

A PATTERN-BASED APPROACH  
TO GAIN SCHEDULING

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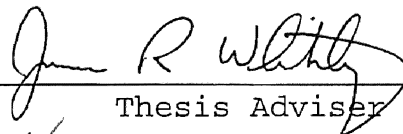
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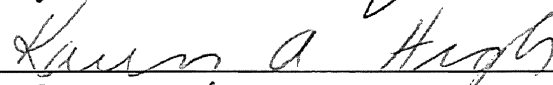
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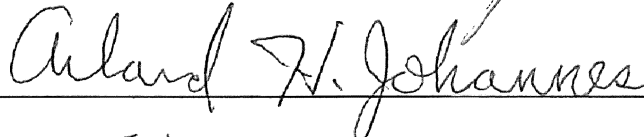
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## PREFACE

Manufacturing industries generate large amounts of process data which can yield beneficial insight on the process. This insight can include normal operating conditions, transient responses, and possible methods for optimization. Furthermore, the process data can provide insight on improving the control system for the process. This paper looks at extending a pattern recognition system into process control. This is accomplished by introducing gain scheduling to a pattern recognition neural network. Based on trained data, the neural network assigns new gains to process controllers to improve their control actions.

A simulation has been conducted using this system and has shown encouraging results for a nonlinear process that was difficult to control with fixed-gain single-input, single-output controllers. This study also looked at changing a variety of parameters that influence the pattern recognition program and the gain scheduling implementation. Pattern-based gain scheduling shows a great deal of promise in advancing pattern-recognition systems in to process control but also ease the use of gain scheduling in a plant.

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## GLOSSARY

ART2 neural network	A specific type of neural network which implements Adaptive Resonance Theory. This neural network was used to perform the clustering and pattern recognition associated with the reported work.
derated	The adjustment of calculated gains by a constant. This was applied to controller gains calculated with tuning relations in order to account for controller coupling.
detuned	The adjustment of a controller from optimum settings to less than optimum control in order to handle process nonlinearities.
gain	The ratio of output to input.
gain scheduling	An adaptive control technique that varies the gain of a controller based on a predetermined table of gains and state of the process being controlled.
IAE	Integral of the Absolute Error - performance measure in this study that calculates the total error between the setpoint value and the actual process value.
IAE_H	The performance measure for the height controller. This is the total error for the height controller during a simulation.
IAE_T	The performance measure for the temperature controller. This is the total error for the temperature controller during a simulation.

KC1	The gain associated with the height controller.
KC2	The gain associated with the temperature controller.
vigilance	The measure of the similarity between one process pattern to another pattern in the ART2 neural network.

## CHAPTER I

### INTRODUCTION

Petroleum refineries, chemical plants, power generating stations, and many other types of manufacturing facilities generate immense amounts of information in the form of sensor data. Competitive pressures dictate the need to find new ways to leverage this information to improve the monitoring, control and optimization of plant operations. To help meet this challenge, a novel approach to the application of gain scheduling is described. This approach uses process patterns to determine the operating state of the process. Based on the operating state of the process, the controller gains are adjusted in order to provide better control performance for nonlinear systems.

#### **The Feedback Control Problem**

Feedback control (Figure 1) is the most heavily utilized form of control in the process industries. This technique uses the error between the desired and actual value of a controlled variable to make adjustments to a manipulated variable. These adjustments drive the controlled variable back to the desired or 'set point' value. Ideally, all controlled variables (flow rates,

temperatures, levels, physical properties, etc.) never deviate from their set point values. Unfortunately, most processes are subject to disturbances which tend to push a controlled variable away from its desired value. The strength of the feedback control approach is the fact that it is insensitive to the source of these disturbances. The response of a feedback control system is driven completely by the difference between the desired and actual value of the controlled variable.

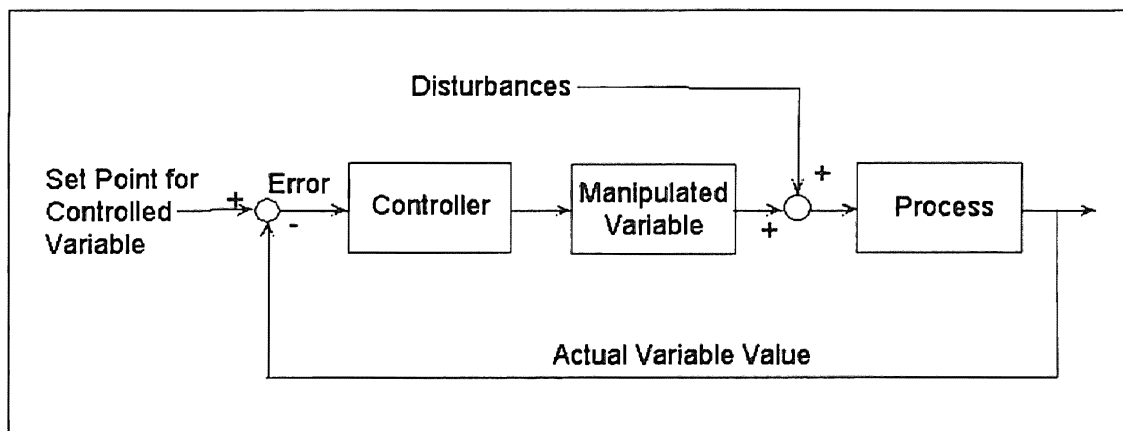


Figure 1: Traditional feedback control loop

The most common feedback controllers make adjustments using either PI (proportional-integral) or PID (proportional-integral-derivative) algorithms [Smith and Corripio, 1985]. These algorithms generate control signals



which are proportional to: 1) the currently measured error (P control), 2) the cumulative or integral error of the measured error (I control), and 3) the current rate of change or derivative of the error (D control, only applicable with PID controllers). A controller is equipped with adjustable weighting factors which determine the contribution of the P, I and D control actions to the final control signal.

The general equation for a PID controller is

$$m(t) = \bar{m} + K_c e(t) + \frac{K_c}{\tau_I} \int e(t) dt + K_c \tau_D \frac{de(t)}{dt} \quad (1)$$

where  $m(t)$  is the control signal to the manipulated variable,  $\bar{m}$  is the nominal or steady-state control signal to the manipulated variable,  $K_c$  is the controller gain, and  $e(t)$  is the error term which is the difference between the actual process value and the set point.  $\tau_I$  is the integral time constant for the controller and  $\tau_D$  is the derivative time constant.  $K_c$ ,  $\tau_I$  and  $\tau_D$  are the adjustable weighting factors mentioned previously.

The performance of a feedback controller is strongly dependent on the values of the controller parameters represented by the adjustable weighting factors. For processes and instruments which are linear and time-invariant, linear systems theory provides an elegant framework to determine suitable feedback control parameter

settings. Unfortunately, most processes exhibit nonlinear behavior and linear systems theory can be applied as an approximation at best. The key result is that in most cases of industrial importance there is not a single best value for a controller parameter. Rather, a range of values are possible which depend on actual operating conditions. Identification and use of optimal controller settings requires adaptive control capability under these conditions.

The goal of implementing a pattern-recognition system is to provide the feedback controller with an adaptive control system. This adaptive control system will use gain scheduling as a method of changing the controller gain,  $K_C$ , to provide enhanced control performance based on the current operating conditions of the process.

### **Thesis Outline**

The thesis is organized as follows. The following chapter will look at gain scheduling. It will contain a review of literature on gain scheduling, its theory, implementation, advantages, and disadvantages. Next, a pattern-recognition system using a neural network will be discussed. In addition, the proposed problem-solving approach is presented. Chapter 4 discusses the mixing tank used as a simulated process for this work. Chapter 5 discusses implementation of the pattern-based approach and development of the gain map for use with the simulation

system. Demonstration results are presented and discussed in Chapter 6. Chapter 7 presents final recommendations and discusses issues for future research.

## CHAPTER II

### BACKGROUND ON GAIN SCHEDULING

#### Introduction

Gain scheduling is an adaptive control method developed to overcome limitations of traditional, fixed-gain PID controllers. Gain scheduling works by changing  $K_C$  on the controller. The technique is typically implemented using gains listed in a predetermined look-up table. A new gain is chosen by a scheduling variable that identifies the current operating condition of the process. Finally, the controller uses this gain to provide better control for the current process conditions.

Gain scheduling works by assigning gains to the controller which take into account changes in the process. All processes are characterized by one or more process gains,  $K_p$ , which changes with the operating conditions. This change occurs due to nonlinearities in the process equations and interactions of process variables. The goal of gain scheduling is to maintain the ratio of the process gain and the controller gain to a constant as shown in Equation 2.

$$K_p(t)K_C(t) = \text{constant} \quad (2)$$

Problems with implementing gain scheduling originate in determining the proper method for scheduling the gains. Current applications (when they exist) use a process variable to monitor the state of the process, yet this method is not applicable for all situations. The approach discussed in this thesis looks at using process pattern sensor data to determine the current state of the system and then assigning  $K_c$  to the controllers.

#### Limitations of Fixed Gain Controllers

Gain scheduling overcomes the limitations of traditional controllers. A fixed-gain PID controller is best at handling linear, time-invariant systems. But, most processes are nonlinear which makes these processes more difficult to control. A feedback controller is implemented and tuned with the adjustable controller parameters for typical operating conditions. At these nominal conditions, the controller is tuned to handle disturbances and set point changes for the expected operating range of the controller.

Unfortunately, this tuning does not provide optimum control. The controller parameters are adjusted in order to give the controller flexibility to handle different operating conditions. Otherwise, the controller would have to be retuned at each operating condition. This would be costly and is impractical. Thus, the controller is designed to provide good control when the process is running near its ideal operating point. Yet, the controller can make

adjustments to the manipulated variable when the system is operating away from the normal operating point. The detuned controller keeps the process running correctly, but the changes may take longer to settle out than they would if the controller was tuned for that operating point.

### Motivation for Gain Scheduling

The motivation to include gain scheduling with a knowledge-based system is enhanced control of the process. The PID controllers found in most CPI plants are tuned loosely so that the controllers will handle a wide range of operating conditions. By tuning the controller loosely, the controller does not respond quickly enough in the one operating condition or it may respond too quickly in another situation.

Gain scheduling provides a quick and convenient method to introduce variable controller gains. Self-tuning controllers could be used instead, but gain scheduling has faster response times (Astrom, 1983). A self-tuning controller must wait for the disturbance to occur, evaluate the changes in the process characteristics, and implement the changes as necessary. Gain scheduling involves monitoring key scheduling variables. If changes occur to these variables, a new gain is implemented.

Another common industry practice is to set problem control loops in manual mode (Andreiev, 1977). Operators

sometimes find that their control over a problem control loop is more effective and reliable than a poorly setup controller. Yet, operating a loop in manual is costly. During upsets, an operator may need to pay more attention to the uncontrolled process loop. This makes him less responsive in emergency situations as he copes with a wide array of instruments and alarms. In addition, placing the control loop in manual forces the operator to supervise it more. As other process conditions change, he must adjust the controller until the desired output is reached. The loop under manual control makes the operator less effective and costs the operation money.

#### Incentives for Gain Scheduling

A number of economic factors drive the development of gain scheduling. Gain scheduling reduces costs in several areas. First, costs for raw materials are reduced. When a plant experiences a change in production, the plant enters a transition period. During this time, new temperatures are set for reactors, distillation columns, etc. A worse case situation is start-up. During this transition time, the plant usually experiences severe swings in process parameters. It takes a long time to get the plant stabilized in some cases. With gain scheduling, a batch reactor system can have a sequence of controller settings

available for each operating step in production or a sequence of gains necessary to make a product.

An example of gain scheduling applied to a polymerization unit by Standard Oil Co. (Indiana) (Whatley and Pott, 1984) yielded significant improvements. The unit was described as uncontrollable, but gain scheduling made the system controllable and reduced temperature variations that had lowered the quality of product. Less raw material was wasted as off-spec product because of the improved control associated with gain scheduling.

Another incentive to use gain scheduling is reduced settling time and improved response. The scheduled gains provide controller settings that drive the process to set point in less time than a fixed-gain controller. This point is especially true if the fixed-gain setting provides sluggish control at the current operating conditions. Again, gain scheduling provides benefits because it changes the gain to fit a variety of operating conditions.

### **Early Work on Adaptive Control**

Early work on adaptive control systems began in the early 1950s by building on control theory of the 1940s. In the 1940s, the Nyquist, Bode, and Evan plots had wide use in the design of linear control systems. But, many systems were non-linear and lacked a truly robust method of control (Mishkin and Braun, 1961). The primary work at this time



was on developing an auto pilot system for high performance aircraft and rockets. The studies found that constant-gain feedback control systems would not work for all operating conditions (Astrom, 1983). In fact, controller constants would only work for one condition, but as aircraft speed and dynamic pressure changed, the controller would not adequately handle flight control.

Since the 1950s, better control theory resulted that benefited adaptive control. Other improvements included developments in system identification and parameter estimation. Seborg (Seborg et. al., 1986) defined two categories of adaptive control problems. The first category of problems involves those where the process changes cannot be directly measured or anticipated. Most adaptive control literature focuses on this area. The second category consists of control problems where process changes can be anticipated or inferred from process measurements. For this type of problem, if the process is well understood, controller settings can be changed in a predetermined manner as the process changes.

From the improvements in process identification and control design, one of the control strategies to emerge was gain scheduling.

## Gain Scheduling

Astrom and Wittenmark (1989) define gain scheduling as a nonlinear feedback using a linear controller whose parameters are changed as a function of operating conditions in a preprogrammed way. The basic approach is shown in Figure 2. Gain scheduling works by looking at process variables which correlate to the process dynamics and adjusts the controller parameters based on a table of controller gains appropriate to the operating conditions.

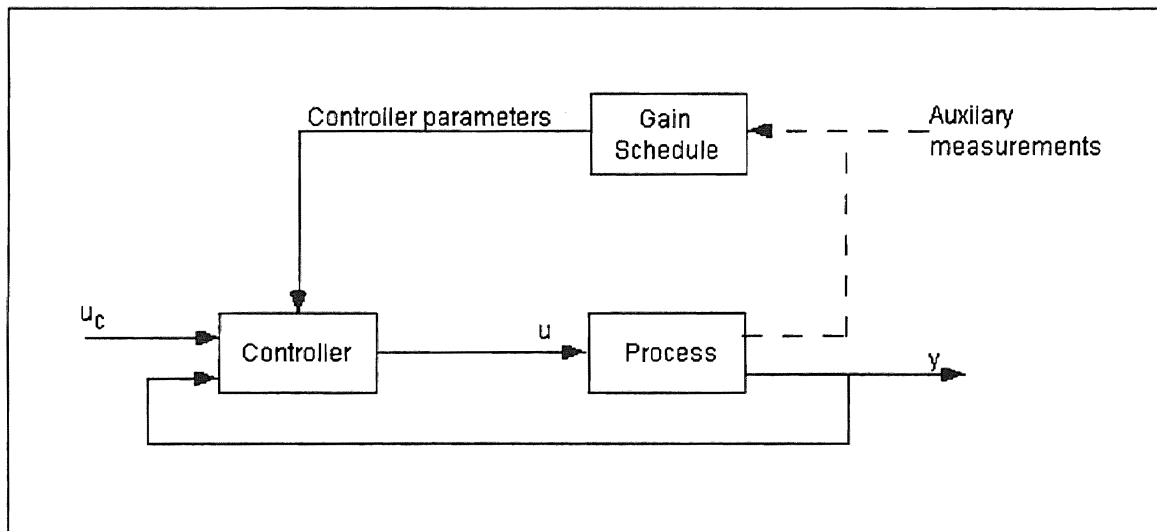


Figure 2: Gain scheduling block diagram (Astrom, 1983)

The gain scheduling design operates by linearizing the design equations for a process at several operating points.

By linearizing the process at a number of operating conditions, feedback control settings can be calculated for a variety of operating conditions. These settings provide better control than possible with a fixed-gain controller.

#### Gain Scheduling Control Benefits

Astrom (1987) lists several advantages to gain scheduling. First, the controller parameters can be adjusted quickly if the operating conditions change (Rugh, 1991; Astrom, 1987). If one of the scheduling variables warrants changing the gain on the controller, the controller is adjusted. This adjustment maintains the proper tuning on the controller for the process conditions. The main limitation to changing the gain depends on the response time of the scheduling variables. If they do not respond quickly to the process, the gain scheduling system will not work properly. Another advantage of the gain scheduling cited by Astrom is that gain scheduling reduces process variations.

Rugh (1991) raises a number of advantages for gain scheduling. First, gain scheduling allows linear design methods to be applied to a nonlinear system at each linearized operating point. In addition, linear control methods are available to design a control system using multi-variable nonlinear equations. Using these advanced design concepts, a robust design for linear systems can be

applied to counter any uncertainties in the plant parameters.

In developing an analytical framework, gain scheduling creates a nonlinear closed-loop system. The scheduling variables cause this. Rugh's objective is to develop an approach in order to use modern nonlinear control theory for measuring performance and stability. The end benefit is to better understand gain scheduling and study ways to alleviate problems with gain scheduling. Rugh develops a number of equations which are applicable for an idealized gain-scheduled controller with state feedback (Rugh, 1991).

#### Gain Scheduling Implementation Problems

Scheduling variable selection represents one of the key problems in developing a suitable gain schedule. What variables are to be used and monitored remain a problem in implementing gain scheduling in the control industry. Normally, scheduling variables are chosen based on the physics of the system. One rule of thumb for selecting scheduling variables has been to use a slow variable as the scheduler (Rugh, 1991; Astrom, 1983; Shamma and Athans, 1991). Scheduling on slow variables adds to the stability of the system as the gain for the controller will not constantly change with fluctuations in the process.

One key variable in a system is production rate and this variable is often incorporated as a scheduling variable

(Astrom, 1983). The flow rate of a process stream strongly influences deadtime, time constants for the controller, and other system responses. These responses are known to be inversely proportional to flow rate. Thus choosing production rate as a scheduling variable does have its merits.

Further limitations cited by Seborg (Seborg *et. al.*, 1986) include the difficulty of relating process changes to variables measured from the plant. If the process contains a long delay time, gain scheduling may be worse than conventional PID control unless some type of delay compensation is employed.

Another serious problem implementing gain scheduling is the selection of a scheduling procedure. The scheduling procedure defines how the gain is adapted based on changes in the scheduling variables. Rugh notes that this issue is rarely addressed in literature. Rugh's analysis of the current control practice is that gain scheduling is an art using simple curve-fitting approaches. In addition, the increased interest in using multi-variable, robust, linear designs has made control laws more complex. Rugh forecasts that scheduling will become more difficult to implement using current standards.

Another drawback listed by Astrom (1983) to gain scheduling is the lack of open-loop compensation. There is no way to correct for a bad schedule with feedback control. Gain scheduling thus can be viewed as a feedback control

system where feedback controls are adjusted by feed forward compensation.

Designing a system implementing gain scheduling requires a great deal of time because controller parameters must be established for a number of operating points. This requires extensive simulation to make sure the correct schedule is created. Unfortunately, simulating the process is not always a viable solution. Finally, gain scheduling is local in nature. The overall performance must be determined by rigorous simulation. By using linear design methods, gain scheduling requires this additional simulation. As more complex gain scheduling designs are implemented, large simulation burdens are expected (Rugh, 1991).

Shamma and Athans (1990; 1991; 1992) present several papers on gain scheduling. Gain scheduling has been found to work in a variety of applications but it lacks a sound theoretical analysis. Without a theoretical basis, no guarantees can be made on the stability of the system on a global (plant-wide) basis. Shamma and Athans clarify this problem by stating that even though the local point designs may have excellent feedback properties, the global gain scheduled design need not have any of these properties, even nominal stability. One cannot assess the *a priori* stability, robustness and performance properties of gain scheduled designs. These properties are analyzed by computer simulation.

## Implementation of Gain Scheduling

Implementation of gain scheduling remains an art at this time. The issue is rarely addressed in literature (Rugh, 1991). The first step in implementing gain scheduling is finding the process variables that reflect the current operating condition of the process. Instead of one scheduling variable, several process variables may be selected as key parameters in the operation of a process. The problem exists in determining the amount of interaction for each variable. A scheduling procedure must be developed from these interactions such as a curve-fitting technique. Further complications arise when complex, multi-variable designs are investigated.

From literature, an important step in setting up a gain scheduling system involves lengthy and costly analysis of the process in question. This problem usually centers on finding the scheduling variables and then determining the gains. A simpler method for scheduling exists if process patterns are used to determine the state of the operation. With this method, the proceeding step should require less time.

With a plant model, a designer selects a set of process conditions which represent the range of plant dynamics. The number of gain settings depends on the dynamic range of the controlled variable and the effect of the manipulated variable on the process (Cardello and San, 1988). From

these, he designs a linear compensator for each process condition (Shamma and Athans, 1991). Gains are then calculated using any of a number of available design techniques. The gains are calculated such that for all frozen values of the parameters give the closed loop system the desirable feedback properties. Since having parameters for each operating point is impossible, the gains are interpolated between operating points. The plant operating parameters are then placed in the gain scheduling table.

Once the gain scheduling table is computed, the stability and performance of the schedule table are evaluated by simulation. This requires extensive computer simulation time. During the simulations, particular emphasis is put on the transition from one operating regime to another to insure the process remains stable. If performance is not satisfactory, more operating conditions are added to the schedule.

Once the gain table is setup and tested, it can be used in actual operation. Gains are selected based on the scheduling variable and the controller is modified.

### **Gain Scheduling Theory**

Work on investigating stability, performance, and system design for gain scheduling systems has lagged behind application. Little work has focused on the theory behind gain scheduling (Shamma and Athens, 1992; Rugh, 1991).



### Scheduling on a Slow Variable

Shamma and Athens (1991) look at one class of gain scheduled control systems called linear parameter-varying plants. This class of plants is important as nonlinear plants can be approximated as a linear parameter-varying plant. The varying parameter in this case is the scheduling variable. Shamma considers a plant having the state-space form of

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}(\theta(t))\mathbf{x}(t) + \mathbf{B}(\theta(t))\mathbf{u}(t), \\ \mathbf{y}(t) &= \mathbf{C}(\theta(t))\mathbf{x}(t). \end{aligned} \tag{3}$$

where  $\mathbf{x}(t)$  is the  $n$ -dimensional state variable,  $\theta(t)$  is a vector of external time-varying parameters such as controller settings (i.e.  $K_C$ ),  $\mathbf{u}(t)$  is the plant input vector, and  $\mathbf{y}(t)$  is the plant output vector.  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{C}$  are process constants. These equations represent a nonlinear plant whose dynamics depend on a vector of time-varying exogenous parameters  $\theta$  which belong to the set  $\theta(t) \in \Theta$ . A set of parameter values  $\{\theta_i\}$ , which represent a range of plant dynamics, are chosen. The control system designer develops a linear, time-invariant controller for each of these points. Between the operating points, the controller gains are interpolated such that the closed loop system has desirable properties such as nominal stability, robustness to unmodeled dynamics and robust performance.

Once this design has been done, the designer has a feedback system which has desirable stability and performance properties at each operating point. But, since these parameters are time-varying, these properties are not guaranteed. Shamma expresses the plant in terms of robust control theory to calculate robust stability and performance requirements. The transition of Figure 3 to Figure 4 shows this transformation.  $\mathbf{H}(\theta)$  represents a finite-dimensional parameter-varying linear system with the following state-space form:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}(\theta(t))\mathbf{x}(t) + \mathbf{B}(\theta(t))\mathbf{e}(t), \\ \mathbf{y}(t) &= \mathbf{C}(\theta(t))\mathbf{x}(t). \end{aligned} \quad (4)$$

where  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{C}$ ,  $\theta(t)$ ,  $\mathbf{x}(t)$  and  $\mathbf{y}(t)$  are defined for Equation 3.  $\mathbf{e}(t)$  is the difference vector of plant inputs to set points. The  $\Delta$  represents a block diagonal stable linear system which depends on uncertainties only. The input/output relationship  $\Delta$  is given by

$$\mathbf{y}'(t) = \int_0^t \Delta(t - \tau)\mathbf{y}(\tau)d\tau. \quad (5)$$

The feedback equations become

$$\dot{\mathbf{x}}(t) = \mathbf{A}(\theta(t))\mathbf{x}(t) + \int_0^t \mathbf{B}(\theta(t))\Delta(t - \tau)\mathbf{C}(\theta(\tau))\mathbf{x}(\tau)d\tau. \quad (6)$$

where  $A$ ,  $B$  and  $C$  have been appropriately redefined. This equation represents a type of linear Volterra integrodifferential equation (VIDE).

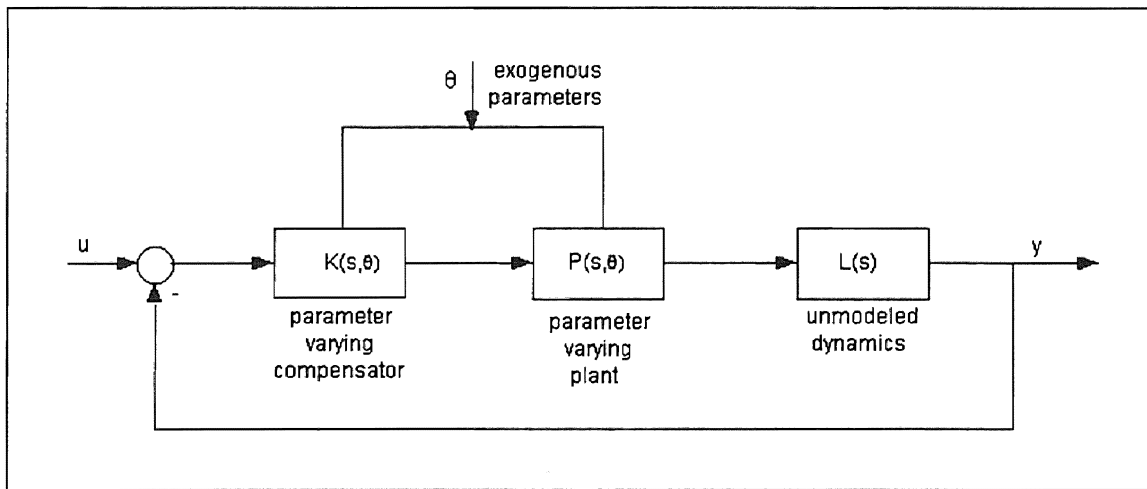


Figure 3: A linear plant scheduling on exogenous parameters (Shamma and Athens, 1991)

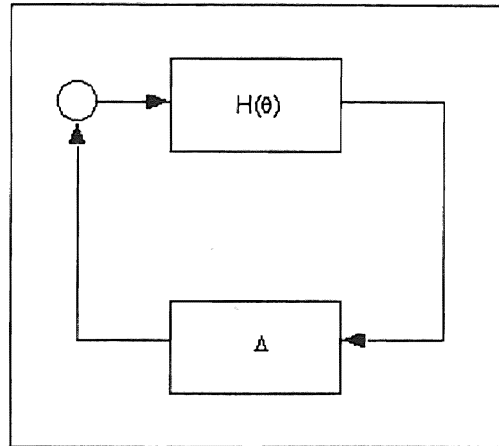


Figure 4: General block diagram for robustness/performance analysis

Shamma proves in his paper that a time-varying linear plant transformed into a linear VIDE is exponentially stable for sufficiently slow time-variations. Thus, robust stability and robust performance are maintained provided that parameter variations are sufficiently slow. This proof quantitatively defines the rule of thumb to 'schedule on a slow variable'.

