# COMPUTER AIDED OPTIMAL DESIGN OF DUCT SYSTEMS 

By<br>FAN WANG<br>Bachelor of Engineering<br>Beijing University of<br>Aeronautics and Astronautics<br>Beijing, P.R.China<br>1986

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Thesis Approved:


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## CHAPTERI

## INTRODUCTION

## General

One of the important problems related to the design of any ventilating or air conditioning system is being able to provide the best duct work. In order to achieve maximum energy efficiency and minimal duct material cost, the best design methods have to be chosen carefully. There are three methods described by the ASHRAE Handbook of Fundamentals (ASHRAE 1989) -- equal friction, static regain, and the T-method. Other methods mentioned by McQuiston (1989) are the balanced capacity and the velocity reduction methods. Among these standard methods, the T-method is the only method that claims to provide the optimal duct design.

The optimized duct design concept is based on minimizing the life cycle cost of the duct system. The duct system optimal design is a form of large scale optimization problem. There are several ways to solve this kind of multi-dimensional problem, but most of them cannot always find the globally optimal point. The T-method uses the ideas of dynamic programming optimization method. If used properly, the dynamic programming method has a good chance of
finding the global minimum. Much research related to duct design has been done before by using the dynamic programming method (Shitzer 1979).

The method of simulated annealing (Kirkpatrick, 1983) is a relatively new general-purpose method of very large scale optimization, which has been attracting significant attention recently. It offers the possibility of finding the global optimal point. This method has been reported to be used in power distribution systems (Chiang et al., 1990), optical areas (Kim, 1990), image processing areas (Carnevali, 1985), electronics (Kirkpatrick, 1983), and biochemistry areas (Prabhakaran, 1985). It was reported to have solved the travelling salesman problem, and the global wiring problem of silicon chips successfully (Kirkpatrick,1983).

Although the T-method has successfully optimized duct network problems, it is unusual for one optimization method to work well for all problems. Also, the T-method has some limitations. For example, the T-method cannot solve problems with the velocity limit constraints and the static pressure limit constraints. This is one of the important reasons to look for alternatives to the Tmethod.

The purpose of this study is to apply the modified simulated annealing method to the duct system optimization problem and compare its results to the T-method. It is therefore necessary to implement the T-method.

## Technical Background

A literature survey was done in the areas of duct design technology and large scale optimization.

## Duct Design Methods

The equal friction method (McQuiston, 1988) is based on making the pressure loss per foot of duct length the same for the entire system. This method will produce a well balanced duct system if all runs of duct are the same. In most practical systems, this is not the case. The short runs will have to be damped to increase the resistance, which will result in a waste of energy. $\Rightarrow$ The balanced capacity method (McQuiston, 1988) involves making the total pressure loss of each path the same under design flow rates. In general, this means that the longest path is sized first, and then the other paths are sized to have the correct pressure loss to match the total pressure at each junction. It is a natural law that all duct systems will balance themselves. The dampers may have to be installed to insure correct flow rates. The system will not be the optimal.

In 1940 Carrier et al. (Tsal, 1988) recommended the static regain duct design method. They thought that this method would be better than the equal friction and the velocity reduction methods. The static regain method saves energy by converting "kinetic energy" into static pressure. The duct section nearest the fan is sized first
by using some additional criteria, and the remaining ducts are sized to have the same total pressure. McQuiston (1988) classified this method as a high velocity duct design method. This method is based on the Bernoulli-Borda equation below:

$$
\begin{equation*}
\Delta P=\left(\frac{V_{1}^{2}}{C}-\frac{V_{2}^{2}}{C}\right)-\left(\frac{\mathrm{k}\left(\mathrm{~V}_{1}-\mathrm{V}_{2}\right)^{2}}{\mathrm{C}}\right) \tag{1.1}
\end{equation*}
$$

$V_{1}, V_{2}$--- velocity of air in the duct ( $\mathrm{m} / \mathrm{s}$ ).
C --- fitting coefficient.
k --- gradual/abrupt expansion loss ratio

This method was critiqued by Tsal et al. (1988). Other disadvantages were discussed by McQuiston (1988).

The velocity reduction method (Tsal, 1986) can be represented below:

$$
\begin{equation*}
v_{j+1}=u V_{j}, \quad j=1,2, \ldots, n-1 \tag{1.2}
\end{equation*}
$$

u --- Reduction factor.
V --- Velocity of the air ( $\mathrm{m} / \mathrm{s}$ ).
j --- The number of duct section.

The velocity reduction method is used primarily for variable air volume systems (VAV), which depends on the VAV boxes to balance the system pressure loss. It consisted practically of making sure that the duct closest to the fan has acceptable velocity, and then reducing the velocities of the downstream ducts so that the
velocities are reduced gradually. This method is some what heuristic and requires a sound background of duct design experience.

The constant velocity method (Tsal, 1986) is a special case of the velocity reduction method, where the reduction factor is 1.0 The velocities of the whole system remain the same. The T-method's starting point is based on this method.

## T-method

The T-method is a special method for duct network optimal design. It uses the ideas of dynamic programming. Because of the complexity of the duct network and its constraints, most of the methods mentioned above are not capable of finding the global minimum of the duct network cost (Tsal, 1987). Dynamic programming has a better chance to find the global minimum.

The first trial of using dynamic programming in duct system design optimization was by Tsal et al. in 1968 (Tsal, 1987). Arkin and Shitzer had also published their works about using dynamic programming design of the duct system (Arkin, 1979). Tsal et al. tried to optimize the velocity of the duct by dynamic programming (1986). They took the first partial derivatives of the objective function, which is always the life cycle cost of the duct network, with respect to velocity for each section of the duct network (Tsal, 1986, Equation 20) to form several equations. They calculated the optimum air velocity of each duct section by solving these partial derivative equations.

In 1989 when the T-method was introduced, the optimum pressure loss ratio is calculated by taking the partial derivatives of the objective function with respect to pressure losses (Tsal, 1989, Equation 1.24). The system is balanced when the fan pressure is distributed optimally by the ratios of T factors which are calculated by using the partial derivatives of the objective function. $T$ factors are the fan pressure distribution factors introduced by Tsal. T factors are calculated by condensing the system into one node. After the fan is selected the fan pressure is distributed by expanding the system. If the system is not balanced, the iteration is needed. This method seems to find the global minimum of the life cycle cost of the duct network. More detailed discussions are made in the following chapter.

## Other Optimization methods

There are several ways to optimize the duct network which have been tried before--the Coordinate Descent Method (Tsal, 1987), Lagrange Multipliers Method (Tsal, 1987), Reduced Gradient Method (Abadie,1969), Quadratic Search Method (Leah, 1987), and Dynamic Programming (Bellman, 1957). These methods were well explained by Tsal and Adler (1987).

## Simulated Annealing Method.

Kirkpatrick (1983) developed a stochastic optimization procedure which is analogous to the statistical thermodynamics of the annealing process of the heated metal with the optimization methods. First, the system is
heated up, then cooled down, and then the temperature is kept at the annealing point for a long time, so that the atoms will line themselves up to form a pure crystal. The pure crystal often contains less energy and has less defects.

By simulating the thermodynamics problem, the introduced pseudo-temperature is the control parameter of the process. When the temperature is high, atoms move in all directions. When the iteration begins, the objective function is allowed to go uphill. The lower the temperature, the longer the iteration will last. Therefore, there is less chance for the system to go in an uphill direction. The possibility for the objective function to go uphill is controlled by the Metropolis Monte Carlo function. The new move is accepted or not with the possibility that its objective function is lower than before, or with the probability $\exp (-\Delta \mathrm{E} / \mathrm{T})$ if the objective function is higher than before. Simulated annealing provides the possibility of finding the global minimum.

## CHAPTER II

## T-METHOD IMPLEMENTATION

## Introduction

The T-method is an optimal fan pressure distribution method. The pressure ratio of two duct sections connected in series are calculated by taking the partial derivative of the objective function with respect to pressure losses.

## Objective function

Tsal (1989) uses life cycle cost as his objective function.

$$
\begin{equation*}
\mathrm{E}=\mathrm{E}_{\mathrm{p}} *(\mathrm{PWEF})+\mathrm{E}_{\mathrm{S}} \tag{2.1}
\end{equation*}
$$

where

$$
\begin{aligned}
E & =\text { life cycle cost of the duct system }(\$) . \\
E_{p} & =\text { annual electric energy cost }(\$) . \\
E_{s} & =\text { Initial cost of the duct system }(\$) .
\end{aligned}
$$

PWEF = present worth escalation factor.
Electric energy cost:

$$
\begin{equation*}
\mathrm{E}_{\mathrm{p}}=\mathrm{Q}_{\mathrm{fan}} \frac{\left(\mathrm{E}_{\mathrm{c}}\right) \mathrm{Y}+\mathrm{E}_{\mathrm{d}}}{10^{5} \eta_{\mathrm{f}} \eta_{\mathrm{e}}} \mathrm{P}_{\mathrm{fan}} \tag{2.2}
\end{equation*}
$$

where

$$
Q_{\text {fan }}=\text { total air flow rate }\left(\mathrm{m}^{3} / \mathrm{s}\right) .
$$

$$
\begin{gather*}
E_{C}=\text { unit energy cost }(\$ / \mathrm{kWh}) . \\
Y=\text { system operation time }(\mathrm{hr} / \text { year }) . \\
E_{d}=\text { energy demand cost }(\$ / \mathrm{kWh}) . \\
P_{\text {fan }}=\text { Fan pressure (Pa.). } \\
\eta_{f}=\text { fan total efficiency. } \\
\eta_{e}=\text { motor total efficiency. } \\
\text { Initial Cost: }  \tag{2.3}\\
E_{S}=S_{d} \pi D L \text { (Round ducts) } \\
E_{S}=2 S_{d}(H+W) L \quad \text { (Rectangular ducts) }
\end{gather*}
$$

Initial Cost:
where

$$
S_{d}=\text { unit duct work cost }\left(\$ / \mathrm{m}^{2}\right)
$$

Present Worth Escalation Factor:

$$
\begin{equation*}
\text { PWEF }=\frac{[(1+\text { AER }) /(1+\text { ARR })] \text { a }-1}{1-[(1+\text { ARR }) /(1+\text { AER })]} \tag{2.5}
\end{equation*}
$$

$A E R=$ annual escalation rate.
AIR $=$ average interest rate.
$\mathrm{a}=$ amortization period.
The objective function can be written in the form of coefficient K , which is the duct characteristic defined by Tsal (1989).

