

RESISTIVITIES OF WATER IN
SELECTED FORMATIONS,
NORTH-CENTRAL
OKLAHOMA

By

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PREFACE

This thesis is an empirical study of the accuracy of artificially derived formation-water resistivities, as compared to the assumed "true" resistivity value of a produced water sample. In addition, this study presents a significant data base of produced-sample resistivity values, enabling the author to arrive at reliable estimates of mean formation-water resistivities for five geologic formations.

The author is grateful to all those individuals and companies who provided assistance and encouragement. Dr. Gary F. Stewart suggested and supervised the investigation, and Dr. Zuhair Al-Shaieb and Dr. J. Leroy Folks served on the author's committee. Data collection was aided by Ms. Dee Ann Dickerson, Sun Production Company; Mr. Joel Nelson of Wil-Mc Oil Corporation; Mr. V. Brown Monnett of Thomas N. Berry & Co.; Mr. Larry Hays of Settlers Energy Corporation; Mr. Dave Jackson of Three Sands Oil; Mr. Pete Mackellar of Mackellar Services; Mr. Tom D. Berry of Berry Operating Company; Dr. Tom Rowland, consulting geologist; and Mr. James Puckette of Ketal Oil Producing Company. Technical assistance was provided by Mr. Johnny Reinschmidt, Sun Production Company.

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CHAPTER I

INTRODUCTION

The need for reliable measurements of formation-water resistivities for well-log analysis has led to considerable research on and experimentation with formation waters. However, most studies deal with effects on resistivity of salts and other impurities dissolved in the solution, calculation of a "true" water resistivity from chemical-composition data, or similar and theoretical problems. In this study an attempt was made to determine the accuracy of log-based methods of calculating water-resistivity, by comparison of log-derived estimates with measurements from samples of formation water. Results of this study add to the data base for exploration and production within the study area.

The general area of study mostly is in Payne County, Oklahoma, but includes portions of Kay, Logan, and Noble Counties, encompassing approximately 60 townships (Figure 1). Five major producing lithologic intervals were studied: the Skinner, Red Fork and Bartlesville sandstones, the "Mississippi Chat" Zone, and the "Mississippi Limestone." A type log of the study area is shown as Figure 2; important stratigraphic units are designated. With excep-

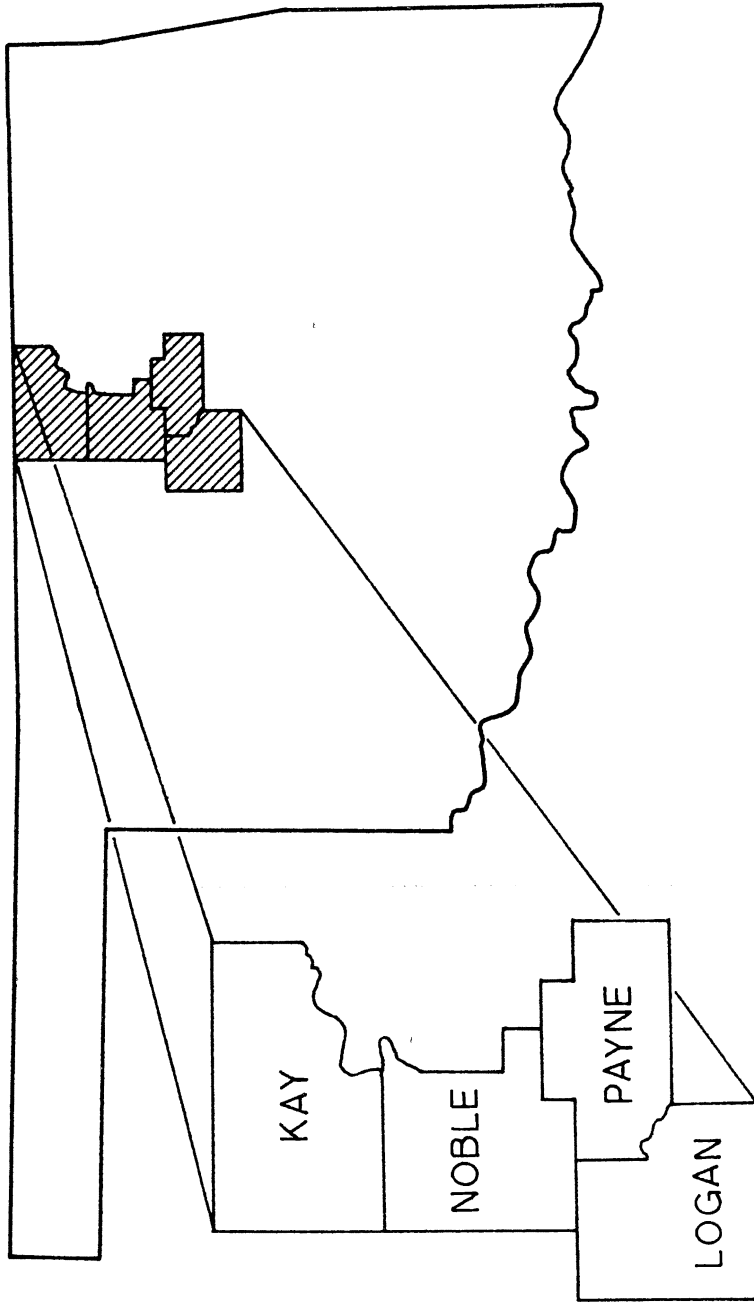


Figure 1. General Location of Study Area

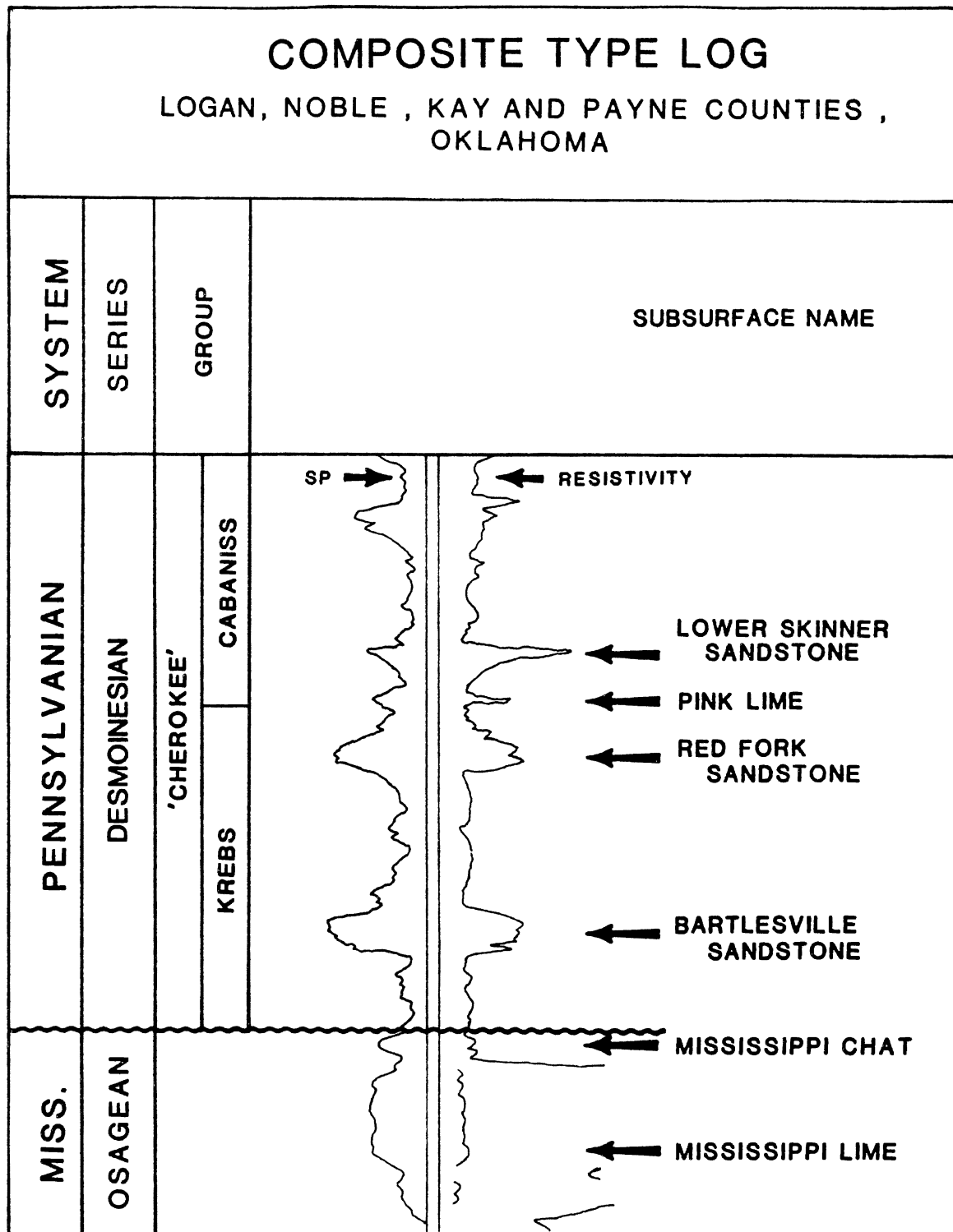


Figure 2. Type Log, Rock-stratigraphic Units of Mississippian and Middle Pennsylvanian Age

tion of the "Mississippi Lime" and the "Chat," all lithostratigraphic units studied are Pennsylvanian sandstones of the "Cherokee Group."

Figures 3 through 7 are well-locality maps for these intervals, showing the locations of wells from which water samples were obtained. Index numbers of wells, names of wells and well-identification, and locations of wells are in Appendix A.

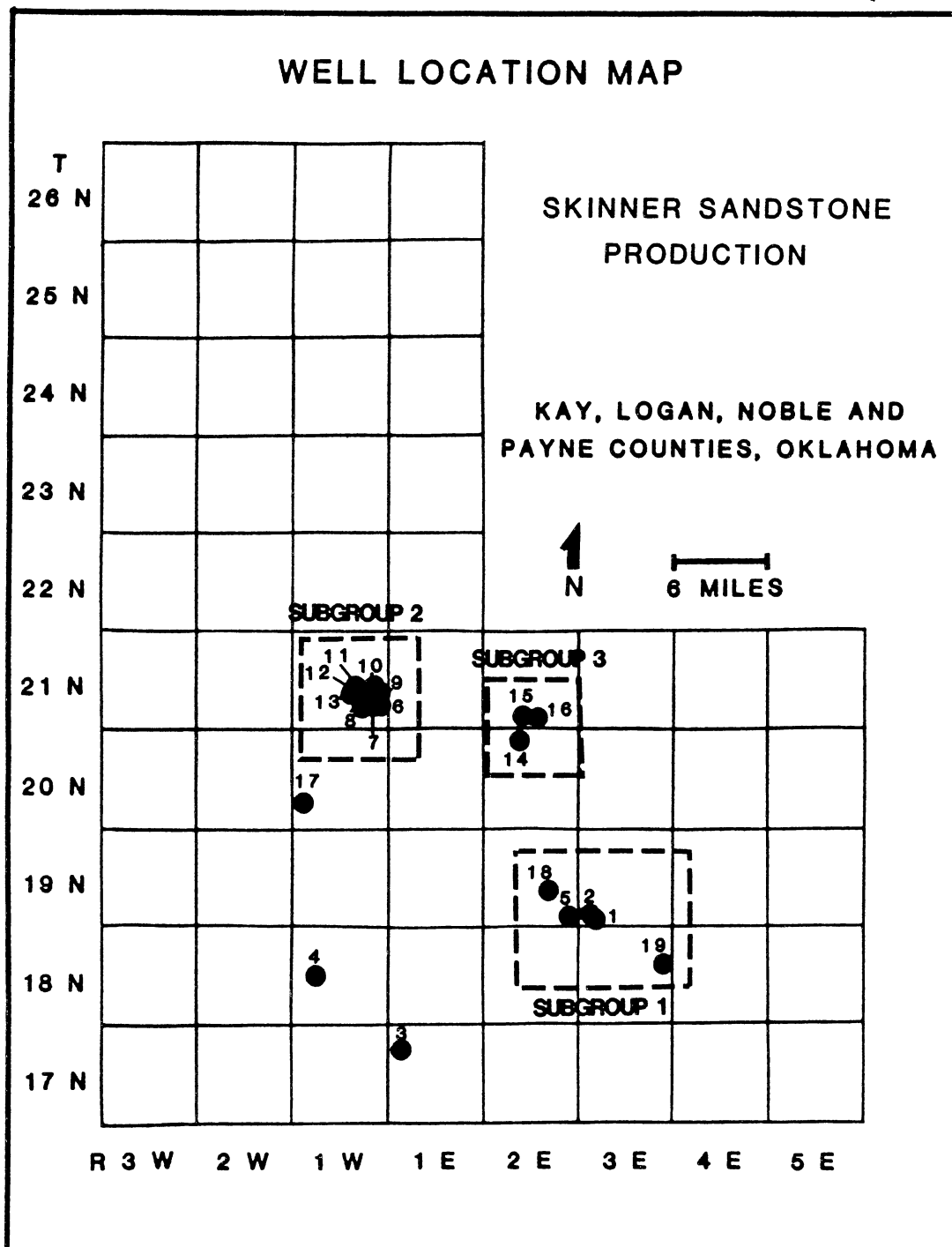


Figure 3. Locations of 19 Wells From Which Estimates Were Made of R_w , Skinner Sandstone

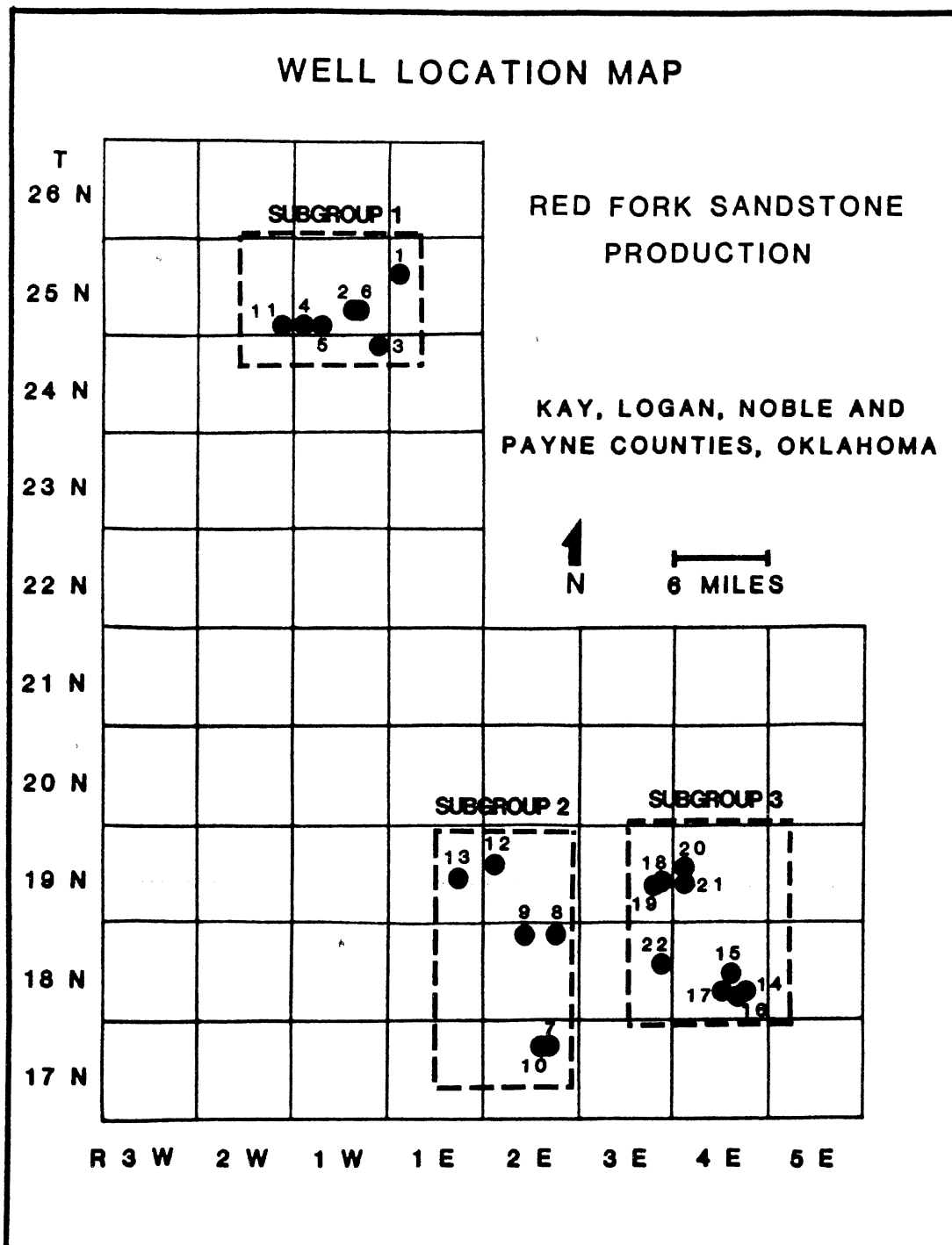


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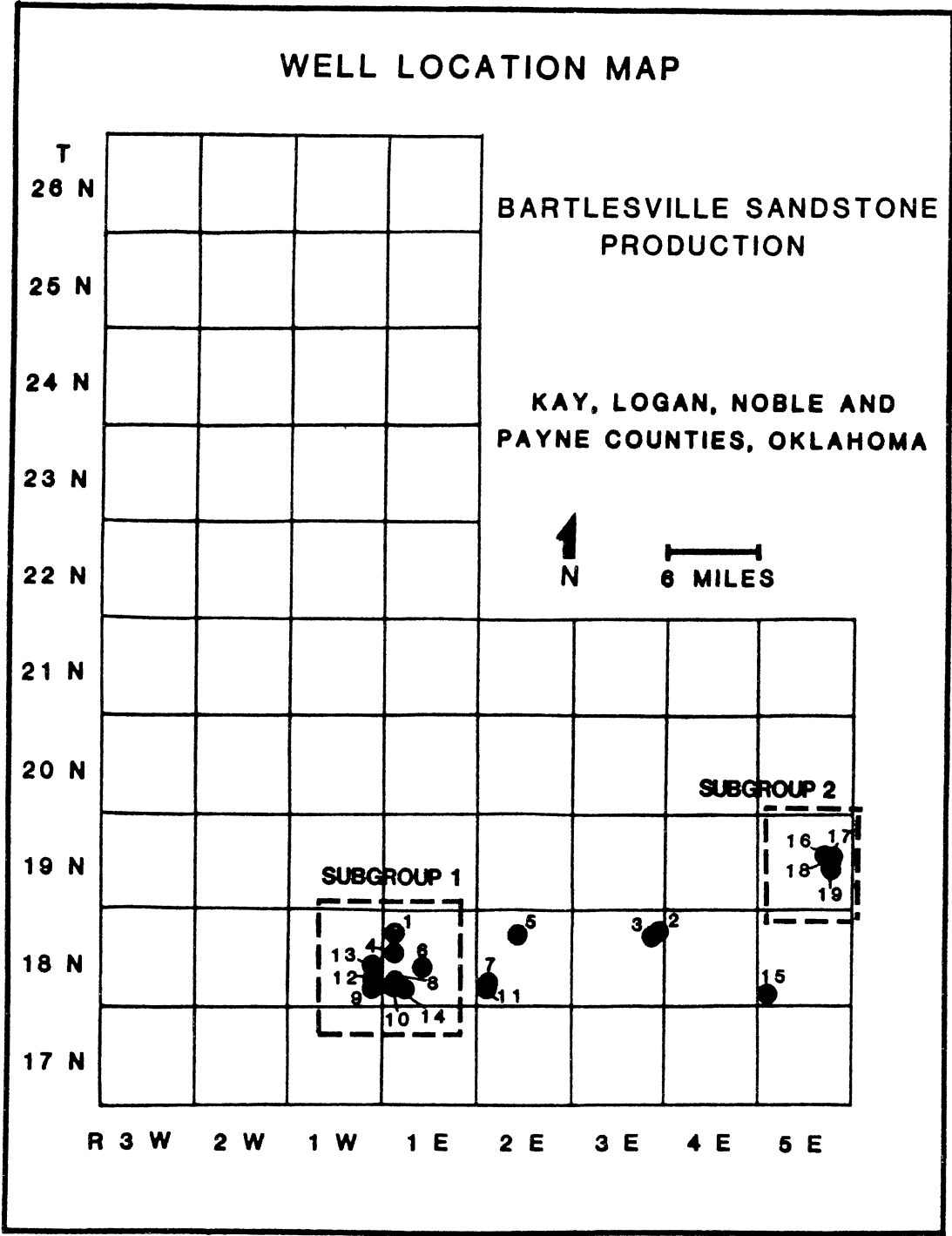


