THE EFFECTS OF CURRICULAR MODIFICATIONS ON MATH FACT FLUENCY RATES

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2009

Submitted to the Faculty of the
Graduate College of the
Oklahoma State University
in partial fulfillment of
the requirements for
the Degree of
DOCTOR OF PHILOSOPHY
May, 2014

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ACKNOWLEDGEMENTS

The completion of this dissertation would not have been possible without the encouragement of my mentors, colleagues, and family members. First, I would like to thank my advisor and dissertation chair, Dr. Brian Poncy, for sharing his knowledge and passion for academic interventions, specifically related to mathematics. His flexibility and willingness to meet at a moments notice to discuss research concerns was greatly appreciated. Many of my skills were refined during our meetings outside of his lectures, and I cannot thank him enough for sharing his knowledge during these times. In addition, I would like to thank Dr. Duhon for providing multiple opportunities to participate in various research projects and professional activities during my graduate training. These experiences further refined my understanding of research methodology within the context of the school setting, which was a crucial component of this dissertation project. I would also like to thank Dr. Terry Stinnett for developing my foundation of skills that are necessary to function as an independent practitioner. I always appreciated his ability to incorporate humor into the learning environment to facilitate my understanding of complex concepts. His support of my dissertation topic and his assistance throughout the preparation of the manuscript is greatly appreciated. Furthermore, I would like to thank Dr. Robert Davis and Dr. Dale Fuqua for their support and novel ideas during the preparation of this dissertation. Without their assistance and perspectives this project would not have been possible.

I would also like to thank the members of my cohort (Alicia Sullivan, Bethany Jordahl, Chelsie Williams, Cristina Villanueva, Jen Ho, Kara Branz, Kellen Vail, Kim Vogel, Lindsey Bardwell, and Sarah Banks) for always making sure that I was aware of the many due dates associated with our graduate training and the dissertation project. Additionally, I would like to thank Greg Schutte for his willingness to assist with the implementation and scoring components of this study despite his hectic third-year schedule. I especially appreciate all of their friendships and for making my graduate training an enjoyable, memorable experience.

Finally, I would like to thank my parents, Scuddy Fontenelle III, Ph.D., and Sheila Fontenelle. I am forever grateful for our countless conversations regarding my graduate experiences and career goals. I have and always will appreciate the support and advice that they both so frequently provide. I would also like to thank my sister, Sarah Fontenelle, Ph.D., and my brother-in-law, Alex Smith, for their encouragement and words of wisdom during the most trying times of my graduate training. They all have an amazing ability to put a positive spin on difficult situations, and I would not have succeeded without their perspectives.

Name: SCUDDY FRANCIS FONTENELLE IV

Date of Degree: MAY, 2014

Title of Study: THE EFFECTS OF CURRICULAR MODIFICATIONS ON MATH FACT FLUENCY RATES

Major Field: EDUCATIONAL PSYCHOLOGY (OPTION: SCHOOL)

Abstract: The current investigation compared the instructional efficiency of an explicit timing intervention between three conditions that varied on the curricular dimension of problem set size. The first goal was to determine if learning rates differ between groups exposed to probe sets containing either a mixture of automatic and non-automatic multiplication problems (Total Condition), non-automatic multiplication problems (Reduced Condition), or a specific ratio of automatic to non-automatic multiplication problems (Ratio Condition). A second goal was to determine if student performance would generalize on 36-problem Reciprocal probe sets. A third goal was to determine which instructional condition facilitates the maintenance of multiplication fact fluency performance the greatest over time. Participants included 73 fourth grade students attending general education at a public school in north central Oklahoma. Student performance was assessed pre-intervention (pre-test), following five weeks of intervention implementation (i.e., posttest), and two weeks following intervention cessation (i.e., maintenance). The total number of digits answered correctly per minute (DCPM) determined fluency performance on each dependent measure (i.e., 100-problem, 36-problem reduced, and 36-problem reciprocal assessments). A doubly repeated-measures multivariate analysis of variance (MANOVA) was utilized and consisted of one within subjects factor (i.e., Pretest, Posttest, and Maintenance) and one between subjects factor (i.e., Total, Reduced, and Ratio Conditions). Collectively, the results indicated that explicit timing was an effective intervention for improving the multiplication fluency performance of students in all treatment conditions. In addition, scores on a two-week follow-up assessment indicated that student fluency performance remained relatively stable over time. The lack of a significant Group x Time interaction suggested the fluency performance of students in each group was similar on multiple assessments over time (i.e., Pretest, Posttest, and Maintenance) on each dependent measure (i.e., 100-problem, 36-problem reduced, and 36-problem reciprocal Assessments). However, the absence of the significant interaction indicated that student fluency performance generalized to the novel presentation of multiplication facts (i.e., reciprocals) for specific conditions.

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CHAPTER I

INTRODUCTION

The majority of school-based referrals are academic in nature, which solidifies the need for effective academic assessment and intervention strategies (Bramlett, Murphy, Johnson, Wallingsford, & Hall, 2002; Harris, Gray, Reese-McGee, & Carroll, 1987). Student performance in mathematics in the United States is concerning as the majority of students in 4^{th} and 8^{th} grade are not performing at the proficient level (National Center for Education Statistics (NCES), 2011). More alarming is the fact that roughly one fifth of 4^{th} graders and one fourth of 8^{th} graders are struggling to perform at the basic level (NCES, 2011). In 2006 the National Mathematics Advisory Panel (NMAP) was established to evaluate methods for improving instruction and performance in the area of mathematics in the United States (NMAP, 2008). NMAP released a final report and recommended that mathematics curricula should target critical skills as early as pre-kindergarten. In addition, the panel suggested that students be able to recall basic math facts automatically to place less strain on working memory during more complex problem-solving activities (NMAP, 2008). Frequent assessment of student performance throughout the elementary years provides educators with important information regarding student progress in relation to educational goals. Ensuring that students are making gains during the elementary years is especially important because the development of lower level skills (i.e., addition and subtraction) is required before students can attempt more complex skills (i.e., multiplication and division) in mathematics (L. Fuchs, Fuchs, Hosp, & Jenkins 2001; Skinner, Fletcher, and Henington, 1996). If students fail to master these skills in early grades they will likely have difficulty with more complex tasks in later grades. Therefore, the ability to prevent and remediate early numeracy skill deficits is important.

Response to Intervention (RTI) is a problem-solving model that focuses on the remediation and prevention of academic skill deficits through the implementation of evidence-based interventions and by continually monitoring student performance (Gresham, 2004, 2005; Heartland Area Education Agency, 2007, National Joint Committee on Learning Disabilities (NJCLD), 2005). Within the RTI approach, a problem is defined as a significant discrepancy between current and expected levels of student performance (Gresham, 2005). An evidence-based intervention is then implemented to reduce this discrepancy, which would be indicative of remediation of the targeted skill (Gresham, 2004, 2005). Within RTI models, at-risk students can be identified early, which allows for remediation prior to the development of significant deficits (Fuchs et al., 2007). Broadly speaking, RTI involves three levels of preventative services (L.S. Fuchs & Fuchs, 2006, Fuchs, Fuchs, & Compton, 2010; NJCLD, 2005). The first level, also referred to as Tier 1 or *primary prevention*, is defined by instructional practices that are delivered to all students within the general education classroom (Fuchs et al., 2010; NJCLD, 2005). The second level, also referred to as Tier 2 or *secondary prevention*, consists of primary prevention non-responders that require more specialized instruction through evidence-based intervention (Fuchs et al., 2010; NJCLD, 2005). The third level, also referred to as Tier 3 or *tertiary prevention*, is reserved for roughly 5% of the general population who fail to respond to primary and secondary prevention (Fuchs et al., 2010; NJCLD, 2005). With emphasis placed on the prevention and remediation of lower level skill deficits (Fuchs et al., 2007; NJCLD, 2005), the development of interventions that facilitate basic math fact accuracy and fluency performance is evident.

Instructional Hierarchy

The Instructional Hierarchy (IH), which was described by Haring and Eaton (1978), pairs appropriate instructional procedures with the topography of student responses. The IH consists of four stages of skill development: (1) Acquisition, (2) Fluency Building, (3) Generalization, (4) and Adaption. Students typically acquire a skill by watching a teacher demonstrate and/or model the skill, by receiving assistance through prompting, and by receiving feedback on responses (Martens & Witt, 2004; Wolery, Bailey, & Sugai, 1988). Fluency building, or the rate at which a student can accurately perform a skill within a certain amount of time, occurs following repeated drill and practice and reinforcement (Martens & Witt, 2004; Daly, Lentz, & Boyer, 1996). Generalization, or the ability to apply a learned skill across settings or situations, is targeted by providing diverse learning situations for the student (Ardoin & Daly, 2007; Martens & Witt, 2004). Adaption, or the ability to modify skills to aid performance in novel situations, is the most complex stage of the IH (Haring & Eaton, 1978; Martens & Witt, 2004). Since the IH can be used to identify the appropriate instructional level of the student, appropriate interventions can be selected to facilitate student performance on each stage (Ardoin & Daly, 2007).

Learning Trials and Student Response Rates

The three-term contingency model is typically used when teaching a new skill and consists of an antecedent, a response, and a consequence (Ferkis, Belfiore, & Skinner, 1997). When targeting skill acquisition, immediate feedback following a response prevents the student from repeatedly practicing inaccurate responding and results in improved performance (Belfiore, Skinner, & Ferkis, 1995; Haring & Eaton, 1978; Skinner, 1998). Increasing accuracy performance results in quicker responding, which increases the number of opportunities to actively respond to an academic stimulus (Skinner, 1998). Increasing the number of opportunities to respond is important because doing so has been shown to improve academic performance (Skinner, 1998; Skinner & Shapiro, 1989). However, increasing response opportunities may require additional time so ensuring that students are actively engaged in accurate and fluent responding is mandatory (Skinner, 1998).

Interventions

Several empirically-supported mathematics interventions have been developed that improve accuracy and/or fluency performance (Houten & Thompson, 1976; McCallum, Skinner, Turner, & Saecker, 2006; Poncy, Skinner, & Axtell, 2010; Skinner, Shapiro, Turco, & Cole, 1992). Cover, Copy, Compare (CCC) improves the accuracy and fluency of student responding on basic math facts. First, CCC requires the student to review several problems and correct answers. Second, the student covers the math fact and answer and attempts to write the problem and answer in a blank space next to the original problem. Lastly, the student uncovers and compares the math fact and correct answer to his/her response, which provides the student with immediate feedback regarding performance (Skinner, McLaughlin, & Logan, 1997; Skinner, Turco, Beatty, & Rasavage, 1989). Taped Problems (TP) is another empirically-supported intervention that improves accurate and fluent responding (McCallum, Skinner, & Hutchins, 2004; McCallum et al., 2006). The TP intervention presents a series of problems and correct answers to students auditorily while they follow along on a corresponding math worksheet. The goal is for the student to write the correct response before the answer is provided by the recording for each problem. TP also provides immediate feedback regarding accuracy performance for each math fact, which is an important component of accuracy building interventions (McCallum et al., 2004; McCallum et al., 2006). Explicit Timing (ET) is an intervention that facilitates fluent responding of basic math facts by explicitly informing students that they will have a specific amount of time to complete as many problems as possible (Houten & Thompson, 1976). ET is an effective method for increasing the fluency performance of students on basic math facts (Houten & Thompson, 1976). Poncy, Skinner, and O'Mara (2006) developed Detect, Practice, Repair (DPR), which is a unique intervention that provides individualized instruction to each student within a classroom. DPR identifies dysfluent problems (Detect) by requiring students to maintain a response time of 1.5 seconds per problem. CCC is then utilized on problems that were incorrect or left blank during the detect phase (Practice). Students are then explicitly timed on problems that were targeted during the practice phase and given immediate feedback on their performance through self-graphing (Repair) (Poncy et al., 2006; Poncy et al., 2010). DPR is an empirically-supported intervention that facilitates accurate and fluent responding on basic math facts (Poncy et al., 2006; Poncy et al., 2010).

Previous research has simultaneously compared interventions to determine if differences in effectiveness or efficiency exist between the two (Codding et al., 2007; Poncy et al., 2007). Codding et al.'s (2007) comparison of ET, CCC, and a control group on basic math fact fluency revealed that all three conditions similarly impacted fluency performance. However, the consideration of initial fluency level revealed that ET was either the most or least effective intervention. Members of the ET condition whose initial fluency levels were in the frustrational range performed lowest when compared to members of the CCC and control group. On the other hand, members of the ET condition whose initial fluency levels were in the instructional range performed better than members of the CCC and control group (Codding et al., 2007). Poncy et al.'s (2007) comparison of TP, CCC, and a control group indicated that both interventions similarly improved math fact accuracy and fluency performance when compared to the control group. Interestingly, CCC was less efficient than TP because it required 30% more instructional time to carry out the intervention (Poncy et al., 2007). The results of the previous investigations provide practitioners with valuable information that can enhance the learning environment for students as well as teachers. More efficient interventions require less instructional time that can be utilized for activities such as additional direct instruction and gathering or organizing teacher materials.