The duct characteristic coefficient K can be calculated by

$$
\begin{equation*}
K=n(\mu)^{0.2} Q^{0.4} L \tag{2.6}
\end{equation*}
$$

n parameter is

$$
\begin{equation*}
n=1 \quad \text { (Round) } \tag{2.7}
\end{equation*}
$$

$$
\begin{equation*}
\mathrm{n}=\frac{1+\mathrm{r}}{(\pi \mathrm{r})^{0.5}} \tag{2.8}
\end{equation*}
$$

where

$$
\begin{align*}
& r=\frac{\text { Height }}{\text { Width }}  \tag{2.9}\\
& \mu \cdots \quad \text { coefficient } \\
& \mu=f L+C D \quad \text { (round) } \\
& \quad \mu=\left(\frac{f}{D_{f}}+C\right) D_{v} \quad \text { (Rectangular) } \tag{2.10}
\end{align*}
$$

where
$L=$ length of the duct (m).
$C=$ fitting coefficient.
$f=$ friction coefficient.
$Q=$ air flow volume ( $\mathrm{m}^{3} / \mathrm{s}$ ).
$r=$ aspect ratio for rectangular duct.
$D_{f}$---- equivalent-by-friction diameter ( $m$ ).

$$
\begin{equation*}
\mathrm{D}_{\mathrm{f}}=\frac{2 \mathrm{HW}}{\mathrm{H}+\mathrm{W}} \tag{2.11}
\end{equation*}
$$

Dv ---- equivalent-by-velocity diameter (m)

$$
\begin{equation*}
D_{v}=1.128\left(H^{*} W\right)^{1 / 2} \tag{2.12}
\end{equation*}
$$

where
$H=$ height of the rectangular duct ( m ).
$\mathrm{W}=$ width of the rectangular duct (m).

From the above equation Tsal found the final objective function. (Tsal, 1989).

$$
\begin{equation*}
E=z_{1}\left(P_{\text {fan }}\right)+z_{2} K(\Delta P)^{-0.2} \tag{2.13}
\end{equation*}
$$

K ---- duct characteristic coefficient
$z_{1}, z_{2}---$ Intermediate variable $z_{1}, z_{2}$ :

$$
\begin{align*}
& z_{1}=\text { Qfan } \frac{\left(\mathrm{E}_{\mathrm{c}}\right) Y}{10^{5} \eta_{\mathrm{e}} \eta_{\mathrm{f}}}(\mathrm{PWEF})  \tag{2.14}\\
& \mathrm{z}_{2}=0.959 \pi\left(\frac{\rho}{\mathrm{~g}}\right)^{0.2} \mathrm{~S}_{\mathrm{d}} \tag{2.15}
\end{align*}
$$

T-method's objective function is reasonable, clear, easy to understand, and easy to take the partial derivatives.

## T Factor

The T-method uses the ideas of the dynamic programming optimization method and other traditional optimization methods. Its optimization relies on the partial derivatives of the objective function. The T-method's objective function can be written as follows, if the duct system has two duct sections connected in series:

$$
\begin{equation*}
E=E_{1}+E_{2} \tag{2.16}
\end{equation*}
$$

$$
E_{1}, E_{2}=\text { the life cycle cost of each section. }
$$

The relationship of pressure losses is

$$
\begin{equation*}
\Delta \mathrm{P}=\Delta \mathrm{P}_{1}+\Delta \mathrm{P}_{2} \tag{2.17}
\end{equation*}
$$

$\Delta P_{1}, \Delta P_{2}=$ pressure loss of each section (Pa.).
In order to calculate the optimal fan pressure distribution factor, the partial derivative of the objective function is taken with ) respect to $\Delta P_{1}$ and $\Delta P_{2}$, and set equal to zero.

$$
\begin{align*}
& \frac{\partial \mathrm{E}}{\partial\left(\Delta \mathrm{P}_{1}\right)}=\mathrm{z}_{1}-0.2 \mathrm{z}_{2} \mathrm{~K}_{1}\left(\Delta \mathrm{P}_{1}\right)^{-1.2}=0 \\
& \frac{\partial \mathrm{E}}{\partial\left(\Delta \mathrm{P}_{2}\right)}=\mathrm{z}_{1}-0.2 \mathrm{z}_{2} \mathrm{~K}_{2}\left(\Delta \mathrm{P}_{2}\right)^{-1.2}=0 \tag{2.18}
\end{align*}
$$

where
$\mathrm{K}_{1}, \mathrm{~K}_{2}$--- intermediate variables (duct characteristic coefficient).

From the partial derivative equations above, we can get the optimal pressure loss ratio of two sections connected in series.

$$
\begin{equation*}
\frac{\Delta \mathrm{P}_{1}}{\Delta \mathrm{P}_{2}}=\left(\frac{\mathrm{K}_{1}}{\mathrm{~K}_{2}}\right)^{0.833} \tag{2.19}
\end{equation*}
$$

Take the reciprocal of each side and add 1 to each side of equation (2.19).

$$
\begin{equation*}
\frac{\Delta \mathrm{P}_{2}}{\Delta \mathrm{P}_{1}}+1=\left(\frac{\mathrm{K}_{2}}{\mathrm{~K}_{1}}\right)^{0.833}+1 \tag{2.20}
\end{equation*}
$$

From equation (2.20)

$$
\begin{equation*}
\mathrm{T}=\frac{\Delta \mathrm{P}_{1}}{\Delta \mathrm{P}_{1}+\Delta \mathrm{P}_{2}}=\left(\frac{\mathrm{K}_{1}^{0.833}}{\mathrm{~K}_{1}^{0.833}+\mathrm{K}_{2}^{0.833}}\right) \tag{2.21}
\end{equation*}
$$

$\mathrm{T}=\mathrm{T}$ factor of T -Method, the optimal ratio of the
pressure losses for two duct sections.

The T factor is calculated by taken the partial derivatives of the objective function. It is the heart of the T-method, which is the optimal fan pressure distribution factor of the two sections or
equivalent sections connected in series. T factor is calculated by finding the K coefficients of every duct sections.

Pressure loss is calculated by the Darcy-Weisbach equation for round and rectangular ducts.

Round:

$$
\begin{equation*}
\Delta P=\left(\frac{f L}{D}+C\right) \frac{V^{2} \rho}{2 g} \tag{2.22}
\end{equation*}
$$

Rectangular:

$$
\begin{align*}
& \Delta P=\left(\frac{f L}{D_{f}}+C\right) \frac{V^{2} \rho}{2 g}  \tag{2.23}\\
& \rho=\text { Air density }\left(\mathrm{kg} / \mathrm{m}^{3}\right) \\
& g=\text { constant }\left(1.0 \mathrm{~kg}-\mathrm{m} /\left(\mathrm{N}-\mathrm{s}^{2}\right) .\right.
\end{align*}
$$

Using $\mu$ coefficient:

$$
\text { Round: } \quad \Delta \mathrm{P}=0.811 \mathrm{~g}^{-1} \mu \rho \mathrm{Q}^{2} \mathrm{D}^{-5}
$$

Rectangular: $\quad \Delta \mathrm{P}=0.811 \mathrm{~g}^{-1} \mu \rho \mathrm{Q}^{2} \mathrm{D}_{\mathrm{v}}{ }^{-5}$
To express the diameter in terms of a pressure loss:

$$
\begin{equation*}
D_{v}=0.959(\mu \rho)^{0.2} Q^{0.4}(g \Delta P)^{-0.2} \tag{2.26}
\end{equation*}
$$

If two duct sections are connected in parallel, there is no pressure distribution problem, just is a balancing problem. T-method just set the pressure losses of these two sections equal.

From previous equations, the equivalent-by-cost diameter $D_{0}$ can be calculated for rectangular duct section.

$$
\begin{equation*}
D_{0}=2(H+W) / \pi \tag{2.27}
\end{equation*}
$$

So

$$
\begin{equation*}
D_{o}=\frac{1+r}{\sqrt{\pi r}} D_{v}=n * D_{v} \quad(\text { Round } n=1) \tag{2.28}
\end{equation*}
$$

The initial cost of duct $E$ is

$$
\begin{align*}
& E_{S}=\pi * D * L * S_{d} \\
& =0.959(\mu \rho)^{0.2} * Q^{0.4} *(g \Delta P)^{-0.2} n \mathrm{~L} \tag{2.29}
\end{align*}
$$

Then, the $K$ coefficient can be calculated alternately:

$$
\begin{equation*}
\mathrm{K}=\mathrm{n} \mu^{0.2} \mathrm{Q}^{0.4} \mathrm{~L} \tag{2.30}
\end{equation*}
$$

K factor or coefficient of each duct section can be calculated by condensing the whole system into one node.

## Condensing

Next is the process of condensing two duct sections connected in series into one node.

$$
\begin{equation*}
K_{1-2}=\left(K_{1}^{0.833}+K_{2}^{0.833}\right)^{1.2} \tag{2.31}
\end{equation*}
$$



Figure 1. Condensing a tee.

Condensing a tee is shown in Figure 1 which contains one node, two children in parallel, and one parent in series:

$$
\begin{align*}
& K_{1-3}=\left(K_{1-2} 2^{0.833}+K_{3}^{0.833}\right)^{1.2}  \tag{2.32}\\
& =\left[\left(K_{1}+K_{2}\right)^{0.833}+K_{3}^{0.833}\right]^{1.2} \tag{2.33}
\end{align*}
$$

From equation 2.33

$$
\begin{equation*}
E=z_{1}\left(P_{\text {fan }}\right)+z_{2} K(\Delta P)^{-0.2} \tag{2.34}
\end{equation*}
$$

The optimum fan pressure can be calculated by taking the derivative of Equation 2.34 with respect to $\Delta \mathrm{P}$, setting to zero, and solving for pressure loss.

$$
\begin{equation*}
\mathrm{P}_{\mathrm{fan}(\mathrm{opt})}=0.26\left(\frac{22}{\mathrm{z1}} \mathrm{~K}^{0.833}+\Delta \mathrm{P} \max \right. \tag{2.35}
\end{equation*}
$$

$\Delta \mathrm{P}_{\text {max }}$----- Maximum additional pressure loss (Pa.).

If fan and motor are preselected, the existing fan pressure is treated as optimum.

## Expansion

This step distributes fan pressure through the system proportional to the T coefficients or T factors.

Duct pressure loss

$$
\begin{equation*}
\Delta P_{i}=\left(P_{i}\right) T_{i} \tag{2.36}
\end{equation*}
$$

Tee coefficient $\quad \mathrm{T}=\left(\frac{\mathrm{K}_{\mathrm{i}}}{\mathrm{K}_{\mathrm{i}-1}}\right)^{0.833}$

$$
\begin{equation*}
\mathrm{K}_{\mathrm{i}}=\mathrm{K}_{\mathrm{S}} \text { at duct section \#i. } \tag{2.37}
\end{equation*}
$$

So

$$
K_{s}=K_{i}
$$

We call $K_{1-i}$ of node $\# K_{t} . K_{t}$ is the $K$ for condensed node.

$$
\begin{equation*}
\mathrm{T}=\left(\frac{\mathrm{K}_{\mathrm{s}}}{\mathrm{~K}_{\mathrm{t}}}\right)^{0.833} \tag{2.38}
\end{equation*}
$$

So we can calculate the pressure loss for each node.

$$
\begin{equation*}
\Delta P=P * T \tag{2.39}
\end{equation*}
$$

$P$ is the pressure at that node. By knowing $\Delta P$, we can find out the optimized duct diameter:

$$
\begin{equation*}
D=0.959(\mu \rho)^{0.2} \quad Q^{0.4}(\mathrm{~g} \Delta \mathrm{P})^{-0.2} \tag{2.40}
\end{equation*}
$$

$$
\begin{equation*}
\frac{2(\mathrm{H}+\mathrm{W})}{\pi}=\mathrm{D} \quad \text { for } \quad \text { rectangular duct } \tag{2.41}
\end{equation*}
$$

After the $D$ is calculated, the pressure loss of each duct is calculated, then the pressure loss of each path is calculated. If the maximum pressure loss of every path is greater than 4 percent different (Tsal, 1989) (or the other percentage) from the fan pressure, iteration is needed. Using the duct diameter $D$ estimated by previous function, the previous calculation can be done again and again until the pressure loss is balanced.

