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AN EXPERIMENTAL EVALUATION OF TWO CURRICULUM DESIGNS FOR TEACHING FIRST-YEAR ALGEBRA IN A NINTH GRADE CLASS

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M. LAVERNE LOMAN

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1961

AN EXPERIMENTAL EVALUATION OF TWO CURRICULUM DESIGNS FOR TEACHING FIRST-YEAR ALGEBRA IN A NINTH GRADE CLASS

APPROVED BY

W. R. Fulton

William H. Huff

Mary Clare Petty

Dora McFarland

Glenn R. Snider

DISSERTATION COMMITTEE

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CHAPTER I

THE PROBLEM: ITS BACKGROUND AND DEFINITION

Introduction

There is a greater, more widespread interest in the improvement of teaching mathematics now than at any other time in recent years. Mathematics itself is growing at an unprecedented rate. Today, due to our tremendous economic and technological growth, nationally, and our international rivalry, we are demanding more mathematical knowledge on the part of more people than ever before. A broader conception of the subject has stimulated new and significant applications which force mathematics to assume a more important part in industry and society. As a result, the social sciences, the natural sciences, and the schools of business administration are indicating their desire for a better command of mathematical principles and techniques. It has thus become necessary that mathematics be taught in such a way that the students realize its increasingly important role in the contemporary world. All of these combine to form the difficult but vital problem of improving mathematics education in our schools. Some progress has been made, but much is yet to be done.

Need for the Study

A few significant curricular studies have either been completed or are now in progress. Yet a good deal of today's activity appears to be somewhat superficial. Large sums of money are being used to bombard teachers and administrators with brochures, bibliographies, reports and experimental classroom materials. Too often, ambitious school administrators expect their teachers to discard immediately their old teaching techniques and materials and adopt a new curriculum design, without fully understanding or appreciating the nature of the change.

Despite all the activity of reform in mathematics education, there has been little research conducted to investigate the effectiveness of these new curriculum designs. Change, in itself, does not assure improvement. For permanent progress, selection should be based, not on opinion alone, but on sound, objective, experimental studies. Thus it was the writer's hope that the present study would supply some evidence of the effectiveness of one of the most prominent of the contemporary algebra programs as compared to the traditional algebra program--evidence that would serve as a basis for future planning.

Definitions

The idea of the traditional algebra program, which shall be denoted by the term "traditional method" in this study, involves both content and point of view in teaching it. There has been little change in the way algebra has been taught since the beginning of the nineteenth century. The study of algebra at that time was a most mechanical, manipulative study, consisting of a set of rules for performing operations on real numbers and manipulations of algebraic expressions. This is the algebra, as presented in the traditional elementary textbook, that we think of today as constituting the traditional algebra program.

The contemporary algebra program, which shall be denoted by the term "discovery method" in this study, presents essentially the same subject matter as the traditional program, but is developed from a different point of view. The text which was used in this study tries specifically to develop a deeper understanding of the structure of algebra through emphasis on precision of language and the method of discovery.

Statement of the Problem

This study was made to investigate the effectiveness of two distinct curriculum designs--the "discovery method" and the "traditional method"--for teaching the basic mathematical concepts of first-year algebra in a ninth grade class.

Hypotheses

Specifically, it is proposed that the following hypotheses be tested in this investigation:

1. There is no significant difference in the achievement of the understanding of basic mathematical concepts between two groups of ninth grade students who complete one year of elementary algebra, one group taught by the "discovery method," the other group taught by the "traditional method."

2. There is no significant difference in the achievement of mathematical abilities between two groups of ninth grade students who complete one year of elementary algebra, one group taught by the "discovery method," the other group taught by the "traditional method."

3. There is no significant difference in the achievement of manipulative skills between two groups of ninth grade students who complete one year of elementary algebra, one group taught by the "discovery method," the other group taught by the "traditional method."

Assumptions

Assumptions basic to the study are:

1. New interpretations and uses of mathematics have increased the demand for knowledge. The wide range of mathematical applications in our society make better instruction in mathematics a necessity.

2. The understanding of basic mathematical concepts and principles is of primary importance in the learning of mathematics and should be attained by all students in the most effective manner.

Delimitations of the Study

This study is limited to data obtained from forty-four ninth grade students enrolled in first-year algebra during the school year 1959-1960, at West Junior High School, Norman, Oklahoma. The description of the sample is presented in Chapter II.

The text used by the group taught by the "discovery method" was developed by the University of Illinois Committe on School Mathematics (hereafter to be denoted by the initials UICSM), primarily for the student of above-average intelligence.[1] The group taught by the "traditional method" used a text designed for the regular first-year algebra class.[2] The study covered the full school year.

The "discovery method" group was taught by the researcher who had received training from the UICSM staff in teaching their material. She was teaching ninth grade mathematics for the first time; her previous experience was in

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[1]University of Illinois Committee on School Mathematics, High School Mathematics, Units 1-4 (Urbana, Illinois: University of Illinois Press, 1959).

