STABILITY SENSITIVITY OF A WEB WRINKLE
ON A CYLINDRICAL ROLLER

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NOMENCLATURE

A	wrinkle amplitude
AR	wrinkle aspect ratio
CD	cross-machine direction
D	flexural rigidity
E	Young's Modulus value
FC	frictional coefficient
h	thickness value
I	area moment of inertia
inf	infinity
M	bending moment
MD	machine direction
n/a	not applicable
mil	0.001 inches
PR	Poisson's Ratio
psi	pounds per square inch
p/i	pounds per inch of width
R	radius
RD	roller diameter
TH	thickness
TN	tension
t	thickness value
v	Poisson's Ratio value
WA	wrap angle

WH	undeformed wrinkle height
WW	wrinkle width
Y	wrinkle width coordinate
YM	Young's Modulus
Z	wrinkle height coordinate

LIST OF SYMBOLS

e_{xx}	x-direction principle strain
e_{yy}	y-direction principle strain
σ	general tensile stress
σ_x	x-direction tensile stress
σ_y	y-direction tensile stress
τ_{xy}	xy-shear stress

CHAPTER I

INTRODUCTION

Many products and materials in common use today are processed or handled in the form of large thin sheets. Examples include plastic film, paper, textiles, and even metals. Materials which are very thin compared to their length or width are referred to as webs. Some webs, plastic film for example, may begin in widths of ten feet or more and several miles of the web may be wound onto a single roll. This form allows for convenient subsequent processing, such as slitting, coating or labeling, and shipment. In order for the web industry to prosper it is not unusual to have web line speeds of many thousands of feet per minute. The web process line speed is heavily dependent upon the web material. Paper, for example is typically wound at speeds much higher than that of plastics because of the permeability of paper. This permeability allows entrained air to escape through the web as it is being wound, giving a harder and higher quality wound roll. As might be expected, the demand for web quality and productivity has presented many problems in web handling. These problems arise from the fact that it is desirable to handle and manipulate a material that might be very fragile. This fragility may be material oriented, such as the ease at which aluminum foil or a web material as thin as 0.00006 inches creases, or product oriented, such as the special handling requirements of photographic film or magnetic media. An ideal web processing line would allow the web to operate at a minimum tension but with maximum web control. In most instances, tension and control are inversely proportional. In addition to this, webs are a planar material. The lack of thickness in a web makes it especially prone to unwanted behavior arising from in-plane compressive or transverse shear forces. Both of these modes of structural loading can lead to localized out-of-plane deformations, or wrinkling. A web wrinkle in the free span (the unsupported region between rollers or guiding devices) is not, in itself, a situation which is damaging to the web. However, a free span wrinkle may well detract from web processes such as coating. The structural problem arises when a web wrinkle encounters a roller or other device which might cause out-of-plane guiding. The presence of a greatly increased section modulus in the web wrinkle creates a resistance to out-of-plane bending. This resistance shows up as increased strains which can cause web material damage or failure. This situation is illustrated in Figure 1. Because the wrinkling problem is related to many of the material and geometric properties of the subject web, an overview of the research in this general area is presented.

The tension in a moving web must be within certain limits for proper operation. A high tension is desirable

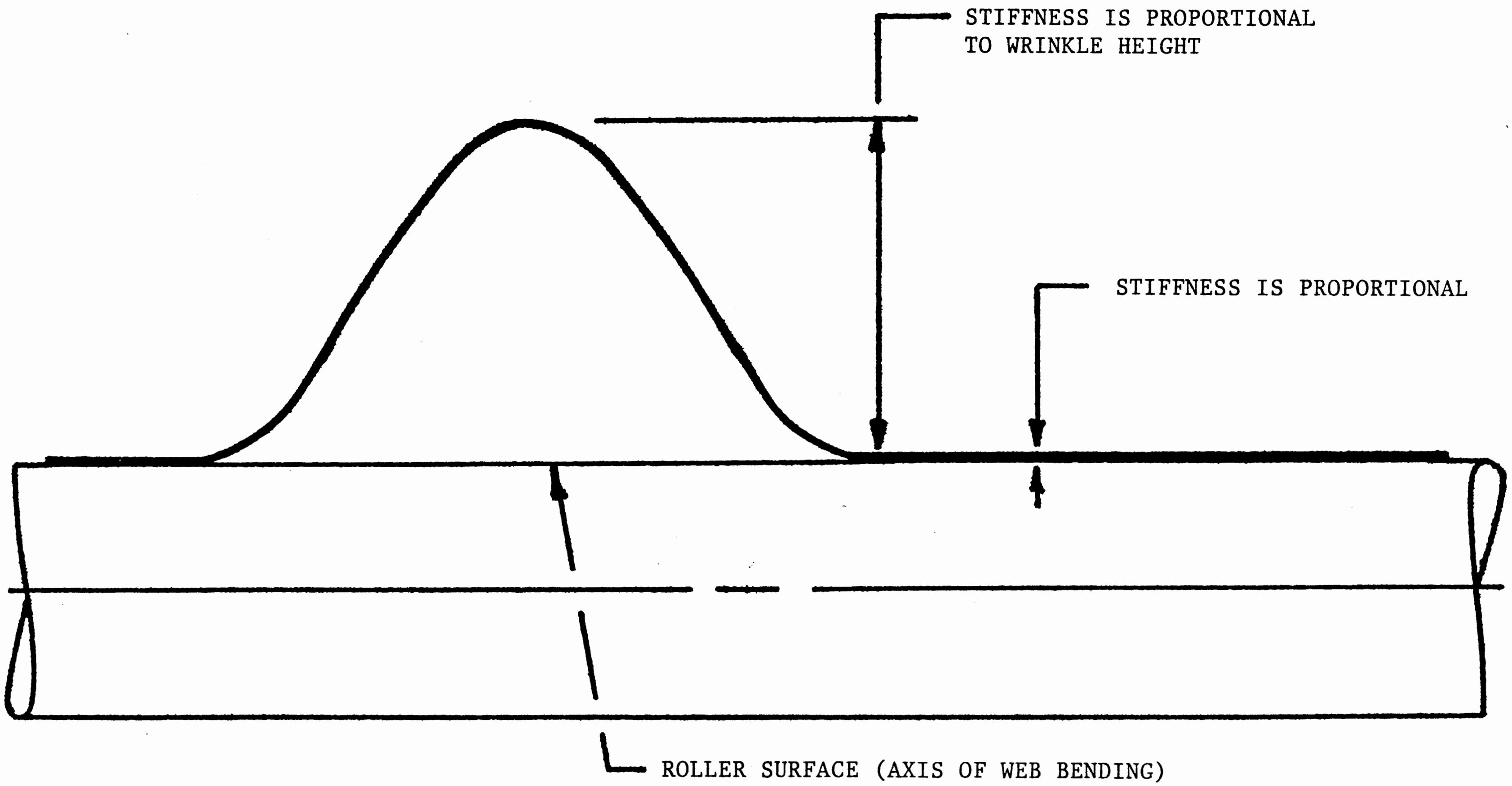
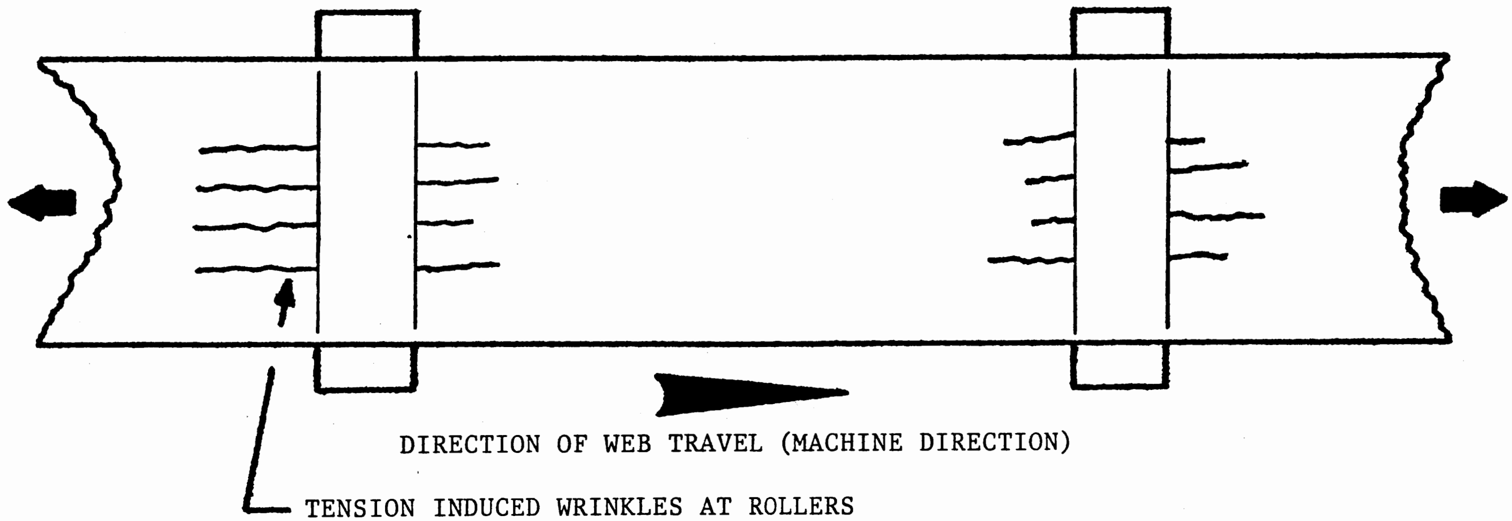


Figure 1. Web Wrinkle Cross Section

for good roller contact, guiding, steering, winding, and dynamic behavior. Too high a tension may cause creeping, plastic yielding, tearing, or wrinkling of the web. This is especially true of plastic webs which are exposed to heat, such as a drying oven during the web processing. In addition, an uneven tension profile in the cross-machine direction (CD) may lead to poor quality coating or winding of the web into large rolls (1). In order to know at what magnitude the tension may be maintained, the process engineer must be familiar with the web material and how it behaves at various tensions. An example of web wrinkle creation is shown in Figure 2.

A considerable amount of research has been conducted dealing with the mechanical properties of paper webs. Hollmark, et al. (2) investigated the mechanical properties of paper sheets as related to the length of the fibers and the degree of adjacent fiber bonding. In a similar study, Williams (3) investigated paper strength as related to fiber bending stiffness, length, and perimeter. Other studies into paper strength have been conducted by Claudio-da-Silva, et al. (4), Pecht and Johnson (5), and Kimura and Shimizu (6). Other various studies have been performed by Seth (7) on paper's resistance to crack propagation, by Fellers and Carlsson (8) on measuring the pure bending properties of paper, and by Pecht and Johnson (9) on the creep of paper. The major area of published research appears to be on the elastic properties of paper. Mann, et al. (10,11),



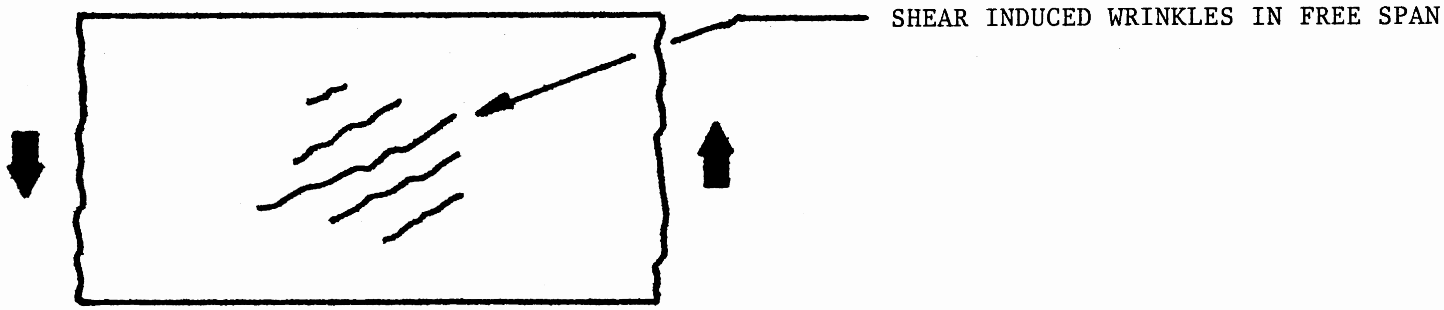


Figure 2. Web Wrinkle Formation

Habeger, et al. (12), Baum and Bornhoeft (13), and Baum, et al. (14) used acoustic wave dispersion techniques to measure many of the three dimensional elastic properties of paper webs. These properties included tensile modulus, shear modulus, and Poisson's ratio. Other similar studies into the theory and measurement of elastic properties were made by Craver and Taylor (15), Papadakis (16), Perkins and Mark (17), Page, et al. (18), Senko and Thorpe (19), and Jones (20). The plastics industry appears to be concerned with a web's mechanical properties as related to the drawing direction of the film. Dhingra, et al. (21) reports that cold rolling of polyethylene, polypropylene, and nylon films may have significant effects on the tensile and yield strength. Similarly, DeVries (22) notes that biaxial stretching of polypropylene, during processing, will greatly increase the material's toughness and impact strength. Polymer properties may also be tailored during the melting and extruding process, as reported by Zeichner and Macosko (23). Several of the instrumentation schemes for measuring web tension and properties include ultrasonics, photo-elasticity, and direct displacement measurement (24) (25) (26). Of course many of these technologies may be assisted by the computer (27) (28).

After the web properties have been assessed, the dynamic behavior of a web, when influenced by various line components, must be studied. A diagram of a simplified web process line is shown in Figure 3. The primary components

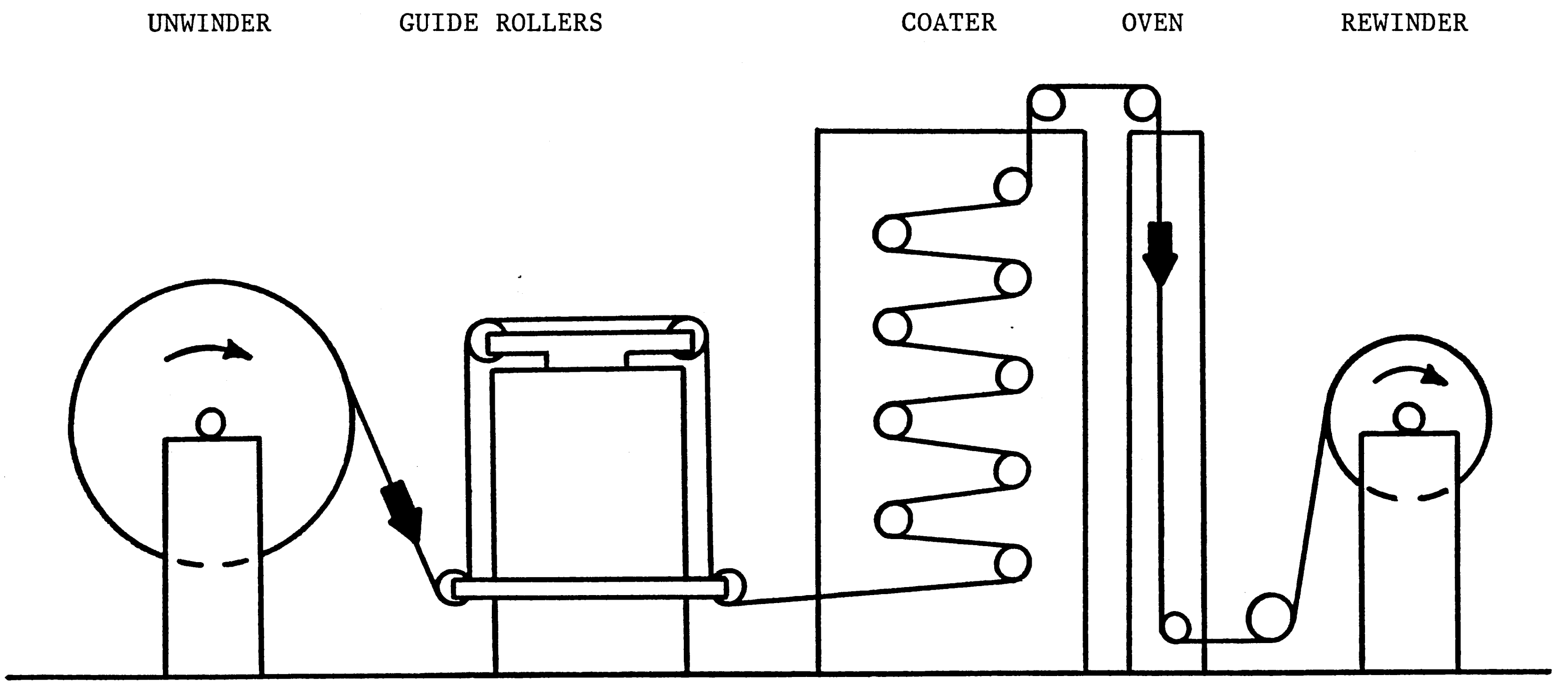


Figure 3. Simplified Web Process Line

are the winder-unwinder, steering and guide rollers, and whatever process may be performed on the web such as coating, slitting, etc. Some of the techniques for measuring properties of a static web have been applied to moving webs. Lu (29) and Baum and Habeger (30), for example, investigated a sonic contact method which relates sonic velocity in the moving web to both machine direction (MD) and CD tensile strength. A similar but contactless method was used by Luukkala, et al. (31) to study the on-line elastic properties of paper webs. Hauptmann and Cutshall (32) studied wet paper webs from a viscoelastic standpoint. Their conclusions were that such webs will be greatly affected by certain vibrational frequencies in the web free spans. These effects can lead to wrinkling and frequency dependent mechanical properties. Jartti and Luukkala (33) also investigated an ultrasonic Doppler shift method for on-line measurement of web speed. The CD web parameters are also important in processing lines. Gess and Segre (34) and Hering (35) contend that sensors and control methods for CD tension profile, roll hardness, moisture content, and moisture free weight are all in demand while Smith (36) notes that CD control systems are being widely developed.

The need for increasing web tension in the drying of textiles has been shown by Westhead (37) and a methodology for real-time web tension measurement has been explored by Al- Sayed (38). The interaction of the web and roller also creates specialized situations. The effect of tension measuring dancer rolls and paired traction rollers, Figure 4, on web dynamics has been modelled by Marhauer (39). Daly (40) reports that traction between a web and rollers is increased with web tension and wrap angle, and that traction as a function of web speed and roll diameter is heavily dependent upon web porosity. Fluid effects of web traction on rollers have also been addressed by Knox and Sweeney (41). Chinick (42) notes that traction and other interaction parameters, such as guiding, are also affected by roll alignment. In some winding or guiding applications, it is customary to have the web pass between two very closely spaced rollers or to have a nip roller apply an out of plane force on the web. This latter application is widely used in the winding operation as shown in Figure 5. Pfeiffer (43) has investigated the strain induced in webs because of the presence of a nip roller.

Rollers are used in process lines not only for tensioning and web support but also for steering and guiding the web. Because a moving web seeks normal entry to a roller, as shown in Figure 6, a steering effect can be produced by adjusting the roller angle relative to the direction of web travel. Shelton and Reid developed mathematical models for idealized webs (44) and tested real webs (45). These models are still used to predict the lateral dynamic behavior of a web when it encounters an in-plane roller. This work was expanded upon by Soong and

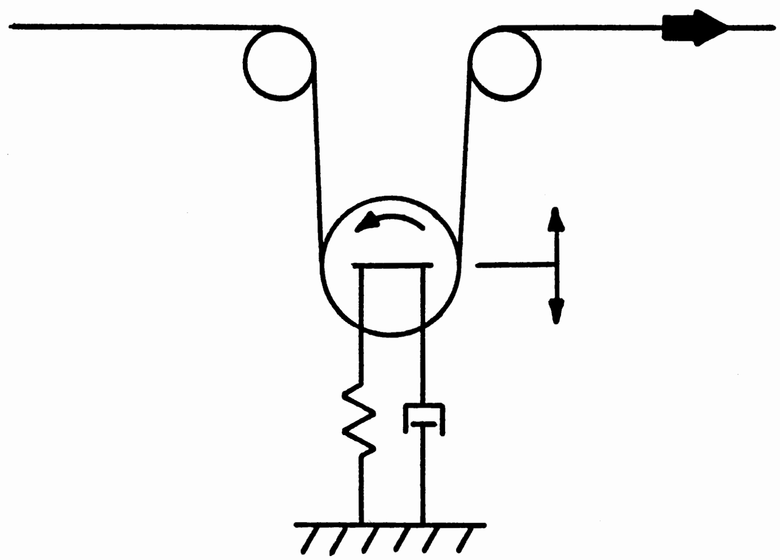


Figure 4. Dancer Roll / Tensioner

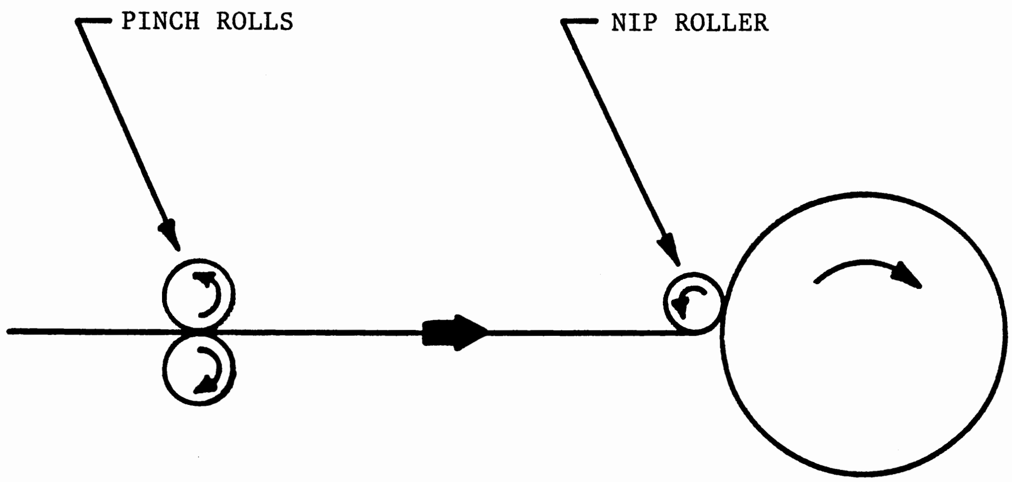
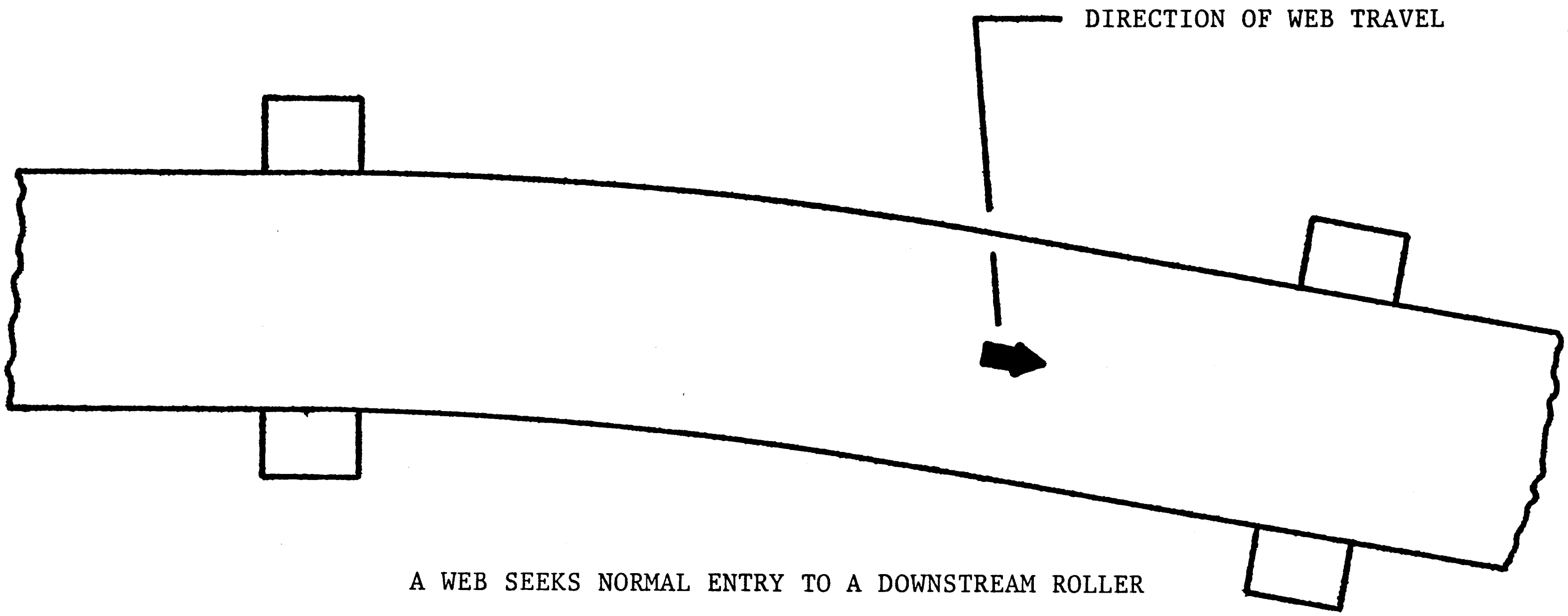


Figure 5. Pinch / Nip Rollers



A WEB SEEKS NORMAL ENTRY TO A DOWNSTREAM ROLLER

Figure 6. Web Steering Effect

Li (46) to include rollers tilted in an out-of-plane orientation. Many of these concepts have been summarized by Pfeiffer (47).

It might be thought that a web is most vulnerable to damage in the free span or where it is being steered. This is not necessarily the case. Pfeiffer (48,49,50,51) has reported on roll defects during winding and unwinding. In winding, air entrapment between web layers can lead to a soft roll which may be subject to damage in handling. In addition, the slippage between the outer and inner web layers in the machine direction, may cause a permanent wrinkling or "starring" pattern in the roll cross section. Slippage in the cross-machine direction can lead to "telescoping" and end damage of the roll. If a roll is not wound tightly enough, layer slippage can occur during the unwind process if the unwind tension is too high. Among others, Daly (52) has indicated that a wound roll consists of inner layers in compression with overlying layers of the web in tension. Frye (53,54,55) has also investigated the effect of winding on roll quality and hardness, while Rand and Eriksson (56) and Hussain and Farrell (57) have specifically addressed the winding problems associated with newsprint. Many other authors have noted on web quality variables and winding, such as Walbaum and Lisnyansky (58,59), Burgeson and Crawford (60), Cox (61), Smith and Meihofer (62), Sjoberg (63), and Green (64).

The Wrinkling Problem

This thesis is concerned with the situation illustrated in Figure 7, namely a machine direction wrinkle encountering a curved roller. As the wrinkle wraps around the roller, there are several possible outcomes. First, for wrinkles of low amplitude, large width, or made of stiff material, and where there is a low traction between the wrinkle and roller, the wrinkle may be pushed back into the plane of the roller with no permanent web damage. Secondly, the web wrinkle may have sufficient section modulus so that it begins behaving like a structural beam or tube and large stresses may be present which result in web damage in the form of a tear or blister. Thirdly, the wrinkle amplitude may be large enough, or the wrinkle width small enough, and the web material may have sufficiently low elastic modulus so that the wrinkle actually collapses on itself causing creasing or tearing of the web. The present investigation will determine how large a wrinkle of assumed cross section may pass over a roller of given diameter and not cause damage to a web having given physical and geometric properties. This problem involves the stability of elastic plates and elastic shells of arbitrary cross section subjected to external tension and bending, and therefore a brief literature review in this area is presented.

Several classic references on structural plates have been authored by Timoshenko and Woinowsky-Kreiger (65) and

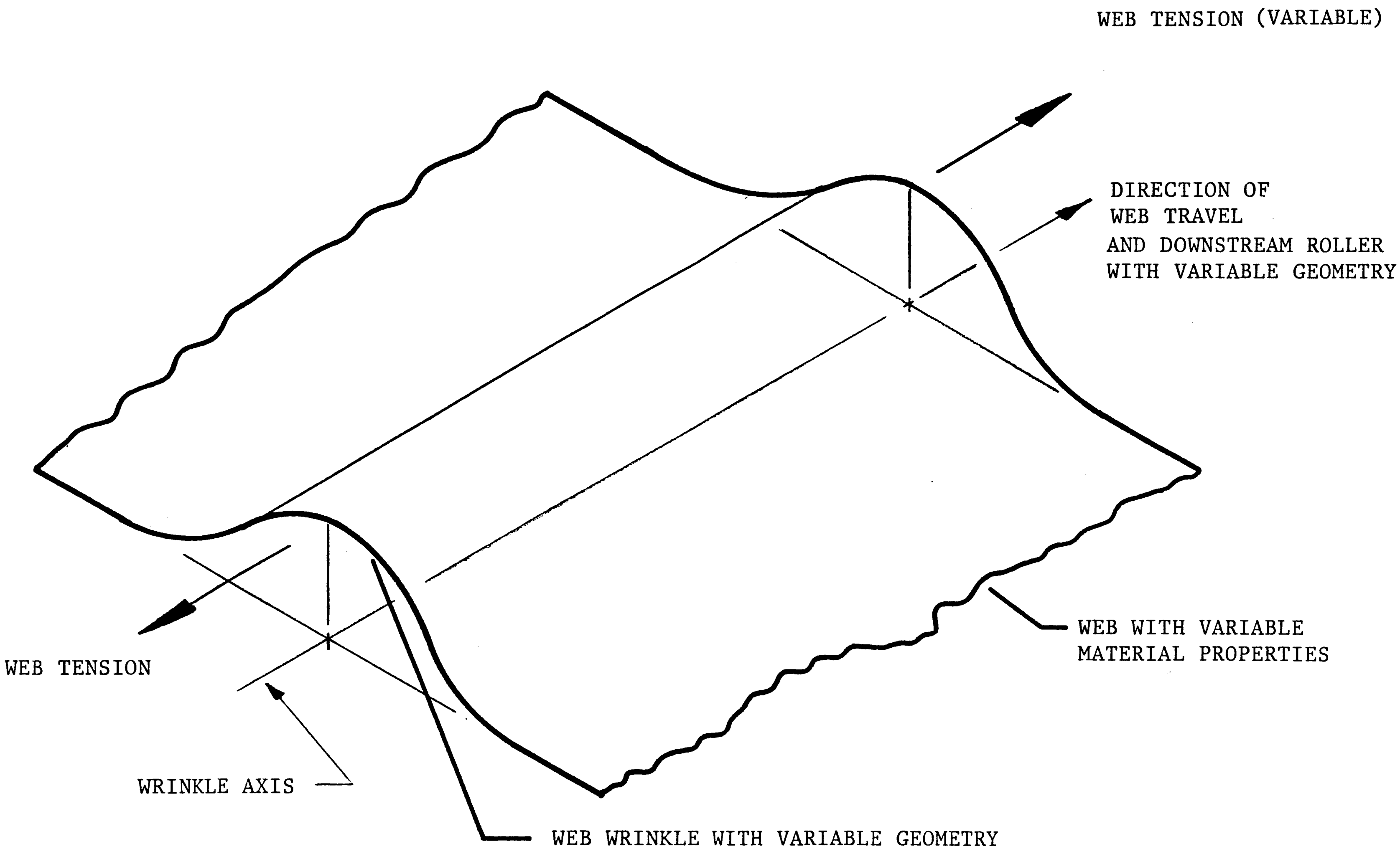


Figure 7. Web Wrinkle

Szilard (66). A few of the many more recent investigations have been published by Caldersmith and Rossing (67), Azimi, et al. (68), Warburton and Edney (69), and Gutierrez and Laura (70). These works are primarily directed to the behavior of large rectangular plates with various edge conditions. Johnson and Urbanik (71) went on to model thin plates with physical nonlinearities to match theory to compressive data from paperboard. A paper by Tvergaard (72) is representative of many in the area of compression of cylindrical panels. Although these works address both small and large deflections, they are primarily concerned with structural materials. Webs, on the other hand, are capable of transmitting very little, if any, bending moment. Instead, webs behave much more like membranes where the primary structural mode is in-plane, such as tension and in-plane shear. Again the literature abounds with papers on large deflections of membranes. These include annular membranes as investigated by Schmidt (73), circular membranes as reported by Kao and Perrones (74) and Storakers (75). Yang and Lu (76) develop equations for hyperelastic membranes, Fenner and Wu (77) allow for membrane inclusions, and Storakers (78,79) uses variation principles and viscoelastic theory for the solution of membranes subjected to lateral pressure. Jones (80) and Seide (81) have altered the classical plate equations to make the plate stiffness zero and solve the resultant set of equations numerically.

There also exists a vast amount of literature on the subject of buckling and as it relates to curved cross sections. Budiansky and Hutchinson (82), Bushnell (83), Babcock (84), and von Karman, et al. (85) are among many papers describing generalized buckling of structures of various shapes and curvature. Among papers dealing specifically with cylindrical shapes are those by von Karman and Tsein (86), Batdorf, et al. (87,88,89), Bijlaard and Gallagher (90), Tamura and Babcock (91) Tvergaard (92), and Zimcik and Tennyson (93). Previous to these papers, however, Donnell (94) had addressed cylinder buckling due to compression and bending in structural tubes and found that experimentation did not agree with the then present theory. Still other studies have been done by Ueng and Sun (95) on inflatable membranes, by Jones and Hennemann (96), Wilson and Orgill (97,98), and Tylikowski (99) on composite and nonlinear cylindrical shells, and Plaut and Johnson (100) and Sinharay and Banerjee (101) on spherical shells. Approaching the shape of the web wrinkle in the present study, but still in the structural regime, are pipes and tubes. These studies include those by Wang and Watson (102) on the equilibrium of elastic cylinders resting on a flat surface, Clark and Reissner (103) on bending of curved tubes, Reissner (104) on bending of curved tubes with internal pressure, Stephens, et al. (105) on tubes with bending and pressure, and Fabian (106) on tubes with bending, pressure, and axial loads. Other associated papers include Rimrott (107) on bending of slit tubes, Seide and

Weingarten (108) on cylindrical buckling due to bending, and Whatham (109) on pipe bend analysis using shell theory. With the wide usage of shells in the aerospace industry, Baker, et al. (110) have compiled many shell equations and solutions for NASA. Still other references address the situation of shells without bending capability, namely membrane shells. These references include Timoshenko (65), Novozhilov (111), Cox (112), and Gol'denveizer (113). Because a web wrinkle is very elastic and only somewhat stable to outside forces, the elastic stability of shells is included with references such as Batdorf (114, 115), Timoshenko and Gere (116), Thompson and Hunt (117), and Budiansky (118).

Summary

Although the literature abounds with papers on the subject of shell behavior, a web wrinkle presents a unique situation. Because the web is so flexible, its original shape may become grossly deformed as it passes over a roller but still does not fail in a structural sense and no creasing or tearing takes place. Such deformations are not in the realm of engineering design for most shells and similarly shaped structures. Secondly, as the web passes over a roller the boundary conditions of friction and web-roller contact at individual points on the web indicate that an iterative solution is necessary to account for subsequent deformations of the web and wrinkle points. For these two main reasons, it seems that a closed form solution using the classical approaches may not be possible. Instead, a solution using numerical methods will allow for the rapid and economical alteration of boundary conditions as well as the physical and material parameters associated with the web wrinkle.

