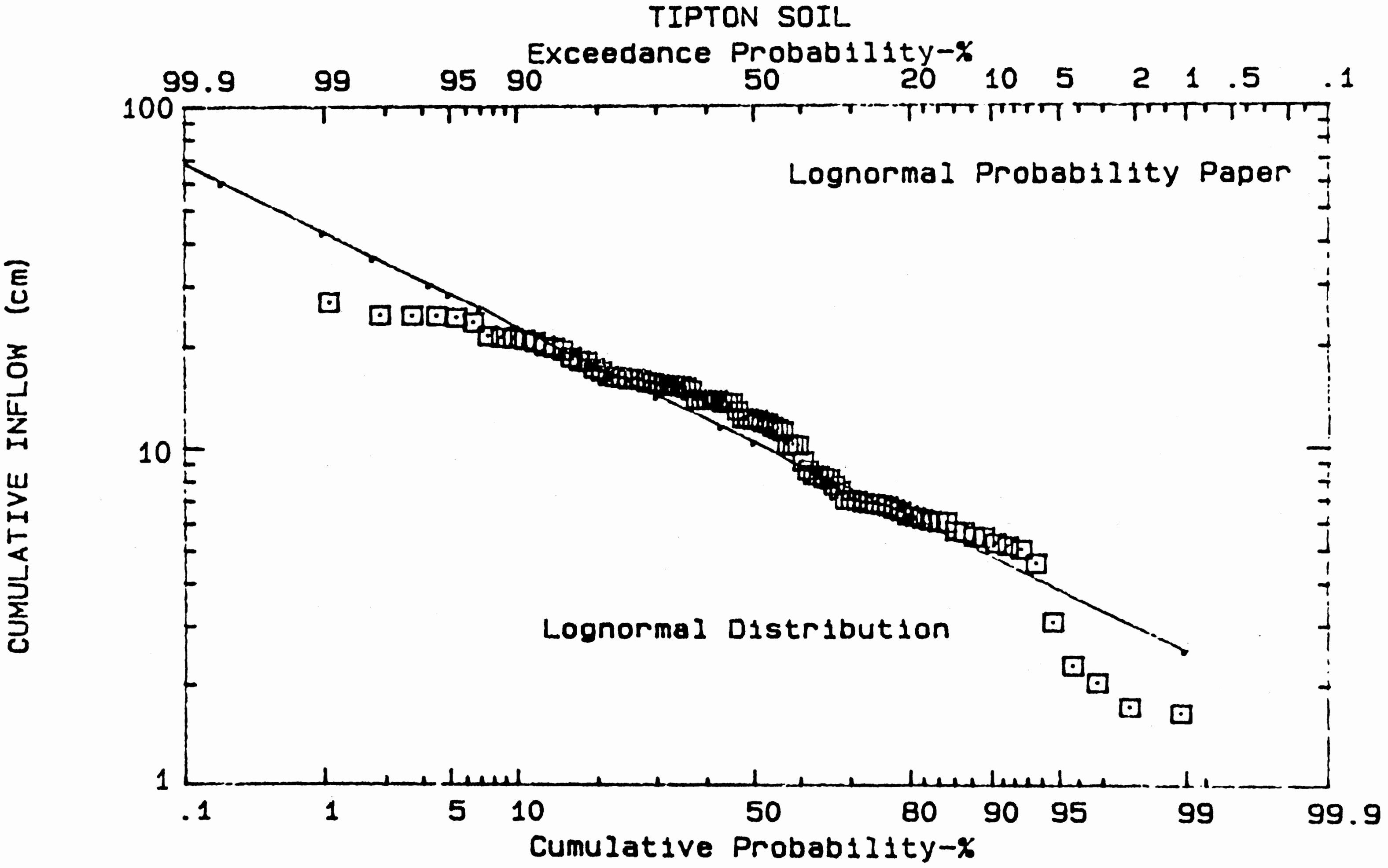

Figure 12. Probability Distribution of Cumulative Inflow After 10 hrs. of Simulation for Tipton Soil

all described by a linear regression of B versus either A or 1/A.

For all four soils an area of convergence in the solution can easily be identified. In at least three cases (Bethany soil (15 and 30 cm) and Tipton soil), the region of convergence is almost reduced to one single point.

Single sets of parameters A and B, noted A* and B*, where the majority of solutions tend to match, were estimated for each soil. Values of A* and B* are compared to A_{avg} and B_{avg} in table 21. The cumulative inflow is then simulated using the extracted parameters, A* and B*. Compared to the simulations done using average parameter values, A_{avg} and B_{avg}, closer results to the average cumulative inflow were obtained when using the parameters A* and B*. Table 22 shows the cumulative inflow values obtained using the parameters A* and B* , and those obtained using the parameters A_{avg} and B_{avg}. Average values of the cumulative inflow are also displayed for comparison purpose.

Although the difference between the averaged value of cumulative inflow and the one obtained using averaged parameters is small for a specified boundary condition of zero matric potential, it seems to be increasing considerably as the value of the matric potential assigned to the boundary condition becomes smaller.

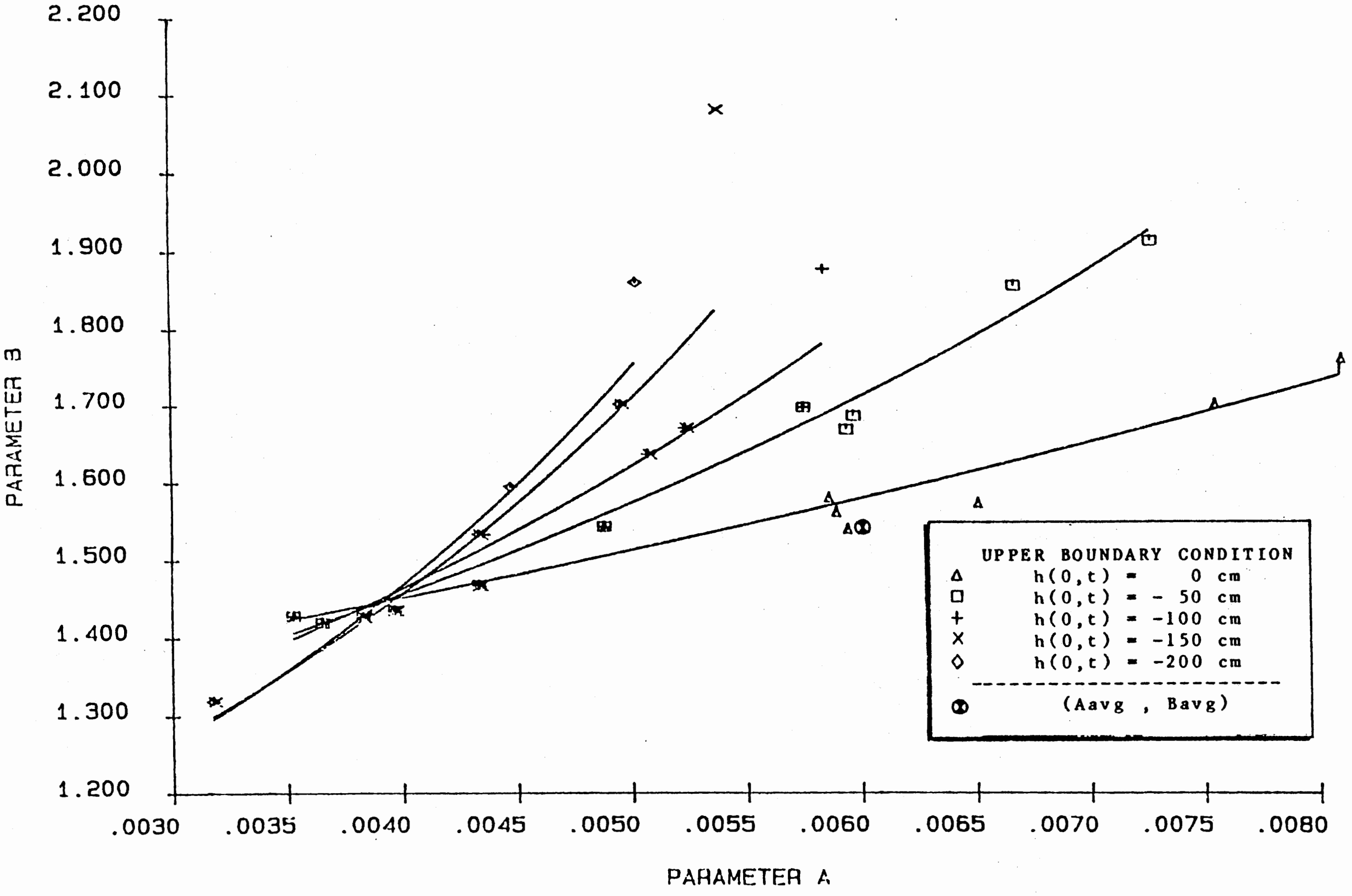

Figure 13. Zone of Convergence in the Solution for Bethany Soil (15 cm)

