# MEASUREMENT AND SCALING OF HYDRAULIC PROPERTIES OF POROUS MEDIA

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1985

Submitted to the Faculty of the
Graduate College of the
Oklahoma State University
in partial fulfillment of
the requirements for
the Degree of
MASTER OF SCIENCE
May, 1989

# MEASUREMENT AND SCALING OF HYDRAULIC PROPERTIES OF POROUS MEDIA

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# PREFACE

In this study a method was developed to describe the spatial variability of the hydraulic conductivity and retention functions of soil. In order to develop this method experimentally determined conductivity and retention functions were obtained for a selected Teller soil. The scaling method developed in this study is based on the fact that both the conductivity and retention functions can be shown to fit a power law function. The exponent of the power law function can then be used to scale the hydraulic functions of the soil. This method was observed to work well for both the experimental data obtained for the Teller soil and other published data.

I would like to express my deepest gratitude to my adviser, Dr. Jan Wagner for his suggestions, encouragement and patience during the course of this study. I would also like to thank the other members of my committee, Dr. Brian J. Carter and Dr. A. H. Johannes for their suggestions and criticism during the course of this work.

Special thanks are due Phillip Ward for his aid in obtaining the particle-size analysis data. Additional thanks are due Dr. Carter for his aid in locating a suitable soil to sample for this study and the use of his laboratory for determining the particle-size analysis

data.

I would also to like thank my friend and colleagues Kris Ravi, Ed Zecchini and Ali Zare for their friendship and aid in completing this work.

I would like to thank my parents Frank and Eula Funk for their constant encouragement and support during the course of this work.

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# CHAPTER I

# INTRODUCTION

The problem of describing the movement of pollutants carried by the flow of water through unsaturated soil and into the groundwater is important in the assessment of groundwater quality. The flow phenomenon encountered in this case is the one of the flow of immiscible fluids through porous media. The relationship describing this phenomena is a nonlinear differential equation obtained by combining Darcy's Law with the equation of continuity for porous media. The solution of this equation requires the knowledge of the functional relationships between the hydraulic properties of the porous media. These hydraulic properties are unsaturated hydraulic conductivity, relative saturation and capillary pressure.

In soils these properties are spatially variable, because soil is a heterogeneous media. It is necessary to characterize this variability before a description of unsaturated flow through soil can be obtained.

One method of describing this spatial variability could be the extension of "similar" media concepts to scale these hydraulic properties. "Similar" media concepts are generally based upon the concept that two media are "similar" if they meet a defined criterion of similitude. By using this criterion and the flow equations, reduced or scaled hydraulic properties can be defined. If the criterion of similitude is met, then the scaled hydraulic functions for different media can be represented by a single function.

These "similar" media concepts can be shown to apply only when the hydraulic functions are represented graphically by parallel curves. This is not often the case, and in this thesis, a method is presented which allows for the scaling of hydraulic functions when they are not parallel. This is achieved by using a defined pore size distribution index as a scaling parameter.

Because there is a limited amount of data available for the description of soil spatial variability, an experimental method was developed to obtain unsaturated hydraulic conductivity and retention function data for the same "undisturbed" soil core. This method was then used to obtain the hydraulic properties of the Teller soil series at two different locations and four different depths at each location. In addition, particle size distribution data was taken on each of the samples. This data was then used to evaluate the different scaling procedures.

Due to the difficulty in obtaining unsaturated hydraulic conductivity data, several methods of calculating unsaturated hydraulic conductivity from retention function data have been proposed. Some of these methods are evaluated using the data obtained in this study. Finally, it has been reported that the pore size distribution index is a function of the particle size distribution, this hypothesis is also evaluated in this thesis.

# CHAPTER II

## A REVIEW OF THE LITERATURE AND FUNDAMENTAL THEORY

### Fundamental Theory

The equations governing the isothermal flow of two immiscible fluids through isotropic porous media can be developed following the approach given by Schiedeggar (1974). First, Darcy's Law is assumed to be valid for both the wetting and nonwetting fluid phases. The wetting fluid phase is defined to be the fluid phase which has a contact angle of less than 90° with the solid phase. Darcy's Law written for both fluids is,

$$q_w = -k(k_w/\mu_w)(\text{grad } p_w - g\rho_w) \tag{2-1}$$

$$q_{nw} = -k(k_{nw}/\mu_{nw})(\text{grad } p_{nw} - g\rho_{nw}) \tag{2-2}$$

in which $q_i$ is the seepage velocity, $k_i$ is the relative permeability, $\mu_i$ is the viscosity, $\rho_i$ is the density and $p_i$ is the pressure of phase i, k is the intrinsic permeability of the porous media and g is the acceleration due to gravity. Next, the equations of continuity for each fluid phase are,

$$-\phi \ \delta(\rho_w S_w)/\delta t = \text{div } (\rho_w q_w) \tag{2-3}$$

$$-\phi \ \delta(\rho_{nw} S_{nw})/\delta t = \text{div } (\rho_{nw} q_{nw}) \tag{2-4}$$

in which t is time, $\phi$ is the porosity of the porous media and $S_i$ is the fractional saturation of each phase. In addition, it is necessary to know how the density of each phase depends on pressure, so

$$\rho_w = \rho_w(p_w) \tag{2-5}$$

$$\rho_{nw} = \rho_{nw}(p_{nw}) . \tag{2-6}$$

The fractional saturations must also sum to one, or

$$S_w + S_{nw} = 1 \tag{2-7}$$

Finally, for immiscible fluid phases, the interfacial tension between the two phases creates a pressure discontinuity at the interface between the two phases. The difference between the two pressures is the capillary pressure ($p_c$) and is a function of saturation so that

$$p_c(S_w) = p_{nw} - p_w . \tag{2-8}$$

These eight equations describe the isothermal flow of two immiscible fluids through an isotropic porous media.

A special case of immiscible flow through porous media occurs when the flow of only one phase is considered and the other phase is a gas with a negligible pressure gradient. This case is encountered when the flow of water and its constituents through soil is considered. For these conditions, water is the wetting fluid, and air is the nonwetting fluid. In developing the equations describing this phenomena, the terms pertaining to the nonwetting phase, in the previously presented equations, are dropped. The pressure of the nonwetting phase is set equal to zero. After dropping the index w and defining the effective permeability $k_e$ by

$$k_e = k_i k. \tag{2-9}$$

The following relationships are obtained:

$$q = k_e/\mu \ (\text{grad } p - g\rho), \tag{2-10}$$

$$\phi \ \delta S/\delta t = - \text{div}(\rho q), \tag{2-11}$$

$$p = p_c(S). \tag{2-12}$$

If the density of water is assumed to be constant, then

$$q = k_e/\mu \ (\text{grad } p_c - g\rho), \tag{2-13}$$

and

$$\phi \ \delta S/\delta t = - \text{div } q. \tag{2-14}$$

By combining Equations (2-13) and (2-14),

$$\phi \ \delta S/\delta t = \text{div } [ \ k_e/\mu \ (\text{grad } p_c - g\rho)]. \tag{2-15}$$

This is a nonlinear differential equation which describes the unsaturated flow of water through a porous media.

In soil physics it is common to define the hydraulic conductivity, K, as,

$$K(S) = k_e \rho g/\mu. \tag{2-16}$$

Soil physicists usually work in terms of the soil flux density (Q), which is related to the seepage velocity by

$$Q = q/\phi. \qquad (2\text{-}17)$$

Darcy's law is usually written as

$$Q = K(S) \text{ grad } \Phi, \qquad (2\text{-}18)$$

where $\Phi$ is the total potential of the fluid defined by

$$\Phi = \psi - z = p_c/\rho g - z, \qquad (2\text{-}19)$$

in which $\psi$ is the matric potential and z is the elevation potential. The equation of continuity is then

$$\delta S/\delta t = \text{div } Q. \qquad (2\text{-}20)$$

Combining the equation of continuity with Darcy's law, as before, one obtains

$$\delta S/\delta t = \text{div}(K \text{ grad } \psi) - \delta K/\delta Z. \qquad (2\text{-}21)$$

Now if, the relationships between K, S and $\psi$ are single valued, no hysteresis, Equation (2-21) can be written as:

$$\delta S/\delta t = \text{div } [K\ d\psi/dS\ \delta S/\delta Z] - dK/dS\ \delta\psi/\delta Z. \qquad (2\text{-}22)$$

If instead of S, $\psi$ is the independent variable, then

$$\delta\psi/\delta t\ \delta S/\delta\psi = \text{div } [K\ \delta\psi/\delta Z] - dK/d\psi\ \delta\psi/\delta Z. \qquad (2\text{-}23)$$

This is equivalent to the equation originally presented by Richards (1931) and is known as the Richards' Equation for flow through porous media.

To solve this nonlinear differential equation, it is necessary to know the relationships between hydraulic conductivity (permeability), capillary pressure (matric potential) and saturation.

In this work the relationship between capillary pressure and saturation, $p_c(S)$, will be referred to as the retention function. Typical retention functions for a few different soils are shown in Figure 1. The relationships between conductivity and saturation, $K(S)$, or capillary pressure, $K(p_c)$, will be known as conductivity functions. Examples of these functions are shown in Figures 2 and 3. Methods of obtaining these relationships will be discussed in Chapter IV.

Unfortunately in soils these hydraulic functions exhibit spatial variability, due to the fact that soil is a heterogeneous not a homogeneous media. Therefore it is necessary to develop methods to account for this variability before one can describe the movement of water and its constituents in soils.

## Similar Media Concepts

Several researchers have proposed methods of scaling or correlating hydraulic properties of porous media (Leverett, 1941; Miller and Miller, 1956; and Corey and Corey, 1966). These methods are based on "similar" media concepts and each method has a different criterion for defining a "similar" media.

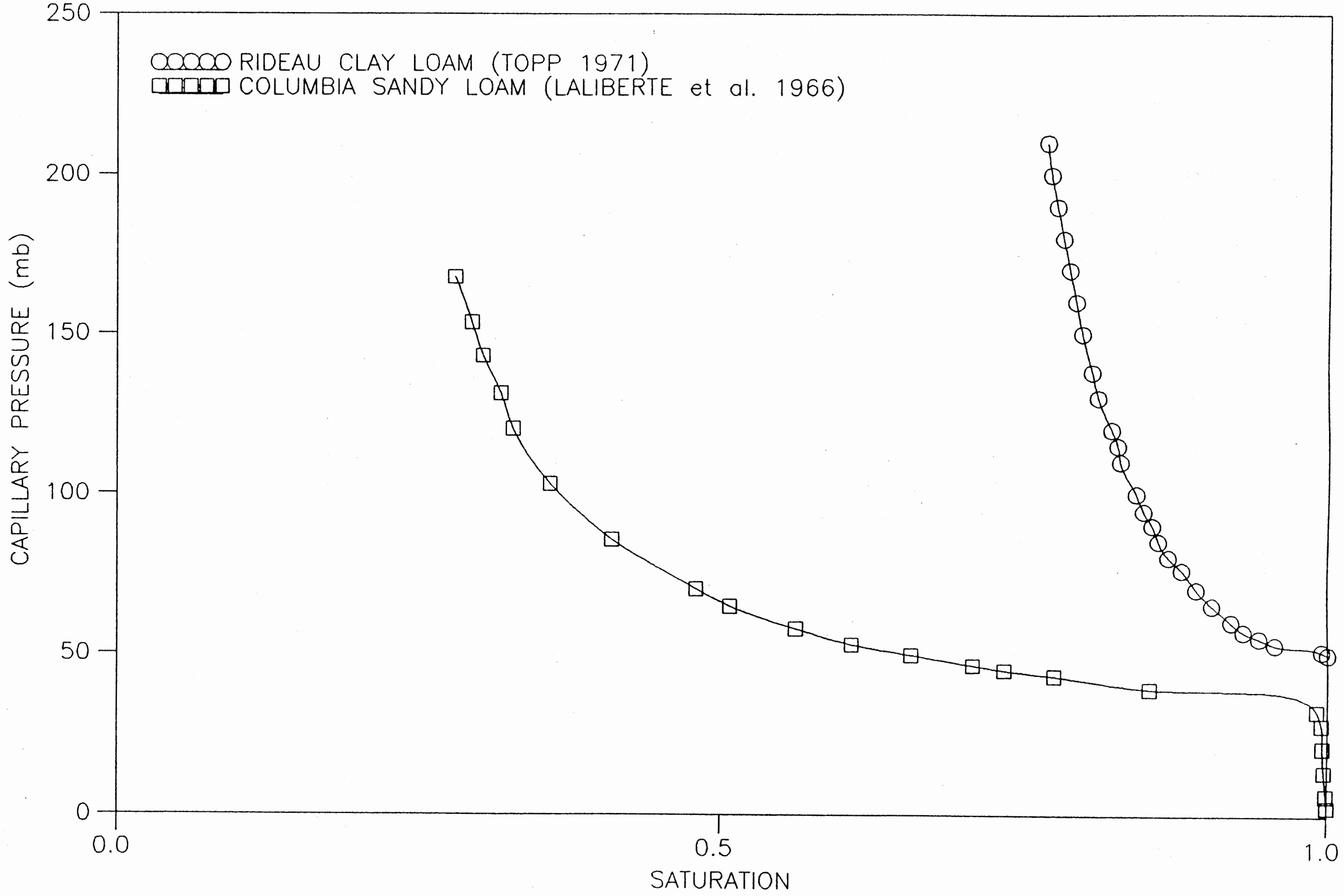

FIGURE 1. Typical Retention Functions

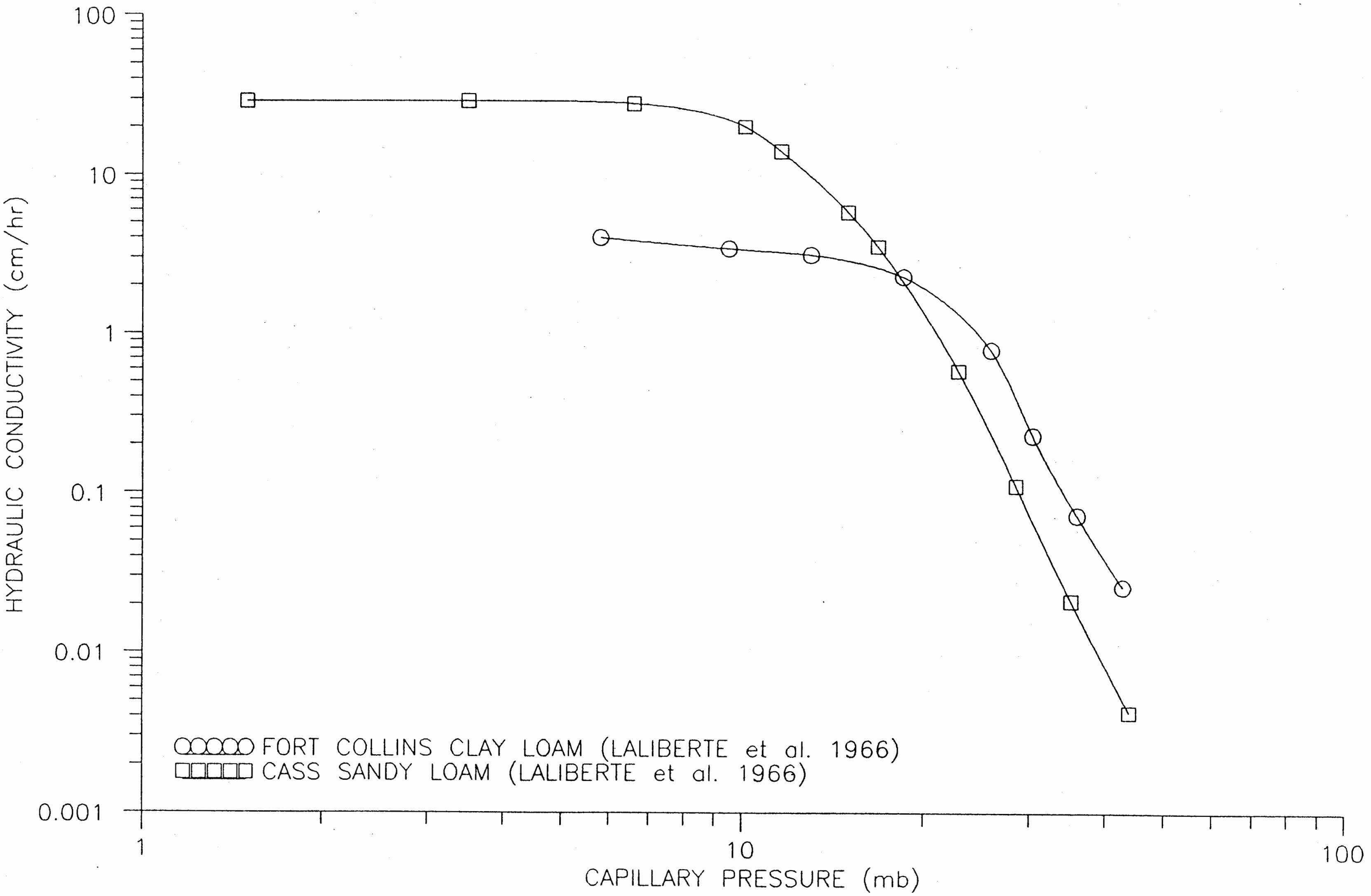

FIGURE 2. Typical Conductivity Functions

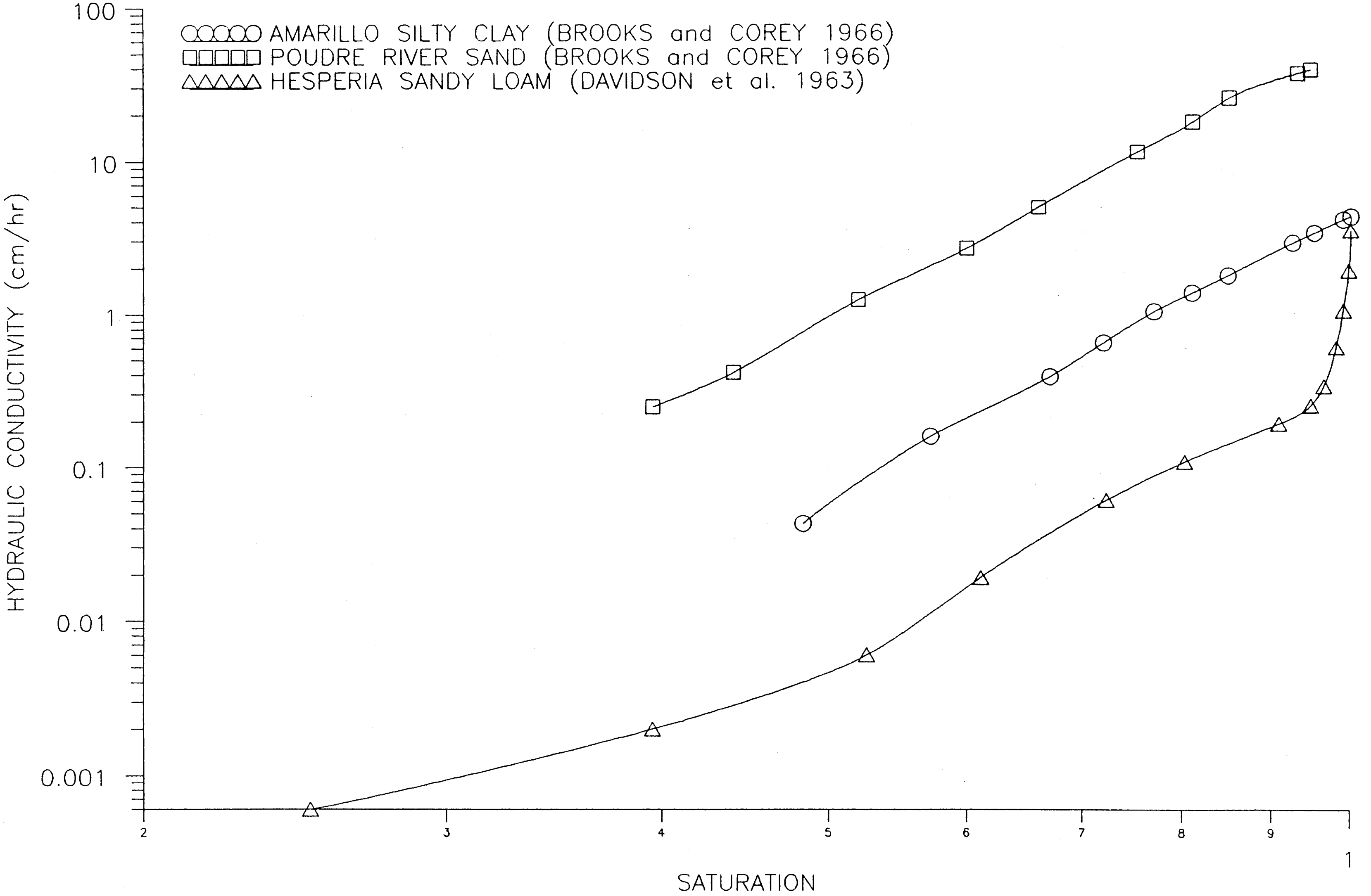

FIGURE 3. Typical Conductivity Functions

The first of these theories was developed by Leverett (1941). In the development of this theory, it is noted that the equation for capillary pressure in a capillary is

$$p_c = 2\sigma\cos\theta/r. \tag{2-24}$$

Since $p_c$ is a function of saturation,

$$p_c(S)r/2\sigma\cos\theta = f(S), \tag{2-25}$$

in which $\sigma$ is interfacial tension, $\theta$ is the contact angle and r is the radius of the capillary. If the pores in the porous media are thought of as capillaries, then r can be thought of as a kind of characteristic length of the porous media.

If a relationship could be found which adequately represents r as a function of measurable physical properties of the media, this relationship could be substituted for r in Equation (2-25). Then, when f(S) is plotted as a function of S, different retention curves for different porous media would coalesce to a single curve.

Leverett found an approximate relationship for r by modeling porous media as a capillary tube bundle. In this case a form of the Hagen-Pouisielle equation is applicable If the Hagen-Pouisielle equation is then substituted into Darcy's law it can be shown that

$$r \propto (K_s/\phi)^{1/2}. \tag{2-26}$$

After substituting this relationship into Equation (2-25), the Leverett J-function is obtained as

$$J(S) = (p_c/\sigma\cos\theta)(K_s/\phi)^{1/2}. \qquad (2\text{-}27)$$

Usually the contact angle term is dropped, and

$$J(S) = (p_c/\sigma)(K_s/\phi)^{1/2} \qquad (2\text{-}28)$$

is the form seen in the literature.

Leverett noted that this theory worked well for clean sands but did not work well when the clay content of the sand was increased.

Another theory was proposed by Miller and Miller (1955) which they call similitude analysis. Their theory is based upon the criterion that different porous media are "similar" if their solid geometries differ only by a constant magnifying factor. They expressed this magnifying factor in terms of a characteristic length ($\lambda$). Using this characteristic length and the flow equations for porous media, they derived the following relationships for the scaled conductivity and retention functions:

$$p_{c\cdot}(S) \equiv \lambda p_c(S)/\sigma, \qquad (2\text{-}29)$$

$$K_{\cdot}(S) \equiv \mu K(S)/\lambda^2. \qquad (2\text{-}30)$$

Where $p_{c\cdot}(S)$ and $K_{\cdot}(S)$ denote scaled functions.

The first tests of this theory (Miller and Miller, 1955; and Klute and Wilkinson, 1958) were performed on sand sieved to different particle size fractions. The characteristic length, $\lambda$, was defined to be the average particle size of each fraction. The results of these studies demonstrated that different retention and conductivity functions did coalesce to a single curve when the scaled functions were plotted.

The problem with this theory is that there is no way to determine the characteristic length in soils. In the next section the procedure used to extend this theory to the description of spatial variability in soils will be discussed.

Another theory was proposed by Corey and Corey (1967). In the development of this theory it was observed that a large amount of experimental data for retention functions fit the following form:

$$S_e = (p_b/p_c)^m \quad \text{for } p_c \geqslant p_b \qquad (2\text{-}31)$$

where $S_e$, the effective saturation, is defined by

$$S_e = (S - S_r)/(1 - S_r). \qquad (2\text{-}32)$$

The parameters $p_b$ and m are implicitly defined by Equation (2-31), which is the equation of a straight line with a slope of -m and an intercept of $m\ln(p_b)$. Brooks and Corey (1964) also noted that the parameter $p_b$ should correspond to the bubbling, or air entry pressure, of the porous media. In addition, they proposed that the parameter m should be a measure of the pore distribution within the porous media and defined it to be a pore size distribution index. The parameter $S_r$ in Equation (2-32) is the residual saturation. Sr is calculated by choosing its value such that when $S_e$ is calculated and plotted versus $p_c$ the best straight line is obtained. Figure 4 illustrates the physical significance of the parameters $p_b$ and $S_r$. From this figure it is seen that $p_b$ is the pressure at which significant desaturation begins, and $S_r$ is the saturation at which increases in $p_c$ causes no further desaturation.

Brooks and Corey (1964) also reported the following power law relationships for the conductivity function:

$$K(S_e) = K_s \, S_e^{\,e}, \qquad (2\text{-}33)$$

and

$$K(p_c) = K_s \, (p_b/p_c)^n \quad \text{for } p_c \geq p_b. \qquad (2\text{-}34)$$

The parameters e and n can be thought of as pore size distribution indices. Brooks and Corey (1964) developed the following relationship for m, n and e:

$$n = em = 2 + 3m. \qquad (2\text{-}35)$$

Using these relationships and Richards' Equation, Corey and Corey (1967) developed the following criterion for "similar" media: two porous media were similar if they have identical pore size distribution indices, $m_1 = m_2$. From Equation (2-35) this implies that, $n_1 = n_2$ and $e_1 = e_2$. Graphically, this means that when plotted on a logarithmic scale, the conductivity functions must be parallel. The retention functions within in the range $S_e \geq S_r$ and $p_c \geq p_b$ must also be parallel.

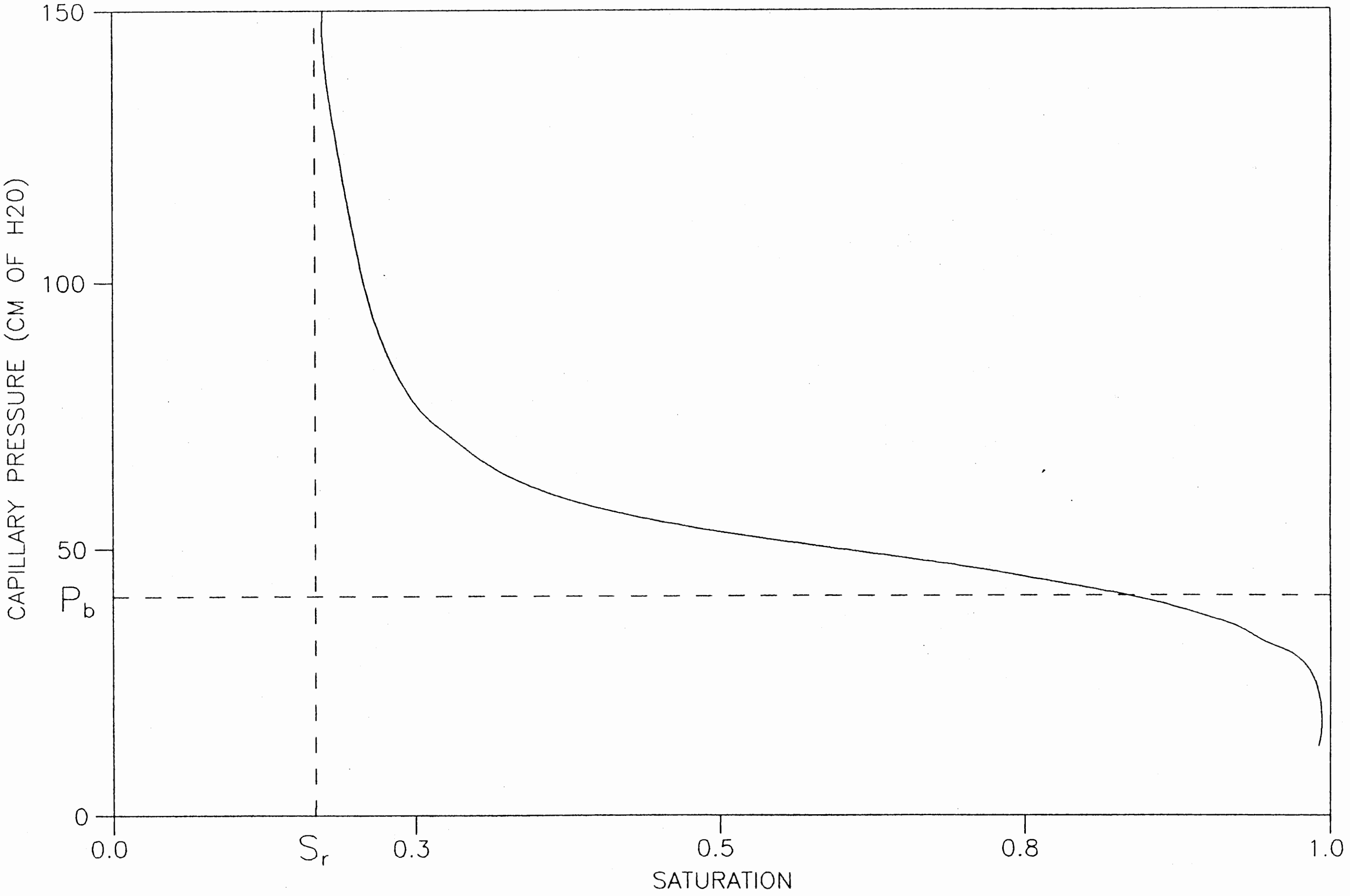

FIGURE 4. Corey and Corey (1966) Scaling Paremeters

Corey and Corey (1967) also defined the following scaled variables:

$$p_{\cdot} = p_c/p_b \qquad (2\text{-}36)$$

$$K_{\cdot} = K/K_s \qquad (2\text{-}37)$$

$$S_{\cdot} = S_e. \qquad (2\text{-}38)$$

If the previously mentioned criterion for similitude is met, then plots of these scaled variables will result in a single curve.

