THEORETICAL AND EXPERIMENTAL STUDIES
OF THERMAL EFEECTS
IN WOUND ROLLS

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PREFACE

Stresses induced in wound rolls due to temperature changes were studied. The study revealed that radial pressures can increase significantly due to temperature changes and the tangential pressures experienced a increase near the inner layers as well as at the outer boundary. The most important aspect of this research is the study of stress profiles in wound rolls as a function of temperature.This requires characterization of the coefficients of thermal expansion for the web and the core materials. In order to predict the stresses induced in rolls due to temperature changes, a thermoelastic code was developed and coded and the model verified experimentally.

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NOMENCLATURE

r	Radius of the roll
r[0]	Outer radius of the core
r[i]	Radius at any location in the roll
p[0] or σ[r]	Radial Pressure at the core
p[i] or σ[t]	Radial Pressure at any location in the roll
dp	Incremental radial pressure
dt	Incremental tangential pressure
$\delta\theta$	Small angle
Et	Tangential Modulus of the web
Er	Radial Modulus of the web
Ec	Modulus of the core material
$\nu_{r\theta}$	Strain in the radial direction due to a stress in the tangential direction
$\nu_{\theta r}$	Strain in the tangential direction due to a stress in the radial direction
ε_r	Strain in the radial direction
ε_θ	Strain in the tangential direction
g^2	Ration of tangential modulus to radial modulus of the web
K1	Springiness factor
K2	Springiness factor
Tw	Winding tension
h	Web caliper
s	Roll radius

CHAPTER I

INTRODUCTION

A web is defined as any material in a continuous flexible strip form. Many products experience a web form sometime during processing. The quality of a wound roll depends upon its storage conditions. A wound roll which is subjected to a temperature different from that at which it was wound, develops a thermal stress field in addition to those stresses which were incurred due to winding. The additional stress may cause defects within the web material or may also cause defects to the wound roll itself. Unless a core failure has occurred, negative circumferential stress is rarely witnessed near the core. Also it would seem if α_r & α_t of the core are larger than α_r & α_t of web in the vicinity of the core that increased circumferential tension could result for an increase in temperature. A decrease in temperature may cause a decrease in the radial stress, which increases the chance of interlayer slip on roll acceleration during unwinding. Also, there is a critical stress at which the radial stress at a radial location can become zero, which has a chance of "layering" (radial separation of the layers) the roll.

Tramposch[1965, 1967] described the problem of the relaxation of internal forces in wound reels, by invoking the theory of viscoelasticity. An isoparametric four-parameter Maxwell - Kelvin model was used to formulate the viscoelastic response. Hakiel [1987] presented a nonlinear model for wound roll stresses. The model predicts the stresses in the rolls due to winding only. Later research focused upon predicting the stress distribution in wound reels of magnetic tape. The main concern was towards insuring the tape reel integrity through use of proper winding tension profiles and correct temperature and humidity conditioning.

He analyzed the effect of high-temperature storage and came to a conclusion that given sufficient time, a reel could approach stress-free conditions.

Connolly [1987] et al., analyzed the effect of temperature differences on wound magnetic tapes and presented the effect of variation in the radial coefficient of thermal expansion in calculation of the thermal stress-field induced in wound tape. He modeled the wound roll as two concentric, thin, right circular cylinders and there are no thermal stresses in the axial direction. He made the following assumptions.

1. The reel of tape behaves as a continuum, which is assumed to be an orthotropic, linearly elastic solid.
2. Radial modulus is a constant.
3. The material properties are independent of temperature and humidity.
4. The temperature change does not vary through the wound tape.
5. The thermal expansitivity of tape, $\alpha_r = \alpha_t$.

He did not consider the anisotropy of the magnetic tape i.e., $\alpha_r \neq \alpha_t$. He considered the radial coefficient of thermal expansion to be a constant and estimated its value from previous papers. He also assumed that the radial modulus was a constant and determined the interlayer pressures for different values of E_r. It was assumed to be a constant because, if E_r were taken to be a function of pressure, it would add to the complexity of the mathematical problem. So, he measured this value immediately after winding and again after some interval of time.

He finally concluded that any mismatch in the coefficient of expansion between the hub and the tape can have a far greater effect on the radial stress.

Willet [1988] et al presented a nonlinear finite difference approach for stress distribution of wound reels of magnetic tape and accounted for variations in temperature and moisture content. He modeled the problem in a similar fashion compared to Connolly[1987] with similar assumptions.

1. The reel of tape behaves as a continuum, which is assumed to be an orthotropic, linearly elastic solid.
2. The radial modulus is a function of interlayer pressure.
3. The material properties are independent of temperature and humidity.
4. The thermal expansitivity of the tape is treated to be constant.

Though much has not been mentioned about the behavior of the tape's thermal expansitivity, he assumed it to be a constant. He developed a second order non-linear differential equation which includes the thermal effects, but did not carry on with the analysis for predicting the tape's behavior due to temperature changes.

In the process of winding, the material of the web is wound on a core of some stiffness with a certain winding tension. A stress field develops in which the most important components are the circumferential or tangential stress and the radial stress. The tangential stress is produced due to the effect of the winding tension applied over the cross-sectional area of the web, and the radial stress between the layers of the web, is produced due to the radial component of the winding tension stress. As a result of the continuous addition of the layers of the web material onto the core, radial stress is accumulated continuously. The radial displacement of the web is small in comparison to the inplane displacement and is neglected. This results in the reduction of the circumferential stress by the core modulus and hence the circumferential strain and simultaneously reduces the build up of the radial pressure.

Research Objective

The objective of the present study is to develop a thermoelastic model which accounts for the induced stress in rolls due to the variations in temperature from first principles, using realistic governing equations and experimentally verify the validity of the model using wound rolls of nickel and aluminum .

Although the topic of thermal stress variation received attention in previous papers, the topic demands further attention. The thermal expansitivity, radial modulus and proper winding tension were inadequately treated and is thoroughly investigated in this research.

In the present study, the effect of thermal stresses on wound rolls was studied. To predict the effect of thermal stresses, a critical study of the material properties of the web was found necessary. The material properties which are very important for analysis are the Young's modulus in the radial direction, usually referred to as radial modulus and the coefficient of thermal expansion in the radial direction.

It is difficult to predict the coefficient of thermal expansion in the radial direction and a reasonable value for the radial coefficient of thermal expansion for the wound rolls was not available. This presented a need for experimental tests for the radial coefficient of thermal expansion, to predict the induced thermal stress field. It has been assumed that the radial coefficient of thermal expansion is a function of temperature and pressure and was proved experimentally. In order to develop a relation between the temperature change, induced pressure and the coefficient of thermal expansion, several experiments were conducted on the Instron 4202 and Instron 8502 using stacks of nickel and aluminum and their radial coefficient of thermal expansion were determined and implemented. In the present research nickel and aluminum were used instead of paper or film because, paper and film do not exhibit thermoelastic response to temperature changes.

An analysis of the thermal stress field created in the wound rolls by a change in temperature has been described. This analysis is sufficient to predict the induced thermal stress field due to temperature changes and can be applied to core geometry and material selection to minimize the adverse effects of thermal stresses in rolls.

In addition to the above analysis, a FORTRAN program has been written for a thermoelastic winding model. The experimental results have been compared with this model. The model is discussed in Chapter III and the source code is presented in Appendix A.

CHAPTER II

CLASSICAL WINDING MODELS

History of wound roll models

Over the past quarter century, several publications have appeared for analyzing the stresses generated during the winding of thin flexible webs, such as magnetic tape, film and paper.

Most of the winding models reported were developed from constitutive equations which describe the behavior of the roll. These constitutive equations were derived from the principles of mechanics and material properties. The roll stresses for each wrap starting from the core to the final roll radius were superposed on the previous stresses, thus arriving at the final stress distribution in the wound roll. The following assumptions were made in most of the winding models.

1. The web material has a stable condition, uniform thickness, width and length.
2. The wound roll is made up of a series of concentric hoops of web material.
3. The body forces on the roll are negligible.
4. There are no gaps or overlaps between the layers of the web.
5. The air floatation effects during winding are ignored.

Gutterman [1959] developed a winding model which assumed isotropic material behavior. He assumed the wound roll to be a linear isotropic thick-walled cylinder. Although some of his assumptions, particularly isotropy, are known to be violated in actual winding applications, the model provided the ground work upon which more advanced winding models are based.

Pfeiffer [1966] showed that the relation of compressive stress vs strain in a paper roll was non-linear using sound wave velocity measurements. He suggested an exponential expression for the radial modulus as a function of radial pressure. Later a number of researchers tried to develop winding models which account for radial modulus non-linearity. The first non-linear orthotropic winding model was introduced by Pfeiffer [1968] and was based upon energy principles.

Hakiel [1987] developed a non-linear winding model for the determination of stresses in wound rolls. He developed a second order differential equation and used the finite difference approximations to solve the equation. This model was chosen as the basis for this research because it allows the most generality in modelling the wound rolls.

The winding models discussed so far were developed entirely from the mechanics formulae and the constitutive properties of the web material. All the models were based upon the equilibrium equations, the compatibility equations, and the stress-strain relationships. All these combined together putforth a second order differential equation, the solution of which yields the stresses in the wound roll. The winding models calculate the roll stresses after each layer has been wound and thus the incremental stresses calculated for each additional layer are added to the previous stresses, thereby developing a fully stressed wound roll.

These models consider the wound roll to be a collection of concentric hoops of web material and disregard the actual spiral geometry. The web material is assumed to have a perfectly uniform thickness, width, and length. The elastic properties of the web except for the radial modulus are assumed to be constant during the addition of the material. As explained the radial modulus is a function of radial pressure.

Hakiel's Winding Model

The winding model developed by Hakiel exactly followed the assumptions of the classical models and the solution method developed in a similar fashion. A second order nonlinear differential equation was developed with suitable boundary conditions and the stress profile determined.

Model Development

Equilibrium Equations

The axisymmetric equilibrium equation in the radial direction is determined from the sum of the forces on a web segment in the radial direction and is shown in Figure 2.1 and Figure 2.2.

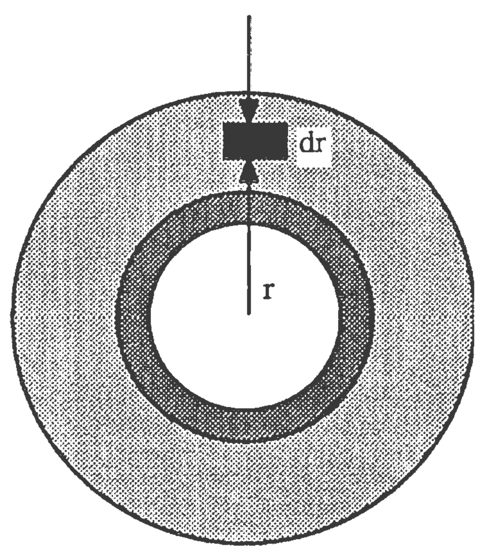

Figure 2.1 Roll of a web showing a small element

For small $d\theta$, the equilibrium equation can be simplified as,

$$\frac{r\, d\sigma_r}{dr} + \sigma_r - \sigma_t = 0 \tag{2.1}$$

Where r is the radial distance from the center, σ_r is the radial or interlayer pressure, and σ_t is the tangential or the circumferential pressure.

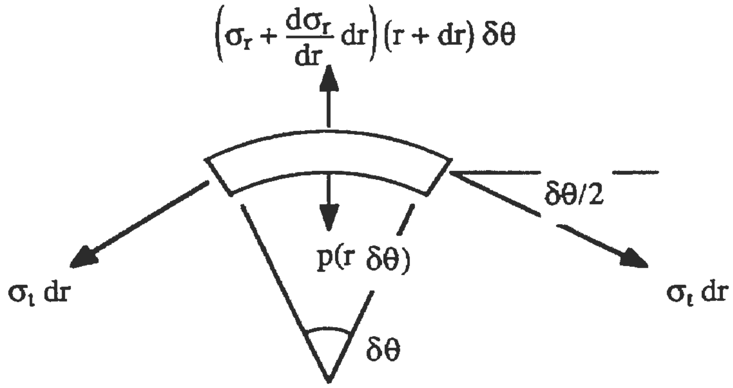

Figure 2.2 Forces acting on a small element of web

Orthotropic Constitutive Relations

The linear orthotropic constitutive relations for strains in cylindrical coordinates are as follows

$$\varepsilon_t = \frac{\sigma_t}{E_t} - \frac{\nu_{rt}}{E_r}\sigma_r \tag{2.2}$$

$$\varepsilon_r = \frac{\sigma_r}{E_r} - \frac{\nu_{tr}}{E_t}\sigma_t \tag{2.3}$$

Where ε_r and ε_t are the radial and tangential strains respectively, σ_r and σ_t are the radial and circumferential stresses, ν_{tr} is the Poisson's ratio representing a deformation in the radial direction due to the application of a stress in the tangential direction, ν_{rt} is the Poisson's ratio representing a deformation in the tangential direction due to a stress applied in the radial direction. E_r and E_t are the radial and circumferential Young's moduli respectively.

The four material constants of Eqn (2.2) can be reduced to three using Maxwell's relation as follows

$$\frac{\nu_{tr}}{E_t} = \frac{\nu_{rt}}{E_r} \tag{2.4}$$

And define that

$$\frac{E_t}{E_r} = \frac{\nu_{tr}}{\nu_{rt}} = g^2 \tag{2.5}$$

Finally we have

$$\nu = \nu_{tr} = g^2 \tag{2.6}$$

Combining Equations (2.2), (2.3), (2.5) and (2.6), yields

$$\varepsilon_r = \frac{1}{E_r}(\sigma_r - \nu\, \sigma_t) \tag{2.7}$$

$$\varepsilon_t = \frac{1}{E_t}(\sigma_t - \nu\, \sigma_r) \tag{2.8}$$

Strain Compatibility Equations

Considering u as the positive outward displacement, the radial deformation due to the addition of each lap of the web material to the winding roll can be represented by Eqn (2.9)

$$\left(u + \frac{du}{dr}dr\right) - u = du \tag{2.9}$$

The radial strain being represented by Eqn (2.10)

$$\frac{du}{dr}\frac{dr}{dr} = \frac{du}{dr} \tag{2.10}$$

The circumferential strain is derived by the comparison of the circumferential length of one layer of the web material before and after the application of a unit radial deformation as shown in Figure 2.3, and is represented by Eqn (2.11)

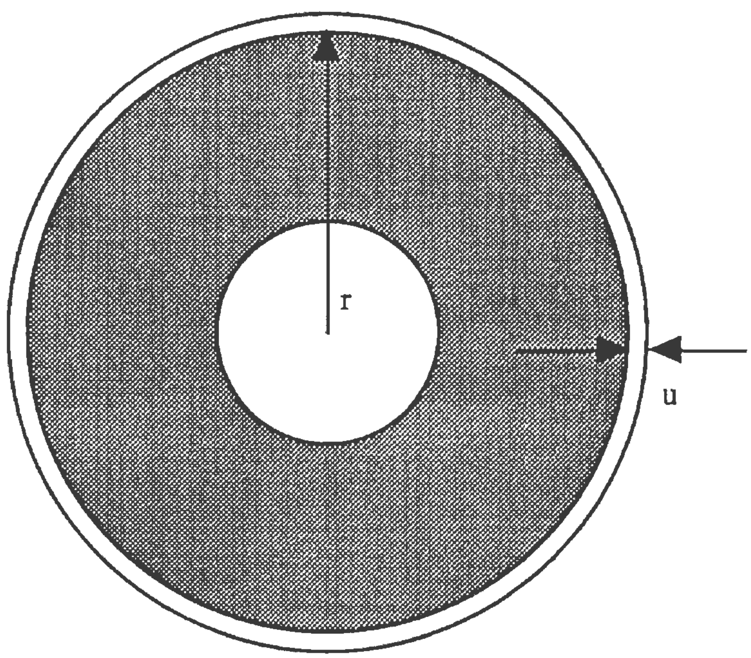

Figure 2.3 Radial displacement of the roll

$$\varepsilon_t = \frac{2\pi(u + r) - 2\pi r}{2\pi r} = \frac{u}{r} \qquad (2.11)$$

ε_t representing the tangential or the circumferential strain. From the above equations the strain compatibility equation can be obtained and may be represented as follows

$$\frac{\partial \varepsilon_t}{\partial r} = \frac{\partial\left(\frac{u}{r}\right)}{\partial r} = \frac{\frac{\partial u}{\partial r}}{r} - \frac{u}{r^2} = \frac{\varepsilon_r}{r} - \frac{\varepsilon_\theta}{r} \qquad (2.12)$$

Rearranging the above equation yields

$$r\left(\frac{\partial \varepsilon_t}{\partial r}\right) = \varepsilon_r - \varepsilon_\theta \qquad (2.13)$$

Governing Differential Equation

Considering Equations (2.7), (2.8), (2.9), (2.13) and solving simultaneously in terms of radial pressure σ_r , a second order differential equation is obtained as follows.

$$r^2 \frac{d^2\sigma_r}{dr^2} + 3r\frac{d\sigma_r}{dr} + \left(1 - \frac{E_t}{E_r}\right)\sigma_r = 0 \qquad (2.14)$$

using Eqn (2.5), Eqn (2.12) can also be written as

$$r^2\frac{d^2\sigma_r}{dr^2} + 3r\frac{d\sigma_r}{dr} + \left(1 - g^2\right)\sigma_r = 0 \tag{2.15}$$

Hakiel proposed a relationship between the radial modulus and the radial pressure, as a second order polynomial, represented by

$$E_r = C_0 + C_1\sigma_r + C_2\sigma^2{}_r \tag{2.16}$$

where the constants C_0 , C_1 , and C_2 are determined by curvefitting the radial modulus versus the radial stress. But a higher order polynomial is used with the first constant being forced to be zero. This can be explained by considering the fact that, when the radial pressure is zero the radial modulus must be zero. When the pressure in the web stack is zero, the peaks on the web surface come in contact and there would not be any stiffness for the stack, which means the radial modulus is zero. Which leads for our assumption of forcing the first constant to zero. This yields the expression for the relation between the radial modulus and the radial stress as given by Eqn (2.17)

$$E_r = C_0 + C_1\sigma_r + C_2\sigma^2{}_r + C_3\sigma^3{}_r \tag{2.17}$$

The solution of Eqn (2.15), a second order nonlinear differential equation required two boundary conditions, one at the at the outer wrap of the roll and the other at the core-roll interface.

