APPLICATION OF FUZZY LOGIC

TO DISTRESS ANALYSIS

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CHAPTER I

INTRODUCTION

All conventional database management systems accept crisp data as input. In these databases, all data processing or data manipulation is based on the crisp set theory. In this theory a characteristic function is used to assign a value of either 1 or 0 to each individual element in the universal set. This way it discriminates between members or non-members of the crisp set under consideration by precisely classifying the elements in the universal set as being included or not included in this crisp set.

However, in real-life, data or queries are not often crisp or precisely defined since we frequently use imprecise properties or qualities using a *fuzzy* language [Li90]. For instance, when we say "Smith is *very young* and *tall*," an unavoidable and uncontrollable functional loss of information occurs if the phrase is modelled through crisp values because crisp values can not incorporate the impreciseness present in such queries.

Keeping this in mind, Dr. Zadeh [Zadeh65] proposed a theory in his paper "Fuzzy Sets" (from which the theory of fuzzy logic emerged). The basis of Zadeh's fuzzy set theory is the human way of thinking that deals with categories that have shady boundaries [Zadeh84]. In his paper, Zadeh argues that most of the sets in the physical world do not have a precisely defined criteria of membership function (or characteristic function) as crisp set theory does. Let us take the example of some linguistic terms such as little, small, large, near, far, about, etc. These words do not have any precise boundaries. If they are used in a database query, the query manipulator can not handle the impreciseness represented by these words. Therefore, fuzzy logic is more suitable in such situations. The following example demonstrates how fuzzy logic can be used in a database query.

```
SELECT *
FROM INDUSTRY-DATA
WHERE AGE = YOUNG
AND
EXPERIENCE = ABOUT(10)
```

In this example, a conventional database manipulator cannot understand such linguistic terms. Even if a numeric limit is defined for such words, resultant tuple(s) are based on exact match or mismatch. This limit can be that YOUNG means 1 to 24 years old, ABOUT(10) can be values 9, 10, 11. Thus, at the time of computing the query, a typical query ignores a 25 year old person with ten years of experience. However, if a fuzzy membership function is defined for such words (YOUNG and ABOUT), then, the membership function gives a grade value ranging from 0 to 1 for the crisp values available in a particular attribute (AGE or EXPERIENCE). Then, these grade values are fuzzyly 'AND'ed (different from conventionally 'AND'ed), giving grade values again ranging from 0 to 1. Finally, the tuple(s) appear on the screen, starting from near

1 and going down near 0 grade values.

The above concept of membership functions and grade values of fuzzy logic can be applied to financial data for classifying (or grading) companies in a financially distressed or non-distressed category. Usually distress analysis is done by such techniques as multiple discriminant analysis (MDA) or regression analysis. Multiple discriminant analysis (MDA) has been generally used for identifying group membership grade based on a profile of attributes. It develops a composite score for observations. The major purpose of discriminant analysis is to establish classification rules and statistical inference of the results.

If we try to model the impreciseness of real-life through conventional databases, an unavoidable and uncontrollable functionality loss occurs. Therefore, a system is needed that can handle impreciseness in the every-day life.

With the given reasoning in mind, there is a need for the introduction of fuzzy logic into financial decision-making. When implemented, the financial decision-making emerges from the conventional modeling, more closely resembling human decision making.

This thesis describes the design and implementation of an extension to the relational query functionality by associating membership grade to derived tuples. It is done by associating membership grade to each atomic value (which is a financial ratio) of the database at the time of querying. Further, fuzzy mathematics is applied on these membership grades to produce the final resultant tuple(s) for forecasting or decision making.

In this thesis, a fuzzy logic approach is used to establish the classification rules (financially distressed or not) for a given set of companies (bankrupt and/or non-bankrupt) by computing a grade value for each tuple (record) using some properly derived membership functions. The retrieval mechanism of the financial data also utilizes fuzzy logic, such that selection or rejection of a tuple is based on fuzzy set theory.

The main objective of this research is selecting proper membership functions for different attributes of a database (these attributes in the financial database represent the performance ratios of a company), and comparing the results with MDA's results.

CHAPTER II

DISTRESS ANALYSIS

Distress analysis, in the financial world, is best viewed as an economic notion for the study of companies with severe liquidity problems. There are several indications of likelihood of financial distress. One source is *cash flow* for the current and the future periods. The benefit of this analysis is that it focuses directly on the financial distress for the time period of interest. Other sources of information about a company's financial distress are its *financial statement*, external variables such as *bond ratings* and *security returns*, and *corporate strategy*, etc.

A univariate approach to predicting financial distress is a method in which a single variable is used for corporate failure prediction. But it is observed that both bankrupt and non-bankrupt companies can have the same good performance on a single selection criterion. For this reason a univariate model can result in inaccurate predictions. Different attempts have been made to combine the information in several financial variables into a single multivariate model because, findings indicate that a company's inclination to fail is increased due to the cumulative effects of different financial variables [Altman83]. Among different multivariate models proposed, *multiple discriminant analysis* (MDA) is one of the widely used method for distress analysis.

2.1 Multiple Discriminant Analysis

The financial statement of a company plays a major role in distress prediction in financial world. Multiple discriminant analysis (MDA) is one of the widely used methods to investigate the financially distressed companies. The concept of multiple discriminant analysis was first introduced by Fisher in 1936 [Fisher36]. Altman in 1968 [Altman68] chose the same technique as the appropriate statistical technique for financial distress analysis.

MDA is a statistical technique used to classify an observation into one of several *a priori* groupings dependent upon the observation's individual characteristics. It is used primarily to distinguish or make predictions in problems where the dependent variables appear in a quantitative form, for example, male or female, bankrupt or non-bankrupt, etc. The discriminant function is of the form:

$$Z = u_1X_1 + u_2X_2 + \dots + u_nX_n$$

This function transforms the individual variable values to a single discriminant score, called Z-score, which is then used to classify the objects,
where,

$$u_1, u_2, \dots, u_n$$

are the discriminant coefficients, and

$$X_1,\ X_2, \ldots, X_n$$

are the independent variables (or the actual variables).

According to Altman [Altman83], the primary advantage of MDA is the potential for analyzing the different variable profiles of the object simultaneously rather than sequentially examining its individual characteristics.

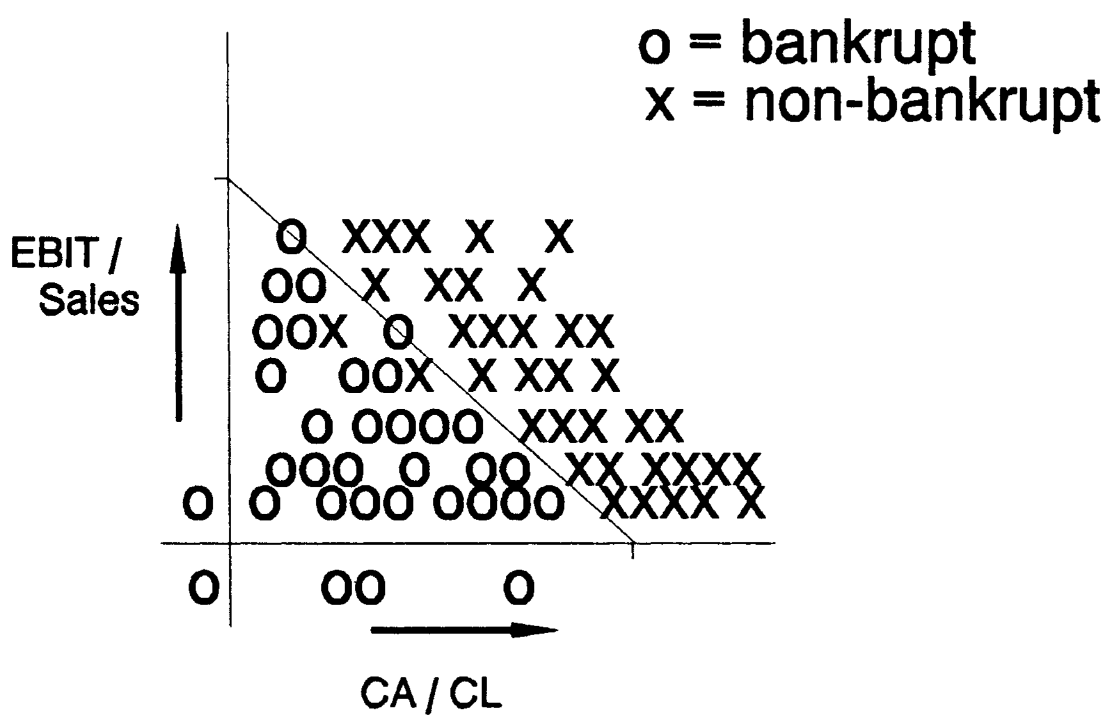

Figure 1. MDA with discriminant line

Fig. 1 shows a two-variable analysis where profitability and liquidity ratios are plotted for healthy and sick companies. The two variables used here are Earnings before Income Tax to Sales ratio (EBIT/Sales) and Current Assets to Current

Liabilities ratio (CA/CL). In this figure 'o' and 'x' represent the bankrupt and non-bankrupt companies. The discriminant model selects the appropriate weights (coefficients) for each ratio and on these basis a separating line is drawn between the two groups (bankrupt and non-bankrupt).

2.2 Calculation of Discriminant Coefficient

The basic equation of MDA is:

$$Z = u_1X_1 + u_2X_2 + \dots + u_nX_n \quad \dots\dots\dots \quad (1)$$

where 'X' represent the independent variables and 'u' represent the discriminant coefficients. In the context of discriminant analysis, this equation is called the *discriminant equation*. The values of 'u' in the discriminant equation are chosen so as to maximize the separation between the two groups. Lindeman [Lindeman80] explains the calculation of the discriminant coefficient 'u' as,

$$u = W^{-1}\, d \quad \dots\dots\dots\dots\dots \quad (2)$$

where,

$$W = \begin{bmatrix} S_{aa} & S_{ab} \\ S_{ba} & S_{bb} \end{bmatrix}$$

here a and b represent the independent variables. So inverse of W can be written as,

$$W^{-1} = \frac{1}{|Det.|} \begin{bmatrix} S_{aa} & -S_{ab} \\ -S_{ba} & S_{bb} \end{bmatrix}$$

$$S_{aa} = \sum a^2 - \frac{(\sum a)^2}{n}$$

$$S_{bb} = \sum b^2 - \frac{(\sum b)^2}{n}$$

where,

$$S_{ab} \;\; or \;\; S_{ba} = \sum (a * b) - \frac{(\sum a * \sum b)}{n}$$

In equation 2 'd' represents the difference between group means on variables a and b as,

$$d = \begin{bmatrix} a'_{group1} & - & a'_{group2} \\ b'_{group1} & - & b'_{group2} \end{bmatrix}$$

Finally, we get the values for each 'u' for equation (1) using the equation number (2). When these discriminant coefficients are multiplied with actual ratios, we get the final Z-score (see eqn. 1). This way companies are ranked on the basis of their Z-score. Then a visual inspection is made to determine an "optimal" cutoff point for predicting a company's bankruptcy or non-bankruptcy.

The fuzzy logic based model presented in this thesis is like the MDA model. A major difference is that we used fuzzy membership grade values in place of discriminant coefficients and also a new parameter of weight is introduced with each ratio (independent variable).

Before we continue further discussion on MDA and how we can use fuzzy logic to improvise on it, we shall take a look at what has been accomplished in these areas. This review is presented in the following chapter.

CHAPTER III

PROBLEM DEFINITION

3.1 Importance of distress analysis

Each year hundreds of thousands companies go bankrupt. Only in 1991, business failures reached a record high of nearly 90,000, up around 45 percent from 1990. At the same time, the liabilities associated with these failures increased around 90 percent, which made around $105 billion in 1991 [DUN&BRAD92].

In the United State and in other countries, businesses are run on the loan obtained from the commercial lending agencies and private investors. And if any business fails, these lending companies and investors have to bear a major loss of their lended money, because very little amount is recovered from the failed company's liquidation. Such loss of billions of dollars of lended money indirectly causes an unhealthy effect on the national economy.

Therefore, loan lending agencies and private investors, before granting the requested loan, evaluate the requesting company's financial information. This evaluation is based on different factors, which includes: loan review, repayment performance, traditional ratios and industry flow-of-funds analysis. These traditional evaluation processes lack the prediction of the future of the company, so there should be some other evaluation processes which can be used to predict a company's future on its current and previous performance which can be helpful for the evaluator decide the granting or refusing the loan request.

3.2 Approaches to distress analysis

There are several approaches used to predict the bankruptcy/non-bankruptcy. Two major classes of prediction models are univariate model and multivariate model. A univariate approach to predict the financial distress involves the use of a single variable in predicting model. There are several assumptions in this approach. The key assumptions are:

* The distribution of the variable for distressed companies differ from the variable for the non-distressed companies.
* This distribution difference can be used for prediction purposes.

But the problem with univariate model is that both classified bankrupt and highly successful non-bankrupt companies can have the same financial performance. Also different variables can imply different prediction for the same company.