### Scheduling Variables

Shamma and Athens (1990) also investigated gain scheduling using two types of scheduling variables. They look at scheduling based on a reference trajectory and scheduling based on plant output. For both methods, Shamma and Athens provide conditions that guarantee that the

overall gain scheduled system will retain the feedback properties of the local linearized points.

Scheduling Based on Reference Trajectory Figure 5 shows the block diagram for scheduling on a reference trajectory. The target trajectory  $\mathbf{r}^*$  is generated by passing a reference signal (set point)  $\mathbf{u}^*$  through a plant model denoted by  $\mathbf{P}_m$ . The control input  $\mathbf{u}$  to the actual plant  $\mathbf{P}$  consists of the reference control  $\mathbf{u}^*$  and a small perturbation control  $\delta\mathbf{u}$  calculated in controller  $\mathbf{K}$ . This represents a perfect case where no modeling errors have occurred. Shamma develops additional block diagrams for cases where unmodeled dynamics are included (Shamma and Athans, 1989).

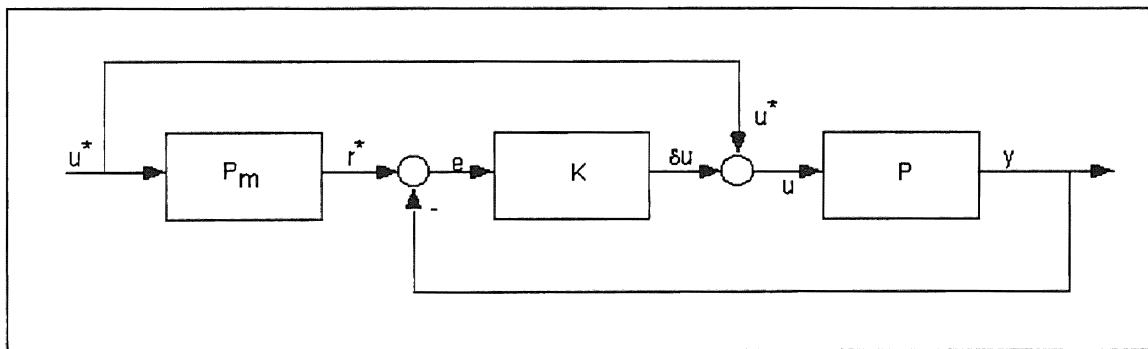


Figure 5: Scheduling on a prescribed reference trajectory (Shamma and Athans, 1990)

Shamma and Athans developed conditions that guarantee the robust stability and robust performance of a global gain scheduled design. The article proves that Figure 5 and derivations containing unmodeled dynamics have guaranteed stability if the reference trajectory changes are slow. This limitation occurs as the gain scheduled design is based on linear time-invariant approximations of the plant. The system is actually nonlinear, so internal stability is local. As nonlinearities approach zero, internal stability approaches global stability. Another restriction on feedback system lies with  $\mathbf{u}^*$  and  $\mathbf{r}^*$ . Reference trajectories cannot excite unmodeled dynamics. If  $\mathbf{u}^*$  contains significant frequencies that disturbs these unmodeled dynamics, stability cannot be guaranteed.

Scheduling on the Plant Output Shamma and Athans (1989) consider a plant model given by

$$\frac{d}{dt} \begin{pmatrix} \mathbf{y} \\ \mathbf{z} \end{pmatrix} (t) = \mathbf{f}(\mathbf{y}(t), \mathbf{z}(t)) + \mathbf{B}\mathbf{u}(t), \quad (7)$$

$$\mathbf{y}(t) \in \mathcal{R}^m, \mathbf{z}(t) \in \mathcal{R}^{n-m}, \mathbf{u}(t) \in \mathcal{R}^m$$

where the plant output is  $\mathbf{y}$ ,  $\mathbf{z}$  is a vector of external parameters, and  $\mathbf{u}$  is the plant input. Shamma makes two assumptions on developing a gain scheduling system based on plant output. First,  $\mathbf{f}: \mathcal{R}^m \times \mathcal{R}^{n-m} \rightarrow \mathcal{R}^m$  is at least twice continuously differentiable over all of  $\mathcal{R}^m \times \mathcal{R}^{n-m}$  and

satisfies  $f(0,0) = 0$ . The second assumption states that unique continuously differentiable functions  $u_{eq}$  and  $z_{eq}$  exist such that

$$\begin{aligned} u_{eq} &: \mathcal{R}^m \times \mathcal{R}^m \\ z_{eq} &: \mathcal{R}^m \times \mathcal{R}^{n-m} \\ \text{and} & \\ 0 &= f(y, z_{eq}(y)) + Bu_{eq}(y). \end{aligned} \tag{8}$$

This assumption states that a family of equilibrium conditions exists based on output  $y$ . Gain scheduling sees these equilibrium conditions as possible operating points.

The next step in the design of a gain scheduling system using plant output is linearizing the plant about a possible operating point  $y_0$  using Equation 9.

$$\begin{aligned} \frac{d}{dt} \begin{pmatrix} y - y_0 \\ z - z_{eq}(y_0) \end{pmatrix} &= Df(y_0, z_{eq}(y_0)) \begin{pmatrix} y - y_0 \\ z - z_{eq}(y_0) \end{pmatrix} \\ &+ B(u - u_{eq}(y_0)) \end{aligned} \tag{9}$$

where  $D$  is the Dini derivative of  $f$ . At each operating point, the designer finds a controller that is based on a local linear time-invariant approximation of Equation 9. This results in a family of linear time-invariant compensators  $(A_k(y_0), B_k(y_0), C_k(y_0))$  parameterized by condition  $y_0$ . This family is used in the control system shown in Figure 6. This set of gain scheduled designs has,

for each of the linearized operating conditions, the desired stability, robustness and performance properties of feedback control. Yet, the actual system has a time-varying scheduling variable evolving under nonlinear dynamics.

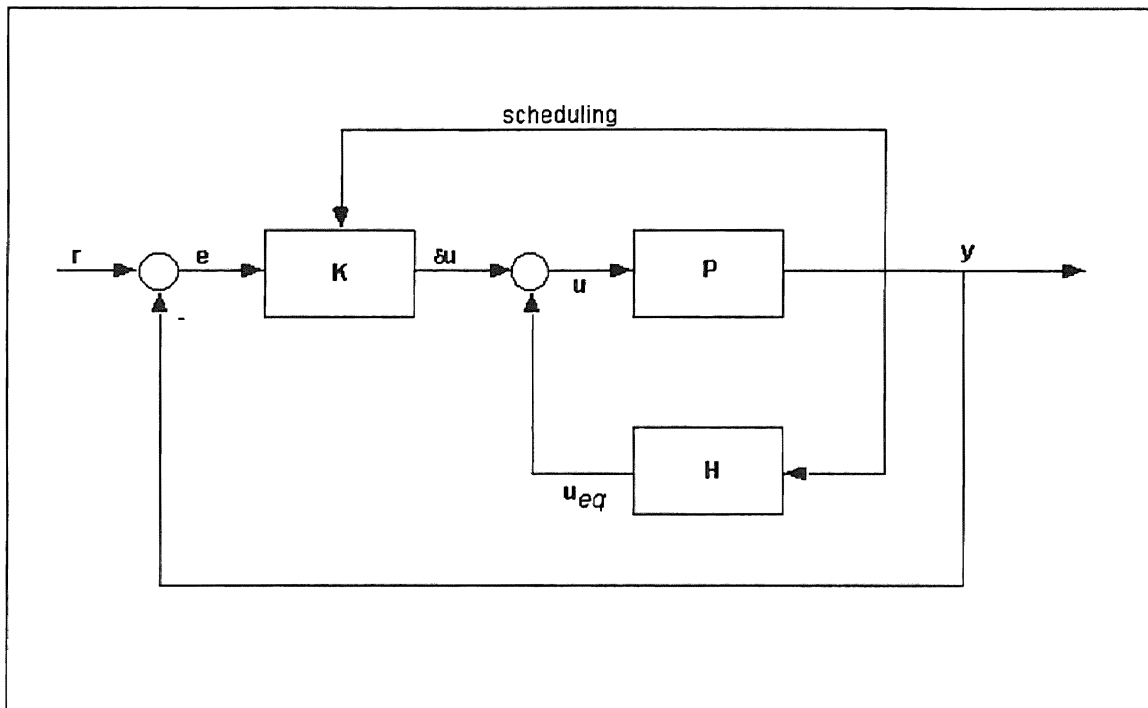


Figure 6: Scheduling on the plant output

Shamma continues his development using Figure 6 and including unmodeled uncertainties in the proofs. Again, slow variations in the scheduling variable are required. The scheduling variable should also capture the plant nonlinearities. Furthermore, the degree of exponential



stability must be large enough to overcome nonlinear function perturbations.

Fast Scheduling Variables Shamma and Athans (1992) reformulated their approach on gain scheduling using linear parameter-varying equations. The aim of their work was to provide guarantees of stability and performance in light of rapidly changing operating conditions. The first change looks at the method of which operating point gains are scheduled. These points ought to be chosen which explicitly address the possibility of rapid variations in the process. This maintains closed-loop stability and the fixed operating point properties remain. Their paper provides an example of this.

It is important to include the possibility of fast parameter variations in the design process. Otherwise, the guaranteed properties of the overall gain scheduled design cannot be established. Shamma emphasizes that theory for linear parameter-varying systems needs to be developed. Included in this development would be modification of robust control design methodologies as well.

The work of Shamma and Athans has paved the way for developing the theoretical basis for gain scheduling. In addition, it is opening up new areas of research and providing tools to improve gain scheduling design.

### Extended Linearization

Rugh (1991) noted that there is a close relation between gain scheduling design and the extended-linearization approach for nonlinear control design. The extended-linearization approach may be viewed as gain scheduling on the basis of state, input, or output variables in a closed-loop system. Rugh also mentions that state variables may be used as substitute gain scheduling variables as they vary more slowly than external parameters.

Rugh summarizes a few key points in making gain scheduling designs based on theory. First, the use of integral-error feedback in the linear control law designs at the scheduling points provide useful performance properties for gain scheduling. Also, problems exist with choosing gains for the feedback loop. A gain-scheduled system can be driven by the time-derivative of the scheduling variable and complicates the decision to choose gains for stability and for rejecting a disturbance in order to preserve performance under scheduling-variable variation.

Finally, Rugh (1991) states that it is not clear from a theoretical viewpoint whether interpolating a control law from individual linear control laws at isolated operating conditions always is superior to interpolating the plant data and computing the corresponding, continuously-parameterized control law as in idealized gain scheduling. This statement has important implications in the gain

scheduled pattern-recognition approach. It is important in the determination of gains for scheduling and application of interpolation techniques to schedule the gains.

### **Applications of Gain Scheduling**

#### Applications

A number of applications of gain scheduling have been made. Early development of gain scheduling included applications in high performance aircraft. In particular, gain scheduling was used as an auto pilot (Seborg et. al., 1986). It was found that monitoring the Mach number and dynamic pressure allowed a suitable schedule to be developed (Astrom, 1987). Gain scheduling has become the predominate method to handle parameter variations in flight control systems. It is used extensively in the design of auto pilot systems for high performance aircraft (Stein, 1980).

This "table look-up" method, where controller settings are stored for a variety of operating conditions, became known as gain scheduling and involves maintaining a constant product between the process gain and the controller gain. For example, for a stable system, the product of the process gain ( $K_p$ ), the controller gain ( $K_c$ ) and other gains in the control loop should be equal to or less than 1.0. If it is greater than 1.0, the system is unstable as oscillations will increase in amplitude instead of damping out (Seborg et. al., 1986).

Gain scheduling was initially limited to the aircraft industry as analog techniques required expensive function generators and multipliers. With the arrival of computer-controlled systems, gain scheduling has become easier to implement (Astrom and Wittenmark, 1989).

At Standard Oil Co. (Indiana), a polymerization unit was considered uncontrollable using classical PID control schemes. Whatley and Pott (1984) describe differences between the traditional and more modern control systems. First, the 'modern' control systems can manipulate multiple inputs versus just one. In addition, new control systems are able to do table look-up. The controller now has the ability to change its strategy as a function of time or as conditions of the process change.

The problem with the polymerization plant centered on narrowing temperature variations in a polymer manufacturing process. The narrower the temperature range, the higher the quality of product produced. The process is also characterized by an ability to run out of control easily with plant upsets.

A detailed analysis of the process was performed. This study included looking at the control valve characteristics, the performance of the heat exchanger system and process variable interaction. A suitable control system was designed for the plant. Part of the new system included a gain-scheduling system with set maximum and minimum gains. The scheduling system changed the gains based upon the

positive temperature difference between inlet and circulating oils which are used to maintain temperature.

As a result of the new control system, significant improvements were obtained. One reactor had improper temperature control. The previous control system allowed this reactor to have a  $\pm 20^{\circ}\text{F}$  temperature range, but the new control system brought the temperature variation to within  $\pm \frac{1}{2}^{\circ}\text{F}$ . During situations that could have lead to a runaway reaction, the new control system handled the upset and the plant did not produce any off-spec product (Whatley and Pott, 1984).

Another application of gain scheduling involves a ship auto pilot (Kallstrom et. al., 1979). The main goal of this system was to reduce drag of the ship which reduces operating costs. Factors influencing the research was to find an auto pilot that could adjust its parameters due to environmental changes. These changes include wind, water currents, and ship movements such as sway and yaw. A stable auto pilot could be developed using high gains but the drawbacks to such a system included excessive rudder movement and increased drag. Although the heart of the Kallstrom auto pilot involves Astrom and Wittenmark's self-tuning regulator, gain scheduling is used in the system.

In particular, the speed of the ship was scheduled based on the rotation rate of the propellers. The gain scheduling affected the results of a Kalman filter. This filter analyzed ship movement. Kallstrom used velocity

scheduling to improve performance of the auto pilot. The parameters of the Kalman filter, the self-tuner and the turning regulator were changed as a function of ship speed. The self-tuner could account for speed changes, but the gain scheduling based on velocity yielded quicker responses than possible with an adapted system. The resulting system was tested on 3 tankers of various sizes and reduced drag in all cases (Kallstrom et. al., 1979).

Another application for gain scheduling has been found for pH control (Astrom, 1987). Astrom (1987) points out that meshing adaptive control systems with gain scheduling has benefits. The primary benefit is that the adaptive control system can be used to create a gain scheduling chart. By storing parameters in a chart, the entire operating range of a process can be stored and used for smooth performance.

Cardello and San (1988) have looked at gain scheduling for a batch bioreactor. They found gain scheduling to be the simplest form of adaptive control to implement. They used oxygen uptake rate (OUR) as the scheduling variable. A look-up table is used to select the gain depending on the OUR measurement. In a comparison of a fixed-gain PID controller and a feedforward-feedback controller, the integral of squared error (ISE) was 20% lower for the gain-scheduled controller than the best feedback controller. Cardello and San found that gain scheduling was an effective method for controlling dissolved oxygen levels in a batch

fermentor. These conclusions are important as the fermentor had large variations in the process load and gain scheduling provided a method for compensating for process dynamics.

March-Leuba *et. al.* (1992) have developed a gain scheduling controller that uses fuzzy-logic to provide adaptive control on a PI controller. The controller controls the fluid level inside U-tube steam generators. The fuzzy-logic circuit analyzes and decides based on the disturbance what action is appropriate for the controller to initiate. The gain-scheduling aspect of the system changes the controller gain based on the temperature of the feed water. The gain is adjusted in a linear function. The addition of this fuzzy/gain-scheduling system to a PI controller leads to smoother and stabler performance (March-Leuba *et. al.*, 1992).

### Industrial Controllers

One application of the gain scheduling has been made in a controller. The Taylor Microscan 1300 controller makes up for the problems of dealing with a nonlinear process (Andreiev, 1977). The need for a controller to have adjustable gains was originated by W. I. Caldwell of Taylor. He noticed that if the controller gain could change, the performance of the system improved markedly. The controller has the ability to change control gain if the process moves outside of a predetermined operating range. The user sets

upper and lower bounds where gain scheduling takes place. Scheduling is based on the percent range of the controller and gains are calculated based on a linear percentage of the base gain (Andreiev, 1977).

### Summary

Gain scheduling has benefits we want to exploit. First, the system provides a controller with the ability to respond quicker in new operating conditions. Gain scheduling needs an enhanced method for scheduling, though. The use of a pattern recognition approach allows gains to be tailored for specific operating conditions. In addition, selecting scheduling variables should become easier as more than one process variable can be incorporated into process patterns.

The possibilities are endless with the emergence of new computer controlled systems. With the ability to store gains, implement them when needed, and even calculate the necessary gains, the future looks bright. The problem remains, how does the computer know when the gain needs to be changed?



## CHAPTER III

### ART2 NEURAL NETWORK

#### Pattern-Based Gain Scheduling Approach

We propose to replace the traditional gain table with a more robust, pattern-based gain map in the manner of Figure 7. As described below, this provides the capability to characterize the process more accurately and significantly improves gain scheduling during periods of transient operation. This work moves the pattern-recognition approach to an application for process control.

Gain tables are typically generated using a single process variable to characterize the process. The pattern recognition approach offers the advantage of matching the gain for a controller to the process conditions. In most industrial operations, several process variables interact and affect the process. By allowing more than one of these variables to determine the gain, gains may be tailored for particular operating conditions. Furthermore, since table entries are generated under steady-state conditions, only a single value of the scheduling variable is used to define each operating point.

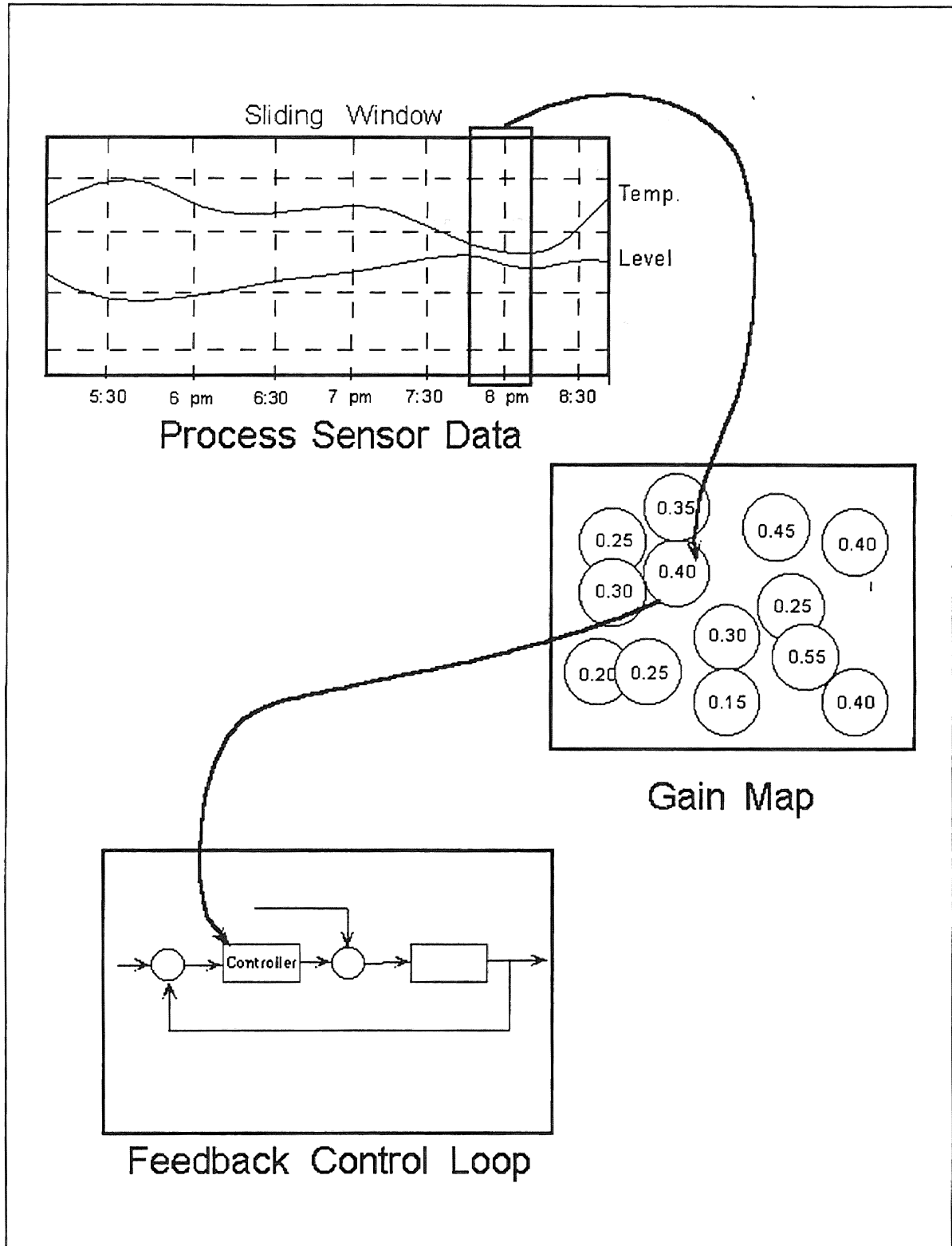


Figure 7: Use of a pattern-based gain map to perform gain scheduling

We argue that the traditional approach to gain scheduling is inappropriate for the process industries due to the large time constants and time delays which are typically encountered. Accurate characterization of a process under these conditions requires consideration of more than one process variable over some finite period of time. We propose to substitute multi-sensor patterns for the traditional single value scheduling variable. We are essentially arguing that more information must be used; consideration of a single point value is inadequate, especially under transient conditions when control is most critical.

The implementation of pattern recognition is straightforward (Figure 8). A neural network learns process patterns which are placed in clusters by the neural network. These clusters represent steady-state operating conditions and have a hyperspherical shape. With each cluster, a controller gain or set of gains for multiple controllers is assigned. This gain is designed to provide enhanced control at the operating point.

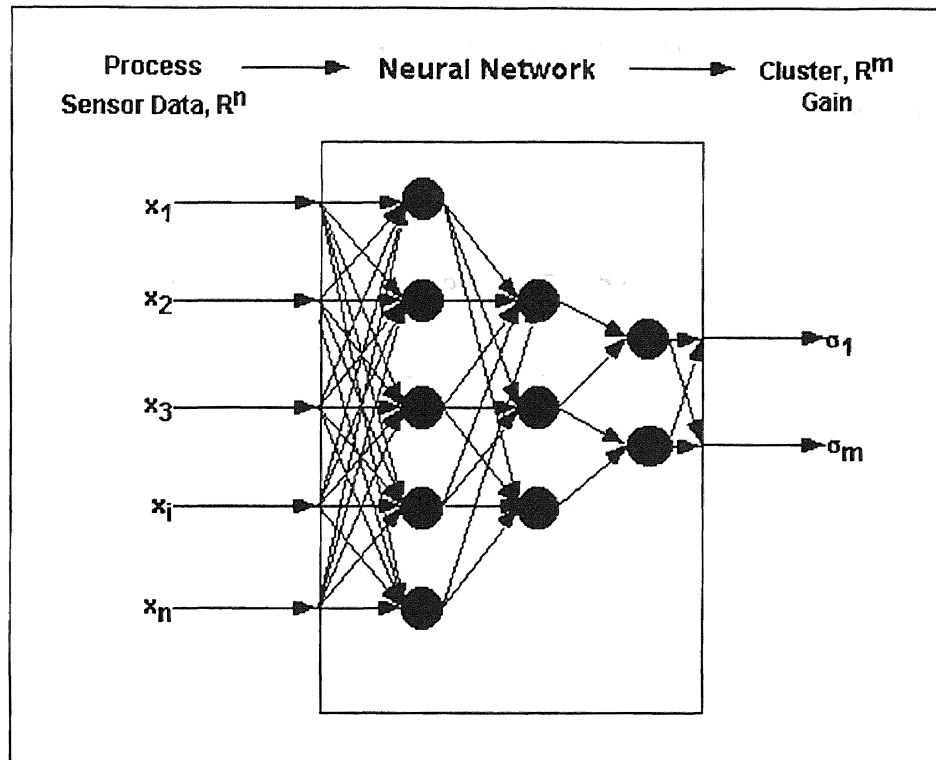


Figure 8: Pattern-recognition gain scheduling approach

The cluster concept has several benefits. It simplifies the problem of interpolation during periods of near steady-state behavior. This is a major practical benefit since many continuous processes operate in such a manner much of the time. More importantly, the use of clusters delays gain changes and gives the control system a chance to respond when the process moves away from steady-state. The size of the clusters determines how far the process must move before a gain change is implemented.

We do not propose to fill the entire map space with clusters but to interpolate when the process operates

between clusters. Key implementation issues which were investigated include cluster density, cluster size, and interpolation rules.

### Implementation Using ART2 Neural Network

The gain map in our implementation is constructed using a technique developed previously to interpret sensor patterns [Whiteley and Davis, 1993a; Whiteley et. al., 1993b]. This technique employs a modified version of the ART2 neural network [Carpenter and Grossberg, 1987]. The Adaptive Resonance Theory 2 (ART2) network is an autonomous learning model based on Grossberg's adaptive resonance theory [1976a; 1976b].

For our problem, the desirable attributes of the ART2 network are the integrated feature extraction/clustering capabilities. The unique combination offered by ART2 provides powerful potential to leverage alternative types of pattern representations.

The patterns used as input to the ART2 network correspond to windows of sensor data as illustrated in Figure 7. The length of the window, data sampling frequency, and number of process variables jointly determine the dimension of the input pattern vectors and the gain map representation space.

In operation, the gain map is used as follows. A sliding window is used to continuously extract the most

recent pattern of operation from the process (Figure 7). This pattern is input to the ART2 network. If the pattern falls within the cluster of one of the gain scheduling prototypes, the corresponding value of the controller gain is used. If the pattern lies outside any clusters, some form of interpolation is applied.

The strength of the technique is the ability to handle transient conditions. The integrated feature extraction capability of the ART2 network is an essential element to providing the desired performance.

#### **ART2 Network for Gain Mapping**

Adaptive Resonance Theory (ART) was developed by Grossberg (Grossberg, 1976a; 1976b) as an autonomous learning model. ART and specifically ART2 was chosen as it addresses the 'stability-plasticity' trade-off. ART has the ability to remain 'plastic' by acquiring new knowledge and 'stable' as ART retains previous knowledge it has learned. ART can determine when new knowledge needs to be learned and still retain previous knowledge whether it occurred the day before or several weeks ago. The clustering ability of ART2 is more complex than others, but it's ability to handle the 'stability-plasticity' trade-off offsets this.

The object of the ART2 network is to 'self-organize pattern recognition codes in response to arbitrary sequences of input patterns'. Figure 9 shows the basic architecture

of the neural network. The ART2 network was originally designed to be an autonomous learning system. With modifications though, it can serve as a pattern recognition system.

The learning operation of the ART2 network begins with the presentation of the input pattern to the network. After processing by the network, the input pattern is compared with each of the existing prototypes in the top layer. The 'winner' in the top layer is the prototype most similar to the input. If the similarity between the 'winner' and the most similar prototype exceeds the vigilance parameter  $\rho$  then the input pattern lies inside the prototype. The network performs learning in order to modify the prototype to be slightly more similar to the input pattern. If the pattern lies outside the cluster, a new prototype is created.