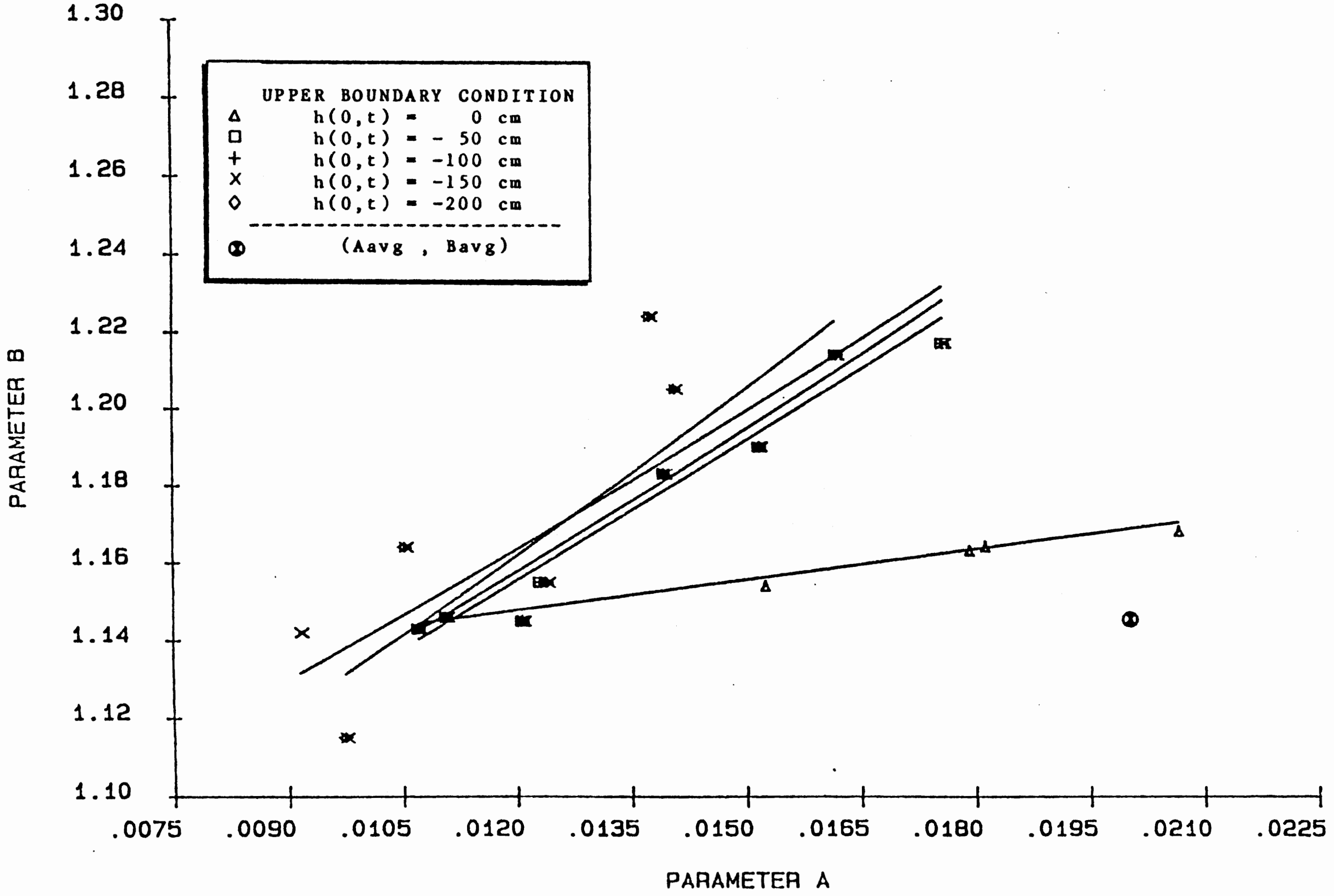

Figure 14. Zone of Convergence in the Solution for Bethany Soil (30 cm)