To overcome such problems, information in several financial variables are combined into a single multivariate model. The dependent variables or the results of these models are used for the distress prediction purposes. The independent variables examined typically have been financial ratios and other company-oriented variables. An in-depth discussion on different multivariate models is given in the next chapter of literature review.

3.3 Human decision making and fuzzy logic

The subject of decision making is the study both of how decision are actually made and how they can be made better or more successful. Much of the focus in the field has been in the area of business management, in which the decision making process is of key importance. Conventionally, decisions like investments, borrowing loans, liquidation, *etc.* depend upon some statistical and probabilistic techniques. But in real life decisions can not be made completely on the basis of some statistical or probabilistic results. Decision makers need to introduce some human decision factors to reach the final decisions. These human factors can be, e.g., close to, less greater than, about near, *etc.* The problems with these human factors are that they are not quantifiable using conventional techniques.

To handle the quantifiable variables in computation, Zadeh [Zadeh65] introduced the concept of fuzzy logic. According to him [Zadeh88]:

> "... unlike classical logic systems, it [fuzzy logic] aims at modeling the imprecise modes of reasoning that play an essential role in the remarkable human ability to make rational decisions in an environment of uncertainty and in imprecision."

While decision making under condition of risk and uncertainty have been modeled by conventional decision theories, fuzzy logic based decision making attempt to deal with the vagueness, or fuzziness inherent in subjective or imprecise determination preference, constraints, and goals [Klir88].

In this thesis, same fuzzy logic concept is introduced within the framework of conventional techniques of distress analysis. In the next section it is explained that how these two techniques can be combined for distress analysis.

3.4 Distress analysis and fuzzy logic

As discussed in chapter 2, distress analysis is an economic notation for study of companies which are in severe liquidity problem. Conventionally, most of the distress analysis models use different financial variables into a single multivariate model. Usually, 'financial ratios' are used as the financial variables in these models.

Conventional distress analysis models are built on some statistical and probabilistic techniques. These probabilistic and statistical techniques are based on crisp set theory. Crisp logic may be defined as logic in which sets conform to a simple dichotomy: either an item belongs to the set or not. Fuzzy logic theory, on the other hand, is a superset of conventional (Boolean or crisp) set theory. It has the capability to handle the concept of partial truth e.g., values which are between "completely true" to "completely false". By incorporating this "degree of truth" concept, fuzzy logic extends traditional sets in two ways. First, these sets are labeled quantatively (using linguistic terms such as "tall," "warm," "slow," "fast," etc.) and have a varying degree of membership. Secondly, any action or output resulting from a premise has a degree of membership that reflects the degree to which that premise is true.

In this thesis we use the same concept of fuzzy logic into our distress analysis model. Here, by using some membership function we calculated the membership grade value (range from zero to one) for each financial ratio of a company. These membership grade values represent that, the closer the value of grade value to one, the greater the contribution of that financial ratio to companies liquidation. Vice versa, the smaller the membership grade value, the lesser the contribution of that financial ratio to a company's liquidation.

These fuzzy membership grade values are further fuzzyfied by using the concept of weights with each of the financial ratio of a company. The concept of weight is introduced in our model for the decision maker to include his/her own experience in decision making and allow him to change or check the performance of the model. Then these fuzzyfied grade values are added and normalized to the scale of zero to one. Similarly these normalized values are calculated for all the companies in our sample data. Then a cutoff point is used to decide manually, upward from where companies show tendency of bankruptcy, downward from where companies show tendency of remaining stable. Now the decision can be made on the basis of these normalized results.

In chapter five, basic concepts of fuzzy logic are explained, and its application to distress analysis is presented in chapter six.

CHAPTER IV

LITERATURE REVIEW

4.1 Fuzzy Databases

Different database models have been proposed using fuzzy logic to control the impreciseness in queries and also in the data. These databases fall in two basic classes. In the first class, the assumed database is conventional (i.e., crisp), and fuzzy sets make its use easier, more "human consistent", by some "add-ons" in the queries to make them more human-like. In the second class, values that are imprecise, uncertain, incomplete or vague in nature, can be included. This impreciseness or incompleteness is represented by using fuzzy sets or possibilistic values [Zadeh78]. Databases belonging to this class can be considered as actual fuzzy databases.

Tahani [Tahani77], concluded that the idea of fuzzy sets can interpret simple imprecise queries and described a method for queries which was based on a data manipulation language called SEQUEL [Chamberlin74]. In his method Tahani associated a grade of membership μ with each tuple but did not provide a query language for the user [Bosc92].

Kacprzyk [Kacprzyk89] used the conventional (non-fuzzy) database and constructed an "add-on" to it which helps the human decision maker retrieve information related to some imprecisely or vaguely defined concept, relations etc. They used Ashton Tate's *Dbase III+* database package and designed *FQUERY III+*, which is a human consistent database querying system based on fuzzy logic with linguistic quantifiers.

Bose [Bosc87,Bosc92] introduced an extension of DBMS querying capabilities in order to allow fuzzy queries against a usual database, i.e., composed of precise facts. They designed SQL^f, an extension to SQL, to introduce fuzzy querying capabilities wherever possible. A sample base block of their formulation is:

SELECT <n/t> <attributes> FROM <relation>

WHERE <fuzzy condition>

Here <n/t> represents quantitative calibration integer or real tuple and <fuzzy condition> represents involvement of a Boolean or fuzzy elementary condition.

Another line (second class) of research in application of fuzzy logic on conventional database is on designing fuzzy databases, where impreciseness and vagueness is incorporated with the data in terms of a fuzzy membership function or possibilistic distribution. In this class, a lot of work has been done and different models have been proposed. But it seems that they mainly focus on the theoretical aspect, while implementation issues have been neglected.

Most of the research on fuzzy databases is done on Codd's [Codd70] relational database model. Zadeh [Zadeh78] designed a relational model, PRUF, capable of processing natural language like propositions. In that work, Zadeh developed rules of fuzzy linguistics used in query translation. Umano *et. al.* [Umano82] developed a fuzzy database system FREEDOM-O based on a fuzzy relational model using the possibility distribution for representing fuzzy data. The data manipulation language provides, with other basic operations, DEFR (Define Fuzzy Relation) and DEFP (Define Fuzzy Predicate) statements. FREEDOM-O was implemented in FSTDSL/FORTRAN.

Prade and Testemale [Prade84] proposed a general model for fuzzy relational database based on Zadeh's possibility theory. They applied the possibility theory with completely unknown values, incomplete data, uncertain information and vague queries involving linguistic terms. They implemented their proposed model using MACLISP.

Zemankova-Leech and A. Kandel [Zemankova84] proposed a possibility distribution - relational model database. Their model is made of three parts;

1. VDB (Value Database),
2. EDB (Explanatory database, and
3. Translational rules.

Both VDB and EDB consist of collections of relations or functions used to compute the degree of membership function of a given data with a user's query. In 'translational rule', along with relational algebraic operations, built-in linguistic translation rules are used to evaluate queries with linguistic modifiers, quantifiers, and composite queries.

Buckles and Petry [Buckles82,Buckles85] proposed a fuzzy relational database model for application in policy analysis. They extended a relational model and introduced similarity relations [Zadeh71], which are an introduction of fuzziness into equivalence relations. The extended operations for relational models are known as relational algebra [Terano92].

4.2 Prediction of Financial Distress

The prediction of financial distress for corporations, universities, and other institutions has been the subject of much interest and research for last 50 years or so. Prediction of financial distress is primarily based on different financial ratios. These ratios are chosen on the bases of their popularity in the literature and their potential relevancy to the study.

The first modern analysis to distress prediction was performed by Beaver [Beaver66]. His method did not use the multi-variable approach rather the empirical test focused upon comparison of means, dichotomous classification tests, and analysis of likelihood ratios. He collected the ratios for 79 firms that failed from 1954 to 1964 and a paired sample of non-failed firms. Selected companies are matched according to asset size and classification code. Among the 30 ratios he used in his model, he found the following six ratios as the 'best':

1. Cash flow / Total Debt
2. Net income / Total Assets
3. Current plus Long-term Liabilities / Total Assets
4. Working Capital to Total Assets
5. Current Ratio

6. No-credit interval

Among these six ratios he found 'cash flow / total debt' as the best ratio. The major finding of his research was that financial ratios have the ability to predict at least five years before failure. The accuracy of his model, one year before bankruptcy , was 88%.

Altman [Altman68] used the multiple discriminant analysis to analyze five financial ratios for bankruptcy prediction. Those ratios were: (1) Working Capital to Total Assets, (2) Retained Earnings to Total Assets, (3) Earnings before Interest and Taxes (EBIT) to Total Assets, (4) Market Value of Equity to Total Debt, and (5) Sales to Total Assets. He selected 34 failed manufacturing firms from the period of 1946 to 1965 and a paired sample of non-failed firms for his model for the same period of time. Pairing was done on the bases of asset size and type of industry. Altman's work reconfirmed the usefulness of ratio analysis and demonstrated the importance of multiple-variable analysis. His model was able to correctly classify 95% of the total sample one year before failure,. The problem with his proposed model was that it did not have the predictive ability beyond 2 years prior to failure. Also, he did not develop the theoretical framework for failure prediction.

Altman *et. al.* [Altman77] developed a model named ZETA, working jointly with a private financial firm. The results of that model were marketed by *ZETA Services, Inc.* (Mountainside, New Jersey). They did not publish the coefficients of their discriminant analysis model. Their ZETA model is an extension of the previous [Altman68] Z-score model. The ZETA study utilized a bankrupt firm sample where the average adjusted asset size, two annual reporting periods prior to failure, was approximately $100 million. And no firm had less than $20 million in assets. With the exception of three (out of 53) firms (which is over 90% correct), every bankrupt firm in their sample of data failed in the prior seven years (1969-1975). An important feature in their model is that the data and footnotes to financial statement scrupulously analyze the most recent changes in financial reporting standards and accepted accounting practices. Also, their model handled both manufacturing and retail firms. The ZETA model consisted of seven variables, including: (1) return of assets, (2) stability of earnings, (3) debt services, (4) cumulative profitability, (5) liquidity, (6) capitalization, and (7) size of the firm. The prediction capabilities of this model were better than the previous Z-score model [Altman68].

Barniv and Raveh [Barniv89] presented a new approach (non-parametric) for identifying financial distress and developed a methodology that overcame some of the problems and shortcomings of the discriminant analysis. Their non-parametric approach provided a continuous scoring system. Their method can be used for scoring observations with the same industry grouping of companies. They tested their model on two widely different industry groupings in the US: industrial (i.e. manufacturing and retail) firms and non-life insurance companies. The variables they utilized in their studies are profitability, liquidity and long-term financial solvency (i.e. financial leverage) ratios. The measure of stability of performance over time was also used. They relate their method to the conventional discriminant analysis as, that both the methods estimate linear combinations of the variables that will maximize a measure of 'separation' between two (or more) given groups. The difference between these two methods lies in the form of the index of separation. Discriminant method is a parametric method and has an explicit solution. It maximizes the ratio between-groups to the within-groups variances. Their proposed method, on the other hand, was non-parametric and used a different quantity to be maximized. The main advantage of the non-parametric approach was that it produced substantially lower misclassifications and resubstitution risks in terms of validation results.

4.3 Current trends in Bankruptcy prediction

Fletcher and Goss [Fletcher93] used the artificial neural networks (ANNs), due to their generalization and abstract mapping capacity, for bankruptcy forecasting. The objective of their study was to illustrate the development of a particular class of ANNs (known as Back-Propagation Neural Networks, or BPNNs) in standard business applications. In their model they determined the optional generalized BPNN model from available historical financial data using a *v-fold* cross-validation technique to estimate the production risk. They compared the results of their findings to the standard Logit Regression (LR), which is also a widely used prediction and classification methodology like discriminant analysis. This comparison was made in terms of (1) minimization of prediction risk, (2) efficiency of the estimation, and (3) maximization of the correctness ratios for unseen test data. With the results mean risk curves are generated to illustrate forecasting empirical probabilities of bankruptcy. The results from their method corresponded closely to other studies which have likewise found neural networks better at extracting information from attributes for forecasting purposes. In their study, they tested thirty-six firms and predicted 32 of them correctly.

Wilson *et. al.* [Wilson93] also used the neural networks for bankruptcy prediction. They used neural networks because of their ability to solve real world problems specially in the area of forecasting and classification decision problems. They compared their study with the multiple discriminant analysis (MDA) under the assumption that neural networks are not subject to restrictions such as normality; thus, they represent a more robust approach than discriminant analysis which requires *a prior* assumptions about the model variables. In their research, they conducted a series of experiments to investigate the effects of the training and testing set composition on network accuracy. The predictive results were compared with the accuracy obtained by classical multiple discriminant analysis method to determine the condition where neural networks models do significantly better predictions. The ratios they used in their model are the same as Altman [Altman68]. Their data consists of the companies who went bankrupt between 1975 to 1982. There were 129 firms, 65 of which went bankrupt during the period, and 64 non-bankrupt firms matched on industries and time period. Their study was based on the Monte Carlo resampling technique to generate multiple subsamples from the original firms in order to gain a better measure of predictive accuracy. To implement the model they used SYSTAT to calculate the coefficients for MDA and BRAINMAKER software to construct and test train a neural networks model. The result of their research outperformed the MDA method in prediction accuracy of both bankrupt and non-bankrupt firms under varying training and testing conditions.

