ECONOMICALLY-BASED MONITORING OF CONTINUOUS FLOW PROCESSES USING CONTROL CHART TECHNIQUES

By

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PREFACE

The objective of this study is to provide optimal economically-based control charts for use in detecting out-of-control conditions when monitoring continuous flow processes.

The economic models of X-bar chart, Moving average chart, and individual chart have been developed for use in monitoring continuous flow processes. The formulation of these models follows the same cost structure as in Duncan's originated economic X-bar Chart model. An optimization procedure is employed to economically design these control charts. The results are then be compared and analyzed.

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CHAPTER I

THE RESEARCH PROBLEM

Purpose

The purpose of this research is to provide sound economically-based control chart techniques for use in detecting out-of-control conditions in continuous flow processes. Three very popular control charts, the X-bar, moving average, and individual charts are employed. An interactive computer program is developed in order to assist practitioners with optimization and evaluation, due to the modeling complexities involved.

The measured quality of manufactured products is always subject to a certain amount of variation as a result of chance. Some stable "system of chance causes" is inherent in any particular scheme of production and inspection. Variation within this stable pattern is inevitable. The reason for this variation outside the stable pattern may be detected and possibly corrected. The objective of process control is to discover and eliminate the causes of variation and waste.

To maintain a state of statistical control for a manufacturing process, several methods are available. Control charts have been widely used. The advantage of using control charts is their ability to indicate assignable causes of quality variation. This allows for the detection and correction of many production problems and brings substantial improvements in quality, or "uniformity about target."

Numerous papers have been published on the exposition, application, modification and economic design of control charts in the last four decades. Most of those papers are only concerned about the characteristics of discrete processes and they are not suitable for use in continuous flow processes because they assume subgroups of size n are sampled together. This makes sence for independent discrete items, but not for highly autocorrelated continuous flow processes. There exists a need to advance the art in process control techniques for continuous flow processes.

Introduction

In recent years, rapidly advancing technology, increasing complexity of operations, and growing competition in the marketplace have made modern industry aware of the necessity to provide, as economically as possible, products which satisfy the requirements of the customers. A company's reputation depends primarily on its ability to deliver a product of satisfactory quality, on time, at an acceptable price to its customers. Process control is a necessary function to help achieve these quality, schedule, and cost objectives.

Attainment of quality requires the performance of a wide variety of identifiable activities or quality functions, such as product design, specification establishment, manufacturing planning, production, inspection, test, sales, service, etc. These functions follow a sequence of events which is depicted as a spiral by Juran (1980). Statistical process control serves an important role in obtaining the information necessary for establishing specifications that can be met and in providing an operational technique for detecting and eliminating production and measurement variation.

An important tool used for statistical process control is the control chart. Control charts are appropriate to help achieve the following purposes:

1. to bring a process under control,
2. to help establish process capability,
3. to monitor and maintain control of a process.

This research is concerned with the optimum design of control charts used to monitor and maintain control of a continuous flow process.

The control chart was originated by Walter A. Shewhart in 1924 (Shewhart, 1931). Since then, many new techniques and variations have been proposed. Some of the more popular control charts used in industry are:

1. X-bar Chart (Mean/Average)
2. Moving Average Chart
3. Individual Chart
4. CUSUM Chart (Cumulative Sum)
5. R Chart (Range)
6. Moving Range Chart
7. s Chart (Standard Deviation)

8. p Chart (Proportion Nonconforming)
9. c Chart (Number of Nonconformities)
10. u Chart (Nonconformities Per Unit)

The first three of these control charts are being applied to continuous flow processes and will be used in this research.

Statistically-Based Control Charts

A certain amount of inherent or natural variation is inevitable in a production process. This inherent variation is the cumulative effect of many small, essentially uncontrollable, causes which are called "chance causes." Major variation due to "assignable causes" usually arises from differences among the 4 M's: machines, materials, men, and methods. Variations due to these factors are relatively large when compared to the variations by chance causes and usually represent unacceptable performance of a process, or an out-of-control condition. One of the objectives of statistical process control is to quickly separate assignable causes from chance causes. Control charts are major tools for performing this function.

X-bar Chart

A survey of many firms performed by Saniga and Shirland (1977) shows that the use of the X-bar chart dominates the use of any other control chart techniques in practice. By summing up the previous and current trends in the theoretical development and application of the X-bar chart, they indicated that X-bar charts will continue to receive further attention because of their fundamental importance in scientific process control.

The procedure for operating an X-bar chart consists of taking samples of size n at regular intervals of h hours and measuring some quality characteristic of interest. The average, X-bar, of such measurements is computed and plotted on an X-bar chart with control limits $k\sigma''/\sqrt{n}$ above and below the mean. If this plotted point falls within the control limits, the underlying process is said to be in a state of statistical control (SOSC); if not, it is judged to be out-of-control (OOC). When in a SOSC, the process is allowed to continue to operate; when OOC, a search is initiated for the cause of the trouble, i.e., for an assignable cause. The process may either be shut down or allowed to run while a search is being made. Once a cause is found, appropriate steps are taken to correct the cause or adjust the process level to the desired value.

The sample size n was suggested to be 4 or 5 by Shewhart. The control limits commonly used are set 3 standard deviations away from the sample average, with a 0.00135 probability beyond either control limit. A 0.00135 probability limit implies that if the process is in a SOSC, a point will fall above the upper limit with a 0.00135 probability. Likewise, the probability of a point falling below the lower limit is 0.00135. That is, the chance of a point falling outside the control limits when the process is in control is very small -- less than three out of a thousand. Therefore, if a point falls outside the control limits, it can be said that variation is produced by an assignable cause.

In general, any multiple of sigma other than the usual 3-sigma can be used to establish the control limits. This choice depends upon the risk that management of the quality function is willing to tolerate; tighter control limits achieved using a smaller multiple of sigma will increase the probability of concluding the process is out of control when it is really in control, while broader control limits will decrease the sensitivity of detection when the process is out of control.

Moving Average Chart

Many schemes other than the Shewhart control chart have been used for plotting data on quality characteristics. For example, a chemical plant may collect data on the results of periodic analyses made to determine the percentages of certain chemical constituents or other properties in its incoming materials, product in process, finished product, or process operating characteristics. Moving averages may then be plotted. The moving average is particularly appropriate in continuous process chemical manufacture when applied to quality characteristics of raw materials and product in process.

The moving average is formed from a time series of individual data values by finding the arithmetic mean of the first n consecutive values, and subsequently dropping the oldest value and adding the newest value to form each successive average. In this way, a point may be plotted on the moving average chart each time a new piece of data is obtained.

A moving average chart is paired with a moving range chart to control the current process. It is set up from past data in the same way as are ordinary charts. Thus, the grand average of the past data (not the mean of the moving averages) and the average range of individual subgroup ranges are first computed. Then, the central line is set equal to the grand average and control limits are set to the equivalent of 3 standard deviations of the sample average above and below the center line. Instead of plotting independent averages, moving averages are plotted on the chart. A point outside the control limits on a moving average chart has the same significance as a point outside control limits on an X-bar chart.

Individual Chart

A popular alternative to the moving average chart is the chart for individual measurements. The individual chart, like the moving average chart, is centered at the grand average of the individual measurements, and control limits are set at the equivalent of 3 process standard deviations above and below this center line. An individual data point outside such limits may be considered as evidence of an assignable cause of variation.

The individual chart is relatively insensitive to small sustained shifts in process average. Although greater sensitivity to such shifts may be gained by the use of narrower limits, such sensitivity is gained only by increasing the chance of false indication of lack of control. For discrete processes, charts for individual measurements are often considered inferior to X-bar or moving average charts because they fail to give as clear a picture of changes in a process or as quick an evidence of assignable causes of variation (Grant and Leavenworth, 1980).

Economic Design of Control charts

Traditionally, control charts have been designed with respect to statistical criteria only. This usually involves selecting the sample size and control limits so that the power of a test to detect a particular shift in the quality characteristic, and the probability of false indication of lack of control, are equal to specified values. The fre- guency of sampling is rarely treated analytically and the practitioner is often given qualitative rather that quantitative guidelines for choosing the sampling interval.

The design of a control chart has economic consequences in that the costs of sampling and testing, the costs associated with investigating out-of-control signals and possibly correcting assignable causes, and the costs of allowing defective products to reach the consumer are all affected by the selection of the control chart parameters. Therefore, it is logical to consider the design of a control chart from an economic viewpoint. In recent years, considerable attention has been devoted to this problem (Vance, 1983).

The pioneer investigation of the economic design of control charts was made by Duncan (1956). He formulated a model to determine the optimal parameters of an X-bar chart. These parameters (the sample size n, the sampling interval h, and the control limit spread k) were derived to maximize the approximate average net income of a process.

Duncan assumed that the process is subject to an assignable cause of variation which is a shift in the process mean. The standard deviation is assumed to remain stable. The time from the start of an in-control process until it goes out-of-control is assumed to follow an exponential distribution. And the process is not shut down while searching for the assignable cause.

An approximation to the optimal design was found which determined the parameters n, h, and k of an X-bar control chart. Since then, numerous works have been developed such as economic design of p charts (Ladany, 1973), economic design of joint X-bar and R charts (Jones and Case, 1980), etc. But none of them are designs for either the moving average chart or the chart for individuals. This research is the first to develop economic designs for the moving average chart and the individual chart.

Control Charts used in Continuous Process Control

In continuous processes, there is not a well defined production unit. Almost any chemical, petroleum, bulk liquid, or otherwise semi-homogenized product is a case of this kind. The application of conventional control chart techniques is always difficult in such cases since the sampling unit is defined in terms of laboratory or laboratory analysis requirements rather than unit of product. Thus, one may sample one liter, one ounce, or one yard of product, while the product is actually produced by the barrel, the ton, or the reel.

The problem is compounded by the fact that the sample may have been taken from a vat or pipeline wherein there is a homogeneous mixture resulting from flow or agitation. To pull n samples instantaneously from a continuous flow process would usually result in ranges of zero, or in the range being an almost pure measure of test variation (Brooks and Case, 1986). The sampling method used in continuous flow processes is shown in Figure 1.1. Note the difference between this sampling method and the sampling method shown in Figure 1.2 which is usually used in discrete processes.

Due to the number of such processes, there is a need to develop appropriate quality control techniques for continuous flow processes.

Summary of Research Objectives

Based on the above discussion, the primary objective of this research is stated as follows:

Objective:

> To provide optimal economically-based control charts for use in detecting out-of-control conditions when monitoring continuous flow processes.

Sample size n = 1

```
          1        1        1        1
          V        V        V        V   ...
|____________________________________________
|<- h ->|<- h ->|<- h ->|<- h ->|  ...
```

V indicates one sample

Figure 1.1 Sampling Method Used in Continuous Flow Processes

Sample size n

```
          n        n        n        n

          V        V        V        V
          .        .        .        .
          .        .        .        .
          .        .        .        .
          V        V        V        V
|         V        V        V        V   ...
|____________________________________________
|<- h ->|<- h ->|<- h ->|<- h ->|  ...
```

V indicates one sample

Figure 1.2 Sampling Method Used in Discrete Processes

In order to accomplish this objective, several subobjectives must be met.

Subobjectives:

1. To establish procedures for the comparison of control chart techniques in a continuous flow process environment.
2. To develop analytical models to evaluate and optimize control charts under a continuous flow process environment.
3. To design and evaluate detection techniques and decision rules for economically-based control charts using:
   a. X-bar Chart,
   b. Moving Average Chart,
   c. Individual Chart.
4. To develop a comprehensive and flexible interactive computer program to provide optimal economic design and evaluation of:
   a. X-bar Chart,
   b. Moving Average Chart,
   c. Individual Chart.

Contribution

This research provides benefits to both theoreticians and practitioners. This study becomes the first of its kind in providing (1) an economic design of the X-bar chart for continuous flow processes, (2) an economic design of the moving average chart for continuous flow processes, (3) an economic design of the individual chart for use in continuous flow processes, and (4) an economically-based comparison among the X-bar chart, moving average chart, and individual chart when they are used in continuous processes. All of these results are new; none are presented in any textbooks or papers on statistical process control.

Practitioners will benefit from this research because it will provide them with practical procedures for designing and evaluating X-bar charts, moving average charts, and individual charts in continuous flow processes. The interative computer program will make the design of economic X-bar charts, moving average charts, and individual charts considerably easier.

CHAPTER II

LITERATURE REVIEW

Introduction

This chapter reviews developments in the literature relevant to the objectives of this research. General support for the research effort has been documented in Chapter I. Other sources which communicate the concepts relating to the objectives of this study will be presented here.

This chapter is divided into three sections. These are:

(1) Statistical process control and control charts

(2) Economic design and optimization of control charts

(3) Process control of continuous flow processes

Statistical Process Control and Control Charts

Quality control is as old as industry itself; statistical quality control is relatively new. The concept of statistical quality control was introduced by Walter A. Shewhart in 1924 (Shewhart, 1931). New techniques have been advanced and developed in various papers and textbooks since then. Statistical process control and acceptance sampling are two major areas of statistical quality control. Statistical process control, the area of interest in this study, concerns itself with guidance during manufacture with the aim of anticipating or identifying process changes in such a way that they may be corrected or modified before they result in adverse effects (Bingham, 1957).

A control chart is a key statistical process control tool used for monitoring and/or analyzing a process. A review of control chart methodology has been published by Gibra (1975). He classifies control charts into five categories as following:

(1) Shewhart control charts and their ramifications

(2) Modifications of Shewhart control charts

(3) Cumulative sum control charts

(4) Acceptance control charts

(5) Multi-characteristic control charts.

In a survey of 173 firms by Saniga and Shirland in 1977, it is shown that Shewhart's original control charts (the X-bar chart, R chart, Sigma chart, p chart, c chart, and u chart) are those most frequently used by industry. The moving average chart is the second most frequently used in industry. Results from this survey also show that the individual chart is one of the more frequently used control chart techniques other than those originally listed in the questionnaire (Saniga and Shirland, 1977).

Shewhart's original design of control charts utilizes 3-sigma control limits, sample sizes of four or five, with the sampling interval decided by user. This design is based upon "empirical-economic" considerations rather than on statistical considerations. Following his work, several techniques have been proposed to improve the performance of control charts. Weiler (1953) demonstrated that the sequential use of runs tests for control charts for the process mean leads to great savings in inspection. Page (1962) suggested that the use of warning limits is better than the use of runs tests. Weindling, et al. (1970) modified Shewhart's chart with a pair of warning limits inside the action limits to increase the sensitivity to small shifts in process mean.

Economic Design and Optimization of Control Charts

Recently, much emphasis has been placed on the economic design of control charts. Gibra (1975) reviews the methodology of the economic design of the X-bar control chart. Montgomery (1980) contains references to earlier work on economic design of control charts. Vance (1983) provides a bibliography for economic design of control charts of the period 1970-1980. Those are good references for the economic design of control charts.

Duncan (1956) proposed a model for the optimum economic design of the X-bar chart. He is the first to propose an economic model to a Shewhart control chart, and to incorporate formal optimization methodology into determining the control chart parameters. An approximation to the optimal control limit spread, sample size, and sampling interval is found to maximize the expected net income per unit of operation time.

Several authors have elaborated on improved optimization methods for Duncan's model. Goel, et al. (1968), developed an algorithm to find the exact optimal solution of Duncan's model by computer. Chiu and Wetherill (1974) have developed a simple, approximate procedure for optimizing Duncan's model.

Several other models have been developed, going beyond Duncan's model. Taylor (1968) developed an economic design of the cumulative sum control chart. Ladany (1973) developed an economic design of the p chart. Saniga (1978) and Jones and Case (1981) have developed the joint economic designs of X-bar and R charts. Other models have been presented by Cowden (1957), Knappenberger and Grandage (1969), Gibra (1971), and Baker (1971). Vance (1986) proposed a general method for determining the economic design of control charts. However, the economic designs of the moving average and individual charts have not yet been developed.

Duncan (1971) later developed an economic design of a situation in which there are multiple assignable causes rather than a single assignable cause. However, Chiu (1973) shows that some of the numerical results in Duncan's paper are wrong. Knappenberger and Grandage (1969) also proposed a model for the economic design of the X-bar chart when there are multiple assignable causes. Both Duncan (1971) and Knappenberger and Grandage (1969) report that a single assignable cause model matches the true multiple assignable cause model in certain ways, producing very good results. Furthermore, Montgomery (1980) concludes that very complex multi-state processes can be satisfactorily approximated by a model containing only a few out-of-control states, provided those states are properly defined.

Chiu and Wetherill (1975) and Saniga and Shirland (1977) report that very few practitioners have implemented economic models for the design of control charts. Montgomery (1980) points out two reasons for the lack of practical implementation of this methodology. One of them is that the mathematical models and their associated optimization schemes are relatively complex and are often presented in a manner that is difficult for the practitioner to understand and use. The availability of computer programs for these models and the development of simplified approximate optimization procedures suitable for manual computation would help alleviate this problem.

Chiu (1975) states that Duncan's model, while perhaps lacking generality, is simple, practical, has received attention, and a considerable amount of work has been developed from it. For this reason, Duncan's model is used as a basis for economic model development in this research.

Process Control of Continuous Flow Processes

Continuous flow processes are different from item-by-item part processes or batch processes (Dunn and Strenk, 1980s). They have received relatively little attention in the literature. The primary difference is that samples of size one are dictated for continuous flow processes. To take n repetitive samples at one time, as is standard with item-by-item part processes, would result in virtually identical or dependent specimens (Brooks and Case, 1986). Another characteristic of continuous flow processes is that they are not characterized by having a well defined production unit (Wortham, 1972) (Dunn and Strenk, 1980s).

To deal with the difficulties caused by the characteristics of continuous flow processes, Freund (1960) suggests the use of the acceptance control chart in batch or continuous processes. The use of exponentially smoothed data in control charts is suggested by Wortham (1972). Juran (1974) recommends the use of moving averages and ranges for continuous processes.

The existing methodology for treating certain kinds of quality control data assumes the existence of normality and independence in the data. Under these conditions the data can be treated simply through the use of available tables and simple calculations. When either independence and/or normality are not present, as is often the case in continuous processes, application of the existing methodology introduces large errors in the analysis of the data and renders conclusions based on them dubious. To cope with these problems, Vasilopoulos and Stamboulis (1978) modify and extend the existing standard methodology of control charts by utilizing the time series analysis approach and by introducing dependence via a second order autoregressive process (AR(2) model) for correlated data. Ermer (1980) also proposes a Time Series Control Chart, developed by the Dynamic

Data System methodology, which takes into account data dependence. Brooks and Case (1986) make use of X-bar and R charts and provide a procedure for checking data independence and present two methods, correction and avoidance, for dealing with autocorrelated data.

Other techniques which can be used for a continuous flow process include: median chart, individual chart, fraction defective chart, and cumulative sum charts. However, there has been no work toward economically comparing or optimizing any of these techniques used for continuous flow processes.

Summary

This chapter presents a survey of the literature on the problems, contributions, and needs relative to the objectives of this research. This survey demonstrates considerable interest in the economic design of control charts. Numerous works have been done for various popular control charts other than the moving average and individual charts. It also indicates that process control techniques have had some application in continuous flow processes; this is a rapidly growing area of industrial interest. Unfortunately, there has been no economic design and evaluation for control charts used in continuous flow processes.

This survey indicates a need for the following:

1. To develop economic models for X-bar, moving average, and individual charts in a continuous flow process environment.

2. To provide a procedure for the economic optimization and comparison of these control chart techniques in a continuous flow process environment.

3. To develop a user-friendly interative computer program to provide economic design of the X-bar, moving average, and individual charts in a continuous flow process environment.

CHAPTER III

MODEL DEVELOPMENT OF ECONOMICALLY-BASED CONTROL CHARTS

Introduction

From the literature review in Chapter II, three different control charts, referred to as the X-bar chart, moving average chart, and individual chart, have been employed in monitoring continuous flow processes. In this chapter, the models for these charts are developed for use in a continuous flow environment. A computer search procedure is then developed to optimize the three decision variables of each of the economically-based control charts. The economic models developed in this research use the same cost structure as Duncan's economically-based X-bar chart (Duncan, 1956) because of its simplicity, flexibility and acceptance (Chiu, 1975).

Assumptions

The basic assumptions of these models are as follows:

1) The production process is characterized by a single in-control state, i.e., the in-control state corresponds to a specific value of the mean of a measurable quality characteristic when no assignable cause is present; the character-

istic is normally distributed.

2) The occurrence of an assignable cause shifts the process mean to a known value.

3) The process standard deviation is assumed to be known and remains constant.

4) The shift in the process mean is instantaneous.

5) The occurrence time of the assignable cause is exponentially distributed. That is, the probability of its non-occurrence before time t when starting from a state of control is $e^{-\lambda t}$ and the probability of its occurrence in the interval t to $t+\Delta t$ is approximately $e^{-\lambda t}\Delta t$. The average time required for the assignable cause to occur is $1/\lambda$.

6) The process is not self correcting. That is, after an assignable cause has occurred, the process can only be brought back to the in-control state by management intervention.

7) The process is not shut down and sampling is continued while the search for the assignable cause is in process.

8) A sample of size 1 is taken from the process at a constant time interval.

9) Sampling inspection is not subject to measurement error.

10) Action is taken when a point falls outside the control limits.

11) The cost of adjustment or repair (including possible shutting down of the process) and the cost of bringing

the process back to a state of control subsequent to the discovery of the assignable cause are constants for the loss-cost function and are not considered.

12) The time required to take and inspect samples and to compute the subgroup results is proportional to the subgroup size.

13) The proportion of product produced outside specifications is increased when an assignable cause occurs.

14) The process is originally centered between the specification limits so that the difference between the average income per hour from operation of the process under control and the average income per hour from operation of the shifted process is the same no matter whether the shift is upward or downward.

Notation

To facilitate the development of the economically-based control chart models, the following notation is introduced and will be used continuously throughtout the research.

n = subgroup size used for the X-bar chart and moving average chart. It is made up of n samples of size 1 each taken h hours apart.

j = number of samples of size 1 each taken from the process while operating at mean $\bar{X}''$. This implies that n-j is the number of samples of size 1 each taken from the process while it is operating at mean $\bar{X}''+\delta\sigma''$, where j = 0, 1, 2, ..., n.

h = sampling interval; samples of size 1 each are taken from the process every h hours.

k = width of control limits on control charts in multiples of the standard deviation of the statistic being plotted.

m = index of the sequential sample number of size 1 each taken h hours apart.

g = sequential subgroup number. On the X-bar chart, the first subgroup of size n following time 0 will be g = 1. The second subgroup of size n following time n*h will be g = 2, etc. On the moving average chart, the first subgroup following time 0 will have a size of 1 and g=1. The second subgroup following time h will have a size of 2 and g=2, etc. The nth subgroup following time nh will have a size of n and g=n. The mth subgroup following time (m-1)h will have a size of n and g=m, when m > n.

r = subgroup number after the shift. The first subgroup of size n following a process shift will be r = 1, etc.

λ = failure rate for the assignable cause to occur, per hour.

σ'' = true process standard deviation.

δ = multiples of σ'' used to determine the magnitude $\delta\sigma''$ of the out-of-control shift in the process mean.

Φ = cumulative probability function of the standard normal distribution;

$$\Phi(X) = \int_{-\infty}^{X} \frac{e^{-Z^2/2}}{\sqrt{2\pi}} \, dZ$$

P = probability of detection when the process mean shifts; $P = 1-\Phi(-k-\delta\sqrt{n}) + \Phi(k-\delta\sqrt{n})$.

Q = probability of not detecting a shift when there is an assignable cause; $Q = 1 - P$.

P' = probability that the first subgroup following a shift will be detected when there is an assignable cause.

Q' = probability that the first subgroup following a shift will not be detected when there is an assignable cause; $Q' = 1 - P'$.

P_j = probability of detection when j samples of size 1 are taken from the process while operating at mean $\bar{X}''$ and n-j samples of size 1 are taken from the process while operating at mean $\bar{X}''+\delta\sigma''$.

Q_j = probability of no detection when j samples of size 1 are taken from the process while operating at mean $\bar{X}''$ and n-j samples of size 1 are taken from the process while operating at mean $\bar{X}''+\delta\sigma''$; $Q_j = 1 - P_j$.

P_{ji} = probability of detection when a shift occurs after the jth sample is taken and i samples are taken from the process while operating at mean $\bar{X}'' + \delta\sigma''$, where j = 0, 1, ..., n-1 and i = 1, 2, ..., n.

Q_{ji} = probability of no detection when a shift occurs after the jth sample is taken and i samples are taken from the process while operating at mean $\bar{X}'' + \delta\sigma''$, where j=0,1,..., n-1 and i = 1, 2, ..., n; $Q_{ji} = 1 - P_{ji}$.

α = probability of a false alarm (i.e., the control chart indicates an OOC indication when the process is in control); $\alpha = 2\Phi(-k)$.

β = proportion of the time a process will be in control.

γ = proportion of the time a process will be out of control; $\gamma = 1 - \beta$.

e = average sampling, inspecting, evaluating, and plotting time for a sample of size 1.

D = average time taken to find an assignable cause after a point has been found to fall outside the control limits.

V_0 = average income per hour accruing from operation of the process under controlled conditions at the standard level $\bar{X}''$.

V_1 = average income per hour accruing from operation at the new level $\bar{X}'' + \delta\sigma''$.

M = the reduction in process hourly income that is attributed to the occurrence of the assignable cause; $M = V_0 - V_1$.

T = average cost of looking for an assignable cause when none exists.

W = average cost of finding an assignable cause when one does exist.

b = cost per subgroup of sampling, inspecting, evaluating, and plotting that is independent of subgroup size.

c = cost per unit of measuring an item of product and other control chart operations directly related to the size of the subgroup.

ENSBSD = expected number of subgroups taken between the time the process shifts out-of-control and subgroup is completed before a shift in the process is detected.

AVGOOCT = average time the process will be out of control before a subgroup is completed which will fall outside of the control limits.

ACL = average cycle length of the process, considering both time in-control and out-of-control.

ENFA = expected number of false alarms per hour of operation.

ACFAC = average cost per hour of finding the assignable cause.

HCMCC = hourly cost of maintaining the control chart.

L = the loss-cost per hour; the minimization of L will result in the maximization of process hourly income(Duncan, 1956).

Model Components and Cycle Length

The general structure of Duncan's economic X-bar chart is adopted for developing models for the X-bar chart, moving average chart and individual chart in this research. The components of his model are (1) the cost of an OOC condition, (2) the cost of false alarms, (3) the cost of finding an assignable cause, and (4) the cost of sampling, inspecting, evaluating, and plotting.

The process starts in-control and is subject to random shifts in the process mean. Once a shift occurs, the process remains there until corrected. The cycle length is defined as the total time from which the process starts in-control, shifts to an OOC condition, has the OOC condition detected, and results in assignable cause identification. A complete cycle length consists of four time intervals as shown in Figure 3.1. These four time intervals are the interval the process is in-control, the interval the process is OOC before the final sample of the detecting subgroup is taken, the interval to sample, inspect, evaluate and plot the subgroup results, and the interval to search for the assignable cause.

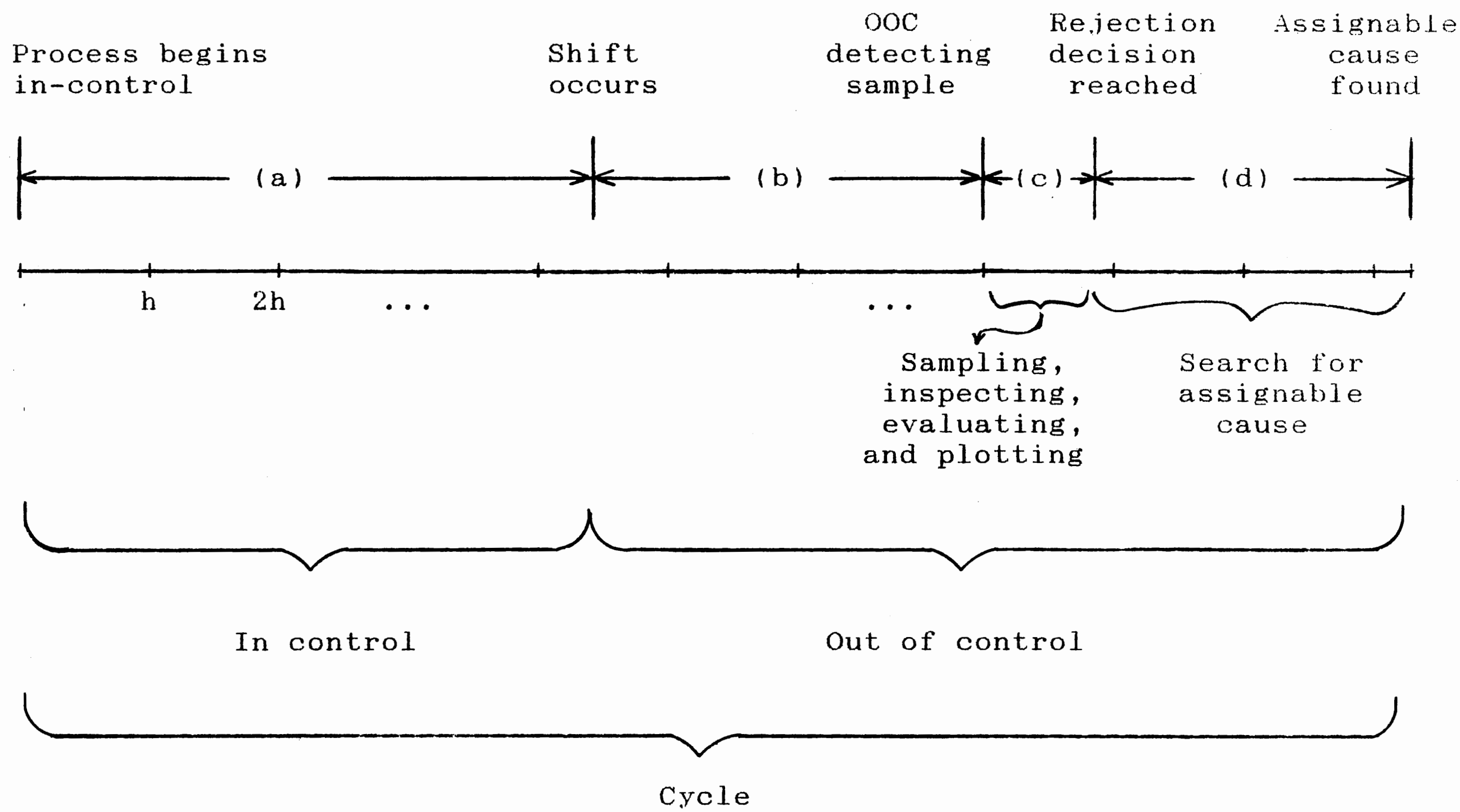


Figure 3.1. Cycle Length

When the average cycle length is determined, then the cost components can be converted to a "per hour of operation" basis. Given associated cost and time parameters, the optimal values of decision variables for each model are then determined using optimization techniques.

Formulation of an Economically-Based X-bar Control Chart

A sample of size 1 is taken every h hours. An X-bar control chart with subgroup size n will have a point plotted on the control chart every nh hours. The sampling and plotting methods for X-bar control charts are illustrated in Figure 3.2.

Average Cycle Length

As illustrated in Figure 3.1, the average cycle length is expressed as follows:

		(a)		(b)
Average cycle length	=	Average in-control time	+	Average time the process is OOC before the detecting subgroup is completed
		(c)		(d)
	+	Time to sample, inspect, evaluate, & plot a subgroup	+	Time to search for the assignable cause

(a) Since the average time for occurrence of the assignable cause is $1/\lambda$, this is the average process in-control time.

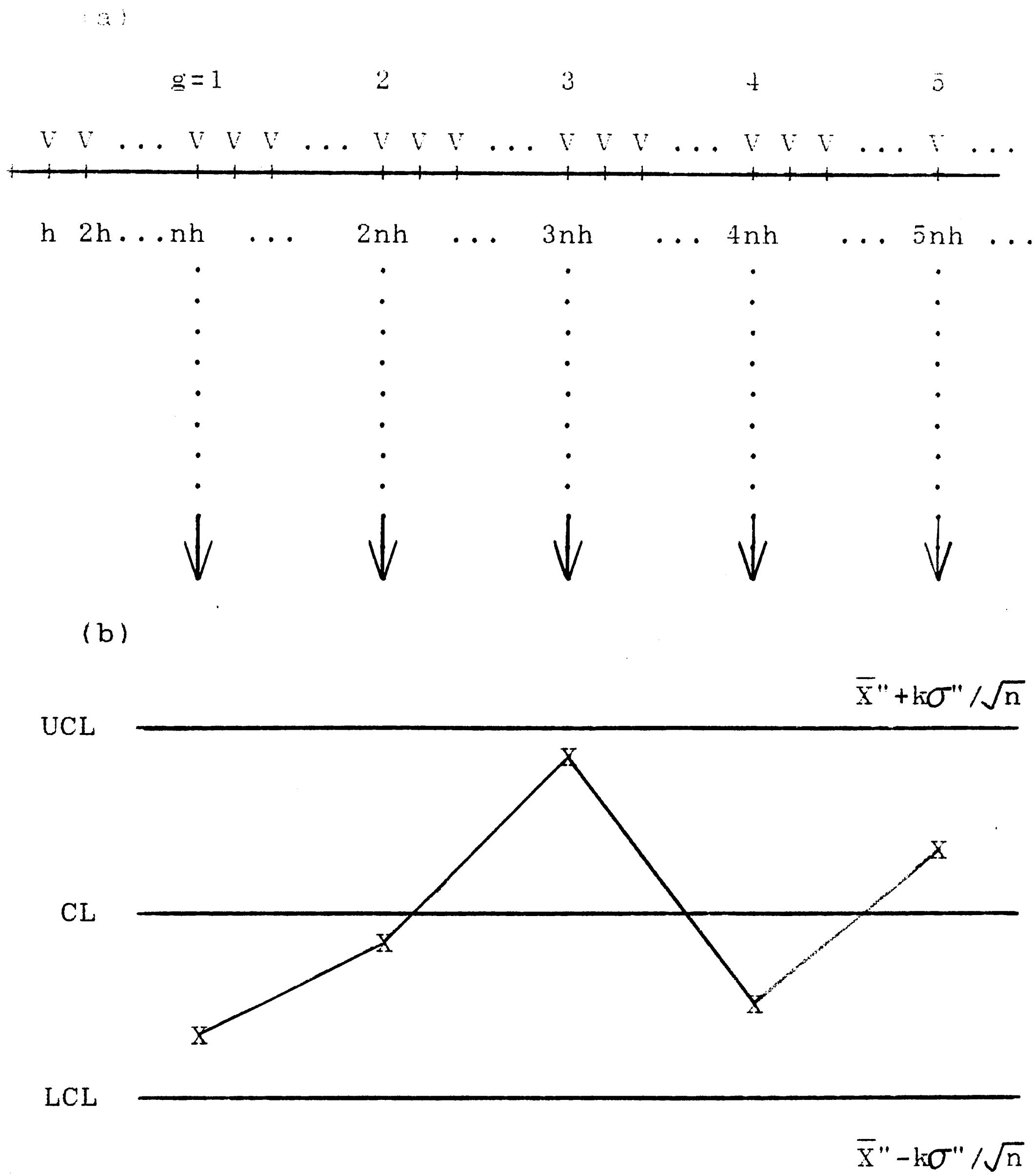


Figure 3.2. (a) Sampling for an X-bar Chart,
(b) Plotting on an X-bar Chart

(b) The average time that the process is OOC before a subgroup is completed which will fall outside the control limits is denoted as AVGOOCT and is derived as follows.