## CHAPTER III

## NUMERICAL STUDY OF DUCT DESIGN METHOD

## Introduction

$\sqrt{ }$ As discussed before, when air flow of each duct section is specified, the duct design involves two major problems. When two duct sections are connected in parallel, there is a pressure balancing problem. When two duct sections are connected in series (Figure 2), there is a pressure distribution problem. Almost all the duct design methods are concerned with these two problems. The way to calculate the pressure distribution ratios in each method is different and is not always obviously observed. The following paragraphs are going to discuss how the pressure distribution ratios are calculated by different duct design methods.)

## Two Sections Connected in Series

Fan pressure can be distributed by introducing the fan pressure distribution factor (FPDF). If two ducts are connected in series (Figure 2), the pressure ratio of these two duct sections can be represented below

$$
\begin{gather*}
\frac{\Delta \mathrm{P}_{1}}{\Delta \mathrm{P}_{2}}=\mathrm{FPDF} \\
\frac{\Delta \mathrm{P}_{1}}{\Delta \mathrm{P}_{1}+\Delta \mathrm{P}_{2}}=\mathrm{FPDF}^{*}
\end{gather*}
$$

These two FPDFs are different in number but are the same in meaning.
where
$\Delta \mathrm{P}_{1}=$ pressure loss of first section (Pa.).
$\Delta P_{2}=$ pressure loss of second section (Pa).


Figure 2. Two duct sections connected in series.

Total pressure loss:

$$
\begin{equation*}
\Delta \mathrm{P}=\Delta \mathrm{P}_{1}+\Delta \mathrm{P}_{2} \tag{3.2}
\end{equation*}
$$

## Two Sections Connected in Parallel

$\sqrt{ }$ If there are only these two sections in the system, their pressure losses have to be the same (Figure 3).


Figure 3. Two duct sections connected in parallel.

$$
\begin{equation*}
\Delta P=\Delta P_{1}=\Delta P_{2} \tag{3.3}
\end{equation*}
$$

The concept of the imaginary section is based on the idea that one subtree of ducts can be viewed as one large imaginary duct section.

The following sections will try to collapse the existing duct design methods, or duct optimization methods into one formula.)

## T-Method

T-method is an optimization method which uses the $T$ factor. $T$ factor is the fan pressure distribution factor. The fan pressure distribution factor (FPDF*) can have any value between zero and one, and the system will still be balanced. The T-method has its own way to calculate the fan pressure distribution factor, given by equation (2.21). In this section, it will be shown that the T-method for determining the T -factor can also be cast as a method which sets the fan pressure distribution factor equal to a ratio of initial costs.

$$
\begin{equation*}
\text { FPDF }^{*}=\mathrm{T}=\frac{\Delta \mathrm{P}_{1}}{\Delta \mathrm{P}_{1}+\Delta \mathrm{P}_{2}}=\frac{\mathrm{E}_{\mathrm{s} 1}}{\mathrm{E}_{\mathrm{s} 1}+\mathrm{E}_{\mathrm{s} 2}} \tag{3.4}
\end{equation*}
$$

Using the technique shown in equations (2.19) through (2.21)

$$
\begin{equation*}
\frac{\Delta \mathrm{P}_{1}}{\Delta \mathrm{P}_{2}}=\frac{\mathrm{Es}_{1}}{\mathrm{Es}_{2}} \tag{3.5}
\end{equation*}
$$

The mathematical analysis is shown below.
The initial cost of one duct section is

$$
\begin{gather*}
E_{S}=\pi^{*} D^{*} L^{*} S_{d} \\
=0.959(\mu \rho)^{0.2 *} Q^{0.4 *}(\mathrm{~g} \Delta P)^{-0.2}{ }_{n L} \tag{3.6}
\end{gather*}
$$

Substituting (3.6) into (3.5) yields

$$
\begin{equation*}
\frac{\Delta \mathrm{P}_{1}}{\Delta \mathrm{P}_{2}}=\frac{\mathrm{E}_{\mathrm{s} 1}}{\mathrm{E}_{\mathrm{s} 2}}=\frac{0.959\left(\mu_{1} \rho\right)^{0.2} \mathrm{Q}_{1}^{0.4}\left(\mathrm{~g} \Delta \mathrm{P}_{1}\right)^{-0.2} \mathrm{n}_{1} \mathrm{~L}_{1}}{0.959\left(\mu_{2} \mathrm{\rho}\right)^{0.2} \mathrm{C}_{2}^{0.4}\left(\mathrm{~g} \Delta \mathrm{P}_{2}\right)^{-0.2} \mathrm{n}_{2} \mathrm{~L}_{2}} \tag{3.7}
\end{equation*}
$$

$\Delta \mathrm{P}_{1}, \Delta \mathrm{P}_{2}$--- Optimal pressure drops of two duct sections connected in series.

Equation (3.7) simplifies to

$$
\begin{equation*}
\frac{\Delta \mathrm{P}_{1}}{\Delta \mathrm{P}_{2}}=\frac{\mathrm{n}_{1}\left(\mu_{1}\right)^{0.2} \mathrm{Q}_{1}^{0.4}\left(\Delta \mathrm{P}_{1}\right)^{-0.2} \mathrm{~L}_{1}}{\mathrm{n}_{2}\left(\mu_{2}\right)^{0.2} \mathrm{Q}_{6}^{0.4}\left(\Delta \mathrm{P}_{2}\right)^{-0.2} \mathrm{~L}_{2}} \tag{3.8}
\end{equation*}
$$

solving (3.8)

$$
\begin{equation*}
\frac{\Delta \mathrm{P}_{1}}{\Delta \mathrm{P}_{2}}=\left(\frac{\mathrm{n}_{1}\left(\mu_{1}\right)^{0.2} \mathrm{Q}_{1}^{0.4} \mathrm{~L}_{1}}{\mathrm{n}_{2}\left(\mu_{2}\right)^{0.2} \mathrm{Q}^{0.4} \mathrm{~L}_{2}}\right)^{0.833} \tag{3.9}
\end{equation*}
$$

from equation (2.30)

$$
\begin{equation*}
\mathrm{K}=\mathrm{n} \mu^{0.2} \mathrm{Q}^{0.4} \mathrm{~L} \tag{3.10}
\end{equation*}
$$

Substitute equation (3.10) into equation (3.9)

$$
\begin{equation*}
\frac{\Delta \mathrm{P}_{1}}{\Delta \mathrm{P}_{2}}=\left(\frac{\mathrm{K}_{1}}{\mathrm{~K}_{2}}\right)^{0.833} \tag{3.11}
\end{equation*}
$$

Using the technique shown in equations (2.19) through (2.21)

$$
\begin{equation*}
\text { FPDF }^{*}=\mathrm{T}=\frac{\Delta \mathrm{P}_{1}}{\Delta \mathrm{P}_{1}+\Delta \mathrm{P}_{2}}=\left(\frac{\mathrm{K}_{1}^{0.833}}{\mathrm{~K}_{1}^{0.833}+\mathrm{K}_{2}^{0.833}}\right)=\frac{\mathrm{E}_{\mathrm{s} 1}}{\mathrm{E}_{\mathrm{s} 1}+\mathrm{E}_{\mathrm{s} 2}} \tag{3.12}
\end{equation*}
$$

Therefore, we reach T-Method's result. Thus, the assertion that the T -factor can also be represented as a ratio of the initial costs is true. This ratio will control the fan pressure distribution to each duct section.

## Equal Friction Method

This method is purely a duct design method without involving any optimization method. The FPDF of this method is equal to the ratio of lengths of two sections connected in series.

This method is based on sizing each duct so that the pressure loss per unit total length is constant.

$$
\begin{equation*}
\frac{\Delta \mathrm{P}_{1}}{\mathrm{~L}_{1}}=\frac{\Delta \mathrm{P}_{2}}{\mathrm{~L}_{2}}=\ldots=\frac{\Delta \mathrm{P}_{\mathrm{n}}}{\mathrm{~L}_{\mathrm{n}}} \tag{3.13}
\end{equation*}
$$

From the formula above, the fan pressure distribution factor of section one can be calculated as

$$
\begin{equation*}
\text { FPDF }^{*}=\frac{\Delta \mathrm{P}_{1}}{\sum_{\mathrm{i}=1}^{\mathrm{n}} \Delta \mathrm{P}_{\mathrm{i}}}=\frac{\mathrm{L}_{1}}{\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{~L}_{\mathrm{i}}} \tag{3.14}
\end{equation*}
$$

This means the equal friction method can be represented in the FPDF* factor.

## Balanced Capacity Method

Balanced Capacity Method is similar to the equal friction method. The difference is that fan pressure distribution factors (FPDF*) are calculated by the longest equivalent length of the subtree.

$$
\begin{equation*}
\text { FPDF }^{*}=\frac{\mathrm{L}_{1}}{\text { Longest (path } 1, \text { path2, } \ldots, \text { pathj) }} \tag{3.15}
\end{equation*}
$$

Velocity Reduction Method

The velocity is reduced by the $u$ factor.

$$
\begin{equation*}
\frac{\mathrm{V}_{1}}{\mathrm{~V}_{2}}=\mathrm{u} \tag{3.16}
\end{equation*}
$$

Recall Darcy-Weisbach equation

$$
\begin{equation*}
\Delta \mathrm{P}=\mathrm{f} \frac{\mathrm{Le} \mathrm{~V}^{2}}{\mathrm{D}} \frac{\rho}{2 \mathrm{~g}} \tag{3.17}
\end{equation*}
$$

Le --- Equivalent length (m).

From equation 3.16 and 3.17 , we get

$$
\begin{equation*}
\frac{\Delta \mathrm{P}_{1}}{\Delta \mathrm{P}_{2}}=\frac{\mathrm{f}_{1} * \operatorname{Le}_{1} * \frac{\left(\mathrm{~V}_{1}\right)^{2}}{\left(\mathrm{D}_{1} * 2 \mathrm{~g}\right)} \rho}{\mathrm{f}_{2} * \operatorname{Le}_{2} * \frac{\left(\mathrm{~V}_{2}\right)^{2}}{\left(D_{2} * 2 g\right)} \rho} \tag{3.18}
\end{equation*}
$$

or

$$
\begin{equation*}
\frac{\Delta \mathrm{P}_{1}}{\Delta \mathrm{P}_{2}}=\frac{\frac{\mathrm{f}_{1} * \mathrm{Le}_{1} *}{\left(\mathrm{D}_{1}\right)}}{\frac{\mathrm{f}_{2} * \mathrm{~L}_{2} *}{\left(\mathrm{D}_{2}\right)}} *\left(\frac{\mathrm{~V}_{1}}{\mathrm{~V}_{2}}\right)^{2} \tag{3.19}
\end{equation*}
$$

Equation 3.19 can be written this way

$$
\begin{equation*}
\frac{\Delta \mathrm{P}_{1}}{\Delta \mathrm{P}_{2}}=\frac{\frac{\mathrm{f}_{1} * L e_{1}}{\left(\mathrm{D}_{1} 1\right)}}{\frac{\mathrm{f}_{2} * L e_{2}}{\left(\mathrm{D}_{2}\right)}} *(\mathrm{u})^{2} \tag{3.20}
\end{equation*}
$$

This means that the velocity reduction method can be described by using the FPDF factor.