Instructional Effectiveness vs. Instructional Efficiency

Learning is typically described in terms of *level* changes, or the number or percentage of correct responses acquired over time (Nist & Joseph, 2008; Skinner, 2008). However, students characterized as having learning problems are capable of learning but typically do so at a slower pace (Skinner et al., 1996). Therefore, these students display a learning *rate* problem (Skinner, 2008). Measures of learning level, or *instructional effectiveness*, do not consider the amount of instructional time required to bring about changes in behavior. Learning rate, or *instructional efficiency*, measures these changes while taking into consideration the amount of time required for learning (Cates et al., 2003; Nist & Joseph, 2008; Skinner, 2008). Interventions that are more efficient, or require less instructional time, are beneficial because they can reduce time required for remediation (Cates et al., 2003; Skinner, 2008).

Cates et al. (2003) investigated the effects of instructional time and learning rates on treatment decision-making in students with spelling deficits. Three spelling interventions were utilized and included interspersal training (IST), high-p sequencing (HPS), and traditional drill and practice (TDP). IST presented a known word following every third unknown word. HPS presented three known words before each unknown word. TDP consisted of presenting six unknown words only. Measures of instructional effectiveness (i.e., cumulative number of words mastered) and instructional efficiency (i.e., number of words mastered per minute of instruction) were analyzed. All three interventions facilitated spelling ability similarly in terms of instructional effectiveness. However, TDP resulted in more learned words per minute of instructional time when compared to the IST and HPS conditions. Learning rates in the IST and HPS conditions were likely decreased due to the inclusion of known words, which resulted in fewer opportunities to respond to unknown items (Cates et al., 2003). Joseph and Nist (2006) replicated the results of the Cates et al. (2003) investigation but measured cumulative word reading acquisition rather than spelling acquisition. The results indicated that TDP resulted in higher rates of learning.

Nist and Joseph (2008) extended the learning rate literature by measuring next-day retention, maintenance, and generalization associated with IST, TDP, and incremental rehearsal (IR). IR consisted of presenting unknown words incrementally nine times among known words. The six unknown words were presented nine times in each session, which held opportunities to respond constant. Measures of instructional effectiveness and instructional efficiency were analyzed. The results suggested that the IR method was more effective, or students recalled more words in this condition. TDP was more efficient, or students recalled more words per minute of instructional time in this condition. In addition, students maintained and generalized more words under the IR condition when compared to TDP and IST. If time is limited and quicker remediation is sought, TDP should be the intervention of choice. If maintenance of skills over time is important, IR may be a better choice (Nist & Joseph, 2008). Skinner (2008) stressed the importance of these results by presenting a hypothetical remediation goal of correctly reading 330 unknown words within sentences. Under the TDP condition, all students would be remediated within five weeks whereas remediation under the IR condition would require at least 11 weeks. Instructionally efficient interventions require much less instructional time to reach remediation goals, which allows students to benefit from general education instruction sooner (Skinner, 2008).

Mediating Variable to Instructional Efficiency: Teacher to Student Ratios

Classwide interventions can be used to efficiently remediate and prevent academic problems across the entire classroom (Hawkins, 2010; Shapiro, 2000). However, the majority of math skill research has focused on intervention effectiveness with individual to small groups of students (Codding et al., 2007; Poncy, Skinner, & Axtell, 2010; Poncy et al., 2007; Skinner et al., 1989). While research suggests the classwide application of mathematics interventions is effective (Axtell et al., 2009; Codding, Chan-Iannetta, Palmer, & Lukito, 2009; Rhymer et al., 2002), limited research exists regarding the most efficient classwide intervention targeting math fact fluency. Future research should investigate learning rate to provide practitioners with information to improve efficiency of practice (Skinner, 2008). Continual evaluation of instructional effectiveness does not provide us with new information such as the amount of instructional time required to learn information (Skinner, 2008). Instructionally efficient classwide interventions are needed, especially since educators are extremely busy and have limited time for individualized intervention. In addition, it is not uncommon for classwide problems to emerge following individual student referrals (Rathvon, 1999). For this reason, efficient class-wide interventions can be beneficial for several reasons. For example, remediation goals can be reached more quickly, which allows the student to benefit from general education instruction more quickly (Skinner, 2008). In addition, classwide academic interventions can develop an environment that increases active academic responding, which may decrease behavior problems (Rathvon, 1999).

Rationale

Empirically-based mathematics interventions have been developed and implemented to remediate early math skill deficits. Emphasis has mainly been placed on improving student learning levels, or the number of accurate responses acquired over time (Nist & Joseph, 2008; Skinner, 2008). Equally important is determining which instructional modifications produce the most efficient method for achieving specific remediation goals. Learning rate considers the instructional time necessary to produce level changes in behavior and can provide useful information regarding quick remediation (Nist & Joseph, 2008; Skinner, 2008). Instructional efficiency research has been conducted but has focused on reading rather than mathematics skill development (Cates et al., 2003; Joseph & Nist, 2006; Nist & Joseph, 2008).

The current investigation examined the efficiency of an explicit timing intervention based on curricular modifications. In particular, instructional efficiency was compared between three conditions that varied on the curricular dimension of problem set size. These conditions are similar to the interventions (i.e., TDP, HPS, and IST) implemented in previous learning rate research (Nist & Joseph, 2008). However, the target skill that was investigated in the current study was fluency rather than accuracy. Problem sets did not consist of known and unknown problems. Instead, sets differed based upon response automaticity (Duhon, Poncy, & Fontenelle, 2012). The primary goal of the current investigation was to determine if learning rate varies between groups exposed to probe sets containing a mixture of automatic and non-automatic problems

(Total Condition), non-automatic problems (Reduced Condition), or a specific ratio of non-automatic to automatic problems (Ratio Condition).

CHAPTER II

REVIEW OF THE LITERATURE

In the United States all children are allowed equal access to educational opportunities (No Child Left Behind Act (NCLB), 2001). This is evident as each state is required to establish meaningful standards to determine whether all students are making gains and meeting expected educational goals. Schools, districts, and states are responsible for improving the academic performance of students, including high and low achievers (NCLB, 2001). In order for schools to receive federal funding, academic skills must be assessed and monitored regularly throughout the school year. The established standards are grade appropriate and become increasingly difficult as students progress through the educational system. For example, kindergarten students may be assessed on letter and number identification fluency whereas third graders may be assessed on reading fluency.

Assessment of student progress throughout the elementary years allows educators and administrators to determine if adequate progress is being made and whether students are performing at expected levels. Assessment of student progress during the elementary years is important because acquisition of lower level skills is critical for students to learn more complex, higher order skills in both reading and math (L. Fuchs, Fuchs, Hosp, &

Jenkins 2001; Skinner, Fletcher, and Henington, 1996). For example, oral reading fluency is a higher order skill that measures a student's ability to rapidly perform lower level skills such as phoneme segmentation, phoneme recoding, and word identification (L. Fuchs et al., 2001). In math, multiplication and long division are higher order skills that involve lower level skills such as addition and subtraction (L. Fuchs & Fuchs, 2005; Skinner et al., 1996). The prevention of early numeracy and literacy skill deficits is crucial during elementary years because without mastery of basic skills students will likely struggle to perform increasingly difficult skills in later grades.

Response to Intervention (RTI) has been defined as a change in behavior or academic performance resulting from evidence-based intervention (Gresham, 2004, 2005). First introduced by Heller, Holtzman, and Messick (1982) (Fuchs, 2003), the concept of RTI was recently signed into law as an alternative method for identifying children with learning disabilities (D. Fuchs & Fuchs, 2006). Prior to the Individuals with Disabilities Education Improvement Act (IDEIA, 2004), students were identified based upon a significant discrepancy between IQ and achievement scores (D. Fuchs & Fuchs, 2006). The IQ-Achievement discrepancy model has also been described as a "wait to fail" approach because identification requires a significant discrepancy that is usually not visible until several years of schooling have passed (National Joint Committee on Learning Disabilities (NJCLD), 2005). Conversely, RTI is considered a problem-solving approach with a focused goal on prevention (Gresham, 2004, 2005; Heartland Area Education Agency, 2007; NJCLD, 2005). Within RTI models, a problem exists when there is a discrepancy between current academic or behavioral performance and what is expected (Gresham, 2005). Evidence-based interventions are then implemented to reduce this discrepancy and facilitate student performance on the target skill (Gresham 2004, 2005). If the student does not respond to intervention, modification or intensification of the intervention is suggested until the instructional conditions under which the child responds are achieved (Gresham, 2004, 2005).

The purpose of RTI is to identify and intervene on at-risk students at an earlier age and to do so through the use of more valid procedures (Gersten & Dimino, 2006). The early identification of at-risk students is important because remediation can begin prior to the emergence of significant deficits (Fuchs et al., 2007). A benefit of RTI is that one can differentiate between poor instruction and an actual disability as reasons for inadequate academic performance (Fuchs, 2003). In other words, poor academic achievement could potentially be the result of poor instruction or a disability. If the majority of students respond to empirically-based instruction, evidence is lent to the fact that a non-responder's disability may be the reason for poor performance (Fuchs, 2003). Other potential benefits of RTI include fewer special education referrals, reduced identification of culturally diverse minority students, reduced labeling within noncategorical approaches, and the inclusion of information to guide individualized education programs (IEP) (Fuchs, Mock, Morgan, & Young, 2003; NJCLD, 2005).

While many variations of the RTI framework exist, these derivations are based on a process that involves three levels, or *tiers*, of preventative services (L.S. Fuchs & Fuchs, 2006; Fuchs, Fuchs, & Compton, 2010; NJCLD, 2005). Tier 1, or *primary prevention*, consists of instruction within the general education classroom that is delivered to all students. Such activities include but are not limited to core instruction, universal screening, curriculum-based assessment, progress monitoring, and differentiated instruction (Fuchs et al., 2010; NJCLD, 2005). Tier 2, or *secondary prevention*, involves more specialized instruction with students that did not respond or progress similarly as peers to tier 1 instruction. Students receiving tier 2 prevention services are provided with research-based instructional programs specific to their needs (Fuchs et al., 2010; NJCLD, 2005). Tier 3, or *tertiary prevention*, is the most intense form of prevention reserved for roughly 5% of the general population who do not respond to tier 1 and tier 2 services (Fuchs et al., 2010; NJCLD, 2005). More intensive instruction is provided to the student over longer periods of time and is progress monitored to determine efficacy (Fuchs et al., 2010). Within some RTI frameworks, a comprehensive evaluation is performed to determine if tier 3 services are needed through special education (L.S. Fuchs & Fuchs, 2006; NJCLD, 2005). In sum, RTI is a multi-tiered system designed "to ensure that quality instruction, good teaching practices, differentiated instruction, and remedial opportunities are available in general education, and that special education is provided for students with disabilities who require more specialized services than what can be provided in general education" (NJCLD, 2005).

The need for mathematics research focusing on principles related to instruction and learning is highly encouraged (National Mathematics Advisory Panel (NMAP), 2008). With RTI models continually gaining support, the need will only become greater as evidence-based interventions are used to facilitate academic performance within the RTI framework (Gresham, 2004). Since early remediation is key to preventing major deficits in later years (Fuchs et al., 2007; NJCLD, 2005), educators and practitioners should have access to interventions that are designed to improve lower level skills such as basic math fact accuracy and fluency. Developing these lower level skills is necessary before attempting higher order operations in higher grades (Fuchs & Fuchs, 2005; Skinner et al., 1996). One of the potential benefits of RTI models is a reduction in special education referrals because at risk students are provided with more individualized instruction (NJCLD, 2005). If evidence-based mathematics interventions are implemented with integrity, at risk students may respond and no longer be considered as a referral concern. Within RTI models, effective classwide interventions could be implemented to benefit all students rather than just those referred for poor academic performance (Gresham, 2004). Therefore, the development of efficient mathematics interventions may facilitate student performance and the instructional environment within all tiers of RTI models.