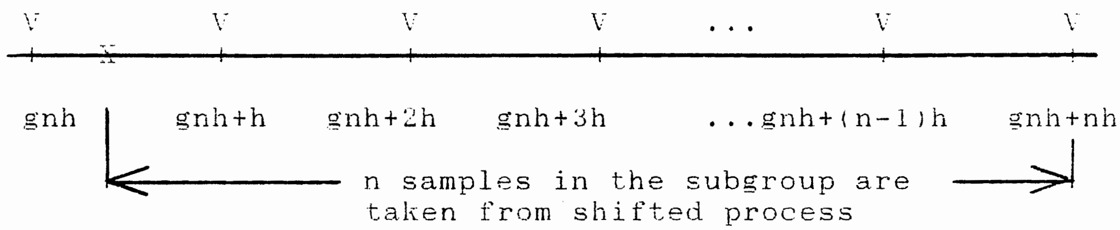
In any time interval, the process has a chance of shifting to the OOC state. The probability of a shift to the OOC condition in the mth interval is

$$\begin{aligned} & P(\text{ shift between } mh \text{ \& } mh+h \text{ }) \\ & = P(\; mh < T \leq (m+1)h \;) \\ & = \int_{mh}^{(m+1)h} \lambda e^{-\lambda t}\, dt \\ & = e^{-\lambda mh} - e^{-\lambda(m+1)h} \quad = e^{-\lambda mh}\,(\,1 - e^{-\lambda h}\,). \end{aligned}$$

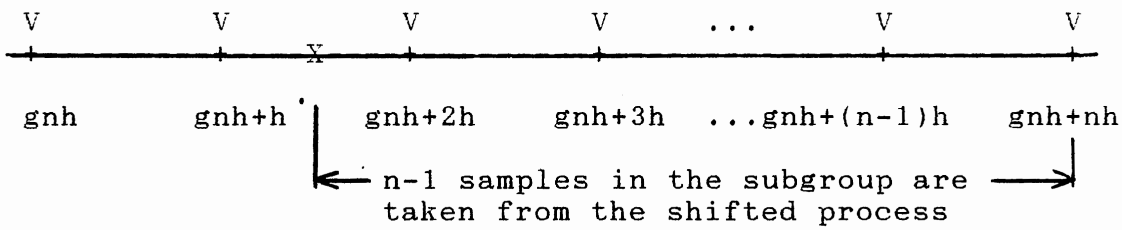
Since the subgroup size of an X-bar chart is n and a sample of size 1 is taken every h hours, it takes nh hours to collect the samples to have a point plotted on the X-bar chart. That is, every nh hours there will be a point plotted on the X-bar chart. The shift can occur any time within this nh hours, so the chance of shifting to the OOC condition can be grouped into n categories listed as follows:

(1) A shift occurs in the time interval immediately after g subgroups have been taken as shown in Figure 3.3(a). The probability that this will occur is

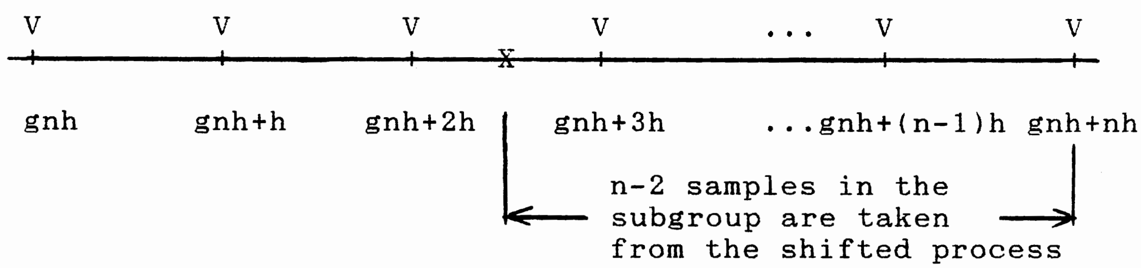
$$\begin{aligned} & P(\text{ shift between } gnh \text{ \& } gnh+h \text{ }) \\ & = P(\; 0<T\leq h \;) + P(\; nh<T\leq nh+h \;) + P(\; 2nh<T\leq 2nh+h \;) + \ldots \\ & = 1 - e^{-\lambda h} + e^{-\lambda nh} - e^{-\lambda(n-1)h} + e^{-\lambda 2nh} - e^{-\lambda(2n+1)h} + \ldots \end{aligned}$$



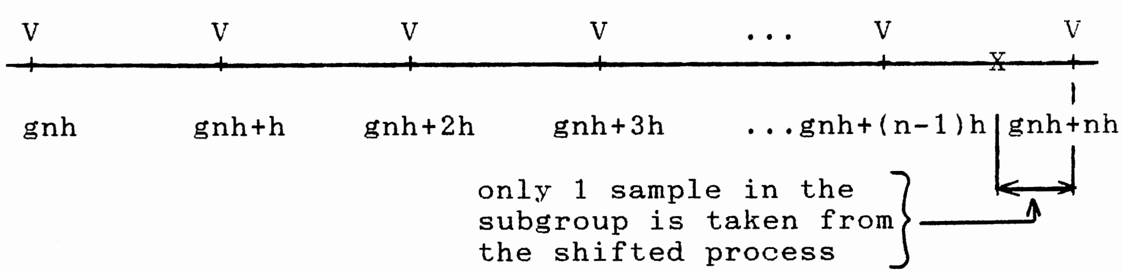
(a) Shift Occurs After g Subgroups Taken



(b) Shift Occurs After g Subgroups and 1 Sample Taken



(c) Shift Occurs After g Subgroups and 2 Samples Taken



(d) Shift Occurs Before g+1 Subgroups Taken

Figure 3.3. Different Conditions Describing Occurrence of a Shift

$$= (1-e^{-\lambda h}) + e^{-\lambda nh} (1-e^{-\lambda h}) + e^{-\lambda 2nh} (1-e^{-\lambda h}) + \ldots$$

$$= (1-e^{-\lambda h}) (1 + e^{-\lambda nh} + e^{-\lambda 2nh} + e^{-\lambda 3nh} + e^{-\lambda 4nh} + \ldots)$$

$$= (1 - e^{-\lambda h}) / (1 - e^{-\lambda nh}).$$

(2) A shift occurs in the time interval after g subgroups and 1 sample taken as shown in Figure 3.3(b). The probability that this will occur is

$$P(\text{shift between } gnh+h \text{ \& } gnh+2h)$$

$$= P(h<T\leq 2h) + P(nh+h<T\leq nh+2h) + P(2nh+h<T\leq 2nh+2h) +\ldots$$

$$= e^{-\lambda h} -e^{-\lambda 2h} +e^{-\lambda(n+1)h} -e^{-\lambda(n+2)h} +e^{-\lambda(2n+1)h} -e^{-\lambda(2n+2)h} +\ldots$$

$$= e^{-\lambda h} \{(1-e^{-\lambda h}) + e^{-\lambda nh} (1-e^{-\lambda h}) + e^{-\lambda 2nh} (1-e^{-\lambda h}) + \ldots\}$$

$$= e^{-\lambda h} (1-e^{-\lambda h}) (1 + e^{-\lambda nh} + e^{-\lambda 2nh} + e^{-\lambda 3nh} + e^{-\lambda 4nh} + \ldots)$$

$$= e^{-\lambda h} (1 - e^{-\lambda h}) / (1 - e^{-\lambda nh}).$$

(3) A shift occurs in the time interval after g subgroups and 2 samples taken as shown in Figure 3.3(c). The probability that this will occur is

$$P(\text{shift between } gnh+2h \text{ \& } gnh+3h)$$

$$= P(2h<T\leq 3h) +P(nh+2h<T\leq nh+3h) +P(2nh+2h<T\leq 2nh+3h)+\ldots$$

$$= e^{-\lambda 2h} -e^{-\lambda 3h} +e^{-\lambda(n+2)h} -e^{-\lambda(n+3)h} +e^{-\lambda(2n+2)h} -e^{-\lambda(2n+3)h} +\ldots$$

$$= e^{-\lambda 2h} \{(1 - e^{-\lambda h}) + e^{-\lambda nh} (1-e^{-\lambda h}) + e^{-\lambda 2nh} (1 -e^{-\lambda h})+\ldots\}$$

$$= e^{-\lambda 2h} (1-e^{-\lambda h}) (1 + e^{-\lambda nh} + e^{-\lambda 2nh} + e^{-\lambda 3nh} + e^{-\lambda 4nh} + \ldots)$$

$$= e^{-\lambda 2h} (1 - e^{-\lambda h}) / (1 - e^{-\lambda nh}).$$

.
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.

(n) A shift occurs in the time interval after g subgroups and n-1 samples are taken, i.e. just before g+1 subgroups are taken, as shown in Figure 3.3(d). The probability that this will occurs is

$$
\begin{aligned}
& P(\text{ shift between } gnh+(n-1)h \;\&\; gnh+nh\ ) \\
&= P(\ nh-h<T\leq nh\ ) + P(\ 2nh-h<T\leq 2nh\ ) + P(\ 3nh-h<T\leq 3nh\ ) + \dots \\
&= e^{-\lambda(n-1)h} - e^{-\lambda nh} + e^{-\lambda(2n-1)h} - e^{-\lambda 2nh} + e^{-\lambda(3n-1)h} - e^{-\lambda 3nh} + \dots \\
&= e^{-\lambda(n-1)h}\{(\ 1-e^{-\lambda h}\ ) + e^{-\lambda nh}(\ 1-e^{-\lambda h}\ ) + e^{-\lambda 2nh}(\ 1-e^{-\lambda h}\ ) + \dots\} \\
&= e^{-\lambda(n-1)h}(\ 1-e^{-\lambda h}\ )(1 + e^{-\lambda nh} + e^{-\lambda 2nh} + e^{-\lambda 3nh} + e^{-\lambda 4nh} + \dots) \\
&= e^{-\lambda(n-1)h}(\ 1 - e^{-\lambda h}\ ) / (\ 1 - e^{-\lambda nh}\ ).
\end{aligned}
$$

When a shift occurs after g subgroups are taken, the g+1st subgroup will have all of the samples taken from the process operating at mean $\bar{\bar{X}}'' + \delta\sigma''$, and the probability that an OOC condition will be detected is

$$
\begin{aligned}
P_0 &= 1 - \Phi(\ k - \delta\sqrt{n}\ ) + \Phi(\ -k - \delta\sqrt{n}\ ) \\
&= P.
\end{aligned}
$$

When a shift occurs after g subgroups and j samples have been taken, the g+1st subgroup will have j samples taken from a process operating at mean $\bar{\bar{X}}''$ and n-j samples from the shifted process. It is known that the sum of normally distributed independent random variables is normally distributed with the mean equal to the sum of the component means. The probability of an OOC condition being detected when j samples of size 1 are taken from the process while operating at mean $\bar{X}''$ and n-j samples of size 1 are taken from the process while operating at mean $\bar{X}''+\delta\sigma''$ is

$$P_j = 1 - \int_{\bar{X}''-k\sigma_{\bar{x}}}^{\bar{X}''+k\sigma_{\bar{x}}} \frac{1}{\sigma_{\bar{x}}\sqrt{2\pi}}\, e^{-((z-\mu)^2/2\sigma_{\bar{x}}^2)}\, dz$$

$$= 1 - \Phi(\ k-(n-j)\delta/\sqrt{n}\ ) + \Phi(\ -k-(n-j)\delta/\sqrt{n}\ ),$$

where

$$\mu = \frac{j\bar{X}'' + (n-j)\ (\ \bar{X}'' + \delta\sqrt{n}\,\sigma_{\bar{x}}\ )}{n}$$

$$= \bar{X}'' + (n-j)\,\delta\,\sigma_{\bar{x}}\ /\ \sqrt{n}$$

$$= \bar{X}'' + (n-j)\,\delta\,\sigma''\ /\ n.$$

The probability that the first subgroup following a shift will detect a process change can be calculated as follows:

$$P' = P_0\, P(\text{shift between } gnh \ \&\ gnh+h)$$

$$+ P_1\, P(\text{shift between } gnh+h \ \&\ gnh+2h)$$

$$+ P_2\, P(\text{ shift between } gnh+2h \ \&\ gnh+3h)$$

$$+ \ldots$$

$$+ P_{n-1}\, P(\text{ shift between } gnh+(n-1)h \ \&\ gnh+nh\ )$$

$$= \sum_{j=0}^{n-1} P_j\, P(\text{ shift between } gnh+jh \text{ and } gnh+(j+1)h)$$

$$= \sum_{j=0}^{n-1} \left(\frac{1- e^{-\lambda h}}{1- e^{-\lambda nh}} \right) e^{-\lambda jh} P_j .$$

The probability that the first subgroup following a shift does not detect the process change is Q' = 1 - P'.

After the occurrence of the assignable cause, the probability that it will be detected right on the rth subgroup taken after the shift is

$$P' \qquad \text{when} \quad r = 1,$$

and

$$Q'Q^{r-2} P \qquad \text{when} \quad r \geq 2.$$

Therefore, the expected number of subgroups taken between the time the process shifts out-of-control and subgroup is completed before a shift in the process is detected is

$$1P' + 2Q'P + 3Q'QP + 4Q'Q^2 P + \ldots + r\, Q'Q^{r-2} P + \ldots$$

$$= P' + Q'P\,(2 + 3Q + 4Q^2 + 5Q^3 + \ldots + rQ^{r-2} + \ldots)$$

$$= P' + Q'P\,(\,1 / (\,1-Q\,)^2\,)\,/Q - Q'P / Q$$

$$= P' + Q'/\,PQ \quad - Q'P / Q$$

The average time of occurrence within an interval between the gth and g+1st subgroups, given an occurrence of the shift in the interval between these subgroups, is

$$\frac{\int_{gnh}^{(g+1)nh} e^{-\lambda t}\lambda(\ t-gnh\ )\ dt}{\int_{gnh}^{(g+1)nh} e^{-\lambda t}\lambda\ dt} = \frac{e^{-\lambda gnh}\int_{0}^{nh} e^{-\lambda T}\lambda T\ dT}{e^{-\lambda gnh}\int_{0}^{nh} e^{-\lambda T}\lambda\ dT}$$

$$= \frac{1-(\ 1+\lambda nh\ )\ e^{-\lambda nh}}{\lambda(\ 1-e^{-\lambda nh})}$$

So, the average time that the process is OOC before a subgroup is completed which will fall outside the control limits is

$$AVGOOCT = nh(\ P'+ Q'/PQ - Q'P/Q\ )$$
$$-\ (\ 1-(\ 1+\lambda nh\ )e^{-\lambda nh})/(\lambda(1-e^{-\lambda nh})).$$

(c) The average sampling, inspecting, evaluating, and plotting time for each sample is e, which is therefore the delay in plotting a subgroup point on the X-bar chart.

(d) The average time taken to locate an assignable cause is D.

Therefore, the average cycle length is

$$ACL = 1/\lambda + AVGOOCT + e + D.$$

The proportion of time a process will be in-control is

$$\beta = (\ 1/\lambda\ )\ /\ ACL,$$

and the proportion of time it will be out-of-control is

$$\gamma = (\ AVGOOCT + e + D\ )\ /\ ACL.$$

Cost Formulation

Based upon the above derivation of average cycle length, formulation of the process average hourly net income is developed as follows:

$$\begin{pmatrix}\text{Process average}\\\text{hourly}\\\text{net income}\end{pmatrix} = \overset{(a)}{\begin{pmatrix}\text{Average hourly}\\\text{in-control}\\\text{income}\end{pmatrix}} + \overset{(b)}{\begin{pmatrix}\text{Average}\\\text{hourly OOC}\\\text{income}\end{pmatrix}}$$

$$- \overset{(c)}{\begin{pmatrix}\text{Average}\\\text{hourly false}\\\text{alarm cost}\end{pmatrix}} - \overset{(d)}{\begin{pmatrix}\text{Average hourly}\\\text{cost of finding an}\\\text{assignable cause}\end{pmatrix}}$$

$$- \overset{(e)}{\begin{pmatrix}\text{Average hourly cost}\\\text{of maintaining}\\\text{the control chart}\end{pmatrix}}$$

(a) Average hourly in-control income

$$= \begin{pmatrix}\text{Proportion of the}\\\text{time the process}\\\text{is in-control}\end{pmatrix} \times \begin{pmatrix}\text{Hourly income}\\\text{from in-control}\\\text{process}\end{pmatrix}$$

$$= \beta V_0 .$$

(b) Average hourly OOC income

$$= \begin{pmatrix}\text{Proportion of}\\\text{the time the}\\\text{process is OOC}\end{pmatrix} \times \begin{pmatrix}\text{Hourly income}\\\text{from OOC}\\\text{process}\end{pmatrix}$$

$$= \gamma V_1 .$$

(c) Average hourly false alarm cost

$$= \begin{pmatrix}\text{Expected number}\\\text{of false alarms}\\\text{per hour}\end{pmatrix} \times \begin{pmatrix}\text{Average cost of searching}\\\text{for an assignable cause}\\\text{when a false alarm occurs}\end{pmatrix}$$

The expected number of false alarms before the process goes out of control will be the probability of a false alarm (α) times the expected number of subgroups taken in the period. This is

$$\alpha \sum_{g=0}^{\infty} g \int_{gnh}^{(g+1)nh} \lambda e^{-\lambda t}\, dt$$

$$= \alpha \sum_{g=0}^{\infty} g \left(e^{-\lambda gnh} - e^{-\lambda (g+1)nh} \right)$$

$$= \alpha \left(1- e^{-\lambda nh} \right) \sum_{g=0}^{\infty} g\, e^{-\lambda gnh}$$

$$= \alpha \left(1- e^{-\lambda nh} \right) e^{-\lambda nh} \left(1 + 2e^{-\lambda nh} + 3e^{-\lambda 2nh} + 4e^{-\lambda 3nh} + \ldots \right)$$

$$= \alpha \left(1- e^{-\lambda nh} \right) e^{-\lambda nh} \left(1/ \left(1- e^{-\lambda nh} \right)^2 \right)$$

$$= \alpha\, e^{-\lambda nh} / \left(1- e^{-\lambda nh} \right).$$

So, the expected number of false alarms per hour of operation will be

$$\text{ENFA} = \left(\alpha\, e^{-\lambda nh} / \left(1- e^{-\lambda nh} \right) \right) / \text{ACL}.$$

Thus, the average hourly false alarm cost = ENFA * T.

(d) Average hourly cost of finding the assignable cause is

$$\text{ACFAC} = \begin{pmatrix}\text{Expected number}\\ \text{of real alarms}\\ \text{per hour}\end{pmatrix} \times \begin{pmatrix}\text{Average cost of searching}\\ \text{for an assignable cause}\\ \text{when a real alarm occurs}\end{pmatrix}$$

$$= (1/\text{ACL}) * W.$$

(e) Average hourly cost of maintaining the control chart is

$$\text{HCMCC} = \begin{matrix}\text{Hourly fixed cost per}\\ \text{subgroup for sampling,}\\ \text{inspecting, evaluating,}\\ \text{and plotting}\end{matrix} + \begin{matrix}\text{Hourly variable cost}\\ \text{per unit for sampling,}\\ \text{inspecting, evaluating,}\\ \text{and plotting}\end{matrix}$$

$= b/(nh) + c/h.$

Therefore, process average hourly net income is

$$I = \beta V_0 + \gamma V_1 - ENFA * T - ACFAC - HCMCC.$$

Since $V_1 = V_0 - M$ and $\beta + \gamma = 1$,

then $I = V_0 - \gamma M - ENFA * T - ACFAC - HCMCC = V_0 - L$,

where

$$L = \text{Loss-Cost} = \gamma M + ENFA * T + ACFAC + HCMCC.$$

In this formulation, minimizing the loss-cost L is equivalent to maximizing average hourly net income.

Formulation of an Economically-Based Moving Average Control Chart

A sample of size 1 is taken every h hours and a subgroup size of n is used to calculate the moving average. When the subgroup number g is less than the subgroup size n, a subgroup size of g will be used, and control limits will be $\bar{X}'' + k\sigma''/\sqrt{g}$. The sampling and plotting methods for the moving average chart are illustrated in Figure 3.4.

Average Cycle Length

As illustrated in Figure 3.1, the average cycle length is expressed as follows:

		(a)		(b)
Average cycle length	=	Average in-control time	+	Average time the process is OOC before the detecting subgroup is completed

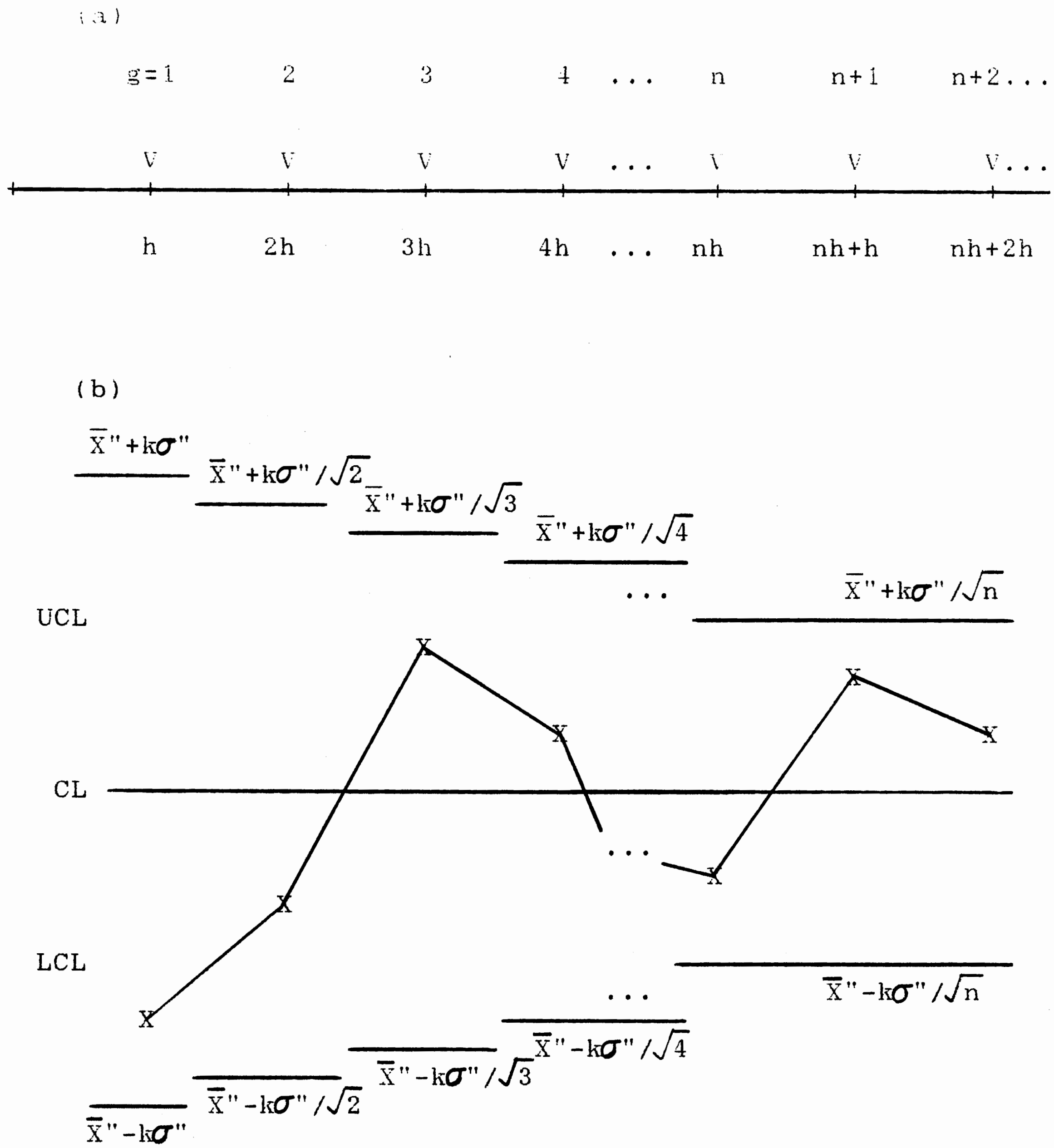

Figure 3.4. (a) Sampling for a Moving Average Chart
(b) Plotting on a Moving Average Chart

	(c)		(d)
+	Time to sample, inspect, evaluate, & plot a subgroup	+	Time to search for the assignable cause

(a) Since the average time for occurrence of the assignable cause is $1/\lambda$, this is the average process in-control time.

(b) The average time that the process is OOC before the detecting subgroup is taken is denoted as AVGOOCT and is derived as follows.

In any time interval, the process has a chance of shifting to the OOC state. The probability of a shift to the OOC condition in the mth interval is

$$P(\text{ shift between } mh \text{ \& } mh+h)$$

$$= P(mh < T \leq (m+1)h)$$

$$= \int_{mh}^{(m+1)h} \lambda e^{-\lambda t} \, dt$$

$$= e^{-\lambda mh} - e^{-\lambda (m+1)h} = e^{-\lambda mh} (1 - e^{-\lambda h}).$$

Since a subgroup size of g is used when the sample number g is less than the subgroup size n, different control limits are used for the first n-1 subgroups. Therefore, the probability of an OOC condition being detected if the shift occurs within the first n-1 subgroups will differ and is derived below. It is known that the average of normally distributed independent random variables is normally distributed with mean equal to the average of the component means and a variance equal to the average of the component variances divided by the number of components. So the probability that an OOC condition will be detected when jth sample is taken and i samples of size 1 are taken from the process while operating at mean $\bar{X}''+\delta\sigma''$ is

$$P_{ji} = 1 - \int_{\bar{X}''-k\sigma''/\sqrt{i+j}}^{\bar{X}''+k\sigma''/\sqrt{i+j}} \frac{e^{-\{(x-\mu_i)/(\sigma''/\sqrt{i+j})\}^2/2}}{\sqrt{2\pi}\,(\sigma''/\sqrt{i+j})}\,dx$$

$$= 1 - \int_{-k-i\delta/\sqrt{i+j}}^{k-i\delta/\sqrt{i+j}} \frac{e^{-z^2/2}}{\sqrt{2\pi}}\,dz$$

$$= 1 - \Phi(k - i\delta/\sqrt{i+j}) + \Phi(-k - i\delta/\sqrt{i+j})$$

and $Q_{ji} = 1 - P_{ji}$,

$$\text{where} \quad \mu_i = \frac{j\bar{X}'' + i(\bar{X}''+\delta\sigma'')}{i + j} = \bar{X}'' + i\delta\sigma''/(i+j).$$

The probability that an OOC condition will be detected when a shift occurs, and the expected number of subgroups taken after occurrence of the shift but before the detecting subgroup is taken, is now derived for the first n-1 subgroups.

(1) If the shift occurs after the j=0 sample, the probability that an OOC condition will be detected on the ith sample is as follows:

$$P_{01} = 1 - \Phi(k-\delta) + \Phi(-k-\delta),$$

$$P_{02} = 1 - \Phi(k-2\delta/\sqrt{2}) + \Phi(-k-2\delta/\sqrt{2}),$$

$$P_{03} = 1 - \Phi(k-3\delta/\sqrt{3}) + \Phi(-k-3\delta/\sqrt{3}),$$

$$\vdots$$

$$P_{0,n-1} = 1 - \Phi(k-(n-1)\delta/\sqrt{n-1}) + \Phi(-k-(n-1)\delta/\sqrt{n-1}),$$

and

$$P_{0,n} = 1 - \Phi(k- n\delta/\sqrt{n}) + \Phi(-k- n\delta/\sqrt{n}) = P.$$

The expected number of subgroups taken after occurrence of the shift but before the detecting subgroup is taken is

$$E_0(t) = 1P_{01} +2P_{02}Q_{01} +3P_{03}Q_{01}Q_{02} + \ldots$$

$$+(n-1)P_{0,n-1}Q_{01}Q_{02}\ldots Q_{0,n-2} +nPQ_{01}\ldots Q_{0,n-1}$$

$$+(n+1)PQQ_{01}\ldots Q_{0,n-1} +(n+2)PQ^2Q_{01}\ldots Q_{0,n-1} + \ldots$$

$$= P_{01} + \sum_{i=2}^{n-1} iP_{0i} \prod_{j=1}^{i-1} Q_{0j} + \prod_{i=1}^{n-1} Q_{0i}\, P\,(n/(1-Q) + Q/(1-Q)^2)$$

$$= P_{01} + \sum_{i=2}^{n-1} iP_{0i} \prod_{j=1}^{i-1} Q_{0j} + \prod_{i=1}^{n-1} Q_{0i}\,(n + Q/ P).$$

(2) If the shift occurs after the j=1 sample, the probability that an OOC condition will be detected on the ith sample is as follows:

$$P_{11} = 1 - \Phi(k-\delta/\sqrt{2}) + \Phi(-k-\delta/\sqrt{2}),$$

$$P_{12} = 1 - \Phi(k-2\delta/\sqrt{3}) + \Phi(-k-2\delta/\sqrt{3}),$$

$$P_{13} = 1 - \Phi(k-3\delta/\sqrt{4}) + \Phi(-k-3\delta/\sqrt{4}),$$

.
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$$P_{1,n-1} = 1 - \Phi(k-(n-1)\delta/\sqrt{n}) + \Phi(-k-(n-1)\delta/\sqrt{n}),$$

and

$$P_{1,n} = 1 - \Phi(k-n\delta/\sqrt{n}) + \Phi(-k-n\delta/\sqrt{n}) = P.$$

The expected number of subgroups taken after occurrence of the shift but before the detecting subgroup is taken is

$$E_1(t) = 1P_{11} + 2P_{12}Q_{11} + 3P_{13}Q_{11}Q_{12} + \ldots$$

$$+(n-1)P_{1,n-1}Q_{11}Q_{12}\ldots Q_{1,n-2} + nPQ_{11}\ldots Q_{1,n-1}$$

$$+(n+1)PQQ_{11}\ldots Q_{1,n-1} + (n+2)PQ^2Q_{11}\ldots Q_{1,n-1} + \ldots$$

$$= P_{11} + \sum_{i=2}^{n-1} iP_{1i} \prod_{j=1}^{i-1} Q_{1j} + \prod_{i=1}^{n-1} Q_{1i} P\,(n/(1-Q) + Q/(1-Q)^2).$$

$$= P_{11} + \sum_{i=2}^{n-1} iP_{1i} \prod_{j=1}^{i-1} Q_{1j} + \prod_{i=1}^{n-1} Q_{1i} (n + Q/P).$$

(3) If the shift occurs after the j=2 sample, the probability that an OOC condition will be detected on the ith sample is as follows:

$$P_{21} = 1 - \Phi(k-\delta/\sqrt{3}) + \Phi(-k-\delta/\sqrt{3}),$$

$$P_{22} = 1 - \Phi(k-2\delta/\sqrt{4}) + \Phi(-k-2\delta/\sqrt{4}),$$

$$P_{23} = 1 - \Phi(k-3\delta/\sqrt{5}) + \Phi(-k-3\delta/\sqrt{5}),$$

.
.
.

$$P_{2,n-1} = 1 - \Phi(k-(n-1)\delta/\sqrt{n}) + \Phi(-k-(n-1)\delta/\sqrt{n}),$$

and

$$P_{2,n} = 1 - \Phi(k-n\delta/\sqrt{n}) + \Phi(-k-n\delta/\sqrt{n}) = P.$$

The expected number of subgroups taken after occurrence of the shift but before the detecting subgroup is taken is

$$E_2(t) = 1P_{21} + 2P_{22}Q_{21} + 3P_{23}Q_{21}Q_{22} + \ldots$$

$$+(n-1)P_{2,n-1}Q_{21}Q_{22}\ldots Q_{2,n-2} + nPQ_{21}\ldots Q_{2,n-1}$$

$$+(n+1)PQQ_{21}\ldots Q_{2,n-1} + (n+2)PQ^2Q_{21}\ldots Q_{2,n-1} + \ldots$$

$$= P_{21} + \sum_{i=2}^{n-1} iP_{2i} \prod_{j=1}^{i-1} Q_{2j} + \prod_{i=1}^{n-1} Q_{2i}\, P\,(n/(1-Q) + Q/(1-Q)^2)$$

$$= P_{21} + \sum_{i=2}^{n-1} iP_{2i} \prod_{j=1}^{i-1} Q_{2j} + \prod_{i=1}^{n-1} Q_{2i}\,(n + Q/P).$$

.
.
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(n-1) If the shift occurs after the j=n-2 sample, the probability that an OOC condition will be detected on the ith sample is as follows:

$$P_{n-2,1} = 1 - \Phi(k-\delta/\sqrt{(n-1)}) + \Phi(-k-\delta/\sqrt{(n-1)}),$$

$$P_{n-2,2} = 1 - \Phi(k-2\delta/\sqrt{n}) + \Phi(-k-2\delta/\sqrt{n}),$$

$$P_{n-2,3} = 1 - \Phi(k-3\delta/\sqrt{n}) + \Phi(-k-3\delta/\sqrt{n}),$$

.
.
.

$$P_{n-2,n-1} = 1 - \Phi(k-(n-1)\delta/\sqrt{n}) + \Phi(-k-(n-1)\delta/\sqrt{n}),$$

and

$$P_{n-2,n} = 1 - \Phi(k-n\delta/\sqrt{n}) + \Phi(-k-n\delta/\sqrt{n}) = P.$$

The expected number of subgroups taken after occurrence of the shift but before the detecting subgroup is taken is

$$E_{n-2}(t) = 1P_{n-2,1} + 2P_{n-2,2}Q_{n-2,1} + 3P_{n-2,3}Q_{n-2,1}Q_{n-2,2} + \ldots$$

$$+ (n-1)P_{n-2,n-1}Q_{n-2,1}\ldots Q_{n-2,n-2} + nPQ_{n-2,1}\ldots Q_{n-2,n-1}$$

$$+ (n+1)PQQ_{n-2,1}\ldots Q_{n-2,n-1} + (n+2)PQ^{2}Q_{n-2,1}\ldots Q_{n-2,n-1} + \ldots$$

$$= P_{n-2,1} + \sum_{i=2}^{n-1} iP_{n-2,i} \prod_{j=1}^{i-1} Q_{n-2,j} + \prod_{i=1}^{n-1} Q_{n-2,i} P(n/(1-Q) + Q/(1-Q)^{2})$$

$$= P_{n-2,1} + \sum_{i=2}^{n-1} iP_{n-2,i} \prod_{j=1}^{i-1} Q_{n-2,j} + \prod_{i=1}^{n-1} Q_{n-2,i} (n + Q/P).$$