## Extension of Similar Media Concepts to Describe Soil Spatial Variability

Of the three presented similar media concepts, only the method presented by Miller and Miller (1955) has been extended for use in the description of soil spatial variability. Following the approach given by Warrick and Nielson (1980), this concept is extended for use in describing soil spatial variability by defining a scaling factor, $\alpha_r$, by

$$\alpha_r = \lambda_r/\lambda_m \qquad (2\text{-}39)$$

where $\lambda_r$ is the characteristic length corresponding to location r, and $\lambda_m$ is a mean characteristic length. Using this scaling factor, the scaled hydraulic properties are

$$p_{c\cdot} = p_c\alpha_r \qquad (2\text{-}40)$$

and

$$K_{\cdot} = K/\alpha_r^2. \qquad (2\text{-}41)$$

These scaling factors are determined by using a statistical "best" fit method. A value for $\alpha_r$ is chosen such that when substituted into Equations (2-40) and (2-41), the data for site r to best coalesce to a mean hydraulic function. This mean hydraulic function is either defined to be the data from one of the sites sampled in the particular study or the mean function obtained by fitting the combined data of all the samples.

This procedure has been employed by several researchers with varying degrees of success. Reichardt et al. (1972) observed that this procedure could not describe the variability in the conductivity and retention functions of different soil types. Therefore, later attempts to scale soil properties have been restricted to specific soil types.

Several researchers have noted that the values of $\alpha_r$ determined from the retention function are different from the values obtained from the conductivity function. Warrick et al. (1977), Simmons et al. (1979), and Russo and Bresler (1980) did report a high degree of correlation between the values obtained by each of the procedures. Rao et al. (1983) reported that there was no correlation between the two scaling factors.

These observations led Sposito and Jury (1985) to define two types similarity. The first, they called macroscopic Miller similitude, which applies when the scaling factors obtained from the retention and conductivity functions are the same. The second, which they called Warrick similitude, occurs when the two scaling factor are different and leads to the following definitions of scaled variables:

$$p_{c\cdot} = \alpha_r p_c, \qquad (2\text{-}42)$$

$$K. = K/\omega_r^2. \qquad (2\text{-}43)$$

In which $\omega_r$ is the scaling factor obtained from the conductivity function. There may or may not be a correlation between the two scaling factors.

Recently, Jury et al. (1987) proposed scaled variables defined as

$$p_{c\cdot} = \alpha_r p_c \qquad (2\text{-}44)$$

and

$$K. = K/\alpha_r^n, \qquad (2\text{-}45)$$

where n is the pore size distribution index defined by Brooks and Corey. They obtained a better description of soil spatial variability, than by using the standard macroscopic Miller scaled properties.

## Summary

One must be able to characterize the spatial variability of the hydraulic functions of soils before an accurate description of water movement through soils can be obtained. It has been shown that the Miller and Miller similar media concept can be used to account for some of this spatial variability in the soil hydraulic properties. No attempt has been made to extend the similar media concepts of Leverett and Corey and Corey to the description of soil spatial variability. No standard criterion has been developed which allows for the extension of Miller and Miller similitude to the description of variability in a particular data set. Finally, no alternative has been developed for use in describing soil spatial variability when this scaling procedure fails.

From this literature review it is apparent that the following areas need to be addressed. First, can the similar media concepts of Leverett and Corey and Corey be extended for use in describing soil spatial variability? Second, what are the criteria which allow for the description of soil spatial variability by similar media concepts? Finally, a method for describing soil spatial variability needs to be developed which will work when similar media concepts fail.

# CHAPTER III

# METHODS OF OBTAINING THE CONDUCTIVITY AND RETENTION FUNCTIONS

## Methods of Obtaining the Retention Function

The most common method of obtaining the retention function is the pressure cell method. In this method, a sample of the porous media is placed in contact with another saturated porous medium which possesses an air entry pressure larger than the maximum capillary pressure imposed during the experiment (Corey 1986). The porous medium under the sample is usually a ceramic plate or plastic membrane which has the desired air entry pressure.

The sample and porous barrier are then put in a pressure cell as shown in Figure 5. The sample is surrounded by the non-wetting phase at a controlled pressure. The wetting phase is in contact with the saturated porous barrier at a controlled pressure. In most cases at the start of the experiment the sample is saturated with the wetting phase. The pressure of both the wetting and non-wetting phases is set to atmospheric pressure, so that $p_c = 0$. Next, a positive capillary pressure is induced upon the sample by either raising the pressure of the non-wetting phase or by lowering the

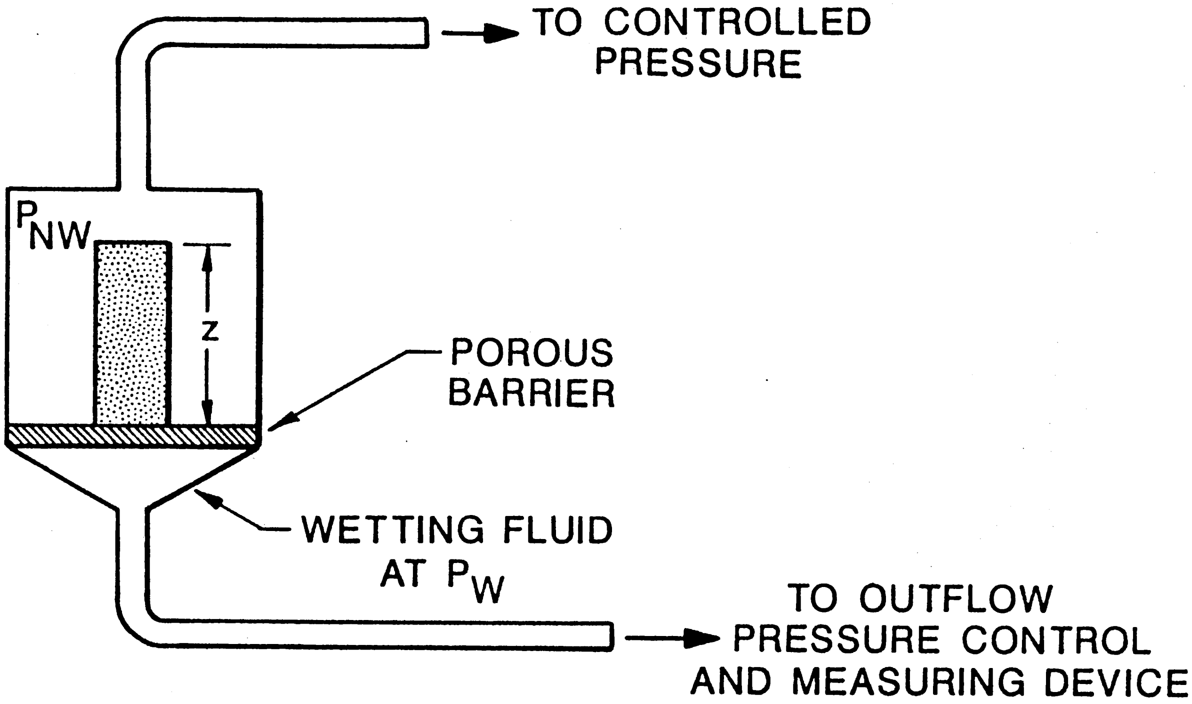

Figure 5. Pressure Cell Apparatus

pressure of the wetting phase. The apparatus is then allowed to attain a steady-state. The change in saturation of the sample is determined either by weighing the sample or by measuring the amount of the wetting phase which was drained from the sample. This procedure is repeated until values of saturation have been obtained for the range of capillary pressures desired.

This procedure requires a large amount of time for the experiment to attain steady-state. Klute (1986) has reported that two to three days are usually required to ensure that steady-state has been achieved.

Su and Brooks (1980) have developed a method of obtaining retention function data, which they claim greatly reduces the amount of time required to obtain retention function data. Their method applies the same principles as the pressure cell method. Except, instead of allowing the wetting phase to come to steady-state for a fixed change in capillary pressure, a specified amount of the wetting phase is removed from the sample and the capillary pressure is allowed to equilibrate.

The equipment needed to conduct the measurements are a pressure cell including a porous barrier, a capillary tube, a buret, a vacuum-pressure regulator and a device for measuring pressure. A schematic diagram of the experimental apparatus is shown in Figure 6.

After the sample and porous barrier are saturated, the pressure cell is connected to the capillary tube and buret

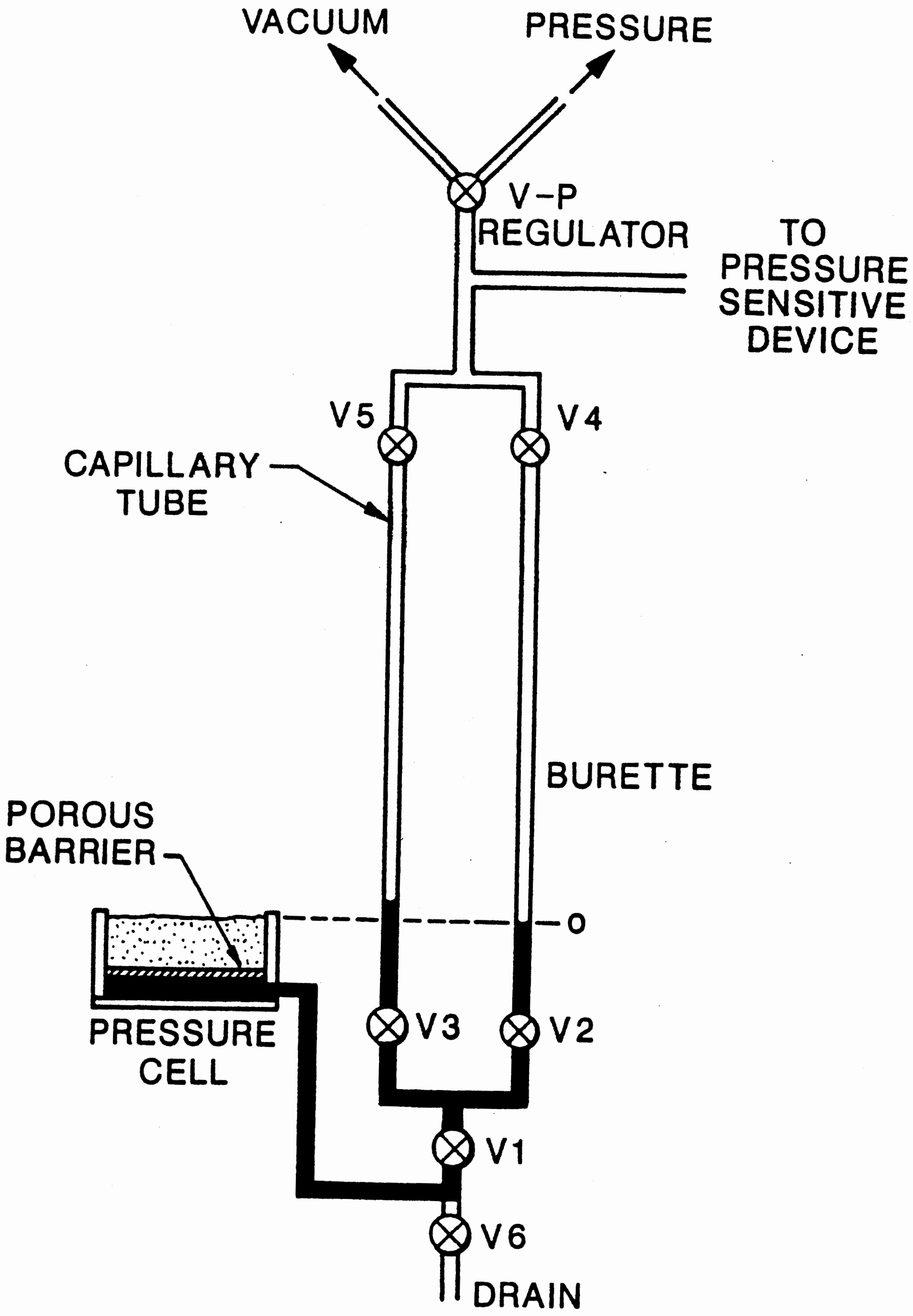

Figure 6. Schematic Diagram of Su and Brooks Apparatus

apparatus which has been filled with the wetting fluid to the zero marks on the buret and capillary tube.
The upper surface of the sample is set at the the height of the zero mark on the buret. The pressure of the wetting phase is reduced by adjusting the vacuum-pressure regulator, and drainage of the wetting fluid is initiated due to the corresponding increase in the capillary pressure. After a specific amount of the wetting fluid has drained into the buret from the sample, the valve connecting the cell with the buret is closed; and the valve connecting the cell with the capillary tube is opened. The wetting fluid then starts draining into the capillary tube causing the meniscus in the tube to rise. Since the diameter of the capillary tube is small, the increase in the elevation of the meniscus in the tube is relatively large for a small amount of drainage. This rise of the level of the wetting fluid in the tube increases the pressure of the wetting fluid in the sample and therefore decreases the capillary pressure in the sample. The rise of fluid in the capillary tube is controlled by manually adjusting the pressure of the wetting phase using the vacuum-pressure regulator, so that water pressure in the porous barrier and sample is equalized. Steady-state is achieved when the meniscus in the capillary tube ceases to move.

By using this method of automatic and manual control of the pressure of the wetting fluid, a steady-state is achieved much more rapidly than by allowing drainage to proceed until the drainage rate approaches zero for a fixed increase in capillary pressure.

The capillary pressure of the sample is calculated by subtracting the elevation of the meniscus in the capillary tube above the zero mark from the pressure head above the meniscus. The pressure head above the meniscus is measured by a pressure sensing device placed between the vacuum-pressure regulator and the capillary tube. The relative saturation is determined by subtracting the volume of water drained into both the buret and capillary tube from the pore volume and dividing by the pore volume. The pore volume is determined by subtracting the dry weight of the sample from the weight of the saturated sample.

Su and Brooks (1980) concluded that by using this method they could determine the retention function of a porous medium much more rapidly than by using the conventional pressure cell method. They also reported that the method provided consistent data which fit a smooth curve.

## Methods of Obtaining the Unsaturated Hydraulic Conductivity Function

### Laboratory Methods

Laboratory methods for determining the unsaturated hydraulic conductivity function of porous media are divided into two types, steady-state methods and unsteady-state methods. All steady-state methods are based on the procedure originally proposed by Richards (1931). Unsteady-state methods are based on the fact that if the time dependence of the seepage velocity (q), capillary pressure ($p_c$) and relative saturation (S) are known. Then the unsaturated hydraulic conductivity can be calculated from Richards' Equation. In this work only the short column steady-state method and the instantaneous profile unsteady-state method will be reviewed, since they are the two methods most commonly used by researchers to evaluate the unsaturated hydraulic conductivity functions. Reviews of other methods are available from the following sources Corey (1986), Klute (1986), Olsen and Daniel (1981), and Scheideggar (1974).

Steady-state methods involve the determination of the seepage velocity and the hydraulic gradient for a system with steady-state one-dimensional flow. If the flow is vertical, the following finite difference form of Darcy's law is applicable:

$$q = -K \{[\Delta(p_c/\rho g)/\Delta z + 1]\}. \tag{3-1}$$

Following the description given by Klute (1972), in the short column method, a sample of the porous medium is held between two saturated porous plates which provide hydraulic contact at the inflow and outflow ends of the sample. The air entry pressure of these two barriers must be greater than the largest value of capillary pressure used in the experiment.

The sample and porous plates are mounted in a conductivity cell consisting of a cylinder to hold the sample and endcaps to hold the barriers in contact with the sample. The sample and barriers are saturated and then connected to a source of a constant head inflow of the wetting fluid. A Mariotte siphon is usually used to provide this constant head flow. The outflow rate is measured to determine the seepage velocity. The hydraulic gradient is measured by the use of tensiometers installed in the sides of the cylinder. The use of tensiometers to measure the hydraulic gradient is necessary due to the uncertainty of the amount of head lost across the end barriers and the contact area between the soil and the barriers. A diagram of a typical short column steady-state conductivity cell is shown in Figure 7.

A tensiometer is a device consisting of a porous sensing probe connected to a pressure sensing element by a tube. The tube is filled with the wetting fluid and the probe is placed in hydraulic contact with the porous media in which the capillary pressure is being determined. The tensiometer works on the same principle as the pressure cell device for measuring the retention function. The wetting fluid in the porous media comes to a steady-state with the fluid in the tensiometer, so that the fluid in the tensiometer is at the same pressure as that in the porous media. This pressure is then measured by the pressure sensing device, which is usually a manometer or pressure

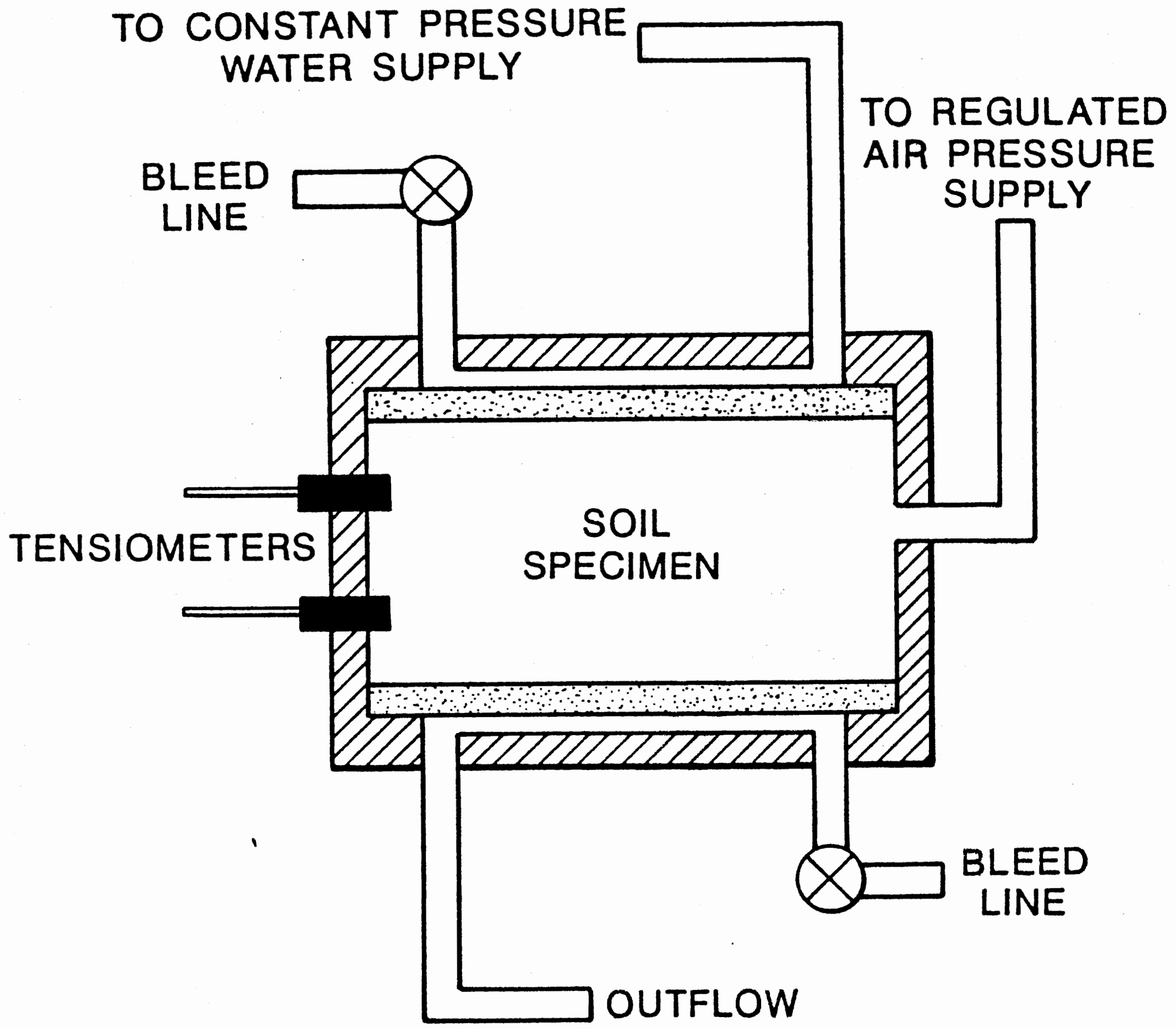

Figure 7. Conductivity Cell

transducer.

After flow is initiated in the sample, the system is allowed to attain a steady-state. The time required for the experiment to attain a steady-state can become quite long especially in the low saturation region. When steady-state is achieved the capillary pressure gradient, $\Delta(p_c/\rho g)$, is determined from the tensiometer readings. The distance between the tensiometers is $\Delta z$. With a knowledge of the seepage velocity, the hydraulic conductivity can be calculated from Equation (3-1). The capillary pressure associated with this value of conductivity is the average of the capillary pressures measured at each of the tensiometers. If a retention function is available for the sample, then the K(S) relationship can also be obtained. If it is desired to obtain the K(S) relationship at the same time as the $K(p_c)$ relationship, then the gamma attenuation or neutron scattering methods for measuring saturation may be used. These methods of measuring saturation are described in detail by Gardner (1986).

Laliberte and Corey (1967) describe a short column method for determining the unsaturated hydraulic conductivity function on "undisturbed" soil samples. In this method a sleeve type soil sampler is used to obtain the soil sample. The inner sleeve of the sampler is the cylinder portion of the short column apparatus. This cylinder has slots cut into its side so that tensiometers can be inserted to measure the hydraulic gradient. By using this method to measure the conductivity of soils a better estimate of the actual field properties of the soil is obtained.

The instantaneous profile unsteady-state procedure for determining the unsaturated hydraulic conductivity relationship is based on the equation of continuity. The relationship can be applied to one dimensional flow and integrated to obtain

$$Q(z_2,t) = Q(z_1,t) - \int(\delta S/\delta t)dz. \qquad (3\text{-}2)$$

The integral is evaluated from $z_1$ to $z_2$. If the saturation distribution $S(z,t)$ is known then the integral term may be evaluated. In addition, if either $Q(z_1,t)$ or $Q(z_2,t)$ is known then the other may be calculated from Equation (3-2). If the capillary pressure distribution $p_c(z,t)$ is also known then the hydraulic gradient at a given z and t can be determined. The ratio of the soil water flux (Q) to the hydraulic gradient at a certain time and location is the hydraulic conductivity at the saturation and capillary pressure at the specific time and location (Klute 1972).

The laboratory apparatus employing the instantaneous profile method for determining conductivity consists of a column fitted with tensiometers for measuring capillary pressures along the soil column. The procedure is to start with the soil at a constant saturation throughout the length of the column. Then the hydraulic conditions at one end of the column are changed. The change may either be the imposition of a constant suction, or imposing a constant inflow of the wetting fluid (Olsen and Daniel 1981). The capillary pressure at each of the tensiometers is continuously monitored and recorded to obtain $p_c(z,t)$. The soil water flux $[Q(z_1,t)]$ is obtained from the knowledge of the constant inflow rate or by the measurement of the outflow rate due to the applied constant suction head. The saturation distribution $[S(z,t)]$ is obtained from a previously determined retention curve for the sample or by means of gamma attenuation or neutron scattering probes along the length of the column. A hydraulic conductivity function may be obtained using the calculation procedure described previously.

Instantaneous profile methods generally are less time consuming and less precise than steady-state determinations of the conductivity function. The decrease in the amount of time required is due to the fact that the system is not required to attain a steady-state. The imprecision of this method is due to increased number of measurements required and the propagation of errors through the calculation procedure.

## Field Methods

Field, or in-situ, measurements of the unsaturated conductivity functions of soils are performed using the instantaneous profile unsteady-state method. The capillary pressure distribution is determined by installing tensiometers at the desired soil depths and locations with in the soil plot. The relative saturation distribution is obtained from gamma attenuation or neutron scattering methods used in the field or by using a retention function determined in the laboratory on soil samples taken at the tensiometer field locations. A known soil water flux is obtained by identifying a "plane of zero flux" in the soil profile. A "plane of zero flux" is identified by locating a depth in the soil profile where the hydraulic gradient is zero. Another method is to begin with the soil saturated to the maximum depth at which measurements are to be taken. Then the soil is allowed to drain, and the flux at the soil surface is zero.

Field methods of determining the conductivity function tend to be more imprecise than laboratory determinations of the conductivity function. Conditions in the field are much more difficult to control than laboratory conditions. Field methods also tend to take more time than laboratory methods.

## Empirical Models for the Calculation of the Conductivity Function from Retention Function Data

Due to the various difficulties involved in determining the hydraulic conductivity function experimentally, numerous methods of calculating the conductivity function from other measurable properties of the porous medium have been proposed. Reviews of these calculation procedures are available from the following sources: Brutsaert (1968), Klute (1972), and Maulem (1986). In this section a brief review of the calculation procedures most often used by researchers will be presented.

Childs and Collis-George (1950) developed a calculation procedure based on the probability of the interconnection of various sized pores within the media. The equation they proposed is

$$K = M \sum_{\sigma=0}^{\sigma=R} \sum_{\rho=0}^{\rho=R} \sigma^2 f(\rho) \Delta r f(\sigma) \Delta r. \qquad (3\text{-}3)$$

In this equation $f(\rho) \Delta r$ is the cross-sectional area of pores of radius $\rho$ to $\rho + \Delta r$, and $f(\sigma) \Delta r$ is the area associated with pores of radius $\sigma$ to $\sigma + \Delta r$. R is the largest pore size which remains full at a corresponding capillary pressure. The pore size distribution function f(r) is calculated from the retention function by using the capillary pressure equation to relate pore radius to capillary pressure (Equation 2-24). The summation is stopped at the pore size R pertaining to the largest pore remaining full at a given capillary pressure. M is a matching factor determined from

$$M = K_0/K_{0c} \qquad (3\text{-}4)$$

in which $K_0$ is the conductivity measured at a specific relative saturation, and $K_{0c}$ is the calculated conductivity from the summation portion of Equation (3-3) corresponding in the previous chapter as

$$S_e = (p_b/p_c)^m \quad \text{for } p_c \geqslant p_b \tag{3-7}$$

$$K = K_s\ (p_b/p_c)^n \quad \text{for } p_c \geqslant p_b \tag{3-8}$$

Laliberte, using experimental data for three disturbed porous media, verified that the pore size distribution indices were related in the following manner:

$$n = 2 + 3m. \tag{3-9}$$

Laliberte also proposed the following equation for calculating values of the saturated hydraulic conductivity from retention function data:

$$Ks = \frac{\phi_e\ \sigma^2}{5\ p_b{}^2} \left[\frac{m}{m + 2}\right] \tag{3-10}$$

where $\phi_e$ is defined by:

$$\phi_e = \phi\ (1-S_r) \tag{3-11}$$

Using this procedure the calculated values of $K_s$ were all within 27% of the experimental values.