School psychologists are aware of the need for academic assessment and intervention as the majority of school-based referrals are related to academic concerns (Bramlett, Murphy, Johnson, Wallingsford, & Hall, 2002; Harris, Gray, Reese-McGee, & Carroll, 1987). Although most academic referrals are related to reading problems (Bramlett et. al, 2002), student performance in mathematics continues to be a concern in the United States. While mathematics performance has improved slightly over the past two decades, 60% of fourth graders and 65% of eighth graders are performing below the proficient level (National Center for Education Statistics (NCES), 2011). In addition, 18% of fourth graders and 27% of eighth graders are not performing at the basic level, which is defined as "partial mastery of prerequisite knowledge and skills that are fundamental for proficient work at each grade" (NCES, 2011, p. 7). These results are alarming and suggest that students will continue to have difficulty with mathematics, especially as concepts become increasingly difficult in higher grades (NCES, 2011).

The National Mathematics Advisory Panel (NMAP) was established in 2006 and given the responsibility of evaluating scientifically based methods for improving mathematics instruction and performance in the United States (NMAP, 2008). NMAP reviewed more than 16,000 research investigations to determine skills necessary for success in algebra. A final report was presented and consisted of the evaluation results as well as recommendations for future practice. NMAP recommended that mathematics curricula focus on building critical foundations gradually and fluidly beginning as early as pre-kindergarten. Furthermore, the report stated that curricula "must simultaneously develop conceptual understanding, computational fluency, and problem solving skills" (NMAP, 2008, p. xix). In addition, the ability to automatically recall basic math facts (i.e., addition, subtraction, multiplication, and division) should be developed because automaticity "frees up working memory for more complex aspects of problem solving" (NMAP, 2008, p. 30). NMAP made these recommendations because without early mathematic skills students will likely struggle with more complex operations like those needed for algebra.

Instructional Hierarchy

Haring and Eaton (1978) described a hierarchy that pairs the topography of student responding to prescriptive instructional procedures. The Instructional Hierarchy

(IH) consists of four stages: (1) Acquisition, (2) Fluency Building, (3) Generalization, (4) and Adaption. In order to perform a desired behavior one must first *acquire* the necessary skill or skills. The goal of this stage is to have the student perform the desired skill accurately and consistently without teacher assistance, and is typically measured by number or percent correct (Martens & Witt, 2004). A new skill is typically acquired by watching someone demonstrate and/or model the skill, by receiving appropriate assistance to be successful (prompting), and by receiving feedback on the response (Martens & Witt, 2004; Wolery, Bailey, & Sugai, 1988). Once the student can accurately perform the skill, one must improve *fluency.* Fluency is a measure of the rate at which a student can accurately perform a skill in a specific amount of time (Martens & Witt, 2004). Skill fluency is increased through drill and practice and providing reinforcement for improvement (Daly, Lentz, & Boyer, 1996). The third stage, *generalization*, refers to the application of a learned skill across activities or situations. To facilitate generalization, students should not only display accurate and fluent responding but also be provided with ample opportunities to practice the skill in diverse situations (Ardoin & Daly, 2007; Martens & Witt, 2004). The final and most complex stage, *adaption* or *skill mastery*, is defined as the ability to modify previously learned skills to facilitate performance in a novel activity. Some suggest that adaption is related to problem solving while others compare this stage to creativity (Haring & Eaton, 1978; Martens & Witt, 2004).

The behavioral components of the IH can also be seen in Response to Intervention (RTI) models: 1) student skills are assessed to determine what they can and cannot do, 2) interventions are developed to facilitate performance on needed skills, and 3) the skill is measured on a regular basis and progress monitored to determine intervention effectiveness (Ardoin & Daly, 2007). Since the IH can be used to determine the skills that should be targeted for each individual child, specific interventions can and should be utilized to increase the specified target behavior (Ardoin & Daly, 2007). For example, if a student performs poorly on a math fluency measure, basic skill assessment may reveal that the student can accurately add numbers but cannot do so quickly. Therefore, an intervention targeting fluency could be developed to improve the student's proficiency of performing basic math facts. While modeling and error correction are best for improving accuracy, fluency is built by drill (i.e., repeated exposure to the stimulus) and would therefore be utilized (Ardoin & Daly, 2007). Based on Haring and Eaton's (1978) model, practitioners can match student response topographies to evidence-based instructional procedures to remediate the target behavior (Daly et al., 1996; Daly, Witt, Martens, & Dool, 1997). Therefore, selecting appropriate interventions that contain essential components targeting specific skills is a critical step in the process of remediating struggling students.

Learning Trials and Student Response Rates

In behavioral terms, when teaching a new skill the procedures included within a three-term contingency model are typically used (Ferkis, Belfiore, & Skinner, 1997). The three-term contingency model consists of an antecedent, a response, and a consequence. For example, a student is presented with a math fact (i.e., antecedent), has the opportunity to respond to the math fact (i.e. response), and is provided with feedback contingent upon the response (i.e. consequence). Once all of these steps have been carried out the student has successfully completed a *learning trial* (Skinner, 1998). When mastering a new task, one must first learn, or acquire, the skills necessary to accurately complete the learning trial (Haring & Eaton, 1978; Wolery, Bailey, Sugai. 1988). Therefore, providing immediate feedback following a student response is an essential component of a learning trial for increasing accurate responding (Skinner, 1998). Belfiore, Skinner, and Ferkis (1995) investigated the effects of trial repetition versus response repetition on sight word accuracy of three elementary students with learning disabilities. Trial repetition consisted of five complete learning trials per day, allowing for one response per trial. Response repetition consisted of five responses within a single learning trial per day. While both conditions increased student sight word reading performance, greater gains were observed in the trial repetition condition. Consequently, simply increasing opportunities to respond may not be as important as increasing the number of completed learning trials (Belfiore et al., 1995). Feedback following each response provides the student with information to perform the skill accurately and does not allow the student to repeatedly practice inaccurate responding (Skinner, 1998).

Quickly recalling accurate information is the next step when mastering a skill (Haring & Eaton, 1978; Poncy, Skinner, & Jaspers, 2007). Increasing accuracy results in quicker responding and ultimately more opportunities to respond, which occurs when a student actively responds following the presentation of an academic stimulus (Skinner, 1998). Increasing opportunities to respond can improve academic performance across the instructional hierarchy (Poncy et al., 2007; Skinner, Bamberg, Smith, & Powell, 1993;

Skinner, Pappas, & Davis, 2005; Skinner, 1998). For example, Skinner and Shapiro (1989) investigated the effects of opportunities to respond on the ability to accurately and fluently read word lists. Comparisons were made between a taped problem condition, which consisted of reading word lists along with an audio recording, and a traditional drill procedure. In these conditions, students were exposed to the randomly assigned word lists twice per session. Also included in the study were continuous assessment and intermittent assessment conditions. In the continuous assessment and intermittent assessment conditions students were assessed on a random list of words assigned to each condition once. Students participated in the Taped Problems, drill procedure, and continuous assessment conditions daily. Students participated in the intermittent condition once every three days (i.e., third, sixth, ninth and twelfth days). The results indicated that increases in the number of opportunities to respond resulted in higher correct oral reading rates in both the taped-problem and drill conditions (Skinner, 1998; Skinner & Shapiro, 1989). Increasing the number of response opportunities not only facilitates fluency but also facilitates maintenance, generalization, and discrimination (Billington, Skinner, & Cruchon, 2004; Martens & Witt, 2004;). Also, accurate and fluent responding requires less effort and is associated with higher rates of reinforcement (Poncy et al., 2007). According to early animal behavior research, organisms engage in behaviors that require less effort and involve higher rates of reinforcement (Herrnstein, 1961). Therefore, the inability to accurately and fluently identify basic math facts may negatively impact a student's willingness to attempt more complex tasks (Skinner, 1998; Skinner et al., 2005).

While accurate and fluent responding is important and an effective means for improving academic performance, increasing opportunities to respond may require additional time (Skinner, 1998). Although increasing time on academic tasks may be considered important and will lead to increases in opportunities to respond, ensuring that the student is engaged in accurate and fluent responding is necessary (Skinner, 1998). Berliner (1990) referred to this as Academic Learning Time (ALT), or "that part of allocated time in a subject-matter area in which a student is engaged successfully in the activities or with the materials to which he or she is exposed, and in which those activities and materials are related to educational outcomes that are valued" (Berliner, 1990, p. 5; Skinner, 1998). When increasing opportunities to respond, considering the amount of time necessary to complete the learning trials results in a measure of learning rate rather than learning level (Skinner, 1998). Increasing rates of student responding within a specified amount of time maximizes student efficiency by increasing opportunities to respond within that time period (Skinner, 1998). Therefore, ensuring that students are actively engaged in academic tasks is very important.

Interventions

The majority of mathematics research has focused on a handful of interventions that have been shown to improve accuracy and/or fluency (Houten & Thompson, 1976; McCallum, Skinner, Turner, & Saecker, 2006; Poncy, Skinner, & Axtell, 2010; Skinner, Shapiro, Turco, & Cole, 1992). Cover, Copy, Compare (CCC) is an intervention that improves accuracy and fluency of student responding on basic math facts (Skinner, Turco, Beatty, & Rasavage, 1989). CCC consists of the following steps: (1) a sheet containing math facts and correct answers is provided to the student, (2) the student is instructed to review these problems and answers, which are located on the left side of the page, (3) the student covers each math fact and answer with a hand, note card, or piece of paper, (4) the student writes the problem and answer in a blank space next to the fact on the right side of the page, and (5) the student removes the note card to compare his/her response to the original problem (Skinner et al., 1989). The student receives immediate feedback regarding performance when the note card is removed (Skinner, McLaughlin, & Logan, 1997). Several variations of CCC have been used to improve accuracy and fluency of math skills including addition (Poncy et al., 2007), subtraction (Codding et al., 2007), multiplication (Codding, Eckert, Fanning, Shiyko, & Solomon, 2007), and division (Lee & Tingstrom, 1994).

A second intervention that has been used to improve accuracy and fluency of math facts is Taped Problems (TP) (McCallum, Skinner, & Hutchins, 2004; McCallum et al., 2006). TP consists of the following steps: (1) a sheet of math facts without the answers is provided to the student, (2) a pre-recorded audio tape presents each problem and answer, and (3) the student is instructed to write the correct response before the tape provides the answer. If the student provides the incorrect response or fails to respond, the student records the correct response following each problem as the tape provides the correct answer. This procedure also provides the student with immediate feedback on each response. The TP intervention has also been implemented to improve accuracy and fluency on various math facts including addition (Miller, Skinner, Gibby, Galyon, Meadows-Allen, 2011), subtraction (Fontenelle, Schutte, & Poncy, 2012), multiplication

(Bliss et al., 2010), and division (McCallum et al., 2004).

A third intervention that improves fluency of basic math facts is Explicit Timing (ET) (Houten & Thompson, 1976). ET consists of two steps: (1) students are provided with a worksheet containing basic math facts and (2) students are explicitly told to complete as many problems as possible in a specified amount of time. Houten and Thompson (1976) were the first to investigate the effectiveness of ET on basic math facts with 20 academically challenged second-graders. During baseline phases, students were instructed to work on math worksheets and were timed covertly by their teacher for 30 min. During the intervention phases, students were explicitly told that they were being timed in 1 min intervals over a 30 min period. A reversal design was utilized in order to determine how timing affected the rate and accuracy of student performance. The class averaged 3.5 and 5.5 correct problems per minute during baseline phases. During intervention phases student performance increased to 10.5 and 11.5 correct problems per minute. Accuracy was above 90% throughout all conditions. The results suggest that ET is an effective method for increasing the fluency of basic math facts (Houten & Thompson, 1976). Throughout the past 35 years researchers have demonstrated ET's effectiveness at improving accuracy and fluency of addition, subtraction, and multiplication math facts (Miller, Hall, & Heward, 1995; Rhymer et al., 2002). Additionally, ET is an effective intervention for both regular and special education students (Miller et al., 1995), African American and Caucasian students (Rhymer, Henington, Skinner, and Looby, 1999), and increasingly difficult problem sets (Rhymer et al., 2002).

Poncy, Skinner, and O'Mara (2006) developed an intervention called Detect, Practice, Repair (DPR) to facilitate accuracy and fluency on basic math facts. DPR consists of the following steps: 1) students work through math facts at a steady pace of 1.5 second intervals to determine which facts are not mastered (Detect), 2) Cover, Copy, Compare (CCC) is then utilized on problems that were not completed during the detect phase (Practice), and 3) students are timed (explicit timing) on the problems addressed in the practice phase and provided with immediate feedback on performance though self-graphing (Repair) (Poncy et al., 2006; Poncy et al., 2010). DPR is unique in the sense that dysfluent problems are identified and intervened on within intervention implementation, which provides individualized intervention to each student within a classroom (Poncy et al., 2010). The DPR intervention has been used to increase fluency of subtraction facts (Poncy et al., 2006), division facts (Axtell, McCallum, Bell, & Poncy, 2009), and multiplication facts (Poncy et al., 2010).