The outer boundary condition is determined by the hoop stress formula and is defined as

$$\sigma_r = \frac{T_w h}{s} \tag{2.18}$$

and note that this is actually the traction condition at the inner surface of the outer layer, where T_w is the winding tension stress or the web line tension, h is the caliper, and s is the outer radius of the outermost wrap at any given time.

The inner boundary condition is obtained by the assumption that the radial deformation of the core must equal that of the roll. The normalized radial deflection of the core is given in terms of the core by the definition of the core modulus E_c

$$u(1) = -d\sigma_r/E_c \tag{2.19}$$

The radial deformation of the roll can be obtained by combining the equation for the tangential strain Eqn (2.4) in terms of the radial deflection $\varepsilon_t = u/r$

$$u(1) = (1/E_t)(\delta\sigma_t + \nu\,\delta\sigma_r) \tag{2.20}$$

where $\delta\sigma_t$ is the incremental in-roll tension caused by the winding on of a single lap onto the roll. Combining Eqn (2.20) with Eqn (2.1) and equating the result with Eqn (2.19) yields Eqn (2.21) for the boundary condition at the core.

$$\left[\frac{d\delta\sigma_r}{dr}\right]|(r=1) = \left[(\frac{E_t}{E_r}) - 1 + \nu\right] d\delta\sigma_r\,|(r=1) \tag{2.21}$$

Eqn's (2.15), (2.18) and (2.21) comprise a complete boundary value problem whose solution is the pressure distribution in the winding roll caused by the winding on of a single lap. The distribution of the in-roll tension stress caused by the winding on of a single lap can be obtained from the pressure distribution by solving Eqn (2.2).

Upon solving his differential equation, Hakiel had to sum the pressures back to previous pressures due to the nonlinearity of the model and arrive at the final stress distribution.

Willet and Poesch's Winding Model

The model developed by Willet and Poesch[1988] is similar to Hakiel's winding model except for the consideration of the temperature variations and associated coefficient of thermal expansion of the web and core material. A second order nonlinear differential equation was developed and the inner boundary condition was modified to include the coefficient of thermal expansion of the core and the tangential coefficient of thermal expansion of the web material. Their analysis was completely devoted to the internal stress distributions of wound reels of magnetic tape using finite difference approach.

Only the differential equation with the boundary conditions is presented here.

Model Presentation

As discussed before, the stress-strain relationship is assumed to be nonlinear and the relation between the stress-strain is as shown in Eqn (2.22)

$$\varepsilon_r = \sum_{i=0}^{n} b_n \sigma_r^{\,n} = b_0 + b_1\sigma_r + b_2\sigma_r^{\,2} + b_3\sigma_r^{\,3} + \ldots.. + b_n\sigma_r^{\,n} \tag{2.22}$$

Using the equations of equilibrium and strain compatibility, solving for the radial stress yields a second order nonlinear differential equation which is represented as follows.

$$r^2 \frac{d^2\sigma_r}{dr^2} - r \frac{d\sigma_r}{dr}\left[E_t v_r \sum_{i=0}^{n} n b_n \sigma_r^{n-1} - (3+v_t)\right] + (1+v_t)\sigma_r$$

$$= E_t(1+v_r)\sum_{i=0}^{n} b_n \sigma_r^n - E_t(\alpha_r - \alpha_t)\Delta T \tag{2.23}$$

Since Eqn (2.23) is nonlinear it is necessary to use iterative solution techniques. To accomplish this, Eqn (2.23) must be linearized by using a set of known values, σ_r', from an initial estimate or the previous iteration. Substituting σ_r' in Eqn (2.23) gives us the desired form of Eqn (2.24)

$$r^2 \frac{d^2\sigma_r}{dr^2} - r \frac{d\sigma_r}{dr}\left[E_t v_r \frac{dg(\sigma_r')}{d\sigma_r'} - (3+v_t)\right] + (1+v_t)\sigma_r$$

$$= E_t(1+v_r)\, g(\sigma_r') - E_t(\alpha_r - \alpha_t)\Delta T \tag{2.24}$$

or more simply

$$r^2 \frac{d^2\sigma_r}{dr^2} + Br \frac{d\sigma_r}{dr} + C\sigma_r = R \tag{2.25}$$

where $B = -\left[r\, E_t v_r \frac{dg(\sigma_r')}{d\sigma_r'} - r\,(3+v_t)\right]$

$C = [1+v_t]$

The differentials in equation in Eqn (2.25) are replaced by a central difference derivative approximations as

$$\frac{d\sigma_r}{dr}\bigg|_{r=rn} = \frac{\sigma_{r\,n+1} - \sigma_{r\,n-1}}{2h} \tag{2.26}$$

$$\frac{d^2\sigma_r}{dr^2}\bigg|_{r=rn} = \frac{\sigma_{r\,n+1} - 2\sigma_{r\,n} + \sigma_{r\,n-1}}{h^2} \tag{2.27}$$

Substituting Eqn's (2.26) and (2.27) in Eqn (2.25) results in

$$\left(\frac{r^2{}_n}{h^2} - \frac{Br_n}{2h}\right)\sigma_{r\,n-1} + (C - 2\frac{r^2{}_n}{h^2})\sigma_{r\,n} + \left(\frac{r^2{}_n}{h^2} + \frac{Br_n}{2h}\right)\sigma_{r\,n+1} = R \tag{2.28}$$

Eqn (2.28) can be rewritten using values of σ_r' from the previous iteration as

$$\left(\frac{r^2{}_n}{h^2}\right)\sigma_{r\,n-1} + (C - 2\frac{r^2{}_n}{h^2})\sigma_{r\,n} + \left(\frac{r^2{}_n}{h^2}\right)\sigma_{r\,n+1} = R + \frac{Br_n}{2h}(\sigma'_{r\,n-1} - \sigma'_{r\,n+1}) \tag{2.29}$$

or

$$a1\ \sigma_{r\,n-1} + a2\ \sigma_{r\,n} + a3\ \sigma_{r\,n+1} = Q \tag{2.30}$$

where $a1 = \frac{r^2{}_n}{h^2}$

$a2 = C - 2\frac{r^2{}_n}{h^2}$

$a3 = \frac{r^2{}_n}{h^2}$

$Q = R + \frac{Br_n}{2h}(\sigma'_{r\,n-1} - \sigma'_{r\,n+1})$

Eqn (2.30) is the governing equation which predicts the wound roll stresses.

Boundary Conditions

The governing equation for this problem is of second order. It is therefore necessary to specify two boundary conditions.

At the outer surface of the roll

$$\delta P|_{r=rn} = \frac{T_w\, h}{r}|_{r=rn} \tag{2.31}$$

which is actually the traction on the inner surface of the outer layer.

At the core - roll interface

$$\sigma_1 = \frac{\left[(\alpha_c - \alpha_t)\Delta T + v_r \sum_{i=1}^{n} b_n \sigma_r'^{\,n} - \frac{r_1 \sigma_2}{E_t h}\right] E_t E_c h}{E_c h - E_t h - E_c r_1} \tag{2.32}$$

Eqn's (2.30), (2.31) and (2.32) represent a complete boundary value problem; and it is important to note that the equations are tridiagonal and the system is symmetric, because a_1 equals a_3 in Eqn (2.30).

CHAPTER III

THERMOELASTIC WINDING MODEL

The wound roll models developed to date do not consider the effect of environmental changes. It is known that after winding and during storage, the wound rolls develop stress changes due to changes in temperature and humidity. These changes may be induced both during and after winding of the web material. In the present study, emphasis was placed on the temperature changes and the resulting thermal stresses, the effects of changes in moisture were not considered.

The objective of the present study is to develop a wound roll model which includes the effect of thermal stresses. Hakiel's wound roll model is taken as a basis and a second order nonlinear differential equation has been developed. Assumptions made in the present analysis are similar to those made in Hakiel's analysis. Willet and Poesch did not consider the nonlinearity of the radial modulus and the nonlinearity of the radial coefficient of thermal expansion, so are taken into account here.

The following undesirable effects of thermal stresses have been noted and are as follows.

Those caused by a temperature rise

1. Wrinkling of the web material near the core due to an increased circumferential compression of inner layers (loss in wound on tension).

2. Increased circumferential tension in the outer wraps of the web, which causes increased distortion.

3. Accelerated stress relaxation; any associated inelastic deformation occurs at a faster rate.

Those caused by a temperature drop

1. A decrease in the radial stress, which increases the chance of interlayer slip on acceleration.

2. Presence of a critical stress at which the radial stress at a radial location can become zero, and cause "layering".

Model Development

The plane-stress, stress-displacement relationship for an orthotropic, linearly thermoelastic solid are

$$\frac{du}{dr} - \alpha_r \Delta T = \left(\frac{\sigma_r - \nu_{r\theta}\sigma_\theta}{E_r} \right) = \varepsilon_r \tag{3.1}$$

$$\frac{u}{r} - \alpha_\theta \Delta T = \left(\frac{\sigma_\theta - \nu_{\theta r}\sigma_r}{E_\theta} \right) = \varepsilon_\theta \tag{3.2}$$

where r represents the radial direction of the roll

θ represents the tangential (machine) direction of the roll

ε_θ and ε_r represent the circumferential and the radial strains respectively

$\nu_{\theta r}$ characterizes the strain in the radial direction produced by a stress in the tangential direction

$\nu_{r\theta}$ characterizes the strain in the tangential direction produced by a stress in the radial direction

σ represents the stress

E represents the Young's modulus or modulus of elasticity

α_θ represent the circumferential coefficient of thermal expansion

α_r represents the radial coefficient of thermal expansion as either a function pressure or temperature

ΔT is the temperature change.

The orthotropic considerations of the material gives the relationship between the Poisson's ratio and the Young's moduli (Maxwell's relationship)

$$\nu_{r\theta} = \nu_{\theta r}\left(\frac{E_r}{E_\theta}\right) \tag{3.3}$$

The equilibrium equation is

$$\frac{d\sigma_r}{dr} + \left(\frac{\sigma_r - \sigma_\theta}{r}\right) = 0 \tag{3.4}$$

From Eqn (3.2) we have

$$\frac{u}{r} = \varepsilon_\theta + \alpha_\theta \Delta T \tag{3.5}$$

Differentiating Eqn (3.5) with respect to 'r' yields

$$\frac{1}{r}\left(\frac{du}{dr}\right) - \frac{u}{r^2} = \frac{1}{r}\left(\varepsilon_r + \alpha_r \Delta T\right) - \frac{1}{r}\left(\varepsilon_\theta + \alpha_\theta \Delta T\right) = \frac{d\varepsilon_\theta}{dr} \tag{3.6}$$

rearranging Eqn (3.6) yields

$$r\left(\frac{d\varepsilon_\theta}{dr}\right) = \left(\varepsilon_r + \alpha_r \Delta T\right) - \left(\varepsilon_\theta + \alpha_\theta \Delta T\right) \tag{3.7}$$

or

$$r\left(\frac{d\varepsilon_\theta}{dr}\right) - \varepsilon_r + \varepsilon_\theta = (\alpha_r - \alpha_\theta)\,\Delta T \tag{3.8}$$

Eqn (3.8) is called the compatibility equation

$$\frac{1}{r}\left(\frac{\sigma_r - \nu_{r\theta}\sigma_\theta}{E_r} + \alpha_r\Delta T\right) - \frac{1}{r}\left(\frac{\sigma_\theta - \nu_{\theta r}\sigma_r}{E_\theta} + \alpha_\theta\Delta T\right) = \frac{d}{dr}\left(\frac{\sigma_\theta - \nu_{\theta r}\sigma_r}{E_\theta}\right) \tag{3.9}$$

but

$$\sigma_\theta = r\left(\frac{d\sigma_r}{dr}\right) + \sigma_r \tag{3.10}$$

Substituting Eqn (3.10) into Eqn (3.9), yields

$$\frac{\left(\sigma_r - \nu_{r\theta}\left(\frac{r\,d\sigma_r}{dr} + \sigma_r\right)\right)}{E_r} + \alpha_r\Delta T - \left(\frac{\frac{r\,d\sigma_r}{dr} + \sigma_r - \nu_{\theta r}\sigma_r}{E_\theta}\right) - \alpha_\theta\Delta T = r\frac{d}{dr}\left(\frac{\frac{r\,d\sigma_r}{dr} + \sigma_r - \nu_{\theta r}\sigma_r}{E_\theta}\right) \tag{3.11}$$

expanding Eqn (3.11) and rearranging yields

$$\frac{r^2}{E_\theta}\left(\frac{d^2\sigma_r}{dr^2}\right) + \frac{d\sigma_r}{dr}\left(\frac{3r - \nu_{\theta r}r}{E_\theta} + \frac{\nu_{r\theta}r}{E_r}\right) + \sigma_r\left(\left(\frac{1 - \nu_{\theta r}}{E_\theta}\right) + \left(\frac{\nu_{r\theta} - 1}{E_r}\right)\right) = \Delta T(\alpha_r - \alpha_\theta) \tag{3.12}$$

substituting Maxwell's relation in Eqn (3.12) yields

$$r^2\frac{d\sigma_r}{dr^2} + r\frac{d\sigma_r}{dr}\left(3 - \nu_{\theta r} + \frac{\nu_{r\theta}E_\theta}{E_r}\right) + \sigma_r\left(1 - \frac{E_\theta}{E_r} + \nu_{\theta r} - \nu_{\theta r}\right) = E_\theta\Delta T(\alpha_r - \alpha_\theta) \tag{3.13}$$

further simplification yields

$$r^2\frac{d\sigma_r}{dr^2} + 3\,r\frac{d\sigma_r}{dr} + \sigma_r\left(g^2 - 1\right) = E_\theta \Delta T(\alpha_r - \alpha_\theta) \tag{3.14}$$

This equation is similar to that developed by Hakiel [1987], but the terms in the right hand side are non zero. The coefficient's of thermal expansion namely α_r and α_θ are not equal because of the effect of orthotropy and also α_c and α_r (1). Eqn (3.14) is the governing differential equation to determine the stress distribution in wound rolls when subjected to temperature changes.

Boundary Conditions

The solution of the above differential equation can be solved with the application of appropriate boundary conditions. The boundary conditions are developed by considering the incremental interlayer pressure developed at any radius r as positive in compression.

Let s be the radius at the outer boundary and h be the thickness of the web.

At the outer boundary

$$\delta\sigma_r|\,(r = s) = \left(\frac{T_w(r = s)}{s}\right) h \tag{3.15}$$

At the core

$$u(1) = \frac{\delta\sigma_r(1)}{E_c} \tag{3.16}$$

where E_c represents the modulus of the core material and T_w represents the winding tension. But from the definition of tangential strain at r = 1 i.e., at the core we have

$$\varepsilon_\theta = \frac{u}{r} - \alpha_\theta \Delta T \tag{3.17}$$

$$u(1) = \left(\frac{1}{E_\theta}\delta\sigma_\theta + \frac{\nu}{E_\theta}\delta\sigma_r\right) + \alpha_\theta \Delta T - \alpha_c \Delta T \tag{3.18}$$

where $\delta\sigma_\theta$ and $\delta\sigma_r$ represent the incremental tangential and radial pressures, and α_c represents the coefficient of thermal expansion of the core material. Substituting $\delta\sigma_\theta$ and $\delta\sigma_r$ for radial and tangential pressures in Eqn (3.10) yields

$$\delta\sigma_\theta = -r\frac{d\delta\sigma_r}{dr} - \delta\sigma_r \tag{3.19}$$

Combining Eqn's (3.18) and (3.19) and simplifying yields

$$\frac{d\delta\sigma_r}{dr} = \delta\sigma_r\left(\frac{E_\theta}{E_c} - 1 + \nu\right) + (\alpha_c - \alpha_\theta)\,E_\theta \Delta T \tag{3.20}$$

The differential Eqn (3.14) along with the two boundary conditions Eqn (3.15) and Eqn (3.20) represents a complete boundary value problem which has been solved numerically.

Solution Method Employed

Considering the fact that the roll consists of N layers of web material and that the radius at the i th layer in the roll can be determined as

$$r(i) = 1 + (i - 1)h \tag{3.21}$$

where r is the normalized radius obtained by dividing the instantaneous radius of the roll by the outer radius of the core. Similarly h the normalized thickness is obtained by dividing the web thickness by the outer radius of the core.

The second order terms in Eqn (3.14) were approximated by a finite difference technique by employing the central difference equations as follows.

$$\left(\frac{d\delta\sigma_r}{dr}\right)_{r=r_i} = \left(\frac{\delta\sigma_r(i+1) - \delta\sigma_r(i-1)}{2h}\right) \tag{3.22}$$

$$\left(\frac{d^2\delta\sigma_r}{dr^2}\right)_{r=r_i} = \left(\frac{\delta\sigma_r(i+1) - 2\delta\sigma_r(i) + \delta\sigma_r(i-1)}{h^2}\right) \tag{3.23}$$

substituting equations (3.22) and (3.23) in equation (3.14) yields

$$r(i)^2\left(\frac{(\delta\sigma_r(i+1)) - 2\delta\sigma_r(i) + \delta\sigma_r(i-1)}{h^2}\right) + 3r(i)\left(\frac{\delta\sigma_r(i+1) - \delta\sigma_r(i-1)}{2h}\right)$$

$$- (g^2 - 1)\,\delta\sigma_r(i) = E_\theta \Delta T(\alpha_r - \alpha_\theta) \tag{3.24}$$

where $\delta\sigma_r(i)$ represents the incremental interlayer pressure caused by the addition of the (i+1) th layer at a particular radius r(i), where i indicates a particular radial location of the roll.

Rearranging and simplifying the above equation conveniently, yields

$$\delta\sigma_r(i+1)\left(1 + \frac{3}{2}\frac{h}{r(i)}\right) + \delta\sigma_r(i)\left(\frac{h^2}{r(i)^2}(1 - g^2) - 2\right) + \delta\sigma_r(i-1)\left(1 - \frac{3\,h}{2r(i)}\right) = K \tag{3.25}$$

where K is a constant if α_r is a function of temperature and is nonlinear if α_r is a function of presure, which is equal to $E_\theta \Delta T(\alpha_r - \alpha_\theta)$,. But α_r assumed to be a function of temperature, because the variation of α_r with respect to temperature is much more simpler and was observed experimentally and eliminates the mathematical complications. Moreover, we obtained a experimental better curve-fit for α_r vs ΔT than for α_r vs Pressure, as shown in Chapter IV.