Figure 5 Locations of 19 Wells From Which Estimates Were Made of Rw, Bartlesville Sandstone

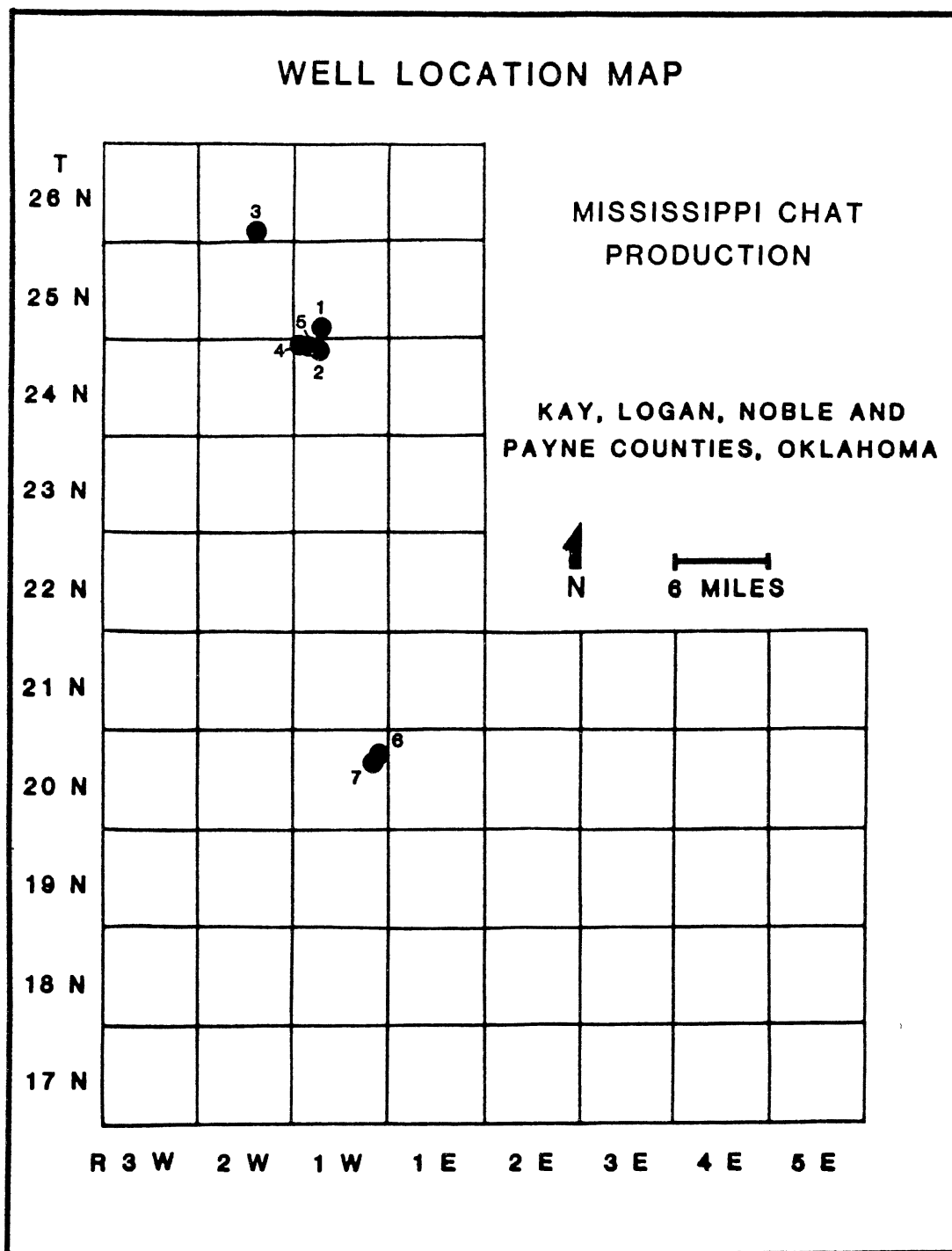


Figure 6. Locations of Seven Wells From Which Estimates Were Made of Rw, "Mississippi Chat"

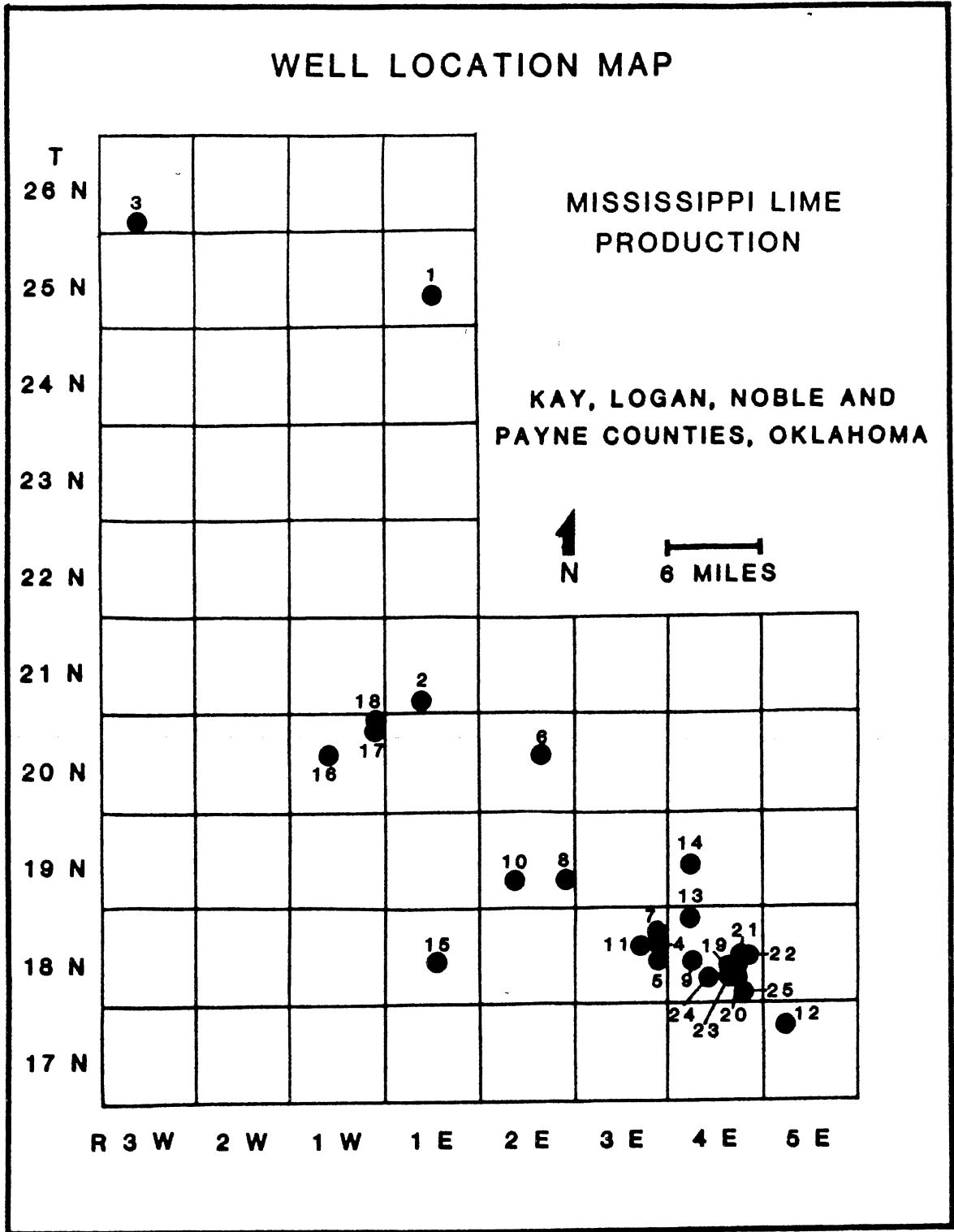


Figure 7 Locations of Seven Wells From Which Estimates Were Made of R_w, "Mississippi Lime"

CHAPTER II

IMPORTANCE OF ACCURATE ESTIMATION OF R_w

One of the most important uses of the formation-water resistivity value, R_w , is in the determination of petroleum saturation. The fundamental relationship between R_w and petroleum saturation is illustrated by the Archie Equation (Asquith, 1982):

$$S_w^n = \frac{F \times R_w}{R_t}$$

S_w = Water saturation, measured in percent.

n = Saturation exponent, generally approximated as 2.

where F = Formation Resistivity Factor, generally approximated as

= $1/\phi^2$ for limestones and dolomites, and

= $0.81/\phi^2$ for consolidated sandstones where ϕ symbolizes porosity, measured in percent, expressed as a decimal fraction.

R_w = Resistivity of formation water, at temperature of the formation.

R_t = True resistivity, as approximated commonly from the deep induction curve of induction-electric logs.

Petroleum saturation, or $S_{O\&g}$, is calculated as $1-S_w$. As this equation is the most commonly used method to make a preliminary estimate of reservoir potential, it is imperative that all factors be as accurate as practicable.

An example of the effect of the R_w variable is illustrated by the following case. Assuming that for a limestone, porosity (ϕ) and deep resistivity (R_t) values read directly and (correctly) from logs, are:

$$\phi = 0.12$$

$$R_t = 5 \text{ ohm-m}$$

and an estimated R_w value of 0.03 ohm-m is used, the $S_{O\&g}$ is determined as follows:

$$F = 1/\phi^2 = 1/0.12^2 = 69.4$$

$$S_w^2 = \frac{69.4 \times 0.03 \text{ ohm-m}}{5 \text{ ohm-m}}$$

$$S_w = 64.5\%$$

$$S_{O\&g} = 1-S_w = 35.5\%$$

If the R_w value of 0.03 ohm-m is an estimate and the true but unknown R_w value is 0.04 ohm-m, then the true $S_{O\&g}$ would be:

$$S_w^2 = \frac{69.4 \times 0.04 \text{ ohm-m}}{5 \text{ ohm-m}}$$

$$S_w = 74.5\%$$

$$S_{O\&g} = 1-S_w = 25.5\%$$

This difference in estimated petroleum saturation could significantly affect subsequent calculations of whether the well would produce oil and gas in profitable quantities. In this example, the R_w values differed only by 0.01 ohm-m, but larger divergence between truth and estimate seems to be rather common.

Methods

Essentially three processes were involved in this investigation: (1) direct water sampling and measurement; (2) calculation of water resistivity by the Spontaneous-potential Method; and (3) calculation of water resistivity by the R_{wa} method, or Apparent Water Resistivity Method.

Direct Water Sampling and Measurement

All water samples were taken from producing wells. In order to obtain a valid sample, the well must be producing from a single, known interval. This information is obtained best from the operating company, as recompletions and perforation of additional zones are common practices, but such "workovers" are not always recorded publicly or reported to the proper authorities. Also, one must assume that the casing and cement are sound -- i.e., no significant ground-water or intraformational contamination has occurred.

Sample-site selection is extremely important. The best sample source is directly from a "bleeder" valve at

the wellhead; such a sample, with any oil removed, represents the most accurate, most nearly unadulterated sample obtainable. However, the physical configurations of some wellheads prohibit the taking of such samples. In these cases, the next best sample is obtained by removing a small volume of water from the base of the separator, from the heater-treater, or from one of the several small drain valves located on either of these vessels. Commonly, even this type of sample is unobtainable, especially on an old tank battery. This leaves only the salt-water tank as a sample source. Due to the circumstances described above, some wells were sampled from the salt-water tank. Because most salt-water tanks are not covered, water contained in the tank can be concentrated by evaporation or diluted by rainfall. Whenever possible, wells where water could be obtained only from the salt-water tank were not sampled immediately after a heavy rain or during extended periods of extremely hot, dry weather.

If the main objective of this study had been to investigate the chemical compositions of water samples, the experimental and sampling techniques would have been considerably more rigid. However, as electrical resistivity was the main consideration, the only requirement (other than obtaining a "pure" sample) was physically to measure the resistivity (actually, conductivity) within a reasonably short time period -- in this case approximately 72 hours (Ostroff, 1965). This time-factor is most important

when dealing with high-sulphate high-bacteria waters; in this study area, sulphate content is relatively low, and comparatively little change in resistivity owing to bacterial action upon sulphates occurs through time. Hence, time relatively is less important in this study are than in some others.

Actual measurement of resistivity of a sample was made with a standard portable conductivity meter. Resistivity is obtained easily of course, as resistivity is the inverse of conductivity. Because of the limited upper range of the conductivity meter, samples were diluted by a ratio of 1:40 (sample to deionized, distilled dilution water.)

Inevitably experimental error occurred, owing to dilution of the sample; additionally, accuracy of the meter varied by approximately 2%. To arrive at a standard error-figure was not practicable; the assumption that all measurements were accurate to within 2 millimhos seems to be stable.

Estimation of R_w by Spontaneous-potential Method

Commonly referred to as " R_w from SP," this method requires the input of several variables from the log and from the log header in order to arrive at an estimate of formation-water resistivity. As with most log-interpretation procedures, this method is subjective; for example, analysts may differ in interpretation of the Static Spontaneous Potential, or measurements of resistivity of mud

filtrate may be incorrect, due to heterogeneity of the mud column. As a result, an R_w value obtained in this manner may differ from the "true" measured R_w and still be correct in procedure and calculation. The R_w -from-SP algorithm used in this study was proprietary, but led to results strongly similar to the following algorithm (after Asquith, 1982, p. 29):

$$R_{mf} \text{ at } 75^\circ\text{F} = R_{mf} \times (R_{mf} \text{ temp}^* + 6.77)/81.77$$

(Correction of R_{mf} to 75° .)

$$K = 60 + (0.133 \times T_f)$$

$$R_{mfe}/R_{we} = 10^{-SSP/K}$$

R_{mfe} formula if R_{mf} at $75^\circ\text{F} < 0.1$:

$$R_{mfe} = (146 \times R_{mf} - 5)/(337 \times R_{mf} + 77)$$

R_{mfe} formula if R_{mf} at $75^\circ > 0.1$:

$$R_{mfe} = 0.85 \times R_{mf}.$$

$$R_{we} = R_{mfe}/(R_{mfe}/R_{we}).$$

R_w at 75° formula if $R_{we} < 0.12$:

$$R_w \text{ at } 75^\circ\text{F} = (77 \times R_{we} + 5)/(146 - 377 \times R_{we})$$

R_w at 75° formula if $R_{we} > 0.12$:

$$R_w \text{ at } 75^\circ\text{F} = -[0.58 - 10^{(0.69 \times R_{we} \text{ exp.}-0.24)}]$$

$$R_w \text{ at formation temperature} = R_w \text{ at } 75^\circ \times \\ 81.77/(T_f + 6.77)$$

* $R_{mf} \text{ temp} = R_{mf}$ at a temperature other than 75°F .
 In line 4 the e subscript (e.g., R_{mfe}) stands for equivalent resistivity.
 K = constant.

All calculations of R_w from the SP were made on a Hewlett-Packard HP41-C calculator: software employed was provided by Dresser Industries Inc.

Estimation of R_w by R_{wa} Method

An alternative to estimation of R_w from the SP method is the " R_{wa} Method" or approximation by apparent water resistivity. This method works best in a fairly thick, "clean" sand where at least one fully water-saturated zone is detectable. Apparent resistivity of formation water (R_{wa}) is derived from the Archie Equation:

$$S_w^2 = \frac{F \times R_w}{R_t}$$

S_w^2 is assumed to be 1; therefore

$$R_{wa} = F/R_t$$

F is approximated as $1/\phi^2$ for limestones and $0.81/\phi^2$ for consolidated sandstones, where ϕ symbolizes porosity. R_t , or "true resistivity," is estimated from the deep induction curve or another curve designed to investigate resistivity of the uncontaminated zone. Figure 8 shows an example of a stratigraphic interval from which R_w was estimated by the R_{wa} Method:

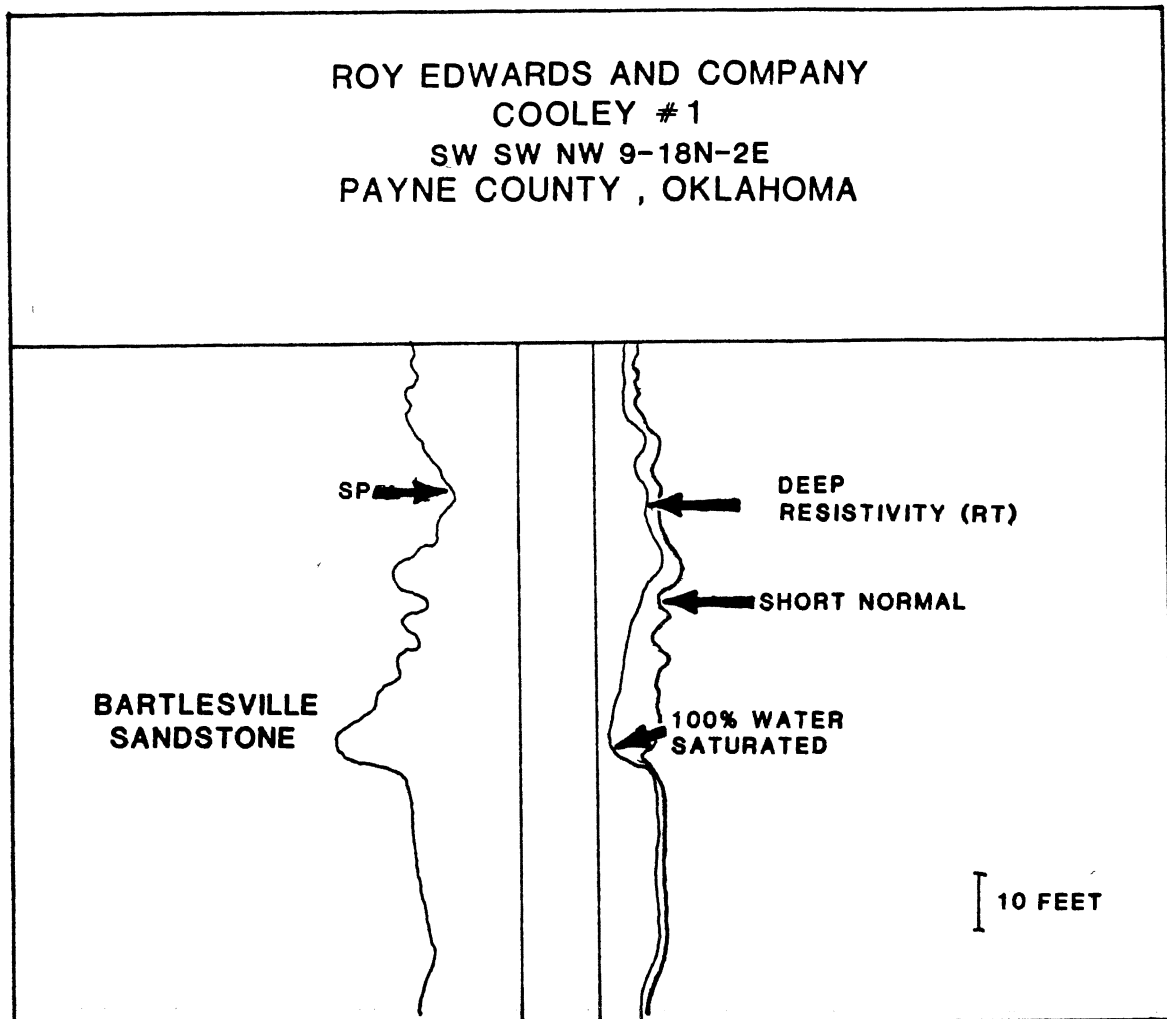


Figure 8. Example of Log for Calculation of R_{wa}

CHAPTER III

PRESENTATION AND INTERPRETATION OF DATA

Format

The sources of data for each interval studied were four: resistivities derived by the SP and R_{wa} methods, and two samples of produced water taken approximately 30 days apart, from which resistivities were measured. The purpose of taking two samples 30 days apart was (1) to test the hypothesis that change exists in overall trend in salinity with time, and (2) to provide what was expected to be the most accurate approximation of true R_w .