Since student performance on basic math facts has been facilitated by all of the previously mentioned interventions, researchers have questioned whether or not interventions differ in effectiveness and efficiency (Codding et al., 2007; Poncy et al., 2007). Codding et al. (2007) compared the effectiveness of ET and CCC on basic math fact fluency of ninety-eight second and third grade students. Students were randomly assigned to either an ET, CCC, or control condition. The results indicated that there was no significant difference between ET, CCC, and the control condition when initial fluency performance was excluded. However, when initial fluency levels were considered, ET was either the most or least effective. For students who were initially at the instructional level for fluency, ET resulted in better performance when compared to the CCC and control conditions. Students whose fluency fell within the frustrational range and were members of the ET condition performed the lowest when compared to members of the CCC and control conditions. These results suggest that intervention effectiveness varies depending upon student fluency skills (Codding et al., 2007).

Poncy et al. (2007) compared the effectiveness of TP and CCC on a 10-year old female student with an intellectual disability. An adapted alternating treatment design across probes was implemented, which allowed the researchers to determine the effects of each intervention independently from one another. The interventions were counterbalanced, each being presented daily in the morning or afternoon. Both interventions similarly improved performance on math fact fluency and accuracy when compared to a control condition. However, CCC required 30% more time to complete when compared to TP. The results of this investigation indicate that TP is a more *efficient* method for improving math fact accuracy and fluency (Poncy et al., 2007). More efficient interventions are beneficial for many reasons. For example, teachers can use this additional time to provide additional direct instruction, engage one-on-one with students, or gather and organize materials.

Instructional Effectiveness vs. Instructional Efficiency

One of the many duties required of school psychologists is to remediate students that display learning problems. In fact, the majority of referrals that school psychologists receive are academic in nature (Bramlett et al., 2002). Learning can be defined as “a relatively permanent change in behavior or behavioral potential brought about by experience…" (Skinner, 2008, p. 309). Rather than defining learning problems as the inability to learn, one should consider that referred students can learn but do so at a slower pace (Skinner et al., 1996). Therefore, students referred for learning problems typically display a learning *rate* problem (Skinner, 2008). However, learning is frequently assessed by looking at changes in behavior, also referred to as learning *level*. Changes in learning level are typically assessed by measuring the number or percentage of correct responses, which is also referred to as *instructional effectiveness* (Nist & Joseph, 2008; Skinner, 2008). Changes in learning level provide information on acquisition over time, but learning level does not take into account instructional time required to bring about these changes. In other words, learning level measures growth in the number of correct responses regardless of the amount of instructional time required to provide instruction. Learning rate assessment, also known as *instructional efficiency*, measures changes in behavior while considering the time required for learning (Cates et al., 2003; Nist & Joseph, 2008; Skinner, 2008). While many academic interventions are instructionally effective, differences may exist between interventions in terms of instructional efficiency (Cates et al., 2003). More efficient interventions require less instructional time and may be preferred by teachers when compared to more time consuming interventions (Cates et al., 2003; Witt, Martens, & Elliott, 1984).

Cates et al. (2003) were interested in the effects of instructional time and learning rates on treatment decision-making. Five second-grade students with spelling deficits were exposed to three spelling interventions including interspersal training (IST), high-p sequencing (HPS), and traditional drill and practice (TDP). The IST intervention consisted of presenting a known word (three total) following every third unknown word (i.e., K1, U1, U2, U3, K2, U4, U5, U6, K3). The HPS intervention exposed the students to six unknown and 18 known words, which presented three known words before each unknown (i.e., K1, K2, K3, U1, K4, K5, K6, U2, etc…). The TDP intervention consisted of presenting six unknown words only. For all conditions, praise was delivered following correct responses while incorrect responses were addressed with an overcorrection procedure to ensure accurate spelling responses. An alternating treatment design was utilized to compare intervention effectiveness on student learning. Students were exposed to two of three randomly selected interventions each session across 12 school days. During each session, students were exposed to two trials in each condition. Dependent measures included the cumulative number of words mastered (instructional effectiveness) and the number of words mastered per minute of instruction (instructional efficiency). Students learned to spell similar amounts of words in each condition, indicating that instructional effectiveness was similar across all conditions. However, when considering instructional efficiency, the TDP condition resulted in more learned words than the IST and HPS conditions for the group of students. At the student level, all but one subject displayed higher learning rates in the TDP condition, which was followed by the IST and HPS conditions respectively. These results suggest that measures of intervention efficiency should be utilized when comparing interventions rather than simply measuring intervention effectiveness (Cates et al., 2003). Including known words in the IST and HPS conditions likely decreased learning rates because more time was spent on unknowns in the TDP condition. In other words, students had fewer opportunities to respond to unknown words in the IST and HPS conditions (Cates et al., 2003).

Joseph and Nist (2006) were interested in replicating and extending the results of the Cates et al. (2003) investigation by measuring cumulative word reading acquisition rather than spelling acquisition and by providing modeling and corrective feedback rather than just corrective feedback. Three intermediate grade (i.e. grades 5 and 6) students with reading deficits participated in the study. Conditions consisted of the HPS, IST, and TDP interventions identical to the Cates et al. (2003) investigation. For all conditions, unknown words were read aloud by the experimenter during a modeling phase. Praise was provided on correct responses and corrective feedback was provided on incorrect responses until the student successfully repeated the word. An alternating treatment design was utilized to compare the effects of each intervention with each student. The study was conducted for eight consecutive days, with each student receiving every condition daily. Students participated in three trials under each condition. Dependent variables included measures of instructional effectiveness and instructional efficiency. Overall, the results indicated that TDP resulted in higher rates of learning, which replicated the findings of Cates et al. (2003) with older students.

Nist and Joseph (2008) sought to extend the learning rate literature by investigating next-day retention, maintenance, and generalization associated with three word reading flashcard instructional methods. Six first-grade students with reading difficulties were exposed to TDP, IST, and incremental rehearsal (IR). The TDP and IST conditions were identical to the Joseph and Nist (2006) study. The IR condition consisted of six unknown words and nine known words. The unknown words were presented incrementally nine times among the known words (i.e., U1, K1, U1, K1, K2, U1, K1, K2, K3, U1, K1, K2, K3, K4...U1, K1, K2, K3, K4, K5, K6, K7, K8, K9). After the first unknown was presented incrementally, the ninth known word was replaced with the first unknown word. During each daily session, this method was utilized until 6 unknowns were presented. Each student participated in three experimental sessions each week. Each instructional method was presented in an alternated and counterbalanced order for each session. The instructional methods were administered in sessions 1 and 2 each week. Sessions 2 and 3 were also initiated with the administration of a retention probe, and was the only activity for session 3. Verbal praise was provided following correct responses while corrective feedback was utilized on incorrect responses. Moreover, opportunities to respond were held constant by presenting the six unknowns nine times in each session. Dependent variables consisted of measures of instructional effectiveness and efficiency. Maintenance and generalization measures were also administered to determine if students recalled information over time and were able to use learned information in another context. The results indicated that the IR method was more effective while the TDP method was more efficient. Furthermore, more words were maintained and generalized under the IR condition. Based on these findings, interventions can be utilized depending upon the specific needs of the student (Nist & Joseph, 2008). When time is limited and efficiency is key, TDP should be utilized. For students who do not respond to TDP or want to maintain skills over time IR may be more beneficial (Nist & Joseph, 2008).

Skinner's (2008) elaboration on the results of the Nist and Joseph (2008) investigation provided a clearer understanding of the importance of instructional efficiency. For example, Skinner (2008) presents a hypothetical remediation goal of correctly reading 330 unknown words within sentences. If the TDP method was utilized, all students would be remediated, or be able to read all 330 words, within 5 weeks. If the IR condition was utilized, remediation would require at least 11 weeks. With TDP requiring 6 fewer weeks for remediation, students could potentially benefit from general education instruction sooner and reduce required resource allocation more quickly when compared to the IR condition (Skinner, 2008). Therefore, learning rate is an important measure that can dramatically reduce time necessary for remediation. The need for additional research and interventions targeting instructional rate is apparent, especially in mathematics.

Mediating Variable to Instructional Efficiency: Teacher to Student Ratios

An important characteristic of math intervention administration is the way in which it is administered. Most math skill research has focused on the effectiveness of CCC, TP, ET, and DPR with individual to small groups of students with learning or behavior problems (Codding et al., 2007; Poncy, Skinner, & Axtell, 2010; Poncy et al. 2007; Skinner et al., 1989). While the results of these studies are important and provide information regarding instructional effectiveness, there is still a need for evidence-based interventions that can be applied at the classwide level (Gresham, 2004). Classwide interventions are efficient and can be used to remediate academic deficits as well as prevent problems across the entire classroom (Hawkins, 2010; Shapiro, 2000). Also, effective classwide interventions can meet the needs of students while making use of available resources within the classroom (Hawkins, 2010). While studies indicate the classwide applications of ET, CCC, TP, and DPR are effective, (Axtell et al., 2009; Codding, Chan-Iannetta, Palmer, & Lukito, 2009; Rhymer et al., 2002) limited research exists comparing the most efficient fact building interventions for multiple students simultaneously.

In Codding et al.'s (2007) classwide comparison of ET and CCC, findings suggested that both interventions similarly impacted performance until initial fluency levels were considered. Initial fluency level determined whether or not a specific intervention (i.e., Explicit Timing) was most or least effective. While this comparison provides important information regarding effective classwide intervention application, it does not provide information regarding learning rate. Poncy et al.'s (2007) comparison of TP and CCC did consider instructional efficiency. The results indicated that both interventions were similarly effective, but TP was more efficient than CCC, requiring nearly 30% less time to implement. However, this particular study involved one student rather than an entire class. The argument has been made that future research should focus on learning rate not only because school psychologists seek answers to basic applied learning questions, but also because doing so will provide practitioners with more relevant information for practice (Skinner, 2008).

Skinner's (2008) argument makes perfect sense, especially when considering teacher to student ratios. Increasing learning rates, or improving instructional efficiency, should be the focus of practice. While intervention research has provided researchers with answers regarding effective math interventions (Houten & Thompson, 1976; McCallum et al., 2004; Skinner et al., 1989; Poncy et al. 2006), the continuation of research that focuses on learning level, or instructional effectiveness, does not provide us with information regarding instructional time (Skinner, 2008). Since teachers are extremely busy and have limited time to run one-on-one interventions, the need for instructionally efficient interventions is apparent. In addition, it is not uncommon that student referrals often result in the discovery of a classwide problem (Rathvon, 1999). Therefore, instructionally efficient classwide interventions are important for many reasons. First, less time is required for remediation, which improves the quality of education for the student more quickly (Skinner, 2008). Second, effective classwide interventions are already more efficient than single-student interventions as less time and labor is required (Rathvon, 1999). Third, classwide interventions eliminate teacher concern that the majority of students lose valuable instruction time while only a few individual students receive one-on-one intervention (Elliott, Turco, & Gresham, 1987). Lastly, the implementation of classwide academic interventions may decrease behavior problems by establishing an environment that promotes academic engagement (Rathvon, 1999). By determining the most instructionally efficient method for classwide mathematics intervention implementation, multiple students may be provided with evidence-based treatments simultaneously to improve performance on early mathematics skills.

Rationale

With the majority of school psychologist referrals related to academics, the need for effective interventions to help remediate struggling students is evident (Bramlett et al., 2002). The shift from IQ-Achievement discrepancy models to RTI models for the identification of children with disabilities only intensifies the need for effective interventions (D. Fuchs & Fuchs, 2006). In addition, interventions that are utilized within RTI models are empirically supported methods for improving student outcomes (Gresham, 2004). As evidenced by the NCES, the majority of students in the United States are struggling with mathematics (NCES, 2011), which solidifies the need for mathematics intervention research.

The development and implementation of empirically-based mathematics interventions has provided researchers and practitioners with answers as to the best method for building early numeracy skills to combat mathematics deficits. Improving a student's learning level, or the number of accurate responses acquired over time, has traditionally been regarded as the most significant outcome measure related to intervention development (Nist & Joseph, 2008; Skinner, 2008). While improving learning level is an important goal of intervention research and practice, determining the most efficient method for reaching desired outcomes is equally important. Measurement of learning rates, which considers the amount of instructional time required for level changes in behavior, can reveal useful information as to the most efficient method for remediating students (Nist, & Joseph, 2008; Skinner, 2008). When looking at long-term remediation goals, drastic differences can be seen between interventions that vary on instructional efficiency (Skinner, 2008). The importance of instructional efficiency has been discussed in multiple studies (Cates et al., 2003; Joseph & Nist, 2006; Nist & Joseph, 2008;). However, these investigations assessed spelling and reading skills, which strengthens the need for learning rate research related to mathematics skill development.