Businesses are run by human, therefore decision making methods should be based on human like principles rather than rigid statistical based techniques. In the recent past different computational approximations have been employed for this purpose, such as artificial intelligence, neural networks etc. Now we move on to a more natural way of thinking that is fuzzy logic.

CHAPTER V

INTRODUCTION TO FUZZY LOGIC

The theory of Fuzzy logic was first introduced by Mr. Lotfi A. Zadeh [Zadeh65]. According to Zadeh [Zadeh80],

> "... the theory of fuzzy sets is the development methodology for the formulation and solution of problems which are too complex or ill-defined to be susceptible to analysis by conventional techniques."

This theory is a superset of conventional (Boolean) set theory, extended to handle the concept of partial truth, e.g., values which are between "completely true" to "completely false". By incorporating this "degree of truth" concept, fuzzy logic extends traditional sets in two ways. First, sets are labeled quantatively (using linguistic terms such as "tall," "warm," "slow," "fast," etc.) and have a varying degree of membership. Secondly, any action or output resulting from a premise has a degree of membership that reflects the degree to which the premise is true. A fuzzy set consists of objects and their respective grades or membership in the set. The grade of membership of an object in the fuzzy set is given by a subjectively defined membership function [Zemankova84].

5.1 Crisp Sets

The word crisp indicates clearly defined boundaries. A crisp set can be represent as:

$$A = \{a, c, d\}$$

This shows that elements a, c, d, in subset A are exact members of a universal set U, or have characteristic value 1: whereas, elements b, e, and f are not members of subset A. They have a characteristic value 0. Characteristic representation of a crisp set is:

$$char_A(x): U \text{ -> } \{0, 1\}$$

Which can be read as, characteristic function of crisp set A gives the grade value 0 or 1 only for variable x for the elements in the universe of discourse U.

5.2 Fuzzy Sets

Fuzzy set theory defines the degree to which element x of set X is included in any subset. The function that gives the degree, to which it is included, is known as the membership function. The degree of inclusion is sometimes called the "extent" or "grade." This grade value is a real number between zero and one. Usually fuzzy sets are represented by ~A, ~B, ~C, etc. where " ~ " known as mark of fuzzyfication. Assume that ~A is a fuzzy set in the universe of discourse U. For any x, which belongs to U, the grade of membership μ of x, which definitely belong to ~A, is 1, where the degree of membership function μ(mu), which does not belong to ~A absolutely, is 0. Apart from those cases, the grade of membership of x, which partly belongs to ~A is in the interval 0 and 1 [Li90].

Let us take an example to represent a fuzzy set: for universe of discourse,

U = {a, b, c, d, e, f},

a fuzzy set ~A can be define as, when

'a' is present in ~A with degree of membership 1.0,

'b' is present in ~A with degree of membership 0.9,

'c' is present in ~A with degree of membership 0.2,

'd' is present in ~A with degree of membership 0.8,

'e' is present in ~A with degree of membership 1.0,

'f' is present in ~A with degree of membership 0.0,

~A = {1/a, 0.9/b, 0.8/d, 1/e}

Because of 0 degree of membership, there is no 'f' (assuming that the threshold value is 0) in the fuzzy set ~A. Characteristic function representation of fuzzy set is

$char_{\sim A}(x)$: U -> [0, 1].

which can be read as, characteristic function of fuzzy set ~A gives the grade value between 0 and 1 for variable x for the elements in the universe of discourse U.

5.3 Operations on Fuzzy Sets

Operations on fuzzy sets are different from the operations on crisp sets. Following are few examples of operations on fuzzy sets and how these are evaluated:

UNION: The union (OR or +) of fuzzy sets ~A and ~B, ~A U ~B,is the fuzzy set defined by the following membership function:

$$\mu_{\sim A U \sim B}(x) = \max \{\mu_{\sim A}(x), \mu_{\sim B}(x)\}$$

Here 'max' mean select maximum grade of the elements of the two membership function.

INTERSECTION: The intersection (AND or .) of fuzzy sets ~A and ~B, ~A $\cap$ ~B,is the fuzzy set defined by the following membership function:

$$\mu_{\sim A \cap \sim B}(x) = \min \{\mu_{\sim A}(x), \mu_{\sim B}(x)\}$$

Here 'min' mean select minimum grade of the elements of the two membership function. These 'min' and 'max' operators are commutative, associative and mutually distributive operators.

COMPLEMENT: The complement of fuzzy set ~A, ~A' is the fuzzy set defined by the following membership function:

$$\mu_{\sim A'}(x) = 1 - \mu_{\sim A}(x) \quad \forall_x \in \sim A$$

$$\mu_{\sim A'}(x) = 1 \quad \forall_x \notin \sim A$$

CONCENTRATION: The elementwise concentrate of fuzzy set ~A is represented by the following notation:

$$CON(\sim A) = \sim A^2$$

This operation is the interpretation of word 'VERY'.

<u>DILATION</u>: The elementwise dilation of fuzzy set A~ is represented by the following notation:

$$DIL(\sim A) = \sim A^{0.5}$$

This operation is the interpretation of word 'APPROXIMATELY'.

There are some other operations defined for fuzzy sets but those are irrelevant to the current discussion.

In the next section we will discuss the different forms of membership functions and how these functions are derived.

5.4 Fuzzy Membership Function

The principal use of a membership function is to determine an element's membership in a fuzzy set. This function takes numeric values and map them onto degree of membership, known as *fuzzy values* represented by numbers between 0 and 1. Membership function is used for describing the procedure of gradual transition of an object from membership to nonmembership. That is a distribution of grades of membership in the universe of discourse.

To come up with a proper membership function for any problem is always a big issue. According to Li [Li90],

> "The concept of membership functions is a cornerstone of theory. But, up to now, their meaning in nature and how membership functions are derived is not yet clear explicitly, ... The problem of practical estimation of membership functions has not been systematically studied in the literature."

This is because a membership function is a matter of definition rather than objective experimentation or analysis. In practice, the shape of a membership function is subjective.

$$\mu_{young}(age) = \begin{cases} 1, & \text{if } age \le 24 \\ \left[1+\left(\dfrac{age-24}{4}\right)^2\right]^{-1}, & \text{if } 25 \le age \le 50 \\ 0, & \text{if } age > 50 \end{cases}$$

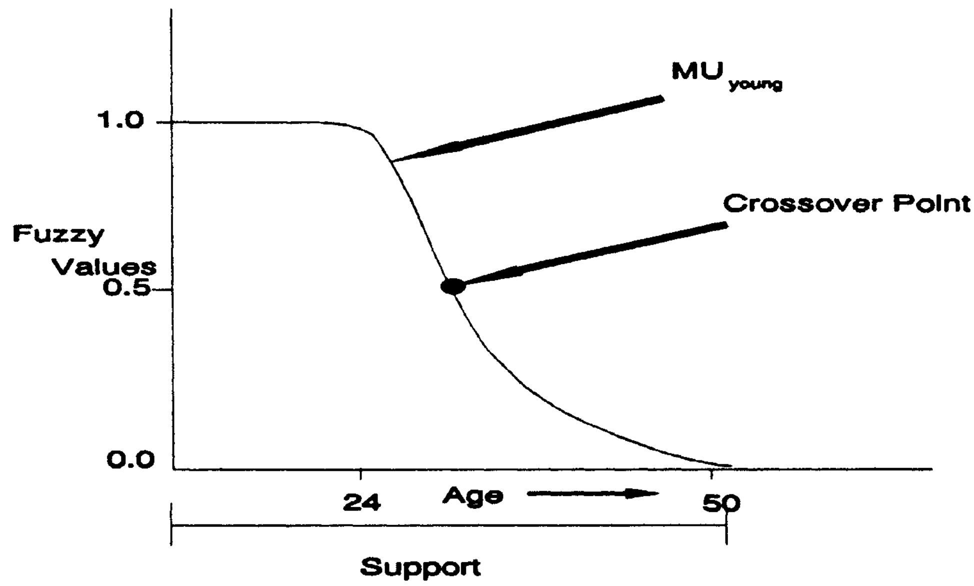

Figure 2: A membership function [Fleschman92]

Generally membership function is described by two attributes, *crossover point* and *support* [Fleischman92]. The crossover point is a point whose degree of membership is 0.5. The *support* is the set of points having a degree of membership greater than 0. Figure 2 shows an example of a membership function for a variable YOUNG.

Membership function in this figure represent that if 'age' is less than or equal to 24 then the grade value will be 1, otherwise the grade value is calculated by the membership function which gives value between 0 and 1.

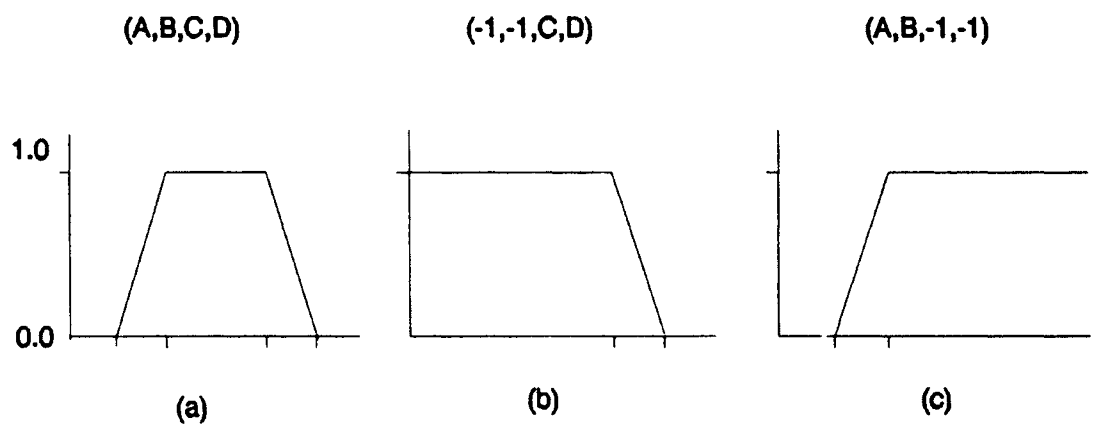

Figure 3. Membership function specification
(a) A complete trapezoid, (b) and (c) trapezoids using two parameters

The process of generating a membership function can be automated by a method known as clustering algorithm [Kamel90]. Clustering algorithm can be used as a tool to generate mapping between fuzzy variables as they apply to a particular attribute.

This algorithm determines the shape of the membership functions based on the clustering of the data.

Another method to represent membership function is by trapezoidal approximation. Trapezoidal membership functions are specified by using 4 parameters (A, B, C, D), which represent their slope-changing points on the X-axis. Figure 3 shows different shapes of a trapezoidal membership function. In order to get the membership grade values, four different values of parameters are mapped on any of these trapezoids. Figure 3(b) and 3(c) give membership grade values only using two of the parameters because they have zero spread on left-hand side and on right-hand side respectively. Among these three shapes of trapezoidal membership function, a particular shape is selected on the bases of the system requirement and upon the input data.

5.5 Applications of Fuzzy Logic

Fuzzy logic has the inherent capabilities of analyzing the data and quickly finding the approximate solutions to the real world problems. The most popular application of fuzzy logic is in the control theory, where data has to be analyzed in real time and solution are suggested and executed in real time. Other examples of fuzzy logic applications are, balancing a vertical beam in a 3D space based on inputs from different sensors, focusing the lens of a movie camera based on the motion of the objects the picture etc. Similarly in the business world a lot of information is flowing concurrently through various channels. These information needs to be analyzed and business decision need to made every moment. Some of these decisions are simpler and can be based on simple numeric information, such as, what quantity of goods should be stocked in a particular warehouse for next two weeks. On the other hand some decisions are much more complex and can not be based on some direct parameters.

In the following chapter a new model is presented based on the principle of fuzzy logic to predict the financial status (bankrupt/non-bankrupt) of a company based on its financial ratios.

CHAPTER VI

PROPOSED FUZZY LOGIC MODEL FOR DISTRESS ANALYSIS

The main focus of this work is to derive and implement a new methodology to predict bankruptcy (and also non-bankruptcy). The new methodology is based on the principles of fuzzy logic. The bankruptcy/non-bankruptcy prediction of this method is based upon a selection of records, using the following equation:

$$\mu = \alpha_{X1}W1 + \beta_{X2}W2 + \gamma_{X3}W3 + \delta_{X4}W4 + \eta_{X5}W5 \quad \ldots\ldots (1)$$

where, μ is the membership function,

X1, X2, X3, X4, and X5 are the independent variables

W1, W2, W3, W4 and W5 are the weights, and

α, β, γ, δ, and η are membership grade values extracted from trapezoidal membership functions.