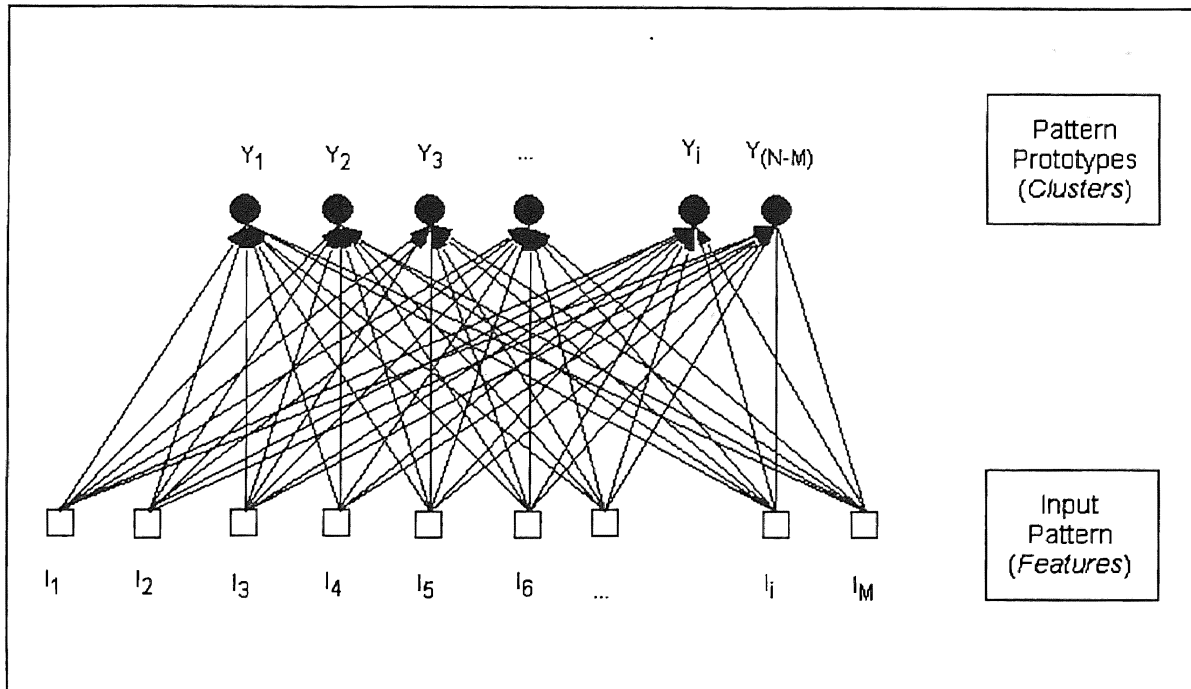


Figure 9: ART2 architecture with two-fully connected layers

Similarity in an ART2 network is based on the pattern direction in the representation space. The clusters associated with prototype can be viewed as hypercones originating from the origin of  $\mathbf{R}^m$ . The ART2 network measures the similarity between a normalized input pattern  $\mathbf{U}_p$  and a cluster prototype  $\mathbf{Z}$  as the  $L^2$  norm of the input pattern vector  $\mathbf{R}$  using Equation 10.

$$R_i = \frac{U_{pi} + cR}{\|U_p\| + \|cP\|} \quad (10)$$

The relationship between  $\|\mathbf{R}\|$  and the angle between  $\mathbf{U}_p$  and  $\mathbf{Z}$  is highly nonlinear and is important to understanding how



the ART2 vigilance parameter affects cluster size and the scale of a gain map. The vigilance parameter  $\rho$  represents the clustering criterion used by ART2. It also provides an indirect measure of the cluster size. The higher the vigilance parameter, the smaller the angle is between the cluster prototype and the input pattern.

### **Summary**

The application of pattern-based recognition builds upon the foundation of Whiteley and Davis's prior work. The move from a 'do/don't know' situation to application as a process controls method is the first step for this approach. Instead of deciding whether a prototype is normal or abnormal, each cluster has a numerical value associated with it. The value is the gain for the controller in the process. We are not limited to just the gains; new integral time constants and derivative time constants can be added as well. The ART2 network modified for gain scheduling presents an advantageous first step for process control applications and ease the implementation of gain scheduling. The following chapter will look at the development of a simulation system to test the pattern recognition system.

## CHAPTER IV

### MIXING TANK SIMULATION

#### Mixing Tank

In order to evaluate the purposed gain scheduling approach, an experimental testbed was necessary. The system chosen is the classical control problem of a mixing tank with three feeds and one exit stream. Figure 10 is a schematic of the mixing tank. The three feeds consist of a hot water stream, a cold water stream, and a disturbance stream used for load testing. The hot stream enters with mass flow rate  $m_h$  (kg/min) and temperature  $T_h$  ( $^{\circ}\text{C}$ ), and the cold stream has mass flow rate  $m_c$  and temperature  $T_c$ . These two streams are the primary feeds to the tank. A disturbance stream provides the ability to load the system and has a mass flow rate  $m_d$  and temperature  $T_d$ .

Hagblom (1992) constructed this experiment. His tank has a constant cross-sectional area  $A$  ( $\text{cm}^2$ ). The exit stream flow rate is controlled by gravity. The exit mass flow rate  $m$  with a temperature of  $T$  flows through a pipe whose outlet is at atmospheric pressure at a level of  $h_0$  (cm) below the bottom of the tank. The hot water and cold

water streams are controlled with single-input, single-output (SISO) controllers via control valve action.

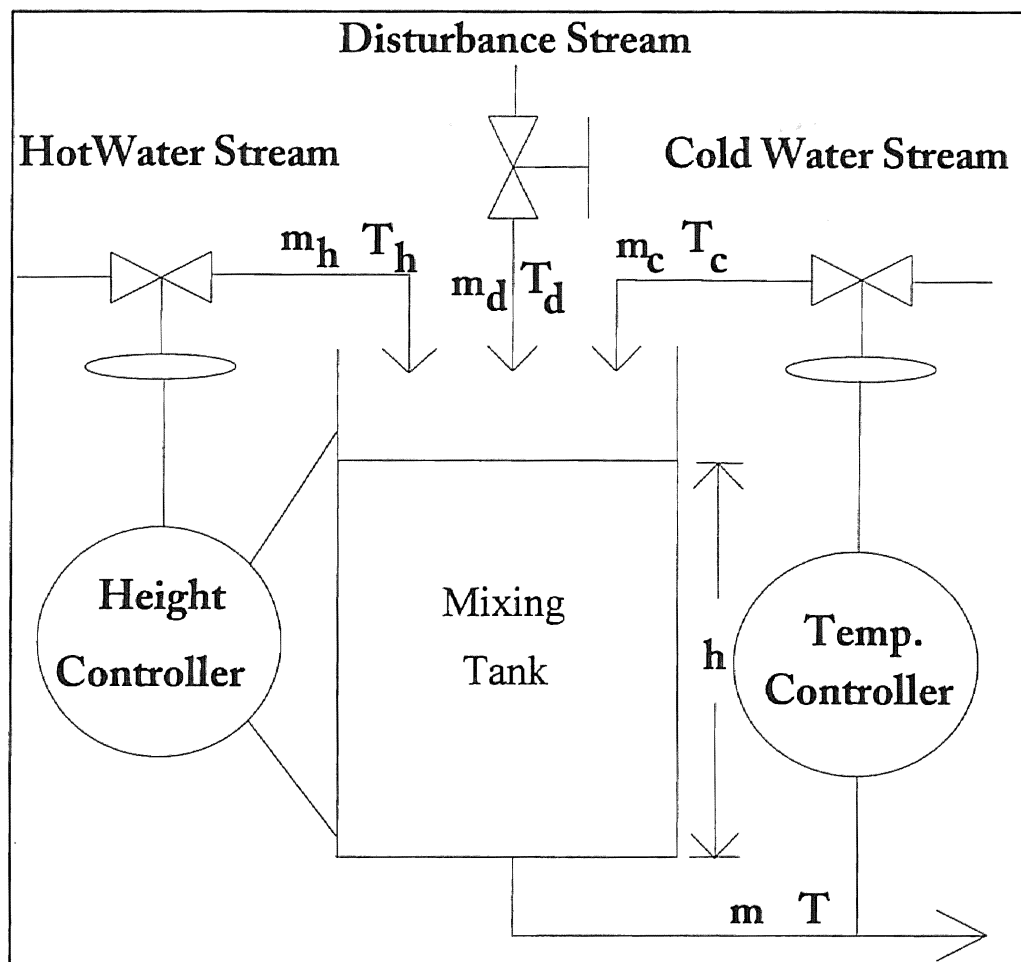


Figure 10: Mixing tank

Hagglom's (1992) paper investigates the limitations of SISO control for this coupled control system. Hagglom compares a SISO controller with a "model-based" controller

designed to take into account nonlinearities and multi-variable characteristics of the mixing tank. His investigation also looked at gain scheduling. Those results will be discussed after a more thorough examination of the mixing tank.

### Numerical Analysis of Mixing Tank

The governing equations for the mixing tank are derived from simple mass and energy balances. The mass balance for the system is

$$\rho A \frac{dh}{dt} + \beta (h + h_0)^{1/2} = m_h + m_c + m_d, \quad (13)$$

and the mass flow rate leaving the tank is governed by

$$m = \beta (h + h_0)^{1/2} \quad (14)$$

assuming a turbulent stream and that the flow characteristic  $\beta$  (kg/min/cm<sup>1/2</sup>) and density  $\rho$  (kg/cm<sup>3</sup>) are constant.

Similarly, an energy balance using the mixing tank contents as the system gives

$$\frac{\partial(\rho V(T - T_{ref}))}{\partial t} = (T_h - T_{ref})m_h + (T_c - T_{ref})m_c + (T_d - T_{ref})m_d - (T - T_{ref})m \quad (15)$$

where  $T_{ref}$  is the reference temperature. Assumptions made with these equations are constant and equal specific heat capacities. Density variations are considered insignificant. If we assume that  $T_{ref}$  is  $0^{\circ}\text{C}$  and separate the volume term into height and area, Equation 15 transforms into

$$\rho A \frac{\partial hT}{\partial t} = T_h m_h + T_c m_c + T_a m_a - T m. \quad (16)$$

Separating the derivative term by parts results in Equation 17.

$$\rho A h \frac{\partial T}{\partial t} + \rho A T \frac{\delta h}{\delta t} = T_h m_h + T_c m_c + T_a m_a - T m \quad (17)$$

The results of Equations 13 and 14 can be combined into Equation 17 to obtain Equation 18.

$$\rho A h \frac{dT}{dt} + (m_h + m_c + m_a)T = m_h T_h + m_c T_c + m_a T_a \quad (18)$$

Hagglom also assumes that perfect mixing takes place inside the tank even though no mechanical agitation is provided. Table I lists constants used in the mixing tank experiment.

TABLE I

## CONSTANTS FOR THE MIXING TANK SYSTEM

$A = 283.5$ $\text{cm}^2$	$\beta = 1.00$ $(\text{kg}/\text{min})/\text{cm}^{1/2}$	$h_0 = 109 \text{ cm}$
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Values in Table I show that  $h_0$  is large compared to  $h$ . Therefore, as Haggblom points out, there is only a weak nonlinearity in Equation 13. Equation 13 makes it appear that there is no coupling of temperature with height. However, since  $m_h$  and  $m_c$  are used to control the temperature and level, coupling will in fact exist. Equation 18 is strongly nonlinear due to the mass flow rates of the streams entering the tank.

### Controller Coupling

As discussed earlier, level and temperature for the mixing tank are coupled. When a control system includes a coupled or 'paired' input, a decision must be made as to which controller will control which manipulated variable. Haggblom (1992) notes that  $m_h$  and  $m_c$  contribute equally to the level of the tank. This result is shown in Equation 13. Yet, in Equation 18, both streams influence the temperature of tank. Haggblom argues that the feed stream that has the greatest effect on the temperature ought to control temperature and leave the other feed stream to control the

height. The decision also depends on the temperature of the feed streams to the mixing tank as shown in the definitions of  $k_{Tmh}$  and  $k_{Tmc}$  in Equation 19. Haggblom (1992) calculated these values as shown in Tables II, III, and IV.

$$\begin{aligned}
 \tau_h &= \frac{2\rho A(\bar{h} + h_o)}{m} & \tau_T &= \frac{2\rho A\bar{h}}{m} & k_{hm} &= \frac{2(\bar{h} + h_o)}{m} \\
 k_{Tmh} &= \frac{\bar{T}_h - \bar{T}}{m} & k_{Tmc} &= \frac{\bar{T}_c - \bar{T}}{m} & k_{Tmd} &= \frac{\bar{T}_d - \bar{T}}{m} \\
 k_{TT_h} &= \frac{m_h}{m} & k_{TT_c} &= \frac{m_c}{m} & k_{TT_d} &= \frac{m_d}{m}
 \end{aligned}
 \tag{19}$$

TABLE II

PROCESS GAINS AND TIME CONSTANTS AT 35°C  
(H = 20 CM AND  $M_D = 0$  KG/MIN)

$\tau_h = 6.44$ min	$\tau_T = 0.50$ min	$k_{hm} = \frac{22.7 \text{ cm}}{\text{kg / min}}$
$k_{Tmh} = \frac{1.41^\circ\text{C}}{\text{kg / min}}$	$k_{Tmc} = \frac{-1.58^\circ\text{C}}{\text{kg / min}}$	$k_{Tmd} = \frac{-1.41^\circ\text{C}}{\text{kg / min}}$
$k_{TT_h} = 0.529$	$k_{TT_c} = 0.471$	$k_{TT_d} = 0$

TABLE III

PROCESS GAINS AND TIME CONSTANTS AT 45°C  
(H = 20 CM AND M<sub>D</sub> = 0 KG/MIN)

$\tau_h = 6.44 \text{ min}$	$\tau_T = 0.50 \text{ min}$	$k_{hm} = \frac{22.7 \text{ cm}}{\text{kg / min}}$
$k_{Tmh} = \frac{0.528^\circ\text{C}}{\text{kg / min}}$	$k_{Tmc} = \frac{-2.46^\circ\text{C}}{\text{kg / min}}$	$k_{Tmd} = \frac{-2.28^\circ\text{C}}{\text{kg / min}}$
$k_{TT_h} = 0.824$	$k_{TT_c} = 0.176$	$k_{TT_d} = 0$

TABLE IV

PROCESS GAINS AND TIME CONSTANTS AT 25°C  
(H = 20 CM AND M<sub>D</sub> = 0 KG/MIN)

$\tau_h = 6.44 \text{ min}$	$\tau_T = 0.50 \text{ min}$	$k_{hm} = \frac{22.7 \text{ cm}}{\text{kg / min}}$
$k_{Tmh} = \frac{2.29^\circ\text{C}}{\text{kg / min}}$	$k_{Tmc} = \frac{-0.704^\circ\text{C}}{\text{kg / min}}$	$k_{Tmd} = \frac{-0.528^\circ\text{C}}{\text{kg / min}}$
$k_{TT_h} = 0.235$	$k_{TT_c} = 0.765$	$k_{TT_d} = 0$

In Table III, the larger absolute value of  $k_{Tmc}$  favors using the cold water stream to control the temperature at the upper end of the operating range. Yet, Table IV shows that the better choice for controlling the temperature is hot water stream,  $m_h$ , when the tank is operated at 25°C. When the mixing tank is operated at 35°C, neither feed stream is better suited to control either process variable.



A Relative Gain Analysis (RGA) (Bristol, 1966) supports these conclusions as well. The relative gain for pairing temperature with the cold water stream flow is defined by

$$\lambda_{Tmc} = \frac{\bar{T} - \bar{T}_c}{T_h - T_c}. \quad (20)$$

Hagglblom's analysis of the relative gain shows that when  $\lambda_{Tmc}$  is greater than 0.5, the temperature should be controlled with the cold water feed stream. This confirms Hagglblom's conclusion that the temperature should be controlled with whichever stream is farthest from the nominal operating temperature. Finally, the relative gain analysis shows that there is no good variable pairing for multiloop SISO control when the nominal operating temperature is near the average of the two feed stream temperatures.

### Simulation Development

The mixing tank was simulated using a package called Simulink by MathWorks. Simulink is an extension of Matlab. The program is suited for simulating dynamic systems. Simulink allows the creation of dynamic simulations using blocks which contain numerical definitions of the simulation. There are two steps to simulating a process with Simulink. First the model must be described. After the model is defined, Simulink analyses it. A detailed

construction is described in a separate technical report (Anderson and Whiteley, 1993).

The resulting simulation is a combination of analog and digital systems. The actual process is a continuous process. Digital controllers were constructed to sample height and temperature at 0.1 minute intervals. The mixing tank simulation running under Matlab used adaptive Runge-Kutta Fourth Order integration to solve the ordinary differential equations associated with the simulation. The minimum step size was set to 0.0001 minutes. The maximum step size was 0.1 minutes. The truncation error used for adaptive step size control was set at 0.0001. The resulting Simulink block diagram is shown in Figure 11. Note, that this does not contain the interface for the neural network used to perform gain scheduling.

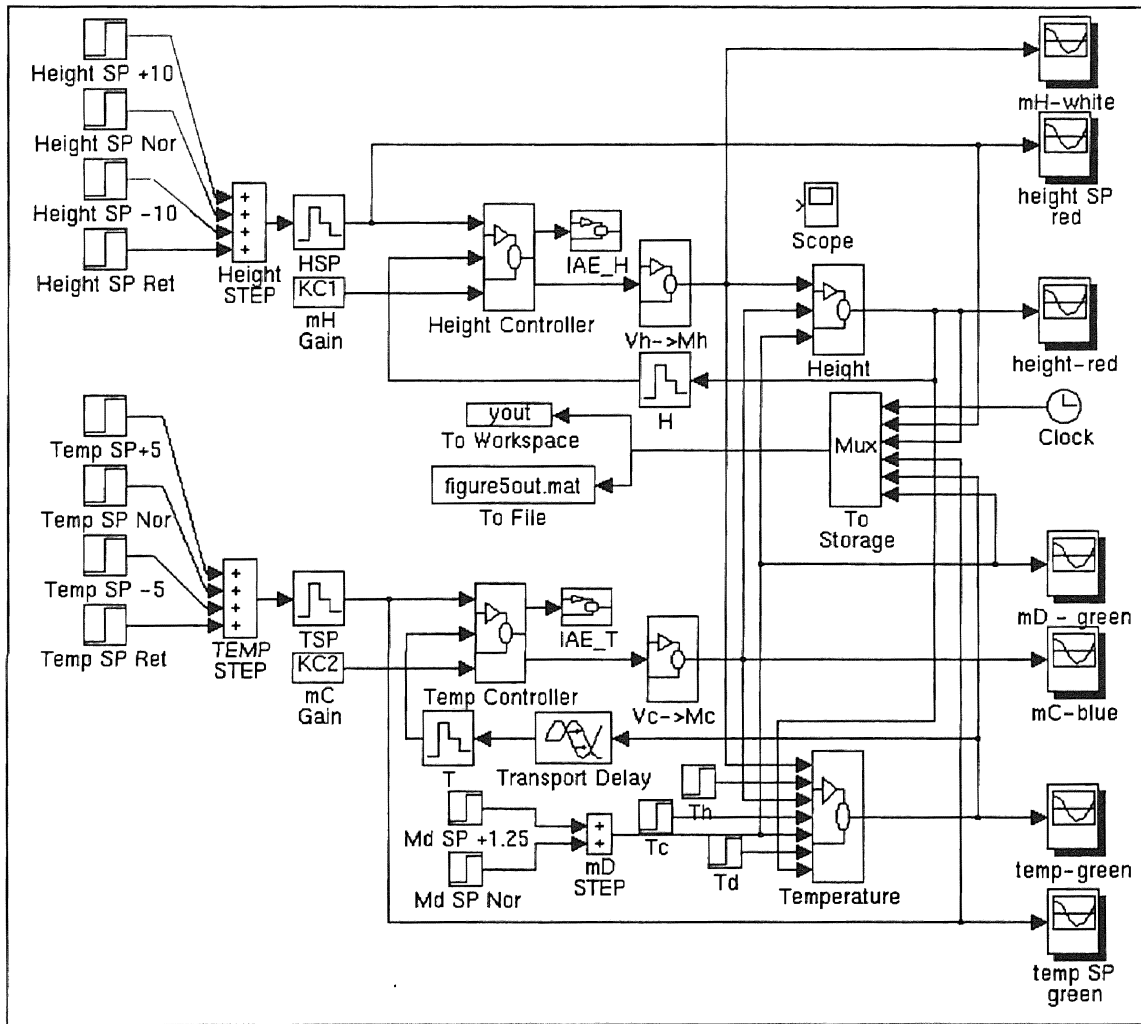


Figure 11: Simulation for Haggblom's mixing tank

### Demonstration of Simulated Mixing Tank

With the mixing tank simulation created, the next step involved verifying the simulation against results documented by Haggblom. Haggblom used an actual apparatus for his mixing tank system. He determined 'best' SISO tuning gains in his study which are used as benchmarks to compare our simulation to his. By comparing his process height, temperature, and flowrate results, a successful duplication was made. A 0.3 minute delay was added to the temperature stream in order to account for delay time caused by incomplete mixing in the tank.

#### Standard Controller Performance Test

Haggblom tested this mixing tank using an experiment that consisted of several setpoint changes and disturbances to measure the performance of the controllers. The simulation which will be described later uses his experimental method as well. Haggblom operated his mixing tank at several operating points. The range for the height was between 10 and 30 cm. Temperature changes ranged from 25 to 45°C. The disturbance stream operated at either 0 or 1.25 kg/min. The main operating points are defined as a level of 20 cm, a temperature of 35°C, and no flow from the disturbance stream. The hot water feed stream has a temperature of 51°C while the cold water stream is at 17°C.

The experiment begins at the nominal operating point and continues for 5 minutes. At 5 minutes, the temperature setpoint is changed from 35°C to 45°C. Ten minutes into the simulation, the temperature setpoint is returned to the nominal operating value. When the mixing tank experiment has ran 15 minutes, the temperature setpoint is lowered to 25°C for 5 minutes. At this point, the temperature setpoint returns to 35°C. Figure 12 shows the setpoint changes implemented in this study.

The next phase of Haggblom's experiment measures the reactions of the height controller to level setpoint changes. Twenty-five minutes into the experiment, the level setpoint is changed from 20 cm to 30 cm. Five minutes later, the setpoint returns to 20 cm. After the experiment has elapsed for 35 minutes, the level setpoint is lowered to 10 cm. Finally, at 40 minutes since the start of the run, the setpoint is returned to 20 cm.

Haggblom's standard experiment ends with testing of the temperature and height controllers' ability to handle disturbances. At 45 minutes into the run, the disturbance stream valve is opened and flows at 1.25 kg/min. This stream has a temperature of 19°C. The stream influences the system for 5 minutes. Afterwards, the experiment runs for 10 minutes at the nominal operating setpoints. Total time for this experiment is 60 minutes.

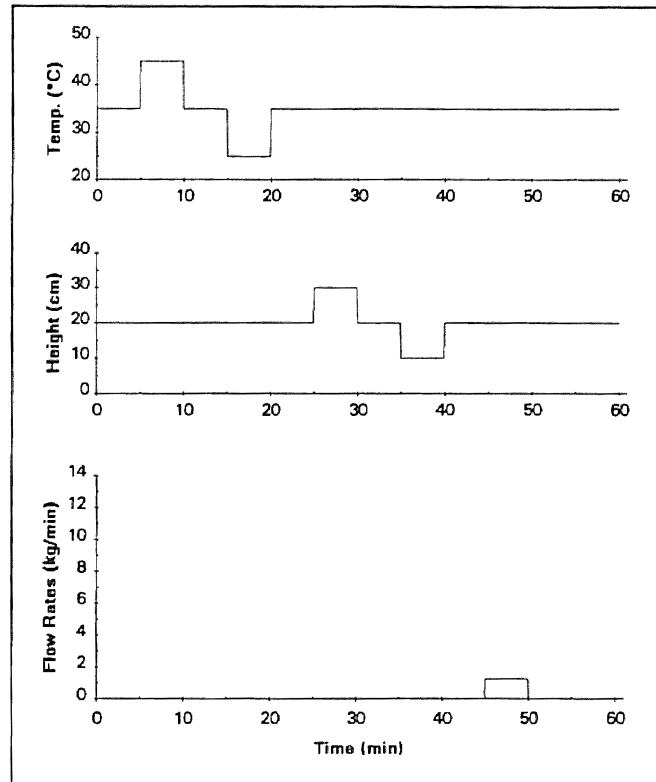


Figure 12: Setpoint changes used at 35°C

Two other experiments are of interest. While the experimental procedure described above is followed, the nominal operating points are changed. In one experiment, the nominal operating temperature is 40°C. Instead of 10°C setpoint changes, the temperature setpoints are changed to 45°C for the higher temperature and 35°C for the lower temperature setpoint. The second experiment uses an nominal operating temperature of 30°C. Like the 40°C experiment, the setpoint is only changed by 5°C. The highest temperature setpoint is 35°C and the lowest setpoint is 25°C. Level setpoint changes are the same in both of these

experiments, and the nominal height remains at 20 cm. Finally, the experiments include the load change outlined earlier. These final two experiments explore controller coupling and the related performance.

### Performance Measure

In order to quantify the performance of Haggblom's control system and the pattern-based gain scheduling approach, the Integral of Absolute Error (IAE) was used. IAE is defined as

$$\text{IAE} = \int_0^{\infty} |e(t)| dt \quad (21)$$

where  $e(t)$  is the difference between the actual process variable and the setpoint. IAE scores were calculated for both the height and temperature controllers. Lower IAE scores indicate better control system performance.

### Baseline Results with Haggblom's Constants

Haggblom's experiment was duplicated using Haggblom's PI values at nominal temperature of 30°C, 35°C, and 40°C. The gain and integral values used in the simulation are found in Table V. These results serve as a guide as to the improvement brought about by the use of the pattern-based gain scheduling system. The goal is for gain scheduling to lower the IAE values of Haggblom's simulation.

TABLE V

CONTROLLER PARAMETERS FOR MULTILoop SISO CONTROL  
(HAGGBLOM, 1992)

Nominal Temperature (°C)	$k_{VhH}$ (V/cm)	$\tau_{h,i}$ (min)	$\tau_{h,d}$ (min)	$k_{VcT}$ (V/°C)	$\tau_{T,i}$ (min)	$\tau_{T,d}$ (min)
30	0.30	2.50	0.00	-0.35	1.00	0.00
35	0.25	2.50	0.00	-0.30	1.00	0.00
40	0.20	2.50	0.00	-0.35	1.00	0.00

The results of the simulations using the values above are shown in Table VI. Figures 13, 14 and 15 show the results for nominal operating temperatures of 30°C, 35°C and 40°C. Table VII contains the IAE scores for these cases. As stated before, the goal of this study is to have lower IAE scores than these.