The purpose of the current investigation was to examine the efficiency of curricular modifications for improving basic math fact fluency. In particular, instructional efficiency was assessed between three conditions receiving the Explicit Timing intervention. The conditions varied based upon the curricular dimension of problem set size, which is similar to the conditions (i.e., TDP, IR, IST) compared in previous learning rate research (Nist & Joseph, 2008). However, the current study differed in the sense that fluency rather than accuracy was targeted. Therefore, problem sets did not consist of known and unknown problems. Rather, problems that comprised the problem sets varied based upon response automaticity (Duhon, Poncy, & Fontenelle, 2012). Duhon et al. (2012) investigated the response times of students on 100 multiplication problems (i.e., all 1x1 facts from 0-9, including reciprocals). Response time was measured on a computer-based program that provided the amount of time (in seconds) required for students to type a response following presentation of each multiplication fact. The results suggested that the majority of students answered specific problems more quickly than others. In other words, students answered 70 of the 100 problems within 5.5 seconds. In addition, 30 of the 100 problems required between 5.5 and 10.5 seconds, which suggests that automaticity performance is better on specific multiplication problems. Therefore, in the current study problems were referred to as "automatic" and "non-automatic" rather than "known" and "unknown" based upon the automaticity levels investigated by Duhon et al. (2012).

The first goal of the current investigation was to determine if learning rates differ between groups exposed to probe sets containing either a mixture of automatic and non-automatic problems (Total Condition), non-automatic problems (Reduced Condition), or a specific ratio of automatic to non-automatic problems (Ratio Condition). The Total set consisted of 100 multiplication problems (i.e., facts including numbers 0-9). The Reduced set consisted of 36 multiplication problems (i.e., facts including numbers 2-9). The Ratio set consisted of a ratio of 36 multiplication problems (i.e., 18 non-automatic and 18 automatic) with differing response latencies. A second goal was to determine if students assigned to the Reduced and Ratio conditions would generalize performance on 36-problem Reciprocal probe sets. In other words, the goal was to determine fluency performance on probe sets that contained problems presented in reverse order (i.e., 36-problem Reciprocal Assessment). A third goal was to determine which instructional condition facilitates the maintenance of multiplication fact fluency performance the greatest over time. Three assessments were conducted to determine fluency performance prior to, post, and two weeks following intervention implementation (i.e., pretest, posttest, maintenance).

Research Questions

Research Question 1: *Which of the three instructional conditions (Total, Reduced, or Ratio) is the most instructionally efficient (i.e., will result in the greatest increase in fluency performance following a specific amount of instructional time) on each of three assessment probe sets (100-problem, 36-problem reduced, and 36-problem reciprocal)?*

It was hypothesized that following a specific amount of instructional time differences would exist between instructional conditions on the 100-problem assessment. In particular, it was expected that students assigned to the Reduced condition would display higher levels of fluency performance when compared to the Ratio and Total conditions, respectively.

Second, it was hypothesized that following a specific amount of instructional time differences would exist between conditions on the 36-problem reduced assessment. In particular, students assigned to the Reduced and Ratio conditions were expected to display higher fluency rates than students assigned to the Total condition.

Lastly, it was hypothesized that differences would exist between conditions on the 36-problem reciprocal assessment following a specified amount of instructional time. In particular, students assigned to the Reduced and Ratio conditions were expected to display higher fluency rates than members of the Total condition.

Research Question 2: *Will the students assigned to the two instructional conditions (Reduced and Ratio) not containing the problems of the differing probe set generalize?*

It was hypothesized that students assigned to the Reduced and Ratio conditions would generalize fluency performance to the 36-problem reciprocal set. Members of the Reduced and Ratio conditions had many opportunities to respond to the non-automatic multiplication. Therefore, it was expected that students would generalize performance when presented with opposite problem sequences on the 36-problem reciprocal assessment.

Research Question 3: *How will the three instructional conditions (Total, Reduced, or Ratio) facilitate the maintenance of fluency performance across time?*

It was hypothesized that differences in the percentage of loss in DCM would exist between instructional conditions on the 100-problem assessment two weeks following intervention cessation. In particular, it was expected that students assigned to the Total condition would maintain fluency performance better than the Reduced and Ratio conditions over time on the 100-problem set.

Second, it was hypothesized that differences in the percentage of loss in DCM would exist between instructional conditions on the 36-problem reduced assessment two weeks following intervention cessation. In particular, members of the Total condition were expected to maintain fluency performance better over time when compared to Reduced and Ratio conditions.

Lastly, it was hypothesized that differences in the percentage of loss in DCM would exist between instructional conditions on the 36-problem reciprocal assessment two weeks following intervention cessation. In particular, it was expected that members of the Total condition would maintain fluency performance better over time when compared to students in the Reduced and Ratio conditions.

CHAPTER III

METHODOLOGY

Participants

Participants included 73 fourth grade students attending general education at a public school in north central Oklahoma. An a priori power analysis was conducted with a small to medium effect size (d = .30) and power of .90 to determine the number of participants required. Both male and female students were allowed to participate. Twenty-five participants were placed into the Total, Reduced, and Ratio conditions using a stratified, random sampling procedure across classrooms.

One student was removed from each the Total and Reduced conditions due to the inability to perform multiplication operations and attrition (i.e., student changed schools), respectively. In addition, the data set was analyzed for outliers by conducting the Cook's Distance (i.e., Cook's D) analysis, which revealed no significant outliers. Informed consent for participation was obtained from the school principal, teachers, and parents of students involved. In addition, approval from both a university's and local public school system's institutional review board was granted for the study. The final number of students included in the data set was 73 (N=73). Students were included in the current investigation through their participation in a schoolwide service project targeting math fluency skills. Students in first through fifth grade participate in the math "Two-A-Days" service project daily. Students participated at their assigned desks within the general education classrooms. Students received folders each day that contained math probes targeting grade-specific skills. Each morning, a schoolwide announcement instructed all students to remove daily probes from the provided folders. In addition, the announcement instructed all students to begin and stop answering questions on each math probe. Students participated in a two-minute timing, took a 10-second break, and participated in a second two-minute timing. Each student received two math probe packets containing basic multiplication facts daily. School psychology graduate students regulated all assessment and intervention procedures for the current study.

Materials

All assessment and intervention data was collected on experimenter-constructed multiplication probes. Each student received a folder that displayed his or her name, teacher, and grade. Math probes were placed within the folder each day prior to assessment and intervention implementation. In addition, student folders contained an additional sheet that displayed a blank graph that students completed for the self-graphing component of the intervention.

Intervention probes. Three probe sets were created for the intervention portion and were unique to each of three conditions (i.e., Total, Reduced, or Ratio). Each contained varying amounts of automatic and non-automatic multiplication facts in a vertical alignment (i.e., see Appendix A). The Total condition probes contained 100 multiplication facts (see Appendix B), which included reciprocals. The Reduced condition probes contained 36 multiplication facts (see Appendix C). The Reduced condition probes did not include mastered facts that contain zeros and ones, which is typical practice in mathematics research (Bliss et al., 2010; Poncy, Skinner, Axtell, 2010; Skinner et al., 1997). Each of the 36 facts had a respective reciprocal (i.e., 2 x 3 and 3 x 2). A random assignment procedure was implemented to determine which problem sequence would be included in the Reduced set (i.e., 2 x3 or 3 x 2). Reciprocals of the selected 36 problems were not included on the Reduced condition probes but were utilized to assess generalization performance on reverse problem sequences during assessment sessions. The Ratio condition probes contained 18 more automatic and 18 less automatic multiplication facts (see Appendix D) based on student response latency, which was investigated by Duhon et al. (2012). Duhon et al. (2012) rank-ordered 100 multiplication problems (i.e., all simple facts 0-9) based upon the amount of time required to respond to each fact. For the current study, the first and last 18 problems of the 36 reduced facts set comprised the Ratio condition probes. The first 18 math facts in the rank order were considered automatic because they were completed within 5.5 seconds while the last 18 math facts are considered non-automatic because they required a between 5.5 and 10.5 seconds to complete. The ratio was achieved by randomly selecting and assigning two non-automatic problems (i.e., 2/3 of problems) for every automatic problem (1/3 of problems). This procedure was carried out until all 18 automatic facts were exhausted. In other words, once automatic problems were randomly selected and assigned to the probe they were excluded from the randomized problem pool (i.e., 18, 17, 16, 15, etc…). Non-automatic problems were selected and assigned in a similar fashion. However, this group of problems was exhausted more quickly (i.e., 18, 16, 14, 12, etc…). Consequently, this problem set was re-randomized, selected, and assigned within the probes before the automatic problems were exhausted. Each sheet of the probe packet contained 24 non-automatic problems and 12 automatic problems. Therefore, the entire 18 non-automatic fact set appeared $2^{2/3}$ times (i.e., 48 problems) while the entire automatic fact set appeared 1.5 times per probe (i.e., 24 problems).

Six forms were created for each condition and contained the appropriate problem sets in randomized order. The Total condition probe packet consisted of six rows of six problems on each of two sheets of paper, totaling 72 problems. Seventy-two out of the 100 multiplication facts were randomly selected for each probe in order to standardize the presentation of materials. The Reduced condition probe packet consisted of six rows of six problems, with each of two sheets containing all 36 problems (72 problems total). The Ratio condition probe packet consisted of six rows of six problems, with each of tow sheets containing 36 problems (i.e., 72 problems total). Each day students received two intervention packets inside their Two-A-Day folder. Each of the daily math probe packets contained the weekday and order number (i.e., Monday 1, Monday 2, etc…) in the top right corner of the page to reduce confusion for the student. Intervention probes were counterbalanced to account for practice and/or fatigue effects.

Assessment probes. Student performance was assessed on three separate occasions. The first assessment was conducted pre-intervention (pre-test). The second assessment was conducted following five weeks of intervention implementation (i.e., posttest). The third and final assessment was conducted two weeks after the posttest to determine maintenance of multiplication fluency skills (i.e., maintenance). Each assessment included three consecutive measures of performance on each dependent variable. In other words, students were assessed on each dependent measure three days in a row at the beginning of the experiment (i.e., pretest), following 5 weeks of intervention (i.e., posttest), and two weeks following intervention cessation (i.e., maintenance). The median of the three scores was then utilized to determine performance for each student on each assessment.

During each assessment session student folders contained three separate packets. The first packet consisted of 72 randomly selected facts out of 100 total problems (see Appendix B). The problems were randomly distributed, with each sheet containing six rows of six problems. A second packet contained 36 multiplication facts (see Appendix C) identical to the Reduced condition multiplication facts. The 36 problems were randomly distributed and presented in six rows of six problems on each sheet within the packet, totaling 72 or more problems. A third packet contained 36 randomly distributed multiplication facts that were the reciprocals from the Reduced condition problem set (see Appendix E). Each probe within the packet contained the 36 randomly distributed reciprocals, totaling 72 or more problems. Each sheet contained the entire set and was presented in six rows of six problems. Students were assessed on these probes to determine if skill generalization occurred following intervention implementation. Assessment probes were counterbalanced to account for practice and/or fatigue effects.

Design

The dependent measures included measures of performance on three separate multiplication probe sets (i.e., 100-problem, 36-problem reduced, and 36-problem reciprocals assessments). The first dependent variable measured student performance on all 100 multiplication facts (i.e., 100-problem assessment). This measure was analyzed to determine if student performance differed between groups that have fewer or no response opportunities to mastered multiplication facts (i.e., Reduced and Ratio Conditions) when compared to a group that had increased opportunities to respond to mastered facts (i.e., Total set with facts containing 0's and 1's). Students were assessed on three separate occasions to determine if differences existed between groups before, immediately after, and two weeks following intervention implementation (i.e., pretest, posttest, and maintenance assessments). The total number of digits answered correctly per minute (DCPM) determined fluency performance on the 100-problem assessment. Students were timed for two minutes on the 100-problem assessment, which measured their performance on two-minute timings. Therefore, the total number of digits answered correctly was divided by two to determine DCPM for each student on the 100-problem assessment.

