# DATABASE DEVELOPMENT AND CORRELATION FOR 

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## DATABASE DEVELOPMENT AND CORRELATION FOR MIXED CONVECTION HEAT TRANSFER IN HORIZONTAL TUBES



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## NOTATION

$C_{p}$ specific heat
$\mathrm{d}_{\mathrm{i}}$ internal diameter
Gr Grashof number, $g \rho^{2} \beta d^{3}\left(T_{w}-T_{b}\right) / \mu^{2}$
g acceleration due to gravity
h heat transfer coefficient
i subscript indicating peripheral position
j subscript indicating radial position
k thermal conductivity
L length
M half the mass flow rate
Nu Nusselt number, hdi/k
Pr Prandtl number, $\mathrm{C}_{\mathrm{p}} \mu / \mathrm{k}$
$\mathrm{q}_{\mathrm{w}}$ heat flux on inside tube wall
$\mathbf{R}$ inside tube radius
r radial co-ordinate
Ra Rayleigh number, GrPr
Re Reynolds number, $D \rho w_{b} / \mu$
S source term
T temperature
$\mathrm{T}_{\mathrm{b}}$ bulk temperature at any cross section
$\mathrm{T}_{\text {in }}$ uniform entrance temperature
$\mathrm{T}_{\mathrm{w}}$ average wall temperature
u velocity component in $\theta$ direction
v velocity component in $r$ direction
w velocity component in z direction
$w_{b}$ average axial velocity at a cross section
z axial coordinate

## GREEK SYMBOLS

B thermal expansion coefficient
T diffusion co-efficient
$\theta$ angular coordinate
$\mu$ viscosity
$\rho$ density
$\emptyset \quad$ a general dependent variable (may be $u, v, w$ or $T$ )

## CHAPTER I

## INTRODUCTION

Laminar flow heat transfer occurs in a wide range of engineering applications in the process and petrochemical industries. In most cases of its occurrence, it dominates the thermal performance of the heat exchanger because of the low heat transfer coefficients usually associated with laminar flow. Examples include heating of viscous oils, for transportation (e.g. lube oil coolers), thermosiphon heat exchangers and even in medical applications like the heating or cooling of blood during surgical operations.

Convection is defined as the transport of mass and energy by bulk fluid motion. It can be brought about in two ways. When a temperature induced density difference acts as the sole driving force for bulk motion, the process is called natural convection. On the other hand, if the bulk fluid motion is brought about by some external mechanical action (pumps or compressors), it is called forced convection. In reality however, the two occur together though in many applications natural convection is small when compared to forced convection. Nevertheless it exists.

The superpositioning of natural convection on forced convection is called mixed convection, and this is what this work deals with. A schematic is shown in Fig. 1 on the next page. So mixed convection may be considered as the general case and natural convection or forced convection may be treated as limiting conditions depending on the circumstances. For example, the natural convection process may dominate a solar collector heat exchange system, whereas forced convection may be the dominant factor in the pumping of a viscous crude.


Figure 1. Flow pattern inside a heated horizontal tube with mixed convection

Tube orientation is important when natural convection effects are pronounced. Many mixed convection applications involve horizontal tubes. In vertical tubes, the velocity due to the buoyant forces is in the same direction as (or exactly opposed to) the flow but this case is axisymmetric.

When fluids are being heated in a horizontal tube the buoyant force causes a circulation upward along the walls of the tube and down at the center as shown in Fig.1. This is the secondary flow and, in combination with the primary or forced flow, sets up a pair of counter-rotating vortices. The secondary flow increases the heat transfer coefficient and distorts the velocity profile. A similar effect is noted in cooling of
fluids except that the direction of rotation of the currents is reversed.
In a horizontal tube, the direction of the buoyancy forces and the flow are perpendicular to one another and so there is no axial symmetry. This situation is described by coupled non-linear partial differential equations and does not lend itself to straight-forward analytical treatment.

The analysis of mixed convection in horizontal tubes may be performed under two boundary conditions. The first is the uniform wall temperature (UWT) case. In this case the wall temperature is constant peripherally as well as axially. This case is approximated commonly in condensers, evaporators and any heat exchanger where one fluid has a very much higher thermal capacity and heat transfer coefficient than the other. This boundary condition can be represented by

$$
\mathrm{T}_{\mathrm{w}}=\text { constant }
$$

where $\mathrm{T}_{\mathrm{w}}$ is the average wall temperature
The temperature profile for this case is shown in Figure 2.
The other commonly encountered case is the uniform heat flux (UHF) case. The axial wall heat flux is constant and independent of $z$ and $\theta$ for that matter. Examples include


UISTANCE IN MEIERS

Figure 2. Temperature profile for the Uniform Wall Temperature (UWT) case
radiative heating and in countercurrent exchangers where the thermal capacitances of the two streams are not very different, i.e., $\mathrm{q}_{\mathrm{o}}{ }^{\prime \prime}=\mathrm{h}\left(\mathrm{T}_{\mathrm{w}}-\mathrm{T}_{\mathrm{b}}\right)=$ constant.

If $h$ is constant (as in fully developed flow), then $T_{W}-T_{b}=$ constant and therefore, $\mathrm{dT}_{\mathrm{w}} / \mathrm{dx}=\mathrm{dT}_{\mathrm{b}} / \mathrm{dx}$. The temperature profile of this case is shown in Figure 3.

In the UHF case, the temperature difference between the wall and the bulk fluid temperature exists throughout the tube and so secondary flow too persists throughout the length of the tube. In the UWT case the secondary flow develops to a maximum intensity and then diminishes to zero as the temperature difference ( $\mathrm{T}_{\mathrm{w}}-\mathrm{T}_{\mathrm{b}}$ ) decreases.

In industry, neither of these cases exists exactly and industrial situations almost always are an intermediate case.

Experimental investigation of simultaneously developing laminar flow and heat transfer profiles of variable property fluids with appreciable buoyancy effects in a uniformly heated horizontal tube has been carried out by Chen (1988). He used mixtures of diethylene glycol (DEG)-water as the test fluid.

A numerical solution to the same problem was developed by Zhang (1990). His analysis is based on the principles of three dimensional parabolic flow and uses a marching procedure. The numerical solution agrees well with the experimental data of Chen (1988) and hence seems to be a valid solution for the problem at hand.

The goal of the present work is to qualify Zhang's (1990) model as a working solution. Following this, the model is to be used over a wide range of cases to create a database. This database will be used in turn to develop a new correlation to evaluate the average heat transfer coefficient along the length of the tube for a given set of


Figure 3 - Temperature profile for the Uniform Heat Flux (UHF) case
conditions. The correlation must reflect the interacting effects of free and forced convection.

Numerous analytical solutions are available in the literature that consider either UHF or UWT, but the majority of them also assume constant property solutions or fully developed flow, neither of which is exactly true in reality. The real world, of course makes no distinction between the various cases mentioned above and the technique that most closely accounts for these situations is the one that approximates the 'real' heat exchange process.

## CHAPTER II

## LITERATURE SURVEY

Laminar flow heat transfer has been studied for over a century (Graetz (1883), Nusselt (1910)). More recently Shah and London (1978) devoted a book to this . The late 60's saw increased interest in this area and since then this field has been the subject of experimental, analytical and numerical investigation. The literature is well stocked with numerous analyses and correlations.

## Experimental investigations

Using water as the test fluid, Petukhov and Polyakhov (1967) studied mixed convection in an electrically heated horizontal tube (approximating the UHF condition). Numerous thermocouples were attached along the length of the tube at various axial and circumferential locations. The tube wall too was rotated to provide more accurate measurements of the circumferential temperature distribution. They used an upstream calming section and hence assumed a fully developed velocity profile. The experiments were performed over the 50 to 2400 range of Reynolds numbers and the 200,000 to $40,000,000$ range of Rayleigh numbers. Physical properties were measured at the axial local bulk temperature. Their results show that the heat transfer coefficient is higher in this case as compared to pure forced convection case.

Chen (1988) conducted an experimental study using a 3.95 m long stainless steel tube $(16.07 \mathrm{~mm}$ ID). A tube rolled into a tubesheet in a shell and tube heat exchanger presents either a square edged entrance or a slightly reentrant configuration. Chen's (1988) test
section was a circular horizontal tube with a square edged entrance. He used an axial entrance chamber to produce a uniform velocity distribution in the test fluid before it entered the test section. The entrance chamber was a 6 inch diameter acrylic plastic cylinder with three perforated plates perpendicular to the cylinder central axis. These perforations helped generate a uniform velocity profile.

The combination of the entrance chamber and the test section produced a square edged entrance for the test section thereby approximating the actual industrial condition. The tube was heated by electrical D-C current flowing through the wall. Thermocouples (either 4 or 8 per station) were placed on the outside surface to measure temperatures at 12 axial locations. Distilled water and diethylene glycol (DEG) solutions were used as the test fluids. The total of 48 experimental runs covered local bulk Reynolds numbers between 121 and 12,400 , Prandtl numbers between 3.5 and 285 and Grashof numbers between 930 and $104,000,000$. His setup is shown on the next page in Fig. 5.

Chen's (1988) data reduction procedure involved the calculation of the overall heat balance, local inside wall temperature and the local inside wall radial heat fluxes. The local heat transfer coefficients were then calculated.

For the overall heat balance, the rate of heat input was calculated from the electrical current and voltage ( $Q_{\text {input }}$ ). The heat absorbed $Q_{\text {output }}$ by the fluid was calculated from the flow rate, specific heat, inlet and outlet temperatures. From $Q_{\text {input }}$ and $Q_{\text {output }}$ the error in heat balance was calculated.

The inside wall temperatures corresponding to each thermocouple location was calculated using a two dimensional relaxation calculation, the details of which are provided in Chen's (1988) thesis.


Figure 5: Chen's (1988) experimental setup and thermocouple layout.

The local heat transfer coefficient is then calculated from the local wall temperature, inside wall heat flux and local bulk temperature using

$$
h_{j i}=q_{j i} /\left(T_{w j i}-T_{b j}\right)
$$

where the subscript ' j ' denotes the station number (axial location ) and ' i ' denotes the peripheral position.

## Analytical Studies

A theoretical analysis of the complete problem including simultaneously developing velocity and temperature profiles, variable property fluids and entrance effects is a forbidding task. Hence various workers have sought simplifications which at the same time steered the problem away from the real situation.

A number of researchers have proposed analytical solutions to the same problem. Prominent among them are Newell and Bergles (1970) and Farris and Viskanta (1969). Shah and London (1978) also summarize various analytical solutions very concisely in their book on laminar flow heat transfer.

It can be generally concluded that the main inadequacy of the theoretical solutions resulting in their failure to predict actual data accurately is their inability to take into account the variable physical properties and their assumption of a fully developed velocity profile.

## Numerical Solutions

Numerical solutions have the ability to incorporate the necessary equations to keep the problem as general as possible. A numerical solution due to Zhang(1990) solves both the problems of simultaneously developing profiles and variable property solutions for the UHF condition. He uses a cylindrical polar coordinate system and corresponding velocity
components. Symmetry about the vertical central plane is assumed and his solution is discussed at greater length in the next chapter.

Flows which are characterized by the absence of reverse flow can be treated as parabolic flows (which is what Zhang(1990) does). An important characteristic of parabolic flow is that downstream pressure fields have little effect on upstream flow conditions. This is tantamount to saying that downstream conditions do not propagate upstream to change conditions upstream,i.e., the flow is 'one-way'. This behaviour of parabolic flow is especially significant in that it allows Zhang (1990) to use a marching procedure starting at the entrance of the tube and working its way downstream across successive cross sectional planes in the direction of flow. This has the attendant benefit of reducing computer memory storage requirements.

Zhang (1990) solves the general differential equation for momentum and energy using the 3 D parabolic flow marching procedure with the following assumptions:
(1) Parabolic flow assumed in the $\mathbf{z}$ (flow) direction. (It may be clarified here that the term "parabolic" refers to the equations and should not be confused as describing the velocity profile. In fact the velocity profile is parabolic only in the limiting case).
(2) Steady state laminar flow
(3) Newtonian fluid with properties independent of pressure.
(4) Energy dissipation is neglected as a heat source. Zhang(1990) converts the general differential equations to a discretization equation and solves these equations to yield the solution. The numerical method calculates the values of a set of dependent variables (velocities, temperature, physical properties, etc.) at a set of chosen grid points. It also evaluates the local values for bulk temperature, average wall temperature and Nusselt number at each of 44 axial stations. It also has the flexibility to print out the temperature fields at any station.

## CHAPTER III

## THE PROGRAM - QUALIFICATION AND VERIFICATION

The study of laminar flow heat transfer in closed conduits was first made by Graetz (1883) and later by Nusselt (1910).

Their main assumptions were-
(1). Incompressible fluid flowing through a circular tube.
(2). Constant physical properties (invariant with temperature).
(3). Fully developed velocity profile and developing temperature profile.
(4). Negligible axial heat conduction and energy sources within the fluid.
(5). Newtonian fluid.

This resulted in the following energy equation,

$$
\frac{\partial^{2} T}{\partial r^{2}}+\frac{1 \partial T}{r \partial r}=\frac{\mu \partial T}{\alpha \partial x}
$$

with the boundary conditions -

$$
\begin{aligned}
& \mathrm{T}=\mathrm{T}_{\mathrm{e}}=\text { constant } \quad \text { for } \mathrm{x} \leq 0 \\
& \mathrm{~T}=\mathrm{T}_{\mathrm{w}} \text { constant at } \mathrm{r}=\mathrm{a} . . . . . . \text { constant wall temp. } \\
& \text { and } \frac{\partial T}{\partial r}=0 \quad \text { at } r=0 \ldots . \text { point of inflection at the axis of symmetry }
\end{aligned}
$$

The velocity profile is parabolic and is given by

$$
\begin{equation*}
u=u_{\max }\left(1-\mathrm{r}^{2} / \mathrm{R}^{2}\right) \tag{3-2}
\end{equation*}
$$

The above problem is referred to as the Graetz - Nusselt problem and its solution is presented in the form called the Graetz series. Sellers, et al. (1956) extended the work of Graetz and their independent solution is discussed later in this section. They considered both the uniform wall temperature and the uniform heat flux cases.