[2]Howard F. Fehr, Walter H. Carnahan, and Max Beberman, Algebra, Course I (Boston: D. C. Heath and Company, 1955).

teaching college mathematics to freshmen students at the University of Oklahoma. The teacher for the "traditional method" group was a regular member of the faculty of West Junior High School. It is recognized that it would have been better if the researcher had taught both groups. Under the conditions, it was impossible to arrange for this to be done. However, the teacher for the "traditional method" group was well-qualified and experienced in the traditional method of teaching.

Background of Research

There are many well-known professional groups now actively engaged in providing possible solutions to the serious problems in mathematics education. Four of the principal groups whose primary objective is the improvement of secondary school mathematics are:

1. The University of Illinois Committee on School Mathematics. This project began in 1951 and since July, 1956, has received support from the Carnegie Corporation. It has developed student texts and teacher guides which have been used for seven years in twelve pilot schools in four states with about 1700 students and in fifty-five schools in twenty-one states with more than 3900 students in 1958-1959. The UICSM is not only working in curricular research but also in training high school teachers in the use of their materials in order that the youngsters using the materials become enthusiastic students who understand mathematics.

2. The Commission on Mathematics. This commission was established in 1955 by the College Entrance Examination Board. Its members were chosen from high school and college teachers of mathematics and from instructors in classes for future mathematics teachers. The Commission has clearly outlined recommendations for a four-year program for high school mathematics. It has prepared a series of excellent monographs especially designed for the in-service training of teachers wishing to become familiar with the relevant mathematical concepts.

3. School Mathematics Study Group. This is a national project supported by the National Science Foundation with headquarters at Yale University. It was established early in 1958 and its members were chosen from university mathematicians and classroom teachers. It has carefully studied the recommendations of other major groups with related objectives and has produced materials for both elementary and secondary levels. It also has a monograph project designed to enable students to gain a better knowledge of the scope of mathematics and to realize that it is an alive and growing subject.

4. The Ball State Teachers College Experimental Program in Geometry and Algebra. This writing object, supported by the college, has been in operation since 1955. Text materials have been written and used in a small number of schools in Indiana. The pupils in the classes have been of average ability and the teachers have had no special training but were under

some supervision from members of the project. The manuscripts have been submitted and the algebra text is expected to be published in 1961. The geometry text was published in 1960 by the Addison-Wesley Publishing Company.

All of these groups state that algebra should be treated as a study of mathematical structure rather than as the development of manipulative skill. This emphasis on the importance of understanding basic principles and concepts of mathematics is not a recent development. It has long been emphasized. In 1940, the Fifteenth Yearbook of the National Council of Teachers of Mathematics stated:

> An understanding of the concepts and principles of mathematics is the key to its successful study. To teach in such a way that the concepts become clear is the hardest and most significant task confronting the teacher of mathematics.[3]

In 1959, the Secondary-School Curriculum Committee of the National Council of Teachers of Mathematics stated:

> . . . at any grade level the principal responsibility of the teacher of mathematics is to do the very best job of which he is capable in helping his pupils acquire fundamental understandings of the basic concepts, principles, and techniques.[4]

Although the committee states that objectives calling for the understanding of basic concepts and principles are

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[3]National Council of Teachers of Mathematics, Fifteenth Yearbook: The Place of Mathematics in Secondary Education, The Final Report of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics (Washington, D. C.: The National Council of Teachers of Mathematics, 1940), p. 57.

[4]"The Secondary Mathematics Curriculum: Report of the Secondary-School Curriculum of the National Council of Teachers of Mathematics," The Mathematics Teacher, LII (May, 1959), 411.

emphasized more than ever and that experience leading to "discovery" of mathematical properties are highly favored for this understanding, there has been little objective evaluation. In effect, there has been no research that has proved there is one best method of teaching algebra.

Two studies were made comparing methods of teaching signed numbers, one by Pauline Lucille Bury,[5] the other by R. E. Michael.[6] Bury taught a unit on directed numbers with emphasis on the "discovery" method. Fifty-five pupils were used in this study, seventeen in the experimental group and forty-five in the control group. The experimental group indicated a definite interest in the use of the "discovery" method. This method seemed useful at all learning levels; however, the reported achievement of the two groups was about the same. Michael, in a study based on fifteen classes with as many teachers, found no outstanding difference in student gain dependent on teaching technique in comparing an inductive approach with an authoritative approach for teaching signed numbers. Gains in computation, skill, generalization ability, and also attitudes toward mathematics were evaluated for classes with similar prior mathematical instruction and for three ability levels within these classes. Slight differences

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[5]Pauline Lucille Bury, "Experimental Introductory Unit with Directed Numerals" (unpublished Master's thesis, Illinois State Normal University, 1956).

[6]R. E. Michael, "The Relative Effectiveness of Two Methods of Teaching Certain Topics in Ninth-Grade Algebra" (unpublished Ed.D. dissertation, Indiana University, 1947).

were found but none that were significant.

A doctoral study by Edwin Hirschi Lewis,[7] involving ten algebra classes, used drill with one group and exploration and discovery with the other. The pupils were tested over a full school year's work by a special test designed to meet the objectives of the experimental group and by the Douglass Survey Test. The experimental method used in this study was found to be definitely superior to the control method when the results were measured by the Douglass Survey Test. The experimental method was superior for above-median students but showed no advantage for below-median students. The individual results on both the special test and the Douglass Survey Test indicate that a large percentage of students showed so little achievement that the time spent was considered to be unjustified.

