ALGORITHM TO AUTOMATICALLY DETECT STRUCTURE SYMMETRY AND APPLY SYMMETRY CONCEPTS IN STRUCTURAL ANALYSIS

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CHAPTER I

INTRODUCTION

1.1 Background

1.1.1 Matrix Analysis of Structures

One of the most important topics in the engineering field is structural analysis. There are many methods for structural analysis which can be classified under three general categories [1] :

- Classical methods
- Approximate methods
- Matrix methods

Classical methods [2] of structural analysis were intended for hand computation and the developers of these tools took great pains to minimize the amount of calculations necessary to solve a given problem, even at the expense of making the methods somewhat unsystematic. Although these difficulties are easily handled by an experienced analyst during hand computation, they make the classical methods unattractive for translation to a computer code.

Approximate methods [2] involve imposing special conditions on a complex structure so that it is sufficiently simplified to allow an approximate hand result to be obtained relatively easily.

Matrix methods currently are the most widely used methods among the producers of the most prominent structural analysis software [4-9]. A major feature that is evident in matrix structural analysis is an emphasis on a systematic approach to the statement of the problem. Matrix notation turns out to be convenient to use in this connection because of its shorthand characteristics. Furthermore, the systematic approach together with matrix notation makes it particularly convenient to translate the statement of the problem to a computer language.

It is also important to recognize that the concepts of matrix analysis of a structure under static load, which is the main concern for most applications, can be extended to the solution of many other classes of structural problems. These classes of problems include dynamic response, material and geometric nonlinearities, inelasticity, instability, and continuous systems (finite element methods). Furthermore, the same concepts can be applied to problems from other areas of engineering, such as geotechnics, hydraulics, and heat transfer, as well as to problems outside of engineering altogether [1].

1.1.2 Time Consumption in Matrix Analysis Methods

Structural analysis may be broken down into five items [3] :

1. *Basic mechanics.* The fundamental relationships of stress and strain, compatibility, and equilibrium.
2. *Finite element mechanics.* The exact or approximate solution of the differential equations of the element
3. *Equation formulation.* The establishment of the governing algebraic equations of the system.
4. *Equation solution.* Computational methods and algorithms.
5. *Solution interpretation.* The presentation of results in a form useful in design.

Matrix methods for structural analysis deal chiefly with items 3 to 5 of the above process. This is the approach to these items that currently seems to be most suitable for automation of the equation-formulation process and for taking advantage of the powerful capabilities of the electronic digital computer in solving large-order systems of equations.

Item 4, equation solution, turns out to be the most time-consuming step in the computer execution time in solving a structure problem [3]. "*For the larger problems common in practice, it has been estimated that 20 to 50 percent of the computer execution time may be devoted to solving sets of linear simultaneous equations. This figure may rise to about 80 percent in dynamic, nonlinear, or structural optimization problems*" [3]. This step involves the solution of a large set of independent simultaneous equations. There are many numerical methods to solve the set of simultaneous equations:

$$[\mathbf{K}]\{\mathbf{Q}\} = \{\mathbf{F}\} \qquad (1.1)$$

Where $\{\mathbf{Q}\}$ is the vector of unknowns, usually the unknown displacements of the structure, $\{\mathbf{F}\}$ is the constant coefficient or the force vector, and $[\mathbf{K}]$ is a positive definite, symmetric, often sparse matrix which is the stiffness matrix of the structure.

The most widely used methods in solving this set of equations in the structural analysis field are the elimination methods, which include, for example, Gaussian Elimination, and The Cholesky Method [1, 3, 19].

For the Gaussian Elimination method, the solution time for a general set of simultaneous equations is proportional to the third power of the number of equations, or the number of degrees of freedom in the structure. That is, the time to solve a structure with n degrees of freedom is proportional to $\{n^3\}$ [1]. In other words, the total number of multiplications or divisions to solve an n by n system through Gaussian elimination equals [21] :

$$n^3/3 + n^2 - n/3 \qquad (1.2)$$

Thus for large n, the sums that give the count of operations are dominated by the first terms and it will be in the order of $\{n^3\}$. It is noted that only multiplications or divisions are counted because these operations are generally much slower than additions or subtractions [21].

For the Cholesky Method, the total number of multiplications or divisions in solving a system of n equations is in the order of the $\{n^2\}$ [20, 21]. Also, the solution time is proportional to

$$[(\beta + 1)(2n - \beta)] \quad (1.3)$$

when using band methods where β is the bandwidth, and is proportional to

$$[2 * (\varphi + n)] \quad (1.4)$$

when using profile methods where φ is the profile which in turn is a function of on $\{n^2\}$ [10, 22].

1.1.3 Optimization Methods in Matrix Analysis

Several methods are used to minimize the solution time in matrix analysis. Some of those methods depend on a mathematical algorithm to decrease the solution time for the set of simultaneous equations [11, 12]. Others depend on reducing the number of degrees of freedom of the structure which is the number of unknowns in the set of equations (1.1) [1, 13]. The latter approach is more related to structural analysis and will be discussed in detail in section 1.3.

1.1.4 Symmetry as One Method of Optimization

Taking advantage of structural symmetry is one method of optimizing matrix methods. In utilizing symmetry, the number of degrees of freedom of the structure is reduced, thus the number of simultaneous equations to be solved using one of the numerical methods is also reduced.

1.2 Structural Symmetry

1.2.1 Definition

Structural symmetry involves both the innate physical symmetry of the structure itself and the symmetry of the load applied thereon. For the structure to be classified as symmetric, one half of the structure must be a mirror image of the other half in terms of three characteristics [1] :

- Configuration .
- Distribution of material properties .
- Arrangement of constraints .

The line that divides one half of the structure from the other half (the line along which the imaginary mirror lies) is referred to as its *axis of symmetry.*

If the loading on one half of the structure is also a mirror image about this axis of the loading on the other half, the loading is also classified as symmetric. It is demonstrated in Figure 1.1 that when a symmetric loading is applied to a symmetric structure, the structure distorts in such a way that its distorted shape remains symmetric also - Figure 1.1b. In other words,

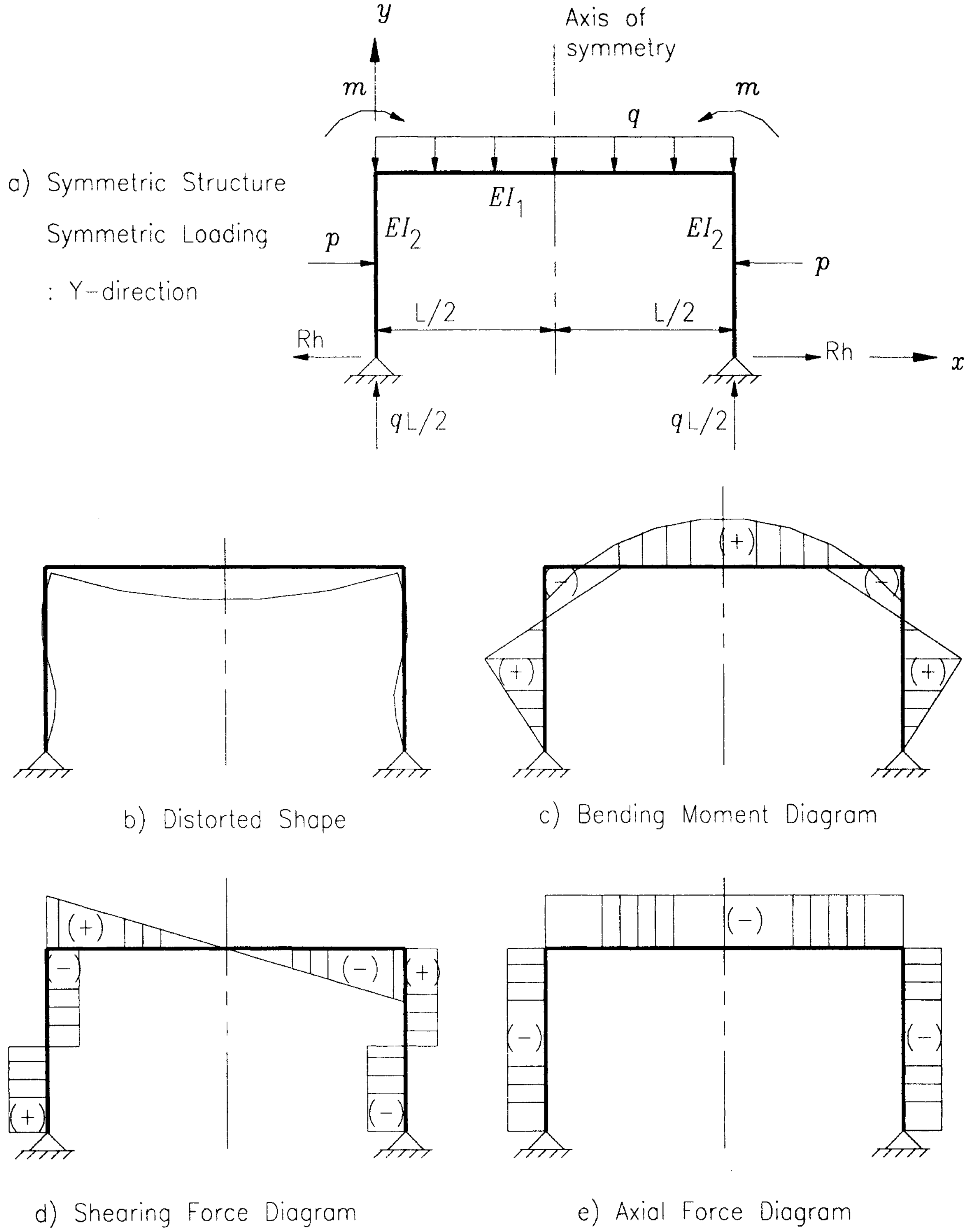


Figure 1.1. Example of A Symmetric Structure With Symmetric Loadings.

with reference to the axis of symmetry, mirror-image distortions occur at corresponding points on each half of the structure. It follows that some of the internal forces (bending moment and axial force) of a symmetric structure subjected to a symmetric loading will also be symmetric [Figure 1.1c and e], while the other internal forces (shear force) will be antisymmetric [Figure 1.1d]. Note that both the transitional reactions parallel and perpendicular to the axis of symmetry (vertical and horizontal reaction) are symmetric on both sides of the structure.

It is also possible for either the structure or the loading, or both, to be antisymmetric in nature. If the loading on one half of a symmetric structure is a reverse mirror image about the axis of symmetry of the loading on the other half, the loading is said to be antisymmetric. It can be shown that when an antisymmetric loading is applied to a symmetric structure, reverse mirror image distortions occur at corresponding points on each half of the structure with reference to the axis of symmetry [Figure 1.2b]. It follows that some of the internal forces (bending moment and axial force) of a symmetric structure subjected to an antisymmetric loading will be antisymmetric [Figure 1.2c and e] and the other internal forces (shear force) will be symmetric [Figure 1.2d]. Note also that both the reactions are

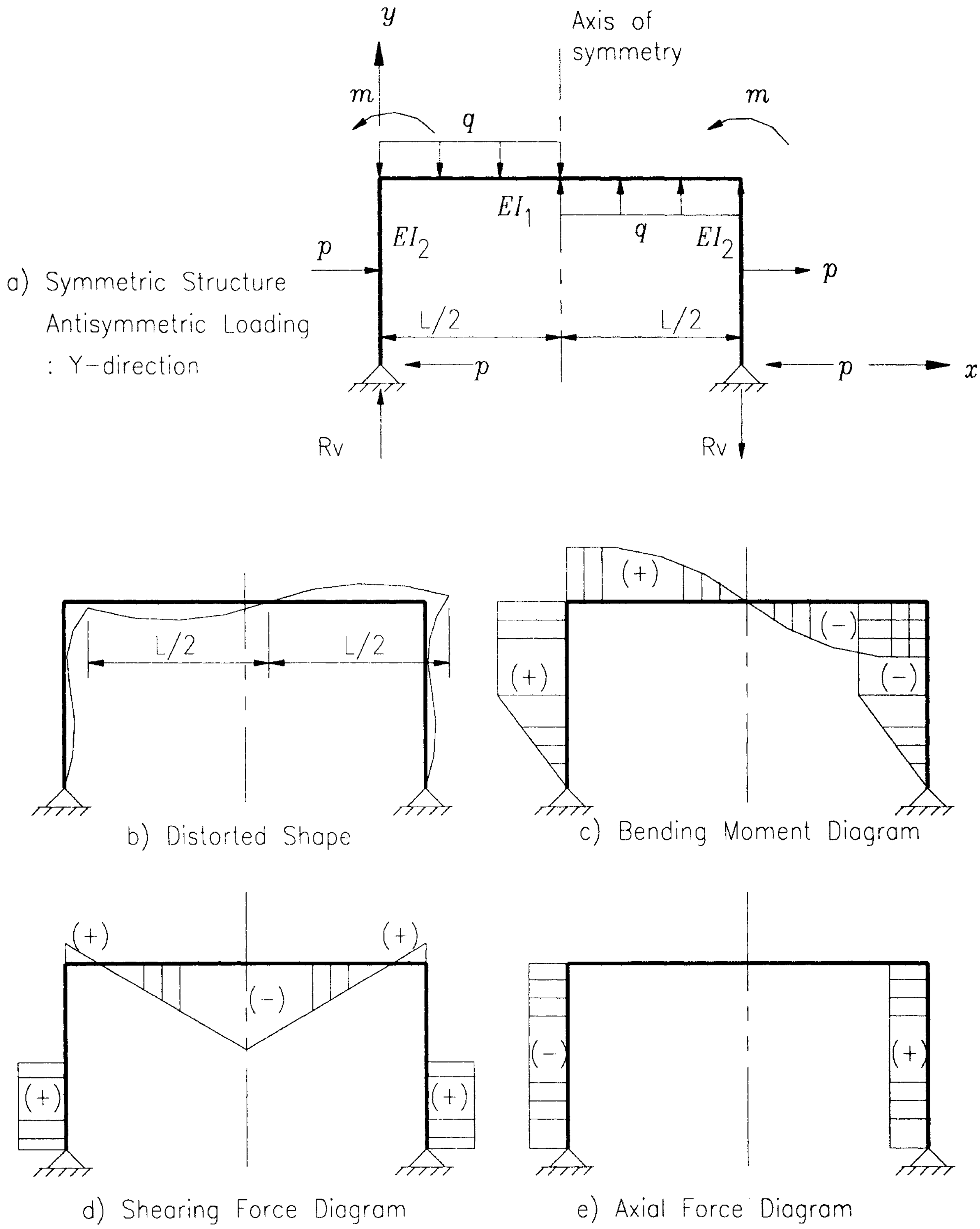


Figure 1.2. Example of A Symmetric Structure With Antisymmetric Loadings.

antisymmetric.

In the case where there is a general loading applied to a symmetric structure, the general loading can be separated into its symmetric and antisymmetric components so that symmetry concepts can be employed, as in Figure 1.3.

1.2.2 Half-Structure Degrees of Freedom on The Axis of Symmetry

By realizing the response patterns of a symmetric structure to a symmetric or an antisymmetric loading -- Figure 1.1 and 1.2 -- the structure can be divided into two halves about the axis of symmetry. Then only the *half* structure need be solved to obtain the responses (deflection, internal forces, and reactions) for the half. The responses for the other half are simply assigned according to the mirror image or reverse mirror image requirements.

To analyze only the *half* structure, the proper degrees of freedom should be provided at the intersection of the axis of symmetry and the structure. For example, considering the case of a symmetric loading on the 2-D frame structure in Figure 1.1, the slope of the deflection curve of the horizontal member to the left of the intersection point with the axis of

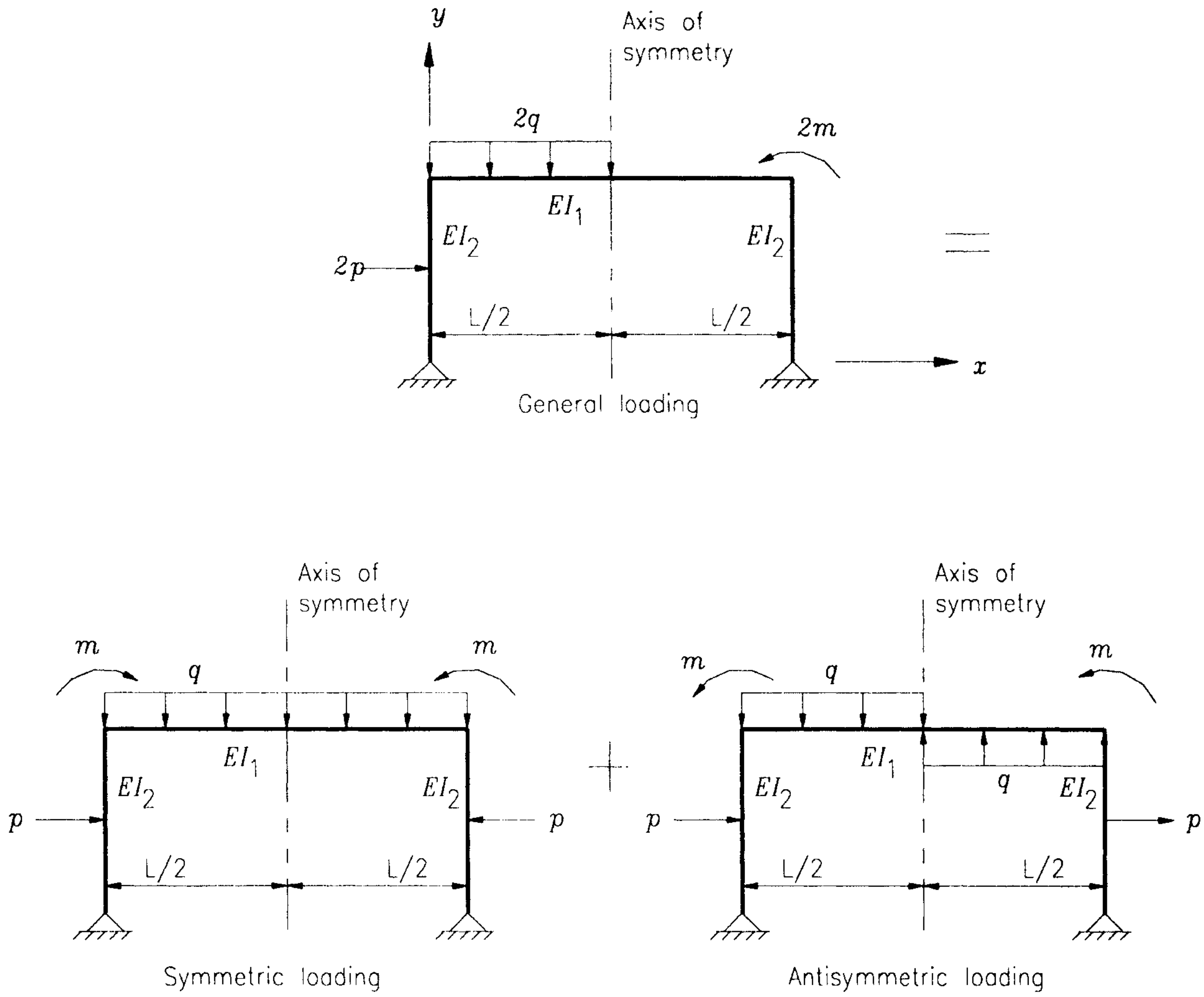


Figure 1.3 Decomposing of General Loadings.

symmetry should be the same as the slope to the right in order to have the mirror image distortion about the axis of symmetry. Thus the slope should be zero to satisfy this condition [Figure 1.1b]. In other words, the rotational degree of freedom at this point should be constrained when considering the *half* structure. Also the transitional degree of freedom perpendicular to the axis of symmetry (on the horizontal direction) should be constrained since this point will not move to the left or to the right. Thus this point will move only parallel to the axis of symmetry which means that the transitional degree of freedom on the vertical direction should be free [Figure 1.1b-e].