CHAPTER II

ANALYTICAL STUDY

The purpose of the analytical study is to produce generated output in the form of stress values and deformed shapes of web wrinkles under a wide variety of physical and geometric parameters. By investigating the stresses created by these parameters it should be possible to make some qualitative and quantitative conclusions concerning the relative sensitivity of the stress about each of the parameters. The output is from a finite element study using the NASTRAN (NAsa STRuctural ANalysis) computer code developed by the National Aeronautics and Space Administration. The version of NASTRAN used is COSMIC release 1985 and 1986. The input to this finite element program is generated by an automatic mesh generation program developed by this author. A source code listing for the mesh generator is given in Appendix A of this thesis. The mesh generator was purposely made to be very general so that a wide variety of cases could be handled with a minimum effort in changing the mesh generator. With minimal input the mesh generator produces web tension, boundary frictional forces, and web-roller constraints which are subsequently used as input to NASTRAN. This approach provides a versatile method for solving many iterations of the web problem simultaneously.

A typical undeformed web wrinkle as produced by the mesh generator and the NASTRAN plotting procedure NASPREV is shown in Figure 8. After discussion with representatives of the industrial members of the Web Handling Research Center at Oklahoma State University, the parameters and their range of values are used as listed in Table I. In a typical web process line, guiding and steering rollers are in the range of two to eight inches in diameter. A 24-inch diameter roller is included in the study to simulate the winding of smaller diameter web rolls. The ranges for wrinkle width and height are such based on personal inspection of wrinkles in both plastic and paper processing lines. Although wrinkles may be encountered outside this range, these values should be representative of a great many wrinkles and the wrinkle aspect ratio (wrinkle height divided by wrinkle width) will be representative of wrinkles outside the given ranges. Web thickness and web tension ranges are the result of consultation with industry representatives. A web-roller frictional coefficient of zero is used to simulate the lower bound of an "air-bearing" roller which uses air pressure to lift the web off of the roller or guide. This is typically used for coated webs which must be steered but which must not come into physical contact with a roller until the coating has dried. A coefficient of infinity is simply defined as that coefficient which allows no web-roller slippage. The two material parameter (Young's modulus and

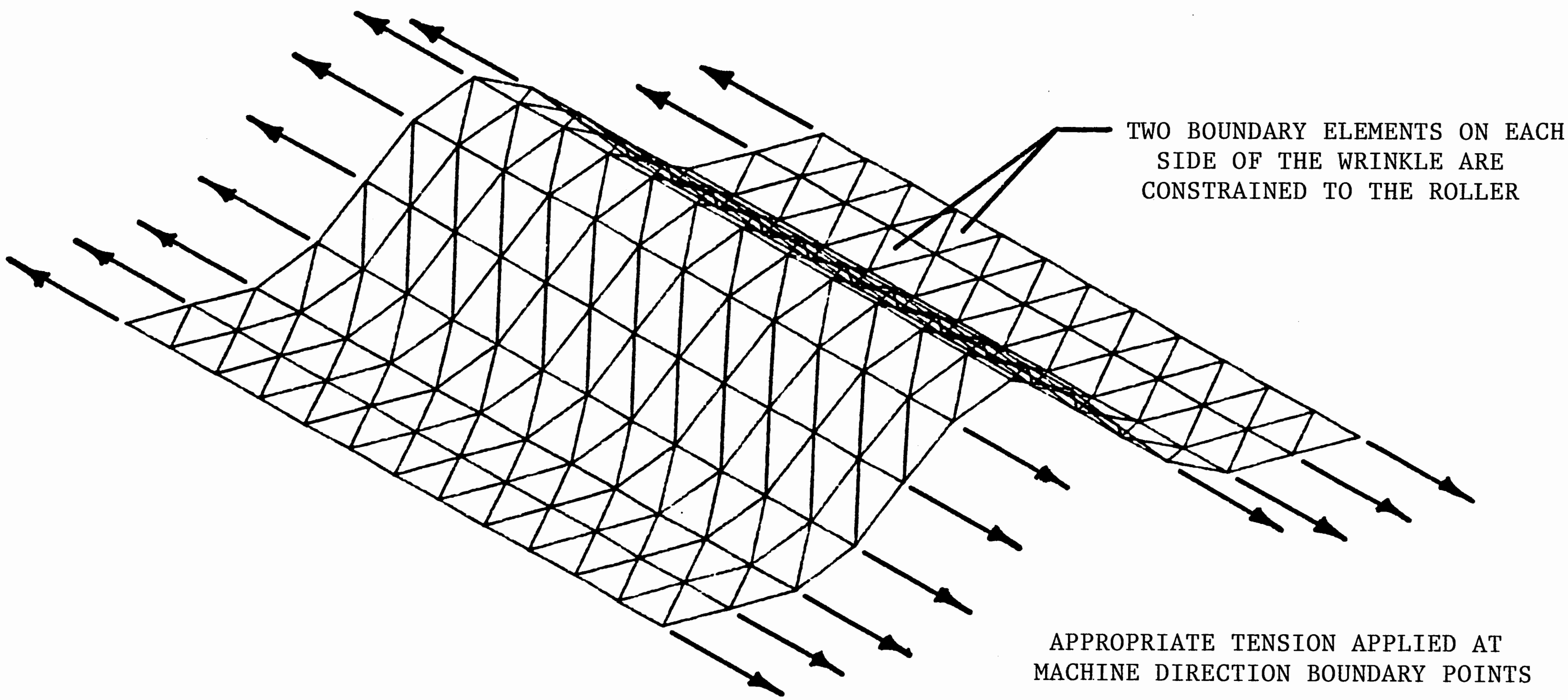


Figure 8. Finite Element Mesh Generator Output

Poisson's ratio) ranges are selected to include polypropylene, polyethylene, polyester, paper, and metals, to name a few.

TABLE I

WRINKLE GEOMETRIC AND MATERIAL VARIABLES

VARIABLE	RANGE
Roller Diameter	2 - 24 inches
Wrinkle Width	0.5 - 6 inches
Wrinkle Height	0.025 - 1 inches
Web Thickness	0.00005 - 0.06 inches
Web-Roller Friction Coefficient	0 - infinity
Tension	0.25 - 50 pounds/inch width
Wrap Angle	0 - 180 degrees
Young's Modulus	2,000 - 30,000,000 psi
Poisson's Ratio	0.01 - 0.5

The finite element model is composed of triangular elements which are capable of both in-plane and bending stiffness. Although a thin membrane can offer little bending resistance, this capability is included in the element to accomodate relatively thick webs. The NASTRAN TRIA2 element was selected and its local coordinate system is shown in Figure 9 and the positive sign convention for stress and displacement is shown in Figure 10 for computed output. The TRIA2 element is a planar element with both in-plane and bending stiffness and a solid homogeneous cross section is assumed. Because the TRIA2 is a planar element, it is also assumed that no change in the element's thickness takes place. Each element is bounded by grid points which may have up to three translational and up to two rotational degrees of freedom in the local element coordinate system. The TRIA2 element does not permit rotations about the axis normal to the element surface. It is necessary to constrain this degree of freedom to zero. With these degrees of freedom and the planar element, it might be thought that this web model is composed of many triangular plates as shown in Figure 11.

The first step in the analyses is to input the material and geometric parameters for a given situation. These parameters typically consist of wrinkle width, wrinkle height, roll diameter, web thickness, Young's modulus, and Poisson's ratio. Initially the web tension is input as zero so as to investigate the deformation of the wrinkle onto the roller. This step is necessary to help determine which web wrinkle points will deform onto the roller surface first. After observation of the deformed shape without tension, an appropriate tension is added to the mesh generator program. After the tension is applied, it is necessary to constrain to the roller surface those points which deformed to that

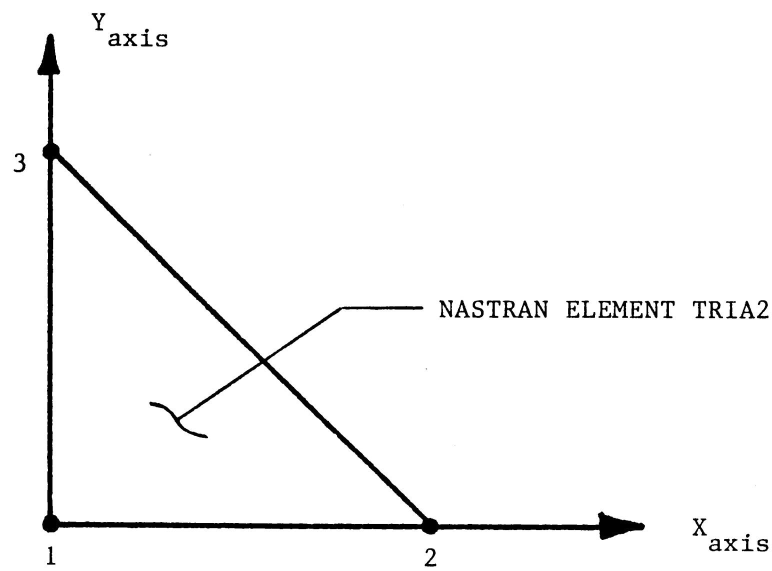


Figure 9. TRIA2 Element Coordinate System

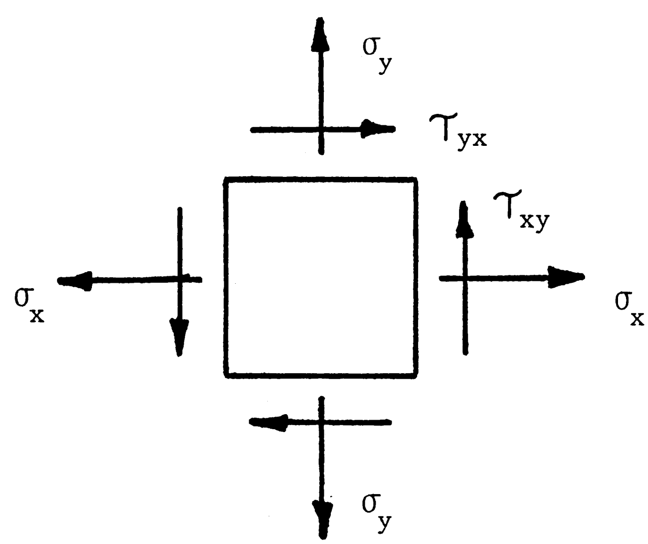


Figure 10. Membrane Element Stresses

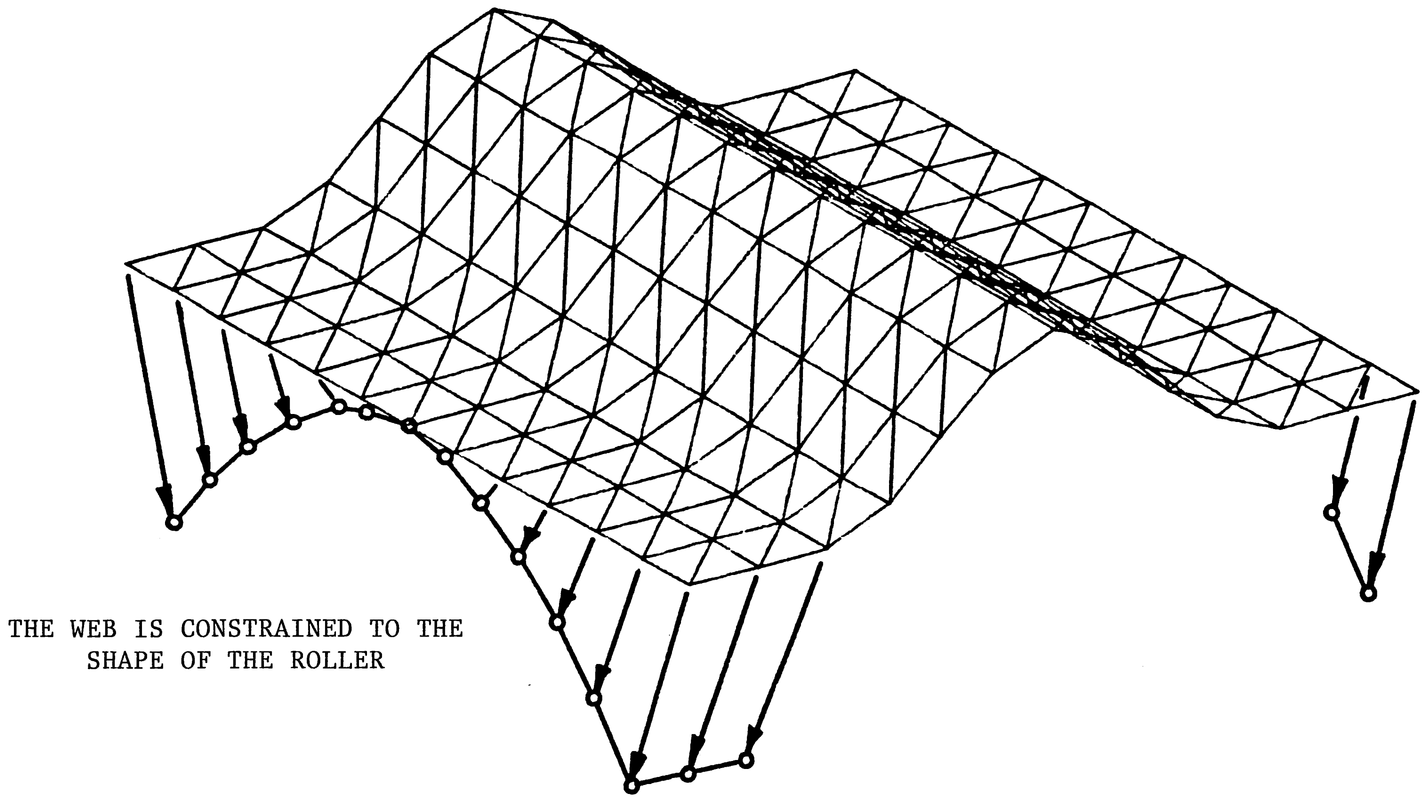


Figure 11. Deformed Wrinkle

surface when there was no tension. Two elements on each side of the wrinkle cross section are arbitrarily chosen to be appropriately constrained to the roller's circular shape causing the wrinkle to deform. This process is illustrated in Figure 11. Both a plot and printed output are obtained at this point for analysis. A typical plot for step one is shown in Figure 12. The printed output consists of the translational and rotational displacement vectors for each grid point, the necessary force applied to constrain the boundary elements to the roller shape, the normal and shear stress at the web material surface, and the principal normal stress and, maximum shear stress. It is easily seen that many points fall below the roller surface due to the boundary constraints and the fact that the interior wrinkle points have not been constrained from deforming below the roller surface.

At this point, the displacement vectors are examined and the necessary constraints applied so that any points which fall below the roller surface are constrained to the roller surface. In addition the boundary forces of constraint are examined and if the necessary force to constrain the point exceeds that which friction can supply, using the frictional coefficient under consideration, then the static frictional force is applied at that point. If the necessary force to constrain the point does not exceed that available from friction, the computed force of constraint is applied to that point. This process is

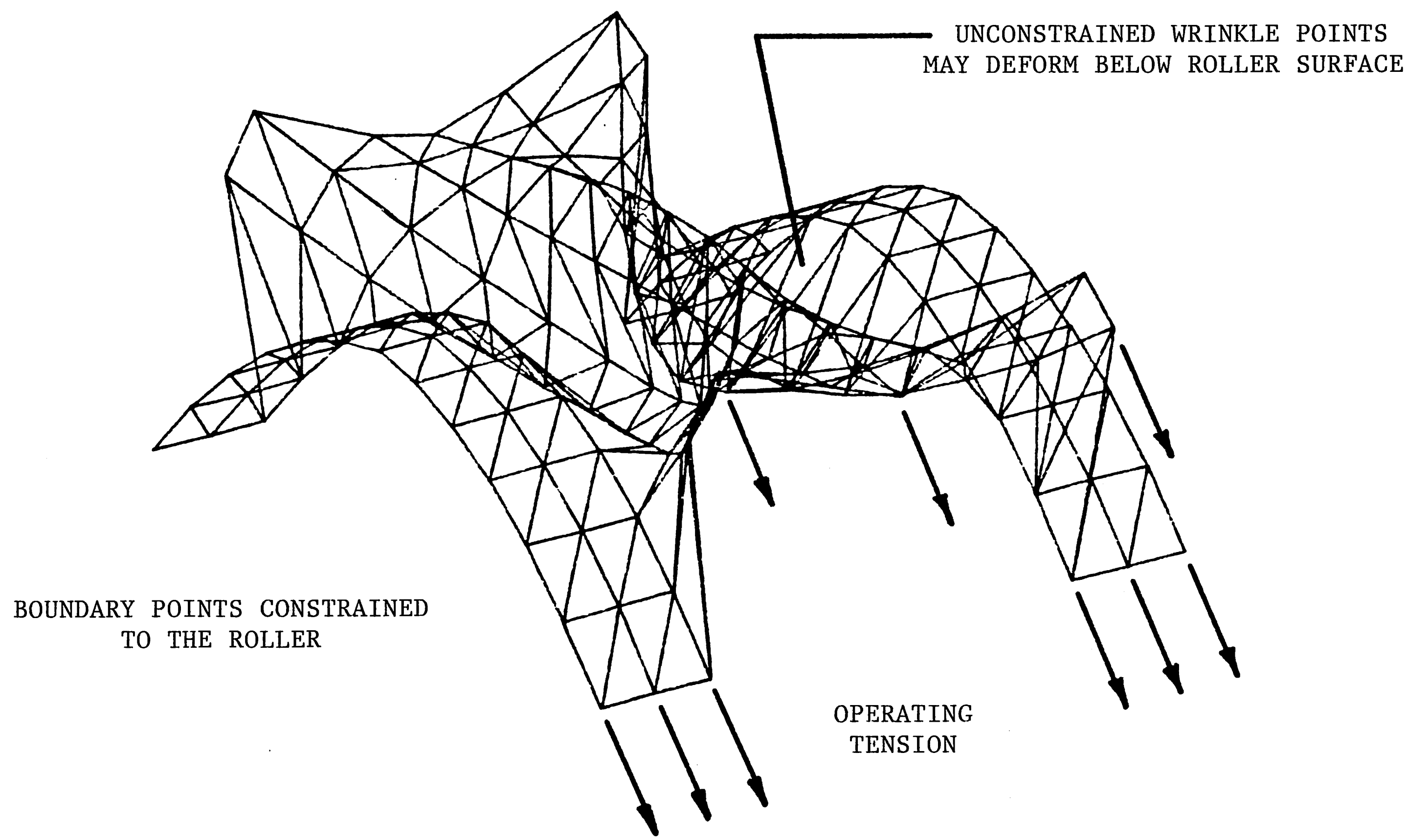


Figure 12. Deformed Wrinkle Without Constraints

illustrated in Figure 13. It should be noted here that the mesh generator also computes the normal force at each boundary grid point due to web tension.

The second step in the analyses is to iteratively constrain the wrinkle points which fall below the roller surface in step one until an equilibrium deformation is produced. This also includes allowing the boundary points to move if the necessary force of constraint exceeds the frictional force available from the roller contact normal force and the chosen frictional coefficient. Both a plot and printed output are obtained at this step for analysis. For thick webs and large roller diameters, step two usually produces an equilibrium condition. However for thin webs, further iteration and constraint of grid points to the roller surface is necessary. It is not unusual to iterate over five times before equilibrium is reached for very thin webs. This iteration of web to roller constraint takes place from the center of the web contact area out toward the machine direction boundaries. This is done after a physical examination of how real web wrinkles behave. This examination produced the wrinkle photo shown in Figure 14. It may be seen that for thin webs the maximum deformation occurs at the web center while at the web machine direction boundaries the wrinkle lifts off of the roller in an effort to conform with the undeformed web wrinkle. This combination produces an "hourglass" shape which is seen to be characteristic of thick or stiff wrinkles.

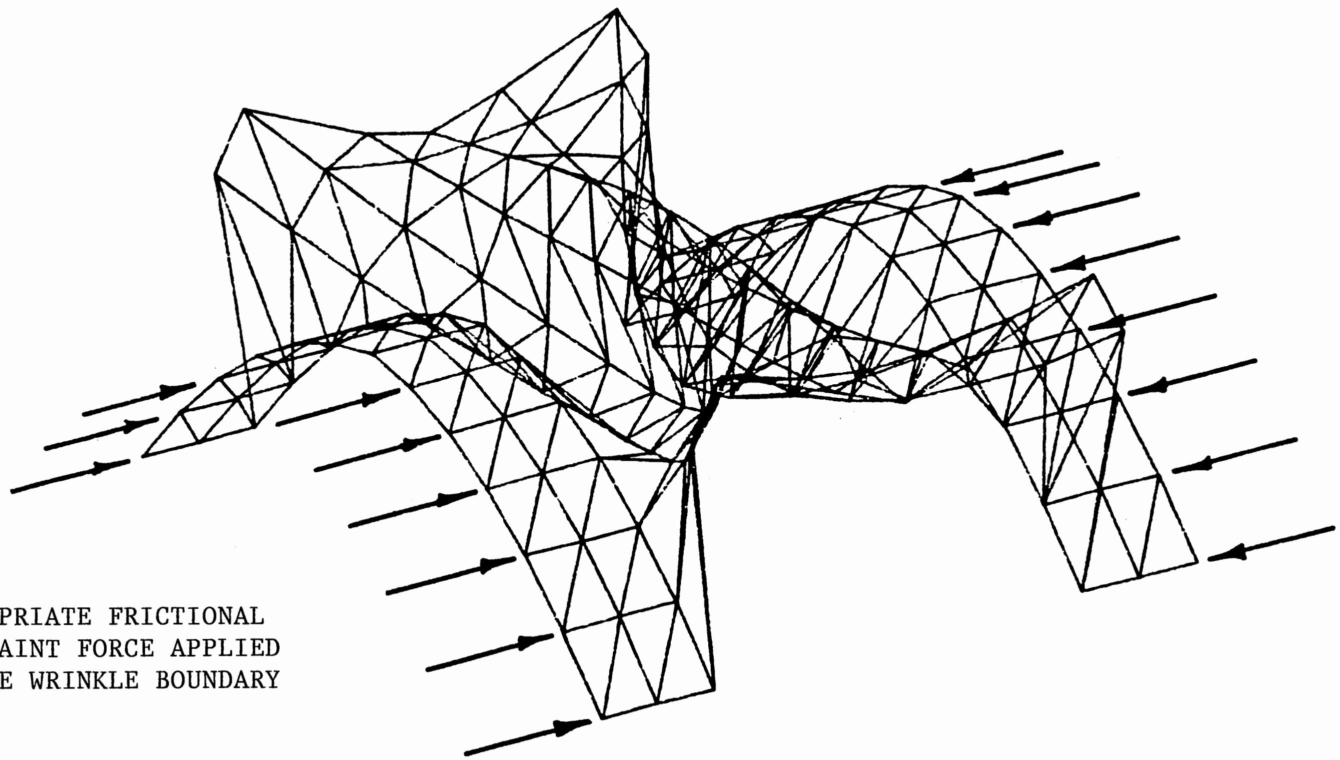


Figure 13. Deformed Wrinkle with Friction

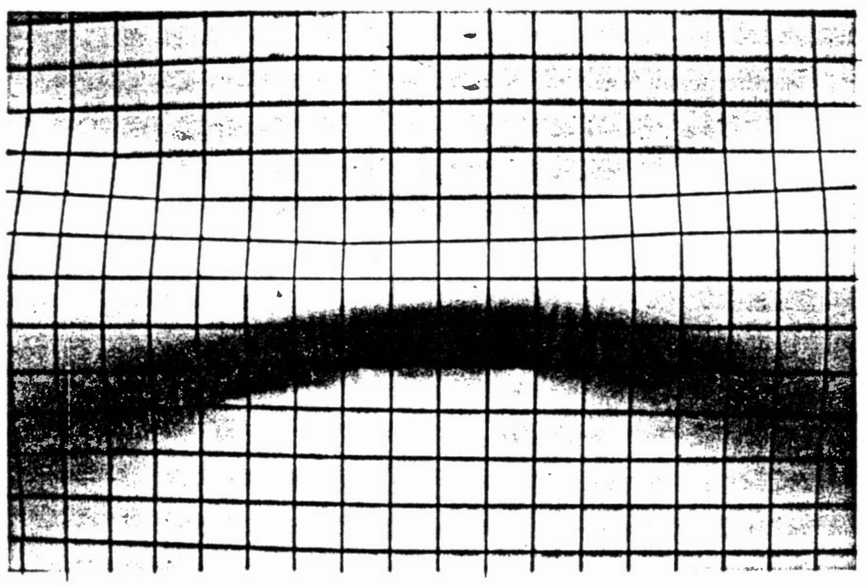


Figure 14. Deformed Wrinkle Photograph

When a stable deformed wrinkle is achieved, a printed output is used to find the maximum normal stresses operating in the web under the given parameters. After investigating the output, it was found that several areas on the wrinkle produced representative stress values. These areas are shown in Figure 15. Although there may be considerable difference in the stress values at these four points for a single computer run, the average of these four values creates a single value which, for different conditions, gives a relatively well behaved measure of that parameter's effect on the stress. The entire procedure is then repeated for the various geometric and material parameters to produce a matrix of results. A flow chart is provided in Figure 16 to more easily show the sequence of events in the analytical study.

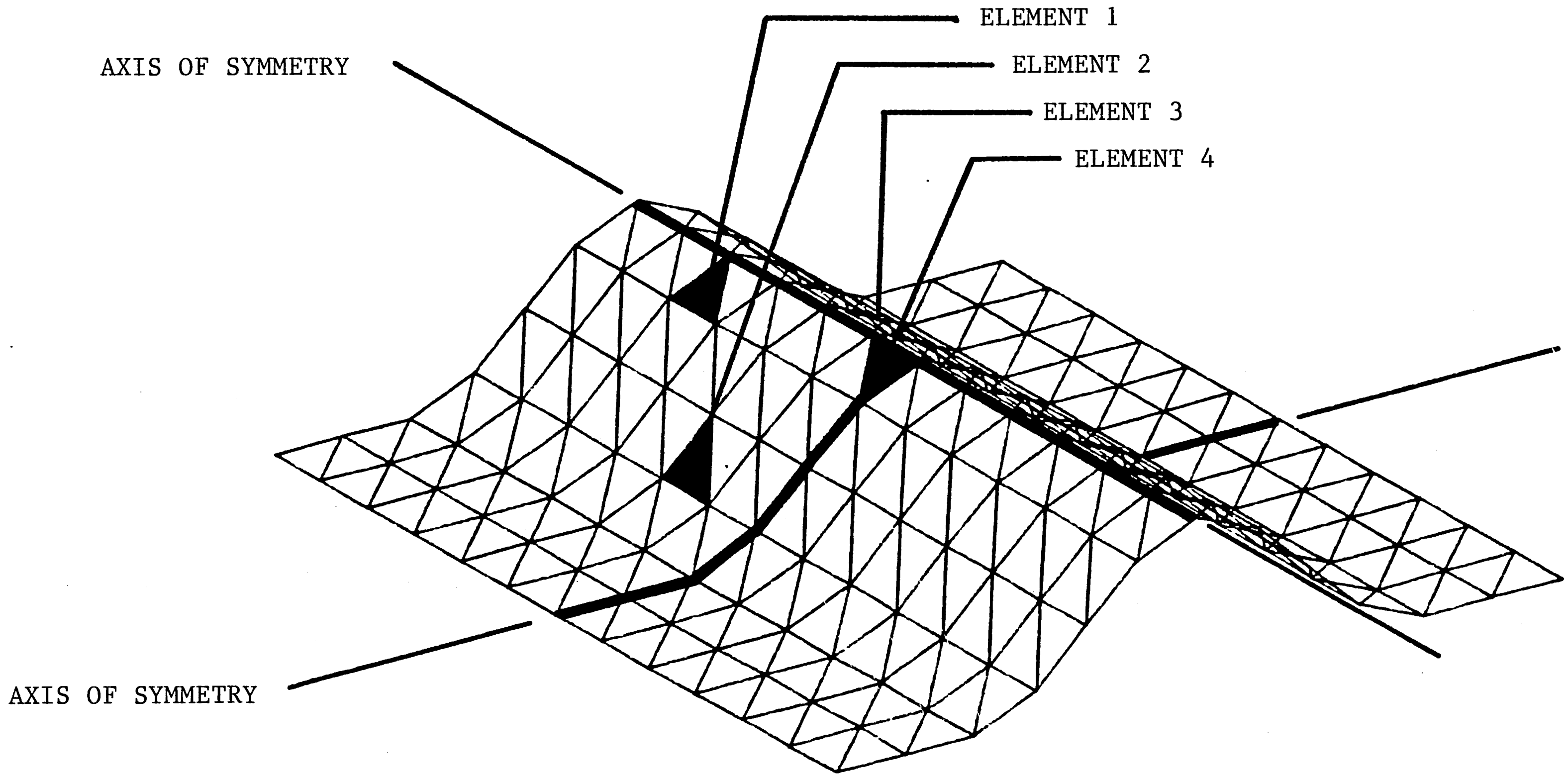


Figure 15. Location of Representative Elements

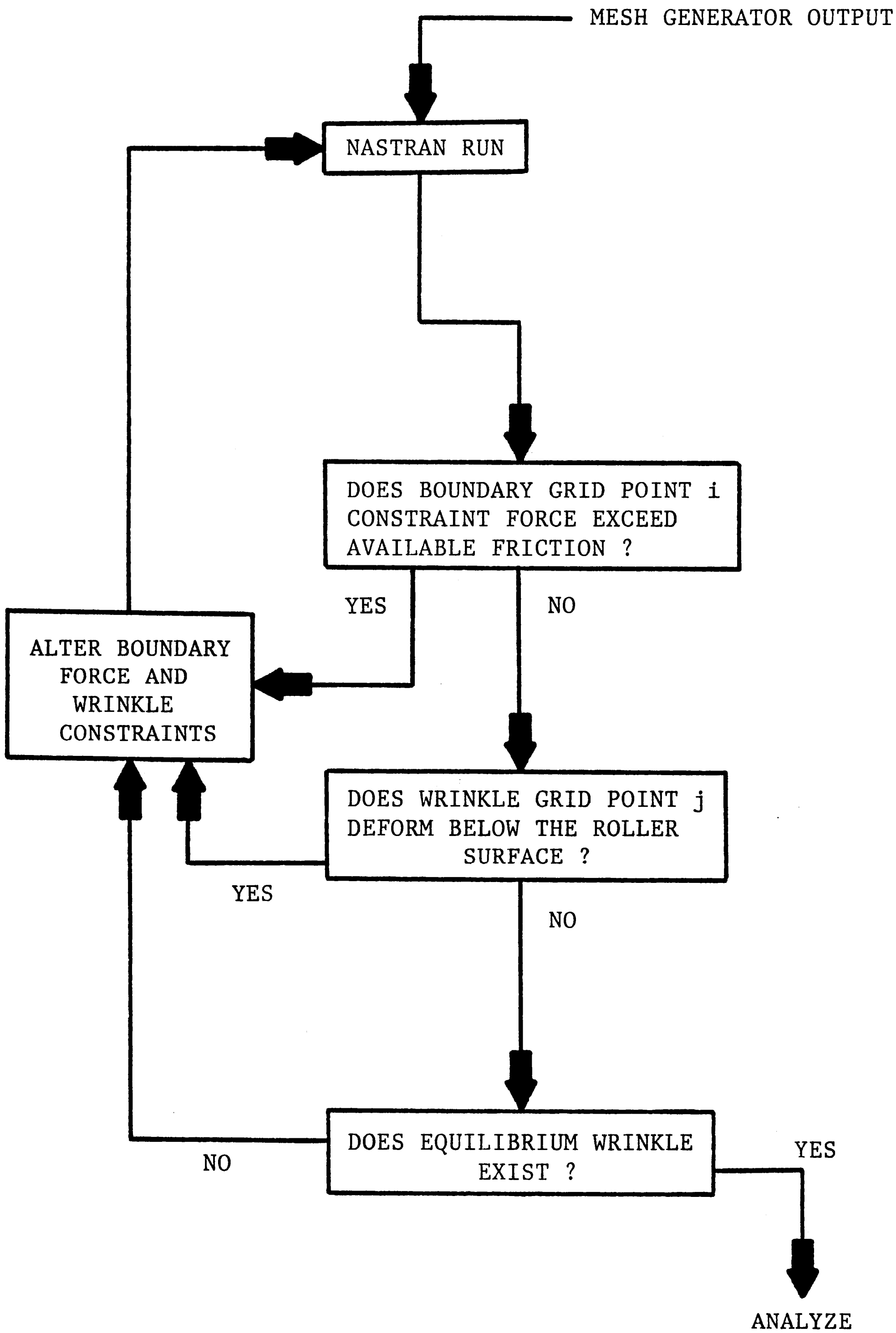


Figure 16. Equilibrium Wrinkle Iteration Diagram

CHAPTER III

ANALYTICAL RESULTS

As stated previously, the results of the analytical study consist of grid point displacements, boundary grid point constraint forces, element stresses, and plots of the deformed wrinkle shapes. The grid point displacements and constraint forces are used to properly constrain the web to the roller and therefore will not be considered further in the results. The analytical study addresses nine parameters. To predict the stress operating in a web wrinkle subject to any combination of these parameters requires detailed knowledge of how each parameter affects the state of stress. For a nine dimensional matrix of solutions, as would exist with these nine parameters, an inordinate number of NASTRAN runs would be required. Instead, a much smaller number of runs, nearly 200, provides enough information to allow a sensitivity analysis to be performed. The results presented are intended to provide insight into the degree to which each parameter contributes to the wrinkle behavior. Any attempt to use the results to accurately predict the stress in a web wrinkle should be done with caution.

Because of the large variation in wrinkle width and roller diameter, it is necessary to vary the number of elements in the cross machine direction as well as the machine direction. For this reason, the relative location of the representative elements previously shown in Figure 15 is maintained regardless of the total number of elements. The wrinkle width and height are varied in specific ratios such that the wrinkle cross section maintains the equation

$$Z = (A/2) * (1 + \cos(Y*W)) \tag{1}$$

In addition, a parameter called the wrinkle aspect ratio (AR) is defined as the wrinkle height divided by the wrinkle width. Other dimensional parameters will be defined as the need arises. The complete stress output data from the nearly 200 computational runs is given in Appendix B. The following results, whether in tabular or graphical form, are extracted from the Appendix B data. Selected deformed wrinkle shape plots are included in the results when appropriate. Those plots not specifically used are given in Appendix C.

The method of investigating the relative sensitivity of each of the nine parameters is to select a given combination of parameters and normalize all stress data to the stress data with that combination. If the normalized stress variations of a given parameter are relatively independent of other parameters, that given parameter is graphically presented by itself. However, if a given parameter is clearly dependent upon other parameters, then that given parameter is graphically presented as a family of curves within the other parameters. There are cases where stress results are presented which are obviously higher than the ultimate stress of nearly any material. These high stresses are used only to investigate the stress relative to some normalized value and are not to imply that the web is capable of surviving the stress. The only assumption to be made is that the web material still behaves in a linearly elastic fashion at these higher stress levels. Because the high level is used only for numerical comparison, this assumption is valid. This method of presentation is more clearly understood as the results are presented. The material parameters are investigated first because of their well behaved result and their independence of other parameters.