The following variables were defined for substitution into Eqn (3.25)

$$A_i = 1 + \frac{3}{2}\frac{h}{r(i)} \tag{3.26a}$$

$$B_i = \frac{h^2}{r(i)^2}\left(1 - g^2\right) - 2 \tag{3.26b}$$

$$C_i = 1 - \frac{3\,h}{2r(i)} \tag{3.26c}$$

Finally we can arrive at an equation by substituting (3.26a), (3.26b) and (3.26c) into (3.25) to obtain

$$\delta\sigma_r(i + 1)A_i + \delta\sigma_r(\,i\,)B_i + \delta\sigma_r(\,i - 1)C_i = K \tag{3.27}$$

Now substituting for i, from one to the maximum number of laps wound onto the core, we can develop a set of simultaneous equations whose solution has been found by Gaussian elimination and back substitution with the help of the boundary conditions.

The solution of Eqn (3.27) with the boundary conditions, gives the incremental pressure distribution in the roll due to the temperature differences. These results are superimposed on the stresses obtained by Hakiel's solution, thus arriving at the final stress distribution in the wound roll.

The flow chart for the computer program is shown in Figure 3.1. The program asks for the input parameters and then runs the Hakiel's Winding problem and when the current layer being wound reaches the outer layer, then the problem is solved to account for the temperature changes. This is a one step solution since the radial modulus and the radial coefficient of thermal expansion are now known because of the winding stresses obtained by Hakiel's program. Then the incremental stresses are summed up with the existing in-roll stresses.

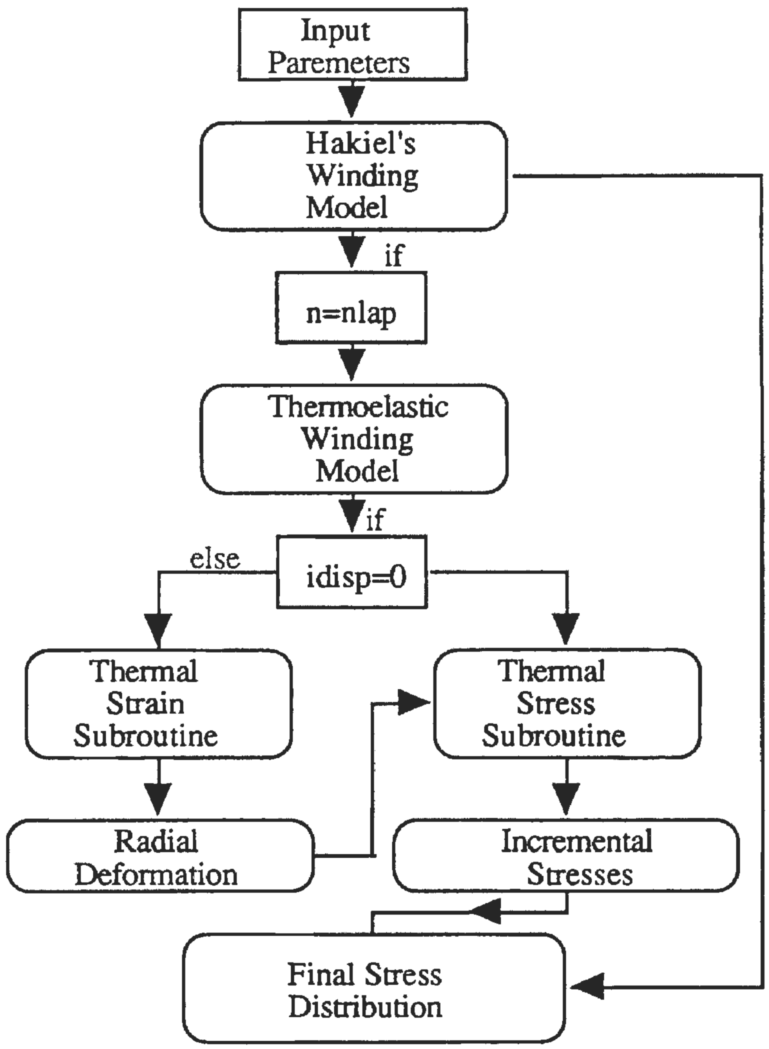

Figure 3.1 Flow chart for computer implementation of the Thermoelastic model

CHAPTER IV

EXPERIMENTAL ANALYSIS

The thermoelastic winding model developed in Chapter III requires the radial modulus vs radial pressure and the radial coefficient of thermal expansion vs radial pressure relationship as its input for the computer program. Experimental analysis was performed to determine the above properties of the web material. The application of the developed thermoelastic model was concentrated towards elastic materials because of the fact that, paper and other polyester films exhibit viscoelastic behavior, when subjected to temperature changes. The web materials used in the present analysis are Nickel 200 and Aluminum

Radial Young's Modulus

In order to determine the radial modulus, experiments were conducted using one inch high stacks of Nickel 200 and Aluminium. The dimensions of the stack of Nickel was 2x2 sq inches and that for Aluminium was 6x6 sq inches. The stack test was performed on the Instron 8502 dynamic testing system Figure 4.1 with the help of LABTECH Note Book software through a data acquisition board connected to an IBM AT compatible. It was assumed that the flat geometry tested in the Instron was equivalent to the web behavior in a wound roll of the circular geometry. The web stack used can have a crossectional area less than the platens of the Instron or can be made larger than the Instron platens.

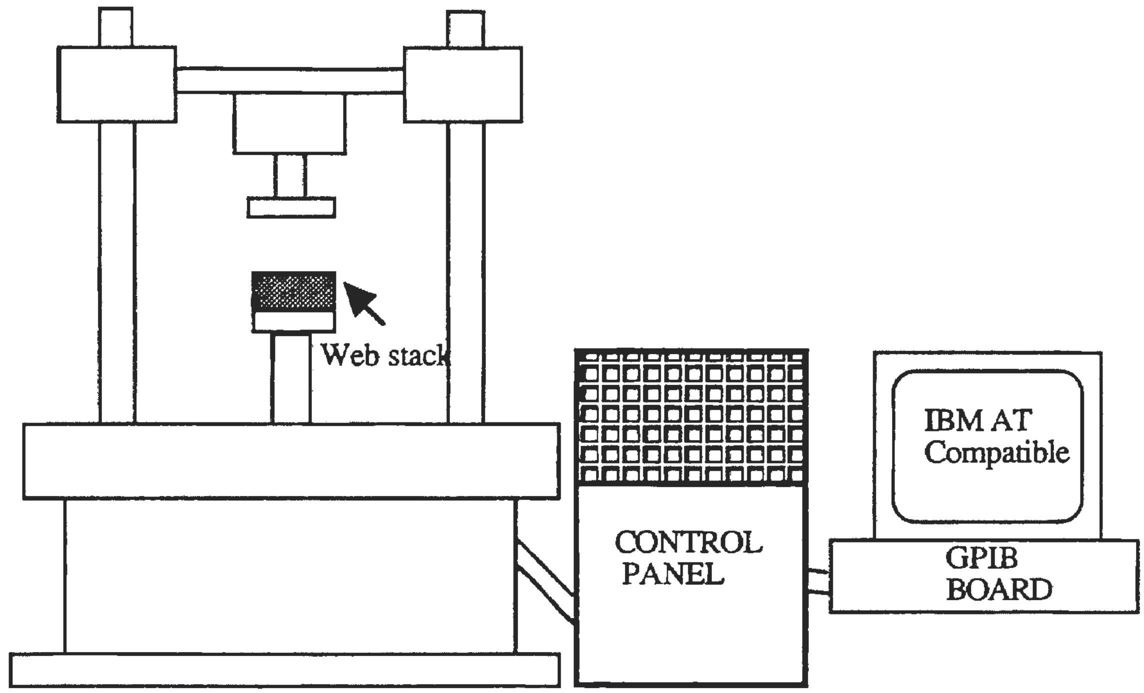

Figure 4.1 Instron 8502 - Testing Machine for Compression and Tension

The relation between the radial stress and strain is obtained by the two modes of operation namely, load control and position control on the Instron. A compressive load was applied on the stack under test. The radial deformation data was obtained by using a strain extensometer and the data was collected by the IBM AT compatible through a GPIB(General Purpose Interface Board). The radial stress was obtained by dividing the applied load by the area of the stack and the strain was obtained by dividing the deflection of the stack by the height of the stack. Several experiments were conducted among which Figure 4.2 and Figure 4.3 exactly represent the nonlinear stress-strain relationship for nickel.

Curve fitting radial stress vs strain yields a higher order equation for radial pressures. Differentiating the equation w.r.t. strain yields the relationship between the radial modulus and the radial strain. Then a relationship between the radial modulus and radial pressure was obtained as shown in Figure 4.4 and Figure 4.5 for nickel and in Figure 4.7 for aluminum. This method is usually employed because it is more accurate over the pressure domain to take a piecewise linear estimation of the stress strain data to produce a plot of radial modulus versus radial pressure.

Figure 4.2 and 4.3 clearly show the nonlinearity as we have seen before for paper and other polyester films. Through a fourth order curve fit, an exact relationship is obtained as represented below.

$$y = M0 + M1*x + M2*x^2 + M3*x^3 + M4*x^4 \qquad (4.1)$$

The equation for Radial modulus as a function of Radial pressure was obtained by differentiating Eqn (4.1) with respect to Strain and then plotting the Radial modulus vs Radial pressure. But in the case of aluminum, a different procedure was adopted. The stress-strain data was estimated linearly using a graphic software and the result gives the radial modulus directly instead of using the tedious method adopted for nickel.

The first coefficient of the curve-fit was forced to zero and curvefitted, thus achieving the final coefficients for the radial modulus as a function of radial pressure. These coefficients are shown in Table VI

The tangential modulus was determined in a similar manner but the tangential modulus was nearly constant with respect to tangential stress which eliminates the need to perform a regression.

TABLE I

COEFFICIENTS FOR STRESS - STRAIN
RELATIONSHIP FOR NICKEL #1

Coefficients	Values of Coefficients
M0	-4081203.1153
M1	41680797
M2	-156759937.72
M3	256050080.64
M4	-252097881.84

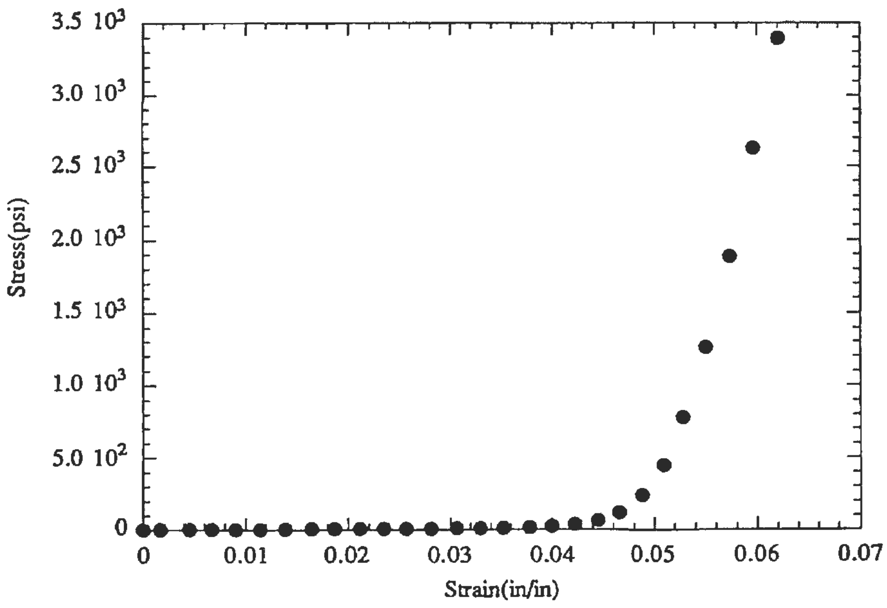

Figure 4.2 Stress Strain relationship for Nickel 200 #1

TABLE II

STRESS - STRAIN RELATIONSHIP FOR
A STACK OF NICKEL #2

Coefficients	Values of Coefficients
M0	6556208.0632
M1	-77459655.342
M2	342497520.26
M3	-671682264.67
M4	492935395.68

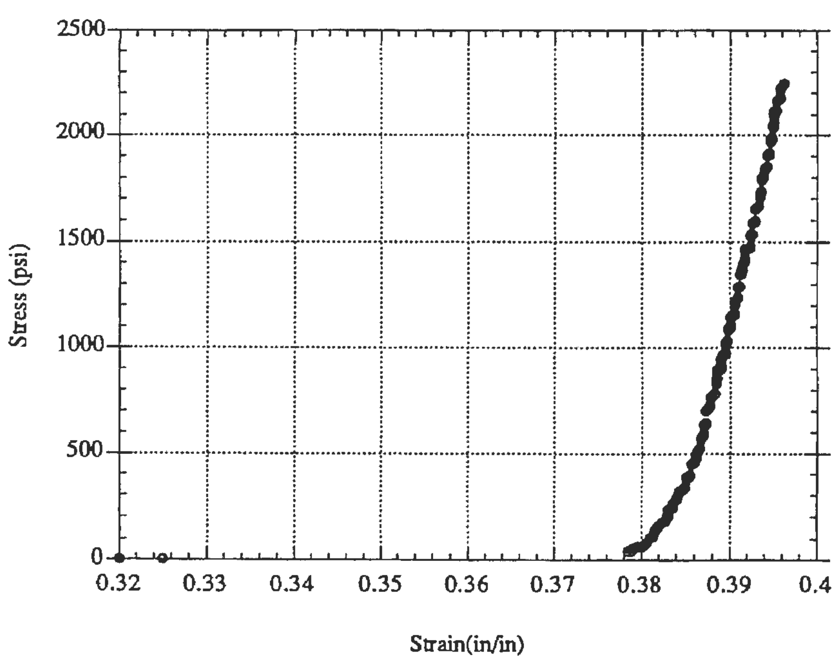

Figure 4.3 Stress Strain relationship for Nickel 200 #2

TABLE III

RADIAL MODULUS AS A FUNCTION OF RADIAL PRESSURE FOR A STACK OF NICKEL #1

Coefficients	Value of Coefficients
M0	114037.00046
M1	104.68004523
M2	-0.033868696
M3	5.76742E-06

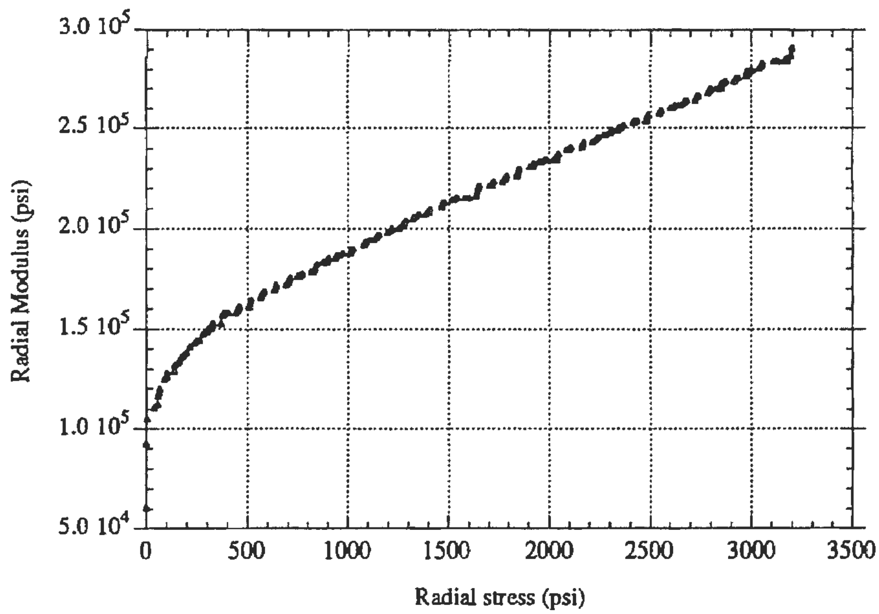

Figure 4.4 Radial Modulus vs Radial pressure for nickel #1

TABLE IV

RADIAL MODULUS AS A FUNCTION OF RADIAL PRESSURE FOR A STACK OF NICKEL #2

Coefficients	Values of Coefficients
M0	331474.12595
M1	243.99655
M2	-0.13498
M3	3.3963E-05

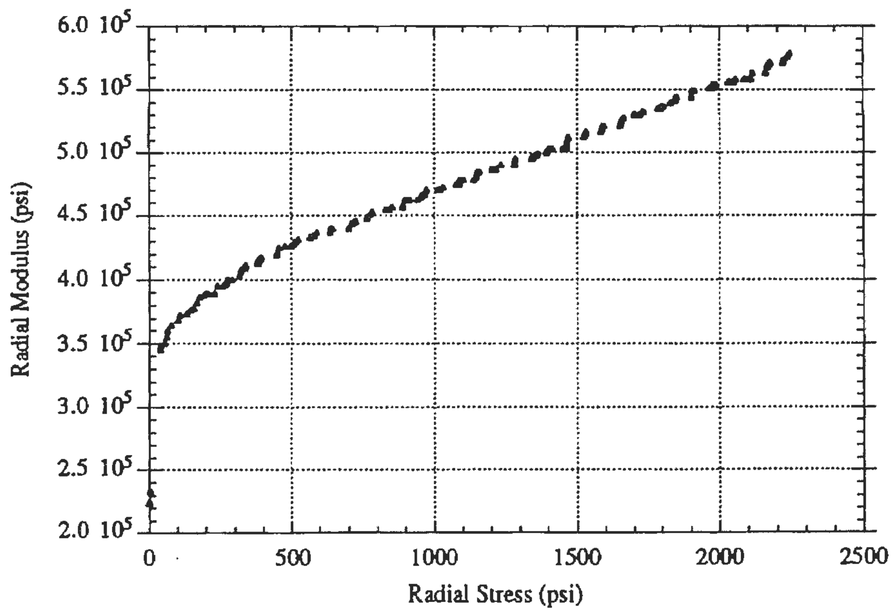

Figure 4.5 Radial Modulus vs Radial pressure for nickel #2

TABLE V

STRESS - STRAIN RELATIONSHIP FOR
A STACK OF ALUMINUM

Coefficients	Values of Coefficients
M0	6556208.0632
M1	-77459655.342
M2	342497520.26
M3	-671682264.67
M4	492935395.68

