DEVELOPMENT OF COMPUTATIONAL ANALYSIS OF WEB TROUGHS AND WRINKLES DUE TO CROWN ROLLERS

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DEVELOPMENT OF COMPUTATIONAL ANALYSIS

OF WEB TROUGHS AND WRINKLES DUE TO

CROWN ROLLERS

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Abstract: A crown roller has a larger radius at the center than at the sides. A web in transit through a manufacturing process must be subjected to tension by driven rollers that also maintain web velocity. If the rollers supporting the web were perfectly cylindrical and if the web was perfect in terms of length and thickness uniformity across its width, then the web tension would develop uniform stress in the direction of web travel. When a web encounters a roller that is not perfectly cylindrical and has a crown radius the internal stress becomes non-uniform. This non-uniformity in stress can induce web instability in free web spans or upon roller surfaces which can be detrimental to web quality. The objective of this research is to develop computational tools using FORTRAN that can predict what level of crown in a roller that a given web can transit without becoming unstable.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF VARIABLES

r	Radius as a variable
a_0 ,a_1	Crown constants
V	Velocity
υ	Poisson's ratio
σ	Stress
ε	Strain
γ	Shear stress
E	Young's modulus
w	Width of the web
t	Thickness of the web
L	Length of the span
R	Radius of the roller
U	Displacement
ω	Rotational velocity

CHAPTER I

INTRODUCTION

Buckling in Web Handling

A web is a material whose length greatly exceeds the width which greatly exceeds the thickness. With such a thin material, failure can occur in many ways since a small force can induce large stresses. Webs are supported intermittently in process machines by rollers. The length of web between any two successive rollers is called a web *span*. When friction is sufficient, a web will enter a roller normal to the axis of rotation as the velocity vectors on the web and roller surfaces will attempt to align. This is called *normal entry* of a web to a roller.

The understanding of the different failure behaviors in thin webs help in reducing waste. As the web moves the buckling of the web anywhere in the process machine can harm the integrity of the final product. The first type of buckling is called *troughs*. A trough will occur when the cross machine direction (CMD) stress in the span before a roller becomes negative enough to begin the buckling process. Troughs may not cause permanent damage but can cause waste in any process which requires the web to be flat. Wrinkling is the second type of web buckling which occurs on rollers rather than in the web spans. Wrinkling is always a concern as it results in inelastic deformation to the web, and waste.

Before web troughs form, a web span can be considered as a flat piece of material and closed form solutions for deformation and stress analyses may be possible. After a web forms troughs, closed form solutions are no longer possible since the web spans are in a post-buckled stage. The use of nonlinear methods is required to determine if the stress is high enough to cause wrinkles.

Crown Rollers

A common occurrence in web handling is a crown roller. This occurs when the center radius of the roller is larger than the sides as shown in Figure I-1. The curve can be approximated as a parabolic curve. This type of roller is intentionally used with belts. The crowned roller will produce steering forces that center the belt on the roller and hence keep the belt centered in a machine. The centering forces are useful in belts but for thin webs can cause the cross machine stress to become very negative and may cause wrinkles.

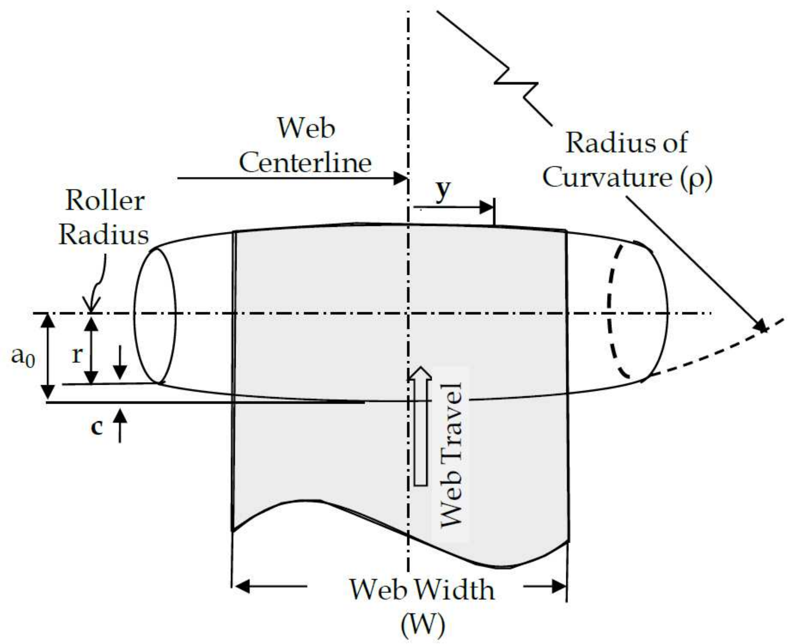

Figure I-1: Crowned Roller

The most common way for a roller to become crowned is during the cutting process of the roller on a lathe. As the cutting tool moves along the roller, the cutting forces deflects the roller which results in the roller radius being larger as the cutting tool moves toward the center of the roller away from the lathe support clamps to make it larger the further away it is from the clamps. With better machining processes the crowning can be kept to a minimum. Machining processes may be too expensive or needless if roller crown is not the cause of the wrinkling.

CHAPTER II

LITERATURE REVIEW

Research on Webs Interacting with a Crown Roller

Beisel [1] and Vaijapurkar [3], previously studied the analysis of webs transiting crowned rollers. Beisel focused mostly on experimental tests and used static steady state finite element analyses to describe the trough and wrinkle behaviors he witnessed. Vaijapurkar followed and focused on the interaction of webs with crowned rollers using finite element explicit simulations.

The *machine direction* (MD) is the direction of web travel through a process machine. The machine direction stress in a web approaching a crowned roller can be developed by studying how the velocity changes along the crown roller. The velocity is calculated using the crown radius as a function of CMD location as shown below. The changing velocity creates a variation in MD strain and stress and is calculated in the equations below. [1]

$$r(y) = a_0 - a_1 * y^2$$

Equation 1: Radius of the Crown Roller

$$V(y) = (a_0 - a_1 * y^2) * \omega$$

Equation 2: Velocity Across the Surface of the Crown Roller

$$V_{avg} = \frac{1}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}} (a_0 - a_1 * y^2) * \omega * dy = \omega\left(a_0 - \frac{a_1 * W^2}{12}\right)$$

Equation 3: Average Velocity of the Surface of Crown Roller

$$\varepsilon(y) = \frac{V(y) - V_{avg}}{V_{avg}}$$

Equation 4: Strain Across the Crown roller

$$\sigma_{md}(y) = E\frac{a_1 * W^2 - 12 * a_1 * y^2}{12 * a_0 - a_1 * W^2} + \frac{T_w}{W * t}$$

Equation 5: Machine Direction Stress Induced by Crown Rollers and Web Tension (Tw)

There are two critical stresses that need to be considered one for troughing and the other for wrinkling. These equations are limited to isotropic webs. The expression (6) is a special case for buckling of a rectangular panel subject to biaxial normal surface tractions. Good and Beisel [8] reduced this expression for a web under tension (σ_{md}) subject to a σ_y CMD compressive traction to:

$$\sigma_{y,Troughs} = -\frac{t * \pi}{L}\sqrt{\frac{\sigma_{md} * E}{3 * (1 - \upsilon^2)}}$$

Equation 6: Troughs Critical Stress

Timoshenko [7] developed the following expression to predict axial buckling of cylindrical shells. Expression (6) and (7) have proven to be useful for predicting troughs and wrinkles in webs. [1, 3, 8]

$$\sigma_{y,wrinkling} = -\frac{E * t}{R * \sqrt{3 * (1 - v^2)}}$$

Equation 7: Wrinkling Critical Stress

Beisel designed and created a roller that could be internally pressurized to provide a variable crown roller. Beisel used this roller to study the onset of trough buckling in webs. The roller was designed with a variable wall thickness such that a parabolic crown as given by expression (1) would develop when pressurized and thus the crown could be varied while the roller was rotating. He performed experiments on polyester webs where the crown level would be slowly increased until troughs would appear in the web span approaching the crowned roller. He then used this roller to perform multiple tests. He confirmed that equation was reasonable and could be used to analyze the problem as long as an effective trough length was input rather than the entire span length. Once the equation was confirmed, he started testing the webs at different tensions. Figure II-1 shows the experimental data Beisel acquired. Note that the data recorded was in the form of the internal pressure (PSI) required to produce a trough. Beisel derived expressions for the coefficients a_0 and a_1 in expression (1) so that the radius variation of his variable crown roller was known as a function of internal pressure (P).

$$a_0 = 1.5 + (1.36145 * 10^{-6} * P + 0.027061774)$$

Equation 8: Constant for the Crown Roller Radius Equation (in)

$$a_1 = 6.01683 * 10^{-8} * P \pm 3.99919 * 10^{-6}$$

Equation 9: Constant for the Crown Roller Radius Equation (1/in)