## The Problem

Zhang numerically (1990) solved the Graetz - Nusselt problem without the assumptions of constant physical properties and fully developed velocity profile and investigated the Uniform Heat Flux (UHF) condition. In a nutshell, Zhang's (1990) study can be described as an analysis of simultaneously developing laminar flow velocity and temperature profiles of variable property fluids in a horizontal heated tube.

The solution to the problem is the solution of the governing differential equations continuity, momentum and energy equation. For example, consider the $z$ component of the momentum equation and the energy equation -

## Energy equation

$\underset{r}{1} \frac{\partial}{\partial \theta}(\rho u T)_{+} \frac{1}{r} \frac{\partial}{\partial r}(\rho r v T)+\frac{\partial}{\partial r}(\rho w T)=\underset{r^{2}}{1} \frac{\partial}{\partial \theta}\left(\underset{C_{p}}{k} \frac{\partial T}{\partial \theta}\right)+\underset{r}{1} \frac{\partial}{\partial r}\left(\underset{C_{p}}{r} \underset{\sim}{k} \frac{\partial T}{\partial r}\right)$

## Z-component momentum equation

$\frac{1}{r} \frac{\partial}{\partial \theta}(\rho \mathrm{pu})_{+}+\frac{1}{\mathrm{r}} \frac{\partial}{\partial r}(\rho \mathrm{rvw})_{+} \frac{\partial}{\partial r}(\rho w w)=\frac{1}{\mathrm{r}^{2}} \frac{\partial}{\partial \theta}\left(\mu \frac{\partial w}{\partial \theta}\right)+\frac{1}{\mathrm{r}} \frac{\partial}{\partial r}\left(\mathrm{r} \mu \frac{\partial w}{\partial r}\right)-\frac{\partial \mathrm{p}}{\partial z}$

Zhang (1990) represented the above two equations (as one could with the other components of the momentum equation ) by the general model

$$
\frac{1}{r} \frac{\partial}{\partial \theta}(\rho u \phi)+\frac{1}{r} \frac{\partial}{\partial r}(\rho r v \phi)_{+} \frac{\partial}{\partial r}(\rho w \phi)=1_{r^{2}}^{1} \frac{\partial}{\partial \theta}\left(\Gamma \frac{\partial w}{\partial \theta}\right)+\frac{1}{r} \frac{\partial}{\partial r}\left(r \Gamma \frac{\partial w}{\partial r}\right)-\frac{\partial p}{\partial z}+S
$$

where $\phi$ denotes the dependent variables $\mathrm{u}, \mathrm{v}, \mathrm{p}, \mathrm{T}$ and w in sequence and $\Gamma$ is the diffusion coefficient. $S$ is the source term which represents any generation terms but can also be used to lump any parameters which do not conform to the format of the above general equation. Table $I$ includes the values of $S$ and $\Gamma$ for several equations.

## Zhang's Numerical Procedure

The general differential equation (equation 3-5) is reduced to the form of algebraic equations as recommended by Patankar (1980). Thus the differential equations are represented by a set of algebraic equations at each of the internal grid points. Then, these are solved to yield the solution of the differential equation. This simplification of differential equations to algebraic equations is what makes numerical methods powerful and widely applicable. Details of the solution procedure are available in Zhang's (1990) thesis.

## Verification

Before proceeding to the generation of the database, Zhang's (1990) program was run for a number of cases to compare against the existing experimental data of Chen (1988). Chen's (1988) experimental fluid was a water - diethylene glycol mixture (DEG solution). The subroutines for physical properties in Zhang's (1990) program used correlations to represent the effect of temperature and concentration on the physical properties.

The runs were made to test the program by varying the heat flux and inlet conditions (temperature, mass flow rate etc.) as per the experimental conditions that Chen(1988) tested.

TABLE 1
$\Gamma$ AND S FOR EACH VARIABLE

| Varable | 「 | Sc | Sp |
| :---: | :---: | :---: | :---: |
| u | $\mu$ | $-\frac{1 \partial \rho}{r \partial \theta}+\frac{2 \mu \partial v}{r^{2} \partial \theta}+\rho g \beta\left(T_{w}-T\right) \sin \theta$ | $-\frac{\mu}{r^{2}}-\frac{\rho v}{r}$ |
| v | $\mu$ | $-\frac{\partial \mu}{\partial r}-\frac{2 \mu \partial u}{r^{2} \partial \theta}+\frac{\rho u^{2}}{r}-\rho g \beta\left(T_{w}-T\right) \cos \theta$ | $-\frac{\mu}{r^{2}}$ |
| T | $k / C p$ | $S c(i, M 1)=\mathrm{q}^{\prime \prime} / \mathrm{Cp}$ |  |
| w | $\mu$ | - dp/dz |  |

For the runs performed the bulk temperature calculated from the heat balance was compared against that calculated from numerical computation. The two compared well. The bulk temperature at any axial position from the heat balance was calculated using

$$
\begin{equation*}
\mathrm{T}_{\mathrm{b}}=\mathrm{T}_{\mathrm{in}}+\left(\mathrm{q}_{\mathrm{w}}\right)\left(\pi \mathrm{d}_{\mathrm{i}} \mathrm{z}\right) / 2(\mathrm{M})\left(\mathrm{C}_{\mathrm{p}}\right) \tag{3-6}
\end{equation*}
$$

where $\mathrm{q}_{\mathrm{w}}$ is the uniform heat flux.
$\mathbf{M}$ is half the mass flow rate (this accounts for the use of the ' 2 ' in the above heat balance equation).

The bulk temperature from the numerical computation was calculated by dividing the calculation domain into a number of smaller control volumes and an energy balance was then applied. This resulted in

$$
\begin{equation*}
\mathrm{T}_{\mathrm{b}}=\iiint_{\mathrm{w}} \operatorname{Tr} d r d \theta / \iint_{\mathrm{wr} d r d \theta} \tag{3-7}
\end{equation*}
$$

where w and T are the computed local values and the double integral represents the integration over the cross sectional area. The calculation domain consists of only a vertical semicircle since symmetry about the vertical plane is assumed. Chen's (1988) experiment used thermocouples symmetrically around the tube and his results justify the assumption of symmetry across the central vertical plane. Data from one of Chen's (1988) runs is listed in Table II to illustrate this point.

Another important parameter was the average inside wall temperature. Once the temperature field at any axial station was calculated, the average inside wall temperature was calculated by taking an arithmetic average of temperatures of the outermost nodes of the field (which coincided with the inside wall) .

Variations of the computed bulk temperatures with length have been shown along with the corresponding data for the heat balance bulk temperature. The numerically

## TABLE II

PERIPHERAL WALL TEMPERATURE VARIATION FOR RUN 2132 $\operatorname{Re}=438 ; \operatorname{Pr}=50.8 ; \operatorname{Gr}=15895 ; \mathrm{Flux}=6880 \mathrm{~W} / \mathrm{m}^{2}$

computed bulk temperature (from equation 3-7) almost coincides with the heat balance bulk temperature (equation 3-6) indicating the validity of the numerical scheme.

Figures $6,7,8$ show these variations for three different runs.
The local peripheral average Nusselt number $\mathrm{Nu}_{\mathrm{z}}$ is defined as

$$
\begin{equation*}
\mathrm{Nu}_{\mathrm{z}}=\mathrm{q}_{\mathrm{w}} \mathrm{~d}_{\mathrm{i}} /\left(\mathrm{k}\left(\mathrm{~T}_{\mathrm{w}, \mathrm{avg}}-\mathrm{T}_{\mathrm{b}}\right)\right) \tag{3-8}
\end{equation*}
$$

where k is the thermal conductivity of the fluid at the local bulk temperature $\mathrm{T}_{\mathrm{b}}, \mathrm{d}_{\mathrm{i}}$ is the inside diameter, and $\mathrm{T}_{\mathrm{w}, \text { avg }}$ is the peripheral mean inside wall temperature of the tube. The variation of Nusselt number along the length has been shown in Figures 9, 10, and 11 for three runs.

The runs are in increasing order of Grashof numbers, the higher Grashof numbers signifying stronger natural convection effects. Zhang (1990) explains that stronger natural convection effects would mean increased variation of peripheral wall temperature and thus considerable variation of peripheral wall heat flux; hence the assumption of uniform heat flux may not be totally accurate. Thus we see higher deviations from experimental results at higher Grashof numbers.

## Fully Developed Flow

An important check on the program was to see whether it would achieve the theoretical asymptotic Nusselt number value of 4.36 [Sellers, et al.(1956)]under conditions of fully developed pure forced convection.with constant properties and constant wall heat flux.

The applicable differential energy equation is equation 3-1. The only difference is that UWT boundary condition ( $\mathrm{T}=\mathrm{T}$ w at $\mathrm{r}=\mathrm{a}$ ) is replaced by

$$
k\left(\frac{\partial T}{(\partial r)_{w}}=q_{w}{ }_{w}=\right.\text { constant }
$$



Figure 6: Run \# 2119 - Comparison of temperature profiles.
Re 1190-1300; Pr 226-206; Gr 980-3480.
A-Experimental data ; B-Predicted data


Figure 7: Run \# 2105 - Comparison of temperature profiles
Re 354-585; Pr 209-128; Gr 3480-12650.
A-From heat balance ; B-Predicted data


Figure 8: Run \# 2107 - Comparison of temperature profiles.
Re 222-452; Pr 220-111; Gr 3450-15700.
A-Experimental data ; B-Predicted data


Figure 9: Run \# 2107 - Comparison of Nusselt number profiles.
Re 222-452; $\operatorname{Pr}$ 220-111; Gr 3450-15700.
A-Experimental data ; B-Predicted data


Figure 10: Run \# 2110 - Comparison of Nusselt number profiles.
Re 1360-1808; $\operatorname{Pr}$ 115-87; Gr 8300-34000.
A-Experimental data ; B-Predicted data


Figure 11: Run \# 2143 - Comparison of Nusselt number profiles.
Re 1155-1555; Pr 20.7-15.2; Gr 22800-82600.
A-Experimental data ; B-Predicted data

This differential equation is solved with a uniform heat flux boundary condition. The separation of variables technique and Sturm - Liouville theory have been used by Sellers et al. (1956) among others to obtain an eigenvalue solution.

The solution is
$N u_{x}=\left[(11 / 48)+0.5 \sum C_{n} R_{n} \exp \left(-2 B_{n}{ }^{2} X^{*}\right)\right]^{-1}$

The values of $B_{n}$ and $C_{n} R_{n}$ are excerpted from Shah and London (1978) and are listed in Table III. Shah and London (1978) have reviewed a number of different solutions to the Graetz-Nusselt problem under various other boundary conditions and non-circular duct geometries.

The theoretical variation of the local Nusselt number under conditions of pure forced convection and fully developed laminar flow are also excerpted from Shah and London (1978) and listed in Table IV. A comparison between the output from Zhang's (1990) program and the theoretically obtained values against dimensionless distance is shown in Figure 12.

After a certain initial length, the agreement between the model and the theoretical analysis is excellent. Close to the entrance of the tube this is not the case. There is a noticeable difference between the computer output and theoretical analysis. This difference is shown magnified on a log-log scale in Fig 13. This region of discrepancy exists to $\mathrm{X}^{*}=0.01$, which is less than 0.3 m for a tube of 0.016 meter diameter and is not of critical importance.

Having passed this test, the program is now considered a qualified and verified tool for the generation of the database.

## TABLE III

## $B_{\mathbf{I}}$ AND $C_{1} R_{\mathbf{R}}$ YALUES TO BE USED IN EQUATION 3-10

| $n$ | $B_{n}^{2}$ | $-C_{n} R_{n}$ |
| :--- | :---: | :---: |
| 1 | 25.6976 | 0.1987 |
| 2 | 83.8617 | 0.0692 |
| 3 | 174.166 | 0.0365 |
| 4 | 296.536 | 0.0230 |
| 5 | 450.947 | 0.0160 |
| 6 | 637.387 | 0.0119 |
| 7 | 855.849 | 0.0092 |
| 8 | 1106.32 | 0.0074 |
| 9 | 1388.32 | 0.0061 |
| 10 | 1703.32 | 0.0051 |
| 11 | 2049.84 | 0.0043 |
| 12 | 2428.36 | 0.0038 |
| 13 | 2838.89 | 0.0033 |
| 14 | 3281.43 | 0.0029 |

## TABLE IV

YARIATION OF NUSSELT NUMBER WITH
DIMENSIONLESS DISTANCE. SHAH AND LONDON (1978)

| $X^{*}$ | $\mathrm{Nu}_{x}$ |
| :---: | :---: |
| 0.000001 | 129.20 |
| 0.000002 | 102.36 |
| 0.000004 | 81.062 |
| 0.000006 | 70.707 |
| 0.000008 | 64.167 |
| 0.000010 | 59.510 |
| 0.000020 | 47.077 |
| 0.000040 | 37.224 |
| 0.000080 | 29.422 |
| 0.000200 | 21.555 |
| 0.000400 | 17.048 |
| 0.000800 | 13.506 |
| 0.002000 | 9.9863 |
| 0.004000 | 8.0200 |
| 0.008000 | 6.5359 |
| 0.020000 | 5.1984 |
| 0.040000 | 4.6213 |
| 0.080000 | 4.3949 |
| 0.200000 | 4.3637 |



Figure 12: Run \# 21AS4 - Comparison of Nusselt number profiles.under conditions of fully developed flow and constant physical properties.
Re 177.1; Pr 12.2
A-Theoretical data [Shah and London (1978)] ; B- Predicted data


Figure 13: Run \# 21AS4 - Variation of Nusselt number with dimensionless distance at the beginning of the tube under conditions of fully developed flow and constant physical properties; $\operatorname{Re} 177.1 ; \operatorname{Pr} 12.2$
Theoretical values [Shah and London (1978)] ; B- Predicted data

## CHAPTER IV

## DATA GENERATION

With a validated numerical program, it was attempted to build a database over a sufficiently wide range of parameters representing industrial interest. This was to be used later to generate an improved heat transfer correlation which reflected the major phenomena (such as developing flow and variable properties) and yet remained succinct enough to be used directly in engineering design.