Two doctoral studies, by Max Sobel[8] and by Nicholas Paul Kushta,[9] placed emphasis on understanding concepts. Sobel compared the concepts of general number, formula, exponent and coefficient in a group of seven schools in New Jersey for four weeks. In each of the seven schools, one

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[7] Edwin Hirschi Lewis, "A Concept Approach to the Teaching of Algebra" (unpublished Ed.D. dissertation, University of Utah, 1956).

[8] Max A. Sobel, "A Comparison of Two Methods of Teaching Certain Topics in Ninth Grade Algebra" (unpublished Ph.D. dissertation, Columbia University, 1954).

[9] Nicholas Paul Kushta, "A Comparison of Two Methods of Teaching Algebra in the Ninth Grade" (unpublished Ph.D. dissertation, University of Chicago, 1958).

class was taught by the experimental method and one class by the control method. His study was initiated to discover the relationships between the learning of these topics and their method of presentation. Two pairs of experimental and control groups were used. The average I.Q. was 100 for one group and between 100-115 for the other. He found that brighter students profited in the learning of these concepts from a teaching method which featured an inductive, concrete, unverbalized approach as opposed to a deductive, abstract, verbalized method of presentation. There was no apparent difference, either in learning or retention, between the two methods for the groups of average I.Q.

Kushta found that when algebra was organized and taught around certain unifying themes instead of the usual topics, students developed as much manipulative skill, while acquiring a greater understanding of the nature of mathematics. In his study, in each of five schools the same teacher taught two classes comparable in predicted success in first semester ninth grade algebra. Both the concept group and the traditional group consisted of 131 students. At the beginning of the seventeen week experiment, each teacher collected data from the school records and administered a specially constructed attitude scale and an instrument for measuring interests. At the end of the experimental period the teacher used the Seattle Algebra Test for the End of the First Half, a test used to measure the understanding of the nature of mathematics, and repeated the use of the attitude scale and interest instrument. Kushta found the scores on the attitude and interest instrument did not differ significantly between the two groups. A slight modification of the concept approach produced scores in manipulative skill equal to those achieved in the traditional approach.

Organization of the Report

This report consists of four chapters. Chapter I contains a statement of the need for the study, a statement of the problem, the hypotheses, basic assumptions, the delimitations of the study, the background of research, and the organization of the report. Chapter II is a presentation of the statistical procedure, including a description of the sample. Chapter III is a presentation and analysis of the data for the study. Chapter IV contains a summary of the study, the conclusions reached as a result of the investigation, recommendations, and implications of the study.

CHAPTER II

STATISTICAL PROCEDURE AND POPULATION

Experimental Procedure

As was indicated in Chapter I, this study involved two groups of ninth grade elementary algebra students from West Junior High School, Norman, Oklahoma. One group, consisting of twenty-six students at the beginning of the study, was taught by the "discovery method." The other group, consisting of twenty-eight students, was taught by the "traditional method." Each of the twenty-six students in the first group was paired with a student in the second group on the basis of California Test of Mental Maturity scores and scores from the Differential Aptitude Tests, Numerical Ability and Verbal Reasoning. There were twenty-two pairs of students who completed the full school year. The scores for these students may be found in Tables I and II. The various test-scores for these two groups, consisting of twenty-two students each, were thus used in the final statistical analysis.

To determine whether or not the effectiveness of the curriculum design depended on various intelligence levels within

TABLE I

PERSONAL DATA FOR INDIVIDUALS TAUGHT BY THE "DISCOVERY METHOD"

Student Number	Differential Aptitude Tests				California Test of Mental Maturity	
	Verbal Reasoning		Numerical Ability			
	Raw Score	Perentile*	Raw Score	Percentile*	Raw Score	Percentile*
1	29	85	26	85	130	98
2	25	75	32	97	126	95
3	31	90	18	60	125	95
4	25	75	9	20	123	90
5	33	95	21	75	112	70
6	25	75	18	60	112	70
7	31	90	25	85	112	70
8	17	50	15	45	111	70
9	25	75	16	50	111	70
10	25	80	21	75	107	60
11	22	70	29	95	104	60
12	15	40	13	35	104	60
13	13	30	9	20	103	50
14	18	55	12	35	100	50
15	17	50	19	65	98	40
16	8	10	2	3	96	40
17	9	15	10	25	95	40
18	19	60	17	55	95	40
19	25	75	15	45	95	40
20	9	15	10	25	93	30

TABLE I--Concluded

Student Number	Differential Aptitude Tests				California Test of Mental Maturity	
	Verbal Reasoning		Numerical Ability			
	Raw Score	Percentile*	Raw Score	Percentile*	Raw Score	Percentile*
21	6	5	14	40	85	10
22	10	20	13	35	84	10
Total	437	...	364	...	2321	...
Mean	19.86	...	16.55	...	105.50	...
S. D.	8.24	...	7.15	...	12.53	...

*Based on national norms.