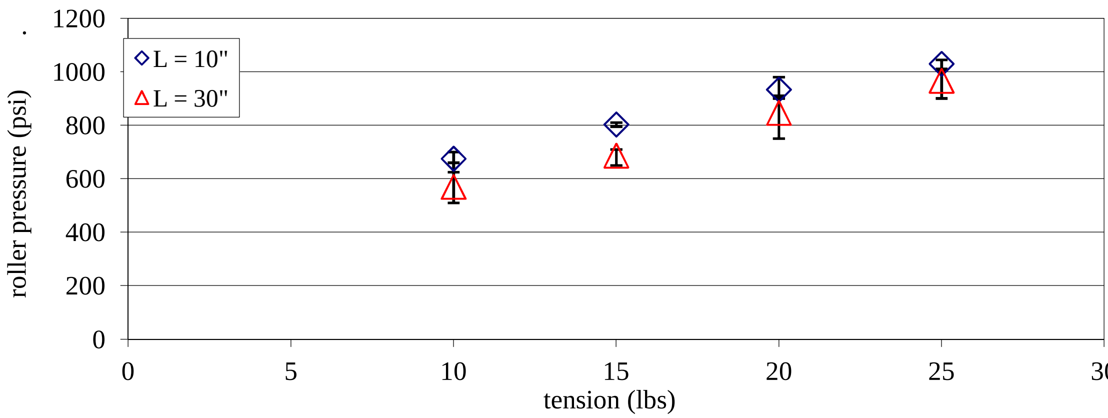

Figure II-1: Troughs Induced in a 92 gage Polyester Web (t=0.00092", E=711,000 psi, ν=0.3, W=6")

Beisel used COSMOS, a commercial finite element code, to model the steady state behavior of the web interacting with the crown roller. He used expression (5) to determine the machine direction stress variation that resulted from a given level of internal pressurization (and hence a_0 and a_1) that had induced troughs in the web. By changing the load applied on each node individually on the roller, the desired machine direction stress across the roller was achieved. Negative CMD stresses resulted from this analysis that were essentially zero at the upstream roller and became negative (compressive) as the web advanced closer to the crowned roller. The CMD stresses reached their highest compressive level just as the web entered the crowned roller. Beisel found that trough buckling was well forecast by the finite element code if the length of the web span subject to compressive stress (L_{eff}) instead of the entire web span length (L) was input to expression (6). When the average compressive stress from the finite element analysis over the effective length became more negative than that given by expression (6), troughs resulted.

The crown levels which could be produced by Beisel's variable crown roller were sufficient to induce troughs in his 92 gage polyester web but not wrinkles. He proceeded to conduct wrinkling tests in this web using rollers with fixed levels of crown machined into them. By using a 0.006" diametric parabolic crown over an 8" roller width and the 92 gage polyester used in the trough tests, the wrinkling could be removed by increasing tension. By starting at high tension and then slowly decreasing the web tension he could find what web tension level was associated with the onset of wrinkling as shown in Figure II-2. Beisel used finite element analysis to model web wrinkling due to roller crown. He used Miller-Hedgepeth wrinkle membrane elements to model the troughed web behavior upstream of the crowned roller. He slowly increased the crown level until the compressive stress was large enough to produce wrinkling per expression (7). An alternative analysis varied the web temperature across the roller width resulting in the machine direction stresses predicted by expression (5) and compressive CMD stresses that were comparable to the first analysis.

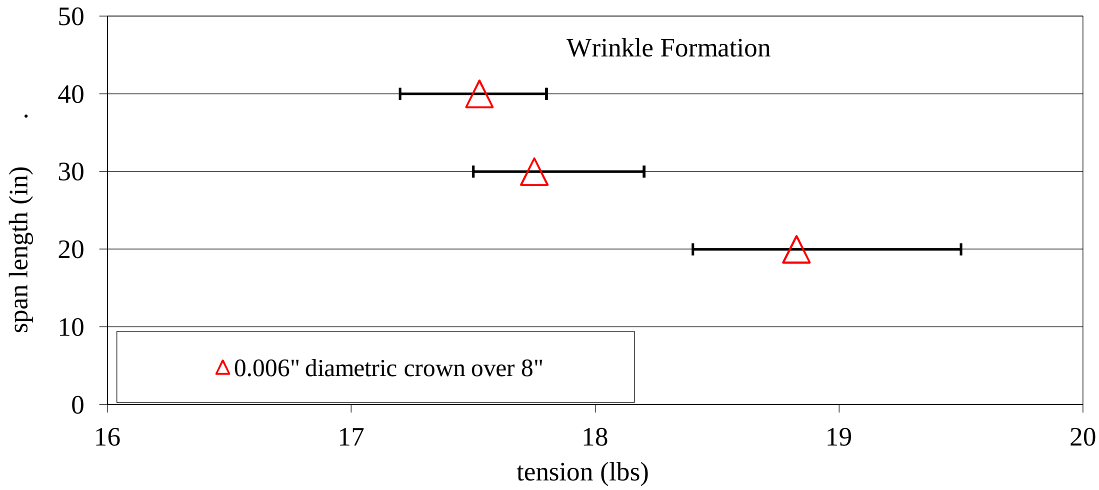

Figure II-2: Wrinkles Induced in a 92 gage Polyester Web (t=0.00092”, E=711,000 psi, ν=0.3, W=6”)

Vaijapurkar [3], used the explicit solution method of Abaqus to study the interaction of webs with crowned rollers. Vaijapurkar modeled several crowned rollers as rigid analytical surfaces in Abaqus and studied the compressive CMD stresses produced in the web by these rollers. These explicit computations were preferable to the steady state calculations run by Beisel because very few assumptions were required to run the explicit simulations. A web velocity was set at one end of the simulated web and the web tension was set at the other end. The simulated contact forces between the web and crown roller then determined what MD and CMD stresses resulted in the web. These simulations required considerable time for execution, several required 48 hours of solution time. Vaijapurkar did achieve success in his simulations in predicting roller crown levels that would induce wrinkles that compared well with Beisel’s test data.

Wrinkling Membranes

Miller and Hedgepeth [4] created a new finite element D matrix (relating the 2D stresses {σ} to strain {ε}) for cases where the element has buckled out-of-plane. This element is useful because it allows the user to model post-buckled behaviors such as troughs with planar finite elements that are computationally efficient. After troughs form the CMD strains in the web are larger than what Hookean elastic constitutive equations would predict. The wrinkle membrane elements have three potential material behaviors which depend on the relative values of the planar principal strains. These behaviors are in the form of the different D matrices shown below.

$$\begin{aligned} D &= D_s : && \varepsilon_1 < 0 \\ D &= D_w : && \varepsilon_1 > 0 \; and \; \nu\varepsilon_1 < -\varepsilon_2 \\ D &= D_T : && Else \end{aligned}$$

Equation 10 The 3 Material Behaviors of the Miller-Hedgepeth Element

$$D_s = 0 \quad : Slack$$

Equation 11 - Slack Behavior (no coupling between stress and strain)

$$D_T = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} : Taut$$

Equation 12 - Taut Behavior (plane stress linear elastic behavior between stress and strain)

$$D_w = \frac{E}{4}\begin{bmatrix} 2(1+P) & 0 & Q \\ 0 & 2(1-P) & Q \\ Q & Q & 1 \end{bmatrix} : Wrinkling$$

$$P = \frac{(\varepsilon_x - \varepsilon_y)}{(\varepsilon_1 - \varepsilon_2)} \qquad Q = \frac{(\gamma_{xy})}{(\varepsilon_1 - \varepsilon_2)}$$

Equation 13 - Wrinkled Behavior (Coupling is now dependent on P and Q and Cartesian and Principal Strains)

Beisel and Vaijapurkar have both shown the Miller-Hedgepeth finite element D matrix are useful in modeling wrinkling of webs on crowned roller whose entering spans have previously buckled and troughed. Both Beisel and Vaijapurkar used the commercial finite element code COSMOS which had several 2D finite elements which allowed the use of the Miller-Hedgepeth wrinkle membrane material behavior.

Research Objective

The objective of this research is to develop stand-alone predictive codes that can be used by practicing engineers to determine what levels of roller crown will induce troughs and wrinkles in a given web. These codes will be useful to determine if new webs can be transported on an existing process machine without troughs or wrinkles. They will also be useful for limiting the setting tolerance levels of roller crown when new rollers are machined for new process machines or refits. These codes will employ the Miller-Hedgepeth material behavior and enforce boundary conditions found in the research of Beisel and Vaijapurkar.

CHAPTER III

TROUGHS ANALYSIS

Introduction

A tool for predicting troughs in webs due to a downstream crowned roller was created by studying the explicit analysis of Vaijapurkar. Equivalent boundary conditions were developed that could be used to model the steady state interaction of a web with a crowned roller using static finite element analysis.

The Kinetic and Kinematic Boundary Conditions for Webs Interacting with Crowned Rollers

Webs are known to approach cylindrical rollers normal to the axis of rotation of the roller. Shelton [9] has used this knowledge to develop models of how webs steer laterality in web process machines. Shelton also found that the frictional forces required to steer a web into “normal entry” to a roller are often very small. Slippage at the entry tangent line of a web to a roller is usually zero or negligible.

What was unknown was how a web approaches a non-cylindrical roller such as a crowned roller. Did normal entry still apply? Vaijapurkar explicit analyses in fact proved that webs do entry crowned rollers normally as shown in Figure III-1. Also any slippage that occurs between a web and a crowned roller is found where the web exits the roller, not at the entry as seen in Figure III-2.

