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CHAPTER I

BACKGROUND OF THE PROBLEM

Introduction

The proper placement of freshman students in college mathematics courses presents a problem which is becoming of increasing importance. In part this is due to the fact that the mathematical backgrounds of high school graduates who now enter college are extremely varied. Less emphasis is being placed on mathematics in the high schools now than formerly, with the result that many students, if they are to succeed in college mathematics, must be given work of a sub-college nature. In order not to penalize those students whose high school preparation is adequate for college mathematics, some means of proper placement of students is necessary. The number and type of mathematics courses appearing on the student's high school transcript do not form adequate criteria because in too many instances a passing mark in high school mathematics does not represent mastery of the minimum essentials necessary to success in college mathematics. While this problem of proper placement exists in other fields also, it is of particular importance in the field of mathematics because of the sequential nature of mathematics courses.

Description of Freshman Mathematics Courses at the University of Oklahoma

In order to look at the problem as it occurs at the University of Oklahoma, a brief survey is needed of the introductory mathematics courses. Beginning students most frequently choose from among remedial mathematics, intermediate algebra, and mathematical analysis I. These and other freshman mathematics courses are described briefly in the following paragraphs.

Mathematics R,[1] or remedial mathematics, is offered to those students whose preparation is very inadequate. At the present time this course meets three hours per week and carries one hour of college credit. Its content consists primarily of the fundamentals of arithmetic and very elementary algebra.

Mathematics 2, or intermediate algebra, is a three-hour course which is approximately the equivalent of the third semester course in high school algebra. It is offered to students who enter college with only one unit of high school algebra. However, students who have had three semesters of high school algebra are permitted to take mathematics 2 and receive college credit for it.

Mathematics 21, or mathematical analysis I, which carries five hours credit, is the first half of an integrated course which covers much of the material usually presented in the standard sequence of college freshman mathematics courses. Mathematics 2 is a prerequisite for this course.

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[1]Designated in the University catalog as mathematics A. The letter R will be used in referring to this course, to avoid any possible confusion with the letter grade A.

Other courses which are concerned to a smaller degree in this study are the following: Mathematics 1 is the equivalent of the usual high school course in plane geometry and is offered to students who enter college without high school credit in plane geometry. This course meets five hours per week for three hours credit. Mathematics 4 is a course in elementary algebra for business students. This course has since been discontinued. Mathematics 5 is the usual standard three-hour course in college algebra. Mathematics 6 is a three-hour course in plane trigonometry, and mathematics 7 is a two-hour course in solid geometry which is offered to students who enter college without high school credit in solid geometry.

Past Practice at the University of Oklahoma

At the Universitv of Oklahoma various methods have been used by the department of mathematics to meet the problem of proper placement of students. At one time students were placed in mathematics courses on a temporary basis for a period of approximately two weeks. At the end of this period of time they were given a test, the results of which were used to determine whether a student should remain in the course in which he had enrolled, or would profit more by changing to either the next higher course or the next lower course in the sequence. Such changes in enrollment were not made compulsory, but in nearly all instances students followed the advice given them.

Later it became the practice to give all entering freshmen a placement test in mathematics previous to their enrollment. These tests were graded by members of the department of mathematics and the advice given students upon their enrollment was based at least in part on their performance on this test. When increased enrollment made this system too cumbersome, a placement test was devised which could be scored by machine and some of the detail work involved in the administration and scoring of the test was handled by the personnel of the guidance center and the office of machine accounting.

Current Practice at the University of Oklahoma

Previous to the time of enrollment each beginning freshman is given a group of tests including a mathematics placement test, the Ohio State Psychological Examination (abbreviated O S P E), and the Iowa High School Content Examination (abbreviated I H S C). The mathematics placement test which is now in use comprises forty multiple-choice items each of which has five possible responses. This test is machine scored and each student's score on the test is made available to his adviser at the time of enrollment. For each student, the number of correct responses on the test is printed on an adviser's card, along with information on the student's high school record and his decile rankings on the O S P E and the I H S C. Decile rankings are listed for both the reading section and the total score on the O S P E. Decile rankings on the I H S C are listed for the total or composite score as well as for each of the four sections of the test; namely, English, mathematics, science, and history.

Any student whose curriculum does not include further courses in mathematics may ignore the mathematics placement test score. Thus, credit in intermediate algebra taken in high school is acceptable in spite of a score of zero on the mathematics placement test.

Each semester the mathematics department recommends a grouping of potential mathematics students into three brackets, based on cumulative experience and the number of sections of each course offered. For example, in the fall semester of 1952 the recommended placement was as follows: Students who scored ten or fewer correct items on the placement test were advised to take remedial mathematics. Students who scored between eleven and twenty correct items, inclusive, were advised to enroll in mathematics 2. Students who scored above twenty correct items were free to enroll in any mathematics course for which they had the prerequisites. For those students in this group who had high school credit in intermediate algebra, the course usually enrolled in was either mathematics 21 or mathematics 5.

In spite of these procedures which attempt to place students in the proper mathematics course, those who are concerned with the placement of freshman students are not sure that the methods used are as effective as possible. Whenever a student fails a mathematics course other than remedial mathematics, the question is asked, "Could this failure have been avoided by starting the student at a lower point in the sequence of mathematics courses?" From the departmental point of view the experience of one semester might suggest changing the separation points between remedial mathematics and mathematics 2, or between mathematics 2 and mathematics 21. Some advisers ignore the score made by the student on the mathematics placement test and enroll him on the basis of other available information. Students who are improperly placed either fail in their first mathematics course if placed too high, or they are needlessly delayed in their collegiate progress if placed too low. The advisers in general and the staff of the mathematics department in particular are anxious that the whole placement procedure be studied with the hope of making it more effective.

Definition of Proper Placement

In selecting a point at which to begin this study, a working definition of proper placement must first be given. Failures are often cited as examples of improper placement, but it is well known that improper application after enrollment, illness, or poor teacher-student relationship may also be contributing factors to failure. Likewise no student whose average grade is D would ever graduate, or for that matter be accepted by any degree-granting college. Thus a grade of D represents improper progress and may be an indication of improper placement. In the engineering school at the University of Oklahoma a student who makes a grade of D in a mathematics course is required to repeat the course, even though the grade of D is accepted as a passing grade by the mathematics department.

At the other extreme, for a student to receive a course grade of A, particularly in remedial mathematics, may be an indication of improper placement. Often teachers will urge such a student to audit the next higher course, with a view to avoiding unnecessary delay in his progress. Yet it would be foolhardy to argue that all students who receive a grade of A in a given course are misplaced, especially since some teachers would be inclined to give a grade of A to the "best in the class" as an incentive to the other members of the class. Obviously, any definition which may be made of proper placement is arbitrary. It shall be assumed in this study that students who make a final grade of B, C, or D in a given mathematics course are properly placed in that course. In setting up this definition grades are accepted at face value. No attempt is made to study comparative grading systems used by different instructors. Grades of F, D, C, B, and A are assigned, respectively, zero, one, two, three, and four grade points per semester hour.

Statement of the Problem

The larger area within which the problem of this study is defined is the proper placement of college freshmen in mathematics courses at the University of Oklahoma. The specific problem which is attacked is two-fold and may be outlined as follows:

In the first part of the study an examination is made of the various criteria which are at the disposal of advisers at the time of enrollment. Among these are the results of the mathematics placement test, information on the high school record of the student, and his decile rankings on the O S P E and the I H S C. These are examined with a view to finding relationships between each of them and the final grade made by the student in the first college mathematics course in which he is enrolled. It is hoped that the discovery of any such relationships may lead to means of improving the procedure used to place beginning freshmen in mathematics courses. This part of the study may be divided into two main categories. The first is that of finding relationships among the variables on which data are available. The second is, on the basis of the relationships found, to choose those variables which appear best to serve the purpose of proper placement of students and to determine the particular function or functions of these variables which should be used in the process of placement. The first category is subordinate to the second but it is important in order that an intelligent selection may be made of the variables to be used in the second category.

With reference to the first category, specific questions which may be asked are:

1. How do the general grade point averages and mathematics grades of mathematics students who withdrew from the University compare with similar data for those students who continued their college career?

2. How does the performance of students who withdrew from a mathematics course compare with that of students who remained in the course through the entire semester?

3. On the basis of various criteria which are available, how do students in the three major freshman mathematics groups compare with students who have taken no work in college mathematics?

With reference to the second category, it is hoped that answers to the following questions may be found:

1. Which variables, if any, could be used, either in conjunction with the placement test or as a substitute for it, as a means of improving the placement of students in mathematics courses?

2. In what way can these variables be used to best advantage as a means of separating mathematics students into remedial mathematics, mathematics 2, and mathematics 21 groups?

The second part of the study consists of a survey of the responses made to each of the forty items of the mathematics placement test by freshman students who entered the University in September, 1952.

This placement test is used to determine whether a beginning freshman student should be advised to take remedial mathematics, mathematics 2, or mathematics 21. The problem in this part of the study is to determine the relative value of the forty items of the placement test in separating students into these three groups. The importance of this problem stems from the fact that while a particular item may be an excellent one to use in an achievement test, it does not follow necessarily that it is a good item to use on a placement test, the primary purpose of which is to divide students into ability groups. The information gained from this part of the study may be used by the department of mathematics when making future revisions of the placement test.

Purposes of the Study

The purposes of this study may be enumerated as follows:

1. To provide advisers with a tool for the better placement of beginning freshman students in mathematics courses.

2. Through improved placement of students to aid in the reduction of the number of failures in freshman mathematics courses, particularly in mathematics 2.

3. To provide information to the staff of the mathematics department on the performance on the basis of various criteria of the students enrolled in remedial mathematics, mathematics 2, and mathematics 21, and to provide a comparison of the performance of these students with that of those students who do not take courses in college mathematics.

4. To aid in improving the efficiency of the mathematics

placement test by pointing out those items which should be retained and those items which should be replaced because of lack of effectiveness in separating students into ability groups in mathematics.

Overview of the Study

Chapter I presents the background of the problem of proper placement of freshman students in mathematics courses as it relates to practice at the University of Oklahoma. The major findings of other related studies are shown in Chapter II.

The remainder of the study is concerned with the proper placement of freshman mathematics students at the University of Oklahoma and with the improvement of the currently used mathematics placement test. The study is based upon the performance with respect to the mathematics placement test, the O S P E, the I H S C, and grade point averages, both in high school and in college, of the students who entered the University of Oklahoma as freshmen in September, 1952. This particular class was chosen in order to make the study as current as possible and at the same time allow ample time for the completion of at least one college mathematics course by most of the students involved.

In Chapter III are given the means and probable errors of the means of distributions of the various groups of mathematics students on the basis of the variables available. Also, distribution charts are shown which give a visual picture of the relationships between grades in college mathematics and each of the other variables. Chapter IV presents comparisons, on the basis of different variables, among the several groups of mathematics students and non-mathematics students.[1] In Chapter V are presented the Pearson product moment coefficients of correlation, between all pairs of variables, for each of the three major groups of mathematics students concerned in the study. Regression equations and discriminant functions, based upon a selection of the variables, are shown in Chapter VI. Distributions of responses to placement test items, by each of the three groups of mathematics students and by non-mathematics students, are shown in Chapter VII. In addition, measures of the discrimination value and difficulty level and biserial coefficient of correlation, for each test item, are presented in this chapter. The more important formulas used in these developments are shown for reference in Appendix I.

Summary of Chapter I

In this introductory chapter the problem with which this study is concerned has been defined. The background of the problem as it relates to freshman mathematics courses offered at the University of Oklahoma has been presented. A working definition of proper placement of students in mathematics courses has been set up and some of the aspects of the problem which are investigated in the sequel have been pointed out.

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[1]Since data for this study were gathered during the second semester of the school year 1953-54, all members of the freshman class of 1952 who had not taken at least one course in college mathematics by the end of the first semester of the year 1953-54 are referred to as "non-mathematics students."

CHAPTER II

LITERATURE IN THE FIELD

Introductory Statement

Much of the literature in the field of placement of students in college mathematics courses exists in the form of articles which have been published in various journals. An examination of some of these articles reveals the fact that there are fundamentally two types of problems to be considered. These are: (1) placement of students in those courses in which they may be expected to do their best work, and (2) having placed a student in a given course, to predict with some degree of accuracy his chances for success in that course. While the majority of the articles found in the literature appear to deal with the second of these two problems, the primary concern of this study is the first problem; namely, that of proper placement of students. The two problems are closely related, but the means which appear to have been used most successfully in their treatment are somewhat different. The purpose of this chapter is to summarize some of the findings of those who have done research work in this field and to point out their more important conclusions; particularly those conclusions on which there seems to be fairly general agreement.

Findings of Other Investigators

On the basis of various research studies which have been conducted, there appears to be ample justification for the use of a placement test. Irick[1] found that the score made on a mathematics placement test was the best single factor for predicting a student's success in a first course in college mathematics. Next in order in predictive value were found to be high school grades, rank in high school graduating class, and the number of semesters' work done in mathematics in high school. Scores made by students on an English placement test and on a psychological test were found to be of relatively little value.

The conclusions from another study conducted at Purdue University by Remmers and Geiger[2] were quite similar. The best predictive measures were found to be the scores on a group of orientation tests and relative position in the high school graduating class. The orientation tests used consisted of the Iowa Mathematics Training Test, the Purdue Placement Test in English, and the American Council on Education Psychological Examination. It was found, however, that after the first semester college grade point averages constituted the best single measure of prediction of scholastic success.

Held[3] reports on a placement test which was used at the University of Pittsburgh for the purpose of dividing freshman students into two course groups. Before the placement test was introduced, the division

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[1]P. E. Irick, "A Study of Factors Related to Engineering Mathematics at Purdue," (Master's Thesis, Purdue University, 1945), reported in Mathematics Teacher, XLI (December, 1948), 351.

[2]H. H. Remmers and H. E. Geiger, "Predicting Success and Failure of Engineering Students in the Schools of Engineering in Purdue University," Studies in Higher Education, XXXVIII (May, 1940), 10-19.

[3]Omar C. Held, "A College Mathematics Placement Test," Journal of Higher Education, XIII (January, 1942), 39-40.

was made on the basis of the number of units of high school mathematics presented for admission. Held indicates that the percentage of failure in mathematics courses was reduced from 21 per cent in the first semester of the school year 1936-37, prior to the introduction of the placement test in 1937, to 6 per cent for the first semester of the year 1939-40.

Bromley and Carter[1] found a coefficient of correlation of .35 between final course grades in mathematics and a proficiency test in mathematics. A correlation coefficient of .40 was found between course grades in mathematics and the rank of the students in their high school graduating classes. Coefficients of correlation between course grades in mathematics and the American Council on Education Psychological Test were found to be considerably lower. Correlation coefficients for the total score, the Q (quantitative) score, and the L (literature) score were .24, .28, and .16, respectively.

Barrett[2] found that in only two of six comparisons did final grades in mathematics courses correlate more highly with the Q score of the American Council on Education Psychological Test than with the L score. The total score on the American Council on Education Psychological Test was found to correlate with grades in mathematics neither consistently higher nor consistently lower than the Q score. A conclusion of this study was that the Q score should not be used as a differential

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[1]Ann Bromley and G. C. Carter, "Predictability of Success," Journal of Educational Research, XLIV (October, 1950), 148-50.

[2]D. M. Barrett, "Differential Value of Q and L Scores on A C E for Predicting Achievement in College Mathematics," Journal of Psychology, XXXIII (April, 1952), 205-7.

predictor of achievement in college mathematics courses.

In a study which was conducted at the University of Oregon, Douglass and Michaelson[1] reached the following conclusion:

> Prediction of success of students in college mathematics cannot be made with any high degree of accuracy from knowledge of the amount of high school training in mathematics, the average high school mark in mathematics, the average mark in all high school subjects, rank on the Psychological Examination of the American Council on Education, or any combination of these variables. The best prediction that can be made is secured from the average high school mark in all subjects.

However, a later study by Kossack,[2] also conducted at the University of Oregon, apparently does not bear out this conclusion. Kossack determined a linear function of five variables by means of which an achievement score in mathematics could be determined for each individual. This achievement score was used both for the purpose of placing the individual in a mathematics course and for predicting his most probable grade in this course. The five variables used were: (1) placement test score, (2) high school mathematics score, (3) psychological decile, (4) scholastic decile based upon high school record, and (5) the number of years elapsed since graduation from high school. The last three variables were found to contribute comparatively little to the accuracy of the achievement score, and consequently only the placement test score and the high school mathematics score were retained. It was claimed that approximately two-thirds of all students actually received their predicted grade

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[1]Harl R. Douglass and Jessie H. Michaelson, "The Relation of High School Mathematics to College Marks and of Other Factors to College Marks in Mathematics," School Review, XLIV (October, 1936), 615-19.

[2]C. F. Kossack, "Mathematics Placement at the University of Oregon," American Mathematical Monthly, XLIX (April, 1942), 234-37.

and that 95 per cent of all students received grades which were within one grade level of the predicted grade.

Summary of Chapter II

The purpose of this chapter is to point out the major conclusions of others who have worked in the field of placement of freshman students in college mathematics courses. The conclusions on which there appears to be fairly general agreement are the following:

1. College grade point averages serve best as a predictor of success in college mathematics courses, though they obviously cannot be used in placing beginning college freshman students.

2. A placement test in mathematics is an effective instrument for the proper placement of mathematics students and for reducing the percentage of failure in freshman mathematics courses.

3. Tests of general ability, such as psychological tests, are of value but are not as effective as a mathematics test in placing students in the proper mathematics course.

CHAPTER III

PROCUREMENT OF DATA AND DETERMINATION OF DISTRIBUTIONS

Sources of Data and Methods of Recording

Through the cooperation of the director of the office of machine accounting, duplicate International Business Machines (hereafter abbreviated I B M) cards were obtained for all students who entered the University of Oklahoma as freshmen in September, 1952. Each of these cards, 1832 in number, was punched with an identification consisting of the name and route number of the individual, the latter being used for purpose of filing and identification by the University. Other information transferred from the permanent files of the University included decile rankings on both the Reading Score and the Total Score of the Ohio State Psychological Examination (abbreviated O S P E) and decile rankings on the Iowa High School Content Examination (abbreviated I H S C). Separate rankings were listed for the total or composite score and for each of the four divisions of the examination; namely, English, mathematics, science, and history.

Since the route numbers punched into the cards were not consecutive, the cards were arranged in alphabetical order and numbered in sequence from one to 1832. These numbers were not actually punched into the cards, but it was thought that for the purpose of this study this means of identification would be advantageous.

The next step was to obtain information regarding the academic record of each of the students involved. Permission was obtained from the director of registration and other personnel in the office of admissions and records to transcribe these data from the permanent records of the University. This information was gathered during the first part of the second semester of the school year 1953-54, immediately after the students' grades for the first semester of that year had been placed on the permanent records. Thus the period of time required by this study consisted of the entire school year 1952-53, the first semester of the year 1953-54, and the intervening summer term. It seemed reasonable to assume that nearly all students who were going to take any mathematics during their college career would take at least one course at some time during the first three semesters.

A record was made on each student's card of the first mathematics course in which he enrolled at the University and of the final grade made in that course. Also, grade point averages for all work taken at the University were computed and recorded. In most instances high school records were available and in all such cases general high school grade point averages and grade point averages in high school mathematics courses were determined and recorded.

Since it is possible to record digits from zero to nine, inclusive, in each column of the I B M card, a somewhat finer subdivision was obtained in recording grade point averages by assigning values of zero, two, four, six, and eight to grades of F, D, C, B, and A, respectively. The intervening digits were used to represent intermediate grade point averages. Thus the figure which was recorded in each case was the digit nearest to the grade point average multiplied by two. However, in all final results in this study, the actual grade point averages are listed.

Three columns were used on each I B M card to record the student's performance on the University of Oklahoma mathematics placement test.[1] The student's raw score was punched into two columns and the third column was used to record his decile rank. A remainder of forty columns was used in the test item analysis study, which will be described later.

The I B M cards were next examined for completeness of data. Only those cards corresponding to students who had taken at least one course in college mathematics and for whom complete data were available on high school records, on the O S P E, and on the I H S C, were used in this part of the study. The number of cards with complete data was 1081 and these were grouped on the basis of enrollment in college mathematics courses. This grouping is shown in Table 1. Separate listings are shown for students whose permanent records were found in the current files in the office of admissions and records and for those whose records were found in the inactive files. The latter group included those students who had dropped out of the University during their first or second semester, or who had transferred to a different school. Students whose records indicated enrollment in more than one mathematics course were classified

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[1]This test has been developed by the mathematics department of the University and is administered to all freshmen by the personnel of the guidance center. The mathematics placement test papers for the members of the freshman class of 1952 were made available for use in this study through the cooperation of the guidance center.

TABLE 1

DISTRIBUTION BY FINAL GRADES IN MATHEMATICS COURSES OF ALL STUDENTS FOR WHOM COMPLETE DATA WERE AVAILABLE

First Mathematics Course in which Student Enrolled: Number	Name	Record File	Number of Students Making Final Grade in Math. Course: A	B	C	D	F	W	Total
R	Remedial Mathematics	Active	30	65	81	30	16	10	232
		Inactive	4	24	37	34	61	7	167
	Total		34	89	118	64	77	17	399
1	Plane Geometry	Active	1	2	0	0	0	0	3
		Inactive	0	0	0	1	1	1	3
	Total		1	2	0	1	1	1	6
2	Intermediate Algebra	Active	24	66	108	45	32	13	288
		Inactive	2	13	28	32	37	8	120
	Total		26	79	136	77	69	21	408
4	Business Algebra	Active	13	8	7	2	1	1	32
		Inactive	0	0	3	0	2	0	5
	Total		13	8	10	2	3	1	37
5	College Algebra	Active	2	3	5	5	1	3	19
		Inactive	0	0	1	0	3	1	5
	Total		2	3	6	5	4	4	24
6	Trigonometry	Active	0	2	3	5	2	0	12
		Inactive	0	0	0	0	1	0	1
	Total		0	2	3	5	3	0	13
7	Solid Geometry	Active	0	2	0	0	0	0	2
		Inactive	0	0	0	0	1	0	1
	Total		0	2	0	0	1	0	3
21	Mathematical Analysis I	Active	36	40	48	18	16	1	159
		Inactive	7	6	7	3	7	2	32
	Total		43	46	55	21	23	3	191
	Total	Active	106	188	252	105	68	28	747
		Inactive	13	43	76	70	113	19	334
	Total		119	231	328	175	181	47	1081

on the basis of the first course in which they had enrolled. The small numbers indicated for some courses do not imply that enrollments are small in these courses, but indicate that few students enroll in them as their first course in college mathematics. In Table 1, the letters A, B, C, D, and F represent final course grades in mathematics and W stands for withdrawal from the course with a passing mark at the time of withdrawal.

Treatment of Withdrawals

It was found that in many instances of withdrawal the student had, at the time of withdrawal, enrolled in the next lower course in the sequence of mathematics courses. In all such cases, a notation was made on the card, but no record of withdrawal was punched into the card. The enrollment of all students who changed from one mathematics course to another was included in the course to which the change was made and excluded from the course from which the change was made. A record of withdrawal was punched into a student's card only if the student did not enroll in another mathematics course at the time of withdrawal. Such changes in enrollment are ordinarily made during the first week or two of the semester and should be considered as a normal part of the enrollment procedure. In punching the cards, no distinction was made between those students who entered a course late as a result of such a change in their enrollment and those who had enrolled in the course originally.

Relation between Mathematics Course Grades and Drop-out Rate

As might be expected, the incidence of withdrawal from the University was found to be least among those students who made high grades in mathematics and greatest among students making low grades. Of those students who made a final grade of A in the first college mathematics course in which they enrolled, approximately 90 per cent returned to the University for the school year 1953-54, while only 38 per cent of the students who failed their first mathematics course returned after their first year. Of the seventy-seven students who failed in remedial mathematics, sixty-one, or 79 per cent, did not return for a second year. However, of the thirty-four students who received a grade of A in remedial mathematics, only four, or 12 per cent, failed to return. A similar but less marked relationship between grades made in mathematics and continuance of a college career may be observed in the case of students enrolled in mathematics 2 and mathematics 21. In the other mathematics courses, this pattern, though present, is less well defined because of the relatively small number of students involved. The percentage of drop-out among the students who withdrew from remedial mathematics and mathematics 2 was lower than that of students who made a grade of F, but appears to be quite comparable to that of students who made a grade of D. As indicated by other parts of this study, the students enrolled in mathematics 21 were, almost without exception, superior students and a low drop-out rate would be expected among them.

Relation between College Grade Point Averages and Drop-out Rate

Table 2 shows the relation between college grade point averages and drop-out rate from the University for students enrolled in remedial mathematics, mathematics 2, and mathematics 21. As might be expected, a large percentage of the low grades in mathematics was made by students

TABLE 2

RELATION BETWEEN COLLEGE GRADE POINT AVERAGES AND DROP-OUT RATE FOR STUDENTS ENROLLED IN REMEDIAL MATHEMATICS, MATHEMATICS 2, AND MATHEMATICS 21

Mathematics Course		Records in Active Files		Records in Inactive Files	
Course Number	Final Grade	Number of Students	College Grade Point Average	Number of Students	College Grade Point Average
R	A	30	2.783	4	2.125
R	B	65	2.315	24	1.750
R	C	81	1.957	37	1.432
R	D	30	1.950	34	1.132
R	F	16	1.531	61	0.664
R	W	10	2.150	7	2.071
Total		232	2.142	167	1.179
2	A	24	3.229	2	3.500
2	B	66	2.667	13	2.192
2	C	108	2.190	28	1.643
2	D	45	2.022	32	1.219
2	F	32	1.703	37	0.824
2	W	13	2.115	8	1.625
Total		288	2.303	120	1.366
21	A	36	3.431	7	3.714
21	B	40	2.775	6	2.917
21	C	48	2.406	7	1.929
21	D	18	2.111	3	1.833
21	F	16	1.625	7	1.143
21	W	1	2.000	2	2.500
Total		159	2.616	32	2.359

whose general grade point averages were also low. The drop-out rate among these students was greater than among those who made relatively higher grades. Table 2 indicates that the grade point averages of those students who were enrolled in remedial mathematics and mathematics 2 and who did not return to the University for the school year 1953-54 were approximately one grade point lower than the averages of the students who returned after their first year at the University. Also, as is shown by this table, there exists a close correlation between college grade point averages and grade points received in mathematics courses. Obviously, however, factors other than grade point averages often enter into a student's decision to leave college.