(n) If the shift occurs after the j=n-1 sample, the probability that an OOC condition will be detected on the ith sample is as follows:

$$P_{n-1,1} = 1 - \Phi(k-\delta/\sqrt{n}) + \Phi(-k-\delta/\sqrt{n}),$$

$$P_{n-1,2} = 1 - \Phi(k-2\delta/\sqrt{n}) + \Phi(-k-2\delta/\sqrt{n}),$$

$$P_{n-1,3} = 1 - \Phi(k-3\delta/\sqrt{n}) + \Phi(-k-3\delta/\sqrt{n}),$$

$$\vdots$$

$$P_{n-1,n-1} = 1 - \Phi(k-(n-1)\delta/\sqrt{n}) + \Phi(-k-(n-1)\delta/\sqrt{n}),$$

and

$$P_{n-1,n} = 1 - \Phi(k-n\delta/\sqrt{n}) + \Phi(-k-n\delta/\sqrt{n}) = P.$$

The expected number of subgroups taken after occurrence of the shift but before the detecting subgroup is taken is

$$E_{n-1}(t) = 1P_{n-1,1} + 2P_{n-1,2}Q_{n-1,1} + 3P_{n-1,3}Q_{n-1,1}Q_{n-1,2} + \ldots$$

$$+ (n-1)P_{n-1,n-1}Q_{n-1,1}\ldots Q_{n-1,n-2} + nPQ_{n-1,1}\ldots Q_{n-1,n-1}$$

$$+ (n+1)PQQ_{n-1,1}\ldots Q_{n-1,n-1} + (n+2)PQ^2Q_{n-1,1}\ldots Q_{n-1,n-1} + \ldots$$

$$= P_{n-1,1} + \sum_{i=2}^{n-1} iP_{n-1,i}\prod_{j=1}^{i-1} Q_{n-1,j} + \prod_{i=1}^{n-1} Q_{n-1,i}\, P\{n/(1-Q)+Q/(1-Q)^2\}$$

$$= P_{n-1,1} + \sum_{i=2}^{n-1} iP_{n-1,i}\prod_{j=1}^{i-1} Q_{n-1,j} + \prod_{i=1}^{n-1} Q_{n-1,i}\,(n + Q/P).$$

Therefore, the expected number of subgroups taken between the time the process shifts out-of-control and subgroup is completed before a shift in the process is detected is

$$\text{ENSBSD} = \begin{pmatrix}\text{Probability a shift}\\ \text{occurs within the}\\ \text{ith interval}\end{pmatrix} \times \begin{pmatrix}\text{Expected number of} & \text{OOC occurrence}\\ \text{subgroup taken before an} & \text{during the}\\ \text{OOC condition is detected} & \text{ith interval}\end{pmatrix}$$

$$= P(0<t\le h)E_0(t) + P(h<t\le 2h)E_1(t) + P(2h<t\le 3h)E_2(t) + \ldots$$

$$+ P((n-1)h<t\le nh)E_{n-1}(t) + \ldots$$

$$= (1-e^{-\lambda h})E_0(t) + (e^{-\lambda h}-e^{-2\lambda h})E_1(t) + (e^{-2\lambda h}-e^{-3\lambda h})E_2(t)$$

$$+ \dots + P((n-1)h < t \leq \infty)\, E_{n-1}(t)$$

$$= (1-e^{-\lambda h})\{P_{01} + \sum_{i=2}^{n-1} iP_{0i} \prod_{j=1}^{i-1} Q_{0j} + \prod_{i=1}^{n-1} Q_{0i} (n + Q/P)\}$$

$$+ e^{-\lambda h}(1-e^{-\lambda h})\{P_{11} + \sum_{i=2}^{n-1} iP_{1i} \prod_{j=1}^{i-1} Q_{1j} + \prod_{i=1}^{n-1} Q_{1i} (n + Q/P)\}$$

$$+ \dots$$

$$+ e^{-(n-2)\lambda h}(1-e^{-\lambda h})$$

$$\{P_{n-2,1} + \sum_{i=2}^{n-1} iP_{n-2,i} \prod_{j=1}^{i-1} Q_{n-2,j} + \prod_{i=1}^{n-1} Q_{n-2,i} (n + Q/P)\}$$

$$+ e^{-(n-1)\lambda h}$$

$$\{P_{n-1,1} + \sum_{i=2}^{n-1} iP_{n-1,i} \prod_{j=1}^{i-1} Q_{n-1,j} + \prod_{i=1}^{n-1} Q_{n-1,i} (n + Q/P)\}$$

$$= (1-e^{-\lambda h}) \sum_{k=0}^{n-2} e^{-k\lambda h} \{P_{k,1} + \sum_{i=2}^{n-1} i\, P_{k,i} \prod_{j=1}^{i-1} Q_{k,j} + \prod_{i=1}^{n-1} Q_{k,i} (n + Q/P)\}$$

$$+ e^{-(n-1)\lambda h}\{P_{n-1,1} + \sum_{i=2}^{n-1} iP_{n-1,i} \prod_{j=1}^{i-1} Q_{n-1,j} + \prod_{i=1}^{n-1} Q_{n-1,i} (n + Q/P)\}$$

The average time of occurrence of a shift within an interval between the gth and g+1st subgroups, given that the occurrence is in that interval between subgroups, is

$$\frac{\int_{gh}^{(g+1)h} e^{-\lambda t} \lambda (t - gh)\, dt}{\int_{gh}^{(g+1)h} e^{-\lambda t} \lambda\, dt} = \frac{e^{-\lambda gh} \int_0^h e^{-\lambda T} \lambda T\, dT}{e^{-\lambda gh} \int_0^h e^{-\lambda T} \lambda\, dT}$$

$$= \frac{1 - (1+\lambda h)\, e^{-\lambda h}}{\lambda (1 - e^{-\lambda h})}$$

So, the average time the process will be out of control before a subgroup is completed which will fall outside of the control limits is

$$AVGOOCT = h \times ENSBAD - (1 - (1+\lambda h)\, e^{-\lambda h}) / (\lambda (1-e^{-\lambda h})).$$

(c) The average sampling, inspecting, evaluating, and plotting time for each sample is e, which is therefore the delay in plotting a subgroup point on the X-bar chart.

(d) The average time taken to locate an assignable cause is D.

Therefore, the average cycle length is

$$ACL = 1/\lambda + AVGOOCT + e + D.$$

The proportion of the time a process will be in-control is

$$\beta = (1/\lambda) / ACL,$$

and the proportion of time it will be out-of-control is

$$\gamma = (AVGOOCT + e + D) / ACL.$$

<u>Cost Formulation</u>

Based upon the above derivation of average cycle length, formulation of the process average hourly net income is developed as follows:

$$\begin{matrix} & & \text{(a)} & & \text{(b)} \\ \text{Process average hourly net income} & = & \text{Average hourly in-control income} & + & \text{Average hourly OOC income} \end{matrix}$$

$$\begin{matrix} & \text{(c)} & & \text{(d)} \\ - & \text{Average hourly false alarm cost} & - & \text{Average hourly cost of finding an assignable cause} \end{matrix}$$

$$\begin{matrix} & \text{(e)} \\ - & \text{Average hourly cost of maintaining the control chart} \end{matrix}$$

(a) Average hourly in-control income

$$= \begin{pmatrix} \text{Proportion of the} \\ \text{time the process} \\ \text{is in-control} \end{pmatrix} \times \begin{pmatrix} \text{Hourly income} \\ \text{from in-control} \\ \text{process} \end{pmatrix}$$

$$= \beta v_0 .$$

(b) Average hourly OOC income

$$= \begin{pmatrix} \text{Proportion of} \\ \text{the time the} \\ \text{process is OOC} \end{pmatrix} \times \begin{pmatrix} \text{Hourly income} \\ \text{from OOC} \\ \text{process} \end{pmatrix}$$

$$= \gamma v_1 .$$

(c) Average hourly false alarm cost

$$= \begin{pmatrix} \text{Expected number} \\ \text{of false alarms} \\ \text{per hour} \end{pmatrix} \times \begin{pmatrix} \text{Average cost of searching} \\ \text{for an assignable cause} \\ \text{when a false alarm occurs} \end{pmatrix}$$

The expected number of false alarms before the process goes out of control will be the probability of a false alarm (α) times the expected number of samples taken in the period. This is

$$\alpha \sum_{g=0}^{\infty} g \int_{gh}^{(g+1)h} \lambda e^{-\lambda t} dt$$

$$= \alpha \sum_{g=0}^{\infty} g \left(e^{-\lambda gh} - e^{-\lambda(g+1)h} \right)$$

$$= \alpha (1- e^{-\lambda h}) \sum_{g=0}^{\infty} g e^{-\lambda gh}$$

$$= \alpha (1- e^{-\lambda h}) e^{-\lambda h} (1 + 2e^{-\lambda h} + 3e^{-\lambda 2h} + 4e^{-\lambda 3h} +...)$$

$$= \alpha (1- e^{-\lambda h}) e^{-\lambda h} (1/ (1- e^{-\lambda h})^{2})$$

$$= \alpha e^{-\lambda h} / (1- e^{-\lambda h}).$$

So, the expected number of false alarms per hour of operation will be

$$ENFA = (\alpha e^{-\lambda h} / (1- e^{-\lambda h})) / ACL.$$

Thus, the average hourly false alarm cost = ENFA * T.

(d) Average hourly cost of finding the assignable cause is

$$ACFAC = \begin{pmatrix} \text{Expected number} \\ \text{of real alarms} \\ \text{per hour} \end{pmatrix} \times \begin{pmatrix} \text{Average cost of searching} \\ \text{for an assignable cause} \\ \text{when a real alarm occurs} \end{pmatrix}$$

$$= (1/ACL) * W.$$

(e) Average hourly cost of maintaining the control chart is

$$HCMCC = \begin{matrix} \text{Hourly fixed cost per} \\ \text{subgroup for sampling,} \\ \text{inspecting, evaluating,} \\ \text{and plotting} \end{matrix} + \begin{matrix} \text{Hourly variable cost} \\ \text{per unit for sampling,} \\ \text{inspecting, evaluating,} \\ \text{and plotting} \end{matrix}$$

$$= b/h + c/h.$$

Therefore, process average hourly net income is

$$I = \beta V_0 + \gamma V_1 - ENFA * T - ACFAC - HCMCC.$$

Since $V_1 = V_0 - M$ and $\beta + \gamma = 1$,

then $I = V_0 - \gamma M - ENFA * T - ACFAC - HCMCC = V_0 - L$,

where

$$L = \text{Loss-Cost} = \gamma M + ENFA * T + ACFAC + HCMCC.$$

In this formulation, minimizing the loss-cost L is equivalent to maximizing average hourly net income.

Formulation of an Economically-Based Individual Control Chart

A sample of size 1 is taken every h hours and is plotted on the individual control chart. The sampling and plotting methods for the individual control chart are illustrated in Figure 3.5.

Average Cycle Length

As illustrated in Figure 3.1, the average cycle length is expressed as follows:

Average cycle length = (a) Average in-control time + (b) Average time the process is OOC before the detecting subgroup is completed + (c) Time to sample, inspect, evaluate, & plot a subgroup + (d) Time to search for the assignable cause

(a) Since the average time for occurrence of the assignable cause is $1/\lambda$, this is the average process in-control time.

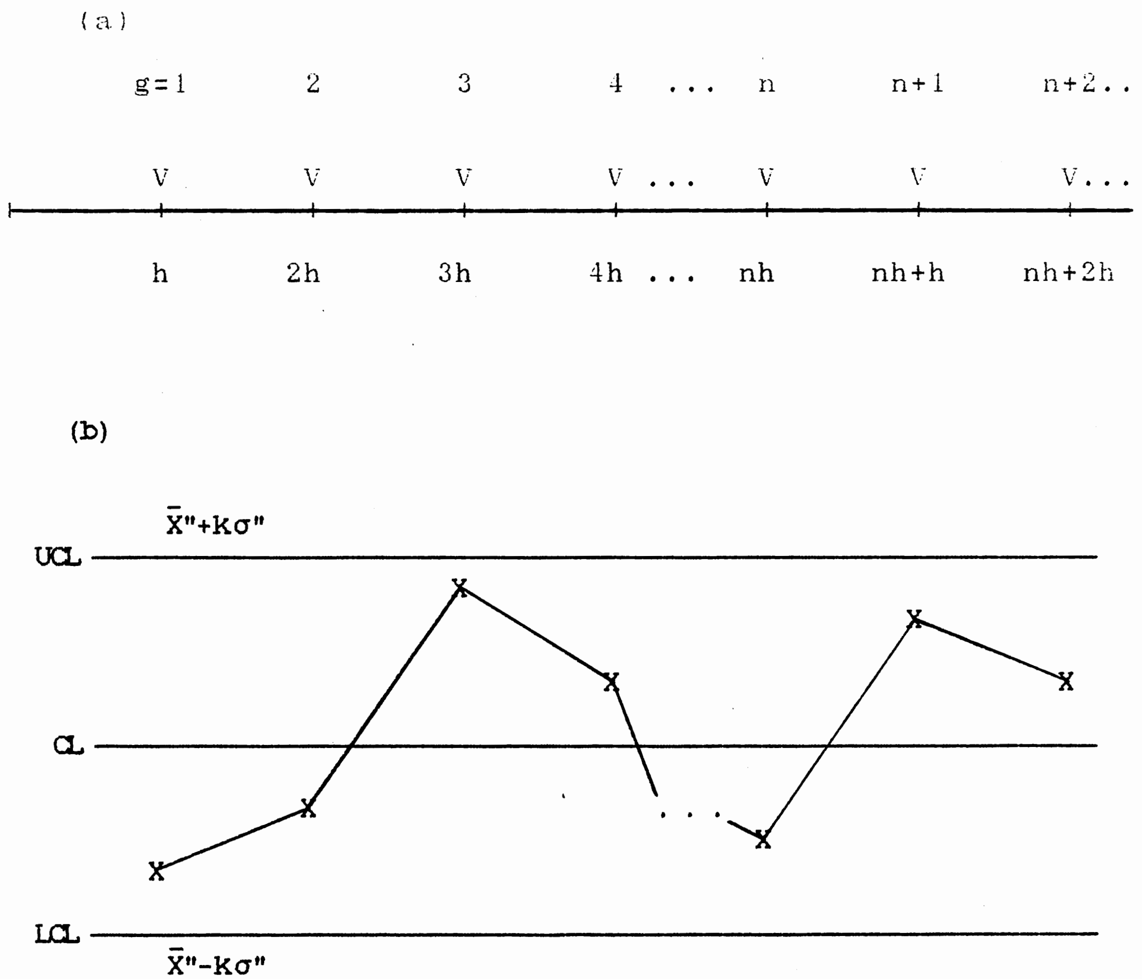

Figure 3.5. (a) Sampling for an Individual Chart
(b) Plotting on an Individual Chart

(b) The average time that the process is OOC before the detecting subgroup is taken is denoted as AVGOOCT and is derived as follows.

After the occurrence of an assignable cause, the probability that it will be detected is

$$P = 1 - \int_{\bar{X}''-k\sigma''}^{\bar{X}''+k\sigma''} \frac{1}{\sigma''\sqrt{2\pi}} e^{-((z-\mu)^2 / 2\sigma''^2)} dz$$

$$= 1 - \Phi(k-\delta) + \Phi(-k-\delta),$$

where $\mu = \bar{X}''+\delta\sigma''$.

The probability of no detection after the process mean shifts is $Q = 1 - P$.

Therefore, the expected number of subgroups taken between the time the process shifts out-of-control and subgroup is completed before a shift in the process is detected is

$$1P + 2QP + 3Q^2 P + 4Q^3 P + \ldots + r Q^{r-1} P + \ldots$$

$$= P (1 + 2Q + 3Q^2 + 4Q^3 + 5Q^4 + \ldots + rQ^{r-1} + \ldots)$$

$$= P (1 / (1-Q)^2)$$

$$= P / P^2 = 1 / P.$$

The average time of occurrence within an interval between the gth and g+1st subgroups, given an occurrence of the shift in the interval between these subgroups, is

$$\frac{\int_{gh}^{(g+1)h} e^{-\lambda t} \lambda (t-gh) \, dt}{\int_{gh}^{(g+1)h} e^{-\lambda t} \lambda \, dt} = \frac{e^{-\lambda gh} \int_0^h e^{-\lambda T} \lambda T \, dT}{e^{-\lambda gh} \int_0^h e^{-\lambda T} \lambda \, dT}$$

$$= \frac{1 - (1+\lambda h) e^{-\lambda h}}{\lambda (1- e^{-\lambda h})}$$

So, the average time that the process is OOC before a subgroup is taken which is destined to fall outside the control limits is

$$\text{AVGOOCT} = h / P - (1 -(1+\lambda h) e^{-\lambda h}) / (\lambda (1-e^{-\lambda h})).$$

(c) The average sampling, inspecting, evaluating, and plotting time for each sample is e, which is therefore the delay in plotting a subgroup point on the X-bar chart.

(d) The average time taken to locate an assignable cause is D.

Therefore, the average cycle length is

$$\text{ACL} = 1/\lambda + \text{AVGOOCT} + e + D.$$

The proportion of time a process will be in-control is

$$\beta = (1/\lambda) / \text{ACL},$$

and the proportion of time it will be out-of-control is

$$\gamma = (\text{AVGOOCT} + e + D) / \text{ACL}.$$

Cost Formulation

Based upon the above derivation of average cycle length, formulation of the process average hourly net income is developed as follows:

$$\underset{\text{net income}}{\text{Process average hourly}} = \underset{\text{income}}{\overset{\text{(a)}}{\text{Average hourly in-control}}} + \underset{\text{income}}{\overset{\text{(b)}}{\text{Average hourly OOC}}}$$

$$- \underset{\text{alarm cost}}{\overset{\text{(c)}}{\text{Average hourly false}}} - \underset{\text{assignable cause}}{\overset{\text{(d)}}{\text{Average hourly cost of finding an}}}$$

$$- \underset{\text{the control chart}}{\overset{\text{(e)}}{\text{Average hourly cost of maintaining}}}$$

(a) Average hourly in-control income

$$= \left(\begin{array}{c}\text{Proportion of the}\\ \text{time the process}\\ \text{is in-control}\end{array}\right) \times \left(\begin{array}{c}\text{Hourly income}\\ \text{from in-control}\\ \text{process}\end{array}\right)$$

$$= \beta v_0 .$$

(b) Average hourly OOC income

$$= \left(\begin{array}{c}\text{Proportion of}\\ \text{the time the}\\ \text{process is OOC}\end{array}\right) \times \left(\begin{array}{c}\text{Hourly income}\\ \text{from OOC}\\ \text{process}\end{array}\right)$$

$$= \gamma v_1 .$$

(c) Average hourly false alarm cost

$$= \left(\begin{array}{c}\text{Expected number}\\ \text{of false alarms}\\ \text{per hour}\end{array}\right) \times \left(\begin{array}{c}\text{Average cost of searching}\\ \text{for an assignable cause}\\ \text{when a false alarm occurs}\end{array}\right)$$

The expected number of false alarms before the process goes out of control will be the probability of a false alarm (α) times the expected number of subgroups taken in the period. This is

$$\alpha \sum_{g=0}^{\infty} g \int_{gh}^{(g+1)h} \lambda e^{-\lambda t} dt$$

$$= \alpha \sum_{g=0}^{\infty} g \left(e^{-\lambda gh} - e^{-\lambda (g+1)h} \right)$$

$$= \alpha (1- e^{-\lambda h}) \sum_{g=0}^{\infty} g e^{-\lambda gh}$$

$$= \alpha (1- e^{-\lambda h}) e^{-\lambda h} (1 + 2e^{-\lambda h} + 3e^{-\lambda 2h} + 4e^{-\lambda 3h} + \ldots)$$

$$= \alpha (1- e^{-\lambda h}) e^{-\lambda h} \{ 1/ (1- e^{-\lambda h})^2 \}$$

$$= \alpha e^{-\lambda h} / (1- e^{-\lambda h}).$$

So, the expected number of false alarms per hour of operation will be

$$\text{ENFA} = (\alpha e^{-\lambda h} / (1- e^{-\lambda h})) / \text{ACL}.$$

Thus, the average hourly false alarm cost = ENFA * T.

(d) Average hourly cost of finding the assignable cause is

$$\text{ACFAC} = \begin{pmatrix} \text{Expected number} \\ \text{of real alarms} \\ \text{per hour} \end{pmatrix} \times \begin{pmatrix} \text{Average cost of searching} \\ \text{for an assignable cause} \\ \text{when a real alarm occurs} \end{pmatrix}$$

$$= (1/\text{ACL}) * W.$$

(e) Average hourly cost of maintaining the control chart is

$$\text{HCMCC} = \begin{array}{c} \text{Hourly fixed cost per} \\ \text{subgroup for sampling,} \\ \text{inspecting, evaluating,} \\ \text{and plotting} \end{array} + \begin{array}{c} \text{Hourly variable cost} \\ \text{per unit for sampling,} \\ \text{inspecting, evaluating,} \\ \text{and plotting} \end{array}$$

$$= b/h + c/h.$$

Therefore, process average hourly net income is

$$I = \beta V_0 + \gamma V_1 - ENFA * T - ACFAC - HCMCC.$$

Since $V_1 = V_0 - M$ and $\beta + \gamma = 1$,

then $I = V_0 - \gamma M - ENFA * T - ACFAC - HCMCC = V_0 - L,$

where

$$L = \text{Loss-Cost} = \gamma M + ENFA * T + ACFAC + HCMCC.$$

In this formulation, minimizing the loss-cost L is equivalent to maximizing average hourly net income.

Optimization Search Method

The goal in optimizing the economically-based X-bar chart, moving average chart, and individual chart is to find the optimal combination of values of decision variables, minimizing the loss-cost L and hence maximizing the average hourly net income of the process. Since L is a very complicated function of the decision variables n, h, and k, there exists no analytically explicit optimal solution. Therefore, multidimensional direct search techniques must be used for optimization of the models.

The direct search technique employed in this research is the Nelder and Mead algorithm (Nelder and Mead, 1965) (Kuester and Mize, 1973), which is simple and robust. This technique is a procedure for finding the minimum of a multivariable unconstrained function. It consists of evaluating a function of n variables at the (n+1) vertices of a general simplex. The simplex is then moved away from the largest function value by replacing the vertex having this value with one located by reflection through the centroid of the other vertices. Extension or contraction is then applied depending on the contours of the response surface. This continues until either the specified number of trials has been used, the function values differ among themselves by less than a specified amount, or the coordinates of the function are changing by less than a specified amount. Derivatives are not required.

In this research, the decision variable n is an integer variable. In order to find an optimal solution with an integer value of n, the following search method is employed.

1. With a starting point and step sizes, do a three variables direct search employing the Nelder and Mead algorithm to find the optimal point of real values n, h, and k.
2. This real number n is truncated to an integer value and treated as a constant, while a two variables direct search is performed on the decision variables h and k. The optimal point (h,k) found with this integer value n is treated as the best solution so far.
3. Do a line search employing the Nelder and Mead algorithm along integer values of n to find the minimum loss-cost. For each value of n considered, optimize the values of h and k. The minimum point found, with its associated integer value n, is the

decision variable set (n, h, k) for the economically-based control chart.

Summary

The economically-based X-bar chart, moving average chart, and individual chart for a continuous flow process are developed in this chapter using Duncan's approach to the economic design of an X-bar control chart for a discrete process. The mathematical development and derivation of net hourly income for the X-bar chart, moving average chart, and individual chart are discussed. The models developed in this chapter consider the characteristics of a continuous flow process, resulting in sampling methods which are different from discrete processes.

Then an optimization procedure is developed to find the decision variables n, h, and k needed to construct the control chart and minimize the loss-cost function. The Nelder and Mead direct search algorithm is employed in this optimization procedure.

CHAPTER IV

USING THE INTERACTIVE COMPUTER PROGRAM

Introduction

This chapter illustrates the use of an interactive computer program which permits easy utilization of the economic design of the X-bar chart, moving average chart, and individual chart presented in the previous chapter. The actual FORTRAN program is well documented and appears in Appendix A. It has been implemented on an IBM 3081D using various time share terminals and an IBM PC.

The entire program is interactive, and the user is prompted for all necessary inputs by the computer. Many typical and/or often-used values of inputs are pre-programmed. These values are presented to the user for either verification or change. Only when a set of inputs has been checked by the program and verified by the user does the program continue.

When several values are to be entered, they only need to be separated by a comma or a space. Integer values are usually entered without a decimal point; however, a decimal may be included. With the prompting and verification feature, the input mechanism is virtually self-explanatory, as long as the user understands the terms being input and their mathematically feasible range.

In the remainder of this chapter, actual interactive output is interspersed with comments and explanations. All computer outputs shown are automatically generated by the computer except for the input values which follow a question mark (?). These question marks remind the user to enter the input values.

Overview

The interactive computer program provides the capability for the following activities:

(1) Design an economically-based X-bar chart.

(2) Design an economically-based moving average chart.

(3) Design an economically-based individual chart.

(4) Evaluation of economically-based X-bar chart.

(5) Evaluation of economically-based moving average chart.

(6) Evaluation of economically-based individual chart.

Designing an economically-based X-bar chart or moving average chart refers to the selection of the subgroup size n, sampling interval h, and width of control limits k needed to maximize the expected income per hour. Designing an economically-based individual chart refers to the selection of the sampling interval h and width of control limits k needed to maximize the expected income per hour.

Evaluation of an economically-based X-bar chart, moving average chart, and individual chart refers to the calculation of loss cost for a specified set of decision variables for the X-bar chart, moving average chart, and individual chart, respectively.

It is noted that in this interactive computer program, the user can choose the option for designing the economically-based Duncan's X-bar chart and/or evaluating a specified Duncan's X-bar chart for a discrete process. These functions are included in this interactive computer program because Duncan's model is used to verify the computer program and search procedure in this research.

The program begins by presenting the main option menu (M.1). The user has entered a "1," indicating a desire to design an economically-based control chart.

```
************************
***    MAIN  MENU    ***
************************

1. DESIGN OF ECONOMICALLY-BASED CONTROL CHARTS,
2. EVALUATION OF ECONOMICALLY-BASED CONTROL CHARTS,
3. EXIT THE PROGRAM.

==> ENTER THE OPTION NUMBER PLEASE!
?
1
```

(M.1)

Design of an Economically-Based Control Chart

After the economically-based design of a control chart is selected, the control chart design option menu (M.2) is presented. A selection of "1" from this menu leads to the design of an economically-based X-bar chart.

```
***************************************************
* CONTROL CHART DESIGN FOR CONTINUOUS FLOW PROCESSES *
***************************************************

1. ECONOMICALLY-BASED DESIGN OF X-BAR CHART,
2. ECONOMICALLY-BASED DESIGN OF MA    CHART,
3. ECONOMICALLY-BASED DESIGN OF I     CHART,
4. DESIGN OF DUNCAN'S  ECONOMIC X-BAR CHART
         ( FOR DISCRETE PROCESS ),
5. RETURN TO MAIN MENU.

==> ENTER THE OPTION NUMBER PLEASE!
?
1
```

(M.2)

Designing an Economically-Based X-bar Chart

In economically-based X-bar chart design, the program prompts the user to enter the shift value, occurrence rate of the assignable cause, and cost and risk parameters. Then the program prints these input data for verification by the user. Only after the user confirms the validity of the inputs does the program continue.

```
==> FOR ECONOMIC X-BAR CHART DESIGN, INPUT VALUES OF
==> DELTA, LAMBDA, M, E, D, T, W, B, C
?
2.0  0.01  100  0.05  2.0  50  2.5  0.5  0.1

** VALUES RECEIVED ARE AS FOLLOWS:
DELTA =      2.0000        LAMBDA =     0.0100
  M   =    100.0000          E    =     0.0500
  D   =      2.0000          T    =    50.0000
  W   =      2.5000          B    =     0.5000
  C   =      0.1000

*** ARE THESE DATA CORRECT? ***
==> PLEASE ENTER 1 = YES, 2 = NO  <<<
?
2

==> FOR ECONOMIC X-BAR CHART DESIGN, INPUT VALUES OF
==> DELTA, LAMBDA, M, E, D, T, W, B, C
?
2.0  0.01  100  0.05  2.0  50  25  0.5  0.1
```

```
** VALUES RECEIVED ARE AS FOLLOWS:
DELTA =      2.0000          LAMBDA =      0.0100
  M   =    100.0000            E    =      0.0500
  D   =      2.0000            T    =     50.0000
  W   =     25.0000            B    =      0.5000
  C   =      0.1000

*** ARE THESE DATA CORRECT? ***
==> PLEASE ENTER 1 = YES, 2 = NO  <<<
?
1
```

When the parameters and cost values have been entered and confirmed, a starting point is suggested by the program for running the Nelder and Mead direct search method, and the user is prompted for acceptance or rejection of this starting point. If the user desires to start from a different point, then a selection of "2" is entered and the program prompts the user for entering a new starting point. Once this selected starting point has been suggested by the program and confirmed by the user, search step sizes of n, h, and k are suggested by the program. Here, the user desires to input different step sizes. The new inputs are prompted for confirmation from the user.

```
*** THE FOLLOWING STARTING POINT IS SUGGESTED
*** FOR ECONOMIC OPTIMIZATION OF X-BAR CHART.
    N =  5    H =    1.00  K =    3.00

 ==>  DO YOU ACCEPT THIS POINT?
 ==>  ENTER 1 = YES, 2 = NO. <<<
 ?
 2

*** INPUT THE STARTING POINT YOU WANT ***
==> KEY IN THE VALUE FOR N,H,K
?
4  2  2

*** STARTING POINT SELECTED IS N =    4,  H =   2.00,AND  K =   2.00
*** ARE THEY CORRECT?    ==> ENTER 1 = YES, 2 = NO
?
1
```

```
*** THE FOLLOWING STEP SIZES ARE SUGGESTED
*** FOR ECONOMIC OPTIMIZATION OF X-BAR CHART.
    N =  1.00   H =   0.50  K =   0.50

 ==>  DO YOU ACCEPT THESE STEP SIZES?
 ==>  ENTER 1 = YES, 2 = NO. <<<
 ?
 2

*** INPUT THE STEP SIZES YOU WANT ***
==> ENTER STEP SIZES OF N,H,K
?
1  1  1

*** STEP SIZES ENTERED ARE N = 1.00,  H =  1.00,AND  K =  1.00
*** ARE THEY CORRECT?   ==> ENTER 1 = YES, 2 = NO
?
1
```

The optimization is performed after the starting point and the step sizes of n, h, and k have been confirmed. The optimal point values, the search iteration for an integer number of n, and the optimal design of economically-based X-bar chart and its associated hourly loss-cost are printed.

```
*** THE OPTIMAL POINT FOUND IS  ***

 N =  5.0466, H =  0.1996, K =  3.1263,  LOSS COST =       4.433005

*** OPTIMIZATION ITERATIONS ***

 I    N     H          K          LOSS COST
---------------------------------------------

 1    5   0.2021    3.0962        4.431999
 2    6   0.1836    3.0923        4.452322
 3    4   0.2434    2.9791        4.451152
===========================================================

*** THE OPTIMAL X-BAR CHART DESIGN IS
         N =    5,    H =    0.20206,     K =   3.09624
*** THE MINIMUM LOSS COST PER HOUR IS      4.431999

===========================================================
```

Designing an Economically-Based Moving Average Chart

A selection of "2" from menu (M.2) leads to the design of an economically-based moving average chart. The interactive procedure and the input parameters generally follow those in designing an economically-based X-bar chart. The format of the resulting listing also follows the one for economically-based X-bar control chart.

```
*****************************************************
* CONTROL CHART DESIGN FOR CONTINUOUS FLOW PROCESSES *
*****************************************************

1. ECONOMICALLY-BASED DESIGN OF X-BAR CHART,
2. ECONOMICALLY-BASED DESIGN OF MA    CHART,
3. ECONOMICALLY-BASED DESIGN OF I     CHART,
4. DESIGN OF DUNCAN'S  ECONOMIC X-BAR CHART
        ( FOR DISCRETE PROCESS ),
5. RETURN TO MAIN MENU.

==> ENTER THE OPTION NUMBER PLEASE!
?
2

==> FOR ECONOMIC MA CHART DESIGN, INPUT VALUES OF
==> DELTA, LAMBDA, M, E, D, T, W, B, C
?
2.0  0.01  100  0.05  2.0  50  25  0.5  0.1

** VALUES RECEIVED ARE AS FOLLOW:
DELTA =      2.0000         LAMBDA =      0.0100
  M   =    100.0000           E    =      0.0500
  D   =      2.0000           T    =     50.0000
  W   =     25.0000           B    =      0.5000
  C   =      0.1000

*** ARE THESE DATA CORRECT? ***
==> PLEASE ENTER 1 = YES, 2 = NO  <<<
?
1

*** THE FOLLOWING STARTING POINT IS SUGGESTED
*** FOR ECONOMIC OPTIMIZATION OF MA CHART.
     N =  5      H =    1.00      K =   3.00

 ==>  DO YOU ACCEPT THIS STARTING POINT?
 ==>  ENTER 1 = YES, 2 = NO. <<<
 ?
 1


*** THE FOLLOWING STEP SIZES ARE SUGGESTED
*** FOR ECONOMIC OPTIMIZATION OF MA CHART.
     N =  1.00   H =   0.50  K =   0.50

 ==>  DO YOU ACCEPT THESE STEP SIZES?
 ==>  ENTER 1 = YES, 2 = NO. <<<
 ?
 1
```

```
*** THE OPTIMAL POINT FOUND IS  ***

 N =  5.0017, H =  0.4860, K =  3.1278,  LOSS COST =        4.887933

*** OPTIMIZATION ITERATIONS ***

 I  N     H          K          LOSS COST

 1  5   0.4699    3.2181         5.125457
 2  6   0.4392    3.1973         5.240192
 3  4   0.4841    3.1632         5.000238
 4  3   0.5391    3.0939         4.895590
 5  2   0.5654    2.9055         4.955134

==========================================================

*** THE OPTIMAL MA CHART DESIGN IS
         N =    3      H =      0.5391       K =    3.0939
*** THE MINIMUM LOSS COST PER HOUR IS        4.895590

==========================================================
```