The data are presented by stratigraphic interval, in the forms of histograms, raw-data compilations and statistical analyses. A critical assumption that underlies all that follows is that the samples considered here were random samples. The wells from which samples were collected were not selected deliberately by locality but rather where permission was granted to do so. Thus no premeditated design or deliberate pattern underlay the gathering of data, and the argument for randomness is made primarily on this basis.

Statistical Analysis

Treatment of Raw Data

All raw data were organized in tabular format, with resistivity values arranged horizontally so that the reader can compare the four estimates quickly (Table I, for example).

Upon simple inspection of the raw data shown in Table I several questions of practical importance arise (and this set of questions is applicable to samples of water from the Red Fork, Bartlesville, Mississippi Chat and Mississippi Lime formations):

1. Do values of R_w differ significantly through time? Specifically, are the R_w -values of Produced Sample 2 significantly different from those of Produced Sample 1?
2. If not, do Produced Samples 1 and 2 represent essentially the same population?
3. Are R_w -values determined from produced samples significantly different from those estimated by calculation of R_w from the SP curve?
4. If so, what is the most likely source of error?
5. Are R_w -values determined from produced samples significantly different from those estimated by calculation of R_w by the R_{wa} method?
6. If so, what is the most likely source of error?
7. Are R_w -values estimated from the SP curve significantly different from those estimated by the R_{wa} method?

TABLE I
RESISTIVITIES OF FORMATION WATER,
SKINNER SANDSTONE

Well No.	Produced Samples		R_w from SP	R_w from R_{wa}
	No. 1	No. 2		
1.	0.041	0.038	0.111	0.041
2.	0.038	0.038	0.089	0.040
3.	0.036	0.036	0.063	0.063
4.	0.036	0.035	0.138	0.024
5.	0.040	0.038	0.087	0.072
6.	0.034	0.035	0.035	0.032
7.	0.035	0.035	0.047	0.095
8.	0.036	0.036	0.065	0.062
9.	0.037	0.046	0.034	0.088
10.	0.045	0.047	0.075	0.074
11.	0.035	0.035	0.054	0.058
12.	0.035	0.035	0.088	0.066
13.	0.034	0.036	0.065	0.073
14.	0.035	0.035	0.040	0.034
15.	0.044	0.041	0.032	0.014
16.	0.035	0.035	0.098	0.014
17.	0.050	0.051	0.098	no data
18.	0.041	0.041	0.060	no data
19.	0.040	0.041	0.134	0.069

^aResistivities in ohm-meters, corrected to 100°F.

^bLocations of wells shown in Figure 3. Names of wells shown in Appendix A.

8. If so, what is the most likely source of error?

In order to estimate the likelihood that means from various groups of samples (the principal variable of interest here) are equal, or in other words, that they represent the same population, the variances and means of samples were compared, and association of the two samples was estimated. For any pair of samples under evaluation the working hypotheses were:

1. Ho: True mean, population represented by Sample 1 = true mean, population represented by Sample 2.
2. Ha: True mean, population represented by Sample 1 \neq true mean, population represented by Sample 2.
3. Ho: Variance, population represented by Sample 1 = variance, population represented by Sample 2.
4. Ha: Variance, population represented by Sample 1 \neq variance, population represented by Sample 2.
5. Ho: Populations represented by Samples 1 and 2 are associated (that is, variation in population represented by Sample 1 is accounted for significantly, or can be explained significantly by, variation in population represented by Sample 2.)
6. Ha: Populations represented by Samples 1 and 2 are not associated significantly.

Statistical Estimates of Parameters

Histograms of combined first- and second-sample measured-resistivity values, and of SP and R_{wa} -derived

resistivity values are shown in Figures 9, 10, and 11. Three such histograms were prepared for each stratigraphic interval. Each histogram shows frequency on the vertical axis and estimates of resistivity on the horizontal axis.

As implied in the list of six working hypotheses shown above, populations of the type under discussion here can differ in two fundamental ways: variances and/or means. Hypotheses were in this manner: Equality of variances was tested by the variance-ratio test (F-test), equality of means was tested by Student's t-test, and association was tested by evaluation of the correlation coefficient (Folks, 1981, p. 290-292, 151-155, 199-205; Sokal and Rohlf, 1969, p. 181-185, 143-145, 220-223, 332, 498-508, 516; Rohlf and Sokal, 1969, p. 159, 168, 224; Hewlett-Packard, 1980, p. 10-13).

Statistical estimates of population-parameters are set out in tables consisting of sample size, mean, variance, correlation coefficient (r), coefficient of determination (r^2), F-value, critical F-value ($F\#$), degrees of freedom of F, Student's-t value (t), critical value of t ($t\#$), and degrees of freedom of Student's t (Table II, for example). For the Skinner sandstone, and for all other formations evaluated in this study, summary statistics are reported in the style of Table II.

General formulae (after Sokal and Rohlf, 1969, p. 49 ff. and p. 181 ff.) for the mean and variance of a sample are:

PRODUCED SAMPLE WATER RESISTIVITIES:
SKINNER SANDSTONE

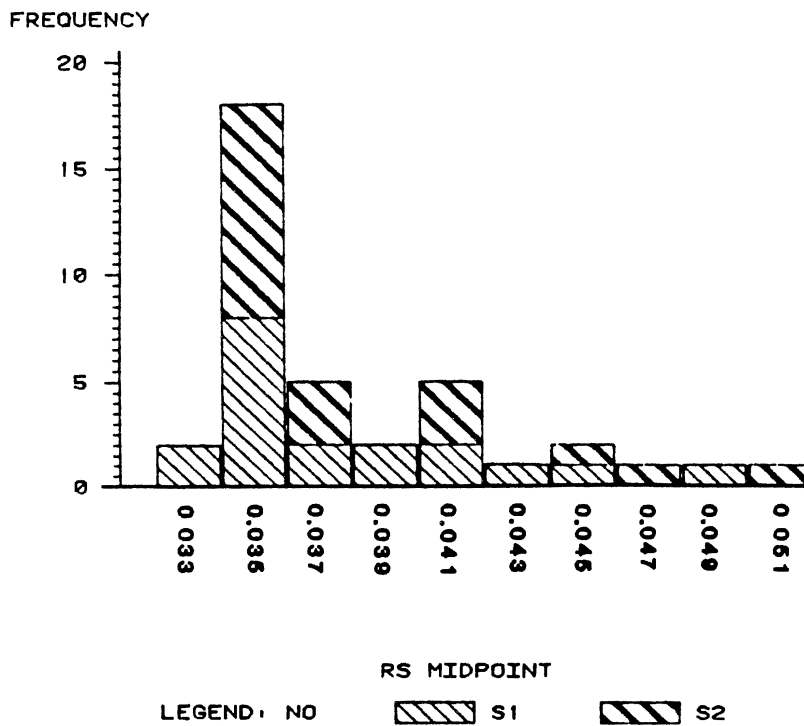


Figure 9. Histogram, Resistivities of Produced-Water Samples, Skinner Sandstone. Class Interval 0.002 Ohm-m. Lowest Class Boundary 0.0325 Ohm-m. Class Marks Rounded to Two Significant Digits

SP DERIVED WATER RESISTIVITIES:
SKINNER SANDSTONE

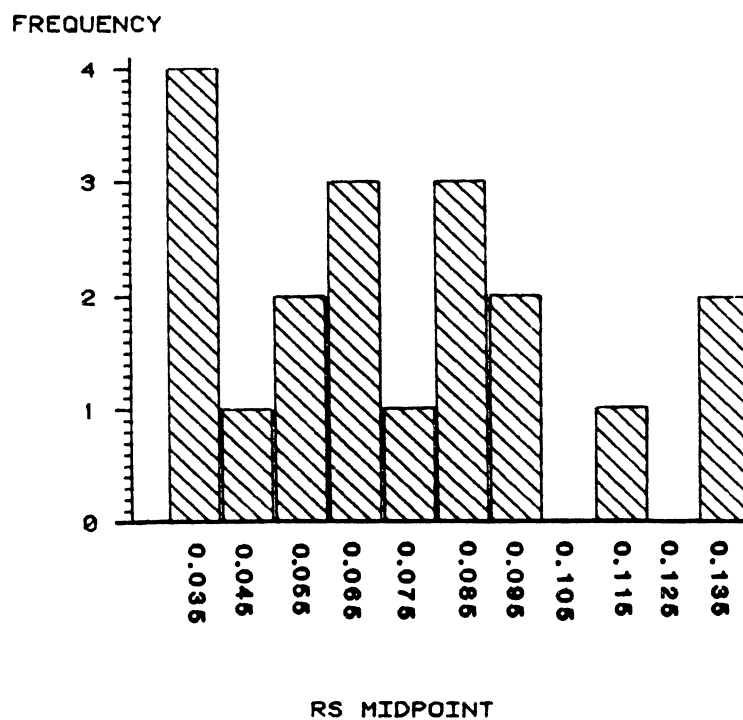


Figure 10. Frequency Histogram, SP-derived Water Resistivities, Skinner Sandstone Class Interval 0.01 Ohm-m Lowest Class Boundary 0.0305 Ohm-m Class Marks Rounded to Two Significant Digits

RWA DERIVED WATER RESISTIVITIES: SKINNER SANDSTONE

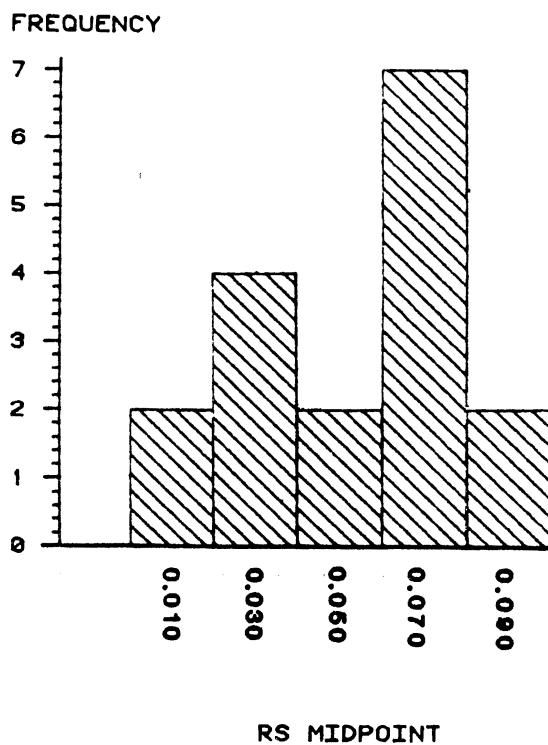


Figure 11 Frequency Histogram, Rwa-derived Water Resistivities, Skinner Sandstone. Class Interval 0.02 Ohm-m. Lowest Class Boundary 0.005 Ohm-m. Class Marks Rounded to Two Significant Digits

TABLE II
SUMMARY STATISTICS: SKINNER SANDSTONE

	Produced Sample 1	Produced Sample 2	Mean, Produced Samples	R_{wsp}	R_{wa}
n	19	19	19	17	17
Mean	.0382632	.0386316	.0387	.0739	.0541
Variance	.000019	.0000227	.0000026	.0011	.0006

	Prod. Smpl. 1 cf. 2	Mean, Prod. Smpls. cf. R_{wsp}	Mean, Prod. Smpls. cf. R_{wa}	R_{wsp} cf. R_{wa}
r	.8537*	.1456	.1530	.1458
r#	.456	.456	.482	.482
r^2	.7288	.0312	.0234	.02127
F	1.1889	38.45*	51.90*	1.83
F#	~2.22	~2.22	2.33	2.33
df	18,18	18,18	16,16	16,16
t	.064	4.91*	2.65*	-2.11*
t#	2.10	2.10	~2.03	~2.03
df	18	18	34	34

o Mean (\bar{X}) = $1/n (\sum X)$, where X is a single observation and n is the sample size.

o Variance (s^2) = $\sum(X-\bar{X})^2 /n-1$.

The correlation coefficient, r , is a measurement of association between two variables. It can range from +1 to -1, where +1 and -1 signify complete positive and negative association respectively, and zero signifies no association at all. The coefficient of determination, r^2 , is a measurement of the fraction of variation in one variable that is determined by variation in the other (Sokal and Rohlf, 1969, p. 498-504).

The F-statistic is calculated as $(s^2 \text{ (the larger)})/(s^2 \text{ (the smaller)})$. If the variances are independent estimates of the same quantity, the F-statistic will be near 1. Of course, some amount of error is introduced by one's sampling scheme and from other sources of error; therefore the meaning of an F-statistic greater than 1 must be evaluated in terms of probability. Effectively, the F-statistic tests the working hypothesis that samples have been drawn from one population or from two populations that have identical variances. After testing of the hypothesis $\sigma_1^2 = \sigma_2^2$ (variances of the two populations), then specific methods for evaluation of $\mu_1 = \mu_2$ (means of the two populations) follow, which involves Student's t-test (material based on Sokal and Rohlf, 1969, p. 181-182). All statistical tests were conducted with the alpha-level being 0.05. In Table II and all similar tables, significance of the sample-

statistic r , F or t at the alpha-level of 0.05 is indicated by an asterisk.

Skinner Sandstone

Table I shows the basic data for samples of formation-water resistivity from 19 wells that produce oil and gas from the Skinner sandstone. With exception of Well 9, no glaring differences in resistivity of produced-water samples exist. Resistivity of produced-water samples ranges from 0.034 to 0.050 ohm-m. Figure 9 shows a modal-class mark of 0.035 ohm-m; clearly the sample-distribution is skewed right. SP-derived resistivities range from 0.032 ohm-m to 0.138 ohm-m, with a multimodal distribution (Figure 10). R_{wa} -derived resistivities range from 0.014 ohm-m to 0.095 ohm-m. The modal class has class mark of 0.07 ohm-m (Figure 11).

Figures 9, 10, and 11 suggest that samples do not represent random collections from normally distributed populations. As described previously, samples of produced water were collected where possible; the option to sample selectively was not available. Neither were samples of R_w calculated from SP curves or by the R_{wa} method knowingly selective. Nevertheless, configurations of histograms in Figures 9, 10 and 11 indicate that either the variables are not distributed normally, as measured, or the samples were biased, or both.

If the variables measured are not distributed normally, the underlying set of reasons is not known to me. If the samples are biased, then the sources of bias were not recognized. Because much testing for detection of differences in parameters of populations is based on the assumption that the variable or variables are distributed normally, the fundamental choices at hand are these: (a) Conclude that the samples are drawn from non-normal distributions and abandon further efforts of analysis that would be based on such an assumption. Conclude that the populations sampled are (or are not) distributed normally but (and) that the samples are (or are not) random, and abandon further efforts of analysis that would be based on such assumptions. (c) Conclude that the populations sampled are (or are not) distributed normally but (and) that the samples are (or are not) random, but carry out analyses nevertheless, in belief that results of quantitative analyses would be better approximations of truth than qualitative analyses, conducted simply by inspection of the data. Option (c) was exercised in attempt to test working hypotheses 1 through 6, above. This procedure applies as well to testing of the Red Fork, Bartlesville, Mississippi Chat and Mississippi Lime formations.

Summary statistics from sampling of R_w of the Skinner Sandstone are set out in Table II. Included in the table are means, variances, coefficients of correlation and determination, and F- and t-tests. Working hypotheses 1

through 6 are shown in Tables III through VI, for convenience of the reader.

Testing (see "Evidence," Table III) indicates that resistivity of formation water in the Skinner sandstone, as estimated from samples of produced water, does not vary significantly through time.

Figure 12 shows a scattering of points in a generally elliptical pattern, with a fitted line of positive slope. Produced-water samples seem to be positively correlated additional evidence suggestive that one population was sampled.

Table IV shows evidence that estimates of formation-water resistivity, drawn from samples of produced water and calculated from the SP curve, apparently do not estimate the same quantity. As indicated in Figure 13, variation among estimates of R_w , calculated from the SP curve, is much greater than that observed in produced-water samples (see also Table II for comparison of variances). R_w calculated from the SP curve is a poor estimate of formation-water resistivity. The Skinner Sandstone tends to be a thin formation, which suppresses the SP curve. Thin-bed effects were corrected. Probably the R_{wsp} measurements were affected adversely by shale, and/or in some of the wells sampled, water saturation was not 100%.

Comparison of R_w calculated by the R_{wa} method, with R_w from produced-water samples leads to the conclusion that the R_{wa} method was an inefficient means of estimating R_w .

TABLE III

SKINNER SANDSTONE: TESTS OF HYPOTHESES ABOUT PARAMETERS
PRODUCED-WATER SAMPLE 1 CF. PRODUCED-WATER SAMPLE 2

-
1. Ho: True mean, population represented by Sample 1 = true mean, population represented by Sample 2.
 2. Ha: True mean, population represented by Sample 1 \neq true mean, population represented by Sample 2.
 3. Ho: Variance, population represented by Sample 1 = variance, population represented by Sample 2.
 4. Ha: Variance, population represented by Sample 1 \neq variance, population represented by Sample 2.
 5. Ho: Populations represented by Samples 1 and 2 are associated (that is, variation in population represented by Sample 1 is accounted for significantly, or can be explained significantly by, variation in population represented by Sample 2).
 6. Ha: Populations represented by Samples 1 and 2 are not associated significantly.
-

Working Hypothesis	Decision	Evidence (Table II)
1	Not rejected	Student's t not significant.
2	Rejected	
3	Not rejected	F-statistic not significant.
4	Rejected	
5	Not rejected	Correlation coefficient significant.
6	Rejected	

TABLE IV

SKINNER SANDSTONE: TESTS OF HYPOTHESES ABOUT PARAMETERS
PRODUCED-WATER SAMPLES CF. R_w CALCULATED FROM SP CURVES

-
1. Ho: True mean, population represented by Sample 1 = true mean, population represented by Sample 2.
 2. Ha: True mean, population represented by Sample 1 \neq true mean, population represented by Sample 2.
 3. Ho: Variance, population represented by Sample 1 = variance, population represented by Sample 2.
 4. Ha: Variance, population represented by Sample 1 \neq variance, population represented by Sample 2.
 5. Ho: Populations represented by Samples 1 and 2 are associated (that is, variation in population represented by Sample 1 is accounted for significantly, or can be explained significantly by, variation in population represented by Sample 2).
 6. Ha: Populations represented by Samples 1 and 2 are not associated significantly.
-

Working Hypothesis	Decision	Evidence (Table II)
1	Rejected	Student's t significant.
2	Not rejected	
3	Rejected	F-statistic significant.
4	Not Rejected	
5	Rejected	Correlation coefficient not significant.
6	Not rejected	

TABLE V

SKINNER SANDSTONE: TESTS OF HYPOTHESES ABOUT PARAMETERS
PRODUCED-WATER SAMPLES CF. R_w CALCULATED BY R_{WA} METHOD

-
1. H_0 : True mean, population represented by Sample 1 = true mean, population represented by Sample 2.
 2. H_a : True mean, population represented by Sample 1 \neq true mean, population represented by Sample 2.
 3. H_0 : Variance, population represented by Sample 1 = variance, population represented by Sample 2.
 4. H_a : Variance, population represented by Sample 1 \neq variance, population represented by Sample 2.
 5. H_0 : Populations represented by Samples 1 and 2 are associated (that is, variation in population represented by Sample 1 is accounted for significantly, or can be explained significantly by, variation in population represented by Sample 2).
 6. H_a : Populations represented by Samples 1 and 2 are not associated significantly.
-

Working Hypothesis	Decision	Evidence (Table II)
1	Rejected	Student's t significant.
2	Not rejected	
3	Rejected	F-statistic significant.
4	Not Rejected	
5	Rejected	Correlation coefficient not significant.
6	Not rejected	