## Conclusion

Most of the duct design method can be represented in the calculation of fan pressure distribution factor. The different methods have different FPDF factors. Therefore, they have different results.

CHAPTER IV

## NUMERICAL PROCEDURE AND ANALYSIS OF OPTIMAL PRESSURE DISTRIBUTION METHOD

$\int$ (Duct system optimization methods should be capable of finding the minimum of the system cost, balancing the system, selecting the fan, and distributing the fan pressure to the system properly.

The basic idea of optimal pressure distribution (OPD) method
jalance the duct network pressure losses of each path, 2) to select the optimal fan pressure, and 3) to distribute the fan pressure in the proportion of the optimal pressure distribution ratio. The optimal pressure distribution ratios are chosen by the modified simulated annealing method rather than calculated by T-method.)

## OPD Factor

We borrowed the T-Method's ideas of optimizing the ratios of the pressure losses of the duct sections instead of optimizing the velocities or the pressure losses directly. We call these ratios Fan Pressure Distribution Ratios. Although the purpose of duct optimization is to find the optimal duct sizes, we cannot optimize the duct sizes explicitly. The sizes of duct sections are dependent on each other because the pressure losses of each paths have to be
balanced. In order to provide simulated annealing method independent variables, the OPD factor is introduced. OPD factor is the ratio of pressure losses of two duct sections or one duct section and a duct subtree.

Consider a two sections duct system where the fan pressure has already been selected. If two ducts are in parallel, there is a balancing problem. These two sections have to have the same pressure losses. There is no optimization problem if the fan pressure has already been chosen. If two ducts are in series, there is no balancing problem, but there is a pressure distribution problem. The cost of the system is related to how the fan pressure is distributed. This is an optimization problem.

In OPD method, the optimum pressure ratio is

$$
\frac{\Delta \mathrm{P}_{1}}{\Delta \mathrm{P}_{2}+\Delta \mathrm{P}_{1}}=\mathrm{OPDF}
$$

where
$\Delta P_{1}, \Delta P_{2}$-- The pressure losses of first and second section in series (Pa.).

OPDF -- Optimal Pressure Distribution Factor.

The fan pressure can be calculated as

$$
P_{\text {fan }}=\Delta P_{1}+\Delta P_{2}
$$

If there are only two duct sections or one duct and one subtree of ducts in the system.

## Objective Function

We borrowed T-method's objective function, for the purpose of comparison. The objective function includes the initial cost and the energy cost of the system.

The objective function is

$$
\begin{equation*}
E=E_{p}(P W E F)+E_{s} \tag{4.2}
\end{equation*}
$$

Electric energy cost:

$$
\begin{equation*}
E_{p}=Q_{f a n} \frac{\left(E_{c}\right) Y+E_{d}}{10^{5} \eta_{f} \eta_{e}} P_{f a n} \tag{4.3}
\end{equation*}
$$

Present Worth Escalation Factor (PWEF):

$$
\begin{equation*}
\text { PWEF }=\frac{[(1+\mathrm{AER}) /(1+\mathrm{AR})]^{\mathrm{a}}-1}{1-[(1+\mathrm{AR}) /(1+\mathrm{AER})]} \tag{4.4}
\end{equation*}
$$

AER = annual escalation rate (\%).
AIR = annual interest rate (\%).
$\mathrm{a}=$ amortization period (year).

The initial cost (Es) can be calculated as

$$
\begin{equation*}
E_{S}=S_{d} \pi D L \quad \text { (Round) } \tag{4.5}
\end{equation*}
$$

$$
\begin{equation*}
E_{S}=2 S_{d}(H+W) L \quad \text { (Rectangular) } \tag{4.6}
\end{equation*}
$$

where
$D=$ duct diameter ( $m$ ).

$$
\begin{aligned}
& H=\text { duct height }(\mathrm{m}) . \\
& \mathrm{W}=\text { duct width }(\mathrm{m}) . \\
& \eta_{f}=\text { fan efficiency }(\%) . \\
& \eta_{e}=\text { fan motor efficiency }(\%) . \\
& P_{\text {fan }}=\text { fan pressure }(\mathrm{Pa} .) . \\
& Q_{f a n}=\text { fan flow rate }\left(\mathrm{m}^{3} / \mathrm{s}\right) .
\end{aligned}
$$

The related economic factors can be estimated from the following sources (Tsal, 1989):

Duct price per unit ( $\mathrm{S}_{\mathrm{d}}$ )---"Sheet Metal Estimating"
Annual escalation rate (AER)---"Utility Costs Forecasting"
Amortization period (a) --- Expected Life time of duct system.
Energy demand cost ( $\mathrm{E}_{\mathrm{d}}$ ) --- "Electric Power Annual"
Energy unit cost ( $\mathrm{E}_{\mathrm{c}}$ ) --- "Electric Power Annual"

The life cycle cost $E$ is calculated one by one of each duct section. The total cost is calculated by adding all the $\mathrm{E}_{\mathrm{S}}$ together. The fitting cost, fan cost, heating and cooling coil cost are considerate as constant, and not included in the objective function.

## Fundamental Equations

The basic equation is the Darcy-Weisbach Equation. The total pressure loss of the flow in closed duct can be calculated as follows: (Wright,1945)

$$
\begin{equation*}
\Delta P=f \frac{L V^{2}}{D} \frac{\rho}{2 g} \tag{4.7}
\end{equation*}
$$

where
$\Delta P=$ head loss due to friction (Pa.).
$L=$ conduit length ( m ).
$D=$ conduit inside diameter $(\mathrm{m})$.
$V=$ fluid velocity $(\mathrm{m} / \mathrm{s})$.
$\mathrm{g}=$ acceleration due to gravity $\left(1.0 \mathrm{~kg}-\mathrm{m} / \mathrm{N}-\mathrm{s}^{2}\right)$.
$\rho=$ air density $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$
$\mathrm{f}=$ friction factor.

Because the air velocity is relatively low, the air flow rate can be calculated as

$$
\begin{equation*}
Q=V * A \tag{4.8}
\end{equation*}
$$

$$
\begin{aligned}
& Q=\text { air flow rate }\left(\mathrm{m}^{3} / \mathrm{s}\right) \\
& A=\text { area of the duct cross-section }\left(\mathrm{m}^{2}\right) .
\end{aligned}
$$

From equation 4.8, the following relationship can be found:

$$
\begin{array}{r}
\mathrm{V}=\frac{4}{\pi} \frac{\mathrm{Q}}{\mathrm{D}^{2}} \\
\mathrm{~V}=\mathrm{Q} /(\mathrm{H} * \mathrm{~W}) \tag{4.9b}
\end{array}
$$

For rectangular ducts, the duct width can be interpreted in terms of an equivalent-by-velocity diameter by equating Equations 4.9 a and 4.9 b .

From 4.7 and 4.9, the equivalent-by-velocity diameter can be calculated

$$
\begin{equation*}
D_{v}=\left(\left(\frac{4}{\pi}\right)^{2} f L e * \rho(2 g \Delta P)^{-1} Q^{2}\right)^{1 / 5} \tag{4.10}
\end{equation*}
$$

where
$D_{v}=$ equivalent-by-velocity diameter ( $m$ ).
Le = equivalent length ( m ).
$\Delta \mathrm{P}=$ pressure loss of this duct section ( Pa ).
$\pi=3.1415926535898$

For the rectangular duct

$$
\begin{equation*}
V=Q /\left(H^{*} W\right) \tag{4.11}
\end{equation*}
$$

From equation 4.9 and 4.11 , we can get the equivalent velocity diameter $D_{V}$ of the rectangular duct section:

$$
\begin{equation*}
\mathrm{D}_{\mathrm{v}}=\sqrt{\left(\left(\frac{4}{\pi}\right) * \mathrm{H} * \mathrm{~W}\right)} \tag{4.12}
\end{equation*}
$$

and the equivalent-by-friction diameter for the rectangular duct is:

$$
\begin{equation*}
\mathrm{D}_{\mathrm{f}}=2 *\left(\frac{\mathrm{H} * \mathrm{~W}}{\mathrm{H}+\mathrm{W}}\right) \tag{4.13}
\end{equation*}
$$

We can also find the following relationship.

$$
\begin{equation*}
\mathrm{W}=\pi \frac{\mathrm{D}^{2}}{4 \mathrm{H}} \tag{4.14}
\end{equation*}
$$

Reynold's number is

$$
\begin{equation*}
\mathrm{Re}=\frac{\mathrm{D}_{\mathrm{f}} * \mathrm{~V}}{\mu} \tag{4.15}
\end{equation*}
$$

where
$\mu=$ viscosity.

By using Altshul's equation (Tsal,1989), we can calculate friction factor

$$
\begin{equation*}
\mathrm{f}=0.11\left(\frac{\varepsilon}{\mathrm{D}_{\mathrm{f}}}+\frac{68}{\operatorname{Re}}\right) \tag{4.16}
\end{equation*}
$$

$\varepsilon=$ roughness of the duct material.

Pressure balancing is a natural law of all duct systems. If the designers do not balance the system, the system will balance itself. In which case the air flow of each duct will be different than the designed flow rate. Some room probably has too much of a supply of air, but some room does not have enough supply of air.

If the system (Figure 4) is balanced, three equations have to be satisfied (ASHRAE, 1989).


Figure 4. Five duct sections system.

$$
\begin{aligned}
\Delta P_{1} & =\Delta P_{2} \\
\Delta P_{4} & =\Delta P_{3}+\Delta P_{2} \\
\text { or } \quad P_{4} & =\Delta P_{3}+\Delta P_{1}
\end{aligned}
$$

$$
\begin{aligned}
& P_{f a n}=\Delta P_{5}+\Delta P_{3}+\Delta P_{1} \\
\text { or } \quad & P_{f a n}=\Delta P_{5}+\Delta P_{3}+\Delta P_{2}
\end{aligned}
$$

or $\quad P_{f a n}=\Delta P_{4}+\Delta P_{5}$
The optimal system should not rely on the damper to balance the pressure. The ducts should be sized properly in order to have the appropriate pressure losses.

The fan pressure is distributed as the ratios of optimal pressure distribution factor (OPDF), where the OPD factor is selected by simulated annealing method.

$$
\begin{equation*}
\Delta \mathrm{P}_{\mathrm{i}}=\mathrm{OPDF}_{\mathrm{i}}\left(\mathrm{P}_{\mathrm{fan}}-\sum_{\text {Root }}^{\text {Present section }} \Delta \mathrm{P}\right) \tag{4.17}
\end{equation*}
$$

$i=$ the duct section number of each path.
$\Delta \mathrm{P}_{\mathrm{i}}=$ the pressure losses of each section.

The optimal pressure distribution factor is first calculated by

$$
\begin{equation*}
\mathrm{OPDF}_{\mathrm{i}}=\frac{\mathrm{Le}_{\mathrm{i}}}{\sum_{\text {Present }}^{\text {Leaf of Subree }} \mathrm{Le}} \tag{4.18}
\end{equation*}
$$

Le $e_{i}=$ equivalent length of number i section.
$\sum L e=$ the longest equivalent length of the tree.

Because of the definitions above, all OPDFs are independent themselves. If you change one, the rest do not have to change, and
the system will still be balanced. So the simulated annealing method is able to change the OPD factor without unbalancing the system.