Young's Modulus

The specific data used to investigate the contribution of Young' modulus is presented in Table II. The effect is relatively independent of roller diameter, wrinkle height, and web thickness. The data is normalized to unity at a value of 300,000 psi. The normalized data is presented in Table III, while the graphical result is shown in Figure 17. Several representative deformed wrinkle plots are shown in Figures 18, 19, 20, and 21. It may be seen that as the elastic modulus is decreased, the wrinkle attempts to collapse onto the roller, toward the centerline of the

TABLE II

YOUNG'S MODULUS VARIATION DATA

Roller Diameter 4" — Poisson's Ratio 0.3
Wrinkle Height 0.33" — Friction Coefficient inf
Wrinkle Width 1.05" — Thickness 10 mils
Tension 0.5 pounds/inch/mil

Modulus (psi)	Element and Stress (psi) 1	2	3	4
2,000	713	525	682	810
10,000	761	1190	1385	1575
50,000	2476	2419	3870	5666
100,000	3152	7956	7034	8927
300,000	7117	26682	19668	23320
1,000,000	28590	52338	44018	68911
30,000,000	859,691	1,565,900	1,281,240	2,011,950

Roller Diameter 24" — Poisson's Ratio 0.3
Wrinkle Height 0.3" — Friction Coefficient inf
Wrinkle Width 6.28" — Thickness 3 mils
Tension 1.67 pounds/inch/mil

Modulus (psi)	Element and Stress (psi) 1	2	3	4
2,000	193	174	189	197
10,000	199	187	203	211
50,000	452	419	516	551
100,000	1047	1004	1126	1240
300,000	4720	1764	3145	1987
1,000,000	9040	7387	9263	7414
30,000,000	216,960	204,993	210,140	182,419

TABLE III

YOUNG'S MODULUS RELATIVE STRESS RESULTS

Roller Diameter 4"

Modulus's (psi)	Average Stress (psi)	Relative Stress
2,000	683	0.04
10,000	1228	0.06
50,000	3608	0.19
100,000	6767	0.35
300,000	19197	1.0
1,000,000	48464	2.52
30,000,000	1,429,695	74.5

Roller Diameter 24"

Modulus (psi)	Average Stress (psi)	Relative Stress
2,000	188	0.06
10,000	200	0.07
50,000	485	0.17
100,000	1104	0.38
300,000	2904	1.0
1,000,000	8276	2.85
30,000,000	203,628	70.1

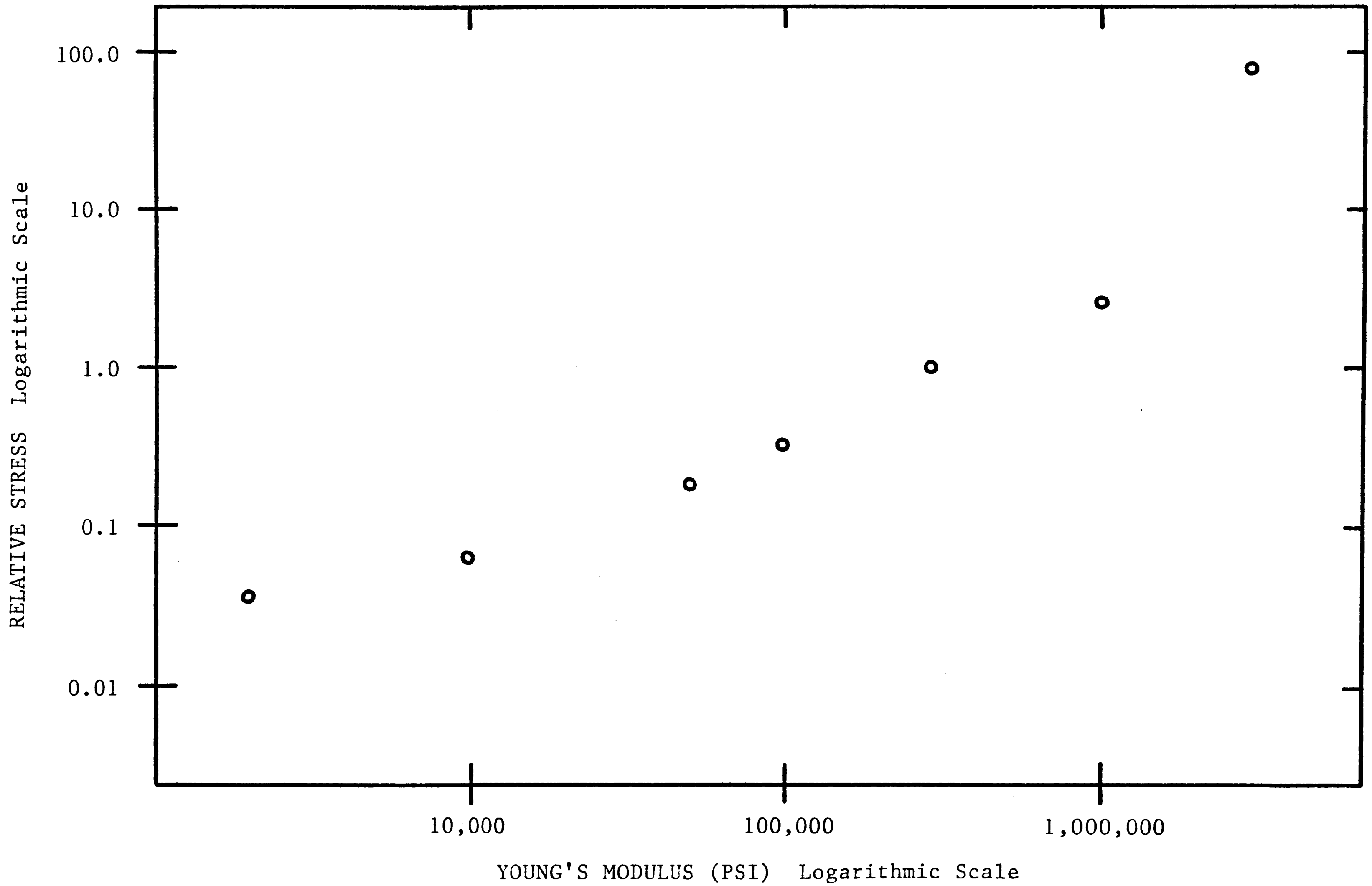


Figure 17. Relative Stress vs Young's Modulus

RD=24"
WW=1.05"
TN=50 p/i
YM=300,000 psi
WA=62 degrees

WH=0.33"
TH=10 mils
PR=0.3
FC=inf

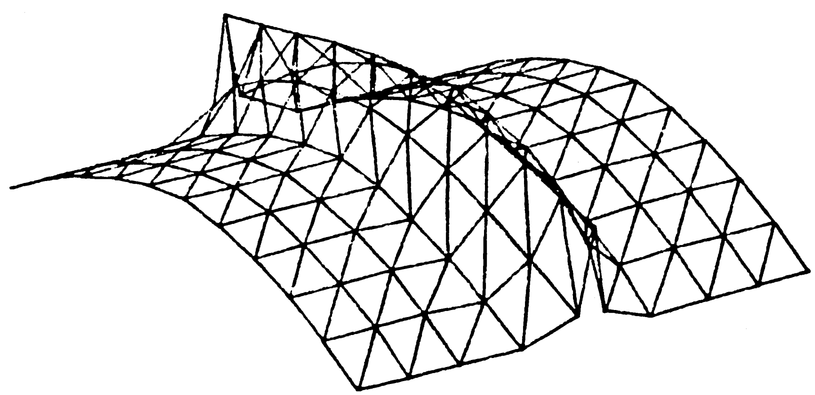


Figure 18. Deformed Wrinkle Shape (Young's Modulus)

RD=4"
WW=1.05"
TN=5 p/i
YM=50,000 psi
WA=61 degrees

WH=0.333"
TH=10 mils
PR=0.3
FC=inf

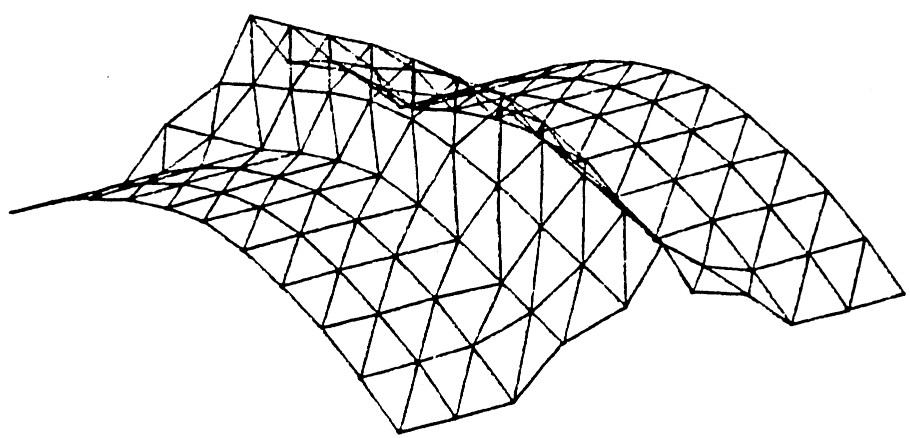


Figure 19. Deformed Wrinkle Shape (Young's Modulus)

RD=4"
WW=1.05"
TN=5 p/i
YM=100,000 psi
WA=61 degrees

WH=0.333"
TH=10 mils
PR=0.3
FC=inf

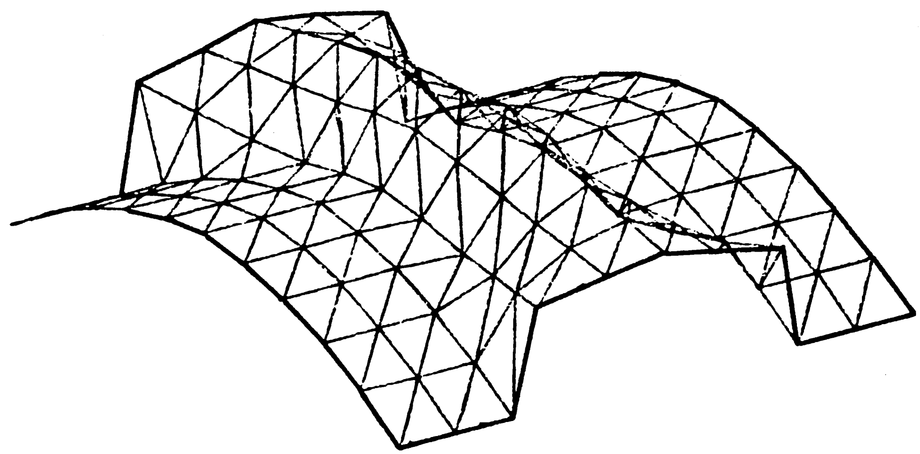


Figure 20. Deformed Wrinkle Shape (Young's Modulus)

RD=4"
WW=1.05"
TN=5 p/i
YM=300,000 psi
WA=61 degrees

WH=0.333"
TH=10 mils
PR=0.3
FC=inf

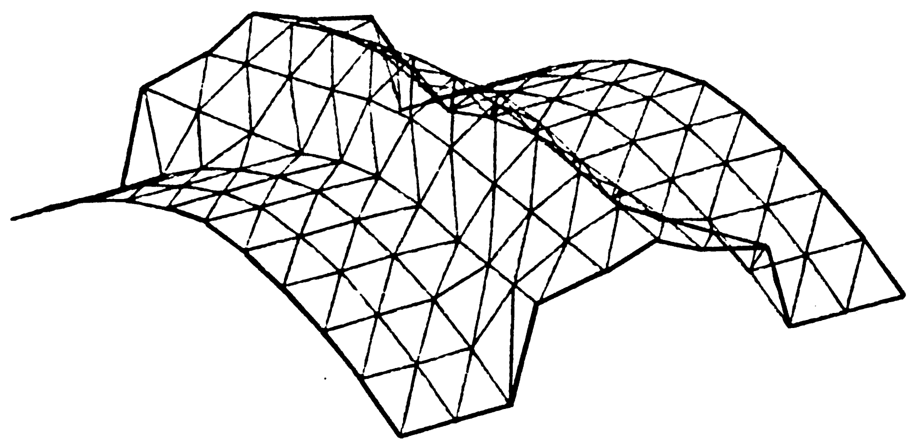


Figure 21. Deformed Wrinkle Shape (Young's Modulus)

wrinkle, with a corresponding decrease in stress. Conversely, a higher elastic modulus allows the wrinkle to more closely maintain its original shape with an increase in stress. As will be seen in subsequent deformation plots, this behavior is very similar to that of the web thickness. For a flat member, the flexural rigidity is given by

$$D = (Eh^3) \;/\; (12(1-v^2)) \tag{2}$$

and is responsible for stress arising due to bending in the cross machine direction. Also, for a linearly elastic isotropic material

$$\sigma_x = (E(e_{xx} + ve_{yy})) \;/\; (1 - v^2) \tag{3}$$

which is responsible for the stress due to tension. In this case, the stress term due to the Poisson effect is negligible because the unconstrained wrinkle points are free to displace in the cross machine (y) direction producing little or no 'yy' strain. The relative stress variation due to changes in Young's Modulus is consistent with both of these equations.

Poisson's Ratio

The specific data used to investigate the contribution of Poisson's ratio is presented in Table IV. As with Young's modulus, the effect appears to be independent of web

TABLE IV

POISSON'S RATIO VARIATION DATA

Roller Diameter 4"	Young's Modulus 300,000(psi)
Wrinkle Height 0.33"	Friction Coefficient inf
Wrinkle Width 1.05"	Thickness 10 mils
Tension 0.5 pounds/inch/mil	

Poisson's Ratio	Element and Stress (psi) 1	2	3	4
0.01	7487	27998	20631	23640
0.05	7345	27695	20454	23470
0.1	7223	27372	20286	23308
0.3	7117	26682	19668	23320
0.5	8778	21140	17648	23847

Roller Diameter 8"	Young's Modulus 300,000(psi)
Wrinkle Height 0.667	Friction Coefficient inf
Wrinkle Width 2.09"	Thickness 20 mils
Tension 0.25 pounds/inch/mil	

Poisson's Ratio	Element and Stress (psi) 1	2	3	4
0.1	4681	1930	9006	23995
0.3	4417	2049	8927	24194
0.5	4383	2426	9021	25876

geometry. The data is normalized to unity at a value of 0.3 and is presented in Table V. The graphical result is shown in Figure 22. Several points are noteworthy concerning the data. First, equations (2) and (3) indicate that the stress level should behave proportional to linear variations and second degree variations of Poisson's ratio. The data indicates that the opposite is happening. The second point to be noticed though, is that the relative variation in the data is quite small, only several percent. Taking the plotted relative stress to be the average of the values in Table V, it can be seen that the variation of the relative stress is well within the standard deviation of the data. In other words, although equations (2) and (3) appear to be violated, the results of the computer runs for Poisson's ratio are inconclusive.

Wrinkle Height

After observation of the results concerning wrinkle height, wrinkle width, and roller diameter, it seems apparent that the stress and deformed wrinkle shapes are dependent upon both wrinkle height and roller diameter. After further observation it becomes clear that a ratio of the two could best be used as an additional dimensionless parameter. The ratio of roller diameter to wrinkle height is referred to as the wrinkle height ratio. A small wrinkle height ratio indicates a wrinkle of high amplitude passing over a small diameter roller. The specific data used to

TABLE V

POISSON'S RATIO RELATIVE STRESS RESULTS

Roller Diameter 4"

Poisson's Ratio	Average Stress (psi)	Relative Stress
0.01	19939	1.04
0.05	19741	1.03
0.10	19547	1.02
0.30	19197	1.00
0.50	17853	0.93

Roller Diameter 8"

Poisson's Ratio	Average Stress (psi)	Relative Stress
0.1	9903	1.00
0.3	9897	1.00
0.5	10427	1.05

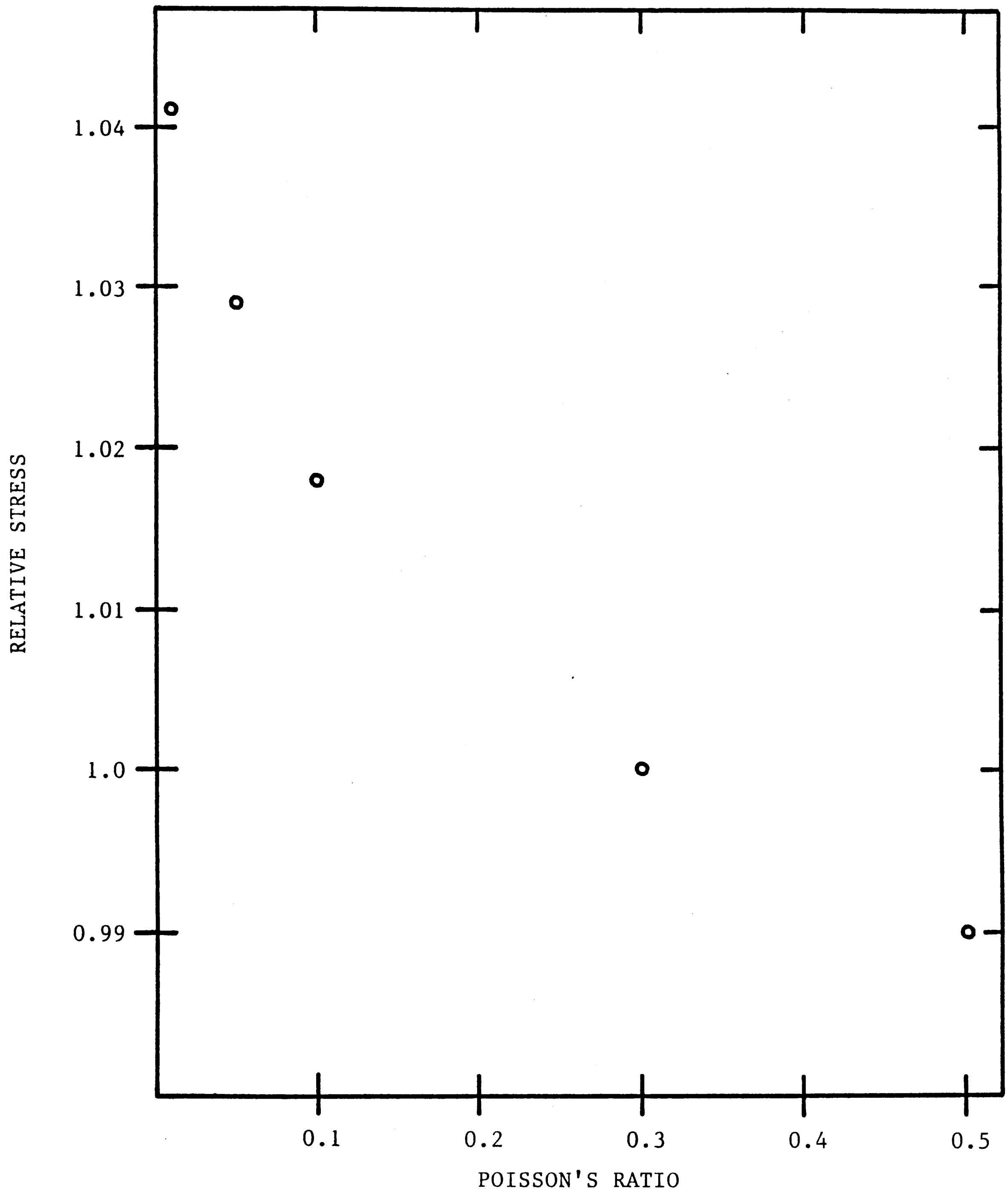


Figure 22. Relative Stress vs Poisson's Ratio

investigate the effect of wrinkle height ratio is presented in Table VI. The data is normalized to unity at a wrinkle height ratio value of 80, with a 24-inch diameter roller and a 0.3-inch high wrinkle. This normalized data is shown in Table VII. For the four-inch diameter data, there are two sets of wrinkle height ratios. This is because of an investigation to determine if the relative stress is independent of wrinkle width. From this data, this does appear to be the case. The graphical result is presented in Figure 23. As might be expected, the relative stress becomes higher as smaller wrinkle height ratios are encountered. This indicates that for wrinkles passing over a given diameter roller, a higher amplitude wrinkle will encounter a higher stress. Given a wrinkle with sufficient rigidity so as not to collapse onto the roller, this result is expected due to the increased section modulus of the wrinkle cross section. An interesting observation is that for a wrinkle height ratio between 12 and 80, the relative stress is nearly a linear function and only increases by about a factor of 2. At ratios lower than 12, the relative stress increases very rapidly, at least for the two-inch and four-inch diameter rollers. The reason that the relative stress is not investigated for the 8 and 24-inch diameter rollers, at wrinkle height ratios lower than 12 and 24 respectively, is due to the very large amplitude wrinkle which will exist in the web. Observation of polypropylene, in a width as much as ten feet and under relatively low

TABLE VI

WRINKLE HEIGHT RATIO VARIATION DATA

Thickness 3 mils | Poisson's Ratio 0.3
Young's Modulus 30,000(psi) | Friction Coefficient inf
Tension 1.67 pounds/inch/mil

Wrinkle Height Ratio	Element and Stress (psi) 1	2	3	4
Roller Diameter 24"				
80	4720	1764	3145	1987
40	4580	926	3154	7065
24	3843	2344	4962	9797
Roller Diameter 8"				
80	4286	1266	8468	9734
24	6729	4766	10151	8076
12	6353	11529	16399	5979
Roller Diameter 4"				
80	4940	2195	5429	7557
24	7395	4703	14174	16516
12	11230	9710	26947	25471
6	31389	21271	46999	42065
4	40851	31551	66303	59306
80	3669	2377	6218	9823
24	5466	14454	11515	11828
12	8024	3092	20686	16715
6	11567	57732	36601	27516

TABLE VI (Continued)

Thickness 3 mils | Poisson's Ratio 0.3
Young's Modulus 30,000(psi) | Friction Coefficient inf
Tension 1.67 pounds/inch/mil

Wrinkle Height Ratio	Element and Stress (psi) 1	2	3	4
Roller Diameter 2"				
80	8135	2568	4685	9735
40	8751	2616	6406	15068
24	12287	3408	10155	21998
12	18086	8321	16737	26696
6	29404	18474	32846	39017
3	48958	44354	69267	68744

TABLE VII

WRINKLE HEIGHT RATIO RELATIVE STRESS RESULTS

Wrinkle Height Ratio	Average Stress (psi)	Relative Stress
Roller Diameter 24"		
80	2904	1.0
40	3931	1.35
24	5237	1.8
Roller Diameter 8"		
80	4418	1.52
24	7431	2.56
12	10065	3.47
Roller Diameter 4"		
80	5030	1.73
24	10697	3.68
12	18340	6.32
6	35431	12.2
4	49503	17.1
80	5522	1.9
24	10816	3.72
12	12129	4.18
6	33354	11.5

TABLE VII (Continued)

Wrinkle Height Ratio	Average Stress (psi)	Relative Stress
Roller Diameter 2"		
80	6281	2.16
40	8210	2.83
24	11962	4.12
12	17460	6.01
6	29935	10.3
3	57831	19.9

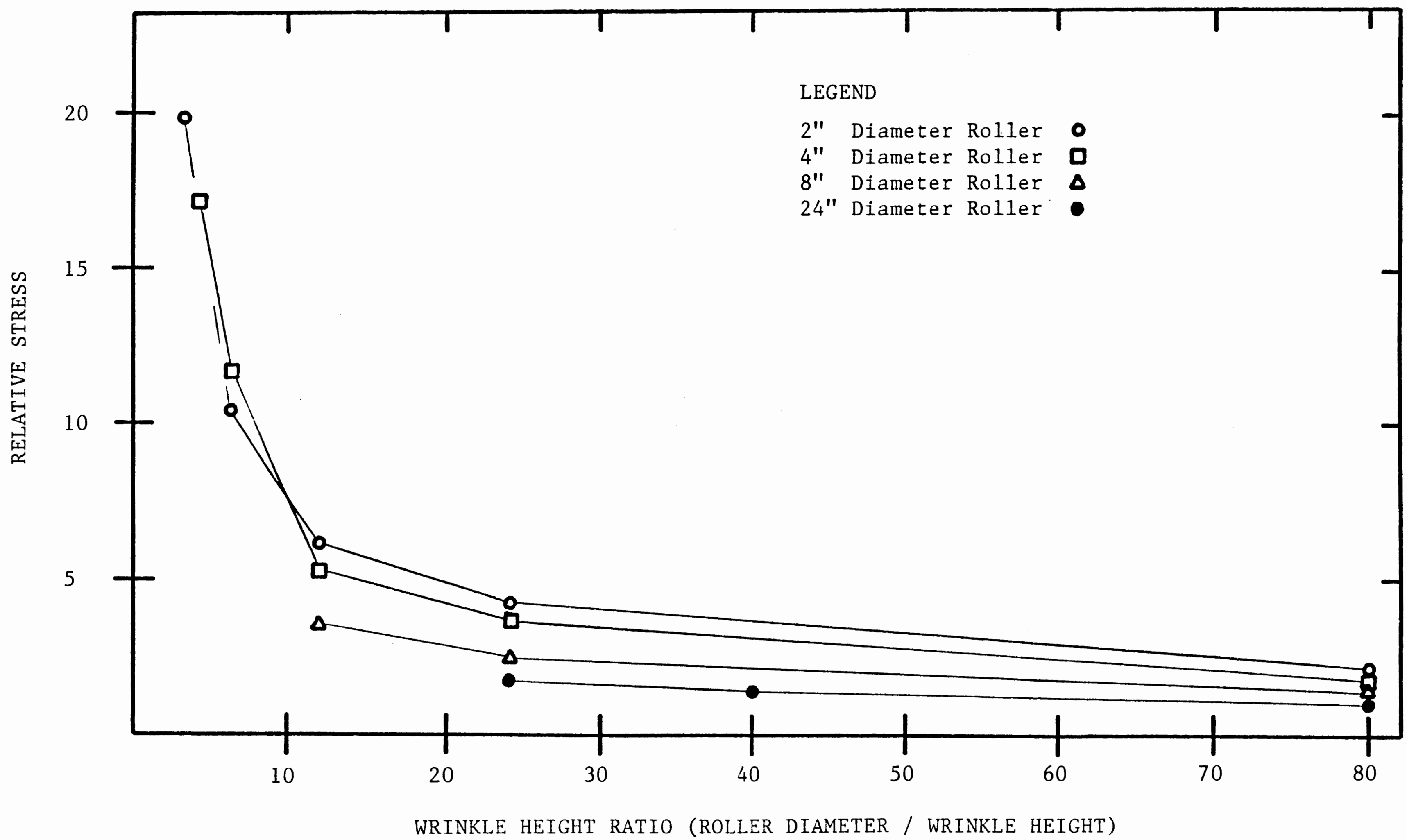


Figure 23. Relative Stress vs Wrinkle Height Ratio

tension, did not produce wrinkles of that amplitude. Several representative deformed wrinkle shapes are shown in Figures 24, 25, and 26. The shapes are for a given diameter roller with increasingly higher wrinkles. Therefore, the progression of shapes represents decreasing wrinkle height ratios. It is interesting to note the bifurcation of the wrinkle at the wrinkle mid-line in Figure 26. This bifurcation, coupled with the collapse of the wrinkle onto the roller, is again indicative of thin web wrinkles or webs with a low elastic modulus.

Wrinkle Width

As previously mentioned, wrinkle height and wrinkle width seem to show a dependence on each other, insofar as the relative stress is concerned. The wrinkle aspect ratio (wrinkle height divided by width) is a dimensionless parameter which is useful in analyzing the results. The specific data used to investigate the effect of wrinkle aspect ratio, along with the normalized data, is shown in Table VIII. The data is normalized to unity at an aspect ratio value of 0.048. The graphical results are presented in Figure 27. Comparison of the data in Tables VII and VIII, for a two inch diameter roller, will show that the wrinkle height ratio is inversely proportional to the aspect ratio. However, the wrinkle aspect ratio plot for a two-inch roller shows a linear increase in relative stress while the wrinkle height ratio data shows a higher degree

RD=8"
WW=2.09"
TN=20 p/i
YM=300,000 psi
WA=45 degrees

WH=0.1"
TH=20 mils
PR=0.3
FC=inf

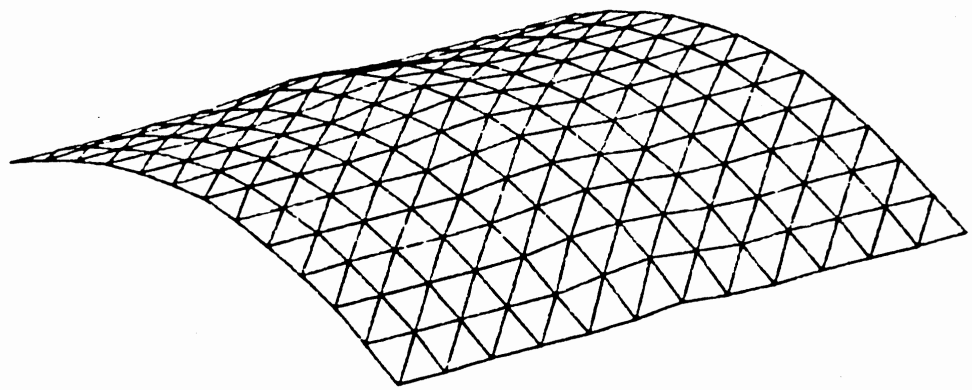


Figure 24. Deformed Wrinkle Shape (Wrinkle Height Ratio)

RD=4"
WW=1.05"
TN=5 p/i
YM=300,000 psi
WA=50 degrees

WH=0.05"
TH=20 mils
PR=0.3
FC=inf

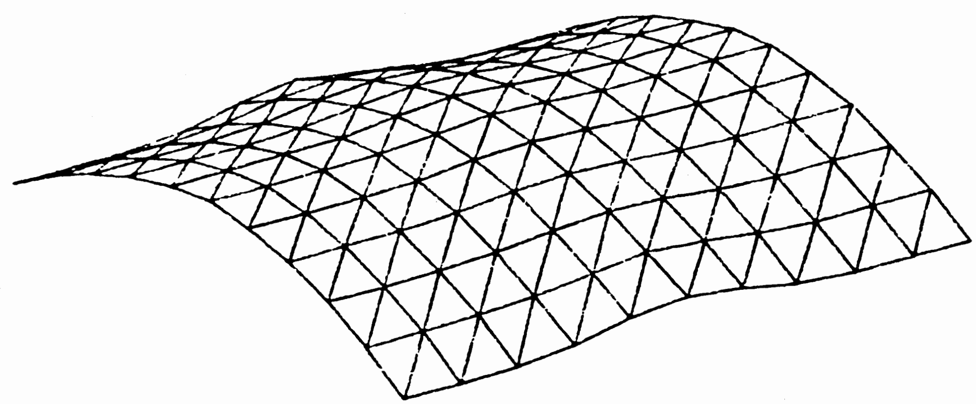


Figure 25. Deformed Wrinkle Shape (Wrinkle Height Ratio)

RD=4"
WW=1.05"
TN=5 p/i
YM=300,000 psi
WA=53 degrees

WH=0.167"
TH=3 mils
PR=0.3
FC=inf

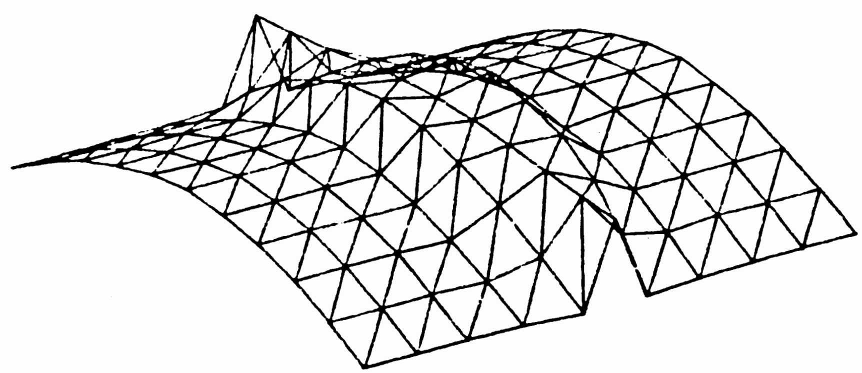


Figure 26. Deformed Wrinkle Shape (Wrinkle Height Ratio)

TABLE VIII

WRINKLE ASPECT RATIO VARIATION DATA
AND RELATIVE STRESS RESULTS

Roller Diameter	2"	Wrinkle Width	0.52"
Thickness	3 mils	Poisson's Ratio	0.3
Young's Modulus	300,000(psi)	Friction Coefficient	inf

Tension 1.67 pounds/inch/mil

Wrinkle Aspect Ratio	Element and Stress (psi) 1	2	3	4
0.048	8135	2568	4685	9735
0.095	8751	2616	6406	15068
0.159	12287	3408	10155	21998
0.319	18086	8321	16737	26696
0.636	29404	18474	32846	39017
1.27	48958	44354	69267	68744

	Average Stress (psi)	Relative Stress
0.048	6281	1.0
0.095	8210	1.3
0.159	11962	1.9
0.319	17460	2.8
0.636	29935	4.8
1.27	57831	9.2

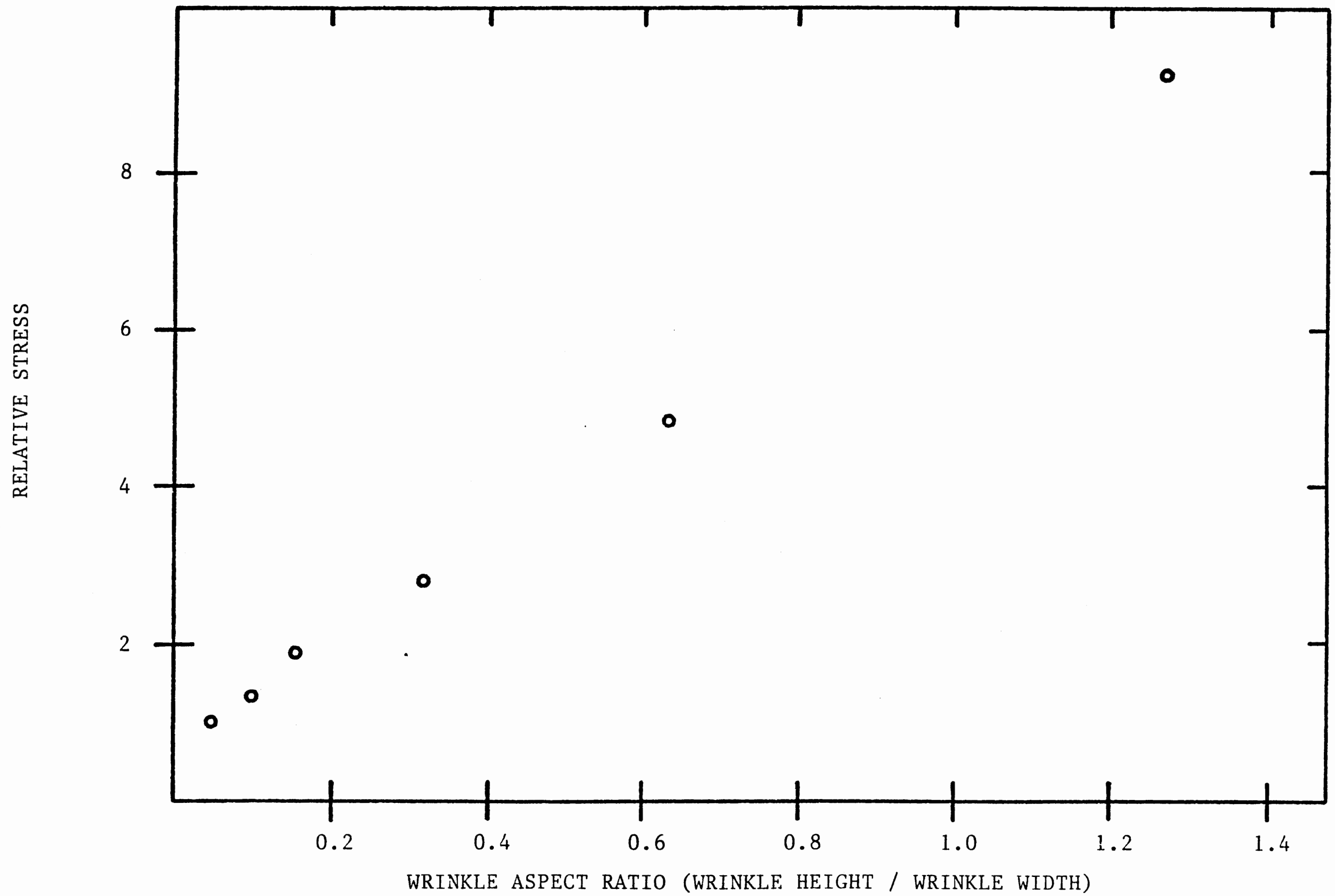


Figure 27. Relative Stress vs Wrinkle Aspect Ratio

behavior. Because of this, it is important to consider these two ratios separately, although the wrinkle height appears in both of them. What is most important in the aspect ratio results, are the deformed wrinkle shapes shown in Figures 28, 29, 30, and 31. Figures 28 and 29 show that a wrinkle will behave structurally, with no adverse effects except possibly high stress, up to an aspect ratio of 0.319. However, for aspect ratios of 0.636 and higher, the wrinkle begins to fold over on itself as shown in Figures 30 and 31. This is seen from the fact that the wrinkle cross section is exhibiting creasing at the machine direction ends. In addition to this, it may be observed that the wrinkle is also creasing on its cross machine direction sides. The material considered in these deformed shapes has a relatively high elastic modulus which is preventing the wrinkle from totally collapsing onto the roller. The sharpness of the creasing may well be due to the type of finite element and the element size and spacing. A much larger number of elements in this area would give a better indication of the sharpness of the creasing. This fact aside, the behavior shown in Figures 30 and 31 indicate that the sides of the web wrinkle pass through the vertical and exhibit a trapezoidal shape rather than the original, well-behaved sinusoidal shape. This progression of shapes provides considerable insight into the importance of keeping not only the wrinkle height as small as possible but also the aspect ratio.