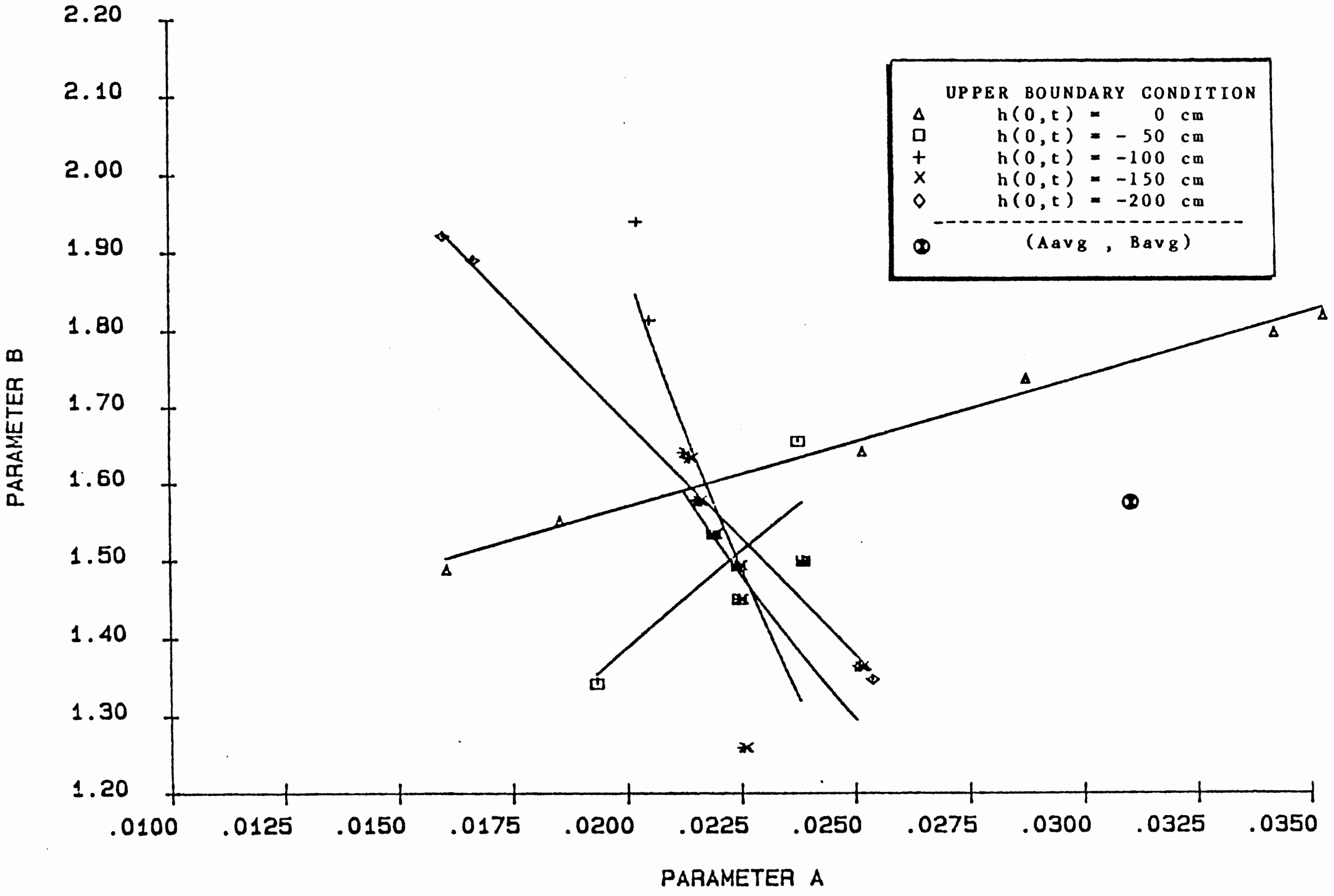

Figure 15. Zone of Convergence in the Solution for Konawa Soil

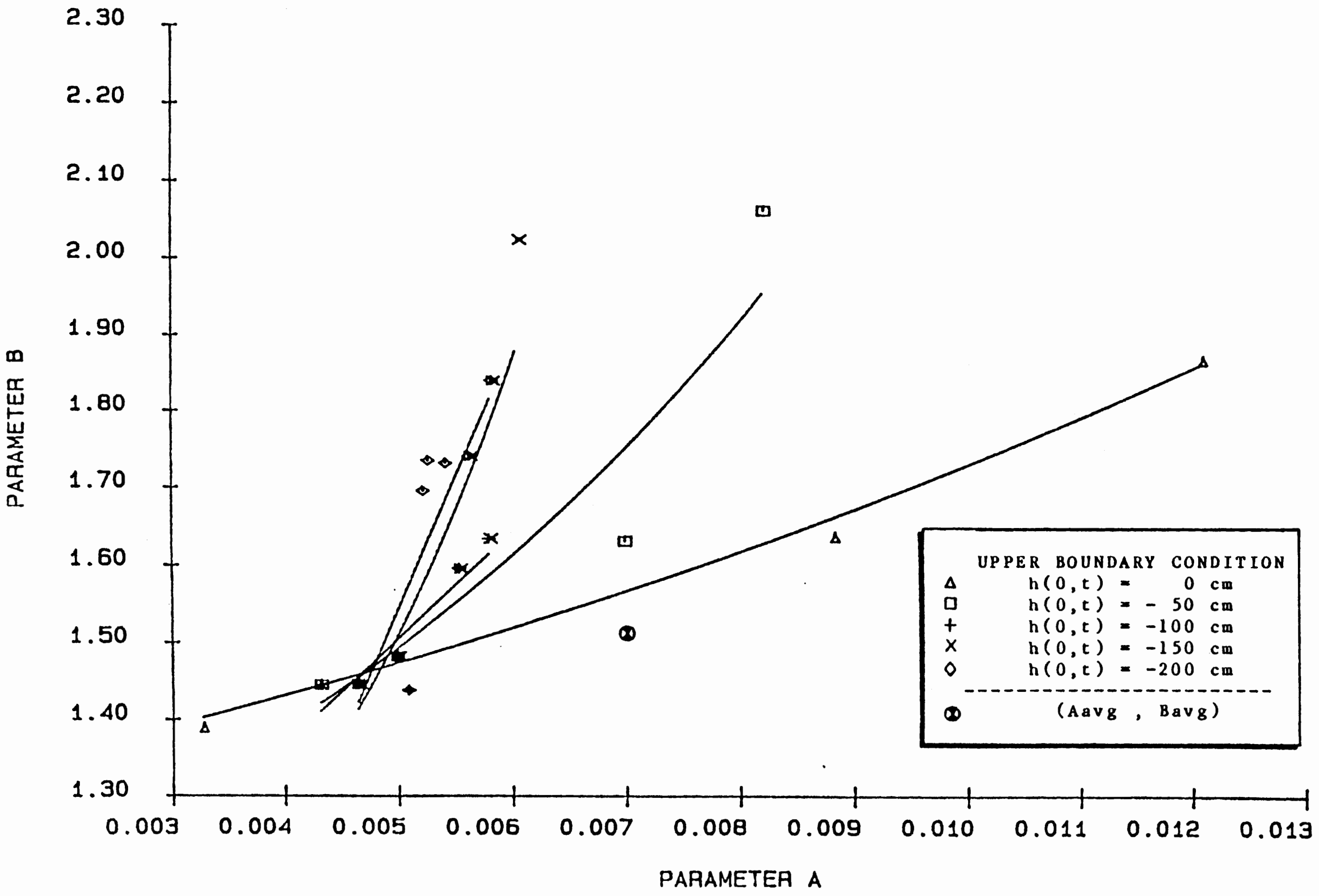

Figure 16. Zone of Convergence in the Solution for Tipton Soil

TABLE 21

EXTRACTED PARAMETERS A^* AND B^* COMPARED TO THE AVERAGE PARAMETERS A_{avg} AND B_{avg})

Soil	A^*	B^*	A_{avg}	B_{avg}
Bethany-15 cm	.00385	1.440	.006	1.543
Bethany-30 cm	.01070	1.144	.020	1.145
Konawa	.02185	1.535	.031	1.576
Tipton	.00475	1.475	.007	1.512

TABLE 22

COMPARISON OF AVERAGE CUMULATIVE INFLOW TO THOSE OBTAINED USING (A*,B*) AND (Aavg,Bavg)

Soil	Boundary Conditions (*)				
	BC#1	BC#2	BC#3	BC#4	BC#5
Bethany-15 cm					
Avg. Cumulative Inflow	8.40	5.67	4.21	3.25	2.59
Inflow Using A*, B*	8.16	5.67	4.31	3.40	2.76
Inf. Using Aavg, Bavg	7.99	4.81	3.24	2.34	1.75
Bethany-30 cm					
Avg. Cumulative Inflow	5.36	2.31	1.59	1.22	.98
Inflow Using A*, B*	5.47	2.32	1.60	1.22	.99
Inf. Using Aavg, Bavg	4.35	1.39	.88	.65	.51
Tipton					
Avg. Cumulative Inflow	12.19	7.59	5.46	4.10	3.16
Inflow Using A*, B*	12.38	7.87	5.64	4.25	3.33
Inf. Using Aavg, Bavg	11.32	6.05	3.88	2.72	2.02
Konawa					
Avg. Cumulative Inflow	13.51	2.34	1.09	.66	.44
Inflow Using A*, B*	13.04	2.45	1.12	.67	.45
Inf. Using Aavg, Bavg	12.51	1.47	.62	.35	.24

(*) BC#1: h(0,t) = 0 cm
BC#2: h(0,t) = -50 cm
BC#3: h(0,t) = -100 cm
BC#4: h(0,t) = -150 cm
BC#5: h(0,t) = -200 cm

Extracted and Averaged Parameters Relationships

An attempt to relate the extracted parameters, A^* and B^*, to the averaged ones, A_{avg} and B_{avg}, was made by fitting different regression models to the four sets of parameters obtained for each soil. For both A and B parameters it was found that a linear type of model with null constant term is very well describing the relationship between the extracted and the averaged values.

A Parameter

A linear model relating A^* to A_{avg} was estimated as

$$A^* = .655 * A_{avg} \tag{8}$$

with a standard error of .043 and R^2 of .987. Table 23 is the ANOVA table for this relationship which is plotted in figure 17.

TABLE 23

ANALYSIS OF VARIANCE TABLE FOR
THE REGRESSION EQUATION (8)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F Ratio	P(2 Tail)
Regression	.001	1	.001	227.782	.001
Residual	.000	3	.000		

B Parameter

Contrary to A^* which was found significantly different from the average A_{avg}, B^* is almost equal to B_{avg}. The estimated model relating the latter parameters is

$$B^* = .967 * B_{avg} \tag{9}$$

with a standard error of .013 and R^2 of 0.999. The analysis of variance is given in table 24. Figure 18 is a plot of the relationship.