## The Computer Experiment

In effect a "computer experiment" was performed using Zhang's (1990) model over a range of dimensionless groups. For this, choice of test fluid was important. Many industrial fluids such as heavy oils have viscosities high enough to keep them in the laminar region even at moderately high flow rates. Heat transfer rates too in such cases are relatively low and so design of large surfaces for such fluids implies careful and accurate thermal sizing with a minimum of overdesign.

The test fluid should also have physical properties which are typically strong functions of temperature so that the effect of variable properties may be reflected in the correlation. Glycerine-water is one such example and was chosen. It is viscous enough to keep it in the laminar region at moderate flow rates and also to satisfy the above requirement. In fact its viscosity was over an order of magnitude more at the bulk temperature as compared to the wall at the wall for some runs.

Thermodynamic and transport properties of most fluids vary with temperature and this will cause the properties to vary not only along the length of the tube but also in the
radial direction. For many liquids, Cp and $\mathbf{k}$ are not strong functions of temperature. Viscosity however varies very significantly.

Even small changes of density with temperature are enough to produce strong natural convection effects. Viscosity however varies very significantly. Viscosity variations of $100 \%$ glycerine as a function of temperature is shown in Figure 15.

The density changes cause the natural convection and secondary flow. Constant density implies absence of natural convection effects, and results from a run with constant properties are shown in Fig. 14 to show the inadequacy of the constant property assumption.

The classic Sieder-Tate (1936) factor $\left(\mu_{\mathrm{b}} / \mu_{\mathrm{w}}\right)^{0.14}$ has been widely accepted in literature to account for the effect of the variation of viscosity with temperature on pressure drop and heat transfer coefficient. The factor is a ratio of the bulk viscosity to the viscosity at the wall and is greater than one for liquids being heated and less than one for liquids being cooled. The viscosity-temperature relationship of glycerine is shown in Fig. 15 and the high temperature sensitivity of the $100 \%$ solution can be clearly observed.

Having so far realized that physical properties play an important role in the solution of the problem, an attempt was made to get 'good' data to incorporate in the program.

Most physical properties of glycerine-water mixtures were available in Gallant (1968), though not in the form of correlations. Properties were also referenced from Lawrie (1928) and Segur(1951). For implementation into the computer these curves had to be reduced to correlations. For this purpose a nonlinear regression procedure was carried out using an available software package(NLR). Data for all physical properties was regressed and correlations were developed. It may be mentioned here that the physical properties only need to be representative and not exact values. Of course the correlations must be well behaved and not display any abnormalities or sharp deviations within the range. This work does not attempt to solve the problem


Figure 14: Run \# R60×30x01 - Comparison of Nusselt number profiles.
Re 1155-1555; Pr 20.7-15.2; Gr 22800-82600.
A- Variable property run ; B-Constant property run


Figure 15. Variation of the viscosity of $100 \%$ glycerine with temperature
specifically for the case of glycerine and water. Hence even "ball park" values would have sufficed.

Most of the correlations are five or six parameter functions. These correlations were introduced into Zhang's (1990) program as subroutines. Some minor modifications were also made in Zhang's (1990) program to make the program a little more interactive and also output the values of dimensionless numbers at each station. This was done to aid both in getting a feel for the variation of the dimensionless groups and in the development of the correlation.

The 'computer experiment' was now ready to be performed. A range of values of parameters had to be defined within which the experiment was to be performed. These were defined in terms of the Prandt and Grashof numbers. The Reynolds number of course is bounded by the laminar flow condition. A limit on the Grashof number was reached because of the properties of the glycerine- water mixture. The Grashof number limit was extended by increasing the diameter of the pipe. Larger diameters required finer circumferential grids to keep the program stable. This implied more computer time. The computer time also went up steeply with tighter grid spacing. It was also necessary to recognize the various upper temperature limits of the physical property correlations of the water-glycerine mixtures to avoid erratic behaviour.

Computer runs were performed for cases varying the temperature, glycerine concentration, wall heat flux, flow rate and diameter and the results from these constituted the database.

## Grid Selection

Grid density was an important parameter to be selected. The program can easily be made interactive to accept any given choice of circumferential grid. Grid size is related to both the criterion of convergence and the computer time to achieve convergence. The
grids in ' $r$ ' and ' $\theta$ ' direction varied from $15 \times 15$ to $23 \times 23$. The number of axial locations was fixed by Zhang as 44.

The computer time needed by the $15 \times 15$ and $23 \times 23$ grids was very different. The $15 \times 15$ required just 7 minutes of C.P.U time on the VAX 6320, whereas the $23 \times 23$ grid required 60 minutes on the same system. The Nusselt number variation along the length of the tube for a particular run using $15 \times 15$ and $23 \times 23$ grids is shown in Fig 16. The difference between the two grid sizes in terms of Nusselt number is minimal; nevertheless most of the runs were carried out using $23 \times 23$ grids. For some runs the $15 \times 15$ grid would cause the run to become unstable resulting in wildly fluctuating values. Also for tube diameters 0.032 m or higher, even a $23 \times 23$ grid was insufficient and a $27 \times 27$ grid was employed. This was how the database was created and as Bell (1990) has suggested the database was kept as dense as possible to minimize local discontinuities within the manifold. He also emphasized the need for physical meaning in a correlation and not mere mathematical manipulation.

Some observstions about the runs

For each run, the temperature profile and the variation of Nusselt number with tube length were plotted. The Nusselt number showed the initial sharp fall and then the expected increase as natural correction effects began to take effect. The data gathered from all the runs constitute the database. Some data indicating details of the runs and the important parameters considered are shown in Table V.

For some runs the viscosity of glycerine (at high concentrations) changes steeply in the high temperature range and the flow rate had to be carefully chosen to keep the runs within the laminar region.


Figure 15: Run \# R60x30x01 - Comparison of Nusselt number profiles. Re 1155-1555; Pr 20.7-15.2; Gr 22800-82600. A- $23 \times 23 \times 44$ grid run ; B- $15 \times 15 \times 44$ grid run

The flux for low concentrations of glycerine had to be kept high to see marked natural convection effects. This is probably due to the fact that the specific heat (and thermal conductivity) of low concentrations of glycerine is high (approaching that of water for very low concentrations) and a higher flux was needed to cause a significant temperature difference between wall and bulk fluid and thus cause a density difference driving force.

## TABLE V

IMPORTANT PARAMETERS OF THE DATABASE

| Run <br> Number | Heat <br> Flux <br> W/m2 | Bulk <br> Temp. <br> in C | Bull <br> Temp. <br> out C | Average <br> Reynolds <br> Number | Average <br> Pranda <br> Number | Average <br> Grashof <br> Number |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| R20X40X01 | 1500 | 40.0 | 52.0 | 746.2 | 5.2 | 43543 |
| R40X30X01 | 12200 | 30.0 | 51.3 | 1831 | 118 | 12235 |
| R40X60X02 | 7000 | 60.0 | 83.7 | 1755.8 | 6.1 | 211928 |
| R40X20X02 | 7000 | 20.0 | 44.6 | 726.7 | 14.9 | 49083 |
| R100X100X02 | 6000 | 100 | 114.2 | 304.9 | 78.5 | 4324 |
| R60X40X01 | 6000 | 40.0 | 56.6 | 626.6 | 25.8 | 22468 |
| R100X50X02 | 17000 | 50.0 | 71.6 | 90.2 | 5197 | 289.7 |
| R60X50X01 | 12200 | 50.0 | 61.9 | 2202 | 206 | 60492 |
| R20X85X03 | 250 | 85.0 | 89.0 | 1431 | 1.3 | 17404 |
| R60X30X01 | 12200 | 30.0 | 42.2 | 1153 | 39.3 | 21000 |
| R20X20X03 | 2000 | 20.0 | 36.1 | 583.7 | 6.9 | 22740 |
| R75X50X01 | 12200 | 50.0 | 63.5 | 718.5 | 65.6 | 9937 |
| R60X50X02 | 10200 | 50.0 | 62.2 | 1602.5 | 20.9 | 54967 |
| R40X60X03 | 5600 | 60.0 | 83.7 | 1268.1 | 6.1 | 184300 |
| R80X20X01 | 12200 | 20.0 | 34.2 | 160.2 | 263 | 810.9 |
| R20X40X01 | 1500 | 40.0 | 52.0 | 673.5 | 5.3 | 49095 |
| R60X30X02 | 18300 | 30.0 | 48.2 | 1195 | 34.3 | 34913 |
| R60X30X03 | 22400 | 30.0 | 52.2 | 1276.4 | 32.1 | 45065 |
| R60X30X04 | 44800 | 30.0 | 73.1 | 1756.8 | 32.3 | 124940 |
| R60X30X05 | 33600 | 30.0 | 62.8 | 1508.3 | 27.2 | 79388 |

## CHAPTER V

## DEVELOPMENT OF THE CORRELATION

This section deals with the development of a heat transfer correlation using the data available from the database. The other correlations for the simultaneously developing temperature and velocity profiles under uniform heat flux (UHF) were by Chen (1988) and Zhang (1990). Both these yielded local Nusselt numbers and thus would be of limited use to the heat exchanger designer. Nevertheless these correlations were integrated over the length of the tube and were tested to see whether values close to the database values were obtained.

## Format followed by Zhang and Chen

Both correlations however do extrapolate to the theoretical asymptotic Nusselt number value of 4.364 as L approaches infinity (and if natural convection is omitted). A note on the basic format of the two correlations is in order.

The correlations take into account the contributions by forced convection, natural convection, entrance effect and variable property solutions. Assuming that forced convection and natural convection components are additive, the basic format of both Chen's (1988) and Zhang's (1990) correlations are :

$$
\begin{equation*}
\mathrm{Nu}=\left[4.364+\mathrm{C} 1 \mathrm{Re}^{\mathrm{c} 2} \mathrm{Pr}^{\mathrm{c} 3}\left(\mathrm{~d}_{\mathrm{i}} / \mathrm{L}\right)^{\mathrm{c} 4}+\mathrm{C} 5(\mathrm{GrPr})^{\mathrm{c} 6}\right]\left(\mu_{\mathrm{b}} / \mu_{\mathrm{w}}\right)^{0.14} \tag{5-1}
\end{equation*}
$$

The first term of the expression is the theoretical asymptotic value for pure fully developed forced convection with constant heat flux. This calculation can be obtained from any convection text such as $\operatorname{Kays}(1966)$. The second term involving the

Reynolds and Prandtl numbers is the developing forced convection term (which decays as $\mathrm{L}-->\infty$ ). Chen (1988) introduces an exponential function of the type $[1+\mathrm{A} \exp (-$ $\left.\left.\mathrm{BX} / \mathrm{d}_{\mathrm{i}}\right)\right]$ to account for the entrance effect on forced convection.

The third term is the natural convection term and is a function of Gr and Pr. Zhang (1990) argues that axial position has little effect on natural convection and hence does not introduce an entrance effect term to augment the natural convection term. Chen (1988) however introduces an exponential function ( $1-\exp \left(-\mathrm{CX} / \mathrm{d}_{\mathrm{i}}\right)$ ) to account for the entrance effect on natural convection.

The conventional Sieder-Tate viscosity correction factor $\left(\mu_{\mathrm{b}} / \mu_{\mathrm{w}}\right)^{0.14}$ has been included in both correlations.

Chen's (1988) correlation for the local Nusselt number is

$$
\begin{align*}
& \mathrm{Nu}_{\mathrm{x}}=\left[4.364+0.00106 \operatorname{Re}^{0.81} 1_{\mathrm{Pr}^{0} \cdot 25}^{\left(1+14.0 \exp \left(-0.063 \mathrm{X} / \mathrm{d}_{\mathrm{i}}\right)\right)+}\right. \\
& \left.0.268(\mathrm{Gr} \mathrm{Pr})^{0.25}\left(1-\exp \left(-0.042 \mathrm{X} / \mathrm{d}_{\mathrm{i}}\right)\right)\right]\left(\mu_{\mathrm{b}} / \mu_{\mathrm{w}}\right) 0.14 \tag{5-2}
\end{align*}
$$

which is base on data taken over the following range of parameters
$121<\operatorname{Re}<2100$
$3.5<\operatorname{Pr}<282.4$
$930<\mathrm{Gr}<67300$
Zhang's (1990) correlation is
$N u_{x}=\left[4.364+0.1 \operatorname{Re}^{0.387{ }_{P_{T}} 0.415}\left(\mathrm{~d}_{\mathrm{i}} / \mathrm{L}\right)^{0.147}+0.11(\mathrm{GrPr}){ }^{0.3}\right]\left(\mu_{\mathrm{b}} / \mu_{\mathrm{w}}\right)^{0.14}$
which is based on calculations covering the following range of parameters
$1<\left(\mu_{\mathrm{b}} / \mu_{\mathrm{w}}\right)<5$
$4<\operatorname{Pr}<270$
$100<\operatorname{Re}<2500$
$1500<\mathrm{Gr}<200,000$
$50<L / d_{i}<300$

Both Zhang's (1990) and Chen's (1988) correlations were integrated assuming constant $\mathrm{Re}, \mathrm{Pr}$ and Gr over the length of the tube and these expressions are listed below.

Chen's average Nusselt number is given by

$$
\begin{align*}
N u & {\left[4.364+0.00106 \operatorname{Re}^{0.8} \operatorname{Pr}^{0.6}\left\{1+222\left(d_{i} / L\right)-222\left(d_{i} / L\right)\right.\right.} \\
& \exp \left(-0.063\left(L / d_{i}\right)\right\}+0.268(G r \operatorname{Pr}) 0.25 \\
& \left.23.8\left(d_{i} / L\right) \exp \left(-0.042\left(L / d_{i}\right)\right\}\right]\left(\mu_{b} / \mu_{w}\right) \tag{5-4}
\end{align*}
$$

Zhang's average Nusselt number is given by

$$
\begin{align*}
& \mathrm{Nu}=\left[4.364+0.1172 \operatorname{Re}^{0.387} \operatorname{Pr}^{0.415}\left(\mathrm{~d}_{\mathrm{i}} / \mathrm{L}\right)^{0.147}+0.11(\mathrm{GrPr})^{0.3}\right] \\
& \quad\left(\mu_{\mathrm{b}} / \mu_{\mathrm{w}}\right)^{0.14} \tag{5-5}
\end{align*}
$$

These average Nusselt number equations were evaluated for the database cases and there was considerable difference between the values obtained and that from the database. The absolute arithmetic average error was 41.3 percent.