TABLE II

PERSONAL DATA FOR INDIVIDUALS TAUGHT BY THE "TRADITIONAL METHOD"

Student Number	Differential Aptitude Tests				California Test of Mental Maturity	
	Verbal Reasoning		Numerical Ability			
	Raw Score	Percentile*	Raw Score	Percentile*	Raw Score	Percentile*
1	23	70	26	85	121	90
2	29	85	29	95	117	80
3	27	85	24	80	116	80
4	23	70	17	55	115	80
5	19	60	15	45	113	70
6	25	80	24	80	113	70
7	22	70	25	85	113	70
8	12	25	18	60	109	70
9	28	85	28	90	109	70
10	19	60	10	25	107	60
11	23	70	19	65	105	60
12	19	55	19	65	105	60
13	10	20	17	55	103	50
14	18	55	10	25	99	40
15	16	45	20	70	99	40
16	8	10	5	10	97	40
17	14	35	11	30	96	40
18	22	70	20	70	95	40
19	23	70	10	25	95	40
20	14	35	18	60	95	40

TABLE II--Concluded

Student Number	Differential Aptitude Tests				California Test of Mental Maturity	
	Verbal Reasoning		Numerical Ability			
	Raw Score	Percentile*	Raw Score	Percentile*	Raw Score	Percentile*
21	20	60	16	50	94	40
22	13	30	19	65	93	30
Total	427	...	400	...	2309	...
Mean	19.41	...	18.18	...	104.95	...
S. D.	5.80	...	6.33	...	8.82	...

*Based on national norms.

the groups, each group was further divided into three subgroups--the upper, middle and lower one-third--based on the California Test of Mental Maturity scores. To meet the criterion of equal sample sizes in each cell, one pair of students (number fourteen) was randomly deleted from this analysis. The upper range of scores was 130-112, the middle range was 111-98, and the lower range was 97-84.

Description of Sample

During the school year, 1959-1960, West Junior High School employed a three-track program in the ninth grade mathematics curriculum. One track consisted of a geometry class for accelerated students, another track consisted of four classes of regular first-year algebra, and the third track consisted of one general mathematics class for the slower students. The samples for this study were taken from the middle track--the four classes of first-year algebra. Two of the four classes of first-year algebra were chosen at random to participate in this study. It was decided by lot which class was to be taught by the researcher using the UICSM materials and techniques, and which class was to be taught by a member of the West Junior High School faculty using the traditional method of instruction and state adopted text.

Since all of the students in the middle track were required to take first-year algebra, it was assumed that each student had an equal opportunity of being enrolled in any one of the four classes. Since none of the students knew that such a study was to be conducted, it was assumed that the students in each class were randomly representative of all the ninth grade students enrolled in first-year algebra.

To test the validity of this assumption, the sample was submitted to tests for normality and randomness. The normality of the distribution of the California Test of Mental Maturity scores for each of the included groups, as given in Tables I and II, was determined by use of the chi-square test of significance with scores placed in six categories and tested for difference from normal distribution. A value of 2.34 was obtained for the "discovery method" group and 3.37 for the "traditional method" group, neither value being significant at the 0.05 level of significance.

Table III, Correlation Coefficients for the Two Groups, gives the correlation between the various test-scores for each group. A chi-square test of significance was computed to determine whether or not the correlation coefficients could be pooled. A value of 23.66 was found for the group taught by the "discovery method" and a value of 19.66 was found for the group taught by the "traditional method," neither value being significant at the 0.05 level of significance. Hence the Fisher's z-test of significance of the difference between the two pooled correlation coefficients was made, giving a z-score of 0.801 which is not significant at the 0.05 level of significance. The assumption that the two groups were random samples drawn from a common population was thus verified.

TABLE III

CORRELATION COEFFICIENTS FOR THE TWO GROUPS

Tests*	Group: "Discovery Method"	Group: "Traditional Method"
DATV-DATN	.648	.636
DATV-CTMM	.773	.566
DATV-Pret	.780	.687
DATV-Post	.714	.719
DATV-Coop	.613	.506
DATV-STEP	.739	.769
DATN-CTMM	.506	.593
DATN-Pret	.753	.575
DATN-Post	.562	.573
DATN-Coop	.340	.210
DATN-STEP	.594	.502
CTMM-Pret	.731	.393
CTMM-Post	.766	.248
CTMM-Coop	.622	.550
CTMM-STEP	.750	.602
Pret-Post	.662	.580
Pret-Coop	.681	.306
Pret-STEP	.727	.552
Post-Coop	.610	.141
Post-STEP	.902	.623
Coop-STEP	.630	.368
Pooled correlation coefficient	.694	.526

*DATV - Differential Aptitude Tests, Verbal Reasoning
DATN - Differential Aptitude Tests, Numerical Ability
CTMM - California Test of Mental Maturity
Pret - Test of Understanding Basic Mathematical Concepts, Pre-test
Post - Test of Understanding Basic Mathematical Concepts, Post-test
Coop - Cooperative Elementary Algebra Test, Form T
STEP - Sequential Tests of Educational Progress, Mathematics Test, Level III

Tests of Hypotheses

Two statistical procedures were utilized in testing the three hypotheses. The t-test was used to test each hypothesis when comparing the two groups as a whole. An analysis of variance was made to test each hypothesis for the three intelligence levels within each of the two groups.