RD=4"
WW=0.524"
TN=5 p/i
YM=300,000 psi
WA=26 degrees

WH=0.05"
TH=3 mils
PR=0.3
FC=inf

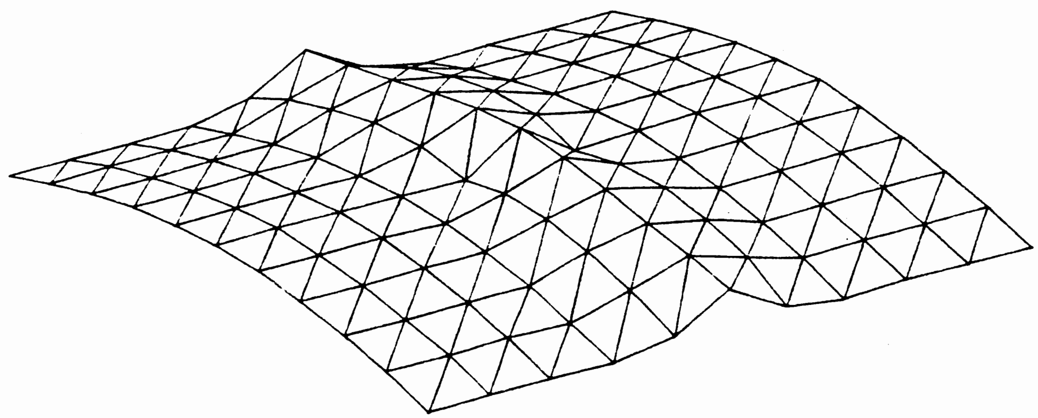


Figure 28. Deformed Wrinkle Shape (Wrinkle Aspect Ratio)

RD=4"
WW=0.524"
TN=5 p/i
YM=300,000 psi
WA=31 degrees

WH=0.167"
TH=3 mils
PR=0.3
FC=inf

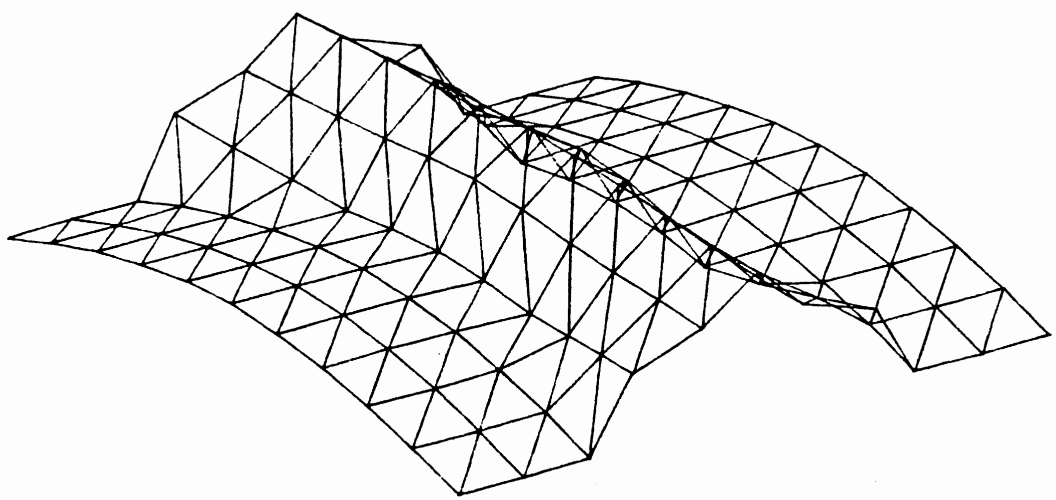


Figure 29. Deformed Wrinkle Shape (Wrinkle Aspect Ratio)

RD=4"
WW=0.524"
TN=5 p/i
YM=300,000 psi
WA=42 degrees

WH=0.333"
TH=3 mils
PR=0.3
FC=inf

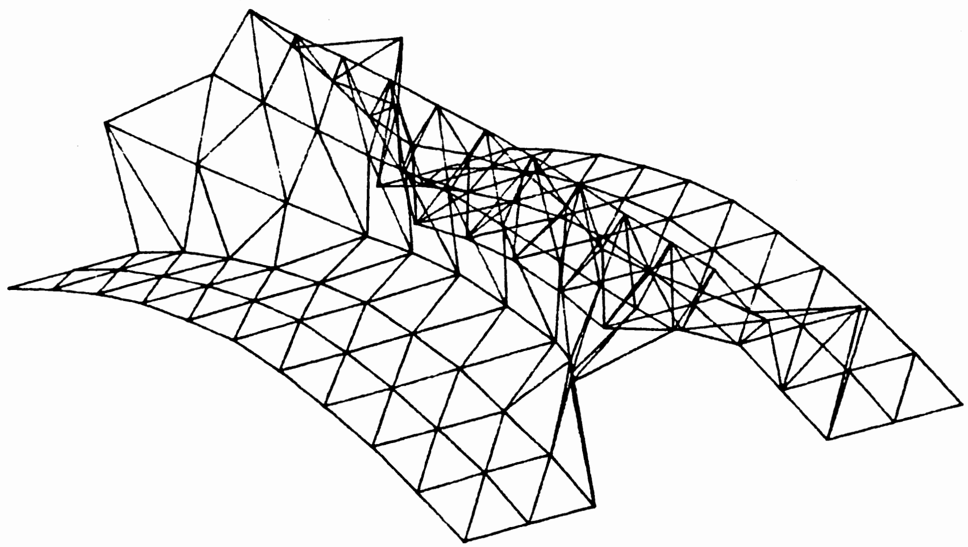


Figure 30. Deformed Wrinkle Shape (Wrinkle Aspect Ratio)

RD=4"
WW=0.524"
TN=5 p/i
YM=300,000 psi
WA=41 degrees

WH=0.667"
TH=3 mils
PR=0.3
FC=inf

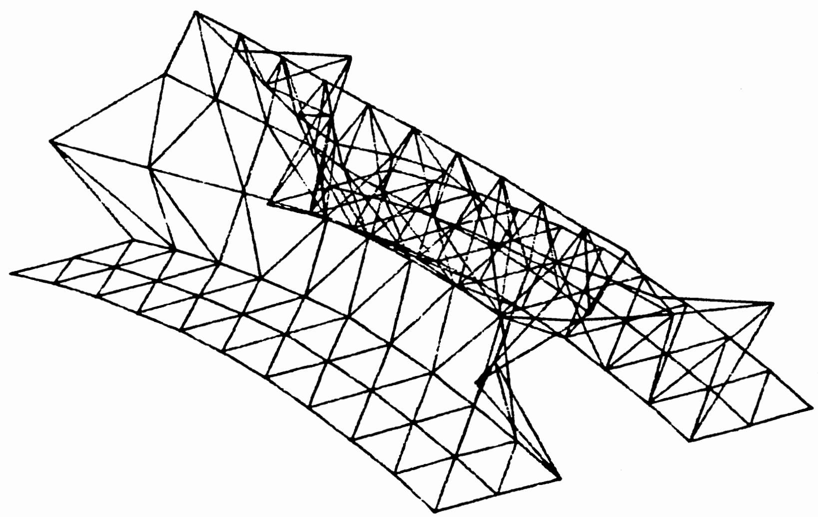


Figure 31. Deformed Wrinkle Shape (Wrinkle Aspect Ratio)

Roller Diameter

For a sensitivity analysis of an existing web processing line, the wrinkle height ratio and wrinkle aspect ratio are sufficient because at least one of the three parameters (roller diameter, wrinkle height, wrinkle width) will be known or can be approximated. However, from a design standpoint, all three of these parameters may be free to vary. With three unknown or variable quantities, it is necessary to provide information concerning the third unknown, namely roller diameter. The specific data used to investigate the roller diameter effect is contained in Tables VI, VII, and VIII and will not be repeated here. The graphical result is presented in Figure 32. The data in Figure 32 is normalized to a roller diameter of 24 inches, a wrinkle height ratio of 80, and a wrinkle aspect ratio of 0.048. For a roller diameter of 4 to 24 inches, a linear increase in relative stress is seen. At a roller diameter of two inches, a higher degree behavior is observed. Assuming a stable or structural cross section for the web wrinkle, a linear behavior can be explained by the fundamental beam bending relationship

$$(1 / R) = M / (EI) \tag{4}$$

wherein the substitution

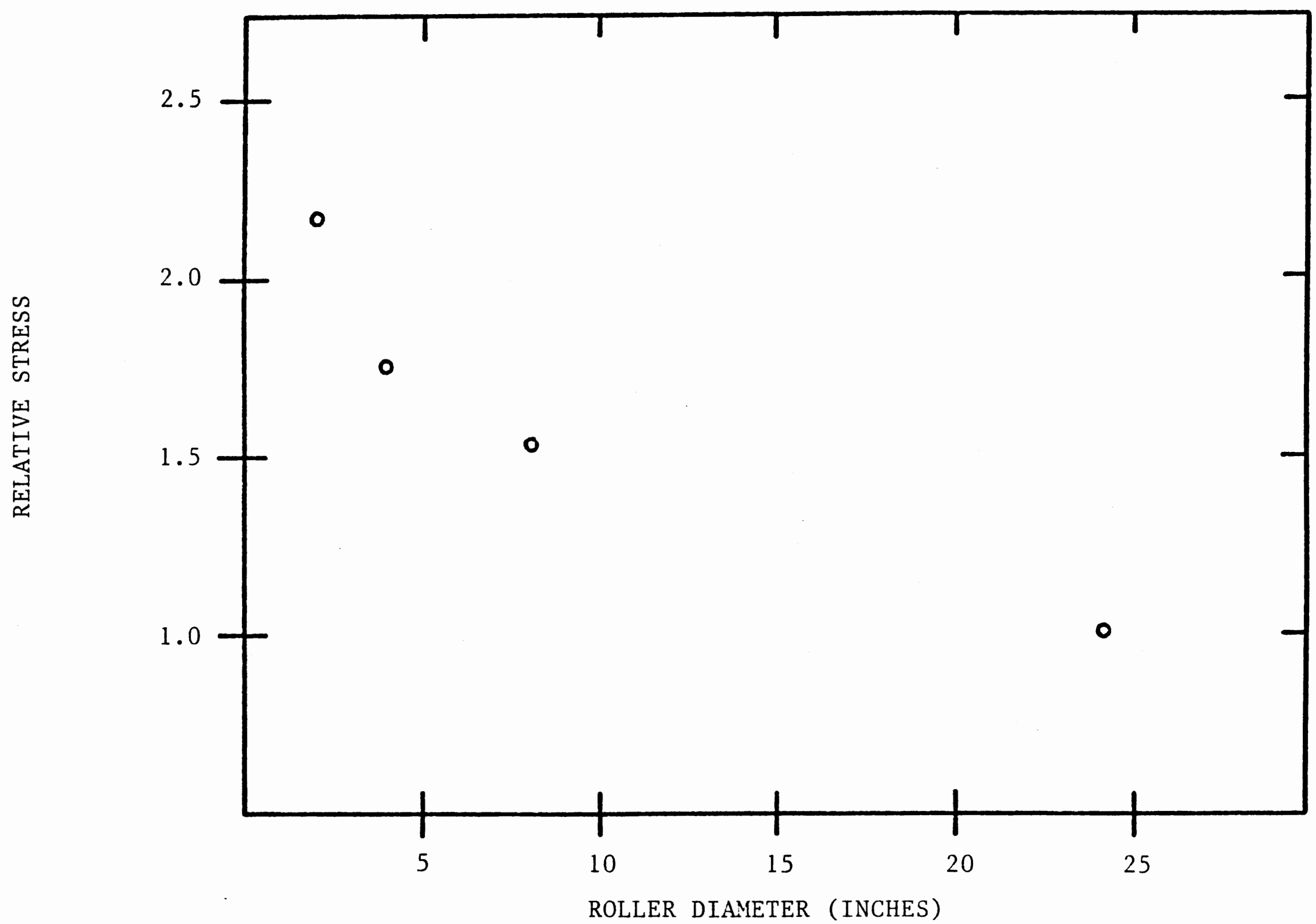


Figure 32. Relative Stress vs Roller Diameter

$$\sigma = (MC) / I \tag{5}$$

is made, resulting in

$$\sigma = (tE) / (2R) \tag{6}$$

This equation exhibits a linear relationship between stress and radius of curvature for the cross section. This equation also explains the linear behavior between stress and Young's modulus as previously shown. The above relationships are for beam bending in the linear region only. Many of the stresses which are compared for their relative contribution are of a magnitude such that they are in the nonlinear plastic region of most materials. It should be remembered that the absolute stress which is being investigated has little physical meaning. Instead, its relative magnitude is what is being investigated. It has been previously stated that it is assumed the web material behaves linearly at all stress levels. In reality, should the web wrinkle material experience plastic behavior, then for all practical cases the web has failed in a structural sense.

Tension

The tension parameter is presented not as an absolute tension, but instead normalized to web thickness. That is the units for tension are pounds per linear inch of web width per mil of web thickness. This removes the web thickness variation in the tension data and provides the results as the same stress, due to tension, in all webs regardless of the absolute tension. The specific data used in the tension analysis is in Table IX, and the normalized data is in Table X. The graphical result is shown in Figure 33. For tensions up to approximately one pound per inch per mil, the relative stress increases linearly but not in proportion to the tension. It should be remembered that one pound per inch per mil will induce a stress of 1,000 psi. In most cases this stress level is low compared to the overall stress in the wrinkle. This is due to the fact that the majority of the operating stress is from the wrinkle wrapping around the roller. Above the one pound per inch per mil region, the stress rises quickly because in this region the operating stress in the web is much more dependent on the tension than on the roller wrap. It would be expected that at a still higher tension, the operating stress would become linearly dependent on the tension and be relatively independent of all other factors. This observation is enforced because of the deformed wrinkle shapes shown in Figures 34, 35, 36, and 37. Although the tension in Figure 37 is ten times that in Figure 34, the only perceivable difference is a slightly larger deformation onto the roller, shown in Figure 37.

TABLE IX

TENSION AND THICKNESS VARIATION DATA

Roller Diameter 4"	Poisson's Ratio 0.3
Wrinkle Height 0.33"	Friction Coefficient inf
Wrinkle Width 1.05"	Young's Modulus 300,000(psi)

Tension is in pounds per inch width per mil thickness

Tension	Element and Stress (psi)			
	1	2	3	4
0.025	8593	15664	12867	20195
0.1	8582	15687	13069	20482
0.25	10519	13343	18089	30917
0.5	7117	26682	19668	23320
1.0	8247	25281	20622	25030
1.67	11230	9710	26947	25471
2.0	10670	22527	21714	28540
5.0	18489	14214	27846	39934
5.0	19468	11485	27564	40216
10.0	23966	15730	32668	46621
20.0	35126	24886	42937	58036
100	128,649	99228	126,543	156,244

TABLE IX (Continued)

Roller Diameter 8"	Poisson's Ratio 0.3
Wrinkle Height 0.67"	Friction Coefficient inf
Wrinkle Width 2.09"	Young's Modulus 300,000(psi)

Tension is in pounds per inch width per mil thickness

Tension	Element and Stress (psi) 1	2	3	4
0.25	4417	2049	8927	24149
0.5	1183	12022	14394	12299
1.67	6353	11529	18089	30917
5.0	22267	7147	29074	3568

TABLE X

TENSION AND THICKNESS RELATIVE STRESS RESULTS

Tension	Average Stress (psi)	Relative Stress
Roller Diameter 4"		
0.025	14330	0.78
0.1	14455	0.79
0.25	18217	0.99
0.5	19197	1.05
1.0	19795	1.08
1.67	18340	1.00
2.0	20863	1.14
5.0	25121	1.37
10.0	29746	1.62
20.0	40246	2.19
100	127,666	6.96
Roller Diameter 8"		
0.25	9897	0.98
0.5	9975	0.99
1.67	10065	1.0
5.0	15514	1.54

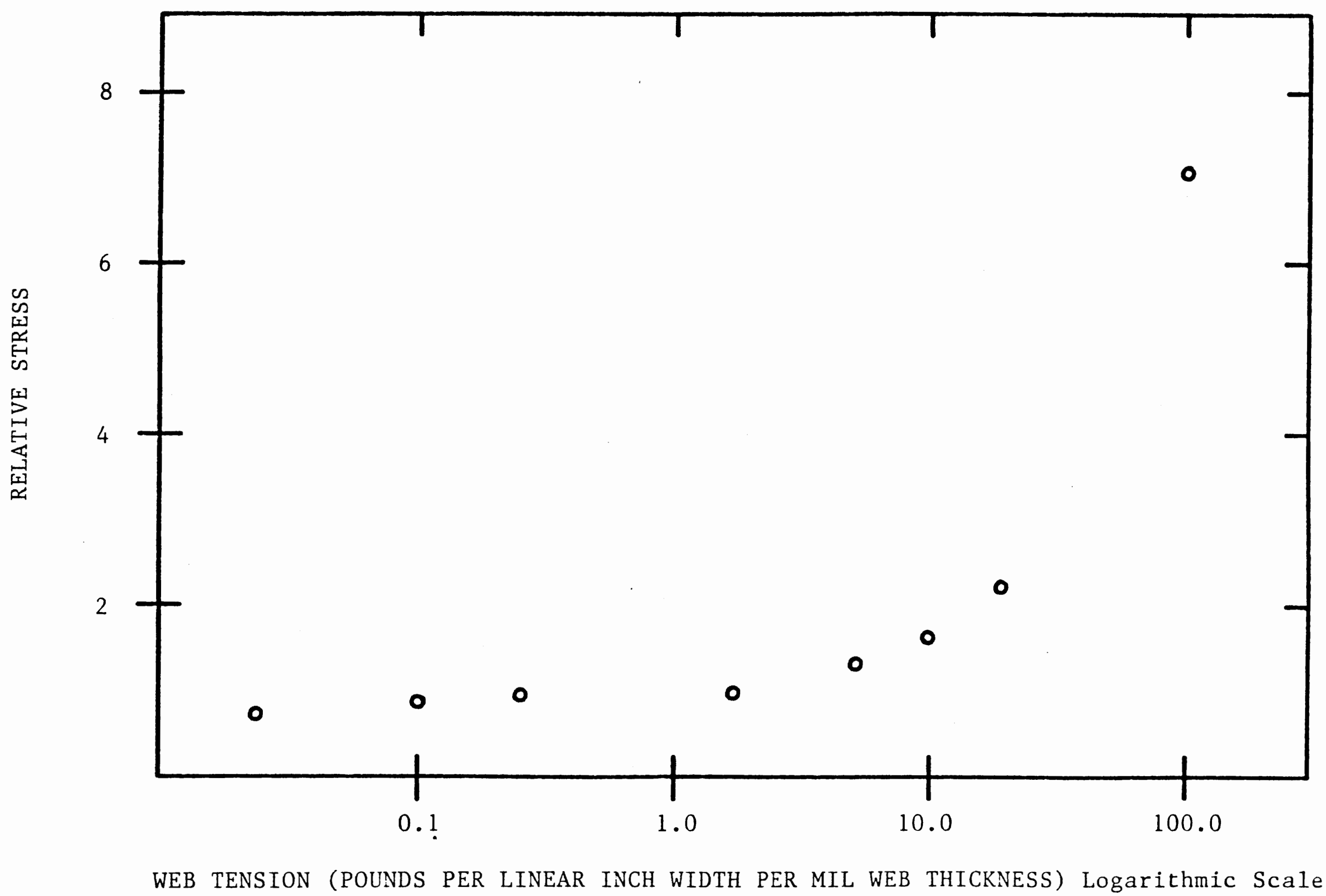


Figure 33. Relative Stress vs Web Tension

RD=4"	WH=0.333"
WW=0.524"	TH=10 mils
TN=1 p/i	PR=0.3
YM=300,000 psi	FC=inf
WA=61 degrees	

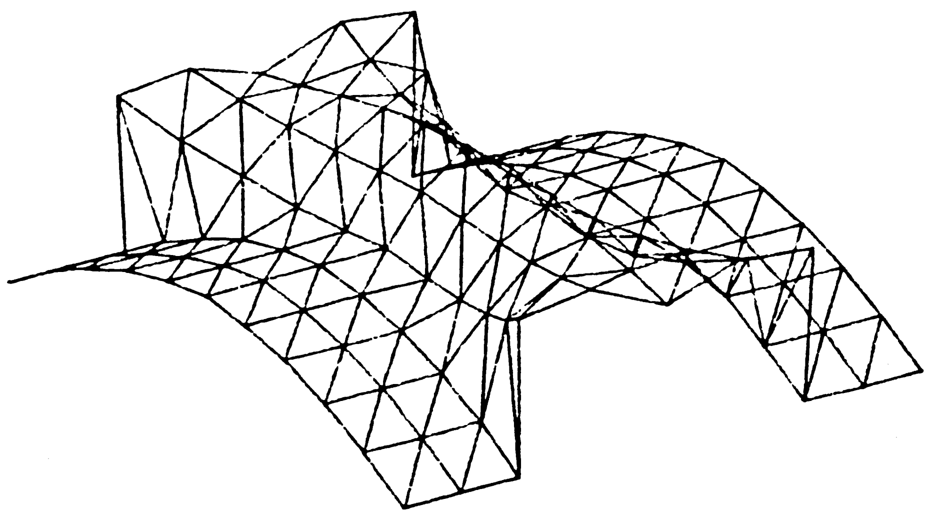


Figure 34. Deformed Wrinkle Shape (Web Tension)

RD=4"
WW=1.05"
TN=5 p/i
YM=300,000 psi
WA=61 degrees

WH=0.333"
TH=20 mils
PR=0.3
FC=inf

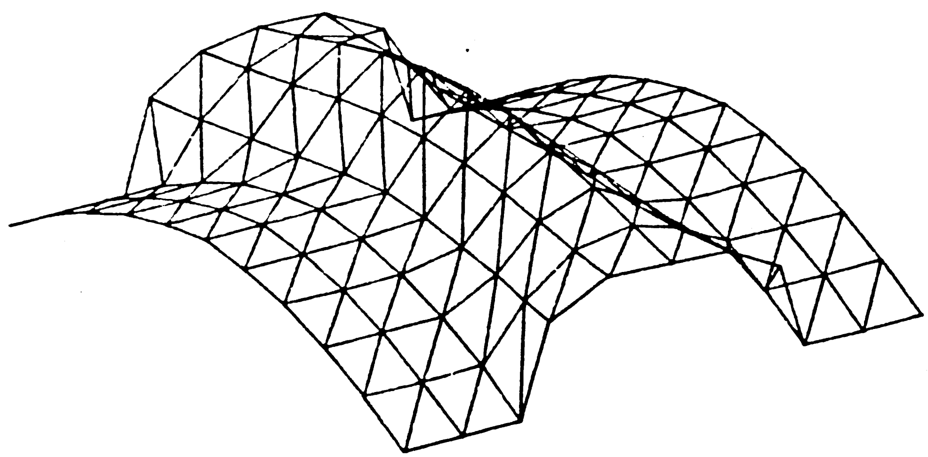


Figure 35. Deformed Wrinkle Shape (Web Tension)

RD=4"
WW=1.05"
TN=5 p/i
YM=300,000 psi
WA=61 degrees

WH=0.333"
TH=10 mils
PR=0.3
FC=inf

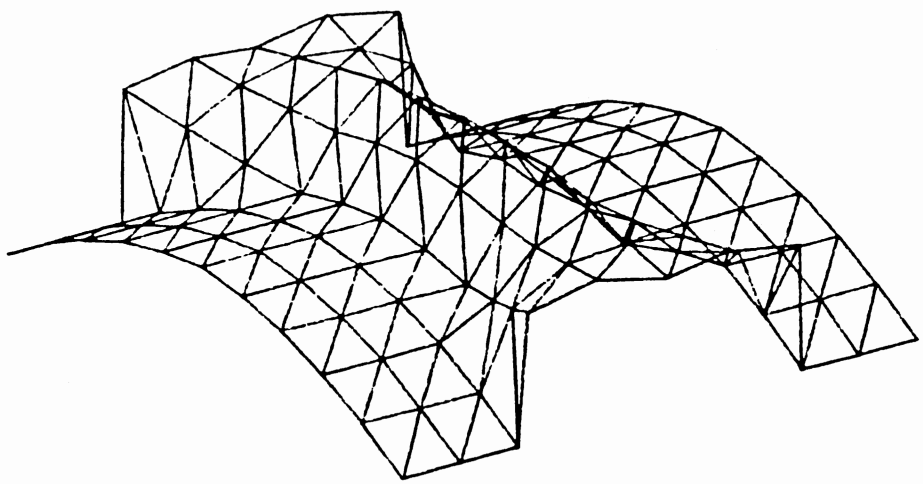


Figure 36. Deformed Wrinkle Shape (Web Tension)

RD=4"
WW=1.05"
TN=10 p/i
YM=300,000 psi
WA=61 degrees

WH=0.333"
TH=10 mils
PR=0.3
FC=inf

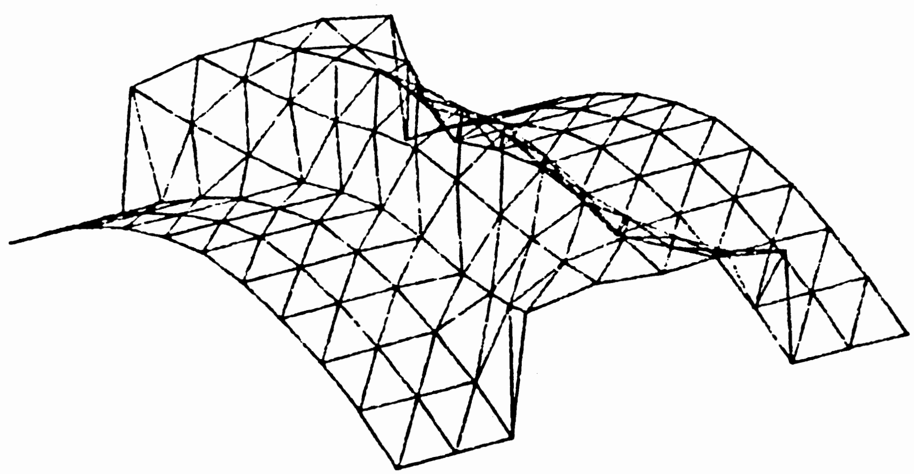


Figure 37. Deformed Wrinkle Shape (Web Tension)

Thickness

For a variety of web thicknesses, zero tension is the only value which would provide relative stress data concerning web thickness alone. The lowest value of tension considered in this investigation is 0.025 pounds per inch per mil. This gives rise to a stress of only 25 psi which is not a realistically induced stress, especially because webs are transported because of tension. Secondly, higher tension is preferred for better web steering. Similar to the reasoning for wrinkle height and width, and roller diameter, the information presented for web tension would be sufficient for analysis. In a design situation, the web thickness is governed by the end user or the application for which the web is to be used. For line parameters such as roller diameter or wrap angle, the designer may have the freedom to vary dimensions. This freedom does not exist for a line which is set up for a given web proccess. For these reasons, the web thickness parameter, at a zero tension, will not be investigated.

Wrap Angle

The specific data used to present the wrap angle variation and the normalized data is in Table XI. The graphical result is shown in Figure 38. The specific data used has been extracted from other than the four representative areas on the web. The reason for this is

TABLE XI

WRAP ANGLE VARIATION DATA AND
RELATIVE STRESS RESULTS

Roller Diameter	4"	Wrinkle Width	1.05"
Thickness	1 mil	Poisson's Ratio	0.3
Young's Modulus	300,000(psi)	Friction Coefficient	inf
	Tension 5.0 pounds/inch/mil		

Wrap Angle (Degrees)	Element and Stress (psi) 1	2	3	4
0	5000	5000	5000	5000
60	n/a	n/a	n/a	40216
120	n/a	n/a	n/a	53659
180	n/a	n/a	n/a	61216

	Average Stress (psi)	Relative Stress
0	5000	0.12
60	40216	1.0
120	53659	1.33
180	61216	1.52

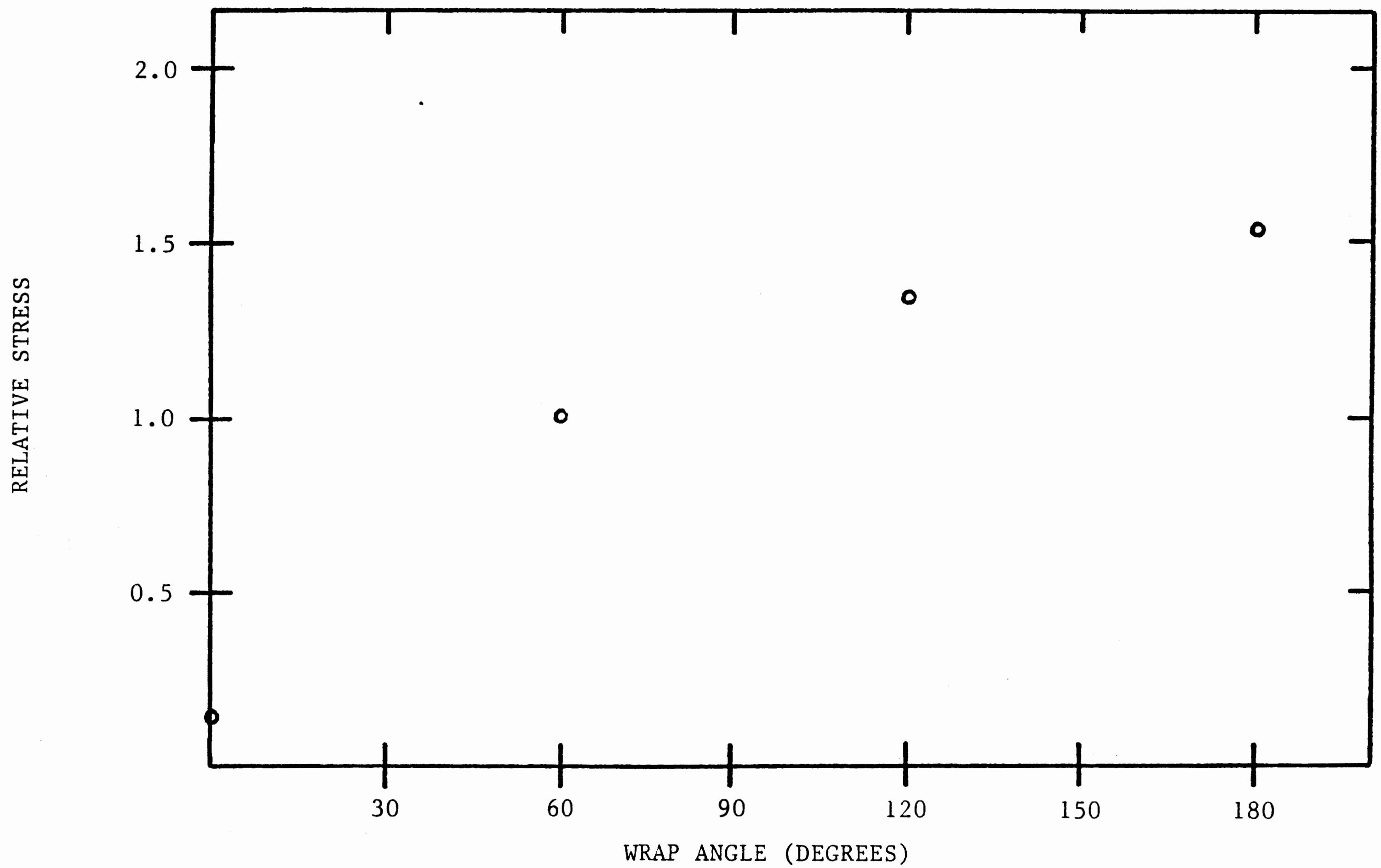


Figure 38. Relative Stress vs Wrap Angle

because at the high wrap angle of 180 degrees, several of the representative points are in contact with the roller. To properly represent the stress at a grid point, the point must be free of applied forces and constraints. The applied forces and constraints will often induce high localized stress which gives an erroneous interpretation of the actual computed stress. This is the reason that the data in Table XI consists of only one grid point stress. That point occurs at the intersection of the wrinkle axes of symmetry as shown previously in Figure 15. A second reason for not averaging the four representative stress values, in this instance, may be seen from Figures 39 through 42. Specifically, in Figure 41, there exists a bifurcation of the wrinkle in its central portion in the machine direction. In this area the stress is much higher than that in the regions where the wrinkle does not bifurcate. To average the stress in this case would give an inaccurate indication. It should be noted that this is the only case in which the representative stress values have not been used.