The second dependent variable measured student performance on the 36-problem reduced set of multiplication facts (i.e., 36-problem reduced assessment). This measure was analyzed to determine if fluency performance differed between a group with fewer opportunities to respond to non-automatic problems (i.e., Total Condition) when compared to groups that were exposed only to the non-automatic problems (i.e., Reduced and Ratio Conditions). This measure was included to investigate if students who receive fewer opportunities to respond to non-automatic problems displayed lower fluency rates when compared to those who solely receive instruction on the non-automatic problem set. Students were assessed on three separate occasions to determine if differences existed between groups before, immediately after, and two weeks following intervention implementation. The total number of digits answered correctly per minute (DCPM) determined fluency performance on the 36-problem reduced assessment. Students were timed for two minutes on the 36-problem reduced assessment. Therefore, the total number of digits answered correctly was divided by two to determine DCPM for each student on the 36-problem reduced assessment.

The third dependent variable measured student performance on the 36-problem reciprocal set of multiplication facts. This measure was analyzed to determine if fluency performance differed between groups that receive no opportunities to respond to multiplication fact reciprocals (i.e., Reduced and Ratio Conditions) when compared to a group that is provided with response opportunities on reciprocal problems (i.e., Total Conditions). Analysis of this measure also indicated whether or not members of the Reduced and Ratio conditions generalize performance to problems that are excluded from intervention probe sets. Students were assessed on three separate occasions to determine if differences existed between groups before, immediately after, and two weeks following intervention implementation. The total number of digits answered correctly per minute (DCPM) determined fluency performance on the 36-problem reciprocal assessment. Students were timed for two minutes on the 36-problem reciprocal assessment, which measured student performance on two-minute timings. Therefore, the total number of digits answered correctly was divided by two to determine DCPM for each student on the 36-problem reciprocal assessment.

Statistical Analysis

The current investigation utilized a doubly repeated-measures multivariate analysis of variance (MANOVA) and consisted of one within subjects factor (i.e., Pretest, Posttest, and Maintenance) and one between subjects factor (i.e., Total, Reduced, and Ratio Conditions). Student fluency rate was assessed on three separate dependent measures of performance (i.e., 100-problem, 36-problem Reduced, and 36-problem Reciprocal Assessments).

Measures of Reliability

A second experimenter rescored 25% of the assessment math probes collected during the study to determine inter-scorer agreement (IA). IA was calculated by dividing actual agreements on DCM scores by total number of possible agreements (agreements + disagreements) on each probe. Multiplying the result by 100 provided the percentage agreement between experimenters to determine reliability of scoring. The average of the IA probes was calculated and represents the overall IA score, which resulted in 97% agreement between scorers. In addition, the same formula was utilized to determine the adherence to procedures (see Appendix H) required during each assessment session. The overall agreement between observers was 99%. Lastly, inter-observer agreement was calculated for adherence to procedural integrity for all intervention sessions (see Appendix G). The overall agreement between observers was 96%.

Procedures

Students within four separate classrooms were randomly assigned to each of three conditions (Total, Reduced, or Ratio) using a stratified random sampling procedure across classrooms. This sampling procedure was used due to the heterogeneous nature of the population (Hinkle, Wiersma, & Jurs, 2003). Students' initial fluency performance was determined with the 100-problem pretest assessment. Next, students were randomly assigned to each of three conditions (i.e., Total, Reduced, or Ratio Conditions), and group means were assessed to determine if differences existed. The group means did not differ and were considered equal.

*Intervention session procedures***.** Prior to each intervention session students were instructed through a schoolwide announcement to take out their math Two-A-Day folders. For intervention sessions, folders contained two probe packets that consisted of randomized problem sets characteristic of the group to which each student is randomly assigned (i.e., Total, Reduced, or Ratio). Explicit timing and self-graphing were utilized to improve behavioral responding on intervention probes (Poncy, Duhon, Lee, & Key, 2010; Rhymer, Dittmer, Skinner, & Jackson, 2000; Stotz, Itoi, Konrad, & Alber-Morgan, 2008). Students were instructed to graph their performance throughout intervention implementation. The experimenter taught all participants how to self-graph their performance during the initial intervention sessions. Students were instructed to color in their bar graph, which visually represented their performance from the previous intervention session. This graphing sheet allowed students to track their progress throughout the study.

Next, an office assistant read the instructions to the students over the schoolwide intercom for math Two-A-Days (see Appendix F). Students were instructed to begin answering math facts and to continue for two minutes. Following two minutes, students were instructed to take a break and were given ten seconds to get the second probe packet out of the folder. Following ten seconds, the announcement instructed students to begin answering math facts until they were told to stop. Students were instructed to put their pencils down following a second two-minute timing. Two-minute timings on single skill probes allow practitioners to be confident "in comparing performance across students, indexing an individual student's performance relative to some criterion, and the construct they are measuring" (Hintze, Christ, & Keller, 2002).

Intervention sessions were conducted in the morning daily for 24 consecutive school days. Each intervention session consisted of two 2-minute timings, which resulted in a total of 96 minutes of instructional time. Intervention probes were scored daily in order to ensure accurate responding and graph completion.

Assessment session procedures. Prior to each assessment session students were provided with their math Two-A-Day folders by a school psychology graduate student. For assessment sessions, folders contained three packets of counterbalanced probes that consisted of randomized problem sets (100-problem, 36-problem reduced, and 36-problem reciprocal assessments). The instructions for the assessment sessions were announced to each classroom individually by the experimenter (see Appendix G) to ensure standardization of the assessment procedures, and were very similar to the intervention procedures. Students were instructed to begin answering math facts and to continue for two minutes. Following two minutes, students were instructed to take a break and were given ten seconds to get the second probe packet out of the folder. Following ten seconds, the experimenter instructed students to begin answering math facts until they were told to stop. Assessment sessions included an additional two-minute timing (i.e., 3 total dependent measures). Students were given a 10-second break following the second two-minute timing. After a third two-minute timing students were instructed to place their pencils on their desks.

A total of three assessment sessions were conducted throughout the current investigation. Assessment sessions were conducted in the morning for three consecutive school days. Each assessment session consisted of three 2-minute timings. A pre-test assessment was conducted prior to intervention implementation in order to determine pre-intervention performance and to establish the cells for each condition. A second assessment, the posttest assessment, was conducted immediately following 96 minutes of instruction time, or 5 weeks of intervention implementation (i.e., 4 minutes per day across 24 school days). A third assessment, the maintenance assessment, was conducted two weeks following the posttest assessment to determine performance following intervention cessation (i.e., performance maintenance).

CHAPTER IV

RESULTS

The purpose of the current study was to determine if learning rates differ between groups exposed to multiplication probe sets containing either a mixture of automatic and non-automatic problems (Total Condition), non-automatic problems (Reduced Condition), or a specific ratio of automatic to non-automatic problems (Ratio Condition). A second goal was to determine if students assigned to the Reduced and Ratio conditions would generalize performance on the 36-problem reciprocal assessment. A third goal was to determine which instructional condition facilitated the maintenance of fluency performance the greatest over time.

Data for the current study were analyzed using a doubly repeated-measures multivariate analysis of variance (MANOVA), which is appropriate when several correlated dependent variables are measured across time or for each treatment (Stevens, 2009). Three dependent measures of performance (i.e., 100-problem, 36-problem reduced, and 36-problem reciprocal assessments) were measured within subjects over time (i.e., Pretest, Posttest, and Maintenance), with one between subjects factor (i.e., Total, Reduced, and Ratio conditions).

The results of the analysis did not reveal a significant two-way interaction between time (i.e., Pretest, Posttest, and Maintenance) and group (i.e., Total, Reduced, and Ratio conditions): Wilks' Lambda = .915, F (12, 365.4) = 1.04, p = .411, η^2 = .029. However, the results indicated a significant main effect for group (i.e., Total, Reduced, and Ratio conditions): Wilks' Lambda = .807, F (6, 136) = 2.564, p = .022, η^2 = .102. In addition, the results revealed a significant main effect for time (i.e., pretest, posttest, and maintenance): Wilks' Lambda = .284, F (6, 276) = 40.263, p = .000, η^2 = .467. The results of the doubly repeated-measures MANOVA for mean fluency scores are presented in Table 1.

Table 1.

Doubly Repeated-Measures Multivariate Analysis of Variance for Mean Fluency Scores

Source	Value	*F*	Hypothesis *df*	Error *df*	Sig.	($\eta\rho^2$)
Between Subjects						
Group	.807	2.564	6.00	136.00	.022*	.102
Within Subjects						
Time	.284	40.263	6.00	276.00	.000**	.467
Time x Group	.915	1.040	12.00	365.405	.411	.029

*Note: *p < .05. **p < .01*

Univariate tests were conducted by running an individual multivariate analysis of variance (MANOVA) for each dependent variable to further assess the significant main effects of Group and Time from the doubly repeated-measures MANOVA. The results of the individual MANOVAs for the main effect of Group did not reveal significant differences on each dependent variable. However, the results of the individual MANOVAs for the main effect of Time did reveal significant differences on each dependent measure. In particular, significant differences were observed on the 100-problem assessment (F (1.38, 96.84) = 167.65, p = .000), 36-problem reduced assessment (F (1.25, 87.53) = 151.01, p = .000), and 36-problem reciprocal assessment (F (1.24, 86.92) = 137.12, p = .000). The results of the individual MANOVAs conducted to assess univariate significance for mean fluency scores are presented in Table 2.

Table 2.

Univariate Assessment with Individual MANOVAs for Mean Fluency Scores

Source	*F*	Hypothesis *df*	Error *df*	Sig.
Group by Assessment				
100-Problem	.126	2	70	.881
36-Problem Reduced	.268	2	70	.765
36-Problem Reciprocal	.220	2	70	.803
Time by Assessment				
100-Problem	167.65	1.38	96.84	.000**
36-Problem Reduced	151.01	1.25	87.53	.000**
36-Problem Reciprocal	137.02	1.24	86.92	.000**

Note: $*p < .05$. $**p < .01$

Pairwise comparisons were calculated and observed in order to obtain additional information regarding the significant univariate tests for the main effect of Time. On the 100-problem assessment, posttest and maintenance scores were significantly better than pretest scores for all groups (i.e., Total, Reduced, and Ratio conditions). However, scores on the maintenance assessment were not significantly different from posttest scores for all groups, which indicates that student performance remained relatively stable following a two-week delay. Please see Table 3 and Figure 1 for descriptive statistics and visual analysis of the group means on the 100-problem assessment, respectively.

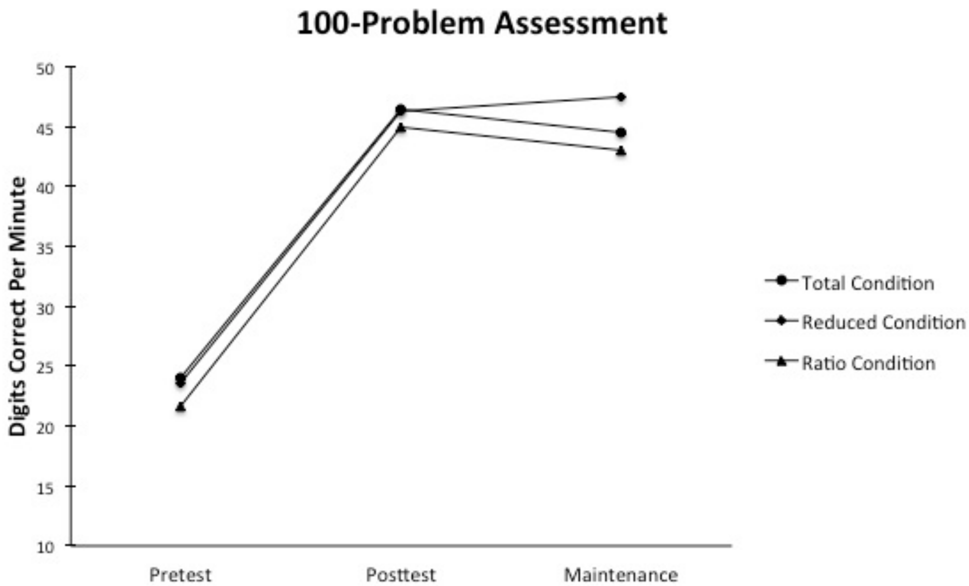
Table 3.

Means and Standard Deviations for Groups on 100-Problem Assessment

100-Problem Assessment							
		Pretest		Posttest		Maintenance	
Group	n	*M*	SD	*M*	SD	*M*	SD
Total	24	24.00	13.52	46.38	22.04	44.50	21.06
Reduced	24	23.58	13.49	46.22	23.35	47.44	24.52
Ratio	25	21.65	10.39	44.93	20.05	43.04	21.29

Figure 1.