In both remedial mathematics and mathematics 2 there appears to be a very close relation between grade point averages and drop-out rate. However, the drop-out rate of mathematics 21 students was much lower than that for either of the other two courses and was more nearly evenly distributed among the various grade groups. The general grade point averages of mathematics 21 students were considerably higher than those of students in remedial mathematics and mathematics 2. Also, there was a smaller difference between the grade point averages of mathematics 21 students who dropped out after their first year and those who returned for a second year. In the case of students who made grades of A or B in mathematics 21, this difference was reversed; the students who dropped out had higher grade point averages than those who returned. Of the seven mathematics 21 students whose records were found in the inactive files, only one had a grade point average lower than that of the thirty-six students with a final grade of A who returned. Among the six students with a grade of B who dropped out, three had grade point averages lower than the grade point averages of the forty students with a grade of B who remained.

Means and Probable Errors of the Distributions

The means and probable errors of the distributions on grade point averages, placement test scores, the I H S C, and the O S P E, for students enrolled in remedial mathematics, mathematics 2, and mathematics 21 are shown in Tables 3, 4, and 5, respectively. Similar data are listed in Table 6 for students enrolled in mathematics courses numbered 1, 4, 5, 6, and 7. Since comparatively few students enrolled in courses numbered 1, 4, 5, 6, and 7 as their first course in college mathematics, no attempt was made to list means and probable errors of the distributions for grade groups in these courses.

In all tables in which deciles are listed, the position of an individual in a given decile is assumed to be at the mid-point of the interval represented by the decile. Thus the scaled position of an individual in the tenth decile is 9.5 deciles and the position of an individual in the first decile is 0.5 deciles. The positions of the means listed are therefore, wherever deciles are used, 0.5 deciles below the means as computed from the I B M cards.

As might be expected, the means of the various distributions for the remedial mathematics students were found to be lower than the corresponding means of most other groups and the means for students enrolled in mathematics 2 and mathematics 21 were successively higher. At the same time, the probable errors of the distributions showed a tendency to become

TABLE 3

MEANS AND PROBABLE ERRORS OF DISTRIBUTIONS ON GRADE POINT AVERAGES, PLACEMENT TEST SCORES, I H S C, AND O S P E, FOR STUDENTS ENROLLED IN REMEDIAL MATHEMATICS

Variable		Final Course Grade						Total
		A	B	C	D	F	W	
Average on College Gr. Pts.	Mean	2.706	2.163	1.792	1.516	0.857	2.118	1.742
	P E	0.602	0.410	0.442	0.473	0.492	0.498	0.580
Average on H S Gr. Pts.	Mean	3.132	2.815	2.674	2.516	2.201	2.706	2.626
	P E	0.386	0.448	0.417	0.426	0.439	0.385	0.464
Gr. Pt. Ave. on H S Mathematics	Mean	2.868	2.545	2.288	2.172	1.747	2.147	2.272
	P E	0.557	0.690	0.534	0.489	0.537	0.501	0.580
Raw Score on Placement Test	Mean	8.471	8.157	6.949	6.234	6.325	6.882	7.110
	P E	1.985	2.055	1.991	2.175	1.979	1.961	2.104
Decile Rank on Placement Test	Mean	3.000	2.792	2.339	2.141	2.071	2.324	2.412
	P E	0.915	0.987	0.895	0.879	0.838	0.809	0.973
Decile Rank on I H S C English	Mean	5.471	4.567	3.754	3.438	2.591	3.441	3.793
	P E	1.960	1.768	1.673	1.796	1.511	1.714	1.807
Decile Rank on I H S C Math.	Mean	3.588	3.208	2.508	1.875	1.201	1.735	2.370
	P E	1.372	1.357	1.109	1.102	0.713	1.207	1.253
Decile Rank on I H S C Science	Mean	4.853	3.904	3.508	2.719	2.084	1.676	3.232
	P E	1.821	1.721	1.692	1.575	1.236	1.029	1.705
Decile Rank on I H S C History	Mean	4.794	3.702	3.153	2.750	2.097	1.618	3.081
	P E	1.890	1.862	1.701	1.633	1.431	1.243	1.698
Decile Rank on I H S C Total	Mean	4.559	3.354	2.644	2.094	1.331	1.441	2.573
	P E	1.821	1.555	1.350	1.160	1.016	1.234	1.495
Decile Rank on O S P E Total	Mean	5.265	3.994	3.169	2.563	2.110	3.028	3.227
	P E	1.701	1.530	1.386	1.405	1.328	1.570	1.569
Decile Rank on O S P E Reading	Mean	5.206	3.949	3.398	2.656	2.071	2.735	3.247
	P E	1.840	1.543	1.447	1.461	1.284	1.612	1.601
Number of Students		34	89	118	64	77	17	399

TABLE 4

MEANS AND PROBABLE ERRORS OF DISTRIBUTIONS ON GRADE POINT AVERAGES, PLACEMENT TEST SCORES, I H S C, AND O S P E, FOR STUDENTS ENROLLED IN MATHEMATICS 2

Variable		Final Course Grade						Total
		A	B	C	D	F	W	
Average on College Gr. Pts.	Mean	3.250	2.589	2.063	1.688	1.232	1.929	2.027
	P E	0.365	0.382	0.418	0.445	0.509	0.573	0.579
Average on H S Gr. Pts.	Mean	3.462	3.232	2.860	2.623	2.529	2.571	2.608
	P E	0.516	0.363	0.430	0.407	0.398	0.523	0.461
Gr. Pt. Ave. on H S Mathematics	Mean	3.462	3.203	2.706	2.429	2.297	2.024	2.688
	P E	0.503	0.444	0.449	0.554	0.475	0.654	0.570
Raw Score on Placement Test	Mean	17.923	16.000	14.787	14.584	13.188	13.286	14.836
	P E	2.632	1.982	1.971	2.666	2.266	1.943	2.359
Decile Rank on Placement Test	Mean	7.038	6.525	6.022	5.864	5.225	5.310	5.980
	P E	0.777	0.838	0.873	1.070	0.911	0.945	0.971
Decile Rank on I H S C English	Mean	7.192	5.563	4.919	4.266	3.949	4.167	4.860
	P E	1.529	1.771	1.707	1.826	1.618	1.733	1.806
Decile Rank on I H S C Math.	Mean	7.500	6.867	6.029	4.903	3.964	3.024	5.566
	P E	1.138	1.191	1.229	1.534	1.614	1.390	1.589
Decile Rank on I H S C Science	Mean	7.385	5.589	5.713	4.695	4.558	3.786	5.306
	P E	1.404	1.687	1.804	1.932	2.005	1.560	1.887
Decile Rank on I H S C History	Mean	5.692	4.867	4.493	3.565	3.442	3.071	4.213
	P E	1.673	1.869	1.733	1.710	1.819	1.552	1.821
Decile Rank on I H S C Total	Mean	7.269	5.576	5.140	3.968	3.601	2.833	4.757
	P E	1.047	1.612	1.541	1.611	1.715	1.255	1.714
Decile Rank on O S P E Total	Mean	7.231	5.082	4.213	3.968	3.138	3.357	4.301
	P E	1.394	1.554	1.506	1.551	1.532	1.305	1.651
Decile Rank on O S P E Reading	Mean	7.192	5.373	4.684	4.383	3.428	3.310	4.618
	P E	1.409	1.712	1.534	1.641	1.558	1.225	1.690
Number of Students		26	79	136	77	69	21	408

TABLE 5

MEANS AND PROBABLE ERRORS OF DISTRIBUTIONS ON GRADE POINT AVERAGES, PLACEMENT TEST SCORES, I H S C, AND O S P E, FOR STUDENTS ENROLLED IN MATHEMATICS 21

Variable		Final Course Grade A	B	C	D	F	W	Total
Average on College Gr. Pts.	Mean	3.477	2.794	2.345	2.071	1.478	2.333	2.573
	P E	0.251	0.341	0.301	0.281	0.378	0.573	0.523
Average on H S Gr. Pts.	Mean	3.640	3.217	3.127	2.905	2.913	3.167	3.215
	P E	0.275	0.515	0.365	0.437	0.339	0.421	0.387
Gr. Pt. Ave. on H S Mathematics	Mean	3.779	3.304	3.082	2.952	2.804	3.000	3.243
	P E	0.245	0.341	0.425	0.440	0.442	0.551	0.438
Raw Score on Placement Test	Mean	31.465	28.130	24.891	24.810	23.043	23.333	26.895
	P E	2.644	3.639	2.852	2.014	2.210	0.841	3.479
Decile Rank on Placement Test	Mean	9.360	9.043	8.773	8.881	8.413	8.500	8.935
	P E	0.234	0.740	0.567	0.388	0.441	0.000	0.563
Decile Rank on I H S C English	Mean	8.035	7.152	6.227	6.167	6.326	5.833	6.835
	P E	1.296	1.510	1.887	1.708	1.965	1.770	1.746
Decile Rank on I H S C Math.	Mean	9.477	9.043	8.682	8.310	8.630	8.500	8.893
	P E	0.102	0.557	0.702	0.823	0.752	0.551	0.658
Decile Rank on I H S C Science	Mean	8.779	8.174	7.445	6.357	6.457	7.167	7.678
	P E	0.840	1.062	1.567	1.491	1.795	1.770	1.466
Decile Rank on I H S C History	Mean	6.802	5.870	5.518	5.690	5.022	6.833	5.867
	P E	1.743	1.706	1.871	1.774	2.007	1.682	1.850
Decile Rank on I H S C Total	Mean	8.826	8.000	7.264	6.929	6.587	7.167	7.673
	P E	0.678	1.031	1.391	1.293	1.744	1.386	1.327
Decile Rank on O S P E Total	Mean	7.733	6.804	5.773	5.024	5.674	5.833	6.369
	P E	1.267	1.474	1.497	1.594	1.718	1.770	1.610
Decile Rank on O S P E Reading	Mean	8.291	7.457	6.482	5.643	6.022	6.500	6.976
	P E	1.123	1.304	1.567	1.695	1.564	1.474	1.555
Number of Students		43	46	55	21	23	3	191

TABLE 6

MEANS AND PROBABLE ERRORS OF DISTRIBUTIONS ON GRADE POINT AVERAGES, PLACEMENT TEST SCORES, I H S C, AND O S P E, FOR STUDENTS ENROLLED IN MATHEMATICS COURSES 1, 4, 5, 6, AND 7

Variable		Mathematics Course Number				
		1	4	5	6	7
Average on College Gr. Pts.	Mean	2.083	2.689	2.313	2.346	1.833
	P E	0.658	1.056	1.045	0.426	0.885
Average on H S Gr. Pts.	Mean	3.259	3.122	3.146	3.077	2.833
	P E	0.424	0.354	0.957	0.586	0.318
Gr. Pt. Ave. on H S Mathematics	Mean	3.000	3.081	3.188	3.038	2.667
	P E	0.551	0.935	1.135	0.584	0.159
Raw Score on Placement Test	Mean	12.167	22.784	23.042	21.231	25.000
	P E	4.110	3.371	4.510	3.844	3.350
Decile Rank on Placement Test	Mean	5.333	8.230	8.208	7.962	8.833
	P E	1.479	0.936	1.313	1.078	0.636
Decile Rank on I H S C English	Mean	3.667	5.500	6.708	5.423	6.167
	P E	1.257	1.637	1.966	1.724	0.841
Decile Rank on I H S C Math.	Mean	3.667	8.203	7.917	6.500	8.500
	P E	2.039	1.118	1.305	2.229	0.551
Decile Rank on I H S C Science	Mean	3.500	5.743	6.875	7.192	6.167
	P E	2.119	1.791	1.814	1.574	2.717
Decile Rank on I H S C History	Mean	3.833	5.122	6.417	5.808	2.187
	P E	1.393	1.749	2.078	1.743	1.146
Decile Rank on I H S C Total	Mean	3.167	6.365	7.250	6.346	6.167
	P E	1.808	1.514	1.766	1.772	0.841
Decile Rank on O S P E Total	Mean	4.167	5.473	5.917	5.423	5.500
	P E	1.898	1.564	2.004	1.784	0.954
Decile Rank on O S P E Reading	Mean	4.167	5.851	6.083	5.577	3.500
	P E	1.808	1.360	1.917	1.879	1.908
Number of Students		6	37	24	13	3

smaller for the more advanced courses. One should expect, therefore, that any prediction of performance based upon these variables could be made more accurately for the students in the higher courses than for those in the lower ones. With respect to all variables except one, the means of the students in mathematics 2 were higher than the corresponding means for the remedial mathematics students. The high school grade point averages of the two groups represent the one exception to this statement. With respect to this variable the two groups had almost identical means.

As indicated by the probable errors of the various distributions, the groups of students enrolled in remedial mathematics and mathematics 2 showed approximately the same amount of "scatter" about the mean with respect to nearly all variables. Distributions of the group of mathematics 21 students showed less "scatter" with respect to most variables than did the corresponding distributions of either of the other two groups. Interquartile ranges computed from the data for the remedial mathematics and mathematics 2 groups showed a certain degree of overlap on all variables except the Oklahoma University mathematics placement test and the mathematics section of the I H S C. Since the students in these two groups were originally segregated largely on the basis of their performance on the placement test, it is to be expected that there should be no overlapping of the placement test interquartile ranges for the two groups. The fact that the division of the students into course groups on the basis of the placement test produced a better degree of separation on the mathematics section of the I H S C than on any of the other variables used is an indication that these two tests probably have numerous elements in common and that either test, or a combination of both, might be expected to serve as the best means available for separating students into approximately homogeneous groups with respect to mathematical ability. The separation on the basis of tests of general ability was less distinct. For example, with respect to the O S P E total score, the amount of overlap between interquartile ranges was 2.146 deciles for the remedial mathematics and mathematics 2 groups and 1.193 deciles for the mathematics 2 and mathematics 21 groups.

The means of the various distributions for students who withdrew from remedial mathematics, mathematics 2, and mathematics 21 were quite consistently lower than the corresponding means for the groups from which they withdrew, but they were generally higher than the corresponding means of the failing students within that group. The students who withdrew from remedial mathematics showed a somewhat better performance on the placement test and on the mathematics section of the I H S C than did the students who failed the course.

Placement Test Score Distributions by Final Course Grades

Tables 7, 8, and 9 show the distributions of placement test scores, by final course grades, for students enrolled in remedial mathematics, mathematics 2, and mathematics 21, respectively. A general trend toward higher final course grades is indicated among the students who made relatively high placement test scores, though many individual exceptions may be noted. In a number of instances, students were enrolled in mathematics courses other than those indicated by their placement test scores. These individual cases will be discussed in the next section.

It might be pointed out that no attempt was made to determine the degree of uniformity which exists among the grading scales used by the various instructors. However, members of the department of mathematics are well aware of the problems of uniform grading, as evidenced by the fact that for some time it has been their custom to develop and

TABLE 7

PLACEMENT TEST SCORE DISTRIBUTION, BY FINAL COURSE GRADES, OF STUDENTS ENROLLED IN REMEDIAL MATHEMATICS

Placement Test		Number of Students Earning Final Course Grade						Total	Cumulative Total
Raw Score	Decile	A	B	C	D	F	W		
0	1			2	5	4		11	11
1	1	1	2	4	2		2	11	22
2	1	1	1	4	3	4		13	35
3	1		1	5	4	9	1	20	55
4	1	1	5	11	3	4	1	25	80
5	2		5	10	10	6		31	111
6	2	2	12	7	5	11	1	38	149
7	3	5	7	21	6	9	3	51	200
8	3	7	18	21	4	7	4	61	261
9	4	7	10	12	11	15	2	57	318
10	4	7	16	7	7	5	2	44	362
11	5		6	9	4		1	20	382
12	5		1	2		3		6	388
13	6			1				1	389
14	6	2	1	2				5	394
15	7							0	394
16	7		2					2	396
17	8	1	1					2	398
18	8		1					1	399
Total		34	89	118	64	77	17	399	399

TABLE 8

PLACEMENT TEST SCORE DISTRIBUTION, BY FINAL COURSE GRADES, OF STUDENTS ENROLLED IN MATHEMATICS 2

Placement Test		Number of Students Earning Final Course Grade						Total	Cumulative Total
Raw Score	Decile	A	B	C	D	F	W		
2	1				1			1	1
3	1							0	1
4	1							0	1
5	2							0	1
6	2					1	1	2	3
7	3		1	1	1	2		5	8
8	3		1	1	2	1		5	13
9	4			2	2	1		5	18
10	4			2		2		4	22
11	5		3	8	2	5	2	20	42
12	5	1	6	19	13	20	7	66	108
13	6	2	4	15	12	15	5	53	161
14	6	4	7	19	9	7	1	47	208
15	7	1	9	17	11	5		43	251
16	7		12	12	4	2	1	31	282
17	8	4	11	13	3	2	1	34	316
18	8	2	7	12	10	3	2	36	352
19	8	2	9	11	1	1	1	25	377
20	8	7	7	1	2	1		18	395
21	9	1	1	1				3	398
22	9		1					1	399
23	9	1		1	1			3	402
24	9			1	1			2	404
25	9							0	404
26	10				1			1	405
27	10							0	405
28	10				1			1	406
29	10							0	406
30	10							0	406
31	10	1				1		2	408
Total		26	79	136	77	69	21	408	408

TABLE 9

PLACEMENT TEST SCORE DISTRIBUTION, BY FINAL COURSE GRADES, OF STUDENTS ENROLLED IN MATHEMATICS 21

Placement Test		Number of Students Earning Final Course Grade						Total	Cumulative Total
Raw Score	Decile	A	B	C	D	F	W		
10	4		1					1	1
11	5							0	1
12	5							0	1
13	6							0	1
14	6		1	1				2	3
15	7					1		1	4
16	7			1				1	5
17	8			1				1	6
18	8							0	6
19	8			2		1		3	9
20	8		1	2	1	2		6	15
21	9		2	3	3	3		11	26
22	9	1	1	5		3	1	11	37
23	9	1		8	3	3	1	16	53
24	9	1	3	2	4	5		15	68
25	9	3	3	5	1	2	1	15	83
26	10		1	9	5			15	98
27	10	1	8	2	1			12	110
28	10	2	2	2				6	116
29	10	2	4	3	1	2		12	128
30	10	4	3	3	1	1		12	140
31	10	3	3	1				7	147
32	10	3	2	3	1			9	156
33	10	6	2	2				10	166
34	10	8	6					14	180
35	10	4	1					5	185
36	10	1	1					2	187
37	10	1	1					2	189
38	10	2						2	191
Total		43	46	55	21	23	3	191	191

use uniform final examinations in all sections of remedial mathematics and mathematics 2.

Performance of Students not Enrolled According to Placement Test Score

Table 7 reveals the fact that a number of remedial mathematics students would have been placed in mathematics 2 by a strict adherence to placement as indicated by the placement test score. An individual check made of the records of these students showed that of the twenty who made a score of 11 on the placement test, one had originally enrolled in mathematics 2 and one in mathematics 1. Of the twelve students who made scores of 12, 13, or 14 on the placement test, six had originally enrolled in mathematics 2. The records of the five students who made scores of 16, 17, or 18 did not show previous enrollment in mathematics 2, indicating that either they had been advised to enroll in remedial mathematics or they did so on their own volition. Thirteen students with placement test scores of 21 or above, which should normally have placed them in mathematics 21, were enrolled in mathematics 2. An individual check of the records of these students showed that only four had originally enrolled in mathematics 21.

Table 8 indicates that twenty-two individuals with placement test scores of 10 or less were enrolled in mathematics 2. The placement test scores of these students should normally have placed them in remedial mathematics. Individual grade point averages and performance on the I H S C and the O S P E are shown for each of these students in Table 10. Certainly some of these twenty-two students had been advised to enroll in remedial mathematics, since the permanent records of three of them

TABLE 10

PERFORMANCE OF STUDENTS ENROLLED IN MATHEMATICS 2 WHOSE PLACEMENT TEST SCORE PLACED THEM IN REMEDIAL MATHEMATICS

Student Number	Placement Test Score	Grade in Math. 2	Grade Point Average		I H S C Decile					O S P E Decile	
			High School	College	Eng.	Math.	Sci.	Hist.	Total	Total	Reading
452	7	B	2.5	2.0	1	5	3	1	1	2	2
711	8	B	3.5	3.5	9	6	4	7	7	8	7
1354	7	C	3.0	1.5	2	4	2	3	2	3	3
1057	8	C	3.0	2.0	6	9	7	7	8	5	6
1006	9	C	4.0	2.5	9	4	6	8	7	7	8
585	9	C	2.0	2.0	4	3	7	2	3	2	2
1078	10	C	3.5	2.0	8	5	9	7	7	9	9
33	10	C	3.5	2.0	3	5	2	3	3	1	2
1799	2	D	3.0	2.5	7	2	8	5	4	2	2
671	7	D	2.5	1.5	7	7	9	8	8	3	2
644	8	D	2.0	1.5	4	3	3	2	2	3	5
1627	8	D	2.5	1.5	10	2	4	5	6	7	6
1100	9	D	2.5	2.0	1	2	4	1	1	6	6
1223	9	D	2.0	1.0	1	7	6	1	3	3	3
843	6	F	2.5	1.0	5	2	5	2	2	1	1
1142	7	F	2.0	1.0	2	1	1	1	1	1	1
1736	7	F	2.5	1.0	5	5	9	3	5	2	3
1778	8	F	3.0	2.0	7	4	6	3	4	2	3
372	9	F	2.0	2.0	3	2	4	1	2	2	2
550	10	F	2.0	0.5	1	2	1	1	1	1	1
938	10	F	2.0	1.5	5	2	1	1	1	2	2
555	6	W	2.0	1.5	7	3	6	7	5	6	7
Means	7.9	D+	2.6	1.7	4.4	3.4	4.4	3.1	3.3	3.0	3.3

TABLE 11

PERFORMANCE OF STUDENTS ENROLLED IN MATHEMATICS 21 WHOSE PLACEMENT TEST SCORE PLACED THEM IN MATHEMATICS 2

Student Number	Placement Test Score	Grade in Math. 21	Grade Point Average		I H S C Decile					O S P E Decile	
			High School	College	Eng.	Math.	Sci.	Hist.	Total	Total	Reading
743	10	B	3.5	3.0	5	5	8	7	6	6	6
1682	14	B	3.5	2.5	6	9	9	6	8	4	5
1155	20	B	3.0	2.5	10	9	8	6	10	9	10
1000	14	C	2.5	2.0	5	8	7	1	4	9	8
182	16	C	3.5	2.5	1	5	1	1	1	3	4
691	17	C	3.0	2.0	4	6	9	2	5	7	7
603	19	C	3.0	1.5	2	9	10	3	6	3	2
332	19	C	2.5	2.5	7	10	6	8	8	5	6
378	20	C	4.0	2.5	8	8	8	7	8	6	9
754	20	C	3.5	2.0	2	10	8	3	6	3	4
1201	20	D	3.5	2.5	6	9	9	10	9	4	5
1052	15	F	3.0	2.0	10	9	10	9	10	8	6
1064	19	F	2.5	1.5	9	10	10	10	10	9	8
485	20	F	3.0	2.0	10	10	10	8	10	8	8
1455	20	F	3.0	1.5	6	9	8	3	7	7	6
Means	17.5	C-	3.1	2.2	5.6	7.9	7.6	5.1	6.7	5.6	5.8

(students with numbers 585, 1057, and 1223) indicate that their original enrollment had been in remedial mathematics and that they had dropped that course and entered mathematics 2 instead. It would be difficult to determine how many of these students enrolled in mathematics 2 contrary to advisement, but a comparison of their individual performance with the means of all students enrolled in mathematics 2, as shown in Table 4, indicates that in nearly all instances the advice given them should have been to enroll in remedial mathematics. In particular, it is seen that approximately one-third of this group of students made a failing grade in mathematics 2 and nearly one-third made a grade of D.

As shown in Table 9, there were fifteen students enrolled in mathematics 21 who made scores of 20 or less on the mathematics placement test. These scores should normally have placed these students in mathematics 2. The individual grade point averages and performance on the I H S C and the O S P E for these students is shown in Table 11. With the exception of students numbered 182, 691, and 743, the scores made by the fifteen students on the mathematics section of the I H S C compare favorably with the mean listed for the mathematics 21 group in Table 5. The four students who made failing grades in mathematics 21 all made scores on the mathematics section of the I H S C which placed them in the ninth and tenth deciles. Each of these four students except the last one listed made similarly high scores on the other tests. These tests would therefore have been of no aid in averting their failure in mathematics 21. However, they would have been placed in mathematics 2 by a strict adherence to the "breaking point" between scores of 20 and 21 on the mathematics placement test. The permanent records of the fifteen students listed in Table 11 contained no evidence to indicate that their original enrollment had been in mathematics 2.