Friction Coefficient

The specific data used to investigate the frictional coefficient effect is presented in Table XII and the result is shown in Figure 43. The data exhibits an exponential behavior beginning with a frictional coefficient of zero, which represents total collapse of the wrinkle onto the roller. At this condition, the only stress present is from

RD=4"	WH=0.05"
WW=1.05"	TH=20 mils
TN=5 p/i	PR=0.3
YM=300,000 psi	FC=inf
WA=151 degrees	

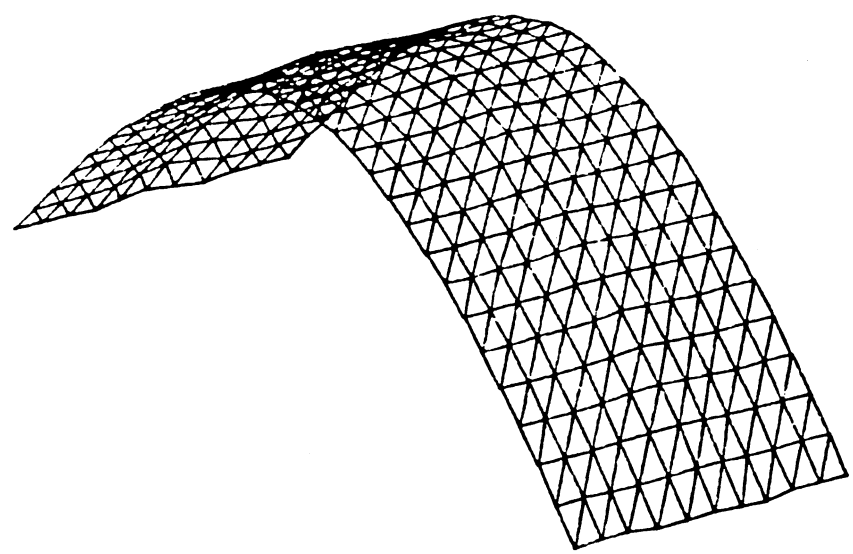


Figure 39. Deformed Wrinkle Shape (Wrap Angle)

RD=4"	WH=0.333"
WW=1.05"	TH=20 mils
TN=5 p/i	PR=0.3
YM=300,000 psi	FC=inf
WA=180 degrees	

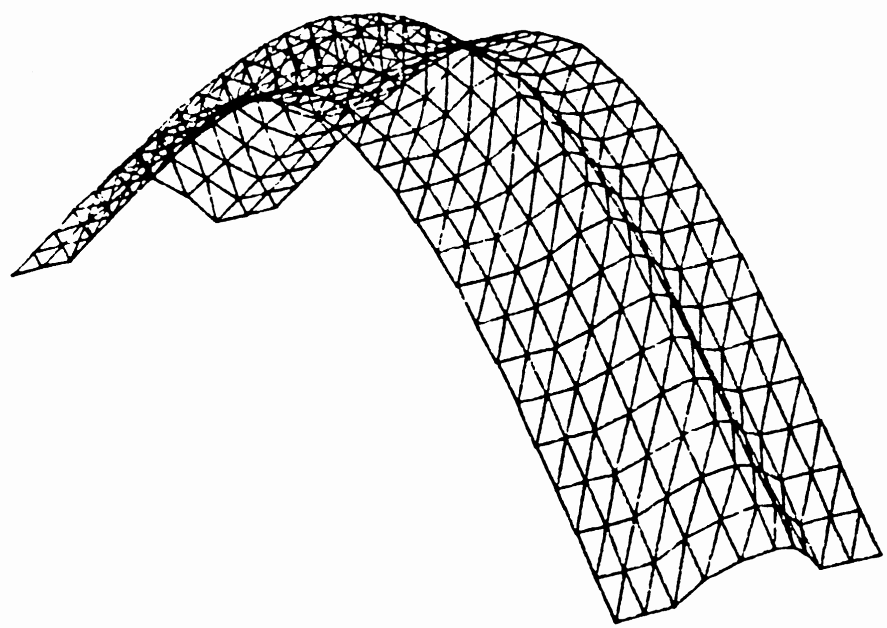


Figure 40. Deformed Wrinkle Shape (Wrap Angle)

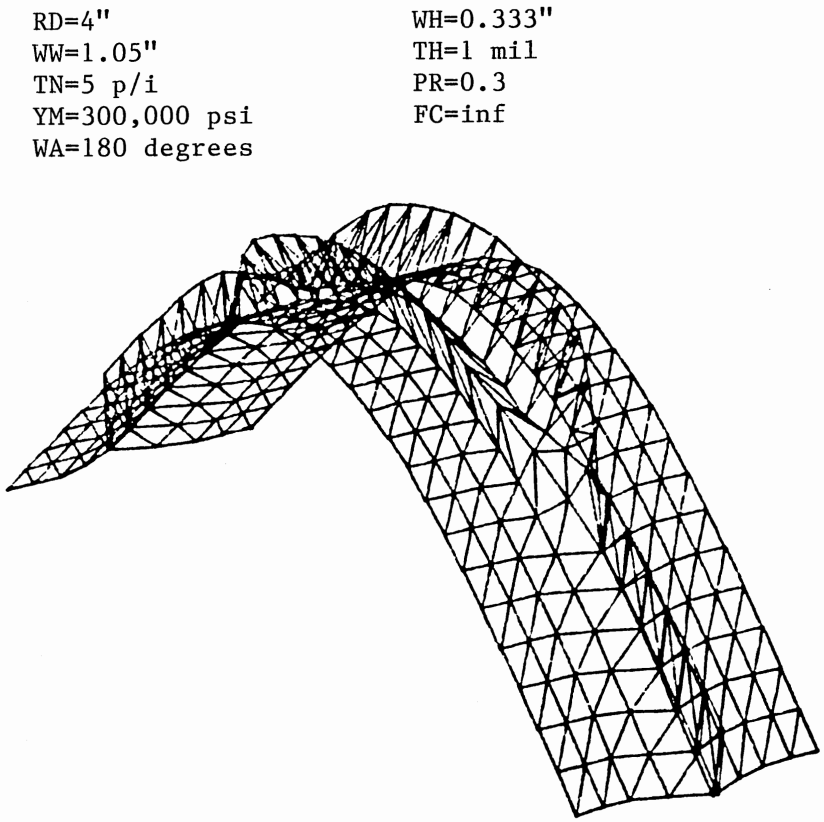


Figure 41. Deformed Wrinkle Shape (Wrap Angle)

RD=4"　　WH=0.333"
WW=1.05"　　TH=3 mils
TN=5 p/i　　PR=0.3
YM=300,000 psi　　FC=inf
WA=180 degrees

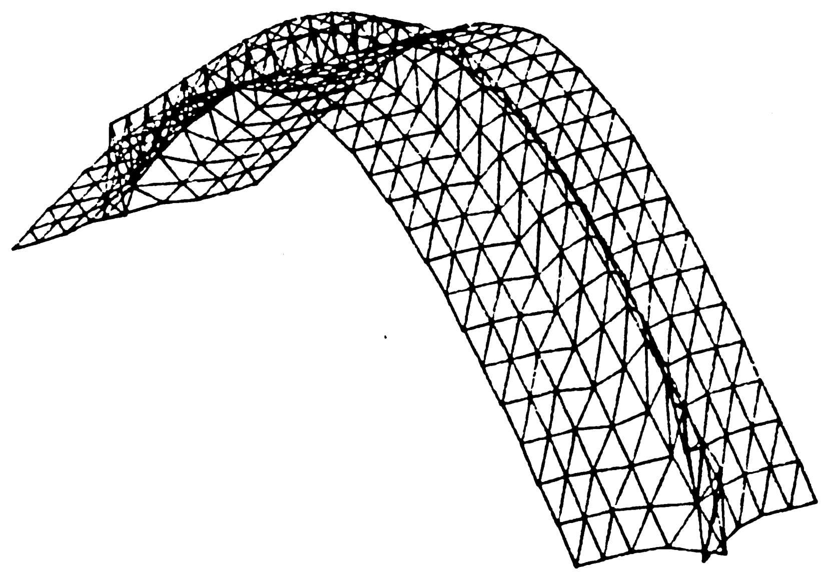


Figure 42. Deformed Wrinkle Shape (Wrap Angle)

TABLE XII

FRICTIONAL COEFFICIENT VARIATION DATA
AND RELATIVE STRESS RESULTS

Roller Diameter	8"	Wrinkle Width	2.09"
Thickness	1 mil	Poisson's Ratio	0.3
Young's Modulus	300,000(psi)	Wrinkle Height	0.333"

Tension 5.0 pounds/inch/mil

Frictional Coefficient	Element and Stress (psi) 1	2	3	4
inf	18614	5215	16869	21956
0.5	16681	4799	15712	18845
0.25	16584	4851	15648	18691
0.15	11465	5276	16371	21434
0.05	11469	5294	16365	21448
0.0	5038	5038	5038	5038

	Average Stress (psi)	Relative Stress
inf	15664	1.0
0.5	14009	0.89
0.25	13944	0.89
0.15	13637	0.87
0.05	13644	0.87
0.0	5038	0.32

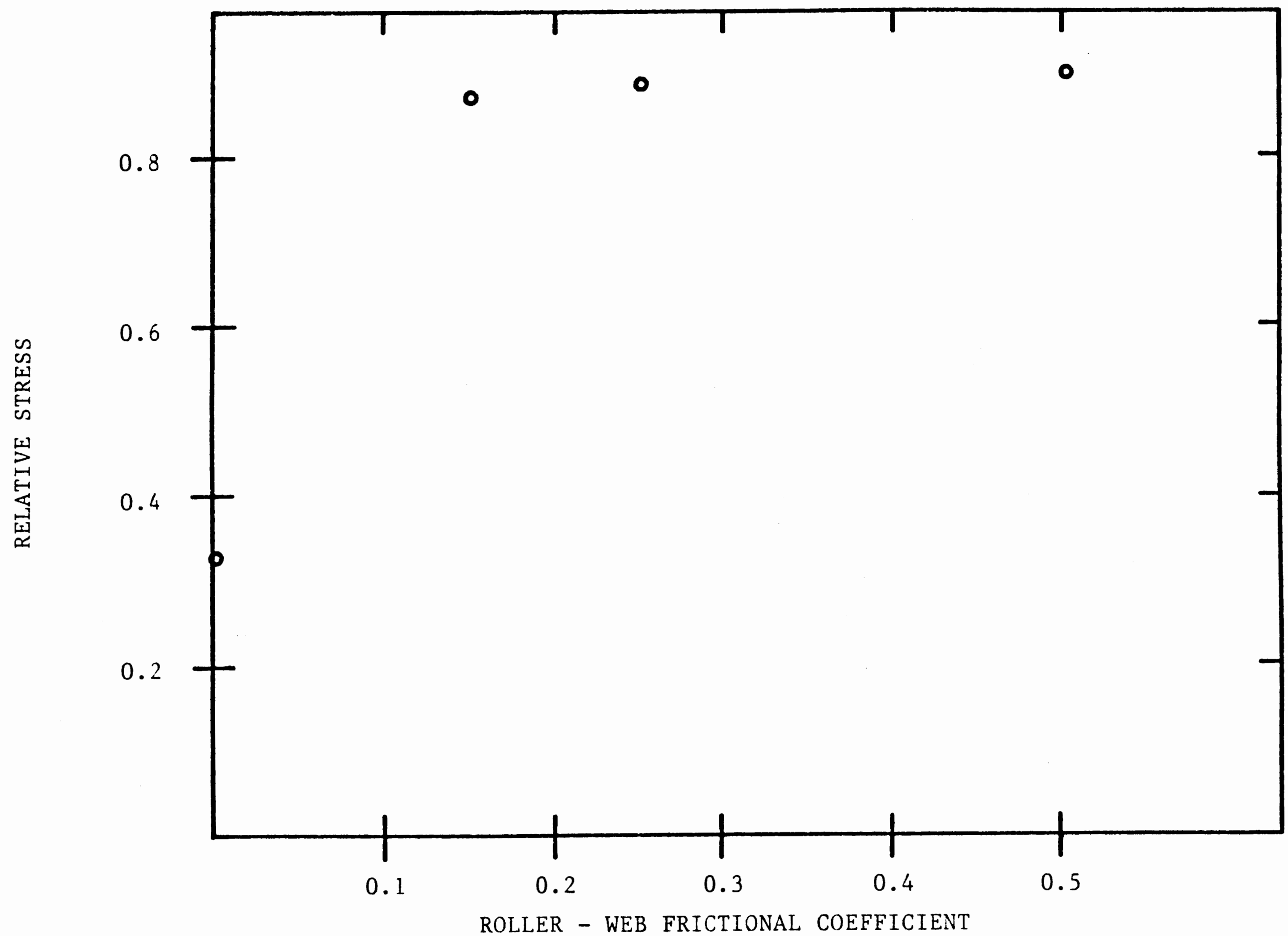


Figure 43. Relative Stress vs Frictional Coefficient

the applied tension. The displacement vectors which are included in the NASTRAN output indicate that a one mil thick web will collapse onto the roller if the frictional coefficient is between 0.15 and 0.25. Below a frictional coefficient value of 0.15, all of the points are constrained to the roller surface, while above a frictional coefficient value of 0.25, the wrinkle partially or fully retains its original sinusoidal shape. This also indicates that thicker web wrinkles may collapse onto the roller with frictional coefficients larger than 0.25 because of their increased stiffness. This behavior may be more clearly seen in Figures 44 through 46. The effect of the frictional coefficient, on the stability of the deformed wrinkle shape, is also presented in the subsequent section.

Deformed Wrinkle Stability

To this point, the results are presented for wrinkle stability as they relate to stress. It is possible for a wrinkle to remain in a benign state of stress, that is one where failure or unwanted behavior is not present, and yet the wrinkle will not be acceptable. This situation may arise in winding where any result except flattening of the wrinkle is unacceptable. Other examples might include materials which are especially susceptible to permanent creasing such as metal foils. For a wrinkle to pass over a roller and not be adversely effected, it is necessary for the wrinkle to pass both a stress criteria and a deformed

RD=8"
WW=2.09"
TN=5 p/i
YM=300,000 psi
WA=48 degrees
WH=0.333"
TH=1 mil
PR=0.3
FC=0.5

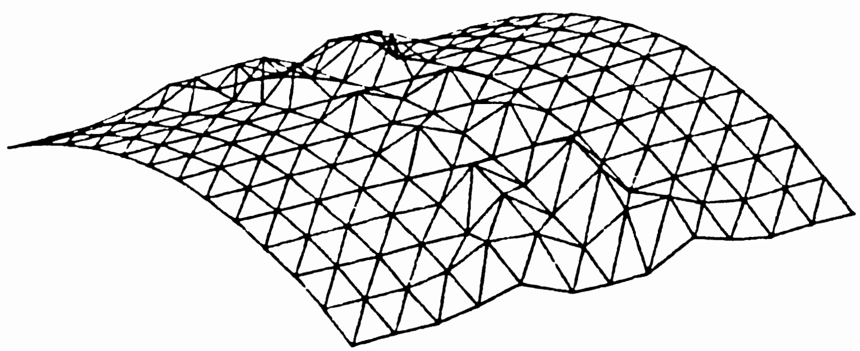


Figure 44. Deformed Wrinkle Shape (Frictional Coefficient)

RD=8"
WW=2.09"
TN=5 p/i
YM=300,000 psi
WA=48 degrees

WH=0.333"
TH=10 mils
PR=0.3
FC=0.5

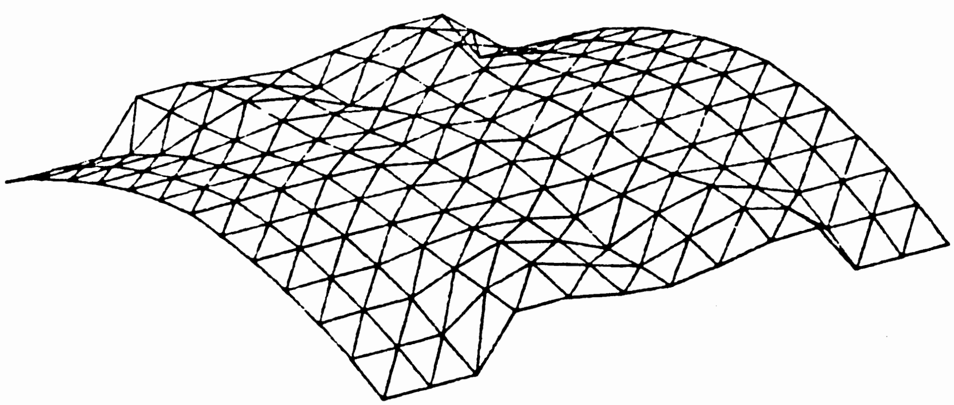


Figure 45. Deformed Wrinkle Shape (Frictional Coefficient)

RD=8"
WW=2.09"
TN=5 p/i
YM=300,000 psi
WA=48 degrees

WH=0.333"
TH=10 mils
PR=0.3
FC=0.25

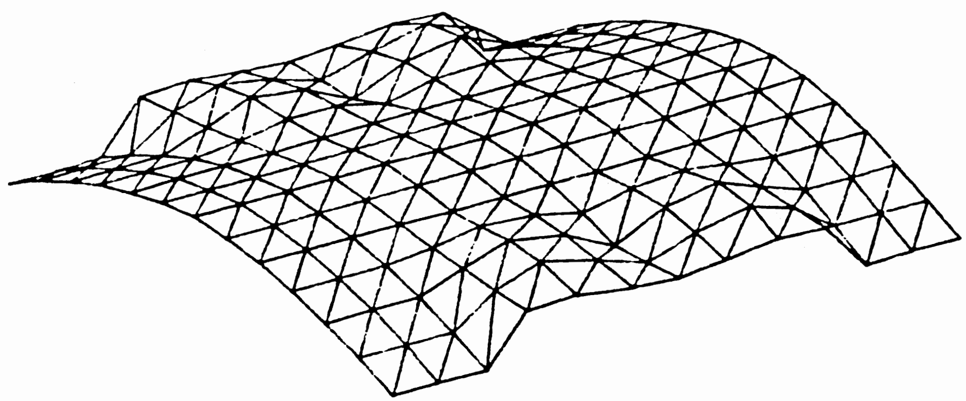


Figure 46. Deformed Wrinkle Shape (Frictional Coefficient)

shape criteria.

A visual inspection of all the deformed wrinkle shapes associated with this study is used to quantitatively determine which wrinkle parameters lead to an acceptable deformed wrinkle. It is apparent that the two parameters which are most significant in maintaining wrinkle shape stability are the wrinkle aspect ratio and the material flexural rigidity. These two parameters are compared to establish a criteria for the deformed wrinkle shapes. As may be seen from Equation (2), the flexural rigidity is dependent upon Young's modulus , web thickness, and Poisson's ratio. Because variations in Poisson's ratio have much less influence on the relative stress than Young's modulus or the thickness, its variations will not be specifically addressed. It should seem apparent that a wrinkle with high flexural rigidity should be able to withstand a higher aspect ratio, without deforming into an adverse shape, much better than a wrinkle of low flexural rigidity.

The specific data used to establish the deformed shape criteria is not repeated here because of its quantity. However, the results of the data are presented in Table XIII. Although the data is somewhat incomplete, it does show a definite trend toward stable shapes at a higher flexural rigidity and a lower aspect ratio. For all aspect ratios it also seems apparent that as the coefficient of friction is lowered, a wrinkle with a lower rigidity should be stable

TABLE XIII

DEFORMED SHAPE STABILITY CRITERIA

Aspect Ratio	Minimum Rigidity* for Stability — Frictional Coefficient 0.25	0.5	Infinity
0.048	2.75E-5	2.75E-5	2.75E-5
0.095	7.42E-4	7.42E-4	7.42E-4
0.159	2.75E-5	2.75E-5	7.42E-4
0.318	n/a	n/a	0.027
0.636	n/a	n/a	0.027
1.27	n/a	n/a	0.027

* The numerical value for rigidity is $Eh^3 / (12(1-v^2))$

for a given aspect ratio. This is because of the tendency of the wrinkle to more easily return to its original flat shape. Several examples of unstable wrinkles are shown in Figures 47, 48, and 49 while several examples of stable wrinkles are shown in Figures 50, 51, and 52 as well in many other figures contained in this study.

Buckling Analysis

For all of the previous analyses, it has been assumed that the web wrinkle behaves in a static sense. All structures may be theoretically loaded up to the point where yielding of the material takes place. For many structural shapes, however, it is possible to have an elastic instability take place at a loading level well below that of the yield point of the material. Among such structural shapes are thin plates and webs. For such shapes, the amount of external loading may be increased up to a point where a sudden decrease in the elastic strain energy of the plate and a sudden decrease in the potential energy of the applied loads takes place. At this point, the deformed shape of the structure may be significantly different from the deformed shape due to the static loading. Such behavior is termed buckling and is investigated for web wrinkles.

The NASTRAN buckling analysis computes eigenvalues which are factors by which the static or prebuckling state of stress is multiplied to produce buckling. Because the buckling analysis uses the prebuckling state of stress, the

RD=4"
WW=1.05"
TN=5 p/i
YM=300,000 psi
WA=53 degrees

WH=0.167"
TH=10 mils
PR=0.3
FC=inf

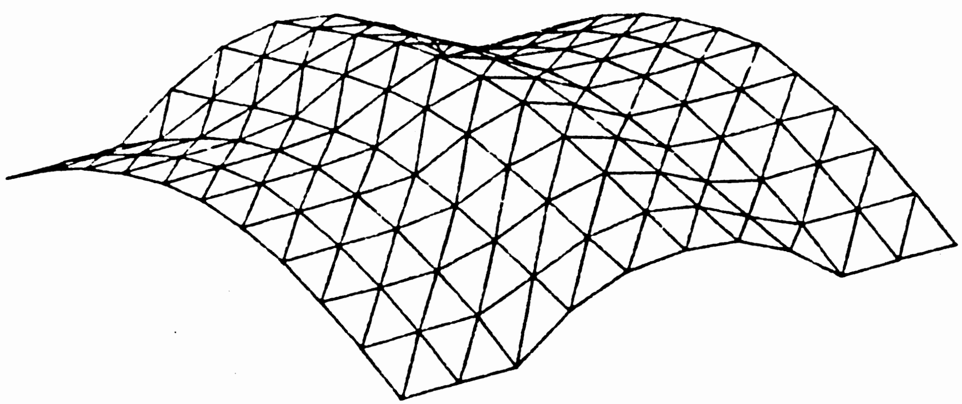


Figure 47. Deformed Wrinkle Shape (Wrinkle Stability)

RD=24"　　　　WH=0.6"
WW=6.28"　　　TH=60 mils
TN=5 p/i　　　PR=0.3
YM=300,000 psi　FC=0.5
WA=41 degrees

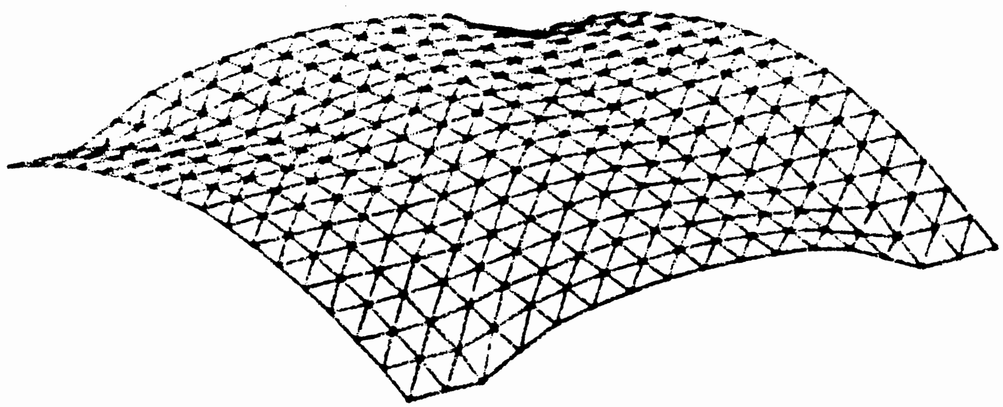


Figure 48. Deformed Wrinkle Shape (Wrinkle Stability)

RD=24"
WW=6.28"
TN=5 p/i
YM=300,000 psi
WA=42 degrees

WH=1"
TH=60 mils
PR=0.3
FC=0.5

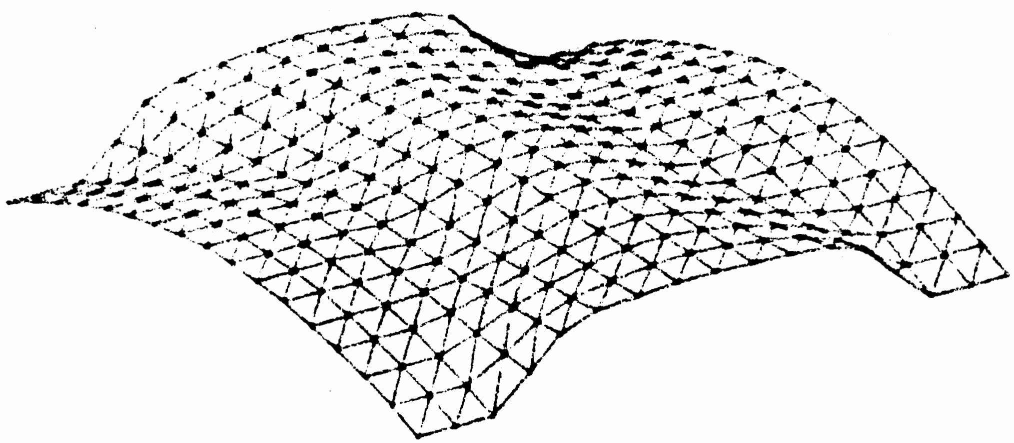


Figure 49. Deformed Wrinkle Shape (Wrinkle Stability)

RD=2"
WW=0.524"
TN=5 p/i
YM=300,000 psi
WA=106 degrees

WH=0.667"
TH=10 mils
PR=0.3
FC=inf

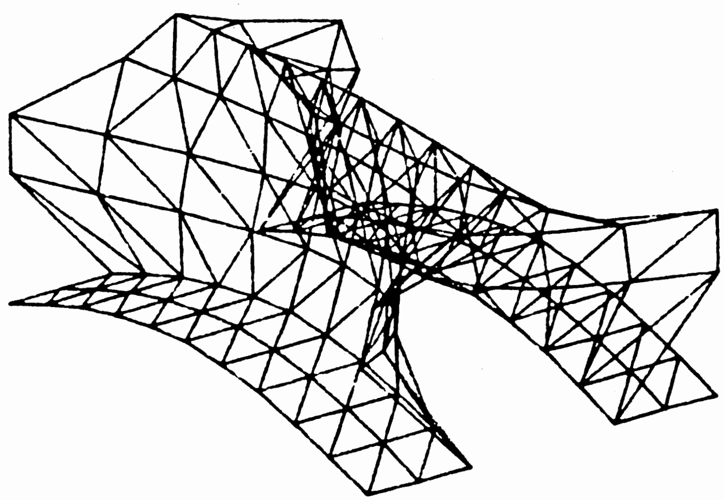


Figure 50. Deformed Wrinkle Shape (Wrinkle Stability)

RD=4"
WW=1.05"
TN=5 p/i
YM=50,000 psi
WA=61 degrees

WH=0.333"
TH=10 mils
PR=0.3
FC=inf

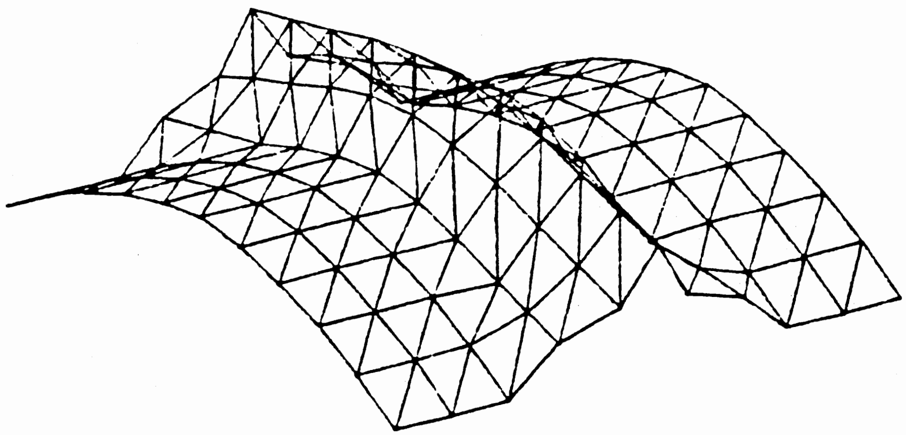


Figure 51. Deformed Wrinkle Shape (Wrinkle Stability)

RD=4"
WW=1.05"
TN=50 p/i
YM=300,000 psi
WA=62 degrees

WH=0.333"
TH=10 mils
PR=0.3
FC=inf

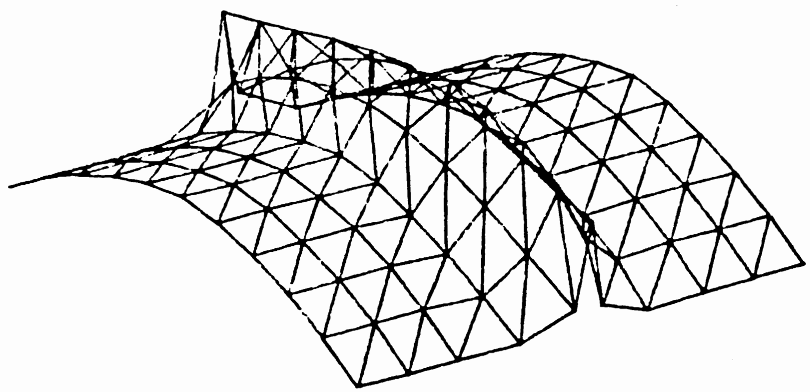


Figure 52. Deformed Wrinkle Shape (Wrinkle Stability)

statically enforced displacement condition must be translated into an equivalent loading condition. One of the output parameters from the previous analyses is the necessary force required to constrain the wrinkle boundary points onto the roller. This force set, combined with the web tension force set, is the prebuckling loading condition which is used in the buckling analysis. The web wrinkle buckling behavior of the deformed shapes shown in Figures 35 and 47 is presented. These two wrinkles are for web thicknesses of 20 mils and 10 mils respectively. The buckling behavior of a three mil thick web was computed, however the deformed shapes appear as a crumpled sheet of paper with no physical interpretation. For most structures, only the lowest valued eigenvalue has importance. This is due to the fact that most structures are loaded with ever increasing loads which will cause failure at the lowest eigenvalue. For the deformed web wrinkle shapes, all eigenvalues are investigated between zero and one. This is because a combination of the material and geometric parameters could create a loading condition which is larger in value than the smallest loading condition which will cause buckling. An eigenvalue of zero is interpreted as a case where no load is applied to the web, whereas an eigenvalue of one signifies the static loading condition. Although negative eigenvalues are computed, they are ignored because a web can not be transported around a roller with a negative tension, or compressive machine direction load.

The smallest and largest valued eigenvalue buckled shapes, for the web wrinkle shown in Figure 35, are shown in Figures 53 and 54. Similarly, the smallest and largest valued eigenvalue buckled shapes, for the web wrinkle shown in Figure 47, are shown in Figures 55 and 56. The remainder of the buckled shapes are in the latter portion of Appendix C. The response of the buckled shapes may be due to the loading condition of the wrinkle. From Timoshenko (65) the equation for the response of a thin plate to applied moments is

$$w(x,y) = -\frac{M_x - vM_y}{2D(1-v^2)}\,x^2 - \frac{M_y - vM_x}{2D(1-v^2)}\,y^2 \qquad (7)$$

If the direction of the moments about the x and y axes is the same, then a state of synclastic bending occurs and the deformed shape is that of a paraboloid or ellipsoid of revolution. The magnitude of the deformation is governed by the magnitude of the applied moments. Should the applied moments be in the opposite direction to each other, then a state of anticlastic bending occurs and the deformed shape is a hyperbolic paraboloid or saddle shape. This latter condition is sometimes termed the "potato chip" effect and may be seen in the buckled shapes. It may be readily seen that the buckled shapes require that a portion of the web boundary lifts off the roller. This condition is inconsistent with the physical wrap of the web around the

Eigenvalue = 0.522

For Static Wrinkle Shape See Figure 35

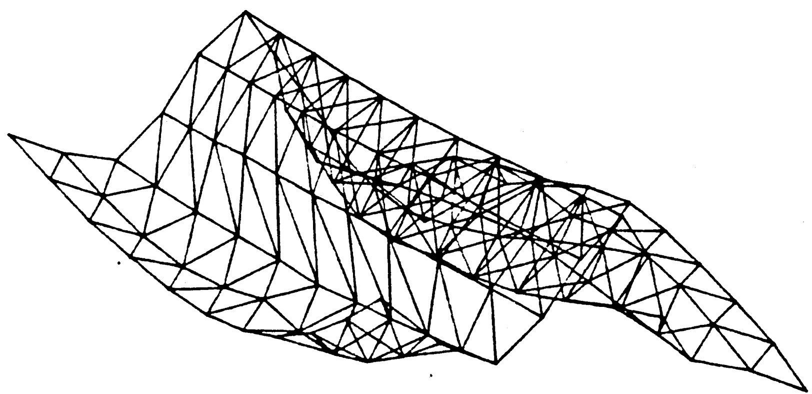


Figure 53. Buckled Wrinkle Shape

Eigenvalue = 0.959

For Static Wrinkle Shape See Figure 35

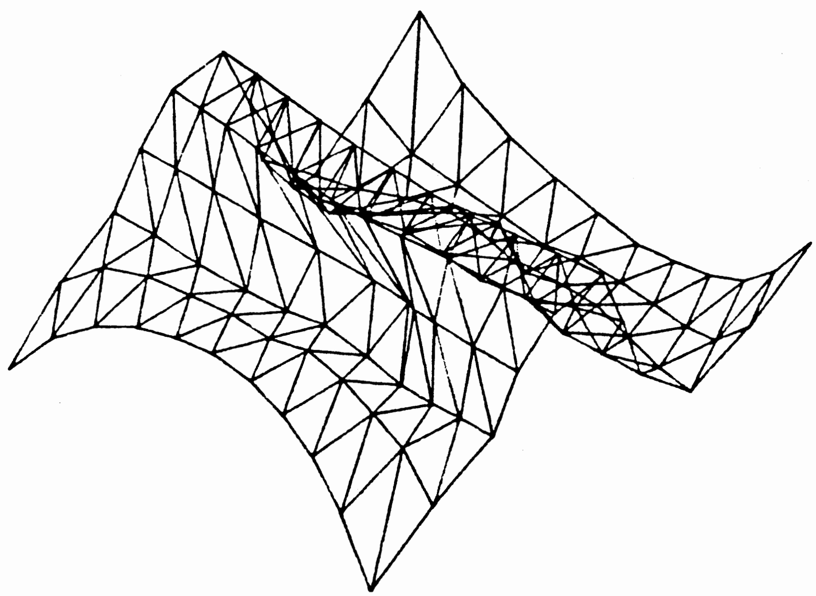


Figure 54. Buckled Wrinkle Shape

Eigenvalue = 0.749

For Static Wrinkle Shape See Figure 47

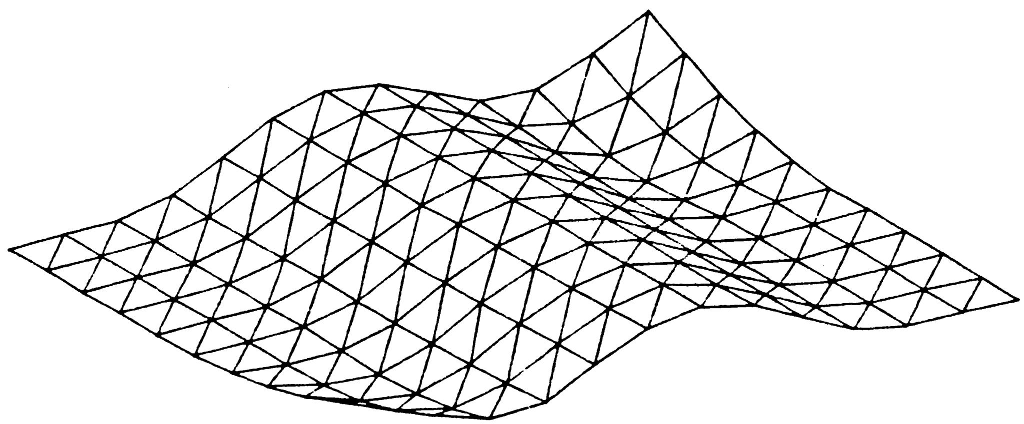


Figure 55. Buckled Wrinkle Shape

Eigenvalue = 0.969

For Static Wrinkle Shape See Figure 47

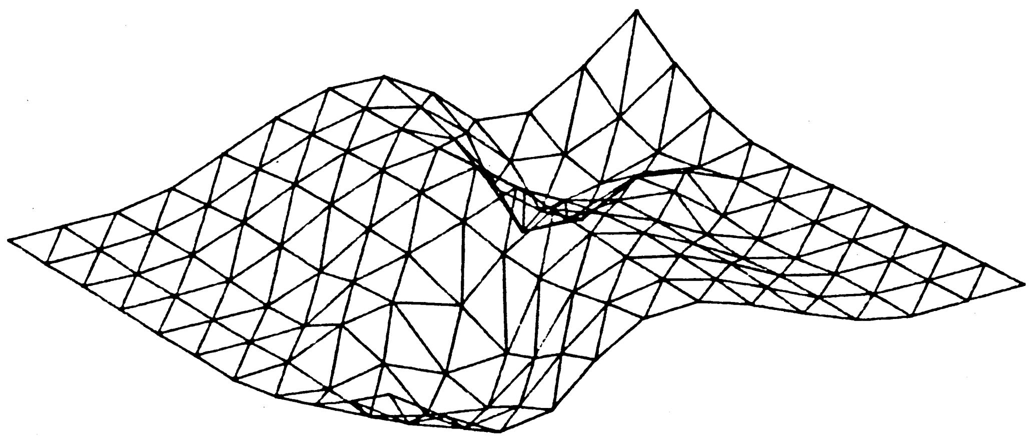


Figure 56. Buckled Wrinkle Shape

roller. This tends to indicate that, although mathematically viable, the buckled wrinkle shapes violate the physical boundary conditions present. From this it also seems apparent that the deformed wrinkle shapes obtained in the static analyses are viable shapes because they satisfy the necessary boundary constraints. This is not to imply that any wrinkles in any web material will not experience buckling. Instead, the wrinkles in the web material in this study do not show a buckling tendency.

