COMPARISON OF THE PERFORMANCE OF PASSIVE AND SEMI-ACTIVE SUSPENSION SYSTEMS

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NOMENCLATURE

b	damping slope for relative displacement dependent daming
B	damping coefficient of cubic damping
c	damping coefficient
f	frequency [cycle/sec]
h	amplitude of road roughness
J	cost function
J_c	cost function for concrete-slab road
J_{cr}	weighted cost function
J_r	cost function for rough road
k_s	suspension stiffness
k_u	tire stiffness
λ	road wave length
m_s	sprung mass
m_u	unsprung mass
PI	performance index
ρ	weighting constant
t	time [sec]
v	vehicle velocity [m/sec]
ω	angular frequency [rad/sec]
x_o	road roughness
x_1	axle displacement
x_2	body displacement

z	suspension deflection or relative displacement
ζ	damping ratio

CHAPTER I

INTRODUCTION

Road vehicles are subject to the excitation caused by the irregular road roughness. The transmitted force usually causes passengers to feel uncomfortable. The primary function of a suspension in ground vehicles is the isolation of the vehicle body (sprung mass) from the road excitation. Until recently, vibration control was achieved by using passive devices, such as springs and dampers. A suspension system using only passive devices does not require any energy input; it controls the effects of an irregular road profile by storing energy in springs and by dissipating energy with dampers. Because the passive suspension system has fixed characteristics, the system has distinct performance limitations.

Although vibration isolation is the main objective of suspension systems, there are other performance requirements. One is the suspension deflection limitation (workspace restriction); suspension stroke should be within the allowable workspace for suspension deflection. Also, the road contact force is another performance requirement; the suspension should not cause the tire to lose contact with the road for driving safety. There are other suspension criteria, but ride comfort, allowable suspension deflection, and road holding are primary criteria. In this paper I will only consider a ride quality and workspace restriction as the performance criteria.

The performance criteria conflict with each other; "soft" suspensions yield better ride quality, but have more chances to hit the stops. "Stiff" suspensions ensure the suspension deflection to be small at the cost of poorer ride comfort. An optimal suspension involves a trade-off between a comfortable ride within an allowable workspace.

In the past, many attempts were made to improve suspensions with control systems which alter the parameters of suspension elements of the system. There are two types of alternative suspension systems: active suspension system and semi-active suspension system. An active suspension system is a control system which can supply energy to the system, while a semi-active suspension system cannot supply energy to the system but can adjust the parameters of the damper.

Active suspension systems use a high power, high speed device such as a hydraulic cylinder and electrohydraulic valve combination to generate suspension forces. Various measuring and sensing devices like accelerometers, force transducers, and potentiometers are additionally necessary for an active suspension. The suspension force could be a function of many variables which are measured. In contrast, passive suspension systems are restricted to generating forces in response to relative motion between sprung and unsprung masses. The ability of active systems to modulate forces according to conditions leads to better performance. However, there are some disadvantages of active suspension. First, the implementation of active suspensions is quite complex because it requires many elements. Second, it requires considerable energy consumption, so running cost is high. Third, active systems tend to be less reliable because of their complexity.

Semi-active suspension systems have been introduced and developed as a compromise between active and passive suspensions with a hope of approaching active suspension performance, while maintaining simplicity, and energy saving. A semi-active suspension requires an adjustable damper which can yield changeable damping force. Adjustable dampers can be realized by employing a valve which controls the flow of fluid in a damper.

Research papers show that the performance of active control system is better than that of semi-active and passive control system. However, it is stillworthwhile to study semi-active suspension systems because they require only small amount of power supply and are simpler and more reliable than active suspension systems. With a good operating strategy, the performance of a semi-active control system might be much better than that of passive systems, perhaps almost as good as an active system.

In general, a semi-active suspension is inherently non-linear, so it is impossible to apply the linear control theory; computer simulation is unavoidable. Numerical simulation was used to study several types of nonlinear damping configurations and logically controlled damping. The results are compared with the performance of a reference passive system.

Describing road roughness is an important input into vehicle suspension studies. The idea of describing the road as a continuous random excitation is common. The road displacement is described by a type of integrated white noise power spectral density function. In other words, the velocity of road roughness is assumed to be white noise, so velocity magnitudes of road input are same for every frequency. Many papers dealing with the description of the road roughness show that the white noise assumption agrees with many real roads. However, this kind of road model might sometimes lead to unfavorable conclusions for other typical roads.

This paper selects two typical simple road models and adapts a two degree-of-freedom quarter-car model for analysis. Suspension performance is a weighted summation of root-mean-square body acceleration or body jerk and suspension deflection amplitude. Computer simulation is the tool to analyze various suspension systems. Passive linear damping, nonlinear damping, and semi-active damping which is based on absolute velocity feedback active damping are simulated and compared with each other.

CHAPTER II

LITERATURE REVIEW

Vibration isolation is the main objective of the vehicle suspension system, which has been studied extensively. Vibration control is achieved by using passive devices such as springs, shock absorbers, and masses in many cases. These passive elements control the vibration by storing or dissipating the energy associated with the vibration motion; no energy input is required. But a passive system has a certain performance limitation because it controls vibration only by storing and dissipating energy in passive devices .

Two alternative control schemes, active control and semi-active control, have been studied by many researchers. Active control systems can supply and dissipate energy independent of the energy previously stored by the suspension. An active system may generate forces which are a function of any state variables of the system. Semi-active control systems were developed for the hope of overcoming the disadvantages of active systems and approaching the performance of active systems. Semi-active systems control the vibration problems by varying the characteristics of passive devices. Though semi-active systems do not supply energy to the suspension systems, they need small amount of energy to adjust the characteristics of passive devices. Many researchers have studied these three vibration control systems, but the semi-active systems need further studies and research.

Description of Road Roughness

The description of road roughness is one of the important aspects in the study of vehicle suspensions. Integrated inches per mile of the road irregularities can be an index of roads. The index for the best roads is about 80 in/mile while the index obtained on main roads ranges from 100 to 250 in/mile [1]. Bastow relates the integrated road roughness to the amplitudes to judge whether the road is smooth or rough at the normal vehicle speed.

Dodds and Robson [2] show that typical road surfaces may be considered as realizations of homogeneous and isotropic two-dimensional Gaussian random processes. They described road surface roughness by a single spectral density function. For example, very good principal roads have the spectral density range of $2 \sim 8 \times 10^{-6}\ m^3$/cycle, while the spectral density of average principal roads ranges $32 \sim 128 \times 10^{-6} m^3$/cycle.

Thompson [3] used an integrated white noise road description, and Sharp and Hassan [4] used a displacement spectral density function to represent a road. It is common to model the road vertical velocity as white noise; this white velocity noise road model is used by many authors [3,5,6,7,8]. Also, the road velocity white noise assumption is valid for real roads in the interesting frequency range.

Active Control Systems

Full state feedback control, absolute body velocity feedback control, and LQG method control are compared in reference [8] for a two degree-of-freedom quarter-car model. Hedrick et. al. [8] show that the feedback of unsprung mass velocity causes high frequency "harshness", and does not affect the low frequency performance much. So the absolute velocity feedback yields all the nice properties of the full state feedback design without causing the high frequency harshness problem.

Redfield and Karnopp [6] studied the frequency response of body acceleration, suspension deflection, and road contact force with varying suspension parameters. Also, Redfield and Karnopp showed RMS responses of performance to variable suspension parameters.

Karnopp [7] studied the optimization of a single degree-of-freedom system under white noise base velocity excitation, using analytical expressions for mean square response quantities. Karnopp [9] also studied optimal feedback law applied systems which involve two often used criteria: one concerns with ride comfort and road contact force variation, the other concerns with ride comfort and main suspension deflection. He showed how optimal suspension parameters change as the weighting of ride comfort and road contact force or suspension deflection is varied through the use of symmetric root locus techniques. Karnopp [10] studied limitations result from a state variable feedback control in two degree-of-freedom suspension systems.

Sharp and Hassan [4] studied a full-state feedback active system, a limited-state feedback active system, and compared discomfort parameter (the root-mean-square value of the ISO 2631 weighted vertical body acceleration) with passive and semi-active systems. For other active suspension control systems, please see references [3,8,11,12].

Semi-active Control Systems

Even though performances of active suspension systems are excellent, some disadvantages exist in active suspension systems, such as high running cost, difficulty to implement, and complexity. Crosby and Karnopp [5] proposed a new semi-active suspension concept. The semi-active force generator can respond to general feedback signals to control the vibration without any external power for the suspension system. Crosby and Karnopp presented physical embodiments of a semi-active controller and compared them with hardware devices used in active and passive vibration control systems. Their computer simulation results show that the performance of semi-active control systems is comparable to that of fully active control systems.

Karnopp and Margolis [13] proposed a new concept involving variable spring stiffness and damping. They discussed how frequency response changes according to parameter variation for a single degree-of-freedom system. Karnopp [14] presented many possible ways to create semi-active dampers with hydraulic devices and electromagnetic devices through bond graph manipulations. Margolis [15] presented a model which included both the heave and pitch motions, and compared the performance of passive, active, and semi-active suspensions. He investigated absolute damping and state variable feedback control for both active and semi-active systems. Margolis studied frequency response for active and semi-active system subject to more realistic control signals -- non-ideal body velocity measurement and no acceleration feedback [16]. He compared passive, active, and semi-active suspension performance which included sprung mass isolation and unsprung mass controlling for a two degree-of-freedom model [17].

Sharp and Hassan [4] compared the performance of passive systems, active systems which have control parameters obtained by using optimal control theory, and semi-active systems which use on-off switching control to follow the control scheme of the fully active systems. Cheok et. al. [18] described the modeling and formulation of an optimal control suspension reference model, and experimented a suspension model with a microcomputerized optimal model-following variable air damper. Hrovat et. al. [19] developed a two degree-of-freedom model to optimize a quadratic performance index reflecting workspace limitations and ride quality requirements for passive, active, and semi-active suspension systems.

CHAPTER III

PRELIMINARIES

Road Configuration

Dodds et. al. showed that roads have profiles of random roughness which could be considered as realizations of homogeneous and isotropic two-dimensional Gaussian random processes [2]. Also, there is the Parkhilovskii assumption that the roll and vertical motions of the road undulation are uncorrelated. Usually the velocity of road model is simplified as a white noise process used with various analytical methods.

Here, I illustrate some methods for the representation of road roughness for numerical simulation. Karnopp, Crosby and Harwood simplified the road input to a white noise so that the velocity of road roughness has an approximately white spectral density for frequencies above 0.8 Hz up to a cutoff frequency of 15 Hz. The white noise was generated by selecting Gaussian random numbers and using the numbers as constant input amplitudes over sampling times of 0.005 sec [5]. Sharp and Hassan generated a white noise road input by adding together 60 sine waves with frequencies 1/4, 1/2, 3/4, ..., 15 Hz, by choosing the proper amplitudes to represent the assumed spectral density and by determine phases with a random number generator. So the road input has the profile an approximately Gaussian probability density [4]. In these two examples, we should notice that the frequency range they used is from 0.25 Hz to 15 Hz. These models are well supported by the fact that the frequency range of 1 Hz to 10 Hz is considered as the most important range for the ride quality [8]. Khulief and Sun modeled a road surface to a single bump with 0.2 m height and 0.4 m width, which was simplified to a sinusoidal function [20].

Although the white-noise random-process models for roads are widely used and easy to manipulate, it is doubtful that these models can properly represent real roads. It is preferable to select particular road conditions which a vehicle frequently encounters. One common road surface to be considered might be a concrete-slab road (e.g. interstate freeway). It can be represented by a saw-tooth wave form function with a wavelength of approximately 6.3 m and an assumed amplitude of 6 mm (refer to Figure 1.). The choice of wavelength of the road is reasonable because it is based on the common road construction convention [2].

As a form of external excitation, the road undulation should be related with the spring constant of a tire and the speed of the vehicle. On a highway, the vehicle speed could be assumed to be a constant speed of about 50 mi/hr

($\cong$ 80 km/hr). To make calculation easy, the saw-tooth function is represented as an appropriate Fourier series. With this approach, road roughness can be converted to an external force which applies to the unsprung mass of vehicle.

However, the frequencies exerted to a high speed vehicle by the saw-tooth road are quite high, so we should choose another vehicle speed or road model to cover the entire frequency range interested, i.e. 0.1 to 15 Hz. An alternative road model is made to contain several frequencies of 0.5, 1, 5, 10, and 15 Hz. I selected the two natural frequencies (i.e. 1 and 10 Hz) and those neighborhood frequencies. for the rest of the paper, I will call this alternative road model "a rough road model" by assigning high amplitudes for each frequency and their velocity amplitudes are chosen to be identical for each frequency.

Concrete-Slab Road Model

Concrete-slab roads can be assumed to have a form of saw-tooth wave form function which has a wavelength of 6.3 m and an amplitude of 0.006 m; the speed of vehicle is assumed to be a constant speed of 80 km/hr.

Let a wavelength be λ [m], an amplitude be h [m] and a vehicle speed be v [m/sec], then the fundamental frequency, ω_o [rad/sec], of the road input can be expressed :

$$v = \lambda f_o \tag{1}$$

$$\omega_o = 2\pi f_o \tag{2}$$

so, with the vehicle parameters which will be given later, ω_o becomes 22 [rad/sec].

With this fundamental frequency, the saw-tooth wave form function can be expressed as a Fourier series expansion as follows :

$$\begin{aligned} x_o(t) = h/2 &-h/\pi\cdot\sin(\omega_o t) -h/(2\pi)\cdot\sin(2\omega_o t) \\ &-h/(3\pi)\cdot\sin(3\omega_o t) \cdots\cdots -h/(n\pi)\cdot\sin(n\omega_o t)\cdots \end{aligned} \tag{3}$$

It is noticeable that the velocities of the saw-tooth wave form road excitation are identical for each frequency. You can find it with ease, if you take the time derivative of the road roughness function. It becomes:

$$\begin{aligned} \dot{x}_o(t) = &- \omega_o h/\pi\cdot\cos(\omega_o t) - \omega_o h/\pi\cdot\cos(2\omega_o t) \\ &- \omega_o h/\pi\cdot\cos(3\omega_o t) \cdots\cdots - \omega_o h/\pi\cdot\cos(n\omega_o t) \end{aligned} \tag{4}$$

The coefficients of cosine terms of each frequency are same, so the spectral density of the velocity of the road input has the white noise property in the sense of the amplitudes of each frequency are identical. The concrete-slab road model also has similar properties with other white noise models adapted by other researchers.

Let's assume that usual vehicle velocity range is from 20 km/hr to 80 km/hr. If we take 18 terms from the Fourier series of saw-tooth function, the frequency range of road excitation would be 0.88 Hz to 15.9 Hz at the vehicle velocity of 20 km/hr. Thus 18 will be assigned to n of the equation (3) so as to cover most of the frequency range of 0.1 Hz to 15 Hz. The road excitation frequency range of a high speed vehicle shifts to higher range, for example, 3.53 Hz to 63.49 Hz for 80 km/hr. Thus we can observe that the higher speed the vehicles have, the higher frequency range excitation they have.

Rough Road Model

A rough road model can be realized by a linear combination of high amplitude sinusoidal functions of several selected frequencies. Because the transmissibility of workspace is important at the natural frequencies (1.0 and 10 Hz), I will include these frequencies in my rough road model. The body acceleration response is also important at the natural frequencies, and the response tendency of the frequencies higher than the sprung mass natural frequency is quite different from that of lower frequencies. Thus I included neighbor frequencies 0.5, 5, and 15 Hz to yield more accurate ride quality response. A constant vehicle speed of 20 km/hr can be an appropriate choice for a rough road model. Let the amplitude of 1 Hz term be h, then the roughness function will be given by equation (5). The value of h is selected to be 0.05 [m], and phase angles are assigned to be $-\pi/6$, $\pi/2$, $\pi/6$, $-\pi/2$, and 0 for ϕ_1, ϕ_2, ϕ_3, ϕ_4, and ϕ_5, respectively. Figure (2) shows how the rough road model looks like.

$$x_o(t) = 2h\cdot\sin(0.5\omega t + \phi_1) + h\cdot\sin(\omega t + \phi_2) + (1/5)h\cdot\sin(5\omega t + \phi_3) + (1/10)h\cdot\sin(10\omega t + \phi_4) + (1/15)h\cdot\sin(15\omega t + \phi_5) \quad (5)$$

Here, $\omega = 2\pi$ [rad/sec].

Multi-criteria of Performances

A good suspension design should satisfy a number of conflicting desires. A suspension system should isolate the body motion from the external roadway disturbances, which is the principal objective of the suspension system. However, there are subsidiary requirements: one is the limitation of the relative displacement between the body and the tire known as a "workspace" or "suspension travel" which is desired to be always within a certain limitation, and another requirement is the wheel-road contact force which should be as constant as possible.

From the studies of linear system with harmonic excitation, we can say that the isolation of a mass can be judged by looking at the transmissibility between the sprung mass motion and the roadway motion. For one degree-of-freedom system, when the frequency ratio is less than $\sqrt{2}$, the transmissibility decreases as the damping ratio increases; when the frequency ratio is greater than $\sqrt{2}$, the transmissibility increases as the damping ratio increases. Transmitted accelerations are larger in the high frequency range than in the low frequency range, so it is plain that a smaller damping suspension is better for good ride quality. This tendency can be extended to higher degree-of-freedom systems.

The problem of vehicle suspension system control lies on the fact that the criteria conflict with each other; the workspace amplitude decreases but the ride quality becomes worse as the suspension becomes stiffer. So it is obvious that there is no one system which both gives the best ride quality and also maintains minimum workspace. Generally, an optimal suspension design must compromise somehow between those conflicting desires, specially between the ride quality and the workspace limitation. An optimal suspension system will show a minimum acceleration transmissibility for a given allowable workspace deflection. As the suspension becomes stiffer, the workspace deflection decreases and the body acceleration increases; so the ride quality become worse than the optimum suspension. Furthermore, if either the speed of vehicle increases or the roadway becomes rougher, the minimum acceleration will be larger for the same allowable workspace deflection limitation. For this new situation, the optimal suspension characteristics should be changed to another optimal point. Thus, we cannot design a suspension system which is optimal to all roads. The best way we can do is to design a optimal spension system to the road models that a vehicle frequently encounters.

Evaluation of Performance

We could get an optimal performance by doing some trade-off between criteria conflicting each other. Among performance criteria, ride comfort and workspace amplitude are more important than any other criterion. So the most desirable suspension performance is to keep a small body acceleration or jerk as possible and a workspace amplitude within the limitation. However, it is not usually possible to keep these two values small for every road. For low-frequency and high-amplitude roads, i.e. rough roads, the workspace criterion is more important than the ride comfort, and for high frequency and low amplitude roads, i.e. good quality roads, the ride comfort is more important because the workspace problem seldom happens on good roads.

Many researchers applied frequency analysis as a tool to judge suspension performances [5,6,8,10,11,15-17,21-23]. This is valid for linear systems at any time and very useful to study the effect of each suspension elements, but not always valid for nonlinear systems like nonlinear damping systems and semi-active systems.

Root-mean-square (RMS) values might be a possible measure of the performance. In this paper, RMS values of body jerk or acceleration and workspace amplitude are calculated for the two road models described above. A good suspension system should meet criteria such as low RMS body jerk (or acceleration) and low RMS workspace amplitude.

Introducing a cost function, J, makes the comparison easy. The cost function is a weighted sum of root-mean-square values of the representative variables of the criteria chosen for comparison. In other words, J is a weighted sum of RMS body acceleration (or jerk) and RMS workspace. The cost function can be expressed as equation (6) for the case of that body acceleration and suspension deflection are considered as the performance criteria.

$$J = \lim_{T \to \infty} \frac{1}{T} \int_0^T [\, \rho \sqrt{\ddot{x}_2^2} + \sqrt{(x_2 - x_1)^2}\,] dt \qquad (6)$$

where ρ is a weighting to emphasize one of the performance , x_1 is unsprung mass displacement, and x_2 is sprung mass displacement.

Two Degree-of-Freedom Vehicle Modeling

Dahlberg treated a vehicle suspension system as a five degree-of-freedom plane linear model that is a half-car model [24]. Also, there are quarter-car models which can be treated as one degree-of-freedom model, or as two degree-of-freedom model which includes the unsprung mass and spring.

In order to predict most accurately the effect of a certain suspension condition, a full-car model is better than other models. However, the full-car model is very complex and likely to lead us to simulation problems and errors. In addition, it is not easy to recognize important results from the complex model, making design interpretation almost impossible. On the other hand, a one degree-of-freedom quarter-car model is too simple to derive useful design concepts for real suspension systems.

Since the essential trends of suspension system performance could be shown with a two degree-of-freedom quarter-car model, it is reasonable to choose a two degree-of-freedom representation of a quarter-car model for simplicity.

The following system equations might be formulated by inspection of the passive system model (Figure 3). The coordinates of the displacements are from each static equilibrium position.

$$\begin{aligned} m_s\ddot{x}_2 + c(\dot{x}_2 - \dot{x}_1) + k_s(x_2 - x_1) &= 0 \\ m_u\ddot{x}_1 + c(\dot{x}_1 - \dot{x}_2) + k_s(x_1 - x_2) + k_u x_1 &= k_u x_o \end{aligned} \tag{7}$$

Let $x_3 = \dot{x}_1$ and $x_4 = \dot{x}_2$, then the system equations can be written as state space equations,

$$\dot{X} = AX + Bx_o \tag{8}$$

where

$$X = [x_1 \quad x_2 \quad x_3 \quad x_4]^T,$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_1^2-\omega_{21}^2 & \omega_{21}^2 & -2e\zeta\omega_{21} & 2e\zeta\omega_{21} \\ \omega_2^2 & -\omega_2^2 & 2\zeta\omega_2 & -2\zeta\omega_2 \end{bmatrix}$$

$$\omega_1 = \sqrt{k_u/m_u}\ , \quad \omega_{21} = \sqrt{k_s/m_u}\ , \quad \omega_2 = \sqrt{k_s/m_s},$$

$$\zeta = c/(2\sqrt{m_s k_s}), \quad e = \sqrt{m_s/m_u}$$

$B = [0 \quad 0 \quad k_u/m_u \quad 0]^T$

Computer Program and Numerical Stability

This paper deals with nonlinear systems, and frequency analysis is not valid for nonlinear systems. Computer simulation is one of the best way to analyze nonlinear systems. The simulation program written in this paper consists of several Pascal procedures which are for the selection of road type, choice of damping scheme, system equations, Runge-Kutta solution routine, and RMS outputs.

The program has three kinds of road input which are a single tone sinusoidal excitation, a rough road model, and a good quality road model (concrete-slab road). A single tone sinusoidal excitation exists mainly for a frequency analysis. The rough road model and the good quality model are used for the analysis of RMS performances.