TABLE VI
RESISTIVITIES OF FORMATION WATER,
RED FORK SANDSTONE

Well No	Produced Samples		SP Method	R _{wa} Method
	No 1	No 2		
1	.035	P&A	083	012
2	036	.035	040	054
3	035	.035	X	Y
4	044	034	050	027
5.	038	.035	079	023
6.	035	.036	082	016
7.	042	040	X	Y
8	038	038	051	023
9	038	.036	.085	022
10	.041	040	040	.018
11.	036	037	.095	026
12	036	036	096	022
13	038	038	174	025
14	041	.041	.770	051
15	040	.040	X	Y
16	039	.030	084	059
17	045	046	156	035
18	040	042	084	Y
19	.040	042	059	Y
20.	.040	040	070	Y
21	041	041	082	Y
22.	044	043	094	.042

Resistivities in Ohm-m , corrected to 100°F
X Resistivity logs not available
Y Porosity logs not available
P&A Plugged and abandoned

PRODUCED SAMPLE 1 VS. PRODUCED SAMPLE 2
SKINNER SANDSTONE
RESISTIVITY IN OHM-METERS

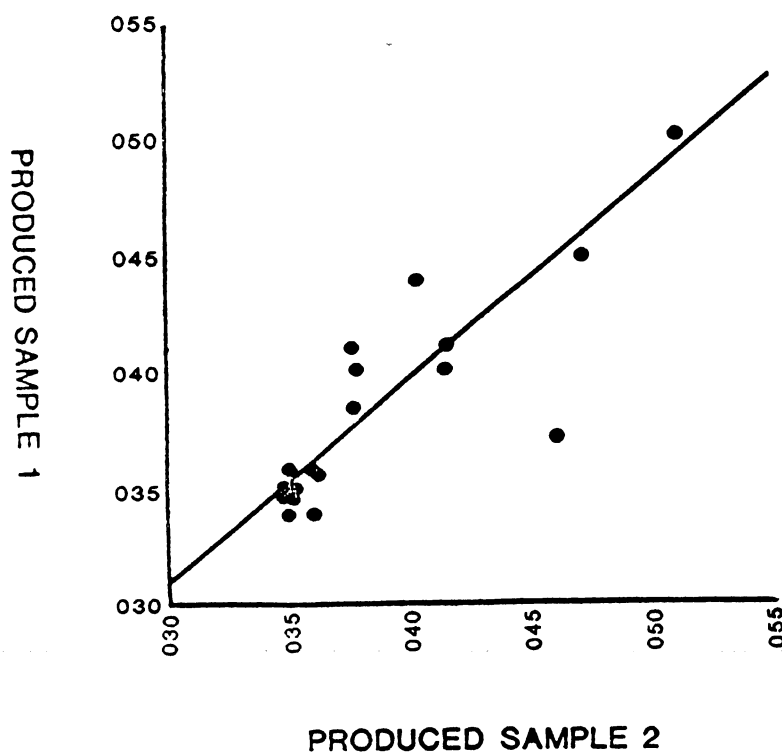


Figure 12. Scatter Diagram, Skinner Sandstone, Resistivities of Produced-water Samples 1 and 2. Correlation Coefficient 0.8537. Coefficient of Determination 0.7288, Indicating That About Three-fourths of the Variation in One Variable Can Be Accounted for by Variation in the Other (Sokal and Rohlf, 1969, p 503)

\bar{X} OF PRODUCED SAMPLE 1 AND 2 VS. RW FROM THE SP
SKINNER SANDSTONE
RESISTIVITY IN OHM-METERS

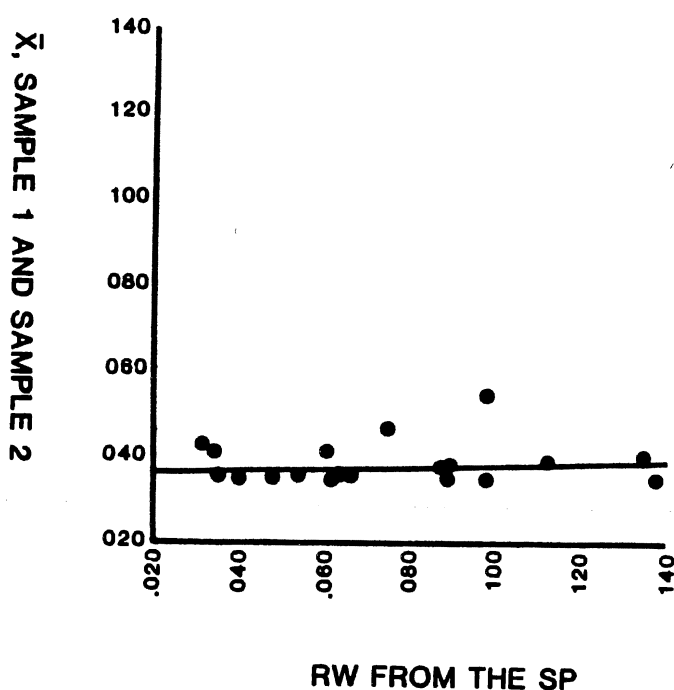


Figure 13 Scatter Diagram, Skinner Sandstone, Mean Resistivity of Produced-water Samples 1 and 2 cf. Resistivities Estimated From SP Curve Correlation Coefficient 0.1456 Coefficient of Determination 0.0312, Indicating That Only a Few Percent of the Variation in One Variable Can Be Accounted for by Variation in the Other (Sokal and Rohlf, 1969, p. 503)

Working hypotheses that asserted equality of means, variances and significant correlation of variables were rejected (Table V, under "Evidence"). The greater scatter of points along the X-axis of Figure 14 is evidence of comparatively large variation among estimates of R_w by the R_{wa} method (see also Table II for comparison of sample variances). Estimates of R_w from R_{wa} are too large (0.054 cf. 0.039), probably because of effects of thin beds and residual petroleum saturation, and consequent overestimation of resistivity in rocks presumed to be totally water-saturated.

Table II shows evidence from comparison of R_w calculated from the SP curve and by the R_{wa} method (R_{wsp} cf. R_{wa}). The correlation coefficient is not significant ($r = 0.1458$ cf. $r(\text{critical}) = 0.482$) and the means are significantly different, but evidence does not require rejection of the hypothesis of equal variances. In terms of practical application these statistics have limited usefulness, for as has been pointed out above, estimates of R_w from R_{wsp} and R_{wa} are not likely to be close to the "true" R_w of the Skinner Sandstone -- about 0.04 ohm-m.

In brief, estimation of formation-water resistivity of Skinner sandstone from the SP curve or by apparent water resistivity was inaccurate and imprecise.

\bar{X} OF PRODUCED SAMPLE 1 AND 2 VS. RW FROM RWA
SKINNER SANDSTONE

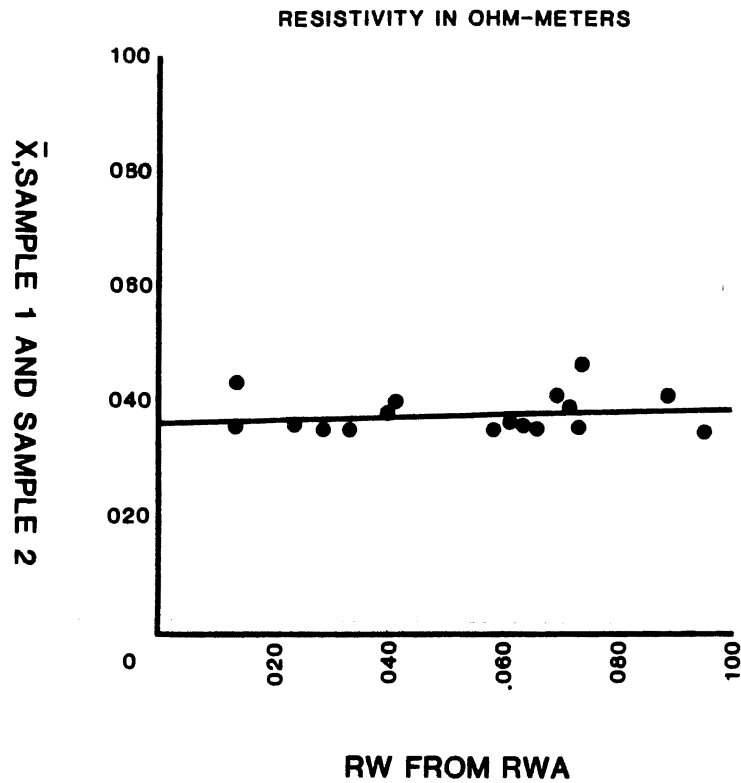


Figure 14. Scatter Diagram, Skinner Sandstone, Mean Resistivity of Produced-water Samples 1 and 2 cf. Resistivities Estimated by Rwa Method Correlation Coefficient 0.153 Coefficient of Determination About 0.02, Indicating That Only a Few Percent of the Variation in One Variable Can Be Accounted for by Variation in the Other (Sokal and Rohlf, 1969, p. 503)

Red Fork Sandstone

Table VI shows resistivities of samples of formation water and calculated R_w values from 22 wells that produced petroleum from Red Fork Sandstone. Only well 4 shows a large difference in R_w measured from produced-water samples. Resistivity ranges from 0.03 to 0.046 ohm-m. Figure 15 shows a modal class mark of 0.035 ohm-m; the distribution is skewed right.

SP-derived resistivities range from 0.040 to 0.77 ohm-m, with a modal value of 0.088 ohm-m (Figure 16). R_{wa} -derived resistivities are smaller measurements on the whole, ranging from 0.012 to 0.059 ohm-m; the modal class has a class mark of 0.025 ohm-m (Figure 17).

Summary statistics from sampling of R_w of the Red Fork Sandstone are set out in Table VII. Working hypotheses 1 through 6 are shown in Tables VIII, IX, and X.

Analysis of samples of produced water shows evidence that resistivity does not vary significantly across time; the two sets of samples seem to represent one population (see "Evidence," Table VIII). Figure 18 is a cross-plot of resistivities, produced-water samples 1 and 2. Points are scattered in a "loose" ellipse, with a fitted line of positive slope.

Table IX shows results of comparison of R_w from produced-water samples and R_w calculated from the SP curve. Rejection of hypotheses 1, 3 and 5 is taken as strong evidence that the two methods of estimating formation-water

PRODUCED SAMPLE WATER RESISTIVITIES: REDFORK SANDSTONE

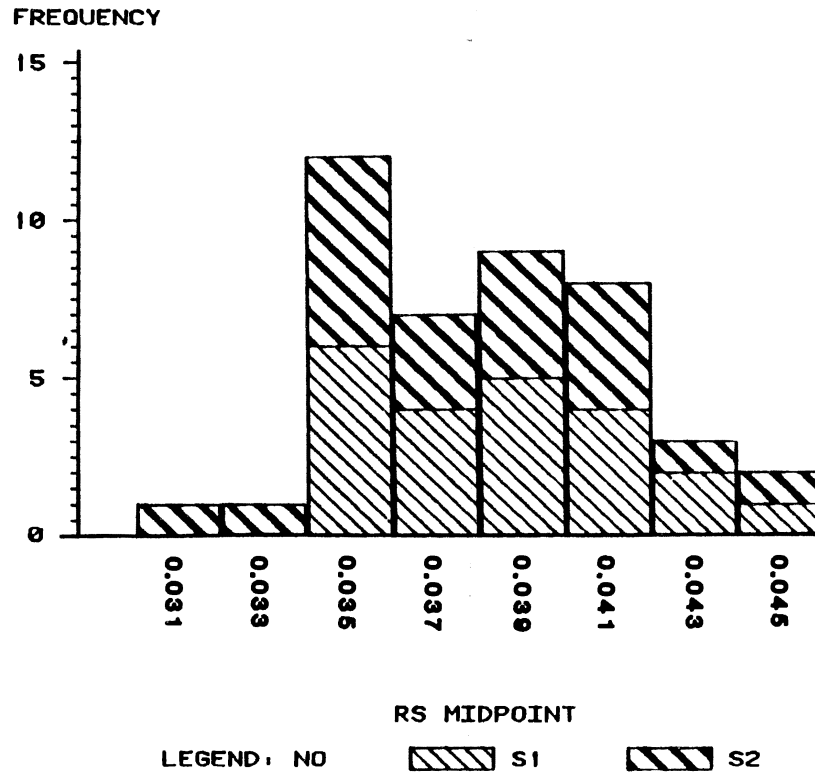


Figure 15 Histogram, Resistivities of Produced-water Samples, Red Fork Sandstone
 Class Interval 0.002 Ohm-m Lowest
 Class Boundary 0.0305 Ohm-m Class
 Marks Rounded to Two Significant
 Digits

SP DERIVED WATER RESISTIVITIES: REDFORK SANDSTONE

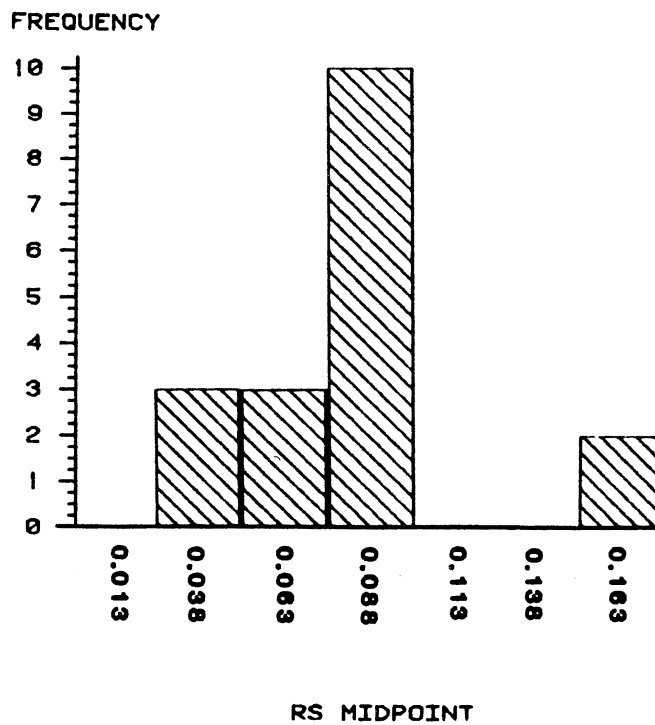


Figure 16 Histogram, SP-derived Water Resistivities, Red Fork Sandstone Class Interval 0.025 Ohm-m Lowest Class Boundary 0.0255 Ohm-m

RWA DERIVED WATER RESISTIVITIES:
REDFORK SANDSTONE

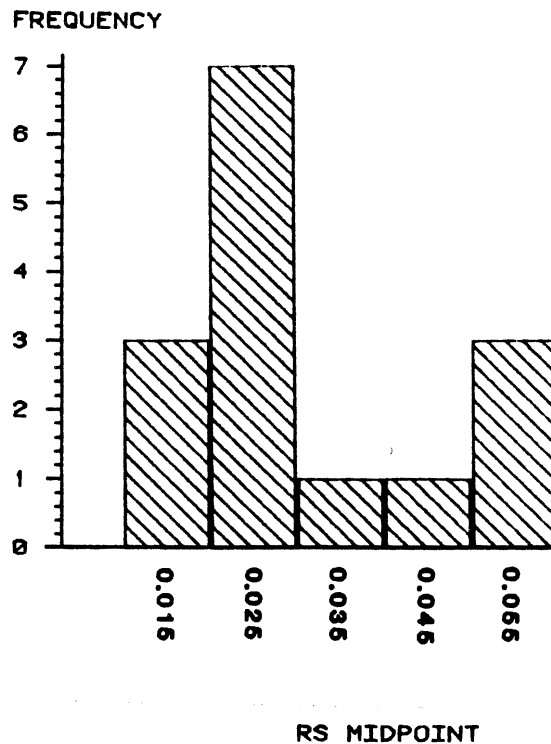


Figure 17. Histogram, Rwa-derived Water Resistivities, Red Fork Sandstone Class Interval 0.01. Lowest Class Boundary 0.0105 Ohm-m. Class Marks Rounded to Two Significant Digits

TABLE VII

SUMMARY STATISTICS: RED FORK SANDSONTE

	Produced Sample 1	Produced Sample 2	Mean, Produced Samples	R_{wsp}	R_{wa}
n	21	21	14	15	14
Mean	.0384	.0383	.0384	.1319	.0316
Variance	.000084	.000014	.0000102	.0326	.0002

	Prod. Smpl. 1 cf. 2	Means Prod. Smpls. cf. R_{wsp}	Means Prod. Smpls. cf. R_{wa}	R_{wsp} cf. R_{wa}
r	.5897*	.2218	.0663	.3789
r#	.433	.468	.532	.514
r^2	.3478	.0492	.0044	.1436
F	1.63	3044*	19.53*	163*
F#	~2.12	~2.28	~2.58	2.46
df	20,20	17,17	13,13	14,14
t	-.8286	2.3023*	-2.6237*	-2.1346*
t#	~2.02	~2.02	~2.03	~2.04
df	41	38	34	32

TABLE VIII

RED FORK SANDSTONE: TESTS OF HYPOTHESES ABOUT PARAMETERS
PRODUCED-WATER SAMPLE 1 CF. PRODUCED-WATER SAMPLE 2

-
1. Ho: True mean, population represented by Sample 1 = true mean, population represented by Sample 2.
 2. Ha: True mean, population represented by Sample 1 \neq true mean, population represented by Sample 2.
 3. Ho: Variance, population represented by Sample 1 = variance, population represented by Sample 2.
 4. Ha: Variance, population represented by Sample 1 \neq variance, population represented by Sample 2.
 5. Ho: Populations represented by Samples 1 and 2 are associated (that is, variation in population represented by Sample 1 is accounted for significantly, or can be explained significantly by, variation in population represented by Sample 2).
 6. Ha: Populations represented by Samples 1 and 2 are not associated significantly.
-

Working Hypothesis	Decision	Evidence (Table VII)
1	Not Rejected	Student's t not significant.
2	Rejected	
3	Not Rejected	F-statistic not significant.
4	Rejected	
5	Not Rejected	Correlation coefficient significant.
6	Rejected	

TABLE IX

RED FORK SANDSTONE: TESTS OF HYPOTHESES ABOUT PARAMETERS
PRODUCED-WATER SAMPLES CF. R_w CALCULATED FROM SP CURVES

-
1. Ho: True mean, population represented by Sample 1 = true mean, population represented by Sample 2.
 2. Ha: True mean, population represented by Sample 1 \neq true mean, population represented by Sample 2.
 3. Ho: Variance, population represented by Sample 1 = variance, population represented by Sample 2.
 4. Ha: Variance, population represented by Sample 1 \neq variance, population represented by Sample 2.
 5. Ho: Populations represented by Samples 1 and 2 are associated (that is, variation in population represented by Sample 1 is accounted for significantly, or can be explained significantly by, variation in population represented by Sample 2).
 6. Ha: Populations represented by Samples 1 and 2 are not associated significantly.
-

Working Hypothesis	Decision	Evidence (Table VII)
1	Rejected	Student's t significant.
2	Not rejected	
3	Rejected	F-statistic significant.
4	Not Rejected	
5	Rejected	Correlation coefficient not significant.
6	Not rejected	

TABLE X

RED FORK SANDSTONE: TESTS OF HYPOTHESES ABOUT PARAMETERS
PRODUCED-WATER SAMPLES CF. R_W CALCULATED BY R_{WA} METHOD

-
1. Ho: True mean, population represented by Sample 1 = true mean, population represented by Sample 2.
 2. Ha: True mean, population represented by Sample 1 \neq true mean, population represented by Sample 2.
 3. Ho: Variance, population represented by Sample 1 = variance, population represented by Sample 2.
 4. Ha: Variance, population represented by Sample 1 \neq variance, population represented by Sample 2.
 5. Ho: Populations represented by Samples 1 and 2 are associated (that is, variation in population represented by Sample 1 is accounted for significantly, or can be explained significantly by, variation in population represented by Sample 2).
 6. Ha: Populations represented by Samples 1 and 2 are not associated significantly.
-

Working Hypothesis	Decision	Evidence (Table XII)
1	Rejected	Student's t significant.
2	Not rejected	
3	Rejected	F-statistic significant.
4	Not Rejected	
5	Rejected	Correlation coefficient not significant.
6	Not rejected	

PRODUCED SAMPLE 1 VS. PRODUCED SAMPLE 2
 REDFORK SANDSTONE
 RESISTIVITY IN OHM-METERS

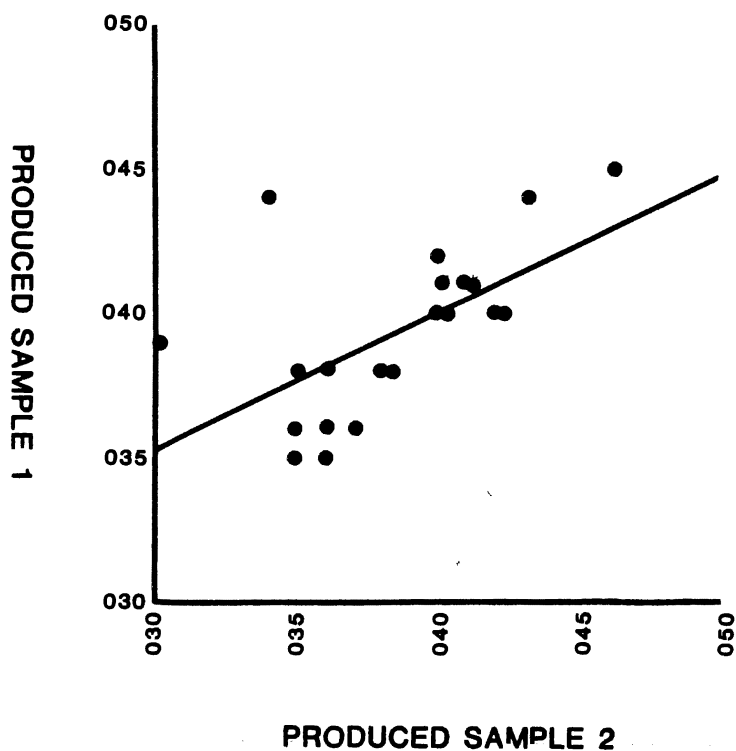


Figure 18. Scatter Diagram, Red Fork Sandstone, Resistivities of Produced-water Samples 1 and 2. Correlation Coefficient 0.5897. Coefficient of Determination 0.3478, Indication That About 35 Percent of the Variation in One Variable Can Be Accounted for by Variation in the Other (Sokal and Rohlf, 1969, p. 503)

resistivity do not approximate the same quantity. The mean and variance of the sample of R_{wsp} are much larger than those of the produced-water sample (Table VII). Figure 19 shows the large difference in variation of the two kinds of samples. Clearly, R_w estimated from the SP curve is greater than the "true" R_w , which is evaluated best from produced water. Because thin-bed effects on SP curves were corrected, in the wells sampled the major source of error probably is the combined effects of shaliness and residual-oil saturation, which tend to suppress the SP curve.