The first hypothesis to be tested was that there is no significant difference in the achievement of the understanding of basic mathematical concepts between the two groups. The two forms of the test developed by the UICSM, Test of Understanding Basic Mathematical Concepts, were administered as pre-test and post-test. The pre-test was given during the first week of classes and the post-test given during the final week. The t-test was used to test the significance of difference in the mean gain of raw scores from the pre-test to the post-test for each group as a whole. An analysis of variance was computed on these difference scores to determine whether or not the effectiveness of a particular curriculum design depended upon the level of intelligence within the groups.

The second hypothesis was that there is no significant difference in the achievement of mathematical abilities between the two groups. To measure the extent to which the assessment of mathematical concepts, abilities, and skills considered essential for the average students had been achieved, the Mathematics Test, Level III, of the Sequential Tests of

Educational Progress was given to both groups during the eighth month of classes. The t-test was used to test the significance of difference between the means of the raw scores for each group as a whole. An analysis of variance was computed on these raw scores for three intelligence levels within each group.

The third hypothesis was that there is no significant difference in the achievement of manipulative skills between the two groups. To measure the achievement of manipulative skills, the Cooperative Elementary Algebra Test, Form T was administered to both groups during the final week. The t-test was used to test the significance of difference between the means of the raw scores for each group as a whole. An analysis of variance was computed on these raw scores for three intelligence levels within each group.

CHAPTER III

PRESENTATION AND ANALYSIS OF DATA

In order to investigate the effectiveness of the "discovery method" as opposed to the "traditional method" for teaching first-year algebra, a comparison of the two groups was desired for three specific areas of achievement. Thus a null hypothesis was stated and tested over each of the three areas.

Hypothesis I

To test the first hypothesis, the UICSM pre-test and post-test, Test of Understanding Basic Mathematical Concepts, were administered to both the "discovery method" group and the "traditional method" group. Table IV, Raw Scores of Test of Understanding Basic Mathematical Concepts for the "Discovery Method" and "Traditional Method" Groups, gives the results of these tests. The reliability for each test was established by the UICSM staff. The reliability for the pre-test was .70 and for the post-test was .83.[1]

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[1]Letter from O. R. Brown, Jr., Evaluator, UICSM, Urbana, Ill., August 2, 1960.

TABLE IV

RAW SCORES OF TEST OF UNDERSTANDING BASIC MATHEMATICAL CONCEPTS FOR THE "DISCOVERY METHOD" AND "TRADITIONAL METHOD" GROUPS

Student Number	"Discovery Method" Group			"Traditional Method" Group		
	Pre-test	Post-test	Difference	Pre-test	Post-test	Difference
1	18	17	-1	14	12	-2
2	17	20	3	19	18	-1
3	15	18	3	19	15	-4
4	12	20	8	10	13	3
5	16	16	0	12	13	1
6	13	14	1	15	13	-2
7	16	20	4	11	11	0
8	16	12	-4	14	10	-4
9	10	12	2	14	18	4
10	16	18	2	12	14	2
11	13	15	2	14	18	4
12	13	8	-5	13	18	5
13	10	11	1	6	9	3
14	11	8	-3	12	9	-3
15	12	14	2	12	14	2
16	7	13	6	8	7	-1
17	10	11	1	12	9	-3
18	15	13	-2	13	17	4
19	12	11	-1	13	13	0
20	9	11	2	14	11	-3

TABLE IV--Concluded

Student Number	"Discovery Method" Group			"Traditional Method" Group		
	Pre-test	Post-test	Difference	Pre-test	Post-test	Difference
21	8	10	2	12	13	1
22	8	6	-2	12	13	1
Total	277	298	21	281	288	7
Mean	12.59	13.55	0.95	12.77	13.09	0.32
S. D.	3.22	4.07	3.09	2.88	3.26	2.83

The t-test was used to determine whether or not a significant difference occurred between the groups in the mean gain from the pre-test scores to the post-test scores, after first testing for equality of variances. A value of 0.89 was obtained and was not significant at the 0.05 level of significance. This would tend to imply that there was no significant difference in the achievement of the understanding of basic mathematical concepts between the two groups.

To determine whether or not the understanding of basic concepts depended upon the level of intelligence within the groups, each group was divided into three equal subgroups--the upper, middle, and lower one-third--on the basis of the California Test of Mental Maturity scores and an analysis of variance was computed. The results may be found in Table V, Difference of Scores From Pre-test to Post-test to Test the Understanding of Basic Mathematical Concepts for Three Intelligence Levels Within the Two Groups, and Table VI, Analysis of Variance of Difference Scores from Pre-test to Post-test for Three Intelligence Levels Within the Two Groups. The only value of F found to be significant was that of interaction. After first testing for equality of variances by the Bartlett technique, the t-test was used to test the significance of the difference between the various means. The only t-score found to be significant was between the two groups at the upper one-third intelligence level. This value was 2.19 which is significant at the 0.05 level of significance.