Included damping schemes in this program are linear passive damping, asymmetric damping, cubic damping, and semi-active damping. Linear damping ratio, rebound and compression damping ratio of asymmetric damping, and damping ratio of cubic damping can be arbitrary input.

The differential equations of this system are solved by Runge-Kutta method. A time step is very important for stable solutions and a fast computation. If a stepsize is greater than the stepsize which is critical for a numerical stability, the solutions blow up and the program will be terminated with overflows. On the other hand, if a stepsize is much smaller than the critical stepsize (maximum stepsize to be numerical stable), the computation might take a long time to yield only slightly improved accuracy. One should find the critical stepsize for numerical stability, and after that assign a stepsize smaller than the critical stepsize for the required accuracy.

During the development of the program there were sometimes "runtime errors", so I traced the cause of errors and found no coding errors. However, I found that the numerical stability depends not only the natural frequency, but also on the damping ratio of the system; numerical instability caused the "runtime errors". Through a lot of simulation with different damping ratios and stepsize, the stabilizing stepsize for each damping ratio was found. Numerical stability is not sensitive to input parameters like external frequencies or amplitudes.

There are two kinds of numerical instability: first, exponentially growing unstable solutions, second, oscillating solutions with growing amplitude. Sometimes, the second case is not easy to be checked because of slow growth, but if one prolongs the calculation time the growing amplitude is noticeable.

The stabilizing stepsizes for each damping ratio are listed in Table (1) and plotted in Figure (4). There are three columns in the Table (1), i.e. damping ratio, stable stepsize and unstable stepsize. The stepsizes in the column of the stable stepsize guarantee that the solution will be numerically stable for an equation whose damping ratio is less than listed in the damping ratio column, but they are not the largest stepsizes to ensure a numerical stability. Also, the stepsizes in the column of the unstable stepsize surely make the solutions be numerically unstable, but not the smallest stepsize. In order to yield numerically stable solutions, one should not choose a stepsize larger than the stepsize in the stable stepsize column. However, it does not guarantee accurate results.

CHAPTER IV

PASSIVE SYSTEMS

Frequency Responses of Passive System

Frequency analysis can guide us to select a good performance passive suspension system for both a good quality and a rough road. When the input to the system is the road roughness, transfer functions of the system are as follows :

$$G(s) = C(sI-A)^{-1}B + D \tag{9}$$

where

A and B are same as in equation (8),

I is a 4 by 4 unit matrix,

$C = [\ 0\ \ 0\ \ 0\ \ 1\ ]$ for evaluation of body velocity

$\quad\ [-1\ \ 1\ \ 0\ \ 0\ ]$ for evaluation of workspace

$D = 0$.

If we substitute $j\omega$ to the Laplace transform operator, s, we can plot Bode diagrams of frequency versus the amplitude of transfer function. Hedrick et. al. remarked that the frequency range of 1 to 10 Hz is important for the ride quality, so it is not necessary to have high frequency (e.g. above 20 Hz) responses [8].

A white noise has same powers for every frequency, so frequency responses with a white noise input are easy to comprehend actual magnitude output responses. Because the velocity of real roads are nearly white, a road velocity input makes analysis easy. For linear systems, we know that:

$$|\ddot{x}_2/\dot{x}_o| = |\omega\dot{x}_2/\dot{x}_o| = |\dot{x}_2/x_o| \tag{10}$$

and

$$|z/\dot{x}_o| = |z/(\omega x_o)| \tag{11}$$

Using the above two equations (10,11), it is possible to get frequency responses of road velocity input with only multiplying 1 and $1/\omega$ to the transfer function which is given with road displacement input.

The following numerical data for a typical passenger car are used for analysis.

m_s = 240 Kg $\qquad$ k_s = 16000 N/m

m_u = 36 Kg $\qquad$ k_u = 160000 N/m

c = 1176 Ns/m (reference case)

Performances are studied with various damping ratios in the range of 0.1 to 100 Hz, but the range of 0.1 to 15 Hz is mainly considered. Figure (5) is the frequency response of body acceleration by road velocity input. The result shows that the body acceleration is reduced as damping increase in the 1 Hz range which is the natural frequency of sprung mass, but increased at higher frequencies. The reason of this result is that the stiffer suspension transmits more road input to the vehicle body in high frequency range. Also, at the unsprung mass resonant frequency (about 10 Hz), body acceleration is not dependent of the damping ratio. This is because the sprung mass acceleration is independent of unsprung mass variables at the unsprung natural frequency ($\omega = \sqrt{k_u/m_u}$). This interesting property of automotive suspensions can be found in the Hedrick's paper [8]. Adding the two equations of suspension system gives

$$m_s\ddot{x}_2 + m_u\ddot{x}_1 = k_u(x_o-x_1) \tag{12}$$

Transforming and setting initial conditions to zero yields:

$$m_s\ddot{x}_2(j\omega) + (k_u - m_u\omega^2)x_1(j\omega) = k_u x_o(j\omega) \tag{13}$$

If we substitute $\omega = \sqrt{k_u/m_u}$ to equation (13), we find that the sprung mass acceleration is independent of the displacement of unsprung mass, x_1, at the unsprung mass natural frequency. If we define,

$$H_A(j\omega) \equiv \ddot{x}_2(j\omega)/\dot{x}_o(j\omega) \tag{14}$$

then we have the magnitude of acceleration transfer function at the unsprung natural frequency,

$$|H_A| = (k_u/\sqrt{k_u/m_u})/m_s = \sqrt{k_u m_u}/m_s \tag{15}$$

Substituting the numerical values for typical passenger car given later to the equation (15) yields $|H_A| = 10$; this result agrees with the acceleration frequency plot in Figure (5). Redfield and Karnopp also show a similar frequency response in Ref. [6]. In order to reduce the body acceleration at the unsprung mass natural frequency, the spring constant of unsprung mass should be reduced for given sprung and unsprung masses. We can learn from frequency response of body acceleration that a soft suspension gives better ride quality than stiff suspension if frequencies of road excitation are evenly distributed.

Frequency response of workspace by road velocity input is on Figure (6), which is also similar to the result showed in Ref. [6]. Two peaks appear at each natural frequencies, but the peaks dies out as damping increases. The workspace response shows that increased damping reduces the suspension deflection amplitude for all of the frequency range, but there are no big differences by changing damping in low (i.e. less than 0.5 Hz) and high (i.e. greater than 12 Hz) frequencies. However, we can conclude that higher damping reduces the workspace amplitude for any frequency range.

Selection of Reference Passive System

Frequency analysis of passive system tells us that higher damping could improve the performance of workspace amplitude, but hurts the performance of ride quality. So we cannot expect that a suspension system simultaneously improve both two criteria, but we can do some trade-off between them. The damping ratio of about 0.3 can be a good passive suspension system damping ratio which satisfies those two criteria; the performance of this system is compared with other suspension systems. This reference passive system damping ratio is chosen to yield the minimum value of the cost function given by equation (6) with a weighting of 0.03 for sprung mass acceleration or 0.003 for sprung mass jerk (i.e. $\dddot{x}_2$). Let's define the cost function of the concrete-slab road as J_c and that of the rough road as J_r. Assigning 0.8 for the weighting of concrete-road model and 0.2 for that of rough road model, then the weighted cost function, J_{cr}, can be expressed the equation given by (16).

$$J_{cr} = 0.8J_c + 0.2J_r \qquad (16)$$

If we divide J_{cr} with minimum value of it, we can normalize the weighted cost function. If we define a performance index, PI, to be the normalized weighted cost function, it is easy to compare the performance between other damping schemes. Figure (7) is the plot of performance index for a passive system as a function of nominal damping ratio, ζ.

The numerical simulation is done with the typical two road conditions above mentioned and the results are taken root-mean-square values. To avoid the inaccuracy of the numerical calculation, the results within two second are discarded and results 2 to 10 second are used to yield the root-mean-square values. The results of several outputs are tabulated with respect to the nominal damping ratio which is simply defined as equation (17),

$$\zeta = c/2\sqrt{m_s k_s}. \qquad (17)$$

The results in Table (2) and (3) show that a better isolation (ride quality) response can be achieved at low damping ratio and the suspension deflection decreases as damping increases. Similar simulation is done with different vehicle speeds for the concrete-slab road model; Figure (8) - (10) are RMS plots for body acceleration, jerk, and workspace, respectively. Results show that if the vehicle speed is increased both the ride quality and workspace performance become worse.

CHAPTER V

NONLINEAR DAMPING PERFORMANCE

Real shock absorbers are usually nonlinear; they have static friction also known as the "stiction", rubber mounts, and different characteristics in compression and rebound. Also, the damping coefficients vary with the temperature. So it is useful to consider nonlinear damping systems and compare responses with those of linear damping systems.

Asymmetric Damping

At first, asymmetric damping systems are taken into account. Asymmetric damping systems are assumed to have different damping coefficients according to the relative velocity between sprung and unsprung masses. Vehicle dynamics engineers use "compression" for the negative relative velocity ($\dot{z} \equiv \dot{x}_2 - \dot{x}_1$) motion and "rebound" for the positive relative velocity. Simulation is done for the cases of one of the compression or rebound damping ratio is positive values (varying from 0 to 2) and the other is 0. The nominal damping ratio is defined as equation (17).

To simulate the asymmetric damping systems, the relative velocity is monitored while solving the differential equation set and if it is positive, ζ is set to a value and if it is negative then ζ is set to 0 for the rebound damping case. Similarly, the compression damping cases can be simulated.

The results are calculated as the form of RMS values calculated from 2 [sec] to 10 [sec]. The simulated RMS workspace, body acceleration, and body jerk are tabulated in Table (4) - (7) for compression and rebound damping. Also, performance index is drawn as a function of nonzero damping ratio in Figure (11) to be compared with reference passive system performance. The results of asymmetric damping show that the performance is worse than the performances of reference linear damping (ζ=0.3) case. However, we could learn from these results that, for previous mentioned typical road surfaces, nonzero damping for rebound motion reduces the amplitude of relative displacement and RMS body jerk, but nonzero damping for compression motion reduces the RMS body acceleration (refer to Table 4-7). Figure (11) shows that the performance of compression damping system is better than rebound damping system for the typical road input.

Relative Displacement Dependent Damping

We could think about a damping scheme of which the nominal damping ratio, ζ (defined as before), depends on the relative displacement (x_2-x_1). Let's take an asymmetric linear relative displacement dependent damping case. This case is almost similar to the asymmetric damping case except damping ratio varies due to the sign of relative displacement rather than of relative velocity. Let's call "compression" for the case of a negative relative displacement and "rebound" for the case of a positive relative displacement. Simulation is done nominal damping ratios of 0 to 2 for one and 0 for the other, similar to the case of asymmetric damping simulation above.

During the simulation, the sign of relative displacement is monitored, if it is positive the damping ratio is set to a value (less than 2) and if negative the damping ratio is set to 0 for the case of nonzero rebound damping. The opposite case is also simulated.

The results are expressed as a form of RMS value calculated during the time duration of 2 - 10 second. The performances are compared with the reference linear damping case (ζ=0.3). The numerical results reveal that the RMS values of relative displacement dependent asymmetrical damping are not satisfactory compared to those of reference case. With this kind of damping scheme, the workspace amplitude tends to be greater and the vibration isolation tends to be worse than those of reference damping system. The results of this damping scheme are not included in this paper.

Also, we can think about a damping scheme such as the nominal damping ratio, ζ (same as before), is a function like equation (18). This kind of damping characteristic could be occurred in real systems such as the middle zone fluid flow area of shock absorbers is quite wide and the end zone fluid flow area of shock absorbers is narrow.

$$\zeta = b|x_1 - x_2| \tag{18}$$

Simulation is done with varying the constant value of "b" of the equation (18) from 0 to 45. The relative displacement is calculated with previous time step conditions, with the value the damping ratio is changed and the system equation set is solved with new damping ratio and so on. As before, the RMS values during 2 - 10 second are calculated with different "b" values. The performance index is plotted in Figure (12) as a function of the constant "b" of the equation (18). RMS values of body acceleration, body jerk and workspace are tabulated in Table (8) and (9) for the typical road inputs.

The workspace RMS amplitude decreases monotonically as "b" increases; this result agrees with the general concept of higher damping reducing workspace amplitude. The minimum RMS, acceleration and jerk are at 5 for the concrete-slab road input and 10 for acceleration and 0 for jerk for the rough road input. So we could say that better ride quality can be achieved with small "b". Comparing with the reference linear system, this system could be better in ride quality sense, but the rattle space limitation could be violated more easily. Performance index plot (Figure 12) shows that this system is not better than the reference passive system. However, if "b" is about 20, the performance is nearly same as the performance of reference passive system.

Cubic Damping

More realistic damping has the characteristics of cubic damping, it yields a damping force which is proportional to the 3rd power of relative velocity between sprung and unsprung masses. It can be written as the equation (19) below,

$$F_d = B\dot{z}^3 \qquad (19)$$

where $\dot{z}$ represents relative velocity.

RMS body accelerations, body jerks, and workspace amplitudes as a function of "B" are results of the simulation of the time duration of 2 to 10 second. The range of cubic damping coefficient, "B", is considered from 0 to 3.

From the results of saw-tooth road model in Table 10, it is shown that the minimum workspace RMS value is 2.725×10^{-3} at which "B" is 3, which is about 30 % greater than that of reference linear system. The minimum RMS values of body acceleration and jerk are 0.318 and 19.4 where "B" is 0.2 and 0.1, respectively. These values are quite less than those of reference linear system. Generally, we can say that the workspace amplitude decreases and the acceleration and the jerk increases as a value of "B" increases.

The result of performance index in Figure 13 shows that the cubic damping scheme can improve the performance if "B" is taken from 0.2 to 2.5. Also, the best performance of cubic damping system can be achieved at which "B" is about 0.7.

CHAPTER VI

SEMI-ACTIVE DAMPING PERFORMANCE

Active Damping

Because semi-active control systems are based on active control systems, one should know about active control systems first to study a semi-active system. Active suspension systems can supply energy to the suspension system, and dissipate energy from the system. A force generator like a servomechanism could be an active damping device which can produce force of a function of any state variables. One possible feedback can be the absolute body velocity feedback. In fact, the same effect can be achieved only with passive devices if we have an inertial ground to fix a damper to a sprung mass as shown in Figure (14). However, this kind of damping scheme cannot be implemented onto real vehicle suspension systems because we cannot have an inertial ground. This fictitious damping system is referred as "sky-hook" damping.

Performance of Sky-hook Damping System

Active control of absolute body velocity feedback can be studied by the analysis of the "sky-hook" damping.

Frequency analysis can help to choose a feedback gain of absolute body velocity. System equations for the sky-hook damping system are given by equation (20).

$$m_s\ddot{x}_2 + c\dot{x}_2 + k_s(x_2 - x_1) = 0$$
$$m_u\ddot{x}_1 + k_s(x_1 - x_2) + k_u x_1 = k_u x_o \qquad (20)$$

Let $x_3 = \dot{x}_1$ and $x_4 = \dot{x}_2$, then the system equations can be written by state space equations,

$$\dot{X} = AX + Bx_o \qquad (21)$$

where

$$X = [x_1 \quad x_2 \quad x_3 \quad x_4]^T,$$
$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_1^2-\omega_{21}^2 & \omega_{21}^2 & 0 & 0 \\ \omega_2^2 & -\omega_2^2 & 0 & -2\zeta\omega_2 \end{bmatrix},$$
$$\omega_1 = \sqrt{k_u/m_u}\ , \quad \omega_{21} = \sqrt{k_s/m_u}\ , \quad \omega_2 = \sqrt{k_s/m_s},$$
$$\zeta = c/(2\sqrt{m_s k_s}),$$
$$B = [0 \quad 0 \quad k_u/m_u \quad 0]^T$$

When the input to the system is vertical road displacement, the transfer function of the system is as follows:

$$G(s) = C(sI-A)^{-1}B + D \qquad (22)$$

where

A and B are given in equation (21),

I is a 4 by 4 unit matrix,

C = [0 0 0 1] for evaluation of body velocity

= [-1 1 0 0] for evaluation of workspace

D = 0.

Using the relations given by equations (10) and (11), it is possible to get the body acceleration and workspace responses of the road velocity input. Multiplying 1 and $1/\omega$ to the magnitudes of transfer function gives the frequency responses of road velocity input.

Figure (15) and (16) are the frequency response of workspace and body acceleration of sky-hook damping system of which numerical parameters are same as those of passive system given earlier. Figure (15) shows that the body acceleration decreases as the damping of sky-hook system increases near the natural frequency of sprung mass, but changes little lower and higher frequency ranges. Especially, a sky-hook damping scheme transmits energy quite much to the sprung mass at the unsprung mass resonant frequency regardless of the damping ratio. This is why there is no damper to retard the motion of unsprung mass and the road excitation is transmitted to the sprung mass through the suspension spring without any excitation attenuation.

Figure (16) shows that the workspace response is more complicated. At the sprung mass resonant frequency, higher damping decreases the suspension deflection, while lower damping decreases the suspension deflection in the lower frequency range. However, in the higher frequency range there is unnoticeable differences between the responses of different damping. The maximum magnitude is at the unsprung mass resonant frequency, which cannot be reduced by changing the damping ratio of sky-hook systems.

The damping ratio of 0.3 or 0.5 in this scheme can give a good compromise for the two performance criteria for the road input which contains wide frequency range. This result also implies that the active control of absolute body velocity feedback gain of 0.3 or 0.5 gives the best performance.

Performance of 2 Inertial Ground Damping

We can think about an active control system of which the absolute sprung and unsprung mass velocity feedback. This system is same as the system as drawn in Figure (17), where the velocity feedback gains are same as the damping coefficients. The two independent inertial grounds support each mass and damper, so each damper can control the motion of mass by a control force of a function of each mass velocity only. The system equation is given by equation (23).

$$\begin{aligned} m_s\ddot{x}_2 + c_2\dot{x}_2 + k_s(x_2 - x_1) &= 0 \\ m_u\ddot{x}_1 + c_1\dot{x}_1 + k_s(x_1 - x_2) + k_u x_1 &= k_u x_o \end{aligned} \qquad (23)$$

where c_1 and c_2 are the damping coefficients of unsprung mass and sprung mass, respectively.

Let $x_3 = \dot{x}_1$ and $x_4 = \dot{x}_2$, the system equations can be written by state space equation same as equation (21), where

$$X = [x_1 \quad x_2 \quad x_3 \quad x_4]^T,$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_1^2-\omega_{21}^2 & \omega_{21}^2 & -2\zeta_1\omega_1 & 0 \\ \omega_2^2 & -\omega_2^2 & 0 & -2\zeta_2\omega_2 \end{bmatrix},$$

$$\omega_1 = \sqrt{k_u/m_u}\ , \quad \omega_{21} = \sqrt{k_s/m_u}\ , \quad \omega_2 = \sqrt{k_s/m_s},$$

$$\zeta_1 = c_1/(2\sqrt{m_u k_u}), \quad \zeta_2 = c_2/(2\sqrt{m_s k_s})$$

$$B = [0 \quad 0 \quad k_u/m_u \quad 0]^T$$

Figure (18) to (23) are the frequency analysis results of 2 inertial grounded damping scheme or active control system with sprung and unsprung mass velocity feedback. The results show that the damper attached to the sprung mass affects on the acceleration and workspace in the lower frequency range, while the damper attached to the unsprung mass affects on those in the higher frequency range.

In the case of body acceleration, larger sprung mass damping decreases body acceleration only near the sprung mass resonant frequency, but does not much change it higher (e.g. greater than 10 Hz) and very low (e.g. less than 0.2 Hz) frequency ranges. Small unsprung mass damping can arise a peak at the unsprung mass natural frequency, but the peak dies out with increased damping. Only for the body acceleration criterion, larger sprung and unsprung damping is preferable.

The results of workspace show that the sprung mass damping is responsible for the response in the lower frequency range, while the unsprung mass damping is responsible for the response at unsprung mass natural frequency. Larger sprung mass damping increases workspace amplitude in the low (less than 0.5 Hz) frequency range, and decreases it near the sprung mass natural frequency. However, there is not noticeable change in the higher frequency range. The unsprung mass damping is mainly responsible for the response at the unsprung mass natural frequency. Small damping can yield a peak at the unsprung mass natural frequency, but it dies out with slightly increasing unsprung damping. The responses in the lower frequency range are not changed with different unsprung mass damping.

For road excitations which contain only higher frequencies (e.g. greater than 5 Hz), higher damping for each mass will be good for the both performance criteria. Large damping for the unsprung mass and small damping for the sprung mass is preferable for the roads which contain only low (e.g. less than 0.5 Hz) frequencies. 0.3 for the sprung mass damping and 0.5 for the unsprung mass damping can be a good trade-off selection for the road excitations which contain wide frequency range.

Semi-Active Damping

The semi-active suspension concept is derived from active suspension; it differs from active suspension systems in having no energy input. So it is natural that the performance of semi-active suspensions be worse than the performance of active suspensions. However, the advantages of semi-active suspensions are simplification of the implement, reduction of running cost, and more liability.

The strategy of semi-active suspension is performing an active control force by a adjustable passive damper. Because semi-active suspensions use a passive device, it is not possible to follow an active damping force at any time. The best way is setting the damping 0 when the active damper generates energy. This logic can be done by monitoring the velocities of sprung mass and unsprung mass. Let F_d be the expecting force to be supplied by an damper and v_s and v_u be the velocity of sprung and unsprung mass velocity, respectively. The passive device limitation is given by :

$$F_d(v_s - v_u) \geq 0 \qquad (24)$$

Here, F_d can have different forms depend on the active control strategy which the semi-active control follows. If a semi-active control intends to follow absolute body velocity feedback control, F_d will be given as equation

(25).

$$F_d = Kv_s \tag{25}$$

where K is the feedback gain.

Equation (24) implies that the power associated with F_d should be always dissipated for semi-active systems. Thus, when the relative velocity between sprung mass and unsprung mass is positive, F_d must be positive, and when the relative velocity is negative, F_d must be negative. "Tensile" and "compressive" are commonly used in the vehicle dynamic field for the case of $F_d > 0$ and $F_d < 0$, respectively. If we denote F_s for the actual damping force generated by a semi-active damper,

$$F_s = F_d \ , \text{ if } F_d(v_s - v_u) > 0 \tag{26}$$

$$F_s = 0 \ , \text{ if } F_d(v_s - v_u) < 0 \tag{27}$$

Simulation results for the semi-active system which follows body velocity feedback control are tabulated in Table 12 and 13. The semi-active damper is assumed to have the ability to adjust the nominal damping ratio from 0 to 2. Results show that there is no firm relationship between damping ratio and output RMS values, but large damping tends to increase body acceleration and jerk so ride quality becomes worse. Semi-active control performance is much improved for the concrete-slab road model, but deteriorated for the rough road model. Performance index plot (Figure 24) shows that semi-active control can yield better performance than the passive reference system. When the damping ratio of "sky-hook" system is 0.6 or 0.7, the performance is slightly better than that of reference system. However, much improved performance might be expected by selecting proper feedback gains for different road input or by following other efficient active control systems.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

Generally, higher damping reduces the suspension deflection and increases body acceleration and body jerk regardless of whatever damping scheme is applied. Thus, we cannot find a system which improves ride quality and reduces the suspension travel amplitude simultaneously. The best we can do is to find a good optimal condition which both satisfies the workspace limitation and minimizes the body acceleration as much as possible.