It was now attempted to develop a new improved correlation that predicted values more closely but over a wider range. It was also intended that the new correlation extrapolate logically to the theoretical limiting conditions. In other words the new correlation ought to predict the theoretical asymptotic value for an infinitely long tube (condition of fully developed velocity and temperature profile) with no natural convection.

It was also desired to keep the new correlation as simple and succinct as posible for direct application to design. The physical properties were to be evaluated at the mean bulk temperature.

## Palen and Taborek's Correlation

Palen and Taborek (1985) developed a correlation for the average Nusselt number over the length of the tube for the UWT case. This correlation was developed after examining data on hydrocarbon oils. They settled on a basic format :

$$
\begin{equation*}
\mathrm{Nu}=\mathrm{C}+\mathrm{A}(\mathrm{Re})^{\mathrm{n}}(\mathrm{Pr})^{\mathrm{p}}\left(\mathrm{~d}_{j} / \mathrm{L}\right)^{\mathrm{r}}\left(\mu_{\mathrm{b}} / \mu_{\mathrm{w}}\right)^{\mathrm{s}} \tag{5-6}
\end{equation*}
$$

They argue that since the effect of Grashof number is to impart an additional velocity component to the flow, a modified Reynolds number would account for the effect of natural convection.

They developed the following equation.

$$
\begin{equation*}
\mathrm{Nu}=2.5+4.55\left(\mathrm{Re}^{*}\right)^{0.37}\left(\mathrm{~d}_{\mathrm{i}} / \mathrm{L}\right)^{0.37}(\mathrm{Pr})^{0.17}\left(\mu_{\mathrm{b}} / \mu_{\mathrm{w}}\right)^{0.14} \tag{5-7}
\end{equation*}
$$

where $\mathrm{Re}^{*}$ the modified Reynolds number is

$$
\begin{equation*}
\operatorname{Re}^{*}=\operatorname{Re}+0.8 \mathrm{Gr}^{0.5} \exp \left(-42 / \mathrm{Gr}^{2}\right) \tag{5-8}
\end{equation*}
$$

The range of validity for the Palen and Taborek(1985) correlation is stated to be$0<\left(\mu_{\mathrm{b}} / \mu_{\mathrm{w}}\right)<55$
$20<\operatorname{Pr}<10,000$
$0.1<\operatorname{Re}<2000$
$0<\mathrm{Gr}<30,000,000$

## The Modified Palen-Taborek Correlation

The Palen - Taborek (1985) correlation yields an average value of the Nusselt number along the length of the tube. It also provides a very compact form convenient to use in design and was thus deemed worthy of further investigation. It also yields accurate results for the constant wall temperature case. It was applied to the uniform heat flux case by first modifying it to extrapolate accurately to the fully developed pure forced
convection case, i.e. the factor 4.364 was introduced. On changing the exponent of the Grashof number in the Palen-Taborek(1985) correlation to 0.55 the resulting correlation was tested to compare results with the database.

The final form of the modified Palen-Taborek equation is
$\mathrm{Nu}=4.364+4.55\left(\operatorname{Re}^{*}\right)^{0.37}(\mathrm{D} / \mathrm{L})^{0.37}(\operatorname{Pr})^{0.17}\left(\mu_{\mathrm{b}} / \mu_{\mathrm{w}}\right)^{0.14}$
where $\mathrm{Re}^{*}=\mathrm{Re}+\mathrm{Gr}{ }^{0.55} \exp \left(-42 / \mathrm{Gr}^{2}\right)$

This correlation is based on data over the following ranges of parameters
$5<\operatorname{Pr}<2000$
$20<\operatorname{Re}<2100$
$80<\mathrm{Gr}<300,000$
This comparison between the modified Palen-Taborek correlation and the database values is shown in Figure 17.


Figure 17: Comparison of database Nusselt number with Nusselt number obtained from the new [modified Palen-Taborek (1985)] correlation.

## CHAPTER VI

## SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

In summary this work consisted of three tasks. The first was to qualify and validate a previously developed computer model and prove it to be a worthy tool in the investigation of mixed convection heat transfer in the laminar regime under conditions of uniform heat flux. This was done by comparison with experiments performed by Chen(1988). The program performed the task and predicted Nusselt numbers along the length of uniformly heated horizontal tube with good accuracy. Importantly the program extrapolated accurately to the asymptotic Nusselt number value of 4.36 for the conditions of pure forced convection.

The second task was to build a database from this program in a field representing industrial interest. This included running the program for various conditions of heat flux, temperature, diameter and various values of the dimensionless groups $\mathrm{Re}, \mathrm{Pr}$, and Gr. The database was kept as dense as possible to minimize local discontinuities.

The final task was to utilize this database to develop a new heat transfer correlation. This was intended to be not only accurate but physically meaningful by extrapolating logically to the limiting conditions. The leading term of the Palen-Taborek (1985) correlation was modified and proved adequate for this purpose.

The neglect of natural convection (in horizontal tubes under laminar flow conditions) as being insignificant may result in seriously erroneous results. The assumption of fully developed flow and the use of $\mathrm{Nu}=4.36$ in the laminar regime may result in considerable overdesign. In fact the Nusselt number ranged from 12 to over 20 in most cases this
database covered. The above two points among others have been conclusively established in this thesis.

The program ZHANG.FOR may also be used to yield heat transfer coefficients for any particular fluid, under a given set of conditions (though not the primary objective of the program). This may be carried out by inserting appropriate correlations for the thermodynamic and transport properties of the fluid flowing and running the program at the desired operational condition.

Having established the usefulness of the program as a tool for investigating mixed convection, it is recommended that this program be used to study the same phenomenon in vertical tubes. This case would be axisymmetric, but would have to take into account the gravity term collinear to the velocity term owing to the tube orientation. The correlation developed from this thesis (Equation 5-9) can be used to design exchangers operating under uniform heat flux (eg., when the thermal capacity of the two streams is about the same, or in cases of radiative heating), but industrial cases seldom follow uniform heat flux or uniform wall temperature exclusively. So the Palen-Taborek (1985) correlation for uniform wall temperature (Eq.5-7) may be used in conjunction with the correlation developed in this thesis (Eq.5-9) to predict heat transfer in " actual" industrial cases. Of course this may require gathering of data under true industrial conditions.