Nielson et al. (1960) compared the methods of Childs and Collis-George with the one proposed by Marshall. They reported that the method of Childs and Collis-George gave results that better fit the experimental data and concluded that the better results were probably due to the use of a matching factor in the Childs and Collis-George method.

Jackson et al. (1965) compared the methods of Childs and Collis-George, Marshall and Millington and Quirk. They concluded that if the method given by Millington and Quirk was modified by the use of a matching factor, it gave the best results of the three methods.

Kunze et al. (1968) and Green and Corey (1971) made further comparisons of the three methods. They also concluded that, if the Millington and Quirk method was modified by the use of matching factor, it would give the best results of the three methods.

Brust et al. (1968) compared the modified Millington and Quirk method with the method proposed by Laliberte. Brust reported that for his data for a clay loam soil the method of Laliberte gave the better results.

## Summary

In this chapter a review of methods of obtaining the conductivity and retention functions has been given. It has been noted that the Su and Brooks (1980) method for obtaining retention function data is the least time consuming and has been reported to give consistent data which fit a smooth curve. The short column steady-state method of obtaining the conductivity function seems to be the most precise and least complicated method of obtaining conductivity data. If "undisturbed" core samples are used, as described in the Laliberte and Corey method, the experimental results should be a better representation of the properties actually encountered in the field. The empirical method of calculating the conductivity function from retention function data proposed by Laliberte is easy to apply and is reported to give results that fit experimental data as well as any other method reviewed.

# CHAPTER IV

# DEVELOPMENT OF SCALING METHOD

## Availability of Data

Data describing the spatial variability of the retention and conductivity functions of soils is reported by, Nielson et al. (1973), Kiesling (1974) and Hornsby et al. (1983). Kiesling (1974) reports only the retention function and values of the saturated hydraulic conductivity for a Teller soil series. This data would only be useful in the determination of the scaling parameters of the retention function and would not be useful in determining a possible relationship between the scaling factors of the retention and conductivity functions. Nielson (1973) reports both the retention and unsaturated hydraulic conductivity functions for a Panoche soil series. The physical data for this study was published separately from the paper by Nielson et al. (1973) which reports the results of the study and could not be obtained in time to be considered in this thesis.

Hornsby et al. (1983) reports the methods used in a regional field study of the spatial variability of the physical properties of soils. This study was performed for eighteen different soil series. The actual results of this study were reported in seven separate soil series reports. As part of this study, data for the conductivity and retention functions were reported for most of the soil series studied.

The conductivity function data were taken in-situ by the instantaneous profile field method. A considerable amount of this data exhibited a large amount of scatter. This was probably due to difficulties in controlling the field conditions and inadequate documentation of soil variability (Wilding et. al. 1983). Another problem with the data in this study is that a large amount of the conductivity data was evaluated using laboratory measured retention function data. It is questionable how well laboratory retention function data can be extended to describe the saturation distribution function in the field.

A portion of the data taken in this study did utilize field methods for evaluating the saturation distribution function. Most of this data is available in the series reports by Bruce et al. (1983) for a Cecil soil series, Dane et al. (1983) for Troup and Lakeland soil series and Nofziger et al. (1983) for Bethany, Konawa and Tipton soil series. Since for this data, both the capillary pressure and relative saturation distributions are known, it is possible to obtain a field measured retention function. Adequate descriptions of the retention function of the soil using this data are questionable, since it is unlikely that the field conditions of the soil could be controlled well enough to eliminate hysteresis in the hydraulic functions of the soil.

The $p_c(S)$ and $K(p_c)$ relationships are not unique, but depend on the saturation history of the porous media. These functions are hysteresis functions. Despite the experimental limitations, data reported entirely utilizing field methods is much more useful than data taken partly in the field and partially in the laboratory. It would be useful to have data taken entirely in the laboratory on "undisturbed" soil samples with the retention and conductivity function data taken on the same soil core.

Since the data taken in the regional field study described by Hornsby et al. (1983) is all that is available, it will be used in this chapter to illustrate the scaling principles presented in this chapter. The data taken by Nofziger on the Bethany soil series appears to exhibit the least amount of scatter and will be used in this chapter.

## Evaluation of Scaling Methods

All of the scaling procedures presented in Chapter II are the same in the following respect. They rely on the use of a single scaling factor which, when multiplied by the variable to be scaled, reduces all of the separate functional relationships to a single scaled relationship. In other words, the functional relationships must only differ by a single scaling factor. This implies that for hydraulic functions to be scaled by any of the methods described earlier, they must be parallel functions. Thus the following criterion of similitude is applicable to all of the scaling methods: for two media to be similar, they must exhibit parallel retention and conductivity functions.

Although the different scaling methods may have additional criteria of similitude, they must at least meet the criterion stated above. This criterion was stated by Corey and Corey (1966) as applicable to their similar media concept. Miller (1980) also mentions it as a criterion for the Miller and Miller scaling procedure.

It can be shown that similar media concepts of Corey and Corey and the one of Leverett are closely related. By rearranging the equation presented by Laliberte (1968) for calculating the saturated hydraulic conductivity (Equation 3-10) one obtains:

$$\frac{1}{p_b} = \sigma \left[ \frac{K_s}{\phi} \right]^{1/2} \left[ \frac{5}{1-S_r} \right]^{1/2} \left[ \frac{m+2}{m} \right]^{1/2} . \qquad (4-1)$$

Looking at the relationship in terms of the Corey and Corey scaling factor for the retention function of $1/p_b$, it is seen that the Leverett scaling factor of $(K_s/\phi)^{1/2}$ is related to the Corey and Corey scaling factor. In addition both methods rely on the use of $1/K_s$ to scale the conductivity function. This similarity of the two methods has also been noted by Corey (1986). Due to this similarity one expects that when one method applies the other should also be just as applicable.

To determine whether these similar media concepts apply to a particular soil it is necessary to plot the conductivity and retention functions and determine whether the function are indeed parallel. Plots of the retention and conductivity functions of a Bethany series at several different soil depths are shown in Figures 8 and 9.

The conductivity and retention functions for the various depths are not represented by parallel curves. Therefore, they are not scalable by any of the different similar media concepts.

These functions were scaled by the methods of Leverett and Corey and Corey to illustrate what occurs when hydraulic properties of dissimilar porous media are scaled. Figure 10 shows the results of Corey and Corey scaling on the retention functions. The curves all converge on a common point at $p_c/p_b = 1$. They then spread out as $p_c$ increases due to the differences in slope. If they had a common slope, they would have reduced to a single curve.

Figure 11 shows the results of scaling the retention curve by the method proposed by Leverett. It appears for this data that the Leverett method did a better job of scaling than did the Corey and Corey method. The method still does not describe the variability due to the differences in slope, as shown by the way the two curves that have the largest difference in slope diverge from the rest of the curves.

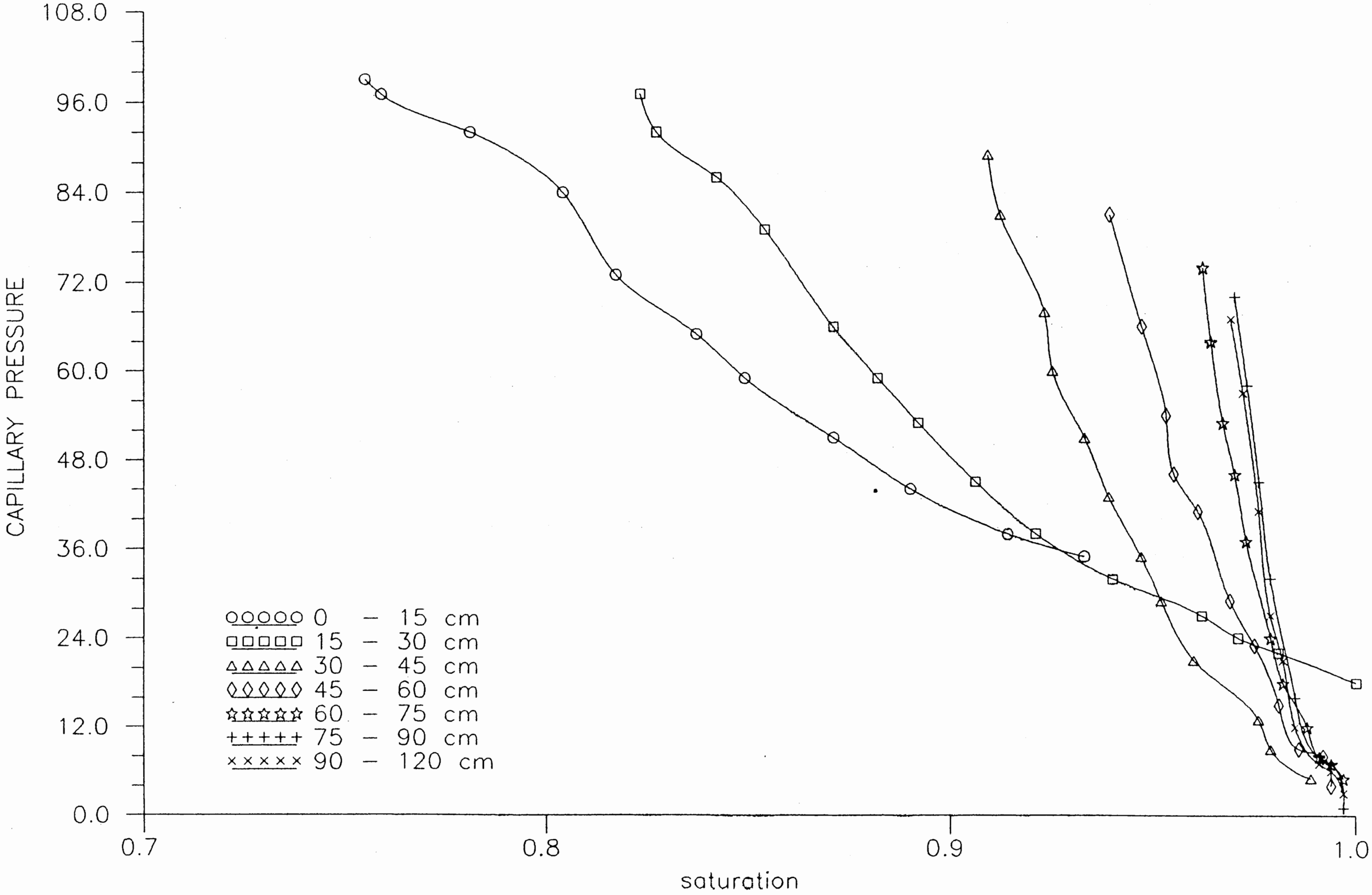

FIGURE 8. $P_c$ vs. S For Bethany Soil, (Nofziger et al. 1983)

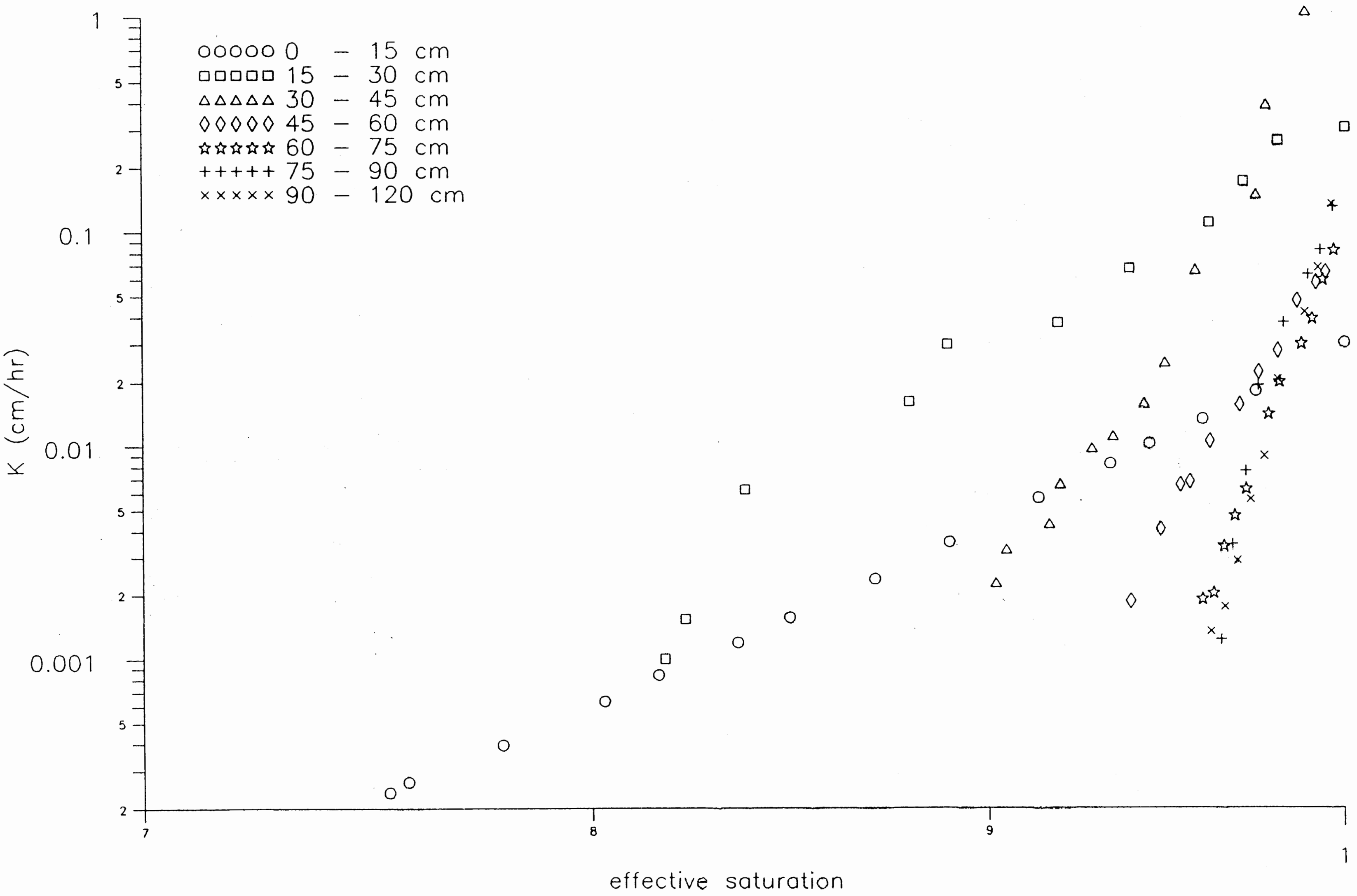

FIGURE 9. Conductivity Functions for Bethany Soil (Nofziger et al. 1983)

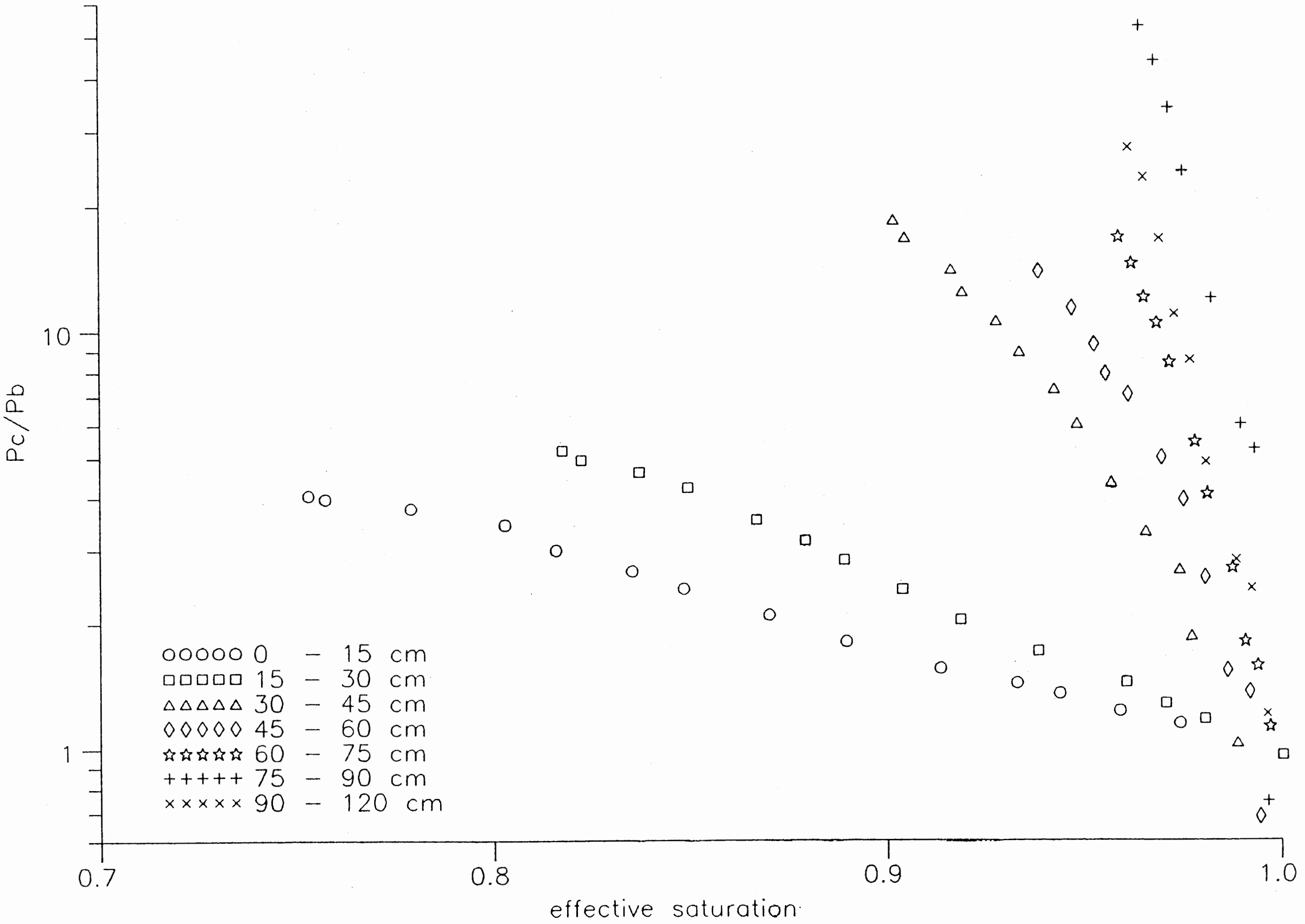

FIGURE 10. Corey and Corey Scaled Retention Function
Bethany Soil (Nofziger et al. 1983)

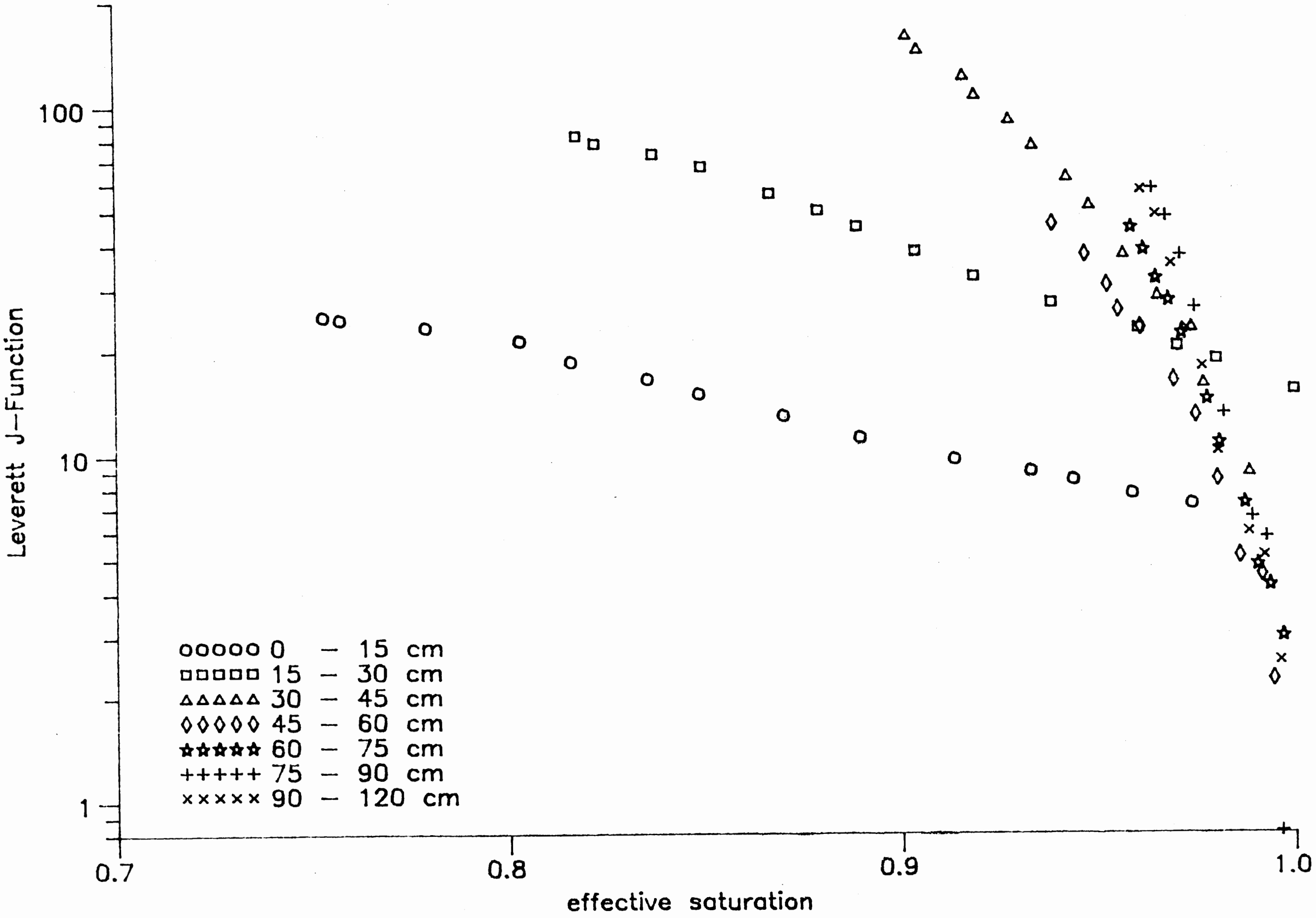

FIGURE 11. Leverett J–Function for Bethany Soil (Nofziger et al. 1983)

Figure 12 illustrates the results of plotting $K/K_s$ versus saturation as required by both the Leverett and Corey and Corey scaling procedures. Again, the curves come together at a common point and diverge from there due to the slope differences.

There was no attempt made to scale this data by the proposed methods of extending Miller and Miller scaling for the description of soil spatial variability. This was due to the fact that it has been shown that the method will not work because the curves are not parallel. Had these methods been applied the following would have occurred. The procedure defines some average function which could be represented by a curve on the graph. All of the curves for the individual depths would intersect the curve of the average function. This intersection would occur at the midpoint of each of curves for the individual depths. This occurs because the scaling factor is defined to be the constant which when multiplied by the variable to be scaled minimizes the sum of the squares of the deviation from the mean function. This means that the only difference between this method and that of Corey and Corey is that, instead of all the curves coming together at a common point as in the Corey and Corey method, they all intersect a common mean curve.

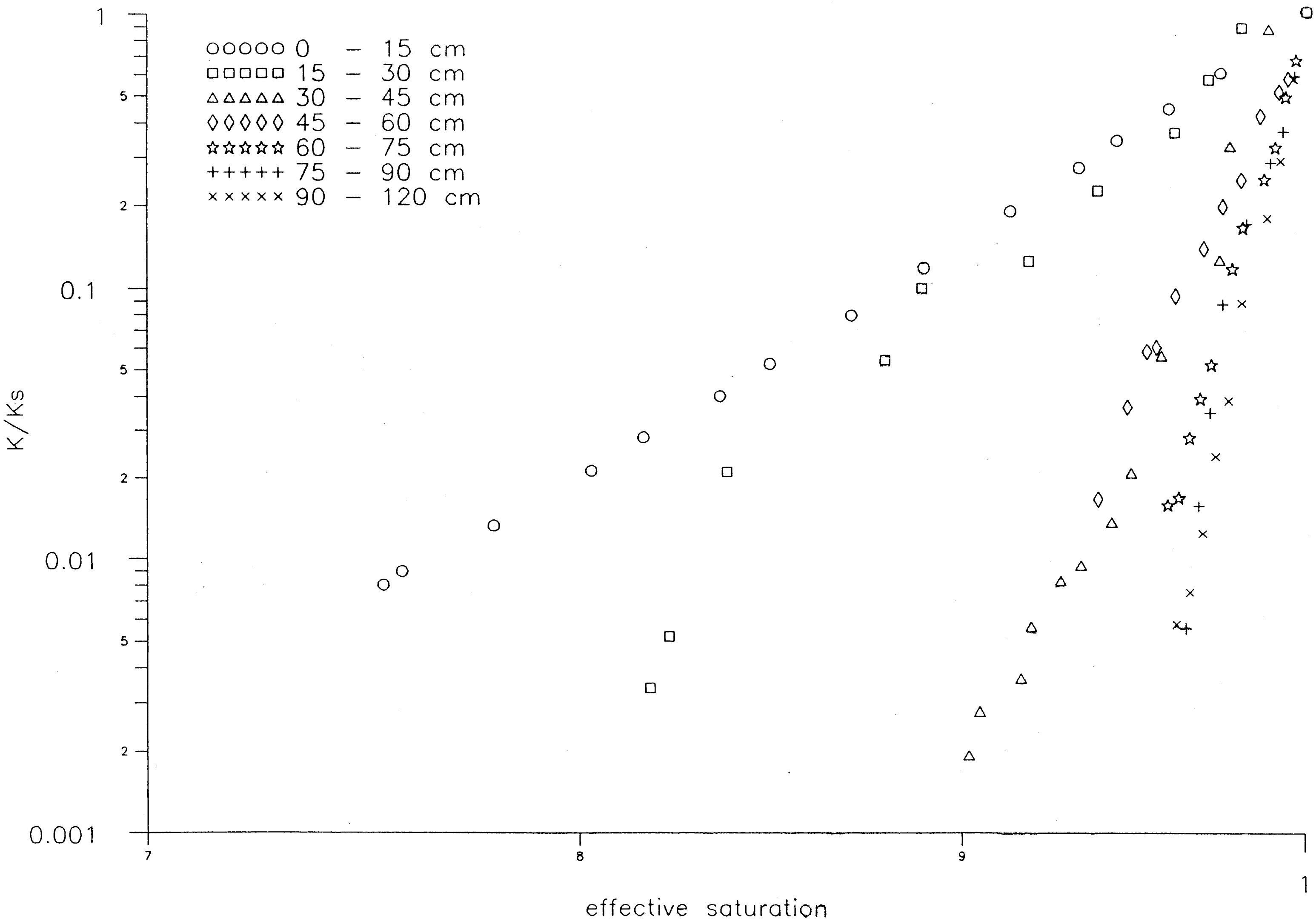

FIGURE 12. $K/K_s$ vs. $S_e$ for Bethany Soil (Nofziger et al. 1983)

## Development of a New Scaling Method

None of the current methods of scaling are able to describe the spatial variability of the soil hydraulic functions due to the variability of the slopes of these functions. It is necessary to develop a new scaling method which will work when the criterion of same slopes is not met. This new method should have the capability of translating the functional curves as the previously described methods do, and it must also be capable of rotating the functions so that they become parallel.

As noted by Brooks and Corey (1964), for a large amount of retention function data, the portion of the curve for $p_c \geqslant p_b$ fits a power law function such that

$$S_e = (p_b/p_c)^m. \qquad (4\text{-}2)$$

In addition a large amount of conductivity function data were observed to fit the following power law function

$$S_e = (K/K_s)^{1/e}. \qquad (4\text{-}3)$$

Brooks and Corey (1964) defined m and e to be indices of the pore size distribution of the porous media. It is also noted that m and 1/e are the slopes of the conductivity and retention functions when they are plotted in terms of Corey and Corey scaled variables on a logarithmic scale.