Designing an Economically-Based Individual Chart

A selection of "3" from menu (M.2) leads to the design of an economically-based individual chart. The interactive procedure and the input parameters are almost the same as those of designing an economically-based X-bar chart. The only difference is that the value of n is not needed when selecting the starting point and step sizes. The format of the resulting listing is very similar to that of economically-based X-bar chart design.

```
*******************************************************
* CONTROL CHART DESIGN FOR CONTINUOUS FLOW PROCESSES *
*******************************************************

1. ECONOMICALLY-BASED DESIGN OF X-BAR CHART,
2. ECONOMICALLY-BASED DESIGN OF MA    CHART,
3. ECONOMICALLY-BASED DESIGN OF I     CHART,
4. DESIGN OF DUNCAN'S  ECONOMIC X-BAR CHART
         ( FOR DISCRETE PROCESS ),
5. RETURN TO MAIN MENU.

==> ENTER THE OPTION NUMBER PLEASE!
?
3
```

```
==> FOR ECONOMIC I CHART DESIGN, INPUT VALUES OF
==> DELTA, LAMBDA, M, E, D, T, W, B, C
?
2.0  0.01  100  0.05  2.0  50  25  0.5  0.1

** VALUES RECEIVED ARE AS FOLLOW:
DELTA =      2.0000         LAMBDA =      0.0100
  M   =    100.0000           E    =      0.0500
  D   =      2.0000           T    =     50.0000
  W   =     25.0000           B    =      0.5000
  C   =      0.1000

*** ARE THESE DATA CORRECT? ***
==> PLEASE ENTER 1 = YES, 2 = NO  <<<
?
1


*** THE FOLLOWING STARTING POINT IS SUGGESTED
*** FOR ECONOMIC OPTIMIZATION OF I CHART.
        H =    1.00    AND  K =    3.00

 ==>  DO YOU ACCEPT THIS STARTING POINT?
 ==>  ENTER 1 = YES, 2 = NO. <<<
 ?
 1


*** THE FOLLOWING STEP SIZES ARE SUGGESTED
*** FOR ECONOMIC OPTIMIZATION OF I CHART.
        H =   0.50  AND  K =   0.50

 ==>  DO YOU ACCEPT THESE STEP SIZES?
 ==>  ENTER 1 = YES, 2 = NO. <<<
 ?
 1
==========================================================

*** THE OPTIMAL INDIVIDUAL CHART DESIGN IS
         H =     0.6580       K =     2.5277
*** THE MINIMUM LOSS COST PER HOUR IS      5.764155

==========================================================
```

Designing an Economically-Based Duncan's X-bar Chart

A selection of "4" from menu (M.2) leads to the design of an economically-based Duncan's X-bar chart for a discrete process. The interactive procedure and the input parameters are the same as those for designing an economically-based X-bar chart for a continuous process. The format of the resulting listing also is the same as the one for an economically-based X-bar control chart. Note again that this design is used for a discrete process, although the heading says the design is for continuous flow processes.

```
*****************************************************
* CONTROL CHART DESIGN FOR CONTINUOUS FLOW PROCESSES *
*****************************************************

1. ECONOMICALLY-BASED DESIGN OF X-BAR CHART,
2. ECONOMICALLY-BASED DESIGN OF MA    CHART,
3. ECONOMICALLY-BASED DESIGN OF I     CHART,
4. DESIGN OF DUNCAN'S  ECONOMIC X-BAR CHART
          ( FOR DISCRETE PROCESS ),
5. RETURN TO MAIN MENU.

==> ENTER THE OPTION NUMBER PLEASE!
?
4

==> FOR DUNCAN'S X-BAR CHART DESIGN, INPUT VALUES
==> OF DELTA, LAMBDA, M, E, D, T, W, B, C
?
2.0  0.01  100  0.05  2.0  50  25  0.5  0.1

** VALUES RECEIVED ARE AS FOLLOWS:
DELTA =      2.0000        LAMBDA =      0.0100
  M   =    100.0000          E    =      0.0500
  D   =      2.0000          T    =     50.0000
  W   =     25.0000          B    =      0.5000
  C   =      0.1000

*** ARE THESE DATA CORRECT? ***
==> PLEASE ENTER 1 = YES, 2 = NO  <<<
?
1


*** THE FOLLOWING STARTING POINT IS SUGGESTED FOR
*** ECONOMIC OPTIMIZATION OF DUNCAN'S X-BAR CHART
     N =  5   H =   1.00  K =   3.00

 ==>  DO YOU ACCEPT THIS STARTING POINT?
 ==>  ENTER 1 = YES, 2 = NO. <<<
 ?
 1


*** THE FOLLOWING STEP SIZES ARE SUGGESTED FOR
*** ECONOMIC OPTIMIZATION OF DUNCAN'S X-BAR CHART

     N =  1.00   H =   0.50  K =   0.50

 ==>  DO YOU ACCEPT THESE STEP SIZES?
 ==>  ENTER 1 = YES, 2 = NO. <<<
 ?
 1

*** THE OPTIMAL POINT FOUND IS   ***

 N =  4.8188, H =  1.4222, K =  3.0357,  LOSS COST =       4.012063

*** OPTIMIZATION ITERATIONS ***

 I    N     H         K          LOSS COST
--------------------------------------------
```

```
 1    4   1.2945    2.9651          4.036273
 2    5   1.3961    3.0432          4.013794
 3    6   1.4869    3.2488          4.047831
==========================================================
*** THE OPTIMAL DUNCAN'S X-BAR CHART DESIGN IS
        N =    5      H =     1.39608      K =   3.04322
*** THE MINIMUM LOSS COST PER HOUR IS       4.013794
==========================================================
```

A selection of "5" from control chart design menu (M.2) returns the control to the main menu (M.1).

```
******************************************************
* CONTROL CHART DESIGN FOR CONTINUOUS FLOW PROCESSES *
******************************************************

1. ECONOMICALLY-BASED DESIGN OF X-BAR CHART,
2. ECONOMICALLY-BASED DESIGN OF MA    CHART,
3. ECONOMICALLY-BASED DESIGN OF I     CHART,
4. DESIGN OF DUNCAN'S  ECONOMIC X-BAR CHART
         ( FOR DISCRETE PROCESS ),
5. RETURN TO MAIN MENU.

==> ENTER THE OPTION NUMBER PLEASE!
?
5

        ***********************
        ***    MAIN  MENU   ***
        ***********************

1. DESIGN OF ECONOMICALLY-BASED CONTROL CHARTS,
2. EVALUATION OF ECONOMICALLY-BASED CONTROL CHARTS,
3. EXIT THE PROGRAM.

==> ENTER THE OPTION NUMBER PLEASE!
?
2
```

Evaluation of an Economically-Based Control Chart

A selection of "2" from the main menu (M.1) indicates a desire to evaluate an economically-based X-bar chart, moving average chart, individual chart, or Duncan's X-bar chart. Once accessed, the control chart evaluation menu (M.3) is presented. A selection of "1" from this menu leads to the evaluation of an economically-based X-bar chart.

```
*********************************************************
* CONTROL CHART EVALUATION FOR CONTINUOUS FLOW PROCESSES *
*********************************************************

1. EVALUATION OF ECONOMICALLY-BASED X-BAR CHART,
2. EVALUATION OF ECONOMICALLY-BASED MA    CHART,
3. EVALUATION OF ECONOMICALLY-BASED I     CHART,
4. EVALUATION OF DUNCAN'S  X-BAR CONTROL  CHART                (M.3)
          ( FOR DISCRETE PROCESS ),
5. RETURN TO MAIN MENU.

==> ENTER THE OPTION NUMBER PLEASE!
?
1
```

Evaluation of an Economically-Based X-bar Chart

Once in the evaluation of an economically-based X-bar chart, the program prompts the user to enter the values of the shift amount, mean occurrence rate of the shift, and relevant cost and risk parameters. After those values have been entered, the program prints out the values for confirmation by the user.

```
==> FOR EVALUATION OF ECONOMIC X-BAR CHART, INPUT
==> VALUES OF DELTA, LAMBDA, M, E, D, T, W, B, C
?
2.0  0.01  100  0.05  2.0  50  25  0.5  0.1

** VALUES RECEIVED ARE AS FOLLOWS:
DELTA =      2.0000          LAMBDA =      0.0100
  M   =    100.0000            E    =      0.0500
  D   =      2.0000            T    =     50.0000
  W   =     25.0000            B    =      0.5000
  C   =      0.1000

*** ARE THESE DATA CORRECT? ***
==> PLEASE ENTER 1 = YES, 2 = NO  <<<
?
1
```

Upon verification of the values, a message to enter the specific design of the economically-based X-bar chart is printed. The design values of n, h, and k are then entered by user. When entered values of n, h, and k are printed by the program and verified by the user, the evaluation of an economically-based X-bar chart is performed and the resulting loss-cost is printed.

```
*** FOR THE SPECIFIC X-BAR CHART TO EVALUATE **
==> INPUT THE VALUES OF N, H, AND K
?
5  0.1792  3.0782

*** VALUES ENTERED ARE N =  5.00     H =  0.1972     K =  3.0782
*** ARE THEY CORRECT?    ==> ENTER 1 = YES, 2 = NO
?
1

==========================================================

*** THE SPECIFIED X-BAR CHART DESIGN IS
         N =   5.00,    H =    0.19720,    K =   3.07820
*** AND THE HOURLY LOSS COST FOR THIS DESIGN IS   4.43298

==========================================================
```

Evaluation of an Economically-Based

Moving Average Chart

A selection of "2" from menu (M.3) leads to an evaluation of an economically-based moving average chart. The interactive procedure and evaluation output are the same as those used in the evaluation of an economically-based X-bar chart and are illustrated as follows:

```
**********************************************************
* CONTROL CHART EVALUATION FOR CONTINUOUS FLOW PROCESSES *
**********************************************************
```

```
1. EVALUATION OF ECONOMICALLY-BASED X-BAR CHART,
2. EVALUATION OF ECONOMICALLY-BASED MA    CHART,
3. EVALUATION OF ECONOMICALLY-BASED I     CHART,
4. EVALUATION OF DUNCAN'S  X-BAR CONTROL  CHART
          ( FOR DISCRETE PROCESS ),
5. RETURN TO MAIN MENU.

==> ENTER THE OPTION NUMBER PLEASE!
?
2

==> FOR EVALUATION OF ECONOMIC MA CHART, INPUT
==> VALUES OF DELTA, LAMBDA, M, E, D, T, W, B, C
?
2.0  0.01  100  0.05  2.0  50  25  0.5  0.1


** VALUES RECEIVED ARE AS FOLLOWS:
DELTA =      2.0000         LAMBDA =     0.0100
  M   =    100.0000           E    =     0.0500
  D   =      2.0000           T    =    50.0000
  W   =     25.0000           B    =     0.5000
  C   =      0.1000

*** ARE THESE DATA CORRECT? ***
==> PLEASE ENTER 1 = YES, 2 = NO  <<<
?
1

*** FOR THE SPECIFIC MA CHART TO EVALUATE **
==> INPUT THE VALUES OF N, H, AND K
?
3  0.5  3

*** VALUES ENTERED ARE N =  3.00     H =  0.5000     K =  3.0000
*** ARE THEY CORRECT?    ==> ENTER 1 = YES, 2 = NO
?
2

*** FOR THE SPECIFIC MA CHART TO EVALUATE **
==> INPUT THE VALUES OF N, H, AND K
?
3  0.5273  3.1138

*** VALUES ENTERED ARE N =  3.00     H =  0.5273     K =  3.1138
*** ARE THEY CORRECT?    ==> ENTER 1 = YES, 2 = NO

==========================================================

*** THE SPECIFIED MA   CHART DESIGN IS

         N =   3.00,    H =    0.52730,    K =   3.11380
*** AND THE HOURLY LOSS COST FOR THIS DESIGN IS      4.654410

==========================================================
```

Evaluation of an Economically-Based Individual Chart

A selection of "3" from menu (M.3) leads to an evaluation of an economically-based individual chart. The interactive procedure and evaluation output are very similar to those used in the evaluation of an economically-based X-bar chart. The only difference is that the value of n is not needed in this scheme. The interactive procedure and the resulting listing are shown below.

```
*********************************************************
* CONTROL CHART EVALUATION FOR CONTINUOUS FLOW PROCESSES *
*********************************************************

1. EVALUATION OF ECONOMICALLY-BASED X-BAR CHART,
2. EVALUATION OF ECONOMICALLY-BASED MA    CHART,
3. EVALUATION OF ECONOMICALLY-BASED I     CHART,
4. EVALUATION OF DUNCAN'S  X-BAR CONTROL  CHART
         ( FOR DISCRETE PROCESS ),
5. RETURN TO MAIN MENU.

==> ENTER THE OPTION NUMBER PLEASE!
?
3

==> FOR EVALUATION OF ECONOMIC I  CHART, INPUT
==> VALUES OF DELTA, LAMBDA, M, E, D, T, W, B, C
?
2.0  0.01  100  0.05  2.0  50  25  0.5  0.1

** VALUES RECEIVED ARE AS FOLLOWS:
DELTA =      2.0000        LAMBDA =      0.0100
  M   =    100.0000          E    =      0.0500
  D   =      2.0000          T    =     50.0000
  W   =     25.0000          B    =      0.5000
  C  =     0.1000

*** ARE THESE DATA CORRECT? ***
==> PLEASE ENTER 1 = YES, 2 = NO  <<<
?
1

*** FOR THE SPECIFIC I  CHART TO EVALUATE **
==> INPUT THE VALUES OF  H AND K
?
0.658  2.5277

*** VALUES ENTERED ARE H =  0.6580     K =  2.5277
*** ARE THEY CORRECT?   ==> ENTER 1 = YES, 2 = NO
?
1

=========================================================

*** THE SPECIFIED INDIVIDUAL CHART DESIGN IS
         H =    0.65800,     K =   2.52770
*** AND THE HOURLY LOSS COST FOR THIS DESIGN IS      5.764154

=========================================================
```

Evaluation of an Economically-Based Duncan's X-bar Chart

A selection of "4" from menu (M.3) leads to the evaluation of an economically-based Duncan's X-bar chart for a discrete process. The interactive procedure and evaluation output are the same as those in the evaluation of an economically-based X-bar chart and are illustrated as follows:

```
********************************************************
* CONTROL CHART EVALUATION FOR CONTINUOUS FLOW PROCESSES *
********************************************************

1. EVALUATION OF ECONOMICALLY-BASED X-BAR CHART,
2. EVALUATION OF ECONOMICALLY-BASED MA    CHART,
3. EVALUATION OF ECONOMICALLY-BASED I     CHART,
4. EVALUATION OF DUNCAN'S  X-BAR CONTROL  CHART
          ( FOR DISCRETE PROCESS ),
5. RETURN TO MAIN MENU.

==> ENTER THE OPTION NUMBER PLEASE!
?
4

==> FOR EVALUATION OF DUNCAN'S X-BAR CHART, INPUT
==> VALUES OF DELTA, LAMBDA, M, E, D, T, W, B, C
?
2.0  0.01  100  0.05  2.0  50  25  0.5  0.1

** VALUES RECEIVED ARE AS FOLLOWS:
DELTA =      2.0000        LAMBDA =      0.0100
  M   =    100.0000          E    =      0.0500
  D   =      2.0000          T    =     50.0000
  W   =     25.0000          B    =      0.5000
  C   =      0.1000

*** ARE THESE DATA CORRECT? ***
==> PLEASE ENTER 1 = YES, 2 = NO  <<<
?
1

*** FOR THE SPECIFIC DUNCAN'S X-BAR CHART TO EVALUATE   **
==> INPUT THE VALUES OF N, H, AND K
?
5  1.419  3.095

*** VALUES ENTERED ARE N =  5.00     H =  1.4190     K =  3.0950
*** ARE THEY CORRECT?    ==> ENTER 1 = YES, 2 = NO
?
1

========================================================

*** THE SPECIFIED DUNCAN'S X-BAR CHART DESIGN IS
         N =   5.00,    H =     1.41901,     K =   3.09499
*** AND THE HOURLY LOSS COST FOR THIS DESIGN IS      4.013004

========================================================
```

Option "5" of the evaluation of an economically-based control chart menu (M.3) is employed to return control to the main menu (M.1). In the main menu, a selection of "3" terminates the execution of the interactive computer program.

```
*********************************************************
* CONTROL CHART EVALUATION FOR CONTINUOUS FLOW PROCESSES *
*********************************************************

1. EVALUATION OF ECONOMICALLY-BASED X-BAR CHART,
2. EVALUATION OF ECONOMICALLY-BASED MA    CHART,
3. EVALUATION OF ECONOMICALLY-BASED I     CHART,
4. EVALUATION OF DUNCAN'S  X-BAR CONTROL  CHART
          ( FOR DISCRETE PROCESS ),
5. RETURN TO MAIN MENU.

==> ENTER THE OPTION NUMBER PLEASE!
?
5

        ************************
        ***    MAIN  MENU    ***
        ************************

1. DESIGN OF ECONOMICALLY-BASED CONTROL CHARTS,
2. EVALUATION OF ECONOMICALLY-BASED CONTROL CHARTS,
3. EXIT THE PROGRAM.

==> ENTER THE OPTION NUMBER PLEASE!
?
3
```

Summary

Nearly every feature of the interactive computer program of this research has been illustrated in this chapter. Several examples are given which describe the capabilities of this program. The interactive feature and its convenience make this computer program a useful tool for designing and evaluating economically-based control charts in a continuous flow process environment.

CHAPTER V

RESULTS, COMPARISON AND ANALYSIS

Introduction

This chapter is used to provide an economic comparison and analysis among the X-bar chart, moving average chart, and individual chart when they are used to monitor continuous flow processes.

The economic formulations of an X-bar chart, moving average chart, and individual chart are developed in Chapter III, and their optimization and evaluation are covered in Chapter IV. In order to economically compare the optimal design of an X-bar chart, moving average chart, and individual chart, 25 examples originally used by Duncan (1956), listed in Table 5.1, are employed in this chapter. The total costs of operating the process when using optimal designs of the X-bar chart, moving average chart, and individual chart are compared based on these 25 examples.

Comparison of Results for Duncan's Model

In order to first verify the search procedure developed in Chapter III, 25 examples are rerun based on Duncan's economically-based X-bar chart design without simplification.

TABLE 5.1

COST AND RISK FACTORS AND PARAMETERS FOR 25 EXAMPLES

Example #	Delta	Lambda	Assumed Cost and Risk Factors M	e	D	T	W	b	c	Remarks on Parameters, Costs and Risk Factors
1	2	.01	100	.05	2	50	25	.5	.1	Basis
2	2	.02	100	.05	2	50	25	.5	.1	Same as #1 except Lambda increased
3	2	.03	100	.05	2	50	25	.5	.1	Ditto
4	2	.02	50	.05	2	50	25	.5	.1	Same as #1 except M decreased
5	2	.01	1000	.05	2	50	25	.5	.1	Same as #1 except M increased
6	2	.01	10000	.05	2	50	25	.5	.1	Ditto
7	2	.01	100	.50	2	50	25	.5	.1	Same as #1 except e increased
8	2	.01	100	.05	20	50	25	.5	.1	Same as #1 except D increased
9	2	.01	100	.05	2	5	2.5	.5	.1	Same as #1 except T & W decreased
10	2	.01	100	.05	2	500	250	.5	.1	" " " increased
11	2	.01	100	.05	2	5000	2500	.5	.1	Ditto
12	2	.01	100	.05	2	50	25	5.	.1	Same as #1 except b increased
13	2	.01	100	.05	2	50	25	.5	1	Same as #1 except c increased
14	2	.01	100	.05	2	50	25	.5	10	Ditto
15	2	.01	1000	.05	2	50	25	.5	1	Same as #1 except M & c increased
16	1	.01	12.87	.05	2	50	25	.5	.1	Same as #1 except Delta & M decreased
17	1	.01	128.7	.05	2	50	25	.5	.1	Same as #16 except M increased
18	1	.01	12.87	.05	2	500	250	.5	.1	Same as #16 except T & W increased
19	1	.01	12.87	.05	2	50	25	5.	.1	Same as #16 except b increased
20	1	.01	12.87	.05	2	50	25	.5	1	Same as #16 except c increased
21	.5	.01	2.25	.05	2	50	25	.5	.1	Same as #1 except Delta & M decreased
22	.5	.01	225	.05	2	50	25	.5	.1	Same as #21 except M incresaed
23	.5	.01	2.25	.05	2	500	250	.5	.1	Same as #21 except T & W increased
24	.5	.01	2.25	.05	2	50	25	5.	.1	Same as #21 except b increased
25	.5	.01	2.25	.05	2	50	25	.5	1	Same as #21 except c increased

Results of these 25 examples are listed in Table 5.2 along with Duncan's results and results from Goel, et al. (1968) and the recalculated loss-cost values from their design parameters n, h, and k.

When Duncan developed the economic X-bar chart model, computers were not yet available. In order to solve the complicated loss-cost function, he had to make some assumptions and sacrificed the accuracy of the solution. The approximate procedure he generated has the following assumptions: (1) terms like $\alpha T/h$ and $\lambda\{(1/P-1/2+\lambda h/12)h+en+D\}$ are neglected, (2) λ is assumed small and all terms in an equation having a smaller order of magnitude than the principal term are also neglected, (3) no account is taken of the values of the cost and risk factors e, D, and W. One other simplification he made is that the average time of occurrence within an interval between samples, given an occurrence of the shift in the interval between these samples, be changed from $\{1-(1+\lambda h)e^{-\lambda h}\}/\{\lambda(1-e^{-\lambda h})\}$ to $h/2 - \lambda h^2/12$, which eliminates terms of order $\lambda^3 h^4$ and higher.

As pointed out by Duncan, some of the limitations of the approximate procedure are: (1) for relatively high values of e, the approximate procedure does not appear to work as well as for relatively low values, and (2) approximate optima for δ=0.5 had to be determined very roughly, owing to difficulties in interpolation and extrapolation of the curves by graphical methods.

Goel, et al., developed an algorithm which consists of

TABLE 5.2

COMPARISON OF RESULTS BY THE SEARCH PROCEDURE OF THIS RESEARCH, DUNCAN'S APPROXIMATE AND EXACT METHODS, AND THE GENERAL METHOD OF GOEL, ET AL.

Example	Optimum Design by Exact Function				Approximate & Exact Design by Duncan					General Solution by Goel, et al.				
Number	n	h	k	Loss Cost	n	h	k	Loss Cost	(1)L	n	h	k	Loss Cost	(2)L
1	5	1.4032	3.0853	4.0128	5	1.3	3.2	4.0213	4.0211	5	1.41	3.08	4.0138	4.0128
					*5	1.4	3.1	4.0135	4.0130					
2	5	1.0216	3.0787	6.9460	5	.9	3.2	6.9655	6.9650	5	1.02	3.08	6.9477	6.9460
3	4	.7832	2.9366	9.5924	5	.8	3.2	9.6239	9.6225	4	.78	2.94	9.5947	9.5924
4	5	1.4617	3.0713	4.1527	5	1.3	3.2	4.1639	4.1642			(3)		
5	4	.4050	2.9574	26.9753	5	.3	3.2	27.5667	27.5625	4	.41	2.95	26.9763	26.9755
					*4	.4	3.0	26.9809	26.9802					
6	2	.0913	2.6914	228.8060	5	.1	3.2	236.8273	236.8266			(3)		
7	2	.9385	2.6856	5.4005	5	1.3	3.2	6.0886	6.0884	2	.94	2.69	5.4116	5.4006
					*2	.9	2.7	5.4035	5.4016					
8	5	1.6554	3.0575	18.3716	5	1.3	3.2	18.4078	18.4076	5	1.62	3.05	18.3728	18.3719
					*6	1.8	3.2	18.3655	18.3993					
9	3	1.2650	2.2082	3.6087	4	1.3	2.4	3.6214	3.6210			(3)		
10	6	1.4572	3.6731	6.3670	6	1.3	3.8	6.5937	6.3826	6	1.45	3.67	6.3705	6.3670
					*6	1.4	3.7	6.3693	6.3685					
11	8	1.7944	4.2499	28.2866	7	1.3	4.4	28.3416	28.3585			(3)		
12	6	3.4650	2.8777	5.8669	7	3.3	3.2	5.8986	5.8984	6	3.47	2.88	5.8695	5.8670
					*6	3.4	2.9	5.8685	5.8674					
13	3	2.5963	2.4243	5.6313	2	1.6	2.6	5.9413	5.9383			(3)		
					*3	2.6	2.4	5.6362	5.6323					
14	1	4.6928	1.4424	9.8733	2	7.0	1.4	10.1074	10.0694	1	4.66	1.46	9.9099	9.8740
15	3	.8120	2.4257	31.7500	2	.5	2.6	32.4165	32.4123			(3)		
					*3	.8	2.4	31.7619	31.7577					

TABLE 5.2 (Continued)

16	14	5.4897	2.6754	1.4159	17	5.6	2.8	1.4282	1.4269	14	5.47	2.68	1.4188	1.4159
					*14	5.4	2.7	1.4178	1.4161					
17	11	1.4552	2.5962	6.2759	17	1.9	2.8	6.4750	6.4735			(3)		
					*12	1.6	2.6	6.2666	6.2775					
18	21	7.1429	3.3953	3.6409	22	6.0	3.5	3.6583	3.6568	21	7.23	3.39	3.6429	3.6409
					*20	6.1	3.4	3.6479	3.6463					
19	18	11.0205	2.5451	1.9551	22	10.6	2.7	1.9743	1.9728	18	11.02	2.56	1.9578	1.9551
20	8	12.3708	1.8864	2.4207	8	10.4	2.0	2.4505	2.4391			(3)		
					*8	12.	1.9	2.4334	2.4213					
21	38	23.5481	2.1582	.8308	46	22.	2.3	.9497	.8407	38	23.45	2.21	.8370	.8313
22	20	1.2541	2.1053	13.5571	46	2.2	2.3	15.2590	15.2647	21	1.30	2.11	13.5715	13.5591
23	1	69.9948	5.3228	2.2586	74	25.	3.1	2.9043	2.7989			(3)		
24	45	37.4977	2.0253	.9772	55	30.	2.3	1.0042	1.0001			(3)		
25	1	69.9967	.00005	1.2036	20	44.	1.3	1.4160	1.3731	12	54.32	1.13	1.3265	1.2857

Note: (1) L are the recalculated loss-cost values for Duncan's design parameters n, h, and k
(2) L are the recalculated loss-cost values for Goel's design parameters n, h, and k
(3) Values not available
* Exact optima design by Duncan

solving an implicit equation in the design of variables n and k, and an explicit equation for h. But Duncan's simplification is still applied when calculating the average time of occurrence within an interval between samples, given an occurrence of the shift in the interval between these samples.

The recalculated loss-cost values for Duncan's design parameters are somewhat different from his reported values. In most cases the recalculated values are lower, but two values are higher. The loss-cost of the exact design for example #8 is reported as 18.3655 but is recalculated as 18.4007 by Goel, et al. and as 18.3993 from the exact loss-cost function. The recalculated loss-cost values for design parameters of Goel, et al. are lower than their reported values.

From Table 5.2, it can be noted that, in all cases considered, the search procedure of this research yields lower loss-costs than Duncan's approximate method and the algorithm of Goel, et al. It shows that the search procedure used in this research is adequate.

Economic Comparison Among Designs of the Economically-Based X-bar Chart, Moving Average Chart, and Individual Chart

To provide an economic comparison among the economically-based X-bar chart, moving average chart, and individual chart, the 25 examples shown in Table 5.1 are considered.

To assure proper comparison among optimal designs of the X-bar chart, moving average chart, and individual chart, the exact same termination conditions of the search procedure are used for each. The optimal design values of parameters n, h, and k and their relative loss-cost for each of these 25 examples when used to monitor a continuous flow process are listed in Table 5.3.

Based on the results listed in Table 5.3, it can be noted that:

(1) In all cases, the X-bar chart is superior to the individual chart.

(2) In most cases, the X-bar chart is better than the moving average chart. When unit cost of inspection and charting (c) are equal to 1, the moving average chart yields a lower loss-cost than the X-bar chart does.

(3) The moving average chart yields a lower loss-cost than the individual chart does in 23 out of 25 examples.

Effect of Variation in Risk and Cost Factors on the Optimum Design

As mentioned by Duncan (1956), the values assigned to the cost and risk factors of the 25 examples listed in Table 5.1 cover a wide range of possibilities and are believed to be, relative to each other, generally typical of industrial

TABLE 5.3

COMPARISON OF RESULTS OF AN ECONOMICALLY-BASED X-BAR CHART, MOVING AVERAGE CHART, AND INDIVIDUAL CHART

Example	X-bar Chart				Moving Average Chart				Individual Chart		
#	n	h	k	Loss Cost	n	h	k	Loss Cost	h	k	Loss Cost
1	5	.2002	3.0904	4.432098	3	.5264	3.0905	4.895355	.6580	2.5277	5.764150
2	5	.1474	3.0627	7.424130	3	.3855	3.0856	8.058315	.4843	2.5192	9.227827
3	5	.1229	3.0796	10.094597	3	.3302	3.0533	10.848398	.4108	2.5103	12.216425
4	5	.2128	3.0822	4.561221	3	.5645	3.0800	5.004049	.7040	2.5078	5.813059
5	5	.0669	3.0527	27.266896	3	.1655	3.0856	28.743986	.2092	2.5165	31.555852
6	5	.0205	3.0954	223.068141	3	.0582	3.0138	227.888678	.0674	2.5115	236.734126
7	5	.2038	3.1016	4.851258	3	.5417	3.0854	5.312092	.6689	2.5324	6.174291
8	5	.2322	3.0395	18.748866	3	.6252	3.0307	19.129697	.7906	2.4941	19.781686
9	3	.3389	2.1595	3.940685	2	.7390	2.1033	4.118197	1.0257	1.6687	4.125784
10	6	.1696	3.7408	6.821819	4	.4361	3.8166	7.506873	.4170	3.2021	10.550546
11	8	.1566	4.3423	28.730876	5	.4425	4.4194	29.502533	.3120	3.7468	36.125067
12	12	.2454	2.6330	6.803078	2	2.2165	2.1680	8.345285	2.9573	1.7318	8.454972
13	2	.8085	2.3916	6.373186	2	1.0407	2.5922	6.069582	1.2813	2.1997	6.708098
14	1	4.7241	1.4352	9.873311	1	4.7454	1.4288	9.873538	4.6020	1.4518	9.874178
15	2	.2579	2.3748	33.504571	2	.3162	2.6099	32.523485	.3903	2.2146	34.611504
16	13	.2820	2.7466	1.649899	6	1.0582	2.9169	1.986716	1.7808	2.1382	3.022552
17	13	.0881	2.7320	6.560448	6	.3151	2.9314	7.694635	.4932	2.1544	11.435418
18	19	.2430	3.4450	3.893115	10	.8892	3.7335	4.312545	1.3044	2.8409	7.19351
19	22	.3991	2.4419	2.376788	3	6.1468	1.7421	3.554685	33.0815	.0006	3.530940
20	5	1.6192	1.8021	2.843942	5	2.0566	2.5259	2.579654	4.2219	1.6966	3.335315
21	30	.5366	2.1901	.950721	13	2.5484	2.7129	1.094950	69.9799	.0006	1.190798
22	32	.0384	2.3001	12.642176	14	.1691	2.8373	14.698421	6.9545	.0001	18.766293
23	117	69.9878	7.9974	2.251505	8	69.9916	7.9318	2.258572	68.6213	4.0514	2.258814
24	2	44.5673	.0001	1.229450	23	69.9999	.00011	1.254989	69.9873	.0021	1.255219
25	2	43.4797	.0002	1.198600	8	6.8811	2.1239	1.309174	69.9868	.0009	1.203665

costs and risks. The effect of variation in risk and cost factors on the optimum design may be gleaned from a study of results listed in Table 5.3.

From Table 5.3, conclusions can be generated as follows:

1. When the rate of occurrence of assignable causes (λ) increases the interval between samples (h) decreases on all three charting techniques. That is, the higher the probability that assignable causes will occur, the shorter the time between samples.
2. The loss rate M has the same effect as λ on the interval between samples (h). When M is relatively small, h should be large; when M is relatively large, h should be small.
3. The effect of delay factor (e) is not quite so clear. An increase in e causes a decrease in h and k on all three charting techniques.
4. The average search time for an assignable cause (D) has a moderate effect on h and k in all three charting techniques. When D increases, h increases and k decreases.
5. Variation in the cost of looking for trouble when none exists (T) and the accompanying variation in the average cost of looking for trouble when it does exist (W) affect all elements of design on all three charting techniques. For high values of T and W, the optimum designs call for taking large samples,

at small intervals between samples, and with control limits at high multiples of sigma. But when the shift of the process mean is equal to 0.5 sigma, increases in values of T and W cause the interval between samples h to increase on designs of X-bar chart and moving average chart.

6. Variation in the unit cost of inspection and charting (c) affects all three design elements on all three charting techniques. A large c leads to a small sample subgroup size, tighter control limits, and a longer interval between samples.
7. Variation in the cost of sampling, testing, and charting (b) affects design parameters n, h, and k. Variation in b has little effect on the individual chart design when delta equals 0.5.