Distributions of Students on Basis of I H S C

Tables 12, 13, and 14 show distributions of students enrolled in remedial mathematics, mathematics 2, and mathematics 21, in which final course grades in mathematics are plotted against decile rankings on the mathematics section of the I H S C. An examination of Table 12

TABLE 12

I H S C MATHEMATICS SECTION DECILE DISTRIBUTION, BY FINAL COURSE GRADES, OF STUDENTS ENROLLED IN REMEDIAL MATHEMATICS

I H S C Math. Section Decile	Number of Students Earning Final Course Grade						Total	Cumulative Total
	A	B	C	D	F	W		
1	2	9	22	25	45	9	112	112
2	8	22	33	18	19	3	103	215
3	4	17	21	6	7	2	57	272
4	8	12	19	8	4	2	53	325
5	3	14	17	4	1		39	364
6	4	4	2	2	1		13	377
7	3	6	2				11	388
8	1	3	1			1	6	394
9	1	2	1	1			5	399
10							0	399
Total	34	89	118	64	77	17	399	399

reveals that of the seventy-four students enrolled in remedial mathematics whose decile ranks were above the fourth decile, only two students received a failing grade in remedial mathematics, while Table 13 shows that of the sixty-eight students enrolled in mathematics 2 whose decile ranks were below the fourth decile, only ten received a final grade higher than D in mathematics 2. Consequently, students whose scores placed them in the fourth decile might be considered as belonging to a border-line area between remedial mathematics and mathematics 2, with the final decision as to their enrollment being made to depend at least in part on their performance on other tests.

Table 13 indicates that of the 130 mathematics 2 students who were

TABLE 13

I H S C MATHEMATICS SECTION DECILE DISTRIBUTION, BY FINAL COURSE GRADES, OF STUDENTS ENROLLED IN MATHEMATICS 2

I H S C Math. Section Decile	Number of Students Earning Final Course Grade						Total	Cumulative Total
	A	B	C	D	F	W		
1				3	9	3	15	15
2		2	3	8	10	6	29	44
3			5	9	6	4	24	68
4		3	12	4	10	1	30	98
5	3	6	22	15	10	3	59	157
6	5	12	17	9	12		55	212
7		13	29	17	3	4	66	278
8	3	21	31	4	3		62	340
9	11	15	14	7	6		53	393
10	4	7	3	1			15	408
Total	26	79	136	77	69	21	408	408

in deciles 8, 9, and 10 of the mathematics section of the I H S C only nine made a final grade of F in mathematics 2. Also, Table 14 shows that, of the eight mathematics 21 students with decile ranks of 7 and below on the I H S C mathematics section, four made a final grade of D or F in mathematics 21. These facts seem to indicate that, if the mathematics section of the I H S C were to be taken as the criterion for separation of students into mathematics 2 and mathematics 21 groups, the separation might be made between the seventh and eighth deciles.

Distributions of Students on Basis of O S P E

Tables 15, 16, and 17 show distributions of students enrolled

TABLE 14

I H S C MATHEMATICS SECTION DECILE DISTRIBUTION, BY FINAL COURSE GRADES, OF STUDENTS ENROLLED IN MATHEMATICS 21

I H S C Math. Section Decile	Number of Students Earning Final Course Grade						Total	Cumu-lative Total
	A	B	C	D	F	W		
1							0	0
2							0	0
3							0	0
4				1			1	1
5		1	1				2	3
6			1		2		3	6
7			1	1			2	8
8			7		1	1	9	17
9	1	16	20	16	10	1	64	81
10	42	29	25	3	10	1	110	191
Total	43	46	55	21	23	3	191	191

in remedial mathematics, mathematics 2, and mathematics 21, in which final course grades are plotted against decile rankings on the O S P E total score. Each of these three groups shows a greater "spread" with respect to the O S P E total score than it did with respect to the mathematics section of the I H S C, as was indicated by Tables 12, 13, and 14. In both remedial mathematics and mathematics 2, approximately one-half of the students with a decile rank of 1 or 2 on the O S P E total score made a final course grade in mathematics of F or D. More than one-half of the failures in remedial mathematics occurred among the students who ranked in the first and second deciles on the O S P E total score. In mathematics 2, almost one-third of all failures are similarly accounted

TABLE 15

O S P E TOTAL DECILE DISTRIBUTION, BY FINAL COURSE GRADES, OF STUDENTS ENROLLED IN REMEDIAL MATHEMATICS

O S P E Total Decile	Number of Students Earning Final Course Grade						Total	Cumulative Total
	A	B	C	D	F	W		
1		5	16	19	35	5	80	80
2	4	17	28	14	11	3	77	157
3	5	14	20	9	9		57	214
4	3	7	12	5	9	3	39	253
5	4	20	18	8	4	1	55	308
6	4	10	14	6	4	3	41	349
7	3	3	4		3	1	14	363
8	6	9	4	1	2	1	23	386
9	2	2	1	2			7	393
10	3	2	1				6	399
Total	34	89	118	64	77	17	399	399

TABLE 16

O S P E TOTAL DECILE DISTRIBUTION, BY FINAL COURSE GRADES, OF STUDENTS ENROLLED IN MATHEMATICS 2

O S P E Total Decile	Number of Students Earning Final Course Grade						Total	Cumulative Total
	A	B	C	D	F	W		
1		3	12	7	13	2	37	37
2		6	17	9	16	2	50	87
3	1	7	16	16	8	7	55	142
4	1	9	15	9	9	5	48	190
5	2	14	20	11	10		57	247
6	5	12	26	10	4	3	60	307
7	2	8	18	7	3	1	39	346
8	1	12	6	3	3		25	371
9	8	5	4	3	3	1	24	395
10	6	3	2	2			13	408
Total	26	79	136	77	69	21	408	408

TABLE 17

O S P E TOTAL DECILE DISTRIBUTION, BY FINAL COURSE GRADES, OF STUDENTS ENROLLED IN MATHEMATICS 21

O S P E Total Decile	Number of Students Earning Final Course Grade						Total	Cumulative Total
	A	B	C	D	F	W		
1	1		1	1	1		4	4
2			2	1	2		5	9
3		2	5	2			9	18
4	1	6	6	5	3	1	22	40
5	2	3	4	1	4	1	15	55
6	1	5	9	3	2		20	75
7	6	5	8	4	2		25	100
8	10	8	10	1	4		33	133
9	9	8	9	2	3		31	164
10	13	9	1	1	2	1	27	191
Total	43	46	55	21	23	3	191	191

TABLE 18

HIGH SCHOOL MATHEMATICS GRADE POINT DISTRIBUTION, BY FINAL COURSE GRADES, OF STUDENTS ENROLLED IN REMEDIAL MATHEMATICS

H S Math. Grade Point Average	Number of Students Earning Final Course Grade						Total	Cumulative Total
	A	B	C	D	F	W		
0.5							0	0
1.0	2	5	13	8	34	3	65	65
1.5	1	9	18	9	5		42	107
2.0	7	21	26	20	19	8	101	208
2.5		19	30	12	8	3	72	280
3.0	13	18	19	12	8	2	72	352
3.5	6	5	5	1	2		19	371
4.0	5	12	7	2	1	1	28	399
Total	34	89	118	64	77	17	399	399

TABLE 19

HIGH SCHOOL MATHEMATICS GRADE POINT DISTRIBUTION, BY FINAL COURSE GRADES, OF STUDENTS ENROLLED IN MATHEMATICS 2

H S Math. Grade Point Average	Number of Students Earning Final Course Grade						Total	Cumulative Total
	A	B	C	D	F	W		
0.5							0	0
1.0			1	7	7	5	20	20
1.5	1	2	16	8	7	5	39	59
2.0	1	5	21	20	18	5	70	129
2.5	4	9	38	16	18	2	87	216
3.0	2	29	27	10	14	1	83	299
3.5	3	11	19	12	4		49	348
4.0	15	23	14	4	1	3	60	408
Total	26	79	136	77	69	21	408	408

TABLE 20

HIGH SCHOOL MATHEMATICS GRADE POINT DISTRIBUTION, BY FINAL COURSE GRADES, OF STUDENTS ENROLLED IN MATHEMATICS 21

H S Math. Grade Point Average	Number of Students Earning Final Course Grade						Total	Cumulative Total
	A	B	C	D	F	W		
0.5							0	0
1.0							0	0
1.5			1	1			2	2
2.0		1	6	1	6	1	15	17
2.5	1	5	8	7	5		26	43
3.0	3	15	16	5	7	1	47	90
3.5	10	15	16	4	2		47	137
4.0	29	10	8	3	3	1	54	191
Total	43	46	55	21	23	3	191	191

for. The high percentage of failure in mathematics 2 among students ranking in the first and second deciles on the O S P E total score indicates that advisers should be particularly cautious in placing students who have a very low O S P E total score in any mathematics course higher than remedial mathematics.

Distributions of Students on Basis of High School Mathematics Record

Tables 18, 19, and 20 show distributions for the same groups of students in which final course grades in college mathematics are tabulated against high school grade point averages in mathematics. These tables indicate that although there is a positive correlation between mathematics grades received in high school and those received in college, high school mathematics grades show a smaller "spread" than do college mathematics grades for the same groups of students. This is particularly true of the group of students enrolled in mathematics 21. In this group only forty-three of the 191 students had less than an average of B in their high school mathematics. Their college grades in mathematics showed a much greater distribution.

Summary of Chapter III

In this chapter have been presented the means and probable errors of the distributions, on the basis of those variables on which data were available, for the various groups of mathematics students. Special emphasis has been placed on groups of students enrolled in remedial mathematics, mathematics 2, and mathematics 21, as these are the groups which will be of primary interest in the sequel. However, for purpose of comparison, data were given also for those students who enrolled in other courses as their first college mathematics course. Data were given for students who withdrew from a mathematics course without re-enrolling in another course. This was done to make possible a comparison of their performance with that of the groups from which they withdrew.

Also, distribution charts were shown which indicate the relationship between selected variables and final course grades in remedial mathematics, mathematics 2, and mathematics 21. While such charts were originally made showing the relation between final course grades and all other variables investigated, only those distribution charts are shown here which involve variables that were used later in the development of regression equations and discriminant functions.

CHAPTER IV

COMPARISON OF MATHEMATICS AND NON-MATHEMATICS STUDENTS

Introduction

In this and succeeding chapters all students who made a final course grade of W in college mathematics are removed from further consideration. Data for these students were included in the material of Chapter III to make possible comparisons in the performance, with respect to various variables, of the students who withdrew and that of the groups from which they withdrew. In this chapter, comparisons are made between groups of mathematics students who remained throughout the entire semester in the mathematics course in which they had enrolled and students who did not take at least one mathematics course previous to the time when data were gathered for this study. Students belonging to the latter group have been designated in Chapter I as "non-mathematics students." The number of non-mathematics students involved in the various distributions varied slightly with different variables because complete data were not available for all students. Consequently the comparison groups of non-mathematics students for the mathematics placement test, the I H S C, and the O S P E consisted of 526, 523, and 529 students, respectively.

Comparative Means and Probable Errors of the Distributions

In Table 21 are presented the means and the probable errors of the distributions, on the basis of the mathematics placement test, the I H S C, and the O S P E, for non-mathematics students and for students in remedial mathematics, mathematics 2, and mathematics 21 who remained in their respective mathematics courses throughout the entire semester. These data are presented primarily for two reasons. First, they are of value in showing how these four groups of students compare with one another on the basis of the various variables. Secondly, in Chapter VI certain variables are chosen as the variables to be used in the development of regression equations and discriminant functions. The choice of variables is made primarily on the basis of two criteria. These are: (1) the correlations shown by each of the three major mathematics groups between each of the variables and final course grades in mathematics and (2) the degree of separation shown between pairs of the three mathematics groups with respect to each of the variables. A measure of this degree of separation is found in the interquartile ranges, which are easily computed from the means and probable errors of the distributions.

With respect to all variables listed in Table 21, the means for the non-mathematics students were found to lie between the corresponding means for the remedial mathematics and mathematics 2 groups. Also, in all cases, the performance of the combined mathematics groups was superior to that of the non-mathematics students. This is indicated by the fact that the means for the non-mathematics students were lower than the corresponding means for the entire group, shown in the column headed "Total."

In all respects the mathematics 21 students showed a considerable degree of superiority over the entire group. This superiority ranged from a difference of slightly less than two deciles on the history section

TABLE 21

COMPARATIVE MEANS AND PROBABLE ERRORS OF DISTRIBUTIONS ON PLACEMENT TEST, I H S C, AND O S P E, FOR STUDENTS ENROLLED IN REMEDIAL MATHEMATICS, MATHEMATICS 2, AND MATHEMATICS 21, AND NON-MATHEMATICS STUDENTS

Variable		Mathematics Course Number R	Mathematics Course Number 2	Mathematics Course Number 21	Non-Mathematics Students	Total
Raw Score on Placement Test	Mean	7.120	14.910	26.963	11.738	13.308
	P E	2.741	2.367	3.492	4.944	5.254
Decile Rank on Placement Test	Mean	2.395	6.017	8.941	4.300	4.851
	P E	0.931	0.967	0.566	1.872	1.903
Decile Rank on I H S C English	Mean	3.809	4.898	6.835	4.743	4.810
	P E	1.804	1.501	1.744	2.128	1.943
Decile Rank on I H S C Math.	Mean	2.398	5.704	8.899	3.640	4.527
	P E	1.250	1.545	0.658	2.026	2.112
Decile Rank on I H S C Science	Mean	3.301	5.389	7.686	3.420	4.446
	P E	1.702	1.887	1.460	1.965	2.086
Decile Rank on I H S C History	Mean	3.147	4.275	5.851	3.506	3.912
	P E	1.774	1.825	1.850	1.430	1.938
Decile Rank on I H S C Total	Mean	2.623	4.862	7.681	3.513	4.166
	P E	1.496	1.708	1.325	2.056	2.044
Decile Rank on O S P E Total	Mean	3.233	4.353	6.378	4.107	4.243
	P E	1.569	1.661	1.606	1.976	1.862
Decile Rank on O S P E Reading	Mean	3.296	4.714	6.984	4.086	4.423
	P E	1.599	1.701	1.556	2.022	1.934
Number of Students		382	387	188	*	**

* Placement Test: 526; I H S C: 523; O S P E: 529.
**Placement Test: 1483; I H S C: 1480; O S P E: 1486.

of the I H S C to a difference of more than four deciles on both the Oklahoma University mathematics placement test and the mathematics section of the I H S C.

The only distributions which showed no overlap between the interquartile ranges computed for each of the three mathematics groups were the distributions for the mathematics placement test and the mathematics section of the I H S C. With respect to all other variables except the history section of the I H S C, a slightly better degree of separation was shown by the interquartile ranges between students in remedial mathematics and mathematics 2 than by the interquartile ranges between students in mathematics 2 and mathematics 21.

The probable errors of the distributions shown in Table 21 indicate that with respect to all variables except the history section of the I H S C, the variability of the group of non-mathematics students was somewhat greater than that of each of the three groups of mathematics students.

Comparative Distributions on Various Criteria

The distribution charts which are presented in Tables 22 to 29, inclusive, serve a dual purpose. First, the totals listed in each table serve as a means of comparison of the performance of mathematics students and that of non-mathematics students on the various tests. Secondly, these tables serve as a visual means of showing the relationship which exists between final course grades in each of the three major mathematics groups and some of the other variables investigated. This visual picture will be of value in the determination, in Chapter VI, of those variables which should be used in setting up regression equations and discriminant functions.

In Table 30, which summarizes the material presented in Tables 22 to 29, inclusive, comparison of the performance of the three mathematics groups and the group of non-mathematics students is facilitated by listing, for each of the variables, the percentages of each group which fall in the various deciles of the distribution.

Summary of Chapter IV

A comparison of the means of the distributions on the placement test, the I H S C, and the O S P E, for the three groups of mathematics students and the non-mathematics students, indicates that in all instances the performance of the non-mathematics students was superior to that of the remedial mathematics students, but inferior to that of the group of students in mathematics 2. The performance of the mathematics 21 group of students was found to be superior in all respects to that of each of the other three groups of students. On all tests except the history section of the I H S C each of the three groups of mathematics students showed a smaller degree of variability than did the group of non-mathematics students.

TABLE 22

PLACEMENT TEST DECILE DISTRIBUTION CHART FOR STUDENTS ENROLLED IN REMEDIAL MATHEMATICS, MATHEMATICS 2, AND MATHEMATICS 21, AND NON-MATHEMATICS STUDENTS

Math. Course		Number of Students in Placement Test Decile										Total
Course Number	Final Grade	1	2	3	4	5	6	7	8	9	10	
21	A									6	37	43
21	B				1		1		1	9	34	46
21	C						1	1	5	23	25	55
21	D								1	11	9	21
21	F							1	3	16	3	23
Total					1		2	2	10	65	108	188
2	A					1	6	1	15	2	1	26
2	B			2		9	11	21	34	2		79
2	C			2	4	27	34	29	37	3		136
2	D	1		3	2	15	21	15	16	2	2	77
2	F		1	3	3	25	22	7	7		1	69
Total		1	1	10	9	77	94	73	109	9	4	387
R	A	3	2	12	14		2		1			34
R	B	9	17	25	26	7	1	2	2			89
R	C	26	17	42	19	11	3					118
R	D	17	15	10	18	4						64
R	F	21	17	16	20	3						77
Total		76	68	105	97	25	6	2	3			382
Total, Math. Students		77	69	115	107	102	102	77	122	74	112	957
Non-Math. Students		76	65	66	52	54	59	34	50	49	21	526
Total, All Students		153	134	181	159	156	161	111	172	123	133	1483

TABLE 23

I H S C ENGLISH DECILE DISTRIBUTION CHART FOR STUDENTS ENROLLED IN REMEDIAL MATHEMATICS, MATHEMATICS 2, AND MATHEMATICS 21, AND NON-MATHEMATICS STUDENTS

Math. Course		Number of Students in I H S C English Decile										Total
Course Number	Final Grade	1	2	3	4	5	6	7	8	9	10	
21	A			1	1	4	1	3	3	11	19	43
21	B		2	1	2	4	3	6	5	13	10	46
21	C	2	4	3	5	7	2	8	3	10	11	55
21	D	1		2	3		3	1	5	4	2	21
21	F	1	3		1	2	2	3	1	5	5	23
Total		4	9	7	12	17	11	21	17	43	47	188
2	A		1	1		3	4		3	8	6	26
2	B	5	4	6	7	10	10	14	4	10	9	79
2	C	9	14	15	11	18	17	17	22	6	7	136
2	D	10	10	9	11	8	5	11	3	6	4	77
2	F	8	11	7	7	16	7	6	2	2	3	69
Total		32	40	38	36	55	43	48	34	32	29	387
R	A	2	5	3	2	1	2	7	3	6	3	34
R	B	8	13	8	8	12	11	12	[illegible]	5	5	89
R	C	20	17	15	10	20	12	11	6	4	3	118
R	D	15	10	8	6	11	2	1	6	3	2	64
R	F	25	13	12	12	3	5	3		3	1	77
Total		70	58	46	38	47	32	34	22	21	14	382
Total, Math. Students		106	107	91	86	119	86	103	73	96	90	957
Non-Math. Students		89	56	52	29	53	38	46	45	44	71	523
Total, All Students		195	163	143	115	172	124	149	118	140	161	1480

TABLE 24

I H S C MATHEMATICS DECILE DISTRIBUTION CHART FOR STUDENTS ENROLLED IN REMEDIAL MATHEMATICS, MATHEMATICS 2, AND MATHEMATICS 21, AND NON-MATHEMATICS STUDENTS

Math. Course		Number of Students in I H S C Math. Decile										Total
Course Number	Final Grade	1	2	3	4	5	6	7	8	9	10	
21	A									1	42	43
21	B					1				16	29	46
21	C					1	1	1	7	20	25	55
21	D				1			1		16	3	21
21	F						2		1	10	10	23
Total					1	2	3	2	8	63	109	188
2	A					3	5		3	11	4	26
2	B		2		3	6	12	13	21	15	7	79
2	C		3	5	12	22	17	29	31	14	3	136
2	D	3	8	9	4	15	9	17	4	7	1	77
2	F	9	10	6	10	10	12	3	3	6		69
Total		12	23	20	29	56	55	62	62	53	15	387
R	A	2	8	4	8	3	4	3	1	1		34
R	B	9	22	17	12	14	4	6	3	2		89
R	C	22	33	21	19	17	2	2	1	1		118
R	D	25	18	6	8	4	2			1		64
R	F	45	19	7	4	1	1					77
Total		103	100	55	51	39	13	11	5	5		382
Total, Math. Students		115	123	75	81	97	71	75	75	121	124	957
Non-Math. Students		147	83	39	39	44	37	28	43	33	30	523
Total, All Students		262	206	114	120	141	108	103	118	154	154	1480

TABLE 25

I H S C SCIENCE DECILE DISTRIBUTION CHART FOR STUDENTS ENROLLED IN REMEDIAL MATHEMATICS, MATHEMATICS 2, AND MATHEMATICS 21, AND NON-MATHEMATICS STUDENTS

Math. Course		Number of Students in I H S C Science Decile										Total
Course Number	Final Grade	1	2	3	4	5	6	7	8	9	10	
21	A					1	1	3	4	5	29	43
21	B				2	1		7	6	11	19	46
21	C	2	1	1	2	1	3	6	11	11	17	55
21	D		2		1	2	3	4	1	8		21
21	F	1	1	1		5	2	1	3	4	5	23
Total		3	4	2	5	10	9	21	25	39	70	188
2	A			2			5	4	3	3	9	26
2	B	3	6	6	7	6	14	13	7	11	6	79
2	C	6	14	4	15	16	11	14	19	27	10	136
2	D	6	16	6	8	4	7	8	8	11	3	77
2	F	11	7	4	14	5	4	4	5	11	4	69
Total		26	43	22	44	31	41	43	42	63	32	387
R	A	1	5	5	5	3	3	1	5	4	2	34
R	B	9	21	9	11	7	12	7	5	6	2	89
R	C	23	22	9	20	12	8	9	8	6	1	118
R	D	19	18	4	5	1	10	3	3	1		64
R	F	25	23	9	12	3	3				2	77
Total		77	89	36	53	26	36	20	21	17	7	382
Total, Math. Students		106	136	60	102	67	86	84	88	119	109	957
Non-Math. Students		137	108	40	61	28	32	30	25	29	33	523
Total, All Students		243	244	100	163	95	118	114	113	148	142	1480

TABLE 26

I H S C HISTORY DECILE DISTRIBUTION CHART FOR STUDENTS ENROLLED IN REMEDIAL MATHEMATICS, MATHEMATICS 2, AND MATHEMATICS 21, AND NON-MATHEMATICS STUDENTS

Math. Course		Number of Students in I H S C History Decile										Total
Course Number	Final Grade	1	2	3	4	5	6	7	8	9	10	
21	A	3		1	1	5	5	4	6	7	11	43
21	B	1	3	5	3	4	5	8	6	6	5	46
21	C	5	2	6	3	7	6	6	7	7	6	55
21	D	1	2	1	2	2	1	4	4	2	2	21
21	F	1	5	2	1	3	2	1	3	2	3	23
Total		11	12	15	10	21	19	23	26	24	27	188
2	A		3	3		3	6	1	3	6	1	26
2	B	8	5	11	9	10	7	9	5	8	7	79
2	C	11	18	20	11	16	18	19	8	8	7	136
2	D	17	11	5	12	12	5	4	8	1	2	77
2	F	15	11	12	7	6	3	8	1	1	5	69
Total		51	48	51	39	47	39	41	25	24	22	387
R	A	3	4	4	3	5	3	4	1	4	3	34
R	B	22	9	12	8	9	6	10	5	5	3	89
R	C	32	19	14	13	13	9	6	3	8	1	118
R	D	20	15	7	3	7	4	1	5	1	1	64
R	F	39	9	7	8	4	4	3	2	1		77
Total		116	56	44	35	38	26	24	16	19	8	382
Total, Math. Students		178	116	110	84	106	84	88	67	67	57	957
Non-Math. Students		149	69	60	38	47	32	46	34	25	23	523
Total, All Students		327	185	170	122	153	116	134	101	92	80	1480

TABLE 27

I H S C TOTAL DECILE DISTRIBUTION CHART FOR STUDENTS ENROLLED IN REMEDIAL MATHEMATICS, MATHEMATICS 2, AND MATHEMATICS 21, AND NON-MATHEMATICS STUDENTS

Math. Course		Number of Students in I H S C Total Decile										Total
Course Number	Final Grade	1	2	3	4	5	6	7	8	9	10	
21	A						1	3	2	12	25	43
21	B				1		6	4	8	11	16	46
21	C	1		1	2	4	6	7	9	12	13	55
21	D		1			2	4	1	6	5	2	21
21	F	1		2	2	1	1	5	2	4	5	23
Total		2	1	3	5	7	18	20	27	44	61	188
2	A				1	1	2	8	6	3	5	26
2	B	5	1	5	11	8	12	13	10	9	5	79
2	C	6	6	21	14	9	21	29	17	11	2	136
2	D	11	9	9	11	10	8	10	5	4		77
2	F	13	10	10	9	7	6	6	3	3	2	69
Total		35	26	45	46	35	49	66	41	30	14	387
R	A	3	5	4	3	2	8	3	1	2	3	34
R	B	17	15	12	12	10	10	5	5	3		89
R	C	31	26	18	13	11	9	7	3			118
R	D	26	9	11	8	5	3	2				64
R	F	51	12	2	6	2	2	2				77
Total		128	67	47	42	30	32	19	9	5	3	382
Total, Math. Students		165	94	95	93	72	99	105	77	79	78	957
Non-Math. Students		168	64	47	56	26	29	38	21	40	34	523
Total, All Students		333	158	142	149	98	128	143	98	119	112	1480

TABLE 28

O S P E TOTAL DECILE DISTRIBUTION CHART FOR STUDENTS ENROLLED IN REMEDIAL MATHEMATICS, MATHEMATICS 2, AND MATHEMATICS 21, AND NON-MATHEMATICS STUDENTS