A trapezoidal membership function has four parameters (A, B, C, D), as shown in Fig. 1, where $A \leq B \leq C \leq D$. Since our simulation is based upon sorted data, the trapezoid shown in Fig. 4 has been chosen as the membership function. Using this trapezoidal function, membership grade values α, β, γ, δ, and η, are calculated for all ratios (X1, X2, ..., X5) available as raw data. For simulation purpose, these ratios are sorted individually. Each sorted ratio helps our simulator select a point where the value of the ratio become greater or equal to 0 (parameter A shown in Fig. 4, or POINT1 shown in Table 2 of Appendix B). Similarly, a search is made for the points where each ratio becomes greater or equal to one.

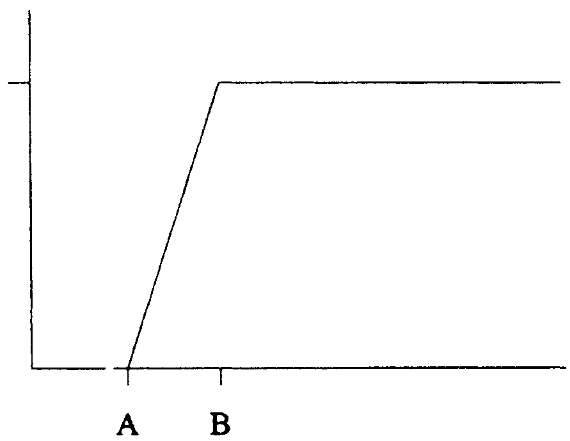

Figure 4. Trapezoidal membership function used for calculating membership grades α, β, γ, δ, and η.

W1, W2, W3, W4, and W5 in the above equation represent different weight factors associated with each membership grade value. These weights are multiplied with membership grade values to calculate the contribution of the ratio towards a company's bankruptcy (or non-bankruptcy).

The values of these weights depend upon the nature and the domain of the data. These weights can be different for data collected in different regions of the world, in different time periods, or different kinds of businesses (manufacturing, retail, banking, lending companies, etc.). The value of each weight (W1, W2, W3, W4, or W5) for its corresponding ratio (X1, X2, X3, X4, or X5) represents the contribution of that ratio in a company's bankruptcy. The larger the value of a weight, the greater the contribution of that ratio towards bankruptcy.

After multiplying the ratios with their respective weights, the resultant values are accumulated to get the final μ. Similarly, the membership value μ is calculated for all the companies in the database. These μ's are then sorted in ascending order. The middle record in the sorted ratios is selected as the threshold point, which distinguishly separates bankrupt from the non-bankrupt companies.

6.1 Implementation Tools

The implementation of the above method was carried out using a popular database package, *Foxbase+*. We selected this package for the following reasons:

* Most of the data manipulation tools are already provided in the package. Therefore the implementation was focused more on methodology than the data manipulation.
* The data provided by the companies is accessible with this package using minimum effort. and,

* It is easier for an end-user to analyze/modify data using this package than creating similar routines in a high-level programming language high-level languages such as C, ADA, Pascal, etc.

6.2 Organization of the Data

For our simulation only 2 database files (*.dbf) are used, known as Table 1 and Table 2 (Both of these tables are presented in Appendices B).

Table 1 has the following fields:

* X1, X2, X3, X4 and X5 are actual ratios. These have 9 digits width including 6 decimal and 2 integer digits.
* Alpha, Beta, Gamma, Delta, and Eta are membership grades, and have the same field width as X1, ..., X5.
* Status, a single digit field represents bankrupt company as 0 and non-bankrupt as 1.
* MU, a 9 digits width field which stores the final result,

Table 2 includes the following fields:

* Ratios, stores the different names of ratio (like X1, X2,..., X5) and has only 2 digits width.
* POINT1 and POINT2 are two fields of 9 digits width with 2 integer and 6 decimal places that store the crossover points for all ratios where the ratios touch or crossover 0 or 1.
* Finally, the Weight field, which is a 5 digit field with 2 digits for decimal and

2 digits for integer and stores the weight for the respective ratios.

There are 65 bankrupt and 64 non-bankrupt companies in the sample data which make 129 records as show in the Table 1. A total of 5 records in Table 2 exist showing one per ratio. Bankrupt companies are represented with status as 0 and non-bankrupt companies with status as 1.

6.3 User Interface

Our entire program is menu driven. When the program is executed, the main menu appears on the screen. The main menu has the following options:

A. START SIMULATION

This option causes the simulation to begin by sorting all ratios (X1, X2, ..., X5) in Table 1 one after another and extracting the points POINT1 and POINT2 for each ratio (See Appendix A for cutoff points POINT1 and POINT2 for all ratios). These points are then copied in front of their respective ratios, in Table 2.

B. CALCULATE MU

Most of the calculations of the simulator are performed choosing this option. Here, we calculate the membership grade values for each of the ratios in Table 1. For the ratios X1, X2, X3, X4, and X5 we calculate membership grade values Alpha, Beta, Gamma, Delta, and Eta respectively. After calculating the grade values, each of these grade value is multiplied with its respective weight (available in Table 2). Then these product values are summed and divided by the total of all the weights to get the final

μ. Division of μ with the total of all the weights is to normalize the μ. The equation to calculate these grade values is:

$$\alpha = (X1 - POINT1)/(POINT2 - POINT1) \quad \ldots\ldots\ldots \quad (2)$$

where, α is the membership grade value,

X1 is the value of ratio X1, and

POINT1 and POINT2 are crossover points for ratio X1 in Table 2.

Similar equations are used for calculating β, γ, δ, and η membership grade values with ratio X1 changed to X2, X3, X4, and X5 with their respective crossover points, POINT1 and POINT2, from Table 2.

After calculating α, β, γ, δ, and η for all the ratios in Table 1, the grade values are copied in front of each ratio in Table 1. Subsequently, these grade values are multiplied with their respective weights available in Table 2 and the resultant values are added together producing the final μ value.

C. HELP

This option assists the user in understanding the working of the simulator.

D. CHANGE WEIGHTS

This option allows the user to change the weights in Table 2, depending upon the data requirements. The new weights are then copied to the weight column of the Table 2.

E. CLEAN TABLE 2

This option allows the user to empty the POINT1 and POINT2 fields for the new raw data in Table 2. After executing, this option the user must choose option A on the main menu.

F. VIEW TABLE 1

This option allows the user to view the data in the file Table 1.

G. VIEW TABLE 2

This option allows the user to view the data in the file Table 2.

H. PRINT TABLE 1

This option allows the user to print the data in the file Table 1.

I. PRINT TABLE 2

This option allows the user to print the data in the file Table 2.

J. CORRECT RESULTS

In this option, the results from simulation are displayed on the screen. To achieve these results, the middle record is selected in Table 1 (already sorted on μ) as the threshold point (65th record out of 129 records). The threshold μ in our simulation is 0.315037. All records above this point are counted on the basis of 'status' field. We found that there are 59 records with status 0 (bankrupt) and 5 records with status 1 (non-bankrupt). Similarly, for the rest of the table (after 59th record), the percentage is calculated for bankrupt and non-bankrupt companies. Results of these calculations are given in next chapter.

K. STATUS OF A COMPANY

This option requests the user to enter the five ratios of the company to be examined. The simulator calculates the membership grade values α, β, γ, δ, and η for these ratios using their corresponding POINT1 and POINT2 from the Table 2 (See Graphs in Appendix A). These grade values are then multiplied with the weights of Table 2 resulting in the final value of μ. The simulator then checks the table 1 and compares the resulting μ with the other μ's available in Table 1. This option checks where the new μ is located in the list and gives the percentage possibility of bankruptcy of the company, depending on its position in the Table 1 (See Appendix B).

6.4 Working of the Simulator

Let us take an example of ratio X1 from Table 1. This ratio is first sorted in ascending order. Then, ignoring all the negative values in this attribute, a value is selected, known as POINT1 in Table 2. In Table 2, POINT1 for ratio X1 is 0.001100 as shown in the Appendix B. A search is then initiated for the POINT2 for the same ratio X1. The value selected this time is that which is greater than or equal to 1 for the same ratio X1.

As there is no value greater than or equal to 1 in ratio X1, the value of POINT2 is the last value of this sorted ratio. The value of POINT2 for the ratio X1 from Table 1 is 0.667400. Similarly sorting the ratios X2, X3, X4, and X5 and selecting the crossover points gives us the values of POINT1 and POINT2 respectively as:

for X2: POINT1 is 0.008500 and POINT2 is 0.697000,

for X3: POINT1 is 0.000600 and POINT2 is 0.299600,

for X4: POINT1 is 0.045300 and POINT2 is 1.006900,

and

for X5: POINT1 is 0.145100 and POINT2 is 1.031900.

Both of these points (POINT1 and POINT2) for different ratios are available, with their corresponding ratios, in Table 2.

Now to pick a value of X1 we choose 0.392200 from the first record (shown as shaded record in Table 1 and 3 of Appendix B). Placing these values in equation 2 results in:

$$\alpha = (0.392200 - 0.001100) / (0.667400 - 0.001100)$$
$$= 0.586973$$

Similarly, for the value of ratio X2 for the same record, β will be:

$$\beta = (0.377000 - 0.008500) / (0.697000 - 0.008500)$$

= 0.5363803

In the same manner γ, δ, and ζ are calculated for the ratios X3, X4 and X5 for the same record, yielding the following results:

$\gamma = 0.438127$

$\delta = 1.000000$

$\eta = 1.000000$

Here, the values of δ and ζ are 1 because in record number 1 the values of X4 and X5 are equal to or greater than 1.

Now we read the weights for all the ratios from Table 2. As shown in Table 2, for ratios X1, X2, X3, X4, and X5 we have weights 1, 6, 2, 2, and 0 respectively. We multiply these weights with their corresponding membership grades and then summed all the resultant values. To normalize the final μ, the summed value is then divided by the total of all the weights. For the first record in Table 1 (shown as shaded record), μ is calculated as:

$$\mu = \frac{(0.586973 * 1) + (0.536383 * 6) + (0.438127 * 2) + (1.00 * 2) + (1.00 * 0)}{(1 + 6 + 2 + 2 + 0)}$$

$= 0.607412$

In the same way, μ is calculated for all of the records in Table 1 The final table with all ratios, grade values, and MU is sorted on μ to make Table 1 ready for prediction.

CHAPTER VII

EXPERIMENTAL DESIGN AND RESULTS

7.1 Financial Ratios

Financial ratios are used for different kinds of purposes in business world. Some of these include: assessing the ability of a firm to pay its debts, evaluation of the business (measuring the profitability, liquidity, and solvency etc.), and managerial success etc. In our study, we used the financial ratios to assess the likelihood of a firm's failure (or bankruptcy). There is considerable debate in the literature as to which ratios are the most useful for predicting a firm's failure [Barnes87]. For our research we chose the same ratios which Altman [Altman68] used for his Z-score model. The Z-score model is a classic example of the use of financial ratios for bankruptcy predictions since these are still used on a fairly widespread basis amongst researchers and practitioners [Wilson93]. The ratios used in Z-score model are:

X_1: Working Capital / Total Assets

X_2: Retained Earning / Total Assets

X_3: Earning Before Interest and Taxes (EBIT) / Total Assets

X_4: Market Value of Equity / Total Debt

X_5: Sales / Total Assets

7.2 Data Collection

The same five financial ratios (as given in last section) have been used in our research. The sample data used in this thesis is same as Wilson and Sharda [Wilson93] used for their research on bankruptcy prediction using neural networks.

There are total 129 companies' financial ratios in the sample data which went bankrupt or remained non-bankrupt between 1975 to 1982. Among 129 companies there are 64 non-bankrupt companies and 65 bankrupt companies. The data is collected from the *Moody's Industrial Manuals*. Data used for bankrupt firms is from the last financial statements issued before the companies declared bankruptcy [Wilson 93]. Sample data used in this research is shown in Table 3 of Appendix B.

In order to check the effect of non-bankrupt companies' data on bankrupt companies, we arranged the data in 3 different data sets. The first data set was a 50/50 percent proportion of non-bankruptcy to bankruptcy cases. Similarly, second and third data sets were 80/20 (80% non-bankrupt and 20% bankrupt) and 90/10 (90% non-bankrupt and 10% bankrupt) percent proportion respectively. The data is arranged in this way to study the effect of different proportions of data on the predictive performance of our model.

7.3 Implementation Results

As discussed in Section 7.2, the financial data is arranged into three different data sets. This is shown in Table 4 (Appendix B) that for all three data sets the performance of conventional MDA model is better than the presented fuzzy logic based model for predicting non-bankruptcy.

Comparing the predictive performance of both models, for bankruptcy prediction, shows the better prediction power of presented fuzzy logic based model. For the data set 50/50, where 50 percent companies are non-bankrupt and 50 percent companies are bankrupt, our model performed better as compared to the MDA model (92.17% vs. 82.76%). Similarly for the second data set where data is arranged as 80 percent non-bankrupt companies and 20 percent bankrupt companies, fuzzy logic based distress analysis model still gives better results (76.12% vs. 60.92%). But for the last 90/10 data set, where non-bankrupt companies are 90 percent and bankrupt companies are 10 percent, conventional MDA model gives little better results than our model (See Table 4 of Appendix B).