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## APPENDIX

PROGRAM SOURCE CODE

```
C ***************************************************************************
C *
    A PROGRAM FOR LAMINAR MIXED CONVECTION HEAT TRANSFER INSIDE *
C * A PROGRAM FOR LAMINAR MIXED CONVECTION HEAT TRANSFER INSIDE * **
C * *
C * AUTHOR : CHANGLIN ZHANG *
C * INSTALLATION: OKLAHOMA STATE UNIVERSITY *
C * DATA : FALL 1989 *
C * LANGUAGE : FORTRAN 77 *
* REFERENCE : PATANKAR, 1984 *
C *********************************************************************
    PROGRAM MAIN
THIS PROGRAM IS A GENERAL PROGRAM
    WE CAN USE CONSTANT HEAT FLUX QW1
        OR VARIABLE HEAT FLUX QW(K,I)
        INPUT OF VARIABLE FLUX IN TUBE
                                VARIABLE FLUX FROM A FILE "FLUX.DAT"
    WE CAN USE 3 GRID SYSTEMS: 115X44
                                    19x19X44
                                    OR 21X21X44
                APRIL 2, }1990\mathrm{ MONDAY
    FOR A SPECIFIC RUN, WE NEED TO CHANGE:
            1) PRINTOUT FILE AROUN LINE }20
            2) PLOTING FILE IN TUBE
            3) OPERATING CONDITIONS: RM, QW1, X1, TIN, WIN, DPDZ
C******************************************************************
    INCLUDE 'ZHANG.CMN'
C***********************************************************************
C WRITE(6,*)'ENTER MASS FRACTION OF GLYCERINE'
C READ (5,*)X1
```

```
    SMAX=0.
    SSUM=0.
    RETURN
    END
```



```
    SUBROUTINE SOLVE
C******************************************************************
    INCLUDE 'ZHANG.CMN'
    ISTF=IST-1
    JSTF=JST-1
    IT1=L2+IST
    IT2=L3+IST
    JT1=M2+JST
    JT2=M3+JST
C******************************************************************
    DO 999 NT=1,NTIMES(NF)
    NFF=NF
    DO }999\mathrm{ N=NF,NFF
C-----------------------------MBLK
    PT(ISTF)=0.
    QT(ISTF)=0.
    DO 11 I=IST,L2
    BL=0.
    BLP=0.
    BLM=0.
    BLC=0.
    DO 12 J=JST,M2
    BL=BL+AP(I,J)
    IF(J.NE.M2) BL=BL-AJP(I,J)
    IF(J.NE.JST) BL=BL-AJM(I,J)
    BLP=BLP+AIP(I,J)
    BLM=BLM+AIM(I,J)
    BLC=BLC+CON(I,J) +AIP(I,J)*F(I+1,J,N) +AIM(I,J)*F(I-1,J,N)
        1 +AJP(I,J)*F(I,J+1,N)+AJM(I,J)*F(I,J-1,N)-AP(I,J)*F(I,J,N)
    12 CONTINUE
    DENOM=BL-PT(I-1)*BLM
    IF(ABS(DENOM/BL).LT.1.E-10) DENOM=1.D30
    PT(I)=BLP/DENOM
    QT(I)=(BLC+BLM*QT(I-1))/DENOM
    11 CONTINUE
    BL=0.
    DO 13 II=IST,L2
    I=IT1-II
            BL=BL*PT(I)+QT(I)
            DO 13 J=JST,M2
    13F(I,J,N)=F(I,J,N)+BL
    PT(JSTF)=0.
    QT(JSTF)=0.
    DO 21 J=JST,M2
    BL=0.
BLP=0.
BLM=0 .
BLC=0.
DO 22 I=IST,L2
BL=BL+AP(I,J)
IF(I.NE.L2) BL=BL-AIP(I,J)
IF(I.NE.IST) BL=BL-AIM(I,J)
BLP=BLP+AJP(I,J)
```

```
            BLM=BLM+AJM(I,J)
            BLC=BLC+CON(I,J)+AIP(I,J)*F(I+1,J,N)+AIM(I,J)*F(I-1,J,N)
            1 +AJP(I,J)*F(I,J+1,N)+AJM(I,J)*F(I,J-1,N)-AP(I,J)*F(I,J,N)
        22 CONTINUE
            DENOM=BL-PT(J-1)*BLM
            IF(ABS(DENOM/BL).LT.1.E-10) DENOM=1.D30
            PT(J)=BLP/DENOM
            QT(J)=(BLC+BLM*QT(J-1))/DENOM
    21 CONTINUE
            BL=0 .
            DO 23 JJ=JST,M2
            J=JT1-JJ
            BL=BL*PT(J)+QT(J)
            DO 23 I=IST,L2
    23F(I,J,N)=F(I,J,N)+BL
    10 CONTINUE
C-------------------
    PT(ISTF)=0.
    QT(ISTF)=F(ISTF,J,N )
    DO 70 I=IST,L2
    50 DENOM=AP(I,J)-PT(I-1)*AIM(I,J)
            PT(I)=AIP(I,J)/DENOM
            TEMP=CON(I,J)+AJP(I,J)*F(I,J+1,N)+AJM(I,J)*F(I,J-1,N)
            QT(I)=(TEMP+AIM(I,J)*QT(I-1))/DENOM
    70 CONTINUE
            DO 80 II=IST,L2
            I=IT1-II
    80 F(I,J,N)=F(I+1,J,N)*PT(I)+QT(I)
    90 CONTINUE
    DO 190 JJ=JST,M3
    J=JT2-JJ
    PT(ISTF)=0.
    QT(ISTF)=F(ISTF,J,N
    DO 170 I=IST,L2
    150 DENOM=AP(I,J)-PT(I-1)*AIM(I,J)
    PT(I)=AIP(I,J)/DENOM
    TEMP=CON(I,J) +AJP(I,J)*F(I,J+1,N) +AJM(I,J)*F(I,J-1,N)
    QT(I)=(TEMP+AIM(I,J)*QT(I-1))/DENOM
    170 CONTINUE
    DO }180\mathrm{ II=IST,L2
    I=IT1-II
    180 F(I,J,N)=F(I+1,J,N)*PT(I)+QT(I)
    190 CONTINUE
        DO 290 I=IST,L2
        PT(JSTF)=0.
        QT(JSTF)=F(I,JSTF,N)
    250 DO 270 J=JST,M2
    DENOM=AP(I,J)-PT(J-1)*AJM(I,J)
    PT(J)=AJP(I,J)/DENOM
    TEMP=CON(I,J) +AIP(I,J)*F(I+1,J,N) +AIM(I,J)*F(I-1,J,N)
    QT(J)=(TEMP+AJM(I,J)*QT(J-1))/DENOM
    270 CONTINUE
    DO 280 JJ=JST,M2
    J=JT1-JJ
    280 F(I,J,N)=F(I,J+1,N)*PT(J)+QT(J)
    290 CONTINUE
```

```
    DO 390 II=IST,L3
    I=IT2-II
    PT(JSTF)=0.
    QT(JSTF)=F(I,JSTF,N)
    350 DO 370 J=JST,M2
    DENOM=AP(I,J)-PT(J-1)*AJM(I,J)
    PT(J)=AJP(I,J)/DENOM
    TEMP=CON(I,J)+AIP(I,J)*F(I+1,J,N)+AIM(I,J)*F(I-1,J,N)
    QT(J)=(TEMP+AJM(I,J)*QT(J-1))/DENOM
    370 CONTINUE
    DO 380 JJ=JST,M2
    J=JT1-JJ
    380 F(I,J,N)=F(I,J+1,N)*PT(J)+QT(J)
    390 CONTINUE
C*******************************************
    999 CONTINUE
        DO 400 J=2,M2
        DO 400 I=2,L2
        CON(I,J)=0.
        AP(I,J)=0.
    400 CONTINUE
        RETURN
        END
```



```
    SUBROUTINE SETUP
C*******************************************************************
    INCLUDE 'ZHANG.CMN'
C********************************************************************
    1 FORMAT('1',14X,'COMPUTATION IN CARTESIAN COORDINATES')
    2 FORMAT('1',14X,'COMPUTATION FOR AXISYMMETRIC SITUATION')
    3 FORMAT('1',14X,'COMPUTATION IN POLAR COORDINATES')
    4 FORMAT(14X,38(1H*),//)
        DATA LISFIL,INPUTF,SAVEF/'R40x60x03.SSS','USER.DAT','USER.DAT'/
        DATA ZERO/O.0/
        DATA NFMAX,NP,NRHO,NGAM/LIV,LIV1,LIV2,LIV3/
        DATA LSTOP,LSOLVE,LPRINT/1*.FALSE.,LV*.FALSE.,LV*.FALSE./
        DATA LINPUT,LSAVE/LV*.FALSE.,LV*.FALSE./
        DATA LBLK/LV*.TRUE./
        DATA MODE,LAST,TIME,ITER/1,5,0.,0/
        DATA RELAX,NTIMES/LV*1.,LV*1/
        DATA DT,IPREF,JPREF,RHOCON/1.D+10,1,1,1043./
C----------------
    L2=L1-1
    L3=L2-1
    M2=M1-1
    M3=M2-1
    X(1)=XU(2)
    DO 5 I=2,L2
    5 X(I)=0.5*(XU(I+1)+XU(I))
        X(L1)=XU(L1)
        Y(1)=YV(2)
        DO 10 J=2,M2
    10 Y(J)=0.5*(YV(J+1)+YV(J))
            Y(M1)=YV(M1)
            DO 15 I=2,L1
    15 XDIF(I)=X(I)-X(I-1)
    DO 18 I=2,L2
    18 XCV(I)=XU(I+1)-XU(I)
    DO 20 I=3,L2
```

```
2 0
    XCVS(I)=XDIF(I)
    XCVS (3)=XCVS (3)+XDIF(2)
    XCVS(L2)=XCVS(L2)+XDIF(L1)
    DO 22 I=3,L3
    XCVI(I)=0.5*XCV(I)
22 XCVIP(I)=XCVI(I)
    XCVIP(2)=XCV (2)
    XCVI(L2)=XCV(L2)
    DO 35 J=2,M1
35 YDIF(J)=Y(J)-Y(J-1)
    DO 40 J=2,M2
40 YCV(J)=YV(J+1)-YV(J)
    DO 45 J=3,M2
45 YCVS(J)=YDIF(J)
    YCVS ( 3)=YCVS ( 3)+YDIF ( 2)
    YCVS(M2)=YCVS(M2)+YDIF(M1)
    IF(MODE.NE.1) GO TO }5
    DO 52 J=1,M1
    RMN(J)=1.0
52R(J)=1.0
    GO TO 56
55 DO 50 J=2,M1
50R(J)=R(J-1)+YDIF(J)
    RMN(2)=R(1)
    DO 60 J=3,M2
60 RMN(J)=RMN(J-1)+YCV (J-1)
    RMN(M1)=R(M1)
56 CONTINUE
    DO 57 J=1,M1
    SX(J)=1.
    SXMN(J)=1.
    IF(MODE.NE.3) GO TO 57
    SX(J)=R(J)
    IF(J.NE.1) SXMN(J)=RMN(J)
57 CONTINUE
    DO 62 J=2,M2
    YCVR(J)=R(J)*YCV (J)
    ARX(J)=YCVR(J)
    IF(MODE.NE.3) GO TO 62
    ARX(J)=YCV(J)
62 CONTINUE
    DO 64 J=4,M3
64 YCVRS(J)=0.5*(R(J)+R(J-1))*YDIF(J)
    YCVRS ( 3)=0.5*(R(3)+R(1))*YCVS (3)
    YCVRS (M2) =0.5*(R(M1)+R(M3))*YCVS (M2)
    IF(MODE.NE.2) GO TO 67
    DO 65 J=3,M3
    ARXJ (J)=0.25*(1.+RMN(J)/R(J))*ARX(J)
65 ARXJP(J)=ARX(J)-ARXJ (J)
    GO TO 68
6 7 \text { DO 66 J=3,M3}
    ARXJ (J)=0.5*ARX(J)
66 ARXJP(J)=ARXJ(J)
6 8 ~ A R X J P ( 2 ) = A R X ( 2 )
    ARXJ (M2)=ARX(M2)
    DO 70 J=3,M3
    FV(J)=ARXJP(J)/ARX(J)
70 FVP(J)=1.-FV(J)
    DO 85 I=3,L2
    FX(I)=0.5*XCV(I-1)/XDIF(I)
```

```
    85 FXM(I)=1.-FX(I)
    FX(2)=0.
    FXM(2)=1.
    FX(L1)=1.
    FXM(L1)=0.
    DO 90 J=3,M2
    FY(J)=0.5*YCV(J-1)/YDIF(J)
    90 FYM(J)=1.-FY(J)
    FY(2)=0.
    FYM(2)=1.
    FY(M1)=1.
    FYM(M1)=0.
CON,AP,U,V,RHO,PC AND P ARRAYS ARE INITIALIZED HERE
    DO }95\textrm{J}=1,\textrm{M}
    DO 95 I=1,L1
    PC}(I,J)=0
    U(I,J)=0.
    V(I,J)=0.
    CON(I,J)=0.
    AP(I,J)=0.
    RHO(I,J)=RHOCON
    P(I,J)=0.
    95 CONTINUE
    OPEN(UNIT=1,FILE=LISFIL,STATUS='NEW')
    IF(MODE.EQ.1) WRITE (1,1)
    IF(MODE.EQ.2) WRITE (1,2)
    IF(MODE.EQ.3) WRITE (1,3)
    WRITE (1,4)
    RETURN
    ENTRY SETUP2
COEFFICIENTS FOR THE U EQUATION-
    NF=1
    IF(.NOT.LSOLVE(NF)) GO TO 100
    IST=3
    JST=2
    CALL GAMSOR
    REL=1.-RELAX(NF)
    DO 102 I=3,L2
    FL=XCVI(I)*V(I, 2)*RHO(I, 1)
    FLM=XCVIP(I-1)*V(I-1,2)*RHO(I-1,1)
    FLOW=R(1)*(FL+FLM)
    DIFF=R(1)*(XCVI(I)*GAM(I,1)+XCVIP(I-1)*
    +GAM(I-1,1))/YDIF(2)
    CALL DIFLOW
    102 AJM(I,2)=ACOF+MAX(ZERO,FLOW)
    DO 103 J=2,M2
    FLOW=ARX(J)*U(2,J)*RHO(1,J)
    DIFF=ARX(J)*GAM(1,J)/(XCV (2)*SX(J))
    CALL DIFLOW
    AIM(3,J)=ACOF+MAX ( ZERO,FLOW)
    DO 103 I=3,L2
    IF(I.EQ.L2) GO TO 104
    FL=U(I,J)*(FX(I)*RHO(I,J)+FXM(I)*RHO(I-1,J))
    FLP=U(I+1,J)*(FX(I+1)*RHO(I+1,J)+FXM(I+1)*RHO(I,J))
    FLOW=ARX(J)*0.5*(FL+FLP)
    DIFF=ARX(J)*GAM(I,J)/(XCV(I)*SX(J))
    GO TO 105
    104 FLOW=ARX(J)*U(L1,J)*RHO(L1,J)
    DIFF=ARX(J)*GAM(L1,J)/(XCV(L2)*SX(J))
```

```
    105 CALL DIFLOW
        AIM(I+1,J)=ACOF+MAX(ZERO,FLOW)
        AIP(I,J)=AIM(I+1,J)-FLOW
        IF(J.EQ.M2) GO TO 106
        FL=XCVI(I)*V(I,J+1)*(FY(J+1)*RHO(I,J+1)+FYM(J+1)*RHO(I,J))
        FLM=XCVIP(I-1)*V(I-1,J+1)*(FY(J+1)*RHO(I-1,J+1)+FYM(J+1)*
    1 RHO(I-1,J))
        GM=GAM(I,J ) *GAM(I,J+1)/(YCV (J ) *GAM(I,J+1) +YCV(J+1 ) *GAM(I,J )+
    1 1.0E-30)*XCVI(I)
        GMM=GAM(I-1,J ) *GAM(I-1,J+1)/( YCV (J ) *GAM (I-1,J+1 ) +YCV(J+1 )*
    1 GAM(I-1,J)+1.E-30)*XCVIP(I-1)
        DIFF=RMN(J+1)*2.*(GM+GMM)
        GO TO 107
    106 FL=XCVI(I)*V(I,M1)*RHO(I,M1)
        FLM=XCVIP(I-1)*V(I-1,M1)*RHO(I-1,M1)
        DIFF=R(M1)*(XCVI(I)*GAM(I,M1)+XCVIP(I-1)*
    +GAM(I-1,M1))/YDIF(M1)
    107 FLOW=RMN(J+1)*(FL+FLM)
        CALL DIFLOW
        AJM(I,J+1)=ACOF+MAX(ZERO,FLOW)
        AJP(I,J)=AJM(I,J+1)-FLOW
        VOL=YCVR(J)*XCVS(I)
        CON(I,J)=CON(I,J)*VOL+FU(I,J)*F1(I,J,NF)
        AP(I,J)=(FU(I,J)-AP(I,J)*VOL+AIP(I,J)+AIM(I,J)+AJP(I,J)
    1+AJM(I,J))/RELAX(NF)
        CON(I,J)=CON(I,J)+REL*AP(I,J)*U(I,J)
        DU(I,J)=VOL/(XDIF(I)*SX(J))
        CON(I,J)=CON(I,J)+DU(I,J)*(P(I-1,J)-P(I,J))
        DU(I,J)=DU(I,J)/AP(I,J)
    103 CONTINUE
    CALL SOLVE
    100 CONTINUE
COEFFICIENTS FOR THE V EQUATION---------------------------------------
    NF=2
    IF(.NOT.LSOLVE(NF)) GO TO 200
    IST=2
    JST=3
    CALL GAMSOR
    REL=1.-RELAX(NF)
    DO 202 I=2,L2
    AREA=R(1)*XCV(I)
    FLOW=AREA*V(I, 2)*RHO(I,1)
    DIFF=AREA*GAM(I,1)/YCV (2)
    CALL DIFLOW
202 AJM(I, 3)=ACOF+MAX(ZERO,FLOW)
    DO 203 J=3,M2
    FL=ARXJ(J)*U(2,J)*RHO(1,J)
    FLM=ARXJP(J-1)*U(2,J-1)*RHO(1,J-1)
    FLOW=FL+FLM
    DIFF=(ARXJ (J)*GAM (1,J) +ARXJP(J-1)*GAM(1,J-1))
    +/(XDIF(2)*SXMN(J))
    CALL DIFLOW
    AIM(2,J)=ACOF+MAX(ZERO,FLOW)
    DO 203 I=2,L2
    IF(I.EQ.L2) GO TO 204
    FL=ARXJ (J)*U(I+1,J)*(FX(I+1)*RHO(I+1,J)+FXM(I+1)*RHO(I,J))
    FLM=ARXJP(J-1)*U(I+1,J-1)*(FX(I+1)*RHO(I+1,J-1)+FXM(I+1)*
    1 RHO(I,J-1))
    GM=GAM(I,J ) *GAM (I+1,J)/(XCV (I ) *GAM (I+1,J )+XCV (I+1) *GAM(I,J )+
    1 1.E-30)*ARXJ(J)
```

```
        GMM=GAM(I,J-1) *GAM(I+1,J-1)/(XCV(I) *GAM(I+1,J-1) +XCV (I+1)*
    1 GAM(I,J-1)+1.0E-30)*ARXJP(J-1)
        DIFF=2.*(GM+GMM)/SXMN(J)
    GO TO 205
    204 FL=ARXJ(J)*U(L1,J)*RHO(L1,J)
    FLM=ARXJP(J-1)*U(L1,J-1)*RHO(L1,J-1)
    DIFF=(ARXJ (J)*GAM(L1,J)+ARXJP(J-1) *GAM(L1,J-1))
    +/(XDIF(L1)*SXMN(J))
    205 FLOW=FL+FLM
    CALL DIFLOW
    AIM(I+1,J)=ACOF+MAX (ZERO, FLOW)
    AIP(I,J)=AIM(I+1,J)-FLOW
    IF(J.EQ.M2) GO TO 206
    AREA=R(J)*XCV (I)
    FL=V(I,J)*(FY(J)*RHO(I,J)+FYM(J)*RHO(I,J-1))*RMN(J)
    FLP=V(I,J+1)*(FY(J+1)*RHO(I,J+1)+FYM(J+1)*RHO(I,J ) *RMN(J+1)
    FLOW=(FV(J)*FL+FVP(J)*FLP)*XCV(I)
    DIFF=AREA*GAM(I,J)/YCV(J)
    GO TO 207
    206 AREA=R(M1)*XCV(I)
    FLOW=AREA*V(I,M1)*RHO(I,M1)
    DIFF=AREA*GAM(I ,M1)/YCV(M2)
    207 CALL DIFLOW
    AJM(I,J+1)=ACOF+MAX(ZERO,FLOW)
    AJP(I,J)=AJM(I,J+1)-FLOW
    VOL=YCVRS(J)*XCV(I)
    SXT=SX(J)
    IF(J.EQ.M2) SXT=SX(M1)
    SXB=SX(J-1)
    IF(J.EQ.3) SXB=SX(1)
    CON(I,J)=CON(I,J)*VOL+FU(I,J)*F1(I,J,NF)
    AP(I,J)=(FU(I,J)-AP(I,J)*VOL+AIP(I,J) +AIM(I,J) +AJP(I,J)
    1+AJM(I,J))/RELAX(NF)
    CON(I,J)=CON(I,J)+REL*AP(I,J)*V(I,J)
    DV(I,J)=VOL/YDIF(J)
    CON(I,J)=CON(I,J)+DV(I,J)*(P(I,J-1)-P(I,J))
    DV(I,J)=DV(I,J)/AP(I,J)
    203 CONTINUE
    CALL SOLVE
    200 CONTINUE
COEFFICIENTS FOR THE PRESSURE CORRECTION EQUATION---------------------
    NF=3
    IF(.NOT.LSOLVE(NF)) GO TO 500
    IST=2
    JST=2
    CALL GAMSOR
    SMAX=0.
    SSUM=0.
    DO 390 J=2,M2
    DO 390 I=2,L2
    VOL=YCVR(J ) *XCV (I )
390 CON(I,J)=CON(I,J)*VOL
    DO 402 I=2,L2
    ARHO=R(1)*XCV (I)*RHO(I,1)
    CON(I, 2)=CON(I, 2)+ARHO*V(I, 2)
402 AJM(I, 2)=0.
    DO 403 J=2,M2
    ARHO=ARX(J)*RHO (1,J)
    CON(2,J)=CON(2,J)+ARHO*U(2,J)
    AIM(2,J)=0.
```

```
    DO 403 I=2,L2
    IF(I.EQ.L2) GO TO 404
    ARHO=ARX(J)*(FX(I+1)*RHO(I+1,J) +FXM(I+1)*RHO(I,J))
    FLOW=ARHO*U(I+1,J)
    CON(I,J)=CON(I,J)-FLOW
    CON(I+1,J)=CON(I+1,J)+FLOW
    AIP(I,J)=ARHO*DU(I+1,J)
    AIM(I+1;J)=AIP(I,J)
    GO TO 405
    404 ARHO=ARX(J)*RHO(L1,J)
    CON(I,J)=CON(I,J)-ARHO*U(L1,J)
    AIP(I,J)=0.
    405 IF(J.EQ.M2) GO TO 406
    ARHO=RMN(J+1)*XCV (I)*(FY(J+1)*RHO(I,J+1)+
    +FYM(J+1)*RHO(I,J))
    FLOW=ARHO*V(I,J+1)
    CON(I,J)=CON(I,J)-FLOW
    CON(I,J+1)=CON(I,J+1)+FLOW
    AJP(I,J)=ARHO*DV(I,J+1)
    AJM(I,J+1)=AJP(I,J)
    GO TO 407
    406 ARHO=RMN(M1)*XCV(I)*RHO(I,M1)
    CON(I,J)=CON(I,J)-ARHO*V(I,M1)
    AJP(I,J)=0.
    407 AP(I,J)=AIP(I,J)+AIM(I,J)+AJP(I,J)+AJM(I,J)
    PC}(I,J)=0
    SMAX=MAX(SMAX,ABS (CON(I,J)))
    SSUM=SSUM+CON(I,J)
    403 CONTINUE
    CALL SOLVE
COME HERE TO CORRECT THE PRESSURE AND VELOCITIES----------------------
    DO 501 J=2,M2
    DO 501 I=2,L2
    P(I,J)=P(I,J)+PC(I,J)*RELAX(NP)
    IF(I.NE.2) U(I,J)=U(I,J)+DU(I,J)*(PC(I-1,J)-PC(I,J))
    IF(J.NE.2) V(I,J)=V(I,J)+DV(I,J)*(PC(I,J-1)-PC(I,J))
    5 0 1 ~ C O N T I N U E ~
    5 0 0 ~ C O N T I N U E ~
COEFFICIENTS FOR TEMPERATURE EQUATIONS----------------------------------------
    IST=2
    JST=2
    NF=4
    IF(.NOT.LSOLVE(NF)) GO TO 400
    CALL GAMSOR
    REL=1.-RELAX(NF)
    DO 452 I=2,L2
    AREA=R(1)*XCV(I)
    FLOW=AREA*V(I, 2)*RHO(I,1)
    DIFF=AREA*GAM(I,1)/YDIF(2)
    CALL DIFLOW
    452 AJM(I,2)=ACOF+MAX(ZERO,FLOW)
    DO 453 J=2,M2
    FLOW=ARX(J)*U(2,J)*RHO(1,J)
    DIFF=ARX(J)*GAM(1,J)/(XDIF(2)*SX(J))
    CALL DIFLOW
    AIM(2,J)=ACOF+MAX(ZERO,FLOW)
    DO 453 I=2,L2
    IF(I.EQ.L2) GO TO 454
    FLOW=ARX(J)*U(I+1,J)*(FX(I+1)*RHO(I+1,J)+
    +FXM(I+1)*RHO(I,J))
```

```
        DIFF=ARX(J)*2.*GAM(I,J)*GAM(I+1,J)/((XCV(I)*GAM(I+1,J)+
        + XCV(I+1)*GAM(I,J)+1.0E-30)*SX(J))
        GO TO 455
    454 FLOW=ARX(J)*U(L1,J)*RHO(L1,J)
        DIFF=ARX(J)*GAM(L1,J)/(XDIF(L1)*SX(J))
    455 CALL DIFLOW
    AIM(I+1,J)=ACOF+MAX(ZERO,FLOW)
    AIP(I,J)=AIM(I+1,J)-FLOW
    AREA=RMN(J+1)*XCV (I)
    IF(J.EQ.M2) GO TO 456
    FLOW=AREA*V(I,J+1)*(FY(J+1)*RHO(I,J+1)+FYM(J+1)*RHO(I,J))
    DIFF=AREA* 2.*GAM(I,J)*GAM(I,J+1)/(YCV (J)*GAM(I,J+1)+
    + YCV(J+1)*GAM(I,J)+1.0E-30)
    GO TO 457
    456 FLOW=AREA*V(I,M1)*RHO(I,M1)
    DIFF=AREA*GAM(I ,M1)/YDIF (M1)
    4 5 7 \text { CALL DIFLOW}
        AJM(I,J+1)=ACOF+MAX(ZERO,FLOW)
        AJP(I,J)=AJM(I,J+1)-FLOW
    453 CONTINUE
C-------------MODIFY THE BOUNDARY COEFFICIENTS-----------------------
    OMEGA=4./3.
    OMEGAM=OMEGA-1.
    DO 470 I=2,L2
    AREAM2=RMN(M2)*XCV(I )
    AREAM1=RMN(M1) *XCV(I)
    AJP(I,M2)=OMEGA*AJP(I,M2)
    AJP(I,M1)=AJP(I,M2)/AREAM1
    AJM(I,M1)=OMEGAM*AJM(I,M2)/AREAM2
    AJM(I,M2) =AJM(I,M2)*(1.+OMEGAM*AREAM1/AREAM2)
    470 CONTINUE
CONSTRUCTE AP AND CON-----------------------------------------------
            DO 475 J=2,M2
            DO 475 I=2,L2
            VOL=YCVR(J)*XCV (I )
```



```
            AP(I,J)=FU(I,J)-AP(I,J)*VOL+AIP(I,J)+AIM(I,J)+
        1AJP(I,J)+AJM(I,J)
    475 CON(I,J)=CON(I,J)
C------------REMOVE BOUNDARY VALUES FROM EQUATIONS-------------
    DO 480 I=2,L2
    AP(I,M1)=AJP(I,M1)-AP(I,M1)
    AP(I,M2)=AP(I,M2)-AJP(I,M2)*(AJP(I,M1)+AJM(I,M1))/AP(I,M1)
    AJM(I,M2)=AJM(I,M2)-AJP(I,M2)*AJM(I,M1)/AP(I,M1)
    CON(I,M2)=CON(I,M2)+CON(I,M1)*AJP(I,M2)/AP(I,M1)
    480 AJP(I,M2)=0.
C--------UNDER RELAX AND SOLVE THE EQUATION-----------------------
    DO 482 J=2,M2
    DO 482 I=2,L2
    AP(I,J)=AP(I,J)/RELAX(NF)
    482\operatorname{CON}(I,J)=\operatorname{CON}(I,J)+REL*AP(I,J)*F(I,J,NF)
CALL SOLVE
C-----------CALCULATE THE BOUNDARY VALUES
    DO 485 I=2,L2
    F(I,M1,NF)=(AJP(I,M1)*F(I,M2,NF)+
    1AJM(I,M1)*(F(I,M2,NF)-F(I,M3,NF))+CON(I,M1))/AP(I M1)
        CON(I,M1)=0.
        AP(I,M1)=0.
    485 CONTINUE
```

```
    400 CONTINUE
C
C
COEFFICIENTS FOR OTHER EQUATIONS
        IST=2
        JST=2
        DO 600 N=5,NFMAX
        NF=N
        IF(.NOT.LSOLVE(NF)) GO TO 600
        CALL GAMSOR
        REL=1.-RELAX(NF)
        DO 602 I=2,L2
        AREA=R(1) * XCV (I)
        FLOW=AREA*V(I, 2)*RHO(I,1)
        DIFF=AREA*GAM(I,1)/YDIF(2)
        CALL DIFLOW
    602 AJM(I, 2)=ACOF+MAX(ZERO,FLOW)
    DO 603 J=2,M2
    FLOW=ARX(J)*U(2,J)*RHO(1,J)
    DIFF=ARX(J)*GAM(1,J)/(XDIF(2)*SX(J))
    CALL DIFLOW
    AIM (2,J)=ACOF+MAX (ZERO,FLOW)
    DO 603 I=2,L2
    IF(I.EQ.L2) GO TO 604
    FLOW=ARX(J)*U(I+1,J)*(FX(I+1)*RHO(I+1,J)+
    +FXM(I+1)*RHO(I,J))
    DIFF=ARX(J)*2.*GAM(I,J)*GAM(I+1,J)/
    +((XCV(I)*GAM(I+1,J)+XCV(I+1)*GAM(I,J)+1.0E-30)*SX(J))
    GO TO 605
    604 FLOW=ARX(J)*U(L1,J)*RHO(L1,J)
    DIFF=ARX(J)*GAM(L1,J)/(XDIF(L1)*SX(J))
    605 CALL DIFLOW
    AIM(I+1,J)=ACOF+MAX(ZERO,FLOW)
    AIP(I,J)=AIM(I+1,J)-FLOW
    AREA=RMN(J+1)*XCV (I)
        IF(J.EQ.M2) GO TO 606
    FLOW=AREA*V(I,J+1)*(FY(J+1)*RHO(I,J+1)+FYM(J+1)*RHO(I ,J))
    DIFF=AREA*2.*GAM(I,J)*GAM(I,J+1)/(YCV(J)*GAM(I,J+1)+
        + YCV(J+1)*GAM(I,J)+1.0E-30)
    GO TO 607
    606 FLOW=AREA*V(I,M1)*RHO(I,M1)
    DIFF=AREA*GAM(I ,M1)/YDIF(M1)
    607 CALL DIFLOW
    AJM(I,J+1)=ACOF+MAX(ZERO,FLOW)
    AJP(I,J)=AJM(I,J+1)-FLOW
    VOL=YCVR(J)*XCV(I)
    CON(I,J)=CON(I,J)*VOL+FU(I,J)*F1(I,J,NF)
    AP(I,J)=(FU(I,J)-AP(I,J)*VOL+AIP(I,J)+AIM(I,J)+AJP(I,J)
    1+AJM(I,J))/RELAX(NF)
        CON(I,J)=CON(I,J)+REL*AP(I,J)*F(I,J,NF)
    603 CONTINUE
    CALL SOLVE
C***********************************************************
    600 CONTINUE
        ITER=ITER+1
        IF(ITER.GE.LAST) LSTOP=.TRUE.
        RETURN
        END
<cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc
```