Comparison of R_w , calculated by the R_{wa} method, with R_w from produced-water samples shows that the R_{wa} method was not effective. Working hypotheses 1, 3 and 5 of Table X assert equality of (population) means and positive correlation of R_w calculated by the R_{wa} method with R_w from produced-water samples. These hypotheses were rejected (Table X; see "Evidence"). The much greater variation in R_w computed by the R_{wa} method and the poor correlation of the two samples are shown in Figure 20. The mean of R_w derived from R_{wa} is less than that of produced-water samples (0.0316 cf. 0.0384); the correct explanation for this is not understood at this time. Explanations involve the suspected underestimation of true formation resistivity, overestimation of true porosity, or an incorrect equation for the formation factor.

\bar{X} OF PRODUCED SAMPLE 1 AND 2 VS. RW FROM THE SP
 REDFORK SANDSTONE

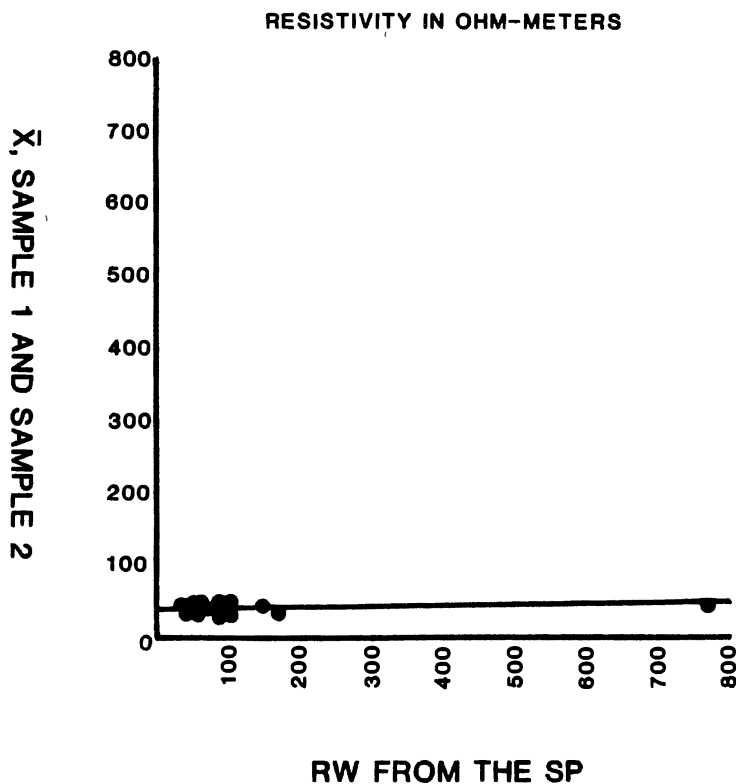


Figure 19 Scatter Diagram, Red Fork Sandstone, Mean Resistivity of Produced-water Samples 1 and 2 cf. Resistivities Estimated From SP Curve Correlation Coefficient 0.2218 Coefficient of Determination 0.0492, Indicating That Only a Few Percent of the Variation in One Variable Can Be Accounted for by Variation in the Other (Sokal and Rohlf, 1969, p. 503)

\bar{X} OF PRODUCED SAMPLE 1 AND 2 VS. RW FROM RWA
 REDFORK SANDSTONE
 RESISTIVITY IN OHM-METERS

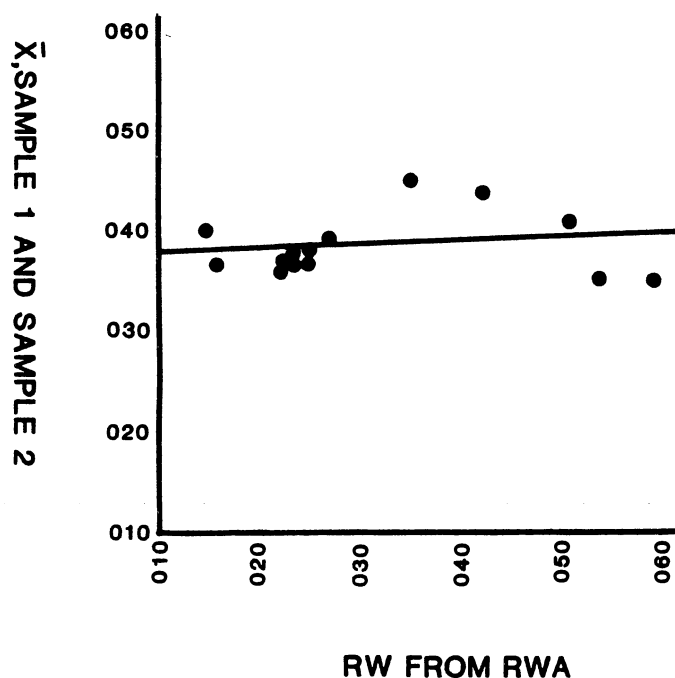


Figure 20 Scatter Diagram, Red Fork Sandstone, Mean Resistivity of Produced-water Samples 1 and 2 cf. Resistivities Estimated by Rwa Method. Correlation Coefficient 0 0663 Coefficient of Determination 0 0044, Indication That Effectively None of the Variation in One Variable Can Be Accounted for by Variation in the Other (Sokal and Rohlf, 1969, p 503)

Statistics for comparison of R_w computed from the SP curve and by the R_{wa} method are in Table VII. The t- and F-tests indicate strongly that the population means and variances are different. The correlation coefficient is not significant ($r \sim 0.4$ cf. $r(\text{critical}) \sim 0.5$).

As conducted in this study, R_w of the Red Fork Sandstone, derived from the SP curve or by the R_{wa} method is likely to overestimate and underestimate, respectively, R_w measured from produced-water samples, the mean of which should be close to the truth.

Bartlesville Sandstone

Resistivities of samples of formation water from the Bartlesville Sandstone and calculated values of R_w from 19 oil wells are shown in Table XI. Of this set, samples from well 1 show abnormally large resistivity. Clearly the samples were contaminated by fresh water. Resistivity of water from wells 15, 16, 17 and 18 was greater than average. Excluding well 1, measurement of produced-water resistivity ranged from 0.035 to 0.067 ohm-m. Figure 21 is a histogram of the produced-water samples; the frequency distribution is skewed-right, with a modal class of 0.038 ohm-m (class mark).

SP-derived resistivities ranged from 0.034 to 0.257 ohm-m. The frequency distribution is skewed right, with observations concentrated in the range from about 0.05 ohm-m to 0.075 ohm-m (Figure 22). By and large, R_{wa} -derived

PRODUCED SAMPLE WATER RESISTIVITIES: BARTLESVILLE SANDSTONE

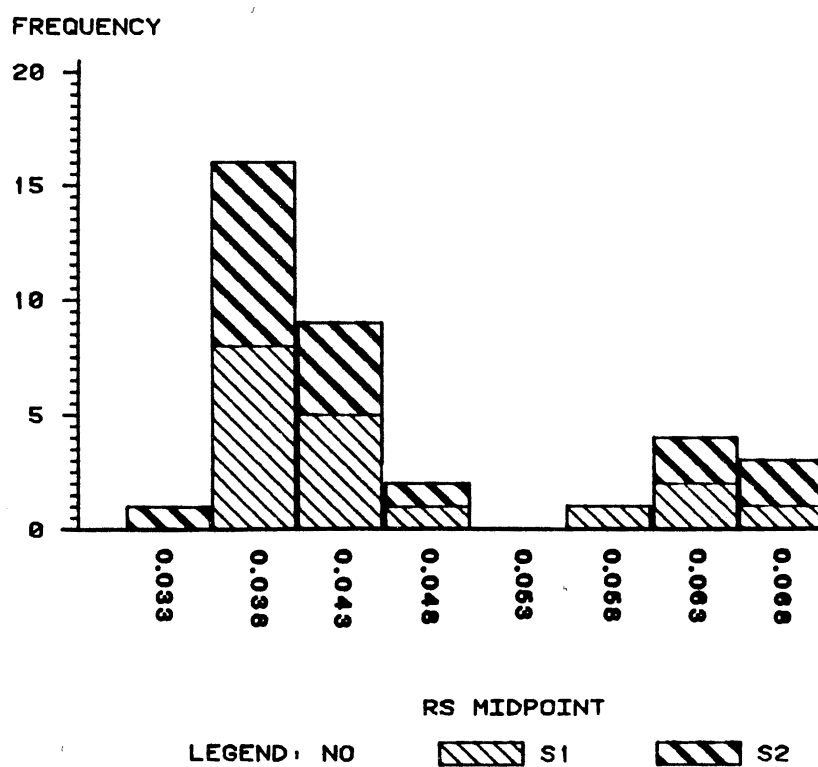


Figure 21. Histogram, Resistivities of Produced-water Samples, Bartlesville Sandstone. Class Interval 0.005 Ohm-m
Lowest Class Boundary 0.0305 Ohm-m

SP DERIVED WATER RESISTIVITIES:
BARTLESVILLE SANDSTONE

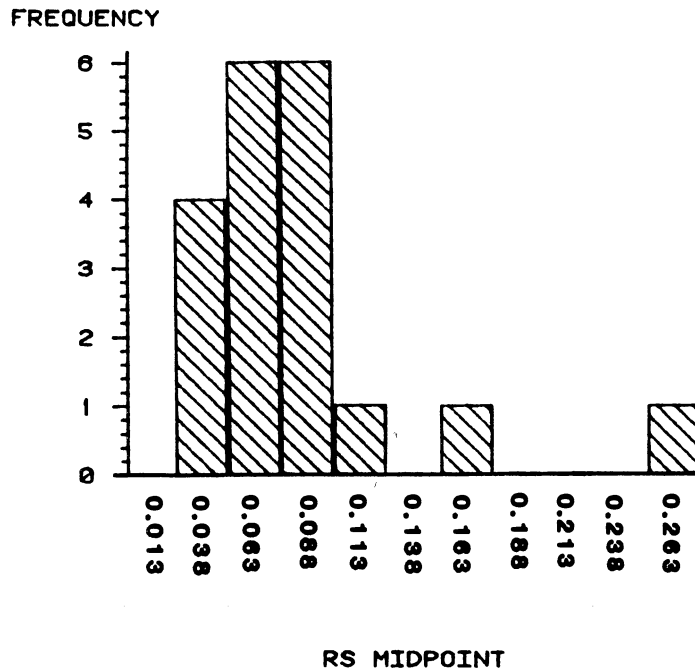


Figure 22. Histogram, SP-derived Water Resistivities, Bartlesville Sandstone Class Interval 0.025 Ohm-m. Lowest Class Boundary 0.0255 Ohm-m

resistivities were less than produced-water or SP-derived resistivities; the range is 0.015 to 0.083 ohm-m (Table XI). Class mark of the modal class is 0.025 ohm-m; the distribution is skewed right (Figure 23).

In Table XII summary statistics of R_w of the Bartlesville Sandstone are shown. Working hypotheses 1 through 6 are set out in Tables XIII, XIV and XV.

Resistivities or produced water seem not to vary significantly through time (Compare means and variances, and refer to t , F and r , Table XII). To infer that the two sets of samples came from one population seems to be dependable (see "Evidence," Table XIII). The two sets of resistivities are cross-plotted in Figure 24. By simple inspection the degree of fit is good; the correlation coefficient is about 0.97.

Comparison of produced-water samples and R_w computed from the SP curve is shown in Table XIV. Rejection of working hypotheses 1, 3 and 5 is interpreted as evidence that R_w computed from SP curves and R_w measured from produced water do not estimate the same measurement. The mean and variance of R_w calculated from the SP curve are markedly greater than those of the produced-water sample (Table XII). Figure 25 illustrates the much larger variation of estimates computed from the SP curve. R_w calculated from the SP curve is likely to overestimate the true value, and scatter is certain to be greater than measurements from produced water. Because the adverse

TABLE XI
RESISTIVITIES OF FORMATION WATER,
BARTLESVILLE SANDSTONE

Well No	Produced Samples		SP Method	R_{wa} Method
	No 1	No 2		
1	23 325	19 865	034	034
2.	044	042	098	022
3	042	041	083	027
4	037	034	049	030
5	043	038	078	037
6	038	037	058	039
7	042	040	076	063
8	037	040	068	053
9	035	036	.257	030
10	037	.037	054	046
11	045	039	107	083
12	038	038	086	055
13	036	039	076	.044
14	039	038	154	015
15	067	066	X	Y
16	059	061	092	Y
17	064	.067	063	Y
18	.061	060	049	Y
19	049	049	034	Y

Resistivities in ohm-m , corrected to 100°F
X Resistivity logs not available
Y Porosity logs not available

RWA DERIVED WATER RESISTIVITIES: BARTLESVILLE SANDSTONE

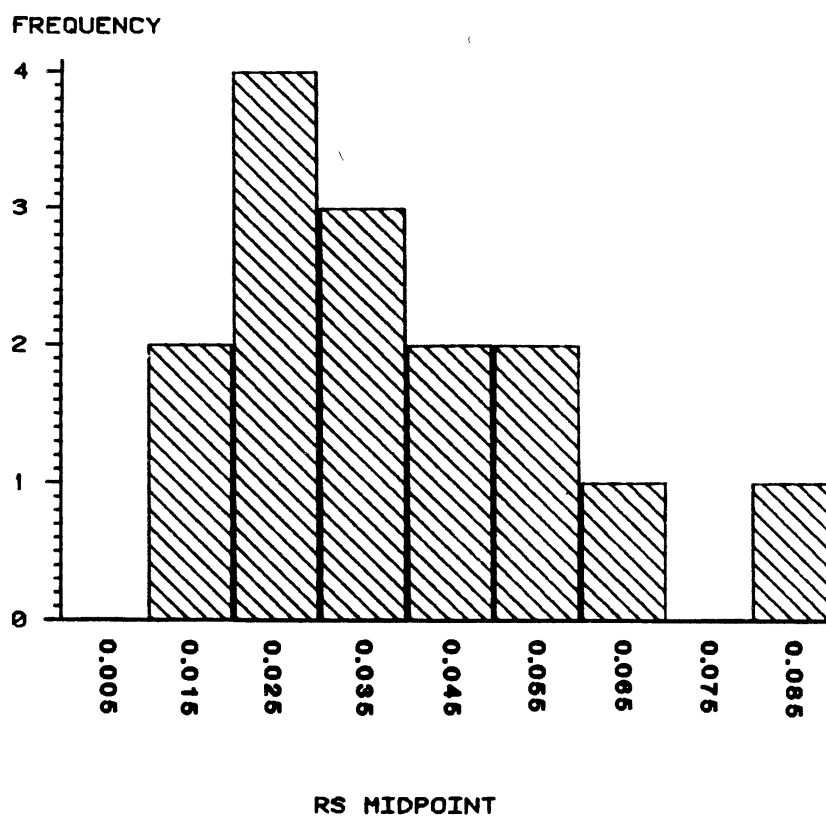


Figure 23. Histogram, Rwa-derived Water Resistivities, Bartlesville Sandstone. Class Interval 0.01 Ohm-m. Lowest Class Boundary 0.0105 Ohm-m. Class Marks Rounded to Two Significant Digits

TABLE XII

SUMMARY STATISTICS: BARTLESVILLE SANDSTONE

	Produced Sample 1	Produced Sample 2	Mean, Produced Samples	R_{wsp}	R_{wa}
n	18	18	17	17	13
Mean	.0452	.0446	.0436	0.861	.0418
Variance	.0001	.0001	.0001	.0027	.0003

	Prod. Smpl. 1 cf. 2	Mean, Prod. Smpls. cf. R_{wsp}	Mean, Prod. Smpls. cf. R_{wa}	R_{wsp} cf. R_{wa}
r	.9734*	.2740	.2066	.1888
r#	.468	.482	.553	.532
r^2	.9475	.0751	.0427	.0358
F	1	27.0*	52.08*	10.67*
F#	~2.28	2.33	2.69	~2.58
df	17,17	16,16	12,12	13,13
t	1.027	3.045*	-.7048	-2.875*
t#	2.11	2.11	2.04	2.04
df	17	17	30	30

TABLE XIII

BARTLESVILLE SANDSTONE: TESTS OF HYPOTHESES ABOUT
PARAMETERS: PRODUCED-WATER SAMPLE 1 CF.
PRODUCED-WATER SAMPLE 2

-
1. Ho: True mean, population represented by Sample 1 = true mean, population represented by Sample 2.
 2. Ha: True mean, population represented by Sample 1 \neq true mean, population represented by Sample 2.
 3. Ho: Variance, population represented by Sample 1 = variance, population represented by Sample 2.
 4. Ha: Variance, population represented by Sample 1 \neq variance, population represented by Sample 2.
 5. Ho: Populations represented by Samples 1 and 2 are associated (that is, variation in population represented by Sample 1 is accounted for significantly, or can be explained significantly by, variation in population represented by Sample 2).
 6. Ha: Populations represented by Samples 1 and 2 are not associated significantly.
-

Working Hypothesis	Decision	Evidence (Table VII)
1	Not Rejected	Student's t not significant.
2	Rejected	
3	Not Rejected	F-statistic not significant.
4	Rejected	
5	Not Rejected	Correlation coefficient significant.
6	Rejected	