From the above discussion it is evident that our model still performs better in predicting bankruptcy when the input data contains a better proportion of non-bankrupt companies' data.

Assigning appropriate weights to each ratio in our model is highly notable aspect of our model. Therefore a considerable amount of time is spent to selecting the proper weights for their respective ratios. Combination of weights showing the best performance in our model are shown in second table of Appendix B with their respective ratios. The values of these weights suggest the relative importance of their corresponding ratio in forecasting i.e. the greater the weight, the larger the contribution of that ratio. As show in the table 2 (See Appendix B) that the optimum weights assigned to ratios X1, X2, X3, X4, and X5 are 1, 2, 2, 6, and 0 respectively. These values show that the ratio X4 plays a major role in predicting bankruptcy/non-bankruptcy in our model. Also it is observed that the contribution of the ratio X5 is negligible, therefore zero weight was assign to it. This observation was conformed by the fact that a increased weight of ratio X5 resulted in decreased performance of our model. The usefulness other ratios X1, X2, and X3 was observed along the same line.

CHAPTER VIII

SUMMARY AND FUTURE WORK

8.1 Summary

The most significant contribution of our research is to present a fuzzy logic application approach to the distress analysis. A simulator is designed to predict the bankruptcy/non-bankruptcy for given companies. Sample data used in this simulator contains total 129 companies, 64 non-bankrupt and 65 bankrupt. This data is collected from the period 1975 to 1982 [Wilson93]. The companies included in the data represent both manufacturer and retailer businesses.

This work has compared the predictive capabilities of proposed fuzzy logic based distress analysis model with that of classical multiple discriminant analysis (MDA) model within the context of forecasting companies bankruptcies on the basis of their selected financial ratios. Concept of flexible weights is introduced in our model, which lets the user enhance or degrade the contribution of any of these financial ratios to improve the predictive capabilities. A step by step working of the simulator has been discussed in chapter VI.

The preliminary evaluation successfully demonstrated that an application of fuzzy logic to distress analysis can provide a more accurate and flexible prediction of a company's bankruptcy.

8.2 Future Work

There are several directions where future research endeavors might pursue.

First, as we discussed in the chapter four of literature review that there are different conventional multivariate models. Researchers, in these models, use different financial ratios in different numbers to get better results. In our distress analysis model we used only five financial ratios. This number can be increased to more than five ratios to get better results than our results, both for bankrupt and non-bankrupt cases.

Second, conventionally for different business situations (like: place of business, time period of business, size of business and kind of business, *etc.*), different distress models are designed. Instead of designing a complete different model for a new business situation, performance of our model can be checked by changing the cutoff points and corresponding weights for different ratios.

Third, there are different multivariate models for distress analysis (e.g. multiple regression model, non-parametric model, Logit or Probit *etc.*). The performance of our proposed fuzzy logic based distress analysis model can be compared statistically with these distress analysis models for in-depth understanding of performance of our model.

Fourth, our model can be used for purposes other than distress prediction. Situations where decisions are made on the basis of different criteria, our model can be used. The financial ratios in our model can be replaced with different factors of that particular situation. Such situations can be prediction of a company's growth, prediction of a new product success, *etc*.

Fifth, the potential problem with our model is that it is not designed to predict bankruptcy or non-bankruptcy several years before time. So this is a good field of research for future work to change this model for predicting the bankruptcy/non-bankruptcy before time. For this the initial raw data should also contain the data of several years.

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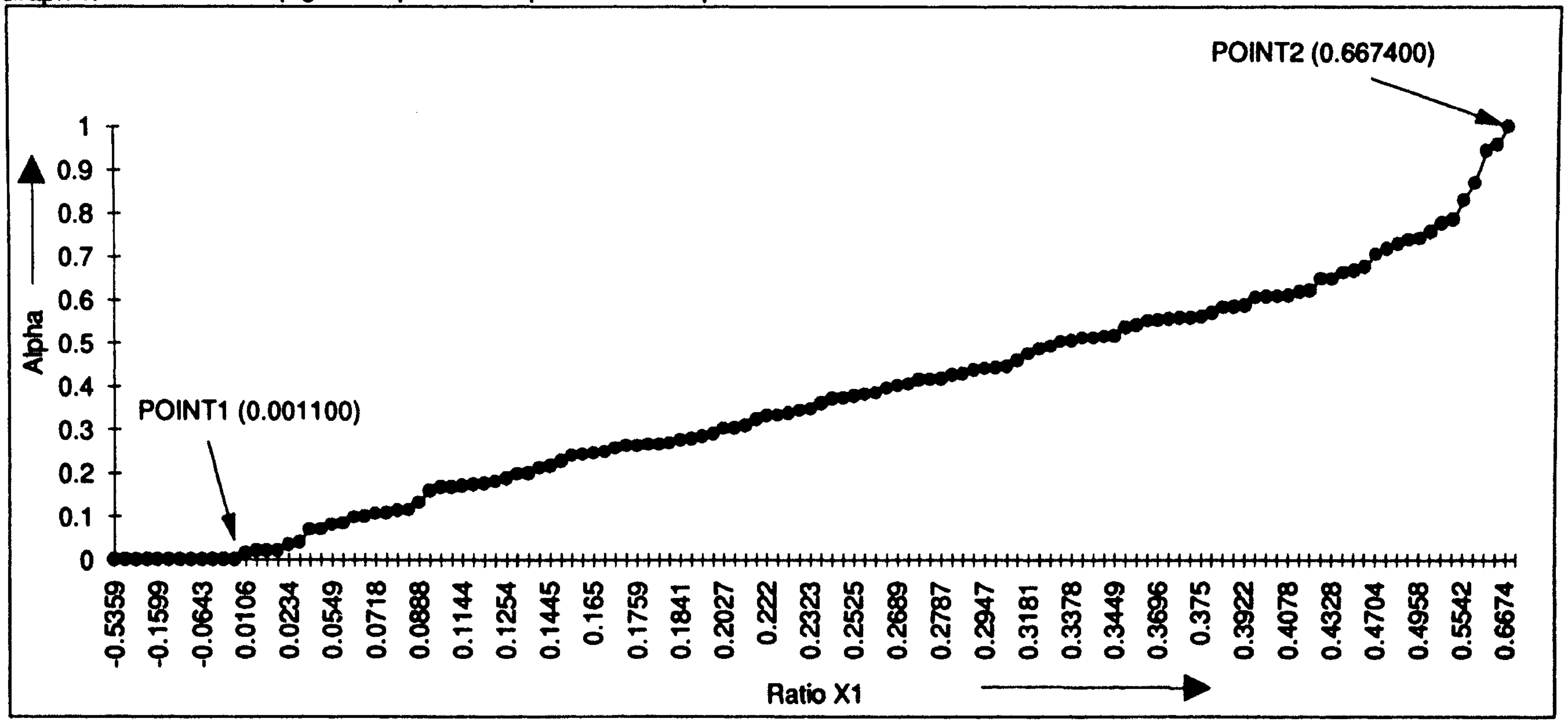
APPENDICES