When considering the case of antisymmetric loading on the 2-D frame example, The structure undergoes a reverse mirror image distortion. For this condition to be true, the transitional degree of freedom parallel to the axis of symmetry (on the vertical direction) should be constrained while the other two degrees of freedom are free.

Thus the two half structure problems in Figure 1.4 represent the entire structure and the general loading. It is noted that the boundary conditions assigned to the two half-structures are the models for the responses under symmetric and antisymmetric conditions.

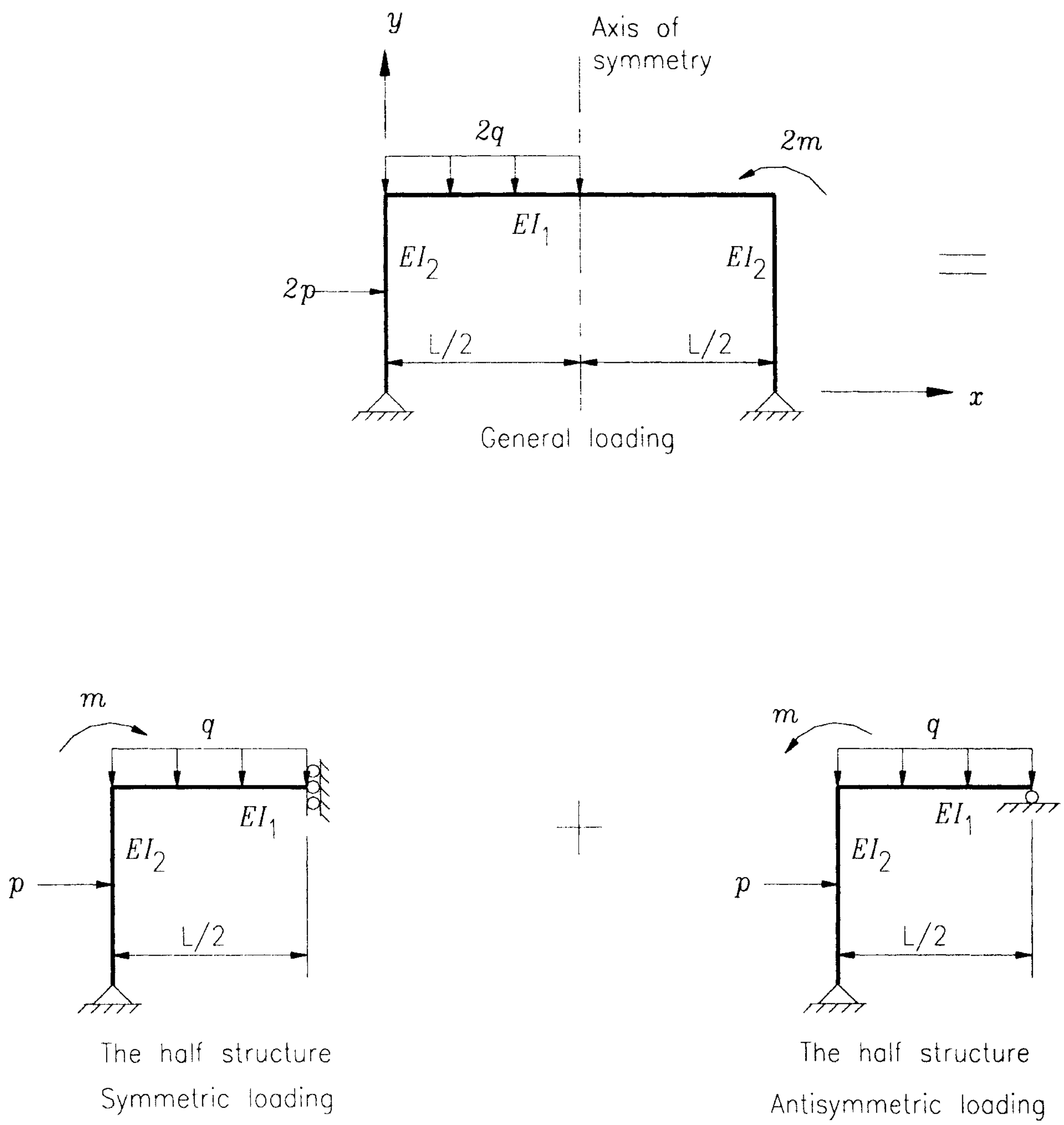


Figure 1.4 Degrees of Freedom at the New Joints.

1.2.3 Analysis Economics in Utilizing Symmetry

If the loading on a symmetric structure is purely symmetric or antisymmetric, the use of symmetry conditions reduces the general problem to a single problem of about half the original size. For the case where a general loading is applied to a symmetric structure, by taking advantage of conditions of symmetry and antisymmetry, a given problem can be reduced to two problems, each roughly half the size of the original.

It must be asked, "*Is there any advantage to working two problems of one half-size as opposed to a single full-size problem* ?" The answer is yes, for the following important reasons:

1- The solution time for a general set of simultaneous equations is proportional to the third power of the number of equations if using Gaussian Elimination method [1] and roughly to twice the second power of the number of equations if the Cholesky decomposition method is used [10]. That is, time to solve a structure with n degrees of freedom is equal to

$t = K\,(n^3)$ Gaussian Elimination

$t = K\,(2n^2)$ Cholesky Decomposition

and the time to solve two structures with $\frac{n}{2}$ degrees of freedom is equal to

$$t = 2\,K\left(\frac{n}{2}\right)^3 = \frac{1}{4}\,K\,n^3 \qquad \text{Gaussian Elimination}$$

$$t = 2\,K\left[2\left(\frac{n}{2}\right)^2\right] = K\,n^2 \qquad \text{Cholesky Decomposition}$$

Thus the time saving in the solution of simultaneous equations is 75 percent for Gaussian Elimination and 50 percent for Cholesky Decomposition.

2- The computer storage usage may be cut about in half, because each problem will have about half the number of degrees of freedom as the original.

1.3 Literature Review

1.3.1 Optimization in Matrix Methods

As mentioned before, much research has been done regarding optimizing matrix methods in structural analysis. Some optimization schemes are concerned with the method of solution of the simultaneous equations and some of them with minimizing the number of degrees of freedom.

1.3.1.1 Optimization With Respect to Simultaneous Equations Solution

By recognizing that the presence of zero terms in the stiffness matrix [K] can be predicted, solution schemes that facilitate the treatment of banded matrices have been presented [1, 3, 14]. The matrix [K] is stored with the dimensions (n * HBW) instead of (n * n), where "n" is the number of degrees of freedom of the structure and "HBW" is the half band width of the matrix [K]. Thus, the computer storage requirements can be reduced for the stiffness matrix and also the solution time for the equations can be reduced. The value of the half band width depends on the way of numbering the degrees of freedom in the structure.

Another way of solving the set of simultaneous equations is the *wave front solution* or the *frontal solution* [14]. This method does not work with all of the stiffness equations of the structure at one time, but reduces them by blocks. The effects of reducing one block are carried over to succeeding blocks by following the Gaussian (or other) elimination method. This idea may be extended a step further to the consideration of stiffness equations element-by-element.

An algorithm for reducing the bandwidth and profile (which refers to the prediction of zeros in the stiffness matrix) of a sparse matrix is presented in Reference 11. This paper presents a new technique for reducing the bandwidth and profile of a symmetric and sparse matrix. A graph representation of the matrix is decomposed into a group of isolated sets by general level structures. These are exploited to construct a maximal-depth partitioned structure, each level of which has as equal a width as possible. Then, the vertices of the partitioned structure are numbered consecutively.

A new algorithm for reducing the profile and root-mean-square wavefront of sparse matrices with a symmetric structure is presented in Reference 12. In this algorithm, the goal is to minimize the storage requirement for the profile scheme. The authors say "This algorithm is fast, simple and useful in engineering analysis where it can be employed to derive efficient orderings for both profile and frontal solution schemes."

From the previous discussion it is seen that optimization by developing and modifying methods of solving a set of simultaneous equations is somewhat unrelated to structural engineering and to the behavior of the structure but more related to linear algebra. These methods can be employed to solve simultaneous equations which are associated with a sparse and symmetric matrix. An example for which these methods are most likely to be employed is the finite element method.

1.3.1.2 Optimization by Reducing The Number of Degrees of Freedom

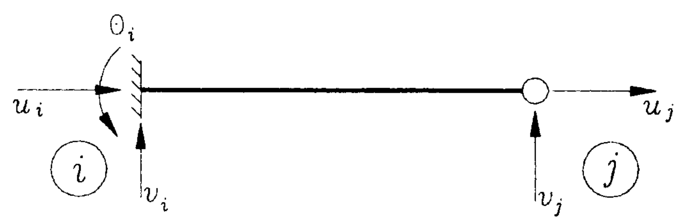
The following methods take advantage of the behavior of the structures under a set of specific conditions.

The first method is to apply the *modified member stiffness coefficients* to structure members that have predefined end degrees of freedom. For example, the local member stiffness matrix for a 2-d frame member is:

• For a member fixed-fixed

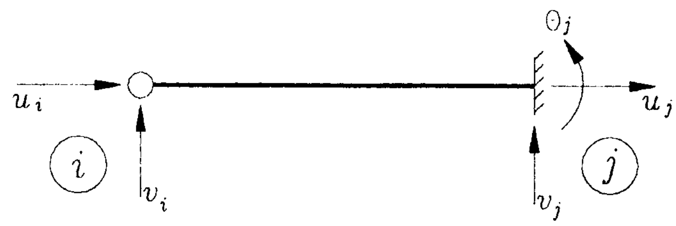
$$\begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ \theta_i \\ u_j \\ v_j \\ \theta_j \end{Bmatrix}$$

•For a member fixed-pinned



$$
\begin{bmatrix}
\frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & - \\
0 & \frac{3EI}{L^3} & \frac{3EI}{L^2} & 0 & -\frac{3EI}{L^3} & - \\
0 & \frac{3EI}{L^2} & \frac{3EI}{L} & 0 & -\frac{3EI}{L^2} & - \\
-\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & - \\
0 & -\frac{3EI}{L^3} & -\frac{3EI}{L^2} & 0 & \frac{3EI}{L^3} & - \\
- & - & - & - & - & -
\end{bmatrix}
\begin{Bmatrix} u_i \\ v_i \\ \theta_i \\ u_j \\ v_j \\ \theta_j \end{Bmatrix}
$$

•For a member pinned-fixed



$$
\begin{bmatrix}
\frac{EA}{L} & 0 & - & -\frac{EA}{L} & 0 & 0 \\
0 & \frac{3EI}{L^3} & - & 0 & -\frac{3EI}{L^3} & \frac{3EI}{L^2} \\
- & - & - & - & - & - \\
-\frac{EA}{L} & 0 & - & \frac{EA}{L} & 0 & 0 \\
0 & -\frac{3EI}{L^3} & - & 0 & \frac{3EI}{L^3} & -\frac{3EI}{L^2} \\
0 & \frac{3EI}{L^2} & - & 0 & -\frac{3EI}{L^2} & \frac{3EI}{L}
\end{bmatrix}
\begin{Bmatrix} u_i \\ v_i \\ \theta_i \\ u_j \\ v_j \\ \theta_j \end{Bmatrix}
$$

Thus by deleting the predefined degrees of freedom (θ_i for a member pinned-fixed and θ_j for a member fixed-pinned), the number of degrees of freedom of the entire structure can be reduced.

The second method used in optimization is to *introduce constraints in deformations* (often axial for frame members or membrane-type for planer members). When analyzing plane and space frames, it is usually found that the axial stiffnesses for beams is larger than that for columns due to the larger dimensions for the beams which sometimes include the area of the slabs as well as the area of the beam. Thus, by eliminating the axial deformations of the beams the number of degrees of freedom can be reduced. This idea can be extended -- with little loss of modeling accuracy in imposing the constraints -- to eliminate the axial deformations for all the members since large axial stiffness mixed with small flexural and torsional stiffness can sometimes cause a significant loss of numerical accuracy. Previous investigations have shown the basic idea of introducing axial constraints in static analysis [15], dynamic analysis [16], stability analysis [17], and non linear analysis [13].

1.3.2 Optimization Using Symmetry Concepts

An approach is suggested by Meyers [1] to take advantage of conditions of symmetry. This approach depends on the user to physically reduce the symmetric structure problem to half its original size and provide the appropriate input for half of the structure twice (first for symmetric loading component and second for antisymmetric loading component). Thus, in this approach, there will be savings in the solution time and in the computer storage requirements. Moreover, the input requirements will also be cut about in half.

This approach, though, will bring up a disadvantage that might cancel out the first advantage of conditions of symmetry and antisymmetry: "time saving". With a more complicated symmetric structure and more complicated general loading, the user must prepare the required input with the correct number of joints, number of members, member properties (especially for those members on the axis of symmetry), degrees of freedom for the additional nodes on the axis of symmetry, and the symmetric and antisymmetric loading for half the structure to be solved. The user also must provide this information twice for the symmetric and antisymmetric cases.

After obtaining the results for the half of the structure, the user must find the results for the second half of the structure for the symmetric and antisymmetric cases and combine both together. All these calculations will take much more time than if the user would work with the original total structure, and also will increase the probabilities of making human errors in input or output data. It is noted also that automated node and member generating schemes for the input data may be interrupted by cutting the structure in half and keying in the half structure may actually take longer than the entire structure.

There are many published works that talk about the symmetry concepts of a structure and how to take advantage of these features [1, 3, and 12] . However, in all of this literature, the structure being symmetric is taken as a given fact. In none of them is the symmetry of a structure detected automatically.

1.3.3 Currently Used Software

A phone-call survey of the producers of the most prominent structural analysis software [4-9] has been undertaken. All of these programs do not have the ability to automatically detect symmetry of a structure and depend on the user to prepare the input data in a way that takes advantage of the symmetry concepts. Further, the user has to combine results from two half-analyses too. The problems with this dependence on the user were discussed in section 1.3.2.

1.4 Objectives and Scope

1.4.1 Objectives

To take full advantage of the conditions of symmetry, an algorithm is to be developed to automatically detect the symmetry of a structure. The algorithm should take the general data for the entire problem and do several tests to detect symmetry of the structure. If the structure is symmetric, then the algorithm should solve it twice (i.e. for the symmetric and antisymmetric cases) for any type of general loading. These advantages are summarized as :

- Saving in solution time,
- Saving in computer storage requirements,
- No extra time needed for preparing the input data or interpolating the output data,
- Decreasing the probability of human errors.

After solving and obtaining the results for each case, the results are expanded for the second half and then combined to get the output results for the entire structure. Then a benchmark study on time and computer storage saving is done to test the algorithm.

1.4.2 Scope of Work

In this research, an algorithm is presented to automatically detect the symmetry of 2-D frame structures for any type of general loading. Only the symmetric structure subjected to a general loading will be considered in this research. The antisymmetric structure is not considered since it is uncommon to encounter this possibility. The algorithm will detect the symmetry only if the axis of symmetry is parallel to the global Y-axis of the structure (which is the common case for 2-D structures). If the structure is symmetric, the algorithm will solve *half* of the structure twice for the symmetric and antisymmetric loading components and then combine the results from both cases and output the final result for the entire structure.

CHAPTER II

AUTOMATED SYMMETRY ANALYSIS

2.1 Detecting Symmetry

The steps to detect symmetry start after identifying the structure's geometry, member locations and properties, and supports. The idea is to assume that the structure is symmetric along an axis parallel to the global X-Y-, or Z-axes and then do a series of checks on the structure to test all requirements for the assumption to hold true. These requirements have to do with :

- Configuration. The joints and members should be distributed so that one half of the structure is a mirror image of the other half.
- Members' stiffnesses (axial, bending, and torsional). These should be distributed in a mirror image fashion on the two halves of the structure.

- Constraints. The structure constraints on the two halves of the structure should be mirror image of one another.

If the result of any of these checks fails, then the structure is no longer symmetric about the axis under consideration and no further checks are made with respect to that axis. In other words, if the result of all of these checks is true, then the structure can be identified as symmetric along the axis under consideration. It is noted that a tolerance of 0.1 percent is used for any of the numerical values check (joint coordinates and members' stiffnesses).

To illustrate the procedures, an example shown in Figure 2.1 is considered. The structure has an axis of symmetry about line A-A parallel to Y-axis. The same procedures can be followed for symmetry about an axis parallel to the X-, or Z-axes.