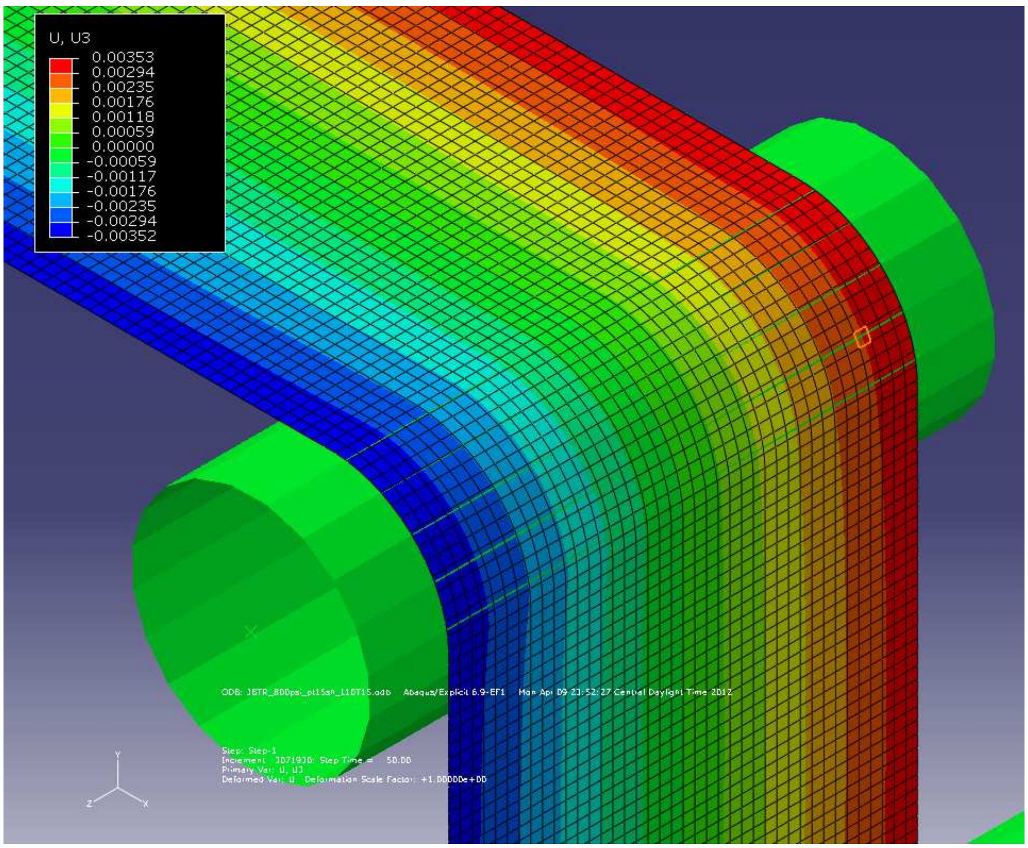

Figure III-1: CMD Deformation of a Web Transiting a Crown Roller (a_o=1.528 in, a_1=0.0000441 1/in, 92 gage PET, E=712000 PSI, L=10 in, w=6 in, T_{web}=15 lbs, υ=0.3)

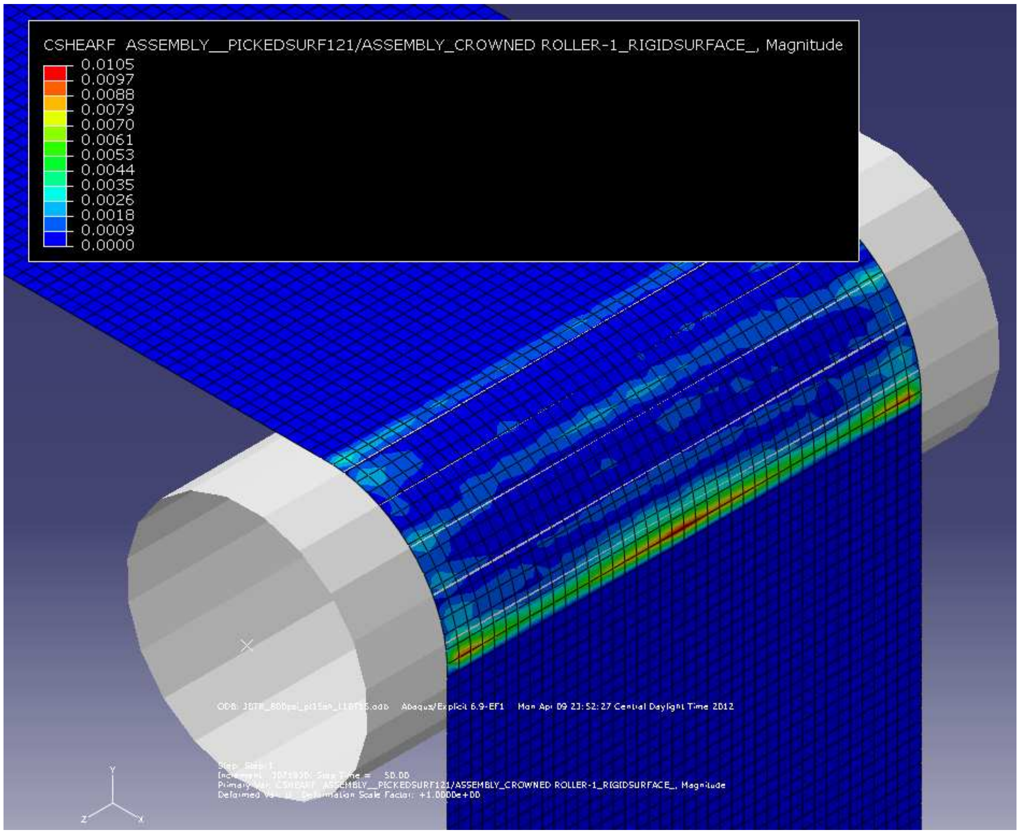

Figure III-2: Slippage Forces Web/Crown Roller for a Web Transiting a Crown Roller (a_o=1.528 in, a_1=0.0000441 1/in, 92 gage PET, E=712000 PSI, L=10 in, w=6 in, T_{web}=15 lbs, υ=0.3)

The explicit analyses also showed the MD stresses predicted by expression (5) are witnessed by the web on the roller. Also apparent from the explicit analyses were in-plane shear stresses as shown in Figure III-3.

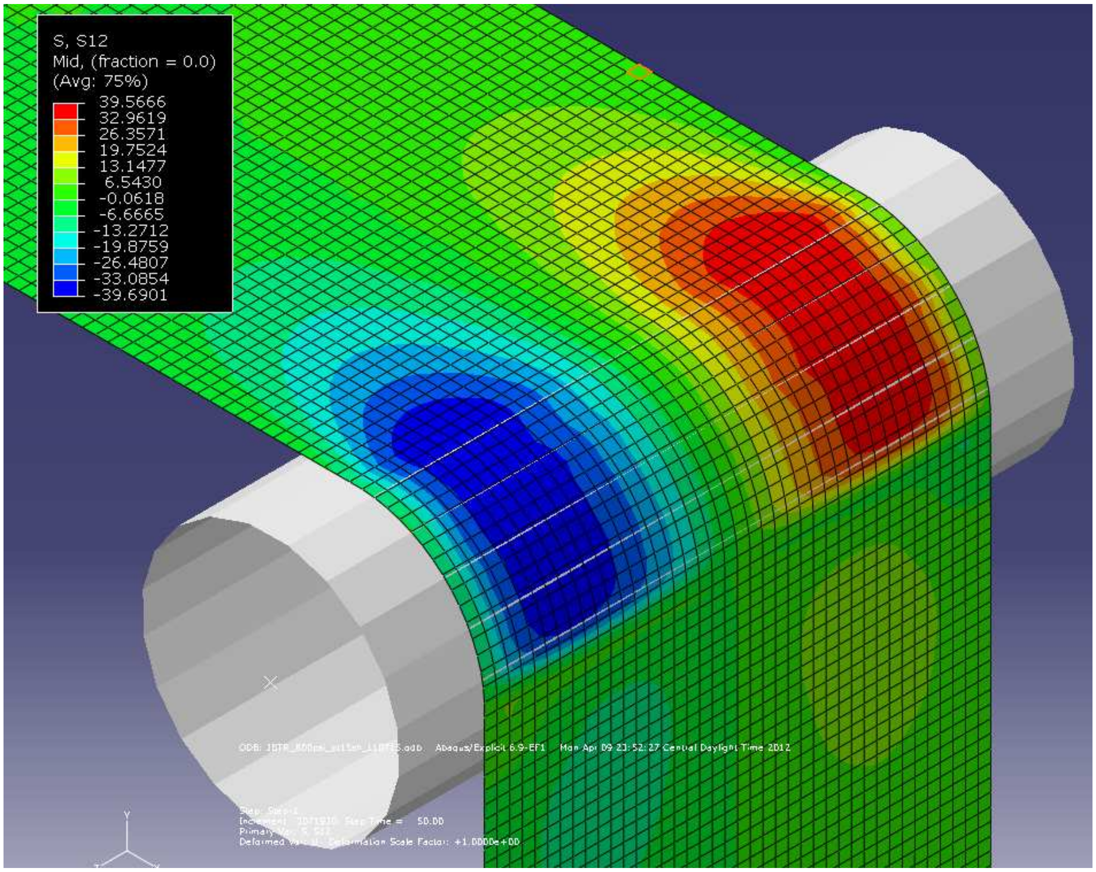

Figure III-3: Shear Stresses in a Web Transiting a Crown Roller (ao=1.528 in, a1=0.0000441 1/in, 92 gage PET, E=712000 PSI, L=10 in, w=6 in, Tweb=15 lbs, υ=0.3)

What was unknown was that the MD stresses and the shearing stresses act together to produce the normal entry boundary condition.