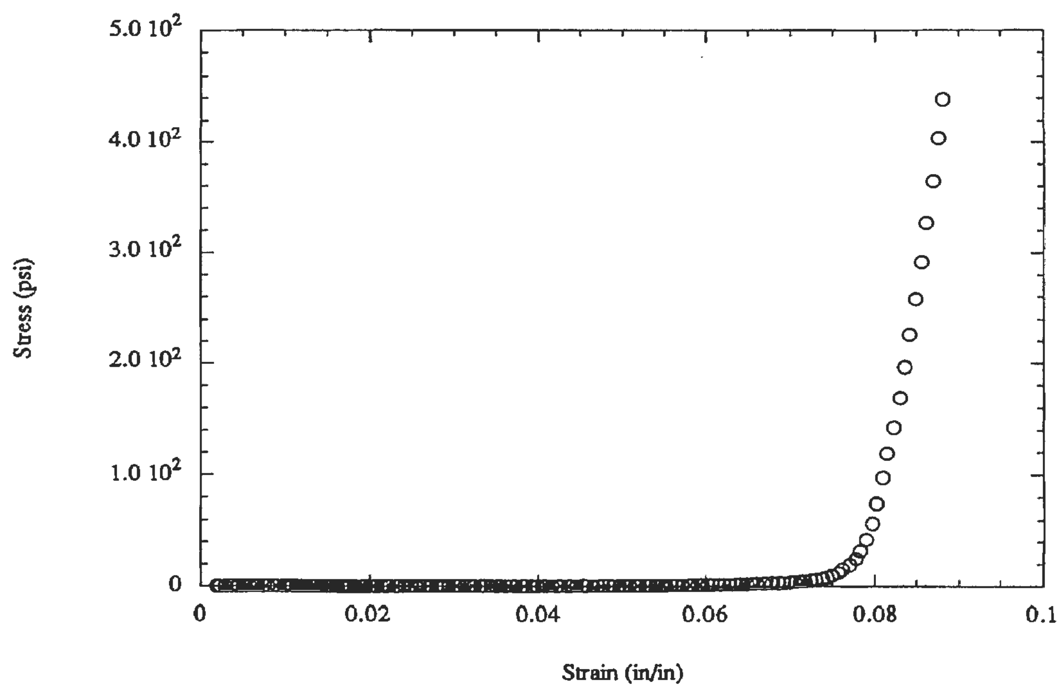

Figure 4.6 Stress - Strain relation for a stack of aluminum

TABLE VI

RADIAL MODULUS AS A FUNCTION OF RADIAL PRESSURE FOR A STACK OF ALUMINUM

Coefficients	Values of Coefficients
M0	4.58002e-06
M1	1473.999
M2	-15.94299
M3	0.05731

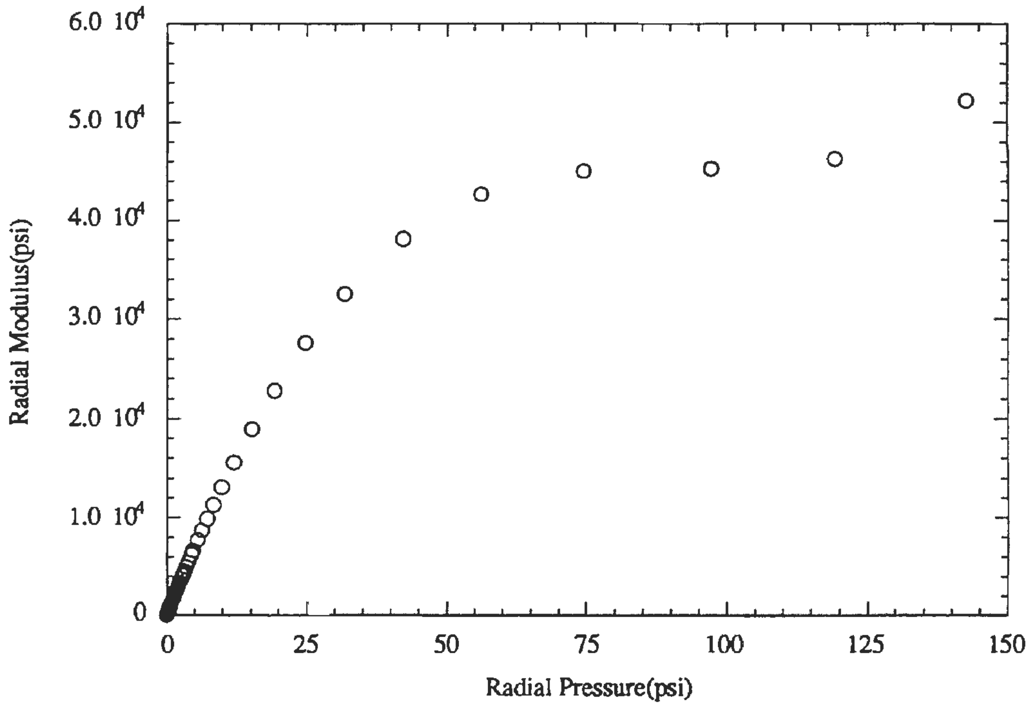

Figure 4.7 Radial Modulus vs Radial Pressure for a stack of aluminum

Radial Coefficient of Thermal Expansion

The material properties of the web material are important for design considerations of which the coefficient of thermal expansion in the radial direction is very important. Several experiments were conducted to estimate its value. The web stack was placed in a temperature chamber and loaded to a desired value by the load cell of the Instron. The temperature of the chamber can be raised or lowered according to the desired value. Now keeping the initial load constant, the temperature of the chamber was raised from ambient to a certain set point maximum in small increments and measuring the induced load at each increment. Considerable amount of time was allowed in measuring the data in each temperature step in order to insure that the chamber attains equilibrium temperature.

This procedure was repeated for the Platens of the Instron alone to consider the induced load due to the expansion of the Platens. The radial coefficient of thermal expansion was measured by the difference of the pressure after and before temperature rise, divided by the radial modulus and the temperature difference.

The real stress was proportional to the strain and is equated as follows.

$$\sigma_{rcal} = E_r \, \alpha_r \, \Delta T \qquad (4.8)$$

The terms of Eqn (4.8) indicate the thermal strain resulting from radial coefficient of expansion and the temperature rise. Now when the stack is subjected to a temperature change, there is a corresponding strain due to the expansion or contraction. Thus the thermal strain can be represented by the quotient of difference in pressure of the stack before and after a temperature change. This can be represented as

$$\frac{\sigma_{real}}{E_T} = \alpha_T \, \Delta T \qquad (4.9)$$

or

$$\frac{\sigma_{final} - \sigma_{initial}}{E_T \, \Delta T} = \alpha_T \qquad (4.10)$$

The coefficient of expansion can be obtained from Eqn (4.10) and the experimental data obtained was as follows.

Experimental Determination of Radial

coefficient of thermal expansion

An important part of the experiment to be discussed is the coefficient of thermal expansion of the platens.

Initially the experiment was conducted to determine the induced load due to the expansion of the platens only. The two platens were brought close enough and an initial load was applied. Then the temperature of the chamber was raised from ambient to a set point. Sufficient time was allowed so that the platens attain the same temperature as the thermocouple of the chamber. Thus the temperature was raised in steps of 10^{o} Fahrenheit and the load was measured.

Then the web stack was placed between the platens and the same initial load applied and the procedure repeated. The difference of the induced load due to the expansion of the

Platens and web and the Platens alone gives the induced load due to the expansion of the web material. The data is presented in Table VII

Results for Nickel

TABLE VII

INDUCED PRESSURE DUE TO TEMPERATURE RISE FOR 1" STACK OF WEB (NICKEL)

S.No	Load (lbs)	Temperature difference (^{0}F)	Pressure (psi) (load/area)
1	26.8	70	136.4912
2	46.2	80	235.294
3	68.3	90	347.849
4	86.7	100	441.559
5	125.6	110	639.675
6	156.2	120	795.520

$$\sigma_{real} = E_r \, \alpha_r \, \Delta T \tag{4.11}$$

The Radial modulus was determined from Eqn (4.3). Using data from Table VII, and the equation for radial modulus, coefficient of thermal expansion was determined.

An important note to be discussed is the area of crossection of the Platens used to apply the compressive load on the web stack. Since the dimensions of the Nickel stack was 2x2 sq inches, it was found necessary to use special platens for the Instron whose area of crossection was lesser than the web dimensions. This would circumvent any possible errors in the observations due to the edge effects of the web stack.

The Platens used for stack of 0.5 inches diameter for Nickel. Thus the data in Table VII is plotted as shown in Figures 4.8 - 4.10 and the results curve - fitted to an exponential expression in Figures 4.11 - 4.12.

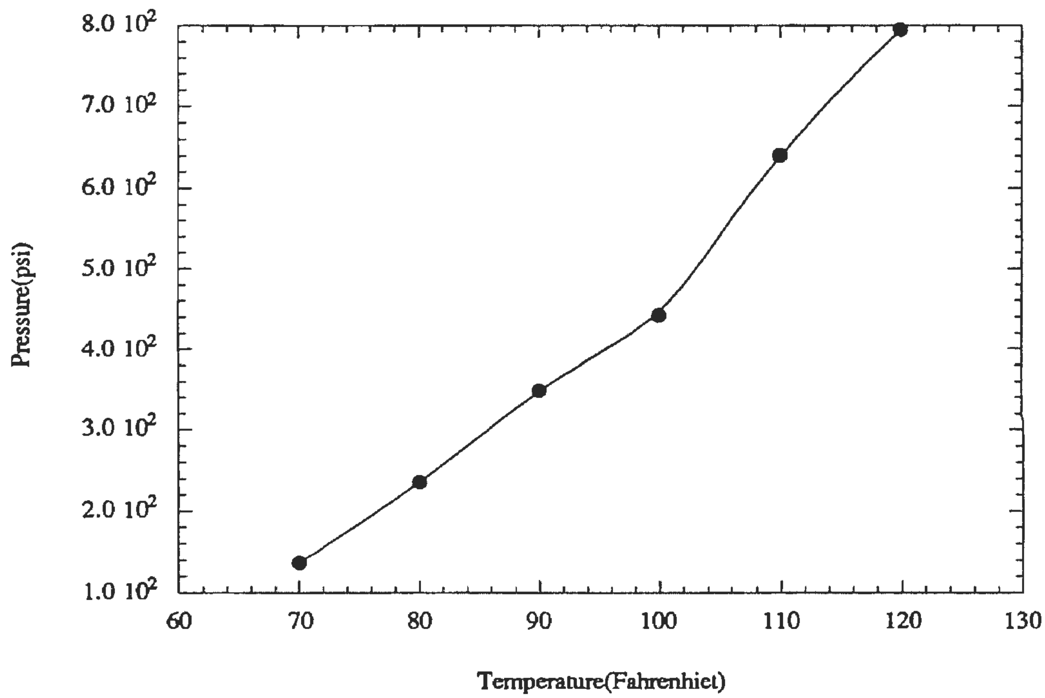

Figure 4.8 Linear Pressure - Temperature relationship for Nickel

It is evident that the radial modulus is a function of radial pressure and Eqn (4.10) is affected due to this fact. The nonlinear effect of the radial modulus is observed in Figure 4.8 and it is clear that the radial coefficient of thermal expansion is a function of radial pressure and also a function of temperature as shown in Figure 4.9.

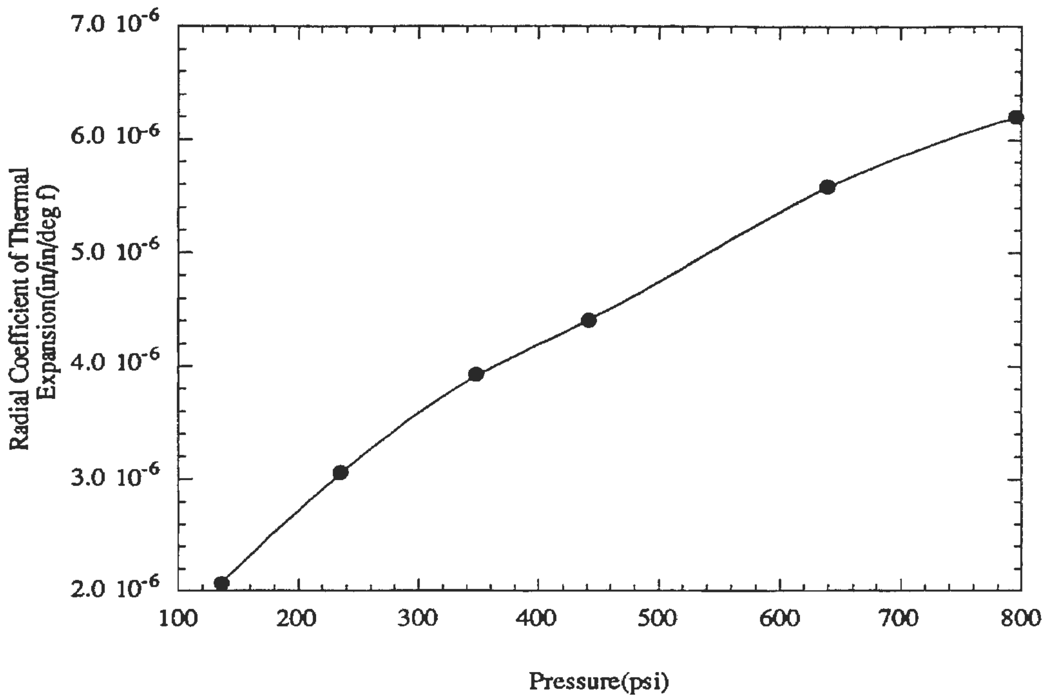

Figure 4.9 Radial coefficient of thermal expansion as a function of Pressure

One can notice from the fact that, as the temperature rises, the induced pressure increases and as a result, we can observe an increase in the radial coefficient of thermal expansion. But after a certain temperature, and a certain pressure the radial coefficient of expansion tries to flatten and reach a constant value and in our case, it is the tangential coefficient of expansion of the web material under test. This is shown in Figure 4.11. An exponential expression was proposed which clearly fits the experimental data points.

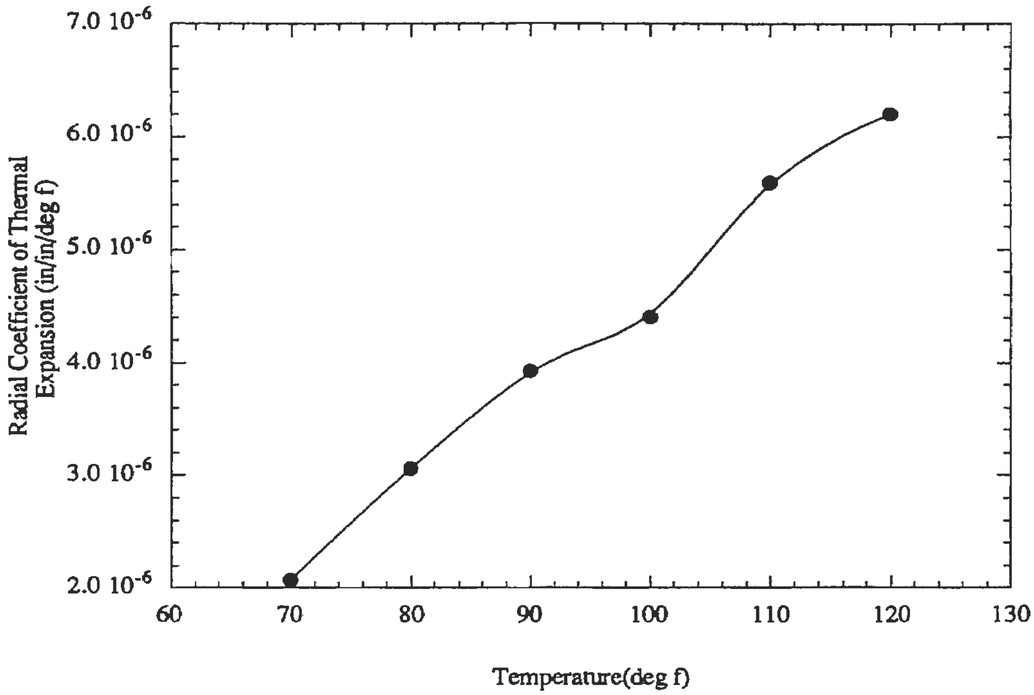

Figure 4.10 Radial Coefficient of Thermal Expansion as a function of Temperature

From the above graphs, a relationship between the radial coefficient of thermal expansion and the induced pressure and temperature was determined and it is observed to be a exponential curve-fit.

$$y = c0 * (1 - \exp(- c1*x)) + c2 \tag{4.12}$$

Where $c0 = 7.4e-06$

$c1 = 0.0041105$

$c2 = 0.0$

Hence using Eqn (4.12) with the above constants and assuming a pressure range from Table VII, a radial coefficient of thermal expansion vs pressure relation is achieved as shown in Figure 4.11. Thus it can be seen that a good relation is achieved between the proposed model and the test data.