Math. Course		Number of Students in O S P E Total Decile										Total
Course Number	Final Grade	1	2	3	4	5	6	7	8	9	10	
21	A	1			1	2	1	6	10	9	13	43
21	B			2	6	3	5	5	8	8	9	46
21	C	1	2	5	6	4	9	8	10	9	1	55
21	D	1	1	2	5	1	3	4	1	2	1	21
21	F	1	2		3	4	2	2	4	3	2	23
Total		4	5	9	21	14	20	25	33	31	26	188
2	A			1	1	2	5	2	1	8	6	26
2	B	3	6	7	9	14	12	8	12	5	3	79
2	C	12	17	16	15	20	26	18	6	4	2	136
2	D	7	9	16	9	11	10	7	3	3	2	77
2	F	13	16	8	9	10	4	3	3	3		69
Total		35	48	48	43	57	57	38	25	23	13	387
R	A		4	5	3	4	4	3	6	2	3	34
R	B	5	17	14	7	20	10	3	9	2	2	89
R	C	16	28	20	12	18	14	4	4	1	1	118
R	D	19	14	9	5	8	6		1	2		64
R	F	35	11	9	9	4	4	3	2			77
Total		75	74	57	36	54	38	13	22	7	6	382
Total, Math. Students		114	127	114	100	125	115	76	80	61	45	957
Non-Math. Students		97	75	60	46	64	33	41	33	43	37	529
Total, All Students		211	202	174	146	189	148	117	113	104	82	1486

TABLE 29

O S P E READING DECILE DISTRIBUTION CHART FOR STUDENTS ENROLLED IN REMEDIAL MATHEMATICS, MATHEMATICS 2, AND MATHEMATICS 21, AND NON-MATHEMATICS STUDENTS

Math. Course		Number of Students in O S P E Reading Decile										Total
Course Number	Final Grade	1	2	3	4	5	6	7	8	9	10	
21	A			1		2	3	1	3	14	19	43
21	B			1		5	8	3	7	7	15	46
21	C	1	2	1	4	9	5	5	8	14	6	55
21	D		3	1	1	5	1		6	3	1	21
21	F		2	1	2	2	3	4	4	3	2	23
Total		1	7	5	7	23	20	13	28	41	43	188
2	A				3	3	1	4	3	5	7	26
2	B	5	7	4	5	11	12	13	8	9	5	79
2	C	5	16	16	16	21	22	15	15	7	3	136
2	D	9	5	13	4	13	13	7	9	2	2	77
2	F	12	13	7	11	9	4	7	4	2		69
Total		31	41	40	39	57	52	46	39	25	17	387
R	A	1	4	5	2	6	2	2	6	2	4	34
R	B	8	15	10	15	10	14	9	3	3	2	89
R	C	16	22	19	19	13	11	10	6	2		118
R	D	19	10	11	12	3	2	4	1	1	1	64
R	F	34	13	8	10	4	4	2	2			77
Total		78	64	53	58	36	33	27	18	8	7	382
Total, Math. Students		110	112	98	104	116	105	86	85	74	67	957
Non-Math. Students		109	73	51	53	49	39	34	38	45	38	529
Total, All Students		219	185	149	157	165	144	120	123	119	105	1486

TABLE 30

PERCENTAGE DISTRIBUTION CHART, BY DECILES, ON PLACEMENT TEST, I H S C, AND O S P E, FOR THREE GROUPS OF MATHEMATICS STUDENTS AND NON-MATHEMATICS STUDENTS

Variable	Group	Percentage of Group in Each Decile									
		1	2	3	4	5	6	7	8	9	10
Placement Test	21				0.5		1.1	1.1	5.3	34.6	57.4
	2	0.3	0.3	2.6	2.3	19.9	24.3	18.9	28.2	2.3	1.0
	R	19.9	17.8	27.5	25.4	6.5	1.6	0.5	0.7		
	N*	14.4	12.4	12.5	9.9	10.3	11.2	6.5	9.5	9.3	4.0
I H S C English	21	2.1	4.8	3.7	6.4	9.0	5.9	11.2	9.0	22.9	25.0
	2	8.3	10.3	9.8	9.3	14.2	11.1	12.4	8.8	8.3	7.5
	R	18.3	15.2	12.0	9.9	12.3	8.4	8.9	5.8	5.5	3.7
	N*	17.0	10.7	9.9	5.6	10.1	7.3	8.8	8.6	8.4	13.6
I H S C Math.	21				0.5	1.1	1.6	1.1	4.3	33.5	58.0
	2	3.1	5.9	5.2	7.5	14.5	14.2	16.0	16.0	13.7	3.9
	R	27.0	26.2	14.4	13.4	10.2	3.4	2.9	1.3	1.3	
	N*	28.1	15.9	7.5	7.5	8.4	7.1	5.4	8.2	6.3	5.7
I H S C Science	21	1.6	2.1	1.1	2.7	5.3	4.8	11.2	13.3	20.7	37.2
	2	6.7	11.1	5.7	11.4	8.0	10.6	11.1	10.9	16.3	8.3
	R	20.2	23.3	9.4	13.9	6.8	9.4	5.2	5.5	4.5	1.8
	N*	26.2	20.6	7.6	11.7	5.4	6.1	5.7	4.8	5.6	6.3
I H S C History	21	5.9	6.4	8.0	5.3	11.2	10.1	12.2	13.8	12.8	14.3
	2	13.2	12.4	13.2	10.1	12.1	10.1	10.6	6.5	6.2	5.7
	R	30.4	14.7	11.5	9.2	9.9	6.8	6.3	4.2	5.0	2.1
	N*	28.5	13.2	11.5	7.3	9.0	6.1	8.8	6.5	4.8	4.4
I H S C Total	21	1.1	0.5	1.6	2.7	3.7	9.6	10.6	14.3	23.4	32.4
	2	9.0	6.7	11.6	11.9	9.0	12.7	17.1	10.6	7.8	3.6
	R	33.5	17.5	12.3	11.0	7.9	8.4	5.0	2.3	1.3	0.7
	N*	32.1	12.2	9.0	10.7	5.0	5.5	7.3	4.0	7.6	6.5
O S P E Total	21	2.1	2.7	4.8	11.2	7.4	10.6	13.3	17.6	16.5	13.8
	2	9.0	12.4	12.4	11.1	14.7	14.7	9.8	6.5	5.9	3.4
	R	19.6	19.4	14.9	9.4	14.1	9.9	3.4	5.8	1.8	1.6
	N*	18.3	14.2	11.3	8.7	12.1	6.2	7.8	6.2	8.1	7.0
O S P E Reading	21	0.5	3.7	2.7	3.7	12.2	10.6	6.9	14.9	21.8	22.9
	2	8.0	10.6	10.3	9.8	14.7	13.4	12.1	10.1	6.5	4.4
	R	20.4	16.8	13.9	15.2	9.4	8.6	7.1	4.7	2.1	1.8
	N*	20.6	13.8	9.6	9.9	9.3	7.4	6.4	7.2	8.5	7.2

* Non-Mathematics Students

CHAPTER V

DETERMINATION OF COEFFICIENTS OF CORRELATION

Introduction

In order to determine as clearly as possible the relationships which can best be used as an aid in the proper placement of freshman students in mathematics courses, an examination of coefficients of correlation between all pairs of variables will be helpful. In this chapter are presented coefficients of correlation between all pairs of variables for groups of students enrolled in remedial mathematics, mathematics 2, and mathematics 21. The means and standard deviations which were used in the computation of these coefficients of correlation are also listed for reference. A selection is made of four variables which are to be used in Chapter VI for the determination of regression equations and discriminant functions. On the basis of these selected variables, a re-computation is made of the means, standard deviations, and coefficients of correlation for the groups of students in remedial mathematics, mathematics 2, and mathematics 21 who are properly placed according to the definition of proper placement which was given in Chapter I.

Means and Standard Deviations for Three Groups of Mathematics Students

In Table 31 are presented the means and standard deviations of distributions on all variables on which data were available for the

TABLE 31

MEANS AND STANDARD DEVIATIONS OF DISTRIBUTIONS ON GRADE POINT AVERAGES, PLACEMENT TEST SCORES, I H S C, AND O S P E, FOR GROUPS OF STUDENTS ENROLLED IN REMEDIAL MATHEMATICS, MATHEMATICS 2, AND MATHEMATICS 21

Variable		Mathematics Course Number		
		R	2	21
Average on College Grade Points	Mean	1.725	2.032	2.577
	S D	0.859	0.858	0.774
Average on College Mathematics Grade Points	Mean	1.840	1.783	2.346
	S D	1.240	1.157	1.281
Average on High School Grade Points	Mean	2.626	2.868	3.215
	S D	0.687	0.676	0.565
Average on High School Mathematics Grade Points	Mean	2.272	2.724	3.247
	S D.	0.859	0.823	0.645
Raw Score on Placement Test	Mean	7.120	14.910	26.963
	S D	4.064	3.509	5.177
Decile Rank on Placement Test	Mean	2.395	6.017	8.942
	S D	1.380	1.433	0.839
Decile Rank on I H S C English	Mean	3.809	4.898	6.851
	S D	2.675	2.226	2.586
Decile Rank on I H S C Mathematics	Mean	2.398	5.704	8.899
	S D	1.854	2.291	0.976
Decile Rank on I H S C Science	Mean	3.301	5.389	7.686
	S D	2.524	2.797	2.164
Decile Rank on I H S C History	Mean	3.147	4.275	5.851
	S D	2.629	2.706	2.141
Decile Rank on I H S C Total	Mean	2.623	4.862	7.681
	S D	2.218	2.532	1.965
Decile Rank on O S P E Total	Mean	3.233	4.353	6.378
	S D	2.326	2.462	2.381
Decile Rank on O S P E Reading	Mean	3.296	4.715	6.984
	S D	2.370	2.522	2.307
Number of Students		382	387	188

groups of students enrolled in remedial mathematics, mathematics 2, and mathematics 21. As is indicated by the standard deviations listed in Table 31, the variability of college mathematics grade point averages is in all instances greater than the variability of general college grade point averages for the same group. This result is to be expected, since the college grade point averages do not represent means of individual grades, as do the college mathematics grade point averages, but rather the means of the grade point averages of the individual students.

Coefficients of Correlation for Three Groups of Mathematics Students

Using the means and standard deviations of the various distributions listed in Table 31, Pearson product moment coefficients of correlation between all pairs of variables were computed for the groups of students in remedial mathematics, mathematics 2, and mathematics 21. These coefficients, shown in Table 32, are listed in groups of three. In each group, reading from top to bottom, are found the coefficients of correlation for the groups of students in remedial mathematics, mathematics 2, and mathematics 21. All correlation coefficients listed are positive. They were computed by means of I B M machinery, using the formulas listed in Appendix I.

Coefficients of correlation of the different variables with the mathematics placement test were computed using both deciles and raw scores. In the distributions which showed a relatively high concentration of scores in the lower or upper deciles, as in remedial mathematics and mathematics 21, the raw scores showed somewhat higher coefficients of correlation than did the decile rankings. The upper deciles in par-

TABLE 32

CORRELATION COEFFICIENTS BETWEEN ALL PAIRS OF VARIABLES FOR STUDENTS ENROLLED IN REMEDIAL MATHEMATICS, MATHEMATICS 2, AND MATHEMATICS 21

Variable			X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}*
	Coll.	R	.221	.246	.311	.424	.322	.281	.411	.384	.373	.624	.378	.382
X_1	Math.	2	.333	.322	.297	.476	.231	.232	.371	.365	.335	.636	.428	.411
	Grade	21	.328	.535	.247	.345	.385	.186	.379	.343	.372	.797	.482	.419
	Place.	R		.964	.080	.312	.112	.109	.168	.102	.120	.128	.184	.167
X_2	Test	2		.953	.098	.441	.198	.119	.261	.202	.204	.199	.145	.142
	Decile	21		.821	.279	.526	.200	.198	.345	.285	.321	.296	.211	.119
	Place.	R			.094	.314	.124	.131	.187	.129	.145	.162	.215	.197
X_3	Test	2			.104	.422	.185	.107	.246	.194	.194	.199	.139	.127
	Raw Score	21			.355	.533	.271	.213	.414	.385	.407	.498	.413	.283
	I H S C	R				.217	.396	.614	.760	.642	.552	.461	.203	.341
X_4	English	2				.328	.435	.566	.769	.659	.620	.444	.263	.381
	Decile	21				.358	.449	.560	.814	.675	.588	.393	.187	.214
	I H S C	R					.403	.264	.513	.187	.238	.180	.168	.100
X_5	Math.	2					.449	.306	.615	.395	.413	.368	.322	.259
	Decile	21					.380	.184	.503	.335	.321	.308	.215	.119
	I H S C	R						.524	.731	.432	.454	.253	.091	.125
X_6	Science	2						.441	.727	.447	.484	.285	.194	.255
	Decile	21						.462	.745	.474	.456	.438	.201	.217
	I H S C	R							.839	.514	.517	.321	.193	.284
X_7	History	2							.788	.526	.520	.406	.256	.335
	Decile	21							.761	.466	.450	.267	.070	.257
	I H S C	R								.617	.598	.404	.218	.296
X_8	Total	2								.664	.663	.459	.316	.387
	Decile	21								.654	.612	.487	.206	.272
	O S P E	R									.857	.508	.246	.298
X_9	Total	2									.869	.525	.349	.435
	Decile	21									.893	.467	.298	.310
	O S P E	R										.474	.276	.289
X_{10}	Reading	2										.513	.323	.403
	Decile	21										.499	.311	.304
	College	R											.434	.530
X_{11}	Grade	2											.516	.577
	Points	21											.486	.467
	H. S. Math.	R												.769
X_{12}	Grade	2												.809
	Points	21												.759

*X_{13}: High School Average Grade Points

ticular represent a comparatively large range of raw scores. In the mathematics 2 group, however, very little difference may be observed between the correlation coefficients computed from the raw scores and those determined from the decile groupings. The range of raw scores included in each decile may be seen by reference to Tables 7, 8, and 9.

The coefficients of correlation between college mathematics course grades and the other variables are found by reading the entries across the top of Table 32. It is seen at once that college mathematics grades appear to correlate more highly with general college grade points than with any of the other variables. Unfortunately, however, this variable cannot be used as a criterion of performance of college freshmen in mathematics courses.

The coefficients of correlation listed in Table 32 are based upon all students in each group who received a final course grade in mathematics of A, B, C, D, or F. It would be extremely difficult to determine how many of the students who made a grade of F in each group did so because of lack of proper mathematical background for the course in which they were enrolled and how many failed for other reasons, such as illness or lack of application. Certainly some of the students who made a grade of F in a given course did so because of lack of preparation and should have been placed in the next lower course in the sequence. Also, it is likely that some students who made a course grade of A had not been working to capacity and might have profited more by taking the next higher course.

Designation of Groups by Letters L, M, and U

The comments of the last paragraph suggest the possibility that a large percentage of the students who made grades of either A or F might be thought of as "misplaced persons," while the students who made grades of B, C, or D and who were previously defined as "properly placed persons" might be considered as being more nearly typical of the group to which they belong. This "typical group" of students was designated by the letter M. Similarly, the "A," or upper, students were designated by the letter U, and the "F," or lower, students, by the letter L. By this means, the students in remedial mathematics were divided into three groups: R-U, R-M, and R-L. Groups in mathematics 2 and mathematics 21 were designated similarly.

Means and Standard Deviations for Groups L, M, and U

Tables 33 and 34 show the means and standard deviations of the distributions on the basis of all variables, for the nine groups of students in remedial mathematics, mathematics 2, and mathematics 21. Of significance, in these tables, is the fact that with respect to all variables except the mathematics placement test and the mathematics section of the I H S C, the means of the group R-U are higher than the means of the group 2-L and the means of the group 2-U are higher than the corresponding means of the group 21-L. One would expect this to be true of the mathematics placement test, since the students were originally grouped largely on the basis of the placement test. The fact that it is also true of the mathematics section of the I H S C indicates that a close agreement would be likely to exist between segregations based upon each of the two tests.

Coefficients of Correlation for Groups R-M, 2-M, and 21-M

In Table 35 are listed the coefficients of correlation between

TABLE 33

MEANS AND STANDARD DEVIATIONS OF DISTRIBUTIONS ON PLACEMENT TEST AND I H S C, FOR GROUPS L, M, AND U, OF STUDENTS ENROLLED IN REMEDIAL MATHEMATICS, MATHEMATICS 2, AND MATHEMATICS 21

Group		Mathematics Placement Test		Iowa High School Content Examination				
		Raw Scores	Dec-iles	English	Mathe-matics	Science	History	Total
R-L	Mean	6.325	2.071	2.591	1.201	2.084	2.097	1.331
	S D	2.934	1.242	2.240	1.057	1.832	2.121	1.506
R-M	Mean	7.177	2.441	3.946	2.589	3.452	3.238	2.747
	S D	3.137	1.392	2.611	1.840	2.522	2.397	1.359
R-U	Mean	8.471	3.000	5.471	3.588	4.853	4.794	4.559
	S D	1.943	1.356	2.906	2.034	2.700	2.802	2.700
R	Mean	7.120	2.395	3.809	2.398	3.301	3.147	2.623
	S D	4.064	1.380	2.675	1.854	2.524	2.629	2.218
2-L	Mean	13.188	5.225	3.949	3.964	4.558	3.442	3.601
	S D	3.359	1.350	2.399	2.393	2.972	2.697	2.542
2-M	Mean	15.062	6.113	4.918	5.956	5.408	4.346	4.945
	S D	3.281	1.386	2.648	2.070	2.716	2.662	2.419
2-U	Mean	17.923	7.038	7.192	7.500	7.385	5.692	7.269
	S D	3.902	1.152	2.267	1.687	2.081	2.465	1.552
2	Mean	14.910	6.017	4.898	5.704	5.389	4.275	4.862
	S D	3.509	1.433	2.225	2.291	2.797	2.706	2.532
21-L	Mean	23.043	8.413	6.326	8.630	6.457	5.022	6.587
	S D	3.276	0.654	2.913	1.115	1.661	2.975	2.585
21-M	Mean	26.097	8.893	6.533	8.746	7.533	5.672	7.484
	S D	4.803	0.919	2.599	1.035	2.146	2.664	1.899
21-U	Mean	31.465	9.360	8.035	9.477	8.779	6.802	8.826
	S D	3.920	0.347	1.921	0.151	1.245	2.584	1.005
21	Mean	26.963	8.941	6.835	8.899	7.686	5.851	7.681
	S D	5.177	0.839	2.586	0.976	2.164	2.141	1.965

TABLE 34

MEANS AND STANDARD DEVIATIONS OF DISTRIBUTIONS ON GRADE POINT AVERAGES AND O S P E, FOR GROUPS L, M, AND U, OF STUDENTS ENROLLED IN REMEDIAL MATHEMATICS, MATHEMATICS 2, AND MATHEMATICS 21

Group		College Grade Point Average		High School Grade Point Average		Ohio State Psychological Test	
		Mathematics	General	Mathematics	General	Reading	Total
R-L	Mean	0.000	0.857	1.747	2.201	2.071	2.110
	S D	0.000	0.729	0.796	0.651	1.903	1.969
R-M	Mean	2.092	1.849	2.347	2.683	3.481	3.297
	S D	0.746	1.392	1.608	0.648	2.216	2.201
R-U	Mean	4.000	2.706	2.868	3.132	5.206	5.265
	S D	0.000	0.892	0.826	0.572	2.728	2.522
R	Mean	1.840	1.725	2.272	2.626	3.296	3.233
	S D	1.240	0.859	0.859	0.687	2.370	2.326
2-L	Mean	0.000	1.232	2.297	2.529	3.428	3.138
	S D	0.000	0.754	0.704	0.590	2.310	2.271
2-M	Mean	2.007	2.113	2.759	2.896	4.753	4.380
	S D	0.731	1.448	1.599	0.642	2.439	2.311
2-U	Mean	4.000	3.250	3.462	3.462	7.192	7.231
	S D	0.000	0.541	0.746	0.898	2.089	2.066
2	Mean	1.783	2.032	2.724	2.868	5.215	4.853
	S D	1.157	0.858	0.823	0.676	2.522	2.462
21-L	Mean	0.000	1.478	2.804	2.913	6.022	5.674
	S D	0.000	0.560	0.655	0.502	2.318	2.547
21-M	Mean	2.205	2.467	3.143	3.123	6.705	6.033
	S D	0.711	1.076	1.211	0.544	2.369	2.326
21-U	Mean	4.000	3.477	3.779	3.640	8.291	7.733
	S D	0.000	0.372	0.363	0.407	1.665	1.878
21	Mean	2.346	2.577	3.247	3.215	7.484	6.878
	S D	1.281	0.774	0.645	0.565	2.307	2.381

TABLE 35

CORRELATION COEFFICIENTS BETWEEN GRADES IN COLLEGE MATHEMATICS AND A SELECTED GROUP OF VARIABLES, FOR GROUPS R-M, 2-M, AND 21-M

	Variable	Group	X_3 Placement Test Raw Score	X_5 I H S C Math. Decile	X_9 O S P E Total Decile	X_{12} High School Mathematics Grade Points
	College	R-M	.233	.271	.245	.183
X_1	Mathematics	2-M	.158	.349	.177	.354
	Grade Points	21-M	.289	.254	.280	.217
	Placement	R-M		.309	.102	.165
X_3	Test	2-M		.322	.100	.018
	Raw Score	21-M		.548	.303	.247
	I H S C	R-M			.037	.003
X_5	Mathematics	2-M			.276	.226
	Decile	21-M			.259	.101
	O S P E	R-M				.156
X_9	Total	2-M				.311
	Decile	21-M				.187

college mathematics grade point averages and a selected group of other variables, for the groups R-M, 2-M, and 21-M. These coefficients of correlation were computed using some of the means and standard deviations of distributions which were shown in Tables 33 and 34. Coefficients of correlation are shown, for each of the three groups, between grade point averages in college mathematics and those variables which are used in Chapter VI in the determination of regression equations and discriminant functions. In most instances these coefficients of correlation are quite comparable to the corresponding coefficients shown in Table 32 for groups of students enrolled in remedial mathematics, mathematics 2, and mathematics 21.

Summary of Chapter V

In this chapter were presented the means and standard deviations of the distributions, on the basis of all variables on which data were available, for groups of students enrolled in remedial mathematics, mathematics 2, and mathematics 21. These means and standard deviations were used in the computation of coefficients of correlation, on all pairs of variables, for each of the three groups of mathematics students.

In each of the three major groups of mathematics students, the lower, middle, and upper groups of students were defined by means of the letters L, M, and U, respectively. Means and standard deviations, on the basis of all variables, were determined for each of the nine groups of students defined in this manner. Coefficients of correlation were presented for groups R-M, 2-M, and 21-M on the basis of those variables which are selected in Chapter VI for use in the determination of regression equations and discriminant functions.

CHAPTER VI

DEVELOPMENT OF REGRESSION EQUATIONS AND DISCRIMINANT FUNCTIONS

Introduction

Two distinct problems present themselves at this point. One is the problem of predicting as accurately as possible the final course grade in college mathematics of a given individual. This may be done by means of setting up regression equations which give, on the basis of a chosen set of variables, the final grade most likely to be made by the individual. Different regression equations may be developed for each group, such as remedial mathematics, mathematics 2, and mathematics 21. A basic assumption, which is made in applying any of these regression equations, is that the individual actually belongs to the group for which the regression equation was developed. This leads to the second problem; namely, that of properly identifying the individual with one of the groups. The tool which is used in this study in an attempt to solve this problem is the discriminant function which was developed by Fisher.[1]

Ideally, one might hope to divide students into approximately homogeneous groups with respect to mathematical background and ability in such a way that no student will fail because of having been placed in

[1]R. A. Fisher, "The Use of Multiple Measurements in Taxonomic Problems," Annals of Eugenics VII (1936), 179-188. For a brief discussion of the discriminant function, see Appendix II.

the wrong group. At the other extreme, it would be desirable to avoid placing a student in a course in which he merely reviews material which he has studied previously. Actually, while it appears to be impossible to eliminate misplacements entirely, one of the major purposes of this study is to aid in finding means by which their number may be reduced to a minimum.

Criteria Used in Choice of Variables

Among the criteria to be kept in mind in making a choice of the variables to be used in setting up a regression equation or a discriminant function are the following:

1. The coefficient of correlation between college mathematics grades and the variable chosen should be as high as possible.

2. The intercorrelations between all pairs of the variables chosen should be as low as possible, in order to insure the least amount of overlap.

3. For each variable chosen, the amount of separation between the distributions for remedial mathematics, mathematics 2, and mathematics 21 should be a maximum. This implies that, with respect to each variable, the means for the three distributions should be well separated and that the standard deviations, or the interquartile ranges, should be small.

While the first of these criteria is of particular importance in setting up a regression equation, the third is of greater significance in determining a discriminant function. It is not necessarily true, therefore, that the variables which appear to be best suited to the solution of one of the two problems are also the best ones to use in the solution of the other. In the case of either the regression equation or the discriminant function, it is desirable, from the standpoint of simplicity, to keep the number of variables used to a minimum. It is possible that the increase in reliability gained through the use of a larger number of variables is not sufficient to justify the additional computation involved.

Selection of Variables to be Used

An examination of Table 32 indicates that college grade points constitute the best single predictor of final grades in college mathematics. Since this criterion cannot be used in predicting grades of entering college freshmen, it was decided to use the mathematics placement test, the mathematics section of the I H S C, the O S P E total, and grade point averages in high school mathematics, as the independent variables in the regression equations. Reasons for the selection of these variables are given in the following paragraphs.