The steps are described subsequently:

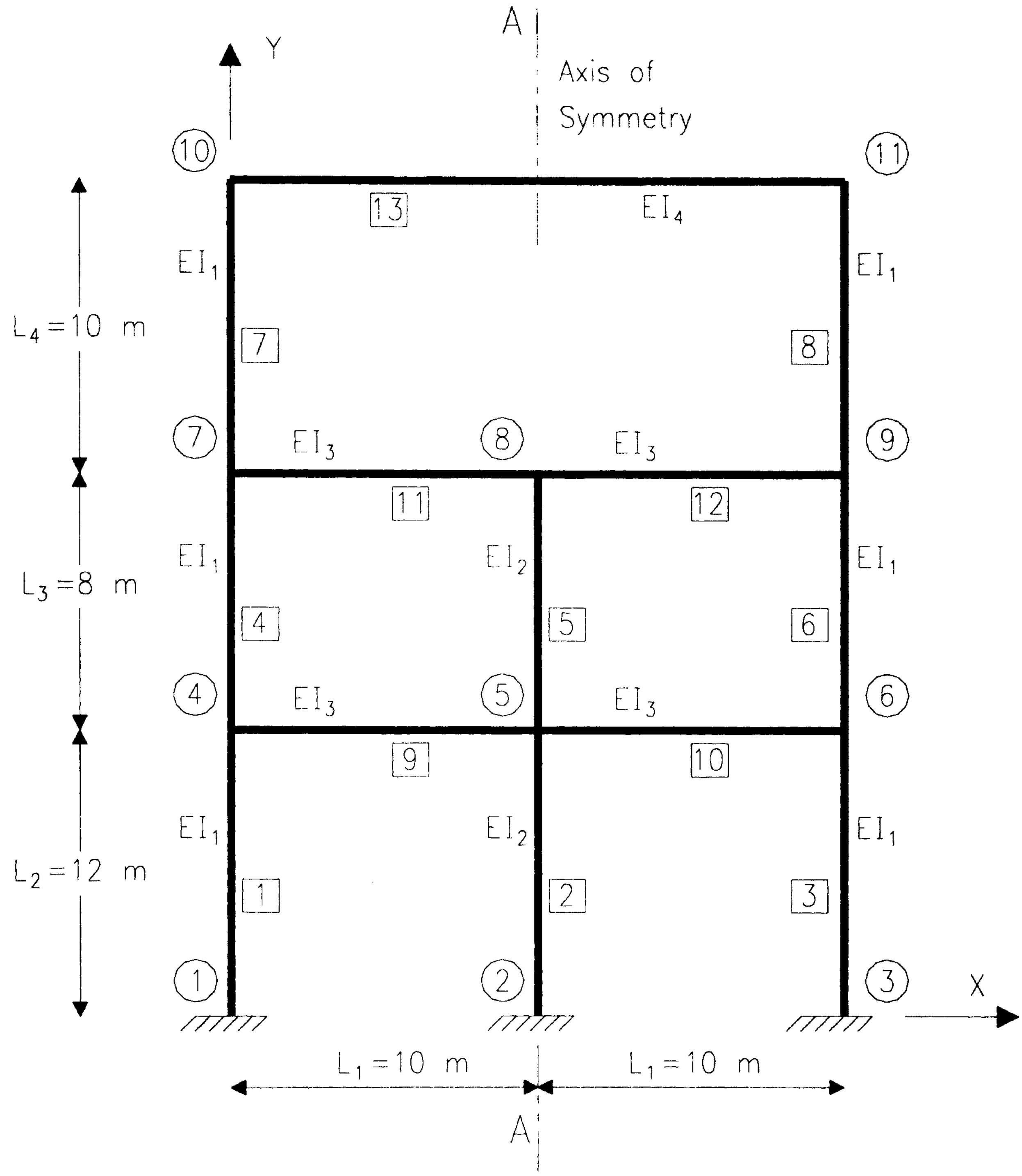


Figure 2.1 : Structure Example.

2.1.1 The Location of The Axis of Symmetry

For a 2-D plane symmetric structure, there must be a line that divides the structure into two halves in such a way that each half must be a mirror image to the other one about that line. This line is called *the axis of symmetry* of the structure. This axis must meet certain criteria, it must cross the mid-coordinate of joints with respect to the direction perpendicular to the axis of symmetry - for the example in hand, the mid-X-coordinate of the joints. This axis must also cross the geometric C.G. of all the joints with respect to the same direction. The mid-X-coordinate is calculated and is equal to:

(maximum X-coordinate + minimum X-coordinate) / 2

The X-geometric C.G. is calculated and is equal to

$$\frac{\sum_{i=1}^{n} X_i}{n}$$

where n = total number of structure joints

X_i = X-coordinate of joint number i

The mid-X-coordinate and the X-geometric C.G. of the joints must be equal in order for the axis of symmetry to exist which will pass through both locations. This is a necessary but not a sufficient condition for the overall structure to be symmetric. Thus if the two locations are the same, then the assumption that the structure is symmetric will still hold true. Figure 2.2 demonstrates the case when both locations are not the same and the location of the axis of symmetry when they are equal.

2.1.2 Check Joint Configuration Symmetry

For the structure to be symmetric, the joint pattern on one side of the axis of symmetry must be a mirror image of that on the other side. This means that for each joint on one side of the axis of symmetry, there must be a joint on the other side with the same coordinates in the direction parallel to the axis of symmetry and perpendicular coordinates the same amount to the right of the axis of symmetry as the first point is to the left.

Another requirement for joint configuration symmetry is that the number of joints on one side of the axis of symmetry must be equal to that on the other side. This number- with the number of joints that are on the axis of symmetry- will be used in calculating the number of joints of the half structure to be analyzed if the structure is symmetric.

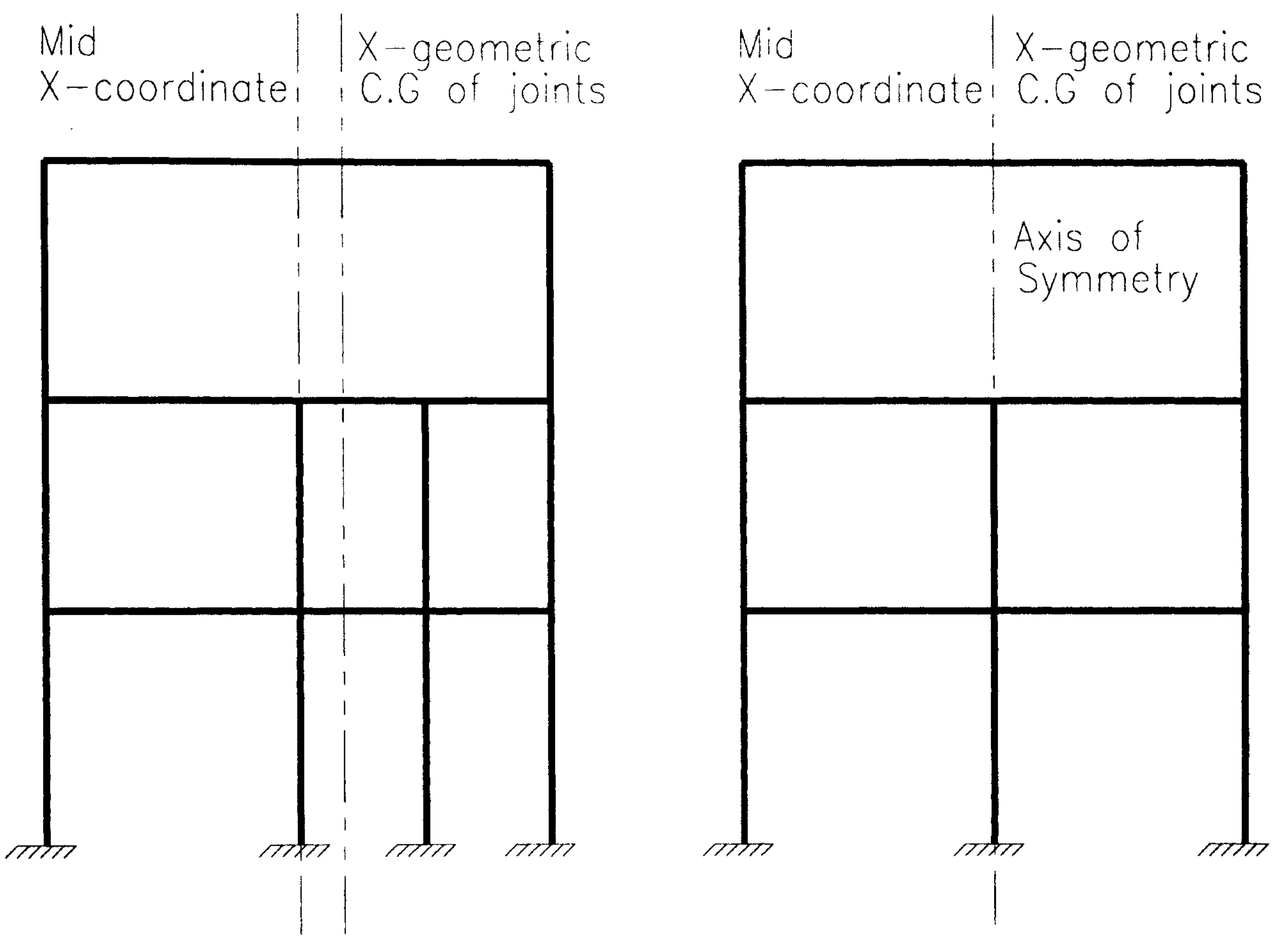


Figure 2.2 : Mid X-coordinate and X-geometric C.G. of Joints.

Figure 2.3 demonstrates that the check for joint symmetry must be done by comparing the X- and Y-coordinates for each joint of the structure and not only by comparing the number of joints on one side of the structure to that on the other side.

2.1.3 Check Member Configuration Symmetry

The second requirement for configuration symmetry has to do with the member locations and connectivity. For the structure to be symmetric, for each member connecting two joints on one side of the structure, there must be a member on the other side that is connected to the two joints which form a mirror image of the first two joints. This means that the two ends for each member to the left of the axis of symmetry, for example, must be checked to see if the two mirror-image joints on the right of that axis are connected with a member. The top case in Figure 2.4 demonstrates the need for this test where there is no symmetry in member connectivity. It is noted that once a member on the right half has been picked up as the mirror image for a member on the left half, it will not checked again for any other member on the left half. Thus the case where there are two members attached to the same two joints can be detected. An additional test must be made to make sure that the number of members that lie completely on one

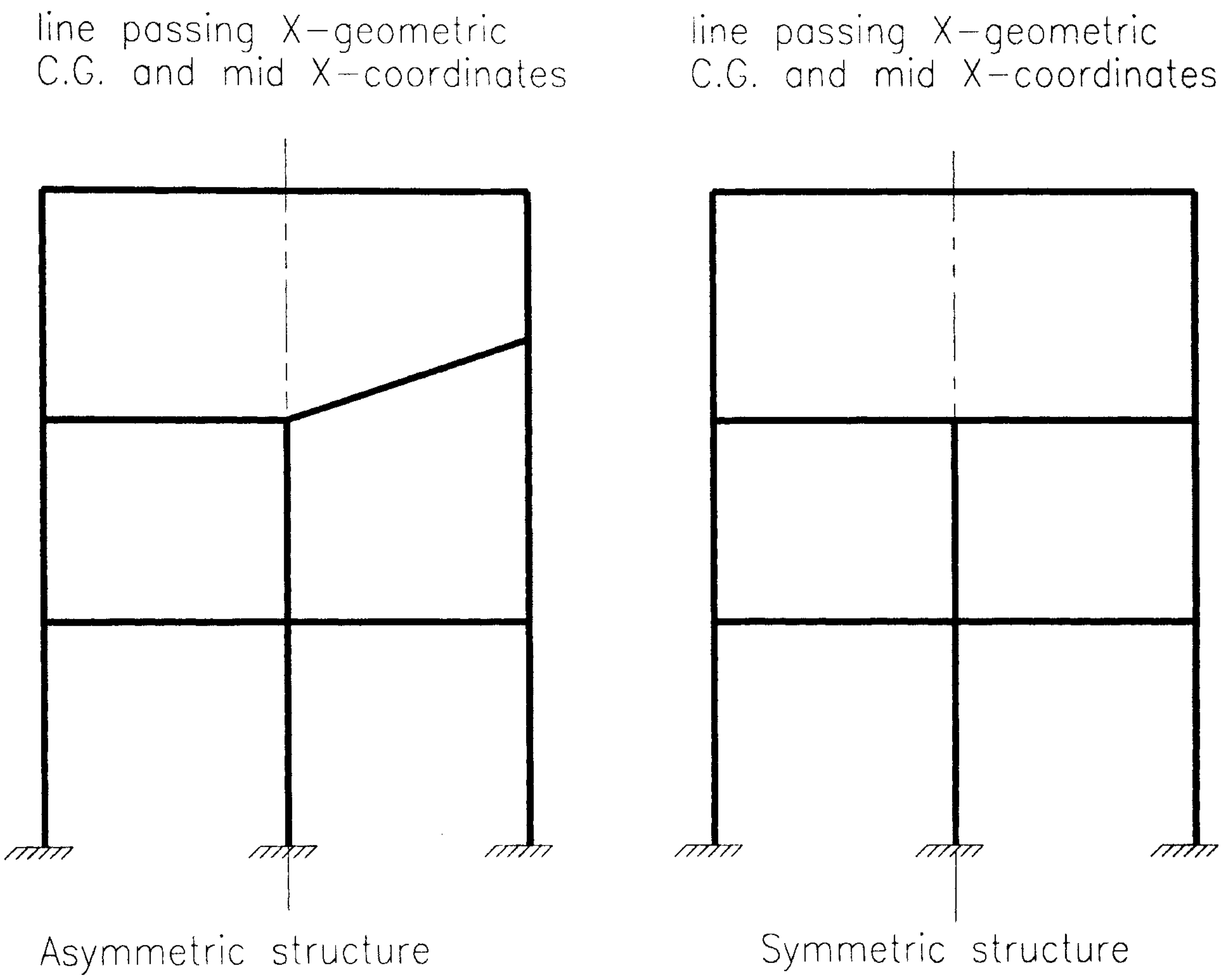


Figure 2.3 : Check Joint Symmetry Using Joint Coordinates.

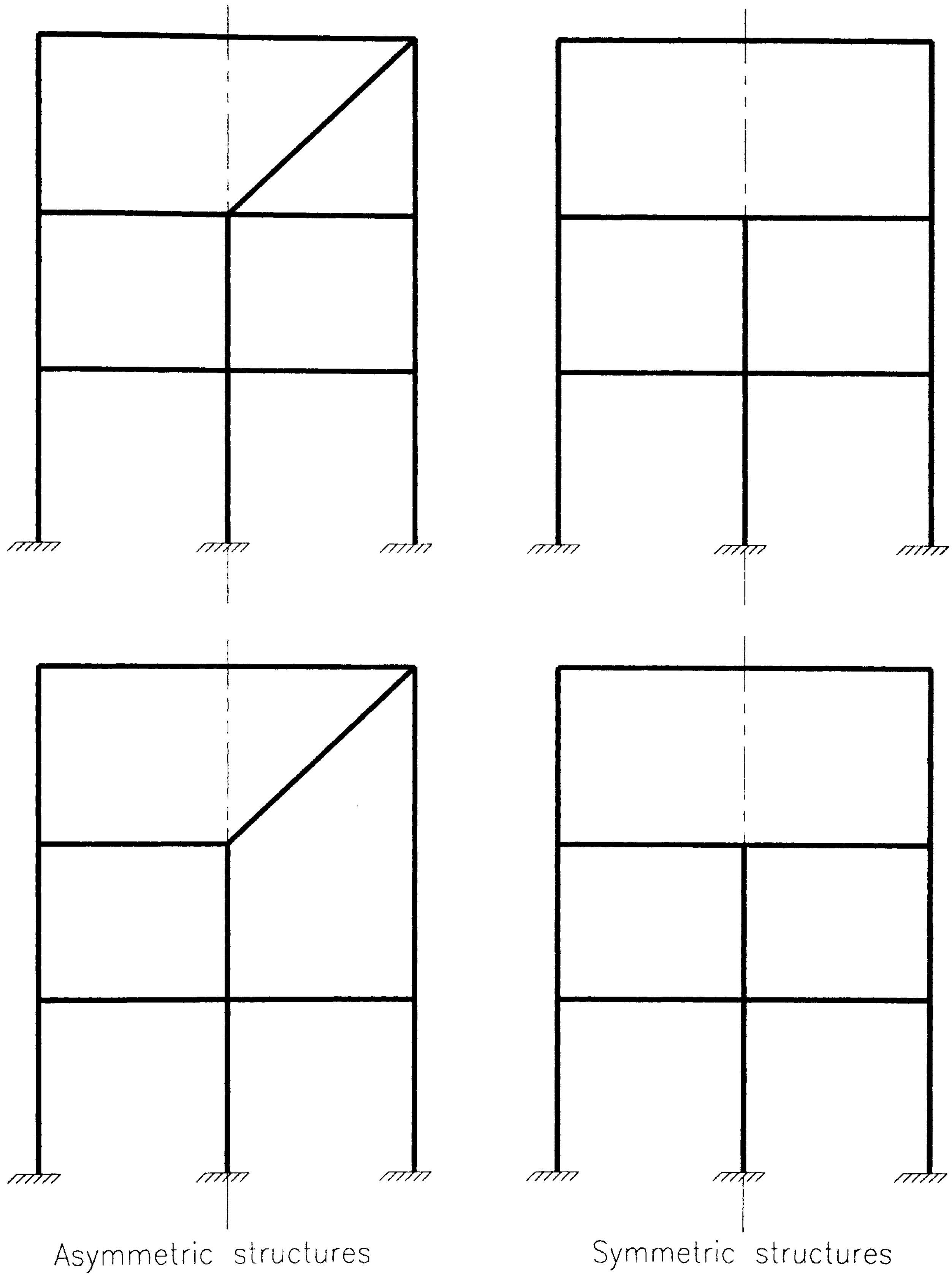


Figure 2.4: Check Member Symmetry.

side of the structure is equal to that of those which lie on the other side. The bottom case in Figure 2.4 demonstrates the need for this test where for every member on the left half there is a mirror image one on the right half plus an additional member that lie on the right half. These two tests are necessary and sufficient for member configuration symmetry.

If the joints and member configuration symmetry requirements are satisfied, the first requirement for the structure to be symmetric, which has to do with the configuration, is satisfied and the assumption that the structure is symmetric will not be violated.