TABLE 24

ANALYSIS OF VARIANCE TABLE FOR
THE REGRESSION EQUATION (9)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F Ratio	P(2 Tail)
Regression	7.910	1	7.911	5451.524	.000
Residual	.004	3	.001		

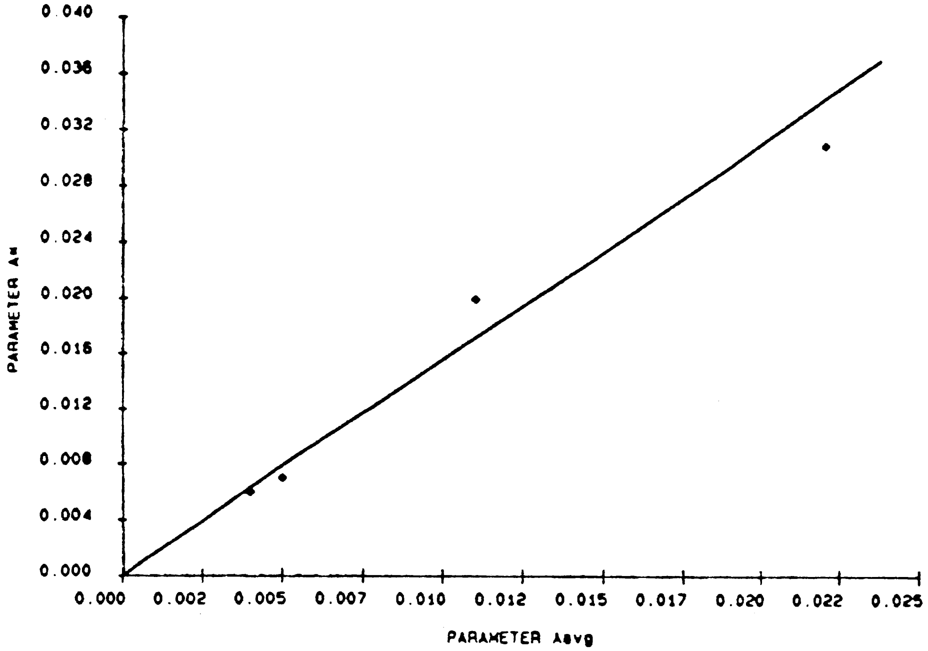

Figure 17. Plot of Extracted Versus Average Values of A Parameter.

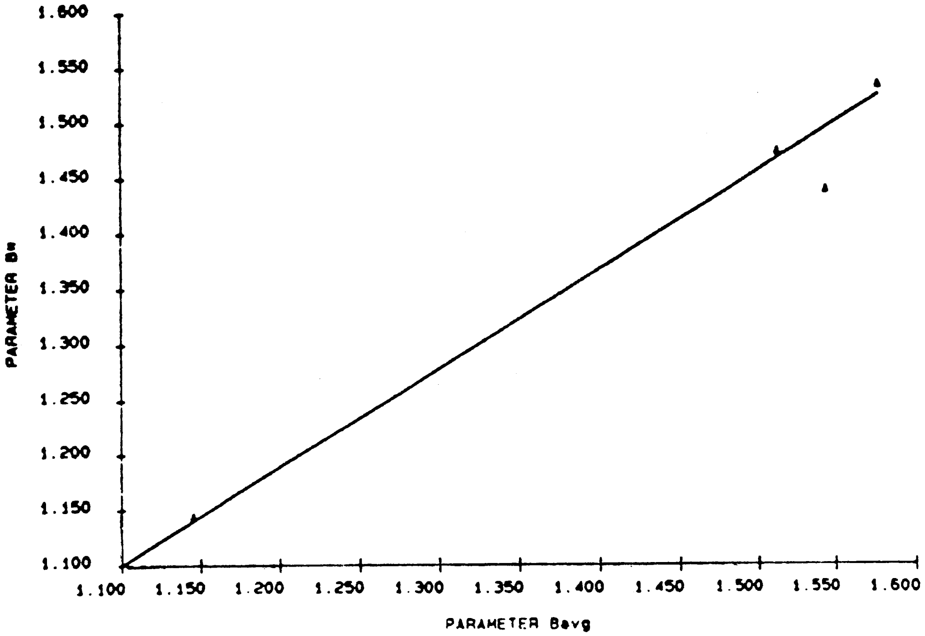

Figure 18. Plot of Extracted Versus Average Values of B Prameter.

CHAPTER VI

SUMMARY AND RECOMMENDATIONS

Summary

The hydraulic model proposed by Van Genuchten (1980) was used to describe the relationships between the hydraulic properties of three soil series. The model parameters were estimated using a nonlinear least squares fitting procedure. Analyzing the random variability of the parameters obtained from fitting 168 sets of water content-matric potential data, it was found to be best described by a lognormal type of distribution.

The cumulative inflow was then computed for 1000 sets of generated bivariate parameters for each soil. A lognoramal distribution was also found to well describe the cumulative inflow variability.

Considering the flow parameters as random variables yields flow values different from those obtained using averaged parameters. The absolute difference in flow was found to be increasing as the simulation time increased.

Considering five different boundary conditions at the upper surface of the soil profile, it has been found that parameters giving values of the cumulative inflow around the mean converge in the A, B plane. This region of convergence in the solution is distinctly different from the average point of parameters.

Recommendations

Although an attempt to relate the parameters A* and B* extracted from the convergence region to the averaged ones was made in this study, higher number of samples should be considered for a more accurate estimation of this relationship. A more complicated study can be conducted where the remaining parameters incorporated in the function describing the soil-water characteristic curve will be considered random variables as well.

BIBLIOGRAPHY

Abramowitz M. and Stegun I. A., 1972. Handbook of Mathematical Functions, Dover Publications, N. Y. 1046 pp.

Andersson J. and Shapiro A. M., 1983. Stochastic analysis of one-dimentional steady state unsaturated flow: a comparison of Monte Carlo and Perturbation methods, Water Res. Res., 15(4):867-872.

Ahuja L. R., Green R. E., Chong S. K., and Nielsen D. R., 1980. A simplified functions approach for determining soil hydraulic conductivities and water characteristics in situ, Water Res. Res., 16(5):947-953.

Babu D. K. and Van Genuchten M. T., 1979. An initial value problem for the horizontal infiltration of water, Water Res. Res., 15(4):867-872.

Boersma L., 1972. Soil water, ASA, SSSA, Madison, Wisconsin, 175 pp.

Bresler E. and Dagan G.,1983. Unsaturated flow in spatially variable fields, 2. Application of water flow models to various fields, Water Res. Res., 19(2):421-428.

Buckingham E., 1907. Studies on the movement of soil moisture, Bull. 38, U. S. D. A., Bur. Soils, Washington, D. C.

Campbell G. S., 1985. Soil physics with Basic, Development in Soil Science 14, Elsevier, New York, 150 pp.

Cosby B. J., Hornberger G. M., Clapp R. B., and Ginn T. R., 1984. A statistical exploration of the relationships of soil moisture characteristics to physical properties of soils, Water Res. Res., 20(6):682-690.

Dagan G. and Bresler E., 1983. Unsaturated flow in spatially variable fields, 1. Derivation of models of infiltration and redistribution, Water Res. Res., 19(2):413-420.

El-Kadi A., 1985. On estimating the hydraulic properties of soil, 1. Comparison between forms to estimate the water characteristic function, Adv. Water Resources, 8(3):136-147.

Fok Yu-Si, 1987. Evolution of algebraic infiltration equations, Proc. Int. Conf. on Infiltration Development and Application, p. 38-49, Jan. 1987, Honolulu, Hawaii.

Freeze A., 1969. The mechanism of natural ground-water recharge and discharge, 1. One-dimentional vertical unsteady unsaturated flow above a recharging or discharging ground-water flow system, Water Res. Res., 5(1):153-171.

Greminger P. J., Sud Y. K., and Nielsen D. R., 1985. Spatial variability of field-measured soil-water characteristics, Soil Sci. Soc. Am. J., 49(5):1075-1082.

Gupta R. P., 1972. Theory of unsaturated flow of water through soils, Advanced Soil Physics, Proc. Summer Institute in soil physics, May-June 1968, pp. 107-113, Council of Ag. Res., New Delhi.

Haan C. T., 1977. Statistical methods in hydrology, Iowa State Univ. Press, Ames, Iowa, 378 pp.

Haan C. T., 1987. The effect of infiltration parameter uncertainty on runoff volume estimates, Proc. Int. Conf. on infiltration develop. and application, 6-9 Jan. 1987, Honolulu, Hawaii, pp. 262-271.