For all three of the major groups of mathematics students, the variables which showed the greatest separation between distributions were the mathematics placement test, the I H S C total, and the mathematics section of the I H S C. This statement may be verified by an examination of the means and probable errors of the various distributions listed in Table 21, or by reference to Figures 1, 2, and 3. Figure 1 shows graphically the percentage distribution of mathematics placement test scores, by deciles, for groups of students enrolled in remedial mathematics, mathematics 2, and mathematics 21. For comparison, the percentage dis-

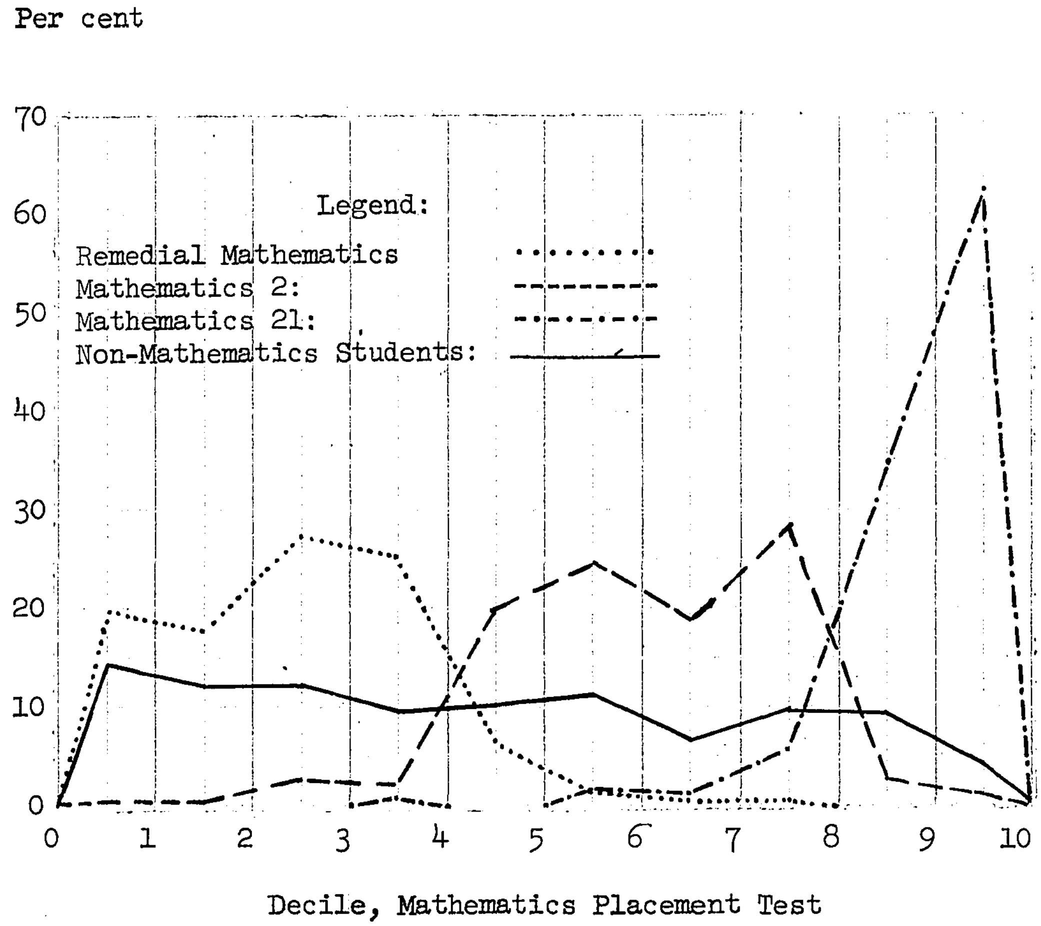

Figure 1. Percentage Distribution of Placement Test Scores, by Deciles, for groups of Students in Remedial Mathematics, Mathematics 2, and Mathematics 21.

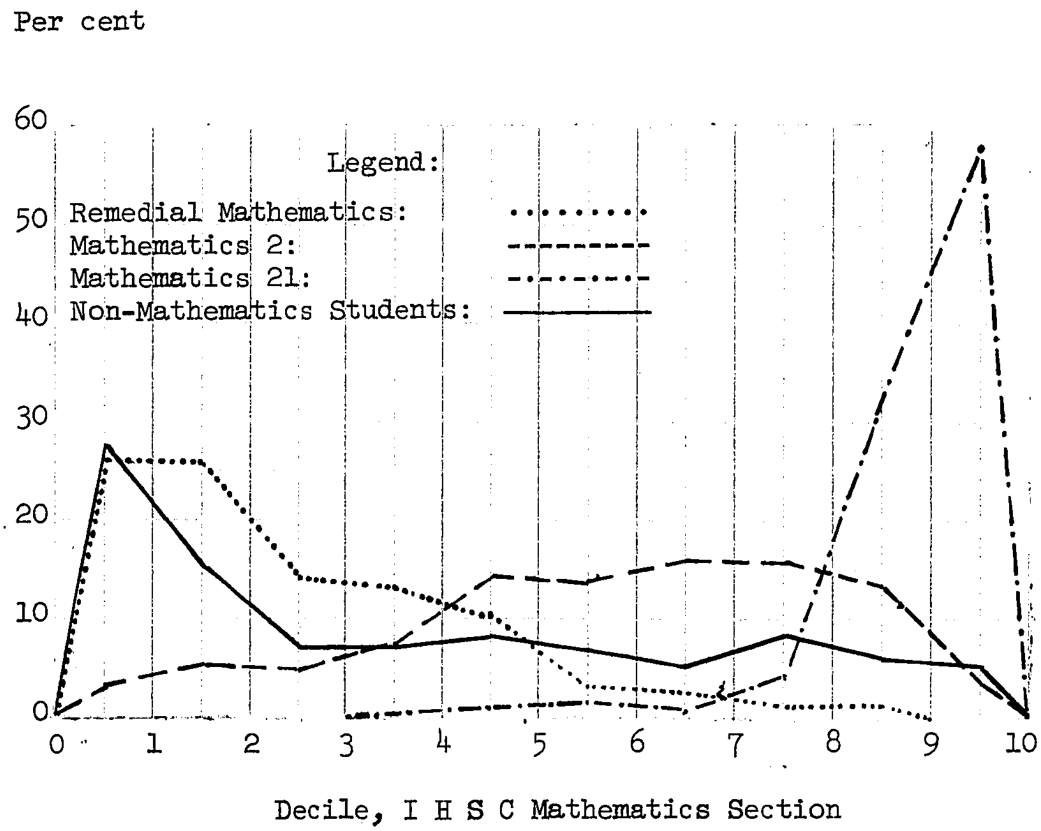

Figure 2. Percentage Distribution, by Deciles, of Scores on Mathematics Section of Iowa High School Content Examination, for groups of Students in Remedial Mathematics, Mathematics 2, and Mathematics 21.

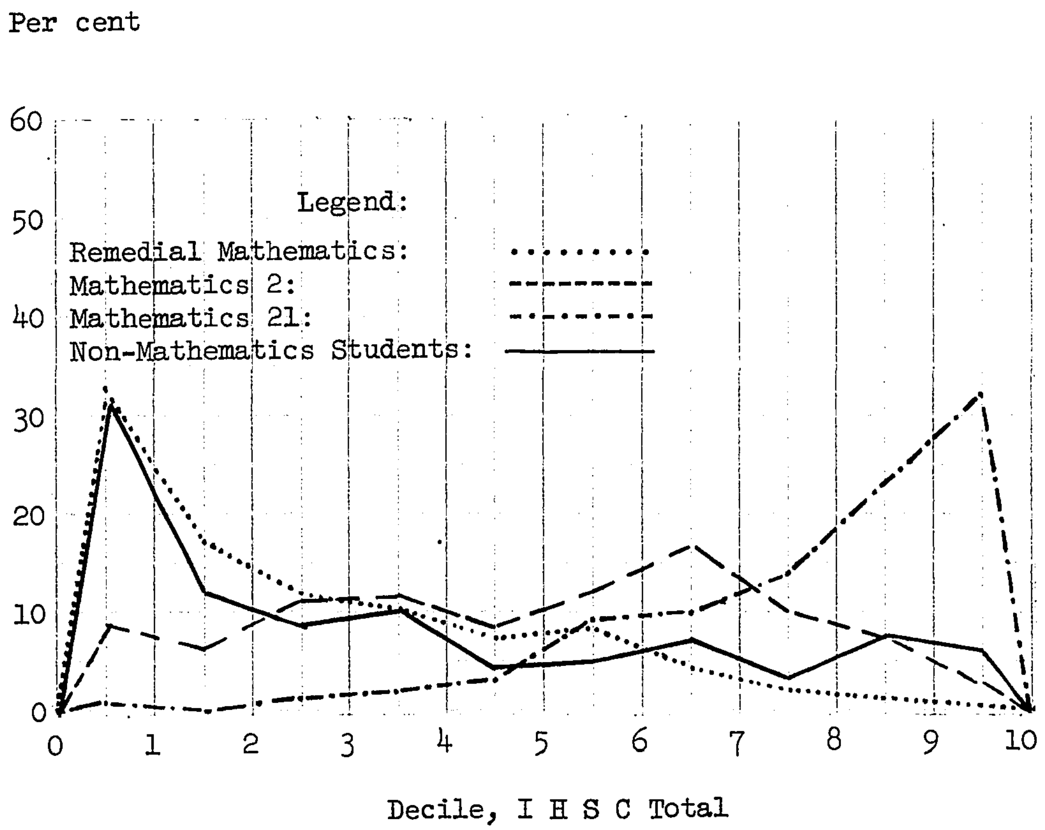

Figure 3. Percentage Distribution, by Deciles, of Iowa High School Content Examination Total, for groups of Students in Remedial Mathematics, Mathematics 2, and Mathematics 21.

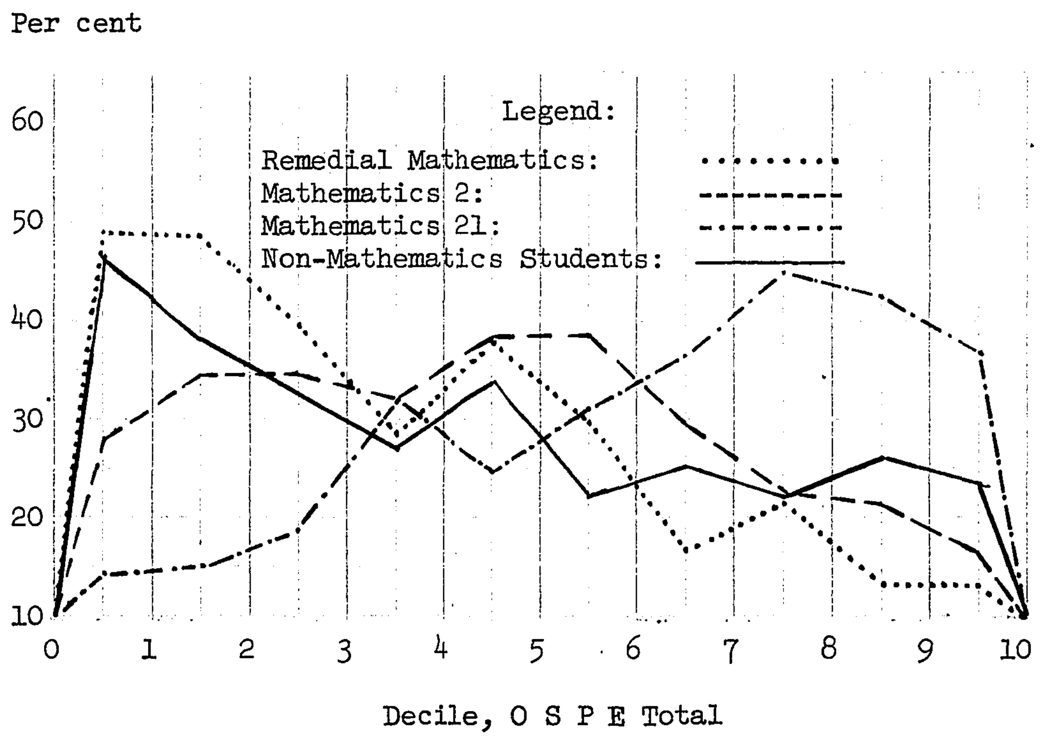

Figure 4. Percentage Distribution, by Deciles, of Ohio State Psychological Examination Total, for groups of Students in Remedial Mathematics, Mathematics 2, and Mathematics 21.

tribution of non-mathematics students is also shown. Figures 2 and 3 show similar distributions, for the same groups of students, on the mathematics section of the I H S C, and the I H S C total, respectively.

In deciding upon a measure of general ability to use in the determination of regression equations and discriminant functions, the choice was made from the I H S C total, the O S P E total, and the O S P E reading section. The fact that the I H S C total and the I H S C mathematics section have a comparatively high intercorrelation indicates that probably little would be gained by the use of both variables. On comparing coefficients of correlation of each of these two tests with either the total or the reading section of the O S P E, the mathematics section of the I H S C appears to have a lower intercorrelation, and consequently a smaller degree of overlap, with the O S P E, than does the I H S C total. For these reasons, it was decided to use the O S P E rather than the I H S C total as a measure of general ability. The O S P E total was decided upon, rather than the reading section, though the latter could probably have served as well, since their intercorrelation is high, and they have approximately the same correlation with college mathematics grades. Figure 4 shows graphically the percentage distribution, by deciles, of the O S P E total for each of the three groups of mathematics students, as well as for non-mathematics students.

High school mathematics grade points and high school average grade points show a relatively high intercorrelation, indicating that little would be gained by using both in a regression equation. Both are open to the objection that they show little separation between the distributions for remedial mathematics, mathematics 2, and mathematics 21.

Because of their slightly higher correlation with college mathematics grades, it was decided to use high school mathematics grades, rather than high school average grade points, as a fourth variable. Also, placement test raw scores were decided upon, rather than deciles, because of the somewhat higher correlation shown with college mathematics grades for the remedial mathematics and mathematics 2 groups. Consequently, the variables decided upon were X_3 (mathematics placement test raw score), X_5 (decile on mathematics section of I H S C), X_9 (O S P E total decile), and X_{12} (high school mathematics grade points). (See Table 33.)

Determination of Regression Equations

To prevent errors in computation, two methods were used to determine the regression equations. One of the methods, described by Garrett,[1] employs partial coefficients of correlation. These partial coefficients were useful also in determining multiple coefficients of correlation which will be discussed later. The other method of determining regression equations, used by Johnson,[2] applies the Doolittle simultaneous linear equations. Table 36 shows the regression equations, written in terms of the standard deviations of the variables as units of measurement. When the equations are written in this form, the coefficients are abstract dimensionless numbers and show the relative weights attached to each of the variables.

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[1]Henry E. Garrett, Statistics in Psychology and Education (New York: Longmans, Green & Co., 1926) pp. 221-265.

[2]Palmer O. Johnson, Statistical Methods in Research (New York: Prentice-Hall, 1949) pp. 327-343.

TABLE 36

REGRESSION EQUATIONS, IN TERMS OF STANDARD DEVIATIONS, BASED UPON FOUR VARIABLES, FOR STUDENTS ENROLLED IN REMEDIAL MATHEMATICS, MATHEMATICS 2, AND MATHEMATICS 21

Course	Predicted Grade Point Average		Placement Test Raw Score	I H S C Math. Decile	O S P E Total Decile	H. S. Math. Grade Point Average
Remedial Mathematics	X	=	$0.061(\frac{X_3}{\sigma_3})$	$+ 0.314(\frac{X_5}{\sigma_5})$	$+ 0.256(\frac{X_9}{\sigma_9})$	$+ 0.248(\frac{X_{12}}{\sigma_{12}})$
Mathematics 2	X	=	$0.143(\frac{X_3}{\sigma_3})$	$+ 0.275(\frac{X_5}{\sigma_5})$	$+ 0.134(\frac{X_9}{\sigma_9})$	$+ 0.273(\frac{X_{12}}{\sigma_{12}})$
Mathematics 21	X	=	$0.337(\frac{X_3}{\sigma_3})$	$+ 0.067(\frac{X_5}{\sigma_5})$	$+ 0.102(\frac{X_9}{\sigma_9})$	$+ 0.298(\frac{X_{12}}{\sigma_{12}})$

Coefficients of Multiple Correlation, Partial Standard Deviations, and Probable Errors of Estimate

Using the regression equations of Table 36, the coefficients of multiple correlation for remedial mathematics, mathematics 2, and mathematics 21 were found to be respectively 0.585, 0.586, and 0.618, as shown in Table 37. These coefficients represent the correlation between the actual raw scores made on the mathematics placement test and the raw scores as predicted by the regression equations of Table 36. The partial standard deviations listed in Table 37 represent a measure of the variability of grade point averages in mathematics courses with the influence of variables X_3, X_5, X_9, and X_{12} held constant. These partial standard deviations may be compared with the standard deviations listed for college mathematics grade point averages in Table 31. These standard deviations are 1.240, 1.157, and 1.281, for remedial mathematics, mathematics

TABLE 37

COEFFICIENTS OF MULTIPLE CORRELATION, PARTIAL STANDARD DEVIATIONS, AND PROBABLE ERRORS OF ESTIMATE OF PREDICTED MATHEMATICS GRADE POINT AVERAGES FOR REMEDIAL MATHEMATICS, MATHEMATICS 2, AND MATHEMATICS 21

Course	Coefficient of Multiple Correlation	Partial Standard Deviation	Probable Errors of Estimate, Based Upon: X_3, X_5, X_9, and X_{12}	X_3	X_5	X_9	X_{12}
Remedial Mathematics	0.585	1.006	0.678	0.810	0.758	0.773	0.775
Mathematics 2	0.586	0.938	0.633	0.739	0.686	0.726	0.705
Mathematics 21	0.618	1.007	0.679	0.730	0.811	0.811	0.757

2, and mathematics 21 respectively. In each group, the distribution is observed to be narrowed down by this process of "partialing out" the effect of other variables, though the extent of this narrowing down is not sufficient to enable one to predict, as accurately as one might wish, the mathematics grade point average of a given individual. In each of the three groups of mathematics students, the probable error of prediction is of the order of two-thirds of a grade point. This statement means that, on the average, in fifty per cent of all cases the true grade point average lies within an interval which extends approximately two-thirds of a grade point in either direction from the predicted value and in the remaining fifty per cent of all cases the true grade point average lies outside this interval.

For purpose of comparison, the probable errors of estimate, based upon each of the four variables individually, were computed. These probable errors of estimate are also listed in Table 37. The maximum reduction effected in the probable error of estimate by the use of four variables instead of one is .132 grade points. If a single variable were to be used, the best prediction for students in mathematics 21 could be obtained by use of the placement test, while for students in remedial mathematics and mathematics 2, the mathematics section of the I H S C appears to be the best predictor. These facts suggest the possibility that a prediction based upon these two variables alone might be almost as reliable as one based upon all four variables. Therefore, the coefficients of multiple correlation, the partial standard deviations, and the probable errors of estimate were computed using variables X_3, X_5, and X_9 and also using only variables X_3 and X_5. These data are shown in Table 38. To facilitate comparison, the corresponding data, using all four independent variables, are repeated from Table 37.

From Table 38 it appears that the drop in the coefficient of multiple correlation and the consequent increase in the probable error of estimate, due to a reduction in the number of independent variables used, is greatest for the remedial mathematics group and is least for the group of mathematics 21 students. Also, a comparison with the probable errors of estimate shown in Table 37 indicates that the probable error of estimate for mathematics 21 is only very slightly less if variables X_3 and X_5 are used, than if only X_3 is used. The difference between the probable errors is 0.003, which is negligible. A similar comparison for remedial mathematics and mathematics 2 reveals that, for these two courses, a prediction based upon X_3 and X_5 is only very slightly

TABLE 38

COMPARISON OF COEFFICIENTS OF MULTIPLE CORRELATION, PARTIAL STANDARD DEVIATIONS, AND PROBABLE ERRORS OF ESTIMATE, USING FOUR, THREE, AND TWO INDEPENDENT VARIABLES

Variables Used	Mathematics Course	Coefficient of Multiple Correlation	Partial Standard Deviation	Probable Error of Estimate
X_3, X_5, X_9, X_{12}	R	0.585	1.006	0.678
X_3, X_5, X_9	R	0.535	1.048	0.707
X_3, X_5	R	0.440	1.113	0.751
X_3, X_5, X_9, X_{12}	2	0.586	0.938	0.633
X_3, X_5, X_9	2	0.529	0.981	0.662
X_3, X_5	2	0.495	1.005	0.678
X_3, X_5, X_9, X_{12}	21	0.618	1.007	0.679
X_3, X_5, X_9	21	0.557	1.064	0.717
X_3, X_5	21	0.540	1.078	0.727

better than one based upon X_5 alone. Since the mathematics placement test appears to be the better criterion in the prediction of grades in mathematics 21, while the mathematics section of the I H S C shows a lower probable error in predicting grades in remedial mathematics and mathematics 2, regression equations were computed which are based upon both variables.

Regression Equations based upon two Variables

Table 39 shows regression equations, based upon the placement test raw score and the decile rank on the I H S C mathematics section,

TABLE 39

REGRESSION EQUATIONS, IN TERMS OF STANDARD DEVIATIONS, BASED UPON TWO VARIABLES, FOR STUDENTS ENROLLED IN REMEDIAL MATHEMATICS, MATHEMATICS 2, AND MATHEMATICS 21

Course	Predicted Grade Point Average		Placement Test Raw Score		I H S C Mathematics Decile
Remedial Mathematics	X	=	$0.156\left(\frac{X_3}{\sigma_3}\right)$	+	$0.477\left(\frac{X_5}{\sigma_5}\right)$
Mathematics 2	X	=	$0.172\left(\frac{X_3}{\sigma_3}\right)$	+	$0.479\left(\frac{X_5}{\sigma_5}\right)$
Mathematics 21	X	=	$0.574\left(\frac{X_3}{\sigma_3}\right)$	+	$0.107\left(\frac{X_5}{\sigma_5}\right)$

for the three major mathematics groups. The coefficients in the equations shown in Table 39 indicate that, in computing the predicted mathematics grade point averages of students in remedial mathematics and mathematics 2, approximately equal weights may be assigned to the raw score on the placement test. For the same two groups, the weights assigned to the mathematics section of the I H S C are also nearly equal and are in both cases approximately three times the weights assigned to the raw scores made on the placement test. However, in the case of mathematics 21, the weight of the placement test is approximately five times the weight of the I H S C mathematics section. The fact that the mathematics 21 group shows a high concentration at the upper end of the distribution on the mathematics section of the I H S C accounts at least in part for the greater emphasis which the regression equation places on the placement test. It should be emphasized that the relative weights assigned to the two tests have been determined for students who were separated on the basis of the mathematics placement test and that they may therefore be expected to change if other means of separation are used.

Determination of Discriminant Functions

As indicated previously, the discriminant function is the tool used in this study in an attempt to solve the problem of properly identifying a given individual with one of the three groups of mathematics students. The development and application of this function is discussed by Johnson.[1] Jackson[2] reports on the use of the discriminant function in separating students into ability groups.

It was decided to determine a discriminant function which would distinguish between the "typical groups" R-M and 2-M which were defined on page 66. Similarly, a function is to be found which sets up a line of demarcation between the "typical groups" of students in mathematics 2 and mathematics 21.

Although correlations enter into the computation of a discriminant function, a more important factor to consider is the degree of separation produced between groups by the variables used in setting up the function. The data of Tables 33 and 34 may be used to determine the de-

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[1]Palmer O. Johnson, Statistical Methods in Research (New York: Prentice-Hall, 1949) pp. 343-357.

[2]Robert Jackson, "The Selection of Students for Freshman Chemistry by Means of Discriminant Functions." Journal of Experimental Education, XVIII (March, 1950), 209-214.

gree of separation produced by different criteria on the "typical groups" R-M, 2-M, and 21-M. For variables X_3, X_5, X_9, and X_{12}, which were used in the determination of regression equations, these data are shown graphically in Figures 5, 6, 7, and 8, respectively. As should be expected, X_3, or the mathematics placement test, shows the best separation between pairs of the three groups. Variable X_5, which is the mathematics section of the I H S C, shows no overlap between the interquartile ranges of the groups R-M, 2-M, and 21-M. Variable X_9, which is the O S P E total decile, shows a better degree of separation, within each course group, among those students making high, average, or failing grades, than it does among the groups R-M, 2-M, and 21-M. It might therefore be expected to serve better as a means of separating grade groups within a given course group than as a criterion for placement in the proper course group. Of the four variables considered, X_{12}, or high school mathematics grade point averages, shows the greatest degree of overlap between pairs of interquartile ranges of the groups R-M, 2-M, and 21-M.

Discriminant functions were determined first in terms of the four variables X_3, X_5, X_9, and X_{12}. Then, since X_{12} appeared to be the least effective of the four variables in distinguishing among the three groups, discriminant functions were written in terms of X_3, X_5, and X_9. Finally, X_9 was dropped, and functions were written in terms of only X_3 and X_5. These discriminant functions are shown in Table 40.