2.1.4 Check Member Stiffness Symmetry

The second requirement for symmetry has to do with the members' stiffnesses. For the structure to be symmetric, stiffnesses should be distributed in a mirror image fashion between the two halves of the structure. This means the stiffnesses (axial stiffness EA, bending stiffnesses EI_z and EI_y, and torsional stiffness GJ) for each member on one side of the structure must be equal to that for the mirror-image member on the other side. Another alternative for this check is to compare the members' properties only - A, I_z, I_y and J, but for the general case where there might be a combination of two or more materials in the structure, stiffnesses represent the actual behavior of the member inside the structure. It is to be noted here that the members on the axis of symmetry need not be checked for stiffness since there is only one set of those members.

2.1.5 Check Support Symmetry

The last step in symmetry detection is to check the distribution of constraints in the structure. For the structure to be symmetric, constraints on one side of the axis of symmetry must be the mirror image of those on the other side. This check is done by comparing the free and constrained degrees of freedom for the support joints on each half of the structure.

After this step, all the requirements for the structure to be symmetric have been checked. In other words, If the results for these checks are true, then the structure is symmetric and necessary data structures must be set up to take advantage of the symmetry concepts.

2.2 Preparing Data Structures for The Half-Structure Analysis

As discussed in section 1.2, to take advantage of symmetry concepts, only one half of the structure will be analyzed twice under symmetric and antisymmetric loading conditions. This half structure must exactly represent the behavior and response for this half inside the entire structure and must be in turn a mirror image or reverse mirror image for the other half. For the analysis to be correct, several data structures must be set up for this half. The left half will be considered for analysis. This is not necessary but is consistent with the usual fashion of numbering the joints from left to right.

2.2.1 Members Crossing The Axis of Symmetry

When the axis of symmetry intersects with a member at mid-length (member number 13 in the example shown in Figure 2.1), This member should be cut at this location and each half of the member will be part of each half of the structure. Thus when analyzing the left half of the structure, the start and end node for that member should be modified to account for this process.

The members for the half structure will be the members that are located completely on the left half, the members that are on the axis of symmetry, and one member for each member that crosses the axis of symmetry. The number of these members will always be greater than half the number of members for the entire structure.

2.2.2 Number of Joints for The Half-Structure

For each member that crosses the axis of symmetry, a new node must be added to the half structure located at the intersection of this member with the axis of symmetry - at mid length of the member. The coordinate of this node can be found from the coordinates of the two end nodes of the entire member. The number of joints for the half structure will be equal to the number of joints that have X-coordinates equal to or less than the X-coordinate of the X-geometric C.G. of the joints of the entire structure in addition to one node per each member crossing the axis of symmetry.

2.2.3 Assigning Free and Constrained Degrees of Freedom for New Nodes

This step depends on the type of structure (beam, 2-D or 3-D frames, 2-D or 3-D trusses or grids). The reason is that each type of structure has its own order in numbering the transitional and rotational degrees of freedom. In addition, each transitional and rotational degree of freedom will respond differently for the symmetric and antisymmetric loading conditions according to whether it is parallel or perpendicular to the axis of symmetry.

To develop a feeling for this condition, the discussion in section 1.2.3 and the responses of the 2-D frame example in Figures 1.1 and 1.2 should be recalled. Figure 2.5 demonstrates the free and constrained degrees of freedom for each structure type separately. The usual fashion of numbering the degrees of freedom is used. Each structure is assumed to have an axis of symmetry parallel to Y-axis for 2-D structure types or a plane of symmetry parallel to YZ-plane (perpendicular to X-axis) for 3-D structure types. The responses for each structure type, which are the bases in assigning free or constrained degrees of freedom for new nodes, can be demonstrated as in Figures 1.1 and 1.2 for a 2-D frame structure [1]. The free and constrained degrees of freedom for the new nodes are summarized in Table 2.1.

It is noted that when the member crossing the axis of symmetry is not perpendicular to that axis, the free and constrained degrees of freedom for the new node are differ from that presented in Figure 2.5 and Table 2.1. A detailed discussion about this case is presented in section 4.4.5.

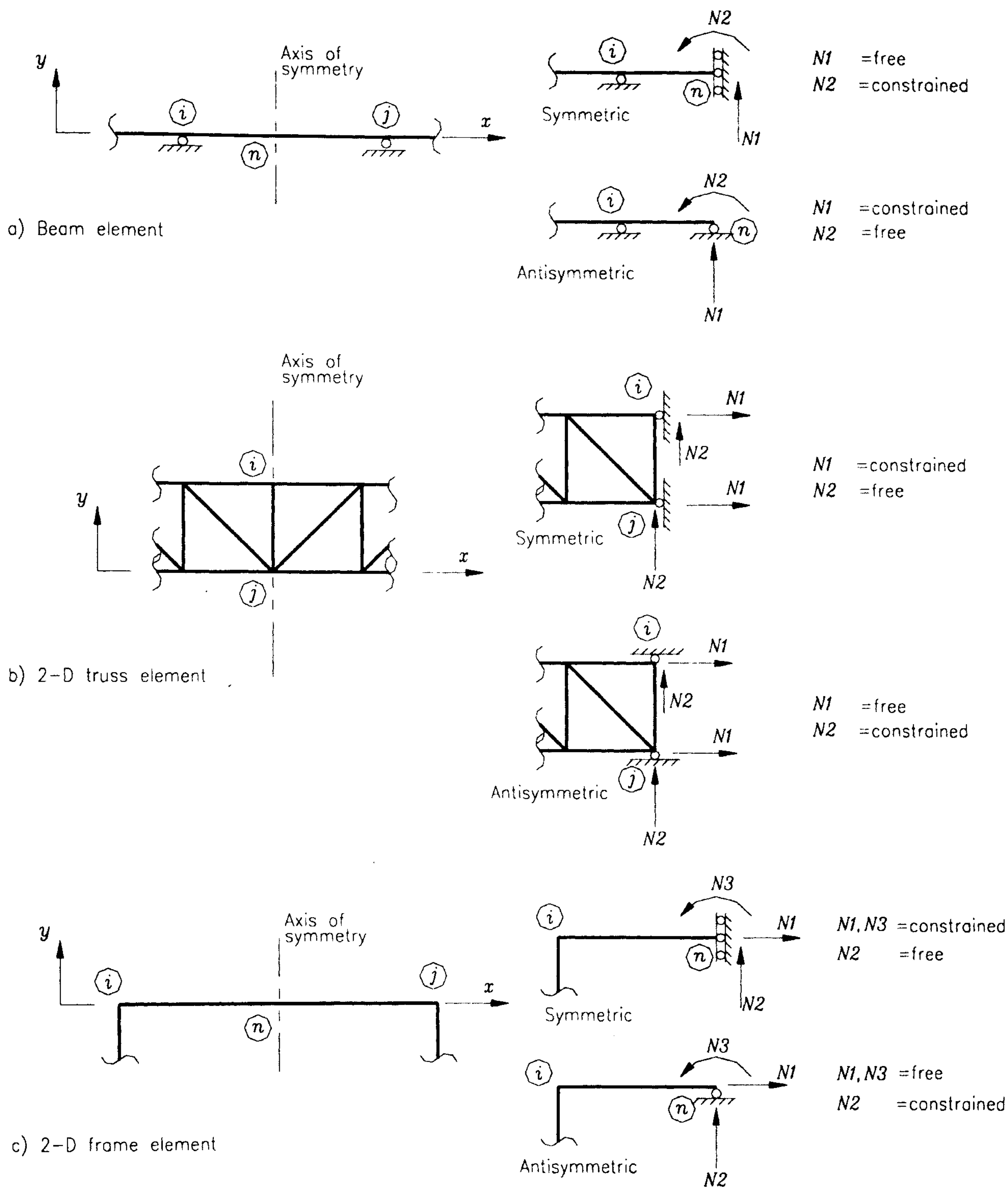


Figure 2.5 : Degrees of Freedom for New Nodes (n).

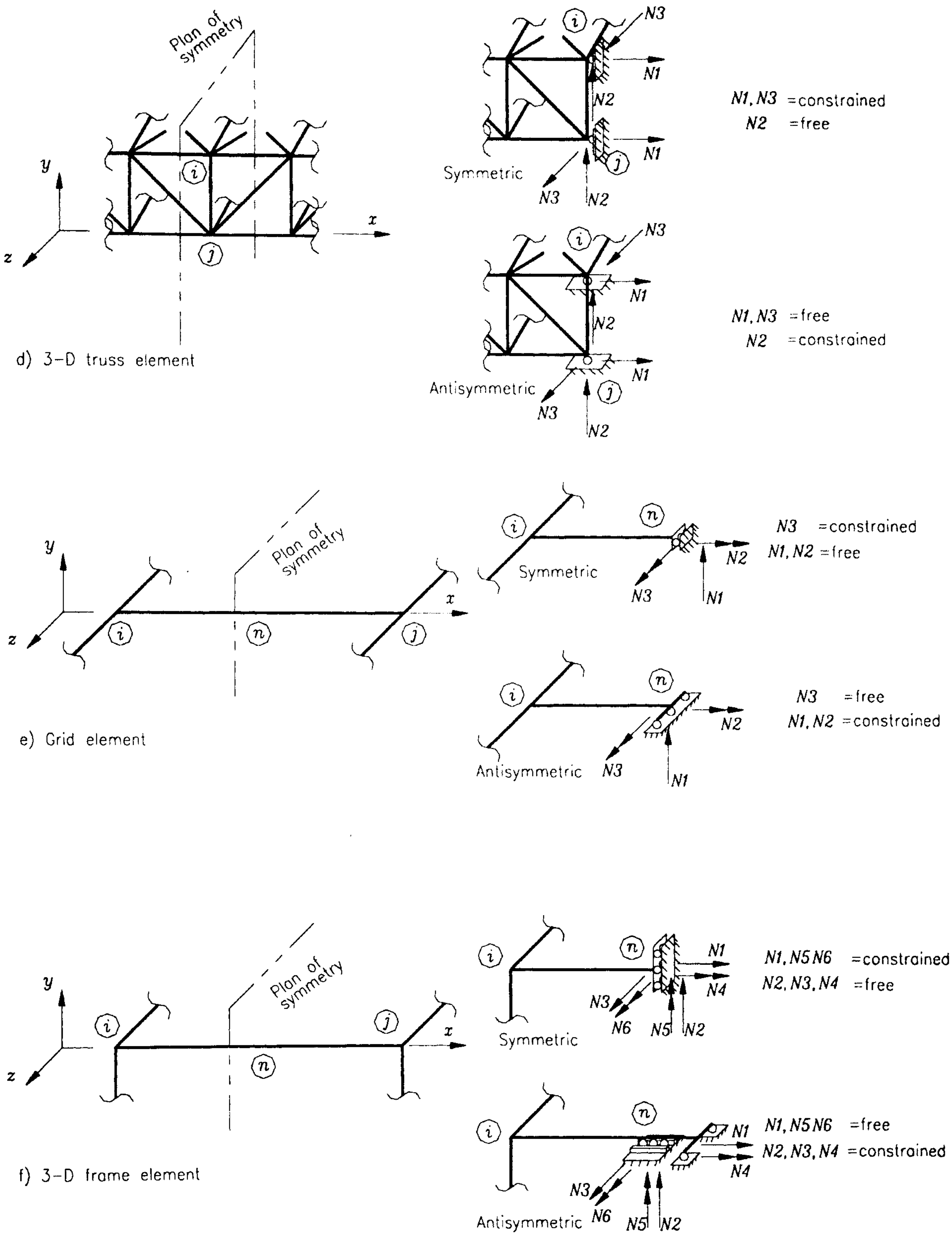


Figure 2.5 (continued) : Degrees of Freedom for New Nodes (n).

Table 2.1 : Free (f) and Constrained (c) Degrees of Freedom for New Nodes.

Degree of Freedom	Symmetric loads					
	Beam	2D truss	2D frame	3D truss	Grid	3D frame
N1	f	c	c	c	f	c
N2	c	f	f	f	f	f
N3			c	c	c	f
N4						f
N5						c
N6						c

Degree of Freedom	Antisymmetric loads					
	Beam	2D truss	2D frame	3D truss	Grid	3D frame
N1	c	f	f	f	c	f
N2	f	c	c	c	c	c
N3			f	f	f	c
N4						c
N5						f
N6						f

2.2.4 Members on The Axis of Symmetry

In order to cut the symmetric structure into exactly two halves, the members on the axis of symmetry should be treated differently. Each member on the axis of symmetry is divided into two members one on each half of the structure so the symmetric requirements can be satisfied from configuration point of view (the discussion in section 2.1 should be recalled).

For the second requirement, which has to do with member stiffness, the stiffness for each member on the one half of the structure should be exactly the same as the mirror image-member on the second half. Thus the stiffness for each member on the axis of symmetry should be divided into two halves one for each half of the structure. Since the member stiffness includes *EA*, *EIz*, *Ely*, *GJ*, and since the material properties *E*, and *G* are the same for the member, then each half of the structure can take half the member's properties *A*, *Iz*, *Iy*, and *J*. Thus when the local stiffness matrix [**k**] is calculated for each member while solving the half structure, each member on the axis of symmetry will have half the stiffness values of the original member as if it is in the entire structure, and the second requirement for symmetry concepts will be satisfied.

For the constraints requirements, if one end of the member on the axis of symmetry is a support joint, then when splitting this member between the two halves of the structure, the constrained degrees of freedom of the original member should be added to the constrained degrees of freedom at the same end for both members depending on the symmetric and antisymmetric loading conditions. As an example for this condition, the 2-D truss structure in Figure 2.6a is featured. The lower joint for the member on the axis of symmetry is a support joint in which the vertical degree of freedom is constrained. Since under the symmetric loading conditions the horizontal degree of freedom for the joint on the axis of symmetry is constrained -- as discussed in section 2.2.3 and Figure 2.5b -- then this node is constrained in both directions under this loading condition. Under antisymmetric loading conditions, this node is already constrained in the vertical direction, thus no modification should be made for this node.

Figure 2.6 demonstrates how to handle members on the axis of symmetry [1]. The discussion about introducing degrees of freedom for the new nodes in section 2.2.3 should be recalled.

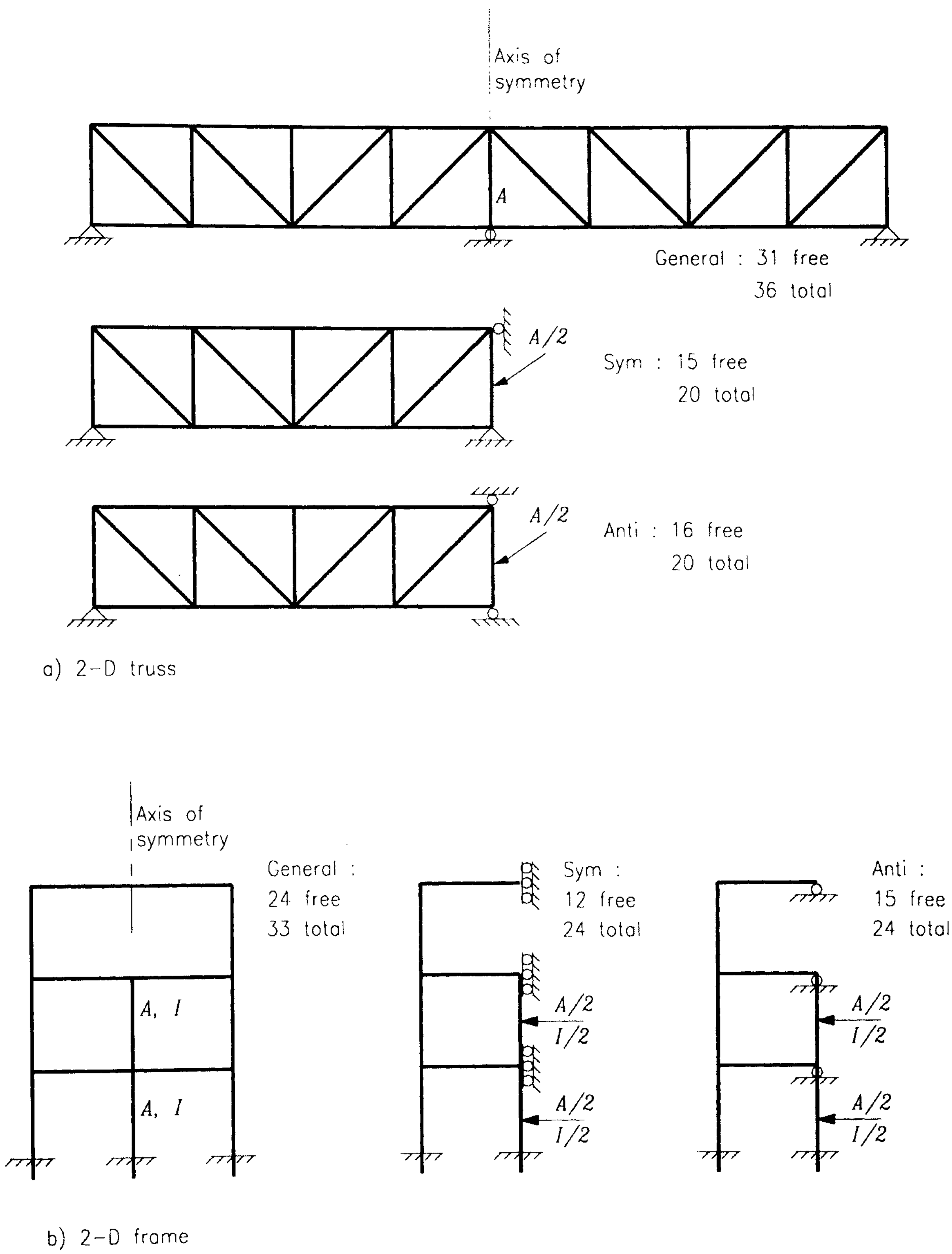


Figure 2.6 : Members on the Axis of Symmetry.

2.2.5 Degrees of Freedom For The Half-Structure

Up to this point, all the necessary data needed to determine the size of the half structure are prepared. These data are the number of joints and constrained and free degrees of freedom at each joint. The total number of degrees of freedom and the total number of reactions for the half structure can be calculated for both the symmetric and antisymmetric loading conditions. Thus the size of the two problems to be solved instead of one problem can be determined.