TABLE V

DIFFERENCE OF SCORES FROM PRE-TEST TO POST-TEST TO TEST THE UNDERSTANDING OF BASIC MATHEMATICAL CONCEPTS FOR THREE INTELLIGENCE LEVELS WITHIN THE TWO GROUPS

Intelligence Level	Student Number	Group: "Discovery Method"	Group: "Traditional Method"	Sum and Mean for Intelligence Level
Upper	1	-1	-2	
	2	3	-1	
	3	3	-4	
	4	8	3	
	5	0	1	
	6	1	-2	
	7	4	0	
	Sum	18	-5	13
	Mean	2.57	-0.71	0.93
Middle	8	-4	-4	
	9	2	4	
	10	2	2	
	11	2	4	
	12	-5	5	
	13	1	3	
	15	2	2	
	Sum	0	16	16
	Mean	0	2.29	1.14

TABLE V--Concluded

Intelligence Level	Student Number	Group: "Discovery Method"	Group: "Traditional Method"	Sum and Mean for Intelligence Level
Lower	16	6	-1	
	17	1	-3	
	18	-2	4	
	19	-1	0	
	20	2	-3	
	21	2	1	
	22	-2	1	
	Sum	6	-1	5
	Mean	0.86	-0.14	0.36
Sum for Groups		24	10	34
Mean for Groups		1.14	0.48	0.81

TABLE VI

ANALYSIS OF VARIANCE OF DIFFERENCE SCORES FROM PRE-TEST TO POST-TEST FOR THREE INTELLIGENCE LEVELS WITHIN THE TWO GROUPS

Source of Variation	Sum of Squares	df	Mean Square	F
Level of Intelligence	4.62	2	2.31	...
Method of Instruction	4.67	1	4.67	...
Interaction	54.90	2	27.45	3.50*
Between Groups	64.19	5	12.84	1.64
Within Groups	282.29	36	7.84	
Total	346.48	41		

*Significant at 0.05 level of significance.

A significant difference in the achievement of the understanding of basic mathematical concepts at the upper one-third level of intelligence thus appeared to present the only difference between the two groups.

Using the UICSM tests as criterion, the first hypothesis, that there is no significant difference in the achievement of the understanding of basic mathematical concepts between the group taught by the "discovery method" and the group taught by the "traditional method," was accepted for the two groups as a whole. It was rejected, however, for the upper one-third level of intelligence.

Hypothesis II

The Mathematics Test, Level III, of the Sequential Tests of Educational Progress was designed to measure the extent to which the attainment of mathematical concepts, abilities, and skills considered essential for the average student have been achieved. It was administered to both the "discovery method" group and the "traditional method" group in the eighth month of classes for purposes of testing the second hypothesis. The raw scores are listed in Table VII.

On testing for equality of variances, there appeared to be a significant difference in variances between the two groups. The number of degrees of freedom for heterogeneous variance was then computed and the t-test was used to test whether or not there was a significant difference between the means of the raw scores of the two groups. A value of 0.36 was obtained which is not significant at the 0.05 level of significance. Hence there was no apparent significant difference in the achievement of mathematical abilities between the two groups.

To determine whether or not the attainment of mathematical abilities depended upon the level of intelligence within the groups, an analysis of variance was made for the three intelligence levels within the groups. The results are given in Tables VIII and IX. After testing for equality of variance by the Bartlett technique, there appeared to be no significant difference in the achievement of mathematical

TABLE VII

RAW SCORES OF THE COOPERATIVE ELEMENTARY ALGEBRA TEST AND THE MATHEMATICS TEST OF THE SEQUENTIAL TESTS OF EDUCATIONAL PROGRESS FOR THE "DISCOVERY METHOD" AND "TRADITIONAL METHOD" GROUPS

Student Number	Cooperative Elementary Algebra Test		Mathematics Test of the STEP	
	"Discovery Method" Group	"Traditional Method" Group	"Discovery Method" Group	"Traditional Method" Group
1	30	28	42	39
2	31	22	43	44
3	26	12	40	40
4	5	24	41	36
5	14	20	44	40
6	11	13	32	35
7	25	19	41	32
8	10	19	35	28
9	7	15	30	37
10	16	15	40	32
11	15	17	40	35
12	9	14	27	36
13	4	6	27	31
14	14	28	23	29
15	17	0	40	36

TABLE VII--Concluded

Student Number	Cooperative Elementary Algebra Test		Mathematics Test of the STEP	
	"Discovery Method" Group	"Traditional Method" Group	"Discovery Method" Group	"Traditional Method" Group
16	8	4	34	28
17	10	8	27	29
18	5	15	30	36
19	16	18	32	39
20	7	12	25	31
21	9	10	24	30
22	6	8	14	24
Total	295	327	731	747
Mean	13.41	14.86	33.23	33.95
S. D.	8.08	7.27	8.06	4.94

TABLE VIII

RAW SCORES OF THE MATHEMATICS TEST OF THE SEQUENTIAL TESTS OF EDUCATIONAL PROGRESS TO TEST THE ATTAINMENT OF MATHEMATICAL ABILITY FOR THREE INTELLIGENCE LEVELS WITHIN THE TWO GROUPS