The total equivalent length equals

$$
\begin{equation*}
L e=L+D * C / f \tag{4.19}
\end{equation*}
$$

Le = equivalent length (m).
$\mathrm{D}=$ diameter of the duct ( m ).
$C=$ loss coefficient.
$\mathrm{f}=$ friction factor.
$L=$ the original length of the duct (m).

## Optimization Procedure

OPD method's optimization procedure is much different than the T-method. It starts from choosing the OPDF factors of the duct system. The computer uses a random number generator to determine which OPDF factors should be changed, and which directions the OPDF should change. If the OPDF picked by computer cannot be changed (Terminal node, presized section or the other constraints) or the direction of the change is wrong, another set of random numbers will be needed. After the OPDF is changed, new duct sizes are calculated, and a new life cycle cost of the duct system is calculated. If this new life cycle cost is lower than the previous one, the change will be kept. If the new life cycle cost is higher than the previous one, the change is kept with the possibility of $\exp (-$
$\Delta \mathrm{E} / \mathrm{T})$. T is the pseudo-temperature introduced by simulated annealing method. T will become smaller and smaller until there are no more changes being accepted. The final life cycle cost of the system is the result.

## Fan Pressure Optimization

The fan pressure is calculated differently than the T-method. The fan pressure is not calculated by simulated annealing also. It is calculated by one dimensional minimization method called the Golden Search Method. The relationship between fan pressure and the system life cycle cost is different than the duct sizes. So one dimensional optimization procedure is used to find out the optimal fan pressure. The golden search is used to find out the optimum fan pressure. If the fan has already selected, the fan should work at the maximal efficient point, that pressure should be the optimum fan pressure. If the fan pressure is given by the user, it will not be changed during the optimization.

## CHAPTER V

## DUCT TREE DATA STRUCTURE

The optimal fan pressure distribution method was calculated in C computer language. C gives more feasibility to design those very complex and very large systems. The tree structure programmed in C makes the programming much more logical than the spread sheet.

## Tree presentation

The tree like duct system can be presented in tree data structure. The physical connection between two duct sections can be represented in logical connection between two data structures. So a tree structure of duct network can be exactly duplicated in the machine memory. This will benefit the simulation of duct system.

## Data Structure of "DUCT TREE"

The data structure of one duct section is represented as below.
typedef struct duct_section \{ double V,L,Q,DPz,Dz,C,Df,Dv,D;
double f,DP,DPmax;
double Pup,Pdn,DPt,DPr,DPp;

```
double Le,OPDF;
double H,W;
    int ch1,ch2,presized;
    } duct_section;
```

It contains most of the important information of this duct section for further calculation or output.

Also there is a shell or connecter for this duct section data structure.
typedef struct node \{
int i;
duct_section *D;
struct node *prev;
struct node *ch1;
struct node *ch2;
\} node;

It contains a pointer to the duct section, a duct number, a pointer to the previous duct section, and two pointers to the child sections.

## Traversal of Duct Tree

Theoretically, traverse of the tree can be done in two ways: depth first traverse and breadth first traverse. Breadth first traverse searches the node in a certain depth --- a certain number of layers. This has nothing to do with the physical duct system. So we use the other way, depth first traverse. It depends on if the children are processed first, or parents are processed first. The
depth first traversal of duct tree can be classified in preorder, inorder, and postorder. Preorder traversal of the tree precesses the parent node first, and then goes to the children node. Postorder is the reverse.of the preorder. The parent node is processed last, the child nodes are processed first. The inorder traversal of the data structure tree processes the nodes from left to right (or from right to left). If the calculation starts from fan to terminals, the preorder traversal of the duct tree is the best choice. If the calculation starts from terminal to fan, the postorder traversal of the duct tree is needed.

Recursion, which uses the hardware stack, is faster than the iteration method, and the source code is shorter also. Therefore we used the recursive function to traverse the duct tree.

## Preorder Traversal

The fan pressure should be distributed from the duct section closest to the fan to the terminal or from root to leaf. A preorder traversal of the duct tree is needed. A preorder traversal example shown below is a function called recursive which calculates the pressure loss of each duct section.

```
dstri_pres(H,Pfan)
node *H;
double Pfan;
{
double Dv5,f,Q,Le,dP;
```

```
if( \(\mathrm{H}==\mathrm{NULL}\) )
        return;
    if(H->D->presized != TRUE)
    H->D->DP = Pfan * H->D->OPDF;
\(f=H->D->f ;\)
\(L e=H->D->L e ;\)
\(Q=H->D->Q ;\)
\(d P=H->D->D P-H->D->D P z ;\)
    if(dP <= 0.0)
        dP = 0.01;
    Dv5 = 16./(PI * PI) * \(f\) * Le
            * DENSITY * (Q * Q)/(2. * GC * dP);
    if(H->D->presized == FALSE)
        H->D->Dv = pow(Dv5,0.2);
    dstri_pres(H->ch1,(Pfan - H->D->DP));
    dstri_pres(H->ch2,(Pfan - H->D->DP));
\}
```


## Postorder Traversal

The biggest pressure loss of each path of the duct network should be calculated from the terminal to fan or from leaf to root. A postorder traversal of the duct tree network is used. This function is a recursive function and returns a value of biggest pressure loss of the whole duct network.

```
double calc_DP(H,DP,biggest)
node *H;
double DP,biggest;
{
    if(H == NULL)
        return(max(biggest,DP));
    biggest = calc_DP(H->ch1,H->D->DP + DP,biggest);
    biggest = calc_DP(H->ch2,H->D->DP + DP,biggest);
    if((H->ch1 == NULL)&&(H->ch2 == NULL))
        biggest = max(biggest, DP);
    return(biggest);
}
```


## CHAPTER VI

## RESULTS, DISCUSSIONS AND CONCLUSIONS

The mathematical model developed in this work was programmed using C language. The computer program worked under both DOS and UNIX system, on both IBM 386 and RISC System 6000. This program is capable of minimizing the life cycle cost of both rectangular and round duct systems. Another program was developed to implement the T-method for comparison.

Both of these two programs can solve the supply-return system problems. Both programs have solved the example problem in the ASHRAE Handbook of Fundamental 1989 and Tsal's Five duct sections example problem. The computer output can be found in the Appendix.

## Results of Simulated Annealing

From Figure 5 to Figure 8, the plots show how the simulated annealing method worked to minimize the life cycle cost of the ASHRAE example problem. The modified simulated annealing method started from the results of balanced capacity method. The air velocities shown on the figures are the air velocities of the longest paths of the system.


Figure 5. ASHRAE Example ( $\mathrm{V}=7.5 \mathrm{~m} / \mathrm{s}$ )

In ASHRAE example, because the fan is preselected both the life cycle cost of the duct system and the total duct surface areas can be the objective function. They have a constant relation.


Figure 6. ASHRAE Example ( $V=7.5 \mathrm{~m} / \mathrm{s}$ )

A different starting point was used to test if the lowest life cycle cost of the duct system has any relationship with the starting point.


Figure 7. ASHRAE Example ( $V=4.5 \mathrm{~m} / \mathrm{s}$ )


Figure 8. ASHRAE Example ( $V=4.5 \mathrm{~m} / \mathrm{s}$ )

We found that the lowest cost has nothing to do with the starting point. The objective function started from different starting points and terminated at the same result.

We observed the significant changes of the total duct surface areas, of the ASHRAE example (Figure 5 and Figure 7). And also we found that the objective function and the total duct surface areas has a very close relationship. Because the fan pressure is preselected, the duct surface areas are actually the objective function.


Figure 9. ASHRAE Example Final Calculation ( $V=7.5 \mathrm{~m} / \mathrm{s}$ )


Figure 10. ASHRAE Example Final Calculation ( $\mathrm{V}=4.5 \mathrm{~m} / \mathrm{s}$ )

Figure 9 and Figure 10 are the detailed analyses of the annealing procedures. We can see that the objective function oscillated violently even at the final stage of the calculation. Perspectively, the objective function moved towards the global minimum. It started at $\$ 8692$ and ended at $\$ 7528$. The duct surface area reduced from 211 square meters to 180.12 square meters. The surface area is $14.8 \%$ smaller than the ASHRAE handbook example (211 square meters).


Fan Pressure $=835 \mathrm{~Pa}$.

Figure 11. Duct Surface Area of Different Design Method.

Figure 11 shows the comparison of the OPD method, the Tmethod and the ASHRAE example. Because the fan pressures used by different methods are the same, the duct surface areas can represent the cost of the system, which is the objective function. "Aft Rd" and "Bf Rd" stand for "After rounding" and "Before Rounding". We noticed the size rounding does not make too much difference of the total duct surface area.


Figure 12. Pressure Changes of the Duct Sections.

Figure 12 shows the changes of pressure loss of each duct section. The OPD method is compared with the equal capacity method which is the starting point of the OPD method. The negative pressure losses refer to the return duct sections. The preselected duct sections' pressure losses are kept the same during the calculation.

## Results of Golden Search Method

The golden search method was used to find out the optimum fan pressure. It solved an example problem given by Tsal (1989). The result is presented in Figure 13 and Figure 14. The life cycle cost
went down quickly and stopped at the bottom of the objective function while the fan pressure changed.


Figure 13. Life Cycle Cost vs. Fan Pressure
Five Sections Ductwork.

Figure 13 shows when the fan pressure increases the objective function goes down hill. But from a detailed look of the iteration, we will find that the objective function will goes uphill if we continue to increase the fan pressure (Figure 14).


Figure 14. Details of Cost vs. Fan Pressure
Five Section Ductwork.

The computer program stopped at the lowest point of the pressure-cost curve. In this Five Duct Sections problem, if we continue to increase fan pressure, the life cycle cost of the duct system will increase rapidly.

## Comparison of Pressure Losses

Without changing the fan pressure, how is the duct material saved? The answer is the fan pressure distributed more optimally. The pressure saved by the shorter duct sections which have the
relatively high fitting resistance is used to shrink the longer duct sections' diameters. We can find the significant duct diameter changes are made by the computer programs in Figure 15.

Duct Diameters of Different Methods (ASHRAE Example)


Figure 15. Duct Diameters of Different Methods ASHRAE Example

The ASHRAE example problem was solved by T-method and the OPD method. We got little different results from Tsal's (1989) because we used different computer methods than Tsal. There is a round off error. There is a small difference between the T-method's results and OPD method's results. But they got the similar total duct surface areas (Figure 12). From Figure 15 we can see that the
biggest pressure changes happened in the return duct sections. Most of the return duct sections have long lengths but less fitting frictions in ASHRAE example. An increase in the velocities of these duct sections can decrease the cross-sections of the ducts; therefore, the surface areas of the duct system will decrease. However, the increase of the pressure loss of the return system will decrease the velocity of the supply system, because the fan is fixed. We can see from Figure 15 that most of the duct pressure losses are reduced. Also we can find that not all of the duct pressure losses decreased. Sections 13 and 14 have relatively low fitting frictions compared to their lengths; therefore their pressure losses increase to reduce the surface area. Sections 18 and 19 have large fitting resistance but relatively short lengths; therefore the pressure losses at these two sections decline to save the energy for the other duct sections.