The relatively small coefficients of X_9 and X_{12} in the discriminant functions of Table 40 indicate that little advantage would be gained through the use of all four variables. Consequently, distribution charts using variables X_3 and X_5 were made for each of the upper, middle,

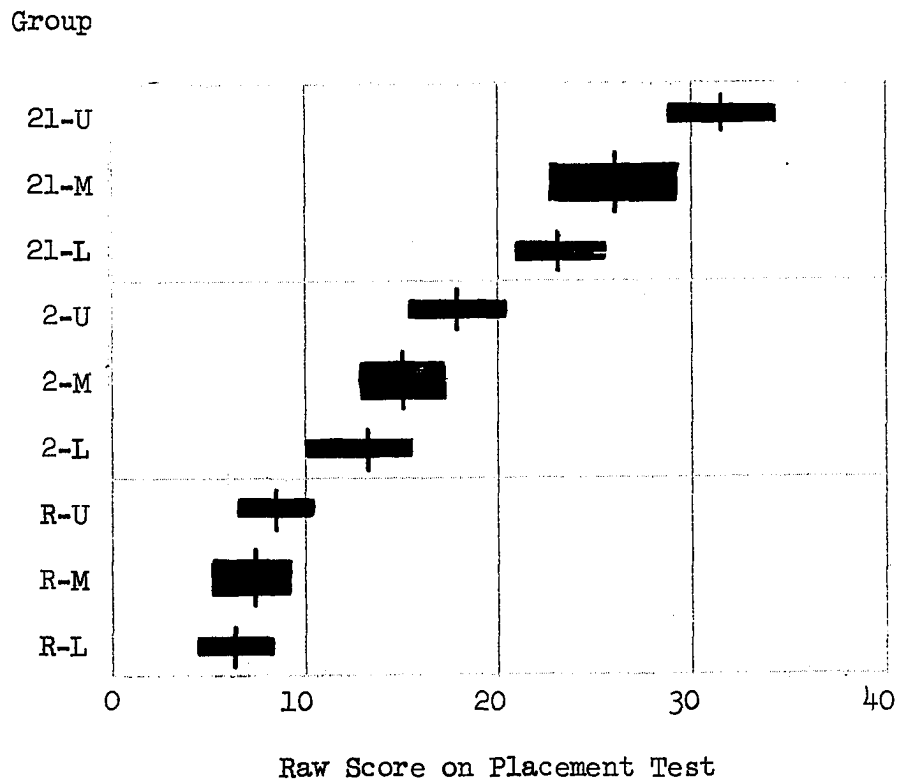

Figure 5. Means and Interquartile Ranges of Distributions on Raw Scores of Placement Test, for Students in Remedial Mathematics, Mathematics 2, and Mathematics 21.

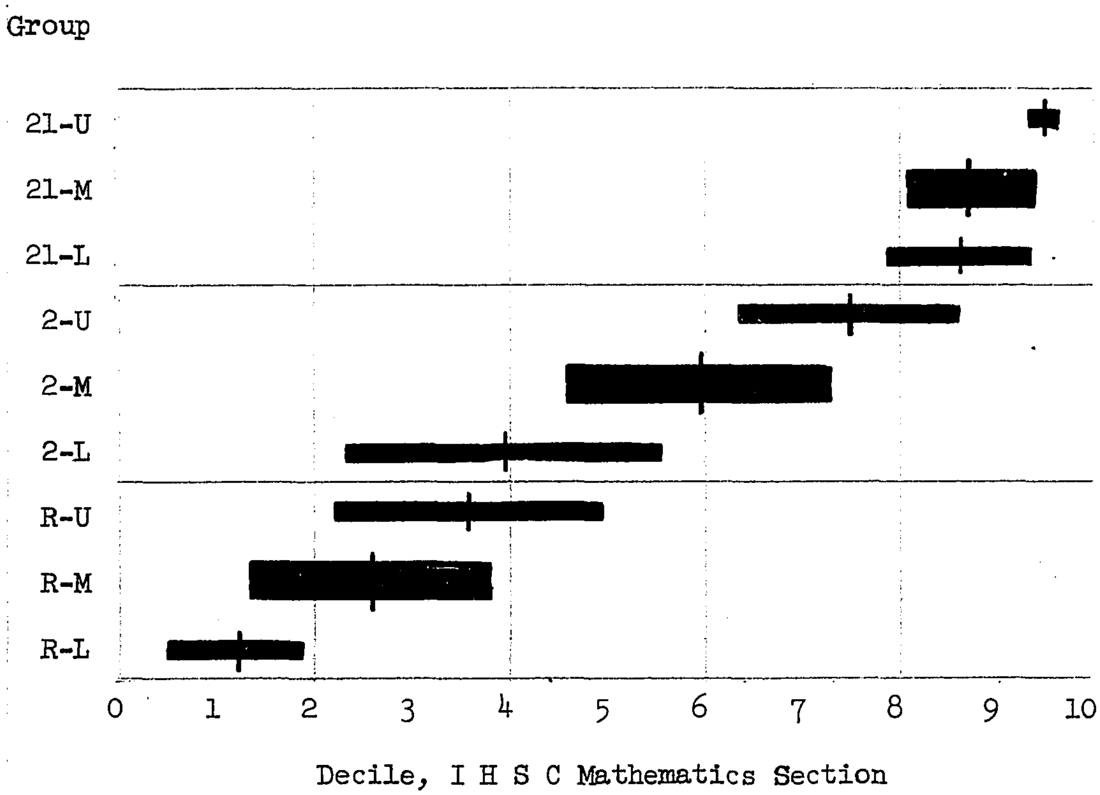

Figure 6. Means and Interquartile Ranges of Decile Ranks on Mathematics Section of Iowa High School Content Examination, for Students in Remedial Mathematics, Mathematics 2, and Mathematics 21.

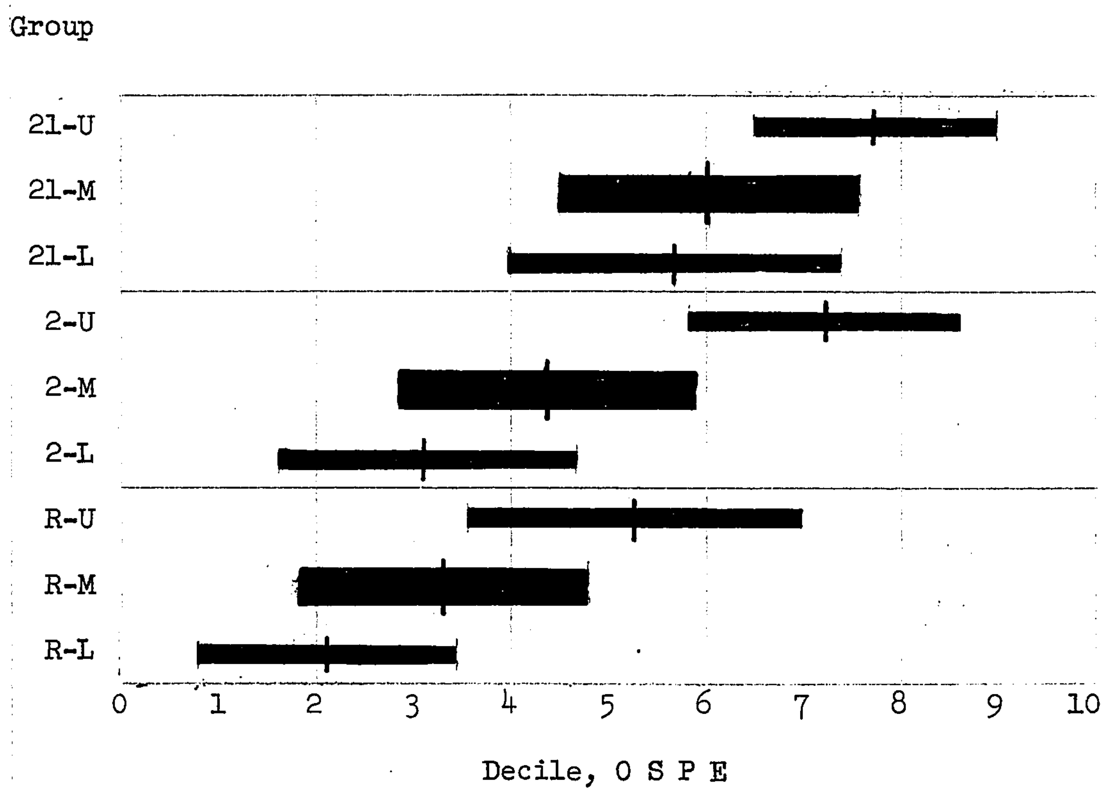

Figure 7. Means and Interquartile Ranges of Decile Rankings on O S P E, for Students in Remedial Mathematics, Mathematics 2, and Mathematics 21.

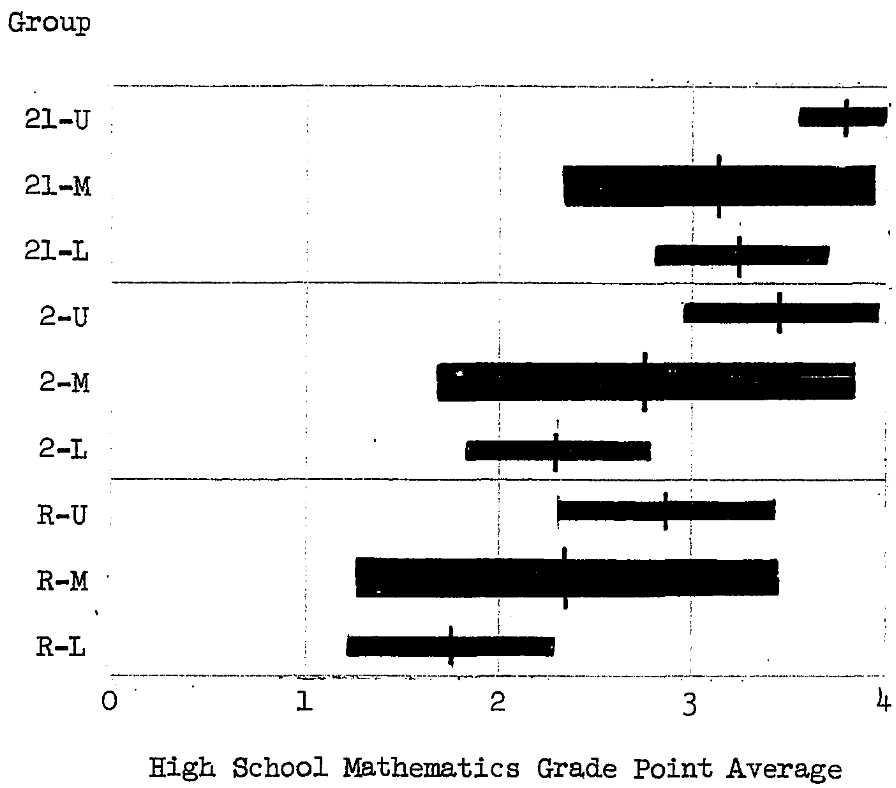

Figure 8. Means and Interquartile Ranges of High School Mathematics Grade Point Averages for Students in Remedial Mathematics, Mathematics 2, and Mathematics 21.

TABLE 40

DISCRIMINANT FUNCTIONS FOR GROUPS R-M, 2-M, AND 21-M

Variables Used	Function Based Upon Groups	Discriminant Function								
		Placement Test Raw Score		I H S C Math. Decile		O S P E Total Decile		H.S.Math. Gr. Pt. Average		
X_3, X_5, X_9 and X_{12}	R-M and 2-M	$1000X_3$	+	$798X_5$	+	$42X_9$	+	$209X_{12}$	-	15,692
X_3, X_5, X_9 and X_{12}	2-M and 21-M	$1000X_3$	+	$444X_5$	+	$20X_9$	+	$52X_{12}$	-	25,426
X_3, X_5, X_9	R-M and 2-M	$1000X_3$	+	$919X_5$	-	X_9			-	15,514
X_3, X_5, X_9	2-M and 21-M	$1000X_3$	+	$512X_5$	-	$69X_9$			-	25,087
X_3 and X_5	R-M and 2-M	$1000X_3$	+	$805X_5$					-	14,991
X_3 and X_5	2-M and 21-M	$1000X_3$	+	$472X_5$					-	24,281

and lower groups in remedial mathematics, mathematics 2, and mathematics 21. The functions $1000X_3 + 800X_5 - 15{,}000$ and $1000X_3 + 500X_5 - 24{,}500$ represent close approximations to the functions listed in Table 40 for the variables X_3 and X_5. When these functions are equated to zero, the equations may be written in the forms $5X_3 + 4X_5 = 75$ and $2X_3 + X_5 = 49$. The lines having these equations are drawn in the distribution charts of

Tables 41 to 49, inclusive, in order to give a visual picture of the separations produced by these discriminant functions.

Geometric Interpretation of the Discriminant Function

Where four variables are used, the discriminant function, equated to zero, may be interpreted geometrically as representing a hyperplane in a space of four dimensions. Each individual is assigned a position in this space, which is determined by the four coordinates X_3, X_5, X_9, and X_{12}. This hyperplane sets up a division by means of which all individuals are identified as belonging to one of two groups. A given individual is identified with the upper or lower of the two groups which are separated in this way, according as the discriminant function is positive or negative when evaluated for the individual. A similar geometric interpretation is possible, in spaces of three and two dimensions, respectively, when the number of variables is reduced to three (X_3, X_5, and X_9) and two (X_3 and X_5).

Separations Produced by Discriminant Functions

Tables 41 to 49, inclusive, represent distribution charts, on the basis of variables X_3 (mathematics placement test raw score) and X_5 (decile rank on the I H S C mathematics section) for the nine groups of students enrolled in remedial mathematics, mathematics 2, and mathematics 21. In all of these tables, the discriminating lines having equations $5X_3 + 4X_5 = 75$ and $2X_3 + X_5 = 49$ are shown.

Table 41 indicates that all students who received a grade of F in remedial mathematics would have been placed in remedial mathematics by the discriminant function. Of this group of students who made a fail-

TABLE 41

DISTRIBUTION CHART, ON BASIS OF VARIABLES X_3 AND X_5, FOR GROUP R-L, WITH DISCRIMINATING LINES SUPER-IMPOSED

X_3	X_5										Total
	1	2	3	4	5	6	7	8	9	10	
38											
37											
36											
35											
34											
33											
32											
31											
30											
29											
28											
27											
26											
25											
24											
23											
22											
21											
20											
19											
18											
17											
16											
15											
14											
13											
12	3										3
11											
10	3			1		1					5
9	6	5	3	1							15
8	5	1	1								7
7	6	2	1								9
6	6	2	1	1	1						11
5	2	3		1							6
4	2	2									4
3	5	3	1								9
2	3	1									4
1											
0	4										4
Total	45	19	7	4	1	1					77

TABLE 42

DISTRIBUTION CHART, ON BASIS OF VARIABLES X_3 AND X_5, FOR GROUP R-M WITH DISCRIMINATING LINES SUPER-IMPOSED

X_3	X_5: 1	2	3	4	5	6	7	8	9	10	Total
38											
37											
36											
35											
34											
33											
32											
31											
30											
29											
28											
27											
26											
25											
24											
23											
22											
21											
20											
19											
18		1									1
17									1		1
16		1		1							2
15											
14		1		1	1						3
13				1							1
12		2		1							3
11	1	3	3	3	3	3	1	1	1		19
10	4	5	3	5	5	3	2	3			30
9	5	8	8	5	5		1		1		33
8	9	10	11	7	4		1		1		43
7	8	8	6	1	9		2				34
6	5	8	3	5	2		1				24
5	5	10	3	3	3	1					25
4	6	8	3	1		1					19
3	4	2	1	2	1						10
2	2	3	2	1							8
1	2	2	1	2	1						8
0	5	1			1						7
Total	56	73	44	39	35	8	8	4	4		271

TABLE 43

DISTRIBUTION CHART, ON BASIS OF VARIABLES X_3 AND X_5, FOR GROUP R-U, WITH DISCRIMINATING LINES SUPER-IMPOSED

X_3	X_5 1	2	3	4	5	6	7	8	9	10	Total
38											
37											
36											
35											
34											
33											
32											
31											
30											
29											
28											
27											
26											
25											
24											
23											
22											
21											
20											
19											
18											
17					1						1
16											
15											
14				1			1				2
13											
12											
11											
10		1	2		2	2					7
9	1	3		2		1					7
8		2		2		1	2				7
7	1			3					1		5
6			1					1			2
5											
4			1								1
3											
2		1									1
1		1									1
0											
Total	2	8	4	8	3	4	3	1	1		34

TABLE 44

DISTRIBUTION CHART, ON BASIS OF VARIABLES X_3 AND X_5, FOR GROUP 2-L, WITH DISCRIMINATING LINES SUPER-IMPOSED

X_3	X_5: 1	2	3	4	5	6	7	8	9	10	Total
38											
37											
36											
35											
34											
33											
32											
31						1					1
30											
29											
28											
27											
26											
25											
24											
23											
22											
21											
20									1		1
19						1					1
18					1		1		1		3
17							1		1		2
16				1	1						2
15			1			2		1	1		5
14	2			1	1	2		1			7
13	1	3	3	4	1	1		1	1		15
12	5	3	2	1	3	4	1		1		20
11				2	2	1					5
10		2									2
9		1									1
8				1							1
7	1				1						2
6		1									1
5											
4											
3											
2											
1											
0											
Total	9	10	6	10	10	12	3	3	6		69

TABLE 45

DISTRIBUTION CHART, ON BASIS OF VARIABLES X_3 AND X_5, FOR GROUP 2-M, WITH DISCRIMINATING LINES SUPER-IMPOSED

X_3	X_5										Total
	1	2	3	4	5	6	7	8	9	10	
38											
37											
36											
35											
34											
33											
32											
31											
30											
29											
28										1	1
27											
26			1								1
25											
24							1		1		2
23		1				1					2
22								1			1
21									1	1	2
20						2		4	2	2	10
19					2	6	3	4	4	2	21
18	1	1			6	2	7	7	4	1	29
17			1		5	2	4	6	7	2	27
16		1	1	1	2	6	11	3	3		28
15	1		1		6	7	7	12	3		37
14		1	2	7	4	2	9	7	2	1	35
13		3	1	4	5	4	6	4	4		31
12	1	3	3	4	6	5	7	5	3	1	38
11			2	1	4		2	3	1		13
10					2						2
9		1	1	1			1				4
8		1	1			1			1		4
7				1	1		1				3
6											
5											
4											
3											
2		1									1
1											
0											
Total	3	13	14	19	43	38	59	56	36	11	292

TABLE 46

DISTRIBUTION CHART, ON BASIS OF VARIABLES X_3 AND X_5, FOR GROUP 2-U, WITH DISCRIMINATING LINES SUPER-IMPOSED

X_3	X_5: 1	2	3	4	5	6	7	8	9	10	Total
38											
37											
36											
35											
34											
33											
32											
31									1		1
30											
29											
28											
27											
26											
25											
24											
23										1	1
22											
21										1	1
20								1	6		7
19						1			1		2
18								1	1		2
17						1			2	1	4
16											
15					1						1
14					1	2		1			4
13					1	1					2
12										1	1
11											
10											
9											
8											
7											
6											
5											
4											
3											
2											
1											
0											
Total					3	5		3	11	4	26

TABLE 47

DISTRIBUTION CHART, ON BASIS OF VARIABLES X_3 AND X_5, FOR GROUP 21-L, WITH DISCRIMINATING LINES SUPER-IMPOSED

X_3	X_5: 1	2	3	4	5	6	7	8	9	10	Total
38											
37											
36											
35											
34											
33											
32											
31											
30										1	1
29										2	2
28											
27											
26											
25									1	1	2
24						1			2	2	5
23									2	1	3
22						1			2		3
21								1	1	1	3
20									1	1	2
19										1	1
18											
17											
16											
15									1		1
14											
13											
12											
11											
10											
9											
8											
7											
6											
5											
4											
3											
2											
1											
0											
Total						2		1	10	10	23

TABLE 48

DISTRIBUTION CHART, ON BASIS OF VARIABLES X_3 AND X_5, FOR GROUP 21-M WITH DISCRIMINATING LINES SUPER-IMPOSED

X_3	X_5										Total
	1	2	3	4	5	6	7	8	9	10	
38											
37										1	1
36										1	1
35										1	1
34										6	6
33										4	4
32									2	4	6
31									1	3	4
30									4	3	7
29									4	4	8
28									2	2	4
27									3	8	11
26									9	6	15
25								2	4	3	9
24									6	3	9
23							1	2	6	2	11
22								1	5		6
21				1			1		2	4	8
20								1	2	1	4
19									1	1	2
18											
17						1					1
16					1						1
15											
14								1	1		2
13											
12											
11											
10					1						1
9											
8											
7											
6											
5											
4											
3											
2											
1											
0											
Total				1	2	1	2	7	52	57	122

TABLE 49

DISTRIBUTION CHART, ON BASIS OF VARIABLES X_3 AND X_5, FOR GROUP 21-U, WITH DISCRIMINATING LINES SUPER-IMPOSED

X_3	X_5: 1	2	3	4	5	6	7	8	9	10	Total
38										2	2
37										1	1
36										1	1
35										4	4
34										8	8
33										6	6
32										3	3
31										3	3
30										4	4
29										2	2
28										2	2
27										1	1
26											
25									1	2	3
24										1	1
23										1	1
22										1	1
21											
20											
19											
18											
17											
16											
15											
14											
13											
12											
11											
10											
9											
8											
7											
6											
5											
4											
3											
2											
1											
0											
Total									1	42	43

ing grade in remedial mathematics, three had placement test scores above ten. Two of these three students had originally enrolled in remedial mathematics, while one had enrolled in mathematics 2 and then changed his enrollment to remedial mathematics. Table 42 shows that while thirty students in group R-M made scores above ten on the placement test, twenty-five would have been placed in mathematics 2 by the discriminant function. This fact indicates that the discriminant function is a somewhat better criterion for placement in the case of these students than is the placement test alone. Table 43 shows that there were three students in the group R-U whose placement test score should have placed them in mathematics 2. Enrollment in mathematics 2 is also indicated for these three students by the discriminant function.

Table 44 indicates that the discriminant function would have placed in remedial mathematics approximately one-third of the students who made failing grades in mathematics 2. Of the remaining two-thirds, about one-third might be classed as border-line cases who would have been placed in remedial mathematics with a slight upward shift of the discriminating line. An examination of Table 45 shows that of the twenty-six students in group 2-M who would have been placed in remedial mathematics by the discriminant function, thirteen, or one-half, made a grade of D in mathematics 2, ten made a grade of C, and three made a grade of B.

Tables 47, 48, and 49 show that there would have been little change in the enrollment of mathematics 21 students had they been enrolled on the basis of the discriminant function. This result is to be expected because of the relatively high rank of most mathematics 21 students on the I H S C mathematics section.

Future Revision of the Discriminant Functions

Because the placement test served as the original basis of separation, more weight is placed on it by the discriminant function than on the mathematics section of the I H S C. This is particularly true of the line which separates students into mathematics 2 and mathematics 21 groups, since more than ninety per cent of all students in mathematics 21 ranked in the ninth and tenth deciles on the mathematics section of the I H S C. Because of this factor of pre-selection of students on the basis of performance on the placement test, it is apparent that greater weight should be placed on the variable X_5. Since the discriminant function separating students in remedial mathematics from those in mathematics 2 was determined to be approximately $5X_3 + 4X_5 - 49$, it might be suggested that the separation be made initially on the basis of a discriminant function in which the coefficients of X_3 and X_5 are made equal. A future re-determination of the discriminant function might be undertaken to determine the coefficients which would reflect more nearly the relative values which should be expected without the disturbing factor of pre-selection on the basis of only one of the variables.

Since the variable X_5 has a very narrow range of values for students in mathematics 21, very little change would result in placement if the coefficients were also made equal in the discriminant function which separates students in mathematics 21 from those in mathematics 2. If the function $X_3 + X_5 - K$, where K is a properly chosen constant, can be used for the purpose of making placements in all three groups, the resulting advantage in simplicity is apparent. On this basis, a student would be placed in remedial mathematics, mathematics 2, or mathematics

21, depending on the value of a composite score which is determined by adding his raw score on the mathematics placement test and his decile rank on the mathematics section of the I H S C.

Composition of Groups, Based on Various Discriminant Functions

Table 50 shows the changes which would be produced, by the application of various discriminant functions, in the composition of the groups of students in remedial mathematics and mathematics 2. Since these students were placed originally on the basis of the mathematics placement test alone, the division is shown for various values of a discriminant function based upon the variable X_3 alone. These divisions may be compared with those produced by a discriminant function which is based upon both X_3 and X_5. Table 50 shows, for example, that approximately the same division is produced, so far as the total number of students in each course is concerned, by the two functions $X_3 - 12$ and $X_3 + X_5 - 16$. However, the function $X_3 + X_5 - 16$ places in remedial mathematics twenty-five of the sixty-nine students who made a grade of F in mathematics 2, as compared with twelve of the same group of failing students placed in remedial mathematics by the function $X_3 - 12$. Similar comparisons may be made for the other functions listed. It is seen in most instances that of the students transferred to remedial mathematics from mathematics 2 by the function $X_3 + X_5 - K$, a larger percentage comes from the group who failed in mathematics 2, than is the case when the function $X_3 - K$ is used. In the same way, Table 51 shows the effect of using various discriminant functions to separate the groups of mathematics 2 and mathematics 21 students.