In this paper, I considered that a better performance yields a smaller cost function value given by equation (6) which emphasizes the ride quality. The cost functions are calculated for the two typical road inputs, and the properly weighted summation of these two cost functions yields a combined cost function as written in equation (16). The combined cost function divided by that of the reference passive system represents the performance index. Thus, if the performance index of a system is less than unity, the performance is better than that of the reference system.

The study of frequency responses of the passive system gives an idea of choosing a damping ratio with which a suspension system could compromise the performance criteria; such damping ratio might be 0.3 (refer to Figure 5,6). This value is confirmed by the performance index plot in Figure 7. So I can conclude that the passive system of which the damping ratio is 0.3 has the best performance for the typical road inputs.

The performance of asymmetric damping is worse than that of the reference system. However, we can see that damping on compression motion yields better performance than damping on rebound motion (see Figure 11). The performance of relative displacement dependent damping is not as good as the reference performance, but quite close to it when "b" is about 20 (refer to Figure 12). Figure 13 shows that the cubic damping can improve the performance where "B" is from 0.2 to 2.5. The best performance of cubic damping system can be achieved at which "B" is about 0.7.

Semi-active control which follows the body velocity feedback control shows performance improvement at which the "skyhook" damping ratio is 0.6 and 0.7 (from Figure 24). However, the performance of semi-active system is not better than that of cubic damping system. So we can conclude that a cubic damping scheme could yield better performance with proper parameter choice.

However, the semi-active system we studied is not the best one of semi-active systems. Future study might yield better results for semi-active systems if nonlinear control concept is applied to find the control parameters for semi-active systems rather than on-off switching to follow the objective active control. Also, we see the advantages of cubic damping scheme, so studies should be extended to cubic damping characteristics and its implementation. Another beneficial research is to study a system which can exert force to sprung mass only or unsprung mass only or to both. This system has some hard nonlinearity like "backlash" or "threshold" so it is not easy to analyze, but it could improve ride quality without any violation of workspace limitation.

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APPENDIX A

TABLES

TABLE 1

NUMERICALLY STABLE STEPSIZE FOR RUNGE-KUTTA

zeta	stable	unstable
0.1	0.04	0.045
0.25	0.04	0.045
0.5	0.04	0.045
0.75	0.04	0.045
1.0	0.04	0.045
1.05	0.03	0.035
1.1	0.025	0.03
1.2	0.02	0.025
1.3	0.02	0.025
1.4	0.015	0.02
1.5	0.015	0.02
1.75	0.01	0.015
2.0	0.01	0.015
2.5	0.008	0.01
3.0	0.006	0.008
4.0	0.004	0.006
5.0	0.004	0.006
6.0	0.003	0.004
7.0	0.003	0.004
8.0	0.002	0.003
10.0	0.002	0.003
15.0	0.001	0.002
20.0	0.001	0.002

TABLE 2

PASSIVE SYSTEM RMS RESPONSE OF SAW-TOOTH ROAD

zeta	workspace	acceleration	jerk
0.000	6.1082E-03	4.0033E-01	2.3939E+01
0.100	3.0244E-03	3.3769E-01	2.0843E+01
0.200	2.3220E-03	3.9026E-01	2.4066E+01
0.300	2.0852E-03	4.5122E-01	2.7352E+01
0.400	1.9665E-03	5.1260E-01	3.0313E+01
0.500	1.8904E-03	5.7162E-01	3.2905E+01
0.600	1.8345E-03	6.2760E-01	3.5163E+01
0.700	1.7905E-03	6.8070E-01	3.7136E+01
0.800	1.7546E-03	7.3138E-01	3.8872E+01
0.900	1.7247E-03	7.8020E-01	4.0415E+01
1.000	1.6994E-03	8.2763E-01	4.1807E+01
1.100	1.6778E-03	8.7409E-01	4.3089E+01
1.200	1.6590E-03	9.1988E-01	4.4302E+01
1.300	1.6426E-03	9.6526E-01	4.5495E+01
1.400	1.6281E-03	1.0105E+00	4.6725E+01
1.500	1.6152E-03	1.0557E+00	4.8058E+01
1.600	1.6037E-03	1.1011E+00	4.9572E+01
1.700	1.5935E-03	1.1471E+00	5.1349E+01
1.800	1.5845E-03	1.1940E+00	5.3470E+01
1.900	1.5772E-03	1.2424E+00	5.6007E+01

NOTE : Vehicle Speed = 80 km/hr

TABLE 3

PASSIVE SYSTEM RMS RESPONSE OF ROUGH ROAD

zeta	workspace	acceleration	jerk
0.000	8.7632E-02	5.8369E+00	9.0480E+01
0.100	6.1006E-02	4.3654E+00	1.1202E+02
0.200	5.0228E-02	4.1266E+00	1.4401E+02
0.300	4.1406E-02	3.9941E+00	1.6289E+02
0.400	3.4930E-02	3.9957E+00	1.7601E+02
0.500	3.0228E-02	4.0967E+00	1.8627E+02
0.600	2.6740E-02	4.2616E+00	1.9495E+02
0.700	2.4073E-02	4.4659E+00	2.0274E+02
0.800	2.1969E-02	4.6932E+00	2.0995E+02
0.900	2.0264E-02	4.9324E+00	2.1677E+02
1.000	1.8849E-02	5.1761E+00	2.2326E+02
1.100	1.7651E-02	5.4188E+00	2.2945E+02
1.200	1.6619E-02	5.6569E+00	2.3532E+02
1.300	1.5720E-02	5.8876E+00	2.4088E+02
1.400	1.4926E-02	6.1094E+00	2.4611E+02
1.500	1.4219E-02	6.3209E+00	2.5102E+02
1.600	1.3585E-02	6.5218E+00	2.5563E+02
1.700	1.3015E-02	6.7120E+00	2.6000E+02
1.800	1.2501E-02	6.8927E+00	2.6429E+02
1.900	1.2041E-02	7.0669E+00	2.6882E+02

NOTE : Vehicle Speed = 20 km/hr

TABLE 4

ASYMMETRIC DAMPING SYSTEM RMS RESPONSE 1

zeta	workspace	acceleration	jerk
0.000	6.1082E-03	4.0033E-01	2.3939E+01
0.100	4.7280E-03	3.3087E-01	2.4540E+01
0.200	4.6639E-03	3.5573E-01	2.9203E+01
0.300	4.5384E-03	3.8829E-01	3.3475E+01
0.400	4.5340E-03	4.2088E-01	3.6507E+01
0.500	4.7772E-03	4.5959E-01	4.0035E+01
0.600	5.1298E-03	4.9944E-01	4.3585E+01
0.700	5.5077E-03	5.3603E-01	4.6829E+01
0.800	5.8690E-03	5.6960E-01	4.9897E+01
0.900	6.3013E-03	5.9801E-01	5.2628E+01
1.000	6.6930E-03	6.2429E-01	5.5022E+01
1.100	7.1120E-03	6.4766E-01	5.7306E+01
1.200	7.5175E-03	6.6955E-01	5.9436E+01
1.300	7.8962E-03	6.9065E-01	6.1412E+01
1.400	8.1854E-03	7.1351E-01	6.3006E+01
1.500	8.4825E-03	7.3515E-01	6.4805E+01
1.600	8.8169E-03	7.5491E-01	6.5483E+01
1.700	8.9573E-03	7.8699E-01	6.7397E+01
1.800	9.1743E-03	8.1453E-01	6.9633E+01
1.900	9.1827E-03	8.5815E-01	7.2598E+01

NOTE : Vehicle Speed = 80 km/hr
Input Road : Saw-tooth Road
Cheta is for Compression.
Rebound cheta is 0.

TABLE 5

ASYMMETRIC DAMPING SYSTEM RMS RESPONSE 2

zeta	workspace	acceleration	jerk
0.000	8.7632E-02	5.8369E+00	9.0480E+01
0.100	7.0569E-02	4.7338E+00	1.0574E+02
0.200	7.0272E-02	4.7415E+00	1.4371E+02
0.300	6.9736E-02	4.7433E+00	1.7042E+02
0.400	6.8903E-02	4.7399E+00	2.0553E+02
0.500	6.7559E-02	4.7028E+00	2.2685E+02
0.600	6.6233E-02	4.6536E+00	2.4265E+02
0.700	6.2534E-02	4.5736E+00	2.6851E+02
0.800	6.1510E-02	4.5743E+00	2.8446E+02
0.900	5.9746E-02	4.6034E+00	3.1142E+02
1.000	5.8472E-02	4.6200E+00	3.3041E+02
1.100	5.9877E-02	4.6572E+00	3.3294E+02
1.200	5.9719E-02	4.6763E+00	3.4448E+02
1.300	6.0066E-02	4.7304E+00	3.5481E+02
1.400	6.2060E-02	4.8349E+00	3.6240E+02
1.500	6.4476E-02	4.9285E+00	3.7112E+02
1.600	6.6803E-02	5.0565E+00	3.7581E+02
1.700	6.9488E-02	5.1673E+00	3.8372E+02
1.800	7.2919E-02	5.3541E+00	3.8745E+02
1.900	7.5035E-02	5.5724E+00	3.8860E+02

NOTE : Vehicle Speed = 20 km/hr
Input Road : Rough Road
Cheta is for Compression.
Rebound cheta is 0.

TABLE 6

ASYMMETRIC DAMPING SYSTEM RMS RESPONSE 3

zeta	workspace	acceleration	jerk
0.000	6.1082E-03	4.0033E-01	2.3939E+01
0.100	4.5883E-03	3.4731E-01	2.2976E+01
0.200	4.3525E-03	3.9837E-01	2.7336E+01
0.300	4.3626E-03	4.5243E-01	3.0525E+01
0.400	4.2018E-03	5.0278E-01	3.5972E+01
0.500	4.2273E-03	5.5118E-01	3.9792E+01
0.600	4.2614E-03	5.9478E-01	4.4141E+01
0.700	4.3613E-03	6.3216E-01	4.8181E+01
0.800	4.5205E-03	6.6491E-01	5.1528E+01
0.900	4.8017E-03	6.9016E-01	5.0941E+01
1.000	5.1831E-03	7.1307E-01	4.9609E+01
1.100	5.6671E-03	7.3611E-01	4.6766E+01
1.200	6.1817E-03	7.6420E-01	4.4182E+01
1.300	6.5862E-03	7.9547E-01	4.5380E+01
1.400	6.9936E-03	8.2807E-01	4.6565E+01
1.500	7.3985E-03	8.6154E-01	4.7959E+01
1.600	7.7998E-03	8.9424E-01	4.9530E+01
1.700	8.2032E-03	9.2713E-01	5.1235E+01
1.800	8.5784E-03	9.6107E-01	5.3421E+01
1.900	8.8966E-03	1.0022E+00	5.6221E+01

NOTE : Vehicle Speed = 80 km/hr
Input Road : Saw-tooth Road
Cheta is for Rebound.
Compression cheta is 0.

TABLE 7

ASYMMETRIC DAMPING SYSTEM RMS RESPONSE 4

zeta	workspace	acceleration	jerk
0.000	8.7632E-02	5.8369E+00	9.0480E+01
0.100	6.8648E-02	4.6175E+00	1.1000E+02
0.200	6.9355E-02	4.7079E+00	1.5686E+02
0.300	7.1715E-02	4.8514E+00	1.9310E+02
0.400	7.2457E-02	4.8690E+00	2.1327E+02
0.500	7.2588E-02	4.8287E+00	2.1834E+02
0.600	7.1083E-02	4.8420E+00	2.4945E+02
0.700	7.1018E-02	4.8389E+00	2.5397E+02
0.800	7.1065E-02	4.8595E+00	2.5986E+02
0.900	6.9273E-02	4.8824E+00	2.7320E+02
1.000	6.9258E-02	4.9758E+00	2.8120E+02
1.100	7.0687E-02	5.0561E+00	2.8117E+02
1.200	7.0421E-02	5.1614E+00	2.9006E+02
1.300	7.0469E-02	5.2729E+00	2.9191E+02
1.400	7.0545E-02	5.4072E+00	2.8500E+02
1.500	7.1132E-02	5.5160E+00	2.8812E+02
1.600	7.0821E-02	5.7130E+00	2.9908E+02
1.700	7.0829E-02	5.8383E+00	3.0378E+02
1.800	7.1054E-02	5.9788E+00	3.1103E+02
1.900	7.1848E-02	6.1325E+00	3.0874E+02

NOTE : Vehicle Speed = 20 km/hr
Input Road : Rough Road
Cheta is for Rebound.
Compression cheta is 0.

TABLE 8

RELATIVE DISPLACEMENT DEPENDENT DAMPING
RMS RESPONSE OF SAW-TOOTH ROAD

b	workspace	acceleration	jerk
0	6.1082E-03	4.0033E-01	2.3939E+01
5	4.8733E-03	3.1211E-01	2.0281E+01
10	4.6325E-03	3.1534E-01	2.3323E+01
15	4.2979E-03	3.2433E-01	2.6094E+01
20	3.9962E-03	3.3642E-01	2.8359E+01
25	3.7503E-03	3.5054E-01	3.0573E+01
30	3.5469E-03	3.6503E-01	3.2519E+01
35	3.3759E-03	3.7896E-01	3.4161E+01
40	3.2322E-03	3.9228E-01	3.5575E+01
45	3.1109E-03	4.0504E-01	3.6868E+01

NOTE : Vehicle Speed = 80 km/hr
b is defined in the paper.

TABLE 9

RELATIVE DISPLACEMENT DEPENDENT DAMPING
RMS RESPONSE OF ROUGH ROAD

b	workspace	acceleration	jerk
0	8.7632E-02	5.8369E+00	9.0480E+01
5	5.7725E-02	4.6426E+00	1.7503E+02
10	4.6631E-02	4.5206E+00	1.9388E+02
15	3.9985E-02	4.5507E+00	2.1128E+02
20	3.5347E-02	4.6346E+00	2.2604E+02
25	3.2359E-02	4.7807E+00	2.3873E+02
30	3.0011E-02	4.9078E+00	2.4303E+02
35	2.8279E-02	5.0660E+00	2.5565E+02
40	2.6964E-02	5.2383E+00	2.7075E+02
45	2.5666E-02	5.3538E+00	2.7862E+02

NOTE : Vehicle Speed = 20 km/hr
b is defined in the paper.

TABLE 10

CUBIC DAMPING RMS RESPONSE OF SAW-TOOTH ROAD

B	workspace	acceleration	jerk
0.000	6.1082E-03	4.0033E-01	2.3939E+01
0.100	4.6410E-03	3.1926E-01	1.9421E+01
0.200	4.3579E-03	3.1790E-01	1.9986E+01
0.300	4.1401E-03	3.2021E-01	2.0725E+01
0.400	3.9630E-03	3.2410E-01	2.1550E+01
0.500	3.8157E-03	3.2890E-01	2.2419E+01
0.600	3.6910E-03	3.3428E-01	2.3308E+01
0.700	3.5837E-03	3.4003E-01	2.4206E+01
0.800	3.4904E-03	3.4601E-01	2.5104E+01
0.900	3.4082E-03	3.5214E-01	2.5999E+01
1.000	3.3353E-03	3.5835E-01	2.6888E+01
1.100	3.2702E-03	3.6460E-01	2.7771E+01
1.200	3.2118E-03	3.7085E-01	2.8646E+01
1.300	3.1590E-03	3.7710E-01	2.9513E+01
1.400	3.1112E-03	3.8331E-01	3.0373E+01
1.500	3.0678E-03	3.8948E-01	3.1225E+01
1.600	3.0282E-03	3.9560E-01	3.2071E+01
1.700	2.9921E-03	4.0167E-01	3.2909E+01
1.800	2.9591E-03	4.0768E-01	3.3740E+01
1.900	2.9289E-03	4.1363E-01	3.4564E+01
2.000	2.9012E-03	4.1951E-01	3.5382E+01
2.100	2.8758E-03	4.2533E-01	3.6194E+01
2.200	2.8525E-03	4.3109E-01	3.7000E+01
2.300	2.8312E-03	4.3678E-01	3.7800E+01
2.400	2.8117E-03	4.4241E-01	3.8594E+01
2.500	2.7937E-03	4.4797E-01	3.9382E+01
2.600	2.7774E-03	4.5347E-01	4.0165E+01
2.700	2.7624E-03	4.5891E-01	4.0942E+01
2.800	2.7487E-03	4.6428E-01	4.1713E+01
2.900	2.7362E-03	4.6959E-01	4.2479E+01
3.000	2.7248E-03	4.7485E-01	4.3240E+01

NOTE : Vehicle Speed = 80 km/hr
B is defined in the paper.

TABLE 11

CUBIC DAMPING RMS RESPONSE OF ROUGH ROAD

B	workspace	acceleration	jerk
0.000	8.7632E-02	5.8369E+00	9.0480E+01
0.100	4.9957E-02	4.1637E+00	2.0503E+02
0.200	4.2887E-02	4.0214E+00	2.3290E+02
0.300	3.9382E-02	3.9765E+00	2.4578E+02
0.400	3.7096E-02	3.9654E+00	2.5419E+02
0.500	3.5379E-02	3.9697E+00	2.6080E+02
0.600	3.3991E-02	3.9824E+00	2.6654E+02
0.700	3.2821E-02	4.0001E+00	2.7180E+02
0.800	3.1809E-02	4.0209E+00	2.7669E+02
0.900	3.0917E-02	4.0437E+00	2.8129E+02
1.000	3.0120E-02	4.0676E+00	2.8563E+02
1.100	2.9401E-02	4.0923E+00	2.8972E+02
1.200	2.8746E-02	4.1174E+00	2.9360E+02
1.300	2.8144E-02	4.1425E+00	2.9729E+02
1.400	2.7588E-02	4.1678E+00	3.0082E+02
1.500	2.7070E-02	4.1929E+00	3.0422E+02
1.600	2.6587E-02	4.2179E+00	3.0751E+02
1.700	2.6132E-02	4.2428E+00	3.1072E+02
1.800	2.5704E-02	4.2676E+00	3.1387E+02
1.900	2.5298E-02	4.2922E+00	3.1698E+02
2.000	2.4911E-02	4.3167E+00	3.2006E+02
2.100	2.4543E-02	4.3410E+00	3.2311E+02
2.200	2.4190E-02	4.3651E+00	3.2612E+02
2.300	2.3851E-02	4.3891E+00	3.2909E+02
2.400	2.3525E-02	4.4127E+00	3.3198E+02
2.500	2.3210E-02	4.4361E+00	3.3476E+02
2.600	2.2907E-02	4.4590E+00	3.3740E+02
2.700	2.2613E-02	4.4815E+00	3.3985E+02
2.800	2.2329E-02	4.5033E+00	3.4206E+02
2.900	2.2054E-02	4.5245E+00	3.4397E+02
3.000	2.1788E-02	4.5449E+00	3.4554E+02

NOTE : Vehicle Speed = 20 km/hr
B is defined in the paper.

TABLE 12

SEMI-ACTIVE DAMPING RMS RESPONSE
OF SAW-TOOTH ROAD

zeta	workspace	acceleration	jerk
0.000	6.1082E-03	4.0033E-01	2.3939E+01
0.100	5.2647E-03	3.4209E-01	2.1562E+01
0.200	4.9200E-03	3.1744E-01	1.9930E+01
0.300	4.9388E-03	3.1939E-01	2.0686E+01
0.400	4.9443E-03	3.1904E-01	2.0949E+01
0.500	5.0762E-03	3.4518E-01	2.7187E+01
0.600	4.9248E-03	3.1807E-01	2.2344E+01
0.700	4.9518E-03	3.2620E-01	2.4667E+01
0.800	4.9335E-03	3.1755E-01	2.2325E+01
0.900	4.9713E-03	3.4514E-01	3.0539E+01
1.000	5.0235E-03	3.5541E-01	3.3362E+01
1.100	5.2746E-03	3.7229E-01	3.8027E+01
1.200	5.0728E-03	3.3736E-01	3.0112E+01
1.300	5.2498E-03	3.7477E-01	3.8945E+01
1.400	5.2210E-03	3.6215E-01	3.6522E+01
1.500	5.2910E-03	3.6247E-01	3.6890E+01
1.600	5.1961E-03	3.8263E-01	4.0496E+01
1.700	5.0972E-03	3.6561E-01	3.7273E+01
1.800	5.3584E-03	3.8659E-01	4.3614E+01
1.900	5.2318E-03	3.9821E-01	4.5698E+01

NOTE : Vehicle Speed = 80 km/hr
Based on body velocity feedback control.
Cheta is that of "skyhook" damping.

TABLE 13

SEMI-ACTIVE DAMPING RMS RESPONSE
OF ROUGH ROAD

zeta	workspace	acceleration	jerk
0.000	8.7632E-02	5.8369E+00	9.0480E+01
0.100	6.4679E-02	4.3869E+00	1.6201E+02
0.200	6.0711E-02	4.2634E+00	2.3826E+02
0.300	5.8446E-02	4.1521E+00	2.5300E+02
0.400	5.2052E-02	3.8099E+00	2.9666E+02
0.500	5.0280E-02	3.9763E+00	3.3926E+02
0.600	4.9515E-02	3.6663E+00	2.9680E+02
0.700	4.7907E-02	3.7357E+00	2.6457E+02
0.800	4.7896E-02	3.9287E+00	2.7130E+02
0.900	4.8185E-02	3.9840E+00	3.0849E+02
1.000	4.8738E-02	3.9115E+00	2.7884E+02
1.100	5.0541E-02	3.9196E+00	3.1308E+02
1.200	5.4460E-02	4.1159E+00	3.9096E+02
1.300	5.5214E-02	4.4283E+00	4.5618E+02
1.400	5.3625E-02	4.1655E+00	3.9699E+02
1.500	5.5393E-02	4.3579E+00	4.0983E+02
1.600	5.2538E-02	4.4637E+00	4.3210E+02
1.700	5.6167E-02	4.0911E+00	3.5076E+02
1.800	5.3680E-02	3.8802E+00	3.1260E+02
1.900	5.5941E-02	4.1399E+00	3.4916E+02

NOTE : Vehicle Speed = 20 km/hr
Based on body velocity feedback control.
Cheta is that of "skyhook" damping.