## Comparison to T-Method

Both the T-Method and the OPD method can find the global minimum of the objective function. Their results validated each other. The T-Method has less iterations but relies on the partial derivatives of the objective function, and hence requires an objective function with analytically differentiable partial derivatives. The OPD method has more calculations but is more flexible to add constraints to without changing the mathematics model too much. Besides the constraints the T-Method can solve, the OPD method can solve the additional constraints, like air velocity
limit, and static pressure limit or the other critical constraints. The changes required to add the new constraints to the software are small. It is more flexible to meet the future challenge of the new constraints brought by new control technology and the VAV system. Another improvement which the OPD method made is that the OPD method's objective function can be life cycle cost, or something else. Many kinds of economic analysis models can be used as the objective function. This gives the OPD method great advantage over T-method in business application.

## Different Starting Point

We have tried to work on the same problem from different starting points. The results (Figure 5 and Figure 7) show that the simulated annealing method is able to reach the same answer (global minimum) from two different starting points.

## Conclusions

Based on this study, the following conclusions have been developed:
(1) Both the OPD method and the T-method find essentially the same minimum of the objective function. The closeness of the minimum points strongly suggests that both methods have found the global minimum.
(2) As described by Tsal, the T-method has not been shown able to incorporate constraints such as air velocity limits or static pressure limits. Further more, it seems unlikely that such constraints
can be incorporated without fundamentally changing the method. The addition of a penalty function would add multiple singularities to the objective function, rendering the analytical partial derivatives indeterminate. On the other hand, the OPD method can easily incorporate such constraints.
(3) The OPD method's objective function is not limited to life cycle cost. It can be modified without changing the method itself. For example, the objective function could be the first cost of the system, including the fan.
(4) Most of the existing duct design methods can be cast as methods for determining the fan pressure distribution ratios of the ducts. This might be useful for future studies.
(5) The OPD method is a good alternative to the T-method.

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APPENDIXES

APPENDIX A
ASHRAE EXAMPLE SCHEMATIC


APPENDIX B
ASHRAE EXAMPLE COMPUTER OUTPUT OPD METHOD


| T-0.000000 |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |
| Noto | 7 | 0 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |  |
| $v$ | 3.038 | 3.832 | 0.623 | 0.276 | 10.209 | 12.100 | 15.267 | 14.488 | 11.089 |  |
| opor | 0 | O | 0.004 | 0.616 | 1 |  | 0.467 | 0.155 | 1 |  |
| 16 | 12.119 | 71.700 | 22.006 | 30.447 | 25.47 | 16.304 | 12.040 | 5.186 | 16.111 |  |
| $L$ | 4.27 | 1.22 | 7.62 | 13.72 | 0.10 | 0.71 | 10.67 | 4.57 | 0.75 |  |
| 0 | 0.20 | 0.22 | 0.56 | 0.56 | 0.47 | 0.47 | 0.94 | 1.51 | 0.18 |  |
| $0{ }^{2}$ | 23 | 37.6 | 0 | 0 | O | 0 | 0 | 0 | 0 |  |
| 08 | 0.303 | 0.303 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| c | 1.09 | 5.3 | 1.20 | 2.19 | 1.79 | 1.12 | 0.12 | 0.04 | 1.27 |  |
| 0 | 0.303 | 0.308 | 0.263 | 0.28 | 0.211 | 0.19 | 0.240 | 0.314 | 0.129 |  |
| 0 | 0.306 | 0.305 | 0.280 | 0.209 | 0.241 | 0.222 | 0.23 | 0.3 Ca | 0.148 |  |
| 1 | 0.029 | 0.023 | 0.022 | 0.022 | 0.023 | 0.023 | 0.022 | 0.02 | 0.026 |  |
| 0 | 36.00 | 05.012 | 05.003 | 137.089 | 176.353 | 170.582 | 140.441 | 42.480 | 237.307 |  |
| DP9me | 30.00 | 05.012 | 15.012 | 0.012 | 0 | 0 | 0 | 15.012 | 0 |  |
| का | 0 | 0 | 7 | 0 | 0 | 0 | 11 | 10 | 0 |  |
| ar | 0 | 0 | 0 | 0 | 0 | 0 | 12 | 13 | 0 |  |
| 0 | 0.308 | 0.309 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| H | 0 | 0 | 0.25 | 0.25 | 0.250 | 0.254 | 0.259 | 0.254 | 0.152 |  |
| $w$ | 0 | 0 | 0.256 | 0.256 | 0.10 | 0.158 | 0.249 | 0.41 | 0.113 |  |
| Mano ${ }^{\text {ar }}$ | E-C | Cn $A$ | 114093 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
| T 0.0 .000000 |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
| Hoco | 16 | 17 | 10 | 10 |  |  |  |  |  |  |
| $v$ | 12.220 | 11.704 | 4.074 | 4.740 |  |  |  |  |  |  |
| OFOF | 1 | 0.351 | 0.201 | 0.114 |  |  |  |  |  |  |
| 6 | 11.042 | 11.7 | 147.013 | 108.11 |  |  |  |  |  |  |
| L | 6.1 | 0.14 | 7.01 | 3.66 |  |  |  |  |  |  |
| 0 | 0.10 | 0.36 | 1.09 | 1.09 |  |  |  |  |  |  |
| DP2 | 0 | 0 | 10 | 12.5 |  |  |  |  |  |  |
| 02 | 0 | 0 | 0 | 0 |  |  |  |  |  |  |
| C | 1.00 | 0.34 | 4.28 | 3.06 |  |  |  |  |  |  |
| 0 | 0.118 | 0.179 | 0.616 | 0.629 |  |  |  |  |  |  |
| 0 | 0.137 | 0.203 | 0.684 | 0.71 |  |  |  |  |  |  |
| 1 | 0.026 | 0.024 | 0.010 | 0.010 |  |  |  |  |  |  |
| D | 240.507 | 127.716 | 76.706 | 65.362 |  |  |  |  |  |  |
| DPmen | 0 | 0 | 05.012 | 107.512 |  |  |  |  |  |  |
| का | 0 | 18 | 19 | 10 |  |  |  |  |  |  |
| al | 0 | 16 | 17 | 0 |  |  |  |  |  |  |
| 0 | 0 | 0 | 0 | 0 |  |  |  |  |  |  |
| H | 0.152 | 0.203 | 0.61 | 0.61 |  |  |  |  |  |  |
| W | 0.007 | 0.16 | 0.62 | 0.649 |  |  |  |  |  |  |
| Pman 0 | E.C | mat | .11409 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
| T = 777.0000 |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
| Now | 1 | 2 | 3 | 4 | 6 | 6 |  |  |  |  |
| $v$ | 13.202 | 0.080 | 10.18 | 2.526 | 12.029 | 12.104 |  |  |  |  |
| OFOF | 1 | 1 | 0.183 | 0 | 0.096 | 0.207 |  |  |  |  |
| 16 | 27.109 | 35.244 | 11.351 | 17.107 | 36.003 | 17.541 |  |  |  |  |
| L | 24.30 | 23.77 | 7.31 | 1.52 | 20.42 | 11.00 |  |  |  |  |
| 0 | 0.71 | 0.24 | 0.04 | 0.64 | 0.04 | 1.00 |  |  |  |  |
| OP7 | 0 | 0 | 0 | 25 | 0 | 68 |  |  |  |  |
| 02 | 0 | 0 | 0 | 0 | 0 | 0 |  |  |  |  |
| c | 0.23 | 1.50 | 0.24 | 0.52 | 1.06 | 0.24 |  |  |  |  |
| 0 | 0.262 | 0.103 | 0.343 | 0.61 | 0.304 | 0.445 |  |  |  |  |
| O | 0.283 | 0.103 | 0.343 | 0.680 | 0.304 | 0.445 |  |  |  |  |
| 1 | 0.021 | 0.024 | 0.02 | 0.02 | 0.021 | 0.010 |  |  |  |  |
| 0 | 233.160 | 232.432 | 41.910 | 27.185 | 245.62 | 120.424 |  |  |  |  |
| DPmax | 0 | 0 | 0 | 27.183 | 27.186 | 02.105 |  |  |  |  |
| का | 0 | 0 | 1 | 0 | 4 | 3 |  |  |  |  |
| ar | 0 | 0 | 2 | 0 | 0 | 6 |  |  |  |  |
| 0 | 0.254 | 0.170 | 0.353 | 0 | 0.305 | 0.457 |  |  |  |  |
| H | 0 | 0 | 0 | 0.61 | 0 | 0 |  |  |  |  |
| W | 0 | 0 | 0 | 0.61 | 0 | 0 |  |  |  |  |
| Pmon $=3{ }^{\text {c }}$ | $E=7$ | 100 A | . 6370 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
| T-77.0000 |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
| Nodo | 7 | 0 | 0 | 10 | 11 | 12 | 13 | 14 | 15 |  |
| $v$ | 3.832 | 3.832 | 0.623 | 0.276 | 10.284 | 12.100 | 15.257 | 14.488 | 11.080 |  |
| OPDF | 0 | 0 | 0.094 | 0.619 | 1 | 1 | 0.457 | 0.155 | 1 |  |
| 10 | 12.110 | 71.780 | 22.086 | 39.447 | 25.47 | 10.394 | 12.040 | 5.185 | 16.111 |  |
| L | 4.27 | 1.22 | 7.62 | 13.72 | 0.16 | 6.71 | 10.67 | 4.57 | 0.75 |  |
| 0 | 0.29 | 0.20 | 0.50 | 0.56 | 0.47 | 0.47 | 0.04 | 1.51 | 0.19 |  |
| $0 \cdot 2$ | 23 | 37.5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| ${ }^{2}$ | 0.305 | 0.303 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| c | 1.04 | 6.3 | 1.26 | 2.10 | 1.73 | 1.10 | 0.12 | 0.04 | 1.27 |  |
| 0 | 0.305 | 0.303 | 0.255 | 0.26 | 0.211 | 0.10 | 0.248 | 0.314 | 0.120 |  |
| 0 | 0.303 | 0.308 | 0.288 | 0.294 | 0.241 | 0.222 | 0.28 | 0.364 | 0.140 |  |
| 1 | 0.023 | 0.023 | 0.022 | 0.022 | 0.023 | 0.023 | 0.022 | 0.02 | 0.026 |  |
| 0 | 36.00 | 15.012 | 05.008 | 137.904 | 176.353 | 179.682 | 146.441 | 42.400 | 237.307 |  |
| OPmin | 36.09 | 15.012 | 15.012 | 05.012 | 0 | 0 | 0 | 15.012 | 0 |  |
| को | 0 | 0 | 7 | 9 | 0 | 0 | 11 | 10 | 0 |  |
| ct | 0 | 0 | 8 | 0 | 0 | 0 | 12 | 13 | 0 |  |
| 0 | 0.309 | 0.306 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| H | 0 | 0 | 0.254 | 0.254 | 0.254 | 0.254 | 0.254 | 0.254 | 0.152 |  |
| W | 0 | 0 | 0.254 | 0.270 | 0.178 | 0.152 | 0.254 | 0.406 | 0.127 |  |
| Pancos | E. 73 | 100 A | 262700 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |


| T-77.000000 |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Nomen | 16 | 17 | 10 |  |  |  |  |  |  |  |
| $v$ | 12.820 | 11.004 | 4.074 | 4.740 |  |  |  |  |  |  |
| Orof |  | 0.351 | 0.201 | 0.114 |  |  |  |  |  |  |
| 6 | 11.042 | 11.7 | 147.019 | 100.81 |  |  |  |  |  |  |
|  | 0.1 | 0.14 | 7.01 | 3.60 |  |  |  |  |  |  |
| 0 | 0.10 | 0.21 | 1.80 | 1.02 |  |  |  |  |  |  |
| OP\% | $\bigcirc$ |  | 10 | 12.6 |  |  |  |  |  |  |
| $\frac{08}{6}$ | 1.0 | 0.4 | 4.21 | 3.08 |  |  |  |  |  |  |
| a | 0.110 | 0.17 | 0.016 | 0.029 |  |  |  |  |  |  |
| ${ }_{0}$ | 0.137 | 0.203 | 0.004 | 0.71 |  |  |  |  |  |  |
| 1 | 0.023 | 0.024 | 0.010 | 0.010 |  |  |  |  |  |  |
| op | 240.607 | 127.716 | 70.708 | 65.302 |  |  |  |  |  |  |
| OPmax | 0 |  | 08.012 | 107.512 |  |  |  |  |  |  |
| क1 | 0 | 18 | 14 | 10 |  |  |  |  |  |  |
| cor | $\bigcirc$ | 16 | 17 |  |  |  |  |  |  |  |
| $\frac{\mathrm{O}}{\mathrm{H}}$ | $0.16{ }^{\circ}$ | 0.209 | 0.01 | 0.01 |  |  |  |  |  |  |
| $w$ | 0.127 | 0.153 | 0.61 | 0.60 |  |  |  |  |  |  |
| Pton | E. 7 | 100 A | . 63700 |  |  |  |  |  |  |  |