TABLE VI

IAE RESULTS FOR FIXED GAIN SISO CONTROL

Nominal Operating Temperature °C	Fixed Gain	
	IAE for Height Controller	IAE for Temperature Controller
30	114.76	101.99
35	95.28	66.96
40	52.25	32.74



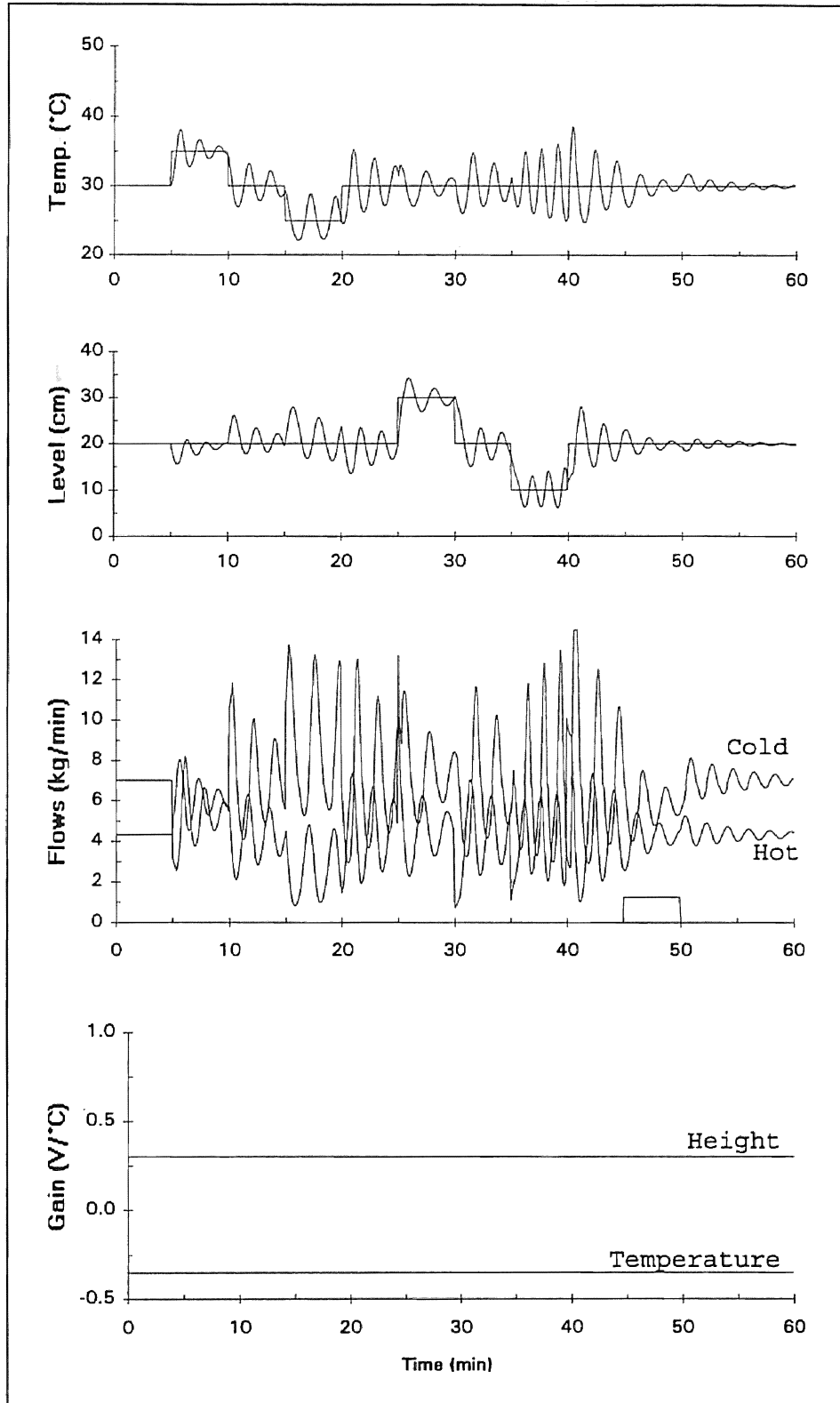


Figure 13: Haggblom's baseline results at 30°C

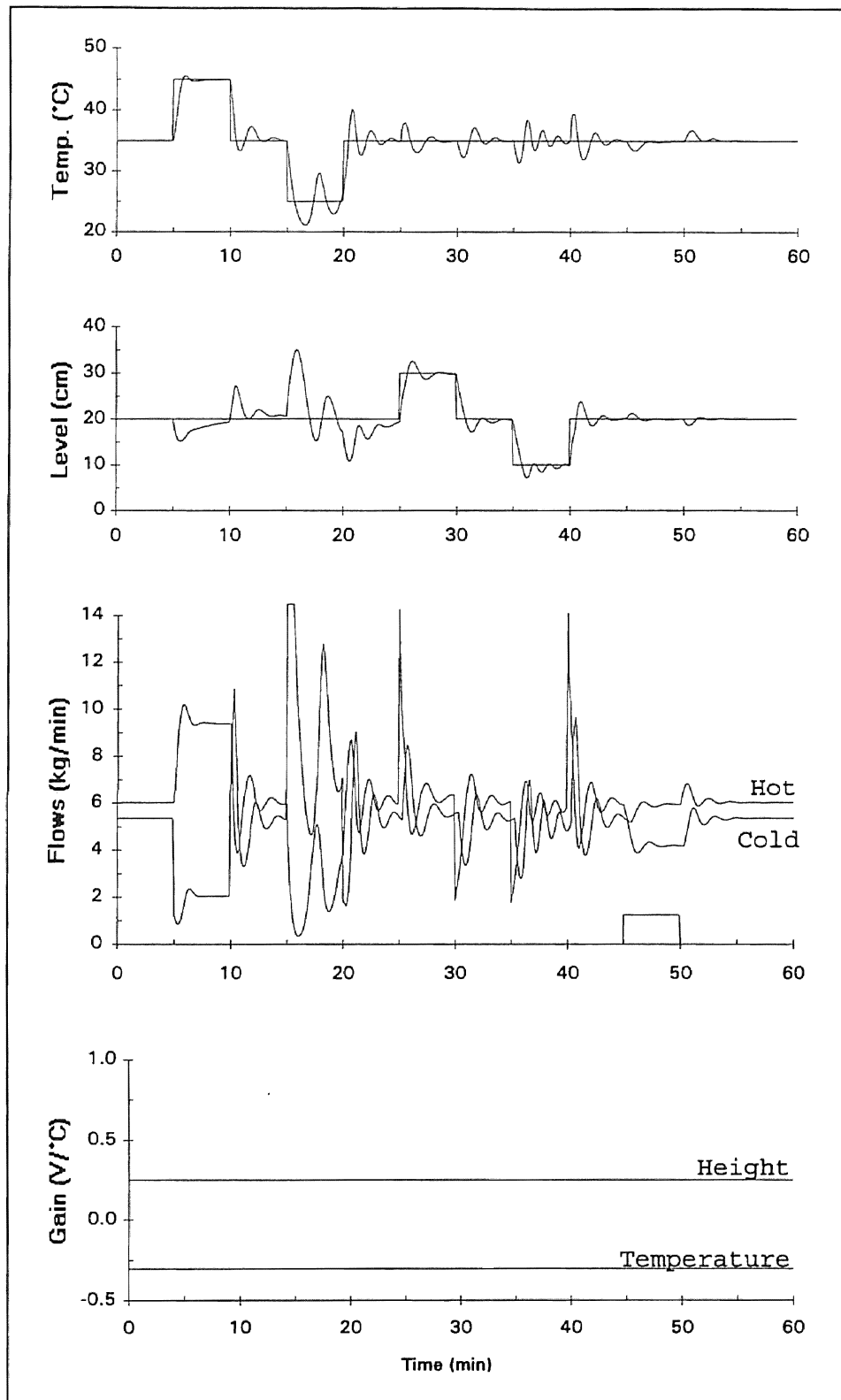


Figure 14: Haggblom's baseline results at 35°C

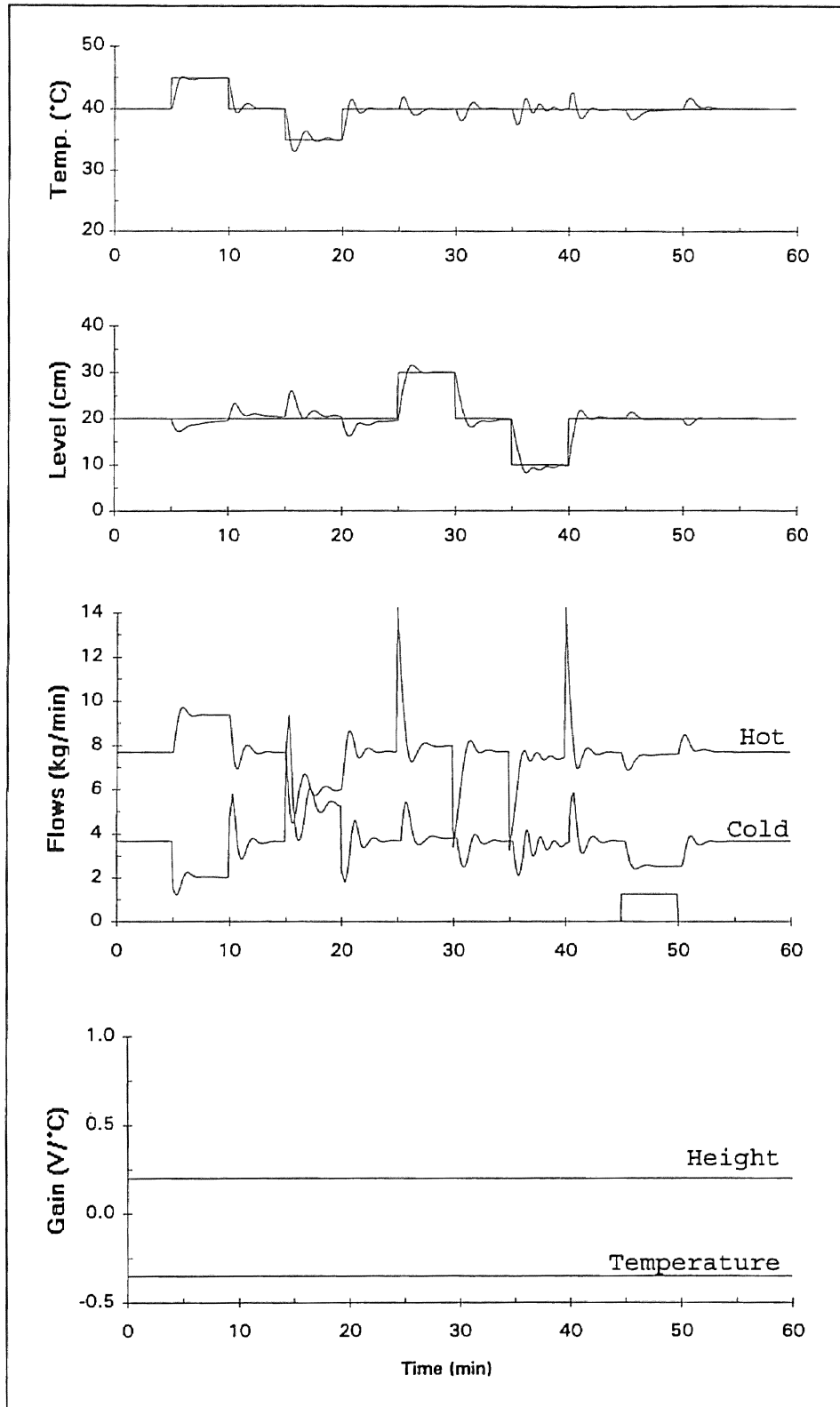


Figure 15: Haggblom's baseline results at 40°C

In Figure 13, the simulation operates where the controllers are not properly paired. The height controller should manipulate the cold water stream while the temperature controller should handle the hot water stream. Since this is not true, the control system has trouble controlling the system. With setpoint changes and load disturbances, the system undergoes oscillation. Although the oscillation is dampened, the control system is ineffective for this nominal temperature and controller pairing. It appears that a lower gain on the controllers is necessary.

Figure 14 represents control system responses at 35°C. The relative gain analysis at this point concludes that either pairing of manipulated/controlled variables would result in effective control. In this case, the height controller controls the hot water stream and the temperature controller manipulates the cold water flow rate. The other controller pairing is possible, but this study uses the pairing Haggblom chose. The results for setpoint changes are better than those in Figure 13. The mass flow rates do oscillate, but the system quickly returns to setpoint after each change. Load disturbances are also handled better. Height control action is slow for temperature setpoint changes, though. The controller fails to return the level back to setpoint.

Finally, at 40°C, the controller pairing is correct. The results shown in Figure 15 indicate a well-controlled

system. The mass flow rates are smooth and the control systems quickly return the system back to setpoint after changes.

The Integral of the Absolute Error (IAE) scores in Table VI further exemplify the problems of a fixed-gain system. The IAE errors for both the temperature and height controllers emphasize that fixed gain controllers cannot control the system over a range of operating conditions without some degradation. As the nominal operating temperature rises, the controllers have more favorable pairings. Thus, at 40°C, the IAE scores are relatively low. One note, the IAE scores for 35°C are higher as the temperature setpoints change by 10°C instead of 5°C for 30°C and 40°C.

### Hagglblom's Gain Scheduling

Hagglblom suggested a method to implement gain scheduling using the coupled SISO controllers. Hagglblom explained gain scheduling as a method of keeping the controller gains inversely proportional to the process gains. Hagglblom (1991; 1992) adjusted the controller gains according to Equations 22 and 23.

$$\bar{k}_{vHh} = k_{vHh} \left( \frac{\bar{h} + h_o}{h + h_o} \right)^{1/2} \left( \frac{\bar{v}_H}{v_H} \right)^{(1/\gamma_h)-1} \quad (22)$$

$$\bar{k}_{vct} = k_{vct} \left( \frac{\bar{T} - \bar{T}_c}{T - \bar{T}_c} \right) \left( \frac{\bar{h} + h_o}{h + h_o} \right)^{-1/2} \left( \frac{\bar{v}_c}{v_c} \right)^{(1/\gamma_c)-1} \quad (23)$$

The controller gains  $k_{vhh}$  and  $k_{vct}$  are the controller gains at the nominal operating point,  $\bar{v}_h$  and  $\bar{v}_c$  are the nominal or steady-state input voltages to the control valves, and  $v_h$  and  $v_c$  are the actual input voltages to the control valves. Haggblom's experiment involved using PID controllers operating at a nominal operating point of 35°C and 20 cm. He noted that the performance was worse than an experiment utilizing a fixed-gain PID controller. Haggblom stated that better performance was possible using different equations, yet he showed no clear method of implementing this. Finally, Haggblom added that Equations 22 and 23 need to be multiplied by  $(T - \bar{T}_c) / (\bar{T} - \bar{T}_c)$ . This modification is necessary if controller gains are assumed to be inversely proportional to the process gains when the other control variable is perfectly controlled. His experiment indicated that this would further degrade the performance.

Our study was unable to duplicate Haggblom's gain scheduling results. The calculated scheduling gains caused the simulation to go unstable and our results did not match his. However, since Haggblom reported his efforts were unsatisfactory anyway, we did not pursue this further.

## Optimized Gain Scheduling

In order to gain a better idea of what controller gains provide the best control performance, setpoint changes and load disturbances were introduced to the mixing tank. These tests determined the controller gains that provided the optimum control performance. These gains provide insight into the gains needed for gain scheduling. For example, at a nominal operating point of 30°C and 20 cm., Haggblom's experiment has 6 possible process changes from the nominal operating point. First, the temperature setpoint can be increased or decreased, the height setpoint can be raised or lowered, or a disturbance stream can be introduced or taken away. Each of these setpoint changes has gains associated with them that improve control performance. These gains allow the controllers to reach the new operating conditions with the smallest IAE error.

### Testing Procedure

The simulation was modified to run for a 10 minute interval. During this time, a setpoint or disturbance change was introduced to the system 1 minute from the start of the simulation. For each run, new controller gains were used. At the end of the run, the IAE results for both controllers were recorded. Table VII shows the results of a test using different gains for a specific setpoint change.

The gains were changed in order to find the lowest IAE scores for both the height and temperature controllers (IAE\_H and IAE\_T, respectively). As shown in Table VII, the chosen controller gains are 0.40 for KC1 (the height controller gain) and -0.225 for KC2 (the temperature controller gain) for the level setpoint change from 20 cm to 30 cm at a temperature of 40°C.

TABLE VII

IAE RESULTS FOR SETPOINT CHANGE  
FROM 20 CM TO 30 CM  
AT 40°C

			KC1							
			0.20	0.25	0.35	0.40	0.50	0.60	0.75	1.00
KC2	-0.15	IAE_H		5.24			4.89		5.05	
		IAE_T		2.77			2.87		2.87	
	-0.20	IAE_H		5.19			4.89		5.05	5.13
		IAE_T		2.58			2.74		2.75	5.74
	-0.225	IAE_H				4.87	4.89			
		IAE_T				2.65	2.67			
	-0.25	IAE_H		5.16	4.90	4.87	4.89	5.00	5.05	4.13
		IAE_T		2.44	2.55	2.59	2.64	2.65	2.66	2.65
	-0.275	IAE_H				4.86	4.88			
		IAE_T				2.59	2.63			
	-0.35	IAE_H	5.53				4.96			5.12
		IAE_T	2.17				2.97			2.95

Although the IAE\_T error is not minimized in this case, these gains are selected as a compromise between control response and minimum IAE error. Since the mixing tank is a coupled system, reducing the height IAE error may lead to



increasing the temperature IAE error. The goal of this phase of the research was finding the controller gain settings that minimized both IAE scores. When either IAE score could not be minimized, gains were chosen that generated good response curves to setpoint changes.

Other criteria used in selecting acceptable gains included discarding gains that produced excessive oscillatory responses. Often these gains produced situations where the control valves experienced ringing. Ringing refers to the rapid opening and closing of a control valve. Figure 16 contains an example of ringing during a simulation run. The hot water mass flow rate is repeatedly opened and closed quickly as the height controller tries to maintain the level. Excessive ringing leads to valve wear.

Finally, if either controller IAE scores were not minimized for a process change, gains were chosen that reduced tailing. Tailing occurs when the integral (I) response in the PI controller is too sluggish. The integral time is the amount of time an integral controller takes to reproduce the effect of a proportional controller constant. Thus, an integral time of 6 minutes indicates that the integral action of the controller takes 6 minutes to make the same control action of a proportional controller. If the integral constant is large, the controller is slow to remove the error that remains after proportional control. Tailing was a problem particularly for the height

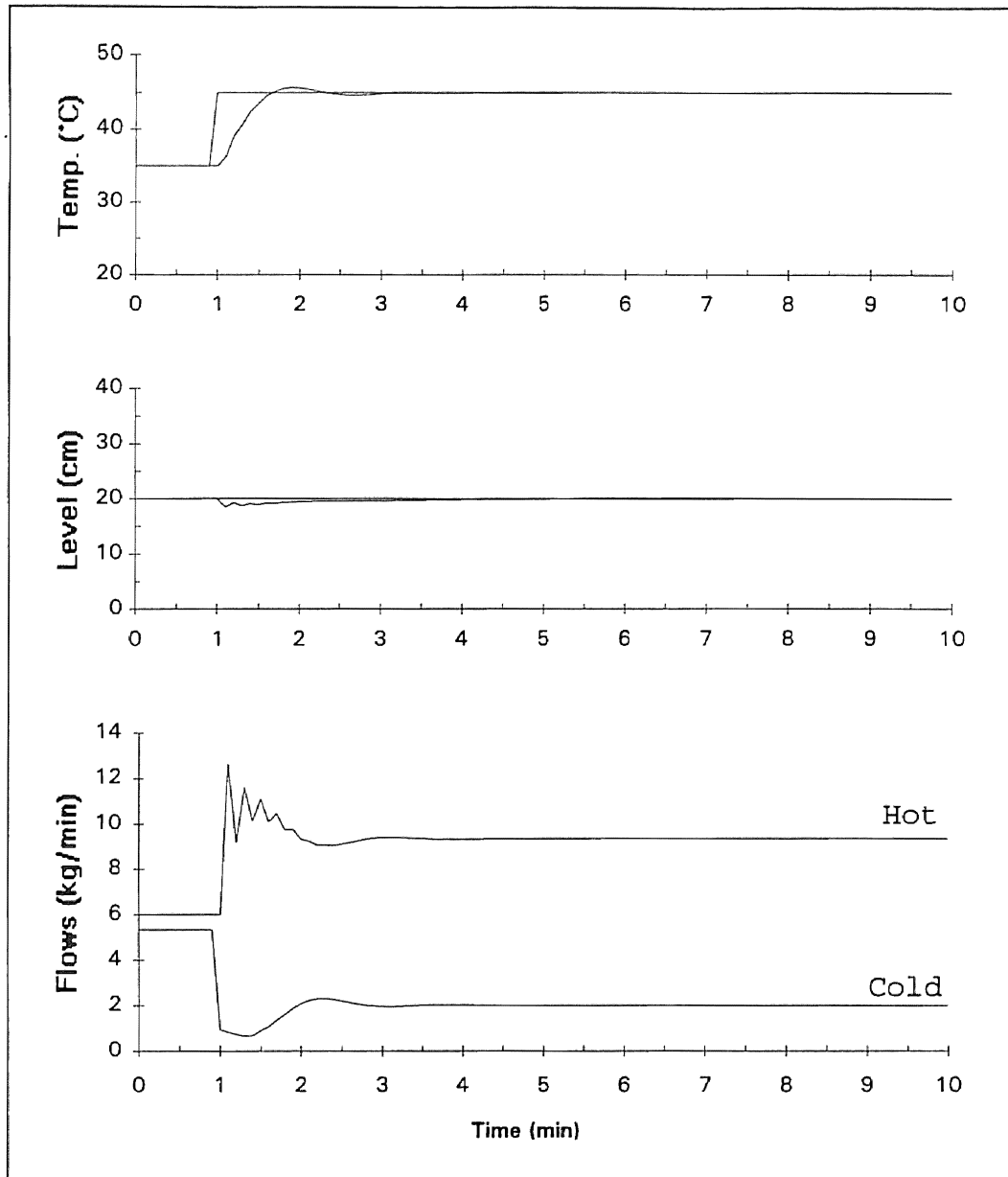


Figure 16: Valve ringing due to high gains

controller. Figure 17 shows an example of tailing. The level controller has trouble returning to setpoint after the temperature setpoint is changed. As a result, the height deviates from setpoint for over 3 minutes. The gains chosen

for each setpoint run tried to keep the effect of tailing to a minimum.

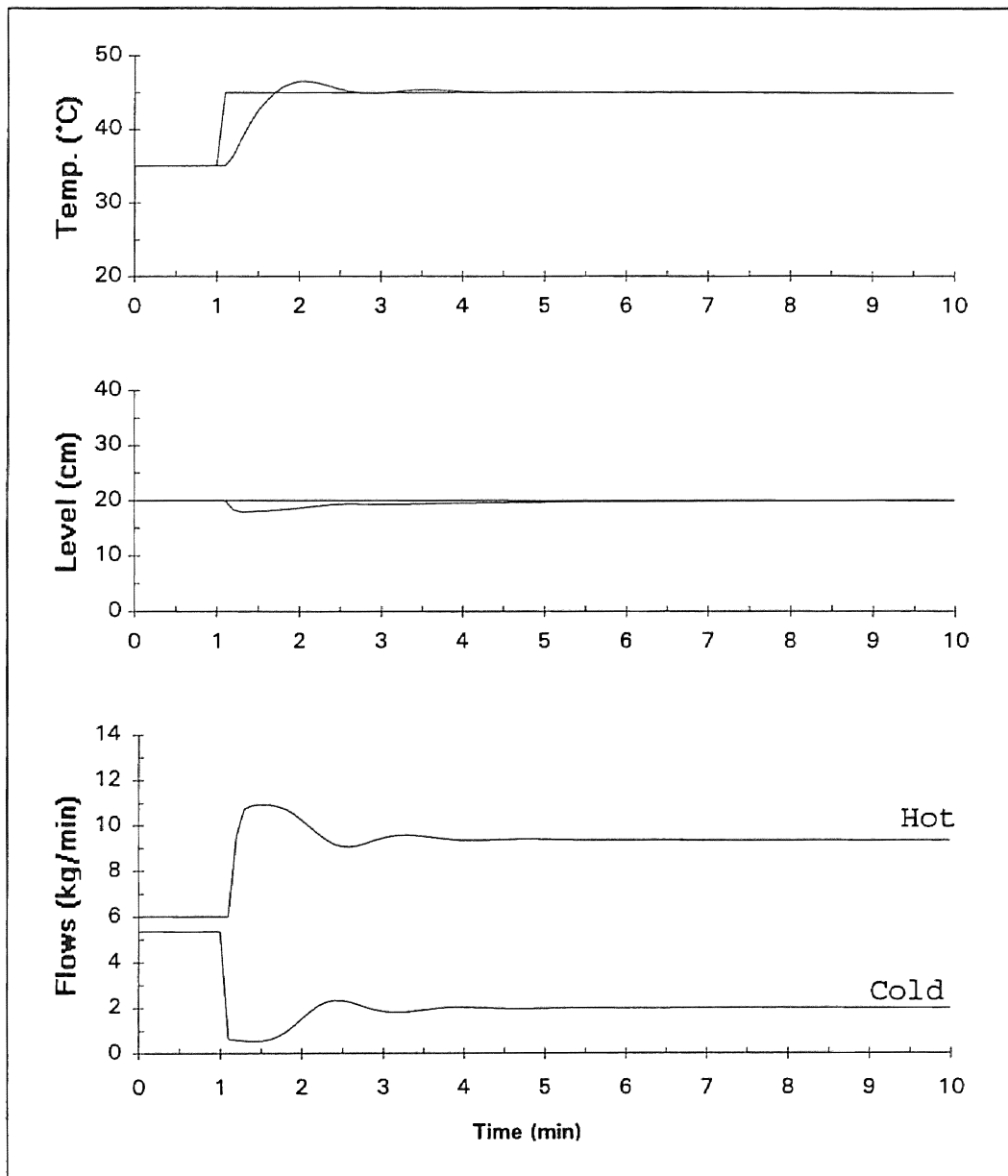


Figure 17: Tailing on level control

### Results of Gain Determination

Simulations were performed for setpoint and disturbance changes at nominal temperatures of 30°C, 35°C, and 40°C. Tables VIII, IX and X lists the optimized gains found in this study.

TABLE VIII

OPTIMIZED GAINS FOR SETPOINT  
AND DISTURBANCE CHANGES FOR  
THE MIXING TANK AT 30°C  
(LOW RELATIVE GAIN)

Height Setpoint cm	Temperature Setpoint °C	Disturbance Setpoint kg/min	Height Gain V/°C	Temp. Gain V/°C
20	30 -> 35	0	1.250	-0.175
20	35 -> 30	0	1.500	-0.175
20	30 -> 25	0	2.000	-0.137
20	25 -> 30	0	1.750	-0.150
20 -> 30	30	0	0.650	-0.100
30 -> 20	30	0	0.650	-0.150
20 -> 10	30	0	0.250	-0.200
10 -> 20	30	0	0.250	-0.125
20	30	0 -> 1.25	0.100	-0.550
20	30	1.25 -> 0	0.100	-0.500

Tables VIII, IX and X show a number of trends in the search for optimum gain. First, for temperature setpoint changes, a high gain was used. These values are common for all three nominal operating points. The values of these

gains decreased as the controllers became better paired as the relative gain for the temperature controller increased. The increased operating temperature of the mixing tank at 40°C raises the relative gain for the temperature controller to best control the flow rate of the cold water stream. In addition, the temperature controller gain increased in value as the controllers pairing improved. The tuning for the controller could be set more aggressively as the system approaches an operating region where each controller manipulates the correct variable.