The Mathematical Model

The results from the stress portion and the deformed shape portion of the study are combined to produce a single computer model. This model assumes that a linear relationship occurs between adjacent data point in the stress portion of the study. The model also uses the deformed shape information from Table XIII to determine whether a desirable deformed shape will be produced. To facilitate its use, the model is written in Microsoft Basic 3.2 which makes the model useful for many personal computers.

To determine the approximate accuracy of the computer model, it is run for 27 random cases utilizing the nine material and geometric parameters for which numerical data is available from the static analyses. These stress results are checked against the stress results from the NASTRAN computer runs. The difference in the stress values between the two sets of results gives an indication of the computer model accuracy. The root mean square average of the 27 stress differences is 27.3% with a standard deviation of 21.9%. Of the 27 cases, only six stress differences lie outside of the standard deviation. The algebraic mean of the differences, accounting for the sign of the stress differences, is +3.8%. This means that the computer model, generated from the static wrinkle analyses, predicts a slightly higher stress value than NASTRAN. A complete listing of the computer model is provided in Appendix D.

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

For a web wrinkle wrapped around a cylindrical roller, there exist predictable stresses and somewhat predictable deformed wrinkle shapes. For each of the nine parameters considered, the average stress of representative points on the web wrinkle are as follows:

1. Young's Modulus: The stress behaves in proportion to the parameter with a linear or slightly higher degree behavior.
2. Poisson's Ratio: The stress behaves inversely proportional to the parameter however, the difference in the stress is only several percent. It is entirely possible that the actual behavior is not clearly shown with such a small difference.
3. Wrinkle Height: The parameter is more meaningful when divided by the roller diameter to form the wrinkle height ratio. For large wrinkle height ratios, the stress behaves inversely proportional and linear, however for small wrinkle height ratios the stress is inversely proportional with a higher degree function.
4. Wrinkle Width: The parameter is more meaningful when

used as the denominator of the wrinkle aspect ratio. The stress behaves in proportion to the aspect ratio and in a linear fashion.

5. Roller Diameter: The parameter is most meaningful when the wrinkle geometry is unknown. Otherwise the roller diameter should be incorporated into the wrinkle height ratio. For constant wrinkle height and wrinkle aspect ratios, the stress behaves inversely proportional with a higher degree than that of a linear function.
6. Web Tension: The parameter is most meaningful when used in conjunction with the web thickness so that it is a measure of the tension induced stress in the web. At low parameter values, the stress behaves only somewhat linearly proportional with little increase, while at high parameter values the stress increases very rapidly and overshadows the stress induced by roller wrap.
7. Web Thickness: The parameter is most meaningful when used in conjunction with web tension. The stress behavior has been previously described.
8. Wrap Angle: The stress behaves in proportion to wrap angle for larger wrinkle aspect ratios and stiffer webs, and the parameter should reach a maximum value for a web wrinkle which collapses onto the roller.

9. Friction Coefficient: The parameter behaves in an exponential fashion with little change in magnitude for higher parameter values. The higher parameter values keep the wrinkle from collapsing onto the roller.

The deformed wrinkle shapes obtained in the study satisfy all of the static and boundary conditions necessary. For the specific web wrinkle geometry and material characteristics investigated in the present study, the buckled wrinkle shapes violate the necessary boundary conditions and therefore are not viable shapes for the wrinkles considered.

In summary, the data indicates that web rigidity should be as high as possible, primarily by the web thickness and secondarily by Young's modulus. For a given wrinkle width, the wrinkle height should be a minimum, and for a given wrinkle height, the wrinkle width should be as large as possible. For a given wrinkle height, the roller encountered should be as large as possible, and the roller should have a coefficient of friction as low as possible. The web stress induced by tension should be as low as possible, and the amount of web wrap on the roller should be kept as low as possible. The Poisson's ratio of the subject web has minimal effect on the web behavior.

The present study assumes a web material which is both isotropic and homogeneous. In addition, the web wrinkle is assumed to approach the roller perfectly normal to the roller. Considerable insight could be gained into the behavior of real web wrinkles by investigating the dependency of stress and deformation behavior on the anisotropy and non-homogeneity of the web material. The understanding of real web wrinkle behavior could also be increased by investigating the dependency of stress and deformed wrinkle behavior on the angle of incidence between the wrinkle axis and the roller axis. Finally, all of the preceeding recommendations for future research would be aided by knowledge of the process of wrinkle formation and transport in a moving web.

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APPENDIXES

APPENDIX A

AUTOMATIC MESH GENERATOR

```
C      -----VARIABLE DEFINITION--LIST
C       BCINC  = NO. OF BOUNDARY INCREMENTS EACH SIDE WRINKLE
C       CDINC  = NO. OF FULL WRINKLE INCREMENTS (-PI TO PI)
C       BCPT   = NO. OF BOUNDARY GRID POINTS EACH SIDE
C       CDPT   = NO. OF FULL WRINKLE GRID POINTS (-PI TO PI)
C       SECPT  = TOTAL SECTION GRID POINTS (WRINKLE & BOUNDARY)
C       TOTPT  = TOTAL GRID POINTS IN MODEL
C       WEBINC= NO. OF INCREMENTS IN HALF WRINKLE
C       WEBAX  = GRID POINT NUMBER OF X=0,Y=0
       DIMENSION Y(1110), ZZ(1110)
C      AMPFAC = WRINKLE AMPLITUDE FACTOR (HEIGHT/2PI)
C      RADFAC = ROLLER RADIUS / WRINKLE AMPLITUDE
C      T = WEB THICKNESS
C      E = MATERIAL MODULUS OF ELASTICITY
C      NU = POISSON'S RATIO FOR WEB MATERIAL
C      RHO = MATERIAL MASS DENSITY
C      ROLINC = NO. OF WEB INCREMENTS CONFINED TO ROLLER
C      ROLARC = ANGLE OF 1 INCREMENT ON ROLLER
C      MDCLPT = GRID POINT NO. OF CENTER POINT AT Y=-MAX
       DIMENSION XPT(4000), YPT(4000), ZPT(4000), XSPC(4000)
C      ROLANG = HALF OF ANGLE OF WEB/ROLL CONTACT(RAD)
       DIMENSION ZSPC(4000)
       INTEGER BCINC,CDINC,BCPT,CDPT,SECPT,TOTPT,WEBINC
       INTEGER WEBAX,TOTEL,SECINC,ELENO,G1,G2,G3
       INTEGER ROLINC
       REAL L,NU,MODEL
       READ(5,*) AMP,R,BCINC,CDINC,TPI
       READ(5,*) T,E,NU,RHO
       PI=3.14159
       W=3.0
       CIRCUM=(R*PI)/8.0
C      FIND WRINKLE ARC LENGTH
       DELY=(PI/W)/1000.0
       L=0.0
       ZZ(1)=0.0
       Y(1)=-PI/W
       DO 100 I=1,1000
       Y(I+1)=Y(1)+(I*DELY)
       ZZ(I+1)=(AMP/2.0)*(1.0+COS((Y(I+1))*W))
       DELL=SQRT(((ZZ(I+1)-ZZ(I))**2)+((Y(I+1)-Y(I))**2))
       L=L+DELL
   100 CONTINUE
       TOTL=2.0*L
C      END
       WEBDEL=TOTL/CDINC
       BCPT=BCINC
       SECINC=CDINC+(2*BCINC)
       MDINC=SECINC
       CDPT=CDINC+1
       MDPT=MDINC+1
```

```
      SECPT=(2*BCPT)+CDPT
      TOTPT=SECPT*MDPT
      TOTEL=2*SECINC*MDINC
      TOTT=TPI*WEBDEL*SECINC
      PTT=TOTT/SECPT
      DELDEL=(TPI*WEBDEL)/(E*T)
      MDDEL=WEBDEL+DELDEL
      YPT(BCPT+1)=-PI/W
      YPT(BCPT+CDPT)=PI/W
      ZPT(BCPT+1)=0.0
      ZPT(BCPT+CDPT)=0.0
      WEBINC=CDINC/2
      YBEG=BCPT+2
      YSTOP=BCPT+WEBINC
      WEBAX=(BCPT+1)+(WEBINC)
      DO 130 N=YBEG,YSTOP
      DO 110 I=1,2000
      THET=0.001*I
      YPT(N)=(WEBDEL*COS(THET))+YPT(N-1)
      ZPT(N)=(WEBDEL*SIN(THET))+ZPT(N-1)
      ZTEST=(AMP/2.0)*(1.0+COS((YPT(N))*W))
      ZDEL=0.001*WEBDEL
      ZERROR=ZTEST-ZDEL
      IF(ZPT(N).GE.ZERROR)GO TO 130
  110 CONTINUE
  130 CONTINUE
      YPT(WEBAX)=0.0
      ZPT(WEBAX)=AMP
      DO 150 I=1,WEBINC
      YPT(I+WEBAX)= -YPT(WEBAX-I)
      ZPT(I+WEBAX)= ZPT(WEBAX-I)
  150 CONTINUE

C     ESTABLISH X COORDINATE OF ALL POINTS
      DO 190 NX=1,MDPT
      DO 170 NN=1,SECPT
      XPT(NN+((NX-1)*SECPT))=(NX-1)*WEBDEL
  170 CONTINUE
  190 CONTINUE
C     END
C     ESTABLISH Z COORDINATE OF BOUNDARY POINTS
      DO 220 NZ=1,BCPT
      DO 210 NN=1,MDPT
      ZPT(NZ+((NN-1)*SECPT))=0.0
  210 CONTINUE
  220 CONTINUE
      JPT=(BCPT+CDPT)+1
      JJPT=(2*BCPT)+CDPT
      JNPT=JPT-1
      DO 240 NJ=JPT,JJPT
      DO 230 NN=1,MDPT
      ZPT(NJ+((NN-1)*SECPT))=0.0
  230 CONTINUE
  240 CONTINUE
C     END
C     ESTABLISH Y COORDINATE OF BOUNDARY POINTS
```

```
      DO 260 NY=1,BCPT
      DO 250 NN=1,MDPT
      YPT(NY+((NN-1)*SECPT))=-(PI/W)-((BCPT+1-NY)*WEBDEL)
  250 CONTINUE
  260 CONTINUE
      DO 280 NY=JPT,JJPT
      DO 270 NN-1,MDPT
      YPT(NY+((NN-1)*SECPT))=(PI/W)+((NY+1-JPT)*WEBDEL)
  270 CONTINUE
  280 CONTINUE
C     END
      JY=BCPT+CDPT
      JN=BCPT+1
C     ESTABLISH Y&Z COORDINATES OF WRINKLE POINTS
      DO 310 NN=JN,JY
      DO 290 NY=1,MDINC
      YPT(NN+(NY*SECPT))=YPT(NN)
      ZPT(NN+(NY*SECPT))=ZPT(NN)
  290 CONTINUE
  310 CONTINUE
C     END
C      WRITE GRID AND ELEMENTS
   10 FORMAT(T2,'GRID',T9,I4,T17,'0',T25,F8.4,T33,F8.4,
     CT41,F8.4,T49,'0',T57,'6')
      DO 400 II=1,TOTPT
      WRITE(6,10) II,XPT(II),YPT(II),ZPT(II)
  400 CONTINUE
      DO 440 NN=1,MDINC
      DO 420 NS=1,SECINC
      ELENO=(NS+((NN-1)*2*SECINC))

      G1=(NS+((NN-1)*SECPT))
      G2=(NN*SECPT)+NS
      G3=((NN-1)*SECPT)+(NS+1)
      WRITE(6,30) ELENO,G1,G2,G3
   30 FORMAT(T2,'CTRIA2',T9,I4,T17,'1',T25,I4,T33,I4,T41,I4,T49,'0.0')
  420 CONTINUE
  440 CONTINUE
      DO 480 NN=1,MDINC
      DO 460 NS=1,SECINC
      ELENO=(((2*NN)-1)*SECINC)+NS
      G2=((NN-1)*SECPT)+(NS+1)
      G3=(NN*SECPT)+NS
      G1=(NN*SECPT)+(NS+1)
      WRITE(6,30) ELENO,G1,G2,G3
  460 CONTINUE
  480 CONTINUE
C      END
      ROLINC=MDINC-(2*BCINC)
      ROLARC=2.0*(ARSIN((MDDEL/2.0)/R))
      MDCLPT=1+(MDINC*SECPT/2)
      MD2INC=MDINC/2
C     COMPUTE CONSTRAINT POINTS ALONG BOUNDARY
      DELZ=0.0
      DO 510 NY=1,MD2INC
      DZ=MDDEL*SIN(NY*ROLARC)
```

```
      DELZ=DELZ+DZ
      DO 500 NN=1,JN
      XSPC((NN-1)+MDCLPT+(NY*SECPT))=NY*DELDEL
      XSPC((NN-1)+MDCLPT-(NY*SECPT))=-(NY*DELDEL)
      ZSPC((NN-1)+MDCLPT+(NY*SECPT))=-DELZ
      ZSPC((NN-1)+MDCLPT-(NY*SECPT))=-DELZ
      NGRID=((NN-1)+MDCLPT+(NY*SECPT))
      WRITE(6,40)NGRID,XSPC(NGRID),NGRID,ZSPC(NGRID)
      WRITE(6,45)NGRID
      NGRID=((NN-1)+MDCLPT-(NY*SECPT))
      WRITE(6,40)NGRID,XSPC(NGRID),NGRID,ZSPC(NGRID)
      WRITE(6,45)NGRID
   40 FORMAT(T2,'SPC',T9,'100',T17,I4,T25,'1',T33,F8.5,
     CT41,I4,T49,'3',T57,F8.5)
   45 FORMAT(T2,'SPC',T9,'100',T17,I4,T25,'26')
      NGRID=(NN-1)+MDCLPT
      WRITE(6,50)NGRID
   50 FORMAT(T2,'SPC',T9,'100',T17,I4,T25,'123456')
  500 CONTINUE
  510 CONTINUE
      DELZ=0.0
      DO 540 NY=1,MD2INC
      DZ=MDDEL*SIN(NY*ROLARC)
      DELZ=DELZ+DZ
      DO 530 NN=JNPT,JJPT
      XSPC((NN-1)+MDCLPT+(NY*SECPT))=NY*DELDEL
      XSPC((NN-1)+MDCLPT-(NY*SECPT))=-(NY*DELDEL)
      ZSPC((NN-1)+MDCLPT+(NY*SECPT))=-DELZ
      ZSPC((NN-1)+MDCLPT-(NY*SECPT))=-DELZ
      NGRID=((NN-1)+MDCLPT+(NY*SECPT))
      WRITE(6,40)NGRID,XSPC(NGRID),NGRID,ZSPC(NGRID)
      WRITE(6,45)NGRID
      NGRID=((NN-1)+MDCLPT-(NY*SECPT))
      WRITE(6,40)NGRID,XSPC(NGRID),NGRID,ZSPC(NGRID)
      WRITE(6,45)NGRID
      NGRID=(NN-1)+MDCLPT
      WRITE(6,50)NGRID
  530 CONTINUE
  540 CONTINUE
C       END
      WRITE(6,53) T
   53 FORMAT(T2,'PTRIA2',T9,'1',T17,'5',T25,F8.5)
      WRITE(6,54) E,NU
   54 FORMAT(T2,'MAT1',T9,'5',T17,E8.1,T33,F8.4,T41,'0.00118')
C       WRITE TENSION FORCES
      ROLANG=ROLARC*MD2INC
   60 FORMAT(T2,'FORCE',T9,'200',T17,I6,T25,'0',T33,F8.4,
     CT41,F8.4,T57,F8.4)
      YS=YBEG+CDINC-2
      DO 750 NN=1,SECPT
      NG=NN
      XN1=-COS(ROLANG)
      XN3=-SIN(ROLANG)
      WRITE(6,60) NG,PTT,XN1,XN3
      NG=(SECPT*MDINC)+NN
      XN1=COS(ROLANG)
```

```
      WRITE(6,60) NG,PTT,XN1,XN3
  750 CONTINUE
C       END
C       WRITE PHYSICAL PARAMETERS
      XNORM=(SIN(ROLARC))*TOTT
      DIA=2.0*R
      WRAP=MDINC*ROLARC*180.0/PI
      WRITE(6,70) AMP
      WRITE(6,71) DIA
      WRITE(6,72) WRAP
      WRITE(6,73) TPI
      WRITE(6,74) T
      WRITE(6,75) MDINC
      WRITE(6,76) MDCLPT
      WRITE(6,77) ROLARC
      WRITE(6,78) XNORM
   70 FORMAT(T2,'$',T10,'WRINKLE AMPLITUDE',F8.4)
   71 FORMAT(T2,'$',T10,'ROLLER DIAMETER',F8.3)
   72 FORMAT(T2,'$',T10,'WRAP ANGLE',F8.3)
   73 FORMAT(T2,'$',T10,'TENSION-POUNDS/INCH',F8.3)
   74 FORMAT(T2,'$',T10,'THICKNESS',F8.5)
   75 FORMAT(T2,'$',T10,'MD ELEMENTS',I6)
   76 FORMAT(T2,'$',T10,'MD CL POINT NO.',I6)
   77 FORMAT(T2,'$',T10,'ROLARC(RADS)',F8.5)
   78 FORMAT(T2,'$',T10,'BOUNDARY NORMAL REACTION',F8.5)
C       END
      WRITE(6,80)
   80 FORMAT(T2,'$345678123456781234567812345678123456781234567812345
     6')
C     INSERT FRICTION FORCE AND ADDITIONAL CONSTRAINTS HERE
C     END FRICTION
C     BEGIN ADDITIONAL CONSTRAINTS
      WRITE(6,55)
   55 FORMAT(T2,'ENDDATA')
      STOP
      END
```

APPENDIX B

DATA

RD=24" WW=6.28" YM=300,000psi
WA=41 degrees TN=5 p/i

	ELEMENT AND STRESS (PSI)			
	1	2	3	4
FC=0.5				
PR=0.3				
WH=0.3"				
TH=				
3 mils	4720	1764	3145	1987
10 mils	2094	640	1593	3775
20 mils	2439	758	1556	3681
40 mils	1208	855	1492	2673
50 mils	1110	1097	1242	2341
60 mils	1093	1275	1243	2177
FC=0.5				
PR=0.3				
WH=0.6"				
TH=				
3 mils	4580	926	3154	7065
10 mils	2406	317	1989	4343
20 mils	2145	622	1669	4511
30 mils	1178	1401	1993	3881
40 mils	1625	1698	1424	3480
50 mils	1702	1804	1542	3333
60 mils	1642	1948	1493	3497
FC=0.5				
PR=0.3				
WH=1.0"				
TH=				
3 mils	3843	2344	4962	9797
10 mils	2453	1538	3979	8062
20 mils	1558	620	2510	4967
40 mils	2697	3380	3062	4253
50 mils	3171	3421	2836	4483
60 mils	2568	2824	5741	4296
FC=0.5				
PR=0.1				
WH=0.3"				
TH=				
40 mils	1235	755	1431	2463
60 mils	1118	1102	1342	2568
FC=0.5				
PR=0.1				
WH=0.6"				
TH=				
40 mils	1521	1673	1649	3222
60 mils	1534	1847	1442	3846

FC=0.5				
PR=0.1				
WH=1.0"				
TH=				
40 mils	2564	3471	2896	4133
60 mils	2453	3004	5119	4516
FC=0.5				
PR=0.5				
WH=0.3"				
TH=				
40 mils	1239	1133	1635	3079
60 mils	1144	1653	1155	2336
FC=0.5				
PR=0.5				
WH=0.3"				
TH=				
40 mils	1898	1959	1538	3904
60 mils	1898	2354	1905	3352
FC=0.5				
PR=0.5				
WH=1.0"				
TH=				
40 mils	3130	3581	3406	4630
60 mils	2969	2987	6865	4500
FC=0.25				
PR=0.3				
WH=0.3"				
TH=				
30 mils	1718	751	1282	3197
40 mils	1195	854	1522	2711
50 mils	1100	1108	1249	2368
60 mils	1080	1286	1275	2153
FC=0.25				
PR=0.3				
WH=1.0"				
TH=				
40 mils	3267	3614	3302	4818
50 mils	3257	3441	2792	4482
60 mils	3064	3029	6686	4597
FC=inf				
PR=0.3				
WH=0.3"				
TH=				
30 mils	1855	1081	1856	3589
40 mils	1480	596	1583	3696
50 mils	1368	384	1378	3867
60 mils	1348	587	1536	4038

FC=inf
PR=0.3
WH=0.6"
TH=

30 mils	1015	2309	2966	3292
40 mils	1472	1862	3002	3991
50 mils	1842	1535	3044	4689
60 mils	2081	1269	3028	5307

FC=inf
PR=0.3
WH=1.0"
TH=

30 mils	807	3439	3608	2654
40 mils	1710	3307	3898	3554
50 mils	2339	3138	4238	4538
60 mils	2794	2902	4522	5512

RD=8" WW=2.09" WA=50 degrees
TN=5 p/i YM=300,000 psi PR=0.3

FC=inf
WH=0.1"
TH=

1 mil	6811	4513	8468	9734
3 mils	4286	1266	5563	6558
10 mils	3665	772	4072	5843
20 mils	3294	949	3685	6004

FC=inf
WH=0.333"
TH=

1 mil	18614	5215	16869	21956
3 mils	6729	4766	10151	8076
10 mils	4027	6745	8113	9399

FC=inf
WH=0.667"
TH=

1 mil	22267	7147	29074	3568
3 mils	6353	11529	16399	5979
10 mils	1183	12022	14394	12299
20 mils	4417	2049	8927	24194

FC=0.5
WH=0.1"
TH=

1 mil	6605	4435	8269	9518
10 mils	2657	1289	3865	6115

FC=0.5				
WH=0.333"				
TH=				
1 mil	16681	4799	15712	18845
10 mils	7454	3712	6712	11268
FC=0.25				
WH=0.1"				
TH=				
1 mil	6538	4489	8085	9611
10 mils	2724	1211	4099	5933
FC=0.25				
WH=0.333"				
TH=				
1 mil	16584	4851	15648	18691
10 mils	7274	3683	7492	10865

RD=8" WW=2.09" WA=50 degrees
PR=0.3 FC=inf YM=300,000 psi

TN=10 p/i				
WH=0.1"				
TH=				
1 mil	11387	9592	13095	14667
10 mils	3766	703	4449	6499
20 mils	4866	1603	4733	7122
TN=20 p/i				
WH=0.1"				
TH=				
1 mil	20456	19901	22630	24122
10 mils	4947	1722	5522	7595
20 mils	4866	1603	4733	7122
TN=10 p/i				
WH=0.667"				
TH=				
1 mil	28479	11678	34212	41127
10 mils	46230	0	2503	17407
20 mils	51039	0	7530	18018
TN=20 p/i				
WH=0.667"				
TH=				
1 mil	40140	21086	44491	52018
10 mils	47370	0	3731	18622
20 mils	51494	0	8127	18620

RD=4" WW=1.05" TN=5 p/i
FC=inf PR=0.3 YM=300,000 psi

WA=55 degrees				
WH=0.05"				
TH=				
1 mil	7691	5132	8680	11260
3 mils	4940	2195	5429	7557
10 mils	3873	1342	4514	6940
20 mils	3530	2060	5125	7348
WA=55 degrees				
WH=0.167"				
TH=				
1 mil	13581	7207	16589	20438
3 mils	7395	4703	14174	16516
10 mils	5063	9332	10752	15604
20 mils	6189	4638	9916	18057
WA=55 degrees				
WH=0.333"				
TH=				
1 mil	19468	11485	27564	40216
3 mils	11230	9710	26947	25471
10 mils	7117	26682	19668	23320
20 mils	10519	13343	18089	30917
WA=105 degrees				
WH=0.05"				
TH=				
1 mil	7455	9538	6285	13119
3 mils	6653	8561	35270	14619
10 mils	4414	6117	2698	9208
20 mils	4858	6452	3276	9794
WA=105 degrees				
WH=0.167"				
TH=				
1 mil	18710	21040	10145	31158
3 mils	10706	17622	7765	27243
10 mils	9032	15770	7704	26951
20 mils	9591	16248	7089	27672
WA=122 degrees				
WH=0.333"				
TH=				
1 mil	30329	31223	14896	53659
3 mils	14998	32857	14670	51299
10 mils	14795	27902	18191	50029
20 mils	16367	28945	15072	52342

WA=152 degrees
WH=0.05"
TH=

3 mils	3293	3046	6291	9997
20 mils	3038	2805	6023	10113

WA=180 degrees
WH=0.333"
TH=

3 mils	10170	11406	27280	53242
20 mils	7719	9652	26141	54397

RD=4" WW=1.05" TN=5 p/i
FC=inf PR=0.3 YM=300,000 psi
WA= 70 degrees

WH=0.333"
TH=

0.5 mil	23966	15730	32668	46621
0.25 mil	35126	24886	42937	58036
0.05 mil	128649	99228	126543	156244

RD=4" WW=1.05 TH=10 mils
FC=inf WH=0.333" YM=300,000 psi
WA=61 degrees PR=0.3

TN=

0.25 p/i	8593	15664	12867	20195
1.0 p/i	8582	15687	13069	20482
10.0 p/i	8247	25281	20622	25030
20.0 p/i	10670	22527	21714	28540
50.0 p/i	18489	14214	27846	39934

RD=4" WW=1.05" TH=10 mils
FC=inf WH=0.333" YM=300,000 psi
WA=61 degrees TN=5 p/i

PR=

0.01	7487	27998	20631	23640
0.05	7345	27695	20454	23470
0.1	7223	27372	20286	23308
0.5	8778	16140	14794	23847

RD=4" WW=1.05" TH=10 mils
FC=inf WH=0.333" PR=0.3
WA=65 degrees TN=5 p/i

YM=				
2,000 psi	713	525	682	810
10,000 psi	761	1190	1385	1575
50,000 psi	2476	2419	3870	5666
100,000 psi	3152	7956	7034	8927
1,000,000 psi	28590	52338	44018	68911
30,000,000 psi	859691	1.566E6	1.281E6	2.01E6

RD=4" WW=1.05" TH=3 mils
FC=inf TN=5 p/i PR=0.3
YM=300,000 psi

WH=0.667" WA=84 degrees	31389	21271	46999	42065
WH=1.0" WA=112 degrees	40851	31551	66303	59306

RD=2" WW=0.524" PR=0.3
FC=inf TN=5 p/i WA=55 degrees
YM=300,000 psi

TH=3 mils WH=				
0.025"	8135	2568	4685	9735
0.05"	8751	2616	6406	15068
0.083"	12287	3408	10155	21998
0.167"	18086	8321	16737	26696
0.333"	29404	18474	32846	39017
0.667"	48958	44354	69267	68744

TH=10 mils WH=				
0.025"	3762	1640	3356	6010
0.05"	3177	1381	3423	8406
0.083"	3458	4631	5800	10851
0.167"	4692	11183	11926	14472
0.333"	7282	22952	22415	21068
0.667"	15658	42039	39852	34090

APPENDIX C

DEFORMED WRINKLE SHAPES

RD=24"
WW=6.28"
TN=5 p/i
YM=300,000 psi
WA=40 degrees

WH=0.3"
TH=60 mils
PR=0.3
FC=inf

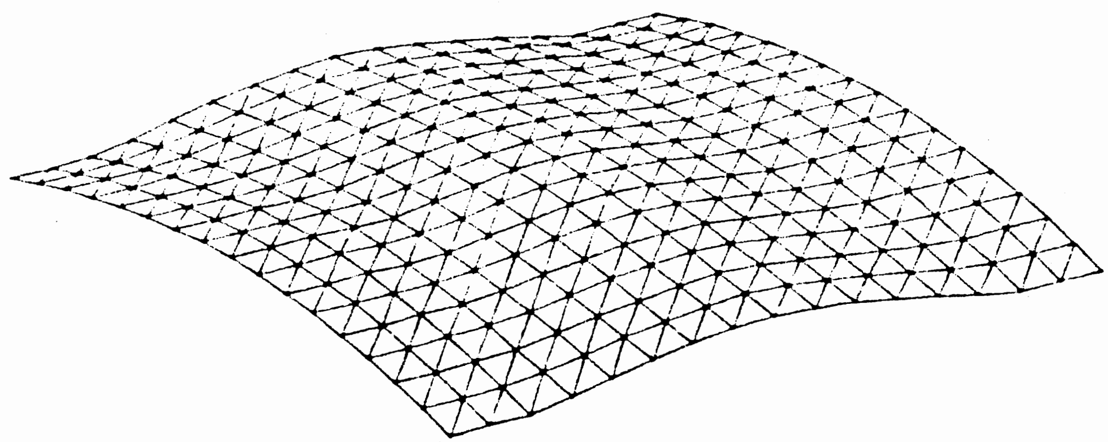

Figure 57. Deformed Wrinkle Shape (24" Roller)

RD=24"
WW=6.28"
TN=5 p/i
YM=300,000 psi
WA=40 degrees

WH=0.3"
TH=30 mils
PR=0.3
FC=0.25

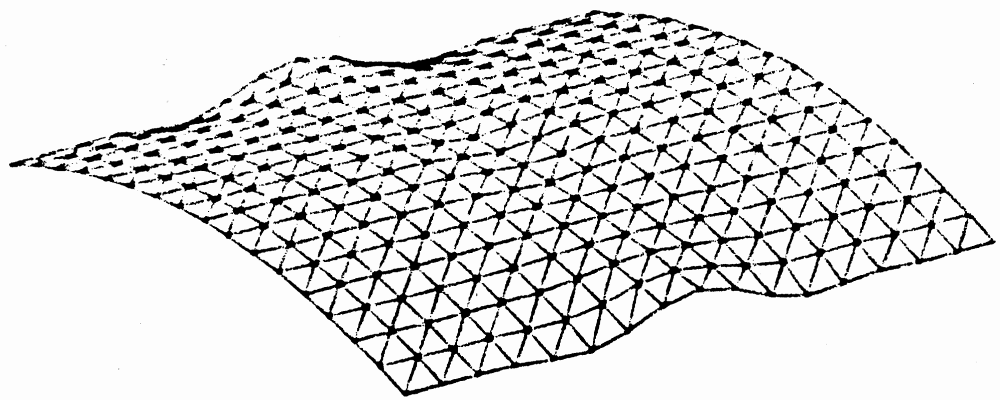

Figure 58. Deformed Wrinkle Shape (24" Roller)

RD=24"
WW=6.28"
TN=5 p/i
YM=300,000 psi
WA=40 degrees

WH=0.3"
TH=40 mils
PR=0.3
FC=0.25

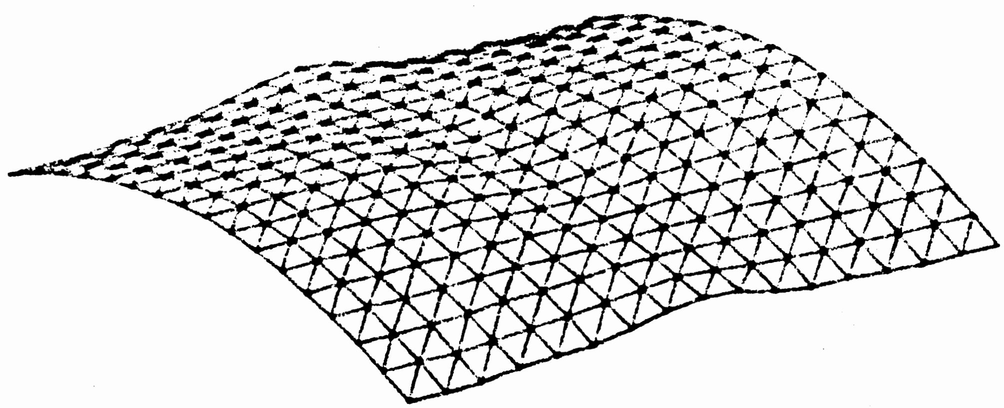

Figure 59. Deformed Wrinkle Shape (24" Roller)

RD=24"	WH=0.3"
WW=6.28"	TH=50 mils
TN=5 p/i	PR=0.3
YM=300,000 psi	FC=0.25
WA=40 degrees	

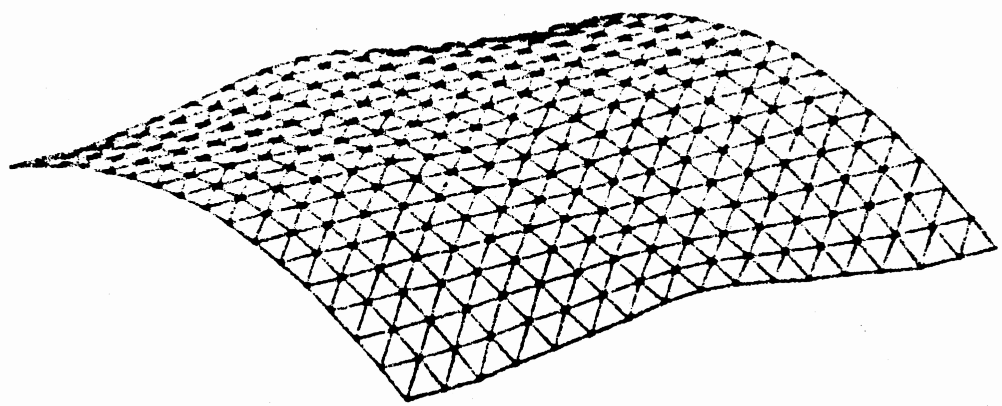

Figure 60. Deformed Wrinkle Shape (24" Roller)