APPENDIX B

FIGURES

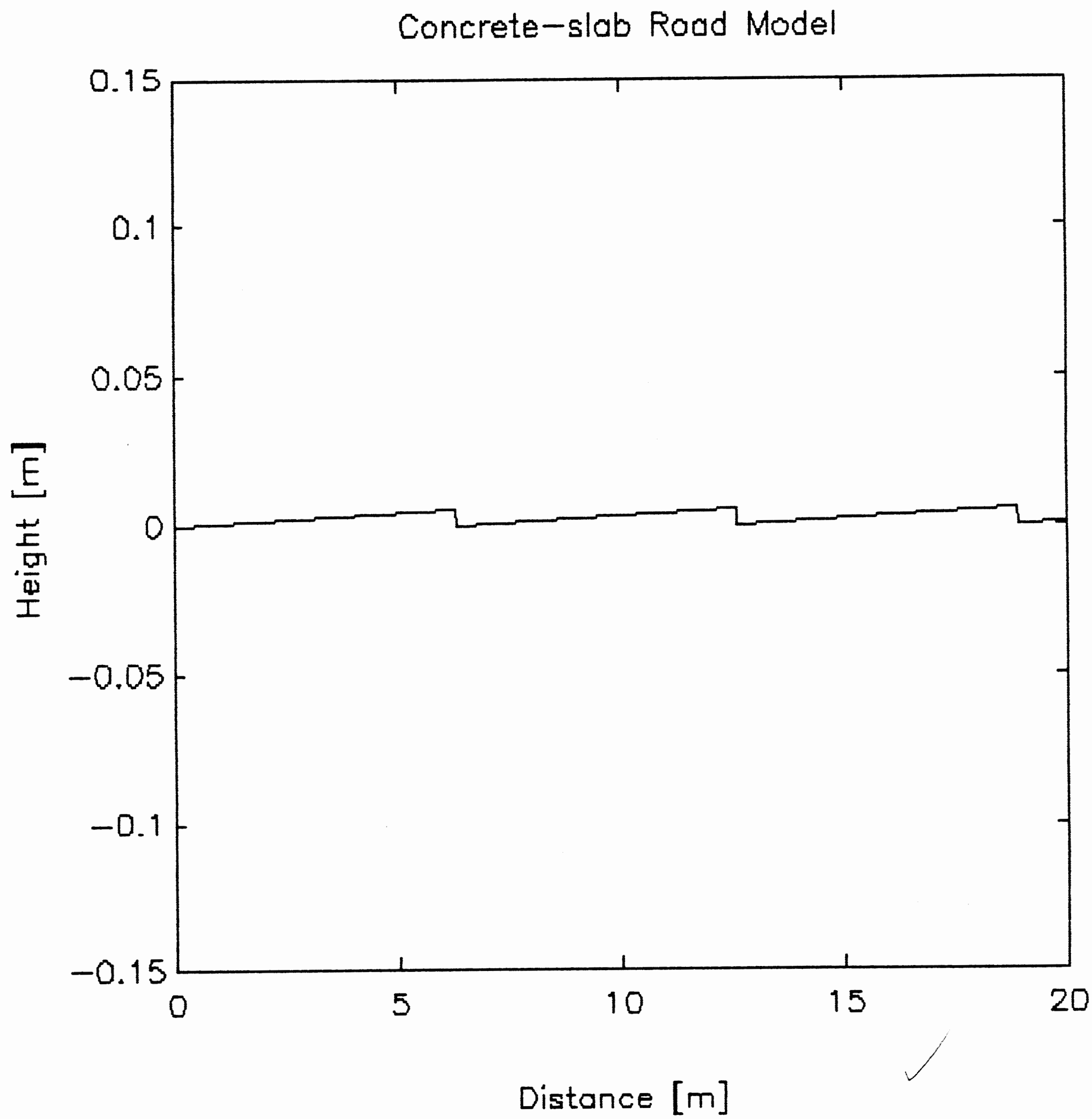

Figure 1. Configuration of Concrete-slab Road Model

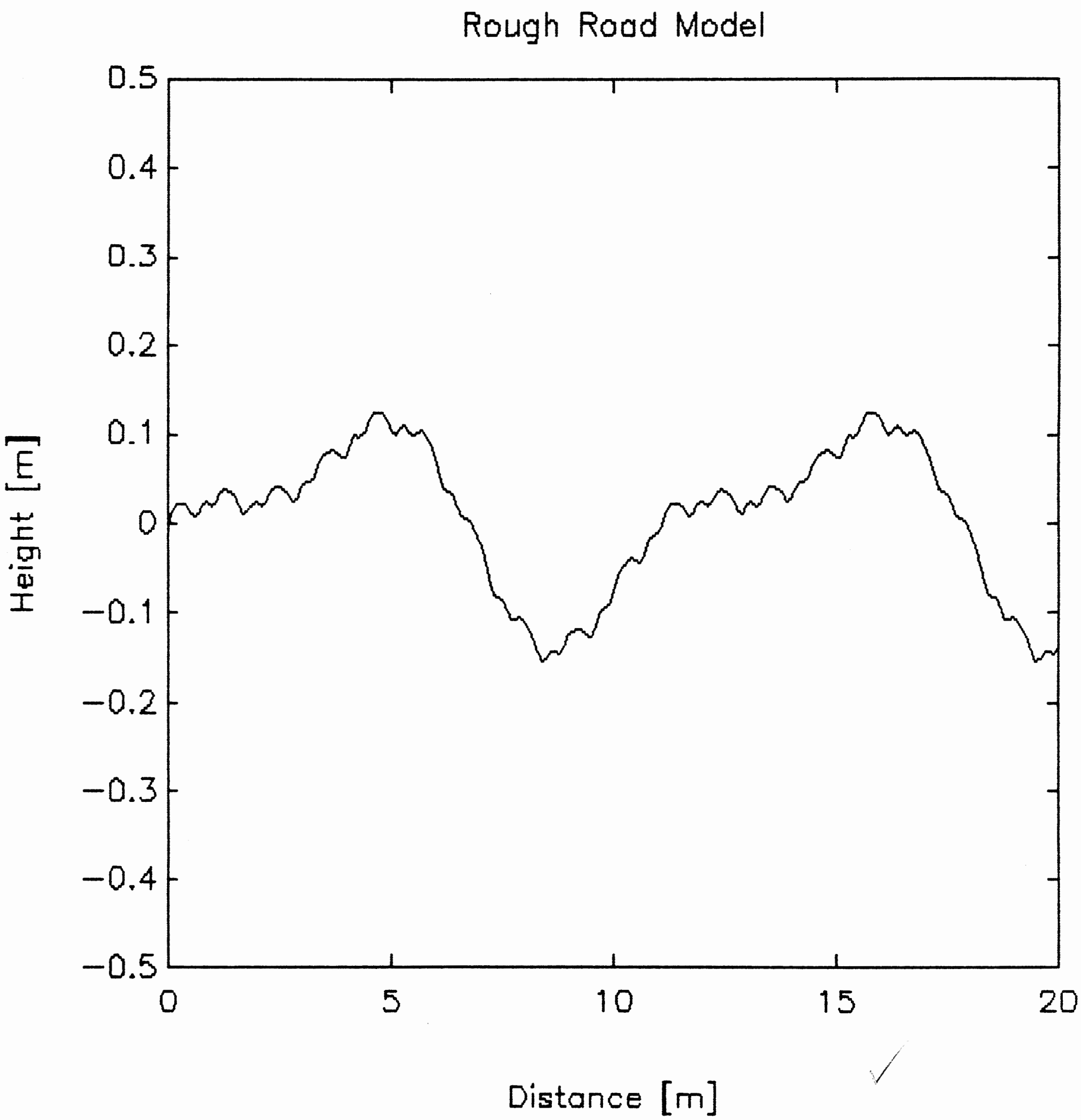

Figure 2. Configuration of Rough Road Model

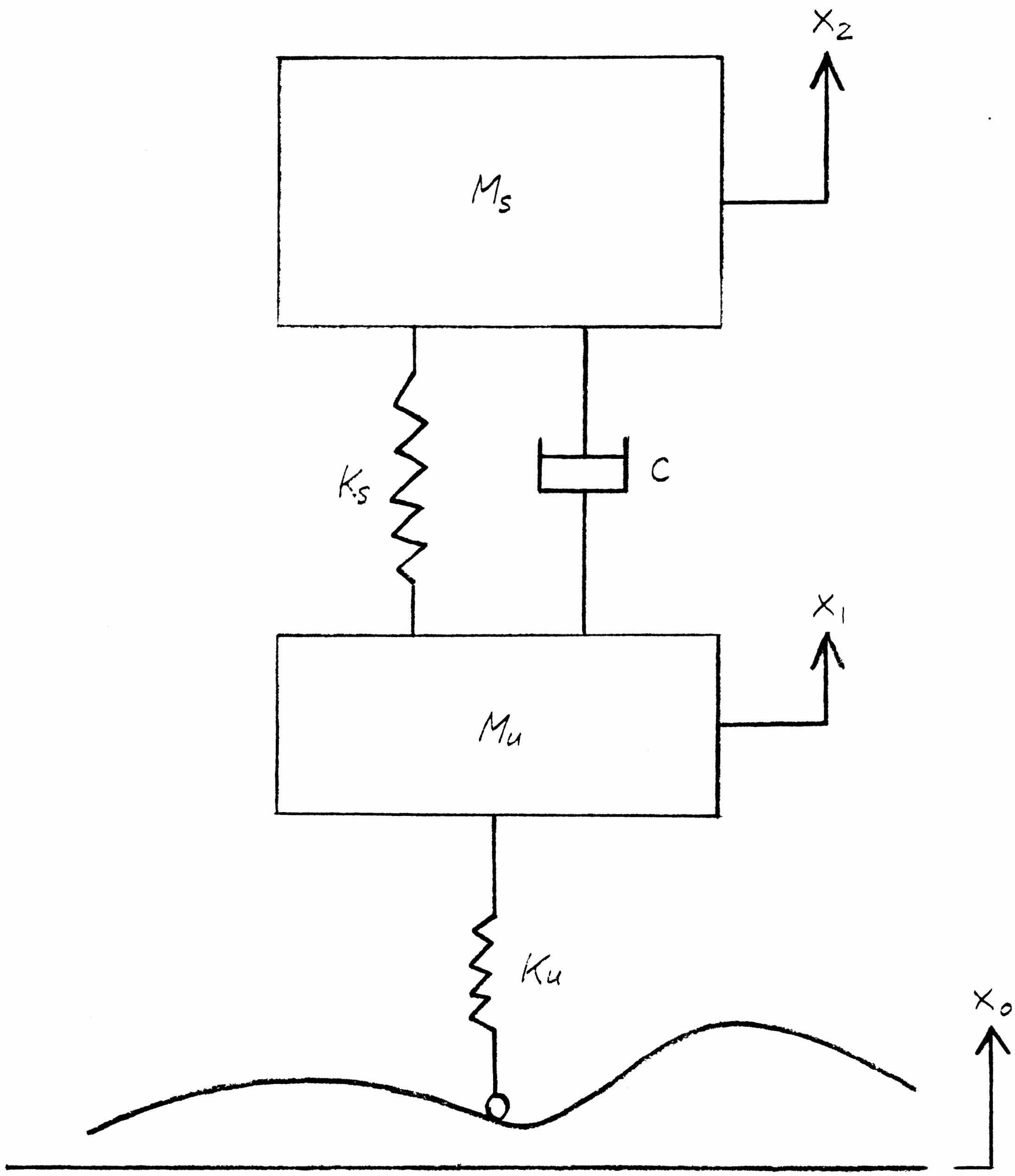

Figure 3. Passive Suspension Model

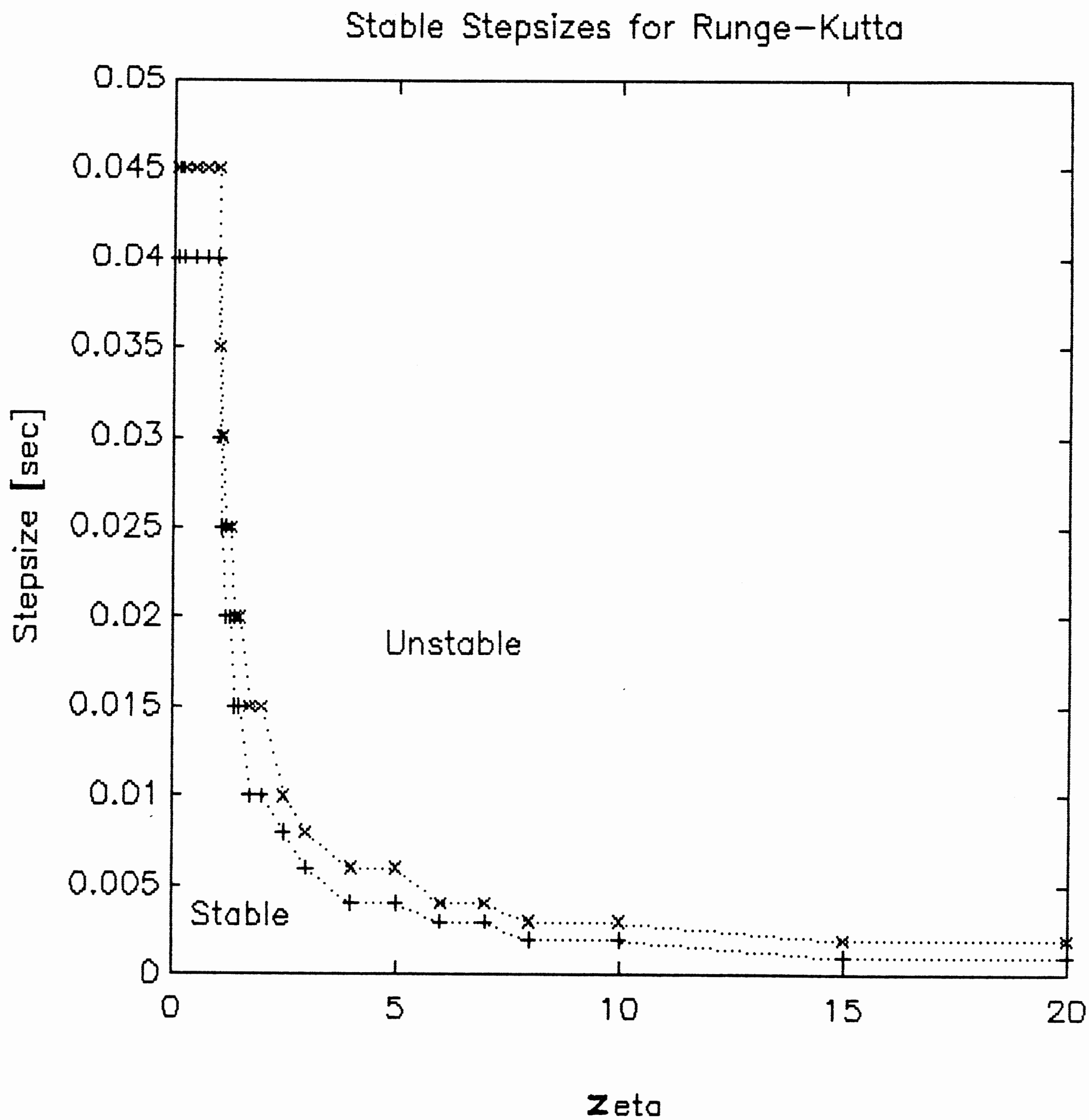

Figure 4. Stable Stepsizes for Runge-Kutta

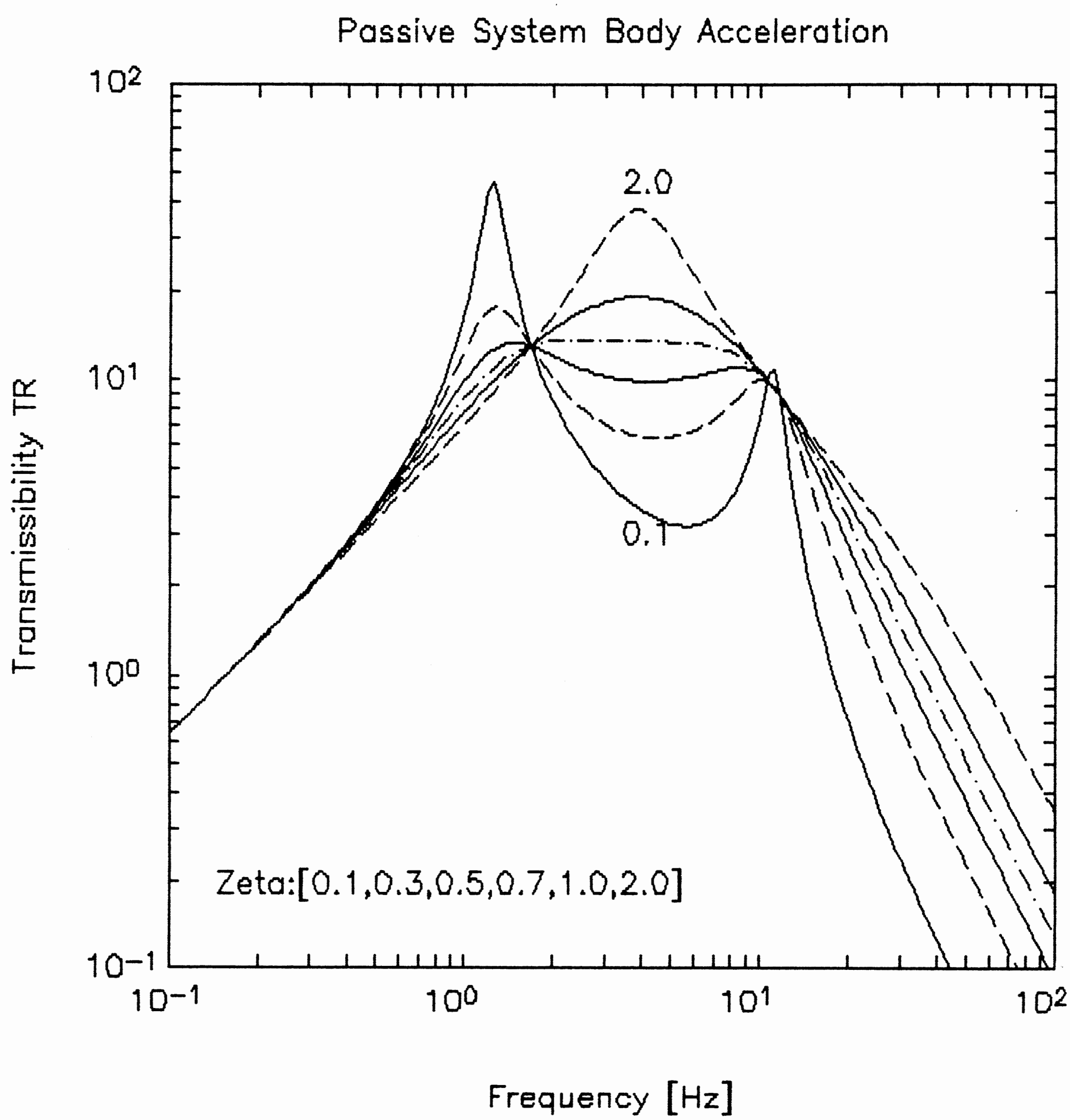

Figure 5. Passive System Frequency Response for Body Acceleration

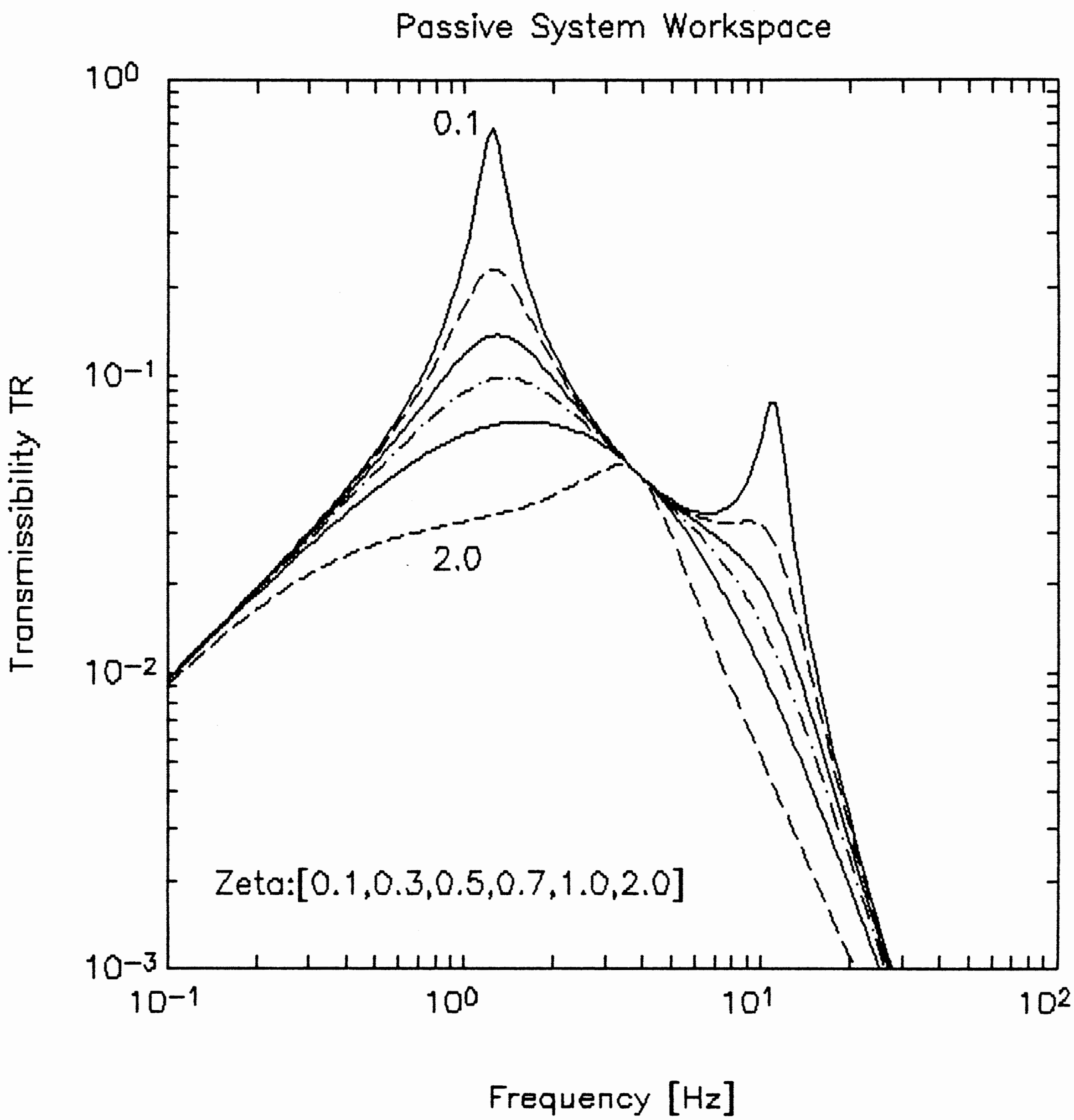

Figure 6. Passive System Frequency Response for Workspace