Analysis of Loss-cost Surface

The loss-cost surfaces of example number 2 for the economically-based X-bar chart, moving average chart, and individual chart are shown in Figure 5.1, Figure 5.2, and Figure 5.3, respectively. The subgroup size n of the X-bar chart and moving average chart in those figures is set equal to 5. A global minimum exists for each charting technique. From those figures, it also shows that the loss-cost surfaces curve substantially along the design parameter k on all three charting techniques. That is, control limit width k strongly affects the loss-cost when n is fixed, while time

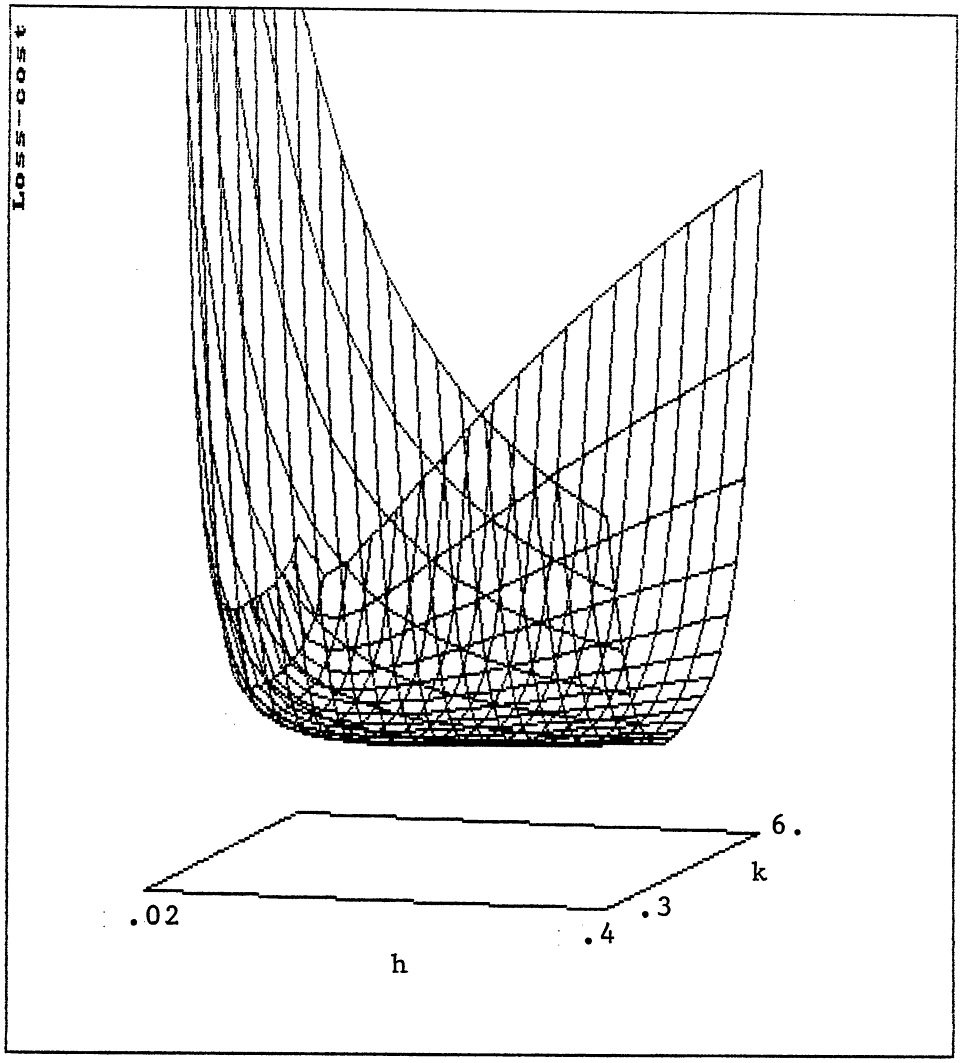

Figure 5.1.1. Loss-cost Surface of Economically-based X-bar Chart When n=5

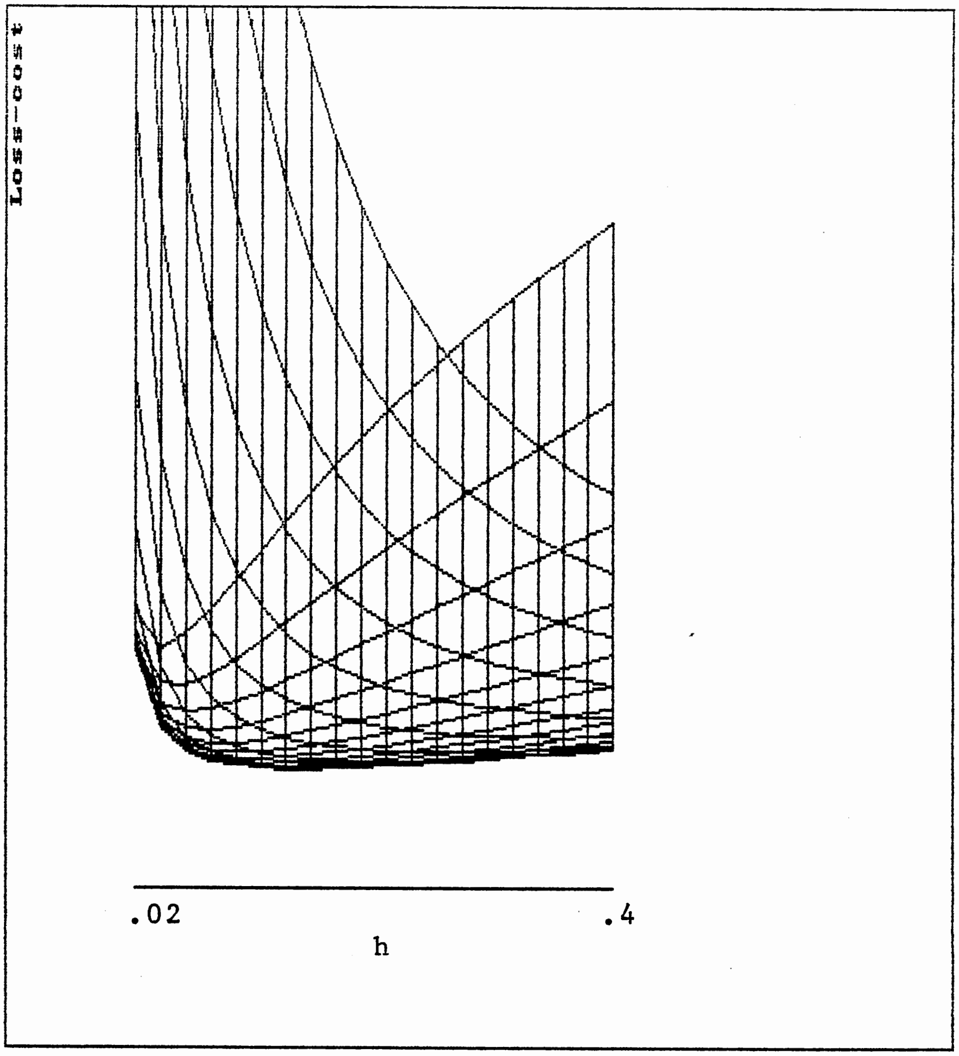

Figure 5.1.2. Loss-cost Surface of Economically-based X-bar Chart When n=5, Viewed from h Plane

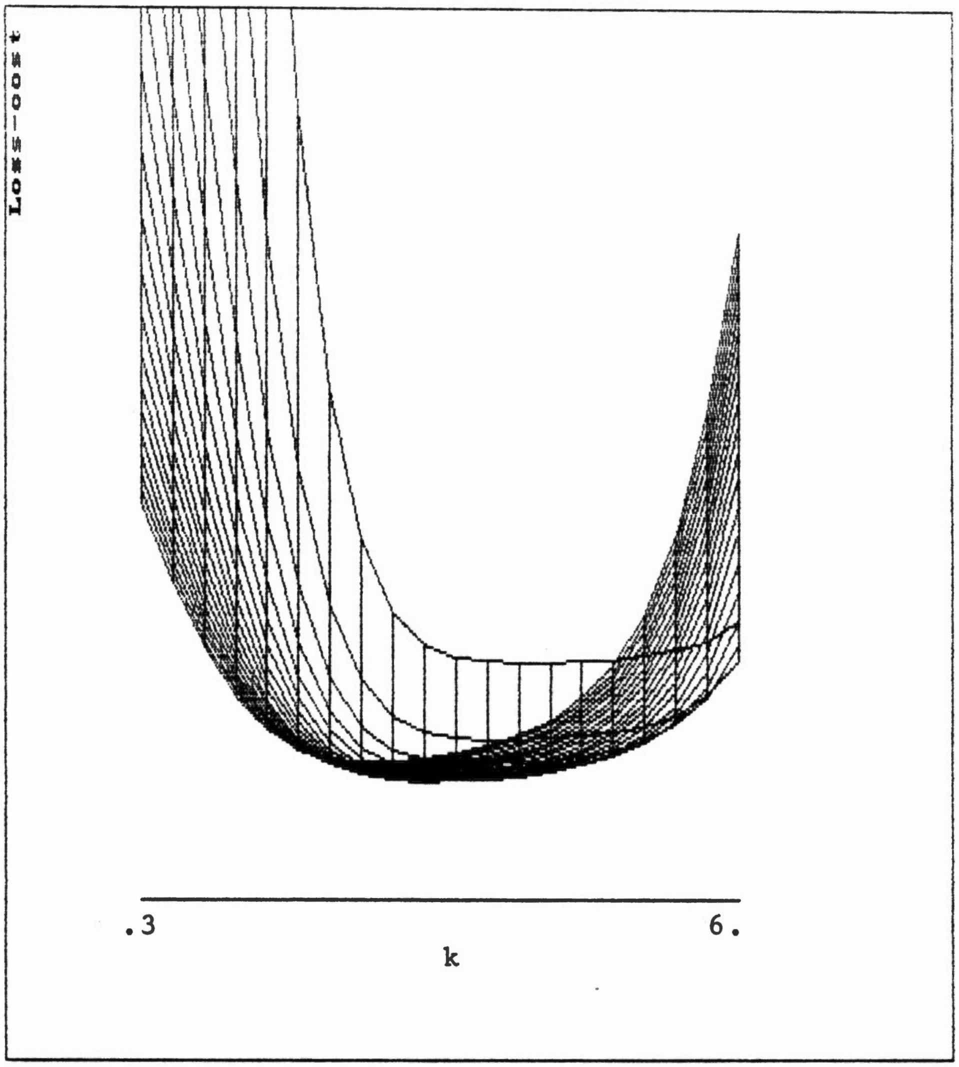

Figure 5.1.3. Loss-cost Surface of Economically-based X-bar Chart When n=5, Viewed from k Plane

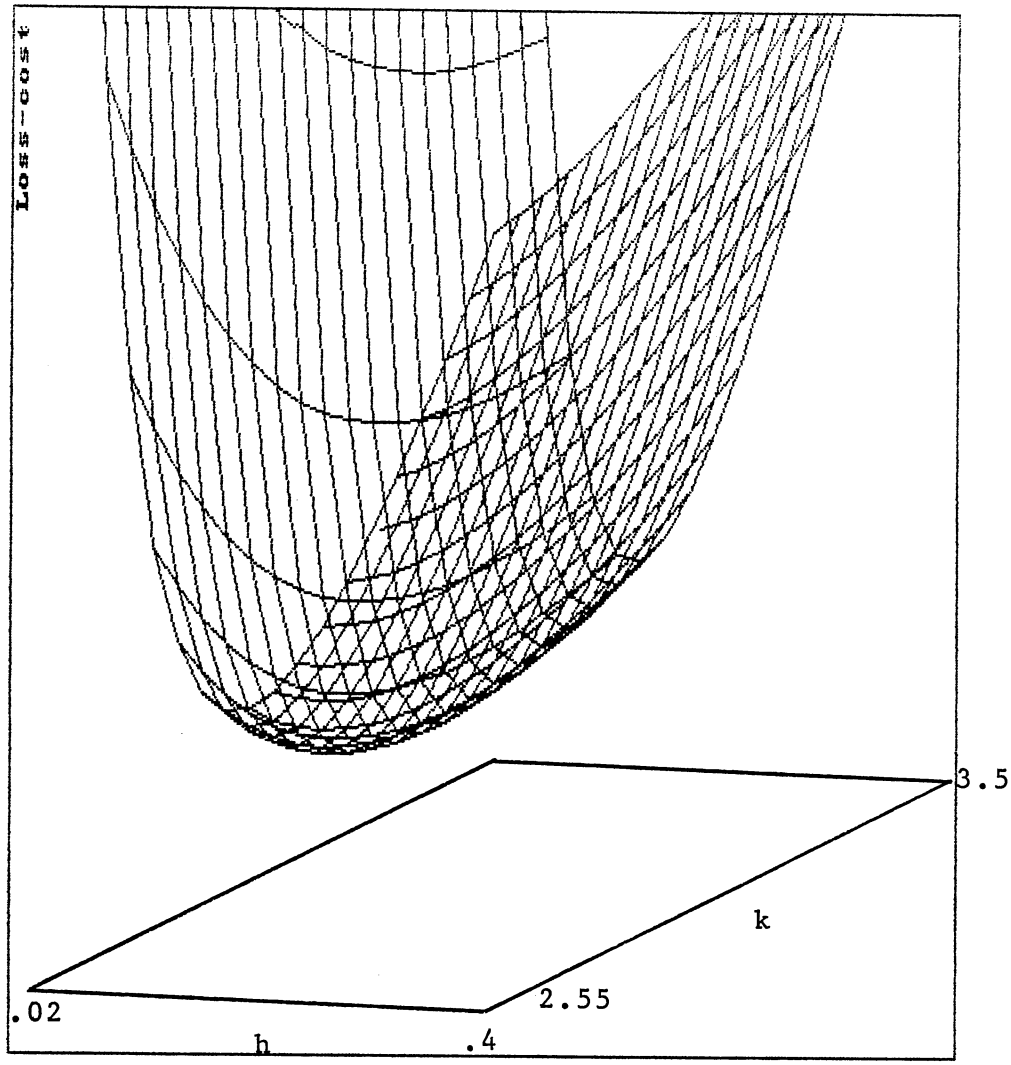

Figure 5.1.4. Loss-cost Surface of Economically-based X-bar Chart When n=5, Enlarged View

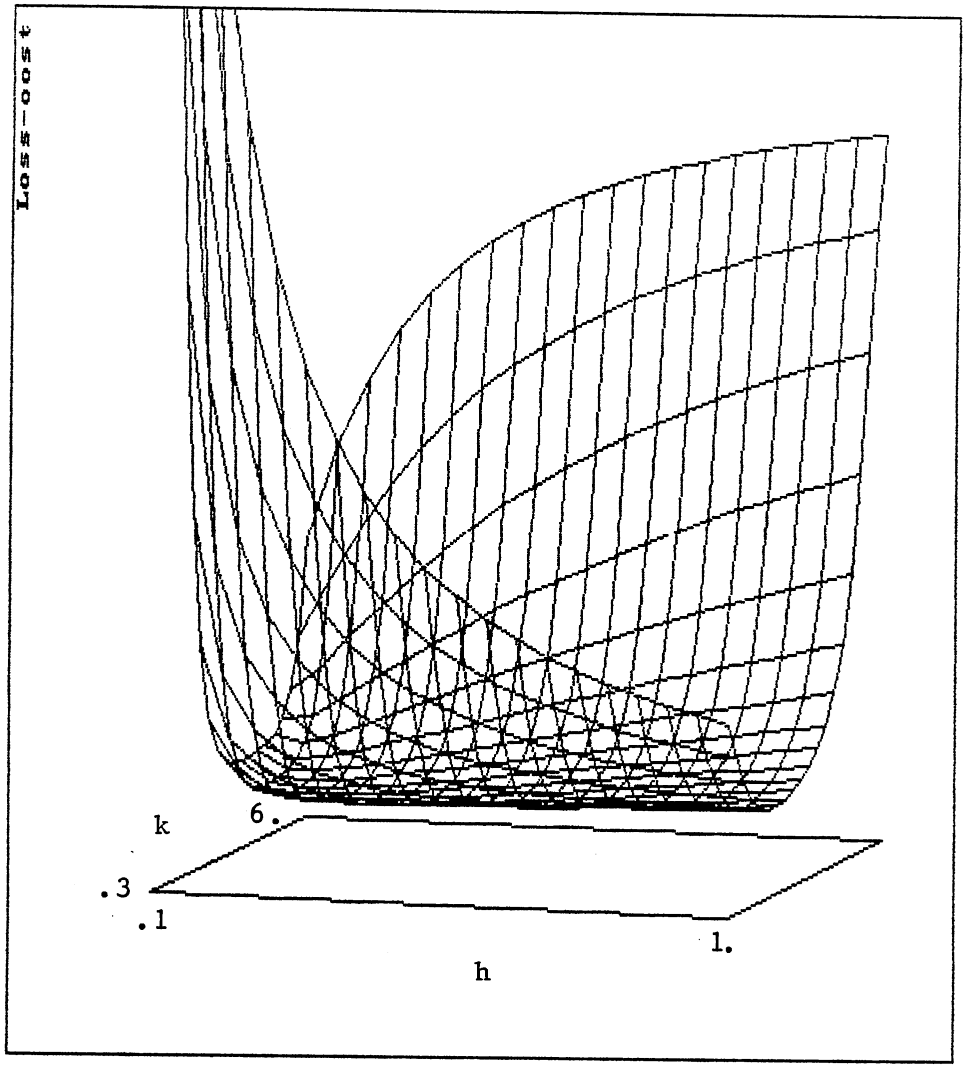

Figure 5.2.1. Loss-cost Surface of Economically-based Moving Average Chart When n=5

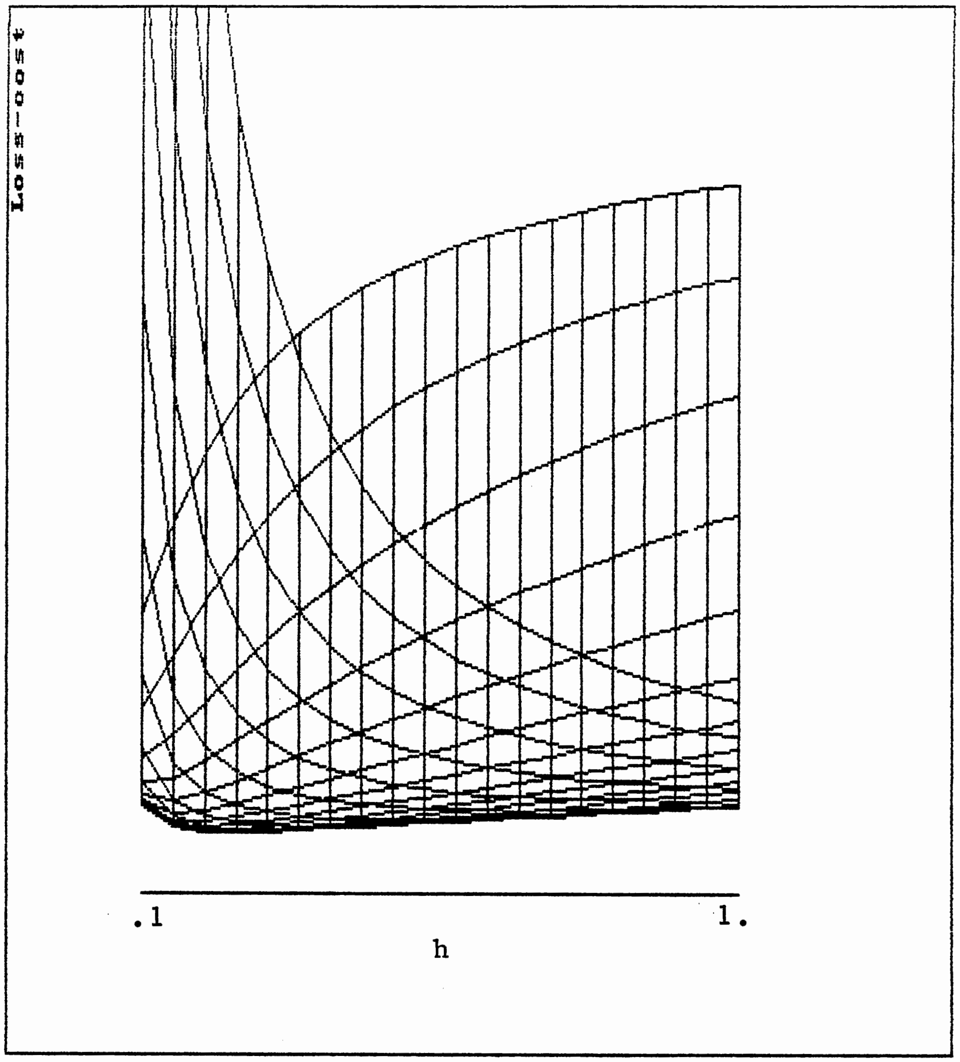

Figure 5.2.2. Loss-cost Surface of Economically-based MA Chart When n=5, Viewed from h Plane

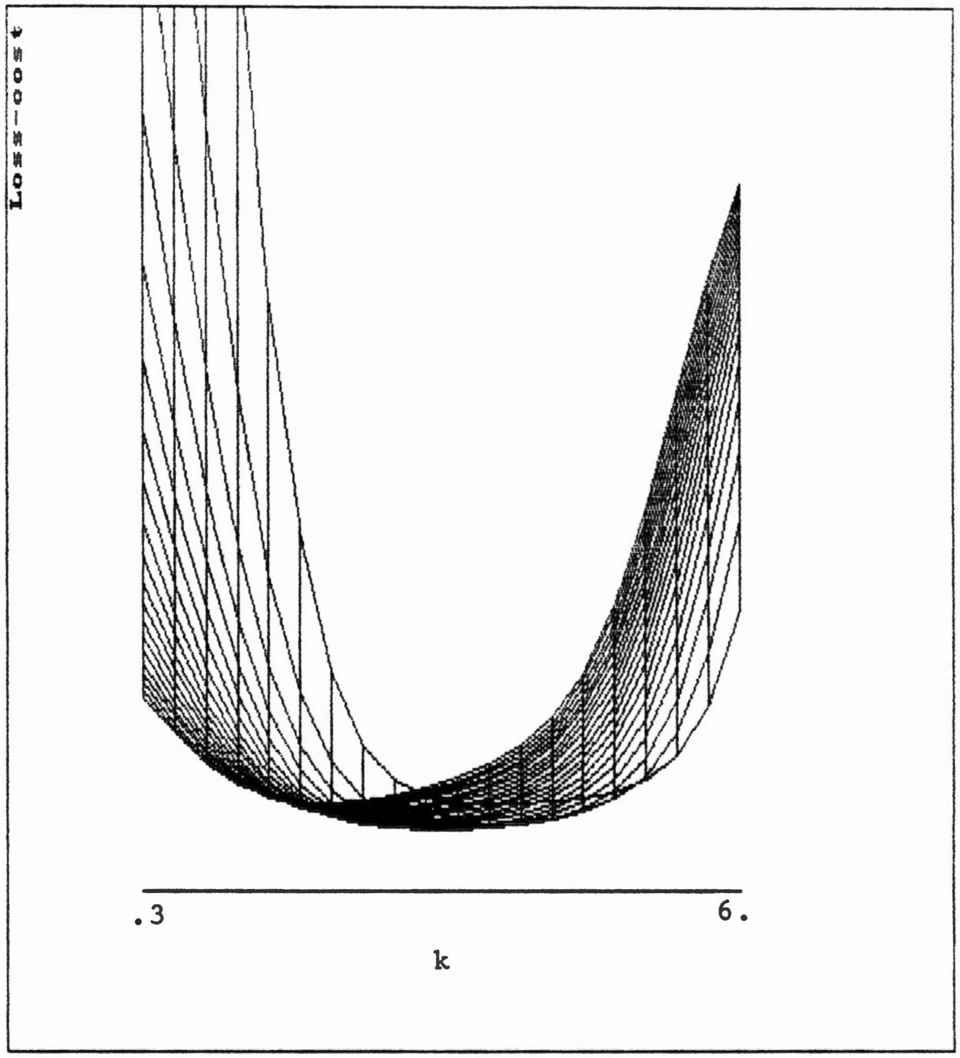

Figure 5.2.3. Loss-cost Surface of Economically-based MA Chart When n=5, Viewed from k Plane

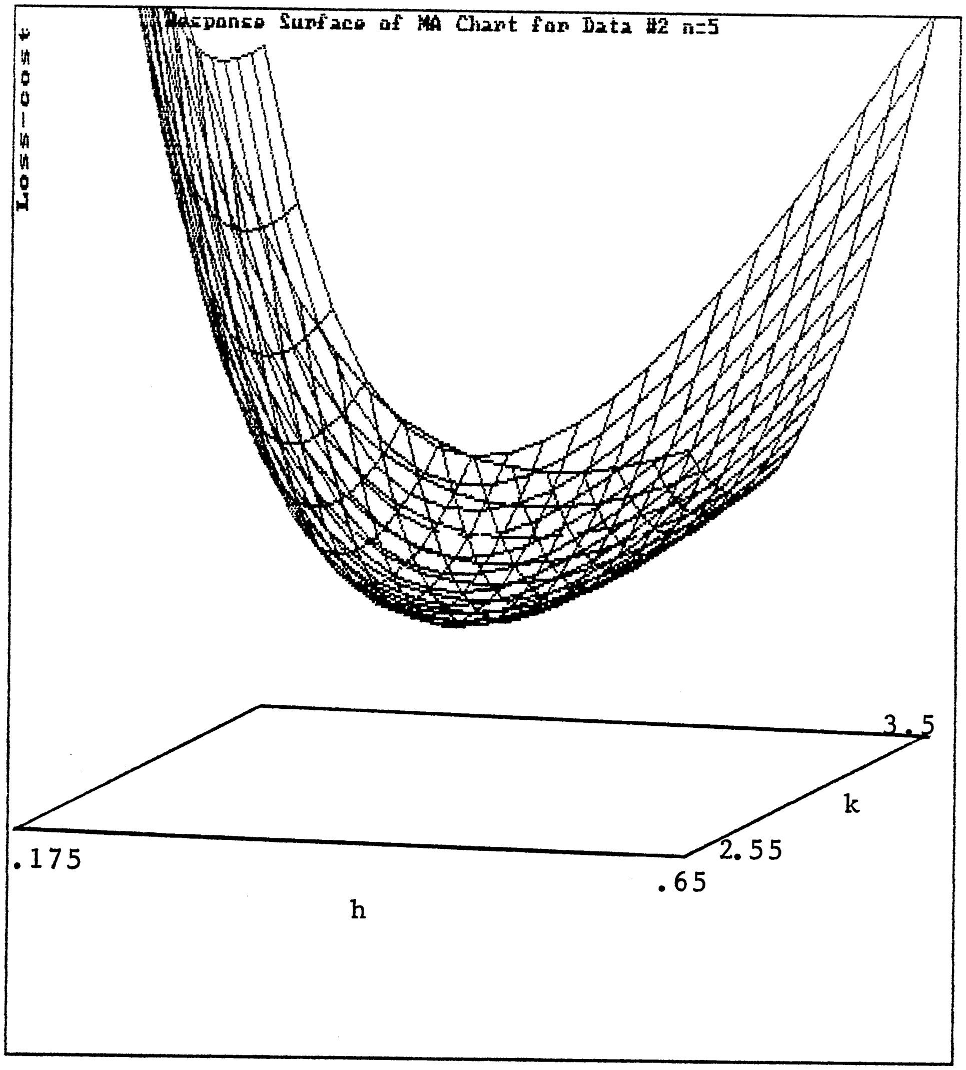

Figure 5.2.4. Loss-cost Surface of Economically-based MA Chart When n=5, Enlarged View

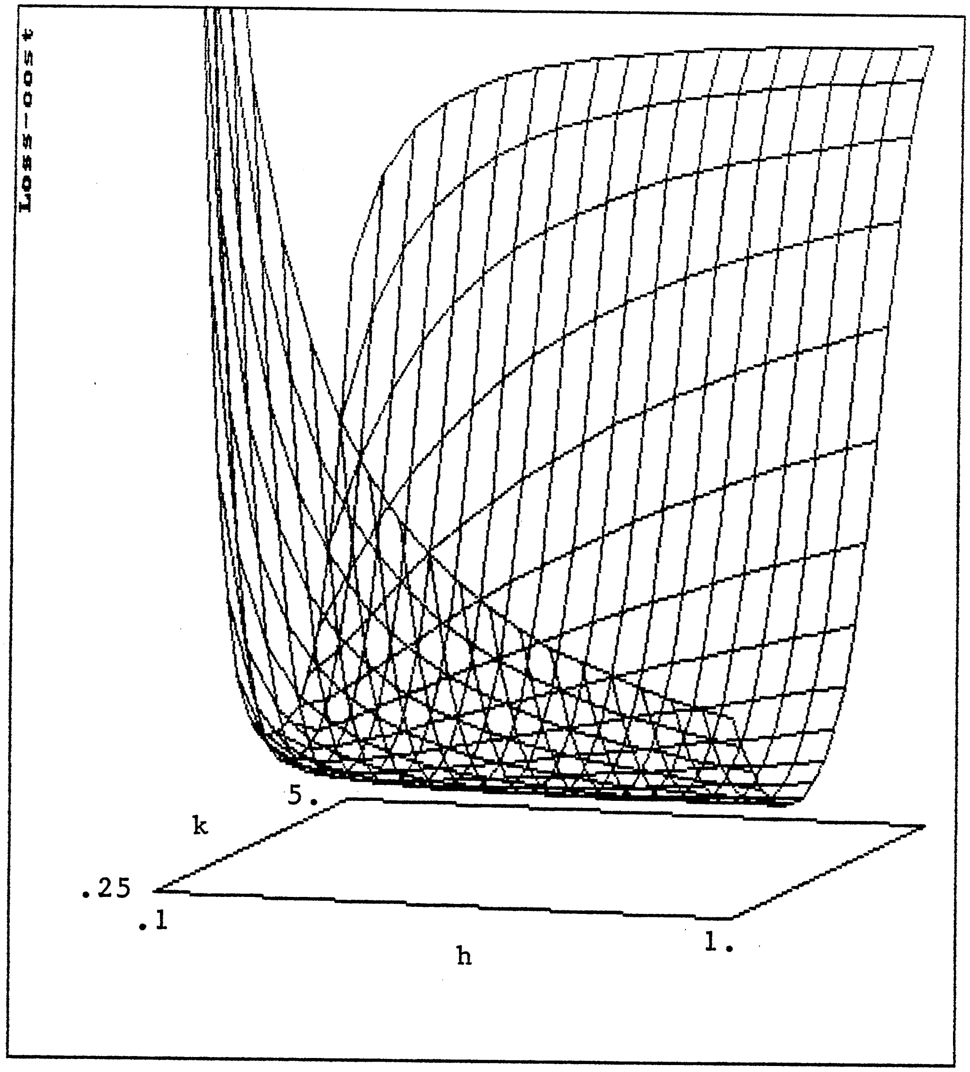

Figure 5.3.1. Loss-cost Surface of Economically-based Individual Chart

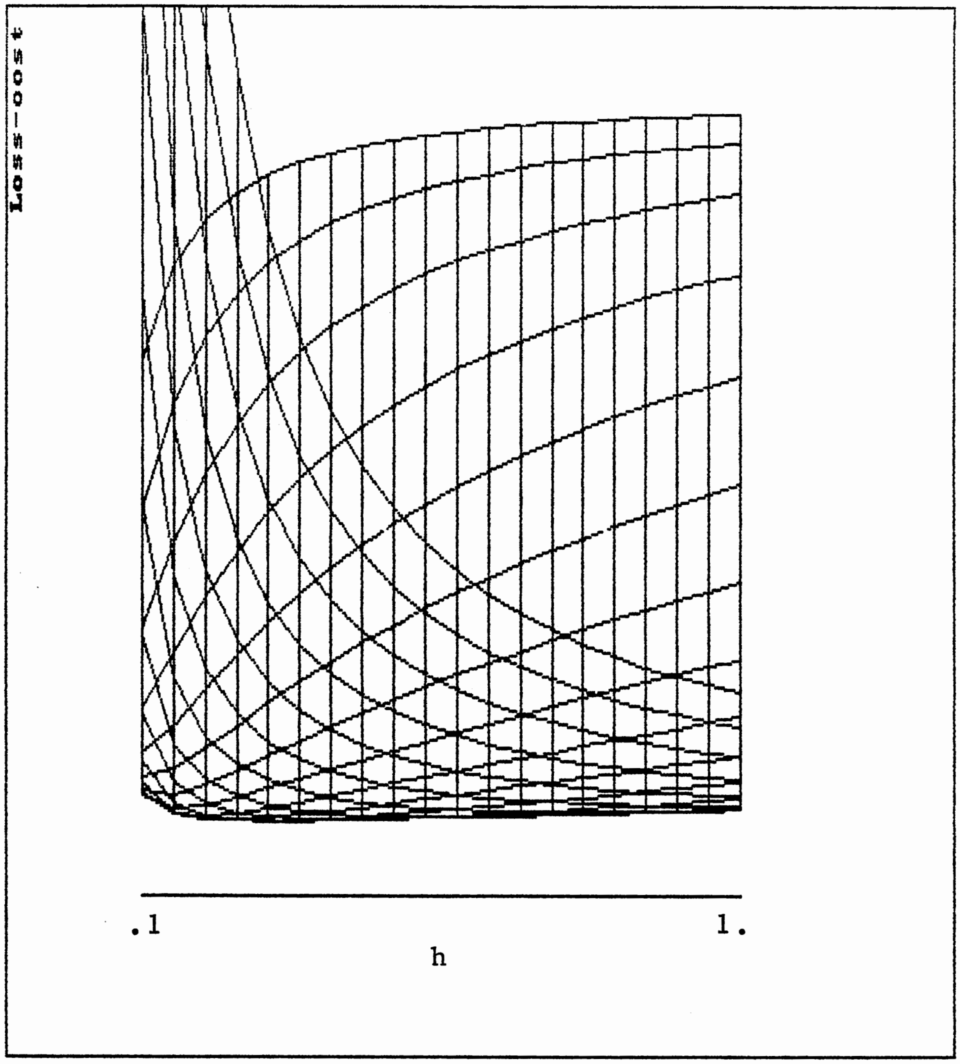

Figure 5.3.2. Loss-cost Surface of Economically-based Individual Chart, Viewed from h Plane