TABLE IX

OPTIMIZED GAINS FOR SETPOINT  
AND DISTURBANCE CHANGES FOR  
THE MIXING TANK AT 35°C  
(RELATIVE GAIN = 0.50)

Height Setpoint cm	Temperature Setpoint °C	Disturbance Setpoint kg/min	Height Gain V/°C	Temp. Gain V/°C
20	35 -> 45	0	1.000	-0.350
20	45 -> 35	0	1.000	-0.240
20	35 -> 25	0	1.500	-0.125
20	25 -> 35	0	1.350	-0.185
20 -> 30	35	0	0.650	-0.175
30 -> 20	35	0	0.850	-0.175
20 -> 10	35	0	0.400	-0.150
10 -> 20	35	0	0.250	-0.175
20	35	0 -> 1.25	0.150	-0.400
20	35	1.25 -> 0	0.150	-0.400

TABLE X

OPTIMIZED GAINS FOR SETPOINT  
AND DISTURBANCE CHANGES FOR  
THE MIXING TANK AT 40°C  
(HIGH RELATIVE GAIN)

Height Setpoint cm	Temperature Setpoint °C	Disturbance Setpoint kg/min	Height Gain V/°C	Temp. Gain V/°C
20	40 -> 45	0	1.000	-0.425
20	45 -> 40	0	1.000	-0.300
20	40 -> 35	0	1.250	-0.200
20	35 -> 40	0	1.100	-0.250
20 -> 30	40	0	0.400	-0.250
30 -> 20	40	0	0.870	-0.225
20 -> 10	40	0	0.250	-0.250
10 -> 20	40	0	0.250	-0.250
20	40	0 -> 1.25	0.150	-0.650
20	40	1.25 -> 0	0.150	-0.500

Other trends can be observed for the height setpoint tests. The height setpoint changes are split into two regions. For operating changes from 20 cm to 30 cm, the mass of fluid inside the mixing tank is sufficient to dampen changes in temperature. Higher gains are possible on the height controller.

For changes that increased the operating level of the tank, Tables VIII, IX and X show higher height controller gains than those where the operating height is 10 cm. At 10 cm, the tank is nearly empty. With little mass inside the tank, temperature changes occur rapidly. Couple this situation with a nominal temperature of 30°C (i.e. the controllers are not properly paired) and the stability of

the system becomes a concern. Thus, lower gains for both the height and temperature controller are required. As shown before, the gain settings do increase as the relative gain favors the proper controller pairing.

Finally, the tables show that controller settings for disturbances are particularly low for the height controller. These values indicate that the temperature controller handles changes in load the best. The temperature gains are higher than the gains found for the setpoint tests.

It is evident that the coupling of the controller influences the gain setting for the controllers. Tables VIII, IX and X show that as the nominal operating temperature increases and the controllers are properly paired, the height gain decreases. The temperature gain increases with increasing temperature. For height setpoint changes, the effect of temperature is less on the height controller gains. Yet, temperature increases deem increases in the temperature controller gain. Disturbances are handled best by making the height controller passive to load changes. A higher gain for the temperature controller is indicated.

#### **Comparison of Fixed-Gain and Optimum Gain**

A comparison of a fixed-gain system and a system using optimized gains allows a benchmark to be established for future work with the pattern-based gain scheduling system.

The process results using optimized gains for every setpoint change set a lower limit for the IAE scores while Haggblom's gains set an upper limit for performance.

Table XI contains the results of Haggblom's system and the optimized gain system. The most dramatic improvement in IAE scores occurs when the system runs at a nominal temperature of 30°C. Significant improvements are made in reducing the IAE scores for both controllers. Using the optimized gains allow the control system to better handle operating conditions where the controllers are not paired correctly. For the other operating temperatures, the benefits of gain scheduling include reducing the height IAE score. Gain-scheduled controllers reduced temperature errors for 35°C and 40°C, but the differences in the IAE scores are less dramatic than the improvement in IAE scores for the height controllers.

TABLE XI

COMPARISON OF FIXED-GAIN AND  
OPTIMIZED GAIN-SCHEDULING  
IAE RESULTS

			Height IAE	Temp. IAE
Nominal Temp.	30 °C	Fixed-Gain	114.76	101.99
		Gain-Scheduled	35.30	42.98
	35 °C	Fixed-Gain	95.28	66.96
		Gain-Scheduled	34.23	56.53
	40 °C	Fixed-Gain	52.25	32.74
		Gain-Scheduled	27.86	31.70



Figures 18, 19 and 20 show the results of Haggblom's experiment using the optimum gain scheduling values at nominal temperatures of 30°C, 35°C and 40°C, respectively. If compared with Figures 13, 14 and 15, optimal gain scheduling significantly reduced the oscillations of the manipulated variables.

Significant improvement occurred for 30°C. Haggblom's fixed-gain system oscillated with each setpoint change. The mixing tank does not reach steady-state with changes in setpoint. Gain scheduling at 30°C allows the system to reach the new setpoints before a new change is introduced.

At 35°C (Figure 19) little deviation in the height takes place when a temperature setpoint change occurs. In Figure 14, Haggblom's fixed-gain controllers do not return the height to its setpoint quickly. Gain scheduling reduced the oscillations that occurred when the setpoint was changed. The results at 40°C (Figure 20) are similar. The height deviations are reduced, while the temperature responses are similar between the fixed-gain and gain-scheduled systems.

The optimized gain-scheduling IAE scores represent the approximate lowest possible scores. The expected pattern-recognition gain scheduling results should be between the fixed-gain and gain-scheduled results in Table XI.

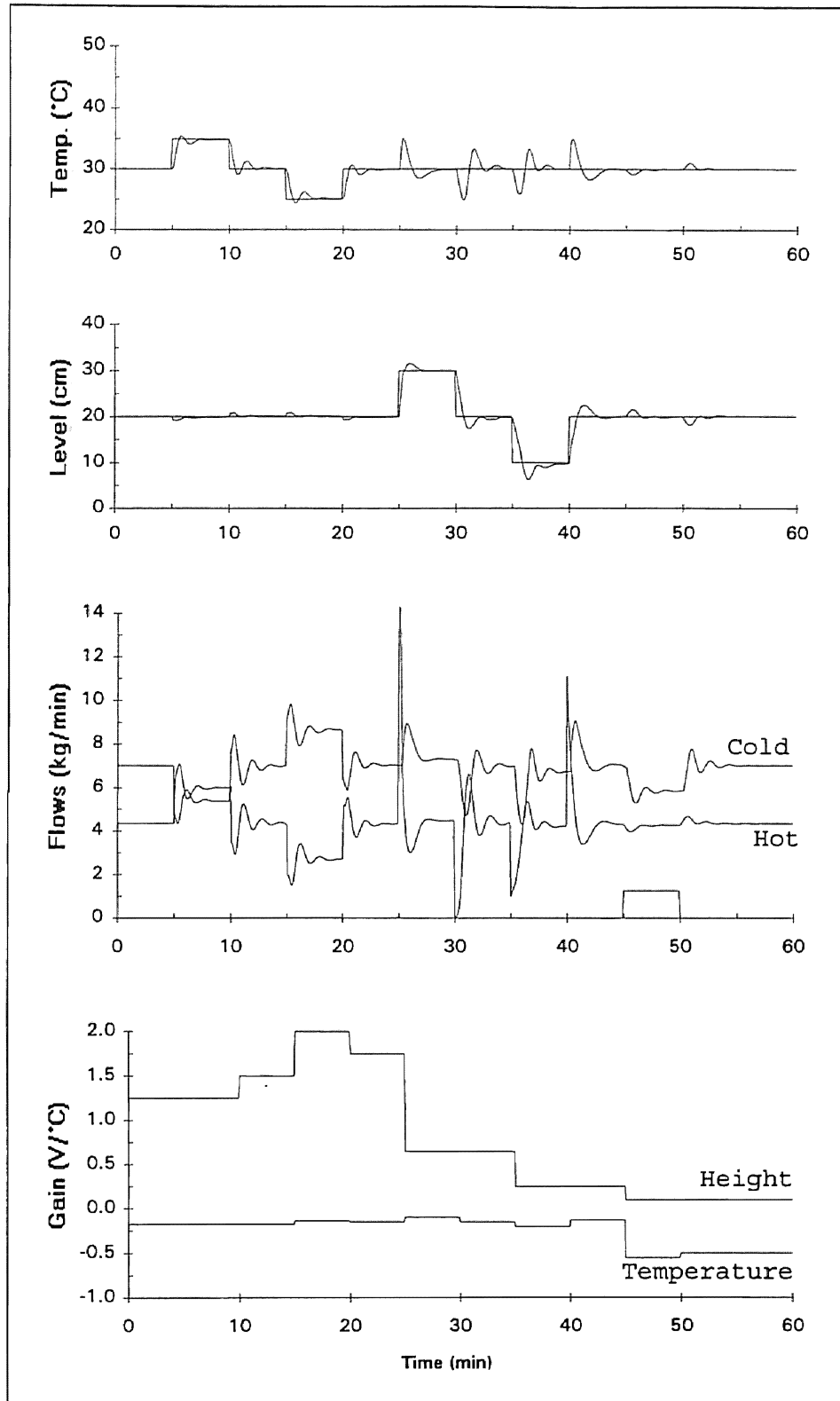


Figure 18: Performance of optimum-gain scheduling at 30°C

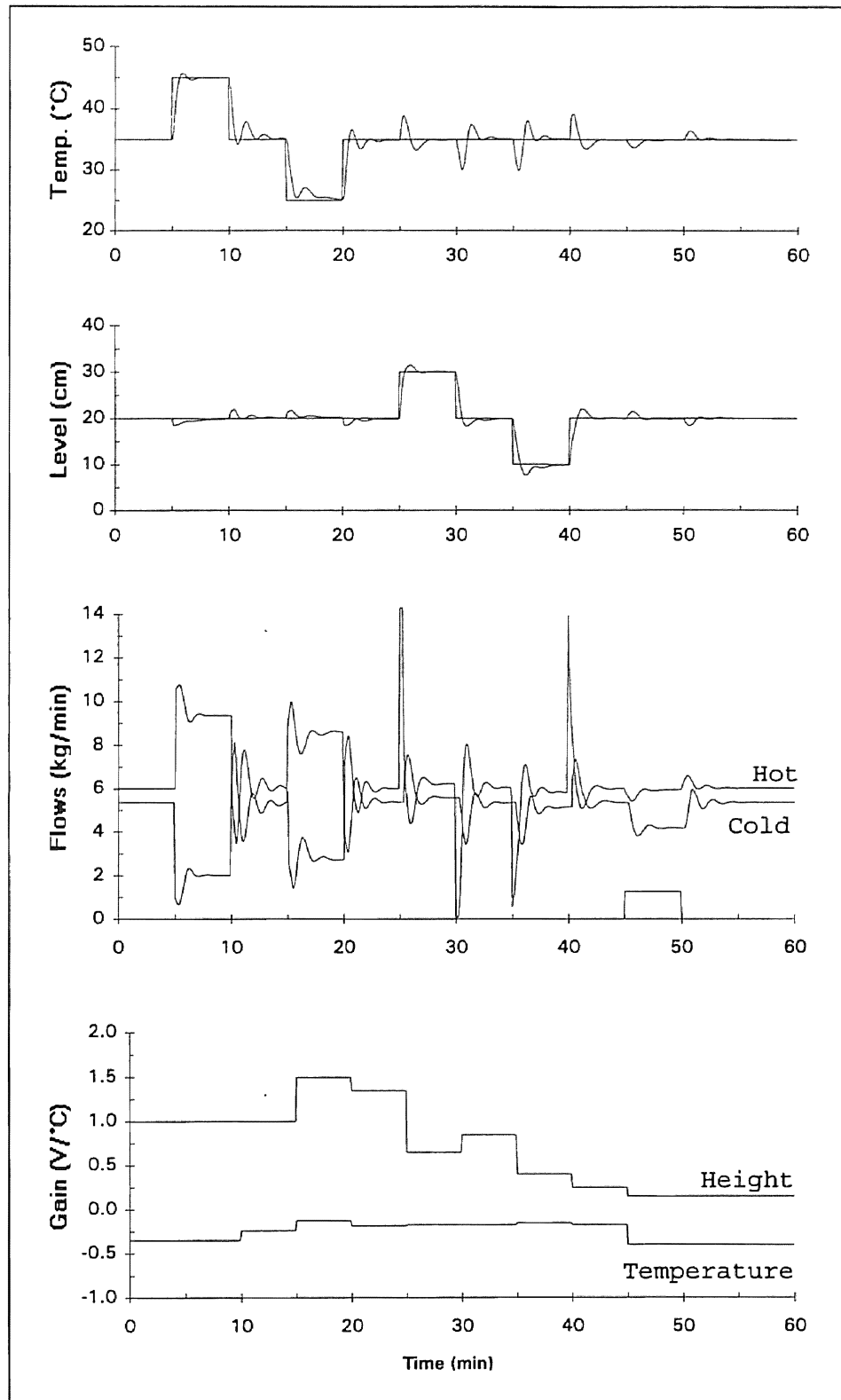


Figure 19: Performance of optimum-gain scheduling at 35°C

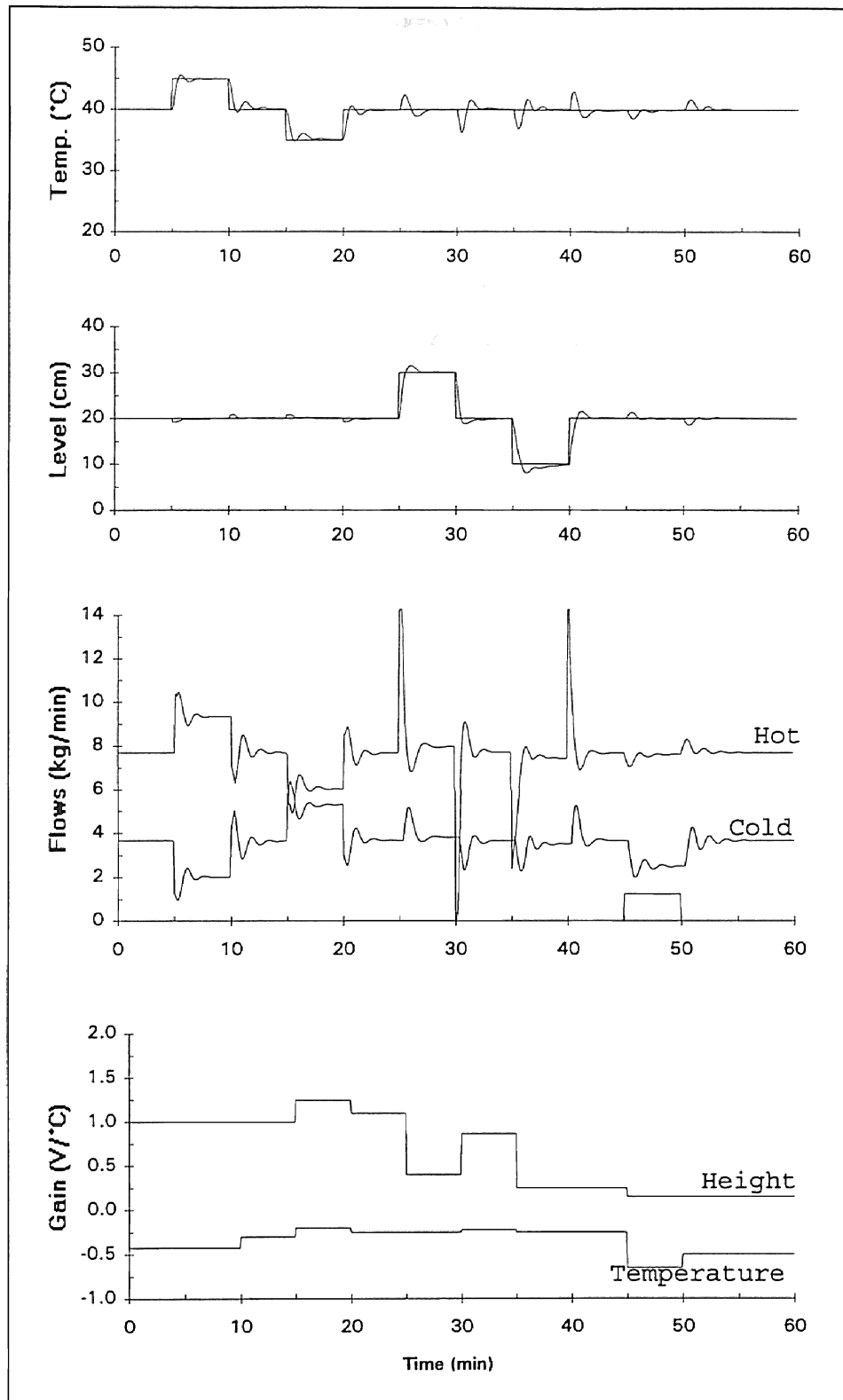


Figure 20: Performance at optimum-gain scheduling at 40°C

### Summary

This chapter details the mixing tank system and the equations governing it. From these equations, a simulation was developed and a control system was created. The simulation was compared with the results of Haggblom and tweaked to reproduce his values as accurately as possible. Once the simulation was verified, simulations were run using his values and the errors were recorded. In addition, a set of runs were made using optimum gains. These optimum gains set the lower limit on the performance scores. These errors serve as a guide for the pattern-recognition work that follows. The next chapter looks at pattern-based gain scheduling and the improvement possible with it.

## CHAPTER V

### PATTERN-BASED GAIN SCHEDULING

#### FOR THE MIXING TANK

##### Introduction

This chapter looks at the development of the pattern-recognition gain scheduling system. The first issue addressed is determination of the number of operating points to schedule the system for. Second, a general purpose method is developed to establish the gains to implement with the pattern-recognition system. The chapter will conclude with discussion of the pattern recognition to Simulink interface. This interface allows the pattern recognition system to see process data and transfer the controller gains back to the simulation.

##### Gain Clusters

As discussed earlier, the gain map contains the pattern prototypes the neural network learns. Each of these prototypes are associated with controller gains designed to improve control system performance when the system near these points.

One method of envisioning fixed-gain control is to consider a gain map consisting of one gain cluster of infinite size. The gain associated with this cluster is assigned to the controller no matter what operating conditions exist in the process. On the other extreme, a gain map could consist of an infinite number of clusters representing each possible operating point. Gains would be assigned to each individual point, yet, many regions of the gain map would have the same gain. It is not the goal of this study to completely cover the gain map with clusters. Interpolation techniques will be used to schedule the gain if the process is operating between the learned prototypes.

The number of clusters used to form the gain map is important. First, if the plant is slightly nonlinear, fewer scheduling points or clusters should be required. If the plant is highly nonlinear, though, a higher number of clusters are needed in order to provide the proper gains to the controllers.

Thus, gain clusters are chosen to represent normal steady-state operating conditions. These operating points are at the center of each cluster. Since the cluster has a radius, a buffer will exist to account for slight variations in operations. Once the process conditions move out of a cluster, interpolation will be used until it moves into another cluster.

## Gains Calculations for Scheduling

Since the ART2 neural network learns with steady-state process data and gains are associated with each of the learned pattern prototypes, a method must be found to calculate the gains for each cluster. The study to find optimum gains looked at the range of gains that provided stable control. It is desired to find an analytical method to determine controller gains.

One of the problems in finding the best gains to associate with each pattern prototype is calculating the gains to handle a variety of setpoint changes. With Haggblom's experiment, 6 possible changes are possible from the nominal operating point. Figures 18, 19 and 20 indicate that different gains work best for specific operations such as a temperature setpoint change, height setpoint change, or load disturbance. Thus, gains must be found which handle possible system changes from the normal steady-state operating points.

While the values found in the optimized gain study could be used to develop gains for the neural network clusters, our goal is to use a standard industry technique to compute the gains. One reason for this selection is that it provides a beneficial and easy method of setting the gains based on process characteristics. Since the neural network works with knowledge acquired by learning past process data, the process characteristics can be determined



by examining that data. In addition, methods of controller design for processes with first-order responses with deadtime are widely known (Smith and Corripio, 1975; Miller et. al., 1967; Murrill and Smith, 1966). The most widely used methods for designing PI/PID feedback control loops include the Integral of Absolute Error (IAE), the Integral of Error Squared (ISE) and Integral of the Absolute Error multiplied by Time (ITAE) tuning relations (Seborg et. al., 1989).

### Tuning Relations

The IAE, ISE and ITAE tuning relations are based on the work of Ziegler and Nichols (1942). Their work looked at developing tuning constants based on the open-loop response of a process to a step change in the manipulated variable (Miller et. al., 1967). Many processes can be approximated by a first order lag described by Equation 24.

$$\frac{P(s)}{M(s)} = \frac{Ke^{-\theta s}}{\tau s + 1} \quad (24)$$

where  $P(s)$  is the process output,  $M(s)$  is the manipulated variable input into the process,  $K$  is the process gain,  $\theta$  is the deadtime and  $\tau$  is the time constant (Seborg et. al., 1989). Miller (Miller et. al., 1967) shows methods to calculate  $\theta$ ,  $\tau$ , and  $K$  from process data. With these values, tuning constants are found by using equations for IAE, ISE

and ITAE and specific constants for each method. These design relations only work when deadtime is present in the process which is the case for the temperature control loop.

The final type of design relation examined in this study is the Direct Synthesis approach (Seborg, et. al., 1989). Equations have been developed to calculate the controller gain, integral time, and derivative time based on a desired trajectory for the controlled variable. Direct synthesis can be used with processes with dead time, but the gains calculated must be reduced (Seborg, et. al., 1989).

#### Use of Controller Design Relations for Gain Scheduling

The necessary process characteristics were found for the mixing tank using Equations 13, 18 and 19. Appendix A shows the calculation of the height and temperature controller parameters. A comparison was developed which looked at the range of controller values possible for each of the nominal operating points. These values are shown in Table XII and XIII. Table XII shows the calculations for height controller gain based on the direct synthesis method. Temperature controller gains are shown in Table XIII using the IAE method and ITAE method for both setpoint changes and disturbance rejection.

TABLE XII

RANGE OF HEIGHT GAINS FOUND IN OPTIMUM GAIN STUDIES  
AND CALCULATED BY THE DIRECT SYNTHESIS METHOD

Height cm	Temperature °C	Optimum Height Gain - Ranges			Direct Synthesis Method Gain
		Height Changes	Temperature Changes	Disturbances	
10	30	0.250			0.51
	35	0.250			0.41
	40	0.250			0.35
20	25		1.350 - 1.750		0.69
	30	0.250 - 0.650	1.250 - 2.000	0.100	0.50
	35	0.400 - 0.650	1.000 - 1.500	0.150	0.40
	40	0.250 - 0.400	1.000 - 1.250	0.150	0.34
	45		1.000		0.30
30	30	0.650			0.48
	35	0.850			0.39
	40	0.870			0.33

TABLE XIII

RANGE OF TEMPERATURE GAINS FOUND IN OPTIMUM GAIN STUDIES  
AND CALCULATED BY THE IAE AND ITAE METHODS

Height cm	Temp. °C	Optimum Temp Gain - Ranges			Calculated Methods			
		Height Changes	Temp. Changes	Disturbances	IAE Method		ITAE Method	
					Setpoint	Disturbance	Setpoint	Disturbance
10	30	-0.125			-0.22	-0.29	-0.17	-0.25
	35	-0.175			-0.20	-0.25	-0.15	-0.22
	40	-0.250			-0.21	-0.26	-0.16	-0.23
20	25		-0.150		-0.55	-0.77	-0.44	-0.67
	30	-0.200 - -0.100	-0.175 - -0.137	-0.550 - -0.500	-0.40	-0.55	-0.32	-0.40
	35	-0.175 - -0.150	-0.350 - -0.125	-0.400	-0.35	-0.49	-0.28	-0.42
	40	-0.250	-0.425 - -0.200	-0.650 - -0.500	-0.35	-0.49	-0.29	-0.42
	45		-0.300 - -0.240		-0.51	-0.70	-0.40	-0.61
30	30	-0.150			-0.55	-0.80	-0.45	-0.69
	35	-0.175			-0.49	-0.71	-0.40	-0.61
	40	-0.225			-0.51	-0.74	-0.41	-0.64

Height controller gain is calculated with the direct synthesis method. This method was chosen as the height response exhibits little or no deadtime. Temperature controller gain is determined using the ITAE setpoint method as this process loop has 0.3 minutes of deadtime.

Stability is a concern with any gains selected for the gain map. Gains used by the neural network must be able to deliver stable setpoint changes and disturbance control for each steady-state operating point. From the optimum gain studies, acceptable values for gains are known. For conservative gains, the lower values of the optimum gain experiments were used for comparison with values calculated by the tuning methods.

Gains calculated for the height controller by the direct synthesis method generally fall within the range of the conservative values found in the optimum gain studies with the exception of the 10 cm level. The direct synthesis method also calculated height controller gains higher than range of gains found for the disturbance tests. Having a higher gain for disturbance rejection will degrade control performance, but a compromise must be made. Furthermore, less accurate control will occur with temperature setpoint changes. The optimum gain studies found that the best gains for the height controller during temperature setpoint changes are in excess of 1. With the compromise of using the more conservative gains, the height controller will not be as aggressive as it can be.

Gains calculated by the IAE and ITAE methods for the temperature controller do not fall into the ranges found by the optimum gain study. Instead, the calculated gains are approximately 100% high. It was decided that the method for calculating the gains would require derating the calculated gain by multiplying it by a constant factor of 0.5. The derated values of the ITAE method matched the range of values found in the optimum gains better. The comparison of the derated ITAE gains and the optimum gain study is shown in Table XIV. Furthermore, the mixing tank is a coupled-system. Derating provides a degree of insurance for stability for the system.

#### **Simulink/ART2 Interface**

Additional work was needed to modify Whiteley's implementation of the ART2 Network (Whiteley, 1991). The following section discusses the additional modifications to Whiteley's program and development of an interface to transfer process data from Simulink/Matlab to the neural network. Figure 21 is the Simulink block diagram for a nominal temperature of 35°C and includes the block for the neural network.