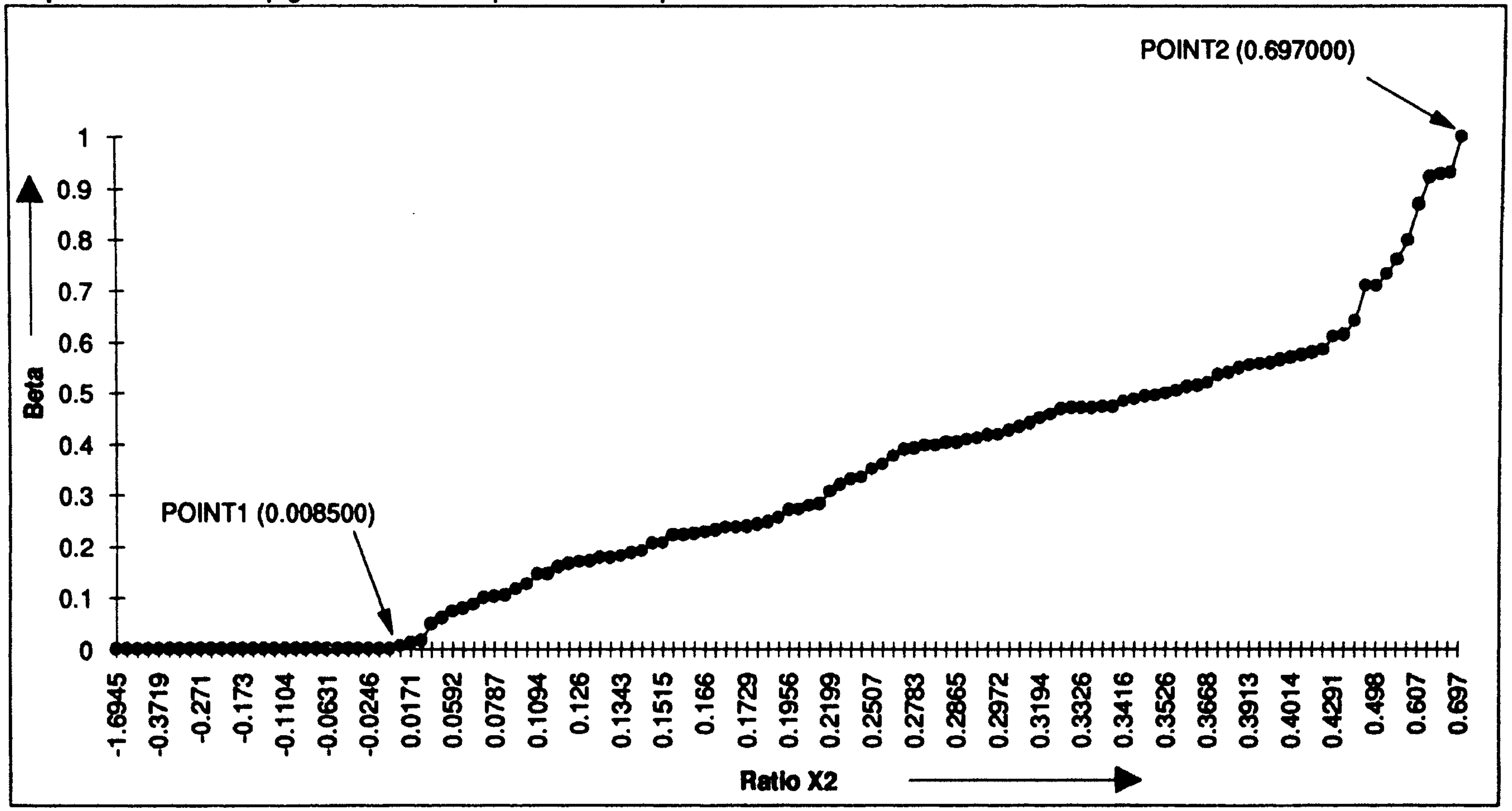
APPENDIX A

GRAPHS

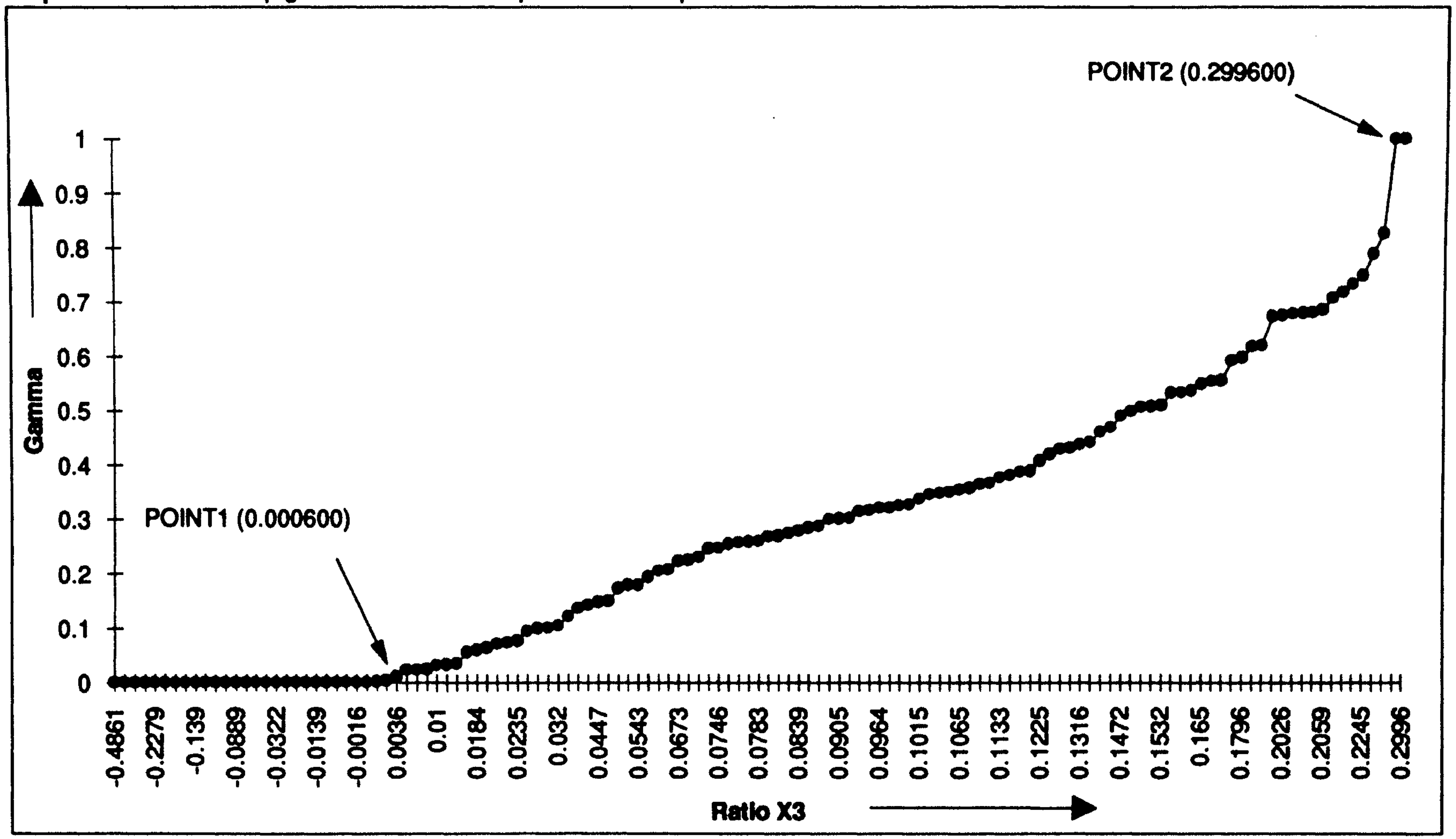
Graph 1. Membership grade Alpha and trapezoidal cutoff points for ratio X1



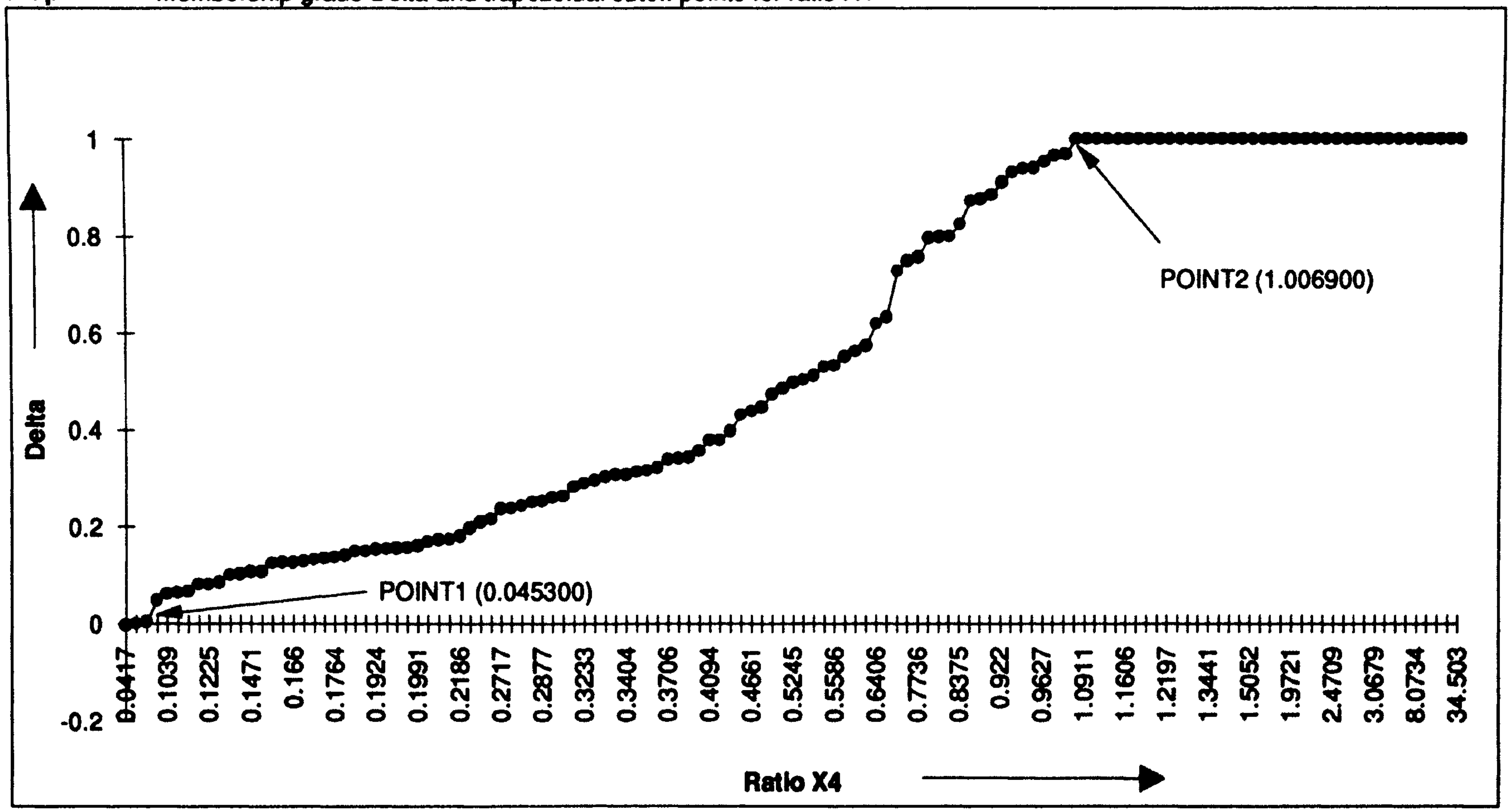
Graph2. Membership grade Beta and trapezoidal cutoff points for ratio X2



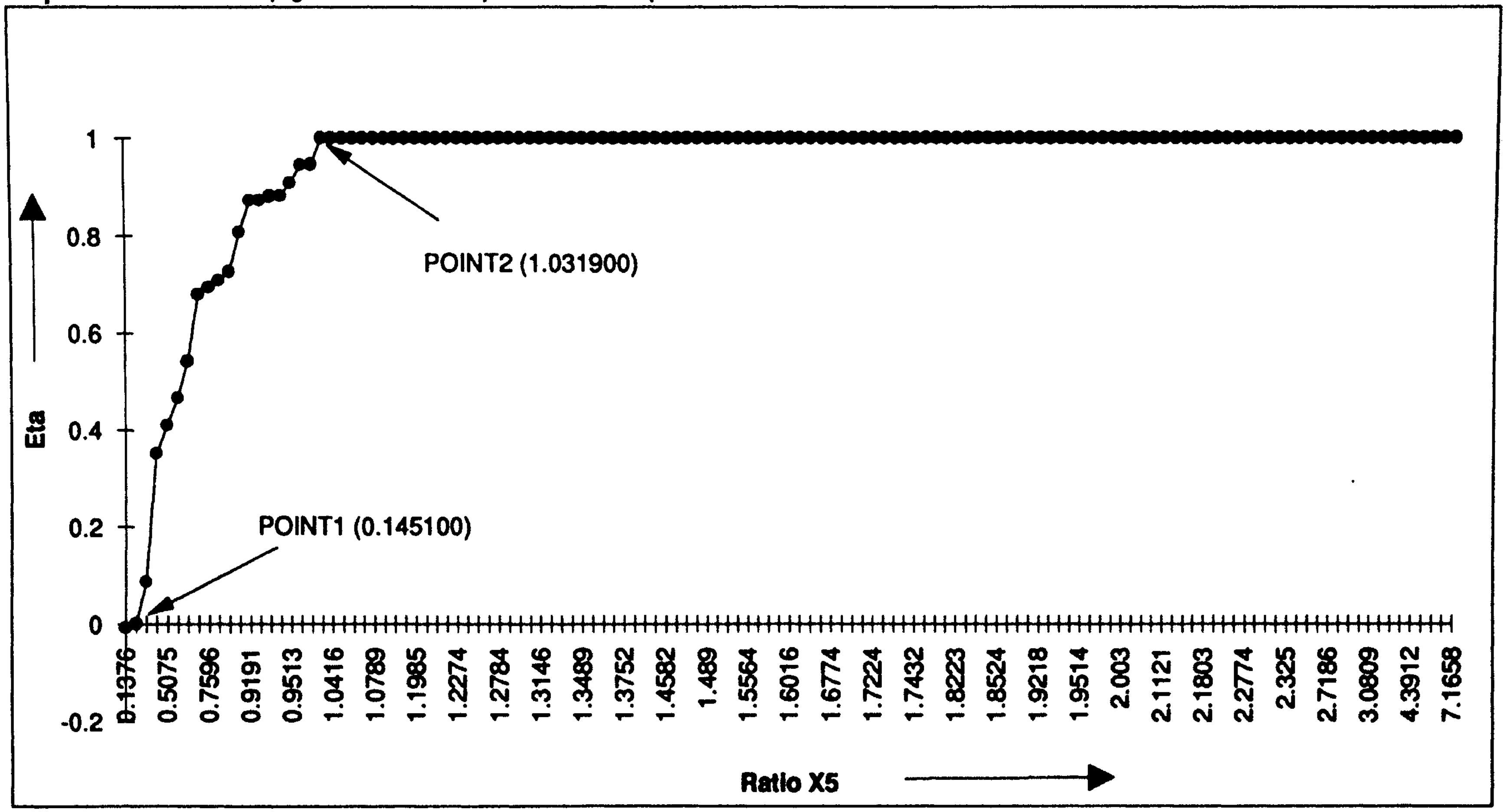
Graph3. Membership grade Gamma and trapezoidal cutoff points for ratio X3



Graph 4. Membership grade Delta and trapezoidal cutoff points for ratio X4



Graph 5. Membership grade Eta and trapezoidal cutoff poins for ratio X5



APPENDIX B

TABLES

Table 1. Financial ratios, membership grade values, status and the final MU (sorted)

REC. No.	X1	ALPHA	X2	BETA	X3	GAMMA	X4	DELTA	X5	ETA	STA	MU
107	-0.080100	0.000000	-0.083500	0.000000	0.003600	0.010033	0.048100	0.002912	0.773000	0.708051	0	0.002354
115	-0.090100	0.000000	-0.271000	0.000000	0.001400	0.002676	0.147300	0.106073	2.506400	1.000000	0	0.019773
102	0.014700	0.020411	-0.144300	0.000000	-0.049800	0.000000	0.143100	0.101705	6.514500	1.000000	0	0.020347
73	-0.159900	0.000000	-0.501800	0.000000	-0.088900	0.000000	0.174800	0.134671	2.160800	1.000000	0	0.024486
113	-0.129400	0.000000	0.008500	0.000000	-0.097100	0.000000	0.176400	0.136335	1.311300	1.000000	0	0.024788
101	0.066400	0.098004	-0.126600	0.000000	-0.155600	0.000000	0.147100	0.105865	3.619200	1.000000	0	0.028158
98	-0.310700	0.000000	-0.878000	0.000000	-0.296900	0.000000	0.194500	0.155158	1.049300	1.000000	0	0.028211
99	0.076600	0.113312	-0.073400	0.000000	0.007600	0.023411	0.168100	0.127704	1.078900	1.000000	0	0.037777
84	0.078000	0.115413	-0.245100	0.000000	0.062700	0.207692	0.045300	0.000000	0.145100	0.000000	0	0.048254
61	0.010600	0.014258	0.020000	0.016703	0.022600	0.073579	0.188700	0.149126	1.227400	1.000000	1	0.050899
120	0.088800	0.131622	-0.037100	0.000000	0.019700	0.063880	0.193100	0.153702	1.376700	1.000000	0	0.051526
80	-0.035700	0.000000	-0.981400	0.000000	-0.003100	0.000000	0.329100	0.295133	2.108800	1.000000	0	0.053661
105	0.054900	0.080744	0.059200	0.073638	-0.227900	0.000000	0.091300	0.047837	1.601600	1.000000	0	0.056204
124	0.252500	0.377308	-0.173000	0.000000	-0.486100	0.000000	0.165600	0.125104	1.444100	1.000000	0	0.057047
122	0.001100	0.000000	-0.063100	0.000000	-0.222500	0.000000	0.389100	0.357529	1.768000	1.000000	0	0.065005
54	0.027200	0.039172	0.050300	0.060712	0.018400	0.059532	0.141300	0.099834	1.200800	1.000000	1	0.065652
77	0.114400	0.170044	-0.019400	0.000000	0.007400	0.022742	0.294000	0.258631	1.573400	1.000000	0	0.066617
106	-0.535900	0.000000	-0.348700	0.000000	-0.032200	0.000000	0.459500	0.430740	0.919100	0.872801	0	0.078316
93	-0.064300	0.000000	0.109400	0.146550	-0.123000	0.000000	0.172500	0.132280	1.375200	1.000000	0	0.103987
125	0.318100	0.475762	-0.109300	0.000000	-0.085700	0.000000	0.375500	0.343386	1.978900	1.000000	0	0.105685
68	0.107000	0.158937	0.078700	0.101961	0.043300	0.142809	0.108300	0.065516	1.205100	1.000000	0	0.107941
108	0.329400	0.492721	0.017100	0.012491	0.037100	0.122074	0.287700	0.252080	3.138200	1.000000	0	0.119634
103	0.132100	0.196608	0.068600	0.087291	0.000800	0.000669	0.354400	0.321443	2.322400	1.000000	0	0.124053
65	0.047100	0.069038	0.150600	0.206391	-0.015000	0.000000	0.103900	0.060940	0.625300	0.541498	0	0.129933
95	0.047800	0.070089	0.063200	0.079448	-0.001600	0.000000	0.474400	0.446235	1.892800	1.000000	0	0.130841
114	0.202700	0.302566	-0.116900	0.000000	-0.026100	0.000000	0.596500	0.573211	0.789200	0.726319	0	0.131726
118	0.023400	0.033468	-0.024600	0.000000	0.032000	0.105017	0.640600	0.619072	1.109100	1.000000	0	0.134695
111	-0.277200	0.000000	0.161900	0.222803	-0.030200	0.000000	0.122500	0.080283	2.325000	1.000000	0	0.136126
70	0.161100	0.240132	0.095400	0.126216	0.030700	0.100669	0.211300	0.172629	1.452900	1.000000	0	0.140366
78	0.404400	0.605283	-0.187800	0.000000	0.076800	0.254849	0.284600	0.248856	1.348900	1.000000	0	0.146609
129	0.249600	0.372955	0.126000	0.170661	-0.247400	0.000000	0.166000	0.125520	3.095000	1.000000	0	0.149815
53	0.344900	0.515984	0.127000	0.172113	-0.008300	0.000000	0.105900	0.063020	0.861100	0.807397	1	0.152246