Haverkamp R., Vauclin M., Touma J., Wierenga P. J., and Vachaud G., 1977. A comparison of numerical simulation models for one-dimentional infiltration, Soil Sci. Soc. Am. J., 41:285-294.

Haverkamp R. and Vauclin M., 1981. A comparative study of three forms of the Richards equation used for predicting one-dimentional infiltration in unsaturated soil, Soil Sci. Soc. Am. J., 45:13-20.

Hillel D., 1971. Soil and water, Academic Press, New York, 288 pp.

Hillel D., 1977. Computer simulation of soil-water dynamics, International Development Research Centre, Ottawa, Canada, 224 pp.

Hubert A., 1978. Relation entre la conductivité hydraulique et la structure des milieux poreux, Ann. Agronomique, 29(5):475-498.

Jones A. J. and Wagenet R. J., 1984. In situ estimation of hydraulic conductivity using simplified methods, Water Res. Res., 20(11):1620-1626.

Khaleel R. and Yeh T. C., 1985. A Galerkin finite-element program for simulating unsaturated flow in porous media, Ground Water, 23(1):90-96.

Kirby J. M., 1985. A note on the use of a simple numerical model for vertical unsaturated fluid flow, Soil Science, 139(5):462-467.

Klute A., 1973. Soil water flow theory and its application in field situation, Field Soil Water Regime, SSSA Special Publication No. 5, pp. 9-35, Madison, Wisconsin.

Kool J. B., Parker J. C., and Van Genuchten M. T., 1985. Determining soil hydraulic properties from one-step outflow experiment by parameter estimation: I. Theory and numerical studies, Soil Sci. Soc. Am. J., 49:1348-1354.

Matalas N. C., 1967. Mathematical Assessment of Synthetic Hydrology, Water Res. Res., 3(4):937-945.

Morel-Seytoux H. J. and Vauclin M., 1983. Superiority of two-phase formulation for infiltration, Proc. Nat. Conf. on Advances in Infiltration, Dec. 12-13, 1983, Chicago, Illinois, ASAE Publication 11-83, pp. 34-47.

Morel-Seytoux H. J. and Billica J. A., 1985. A two-phase numerical model for prediction of infiltration: applications to a semi-infinite soil column, Water Res. Res., 21(4):607-615.

Naney J. W., Ahuja L. R., and Barnes B. B., 1983. Variability and interrelation of soil-water and some related properties in small watershed, Proc. of the Nat. Conf. on Advances in Infiltration, Dec. 12-13, 1983, Chicago, Illinois, ASAE Publication 11-83, pp. 92-101.

Nofziger D. L., Williams J. R., Hornsby A. G., and Wood A. L., 1983. Physical characteristics of soils of the Southern Region - Bethany, Konawa, and Tipton series, Southern Cooperative Series, Bulletin No. 265, Nov. 1983.

Nofziger D. L., 1985. Interactive simulation of one-dimentional water movement in soils: User's Guide, University of Florida, Institute of Food and Agricultural sciences, Gainesville, Florida, 56 pp.

Nofziger D. L., 1988. Personal Program, Agronomy Department, Oklahoma State University, Stillwater.

Philip J. R., 1980. Field heterogeneity: some basic issues, Water Res. Res., 16(2):443-448.

Philip J. R., 1984. Aspect of quasilinear infiltration from surface sources especially the case alpha=0, Water Res. Res., 20(5):633-635.

Philip J. R., 1986. Steady infiltration from spheroidal cavities in isotopic and anisotropic soils, Water Res. Res., 22(13):1874-1880.

Richie E. B., Schnabel R. R., and Hoover J. R., 1987. Estimation of soil-water parameters from unsteady-state flow, Proc. Int. Conf. on Infiltration Development and Application, 6-9 Jan. 1987, Honolulu, Hawaii, pp. 377-387.

Russo D. and Bresler E., 1980. Scaling soil hydraulic properties of a heterogeneous field, Soil Sci. Soc. Am. J., 44:681-684.

Russo D. and Bresler E., 1980. Field determination of soil hydraulic properties for statistical analyses, Soil Sci. Soc. Am. J., 44:697-702.

Russo D. and Bresler E., 1981. Soil hydraulic properties as stochastic processes: I. An analysis of field spatial variability, Soil Sci. Soc. Am. J., 45:682-687.

Russo D. and Bresler E., 1982. A univariate versus a multivariate parameter distribution in a stochastic-conceptual analysis of unsaturated flow, Water Res. Res., 18(3):483-488.

Sharma M. L., Barron R. J. W., and DeBoer E. S., 1983. Spatial structure and variability of infiltration parameters, Advances in infiltration, Proc. Nat. Conf. on Advances in Infiltration, Dec. 12-13, 1983, Chicago, Illinois, ASAE Publication 11-83, pp. 113-121.

Sharma M. L., Luxmoore R. J., DeAngelis R., Ward R. C., and Yeh G. T., 1987. Subsurface water flow simulated for hillslopes with spatially dependent soil hydraulic characteristics, Water Res. Res., 23(8):1523-1530.

Smith R. E. and Hebbert R. H. B., 1979. A Monte Carlo analysis of hydrologic effects of spatial variability of infiltration, Water Res. Res., 15(2):419-429.

Sposito G. and Jury W. A., 1985. Inspectional analysis in the theory of water flow through unsaturated soil, Soil Sci. Soc. Am. J., 49(4):791-798.

Sposito G., 1986. The "Physics" of soil physics, Water Res. Res., 22(9):83S-88S.

Stephens D. B., 1985. A field method to determine unsaturated hydraulic conductivity using flow nets, Water Res. Res., 21(1):45-50.

Swartzendruber D., 1987. A quasi-solution of Richards' equation for the downward infiltration of water into soil, Water Res. Res., 23(5):809-817.

Vauchaud G., Dancette C., Sonko S., and Thony J. L., 1978. Méthodes de la charactérisation hydrodynamique in situ d'un sol non saturé, Ann. Agronomique, 29(1):1-36.

Van Genuchten M. Th., 1878. Mass transport in saturated-unsaturated media: one-dimentional solutions, Water Res. Program, Princeton Univ. Techn. Rept. 78-WR-11, Princeton, N. J.

Van Genuchten M. Th., 1980. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils, Soil Sci. Soc. Am. J., 44:892-898.

Warrick A. W., Biggar J. W., and Nielsen D. R., 1971. Simultaneous solute and water transfer for unsaturated soil, Water Res. Res., 7(5):1216-1225.

Warrick A. W., 1983. Parameters in infiltration equations, Advances in Infiltration, Proc. Nat. Conf. on Advances in Infiltration, Dec. 12-13, 1983, Chicago, Illinois, ASAE Publication 11-83, pp. 69-81.

Yeh T. C. J., Gelhar L. W., and Gutjahr A. L., 1985. Stochastic analysis of unsaturated flow in heterogeneous soils, 1. Statistically isotropic media, Water Res. Res., 21(4):447-456.

Yeh T. C. J., Gelhar L. W., and Wierenga P. J., 1986. Observation of spatial variability of soil-water pressure in a field soil, Soil Sci., 142(1):7-12.

Zachmann D. W., DuChateau P. C., and Klute A., 1981. The calibration of the Richards flow equation for a draining column by parameter identification, Soil Sci. Soc. Am. J., 45:1012-1015.

APPENDIXES

APPENDIX A

SITE AND PROFILE DESCRIPTIONS OF SOILS

Site and Profile Descriptions for Bethany Soil

Sites 1, 2, and 3

Location: 479.9 m north and 192.9 m west of SE corner of Section 16, t 19 N., R. 2E, Payne county, Oklahoma.

Classification: Pachic Argiustoll, fine, mixed, thermic.

Topography: Slightly concave, nearly level.

Vegetation: Wheat, experiment station.

Soil profile: See table 25.

Described by: Earl C. Nance and Joe Williams.

Site 4

Location: 21.3 m west and 416 m south of the northeast corner of Section 11, T. 19N, R. 1E, Payne county, OK.

Classification: Pachic Paleustoll, fine, mixed, thermic.