Observed Group Mean Performance on 100-Problem Assessment



On the 36-problem reduced assessment, posttest and maintenance scores were significantly better than pretest scores for all groups (i.e., Total, Reduced, and Ratio Conditions). In addition, a significant difference was observed between posttest and maintenance scores for students assigned to the Reduced condition. This difference suggests that student performance significantly declined following a two week delay from intervention. Scores on the maintenance assessment were not significantly different from posttest scores for the Total and Ratio conditions, which indicates that student performance remained relatively stable following a two-week delay. Please see Table 4 and Figure 2 for descriptive statistics and visual analysis of the group means on the 36-problem reduced assessment, respectively.

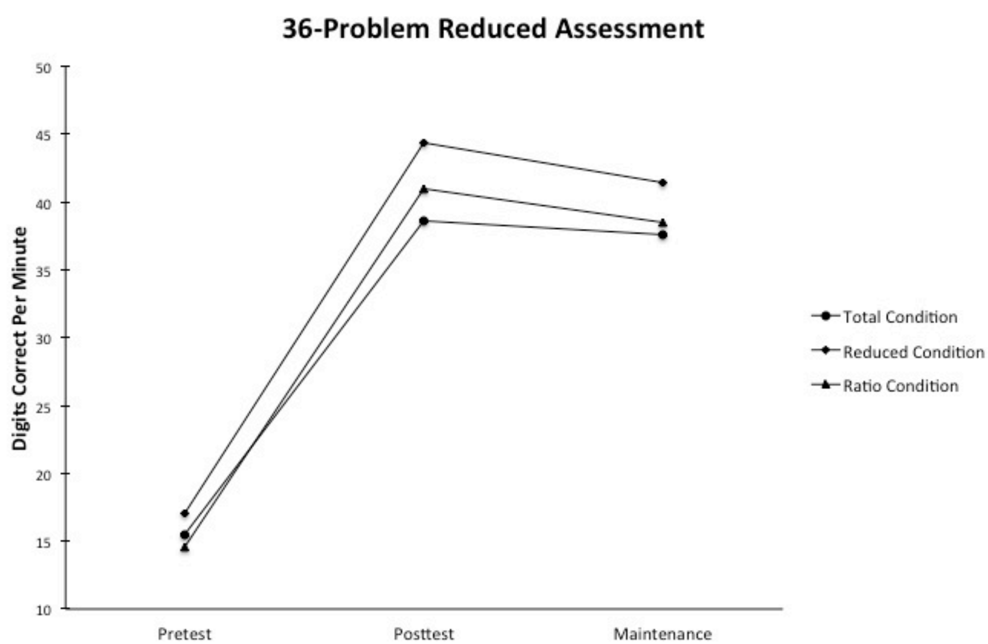
Table 4.

Means and Standard Deviations for Groups on 36-Problem Reduced Assessment

36-Problem Reduced Assessment							
		Pretest		Posttest		Maintenance	
Group	n	*M*	SD	*M*	SD	*M*	SD
Total	24	15.5	10.18	38.58	21.97	37.58	23.21
Reduced	24	17.04	12.38	44.32	26.17	41.48	26.00
Ratio	25	14.58	7.63	40.96	22.25	38.44	22.80

Figure 2.

Observed Group Mean Performance on 36-Problem Reduced Assessment



On the 36-problem reciprocal assessment, posttest and maintenance scores were significantly better than pretest scores for all groups (i.e., Total, Reduced, and Ratio Conditions). However, scores on the maintenance assessment were not significantly different from posttest scores, which indicates that student performance remained relatively stable following a two-week delay. Please see Table 5 and Figure 3 for descriptive statistics and visual analysis of the group means for the 36-problem reciprocal assessment, respectively.

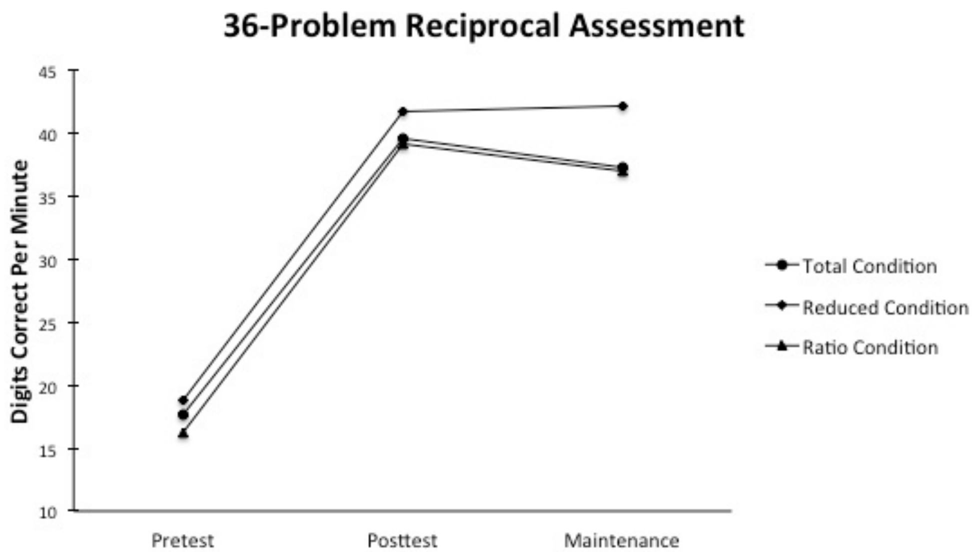
Table 5.

Means and Standard Deviations for Groups on 36-Problem Reciprocal Assessment

		36-Problem Reciprocal Assessment					
		Pretest		Posttest		Maintenance	
Group	n	*M*	SD	*M*	SD	*M*	SD
Total	24	17.68	12.17	39.58	24.91	37.23	23.25
Reduced	24	18.75	13.38	41.67	25.15	42.19	26.07
Ratio	25	16.25	8.02	39.15	20.11	36.96	21.87

Figure 3.

Observed Group Mean Performance on 36-Problem Reciprocal Assessment



CHAPTER V

DISCUSSION

The primary purpose of the current investigation was to determine if modifications to multiplication probes affects student learning rates within the context of an explicit timing intervention. A second goal was to determine if students assigned to specific conditions would generalize multiplication fluency performance to assessment probes that contained problems presented in reverse order (i.e., 36-problem Reciprocal Assessment). A final goal of the study was conducted to determine if student multiplication fact fluency performance would maintain two-weeks following intervention cessation.

In the current study, three mutually exclusive groups of students were exposed to multiplication fact probes that were comprised of automatic and non-automatic problems (Total Condition), non-automatic problems (Reduced Condition), or a specific ratio of facts that differed in automaticity based on response latency (Ratio Condition). Collectively, the results indicated that explicit timing was an effective intervention for improving the fluency performance of students in all treatment conditions. In addition, scores on a two-week follow-up assessment indicated that student fluency performance remained relatively stable over time. The lack of a significant Group x Time interaction suggested the fluency performance of students in each group was similar on multiple assessments over time (i.e., Pretest, Posttest, and Maintenance) on each dependent measure (i.e., 100-problem, 36-problem reduced, and 36-problem reciprocal Assessments). However, the absence of the significant interaction indicated that student fluency performance generalized to the novel presentation of multiplication facts (i.e., reciprocals) for specific conditions.

First, it was hypothesized that significant differences would exist between treatment conditions following 5 weeks of intervention on each dependent measure. On the 100-problem assessment, it was expected that students assigned to the Reduced condition would display higher levels of fluency performance when compared to the Ratio and Total conditions, respectively. On the 36-problem reciprocal and 36-problem reduced assessments, students assigned to the Reduced and Ratio conditions were expected to display higher fluency rates than students assigned to the Total condition. The absence of a significant Group x Time interaction indicates that students performed similarly, regardless of condition assignment. In other words, students who practiced 100 problems, 36 reduced problems, or a ratio of 36 reduced problems performed similarly on each dependent measure initially, after intervention, and two-weeks following intervention cessation.

Second, it was hypothesized that students assigned to the Reduced and Ratio conditions would perform similarly when assessed on the same multiplication facts presented in reverse order (36-problem reciprocal assessment). The lack of a significant

Group x Time interaction confirms the hypothesis that these students' fluency performance would generalize when problems were presented in reverse order, which is considered a form of stimulus generalization (Poncy et al., 2010). Students assigned to the Reduced and Ratio conditions were exposed to multiplication problems that were presented in one specific order during all intervention sessions. However, their performance on the 36-problem reciprocal assessment was nearly the same as the group of students who were exposed to all problems, including reciprocals, during intervention sessions (i.e., Total Condition).

Third, it was hypothesized that students assigned to the Total condition would maintain fluency performance better than those assigned to the Reduced and Ratio conditions over time on the 100-problem, 36-problem reduced, and 36-problem reciprocal assessments. The results indicated that students in all groups maintained their fluency performance following a two-week delay, during which the intervention sessions were discontinued. In particular, relatively small decreases were observed for the Total condition (i.e., 4%, 3%, and 2%) and Ratio condition (i.e., 5%, 6%, and 6%) from the posttest to maintenance assessment on the 100-problem, 36-problem reduced, and 36-problem reciprocal assessments, respectively. For the Reduced condition, performance decreased roughly 6% from the posttest to maintenance assessment on the 36-problem reduced assessment; however, members of the Reduced condition increased performance roughly 3% and 1% on the 100 Total Assessment and 36-problem reciprocal assessment, respectively.

The results of the current study also support previous findings that student performance on mathematics operations is enhanced following explicit timing instruction (Houten & Thompson, 1976; Miller et al., 1995; Rhymer et al., 2002). In the current study, a significant main effect for Time indicated that student performance significantly increased following ninety-six minutes of instructional time for all conditions on each dependent measure. In particular, significantly large increases were observed for the Total condition (i.e., 93%, 148%, and 124%), Reduced Condition (i.e., 96%, 160%, and 122%), and Ratio condition (i.e., 107%, 180%, and 140%) from pretest to posttest on the 100-problem, 36-problem reduced, and 36-problem reciprocal assessments, respectively. Although not significantly different from one another, the greatest percentages of gains in student performance were observed in the Ratio, Reduced, and Total conditions, respectively.

Implications for Practice

A main purpose of academic intervention research is to determine the most effective method for remediating students struggling in specific areas. The majority of effective interventions targeting math fact fluency (i.e., Cover-Copy-Compare, Taped Problems, Explicit Timing) have repeatedly been supported in the literature (Houten & Thompson, 1976; McCallum et al., 2006; Poncy et al., 2007). More recently, however, emphasis has been placed on determining the most *efficient* methods for improving student performance for various academic tasks (Cates et al., 2003; Joseph & Nist, 2006; Poncy et al., 2007; Nist and Joseph, 2008; Skinner, 2008). For example, all measured the instructional efficiency of early literacy task In other words, research is focusing on modifying specific intervention components in order to determine the quickest method for remediating students displaying academic skill deficits.

The current study added to the instructional efficiency literature base by investigating the effects of mathematics probe modifications on student learning rates within an explicit timing intervention. The rationale was that increasing the number of opportunities to respond to non-automatic multiplication facts (i.e., Reduced and Ratio conditions) would result in higher rates of student learning following intervention when compared to those who practiced automatic and non-automatic facts (i.e., Total condition) for the same amount of time. The results indicated that practicing only non-automatic facts (i.e., Reduced and Ratio conditions) similarly enhances student learning rates when compared to students who practiced a mixture of automatic and non-automatic problems (i.e., Total condition).

In addition, the generalization of academic skills is an important goal of intervention research, but is less frequently represented in the literature when compared to generalization strategies for behavioral concerns (Skinner & Daly, 2010). Previous literature suggests that student fluency performance generalizes to addition problems presented in reverse order following exposure to the Taped Problem intervention (Miller, Skinner, Gibby, Galyon, & Meadows-Allen, 2011). Similarly, the results of the current study indicated that student fluency performance generalized to multiplication facts presented in reverse order. Students who were assigned to the Reduced and Ratio conditions were exposed to 36 multiplication facts that were presented in one particular order (i.e., 6 x 9 and 5 x 8), and did not practice the reciprocals (i.e., 9 x 6 and 8 x 5) to these problems. However, when members of these two conditions were assessed solely on the reciprocal problems, their posttest fluency performance was significantly greater than pretest scores.

These results provide practitioners with valuable information that can be applied to intervention development and consultation. First, when targeting multiplication fact fluency with explicit timing, school psychologists and teachers can simply create math probes that contain thirty-six multiplication facts rather than lengthy probes containing up to one hundred facts. Teaching this 36-problem reduced set is just as effective for increasing multiplication fluency performance. In addition, students became fluent on multiplication problems that were not practiced (i.e., reciprocals), which supported the effectiveness of teaching reduced multiplication fact sets to enhance generalization.