A Steady State FEA Model for the Prediction of Troughs

It was found the shearing stresses in the web at the entry of the crown roller, Figure III-3 for example, were well modeled by the expression:

$$\tau_{xy} = C_1\left(y - \frac{8}{w^4}y^5 - \frac{2}{w^2}y^3\right)$$

Equation 14 The Shearing Force Added at the Edge to Enforce Normal Entry.

Where y is a CMD coordinate originating at the widthwise center of the web and C_1 is a constant. A finite element model was developed with the boundary conditions shown in Figure III-4.

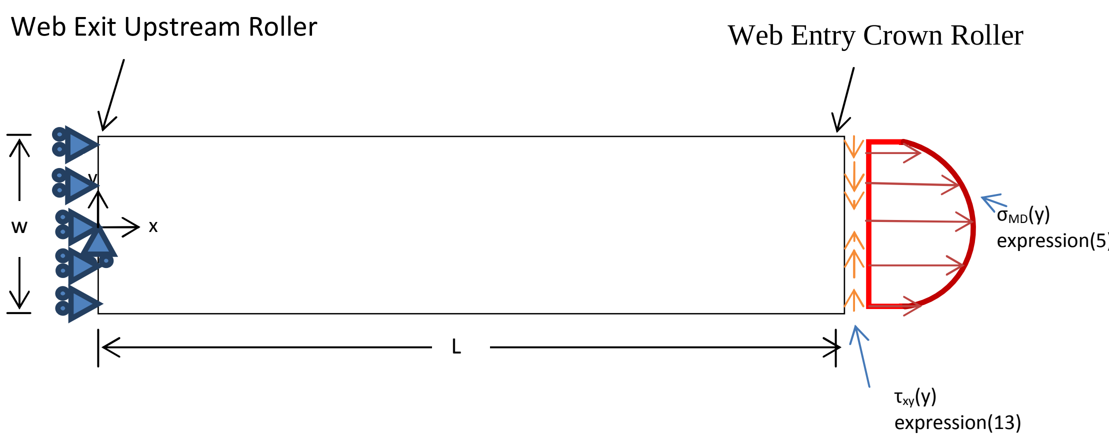

Figure III-4: Steady State Static FE Model

Note that no attempt is made to enforce normal entry of the web to the crown roller with kinematic boundary conditions. Instead the constant C_1 in expression (14) is varied until the web slope across the web width approaches zero and normal entry is attained. A flow chart of the code that accomplishes this is shown in Figure III-5. Elastic plane stress quadrilateral elements are used to model the web prior to any trough buckling.

This code executes by computing the slopes at entry for two unique values of C_1 in expression (14). Since this is linear analysis a third value of C_1 can then be projected to minimize the slope error. The slope at each CMD nodal location must be estimated from lateral CMD deformation output by the code near the exit of the web span.

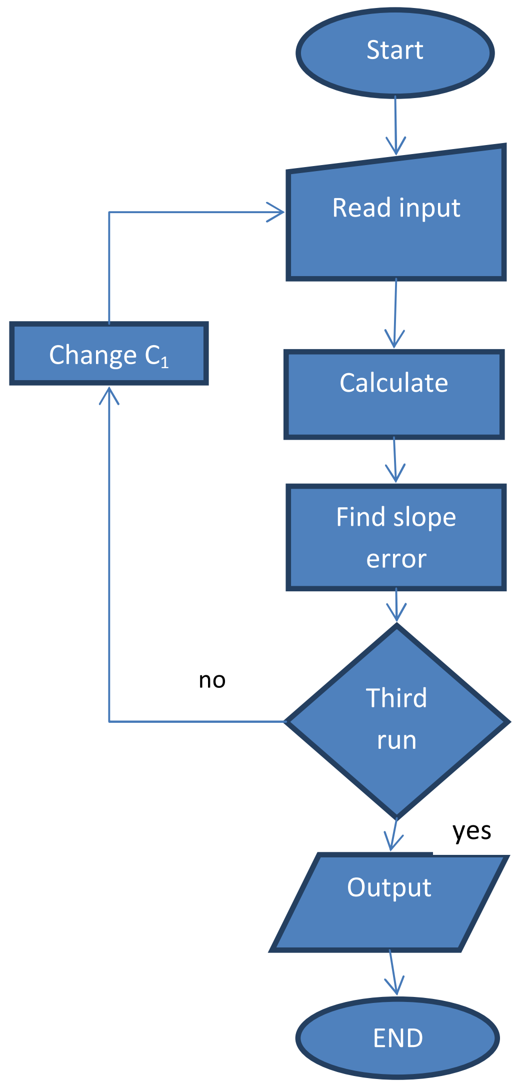

Figure III-5: Flow Chart for FEA Code Used to Predict Web Stresses Prior to Trough Buckling

After the code has succeeded in producing normal entry of the web to the crown roller the two dimensional stresses (σ_x, σ_y, τ_{xy}) are known throughout the web span entering the crown roller. Even with the slightest roll crown (i.e. $a_1>0$) negative CMD stresses (σ_y) are witnessed immediately upstream of the crown roller. The CMD stresses are most negative at the web center (y=0) and dissipate with the distance upstream of the crown roller in the entering web span.

The Trough Instability Criteria

Timoshenko [7] developed a buckling criterion for a rectangular panel subjected to a biaxial normal surface traction shown in Figure III-6.

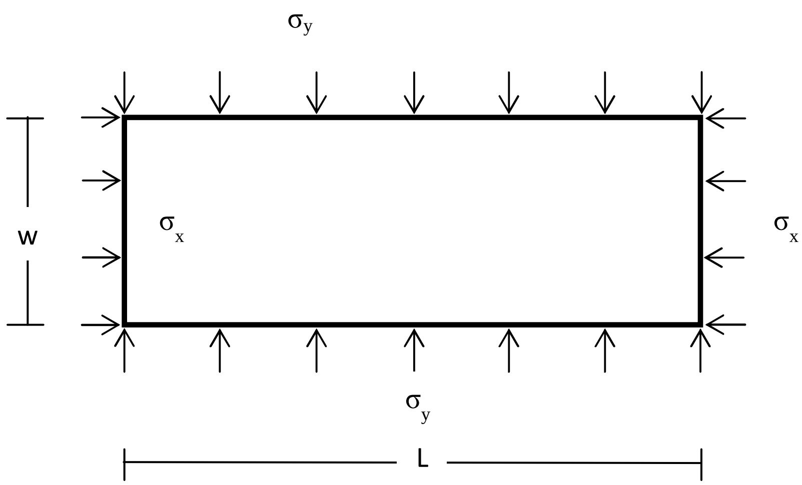

Figure III-6: Rectangular Plate Subject to Biaxial Traction

Timoshenko assumed the four edges were simply supported. Later Good and Beisel [8] simplified Timoshenko's development for a web span subjected to a tensile x direction stress and predicted the critical CMD σ_y compressive stress that would produce trough buckling:

$$\sigma_{y,Troughs} = -\frac{t * \pi}{L}\sqrt{\frac{\sigma_{md} * E}{3 * (1 - \upsilon^2)}}$$

Equation 15: Troughs Critical Stress

Note the critical buckling stress is independent of the web width (w). Later Beisel [1] concluded expression (15) was still useful when compressive stresses were only present over part of the web span. In those cases Beisel suggested that L, the span length, in expression (15) be replaced with $L_{effective}$, the length of the span subject to compressive stress. Beisel also suggested comparing the average CMD σ_y compressive stress to the critical buckling stress from expression (15) where $L_{effective}$ = L to predict the onset of trough buckling.

Predicting Web Troughs Due to Roller Crown

With the steady state static finite element model for predicting web stresses in the entry span to a crown roller and trough failure criteria the level of crown required to induce troughs in a web can be predicted. For a particular test case a crown level (a_1) is assumed. The finite element code is executed and the CMD (σ_y) stresses are known spatially in the entry span. The code then assesses the effective portion or length of the web span upstream of the crown roller that is subject to compressive CMD stresses ($L_{effective}$) and the compressive stresses are averaged over that length. That average stress is then compared to the critical trough buckling stress from expression (15) with

$L=L_{effective}$. If this average compressive stress is more negative than that given by expression (15), troughs exist. Then a search routine ensues where the crown level (a_1) is varied until the average compressive stress over the effective length becomes equal to that given by the trough buckling criteria. Now the crown level (a_1) to induce troughs in a given web is known.

Application

First a set of input properties are selected.

Web Width (w)	6 in.
Web Thickness (t)	0.00092 in.
Young's Modulus (E)	712000 PSI
Poisson's Ratio (υ)	0.3
Length (L)	10 in.
Tension	15 lbs
a_o	1.5275 in.
a_1	$4.11 * 10^{-5}$ 1/in.

Table III-1

The finite element code is now executed for two arbitrary values of C_1 (0 and 5) in expression (14). A third value of C_1 is then projected that will minimize the slope error. The slopes across the web width are shown in Figure III-7 and the slope summed absolute slopes across the web width at the web entry to the crowned roller are given in Table III-2.