TABLE XIV

BARTLESVILLE SANDSTONE: TESTS OF HYPOTHESES ABOUT
PARAMETERS: PRODUCED-WATER SAMPLES CF.
 R_W CALCULATED FROM SP CURVES

-
1. Ho: True mean, population represented by Sample 1 = true mean, population represented by Sample 2.
 2. Ha: True mean, population represented by Sample 1 \neq true mean, population represented by Sample 2.
 3. Ho: Variance, population represented by Sample 1 = variance, population represented by Sample 2.
 4. Ha: Variance, population represented by Sample 1 \neq variance, population represented by Sample 2.
 5. Ho: Populations represented by Samples 1 and 2 are associated (that is, variation in population represented by Sample 1 is accounted for significantly, or can be explained significantly by, variation in population represented by Sample 2).
 6. Ha: Populations represented by Samples 1 and 2 are not associated significantly.
-

Working Hypothesis	Decision	Evidence (Table XII)
1	Rejected	Student's t significant.
2	Not rejected	
3	Rejected	F-statistic significant.
4	Not Rejected	
5	Rejected	Correlation coefficient not significant.
6	Not rejected	

TABLE XV

BARTLESVILLE SANDSTONE: TESTS OF HYPOTHESES ABOUT
 PARAMETERS: PRODUCED-WATER SAMPLES CF.
 R_W CALCULATED R_{WA} METHOD

-
1. Ho: True mean, population represented by Sample 1 = true mean, population represented by Sample 2.
 2. Ha: True mean, population represented by Sample 1 \neq true mean, population represented by Sample 2.
 3. Ho: Variance, population represented by Sample 1 = variance, population represented by Sample 2.
 4. Ha: Variance, population represented by Sample 1 \neq variance, population represented by Sample 2.
 5. Ho: Populations represented by Samples 1 and 2 are associated (that is, variation in population represented by Sample 1 is accounted for significantly, or can be explained significantly by, variation in population represented by Sample 2).
 6. Ha: Populations represented by Samples 1 and 2 are not associated significantly.
-

Working Hypothesis	Decision	Evidence (Table XII)
1	Not rejected	Student's t not significant.
2	Rejected	
3	Rejected	F-statistic significant.
4	Not Rejected	
5	Rejected	Correlation coefficient not significant.
6	Not rejected	

PRODUCED SAMPLE 1 VS. PRODUCED SAMPLE 2
 BARTLESVILLE SANDSTONE
 RESISTIVITY IN OHM-METERS

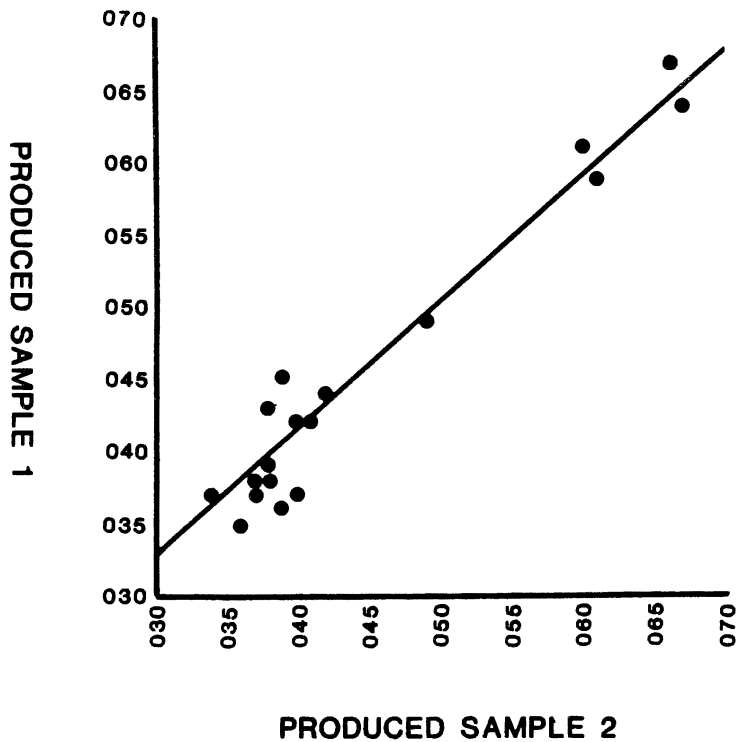


Figure 24 Scatter Diagram, Bartlesville Sandstone, Resistivities of Produced-water Samples 1 and 2 Correlation Coefficient 0.9734 Coefficient of Determination 0.9475, Indicating That 90 to 95 Percent of the Variation in One Variable Can Be Accounted for by Variation in the Other (Sokal and Rohlf, 1968, p. 503)

\bar{X} OF PRODUCED SAMPLE 1 AND 2 VS. RW FROM THE SP
BARTLESVILLE SANDSTONE
RESISTIVITY IN OHM-METERS

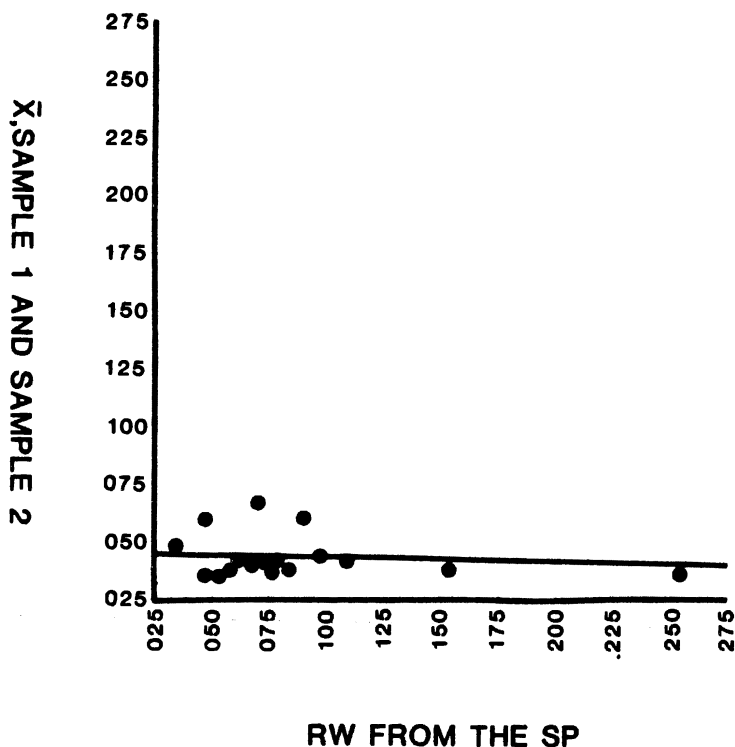


Figure 25 Scatter Diagram, Bartlesville Sandstone, Mean Resistivity of Produced-water Samples 1 and 2 cf Resistivities Estimated From SP Curve. Correlation Coefficient 0.02740 Coefficient of Determination 0.0751, Indicating That Only Several Percent of the Variation in One Variable Can Be Accounted for by Variation in the Other (Sokal and Rohlf, 1949, p 503)

effects of thin beds were corrected, the major sources of error probably are residual-oil saturation and shaliness of sandstone, which tend to elevate resistivity and suppress the SP curve.

R_w computed by the R_{wa} method yielded a sample mean close to that of produced-water samples, but greater variation (Table XII). Although the (true) means of populations represented by these variables seem not to be significantly different, the variances are almost surely quite different (Tables XII, XV), as indicated by inspection of scatter of points along the X-axis relative to scatter along the Y-axis, in Figure 26. Rejection of working hypothesis 3 (Table XV; see "Evidence") permits the conclusion that true means of the populations sampled (R_w measured from produced water and R_w measured by the R_{wa} method) are not the same quantity. The mean of the sample of R_w computed by the R_{wa} method is slightly less than that of R_w measured from produced water (about 0.042 ohm-m cf. 0.045 ohm-m, Table XII). Probably this is due to the exclusion of records from wells 15 through 19 (Table XI), but errors inherent in the interpretation of resistivity and porosity from the wireline logs may have been a factor.

Table XII shows summary statistics of comparison of R_w calculated from the SP curve and by the R_{wa} method (R_{wsp} cf. R_{wa}). The F- and t-statistics are quite significant; the means and variances estimated by the two methods are markedly different. The correlation coefficient is not

\bar{X} OF PRODUCED SAMPLE 1 AND 2 VS. RW FROM RWA
BARTLESVILLE SANDSTONE
RESISTIVITY IN OHM-METERS

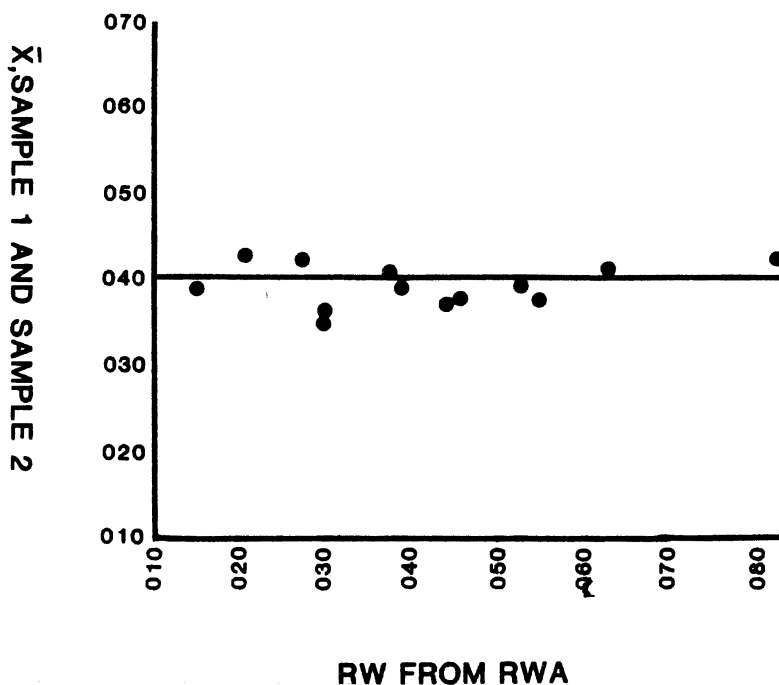


Figure 26. Scatter Diagram, Bartlesville Sandstone, Mean Resistivity of Produced-water Samples 1 and 2 cf. Resistivities Estimated by Rwa Method. Correlation Coefficient 0.2066. Coefficient of Determination About 0.043, Indicating That Only a Few Percent of the Variation in One Variable Can Be Accounted for by Variation in the Other (Sokal and Rohlf, 1969, p 503)

significant ($r \sim 0.2$ cf. $r(\text{critical}) \sim 0.5$). In brief, both methods of measuring R_w diverge and diverge differently from the "true" produced-water R_w of Bartlesville Sandstone.

Mississippi Chat

Table XVI is composed of measurements of R_w from seven wells that produced oil from the Mississippi Chat. Inspection of R_w of produced-water samples shows little evidence of difference from well to well or time to time. Range of measurements is 0.034 to 0.044 ohm-m. Figure 27 shows a slightly skewed-right frequency distribution, with modal class mark of 0.037 ohm-m.

R_w derived from the SP curve is generally a larger number in a well-by-well comparison. R_{wsp} ranges from 0.033 to 0.099 ohm-m (Table XVI). Plotted as a frequency distribution, the few samples indicate a skewed-right tendency (Figure 28). R_w computed by the R_{wa} method is less than R_{swp} , on the whole, ranging from 0.028 to 0.061 ohm-m (Table XVI); the frequency distribution of this small sample suggests that the distribution of this variable tends to be skewed right (Figure 29).

Table XVII is composed of summary statistics of R_w , of the Mississippi Chat. Tables XVIII, XIX and XX show evaluation of working hypotheses 1 through 6.

With regard to practical application of R_w in calculating water saturation of the Mississippi Chat,

TABLE XVI
RESISTIVITIES OF FORMATION WATER,
MISSISSIPPI CHAT

Well No	Produced Samples		SP Method	R _{wa} Method
	No. 1	No. 2		
1	034	.038	.050	051
2	038	039	053	028
3	040	.037	033	048
4	034	.038	099	039
5	037	.038	076	043
6	038	044	045	033
7	041	041	067	061

Resistivities in ohm-m , corrected to 100°F

PRODUCED SAMPLE WATER RESISTIVITIES: MISSISSIPPI CHAT

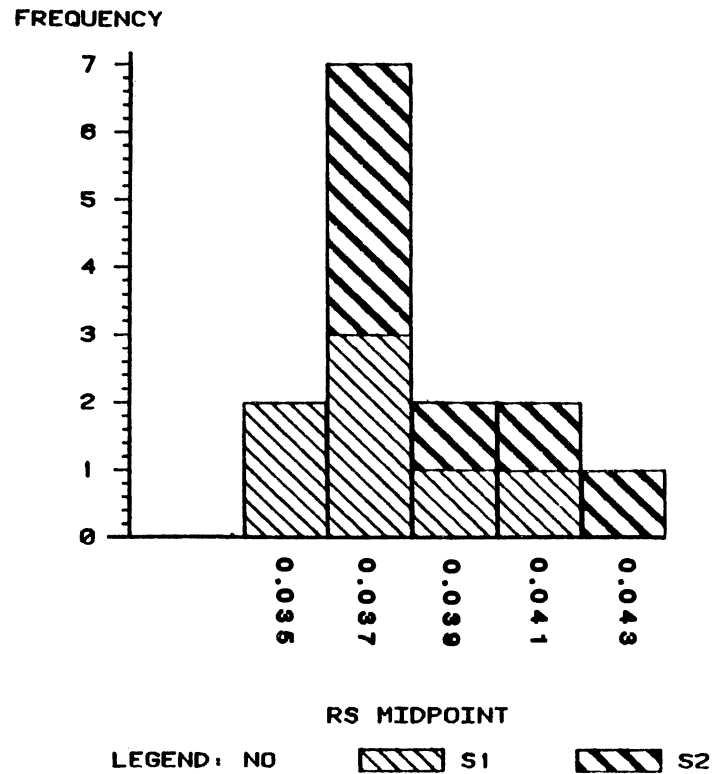


Figure 27 Histogram, Resistivities of Produced-water Samples, Mississippi Chat
 Class Interval 0.002 Ohm-m Lowest
 Class Boundary 0.0345 Ohm-m Class
 Marks Rounded to Two Significant
 Digits

SP DERIVED WATER RESISTIVITIES: MISSISSIPPI CHAT

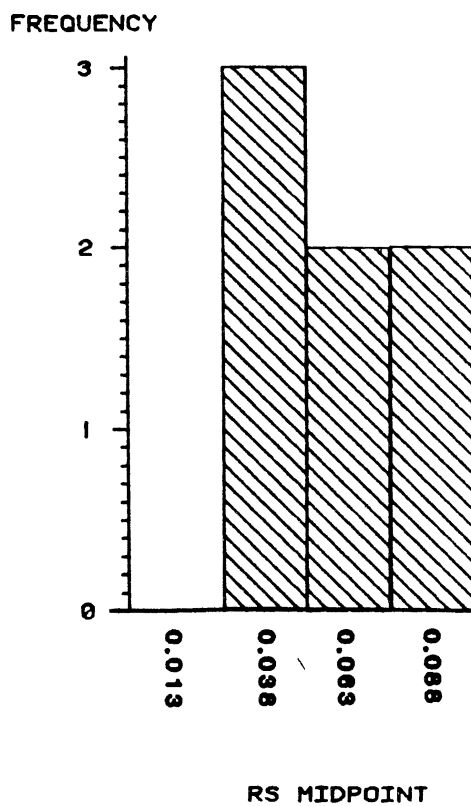


Figure 28. Histogram, SP-derived Water Resistivities, Mississippi Chat
Class Interval 0.025 Ohm-m
Lowest Class Boundary 0.0255 Ohm-m

TABLE XVII

SUMMARY STATISTICS: MISSISSIPPI "CHAT"

	Produced Sample 1	Produced Sample 2	Mean, Produced Samples	R_{wsp}	R_{wa}
n	7	7	7	7	7
Mean	.0374	.0393	.0384	.0604	.0433
Variance	.0000073	.0000058	.0000044	.0005	.0001
	Prod. Smpl. 1 cf. 2	Mean, Prod. Smpls. cf. R_{wsp}	Mean, Prod. Smpls. cf. R_{wa}	R_{wsp} cf. R_{wa}	
r	.3085	.3928	.0735	.0062	
r#	.754	.754	.754	.754	
r^2	.0952	.1543	.0054	.000038	
F	1.27	113.4*	22.69*	5.0*	
F#	4.28	4.28	4.28	4.28	
df	6,6	6,6	6,6	6,6	
t	1.625	2.541*	1.16	-1.8357	
t#	2.45	2.45	2.45	2.45	
df	6	6	6	6	

RWA DERIVED WATER RESISTIVITIES: MISSISSIPPI CHAT

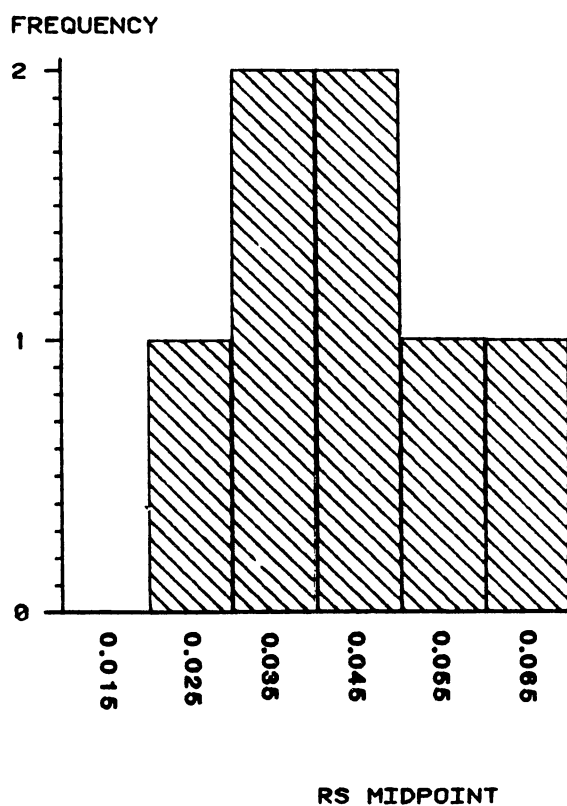


Figure 29 Histogram, Rwa-derived Water Resistivities, Mississippi Chat Class Interval 0.01 Ohm-m Lowest Class Boundary 0.0205 Ohm-m. Class Marks Rounded to Two Significant Digits

TABLE XVIII

MISSISSIPPI "CHAT": TESTS OF HYPOTHESES ABOUT
PARAMETERS: PRODUCED-WATER SAMPLE 1 CF.
PRODUCED-WATER SAMPLE 2

-
1. Ho: True mean, population represented by Sample 1 = true mean, population represented by Sample 2.
 2. Ha: True mean, population represented by Sample 1 \neq true mean, population represented by Sample 2.
 3. Ho: Variance, population represented by Sample 1 = variance, population represented by Sample 2.
 4. Ha: Variance, population represented by Sample 1 \neq variance, population represented by Sample 2.
 5. Ho: Populations represented by Samples 1 and 2 are associated (that is, variation in population represented by Sample 1 is accounted for significantly, or can be explained significantly by, variation in population represented by Sample 2).
 6. Ha: Populations represented by Samples 1 and 2 are not associated significantly.
-

Working Hypothesis	Decision	Evidence (Table VII)
1	Not Rejected	Student's t not significant.
2	Rejected	
3	Not Rejected	F-statistic not significant.
4	Rejected	
5	Rejected	Correlation coefficient not significant.
6	Not rejected	

TABLE XIX

MISSISSIPPI "CHAT": TESTS OF HYPOTHESES ABOUT
 PARAMETERS: PRODUCED-WATER SAMPLES CF.
 R_w CALCULATED FROM SP CURVES

-
1. Ho: True mean, population represented by Sample 1 = true mean, population represented by Sample 2.
 2. Ha: True mean, population represented by Sample 1 \neq true mean, population represented by Sample 2.
 3. Ho: Variance, population represented by Sample 1 = variance, population represented by Sample 2.
 4. Ha: Variance, population represented by Sample 1 \neq variance, population represented by Sample 2.
 5. Ho: Populations represented by Samples 1 and 2 are associated (that is, variation in population represented by Sample 1 is accounted for significantly, or can be explained significantly by, variation in population represented by Sample 2).
 6. Ha: Populations represented by Samples 1 and 2 are not associated significantly.
-

Working Hypothesis	Decision	Evidence (Table XII)
1	Rejected	Student's t significant.
2	Not rejected	
3	Rejected	F-statistic significant.
4	Not Rejected	
5	Rejected	Correlation coefficient not significant.
6	Not Rejected	

TABLE XX

MISSISSIPPI "CHAT": TESTS OF HYPOTHESES ABOUT
PARAMETERS: PRODUCED-WATER SAMPLES CF. R_W
CALCULATED BY R_{WA} METHOD

-
1. Ho: True mean, population represented by Sample 1 = true mean, population represented by Sample 2.
 2. Ha: True mean, population represented by Sample 1 \neq true mean, population represented by Sample 2.
 3. Ho: Variance, population represented by Sample 1 = variance, population represented by Sample 2.
 4. Ha: Variance, population represented by Sample 1 \neq variance, population represented by Sample 2.
 5. Ho: Populations represented by Samples 1 and 2 are associated (that is, variation in population represented by Sample 1 is accounted for significantly, or can be explained significantly by, variation in population represented by Sample 2).
 6. Ha: Populations represented by Samples 1 and 2 are not associated significantly.
-

Working Hypothesis	Decision	Evidence (Table VII)
1	Not Rejected	Student's t not significant.
2	Rejected	
3	Rejected	F-statistic not significant.
4	Not Rejected	
5	Rejected	Correlation coefficient not significant.
6	Rejected	

resistivities of produced-water samples apparently do not vary significantly through time. (Compare means and variances, and refer to t and F , Table XVII; see "Evidence," Table XVIII.) In the absence of direct measurement, R_w generally is rounded to the third decimal place for calculation of water saturation.) The two samples probably were drawn from one population, but the correlation coefficient is not significant (Table XVIII), indicating that variation in one variable does not account for much variation in the other (apparently, only about 10 percent (Table XVII and Figure 30)). The reasons for this difference are not understood by the writer. Periodic mixture of waters from some other formation with that of the Mississippi Chat could account for the differential variation, but that process would be unlikely to affect several wells, as suggested by the almost "random" plot of points in Figure 30.