TABLE XIV

TEMPERATURE CONTROLLER GAINS CALCULATED  
WITH DERATED ITAE METHOD

Height cm	Temp. °C	Optimum Temp Gain - Ranges			ITAE Setpoint Method
		Height Changes	Temperature Changes	Disturbances	
10	30	-0.125			-0.09
	35	-0.175			-0.08
	40	-0.250			-0.08
20	25		-0.150		-0.22
	30	-0.200 - -0.100	-0.175 - -0.137	-0.550 - -0.500	-0.16
	35	-0.175 - -0.150	-0.350 - -0.125	-0.400	-0.14
	40	-0.250	-0.425 - -0.200	-0.650 - -0.500	-0.15
	45		-0.300 - -0.240		-0.20
30	30	-0.150			-0.23
	35	-0.175			-0.20
	40	-0.225			-0.21

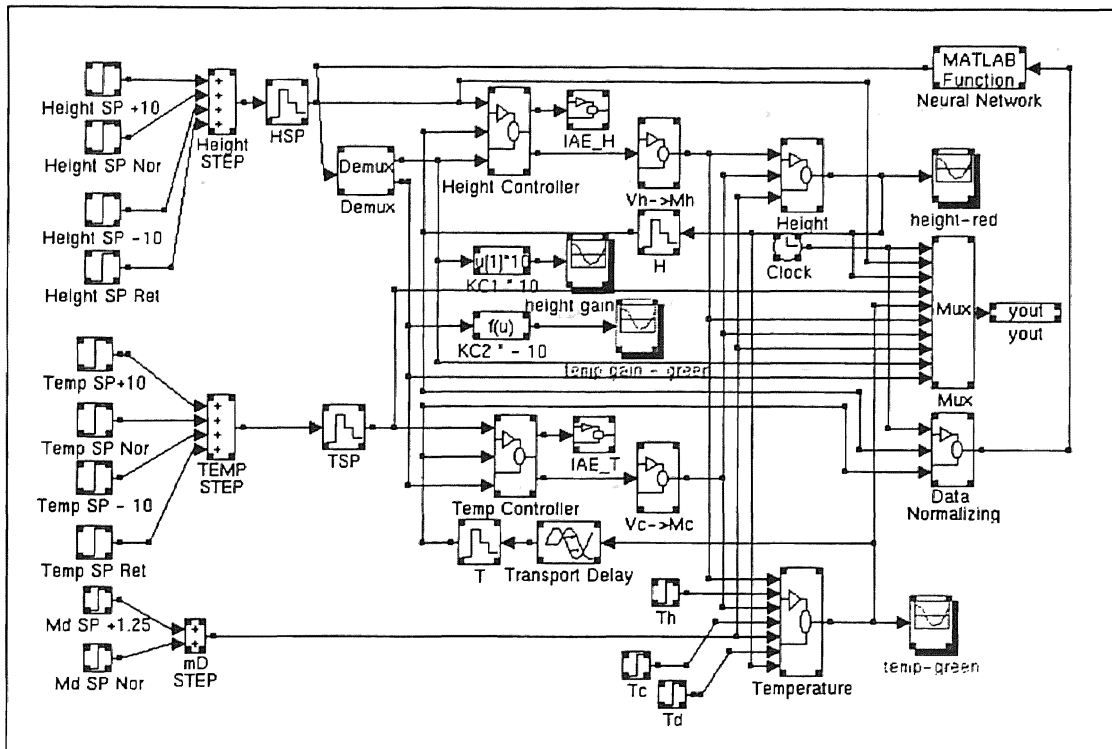


Figure 21: Simulink simulation with ART2 interface

### Simulink/ART2 Interface

An interface was needed to transfer the process data to the neural network. Several methods were investigated. Matlab allows the incorporation of C and FORTRAN code inside functions. The solution to implementing an interface uses a MatLab function call to send the current temperature, height, and time to an intermediate program that maintains a data file containing the process pattern. The intermediate program keeps the past data based on the window length used by the neural network. In addition to storing this data, the program transfers the process data to the ART2 network.



A separate report (Anderson and Whiteley, 1993) contains the program Quest which is the Simulink/Matlab interface. The Quest program sends the process data to the neural network program. Quest receives the output of the neural network program. This output is the gains for the height and temperature controllers in the Simulink simulation.

### ART2 Neural Network Modifications

Whiteley's neural network program was modified as well. The modified program reads in a preprocessed weight file that contains the top-down (TD) and bottom-up (BU) weights for the ART2 neural network. The TD and BU weights define the prototype vectors which locates the clusters. Data from the Quest program is the input to the neural network. The gain file generated by the learning program is read into the neural network and compared with the most similar and the next most similar clusters found by the neural network.

At this point, the gains may be interpolated or the gains of the most similar cluster are assigned and returned to the Quest program. If the pattern falls within one of the learned pattern clusters, the gains for that cluster are returned to the controllers. The determination of whether interpolation takes place depends on the ART2 vigilance parameter  $\rho$ . Once the gains are passed to the MATLAB function, the gains are sent to the controllers.

### Interpolation Methods

Once the neural network has determined which clusters are similar to the process pattern it is classifying, the program has the option of assigning the gains based on the most similar cluster gains or by interpolation. Gain scheduling uses interpolation to smooth gain changes between the normalized operating points (Rugh, 1991; Shamma and Athans, 1992). That is our goal here as well. This study looked at two types of interpolation, linear and quadratic.

The basis for interpolation is using the similarity between the current process pattern and the learned pattern prototypes most similar to the process pattern. Whiteley (1991) discusses the similarity measure and cluster similarity between a normalized input pattern and the prototype pattern the neural network program learns.

Linear Interpolation The first method for interpolation is a linear function. Linear interpolation simply implements the lever rule for determining the gain between two points. Figure 22 is a diagram of the similarity as distances between cluster centers.  $d_1$  and  $d_2$  represent the difference between the similarity of the process pattern and the nearest pattern prototype less the cluster criterion  $\rho$ . The interpolated gain is calculated with Equation 25.

$$K_c = \frac{K_{c1} * d_2 + K_{c2} * d_1}{d_1 + d_2} \quad (25)$$

where  $K_c$  is the interpolated gain,  $K_{c1}$  is the gain assigned to the closest cluster,  $K_{c2}$  is the gain assigned to the next closest cluster,  $d_1$  is the similarity of the closest pattern minus the cluster radius  $\rho$  and  $d_2$  is the similarity of the second closest pattern minus the cluster radius  $\rho$ .

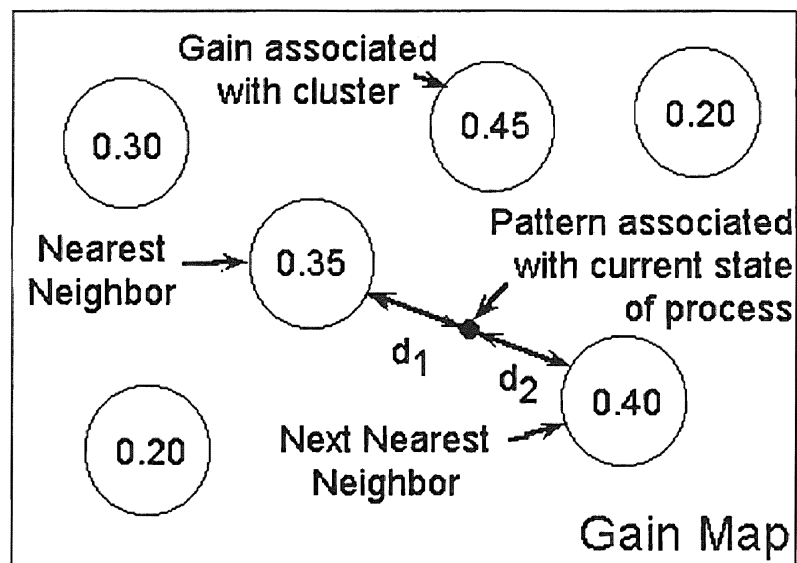


Figure 22: Interpolation between clusters

A disadvantage of this technique is that there is no weighting to favor the gain at the destination operating conditions. A weighting which favored the expected

operating state would potentially allow improved controller performance. As the process moves into the new operating state, weighting provides a method of giving the controller a gain more suitable to the new operating point. The chief advantage of linear interpolation is the ease of implementation.

Quadratic Interpolation The second method for interpolation employs quadratic interpolation. Similarity between the process pattern and the cluster is actually a quadratic function of the angle between the process sensor pattern and the cluster (Whiteley et. al., 1993]. The function used to find the quadratic distance between the process pattern and the learned pattern is

$$d_i = \sqrt{1.00000 - \rho} + \sqrt{\rho - S_i} \quad (26)$$

where  $d_i$  is the distance from the pattern to the cluster,  $\rho$  is the radius of the pattern cluster and  $S_i$  is the similarity of the cluster to the pattern.  $d_i$  is calculated and used in Equation 24 to determine the process gain.

The motivation for the quadratic method is to add additional weight to the gain found in the cluster the process is moving toward. The distance  $d_i$  initially favors the second nearest neighbor when interpolation begins. Thus, the gain of the expected new operating cluster has additional weight in the gain calculations. One drawback to

this approach is the gain of the previous nearest neighbor is favored as the process arrives at its new operating state.

### **Summary**

This chapter has looked at the development of the pattern-recognition gain scheduling approach for the mixing tank. Gain cluster selection was discussed as well as the method by which gains were calculated. Finally, the actual implementation was addressed. The following chapter will look at the results of the interpolation methods as well as the effects of changing several ART2 parameters on process control.

## CHAPTER VI

### RESULTS AND DISCUSSIONS

#### Pattern-Based Gain Scheduling Results

A variety of conditions were tested to examine the performance of the mixing tank using the pattern-based gain scheduling system. Among the issues investigated included using different numbers of clusters, varying cluster size, interpolation of the gains, and window length.

#### Number of Clusters

The number of clusters learned by the network and in scheduling the process system is important. The number of clusters used for scheduling depends on the nonlinearity of the process. In addition, the number of scheduling clusters also influences the interpolation of gains. Fewer clusters obviously require more interpolation for better control system performance.

If controller gain is assigned based on the gain of the nearest neighbor, the controller gain may not be the optimum gain for the current operating point when few clusters exist in the gain map. Assigning gains using the gain associated

with the most similar cluster to the current process pattern is called the winner technique. The effect of cluster number on the winner technique is important to control system operation. If the process is nonlinear, a situation may exist where improper or at least nonoptimum gains used by the controllers. The more clusters learned, the more likely the control system will have better gains to use.

Interpolation provides some benefits over the winner method when few gain clusters are used. Gain scheduling is normally implemented with some type of interpolation between the linearized points. This is done as the process will not necessarily operate precisely at the linearized points and linearizing the gains between these points will provide the appropriate gain at that situation. Interpolation allows the gain to change when the process is not inside a learned cluster. Thus, the gain will change to reflect current conditions. If the system is operating at the midpoint between two clusters, the gains are interpolated between those gains rather than assigned the gain of the nearest neighbor.

With the ART2 network, we don't have the luxury of linearizing the gain based on current operating values such as temperature or level. Instead, current process conditions are compared with a set of learned patterns. Information available at this point is the similarity of the process pattern and the learned patterns. The similarity

measure is a quadratic function of the angle between the process pattern and the learned prototypes.

Figures 23 and 24 show the results and gain change profiles for systems using linear and quadratic interpolation, respectively. These gain changes alter the process responses slightly to reflect the current operating conditions. This is beneficial if the system operates between the learned operating conditions. Sudden changes in the gains occur when the ART2 network has determined that the second nearest neighbor has changed. Thus, new values are introduced into the calculations which alters the gains used in the calculation. The effects of the similarity should be negligible at this point as the prototypes chosen as the second nearest neighbors have fairly similar similarity values.

The similarity is based on the vigilance  $\rho$  set by the user. When the winner method is used, vigilance has no influence on the gain selected. The gain selected is the gain associated with the nearest neighbor. When interpolation is desired, though, a measure of the similarity between the process pattern and all the learned prototypes is available. Vigilance enters when trying to decide the appropriate point that interpolation is needed. If the process pattern meets or exceeds the vigilance set, it is assumed that the learned cluster and the process conditions are nearly identical and that associated gain is appropriate for the conditions.



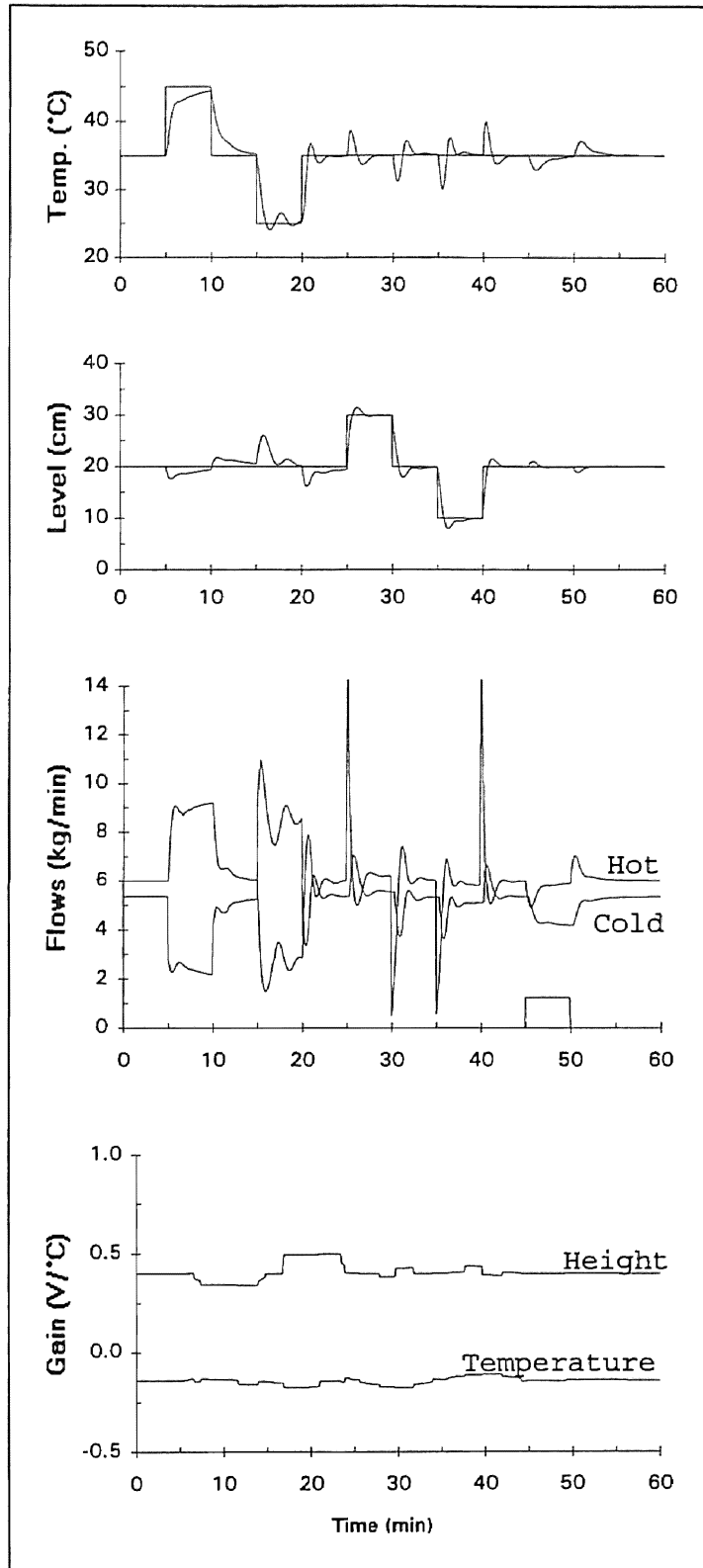


Figure 23: Linear interpolation with  $\rho = 0.999999$  and 9 clusters in the gain map

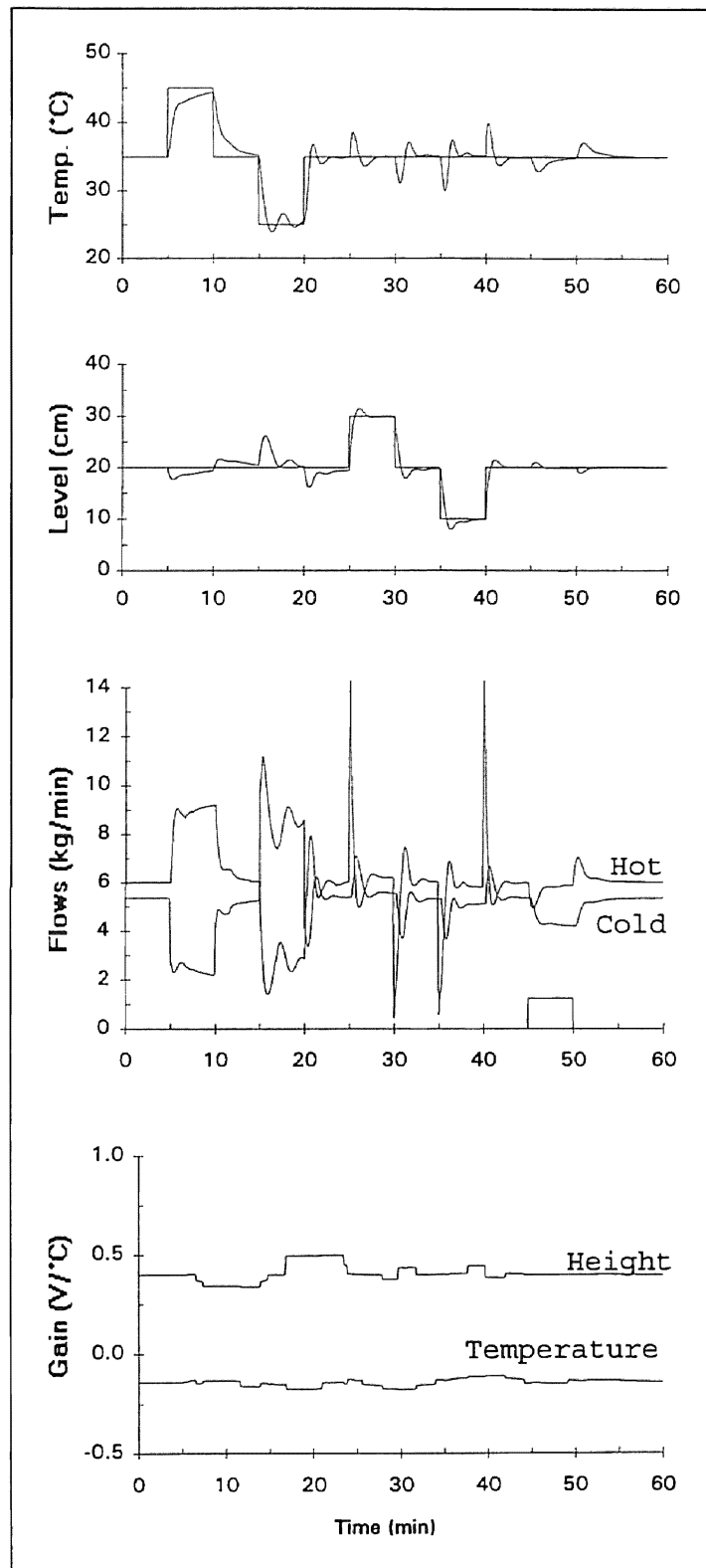


Figure 24: Quadratic interpolation with  $\rho = 0.999999$  and 9 clusters in the gain map

To investigate the influence of the number of clusters on the ART2 and gain scheduling, test runs were ran using gain maps containing 5, 9, 13 and 25 steady-state prototypes. Clusters were spaced at 5°C and 5 cm. intervals for the 9, 13 and 25 cluster runs. The vigilance parameter which determines the size of the clusters ranged in value from 0.99 to 0.999999. The 0.999999 value produced situations where interpolation was always required. Again, the cluster size had no influence for winner results.

An additional test was developed to examine the performance of the pattern-based gain scheduling system in response to load disturbances. This test differs from Haggblom's test (Figure 12) by subjecting the system to a variety of loads. The disturbance test looks at the ability of the system to handle changes in the disturbance feed rate and temperature. Figure 25 shows the disturbance test sequence. The disturbance feed rate is increased to 1.5 kg/hr initially. A second disturbance change increases the feed rate to 3.0 kg/hr. During this increased feed rate, the temperature of the disturbance stream is changed by  $\pm 5^{\circ}\text{C}$ .

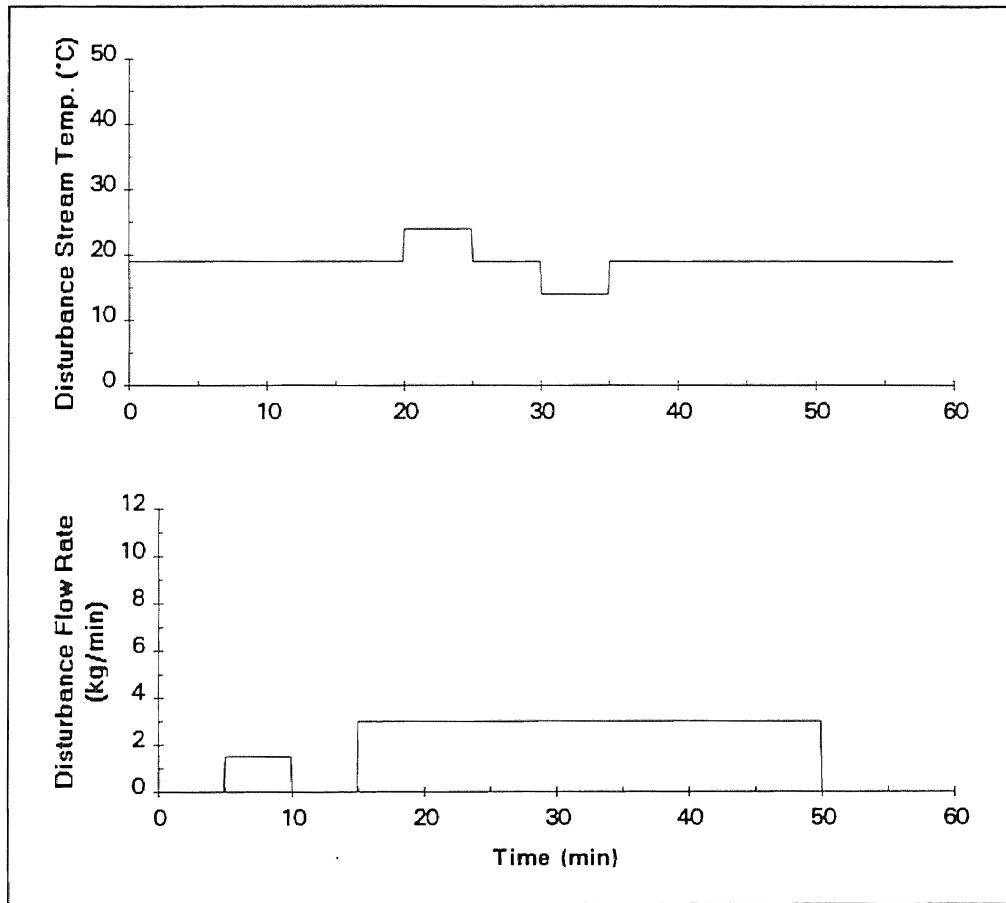


Figure 25: Disturbance test

## Evaluation Results

30°C Runs Tables XV and XVI show the results for the mixing tank at a nominal operating temperature of 30°C using different numbers of clusters. Figures 26 and 27 show the results for pattern-based gain scheduling using 25 clusters and the winner method. The '5+' represents the 5 clusters and gains selected in a cross centered on the operating point of 35°C and 20 cm. The other points used for the '5+' runs are shown in Table XVII. Tables XVIII, XIX, XX and XXI contain the gains and steady-state operating points used to create the gain map with differing number of clusters.

TABLE XVI

RESULTS FOR OPERATIONS AT 30°C AND WINNER METHOD

Number of Clusters	Setpoint Test		Disturbance Test	
	IAE <sub>H</sub>	IAE <sub>T</sub>	IAE <sub>H</sub>	IAE <sub>T</sub>
5 +	43.35	53.34	12.68	28.52
5 X	45.63	56.71	13.08	31.45
9	43.32	53.66	12.68	28.52
13	43.05	56.64	12.68	28.52
25	42.59	56.35	12.68	28.52
Hagblom	114.76	101.99	28.30	26.43

TABLE XVII

RESULTS FOR OPERATIONS AT 30°C AND LINEAR INTERPOLATION  
METHOD,  $\rho = 0.99992$

Number of Clusters	Setpoint Test		Disturbance Test	
	IAE_H	IAE_T	IAE_H	IAE_T
5 +	44.10	53.05	12.77	28.79
5 X	43.82	53.51	12.96	30.31
9	43.47	53.56	12.74	28.63
13	42.88	55.43	12.66	28.55
25	43.14	56.28	12.66	28.55

TABLE XVII

GAINS USED FOR THE '5+' CLUSTER RUN

Steady-State Operating Point	Height Gain	Temperature Gain
35 °C - 15 cm	0.41	-0.11
30 °C - 20 cm	0.50	-0.16
35 °C - 20 cm	0.40	-0.14
40 °C - 20 cm	0.34	-0.29
35 °C - 25 cm	0.40	-0.17

TABLE XVII

GAINS USED FOR THE '5X' CLUSTER RUN

Steady-State Operating Point	Height Gain	Temperature Gain
30 °C - 15 cm	0.50	-0.12
40 °C - 15 cm	0.35	-0.23
35 °C - 20 cm	0.40	-0.14
30 °C - 25 cm	0.49	-0.19
40 °C - 25 cm	0.34	-0.35

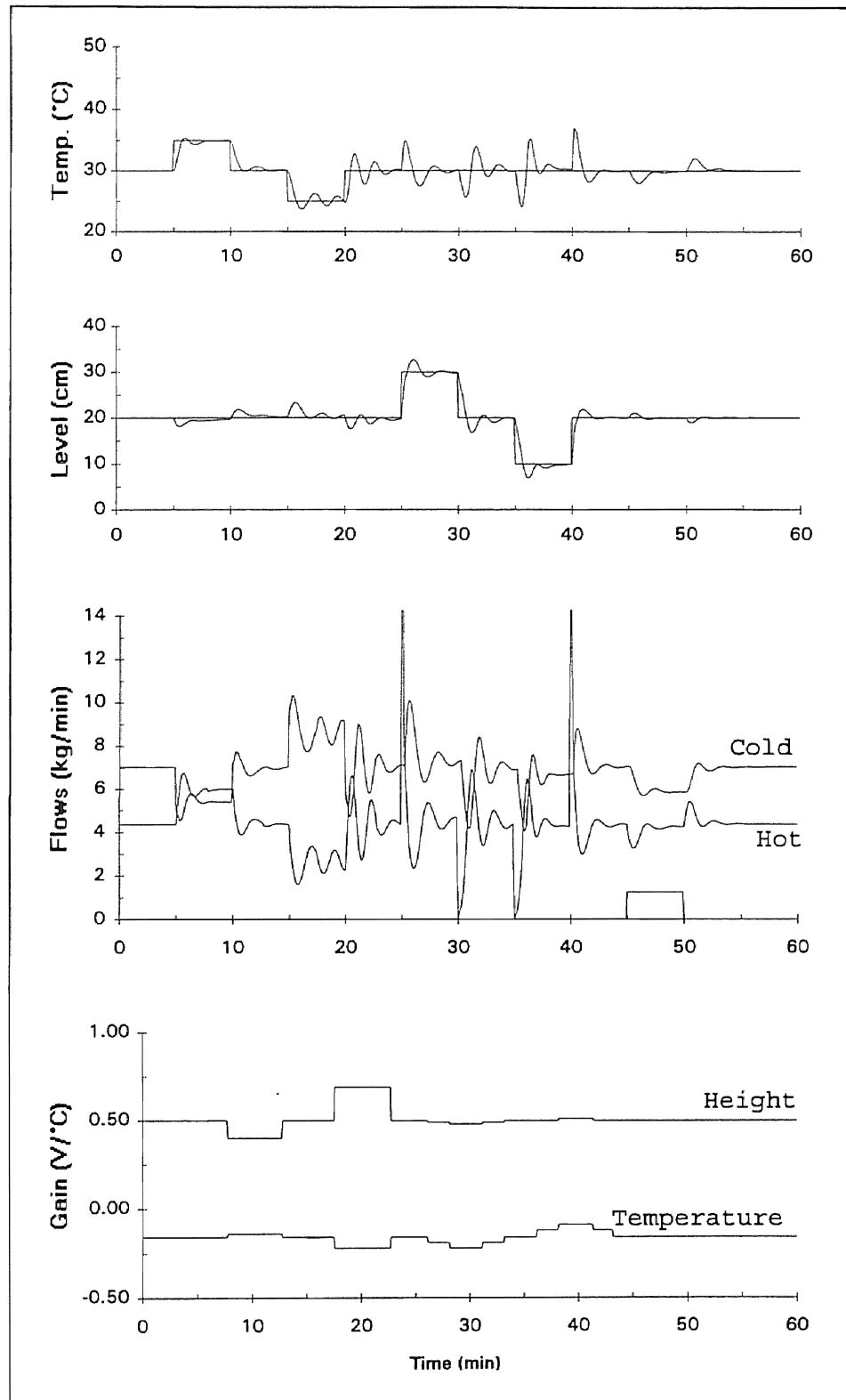


Figure 26: Results of setpoint test using 25 clusters at 30°C and the winner method