APPENDIX C
FIVE DUCT SECTION PROBLEM COMPUTER OUTPUT OPD METHOD


APPENDIX D

ASHRAE EXAMPLE COMPUTER OUTPUT

T-METHOD



| nexation? |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| .1........................................................................ |  |  |  |  |  |  |  |  |  |  |
| Noce: 1 | 2 | 3 | 4 | 5 | 6 |  |  |  |  |  |
| V : $12 . \mathrm{Cl}$ | 8.72 | 10.64 | 2.520 | 12.52 | 11.764 |  |  |  |  |  |
| L: 24.350 | 23.77 | 7.31 | 1.52 | 20.42 | 11.09 |  |  |  |  |  |
| 0: 0.710 | 0.24 | 0.94 | 0.99 | 0.04 | 1.00 |  |  |  |  |  |
| DFFE: 0.00 | 0 | 0 | 26 | 0 | 55 |  |  |  |  |  |
| 02: 0.00 | 0 | 0 | 0 | 0 | 0 |  |  |  |  |  |
| C: 0.220 | 1.59 | 0.24 | 0.52 | 1.06 | 0.24 |  |  |  |  |  |
| Of: 0.2:37 | 0.107 | 0.333 | 0.61 | 0.300 | 0.461 |  |  |  |  |  |
| Or. 0.25 | 0.107 | 0.336 | 0.689 | 0.300 | 0.451 |  |  |  |  |  |
| t. 0.081 | 0.024 | 0.02 | 0.02 | 0.021 | 0.010 |  |  |  |  |  |
| DF: 200 C | 210.783 | 40.782 | 27.101 | 227.05 | 116.101 |  |  |  |  |  |
| mu: 0.58 | 0.669 | 0.23 | 0.303 | 0.749 | 0.332 |  |  |  |  |  |
| Ka: 10.00 | 13.047 | 5.314 | 0 | 12.804 | 12.270 |  |  |  |  |  |
| Kt 10.0 | 13.047 | 40.02 | 0 | 18.804 | 79.383 |  |  |  |  |  |
| T: 1.000 | 1 | 0.112 | 0 | 1 | 0.211 |  |  |  |  |  |
| DPman: $0 . C$ | 0 | 0 | 27.101 | 27.101 | 12.101 |  |  |  |  |  |
| P2: 200.6 | 200.811 | 250.4 | 27.181 | 266.4 | 372.741 |  |  |  |  |  |
| Ptin: 0.00 | 0 | 200.611 | 0 | 27.101 | 250.4 |  |  |  |  |  |
| Opt 200.6 | 200.611 | 46.788 | 27.191 | 228.200 | 116.342 |  |  |  |  |  |
| DPr. 200.0 | 200.611 | 46.780 | 0.1 | 220.200 | 61.342 |  |  |  |  |  |
| का: 0 | 0 | 1 | 0 | 4 | 3 |  |  |  |  |  |
| de 0 | 0 | 2 | 0 | 0 | 3 |  |  |  |  |  |
|  | 373.720 | 102.53 | 371.381 | Y4.1श1 | 11.101 |  |  |  |  |  |
| Dreat 08 | . 0.006 | 0 | 1.48 | - 0 | 0 |  |  |  |  |  |
| 0: 08:57 | 0.107 | 0.336 | 0 | 0.300 | 0.451 |  |  |  |  |  |
| H: 0.000 | 0 | 0 | 0.61 | 0 | 0 |  |  |  |  |  |
| w: 0.006 | 0 | 0 | 0.609 | 0 | 0 |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
| …........................................................................... |  |  |  |  |  |  |  |  |  |  |
| Noto: 7 | d | O\| | 10 | 11 | 12 | 13 | 14 | 15 | 16 |  |
| V : 3.69 | 3.032 | 0.306 | 8.467 | 10.168 | 11.041 | 14.007 | 12.007 | 10.203 | 11.725 |  |
| L: 4.270 | 1.22 | 7.62 | 13.72 | 0.16 | 6.71 | 10.07 | 4.67 | 0.75 | 6.1 |  |
| 0: 0.200 | 0.20 | 0.56 | 0.56 | 0.47 | 0.47 | 0.94 | 1.51 | 0.19 | 0.10 |  |
| DP2: 26.00 | 37.6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 02: 0.30 | 0.305 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| C: 1.040 | 6.3 | 1.20 | 2.10 | 1.78 | 1.10 | 0.12 | 0.04 | 1.27 | 1.00 |  |
| Dot. 0.506 | 0.305 | 0.250 | 0.257 | 0.212 | 0.192 | 0.251 | 0.329 | 0.136 | 0.125 |  |
| Or: 0.30 | 0.305 | 0.292 | 0.201 | 0.243 | 0.224 | 0.244 | 0.310 | 0.154 | 0.144 |  |
| t. 0.009 | 0.023 | 0.022 | 0.022 | 0.023 | 0.023 | 0.022 | 0.02 | 0.026 | 0.026 |  |
| DP: 3.CC | 05.012 | 00.712 | 144.602 | 171.522 | 171.31 | 137.652 | 31.125 | 104.648 | 104.163 |  |
| mu: 0.AI | 1.644 | 0.659 | 0.070 | 0.673 | 0.448 | 0.204 | 0.125 | 0.40 | 0.333 |  |
| K: 0.000 | 0 | 0.080 | 12.23 C | 7.157 | 4.014 | 0.103 | 4.203 | 4.017 | 2.109 |  |
| KL 0.000 | 0 | 6.080 | 20.612 | 7.157 | 4.016 | 24.300 | 52.764 | 4.917 | 2.097 |  |
| T: 0.000 | 0 | 1 | 0.042 | 1 | 1 | 0.443 | 0.121 | 1 | 1 |  |
| DFmax: 30. | 85.012 | 05.012 | 85.012 | 0 | 0 | 0 | 85.012 | 0 | 0 |  |
| Pra: 0.09 | 05.012 | 105.653 | 300.09 | 172.480 | 172.451 | 300.84 | 341.01 | 103.724 | 193.724 |  |
| Path: 40.06 | 0 | 05.012 | 165.539 | 0 | 0 | 172.450 | 309.04 | 0 | 0 |  |
| Of: 36.C | 35.012 | 60.541 | 144.387 | 172.400 | 172.400 | 137.454 | 31.07 | 193.724 | 193.724 |  |
| Depr 36.Cl | 15.012 | 00.541 | 144.307 | 172.486 | 172.480 | 137.454 | 31.07 | 193.724 | 103.724 |  |
| chl: 0 | 0 | 7 | 0 | 0 | 0 | 11 | 10 | 0 | 0 |  |
| O2: 0 | 0 | 3 | 0 | 0 | 0 | 12 | 13 | 0 | 0 |  |
| Opp: 417.8 | 466.682 | 360.54 | 200.020 | 464.310 | 464.108 | 292.709 | 155.146 | 466.086 | 465.601 |  |
| DPem: 4.7 | . 3.209 | 0 | 0 | -2.061 | . 1.840 | 0 | -0 | -3.127 | . 3.342 |  |
| 0: 0.306 | 0.309 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| H: 0.000 | 0 | 0.259 | 0.254 | 0.254 | 0.254 | 0.254 | 0.254 | 0.152 | 0.152 |  |
| w: 0.000 | 0 | 0.263 | 0.881 | 0.102 | 0.159 | 0.240 | 0.460 | 0.122 | 0.107 |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
| neretion 2 |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
| Noco: 17 | 10 | 19 |  |  |  |  |  |  |  |  |
| V : $12.3{ }^{\circ}$ | 4.817 | 4.377 |  |  |  |  |  |  |  |  |
| L: 0.140 | 7.01 | 3.66 |  |  |  |  |  |  |  |  |
| 0: 0.300 | 1.09 | 1.08 |  |  |  |  |  |  |  |  |
| Dp:z: 0.00 | 10 | 12.5 |  |  |  |  |  |  |  |  |
| DE: 0.000 | 0 | 0 |  |  |  |  |  |  |  |  |
| C: 0.340 | 4.21 | 3.06 |  |  |  |  |  |  |  |  |
| 0t: 0.173 | 0.619 | 0.654 |  |  |  |  |  |  |  |  |
| Dr: 0.10 | 0.701 | 0.744 |  |  |  |  |  |  |  |  |
| f. 0.084 | 0.019 | 0.019 |  |  |  |  |  |  |  |  |
| DP: 147.a | 75.145 | 40.876 |  |  |  |  |  |  |  |  |
| mu: 0.31 | 3.133 | 2.341 |  |  |  |  |  |  |  |  |
| Kt: 5.62 | 12.797 | 6.319 |  |  |  |  |  |  |  |  |
| KEt 15.40 | 80.004 | 100.872 |  |  |  |  |  |  |  |  |
| $T$ T 0.432 | 0.109 | 0.098 |  |  |  |  |  |  |  |  |
| DPmax: 0.C | 05.012 | 107.512 |  |  |  |  |  |  |  |  |
| Pup: 31.0 | 414.6 | 482.259 |  |  |  |  |  |  |  |  |
| Poti: 1027 | 341.01 | 414.5 |  |  |  |  |  |  |  |  |
| DPEt 147.2 | 73.401 | 47.753 |  |  |  |  |  |  |  |  |
| DFT. 1472 | 63.401 | 35.250 |  |  |  |  |  |  |  |  |
| ch1: 15 | 14 | 18 |  |  |  |  |  |  |  |  |
| cre: 16 | 17 | 0 |  |  |  |  |  |  |  |  |
| DPp: 271.4 | 124.021 | 41.876 |  |  |  |  |  |  |  |  |
| Dpeat 0.0. | 0 | 0 |  |  |  |  |  |  |  |  |
| D: 0.000 | 0 | 0 |  |  |  |  |  |  |  |  |
| H: 0.203 | 0.61 | 0.61 |  |  |  |  |  |  |  |  |
| W: 0.15 | 0.627 | 0.7091 |  |  |  |  |  |  |  |  |
| DPoxmex - 44.723 Ep $=1510.761400$ Es -5314.032027 |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |




APPENDIX E
FIVE DUCT SECTION PROBLEM COMPUTER OUTPUT
T-METHOD

|  | Herreven 0 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Nocio | -1 | 2 | 3 | 4 | 5 |  |
| $v$ | 7.6 | 7.6 | 10.750 | 7.6 | 7.5 |  |
| L | 14 | 12 | 0 | 10 | 10.81 |  |
| 0 | 0.7 | 0.22 | 0.08 | 0.6 | 1.42 |  |
| DP\% | 25 | 37.5 | 0 | 0 | 37.6 |  |
| 03 | 0 | 0 | 0.23 | 0 | 0 |  |
| c | 0.0 | 0.05 | 0.10 | 0.65 | 1.5 |  |
| 0 | 0.3 | 0.103 | 0.33 | 0.801 | 0.401 |  |
| 0 | 0.278 | 0.161 | 0.33 | 0.22 | 0.442 |  |
| 1 | 0.010 | 0.019 | 0.010 | 0.010 | 0.010 |  |
| D | 81.009 | 08.269 | 44.478 | 67.166 | 114.008 |  |
| mu | 0.501 | 0.354 | 0.211 | 0.403 | 1.113 |  |
| $\mathrm{K}_{0}$ | 12.407 | 5.310 | 0 | 10.620 | 23.286 |  |
| 18 | 12.407 | 6.310 | 17.1816 | 10.636 | 80.259 |  |
| 1 | 1 | 1 | 0 |  | 0.450 |  |
| DPmin | 25 | 37.5 | 11.072 | 0 | 11.472 |  |
| AP | 180.411 | 100.411 | 234.089 | 234.689 | 402.189 |  |
| Ptin | 0 | 0 | 180.411 | 0 | 234.889 |  |
| Of | 180.411 | 100.411 | 44.472 | 234.019 | 167.309 |  |
| Ofi | 105.411 | 182.011 | 44.472 | 2\%.003 | 12..609 |  |
| al | 0 | 0 | 1 | 0 | 3 |  |
| de | 0 | 0 | 2 | - 0 | 4 |  |
| Op? | 240.300 | 257.746 | 130.479 | 171.172 | 114.000 |  |
| DPer | 101.810 | 144.430 | of | 231.013 | 0 |  |
| D | 0 | 0.161 | 0.33 | 0.22 | 0.442 |  |
| H | 0.2 Fa | 0 | 0 | 0 | 0.4 |  |
| $w$ | 0.307 | 0 | 0 | 0 | 0 |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| - Meraten 1 |  | .......... | -......... | ........ |  |  |
|  |  |  |  |  |  |  |
| -1.0.0...................... |  |  | -........... | ........... |  |  |
| Nose | TI | 2 | 3 | 4 | 5 |  |
| $v$ | 11.409 | 10.776 | 10.756 | 13.107 | 0.265 |  |
| L | 14 | 12 | ${ }^{\circ}$ | 16 | 10.01 |  |
| 0 | 0.7 | 0.22 | 0.02 | 0.5 | 1.42 |  |
| $00_{2}$ | 26 | 37.6 | 0 | 0 | 37.5 |  |
| $\mathrm{D}_{2}$ | 0 | 0 | 0.33 | - | 0 |  |
| c | 0.0 | 0.65 | 0.10 | 0.65 | 1.6 |  |
| 0 | 0.247 | 0.161 | 0.33 | 0.22 | 0.442 |  |
| Or | 0.279 | 0.169 | 0.33 | 0.22 | 0.437 |  |
| 1 | 0.022 | 0.025 | 0.02 | 0.022 | 0.010 |  |
| 0 | 187.308 | 200.881 | 46.09 | 230.642 | 150.175 |  |
| mu | 0.57 | 0.300 | 0.223 | 0.502 | 1.044 |  |
| Ks | 12.248 | 5.440 | O | 10.663 | 22.009 |  |
| 18 | 12.249 | 6.440 | 17.609 | 10.603 | 58.813 |  |
| $r$ |  | 1 | 0 | 1 | 0.467 |  |
| Demen | 25 | 37.5 | 6.40 | 0 | 121.00 |  |
| P | 100.019 | 100.016 | 236.009 | 236.090 | 402.023 |  |
| Pot | 0 | 0 | 100.010 | 0 | 238.009 |  |
| On | 100.016 | 100.016 | 46.00 | 236.000 | 105.926 |  |
| Opt | 165.019 | 152.510 | 40.09 | 236.089 | 128.423 |  |
| का | 0 | 0 | 1 | 0 | 3 |  |
| di | 0 | 0 | 2 | 0 | 4 |  |
| DPP | 303.461 | 416.037 | 208.153 | 307.017 | 150.176 |  |
| OPOM | 0.464 | .13.112 | 0 | 5.107 | 0 |  |
| 0 | 0 | 0.106 | 0.33 | 0.22 | 0.437 |  |
| H | 0.254 | 0 | 0 | 0 | 0 |  |
| $w$ | 0.24 | 0 | 0 | 0 | 0 |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| nerasen 2 1 , |  |  |  |  |  |  |
|  |  |  | -0.0.0. | -......... |  |  |
| Noco | 1 | 2 | 3 | 4 | 5 |  |
| $v$ | 11.576 | 10.259 | 10.750 | 13.157 | 9.409 |  |
| $L$ | 14 | 12 | 6 | 16 | 18.81 |  |
| 0 | 0.7 | 0.22 | 0.02 | 0.5 | 1.42 |  |
| OP2 | 29 | 37.5 | 0 | 0 | 37.5 |  |
| D2 | 0 | 0 | 0.33 | 0 | 0 |  |
| c | 0.0 | 0.69 | 0.18 | 0.65 | 1.5 |  |
| 0 | 0.246 | 0.105 | 0.33 | 0.22 | 0.437 |  |
| 0 | 0.277 | 0.169 | 0.33 | 0.22 | 0.427 |  |
| 1 | 0.022 | 0.024 | 0.02 | 0.022 | 0.019 |  |
| OP | 18.011 | 100.5c9 | 46.09 | 238.090 | 165.03 |  |
| mi | 0.560 | 0.401 | 0.223 | 0.502 | 1.037 |  |
| K | 12.244 | 5.454 | 0 | 10.563 | 22.08 |  |
| 18 | 12.244 | 6.454 | 17.607 | 10.583 | 50.764 |  |
| $T$ | 1 | , | 0 | 1 | 0.457 |  |
| Ormen | 25 | 37.6 | 6.40 | 0 | 121.08 |  |
| Po | 190.037 | 180.037 | 237.017 | 237.017 | 402.009 |  |
| Pot | 0 | 0 | 190.037 | 0 | 237.017 |  |
| DP? | 180.037 | 100.037 | 46.08 | 237.017 | 165.781 |  |
| Of | 165.037 | 152.537 | 46.98 | 237.017 | 128.291 |  |
| al | 0 | 0 | 1 | 0 | 3 |  |
| at | 0 | 0 | 2 | 0 | , |  |
| OPP | 401.921 | 402.574 | 212.01 | 401.028 | 165.03 |  |
| Drox | 0.887 | 0.235 | 0 | 0.08 | 0 |  |
| 0 | 0 | 0.165 | 0.33 | 0.22 | 0.437 |  |
| H | 0.254 | 0 | 0 | 0 | 0 |  |
| w | 0.230 | 0 | 0 | 0 | 0 |  |
| DPcunion a 0.007 Ep $=733.141008$ E |  |  | 2304.531517 |  |  |  |
|  |  |  |  |  |  |  |



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| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
|  | 1.77 |  |  |  |  |  |
|  | -0.0.0.0. | .......... | ............ | .a.o...... |  |  |
| Nato | 1 | 2 | 3 | 4 | 6 |  |
| $V$ | 11.81 | 10.241 | 10.760 | 13.158 | 0.484 |  |
| 6 | 14 | 12 | B | 10 | 18.81 |  |
| 0 | 0.7 | 0.28 | 0.08 | 0.6 | 1.42 |  |
| OP1 | 23 | 37.6 | 0 | 0 | 37.6 |  |
| $\mathrm{D}_{5}$ | 0 | 0 | 0.29 | 0 | 0 |  |
| C | 0.0 | 0.03 | 0.18 | 0.03 | 1.6 |  |
| 0 | 0.246 | 0.106 | 0.33 | 0.22 | 0.437 |  |
| 0 | 0.277 | 0.108 | 0.33 | 0.22 | 0.437 |  |
| 1 | 0.022 | 0.024 | 0.02 | 0.022 | 0.010 |  |
| 0 | 180.078 | 10.085 | 46.08 | 238.104 | 10.1.27 |  |
| mu | 0.508 | 0.401 | 0.229 | 0.502 | 1.038 |  |
| K | 12.248 | 6.484 | 0 | 10.803 | 22.057 |  |
| 10 | 12.244 | 6.464 | 17.607 | 10.508 | 60.74 |  |
| $T$ | 1 | 1 | 0 | 1 | 0.417 |  |
| Crimer | 23 | 37.6 | 4.48 | 0 | 121.00 |  |
| n | 180.087 | 100.097 | 237.017 | 337.017 | 402.809 |  |
| Fth | 0 | 0 | 100.037 | 0 | 237.017 |  |
| Din | 100.037 | 100.037 | 46.08 | 237.017 | 165.701 |  |
| OfP | 108.037 | 152.637 | 46.00 | 237.017 | 123.801 |  |
| cti | 0 | 0 | 1 | 0 | 3 |  |
| ${ }^{4}$ | 0 | 0 | 2 | 0 | 4 |  |
| Op | 401.821 | 408.574 | 218.01 | 401.020 | 165.09 |  |
| Dram | 0.187 | 0.235 | 0 | 0.80 | 0 |  |
| 0 | 0 | 0.178 | 0.33 | 0.19 | 0.438 |  |
| H | 0.254 | 0 | 0 | 0 | 0 |  |
| W | 0.234 | 0 | 0 | 0 | 0 |  |
| DPCame | 7 Ep- | 1410218 | 2304.3151 |  |  |  |
| Pan | F10 | 729 12 | 157.733 A | SHCN |  |  |
| …..... | .......... | -......... | -.......... | ......... |  |  |
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| Cathtencenset. |  |  |  |  |  |  |

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VITA
Fan Wang
Candidate for the Degree of
Master of Science
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Thesis: COMPUTER AIDED OPTIMAL DESIGN OF DUCT SYSTEMS
Major Field: Mechanical Engineering
Biographical:
Personal Data: Born in Beijing, P.R. China, January 30, 1965 the son of Mr. Geng Wang and Mrs. Min Wen.

Education: Graduated from High School Attached to Beijing University, Beijing, P.R.China, September, 1982; received the Bachelor of Science in Aerospace Engineering degree from Beijing University of Aeronautics and Astronautics, Beijing, P.R.China, 1986; completed the requirements for the Master of Science degree at Oklahoma State University, July, 1991.

Professional Experience: Design Engineer, QingYun Instrument Factory, Department of Aerospace of China, Beijing, P.R.China, 1986-1989; Research Assistant, Oklahoma State University, Stillwater, Oklahoma, 1989-1991;