127	0.177700	0.265046	0.089100	0.117066	0.069500	0.230435	0.192400	0.152974	1.687100	1.000000	0	0.157660
100	0.389900	0.583521	0.080900	0.105156	0.044700	0.147492	0.218600	0.180220	0.927300	0.882048	0	0.169989
116	-0.375700	0.000000	-1.694500	0.000000	-0.450400	0.000000	1.219700	1.000000	2.268500	1.000000	0	0.181818
66	0.277000	0.414078	-0.041700	0.000000	0.090400	0.300334	0.524500	0.498336	1.938000	1.000000	0	0.182856
88	0.118600	0.176347	0.184900	0.256209	-0.071800	0.000000	0.211700	0.173045	0.137600	-0.008457	0	0.187245
69	0.193600	0.288909	0.077800	0.100654	-0.183000	0.000000	0.653100	0.632072	2.426300	1.000000	0	0.196089
97	0.268900	0.401921	0.172900	0.238780	0.028700	0.093980	0.122400	0.080179	0.927700	0.882499	0	0.198447
126	0.125400	0.186553	0.195600	0.271750	0.007900	0.024415	0.207300	0.168469	1.489000	1.000000	0	0.200257
92	0.442200	0.662014	0.137900	0.187945	0.010400	0.032776	0.246000	0.208715	1.249400	1.000000	0	0.206606
110	0.175900	0.262344	0.134300	0.182716	0.094600	0.314381	0.195500	0.156198	1.921800	1.000000	0	0.209073
94	0.297500	0.444845	-0.371900	0.000000	-0.139000	0.000000	0.962700	0.954035	2.277400	1.000000	0	0.213901
79	0.278700	0.416629	0.176700	0.244299	0.030500	0.100000	0.179700	0.139767	5.300300	1.000000	0	0.214723
112	0.255100	0.381210	-0.344200	0.000000	-0.110800	0.000000	1.221200	1.000000	2.281500	1.000000	0	0.216474
96	0.071800	0.106108	0.042200	0.048947	0.000600	0.000000	3.296400	1.000000	2.133100	1.000000	0	0.218163
74	0.112300	0.166892	0.228800	0.319971	0.010000	0.031438	0.188400	0.148814	2.718600	1.000000	0	0.222475
89	0.361700	0.541198	0.131200	0.178214	0.041300	0.136120	0.370600	0.338290	2.189000	1.000000	0	0.232664
119	0.357900	0.535495	0.151500	0.207698	0.081200	0.269565	0.199100	0.159942	1.458200	1.000000	0	0.240063
81	-0.017900	0.000000	-0.290200	0.000000	0.098400	0.327090	2.284800	1.000000	2.180300	1.000000	0	0.241289
128	0.240900	0.359898	0.166000	0.228758	0.074600	0.247492	0.251600	0.214538	1.852400	1.000000	0	0.241501
83	0.226000	0.337536	0.162000	0.222948	0.096500	0.320736	0.273700	0.237521	1.919900	1.000000	0	0.253794
91	0.232300	0.346991	0.109500	0.146696	0.105400	0.350502	0.466100	0.437604	0.919300	0.873027	0	0.254852
121	0.284500	0.425334	0.203800	0.283660	0.017100	0.055184	0.335700	0.301997	1.325800	1.000000	0	0.258332
117	0.342400	0.512232	-0.110400	0.000000	0.054100	0.178930	1.505200	1.000000	1.041600	1.000000	0	0.260917
67	0.495800	0.742458	0.219900	0.307044	0.021900	0.071237	0.126700	0.084651	3.030500	1.000000	0	0.263318
104	0.203900	0.304367	-0.047600	0.000000	0.126300	0.420401	0.896500	0.885191	1.045700	1.000000	0	0.265050
14	0.180100	0.268648	0.163500	0.225127	0.090800	0.301672	0.409400	0.378640	0.456600	0.351263	1	0.270912
57	0.407800	0.610386	0.131600	0.178794	0.109500	0.364214	0.323300	0.289101	1.822600	1.000000	1	0.271799
123	0.120900	0.179799	0.282300	0.397676	-0.011300	0.000000	0.315700	0.281198	2.321900	1.000000	0	0.284387
109	0.505600	0.757166	-0.195100	0.000000	0.202600	0.675585	0.538000	0.512375	1.951400	1.000000	0	0.284826
76	0.270200	0.403872	0.140200	0.191285	0.166800	0.555853	0.271700	0.235441	2.112100	1.000000	0	0.284925
86	0.344000	0.514633	0.172500	0.238199	0.138600	0.461538	0.277500	0.241473	2.003000	1.000000	0	0.304532
87	0.175600	0.261894	0.123300	0.166739	0.104600	0.347826	0.746800	0.729513	1.677400	1.000000	0	0.310637
30	0.222000	0.331532	0.179700	0.248656	0.152600	0.508361	0.345900	0.312604	1.723700	1.000000	1	0.315037
64	0.292100	0.436740	0.239000	0.334786	0.067300	0.223077	0.340200	0.306676	0.759600	0.692941	1	0.318633
56	0.217000	0.324028	0.250700	0.351779	0.082600	0.274247	0.340400	0.306884	1.988900	1.000000	1	0.326997

24	0.014100	0.019511	0.236600	0.331300	0.090500	0.300669	0.586300	0.562604	1.465100	1.000000	1	0.339441
72	0.265300	0.396518	0.268300	0.377342	0.023500	0.076589	0.511800	0.485129	1.835000	1.000000	0	0.344001
60	0.073200	0.108210	0.352600	0.499782	0.058700	0.194314	0.234900	0.197171	1.743200	1.000000	1	0.353625
44	0.190400	0.284106	0.201100	0.279739	0.132900	0.442475	0.558600	0.533798	1.662300	1.000000	1	0.355917
49	0.163000	0.242984	0.355500	0.503994	0.011000	0.034783	0.373000	0.340786	2.830700	1.000000	1	0.365281
8	0.337800	0.505328	0.013000	0.006536	0.236600	0.789298	2.470900	1.000000	1.223000	1.000000	1	0.374831
71	0.373200	0.558457	0.348300	0.493537	-0.013900	0.000000	0.348300	0.315100	1.822300	1.000000	0	0.377262
3	0.165000	0.245985	0.119200	0.160784	0.203500	0.678595	0.813000	0.798357	1.670200	1.000000	1	0.378600
32	0.277600	0.414978	0.256700	0.360494	0.161200	0.537124	0.296800	0.261543	1.890400	1.000000	1	0.379570
13	0.207300	0.309470	0.361100	0.512128	0.147200	0.490301	0.041700	-0.003744	1.198500	1.000000	1	0.395941
82	0.406700	0.608735	0.297200	0.419317	0.045400	0.149833	0.500070	0.472931	2.063100	1.000000	0	0.397288
35	0.186200	0.277803	0.168700	0.232680	0.129800	0.432107	0.949800	0.940620	4.954800	1.000000	1	0.401758
16	0.230400	0.344139	0.296000	0.417574	0.122500	0.407692	0.410200	0.379472	3.080900	1.000000	1	0.402174
85	0.342200	0.511932	0.286500	0.403776	0.077800	0.258194	0.530000	0.504056	1.556400	1.000000	0	0.405372
11	0.470400	0.704337	0.277200	0.390269	0.096400	0.320401	0.426800	0.396735	1.931700	1.000000	1	0.407293
20	0.368400	0.551253	0.391300	0.555991	0.052400	0.173244	0.165800	0.125312	1.153300	1.000000	1	0.407665
75	0.369600	0.553054	0.291700	0.411329	0.062100	0.205686	0.555400	0.530470	1.732600	1.000000	0	0.408485
52	0.066900	0.098754	0.290400	0.409441	0.097800	0.325084	0.765900	0.749376	4.391200	1.000000	1	0.427665
26	0.014000	0.019361	0.286200	0.403341	0.074100	0.245819	30.648600	1.000000	1.960600	1.000000	1	0.448277
6	0.141500	0.210716	0.386800	0.549455	0.068100	0.225753	0.575500	0.551373	1.057900	1.000000	1	0.460154
21	0.152700	0.227525	0.334400	0.473348	0.078300	0.259866	0.773600	0.757384	1.504600	1.000000	1	0.463828
90	0.116200	0.172745	0.302600	0.427160	0.086300	0.286622	0.922000	0.911710	0.951300	0.909111	0	0.466579
2	0.057400	0.084496	0.278300	0.391866	0.116600	0.387960	1.344100	1.000000	0.221600	0.086265	1	0.473783
23	0.112600	0.167342	0.307100	0.433696	0.083900	0.278595	1.342900	1.000000	1.573600	1.000000	1	0.484246
38	0.172500	0.257241	0.323800	0.457952	0.104000	0.345819	0.884700	0.872920	0.557600	0.465156	1	0.494767
39	0.295500	0.441843	0.195900	0.272186	0.224500	0.748829	1.160600	1.000000	1.847800	1.000000	1	0.506602
59	0.184100	0.274651	0.334400	0.473348	0.085700	0.284615	2.123000	1.000000	2.168600	1.000000	1	0.516724
62	0.639800	0.958577	0.172300	0.237908	0.201900	0.673244	34.503201	1.000000	1.138800	1.000000	1	0.521138
15	0.177800	0.265196	0.366800	0.520407	0.077900	0.258528	0.974200	0.965994	0.507500	0.408660	1	0.530607
51	0.479200	0.717545	0.349500	0.495280	0.107600	0.357860	0.810500	0.795757	1.722400	1.000000	1	0.545132
29	0.133200	0.198259	0.407700	0.579811	0.054300	0.179599	1.492100	1.000000	1.482600	1.000000	1	0.548757
58	0.286400	0.428186	0.282300	0.397676	0.185600	0.618729	2.770900	1.000000	2.773000	1.000000	1	0.550155
63	0.375000	0.561159	0.332600	0.470733	0.129000	0.429431	0.948700	0.939476	1.252900	1.000000	1	0.556670
12	0.580400	0.869428	0.333100	0.471460	0.081000	0.268896	1.196400	1.000000	1.357200	1.000000	1	0.566907
47	0.294700	0.440642	0.392400	0.557589	0.110400	0.367224	0.941000	0.931468	1.356800	1.000000	1	0.580324