C_1	Slope Error
0.00E+00	-2.35E-03
5.00E+00	-2.74E-03
-3.03E+01	2.96E-16

Table III-2

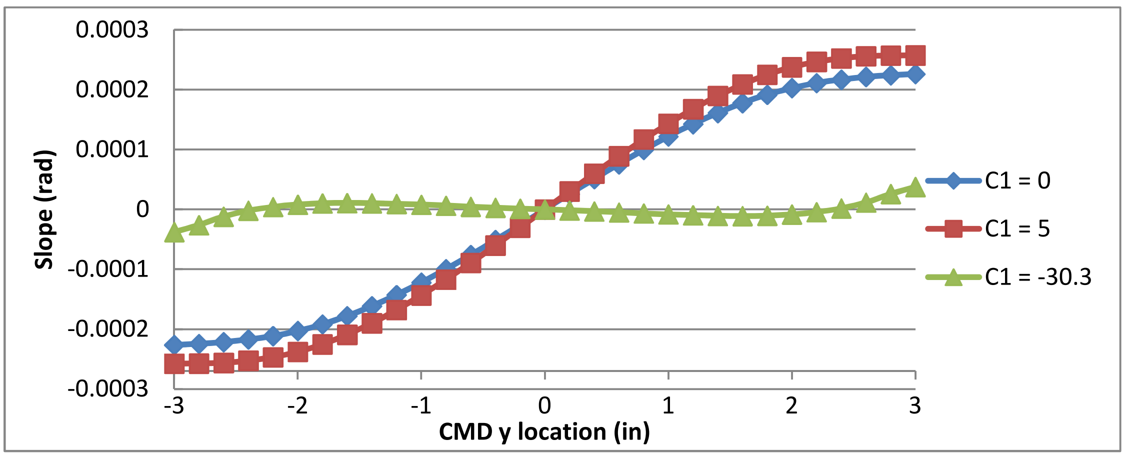

Figure III-7: Slopes in the Web at the Entry to the Crowned Roller

Next from the output of the finite element code the CMD (σ_y) stresses are studied on the web centerline (y=0) as shown in Figure III-8.

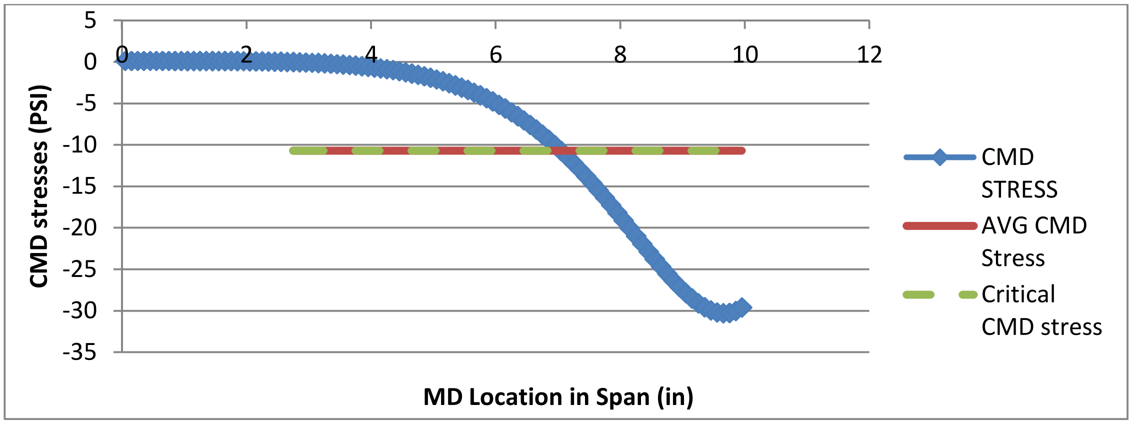

Figure III-8: CMD Stresses

The average stress in this case is -10.7 PSI returning to the failure criteria (15) the compressive stress required to induce troughs is -10.7 PSI. Thus for this level of crown (a_1) troughing would be predicted. This is linear analysis and thus two arbitrary values of a_1 can be analyzed to project a third value of a_1 which will produce an average compressive stress over the effective length that will equal the trough bucking stress given by expression (15). This is the level of crown (a_1) needed to produce troughs in a web span of given span length, width and thickness with given material properties subject to a given web tension level.

Comparison to Previous Research

Beisel conducted tests and Vaijapurkar conducted analyses for the following cases.

Web Width (w)	6 in.
Web Thickness (t)	0.00092 in.
Young's Modulus (E)	712000 PSI
Poisson's Ratio (υ)	0.3
Length (L)	10, 30 in.
Tension	10, 15, 20, 25 lbs
a_o	1.5275 in.

Table III-3

A comparison of the results from the code developed here to the previous results is given in Figure III-9.

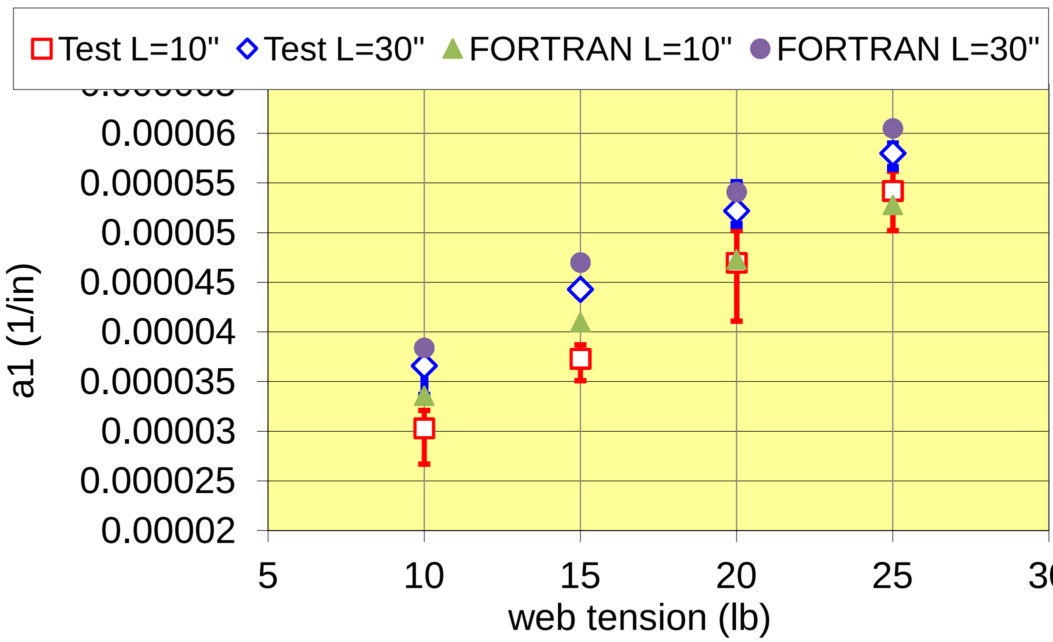

Figure III-9: Comparison of Results

The results above show fine comparison with all previous results. All results show less dependence on length of the entering span than the web tension.

The FORTRAN code executes in 6 seconds which is very favorable compared to the 48 hour explicit simulations conducted by Vaijapurkar.

CHAPTER IV

WRINKLING ANALYSIS

Introduction

A tool for predicting if the crown level of a roller will cause wrinkles in a web was created by studying the explicit analysis of Vaijapurkar. The problem was also analyzed using Cosmos to create the boundary conditions that will be used in a static finite element analysis to model the steady state interaction between the crowned roller and web.

A Steady State FEA Model for the Prediction of Wrinkles

The same kinematic and kinetic boundary conditions as shown in Figure III-4, the MD traction from expression (5) and the CMD shearing force in expression (14) was used in the wrinkling prediction code. Again there is no attempt to enforce normal entry of the web using kinematic boundary conditions. The constant C_1 in expression (14) is varied until normal entry is attained. The code uses elastic plane stress quadrilateral elements that can react to the CMD compressive stress at entry and exit of the web span. The remaining elements of the web are the wrinkling membrane quadrilateral elements of

Miller and Hedgepeth described in the *Literature Survey*.

As the code is a nonlinear analysis it can no longer be assumed that the C_1 can be projected from two previous known values of C_1 and slope error. C_1 is estimated by assuming it is linear. This will get C_1 to be closer to the correct C_1, which is associated with normal entry, where it is varied slightly and the same linear analysis is done once more. The flow chart for the code is shown in Figure IV-1.