Intelligence Level	Student Number	Group: "Discovery Method"	Group: "Traditional Method"	Sum and Mean for Intelligence Level
Upper	1	42	39	
	2	43	44	
	3	40	40	
	4	41	36	
	5	44	40	
	6	32	35	
	7	41	32	
	Sum	283	266	549
	Mean	40.43	38.00	39.22
Middle	8	35	28	
	9	30	37	
	10	40	32	
	11	40	35	
	12	27	36	
	13	27	31	
	15	40	36	
	Sum	239	235	474
	Mean	34.14	33.57	33.86

TABLE VIII--Concluded

Intelligence Level	Student Number	Group: "Discovery Method"	Group: "Traditional Method"	Sum and Mean for Intelligence Level
Lower	16	34	28	
	17	27	29	
	18	30	36	
	19	32	39	
	20	25	31	
	21	24	30	
	22	14	24	
	Sum	186	217	403
	Mean	26.57	31.00	28.79
Sum for Groups		708	718	1426
Mean for Groups		33.71	34.19	33.95

TABLE IX

ANALYSIS OF VARIANCE OF SCORES MADE ON THE MATHEMATICS TEST OF THE SEQUENTIAL TESTS OF EDUCATIONAL PROGRESS FOR THREE INTELLIGENCE LEVELS WITHIN THE TWO GROUPS

Source of Variation	Sum of Squares	df	Mean Square	F
Level of Intelligence	761.47	2	380.74	15.36*
Method of Instruction	2.38	1	2.38	.....
Interaction	88.05	2	44.03	1.78
Between Groups	851.90	5	170.38	6.88*
Within Groups	892.00	36	24.78	
Total	1743.90	41		

*Significant at 0.05 level of significance.

abilities between the two groups at any of the three levels of intelligence within the groups. The only significant value of F was found to be between the levels of intelligence within the groups.

Using the Mathematics Test of the Sequential Tests of Educational Progress as criterion, the second hypothesis, that there is no significant difference in the achievement of mathematical abilities between the group taught by the "dis overy method" and the group taught by the "traditional method," was accepted in its entirety.

Hypothesis III

The Cooperative Elementary Algebra Test, Form T, was given to both the "discovery method" group and the "traditional method" group in the final week of classes for purposes of testing the third hypothesis. The raw scores are listed in Table VII.

After testing for equality of variances, the t-test was used to test whether or not there was a significant difference between the means of the raw scores of the two groups. A value of 0.62 was obtained which is not significant at the 0.05 level of significance. This would tend to imply that there was no significant difference in the achievement of manipulative skills between the two groups.

To determine whether or not the attainment of manipulative skills depended upon the level of intelligence within the groups, an analysis of variance was made for the three intelligence levels within the groups. The results are given in Tables X and XI. After first testing for equality of variances by the Bartlett technique, the only value of F found to be significant was, again, not between the groups but between the levels of intelligence. In each group, the students in the upper level of intelligence appeared to do significantly better than those in the middle level or the lower level. However, there appeared to be no significant difference in the achievement of manipulative skills between the two groups at any of the three levels of intelligence within the groups.

TABLE X

RAW SCORES OF COOPERATIVE ELEMENTARY ALGEBRA TEST TO TEST THE ACHIEVEMENT OF MANIPULATIVE SKILL FOR THREE INTELLIGENCE LEVELS WITHIN THE TWO GROUPS

Intelligence Level	Student Number	Group		Sum and Mean for Intelligence Level
		"Discovery Method"	"Traditional Method"	
Upper	1	30	28	
	2	31	22	
	3	26	12	
	4	5	24	
	5	14	20	
	6	11	13	
	7	25	19	
	Sum	142	138	280
	Mean	20.29	19.71	20.00
Middle	8	10	19	
	9	7	15	
	10	16	15	
	11	15	17	
	12	9	14	
	13	4	6	
	15	17	0	
	Sum	78	86	164
	Mean	11.14	12.29	11.71

TABLE X--Concluded

Intelligence Level	Student Number	Group: "Discovery Method"	Group: "Traditional Method"	Sum and Mean for Intelligence Level
Lower	16	8	4	
	17	10	8	
	18	5	15	
	19	16	18	
	20	7	12	
	21	9	10	
	22	6	8	
	Sum	61	75	136
	Mean	8.71	10.71	9.71
Sum for Groups		281	299	580
Mean for Groups		13.38	14.24	13.81

TABLE XI

ANALYSIS OF VARIANCE OF COOPERATIVE ELEMENTARY ALGEBRA TEST SCORES FOR THREE INTELLIGENCE LEVELS WITHIN THE TWO GROUPS

Source of Variation	Sum of Squares	df	Mean Square	F
Level of Intelligence	832.77	2	416.39	10.30*
Method of Instruction	7.72	1	7.72	.....
Interaction	11.99	2	6.00	.....
Between Groups	852.48	5	170.50	4.22*
Within Groups	1456.00	36	40.44	
Total	2308.48	41		

*Significant at 0.05 level of significance.