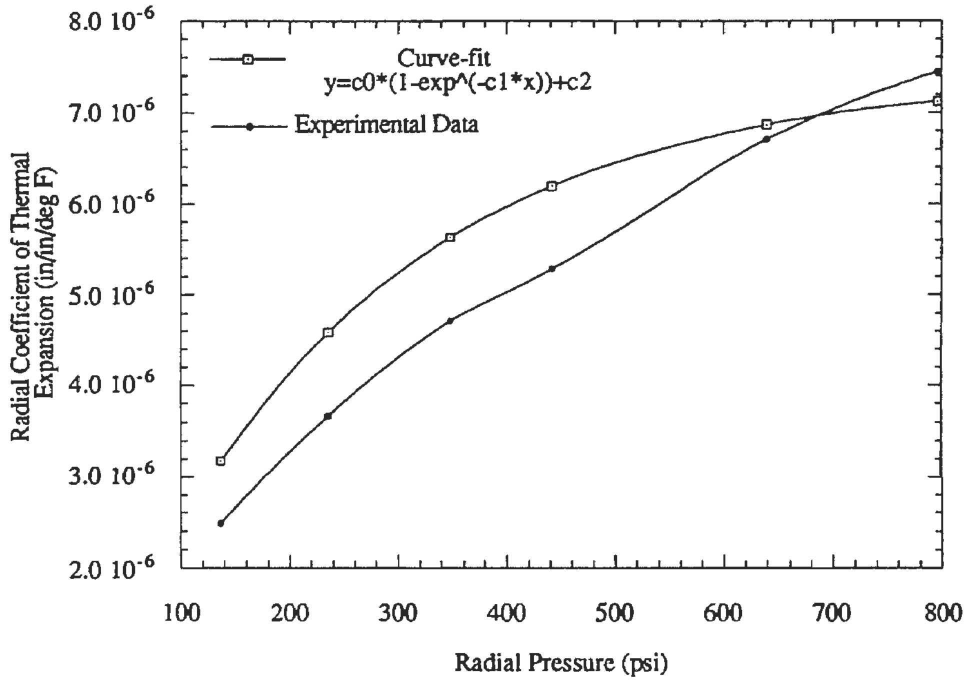

Figure 4.11 Relationship between Radial coefficient of thermal expansion and Radial Pressure for Nickel

A similar correlation is shown in Figure 4.12 for radial coefficient of thermal expansion as a function of temperature. Eqn (4.12) predicts the coefficient of expansion data for different pressures and different temperatures. This expression is incorporated into the thermoelastic winding model in the computer program, and the coefficients must be given depending on the web material under test. These coefficients must be determined experimentally.

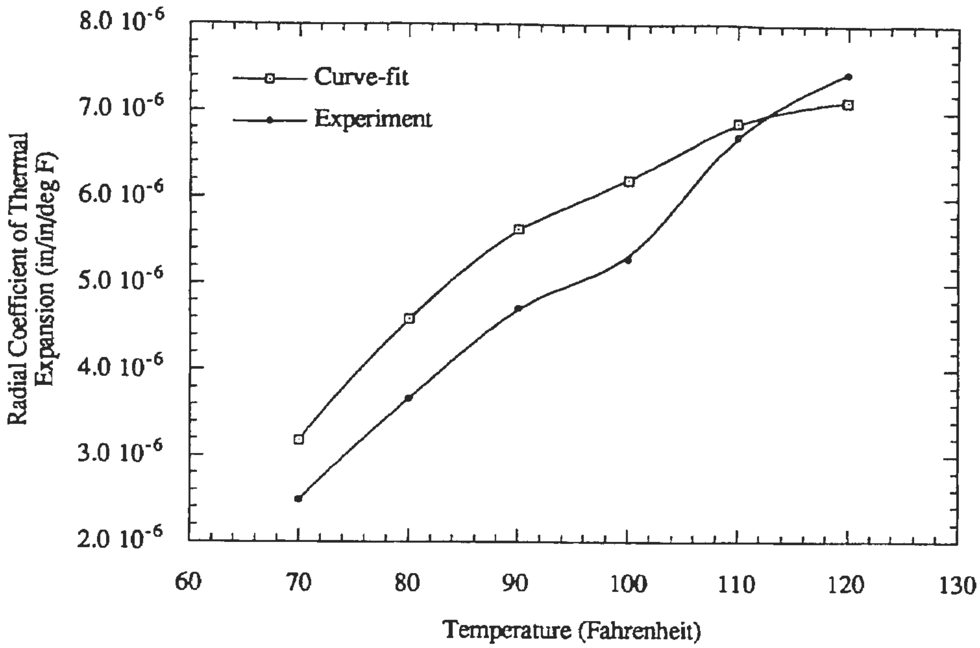

Figure 4.12 Relationship between theradial coefficient of thermal expansion and temperature for Nickel

Results for Aluminum

Similar procedure was adopted for determination of the radial coefficient of thermal expansion for aluminum as was done for nickel. The diameter of Platens used for test was 6.0 inches and the test was conducted on the Instron Dynamic Testing System. The results are presented in Figures 4.13 - 4.15. The relation between radial coefficient of thermal expansion and radial pressure was curvefitted and the relation was incorporated into the thermoelastic model on the computer. The program was run using this relation which achieved compatible results from both theory and through experiments.

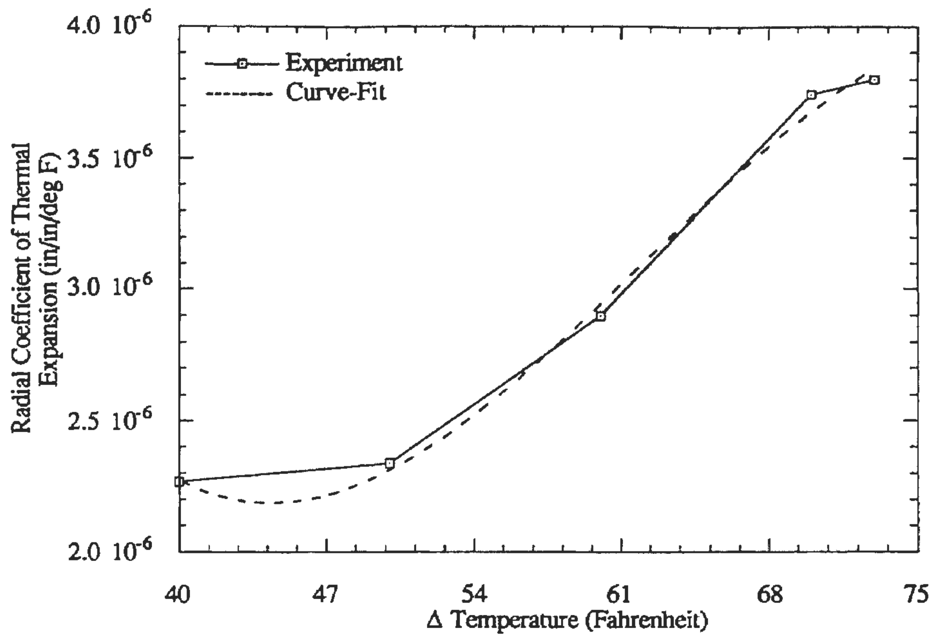

Figure 4.13 Radial Coefficient of Thermal Expansion as a function of Temperature

TABLE VIII

COEFFICIENTS FOR RADIAL COEFFICIENT OF THERMAL EXPANSION AS A FUNCTION OF TEMPERATURE

Coefficients	Values of Coefficients
c0	1.79e-05
c1	-8.72e-07
c2	1.53e-08
c3	-8.23e-11

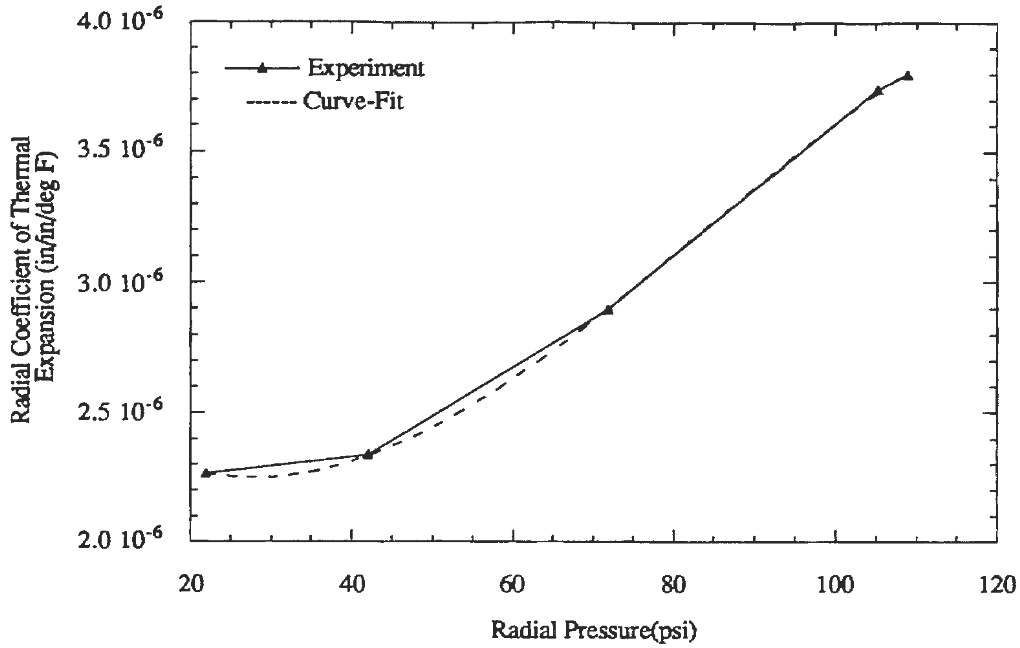

Figure 4.14 Radial Coefficient of Thermal Expansion as a function of Pressure

TABLE IX

COEFFICIENTS FOR RADIAL COEFFICIENT OF THERMAL EXPANSION AS A FUNCTION OF PRESSURE

Coefficients	Values of Coefficients
c0	2.66e-06
c1	-3.15e-08
c2	6.78e-10
c3	-2.68e-12

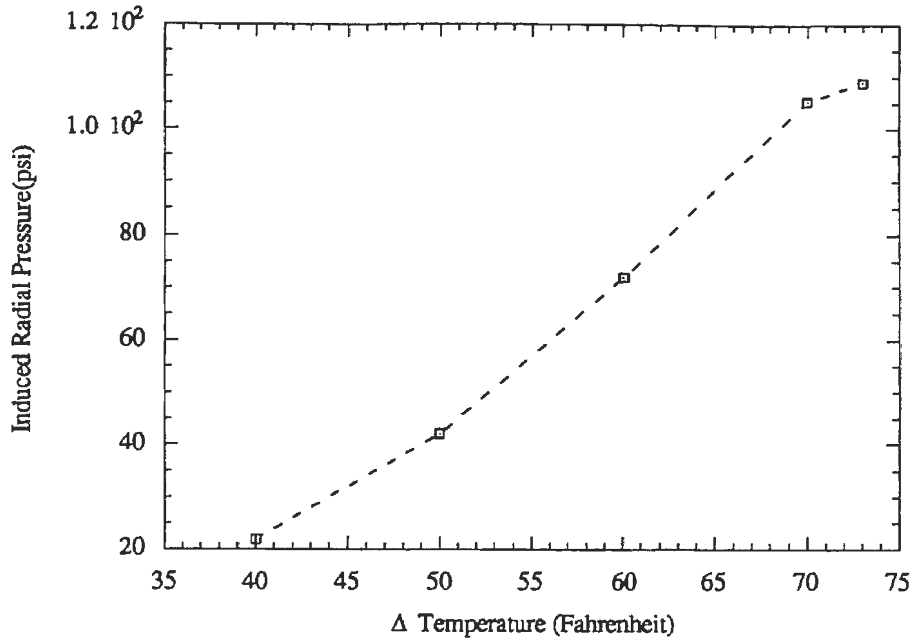

Figure 4.15 Pressure - Temperature relation for aluminum

Pull Tab Calibration

Sandwich type pull tabs were used to measure the radial stress induced in the wound rolls due to the winding tension. To get precise results from the winding experiments, careful calibration of the pull tabs is very important. The pull tab was placed at a desired radial location because both from theory and experiments it was clear that the calibration of the curve of the pull tab is a function of its position in the stack. The tab's position along the thickness of the web stack should be determined by the position in which it will be inserted in the winding roll.

Since the width of the web was only 2 inches, special guides were used to prevent the web from walking away from the winding roller, which may cause telescoping. The guides were fixed onto the core with the help of screws, but this caused a disadvantage that the pull tabs could not be placed near the core. Four tabs were placed in the Nickel roll and their position was noted and four pull tabs were placed in the stack for calibration at the same radial location as were placed in the roll.

Figure 4.16 shows the calibration curve of one of the four pull tabs used in these experiments for nickel. The points show the average results of three pulls and the dotted line is the curve fit line which is a linear function Y = A*X. The error bars along the Y - axis are the 95% probability of finding the actual stresses. Similarly Figure 4.17 shows one of the calibration curve for one pull tab for aluminum.

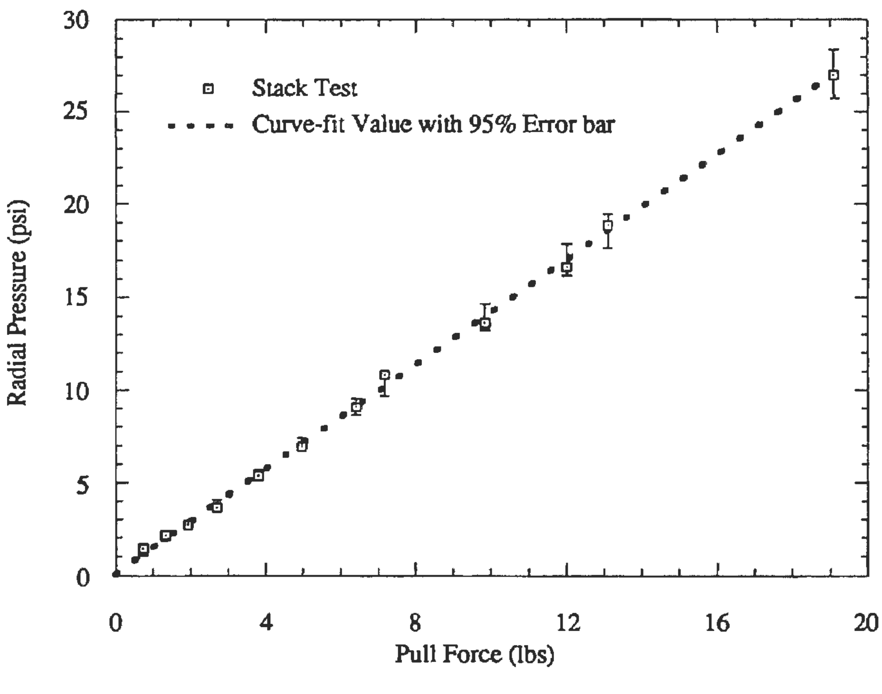

Figure 4.16 Calibration Curve for a 1" high stack of Nickel using pull tabs

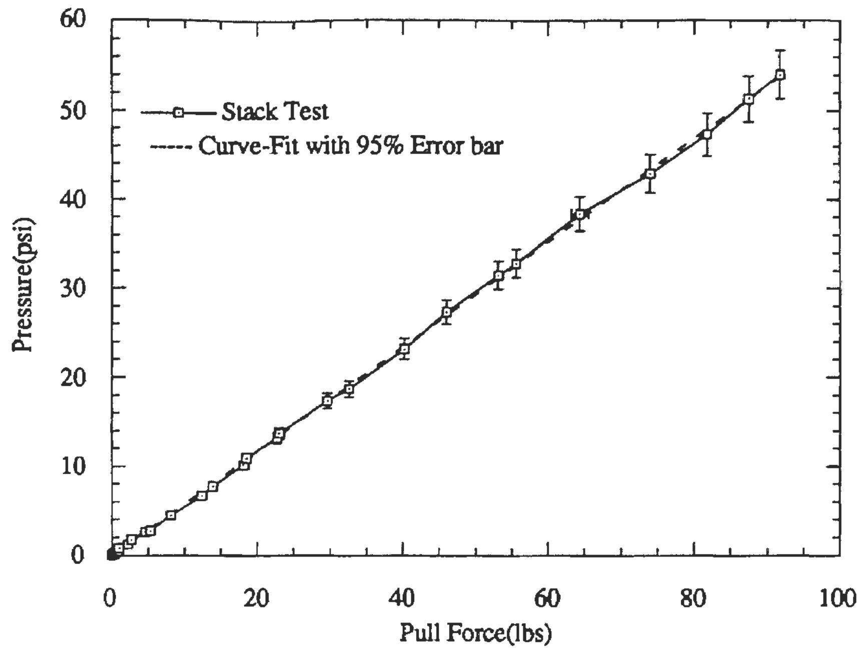

Figure 4.17 Calibration Curve for a 1" high stack of Aluminum using pull tabs

CHAPTER V

RESULTS AND DISCUSSIONS

Validation of the results obtained by experiments were carried out by the computer code developed as was explained in chapter III. It is interesting to note that the radial pressure increases as a result of a rise in temperature. This is because if there is a temperature rise, a stress field develops near the core due to the expansion of the core and it propagates throughout the roll and as a result, there will be a increase of radial pressure inside the roll. The core roll interface will be experiencing the same stress as long as there is no mismatch between the radial coefficient of thermal expansion. This resulted in an increased compression in the inner layers.

It was verified theoretically by the thermoelastic model that an increase in temperature causes an increase in radial pressure. The theoretical results were very much in accordance with the experimental results and are presented later.The radial coefficient of thermal expansion was considered as a function of temperature and similarly, the radial modulus was considered as a function of radial pressure.The numerical data for input is shown in Table 5.1 for nickel 200 and in Table 5.2 for aluminum

Winding Results

Nickel 200

TABLE X

INPUT DATA FOR THERMOELASTIC MODEL
FOR NICKEL 200

Web Thickness	0.0065 (inches)
Initial Roll Radius	1.75 (inches)
Final Roll Radius	3.0 (inches)
Inner Radius of Core	1.5 (inches)
Modulus of Core Material	1.1 e+07 (psi)
Poisson's Ratio of Core	0.33
Tangential Modulus of Web	3.1e+07 (psi)
Poisson's Ratio of Web	0.01
Radial Modulus of Web E_r (P) = c0+c1*P+c2*P^2+c3*P^3	c0 = 114037.0 c1 = 104.68 c2 = -0.0338 c3 = 5.767e-06
Radial Coefficient of Thermal Expansion α_r (P) = c0* (1 - e^(-c1*P)) + c2	c0 = 7.4e-06 c1 = 0.0041105 c2 = 0.0

TABLE X (Continued)

Tangential Coefficient of Thermal Expansion α_t	7.43e-06 in/in/^{o} F
Coefficient of Expansion of Core α_c	6.43e-06 in/in/^{o} F
Temperature Difference ΔT	0.0 - 40.0 ^{o} F
Winding Tension T_W	70 psi

A winding tension of 70 psi was used and as shown the program was run for different temperature changes.