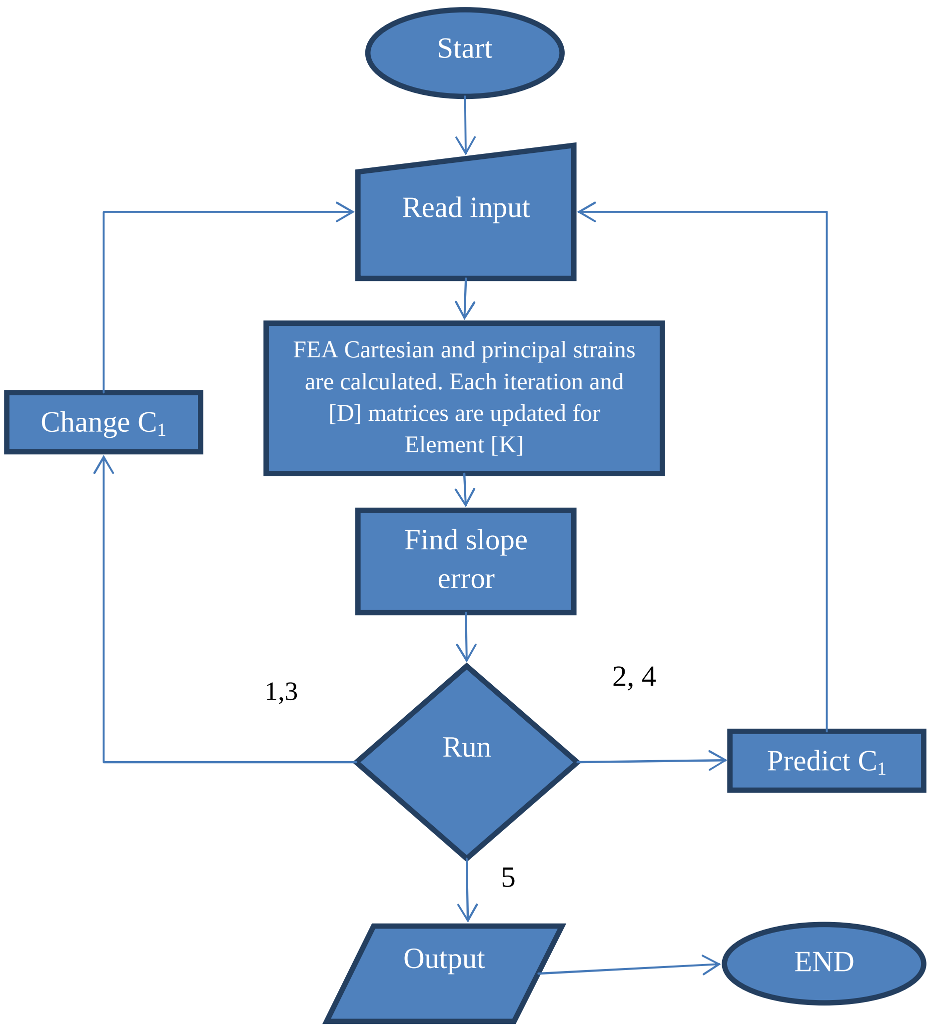

Figure IV-1: Flow Chart for FEA Code Used to Predict Web Stresses Prior to Wrinkling

After the code has produced normal entry the two dimensional stresses near the crowned roller are known. The buckling criteria for wrinkling only requires the CMD σ_y stress at the web center (y=0) near the entrance to the crowned roller and known roller and web properties.

The Wrinkling Instability Criteria

Timoshenko [7] developed the buckling criterion for a thin cylindrical isotropic shell:

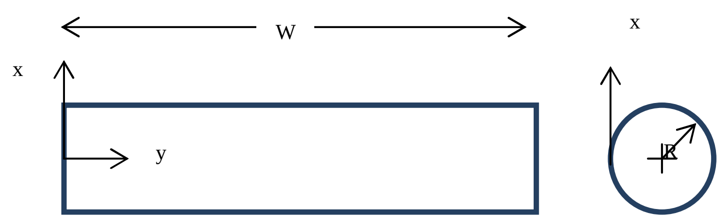

Figure IV-2: Thin Cylindrical Dimensions

$$\sigma_{y,wrinkling} = -\frac{E * t}{R * \sqrt{3 * (1 - \upsilon^2)}}$$

Equation 16: Wrinkling Critical Stress

The shell is formed by the conformance of the web to a roller. Wrinkling should occur when the compressive stresses σ_y near the center of the web (y=0) when it begins to take the shape of the roller are more negative than given by expression (16). Note that the critical buckling stress is not dependent on the web length (L) or the machine direction stress σ_x as the trough buckling criteria was in expression (15).

Predicting Web Wrinkling Due to Roller Crown

With the steady state static finite element model for predicting web stresses near the entrance to a crown roller and wrinkling failure criteria it can be predicted if a certain crown level will produce web wrinkling. For a particular test case a crown level (a_1) is assumed. The finite element code is executed and the CMD (σ_y) stresses are known near the web entrance to the crown roller. That stress is then compared to the critical wrinkling buckling stress from expression (16) with R = a_0. The CMD location where wrinkling will begin is at the CMD center (y=0) and a_0 will be the radius at that point. If this compressive stress is more negative than that given by expression (16), wrinkling will occur.

Application

First a set of input properties are selected.

Web Width (w)	6 in.
Web Thickness (t)	0.00092 in.
Young's Modulus (E)	712000 PSI
Poisson's Ratio (υ)	0.3
Length (L)	30, 40 in.
Tension	18.6 lbs
a_o	1.45 in.
a_1	0.000188 1/in.

Table IV-1

For the test cases herein the critical buckling stress per expression (15) is -273 PSI.

The finite element code is now executed for two arbitrary values of C_1 (0 and -0.005) in expression (14). A third value of C_1 is then projected to minimize error, input to the code, and the slope error is calculated. The code then takes the third value of C_1 and varies it to produce a fourth value of C_1. A fifth value of C_1 is then projected, as was done with the third value, to minimize the slope across the width of the web. The slopes across the web width are shown in Figure IV: 3-4 and the summed slopes across half of the web width at the web entry to the crowned roller are given in Table IV: 2-3 for the 30” and 40” span lengths.

C1	Slope error
0	4.76E-03
-0.005	4.66E-03
-0.249	-6.07E-04
-0.254	-7.13E-04
-0.2205	5.10E-06

Table IV-2: L=30 in

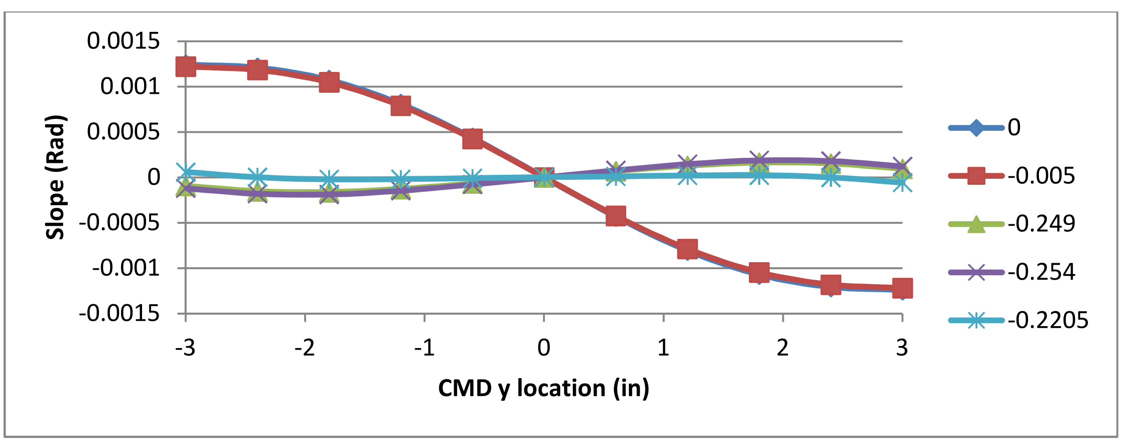

Figure IV-3: Slopes in the Web at the Entry to the Crowned Roller (L=30 in)

Con2	Slope error
0	0.0052328
-0.005	0.0051116
-0.216	0.0002139
-0.221	9.768E-05
-0.225	1.339E-06

Table IV-3: L=40 in

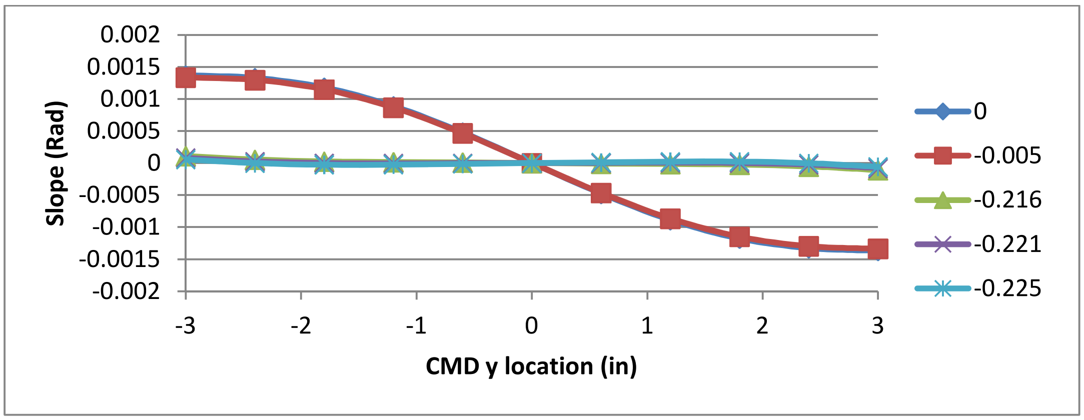

Figure IV-4: Slopes in the Web at the Entry to the Crowned Roller (L=40 in)

A comparison between the results obtained from the FORTRAN code developed here and Cosmos is given in Figures IV: 5-8.