RD=24"
WW=6.28"
TN=5 p/i
YM=300,000 psi
WA=40 degrees

WH=0.3"
TH=60 mils
PR=0.3
FC=0.25

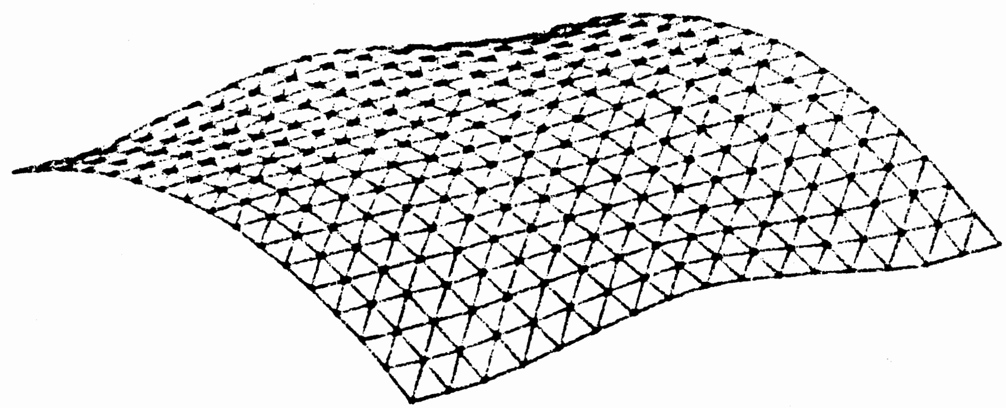

Figure 61. Deformed Wrinkle Shape (24" Roller)

RD=24"
WW=6.28"
TN=5 p/i
YM=300,000 psi
WA=42 degrees

WH=1.0"
TH=40 mils
PR=0.5
FC=0.25

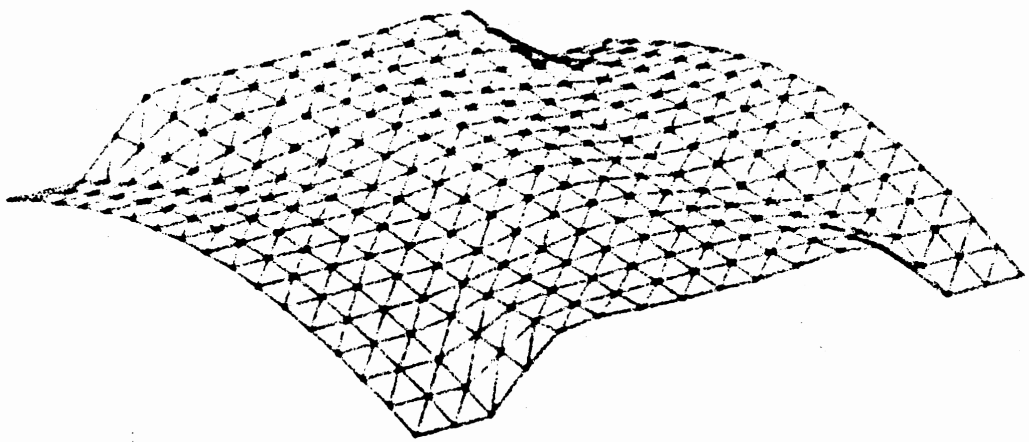

Figure 62. Deformed Wrinkle Shape (24" Roller)

RD=24"
WW=6.28"
TN=5 p/i
YM=300,000 psi
WA=42 degrees

WH=1.0"
TH=50 mils
PR=0.3
FC=0.25

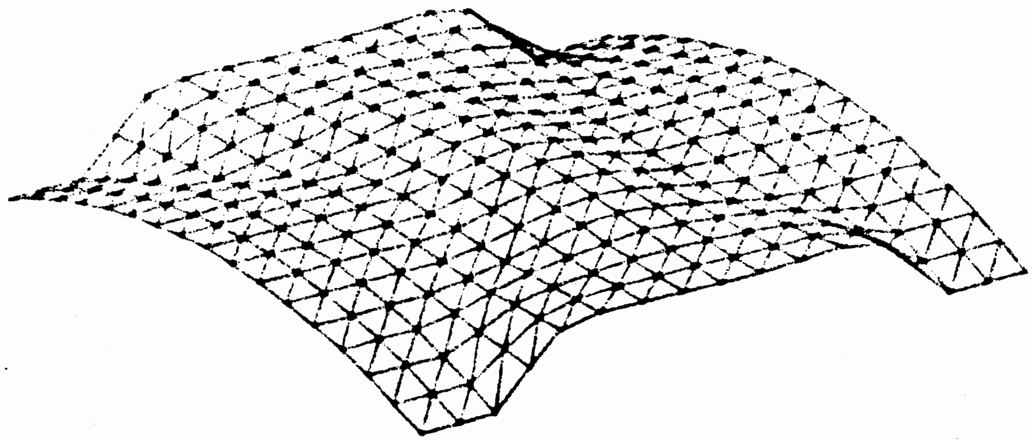

Figure 63. Deformed Wrinkle Shape (24" Roller)

RD=24"
WW=6.28"
TN=5 p/i
YM=300,000 psi
WA=40 degrees

WH=0.3"
TH=40 mils
PR=0.3
FC=0.5

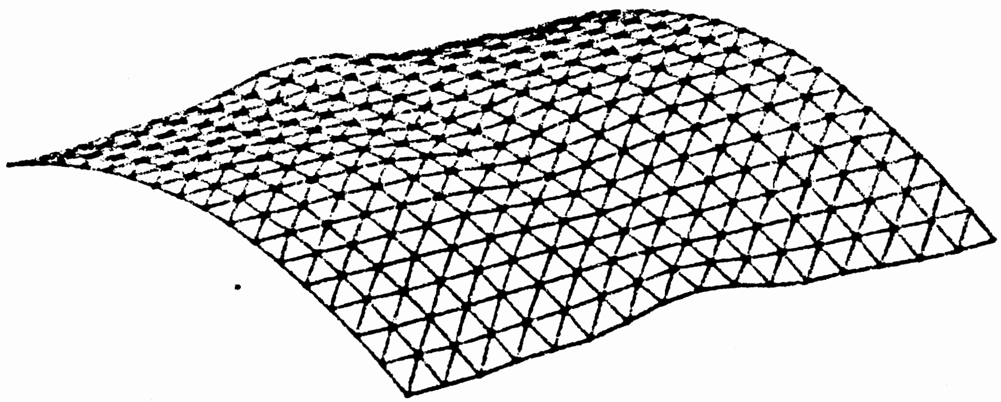

Figure 64. Deformed Wrinkle Shape (24" Roller)

RD=24"
WW=6.28"
TN=5 p/i
YM=300,000 psi
WA=40 degrees

WH=0.3"
TH=50 mils
PR=0.3
FC=0.5

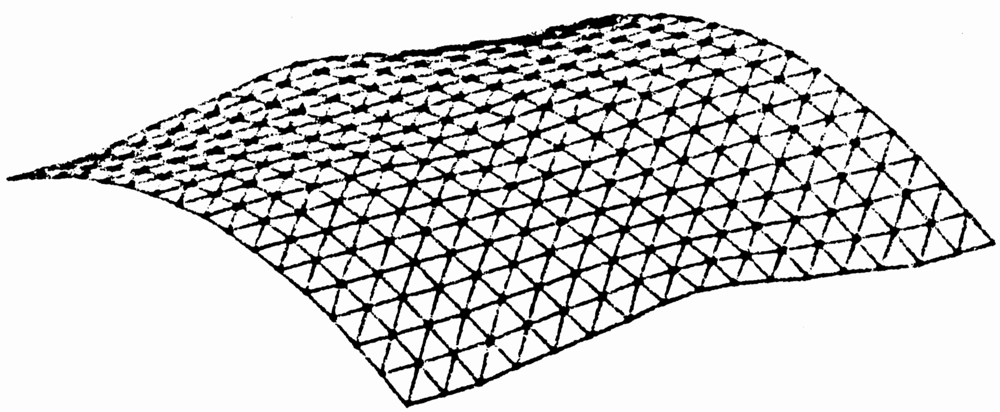

Figure 65. Deformed Wrinkle Shape (24" Roller)

RD=24"
WW=6.28"
TN=5 p/i
YM=300,000 psi
WA=40 degrees

WH=0.3"
TH=60 mils
PR=0.3
FC=0.5

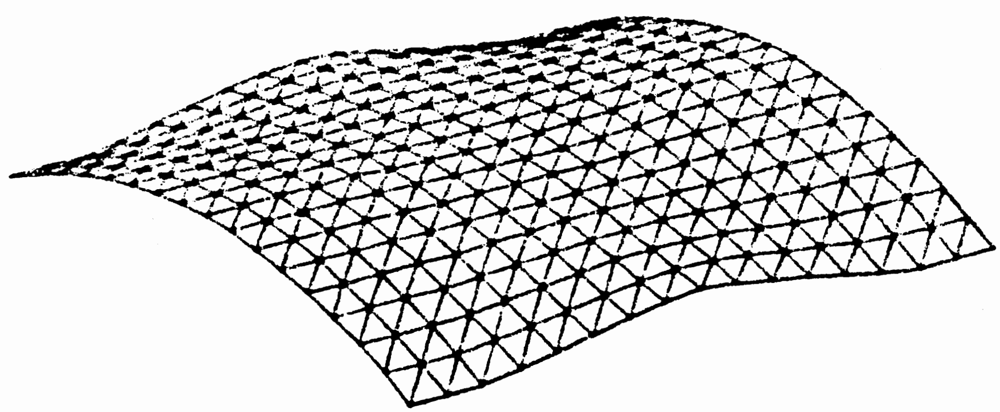

Figure 66. Deformed Wrinkle Shape (24" Roller)

RD=24" WH=0.6"
WW=6.28" TH=40 mils
TN=5 p/i PR=0.3
YM=300,000 psi FC=0.5
WA=41 degrees

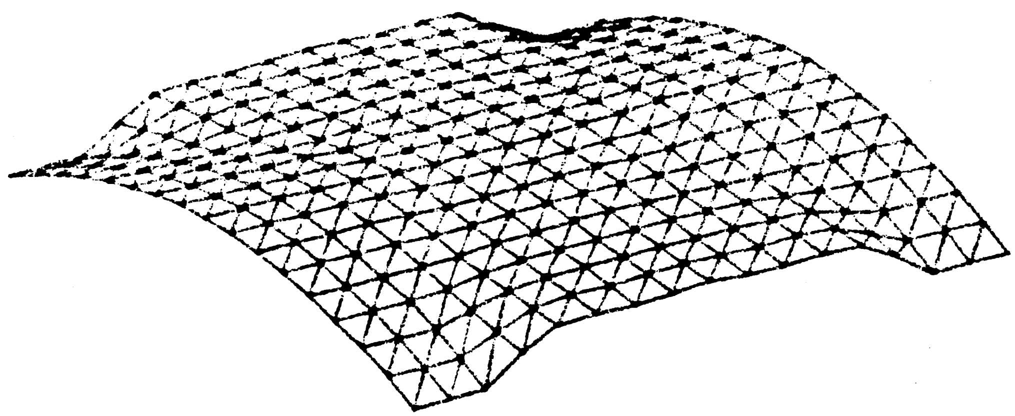

Figure 67. Deformed Wrinkle Shape (24" Roller)

RD=24"
WW=6.28"
TN=5 p/i
YM=300,000 psi
WA=41 degrees

WH=0.6"
TH=50 mils
PR=0.3
FC=0.5

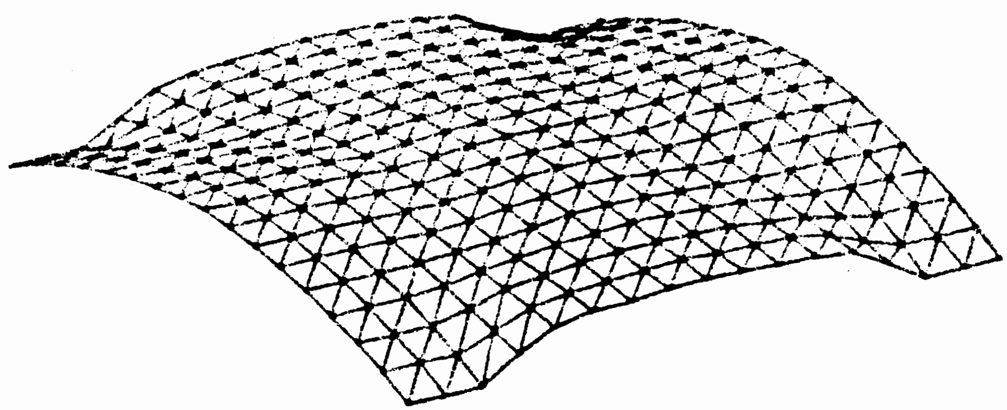

Figure 68. Deformed Wrinkle Shape (24" Roller)

RD=24"
WW=6.28"
TN=5 p/i
YM=300,000 psi
WA=42 degrees

WH=1.0"
TH=40 mils
PR=0.3
FC=0.5

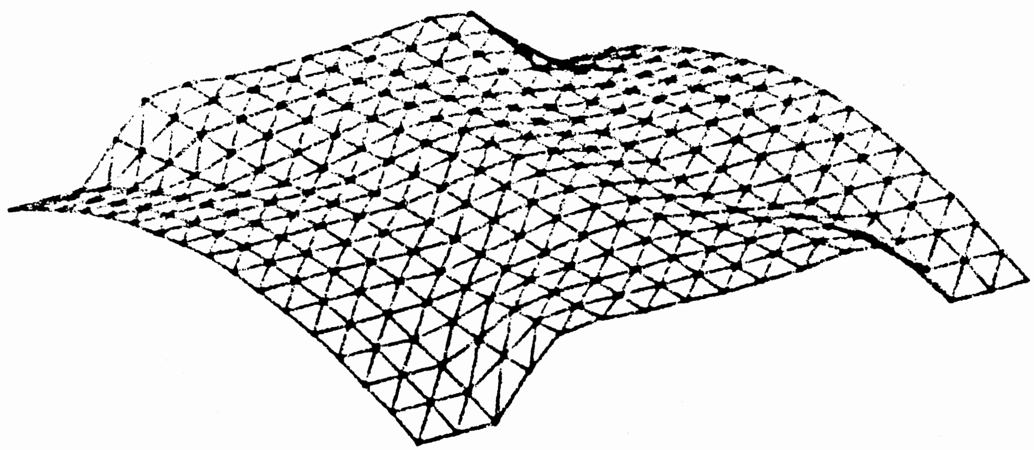

Figure 69. Deformed Wrinkle Shape (24" Roller)

RD=24"
WW=6.28"
TN=5 p/i
YM=300,000 psi
WA=42 degrees

WH=1.0"
TH=50 mils
PR=0.3
FC=0.5

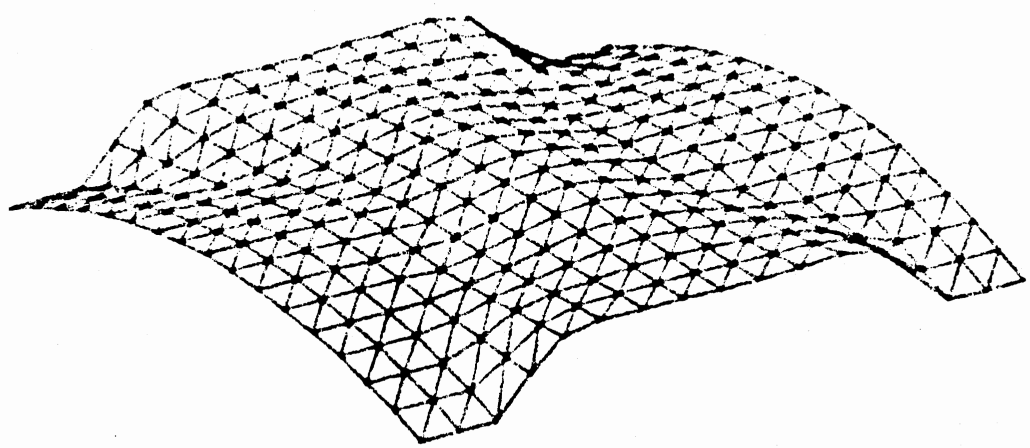

Figure 70. Deformed Wrinkle Shape (24" Roller)

RD=24"	WH=1.0"
WW=6.28"	TH=60 mils
TN=5 p/i	PR=0.3
YM=300,000 psi	FC=0.5
WA=42 degrees	

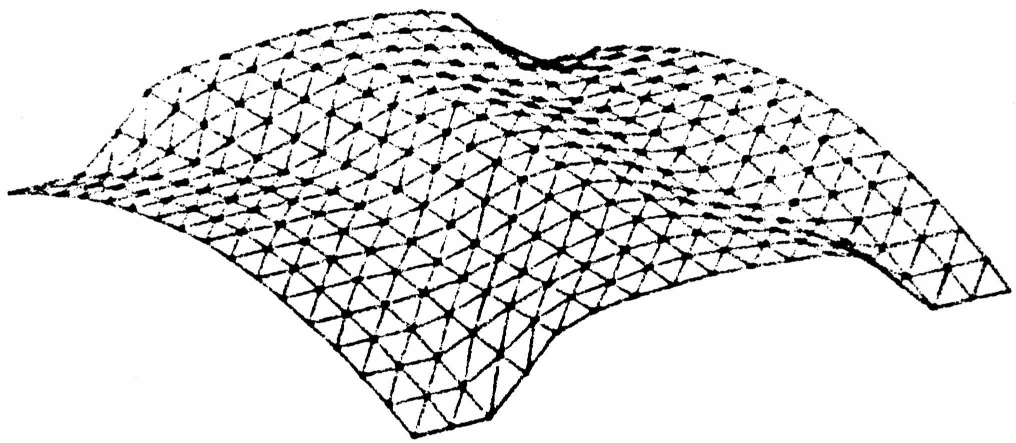

Figure 71. Deformed Wrinkle Shape (24" Roller)

RD=4"
WW=1.05"
TN=5 p/i
YM=300,000 psi
WA=54 degrees

WH=0.167"
TH=1 mil
PR=0.3
FC=inf

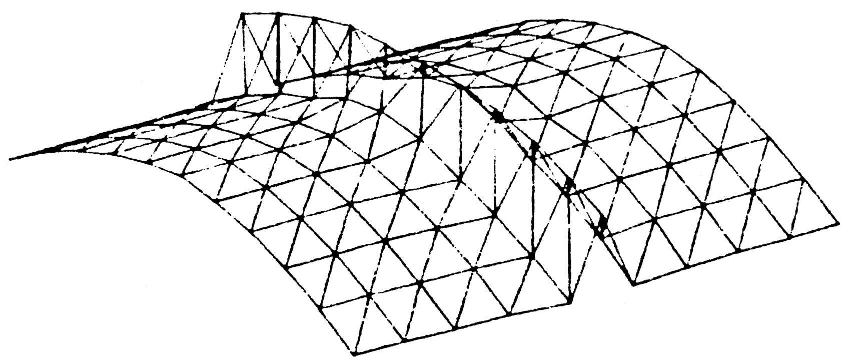

Figure 72. Deformed Wrinkle Shape (4" Roller)

RD=4"
WW=1.05"
TN=5 p/i
YM=300,000 psi
WA=53 degrees

WH=0.167"
TH=10 mils
PR=0.3
FC=inf

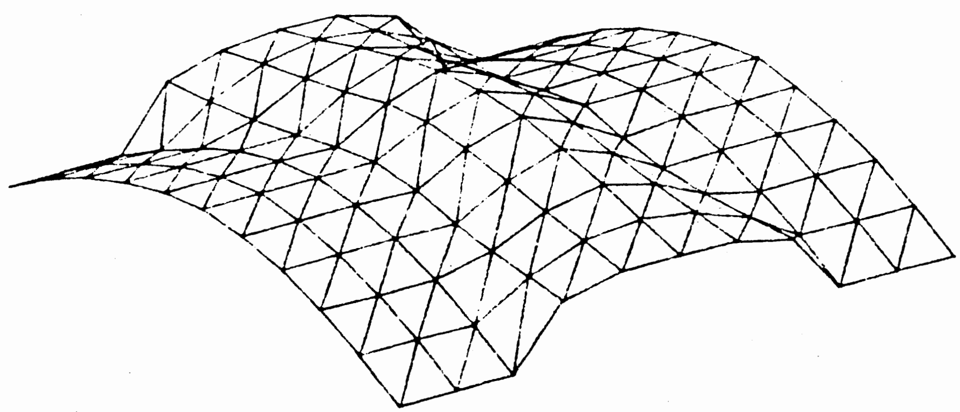

Figure 73. Deformed Wrinkle Shape (4" Roller)

RD=2"
WW=0.524"
TN=5 p/i
YM=300,000 psi
WA=38 degrees

WH=0.025"
TH=3 mils
PR=0.3
FC=inf

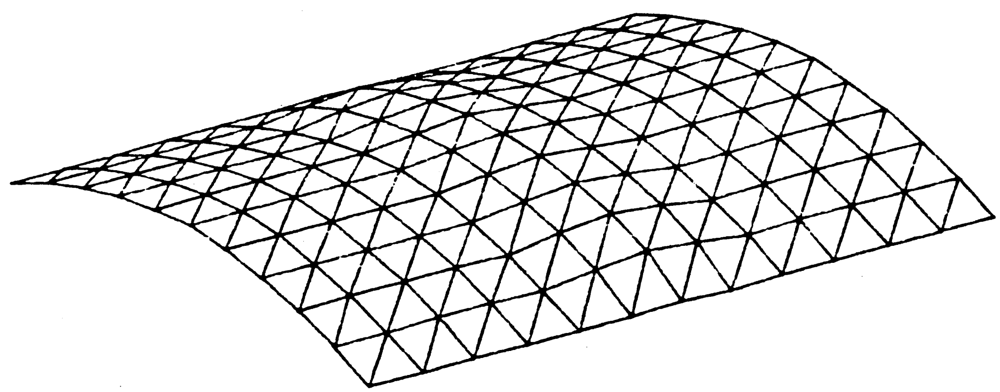

Figure 74. Deformed Wrinkle Shape (2" Roller)

RD=2"
WW=0.524"
TN=5 p/i
YM=300,000 psi
WA=38 degrees

WH=0.025"
TH=10 mils
PR=0.3
FC=inf

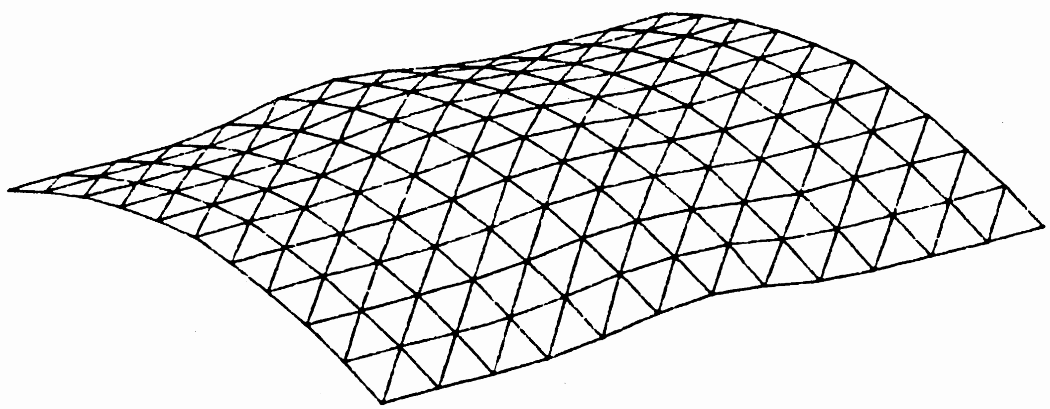

Figure 75. Deformed Wrinkle Shape (2" Roller)

RD=2"	WH=0.05"
WW=0.524"	TH=3 mils
TN=5 p/i	PR=0.3
YM=300,000 psi	FC=inf
WA=39 degrees	

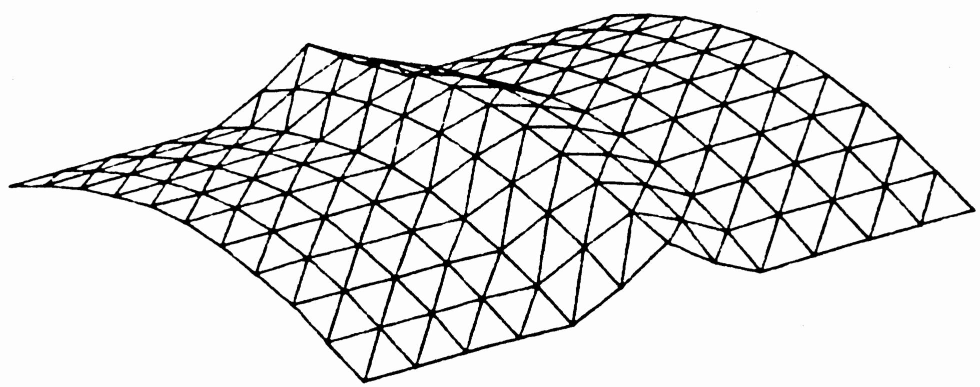

Figure 76. Deformed Wrinkle Shape (2" Roller)

RD=2"
WW=0.524"
TN=5 p/i
YM=300,000 psi
WA=38 degrees

WH=0.05"
TH=10 mils
PR=0.3
FC=inf

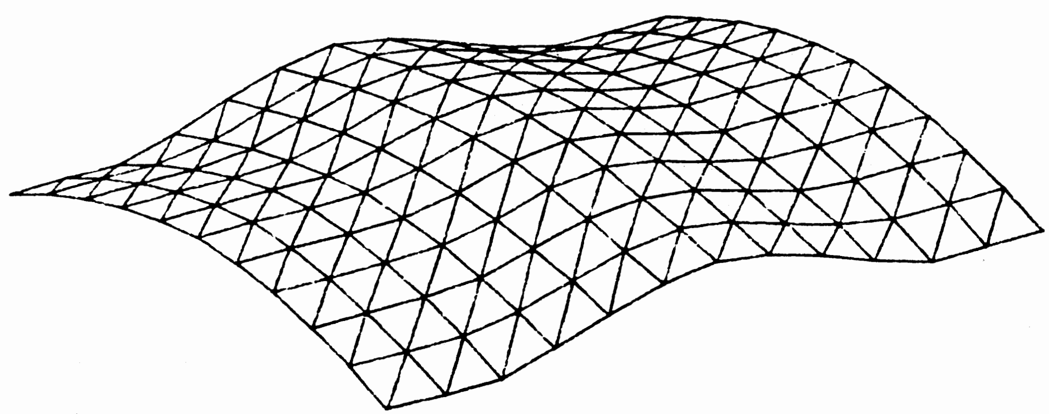

Figure 77. Deformed Wrinkle Shape (2" Roller)

RD=2"
WW=0.524"
TN=5 p/i
YM=300,000 psi
WA=40 degrees

WH=0.083"
TH=3 mils
PR=0.3
FC=inf

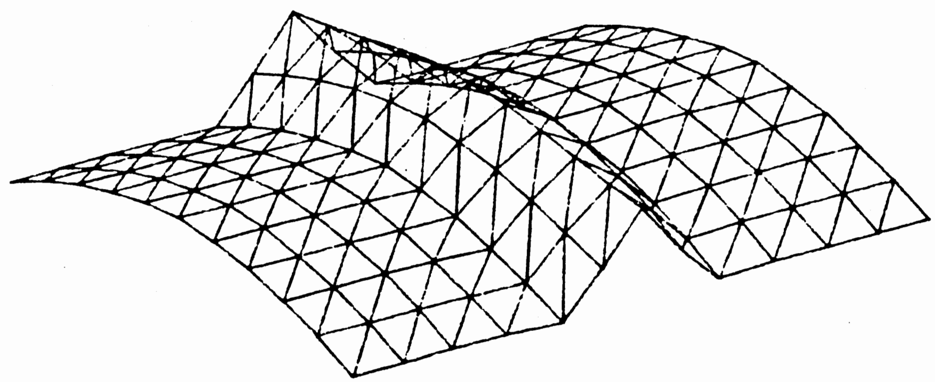

Figure 78. Deformed Wrinkle Shape (2" Roller)

RD=2"
WW=0.524"
TN=5 p/i
YM=300,000 psi
WA=46 degrees

WH=0.167"
TH=10 mils
PR=0.3
FC=inf

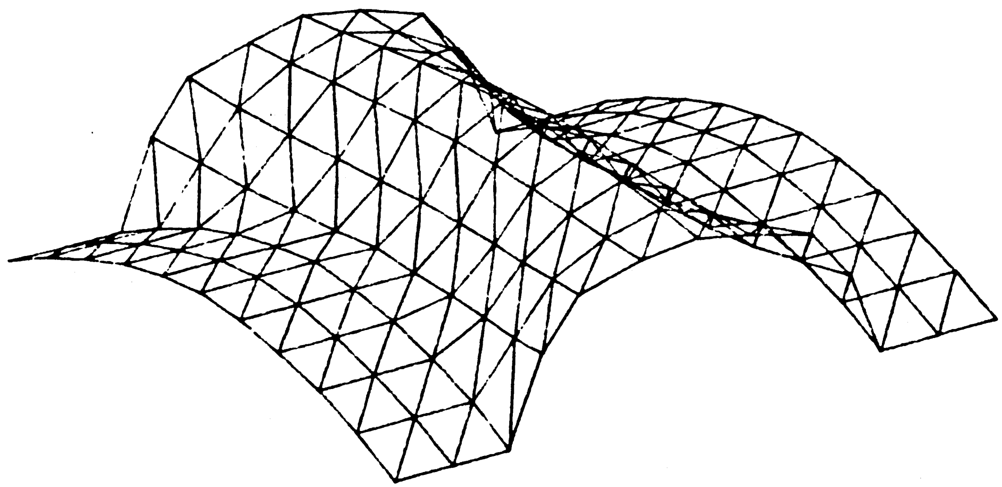

Figure 79. Deformed Wrinkle Shape (2" Roller)

RD=2"
WW=0.524"
TN=5 p/i
YM=300,000 psi
WA=63 degrees

WH=0.333"
TH=10 mils
PR=0.3
FC=inf

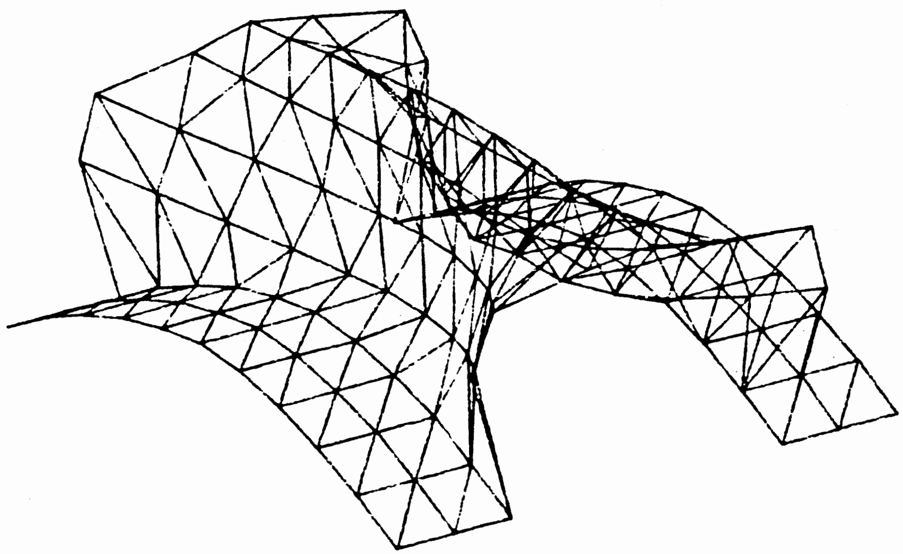

Figure 80. Deformed Wrinkle Shape (2" Roller)