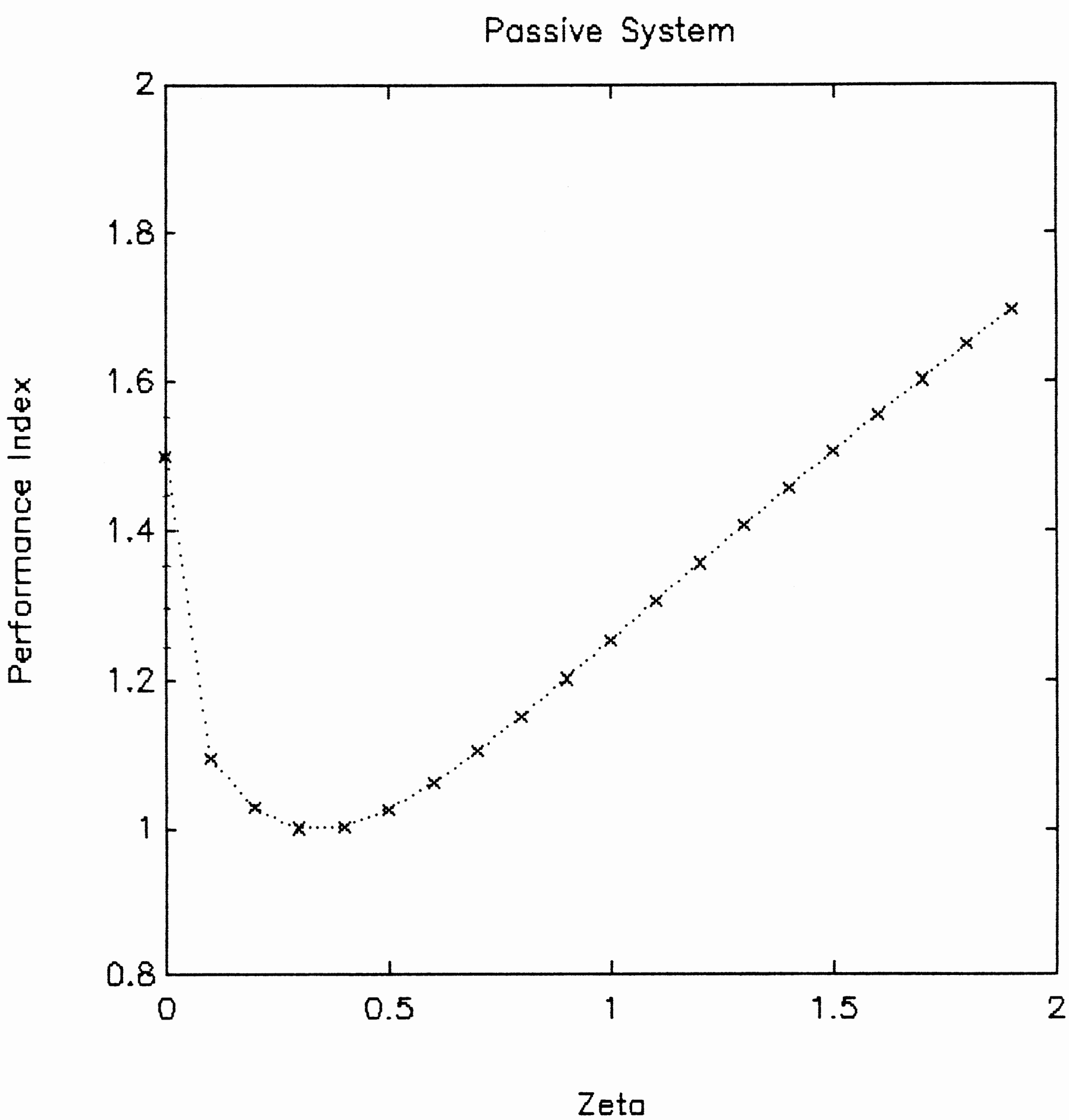

Figure 7. Performance Index for Passive System

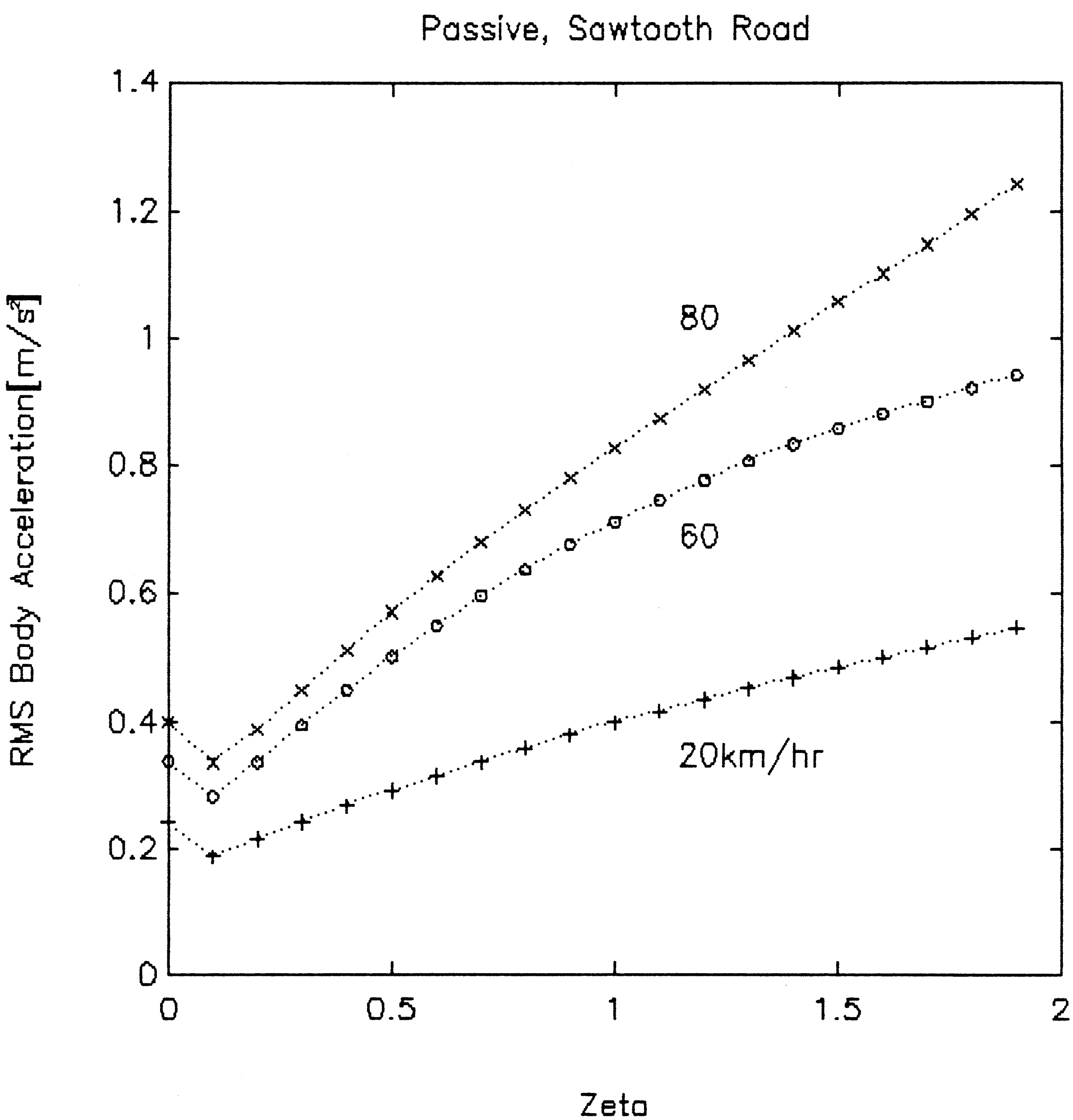

Figure 8. Passive System RMS Body Acceleration Response

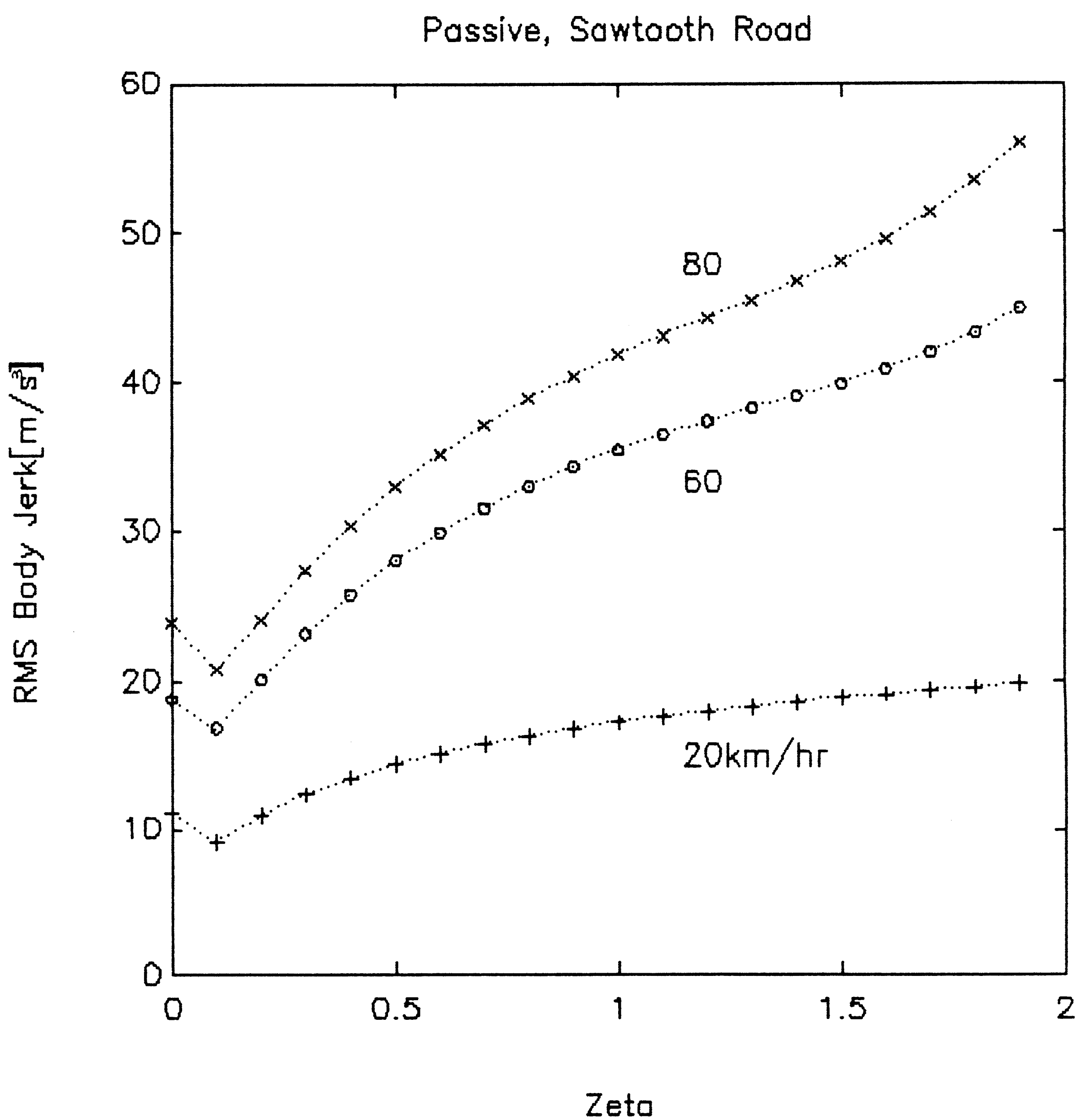

Figure 9. Passive System RMS Body Jerk Response

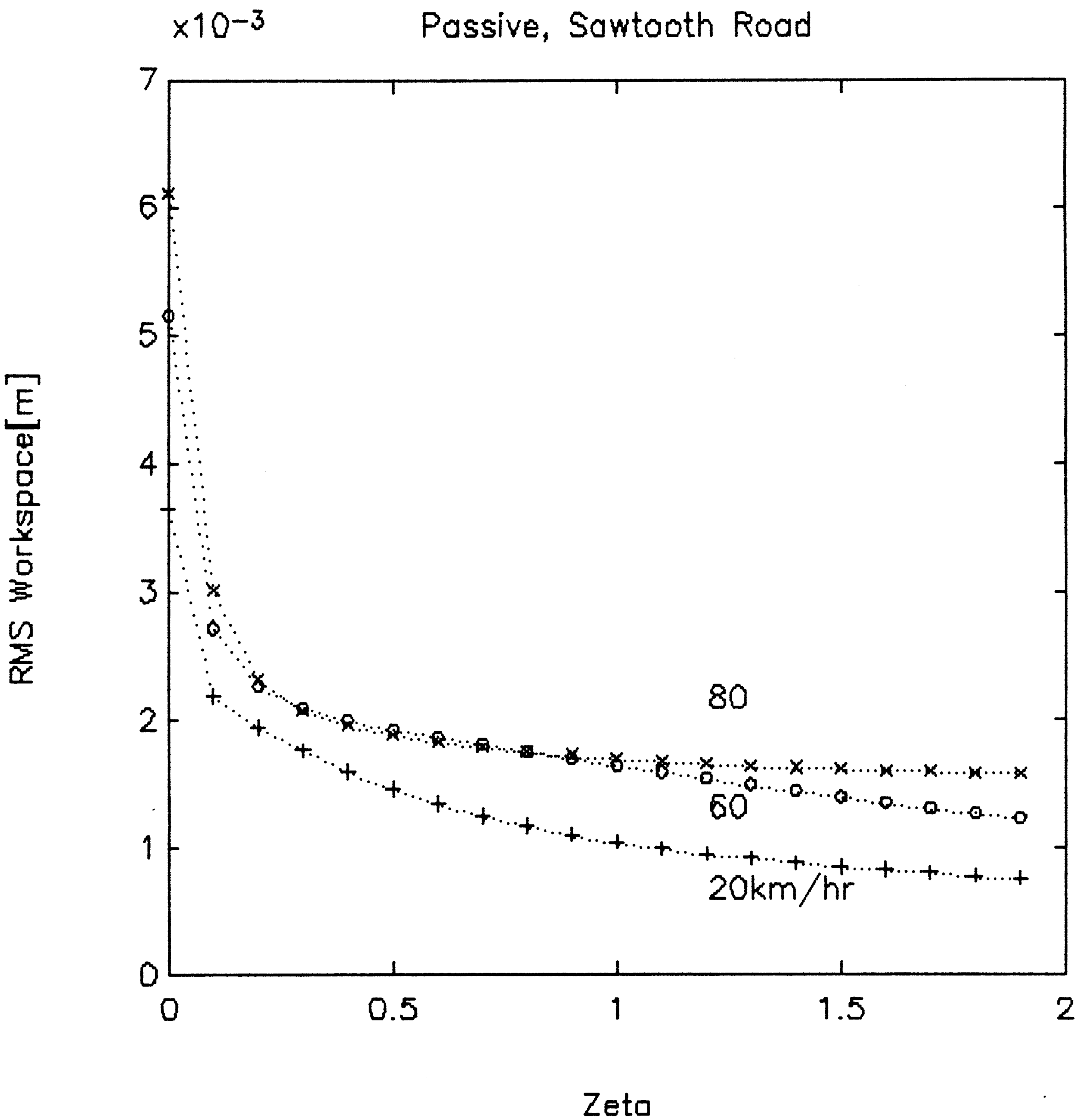

Figure 10. Passive System RMS Workspace Response

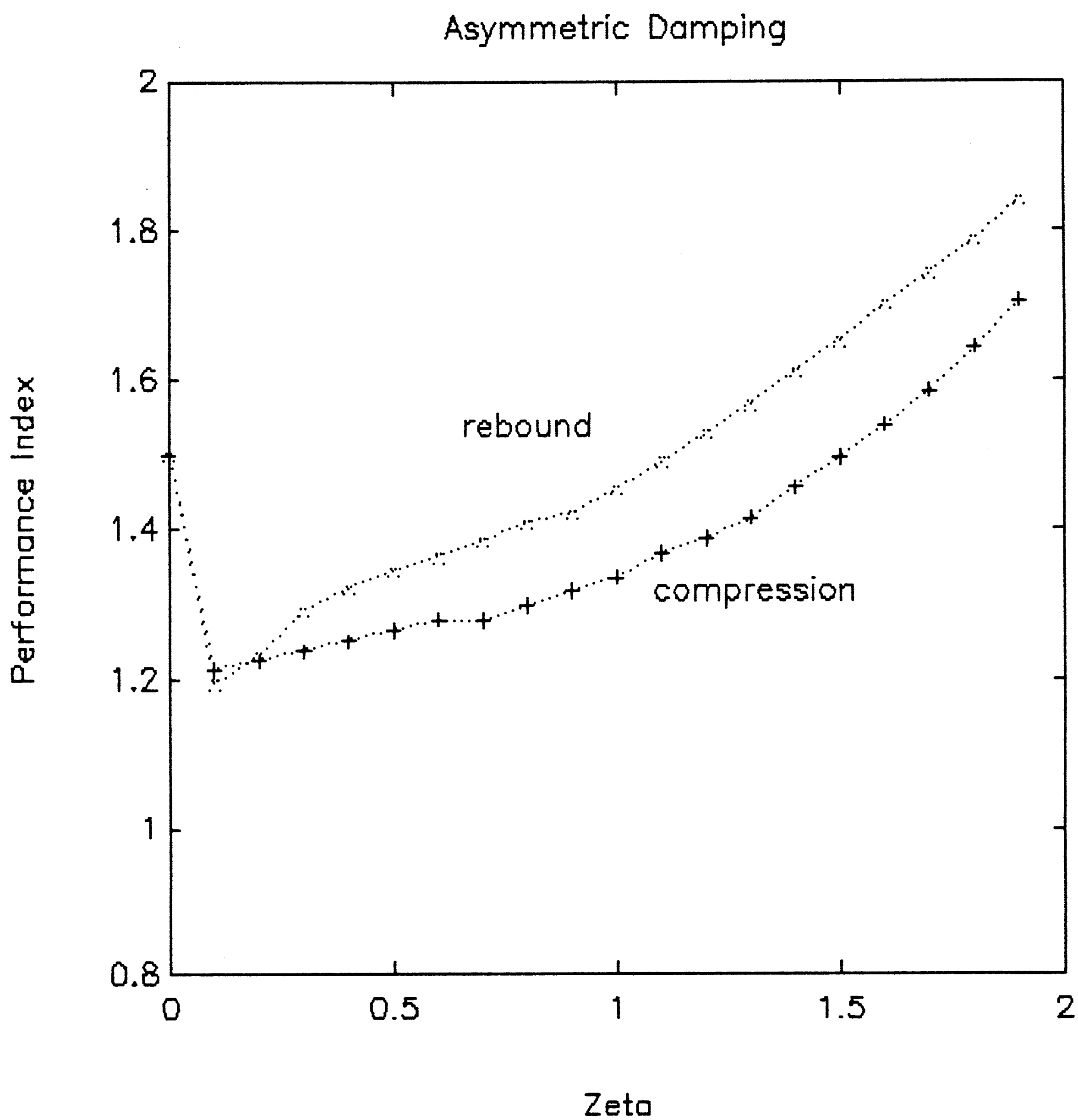

Figure 11. Performance Index for Asymmetric Damping

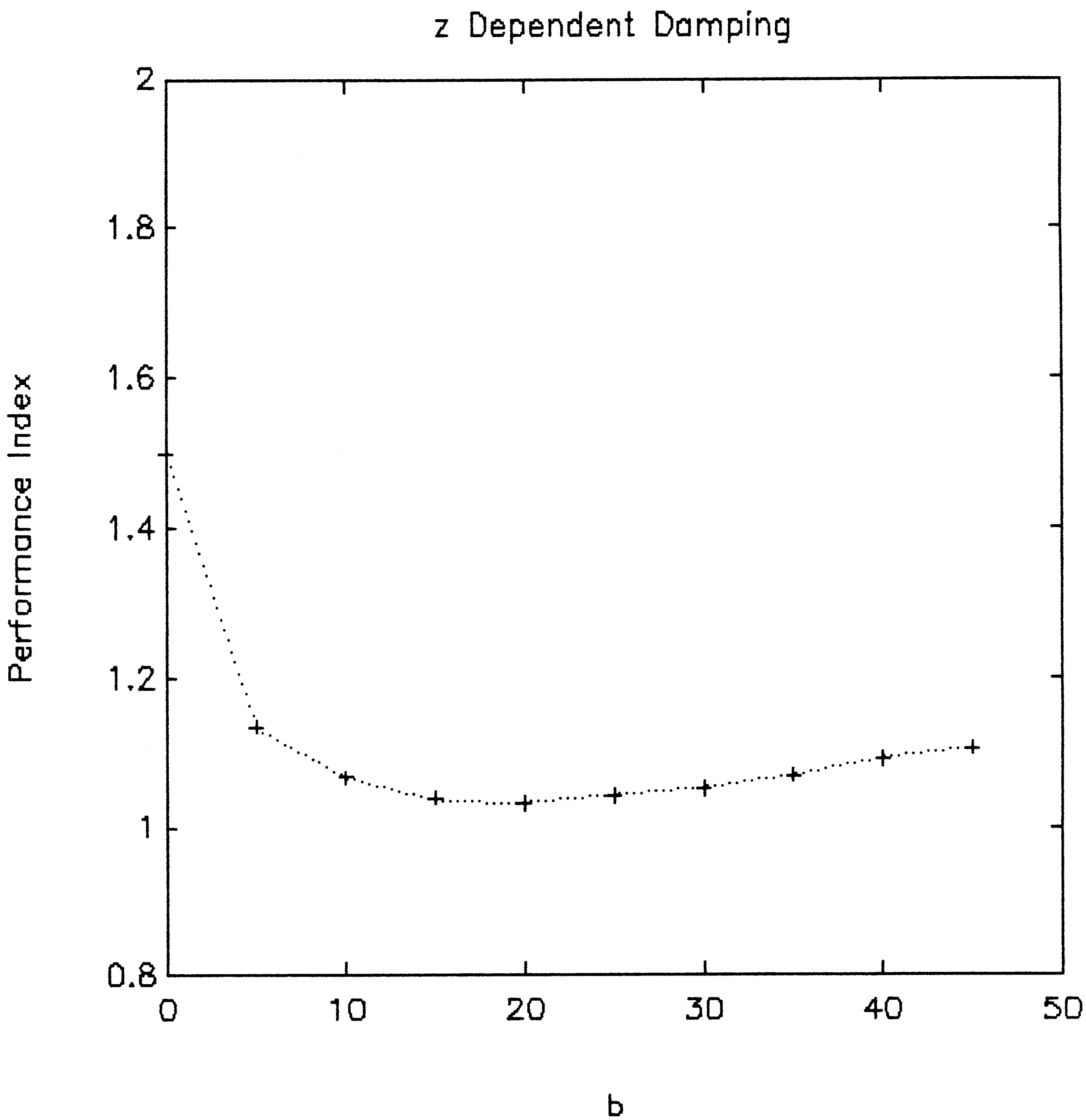

Figure 12. Performance Index for Relative Displacement Dependent Damping

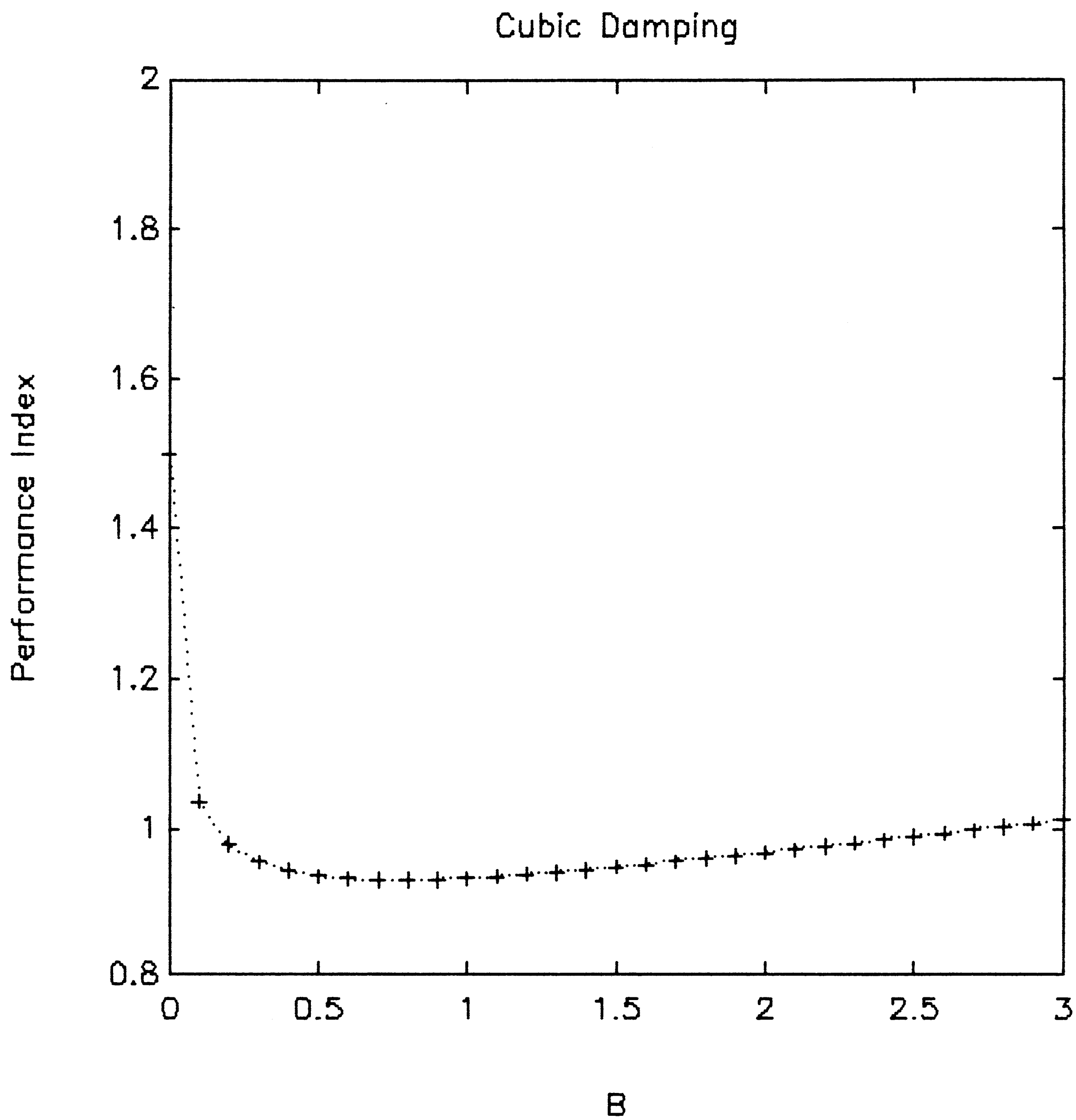