It should be noted here that if the number of free degrees of freedom of the half structure under symmetric loading conditions is added to that under antisymmetric loading conditions, exactly the same number of free degrees of freedom of the original structure will be yielded unless there are new nodes added to the half structure. For example, in the 2-D truss in Figure 2.6a, the free degrees of freedom of the entire structure is 31 while for symmetric and antisymmetric conditions are 15, and 16 respectively. That is true for any structure type.

The reason for this equality in the total number of free degrees of freedom is (recall the discussion on section 2.2.3 and Table 2.1) at a joint on the axis of symmetry, the free and constrained degrees of freedom under symmetric loading conditions are switched to constrained and free degrees of freedom under antisymmetric loading conditions respectively. Thus the total free degrees of freedom at this joint before dividing the structure to two halves is equal to the sum of free degrees of freedom at this joint for the two cases of symmetric and antisymmetric conditions. This is true unless there are members that cross the axis of symmetry. In this case, a new node is introduced at the intersection of each of these members with the axis of symmetry. Thus extra free degrees of freedom (equal to the number of new nodes multiplied by the degrees of freedom at each node) are added to that of the entire structure and then split between the symmetric and antisymmetric loading conditions. For example, in the 2-D frame in Figure 2.6, which is the same as the example introduced at the beginning of this chapter, the free degrees of freedom for the symmetric and antisymmetric loading conditions are 12 and 15 respectively, which equal to the free degrees of freedom for the general case 24 in addition to 3, since there is only one new node added to the half structure.

2.3 Decomposing Loads Into Symmetric and Antisymmetric Components

As discussed in section 1.2.1, a general loading applied to a symmetric structure can be separated into its symmetric and antisymmetric components so that symmetry concepts can be employed. This separation process depends on the type of loading (concentrated, distributed, and moment load), the direction of the load (in X-, Y-, or Z-direction), the start and end node for the loaded member, and the orientation of the axis of symmetry (parallel to global X-, Y-, or Z-axis or with a general orientation). Only a two-dimensional frame structure with axis of symmetry parallel to the global Y-axis is considered in this research.

Figure 2.6 shows some examples for decomposing general loads to symmetric and antisymmetric components. More examples and the bases for this process can be found in Reference [1].

It is noted here that since only one half of the symmetric structure will be solved, only the load components on this half will be calculated. For example, for the structure shown in Figure 2.1, only the left half will be

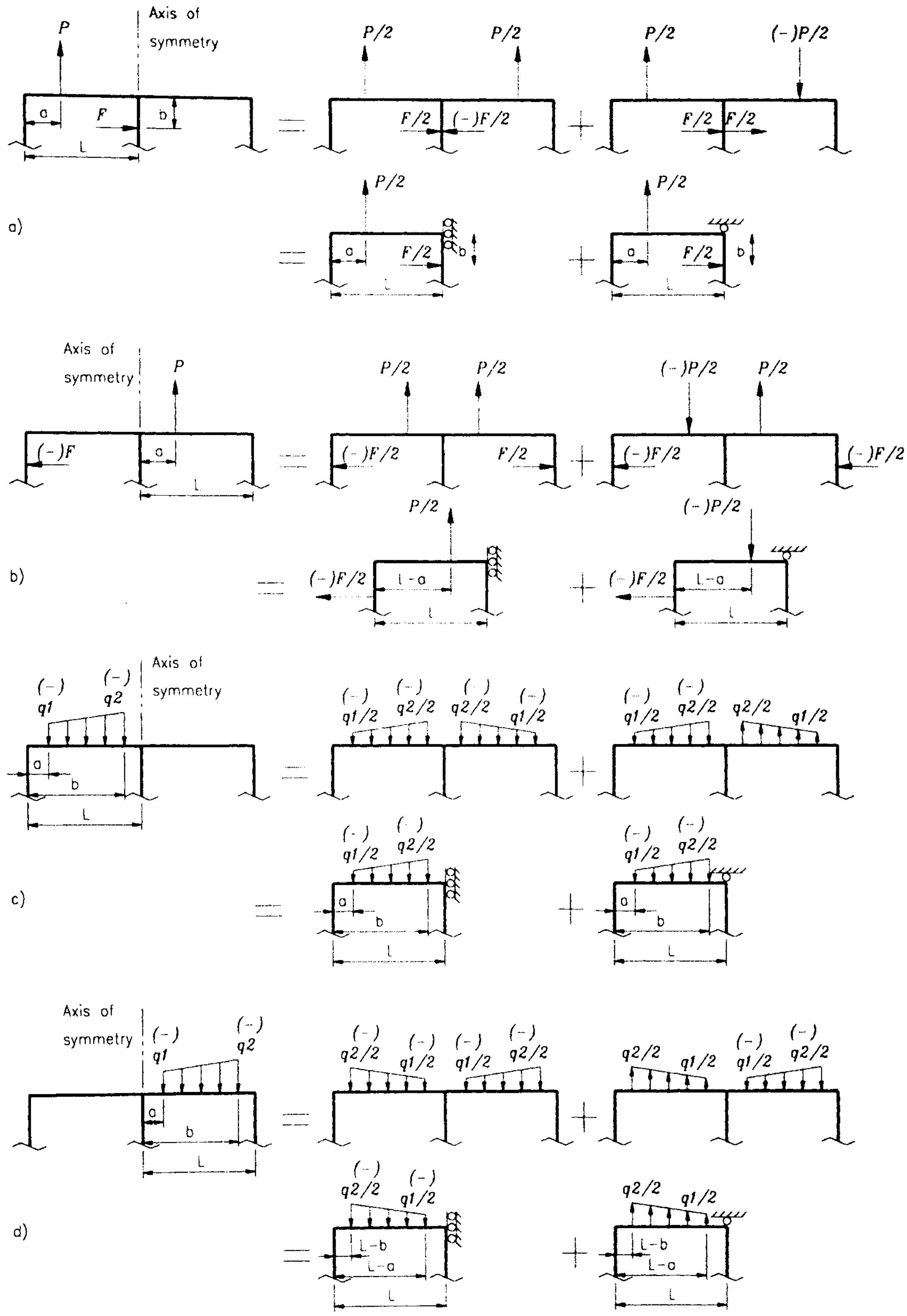


Figure 2.7 : Decomposing General Loads into Symmetric and Antisymmetric Components.

solved: once each for the symmetric and antisymmetric cases. Thus the symmetric and antisymmetric load components will be calculated and stored for the left half only.

2.4 Solving The Half Structure for The Symmetric and Antisymmetric Load Components

All the necessary data are now available to solve the half structure twice: once each for the symmetric and antisymmetric loading components. The original problem is now divided into two half-problems each with about half the number of degrees of freedom as the original. The solution process can be carried out through a procedure inside the program -- discussed in Chapter three -- and the results for the left half can be obtained for the two cases.

CHAPTER III

TIME EFFICIENCY AND BENCHMARK STUDY

3.1 The Computer Program Used in Analysis

A computer program was created to do matrix structural analysis using the stiffness method. The program was written in QuickBASIC 4.5 and is capable of solving several types of structures (beams, two- and three-dimensional trusses, two- and three-dimensional frames, and grids) under any type of load (point load, concentrated moment, uniform load, and distributed load). The main steps for the program are:

- **Reading Data:** Joint coordinates, member location and properties, support conditions, and load values are the data needed to solve any structure. These data can be entered to the program interactively or by reading from an input file. The input file approach is used in this program in order to be able to compare the time used for each step without depending in the speed of the user responses.

• **Data Structure:** This step includes computing the data and preparing the data structures necessary for the solution process. The data to be calculated are the total number of degrees of freedom, the total number of reactions, and the half band width for the global stiffness matrix of the structure. Declaring the necessary arrays used by the program is then done using the computed data.

• **Stiffness Assembly:** In this step, the global stiffness matrix is calculated for each member and the global stiffness matrix for the free degrees of freedom of the entire structure is assembled using the direct stiffness assembly method [1, 3]. Also the load vector is assembled in this step using the load data for each joint and member.

• **Solution of Equations:** This step is the solution of the set of simultaneous equations to find the unknown vector which is the displacement at each joint. The solution method used in this program is the Cholesky's Decomposition Method with a symmetric banded matrix which is the case for the stiffness matrix of a linearly elastic structure [18-21]. This method is sometime called *Cholesky's square root method* [19] which is particularly well suited for structural analysis programs and used by the most prominent software [4-9]. Also, this method is one of the most time-efficient methods in solving a set of simultaneous equation [21].

- **Output The Results:** In this step the results are calculated and saved to output files. These results are the displacement at each joint, the reactions at supports, and local member forces.

The program is modified by adding the code corresponding to the algorithm presented in chapter two. A new main step is added and two steps are modified.

- **Symmetry Detection** : As presented in Chapter two, all the checks to detect symmetry for two-dimensional plane structures with axis of symmetry parallel to the global Y-axis are performed in this step. Only one failed check is sufficient to end this step and stop doing further tests for symmetry.

- **Data Structure:** This step is modified to prepare the data required to solve the symmetric structure as presented in Chapter two. If the structure is not symmetric, then the data will be prepared as discussed earlier in this Chapter.

- **Output the Results:** This step is also modified, for the symmetric structures, to expand the results for the unanalyzed half-structure and to combine the results from the two cases of symmetric and antisymmetric loading conditions for the entire structure which will be discussed in the following sections.

It is noted here that the main steps presented earlier in this Chapter, before adding and modifying to account for symmetry concepts, are the standard steps for professional-level programs. These steps will be used for a time efficiency and benchmark study with and without using the new algorithm.

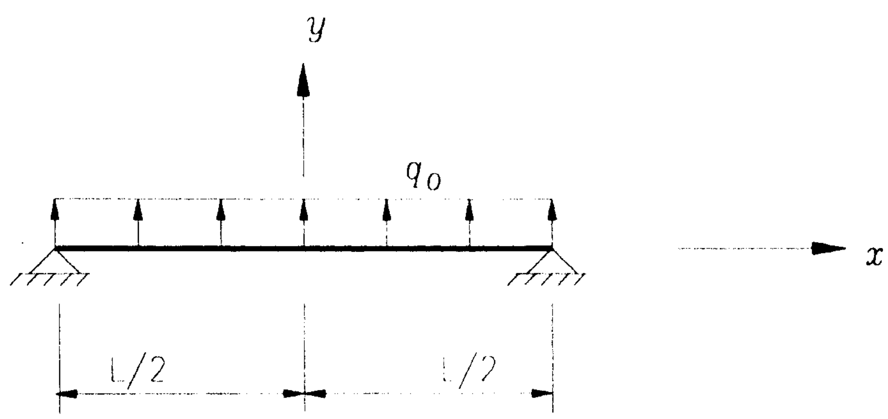
3.2 Results for The Entire Structure Under General Loading

3.2.1 Expanding Results for The Unanalyzed Half-Structure

After solving the half structure twice for the symmetric and antisymmetric loading conditions, the results for the second half structure should be calculated for the two cases. These results include the displacements of the joints, reactions at the supports, and internal forces (axial force, shearing force, and bending moment) for the members. The calculation process for the results of the second half depends on the type of structure (beam, 2-D frame, 3-D frame, 2-D truss, 3-D truss, or grid), the orientation of the axis or plane of symmetry with respect to the global axes, sign convention adopted for the loading, and the loading conditions (symmetric or antisymmetric loading).

Only a 2-D structure type with axis of symmetry parallel to the global Y-axis is considered in this research, although the concepts of calculating the results for the second half structure can be followed in all other cases. The sign convention used is that the forces and transitional degrees of freedom are positive if directed from left to right and from bottom to top, while moments and rotational degrees of freedom are positive if rotating counterclockwise.

To develop the mathematical bases for calculating the results for the second half, a simple symmetric structure with symmetric loading is considered below [1].



From elementary bending theory [2] and neglecting the interaction of axial and flexural modes of behavior,

$$\frac{d^4 v}{dx^4} = \frac{q_o}{EI}$$

Assuming constant EI and integrating gives

$$-\frac{V}{EI} = \frac{d^3 v}{dx^3} = \frac{q_o x}{EI} + C_1$$

$$-\frac{M}{EI} = \frac{d^2 v}{dx^2} = \frac{q_o x^2}{2EI} + C_1 x + C_2$$

$$\frac{dv}{dx} = \frac{q_o x^3}{6EI} + \frac{C_1 x^2}{2} + C_2 x + C_3$$

$$v = \frac{q_o x^4}{24EI} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$$

Enforcing boundary conditions gives

$$M\left(\pm\frac{L}{2}\right) = 0 \qquad \rightarrow \qquad \frac{d^2 v}{dx^2}\left(\pm\frac{L}{2}\right) = 0 \qquad \Rightarrow$$

$$\frac{q_o L^2}{8EI} \pm C_1 \frac{L}{2} + C2 = 0$$

$$C_1 = 0, \qquad C_2 = -\frac{q_o L^2}{8EI}$$

$$v\left(\pm\frac{L}{2}\right) = 0 \qquad \Rightarrow$$

$$\frac{q_o L^4}{384EI} - \frac{q_o L^2}{16EI} \cdot \frac{L^2}{4} \pm C_3 \frac{L}{2} + C_4 = 0$$

$$C_3 = 0, \qquad C_4 = \frac{5 q_o L^4}{384EI}$$

Backsubstituting, we see that where

$q(x) = q_o$ mathematically even

we find

shear $V = -q_o x$ mathematically odd

moment $M = -\frac{q_o}{2}\left(x^2 - \frac{L^2}{4}\right)$ mathematically even slope $\frac{dv}{dx} = \frac{q_o}{EI}\left(\frac{x^3}{6} - \frac{xL^2}{8}\right)$ mathematically odd

and

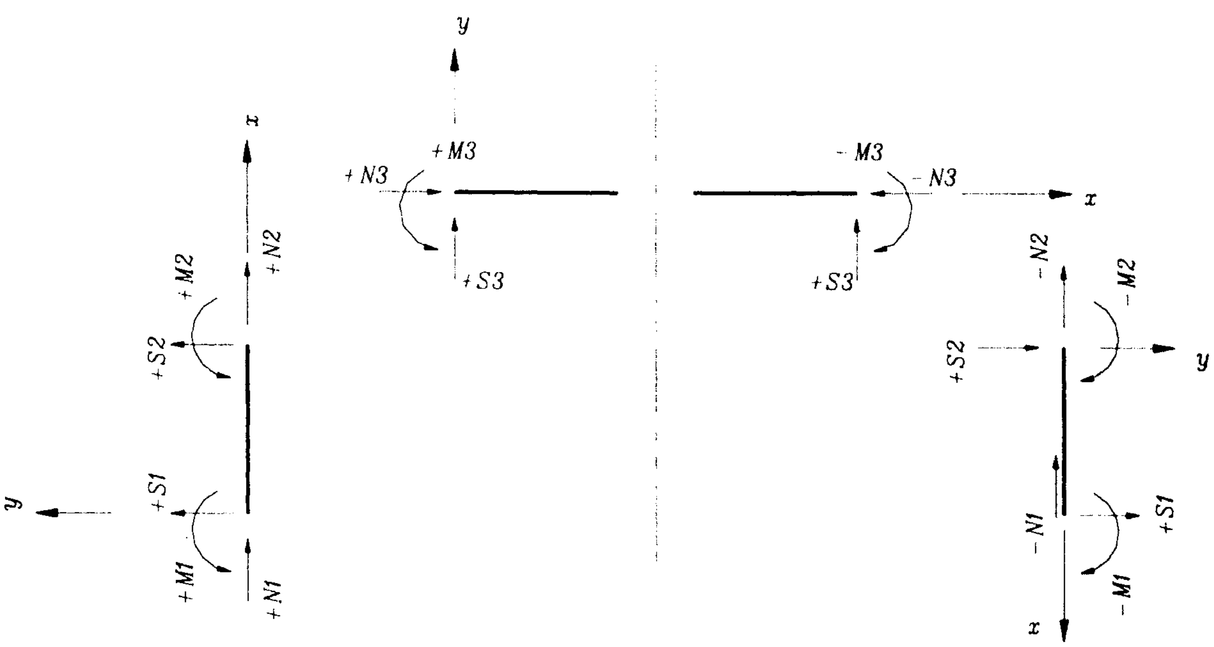
Y-deflection $v = \frac{q_o}{EI}\left(\frac{x^4}{24} - \frac{x^2L^2}{16} + \frac{5L^4}{384}\right)$ mathematically even

Thus although all the preceding functions are physically symmetric, some are mathematically even and some are odd. The same analysis can be conducted for antisymmetric loading. Considering the two cases of loading separately we can calculate the results for the second half as follows.

3.2.1.1 Symmetric Loading:

The structure shown in Figure 3.1 has an axis of symmetry parallel to the Y-axis. It is assumed that the structure is subjected to a symmetric loading and the results obtained from solving the left half of the structure are all positive. If the members are separated, then the results for the second half are as shown in the figure. Note that the sign convention for the deflection and reactions are with respect to global axes while that for some of the internal forces (shearing and normal forces) are with respect to member local axes. Thus the choice of start and end nodes for each member plays an important part in this calculation process. For example, if the local

X-axis for the vertical member on the right side of the structure in Figure 3.1 is directed downward as shown below,



then the signs chosen for only the axial and shear internal forces for this member should be flipped although the directions will be the same in both cases. Note that the bending moment has the same sign in both cases since the direction of the Z-axis -- from which the sign is determined -- does not depend on the orientation of the member local axis for two-dimensional structures.