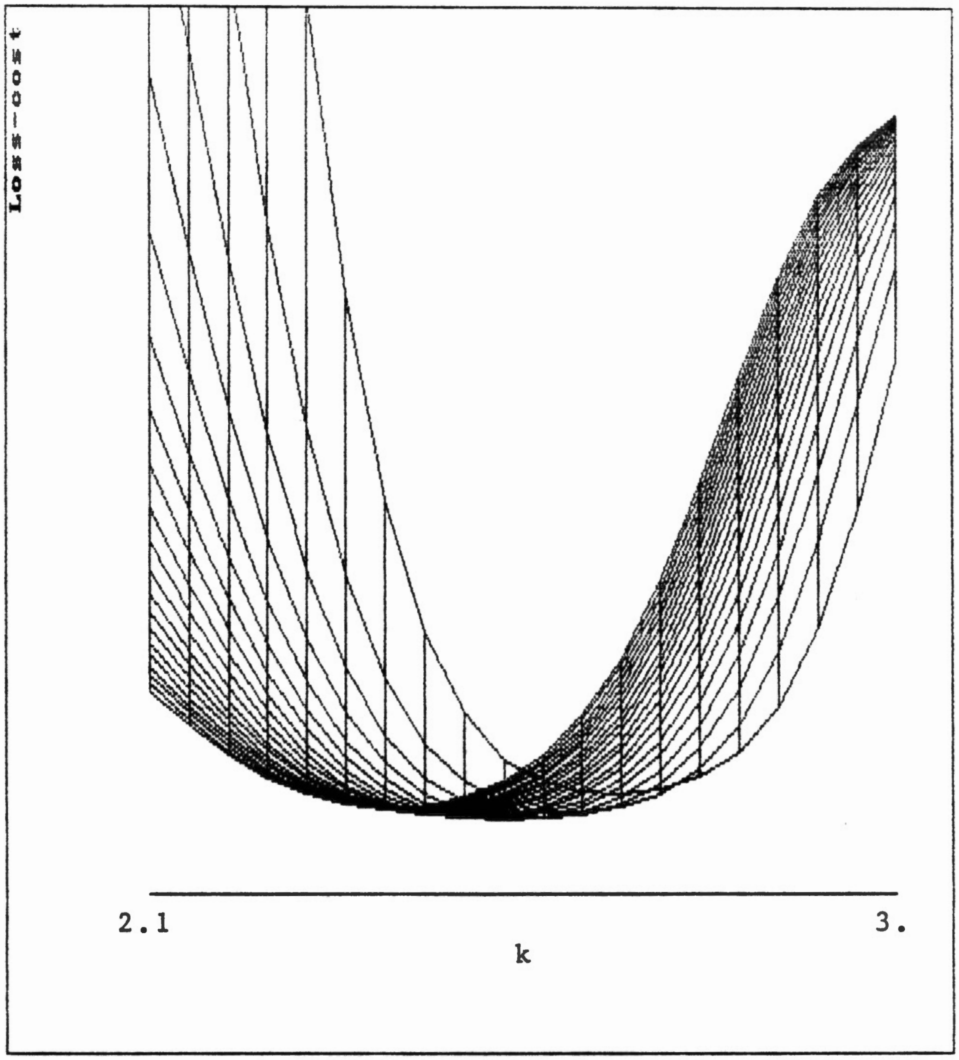

Figure 5.3.3. Loss-cost Surface of Economically-based Individual Chart, Viewed from k Plane

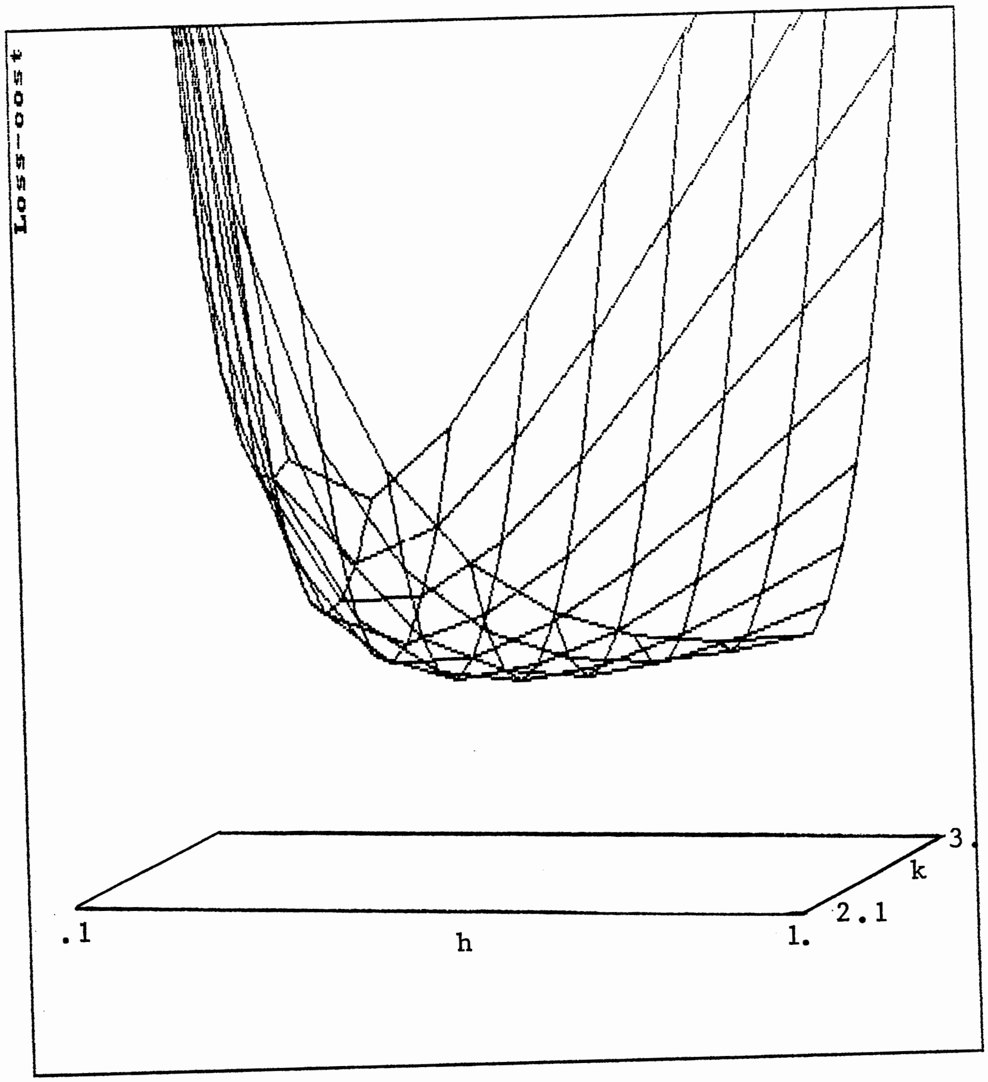

Figure 5.3.4. Loss-cost Surface of Economically-based Individual Chart, Enlarged View

interval h is not a big factor when compared to k.

The variation in h, k, and loss-cost as subgroup size n is varied from 1 to 12 on the economically-based X-bar chart and moving average chart are shown in Figure 5.4 and Figure 5.5, respectively. Example number 2 from Duncan's paper is used to get the results which are plotted on these two figures. These figures show that when subgroup size n is increased, the time interval h of the optimal design is decreased and control limit width k of the optimal design is increased. The loss-cost increases greatly when subgroup size is less than 2 for both charting techniques. When n increases from the optimal subgroup size, the increase in loss-cost of the moving average chart is larger than that of the X-bar chart.

Summary

The search procedure of Chapter III is verified through a comparison between this research, Duncan's approximate and exact methods, and the results of Goel, et al. The results of this comparison are shown in Table 5.2.

An economic comparison among the X-bar chart, moving average chart, and individual chart is performed. Duncan's 25 examples are used in this comparison. The results of this comparison are shown in Table 5.3. An analysis of these results shows that the X-bar chart design has the lowest loss-cost in almost all examples. The moving average chart is considered to be the second most economical chart-

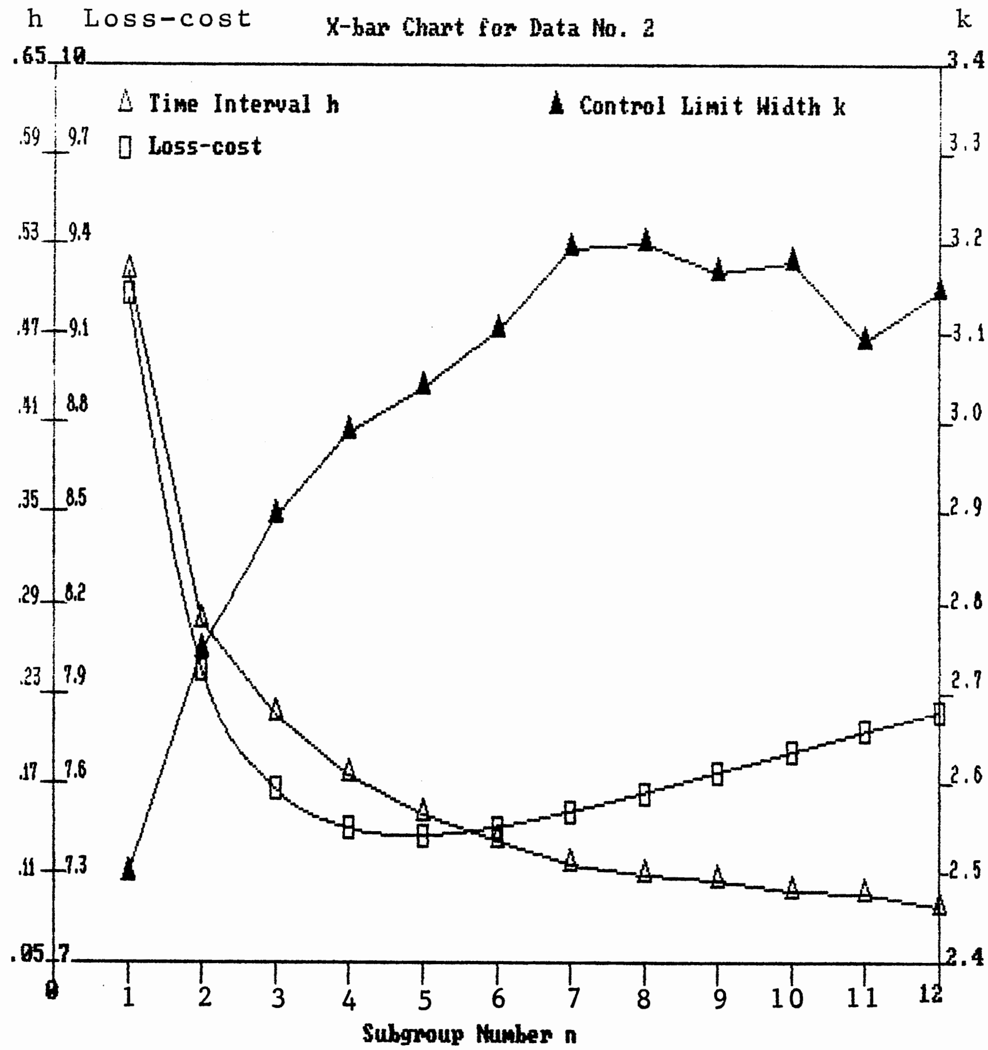

Figure 5.4. Optimum Values of h, k, and Loss-cost for Varying Subgroup Size on an X-bar Chart

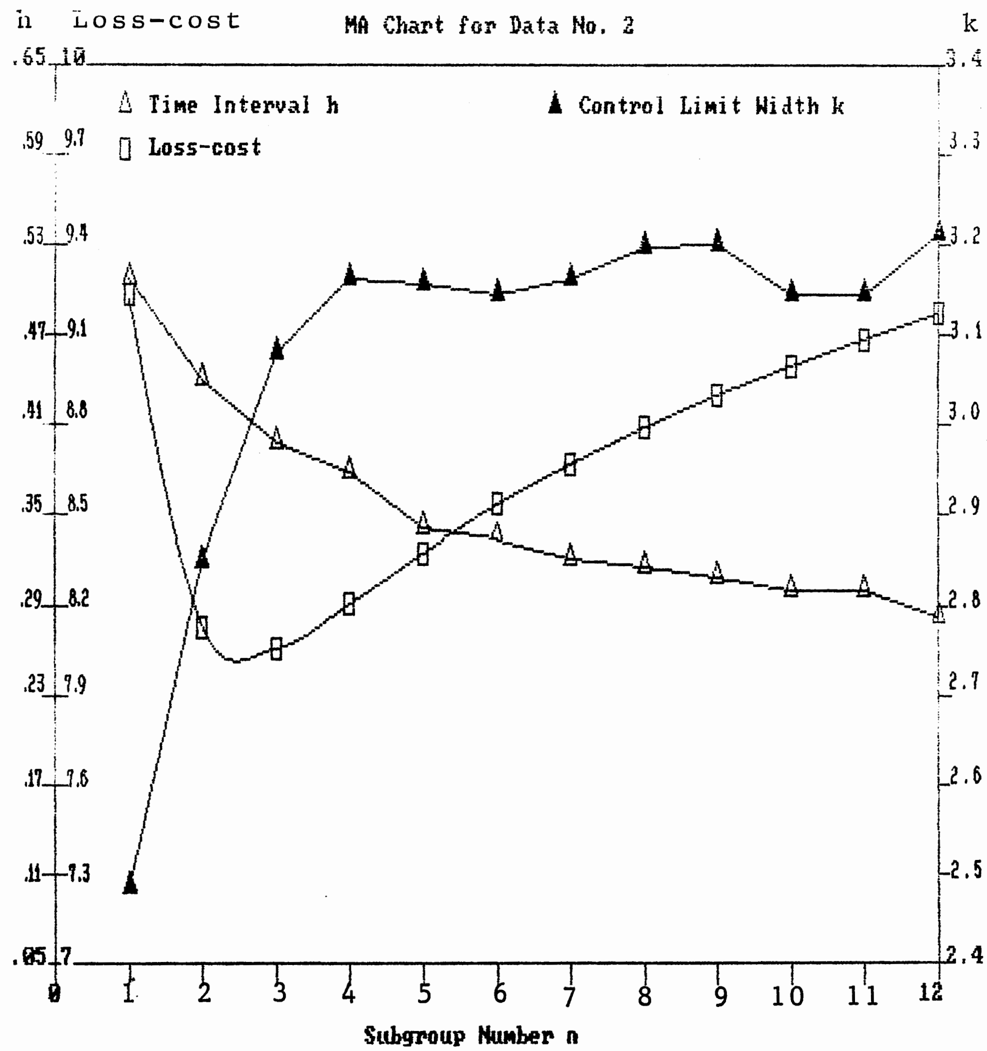

Figure 5.5. Optimum Values of h, k, and Loss-cost for Varying n on a Moving Average Chart

ing technique when used to monitor continuous flow processes. The effects of variation in cost and risk factors on optimal design are also gleaned from Table 5.3.

The loss-cost surfaces are analyzed in this chapter. The result shows that when subgroup size is fixed, the loss-cost changes greatly when control limit width k is varied from its optimal value. The time interval h does not affect the loss-cost much when it is increased from its optimal value. But a decrease in h from the optimal value causes a large increase in loss-cost.

CHAPTER VI

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

This chapter summarizes all the steps carried out in order to fulfill the objective and all subobjectives of this research. Conclusions from this research are then provided. Finally, recommendations for future work and possible extensions of this research are outlined.

Summary

Chapter I of this research provides the problem statement. Introduction of the charting techniques, economically-based control chart designs, and the characteristics of continuous flow processes are given. The research objective which involves primary and secondary objectives is then identified. An extensive literature survey of economically-based control chart design and the techniques used to monitor continuous flow processes is given in Chapter II. Chapter III develops the economic models of the X-bar chart, moving average chart, and individual chart to be used in monitoring continuous flow processes. A comprehensive, interactive computer program has been developed and described in Chapter IV. An economic comparison among the X-bar chart, moving average chart, and individual chart and the analysis of these three models is provided in Chapter V.

In order to fulfill the objective and all the subobjectives, the following accomplishments have been achieved:

1. A procedure of economic comparison among the X-bar chart, moving average chart, and individual chart has been established in a continuous flow process environment.
2. An analytical model to optimize and evaluate the X-bar chart, moving average chart, and individual chart has been developed under a continuous flow process environment. This model follows the same cost structure as Duncan's original economic X-bar chart model.
3. Sensitivity analyses of the economically-based X-bar chart, moving average chart, and individual chart design have been performed.
4. An interactive and comprehensive computer program has been developed and described. This program implements the economically-based design and evaluation of the (1) X-bar chart, (2) moving average chart, (3) individual chart, in a continuous flow process environment, and (4) Duncan's economic X-bar chart model for a discrete process.

Conclusions and Recommendations

Based on the results obtained in this research, the optimal design of the X-bar chart is always superior to the optimal individual chart when monitoring continuous flow processes and is superior to the optimal design of the moving average chart except in some cases in which the unit cost of inspection and charting (c) is equal to 1. In most conditions, the optimal design of a moving average chart is better than an optimal individual chart.

Possible further work with respect to economically-based control chart techniques when monitoring continuous flow processes are as follows:

1. The same techniques developed in this research can be extended to other control chart methods such as: the CUSUM chart, s chart, etc.
2. Multiple assignable causes may be considered in an extension to this research. In this study, a single assignable cause is assumed.
3. Out-of-control decision methods such as the runs rules of AT&T can be applied in the detection of an OOC condition. A point outside the control limits is the only OOC condition in this research.
4. Average run length is another criterion besides cost which can be used to evaluate the performance of control chart techniques.

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APPENDIX

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C***************************************************************
C                                                              *
C   PURPOSE:                                                   *
C        THIS INTERACTIVE PROGRAM IS DESIGNED TO PERFORM       *
C   ECONOMIC DESIGN AND EVALUATION OF X-BAR CHART, MOVING      *
C   AVERAGE CHART, AND INDIVIDUAL CHART FOR USE IN CONTINUOUS  *
C   FLOW PROCESS ENVIRONMENT AND ECONOMIC DESIGN AND EVALUATION *
C   OF DUNCAN'S MODEL FOR DISCRETE PROCESS.                    *
C                                                              *
C   AUTHOR: TONG-YUAN KOO                                      *
C           SCHOOL OF INDUSTRIAL ENGINEERING AND MANAGEMENT    *
C           OKLAHOMA STATE UNIVERSITY                          *
C                                                              *
C   DISSERTATION ADVISOR: DR. KENNETH E. CASE                  *
C                                                              *
C***************************************************************
C                                                              *
C   DEFINITION OF SUBROUTINES:                                 *
C                                                              *
C      ECDSGN -- PROMPT THE USER FOR SELECTION OF ECONOMIC     *
C                DESIGN OF X-BAR CHART, MOVING AVERAGE CHART,  *
C                INDIVIDUAL CHART, AND DUNCAN'S X-BAR CHART.   *
C                                                              *
C      ECEVAL -- PROMPT THE USER FOR SELECTION OF ECONOMIC     *
C                EVALUATION OF X-BAR CHART, MOVING AVERAGE CHART,*
C                INDIVIDUAL CHART, AND DUNCAN'S X-BAR CHART.   *
C                                                              *
C      ECXBAR -- INPUT COST AND RISK PARAMETERS AND CALL OPXBAR *
C                TO PERFORM THE ECONOMIC DESIGN OF X-BAR CHART. *
C                                                              *
C      ECMA   -- INPUT COST AND RISK PARAMETERS AND CALL OPMA  *
C                TO PERFORM THE ECONOMIC DESIGN OF MOVING      *
C                AVERAGE CHART.                                *
C                                                              *
C      ECIND  -- INPUT COST AND RISK PARAMETERS AND CALL OPIND *
C                TO PERFORM THE ECONOMIC DESIGN OF INDIVIDUAL  *
C                CHART.                                        *
C                                                              *
C      ECDUNC -- INPUT COST AND RISK PARAMETERS AND CALL OPDUNC *
C                TO PERFORM THE ECONOMIC DESIGN OF DUNCAN'S    *
C                X-BAR CHART FOR DISCRETE PROCESS.             *
C                                                              *
C      OPXBAR -- PERFORM THE ECONOMIC DESIGN OF X-BAR CHART.   *
C                                                              *
C      OPMA   -- PERFORM THE ECONOMIC DESIGN OF MOVING AVERAGE *
C                CHART.                                        *
C                                                              *
C      OPIND  -- PERFORM THE ECONOMIC DESIGN OF INDIVIDUAL CHART.*
C                                                              *
C      OPDUNC -- PERFORM THE ECONOMIC DESIGN OF DUNCAN'S X-BAR *
C                CHART.                                        *
C                                                              *
C      EVXBAR -- PERFORM THE ECONOMIC EVALUATION OF X-BAR CHART. *
C                                                              *
C      EVMA   -- PERFORM THE ECONOMIC EVVLUATION OF MOVING     *
C                AVERAGE CHART.                                *
C                                                              *
C      EVIND  -- PERFORM THE ECONOMIC EVALUATION OF INDIVIDUAL *
C                CHART.                                        *
C                                                              *
C      EVDUNC -- PERFORM THE ECONOMIC EVALUATION OF DUNCAN'S   *
C                X-BAR CHART FOR DISCRETE PROCESS.             *
C                                                              *
C      NELMIN -- PERFORM THE NELDER AND MEAD DIRECT SEARCH     *
C                ALGORITHM TO FIND THE OPTIMUM POINT.          *
C                                                              *
C      ================================================        *
C                                                              *
C   DEFINITION OF FUNCTIONS:                                   *
C                                                              *
C      FUNCT3 -- X-BAR CHART MODEL FOR USE IN 3-DIMENSIONAL    *
C                OPTIMIZATION.                                 *
```

```
C                                                                          *
C       FUNCT2 -- X-BAR CHART MODEL FOR USE IN 2-DIMENSIONAL               *
C                 OPTIMIZATION OVER H AND K.                               *
C                                                                          *
C       FNMA3  -- MOVING AVERAGE CHART MODEL FOR USE IN                    *
C                 3-DIMENSIONAL OPTIMIZATION.                              *
C                                                                          *
C       FNMA2  -- MOVING AVERAGE CHART MODEL FOR USE IN                    *
C                 2-DIMENSIONAL OPTIMIZATION OVER H AND K.                 *
C                                                                          *
C       FNIND2 -- INDIVIDUAL CHART MODEL FOR USE IN 2-DIMENSIOAL           *
C                 OPTIMIZATION OVER H AND K.                               *
C                                                                          *
C       FNDUN3 -- DUNCAN'S X-BAR CHART MODEL FOR USE IN                    *
C                 3-DIMENSIONAL OPTIMIZATION.                              *
C                                                                          *
C       FNDUN2 -- DUNCAN'S X-BAR CHART MODEL FOR USE IN                    *
C                 2-DIMENSIONAL OPTIMIZATION OVER H AND K.                 *
C                                                                          *
C       DNML   -- FUNCTION TO COMPUTE THE CUMULATIVE DISTRIBUTION          *
C                 P(Y<=X) OF A RANDOM VARIABLE Y HAVING A                  *
C                 STANDARD NORMAL DISTRIBUTION.                            *
C                                                                          *
C       ==================================================                 *
C                                                                          *
C    DEFINITION OF VARIABLES:                                              *
C                                                                          *
C      DELTA  -- MAGNITUDE OF THE OOC SHIFT IN THE PROCESS MEAN            *
C                IN MULTIPLES OF PROCESS STANDARD DEVIATION                *
C      LAMBDA -- FAILURE RATE FOR THE ASSIGNABLE CAUSE TO OCCUR,           *
C                PER HOUR                                                  *
C      EM     -- THE REDUCTION IN THE PROCESS HOURLY INCOME THAT           *
C                IS ATTRIBUTED TO THE OCCURRENCE OF THE                    *
C                CAUSE                                                     *
C      E      -- AVERAGE SAMPLING, INSPECTING, EVALUATING, AND             *
C                PLOTTING TIME FOR A SAMPLE OF SIZE 1                      *
C      D      -- AVERAGE TIME TAKEN TO FIND AN ASSIGNABLE CAUSE            *
C                AFTER A POINT HAS BEEN FOUND TO FALL OUTSIDE THE          *
C                CONTROL LIMITS                                            *
C      T      -- AVERAGE COST OF LOOKING FOR AN ASSIGNABLE CAUSE           *
C                WHEN NONE EXISTS                                          *
C      W      -- AVERAGE COST OF FINDING AN ASSIGNABLE CAUSE WHEN          *
C                ONE DOES EXIST                                            *
C      B      -- COST PER SUBGROUP OF SAMPLING, INSPECTING,                *
C                EVALUATING, AND PLOTTING THAT IS INDEPENDENT              *
C                OF SUBGROUP SIZE                                          *
C       C     -- COST PER UNIT OF MEASURING AN ITEM OF PRODUCT             *
C                AND OTHER CONTROL CHART OPERATIONS DIRECTLY TO            *
C                SIZE OF THE SUBGROUP                                      *
C      ULMT   -- STANDARDIZE UPPER CONTROLd!YMIT                           *
C      DLMT   --  "     "     LOWER    "      "                            *
C      PJ     -- PROB. OF DECT. FOR N-J SAMPLES OUT OF CONTROL             *
C      PPRIME -- P'                                                        *
C      PJI    -- PROBABILITY OF DETECTION WHEN A SHIFT OCCURS              *
C                AFTER THE jTH SAMPLE IS TAKEN AND i SAMPLES ARE           *
C                TAKEN FROM THE PROCESS WHILE OPERATING AT                 *
C                SHIFTED PROCESS                                           *
C      QJI    -- 1 - PJI                                                   *
C      AOOCT  -- AVERAGE OUT OF CONTROL TIME                               *
C      ACL    -- AVERAGE CYCLE LENGTH                                      *
C      ENFA   -- EXPECTED NUMBER OF FALSE ALARMS PER OPERATION             *
C                HOUR                                                      *
C      ACFAC  -- AVERAGE COST OF FINDING THE ASSIGNABLE CAUSE              *
C                PER HOUR                                                  *
C      HCMCC  -- HOURLY COST OF MAINTAINING THE CONTROL CHART              *
C      ELOSSC -- LOSS COST PER HOUR OF OPERATION                           *
C      PNOT   -- PROB. OF OUT OF CONTROL DETECTION WHEN SHIFT              *
C                OCCURRED                                                  *
C      QNOT   -- PROB. OF NO DETECTION WHEN SHIFT OCCURRED                 *
C      ENSBSD -- EXPECTED NUMBER OF SUBGROUPS TAKEN BETWEEN THE            *
C                TIME THE PROCESS SHIFTS OOC AND SUBGROUP IS               *
C                COMPLETED BEFORE A SHIFT IN THE PROCESS IS                *
```

```
C               DETECTED                                            *
C                                                                   *
C********************************************************************
C
C    MAIN PROGRAM
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON / LOGIO / IR,IW
C
C----ASSIGN LOGICAL INPUT AND OUTPUT UNIT NUMBER
C
      IR=5
      IW=6
C
C----PROMPT THE MAIN MENU
C
  1   WRITE(IW,10)
  10  FORMAT(1H1,12X,24(1H*),/,13X,'***    MAIN  MENU     ***',/,13X,24(
     *1H*),/,/,5X,'1. DESIGN OF ECONOMICALLY-BASED CONTROL CHARTS,',/,
     *         5X,'2. EVALUATION OF ECONOMICALLY-BASED CONTROL CHARTS,'
     *      ,/,5X,'3. EXIT THE PROGRAM.',/,/,
     *         5X,'==> ENTER THE OPTION NUMBER PLEASE!')
C
      READ(IR,*) MENU
      GO TO (100,200,300) MENU
      WRITE(IW,20)
  20  FORMAT(/,5X,'??? ENTERED NUMBER ERROR ???  TRY IT AGAIN!')
      GO TO 1
 100  CALL ECDSGN
      GO TO 1
 200  CALL ECEVAL
      GO TO 1
 300  STOP
      END
C
C********************************************************************
C
      SUBROUTINE ECDSGN
C
C********************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON / LOGIO / IR,IW
C
C----PROMPT THE CONTROL DESIGN MENU
C
  1   WRITE(IW,10)
  10  FORMAT(1H1,4X,54(1H*),/,
     * 5X,'* CONTROL CHART DESIGN FOR CONTINUOUS FLOW PROCESSES *',/,
     * 5X,54(1H*),/,/,
     *         5X,'1. ECONOMICALLY-BASED DESIGN OF X-BAR CHART,',/,
     *         5X,'2. ECONOMICALLY-BASED DESIGN OF MA    CHART,',/,
     *         5X,'3. ECONOMICALLY-BASED DESIGN OF I     CHART,',/,
     *         5X,'4. DESIGN OF DUNCAN''S ECONOMIC  X-BAR CHART,',/,
     *         5X,'           ( FOR DISCRETE PROCESSES )',/,
     *         5X,'5. RETURN TO MAIN MENU.',/,/,
     *         5X,'==> ENTER THE OPTION NUMBER PLEASE!')
C
      READ(IR,*) MENU
      GO TO (100,200,300,400,500) MENU
      WRITE(IW,20)
  20  FORMAT(/,5X,'??? ENTERED NUMBER ERROR ???  TRY IT AGAIN!')
      GO TO 1
 100  CALL ECXBAR
      GO TO 1
 200  CALL ECMA
      GO TO 1
 300  CALL ECIND
      GO TO 1
 400  CALL ECDUNC
      GO TO 1
 500  RETURN
```

```
      END
C
C***************************************************************
C
      SUBROUTINE ECEVAL
C
C***************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON / LOGIO / IR,IW
C
  1   WRITE(IW,10)
  10  FORMAT(1H1,4X,58(1H*),/,
     * 5X,'* CONTROL CHART EVALUATION FOR CONTINUOUS FLOW PROCESSES *',
     * /,5X,58(1H*),/,/,
     *          5X,'1. EVALUATION OF ECONOMICALLY-BASED X-BAR CHART,',/,
     *          5X,'2. EVALUATION OF ECONOMICALLY-BASED MA    CHART,',/,
     *          5X,'3. EVALUATION OF ECONOMICALLY-BASED I     CHART,',/,
     *          5X,'4. EVALUATION OF DUNCAN''S  X-BAR CONTROL  CHART,',/
     *          ,5X,'           ( FOR DISCRETE PROCESSES )',/,
     *          5X,'5. RETURN TO MAIN MENU.',/,/,
     *          5X,'==> ENTER THE OPTION NUMBER PLEASE!')
      READ(IR,*) MENU
      GO TO (100,200,300,400,500) MENU
      WRITE(IW,20)
  20  FORMAT(/,5X,'??? ENTERED NUMBER ERROR ???  TRY IT AGAIN!')
      GO TO 1
  100 CALL EVXBAR
      GO TO 1
  200 CALL EVMA
      GO TO 1
  300 CALL EVIND
      GO TO 1
  400 CALL EVDUNC
      GO TO 1
  500 RETURN
      END
C
C***************************************************************
C
      SUBROUTINE ECXBAR
C
C***************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON / LOGIO / IR,IW
      REAL*8 LAMBDA
      COMMON / EC1 / LAMBDA
      COMMON / EC2 / N,NOPT
      COMMON / EC3 / H,EK,HOPT,EKOPT,FOPT,STEP(20)
      COMMON / EC4 / DELTA,B,C,D,E,EM,T,W
C
C-------INPUT COST AND RISK PARAMETERS
C
  101 WRITE(IW,110)
  110 FORMAT(/,5X,'==> FOR ECONOMIC X-BAR CHART DESIGN, INPUT VALUES OF
     *',/,5X,'==> DELTA, LAMBDA, M, E, D, T, W, B, C')
      READ(IR,*)DELTA,LAMBDA,EM,E,D,T,W,B,C
C
C-------ECHO PRINT OUT INPUT DATA
C
  115 WRITE(IW,120)DELTA,LAMBDA,EM,E,D,T,W,B,C
  120 FORMAT(/,5X,'** VALUES RECEIVED ARE AS FOLLOWS:',/,
     *  5X,'DELTA = ',F10.4,7X,' LAMBDA = ',F10.4,/,
     *  5X,'  M   = ',F10.4,7X,'   E     = ',F10.4,/,
     *  5X,'  D   = ',F10.4,7X,'   T     = ',F10.4,/,
     *  5X,'  W   = ',F10.4,7X,'   B     = ',F10.4,/,
     *  5X,'  C   = ',F10.4,/,/,5X,'*** ARE THESE DATA CORRECT? ***',
     */,5X,'==> PLEASE ENTER 1 = YES, 2 = NO  <<<')
      READ(IR,*) INQUR
      GO TO (125,101),INQUR
      GO TO 115
```

```
C
C------SELECT THE STARTING POINT
C
  125 N=5
      H=1.DO
      EK=3.DO
  127 WRITE(IW,130)N,H,EK
  130 FORMAT(//,5X,'*** THE FOLLOWING STARTING POINT IS SUGGESTED',
     * /,5X,'*** FOR ECONOMIC OPTIMIZATION OF X-BAR CHART.',
     * /,5X,'      N = ',I2,'    H = ',F6.2,'   K = ',F6.2,/,/,
     *   5X,' ==>  DO YOU ACCEPT THIS POINT?',/,
     *   5X,' ==>  ENTER 1 = YES, 2 = NO. <<<')
      READ(IR,*) INQUR
      GO TO (135,170),INQUR
      GO TO 127
C
C------SELECCT THE STEP SIZES
C
  135 STEP(1)=0.5
      STEP(2)=0.5
      STEP(3)=1.0
  137 WRITE(IW,139)STEP(3),STEP(1),STEP(2)
  139 FORMAT(//,5X,'*** THE FOLLOWING STEP SIZES ARE SUGGESTED',
     *  /,5X,'*** FOR ECONOMIC OPTIMIZATION OF X-BAR CHART.',
     *  /,5X,'      N = ',F5.2,'    H = ',F6.2,'   K = ',F6.2,/,/,
     *    5X,' ==>  DO YOU ACCEPT THESE STEP SIZES?',/,
     *    5X,' ==>  ENTER 1 = YES, 2 = NO. <<<')
      READ(IR,*) INQUR
      GO TO (145,180),INQUR
      GO TO 137
C
C------PERFORM THE ECONOMICALLY-BASED X-BAR CHART DESIGN
C
  145 CALL OPXBAR
C
C-------PRINT OUT OPTIMAL X-BAR CHART DESIGN
C
  146 WRITE(IW,150)NOPT,HOPT,EKOPT,FOPT
  150 FORMAT(/,5X,58(1H=),
     *   /,5X,'*** THE OPTIMAL X-BAR CHART DESIGN IS ',
     *   /,5X,'N = ',I4,',',3X,' H = ',F10.5,',',3X,'  K =',F10.5,
     *   /,5X,'*** THE MINIMUM LOSS COST PER HOUR IS',F14.6,/,
     *   /,5X,58(1H=))
      RETURN
C
C------RESELECT STARTING POINT
C
  170 WRITE(IW,175)
  175 FORMAT(/,5X,'*** INPUT THE STARTING POINT YOU WANT ***',/,
     *            5X,'==> KEY IN THE VALUE FOR N,H,K')
      READ(IR,*)N,H,EK
  185 WRITE(IW,190)N,H,EK
  190 FORMAT(/,5X,'*** STARTING POINT SELECTED IS N =',I4,5X,
     * 'H =',F8.4,5X,'K =',F8.4,/
     * ,5X,'*** ARE THEY CORRECT?   ==> ENTER 1 = YES, 2 = NO')
      READ(IR,*) INQUR
      GO TO (135,170),INQUR
      GO TO 185
C
C-------RESELECT STEP SIZES OF N, H, K
C
  180 WRITE(IW,195)
  195 FORMAT(/,5X,'*** INPUT THE STEP SIZES YOU WANT ***',/,
     *            5X,'==> ENTER STEP SIZES OF N,H,K')
      READ(IR,*)STEP(3),STEP(1),STEP(2)
  196 WRITE(IW,197)STEP(3),STEP(1),STEP(2)
  197 FORMAT(/,5X,'*** STEP SIZES ENTERED ARE N =',F5.2,5X,
     * 'OF H =',F6.2,5X,'OF K =',F6.2,/
     *          ,5X,'*** ARE THEY CORRECT?   ==> ENTER 1 = YES, 2 = NO')
      READ(IR,*) INQUR
      GO TO (145,180),INQUR
      GO TO 196
```