Figure 13. Performance Index for Cubic Damping

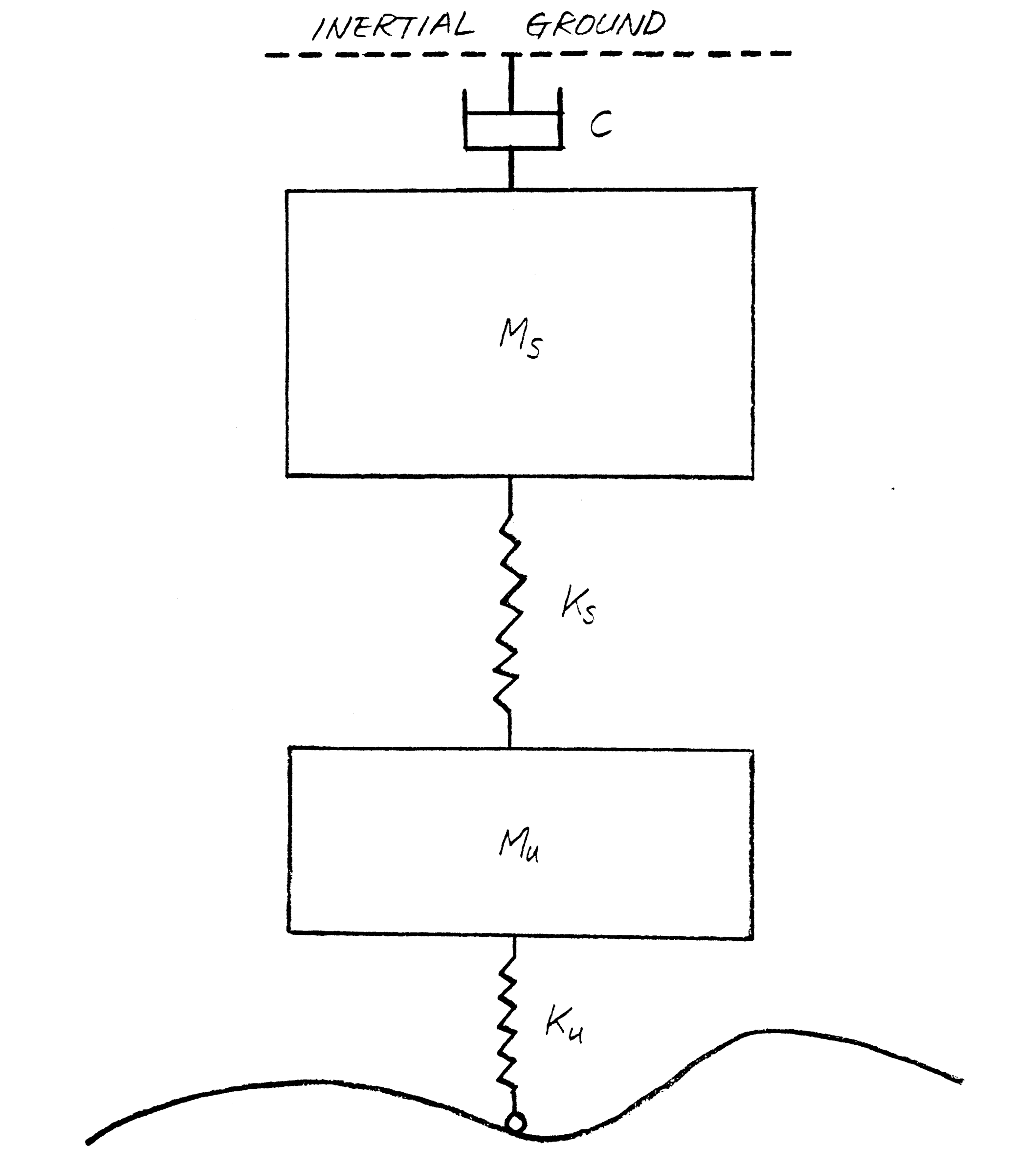

Figure 14. Skyhook Damping Model

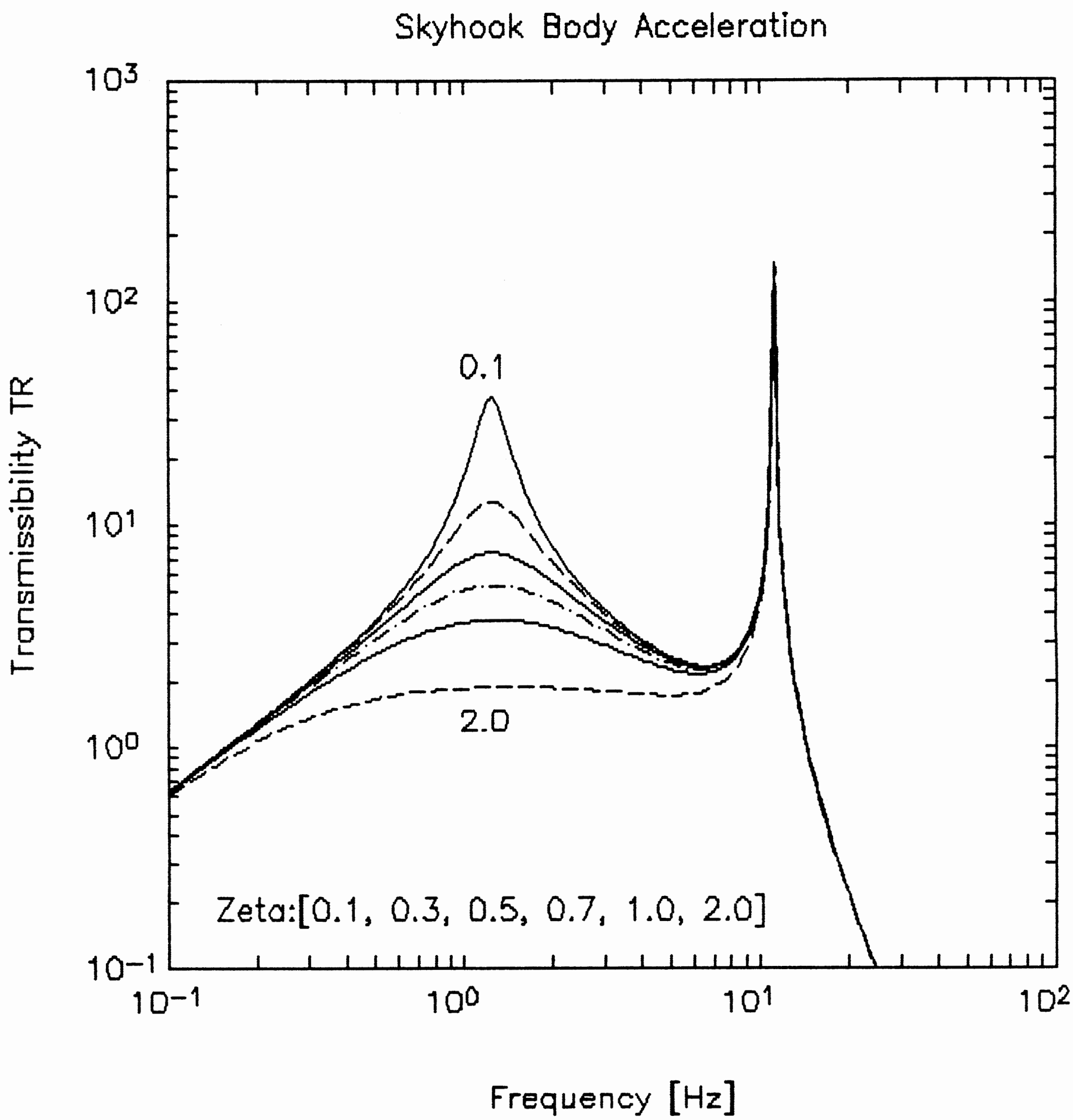

Figure 15. Skyhook Damping System Frequency Response for Body Acceleration

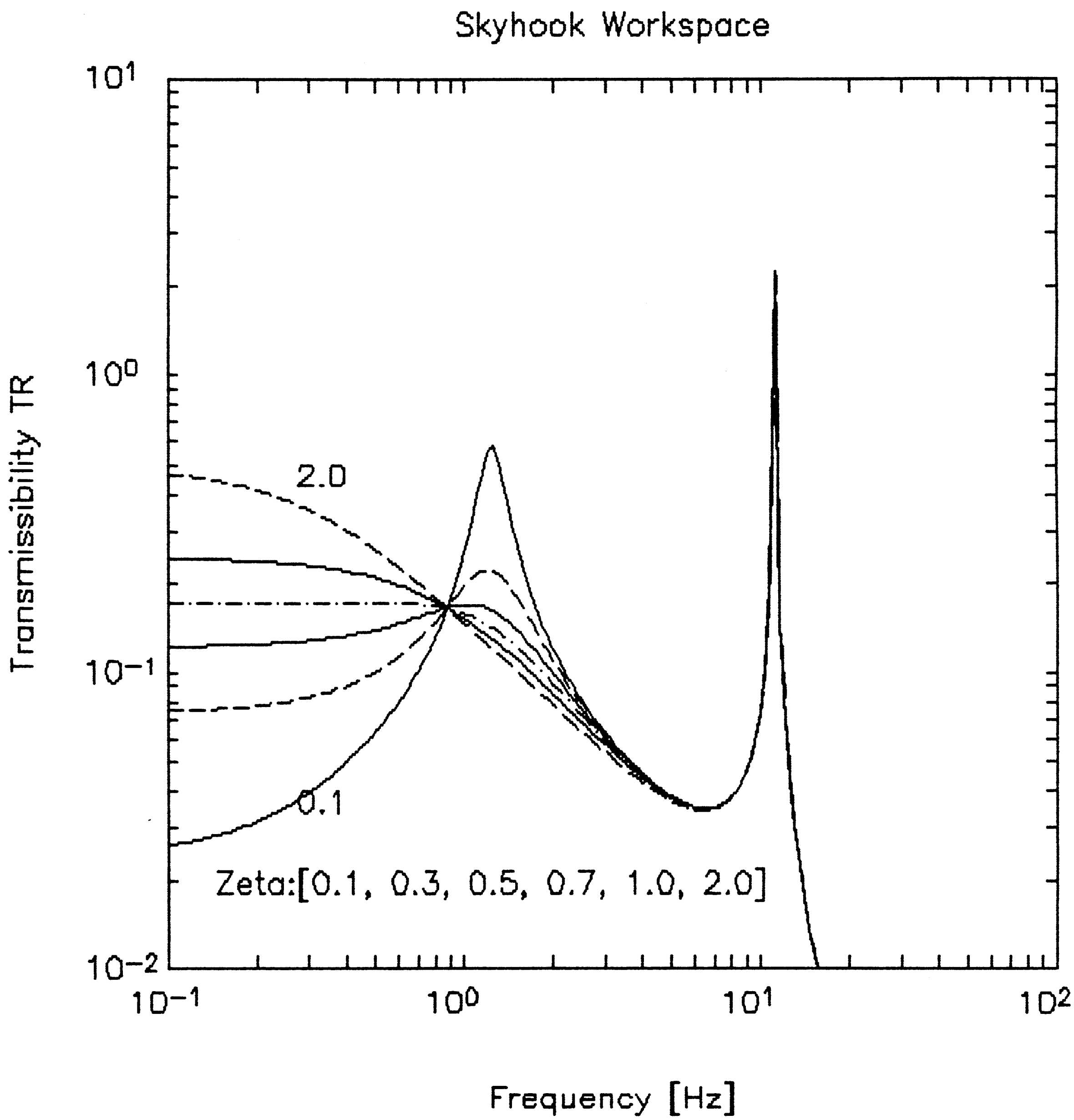

Figure 16. Skyhook Damping System Frequency Response for Workspace

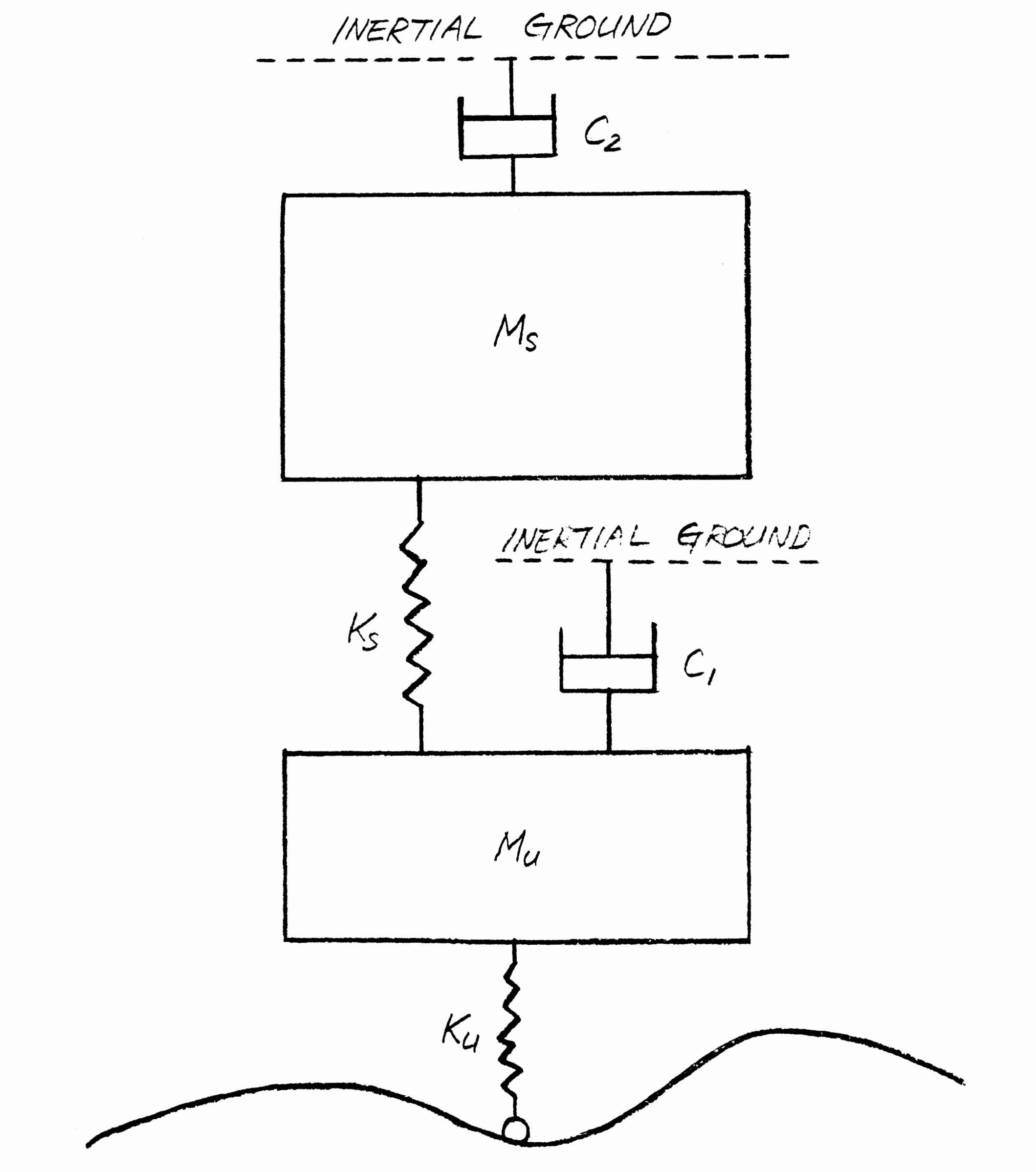

Figure 17. Two Inertial Grounded Damping Model

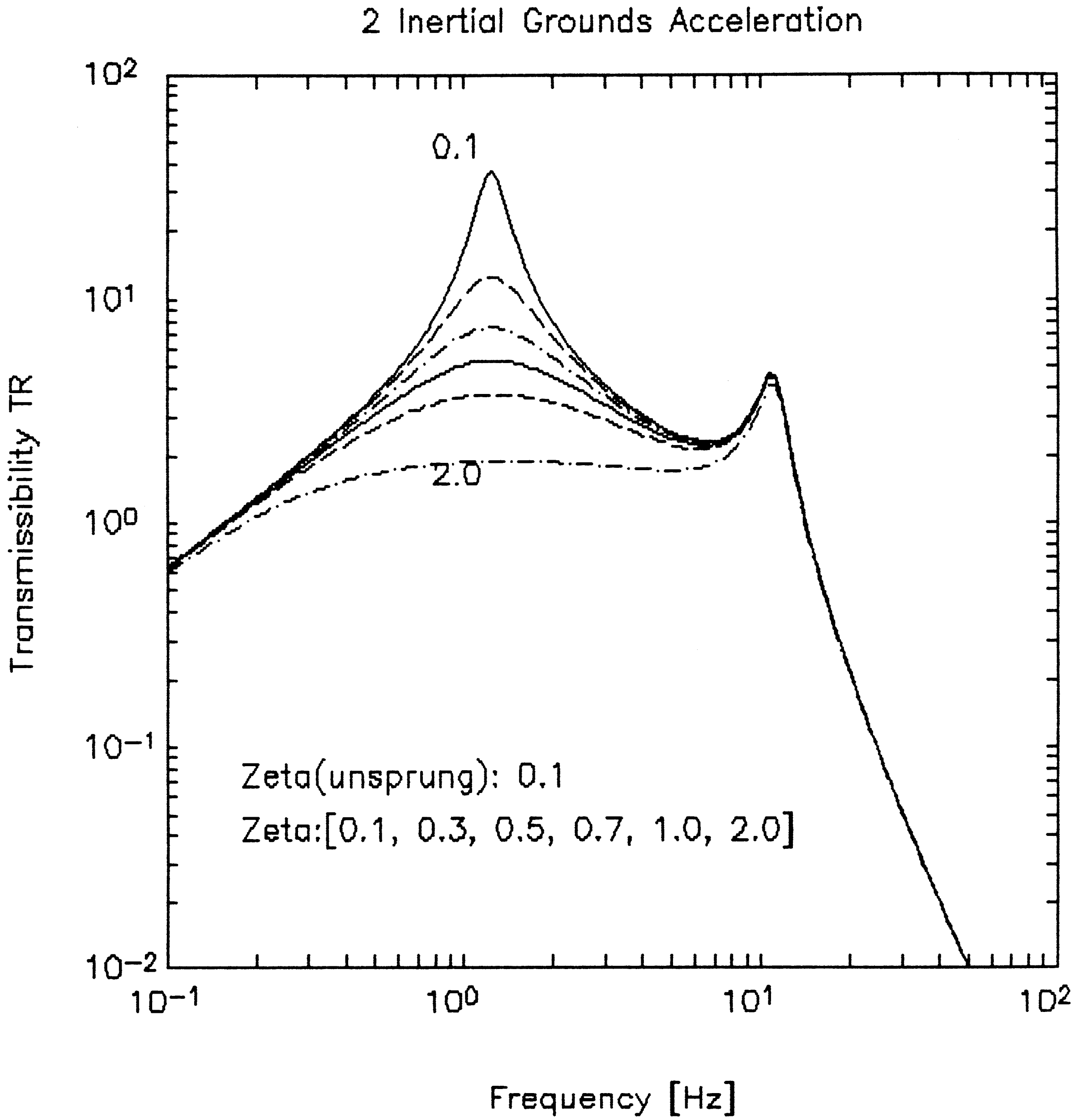

Figure 18. Two Inertial Grounded Damping System Frequency Response for Body Acceleration 1