3.2.1.2 Antisymmetric Loading:

If it is assumed that the same structure is subjected to an antisymmetric loading and the results of the left half are all positive as

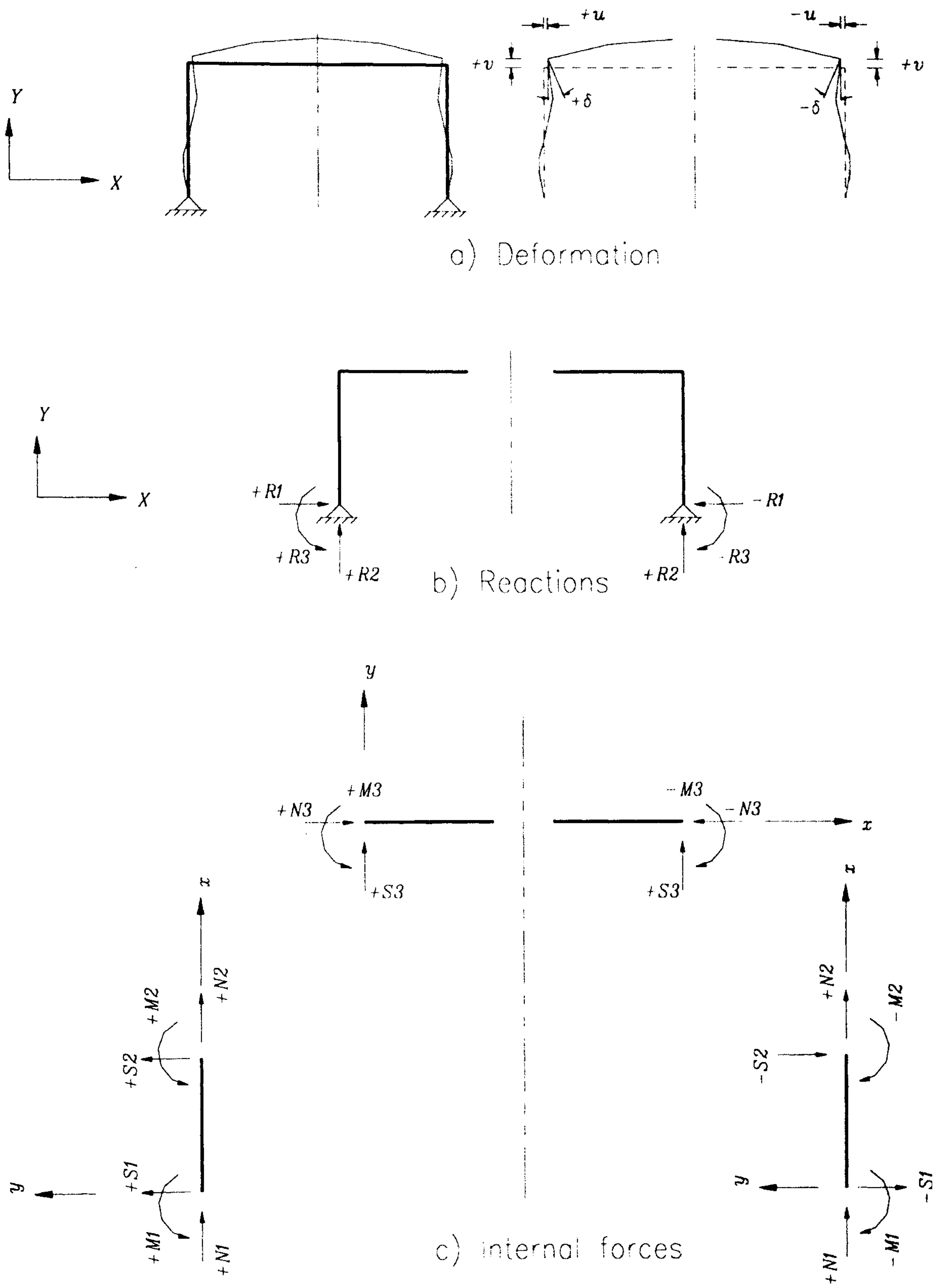


Figure 3.1 : Second Half Results Under Symmetric Loads.

shown in Figure 3.2, then the results for the second half are the reverse mirror image for that of the left half as shown in the Figure. It is noted also that the signs chosen for the results are determined by the orientation of the global axes and local axes for each member.

3.2.2 Combining Symmetric and Antisymmetric Loading Cases

After obtaining the results for the entire structure for the two cases of symmetric and antisymmetric loading components, the two results should be added to give the final result for the structure under the general loading case. Since it is assumed that the material of the structure under any stage of loading is linear and elastic, the combination process is purely an algebraic addition for the deflections, reactions, and internal forces [1].

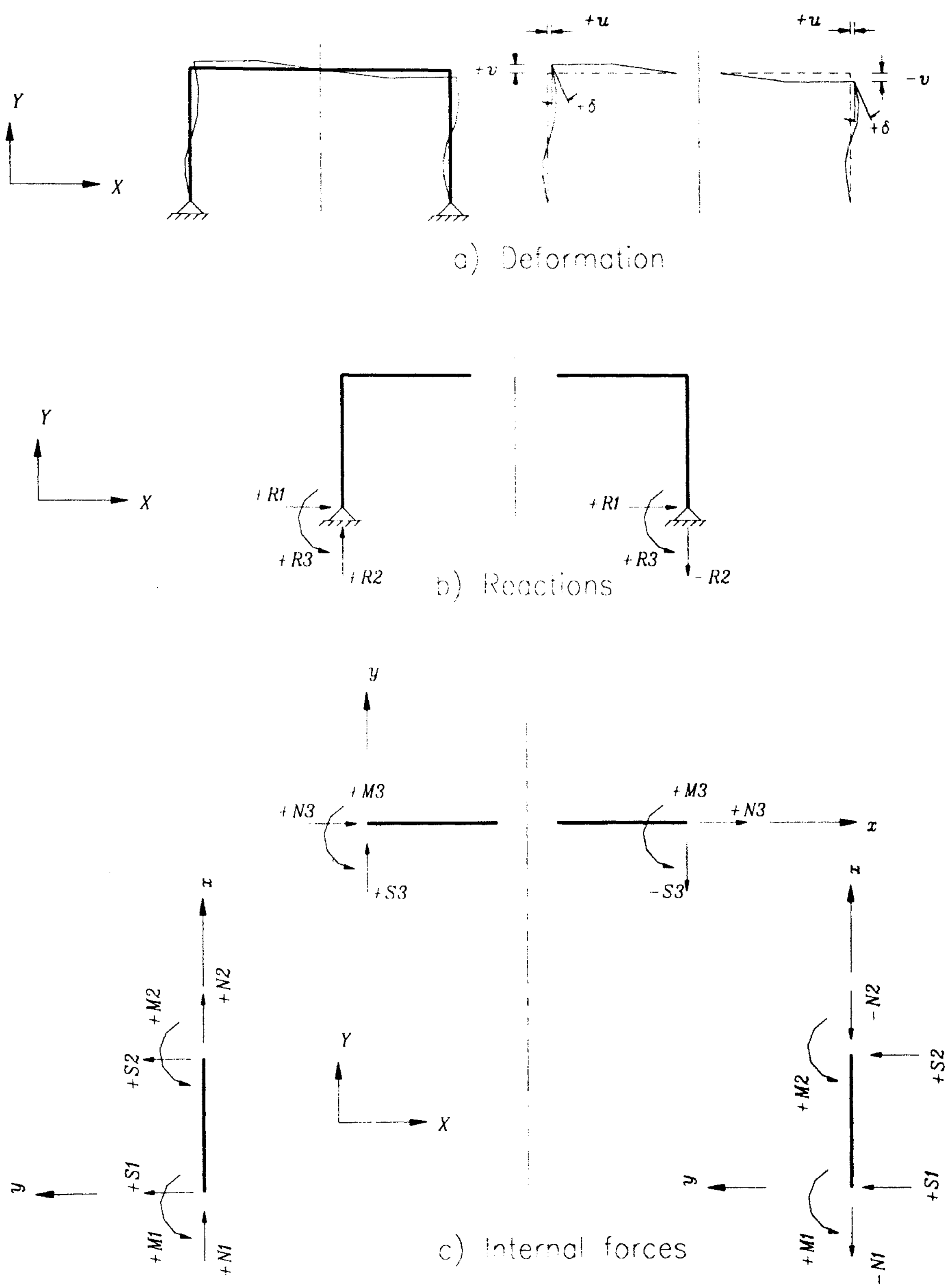
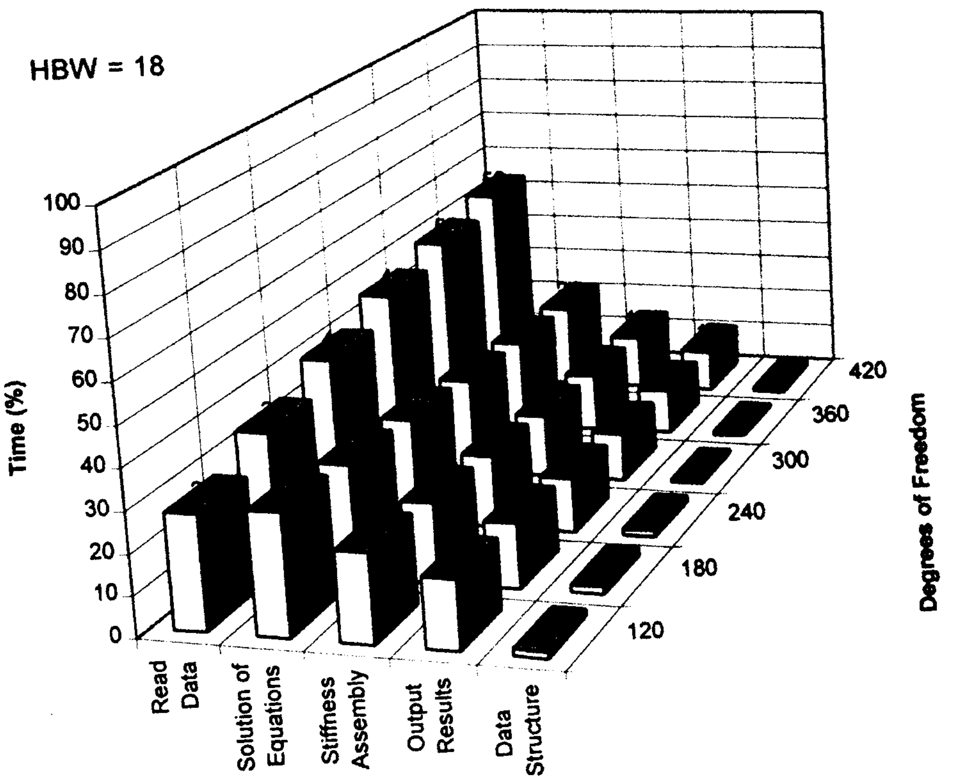


Figure 3.2 : Second Half Results Under Antisymmetric Loads.

3.3 Differential Time Consumption for a Structure Analysis

3.3.1 Time Consumption Without Using Symmetry Concepts

A study has been done to determine the time consumed in solving a structure analysis problem with and without using the symmetry algorithm. Figure 3.3 shows the time consumed by each main part of the program without using the symmetry concepts for different degrees of freedom of the structure for two different values of half band width. It is shown from the Figure that for a larger half band width (HBW equal to 30), the time used in solving the simultaneous equation is 49 percent for smaller number of degrees of freedom and decreases to 38 percent for larger number of degrees of freedom. For HBW equal to 18, the time used by the same step is 30 and 21 percent for smaller and larger values of degrees of freedom respectively. The reason for this is for a larger HBW, the size of the structure stiffness matrix is larger than that for a smaller HBW for the same number of degrees of freedom. Since the time of solving the simultaneous equations depends on the number of degrees of freedom and on the half band width of the stiffness matrix (as discussed in section 1.1.2), then that time will be more dominant for a larger HBW than for a smaller HBW. If comparing two



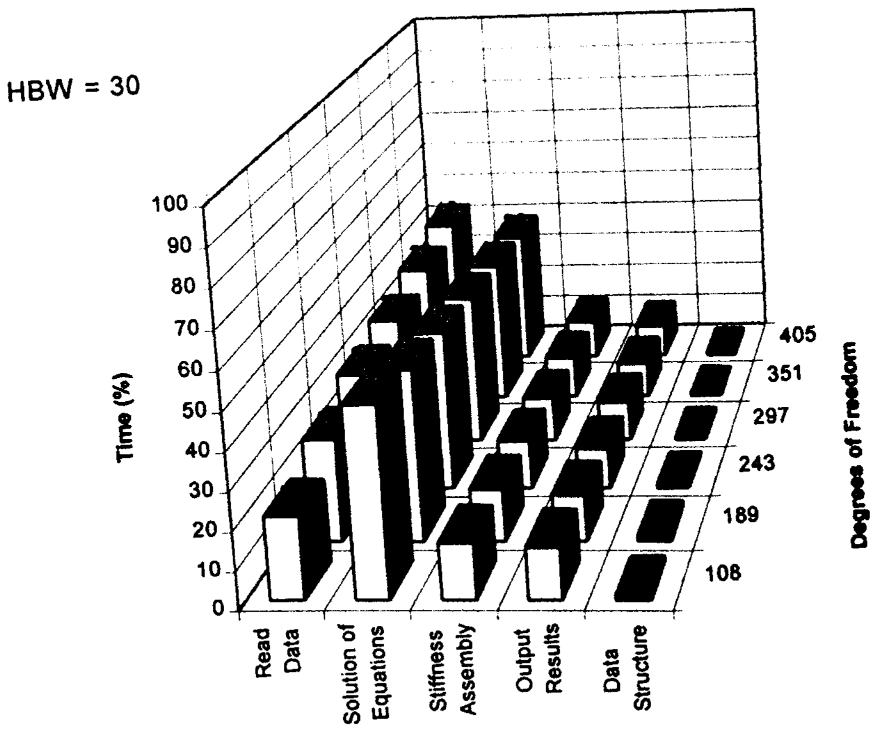


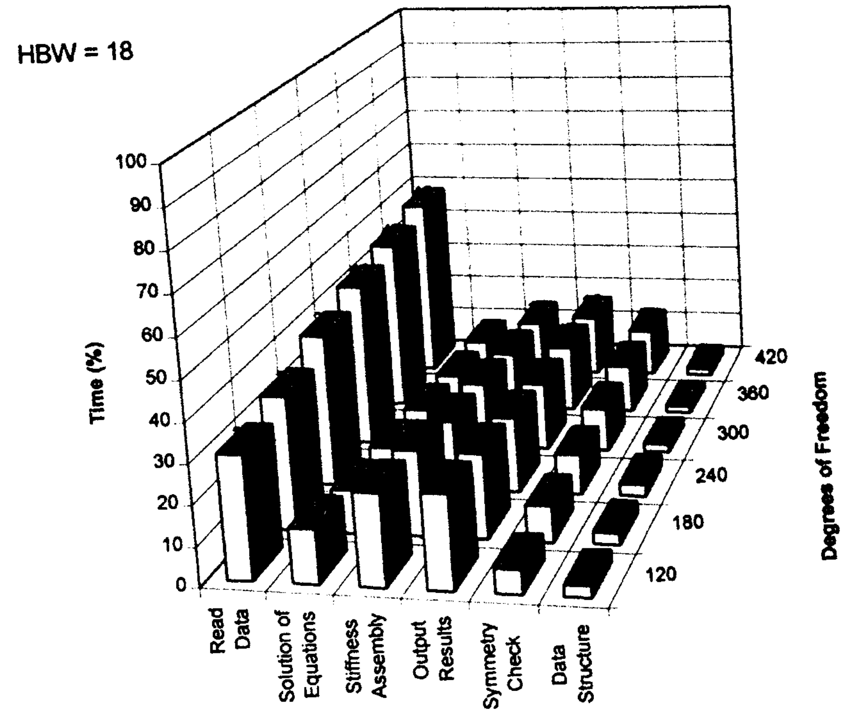
Figure 3.3 : Time Consumption Without Using Symmetry.

problems with about the same number of degrees of freedom but different HBW (structure with 300 DOF and 18 HBW verses one with 297 DOF and 30 HBW as shown in Figure 3.3), the time used by the *read data* step is more dominant for the first structure than the second one (46 percent verses 35 percent). The two times have the same value in both cases (since each of the two problems has almost the same size), but the time used for the solution of simultaneous equations is reduced significantly for the first structure.

The reason for the previous discussion is that it is expected that the saving in total solution time of the problem after applying the symmetry algorithm will be significant for problems with smaller DOF and larger HBW. For example, the time for solving the simultaneous equations for the structure with 108 DOF and 30 HBW in Figure 3.3 is dominant and equal to 49 percent of the total time. Thus savings in this step will result in a significant savings in the total solution time of the problem.

3.3.2 Time Consumption After Using Symmetry Concepts

Figure 3.4 shows the differential time consumption for the analysis program after using the symmetry algorithm. It is shown that the time used in solving the simultaneous equations has dropped significantly for both



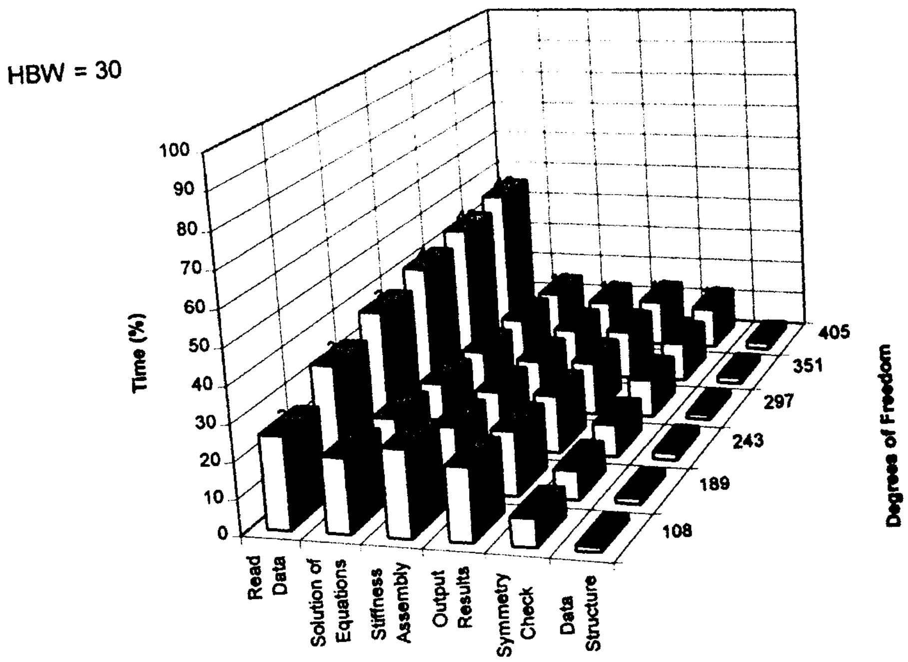


Figure 3.4 : Time Consumption With Symmetry.

two values of HBW. Also, the *read data* step is dominant in both cases. A new step, *symmetry check,* is added to the main steps, as discussed earlier in this chapter. The time consumed by this new step, in addition to the increase in time consumption in the *data structure* and *output results* steps, will result in decreasing the time savings from the solution of simultaneous equations.