Using the Cooperative Elementary Algebra Test as criterion, the third hypothesis, that there is no significant difference in the achievement of manipulative skills between the group taught by the "discovery method" and the group taught by the "traditional method," was accepted in its entirety.

CHAPTER IV

SUMMARY AND CONCLUSIONS

Summary

The purpose of this study was to investigate the effectiveness of two distinct curriculum designs, called the "discovery method" and the "traditional method," for teaching the basic mathematical concepts of first-year algebra in a ninth grade class. This purpose was accomplished by comparing statistically three specific areas of achievement of forty-four students enrolled in first-year algebra.

The following hypotheses were tested in this investigation:

1. There is no significant difference in the achievement of <u>the understanding of basic mathematical concepts</u> between two groups of ninth grade students who complete one year of elementary algebra, one group taught by the "discovery method," the other group taught by the "traditional method."

2. There is no significant difference in the achievement of <u>mathematical abilities</u> between two groups of ninth grade students who complete one year of elementary algebra,

one group taught by the "discovery method," the other group taught by the "traditional method."

3. There is no significant difference in the achievement of manipulative skills between two groups of ninth grade students who complete one year of elementary algebra, one group taught by the "discovery method," the other group taught by the "traditional method."

Four classes of first-year algebra were taught in the ninth grade at West Junior High School, Norman, Oklahoma, in 1959-1960. This was the middle track of a three-track program. Two classes were chosen at random and on the basis of tests made on California Test of Mental Maturity scores and correlation coefficients for the various test-scores, it was assumed that each was a random sample and representative of all ninth grade students taking first-year algebra at West Junior High. It was decided by lot which class was to be taught by the "discovery method" by the researcher, the other class to be taught by the "traditional method" by a regular member of the faculty. Tests were made to verify the normality of the distribution of the California Test of Mental Maturity scores for each group. The students were paired on the basis of these scores and scores for the Differential Aptitude Tests, Numerical Ability and Verbal Reasoning. There was a total of twenty-two pairs of students completing the full school year. The final statistical analysis was made using test-scores for these twenty-two pairs of students. The twenty-two students in the group taught by the "traditional method" used a traditional state adopted elementary algebra textbook, the twenty-two students in the group taught by the "discovery method" used the text developed by the UICSM. By random numbers, one student was omitted in order to divide each group into three equal subgroups--the upper, middle, and lower one-third intelligence levels.

A comparison of the two groups was made on the mean gain from the pre-test scores to the post-test scores for the UICSM tests, Test of Understanding Basic Mathematical Concepts. The first hypothesis, that there is no significant difference in the achievement of the understanding of basic mathematical concepts between the two groups, was accepted for the two groups as a whole but was rejected for the upper one-third intelligence level. A comparison of the groups was made on scores for the Mathematics Test of the Sequential Tests of Educational Progress. The second hypothesis, that there is no significant difference in the achievement of mathematical abilities between the two groups, was accepted in its entirety. A comparison of the groups was made on scores for the Cooperative Elementary Algebra Test. The third hypothesis, that there is no significant difference in the achievement of manipulative skills between the two groups, was accepted in its entirety.

Conclusions

On the basis of the data obtained in this study, the following conclusions are made:

1. The students in the upper one-third intelligence level of the group taught by the "discovery method" achieved significantly more in the understanding of basic mathematical concepts than the students in the upper one-third intelligence level of the group taught by the "traditional method." For the students in the middle and lower intelligence levels, no significant difference between the two groups was found.

2. There was no significant difference found between the two groups in the achievement of mathematical abilities.

3. There was no significant difference found between the two groups in the achievement of manipulative skills.

It would thus seem from these data that the two methods may be expected to produce approximately equivalent gains in all three areas only at the middle and lower intelligence levels.

Recommendations

Although the two algebra programs investigated in this study resulted generally in no real differences between the two groups in acquiring mathematical abilities and manipulative skills, the "discovery method" resulted in greater understanding of basic mathematical concepts for the students in the upper intelligence level and appeared to be at least as effective for those in the middle and lower intelligence levels. Since the understanding of basic principles and concepts is considered to be of primary importance in the learning of mathematics, it is the recommendation of the researcher that further, more extensive experimental studies be made trying a similar approach to instruction in first-year algebra.

Implications

The major implication of this study is that the upper one-third of the students in the middle-track program at West Junior High School would gain a better understanding of basic mathematical concepts through the method of discovery used to teach algebra from the point of view of mathematical structure as opposed to the traditional approach. On the basis of the data obtained in this study, it appears that West Junior High School should separate the upper one-third of their students in the middle-track and teach elementary algebra using the contemporary program, rather than the traditional program.

It is recognized that the evidence produced by a single study has numerous limitations and cannot settle controversial issues with any degree of finality. The value of such an exploratory study comes rather from the addition of evidence, the accumulation of which may eventually point toward definite answers to certain key questions. For permanent progress, we must study programs of action through experimentation, evaluation of results, and the analysis of their implication, such programs not to be limited to a single school nor even a single town. It is hoped that the results of this study will serve as a basis for future planning by school administrators and classroom teachers, and what is more important, that it will instigate and stimulate plans for cooperative team-work research in this area by others.

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