37	0.223400	0.333633	0.393100	0.558606	0.116800	0.388629	1.137100	1.000000	1.752300	1.000000	1	0.587502
50	0.518900	0.777127	0.362700	0.514452	0.101500	0.337458	0.976400	0.968282	0.746600	0.678281	1	0.588665
25	0.413500	0.618940	0.312000	0.440813	0.186100	0.620401	1.174300	1.000000	1.031900	1.000000	1	0.591329
42	0.381300	0.570614	0.319400	0.451561	0.204400	0.681605	2.851300	1.000000	0.985100	0.947226	1	0.603927
33	0.144500	0.215218	0.380800	0.540741	0.178000	0.593311	1.479600	1.000000	1.481100	1.000000	1	0.604208
31	0.372000	0.556656	0.344600	0.488163	0.212400	0.708361	0.888800	0.877184	1.924100	1.000000	1	0.605157
36	0.166300	0.247936	0.429100	0.610893	0.113300	0.376923	1.174500	.1.000000	1.683100	1.000000	1	0.606104
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28	0.493400	0.738856	0.341600	0.483805	0.220000	0.733779	0.814400	0.799813	2.193700	1.000000	1	0.609898
7	0.336300	0.503077	0.331200	0.468700	0.215700	0.719398	3.067900	1.000000	2.089900	1.000000	1	0.614007
55	0.630200	0.944169	0.332400	0.470443	0.152400	0.507692	1.125900	1.000000	1.557900	1.000000	1	0.616565
40	0.554200	0.830107	0.431600	0.614524	0.106500	0.354181	0.837500	0.823835	1.667800	1.000000	1	0.624844
41	0.248900	0.371905	0.401400	0.570661	0.166900	0.556187	1.460900	1.000000	7.165800	1.000000	1	0.628022
43	0.451200	0.675522	0.411400	0.585185	0.114600	0.381271	1.718500	1.000000	1.554300	1.000000	1	0.631743
22	0.414700	0.620741	0.398300	0.566158	0.153200	0.510368	1.314800	1.000000	1.374500	1.000000	1	0.639857
46	0.405800	0.607384	0.449700	0.640813	0.149700	0.498662	1.107600	1.000000	1.742800	1.000000	1	0.677235
10	0.445500	0.666967	0.498000	0.710966	0.095200	0.316388	1.933800	1.000000	1.769600	1.000000	1	0.687776
18	0.667400	1.000000	0.404700	0.575454	0.179600	0.598662	1.006900	1.000000	1.296800	1.000000	1	0.695459
27	0.373500	0.558907	0.498000	0.710966	0.160400	0.534448	1.836600	1.000000	2.379300	1.000000	1	0.717600
5	0.257400	0.384662	0.533400	0.762382	0.165000	0.549833	8.073400	1.000000	1.347400	1.000000	1	0.732602
19	0.325500	0.486868	0.558300	0.798548	0.160000	0.533110	2.288900	1.000000	1.314600	1.000000	1	0.758579
17	0.432800	0.647906	0.513600	0.733624	0.205900	0.686622	1.972100	1.000000	1.319400	1.000000	1	0.765718
4	0.307300	0.459553	0.607000	0.869281	0.204000	0.680268	14.409000	1.000000	0.984400	0.946437	1	0.821434
34	0.390700	0.584722	0.648200	0.929121	0.140800	0.468896	3.048900	1.000000	1.525500	1.000000	1	0.827022
45	0.524800	0.785982	0.643700	0.922585	0.247800	0.826756	6.350100	1.000000	1.254200	1.000000	1	0.906819
48	0.432700	0.647756	0.649400	0.930864	0.299600	1.000000	8.298200	1.000000	1.286500	1.000000	1	0.930267
9	0.487000	0.729251	0.697000	1.000000	0.299400	0.999331	5.438300	1.000000	1.720000	1.000000	1	0.975265

Table 2. Ratios, cutoff points and weights			
RATIOS	POINT1	POINT2	WEIGHT
X1	0.001100	0.667400	1.00
X2	0.008500	0.697000	6.00
X3	0.000600	0.299600	2.00
X4	0.045300	1.006900	2.00
X5	0.145100	1.031900	0.00

Table 3. Raw data with non-bankrupt (1) and bankrupt (0) companies						
REC. No.	X1	X2	X3	X4	X5	STATUS
[redacted]						
2	0.057400	0.278300	0.116600	1.344100	0.221600	1
3	0.165000	0.119200	0.203500	0.813000	1.670200	1
4	0.307300	0.607000	0.204000	14.409000	0.984400	1
5	0.257400	0.533400	0.165000	8.073400	1.347400	1
6	0.141500	0.386800	0.068100	0.575500	1.057900	1
7	0.336300	0.331200	0.215700	3.067900	2.089900	1
8	0.337800	0.013000	0.236600	2.470900	1.223000	1
9	0.487000	0.697000	0.299400	5.438300	1.720000	1
10	0.445500	0.498000	0.095200	1.933800	1.769600	1
11	0.470400	0.277200	0.096400	0.426800	1.931700	1
12	0.580400	0.333100	0.081000	1.196400	1.357200	1
13	0.207300	0.361100	0.147200	0.041700	1.198500	1
14	0.180100	0.163500	0.090800	0.409400	0.456600	1
15	0.177800	0.366800	0.077900	0.974200	0.507500	1
16	0.230400	0.296000	0.122500	0.410200	3.080900	1
17	0.432800	0.513600	0.205900	1.972100	1.319400	1
18	0.667400	0.404700	0.179600	1.006900	1.296800	1
19	0.325500	0.558300	0.160000	2.288900	1.314600	1
20	0.368400	0.391300	0.052400	0.165800	1.153300	1
21	0.152700	0.334400	0.078300	0.773600	1.504600	1
22	0.414700	0.398300	0.153200	1.314800	1.374500	1
23	0.112600	0.307100	0.083900	1.342900	1.573600	1
24	0.014100	0.236600	0.090500	0.586300	1.465100	1
25	0.413500	0.312000	0.186100	1.174300	1.031900	1
26	0.014000	0.286200	0.074100	30.648600	1.960600	1
27	0.373500	0.498000	0.160400	1.836600	2.379300	1
28	0.493400	0.341600	0.220000	0.814400	2.193700	1
29	0.133200	0.407700	0.054300	1.492100	1.482600	1
30	0.222000	0.179700	0.152600	0.345900	1.723700	1
31	0.372000	0.344600	0.212400	0.888800	1.924100	1
32	0.277600	0.256700	0.161200	0.296800	1.890400	1
33	0.144500	0.380800	0.178000	1.479600	1.481100	1
34	0.390700	0.648200	0.140800	3.048900	1.525500	1
35	0.186200	0.168700	0.129800	0.949800	4.954800	1
36	0.166300	0.429100	0.113300	1.174500	1.683100	1
37	0.223400	0.393100	0.116800	1.137100	1.752300	1
38	0.172500	0.323800	0.104000	0.884700	0.557600	1
39	0.295500	0.195900	0.224500	1.160600	1.847800	1
40	0.554200	0.431600	0.106500	0.837500	1.667800	1
41	0.248900	0.401400	0.166900	1.460900	7.165800	1

42	0.381300	0.319400	0.204400	2.851300	0.985100	1
43	0.451200	0.411400	0.114600	1.718500	1.554300	1
44	0.190400	0.201100	0.132900	0.558600	1.662300	1
45	0.524800	0.643700	0.247800	6.350100	1.254200	1
46	0.405800	0.449700	0.149700	1.107600	1.742800	1
47	0.294700	0.392400	0.110400	0.941000	1.356800	1
48	0.432700	0.649400	0.299600	8.298200	1.286500	1
49	0.163000	0.355500	0.011000	0.373000	2.830700	1
50	0.518900	0.362700	0.101500	0.976400	0.746600	1
51	0.479200	0.349500	0.107600	0.810500	1.722400	1
52	0.066900	0.290400	0.097800	0.765900	4.391200	1
53	0.344900	0.127000	-0.008300	0.105900	0.861100	1
54	0.027200	0.050300	0.018400	0.141300	1.200800	1
55	0.630200	0.332400	0.152400	1.125900	1.557900	1
56	0.217000	0.250700	0.082600	0.340400	1.988900	1
57	0.407800	0.131600	0.109500	0.323300	1.822600	1
58	0.286400	0.282300	0.185600	2.770900	2.773000	1
59	0.184100	0.334400	0.085700	2.123000	2.168600	1
60	0.073200	0.352600	0.058700	0.234900	1.743200	1
61	0.010600	0.020000	0.022600	0.188700	1.227400	1
62	0.639800	0.172300	0.201900	34.503201	1.138800	1
63	0.375000	0.332600	0.129000	0.948700	1.252900	1
64	0.292100	0.239000	0.067300	0.340200	0.759600	1
65	0.047100	0.150600	-0.015000	0.103900	0.625300	0
66	0.277000	-0.041700	0.090400	0.524500	1.938000	0
67	0.495800	0.219900	0.021900	0.126700	3.030500	0
68	0.107000	0.078700	0.043300	0.108300	1.205100	0
69	0.193600	0.077800	-0.183000	0.653100	2.426300	0
70	0.161100	0.095400	0.030700	0.211300	1.452900	0
71	0.373200	0.348300	-0.013900	0.348300	1.822300	0
72	0.265300	0.268300	0.023500	0.511800	1.835000	0
73	-0.159900	-0.501800	-0.088900	0.174800	2.160800	0
74	0.112300	0.228800	0.010000	0.188400	2.718600	0
75	0.369600	0.291700	0.062100	0.555400	1.732600	0
76	0.270200	0.140200	0.166800	0.271700	2.112100	0
77	0.114400	-0.019400	0.007400	0.294000	1.573400	0
78	0.404400	-0.187800	0.076800	0.284600	1.348900	0
79	0.278700	0.176700	0.030500	0.179700	5.300300	0
80	-0.035700	-0.981400	-0.003100	0.329100	2.108800	0
81	-0.017900	-0.290200	0.098400	2.284800	2.180300	0
82	0.406700	0.297200	0.045400	0.500070	2.063100	0
83	0.226000	0.162000	0.096500	0.273700	1.919900	0
84	0.078000	-0.245100	0.062700	0.045300	0.145100	0
85	0.342200	0.286500	0.077800	0.530000	1.556400	0

86	0.344000	0.172500	0.138600	0.277500	2.003000	0
87	0.175600	0.123300	0.104600	0.746800	1.677400	0
88	0.118600	0.184900	-0.071800	0.211700	0.137600	0
89	0.361700	0.131200	0.041300	0.370600	2.189000	0
90	0.116200	0.302600	0.086300	0.922000	0.951300	0
91	0.232300	0.109500	0.105400	0.466100	0.919300	0
92	0.442200	0.137900	0.010400	0.246000	1.249400	0
93	-0.064300	0.109400	-0.123000	0.172500	1.375200	0
94	0.297500	-0.371900	-0.139000	0.962700	2.277400	0
95	0.047800	0.063200	-0.001600	0.474400	1.892800	0
96	0.071800	0.042200	0.000600	3.296400	2.133100	0
97	0.268900	0.172900	0.028700	0.122400	0.927700	0
98	-0.310700	-0.878000	-0.296900	0.194500	1.049300	0
99	0.076600	-0.073400	0.007600	0.168100	1.078900	0
100	0.389900	0.080900	0.044700	0.218600	0.927300	0
101	0.066400	-0.126600	-0.155600	0.147100	3.619200	0
102	0.014700	-0.144300	-0.049800	0.143100	6.514500	0
103	0.132100	0.068600	0.000800	0.354400	2.322400	0
104	0.203900	-0.047600	0.126300	0.896500	1.045700	0
105	0.054900	0.059200	-0.227900	0.091300	1.601600	0
106	-0.535900	-0.348700	-0.032200	0.459500	0.919100	0
107	-0.080100	-0.083500	0.003600	0.048100	0.773000	0
108	0.329400	0.017100	0.037100	0.287700	3.138200	0
109	0.505600	-0.195100	0.202600	0.538000	1.951400	0
110	0.175900	0.134300	0.094600	0.195500	1.921800	0
111	-0.277200	0.161900	-0.030200	0.122500	2.325000	0
112	0.255100	-0.344200	-0.110800	1.221200	2.281500	0
113	-0.129400	0.008500	-0.097100	0.176400	1.311300	0
114	0.202700	-0.116900	-0.026100	0.596500	0.789200	0
115	-0.090100	-0.271000	0.001400	0.147300	2.506400	0
116	-0.375700	-1.694500	-0.450400	1.219700	2.268500	0
117	0.342400	-0.110400	0.054100	1.505200	1.041600	0
118	0.023400	-0.024600	0.032000	0.640600	1.109100	0
119	0.357900	0.151500	0.081200	0.199100	1.458200	0
120	0.088800	-0.037100	0.019700	0.193100	1.376700	0
121	0.284500	0.203800	0.017100	0.335700	1.325800	0
122	0.001100	-0.063100	-0.222500	0.389100	1.768000	0
123	0.120900	0.282300	-0.011300	0.315700	2.321900	0
124	0.252500	-0.173000	-0.486100	0.165600	1.444100	0
125	0.318100	-0.109300	-0.085700	0.375500	1.978900	0
126	0.125400	0.195600	0.007900	0.207300	1.489000	0
127	0.177700	0.089100	0.069500	0.192400	1.687100	0
128	0.240900	0.166000	0.074600	0.251600	1.852400	0
129	0.249600	0.126000	-0.247400	0.166000	3.095000	0

Table 4. Comparision of results				
Data sets (%)	Non-bankrupt cases (MDA)	Non-Bankrupt cases (FL)	Bankrupt cases (MDA)	Bankrupt cases (FL)
50/50	94.54	90.63	82.76	92.17
80/20	97.69	94.12	60.91	76.12
90/10	99.13	94.83	54.67	50.00

VITA 2

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Master of Science

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