3.4 Results After Applying The Symmetry Algorithm

Different symmetric structures with different general loading are solved with and without using the symmetry algorithm. The structures used in analysis are shown in Figure 3.5. The joints are numbered in the most efficient way to give minimum Half Band Width for the global structure stiffness matrix, and each type of these structures gives a different value for the HBW. Each structure is then solved for different values of Degrees of Freedom by adding horizontal and vertical members to expand in the vertical direction.

The computer used in running the analysis program and to keep track of time measurements is an IBM compatible with 486DX2 CPU with 66 Mhz. To eliminate the change in time results in using different CPU machines, the percentage in time saving is usually used to compare the time consumption for each main step with and without the symmetry algorithm. The following results are obtained.

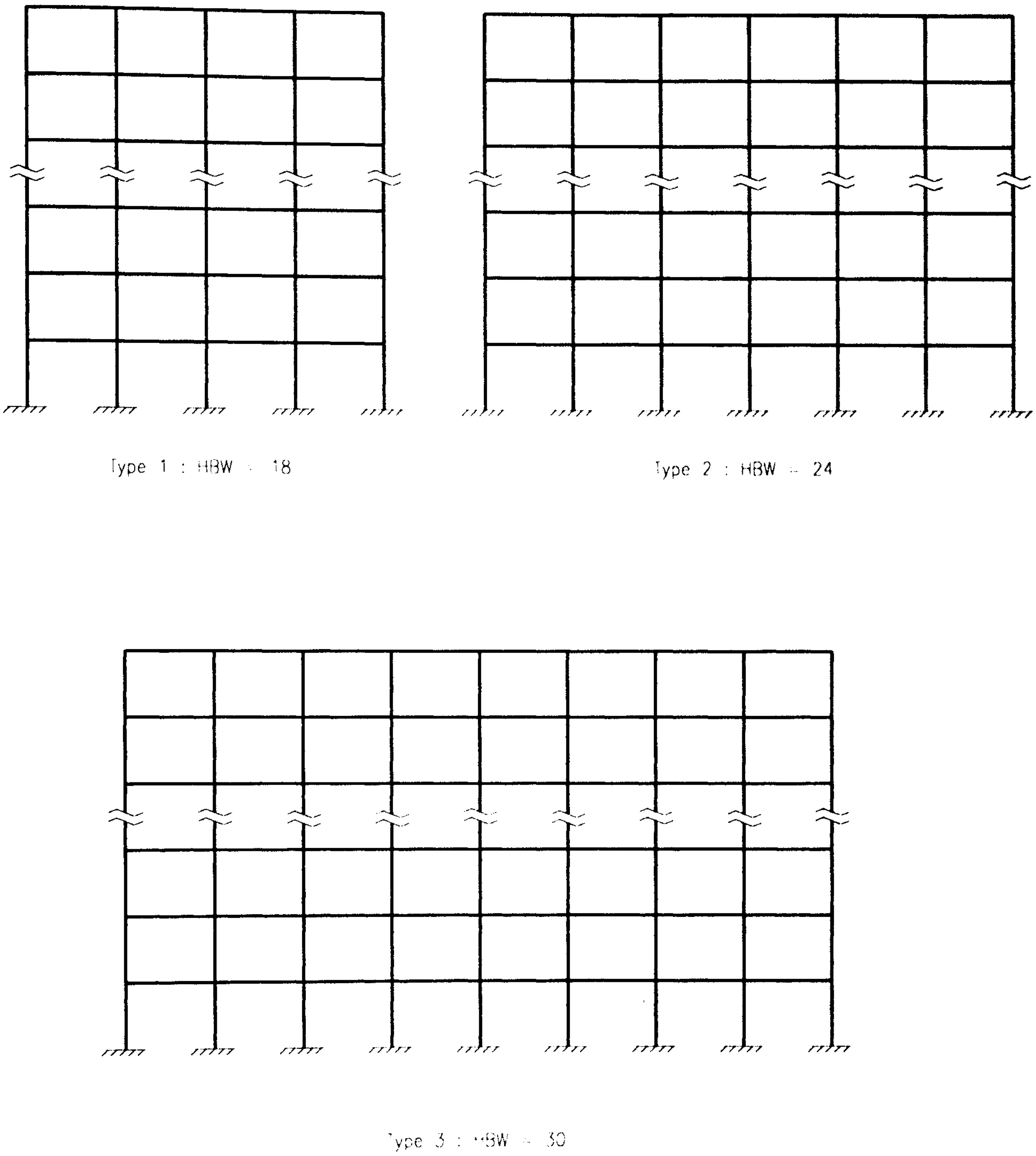


Figure 3.5 : Structures Used in Analysis.

3.4.1 Savings in Simultaneous Equation Solution Time

The time used in solving the set of simultaneous equations with and without using the symmetry algorithm is shown in Figure 3.6. The Figure shows the results for structures with different DOF's and for different HBW's for the stiffness matrix. The solution time after applying symmetry is the sum of the two solution times for symmetric and antisymmetric loading components. It is noted that for structures with the same DOF's, the solution time for a matrix with larger HBW is higher than that for a matrix with smaller HBW's. Also, for matrices with the same HBW's, equation-solution time for a structure with larger DOF is higher than that for a structure with smaller DOF. This is true for both cases with and without using the algorithm which proves that the time for solving the structure stiffness matrix depends on both the DOF and the HBW as discussed in section 1.1.2 [10, 19].

The percentage of time savings is shown in Figure 3.7. The savings in solution time is nearly constant for different DOF's with the same HBW, and increases slightly for higher HBW. The reason for that is the solution time is proportional to $[(\beta+1)(2n-\beta)]$ where β is the HBW and n is the

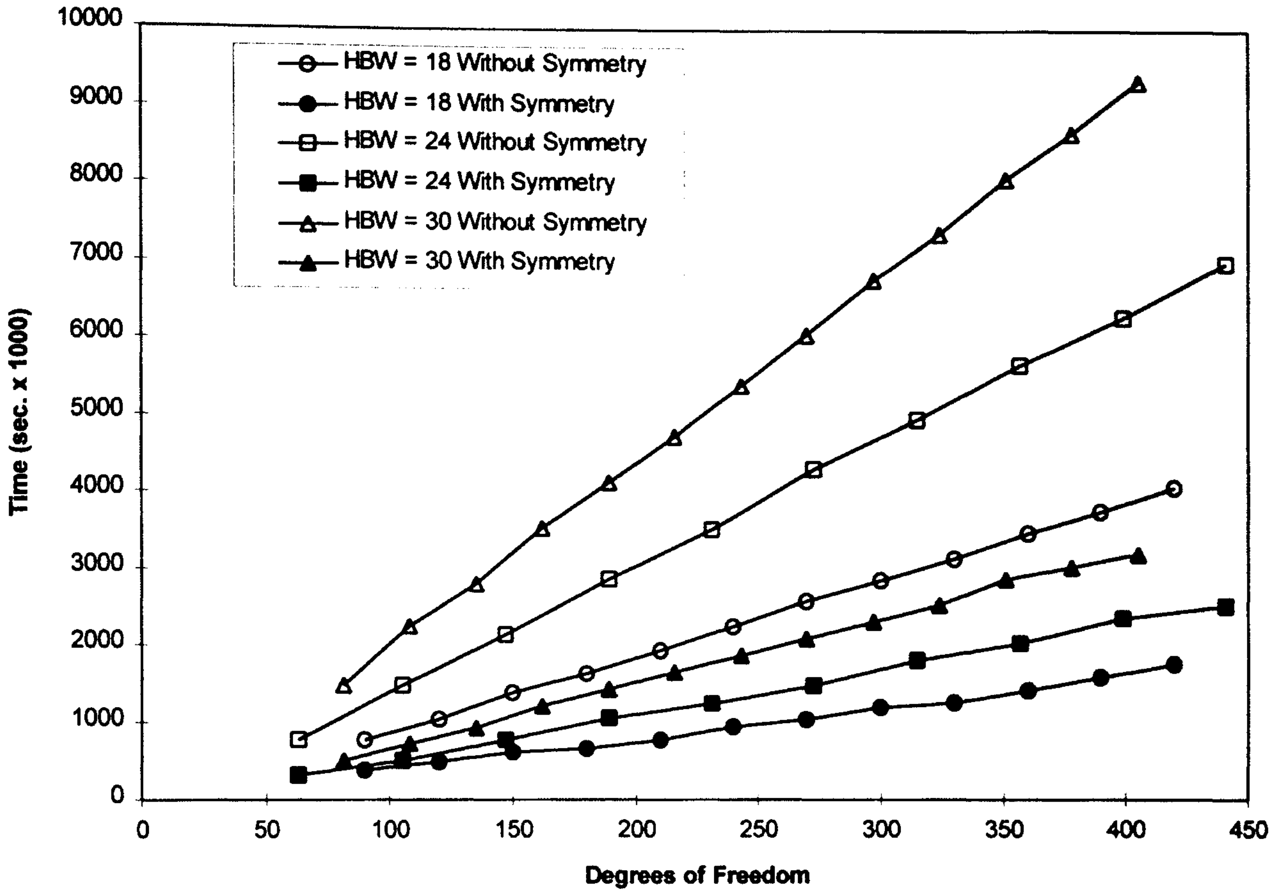


Figure 3.6 : Time Used in Solving Simultaneous Equations.

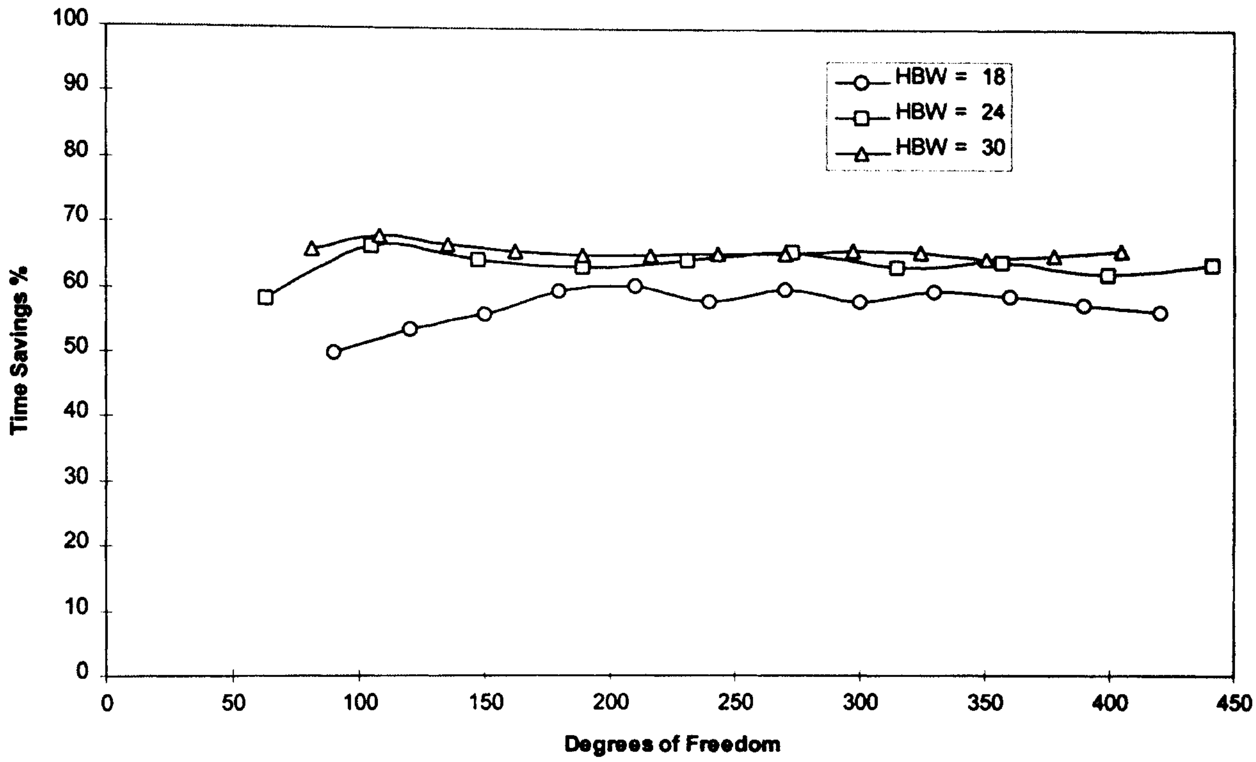


Figure 3.7 : Percentage of Time Saving in Solving Simultaneous Equations.

DOF [10]. Also, when the structure is cut into two halves and only one half is analyzed, both the DOF and the HBW for the half structure is reduced to about 50 percent of the original values of the entire structure. Thus the solution time required to solve the half structure twice is:

$$2\left[\left(\frac{\beta}{2}+1\right)\left(2\frac{n}{2}-\frac{\beta}{2}\right)\right]=\frac{1}{2}(\beta+2)(2n-\beta)$$

Thus since n and β are large comparing to 2, the saving in time is constant and is equal to about 50 percent.

The percentage of time saving is found to be between 55 and 66 percent as shown in Figure 3.7. This saving is different from what was expected before, which was 50 percent, due to the fact that both the DOF's and HBW's for the half-structure are less than that for the entire structure. Also both the DOF's and HBW's for the half-structure under the symmetric loading case are not equal to the corresponding values under the antisymmetric loading case, and any of these values is not exactly equal to half the corresponding values for the entire structure.

3.4.2 Savings in Total Solution Time

The total time used in solving the structure problem with and without applying the symmetry algorithm is shown in Figure 3.8. Also, the Figure shows the results for structures with different DOF's and for different HBW's for the stiffness matrix. The total time is the time starting from reading of the data to output of the result for the two cases with and without using the algorithm. The total solution time using the algorithm includes the symmetry detection, the solution of the half structure twice for symmetric and antisymmetric loading, and expansion and combination of the half-structure results to get results for the complete structure.

The percentage of total solution time savings is shown in Figure 3.9. It is noted that the percentage of savings in total time for a larger value of HBW is about 10 percent for a larger number of DOF and increases to about 25 percent for smaller number of DOF. This results was expected from Figure 3.3 where the time consumed by the solution of equations is dominant for a smaller number of DOF's.

It is noted also that for smaller HBW there is no saving in time. In fact the total solution time using the algorithm is higher than that without

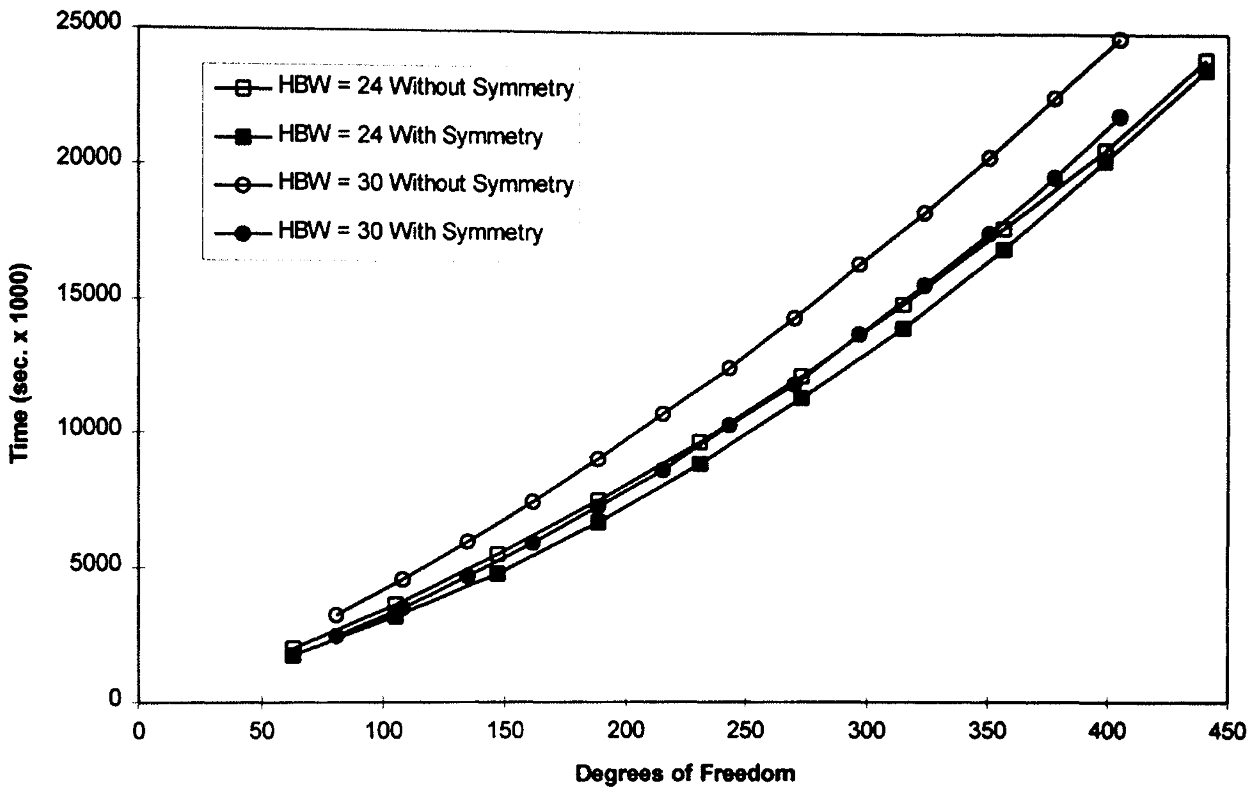


Figure 3.8 : Total Solution Time.