Topography: Slightly concave, nearly level.

Vegetation: Wheat.

Soil profile: See table 26.

Described by: Tom Reinsch.

Site 5

Location: 134.9 m west and 155.4 m south of the northeast corner of SE 1/4, NE 1/4 of section 5, T.

TABLE 25

PROFILE DESCRIPTION FOR BETHANY SOIL SITE 1, 2, AND 3

Horizon	Depth	Description
A1	0-25 cm	Very dark brown (7.5YR 2.5/2) silt loam weak medium subangular blocky breaking to weak fine and medium granular structure; friable when moist; medium acid; abrupt boundary.
B1t	25-33 cm	Very dark brown (7.5YR 2.5/2) heavy silty clay loam;strong coarse subangular blocky structure; firm when moist; clay films on ped surfaces; few fine black bodies; slightly acid; clear boundary.
B21t	33-71 cm	Dark brown (7.5YR 2.5/2) moist silty clay; moderate coarse prismatic breaking to very fine blocky structure; very firm when moist; clay films on ped surfaces; few very fine black bodies and cocretions; slightly acid in upper part and neutral in lower part; roots are mainly on ped surfaces; some evidence of high shrink swell with coatings of less clayey textures on some vertical faces; clear boundary.

TABLE 26

PROFILE DESCRIPTION FOR BETHANY SOIL SITE 4

Horizon	Depth	Description
Ap	0-23 cm	Dark brown (7.5YR3/2); loam; moderate medium subangular-blocky breaking to granular; friable when moist; many roots; many fine and medium pores; clear boundary
B1t	23-58 cm	Dark brown (7.5YR3/3); clay loam; moderate medium subangular blocky breaking to granular; slightly firm; many fine roots; many fine pores; clay film on ped faces; gradual boundary.
B21t	58-90 cm	Dark brown (7.5YR3/3); clay; moderate strong angular blocky; exremely firm; clay films on ped surfaces; many fine pores; clear boundary.

19N, R. 2E, Payne county, Oklahoma.

Classification: Pachic Paleustoll, fine, mixed, thermic.

Topography: Slightly concave, nearly level.

Vegetation: Wheat.

Soil profile: See table 27.

Described by: Tom Reinsch.

Site 6

Location: 158.5 m west and 307.8 m north of the SE corner of SW 1/4, SE 1/4, Sec. 4, T. 12N, R. 8W, Canadian county, Oklahoma.

Classification: Pachic Paleustoll, fine, mixed, thermic.

Topography: Convex, 3% slope.

Vegetation: Native grass.

Soil profile: See table 28.

Described by: Bob Bourlier.

Site and Profile Descriptions for Konawa Soil

Site 1

Location: 762 m feet south and 579.1 m feet west of the NE corner Sec. 36 T18N R2E, Payne County, Oklahoma.

Classification: Fine loamy, mixed, thermic, Ultic Haplustalf.

Physiographic Position: High Terrace summit view level.

Topography: Very gently slopping 2% slope.

Vegetation: Idle Cool season annuals.

TABLE 27

PROFILE DESCRIPTION FOR BETHANY SOIL
SITE 5

Horizon	Depth	Description
Ap	0-20 cm	Dark brown (7.5YR 3/2); silt loam; moderate medium subangular blocky breaking to angular; many commom pores; friable when moist; clear boundary.
B1t	20-46 cm	Dark reddish brown (5YR 3/3); silty clay; coarse medium prismatic breaking to moderate medium angular blocky; firm when moist; clay films on ped surfaces; gradual boundary.
B21t	46-81 cm	Dark reddish brown (5YR 3/3); silty clay; coarse medium prismatic breaking to moderate medium angular blocky; firm when moist; clay films on ped surfaces; many fine random root orientation; black bodies; gradual boundary

TABLE 28

PROFILE DESCRIPTION FOR BETHANY SOIL SITE 6

Horizon	Depth	Description
A11	0-18 cm	Very dark gray (10YR 3/1) silt loam dark grayish brown (10YR 4/2) dry; weak coarse platy breaking to moderate medium granular structure; hard; friable; many fine roots; few worm casts; (pH 6.8) neutral; clear smooth boundary
A12	18-28 cm	Very dark grayish brown (10YR 3/2) silt loam brown to dark brown (10YR 4/3) dry moderate medium granular structure; slightly hard, friable, many fine roots, few worm casts; (pH 6.5) slightly acid; clear smooth boundary.
B1	28-46 cm	Dark brown (7.5YR 3/2) silty clay loam, brown to dark brown (10YR 4/3) dry; moderate fine subangular blocky breaking to moderate medium granular structure; hard, firm; many fine roots; patchy clay flims; about 1% quartz gravel by volume 2 mm to 76 mm in diameters; (pH 6.8) neutral; clear smooth boundary.

Parent materials: Old Alluvium (Pleistocene).

Soil profile: See table 29.

Described by: Jim Frie and Jim Henley.

Site 2

Location: 731.5 m feet West and 281.9 m feet North of the SE corner Sec. 36 T18N R2E, Payne county, Oklahoma.

Classification: Fine loamy, mixed, thermic, Ultic Haplustalf.

Physiographic position: High Terrace, Perkins level.

Topography: Very gently sloping 2% slope.

Vegetation: Bermuda pasture (low condition).

Parent material: Old Alluvium (pleistocene).

Soil profile: See table 30.

Described by: Jim Frie and Jim Henley.

Site 3

Location: South side of north study site 411.5 m S. 7.6 m W. of NE corner of Sec. 10, T. 4N., R. 3E. on O. S. U. Agronomy Research Station, Stratford, OK, Garvin County.

Classification: Fine, loamy, mixed, thermic Ultic Haplustalf.

Topography: Upslope portion of terraced hillside with slope of 3-5%.

Vegetation: Fallow for last 2 years, previously in peanuts.

Soil profile: See table 31.

TABLE 29

PROFILE DESCRIPTION FOR KONAWA SOIL
SITE 1

Horizon	Depth	Description
AP	0-30 cm	Brown (7.5YR 4/4) fine sandy loam; weak fine granular structure,very friable slightly hard; many fine roots; few small bodies of B2t material randomly mixed;very strongly acid; abrupt smooth boundary.
B21t	30-76 cm	Dark reddish brown (5YR 3/1) sandy clay loam. Dark reddish brown (5YR 3/3) ped faces; moderate medium prismatic structure ; friable; very hard; many fine roots; within near continuous clay film; slightly acid; gradual wavey boundary.
B22t	76-99 cm	Yellow red (5YR 4/6) fine sandy loam; reddish brown (5YR 4/4) ped faces; moderate coarse prismatic structure; very friable, few fine faint yellowish red mottles; very thin near continuous clay film on ped faces; neutral; gradual wavy boundary.

TABLE 30

PROFILE DESCRIPTION FOR KONAWA SOIL SITE 2

Horizon	Depth	Description
A1	0-17 cm	Brown (7.5YR 4/4) fine sandy loam; weak fine granular structure,very friable slightly hard; many fine roots; neutral clear smooth boundary.
A2	17-26 cm	Brown 7.5YR 5/4), loamy fine sand; weak very fine granular structure; very friable, slightly hard; many fine roots slightly acid; abrupt smooth boundary.
B21t	26-68 cm	Yellowish red (5YR 4/6) sandy clay loam moderate medium prismatic structure; very hard, friable; common fine roots; thin near continuous clay film on ped faces; neutral; gradual smooth boundary.
B22tb	68-102cm	Strong brown (7.5YR 4/6) sandy clay loam; moderate medium prismatic structure; very hard; friable; few fine roots; thin near continuous clay film on ped faces; few fine distinct reddish brown and strong brown mottles; neutral; gradual smooth boundary.