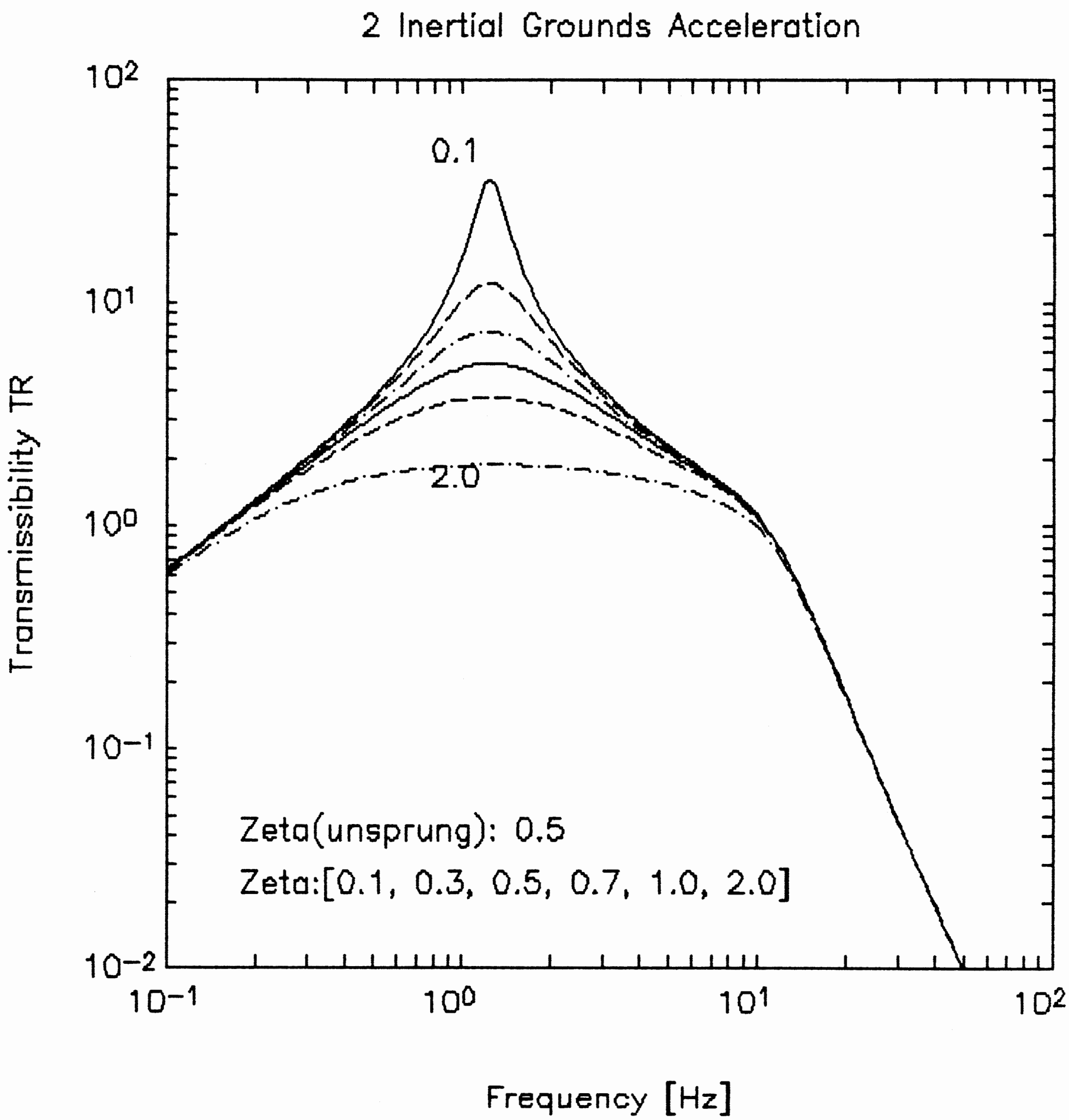

Figure 19. Two Inertial Grounded Damping System Frequency Response for Body Acceleration 2

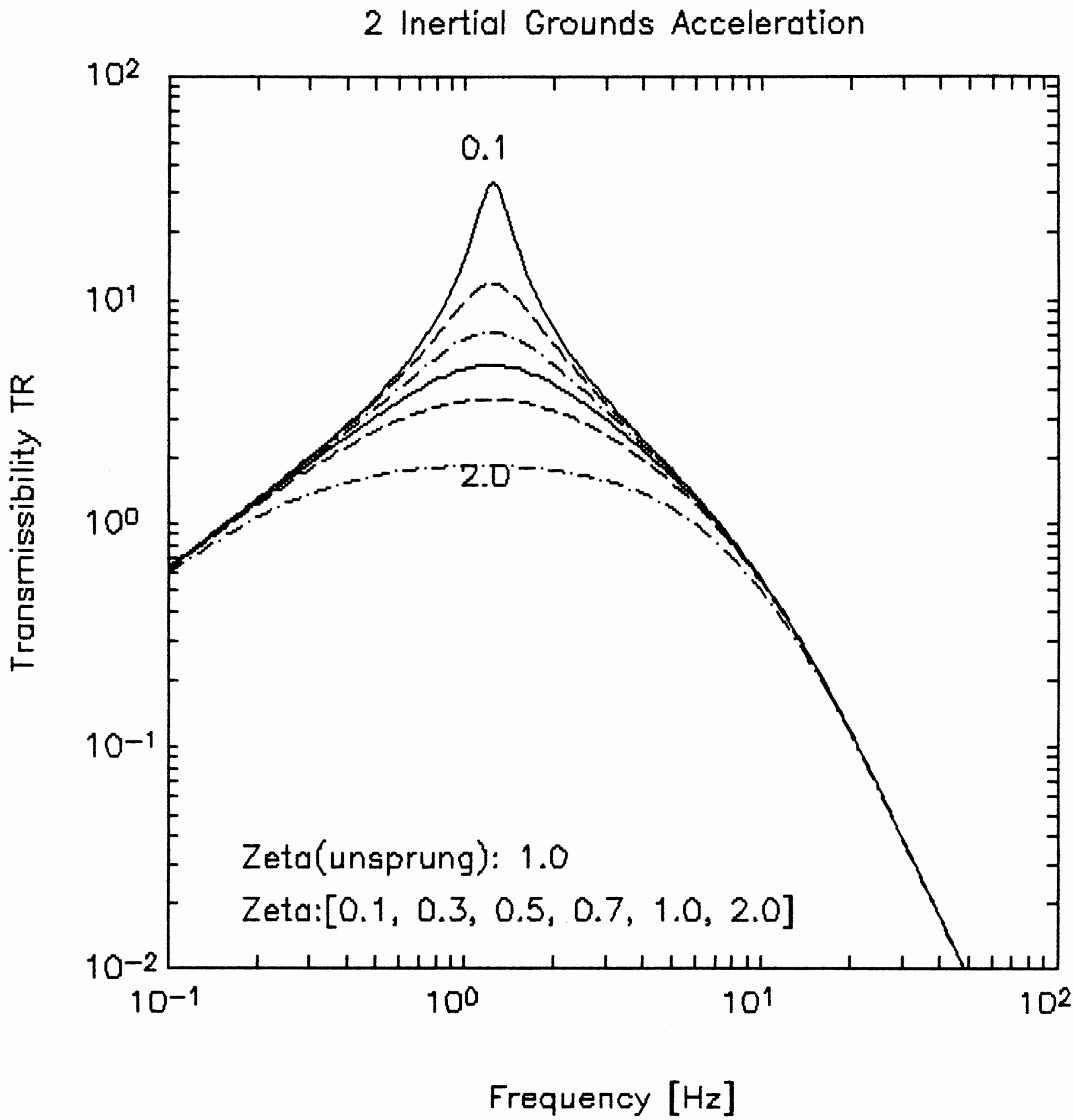

Figure 20. Two Inertial Grounded Damping System Frequency Response for Body Acceleration 3

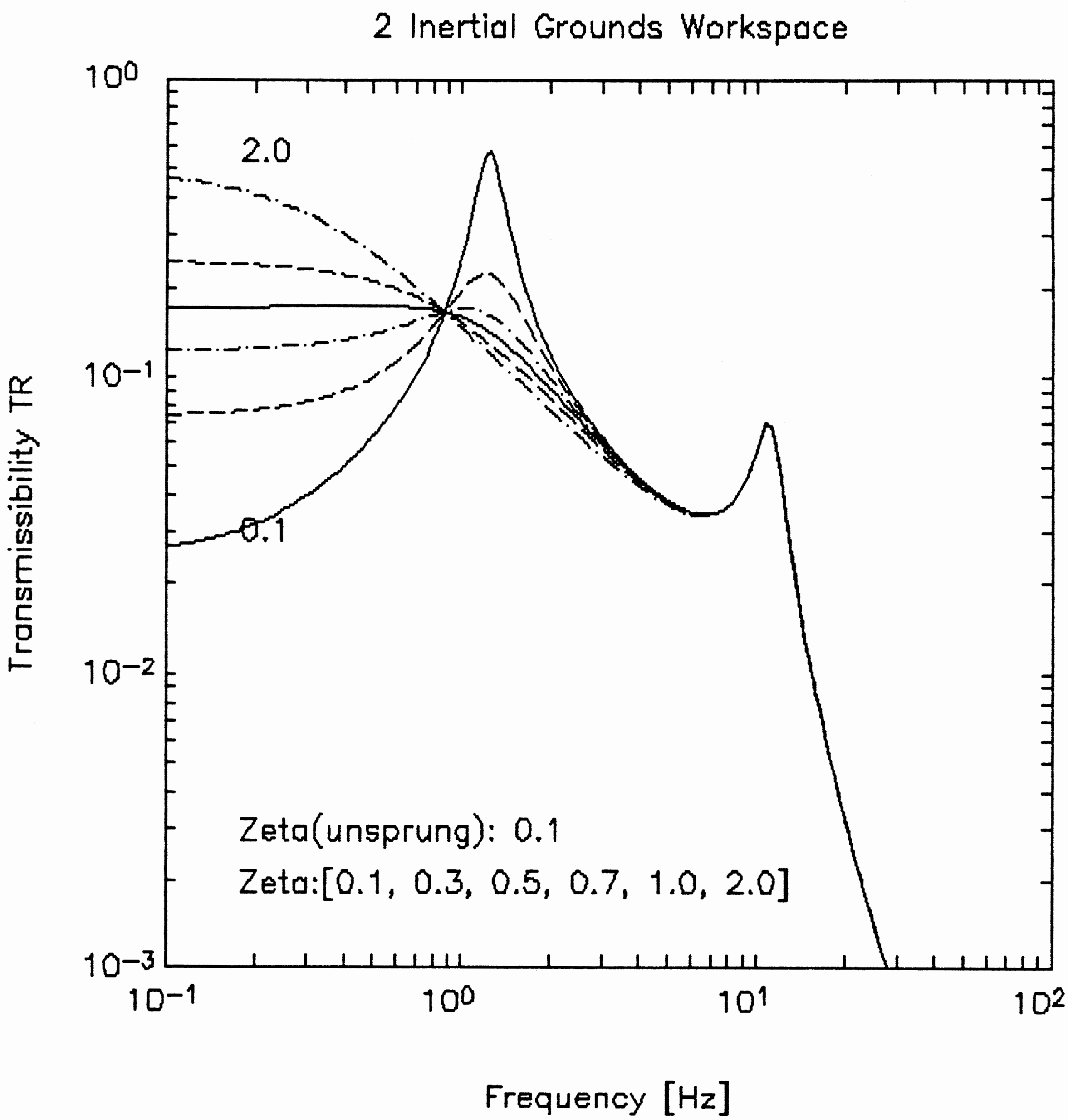

Figure 21. Two Inertial Grounded Damping System Frequency Response for Workspace 1

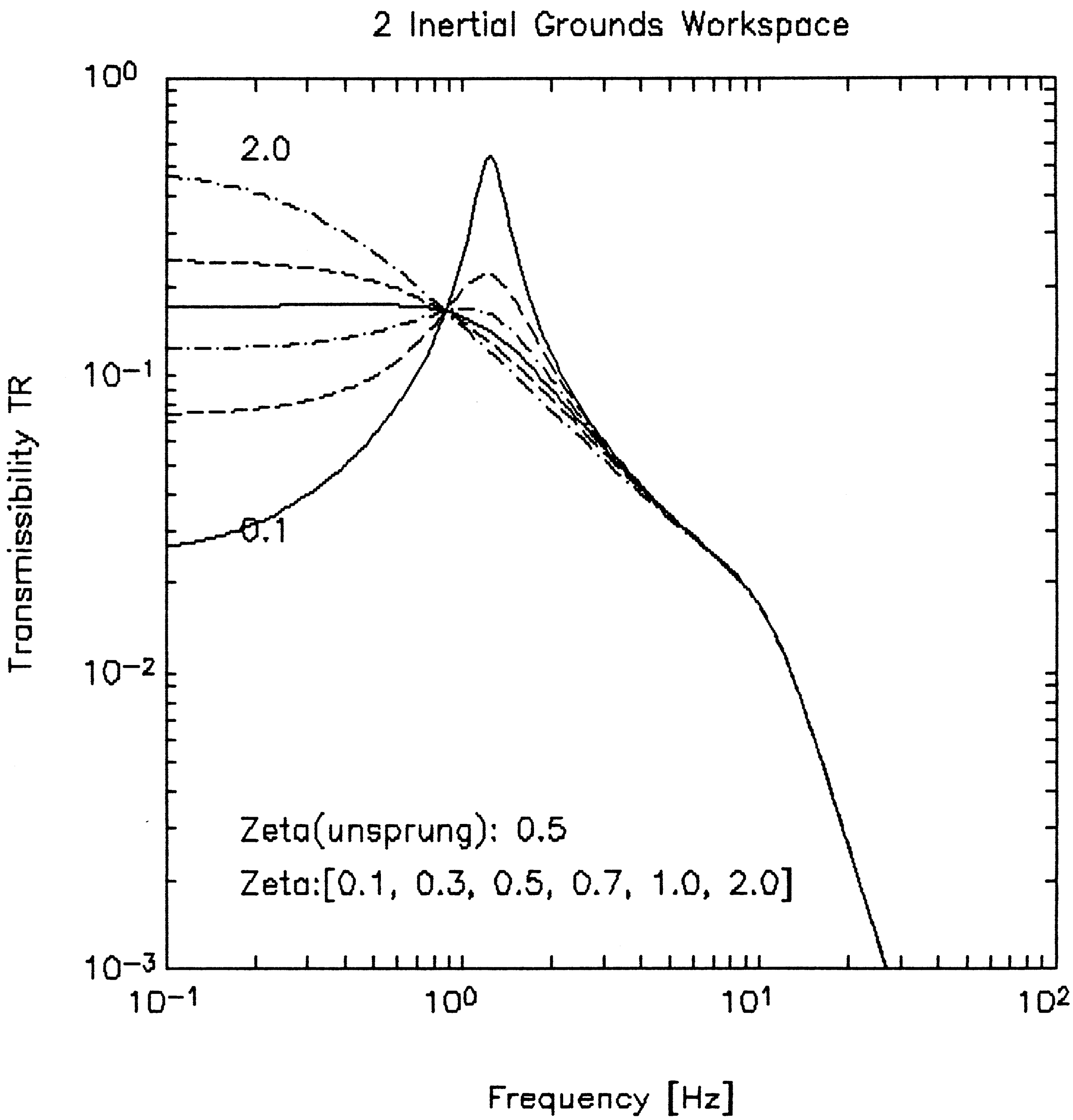

Figure 22. Two Inertial Grounded Damping System Frequency Response for Workspace 2

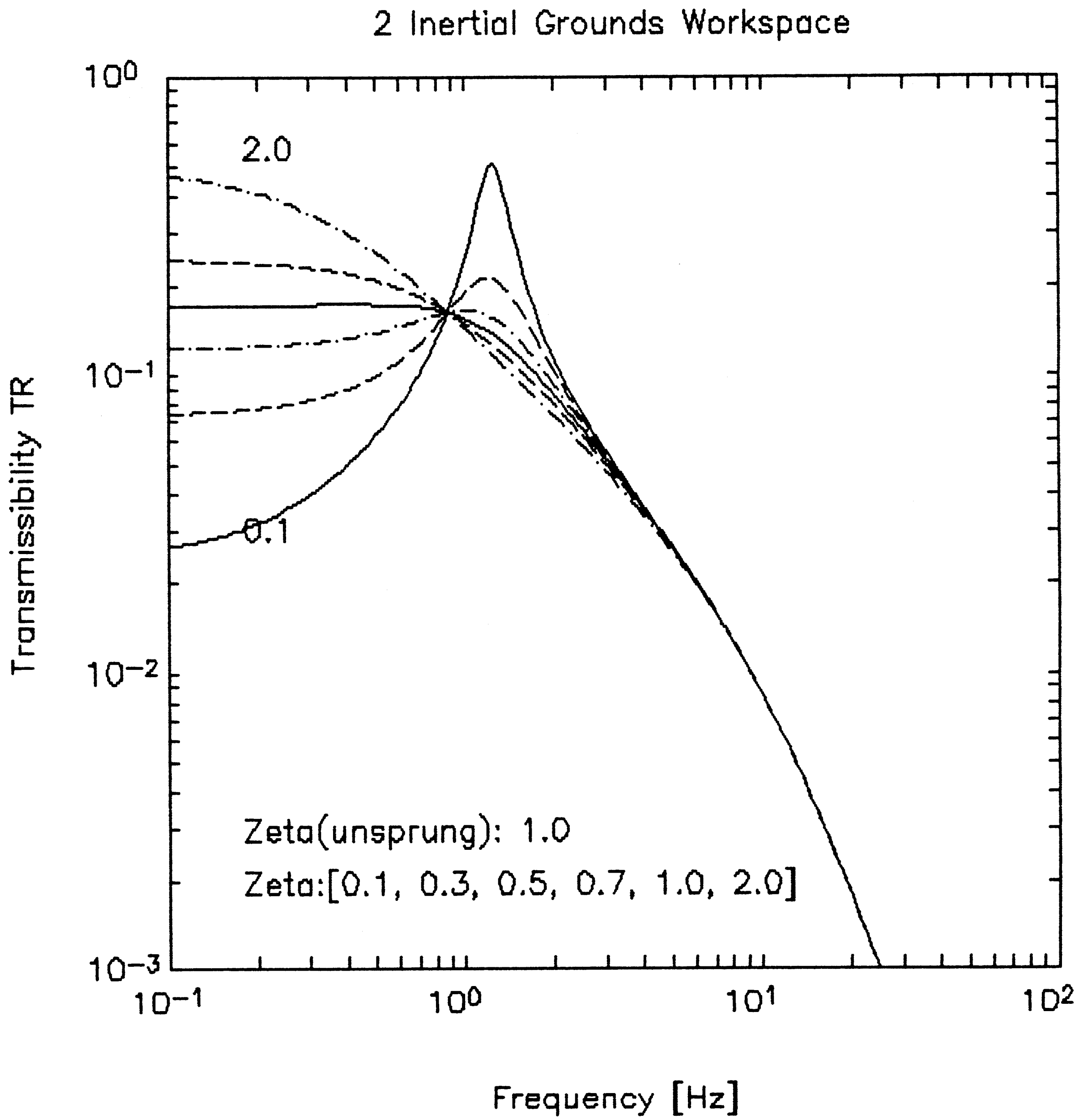

Figure 23. Two Inertial Grounded Damping System Frequency Response for Workspace 3