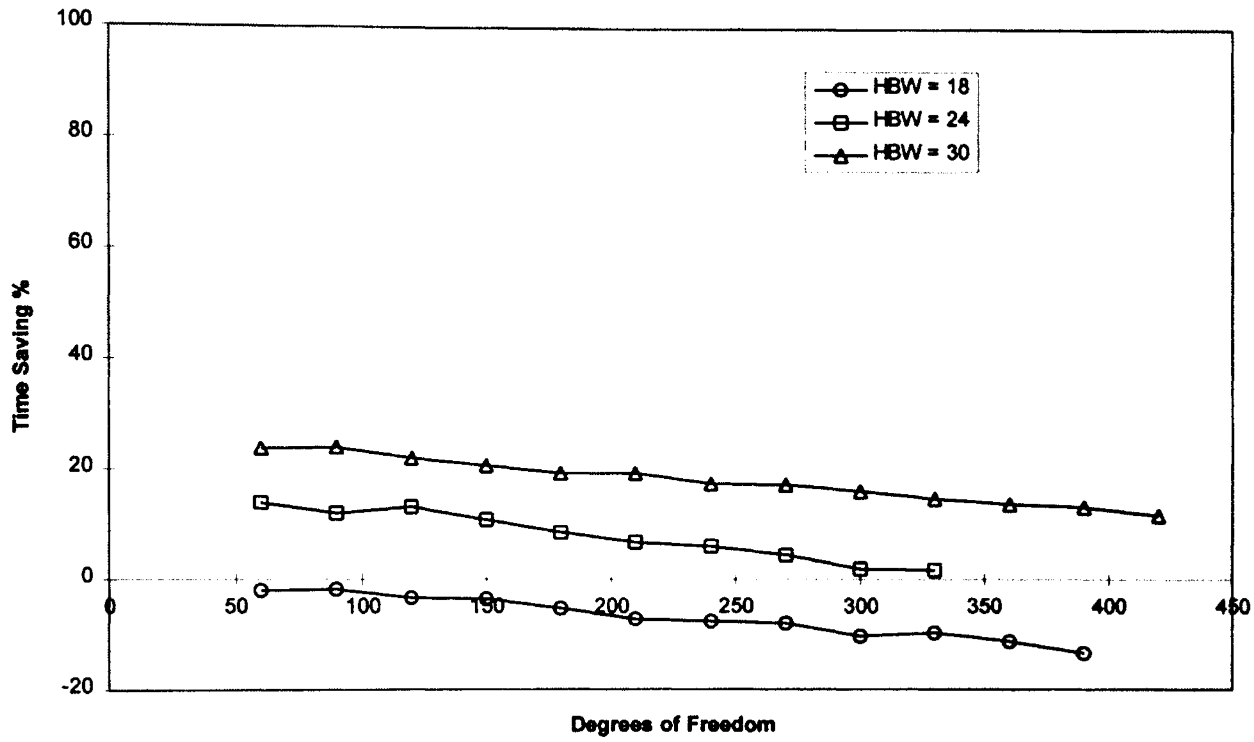


Figure 3.9 : Percentage of Saving in Total Solution Time.

3.4.3 Savings in Computer Storage Requirements

The second benefit of applying symmetry concepts is the reduction in computer storage usage which is basically the maximum number of bytes of computer memory required by one object in the program. Usually the object which has the maximum size is the global stiffness matrix of the structure which has dimensions of number of free degrees of freedom by half band width. This array is usually stored with the maximum precision (usually double precision) provided by the computer in order to minimize the round off error of the several multiplication operations performed in this array [3], although a special advantage of Cholesky's is that greater accuracy can be provided by just using one or two double-precision variables [21]. Thus, the global stiffness matrix can be saved in a single-precision array and more computer storage can be saved.

The number of bytes required by the global stiffness array for different degrees of freedom with and without using the symmetry algorithm is shown in Figure 3.10. For a specific number of free degrees of freedom, the size of the array depends on the half band width of the structure which in turn depends on the way of numbering the joints [1, 3]. The savings in computer storage requirement is shown in Figure 3.11. These

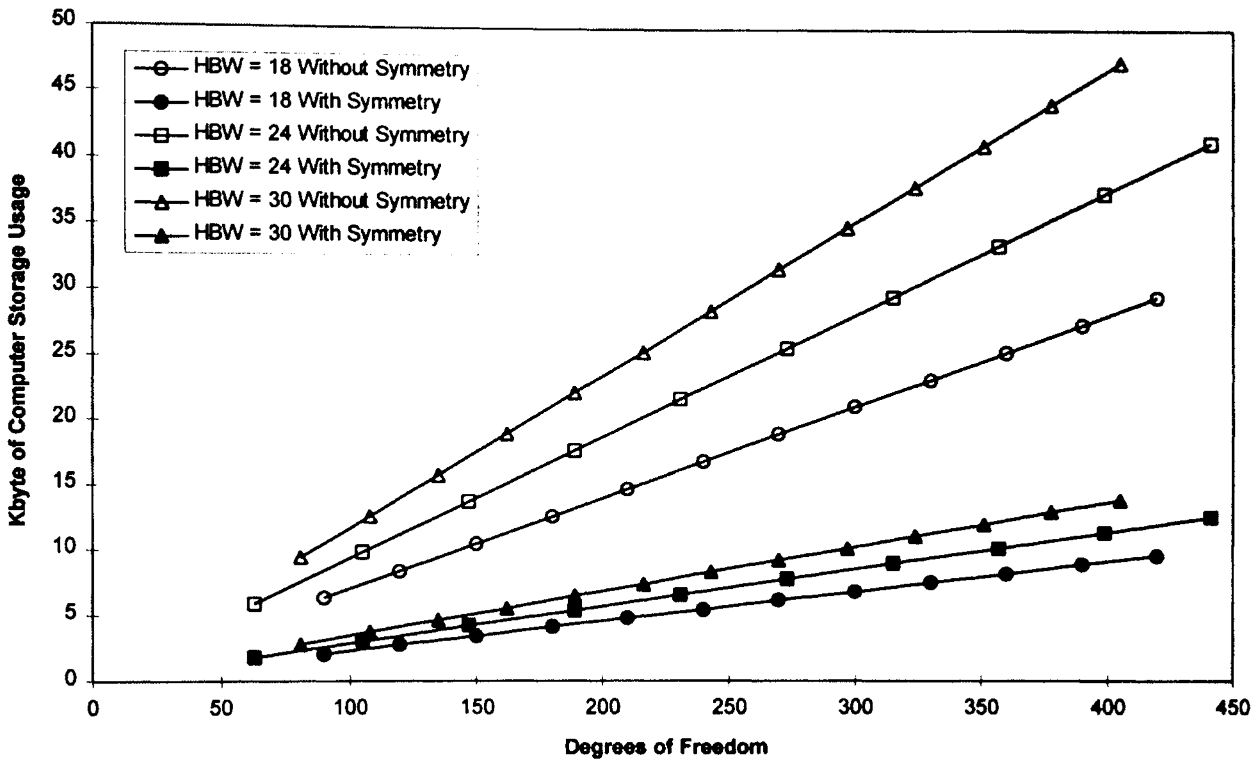


Figure 3.10 : Maximum Computer Storage Usage.

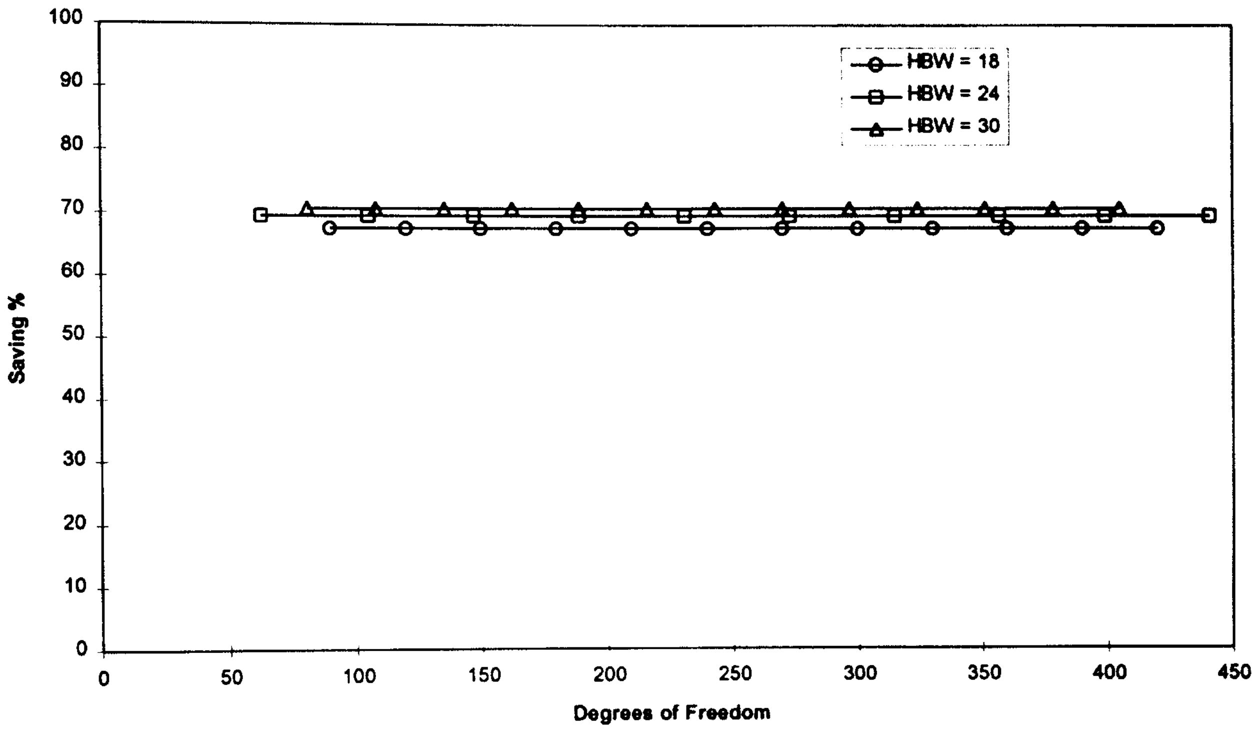


Figure 3.11 : Percentage of Saving in Computer Storage Usage.

savings are plotted for different degrees of freedom and for different values of half band width. It is noted that the saving is constant for a specific value of HBW, ranging from about 67 to 75 percent. These savings in computer storage allows the analysis of large-size symmetric structure problems that cannot be solved without using the symmetry algorithm.

CHAPTER IV

SUMMARY AND CONCLUSIONS

4.1 Summary

Studies show that the most time consuming step in a matrix structural analysis is the solution of the set of simultaneous equations that relate the displacements to the applied forces at the joints. Studies show also that the data structure that takes up the largest part of computer memory is the global stiffness matrix for the entire structure. Much research has been done in order to minimize the time required to solve the simultaneous equations and to minimize the size of the global stiffness matrix. One way of achieving these two tasks is by utilizing the symmetry of a structure, thus reducing the number of degrees of freedom by analyzing only one half of the structure.

In the research reported herein, the focus is on enabling automated utilization of symmetry benefits. An algorithm is presented to automatically detect the symmetry of a structure. The symmetry detection is done by performing several tests on the entire structure to check all the requirements for the structure to be symmetric. The algorithm is able to detect the symmetry of a two-dimensional plane frame structure in the XY-plane with the axis of symmetry parallel to the Y-axis. However, the procedure presented in this research can be applied with little modification to a structure with an axis of symmetry parallel to the global X- or Z-axis.

If the structure is symmetric, only one half of the structure is considered for analysis and several data structures are created to exactly represent the behaviors and responses of that half as if it is in the entire symmetric structure. If the structure is subjected to a general loading, then the general loading is decomposed into its symmetric and antisymmetric components. The half-structure is then analyzed twice for each of the two loading components applied on this half. The results of the two cases are expanded and combined to obtain the analysis of the entire structure under the general case of loading. If the loading on the entire structure is purely symmetric or antisymmetric, then the half structure is analyzed only once and the results for the entire structure are obtained thereby.

4.2 Validation

Several symmetric structures with different numbers of degrees of freedom and subjected to different types of general loading are solved using a computer program that includes the symmetry algorithm (symmetry detection, data structure handling, and decomposing of general loads). The final results obtained were checked by analyzing the same structures with the same program without the algorithm and then checked again using a commercial software [4]. The results were the same for all structures analyzed which proved that the program written for this project, including the symmetry algorithm, gives the correct results.

4.3 Conclusions

By taking advantage of the symmetry of a structure and solving only half of it twice under symmetric and antisymmetric loading components, a reduction in the solution time and in the computer storage requirements is noticeable. The saving in time in the solution of the set of simultaneous equations is higher than that for the total solution time. The cause of this is the time used by the new algorithm to detect symmetry, prepare data structures and loading components, and combine the results.

The percentage of time saving in the solution of simultaneous equations does not depend on the number of degrees of freedom of the entire structure and slightly depends on the shape of the symmetric structure (or the value of the half band width of the stiffness matrix). This time saving is ranging from about 55 to 65 percent of the equation solution time of the entire structure.

The percentage of saving in total solution time depends on the number of degrees of freedom of the entire structure along with the Half band Width of the stiffness matrix. For a structure with higher value of HBW, the saving in total time is about 10 percent for structures with a large number of degrees of freedom and increases to about 25 percent for those with a smaller number of degrees of freedom. For a structure with a smaller value of HBW, the saving in time is not as noticeable and depends greatly on the size of the structure.

The size of the largest array when solving the half structure is about one third of that when solving the entire structure, thus the saving in computer storage usage is about 66 percent, assuming the reduced-size global stiffness matrix is still the largest data structure required.

4.4 Further Research Needed

The algorithm presented must be generalized to address the following issues pertinent to structural symmetry.

4.4.1 Symmetry Detection For Other Types of Structures

Symmetry is possible for three-dimensional structures and other types of two-dimensional structures, e.g. grids. In the 3-D cases, there will be a plane of symmetry instead of an axis of symmetry. Also, there will be more degrees of freedom per each node, thus the half band width and the total number of degrees of freedom of the entire structure will be higher compared to the problem size and saving in total time will be impressive.

4.4.2 Multiple Symmetry

Structures with multiple symmetry are those that have two or more axes or planes of symmetry. For example, two-dimensional structures with two axes of symmetry, one parallel to the global Y-axis and the other parallel to the global X-axis. For three-dimensional structures there might be two or three planes of symmetry, perpendicular to X-, Y-, or Z-axes or a combination of them.

4.4.3 Axis or Plane of Symmetry not Parallel to The Global Axes

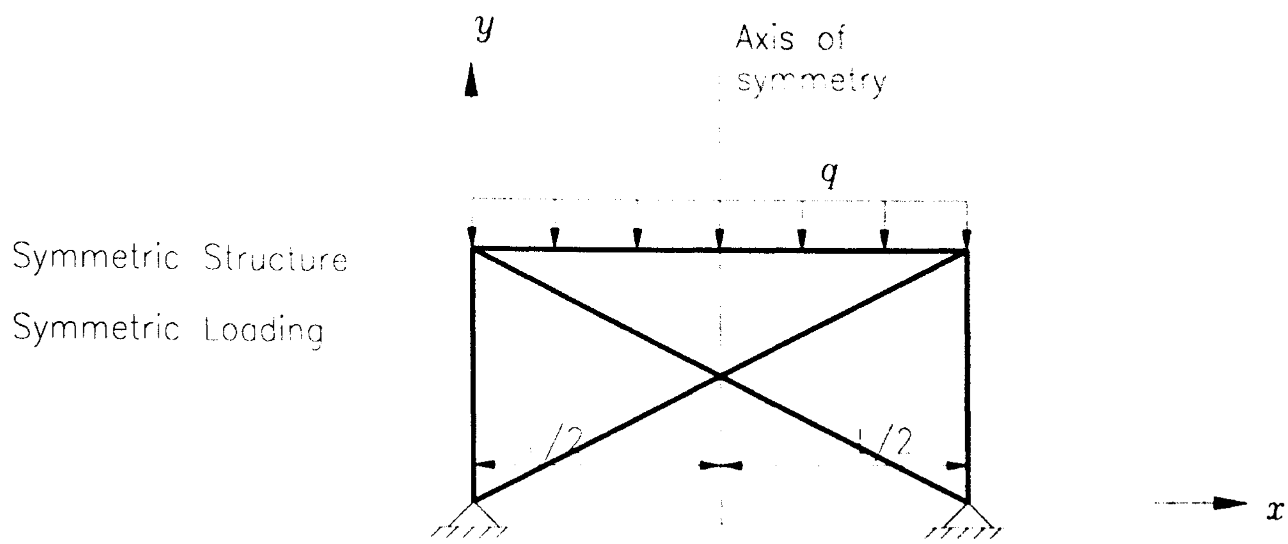
Skewed structures sometimes have axes or planes of symmetry not parallel to the global X-, Y-, or Z-axes. Also there might be more than one axis or a plane of symmetry, or a combination of axes or planes parallel or not parallel to the global axes.

4.4.4 Other Methods for Detecting Symmetry

Structure symmetry might have an effect on the values in the global stiffness matrix or only in the main diagonal of it. Thus utilizing symmetry might be performed by just examining all or some of the values of this matrix and more time can be saved. However, considerable effort would have to be expanded to set up the structure stiffness matrix.

4.4.5 General Symmetry Line Constraints

For members crossing and not perpendicular to the symmetry line, the choice of free and constrained degrees of freedom for the new nodes added at the intersections need to be studied. The structure shown in Figure 4.1 is a symmetric 2-D frame in which there is no joint at the intersection of the two diagonal members. Under symmetric loads, for example, the intersection point for each of the two diagonal members with the axis of



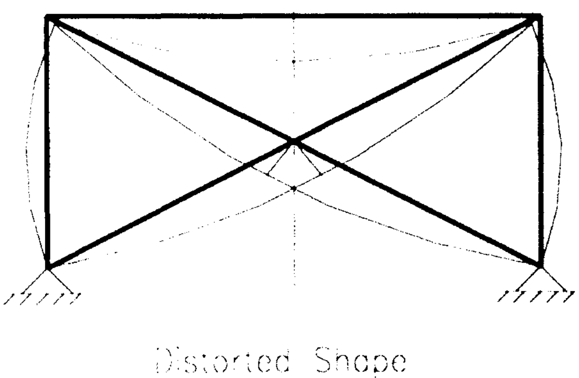


Figure 4.1 General Symmetry Line Constraints.

symmetry will deflect to the right or to the left of that axis. Thus when introducing new node for each member at this point under symmetric loading component, the transitional degree of freedom in the global X-direction is no longer constrained and the slope of the deflection curve for the diagonal member at the intersection is not equal to zero and is not parallel to the local axis of the member itself. Although the responses of the structure will be in a symmetric fashion, but the responses of the half-structure need more study to be predicted.

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