If retention function data, which is well described by Equation (4-1), were plotted as $(p_c/p_b)^m$ as a function of $S_e$ a curve which had a slope of -1 for all values of $p_c \geqslant$

$p_b$ would result. This would be true for all data which was described by Equation (4-1). If this operation were performed on retention function data for all samples taken to describe a soil series, all the retention function data would coalesce to a single curve. This curve would have the properties described previously. This operation was performed on the data for the Bethany soil series. The resulting curve is shown in Figure 13.

It is seen from Figure 13 that by using this method all of the data for the Bethany soil series can be described by a single curve. Since this is the desired result of a scaling procedure, it is proposed that the retention function be scaled using the following scaling functions:

$$p_{c.} = (p_c/p_b)^m \qquad (4\text{-}4)$$

$$S. = S_e. \qquad (4\text{-}5)$$

If the same reasoning is applied to the conductivity function, a similar scaling procedure results. The resulting function for scaled conductivity is

$$K. = (K/K_s)^{1/e}. \qquad (4\text{-}6)$$

The scaled conductivity function would be expected to yield a curve which has a slope of 1. The results of applying this procedure to the Bethany conductivity data are shown in Figure 14. It is seen that the expected results were obtained.

This scaling procedure will work for all data which can

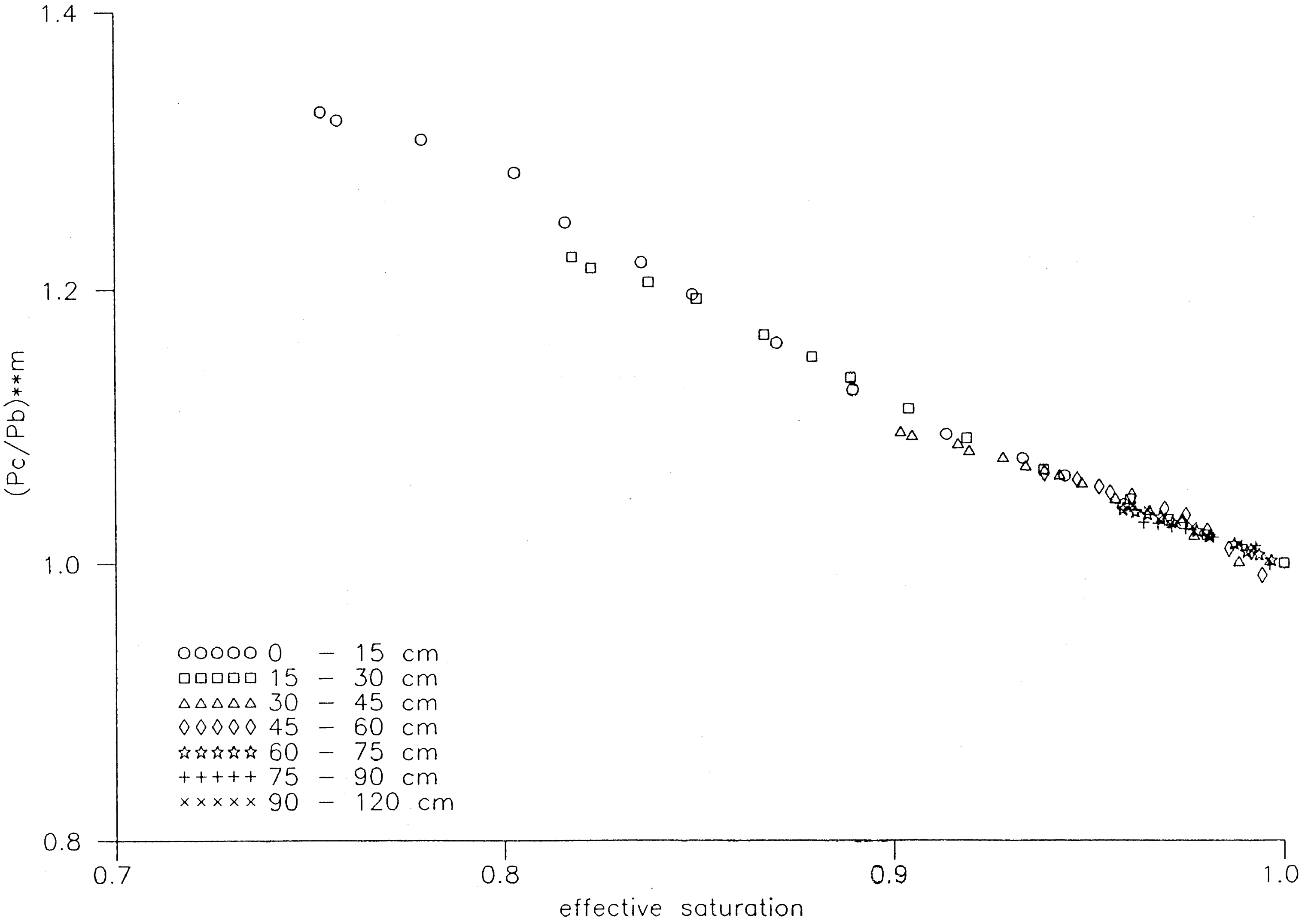

FIGURE 13. Scaled Retention Function, Bethany Soil (Nofziger et al. 1983)

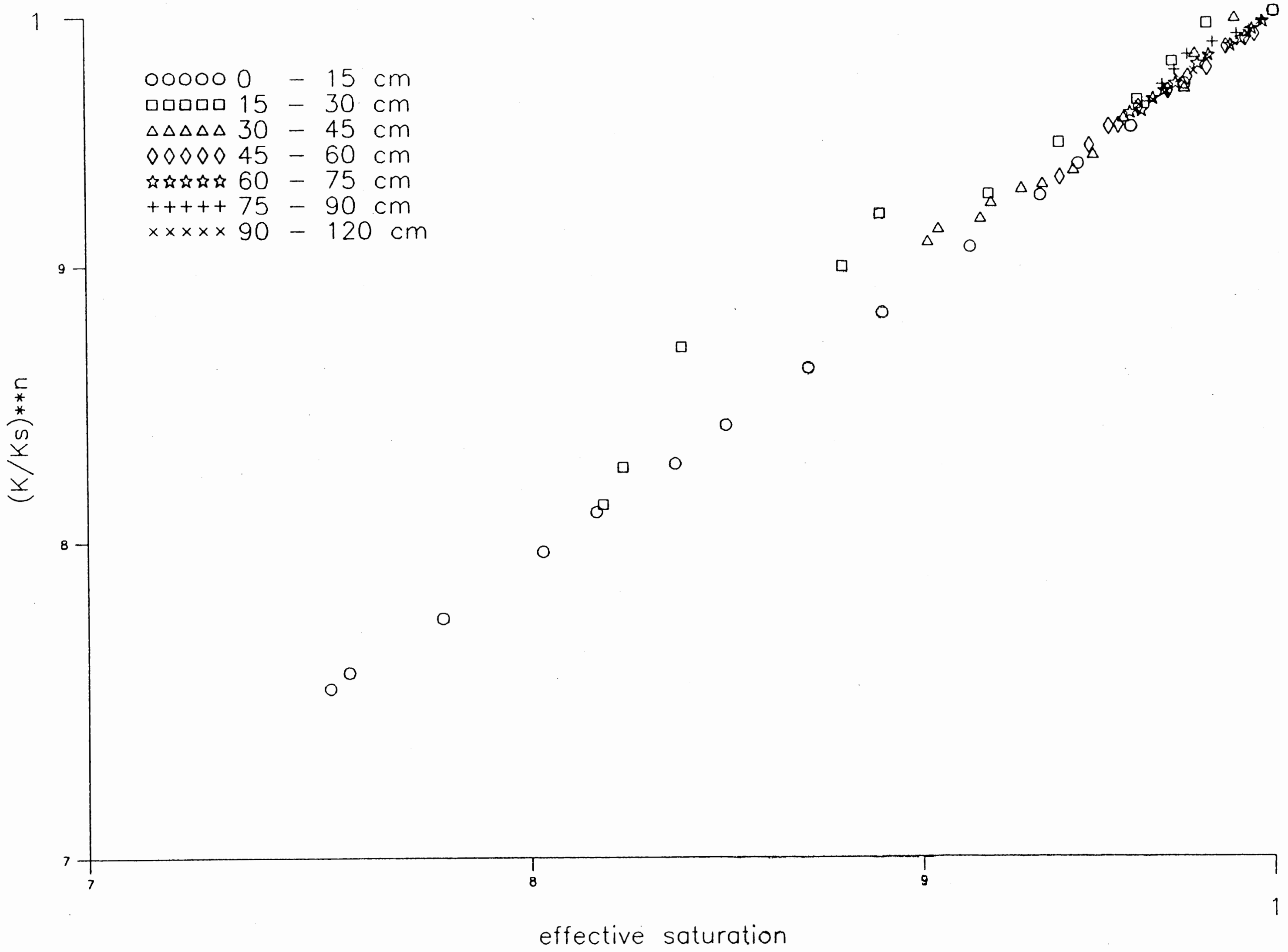

FIGURE 14. Scaled Conductivity Function, Bethany Soil (Nofziger et al. 1983)

be described by the Brooks and Corey equations. In addition it would be expected to apply to all data which meet the following criterion: if a set hydraulic functions deviate from the power law models, they must all deviate in the same manner. Deviations of data from the Brooks and Corey model most frequently occur in the regions of low values of effective saturation and in regions of capillary pressure below the bubbling pressure.

## Conclusions

In the first section of this chapter, it has been noted that only a limited amount of data which describes the spatial variability of the retention and conductivity functions of soils. It is recommended that additional data be obtained so that scaling procedures for description of the spatial variability of soils may be evaluated. In addition it is recommended that this data be taken in the laboratory were experimental conditions may be more strictly controlled. These data should be taken using the same "undisturbed" sample for both the retention and conductivity functions.

In the second section some of the previously proposed scaling procedures are evaluated. It was observed that for any of these scaling procedures to be applicable, the hydraulic functions of the different porous media must exhibit parallel conductivity and retention functions. This is not often the case.

In the third section of this chapter a new scaling method was proposed. This scaling procedure is based on the fact that the conductivity and retention functions of soils can often be fit by a power law function. This proposed scaling procedure utilizes the exponent obtained by fitting the data to the power law function as a parameter for scaling the data.

# CHAPTER V

# DESCRIPTION OF EXPERIMENTAL METHODS

## Introduction

In this chapter a experimental method is described for determining the conductivity and retention functions of soils. In this method both the retention and conductivity function are determined on the same "undisturbed" soil core. The method of obtaining "undisturbed" samples is the sleeve sampler method described by Laliberte and Corey (1967). The retention function is determined by the method proposed by Su and Brooks (1980). The conductivity function is determined by the steady-state short column method described by Klute (1986). The only modification to this method was the utilization of the conductivity cell for undisturbed core samples described by Laliberte and Corey.

These methods were used to determine the conductivity and retention functions of a selected Teller soil. Samples were taken at two sites and four different depths at each site.

## Method of Obtaining "Undisturbed" Soil Samples

The "undisturbed" core samples were obtained using the sleeve sampler method described by Laliberte and Corey. A photograph of the sleeve sampler used is shown in Figure 15.

The procedure for obtaining samples using the sleeve sampler is as follows. The sleeve and spacers are inserted into sampling tube. The cutting blade is attached to the bottom of the sampling tube and an extension is attached to the other end of the tube. The sampler is then driven into the soil, forcing a soil core into the sleeve inside the sampling tube. The sampling tube is then removed from the soil. The inner sleeve containing the soil core is removed from the sampling tube and transported to the laboratory for experimental measurements.

## Method of Obtaining Conductivity Function Data

The conductivity cell used in this method consists of the following:

a sleeve made from 2" acrylic pipe, with slots milled in the side for the insertion of tensiometers,

two ceramic disks for use as porous barriers, (available from Osmonics, Inc., Minnetonka, MN 55343)

two endcaps to hold the ceramic disks in contact with the soil core and to provide a point of attachment for the constant head inflow and outflow sources,

two tensiometers for measurement of capillary pressure, constructed of microporous plastic, (available from Amerace Corp., Hackettstown, NJ 07840),

Figure 15. Photograph of Sleeve Sampler

four retaining rods for holding the cell together. A photograph of the assembled conductivity cell is shown in Figure 16.

The assembled cell and sample are then vacuum saturated by the following procedure. The cell is placed in a vacuum chamber and the chamber is evacuated with a vacuum pump for approximately one hour. Then, while still maintaining the vacuum, water is allowed to flow into the chamber until the cell is covered. The vacuum is then released, and water flows into the evacuated pores in the sample and porous barriers, saturating them. The cell is then removed from the chamber and placed in position for the conductivity measurements.

The apparatus for the measurement of the conductivity function consists of three main sections. The first section is a constant head supply consisting of a mariotte siphon attached to a source of regulated pressure. The second section is the source of pressure measurement. In this apparatus the pressure sensing devices were mercury-water manometers. The final section is the apparatus providing for constant head outflow and outflow measurement. The flow rate is measured in a buret. The pressure at the drip point is controlled by a regulated pressure source. A schematic diagram of this apparatus is shown in Figure 17.

The constant head inflow supply system is connected to

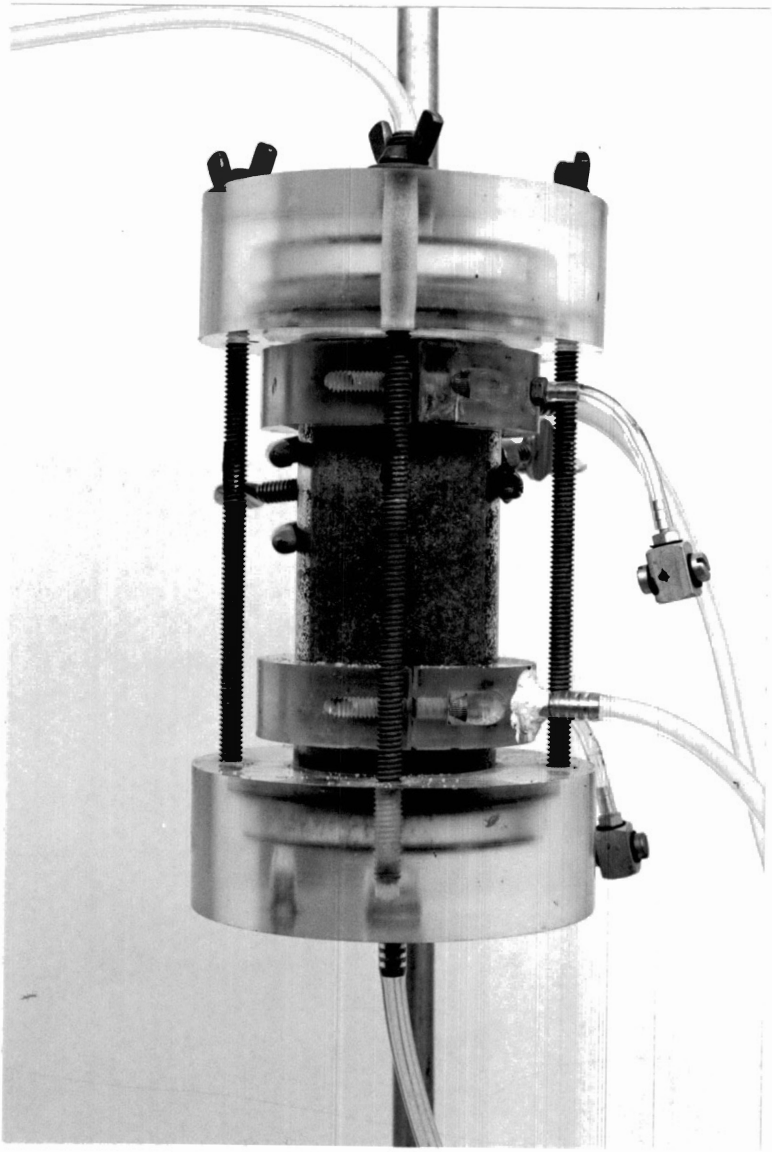

Figure 16. Photograph of Assembled Conductivity Cell

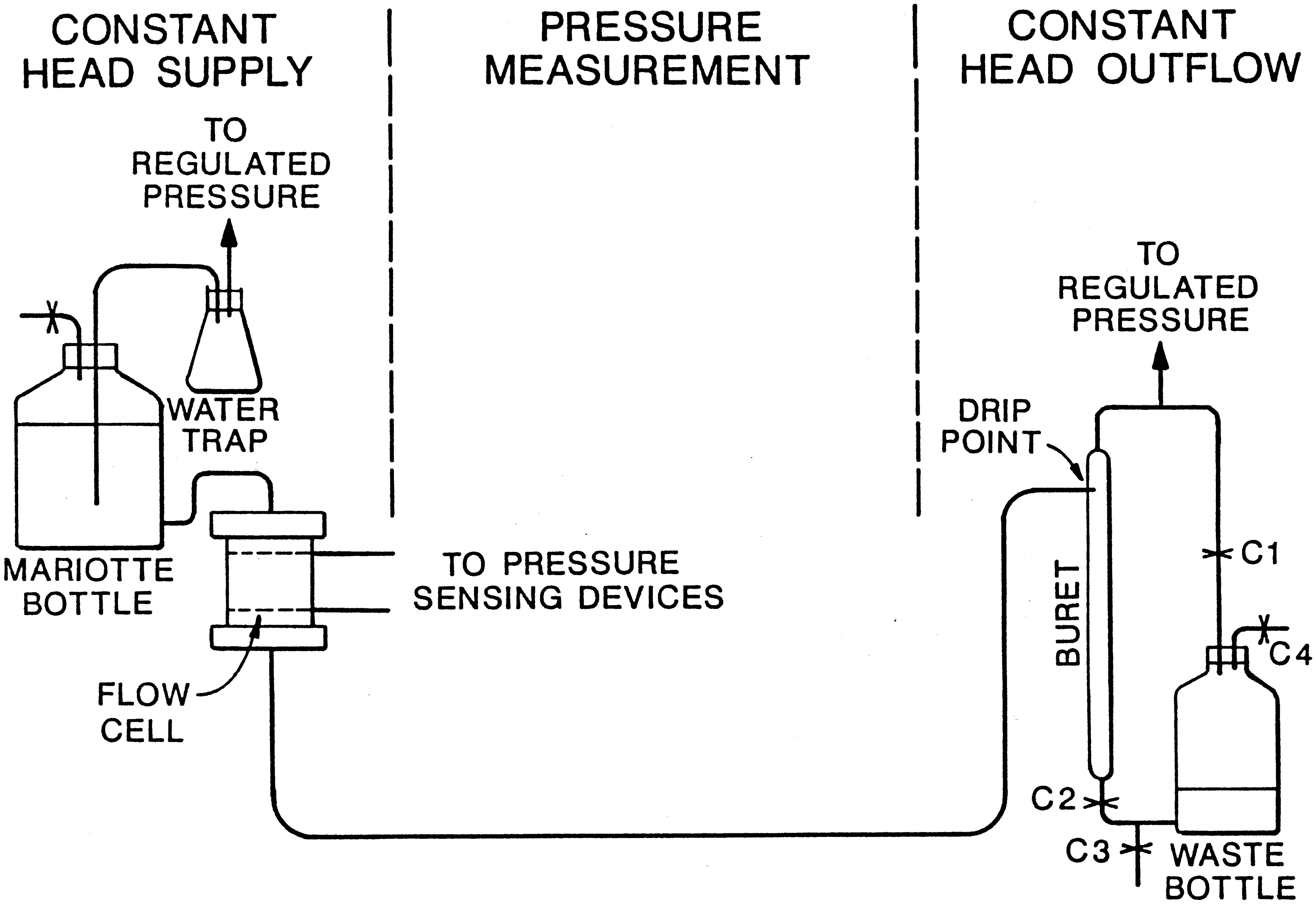

Figure 17. Schematic Diagram of Conductivity Measurement Apparatus

the top of the cell. The outflow system is connected to the bottom of the cell. All the tubing and connections are flushed free of air bubbles, by means of the bleed screws located in the top and bottom end caps. Saturated flow is then established through the sample by releasing the clamps on the inflow and outflow lines. Saturated flow in the sample is obtained by maintaining a positive wetting phase pressure. This is achieved by maintaining the elevation of the constant head inflow source above the top of the sample.

The tensiometers are then connected to the pressure measurement system. For this experiment mercury-water manometers were used. The tubing connecting the tensiometers to the manometers is flushed free of air bubbles.

The system is now allowed to attain a steady-state. The system is assumed to be at steady-state when the pressure read from the manometers remains constant for one hour.

After a steady-state was attained the manometer readings were recorded. The outflow rate was measured using the outflow buret and a stopwatch.

The first data are used to calculate the saturated conductivity of the sample.

To obtain the values of conductivity at unsaturated conditions it is necessary to reduce the pressure of the wetting phase below atmospheric pressure. This is achieved by either connecting the inflow source and outflow to a controlled vacuum source or by raising the elevation of the sample above the elevation of the inflow source. The system is then allowed to re-attain a steady-state and the measurements are repeated. This procedure is repeated until data is obtained for the range of capillary pressures desired. The largest value of capillary pressure for which the conductivity is limited by the bubbling pressure of the porous barriers.

The following calculations are performed to obtain the conductivity as a function of capillary pressure. The conductivity is calculated from the following rearranged form of Darcy's Law:

$$K = (V/A)\ (\Delta p_c/\rho g/\Delta z + 1), \qquad (5\text{-}1)$$

in which V is the outflow rate, A is the cross sectional area of the soil core and $\Delta z$ the distance between the tensiometers. For the cell used in this experiment A is 15.5 $cm^2$ and $\Delta z$ is 7.62 cm. Values of conductivity are associated with the corresponding values of the average of the capillary pressures measured at the tensiometers.

## Method of Obtaining Retention Function Data

The method for obtaining retention function data is the same as the method described by Su and Brooks (1980). Except the same cell used in the conductivity function measurements is used in the retention function measurements.

The procedure is to first saturate the sample and porous barriers by the same method described for conductivity measurements. The outflow of the cell is then connected to the Su and Brooks apparatus. The apparatus used in this experiment is the same as the one shown in Figure 6 of chapter III.

Any air bubbles in the tube connecting the cell to the apparatus are flushed from the system. The buret and capillary tube are drained to the zero mark. The vacuum-pressure regulator is adjusted to zero gauge pressure. Then the elevation of the cell is adjusted, so that the upper surface of the sample is at the same elevation as the zero mark.

Valves V1 and V2 are opened. The vacuum-pressure regulator is adjusted so that pressure in the buret and capillary tube is below atmospheric. This causes drainage of the soil water through the porous barrier into the buret. When a specified amount of water has drained into the buret valve V2 is closed and valve V3 is opened. Water then drains into the capillary tube and the meniscus in the capillary tube rises. This rise is stopped by increasing the pressure above the meniscus with the vacuum-pressure regulator. Care must be taken in adjusting the pressure so that water is not forced back in the sample to avoid hysteresis effects.

The apparatus is allowed to come to steady-state, which is attained when the movement of the meniscus has stopped. The height of the meniscus and amount of water drained into the buret are measured and recorded. The pressure above the meniscus in the tube is measured using the manometer installed between the tube and the regulator. This pressure is also recorded.

After these measurements have been taken, valve V3 is closed and Valve V2 is opened. The pressure in the buret is decreased by adjusting the vacuum-pressure regulator. Another incremental amount of water is drained from the sample. The procedure described previously is repeated and the measurements recorded. This is repeated until data is collected for the range of capillary pressures desired.

The following calculations are performed to obtain a set of data for capillary pressure as a function of volumetric outflow. Capillary pressure is first calculated by subtracting the height of the meniscus above the zero mark in the capillary tube from the pressure head measured above the meniscus by the manometer.

$$p_c = \text{pressure head (cm of } H_2O) - \text{height of meniscus} \quad (5\text{-}2)$$

For each value of capillary pressure the volumetric outflow is calculated by adding the amount of water drained into the buret to the amount of water drained into the capillary tube. The average cross-sectional area of the capillary tube is 0.0184 $cm^2$. The volume of water drained to the capillary tube is calculated from:

$$\text{Height of meniscus (cm)} \times 0.0184 = \text{vol. in tube} \qquad (5\text{-}3)$$

From these calculations a table of values of volumetric outflows and capillary pressures is obtained.

After the last measurement is obtained the weight of the sample and cell is measured and recorded. The sample is then removed from the cell and dried in an oven at 105 C. The weight of The cell less the sample is obtained. After the sample has dried overnight, the weight of the dry sample is also measured and recorded.

From these data the bulk density and porosity of the sample can obtained, and the values of volumetric outflow can be converted to relative saturations. The calculations are carried out as follows. The volume of water in the sample is calculated by first obtaining the weight of water in the sample after the last data point was obtained. This weight is

$$\begin{aligned}&\text{wt. of the cell and "wet" sample} - \text{wt. of the cell}\\ &\quad - \text{wt. of the dry sample} = \text{wt. of water.}\end{aligned} \qquad (5\text{-}4)$$

The total volume of water in the sample is now obtained from

$$\begin{aligned}\text{volume} &= \text{volume drained during measurements}\\ &\quad + \text{wt. of water} \times \rho_w.\end{aligned} \qquad (5\text{-}5)$$

The volumetric outflows are now converted to relative saturations by

$$S = \text{volume of outflow/total volume of } H_2O. \qquad (5\text{-}6)$$

The porosity is now calculated by

$$\phi = \theta_s = \text{volume of water/volume of the cell,} \qquad (5\text{-}7)$$

this formula assumes that there was no shrinkage of the sample during the experiment. The bulk density of the sample ($\rho_b$) is calculated from

$$\rho_b = \rho_p \ (1 - \phi), \qquad (5\text{-}8)$$

in which $\rho_p$ is the particle density assumed to be 2.65 $g/cm^3$.

## Precision of Experimental Methods

The precision of the measurements for the conductivity determination are as follows. The mercury-water manometers were read to a precision of ± 0.025 cm. of Hg (0.863 cm of $H_2O$). This results in a possible error of ± 0.227 in the hydraulic gradient determination. The effect of this imprecision on the value of the conductivity depends on the magnitude of the hydraulic gradient. Since the flow rates were small the measurements of outflow were relatively precise ± 0.01 ml.

The measurements in the retention function experiment were obtained with the following precisions. The manometer again could be read to ± 0.025 cm. of Hg. The elevation of the meniscus in the capillary tube could be determined to ± 0.05 cm of $H_2O$. Volumetric outflow could be measured to a precision of ± 0.025 ml. This results in a possible error of ± 0.91 cm. of $H_2O$ in the capillary pressure. The error in relative saturation is (± 0.025/pore volume of the sample ($cm^3$)).

From this description of the precision of the methods, it is seen that the major source of experimental error is in the measurement of pressure. These imprecisions can cause significant errors in the hydraulic conductivity determination at low values of the hydraulic gradient.

# CHAPTER VI

# RESULTS AND DISCUSSION

## Introduction

In Chapter II a review of the fundamental equations governing immiscible flow of fluids through porous media is presented. It was shown that in order to solve these equations, a knowledge of both the retention and conductivity functions was required. These functions are spatially variable. Several proposed methods of describing this spatial variability by means of "similar" media concepts were presented.

A few experimental methods for determining the hydraulic properties of porous media were examined in Chapter III. In addition, methods of obtaining the conductivity function from retention function data were examined. It was shown that the method proposed by Su and Brooks (1980) was the least time consuming method of obtaining retention function. The short column steady-state method of obtaining conductivity data was observed to a reasonably precise and relatively uncomplicated means of obtaining conductivity data. A method of obtaining "undisturbed" soil cores for laboratory testing was also presented. The use of "undisturbed" samples allows for a better representation of properties encountered in-situ. The method of Laliberte and Corey (1967) for obtaining the conductivity function from retention data was observed to be relatively easy to apply and reproduced experimental data as well as any other method presented.