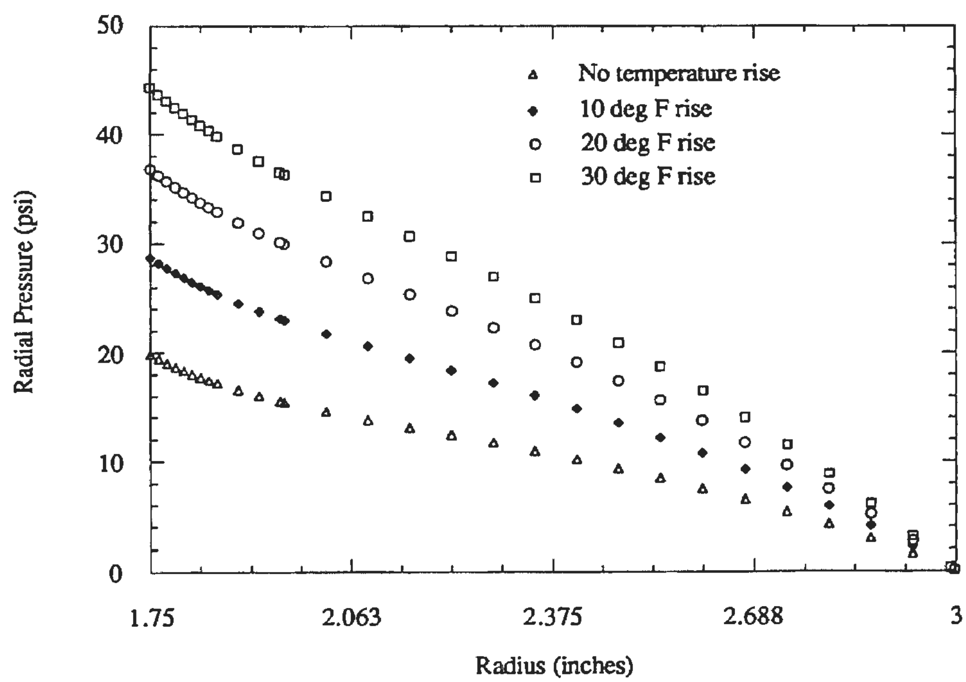

Figure 5.1 Radial Stress profile for different temperatures

It is clearl from Figure 5.1 that there is a increase in the radial stress as resultof a rise in temperature. The nickel roll was centerwound on the winder with a tension of 9 lbs (70 psi). Since the web is only 2 inches wide, a aluminum spool was made and fixedon the core of the winding roll, in order to guide the incoming web.

It is believed that a rise in temperature causes an increase in the tangential pressures at the outer layers as well as in inner layers of the web due to the expansion of the core and the web material. This analysis was carried out to verify that the stress profile follows correctly as was analyzed by Connolly et al.(1987).

It is seen from the above graph that, the radial pressure profile follows our previous discussion but when there is a mismatch between the radial coefficient of expansion of the web and the expansion of the core, there is a rapid increase in radial pressure.

In order to validate the thermoelastic model, Center Winding was carried at the WHRC on the O.S.U Winder. The nickel roll was wound on to a steel core with known material properties and four pull tabs were used at different radial locations. Then immediately after winding, the radial pressures were measured at the four locations using the calibration curves previously obtained for the pull tabs. Then the roll was placed in the temperature chamber and heated from room temperature with desired increments of temperature and each time the pressures measured.

The results showed a good correlation to the theoretical results obtained by the computer program. The theoretical and experimental correlation of the radial pressures are shown in Figures 5.2 - 5.6 for nickel at different temperature increments.

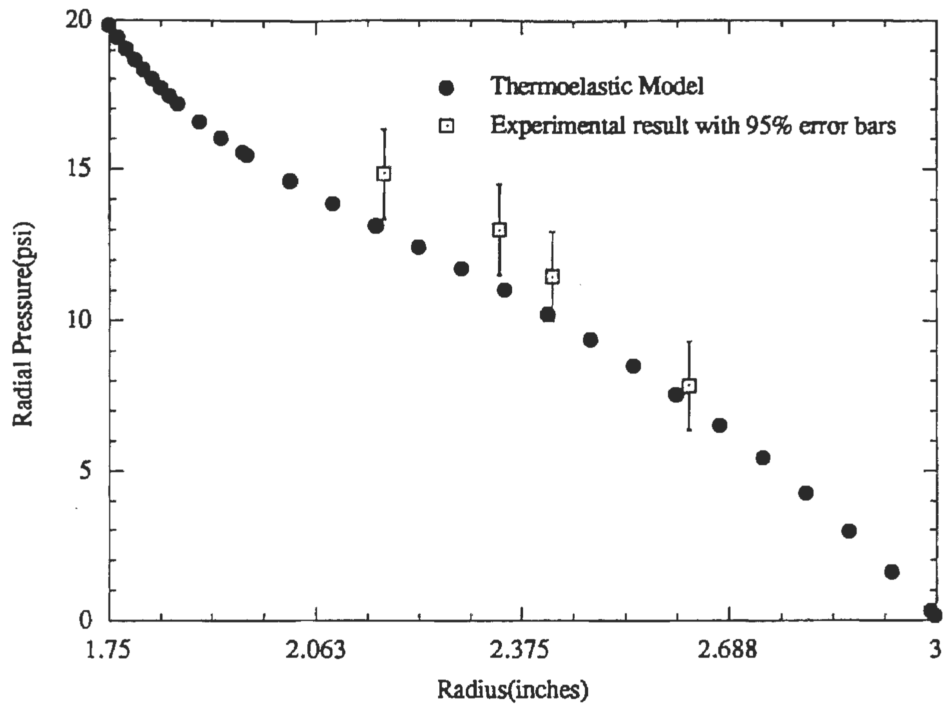

Figure 5.2 Radial Pressure profile for no rise in temperature

The experimental results shown in Figure 5.2 show the average of three pulls with 95% error bars. Similar plots are shown for different temperatures.

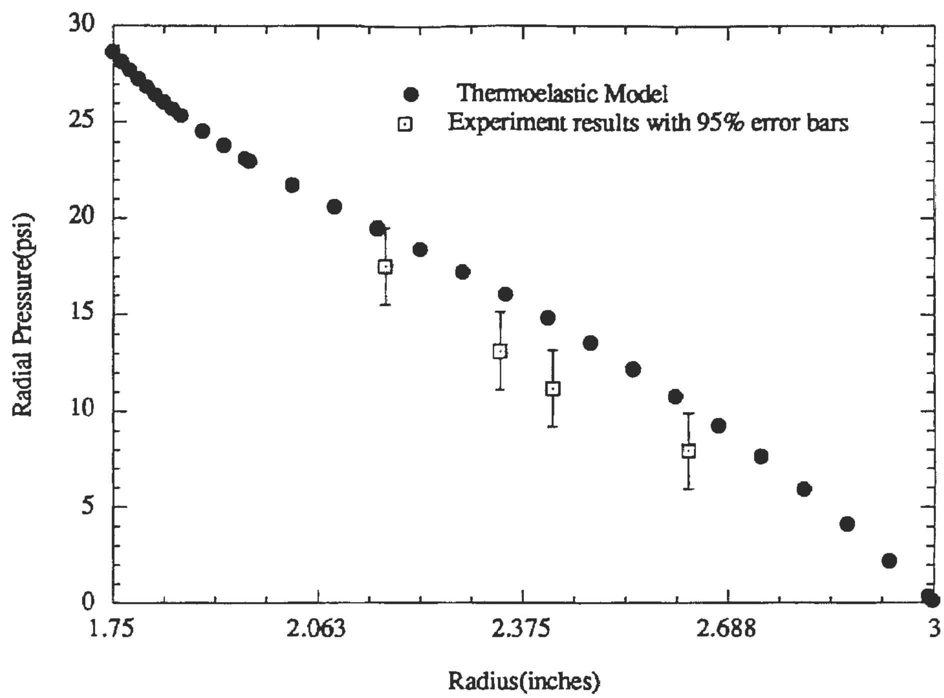

Figure 5.3 Radial Pressure profile for a 10° F rise in temperature

It can be seen from these figures that the radial pressures determined experimentally are less than those predicted by the model. This is due to the fact the web is thick and when the tension is set to 70 psi on the winder, only part of the winding tension is being utilized by the winding web and some part of it is lost in elastic deformation of the web material.

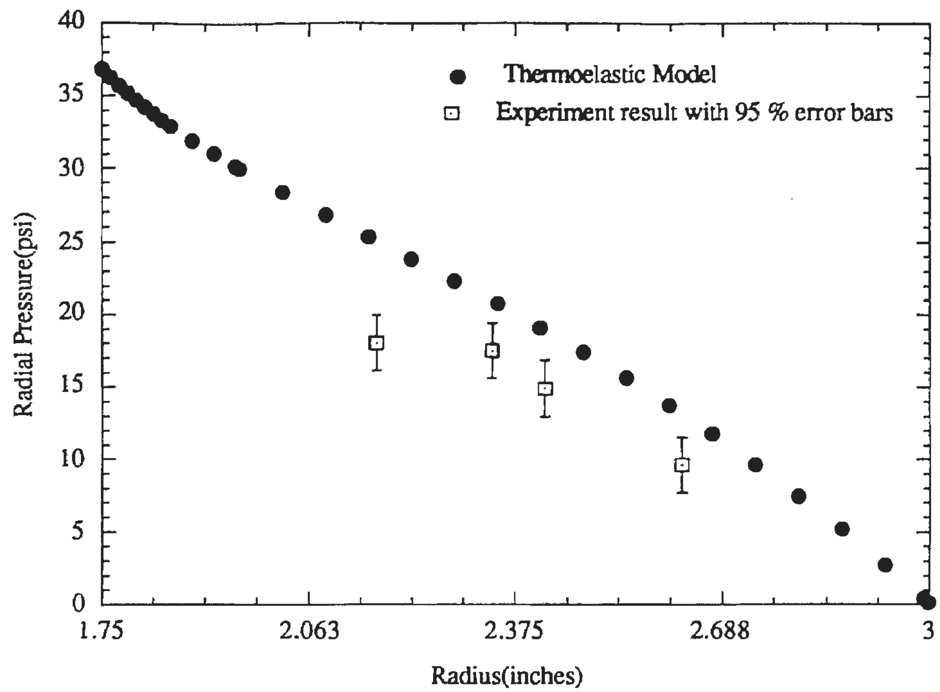

Figure 5.4 Radial Pressure profile at 20^{o}F rise in temperature

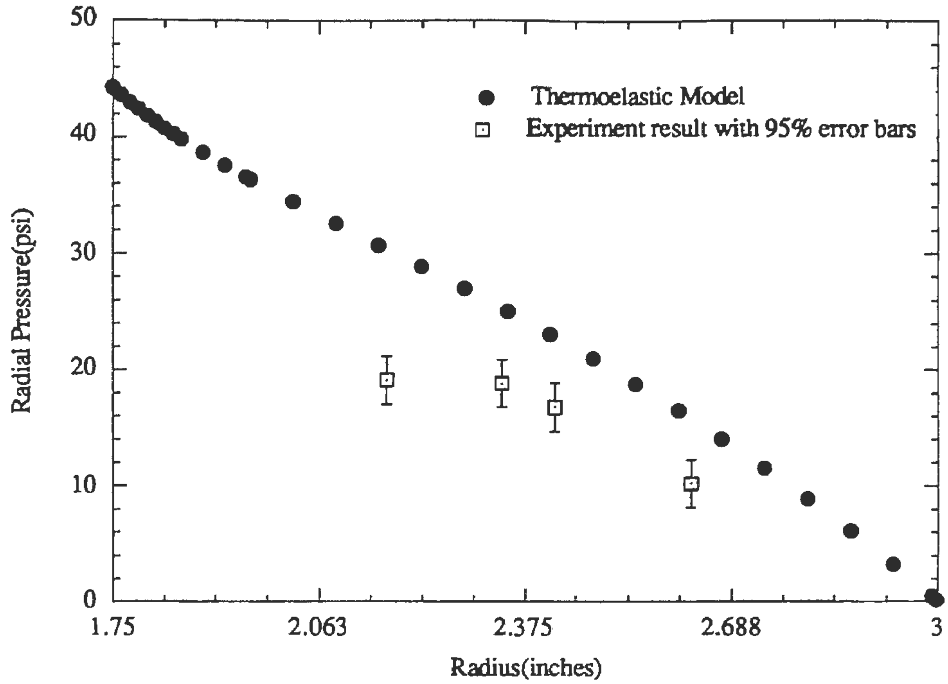

Figure 5.5 Radial Stress profile for a 30^{0} F rise in temperature

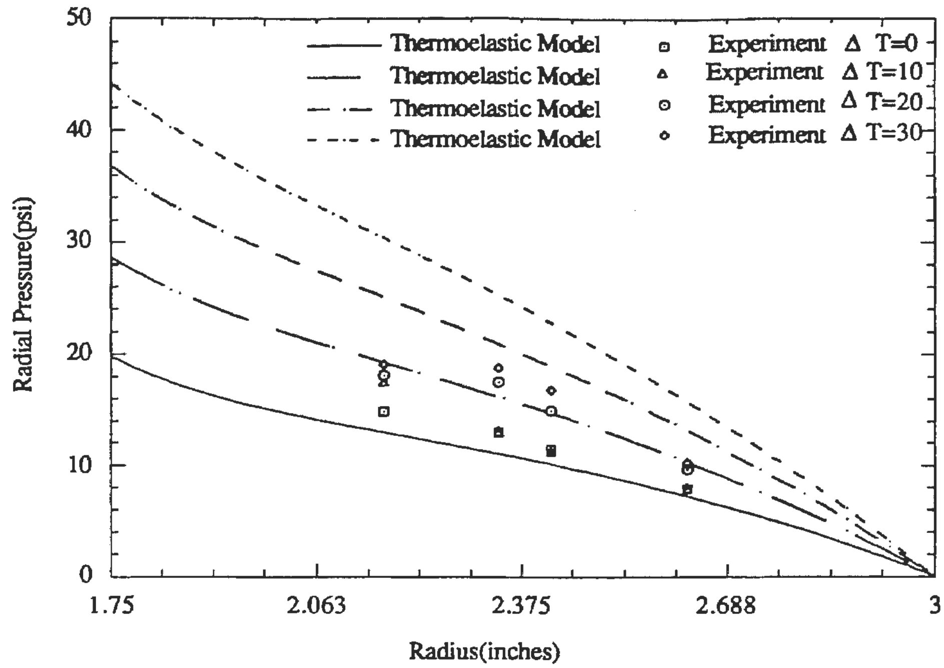

Figure 5.6 Theoretical and Experimental Radial Stress profiles for different rise in temperatures in °Fahrenheit

From Figure 5.6, we can observe that there is a considerable drop in the experimental radial pressures as predicted theoretically. The significance may be due to elastic deformation taking place in the span of entry on to the winding roll or a reduced expansion of the wound roll in the radial direction, after winding and causing a forced temperature change.

Aluminum

TABLE XI

INPUT DATA FOR THERMOELSATIC MODEL
FOR ALUMINUM

Web Thickness	0.002 (inches)
Initial Roll Radius	1.75 (inches)
Final Roll Radius	2.741 (inches)
Inner Radius of Core	1.5 (inches)
Modulus of Core Material	1.1 e+07 (psi)
Poisson's Ratio of Core	0.33
Tangential Modulus of Web	11.5e+06 (psi)
Poisson's Ratio of Web	0.01
Radial Modulus of Web E_r (P) = c0+c1*P+c2*P^2+c3*P^3	c0 = 0.0 c1 = 1473.9999 c2 = -15.943 c3 = 0.057
Radial Coefficient of Thermal Expansion α_r (P) = c0* (1 - e^(-c1*P)) + c2	c0 = 13.1e-06 c1 = 0.0041105 c2 = 0.0
Tangential Coefficient of Expansion of Web α_t	13.1e-06 in/in/ ^{o}F

TABLE 5.2 (Continued)

Coefficient of Expansion of Core α_c	6.43e-06 in/in/ ^oF
Temperature Difference ΔT	Varies from 0 ^oF - 60 ^oF
Winding Tension T_w	1000 (psi)