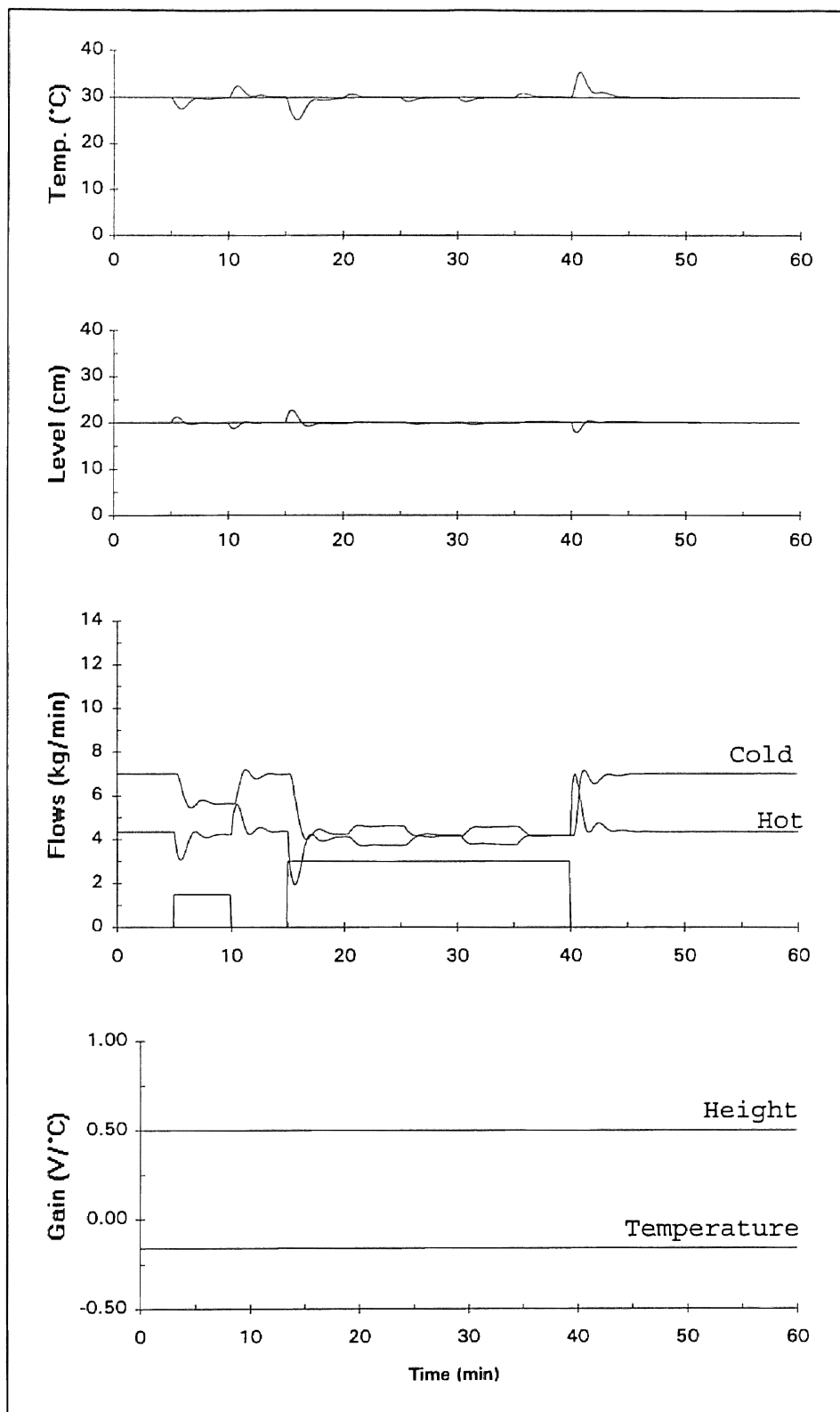


Figure 27: Results of disturbance test with 25 clusters at 30°C and the winner method



TABLE XIX

GAINS USED FOR THE 9 CLUSTER RUN

Steady-State Operating Point	Height Gain	Temperature Gain
30 °C - 15 cm	0.50	-0.12
35 °C - 15 cm	0.41	-0.11
40 °C - 15 cm	0.35	-0.23
30 °C - 20 cm	0.50	-0.16
35 °C - 20 cm	0.40	-0.14
40 °C - 20 cm	0.35	-0.29
30 °C - 25 cm	0.49	-0.19
35 °C - 25 cm	0.40	-0.17
40 °C - 25 cm	0.34	-0.35

TABLE XX

GAINS USED FOR THE 13 CLUSTER RUN

Steady-State Operating Point	Height Gain	Temperature Gain
35 °C - 10 cm	0.41	-0.08
30 °C - 15 cm	0.50	-0.12
35 °C - 15 cm	0.41	-0.11
40 °C - 15 cm	0.35	-0.23
25 °C - 20 cm	0.69	-0.22
30 °C - 20 cm	0.50	-0.16
35 °C - 20 cm	0.40	-0.14
40 °C - 20 cm	0.35	-0.29
45 °C - 20 cm	0.30	-0.40
30 °C - 25 cm	0.49	-0.19
35 °C - 25 cm	0.40	-0.17
40 °C - 25 cm	0.34	-0.35
35 °C - 30 cm	0.39	-0.20

TABLE XXI

GAINS USED FOR THE 25 CLUSTER RUN

Steady-State Operating Point	Height Gain	Temperature Gain
25 °C - 10 cm	0.70	-0.12
30 °C - 10 cm	0.51	-0.09
35 °C - 10 cm	0.41	-0.08
40 °C - 10 cm	0.35	-0.16
45 °C - 10 cm	0.31	-0.22
25 °C - 15 cm	0.70	-0.17
30 °C - 15 cm	0.50	-0.12
35 °C - 15 cm	0.41	-0.11
40 °C - 15 cm	0.35	-0.23
45 °C - 15 cm	0.30	-0.31
25 °C - 20 cm	0.69	-0.22
30 °C - 20 cm	0.50	-0.16
35 °C - 20 cm	0.40	-0.14
40 °C - 20 cm	0.35	-0.29
45 °C - 20 cm	0.30	-0.40
25 °C - 25 cm	0.68	-0.27
30 °C - 25 cm	0.49	-0.19
35 °C - 25 cm	0.40	-0.17
40 °C - 25 cm	0.34	-0.35
45 °C - 25 cm	0.30	-0.49
25 °C - 30 cm	0.67	-0.31
30 °C - 30 cm	0.48	-0.22
35 °C - 30 cm	0.39	-0.20
40 °C - 30 cm	0.33	-0.41
45 °C - 30 cm	0.29	-0.57

TABLE XXII

RESULTS FOR OPERATIONS AT 30°C AND QUADRATIC INTERPOLATION  
METHOD,  $\rho = 0.99992$

Number of Clusters	Setpoint Test		Disturbance Test	
	IAE_H	IAE_T	IAE_H	IAE_T
5 +	44.38	52.82	12.87	29.01
5 X	43.80	53.31	12.95	30.26
9	43.51	53.55	12.74	28.63
13	42.87	55.23	12.66	28.61
25	43.27	56.40	12.66	28.61

As Tables XV and XVI show, the IAE scores for the height and temperature controllers do not change significantly with the number of clusters. For the setpoint change (Hagglblom's) test, the height IAE score varies by 3.4% with the lowest score occurring when 25 clusters are used. These 25 clusters (Figures 26 and 27) effectively cover every possible operating point for the system. The height IAE scores generally decrease as the number of clusters used for pattern recognition increases. Yet, the temperature IAE scores are generally lowest when the number of clusters is kept to a minimum. This trend is supported whether the winner method or interpolation (Tables XXII and XXIII) is used.

The '5+' and '5X' represent a special case where clusters are chosen to determine the effect of different maps using the same number of clusters. The '5+' set contains points that match steady-state operating points

used in the setpoint tracking test. The '5X' set requires the neural network to decide which cluster to use that best represents the current operating point. Thus, the control system does not have the gains matching typical operating conditions. Performance should degrade and this is evident in the results for the winner method. Yet, interpolation reduced the IAE scores when it was employed with the '5X' set. This shows that interpolation is indeed beneficial.

Figure 26 shows the results of the mixing tank running at 30°C with 25 clusters. This control system works far better than the results shown in Figure 13 using Haggblom's fixed-gain controllers.

Figure 27 brings up an important point. The setpoints do not change in this figure. Disturbances are introduced to the mixing tank to gauge the response of the control system using gains associated with the nominal operating point. Unless the change in the system has a long lasting effect on the system (i.e. the control system cannot return it to setpoint) the pattern recognition system most likely will not send new gains to the controllers. Another reason that disturbances do not change the gain is that patterns used for scheduling do not include disturbance variables.

35°C Runs Besides operating the mixing tank where the control system is not properly paired, tests were conducted where either controller was suitable for controlling the height or temperature. At 35°C, Tables XXIII, XXIV and XXV

show that the '5X' set provides poor performance compared to other sets. The IAE scores are again higher than those of set '5+'. Unlike the runs at 30°C, both the height and temperature IAE scores decrease as the number of clusters increase.

The differences in scores between the winner method and the interpolation schemes are small in Tables XXIII, XXIV, and XXV. This trend has been noticed in all runs used to compare the three methods for assigning gains. It is apparent that the disturbance response is not affected by the choice of gain scheduling interpolation.

TABLE XXIII

RESULTS FOR OPERATIONS AT 35°C AND WINNER METHOD

Number of Clusters	Setpoint Test		Disturbance Test	
	IAE_H	IAE_T	IAE_H	IAE_T
5 +	52.24	72.82	13.18	41.28
5 X	51.74	75.24	13.09	41.47
9	52.24	72.84	13.18	41.28
13	51.65	70.87	13.18	41.28
25	51.65	70.87	13.18	41.28
Hagblom	57.37	45.92	17.31	21.07

TABLE XXIV

RESULTS FOR OPERATIONS AT 35°C AND LINEAR INTERPOLATION  
METHOD,  $\rho = 0.99992$

Number of Clusters	Setpoint Test		Disturbance Test	
	IAE_H	IAE_T	IAE_H	IAE_T
5 +	52.55	73.14	13.18	41.12
5 X	51.77	73.73	13.19	41.76
9	52.20	73.67	13.18	41.12
13	51.65	70.73	13.18	41.12
25	51.55	70.76	13.18	41.12

TABLE XXV

RESULTS FOR OPERATIONS AT 35°C AND QUADRATIC INTERPOLATION  
METHOD,  $\rho = 0.99992$

Number of Clusters	Setpoint Test		Disturbance Test	
	IAE_H	IAE_T	IAE_H	IAE_T
5 +	52.63	73.42	13.20	41.11
5 X	51.91	73.87	13.24	41.89
9	52.21	73.90	13.20	41.11
13	51.74	70.97	13.20	41.11
25	51.59	71.02	13.18	41.11

Figures 28 and 29 display the results of the setpoint and disturbance tests using the winner method and 25 clusters. The setpoint test had good results in minimizing the error between the setpoint and actual operating conditions. Height setpoint changes are particularly good. However, the temperature setpoint change from 35°C to 45°C is sluggish. Since gains must be chosen to provide stable,

low oscillatory setpoint changes in this study, the temperature controller gain does not provide the right dynamics to move the system to its new setpoint rapidly. On the other hand, this same gain causes an oscillatory response when going from 35°C to 25°C. Since the process system is nonlinear, an optimum controller gain does not exist at a single operating point.

In Figure 29, the system suffers a slow return of the tank temperature to setpoint after the disturbance stream is turned on and turned off. With the disturbance stream flowing at 3 kg/min, the ART2 network recognizes a pattern change and issues new gains. The effect is temporary though. As the system returns to setpoint, ART2 returns the system to the previous steady-state gains. The mixing tank does not destabilize with this operation.

The gain scheduling system handles operations where either controller is suitable for controlling the height or temperature. Despite the low temperature gains, the results are favorable.

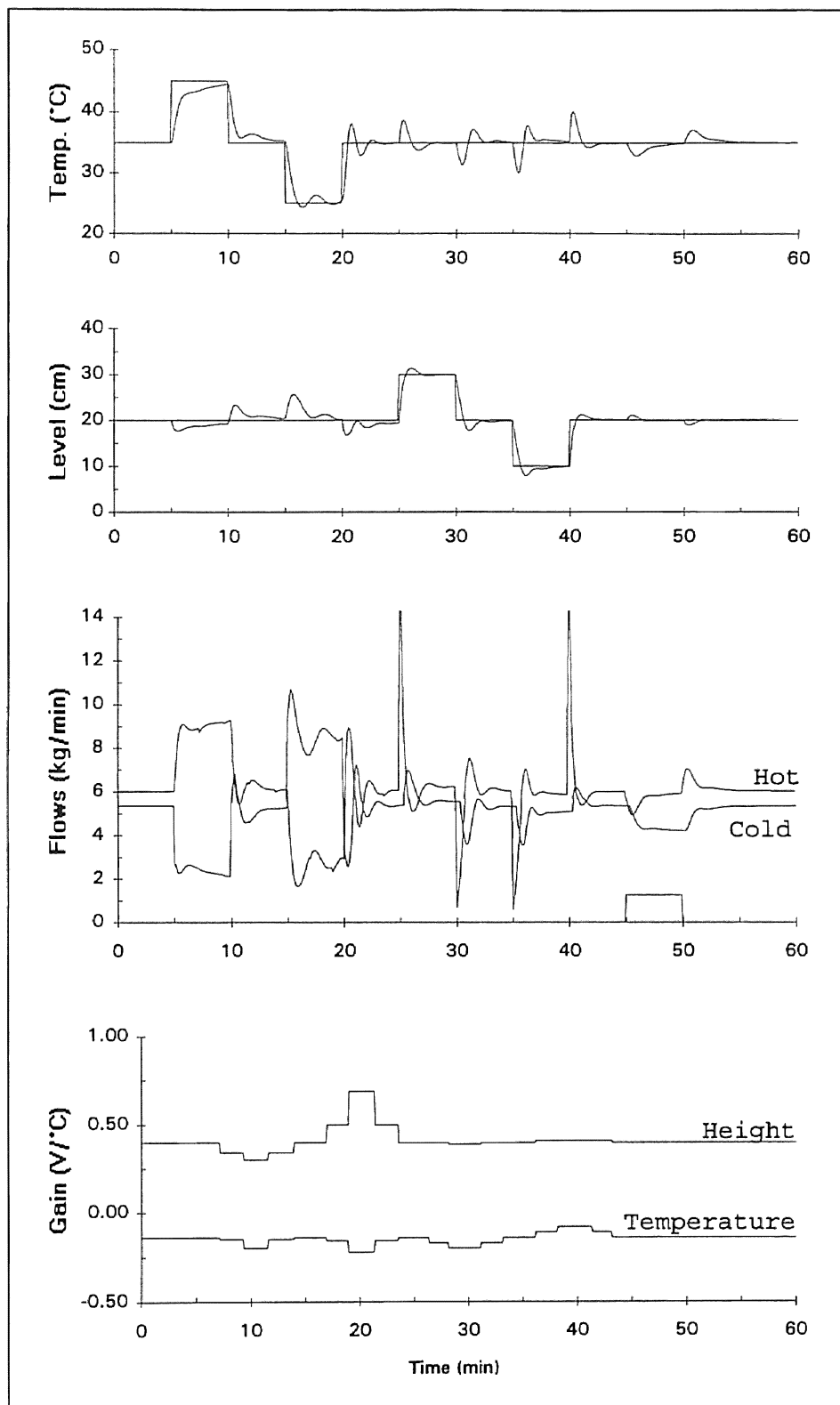


Figure 28: Results of setpoint test using 25 clusters at 35°C and the winner method



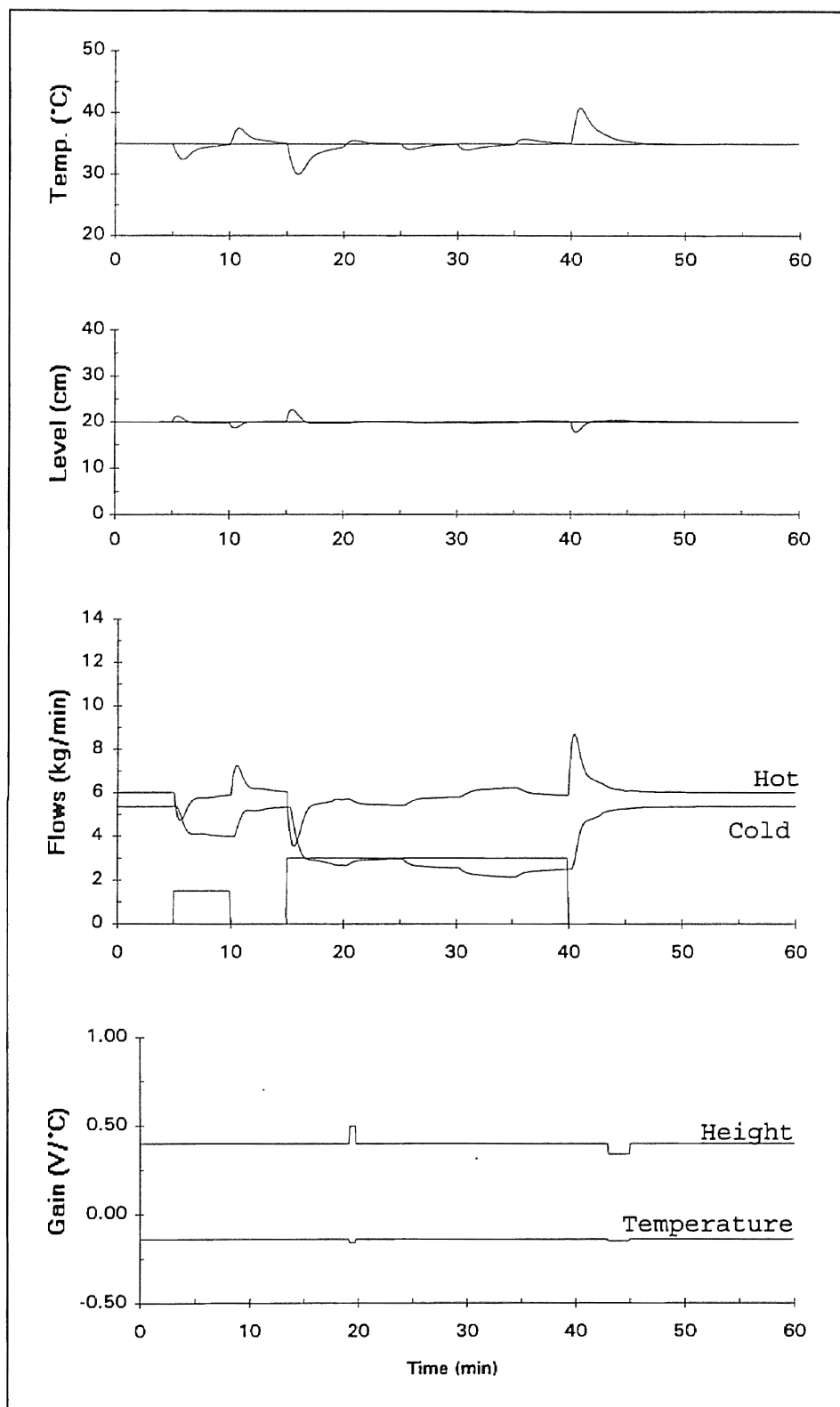


Figure 29: Results of disturbance test with 25 clusters at 35°C and the winner method

40°C Runs Finally, runs were made to determine the effect of operating the system where the controllers and manipulated variables are optimally paired. The results of the gain scheduling tests are presented in Tables XXVI, XXVII and XXVIII. Figures 30 and 31 display the results using 25 clusters in the gain map and the winner method for gain scheduling.

Similar to the 30°C runs, the height IAE scores decrease as the number of clusters decrease. The IAE scores for the temperature controller are better with more clusters. The temperature trends are repeated with the results for the disturbance tests as well.

Comparing the results of the pattern-based gain scheduling approach and Haggblom's fixed gain system (Table XXVI), the pattern-based gain scheduling approach lowers the IAE score on the height controller. Yet, the temperature performance is not as good. While lower IAE scores are possible for one controller, it is possible that the other controller scores will rise.

The differences between the '5+' and '5X' results are very pronounced for the winner method. Since the gains for the '5X' set do not correspond to expected operating points of the system, performance is a little worse. The temperature IAE scores are lower when interpolation is used in the setpoint tests. The temperature IAE disturbance test results with the '5X' are significantly higher than the other values shown in Tables XXVI, XXVII and XXVIII.

It is apparent that the '5X' configuration is not the best for forming the gain map. Clusters should be chosen based on those that reflect the normal operating points of the system. If clusters are chosen that do not represent normal operating conditions, the performance of the system suffers. Interpolation will help in these situations, but the performance is not optimal. A reason for these results is the coupling that exists in the control system.

TABLE XXVI

RESULTS FOR OPERATIONS AT 40°C AND WINNER METHOD

Number of Clusters	Setpoint Test		Disturbance Test	
	IAE_H	IAE_T	IAE_H	IAE_T
5 +	35.86	50.17	15.01	61.40
5 X	35.31	52.94	15.21	67.22
9	35.94	50.25	15.01	61.40
13	36.39	48.59	15.00	58.05
25	36.45	48.60	15.00	58.05
Hagblom	53.25	32.74	17.88	34.49

TABLE XXVII

RESULTS FOR OPERATIONS AT 40°C AND LINEAR INTERPOLATION METHOD,  $\rho = 0.99992$ 

Number of Clusters	Setpoint Test		Disturbance Test	
	IAE_H	IAE_T	IAE_H	IAE_T
5 +	35.41	50.32	14.91	61.73
5 X	35.38	51.07	14.86	62.69
9	35.81	50.48	15.17	62.95
13	36.33	48.55	14.86	58.44
25	36.43	48.49	14.89	58.43

TABLE XXVIII

RESULTS FOR OPERATIONS AT 40°C AND QUADRATIC INTERPOLATION  
METHOD,  $\rho = 0.99992$

Number of Clusters	Setpoint Test		Disturbance Test	
	IAE_H	IAE_T	IAE_H	IAE_T
5 +	35.19	50.42	14.76	61.98
5 X	35.34	51.07	14.83	62.35
9	35.81	50.53	15.17	63.18
13	36.32	48.62	14.82	58.51
25	36.44	48.54	14.87	58.45

Figures 30 and 31 show the results for the mixing tank at 40°C using the setpoint and disturbance tests, respectively. As expected, with proper coupling, the system responses are quick and show minimal deviation from setpoint. The most significant feature shown in the graphs is the slow response of the temperature controller to return the system to setpoint. Again, the gains chosen for gain scheduling limit the response of the system as the gains must be selected to handle several setpoint and load changes. The temperature controller has a difficult time moving the system to new temperature setpoints. But, the settings for height setpoint changes lead to quick responses.

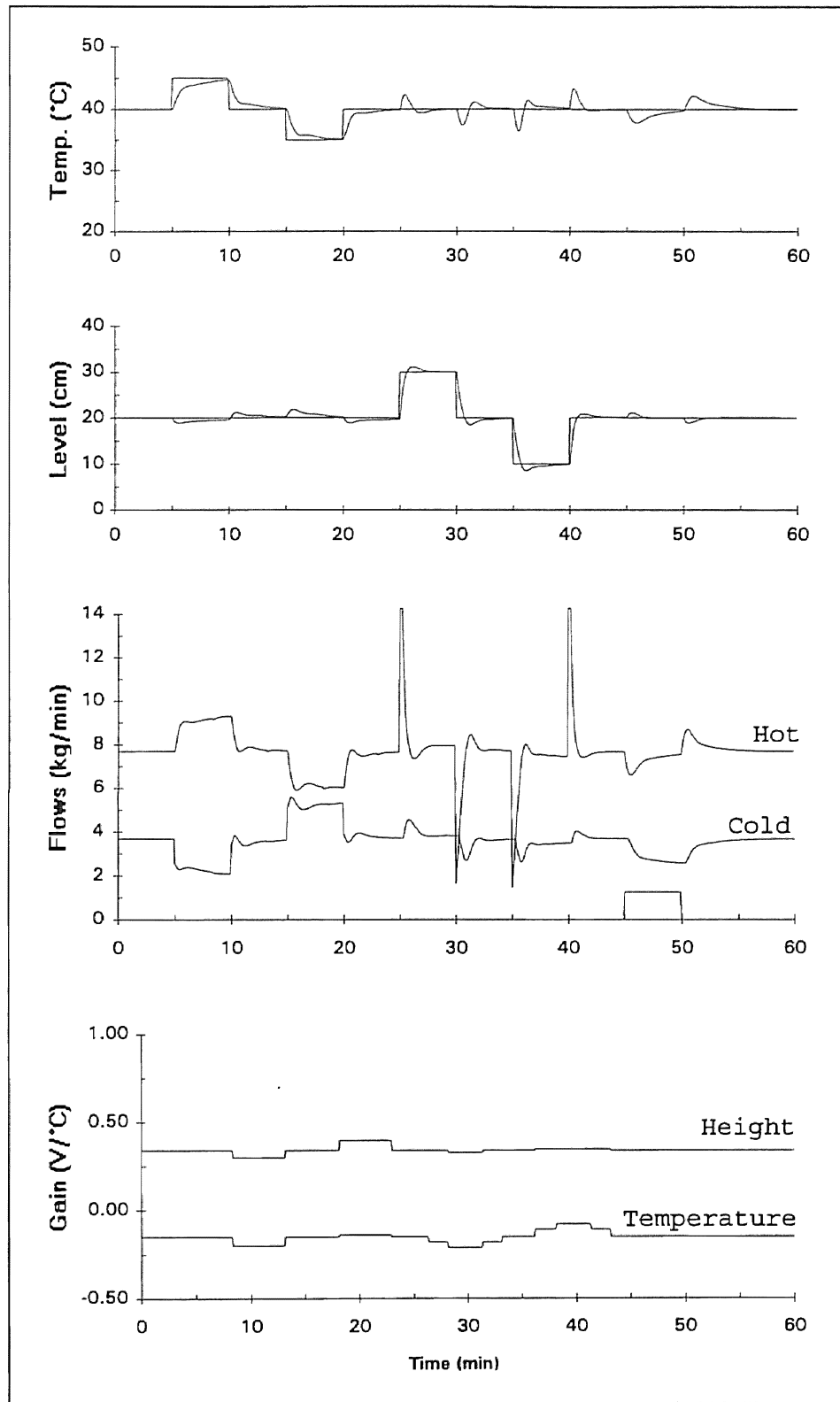


Figure 30: Results of setpoint test using 25 clusters at 40°C and the winner method

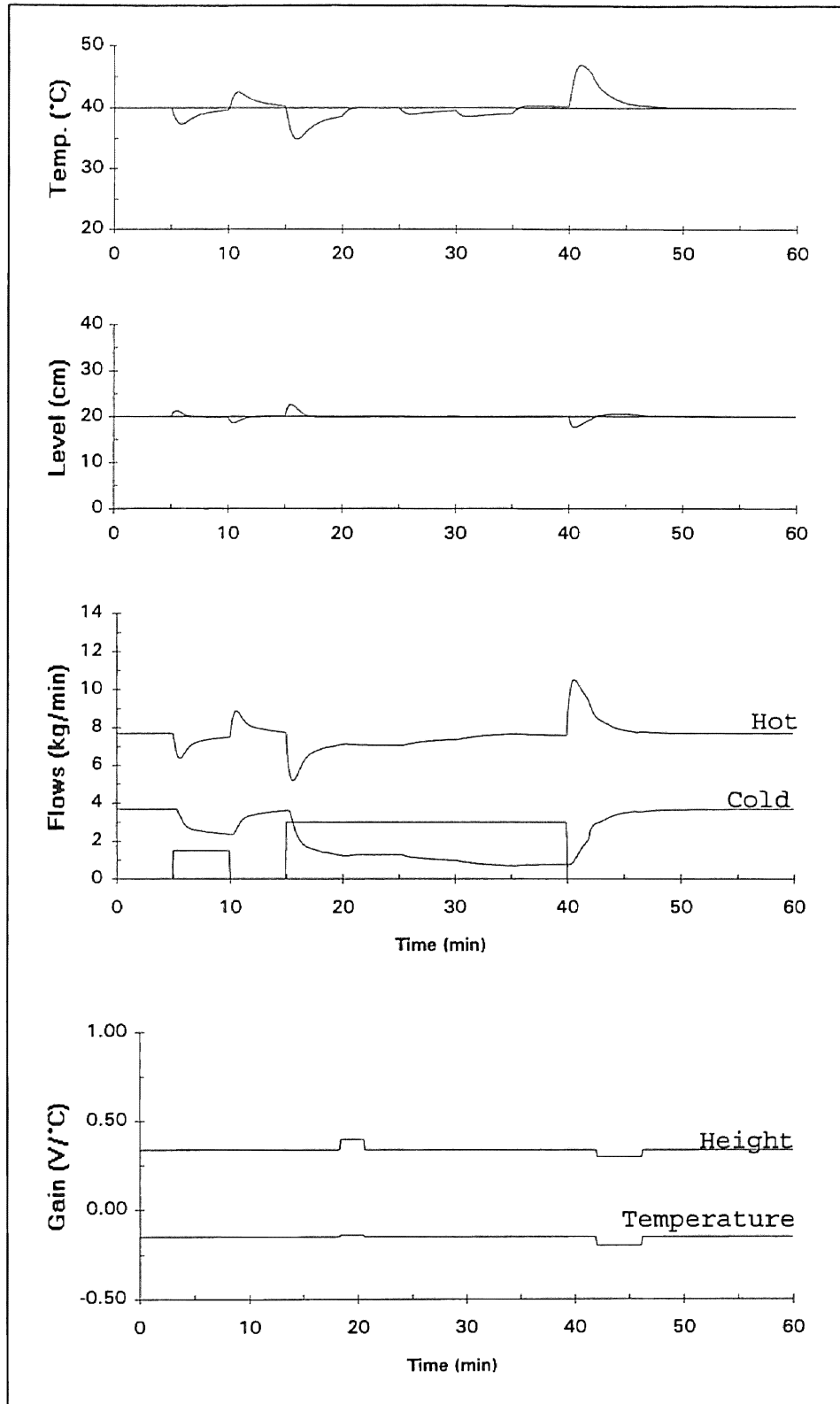


Figure 31: Results of disturbance test with 25 clusters at 40°C and the winner method

The disturbance tests show that some gain scheduling was done. The gain change lasts for a longer time than that shown in Figure 29 at 35°C. The major draw back is the slow return to the temperature setpoint. Height responses are very good and have little deviation. The pattern-based gain scheduling approach reduced the height IAE scores by several points. The temperature control performance was worse for gain scheduling, though. Again, this is attributed to the gains selected for the gain map. Higher temperature gains would reduce the deviation.