In Chapter IV it was shown that there is a shortage of data describing the spatial variability of the conductivity and retention functions of soils. All of the "similar" media concepts were seen to apply only when the hydraulic functions could be represented by parallel curves. A method which allows for the description of soil spatial variability when the curves are not parallel was presented. This method was based on the power law models proposed by Brooks and Corey for the retention and conductivity relationships. This method would allow for the scaling of hydraulic functions which are not scalable by other methods.

In Chapter V the experimental methods used to obtain retention and conductivity functions of Teller soil series are described. These methods allow for the determination of both the retention and conductivity functions on the same "undisturbed" soil core sample.

In this chapter, the experimental conductivity and retention data for the Teller soil series will be presented in the first section. In Appendix A a typifying pedon description of a Teller series soil is presented. The reproducability of the data will then be evaluated. A discussion of how accurately the data represents actual field conditions will be presented.

In the next section the proposed scaling method will be evaluated using the experimental data. An analysis of variance which allows for the statistical determination of whether or not the hydraulic functions are parallel is presented. The analysis of variance procedure is used to evaluate the experimental data.

In the final section the ability of the method proposed by Laliberte et al. (1968) to predict the conductivity function is evaluated. In addition it is shown that the indices e and m appear to be related by a power law function. Finally, it has been suggested in the literature that m is a function of the clay content of the soil (Clapp and Hornberger 1978). This theory is also evaluated using experimental data for the Teller soil together with additional published data.

## Experimental Results

### Conductivity and Retention Function Data

The experimentally determined conductivity data is presented in Table I. This data is shown graphically in Figures 18 and 19. The graphs show that the conductivity remains constant or has a gradual slope until the capillary pressure reaches a limiting value. This pressure should correspond to the bubbling pressure. The conductivity is

## TABLE I

## CAPILLARY PRESSURE - CONDUCTIVITY DATA FOR A SELECTED TELLER OIL

| Site | Depth (cm) | $p_c$ (cm of $H_2O$) | K (cm/hr) | Hydraulic Gradient | V/A (cm/s) |
|---|---|---|---|---|---|
| 1 | 0 -20 | -8.7* | 10.3 | 0.272 | 0.000776 |
| | | 7.6 | 0.678 | 2.74 | 0.000517 |
| | | 25.6 | 0.126 | 3.86 | 0.000135 |
| | | 52.2 | 0.0274 | 3.19 | 0.0000243 |
| | | 62.5 | 0.0148 | 4.08 | 0.0000168 |
| | | 86.4 | 0.00521 | 7.25 | 0.0000105 |
| 1 | 23-43 | -7.0* | 0.241 | 2.51 | 0.000169 |
| | | 23.9 | 0.239 | 2.08 | 0.000138 |
| | | 32.5 | 0.142 | 2.97 | 0.000117 |
| | | 35.9 | 0.0379 | 2.97 | 0.0000313 |
| | | 43.2 | 0.0194 | 1.07 | 0.00000575 |
| | | 46.2 | 0.0161 | 1.62 | 0.00000723 |
| 1 | 53-81 | 0.0 | 12.3 | 0.72 | 0.0381 |
| | | 14.5 | 7.56 | 0.49 | 0.00104 |
| | | 25.6 | 1.61 | 1.16 | 0.000521 |
| | | 37.9 | 0.545 | 1.62 | 0.000246 |
| | | 51.6 | 0.253 | 2.51 | 0.000177 |
| | | 61.9 | 0.126 | 3.41 | 0.000119 |
| | | 77.3 | 0.0333 | 4.32 | 0.0000399 |
| | | 96.1 | 0.0103 | 6.11 | 0.0000176 |
| 1 | 107-127 | -5.0* | 15.5 | 0.575 | 0.00247 |
| | | 5.4 | 7.43 | 0.492 | 0.00102 |
| | | 31.9 | 1.41 | 0.717 | 0.000280 |
| | | 45.2 | 0.464 | 2.74 | 0.000353 |
| | | 58.9 | 0.189 | 2.74 | 0.000144 |
| | | 76.4 | 0.0543 | 1.84 | 0.0000278 |
| 2 | 0 -20 | 5.9 | 0.368 | 0.718 | 0.000034 |
| | | 12.7 | 0.268 | 1.62 | 0.000121 |
| | | 29.5 | 0.155 | 2.29 | 0.0000986 |
| | | 41.9 | 0.113 | 2.29 | 0.0000722 |
| | | 47.8 | 0.082 | 1.84 | 0.0000419 |
| | | 58.0 | 0.00523 | 1.39 | 0.00000202 |
| | | 65.6 | 0.00373 | 1.41 | 0.00000132 |
| 2 | 23-43 | -18.8* | 1.75 | 0.293 | 0.000142 |
| | | 1.2 | 0.416 | 0.948 | 0.000110 |
| | | 11.1 | 0.370 | 0.948 | 0.0000976 |
| | | 19.0 | 0.299 | 0.948 | 0.0000788 |
| | | 36.3 | 0.0812 | 1.82 | 0.0000411 |
| | | 48.6 | 0.0405 | 1.82 | 0.0000205 |
| | | 57.2 | 0.0249 | 1.82 | 0.0000126 |
| 2 | 53-81 | -4.1* | 0.1399 | 2.52 | 0.0000978 |
| | Run 1 | 26.9 | 0.0829 | 2.04 | 0.0000470 |
| | | 34.0 | 0.0658 | 2.04 | 0.0000373 |
| | | 40.1 | 0.0482 | 2.04 | 0.0000273 |

TABLE I Continued

| Site | Depth (cm) | $p_c$ (cm of $H_2O$) | K (cm/hr) | Hydraulic Gradient | V/A (cm/s) |
|---|---|---|---|---|---|
| 2 | 53-81 | 47.7 | 0.0277 | 2.04 | 0.0000157 |
| | | ▲3.5 | 0.0206 | 2.04 | 0.0000117 |
| | | 64.9 | 0.0152 | 2.04 | 0.0000086 |
| | Run 2 | -6.3* | 0.121 | 2.26 | 0.000076 |
| | | 16.5 | 0.0801 | 2.04 | 0.0000454 |
| | | 24.6 | 0.0681 | 2.04 | 0.0000386 |
| | | 29.9 | 0.0551 | 2.04 | 0.0000312 |
| | | 38.7 | 0.0411 | 1.82 | 0.0000208 |
| | | 46.6 | 0.0320 | 1.82 | 0.0000162 |
| | | 53.0 | 0.0225 | 1.60 | 0.0000100 |
| | | 58.3 | 0.0186 | 1.60 | 0.0000083 |
| 2 | 107-127 | -20.8* | 0.803 | 3.19 | 0.00071 |
| | | 11.8 | 0.685 | 2.29 | 0.000436 |
| | | 18.6 | 0.642 | 2.29 | 0.000409 |
| | | 28.0 | 0.546 | 2.29 | 0.000347 |
| | | 36.1 | 0.456 | 2.29 | 0.000290 |
| | | 43.4 | 0.428 | 2.29 | 0.000273 |
| | | 60.3 | 0.254 | 2.97 | 0.000209 |
| | | 69.5 | 0.296 | 2.07 | 0.000171 |
| | | 87.3 | 0.271 | 2.07 | 0.000156 |
| | | 134.3 | 0.0734 | 1.10 | 0.0000255 |
| | | 171.8 | 0.0111 | 1.16 | 0.00000357 |

* Negative values of capillary pressure, indicate a positive wetting phase pressure.

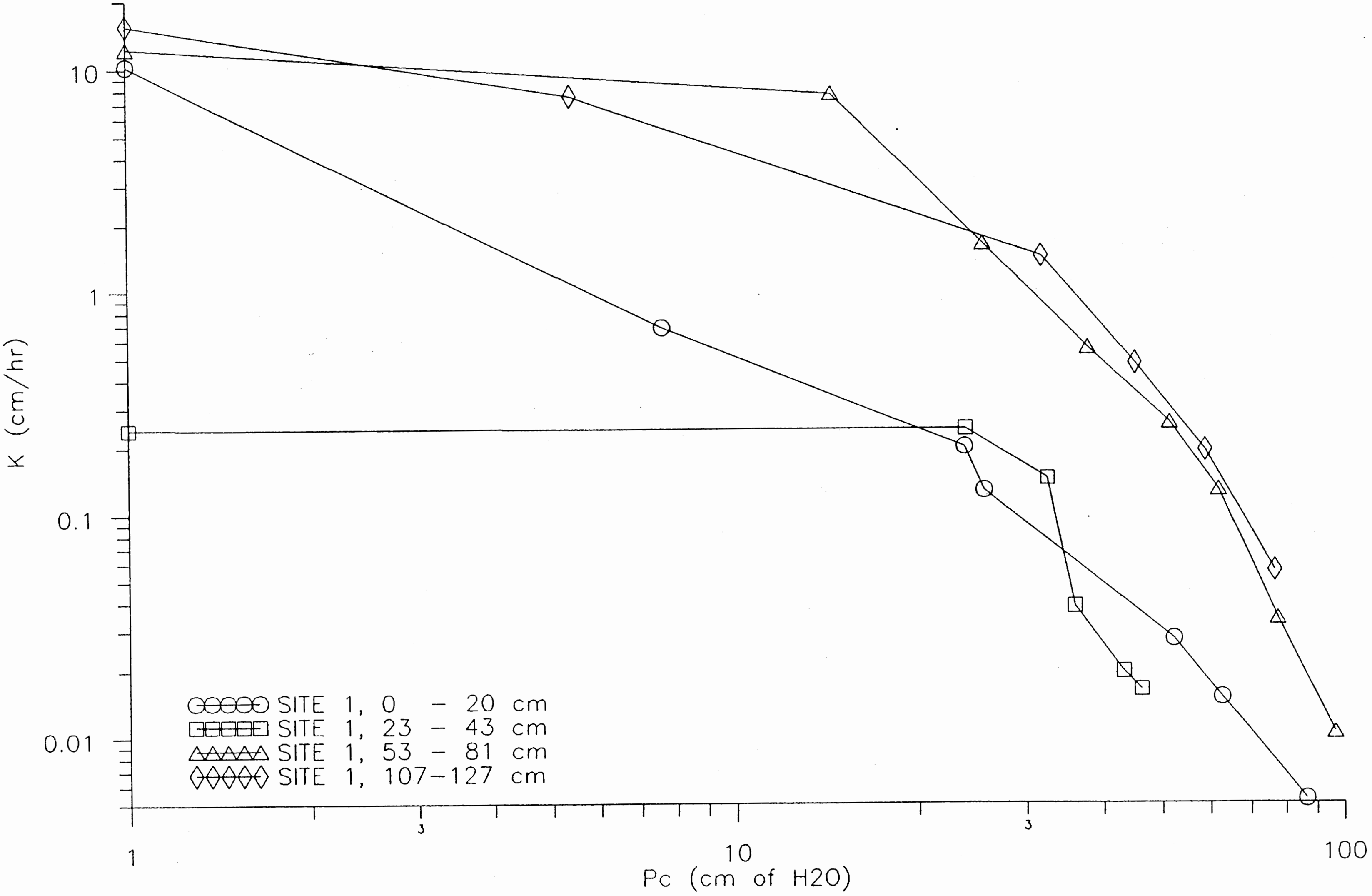

FIGURE 18. Capillary Pressure – Conductivity Data for Site 1 of Teller Soil

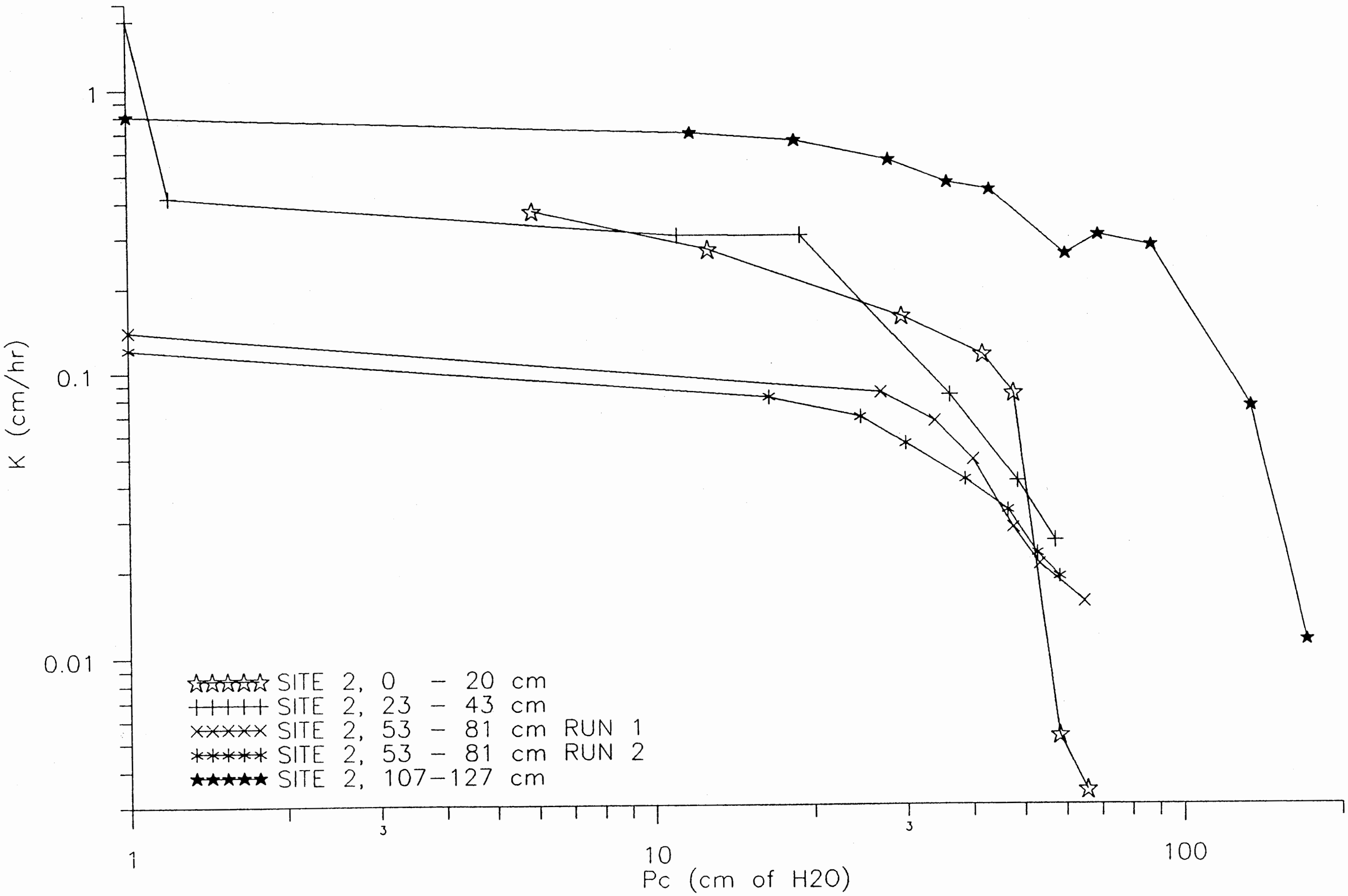

FIGURE 19. Capillary Pressure - Conductivity Data for Site 2 of Teller Soil

then observed to decrease rapidly with increasing capillary pressure. The curves for the samples obtained at 0-20 cm. deep at Site 1 and 23-43 cm. deep at Site 2 are observed to deviate from this pattern. This deviation occurs as a result of high values of the saturated conductivity obtained for these samples. The saturated conductivities obtained at positive wetting phase pressures are plotted at capillary pressure of one in the figures.

Referring to Table I the hydraulic gradients used in calculating the saturated conductivities are low. As observed in Chapter V the imprecision of the pressure measurements could cause an error of ± 0.227 in the hydraulic gradient. This is a possible explanation for the deviations observed in the two samples.

The data obtained from the retention curve measurements are presented in Table II. These data are presented graphically in Figures 20 and 21. The data are observed to be smoother than the conductivity data.

The residual saturations of the retention function data were obtained using the method described by Brooks and Corey (1964). In addition, the porosities and bulk densities of each sample were also obtained by the methods described in Chapter V. Values of residual saturation, porosity and bulk density for each sample are presented in Table III.

To obtain hydraulic conductivity as a function of saturation, it is necessary to fit the retention function

# TABLE II

## RETENTION FUNCTION DATA FOR A SELECTED TELLER SOIL

| Site | Depth (cm) | $p_c$ (cm of $H_2O$) | S |
|---|---|---|---|
| 1 | 0 - 20 | 8.3 | 0.982 |
| | | 17.9 | 0.965 |
| | | 25.8 | 0.949 |
| | | 28.4 | 0.922 |
| | | 29.2 | 0.918 |
| | | 34.3 | 0.902 |
| | | 41.6 | 0.879 |
| | | 48.5 | 0.857 |
| | | 53.4 | 0.842 |
| | | 60.4 | 0.827 |
| | | 65.6 | 0.809 |
| | | 73.5 | 0.792 |
| | | 84.6 | 0.757 |
| | | 107.7 | 0.724 |
| | | 119.0 | 0.712 |
| 1 | 23 - 43 | 14.7 | 1.0 |
| | | 19.7 | 1.0 |
| | | 45.1 | 0.997 |
| | | 52.1 | 0.991 |
| | | 58.8 | 0.969 |
| | | 71.7 | 0.959 |
| | | 90.7 | 0.952 |
| | | 101.3 | 0.945 |
| | | 122.3 | 0.928 |
| 1 | 53 - 81 | 6.2 | 0.986 |
| | | 18.4 | 0.963 |
| | | 27.9 | 0.947 |
| | | 30.8 | 0.936 |
| | | 35.4 | 0.919 |
| | | 38.8 | 0.900 |
| | | 43.8 | 0.875 |
| | | 52.8 | 0.852 |
| | | 60.9 | 0.825 |
| | | 67.1 | 0.807 |
| | | 83.8 | 0.767 |
| | | 97.8 | 0.737 |
| | | 110.8 | 0.715 |
| | | 132.8 | 0.686 |
| 1 | 107 - 127 | 10.2 | 0.987 |
| | | 35.6 | 0.980 |
| | | 36.4 | 0.970 |
| | | 38.5 | 0.955 |
| | | 46.4 | 0.923 |
| | | 60.6 | 0.882 |

TABLE II Continued

| Site | Depth (cm) | $p_c$ (cm of $H_2O$) | S |
|---|---|---|---|
| 1 | 107 - 127 | 69.9 | 0.862 |
| | | 78.8 | 0.838 |
| | | 89.0 | 0.814 |
| | | 95.2 | 0.788 |
| | | 102.9 | 0.767 |
| | | 115.3 | 0.740 |
| | | 125.4 | 0.718 |
| | | 131.5 | 0.699 |
| | | 149.5 | 0.648 |
| | | 163.0 | 0.629 |
| 2 | 0 - 20 | 9.3 | 0.986 |
| | | 42.8 | 0.957 |
| | | 52.7 | 0.936 |
| | | 60.3 | 0.929 |
| | | 72.2 | 0.925 |
| | | 85.2 | 0.918 |
| | | 92.7 | 0.913 |
| 2 | 23 - 43 | 15.4 | 0.991 |
| | | 41.4 | 0.979 |
| | | 59.5 | 0.956 |
| | | 82.0 | 0.941 |
| | | 102.4 | 0.927 |
| | | 132.4 | 0.917 |
| | | 155.2 | 0.910 |
| 2 | 53 - 81 | 13.7 | 0.956 |
| | Run 1 | 38.9 | 0.931 |
| | | 60.8 | 0.918 |
| | | 75.4 | 0.900 |
| | | 106.2 | 0.891 |
| | | 119.3 | 0.876 |
| | | 143.3 | 0.850 |
| | | 169.9 | 0.825 |
| 2 | 53 - 81 | 6.3 | 0.992 |
| | Run 2 | 22.7 | 0.976 |
| | | 37.1 | 0.937 |
| | | 48.6 | 0.924 |
| | | 60.1 | 0.907 |
| | | 71.4 | 0.896 |
| | | 96.8 | 0.881 |
| | | 112.6 | 0.872 |
| | | 147.2 | 0.846 |
| | | 156.0 | 0.841 |
| 2 | 107 - 127 | 11.8 | 0.989 |
| | | 21.8 | 0.982 |
| | | 41.2 | 0.975 |
| | | 58.4 | 0.943 |
| | | 83.0 | 0.893 |

TABLE II Continued

| Site | Depth (cm) | $p_c$ (cm of $H_2O$) | S |
|---|---|---|---|
| 2 | 107 - 127 | 106.3 | 0.850 |
| | | 127.5 | 0.800 |
| | | 133.1 | 0.779 |
| | | 143.1 | 0.756 |
| | | 155.4 | 0.732 |

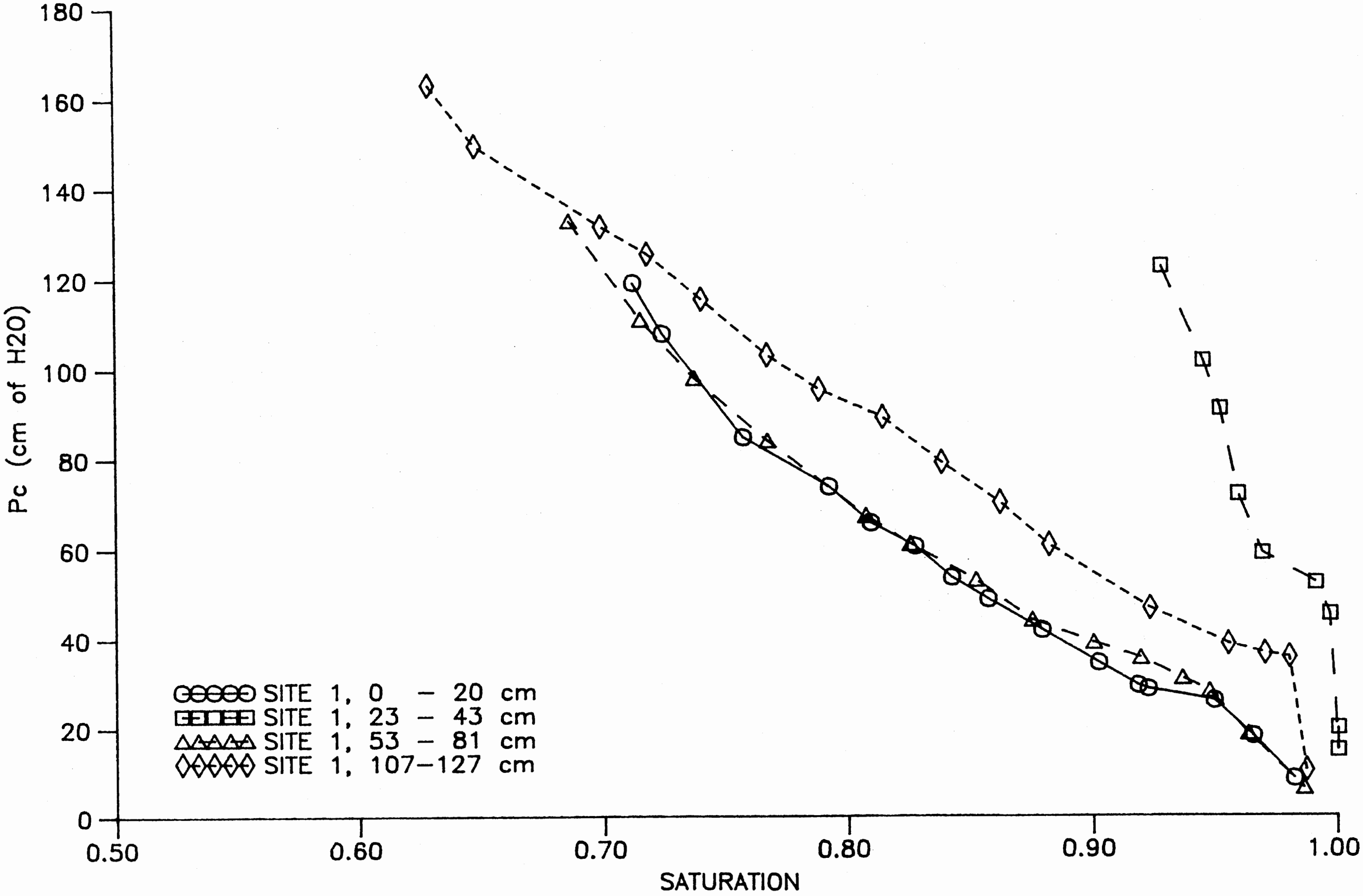

FIGURE 20. Retention Function Data for Site 1 of Teller Soil

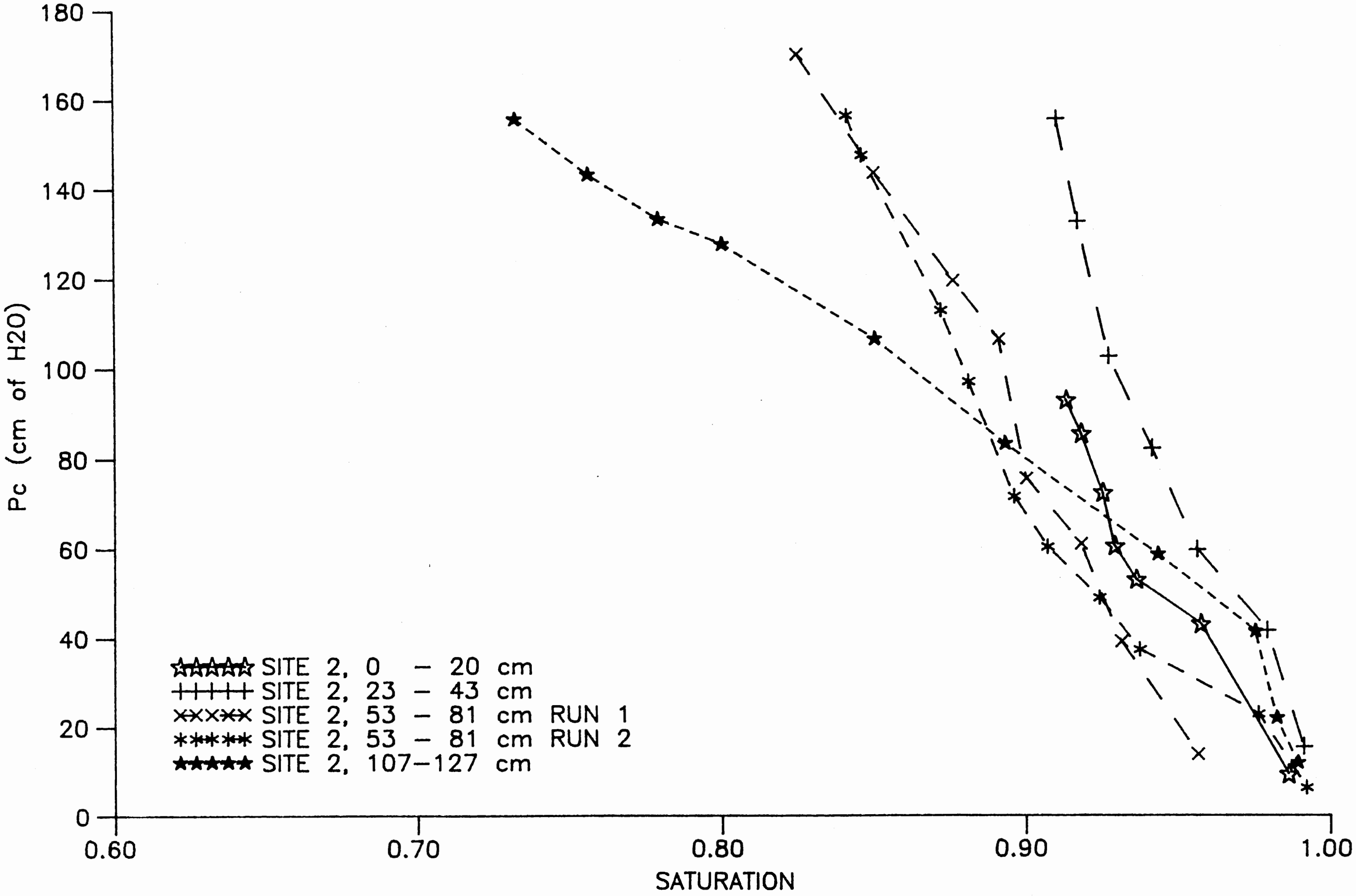

FIGURE 21. Retention Function Data for Site 2 of Teller Soil

## TABLE III

## RESIDUAL SATURATIONS, BULK DENSITIES AND POROSITIES OF A SELECTED TELLER SOIL

| Site | Depth (cm) | $S_r$ | $\phi$ | $\rho_b$ (g/cm$^3$) |
|---|---|---|---|---|
| 1 | 0 -20 | 0.012 | 0.372 | 1.66* |
| | 23-43 | 0.90 | 0.359 | 1.70* |
| | 53-81 | 0.004 | 0.402 | 1.58* |
| | 107-127 | 0.096 | 0.412 | 1.56* |
| 2 | 0 -20 | 0.45 | 0.357 | 1.70* |
| | 23-43 | 0.80 | 0.369 | 1.67* |
| | 53-81 | 0.010 | 0.316 | 1.81* |
| | 53-81 | 0.031 | 0.339 | 1.75* |
| | 107-127 | 0.069 | 0.375 | 1.66* |

* Bulk densities are probably high, due the fact that 100% saturation was probably not achieved in the vacuum saturation procedure.

data to an empirical model. The model proposed by Brooks and Corey (1964) does not describe the portion of the retention curve for $p_c < p_b$ so it is not used. The simplest model which allows for the description of the downward curvature of the retention function at $p_c < p_b$ is the model proposed by Brutsaert (1966). The model is:

$$S_e = a/(a + p_c^{\,b}) \tag{6-1}$$

in which a and b are constants determined by a best fit method. The retention function data were fit to this model and the resulting values of a and b are summarized in Table IV. The resulting values of the coefficients of correlation (r) are also presented in Table IV.