SUBROUTINE SUPPLY

```
C********************************************************************
            INCLUDE 'ZHANG.CMN'
C*******************************************************************
    10 FORMAT('1',26(1H*),3X,A10,3X,44(1H*))
    20 FORMAT(1X,4H I =,I7,8I12)
    30 FORMAT(1X,1HJ)
    40 FORMAT(1X,I2,1P9E12,2)
    50 FORMAT(1X,1H )
    51 FORMAT(1X,'I =',2X,9(I4,5X))
    52 FORMAT(1X,'X =',1P9E9.2)
    53 FORMAT(1X,'TH =',1P9E9.2)
    54 FORMAT(1X,'J =',2X,9(I4,5X))
    55 FORMAT(1X,'Y =',1P9E9.2)
C******************************************************************
    ENTRY UGRID
    XU(2)=0.
    DX=XL/DFLOAT(L1-2)
    DO 1 I=3,L1
    1 XU(I)=XU(I-1)+DX
    YV (2)=0.
    DY=YL/DFLOAT(M1-2)
    DO 2 J=3,M1
    2 YV(J)=YV (J-1)+DY
RETURN
C************************************************
ENTRY PRINT
IF(.NOT.LPRINT(3)) GO TO 80
CALCULATE THE STREAM FUNCTION
    F(2,2,3)=0.
    DO 82 I=2,L1
    IF(I.NE.2) F(I, 2, 3)=F(I-1,2,3)-RHO(I-1,1)*V(I-1,2)
    1*R(1)*XCV(I-1)
    DO 82 J=3,M1
    RHOM=FX(I) *RHO(I,J-1)+FXM(I)*RHO(I-1,J-1)
    82 F(I,J,3)=F(I,J-1,3)+RHOM*U(I,J-1)*ARX(J-1)
    8 0 ~ C O N T I N U E ~
C
    IF(.NOT.LPRINT(NP)) GO TO 90
C
CONSTRUCT BOUNDARY PRESSURES BY EXTRAPOLATION
    DO 91 J=2,M2
    P(1,J)=(P(2,J)*XCVS (3)-P(3,J)*XDIF(2))/XDIF (3)
    91P(L1,J)=(P(L2,J)*XCVS(L2)-P(L3,J)*XDIF(L1))/XXIF(L2)
            DO 92 I=2,L2
            P(I, 1)=(P(I, 2)*YCVS(3)-P(I, 3)*YDIF(2))/YDIF(3)
    92 P(I,M1)=(P(I,M2)*YCVS(M2)-P(I,M3)*YDIF(M1))/YDIF(M2)
            P(1,1)=P(2,1)+P(1,2)-P(2,2)
            P(L1,1) =P(L2,1)+P(L1, 2)-P(L2, 2)
            P(1,M1) =P(2,M1)+P(1,M2)-P(2,M2)
            P(L1,M1) =P(L2 ,M1) +P(L1,M2 )-P(L2,M2)
            PREF=P(IPREF,JPREF)
            DO 93 J=1,M1
            DO 93 I=1,L1
        93 P(I,J)=P(I,J)-PREF
        90 CONTINUE
C
            WRITE (1,50)
            IEND=0
    301 IF(IEND.EQ.LI) GO TO 310
```

```
        IBEG=IEND+1
        IEND=IEND+9
        IEND=MINO(IEND,L1)
        WRITE (1,50)
        WRITE (1,51),(I,I=IBEG,IEND)
        IF(MODE.EQ.3) GO TO 302
        WRITE (1,52),(X(I),I=IBEG,IEND)
        GO TO 301
    302 WRITE (1,53),(X(I),I=IBEG,IEND)
        GO TO 301
    310 JEND=0
        WRITE (1,50)
    311 IF(JEND.EQ.M1) GO TO 320
    JBEG=JEND+1
    JEND=JEND+9
    JEND=MINO(JEND,M1)
    WRITE (1,50)
    WRITE (1,54),(J,J=JBEG,JEND)
    WRITE (1,55),(Y(J),J=JBEG,JEND)
    GO TO 311
    320 CONTINUE
C
    DO 999 N=1,NGAM
    NF=N
    IF(.NOT.LPRINT(NF)) GO TO 999
    WRITE (1,50)
    WRITE (1,10),TITLE(NF)
    IFST=1
    JFST=1
    IF(NF.EQ.1.OR.NF.EQ.3) IFST=2
    IF(NF.EQ.2.OR.NF.EQ.3) JFST=2
    IBEG=IFST-9
    110 CONTINUE
        IBEG=IBEG}+
        IEND=IBEG+8
        IEND=MIN0(IEND,L1)
        WRITE (1,50)
        WRITE (1,20),(I,I=IBEG,IEND)
        WRITE (1,30)
        JFL=JFST+M1
        DO 115 JJ=JFST,M1
        J=JFL-JJ
        WRITE (1,40),J,(F(I,J,NF),I=IBEG,IEND)
    115 CONTINUE
        IF(IEND.LT.L1) GO TO }11
    9 9 9 ~ C O N T I N U E ~
        RETURN
C
    ENTRY INPUT
    OPEN(UNIT=2,FILE=INPUTF,STATUS='OLD')
    DO 410 N=1,NGAM
    NF=N
    IF(.NOT.LINPUT(NF)) GO TO 410
    READ(2,*)
    READ(2,420)((F(I, J,NF),I=1,L1),J=1,M1)
    420 FORMAT(1X,10(E12.5,1X))
    410 CONTINUE
    CLOSE(UNIT=2)
    DO 430 NF=1,5
    DO 430 J=1,M1
```

```
            DO 430 I=1,L1
    430 F1(I,J,NF)=F(I,J,NF)
            DO 440 J=2,M2
    DO 440 I=2,L2
    440 FU(I,J)=YCVR(J)*XCV(I)/DEZ(K)*RHO(I,J)*F1(I,J,5)
    RETURN
C
    ENTRY SAVE
    OPEN(UNIT=3,FILE=SAVEF,STATUS='NEW')
    DO 500 N=1,NGAM
    NF=N
    IF(.NOT.LSAVE(NF)) GO TO 500
    WRITE(3,*)
    WRITE(3,520)((F(I,J,NF),I=1,L1),J=1,M1)
    520 FORMAT(1X,10(1PE12.5,1X))
    500 CONTINUE
    CLOSE(UNIT=3)
    RETURN
C
    ENTRY FILSPC
    PRINT 600,INPUTF
    600 FORMAT(' SPECIFY INPUT DATA FILE:'/4X,'DEFAULT= ',A40)
    READ 610,DUMMY
    610 FORMAT(A40)
    IF(DUMMY.NE.' ') INPUTF=DUMMY
    PRINT 620,SAVEF
    620 FORMAT(' SPECIFY OUTPUT DATA FILE:'/4X,'DEFAULT= ',A40)
    READ 610,DUMMY
    IF(DUMMY.NE.', ) SAVEF=DUMMY
    PRINT 630,LISFIL
    630 FORMAT(' SPECIFY LISTING FILE:'/4X,'DEFAULT= ',A40)
    READ 610,DUMMY
    IF(DUMMY.NE.' ') LISFIL=DUMMY
    RETURN
    END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCL
    SUBROUTINE TUBE
```



```
    INCLUDE 'ŹHANG.CMN'
C************************************************************************
C
C -------------------3-D TUBE FLOW----------------------------------
C
C----------------THEN SOLVE U,V,P SIMUTANOUSLY----------------------
C---------------------AT LAST SOLVE T---------------------------------
C***************************************************************************
    DIMENSION U1(30,30),V1(30,30),T1(30,30),W1(30,30)
    DIMENSION T(ID,JD)
    EQUIVALENCE (F(1,1,4),T(1,1))
    DATA TITLE(1),TITLE(2),TITLE(3),TITLE(4),TITLE(5),
    +TITLE(11)/7H VEL U,7H VEL V,7H STR FN,6H TEMP ,
    +7H W/WBAR,8HPRESSURE/
    DATA RELAX(1),RELAX(2),RELAX(11)/0.5,0.5,0.5/
    DATA RELAX(4)/0.9/
    DATA (LSOLVE(I),I=5,6),(LINPUT(I),LSAVE(I),LPRINT(I),I=1,5)
    +/17*.TRUE./
C DATA LPRINT(1),LPRINT(2)/2*.FALSE./
C DATA LSOLVE(6)/.FALSE./
    DATA LAST/200/
```

```
            DATA (NTIMES(I),I=1,6)/6*3/
            DATA (DEZ(K),K=1,44)/0.0186,0.02,0.0377,0.0377,0.05,0.051,
        +0.06,0.07,0.073,0.06,0.07,0.072,0.101,0.102,0.101,0.101,
        +0.102,0.102,0.101,0.101,0.101,0.102,0.101,0.101,0.101,0.102,
        +0.101,0.101,0.101,0.101,0.101,0.101,0.101,0.103,0.105,0.105,
        +0.105,0.105,0.105,0.105,0.11,0.11,0.11,0.117/
C
    ENTRY GRID
    MODE=3
    PI=3.14159
    L1=15
    M1=15
    R(1)=0.
    THL=PI
    YL=0.008
    CALL UGRID
    RETURN
    ENTRY GRID
    MODE=3
    PI=3.14159
    L1=23
    M1=23
    L3=L1-2
    XU(2)=0.
        DX=PI/DFLOAT(L3)
        DO 101 I=3,L1
    101 XU(I)=XU(I-1)+DX
C---------NONUNIFORM IN R-DIRECTION------------
    YV (2)=0.
    YV ( 3 ) =0.0006
        DY=0.0006
        DO 103 J=4,12
    103 YV(J)=YV(J-1)+DY
        DY=0.0003
        DO 105 J=13,17
    105 YV(J)=YV(J-1)+DY
        DY=0.000083333333333333
        DO 107 J=18,M1
    107 YV(J)=YV(J-1)+DY
        R(1)=0.
        RETURN
C ENTRY GRID
        MODE=3
        PI=3.14159
        L1=21
        M1=21
        L3=L1-2
        XU(2)=0.
        DX=PI/DFLOAT(L3)
        DO 101 I=3,L1
    101 XU(I)=XU(I-1)+DX
        YV}(2)=0
        YV (3)=0.001
        DY=0.001
        DO 103 J=4,6
    103 YV(J)=YV(J-1)+DY
        DY=0.0004
```

```
C DO 105 J=7,11
C 105 YV(J)=YV(J-1)+DY
        DY=0.0002
        DO 107 J=12,M1
    107 YV(J)=YV(J-1)+DY
        R(1)=0.
        RETURN
    ENTRY START
    TIN=60.0
    WIN=0.3
    DO 120 J=1,M2
    DO 120 I=1,L1
    F(I,J,4)=TIN
    F(I,M1,4)=TIN
    F(I,J,5)=WIN
    120 F(I,M1,5)=0.
C---------------RUN #2105----------------------------------
    RM=0.01
    DIA=0.016
    QW1=7000
    RHOCON=1090.
        AMU1=1.4E-2
        X1=0.4
        DPDZ=-400.
        DO 130 J=2,M2
        DO }130\textrm{I}=2,L
        FU(I,J)=YCVR(J)*XCV(I)/DEZ(1)*RHOCON*F(I,J,5)
        F1(I,J,4)=F(I,J,4)
    130 F1(I,J,5)=F(I,J,5)
C---------INPUT VARIABLE HEAT FLUX
C OPEN(UNIT=8,FILE='FLUX.DAT',STATUS='OLD')
C DO 160 K=1,44
C DO 160 I=1,L1
C READ}(8,150) QW(K,I
C 150 FORMAT(1X,1P836E12.5)
C 160 CONTINUE
        RETURN
    ENTRY DENSE
    A1=998.8+207.29*X1-72.103*X1**2
    B1=-0.10357-1.0797*X1+0.42904*X1**2
    Cl=-3.2251E-3+3.4321E-3*X1-4.5246E-4*X1**2
C RHOCON=A1+B1*T0+C1*T0**2
    A1=1:012
    A2 = - 0.4244E-03
    A3=0.23266
    A4=-0.1538E-05
    A5=0.2447E-01
                RHOCON=1000*(A1+A2*T0+A3*X1+A4*T0*T0+A5*X1*X1)
            DO 200 J=1,M1
            DO 200 I=1,L1
                RHO(I,J)=1000*(A1+A2*T(I,J)+A3*X1+A4*T(I,J)*T(I,J)+A5*X1*X1)
C RHO(I,J)=A1+B1*T(I,J)+C1*T(I,J)**2
    200 CONTINUE
        RETURN
C
    ENTRY VISCO
        X1=0.4
```

```
    DO 210 J=1,M1
    DO 210 I=1,L1
    A2=(0.63513+3.0176*X1-0.49609*X1**2)**1.3514
    B2=-0.029276-0.0440815*X1+0.0099051*X1**2
    C2=(1.8238E-6+5.765E-6* X1-2.6245E-6*X1**2)**0.6803
C AMU1=EXP(A2+B2*T0+C2*T0**2)*1.E-3
    IF (X1.LT.0.3) GO TO 776
    IF (X1.LT.0.5) GO TO 777
    IF (X1.LT.0.7) GO TO }77
    IF (X1.LT.0.9) GO TO 779
        AMU(I,J)=1.E-03*(-100.13+(1.0441*T(I,J))-0.00335*T(I,J)*T(I,J))/
    +(-.093+(.006969*T(I,J))+(-0.2586E-03*T(I,J)*T(I,J)))
        AMU1=1.E-03*(-100.13+(+1.0441*T0)-0.00335*T0*T0)/
    +(-.093+(0.006969*T0)+(-0.2586E-03*T0*T0))
        AMU1=(1855.+(-0.63037E+02*T0) +(0.72763*T0*T0)
    + +(-0.28137E-02*T0*T0*T0))*1.E-03
        AMU(I,J)=(1855.+(-0.63037E+02*T(I,J))+(0.72763*T(I,J)*T(I,J))
        + +(-0.28137E-02*T(I,J)*T(I,J)*T(I,J)))*1.E-03
                            SL=6.41E-01
        GO TO 309
    7 7 9
    AMU(I,J)=1.E-03*(-22.411+(0.0919*T(I,J))+.253E-03*T(I,J)*T(I,J))/
    +(-.1466+(-0.3408E-03*T(I,J))+(-0.4764E-03*T(I,J)*T(I,J)))
    AMU1=1.E-03*(-22.411+(0.0919*T0)+.253E-03*T0*T0)/
    +(-.1466+(-0.3408E-03*T0)+(-0.4764E-03*T0*T0))
C }77
        AMU1=(105.97+(-3.3950*T0)+(0.03873*T0*T0)
    + +(-0.15071E-03*T0*T0*T0))*1.E-03
        AMU(I,J)=(105.97+(-3.3950*T(I,J))+(0.03873*T(I,J)*T(I,J))
    + +(-0.15071E-03*T(I,J)*T(I,J)*T(I,J)))*1.E-03
                    SL=6.667E-01
    GO TO 309
778 AMU(I,J)=1.E-03*(-100.+(-. 1429*T(I,J))+.3205E-03*T(I,J)*T(I,J))/
    +(-3.698+(-0.1556*T(I,J))+(-0.6757E-02*T(I,J)*T(I,J)))
    AMU1=1.E-03*(-100.+(-.1429*T0)+.3205E-03*T0*T0)/
    + (-3.698+(-0.1556*T0)+(-0.6757E-02*T0*T0))
C 778 AMU1=(18.963+(-0.54397*T0)+(0.5941E-02*T0*T0)
C + +(-0.22756E-04*T0*T0*T0))*1.E-03
        AMU(I,J)=(18.963+(-0.54397*T(I,J))+(0.5941E-02*T(I,J)*T(I,J))
        +(-0.22756E-04*T(I,J)*T(I,J)*T(I,J)))*1.E-03
            SL=6.0E-01
        GO TO 309
    7 7 7
    AMU(I,J)=1.E-02*(2.793+(-.697*T(I,J))+.228E-02*T(I,J)*T(I,J))/
    +(-.6363+(0.03207*T(I,J))+(-0.689E-01*T(I,J)*T(I,J)))
        AMU1=1.E-02*(2.793+(-.697*T0)+.228E-02*T0*TO)/
    +(-.6363+(0.03207*T0)+(-0.689E-01*T0*T0))
C 777 AMU1=(6.195+(-0.158*T0)+(0.16416E-02*T0*T0)
    + +(-0.61451E-05*T0*T0*T0))*1.E-03
        AMU(I,J)=(6.195+(-0.158*T(I,J))+(0.16416E-02*T(I,J)*T(I,J))
    + +(-0.61451E-05*T(I,J)*T(I,J)*T(I,J)))*1.E-03
                SL=5.667E-01
            GO TO 309
776 AMU(I,J)=1.3E-03*(-566.04+(-2.27*T(I,J))+.0508*T(I,J)*T(I,J))/
    +(-98.54+(-12.702*T(I,J))+(-0.0439*T(I,J)*T(I,J)))
    AMU1=1.3E-03*(-566.04+(-2.27*T0)+.0508*T0*T0)/
    +(-98.54+(-12.702*T0)+(-0.0439*T0*T0))
C 776 AMU1=(2.746+(-0.06309*T0)+(0.6477E-03*T0*T0)
    + +(-0.261E-05*T0*T0*T0))*1.E-03
        AMU(I,J)=(2.746+(-0.06309*T(I,J))+(0.6477E-03*T(I,J)*T(I,J))
    + +(-0.261E-05*T(I,J)*T(I,J)*T(I,J)))*1.E-03
                    SL=4.417E-01
    GO TO 309
```