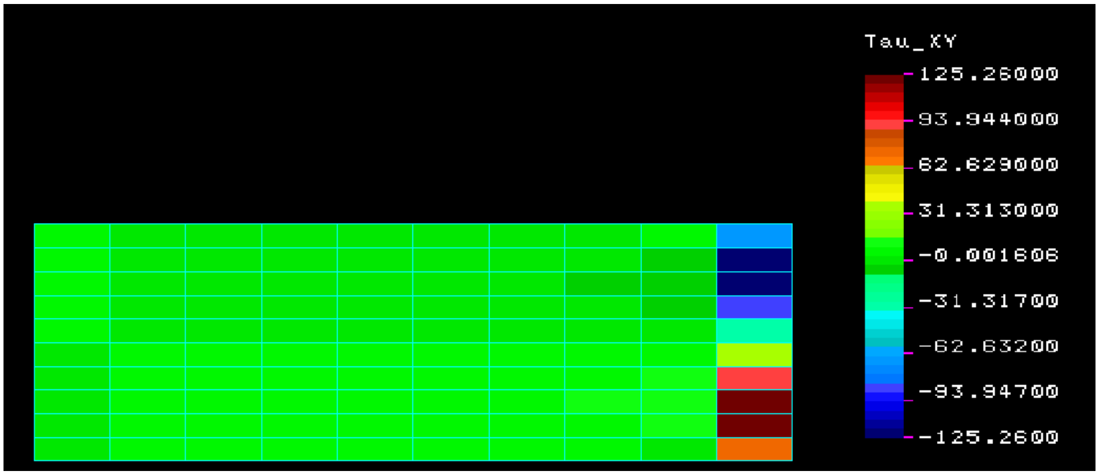

Figure IV-5: Cosmos Shear Stresses (L=20in, w=6in, t=0.00092in, E=712000PSI, v=0.3, T=18.6 lb, a_0=1.45in, a_1=0.000188 1/in)

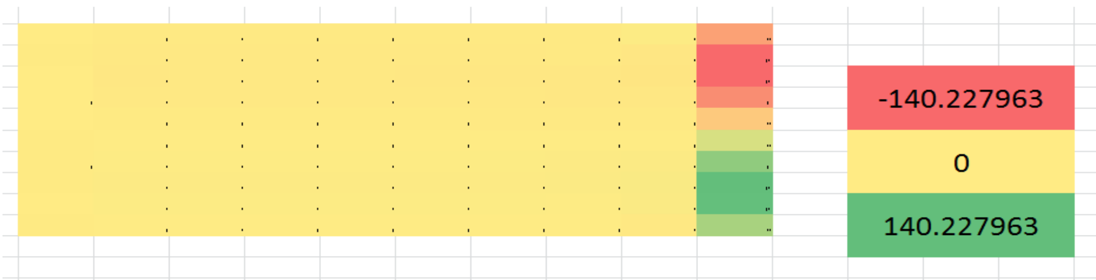

Figure IV-6: Fortran Shear Stresses (L=20in, w=6in, t=0.00092in, E=712000PSI, v=0.3, T=18.6 lb, a_0=1.45in, a_1=0.000188 1/in)

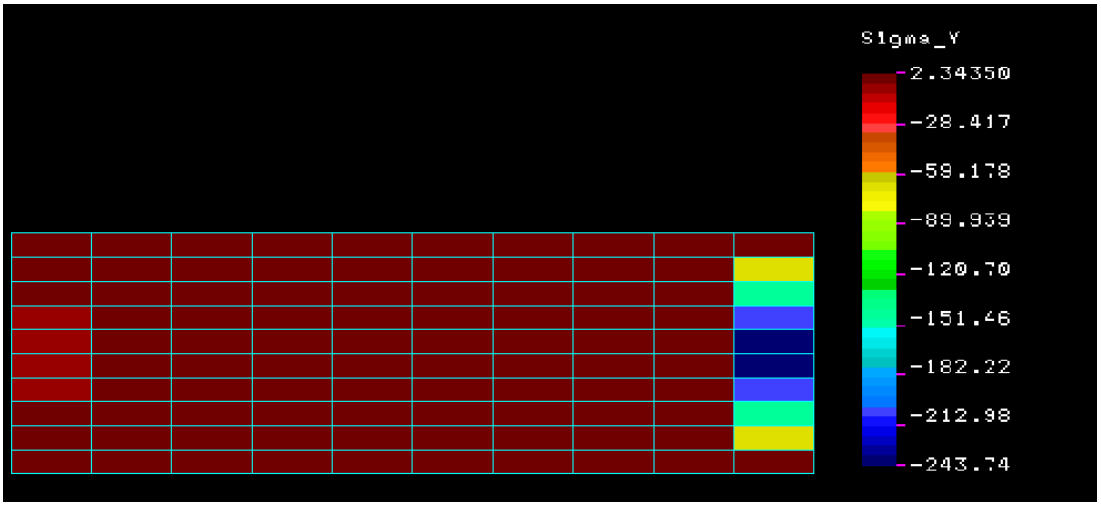

Figure IV-7: Cosmos CMD Stresses (L=20in, w=6in, t=0.00092in, E=712000PSI, v=0.3, T=18.6 lb, a_0=1.45in, a_1=0.000188 1/in)

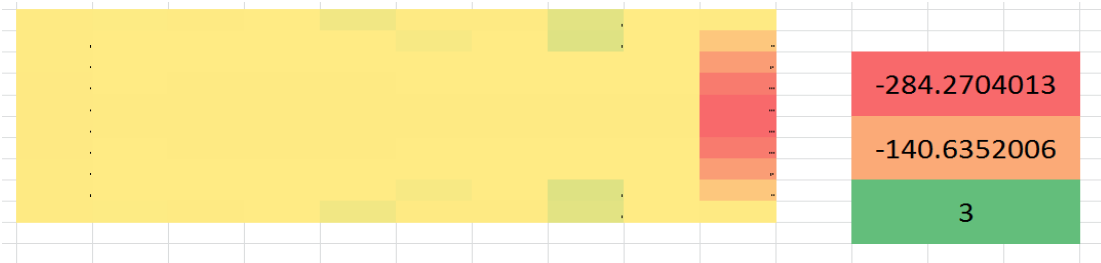

Figure IV-8: Fortran CMD Stresses (L=20in, w=6in, t=0.00092in, E=712000PSI, v=0.3, T=18.6 lb, a_0=1.45in, a_1=0.000188 1/in)

The results indicate that the last column of elements, as it is linear elastic element, contains most of the stresses. This is also the only column that is of interest when determining slope and the compressive stress that is compared to the critical stress.

Comparison to Previous Research

Beisel conducted tests, and Vaijapurkar conducted analyses for the following cases.

Web Width (w)	6 in.
Web Thickness (t)	0.00092 in.
Young's Modulus (E)	712000 PSI
Poisson's Ratio (υ)	0.3
Length (L)	20, 30, 40 in.
Tension	18.6 lbs
a_o	1.45 in.
a_0	.000188 1/in

Table IV-4

Beisel proved that this level of crowning is sufficient for all three lengths to produce wrinkling.

A comparison of the results from the code developed as well as Cosmos and previous results are given in Figure IV: 9-13. It should be noted that the ABAQUS explicit simulations includes dynamic and nonlinear effects. The double peak in CMD compressive stress in Figure IV-10 is due to out-of-plane trough deformation in the entry span. Out-of-plane deformations are not considered by the Cosmos and FORTRAN solutions that account for troughs with the Miller-Hedgepeth elements. The different solution methods produce the different behaviors seen in Figure IV: 9-11. What is important is that the computational efficient static FEA models are capable of providing good estimates of the CMD compressive stresses and can be used to predict wrinkling.

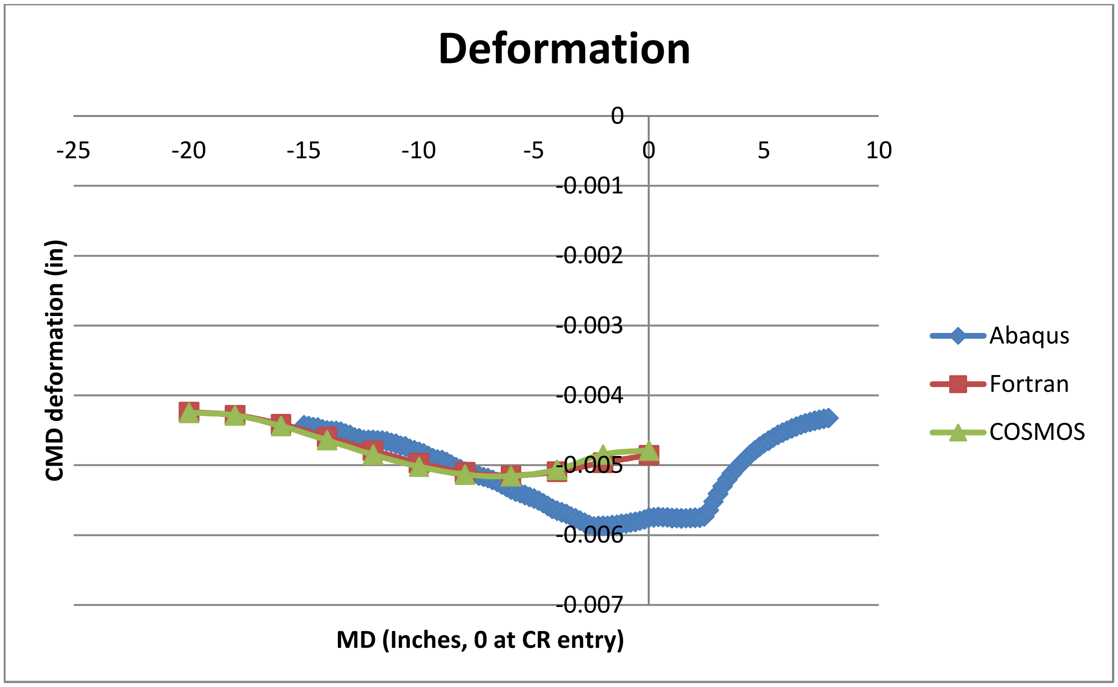

Figure IV-9: CMD deformation of web edge x=0 at Crown roller Entrance (L=20 in). FORTRAN and Cosmos Results are shown only in the Entry Span.