By using the fitted equations for the retention functions the $K(S_e)$ relationships were obtained. These relationships are plotted in Figures 22 and 23.

## Data Reproducability

Replicate measurements of the conductivity and retention functions were performed on the core sample taken at Site 2 at a depth of 53-81 cm. The data obtained from the replicate measurements is shown in Figures 24 and 25.

Figure 24 shows the results of the replicate conductivity measurements. The range of possible values of conductivity due to the imprecision of the hydraulic gradient measurement is also plotted on the graph. It is seen that this possible error could account for some of the

# TABLE IV

## VALUES OF a AND b CONSTANTS FOR A SELECTED TELLER SOIL

| Site | Depth (cm) | a | b | r |
|---|---|---|---|---|
| 1 | 0 -20 | 498 | 1.13 | -0.995 |
| | 23-43 | 87800 | 2.54 | 0.998 |
| | 53-81 | 721 | 1.21 | -0.995 |
| | 107-127 | 3500 | 1.53 | -0.997 |
| 2 | 0 -20 | 233 | 0.842 | -0.997 |
| | 23-43 | 692 | 1.27 | -0.996 |
| | 53-81 | 240 | 0.767 | -0.995 |
| | 107-127 | 4660 | 1.48 | -0.998 |

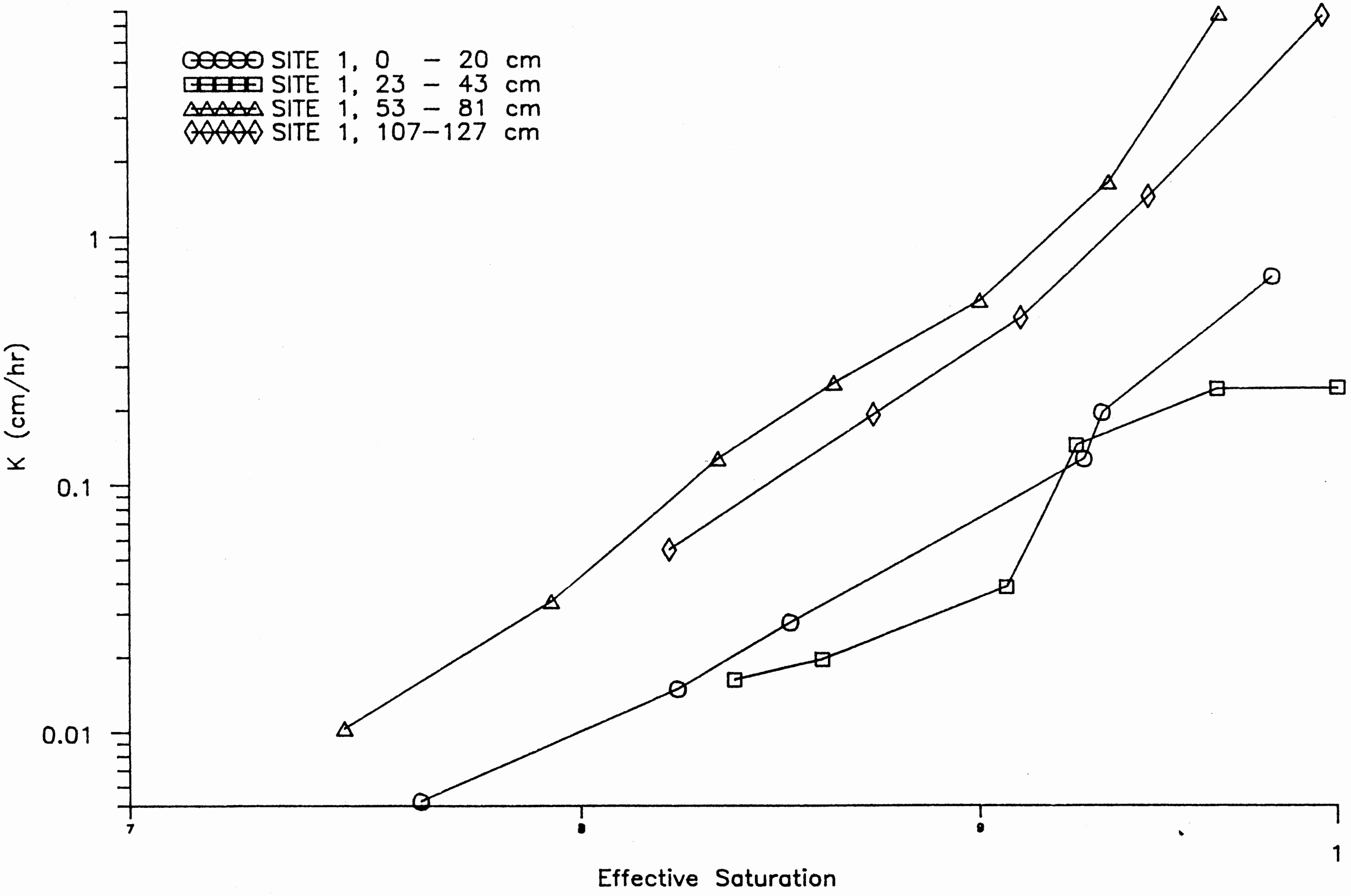

FIGURE 22. K vs. $S_e$ Curves for Site 1 of Teller Soil

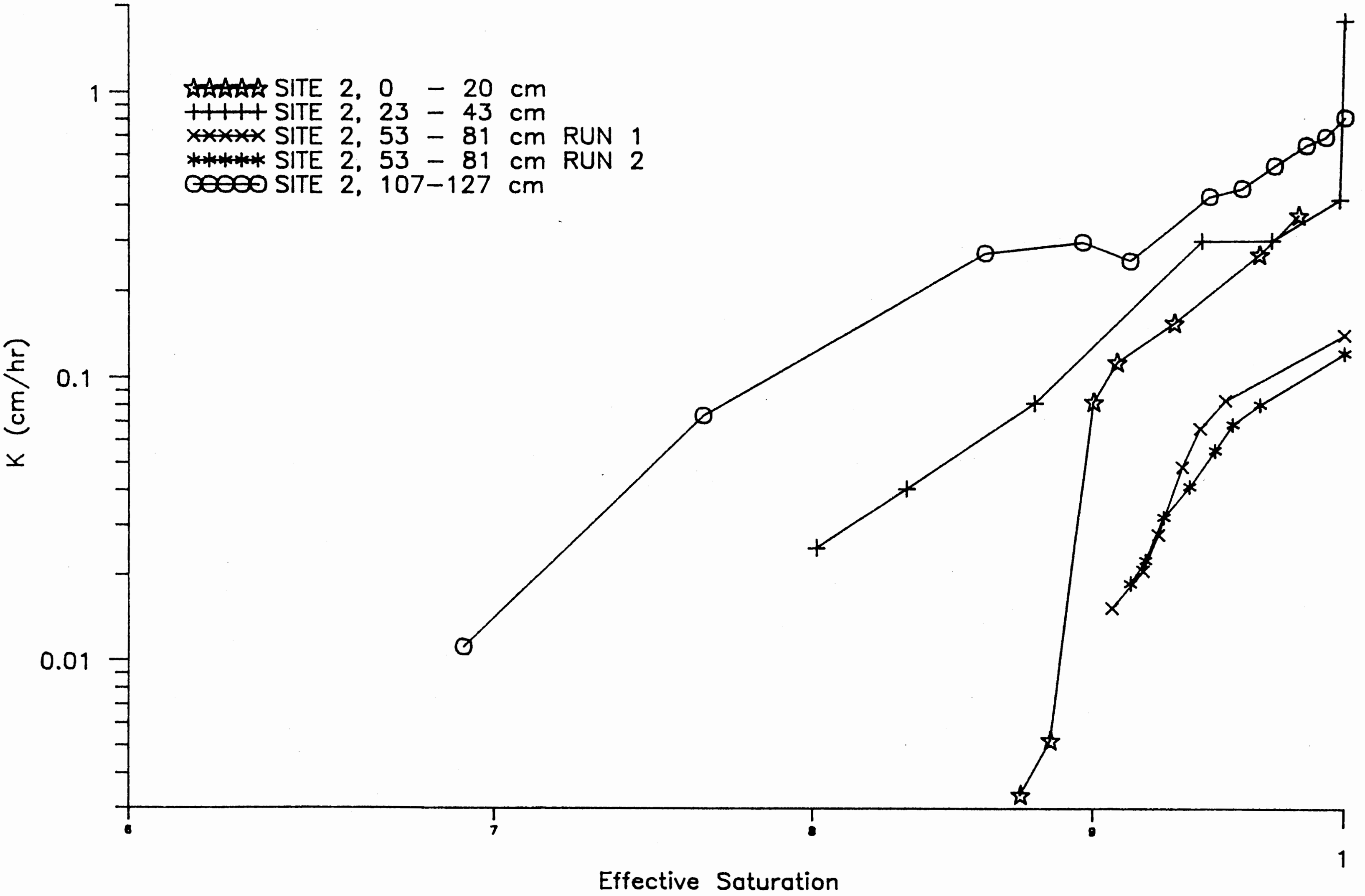

FIGURE 23. K vs. $S_e$ Curves for Site 2 of Teller Soil

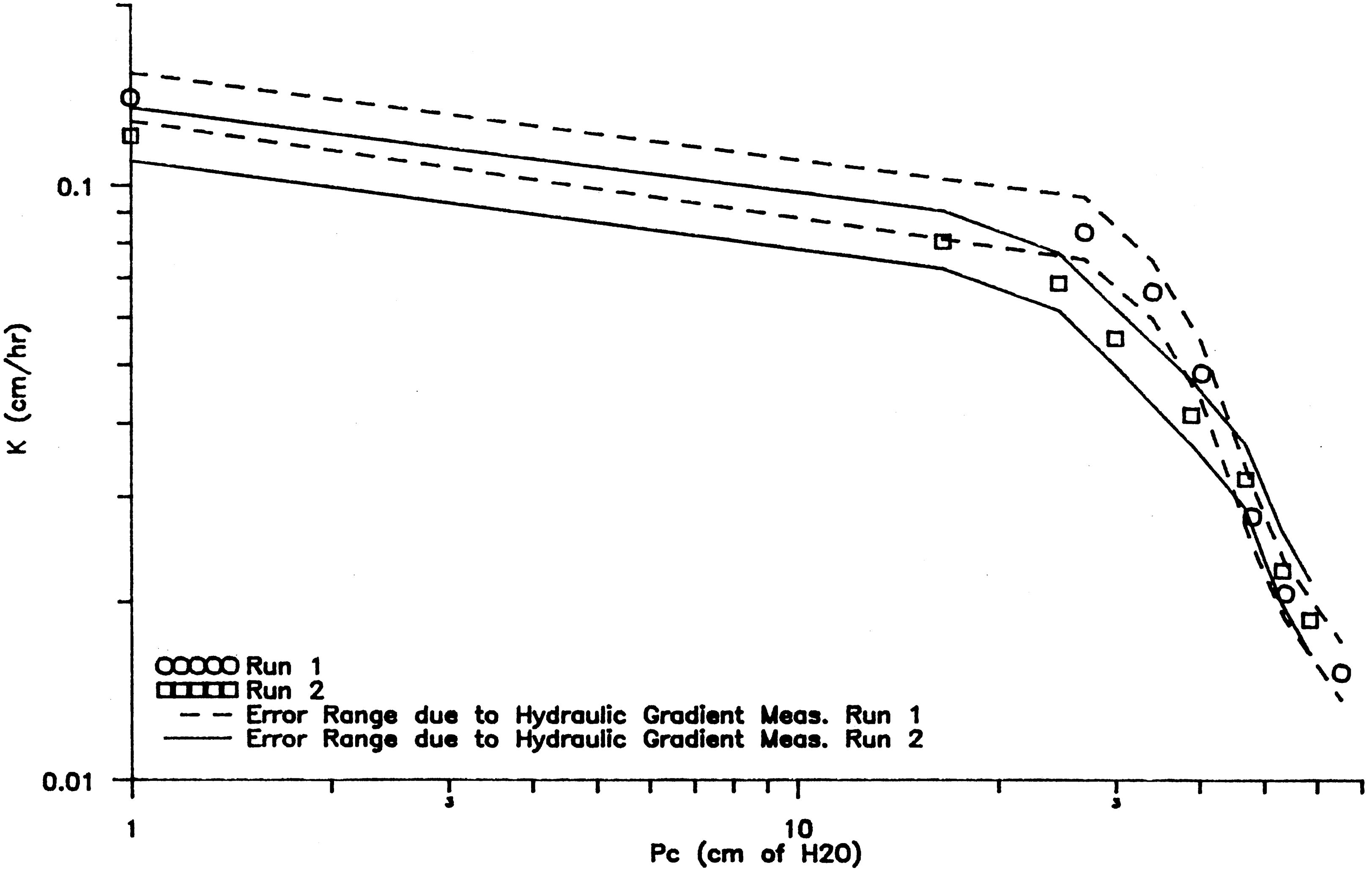

FIGURE 24. Data Reproducability of the Conductivity Function for Teller Soil, Site 2, Depth 53 – 81 cm

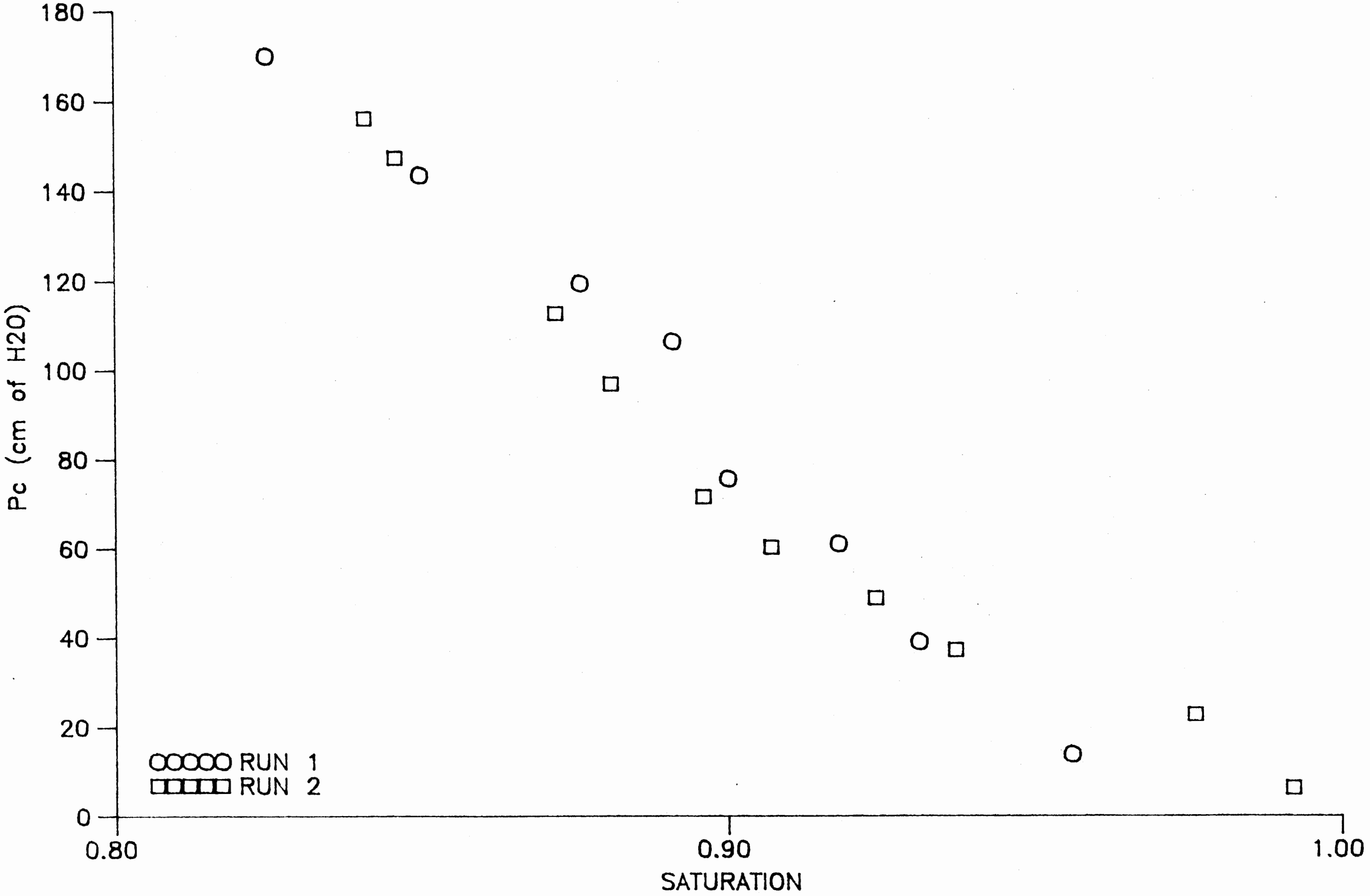

FIGURE 25. Data Reproducability of the Retention Function for Teller Soil, Site 2, Depth 53 – 81 cm.

difference in the two sets of data.

From Figure 24 it is seen that the values of conductivity from the second run are consistently lower than the ones obtained from run one, except for the values obtained at higher capillary pressures. A possible explanation for this observation is that when the sample was resaturated it was not completely saturated with water. This was caused by entrapped air bubbles in the sample, which were not removed when the sample was vacuum saturated. These entrapped air bubbles have the effect of reducing the wetting phase saturation of the sample. Since conductivity is a function of saturation, a reduced value of conductivity is observed. At higher values of capillary pressure the air phase becomes continuous. This removes the effect of the entrapped air bubbles.

In Figure 25, the replicate retention curves are presented. The curve obtained from Run 2 is smoother than the one obtained in Run 1. The scatter observed in the data from run 1 is probably a result of the system not attaining steady-state before the measurements were made.

### Particle-Size Analysis

A particle-size analysis was performed by Phillip Ward on each soil sample utilizing the procedure described by Gee and Bauder (1986). The pipet method was used to obtain the silt and clay fractions. The results of this analysis are presented in Table V.

## TABLE V

## PARTICLE-SIZE ANALYSIS OF BOTH SITES OF A SELECTED TELLER SOIL

| Site | Depth (cm) | % Sand | | | | | % Silt* | | | %Clay |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1mm | 500μ | 250μ | 106μ | 53μ | CRSE | MED | FINE | <2μ |
| 1 | 0 -20 | 0.0 | 1.0 | 19.8 | 33.5 | 13.2 | 18.9 | 4.7 | 1.7 | 7.2 |
| | 23-43 | 0.0 | 1.1 | 14.6 | 24.1 | 8.7 | 19.7 | 5.0 | 1.1 | 25.7 |
| | 53-81 | 0.0 | 0.6 | 15.5 | 29.9 | 9.1 | 27.9 | 6.3 | 0.8 | 9.9 |
| | 107-127 | 0.0 | 1.0 | 18.5 | 33.6 | 15.3 | 17.4 | 3.3 | 2.2 | 8.7 |
| 2 | 0 -20 | 0.0 | 1.3 | 21.1 | 33.2 | 11.3 | 13.1 | 4.2 | 1.2 | 14.6 |
| | 23-43 | 0.0 | 0.8 | 14.9 | 23.4 | 9.9 | 14.2 | 6.7 | 2.4 | 27.7 |
| | 53-81 | 0.0 | 0.8 | 15.1 | 28.9 | 14.4 | 17.8 | 5.6 | 2.3 | 15.0 |
| | 107-127 | 0.1 | 1.3 | 20.3 | 33.4 | 15.2 | 16.8 | 5.5 | 1.9 | 5.5 |

* coarse silt (50-20μ), medium silt (20-5μ), and fine silt (5-2μ)

From these results, the soil texture can be determined. For both sites the 0 - 20, 53 - 81 and 107 - 127 cm. depths are classified as sandy loams. The 23 - 43 cm. depth for both sites is classified as sandy clay loam. These classifications are based on the USDA classification scheme (Gee and Bauder 1986). An increased clay content at the 0 - 20 and 53 - 81 cm. depths at Site 2 is also observed.

## Extrapolation of Laboratory Data to Field Conditions

How well the experimental data represents field conditions encountered during soil drainage is dependent on how much the soil was disturbed when the soil core was obtained.

The soil core can be disturbed by the sleeve sampler in two ways. The first is shear stress occurring at the outer edge of the core when the sampler cuts into the soil. This disturbance affects the soil structure at the edge of the sample. The second disturbance is compaction of the sample. Compaction of the sample is minimized if the core sample is taken at a time when the soil is relatively dry.

The samples obtained from Site 1 were taken when the soil was relatively dry. The soil was wetter when the samples at Site 2 were taken. This could account for the higher bulk densities observed in the Site 2 samples, (Table III), especially at the two lower depths.

Despite these disturbances, these core samples would be expected to maintain some of the structural integrity of the soil.

## Evaluation of Scaling Procedure

### Scaling of Experimental Data

The first step in scaling the experimental data was the determination of the residual saturation. This was done by the extrapolation procedure described by Brooks and Corey (1964). The values obtained for $S_r$ are reported in Table III. It should be noted that the values obtained for all but three of the samples were essentially zero. This is because the retention functions for these samples do not exhibit well defined values of $S_r$. This is either because the samples have a large range of pore sizes or the retention function data were not taken at high enough values of capillary pressure for the function to approach an asymptotic value of $S_r$.

After $S_r$ is determined, values of the $p_b$ and the indices m and e are obtained. This is done by linear regression of ln ($p_c$) as a function of ln ($S_e$). The parameter m is the absolute value of the slope of the regression line. The intercept of the regression line is mln ($p_b$). The index e is the slope of the regression line obtained by regressing values of the ln (K) as a function of ln ($S_e$). The resulting values of these scaling parameters are presented in Table VI.

## TABLE VI

## SCALING PARAMETERS FOR A SELECTED TELLER SOIL

| Site | Depth (cm.) | m | e | $p_b$ (cm $H_2O$) | $K_s$ (cm/hr) |
|---|---|---|---|---|---|
| 1 | 0 -20 | 0.192 | 19.5 | 19.8 | 0.678 |
| | 23-43 | 0.975 | 19.2 | 43.2 | 0.241 |
| | 53-81 | 0.225 | 24.8 | 24.6 | 12.3 |
| | 107-127 | 0.188 | 25.6 | 23.0 | 7.43 |
| 2 | 0 -20 | 0.0823 | 48.8 | 11.3 | 0.368 |
| | 23-43 | 0.369 | 13.6 | 30.7 | 0.416 |
| | 53-81 (run 1) | 0.106 | 27.1 | 27.1 | 0.140 |
| | 53-81 (run 2) | 0.0826 | 22.7 | 17.8 | 0.121 |
| | 107-127 | 0.327 | 10.8 | 60.9 | 0.803 |

The values of the saturated hydraulic conductivities used in scaling the conductivity function are also presented in Table VI. The values used for $K_s$ at the 0 - 20 and 107 - 127 cm. depths at Site 1 and the 23 - 43 cm. depth at Site 2 are not the maximum values of K obtained during the measurements. The maximum values of K for these samples were not used in the calculation of e, because, these maximum values of K deviate from the straight line obtained by plotting K as a function of $S_e$ on a logarithmic scale. These deviations can be seen in Figures 22 and 23. The value of e is only a measure of the pore size distribution of the portion of the curve which fits the power law model. Therefore these values were not used in the calculation of e.

In Figure 26 the scaled conductivity function for the Teller soil is presented. The curve plotted on the graph is the following function:

$$(K/K_s)^{1/e} = S_e. \qquad (6-2)$$

If the conductivity functions were completely described by the power law function, all of the scaled data would lie on this line. The only set of data which appears to deviate significantly from this line is the data for Site 2 at a depth of 0 - 20 cm. Referring back to Figure 23, the data for this sample is seen to deviate from the power law function at the lower values of saturation. This is why this set of data deviates from the expected scaled

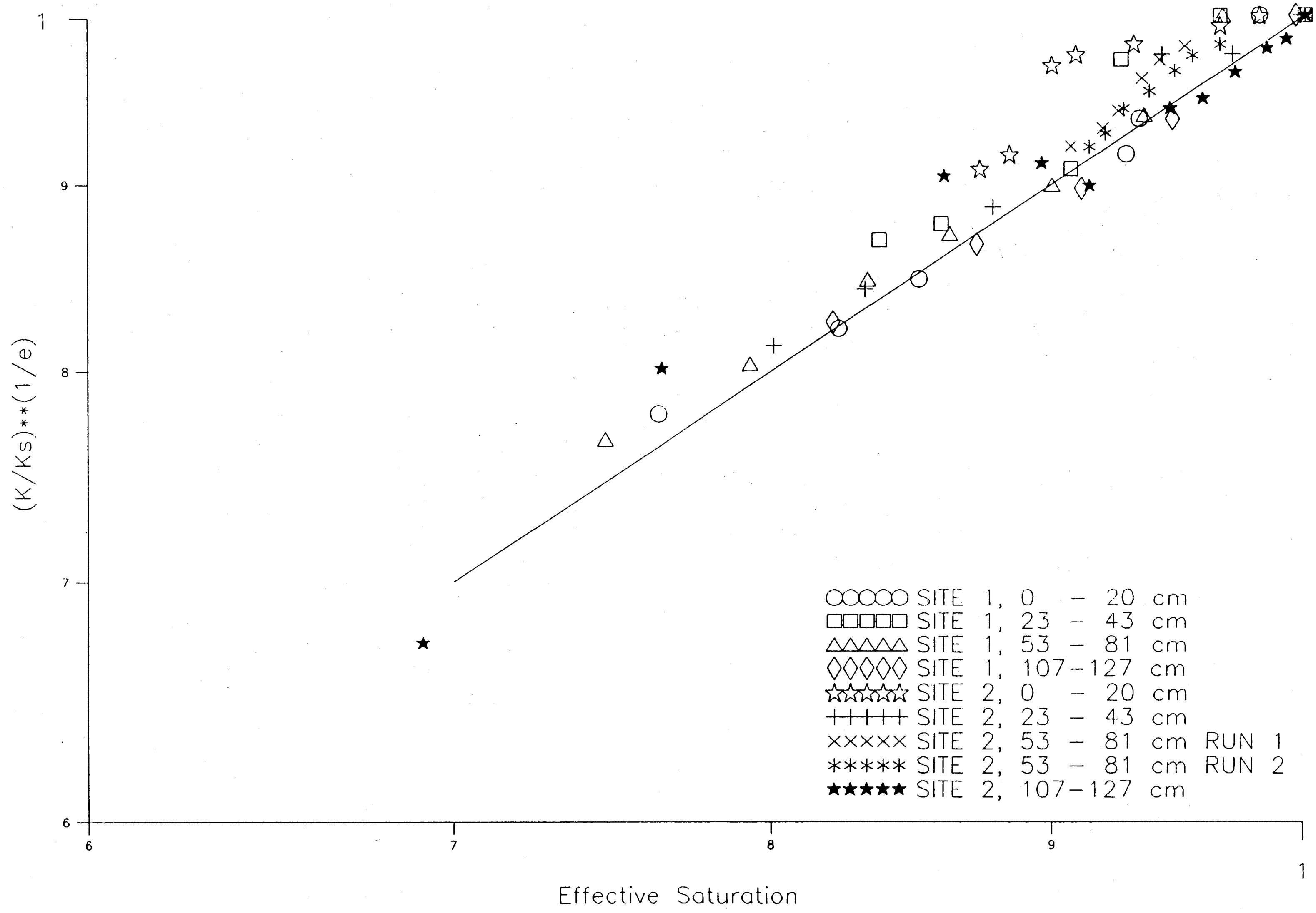

FIGURE 26. Scaled Conductivity Function, Teller Soil

function. The rest of the data are seen to be represented by the scaled function.