TABLE 50

HYPOTHETICAL DIVISION, ON BASIS OF VARIOUS DISCRIMINANT FUNCTIONS, OF STUDENTS IN REMEDIAL MATHEMATICS AND MATHEMATICS 2

Basis of Division	Number of Students Placed in Remedial Mathematics							Number of Students Placed in Mathematics 2						
	Enrollment Unchanged			Transferred by Discriminant Function from:			Total	Enrollment Unchanged			Transferred by Discriminant Function from:			Total
	R-L	R-M	R-U	2-L	2-M	2-U		2-L	2-M	2-U	R-L	R-M	R-U	
Actual Enrollment	77	271	34				382	69	292	26				387
$X_3 - 10$	69	211	24	5	12	0	321	64	280	26	8	60	10	448
$X_3 - 11$	74	241	31	7	14	0	367	62	278	26	3	30	3	402
$X_3 - 12$	77	260	31	12	27	0	407	57	265	26	0	11	3	362
$X_3 - 13$	77	263	31	32	65	2	470	37	227	24	0	8	3	299
$5X_3 + 4X_5 - 75$	77	246	31	25	26	0	405	44	266	26	0	25	3	364
$X_3 + X_5 - 16$	76	242	28	25	25	0	396	44	267	26	1	29	6	373
$X_3 + X_5 - 17$	77	251	31	31	37	0	427	38	255	26	0	20	3	342
$X_3 + X_5 - 18$	77	258	31	39	50	0	455	30	242	26	0	13	3	314

TABLE 51

HYPOTHETICAL DIVISION, ON BASIS OF VARIOUS DISCRIMINANT FUNCTIONS, OF STUDENTS IN MATHEMATICS 2 AND MATHEMATICS 21

Basis of Division	Number of Students Placed in Mathematics 2							Number of Students Placed in Mathematics 21						
	Enrollment Unchanged			Transferred by Discriminant Function from:			Total	Enrollment Unchanged			Transferred by Discriminant Function from:			Total
	2-L	2-M	2-U	21-L	21-M	21-U		21-L	21-M	21-U	2-L	2-M	2-U	
Actual Enrollment	69	292	26				387	23	122	43				188
$X_3 - 19$	66	252	14	1	5	0	338	22	117	43	3	40	12	237
$X_3 - 20$	67	273	16	2	7	0	365	21	115	43	2	19	10	210
$X_3 - 21$	68	283	23	4	13	0	391	19	109	43	1	9	3	184
$2X_3 + X_5 - 49$	67	280	17	2	9	0	375	21	113	43	2	12	9	200
$X_3 + X_5 - 28$	67	269	15	1	6	0	358	22	116	43	2	23	11	217
$X_3 + X_5 - 29$	67	280	17	2	9	0	375	21	113	43	2	12	9	200
$X_3 + X_5 - 30$	68	284	23	5	12	0	392	18	110	43	1	8	3	183
$X_3 + X_5 - 31$	68	288	23	8	17	0	404	15	105	43	1	4	3	171

Performance of Individual Students in Groups L and U

Tables were prepared which show the performance with respect to each of the variables X_3 to X_{13}, inclusive, for all individual students who made final course grades of A or F in remedial mathematics, mathematics 2, and mathematics 21. These tables also show the value of the discriminant function, as evaluated for each student. Since these tables are not of general interest, they are not included here, but are deposited in the mathematics and education branch libraries at the University of Oklahoma.

Summary of Chapter VI

A selection was made in this chapter of those variables which appear to show the greatest degree of separation among the "typical groups" R-M, 2-M, and 21-M. Using these variables, regression equations and discriminant functions were determined for each group. It was found that regression equations and discriminant functions based upon the mathematics placement test and the mathematics section of the I H S C were nearly as effective in producing separations among the three groups as those based upon the four variables which were originally chosen. Distribution charts with discriminating lines based upon these two variables were shown for each of the groups L, M, and U for students in remedial mathematics, mathematics 2, and mathematics 21. Tables were presented which show the effect produced on the composition of the various mathematics groups by the application of different discriminant functions.

CHAPTER VII

ANALYSIS OF MATHEMATICS PLACEMENT TEST ITEMS

Description of Mathematics Placement Test

As previously indicated, the mathematics placement test which is currently in use at the University of Oklahoma consists of forty multiple-choice items, each with five possible responses. The test has been carefully constructed, both as to form and content. The test items cover, primarily, topics taken from elementary arithmetic and first-year high school algebra. Only a few items involve geometric concepts. These are problems of the type that are encountered by the student in junior high school mathematics and require no knowledge of formal plane geometry. A maximum of three hours' time is allowed students when they take the test, though it is expected that the average student will be able to complete it in less than one-half of this time.

Procedure in Obtaining and Recording Data

As a basis for the study of the performance of students on the placement test, each test item was assigned a column on the students' I B M cards and each of these columns was punched with the appropriate digit, one, two, three, four, or five, depending on the response of the student on the corresponding test item. The digit zero was punched into each column corresponding to an item which was omitted, making it possible to separate by machine the cards of those students who did not take the test and those students who failed to respond to some of the test items. These data were determined for each of the grade groups in remedial mathematics, mathematics 2, and mathematics 21, and for those students who did not take any college mathematics. A total of 1211 students was involved in this study. Because the summaries of responses made on individual test items are not of general interest, they are not included here, but are on deposit in the mathematics and education branch libraries at the University of Oklahoma.

Distributions of Responses

A summary of the distributions of responses made by groups R-M, 2-M, and 21-M, is found in Table 52. The numbers of students in the three groups R-M, 2-M, and 21-M are 216, 220, and 92, respectively. These numbers are not the same as those in the groups which were designated in the same way in Chapters V and VI, because of the fact that the original placement test papers were not available for all students. In Table 52 and some of the succeeding tables, the letters R, W, and O are used to represent the number of right responses, the number of wrong responses, and the number of items omitted, respectively.

Discrimination Value of Test Items

The differences between the percentages of correct responses for the various groups were taken as a measure of the ability of a test item to discriminate between groups. These differences are shown in Table 53. Since the placement test is used to discriminate among three groups, the best separation is produced by an item for which the differ-

TABLE 52

DISTRIBUTION OF RESPONSES TO PLACEMENT TEST ITEMS FOR GROUPS 21-M, 2-M, AND R-M

Item No.	Distribution												Percentage Distribution											
	21-M			2-M			R-M			Total			21-M			2-M			R-M			Total		
	R	W	O	R	W	O	R	W	O	R	W	O	R	W	O	R	W	O	R	W	O	R	W	O
1	86	6		159	61		69	135	12	314	202	12	93	7		73	27		31	63	6	60	38	2
2	87	5		137	83		45	159	12	269	247	12	95	5		62	38		21	73	6	51	47	2
3	87	5		174	43	3	111	93	12	372	141	15	95	5		79	20	1	51	43	6	70	27	3
4	79	13		150	69	1	71	132	13	300	214	14	86	14		68	32		33	61	6	57	40	3
5	69	23		107	104	9	56	141	19	232	268	28	75	25		49	47	4	26	65	9	44	51	5
6	87	5		176	41	3	60	135	21	323	181	24	95	5		80	19	1	28	62	10	61	34	5
7	82	10		125	92	3	54	141	21	261	243	24	89	11		57	42	1	25	65	10	50	45	5
8	82	10		168	47	5	86	105	25	336	162	30	89	11		77	21	2	40	49	11	63	31	6
9	79	13		145	70	5	49	122	45	273	205	50	86	14		66	32	2	23	56	21	52	39	9
10	89	3		185	31	4	138	66	12	412	100	16	97	3		84	14	2	64	30	6	78	19	3
11	72	19	1	85	123	12	29	141	46	186	283	59	78	21	1	39	56	5	14	65	21	35	54	11
12	71	21		84	123	13	37	139	40	192	283	53	77	23		38	56	6	17	64	19	36	54	10
13	78	14		119	99	2	30	153	33	227	266	35	85	15		54	45	1	14	71	15	43	50	7
14	40	50	2	17	190	13	5	173	38	62	413	53	44	54	2	8	86	6	2	80	18	12	78	10
15	48	43	1	36	181	3	7	186	23	91	410	27	52	47	1	17	82	1	3	86	11	17	78	5
16	82	10		146	72	2	90	109	17	318	191	19	89	11		66	33	1	42	50	8	60	36	4
17	79	12	1	90	105	25	32	121	63	201	238	89	86	13	1	41	48	11	15	56	29	38	45	17
18	59	30	3	31	164	25	7	148	61	97	342	89	64	33	3	14	75	11	3	69	28	18	65	17
19	69	21	2	53	156	11	18	171	27	140	348	40	75	23	2	24	71	5	8	79	13	26	66	8
20	29	63		27	186	7	18	180	18	74	429	25	32	68		12	85	3	8	84	8	14	81	5

TABLE 52 (Continued)

Item No.	Distribution												Percentage Distribution											
	21-M			2-M			R-M			Total			21-M			2-M			R-M			Total		
	R	W	O	R	W	O	R	W	O	R	W	O	R	W	O	R	W	O	R	W	O	R	W	O
21	56	36		64	124	32	25	127	64	145	287	96	61	39		29	56	15	12	59	29	28	54	18
22	32	54	6	36	138	46	24	127	65	92	319	117	35	59	6	16	63	21	11	59	30	17	61	22
23	55	31	6	44	131	45	41	107	68	140	269	119	60	34	6	20	60	20	19	50	31	26	51	23
24	64	28		105	107	8	33	119	64	202	254	72	70	30		48	48	4	15	55	30	38	48	14
25	26	59	7	18	173	29	12	143	61	56	375	97	28	64	8	8	79	13	6	66	28	11	71	18
26	77	14	1	126	78	16	77	98	41	280	190	58	84	15	1	57	36	7	36	45	19	53	36	11
27	86	6		135	73	12	35	121	60	256	200	72	93	7		61	33	6	16	56	28	48	38	14
28	72	19	1	67	108	45	15	125	76	154	252	122	78	21	1	31	49	20	7	58	35	29	48	23
29	42	46	4	49	138	33	17	134	65	108	318	102	46	50	4	22	63	15	8	62	30	21	60	19
30	57	28	7	50	97	73	30	99	87	137	224	167	62	30	8	23	44	33	14	46	40	26	42	32
31	65	19	8	74	101	45	26	105	85	165	225	138	71	21	8	34	46	20	12	49	39	31	43	26
32	73	18	1	97	107	16	42	115	59	212	240	76	79	20	1	44	49	7	20	53	27	40	46	14
33	29	58	5	17	145	58	4	124	88	50	327	151	32	63	5	8	66	26	2	57	41	9	62	29
34	56	35	1	22	171	27	14	131	71	92	337	99	61	38	1	10	78	12	6	61	33	17	64	19
35	65	23	4	59	130	31	44	109	63	168	262	98	71	25	4	27	59	14	20	51	29	32	49	19
36	23	61	8	15	171	34	10	136	70	48	368	112	25	66	9	7	78	15	5	63	32	9	70	21
37	40	45	7	31	108	81	14	95	107	85	248	195	43	49	8	14	49	37	6	44	50	16	47	37
38	41	42	9	32	112	76	15	97	104	88	251	189	44	46	10	15	51	34	7	45	48	17	47	36
39	22	70		20	168	32	19	135	62	61	373	94	24	76		9	76	15	9	62	29	11	71	18
40	11	79	2	21	169	30	21	119	76	53	367	108	12	86	2	9	77	14	10	55	35	10	70	20

ences in the columns headed (A)-(B) and (B)-(C) are both as large as possible.

Difficulty Level of Test Items

In order to obtain a measure of the relative difficulty of the forty test items, the percentage of correct responses was determined for each item. The difficulty level of the item was determined as the abscissa of that point under the normal probability curve which divides the area under the curve into two parts having a ratio equal to the ratio of the number of students who answered the item incorrectly to the number who answered it correctly. The abscissa of the point of division was determined in two different ways. In the first computation the fraction $R/(R + W)$ was used to represent the fractional part of the group of students answering the item correctly. In the second case the fraction $R/(R + W + O)$ was used. The latter procedure placed a higher difficulty value on the item, since by this method an omission was considered as a wrong answer. The resulting difficulty levels found for the various test items are listed in Table 54. In this table, a difficulty level of zero would imply that the item was answered correctly by fifty per cent of the examinees. It will be observed that approximately one-fourth of the items were answered correctly by more than fifty per cent of all persons taking the test, while the remaining three-fourths of the items were answered correctly by fewer than fifty per cent. Since the purpose of the test is primarily that of separating students into ability groups, it is possible that a somewhat better separation might be obtained by replacing some of the items which lie at the upper end of the difficulty

TABLE 53

DISCRIMINATION VALUE OF PLACEMENT TEST ITEMS

Item Number	Value of $\frac{R}{R + W + O}$ * Group 21-M (A)	Group 2-M (B)	Group R-M (C)	(A)-(B)	(B)-(C)
1	0.935	0.723	0.319	0.212	0.404
2	0.946	0.623	0.208	0.323	0.415
3	0.946	0.791	0.541	0.155	0.277
4	0.859	0.682	0.329	0.177	0.353
5	0.750	0.486	0.259	0.264	0.227
6	0.946	0.800	0.278	0.146	0.522
7	0.891	0.568	0.250	0.323	0.318
8	0.891	0.764	0.398	0.127	0.366
9	0.859	0.659	0.227	0.200	0.432
10	0.967	0.841	0.639	0.126	0.202
11	0.783	0.386	0.134	0.397	0.252
12	0.722	0.382	0.171	0.390	0.211
13	0.848	0.541	0.139	0.307	0.402
14	0.435	0.077	0.023	0.358	0.054
15	0.522	0.164	0.032	0.358	0.132
16	0.891	0.664	0.417	0.227	0.247
17	0.859	0.409	0.148	0.450	0.261
18	0.641	0.141	0.032	0.500	0.109
19	0.750	0.241	0.083	0.509	0.158
20	0.315	0.123	0.083	0.192	0.040
21	0.609	0.291	0.116	0.318	0.175
22	0.348	0.164	0.111	0.184	0.053
23	0.598	0.200	0.190	0.398	0.010
24	0.696	0.477	0.153	0.219	0.324
25	0.283	0.082	0.056	0.201	0.026
26	0.837	0.576	0.356	0.261	0.220
27	0.935	0.614	0.162	0.321	0.452
28	0.783	0.305	0.069	0.478	0.236
29	9.457	0.223	0.079	0.234	0.144
30	0.620	0.227	0.139	0.393	0.088

TABLE 53 (Continued)

Item Number	Value of $\frac{R}{R+W+0}$ * Group 21-M (A)	Group 2-M (B)	Group R-M (C)	(A)-(B)	(B)-(C)
31	0.707	0.336	0.120	0.371	0.216
32	0.793	0.441	0.194	0.352	0.247
33	0.315	0.077	0.019	0.238	0.058
34	0.609	0.100	0.065	0.509	0.035
35	0.707	0.268	0.204	0.439	0.064
36	0.250	0.068	0.046	0.182	0.022
37	0.435	0.141	0.065	0.294	0.076
38	0.446	0.145	0.069	0.301	0.076
39	0.239	0.091	0.088	0.148	0.003
40	0.120	0.095	0.097	0.025	-0.002

$$* \frac{R}{R+W+0} = \frac{\text{(Right Responses)}}{\text{(Right Responses)} + \text{(Wrong Responses)} + \text{(Items Omitted)}}$$

scale by easier items. As indicated by Johnson[1] and others, the best discrimination between groups may be obtained by the use of a test in which the items are approximately at the fifty per cent level of difficulty. In this connection, Brogden[2] says:

> The problem of maximal item difficulty distribution is complex from the theoretical viewpoint. Although it can be seen that with perfectly valid items, their difficulty values

[1] A. Pemberton Johnson, "Notes on a Suggested Index of Item Validity: The U-L Index," Journal of Educational Psychology, XLII (December, 1951), 499-504.

[2] Hubert E. Brogden, "Variation in Test Validity with Variation in the Distribution of Item Difficulties, Number of Items, and Degree of their Intercorrelation," Psychometrika, XI (December, 1946), 197-214.

TABLE 54

DIFFICULTY LEVEL OF PLACEMENT TEST ITEMS FOR 1211 STUDENTS

Item Number	Number Right (R)	Number Wrong (W)	Number Omitted (O)	$\frac{R}{R+W}$	$\frac{R}{R+W+O}$	Difficulty level in Standard Deviations based upon: $\frac{R}{R+W}$	Difficulty level in Standard Deviations based upon: $\frac{R}{R+W+O}$
1	709	464	38	0.604	0.585	-0.265	-0.215
2	607	558	46	0.521	0.501	-0.053	-0.003
3	809	342	60	0.703	0.668	-0.533	-0.434
4	640	516	55	0.554	0.528	-0.135	-0.070
5	518	630	63	0.451	0.428	+0.123	+0.181
6	705	430	76	0.621	0.582	-0.308	-0.207
7	582	564	65	0.508	0.481	-0.020	+0.048
8	745	393	73	0.655	0.615	-0.398	-0.293
9	600	471	140	0.560	0.496	-0.152	+0.010
10	949	214	48	0.816	0.784	-0.900	-0.784
11	415	646	150	0.391	0.343	+0.277	+0.404
12	443	639	129	0.409	0.366	+0.229	+0.342
13	496	601	114	0.452	0.410	+0.120	+0.227
14	140	946	125	0.129	0.116	+1.132	+1.195
15	224	902	85	0.199	0.185	+0.845	+0.897
16	717	442	52	0.619	0.592	-0.302	-0.233
17	465	538	208	0.463	0.384	+0.094	+0.295
18	246	747	218	0.248	0.203	+0.681	+0.831
19	324	782	105	0.293	0.268	+0.545	+0.619
20	188	953	70	0.165	0.155	+0.975	+1.015
21	335	630	246	0.347	0.277	+0.393	+0.592
22	235	711	265	0.248	0.194	+0.679	+0.863
23	326	595	290	0.354	0.269	+0.375	+0.616
24	444	579	188	0.434	0.367	+0.166	+0.340
25	146	851	214	0.146	0.121	+1.052	+1.170
26	603	454	154	0.570	0.498	-0.178	+0.005
27	523	482	206	0.520	0.432	-0.051	+0.171
28	352	584	275	0.376	0.291	+0.316	+0.551
29	240	714	257	0.251	0.198	+0.669	+0.849
30	322	490	399	0.397	0.266	+0.262	+0.625

TABLE 54 (Continued)

Item Number	Number Right (R)	Number Wrong (W)	Number Omitted (O)	$\frac{R}{R+W}$	$\frac{R}{R+W+O}$	Difficulty Level in Standard Deviations based upon: $\frac{R}{R+W}$	$\frac{R}{R+W+O}$
31	385	519	307	0.426	0.318	+0.187	+0.473
32	461	562	188	0.451	0.381	+0.124	+0.303
33	122	734	355	0.143	0.101	+1.069	+1.275
34	235	727	249	0.244	0.194	+0.693	+0.863
35	352	627	232	0.360	0.291	+0.360	+0.551
36	131	797	283	0.141	0.108	+1.075	+1.237
37	219	573	419	0.277	0.181	+0.593	+0.912
38	213	602	396	0.261	0.176	+0.639	+0.931
39	143	849	219	0.144	0.118	+1.062	+1.185
40	118	835	258	0.124	0.097	+1.156	+1.299

should, like points on a yardstick, be equally spaced,* [*When expressed in terms of standard score scale values - not percentage correct.] as the items involve more and more error and thus become less and less valid, it is probable that the optimal distribution involves closer grouping of the difficulty values around the fifty per cent value. The latter value is optimal for a single item and for a group of items which all correlate with the criterion but which do not intercorrelate. The question as to just how closely the items should be grouped around the fifty per cent value or just "how much difference it makes" has no immediately obvious answer for cases intermediate to the two extremes just mentioned.

Percentages of Correct Responses in Distribution Quartiles

In order to determine a third measure of the efficiency of each placement test item, the 739 mathematics students and 472 non-mathematics students were divided into quartiles, as nearly as possible, on the basis of the raw score made on the placement test. In each quartile the percentage of correct responses was determined for each test item. These percentages are recorded in Table 55. In this table, percentages are given rather than the actual number of correct responses, because it was impossible to divide the students into groups of equal numbers on the basis of the raw scores on the placement test. The groups Q_1, Q_2, Q_3, and Q_4 of mathematics students contained 202, 167, 184, and 186 individuals, respectively, while the corresponding groups of non-mathematics students consisted of 119, 117, 115, and 121 individuals. For most test items, a good degree of separation is shown between the groups Q_1, Q_2, Q_3, and Q_4. However, items 36, 39, and 40 show an inversion between the responses of groups Q_2 and Q_3. That is, a higher percentage of correct responses was found among students in the lower of the two middle quartiles than among those in the upper. This inversion may, however, be due to chance, since the number of correct responses to these items was small. Reference to the original tables which, as previously indicated, are on file in the mathematics and education branch libraries at the University of Oklahoma, shows that only 11 per cent of the 1211 students answered item 36 correctly. For items 39 and 40 the percentages of correct responses were found to be 12 per cent and 10 per cent, respectively.

Biserial Coefficients of Correlation

As a final measure of the validity of the various test items, the biserial coefficient of correlation was determined for each item. This is a measure of the correlation of each item with the test as a whole. These biserial coefficients of correlation are shown in Table 56. Formulas used in their computation are found in Appendix I.

TABLE 55

PERCENTAGE OF CORRECT RESPONSES IN EACH QUARTILE OF THE DISTRIBUTIONS OF ALL MATHEMATICS AND NON-MATHEMATICS STUDENTS FOR WHOM ITEM ANALYSIS DATA WERE AVAILABLE

Item Number	Mathematics Students				Non-Mathematics Students				Total			
	Q_1	Q_2	Q_3	Q_4	Q_1	Q_2	Q_3	Q_4	Q_1	Q_2	Q_3	Q_4
1	24.3	44.9	72.3	92.5	21.8	47.9	77.4	90.1	23.4	46.1	74.2	91.5
2	13.4	40.1	60.3	93.6	15.1	32.5	57.4	87.6	14.0	37.0	59.2	91.2
3	45.5	58.1	81.0	90.9	37.8	55.6	75.7	86.8	42.7	57.0	78.9	89.2
4	24.8	47.3	63.6	85.5	13.4	36.8	60.0	88.4	20.6	43.0	62.2	86.6
5	17.3	43.1	51.6	75.3	10.9	23.1	47.0	67.8	15.0	34.9	49.8	72.3
6	17.8	50.3	85.3	94.1	12.6	47.0	67.0	87.6	15.9	48.9	78.2	91.5
7	19.3	37.7	59.8	84.4	13.4	29.1	53.9	83.5	17.1	34.2	57.5	84.0
8	34.7	55.7	81.5	89.8	25.2	47.9	67.8	83.5	31.2	52.5	76.3	87.3
9	15.3	40.7	71.7	87.6	8.4	35.0	53.0	77.7	12.8	38.4	64.5	83.7
10	59.9	73.7	85.9	97.3	53.8	80.3	81.7	94.2	57.6	76.4	84.3	96.1
11	11.4	19.2	41.3	74.2	9.2	19.7	33.0	61.2	10.6	19.4	38.1	69.1
12	13.4	26.3	38.6	73.7	9.2	23.1	38.3	67.8	11.8	25.0	38.5	71.3
13	8.9	21.0	58.2	84.9	5.0	20.5	44.3	80.2	7.5	20.8	52.8	83.4
14	2.5	3.0	7.6	36.6	3.4	4.3	9.6	23.1	2.8	3.5	8.4	31.3
15	1.5	7.8	16.8	54.3	2.5	3.4	13.0	44.6	1.9	6.0	15.4	50.5
16	36.1	56.9	71.2	83.9	24.4	55.6	59.1	82.6	31.8	56.3	66.6	83.4
17	12.4	27.5	41.3	84.4	10.1	24.8	35.7	65.3	11.5	26.4	39.1	76.9
18	4.0	5.4	15.8	60.8	2.5	6.8	11.3	52.1	3.4	6.0	14.0	57.3
19	5.4	15.6	23.9	68.8	5.0	12.8	15.7	62.8	5.3	14.4	20.7	66.4
20	7.9	12.6	10.9	32.3	10.9	14.5	13.0	21.5	9.0	13.4	11.7	28.0

TABLE 55 (Continued)

Item Number	Mathematics Students				Non-Mathematics Students				Total			
	Q_1	Q_2	Q_3	Q_4	Q_1	Q_2	Q_3	Q_4	Q_1	Q_2	Q_3	Q_4
21	9.4	16.8	24.5	65.1	3.4	12.0	30.4	57.0	7.2	14.8	26.8	61.9
22	10.4	18.0	13.6	39.8	2.5	12.8	20.0	36.4	7.5	15.8	16.1	38.4
23	8.9	26.3	19.0	56.5	5.0	27.4	21.7	50.4	7.5	26.8	20.1	54.1
24	11.9	28.1	45.1	68.3	7.6	23.9	42.6	63.6	10.3	26.4	44.1	66.4
25	6.4	6.0	8.2	31.2	3.4	7.7	11.3	19.8	5.3	6.7	9.4	26.7
26	29.2	54.5	57.6	78.0	18.5	38.5	51.3	62.8	25.2	47.9	55.2	72.0
27	9.9	26.9	58.2	93.5	3.4	17.1	41.7	86.8	7.5	22.9	51.8	90.9
28	5.9	13.2	29.3	76.3	3.4	13.7	28.7	57.0	5.0	13.4	29.1	68.7
29	5.0	14.4	26.1	44:1	2.5	11.1	18.3	32.2	4.0	13.0	23.1	39.4
30	9.9	22.7	22.8	60.8	8.4	13.7	26.1	43.8	9.3	19.0	24.1	54.1
31	6.9	21.0	29.3	73.1	8.4	11.1	32.2	71.1	7.5	16.9	30.4	72.3
32	15.8	25.7	42.4	76.3	11.8	22.2	40.0	66.1	14.3	24.3	41.5	72.3
33	1.5	2.4	7.1	30.6	0.0	4.3	7.0	26.4	0.9	3.2	7.0	29.0
34	1.5	9.6	14.7	58.1	1.7	10.3	9.6	46.3	1.6	9.9	12.7	53.4
35	15.8	24.6	25.5	69.4	8.4	8.5	22.6	47.1	13.1	18.0	24.4	60.6
36	2.5	6.6	6.5	31.7	0.0	6.8	6.1	24.0	1.6	6.7	6.4	28.7
37	4.0	12.6	15.8	41.9	4.2	13.7	19.1	33.1	4.0	13.0	17.1	38.4
38	4.0	7.8	14.7	44.1	3.4	6.8	13.9	45.5	3.7	7.4	14.4	44.6
39	6.9	13.8	10.9	23.1	5.0	12.8	7.0	11.6	6.2	13.4	9.4	18.6
40	5.4	14.3	11.4	14.0	5.9	8.5	5.2	10.7	5.6	11.9	9.1	12.7