Second, previous academic intervention research suggests that increasing known items enhances spelling and reading accuracy due to increased rates of reinforcement, but requires additional time and sacrifices efficiency when compared to traditional drill and practice (Cates et al., 2003; Joseph & Nist, 2006; Nist & Joseph, 2008). The conditions in the current study varied from previous instructional efficiency research in the sense that math fluency rather than literacy skill accuracy was targeted. In other words, automatic and non-automatic multiplication facts were manipulated rather than known and unknown literacy items. However, the results of the current study indicated that incorporating automatic items does not require additional time or sacrifice instructional efficiency, and similarly enhances multiplication fact fluency performance. Therefore, teachers and school psychologists can provide students with randomized multiplication probes containing a mixture of non-automatic and automatic facts to increase rates of reinforcement without sacrificing the efficiency of the intervention. This is especially important because students displaying a performance deficit due to task difficulty or lack of interest may respond to intervention better simply by modifying multiplication problem sets.

Third, an explicit timing intervention significantly enhanced student learning rates for students in all conditions, and only required 96 minutes of instructional time (i.e., 4 minutes per day for 24 days). Previous research has demonstrated the effectiveness of explicit timing with various math operations (Houten & Thompson, 1976; Rhymer et al., 1999). The results of the current study indicated a large improvement in multiplication fact fluency performance, and simply required an adult to explicitly tell students their performance would be timed. These results support the effectiveness of the classwide implementation of explicit timing, which can be implemented as a tier 1 strategy within an RTI framework.

Limitations and Future Directions

A major limitation of the current study included the method for assessing student performance on each dependent measure. Traditionally, in mathematics research student performance is assessed by calculating the number of digits answered correctly per minute (i.e., DCPM). However, several subjects in the current study were observed skipping more difficult (i.e., non-automatic) problems during assessment sessions, which may have contributed to the nonsignificant interaction due to the possibility of inflated scores on the Total Set Assessment. Future response latency research may obtain a better estimate of subject performance by timing each subject on a specific set of problems (i.e., 100-problem, 36-problem reduced, and 36-problem reciprocal assessments), and converting their score to DCPM for each assessment (i.e., Pretest, Posttest, and Maintenance). For example, each student could be provided with each measure (i.e., 100-problem, 36-problem reduced, and 36-problem reciprocal assessments), which would contain the respective number of mathematics problems. Next, each student could be timed while attempting every single problem on each probe. Student performance could then be converted to a DCPM score, and group comparisons could then be made based upon this value (i.e., (Total Digits Correct x 60) / Worksheet Completion Time = DCPM score). This would require students to attempt all automatic and non-automatic problems, which would accurately measure DCPM based on response latencies to the multiplication facts.

Second, the only mathematics operation involved in the current study was simple multiplication. Future research should investigate the effects of modifying ratios of automatic and non-automatic facts to enhance fluency performance on other operations. In addition, previous findings suggest that student fluency performance generalizes to the inverse presentation of addition facts (Miller et al., 2011). However, Poncy et al. (2010) found that fluency performance does not generalize from addition problems to related subtraction problems. The observed fluency performance that generalized in the current study supports the findings by Miller et al. (2011), both of which measured student responses to inverse facts that were nearly identical to problems targeted within intervention sessions. Additional generalization studies are needed to determine if student fluency performance remains consistent by simply modifying the ratio of automatic and non-automatic problems on related problems with more difficult skills (i.e., 7 – 4 = 3 to 7 – 3 = 4). Future research should also assess whether fluency gains generalize from one operation to another (i.e., 4 + 5 = 9 and 9 – 5 = 4 or 9 x 5 = 45 and 45 / 9 = 5).

A third limitation included the limited population utilized in the study. All students were enrolled in fourth grade at one particular school in north central Oklahoma. Consequently, there is no evidence regarding the generalizability of the findings to students in different grades in various locations. Future investigations should seek to increase the sample size in order to assess a more diverse population than the one included within the current study. Replication with a larger sample size is also suggested in order to determine if the results remain consistent.

Summary

In summary, school psychologists spend a large amount of time developing and implementing academic interventions to enhance student learning. In particular, mathematics research has typically focused on the impact of intervention on student performance, few investigations have assessed intervention-specific curricular modifications that may enhance the rate at which students learn information. The current study examined the effects of mathematics probe modifications on student learning rates within an explicit timing intervention. Specifically, three groups of students were exposed to multiplication fact probes that were comprised of automatic and non-automatic problems (Total Condition), non-automatic problems (Reduced Condition), or a specific ratio of non-automatic facts that differed in automaticity based on response latency (Ratio Condition).

Overall, the results suggest that the fluency performance of students in all groups similarly improved from pretest to posttest measures. The results suggest that Explicit Timing is an effective intervention for enhancing student fluency performance. In addition, students exposed to a reduced number of multiplication facts (i.e., Reduced and Ratio conditions) performed similarly on generalization measures (i.e., 36-problem Reciprocal Assessment). These results are important for researchers and practitioners who are interested in improving student learning rates through simple curricular modifications within mathematics interventions. However, additional research is necessary to further understand the impact of intervention modification on student learning rates and generalization.

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APPENDICES

Appendix A

Example Probe Construction (i.e., 36 problems per page)

Wednesday 1

1 x 2	9 x 7	2 x 3	4 x 9	8 x 7	4 x 5
2 x 2	0 x 5	9 x 9	1 x 6	0 x 8	5 x 2
1 x 1	2 x 1	3 x 6	6 x 0	4 x 4	8 x 6
0 x 1	6 x 7	9 x 4	4 x 1	3 x 2	6 x 9
6 x 8	5 x 1	6 x 4	2 x 7	8 x 9	9 x 0
1 x 0	4 x 8	7 x 8	0 x 6	3 x 8	9 x 3

Appendix B

Total Problem Set

0 x 0	0 x 1	0 x 2	0 x 3	0 x 4	0 x 5	0 x 6	0 x 7	0 x 8	0 x 9
1 x 0	1 x 1	1 x 2	1 x 3	1 x 4	1 x 5	1 x 6	1 x 7	1 x 8	1 x 9
2 x 0	2 x 1	2 x 2	2 x 3	2 x 4	2 x 5	2 x 6	2 x 7	2 x 8	2 x 9
3 x 0	3 x 1	3 x 2	3 x 3	3 x 4	3 x 5	3 x 6	3 x 7	3 x 8	3 x 9
4 x 0	4 x 1	4 x 2	4 x 3	4 x 4	4 x 5	4 x 6	4 x 7	4 x 8	4 x 9
5 x 0	5 x 1	5 x 2	5 x 3	5 x 4	5 x 5	5 x 6	5 x 7	5 x 8	5 x 9
6 x 0	6 x 1	6 x 2	6 x 3	6 x 4	6 x 5	6 x 6	6 x 7	6 x 8	6 x 9
7 x 0	7 x 1	7 x 2	7 x 3	7 x 4	7 x 5	7 x 6	7 x 7	7 x 8	7 x 9
8 x 0	8 x 1	8 x 2	8 x 3	8 x 4	8 x 5	8 x 6	8 x 7	8 x 8	8 x 9
9 x 0	9 x 1	9 x 2	9 x 3	9 x 4	9 x 5	9 x 6	9 x 7	9 x 8	9 x 9

Appendix C

Reduced Problem Set

2 x 2	2 x 3	4 x 2	2 x 5	6 x 2	7 x 2
2 x 8	9 x 2	3 x 3	4 x 3	3 x 5	3 x 6
3 x 7	8 x 3	9 x 3	4 x 4	4 x 5	4 x 6
7 x 4	4 x 8	4 x 9	5 x 5	5 x 6	7 x 5
5 x 8	9 x 5	6 x 6	7 x 6	6 x 8	9 x 6
7 x 7	7 x 8	9 x 7	8 x 8	9 x 8	9 x 9

Appendix D

Ratio Problem Set

Automatic Facts (Response Latency 2.5-5 seconds)

2 x 2	3 x 3	2 x 5	5 x 5	2 x 3	4 x 2	6 x 2	7 x 2	3 x 5
4 x 5	5 x 6	7 x 5	9 x 5	2 x 8	9 x 2	4 x 3	3 x 7	4 x 4

Non-Automatic Facts (Response Latency > 5 seconds)

5 x 8	6 x 6	9 x 9	8 x 3	9 x 3	7 x 4	6 x 8	3 x 6	4 x 6
4 x 9	7 x 7	9 x 8	8 x 8	9 x 6	4 x 8	7 x 8	9 x 7	7 x 6

Appendix E

36-problem Reduced Reciprocal Set

2 x 2	3 x 2	2 x 4	5 x 2	2 x 6	2 x 7
8 x 2	2 x 9	3 x 3	3 x 4	5 x 3	6 x 3
7 x 3	3 x 8	3 x 9	4 x 4	5 x 4	6 x 4
4 x 7	8 x 4	9 x 4	5 x 5	6 x 5	5 x 7
8 x 5	5 x 9	6 x 6	6 x 7	8 x 6	6 x 9
7 x 7	8 x 7	7 x 9	8 x 8	8 x 9	9 x 9

Appendix F

Two-A-Day Instructions

"If you want strong math muscles, it's important to practice! So get ready, students; it's time for Two-A-Days! Open your folders and turn to the first worksheet for __*(Day)*__. Try to work every problem. Work across the rows, and when you finish a row, move on to the next one. If you come across one you don't know, skip it and go onto the next. Remember to try your best! Get your pencils ready." (*pause for 10 seconds*) "Get set. Begin!"

(Time for two minutes)

"Stop! Time's up! Put your pencils down. Let's take a short break while you are getting your second worksheet for __*(day)*__. Pull it out of your folder and place it on top." (*pause for 10 seconds*). "Get set. Begin!"

(*Time for two minutes)*

"Stop! Put your pencils down and put your worksheets back in your folder. Good job, everyone!"

Appendix G

Assessment Instructions and Integrity Checklist

Date: ________________________	Observer: _____________________
Teacher: ______________________	Grade: _______________________

______ 1.) Have a stopwatch & pencil.

______ 2.) Place assessment probe packet face down on student(s) desks.

______ 3.) Make sure the students have a functional writing utensil.

______ 3.) Have the student(s) put their name and the date on the back of the probe packet.

______ 4.) Read the following directions verbatim,

> *"The sheets on your desk are multiplication facts. When I say 'Begin" start answering the problems. Begin with the first problem and work across the page, then go to the next row. Do each problem then go to the next, if you 'x' out a problem it will be counted as wrong, so try your best on each problem. Turn your packet over. Ready. Begin."*

______ 5.) Stop the student after 2 minutes have elapsed.

______ 6.) Read the detailed directions above for the first assessment only and say, "*Ready. Begin*." for the remaining two assessments and stop student after 2 mins.

Appendix H

Math Two-a-Days Integrity Checklist

Date: ________________________	Observer: _____________________
Teacher: ______________________	Grade: _______________________

_______ 1. Instructions were read over the intercom.

_______ 2. Students were told to begin.

_______ 3. Students were engaged in working on math probes for 2 minutes (without cheating).

_______ 4. Students were told to stop after 2 minutes.

_______ 5. Students were instructed to turn to the second probe.

_______ 6. Steps 2-4 were repeated for the second probe.

Collect this information for the observation as a whole:

_______ Teacher walked around the room observing students completing the math probes.

_______ Students were given clear, specific instructions before and between math probes.

_______ Students were praised for following instructions or classroom expectations.

Appendix I

Institutional Review Board Approval Letter

Oklahoma State University Institutional Review Board

Date: Wednesday, September 05, 2012 Protocol Expires: 9/4/2013

IRB Application No: ED10121

Proposal Title: The Impact of School Wide Tiered Interventions on the Math Fluency and Accuracy Performance of Students

Reviewed and Processed as: **Modification/Continuation**

Status Recommended by Reviewer(s) **Approved**

Principal Investigator(s) :

Gary J Duhon
423 Willard
Stillwater, OK 74078

Approvals are valid for one calendar year, after which time a request for continuation must be submitted. Any modifications to the research project approved by the IRB must be submitted for approval with the advisor's signature. The IRB office MUST be notified in writing when a project is complete. Approved projects are subject to monitoring by the IRB.

☒ The final versions of any printed recruitment, consent and assent documents bearing the IRB approval stamp are attached to this letter. These are the versions that must be used during the study.

Signature Shelia M. Kennison

Shelia Kennison, Chair, Institutional Review Board

Wednesday, September 05, 2012
Date

VITA

SCUDDY FRANCIS FONTENELLE IV

Candidate for the Degree of

Doctor of Philosophy

Dissertation: THE EFFECTS OF CURRICULAR MODIFICATIONS ON MATH FACT FLUENCY RATES

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