In Figure 27 the scaled retention function is shown. The curve plotted on the graph is represented by the following equation:

$$(p_c/p_b)^m = 1/S_e \qquad \text{for } p_c \geqslant p_b. \tag{6-3}$$

All of the scaled data are observed to fall close to this curve, except for the values for scaled capillary pressure less than one. The data for $p_c < p_b$ are the data which give scaled capillary pressures less than one. These data are all seen to have values of effective saturation close to one. They would be well represented by

$$S_e = 1 \quad \text{for } p_c < p_b. \tag{6-4}$$

Which is the same equation presented by Brooks and Corey (1964).

## Statistical Procedure for Determining Equivalence of Slopes

It is useful to have a procedure to determine whether the slopes of the hydraulic functions are equivalent. If the slopes are the same, it is possible to use other less complex scaling procedures.

A statistical procedure for the determination of the equivalence of the slopes of different functions was presented by Volk (1958). This method is based on a

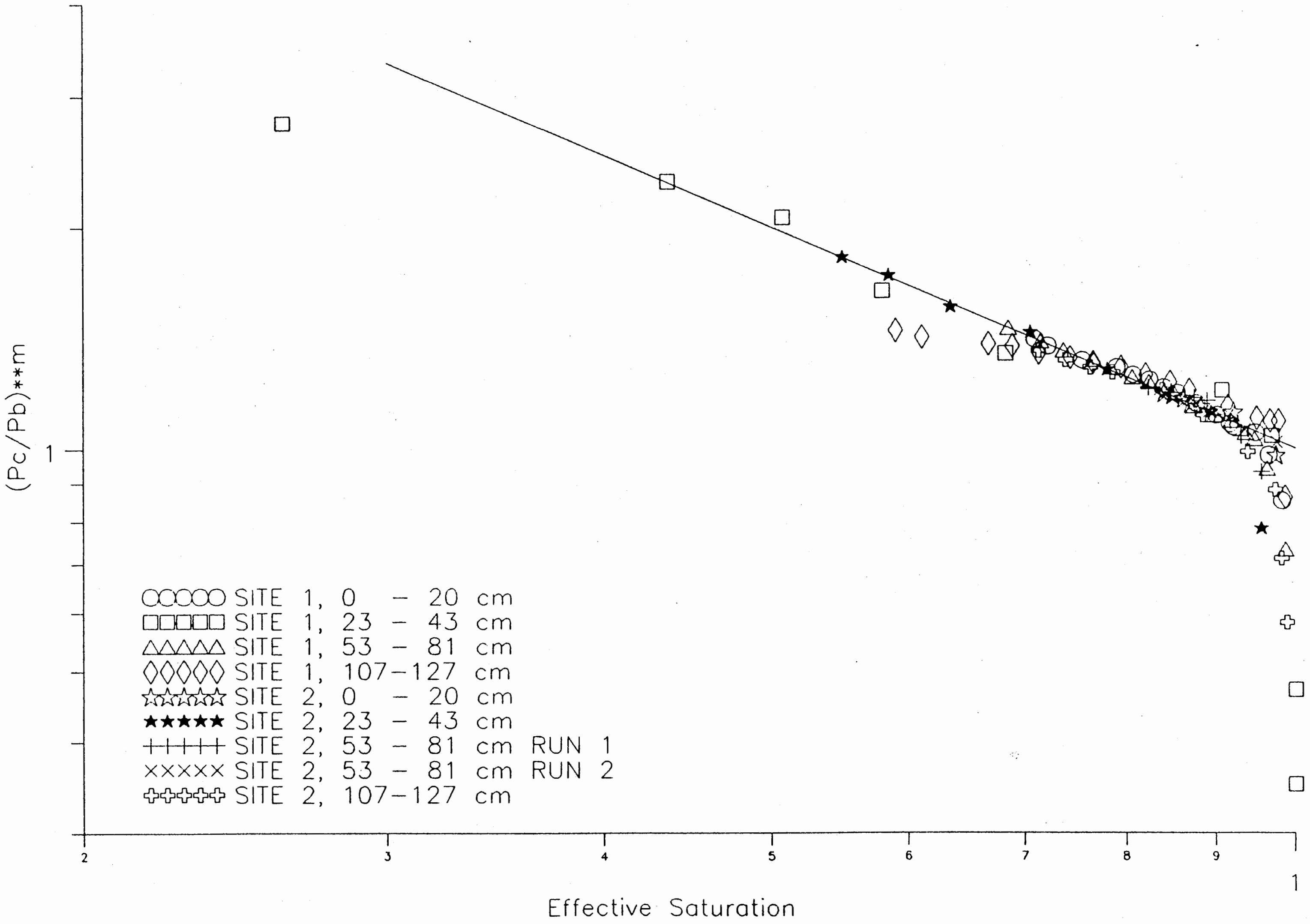

FIGURE 27. Scaled Retention Function, Teller Soil

analysis of variance procedure. It allows for the determination of the answers to the following questions. Will a better correlation result from individual straight lines or from separate straight lines? Should all the lines be drawn with the same slope or with different slopes? Is the displacement in the Y direction between the lines significant?

This analysis of variance procedure was performed on both the conductivity and retention function data. Only the data for the portion of the function which is represented by a straight line on a log-log plot was used in this analysis. The resulting analyses of variances are presented in Tables VII and VIII.

In these tables, the total sum of squares represents the sum of squares of deviation from the best straight line through all the data. The error sum of squares is the sum of squares of deviation from the best straight lines fit to each separate set of data. The between slopes sum of squares is the improvement in the correlation gained by using individual slopes instead of a single pooled slope. The sum of squares in the means correlation row is representative of the best straight line drawn through the means of each set of data. The difference row is the total sum of squares minus the means correlation and between slopes sums of squares.

The row of most interest in the analysis of variance is the between slopes row. The F-value associated with this

## TABLE VII

## ANALYSIS OF VARIANCE FOR THE CONDUCTIVITY FUNCTIONS OF TELLER SOIL

| Source | D.F. | Sum of Squares | Mean Square | F |
|---|---|---|---|---|
| Means Correlation | 6 | 0.1065 | 0.0178 | 10.33* |
| Difference | 1 | 0.0679 | 0.0679 | 39.50* |
| Between Slopes | 7 | 0.0360 | 0.00514 | 2.99* |
| Error | 41 | 0.0705 | 0.00172 | |
| Total | 55 | 0.2809 | | |

* Indicates that the F-value is significant at the 0.05 level of significance.

## TABLE VIII

## ANALYSIS OF VARIANCE FOR THE RETENTION FUNCTIONS OF TELLER SOIL

| Source | D.F. | Sum of Squares | Mean Square | F |
|---|---|---|---|---|
| Means Correlation | 6 | 0.688 | 0.1147 | 18.1* |
| Difference | 1 | 0.089 | 0.089 | 14.0* |
| Between Slopes | 7 | 0.248 | 0.0354 | 5.57* |
| Error | 54 | 0.343 | 0.00635 | |
| Total | 68 | 1.367 | | |

* Indicates that the F-value is significant at the 0.05 level of significance.

row allows for the testing of the following null hypothesis

$H_o$: All the slopes are equivalent

Therefore, if the F-value is significant, the null hypothesis is rejected, and it is concluded that all the slopes are not the same. If this is the case, none of the scaling procedures presented in Chapter II will work. It is then necessary to try the new scaling procedure.

The F-value associated with the means correlation row is an indication of the difference between the means of each data set. If this F-value and the one associated with the between slopes row are not significant, then the data is best represented by a single straight line. The F-value associated with the difference row is an indication of the displacement in the Y direction of the data sets.

Referring to the analysis of variance for the conductivity data in Table VII, the F-value in the between slopes row is 2.99. Using the tables of F-values in Volk (1958), the value obtained for F at 7 and 41 degrees of freedom is 2.24 at the 0.05 level of significance. It can therefore be concluded with greater than 95% confidence that the slopes are not the same. It also should be noted that the other F-values are also significant.

Table VIII is the analysis of variance for the retention function data. The F-value associated with the between slopes row is 5.57. From the table in Volk at 7 and 54 degrees of freedom, the F-value at the 0.05 level of significance is 2.18. So, it is concluded that the slopes are not the same for the retention functions.

From the results of the analysis of variance it is seen that the hydraulic functions of the Teller soil could not be scaled by the procedures described in Chapter II.

## Prediction of Parameters e and m

In addition to the values of parameters e and m obtained for the Teller soil series, values were calculated for five additional data sets. These data sets were for Bethany, Cecil, Tipton and Troup soil series. The data for the Bethany soil was from Sites 4 and 5 reported by Nofziger et al. (1983). The data for the Cecil soil was for the North Plot near Auburn, Alabama, reported by Bruce et al. (1983). The Tipton soil data was from Site 2 reported by Nofziger et al. The Troup soil data was reported by Dane et al. (1983) for the West Plot near Union Springs, Alabama.

All of the data reported in these studies was not used. Data which deviated significantly from a smooth curve or were taken at positive wetting phase pressures was not used. The data which were used is presented in the Appendix B. These data were all obtained by the instantaneous profile field methods. All of these data was taken in the regional field study described by Hornsby et al. (1983).

The calculated values of e and m and the clay contents of these soils are presented in Table IX.

# TABLE IX

## VALUES OF m, e AND CLAY FRACTIONS FOR SOME PUBLISHED DATA

| Source | | m | e | % Clay |
|---|---|---|---|---|
| Bethany Soil Series, Site 5<br>(Nofziger et al. 1983) | | | | |
| | Depth (cm.) | | | |
| | 0 -15 | 0.203 | 16.9 | 25 |
| | 15-30 | 0.123 | 27.4 | 28 |
| | 30-45 | 0.0315 | 65.4 | 37 |
| | 45-60 | 0.0241 | 59.2 | 41 |
| | 60-75 | 0.0137 | 98.0 | 38 |
| | 75-90 | 0.0073 | 137 | 35 |
| | 90-120 | 0.0121 | 131 | 34 |
| Bethany Soil Series, Site 4<br>(Nofziger et al. 1983) | | | | |
| | Depth (cm.) | | | |
| | 0 -15 | 0.264 | 13.7 | 25 |
| | 15-30 | 0.121 | 37.0 | 28 |
| | 30-45 | 0.0322 | 69.9 | 39 |
| | 45-60 | 0.0113 | 131 | 40 |
| | 60-75 | 0.0178 | 97.9 | 37 |
| | 75-90 | 0.0149 | 114 | 34 |
| | 90-120 | 0.0141 | 138 | 33 |
| Cecil Soil Series, North Plot<br>Auburn, AL<br>(Bruce et al. 1983) | | | | |
| | Depth (cm.) | | | |
| | 0 -25 | 0.167 | 10.1 | 29 |
| | 25-50 | 0.753 | 2.8 | 46 |
| | 50-75 | 0.778 | 5.7 | 54 |
| | 75-105 | 0.185 | 13.7 | 47 |
| | 105-160 | 0.0308 | 74.6 | 39 |
| Tipton Soil Series, Site 2<br>(Nofziger et al. 1983) | | | | |
| | Depth (cm.) | | | |
| | 0 -15 | 0.102 | 39.4 | 12 |
| | 30-45 | 0.152 | 28.7 | 16 |
| | 45-60 | 0.202 | 21.1 | 20 |
| | 60-75 | 0.180 | 20.2 | 20 |

TABLE IX continued

| Source | | m | e | % Clay |
|---|---|---|---|---|
| | 75-90 | 0.171 | 20.2 | 21 |
| Troup Series Soil, West Plot<br>Union Springs, AL<br>(Dane et al. 1983) | | | | |
| | Depth (cm.) | | | |
| | 0 -24 | 0.716 | 10.4 | 3 |
| | 24-58 | 0.964 | 8.3 | 3 |
| | 58-92 | 0.817 | 7.7 | 3 |
| | 92-123 | 1.18 | 4.8 | 2 |
| | 123-154 | 0.51 | 6.5 | 6 |

The conductivity function for a large range of relative saturations has been seen to fit a power law function. Therefore, if the value of the exponent can be predicted by the use of retention function data, it would only be necessary to measure retention function data and the saturated hydraulic conductivity to obtain the conductivity function. Of the methods presented in Chapter III for calculating the conductivity function, the method of Laliberte et al. (1968) is the easiest to apply, since it require the least amount of calculations. This method has also been reported to give as accurate results as any of the other methods (Brust et al. 1968). The method of Laliberte et al. (1968) relates the exponent of the retention function to the exponent of the conductivity function as follows:

$$e = (2 + 3m)/m. \qquad (6\text{-}5)$$

In evaluating this method it is useful to plot e as a function of m. This is done in Figure 28. The solid curve on the graph is the relationship predicted by Equation (6-5) method. From this plot it was also observed that the data might be fit well by a power law function.

A power law function was fit to the data and the following equation was obtained:

$$e = 8.6\ m^{-0.6}. \qquad (6\text{-}6)$$

A coefficient of correlation of 0.978 was obtained for this

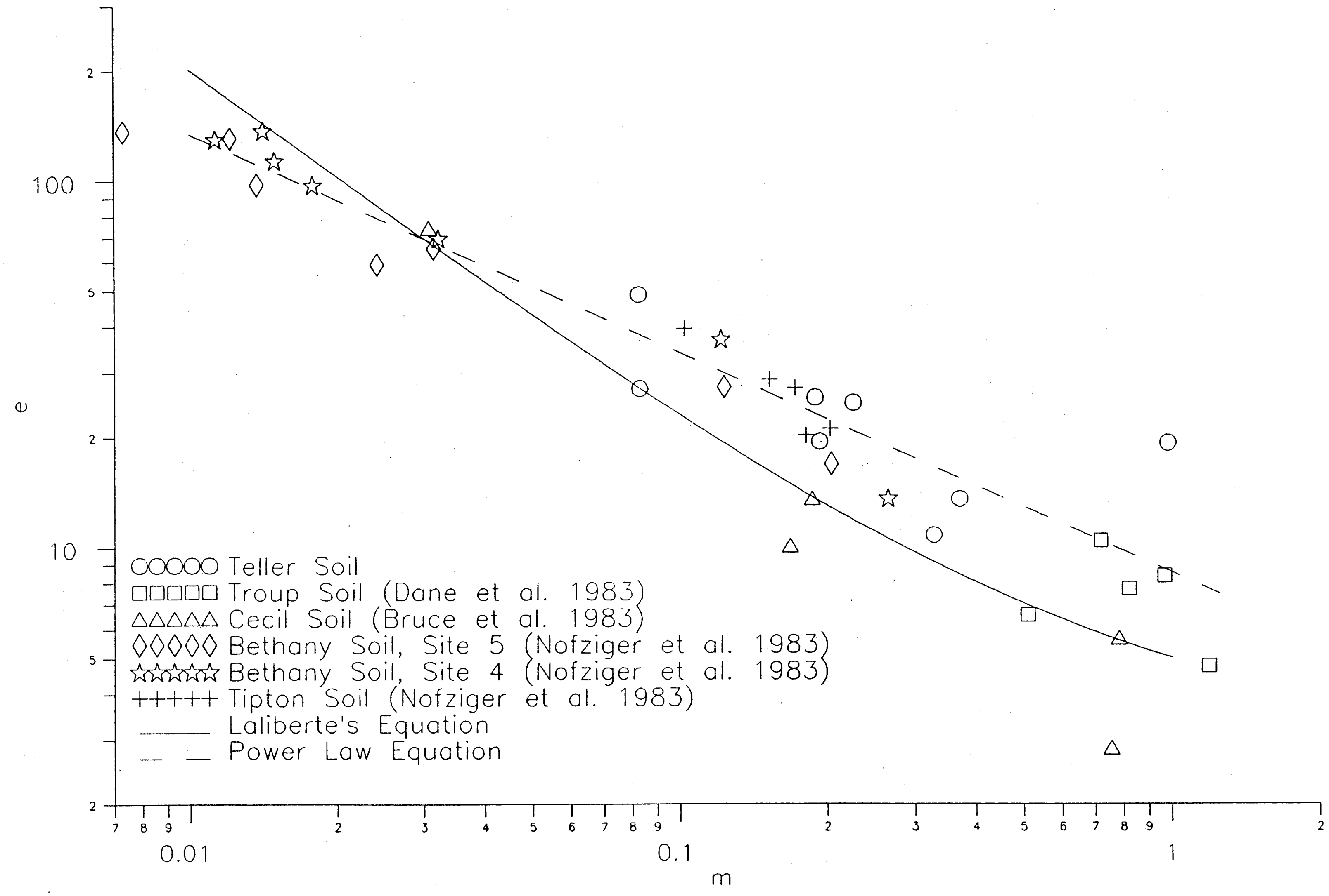

FIGURE 28. e vs. m For Several Experimental Data Sets

fit. This indicates the data are highly correlated by this equation. Equation 6-6 is represented by the dashed curve on the graph.

The values of e predicted by both the method proposed by Laliberte et al. (1968) and the power law equation for the Teller soil data as well as the Bethany, Cecil, Tipton and Troup soils are presented in Table X. The percent errors were calculated by the following equation:

$$\frac{|\text{calculated value} - \text{experimental value}|}{\text{experimental value}} \times 100\ \% \qquad (6\text{-}7)$$

The average percent error for both methods was 33 percent. It is noticed that the largest deviations from the power law model occur for the Cecil soil data. The average error for the power law excluding the Cecil data is 21 percent, while that of Equation (6-5) is still 33 percent. If the data for Cecil soil is excluded the power law model gives better results for this data set than Equation (6-5).

The effect of errors in the prediction of the value of e is illustrated in Figure 29. In this figure the conductivity function data for Teller soil at Site 1 and a depth of 55 - 81 cm. is plotted. In addition the functions predicted by the fitted value of e and the values of e calculated by both Equation (6-5) and the power law model are also plotted. The error in the value of e predicted by Equation (6-5) is 52 percent. The error in the value of e predicted by power law model is 17 percent.

# TABLE X

## COMPARISON OF ACTUAL AND PREDICTED VALUES OF THE SLOPES OF THE CONDUCTIVITY FUNCTIONS

| Soil | Depth (cm.) | e | Predicted e | | | |
|---|---|---|---|---|---|---|
| | | | EQN (6-5) | %ERR | Power Law | %ERR |
| Teller | | | | | | |
| (Site 1) | 0 -20 | 19.5 | 13.4 | 31.3 | 22.7 | 16.8 |
| | 23-43 | 19.2 | 5.1 | 73.4 | 8.6 | 55.3 |
| | 53-81 | 24.8 | 11.9 | 52.0 | 20.7 | 16.6 |
| | 107-127 | 25.6 | 13.6 | 46.9 | 23.0 | 10.1 |
| (Site 2) | 0 -20 | 48.8 | 27.3 | 44.1 | 37.7 | 22.6 |
| | 23-43 | 13.6 | 8.4 | 38.2 | 15.4 | 13.1 |
| | 53-81 | 27.1 | 27.2 | 0.4 | 37.7 | 39.1 |
| | 107-127 | 10.8 | 9.1 | 15.7 | 16.5 | 52.8 |
| Troup | | | | | | |
| | 0 -24 | 10.4 | 5.8 | 44.2 | 10.3 | 0.6 |
| | 24-58 | 8.3 | 5.1 | 38.9 | 8.7 | 3.8 |
| | 58-92 | 7.7 | 5.4 | 29.7 | 9.6 | 24.7 |
| | 92-123 | 4.8 | 4.7 | 1.1 | 7.7 | 61.1 |
| | 123-154 | 6.5 | 6.9 | 6.1 | 12.7 | 94.8 |
| Cecil | | | | | | |
| | 0 -25 | 10.1 | 15.0 | 48.5 | 24.7 | 144 |
| | 25-50 | 2.8 | 5.7 | 101 | 10.0 | 253 |
| | 50-75 | 5.7 | 5.6 | 1.1 | 9.8 | 73.8 |
| | 75-105 | 13.7 | 13.8 | 0.7 | 23.2 | 70.1 |
| | 105-160 | 74.6 | 67.9 | 9.0 | 68.0 | 8.9 |
| Bethany | | | | | | |
| (Site 5) | 0 -15 | 17.0 | 12.9 | 24.1 | 22.0 | 29.8 |
| | 15-30 | 27.4 | 19.3 | 29.6 | 29.7 | 8.5 |
| | 30-45 | 65.4 | 66.5 | 1.7 | 67.1 | 2.6 |
| | 45-60 | 59.2 | 86.0 | 45.3 | 78.7 | 33.1 |
| | 60-75 | 98.0 | 149 | 52.0 | 110 | 12.7 |
| | 75-90 | 137 | 277 | 102 | 161 | 17.6 |
| | 90-120 | 131 | 168 | 28.2 | 119 | 9.5 |
| Bethany | | | | | | |
| (Site 4) | 0 -15 | 13.7 | 10.6 | 22.6 | 18.8 | 37.3 |
| | 15-30 | 37.0 | 19.5 | 47.3 | 30.0 | 18.9 |
| | 30-45 | 69.9 | 65.1 | 6.9 | 66.2 | 5.3 |
| | 45-60 | 131 | 180 | 27.2 | 124 | 5.2 |
| | 60-75 | 97.9 | 115 | 17.5 | 94.4 | 3.5 |
| | 75-90 | 114 | 137 | 20.2 | 105 | 7.8 |
| | 90-120 | 138 | 145 | 5.1 | 108 | 21.3 |
| Tipton | | | | | | |
| | 0 -15 | 39.4 | 22.6 | 42.6 | 33.2 | 15.6 |
| | 30-45 | 28.7 | 16.2 | 43.6 | 26.2 | 9.0 |
| | 45-60 | 21.1 | 12.9 | 38.9 | 22.1 | 4.4 |

TABLE X continued

| Soil | Depth (cm.) | e | Predicted e | | | |
|---|---|---|---|---|---|---|
| | | | EQN (6-5) | %ERR | Power Law | %ERR |
| | 60-75 | 20.2 | 14.1 | 30.2 | 23.6 | 16.7 |
| | 75-90 | 27.3 | 14.7 | 46.2 | 24.3 | 10.6 |

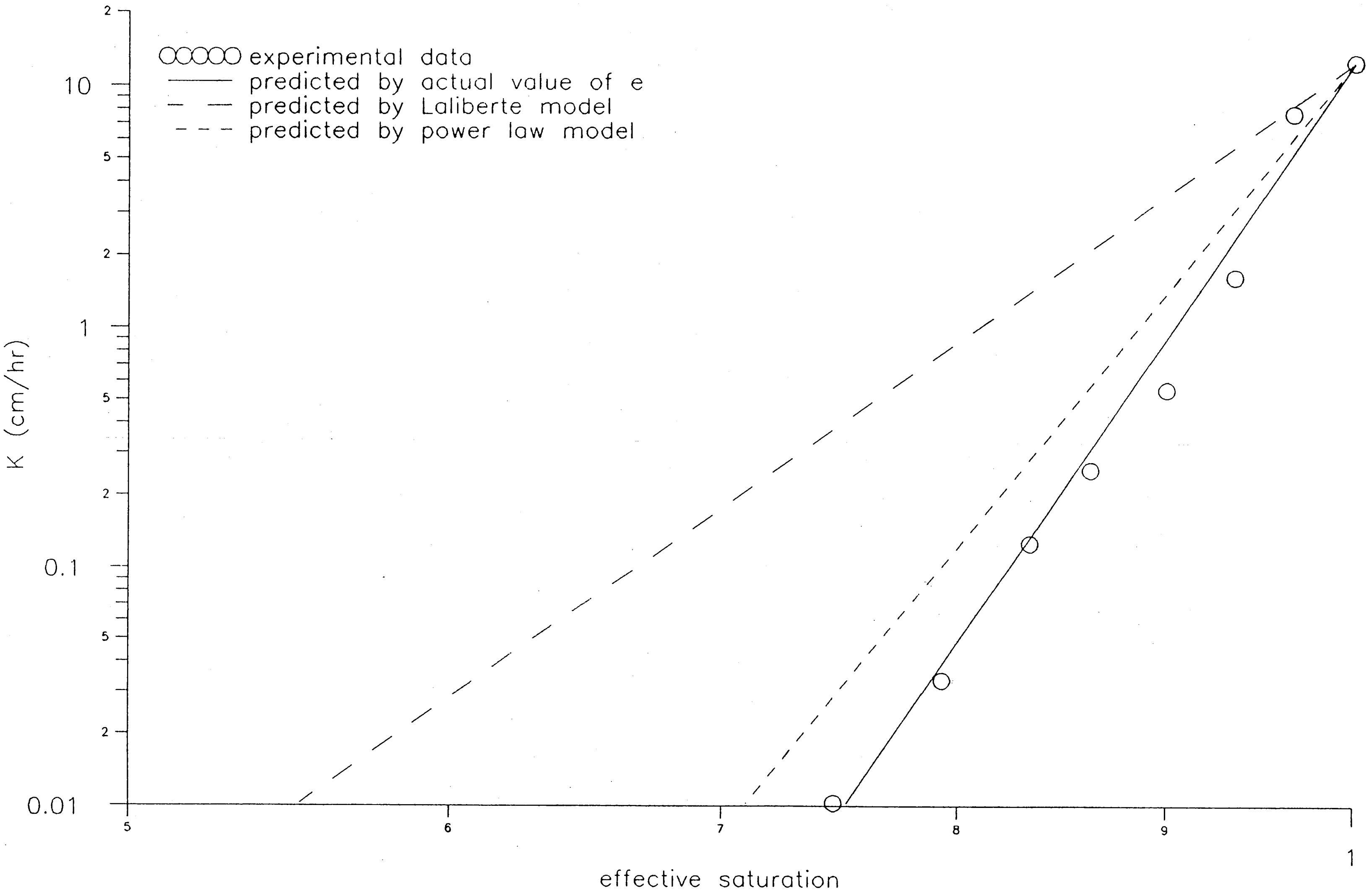

Figure 29. Comparision of conductivities predicted by different methods of calculating e

From Figure 29 it is seen that errors in e cause increasing errors in the predicted conductivity as saturation decreases. In Table XI the experimental and calculated conductivities are compared. From this table it is observed that errors in the predicted values in e can rapidly lead to larger errors in the predicted conductivity.

Clapp and Hornberger (1978) presented results that indicated, that the index m was highly correlated with the clay content of the soil. The values of clay content and m in Table IX are plotted in Figure 30. The curve on the plot represents the correlation obtained by Clapp and Hornberger. The solid stars are the average values obtained by Clapp and Hornberger for each soil texture in their data set.

From Figure 30 it is seen that m may be a function of clay content. But, there are obviously some other factors influencing the value of m. These other factors could be bulk density, silt fraction or some other factor describing the soil fabric that influences the soil structure.