TABLE 56

BISERIAL COEFFICIENTS OF CORRELATION OF TEST ITEMS FOR 1211 STUDENTS

Item Number	Biserial Coefficient of Correlation	Item Number	Biserial Coefficient of Correlation
1	0.643	21	0.648
2	0.711	22	0.513
3	0.494	23	0.548
4	0.614	24	0.577
5	0.543	25	0.542
6	0.682	26	0.475
7	0.635	27	0.786
8	0.562	28	0.744
9	0.649	29	0.524
10	0.466	30	0.588
11	0.642	31	0.689
12	0.613	32	0.608
13	0.741	33	0.695
14	0.645	34	0.816
15	0.746	35	0.591
16	0.490	36	0.689
17	0.668	37	0.543
18	0.799	38	0.678
19	0.744	39	0.293
20	0.351	40	0.198

Summary of Findings on Placement Test

A summary of the findings on the placement test items, with the data on each criterion arranged in sequence, is found in Table 57. In each entry of this table the number preceding the parenthesis is the item number. Each column heading lists the number of the table from which the data in the column are taken. The mean difficulty level, listed in the first column, is the arithmetic average of the two difficulty levels which

TABLE 57

PLACEMENT TEST ITEMS ARRANGED IN ORDER ON BASIS OF FOUR CRITERIA

Mean Difficulty Level (Table 54)	Difference of Percentages of Correct Responses in Upper and Lower Quartiles (Table 55) $Q_4 - Q_1$	Discrimination Value (Table 53)		Biserial Coefficient of Correlation (Table 56)
		(A)-(B)	(B)-(C)	
10(-0.842)	27(83.4)	19(0.509)	6(0.522)	34(0.816)
3(-0.483)	2(77.2)	34(0.509)	27(0.452)	18(0.799)
8(-0.345)	13(75.9)	18(0.500)	9(0.432)	27(0.786)
16(-0.267)	6(75.6)	28(0.478)	2(0.415)	15(0.746)
6(-0.257)	9(70.9)	17(0.450)	1(0.404)	19(0.744)
1(-0.240)	1(68.1)	35(0.439)	13(0.402)	28(0.744)
4(-0.102)	7(66.9)	23(0.398)	8(0.366)	13(0.741)
26(-0.086)	4(66.0)	11(0.397)	4(0.353)	2(0.711)
9(-0.071)	17(65.4)	30(0.393)	24(0.324)	33(0.695)
2(-0.028)	31(64.8)	12(0.390)	7(0.318)	36(0.689)
7(+0.014)	28(63.7)	31(0.371)	3(0.277)	31(0.689)
27(+0.060)	19(61.1)	15(0.358)	17(0.261)	6(0.682)
5(+0.152)	12(59.5)	14(0.358)	11(0.252)	38(0.678)
13(+0.173)	11(58.5)	32(0.352)	16(0.247)	17(0.668)
17(+0.194)	32(58.0)	2(0.323)	32(0.247)	9(0.649)
32(+0.213)	5(57.3)	7(0.323)	28(0.236)	21(0.648)
24(+0.253)	8(56.1)	27(0.321)	5(0.227)	14(0.645)
12(+0.285)	24(56.1)	21(0.318)	26(0.220)	1(0.643)
31(+0.330)	21(54.7)	13(0.307)	31(0.216)	11(0.642)
11(+0.340)	18(53.9)	38(0.301)	12(0.211)	7(0.635)
28(+0.433)	34(51.8)	37(0.294)	10(0.202)	4(0.614)
30(+0.443)	16(51.6)	5(0.264)	21(0.175)	12(0.613)
35(+0.455)	15(48.6)	26(0.261)	19(0.158)	32(0.608)
21(+0.492)	35(47.5)	33(0.238)	29(0.144)	35(0.591)
23(+0.495)	26(46.8)	29(0.234)	15(0.132)	30(0.588)
19(+0.582)	23(46.6)	16(0.227)	18(0.109)	24(0.577)
37(+0.752)	3(46.5)	24(0.219)	30(0.088)	8(0.562)
18(+0.756)	30(44.8)	1(0.212)	38(0.076)	23(0.548)
29(+0.759)	38(40.9)	25(0.201)	37(0.076)	5(0.543)
22(+0.771)	10(38.5)	9(0.200)	35(0.064)	37(0.543)

TABLE 57 (Continued)

Mean Difficulty Level (Table 54)	Difference of Percentages of Correct Responses in Upper and Lower Quartiles (Table 55) $Q_4 - Q_1$	Discrimination Value (Table 53) (A)-(B)	Discrimination Value (Table 53) (B)-(C)	Biserial Coefficient of Correlation (Table 56)
34(+0.778)	29(35.4)	20(0.192)	33(0.058)	25(0.542)
38(+0.785)	37(34.4)	22(0.184)	14(0.054)	29(0.524)
15(+0.871)	22(30.9)	36(0.182)	22(0.053)	22(0.513)
20(+0.995)	14(28.5)	4(0.177)	20(0.040)	3(0.494)
25(+1.111)	33(28.1)	3(0.155)	34(0.035)	16(0.490)
39(+1.123)	36(27.1)	39(0.148)	25(0.026)	26(0.475)
36(+1.156)	25(21.4)	6(0.146)	36(0.022)	10(0.466)
14(+1.163)	20(19.0)	8(0.127)	23(0.010)	20(0.351)
33(+1.172)	39(12.4)	10(0.126)	39(0.003)	39(0.293)
40(+1.227)	40(7.1)	40(0.025)	40(-0.002)	40(0.198)

were computed and recorded in Table 54. Because there is no way of determining whether a particular item was omitted because of lack of familiarity with the principle involved, or for some other reason, it seems reasonable that the true index of difficulty should lie between the two indices listed in Table 54. In the absence of a criterion which would determine the relative weights which should be assigned to these indices, it was decided to give them equal weights in making the entries in the first column of Table 57.

In interpreting the data of Table 57 it should be kept in mind that, with respect to each of the criteria except the mean difficulty level, the best ratings are the highest. The best difficulty levels, for the purpose of discrimination between groups, are those which are not too far removed from zero, which represents the fifty per cent level of difficulty. Those items which rate uniformly low on all four criteria, such as items 39 and 40, apparently do not serve, as well as might be expected, the primary purpose of the test, which is that of producing a separation among the three mathematics groups. Also, a comparison of the columns headed (A)-(B) and (B)-(C) shows that in general a better separation is obtained between mathematics 2 and mathematics 21 than between remedial mathematics and mathematics 2. As might be expected, a large portion of the items which failed to produce an appreciable separation between the groups in remedial mathematics and mathematics 2 were also items which appear close to the upper end of the difficulty scale. In order to make them as effective as the other items in discriminating among the three groups, it appears that a downward revision of their difficulty level might be advisable.

Summary of Chapter VII

The distributions of responses to the mathematics placement test items, for students in groups R-M, 2-M, and 21-M and for non-mathematics students, were presented in this chapter. A measure was given of the degree to which each item discriminates among the three groups of mathematics students. Other measures obtained for all test items, on the basis of item analysis data for 1211 students, included a measure of the difficulty level of each item, the percentage of correct responses in each quartile of the distribution of 1211 students based upon the total number of correct responses, and the biserial coefficient of correlation for each item. A summary of the findings on these various criteria appears in Table 57.

CHAPTER VIII

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Introduction

Some of the major findings of this study are summarized in this chapter. In an attempt to solve the problems stated in Chapter I, data were compiled for 1832 students who entered the University of Oklahoma as beginning freshmen in September, 1952. Among the data used were the University of Oklahoma mathematics placement test, decile ranks on the various sections of both the Iowa High School Content Examination (abbreviated I H S C) and the Ohio State Psychological Examination (abbreviated O S P E), and average grade points, as well as mathematics grade points, both in high school and in college.

Comparisons of Various Groups of Students

Some of the findings which are in the nature of comparisons are the following:

1. There is a marked relation between grades earned by students in the first mathematics course in which they enrolled and their continuance of a college career. Table 1 indicates that of 119 students who earned a grade of A in the first college mathematics course in which they enrolled, 106 were still enrolled after the conclusion of three semesters' work at the University. In contrast, of 181 students who

received a grade of F in the first college mathematics course in which they enrolled, only sixty-eight remained through three or more semesters.

2. Students who withdrew from freshman mathematics courses in which they had enrolled, but did not re-enroll in the next lower course in the sequence, had college grade point averages which closely approximated the grade point averages of the groups from which they withdrew. The mean of grade point averages of the students who withdrew was somewhat greater than 2.0, as may be seen by referring to Table 2.

3. On the basis of all variables investigated, the mean performance of non-mathematics students was found to be superior to that of remedial mathematics students but inferior to that of students in mathematics 2. With respect to all variables, the mean performance of students in mathematics 21 was found to be superior to that of all other groups of students. Comparisons of mathematics and non-mathematics groups of students, on the basis of various criteria, are found in Tables 21 to 30, inclusive.

Criteria Best Suited to the Proper Placement of Students

Regression equations and Fisher discriminant functions were determined, based upon groups of students who made final course grades of B, C, or D, in remedial mathematics, mathematics 2, and mathematics 21. It was found that the best practical separation between these three "typical groups" of students was produced by the use of linear functions of X_3 (raw score on the mathematics placement test) and X_5 (decile rank on the mathematics section of the I H S C), in which the coefficients of the variables are made approximately equal. While a somewhat better separation was produced by the use of discriminant functions involving additional variables, the small increase in the reliability of the results appeared to be insufficient to warrant the use of more complicated functions. Though measures of general ability, such as the O S P E, correlate positively with grades made in mathematics courses, they show a large overlap between the various mathematics groups and therefore do not serve well as a basis of separation into ability groups in mathematics.

Improvement in Placement Through Use of the Discriminant Function

This study appears to indicate that the efficiency of the placement test may be increased by the supplementary use of the mathematics section of the I H S C. Reference to Tables 50 and 51 shows that the use of the functions $X_3 + X_5 - 16$ and $X_3 + X_5 - 30$ would produce comparatively little change in the total number of students assigned to each of the three groups, though it may be desirable, in order to reduce the number of failures in mathematics 2, to use either $X_3 + X_5 - 17$, or $X_3 + X_5 - 18$, as the basis of separation between the remedial mathematics and mathematics 2 groups. Reference to the distribution chart in Table 44 shows that the use of the function $X_3 + X_5 - 17$ would have placed in remedial mathematics thirty-one of the sixty-nine students who made a failing grade in mathematics 2. This reduction in the number of failures in mathematics 2 would have been accomplished with a net increase in enrollment in remedial mathematics from 382 to 427, or an increase of forty-five students.

Improvement of the Placement Test

The study of the mathematics placement test reveals that approximately one-fifth of the forty items of the test were answered correctly by fifty per cent or more of the examinees, while the remaining four-fifths were answered correctly by fewer than fifty per cent. This indicates that an improvement in the discriminating power of the test might be expected if the more difficult items were replaced by easier ones. As shown by the value of the fraction $R/(R + W + O)$, in Table 54, the percentage of correct responses on fourteen items, representing approximately one-third of the test, is 20 per cent or less.

Those items of the placement test in which a particular alternate response seems to be especially popular, such as items 20, 36, 39, and 40, might be re-examined with a view to making them more effective. These are also, in general, the items which show a smaller difference between the percentages of correct responses in the second and third quartiles of the distributions. Table 55 shows a slightly higher percentage of correct responses to items 36, 39, and 40, among students in the second quartile than among those in the third quartile. Questions showing such inversions might be considered for revision or replacement.

The results of this study indicate that the placement test serves better as an instrument for the separation of students in mathematics 2 and mathematics 21, than for separation into remedial mathematics and mathematics 2 groups. The addition to the raw placement test score of the decile rank on the mathematics section of the I H S C tends to make the test more sensitive at the lower levels of ability.

Conclusions

Following are enumerated some of the major conclusions of the foregoing paragraphs:

1. In the placement of students in mathematics courses, the use of the mathematics section of the I H S C in addition to the mathematics placement test results in better placement than the use of the mathematics placement test alone.

2. Slightly better placement is possible through the use of other variables in addition to the mathematics placement test and the mathematics section of the I H S C. However, the use of these additional variables appears to be unwarranted in view of the desirability of a criterion for placement which is simple and easily applied.

3. The mathematics placement test, in its present form, serves better as a means of separating students in mathematics 2 and mathematics 21 than it does in separating students in remedial mathematics and mathematics 2. The test might be made a more effective instrument for separation of students at the lower levels of ability by replacing the items which appear to be high on the difficulty scale by easier items.

Recommendations

In view of the above conclusions, the following recommendations are made:

1. It is recommended that the function $X_3 + X_5 - K$ be used, initially, as a basis for the separation of students into remedial mathematics, mathematics 2, and mathematics 21 groups. Reference to Table 51 shows that little change should be expected in the proportion of students placed in each of the three groups if the functions $X_3 + X_5 - 16$ and $X_3 + X_5 - 30$ are used as a basis of separation.

2. It is recommended that those items of the placement test

be replaced which show a poor separation value, particularly at the upper end of the difficulty scale.

Suggested Future Investigations

While the present study indicates that the best separation may be produced by assigning approximately equal weights to X_3 and X_5, it is possible that a future investigation might indicate the desirability of a change in this ratio. Because the subjects of this study were separated primarily on the basis of the placement test, greater emphasis is placed upon it in the discriminant functions which were derived than would otherwise be the case. For this reason, it is expected that a follow-up study of students segregated on the basis of the function $X_3 + X_5 - K$ would almost certainly indicate an increase in the relative weight of the variable X_5.

The conditions under which the placement test is administered might be given some study, with the purpose of determining optimum conditions. In particular, the effect of guessing on test items might be studied, to determine whether or not guessing should be encouraged. Reference to Table 52 shows that thirteen test items had less than a 20 per cent correct response. If chance alone were operating, and all responses to an item appeared to be equally attractive, one should expect approximately 20 per cent of all responses to be correct. The fact that (see Table 8) 261 of 399 students in remedial mathematics had no better than a 20 per cent correct response indicates the importance of a uniform procedure in dealing with the problem of guessing. Since some students will guess, even if told not to do so, it would seem that the highest degree of uniformity can be attained by encouraging all to guess on those items on which they are in doubt. Further study of this problem might be of value.

There is a possibility that some improvement might be effected by a re-arrangement of the placement test items in the approximate order of difficulty, as indicated by the percentages of correct responses on each item. A few questions of low difficulty level, such as item 10, are probably desirable at the beginning of the test, for psychological reasons, even though their discriminatory value may be low.

A study might be made of the content of the various mathematics courses. An important problem in this connection would be to determine whether there are any gaps which should be filled, particularly in remedial mathematics and mathematics 2.

Case studies might be made of those persons who appear to be misplaced. This is indicated by the fact that, in a number of instances which were discovered, either students were ill-advised or they did not take the advice given them. Data were compiled for individual students who made a course grade of A or F in remedial mathematics, mathematics 2, and mathematics 21, and these are on file in the mathematics and education branch libraries at the University of Oklahoma. An examination of these data indicates that in a number of cases students whose performance on the placement test, on the I H S C, and on the O S P E was uniformly low were placed in mathematics 2. In these instances, the data which were available at the time of enrollment clearly indicated that they should be placed in remedial mathematics.

Concluding Remarks

The investigations made in this study appear to indicate that some improvement is possible in the placing of freshman students in college mathematics courses by the combined use of the mathematics placement test and the mathematics section of the I H S C. The effectiveness of the placement test may also be increased by the revision or replacement of those items of the test which have a low separation value. It is hoped that the investigations made will serve the purpose of aiding in the placement of freshman mathematics students in those courses in which they can do their best work and that it may have indicated some avenues for possible further exploration.

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APPENDIX I

FORMULAS

Some of the more important formulas used in computations involved in this study are shown here for reference.

1. The mean, $\bar{X}$, of a distribution of a group of N individuals on the basis of variable X, is defined by the equation

$$\bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i \qquad (1)$$

2. The standard deviation, σ_X, of a distribution of a group of N individuals on the basis of variable X, is defined by the equation

$$\sigma_X = \frac{1}{N} \sqrt{N \sum_{i=1}^{N} X_i^2 - (\sum_{i=1}^{N} X_i)^2} \qquad (2)$$

3. The probable error, P E, of a distribution of a group of N individuals on the basis of a variable X, is defined as

$$P\,E = .6745\,\sigma_X$$

For a normal distribution, twenty-five per cent of the items in the distribution lie between $\bar{X}$ and $\bar{X} + 1\,P\,E$. Similarly, twenty-five per cent of all cases lie between $\bar{X}$ and $\bar{X} - 1\,P\,E$.

4. The Pearson product moment coefficient of correlation, r_{XY}, between distributions with respect to the variables X and Y for a group

of N individuals, is defined by the equation

$$r_{XY} = \frac{N \sum_{i=1}^{N} X_iY_i - \sum_{i=1}^{N} X_i \sum_{i=1}^{N} Y_i}{\sqrt{N \sum_{i=1}^{N} X_i^2 - (\sum_{i=1}^{N} X_i)^2} \sqrt{N \sum_{i=1}^{N} Y_i^2 - (\sum_{i=1}^{N} Y_i)^2}} \qquad (4)$$

5. The partial coefficient of correlation, $r_{12.3}$, between variables X_1 and X_2, with the influence of variable X_3 held constant, is

$$r_{12.3} = \frac{r_{12} - r_{13} \cdot r_{23}}{\sqrt{1 - r^2_{13}}\sqrt{1 - r^2_{23}}} \qquad (5)$$

Similarly, the partial coefficient of correlation, $r_{12.34}$, between variables X_1 and X_2, with the influence of variables X_3 and X_4 held constant, is

$$r_{12.34} = \frac{r_{12.3} - r_{14.3}r_{24.3}}{\sqrt{1 - r^2_{14.3}}\sqrt{1 - r^2_{24.3}}} \qquad (6)$$

6. The partial standard deviation, $\sigma_{1.23}$, of variable X_1, with the influence of variables X_2 and X_3 held constant, is given by

$$\sigma_{1.23} = \sigma_1\sqrt{1 - r^2_{12}}\sqrt{1 - r^2_{13.2}} \qquad (7)$$

Similarly, the partial standard deviation, $\sigma_{1.234}$, of variable X_1, with the influence of variables X_2, X_3, and X_4 held constant, is given by

$$\sigma_{1.234} = \sigma_1\sqrt{1 - r^2_{12}}\sqrt{1 - r^2_{13.2}}\sqrt{1 - r^2_{14.23}} \qquad (8)$$

7. The coefficient of multiple correlation, R, between a single dependent variable X_1 and $(n - 1)$ independent variables $X_2, \cdot \cdot \cdot, X_n$ is given by

$$R_{(1.23\cdots n)} = \sqrt{1 - \frac{\sigma^2_{1.23\cdots n}}{\sigma^2_1}} \tag{9}$$

8. The biserial coefficient of correlation, r_b, of a particular item in a test is defined by the equation

$$r_b = \frac{M_p - M_t}{\sigma_t} \cdot \frac{p}{y} \tag{10}$$

where M_p = mean criterion score of those passing the item

M_t = mean criterion score of all examinees

σ_t = standard deviation of all total scores

p = proportion passing the item

y = ordinate in unit normal distribution corresponding to p.

APPENDIX II

THE DISCRIMINANT FUNCTION

The purpose of a discriminant function is to discriminate between two different populations on the basis of a number, p, of measurements which are known for each individual in the two populations. Knowing the values of the p measurements for a given individual, it is to be decided with which of the two groups the individual should be associated. If the measurements of individuals in the two groups show a high degree of separation with respect to a single variable, a decision may be based on the individual's measurement on that variable. More often, however, while the means of the measurements of the two groups may be well separated, there is a considerable degree of overlap between measurements for individuals belonging to the two groups. Where such overlap exists, any linear function of one variable obviously places some individuals in the wrong group.

Often the two groups may be found to differ with respect to several variables, each of which may give some indication as to the placement of a given individual in one of the two groups. It is assumed that a certain degree of overlap exists between the two groups with respect to each of the variables; otherwise complete separation would be possible through the use of a linear function of one of the variables.

When measurements with respect to two or more variables are known, a linear function of the variables is sought, the use of which results in a minimum of misplacement of individuals. The term "minimum of misplacement" requires definition. It is obvious that different definitions of this term would result in different discriminant functions. The problem is to determine a function of the form

$$Z = \sum_{i=1}^{p} \lambda_i X_i \qquad (1)$$

in which $X_1, \ldots, X_p$ are the variables measured and $\lambda_1, \ldots, \lambda_p$ are weights assigned to the variables which are to be so determined as to produce the "best" separation between the two groups. In the discriminant function devised by Fisher,[1] the values of the λ_i are so determined that the ratio of the variance of the values of Z between groups to the variance of the values of Z within groups is a maximum. If the two groups are designated as group A, containing N_a individuals, and group B, containing N_b individuals, the function, G, which is to be made a maximum is represented by the expression

$$G = \frac{(\bar{Z}_a - \bar{Z}_b)^2}{\sum_{i=1}^{N_a}(Z_i - \bar{Z}_a)^2 + \sum_{j=1}^{N_b}(Z_j - \bar{Z}_b)^2} . \qquad (2)$$

The numerator of this ratio is the square of the difference between the means of Z for the two groups, and the denominator is the sum of the squares of the deviations from the mean within each of the two groups.

Let $\bar{X}_{ai}$ and $\bar{X}_{bi}$ represent, respectively, the means of group A

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[1] R. A. Fisher, "The Use of Multiple Measurements in Taxonomic Problems," Annals of Eugenics VII (1936), 179-188.

and group B on variable X_i. Then the difference $\bar{Z}_a - \bar{Z}_b$ may be written in the form

$$\bar{Z}_a - \bar{Z}_b = \sum_{i=1}^{p} \lambda_i \bar{X}_{ai} - \sum_{i=1}^{p} \lambda_i \bar{X}_{bi}$$

$$= \sum_{i=1}^{p} \lambda_i (\bar{X}_{ai} - \bar{X}_{bi})$$

$$= \sum_{i=1}^{p} \lambda_i d_i \quad ,$$

where d_i represents the difference between the means of the two groups on the variable X_i. This form shows explicitly the dependence of the numerator of G on the parameters λ_i which are to be determined.

Likewise, the denominator of G may be written in the form

$$\sum_{u=1}^{N_a} (Z_u - \bar{Z}_a)^2 + \sum_{v=1}^{N_b} (Z_v - \bar{Z}_b)^2 = \sum_{i=1}^{p} \sum_{j=1}^{p} \lambda_i \lambda_j S_{ij} \quad ,$$

where S_{ij} is defined as the sum of squares or products of deviations from the specific means within the groups. Thus,

$$S_{ij} = \left[\sum_{u=1}^{N_a} X_{ui} X_{uj} + \sum_{v=1}^{N_b} X_{vi} X_{vj} \right] - \left[\frac{(\sum_{u=1}^{N_a} X_{ui})(\sum_{u=1}^{N_a} X_{uj})}{N_a} + \frac{(\sum_{v=1}^{N_b} X_{vi})(\sum_{v=1}^{N_b} X_{vj})}{N_b} \right] .$$

The function G may now be written in the form

$$G = D^2/S \tag{3}$$

where

$$D = \sum_{i=1}^{p} \lambda_i d_i \tag{4}$$

and

$$S = \sum_{i=1}^{p} \sum_{j=1}^{p} \lambda_i \lambda_j S_{ij} \quad . \tag{5}$$

The function D^2/S, of the variables $X_1, \cdots, X_p$ is made a maximum by proper choice of the coefficients $\lambda_1, \cdots, \lambda_p$. To make the function D^2/S a maximum, its partial derivatives with respect to each of the λ_i are equated to zero:

$$\frac{\partial}{\partial \lambda_i} \frac{D^2}{S} = 0 , \qquad (i = 1, \cdots, p) . \tag{6}$$

These equations are equivalent to

$$\frac{D}{S^2}\left(2S \frac{\partial D}{\partial \lambda_i} - D \frac{\partial S}{\partial \lambda_i}\right) = 0 , \quad (i = 1, \cdots, p) . \tag{7}$$

It follows therefore that

$$\frac{1}{2} \frac{\partial S}{\partial \lambda_i} = \frac{S}{D} \frac{\partial D}{\partial \lambda_i} , \qquad (i = 1, \cdots, p) . \tag{8}$$

Since $\dfrac{\partial S}{\partial \lambda_i} = 2 \sum_{j=1}^{p} \lambda_j S_{ij}$ and $\dfrac{\partial D}{\partial \lambda_i} = d_i$, equation (8) may be written in the form

$$\sum_{j=1}^{p} S_{ij} \lambda_j = \frac{S}{D} d_i , \qquad (i = 1, \cdots, p) . \tag{9}$$

The required values of $\lambda_1, \cdots, \lambda_p$ are determined by the solution of these simultaneous linear equations. These values may then be placed in equation (1) which defines the function Z.

The means of Z for the two groups are next defined:

$$\bar{Z}_a = \sum_{i=1}^{p} \lambda_i \bar{X}_{ai} ,$$

$$\bar{Z}_b = \sum_{i=1}^{p} \lambda_i \bar{X}_{bi} .$$

The constant K is defined by the equation

$$K = \frac{\overline{Z}_a - \overline{Z}_b}{2} \quad .$$

The value of Z may be determined for any individual for whom measurements on the variables $X_1 \cdot \cdot \cdot X_p$ are known. In the application of the discriminant function, if Z_n represents the value of Z when evaluated for the nth individual, that individual is associated with the first or second of the two populations separated by the discriminant function according as $(Z_n - K)$ is negative or positive.