C
DO $210 \mathrm{~J}=1, \mathrm{M1}$
$\mathrm{C} 2=(1.8238 \mathrm{E}-6+5.765 \mathrm{E}-6 * \mathrm{X} 1-2.6245 \mathrm{E}-6 * \mathrm{X} 1 * * 2) * * 0.6803$
DO $210 \mathrm{I}=1, \mathrm{~L} 1$
C 210
210
CONTINUE
RETURN
C
ENTRY SPHT
A3 $=1.027-0.52469 * X 1+0.021435 * X 1 * * 2$
В $3=-2.6187 \mathrm{E}-4+3.8054 \mathrm{E}-3 * \mathrm{X} 1-2.5793 \mathrm{E}-3 * \mathrm{X} 1 * * 2$
$\mathrm{C} 3=-2.3096 \mathrm{E}-7+6.0706 \mathrm{E}-7$ * X1
$\mathrm{CP} 1=4187 . *(\mathrm{~A} 3+\mathrm{B} 3 * \mathrm{~T} 0+\mathrm{C} 3 * \mathrm{~T} 0 * * 2) * 0.78$
DO $220 \mathrm{~J}=1$,M1
DO $220 \mathrm{I}=1$, L1
$C P(I, J)=4187 . *(A 3+B 3 * T(I, J)+C 3 * T(I, J) * * 2) * 0.78$
CONTINUE
RETURN
C
ENTRY CONDC
$\mathrm{WK}=0.56276+1.874 \mathrm{E}-3 * \mathrm{~T} 0-6.8 \mathrm{E}-6 * \mathrm{~T} 0 * * 2$
$\mathrm{DEGK}=0.19589+1.689 \mathrm{E}-4 * \mathrm{TO}-8.1 \mathrm{E}-7 * \mathrm{~T} 0 * * 2$
ALMDA $=0.4052+0.0594 * X 1-8.4 E-4 * T 0$
ALM=ALMDA * (WK-DEGK) * (1-X1) *X1
COND1 $=W K *(1-X 1)+D E G K * X 1-A L M$
A11=193. 87
A12 $=0.2985$
$A 13=-0.16683 \mathrm{E}-03$
A14 $=0.7599$
A15 $=-0.6698$
A16=0. 23439
COND1 $=4.18 \mathrm{E}-03 *(\mathrm{~A} 11+\mathrm{A} 12 * \mathrm{~T} 0+\mathrm{A} 13 * \mathrm{~T} 0 * \mathrm{~T} 0)$ *
$+\quad(\mathrm{A} 14+\mathrm{A} 15 * \mathrm{X} 1+\mathrm{A} 16 * \mathrm{X} 1 * \mathrm{X} 1)$
RETURN
C
ENTRY CONDY
DO $230 \mathrm{~J}=1, \mathrm{M} 1$
DO $230 \mathrm{I}=1$, L1
$W K=0.56276+1.874 \mathrm{E}-3 * T(I, J)-6.8 E-6 * T(I, J) * * 2$
DEGK=0.19589+1.689E-4*T(I,J)-8.1E-7*T(I,J)**2
ALMDA $=0.4052+0.0594 * X 1-8.4 E-4 * T(I, J)$
ALM=ALMDA* (WK-DEGK) * (1-X1) *X1
C $230 \operatorname{COND}(I, J)=W K *(1-X 1)+D E G K * X 1-A L M$
A11=193.87
A12=0. 2985
$A 13=-0.16683 \mathrm{E}-03$
A14=0.7599
$A 15=-0.6698$
A16=0. 23439
$\operatorname{COND}(I, J)=4.18 E-03 *(A 11+A 12 * T(I, J)+A 13 * T(I, J) * T(I, J)) *$
$+(\mathrm{A} 14+\mathrm{A} 15 * \mathrm{X} 1+\mathrm{A} 16 * \mathrm{X} 1 * \mathrm{X} 1)$
230 CONTINUE
RETURN
C