Table XIX shows results of comparison of R_w from produced-water samples and R_w calculated from the SP curve. Working hypotheses 1, 3 and 5 were rejected; the two methods of estimating "true" R_w almost certainly do not refer to the same quantity. The mean and variance of R_{wsp} are much larger than the mean and variance of R_w measured from produced formation water (Table XVII). Figure 31 illustrates clearly the greater variation of R_{wsp} , by large scatter of points along the X-axis.

PRODUCED SAMPLE 1 VS. PRODUCED SAMPLE 2
MISSISSIPPI CHAT

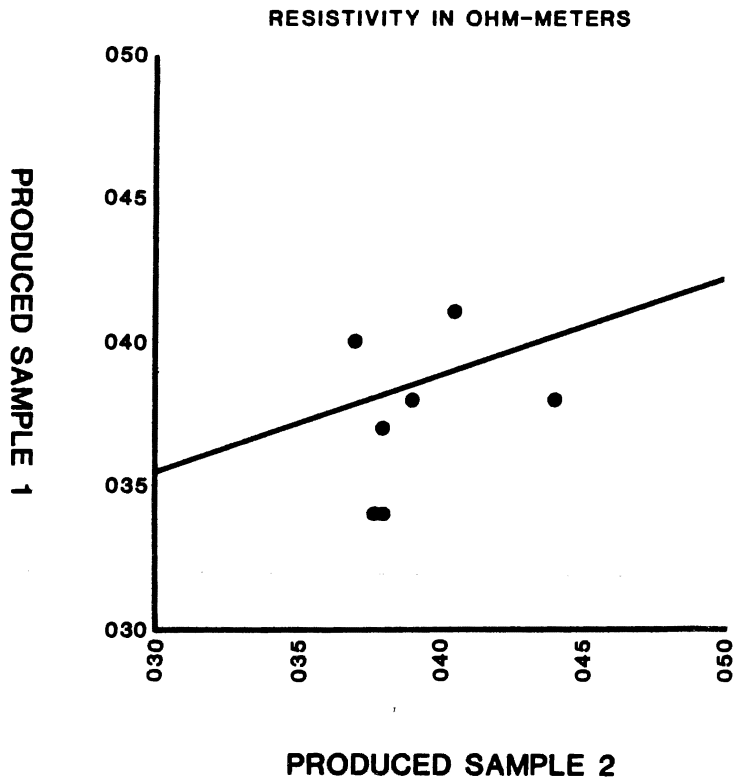


Figure 30. Scatter Diagram, Mississippi Chat, Resistivities of Produced-water Samples 1 and 2. Correlation Coefficient 0 3085. Coefficient of Determination 0 0952, Indicating That Only About 10 Percent of the Variation in One Variable Can Be Accounted for by Variation in the Other (Sokal and Rohlf, 1969, p 503)

\bar{X} OF PRODUCED SAMPLE 1 AND 2 VS. RW FROM THE SP
MISSISSIPPI CHAT
RESISTIVITY IN OHM-METERS

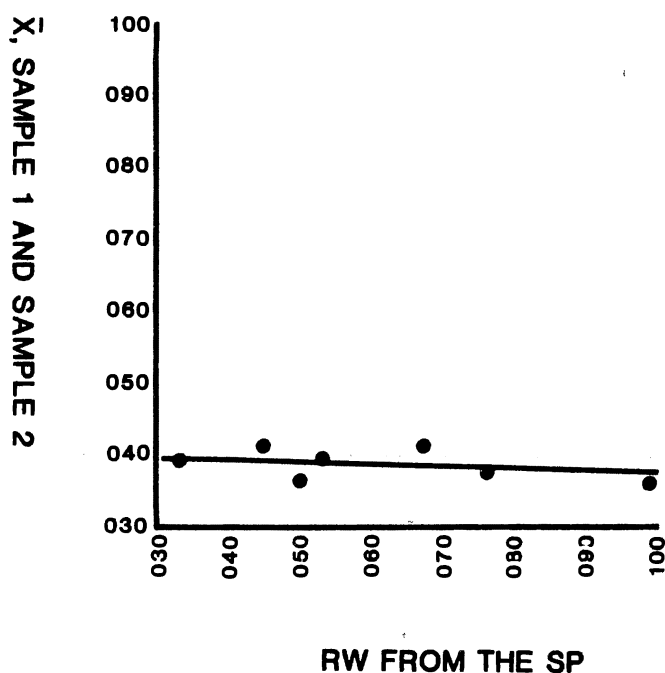


Figure 31. Scatter Diagram, Mississippi Chat, Mean Resistivity of Produced-water Samples 1 and 2 cf. Resistivities Estimated From SP Curve. Correlation Coefficient 0.3928. Coefficient of Determination 0.1543, Indicating That Only About One-Sixth of the Variation in One Variable Can Be Accounted for by Variation in the Other (Sokal and Rohlf, 1969, p 503)

Comparison of R_w calculated by the R_{wa} method with R_w from produced-water samples indicates that for practical purposes the R_{wa} method is reasonably good for approximating the mean (see Tables XVII and XX). Working hypothesis 1 was not rejected; both methods yielded R_w that rounds to 0.04 ohm-m. However, variance of R_w computed by the R_{wa} method is the greater (F-statistic, Table XVII; see also Figure 32, and compare scatter on X-axis with that on Y-axis).

Statistics for comparison of R_w computed from the SP curve and by the R_{wa} method are in Table XVII. The t-test is not significant, but the F-test indicates strongly that the samples represent different populations. The correlation coefficient is practically zero, indicating that the variables simply are not correlated.

As carried out in the course of this work and in treatment of this small sample, R_w of the Mississippi Chat computed from the SP curve is quite likely to overestimate the mean, but on the average, calculation of R_w by the R_{wa} method may yield a useful estimate.

Mississippi Limestone

Table XXI lists resistivities of samples of formation water and calculated R_w values from 25 oil wells in the Mississippi Limestone. Resistivities of water from wells 5, 6 and 8 were considerably different from sampling-time 1 to sampling-time 2. Resistivity ranges from 0.036 to 0.66

\bar{X} OF PRODUCED SAMPLE 1 AND 2 VS. RW FROM RWA
 MISSISSIPPI CHAT
 RESISTIVITY IN OHM-METERS

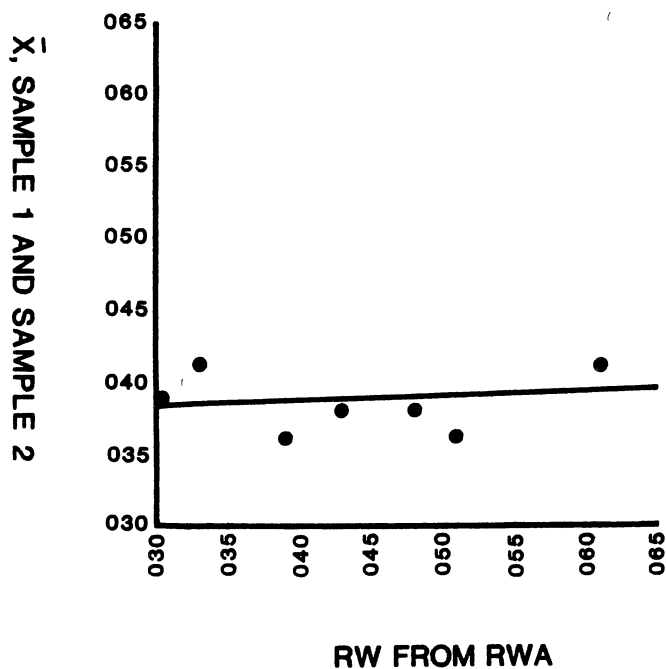


Figure 32 Scatter Diagram, Mississippi Chat, Mean Resistivity of Produced-water Samples 1 and 2 cf. Resistivities Estimated by Rwa Method Correlation Coefficient 0.0735 Coefficient of Determination About 0.005, Indicating That Effectively None of the Variation in One Variable Can Be Accounted for by Variation in the Other (Sokal and Rohlf, 1969, p. 503)

TABLE XXI
RESISTIVITIES OF FORMATION WATER,
MISSISSIPPI LIMESTONE

Well No	Produced Samples		SP Method	R _{wa} Method
	No 1	No 2		
1	039	038	090	182
2	040	040	.059	120
3.	036	037	.033	028
4	042	043	129	249
5.	039	049	107	324
6	048	066	125	185
7	045	046	130	163
8.	048	041	118	200
9.	039	044	100	195
10	039	039	106	322
11.	043	045	153	322
12	047	P&A	139	Y
13	048	046	172	164
14	050	048	038	256
15.	051	054	073	134
16.	036	.037	129	132
17.	042	.038	090	168
18	.039	044	095	172
19.	049	.047	.304	Y
20	046	.048	208	Y
21.	047	.047	X	Y
22.	.046	.047	060	645
23	046	.046	X	Y
24	.047	047	029	Y
25	045	046	X	Y

Resistivities in ohm-m , corrected to 100°F

X Resistivity logs not available

Y Porosity logs not available

P&A Plugged and abandoned

ohm-m. Figure 33 is a histogram of produced-water resistivities. Class mark of the modal class is 0.046 ohm-m; the distribution is slightly skewed to the right.

Resistivities computed from the SP curve range across a large interval, from 0.029 ohm-m to 0.304 ohm-m (Table XXI). The frequency distribution is skewed-right slightly, with a modal class mark of 0.125 ohm-m (Figure 34). R_{wa} measurements range from 0.028 ohm-m to 0.645 ohm-m; the frequency distribution is bimodal (Figure 35). Table XXII shows summary statistics from sampling of R_w . Working hypotheses 1 through 6 are set out in Tables XXIII, XXIV, and XXV.

Resistivities of samples of produced water did not vary significantly from time 1 to time 2 (see "Evidence," Table XXII). A cross-plot of resistivities (Figure 36) is a "loose" ellipse; a best-fit line has positive slope. In Table XXIV R_w from produced-water samples and R_w calculated from the SP curve are compared. Working hypotheses 1, 3 and 5 are rejected, evidence that the two methods of approximating R_w do not have to do with the same quantity. The mean and variance of R_{wsp} are much larger than those of produced-water samples (Table XXII). The disparity is illustrated well in Figure 37 (cf. spread of observations on X-axis with spread on Y-axis and clustering of data-points near 0.10 ohm-m). Obviously, R_w approximated from SP curves is an overestimate of the "true" R_w , measured best from samples of produced formation

PRODUCED SAMPLE WATER RESISTIVITIES:
MISSISSIPPI LIME

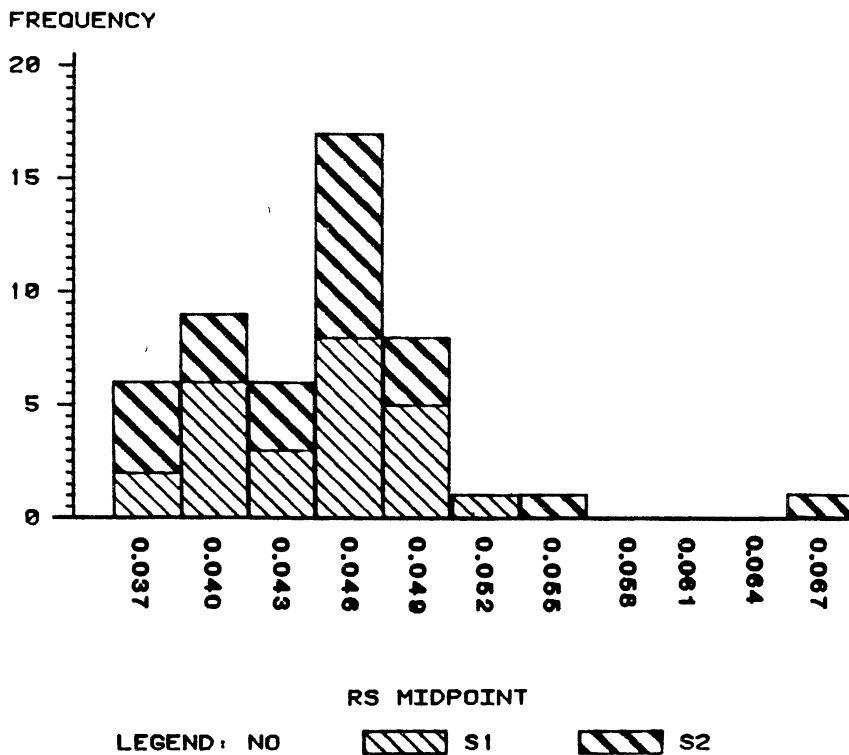


Figure 33 Histogram, Resistivities of Produced-water Samples, Mississippi Limestone
 Class Interval 0 003 Ohm-m Lowest
 Class Boundary 0 0355 Ohm-m

SP DERIVED WATER RESISTIVITIES:
MISSISSIPPI LIME

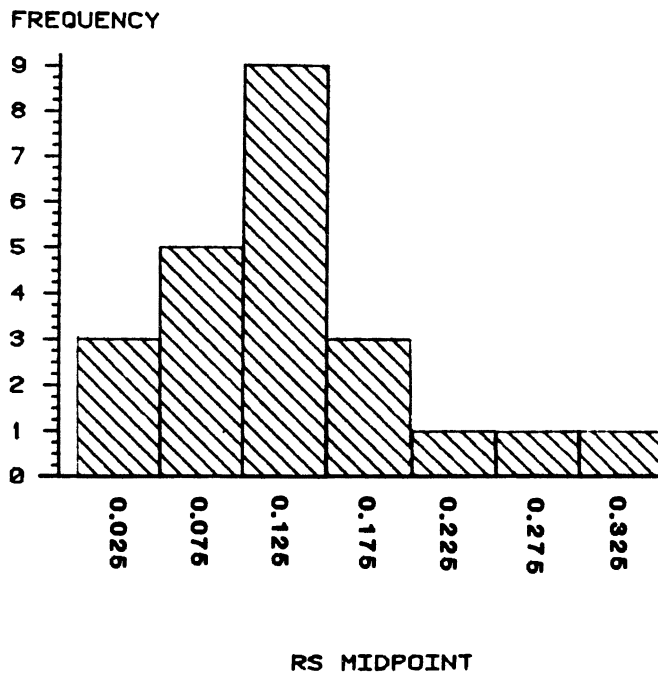


Figure 34. Histogram, SP-derived Water Resistivities, Mississippi Limestone. Class Interval 0.05 Ohm-m. Lower Class Boundary, Second Class 0.0505 Ohm-m. Class Marks Rounded to Two Significant Digits

RWA DERIVED WATER RESISTIVITIES:
MISSISSIPPI LIME

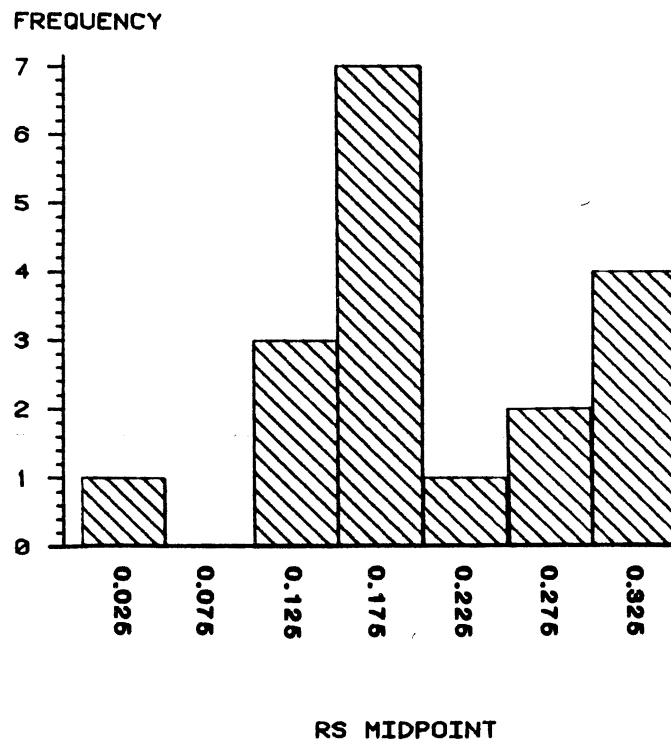


Figure 35. Histogram, Rwa-derived Water Resistivities, Mississippi Limestone. Class Interval 0.05 Ohm-m. Lowest Class Boundary 0 Ohm-m. Class Marks Rounded to Two Significant Digits

TABLE XXII

SUMMARY STATISTICS: MISSISSIPPI LIMESTONE

	Produced Sample 1	Produced Sample 2	Mean, Produced Samples	R_{wsp}	R_{wa}
n	24	24	21	18	18
Mean	.0438	.0451	.0442	.1003	.2256
Variance	.00001	.000038	.00027	.0014	.017

	Prod. Smpl. 1 cf. 2	Mean, Prod. Smpls. cf. R_{wsp}	Mean, Prod. Smpls. cf. R_{wa}	R_{wsp} cf. R_{wa}
r	.6394*	.2007	.2229	.0755
r#	.404	.433	.468	.468
r^2	.4088	.0403	.0497	.0057
F	1.9	147.9*	583*	12.14*
F#	2.01	2.12	~2.28	~2.28
df	20,20	20,20	17,17	17,17
t	.8095	5.41*	6.82*	3.59*
t#	~2.02	~2.02	~2.02	~2.02
df	47	44	40	38

TABLE XXIII

MISSISSIPPI LIMESTONE: TESTS OF HYPOTHESES ABOUT
PARAMETERS: PRODUCED-WATER SAMPLE 1 CF.
PRODUCED-WATER SAMPLE 2

-
1. Ho: True mean, population represented by Sample 1 = true mean, population represented by Sample 2.
 2. Ha: True mean, population represented by Sample 1 \neq true mean, population represented by Sample 2.
 3. Ho: Variance, population represented by Sample 1 = variance, population represented by Sample 2.
 4. Ha: Variance, population represented by Sample 1 \neq variance, population represented by Sample 2.
 5. Ho: Populations represented by Samples 1 and 2 are associated (that is, variation in population represented by Sample 1 is accounted for significantly, or can be explained significantly by, variation in population represented by Sample 2).
 6. Ha: Populations represented by Samples 1 and 2 are not associated significantly.
-