```
      END
C
C*************************************************************
C
      SUBROUTINE OPXBAR
C
C*************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON / LOGIO / IR,IW
      COMMON / EC2 / N,NOPT
      COMMON / EC3 / H,EK,HOPT,EKOPT,FOPT,STEP(20)
      COMMON / SAMPLE / IX
      EXTERNAL FUNCT3
      EXTERNAL FUNCT2
      REAL*8 FMIN(5)
      REAL*8 X(3),XMIN(20),XSEC(20),F
C
C------ASSIGN VARIABL NUMBER, SEARCH STEP, AND TERMINATE VALUE
C
      ND=3
      ICOUNT=500
      REQMIN=0.001
C
C------ASSIGN STARTING POINT
C
      X(1)=H
      X(2)=EK
      X(3)=DFLOAT(N)
C
      CALL NELMIN(FUNCT3,ND,X,XMIN,XSEC,YNEWLO,YSEC,REQMIN,STEP,ICOUNT)
C
C------PRINT OUT THE OPTIMAL POINT FOUND
C
      WRITE(IW,133)XMIN(3),XMIN(1),XMIN(2),YNEWLO
  133 FORMAT(/,5X,'*** THE OPTIMAL POINT FOUND IS  ***',/,/,
     *          5X,' N = ',F7.4,', H = ',F7.4,', K = ',F7.4,
     * ',  LOSS COST = ',F14.6,/)
C
C------ASSIGN VARIABL NUMBER, SEARCH STEP, AND TERMINATE VALUE
C
      ND=2
      ICOUNT=500
      REQMIN=0.001
C
C------ASSIGN STARTING POINT
C
      IX=XMIN(3)
      X(1)=XMIN(1)
      X(2)=XMIN(2)
C
      CALL NELMIN(FUNCT2,ND,X,XMIN,XSEC,YNEWLO,YSEC,REQMIN,STEP,ICOUNT)
C
      I=1
C
C-----PRINT SEARCH ITERATIONS
C
      WRITE(IW,140)
  140 FORMAT(/,5X,'*** OPTIMIZATION ITERATIONS ***',/,/,
     *          5X,' I',T12,'N',T18,'H',T28,'K',T38,'LOSS COST',/,
     *          5X,44(1H-),/)
        WRITE(IW,141)I,IX,XMIN(1),XMIN(2),YNEWLO
  141   FORMAT(5X,I2,T10,I3,T15,F7.4,T25,F7.4,T35,F14.6)
C
      INCR=1
      ITIME=0
C
C-------KEEP THE POINT AS THE BEST OPTIMUM SO FAR
C
 10   FMIN(5)=YNEWLO
      DO 11 L=1,2
          FMIN(L)=XMIN(L)
```

```
 11    CONTINUE
 12    IX=IX+INCR
 15    X(1)=XMIN(1)
       X(2)=XMIN(2)
       ND=2
       ICOUNT=500
       REQMIN=0.001
C
       CALL NELMIN(FUNCT2,ND,X,XMIN,XSEC,YNEWLO,YSEC,REQMIN,STEP,ICOUNT)
C
       I=I+1
C
C-----PRINT SEARCH ITERATIONS
C
       WRITE(IW,141)I,IX,XMIN(1),XMIN(2),YNEWLO
C
       IF (ITIME.EQ.1) GO TO 40
       IF (YNEWLO .GT. FMIN(5)) GO TO 30
         ITIME=1
         FMIN(5)=YNEWLO
         DO 20 L=1,2
            FMIN(L)=XMIN(L)
  20   CONTINUE
       GO TO 12
  30   INCR=-INCR
       IX=IX-2
       ITIME=1
       GO TO 15
  40   IF (YNEWLO.LE.FMIN(5)) GO TO 10
       IXMIN=IX-INCR
       NOPT=IXMIN
       HOPT=FMIN(1)
       EKOPT=FMIN(2)
       FOPT=FMIN(5)
       RETURN
       END
C
C***************************************************************
C
       SUBROUTINE FUNCT3(X,F)
C
C***************************************************************
C
       IMPLICIT REAL*8(A-H,O-Z)
       REAL*8 LAMBDA
       COMMON / EC1 / LAMBDA
       COMMON / EC4 / DELTA,B,C,D,E,EM,T,W
       REAL*8 X(3)
C
       H=X(1)
       DK=X(2)
       DN=X(3)
C
C-------SET LOWER AND UPER BOUNDS FOR H AND K
C-------SET LOWER BOUND FOR N
C
       IF(H.GT.70..OR.H.LE.O.)F=100000000.
       IF(H.GT.70..OR.H.LE.O.)RETURN
       IF(DK.GT.8..OR.DK.LE.O.)F=100000000.
       IF(DK.GT.8..OR.DK.LE.O.)RETURN
       IF(DN.LT.1.)F=100000000.
       IF(DN.LT.1.)RETURN
       IN=X(3)
C
       EXPLH=DEXP(-LAMBDA*H)
       EXPLNH=DEXP(-LAMBDA*H*DN)
       PPRIME=0.DO
C
       DO 10 I=1,IN
          DI=DFLOAT(I)
          ULMT=DK-(DN-DI+1.DO)*DELTA/DSQRT(DN)
          DLMT=-DK-(DN-DI+1.DO)*DELTA/DSQRT(DN)
```

```
         PJ=1.DO-DNML(ULMT)+DNML(DLMT)
         PPRIME=PPRIME+DEXP(-LAMBDA*(DI-1.DO)*H)*PJ
   10 CONTINUE
      PPRIME=PPRIME*(1.DO-EXPLH)/(1.DO-EXPLNH)
      QPRIME=1.DO-PPRIME
      PNOT=1.DO-DNML(DK-DELTA*DSQRT(DN))+DNML(-DK-DELTA*DSQRT(DN))
      QNOT=1.DO-PNOT
C
      AOOCT=DN*H*(PPRIME+QPRIME/(PNOT*QNOT)-QPRIME*PNOT/QNOT)
     *      -(1.DO-(1.DO+LAMBDA*DN*H)*EXPLNH)/(LAMBDA*(1.DO-EXPLNH))
C
      ACL=1.DO/LAMBDA+AOOCT+E+D
      ALPHA=2.DO*(1.DO-DNML(DK))
C
      ENFA= (ALPHA*EXPLNH/(1.DO-EXPLNH))/ACL
C
      ACFAC=W/ACL
C
      HCMCC=B/(DN*H)+C/H
C
      ELOSSC=EM*(1.DO-1.DO/(LAMBDA*ACL))+ENFA*T+ACFAC+HCMCC
      F=ELOSSC
      RETURN
      END
C
C*****************************************************************
C
      SUBROUTINE FUNCT2(X,F)
C
C*****************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 LAMBDA
      COMMON / EC1 / LAMBDA
      COMMON / EC4 / DELTA,B,C,D,E,EM,T,W
      COMMON / SAMPLE / IX
      REAL*8 X(2)
C
      H=X(1)
      DK=X(2)
      DN=DFLOAT(IX)
C
C-------SET LOWER AND UPER BOUNDS FOR H AND K
C-------SET LOWER BOUND FOR N
C
      IF(H.GT.70..OR.H.LE.O.)F=100000000.
      IF(H.GT.70..OR.H.LE.O.)RETURN
      IF(DK.GT.8..OR.DK.LE.O.)F=100000000.
      IF(DK.GT.8..OR.DK.LE.O.)RETURN
      IF(DN.LT.1.)F=100000000.
      IF(DN.LT.1.)RETURN
      IN=IX
C
      EXPLH=DEXP(-LAMBDA*H)
      EXPLNH=DEXP(-LAMBDA*H*DN)
      PPRIME=O.DO
C
      DO 10 I=1,IN
         DI=DFLOAT(I)
         ULMT=DK-(DN-DI+1.DO)*DELTA/DSQRT(DN)
         DLMT=-DK-(DN-DI+1.DO)*DELTA/DSQRT(DN)
         PJ=1.DO-DNML(ULMT)+DNML(DLMT)
         PPRIME=PPRIME+DEXP(-LAMBDA*(DI-1.DO)*H)*PJ
   10 CONTINUE
      PPRIME=PPRIME*(1.DO-EXPLH)/(1.DO-EXPLNH)
      QPRIME=1.DO-PPRIME
      PNOT=1.DO-DNML(DK-DELTA*DSQRT(DN))+DNML(-DK-DELTA*DSQRT(DN))
      QNOT=1.DO-PNOT
C
      AOOCT=DN*H*(PPRIME+QPRIME/(PNOT*QNOT)-QPRIME*PNOT/QNOT)
     *      -(1.DO-(1.DO+LAMBDA*DN*H)*EXPLNH)/(LAMBDA*(1.DO-EXPLNH))
C
```

```
      ACL=1.DO/LAMBDA+AOOCT+E+D
      ALPHA=2.DO*(1.DO-DNML(DK))
C
      ENFA= (ALPHA*EXPLNH/(1.DO-EXPLNH))/ACL
C
      ACFAC=W/ACL
C
      HCMCC=B/(DN*H)+C/H
C
      ELOSSC=EM*(1.DO-1.DO/(LAMBDA*ACL))+ENFA*T+ACFAC+HCMCC
      F=ELOSSC
      RETURN
      END
C
C****************************************************************
C
      SUBROUTINE EVXBAR
C
C****************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON / LOGIO / IR,IW
      REAL*8 LAMBDA
      COMMON / EC1 / LAMBDA
      COMMON / EC4 / DELTA,B,C,D,E,EM,T,W
      REAL*8 X(3)
C
C-------INPUT PARAMETERS
C
  101 WRITE(IW,110)
  110 FORMAT(/,5X,'==> FOR EVALUATION OF ECONOMIC X-BAR CHART, INPUT'
     *  ,/,5X,'==> VALUES OF DELTA, LAMBDA, M, E, D, T, W, B, C')
      READ(IR,*)DELTA,LAMBDA,EM,E,D,T,W,B,C
C
C-------ECHO PRINT OUT INPUT DATA
C
  115 WRITE(IW,120)DELTA,LAMBDA,EM,E,D,T,W,B,C
  120 FORMAT(/,5X,'** VALUES RECEIVED ARE AS FOLLOWS:',/,
     *  5X,'DELTA = ',F10.4,7X,' LAMBDA = ',F10.4,/,
     *  5X,'  M   = ',F10.4,7X,'   E    = ',F10.4,/,
     *  5X,'  D   = ',F10.4,7X,'   T    = ',F10.4,/,
     *  5X,'  W   = ',F10.4,7X,'   B    = ',F10.4,/,
     *  5X,'  C   = ',F10.4,/,/,5X,'*** ARE THESE DATA CORRECT? ***',
     */,5X,'==> PLEASE ENTER 1 = YES, 2 = NO  <<<')
      READ(IR,*) INQUR
      GO TO (127,101),INQUR
      GO TO 115
C
C-------INPUT THE VALUES OF N, H, AND K FOR THE SPECIFIED X-BAR CHART
C
  127 WRITE(IW,175)
  175 FORMAT(/,5X,'*** FOR THE SPECIFIC X-BAR CHART TO EVALUATE **',/,
     *          5X,'==> INPUT THE VALUES OF N, H, AND K')
      READ(IR,*)X(3),X(1),X(2)
C
C-------CONFIRM THE INPUT VALUES
C
  185 WRITE(IW,190)X(3),X(1),X(2)
  190 FORMAT(/,5X,'*** VALUES ENTERED ARE N =',F6.2,5X,
     * 'H =',F8.4,5X,'K =',F8.4,/
     * ,5X,'*** ARE THEY CORRECT?    ==> ENTER 1 = YES, 2 = NO')
      READ(IR,*) INQUR
      GO TO (145,127),INQUR
      GO TO 185
C
  145 CALL FUNCT3(X,F)
C
C-------PRINT OUT LOSS-COST OF SPECIFIED X-BAR CHART
C
  146 WRITE(IW,150)X(3),X(1),X(2),F
  150 FORMAT(/,5X,58(1H=),
     *  /,5X,'*** THE SPECIFIED X-BAR CHART DESIGN IS ',
```

```
     *  /,5X,'N = ',F6.2,',',3X,' H = ',F10.5,',',3X,'  K =',F10.5,
     *  /,5X,'*** AND THE HOURLY LOSS COST FOR THIS DESIGN IS',
     *  F10.5,/,5X,58(1H=))
      RETURN
      END
C
C*****************************************************************
C
      SUBROUTINE ECMA
C
C*****************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON / LOGIO / IR,IW
      REAL*8 LAMBDA
      COMMON / EC1 / LAMBDA
      COMMON / EC2 / N,NOPT
      COMMON / EC3 / H,EK,HOPT,EKOPT,FOPT,STEP(20)
      COMMON / EC4 / DELTA,B,C,D,E,EM,T,W
C
C-----INPUT PARAMETERS
C
  101 WRITE(IW,110)
  110 FORMAT(/,5X,'==> FOR ECONOMIC MA CHART DESIGN, INPUT VALUES OF'
     *  ,/,5X,'==> DELTA, LAMBDA, M, E, D, T, W, B, C')
      READ(IR,*)DELTA,LAMBDA,EM,E,D,T,W,B,C
C
C-----ECHO PRINT OUT INPUT DATA
C
  115 WRITE(IW,120)DELTA,LAMBDA,EM,E,D,T,W,B,C
  120 FORMAT(/,5X,'** VALUES RECEIVED ARE AS FOLLOW:',/,
     *  5X,'DELTA = ',F10.4,7X,' LAMBDA = ',F10.4,/,
     *  5X,'  M   = ',F10.4,7X,'   E    = ',F10.4,/,
     *  5X,'  D   = ',F10.4,7X,'   T    = ',F10.4,/,
     *  5X,'  W   = ',F10.4,7X,'   B    = ',F10.4,/,
     *  5X,'  C   = ',F10.4,/,/,5X,'*** ARE THESE DATA CORRECT? ***',
     */,5X,'==> PLEASE ENTER 1 = YES, 2 = NO  <<<')
C
      READ(IR,*) INQUR
      GO TO (125,101),INQUR
      GO TO 115
C
C-----SELECT STARTING POINT
C
  125 N=5
      H=1.DO
      EK=3.DO
  127 WRITE(IW,130)N,H,EK
  130 FORMAT(/,5X,'*** THE FOLLOWING STARTING POINT IS SUGGESTED',
     * /,5X,'*** FOR ECONOMIC OPTIMIZATION OF MA CHART.',
     * /,5X,'     N = ',I2,6X,'H = ',F6.2,6X,'K = ',F6.2,/,/,
     *   5X,' ==>  DO YOU ACCEPT THIS STARTING POINT?',/,
     *   5X,' ==>  ENTER 1 = YES, 2 = NO. <<<')
      READ(IR,*) INQUR
      GO TO (135,170),INQUR
      GO TO 127
C
C------SELECT STEP SIZES
C
  135 STEP(1)=0.5
      STEP(2)=0.5
      STEP(3)=1.0
  137 WRITE(IW,139)STEP(3),STEP(1),STEP(2)
  139 FORMAT(//,5X,'*** THE FOLLOWING STEP SIZES ARE SUGGESTED',
     * /,5X,'*** FOR ECONOMIC OPTIMIZATION OF MA CHART.',
     * /,5X,'     N = ',F5.2,'    H = ',F6.2,'  K = ',F6.2,/,/,
     *   5X,' ==>  DO YOU ACCEPT THESE STEP SIZES?',/,
     *   5X,' ==>  ENTER 1 = YES, 2 = NO. <<<')
      READ(IR,*) INQUR
      GO TO (145,180),INQUR
      GO TO 137
C
```

```
  145 CALL OPMA
C
C-----PRINT OUT THE OPTIMAL MA CHART DESIGN
C
  146 WRITE(IW,150)NOPT,HOPT,EKOPT,FOPT
  150 FORMAT(/,5X,58(1H=),
     * /,5X,'*** THE OPTIMAL MA CHART DESIGN IS',
     * /,5X,'N = ',I4,6X,'H = ',F10.4,6X,'K =',F10.4,
     * /,5X,'*** THE MINIMUM LOSS COST PER HOUR IS',F14.6,/,
     * /,5X,58(1H=))
      RETURN
C
C------RESELECT STARTING POINT
C
 170  WRITE(IW,175)
  175 FORMAT(/,5X,'*** SELECT THE STARTING POINT YOU WANT ***',/,
     *           5X,'==> KEY IN THE VALUES OF N,H,K')
      READ(IR,*)N,H,EK
  185 WRITE(IW,190)N,H,EK
  190 FORMAT(/,5X,'*** STARTING POINT SELECTED IS  N =',I3,',',3X,
     *     'H =',F6.2,',',3X,'AND  K =',F6.2,/
     *     ,5X,' ARE THEY CORRECT?   ==> ENTER 1 = YES, 2 = NO')
      READ(IR,*) INQUR
      GO TO (135,170),INQUR
      GO TO 185
C
C------RESELECT STEP SIZES OF N, H, K
C
  180 WRITE(IW,195)
  195 FORMAT(/,5X,'*** INPUT THE STEP SIZES YOU WANT ***',/,
     *           5X,'==> ENTER STEP SIZES OF N,H,K')
      READ(IR,*)STEP(3),STEP(1),STEP(2)
  196 WRITE(IW,197)STEP(3),STEP(1),STEP(2)
  197 FORMAT(/,5X,'*** STEP SIZES ENTERED ARE  N =',F5.2,',',3X,
     * ' H =',F6.2,',',3X,'AND  K =',F6.2,/
     *          ,5X,'*** ARE THEY CORRECT?   ==> ENTER 1 = YES, 2 = NO')
      READ(IR,*) INQUR
      GO TO (145,180),INQUR
      GO TO 196
      END
C
C*****************************************************************
C
      SUBROUTINE OPMA
C
C*****************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON / LOGIO / IR,IW
      COMMON / EC2 / N,NOPT
      COMMON / EC3 / H,EK,HOPT,EKOPT,FOPT,STEP(20)
      COMMON / SAMPLE / IX
      EXTERNAL FNMA3
      EXTERNAL FNMA2
      REAL*8 FMIN(5)
      REAL*8 X(3),XMIN(20),XSEC(20),F
C
C------ASSIGN VARIABL NUMBER. SEARCH STEP. AND TERMINATE VALUE
C
      ND=3
      ICOUNT=500
      REQMIN=0.001
C
C------ASSIGN STARTING POINT
C
      X(1)=H
      X(2)=EK
      X(3)=DFLOAT(N)
C
      CALL NELMIN(FNMA3,ND,X,XMIN,XSEC,YNEWLO,YSEC,REQMIN,STEP,ICOUNT)
C
      WRITE(IW,133)XMIN(3),XMIN(1),XMIN(2),YNEWLO
```

```
  133 FORMAT(/,5X,'*** THE OPTIMAL POINT FOUND IS  ***',/,/,
     *          5X,' N = ',F7.4,', H = ',F7.4,', K = ',F7.4,
     * ',  LOSS COST = ',F14.6,/)
C
C------ASSIGN VARIABL NUMBER, SEARCH STEP, AND TERMINATE VALUE
C
      ND=2
      ICOUNT=500
      REQMIN=0.001
C
C------ASSIGN STARTING POINT
C
      IX=XMIN(3)
      X(1)=XMIN(1)
      X(2)=XMIN(2)
C
      CALL NELMIN(FNMA2,ND,X,XMIN,XSEC,YNEWLO,YSEC,REQMIN,STEP,ICOUNT)
C
      I=1
C
C-----PRINT SEARCH ITERATIONS
C
      WRITE(IW,140)
  140 FORMAT(/,5X,'*** OPTIMIZATION ITERATIONS ***',/,/,
     *          5X,' I',T10,'N',T16,'H',T26,'K',T36,'LOSS COST',/)
        WRITE(IW,141)I,IX,XMIN(1),XMIN(2),YNEWLO
  141   FORMAT(5X,I2,T8,I3,T13,F7.4,T23,F7.4,T33,F14.6)
C
      INCR=1
      ITIME=0
C
C-----KEEP THE POINT AS THE BEST OPTIMUM SO FAR
C
 10   FMIN(5)=YNEWLO
      DO 11 L=1,2
         FMIN(L)=XMIN(L)
 11   CONTINUE
 12   IX=IX+INCR
C
C------ASSIGN THE STARTING POINT AND STEP SIZES
C
 15   X(1)=XMIN(1)
      X(2)=XMIN(2)
      ND=2
      ICOUNT=500
      REQMIN=0.001
C
      CALL NELMIN(FNMA2,ND,X,XMIN,XSEC,YNEWLO,YSEC,REQMIN,STEP,ICOUNT)
C
      I=I+1
C
C-----PRINT SEARCH ITERATIONS
C
      WRITE(IW,141)I,IX,XMIN(1),XMIN(2),YNEWLO
C
      IF (ITIME.EQ.1) GO TO 40
      IF (YNEWLO .GT. FMIN(5)) GO TO 30
         ITIME=1
         FMIN(5)=YNEWLO
         DO 20 L=1,2
            FMIN(L)=XMIN(L)
  20  CONTINUE
      GO TO 12
  30  INCR=-INCR
      IX=IX-2
      ITIME=1
      GO TO 15
  40  IF (YNEWLO .LE. FMIN(5)) GO TO 10
      IXMIN=IX-INCR
      NOPT=IXMIN
      HOPT=FMIN(1)
      EKOPT=FMIN(2)
```

```
      FOPT=FMIN(5)
      RETURN
      END
C
C***************************************************************
C
      SUBROUTINE FNMA3(X,F)
C
C***************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 LAMBDA
      COMMON / EC1 / LAMBDA
      COMMON / EC4 / DELTA,B,C,D,E,EM,T,W
      REAL*8 X(3)
C
      H=X(1)
      DK=X(2)
      DN=X(3)
C
C-----SET THE LOWER AND UPPER BOUNDS FOR H AND K
C-----AND LOWER BOUND FOR N
C
      IF(H.GT.70..OR.H.LE.O.)F=100000000.
      IF(H.GT.70..OR.H.LE.O.)RETURN
      IF(DK.GT.8..OR.DK.LE.O.)F=100000000.
      IF(DK.GT.8..OR.DK.LE.O.)RETURN
      IF(DN.LT.1.)F=100000000.
      IF(DN.LT.1.)RETURN
C
      IN=X(3)
C
      EXPLH=DEXP(-LAMBDA*H)
C
      PNOT=1.DO-DNML(DK-DELTA*DSQRT(DN))+DNML(-DK-DELTA*DSQRT(DN))
      QNOT=1.DO-PNOT
      ANSBSD=O.DO
C
      IF (IN.LT.3) GO TO 11
      DO 10 J=1,IN-1
         DJ=DFLOAT(J)
         TEMP=1.DO-DNML(DK-DELTA/DSQRT(DJ))+DNML(-DK-DELTA/DSQRT(DJ))
         QJI=1.DO-TEMP
         DO 8 I=2,IN-1
            DJI=DFLOAT(J+I-1)
            DI=DFLOAT(I)
            IF(DJI.GT.DN) DJI=DN
            PJI=1.DO-DNML(DK-(DI*DELTA)/DSQRT(DJI))+
     *               DNML(-DK-(DI*DELTA)/DSQRT(DJI))
            TEMP=TEMP+DFLOAT(I)*PJI*QJI
            QJI=QJI*(1.DO-PJI)
  8      CONTINUE
         TEMP=TEMP+QJI*(DN+QNOT/PNOT)
         ANSBSD=ANSBSD+TEMP*DEXP(-DFLOAT(J-1)*LAMBDA*H)
 10   CONTINUE
      ANSBSD=ANSBSD*(1.DO-EXPLH)
      TEMP=1.DO-DNML(DK-DELTA/DSQRT(DN))+DNML(-DK-DELTA/DSQRT(DN))
         QJI=1.DO-TEMP
         DO 12 I=2,IN-1
             DI=DFLOAT(I)
             PJI=1.DO-DNML(DK-(DI*DELTA)/DSQRT(DN))+
     *               DNML(-DK-(DI*DELTA)/DSQRT(DN))
            TEMP=TEMP+DI*PJI*QJI
            QJI=QJI*(1.DO-PJI)
 12      CONTINUE
         TEMP=TEMP+QJI*(DN+QNOT/PNOT)
         ANSBSD=ANSBSD+TEMP*DEXP(-(DN-1.DO)*LAMBDA*H)
         GO TO 20
C
C-----IF N < 3
C
 11   IF (IN.LT.2) GO TO 18
```

```
      PO1=1.DO-DNML(DK-DELTA)+DNML(-DK-DELTA)
      P11=1.DO-DNML(DK-DELTA/DSQRT(2.DO))+DNML(-DK-DELTA/DSQRT(2.DO))
      ANSBSD=(1.DO-EXPLH)*(PO1+(1.DO-PO1)*(DN+QNOT/PNOT))
     *        +EXPLH*(P11+(1.DO-P11)*(DN+QNOT/PNOT))
      GOTO 20
C
C-----IF N < 2
C
   18 ANSBSD=1.DO/PNOT
C
   20 AOOCT=H*ANSBSD
     *       -(1.DO-(1.DO+LAMBDA*H)*EXPLH)/(LAMBDA*(1.DO-EXPLH))
C
      ACL=1.DO/LAMBDA+AOOCT+E+D
      ALPHA=2.DO*(1.DO-DNML(DK))
C
      ENFA= (ALPHA*EXPLH/(1.DO-EXPLH))/ACL
C
      ACFAC=W/ACL
C
      HCMCC=(B+C)/H
C
      ELOSSC=EM*(1.DO-1.DO/(LAMBDA*ACL))+ENFA*T+ACFAC+HCMCC
      F=ELOSSC
      RETURN
      END
C
C*****************************************************************
C
      SUBROUTINE EVMA
C
C*****************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON / LOGIO / IR,IW
      REAL*8 LAMBDA
      COMMON / EC1 / LAMBDA
      COMMON / EC4 / DELTA,B,C,D,E,EM,T,W
      REAL*8 X(3)
C
C-------INPUT PARAMETERS
C
  101 WRITE(IW,110)
  110 FORMAT(/,5X,'==> FOR EVALUATION OF ECONOMIC MA CHART, INPUT'
     *  ,/,5X,'==> VALUES OF DELTA, LAMBDA, M, E, D, T, W, B, C')
      READ(IR,*)DELTA,LAMBDA,EM,E,D,T,W,B,C
C
C-------ECHO PRINT OUT AND CONFIRM INPUT DATA
C
  115 WRITE(IW,120)DELTA,LAMBDA,EM,E,D,T,W,B,C
  120 FORMAT(/,5X,'** VALUES RECEIVED ARE AS FOLLOWS:',/,
     *  5X,'DELTA = ',F10.4,7X,' LAMBDA = ',F10.4,/,
     *  5X,'  M   = ',F10.4,7X,'   E    = ',F10.4,/,
     *  5X,'  D   = ',F10.4,7X,'   T    = ',F10.4,/,
     *  5X,'  W   = ',F10.4,7X,'   B    = ',F10.4,/,
     *  5X,'  C   = ',F10.4,/,/,5X,'*** ARE THESE DATA CORRECT? ***',
     * /,5X,'==> PLEASE ENTER 1 = YES, 2 = NO  <<<')
      READ(IR,*) INQUR
      GO TO (127,101),INQUR
      GO TO 115
C
C-------INPUT THE VALUES OF N, H, AND K FOR THE SPECIFIED MA CHART
C
  127 WRITE(IW,175)
  175 FORMAT(/,5X,'*** FOR THE SPECIFIC MA CHART TO EVALUATE **',/,
     * 5X,'==> INPUT THE VALUES OF N, H, AND K')
      READ(IR,*)X(3),X(1),X(2)
C
C-------CONFIRM THE INPUT VALUES
C
  185 WRITE(IW,190)X(3),X(1),X(2)
  190 FORMAT(/,5X,'*** VALUES ENTERED ARE N =',F6.2,5X,
```

```
     * 'H =',F8.4,5X,'K =',F8.4,/
     * ,5X,'*** ARE THEY CORRECT?   ==> ENTER 1 = YES, 2 = NO')
      READ(IR,*) INQUR
      GO TO (145,127),INQUR
      GO TO 185
C
  145 CALL FNMA3(X,F)
C
C-------PRINT OUT LOSS-COST OF SPECIFIED MA CHART
C
  146 WRITE(IW,150)X(3),X(1),X(2),F
  150 FORMAT(/,5X,58(1H=),
     *  /,5X,'*** THE SPECIFIED MA   CHART DESIGN IS',
     *  /,5X,'N = ',F6.2,',',3X,' H = ',F10.5,',',3X,'  K =',F10.5,
     *  /,5X,'*** AND THE HOURLY LOSS COST FOR THIS DESIGN IS',F14.6,/,
     *  /,5X,58(1H=))
      RETURN
      END
C
C*****************************************************************
C
      SUBROUTINE FNMA2(X,F)
C
C*****************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 LAMBDA
      COMMON / EC1 / LAMBDA
      COMMON / EC4 / DELTA,B,C,D,E,EM,T,W
      COMMON / SAMPLE / IX
      REAL*8 X(2),F
C
C  COVERT SINGLE PRECISION VALUES OF X(I) TO DOUBLE PRECISION VALUE
C
      H=X(1)
      DK=X(2)
      DN=DFLOAT(IX)
C
C   SET THE LOWER AND UPPER BOUNDS FOR H AND K
C   AND LOWER BOUND FOR N
C
      IF(H.GT.70..OR.H.LE.O.)F=100000000.
      IF(H.GT.70..OR.H.LE.O.)RETURN
      IF(DK.GT.8..OR.DK.LE.O.)F=100000000.
      IF(DK.GT.8..OR.DK.LE.O.)RETURN
      IF(DN.LT.1.)F=100000000.
      IF(DN.LT.1.)RETURN
      IN=IX
C
      EXPLH=DEXP(-LAMBDA*H)
C
      PNOT=1.DO-DNML(DK-DELTA*DSQRT(DN))+DNML(-DK-DELTA*DSQRT(DN))
      QNOT=1.DO-PNOT
      ANSBSD=O.DO
C
      IF (IN.LT.3) GO TO 11
      DO 10 J=1,IN-1
          DJ=DFLOAT(J)
      TEMP=1.DO-DNML(DK-DELTA/DSQRT(DJ))+DNML(-DK-DELTA/DSQRT(DJ))
      QJI=1.DO-TEMP
      DO 8 I=2,IN-1
          DJI=DFLOAT(J+I-1)
          DI=DFLOAT(I)
          IF(DJI.GT.DN) DJI=DN
          PJI=1.DO-DNML(DK-(DI*DELTA)/DSQRT(DJI))+
     *                DNML(-DK-(DI*DELTA)/DSQRT(DJI))
          TEMP=TEMP+DI*PJI*QJI
          QJI=QJI*(1.DO-PJI)
8      CONTINUE
       TEMP=TEMP+QJI*(DN+QNOT/PNOT)
       ANSBSD=ANSBSD+TEMP*DEXP(-DFLOAT(J-1)*LAMBDA*H)
10  CONTINUE
```

```
      ANSBSD=ANSBSD*(1.DO-EXPLH)
      TEMP=1.DO-DNML(DK-DELTA/DSQRT(DN))+DNML(-DK-DELTA/DSQRT(DN))
      QJI=1.DO-TEMP
      DO 12 I=2,IN-1
           DI=DFLOAT(I)
           PJI=1.DO-DNML(DK-(DI*DELTA)/DSQRT(DN))+
     *               DNML(-DK-(DI*DELTA)/DSQRT(DN))
           TEMP=TEMP+DFLOAT(I)*PJI*QJI
           QJI=QJI*(1.DO-PJI)
 12   CONTINUE
        TEMP=TEMP+QJI*(DN+QNOT/PNOT)
        ANSBSD=ANSBSD+TEMP*DEXP(-(DN-1.DO)*LAMBDA*H)
        GO TO 20
C
C      IF N < 3
C
  11  IF (IN.LT.2) GO TO 18
      PO1=1.DO-DNML(DK-DELTA)+DNML(-DK-DELTA)
      P11=1.DO-DNML(DK-DELTA/DSQRT(2.DO))+DNML(-DK-DELTA/DSQRT(2.DO))
      ANSBSD=(1.DO-EXPLH)*(PO1+(1.DO-PO1)*(DN+QNOT/PNOT))
     *         +EXPLH*(P11+(1.DO-P11)*(DN+QNOT/PNOT))
      GO TO 20
C
C      IF N < 2
C
  18  ANSBSD=1.DO/PNOT
C
  20  AOOCT=H*ANSBSD
     *      -(1.DO-(1.DO+LAMBDA*H)*EXPLH)/(LAMBDA*(1.DO-EXPLH))
C
      ACL=1.DO/LAMBDA+AOOCT+E+D
      ALPHA=2.DO*(1.DO-DNML(DK))
C
      ENFA= (ALPHA*EXPLH/(1.DO-EXPLH))/ACL
C
      ACFAC=W/ACL
C
      HCMCC=(B+C)/H
C
      ELOSSC=EM*(1.DO-1.DO/(LAMBDA*ACL))+ENFA*T+ACFAC+HCMCC
      F=ELOSSC
      RETURN
      END
C
C*****************************************************************
C
      SUBROUTINE ECIND
C
C*****************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON / LOGIO / IR,IW
      REAL*8 LAMBDA
      COMMON / ECIND1 / LAMBDA
      COMMON / ECIND2 / H,EK,HOPT,EKOPT,FOPT,STEP(20)
      COMMON / ECIND3 / DELTA,B,C,D,E,EM,T,W
C
C-----INPUT COST AND RISK PARAMETERS
C
  101 WRITE(IW,110)
  100 FORMAT(/,5X,'==> FOR ECONOMIC I CHART DESIGN, INPUT VALUES OF'
     *  ,/,5X,'==> DELTA, LAMBDA, M, E, D, T, W, B, C')
      READ(IR,*)DELTA,LAMBDA,EM,E,D,T,W,B,C
C
C-----ECHO PRINT OUT INPUT DATA
C
  115 WRITE(IW,120)DELTA,LAMBDA,EM,E,D,T,W,B,C
  120 FORMAT(/,5X,'** VALUES RECEIVED ARE AS FOLLOW:',/,
     *  5X,'DELTA = ',F10.4,7X,' LAMBDA = ',F10.4,/,
     *  5X,'  M   = ',F10.4,7X,'   E    = ',F10.4,/,
     *  5X,'  D   = ',F10.4,7X,'   T    = ',F10.4,/,
     *  5X,'  W   = ',F10.4,7X,'   B    = ',F10.4,/,
```