Eigenvalue = 0.717

For Static Wrinkle Shape See Figure 35

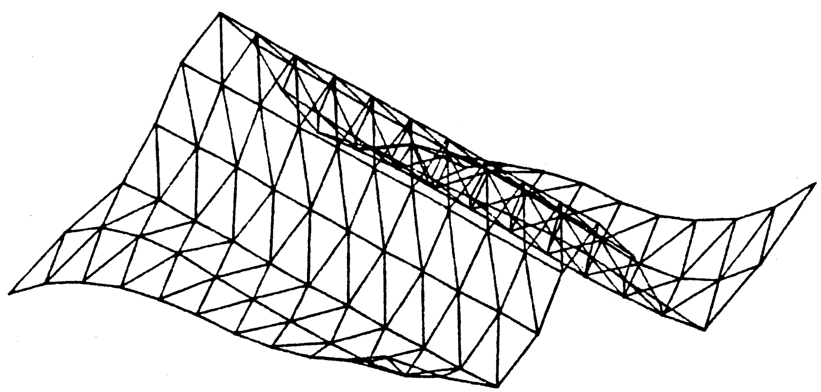

Figure 81. Buckled Wrinkle Shape

Eigenvalue = 0.717

For Static Wrinkle Shape See Figure 35

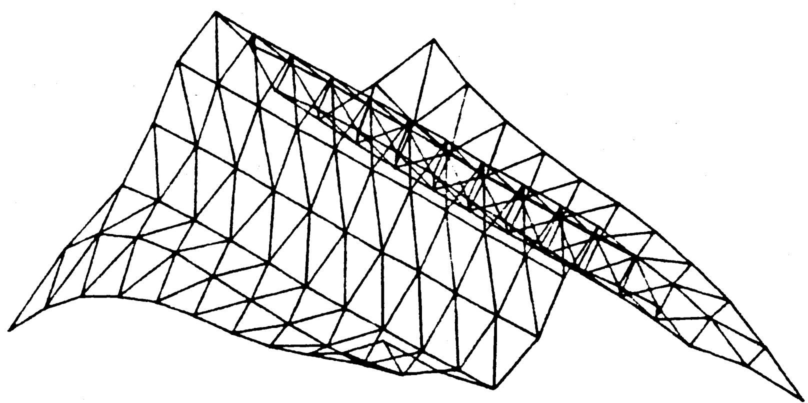

Figure 82. Buckled Wrinkle Shape

Eigenvalue = 0.783

For Static Wrinkle Shape See Figure 35

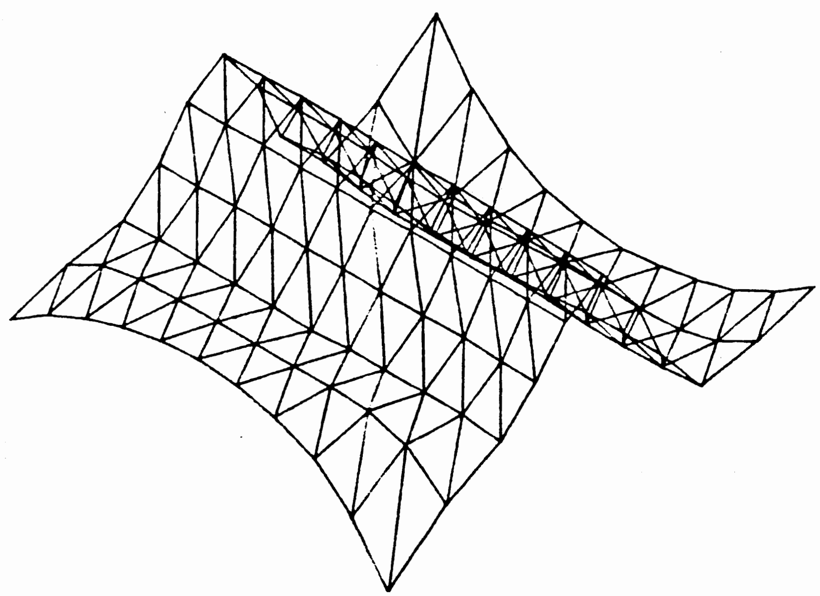

Figure 83. Buckled Wrinkle Shape

Eigenvalue = 0.872

For Static Wrinkle Shape See Figure 47

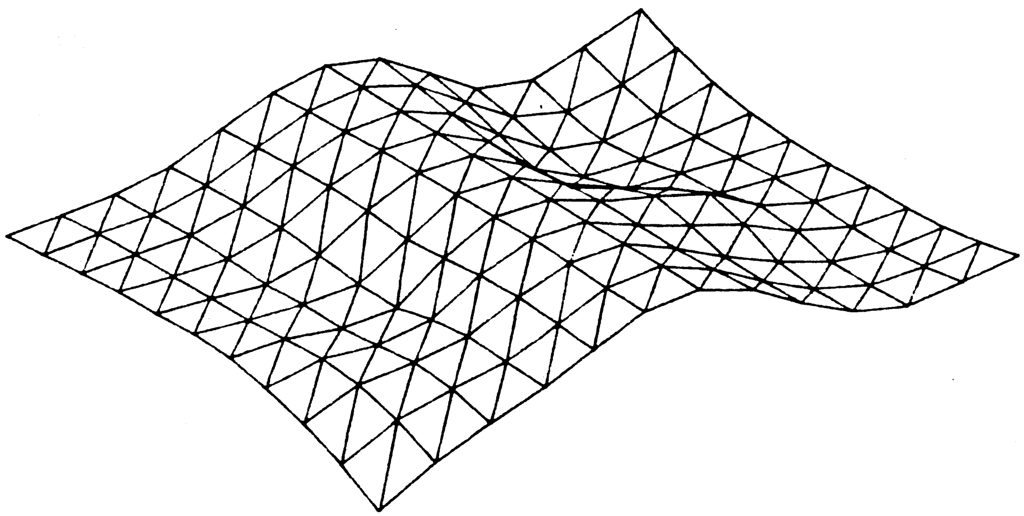

Figure 84. Buckled Wrinkle Shape

Eigenvalue = 0.884

For Static Wrinkle Shape See Figure 47

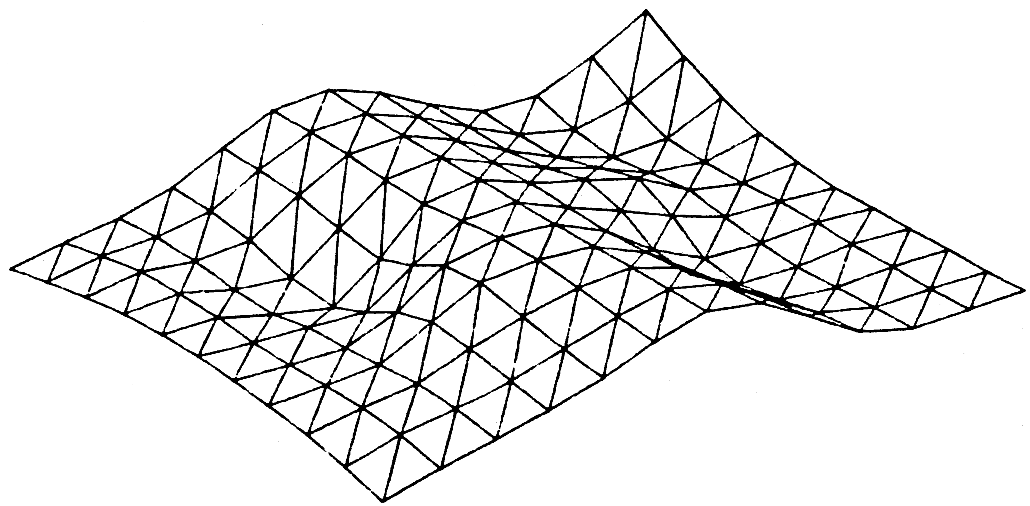

Figure 85. Buckled Wrinkle Shape

Eigenvalue = 0.904

For Static Wrinkle Shape See Figure 47

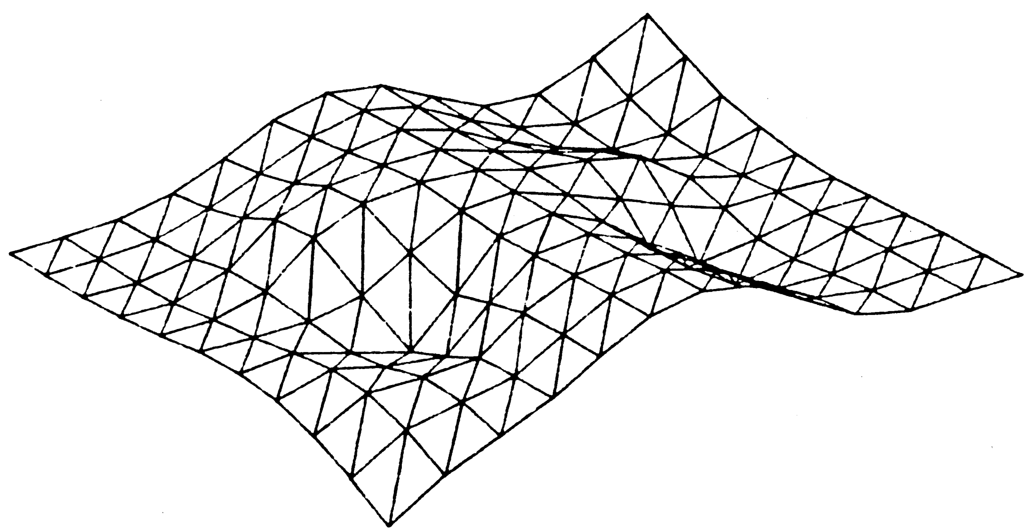

Figure 86. Buckled Wrinkle Shape

APPENDIX D

MATHEMATICAL MODEL

```
10 REM THIS PROGRAM IS WRITTEN IN MICROSOFT BASIC 3.2
20 REM THIS PROGRAM IS THE RESULT OF A WEB WRINKLE ANALYSIS
30 REM AND PROVIDES A MODEL FOR NINE WEB PARAMETERS
40 REM THE KNOWN PARAMETERS ENTERED ARE USED ONLY WHERE THAT
50 REM PARAMETER IS NEEDED FOR RATIOS INVOLVING UNKNOWN PARAMETERS
60 REM THE OUTPUT OF THE MODEL IS A RELATIVE STRESS NUMBER WHICH
70 REM MAY BE APPLIED TO THE KNOWN OR DESIGN STATE OF STRESS
80 CLS
90 PRINT:PRINT:PRINT:PRINT:PRINT:PRINT
100 PRINT TAB(23) "WEB WRINKLE STABILITY ANALYSIS"
110 PRINT TAB(32) "DEVELOPED BY"
120 PRINT TAB(30) "CRAIG  FRIEDRICH"
130 PRINT:PRINT
140 PRINT TAB(24) "WEB HANDLING RESEARCH CENTER"
150 PRINT TAB(16) "SCHOOL OF MECHANICAL & AEROSPACE ENGINEERING"
160 PRINT TAB(26) "OKLAHOMA STATE UNIVERSITY"
170 PRINT TAB(35) "1987"
180 FOR J = 1 TO 1000
190 X=1!*1!
200 NEXT J
210 CLS
220 PRINT:PRINT:PRINT:INPUT;"DO YOU WANT INSTRUCTIONS (Y/N) ";A$
230 IF A$="Y" THEN GOTO 4560
240 IF A$="N" THEN GOTO 260
250 GOTO 210
260 CLS
270 DIM P(11)
280 DIM RP(11)
290 PRINT:PRINT
300 PRINT "   THE FOLLOWING PARAMETERS ARE USED IN THIS PROGRAM"
310 PRINT:PRINT
320 PRINT "   1. YOUNG'S MODULUS  "
330 PRINT "   2. POISSON'S RATIO"
340 PRINT "   3. WRINKLE HEIGHT"
350 PRINT "   4. WRINKLE WIDTH"
360 PRINT "   5. ROLLER DIAMETER"
370 PRINT "   6. TENSION"
380 PRINT "   7. THICKNESS"
390 PRINT "   8. WRAP ANGLE"
400 PRINT "   9. FRICTION COEFFICIENT"
410 I=I+1
420 JJ=JJ+1
430 PRINT
440 INPUT;"   ENTER THE NUMBER OF YOUR UNKNOWN PARAMETER";Q(I)
450 IF Q(I)§1 OR  Q(I)¶9 THEN GOTO 260
460 PRINT
470 INPUT;"   ARE THERE ADDITIONAL UNKNOWN PARAMETER(S)? (Y/N)";A$
480 PRINT
490 IF A$ §¶ "Y" THEN IF A$ §¶ "N" THEN GOTO 470
500 IF A$ = "N" THEN GOTO 520
510 GOTO 410
520 CLS
530 Z$(1)="YOUNG'S MODULUS (PSI)"
```

```
540 Z$(2)="POISSON'S RATIO"
550 Z$(3)="WRINKLE HEIGHT (INCHES)"
560 Z$(4)="WRINKLE WIDTH (INCHES)"
570 Z$(5)="ROLLER DIAMETER (INCHES)"
580 Z$(6)="TENSION (POUNDS/INCH)"
590 Z$(7)="THICKNESS (MILS)"
600 Z$(8)="WRAP ANGLE (DEGREES)"
610 Z$(9)="FRICTION COEFFICIENT"
620 CLS
630 PRINT:PRINT "YOUR UNKNOWN PARAMETERS ARE : ":PRINT
640 FOR II=1 TO I
650 IF Q(II)=1 THEN PRINT Z$(1):INPUT;"     ENTER VALUE";E:PRINT
660 IF Q(II)=2 THEN PRINT Z$(2):INPUT;"     ENTER VALUE";NU:PRINT
670 IF Q(II)=3 THEN PRINT Z$(3):INPUT;"     ENTER VALUE";WH:PRINT
680 IF Q(II)=4 THEN PRINT Z$(4):INPUT;"     ENTER VALUE";WW:PRINT
690 IF Q(II)=5 THEN PRINT Z$(5):INPUT;"     ENTER VALUE";RD:PRINT
700 IF Q(II)=6 THEN PRINT Z$(6):INPUT;"     ENTER VALUE";TN:PRINT
710 IF Q(II)=7 THEN PRINT Z$(7):INPUT;"     ENTER VALUE";TH:PRINT
720 IF Q(II)=8 THEN PRINT Z$(8):INPUT;"     ENTER VALUE";WA:PRINT
730 IF Q(II)=9 THEN PRINT Z$(9):INPUT;"     ENTER VALUE";MU:PRINT
740 NEXT II
750 FOR J=1 TO 1000
760 X=1!*1!
770 NEXT J
780 FOR AA=1 TO JJ
790 IF Q(AA)=1 THEN GOTO 1060
800 NEXT AA
810 FOR BB=1 TO JJ
820 IF Q(BB)=2 THEN GOTO 1330
830 NEXT BB
840 FOR CC=1 TO JJ
850 IF Q(CC)=3 THEN GOTO 1560
860 NEXT CC
870 FOR DD=1 TO JJ
880 IF Q(DD)=4 THEN GOTO 1930
890 NEXT DD
900 FOR FF=1 TO JJ
910 IF Q(FF)=5 THEN GOTO 2260
920 NEXT FF
930 FOR GG=1 TO JJ
940 IF Q(GG)=6 THEN GOTO 2470
950 NEXT GG
960 FOR HH=1 TO JJ
970 IF Q(HH)=7 THEN GOTO 2890
980 NEXT HH
990 FOR LL=1 TO JJ
1000 IF Q(LL)=8 THEN GOTO 3310
1010 NEXT LL
1020 FOR MM=1 TO JJ
1030 IF Q(MM)=9 THEN GOTO 3620
1040 NEXT MM
1050 GOTO 3870
1060 REM YOUNG'S MODULUS RELATIVE STRESS CALCULATIONS
```

```
1070 E(1)=2000!
1080 E(2)=10000!
1090 E(3)=50000!
1100 E(4)=100000!
1110 E(5)=300000!
1120 E(6)=1000000!
1130 E(7)=30000000#
1140 RE(1)=.05
1150 RE(2)=.065
1160 RE(3)=.18
1170 RE(4)=.365
1180 RE(5)=1!
1190 RE(6)=2.685
1200 RE(7)=72.3
1210 CLS
1220 IF E § E(1) OR E ¶ E(7) THEN GOTO 1260
1230 FOR I=1 TO 6
1240 IF E¶=E(I) THEN IF E§=E(I+1) THEN GOTO 1300
1250 NEXT I
1260 PRINT:PRINT "YOUNG'S MODULUS VALUE IS OUT OF RANGE OF THE DATA !!"
1270 PRINT:PRINT
1280 INPUT;"ENTER VALUE 2,000 TO 30,000,000 PSI";E
1290 GOTO 1210
1300 RSE=((RE(I+1)-RE(I))/(E(I+1)-E(I)))*(E-E(I))+RE(I)
1310 RS(1)=RSE
1320 GOTO 810
1330 REM POISSON'S RATIO RELATIVE STRESS CALCULATIONS
1340 NU(1)=.01
1350 NU(2)=.05
1360 NU(3)=.1
1370 NU(4)=.3
1380 NU(5)=.5
1390 RNU(1)=1.04
1400 RNU(2)=1.03
1410 RNU(3)=1.01
1420 RNU(4)=1!
1430 RNU(5)=.99
1440 CLS
1450 IF NU §NU(1) OR NU ¶NU(5) THEN GOTO 1490
1460 FOR I=1 TO 4
1470 IF NU¶NU(I) THEN IF NU§=NU(I+1) THEN GOTO 1530
1480 NEXT I
1490 PRINT:PRINT "POISSON'S RATIO VALUE IS OUT OF RANGE OF THE DATA !!"
1500 PRINT:PRINT
1510 INPUT;"ENTER VALUE 0.01 TO 0.5";NU
1520 GOTO 1440
1530 RSNU=((RNU(I+1)-RNU(I))/(NU(I+1)-NU(I)))*(NU-NU(I))+RNU(I)
1540 RS(2)=RSNU
1550 GOTO 840
1560 REM WRINKLE HEIGHT RATIO RELATIVE STRESS CALCULATIONS
1570 IF RD §¶0! THEN GOTO 1610
1580 CLS
1590 INPUT;"ENTER ROLLER DIAMETER (REQUIRED) ! ";RD
```

```
1600 PRINT:PRINT
1610 HR=RD/WH
1620 HR(1)=3!
1630 HR(2)=4!
1640 HR(3)=6!
1650 HR(4)=12!
1660 HR(5)=24!
1670 HR(6)=40!
1680 HR(7)=80!
1690 RHR(1)=9.2
1700 RHR(2)=9.88
1710 RHR(3)=5.96
1720 RHR(4)=2.73
1730 RHR(5)=1.9
1740 RHR(6)=1.33
1750 RHR(7)=1!
1760 CLS
1770 IF HR§HR(1) THEN GOTO 1820
1780 FOR I=1 TO 6
1790 IF HR¶=HR(I) THEN IF HR§=HR(I+1) THEN GOTO 1880
1800 IF HR¶HR(7) THEN GOTO 1900
1810 NEXT I
1820 PRINT:PRINT "WRINKLE HEIGHT RATIO IS OUT OF RANGE OF DATA !!"
1830 PRINT:PRINT
1840 PRINT"WRINKLE HEIGHT RATIO IS ROLLER DIAMETER / WRINKLE HEIGHT"
1850 PRINT:PRINT
1860 INPUT;"ENTER VALUE GREATER THAN OR EQUAL TO 3";HR
1870 GOTO 1760
1880 RSHR=((RHR(I+1)-RHR(I))/(HR(I+1)-HR(I)))*(HR-HR(I))+RHR(I)
1890 GOTO 1910
1900 RSHR=RHR(7)-((.00825)*(HR-HR(7)))
1910 RS(3)=RSHR
1920 GOTO 870
1930 REM WRINKLE ASPECT RATIO RELATIVE STRESS CALCULATIONS
1940 IF WH§¶0! THEN GOTO 1980
1950 CLS
1960 INPUT;"ENTER WRINKLE HEIGHT (REQUIRED) !";WH
1970 PRINT:PRINT
1980 AR=WH/WW
1990 AR(1)=.048
2000 AR(2)=.095
2010 AR(3)=.159
2020 AR(4)=.319
2030 AR(5)=.636
2040 AR(6)=1.27
2050 RAR(1)=1!
2060 RAR(2)=1.3
2070 RAR(3)=1.9
2080 RAR(4)=2.8
2090 RAR(5)=4.8
2100 RAR(6)=9.2
2110 CLS
2120 IF AR§AR(1) OR AR¶AR(6) THEN GOTO 2160
```

```
2130 FOR I = 1 TO 5
2140 IF AR¶=AR(I) THEN IF AR§=AR(I+1) THEN GOTO 2220
2150 NEXT I
2160 PRINT:PRINT"WRINKLE ASPECT RATIO IS OUT OF RANGE OF THE DATA !!"
2170 PRINT:PRINT
2180 PRINT"WRINKLE ASPECT RATIO IS WRINKLE HEIGHT / WRINKLE WIDTH"
2190 PRINT:PRINT
2200 INPUT;"ENTER VALUE 0.048 TO 1.27";AR
2210 GOTO 2110
2220 RSAR=((RAR(I+1)-RAR(I))/(AR(I+1)-AR(I)))*(AR-AR(I))+RAR(I)
2230 IF AR¶.159 THEN RSAR=1!
2240 RS(4)=RSAR
2250 GOTO 900
2260 REM ROLLER DIAMETER RELATIVE STRESS CALCULATIONS
2270 RD(1)=2!
2280 RD(2)=4!
2290 RD(3)=8!
2300 RD(4)=24!
2310 RRD(1)=2.16
2320 RRD(2)=1.73
2330 RRD(3)=1.52
2340 RRD(4)=1!
2350 CLS
2360 IF RD§RD(1) OR RD¶RD(4) THEN GOTO 2400
2370 FOR I = 1 TO 3
2380 IF RD¶=RD(I) THEN IF RD§=RD(I+1) THEN GOTO 2440
2390 NEXT I
2400 PRINT:PRINT"ROLLER DIAMETER IS OUT OF RANGE OF THE DATA !!"
2410 PRINT:PRINT
2420 INPUT;"ENTER VALUE 2 TO 24";RD
2430 GOTO 2350
2440 RSRD=((RRD(I+1)-RRD(I))/(RD(I+1)-RD(I)))*(RD-RD(I))+RRD(I)
2450 RS(5)=RSRD
2460 GOTO 930
2470 REM TENSION (THICKNESS) RELATIVE STRESS CALCULATIONS
2480 IF TH§¶0! THEN GOTO 2520
2490 CLS
2500 INPUT;"ENTER THICKNESS IN MILS (REQUIRED) !";TH
2510 PRINT:PRINT
2520 P=TN/TH
2530 P(1)=.025
2540 P(2)=.1
2550 P(3)=.25
2560 P(4)=.5
2570 P(5)=1!
2580 P(6)=1.67
2590 P(7)=2!
2600 P(8)=5!
2610 P(9)=10!
2620 P(10)=20!
2630 P(11)=100!
2640 RP(1)=.78
2650 RP(2)=.79
```

```
2660 RP(3)=.985
2670 RP(4)=1.02
2680 RP(5)=1.08
2690 RP(6)=1!
2700 RP(7)=1.14
2710 RP(8)=1.455
2720 RP(9)=1.62
2730 RP(10)=2.19
2740 RP(11)=6.96
2750 CLS
2760 IF P§P(1) OR P¶P(11) THEN GOTO 2800
2770 FOR I = 1 TO 10
2780 IF P¶=P(I) THEN IF P§=P(I+1) THEN GOTO 2860
2790 NEXT I
2800 PRINT:PRINT"TENSION IS OUT OF RANGE OF THE DATA !!"
2810 PRINT:PRINT
2820 PRINT"TENSION IS IN POUNDS PER INCH WIDTH PER MIL THICKNESS"
2830 PRINT:PRINT
2840 INPUT;"ENTER VALUE 0.025 TO 100";P
2850 GOTO 2750
2860 RSP=((RP(I+1)-RP(I))/(P(I+1)-P(I)))*(P-P(I))+RP(I)
2870 RS(6)=RSP
2880 GOTO 960
2890 REM THICKNESS (TENSION) RELATIVE STRESS CALCULATIONS
2900 IF TN§¶0! THEN GOTO 2940
2910 CLS
2920 INPUT;"ENTER TENSION IN POUNDS (REQUIRED) !";TN
2930 PRINT:PRINT
2940 P=TN/TH
2950 P(1)=.025
2960 P(2)=.1
2970 P(3)=.25
2980 P(4)=.5
2990 P(5)=1!
3000 P(6)=1.67
3010 P(7)=2!
3020 P(8)=5!
3030 P(9)=10!
3040 P(10)=20!
3050 P(11)=100!
3060 RP(1)=.78
3070 RP(2)=.79
3080 RP(3)=.985
3090 RP(4)=1.02
3100 RP(5)=1.08
3110 RP(6)=1!
3120 RP(7)=1.14
3130 RP(8)=1.455
3140 RP(9)=1.62
3150 RP(10)=2.19
3160 RP(11)=6.96
3170 CLS
3180 IF P§P(1) OR P¶P(11) THEN GOTO 3220
```

```
3190 FOR I = 1 TO 10
3200 IF P¶=P(I) THEN IF P§=P(I+1) THEN GOTO 3280
3210 NEXT I
3220 PRINT:PRINT"TENSION IS OUT OF RANGE OF THE DATA !!"
3230 PRINT:PRINT
3240 PRINT"TENSION IS IN POUNDS PER INCH WIDTH PER MIL THICKNESS"
3250 PRINT:PRINT
3260 INPUT;"ENTER VALUE 0.025 TO 100";P
3270 GOTO 3170
3280 RSP=((RP(I+1)-RP(I))/(P(I+1)-P(I)))*(P-P(I))+RP(I)
3290 RS(7)=RSP
3300 GOTO 990
3310 REM WRAP ANGLE RELATIVE STRESS CALCULATIONS
3320 IF TH§¶0! THEN GOTO 3350
3330 CLS
3340 INPUT;"ENTER THICKNESS IN MILS (REQUIRED) !";TH
3350 PRINT:PRINT
3360 IF TN§¶0! THEN GOTO 3400
3370 CLS
3380 INPUT;"ENTER TENSION IN POUNDS (REQUIRED) !";TN
3390 PRINT:PRINT
3400 STR=TN/(TH/1000!)
3410 RSTR=STR/40216!
3420 WA(1)=0!
3430 WA(2)=60!
3440 WA(3)=120!
3450 WA(4)=180!
3460 RWA(1)=RSTR
3470 RWA(2)=1!
3480 RWA(3)=1.33
3490 RWA(4)=1.52
3500 CLS
3510 IF WA§WA(1) OR WA¶WA(4) THEN GOTO 3550
3520 FOR I = 1 TO 3
3530 IF WA¶=WA(I) THEN IF WA§=WA(I+1) THEN GOTO 3590
3540 NEXT I
3550 PRINT:PRINT"WRAP ANGLE IS OUT OF RANGE OF THE DATA !!"
3560 PRINT:PRINT
3570 INPUT;"ENTER VALUE 0 TO 180";WA
3580 GOTO 3500
3590 RSWA=((RWA(I+1)-RWA(I))/(WA(I+1)-WA(I)))*(WA-WA(I))+RWA(I)
3600 RS(8)=RSWA
3610 GOTO 1020
3620 REM FRICTION CALCULATIONS
3630 MU(1)=0!
3640 MU(2)=.05
3650 MU(3)=.15
3660 MU(4)=.25
3670 MU(5)=.5
3680 RMU(1)=.32
3690 RMU(2)=.87
3700 RMU(3)=.87
3710 RMU(4)=.89
```

```
3720 RMU(5)=.89
3730 CLS
3740 IF MU§MU(1) THEN GOTO 3780:IF MU¶MU(5) THEN GOTO 3840
3750 FOR I=1 TO 4
3760 IF MU¶=MU(I) THEN IF MU§=MU(I+1) THEN GOTO 3820
3770 NEXT I
3780 PRINT:PRINT"FRICTION COEFFICIENT IS OUT OF RANGE OF DATA !!"
3790 PRINT:PRINT
3800 INPUT;"ENTER VALUE EQUAL OR GREATER THAN 0.0";MU
3810 GOTO 3730
3820 RSMU=((RMU(I+1)-RMU(I))/(MU(I+1)-MU(I)))*(MU-MU(I))+RMU(I)
3830 GOTO 3850
3840 RSMU=((MU-MU(5))*.0571)+RMU(5)
3850 RS(9)=RSMU
3860 GOTO 1050
3870 REM PRINTOUT FINAL VALUES
3880 CLS
3890 FOR OO=1 TO JJ
3900 IF Q(OO)=1 THEN PRINT Z$(1) " RELATIVE STRESS FACTOR IS " RS(1)
3910 IF Q(OO)=2 THEN PRINT Z$(2) " RELATIVE STRESS FACTOR IS " RS(2)
3920 IF Q(OO)=3 THEN PRINT Z$(3) " RELATIVE STRESS FACTOR IS " RS(3)
3930 IF Q(OO)=4 THEN PRINT Z$(4) " RELATIVE STRESS FACTOR IS " RS(4)
3940 IF Q(OO)=5 THEN PRINT Z$(5) " RELATIVE STRESS FACTOR IS " RS(5)
3950 IF Q(OO)=6 THEN PRINT Z$(6) " RELATIVE STRESS FACTOR IS " RS(6)
3960 IF Q(OO)=7 THEN PRINT Z$(7) " RELATIVE STRESS FACTOR IS " RS(7)
3970 IF Q(OO)=9 THEN PRINT Z$(9) " RELATIVE STRESS FACTOR IS " RS(9)
3980 IF Q(OO)=8 THEN PRINT Z$(8) " RELATIVE STRESS FACTOR IS " RS(8)
3990 NEXT OO
4000 TRSF=1!
4010 FOR PP=1 TO JJ
4020 TRSF=TRSF*RS(Q(PP))
4030 NEXT PP
4040 PRINT:PRINT
4050 RSTRESS=TRSF*2900
4060 IF HR¶80 THEN GOTO 4080
4070 GOTO 4130
4080 PRINT"WRINKLE HEIGHT RATIO FACTOR WAS EXTRAPOLATED"
4090 PRINT"THIS MAY BE RESPONSIBLE FOR ANY UNUSUALLY LOW"
4100 PRINT"STRESS VALUES WHICH ARE PREDICTED"
4110 PRINT"THE SMALLEST WRINKLE HEIGHT RATIO RELATIVE STRESS FACTOR,"
4120 PRINT"FOR WHICH COMPUTATIONAL DATA WAS FOUND, IS 1.00"
4130 PRINT
4140 PRINT "AVERAGE STRESS IN WEB IS " RSTRESS
4150 PRINT "TOTAL RELATIVE STRESS FACTOR IS  " TRSF
4160 PRINT:PRINT
4170 PRINT:PRINT"THE FOLLOWING DATA IS NEEDED FOR STABILITY CRITERIA"
4180 PRINT:PRINT"ENTER THE CLOSEST VALUE OF THE WRINKLE ASPECT RATIO"
4190 INPUT;"0.048, 0.095, 0.159, 0.318, 0.636, 1.27 ";AR
4200 PRINT:PRINT"ENTER CLOSEST VALUE FOR THE FRICTIONAL COEFFICIENT"
4210 INPUT;"0.25, 0.50, 1.0 ";MU
4220 IF E§¶0! THEN GOTO 4240
4230 PRINT:PRINT:INPUT;"ENTER YOUNG'S MODULUS ";E
4240 IF TH§¶0! THEN GOTO 4260
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4250 PRINT:PRINT:INPUT;"ENTER THE THICKNESS (MILS) ";TH
4260 RIG=E*((TH/1000!)¢3)
4270 IF AR=.048 THEN IF MU=1! THEN IF RIG¶=.0003 THEN GOTO 4470
4280 IF AR=.048 THEN IF MU=.5 THEN IF RIG¶=.0003 THEN GOTO 4470
4290 IF AR=.048 THEN IF MU=.25 THEN IF RIG¶=.0003 THEN GOTO 4470
4300 IF AR=.095 THEN IF MU=1! THEN IF RIG¶=.00808 THEN GOTO 4470
4310 IF AR=.095 THEN IF MU=.5 THEN IF RIG¶=.00808 THEN GOTO 4470
4320 IF AR=.095 THEN IF MU=.25 THEN IF RIG¶=.00808 THEN GOTO 4470
4330 IF AR=.159 THEN IF MU=1! THEN IF RIG¶=.00808 THEN GOTO 4470
4340 IF AR=.159 THEN IF MU=.5 THEN IF RIG¶=.0003 THEN GOTO 4470
4350 IF AR=.159 THEN IF MU=.25 THEN IF RIG¶=.0003 THE GOTO 4470
4360 IF AR=.318 THEN IF MU=1! THEN IF RIG¶=.3 THEN GOTO 4470
4370 IF AR=.318 THEN IF MU=.5 THEN GOTO 4490
4380 IF AR=.318 THEN IF MU=.25 THEN GOTO 4490
4390 IF AR=.636 THEN IF MU=1! THEN IF RIG¶=.3 THEN GOTO 4470
4400 IF AR=.636 THEN IF MU=.5 THEN GOTO 4490
4410 IF AR=.636 THEN IF MU=.25 THEN GOTO 4490
4420 IF AR=1.27 THEN IF MU=1! THEN IF RIG¶= .3 THEN GOTO 4470
4430 IF AR=1.27 THEN IF MU=.5 THEN  GOTO 4490
4440 IF AR=1.27 THEN IF MU=.25 THEN GOTO 4490
4450 PRINT:PRINT"WRINKLE WILL PROBABLY HAVE AN UNDESIRABLE SHAPE"
4460 GOTO 4510
4470 PRINT:PRINT"WRINKLE WILL PROBABLY BE STABLE"
4480 GOTO 4510
4490 PRINT:PRINT:PRINT"THE DATA IS INCONCLUSIVE FOR THIS COMBINATION"
4500 GOTO 4510
4510 PRINT:PRINT:INPUT"DO YOU WISH TO RUN THE PROGRAM AGAIN (Y/N) ";A$
4520 IF A$="N" THEN GOTO 5040
4530 IF A$="Y" THEN CLEAR: GOTO 260
4540 GOTO 4510
4550 PRINT:PRINT"E N D   O F   P R O G R A M "
4560 REM INSTRUCTIONS
4570 CLS:PRINT:PRINT:PRINT
4580 PRINT"THIS PROGRAM IS COMPOSED OF TWO SECTIONS.  SECTION ONE"
4590 PRINT"ASKS FOR VALUES OF VARIABLE PARAMETERS AND CALCULATES A"
4600 PRINT"RELATIVE STRESS FACTOR TO BE MULTIPLIED BY THE STRESS"
4610 PRINT"ARISING FROM THE BASIC COMBINATION OF PARAMETERS.  THAT"
4620 PRINT"COMBINATION IS"
4630 PRINT"   YOUNG'S MODULUS = 300000 PSI"
4640 PRINT"   POISSON'S RATIO = 0.3"
4650 PRINT"   WRINKLE HEIGHT = 0.3 INCHES"
4660 PRINT"   WRINKLE WIDTH = 6.25 INCHES"
4670 PRINT"   ROLLER DIAMETER = 24 INCHES"
4680 PRINT"   TENSION = 5 POUNDS PER INCH OF WIDTH"
4690 PRINT"   THICKNESS = 3 MILS"
4700 PRINT"   WRAP ANGLE = 60 DEGREES"
4710 PRINT"   FRICTION COEFFICIENT = INFINITY"
4720 PRINT"THE AVERAGE STRESS UNDER THIS SET OF PARAMETERS IS"
4730 PRINT"2900 PSI.  THE RELATIVE STRESS FACTOR COMPUTED SHOULD"
4740 PRINT"BE MULTIPLIED BY THIS STRESS TO PREDICT THE STRESS WITH"
4750 PRINT"THE VARIABLE SET OF PARAMTERS.  THE RELATIVE STRESS"
4760 PRINT"FACTOR MAY ALSO BE USED BY ITSELF TO PREDICT THE"
4770 PRINT"CHANGE IN STRESS DUE TO VARIABLE PARAMETERS"
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4780 PRINT:PRINT:INPUT;"HIT CARRIAGE RETURN TO CONTINUE";A$
4790 CLS
4800 PRINT:PRINT
4810 PRINT"I M P O R T A N T ! ! ! !"
4820 PRINT:PRINT:PRINT
4830 PRINT"TO AVOID COMPUTATIONAL ERROR DUE TO THE REDUNDANT"
4840 PRINT"USE OF RELATIVE STRESS FACTORS, DO NOT USE"
4850 PRINT"THICKNESS AS A VARIABLE IF TENSION"
4860 PRINT"IS USED.  USE OF THESE VARIABLE COMBINATIONS WILL CAUSE"
4870 PRINT"FACTORS TO BE APPLIED TWICE !!!"
4880 PRINT:PRINT:PRINT
4890 PRINT"IF VARIABLE VALUES OUTSIDE THE RANGE OF THE COMPUTATIONAL"
4900 PRINT"DATA IN THE THESIS ARE USED, ERROR DUE TO EXTRAPOLATION"
4910 PRINT"WILL BE PRESENT.  THIS SHOULD BE LOOKED FOR IF UNUSUAL"
4920 PRINT"STRESS VALUES ARE PREDICTED."
4930 PRINT:PRINT:INPUT;"HIT CARRIAGE RETURN TO CONTINUE";A$
4940 CLS
4950 PRINT:PRINT
4960 PRINT"THE SECOND PART OF THE PROGRAM SEES IF THE DEFORMED"
4970 PRINT"WRINKLE SHAPE IS ACCEPTABLE, CALCULATED FROM YOUR"
4980 PRINT"SET OF PARAMETERS.  THIS RESULT SHOULD BE USED"
4990 PRINT"CONSERVATIVELY AND APPLIES TO WRINKLES ENTERING"
5000 PRINT"AND EXITING A ROLLER (NOT FOR WINDING APPLICATIONS)."
5010 PRINT:PRINT:PRINT:PRINT:PRINT
5020 INPUT;"HIT CARRIAGE RETURN TO BEGIN THE PROGRAM";A$
5030 GOTO 260
5040 PRINT:PRINT"E N D  O F  P R O G R A M "
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NOTE: BECAUSE OF PRINTER LIMITATIONS THE FOLLOWING SYMBOLS SHOULD BE INTERPRETED,

§ 'LESS THAN'
¶ 'GREATER THAN'
§¶ 'NOT EQUAL'

VITA

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