TABLE 31

PROFILE DESCRIPTION FOR KONAWA SOIL
SITE 3

Horizon	Depth	Description
AP	0-23 cm	Dark grayish brown (10YR 5/3) loamy fine sand grayish brown (10YR 4/3) moist; weak fine and medium granular structure; soft, very friable slightly acid; clear smooth boundary.
A2	23-36 cm	Light yellowish brown (10YR 6/4) loamy fine sand, yellowish brown (10YR 5/4) moist; weak fine granular structure; soft, very friable; neutral, clear smooth boundary.
B21t	36-53 cm	Yellowish red (5YR 4/6) sandy clay loam yellowish red (5YR 4/6) moist; common fine and medium distinct red (2.5YR 5/6) mottles; moderate medium subangular structure; hard, firm; clay films on ped faces and bridging sand grains; common medium and fine roots; neutral, gradual smooth boundary.
B22t	53-89 cm	Red(2.5YR 4/6) sandy clay loam, dark red (2.5YR 3/6) moist; common fine sand medium distinct yellowish red (5YR 5/6) mottles in upper part; moderate coarse prismatic structure parting to weak medium subangular structure; very hard; firm;clay films on ped faces and bridging sand grains; common medium and fine roots; common worm cast; slightly acid; gradual smooth boundary.

Described by: Vinson Bougard and Larry E. Kichler.

Site 4

Location: North side of south study site 403.9 m S. and 6.1 m W. of NE corner of Sec. 10, T. 4N., R. 3E. Agronomy Research Station, Stratford, Oklahoma, Garvin County.

Classification: Fine loamy, mixed, thermic Ultic Haplustalfs.

Topography: Upslope portion of terraced hillside with slope of 3-5%.

Vegetation: Fallow for last 2 years, previously in peanuts.

Soil profile: See table 32.

Described by: Vinson Bogard and Larry E. Kichler.

Site and Profile Descriptions for Tipton Soil

Site 1

Location: 31 m East and 169 m South of the northwest corner of section 32, T. 1S., R. 18W., Tillman County, OK.

Classification: fine loamy, mixed, thermic, Pachic Argiustoll.

Topography: Slightly concave, nearly level.

Vegetation: Wheat and cotton.

Soil profile: See table 33.

Described by: Earl C. Nance and Tom Reinsch.

TABLE 32

PROFILE DESCRIPTION FOR KONAWA SOIL
SITE 4

Horizon	Depth	Description
Ap	0-23 cm	Dark brown (7.5YR3/2); loam; moderate medium subangular-blocky breaking to granular; friable when moist; many roots; many fine and medium pores; clear boundary
B1t	23-58 cm	Dark brown (7.5YR3/3); clay loam; moderate medium subangular blocky breaking to granular; slightly firm; many fine roots; many fine pores; clay film on ped faces; gradual boundary.
B21t	58-90 cm	Dark brown (7.5YR3/3); clay; moderate strong angular blocky; exremely firm; clay films on ped surfaces; many fine pores; clear boundary.

TABLE 33

PROFILE DESCRIPTION FOR TIPTON SOIL
SITE 1

Horizon	Depth	Description
AP	0-22 cm	Dark brown (7.5YR 3/3) moist; loam; weak fine granular structure; friable; few fine and medium random pores; few fine roots; slightly acid; clear boundary.
A12	22-59 cm	Dark brown (7.5YR 3/2) moist; loam; weak medium subangular blocky breaking to moderate medium and fine granular structure; friable; few earthworm casts many medium vertical pores; few roots slightly acid; clear boundary.
B21t	59-72 cm	Dark reddish brown (5YR 3/3)moist; loam weak coarse prismatic breaking to weak medium subangular blocky structure; friable;thin clay films on ped surfaces and coating sand grains; many medium and fine vertical pores; few fine roots few earthworm casts; neutral; gradual boundary.

Site 2

Location: 8.2 m South and 326.4 m West of the northeast corner of the southeast 1/4 of Sec. 15T. 1S. R. 19W, Tillman County, OK.

Classification: Fine loamy, mixed, thermic, Pachic Argiustolls.

Topography: Linear slope, nearly level.

Vegetation: Cotton - research station.

Soil profile: See table 34.

Described by: Earl C. Nance and Tom Reich.

Site 3

Location: 76.8 m South and 128.9 m West of the northeast corner of the southeast 1/4 of Sec. 25, T. 1S., R. 19W., Tillman County, OK.

Classification: Fine loamy, mixed, thermic, Pachic Argiustolls.

Topography: Linear slope, nearly level.

Vegetation: Sorghum, cotton - research station.

Soil profile: See table 35.

Described by: Earl C. Nance and Ton Reinsch.

TABLE 34

PROFILE DESCRIPTION FOR TIPTON SOIL SITE 2

Horizon	Depth	Description
AP	0-25 cm	Dark brown (7.5YR 3/3) moist; loam; weak fine granular structure; friable; few fine and medium random pores; few fine roots; neutral; clear boundary.
A12	25-60 cm	Dark brown (7.5YR 3/2) moist; loam; weak medium subangular blocky breaking to moderate medium and fine granular structure; friable; few earthworm casts many medium vertical pores; few roots neutral; gradual boundary.
B21t	60-103cm	Reddish brown (5YR 4/3) moist; loam; weak coarse prismatic breaking to weak medium subangular blocky structure; friable;thin clay films on ped surfaces and coating sand grains; many medium and fine vertical pores; few fine roots few earthworm casts; mildly alkaline; few fine CaCO3 concretions; clear boundary.

TABLE 35

PROFILE DESCRIPTION FOR TIPTON SOIL SITE 3

Horizon	Depth	Description
Ap	0-21 cm	Dark brown (7.5YR 3/2) moist; loam; weak fine granular structure; friable; few fine random pores; few fine roots; mildly alkaline; clear boundary.
A12	21-46 cm	Dark brown (7.5YR 3/2) moist; loam; weak coarse subangular blocky breaking to moderate medium and fine granular structure; friable; few earthworm casts many medium vertical pores; few fine roots; moderately alkaline; gradual boundary.
B21t	46-99 cm	Dark reddish brown (5YR 3/3) moist upper; and (5YR 3/4) moist lower; loam; moderate medium prismatic breaking to moderate medium subangular blocky structure; friable; many fine random pores;few fine roots; few earthworm casts; few threads mycelia carbonates; thin clay films on ped surfaces; moderately alkaline; gradual boundary.