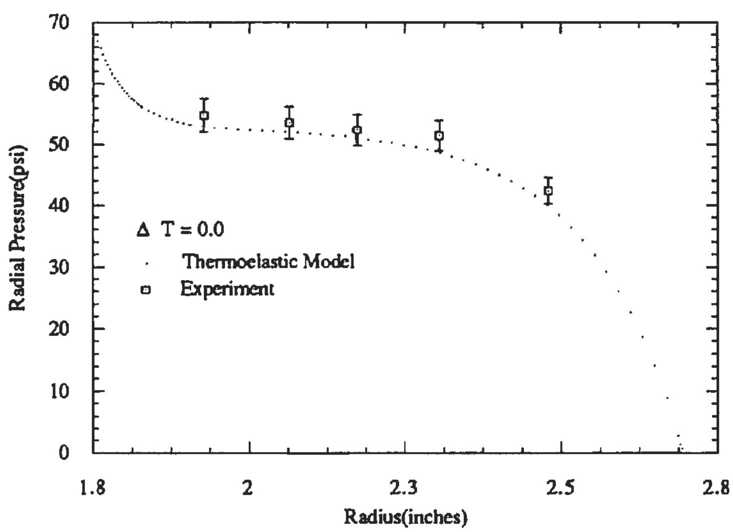

Figure 5.7 Radial Pressure Profile for no rise in Temperature

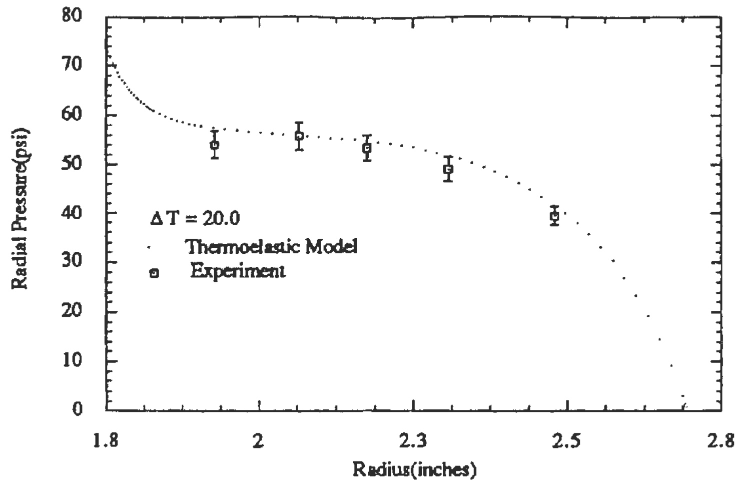

Figure 5.8 Radial Pressure Profile for a 20° F rise in Temperature

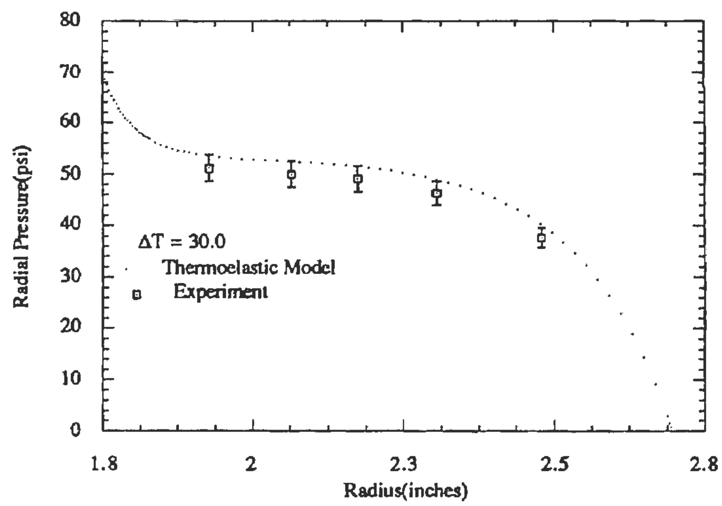

Figure 5.9 Radial Pressure for a 30° F rise in Temperature

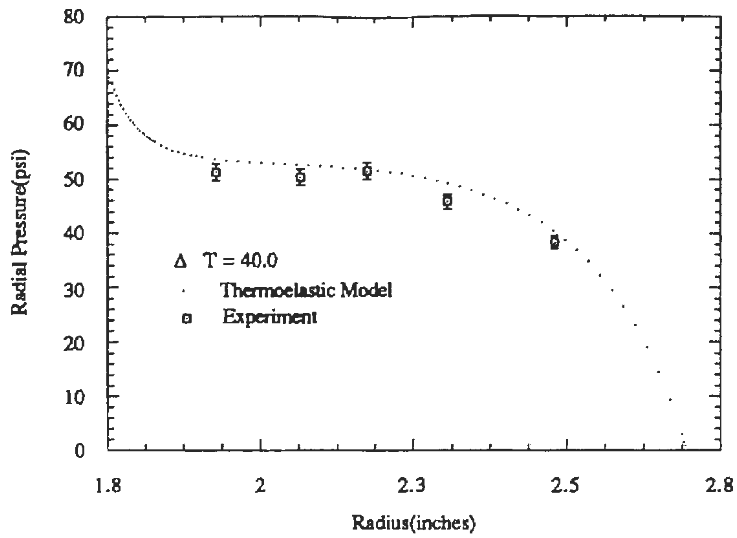

Figure 5.10 Radial Pressure Profile for a 40° F rise in Temperature

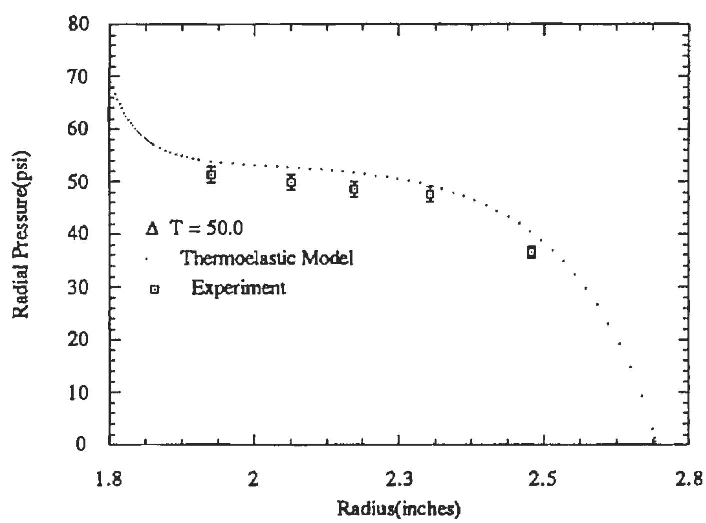

Figure 5.11 Radial Pressure Profile for a 50° F rise in Temperature

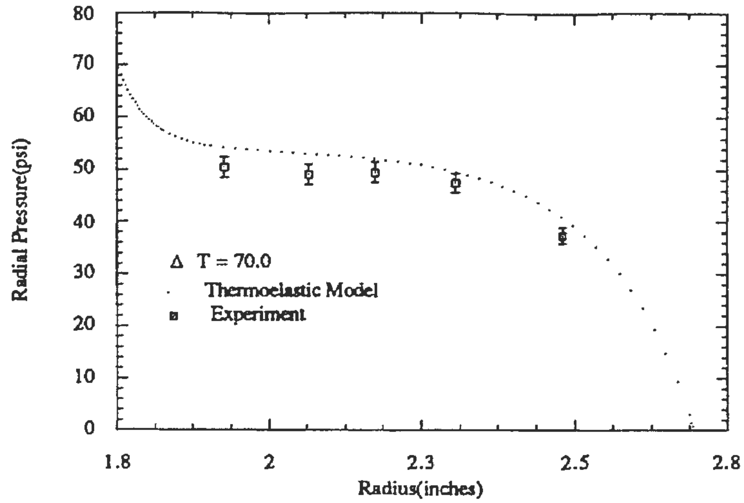

Figure 5.12 Radial Pressure Profile for a 70° F rise in Temperature

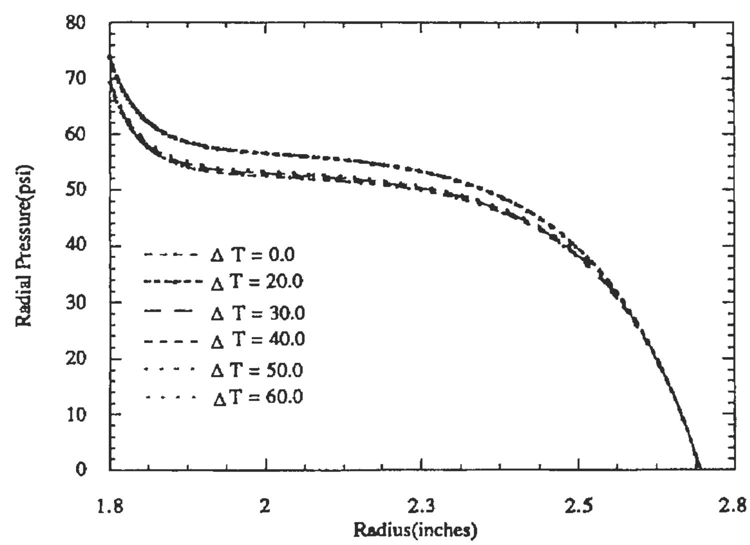

Figure 5.13 Radial Pressure Profiles for different Temperatures

The results shown in the figures 5.7 - 5.12 are the correlation of the Thermoelastic Model with the experimentally determined radial pressures as a function of temperature. They are clearly shown to match with the theoretical model. There is a slight descrepency in the experimental results. This may account for the values of radial coefficient of thermal expansion. We have assumed the radial coefficient of thermal expansion as a function of temperature. But at different temperatures, there is a reduction of radial pressures in the wound roll. The main effect could be a reduction in the expansion in the radial direction as has been proposed theoretically.

Nothing much can be studied about the radial coefficient of thermal expansion as the main idea of this research is to develop a thermoelastic wound roll model rather than to study the effect of temperature and pressure on radial coefficent of expansion.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The parameters that effect the wound roll structure have been shown to be the temperature the rolls are subjected to during and after winding and the radial coefficient of thermal expansion of the core and the web material. The radial modulus as a function of radial pressure plays an important role in the roll stress profile. The developed model agrees well with the previous findings and is easily implemented as a modified differential equation with a non zero right hand vector. Considering the coefficient of radial and tangential thermal expansion of the web with an modified inner boundary condition, and also considering the effect of the coefficient of thermal expansion of the core in the Hakiel's winding model is the basis of the Thermoelastic winding model

The main factors which contribute for the stress distribution are the winding tension as was shown in previous findings and the coefficient of thermal expansions of the web as well as the core with the correct operating temperatures as seen in the present analysis. The coefficient of thermal expansion in the radial direction was a difficult property to be measured, but the present research provides a way for its determination. Though the method employed here may not represent the exact behavior of the wound roll, it provides a realistic way for its response to temperature and pressure.

The radial coefficient of thermal expansion was a factor which affects the wound roll pressures when subjected to temperature changes. But the main purpose of this research was to develop a complete wound roll model which considers the effects of temperature changes. Finally it can be concluded that the theoretical model developed agrees well with the experiments, and the model completely describes the thermal effects in wound rolls.

Future Work

To consider the moisture effects on the wound roll stress distribution is beyond the scope of this thesis. In order to completely predict the hygroscopic effects in wound rolls, the effect of the coefficient of hygroscopic expansion needs to be considered along with the thermal effects. Addition of this capability will allow the model to predict the thermal as well as the moisture effects on wound rolls. Future work should explore explore the stress induced due to hygroscopic expansion or contraction.

The effect of the radial coefficient of thermal expansion needs to be studied in detail and an analytical model needs to be developed which exactly predicts its response to temperature cycles and pressures.