ENTRY BOUND
WSUM=0.
ASUM=0.
TSUM=0.
FRSUM=0.
RMSUM=0.
TWSUM=0.

```
    ERSUM1=0.
    ERSUM2=0.
    ERSUM4=0.
    ERSUM5=0 .
    DO 300 J=2,M2
    DO 300 I=2,L2
    AR=YCVR(J)*THCV(I)
    WSUM=WSUM+F(I,J,5)*AR
    TSUM=TSUM+AR*F(I,J,5)*F(I,J,4)
    FRSUM=FRSUM+F(I,J, 6)*RHO(I,J)*AR
    RMSUM=RMSUM+F(I,J,5)*RHO(I,J)*AR
    ASUM=ASUM+AR
    300 CONTINUE
C******************MASS BALANCE****************************
    IF(.NOT.LSOLVE(6)) GO TO 391
    IF(ITER.LE.2) GO TO 390
    DQ=(RM-RMSUM)/FRSUM
    DPDZ=DPDZ-DQ
    DO 390 J=2,M2
    DO 390 I=2,L2
    390 F(I,J,5)=F(I,J,5)+F(I,J,6)*DQ
    391 CONTINUE
C**************************************************************
    WBAR=WSUM/(ASUM)
    RE=RHOCON*WBAR*DIA/AMU1
        WRITE(6,*)'WBAR=***********',WBAR
        WRITE(1,*)'WBAR=***********', WBAR
    FRE=-2.*DPDZ*DIA/(RHOCON*WBAR**2+1.D-30)*RE
    TBULK=TSUM/(WSUM+1.D-30)
C
C--_-_--FIND ERRORS BETWEEN ITERATIONS-------------------------
C IF(ITER.LE.8) GO TO }31
    DO 366 J=2,M2
    DO 366 I=2,L2
    ERR1=ABS ((F(I,J,1)-U1(I,J))/(U(I,J)+1.E-25))
    ERR2=ABS((F(I,J,2)-V1(I,J))/(V(I,J)+1.E-25))
    ERR4=ABS((F(I,J,4)-T1(I,J))/(F(I,J,4)+1.E-25))
    ERR5=ABS((F(I,J,5)-W1(I,J))/(F(I,J,5)+1.E-25))
    ERSUM1=ERSUM1+ERR1
    ERSUM2=ERSUM2+ERR2
    ERSUM4=ERSUM4+ERR4
    366 ERSUM5=ERSUM5+ERR5
C ERRSUM=ERRSUM/225.
C----------------SYMMETRICAL B.C.----------------------------------
    DO 320 J=2,M2
    U(2;J)=0.
    U(L1,J)=0.
    V(1,J)=V(2,J)
    V(L1,J)=V(L2,J)
    F(1,J,4)=F(2,J,4)
    F(L1,J,4)=F(L2,J,4)
    F(1,J,5)=F(2,J,5)
    F(L1,J,5)=F(L2,J,5)
    F(I,J,6)=F(2,J,6)
    320 F(L1,J,\sigma)=F(L2,J,6)
        DO 330 I=2,L2
    V(I, 2)=0.
    U(I,1)=U(I,2)
    F(I,1,4)=F(I, 2, 4)
    F(I,1,6)=F(I, 2, 6)
```

```
    330 F(I,1,5)=F(I, 2, 5)
    310 CONTINUE
C---------ASSIGN PREVIOUS RESULTS
    DO 420 J=1,M1
    DO 420 I=1,L1
    U1(I,J)=F(I,J,1)
    V1(I,J)=F(I,J,2)
    T1(I,J)=F(I,J,4)
    420 W1(I,J)=F(I,J,5)
C----------SWITCH TO U,V,P,T--------------------------------------------
    372 CONTINUE
        IF(ITER.LE.5) RETURN
    DO 370 NF=5,6
    370 LSOLVE(NF)=.FALSE.
    DO 360 NF=1,3
    360 LSOLVE(NF)=.TRUE.
    IF(ITER.LE.10) GO TO 382
    IF(ERSUM1.GE.1.) RETURN
    IF(ERSUM2.GE.1.) RETURN
    GO TO 383
    382 RETURN
    383 DO 362 NF=1,3
    362 LSOLVE(NF)=.FALSE.
        LSOLVE(4)=.TRUE.
        IF (ERSUM4.EQ.O.) GO TO 386
        IF(ERSUM4.LE.1E-2) LSTOP=.TRUE.
    386 CONTINUE
    DO 352 I=2,L2
    352 TWSUM=TWSUM+F(I,M1,4)
    TW=TWSUM/DFLOAT(L3)
    HTC=QW1/(TW-TBULK+1.D-30)
    ANU=HTC*DIA/COND1
    RETURN
C
    ENTRY OUTPUT
    IF(ITER.NE.O) GO TO 400
        WRITE(6,*)'VELOCITY IN M/SEC; TEMP IN DEG.CENT;FLUX IN W/M**2'
        WRITE(1,*)'VELOCITY IN M/SEC; TEMP IN DEG.CENT;FLUX IN W/M**2'
        WRITE(6,*)'X1=',X1,'VELOCITY=',WIN,'TEMP IN=',TIN,'FLUX Q=',QW1
        WRITE(1,*)'X1=',X1,'VELOCITY=',WIN,'TEMP IN=',TIN,'FLUX Q=',QW1
        PRINT 402,K,TB1
    WRITE(1,402) K,TB1
    402 FORMAT('1','*******************STATION #',I3,'****************',
        + //' TBULK CALCULATED BY HEAT BALANCE=',1P1E12.3)
            PRINT 401
        WRITE(1,401)
401 FORMAT(1X,//
    +' ITER',6X,'SSUM',7X,'ERR1',8X,'ERR2',8X,
    +'ERR4',6X,'DPDZ', 8X,'F.RE', 8X,'TBULK',8X,'TWavg',8x,'NU')
    400 PRINT 403,ITER,SSUM,ERSUM1,ERSUM2,
    +ERSUM4,DPDZ,FRE,TBULK,TW,ANU
                WRITE(1,403) ITER,SSUM,ERSUM1,ERSUM2,
    +ERSUM4,DPDZ,FRE,TBULK,TW,ANU
    403 FORMAT(I6,1P9E12.3)
    IF(.NOT.LSTOP) RETURN
C
    OPEN(UNIT=7,FILE='R40X60X03.PL',STATUS='NEW')
        X1=0.4
    IF(X1.LT.0.3)GOTO 205
```

```
    IF(X1.LT.0.5)GOTO 305
    IF(X1.LT.0.7)GOTO 405
    IF(X1.LT.0.9)GOTO 505
        SL=6.41E-01
        GO TO 6005
        205 SL=4.417E-01
        GO TO 6005
        305 SL=5.667E-01
        GO TO 6005
    405 SL=6.04E-01
    GO TO 6005
    SL=6.667E-01
        PR=CP1*AMU1/COND1
            GR=((DIA**3)*(RHOCON**2)*(SL/RHOCON)* 2.81*(TW-TB1)/(AMU1**2))
            IF(K.NE.1) GO TO 440
            WRITE(7,430)
    430 FORMAT(3X,/4X,' No.', 8X,'THB', 8X,'Tbulk', 8X,'Tw,avg',8X,
    +'Nu',9X,'f.Re',9X,'Ttop',9X,'Tbott',8x,'RE',8X,'PR',8X,'GR'/)
    440 WRITE(7,450)Z,TB1,TBULK,TW,ANU,FRE,F(2,M1,4),F(L2,M1,4),RE,PR,GR
    450 FORMAT(1X,11F12.5)
C
CALL SAVE
DO \(410 \mathrm{~J}=1\), M1
    DO 410 I=1,L1
    410F(I,J,5)=F(I,J,5)/WBAR
    IF(MOD(K,11).NE.0) RETURN
    CALL PRINT
C**************CREATE TEMPERATURE PROFILE OF #44****************
    IF(K.NE.44) GO TO 425
    OPEN(UNIT=4,FILE='TEMP2105.DAT',STATUS='NEW')
    DO 422 J=1,M1
    DO 422-I=1,L1
    X2=Y(J)*SIN(TH(I))
    Y2=Y(J)*COS(TH(I))
    422 WRITE(4,415) X2,Y2,T(I,J)
    415 FORMAT(1X,1P3E12.4)
    425 CONTINUE
    RETURN
C
    ENTRY GAMSOR
    DO 500 J=1,M1
    DO 500 I=1,L1
    GAM(I,J)=AMU(I,J)
            IF(NF.EQ.4) GAM(I,J)=COND(I,J)/CP(I,J)
            GAM (1,J)=0.
            GAM(L1,J)=0.
    500 CONTINUE
    DO 510 J=2,M2
    DO 510 I=2,L2
    IF(NF.NE.1) GO TO 520
            CON(I,J)=(F(I,M1,4)-T(I,J))*(-9.81)*(B1+2.*C1*T(I,J))*
        +SIN(TH(I))+2*AMU(I,J)*(V(I+1,J)-V(I,J))/XDIF(I)/Y(J)**2
            AP(I,J)=-RHO(I,J)*V(I,J)/Y(J)-AMU(I,J)/Y(J)**2
    520 IF(NF.EQ.2) CON(I,J)=-(F(I,M1,4)-T(I,J))*(-9.81)*(B1+
    +2.*C1*T(I,J))*COS(TH(I))+RHO(I,J)*U(I,J)**2/Y(J)-
    +2.*AMU(I,J)*(U(I+1,J)-U(I,J))/XDIF(I)/Y(J)**2
    IF(NF.EQ.2) AP(I,J)=-AMU(I,J)/Y(J)**2
    IF(NF.EQ.4) CON(I,M1)=QW1/CP(I,M1)
    IF(NF.EQ.5) CON(I,J)=-DPDZ
    IF(NF.EQ.6) CON(I,J)=1.
```

510 CONTINUE RETURN
END

# VITA $>$ 

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