Working Hypothesis	Decision	Evidence (Table VII)
1	Not Rejected	Student's t not significant.
2	Rejected	
3	Not Rejected	F-statistic not significant.
4	Rejected	
5	Not Rejected	Correlation coefficient significant.
6	Rejected	

TABLE XXIV

MISSISSIPPI LIMESTONE: TESTS OF HYPOTHESES ABOUT
PARAMETERS: PRODUCED-WATER SAMPLES CF.
 R_W CALCULATED FROM SP CURVES

-
1. Ho: True mean, population represented by Sample 1 = true mean, population represented by Sample 2.
 2. Ha: True mean, population represented by Sample 1 \neq true mean, population represented by Sample 2.
 3. Ho: Variance, population represented by Sample 1 = variance, population represented by Sample 2.
 4. Ha: Variance, population represented by Sample 1 \neq variance, population represented by Sample 2.
 5. Ho: Populations represented by Samples 1 and 2 are associated (that is, variation in population represented by Sample 1 is accounted for significantly, or can be explained significantly by, variation in population represented by Sample 2).
 6. Ha: Populations represented by Samples 1 and 2 are not associated significantly.
-

Working Hypothesis	Decision	Evidence (Table XII)
1	Rejected	Student's t significant.
2	Not rejected	
3	Rejected	F-statistic significant.
4	Not Rejected	
5	Rejected	Correlation coefficient not significant.
6	Not rejected	

TABLE XXV

MISSISSIPPI LIMESTONE: TESTS OF HYPOTHESES ABOUT
PARAMETERS: PRODUCED-WATER SAMPLES CF.
 R_W CALCULATED BY R_{WA} METHOD

-
1. Ho: True mean, population represented by Sample 1 = true mean, population represented by Sample 2.
 2. Ha: True mean, population represented by Sample 1 \neq true mean, population represented by Sample 2.
 3. Ho: Variance, population represented by Sample 1 = variance, population represented by Sample 2.
 4. Ha: Variance, population represented by Sample 1 \neq variance, population represented by Sample 2.
 5. Ho: Populations represented by Samples 1 and 2 are associated (that is, variation in population represented by Sample 1 is accounted for significantly, or can be explained significantly by, variation in population represented by Sample 2).
 6. Ha: Populations represented by Samples 1 and 2 are not associated significantly.
-

Working Hypothesis	Decision	Evidence (Table XII)
1	Rejected	Student's t significant.
2	Not rejected	
3	Rejected	F-statistic significant.
4	Not Rejected	
5	Rejected	Correlation coefficient not significant.
6	Not rejected	

PRODUCED SAMPLE 1 VS. PRODUCED SAMPLE 2
MISSISSIPPI LIME
RESISTIVITY IN OHM-METERS

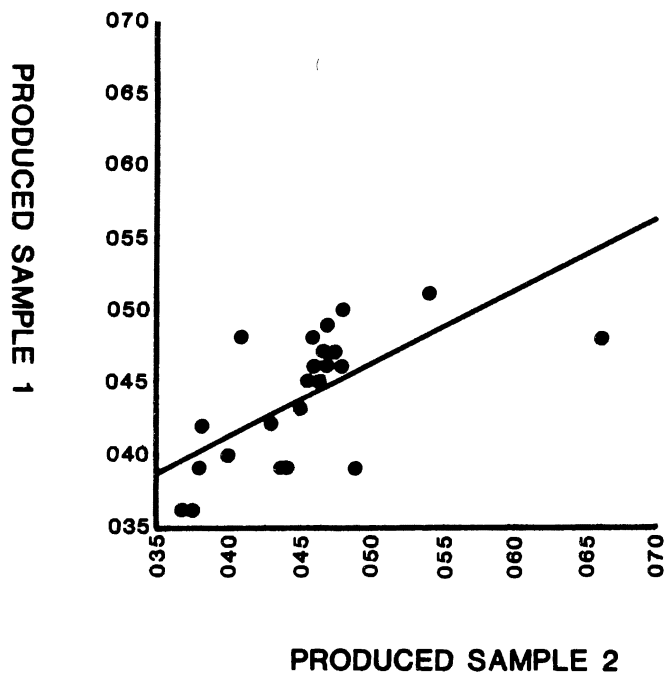


Figure 36. Scatter Diagram, Mississippi Limestone, Resistivities of Produced-water Samples 1 and 2. Correlation Coefficient 0.6394 Coefficient of Determination 0.4088, Indicating That About 40 Percent of the Variation in One Variable Can Be Accounted for by Variation in the Other (Sokal and Rohlf, 1969, p 503)

\bar{X} OF PRODUCED SAMPLE 1 AND 2 VS. RW FROM THE SP
MISSISSIPPI LIME
RESISTIVITY IN OHM-METERS

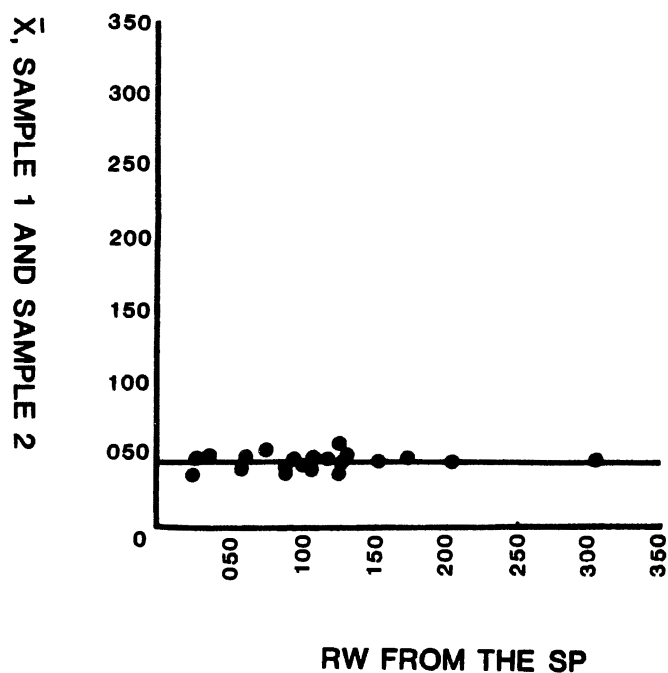


Figure 37 Scatter Diagram, Mississippi Limestone, Mean Resistivity of Produced-water Samples 1 and 2 cf. Resistivities Estimated From SP Curve. Correlation Coefficient 0.2007, Coefficient of Determination 0.0403, Indicating That Only a Few Percent of the Variation in One Variable Can Be Accounted for by Variation in the Other (Sokal and Rohlf, 1969, p. 503)

water. Almost everywhere in the study area the Mississippi Limestone is a thick formation with average porosity probably near 5 percent. Lithology and "tightness" of the formation suppress SP curves considerably, especially where compounded by undetected oil in the rock. Inaccuracy of measurement of R_w by R_{wsp} is inherent in the lithic composition of the Mississippi Limestone.

Computation of resistivity of water in the Mississippi Limestone by the R_{wa} method was ineffective. Comparison of means and variances of R_w of produced water and R_{wa} shows vast differences in estimates of the mean (~ 0.045 cf. ~ 0.23 ohm-m) and variance (~ 0.000025 cf. 0.017 ohm-m). Working hypotheses 1, 3 and 5 of Table XXV assert equality of (population) means and positive correlation of R_{wa} and R_w from produced-water samples. These hypotheses were rejected (see "Evidence," Table XXV). The greater variation of R_{wa} measurements and poor correlation of the variables are apparent from Figure 38, if one compares scatter of data along the X-axis with scatter along the Y-axis. Inasmuch as the reliability of calculations of R_{wa} depend on analysis of reservoir rock that is 100 percent water-saturated, the most probable source of error is a complex of undetected oil in the rock and low porosity, which elevate R_t ("true" resistivity of the formation as a unit), and perhaps an incorrect equation for the formation factor.

**\bar{X} OF PRODUCED SAMPLE 1 AND 2 VS. RW FROM RWA
MISSISSIPPI LIME
RESISTIVITY IN OHM-METERS**

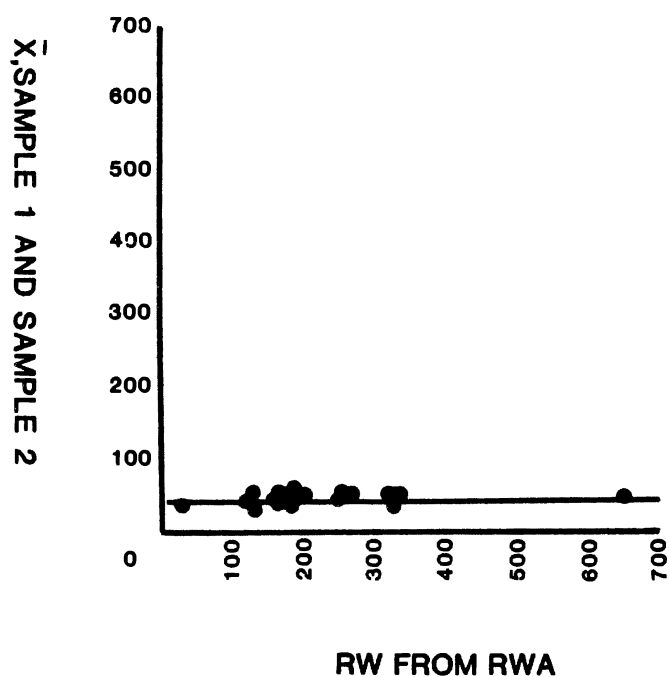


Figure 38. Scatter Diagram, Mississippi Limestone, Mean Resistivity of Produced-water Samples 1 and 2 cf. Resistivities Estimated by Rwa Method. Correlation Coefficient 0.2229. Coefficient of Determination 0.0497, Indicating That Only a Few Percent of the Variation in One Variable Can Be Accounted for by Variation in the Other (Sokal and Rohlf, 1969, p. 503)

In Table XXII statistics for comparison of R_{wsp} and R_{wa} are listed. The t- and F-tests suggest strongly that the population means and variances are different. The correlation coefficient is small and not significant ($r \sim 0.08$ cf. $r(\text{critical}) \sim 0.47$).

As computation of R_w by the R_{wsp} methods was conducted in this work, approximations of R_w are certain to be large overestimations and to lead to serious error if applied in practical work. (Not a likely event in any case.) In terrain where information about R_w of the Mississippi Limestone is insufficient, data should be drawn from analysis of samples of produced water.

CHAPTER IV

CONCLUSIONS

1. Samples of water produced from the formation of interest yield measurements of R_w that are more consistent (have smaller variances) than do approximations of R_w by the R_{wsp} or R_{wa} methods.
2. R_w measured from produced-water samples seems not to vary across time (in this case, a period of a few months).
3. True R_w of all five formations studied is almost certainly in the range of 0.035 - 0.045 ohm-m.
4. Mean resistivities of samples of produced water were markedly smaller than means computed by the R_{wsp} method. Estimated by the R_{wa} method, mean R_w values of the Red Fork and Bartlesville Sandstones were less than means of produced-water samples, whereas mean values of the Skinner Sandstone, Mississippi Chat and Mississippi Limestone were greater. This circumstance indicates that estimates of R_w by the R_{wsp} and by the R_{wa} method (except for the Red Fork and Bartlesville) are quite likely to lead to estimates of water saturation that are too large.
5. In order, chief sources of error in approximations by the R_{wsp} and R_{wa} methods probably were

undetected petroleum in rock assumed to be 100% water-saturated, shakiness of the reservoir, and inaccurate assessment of porosity.

6. R_w approximated by the R_{wsp} method was the most consistently erroneous.

7. Although samples analyzed in this study were relatively small, they show that measurements of R_w from produced-water samples are much to be preferred; approximations of R_w by the R_{wsp} and R_{wa} methods should be relied upon fully only where the analyst has cause to believe that formations are water-saturated and the effects of thin beds and shale are negligible or are corrected.

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APPENDIXES

APPENDIX A

INDEX NUMBERS, OPERATORS, AND LOCATIONS
OF WELLS SAMPLED FOR R_w of
SKINNER SANDSTONE

Well No.	Operator	Well	Location
1.	Earth Energy	Katz 1	31-19N-3E C NW-SE
2.	Earth Energy	Brock 1	31-19N-3E C NW-SW
3.	Wil-Mc	Cohee 1	7-17N-1E NE-SE-SW
4.	Wil-Mc	Schneider 1	10-18N-1W SW-SW-NW
5.	Wil-Mc	Min-Tex State 1	36-19N-2E SE/4
6.	Mackellar	Warren 1-B	25-21N-1W NE-NW
7.	Mackellar	Warren 1	25-21N-1W NW-NW
8.	Mackellar	Warren 2	26-21N-1W C NE-NE
9.	Mackellar	Warren 3-A	24-21N-1W S/2 SW-SE
10.	Mackellar	Warren 4	24-21N-1W C SW-SW-NW
11.	Mackellar	Chenowith 1	23-21N-1W NE SE
12.	Mackellar	Chenowith 2	23-21N-1W NW SE
13.	Mackellar	Chenowith 3	23-21N-1W SW SE
14.	Three Sands	Vitek 1	4-20N-2E NW-NE-NE
15.	Three Sands	Bar-W 1	33-21N-2E SE-NE-SE
16.	Three Sands	H.T. Smith 1-34	34-21N-2E SE-NE-SE
17.	T.N. Berry	Schnurr 2	30-20N-1W NW-NE-NW
18.	T.N. Berry	H.V. Berry 1	26-19N-2E NW-SE-SE
19.	T.N. Berry	Fee 7	13-18N-3E SE-NE-NW-SW

APPENDIX B

INDEX NUMBERS, OPERATORS, AND LOCATIONS
OF WELLS SAMPLED FOR RW OF
RED FORK SANDSTONE

Well No.	Operator	Well	Location
1.	El Dorado	Smotherman 18-1	18-25N-1E C NW-NW-NE
2.	El Dorado	Two Sands 27-9	27-25N-1W NE/4
3.	El Dorado	Getwell 1-1	1-23N-1W C NE-NE-NW
4.	El Dorado	Rolly 5,6,7, 10,13	31-25N-1W
5.	El Dorado	Rolly 2	32-25N-1W NW-NW-NW
6.	El Dorado	Two Sands 27-1	27-25N-1W C SE-SE-NW
7.	Settlers	Wall 2	10-17N-2E C SE-NE
8.	Kopco	Dotter 2	1-18N-2E C W/2 SW-SE
9.	Kopco	Kautz 1,3	4-18N-2E S/2
10.	Berry Op.	Wall 1A	10-17N-2E C SE-SW
11.	El Dorado	Rolly 18	36-25N-2W W/2 SE
12.	Wil-Mc	Church 1	18-19N-2E W/2 NE-SW
13.	Wil-Mc	Amoco-Amerada	23-19N-1E E/2 NW-NW
14.	Sun	Howard 1	26-18N-4E SE-NE-SE
15.	Sun	M.C. Howard 2	22-18N-4E SE-SW-SE
16.	Sun	Holderread 1	26-18N-4E C NW-SW
17.	Sun	Bellis 2	27-18N-4E SE-SE-NE
18.	T.N. Berry	Tully Fisher 1	24-19N-3E NW-NW-NE
19.	T.N. Berry	McVay 1	24-19N-3E SE-NE-NW
20.	Foster	Grant 1	18-19N-4E SE-SE-SW
21.	Foster	Soric 1	19-19N-4E NW-NW-NW
22.	T.N. Berry	Fee 9	13-18N-3E NE-NE-SW

APPENDIX C

INDEX NUMBERS, OPERATORS, AND LOCATIONS
OF WELLS SAMPLED FOR RW OF
BARTLESVILLE SANDSTONE

Well No.	Operator	Well	Location
1.	Bogert	Gloria 1-7	7-18N-1E SE-SE-SE
2.	Settlers	Tucker 4	12-18N-3E SW-SW-NE
3.	Settlers	Tucker 3A	12-18N-3E C NE-SE-SW
4.	Bogert	Minnie	18-18N-1E SE-SE-NE
5.	Kopca	Hale 1/Cooley 1	9-18N-2E SW-SE-NW
6.	Berry Op.	Gripe	3 21-18N-1E C SE/4
7.	Wil-Mc	Blair C-1	30-18N-2E E/2 SW-NE
8.	Wil-Mc	J.O.Williams 1-2	30-18N-1E NE-SE-NW
9.	Wil-Mc	Disney 1	25-18N-1E NE-SE-NE
10.	Wil-Mc	J.O. Williams 3	30-18N-1E N/2 SW-NW
11.	Wil-Mc	Bostion C-1	30-18N-2E SW-SE-NE
12.	Wil-Mc	Cowger 1	25-18N-1W NE-NE-NE
13.	Wil-Mc	Cowger 1-A	24-28N-1W SE-SE-NE
14.	Wil-Mc	Beck 1	30-18N-1E SW/4
15.	Foster	North Hall 1	31-18N-SE SW-NW-NE
16.	Sun	Crow 2	14-19N-SE SE-NE-NE
17.	Sun	Crow 5	14-19N-SE SE-NE-NE
18.	Sun	Crow 6	14-19N-SE SE-SE-NE
19.	Sun	M. Sherman 9	14-19N-SE NW-NE-SE

APPENDIX D

INDEX NUMBERS, OPERATORS, AND LOCATIONS
OF WELLS SAMPLES FOR RW OF
"MISSISSIPPI CHAT"

Well No.	Operator	Well	Location
1.	El Dorado	Rolly South 2	32-25N-1W C NW-NW-SW
2.	El Dorado	Rolly South 5	5-24N-1W NE-NE-NW
3.	El Dorado	Rolly Faith 1	34-26N-2W SE-SE-SE
4.	El Dorado	Rolly South 6	6-24N-1W NE-NW
5.	El Dorado	Rolly South 4	6-24N-1W C NE-NE-NE
6.	Wil-Mc	Guttschalk 1	12-20N-1W W/2E/2NW-SE
7.	Wil-Mc	Bezdichek 1	12-20N-1W NE-SE-SW

APPENDIX E

INDEX NUMBERS, OPERATORS, AND LOCATIONS
OF WELLS SAMPLED FOR RW OF
"MISSISSIPPI LIMESTONE"

Well No.	Operator	Well	Location
1.	El Dorado	Cales	27-25N-1E C N/2 NE-NW
2.	El Dorado	Robinson 33-1	33-21N-1E C NE-NE-SW
3.	El Dorado	Muegge 2	33-26N-3W C SE-NW-SE
4.	Settlers	McKenzie 11	13-18N-3E C SE-NE
5.	Settlers	Busch 1	24-18N-3E C SE-SW
6.	Settlers	Kaleidoscope 1	15-20N-2E NW-NW-NE
7.	Settlers	Randolph 3	12-18N-3E C-NW-SE
8.	Settlers	Telford 3	25-19N-2E SE-SE-SW
9.	Settlers	Shoup 1	20-18N-4E C-SE-SE
10.	Kopco	Peters 1	28-19N-2E NE-SW-SE
11.	Settlers	E.Townsdin 2/ West 1	14-18N-3E E/2
12.	Settlers	Fillmore 1	8-17N-5E C NW-NE
13.	Kopco	Rainwater 1	5-18N-4E SW-SW-SE
14.	Kopco	Thomas 1	20-19N-4E NE-NE-SW
15.	Berry Op.	Owsley 1	22-18N-1E SW-SW-NW
16.	Wil-Mc	Campbell 1	15-20N-1W S/2 NE
17.	Wil-Mc	Bowers A-1	1-20N-1W S/2SW-SW-SE
18.	Wil-Mc	Chase 1	1-20N-1W NE-SW-NE
19.	Sun	Bellis 1	27-18N-4E NE-NE-NE
20.	Sun	Holderread 2,3	26-18N-4E SW/4
21.	Sun	Broyles 2-1	23-18N-4E NE-SW-SE
22.	Sun	Broyles 2-3	23-18N-4E SE-SE-SE
23.	Sun	Broyles 3-2	26-18N-4E NW-SE-NW
24.	Sun	Schutkesting	20-18N-4E NW-SE-NW
25.	Foster	Stufflebeam 1	20-18N-4E NE-NW-NW

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