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APPENDIX

PROGRAM FOR CALCULATING THE STRESSES DEVELOPED IN WOUND ROLLS DUE TO WINDING AND TEMPERATURE CHANGES

```
c    This is a Program for finding the Thermal Stresses induced
c     in Wound Rolls due to temperature Differences
c     Program for calculating the pressure distribution in the
c     center wound rolls that are wound at constant wound-in
c     stress tw, or constant wound-in torque.
c     The program is capable of applying a variable radial modulus
c     and is capable of accounting for core and caliper deformatio
c     The program consists of the main program, seven subroutines
c     one function.  The subroutines and function are as follows:
c
c           ptstrs:  dp(i) for each layer wrapped onto the roll.
c          junk:  Calculates Thermal Stresses in Wound Rolls.
c           strain:  dtstran(i) for tangential strain E.L.W.R.
c            lineq:  solves tri-diagonal matrix eq's.
c           lineqq:  solves tri-diagonal matrix eq's.
c           output:  output file writing.
c        data block:  initialization of variables p(i) and t(i).
c               er:  function that defines the radial modulus
c ****************************************
c     The following variables are all inputs to the program:
c
c              h = web thickness
c             ec = core stiffness
c             et = elastic modulus of the web material in the
c                  tangential direction
c            rmu = Poisson's ratio of the web material
c           rmin = initial radius of the roll (core o.d./2)
c           rmax = final radius of the roll
c          iflag = 1:  constant torque winding
c          iflag = 0:  constant tension winding
c          IFLAG = 2:  POLYNOMIAL TENSION VARIATION
c          idisp = 1:  stress and displacement calculations
c          idisp = 0:  stress calculations only
c
c     Consistent units should be used throughout.
************************** MAIN PROGRAM   ************
c
      IMPLICIT REAL*8 (A-H,O-Z)
      dimension tkeep(2000)

      common /mat1/a(2000,3),b(2000)
      common /mat2/dp(2000)
      common /mat3/p(2000),t(2000),dt(2000)
      common /mat4/ r(0:2000),hh(2000),u(2000)
      common /mat5/c0,c1,c2,c3
      common /mat9/f(2000)
        common /mat6/ y(2000),z(2000),dz(2000)
      common /mat7/ dy(2000)
        common /mat11/q0,q1,q2,q3
        common /mat12/alphc,alpht,delt
      common /strin/ dtstran(2000)

      DATA IO/12/
```

```
      OPEN(IO, FILE='ram.out',status='unknown')
        TAB=CHAR(9)

c
c
c---------- Data Input from console
c

       WRITE(9,*) 'THIS PROGRAM IS AN IMPLEMENTATION OF '
         WRITE(9,*) '              THERMOELASTIC MODEL'
         WRITE(9,*) '   BASED ON HAKIELS WINDING MODEL'
       WRITE(9,*) 'PRODUCED BY DR.J.K.GOOD AND HIS STUDENTS'
       WRITE(9,*) 'AT THE WEB HANDLING RESEARCH CENTER,EN 218,OSU'
       WRITE(9,*) '         STILLWATER,OKLA. 74078   '
      WRITE(9,*) 'ENTER THE FOLLOWING PARAMETERS:'
      WRITE(9,*)
      WRITE(9,*) 'TYPE EITHER:'
      WRITE(9,*) 'ZERO: FOR STRESS CALCULATIONS ONLY'
      WRITE(9,*) ' ONE: FOR STRESS AND DISPLACEMENT CALCULATIONS'
       READ(9,*) IDISP
      IF(IDISP.EQ.0) THEN
      WRITE(IO,88)
     ELSE
      WRITE(IO,90)
      ENDIF
      LP = 1
        write(9,*)'ENTER THE WINDING TENSION'
        read(9,*)tw
        write(9,*)'WEB CALIPER'
      READ(9,*) H
        WRITE(9,*)'INITIAL ROLL RADIUS'
      READ(9,*) RMIN
        write(9,*)'FINAL ROLL RADIUS'
      READ(9,*) RMAX
        write(9,*)'TYPE EITHER:'
        write(9,*)'ZERO:IF THE CORE MODULUS IS KNOWN'
        write(9,*)'ONE:IF THE CORE MODULUS NEEDS TO'
        write(9,*)'    BE COMPUTED'
      READ(9,*) IMOD
      IF(IMOD.EQ.0) THEN
        write(9,*)'ENTER THE CORE MODULUS'
       READ(9,*) EC
      ELSE
        write(9,*)'INNER RADIUS OF CORE'
       READ(9,*) RIC
          write(9,*)'MODULUS OF THE CORE MATERIAL'
       READ(9,*) ECM
          write(9,*)'POISSONS RATIO OF CORE'
       READ(9,*) POIS
       RAT2=(RMIN/RIC)**2
       EC=(ECM/((RAT2+1.0)/(RAT2-1)-POIS))
      ENDIF
        write(9,*)'TANGENTIAL MODULUS OF WEB'
      READ(9,*) ET
```

```
         write(9,*)'POISSONS RATIO OF WEB'
      READ(9,*) RMU
      write(9,*)'ENTER THE CONSTANTS FOR RADIAL MODULUS'
      write(9,*)'CO=?'
      READ(9,*) C0
         write(9,*)'C1=?'
      READ(9,*) C1
         write(9,*)'C2=?'
      READ(9,*) C2
         write(9,*)'C3=?'
      READ(9,*) C3
         write(9,*)'ENTER THE TEMPERATURE DIFF'
      READ(9,*)DELT
         write(9,*)'ENTER THE SET POINT DIFF'
         read(9,*)setpoint
         write(9,*)'ENTER INCREMENT OF TEMPERATURE'
         read(9,*)incr
         write(9,*)'TYPE EITHER'
      write(9,*)'ZERO: FOR CONSTANT RADIAL COEFF OF EXP'
      write(9,*)'ONE:IF RADIAL COEFF OF EXP IS A'
      write(9,*)'    FUNCTION OF TEMPERATURE'
      write(9,*)'TWO: IF RADIAL COEFF OF EXP IS A'
      write(9,*)     'CRAZY FUNCTION'
         READ(9,*)icoeff
         if(icoeff.eq.0) then
         write(9,*)'RADIAL COEFF OF THERMAL EXP'
         READ(9,*)ALPHR
         endif
         if(icoeff.eq.1) then
         write(9,*)'Z0=?'
      READ(9,*)z0
         write(9,*)'z1=?'
         READ(9,*)z1
         write(9,*)'Z2=?'
         READ(9,*)z2
         ALPHR=z0*(1-exp(-z1*delt))+z2
         endif
         if(icoeff.eq.2) then
         write(9,*)'q0=?'
         read(9,*)q0
         write(9,*)'q1=?'
         read(9,*)q1
         write(9,*)'q2=?'
         read(9,*)q2
         write(9,*)'q3=?'
         read(9,*)q3
c        ALPHR=q0+(q1*delt)+(q2*delt*delt)+(q3*delt*delt*delt)
         endif
         write(9,*)'TANGENTIAL COEFF OF EXP OF WEB'
      READ(9,*)ALPHT
         write(9,*)'COEFF OF EXP OF CORE'
      READ(9,*)ALPHC
         write(9,*)'TYPE EITHER'
         write(9,*)'ZERO:IF CONSTANT WINDING TENSION'
```

```
        write(9,*)'ONE: IF CONSTANT TORQUE'
        write(9,*)'TWO: IF VARIABLE TORQUE'
        read(9,*)iflag
        IF(IFLAG.EQ.0)THEN
        WRITE(9,*)'CONSTANT WINDING TENSION'
        READ(9,*)TW

      ENDIF
      IF(IFLAG.EQ.1) THEN
        write(9,*)'CONSTANT TORQUE'
       READ(9,*) TOR
      ENDIF
      IF(IFLAG.EQ.2) THEN
        write(9,*)'ENTER T1'
       READ(9,*) T1
         write(9,*)'ENTER T2'
       READ(9,*) T2
         write(9,*)'ENTER T3'
       READ(9,*) T3
      ENDIF
c
c---------- Initial writing of input data
c

      WRITE(IO,101) H,EC,ET,RMU,RMIN,RMAX,LP,DELT
      WRITE(IO,103) c0,c1,c2,c3
        WRITE(IO,104) z0,z1,z2
      WRITE(IO,102) IFLAG,TOR,ALPHR,ALPHT,ALPHC,TW,T0,T1,T2,T3,Q
 88   format(10x,'io.disp--STRESS vs. RADIUS',
   +  1x,'WITHOUT CALIPER & CORE DEFORMATIION')
 90   format(10x,'io.disp--STRESS vs. RADIUS',
   +  1x,'WITH CALIPER & CORE DEFORMATIION')
101    format(/,'h =',f7.5,/,'ec =',F10.1,/,'et =',f10.1,/,'rmu =',f5.4
   &  ,/,'rmin =',f6.3,/,'rmax =',f6.3,/,'lp =',i2/,'delt = ',f10.3,/)
102    format('iflag =',i2,/,'tor =',f16.8,/,'alphr=',f10.9,/,
   &   'alpht=',f10.9,/,'alphc=',f10.9,/,'tw =',f16.8,/,'Q=',f16.9,/,
   &   'Tw=',F8.3,'+',F8.3,'*RR+',F8.3,'*RR**2+',F8.3,'*RR**3')

103    FORMAT('Er =',f10.3,'+',f8.3,'*P +',f8.3,'*P^2 + ',f8.3,'*P^3')
104     FORMAT('Alpha Rad = ',f10.9,'*','(1-exp(',f8.6,'*delt )+',f5.3)

c
c---------- Initialization of variables
c

      NLAP = (RMAX-RMIN)/H
      hlimt=0.8*h
      r(0)=rmin
      hh(1)=h
      if(iflag .eq. 1) tw=tor/h/r(0)
      if(iflag .eq. 2) tw=T0
      twkeep=tw
      kk = 0
c       tk= et*delt*(alphr-alpht)
```

```
c       ttk=et*delt*(alphc-alpht)

c
c---------- Do Loop 1000 does primary work of the program
c
c---------- Call "ptstrs" to calculate the increment of the radial stress, dp(i)
c---------- Calculate the radial stress after each lap to evaluate Er
c
 1000   do while (r(kk).lt.rmax)

        if (kk/40*40 .eq. kk) write(9,*)'Thermoelastic Winding
     + model - Iteration #',kk,' of ',nlap

        if(iflag.eq.1) tw=tor/r(kk)/h
        XXX=r(kk)-r(0)
        if(iflag .eq. 2) tw=T0+(T1*XXX)+(T2*XXX**2)+(T3*XXX**3)

        dp(kk+1)=h*tw/r(kk)

        dt(kk+1)=tw



        if (idisp.eq.0)  then
         hh(kk+1)=h
        else
         hh(kk+1)=h*(1.0d0-dp(kk+1)/2/er(dp(kk+1)/2)-rmu*tw/et)

        endif
        r(kk+1)=r(kk)+hh(kk+1)
  20    call strs(kk,h,et,ec,rmu,q)
c
c----------  Calculating the p(i) & dt(i)
c

        do 60 i=1,kk+1

       p(i)=p(i)+dp(i)
  60      continue
        goto (73,72) kk+1
         do 70 i= kk,2,-1
           deridp=(dp(i+1)-dp(i-1))/(r(i+1)-r(i-1))
  70       dt(i)= -dp(i)-r(i)*deridp
  72       dt(1)= -dp(1)*(rmu+et/ec)

  73     continue
        if (idisp.eq.0) goto 1002
        if (kk.gt.1) call strain(kk,rmu,et,h,ec,hlimt,lp)
 1002     kk=kk+1
       enddo
       kk=kk-1
       t(kk+1)=tw
       do 80 ii= kk,2,-1
  80   t(ii)=-p(ii)-r(ii-1)*(p(ii+1)-p(ii))/(hh(ii+1)+hh(ii))
```

```
      t(1)= twkeep-p(1)*(rmu+et/ec)
     if (idisp.eq.0)goto 1003
     hmin=h
     do 81 i=1,kk
      if(hh(i).lt.hmin) hmin=hh(i)
  81  enddo
 1003  ii=int((rmax-rmin)/h/6)
c
c--------- Print output to file
c
      call output(kk,twkeep,hmin,idisp,ii,tw,rmin)
c
c--------- calculating Thermal stresses in rolls
c

          write(9,*)'Calculating thermally induced stresses'
          nlap=(rmax-rmin)/h
          kk = nlap

          dy(nlap) = 0.0
        dz(kk+1)=tw
           do while(delt.lt.setpoint)
           write(9,*)'iterating the thermal problem ',delt,'of',setpoint
           call ptstrs(kk,h,et,ec,rmu,tk,ttk)
c         write(9,*)'after iteration #',kk,nlap
           do 89 i=1,kk+1




 89      y(i)=p(i)+dy(i)
          write(9,*)'calculating final stresses'
c        goto (93,92) kk+1
c          do 91 i= kk,2,-1
c            deridy=(dy(i+1)-dy(i-1))/(r(i+1)-r(i-1))
c  91       dz(i)= -dy(i)-r(i)*deridy
c 92        dz(1)= -dy(1)*(rmu+et/ec)
c  93      continue
         do 94 i=1,kk

          z(i)=t(i)+dz(i)
  94   continue

       if (kk.gt.1) call strain(kk,rmu,et,h,ec,hlimt,lp)
           delt=delt+incr
         enddo
     if (idisp.eq.0)goto 96
     hmin=h
     do 97 i=1,kk
      if(hh(i).lt.hmin) hmin=hh(i)
  97  enddo
  96  ii=int((rmax-rmin)/h/6)
     write(9,*)'calculated stresses'
```

```
          call out(kk,twkeep,hmin,idisp,ii,tw,rmin)

       Write(9,*) 'writing io.disp ends!'
       stop
       end

******************   FUNCTION ER      *****************
****************   Radial Modulus Determination   ******
C
      function er(x)
      implicit real*8 (a-h,o-z)
      common /mat5/c0,c1,c2,c3
      er = c0+(C1*x)+(C2*(x**2))+(C3*(x**3))
      return
      end

         function alphr(x)
         implicit real*8 (a-h,o-z)
         common /mat11/q0,q1,q2,q3
      alphr = q0+(q1*x)+(q2*(x**2))+(q3*(x**3))
         return
         end

*********** Subroutine Strs *****************

           subroutine strs(n,h,et,ec,rmu,q)
      implicit real*8 (a-h,o-z)

      common /mat1/ a(2000,3),b(2000)
      common /mat3/ p(2000),t(2000),dt(2000)
      common /mat2/ dp(2000)
      common /mat4/ r(0:2000),hh(2000),u(2000)
      data in/10/
      data io/12/
c     ************************************
c     rk is the coefficient relating to the
c     internal boundary condition
c     ************************************

         rk=1.0d0+hh(1)*(et/ec-1.0+rmu)/r(0)


      go to (2,4,6) n+1
c     n=0 goto 2 to calculate dp(1) when winding lap at r(1)
c     n=1 goto 4 to calculate dp(1), dp(2) when winding lap at r(2)
c     n=3 goto 6 to calculat dp(1), dp(2) & dp(3) when winding lap at r(3)
c     n>=4 goto 8 to form matrix a
c     elements order of the first row is changed to fit sub. lineq( )


c     ****************************
c     Forming the Matrix a
c     ****************************
```

```
 8
      a(1,1)=0.0d0
      a(1,2)=-1.0d0*rk
         a(1,3)=1.0d0
      do 10 i=2,n
      temp=hh(i)/r(i)

      gi2=et/er(p(i))



      a(i,1)=1.0d0-temp*1.5d0
      a(i,2)=temp*temp*(1.0d0-gi2)-2.0d0
 10   a(i,3)=1.0d0+temp*1.5d0

         do 11 i=1,n-1
 11   b(i)=0.0d0
      b(n)=-a(n,3)*dp(n+1)


      call lineqq(n)
      return


c     for the first 3 layers the change of thickness is ignored
 2    return

 4       dp(1)=dp(2)/rk
      return

 6       a2=1.0d0+1.5d0*h/(1.0d0+h)
         b2=h/(1.0d0+h)*h/(1.0d0+h)*(1.0d0-et/er(p(2)))-2.0d0

         c2=1.0d0-1.5d0*h/(1.0d0+h)
         temp=1.0d0*rk*b2+c2

         temp2=-a2*dp(3)
         dp(1)=temp2/temp

         dp(2)=dp(1)*rk
      return
      end


****************** SUBROUTINE PTSTRS ***************
*********** for constructing coefficient matrix a*******
*************used in the subroutine lineq.f *********
c
      subroutine ptstrs(n,h,et,ec,rmu,tk,ttk)
      implicit real*8 (a-h,o-z)
      dimension xx(2000)

      common /mat1/ a(2000,3),b(2000)
         common /mat3/ p(2000),t(2000),dt(2000)
```

```
      common /mat6/ y(2000),z(2000),dz(2000)
      common /mat7/ dy(2000)
         common /mat12/alphc,alpht,delt
      common /mat4/ r(0:2000),hh(2000),u(2000)
         common /mat9/ f(2000)
      data in/10/
      data io/12/
c     ****************************************
c     rk is the coefficient relating to the
c     internal boundary condition
c     ****************************************

      WRITE(9,*)'ENTERED PTSTRS'

         rk=1.0d0+hh(1)*(et/ec-1.0+rmu)/r(0)


c      go to (2,4,6) n+1
c     n=0 goto 2 to calculate dp(1) when winding lap at r(1)
c     n=1 goto 4 to calculate dp(1), dp(2) when winding lap at r(2)
c     n=3 goto 6 to calculat dp(1), dp(2) & dp(3) when winding lap at r(3)
c     n>=4 goto 8 to form matrix a
c     elements order of the first row is changed to fit sub. lineq( )
c     ******************************
c     Forming the right hand vector
c     ******************************
      do 5 i=1,n
         xx(i)=et*delt*(alphr(p(i))-alpht)
 5       continue


c     ******************************
c     Forming the Matrix a
c     ******************************

 8
      a(1,1)=0.0d0
      a(1,2)=-1.0d0*rk
         a(1,3)=1.0d0
      do 10 i=2,n
      temp=hh(i)/r(i)

      gi2=et/er(p(i))


      a(i,1)=1.0d0-temp*1.5d0
      a(i,2)=temp*temp*(1.0d0-gi2)-2.0d0
 10    a(i,3)=1.0d0+temp*1.5d0

c     ******************************
c     Modified right hand vector
c     ******************************
      do 30 i=2,n-1
 30    f(i)=xx(i)
```

```
      f(n)=xx(n)-a(n,3)*dy(n+1)


      f(1)=xx(1)-(h*et*(alphc-alpht)*delt)/r(0)

      call lineq(n)

      return

      end
******* SUBROUTINE STRAIN ***************
       subroutine strain(kk,rmu,et,h,ec,hlimt,lp)
      implicit real*8 (a-h,o-z)
      common /mat2/ dp(2000)
      common /mat3/ p(2000),t(2000),dt(2000)
      common /mat4/ r(0:2000),hh(2000),u(2000)
      common /strin/ dtstran(2000)

c        Calculating incremental tangential strain due to web
c        deformation and thermal expansion and updating radius

 20   do 30 i=kk,1,-1
      dtstran(i)=(dt(i)+rmu*dp(i))/et+(alpht*delt)
      u(i)=r(i)*dtstran(i)
      r(i)=r(i)+u(i)
 30   continue

c        Deformation at the core due to winding tension and
c        thermal expansion of the core
c
         str=alphc*delt
      r(0)=r(0)*(1.0d0-dp(1)/ec-str)

c        ******************************************
c      updating thickness hh(i)
c        ******************************************

      do 31 i=1,kk
        hh(i)=r(i)-r(i-1)
        if (u(i).lt.0.d0 .and. hh(i) .gt. h) then
         iredo=1
         hh(i)=h
        endif
        if (hh(i) .lt. hlimt) then
         iredo=2
         hh(i)=hlimt
        endif
 31   end do
      if (iredo .ne. 0) then
        do 32 i=1,kk
          r(i)=r(i-1)+hh(i)
 32       end do
      endif
      iredo=0
```

```
130   continue

110   format('displ.=',f15.10,' > 0.0, at r=',f9.6,
  +        ' while wound at r=',f9.6)
112   format ('for every 40 layers core radius=',f10.6)
114   format('iredo=',i1,' at',i4,' th while wind at',i4,' layer'/
  +          'recalculate r(i) for hh(',i3,')=',f12.7)


125   format('output r(0),dtstran(i),u(i),r(i),hh(i)')
126   format('i=',i3,4f15.10/)
128   format('r(0)=',f15.10)
      return

* SUBROUTINE LINEQQ  c
      subroutine lineqq(n)
      implicit real*8 (a-h, o-z)
      dimension gam(2000)
      common /mat1/ a(2000,3),b(2000)
      common /mat2/ x(2000)
      bet=a(1,2)
      x(1)=b(1)/bet
      do 10 k=2,n
      gam(k)=a(k-1,3)/bet
      bet=a(k,2)-a(k,1)*gam(k)
      x(k)=(b(k)-a(k,1)*x(k-1))/bet
  10  continue
      do 20 k=n-1,1,-1
  20  x(k)=x(k)-gam(k+1)*x(k+1)
      return
      end
****************** subroutine lineq************

      subroutine lineq(n)
      implicit real*8 (a-h, o-z)
      dimension gam(2000)
      common /mat1/ a(2000,3),b(2000)
         common /mat9/f(2000)
      common /mat7/ x(2000)
      bet=a(1,2)
      x(1)=f(1)/bet
      do 10 k=2,n
      gam(k)=a(k-1,3)/bet
      bet=a(k,2)-a(k,1)*gam(k)
      x(k)=(f(k)-a(k,1)*x(k-1))/bet
  10  continue
      do 20 k=n-1,1,-1
  20  x(k)=x(k)-gam(k+1)*x(k+1)
      return
      end
```

```
***********************************************
***********************************************
******************* SUBROUTINE OUTPUT **********
****************** writes ouput to file *********
c
      subroutine output(kk,twkeep,hmin,idisp,ii,tw,rmin)
      implicit real*8 (a-h,o-z)
      common /mat3/p(2000),t(2000),dt(2000)
      common /mat4/ r(0:2000),hh(2000),u(2000)
      data in/10/
      data io/12/
         TAB=CHAR(9)
      if (idisp.eq.0) then
        write(io,122) kk
      else
        write(io,120) r(0), kk, hmin
      endif

      write(io,118)
      do 78 i=1,ii/2,2
 78     write(io,119) r(i-1),TAB,p(i),TAB,t(i)
      do 79 i=ii/2+1,ii,5
 79     write(io,119) r(i-1),TAB,p(i),TAB,t(i)
      do 80 i=ii+1,kk-1,10
 80     write(io,119) r(i-1),TAB,p(i),TAB,t(i)
      write(io,119) r(kk-1),TAB,p(kk),TAB,t(kk)
      write(io,119) r(kk),TAB,p(kk+1),TAB,t(kk+1)
118   format(5x,'R',12x,'P(r)',10x,'T(r)')
119    format(F8.5,A1,F10.5,A1,F10.5)
120   format(/'after finishing winding the core radius=',f8.5/
     +  'total layers wound onto the roll core=', i4,1x,/
     +  'after finishing winding the smallest caliper=',f8.5//)
122   format(/'total layers wound onto the roll core=', i4//)
123   format(F8.5,A1,F10.5,A1,F10.5)
       if (idisp.eq.0) then
        write(io,122) kk
      else
        write(io,120) r(0), kk, hmin
      endif
      write(io,192)
      do 90 i=1,ii/2,2
 90     write(io,123) r(i-1)/r(0),TAB,p(i)/tw,TAB,t(i)/tw
      do 91 i=ii/2+1,ii,5
 91     write(io,123) r(i-1)/r(0),TAB,p(i)/tw,TAB,t(i)/tw
      do 92 i=ii+1,kk-1,10
 92     write(io,123) r(i-1)/r(0),TAB,p(i)/tw,TAB,t(i)/tw
      write(io,123) r(kk-1)/r(0),TAB,p(kk)/tw,TAB,t(kk)/tw
      write(io,123) r(kk)/r(0),TAB,p(kk+1)/tw,TAB,t(kk+1)/tw
192   format(2x,'N.RADIUS',7x,'N.RADIAL PRESSURE(r)',7x,
     +  'N.TANGENTIAL PRESSURE(r)')
      return
      end
```

```
**************************************************
******************** subroutine output*********

      subroutine out(kk,twkeep,hmin,idisp,ii,tw,rmin)
      implicit real*8 (a-h,o-z)
         common /mat3/p(2000),t(2000),dt(2000)
      common /mat6/y(2000),z(2000),dz(2000)
      common /mat4/ r(0:2000),hh(2000),u(2000)
         common /mat7/ dy(2000)
      data in/10/
      data io/12/
         TAB=CHAR(9)
      if (idisp.eq.0) then
       write(io,122) kk
      else
       write(io,120) r(0), kk, hmin
      endif

      write(io,118)
      do 78 i=1,ii/2,2
 78    write(io,119) r(i-1),TAB,y(i),TAB,z(i)
      do 79 i=ii/2+1,ii,5
 79    write(io,119) r(i-1),TAB,y(i),TAB,z(i)
      do 80 i=ii+1,kk-1,10
 80    write(io,119) r(i-1),TAB,y(i),TAB,z(i)
      write(io,119) r(kk-1),TAB,y(kk),TAB,z(kk)
      write(io,119) r(kk),TAB,y(kk+1),TAB,z(kk+1)
118    format(5x,'R',12x,'Y(r)',10x,'Z(r)')
119     format(F8.5,A1,F10.5,A1,F10.5)
120    format(/'after finishing winding the core radius=',f8.5/
   +    'total layers wound onto the roll core=', i4,1x,/
   +    'after finishing winding the smallest caliper=',f8.5//)
122    format(/'total layers wound onto the roll core=', i4//)
123    format(F8.5,A1,F10.5,A1,F10.5)
       if (idisp.eq.0) then
       write(io,122) kk
      else
       write(io,120) r(0), kk, hmin
      endif
      write(io,192)
      do 90 i=1,ii/2,2
 90    write(io,123) r(i-1)/r(0),TAB,y(i)/tw,TAB,
   +z(i)/tw
      do 91 i=ii/2+1,ii,5
 91    write(io,123) r(i-1)/r(0),TAB,y(i)/tw,TAB,
   +z(i)/tw
      do 92 i=ii+1,kk-1,10
 92    write(io,123) r(i-1)/r(0),TAB,y(i)/tw,TAB,
   +z(i)/tw
      write(io,123) r(kk-1)/r(0),TAB,y(i)/tw,TAB,
   +  z(kk)/tw
      write(io,123) r(kk)/r(0),TAB,y(kk+1)/tw
   +  ,TAB,z(kk)/tw
192    format(5x,'RADIUS',7x,'TH.RADIAL PRESSURE(r)',5x,
```

```
   +  'TH.TANGENTIAL PRESSURE(r)')

      return
      end
**************************************************
c************************************************
c*************************** block data subprogram *********************
c
      block data
      implicit real*8 (a-h, o-z)
      common /mat3/p(2000),t(2000),dt(2000)
      data p/2000*0.0d0/, t/2000*0.0d0/
      end
```

VITA 2

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