```
     *   5X,'  C    = ',F10.4,/,/,5X,'*** ARE THESE DATA CORRECT? ***',
     */,5X,'==> PLEASE ENTER 1 = YES, 2 = NO  <<<')
C
      READ(IR,*) INQUR
      GO TO (125,101),INQUR
      GO TO 115
C
C------SELECT STARTING POINT
C
  125 H=1.DO
      EK=3.DO
  127 WRITE(IW,130)H,EK
  130 FORMAT(//,5X,'*** THE FOLLOWING STARTING POINT IS SUGGESTED',
     * /,5X,'*** FOR ECONOMIC OPTIMIZATION OF I CHART.',
     * /,5X,'        H = ',F6.2,4X,'AND  K = ',F6.2,/,/,
     *   5X,' ==>  DO YOU ACCEPT THIS STARTING POINT?',/,
     *   5X,' ==>  ENTER 1 = YES, 2 = NO. <<<')
      READ(IR,*) INQUR
      GO TO (135,170),INQUR
      GO TO 127
C
C------SELECT SEARCH STEP SIZES OF H AND K
C
  135 STEP(1)=0.5
      STEP(2)=0.5
  137 WRITE(IW,139)STEP(1),STEP(2)
  139 FORMAT(//,5X,'*** THE FOLLOWING STEP SIZES ARE SUGGESTED',
     * /,5X,'*** FOR ECONOMIC OPTIMIZATION OF I CHART.',
     * /,5X,'        H = ',F6.2,'  AND  K = ',F6.2,/,/,
     *   5X,' ==>  DO YOU ACCEPT THESE STEP SIZES?',/,
     *   5X,' ==>  ENTER 1 = YES, 2 = NO. <<<')
      READ(IR,*) INQUR
      GO TO (145,180),INQUR
      GO TO 137
C
  145 CALL OPIND
C
C------PRINT OUT THE OPTIMUM INDIVIDUAL CHART DESIGN
C
  146 WRITE(IW,150)HOPT,EKOPT,FOPT
  150 FORMAT(/,5X,58(1H=),
     *  /,5X,'*** THE OPTIMAL INDIVIDUAL CHART DESIGN IS',
     *  /,5X,'     H = ',F10.4,6X,' K =',F10.4,
     *  /,5X,'*** THE MINIMUM LOSS COST PER HOUR IS',F14.6,/,
     *  /,5X,58(1H=))
      RETURN
C
C-----RESELECT STARTING POINT
C
  170 WRITE(IW,175)
  175 FORMAT(/,5X,'*** INPUT THE STARTING POINT YOU WANT ***',/,
     *          5X,'==> ENTER VALUES OF H,K')
      READ(IR,*)H,EK
  185 WRITE(IW,190)H,EK
  190 FORMAT(/,5X,'*** INPUT VALUES ARE  H =',F8.4,4X,'AND K =',F8.4,/
     *         ,5X,'*** ARE THEY CORRECT?  ==> ENTER 1 = YES, 2 = NO')
      READ(IR,*) INQUR
      GO TO (135,170),INQUR
      GO TO 185
C
C-------RESELECT STEP SIZES OF  H AND K
C
  180 WRITE(IW,195)
  195 FORMAT(/,5X,'*** SELECT THE STEP SIZES YOU WANT ***',/,
     *          5X,'==> INPUT STEP SIZES OF N,H,K')
      READ(IR,*)STEP(1),STEP(2)
  196 WRITE(IW,197)STEP(1),STEP(2)
  197 FORMAT(/,5X,'*** STEP SIZES ENTERED ARE H =',F6.2,5X,
     * 'AND  K =',F6.2,/
     * ,5X,'*** ARE THEY CORRECT?   ==> ENTER 1 = YES, 2 = NO',/)
      READ(IR,*) INQUR
      GO TO (145,180),INQUR
```

```
      GO TO 196
      END
C
C*************************************************************
C
      SUBROUTINE OPIND
C
C*************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON / ECIND2 / H,EK,HOPT,EKOPT,FOPT,STEP(20)
      EXTERNAL FNIND2
      REAL*8 X(2),XMIN(20),XSEC(20),F
C
C------ASSIGN NUMBER OF VARIABLES, SEARCH STEP, AND TERMINATE VALUE
C
      ND=2
      ICOUNT=500
      REQMIN=0.0001
C
C------ASSIGN STARTING POINT
C
      X(1)=H
      X(2)=EK
C
      CALL NELMIN(FNIND2,ND,X,XMIN,XSEC,YNEWLO,YSEC,REQMIN,STEP,ICOUNT)
C
      HOPT=XMIN(1)
      EKOPT=XMIN(2)
      FOPT=YNEWLO
      RETURN
      END
C
C*************************************************************
C
      SUBROUTINE FNIND2(X,F)
C
C*************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 LAMBDA
      COMMON / ECIND1 / LAMBDA
      COMMON / ECIND3 / DELTA,B,C,D,E,EM,T,W
      REAL*8 X(2)
C
      H=X(1)
      EK=X(2)
C
C------ASSIGN LOWER AND UPPER BOUND FOR H AND K
C
      IF(H.GT.70..OR.H.LE.0.)F=100000000.
      IF(H.GT.70..OR.H.LE.0.)RETURN
      IF(EK.GT.8..OR.EK.LE.0.)F=100000000.
      IF(EK.GT.8..OR.EK.LE.0.)RETURN
C
      EXPLH=DEXP(-LAMBDA*H)
C
      P=1.D0-DNML(EK-DELTA)+DNML(-EK-DELTA)
C
      A0OCT=H/P
     *      -(1.D0-(1.D0+LAMBDA*H)*EXPLH)/(LAMBDA*(1.D0-EXPLH))
C
      ACL=1.D0/LAMBDA+A0OCT+E+D
C
      ALPHA=2.D0*(1.D0-DNML(EK))
C
      ENFA= (ALPHA*EXPLH/(1.D0-EXPLH))/ACL
C
      ACFAC=W/ACL
C
      HCMCC=B/H+C/H
C
```

```
      ELOSSC=EM*(1.DO-1.DO/(LAMBDA*ACL))+ENFA*T+ACFAC+HCMCC
      F=ELOSSC
      RETURN
      END
C
C*****************************************************************
C
      SUBROUTINE EVIND
C
C*****************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON / LOGIO / IR,IW
      REAL*8 LAMBDA
      COMMON / ECIND1 / LAMBDA
      COMMON / ECIND3 / DELTA,B,C,D,E,EM,T,W
      REAL*8 X(2)
C
C-------INPUT COST AND RISK PARAMETERS
C
  101 WRITE(IW,110)
  110 FORMAT(/,5X,'==> FOR EVALUATION OF ECONOMIC I  CHART, INPUT'
     *  ,/,5X,'==> VALUES OF DELTA, LAMBDA, M, E, D, T, W, B, C')
      READ(IR,*)DELTA,LAMBDA,EM,E,D,T,W,B,C
C
C-------ECHO PRINT AND CONFIRM INPUT DATA
C
  115 WRITE(IW,120)DELTA,LAMBDA,EM,E,D,T,W,B,C
  120 FORMAT(/,5X,'** VALUES RECEIVED ARE AS FOLLOWS:',/,
     *  5X,'DELTA = ',F10.4,7X,' LAMBDA = ',F10.4,/,
     *  5X,'  M    = ',F10.4,7X,'   E     = ',F10.4,/,
     *  5X,'  D    = ',F10.4,7X,'   T     = ',F10.4,/,
     *  5X,'  W    = ',F10.4,7X,'   B     = ',F10.4,/,
     *  5X,'  C    = ',F10.4,/,/,5X,'*** ARE THESE DATA CORRECT? ***',
     */,5X,'==> PLEASE ENTER 1 = YES, 2 = NO  <<<')
      READ(IR,*) INQUR
      GO TO (127,101),INQUR
      GO TO 115
C
C-------INPUT THE VALUES OF N, H, AND K FOR THE SPECIFIED I CHART
C
  127 WRITE(IW,175)
  175 FORMAT(/,5X,'*** FOR THE SPECIFIC I  CHART TO EVALUATE **',/,
     *         5X,'==> INPUT THE VALUES OF  H AND K')
      READ(IR,*)X(1),X(2)
C
C-------CONFIRM THE INPUT VALUES
C
  185 WRITE(IW,190)X(1),X(2)
  190 FORMAT(/,5X,'*** VALUES ENTERED ARE ',
     * 'H =',F8.4,5X,'K =',F8.4,/
     * ,5X,'*** ARE THEY CORRECT?   ==> ENTER 1 = YES, 2 = NO')
      READ(IR,*) INQUR
      GO TO (145,127),INQUR
      GO TO 185
C
  145 CALL FNIND2(X,F)
C
C-------PRINT OUT LOSS-COST OF SPECIFIC I CHART
C
  146 WRITE(IW,150)X(1),X(2),F
  150 FORMAT(/,5X,58(1H=),
     *  /,5X,'*** THE SPECIFIED INDIVIDUAL CHART DESIGN IS',
     *  /,5X,' H = ',F10.5,',',3X,'  K =',F10.5,
     *  /,5X,'*** AND THE HOURLY LOSS COST FOR THIS DESIGN IS',F14.6,/,
     *  /,5X,58(1H=))
      RETURN
      END
C
C*****************************************************************
C
      SUBROUTINE ECDUNC
```

```
C
C*****************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 LAMBDA
      COMMON / EC1 / LAMBDA
      COMMON / EC2 / N,NOPT
      COMMON / EC3 / H,EK,HOPT,EKOPT,FOPT,STEP(20)
      COMMON / EC4 / DELTA,B,C,D,E,EM,T,W
C
C-------INPUT COST AND RISK PARAMETERS
C
  101 WRITE(IW,110)
  100 FORMAT(/,5X,'==> FOR DUNCAN''S X-BAR CHART DESIGN, INPUT VALUES'
     *  ,/,5X,'==> OF DELTA, LAMBDA, M, E, D, T, W, B, C')
      READ(IR,*)DELTA,LAMBDA,EM,E,D,T,W,B,C
C
C-------ECHO PRINT OUT INPUT DATA
C
  115 WRITE(IW,120)DELTA,LAMBDA,EM,E,D,T,W,B,C
  120 FORMAT(/,5X,'** VALUES RECEIVED ARE AS FOLLOWS:',/,
     *  5X,'DELTA = ',F10.4,7X,' LAMBDA = ',F10.4,/,
     *  5X,'  M   = ',F10.4,7X,'   E    = ',F10.4,/,
     *  5X,'  D   = ',F10.4,7X,'   T    = ',F10.4,/,
     *  5X,'  W   = ',F10.4,7X,'   B    = ',F10.4,/,
     *  5X,'  C   = ',F10.4,/,/,5X,'*** ARE THESE DATA CORRECT? ***',
     */,5X,'==> PLEASE ENTER 1 = YES, 2 = NO  <<<')
      READ(IR,*) INQUR
      GO TO (125,101),INQUR
      GO TO 115
C
C------SELECT STARTING POINT
C
  125 N=5
      H=1.DO
      EK=3.DO
  127 WRITE(IW,130)N,H,EK
  130 FORMAT(//,5X,'*** THE FOLLOWING STARTING POINT IS SUGGESTED FOR',
     * /,5X,'*** ECONOMIC OPTIMIZATION OF DUNCAN''S X-BAR CHART',
     * /,5X,'     N = ',I2,'   H = ',F6.2,'  K = ',F6.2,/,/,
     *   5X,' ==>  DO YOU ACCEPT THIS STARTING POINT?',/,
     *   5X,' ==>  ENTER 1 = YES, 2 = NO. <<<')
      READ(IR,*) INQUR
      GO TO (135,170),INQUR
      GO TO 127
C
C------SELECT STEP SIZES OF N, H, AND K
C
  135 STEP(1)=0.5
      STEP(2)=0.5
      STEP(3)=1.0
  137 WRITE(IW,139)STEP(3),STEP(1),STEP(2)
  139 FORMAT(//,5X,'*** THE FOLLOWING STEP SIZES ARE SUGGESTED FOR',
     * /,5X,'*** ECONOMIC OPTIMIZATION OF DUNCAN''S X-BAR CHART',
     * /,5X,'     N = ',F5.2,'   H = ',F6.2,'  K = ',F6.2,/,/,
     *   5X,' ==>  DO YOU ACCEPT THESE STEP SIZES?',/,
     *   5X,' ==>  ENTER 1 = YES, 2 = NO. <<<')
      READ(IR,*) INQUR
      GO TO (145,180),INQUR
      GO TO 137
C
  145 CALL OPDUNC
C
C-------PRINT OUT OPTIMAL DUNCAN'S X-BAR CHART DESIGN
C
  146 WRITE(IW,150)NOPT,HOPT,EKOPT,FOPT
  150 FORMAT(/,5X,58(1H=),
     *  /,5X,'*** THE OPTIMAL DUNCAN''S X-BAR CHART DESIGN IS',
     *  /,5X,'N = ',I4,6X,'H = ',F10.5,6X,'K =',F10.5,
     *  /,5X,'*** THE MINIMUM LOSS COST PER HOUR IS',F14.6,/,
     *  /,5X,58(1H=))
      RETURN
```

```
C
C------RESELECT STARTING POINT
C
  170 WRITE(IW,175)
  175 FORMAT(/,5X,'*** INPUT THE STARTING POINT YOU WANT ***',/,
     *         5X,'==> KEY IN THE VALUES OF N,H,K')
      READ(IR,*)N,H,EK
  185 WRITE(IW,190)N,H,EK
  190 FORMAT(/,5X,'*** STARTING POINT SELECTED IS N =',I4,',',3X,
     * 'H =',F8.4,',',3X,'AND K =',F8.4,/
     *  ,5X,'*** ARE THEY CORRECT?   ==> ENTER 1 = YES, 2 = NO')
      READ(IR,*) INQUR
      GO TO (135,170),INQUR
      GO TO 185
C
C------RESELECT STEP SIZES OF N, H, K
C
  180 WRITE(IW,195)
  195 FORMAT(/,5X,'*** INPUT THE STEP SIZES YOU WANT ***',/,
     *         5X,'==> ENTER STEP SIZES OF N,H,K')
      READ(IR,*)STEP(3),STEP(1),STEP(2)
  196 WRITE(IW,197)STEP(3),STEP(1),STEP(2)
  197 FORMAT(/,5X,'*** STEP SIZES ENTERED ARE N =',F5.2,',',3X,
     * ' H =',F6.2,',',3X,'AND K =',F6.2,/
     *  ,5X,'*** ARE THEY CORRECT?   ==> ENTER 1 = YES, 2 = NO')
      READ(IR,*) INQUR
      GO TO (145,180),INQUR
      GO TO 196
      END
C
C*****************************************************************
C
      SUBROUTINE OPDUNC
C
C*****************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON / LOGIO / IR,IW
      COMMON / EC2 / NN,NOPT
      COMMON / EC3 / EH,EK,HOPT,EKOPT,FOPT,STEP(20)
      COMMON / SAMPLE / IX
      EXTERNAL FNDUN3
      EXTERNAL FNDUN2
      REAL*8 FMIN(5)
      REAL*8 X(3),XMIN(20),XSEC(20),F
C
C------ASSIGN NUMBER OF VARIABLES, SEARCH STEP, AND TERMINATE VALUE
C
      ND=3
      ICOUNT=500
      REQMIN=0.001
C
C------ASSIGN STARTING POINT
C
      X(1)=EH
      X(2)=EK
      X(3)=DFLOAT(NN)
C
      CALL NELMIN(FNDUN3,ND,X,XMIN,XSEC,YNEWLO,YSEC,REQMIN,STEP,ICOUNT)
C
      WRITE(IW,133)XMIN(3),XMIN(1),XMIN(2),YNEWLO
  133 FORMAT(/,5X,'*** THE OPTIMAL POINT FOUND IS  ***',/,/,
     *          5X,' N = ',F7.4,', H = ',F7.4,', K = ',F7.4,
     * ',  LOSS COST = ',F14.6,/)
C
C------ASSIGN NUMBER OF VARIABLES, SEARCH STEP, AND TERMINATE VALUE
C
      ND=2
      ICOUNT=500
      REQMIN=0.0001
C
C------ASSIGN STARTING POINT
```

```
C
      IX=XMIN(3)
      X(1)=XMIN(1)
      X(2)=XMIN(2)
C
      CALL NELMIN(FNDUN2,ND,X,XMIN,XSEC,YNEWLO,YSEC,REQMIN,STEP,ICOUNT)
C
      I=1
C
C-----PRINT SEARCH ITERATIONS
C
      WRITE(IW,140)
  140 FORMAT(/,5X,'*** OPTIMIZATION ITERATIONS ***',/,/,
     *         5X,' I',T12,'N',T18,'H',T28,'K',T38,'LOSS COST',/,
     *         5X,44(1H-),/)
        WRITE(IW,141)I,IX,XMIN(1),XMIN(2),YNEWLO
  141   FORMAT(5X,I2,T10,I3,T15,F7.4,T25,F7.4,T35,F14.6)
C
      INCR=1
      ITIME=0
C
C------KEEP THE POINT AS THE BEST OPTIMUM SO FAR
C
 10   FMIN(5)=YNEWLO
      DO 11 L=1,2
         FMIN(L)=XMIN(L)
 11   CONTINUE
 12   IX=IX+INCR
 15   X(1)=XMIN(1)
      X(2)=XMIN(2)
      ND=2
      ICOUNT=500
      REQMIN=0.0001
C
      CALL NELMIN(FNDUN2,ND,X,XMIN,XSEC,YNEWLO,YSEC,REQMIN,STEP,ICOUNT)
C
      I=I+1
C
C-----PRINT SEARCH ITERATIONS
C
      WRITE(IW,141)I,IX,XMIN(1),XMIN(2),YNEWLO
C
      IF (ITIME.EQ.1) GO TO 40
      IF (YNEWLO .GT. FMIN(5)) GO TO 30
         ITIME=1
         FMIN(5)=YNEWLO
         DO 20 L=1,2
            FMIN(L)=XMIN(L)
  20  CONTINUE
      GO TO 12
  30  INCR=-INCR
      IX=IX-2
      ITIME=1
      GO TO 15
  40  IF (YNEWLO.LE.FMIN(5)) GO TO 10
      IXMIN=IX-INCR
      NOPT=IXMIN
      HOPT=FMIN(1)
      EKOPT=FMIN(2)
      FOPT=FMIN(5)
      RETURN
      END
C
C******************************************************************
C
      SUBROUTINE FNDUN3(X,F)
C
C******************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 LAMBDA
      COMMON / MAIN1 / IR,IW
```

```
      COMMON / EC1 / LAMBDA
      COMMON / EC4 / DELTA,B,C,D,E,EM,T,W
      REAL*8 X(3)
C
C--CONVERT SINGLE PRECISION VALUES OF X(I) TO DOUBLE PRECISION VALUES
C
      H=X(1)
      DK=X(2)
      DN=X(3)
C
C------ASSIGN UPPER AND LOWER BOUND FOR H AND K
C------AND LOWER BOUND FOR N
C
      IF(H.GT.70..OR.H.LE.O.)F=100000000.
      IF(H.GT.70..OR.H.LE.O.)RETURN
      IF(DK.GT.8..OR.DK.LE.O.)F=100000000.
      IF(DK.GT.8..OR.DK.LE.O.)RETURN
      IF(DN.LT.1.)F=100000000.
      IF(DN.LT.1.)RETURN
C
      IN=X(3)
C
      EXPLH=DEXP(-LAMBDA*H)
      IF(EXPLH.EQ.1.) F=100000000.
      IF(EXPLH.EQ.1.) RETURN
C
      PNOT=1.DO-DNML(DK-DELTA*DSQRT(DN))+DNML(-DK-DELTA*DSQRT(DN))
C
      AOOCT=H/PNOT
     *      -(1.DO-(1.DO+LAMBDA*H)*EXPLH)/(LAMBDA*(1.DO-EXPLH))
C
      ACL=1.DO+LAMBDA*(AOOCT+E*DN+D)
      ALPHA=2.DO*(1.DO-DNML(DK))
C
      ENFA= (LAMBDA*ALPHA*EXPLH/(1.DO-EXPLH))/ACL
C
      ACFAC=W*LAMBDA/ACL
C
      HCMCC=B/H+DN*C/H
C
      ELOSSC=EM*(ACL-1.DO)/ACL+ENFA*T+ACFAC+HCMCC
      F=ELOSSC
      RETURN
      END
C
C******************************************************************
C
      SUBROUTINE EVDUNC
C
C******************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON / LOGIO / IR,IW
      REAL*8 LAMBDA
      COMMON / EC1 / LAMBDA
      COMMON / EC4 / DELTA,B,C,D,E,EM,T,W
      REAL*8 X(3)
C
C------INPUT PARAMETERS
C
  101 WRITE(IW,110)
  100 FORMAT(/,5X,'==> FOR EVALUATION OF DUNCAN''S X-BAR CHART, INPUT'
     *  ,/,5X,'==> VALUES OF DELTA, LAMBDA, M, E, D, T, W, B, C')
      READ(IR,*)DELTA,LAMBDA,EM,E,D,T,W,B,C
C
C------ECHO PRINT OUT INPUT DATA
C
  115 WRITE(IW,120)DELTA,LAMBDA,EM,E,D,T,W,B,C
  120 FORMAT(/,5X,'** VALUES RECEIVED ARE AS FOLLOWS:',/,
     *  5X,'DELTA = ',F10.4,7X,' LAMBDA = ',F10.4,/,
     *  5X,'  M   = ',F10.4,7X,'   E    = ',F10.4,/,
     *  5X,'  D   = ',F10.4,7X,'   T    = ',F10.4,/,
```

```
     *  5X,'  W   = ',F10.4,7X,'   B     = ',F10.4,/,
     *  5X,'  C   = ',F10.4,/,/,5X,'*** ARE THESE DATA CORRECT? ***',
     */,5X,'==> PLEASE ENTER 1 = YES, 2 = NO  <<<')
      READ(IR,*) INQUR
      GO TO (127,101),INQUR
      GO TO 115
C
C-------INPUT THE VALUES OF N, H, AND K FOR
C-------THE SPECIFIED DUNCAN'S X-BAR CHART
C
  127 WRITE(IW,175)
  175 FORMAT(/,5X,'*** FOR THE SPECIFIC DUNCAN''S X-BAR CHART TO EVALUA
     *TE  **',/,5X,'==> INPUT THE VALUES OF N, H, AND K')
      READ(IR,*)X(3),X(1),X(2)
C
C-------CONFIRM THE INPUT VALUES
C
  185 WRITE(IW,190)X(3),X(1),X(2)
  190 FORMAT(/,5X,'*** VALUES ENTERED ARE N =',F6.2,5X,
     * 'H =',F8.4,5X,'K =',F8.4,/
     * ,5X,'*** ARE THEY CORRECT?   ==> ENTER 1 = YES, 2 = NO')
      READ(IR,*) INQUR
      GO TO (145,127),INQUR
      GO TO 185
C
  145 CALL FNDUN3(X,F)
C
C-------PRINT OUT LOSS-COST OF SPECIFIED DUNCAN'S X-BAR CHART
C
  146 WRITE(IW,150)X(3),X(1),X(2),F
  150 FORMAT(/,5X,58(1H=),
     *  /,5X,'*** THE SPECIFIED DUNCAN''S X-BAR CHART DESIGN IS',
     *  /,5X,'N = ',F6.2,',',3X,' H = ',F10.5,',',3X,'  K =',F10.5,
     *  /,5X,'*** AND THE HOURLY LOSS COST FOR THIS DESIGN IS',F14.6,/,
     *  /,5X,58(1H=))
      RETURN
      END
C
C***************************************************************
C
      SUBROUTINE FNDUN2(X,F)
C
C***************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 LAMBDA
      COMMON / MAIN1 / IR,IW
      COMMON / EC1 / LAMBDA
      COMMON / EC4 / DELTA,B,C,D,E,EM,T,W
      COMMON / SAMPLE / IX
      REAL*8 X(2)
C
      H=X(1)
      DK=X(2)
      DN=DFLOAT(IX)
C
C------ ASSIGN UPPER AND LOWER BOUND FOR H AND K
C------ AND LOWER BOUND FOR N
C
      IF(H.GT.70..OR.H.LE.O.)F=100000000.
      IF(H.GT.70..OR.H.LE.O.)RETURN
      IF(DK.GT.8..OR.DK.LE.O.)F=100000000.
      IF(DK.GT.8..OR.DK.LE.O.)RETURN
      IF(DN.LT.1.)F=100000000.
      IF(DN.LT.1.)RETURN
C
      IN=IX
C
      EXPLH=DEXP(-LAMBDA*H)
      PNOT=1.DO-DNML(DK-DELTA*DSQRT(DN))+DNML(-DK-DELTA*DSQRT(DN))
C
      AOOCT=H/PNOT
```

```
     *      .-(1.DO-(1.DO+LAMBDA*H)*EXPLH)/(LAMBDA*(1.DO-EXPLH))
C
      ACL=1.DO+LAMBDA*(AOOCT+E*DN+D)
      ALPHA=2.DO*(1.DO-DNML(DK))
C
      ENFA= (LAMBDA*ALPHA*EXPLH/(1.DO-EXPLH))/ACL
C
      ACFAC=W*LAMBDA/ACL
C
      HCMCC=B/H+DN*C/H
C
      ELOSSC=EM*(ACL-1.DO)/ACL+ENFA*T+ACFAC+HCMCC
      F=ELOSSC
      RETURN
      END
C
C*************************************************************
C
      SUBROUTINE NELMIN(FN,N,START,XMIN,XSEC,YNEWLO,YSEC,
     *                  REQMIN,STEP,ICOUNT)
C
C*************************************************************
C
      REAL*8 START(N),STEP(N),XMIN(N),XSEC(N),YNEWLO,YSEC,REQMIN,
     *       P(20,21),PSTAR(20),P2STAR(20),PBAR(20),Y(20),DN,Z,YLO,
     *       RCOEFF,YSTAR,ECOEFF,Y2STAR,CCOEFF,F,DABIT,DCHK,COORD1,
     *       COORD2
      DATA RCOEFF/1.ODO/,ECOEFF/2.ODO/,CCOEFF/O.5DO/
      KCOUNT=ICOUNT
      ICOUNT=O
C
C------INITIALIZATION
C
      DO 60 I=1,N
         XMIN(I)=O.DO
         XSEC(I)=O.DO
   60 CONTINUE
      YNEWLO=O.DO
      YSEC=O.DO
C
      IF ( REQMIN .LE. O.DO ) ICOUNT=ICOUNT-1
      IF ( N .LE. O ) ICOUNT=ICOUNT-10
      IF ( N .GT. 20 ) ICOUNT=ICOUNT-10
      IF ( ICOUNT .LT. O ) RETURN
C
      DABIT=2.04607D-35
      BIGNUM=1.OD38
      KONVGE=5
      XN=FLOAT(N)
      NN=N+1
C
C-----CONSTRUCTION OF INITIAL SIMPLEX
C
 1001 DO 1 I=1,N
    1 P(I,NN)=START(I)
      CALL FN(START,F)
      Y(NN)=F
      ICOUNT=ICOUNT+1
      DO 2 J=1,N
      DCHK=START(J)
      START(J)=DCHK+STEP(J)
      DO 3 I=1,N
    3    P(I,J)=START(I)
      CALL FN(START,F)
      Y(J)=F
      ICOUNT=ICOUNT+1
    2 START(J)=DCHK
C
C-----SIMPLEX CONSTRUCTION COMPLETE
C
C-----FIND HIGHEST AND LOWEST Y VALUES
C-----YNEWLO (Y(IHI)) INDICATES THE VERTEX
```

```
C-----OF THE SIMPLEX TO BE REPLACED
C
 1000 YLO=Y(1)
      YNEWLO=YLO
      ILO=1
      IHI=1
      DO 5 I=2,NN
          IF ( Y(I) .GE. YLO ) GO TO 4
          YLO=Y(I)
          ILO=I
  4       IF ( Y(I) .LE. YNEWLO ) GO TO 5
          YNEWLO=Y(I)
          IHI=I
  5   CONTINUE
C
C-----PERFORM CONVERGENCE CHECKS ON FUNCTION
C
      DCHK=(YNEWLO+DABIT)/(YLO+DABIT)-1.DO
      IF ( DABS(DCHK) .LT. REQMIN ) GO TO 900
C
      KONVGE=KONVGE-1
      IF ( KONVGE .NE. O ) GO TO 2020
      KONVGE=5
C
C-----CHECK CONVERGENCE OF COORDINATES ONLY
C-----EVERY 5 SIMPLEXES
C
      DO 2015 I=1,N
          COORD1=P(I,1)
          COORD2=COORD1
          DO 2010 J=2,NN
              IF ( P(I,J) .GE. COORD1 ) GO TO 2005
              COORD1=P(I,J)
 2005         IF ( P(I,J) .LE. COORD2 ) GO TO 2010
              COORD2=P(I,J)
 2010     CONTINUE
          DCHK=(COORD2+DABIT)/(COORD1+DABIT)-1.DO
          IF ( DABS(DCHK) .GT. REQMIN ) GO TO 2020
 2015 CONTINUE
      GO TO 900
 2020 IF ( ICOUNT .GE. KCOUNT ) GO TO 900
C
C-----CALCULATE PBAR, THE CENTROID OF THE SIMPLEX VERTICES
C-----EXCEPTING THAT WITH Y VALUE TNEWLO
C
      DO 7 I=1,N
          Z=O.ODO
          DO 6 J=1,NN
              Z=Z+P(I,J)
  6       CONTINUE
          Z=Z-P(I,IHI)
  7   PBAR(I)=Z/FLOAT(N)
C
C-----REFLECTION THROUGH THE CENTROID
C
      DO 8 I=1,N
  8       PSTAR(I)=(1.ODO+RCOEFF)*PBAR(I)-RCOEFF*P(I,IHI)
      CALL FN(PSTAR,F)
      YSTAR=F
      ICOUNT=ICOUNT+1
      IF ( YSTAR .GT. YLO ) GO TO 12
      IF ( ICOUNT .GE. KCOUNT ) GO TO 19
C
C-----SUCCEFUL REFLECTION, SO EXTENSION
C
      DO 9 I=1,N
  9       P2STAR(I)=ECOEFF*PSTAR(I)+(1.ODO-ECOEFF)*PBAR(I)
      CALL FN(P2STAR,F)
      Y2STAR=F
      ICOUNT=IOUNT+1
C
C-------RETAIN EXTENSION OR CONTRACTION
```

```
C
      IF ( Y2STAR .GE. YSTAR ) GO TO 19
   10 DO 11 I=1,N
   11    P(I,IHI)=P2STAR(I)
      Y(IHI)=Y2STAR
      GO TO 1000
C
C-------NO EXTENSION
C
   12 L=0
      DO 13 I=1,NN
         IF ( Y(I) .GT. YSTAR ) L=L+1
   13 CONTINUE
      IF ( L .GT. 1 ) GO TO 19
      IF ( L .EQ. 0 ) GO TO 15
C
C-------CONTRACTION ON THE REFLECTION SIDE OF THE CENTROID
C
      DO 14 I=1,N
   14    P(I,IHI)=PSTAR(I)
      Y(IHI)=YSTAR
C
C-------CONTRACTION ON THE Y(IHI) SIDE OF THE CENTROID
C
   15 IF ( ICOUNT .GE. KCOUNT ) GO TO 900
      DO 16 I=1,N
   16    P2STAR(I)=CCOEFF*P(I,IHI)+(1.0D0-CCOEFF)*PBAR(I)
      CALL FN(P2STAR,F)
      Y2STAR=F
      ICOUNT=ICOUNT+1
      IF ( Y2STAR .LT. Y(IHI) ) GO TO 10
C
C-------CONTRACT THE WHOLE SIMPLEX
C
      DO 18 J=1,NN
         DO 17 I=1,N
            P(I,J)=(P(I,J)+P(I,ILO))*0.5D0
   17       XMIN(I)=P(I,J)
         CALL FN(XMIN,F)
         Y(J)=F
   18 CONTINUE
      ICOUNT=ICOUNT+NN
      IF ( COUNT .LT. KCOUNT ) GO TO 1000
      GO TO 900
C
C-------RETAIN REFLECTION
C
   19 CONTINUE
      DO 20 I=1,N
   20    P(I,IHI)=PSTAR(I)
      Y(IHI)=YSTAR
      GO TO 1000
C
C-------SELECT THE TWO BEST FUNCTION VALUES (YNEWLO AND YSEC)
C-------AND THEIR COORDS. (XMIN AND XSEC)
C
  900 DO 23 J=1,NN
         DO 22 I=1,N
   22       XMIN(I)=P(I,J)
         CALL FN(XMIN,F)
         Y(J)=F
   23 CONTINUE
      YNEWLO=BIGNUM
      DO 24 J=1,NN
         IF ( Y(J) .GE. YNEWLO ) GO TO 24
         YNEWLO=Y(J)
         IBEST=J
   24 CONTINUE
      Y(IBEST)=BIGNUM
      YSEC=BIGNUM
      DO 25 J=1,NN
         IF ( Y(J) .GE. YSEC ) GO TO 25
```

```
         YSEC=Y(J)
         ISEC=J
 25   CONTINUE
      DO 26 I=1,N
         XMIN(I)=P(I,IBEST)
         XSEC(I)=P(I,ISEC)
 26   CONTINUE
      RETURN
      END
C
C*******************************************************************
C
      DOUBLE PRECISION FUNCTION DNML(X)
C
C*******************************************************************
C                                                                  *
C  COMPUTES THE CUMULATIVE DISTRIBUTION FUNCTION P(Y<=X) OF A      *
C   RANDOM VARIABLE Y HAVING A STANDARD NORMAL DISTRIBUTION.       *
C                                                                  *
C*******************************************************************
C                                                                  *
C         THIS FUNCTION IS WRITTEN BY DR. R. J. CRAIG FROM         *
C    "NORMAL FAMILY DISTRIBUTION FUNCTIONS: FORTRAN AND BASIC      *
C    PROGRAM", JOURNAL OF QUALITY TECHNOLOGY, VOL. 16, NO. 4,      *
C    OCTOBER 1984.                                                 *
C                                                                  *
C*******************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      DATA PI /3.141592653589793/
C
      Y=X/DSQRT(2.DO)
      IF(X.LT.O.DO) Y=-Y
      S=O.DO
      DO 1 N=1,37
         RN=DFLOAT(N)
         S=S+DEXP(-RN*RN/25)/N*DSIN(2*N*Y/5)
   1  CONTINUE
      S=S+Y/5
      ERF=2*S/PI
      DNML=(1.DO+ERF)/2
      IF(X.LT.O.DO) DNML=(1.DO-ERF)/2
      IF(X.LT.-8.3DO) DNML=O.DO
      IF(X.GT.8.3DO) DNML=1.DO
      RETURN
      END
```

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