#### Conclusions on the number of clusters

Based on the studies at 30°C, 35°C and 40°C, the more clusters used to schedule the control system, the better the temperature performance. The results for the height controller using different numbers of clusters are mixed. The height IAE scores do not vary more than 3.4% as the number of clusters is changed. If the temperature is the important variable in the process, more gain scheduling clusters are recommended. Finally, gain scheduling clusters should lie on or near normal or expected operating points. As shown with the '5+' and '5X' sets, centering clusters outside the normal operating range of the process produced slightly poorer performance.

### Cluster Size

Tests were run to look at the influence of cluster size on the IAE scores for the height and temperature controllers. Cluster size is important in determining the transition points as the process moves from one state to another. Cluster size is changed by altering the ART2 vigilance parameter  $\rho$ . A large cluster refers to vigilances of 0.999 or lower. If the cluster vigilance is above 0.9999, the cluster has a small radius.

For the winner method, cluster size holds no meaning. The gains chosen for the controllers are based on the nearest neighbor of the process pattern. Thus gain changes only occur when a new nearest neighbor occurs.

Cluster size is important when interpolating.  $\rho$  determines when the system needs to interpolate the gains between the two nearest neighbors of the process pattern. With high values of  $\rho$ , interpolation occurs quickly when the process pattern differs from the learned prototype. Interpolation occurs when the similarity between the process pattern and the nearest neighbor is less than  $\rho$ . When  $\rho$  is set to 0.99, the cluster radii are huge and all clusters overlap. Thus, no interpolation occurs. The result is the winner method.

The cluster size also plays a factor in the interpolation methods as a calculation tool. In Equations 24 and 25, the vigilance determines the distance between the



process pattern and the nearest neighbor. Thus,  $\rho$  influences the gap between the clusters which in turns determines the value of the gain sent to the controller.

The results for interpolation are shown in Table XXIX for the three nominal operating temperatures. By comparing this value with the results for different cluster sizes and either interpolation method, using the nearest neighbor ( $\rho = 0.99$ ) as the gaining cluster produces comparable results to schemes using interpolation. In general, the best temperature IAE scores occurred when the cluster size was large (i.e. the vigilance was equal to or less than 0.999). The height IAE results also decreased with larger cluster radii.

Comparing the linear and quadratic interpolation results shows that linear interpolation produced smaller IAE scores for the temperature controller. The height controller also favors the linear interpolation technique in most cases. Yet, the results between the two interpolation schemes show that the scores differ only slightly. This difference is not readily detectable in an operating plant environment. In fact, the winner method produces similar results as well. Thus, the choice of which interpolation to method to use or whether to use one at all cannot be readily settled.

TABLE XXIX

RESULTS FOR VARYING CLUSTER SIZES USING THE  
SETPOINT TEST AND 9 CLUSTERS

Nominal Operating Temp.	Vigilance $\rho$	Linear Interpolation		Quadratic Interpolation	
		IAE_H	IAE_T	IAE_H	IAE_T
30°C	0.99	35.94	50.25	35.94	50.25
	0.999	35.82	50.44	35.81	50.51
	0.9999	35.81	50.48	35.81	50.54
	0.99999	35.82	50.47	35.81	50.49
	0.999999	35.82	50.46	35.81	50.49
35°C	0.99	43.32	53.66	43.32	53.66
	0.999	43.42	53.55	43.41	53.52
	0.9999	43.47	53.56	43.50	53.53
	0.99999	43.49	53.60	43.57	53.71
	0.999999	43.49	53.60	43.59	53.77
40°C	0.99	52.24	72.84	52.24	72.84
	0.999	52.17	73.44	52.15	73.56
	0.9999	52.19	73.66	52.21	73.88
	0.99999	52.20	73.69	52.08	74.00
	0.999999	52.20	73.70	52.21	74.04

Table XXX contains results using the disturbance test to determine the effects of changing the cluster size. The best results occur when the winner method is assigning gains to the system. As the mixing tanks operating temperature increases, the effect of the changes in cluster size diminish. This may be an indication that our controller gain selection becomes more important as the system enters an area where the controllers are not paired correctly.

TABLE XXX

RESULTS FOR VARYING CLUSTER SIZES USING THE DISTURBANCE TEST, 4 MINUTE WINDOWS AND 9 CLUSTERS

Nominal Operating Temp.	Vigilance $\rho$	Linear Interpolation		Quadratic Interpolation	
		IAE_H	IAE_T	IAE_H	IAE_T
30°C	0.99	15.01	61.40	15.01	61.40
	0.99992	15.67	62.95	15.17	63.18
	0.999999	15.16	62.97	15.14	63.27
35°C	0.99	12.68	28.52	12.68	28.52
	0.99992	12.74	28.63	12.75	28.70
	0.999999	12.74	28.66	12.75	28.87
40°C	0.99	13.18	41.28	13.18	41.28
	0.99992	13.18	41.12	13.20	41.11
	0.999999	13.18	41.14	13.17	41.25

### Window Length

Another influence on the pattern-based gain scheduling system is the window length. Window length influences pattern-recognition primarily, yet it also determines how quickly the gains change to the controllers.

Window length is the time period the ART2 network looks at when analyzing patterns. In this study, 1, 2, 3, 4 and 6 minute windows were investigated. Window length influences pattern recognition in our case as it determines how quickly the system realizes that a new steady-state operating point has been reached. In addition, window length also determines when disturbances have entered the system and how to handle the situation.

A number of benefits are derived from using shorter windows with the mixing tank. First, the pattern-recognition system determines that a new steady-state operating point has been reached in a shorter amount of time. Thus, new gains are applied to the controller quickly that correspond to the new operating conditions. Second, it takes less time for the system to detect that a change in the system has occurred. Since a change in the process takes longer to affect a neural network taught for long periods of time, a change in the process is dampened.

A drawback to using short windows, though, is the fact that a disturbance may cause a change in the process that changes the gains. Depending on the gains at this situation, system response may become less stable. Yet, the possibility exists that better gains may also be substituted into the system and drive the system back to setpoint faster. Thus, the dynamics of the system are important in deciding the length of the pattern window.

Tables XXXI and XXXII contain the results for varying window size and cluster size for the linear and quadratic methods, respectively, using the setpoint test developed by Haggblom. Tables XXXIII and XXXIV are the results using our disturbance test for the linear and quadratic interpolation methods.

TABLE XXXI

RESULTS USING VARYING WINDOWS LENGTHS, THE SETPOINT TEST AND  
LINEAR INTERPOLATION AND 9 CLUSTERS

Nom. Temp.	Vigilance $\rho$	1 minute		2 minute		3 minute		4 minute		6 minute	
		IAE_H	IAE_T	IAE_H	IAE_T	IAE_H	IAE_T	IAE_H	IAE_T	IAE_H	IAE_T
30°C	0.99	44.49	53.21	43.83	53.60	43.49	53.39	43.32	53.66	43.17	53.73
	0.999			43.79	53.28			43.42	53.55	43.17	53.65
	0.9999			43.82	53.27			43.47	53.56	43.19	53.79
	0.99999			43.83	53.29			43.49	53.60	43.19	53.82
	0.999999	44.47	53.21	43.83	53.30	42.73	53.62	43.49	53.60	43.19	53.82
35°C	0.99	52.58	74.13	52.42	73.00	52.26	72.78	52.24	72.84	52.80	73.39
	0.999			52.19	73.54			52.17	73.44	52.66	74.00
	0.9999			52.21	73.71			52.19	73.66	52.66	74.15
	0.99999			52.22	73.73			52.20	73.69	52.66	74.18
	0.999999	52.47	74.27	52.22	73.74	52.22	73.67	52.20	73.70	52.66	74.19
40°C	0.99	35.79	50.04	35.82	50.10	35.91	50.16	35.94	50.25	35.99	50.26
	0.999			35.74	50.37			35.82	50.44	35.94	50.44
	0.9999			35.73	50.48			35.81	50.48	35.99	50.42
	0.99999			35.73	50.49			35.82	50.47	35.99	50.44
	0.999999	35.58	50.28	35.73	50.50	35.77	50.46	35.82	50.46	36.00	50.44

TABLE XXXII

RESULTS USING VARYING WINDOWS LENGTHS, THE SETPOINT TEST AND QUADRATIC INTERPOLATION AND 9 CLUSTERS

Nom. Temp.	Vigilance $\rho$	1 minute		2 minute		3 minute		4 minute		6 minute	
		IAE_H	IAE_T	IAE_H	IAE_T	IAE_H	IAE_T	IAE_H	IAE_T	IAE_H	IAE_T
30°C	0.99	44.49	53.21	43.83	53.60	43.49	53.39	43.32	53.66	43.17	53.73
	0.999			43.78	53.22			43.41	53.52	43.19	53.64
	0.9999			43.83	53.25			43.50	53.53	43.23	53.92
	0.99999			43.87	53.36			43.57	53.71	43.25	54.01
	0.999999	44.51	53.49	43.88	53.40	43.82	53.76	43.59	53.77	43.25	54.03
35°C	0.99	52.58	74.13	52.23	73.94	52.26	72.78	52.24	72.84	52.80	73.39
	0.999			52.23	73.91			52.15	73.56	52.64	74.09
	0.9999			52.22	73.84			52.21	73.88	52.65	74.40
	0.99999			52.42	73.59			52.08	74.00	52.65	74.51
	0.999999	52.43	74.41	52.42	73.00	52.25	73.94	52.21	74.04	52.65	74.55
40°C	0.99	35.79	50.04	35.82	50.10	35.91	50.16	35.94	50.25	35.99	50.26
	0.999			35.73	50.41			35.81	50.51	35.93	50.46
	0.9999			35.72	50.62			35.81	50.54	36.02	50.50
	0.99999			35.71	50.64			35.81	50.49	36.02	50.58
	0.999999	35.54	50.56	35.71	50.65	35.76	50.53	35.81	50.49	36.02	50.61

TABLE XXXIII

RESULTS USING VARYING WINDOWS LENGTHS, THE DISTURBANCE TEST AND LINEAR INTERPOLATION AND 9 CLUSTERS

Nom. Temp.	Vigilance $\rho$	2 minute		4 minute	
		IAE_H	IAE_T	IAE_H	IAE_T
30°C	0.99	12.73	28.72	12.68	28.52
	0.99992	12.74	28.72	12.74	28.63
	0.999999	12.74	28.75	12.74	28.66
35°C	0.99	13.13	40.74	13.18	41.28
	0.99992	13.11	40.85	13.18	41.12
	0.999999	13.10	40.88	13.18	41.14
40°C	0.99	15.08	61.64	15.01	61.40
	0.99992	15.29	62.98	15.67	62.95
	0.999999	15.28	62.99	15.16	62.97

TABLE XXXIV

RESULTS USING VARYING WINDOWS LENGTHS, THE DISTURBANCE TEST  
AND QUADRATIC INTERPOLATION AND 9 CLUSTERS

Nom. Temp.	Vigilance $\rho$	2 minute		4 minute	
		IAE_H	IAE_T	IAE_H	IAE_T
30°C	0.99	12.73	28.72	12.68	28.52
	0.99992	12.73	28.78	12.75	28.70
	0.999999	12.72	28.94	12.75	28.87
35°C	0.99	13.13	40.74	13.18	41.28
	0.99992	13.10	40.92	13.20	41.11
	0.999999	13.06	41.07	13.17	41.25
40°C	0.99	15.08	61.64	15.01	61.40
	0.99992	15.27	63.06	15.17	63.18
	0.999999	15.23	63.09	15.14	63.27

The results in the tables indicate that shorter pattern windows decrease the IAE scores for the controllers. This is especially true for operations at 35°C and 40°C. The shorter windows cause gain changes to take place quicker and thus providing gains best suited for the new operating conditions.

In Tables XXXI and XXXII running the tank at 30°C produces opposite results. Better performance numbers (i.e., lower IAE scores) result for the height controller when longer windows are used. Temperature results improve when the windows are shorter. A possible answer for this occurrence is that the gains for the height controller are not the best. The more likely reason for the discrepancy is that a trade-off is occurring between the height and

temperature controllers. As found in the optimized gain studies, the IAE score for one controller can be lowered at the expense of the other. The improved control at 30°C could be causing this.

Yet, as the window lengths do decrease, the IAE scores in the setpoint tests decrease. The disturbance test results present a different conclusion. In a few cases, using 2 minute windows generates better IAE scores than those of the 4 minute windows. In Table XXXIII at 30°C, the 4 minute window results are slightly lower than the 2 minute window results. Yet, at 35°C, the IAE scores are lowest for the 2 minute windows. One thing to note in these charts is the slight change in the values. The 2 minute and 4 minute results differ only slightly, thus either window size can be used.

Another feature to extract from Tables XXXI, XXXII, XXXIII, and XXXIV is the influence of cluster size on interpolation using different window lengths. Studying the tables shows that larger clusters (i.e.,  $\rho$  0.999 or less) produced lower errors. This is consistent with the results in the cluster size section. The end result of this analysis is that the winner method ( $\rho = 0.99$ ) is the best implementation method for this study using 9 clusters in the gain map.

While a separate table containing the results for a pure winner gain scheduling system is not presented here, the results are contained in the tables included in this



chapter. Using a cluster size of 0.99, the 30°C runs favor long windows for the lowest height IAE score. Yet, the temperature IAE score is lowest when the 1 minute window is used. At 35°C, the best temperature IAE scores occur with a window length of 3 minutes. The 4 minute window score is comparable. The height results are best at these lengths as well. At 40°C, a short window length of 1 or 2 minutes is favored. Both controller IAE scores are minimized.

The results in Table XXXIII and XXXIV provide some more information as to what window length to use. At a nominal operating temperature of 35°C, the best values IAE scores occur with 2 minute windows instead of 4 minute lengths. At the other nominal temperatures, the 4 minute window lengths are favored. As stated before, the differences in the values are slight. Thus, no definite trend can be surmised.

The question of window length ought to fall on to the process characteristics to determine the length of the pattern data window. The characteristic time ( $\tau_h$ ) for the height equation at a nominal height of 20 cm is 6.44 minutes (Hagglom, 1992). The temperature time constant ( $\tau_T$ ) is 0.50 minutes at nominal conditions of 20 cm and 35°C. With the short time constant for the temperature, it takes less time for the system to respond to changes. With this information, a pattern window should be able to monitor the process and detect changes in the system quickly. With this analogy, a short window length is dictated.

At the same time, the window length must be chosen which allows the ART2 network to ignore minor disturbances to the system. A minor disturbance is one where the system does not undergo a change that results in a gain cluster change when no setpoint change has occurred. For this system, such a disturbance would be short deviations in temperature in which the control system returns the process back to setpoint before the network decides that another pattern better matches the current process conditions. The disturbance tests for the mixing tank do not clearly show which window length to use. Based on the setpoint test results, though, a 2 minute window length would satisfactorily handle disturbance rejection and gain scheduling with setpoint changes.

## CHAPTER VII

### CONCLUSIONS

As shown in this paper, the ART2 neural network provides a method to implement gain scheduling with a feedback control loop. It aides in overcoming the disadvantage of nonlinear systems where a PID controller must be detuned to handle a wider operating range with worse performance. Using gain scheduling allows tighter control to be implemented and maintain good performance and maintain robustness. Our gain-scheduling system also provides the advantage of using more than one scheduling variable by analyzing the process patterns and eases the determination of which process variables to use as scheduling variables. The elements of having previous process data and logs of operations make the implementation of gain scheduling a promising prospect for implementing advanced pattern-recognition controls in manufacturing processes.

Based on the results in Chapter 6, the following conclusions have been reached. First, the number of clusters used depends on the nonlinearity of the system. If the process is highly nonlinear, more scheduling prototypes

are needed. This provides the system with an adequate number of scheduling points. Systems where the gain clusters are widely spaced over the operating range, interpolation aides significantly. Our studies have shown that clusters should be chosen to reflect normal operating conditions.

For cluster size, the best results for the interpolation techniques occurred when the cluster size is large. The cluster size should remain smaller than the radius that causes the clusters in the gain map to overlap. This cluster size allows interpolation to take place and produces the best results in this thesis.

The third conclusion found that shorter window lengths are needed. The length of the system is determined by the process dynamics. Based on this study, 2 minute pattern windows appear to offer a balance between detecting setpoint changes and rejecting disturbances that may cause a gain change.

For the interpolation techniques, linear interpolation offers a simpler method of implementing interpolation. Yet, the winner method works equally well for systems with 9 scheduling prototypes. No clear cut decisions can be made on the use of interpolation except when the number of prototypes is small.

While using the gain associated with the most similar pattern prototypes provides good performance, interpolation of the gains is normally used with gain scheduling. Our

studies of two types of interpolation techniques showed that little improvement occurred over the winner-takes-all method for assigning gains. Large clusters also improved control. Yet, the large clusters began favoring the winner method. If the process has a few learned prototypes for scheduling, interpolation provides a beneficial aide. Otherwise, a dense packing of steady-state prototypes favors the winner method. Finally, shorter window times also provide better control of the mixing tank. The system is able to detect setpoint changes quickly while still rejecting the effects of disturbances.

An important conclusion to this work lies with the nature of gain scheduling. Gain scheduling is designed to handle changes in the operating conditions of the process. These changes may range from setpoint changes to the introduction of disturbances. Yet, like traditional gain scheduling, pattern-based gain scheduling relies on monitoring process variables, which in turn are used to drive a feedback control loop. Feedback control doesn't care where the disturbances originate; it is designed to correct for them. When gain scheduling with the pattern-recognition approach, it should be remembered to use the gains for steady-state operating points which handle setpoint changes.

## Recommendations

With the development of the ART2-gain scheduling system, a new door has opened up for process control. A few studies are still required in order to determine specific answers for implementation and theory.

Recommendations for future work include:

- 1) performing a stability analysis of the gain scheduling approach using the ITAE setpoint method to find the gains. The analysis will build upon the work of Shamma and Athens as well as Rugh.
- 2) further work to examine interpolation schemes. Besides the linear and quadratic methods for interpolation, perhaps new schemes such as a center of mass approach would prove beneficial. Further work ought to look at adding weight to the expected new operating point.
- 3) further work on determining the size of the pattern window. This is a factor of process dynamics. The key question is finding a suitable length that prevents the system from reacting to disturbances and one that detects setpoint changes quickly.
- 4) tuning the ART2 network to detect specific setpoint changes such as temperature or height changes for the mixing tank. As demonstrated in Chapter 4, using gains tuned specifically for the setpoint change minimize the

- error generated. A general steady-state gain could then be used to handle disturbances in the system.
- 5) investigation of using gain scheduling with integral time and disturbance time scheduling. This system presents the opportunity to perform these tasks as well.
  - 6) utilization of wavelet pattern representations to recognize steady-state patterns and detect new operating conditions. The work of Raghavan and Whiteley (1993) promises to improve monitoring of sensor patterns.
  - 7) utilization of previous process data to determine the normal operating points of the process. With the ART2 network's ability to cluster the patterns, these clusters can be associated with gains suitable for that operating condition.
  - 8) testing the ART2 neural network's ability to handle process signal noise. In a plant situation, all process data has some type of noise, and the ability of the pattern recognition system to reject process noise is important.

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**APPENDIXES**

APPENDIX A

SIMULINK SIMULATION OF THE MIXING TANK

This appendix provides an example of how gains were calculated for the gain map.

The first step in calculating process characteristics is to find the rate of water leaving the mixing tank. The mass flowrate out of the tank is found using Equation 14. For an operating height of 20 cm, the mass flowrate of the the mixing tank is 11.36 kg/min. Assuming a steady-state system, the mass flowrates for the hot and cold water streams is found by solving Equations 13 and 18 simultaneously. With the inlet hot water stream at 51°C and the inlet cold water stream at 17°C, no disturbance flow, an operating temperature of 35°C and an operating height of 20 cm, the mass flowrate of hot water into the tank is 6.01 kg/min. The cold water flowrate into the tank is 5.34 kg/min.

Once the flowrates into the tank are determined, the input voltage to the hot water and cold water control valves needs to be calculated. These voltages are important in calculating the process characteristics using equations developed by Haggblom (1992). Equations 27 and 28 calculate the input voltages for the hot and cold water streams, respectively.

$$v_H = \alpha_H(m_H - m_{H0})^{\gamma_H} \quad (27)$$

$$v_C = \alpha_C(m_C - m_{C0})^{\gamma_C} \quad (28)$$

Constants for Equations 27 and 28 can be found in Table XXXV. With operating conditions of 35°C and 20 cm, the

voltage sent to the hot water control valve is 7.34 V. The cold water control valve has an input voltage of 7.00 V.

TABLE XXXV

## CONSTANTS FOR THE MIXING TANK SYSTEM

$\alpha_h = 3.965$	$\gamma_h = 0.3446$	$m_{HO} = 0.0484 \text{ kg/min}$
$\alpha_c = 4.212$	$\gamma_c = 0.3244$	$m_{CO} = 0.5510 \text{ kg/min}$

With the input voltages calculated, the process gain relating the hot water control voltage to the temperature is found using Equation 29.

$$K_{hvh} = \left( \frac{2(\bar{h} + h_o)}{\gamma_H \bar{m}} \right) \left[ \left( \frac{1}{\alpha_H} \right)^{\frac{1}{\gamma_H}} \left( \frac{\bar{V}_H}{\alpha_H} \right)^{\frac{1}{\gamma_H} - 1} \right] \quad (29)$$

where  $\bar{V}_H$  is the steady-state input voltage,  $\bar{h}$  is the steady-state height in the tank and  $\bar{m}$  is the steady-state outlet flowrate. At 35°C and 20 cm, the value of  $K_{hvh}$  is 53.59 V/cm.

Another important value for height gain calculation is the time constant. That constant is calculated using Equation 30 and is 6.44 minutes at 35°C and 20 cm.

$$\tau_H = \frac{2\rho A \sqrt{h + h_o}}{\beta} \quad (30)$$

For application of the direct synthesis technique, a settling time  $\tau_c$  was specified at 0.3 minutes. This value was determined based on a step change to the process. Using equations found in Seborg (Seborg *et. al.*, 1989), the controller gain  $K_C$  is found using Equation 31.

$$K_c = \frac{\tau_H}{\tau_c K_{hvh}} \quad (31)$$

$K_C$  for 35°C and 20 cm was 0.40 V/°C.

A variety of methods are possible for calculating the gain for the temperature controller. An example of calculating the gain using the ITAE setpoint method will be shown. The deadtime  $\tau_d$  for this process loop is 0.3 minutes.

First, the process characteristics for the temperature system need to be calculated. Using Equation 32, the gain relating input voltage of the cold water control valve to the mixing tank temperature is -3.34 V/°C.

$$K_{rv_c} = \left[ \frac{1}{\gamma_c} \left( \frac{1}{\alpha_c} \right)^{\frac{1}{\gamma_c}} \left( \frac{\bar{V}_c}{\alpha_c} \right)^{\frac{1}{\gamma_c} - 1} \right] \left( \frac{\bar{T}_c - \bar{T}}{\bar{m}} \right) \quad (32)$$

where  $\bar{V}_c$  is the steady-state voltage to the cold water control valve,  $\bar{T}_c$  is the steady-state temperature of the cold water inlet stream,  $\bar{T}$  is the steady-state temperature of the mixing tank. The temperature time constant is 0.50 minutes at 35°C and 20 cm using Equation 33.

$$\tau_T = \frac{\rho \bar{h}A}{m} \quad (33)$$

With these process values, the ITAE setpoint method calculates a controller gain of -0.28 V/°C using Equation 34.

$$K_{CT} = \frac{1.305 * \left( \frac{\tau_d}{\tau_T} \right)^{-0.959}}{K_{Tvc}} \quad (34)$$

When this gain is derated, the temperature controller gain is multiplied by a constant (0.50). The resulting gain is -0.14 V/°C.

These gains are calculated for a variety of operating points and associated with the steady-state process clusters in the gain map.



✓  
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