## TABLE XI

## COMPARISION OF CONDUCTIVITIES PREDICTED BY DIFFERENT METHODS OF CALCULATING e

| $S_e$ | K | Predicted K | | | |
|---|---|---|---|---|---|
| | | EQN (6-5) | %Err | Power Law | %Err |
| 1.0 | 12.3 | 12.3 | 0.0 | 12.3 | 0.0 |
| 0.97 | 7.56 | 8.56 | 13.2 | 6.55 | 13.4 |
| 0.94 | 1.61 | 5.89 | 266 | 3.42 | 112 |
| 0.90 | 0.545 | 3.51 | 544 | 1.39 | 155 |
| 0.86 | 0.253 | 2.04 | 706 | 0.542 | 114 |
| 0.83 | 0.126 | 1.34 | 963 | 0.260 | 106 |
| 0.79 | 0.0333 | 0.74 | 2230 | 0.094 | 181 |
| 0.75 | 0.0103 | 0.40 | 3790 | 0.032 | 210 |

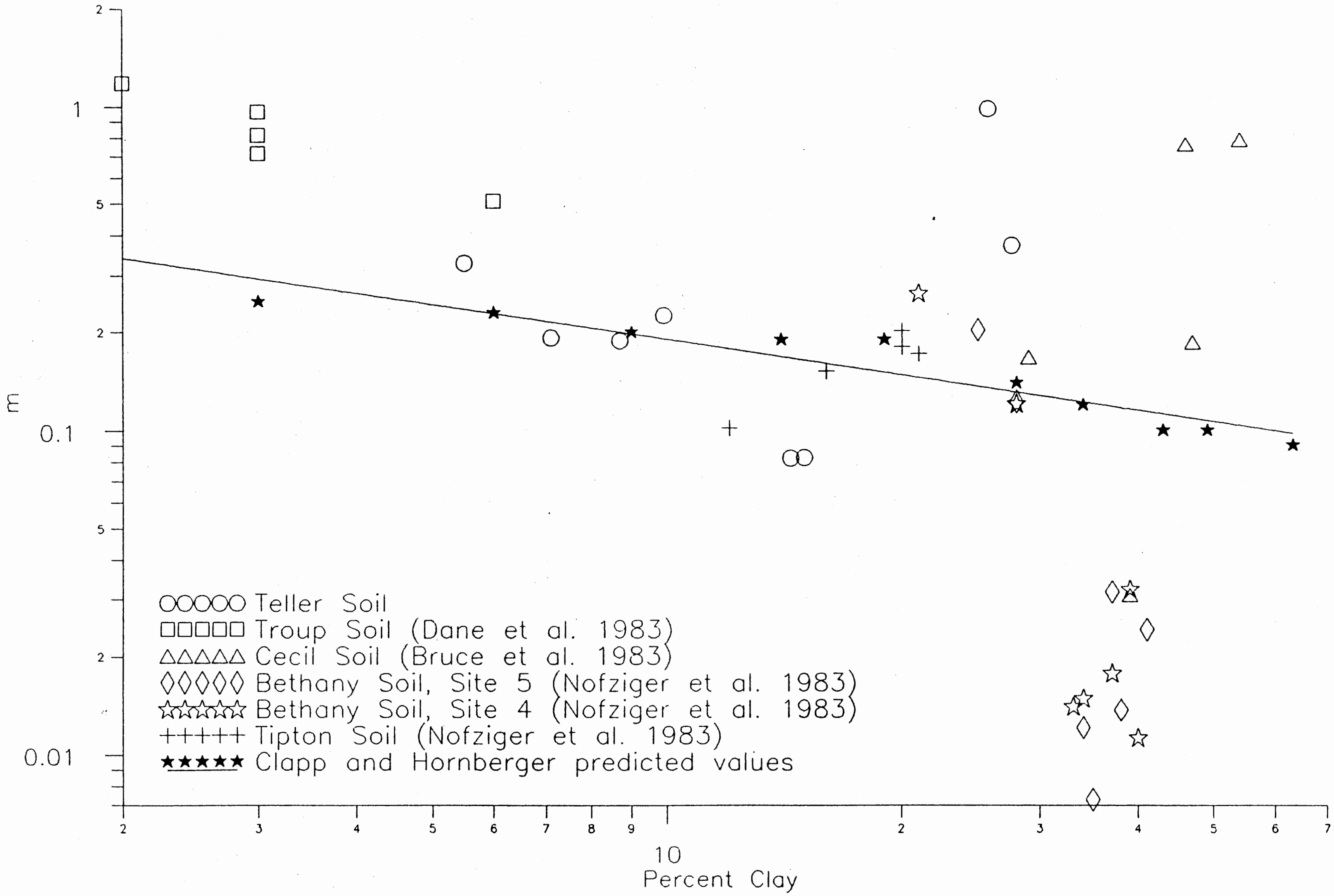

FIGURE 30. m vs. Clay Content For Several Experimental Data Sets

# CHAPTER VII

## CONCLUSIONS AND RECOMMENDATIONS

The hydraulic properties of soil are spatially variable. In order to describe the unsaturated flow of water through soil it is necessary to account for this spatial variability. Previous methods for describing this spatial variability can only be applied if the hydraulic functions are represented by parallel curves.

A new method of describing the spatial variability of the unsaturated conductivity and retention functions was developed. This method is based on the fact that both the conductivity and retention functions exhibit behavior which can be fit to a power law function. The exponent of the power law function can be used to account for nonparallel variability in the hydraulic properties of soils.

There is a limited amount of data available which describes the spatial variability of the hydraulic properties of soils. Because of this an experimental method to determine the conductivity and retention functions for "undisturbed" soil core samples was developed. This method was then used to determine the conductivity and retention functions of a Teller soil series at two different sites and four different depths at each site.

The hydraulic functions of the Teller soil series were observed to not be representable by parallel curves. The new scaling procedure was able to account for most of the spatial variability exhibited by the experimental data. This scaling procedure will work for all data which exhibits power law behavior. It will also work for data which deviates from power law behavior as long as all of the data deviates in the same manner.

In addition the accuracy of two methods for predicting the exponent (e) of the power law describing the conductivity function were evaluated. Both of these methods use the exponent (m) obtained from the retention function to predict the exponent in the conductivity function. There is a high degree of correlation between these exponents. But both methods were observed to have average percent errors of 33 percent in the prediction of e. These errors were shown to lead to large errors in the prediction of the actual values of the conductivity. This indicates that there may be some factor other than m which must be considered in the prediction of the conductivity function.

Finally, it was shown that the exponent m could not be predicted as a function of clay content alone. It is also a function of some other property of the soil. This other property could be the soil fabric.

In order to determine these additional properties of soil, the available data base of conductivity and retention functions of soils must be extended. These data must to be taken in a consistent manner. Since the precision of field methods is questionable, it is recommended that this data be taken on "undisturbed" soil cores by the methods described in this work.

The experimental methods used in this work were shown to have the following major sources of error. First, the pressure measurement is not very precise. Second, it is difficult to determine when the system has attained a steady-state. The first source of error can easily be reduced by using a more precise source of pressure measurement such as pressure transducers. The second source of error could be reduced by using a continuous computer monitoring system to record data and determine when the system has attained a steady-state.

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# APPENDIX A

## TYPIFYING PEDON DESCRIPTION OF A TELLER SOIL SERIES (Henely et al. 1987)

Teller soil is classified as a fine-loamy, mixed, thermic Udic Argiustoll.

| Horizon | |
|---|---|
| A(0 - 15 cm) | Brown (10YR 5/3) dry fine sandy loam, dark brown (10YR 3/3) moist; weak fine and medium granular structure; slightly hard, very friable; medium acid; clear smooth boundary. (0 to 25 cm. thick) |
| A1(15 - 38 cm) | Brown (10YR 4/3) dry fine sandy loam, dark brown (10YR 3/3) moist; moderate medium and fine granular structure; slightly hard, friable; medium acid; gradual smooth boundary. (15 to 38 cm. thick) |
| B1(38 - 51 cm) | Brown (7.5YR 4/4) dry fine sandy loam, dark brown (7.5YR 3/4) moist; compound weak medium subangular blocky and moderate medium granular structure; hard, friable; medium acid; gradual smooth boundary. (0 to 20 cm. thick) |
| B21t(51-81 cm) | Yellowish red (5YR 4/6) dry sandy clay loam, yellowish red (5YR 3/6) moist; moderate medium subangular blocky structure; hard, firm; thin nearly continous clay films on faces of peds; medium acid; gradual smooth boundary. (15 to 51 cm. thick) |
| B22t(81-107 cm) | Yellowish red (5YR 5/6) dry sandy clay loam, yellowish red (5YR 4/6) moist; compound weak coarse prismatic and weak medium subangular blocky structure; hard, firm; patchy clay films on faces of peds; medium acid; gradual smooth boundary. (15 to 51 cm. thick) |
| B23t(107-152 cm) | Yellowish red (5YR 5/6) dry fine sandy loam, yellowish red (5YR 4/6) moist; weak coarse prismatic structure; hard, friable; patchy clay films on faces of peds; medium acid; diffuse smooth boundary. (25 to 76 cm. thick) |

## APPENDIX A continued

| | |
|---|---|
| C(152-178 cm) | Yellowish red (5YR 5/6) dry fine sandy loam, yellowish red (5YR 4/6) moist; massive; slightly hard, friable; medium acid. |

# APPENDIX B

## CONDUCTIVITY AND RETENTION FUNCTION DATA OF PUBLISHED DATA

| Source | Depth (cm.) | Water Content | K (cm/hr) | $p_c$ (cm) |
|---|---|---|---|---|
| Bethany, Site 5<br>(Nofziger et al. 1983) | 0 -15 | 0.351 | 0.000237 | 99 |
| | | 0.353 | 0.000266 | 97 |
| | | 0.363 | 0.000392 | 92 |
| | | 0.374 | 0.000628 | 84 |
| | | 0.380 | 0.000835 | 73 |
| | | 0.389 | 0.001180 | 65 |
| | | 0.395 | 0.001550 | 59 |
| | | 0.405 | 0.002340 | 51 |
| | | 0.414 | 0.003490 | 44 |
| | 15-30 | 0.343 | 0.000994 | 97 |
| | | 0.345 | 0.00153 | 92 |
| | | 0.351 | 0.00618 | 86 |
| | | 0.356 | | 79 |
| | | 0.363 | | 66 |
| | | 0.368 | 0.0159 | 59 |
| | | 0.372 | 0.0293 | 53 |
| | | 0.378 | | 45 |
| | | 0.384 | 0.0366 | 38 |
| | | 0.392 | 0.066 | 32 |
| | | 0.401 | 0.107 | 27 |
| | | 0.405 | 0.166 | 24 |
| | | 0.409 | 0.258 | 22 |
| | | 0.417 | 0.294 | 18 |
| | 30-45 | 0.341 | 0.002578 | 89 |
| | | 0.342 | 0.003722 | 81 |
| | | 0.346 | 0.004901 | 68 |
| | | 0.347 | 0.007526 | 60 |
| | | 0.350 | 0.011063 | 51 |
| | | 0.352 | 0.012600 | 43 |
| | | 0.355 | 0.018149 | 35 |
| | | 0.357 | 0.027744 | 29 |
| | | 0.360 | 0.074678 | 21 |
| | | 0.366 | 0.167620 | 13 |
| | | 0.367 | 0.434656 | 9 |
| | | 0.371 | 1.156000 | 5 |
| | 45-60 | 0.357 | 0.062800 | 4 |
| | | 0.356 | 0.056248 | 8 |
| | | 0.354 | 0.046146 | 9 |
| | | 0.352 | 0.027159 | 15 |
| | | 0.350 | 0.021740 | 23 |

## APPENDIX B continued

| Source | Depth (cm.) | Water Content | K (cm/hr) | $p_c$ (cm) |
|---|---|---|---|---|
| | 45-60 | 0.348 | 0.015264 | 29 |
| | | 0.345 | 0.010252 | 41 |
| | | 0.343 | 0.006647 | 46 |
| | | 0.342 | 0.006435 | 54 |
| | | 0.340 | 0.004017 | 66 |
| | | 0.337 | 0.001834 | 81 |
| | 60-75 | 0.337 | 0.079438 | 5 |
| | | 0.336 | 0.058002 | 7 |
| | | 0.335 | 0.038330 | 8 |
| | | 0.334 | 0.029260 | 12 |
| | | 0.332 | 0.019589 | 18 |
| | | 0.331 | 0.013915 | 24 |
| | | 0.329 | 0.006159 | 37 |
| | | 0.328 | 0.004622 | 46 |
| | | 0.327 | 0.003330 | 53 |
| | | 0.326 | 0.001997 | 64 |
| | | 0.325 | 0.001875 | 74 |
| | 75-90 | 0.327 | 0.061535 | 7 |
| | | 0.326 | 0.036696 | 8 |
| | | 0.324 | 0.018778 | 16 |
| | | 0.322 | 0.007465 | 32 |
| | | 0.321 | 0.003381 | 45 |
| | | 0.320 | 0.001784 | 58 |
| | | 0.319 | 0.001214 | 70 |
| | 90-120 | 0.326 | 0.129754 | 3 |
| | | 0.325 | 0.066212 | 6 |
| | | 0.324 | 0.040933 | 7 |
| | | 0.322 | 0.020101 | 12 |
| | | 0.321 | 0.008782 | 21 |
| | | 0.320 | 0.005488 | 27 |
| | | 0.319 | 0.002836 | 41 |
| | | 0.318 | 0.001722 | 57 |
| | | 0.317 | 0.001319 | 67 |
| Bethany, Site 4 (Nofziger et al. 1983) | 0 -15 | 0.374 | 0.000134 | 105 |
| | | 0.378 | 0.000166 | 98 |
| | | 0.383 | 0.000228 | 85 |
| | | 0.388 | 0.000317 | 77 |
| | | 0.391 | 0.000397 | 72 |
| | | 0.395 | 0.000498 | 69 |
| | | 0.400 | 0.000689 | 62 |
| | | 0.405 | 0.000953 | 58 |
| | | 0.408 | 0.001150 | 53 |
| | | 0.412 | 0.001470 | 49 |
| | | 0.417 | 0.002030 | 46 |

## APPENDIX B continued

| Source | Depth (cm.) | Water Content | K (cm/hr) | $p_c$ (cm) |
|---|---|---|---|---|
| | 0 -15 | 0.421 | 0.002760 | 43 |
| | | 0.426 | 0.003750 | 40 |
| | | 0.432 | 0.005340 | 37 |
| | | 0.434 | 0.006450 | 35 |
| | | 0.438 | 0.008290 | 33 |
| | | 0.444 | 0.011900 | 31 |
| | | 0.452 | 0.020100 | 24 |
| | 15-30 | 0.366 | 0.0056 | 71 |
| | | 0.367 | 0.0062 | 68 |
| | | 0.369 | 0.0097 | 66 |
| | | 0.370 | 0.0105 | 64 |
| | | 0.371 | 0.0194 | 62 |
| | | 0.380 | 0.0373 | 46 |
| | | 0.387 | 0.0416 | 39 |
| | | 0.390 | 0.1190 | 36 |
| | | 0.394 | 0.2430 | 33 |
| | | 0.398 | 0.2970 | 30 |
| | 30-45 | 0.352 | 0.000756 | 83 |
| | | 0.355 | 0.001290 | 66 |
| | | 0.356 | 0.002610 | 60 |
| | | 0.357 | 0.002800 | 58 |
| | | 0.360 | 0.003690 | 47 |
| | | 0.362 | 0.006630 | 41 |
| | | 0.364 | 0.010900 | 35 |
| | | 0.365 | 0.018100 | 32 |
| | | 0.368 | 0.036600 | 24 |
| | | 0.370 | 0.059800 | 20 |
| | | 0.373 | 0.058400 | 15 |
| | | 0.374 | 0.062600 | 12 |
| | | 0.376 | 0.122000 | 10 |
| | | 0.379 | 0.169000 | 2 |
| | 45-60 | 0.353 | 0.001880 | 70 |
| | | 0.355 | 0.004870 | 45 |
| | | 0.357 | 0.008820 | 30 |
| | | 0.358 | 0.015200 | 27 |
| | | 0.359 | 0.028200 | 19 |
| | | 0.361 | 0.041700 | 10 |
| | | 0.362 | 0.093600 | 7 |
| | 60-75 | 0.330 | 0.001020 | 67 |
| | | 0.331 | 0.002670 | 37 |
| | | 0.332 | 0.008090 | 19 |
| | | 0.333 | 0.019300 | 10 |
| | | 0.334 | 0.050000 | 8 |

## APPENDIX B continued

| Source | Depth (cm.) | Water Content | K (cm/hr) | $p_c$ (cm) |
|---|---|---|---|---|
| | 75-90 | 0.316 | 0.000932 | 59 |
| | | 0.317 | 0.001550 | 42 |
| | | 0.318 | 0.002550 | 31 |
| | | 0.319 | 0.003780 | 24 |
| | | 0.320 | 0.006960 | 17 |
| | | 0.321 | 0.009250 | 14 |
| | | 0.322 | 0.017000 | 9 |
| | | 0.323 | 0.040000 | 2 |
| | 90-120 | 0.311 | 0.000868 | 54 |
| | | 0.312 | 0.002090 | 34 |
| | | 0.313 | 0.003890 | 25 |
| | | 0.314 | 0.007330 | 17 |
| | | 0.315 | 0.014400 | 11 |
| | | 0.316 | 0.045800 | 2 |
| Cecil, North Plot<br>Auburn, AL<br>(Bruce et al. 1983) | 0 -25 | 0.334 | 0.000138 | 210 |
| | | 0.336 | | 190 |
| | | 0.339 | | 171 |
| | | 0.342 | | 151 |
| | | 0.345 | | 132 |
| | | 0.348 | 0.000282 | 114 |
| | | 0.353 | | 79 |
| | | 0.355 | | 67 |
| | | 0.363 | 0.002080 | 45 |
| | | 0.385 | 0.005740 | 27 |
| | | 0.392 | 0.008320 | 22 |
| | | 0.400 | 0.017900 | 18 |
| | | 0.410 | 0.022200 | 14 |
| | | 0.422 | 0.026800 | 10 |
| | | 0.437 | 0.047200 | 4 |
| | 25-50 | 0.403 | | 120 |
| | | 0.404 | 0.000411 | 105 |
| | | 0.405 | | 99 |
| | | 0.406 | | 92 |
| | | 0.408 | | 70 |
| | | 0.413 | 0.002070 | 61 |
| | | 0.421 | | 46 |
| | | 0.425 | 0.008160 | 34 |
| | | 0.433 | 0.011800 | 27 |
| | | 0.449 | 0.021400 | 20 |
| | | 0.464 | 0.030400 | 13 |
| | | 0.471 | 0.042200 | 9 |
| | | 0.477 | 0.062200 | 4 |

## APPENDIX B continued

| Source | Depth (cm.) | Water Content | K (cm/hr) | $p_c$ (cm) |
|---|---|---|---|---|
| | 50-75 | 0.497 | 0.0680 | 4 |
| | | 0.491 | 0.0703 | 12 |
| | | 0.479 | 0.0695 | 20 |
| | | 0.464 | 0.0620 | 30 |
| | | 0.456 | 0.0361 | 37 |
| | | 0.452 | 0.00621 | 51 |
| | | 0.446 | 0.00810 | 70 |
| | | 0.442 | 0.000559 | 82 |
| | | 0.440 | 0.000265 | 98 |
| | | 0.439 | 0.000202 | 107 |
| | | 0.438 | 0.000165 | 116 |
| | | 0.437 | 0.000157 | 132 |
| | 75-105 | 0.476 | 0.0480 | 7 |
| | | 0.469 | 0.0387 | 18 |
| | | 0.464 | 0.0304 | 27 |
| | | 0.459 | 0.0217 | 36 |
| | | 0.454 | 0.00266 | 57 |
| | | 0.448 | 0.00358 | 80 |
| | | 0.444 | 0.000398 | 99 |
| | | 0.443 | 0.000318 | 115 |
| | | 0.442 | 0.000266 | 129 |
| | | 0.440 | 0.000152 | 177 |
| | 105-160 | 0.426 | 0.105 | 26 |
| | | 0.412 | 0.0844 | 33 |
| | | 0.407 | 0.0362 | 42 |
| | | 0.402 | 0.0457 | 53 |
| | | 0.400 | 0.00778 | 104 |
| | | 0.397 | 0.00158 | 121 |
| | | 0.396 | 0.00133 | 134 |
| | | 0.395 | 0.00114 | 144 |
| | | 0.393 | 0.000917 | 155 |
| | | 0.392 | 0.000711 | 166 |
| | | 0.391 | 0.000570 | 174 |
| | | 0.389 | 0.000338 | 187 |
| Tipton, Site 2 (Nofziger et al. 1983) | 0 -15 | 0.371 | 0.073641 | 31 |
| | | 0.364 | 0.028325 | 40 |
| | | 0.359 | 0.015343 | 45 |
| | | 0.355 | 0.010229 | 50 |
| | | 0.353 | 0.007672 | 58 |
| | | 0.347 | 0.003866 | 72 |
| | | 0.343 | 0.002144 | 84 |
| | | 0.339 | 0.001345 | 90 |
| | | 0.335 | 0.000904 | 97 |
| | | 0.333 | 0.000689 | 101 |

## APPENDIX B continued

| Source | Depth (cm.) | Water Content | K (cm/hr) | $p_c$ (cm) |
|---|---|---|---|---|
| | 0 -15 | 0.331 | 0.000550 | 103 |
| | | 0.330 | 0.000455 | 107 |
| | | 0.329 | 0.000387 | 109 |
| | | 0.327 | 0.000312 | 114 |
| | | 0.325 | 0.000262 | 119 |
| | | 0.324 | 0.000225 | 124 |
| | | 0.323 | 0.000198 | 129 |
| | | 0.322 | 0.000184 | 131 |
| | 30-45 | 0.296 | 0.323680 | 32 |
| | | 0.287 | 0.164660 | 36 |
| | | 0.282 | 0.100543 | 39 |
| | | 0.278 | 0.098309 | 42 |
| | | 0.276 | 0.065058 | 46 |
| | | 0.270 | 0.031668 | 56 |
| | | 0.264 | 0.020335 | 64 |
| | | 0.260 | 0.010100 | 72 |
| | | 0.257 | 0.006651 | 79 |
| | | 0.254 | 0.004819 | 83 |
| | | 0.252 | 0.004086 | 86 |
| | | 0.251 | 0.003818 | 89 |
| | | 0.249 | 0.003240 | 91 |
| | | 0.247 | 0.002447 | 95 |
| | | 0.246 | 0.002011 | 100 |
| | | 0.245 | 0.001762 | 103 |
| | | 0.243 | 0.001516 | 106 |
| | 45-60 | 0.283 | 1.135894 | 29 |
| | | 0.273 | 0.353686 | 32 |
| | | 0.267 | 0.174922 | 35 |
| | | 0.263 | 0.085147 | 37 |
| | | 0.260 | 0.071844 | 42 |
| | | 0.253 | 0.043138 | 50 |
| | | 0.247 | 0.022047 | 58 |
| | | 0.243 | 0.018088 | 66 |
| | | 0.239 | 0.013947 | 72 |
| | | 0.234 | 0.008358 | 79 |
| | | 0.230 | 0.005480 | 84 |
| | | 0.226 | 0.004912 | 93 |
| | | 0.223 | 0.003345 | 99 |
| | 60-75 | 0.293 | 0.596635 | 26 |
| | | 0.283 | 0.293609 | 28 |
| | | 0.276 | 0.189731 | 31 |
| | | 0.272 | 0.121173 | 35 |
| | | 0.269 | 0.089379 | 39 |
| | | 0.261 | 0.054500 | 45 |
| | | 0.255 | 0.034346 | 53 |

## APPENDIX B continued

| Source | Depth (cm.) | Water Content | K (cm/hr) | $p_c$ (cm) |
|---|---|---|---|---|
| | 60-75 | 0.250 | 0.020173 | 60 |
| | | 0.243 | 0.013483 | 69 |
| | | 0.239 | 0.011070 | 74 |
| | | 0.237 | 0.008651 | 77 |
| | | 0.233 | 0.006162 | 84 |
| | | 0.231 | 0.004404 | 88 |
| | | 0.229 | 0.003606 | 93 |
| | 75-90 | 0.313 | 2.574147 | 23 |
| | | 0.303 | 0.814446 | 24 |
| | | 0.297 | 0.497308 | 25 |
| | | 0.292 | | 28 |
| | | 0.289 | 0.441167 | 32 |
| | | 0.282 | 0.149825 | 38 |
| | | 0.271 | 0.085644 | 51 |
| | | 0.264 | 0.051150 | 59 |
| | | 0.259 | 0.023881 | 64 |
| | | 0.255 | 0.017400 | 70 |
| | | 0.250 | 0.012590 | 81 |
| | | 0.248 | 0.006215 | 84 |
| | | 0.244 | 0.001752 | 87 |
| | | 0.240 | 0.000723 | 89 |
| Troup, West Plot<br>Union Springs, AL<br>(Dane et al. 1983) | 0 -24 | 0.282 | 1.90 | 18 |
| | | 0.265 | 1.54 | 27 |
| | | 0.251 | 1.25 | 34 |
| | | 0.240 | 1.01 | 39 |
| | | 0.231 | 0.823 | 42 |
| | | 0.215 | 0.466 | 47 |
| | | 0.197 | 0.127 | 51 |
| | | 0.145 | 0.018 | 79 |
| | | 0.134 | 0.00372 | 88 |
| | | 0.132 | 0.000918 | 90 |
| | | 0.110 | 0.0000884 | 110 |
| | 24-58 | 0.274 | 3.76 | 15 |
| | | 0.255 | 3.38 | 20 |
| | | 0.239 | 2.79 | 25 |
| | | 0.227 | 2.32 | 28 |
| | | 0.217 | 1.94 | 31 |
| | | 0.199 | 1.16 | 34 |
| | | 0.178 | 0.331 | 38 |
| | | 0.169 | 0.0951 | 40 |
| | | 0.135 | 0.0463 | 51 |
| | | 0.124 | 0.0145 | 58 |
| | | 0.117 | 0.00440 | 71 |

APPENDIX B continued

| Source | Depth (cm.) | Water Content | K (cm/hr) | $p_c$ (cm) |
|---|---|---|---|---|
| | 24-58 | 0.100 | 0.000416 | 73 |
| | 58-92 | 0.307 | 55.7 | 10 |
| | | 0.292 | 35.5 | 11 |
| | | 0.278 | 25.8 | 16 |
| | | 0.265 | 18.3 | 20 |
| | | 0.254 | 13.1 | 22 |
| | | 0.227 | 5.88 | 27 |
| | | 0.184 | 1.55 | 33 |
| | | 0.154 | 0.473 | 36 |
| | | 0.142 | 0.219 | 38 |
| | | 0.138 | 0.136 | 41 |
| | | 0.134 | 0.0752 | 46 |
| | | 0.123 | 0.0363 | 51 |
| | | 0.106 | 0.0138 | 57 |
| | | 0.099 | 0.00483 | 63 |
| | 92-123 | 0.314 | 7.97 | 0 |
| | | 0.311 | 7.87 | 3 |
| | | 0.296 | 6.30 | 9 |
| | | 0.255 | 2.96 | 20 |
| | | 0.208 | 1.16 | 27 |
| | | 0.172 | 0.463 | 30 |
| | | 0.145 | 0.155 | 35 |
| | | 0.130 | 0.0655 | 41 |
| | | 0.120 | 0.0357 | 48 |
| | | 0.104 | 0.0141 | 57 |
| | | 0.098 | 0.00795 | 60 |
| | | 0.095 | 0.00339 | 63 |
| | 123-154 | 0.269 | 1.71 | 7 |
| | | 0.242 | 0.590 | 14 |
| | | 0.209 | 0.205 | 22 |
| | | 0.184 | 0.100 | 31 |
| | | 0.171 | 0.0653 | 37 |
| | | 0.152 | 0.0333 | 43 |

# VITA 2

Lloyd Elbert Funk

Candidate for the Degree of

Master of Science

Thesis: MEASUREMENT AND SCALING OF HYDRAULIC PROPERTIES OF POROUS MEDIA

Major Field: Chemical Engineering

Biographical:

Personal Data: Born in Lubbock, Texas, April 29, 1963, the son of Frank H. and Eula J. Funk.

Education: Graduated from Stamford High School, Stamford, Texas, in May 1981; received Bachelor of Science Degree in Chemical Engineering from Texas Tech University in May 1985; completed requirements for the Master of Science degree in Chemical Engineering in May, 1989.

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