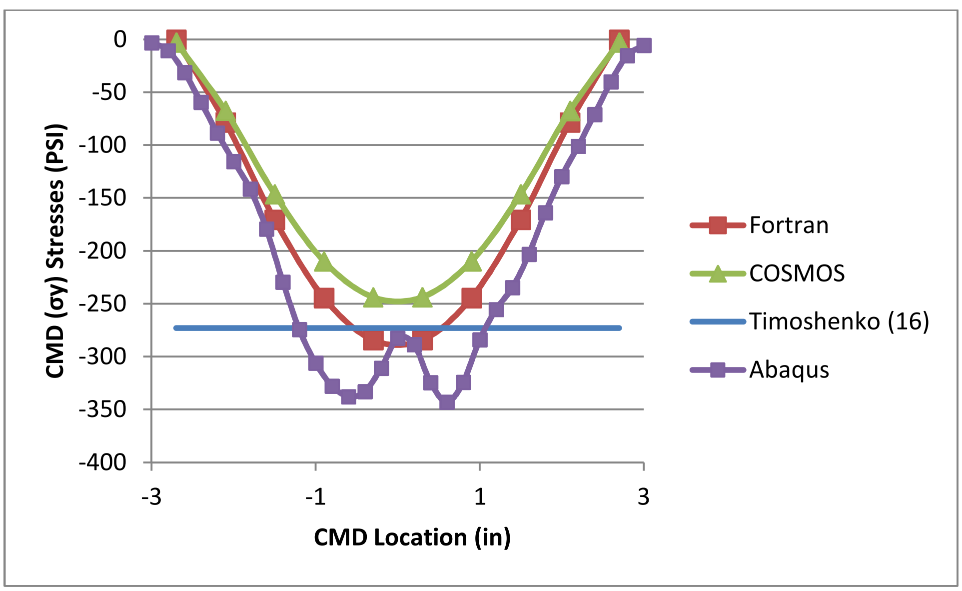

Figure IV-10: CMD Stresses at Crowned Roller Entrance (L=20 in)

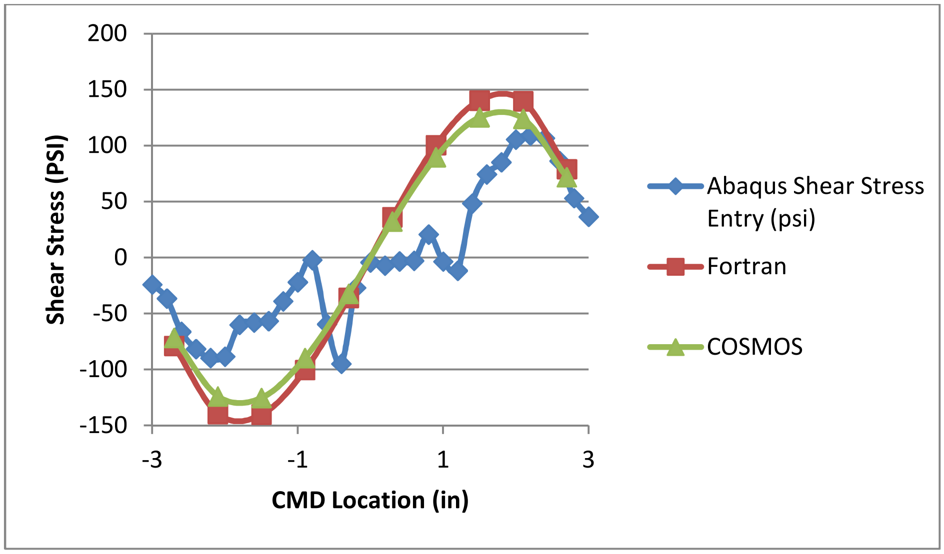

Figure IV-11: Shear Stresses at Crowned roller Entrance (L=20 in)

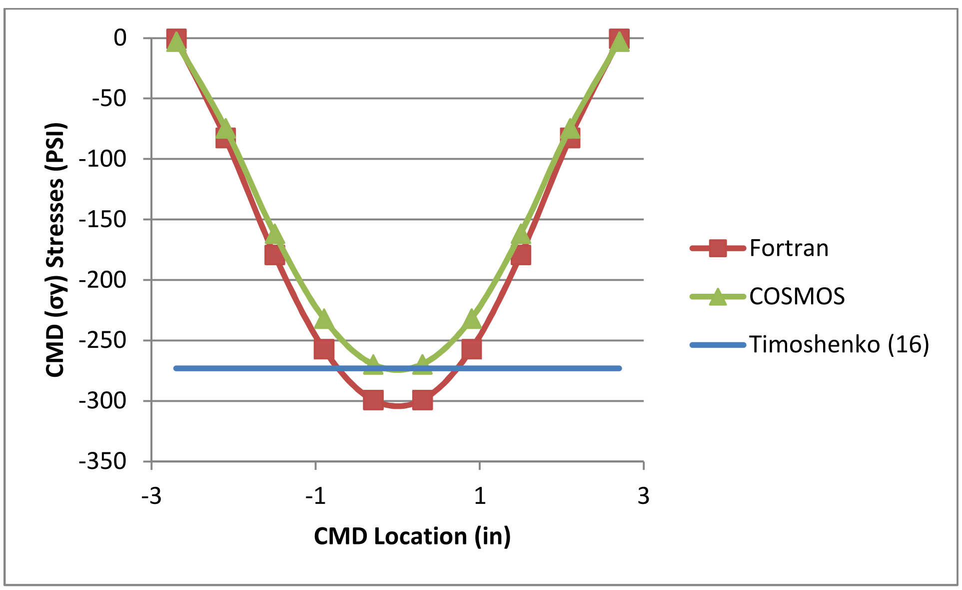

Figure IV-12: CMD Stresses at Crowned Roller Entrance (L=30 in)

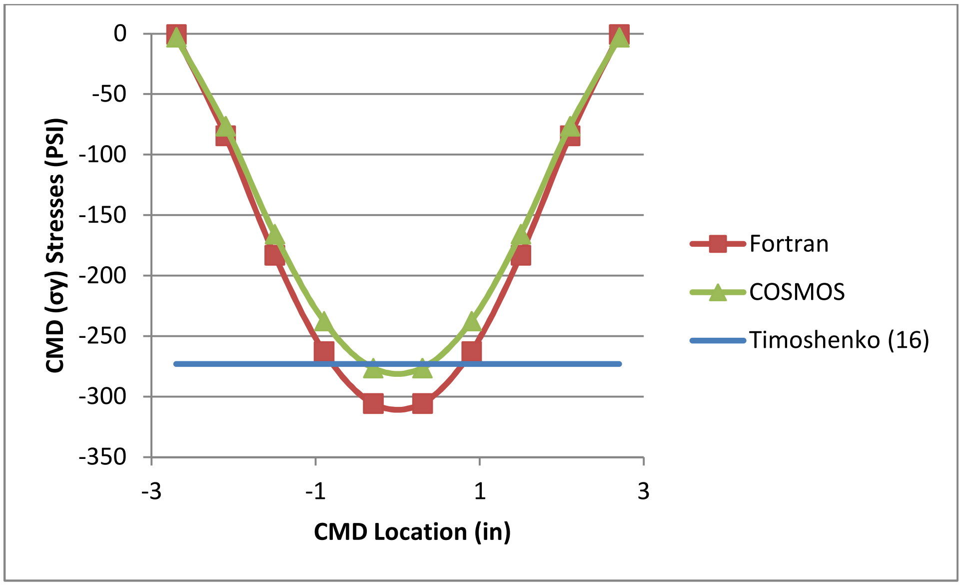

Figure IV-13: CMD Stresses at Crowned Roller Entrance (L=40 in)

The results above show fine comparison with all previous results and prove that static FEA with the correct boundary conditions can be used to predict web wrinkling due to roller crown. The FORTRAN code executes in 3 seconds for these test cases and is very favorable compared to the 48 hour explicit simulations conducted by Vaijapurkar.

CHAPTER V

SUMMARY, CONCLUSION, AND FUTURE WORK

Summary

The use of crowned roller to guide belts and transmit power to line shafts date to early in the industrial revolution. These belts were not susceptible to troughs and wrinkles as thin webs are in process machinery today. The codes developed herein could provide a quick engineering tool to forecast web troughs and wrinkles due to roller crown. They might have value for those who set manufacturing tolerances for rollers to prevent web instability due to roller crown.

As the codes were developed it was assumed that there was sufficient frictional force between the web and crown roller that the roller would induce the differential MD stresses given in the first term of expression (5) and the CMD shear tractions given by expression (14). If the available friction forces are insufficient slippage will occur at the entry of the web to the crown roller and the boundary conditions assumed herein become invalid. This will also inhibit wrinkling as this slippage will diminish the CMD compressive stresses.

Conclusion

1. This research used explicit FEA simulations to seek out steady state boundary conditions that can be used with static analyses that are computationally efficient.
2. This work has shown that combined machine and shear stresses that are exerted by the roller can be used to ensure that webs enter crown roller normally.
3. A linear FEA code combined with a modified trough buckling criteria can successfully predict web troughs resulting from roller crown.
4. A non-linear FEA code was successfully developed that can predict what degree of crown will wrinkle the web. This code required the use of a Miller-Hedgepeth finite element to account for web troughs that precedes wrinkles.

Future Work

The current research addresses the steady state behavior of a web transiting a crown roller. These devices can also be responsible for dynamic lateral behavior of a web in a process machine which has received little attention.

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APPENDICE

The codes developed are proprietary to the Web Handling Research Center of Oklahoma State University. For more information about the Web Handling Research Center see the following web site: http://webhandling.okstate.edu/

VITA

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