APPENDIX B

BIVAR.BAS COMPUTER PROGRAM FOR GENERATION OF BIVARIATE, CORRELATED, LOGNORMALLY DISTRIBUTED VARIABLES

```
10 '******************************************************************
20 '*                                                                *
30 '*  GENERATION OF BIVARIATE. CORRELATED. LOGNORMALLY DISTRIBUTED   *
40 '*                    RANDOM VARIABLES                            *
50 '*                                                                *
60 '******************************************************************
70 'N NUMBER OF OBSERVATIONS TO BE GENERATED
80 'MEAN1. MEAN2 MEANS OF VARIABLES 1 AND 2 RESPECTIVELY
90 'SD1. SD2 STANDARD DEVIATIONS OF VARIABLES 1 AND 2 RESPECTIVELY
100 'R CORRELATION COEFFICIENT BETWEEN THE TWO VARIABLES
110 DIM XRND(200), X(2.200), Z(2.200).V1(200), V2(200)
120 PRINT"INPUT THE MEANS OF VARIABLES 1. 2 (ORIGINAL DATA)"
130 INPUT MEAN1. MEAN2
140 PRINT "INPUT THE STANDARD DEVIATIONS SD1.SD2  (ORIGINAL DATA)"
150 INPUT SD1.SD2
160 PRINT"INPUT THE CORRELATION COEFFICIENT"
170 INPUT R
180 SD1=LOG((SD1/MEAN1)^2+1)
190 SD1=SQR(SD1)
200 SD2=LOG((SD2/MEAN2)^2+1)
210 SD2=SQR(SD2)
220 MEAN1=LOG(MEAN1)-SD1^2/2
230 MEAN2=LOG(MEAN2)-SD2^2/2
240 A1=EXP(SD1^2)-1
250 A2=EXP(SD2^2)-1
260 A3=SD1*SD2
270 R=LOG(1+R*SQR(A1*A2))/A3
280 PRINT"INPUT THE NUMBER OF OBSERVATIONS TO BE GENERATED"
290 INPUT N
300 S1=0
310 S2=0
320 SS1=0
330 SS2=0
340 SS3=0
350 REM eigenvalues
360 L1=1+R
370 L2=1-R
380 REM The A matrix
390 A(1.1)=1/SQR(2)
400 A(2,1)=A(1.1)
410 A(1,2)=A(1.1)
420 A(2.2)=-A(1,1)
430 REM Generation of Z values
440 MEAN=0
450 SD=SQR(L1)
460 GOSUB 1020
470 FOR I=1 TO N
480 Z(1.I)=XRND(I)
490 NEXT I
500 SD=SQR(L2)
510 GOSUB 1000
520 FOR I=1 TO N
530 Z(2.I)=XRND(I)
540 NEXT I
550 REM Transformation to X values
560 FOR I=1 TO N
570 X(1.I)=Z(1.I)*A(1.1)+Z(2.I)*A(1.2)
580 V1(I)=EXP(X(1.I)*SD1+MEAN1)
590 X(2.I)=Z(1.I)*A(1.2)+Z(2.I)*A(2.2)
600 V2(I)=EXP(X(2.I)*SD2+MEAN2)
610 NEXT I
```

```
620 PRINT "ENTER DISK FILE NAME TO STORE GENERATED DATA"
630 INPUT F$
640 OPEN "O", #1, F$
650 FOR I=1 TO N
660 PRINT #1, I, V1(I), V2(I)
670 NEXT I
680 CLOSE #1
690 REM Means and Standard deviations of generated obs.
700 FOR I=1 TO N
710 S1=S1+V1(I)
720 SS1=SS1+(V1(I))^2
730 S2=S2+V2(I)
740 SS2=SS2+(V2(I))^2
750 SS3=SS3+V1(I)*V2(I)
760 NEXT I
770 M1=S1/N
780 M2=S2/N
790 VAR1=(SS1-(S1^2)/N)/(N-1)
800 COV=(SS3-N*M1*M2)/(N-1)
810 VAR2=(SS2-(S2^2)/N)/(N-1)
820 CORR=COV/SQR(VAR1*VAR2)
830 LPRINT"FOR  "::LPRINT N::LPRINT"  GENERATED OBS."
840 LPRINT"             ***ALPHA***  *** n ***"
850 LPRINT"    MEAN:"::LPRINT USING"     ##.####":M1,M2
860 LPRINT"STD. DEV:"::LPRINT USING"     ##.####":SQR(VAR1),SQR(VAR2)
870 LPRINT "Correlation: ":CORR
880 PRINT "WANT TO GENERATE MORE OBSERVATIONS "
890 PRINT "FOR SAME SAMPLE ? Y or N ?"
900 INPUT R$
910 IF R$="Y" OR R$="y" THEN 280
920 IF R$="N" OR R$="n" THEN 940
930 BEEP:GOTO 880
940 PRINT "FOR OTHER SAMPLES ?"
950 INPUT A$
960 IF A$="Y" OR A$="y" THEN 120
970 IF A$="N" OR A$="n" THEN 990
980 BEEP:GOTO 940
990 END
1000 '******************************************************************
1010 '*                                                                *
1020 '*          SUBROUTINE: NORMAL DISTRIBUTION GENERATOR             *
1030 '*                                                                *
1040 '******************************************************************
1050 RANDOMIZE TIMER
1060 FOR I= 1 TO N
1070 IF NRN=1 THEN 1180
1080 R1=2*RND-1
1090 R2=2*RND-1
1100 S=R1^2+R2^2
1110 IF S>=1 THEN 1080
1120 RNN1= R1*SQR((-2*LOG(S))/S)
1130 RNN2= R2*SQR((-2*LOG(S))/S)
1140 XRND(I)=MEAN+RNN1*SD
1150 NRN=NRN+1
1160 IF I>=N THEN 1220
1170 GOTO 1210
1180 XRND(I)=MEAN+RNN2*SD
1190 NRN=0
1200 IF I>=N THEN 1220
1210 NEXT I
1220 RETURN
```

APPENDIX C

SUPPLEMENTARY DATA

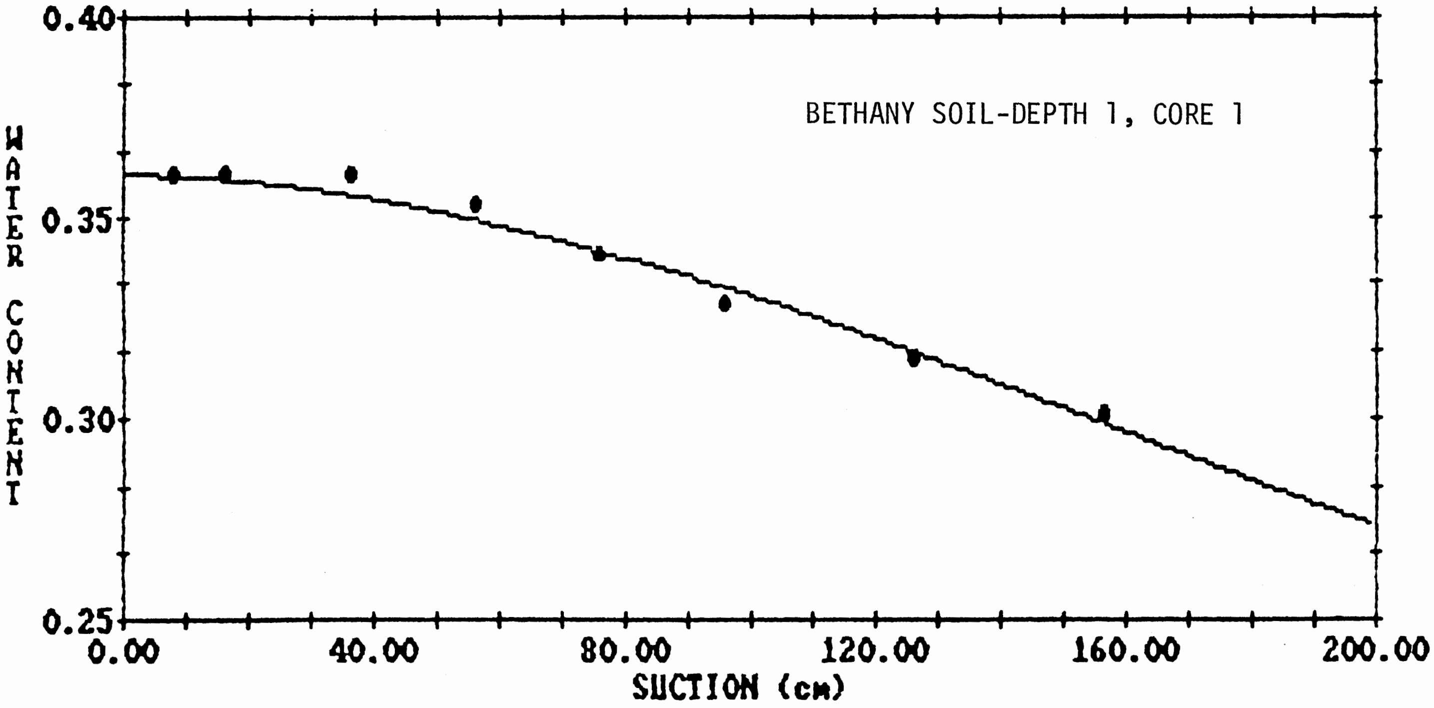

Figure 19. Typical Fit of Equation 5 to |h| and WC Data

VITA

Fethi Ben Jemaa

Candidate for the Degree of

Master of Science

Thesis: IMPACT OF PARAMETER UNCERTAINTY ON ONE-DIMENSIONAL VERTICAL UNSATURATED FLOW OF WATER

Major Field: Agricultural Engineering

Biographical:

Personal Data: Born in Douiret, Tataouine, Tunisia, on March 15, 1963, the son of Ahmed BenJemaa and Khadija Zanzana.

Education: Graduated from Lycée Secondaire Mixte de Tataouine, Tunisia, in 1981; received Engineering Diploma in 1986 from National Agronomical Institute of Tunis; completed the requirements for the Master Degree at Oklahoma State University in July, 1988.

Member: Alpha Epsilon Honor Society.