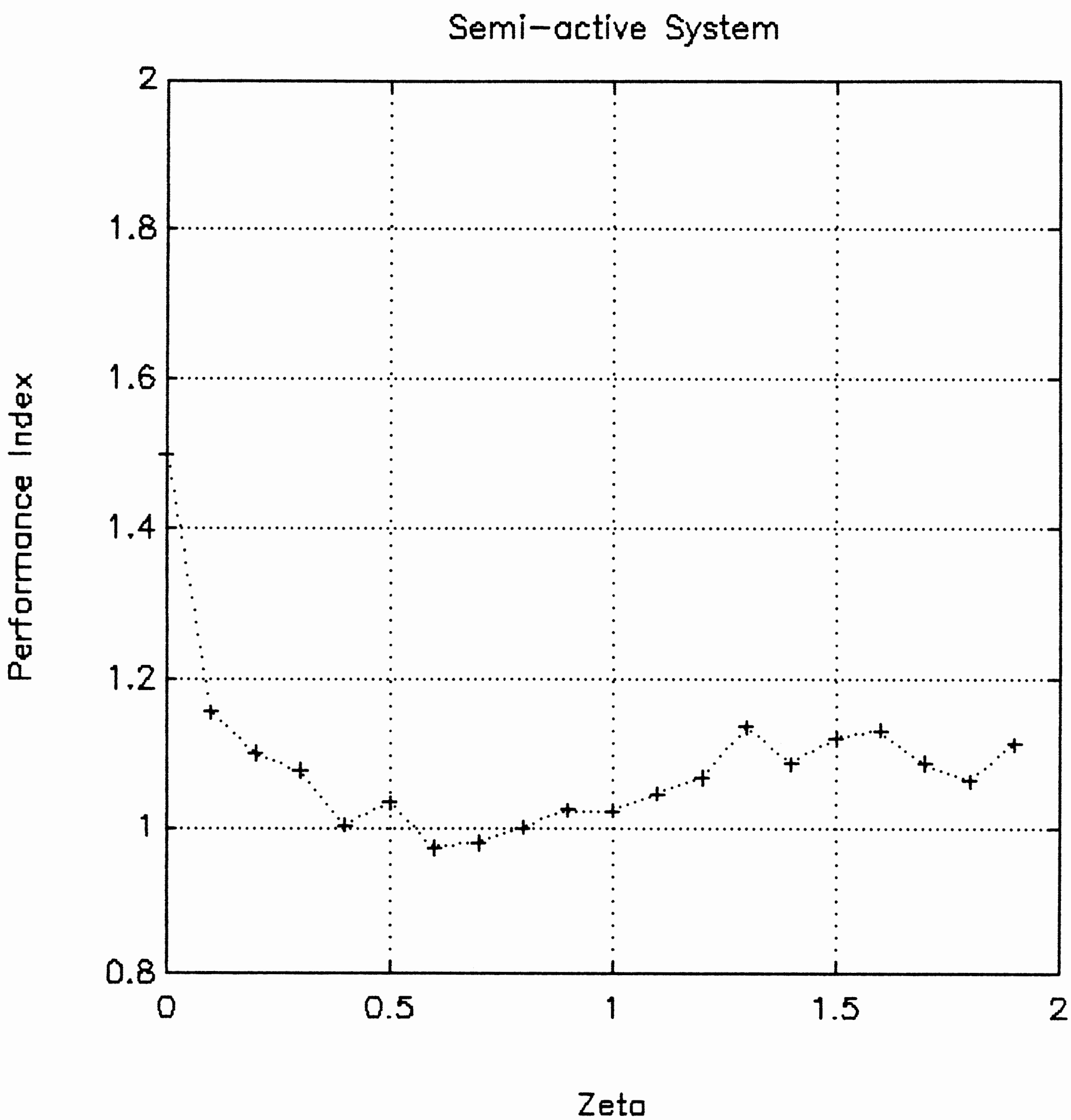

Figure 24. Performance Index for Semi-active System

APPENDIX C

PASCAL PROGRAM

```
{*******************************************************}
{                                                       }
{  Mechanical & Aerospace Engineering                   }
{  Oklahoma State University                            }
{  Stillwater, OK 74075                                 }
{                                     Date : 03/11/91   }
{                                     Programmed by     }
{                                     Minsup Lee        }
{                                                       }
{   Two Degree-of-Freedom Vehicle Suspension Simulation. }
{  ==================================================== }
{                                                       }
{  Procedures                                           }
{  1.Heading         : Program Heading                  }
{  2.Analysis_Input: Input the System Enviornments      }
{  3 Property        : Assign the System Properties     }
{  4.ODE1            : Ordinary Differential Eqution 1  }
{  5.ODE2            : Ordinary Differential Eqution 2  }
{  6.SineExcitation: Sinusoidal Excitation              }
{  7.Roughroad       : Roughroad Excitaiton             }
{  8.SawTooth        : Sawtooth Type Road Excitation    }
{  9.Damping_Scheme: Damping Scheme Selector            }
{ 10.Square          : Squaring variables               }
{ 11.RMS             : Root-Mean-Square Calculator      }
{ 12.RungeKutta      : Calculation Next-Step-Values     }
{ 13.RK_Loop         : Runge-Kutta, etc. Iterator       }
{                                                       }
{ Note : Real time by second can be input.              }
{                                                       }
{*******************************************************}

PROGRAM TwoDOF(infile,outfile);

CONST
     PI   = 3.141592654;
```

```
     Ten_Per_Decade = 1.258925412;
     NO   = 0;
     YES = 1;
     UPPER_LIMIT = 30.0; {Highest interesting frequency
                          ratio is 15.}

VAR
     infile,outfile : TEXT;
     LIMIT,road,i : INTEGER;
     scheme,frequency_analysis,root_mean_square : INTEGER;
     non_linear_scheme,time_response : INTEGER;

     FINAL_TIME,max_height : REAL;
     X0,Y0,U0,V0,wn,c_cubic,c_tension,c_compression :REAL;
     mass1,mass2,damper,spring1,spring2,cheta : REAL;
     rms_x,rms_y,rms_z,rms_v,rms_a,rms_j       : REAL;
     freq_ratio,vehicle_speed,k1,k2            : REAL;
     step_size,TR_x,TR_y,TR_z,TR_v,TR_a,TR_j   : REAL;

{----------------------------------------------------------}
PROCEDURE Heading;

VAR
     i :INTEGER;
begin
     for i := 1 to 25 do writeln('');
     for i := 1 to 57 do write('*');writeln('*');
     write('Two Degree-of-Freedom Vehicle Suspension ');
     writeln('System Simulation');
     writeln('Programmed by Minsup Lee.');
     for i :=1 to 57 do write('*');writeln('*');
     writeln(' ');
end;

{----------------------------------------------------------}
```

```
PROCEDURE Analysis_Input;

VAR
     semi_active,passive,non_linear,i          :INTEGER;
     LOWER_LIMIT : REAL;
     {LIMIT,step_size,scheme,frequency_analysis,road,
     X0,Y0,U0,V0:GLOBAL}

begin
     scheme := 0; non_linear_scheme := 0;
     frequency_analysis := 0; cheta := 0;

     passive     := 1;
     non_linear  := 2;
     semi_active := 3;
     LOWER_LIMIT := 0.1;{Lowest interesting frequency
                         ratio is 0.1}

     writeln('');
     writeln('Input the step size. [i.e. 0.01]');
     readln(step_size);
     writeln('');
     writeln('Input the final time [sec]. [i.e. 10]');
     readln(FINAL_TIME);
     writeln('');
     writeln('Input initial conditions -X0 Y0 U0 V0-.');
     readln(X0,Y0,U0,V0);

     writeln('');
     writeln('Do you want the frequency analysis?');
     writeln('If yes input 1, otherwise 0');
     readln(frequency_analysis);

     if frequency_analysis = NO then begin
        writeln('');
```

```
   writeln('Input vehicle speed [km/hr]. [i.e. 80]');
   readln(vehicle_speed);

   writeln('');
   writeln('Input road condition.');
   writeln('1 : Sawtooth Shape Road');
   writeln('2 : Sinusoidal Road');
   writeln('3 : Rough Road');
   readln(road);

   while NOT((road = 1) OR (road = 2) OR (road = 3))
   do begin
      writeln('');
      writeln('Invalid Input!!!');
      write('INPUT : 1 (sawtooth), 2 (sine)');
      writeln(' 3 (rough)');
      readln(road)
   end;
end;

if frequency_analysis = YES then begin
   road := 2;
   freq_ratio := LOWER_LIMIT
end
else if frequency_analysis = NO then begin
   freq_ratio := LOWER_LIMIT;
end;

writeln('');
writeln('Input damping condition.');
writeln('1 : Passive Control');
writeln('2 : Nonlinear Damping');
writeln('3 : Semi-Active Control');
readln(scheme);
```

```
     if scheme = non_linear then begin
        writeln('');
        writeln('1 : Cubic damping');
        writeln('2 : Asymmetric damping');
        writeln('3 : Displacement dependent damping');
        readln(non_linear_scheme);
     end;
     writeln('');
     while NOT((scheme=1) OR (scheme=2) OR (scheme=3)) do
     begin
        writeln('');
        writeln('Invalid Input!!!');
        write('INPUT : 1 (passive), 2 (nonlinear), ');
        writeln('3 (semi_active)');
        readln(scheme);
     end;

     if scheme = semi_active then begin
        writeln('');
        writeln('Input velocity feedback gains.');
        writeln('For body velocity :');
        readln(k2);
        writeln('For tire velocity :');
        readln(k1);
     end;
end;

{----------------------------------------------------------}
PROCEDURE Property;

     {mass1,mass2,damper,cheta,spring1,spring2,wn:GLOBAL}

begin

     {1/4 Car Model Parameters used by Hedrick or Thompson}
```

```
     mass1 := 36                {28.58};
     mass2 := 240               {Hedrick}{288.9:Thompson};
     {damper := cheta*3920}
                      {980 = damp 0.25,damp 1 = 3920}{1861};
     spring1 := 160000          {155900};
     spring2 := 16000           {1960};
     wn := sqrt(spring2/mass2)

end;

{---------------------------------------------------------}
PROCEDURE ODE1(x,y,vx,vy,m,damper,spring1,spring2,f:REAL;
               VAR ax:REAL);

{ i.e., y represents the deviation from the static        }
{         equilibrium of the sprung mass,                  }
{       x represents samething for the unsprung mass.      }


begin

      ax := f/m +damper*(vy-vx)/m
            -spring1*x/m +spring2*(y-x)/m

end;

{---------------------------------------------------------}
PROCEDURE ODE2(x,y,vx,vy,m,damper,spring1,spring2,f:REAL;
               VAR ay:REAL);

{ Same coordinates as used in PROCEDURE ODE1               }
begin

     {When one degree of freedom is simulated,
      set damper = 0}
```

```
     ay := -damper*(vy-vx)/m -spring2*(y-x)/m

end;

{-----------------------------------------------------------}
PROCEDURE SineExcitation(time:REAL;
                          VAR roughness,force,vr,ar,jr:REAL);

CONST
     phase      = PI/2;

VAR
     w,f,v            : REAL;
     force_amplitude : REAL;
     {spring1,wn,freq_ratio,step_size,max_heght:GLOBAL}

begin

     max_height := 0.006;{m}
     force_amplitude := spring1*max_height;

     w := wn * freq_ratio;

     roughness := max_height * sin(w*time + phase)
                + max_height;
     vr :=  max_height * w * cos(w*time + phase);
     ar := -max_height * w * w * sin(w*time + phase);
     jr := -max_height * w * w * w * cos(w*time + phase);

     force := force_amplitude * sin(w*time + phase);

end;

{-----------------------------------------------------------}
```

```
PROCEDURE RoughRoad(time:REAL;
                    VAR roughness,force,vr,ar,jr:REAL);

CONST
     wavelength = 5.5556;{m}
     max_height = 0.05;{m}

VAR
     w,f,v                          : REAL;
     ph_1,ph_2,ph_3,ph_4,ph_5       : REAL;
     force_amplitude                : REAL;
     {spring1,wn,freq_ratio,step_size:GLOBAL}

begin

     v := vehicle_speed/3.6;
     f := v/wavelength;
     w := 2*PI*f;
     ph_1 :=  -PI/6;
     ph_2 :=   PI/2;
     ph_3 :=   PI/6;
     ph_4 :=  -PI/2;
     ph_5 :=      0;

     roughness :=  2 * max_height * sin(0.5*w*time + ph_1)
                +      max_height * sin(    w*time + ph_2)
            + (1/5)  * max_height * sin(  5*w*time + ph_3)
            + (1/10) * max_height * sin( 10*w*time + ph_4)
            + (1/15) * max_height * sin( 15*w*time + ph_5);

     force := roughness * spring1;

end;

{----------------------------------------------------------}
```

```
PROCEDURE SawTooth(time:REAL;
                        VAR roughness,force,vr,ar,jr:REAL);

CONST
    Slab_h     = 0.006 {m};{road input}
    wavelength = 6.3   {m};{road input}

VAR
     roadx,h,w,t,f,v : REAL;
     {step_size,spring1:GLOBAL}

begin

     h := Slab_h;
     v := vehicle_speed/3.6; {m/sec}
     f := v/wavelength;
     w := 2*PI*f;
     t := time;

     roadx := h/2  - h/PI*sin(w*t)  - h/( 2*PI)*sin( 2*w*t)
            - h/( 3*PI)*sin( 3*w*t) - h/( 4*PI)*sin( 4*w*t)
            - h/( 5*PI)*sin( 5*w*t) - h/( 6*PI)*sin( 6*w*t)
            - h/( 7*PI)*sin( 7*w*t) - h/( 8*PI)*sin( 8*w*t)
            - h/( 9*PI)*sin( 9*w*t) - h/(10*PI)*sin(10*w*t)
            - h/(11*PI)*sin(11*w*t) - h/(12*PI)*sin(12*w*t)
            - h/(13*PI)*sin(13*w*t) - h/(14*PI)*sin(14*w*t)
            - h/(15*PI)*sin(15*w*t) - h/(16*PI)*sin(16*w*t)
           - h/(17*PI)*sin(17*w*t) - h/(18*PI)*sin(18*w*t);

     roughness := roadx;
     force := roughness * spring1;

end;

{-----------------------------------------------------------}
```

```
PROCEDURE Damping_Scheme(vx,vy,vz:REAL);
VAR
     passive,non_linear,semi_active :INTEGER;
     cubic,asymmetric,position      :INTEGER;
     power,sa_cheta                 :REAL;
     {freq_ratio,scheme,cheta,damper:GLOBAL}
begin
     passive    := 1;  non_linear  := 2;  semi_active := 3;

     cubic      := 1;  asymmetric  := 2;  position    := 3;

     if scheme = non_linear then begin
        if non_linear_scheme = cubic then
           cheta := c_cubic*vz*vz;
        if non_linear_scheme = asymmetric then
           if vz <= 0 then cheta := c_compression
           else cheta := c_tension;
     end;

     if scheme = semi_active then begin
        power := vy*vz;

        if power = 0 then cheta := 0
        else sa_cheta := k2*vy/vz;

        if power > 0 then cheta := sa_cheta
        else if power < 0 then cheta := 0;

        if cheta < 0 then cheta := 0
        else if cheta > 2 then cheta := 2;
     end;
     damper := cheta*3920;

end;

{---------------------------------------------------------}
```

```
PROCEDURE Square(step_size,x,y,z,vy,ay,jy:REAL;
                 VAR sq_x,sq_y,sq_z,sq_v,sq_a,sq_j:REAL);

      {step_size:GLOBAL}

begin

      sq_x :=   x*x * step_size;
      sq_y :=   y*y * step_size;
      sq_z :=   z*z * step_size;
      sq_v := vy*vy * step_size;
      sq_a := ay*ay * step_size;
      sq_j := jy*jy * step_size

end;

{----------------------------------------------------------}

PROCEDURE RMS(sum_of_square,
              final_time,start_time : REAL;
              VAR root_mean_square : REAL);

VAR
     t,t0 : REAL;

begin

     t0 := start_time;
     t  := final_time;

     root_mean_square := sqrt(sum_of_square/(t-t0));

end;

{----------------------------------------------------------}
```

```
PROCEDURE RungeKutta(time,d,x,y,vx,vy,ay:REAL;
                     VAR xn,yn,zn,vxn,vyn,ayn,jyn,vzn,
                         roughness,vr,ar,jr:REAL);

VAR
     dx,dy          :REAL;
     dvx,dvy        :REAL;
     old_ay         :REAL;

     n1,n2,n3,n4,p1,p2,p3,p4          :REAL;
     l1,l2,l3,l4,q1,q2,q3,q4          :REAL;
     fx1,fx2,fx3,fx4,fy1,fy2,fy3,fy4  :REAL;
     force                            :REAL;
     {road,mass1,mass2,spring1,spring2,damper:GLOBAL}

begin

     if road = 1 then
        SawTooth(time,roughness,force,vr,ar,jr)
     else if road = 2 then
        SineExcitation(time,roughness,force,vr,ar,jr)
     else if road = 3 then
        Roughroad(time,roughness,force,vr,ar,jr);

     old_ay := ay;

     n1 := x;
     l1 := vx;
     p1 := y;
     q1 := vy;

     ODE1(n1,p1,l1,q1,mass1,damper,spring1,spring2,
          force,fx1);
     ODE2(n1,p1,l1,q1,mass2,damper,spring1,spring2,
          force,fy1);
```

```
n2 := x + l1*d/2;;
l2 := vx + fx1*d/2;
p2 := y + q1*d/2;
q2 := vy + fy1*d/2;
ODE1(n2,p2,l2,q2,mass1,damper,spring1,spring2,
     force,fx2);
ODE2(n2,p2,l2,q2,mass2,damper,spring1,spring2,
     force,fy2);

n3 := x + l2*d/2;;
l3 := vx + fx2*d/2;
p3 := y + q2*d/2;
q3 := vy + fy2*d/2;
ODE1(n3,p3,l3,q3,mass1,damper,spring1,spring2,
     force,fx3);
ODE2(n3,p3,l3,q3,mass2,damper,spring1,spring2,
     force,fy3);

n4 := x + l3*d;;
l4 := vx + fx3*d;
p4 := y + q3*d;
q4 := vy + fy3*d;
ODE1(n4,p4,l4,q4,mass1,damper,spring1,spring2,
     force,fx4);
ODE2(n4,p4,l4,q4,mass2,damper,spring1,spring2,
     force,fy4);

dx := (l1 + 2*l2 + 2*l3 + l4)/6.0*d;
dy := (q1 + 2*q2 + 2*q3 + q4)/6.0*d;
dvx := (fx1 + 2*fx2 + 2*fx3 + fx4)/6.0*d;
dvy := (fy1 + 2*fy2 + 2*fy3 + fy4)/6.0*d;

xn := x + dx;
yn := y + dy;
zn := yn - xn;
```

```
     vxn := vx + dvx;
     vyn := vy + dvy;
     vzn := vyn - vxn;

     ayn := (fy1 + 2*fy2 + 2*fy3 + fy4)/6.0;
     {ayn := dvy/d;       gives same results as above line}
     jyn := (ayn-old_ay)/d;

end;

{----------------------------------------------------------}
PROCEDURE RK_Loop(VAR rms_x,rms_y,rms_z,rms_v,rms_a,rms_j,
                      TR_x,TR_y,TR_z,TR_v,TR_a,TR_j :REAL);

VAR
     x,y,z      : REAL;
     vx,vy,vz   : REAL;
     ax,ay      : REAL;
     jy         : REAL;
     time,d,w   : REAL;
     vr,ar,jr   : REAL;

     sq_x,sq_y,sq_z,sq_v,sq_a,sq_j : REAL;
     sumx,sumy,sumj,sumz,sumv,suma : REAL;
     roughness                     : REAL;
     skip,YET                      : INTEGER;

     STEADY,START_TIME             : REAL;

     {step_size,LIMIT,freq_ratio,wn,cheta:GLOBAL}

begin

     YET  := 0; {skip lines}
     skip := YET;
```

```
d     := step_size;
STEADY := 2.0;

x := X0;   y := Y0;   vx := U0;   vy := V0;
z := 0;    ay:= 0;    jy :=0;     vz := vy-vx;

time:= 0;

sq_x:=0; sq_y:=0; sq_z:=0; sq_v:=0; sq_a:=0; sq_j:=0;
sumx:=0; sumy:=0; sumz:=0; sumv:=0; suma:=0; sumj:=0;
rms_x:=0;rms_y:=0;rms_z:=0;rms_v:=0;rms_a:=0;rms_j:=0;

w := wn * freq_ratio;

while time <= (FINAL_TIME -step_size) do begin
    Damping_Scheme(vx,vy,vz);

    RungeKutta(time,d,x,y,vx,vy,ay,
             x,y,z,vx,vy,ay,jy,vz,roughness,vr,ar,jr);

    time := time + d;
    if (time<STEADY+step_size) and (time>=STEADY) then
       START_TIME := time;

    if (time_response=Yes) AND (time>=STEADY)
       then
       writeln(time:6:3,' ',x:11,' ',y:11,
             ' ',z:11,' ',vy:11,' ',ay:11,' ',jy:11);

    if (time>=STEADY) then
        skip := skip + 1;

    if time >= STEADY then begin
       Square(d,x,y,z,vy,ay,jy,
              sq_x,sq_y,sq_z,sq_v,sq_a,sq_j);
```

```
          sumx := sumx + sq_x;
          sumy := sumy + sq_y;
          sumz := sumz + sq_z;
          sumv := sumv + sq_v;
          suma := suma + sq_a;
          sumj := sumj + sq_j
       end;

    end;
    FINAL_TIME := time;

    RMS(sumx,FINAL_TIME,START_TIME,rms_x);
    RMS(sumy,FINAL_TIME,START_TIME,rms_y);
    RMS(sumz,FINAL_TIME,START_TIME,rms_z);
    RMS(sumv,FINAL_TIME,START_TIME,rms_v);
    RMS(suma,FINAL_TIME,START_TIME,rms_a);
    RMS(sumj,FINAL_TIME,START_TIME,rms_j);

    if frequency_analysis = YES then begin
       {Input : the velocity of road roughness}

       TR_x := rms_x/(max_height*w/sqrt(2));
       TR_y := rms_y/(max_height*w/sqrt(2));
       TR_z := rms_z/(max_height*w/sqrt(2));
       TR_v := rms_v/(max_height*w/sqrt(2));
       TR_a := rms_a/(max_height*w/sqrt(2));
       TR_j := rms_j/(max_height*w/sqrt(2));

       writeln(outfile,'  ',freq_ratio:5:2,'  ',
               TR_x:6:3,'  ',TR_y:6:3,'  ',TR_z:6:3,'  ',
               TR_v:6:3,'  ',TR_a:6:3,'  ',TR_j:6:3);
    end;

end;

{-----------------------------------------------------------}
```

```
begin
       assign(outfile,'fout.out');
       rewrite(outfile);

       Heading;
       Analysis_Input;

       writeln('');

       if scheme = 1 then begin
          writeln('Input cheta.');
          readln(cheta);
       end
       else if scheme = 2 then begin
            if non_linear_scheme = 1 then begin
               writeln('Input cheta for cubic damping.');
               readln(c_cubic)
               end
            else if non_linear_scheme = 2 then begin
                 writeln('Input cheta for compression.');
                 readln(c_compression);
                 writeln('Input cheta for tension.');
                 readln(c_tension);
            end
       end;

       writeln('');
       writeln('Do you want RMS values?');
       writeln('Input 1 for yes, 0 for no.');
       readln(root_mean_square);
       writeln;
       writeln('Do you want time responses?');
       writeln('Input 1 for yes, 0 for no.');
       readln(time_response);
```

```
        for i := 1 to 25 do writeln('');
        writeln('     Calculation is started.');
        write('     Results will be stored in the file');
        writeln(' named "fout."');

        while freq_ratio <= UPPER_LIMIT do begin
              Property;

              RK_Loop(rms_x,rms_y,rms_z,rms_v,rms_a,rms_j,
                      TR_x,TR_y,TR_z,TR_v,TR_a,TR_j);

              if (root_mean_square = YES)
                  then
                 writeln(outfile,rms_z:11,' ',rms_a:11,
                 ' ',rms_j:11);

              if frequency_analysis = YES then
                 freq_ratio := freq_ratio * Ten_Per_Decade
              else if frequency_analysis = NO then
                 freq_ratio := UPPER_LIMIT+1;

        end;
        close(outfile);
end.
```

VITA

MINSUP LEE

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Master of Science

Thesis: COMPARISON OF THE PERFORMANCE OF PASSIVE AND SEMI-ACTIVE SUSPENSION SYSTEMS

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